

Searches for CP violation in multi-body charm decays and studies of radiation damage in the LHCb VELO detector

Shanzhen Chen

A thesis submitted to The University of Manchester
for the degree of Doctor of Philosophy

Particle Physics Group
School of Physics and Astronomy
Faculty of Science & Engineering
September 2017



The University of Manchester



Contents

Contents	3
List of Figures	7
List of Tables	17
1 Abstract	21
2 Declaration	23
3 Copyright	25
4 Acknowledgement	27
5 Introduction	29
6 Theory	31
6.1 The Standard model	31
6.1.1 Electroweak interactions	32
6.1.2 Strong interactions	33
6.2 CP violation in heavy flavour physics	33
6.2.1 Symmetries in particle physics	33
6.2.2 CP violation and CKM matrix	36
6.2.3 Neutral meson mixing	38
6.2.4 Types of CP violations	40
6.2.5 CP violation in multi-body decays	41
6.3 CP violation in charm sector	42
6.3.1 Heavy flavour averaging group fit	44
7 The LHCb experiment	45
7.1 The Large Hadron Collider	45
7.1.1 Running conditions in run-1	47
7.1.2 Running conditions in run-2	48
7.2 The LHCb experiment	48
7.2.1 The Vertex Locator	51

7.2.2	Tracking Stations	54
7.2.3	The dipole magnet	56
7.2.4	Ring Imaging Cherenkov Detectors	57
7.2.5	Calorimeter system	60
7.2.6	Muon system	61
7.2.7	Triggers	63
8	Radiation damage in LHCb vertex locator	65
8.1	Particle detection with silicon detectors	65
8.2	Radiation damage in silicon sensors	69
8.3	The VELO sensors	72
8.4	Monitoring radiation damage of VELO sensors	75
8.4.1	IV scan	76
8.4.2	Charge collection efficiency scan	76
8.5	Determination of VELO sensor operation voltages for 2015 and 2016 operation	80
8.6	Determination of VELO sensor operation voltage for 2017 operation	86
8.6.1	Recommendations for 2017 and 2018 operation	88
8.7	Optimisation of the CCE scan in run-2	90
8.8	Study of cluster finding efficiency of VELO sensors	95
8.8.1	Second metal layer effect of VELO sensors	97
8.8.2	Impact from the geometry of VELO detector	101
8.9	Conclusions	104
9	Searches for CPV in multi-body decays	109
9.1	Techniques for multi-body CP violation searches	110
9.1.1	Binned χ^2	110
9.1.2	Energy test	112
9.2	Fitting the T-value distribution	115
9.3	Sensitivity of energy test	119
9.4	Visualisation of asymmetries	121
9.5	Presence of background	123
9.6	Coordinate selection	126
9.6.1	Distances in Phase-Space	126
9.6.2	Coordinates in three-body decays	126
9.6.3	Coordinates in four-body decays	127
9.7	Parity-even and Parity-odd CP Violation Tests	130
9.8	Implementation of unbinned techniques	134
9.9	Conclusions	135
10	CP violation in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ decays	137
10.1	Introduction	137
10.2	Dataset and signal selection	139
10.2.1	Trigger selection	139

10.2.2	The LHCb centralised selection	140
10.2.3	Offline selection	140
10.2.4	Dalitz plot and mass projections for $D^0 \rightarrow \pi^- \pi^+ \pi^0$. . .	143
10.2.5	Selection of reference decay $D^0 \rightarrow K^- \pi^+ \pi^0$	147
10.3	Sensitivity tests	154
10.3.1	Simulation Samples	154
10.3.2	Studies accounting for selection efficiencies and back-ground effects	156
10.3.3	Variation of metric parameter σ	157
10.4	Systematic uncertainties	169
10.4.1	Background asymmetry	170
10.4.2	Cross-check using a control mode	175
10.4.3	Tracking and detection asymmetries	182
10.5	Results	185
11	CP violation in $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decays	187
11.1	Introduction	187
11.2	Signal selection	189
11.2.1	Trigger selection	189
11.2.2	The LHCb centralised selection	189
11.2.3	Offline selection	191
11.2.4	Background events	198
11.2.5	Selection of reference decay $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$	199
11.3	Sensitivity tests	202
11.3.1	Simulation samples	202
11.3.2	Studies accounting for selection efficiencies	202
11.3.3	Sensitivity study with default metric parameter σ	204
11.3.4	Variation of metric parameter σ	211
11.3.5	P -odd tests	213
11.3.6	Visualise the asymmetric region	214
11.4	Systematic uncertainties	222
11.4.1	Background asymmetries	222
11.4.2	Cross-check using a control mode	228
11.5	Results	232
12	Summary and outlook	235
A	CFE as function of position on sensors 0 to 40 (R-sensors)	239
B	CFE as function of position on sensors 64 to 104 (Φ-sensors)	245
C	CFE versus sensor radius for sensor 0 to 40 (R-sensors)	251
D	CFE versus sensor radius for sensor 64 to 104 (Φ-sensors)	257

E	CFE as function of distances to routing lines and strips on sensors 0 to 40 (R-sensors)	263
F	Track population as function of pseudo-rapidity and sensor radius for sensor 0 to 40 (R-sensors)	269
G	CFE as function of pseudo-rapidity and sensor radius for sensor 0 to 40 (R-sensors)	275
H	Variation of metric parameter σ	281
	References	289

List of Figures

6.1	Box diagrams for $D^0 - \bar{D}^0$ transitions via exchanging intermediated quarks.	39
6.2	Diagrams for $D^0 - \bar{D}^0$ transitions via long range hadronic interactions.	39
6.3	Feynman diagrams for tree level (left) and loop level (right) transitions in $D^0 \rightarrow \pi^+ \pi^-$ decay.	43
6.4	The heavy flavour averaging group (HFLAv) fit combining various searches for direct and indirect CP violation in the charm system.	44
7.1	Schematic view of the LHC showing the eight interaction points and the facilities at the interaction points.	46
7.2	The CERN accelerator complex.	47
7.3	The sub-system pseudo-rapidity coverage of ATLAS, CMS and LHCb experiments, the $b\bar{b}$ pair production angle at 14 TeV and the LHCb coverage, and the population of $b\bar{b}$ pairs as function of pseudo-rapidity and the coverage of LHCb detector and other general purpose detectors.	49
7.4	Illustration of the luminosity levelling employed at LHCb.	50
7.5	The cross-section of LHCb detector.	51
7.6	Schematic view of the VELO detector.	52
7.7	Cross section in the (x, z) plane of the placement of the pile-up sensor modules and VELO modules along the LHCb beam axis, and position of the VELO sensors before and after closing.	53
7.8	The layout of R - and Φ -type sensor strips and routing lines.	54
7.9	Layout of four layers of the TT.	55
7.10	Layout of one IT station.	56
7.11	Layout of OT straw tubes in one module, and layout of three OT stations.	57
7.12	Schematic view of RICH1 and RICH2 detectors.	58
7.13	Reconstructed Cherenkov angle as a function of track momentum in C_4F_{10} radiator.	59

7.14	Kaon identification efficiency and pion misidentification rate as a function of track momentum with two different delta log-likelihood requirements.	60
7.15	The granularity of SPD/PS and ECAL, and the granularity of HCAL.	62
7.16	Schematic side view of the muon stations, and schematic front view of one muon station.	62
7.17	The 2012 trigger data flow, including the manner in which the deferral of events is applied, and the run-2 trigger scheme showing the online detector alignment and calibration stage.	63
8.1	Band gap energies in metals, semiconductors and insulators.	67
8.2	Illustration of p - n junction and depletion region.	68
8.3	Illustration of radiation induced defects in a silicon lattice.	70
8.4	The evolution of effective doping concentration and depletion voltage for n -type silicon as a function of fluence.	71
8.5	Photographs of the innermost region and outermost region of an R -type sensor.	74
8.6	A schematic cross-section of a portion of an R -type sensor with n^+ -on- p structure.	75
8.7	Pulse-shape obtained by combining data from all VELO sensors.	75
8.8	Current measured from IV scans versus date and delivered luminosity from the start of LHCb operations to September 2016.	77
8.9	Schematic illustration of the method for determining the effective depletion voltage in a CCE scan.	78
8.10	Fits to the pedestal-subtracted ADC distributions at different bias voltages for an R sensor, and MPV of the ADC distribution as a function of bias voltage for an R sensor.	79
8.11	EDV with fluence for all sensors before LHC LS1, and the dependence of the CFE on the sensor bias voltage for a downstream R -type sensor at the end of run-1.	81
8.12	Variation of fluence with z -position of the sensor as determined from simulation at 7 TeV and 14 TeV.	83
8.13	Linear fit to the EDV of the inner radius region of R and ϕ n^+ -on- n sensors.	84
8.14	Linear fit to the EDV of the inner radius region of R and ϕ n^+ -on- p sensors.	85
8.15	Predicted EDV with module number for all VELO sensors after a luminosity of 4.4 fb^{-1} grouped according to fluence at the sensor. Recommended operating voltages for the start of run-2 are overlaid.	86
8.16	Comparison of the last EDV versus fluence plot from the 2012 and 2013 CCE scans and the first CCE scan in run-2.	87
8.17	EDV versus fluence plot made with data from October 2016 CCE scan overlaid with Hamburg model simulation plot.	88

8.18	Evolution of depletion voltage of all sensors in five radial regions with time using the Hamburg model.	89
8.19	Prediction of depletion voltage of the innermost edge of all sensors till the end of 2018 operation.	90
8.20	Failed fits in the January 2013 CCE scan as function of sensor number for the different radial regions, and Change in EDV with fluence for all sensors before LS1, with the excluded regions of fluence and EDV.	91
8.21	Failed fits when removing the steps at 10 V, 20 V, 30 V, 40 V, 50 V, 60 V, 70 V and 80 V.	94
8.22	Differences in EDV before and after removing the steps at 10 V, 20 V, 30 V, 40 V, 50 V, 60 V, 70 V and 80 V.	95
8.23	Failed fits and the difference in EDV when the steps at 10 V and 30 V are removed.	96
8.24	CFE as function of position for sensor 36 (R -type sensor) and sensor 100 (Φ -type sensor).	98
8.25	CFE as function of position for sensor 76 (Φ -type sensor) and sensor 97 (Φ -type sensor).	99
8.26	Summaries of CFE versus sensor radius for R -type and Φ -type sensors.	100
8.27	Summaries of CFE versus sensor number for R -type and Φ -type sensors.	101
8.28	CFE as function of position for sensor 32 (R -type sensor) made with physics data taken at the beginning of 2017 operation, and routing line map on a R -type sensor.	102
8.29	CFE as a function of distance to strip and distance to RL for sensor 36 (R -type sensor).	103
8.30	Illustration of second metal layer effect on a R -type sensor	104
8.31	ADC counts for clusters observed on R -type sensors for a range of delivered luminosity, and ADC counts for cluster on R -type sensors where the cluster is associated with a track.	105
8.32	Population of tracks as a function of sensor radius and pseudo-rapidity, CFE as function of sensor radius and pseudo-rapidity, CFE as a function of sensor radius, and CFE as a function of position on a sensor for an R -type sensor at the upstream end of the VELO detector (sensor number 1) using 2016 October CCE scan data.	106
8.33	Same set of plots as in Fig. 8.32. These plots are for an R -type sensor near the pp interaction region (sensor number 10).	106
8.34	Same set of plots as in Fig. 8.32. These plots are for an R -type sensor in the forward region of the VELO (sensor number 31).	107
8.35	Same set of plots as in Fig. 8.32. These plots are for an R -type sensor in the very downstream interaction region of the VELO (sensor number 40).	107

9.1	Dalitz plot of a sample generated with a simple dimensionless resonance model.	112
9.2	p -value versus number of bins in S_{CP} method for simulation samples with CP violation introduced at the 3% and 5% level in one amplitude	113
9.3	An illustration of an isolated system containing particles carrying opposite signed charges.	114
9.4	Examples of T values calculated from simulated data samples and permutation T values. The distributions of permutation T -values are fitted with GEV functions.	116
9.5	Examples of (left) p -value distribution and (right) cumulative p -value distribution from random sampling of the fit parameters of a GEV fit that is used in Fig. 9.4.	118
9.6	T values and p -values versus σ in the energy test method for simulation samples with CP violation introduced at the 3% level in one amplitude.	120
9.7	T_i^{MAX} and T_i^{MIN} distributions and the significance levels set by them.	122
9.8	The T value relative to the T value found from only considering signal events (T_{signal}), when calculated using Eq. 9.3, where background is not removed, and Eq. 9.9, which subtracts (on average) the effect of background.	125
9.9	An example distribution of events in a simplified simulation Monte Carlo $D^0 \rightarrow \pi^+ \pi^- \pi^0$ decay sample in 3D Dalitz space.	127
9.10	Symmetry transformation relationships of the four data samples used in the CP violation tests.	131
10.1	Distribution of Δm with fit overlaid for the selected data set with resolved and merged π^0 candidates in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis. . . .	142
10.2	Dalitz plot of the resolved, merged and combined $D^0 \rightarrow \pi^- \pi^+ \pi^0$ data sample.	143
10.3	Dalitz plots $m_{\pi^- \pi^0}^2$ vs. $m_{\pi^+ \pi^0}^2$ for resolved pions and merged pions samples for Δm signal region events and Δm sidebands in data for the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis.	145
10.4	Dalitz plot projections $m_{\pi^- \pi^0}$, $m_{\pi^+ \pi^0}$, and $m_{\pi^+ \pi^-}$ for resolved pions and merged pion samples in data for the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis showing Δm signal region events and Δm sideband region events.	146
10.5	Dalitz plots $m_{\pi^- \pi^0}^2$ vs. $m_{\pi^+ \pi^0}^2$ for $sWeighted$ signal and $sWeighted$ background for resolved and merged samples in data for the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis.	147
10.6	Dalitz plot projections $m_{\pi^- \pi^0}$, $m_{\pi^+ \pi^0}$, and $m_{\pi^+ \pi^-}$ for resolved pion and merged pion samples in data for the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis showing $sWeighted$ signal, $sWeighted$ background events.	148

