



Collision-energy calibration and monochromatisation studies at FCC-ee

Below list is an initial iteration: more names to be added

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On behalf of the energy calibration, polarisation and monochromatisation working group

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Abstract

The Future Circular Collider (FCC) technical and financial Feasibility Study (FS) includes a work package on Energy Calibration, Polarisation and Monochromatisation (EPOL), which is concerned with the precision determination of the centre-of-mass energy at the e^+e^- machine, FCC-ee. To achieve this goal it is proposed to use resonant depolarisation and possibly spin precession measurements, in conjunction with precise measurement by the detectors of the energy spread and other parameters with physics events. Beam diagnostics, provided by polarimeters and beam-position monitors, play an essential role in monitoring the polarisation level and controlling numerous beam parameters at the interaction point. Specific items of accelerator equipment include polarisation wigglers and depolarising RF-kickers. The target is to achieve a precision commensurate to the remarkable statistical precision achievable in the physics experiments. The possible monochromatisation of colliding beams in view of a measurement of the $e^+e^- \rightarrow H(125)$ process is also being studied, with the specific requirements of this procedure under investigation. A review is presented of the status of these activities at the end of the FS, together with the outlook for future work.

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Contents

1	Introduction	4
2	Overview	4
3	Optics	5
4	Beam polarisation and optimisation	5
4.1	Introduction	5
4.2	Resonances	6
4.3	Harmonic spin bumps for depolarisation reduction	7
4.4	The question of the beam energy	8
5	Wigglers	9
6	Injection of polarised bunches	10
6.1	Generation of polarized bunches	10
6.2	Transmission to end of the LINAC	10
6.3	Transfer line and booster injection	10
6.4	Booster energy ramp	12
6.5	Collider injection	16
7	FCC powering	16
8	Depolarisers System	16
8.1	Placement considerations	17
8.2	Simplified model	17
8.3	Depolarizer design	18
9	Polarimeter	23
9.1	Physics requirements and measurement techniques	23
9.2	Technical specification and challenges	28
9.2.1	Laser	28
9.2.2	Vacuum chambers and magnet	32
9.2.3	Sensors	35
9.2.4	Background mitigation	35
9.2.5	Polarimeter system availability and efficiency	36
10	Polarisation studies at KARA storage rings	38
10.1	KARA Machine Description	38
10.2	RDP measurement set-up	39
10.3	Dependence on the scan direction	40

11 From resonant depolarisation to centre-of-mass energies	40
11.1 Energy losses and their control	42
11.2 Energy losses for pilot and colliding bunches	45
11.3 Dispersion and collision offsets	46
11.3.1 Introduction	46
11.3.2 Beam-beam deflection	47
11.3.3 Opposite sign vertical dispersion measurement	49
11.3.4 Discussion and requirements	50
12 Input from the experiments	50
12.1 The crossing angle α	50
12.2 The longitudinal boost and the collision-energy spread	51
12.3 Relative E_{CM} determination in the Z -resonance scan	52
12.4 Absolute E_{CM} determination	53
13 Expected precision on EW observables from the collision energy and its spread	54
14 Monochromatisation	55
15 Outlook	55
16 Summary	57
17 Acknowledgements	57
References	57

1 Introduction

This document is a supporting note to the discussion on the calibration of the FCC-ee collision energy that can be found in the Final Report of the Feasibility Study. The key sections of the Feasibility Study are also presented here, together with more extensive information on critical aspects of the calibration procedure and strategy. A status report is also given for monochromatisation studies directed at facilitating a measurement of the electron Yukawa coupling.

2 Overview

Excellent knowledge of the collision energy, E_{CM} , is vital for many of the most important measurements that will be performed at FCC-ee, in particular the determination of the Z -resonance parameters, and the mass and width of the W boson. To achieve this goal requires calibrating the mean energy of each beam around the ring, E_{b} , in principle not identical for electrons and positrons but here designated with a single symbol for simplicity. The collision energy E_{CM} can then be calculated, provided there is sufficiently good knowledge of the crossing angle of the two beams and all effects that give rise to local shifts of the energy at each interaction point.

Circular colliders have the unique attribute that transverse polarisation naturally accumulates through the Sokolov–Ternov effect, and the closed orbit spin tune, which is the ratio of the spin precession frequency to the revolution frequency, is, to first order, directly proportional to E_{b} . The closed-orbit spin tune can be directly measured by the procedure of resonant depolarisation (RDP), in which the frequency of a depolarising kicker magnet is adjusted until the polarisation is found to vanish. This technique, which has an intrinsic relative precision of 10^{-6} or better, has been exploited at many facilities, most notably at LEP in scans of the Z resonance [1] and more recently at VEPP4 in the determination of the J/ψ and $\psi(2S)$ masses [2]. Alternatively, in a free spin precession (FSP) measurement the depolariser may be used to rotate the spin vector into the horizontal plane, and the precession frequency measured directly. The self-polarisation of the beams can be used for these measurements, but this will only be possible for Z -pole operation and at energies up to and including the W^+W^- threshold. At energies higher than these, the polarisation level will be too small for RDP and FSP measurements to be practical and the energy scale will have to be determined from physics processes at the experiments, such as $e^+e^- \rightarrow Z(\gamma)$, ZZ and W^+W^- production. Here, the Z and W masses measured with high precision at lower energies with RDP provide a calibration that can then be applied at higher energies.

The calculation of E_{CM} at each interaction point requires good knowledge of the crossing angle of the two beams, which must be measured by the experiments in real time. In addition, it is necessary to account for local energy variations from synchrotron radiation, the RF system and impedance, and to consider the effects of opposite-sign vertical dispersion.

The knowledge of E_{b} at LEP was ultimately limited by the sampling rate of RDP measurements, which were performed outside physics operation with a periodicity of around once per week. The energy was found to vary significantly between measurements due to several effects, for example earth tides and stray ground electric currents [1]. In order to enable the

much greater degree of systematic control than the vastly larger event samples at FCC-ee warrant, the operational strategy will be very different to LEP. Measurements of E_b will be performed several times per hour on non-colliding pilot bunches. In Z running, around 160 pilot bunches will be injected at start of fill, and wiggler magnets will be activated to speed up the polarisation time. One to two hours will be required for the polarisation to build, after which the wigglers will be turned off and physics (colliding) bunches injected. The RF frequency will be continually adjusted to keep the beams centred in the quadrupoles, thus suppressing tide-driven energy changes, which would otherwise be $\mathcal{O}(100\text{ MeV})$. A model will be developed to track residual energy variations between measurements.

3 Optics

The spin-modulation index is defined as

$$B = \frac{\nu_0 \sigma_E}{Q_s} \quad (1)$$

with the closed-orbit spin tune ν_0 , the energy spread σ_E and the synchrotron tune Q_s . It quantifies the strength of synchrotron side bands in proximity to the closed-orbit spin tune. RDP can only be successfully performed if this index is sufficiently low, namely $B \lesssim 1.5$. This gives, for a constant spin tune and energy spread, an upper limit on the acceptable synchrotron tune.

For the FCC-ee Z -pole measurements $\nu_0 = 103.48$, $\sigma_E = 3.9 \times 10^{-4}$ for low intensity ($\approx 10^{10}$ particles per bunch) pilot bunches, and $Q_s = 0.0288$, yielding a spin modulation index of 1.4. Assuming the same bunch intensity and, thus, energy spread, this limits the synchrotron tune to values greater than 0.02691.

At W -pair-production with 80 GeV beam energy, $\nu_0 = 181.55$ and $\sigma_E = 7.0 \times 10^{-4}$, which constrains the lower limit of the synchrotron tune to be approximately 0.0847. In the current optics design $Q_s = 0.081$ and thus the spin modulation index is 1.56, which could reduce the accuracy of RDP measurements at W -energy, compared to the Z -pole. RDP measurements at 80 GeV beam energy would benefit from a lower synchrotron tune than currently foreseen in the latest optics design.

4 Beam polarisation and optimisation

4.1 Introduction

Statistically, the emission of one in every 10^{10} photons through synchrotron radiation (SR) leads to a spin flip of the electron or positron. It has been found in Ref. [3, 4] that the probability of the occurrence of spin flip depends slightly on the initial spin state of the electron (positron), which leads to a maximum theoretical polarisation of 92.4% anti-parallel (parallel) to the dipole magnetic field in lepton storage rings. When polarised, the leptons precess, with the spin precession under electromagnetic fields being described by the T-BMT equation [5, 6] with ν_0 being the number of precessions on the closed orbit, which is named the closed-orbit spin tune. In an error-free flat machine, $\nu_0 = a\gamma$, where $a = (g - 2)/2$ is

the gyromagnetic anomaly and γ is the Lorentz factor. Nevertheless, the correspondence between the measured spin precession frequency using RDP and the actual beam energy will be broken by machine imperfections and energy saw tooth, which will impact the accuracy of energy determination. These systematic errors are currently being studied.

4.2 Resonances

Depolarisation happens at spin-orbit resonances when the spin precession is coherent with the perturbations from synchro-betatron oscillations

$$\nu_0 = k + k_x Q_x + k_y Q_y + k_s Q_s \quad \text{with} \quad k, k_x, k_y, k_s \in \mathbb{N}, \quad (2)$$

where Q_x, Q_y are the horizontal and vertical betatron tunes and Q_s is the synchrotron tune. Meanwhile, the attainable equilibrium polarisation is restricted by the level of orbit distortions. In presence of vertical orbit distortions the invariant spin axis gets tilted, resulting in depolarization effects [7, 8]. Fig. 1 shows the first-order equilibrium polarisations near Z energy using 15 error seeds which induce different levels of vertical orbit distortion. These error seeds are created from an effective model where truncated Gaussian distributions are utilised to apply small random misalignments ($\sigma_{dx,dy,ds} = 200 \text{ nm}$) and angular deviations ($\sigma_{d_\theta, d_\phi, d_\psi} = 2 \mu\text{rad}$) to all quadrupoles, sextupoles and dipoles. No orbit or optics corrections are applied [9]. As shown in Fig. 1, strong depolarisation occurs near first-order spin-orbit resonances, i.e. where Eq. (2) is fulfilled and the polarisation is almost 0. Furthermore, the attainable polarisation level at nominal energy ($\nu_0 = 103.48$) is lower with increasing rms vertical closed orbit deviation. Excellent orbit and optics corrections are, therefore, required for achieving a sufficient level of polarisation. Future studies aim to predict the polarisation level after proper orbit correction and optics tuning, and determine the maximum acceptable closed orbit for achieving a sufficient level of polarisation, which itself remains to be defined. Complementary, recent progress of the optics tuning working group is reported in [10].

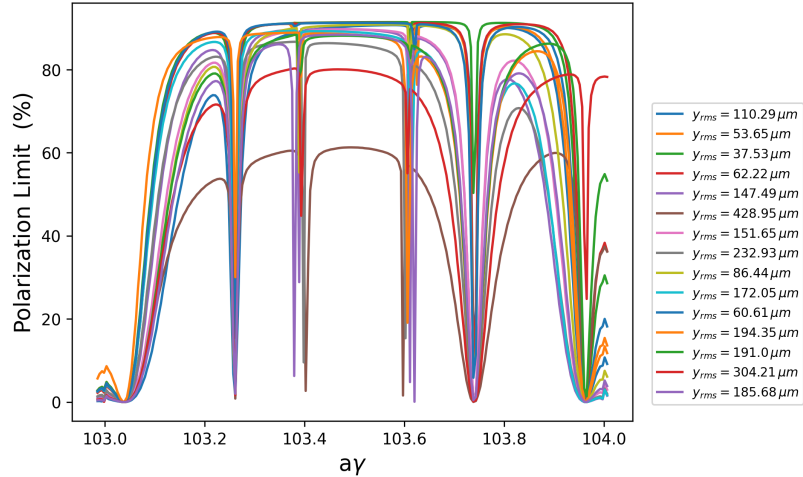


Figure 1: First order energy scan at an energy near the Z ($a\gamma = 103.48$) using Bmad [9].

4.3 Harmonic spin bumps for depolarisation reduction

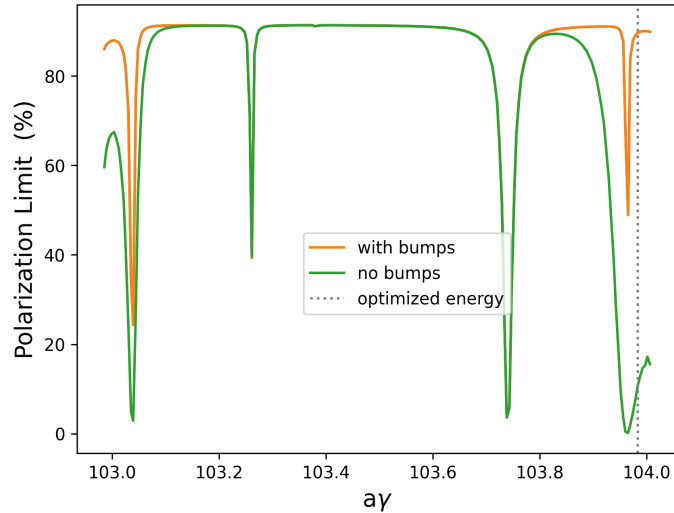


Figure 2: First order energy scan with and without harmonic bumps optimised at 45.82 GeV ($a\gamma = 103.983$) using the Rossmanith-Schmidt scheme in the V22 lattice [9].

After conventional lattice correction, one method to improve the polarisation level and alleviate the effects of misalignments is harmonic closed-orbit spin matching (HCOSM), which uses closed vertical-orbit bumps to manipulate the stable spin direction on the closed orbit ($\hat{n}_0(s)$) and minimise the polarisation loss due to the spin diffusion. Each bump contains three individually powered vertical-orbit correctors, which can be installed next to three consecutive vertically focusing arc quadrupoles. A response matrix can be used to represent the linear contribution of each bump to the target harmonics. In order to correct some specific harmonics of a misaligned lattice, the bump amplitudes are predicted by calculating the inverse of the response matrix.

There are three possible HCOSM schemes that are currently under consideration: the HERA formalism [11, 12], the Rossmanith-Schmidt scheme [13, 14], and the LEP method [15]. Studies of these methods use the FCC-ee V22 lattice together with effective errors resulting in a $72\ \mu\text{m}$ rms vertical closed orbit. Figure 2 [16] shows the first-order equilibrium-polarisation level at an energy near the Z, before and after installing four bumps at 45.82 GeV using the Rossmanith-Schmidt scheme. We note, that at a closed orbit spin tune of 103.0 the polarization does not vanish, which is an artefact from choosing finite energy steps. Employing HCOSM, the polarisation loss near both integer resonances next to the Z pole is significantly improved. This scheme assumes that the vertical orbit information is obtained from BPMs installed at both ends of each dipole (or quadrupole), and does not consider BPM misalignments, resolution or calibration errors. Such extensive instrumentation would be costly, and is, hence, currently the main obstacle to applying HCOSM. Future investigations aim to explore possible HCOSM scheme using fewer BPMs, and test the effectiveness of this scheme in the presence of various lattice errors, BPM misalignments, resolution and calibration errors. In addition, applying HCOSM at W, Higgs and $t\bar{t}$ energies will be explored with the goal of achieving a few percent polarisation at these energies.

4.4 The question of the beam energy

In a perfectly flat circular machine with no solenoid, or synchrotron radiation, the instantaneous spin precession frequency Ω is related to the instantaneous beam energy by

$$\Omega = \Omega_0(1 + a\gamma) \quad (3)$$

where Ω_0 is the revolution frequency. However, energy sawtooth, caused dominantly by synchrotron radiation leads to a different beam energy along the synchrotron storage rings. Averaging over one revolution, the number of spin precessions along the closed orbit is determined by the average beam energy, neglecting misalignments or solenoids, namely

$$\nu_0 = \frac{\langle \Omega \rangle}{\Omega_0} - 1 = a \langle \gamma \rangle \quad (4)$$

where a is the gyromagnetic anomaly. In resonant depolarization measurements, the value of $a\langle\gamma\rangle$ obtained from the average beam energy of the average closed orbit deviates slightly from ν_0 due to machine imperfections. The discrepancy between the closed-orbit spin tune, which is more readily accessible in simulations, and the value of $a\langle\gamma\rangle$ derived from the estimated average energy, is utilized to provide a preliminary assessment of the magnitude of the systematic error.

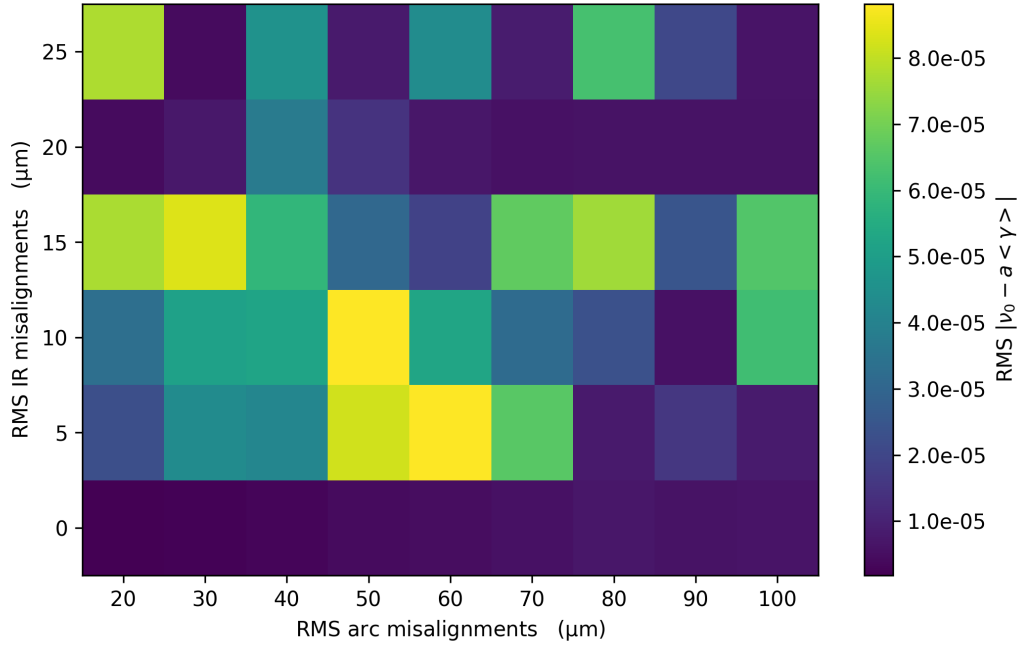


Figure 3: Colormap of the RMS closed-orbit spin tune shift from $a\langle\gamma\rangle$ in the presence of varying levels of arc and IR misalignment

A method has been developed to estimate the average energy by averaging the beam energy over the bending angles. This method yielded results consistent with the closed orbit

spin tune in a perfectly aligned flat machine without solenoids. Fig. 3 shows the RMS closed-orbit spin tune shift from $a\langle\gamma\rangle$ in the presence of varying levels of arc and IR misalignment. So far, the spin tune shift remains below the level of the targeted systematic error. However, the actual magnitude of the current systematic error requires further evaluation, as incorporating additional complex factors could easily push it beyond the precision target.

5 Wigglers

The natural polarisation rise time in the FCC-ee at the Z operation mode is about 250 h, which is too long to wait at the start of physics fills before RDP is performed. Therefore, the pilot bunches must either be injected in a polarised state, or the time needed for polarisation to build up in the FCC-ee must be reduced. The latter approach is the current baseline strategy, as proposed in Ref. [17]. Polarisation wigglers will be used to reduce the rise time such that sufficient transverse polarisation of about 10 % can be achieved in about 100 min. At the start of fill about 100 to 200 pilot bunches will be injected with the polarisation wigglers turned on, but no physics bunches on account of the severe synchrotron radiation during this period of operation. After sufficient polarisation is reached, the wigglers will be ramped down, whilst keeping the polarised pilot bunches. The rest of the machine will then be filled with colliding bunches and RDP measurements will be performed on the pilot bunches at regular intervals.

Requirements on the number and strength of the wigglers have been determined in Ref. [17], taking into account the orbit excursion and synchrotron-radiation power. While reducing the polarisation time, the energy spread is increased, which could lead to polarisation loss and an increased spin-modulation index. Developing wiggler specifications must, therefore, aim to limit the introduced energy spread. The wiggler design for FCC-ee follows the three-pole design of the LEP damping wigglers [18]. The current FCC-ee lattice design foresees the installation of the wigglers in a short straight section downstream of each IP. Wigglers will be grouped in packages of three units, and two packages will be installed in consecutive 16 m long drift-spaces. The required number of polarisation wigglers and first specifications are given in Table 1.

Table 1: Initial specifications for the polarisation wigglers.

Parameter	Value
Number of units per beam	24
Central field B_+ [T]	0.7
Central pole length L_+ [mm]	430
Asymmetry ratio $r = \frac{B_+}{B_-} = \frac{L_-}{L_+}$	6
Critical energy E_c [keV]	968

The alternative approach of injecting polarised beams would have the advantage of making more time available for physics operation. This option is under investigation.

6 Injection of polarised bunches

The current operational scenario foresees the injection of unpolarized pilot bunches, which will naturally polarize over time in the collider rings thanks to the Sokolov-Ternov effect. Even with foreseen wigglers, achieving 5 to 10 % polarization requires roughly 90 min at the Z-pole. During that time, no physics bunches can be circulating in the machine. Since beam energy measurements are foreseen to take place in parallel to physics experiments, this waiting time is required after every beam dump.

Availability studies [19] including failures of systems such as e.g. RF-systems or power converters suggest a great improvement of total integrated luminosity if pre-polarized bunches could be injected into the collider rings. Although no complete study has been performed so far, first thoughts and preliminary results are described here.