10.7	Comparison of background estimated from Δm sidebands and from the $sWeights$ distributed on Dalitz plot projections $m_{\pi^-\pi^0}$, $m_{\pi^+\pi^0}$, and $m_{\pi^+\pi^-}$ for resolved and merged samples in data for the $D^0 \rightarrow \pi^-\pi^+\pi^0$ analysis.	149
10.8	Dalitz plots $m_{\pi^-\pi^0}^2$ versus $m_{\pi^+\pi^0}^2$ for the combined data sample of $D^0 \rightarrow \pi^-\pi^+\pi^0$ events for Δm signal region events and Δm sidebands region events.	150
10.9	Dalitz plot projections $m_{\pi^-\pi^0}$, $m_{\pi^+\pi^0}$, and $m_{\pi^+\pi^-}$ for the combined data sample for the $D^0 \rightarrow \pi^-\pi^+\pi^0$ analysis showing Δm signal region events and Δm sidebands region events.	150
10.10	Comparison of mass projections for D^0 and $\bar{D}^0$ for Δm signal region and Δm sideband in data for the $D^0 \rightarrow \pi^-\pi^+\pi^0$ analysis. The projections show $m_{\pi^-\pi^0}$, $m_{\pi^+\pi^0}$, and $m_{\pi^+\pi^-}$ for both the resolved and merged pion samples combined.	151
10.11	Δm distributions for $D^0 \rightarrow K^-\pi^+\pi^0$ data after BDT selection for resolved pion and merged pion data samples.	151
10.12	$D^0 \rightarrow K^-\pi^+\pi^0$ analysis Dalitz plots $m_{K^-\pi^0}^2$ vs. $m_{\pi^+\pi^0}^2$ for the combined data sample for Δm signal region events and Δm sidebands region events.	152
10.13	Projections of $D^0 \rightarrow K^-\pi^+\pi^0$ analysis selected data sample Dalitz plot: $m_{K^-\pi^0}$, $m_{\pi^+\pi^0}$, and $m_{\pi^+\pi^-}$ for both samples combined.	153
10.14	Distribution of transverse momentum p_T and momentum for the π^+ from $sWeighted D^0 \rightarrow \pi^-\pi^+\pi^0$ signal and $sWeighted D^0 \rightarrow K^-\pi^+\pi^0$ signal data samples.	153
10.15	T value result for a test sample and its permutations T values, and the visualization tool applied to observe the regions which have an asymmetry on the Dalitz plot showing $m^2(\pi^-\pi^0)$ versus $m^2(\pi^+\pi^0)$	155
10.16	The efficiency map obtained with full LHCb simulation of $D^0 \rightarrow \pi^-\pi^+\pi^0$ decay in a square Dalitz plot of the combined resolved and merged pion samples and the corresponding projections on $m_{\pi^+\pi^-}$	156
10.17	The Dalitz plots of the generated toy simulation sample for $D^0 \rightarrow \pi^-\pi^+\pi^0$ decay without and with the efficiency map applied.	157
10.18	T value and permutation T values distribution for toy $D^0 \rightarrow \pi^-\pi^+\pi^0$ simulation samples in $\rho^+(770)$ CP violation samples.	158
10.19	T value and permutation T values distribution for toy $D^0 \rightarrow \pi^-\pi^+\pi^0$ simulation samples in $\rho^-(770)$ CP violation samples.	159
10.20	T value and permutation T values distribution for toy $D^0 \rightarrow \pi^-\pi^+\pi^0$ simulation samples in $\rho^0(770)$ CP violation samples.	162
10.21	T value and permutation T values distribution for one toy $D^0 \rightarrow \pi^-\pi^+\pi^0$ simulation sample (+2% in $\rho^+(770)$ amplitude) for a range of σ_s	163
10.22	T value and permutation T values distribution for one toy $D^0 \rightarrow \pi^-\pi^+\pi^0$ simulation sample (+2° in $\rho^+(770)$ phase) for a range of σ_s	164

10.23	T value and permutation T values distribution for one toy $D^0 \rightarrow \pi^- \pi^+ \pi^0$ simulation sample (+1° in $\rho^+(770)$ phase) for a range of σ s.	165
10.24	T value and permutation T values distribution for one toy $D^0 \rightarrow \pi^- \pi^+ \pi^0$ simulation sample (+2% in $\rho^-(770)$ amplitude) for a range of σ s.	166
10.25	$D^0 \rightarrow \pi^- \pi^+ \pi^0$ T_i value distributions on Dalitz plot and the significance level plots for the same simulation sample (+2% in $\rho^+(770)$ amplitude) with σ from 0.03 (GeV/c ²) ² to 0.2 (GeV/c ²) ² .	167
10.26	$D^0 \rightarrow \pi^- \pi^+ \pi^0$ T_i value distributions on Dalitz plot and the significance level plots for the same simulation sample (+2% in $\rho^+(770)$ amplitude) with σ from 0.3 (GeV/c ²) ² to 1 (GeV/c ²) ² .	168
10.27	Kinematics of D^0 in regions in $m_{\pi^+ \pi^-}^2$ and the helicity angle $\cos \theta_{\pi^+ \pi^-}$.	169
10.28	Comparison of s Weighted backgrounds for D^0 and $\bar{D}^0$ in the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ sample data.	170
10.29	Comparison of background estimated from Δm sidebands for D^0 (blue) and $\bar{D}^0$ (green) in the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis data.	171
10.30	Results from applying the Energy test to the $m(D^0)$ sideband region events in the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis.	172
10.31	S_{CP}^i in bins of $\pi^- \pi^+ \pi^0$ Dalitz plot for the background events, and S_{CP}^i distribution compared with a Gaussian distribution expected for no- CP violation hypothesis.	173
10.32	p -value distributions for symmetric and asymmetric toy MC samples.	173
10.33	Comparison of $D^0 \rightarrow K^- \pi^+ \pi^0$ signals for D^0 and $\bar{D}^0$ on Dalitz plot projections for Δm signal region events.	175
10.34	Comparison of $D^0 \rightarrow K^- \pi^+ \pi^0$ background for D^0 and $\bar{D}^0$ on Dalitz plot projections for Δm sideband events.	175
10.35	T values of the permutation test of $K^- \pi^+ \pi^0$ control mode for subsamples 2, 4 and 7, and the corresponding visualisation on the Dalitz plot $m_{K^- \pi^0}^2$ vs $m_{\pi^+ \pi^0}^2$.	177
10.36	p -values from applying the energy test to the $K^- \pi^+ \pi^0$ control samples.	178
10.37	Distribution of the permutation T value for the $D^0 \rightarrow K^- \pi^+ \pi^0$ data selected with magnet up magnet down polarities.	178
10.38	S_{CP}^i in bins of $D^0 \rightarrow K^- \pi^+ \pi^0$ Dalitz plot for subsamples 2, 4 and 7, and the corresponding S_{CP}^i distribution compared with a Gaussian distribution expected for no- CP violation hypothesis.	180
10.39	p -values from applying the S_{CP} method to the $K^- \pi^+ \pi^0$ control samples.	181
10.40	Kinematics of slow pion for different bins of $D^0 \rightarrow \pi^- \pi^+ \pi^0$ Dalitz plot.	183
10.41	T value distribution after applying selections that removed the asymmetries in the acceptance caused by the magnetic polarity to a sample of $D^0 \rightarrow K^- \pi^+ \pi^0$ decays, and T value distribution of the vetoed events.	184

10.42 $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis results.	185
11.1 Asymmetry between D^0 and $\bar{D}^0$ candidates as a function of the p and p_x of the slow pion in magnetic field up and magnetic field down data samples for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis.	192
11.2 Asymmetry between D^0 and $\bar{D}^0$ candidates as a function of the p_x and p_y of the slow pion in magnetic field up and magnetic field down data samples for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis.	192
11.3 Asymmetries between D^0 and $\bar{D}^0$ candidates as a function of the p versus p_x phase space and the p_x versus p_y phase space of the slow pions after fiducial selections.	193
11.4 The Gaussian fit around K_S^0 mass for slowest $\pi^+ \pi^-$ pairs and the second slowest $\pi^+ \pi^-$ pairs in the selected $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ events.	193
11.5 Δm distribution of 2011 and 2012 data for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis after selection.	195
11.6 $m(D^0)$ distribution of 2011 and 2012 data for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis.	196
11.7 The four possible $\pi^+ \pi^-$ invariant masses in the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ data.	197
11.8 Background events for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis.	198
11.9 Momentum and transverse momentum of D^0 in the reference channel $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$	200
11.10 Δm distribution for $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ decay after selection for 2011 and 2012 data samples.	200
11.11 The distributions of momentum of the slowest pion and 2nd slowest pion from generator level, reconstruction level, and the efficiency between them.	205
11.12 Acceptance of slowest pions versus 2nd slowest pion in LHCb full simulation events for $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$	205
11.13 T value distributions of $D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$ decays simulation samples with 2 degree, 3 degree, 4 degree, and 5 degree phase difference in $D^0 \rightarrow (a_1 \rightarrow \rho \pi(S - wave)) \pi$ mode.	207
11.14 T value distributions of $D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$ decays simulation samples with 2%, 3%, 4%, and 5% amplitude difference in $D^0 \rightarrow (a_1 \rightarrow \rho \pi(S - wave)) \pi$ mode.	208
11.15 T value distributions of $D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$ decays simulation samples with 2 degree, 3 degree, 4 degree, and 5 degree phase difference in $D \rightarrow \rho \rho(D - wave)$ mode.	209
11.16 T value distributions of $D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$ decays simulation samples with 2%, 3%, 4%, and 5% amplitude difference in $D \rightarrow \rho \rho(D - wave)$ mode.	210
11.17 T value and permutation T values distribution for one toy simulation sample ($+3^\circ$ phase asymmetry applied on the $a_1^+ \rightarrow \rho^0 \pi^+$) for a range of σ_s	212

11.18	T value and permutation T values distribution for one toy simulation sample ($+3^\circ$ phase asymmetry applied on the $a_1^+ \rightarrow \rho^0 \pi^+$) for a range of σ s.	213
11.19	T value, p -value, and width of GEV function versus metric parameter σ for toy simulation $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ samples with 3° phase asymmetry and 5% amplitude asymmetry on $a_1^+ \rightarrow \rho^0 \pi^+$, 4° phase asymmetry and 5% amplitude asymmetry on $D^0 \rightarrow \rho \rho$	216
11.20	An example of a parallel coordinate display of simulated $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ decays.	217
11.21	Parallel coordinate plots of simulated $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ samples with only the $D^0 \rightarrow (a_1^+ \rightarrow \rho^0 \pi^+) \pi^-$ (S-wave) decay.	218
11.22	Parallel coordinate plots of simulated $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ with only the $D^0 \rightarrow \rho^0 \rho^0$ (D-wave) decay.	219
11.23	Distribution of permutation T -values fitted with a GEV function for the simulated $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ sample and showing the measured T -value as a vertical line, and local asymmetry significances. . . .	221
11.24	Comparison of background from Δm - D mass sideband for D^0 and $\bar{D}^0$ for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis.	224
11.25	Comparison of background from Δm - D mass sideband for D^0 and $\bar{D}^0$ for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis.	225
11.26	T values distribution results from applying energy test for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ sideband sub-samples.	226
11.27	p -value distributions for $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ sideband background sub-samples that listed in Table. 11.12.	227
11.28	Comparison of reference sample from Δm - D mass for D^0 and $\bar{D}^0$	228
11.29	Comparison of reference sample from Δm - D mass for D^0 and $\bar{D}^0$	229
11.30	p -value distributions for reference channel sub-samples that listed in Table. 11.13.	231
11.31	T value distributions for reference channel sub-samples that listed in Table. 11.13, and permutation T value distributions generated from reference sample.	231
11.32	Distribution of permutation T -values fitted with a GEV function and showing the T -value of the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ data tests as a vertical line, and local asymmetry significances.	233
H.1	T value and permutation T values distribution for one toy simulation sample (5% amplitude asymmetry applied on the $a_1^+ \rightarrow \rho^0 \pi^+$) with σ from $0.2(\text{GeV}/c^2)^2$ to $0.7(\text{GeV}/c^2)^2$	282
H.2	T value and permutation T values distribution for one toy simulation sample (5% amplitude asymmetry applied on the $a_1^+ \rightarrow \rho^0 \pi^+$) with σ from $0.8(\text{GeV}/c^2)^2$ to $1.1(\text{GeV}/c^2)^2$	283
H.3	T value and permutation T values distribution for one toy simulation sample ($+4^\circ$ phase asymmetry applied on the $D^0 \rightarrow \rho \rho$) with σ from $0.2(\text{GeV}/c^2)^2$ to $0.7(\text{GeV}/c^2)^2$	284

H.4	T value and permutation T values distribution for one toy simulation sample ($+4^\circ$ phase asymmetry applied on the $D^0 \rightarrow \rho\rho$) with σ from $0.8(\text{GeV}/c^2)^2$ to $1.1(\text{GeV}/c^2)^2$	285
H.5	T value and permutation T values distribution for one toy simulation sample (5% amplitude asymmetry applied on the $D^0 \rightarrow \rho\rho$) with σ from $0.2(\text{GeV}/c^2)^2$ to $0.7(\text{GeV}/c^2)^2$	286
H.6	T value and permutation T values distribution for one toy simulation sample (5% amplitude asymmetry applied on the $D^0 \rightarrow \rho\rho$) with σ from $0.8(\text{GeV}/c^2)^2$ to $1.1(\text{GeV}/c^2)^2$	287

List of Tables

6.1	List of elementary particles in the standard model.	32
6.2	The electric charge (Q) and weak isospin (T_3^L) for different types of fermions (f) and anti-fermions ($\bar{f}$). The subscripts L and R denote left- and right-handed fermions.	32
6.3	Examples of the relations of non-observables, symmetry transformations, and conservation laws.	35
8.1	Parameters of VELO sensors	72
8.2	Numbers and percentages of failed fits when a single voltage step is removed from the January 2013 CCE scan analysis.	92
8.3	List of CCE scans from 2009 to the start of 2017 operation. The determination of CCE scan steps are described in Sect. 8.7.	96
9.1	A simple dimensionless resonance model of a three-body scalar decay $X \rightarrow abc$. The mass of mother particle X is in arbitrary unit. . . .	111
9.2	Fit parameters of the GEV function, for the fits to T vlaue disributions that are shown in Fig. 9.4.	117
9.3	Coordinates used, and those excluded, in the measurement of $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ decays [15] and coordinates suggested for $D^0 \rightarrow K^+K^-\pi^+\pi^-$ decays. Excluding the mass marked in brackets would reduce to the minimal five coordinates that span the phase-space. .	129
10.1	Stripping selection criteria for the LHCb centralised selection algorithms (stripping lines) that used in this $D^0 \rightarrow \pi^-\pi^+\pi^0$ analysis.	141
10.2	Results of fit to the Δm distributions for the full data in the $D^0 \rightarrow \pi^-\pi^+\pi^0$ analysis after BDT selection shown in Fig. 10.1. $\Delta m_{0\ core}$, $\Delta m_{0\ mid}$ and $\Delta m_{0\ tail}$ are the peak positions of the three Gaussians, σ_{core} , σ_{mid} and σ_{tail} are their widths, and f_{core} and f_{mid} are the signal fractions corresponding to the two narrow Gaussians.	143
10.3	Results of fit to the Δm distributions for $D^0 \rightarrow K^-\pi^+\pi^0$ shown in Fig. 10.11.	152
10.4	List of resonances and their parameters used to generate $D^0 \rightarrow \pi^-\pi^+\pi^0$ simulation samples.	155

10.5	Summary of T values obtained from toy simulation samples of $D^0 \rightarrow \pi^- \pi^+ \pi^0$ with backgrounds generated with a range of CP violation scenarios applied to the $\rho(770)^+$ resonance. $\sigma = 0.2(\text{GeV}/c^2)^2$ is used in all tests.	157
10.6	Summary of T values obtained from toy simulation samples in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis with backgrounds generated with a range of CP violation scenarios applied to the $\rho(770)^-$ resonance. $\sigma = 0.2(\text{GeV}/c^2)^2$ is used in all tests.	160
10.7	Summary of T values obtained from toy simulation samples in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis with backgrounds generated with a range of CP violation scenarios applied to the $\rho(770)^0$ resonance. $\sigma = 0.2(\text{GeV}/c^2)^2$ is used in all tests.	160
10.8	Summary of T values obtained from toy simulation samples in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis generated with a CP violation of $+2\%$ applied to the amplitude of the $\rho(770)^+$ resonance for a range of value of the metric parameter σ	160
10.9	Summary of T values obtained from toy simulation samples in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis generated with a CP violation of $+2^\circ$ applied to the phase of the $\rho(770)^+$ resonance for a range of value of the metric parameter σ	161
10.10	Summary of T values obtained from toy simulation samples in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis generated with a CP violation of $+1^\circ$ applied to the phase of the $\rho(770)^+$ resonance for a range of value of the metric parameter σ	161
10.11	Summary of T values obtained from toy simulation samples in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis generated with a CP violation of $+2\%$ applied to the amplitude of the $\rho(770)^-$ resonance for a range of value of the metric parameter σ	161
10.12	Results from applying the S_{CP} method to the $\pi^- \pi^+ \pi^0$ background	172
10.13	Results from applying the energy test for toys with flavour-averaged and flavour-separated background.	174
10.14	Results from applying the energy test to the $K^- \pi^+ \pi^0$ control samples.	176
10.15	Results from applying the energy test to the $K^- \pi^+ \pi^0$ sample split according to the magnet polarity.	176
10.16	Results from applying the S_{CP} method to the $K^- \pi^+ \pi^0$ control samples	179
10.17	Results for various metric parameter values. The p -values are obtained with the counting method.	185
11.1	Stripping selection criteria for the LHCb centralised selection algorithm (stripping line) that used in this $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis.	190
11.2	Gaussian function fit it parameters of K_S^0 peaks in the selected $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ sample.	194

11.3	Results of fit to the Δm distributions for selected 2011 and 2012 data for the $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ analysis. $\Delta m_{0_{core}}$, $\Delta m_{0_{mid}}$, and $\Delta m_{0_{tail}}$ are the peak position of the three Gaussians, σ_{core} , σ_{mid} , and σ_{tail} are their widths, f_{core} and f_{mid} are the signal fractions of the two narrow Gaussians.	195
11.4	Results of fit to the $m(D^0)$ distributions for selected 2011 and 2012 data for the $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ analysis. $m(D)_{0_{mid}}$, and $m(D)_{0_{tail}}$ are the peak position of the two Gaussians, σ_{mid} , and σ_{tail} are their widths, f_{mid} is the signal fractions of the narrow Gaussian.	196
11.5	Results of fit to the Δm distributions for the selected $D^0 \rightarrow K^-\pi^+\pi^-\pi^+$ events shown in Fig. 11.10. $\Delta m_{0_{core}}$, $\Delta m_{0_{mid}}$, and $\Delta m_{0_{tail}}$ are the peak position of the three Gaussians, σ_{core} , σ_{mid} , and σ_{tail} are their widths, f_{core} and f_{mid} are the signal fractions of the two narrow Gaussians.	201
11.6	List of CLEO-c model resonances and their parameters used to generate $D^0 \rightarrow \pi^-\pi^+\pi^-\pi^+$ simulation samples.	203
11.7	Fit parameters in the slowest pion momentum distribution for all events and for π^+ slowest and π^- slowest from full simulation. . .	204
11.8	T values and p -value for $D^0 \rightarrow \pi^-\pi^+\pi^+\pi^-$ decays simulation samples with phase and amplitude asymmetries in $D^0 \rightarrow (a_1 \rightarrow \rho\pi)\pi$	206
11.9	T values and p -value for $D^0 \rightarrow \pi^-\pi^+\pi^+\pi^-$ decays simulation samples with phase and amplitude asymmetries in $D \rightarrow \rho\rho(D - wave)$	206
11.10	Summary of T values obtained from toy simulation samples of $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ decays generated with a CP violation of $+3^\circ$ applied on the $a_1^+ \rightarrow \rho^0\pi^+$ resonance for a range of values of the metric parameter σ	211
11.11	T values and p -values for $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ decay simulation samples with phase and amplitude asymmetries in $D^0 \rightarrow \rho\rho (P - wave)$. Results from both P -even (default) and P -odd tests are listed. Yellow regions are scenarios where some level of sensitivity can be anticipated.	215
11.12	Results from applying energy test for the $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ sideband sub-samples. A graphical representation of p -values is given in Fig. 11.27	223
11.13	Results from applying energy test for the reference channel sub-samples A graphical representation of p -values is given in Fig. 11.30.	230
11.14	Results for the P -even and P -odd CP violation tests for three different values of the effective radius δ . The p -values obtained with both the counting and GEV fitting methods are given (see text). The counting method is the default method.	234

H.1	Summary of T values obtained from toy simulation samples generated with a CP violation of 5% applied on the $a_1^+ \rightarrow \rho^0 \pi^+$ for a range of values of the metric parameter σ	281
H.2	Summary of T values obtained from toy simulation samples generated with a CP violation of 4° applied on the $D^0 \rightarrow \rho\rho$ resonance for a range of values of the metric parameter σ	281
H.3	Summary of T values obtained from toy simulation samples generated with a CP violation of 5% applied on the $D^0 \rightarrow \rho\rho$ resonance for a range of values of the metric parameter σ	283

1 Abstract

This thesis presents two searches for direct charge-parity (CP) violation in multi-body decays in the charm-sector at LHCb, the development of techniques for performing model-independent searches for direct CP violation in multi-body decays, and the development of studies of radiation damage effects in the LHCb vertex detector.

LHCb is a dedicated experiment to study New Physics in the decays of heavy hadrons at the Large Hadron Collider (LHC) at CERN. The detector includes a high precision vertex detector surrounding the pp interaction region made with silicon strip sensors. Studies of the effects of radiation damage in LHC run-2 for the operation of this detector are presented and the determination of the operational bias voltages of the silicon strip sensors is discussed.

An unbinned model independent technique for CP violation searches in multi-body decays called the energy test is used for the first time. The selection and treatment of the coordinates used to describe the phase-space of the decay are discussed. The conventional model-independent techniques, which test for parity-even CP violation, are modified to create a new approach for testing for parity-odd CP violation.

The first applications of the energy test in CP violation searches are made with $D^0 \rightarrow \pi^- \pi^+ \pi^0$ and $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decays, using data collected by LHCb. The world's best sensitivity to CP violation in both decays is obtained. The first search uses the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ decay data collected in 2012 at a proton-proton centre-of-mass energy of $\sqrt{s} = 8$ TeV corresponding to an integrated luminosity of 2.0 fb^{-1} . The data are found to be consistent with the hypothesis of CP symmetry with a p -value of $(2.6 \pm 0.5)\%$. This is the first CP violation search at LHCb using the π^0 . The second search uses $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decay data from proton-proton collisions collected in 2011 and 2012 at centre-of-mass energies of $\sqrt{s} = 7$ TeV and 8 TeV, corresponding to an integrated luminosity of 3.0 fb^{-1} . The $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ study also presents the first application of the new parity-odd CP violation test. The data are found to be consistent with the hypothesis of CP symmetry with a p -value of $(4.6 \pm 0.5)\%$ in the parity-even case, and marginally consistent with a p -value of $(0.6 \pm 0.2)\%$ in the parity-odd case, corresponding to a significance for CP non-conservation of 2.7 standard deviations.

2 Declaration

This work represents the combined efforts of the LHCb collaboration. Some of the content has been published elsewhere and/or presented to several audiences. No portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning.

3 Copyright

1. The author of this thesis (including any appendices and/or schedules to this thesis) owns certain copyright or related rights in it (the “Copyright”) and he has given The University of Manchester certain rights to use such Copyright, including for administrative purposes.
2. Copies of this thesis, either in full or in extracts and whether in hard or electronic copy, may be made only in accordance with the Copyright, Designs and Patents Act 1988 (as amended) and regulations issued under it or, where appropriate, in accordance with licensing agreements which the University has from time to time. This page must form part of any such copies made.
3. The ownership of certain Copyright, patents, designs, trade marks and other intellectual property (the “Intellectual Property”) and any reproductions of copyright works in the thesis, for example graphs and tables (“Reproductions”), which may be described in this thesis, may not be owned by the author and may be owned by third parties. Such Intellectual Property and Reproductions cannot and must not be made available for use without the prior written permission of the owner(s) of the relevant Intellectual Property and/or Reproductions.
4. Further information on the conditions under which disclosure, publication and commercialisation of this thesis, the Copyright and any Intellectual Property University IP Policy, in any relevant Thesis restriction declarations deposited in the University Library, The University Library’s regulations and in The University’s policy on Presentation of Theses.