6.1 Generation of polarized bunches

Whereas polarized electrons could, in principle, be generated by a polarized source, this is not possible for positrons. Thus, a ring is needed to generate polarized bunches at low energy. Such a Polarization and Damping Ring (PDR) would have some similarities with the damping ring. An important difference is that, in order to achieve significant polarization levels, the wigglers have to be asymmetric with higher fields over shorter distance deflection in one direction (typically inwards) and lower fields over longer regions deflecting in the opposite direction.

The polarization build-up time constants are in the order of minutes for an optimised low energy ring and, thus, significantly shorter than the baseline scheme polarization build-up in the collider prior to filling with physics bunches. As the polarization build-up at low energy is long compared to synchrotron radiation damping times in the damping ring, the latter cannot be used and a dedicated ring is needed. The proposed low energy polarization ring could be used for the generation of both polarized positron and electron bunches.

6.2 Transmission to end of the LINAC

The polarized bunches with a vertical equilibrium polarization are extracted from the PDR and injected into the linac with their vertical polarization. The vertical polarization is maintained throughout the linac.

6.3 Transfer line and booster injection

To date, no complete optics is available for the transfer line (TL) from the end of the linac to the injection point of the booster. The TL lattice has only been matched for the main segments, matching between the segments has not been completed. The TL geometry with all the dipoles that are relevant for spin transport in the absence of imperfections is however available. This information was used for first estimates of the transmission of vertical polarization to the booster injection.

The TL geometry for electrons and positrons is presented in Fig 4. Electrons and positrons share most of the line, only the last sections differ since the two species are injected in

opposite directions into the booster.

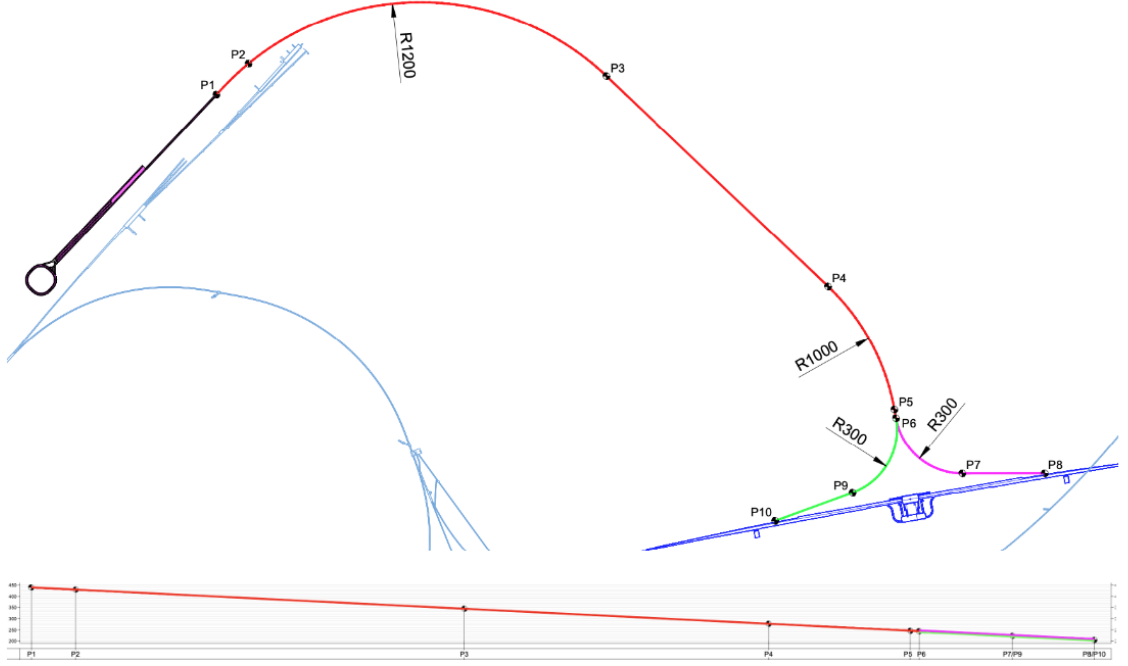


Figure 4: Geometry of the roughly 5.5 km long transfer line from linac to booster. Bottom: vertical cut with a height difference of 230 m between linac and booster.

The transport of polarization through the TL was investigated assuming an initial vertical polarization of the beam [20]. The vertical bends at the beginning of the TLs rotate the polarization away from the vertical direction. Along most of the TL length, the polarization (spin) vector precesses around the vertical axis before the last bends rotate it back the vertical. Due to the spin phase advance between the vertical bends at start and end of the TL, the polarization may not be restored.

At the nominal momentum of 20 GeV, $a\gamma = 45.388$. The total spin phase advance in the horizontal (bending) plane of the TL is $25.2 [2\pi]$ for the part leading to the green branch and $5.0 [2\pi]$ for the magenta branch of the TL (see Fig. 4). The large difference is due to the fact that in the magenta branch, the bending changes sign in the last section and the spin precesses "back". This difference is also reflected in the fact that the total bending in the green branch is larger than 180° while it is around 45° for the green branch (with respect to the direction at the exit of the linac).

By adjusting slightly the energy at the end of the linac (and of course at booster injection) to 19.873 GeV, $a\gamma = 45.099$, the total spin phase advance of both TL segments can be brought close to an integer: $25.04 [2\pi]$ for the green branch and $4.97 [2\pi]$ for the magenta branch. Under such conditions the vertical bends at the end of the TL can effectively restore the initial polarization direction. The results of polarization tracking through the two branches of the TL are presented in Figures 5 and 6: the polarization is very well preserved, with only a loss of a few %. The only side effect of this energy adjustment is a spin tune at injection into the booster that is close to the integer, i.e. $a\gamma = 45.099$.

The estimated bunch energy spread at the exit of the linac is 0.01%, while the shot-to-

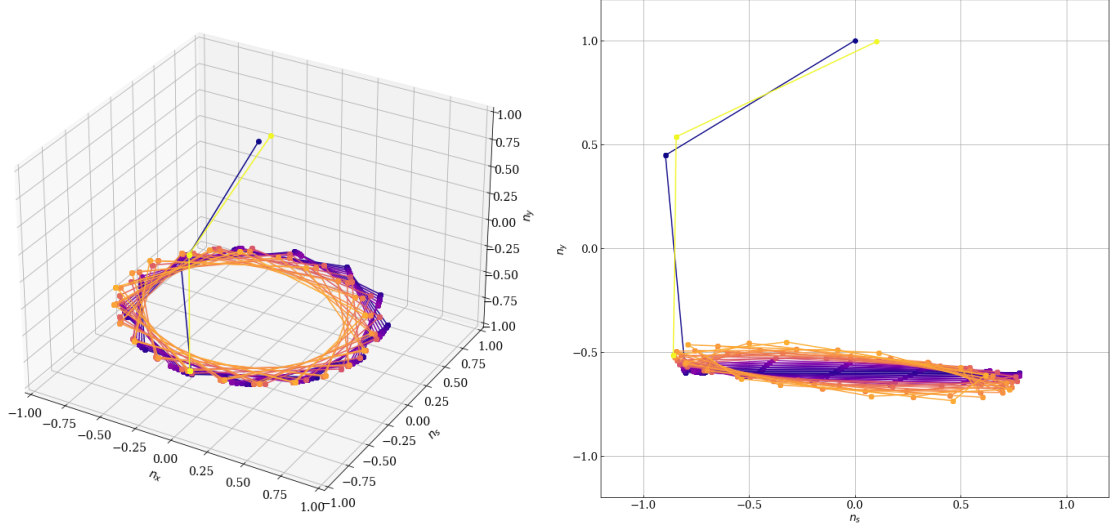


Figure 5: Polarization transport through the green branch of the TL. The polarization is initially vertical (dark blue point). Each point represents the polarization direction in one elements of the line, color-coded from dark blue to yellow. The final orientation of the polarization is given by the yellow point. Left: three dimensional representation (x,s,y). Right: projection in the s-y plane.

shot energy jitter is estimated to 0.03%. Assuming that the later corresponds to 2 standard deviations, the convoluted RMS spin tune spread at the entrance of the TL is estimated to be $\sigma_{a\gamma} = 0.083$. A convolution of the spin tune spread with the polarization transport efficiency as a function of the spin tune results in a $\approx 97\%$ transmission of the polarization.

The combination of horizontal bending on a transfer line with a slope leads to a rotation of the coordinate system. While this effect is usually very small for short lines, it cannot be ignored for transfer lines of many kilometres. This coordination system rotation affects for example the transfer lines from the SPS to the LHC ([21]). For the FCCee TL the effect is small for the magenta branch (due to the reversed bending in the last segment), but it is important for the green branch, with a tilt of the coordinate system of almost 10° degrees. The effect must be properly analysed in future simulations, but the potential impact on the vertical polarization will not exceed a few percent.

Those results will have to be refined once a complete optics becomes available.

The booster injection will have little impact on the vertical polarization: the septa bend horizontally, and the vertical deflection of the injection kicker is very small, with negligible impact on the vertical polarization.

6.4 Booster energy ramp

As the beam energy ramps in an electron (positron) booster ring, so is the closed orbit spin tune ν_0 and the amplitude-dependent spin tune ν_s among beam particles since $\nu_s \approx \nu_0 \approx a\gamma$. The crossing of the underlying spin resonances could lead to beam depolarization. The polarization loss during crossing of a single, isolated spin resonance $\nu_0 = K$ can be estimated

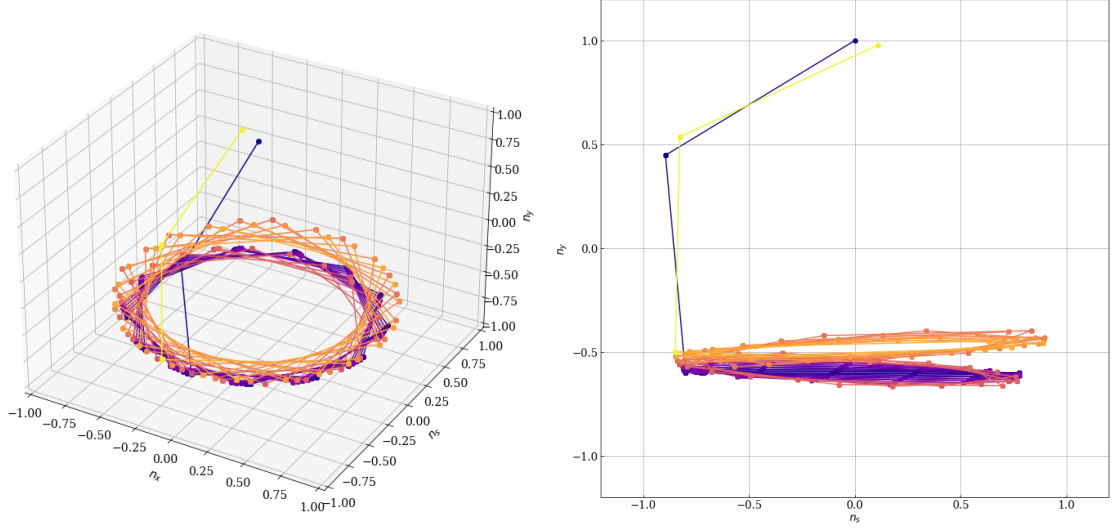


Figure 6: Polarization transport through the magenta branch of the TL. The polarization is initially vertical (dark blue point). Each points represents the polarization direction in one elements of the line, color-coded from dark blue to yellow. The final orientation of the polarization is given by the yellow point. Left: three dimensional representation (x,s,y). Right: projection in the s-y plane.

with the Froissart-Stora formula [22]

$$P_f/P_i = 2e^{-\frac{\pi|\tilde{\omega}_K|^2}{2\alpha}} - 1, \quad (5)$$

P_i and P_f are the beam vertical polarization before and after crossing the resonance, $\tilde{\omega}_K$ is the spin resonance strength, $\alpha = \frac{d\omega}{d\theta}$ is related to the energy ramping rate. There are two families of important spin resonances in this context: the imperfect resonances $\nu_0 = k, k \in \mathbb{Z}$, mainly driven by horizontal magnetic fields due to vertical orbit offsets in quadrupoles and dipole roll errors, and the intrinsic resonances $\nu_0 = k \pm Q_y, k \in \mathbb{Z}$ with Q_y the vertical betatron tune, driven by the horizontal magnetic field due to vertical betatron oscillations in quadrupoles. Adjacent imperfection resonances are spaced by 440 MeV, hundreds of spin resonances of these two families will be crossed in the acceleration of FCC-ee high energy booster (HEB) from injection energy at 20 GeV to extraction energies of 45.6 GeV, 80 GeV, 120 GeV and 182.5 GeV. Since the HEB operates in a fast ramping manner, α is on the order of 0.01, only if the spin resonances are so weak that $|\epsilon| \ll 0.005$ then the depolarization when crossing each resonance is much less than 1%, the beam polarization could be mostly preserved. It is therefore essential to evaluate the spectrum of spin resonances of the HEB lattices.

The spin resonance spectra of two different HEB lattices, i.e., the FODO 90 lattice and the HFD lattice were evaluated [23]. Both feature a super-periodicity of $P = 8$ with interleaved arc and straight sections. Each arc section contains $M = 175$ standard cells, the vertical betatron tunes are $Q_y = 416.29$ and $Q_y = 382.29$ respectively, and the total contribution from these arc sections to the vertical betatron tune is 350 and 320 respectively. These lattices have effectively a very large super-periodicity of $PM = 1400$. Following previous

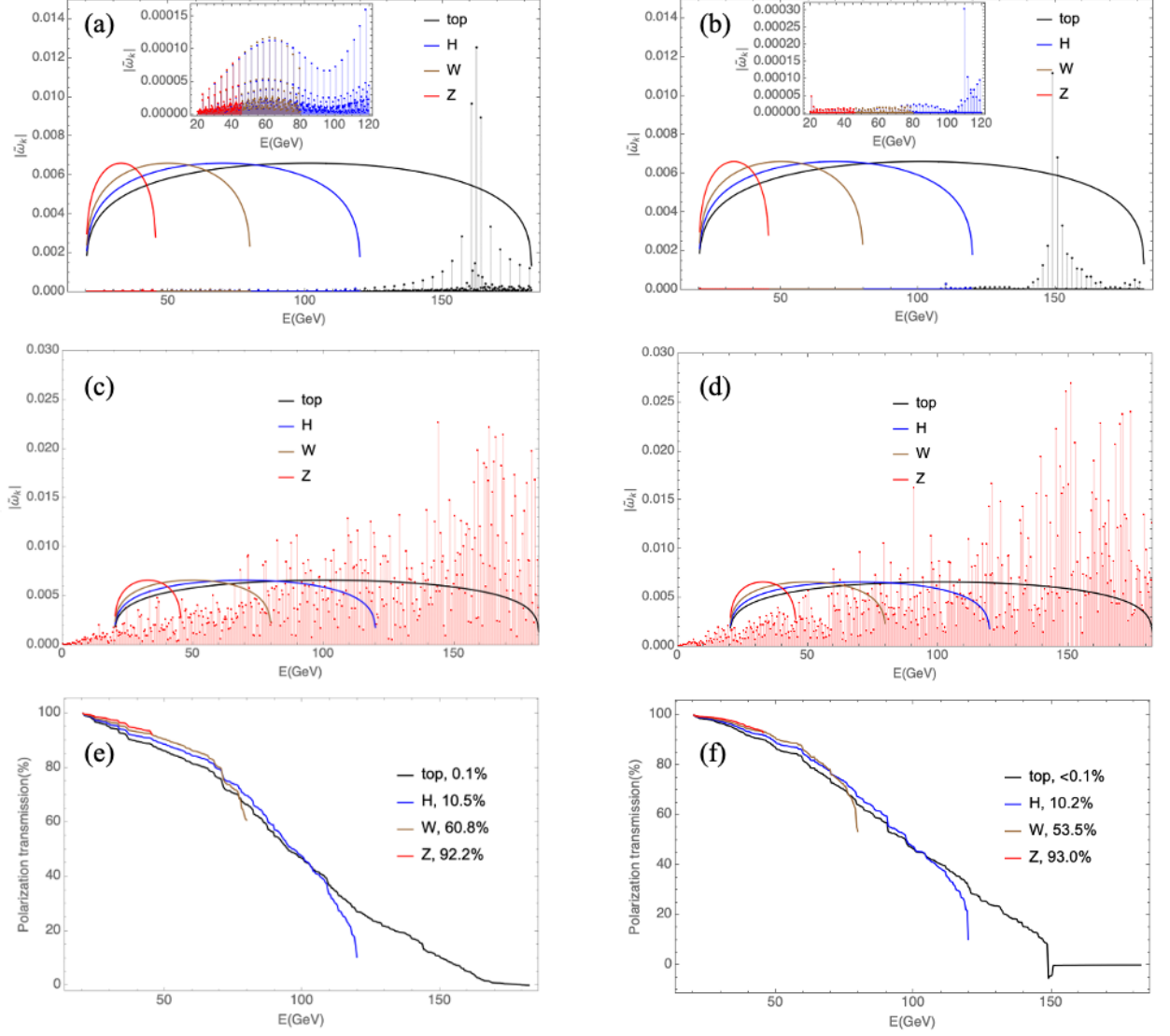


Figure 7: Spin resonance spectra for the two HEB lattices [23, 25]. For the FODO 90 lattice, (a), (c) and (e) are the spectra of intrinsic resonances of the bare lattice, the spectra of the imperfection resonances of an imperfect lattice seed, and the polarization transmission as estimated by repetitive usage of the Froissart-Stora formula. (b), (d) and (f) are the corresponding results for the HFD lattice.

studies for the CEPC booster [24], the super strong intrinsic and imperfection resonances are located near $(kPM \pm \nu_B)/\eta$, $k \in \mathbb{Z}$, η is the fraction of the total bending angle of all standard arc cells in 2π , which equals to 0.9486 for both lattices. Therefore, the first super strong resonances are located near $\nu_0 = 369$ and $\nu_0 = 337$ for these two lattices, corresponding to a beam energy of 162.6 GeV and 148.8 GeV respectively. This suggests strong depolarization in the acceleration to the $t\bar{t}$ energy, while the spin resonances can be relatively weak in the acceleration up to the HZ energy [23].

To evaluate the transmission of beam polarization during the acceleration process, a

cosine shape energy ramping curve was assumed, following

$$E(t) = E_{\text{inj}} + \frac{(E_{\text{ext}} - E_{\text{inj}})}{2} \left(1 - \cos\left(\frac{\pi t}{t_{\text{ramp}}}\right)\right). \quad (6)$$

It should be noted that this is an inefficient ramp compared to a standard parabolic-linear-parabolic (PLP) ramp shape. A PLP is efficient because the ramp is mostly linear and matched to the ratings of the large power converters (ramp and acceleration rates). A cosinus shaped ramp does not make good use of the ramp rate performance, respectively requires much more performant power converters with a significant impact on cost. Since the depolarization mostly occur near the higher energy end with generally stronger spin resonance strengths and a slower energy ramping rate which inevitably decreases to zero, assuming cosine shape ramp shape could provide a reasonable estimates of the polarization transmission, before more detailed energy ramp design is available taking into account various engineering complexities.

The booster parameters are taken from the mid-term report of the FCC Feasibility Study. Imperfect lattices were generated and closed orbit corrections were conducted, for the evaluation of strengths of imperfection resonances. The intrinsic and imperfection spin resonance strengths were evaluated using the DEPOL code [26] for both lattices, as shown in Fig.7.

The coloured curves show the spin resonance strength with 99% polarization transmission. Those spin resonances above these curves lead to more substantial depolarization. Assume there is no correlation between the crossings of multipole spin resonances, the Froissart-Stora formula are repetitively used to estimate the cumulative effects of the depolarization due to the crossing of each intrinsic and imperfection resonances. Up to 120 GeV, intrinsic resonances cause negligible depolarization, and some stronger imperfection resonances together with the slower ramping speed at the higher energies would lead to some depolarization. In the acceleration to 182.5 GeV, more severe depolarization is expected mainly due to the large number of stronger imperfection resonances.

More random imperfect lattice seeds were generated for both lattices for the evaluation of the polarization transmission, as shown in Fig.8. Assuming 100% polarization at the injection energy, the extracted beam polarization is typically about 90% at 45.6 GeV, about 60% at 80 GeV, about 15% at 120 GeV and zero at 182.5 GeV, no major difference between the two HEB lattice is seen.

These studies were based on the error settings and correction schemes still under development, and more comprehensive error sources and more sophisticated error correction schemes are yet to be included. The ramping curve can be in principle optimized to partially reduce the depolarization, but the more realistic PLP ramp yields a bit worse polarization transmission for the Z-mode, compared to the result obtained by assuming the cosine ramp. The polarization transmission was estimated by repetitive usage of the Froissart-Stora formula, and previous comparison between estimates and tracking simulations showed good agreement in the polarization transmission if the depolarization is small, while tracking simulations are necessary to quantitatively evaluate the preserved polarization level after acceleration to higher energies like HZ. In summary, this preliminary study suggests the polarization can be well preserved in the booster acceleration to Z and W energies, if highly polarized beams are prepared from the source and injected into the booster with the proper vertical polarization direction.

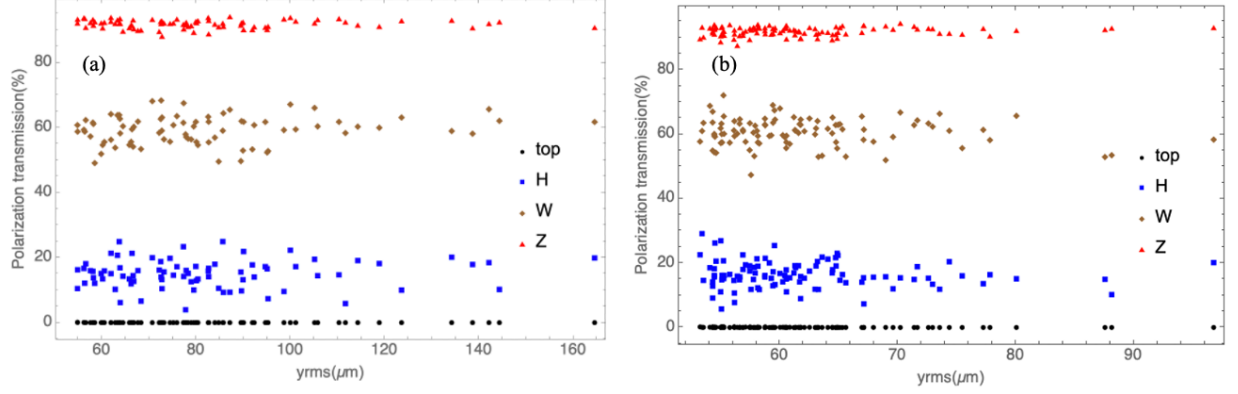


Figure 8: Estimated polarization transmission to different extraction energies [25]. (a) shows the polarization transmission as a function of the vertical rms closed orbit distortion for 79 random seeds of the FODO 90 lattice; (b) shows the corresponding result for 100 random seeds of the HFD lattice.

6.5 Collider injection

No details are available for the geometry of the transfer between booster and collider rings. It has however been agreed that vertical and horizontal bending will not be interleaved, thus no issues are expected at this stage of the study.

7 FCC powering

The main dipole magnets define the beam energy on the central orbit (*i.e.* which is centred on average in all quadrupoles). The stability of the magnetic field must match the desired accuracy of a single energy measurement, at least on the time scale of a single RDP measurement, else the depolarising resonance width will be enlarged by the magnetic field ripple.

For a target accuracy of 10 keV for example, the intrinsic magnetic field stability at 45 GeV should be at the level of 0.2 ppm (excluding uncontrolled external perturbations). This translates into the same stability of the power converters feeding the main dipoles magnets. This stability requirement applies for a time scale of one hour which should be sufficient even for slow RDP scan (see below).

8 Depolarisers System

It is foreseen that the depolarizer systems will be used for all bunches. On the one hand polarized pilot bunches are depolarized regularly for RDP scans. On the other hand, colliding bunches build-up polarization over time and that could spoil physics measurements and therefore they must be depolarized too. Considerations presented here on the depolarizer focus on requirements for RDP scans.

8.1 Placement considerations

In the FCC-ee the polarization is oriented vertically. RDP aims to destroy this polarization by rotating the spin away from the vertical axis which requires excitation of the vertical orbit. To avoid a propagating vertical orbit and forced vertical betatron oscillation through the machine, even for the pilot bunches, this orbit bumps must be closed, realised by either 2 kickers with π phase advance between them, or 3 ones with less stringent phase advance constraints. Furthermore, a closed orbit bump without any dipole magnets in-between resembles a closed spin-orbit bump without any net rotation. The depolarizers must, therefore, be integrated in the arc structure [27].

8.2 Simplified model

A simplified simulations framework is generated to study the RDP process, based on one-turn matrices. A more detailed description is given in [28]. Assuming only a vertical magnetic field, $\vec{\Omega}$ has solely a vertical non-zero component. Further, in the absence of transverse particle motion, the spin motion is simply a precession around the vertical axis with a closed-orbit spin tune ν_0 [29]. The one-turn spin matrix, therefore, becomes

$$\begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}_{n+1} = \begin{pmatrix} \cos 2\pi\nu_0 & 0 & -\sin 2\pi\nu_0 \\ 0 & 1 & 0 \\ \sin 2\pi\nu_0 & 0 & \cos 2\pi\nu_0 \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}_n. \quad (7)$$

The excitation through the depolarizer, before or after every turn, is described by

$$\Delta\vec{S}(n) = \begin{pmatrix} \theta_x(n) \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}_n, \quad (8)$$

where

$$\theta_x(n) = \theta_s \sin(Q_{\text{dep},0} 2\pi n + \tilde{q}_{\text{dep}} \pi n^2), \quad (9)$$

with instantaneous depolarizer tune

$$Q_{\text{dep}}(n) = Q_{\text{dep},0} + \tilde{q}_{\text{dep}} n. \quad (10)$$

We note that ν_0 and θ_s depend on a particle's relative energy deviation $\delta \equiv \Delta E/E$ as $Q_0 \rightarrow \nu_0(1 + \delta)$. At 45.6 GeV beam energy, a single depolariser deflects the particle bunch by about $10 \mu\text{rad}$, and rotates the spin by about $a_e \gamma$ this amount, that is by about 10^{-3} rad in a single passage.

Simulations without synchrotron oscillation, for a single on-energy particle, with initial spin oriented towards the positive vertical direction ($S_{y_{\text{initial}}} = 1$) are performed, assuming a tune sweep velocity, $\tilde{q}_{\text{dep}}$ and kick strength θ_s . The scan is performed in both directions. It can be seen in Fig. 9 that the spin is fully flipped at $Q_{\text{dep}} = \nu_0$.

We note that this simulation framework represents an idealistic, simplified description of the RDP process, neglecting betatron oscillations, detailed machine description, synchrotron radiation, energy drifts, etc. Nevertheless, it can be seen as a starting point to fully understand the RDP process and could be extended in future studies. Results assuming one Gaussian bunch with synchrotron oscillations are given in [28].

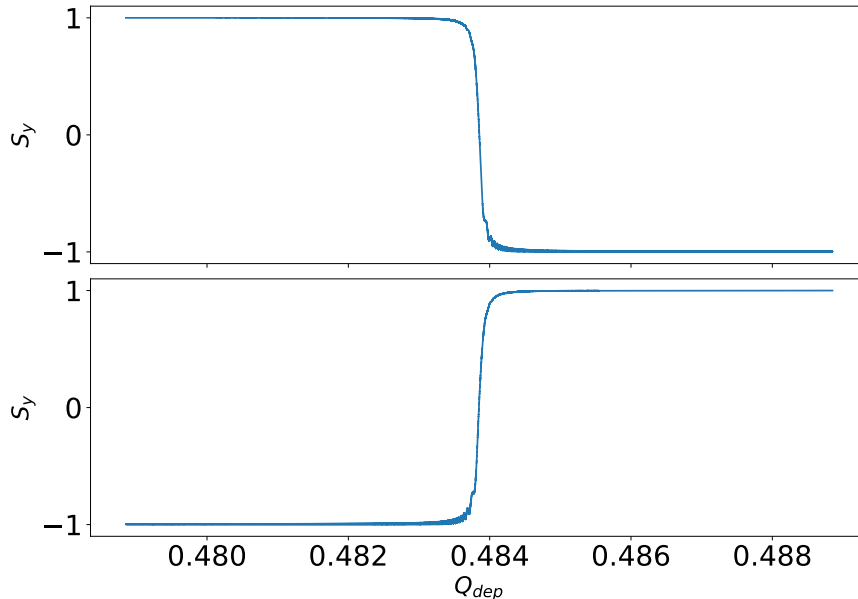


Figure 9: RDP scan for a single on-energy particle with $\tilde{q}_{\text{dep}} = 10^{-8}$ and $\theta_s = 10^{-3}$. The scan is performed from lower to higher depolarizing frequencies (top) or in the opposite direction (bottom).

8.3 Depolarizer design

As depolariser kickers striplines supplied by RF power counter propagating the beam offer both the required bandwidth for bunch selective RDP and the possibility to inject a signal to prevent polarisation build up on the physics bunches as needed. Integrating the system together with the kickers needed for the transverse feedback has been explored and seems feasible with the machine protection considerations related to the strong kicks needed for the RDP scans requiring particular attention.

While increasing the length of a stripline kicker increases the power efficiency, the maximum length permitted for a kicker will be defined by space considerations and the needed bandwidth for bunch selective kicking of the pilot bunches spaced by 100 ns in the gaps of the beam. It is therefore suggested that the basic building block for the kickers has an active length of 1 m with a stay clear aperture determined by beam impedance considerations and the chosen installation region.

The number of kickers needed and their final geometry will be chosen to minimize the required RF power, adapted to the installation locations with a cross-section that is still acceptable for machine protection and beam impedance. For example a total of 16 main kickers (8 per beam) and 8 auxiliary kickers (4 per beam) can be used at 18 mm stay clear aperture using a total of 144 kW RF power for the main kickers to achieve several modulated orbit bumps adding to an equivalent modulated bump as created by two 10 μrad kicks (per beam).

In one does not impose a kicker system in the form of one or multiple closed bumps, but rather **relies on a single kicker**, the performance of the depolariser for the RDP measurements can be reinforced significantly by the FCC-ee optical structure. It should

however be pointed out that this option is not recommended due to the large betatron oscillation of the kicked bunches. The gain from the optical structure, calculated here using the ASPIRRIN program [30] as a modulus of the spin-orbit response function $|F_3|$, can be many thousands and even tens of thousands units as seen in Fig. 10 for the Z-lattice at 45.1358 GeV, corresponding to ν_0 of 102.45 (the responses are twice greater than at the spin tune 102.5 due to twice less detuning from the resonant value of the betatron frequency 222.4).

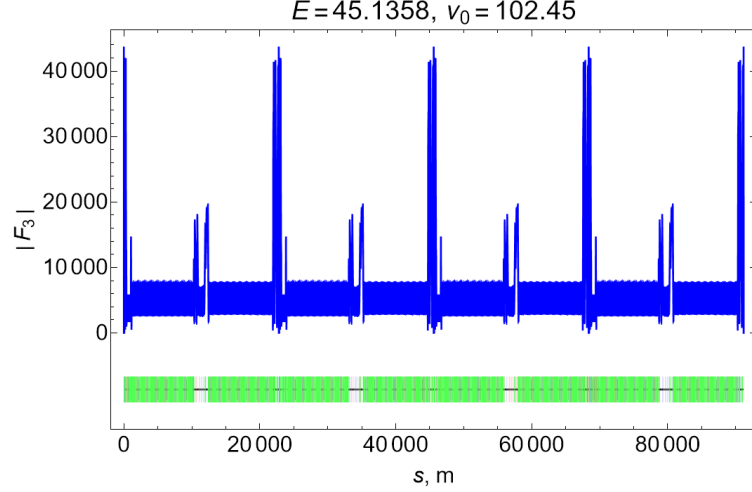


Figure 10: Spin-orbit response function $|F_3|$ for the closed-orbit spin tune 102.45 around the ring.

In some cases, the proximity of the fractional part of the spin tune to the resonant betatron tune of 0.4 clearly manifests itself. In these situations, the maximum response is inversely proportional to the difference of the driven tune to the closed-orbit spin tune. And at other values than at the nearest integer value of the spin tune, such resonant amplification is not observed. It is verified that the full resonance strength, w , is related to the amplitude of the kick-angle θ of the trajectory by the RF kicker by

$$w = |F_3|\theta/2\pi . \quad (11)$$

We assume that the main mode of the RDP procedure should be a continuous monotonic scanning of the kicker frequency with the following considerations.

- The natural width of the spin line, caused by the energy spread of the beam, is about 200 keV at 45 GeV and 1.4 MeV at 80 GeV due to the radiative diffusion of the spin precession and influence of a quadratic non-linearity of the guide field at FCC-ee.
- To determine the spin resonance with an uncertainty much smaller than the width of the spin line, it is proposed to use two kickers simultaneously, each of which selectively affects one of two pilot bunches, scanning their frequencies in opposite directions;
- In general, the main spin resonance may drift in time. As shown by preliminary Monte-Carlo simulation, the proposed method of simultaneous counter-scanning makes

it possible to determine with an accuracy better than 10 keV the position of the spin resonance relative to a certain point in time together with the speed and direction of the collider energy drift.

- Given the accuracy requirements, the relevant scan rate is of the order of 1 keV/s in energy units, or 0.007 Hz/s in the frequency scale. The corresponding resolution of the kicker-synthesizer in terms of the frequency tuning step should be no worse than 10^{-4} Hz, if taken with a margin.
- The depolariser is assumed to be implemented as a stripline that creates a TEM wave propagating towards the beam, as shown in Fig. 11. In this case, the depolarising effect is exerted by the horizontal magnetic and vertical electric fields of the wave together.

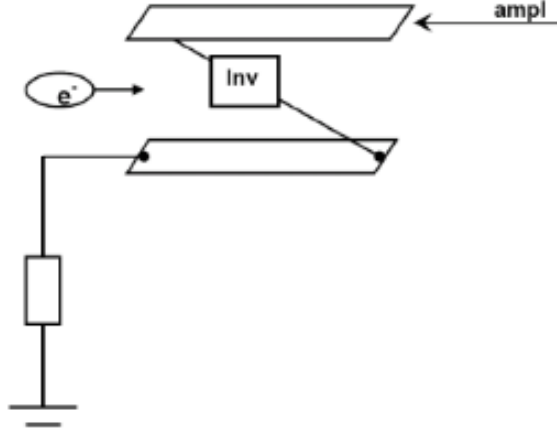


Figure 11: TEM wave-based depolariser: ‘ampl’ is an amplifier of signal from the synthesizer; ‘inv’ is the signal inverter.

The amplitude of the spin resonance strength created with the TEM wave-based depolariser can be written as

$$|w_k| = \frac{\nu_0 B l}{2\pi B\rho} |F^{\nu_0}| = |F^{\nu_0}| \frac{\nu_0 \phi}{2\pi} \propto \frac{\nu_0 U l_d |F^{\nu_0}|}{Ed} \quad (12)$$

where $\nu_0 = E[\text{MeV}]/440.65$ is the spin-tune parameter, B is the amplitude of the TEM wave magnetic field, l is the strip-line length, $B\rho$ is the magnetic rigidity, $|F^\nu|$ is the module of the spin response function at the azimuth of the kicker location, ϕ is the angle of a particle trajectory deflection, and $\nu_0 \phi$ is the spin-rotation angle in one passage through the section with horizontal magnetic field B . We calculate the spin-response function $|F^{\nu_0}|$ using its classical analytic definition [31]. Figures 12 and 13 present the results of our calculations of $|F^{\nu_0}|$ for energies of 45.6 and 80 GeV¹. In accordance with preliminary estimates of the

¹It should be noted that in the ASPIRRIN code [30], the similar function is defined as ν_0 times larger. In this case, the multiplier ν_0 is absent in the corresponding form of Eq. 12.

effectiveness of the depolariser, both kickers can be placed anywhere in the arcs, where the response function, periodic with azimuth, is several units, for example $|F^{\nu_0}|=5$, as is the case in Figs. 12 and 13. Furthermore, the spin response also depends on the beam energy as illustrated in Fig. 14. Comparing Figs. 12 and 13, we can conclude that the spin responses of the same depolariser at energies in the region of 80 GeV and in the region of 45 GeV are of the same order of magnitude in amplitude.

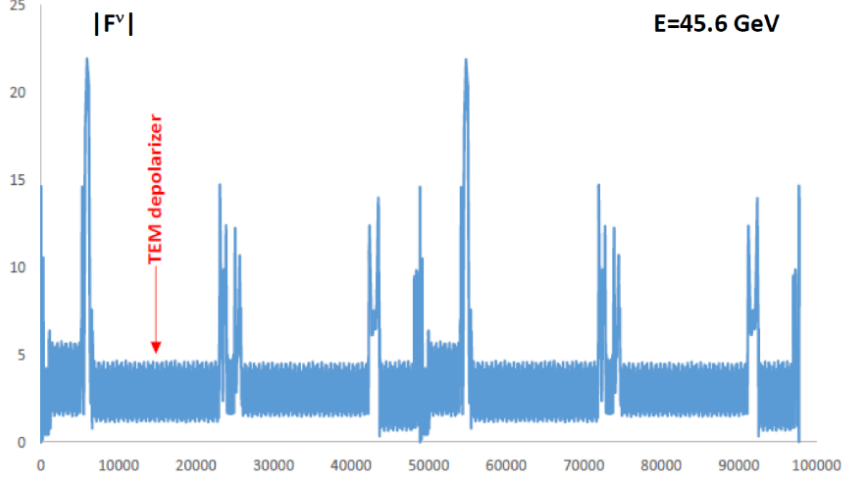


Figure 12: Spin response function $|F^{\nu_0}|$ versus FCC-ee longitudinal position at 45.6 GeV (vertical betatron tune 269.215).

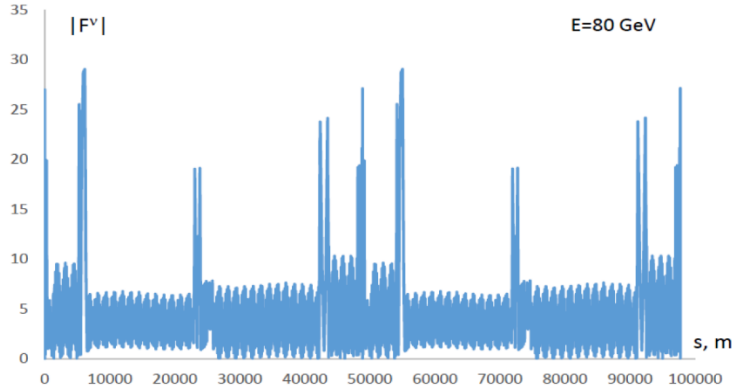


Figure 13: Spin response function $|F^{\nu_0}|$ versus FCC-ee longitudinal position at 80 GeV (vertical betatron tune 269.215).

The forced depolarisation rate is proportional to $|w_k|^2 = \nu^2 |F^{\nu_0}|^2$. The amplifying factor $|F^{\nu_0}|$ significantly grows in the vicinity of the intrinsic spin resonances $\nu_0 \pm Q_y = k$. The choice of technical parameters of the TEM wave-based depolariser, such as the length l_d of the plates, or the amplitude U_d of the voltage on the plates, depends on the harmonic amplitude requirement. RDP scanning should lead to depolarisation of the beam in a time of a few minutes at a scanning rate that provides the required accuracy for a given spin line

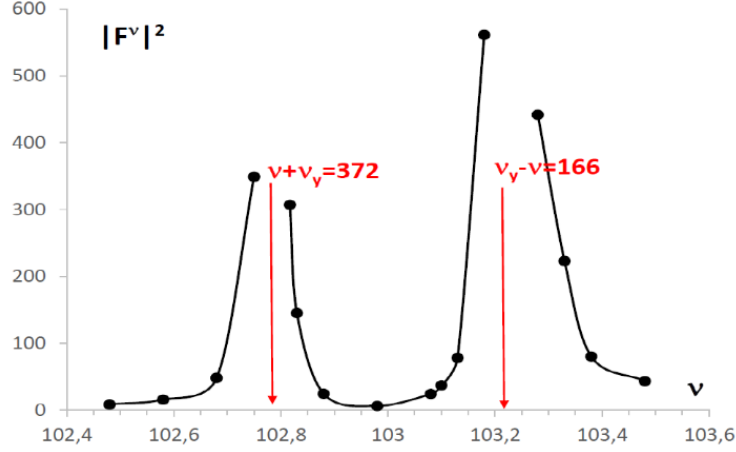


Figure 14: Response function $|F^{\nu_0}|$ of depolariser which is placed at the azimuth=14947 m in the arc (see Fig. 12) versus the spin tune parameter in Z-pole region ($Q_y=269.215$).

width. In our calculations of depolarisation process, the parameter τ_{RFD} is used. This is the characteristic depolarisation time for the selected case, when scanning is not performed, and after turning on the depolariser, its line completely turns out to lie within and remain inside the spin spectral band Δf

$$\tau_{RFD} = \frac{\pi \Delta f}{\omega_0^2 |w_k|^2}, \quad (13)$$

with ω_0 the angular revolution frequency. When modeling a process of RDP scanning, the time dependence of the convolution of the depolariser and spin spectra is taken into account, and τ_{RFD} is used as a scaling parameter. Table 2 gives an example set of depolariser parameters for which the harmonic amplitude $|w_k|$ is of the order of 10^{-5} and the time τ_{RFD} a few ten seconds. Monte Carlo simulations show that these scaling parameters are well suited to achieve better than 10 keV accuracy at a scan rate of approximately 1 keV/s. It will take at least 20 min for a single calibration of the beam energy at a given reference time.

Table 2: Depolariser parameters at the Z-pole, assuming a strip-line design.

Parameter	Value
Beam energy E [GeV]	45.6
Strip-line length l_d [m]	1
Vertical gap between plates d [mm]	20
Amplitude of signal from amplifier [V]	100
Amplitude of voltage between plates [V]	200
Spin response factor $ F^\nu $	5
Spin rotation in one passage $\nu\phi$	2.3×10^{-5}
Harmonic amplitude $ w_k $	1.8×10^{-5}
Spin spectral band Δf [Hz]	1.4
Characteristic time τ_d [s]	38

In an alternative approach using a stronger depolariser, it is proposed to find the spin resonance by the method of adiabatic spin-flip with the preservation of polarisation. For this it is proposed to use a harmonic amplitude of about $w = (1.0 - 1.5) \times 10^{-4}$. In this case, the resonance search time can be reduced to 10-60 s. This stronger depolarisation can be achieved by both lengthening the depolariser plates from 1 to 3 m, and increasing the amplitude of the voltage in the travelling TEM-wave. An extension to this approach is to rotate the spin vector into the horizontal plane and then measure the spin tune from free precession. Achieving this requires a larger value for the spin-harmonic amplitude, up to $w = 1.0 \times 10^{-3}$. The very large modulus of the spin-orbit response function at certain energies (see Fig. 14) indicates that this approach should be feasible.