4 Acknowledgement

So many people helped me during my PhD, without them I can hardly imagine how can I accomplish my thesis. I would like to take the opportunity to thank those who have made this thesis what it is today.

I would like to thank my supervisor, Chris Parkes, for advising and supporting me in many years and for inspiring me in many ways. His wisdom is like a torchlight illuminating my path to particle physics. I would like to thank my co-supervisor, Marco Gersabeck, for his great ideas and for invaluable helps. I would like to thank Jolanta Brodzicka for lots of very useful discussions on physics, and Jon Harrison for leading me to the area of detector analyses.

Thanks to Paula Collins, for advising me on VELO radiation damage analyses. Thanks to Kazu Akiba, for training me to be a VELO operational expert. Thanks to Kurt Rinnert, Alexander Leflat, Karol Hennessy, Eddy Jans, and Cameron Dean for helping me on VELO operation. Thanks to David Hutchcroft for useful discussions on VELO efficiency analysis.

Thanks to Sam Harnew for discussions on coordinates in high-dimensional phase space, Thanks to Eva Gersabeck and Philippe d'Argent for sharing their work on physics models. Thanks to LHCb charm working group convenors for arranging discussions, and to people in charm working group and in LHCb collaboration who advised me on my analyses.

The time I have spent in Manchester have been an great experience, and I would like to thank everyone I shared an office with at Manchester, Alex, Giulio, Suzanne, Kevin, Dominik, Chris, Igor, Lorenzo and Gedas, and other members in Manchester LHCb groups, George, Silvia, Will, for making my time in the group such joyful.

Thanks also goes to the members in LHCb Chinese group, Especially Yanxi, Liupan, Yiming, Zhenwei, Liming, Yuanning, Xuhao, Wenbin, Jiesheng, Jibo, and Xiaorui, for making my life in Geneva much enjoyable. A special thanks to Xuesong, for sharing so many pleasant experiences with me.

Finally, last but by no means least, my parents and family also deserve a great deal of credit for their supports during my PhD. Thanks.

5 Introduction

Particle physics studies fundamental particles and the forces that interact among them. The aim of particle physics is to answer a series of questions related to understanding our universe. One fundamental question about the universe is why it is made of matter rather than anti-matter. According to the Big Bang theory, at the start of the universe, matter and anti-matter were created simultaneously and equally from pure energy. During the evolution of the universe, matter and anti-matter were mostly annihilated with each other. However, a small amount of matter was left from this annihilation process, and this makes up the matter in our universe today. This fact indicates that the rate of matter turning to anti-matter slightly differs from the opposite process, *i.e.* the charge-parity symmetry between matter and anti-matter is slightly broken.

The standard model of particle physics is a theory that tries to explain all the particle physics phenomena that are observed by particle experiments. It has achieved great success in many areas. However, the amount of charge-parity asymmetry that is predicted by the standard model is not enough to explain the amount observed in the universe, therefore new physics that contributes to the charge-parity asymmetry is expected.

Many aspects of particle physics phenomena can be studied with particle colliders, which can create particle collisions at extreme high energy and partially reproduce the situation at the early stage of the universe. The Large Hadron Collider is the highest energy particle collider in the current world, and it hosts four major particle physics experiments on it. At its full operational power, tens of millions of collisions happen per second at each Large Hadron Collider experiment point. This makes the Large Hadron Collider a powerful tool for particle physics studies. The LHCb experiment is an experiment located at the Large Hadron Collider, and has a dedicated design for studying charge-parity violation by looking at the decay of particles containing b and c quarks.

Being such a powerful tool, a huge amount of radiation is expected near the interaction point at each experiment. At LHCb the system that is closest to the interaction point is the vertex locator. The monitoring of the radiation damage effect in the vertex locator is important for maintaining the performance of the detector and the data quality.

In this thesis, both radiation damage effects in the LHCb vertex locator

and searches for charge-parity asymmetries in multi-body decays are studied. Chapter 6 gives a brief overview of the standard model of particle physics, with a focus on flavour physics, and a discussion of physics of the charm-sector. The LHC and LHCb experiment are discussed in Chapter 7. Chapter 8 focusses on the properties of silicon detectors, the design of the LHCb vertex locator, and the effect of radiation damage upon it. The performance of the detector with radiation damage in run-2 and the optimisation of its operational voltage is discussed. In Chapter 9, a discussion of methods for searching for direct charge-parity violations in multi-body decays are given, with a focus on introducing a novel statistics method - the energy test method. The chapter discusses the selection of coordinates when analysing multi-body decays, and presents techniques for measuring both P -even and P -odd CP violation. Two applications of the energy test method to LHCb data are given in Chapter 10 and Chapter 11. The first presents the study of the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ decay, where π^0 s are detected both as two separate photons and as one merged cluster in the calorimeter. The second presents the study of the $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decay, and the first application of the parity-odd test yields a result only marginally consistent with CP symmetry. The final remarks of the thesis and the outlook are given in Chapter 12.

6 Theory

6.1 The Standard model

The standard model (SM) of particle physics is a gauge theory describing three (strong, weak, electromagnetic) of the four known fundamental interactions in the universe. This theory was largely based on the work of Glashow [17], Weinberg [18], and Salam [19], and it has been the cornerstone of particle physics since the mid-1970s. With the discovery of the Higgs boson in 2012 [20, 21], physicists have experimentally confirmed the existence of all elementary particles predicted by SM.

The SM includes two classes of elementary particles, fermions that make up matter, and bosons that mediate interactions between fermions. Fermions have half-integer spin and obey the Pauli exclusion principle. There are two types of fermions, quarks and leptons. Quarks have color charges, and participate in strong interactions. In the SM, there are three generations of quarks. Up-type quarks (up, charm, top) carry electric charge of $(+2/3)e$, and down type quarks carry electric charge of $(-1/3)e$. Leptons do not have color charge, and only participate in electroweak interactions. Electron, muon and tau leptons carry an electric charge of $-e$, while neutrinos are electrically neutral. Bosons have integer spin, and obey Bose-Einstein statistics. There are two types of bosons, gauge bosons and scalar boson. Gauge bosons have non-zero spin, and carry forces, they are the photon (γ , carries electromagnetic force), W and Z bosons, ($W^\pm$, Z , carry weak force), and eight types of gluons (g , carry strong force). Scalar bosons have zero spin, and the only boson of this type is the Higgs boson, which is predicted by the element of the theory explaining the origin of mass of gauge bosons via a spontaneous symmetry breaking mechanism. All elementary particles in SM are listed in Table. 6.1.

The SM is built on the $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ symmetry group transformations. These local gauge invariances generate strong, weak, and electromagnetic interactions via exchanging gauge bosons between the elementary fermions. Each of the gauge groups will be described in the following sections.

Table 6.1: List of elementary particles in the standard model.

	Three generations of fermions			Bosons
	I	II	III	
Quarks	$\begin{pmatrix} u \\ d \end{pmatrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}$	$\begin{pmatrix} t \\ b \end{pmatrix}$	$\gamma, W^\pm, Z, g$
Leptons	$\begin{pmatrix} e \\ \nu_e \end{pmatrix}$	$\begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}$	$\begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$	H

6.1.1 Electroweak interactions

The unified electroweak interactions are governed by the groups $SU(2)_L \otimes U(1)_Y$. $SU(2)_L$ is a non-abelian group with weak isospin, which couples only to left-handed fermions. The subscript L indicate the distinction between left- and right-handed fermions. $U(1)_Y$ is the group of weak hypercharge, and weak hypercharge Y_W is defined by $Y_W = 2(Q - T_3)$, where Q is the electric charge, T_3 is the weak isospin. The electric charge and weak isospin for the different types of fermions can be found in Table. 6.2

Table 6.2: The electric charge (Q) and weak isospin (T_3^L) for different types of fermions (f) and anti-fermions ($\bar{f}$). The subscripts L and R denote left- and right-handed fermions.

Particle	Q_f	$T_{3,L}^f$	$T_{3,R}^f$	$Q_{\bar{f}}$	$T_{3,L}^{\bar{f}}$	$T_{3,R}^{\bar{f}}$
u, c, t	$+2/3$	$+1/2$	0	$-2/3$	0	$-1/2$
d, s, b	$-1/3$	$-1/2$	0	$+1/3$	0	$+1/2$
e^-, μ^-, τ^-	-1	$-1/2$	0	$+1$	0	$+1/2$
ν_e, ν_μ, ν_τ	0	$+1/2$	0	0	0	$-1/2$

The gauge bosons associated to the $SU(2)_L$ group are $W^0/W^\pm$, and the gauge boson for $U(1)_Y$ group is B . The γ and Z bosons are the mass eigenstates of mixed W^0 and B bosons:

$$\begin{pmatrix} \gamma \\ Z \end{pmatrix} = \begin{pmatrix} \cos \theta_W & \sin \theta_W \\ -\sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} B \\ W^0 \end{pmatrix}, \quad (6.1)$$

where θ_W is the weak mixing angle that is given by the couplings of the $SU(2)_L$ (h) and $U(1)_Y$ (g') gauge groups:

$$\cos \theta_W = \frac{g}{(g^2 + g'^2)^{1/2}}, \quad \sin \theta_W = \frac{g'}{(g^2 + g'^2)^{1/2}}. \quad (6.2)$$

This angle also gives the relationship between the masses of the $W^\pm$ and Z bosons,

$$m_Z = \frac{m_W}{\cos \theta_W}. \quad (6.3)$$

6.1.2 Strong interactions

The strong interaction is the mechanism responsible for the strong nuclear force. Quantum chromodynamics (QCD) is the gauge theory that governs the strong interactions among gluons and colored quarks. QCD is described by the non-abelian $SU(3)_C$ gauge group in the SM.

The Lagrangian for QCD is

$$\mathcal{L}_{QCD} = \bar{\psi}_i(i(\gamma^\mu D_\mu)_{ij} - m\delta_{ij})\psi_j - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu}, \quad (6.4)$$

with the gauge invariant gluon field strength tensor

$$G_{\mu\nu}^a(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + gf_{abc}A_\mu^b(x)A_\nu^c(x), \quad (6.5)$$

and the covariant derivative

$$D^\mu = \partial^\mu - igA^\mu(x), \quad (6.6)$$

the Lagrangian is invariant under the transformation of $SU(3)_C$. The calculation of the Lagrangian shows the effective potential between a quark and its anti-quark in a meson contains a term proportional to the distance between them, which leads to the effect of color confinement [22] and asymptotic freedom [23, 24].

6.2 CP violation in heavy flavour physics

Despite the remarkable success of the SM, it still leaves some unexplained phenomena. One important fact is the amount of baryons in the universe predicted by the SM (in the CKM mechanism which is discussed below) is several orders of magnitude less than what is observed by the astronomers [25]. At the start of universe, equal amounts of matter and anti-matter are expected to have been produced, and during the inflation of the universe, matter and anti-matter cooled down and annihilated with each other. Universe is matter dominated today, which mean the symmetry between the process of matter turning into anti-matter and its opposite process, *i.e.* charge-parity (CP) symmetry, is broken. The SM allows CP violation, but the CP violation predicted by the SM is too small to explain the amount of baryons in universe, hence the SM is incomplete and there must be other sources of CP violation. Therefore, CP violation searches can be method to probe for new physics.

6.2.1 Symmetries in particle physics

In particle physics, every differentiable symmetry of the action of a physical system has a corresponding conservation law, this is known as the Noether's theorem. For a certain quantum mechanism operator O , a system is said to

have the symmetry corresponds to O if the Hamiltonian $\mathcal{H}$ is invariant under a O transformation.

$$\mathcal{H}_O \equiv O^\dagger \mathcal{H} O = \mathcal{H}. \quad (6.7)$$

There are four main types of symmetry in particle physics:

- permutation symmetry, *e.g.* indistinguishability of identical particles;
- continuous space-time symmetries, *e.g.* translation, rotation and acceleration;
- discrete symmetries, *e.g.* spatial inversion, time inversion, and charge inversion;
- unitary symmetries (gauge invariances), *e.g.* U_1 (charges), SU_2 (isospin), and SU_3 (color).

If an absolute quantity is fundamentally non-observable, then this quantity is related to an exact symmetry. Otherwise if the quantity can in principle be observed, the symmetry is broken. Examples of the relations of non-observables, symmetry transformations, and conservation laws are listed in Table 6.3

The three discrete symmetries, *i.e.* charge conjugation C , parity P , and time reversal T , are responsible for the explanation of the matter-antimatter asymmetry in the universe, and are described below.

Charge conjugation

The charge conjugation operator C reverses the signs of electric charge, magnetic moment, and color charge of a particle. Charge conjugation operator is an unitary operator, which has the property of $C^2 = 1$. The eigenstate of a charge conjugated wave function ψ is either even ($C = +1$), or odd ($C = -1$). The laws of electromagnetism are invariant under C operation, *i.e.* if in a system, charge q is replaced by $-q$, and the directions of electric and magnetic fields are reversed, the dynamics of the system remain the same.