9 Polarimeter

9.1 Physics requirements and measurement techniques

The stringent demands of the beam-energy measurement at FCC-ee imposes several requirements on the polarimeter system. Most of these requirements are driven by Z-pole operation. The needs for such precision beam-energy information at the W^+W^- and ZH modes need further consideration. At $t\bar{t}$ the polarisation of the pilot bunch cannot be maintained since the beam energy spread is overlapping with depolarisation resonances, nevertheless Z calibration runs are still foreseen to be performed occasionally.

- It will be necessary to perform measurements of equal precision and frequency on both the electron and positron beams. This requires a separate polarimeter for each beam. Considerations of systematic robustness and system availability may motivate installing more than a single pair of polarimeters, perhaps with differing attributes, but this is not yet considered to be part of the baseline plan.
- The polarimeters will be mainly used for resonant depolarisation beam-energy calibration (RDP hereafter) applied to the pilot bunches, providing a precise measurement of the average spin tune of both lepton beams. Given the relation between the average spin tune and the average beam energy, this information will be combined with the boosts measured at each experimental IP by the physics detectors to infer a precise knowledge of the collision energy conditions in each experiment.
- A second task for the polarimeter instruments will be to provide a precise knowledge of the polarisation state of the colliding bunches. Both transverse and longitudinal polarisation levels need to be measured with high precision, this is to insure absence of bias in the expected physics cross-sections at the experiments level.
- The bunch polarisation will be measured through the experimental process of inverse Compton scattering. This type of event will be produced by colliding rather dense laser pulses onto selected lepton bunches of the train. Measurements of the full polarisation vector are required, rather than only the transverse component, as was monitored at LEP. This requirement is motivated by FSP measurements, which are best performed having rotated the polarisation vector to the horizontal plane, and for placing limits on

the size of any longitudinal polarisation that may exist in the colliding physics bunches. Measuring the full polarisation vector necessitates detecting both the scattered electron², and the back-scattered photon. On each bunch crossing we expect about 1000 Compton collision to happen. The Compton pattern obtained with 10 million events (about 3 seconds of data acquisition) is presented in Figs. 15. These patterns are finally fit with a convolution of the Born Compton cross-section with a two dimensional resolution function to extract the polarisation levels [32]. Systematic uncertainties arising from radiative corrections are expected to be of few per-mille at most at 45 GeV [33]. Toy Monte-Carlo studies have been performed at high statistics (100 million scatters) so far to investigate possible fit biases. Results are shown for the most important observables in Fig. 16.

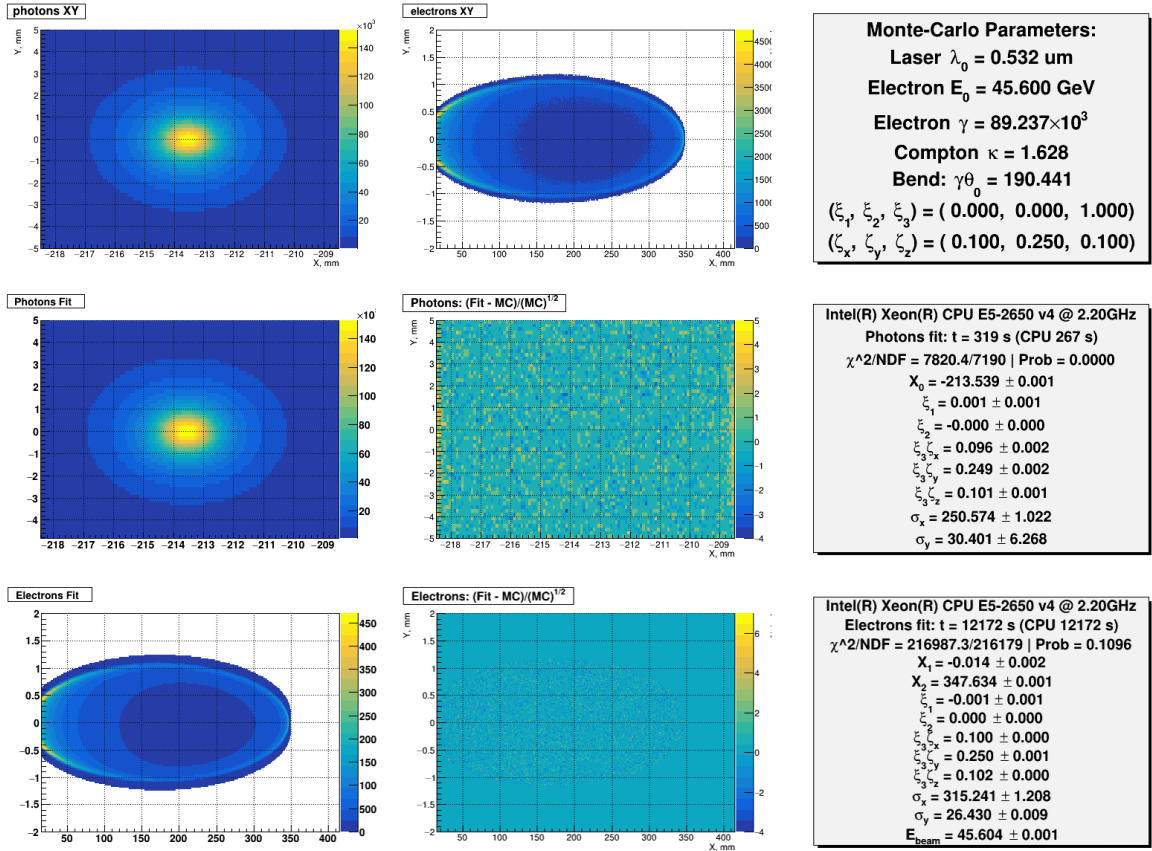


Figure 15: (Top) Simulated distributions of scattered (left) photons and (middle) electrons as they could be measured in a perfect detector. Electron beam emittance and dispersion is included in the simulation. Relevant parameters are shown in the top right box. Fit results are shown on the second line (distribution on the left, residuals in the middle, relevant parameters on the right). Similar results for electrons on the bottom line. Statistical precision on the polarization parameters is within a per-mille.

²For convenience, only the electron polarimeter will be described, with the assumption that the positron polarimeter will have identical attributes.

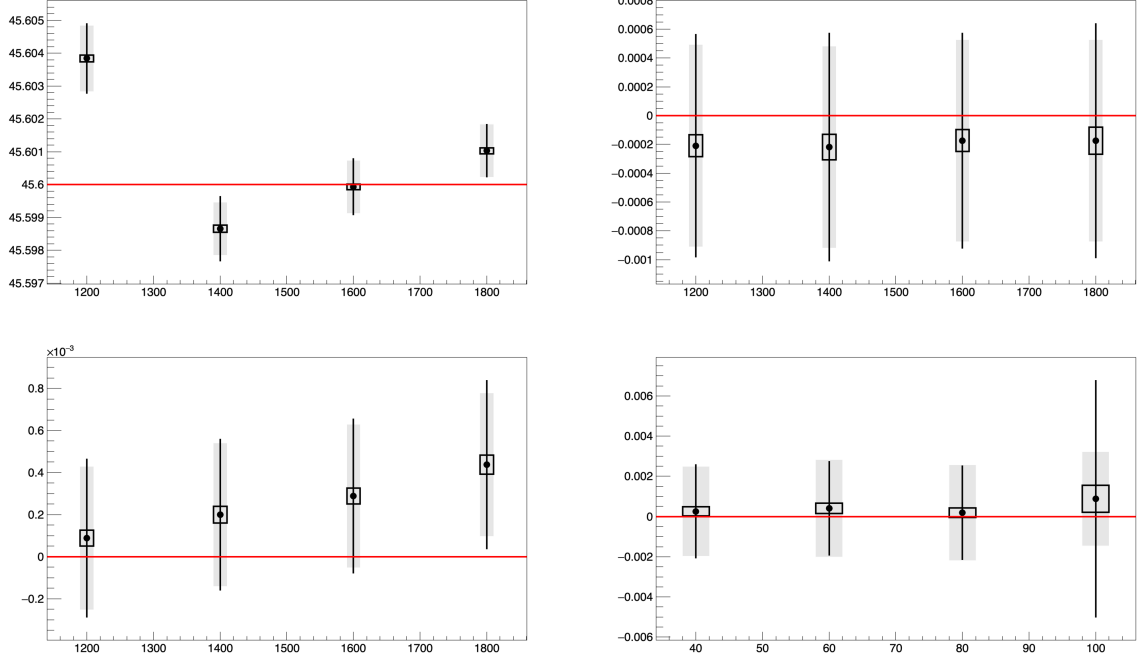


Figure 16: Results of a toy Monte-Carlo (100 experiments of 100 million scatters each) for (Top-left) the direct extraction of beam energy (in GeV) from scattered particles profiles, (Top-right) vertical electron beam polarization measured with electron detector, (Bottom-left) longitudinal electron beam polarization measured with electron detector and (Bottom-right) horizontal electron beam polarization measured with the photon detector. Red-line show the generated value. Dots represent the mean of the fit results and the box its uncertainty. The grey band is the average of the single fit uncertainties, and the black line is the standard deviation of single fit values. The horizontal axis represent different choices of the number of horizontal pixels of the electron detector (photon detector for the bottom-right plot). The electron (photon) detector size is taken to be 400 mm^2 (10 mm^2). Simulated pixel numbers correspond to $(1200 \times 40, 40 \times 40)$, $(1400 \times 60, 60 \times 60)$, $(1600 \times 80, 80 \times 80)$, and $(1800 \times 100, 100 \times 100)$ for the electron detector (horizontal $\times$ vertical axis and photon detector respectively).

- It is also possible to build left-right asymmetries by flipping the helicity of photons in the initial state. This technique is more robust against QED radiative corrections [33] but assumes excellent stability of all experimental parameters. This technique is still under conceptual study. Only preliminary work has been recently done.

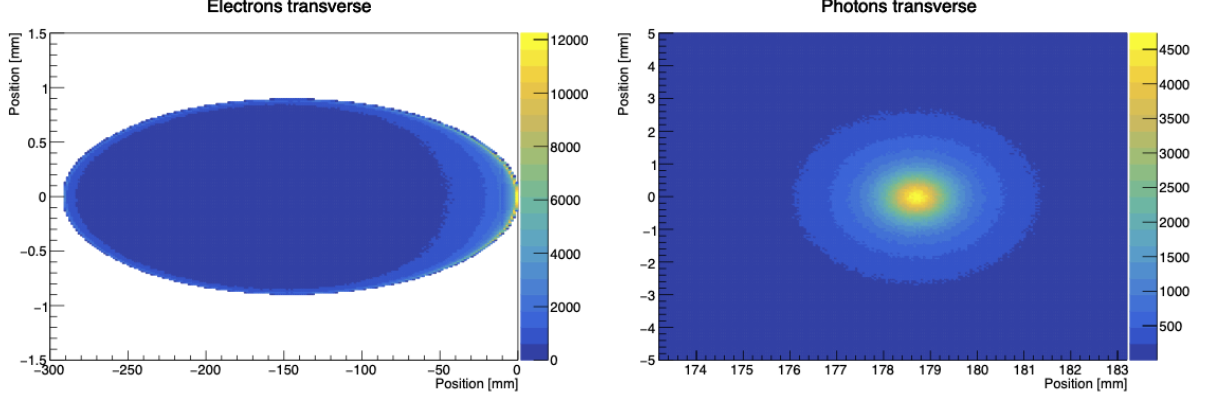


Figure 17: Left: transverse profile of the Compton electrons reaching the extraction windows 96 m from the laser IP. Right: transverse profile of the Compton photons reaching the extraction windows 96 m from the laser IP.

- RDP energy calibration measurements would be performed every 10-15 minutes selecting one bunch of the pilots train. These frequent RDP measurements are essential to guard against any residual energy drifts from uncorrected tide effects or other mechanisms of energy change in the machine [34]. The baseline RDP method of operation would be to change an electromagnetic depolarising signal applied to the selected bunch by the kickers. The sweep excitation of the bunch would happen in steps over several machine turns. Each step will be followed by shining the laser on the selected bunch, for a few seconds of Compton scattering to happen providing the current polarisation level. When the depolarisation occurs, the frequency of the excitation signal at that step of the sweep is then used to provide the beam-energy calibration. A baseline goal for these measurements would be to achieve a 1% statistical precision every second, but this target requires further investigation.
- A complementary way to measure the beam-energy is to perform a so-called free spin precession (FSP) measurement. It is based on the rapid resonant rotation of the electrons bunch' spins into the horizontal plane of the collider ring and following their subsequent free rotation around the equilibrium vertical direction [35]. This type of measurement can be made very quickly (few thousand turns) and could follow directly after the RDP measurement, in which the depolariser will have rotated the polarisation vector into the horizontal plane, rather than actually depolarising the bunch completely. It is possible that in the FSP measurement, the bunch will require an increased laser pulse intensity than during the RDP measurement to reduce statistical errors, but this requires further investigations.
- Monitoring of level of the longitudinal polarisation of the colliding bunches is necessary as any longitudinal polarisation will 'pollute' the electroweak observables at the Z pole.

Preliminary studies suggest that this component of the polarisation vector should be known to 10^{-5} absolute, or known to be less than this level. This extremely demanding requirement is somewhat softened on the assumption that the longitudinal component will always be much smaller than the transverse component, and therefore constraining the latter to, say a few 10^{-4} , will probably therefore be sufficient. The polarimeter should be designed such that 10^{-4} systematic uncertainty is a feasible goal. We retain 10^{-5} as the target for the statistical precision, which should be the average uncertainty over all the colliding bunches during some reasonably short period of time. The choice of parameters for the scattering rate per bunch, and the number of bunches probed at the same time will be decided at a later stage. It will result from a compromise between the ability to keep accurate control and monitoring of the laser polarization under relatively large laser power due to thermal effects, the need to have enough scatters per bunch crossing to keep sensitivity to polarization parameters, especially due to background contributions. RDP and FSP energy calibration using one of the pilot bunches could eventually happen at the same time as for several physics bunch polarization measurement. Nevertheless, feasibility still needs to be investigated.

- Physics places some requirement on the position of the polarimeters. For example, the concerns related to the level of longitudinal polarisation in the colliding bunches mean that the orientation of the polarisation vector, ν_0 , should not be allowed to evolve significantly between the measurement and the experiments, and motivates placing the polarimeters close to one of the interaction regions. However, other considerations might instead point to a location before the beam passes through the RF system (*i.e.* upstream) as being more suitable. Background sources and levels need careful studies prior finalizing the polarimeters location. The question of desired full-time access to the laser source (24h/7d) is also a strong constraint on the polarimeter integration concerning civil engineering. Studies are ongoing to resolve this question and find the best suited position on the rings to install both polarimeters.
- Knowledge of the relative positions of the scattered electrons, back-scattered photons, and electron beam allows for a real-time measurement of the beam energy [32]. This measurement is valuable for many applications, and may be essential for some studies. For example, when running at $E_{\text{CM}} = 125$ GeV it will be essential to track any energy variations of the order of 10^{-4} , in order to keep the collision energy sufficiently close to the pole of the Higgs resonance. According to the polarimeter design outlined in Ref. [36] this precision can be obtained with around 10s of data taking, which is at the higher end of what is required. However a dedicated study still has to be performed at 125 GeV center of mass. The estimates must also be updated once a realistic implementation of the polarimeter is found, since it is expected to slightly depend on the distance between the edge of the detector with the nominal beam position.
- Another requires is being able to switch between energy modes without physical intervention on the polarimeter instrument itself. We would need to extend the polarimeter chamber width with increasing energy range expected from the instrument. Indeed the polarimeter is a spectrometer and the Compton electron pattern enlarges with increasing beam energies. Therefore the much larger size of the needed instrument and the

more stringent constraint on background due to higher SR critical energy may lead us to consider the use of the polarimeter for the $t\bar{t}$ mode operation. It will nevertheless be maintained for some short interleaved Z calibration fills.

9.2 Technical specification and challenges

A schematic of the FCC-ee polarimeter is shown in Fig. 18. A dedicated laser/lepton-beam interaction chamber of two meters provides Compton collisions for the polarisation measurement. The 26 m dipole magnet used as a spectrometer to separate the Compton collision products from the main beam, is already part of the magnets of the collider ring optics. This 2 mrad bending magnet provide both a way to steer the main beam, while separating the Compton electrons from it since they have lost energy during the collision process. On the other hand we can detect the Compton photons that have not been deflected and are coming straight from the laser IP. The Compton scattered electrons and photons will travel under vacuum for about 100 m to be separated in a field-free region before heading out of vacuum through an extraction window to be detected outside. Thanks to the relatively large beam to beam transverse separation at 830 m upstream of the experimental IPs, and the presence of a long field-free region following the last dispersion suppression dipole, this layout can be accommodated in the baseline collider optics design.

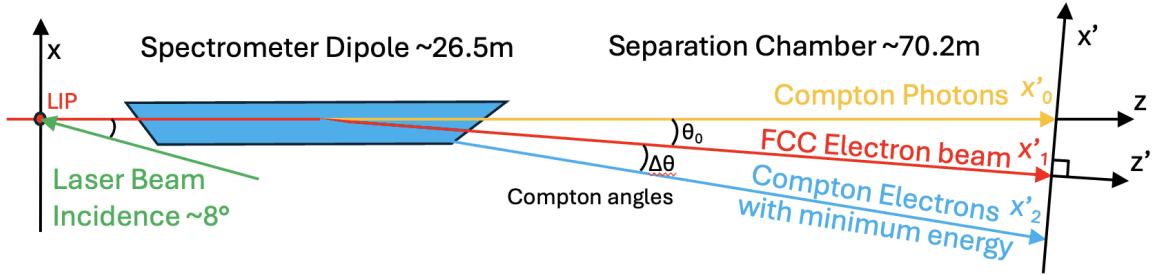


Figure 18: Schematic drawing of the FCC-ee polarimeter. Laser interaction point at xz reference frame. Compton products sensors at $x'z'$ frame, more details in reference [36].

9.2.1 Laser

Physics requirements for the polarimeter mean that the laser system be reliable, remotely operable, and versatile in terms of laser intensity and temporal pattern of train of pulses. It is currently planned to use a laser with a wavelength in the green, as a compromise between the required length of the field-free region and the reliability of the laser system itself. Indeed a UV laser system would provide a much more compact polarimeter but would be much more risky in terms of reliability and versatility. Use of an infrared laser would also provide the reliability and would likely be easier to operate but would induce a twice larger free-field drift to measure the scattered particles with the necessary precision. Control and real-time monitoring of the laser polarisation will be needed with on-line calibrations, for example as already performed at JLAB [37]. However, an optical enhancement cavity was used there, which allowed to more simply use optical reversibility theorems which facilitates the absolute

calibration of the circular laser polarisation [38]. Such a system cannot be used in the case of FCC-ee since one must be able to hit single bunches. A careful conceptual study of the absolute calibration of the laser polarisation has to be realized. The calibration of the circular laser polarization at the interaction point must be ensured at 10^{-3} for pilot-bunch measurements for RDP and FSP. The monitoring of the longitudinal polarisation however demands an accuracy down to a level of 10^{-4} or below that is unprecedented and will require careful evaluation of possible systematic biases. It will also necessitate experimental validation to confirm feasibility and robustness. Attention must be paid to possible major difficulties arising from multiplication of optical elements in the beam line, and the difficulty to calibrate the laser polarisation at the Compton IP. Detailed modeling and experimental studies remain to be done and must be planned for the next phase of the project.

Two possible laser systems are currently under consideration. Initially a Q-switched Nd:YAG laser system at 532 nm delivering few nanosecond pulses triggered at 3kHz was proposed for probing the pilot bunches for RDP and FSP measurements [17]. Operation of this laser with the colliding bunches would require probing a single bunch at a given time. Changing the phase of the trigger would be necessary in order to allow a different bunch to be probed. An alternative choice of laser system, which would not have this complication, would be an Yb based laser system seeded by a mode-locked oscillator. This oscillator would have a frequency that matches a sub-harmonic of the RF frequency of the collider and would be permanently locked to it. It could seed various amplification stages in parallel to deliver laser pulses at 3 kHz (a sub-harmonic of the RF) for the RDP and FSP with pilot bunches and bursts of laser pulses for the colliding bunches possibly at a higher repetition frequency. Optical pulse picking elements would be employed to ensure a proper time structure of the pulses. After amplification, frequency doubling would be implemented to deliver laser pulses at approximately 515 nm. Such a system would typically provide a few to several tens of picosecond laser pulses with a spectral bandwidth of a nanometre, or a little below.

It would be desirable to have a dedicated, radiation-shielded and air-conditioned laser room installed in a bunker near to the Compton interaction point, which will host the laser system and electronics. The latest civil engineering studies point toward an integration of this laser hut in the first big alcove of the arc. To ensure near full time availability of the polarimeter, this room should be accessible 24h/7d during accelerator operation for careful optimization on the laser. Control and monitoring of the laser position, pointing, profile, energy and polarisation will be implemented, with a transport line towards the interaction region, most likely under vacuum (or inert gas) in order to minimize perturbations. Moreover, the implementation of a relay imaging may likely be required. The optics that compose it require careful consideration. The length of this laser transport line will ideally be below 50 m, but will depend on the required crossing angle between the lepton and laser beams, the actual location of the laser room with respect to the interaction point and constraints related to the overall integration and operations of the system.

The choice and optimisation of the laser system depends on the following parameters:

- the pulse duration, its center wavelength and corresponding spectrum [32];
- the choice of the interaction plane and the value of the crossing angle of the two beams will affect the luminosity of the interaction and is therefore coupled to the choice made on the pulse duration;

- the transverse beam size at the interaction point that will affect the luminosity of the interaction and the (in-)homogeneity of the interaction probability among leptons within the bunch, which is also dependent on the crossing angle and the laser pulse duration;
- the single pulse energy of the laser for pilot and colliding bunches, which affects the scattering rates and thus the statistical precision of the measurements;
- the laser polarization in-homogeneity on the wavefront which must not exceed 10^{-4} ;
- the beam quality, for which a specification remain to be set.

This optimisation will also depend on the parameters of the lepton beam at the interaction point.

As demonstrated in Ref. [32], the beam energy can be extracted directly from the measurement of the distribution of the scattered electrons and the position of the photon beam on the detector. It is shown that it may be possible to do so with a relative precision and accuracy of few 10^{-5} . This measurement, being continuous and available in real time, is complementary to those performed with RDP and FSP. It also opens the possibility to monitor beam-energy variations with the same apparatus for pilot and colliding bunches continuously, even at energies beyond the WW threshold, where RDP and FSP measurements are no longer possible. However at higher energies, a significantly larger separation chamber width as well as a larger electron pixel detector will be needed.

The direct energy measurement may be affected by energy spread in the lepton beams and also the spectral bandwidth of the laser. In the absence of any frequency chirp, the RMS relative laser intensity spectral bandwidth is

$$\frac{\sigma_\lambda}{\lambda_0} = \frac{\lambda_0}{4\pi c\sigma_t}, \quad (14)$$

where σ_t is the rms intensity laser pulse duration. This quantity reaches 10^{-5} for $\sigma_t = 10$ ps, thus disfavouring smaller pulse durations. Larger pulse durations may be obtained by either cutting off part of the laser spectrum before amplification or by inducing a time-frequency chirp in the Yb laser pulse. The pulse duration is naturally of a few nanoseconds for Q-switched Nd:YAG technology. The requirements on the absolute stability of the spectrum implies controlling the average laser wavelength within a range of few hundreds of picometres. This may be done with commercial high accuracy wavelength meters [39]. If pursuing the modelock Yb laser oscillator approach, this control could be achieved by locking its spectrum on an atomic clock distributed over an optical fibre network, following the frequency-comb technology [40], as recently made available at CERN by the T-REFIMEVE [41].

The choice of the crossing plane of the lepton and laser beam must satisfy two contradictory requirements. On one hand, the horizontal plane has the disadvantage of being the one in which the synchrotron radiation is generated.

On the other hand, it is desirable to optimise the luminosity of the interaction, which, for a crossing angle in the horizontal plane, has the following dependence:

$$\mathcal{L} = \frac{f_{\text{rep}} N_l N_e}{2\pi\sigma_x\sigma_y\sqrt{1 + \frac{\sigma_z^2}{\sigma_x^2}\tan^2\frac{\theta}{2}}}.$$

Here N_l (N_e) is the number of particles in the laser (lepton) beam, f_{rep} is the frequency of collisions (here 3 kHz), $\sigma_i = \sqrt{\sigma_{i,l}^2 + \sigma_{i,e}^2}$ with $i = x, y, z$ is related to the laser and electron RMS beam sizes in the three coordinates of the reference frame of the lepton beam, and θ is the crossing angle in between the two beams ($\theta = 0$ for contra-propagating beams). In order to avoid large variations over their phase space of the interaction probability, it is advisable to choose the laser beam size to be two to three times larger than that of the leptons. Given the small vertical emittance of the lepton beams at FCC-ee, the vertical lepton beam size is small. Reflecting this ellipticity on the laser beam is possible to some extent thanks to the use of cylindrical lenses. However the Piwinski contribution to the luminosity, $\frac{\sigma_z^2}{\sigma_x^2} \tan^2 \frac{\theta}{2}$ would induce a large luminosity reduction for a crossing in the vertical plane. These considerations lead to the decision to implement crossing angle in *both* the horizontal and vertical planes.

For the Q-switched Nd:YAG laser option, the pulse duration of several nanoseconds imposes that the crossing angle should not exceed a few milliradians, in order to maintain a sufficiently high luminosity. It implies that the laser must be injected through the spectrometer dipole which imposes some additional integration constraints. The shorter pulse of the Yb modelock laser allows for crossing angles of several degrees. The choice of the crossing angle and pulse duration combination also affects the stability of the luminosity due to possible jitters in the laser direction of up to $100 \mu\text{rad}$. Indeed the luminosity variations for given angular fluctuations of the crossing-angle θ , for beams crossing either in the horizontal or vertical planes, reads

$$\frac{\sigma_{\mathcal{L}}}{\mathcal{L}} = \frac{\frac{\sigma_z^2}{\sigma_{x,y}^2} \tan^2 \frac{\theta}{2}}{1 + \cos \theta + \frac{\sigma_z^2}{\sigma_{x,y}^2} (1 - \cos \theta)} \sigma_{\theta}. \quad (15)$$

The numerical calculation is represented in Figure 19. It shows that for $100 \mu\text{rad}$ pointing stability, a relative luminosity fluctuation of about 2.5% is expected for the Q-switch option while it is a factor 100 smaller for the modelock laser option, despite a much larger crossing angle.

The laser arrival of the laser pulse at the Compton interaction point must be correctly adjusted relatively well. The luminosity indeed scales as $\exp\left(-\frac{\delta t^2}{2\tilde{\sigma}^2}\right)$ where $\tilde{\sigma} = \frac{\sqrt{\sigma_x^2 + \sigma_z^2 \tan^2(\theta/2)^2}}{c \tan(\theta/2)}$ and δt is the systematic laser pulse delay with respect to the lepton beam. Fluctuations of arrival time of the laser at the interaction point Δt further induce, in average, a reduction of luminosity of $\frac{\tilde{\sigma}}{\sqrt{\tilde{\sigma}^2 + \Delta t^2}} \approx 1 - \frac{1}{2} \frac{\Delta t^2}{\tilde{\sigma}^2}$. These two contributions are shown in Fig. 20. As expected, it is sufficient to require that the temporal jitter and systematic timing error are both much smaller than the laser pulse duration.

The values of the beam parameters depend on the location of the polarimeters. Close to the IP, the beam size is expected to be $\sigma_{x,e} = 525 \mu\text{m}$ and $\sigma_{y,e} = 11 \mu\text{m}$. The bunch length is expected to be 4.3 mm for pilot bunches and 15.4 mm for bunches in collision. Table 3 shows a tentative set of parameters for both choices of laser, and for both pilot and colliding bunches in the case of the modelock Yb technology. The parameters assume a location for the polarimeter close to the IP, and populations of 10^{10} for the pilot bunches and 25×10^{10} for the colliding bunches. Here ten colliding bunches are probed, but this number can be increased if higher laser power is available. If the polarimeter is located close to the interaction point then the parameters would have to be adjusted to match the reduced horizontal beam size.

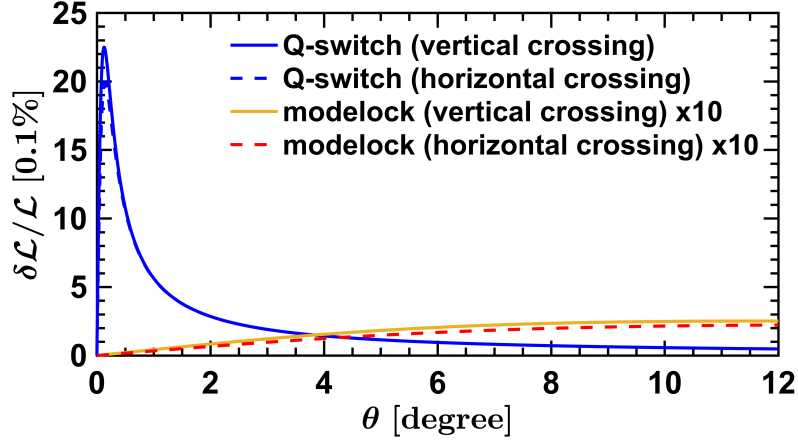


Figure 19: The relative variation of luminosity for the two types of considered lasers ($\times 10$ for the modelock option for better visualization) and their related configuration of interaction region for an angular pointing stability of $100 \mu\text{rad}$. It is shown assuming either beams crossing either in horizontal or vertical planes. Results are similar since effective beam size is dominated by the laser beam size.

The relative luminosity defined as $(1 + \frac{\sigma_z^2}{\sigma_x^2} \tan^2 \frac{\theta}{2})^{-1/2}$ for the two laser options are shown on Fig. 21. The crossing angle for the modelock laser can be adjusted according to integration constraints on a few degrees range.

9.2.2 Vacuum chambers and magnet

Monte Carlo simulation models are under development to evaluate and optimize the expected polarimeter capabilities. It comprises the design of the laser/beam interaction chamber, the separation chamber of almost 100 m, and the detector systems foreseen to record the Compton products. While a preliminary estimation have been performed by the means of a Toy Monte Carlo model as used in Ref. [32]. Complementary models have recently been developed using BDSIM Ref. [42], a Geant4 based software performing Monte-Carlo particle-matter interactions. The design of the instrument is being optimized thanks to these models.

- The laser/beam interaction chamber (LIP), see Fig. 22, will be made out of a typical FCC beam pipe profile, and feature two tapping tubes on a diagonal axis holding viewing ports to allow the laser beam in (and out) the chamber. The design will be very close to the one developed for the SuperKEKB Compton polarimeter [43]. The crossing design angle in the horizontal plane is expected to be (8-12 deg). The vertical angle is not yet defined, but might be similar. This laser interaction chamber will be placed right before the spectrometer dipole magnet. An optical table will be installed underneath the LIP chamber to support the laser mirrors in the most stable way, materially decoupled from the beam pipe which could transport vibration from elsewhere (water cooling, etc.).

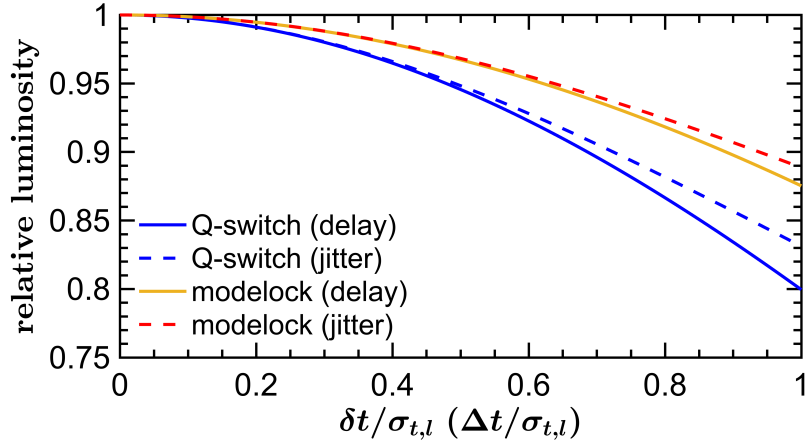


Figure 20: The relative luminosity of the Compton interaction as function of the (solid lines) systematic delay of arrival of the center of the laser pulse at the interaction point with respect to the center of the electron beam; and (dashed lines) temporal jitter of the laser beam for a perfectly well synchronized laser. It is shown for both Q-switch laser (blue lines) and modelock laser (red and orange lines).

- After the LIP the so-called separation chamber starts. The geometry of this chamber basically starts from the FCC profile beam pipe, and goes over almost 100 m with the winglets enlarging along the path. Thanks to the dipole field present over the first 26 m, the Compton photons and electrons will channel through the winglet and be separated from the main beam along the way. For now it is assumed that a standard dipole in the FCC-ee lattice will be adequate, but it is possible that requirements on field homogeneity *etc.* will demand a custom-made solution.
- Following this dipole field region, the Compton products will continue their drift on about 75 m in free field propagation. Along this path the winglets continue to expand transversely up to about 300 mm from the main beam axis. Mu-metal shielding might be needed along this free field region to get rid of the earth geomagnetic field, and other local machine related magnetic fields that might disturb the Compton electron propagation.
- The Compton products will eventually cross the extraction windows to get detected out of vacuum. The thickness of this exit window is a compromise between the extinction ratio of incoming synchrotron radiation (SR) background, and the scattering of the signal from the Compton products generating a loss of information. Due to the very large amount of SR generated by the bunch when crossing a few upstream magnets, a dedicated study is ongoing to optimise the exit window thickness. Together with the use of SR radiation absorbers upstream of the laser interaction point to reduce the contribution of SR photons from upstream magnets ongoing internal reflections and reaching the extraction window. To give the reader an order of magnitude, without absorbers, each electron from the bunch will radiate about 2 photons reaching the extraction window surface. According to the bunch population of the order of 10^{11} ,

Table 3: Preliminary laser parameters for pilot and colliding bunches. Note that single bunch charges are different for pilot and colliding bunches.

Technology	Q-switch	Modelock Yb	Modelock Yb
Bunch type	Pilot	Pilot	Colliding
Repetition frequency	3 kHz	3 kHz	3 kHz
number of targeted bunches	1	1	10
Pulse energy	3 mJ	3 mJ	50 μ J
Average power	9 W	9 W	1.5 W
Pulse duration	3 ns	30 ps	30 ps
Beam width ($\sigma_{x/y,l}$)	1 mm	1 mm	1 mm
Crossing angle	2 mrad	8 deg	8 deg
Scatters per bunch crossing	260	290	94
Scatters per second	8 10^5 /s	9 10^5 /s	28 10^5 /s

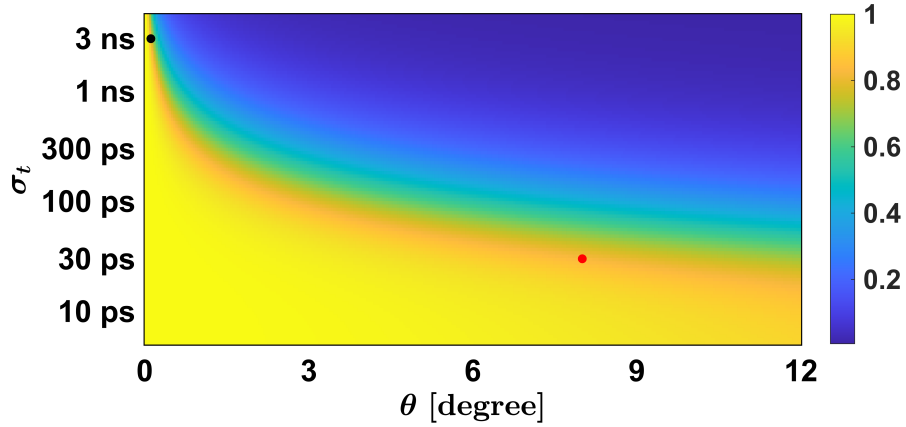


Figure 21: The relative luminosity as function of the laser pulse duration and the crossing angle for interaction with pilot bunches. The solutions listed in Table 3 are shown with the black (red) dots for Q-switch (modelock) lasers.

selecting a few hundreds Compton products into this important SR background seems to be a challenging task. Mitigation of this background will also come from the detector design itself.

- The effect of the polarimeter's vacuum chamber, and the extraction window shape on the collider ring beam-impedance has been evaluated using CST Studio Suite electromagnetic simulation software. First results on the most advanced chamber models have shown a minor contribution of the polarimeter chamber to the overall impedance budget of the FCC-ee machine. As shown in Fig. 24, the impedance fall below 0.01% of the total machine impedance. In turn, the ongoing design is not constrained by such aspects and that allows some welcome freedom for further geometrical optimisations. One of the polarimeter chamber geometries simulated in CST is shown in Fig. 23.

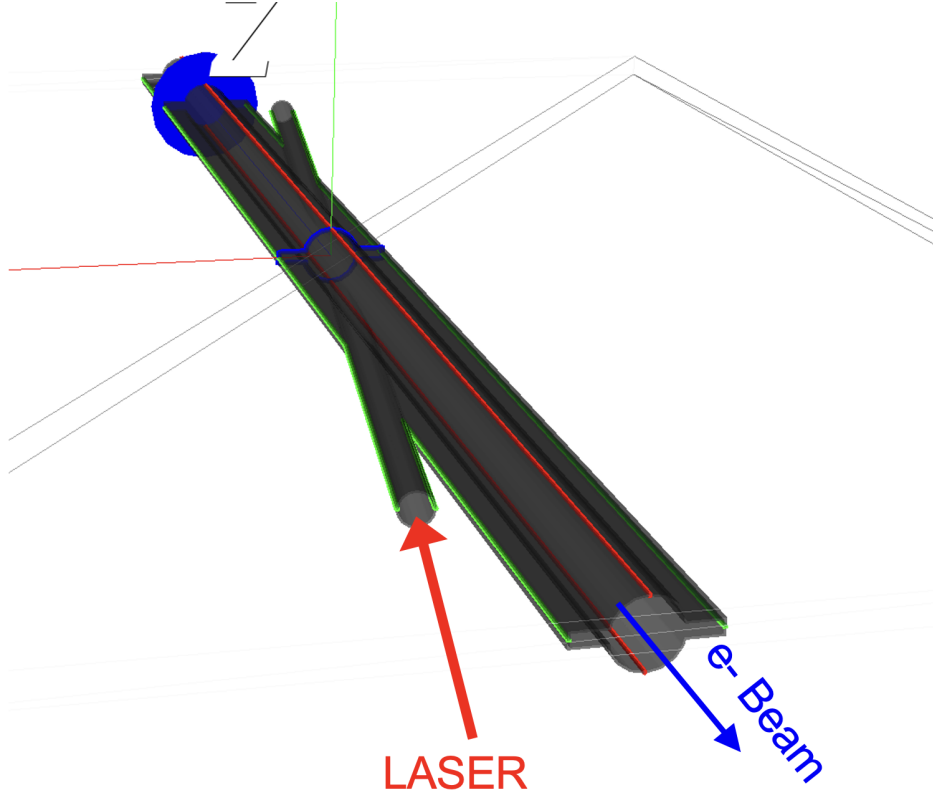


Figure 22: Pre-Design of the laser interaction chamber used for simulations.

9.2.3 Sensors

Two types of detector assemblies are needed for the polarimeter. The first one aims to record the transverse ellipse of the Compton scattered electrons over a surface of about $5 \times 300 \text{ mm}^2$ (for the Z mode). The second one will record the peak shaped distribution from the Compton gammas on a detector of about $10 \times 10 \text{ mm}^2$ transverse area. The baseline design for the Compton polarimeter detectors aim to use pixelated sensors to record precisely both photons and electrons transverse distribution patterns ($18\text{--}50 \mu\text{m}$ pitch). These sensors will be developed thanks to the experience accumulated in calorimeters and silicon detector development for HL-LHC and FCC experiments. No choices have been made concerning the exact type of sensor substrate and design. This will come out of a study benchmarking with existing sensor responses using the appropriate digitisation software to compare them for the polarimeter application needs. Monolithic active pixel sensors (MAPS) recently developed could be a good choice for fitting the electron ellipse pattern into a single sensor unit.

9.2.4 Background mitigation

As introduced before, a strong synchrotron radiation (SR) background is expected to reach the extraction window as shown Fig. 25. For the measurement of the scattered electrons, techniques using multilayered pixelated sensors could be used in order to perform basic tracking over a few layers and get rid of the photon background in the sensors. This kind of mitigation techniques could be an option to improve on the signal to noise ratio of the

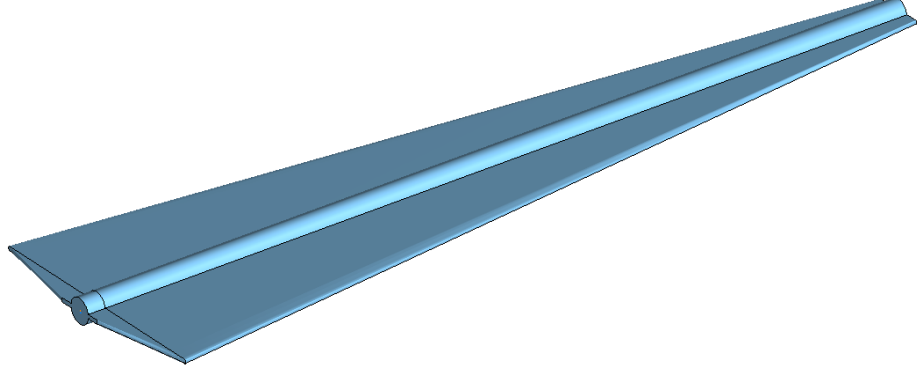


Figure 23: Design geometry of the separation chamber (long side / right) and extraction window (short side / left) as simulated in CST. The FCC-ee lepton beam would circulate from the right side to the left side of the object.

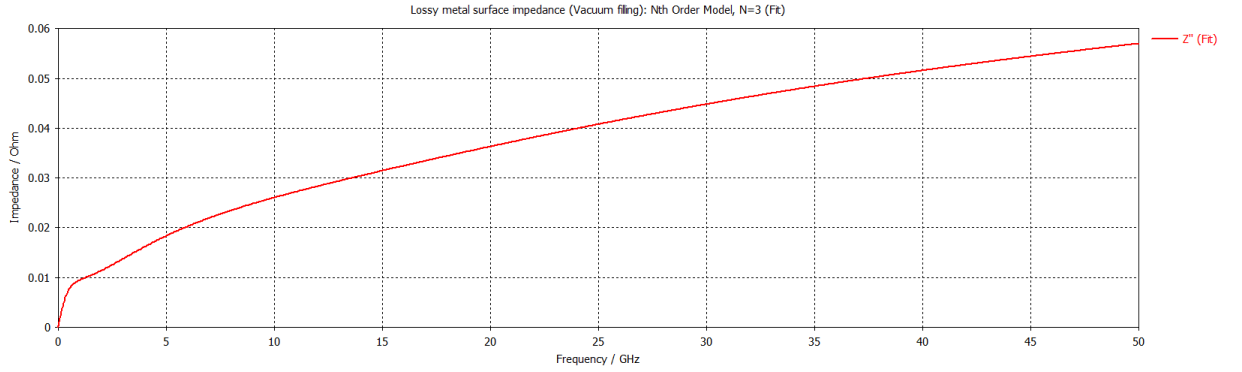


Figure 24: Impedance of the polarimeter separation chamber as simulated using CST.