Fermions would change their charges under C operation, therefore fermions are not eigenstates of C . However, a combination of a fermion and its anti-particle can be an eigenstate of C , and has the relation

$$C_{f\bar{f}} = (-1)^{L+S}, \quad (6.8)$$

where L is the orbital angular momentum and S is the sum of the spins, which in this case can be either 0 ($\frac{1}{2} - \frac{1}{2}$) or 1 ($\frac{1}{2} + \frac{1}{2}$), $L + S = J$ is the total angular momentum. For example, the J/ψ particle contains a c quark and a $\bar{c}$ quark, its total angular momentum is 1, therefore $C_{J/\psi} = -1$.

Table 6.3: Examples of the relations of non-observables, symmetry transformations, and conservation laws.

Non-observables	Symmetry transformation	Conservation laws
Difference between identical particles	Permutation	Bose-Einstein or Fermi-Dirac Statistics
Absolute spatial position	Space translation	Momentum
Absolute spatial direction	Rotation	Angular momentum
Absolute time	Time translation	Energy
Absolute velocity	Lorentz transformation	Lorentz group generators
Absolute left or absolute right	$\vec{r} \rightarrow -\vec{r}$	Parity
Absolute sign of charge	$e^- \rightarrow e^+$	Charge conjugation
Absolute direction of time	$t \rightarrow -t$	Time reversal

Parity

The parity operation P performs a spatial inversion through the origin:

$$P |\psi(\vec{r})\rangle = |\psi(-\vec{r})\rangle. \quad (6.9)$$

Like the charge conjugation operator, the parity operator is also a unitary operator, and satisfies $P^2 = 1$. The eigenstate of parity of the wave function $\psi(\vec{r})$ is either even, $P = +1$, or odd, $P = -1$. For example, the wave function $\psi(x) = \cos kx$ is P -even and $\psi(x) = \sin kx$ is P -odd. Another example is the wave function of an electron in an Hydrogen atom, which can be expressed in terms of spherical harmonics,

$$\Psi(r, \theta, \phi) = R(r)Y_L^m(\theta, \phi), \quad (6.10)$$

where R are radial functions and L, m in the spherical harmonics are the orbital angular momentum of the state and its projection along an axis. Due to the properties of Y_L^m , the wave functions has the parity $P = (-1)^L$.

The parity of a fundamental particle is called the intrinsic parity. The intrinsic parity of fermions is even ($P = +1$) and of anti-fermions is odd ($P = -1$). The parity of a system that contains many particles is the product of the intrinsic parities of the particles multiplied by the parity of the spatial wave function $(-1)^L$.

Electromagnetic interactions are invariant under charge conjugation and parity, therefore the C and P quantum numbers of a particle can be defined. For example, pseudo-scalar mesons have $J^{P(C)} = 0^{-(+)}$, and vector mesons have $J^{P(C)} = 1^{-(-)}$.

Combined CP transformation

The CP transformation combines charge conjugation C and parity P . Under C transformation, all quantum numbers are conjugated, hence particle and anti-particles are interchanged. Under P transformation, the handedness of space is reversed. Under the combined CP transformation, a left-handed fermion is transformed under into a right-handed anti-fermion. The laws of physics would be exactly the same if CP were an exact symmetry.

Time reversal

The time reversal operator T reverses the arrow of time, and it reverses all time dependent quantities, *e.g.* momentum and spin. Time reversal operator is an anti-unitary operator, and obeys $T^2 = -1$.

6.2.2 CP violation and CKM matrix

Although most interactions are invariant under C , P , and T symmetry operations, a theoretical prediction by Yang and Lee [26] and experimental

measurement performed by Wu [27] in mid 1950s proved that the weak interaction violates P symmetry. As argued by Lee, Oehme and Yang [28], non-conservation of P directly leads to the violation of C , which was experimentally proved by Lederman in 1956 [29]. In 1964, the combined CP symmetry was found to be broken in the decays of the neutral Kaon system [30]. The discovery of CP violation led to the development of theories of flavour changing interactions. In 1970, Glashow, Iliopoulos, and Maiani published a model for weak interactions with a lepton-hadron symmetry [31]. This model is known as the GIM mechanism, and it predicted the existence of the fourth quark, which was discovered four years later [32, 33].

The GIM mechanism introduces the weak interaction matrix, which describes the mixed quark states as a ‘rotation’ of flavour (or weak) eigenstates (d', s') w.r.t. mass (or strong) eigenstates (d, s) by θ_c ,

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos \theta_c & \sin \theta_c \\ -\sin \theta_c & \cos \theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}. \quad (6.11)$$

In 1973, even before the confirmation of the existence of the fourth quark, based on the idea of GIM mechanism, Kobayashi and Maskawa predicted a new doublet of quarks, and expressed the couplings between up- and down-type quarks as a ‘3D rotation’ by introducing the Cabibbo-Kobayashi-Maskawa (CKM) matrix [34].

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \text{ with } V_{CKM}^\dagger V_{CKM} = 1. \quad (6.12)$$

The matrix elements of the CKM matrix V_{ij} give the coupling strength of the weak flavour-changing charged current mediated by $W^\pm$ bosons. As the CKM matrix elements are connected to the probability of transitions, the matrix has to be unitary, which gives the relation:

$$\sum_j V_{ij} V_{jk}^* = \delta_{ik}. \quad (6.13)$$

Imposing the constraints of unitarity, and ignoring the unobservable phases, there are four free parameters for a 3×3 matrix. Therefore, the CKM matrix can be parametrised by three rotation angles θ_{12} , θ_{13} , θ_{23} , and a CP -violating phase δ , as shown in Eq. 6.14,

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (6.14)$$

here the notations $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ are used.

It is known experimentally that the three rotation angles are found to have a certain hierarchy, $s_{13} < s_{23} < s_{12} < 1$. To demonstrate the hierarchy, the CKM matrix can also be expressed in the Wolfenstein parametrisation as a series expansion with three real parameters and one imaginary one. The fifth order of the CKM matrix in the Wolfenstein parametrisation is

$$V_{CKM(5)} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda + A\lambda^5(\frac{1}{2} - \rho - i\eta) & 1 - \frac{\lambda^2}{2} - \frac{\lambda^4}{8}(1 + 4A^2) & A\lambda^2 \\ A\lambda^3(1 - (1 - \frac{\lambda^2}{2})(\bar{\rho} + i\bar{\eta})) & -A\lambda^2 + A\lambda^4(\frac{1}{2} - \rho - i\eta) & 1 - \frac{1}{2}A^2\lambda^4 \end{pmatrix}, \quad (6.15)$$

where $\lambda = s_{12} = \sin \theta_C \approx 0.22$, θ_C is the Cabibbo angle of the 2×2 Cabibbo mixing matrix, $A = s_{23}/s_{12}^2$, $\rho = s_{13} \cos \delta / s_{12}s_{23}$, and $\eta = s_{13} \sin \delta / s_{12}s_{23}$.

The magnitudes of CKM matrix elements can be determined using a global fit to all available measurements, the current best fit [1] is

$$|V_{CKM}| = \begin{pmatrix} 0.97434^{+0.00011}_{-0.00012} & 0.22506 \pm 0.00050 & 0.00357 \pm 0.00015 \\ 0.22492 \pm 0.00050 & 0.97351 \pm 0.00013 & 0.0411 \pm 0.0013 \\ 0.00875^{+0.00032}_{-0.00033} & 0.0403 \pm 0.0013 & 0.99915 \pm 0.00005 \end{pmatrix}. \quad (6.16)$$

The imaginary component of the CKM matrix is responsible for CP violation and is discussed further below.

6.2.3 Neutral meson mixing

A neutral meson can transit to its CP conjugate state and vice versa, this process is known as neutral meson mixing. Mixing can happen via exchange of intermediate quarks (box diagram, Fig. 6.1), and this contribution is dominant in the mixing in the Kaon [30] and B [35, 36] sectors.

For the D^0 meson, transitions through the equivalent box diagrams contribute with a factor of $V_{ci}^* V_{ui}$, where i is d , s , or b . The mass difference between d and s quarks is relatively small, hence transitions via d or s quarks is GIM suppressed. The factor $V_{cb}^* V_{ub}$ is $\propto \lambda^5$, which is negligible, thus transitions via a b quark is CKM suppressed. Therefore, the contribution of the box diagram to D^0 meson mixing is small.

Another contribution to the mixing between neutral mesons is long range hadronic interactions (Fig. 6.2). This is the dominating contribution to D^0 meson mixing, for example D^0 mesons can decay via a singly-Cabibbo-suppressed transition to the CP eigenstates $K^+ K^-$ or $\pi^+ \pi^-$, then a $\bar{D}^0$ meson is recreated through the recombination of $K^+ K^-$ or $\pi^+ \pi^-$.

For a neutral meson P^0 , at time $t = 0$, the mass eigenstate is a superposition of flavour eigenstates P^0 and $\bar{P}^0$,

$$|\psi(0)\rangle = a(0) |P^0\rangle + b(0) |\bar{P}^0\rangle, \quad (6.17)$$

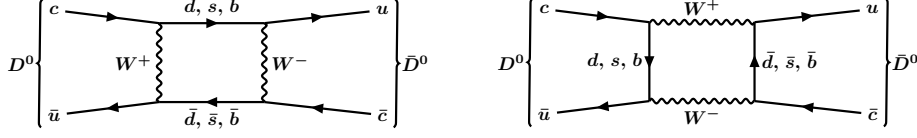


Figure 6.1: Box diagrams for $D^0 - \bar{D}^0$ transitions via exchanging intermediated quarks.

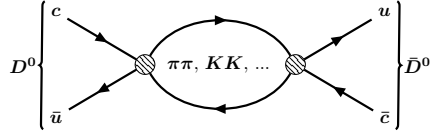


Figure 6.2: Diagrams for $D^0 - \bar{D}^0$ transitions via long range hadronic interactions.

which can evolve in time as the factors a and b are time dependent

$$|\psi(t)\rangle = a(t) |P^0\rangle + b(t) |\bar{P}^0\rangle + c_1(t) |f_1\rangle + c_2(t) |f_2\rangle + \dots \quad (6.18)$$

The wave function can be written as

$$|\psi\rangle = \begin{pmatrix} |a(t)\rangle \\ |b(t)\rangle \end{pmatrix}, \quad (6.19)$$

hence the time dependent Schrödinger equation can be defined as

$$i \frac{\partial \Psi}{\partial t} |\psi\rangle = \mathcal{H} |\psi\rangle = (\mathbf{M} - \frac{i}{2} \mathbf{\Gamma}) |\psi\rangle, \quad (6.20)$$

where Hamiltonian $\mathcal{H}$, Hermitian mass matrix $\mathbf{M}$ and Hermitian decay width matrix $\mathbf{\Gamma}$ are 2×2 matrices

$$\mathcal{H} = \begin{pmatrix} M & M_{12} \\ M_{12}^* & M \end{pmatrix} + \begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{pmatrix}. \quad (6.21)$$

By diagonalising the Hamiltonian, the mass eigenstates of neutral mesons can be expressed as linear combination of the two flavour eigenstates.

$$|P_1\rangle = p|P^0\rangle + q|\bar{P}^0\rangle, \quad |P_2\rangle = p|P^0\rangle - q|\bar{P}^0\rangle. \quad (6.22)$$

where p and q are complex parameters which satisfying $|p|^2 + |q|^2 = 1$. In general, the flavour eigenstates $|P^0\rangle$ and $|\bar{P}^0\rangle$ do not coincide with mass eigenstates $|P_1\rangle$ and $|P_2\rangle$. The two mass eigenstates could have different mass m_1 and m_2 and decay with Γ_1 and Γ_2 . The ratio of p and q is

$$\frac{p}{q} = \sqrt{\frac{\mathcal{H}_{12}}{\mathcal{H}_{21}}} = \sqrt{\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}}. \quad (6.23)$$

If $p = q = \frac{1}{\sqrt{2}}$, the mesons P_1 and P_2 are pure CP eigenstates.

The time evolution of the eigenstates can be written as

$$|P_1(t)\rangle = |P_1\rangle e^{-i(m_1 - \frac{i}{2}\Gamma_1)t}, \quad |P_2(t)\rangle = |P_2\rangle e^{-i(m_2 - \frac{i}{2}\Gamma_2)t}. \quad (6.24)$$

6.2.4 Types of CP violations

Given the Hamiltonian $\mathcal{H}$ governing the weak interactions, the decay amplitudes of a meson P and its CP conjugate particle $\bar{P}$ decay to a final state f and its CP conjugate state $\bar{f}$ are defined as

$$\begin{aligned} \mathcal{A}_f &= \langle f|\mathcal{H}|P\rangle, & \bar{\mathcal{A}}_f &= \langle f|\mathcal{H}|\bar{P}\rangle, \\ \mathcal{A}_{\bar{f}} &= \langle \bar{f}|\mathcal{H}|P\rangle, & \bar{\mathcal{A}}_{\bar{f}} &= \langle \bar{f}|\mathcal{H}|\bar{P}\rangle. \end{aligned} \quad (6.25)$$

There are three types of CP violation, CP violation in decay, CP violation in mixing, and CP violation in the interference between mixing and decay.