Compton profiles. If it is not sufficient, technologies of lower spatial resolution but much less sensitive to SR photons such as the ones based on Cherenkov radiation by charged particles, could be another way of performing the Compton electron detection with greater selecting power, as initially planned at ILC [44]. Such a device would allow to perform energy calibration like RDP and FSP, but might be weak for direct energy measurement techniques and precise knowledge of the polarisation vector. Other sources of background like beam-gas Bremsstrahlung, Compton interaction on the thermal photons and backgrounds coming from nearby experiment's IP will also deserve careful consideration.

9.2.5 Polarimeter system availability and efficiency

The Compton polarimeter availability for RDP energy calibration is required to reach 95% for the Z and WW modes of operation. Such an unprecedented goal is a real challenge and work is still on going to make sure we can reach such instrument uptime.

Up to now the most critical part of the polarimeter has been identified to be the laser source and the transport of the laser beam toward the interaction point with the lepton beams, with the related high accuracy monitoring of the laser polarization to ensure the level of systematic uncertainty. Multiple radiation-compatible systems will have to be developed

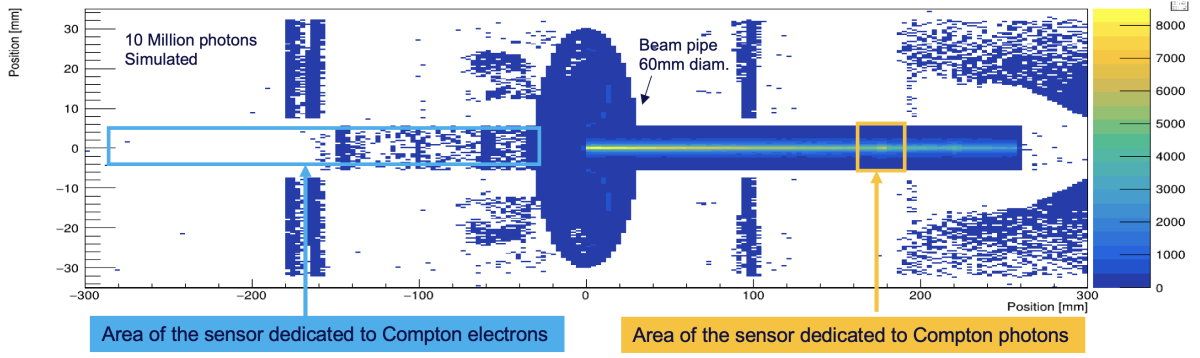


Figure 25: Synchrotron radiation background expected in the transverse plane at the location of the polarimeter sensors.

to monitor the circular polarisation level along the laser transport line. The objective is to have a fully remotely controlled system, where we could steer, monitor and re-adjust each of the optical elements. Radiation levels expected in the tunnel represent a major constraint since off-the-shelf optical devices are not radiation hard.

Despite modern laser technology having gained in robustness and performance, operational constraints at FCC-ee and in general in accelerator environments are specific. In particular, some parts of the foreseen laser system may require interventions from time to time to ensure the performance and operational constraints are met. For reference, experience at the LEP and other facilities have proven that full time access to the laser hutch while the beam is circulating is a must to allow a smooth set-up of the instrument to the best working conditions and guarantee system availability. For that purpose, a dedicated tunnel to access the polarimeter laser hutches on a 24h/7d basis while the machine is running is foreseen. Unfortunately, the feasibility study civil engineering proposal does not allow such a tunnel to be bored for costing reasons.

In order to overcome this problem the best option would be to develop a mock-up system of the full laser line in the pre-TDR phase, and attempt to run it over long periods with desired circular polarisation precision to gain experience. That would be a way to optimise the whole set of remotely controlled actuators in order to be able to assert the best up-time performance for the FCC polarimeter laser system. Nevertheless, the unavoidable downtime due to access needs of the polarimeter laser still needs to be evaluated for a laser system similar to that of FCC-ee polarimeters.

We need to stress again here that without this full time tunnel access to the laser, the optimum polarimeter availability (95%) is not yet guaranteed at present and the impact of energy calibration for physics should be assessed in the future.

A possible solution to overcome this would be to invest in redundancy, having polarimeters in two experimental IPs for each beam. In case one is failing we could still rely on the second instrument before having a long enough access to fix any problem. Another possible redundancy level could be to have a duplication of laser systems in each hutch and to be able to switch between two laser sources by remotely inserting a mirror on the optical path. But in any case the commissioning of the system without access still requires dedicated studies, if it is not revealed to become a showstopper for the required availability of the energy

calibration.

10 Polarisation studies at KARA storage rings

Beam-energy measurements based on resonant depolarisation (RDP) were carried out at various previous collider rings, including VEPP-2M [45], VEPP-4M [46, 47], CESR [48], and LEP [49]. The change in polarisation occurs when the slowly varied excitation frequency f_{dep} of the depolariser, modulo an integer multiple of the revolution frequency f_{rev} , coincides with the spin precession frequency $f_{\text{spin}} = f_{\text{rev}}\nu_0$,

$$f_{\text{dep}} = (k \pm \nu_0) f_{\text{rev}} \quad \text{with } k \in \mathbf{Z} , \quad (16)$$

where the ideal spin tune ν_0 depends on the average beam energy, or Lorentz factor γ , according to

$$\nu_0 = a_e \gamma , \quad (17)$$

with $a_e = (g_e - 2)/2 = 1.159652... \times 10^{-3}$ the anomalous magnetic moment of the electron (with g_e the g-factor). The studies presented here have been carried out at Karlsruhe Research Accelerator.

10.1 KARA Machine Description

Located at Karlsruhe Institute of Technology (KIT), KARA is a synchrotron light source and test facility with a circumference of 110 m and an 500 MHz RF system. KARA is a machine ramping in beam energy from 0.5 to 2.5 GeV. It features a four-fold super-periodicity, where each sector consists of two double bend achromat (DBA) cells, with straight sections in-between, hosting, the RF system, injection magnets or other insertion devices. The double DBA optics for one quarter of the KARA ring is shown in Fig. 26.

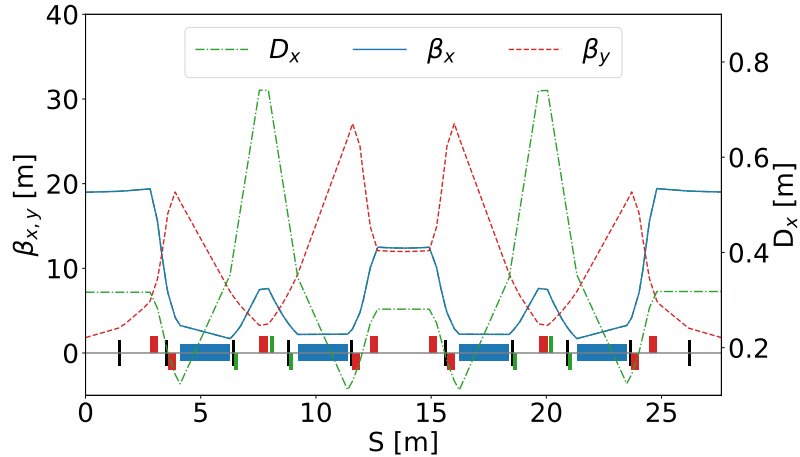


Figure 26: β -functions and lattice of the first quarter of KARA light source. Focusing (defocusing) elements are shown above (below) the horizontal axis.

No polarimeter is currently installed in KARA, however, depolarization can be detected through a sudden change in the beam lifetime, and in the associated local beam losses. The reason is that the KARA beam lifetime is dominated by the Touschek effect, and the underlying Möller cross section and resulting local beam losses depend on the degree of beam polarisation. In a high-energy storage ring, beam polarisation increases the Touschek lifetime, by up to 23% [50, 51]. More details have already been reported in [52, 53]. Complementary to RDP scans, measurements characterizing the machine orbit and optics have been performed where a detailed summary is given in [54].

10.2 RDP measurement set-up

Measurements presented here were performed at 2.5 GeV beam energy, where the Sokolov-Ternov polarization time is roughly 10 min. In order to achieve sufficient level of polarization to perform RDP scans it is found necessary to wait at least 20 min in-between scans. The total beam current during measurements of 35 to 65 mA has been distributed over roughly 30 bunches with a filling pattern shown in Fig. 27. From the average reading of the BPMs a rms closed orbit of 125 and 125 μ m has been measured, respectively, for the horizontal and vertical plane.

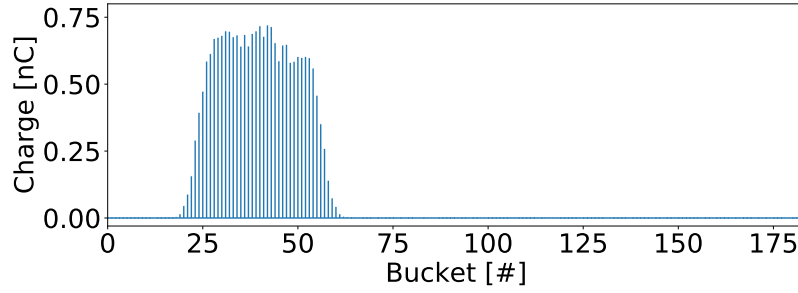


Figure 27: Bunch filling pattern during optics measurements.

Figure 28 presents a typical scan result, where the depolariser frequency is swept from 1.705 to 1.735 MHz over 650 s and we observe the change in the local loss rate. The depolarising frequency, f_{dep} , at which the loss rate changes gives the spin tune via Eq. (16), so that f_{dep} can be converted to an equivalent beam energy E using Eq. (17). For the example of Fig. 28, f_{dep} is 1.7139 MHz, with an equivalent beam energy of roughly 2.48 GeV. To simplify the plots, we omit plotting the frequency and only show the corresponding beam energy. Furthermore, since initial measurements indicated a beam energy of about 2.48 GeV, we now vary the depolariser frequency only from 1.705 to 1.725 MHz. The measured loss rates as a function of E are fitted to

$$F(E) = F_0 + (h/2) \operatorname{erf}((E - E_0)a) + bE + cE^2, \quad (18)$$

where the fit parameter E_0 corresponds to the centre of the step, and, hence, to the inferred beam energy. For the fitting, we used the Python function *curve_fit*, available from the *scipy* package [55], which implements a nonlinear least-squares method based on the Levenberg-Marquardt algorithm. The parameters a , b , c , h and F_0 are also determined by this fit,

including the respective errors. To suppress these numerical uncertainties, we introduced a moving average over 3 successively measured loss-rate data for further analysis, resulting in a very low fitting error in the order of a few keV (10^{-6} relative accuracy).

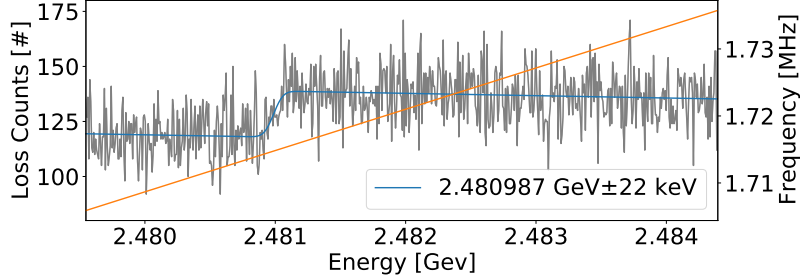


Figure 28: Typical depolarising scan result at KARA. Measured local beam losses versus the energy corresponding to the depolariser frequency. The stored beam current was approximately 35 mA. The orange line shows the applied Frequency in MHz (see right axis).

10.3 Dependence on the scan direction

The dependence of the result on the scan direction is of interest with regard to systematic errors. We executed measurements where the frequency either increased or decreased, while scanning the same range between 1.705 and 1.725 MHz. Here, we present results for scanning in either directions with a 400 s scan duration. Loss rates from these two scans are displayed in the top and bottom pictures of Fig. 29, respectively. Notably, the scan towards higher frequencies yields a significantly steeper change of loss counts, than the scan in the other direction. These findings are consistent with FCC simulations [56] for the case of a negative energy drift. When scanning towards higher frequencies the beam energy is drifting towards the depolarising frequency (on-coming), and, thus, the time required to complete the resonant depolarisation process is shortened compared to sweeping towards lower frequencies. For the latter case, the depolariser frequency is catching-up with the energy drift. Furthermore, we note that the beam energy inferred from the (later) downward scanning is 14 keV lower than for the (earlier) upward scan, which also hints at a negative energy drift.

11 From resonant depolarisation to centre-of-mass energies

The base measurement in the FCC-ee centre-of-mass determination is that of the spin tune by RDP as described in Section 8 ³. The spin tune is directly related to beam energy

$$\nu_0 = E[\text{MeV}]/440.6485 \quad (19)$$

for a perfectly flat machine without solenoids. This average energy determination is independent of the location of either the depolariser or the polarimeter, since it represents the

³The discussion in this section is equally applicable for a spin tune measured by FSP.

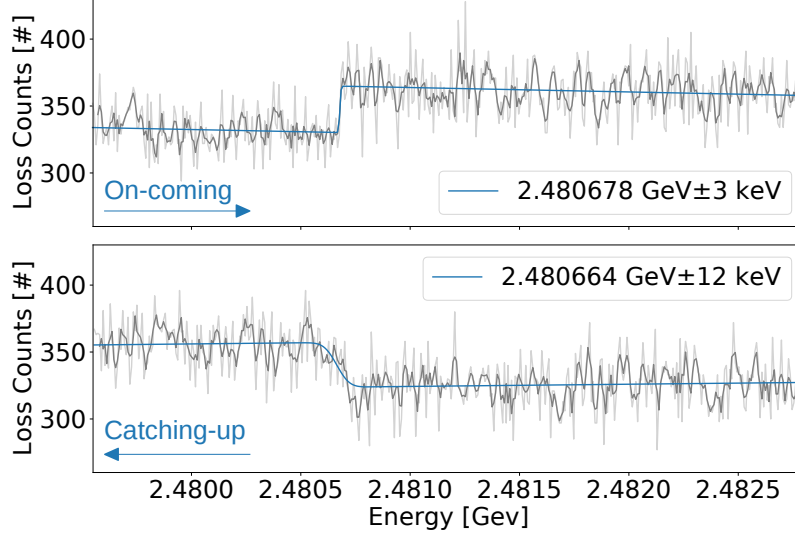


Figure 29: Scan results for increasing (top) and decreasing depolariser frequency (bottom) with the respective fit results shown. The different shape of the fitted function is consistent with a downward drift in beam energy.

average over the guiding magnetic field. We call $E_{\text{RDP}}^{\pm}$ the energy obtained assuming a perfect machine by RDP. The relationship between $E_{\text{RDP}}^{\pm}$ and the correct beam-energy average, $\langle E_b^{\pm} \rangle$, comprises at least two sources of corrections and uncertainties:

- The corrections to spin tune due to alignment and magnetic imperfections in the lattice;
- The corrections to the RDP frequency due to the interference of spin resonances with the depolariser-induced resonance.

Both of these corrections are potentially spin-tune dependent, leading to possible point-to-point uncertainties. They must be studied, on one hand, by performing spin tracking simulations, where first results are given in Sec. 4. On the other hand, detailed simulations of the RDP process must be performed, and evaluated with the goal of finding workarounds to minimise or evaluate their effect and the resulting uncertainties. The result of these investigations could lead to modified experimental procedures or operating energies.

In order to go from the determination of $\langle E_b^{\pm} \rangle$, to the centre-of-mass determination at each of the interaction regions, the following corrections and sources of uncertainties must be considered:

- The energy losses and gains around the ring, which include the losses from synchrotron radiation (40 MeV per turn at the Z pole) and the losses from longitudinal impedance (a few MeV per turn, depending on bunch population and bunch length);
- The beamstrahlung losses, which depend on the exact bunch population and thus are luminosity dependent;
- The combined effect of beam collision offsets and parasitic opposite sign vertical dispersion (OSVD);

- The beam-beam collective electromagnetic interaction effects, which modify the crossing angle and the centre-of-mass energy.

This section discusses these items, with the exception of the electromagnetic interaction effects, which are addressed in Sec. 12.

11.1 Energy losses and their control

Considering that there is only one common RF section for the electron and the positron ring, it is useful to define $E_0^\pm$ as the average energy of the electron or positron beam in that RF section. If the energy losses are distributed evenly around the ring, (or according to the four-fold symmetry) this number is nearly equal to $\langle E_b^\pm \rangle$. In FCC-ee with the electron and positrons circulating in different magnetic lattices the energies of the two beams can be different.

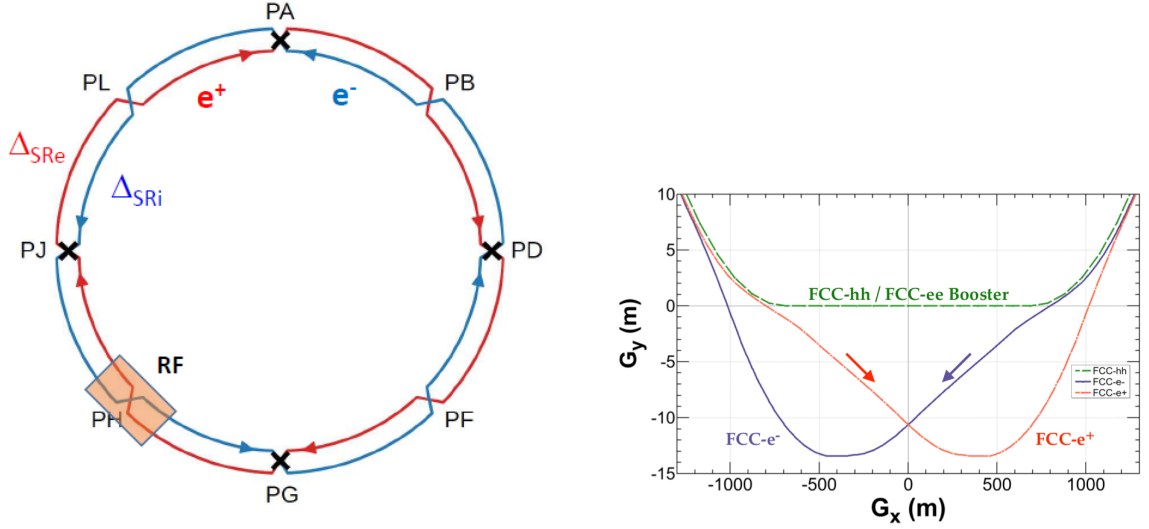


Figure 30: Left: Overview of the energy losses around the FCC-ee ring. Electrons circulate anti-clockwise and positrons clockwise. The beams enter the interaction regions from the inside. Δ_{SR_e} and Δ_{SR_i} correspond to the energy losses along the external and outside rings. Plot adapted from [57]. Right: Layout of the asymmetric layout of interaction regions with the beams entering from the inner side and outgoing on the external side with a larger deflection.

The approximate size of the energy losses at the Z pole is as follows:

- The overall SR loss is $\Delta_{SR} \approx 39$ MeV at the Z-pole with 45.6 GeV;
- The losses due to longitudinal impedance are summarised in Fig. 31, ranging from a fraction of MeV to 5 MeV, depending on bunch intensity and bunch length;
- The difference between the energy loss in the inside and external rings is commensurate with the beam crossing angle, $(\Delta E_{SRi} + \Delta E_{LI}) - (\Delta E_{SRe} + \Delta E_{LIe}) \approx \frac{\alpha}{2\pi} \times 8\Delta_{SR} \simeq$

0.19 MeV, with energy losses from synchrotron radiation (SR) or longitudinal impedance (LI) from the inside (index i) or external (index e) octants respectively, or from the beamstrahlung (BS);

- Further energy losses arise for colliding bunches from beamstrahlung and are up to 0.31 MeV per IP and per beam at the Z-pole.

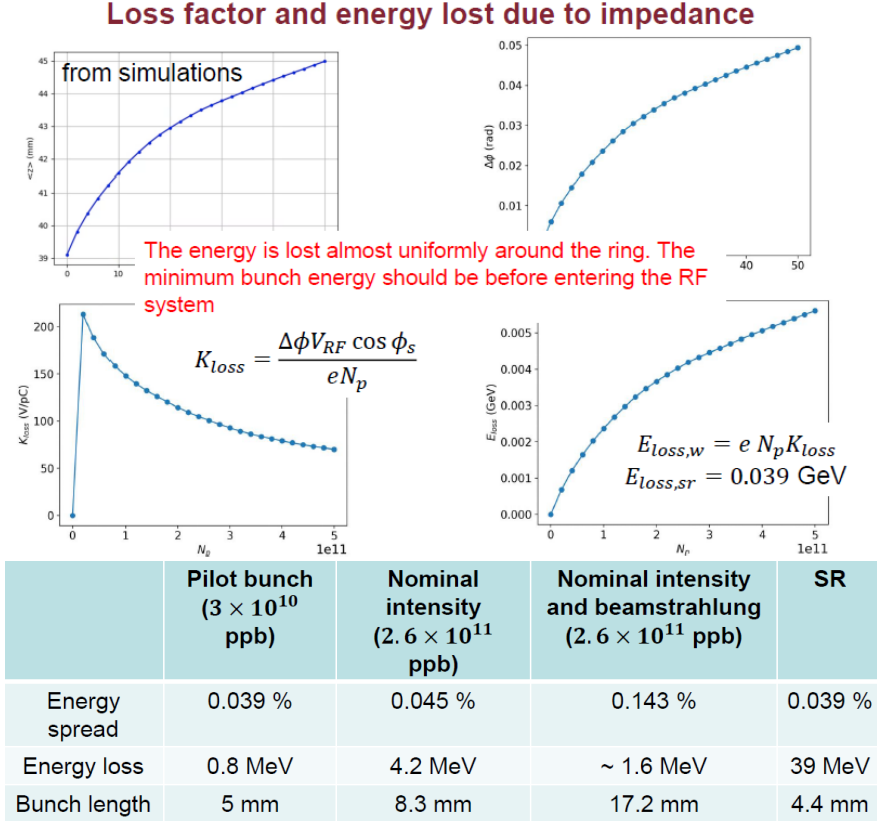


Figure 31: Top: Calculation of energy losses from longitudinal impedance as a function of energy, beam intensity and bunch length. Bottom: summary for two categories of pilot bunch (polarisation and orbit pilot bunches) and for the colliding bunches. (from [58])

The energy losses around the ring are sketched on Fig. 30. The total energy loss around each ring is equal to the energy gain from the RF-cavities

$$\Delta E_{RF} = 4(\Delta E_{SRi} + \Delta E_{LIi}) + 4(\Delta E_{SRe} + \Delta E_{LLe}) + 4\Delta E_{BS}. \quad (20)$$

In practice it is be difficult to separate the longitudinal impedance and synchrotron radiation distributions, so we define $\Delta E_{Arc,i(e)}$ as the energy loss along the inner or outer arc respectively, keeping in mind that these numbers will be different for nominal intensity colliding bunches, low-intensity polarisation pilot bunches or pilot bunches with higher intensity (orbit pilot bunches). The beam energies at the IPs are expressed as

$$E_J^- = E_0^- + \Delta E_{RF}/2 - \Delta E_{Arc,i} - \Delta E_{BS}/2, \quad (21)$$

$$E_J^+ = E_0^+ + \Delta E_{\text{RF}}/2 - 4\Delta E_{\text{Arc},i} - 3\Delta E_{\text{Arc},e} - 7\Delta_{\text{BS}}/2 , \quad (22)$$

$$E_A^- = E_0^- + \Delta E_{\text{RF}}/2 - 2\Delta E_{\text{Arc},i} - \Delta E_{\text{Arc},e} - 3\Delta_{\text{BS}}/2 , \quad (23)$$

$$E_A^+ = E_0^+ + \Delta E_{\text{RF}}/2 - 3\Delta E_{\text{Arc},i} - 2\Delta E_{\text{Arc},e} - 5\Delta_{\text{BS}}/2 . \quad (24)$$

The energies at points G (resp. D) are similar to those for J (resp. A) above, and are obtained by exchanging the positive and negative particles. It follows from these equations, eliminating the total energy loss using Eq. 20, that

$$E_A^+ + E_A^- = E_J^+ + E_J^- = E_0^+ + E_0^- + \Delta E_{\text{Arc},e} - \Delta E_{\text{Arc},i}, \quad (25)$$

assuming that all energy losses are identical between similar arcs, and beamstrahlung at the IPs too, and hence, **all centre-of-mass energies are the same**. This is an important feature of the FCC-ee design which requires only one RF section for operation at the energies of the Z-pole, s-channel Higgs, and up to W-pair-production. Of course, achieving the desired precision will require careful assessment, by means of calculations and measurements, of all unavoidable variations from the above assumption. For example, the fact that SR is proportional to E^4 amounts to a few keV correction, as can be seen in Fig. 32 [59]. Furthermore, other effects resulting from energy losses in quadrupoles and other lattice elements must be considered, since misalignment and field errors are unique for each element, resulting in variations between the electron and positron machines, and from one octant to another.

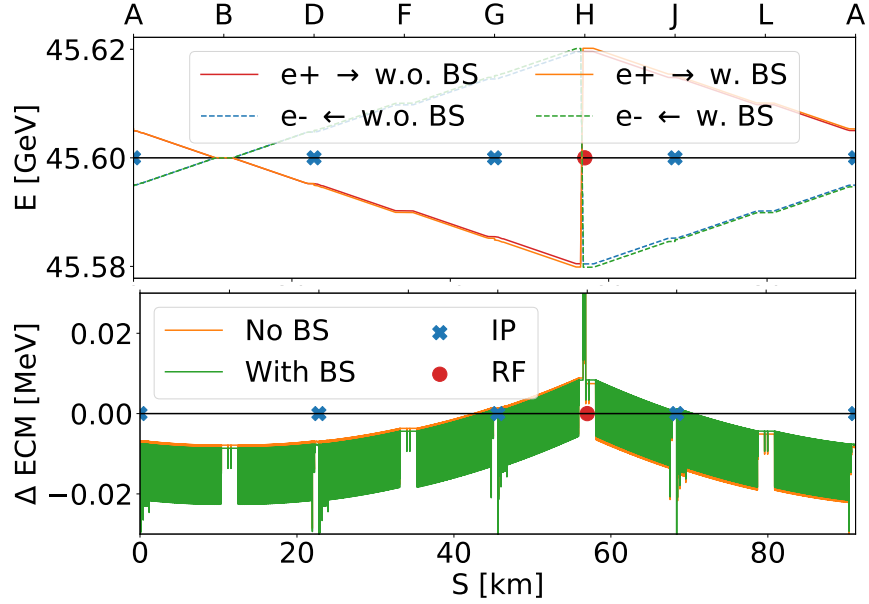


Figure 32: Beam energies (top) and E_{CM} (bottom) at the Z-pole with and without beamstrahlung (BS) at the IPs with a RF-section in PH [59].

A second observation is also of interest, namely

$$E_J^+ - E_J^- = E_0^+ - E_0^- - 3\Delta E_{\text{Arc},i} - 3\Delta E_{\text{Arc},e} - 3\Delta_{\text{BS}} , \quad (26)$$

$$E_A^+ - E_A^- = E_0^+ - E_0^- - \Delta E_{Arc,i} - \Delta E_{Arc,e} - \Delta_{BS} . \quad (27)$$

As previously the boosts for points G (resp. D) are similar to those for J (resp. A), by exchanging the positive and negative particles. It is remarkable that the boost in point J and G represents, in addition to the (measured) difference between the average energies of the two beams, 3/4 (resp 1/4) of the total energy losses around the ring. The difference in boosts between the two points J and G (resp. A and D) represent 3/4 (resp 1/4) of the sum of e^+ and e^- losses. The boosts can be measured from studying the topology of di-fermion events in the experiments, as summarised in chapter 8 of ref. [17] and Sec. 12. with a precision of $\pm 1.5 \cdot 10^{-6}$ on the boost $x_\gamma \simeq (E^+ - E^-)/(E^+ + E^-)$.

11.2 Energy losses for pilot and colliding bunches

Continuous energy losses around the lattice occur from Synchrotron Radiation (SR), beamstrahlung and longitudinal impedance. The Colliding and non-colliding beams, having the same revolution frequency, and travelling through the same magnetic channel, have the same **average** energy, it's being given by the average of the energy over the guiding magnetic field. This **revolution frequency constraint**, RFC, for the case of a storage ring with RF acceleration situated in one single location, compensates many of the small differences arising between colliding and non-colliding beams when comparing the spin precession frequency to the collision energy. However, while SR losses are identical for pilot and colliding bunches, (40 MeV at 45.6 GeV), the beams do not have the same local energy.

Beamstrahlung at the collision points has several consequences.

Beamstrahlung energy losses have been estimated by D. Shatilov in [17] to be as large as 620 keV at full intensity at the Z pole. It was noted also that in itself it does not affect the center of mass energy, this energy loss being compensated by RF acceleration, given the RFC constraint.

Beamstrahlung modifies the energy spread and the bunch length. Longitudinal Impedance losses increase with increasing bunch intensity, and decrease with increasing bunch length (see Fig.31. The main contribution to longitudinal impedance stems from resistive wall impedance, which has recently been estimated at the Z-pole to be 0.8 MeV and 4.2 MeV per revolution, respectively, for 3×10^{10} and 2.6×10^{11} non-colliding bunch population. The Beamstrahlung-induced colliding bunch lengthening reduces the latter number from 4.2 MeV down to 1.6 MeV Further energy losses arise for colliding bunches from beamstrahlung and are about 0.31 MeV per IP at the Z-pole The sum of all losses are compensated by the RF-cavities, where one crucial requirement is that all RF-cavities must be located in the same straight section.

The RFC constraint only works, of course, for a perfect machine, where the energy losses are spread symmetrically around the ring and at the same location for positrons and electrons. Given that the effects in question are as large as several MeV, i.e. three orders of magnitude larger than the target accuracy, it is essential to devise means of diagnosis and monitoring.

In equations 26 and 27 it was stressed that the centre of mass boost at the four IPs can be used to measure the energy losses.

The measurement of the average centre of mass boost in each of the IPs can be done using muon pairs as described in [17](Sec 8.1). The quantity $x_{\text{boost}} = \frac{E^- - E^+}{E^- + E^+}$ can be determined on an event by event basis⁴ and the average extracted from the distribution shown in Fig. ?? . For a sample of 10^6 muon pair events and for a boost of 10^{-3} ,

$$\langle x_{\text{boost}} \rangle = (0.9991 \pm 0.0015) \times 10^{-3}, \quad (28)$$

which corresponds to a measurement of the energy difference with a precision of 140 keV. Such a sample can be collected in 600 seconds in each experiment at the Z pole in FCC-ee with the Feasibility Study luminosity of $140 \times 10^{34} / \text{cm}^2 / \text{s}$. At the 88.6 and 93.9 GeV energy points, (spin tunes of 100.3 and 106.5 respectively), where the cross-section is lower, the required time to achieve this precision is longer, 2500 and 1600 seconds respectively. Integrated over a 'typical 8hr shift' (resp. the full duration of the run, with exposures of 10/ab, 31/ab, and 10/ab) in each experiment at the considered scan points, samples of 17, 85 and 29×10^6 (resp. 3, 43 and 5×10^9) muon pairs will be collected, providing statistical precisions on the local boost of 32, 15 and 26 keV (resp. 2.5, 0.7 and 2 keV) respectively. This high statistical precision on the boosts will be an effective way to monitor the stability of the system, and identify possible variations in the energy losses, and, by studying correlations with luminosity and other environmental parameters, help identify the sources of the energy losses.

NB: Here the measurement of the bunch length using the time distribution of interaction events (using any annihilation event) could be mentioned. Since all Z decays can be used, a resolution of 0.1% of 30 ps can be obtained in less than one minute.

11.3 Dispersion and collision offsets

11.3.1 Introduction

A potentially significant source of bias in the centre-of-mass energy determination arises from the combined effect of collision offset and opposite sign vertical dispersion (OSVD). For head-on collisions, *i.e.* without a crossing angle, the centre-of-mass energy shift is [17]

$$\Delta E_{CM} = -2u_0 \frac{\sigma_E(D_{u,B1} - D_{u,B2})}{E_0(\sigma_{B1}^2 + \sigma_{B2}^2)}, \quad (29)$$

where the dispersion at the IP for both beams is $D_{u,B1,B2}$, and the beam-energy spread is σ_E assumed to be identical between electrons and positrons. Furthermore, are beam sizes $\sigma_{B1,B2}$, reference energy is E_0 and the transverse offset u_0 . Assuming 1 μm OSVD at the IP and 0.13 % energy spread at 45.6 GeV beam energy, which are indicative values consistent with recent tuning studies, the collision energy is shifted by roughly 100 keV per nm offset ($\approx 0.03 \sigma_y$). Reducing the centre-of-mass energy shift, therefore, requires minimising the collision offsets at the IPs and the OSVD. Because the beams collide at an angle in the horizontal plane, each beam samples the full energy distribution of the other; therefore the

⁴For historical reasons this quantity is noted x_γ in [17]

horizontal opposite sign dispersion creates a much smaller potential shift in the the centre-of-mass energy. We therefore restrict our discussion to vertical offsets.

The operational strategy proposed in Ref. [17] included regular (about once per hour) luminosity scans (varying the vertical collision offset) to measure the luminosity maximum and a possible vertical collision offset. By performing the same measurement for different RF frequencies, a measurement of the vertical dispersion at the IP can be obtained. At LEP this procedure was executed during data taking with large collisions offsets that reduced the luminosity by half. At FCC-ee this procedure might be risky because of the high beam charges and beam-beam tune shift, meaning that a reduced amplitude of collision offset should be employed.

11.3.2 Beam-beam deflection

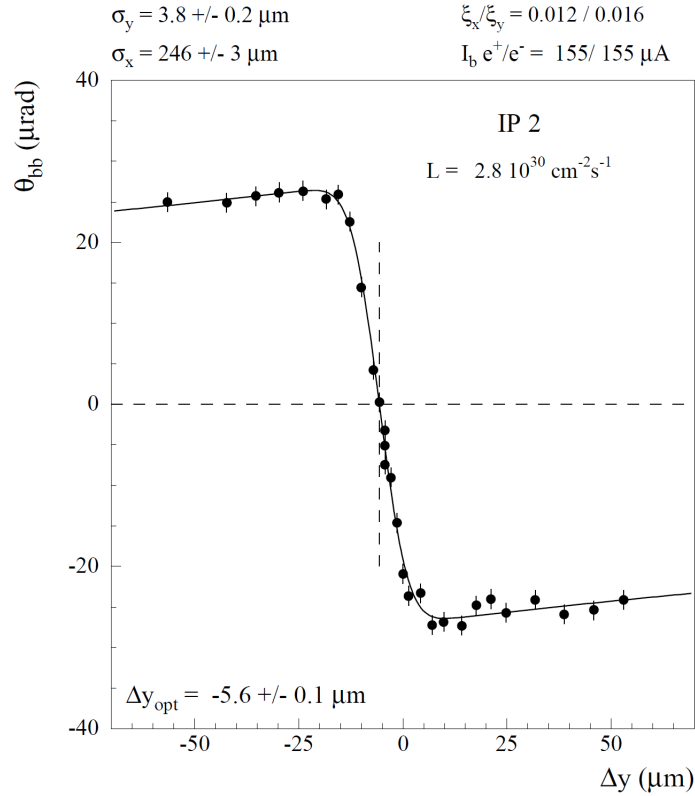


Figure 33: Beam-beam deflection scan performed at LEP in 1996 [60].

Giving the potential difficulty with luminosity scans, an alternative proposal is to measure the beam-beam deflection, rather than measuring the collision offset from a scan of the same parameter. The high bunch charge and small beam sizes open the possibility to monitor with great sensitivity the collision offsets, as had also been demonstrated at LEP [60]. An example measurement using this technique, showing the beam-beam kick versus beam separation is presented in Fig. 33. In the limit of collision offsets that are small compared with the beam

size, the beam-beam kick (θ_{bb}) is proportional to the separation u according to

$$\Delta\theta_{bb} = -\frac{4\pi\xi_u}{\beta_u^*}u. \quad (30)$$

Given typical FCC-ee parameters of beam-beam tune shift $\xi_y = 0.14$, $\beta_y^* = 0.8$ mm and $\sigma_y = 30$ nm, a vertical collision offset of 10% of the vertical beam size (3 nm) will lead to a beam-beam deflection angle of $6.6\mu\text{rad}$. The layout for a beam-beam deflection experiment is sketched on Fig. 34. We note that a beam-beam collision offset creates a distinctive pattern of movements of the beams, resulting in an orbit change.

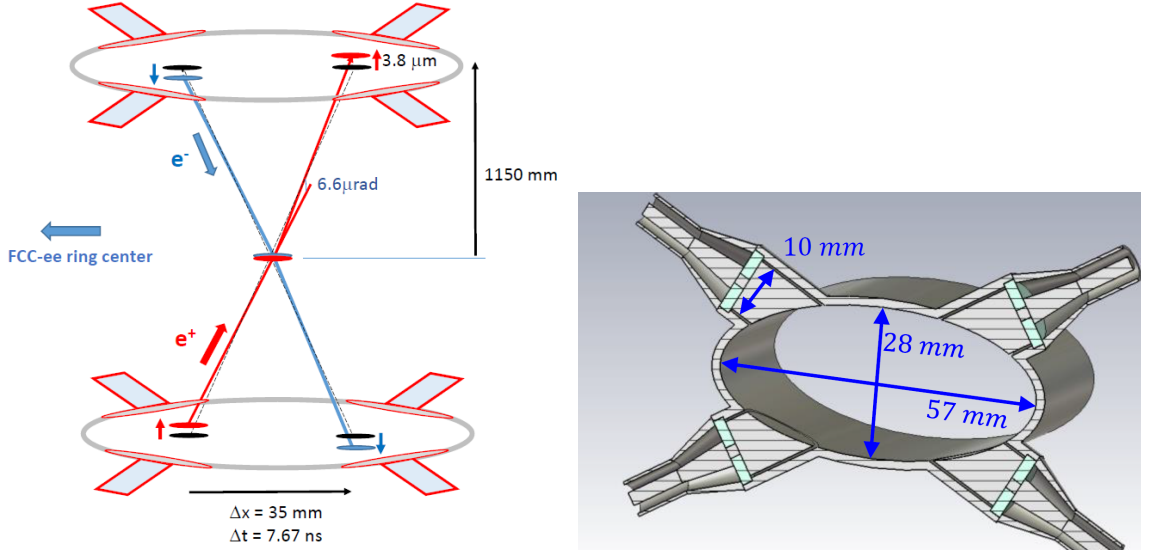


Figure 34: Sketch of the principle of the beam-beam deflection measurement at FCC-ee, by comparison of the vertical displacement of the colliding beams with respect to high intensity pilot bunches, as measured in the same Beam Position Monitors, shown on the right.

The intention is to measure the beam movements using the recently proposed LumiCal BPM pickups [61] located 1.15 m away from the interaction points, which could be installed directly behind the experimental luminosity monitors, as shown in Fig. 35. The beam-beam kick will lead to a shift in the beam position of $3.7\mu\text{m}$ in each of the two BPMs. Assuming a BPM resolution of $1\mu\text{m}$ for 10^8 bunch passages, which can be acquired in 3 seconds for the 10^4 bunches of each beam, the relative difference of $7.6\mu\text{m}$ between the e^+ and e^- bunches induced by the considered collision offset can be measured with a precision of $1\mu\text{m}$, giving a precision of 0.4 nm on the collision offset (1.3 percent of the beam size).

In isolation this measurement is of limited use, since the two beams are circulating in two different rings with different imperfections. Hence a normalisation is necessary. Here two methods are under consideration:

1. Perform occasional studies of the relation between luminosity and the beam-beam collision offset using beam separator scans. This will provide a secure way of verifying that the maximum luminosity corresponds to a null collision offset. Separator scans have been discussed in [17].

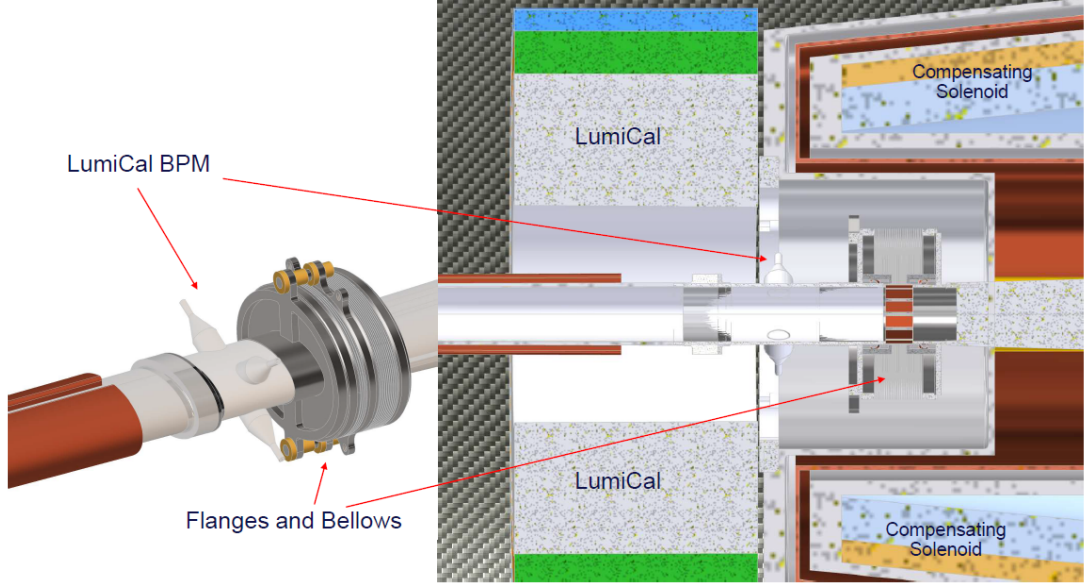


Figure 35: Proposed Lumical BPM location [61, 62].

2. Perhaps more promising is the use of dedicated, non-colliding, ‘orbit’ pilot bunches, with an intensity somewhat lower than that of the colliding bunches to ensure a similar longitudinal impedance. With these the beam position can be measured in the absence of beam-beam collisions. Assuming that 100 such bunches circulate in the machine, the time necessary to obtain a measurement of the collision offset with the above precision is about 15 minutes.