CP violation in decay

The CP violation in decay, or direct CP violation, occurs if the physical amplitude for the transition $P \rightarrow f$ differ from $\bar{P} \rightarrow \bar{f}$,

$$\frac{|\mathcal{A}_f|}{|\bar{\mathcal{A}}_{\bar{f}}|} \neq 1. \quad (6.26)$$

For charged mesons and baryons, this is the only type of CP violation that can occur.

CP violation in mixing

For neutral mesons, CP violation can occur not only in decay, but also in mixing. CP violation occurs if

$$\frac{|p|}{|q|} \neq 1. \quad (6.27)$$

CP violation in interference between decay and mixing

If neutral meson decays to CP eigenstates, *i.e.* the final states are common for both P^0 and $\bar{P}^0$ decays, CP violation can also happen in the interference between a decay without mixing, $P \rightarrow f$, and a decay with mixing, $P \rightarrow \bar{P} \rightarrow f$. We define

$$\lambda_f = \frac{q}{p} \frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f}, \quad (6.28)$$

then CP violation in the interference occurs if

$$\text{Im}(\lambda_f) \neq 0. \quad (6.29)$$

CP violation in both mixing and the interference between decay and mixing are called indirect CP violation.

6.2.5 CP violation in multi-body decays

The high branching fractions of two-body decays make them a good place to search for CP violation. However, the study of CP violation in a two-body decay is limited to the measurement of a single asymmetry number. In multi-body decays particles can decay via intermediate states called resonances [25]. These resonance particles can decay strongly and have very short lifetimes, typically on the order of 10^{-23} seconds. The width of the resonance, Γ , is related to its mean lifetime by

$$\Gamma = \frac{\hbar}{\tau},$$

where $\hbar$ is the reduced Planck constant.

Consider a two-body pseudo-scalar decay, a mother particle X decays to two daughter particles A and B , where P has the mass M and A B have masses m_1 m_2 respectively, then in the P rest frame, the energy and absolute value of momentum of the daughter particles are

$$E_{1(2)} = \frac{M^2 - m_{2(1)}^2 + m_{1(2)}^2}{2M}, \quad (6.30)$$

$$|\mathbf{p}_1| = |\mathbf{p}_2| = \frac{((M^2 - (m_1 + m_2)^2)(M^2 - (m_1 - m_2)^2))^{1/2}}{2M}. \quad (6.31)$$

These values are determined with no free parameters, therefore no phase space can be defined. The partial decay rate can be defined as

$$d\Gamma = \frac{1}{32\pi^2} |\mathcal{M}|^2 \frac{\mathbf{p}_1}{M^2} d\Omega. \quad (6.32)$$

where $\mathcal{M}$ is the Lorentz-invariant matrix element, which contains dynamic factors of the decay process. The quantity $d\Omega = d\phi_1 d(\cos \theta_1)$ is the solid angle of A .

Consider a three-body pseudo scalar decay, $X \rightarrow ABC$, define the invariant masses of mother and daughter particles to be M and m_1, m_2, m_3 , the momentum to be P and p_1, p_2, p_3 , if the momentum of a pair of daughter particles is defined as $p_{ij} = p_i + p_j$, the invariant mass has the form $m_{ij}^2 = p_{ij}^2$, then the relation between invariant masses can have the form

$$m_{12}^2 + m_{23}^2 + m_{13}^2 = M^2 + m_1^2 + m_2^2 + m_3^2, \quad (6.33)$$

and

$$m_{12} = (P - p_3)^2 = M^2 + m_3^2 - 2ME_3 \quad (6.34)$$

where E_3 is the energy of particle C in the X rest frame. In this frame, the momenta of the three daughter particles lies in a plane, which requires two free parameters to describe the kinematic phase space. The decay rate of such a decay can be written as

$$d\Gamma = \frac{1}{(2\pi)^3 \times 32M^3} |\mathcal{M}|^2 dm_{12}^2 dm_{23}^2. \quad (6.35)$$

A natural choice of the parameters to describe the phase space is to use m_{12}^2 and m_{23}^2 . The scatter plot of m_{12}^2 versus m_{23}^2 can fully describe the decay kinematics, and is known as a Dalitz plot. The presence of resonances in the decay can enhance the decay cross-section in certain regions of phase space, therefore resonant structures can be observed on the Dalitz plot.

A common approach to describe resonance amplitudes in multi-body decays is the isobar model. In this the total amplitude is the sum of a set of contributing amplitudes each with a complex coefficient

$$\mathcal{A}(\vec{x}) = a_{nr} e^{i\Phi_{nr}} + \sum a_r e^{i\Phi_r} \mathcal{A}_r(\vec{x}), \quad (6.36)$$

where $\vec{x}$ represents the position in the phase space, $a_r e^{i\Phi_r}$ describes the complex amplitude of each resonance, and $a_{nr} e^{i\Phi_{nr}}$ describes the non-resonant contribution. $\mathcal{A}_r(\vec{x})$ contains information about the decay dynamics.

The rich structure of interfering amplitudes enables the search for CP violation in multi-body decays to have access to the contributions from different resonances.

6.3 CP violation in charm sector

In the charm sector, the standard model CP violation, either direct or indirect, can be observed in Cabibbo-suppressed transitions. Amplitudes from different processes can interfere and generate CP asymmetries. For example, the CP violation in the decay of $D^0 \rightarrow \pi^+ \pi^-$ can occur via a singly Cabibbo-suppressed $c \rightarrow du\bar{d}$ transition with an admixture from a $c \rightarrow ug$ gluonic-penguin transition. Feynman diagrams of these transitions can be found in Fig. 6.3.

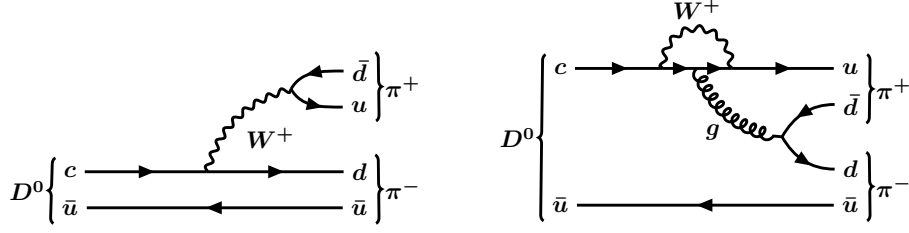


Figure 6.3: Feynman diagrams for tree level (left) and loop level (right) transitions in $D^0 \rightarrow \pi^+ \pi^-$ decay.

The CP asymmetry is defined as the difference between D and $\bar{D}$ partial widths divided by the sum of them

$$A_{CP} = \frac{\Gamma(D) - \Gamma(\bar{D})}{\Gamma(D) + \Gamma(\bar{D})}. \quad (6.37)$$

In the case of charged D meson decays, A_{CP} measures direct CP violation, and for neutral D mesons, A_{CP} measures the combination of direct and indirect CP violation.

In neutral charm meson decays, the mass eigenstates can be denoted as

$$|D_1\rangle = p |D^0\rangle + q |\bar{D}^0\rangle, \quad |D_2\rangle = p |D^0\rangle - q |\bar{D}^0\rangle. \quad (6.38)$$

in the absence of CP violation, D_1 is CP -even and D_2 is CP -odd. The weak phase ϕ is defined as $\text{Arg}(q/p)$. With the masses and decay widths for the mass eigenstates, mixing parameters x and y can be introduced,

$$x = \frac{m_1 - m_2}{\Gamma}, \quad (6.39)$$

$$y = \frac{\Gamma_1 - \Gamma_2}{2\Gamma}, \quad (6.40)$$

where $\Gamma = \frac{\Gamma_1 + \Gamma_2}{2}$ is the averaged value of decay widths.

For decays of neutral charm mesons to CP eigenstates, the direct part and indirect part of CP asymmetries [25] can be written as

$$a_{CP}^{dir} = \frac{|\mathcal{A}_{D^0 \rightarrow f}|^2 - |\mathcal{A}_{\bar{D}^0 \rightarrow f}|^2}{|\mathcal{A}_{D^0 \rightarrow f}|^2 + |\mathcal{A}_{\bar{D}^0 \rightarrow f}|^2}, \quad (6.41)$$

$$a_{CP}^{ind} = \frac{1}{2} \left[\left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right) x \sin \phi - \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right) y \cos \phi \right]. \quad (6.42)$$

6.3.1 Heavy flavour averaging group fit

The fit of the results from various direct and indirect CP violation searches in the charm sectors are shown in Fig. 6.4. The small size of the SM CP violation means that searches for CP violation in the charm sector can provide powerful constraints on the possible contributions from physics beyond the SM.

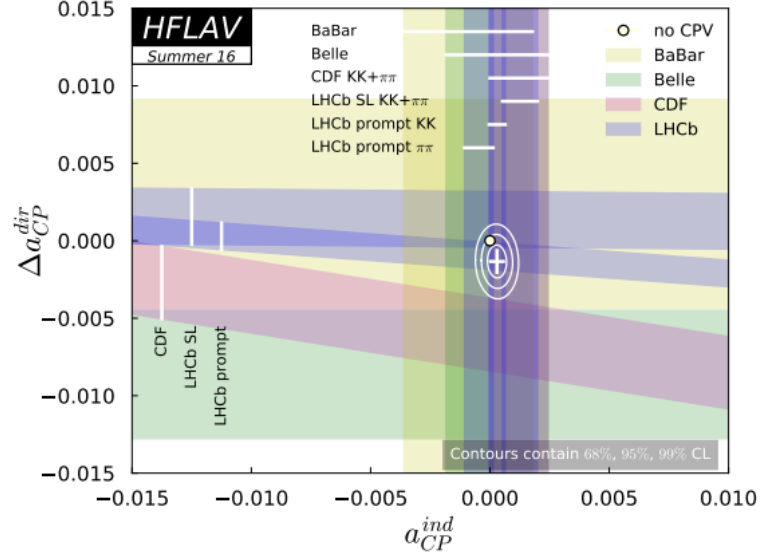


Figure 6.4: The heavy flavour averaging group (HFLAV) fit from summer 2016. This fit combines various searches for direct and indirect CP violation in the charm system. a_{CP}^{ind} is a universal indirect CP violation parameter, while Δa_{CP}^{dir} measures direct CP violation in $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow K^+K^-$ decays. Figure taken from Ref. [1]

7 The LHCb experiment

The LHCb (Large Hadron Collider beauty) experiment is one of the four major experiments at the Large Hadron Collider (LHC). The primary goal of LHCb is to measure CP violation parameters and the interactions of heavy flavour particles. In this chapter, the main features of the LHC accelerator and the subsystems of LHCb detector will be discussed.

7.1 The Large Hadron Collider

The LHC is a hadron synchrotron at the European Organisation for Nuclear Research (CERN) built on the border between France and Switzerland near the city of Geneva. It is the world's largest and highest energy collider. It acts as a proton-proton collider for most of its operational time, and also serves as a heavy ion collider for short periods. The LHC is installed in a circular tunnel of 26.7 km circumference, which is located between about 50 and 175 metres underground. The tunnel was used to host the Large Electron-Positron (LEP) accelerator before the LHC.

The LHC can be divided into eight 3.3 km long sectors, each sector is made of a curved part and one long straight section. The straight sections are called insertions. At the LHC each insertion has a unique function. The interaction points (IP) for LHC experiments are located at the centre of insertion regions. IP1, IP2, IP5, and IP8 are used for hosting particle experiments. The proton beams are accelerated by superconducting radio frequency cavities (RF) at IP4. The beams are cleaned by removing particles with large longitudinal or transversal amplitudes at IP3 and IP7. And the beam dump is located at IP6. The layout of the insertions are shown in Fig. 7.1. In the LHC, two particle beams are contained in two separate pipes throughout most of the ring, and are gathered in a single pipe around the interaction regions. The beams are guided in the tunnel by the superconducting magnets, which are cooled by liquid helium, and operate at 1.9 K. Since the LHC is a proton-proton collider the magnetic fields are oriented oppositely for the two beam pipes.

The particles in beam pipe are stored as bunches rather than a continuous particle flow. Each storage ring can have a maximum of 2808 bunches, the shortest collision period is 25 ns, bunches are separated by about 7.5 m in the

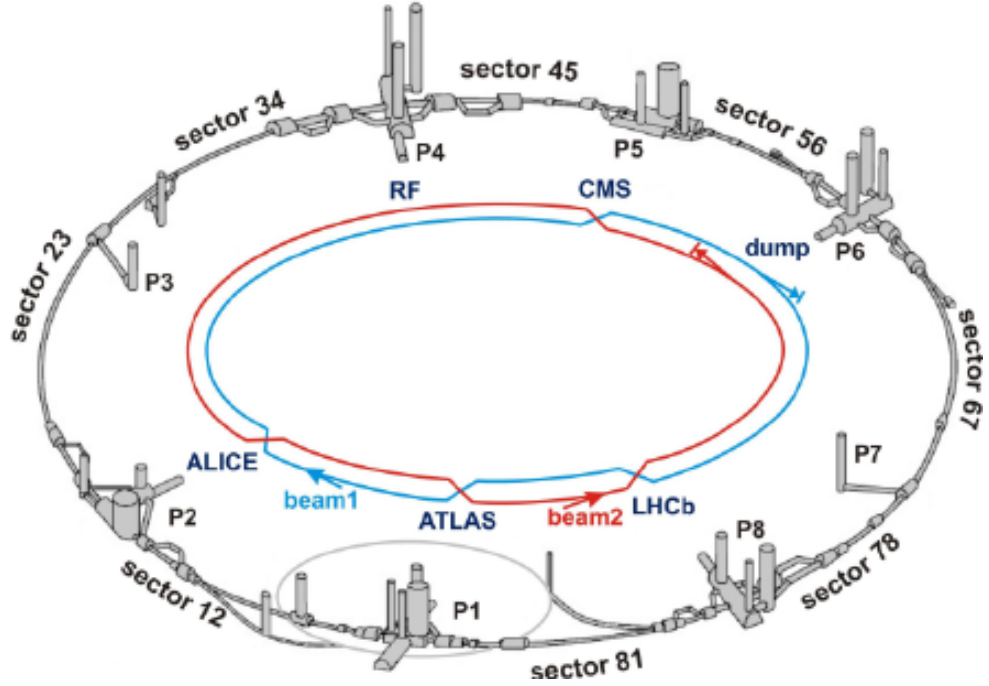


Figure 7.1: Schematic view of the LHC showing the eight interaction points and the facilities at the interaction points. Figure reproduced from [2].

beam pipe.

Before the particles are injected into the main acceleration ring of the LHC, they need to pass through a series of acceleration facilities to boost their energy step by step. The CERN accelerator complex is shown in Fig. 7.2, hydrogen atom gas is injected into the linear particle accelerator (LINAC 2), where electrons are stripped off, and protons are accelerated to 50 MeV. Protons from LINAC 2 feed the Proton Synchrotron Booster (PSB), where the energy of protons is boosted to 1.4 GeV. Then protons are injected into the Proton Synchrotron (PS), and are accelerated to 26 GeV. Finally, the Super Proton Synchronous Accelerator (SPS) boosts the proton energy to 450 GeV before they are injected to the LHC.

There are four major experiments at the LHC: ALICE, ATLAS, CMS and LHCb. ATLAS [37] and CMS [38] are general-purpose detectors, which investigate a wide range of physics, from the search for the Higgs boson to extra dimensions and particles that could make up dark matter. ATLAS is located at IP1 and CMS is located at IP5. ALICE [39] is a detector with dedicated design for studying the physics of strongly interacting matter at extreme energy densities with heavy-ion collisions, and is located at IP2. LHCb [8], which is hosted at IP8, is designed specifically to investigate the slight differences between matter and antimatter by studying the decays of hadrons containing b

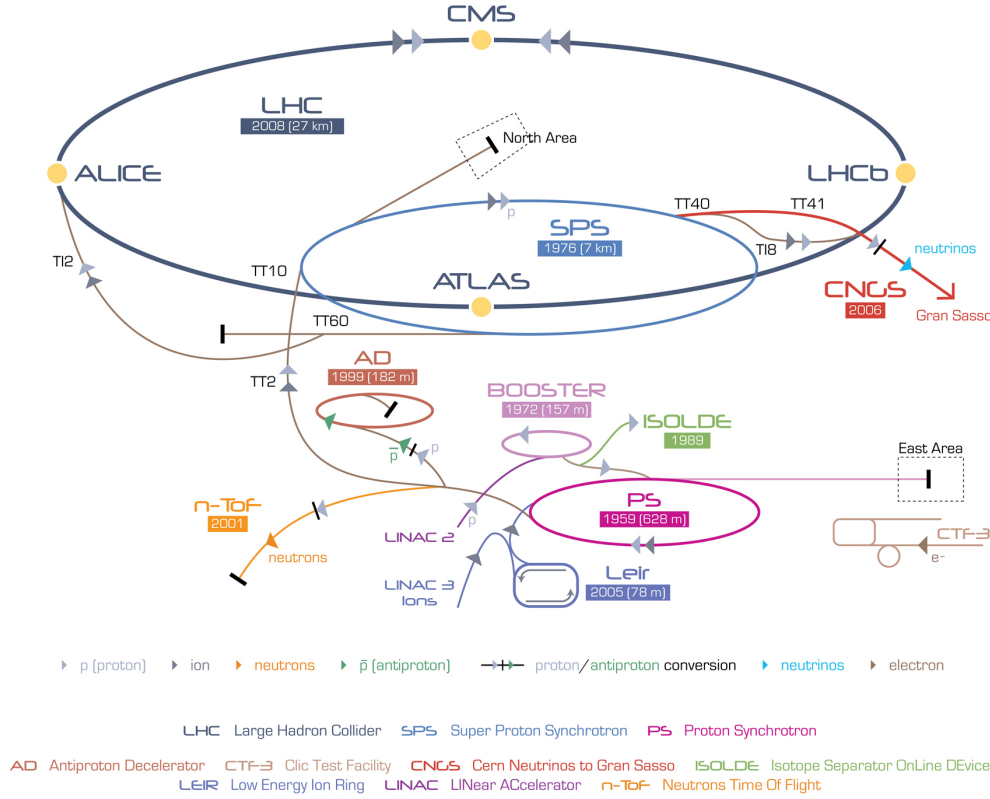


Figure 7.2: The CERN accelerator complex. The protons are injected to LHC main storage ring through LINAC 2 - PSB - PS - SPS - LHC. Figure taken from [3]

and c quarks. However, it has subsequently developed a wide ranging physics programme including electroweak physics, long-lived particle searches and heavy-ion physics by exploiting its unique features. There are three additional experiments that use the LHC collisions and share the interaction points with the major experiments. The TOTEM experiment [40], which shares the interaction point with CMS, is designed to measure the total cross section, elastic scattering, and diffractive processes. The LHCf experiment [41], which shares the interaction point with ATLAS, is used to detect particles generated in the very forward region and to simulate cosmic ray particles. The MoEDAL experiment [42] shares the interaction point with LHCb and is used for searching for magnetic monopoles.

7.1.1 Running conditions in run-1

Beams were first circulated in the LHC in 2008, and following recovery from an accident, LHC Run-1 operations started at the end of 2009. After careful

commissioning in 2010, the LHC had its major run-1 physics data taking from 2011 to 2013. In the 2011 and 2012 proton-proton collision runs, the collision energies were $\sqrt{s}=7$ and 8 TeV, respectively. In run-1, the LHC did not reach its full capacity, the bunch spacing in run-1 was 50 ns, and the maximum bunches in one beam pipe was 1380. The total recorded luminosities from proton-proton collisions in 2011 were 5.08 fb^{-1} for ATLAS, 5.55 fb^{-1} for CMS, 1.1 fb^{-1} for LHCb, and in 2012 were 21.3 fb^{-1} for ATLAS, 21.8 fb^{-1} for CMS, 2.1 fb^{-1} for LHCb.

7.1.2 Running conditions in run-2

The LHC started its run-2 operation from June 2015, and expects to continue operations until the end of 2018. In run-2 the LHC has doubled its collision frequency compared to run-1, reducing the inter-bunch time to 25 ns, and the collision energy in run-2 has reached $\sqrt{s}=13$ TeV.

7.2 The LHCb experiment

The primary physics goal of LHCb experiment is to explore the area of heavy flavour physics, notably the behaviour of hadrons containing beauty and charm quarks. The LHCb experiment has made many precise measurements of heavy flavour properties, and has made several discoveries of new particles and many new decay modes. Highlights includes the significant improvement in the precision of the determination of the measurement of the CKM matrix angle γ [43], the first single experiment 5σ observation of charm oscillations [44], the first observation of a baryon contain two charm quarks [45], the discovery of the penta-quark [46], and discovery of the $B_s \rightarrow \mu\mu$ decay [47].

LHC, as a hadron collider, produces b and c quarks mainly through a gluon-gluon fusion process. Since the mass of b and c quarks are much smaller than the collision energy, the $b\bar{b}$ and $c\bar{c}$ pairs are predominantly boosted in the lab frame and produced in the forward region (see Fig. 7.3). To accommodate this fact, LHCb is designed as a single-armed forward spectrometer. The pseudo-rapidity region covered by LHCb is $2 < \eta < 5$, where

$$\eta = -\ln \tan \frac{\theta}{2}, \quad (7.1)$$

and θ is the polar angle with respect to the beam line. A comparison of the pseudo-rapidity coverage from LHCb and other general purpose detectors is shown in Fig.7.3.

The instantaneous luminosity at collision point is defined as

$$L \approx \frac{N_1 N_2 f_{bc}}{4\pi\epsilon\beta^*}, \quad (7.2)$$

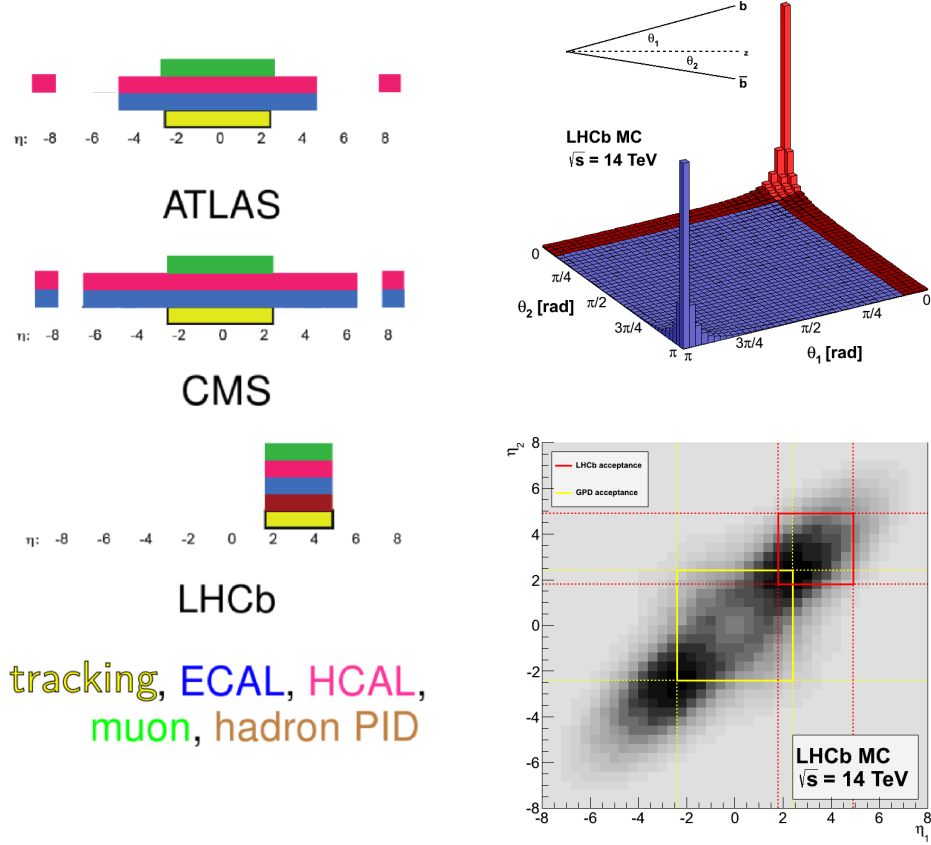


Figure 7.3: (left) The sub-system pseudo-rapidity coverage of ATLAS, CMS and LHCb experiments. (top right) The $b\bar{b}$ pair production angle at 14 TeV and the LHCb coverage (red). LHCb covers 27% of b or $\bar{b}$ quarks with a small coverage of angle. (bottom right) The population of $b\bar{b}$ pairs as function of pseudo-rapidity and the coverage of LHCb detector and other general purpose detectors.

where N_1 and N_2 are the number of particles per bunch in beam 1 and beam 2, f_{bc} is the bunch clock frequency, for 25 ns bunch spacing, $f_{bc} = 40.0789$ MHz, ϵ is the beam transverse emittance and is defined as the average spread of particle coordinates in position-and-momentum phase space and has the dimension of length. A typical value of emittance at LHC is $2.5 \mu\text{m}$. β^* is the beta function at an interaction point, and is related to the transverse size of the particle beam along the nominal beam trajectory. If β^* is low, the beam is narrower and squeezed. If β^* is high, the beam is wide and straight. ATLAS and CMS experiments utilise the maximum collision luminosity, the beams are squeezed

as much as possible, and β^* is typically about 0.4 m which will lead to many pile-up events. Typically, the instantaneous luminosity for ATLAS and CMS can reach $1 - 2 \times 10^{34} \text{cm}^{-2} \text{s}^{-1}$ for 25 ns bunch-spacing beams, and there will be about 30 to 60 collisions in each bunch crossing. The LHCb detector is designed to operate at lower luminosities than ATLAS and CMS. A typical β^* value is 3 m, which not only reduced the number of collisions per bunch crossing to between 1 and 2, but also enabled the instantaneous luminosity to be as constant as possible. Typically LHCb operates at an instantaneous luminosity of $2 - 5 \times 10^{32} \text{cm}^{-2} \text{s}^{-1}$. This is achieved by the luminosity levelling technique. A transverse offset is introduced between the two LHC beams at the LHCb interaction point, and the offset is reduced during the fill to keep the instantaneous luminosity at the desired constant value. This is illustrated in Fig. 7.4.