11.3.3 Opposite sign vertical dispersion measurement

The OSVD can be directly estimated by varying both beam energies with a modification of the revolution frequency. However, although this method was successful at LEP, the feasibility of performing this procedure during physics operation at FCC-ee remains to be demonstrated. Furthermore, this could lead to a cross-talk between the IPs, due to their individual collision offsets and OSVD, which remains to be investigated. For a modification of the energy by 10^{-3} , the above measurement of collision offsets with a precision of 0.4 nm corresponds to a measurement of OSVD with a precision of $\pm 0.6 \mu\text{m}$, or altogether a precision on the combined effect of collision offsets and OSVD on the centre-of-mass energy with an accuracy of 20 keV every 30 minutes. If a four times smaller relative excursion of energy of 2.5×10^{-4} is preferred, the same result will be obtained after 16 such measurements, providing a precision of 20 keV on the centre-of-mass energy every 8 hours, which would constitute a perfectly acceptable solution.

It has recently been proposed to use the horizontal orbit correctors to modify the path length, as an alternative to adjusting the RF frequency. This technique could only be performed on non-colliding pilot bunches. It would therefore be necessary to assume that these pilots possess the same OSVD as the colliding bunches.

11.3.4 Discussion and requirements

The above estimates assume implicitly that the movements of the beam are independent between interaction points, which is certainly not the case, since the beam-beam interaction generated by a collision offset at one IP will certainly generate a beam motion all around the ring, and in particular at the other IPs. Deciphering the source of the observed beam-beam kicks and thus correcting them will require the development of a correction procedure involving all four IPs, probably using as input the BPM readings of both colliding and non-colliding bunches. This procedure is likely to require dedicated correction knobs to adjust both the vertical beam positions and the vertical dispersion at the IP with a resolution of, typically, one tenth of the expected statistical precision, *i.e.* 0.04 nm (position) and 0.4 μm (dispersion).

In conclusion, the beam-beam deflection measurements give access to high-precision measurements of the collision offsets, as well as a unique measurement of the opposite sign vertical dispersion at the interaction points. The availability of high precision BPMs at a location where they can measure both beams at once is essential. The feasibility of regular energy shifts of 2.5×10^{-4} will be required for the measurement of OSVD.

12 Input from the experiments

The experiments operating at FCC-ee will themselves provide measurements that are essential input to the calibration of the collision energy and related quantities. A full discussion of these measurements can be found in Ref. [17]. Here, a brief summary is given, together with some recent updates. The principal data set for performing these measurements is the very large sample of dimuon events that each experiment will collect, arising from the process $e^+e^- \rightarrow \mu^+\mu^-(\gamma)$, where γ indicates the possible presence of initial-state radiation (ISR). Analysis of the topology of these events, constrained by the total energy and momentum conservation in the final state, allows several important quantities to be determined. This analysis is, in general, based on the knowledge of the muon directions, which in turn imposes demands on the performance of the tracking system.

12.1 The crossing angle α

The nominal value of the crossing angle is $\alpha = 30 \text{ mrad}$, but the true value must be determined throughout data-taking so that the collision energy can be calculated to the required precision. At the Z pole, more than 10^6 dimuon events will be collected every 10 minutes in each detector, which will allow this parameter to be measured with a statistical uncertainty of 0.3 μrad , which is sufficient for the physics goals, since a precision of 15 μrad leads to an uncertainty of 10 keV on E_{CM} . The statistical precision will be worse at higher energies, where the production rate is lower, but will not compromise the physics measurements that are targeted in these regimes.

There is an important subtlety in the crossing-angle determination that must be accounted for. The electron and positron bunches experience mutual electric and magnetic fields that accelerate (decelerate) the bunches before (after) the collision and also increase (decrease) the crossing angle. The collision energy is invariant, but the change in crossing angle

from this effect (estimated to be a relative 0.6% modification) must be known so that the measured crossing angle can be corrected back to the unaffected quantity and used together with beam energies as determined from RDP to calculate E_{CM} .

The magnitude of the variation in α depends on parameters such as the bunch population and the spread in collision energy $\delta_{E_{\text{CM}}}$. It is found empirically, from simulation studies, that α is proportional to $\mathcal{L}^{1/2} / \delta_{E_{\text{CM}}}^{1/6}$. By measuring $\mathcal{L}$, α , and $\delta_{E_{\text{CM}}}$ from dimuon events for different bunch intensities, it will be possible to extrapolate to zero intensity and determine the value of α in the absence of these effects. A good opportunity to perform these measurements would be in the period that top-up injection is taking place. It is therefore important that the detector can operate during this period and that the beams are stable. A simulated study of the measurement of α against $\mathcal{L}^{1/2} / \delta_{E_{\text{CM}}}^{1/6}$ is presented in Fig. 36.

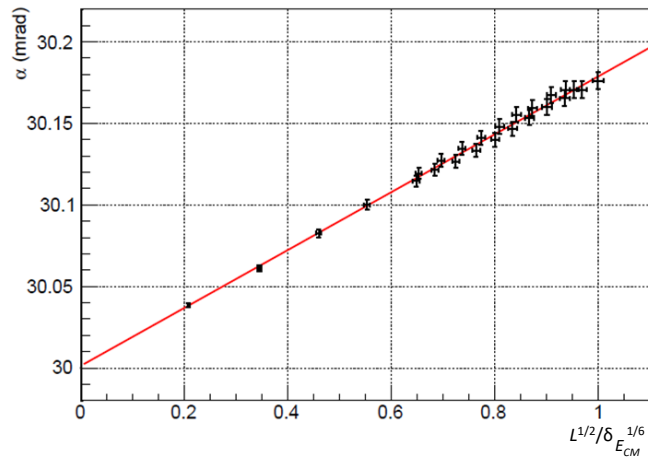


Figure 36: Change in the measured crossing-angle α vs. $\mathcal{L}^{1/2} / \delta_{E_{\text{CM}}}^{1/6}$, at various points during the top-up injection. Extrapolation down to $\mathcal{L}^{1/2} / \delta_{E_{\text{CM}}}^{1/6} = 0$ allows the crossing-angle to be determined in the absence of bunch-bunch effects [17].

12.2 The longitudinal boost and the collision-energy spread

The dimuon topology allows the longitudinal boost to be determined on an event-by-event basis. When averaged over a suitable sample size, this provides invaluable information to constrain the model of the energy loss around the ring and to calculate the local collision energy at each interaction point. The width of this distribution (Fig. 37) is a measure of $\delta_{E_{\text{CM}}}$, which is an essential input to the measurement of certain observables, such as the Z mass and widths. Again, the foreseen statistical precision on these quantities is excellent. For example, the energy spread can be measured to one part in a thousand with one million dimuon events. Recent work [63] has investigated how sensitive the determination of $\delta_{E_{\text{CM}}}$ is to the knowledge of the ISR corrections in dimuon production. The conclusion is that the measurement is robust; even if it is assumed that the second-order corrections from ISR are unknown (which is not the case), the resulting bias on the extraction of $\delta_{E_{\text{CM}}}$ is far smaller than the statistical uncertainty.

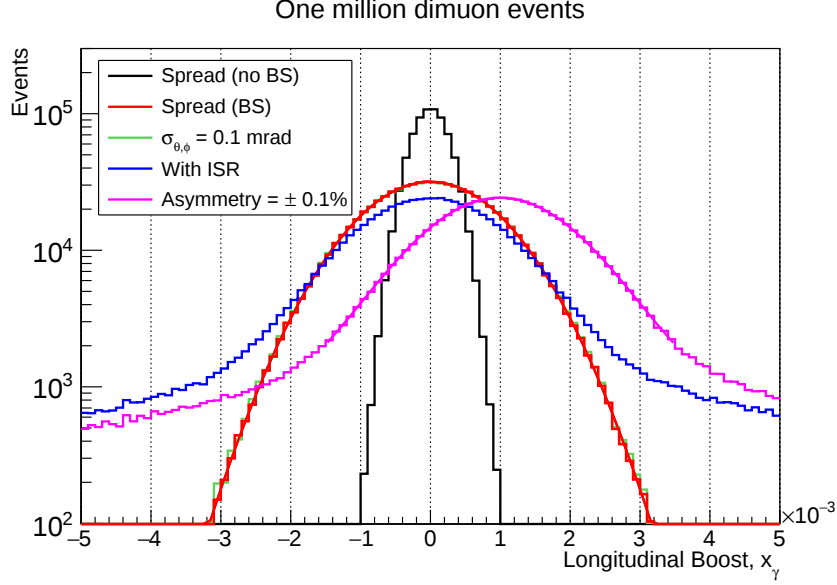


Figure 37: Fitted value of longitudinal boost from one million dimuon events at one of the FCC-ee IPs [17]. Once the ISR is unfolded, this distribution can be used to measure the energy spread. The magenta line shows the impact of a centre-of-mass boost on the distribution. The shift can be measured with a statistical precision of 40 keV. The other curves indicate the impact of beamstrahlung, angular resolution on the track directions, and ISR.

12.3 Relative E_{CM} determination in the Z -resonance scan

The reconstructed peak position of the dimuon invariant-mass distribution provides an excellent proxy for the collision energy. The difference in this reconstructed position between the points of the Z -resonance scan provides a measure of the change in collision energy, which is a critical input for several analyses, in particular the measurement of the Z boson width. The distribution is fitted in bins of the polar angle for back-to-back events. An example fit is shown in Fig. 38 (left). The statistical precision on this pseudo- E_{CM} measurement, when summing the samples from four experiments, is around 20 keV for each of the two off-peak running points, assuming the momentum resolution of the IDEA detector. The momentum scale stability must be controlled at this level, so that the detector does not introduce a bias in the measurement larger than the statistical precision.

The field stability can be tracked with NMR probes and the momentum scale can be directly monitored through the reconstruction of low-mass resonances. However, even with a perfect detector, there is a bias in the pseudo- E_{CM} measurement in the Z resonance scan arising from ISR/FSR effects, and from the product of the Breit–Wigner shape of the resonance and the Gaussian distribution of the energy spread of the colliding beams. The value of this bias varies by about 8 MeV when going from $E_{\text{CM}} = 87.9$ to 94.3 GeV, as can be seen in Fig. 38 (right). This difference must be corrected for in the measurement, which requires a good understanding of the ISR/FSR effects. In a generator-level study, disabling ISR/FSR changes the difference in the bias between the two off-peak points by around 500 keV. Therefore, the theoretical prediction of these ISR/FSR effects to the 1%

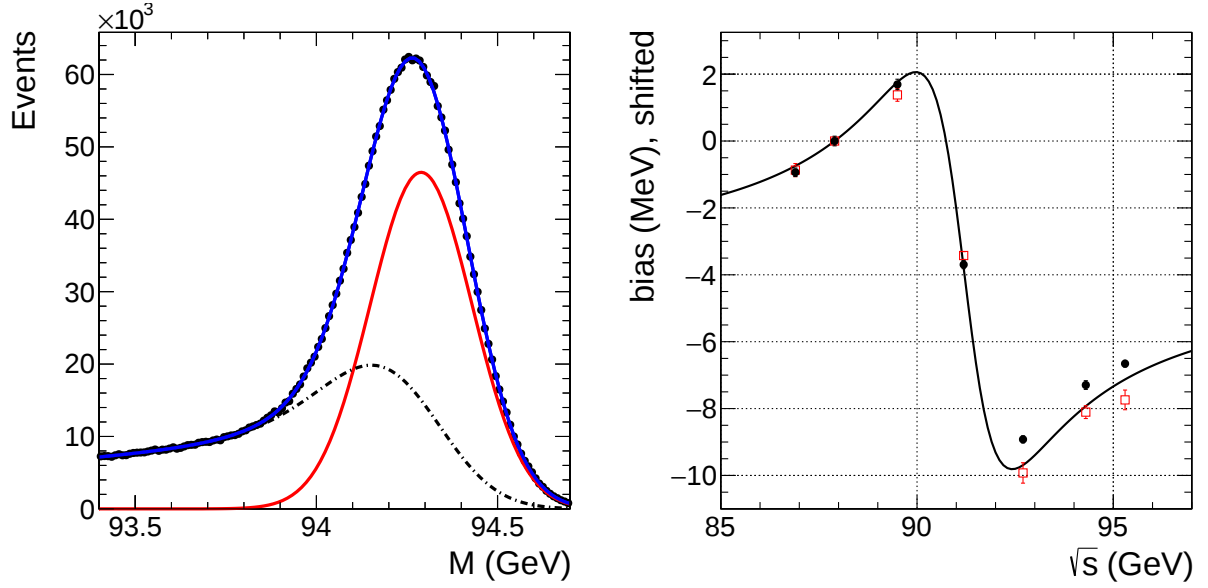


Figure 38: Left: Example fit to the dimuon invariant-mass distribution at $E_{\text{CM}} = 94.3 \text{ GeV}$, where the peak is modelled with the superposition (blue) of a Gaussian function (red) and two exponential functions (black). Right: The difference (bias) between the E_{CM} extracted from the dimuon invariant-mass fit and the true value. Results are shown for the simulated performance of the IDEA detector (red points) and for the expected dependence, including (black points) or not (black curve) ISR/FSR effects. Each set of results also has an overall offset of a few MeV, which is energy independent and, hence, not relevant for the determination of the Z width. A single shift has been applied to the bias, to correct for these offsets, so that the bias is zero at 87.9 GeV , for all sets of results.

level would be sufficient to render their impact negligible for the Z -width measurement.

12.4 Absolute E_{CM} determination

At collision energies above the Z boson mass, the dimuon events may be used to provide an absolute measurement of E_{CM} . Radiative returns, in which the emission of an initial-state photon means that the dimuon has the Z mass, allows for the calibration of events unaffected by ISR. The method can be extended to also include multihadron final states. This method can be calibrated at the W -pair production threshold with RDP, and is of great value for physics studies in the regime where no RDP is possible, i.e., collision energies above 200 GeV . This approach also provides a useful complementary measurement of E_{CM} in the intermediate energies where RDP is possible but challenging. The foreseen statistical uncertainty is around 280 keV for 6 ab^{-1} of integrated luminosity at $E_{\text{CM}} = 125 \text{ GeV}$ and 260 keV for 20 ab^{-1} at $E_{\text{CM}} = 160 \text{ GeV}$. The performance of the tracking system must be sufficiently good that the precision is not compromised.

13 Expected precision on EW observables from the collision energy and its spread

Several of the most important electroweak observables are expected to have a dominant or significant systematic uncertainty associated with the knowledge of the collision energy and collision-energy spread. The collision-energy uncertainties can be classed in three distinct categories, itemised below. These uncertainties propagate to the physics results in an observable-dependent manner, as discussed in Ref. [17].

- Uncertainties that are fully correlated between measurements propagate to the knowledge of the absolute energy scale. Examples include the values of the anomalous magnetic moment and the mass of the electron, the frequency of the RF system, and any other systematic biases that occurs at all times and all energies. At this stage in the studies, it is estimated that this uncertainty will be around 100 keV on the collision energy at the Z pole and 300 keV at the W^+W^- threshold. This contribution is expected to be the dominant systematic uncertainty in the measurements of the and boson masses.
- A point-to-point contribution comprises biases that occur at all times, or lead to an average shift, but are different for each energy setting. The principal method of determining this uncertainty will be based on the dimuon invariant-mass distribution, as reconstructed by the experiments. The estimated magnitude of this uncorrelated uncertainty is 20 keV for each off-peak point of the Z -resonance scan. The understanding gained at the Z pole and complementary measurements will lead to a corresponding uncertainty of around 100 keV at the W^+W^- threshold. The point-to-point uncertainty is expected to be the dominant contribution in the measurement of the Z width.
- The uncertainty on each individual RDP measurement is dominated by an uncertainty that is set by the frequency of the polarimeter sampling or the size of the energy bins where the depolarisation can be located. A reasonable estimate of this uncertainty is 200 keV at the Z pole and 300 keV at the W^+W^- threshold. As this component is statistical in nature, its impact decreases with the square-root of the number of events. As it is planned to collect $\sim 10^4$ measurements at each energy point, the final uncertainty from this source will be essentially negligible, compared to other contributions. However, the importance of making each measurement as precise as possible, and of collecting the largest possible number of measurements, will become more evident when the data set is split into smaller samples to perform systematic checks.

The contributions from each uncertainty category, and their quadratic sum, are listed in Table 4 for several key electroweak observables. This table also shows the contribution from the uncertainty in the knowledge of the energy spread, which affects quantities with a strong quadratic dependence on the collision energy. Observables that are most susceptible to this uncertainty include the Z cross section and the Z width. A collision-energy spread of 70 MeV, determined with a precision of ± 0.05 MeV, leads to a sub-dominant systematic uncertainty in the measurement of these observables.

Table 4: Current projected E_{CM} -related uncertainties on selected electroweak observables.

Uncertainty	Observable				
	m_Z (keV)	Γ_Z (keV)	$\sin^2 \theta_W^{\text{eff}} (\times 10^{-6})$	$\frac{\Delta\alpha_{\text{QED}}(m_Z^2)}{\alpha_{\text{QED}}(m_Z^2)} (\times 10^{-5})$	m_W (keV)
Absolute	100	2.5	–	0.1	150
Point-to-point	14	11	1.2	0.5	50
Sample size	1	1	0.1	–	3
Energy spread	–	5	–	0.1	–
Total E_{CM} -related	101	12	1.2	0.5	158
FCC-ee statistical	4	4	2	3	180

With the current expectations, it will be possible to reduce the uncertainty from energy-related quantities by an order of magnitude or more with respect to what was achieved at LEP, such that they will be smaller than, or similar to, the statistical uncertainty for all observables apart from m_Z . Indeed, the entries in Table 4 for the E_{CM} -related systematic uncertainties can be compared to the corresponding LEP values of 1.7 MeV for m_Z , 1.2 MeV for Γ_Z , and 9 MeV for m_W .

14 Monochromatisation

To be updated by Jacqueline and Guy in consultation with Angeles

Although it is not yet part of the baseline run plan of FCC-ee, studies are ongoing concerning the possibility of operating at $E_b = 62.5$ GeV in order to be sensitive to s -channel Higgs production and thus measure the electron Yukawa coupling. In this mode of operation, the collision energy spread due to synchrotron radiation is approximately 50 MeV, to be compared to the width of the resonance which is 4.2 MeV. In order to attain the necessary sensitivity to the very low cross section it is therefore necessary to implement a monochromatisation scheme in which the natural energy spread is reduced by about a factor 10, while preserving the luminosity at a sufficient level.

The monochromatisation schemes under current consideration involve introducing opposite sign dispersion at the interaction point, or a local chromaticity, which could possibly require modification of the final-focus optics. More details are given in Ref. [64].

15 Outlook

The studies performed before and during the Feasibility Study established a baseline scheme for calibration of the collision energy, ensuring that the physics goals of FCC-ee can be met. Nevertheless, these studies must be refined in certain areas and alternative schemes should be considered to further improve performance and operational efficiency.

The absolute uncertainties in E_{CM} arising from RDP are the dominant systematic uncertainties in the measurements of the Z and W masses. Future studies will investigate whether

these uncertainties can be reduced beyond the currently assumed values: 100 keV at the Z pole and 300 keV at the W^+W^- threshold.

The measurements of energy-related quantities made by the experiments using dimuon events are a critical ingredient in the E_{CM} calibration, at all centre-of-mass energies. Recently, several of these studies have been deepened to validate their robustness with respect to the uncertainties in the knowledge of higher-order ISR/FSR effects; this work will be extended further. It is also important to consolidate the strategy for understanding how the measurement of the crossing angle is affected by changes in the bunch intensity. The impact of detector performance and the interplay with alignment studies will be another focus of attention. Finally, the use of other categories of physics events, beyond dimuons, will be investigated.

It is important to have a reliable procedure to accurately translate the mean beam energy, measured by RDP, to the local collision energy, relevant for the physics measurements. Full simulations of this procedure will be conducted, at each interaction point, incorporating the in-situ measurement of the longitudinal boosts of the collisions and the knowledge of the machine impedances. Attention will also be paid to the control of energy shifts from possible dispersion effects at each interaction point, as well as to the related requirements on the precision of the system of beam-position monitors.

More detailed simulations of the level and lifetime of transverse polarisation will be performed, in parallel with changes to account for any evolution in the proposed optics of the accelerator. A deeper understanding will be sought of any effects that might bias the assumed proportionality between the spin tune and the mean beam energy. It will be particularly important to monitor the expected level of polarisation at the W^+W^- threshold and the RDP strategy in this challenging regime. Detailed technical designs will be made of the polarimeter and depolariser systems.

So far, the baseline strategy to get transversally polarised pilot bunches has been to inject unpolarised beams and to stimulate the growth of polarisation by activating wigglers at the start of each fill. This is a robust approach but has the disadvantage of introducing dead-time, during which no collisions are possible. To overcome this inconvenience, studies will investigate the possibility of injecting already-polarised pilot bunches, for which the design of the injection system must be modified. Simulations will be required to validate that the bunches retain their polarisation throughout the injection step and while travelling in the booster ring.

Further investigations will also take place regarding the feasibility of the electron Yukawa measurement. In particular, new and refined schemes will be investigated with the aim of improving the monochromatisation of the collision energy. It will also be necessary to develop and simulate a procedure to monitor and adjust the collision energy in real time, to ensure that it remains centred at the Higgs pole. Further physics studies will be performed to improve the signal yield and the signal-to-background discrimination. Investigations will be performed to evaluate if the significance of the signal can be improved with differential measurements, accounting for the expected correlations between the monochromatisation and the longitudinal coordinate of the e^+e^- collision.

16 Summary

A strategy has been developed for the calibration of the centre-of-mass energy, and energy spread, with a precision that allows for a precise measurement of many important electroweak observables at FCC-ee. Further improvements will allow for even higher sensitivity to be gained, in particular for the Z width. This strategy has important consequences for machine operation, the necessary instrumentation and the measurements that are expected from the physics experiments themselves. Requirements are being formulated for critical items such as the depolariser and polarimeters. A set of goals is being defined for the next phase of the project.

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