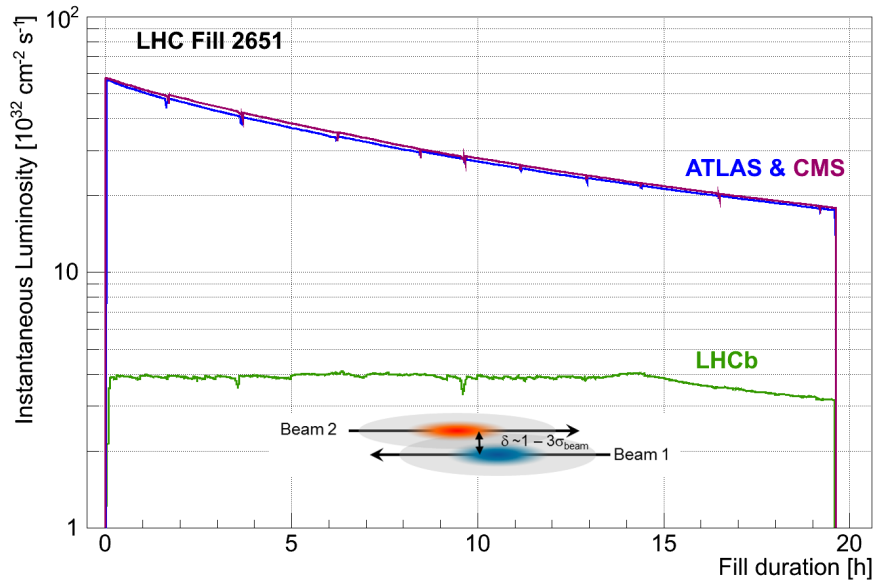


Figure 7.4: Illustration of the luminosity levelling employed at LHCb. The instantaneous luminosities for ATLAS and CMS decay exponentially, but for LHCb this is constantly levelled. The levelling is tuned off after beams cannot be kept separated any more after 14 hours of the fill. Figure taken from [4]

The LHCb detector is situated at point 8 of the LHC ring. The experimental cavern was previously occupied by the DELPHI experiment at LEP. The layout of the LHCb sub-detectors is shown in Fig. 7.5. The interaction point is located inside the vertex locator (VELO) on the left side of Fig. 7.5. Since b - and c -hadrons are highly boosted in the lab frame, and have ps level lifetimes, they

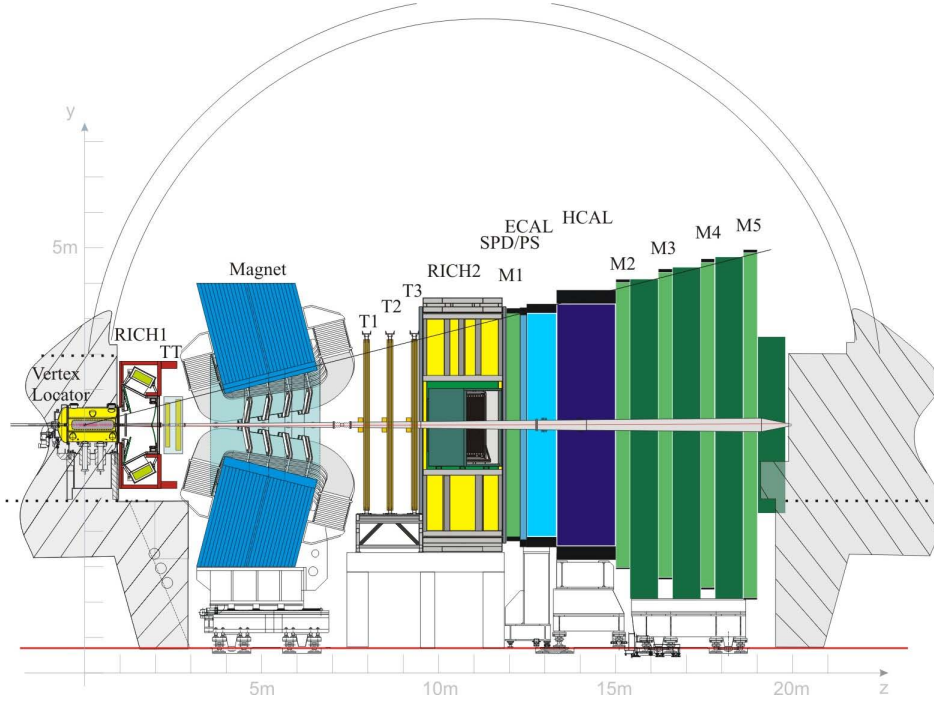


Figure 7.5: The cross-section of LHCb detector.

can fly several millimetres before decaying. At LHCb, the distance between the b - and c -hadron decay vertex and the proton-proton interaction point can be precisely measured by the VELO, which is a powerful tool that discriminates between signal and background events. After leaving the VELO, particles produced in the collisions pass through the first ring imaging Cherenkov detector (RICH) and the Tracker Turicensis (TT) before entering the magnet. The magnet bends the particle tracks, which then transverse three tracking stations (T1 - T3) and the second RICH. Tracking system (TT + tracking stations) together with the magnet provide measurements of the momenta of charged particles, while the first and second RICH provide particle identification of low-momentum and high-momentum tracks, respectively. After the tracking stations, most of the electrons, photons, and hadrons deposit their energy in electromagnetic and hadronic calorimeters. The muon system is used to identify and trigger on muons in the events. Descriptions of the sub-detectors are given in the sections below.

7.2.1 The Vertex Locator

The vertex locator is the detector that is closest to the collision point among all the LHC detectors. It consists of 42 silicon sensor modules arranged along the z -axis. The first active strip is at only 8.2 mm away from beam. The VELO

is operated in a secondary vacuum vessel, which is separated by a $300\ \mu\text{m}$ aluminum *RF*-foil from the LHC beam vacuum. The 42 silicon sensor modules are mounted on two retractable halves. To protect the detector, the VELO modules are retracted and separated during beam injection or when the beam is not stable. VELO modules are cooled by a bi-phase CO_2 cooling system, and VELO modules operate at about $-10\ ^\circ\text{C}$. A schematic view of the VELO is shown in Fig. 7.6.

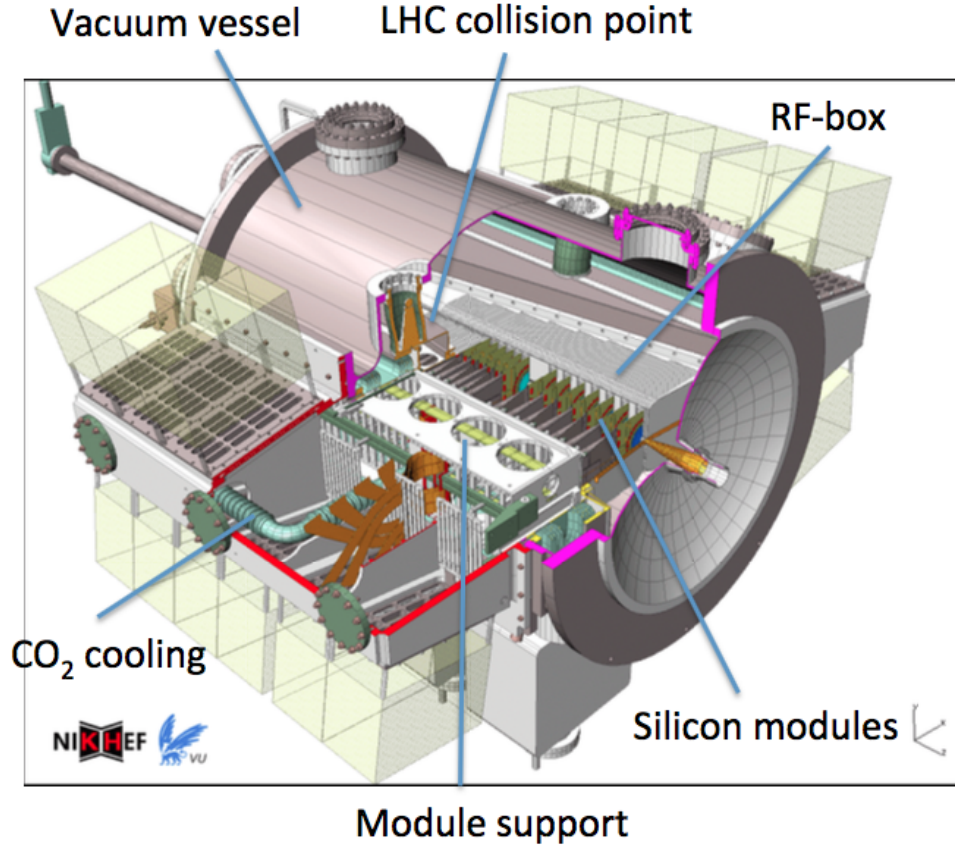


Figure 7.6: Schematic view of the VELO detector.

Each VELO module contains one *R*-type sensor which provides measurements of radial distance from the beam axis and one Φ -type sensor which measures the azimuthal coordinate around the beam. By combining the measurements from *R*- and Φ -type sensors, 3 D hit points can be determined on the VELO modules. A VELO track can be reconstructed by connecting the VELO hit points. The magnetic field inside the VELO is negligible, which enables the reconstruction of tracks inside the VELO with straight lines. In addition to the 42 VELO modules, four *R*-type sensors are placed upstream of the VELO modules. These four sensors are pile-up sensors, which serve as a veto system

to distinguish between bunch crossings with a single or with multiple visible pp interactions. The distribution of VELO modules is shown in Fig. 7.7.

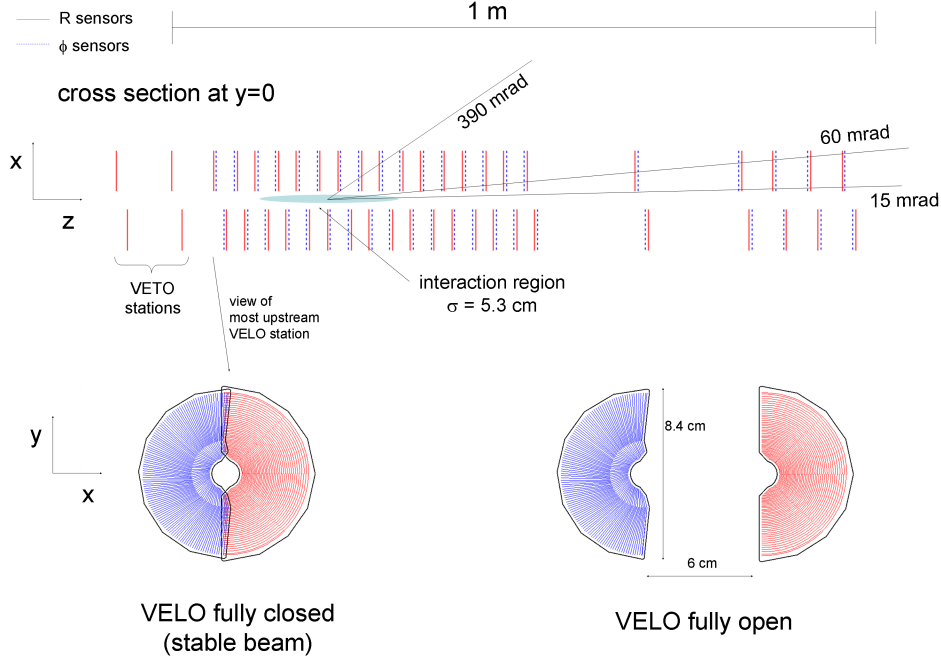


Figure 7.7: (top) Cross section in the (x, z) plane of the placement of the pile-up sensor modules and VELO modules along the LHCb beam axis. (bottom) Position of the VELO sensors before and after closing. Figures from Reference [5].

Both R - and Φ -type sensors are $300\text{ }\mu\text{m}$ thick silicon strip sensors. Among the 84 sensors, 82 of them are n^+ implants on n -doped silicon strips, and are called n^+ -on- n sensors. The R - and Φ -type sensors at the most upstream are n^+ implants on p -doped silicon strips, and are used for testing this technology as an upgrade option. There are 2048 strips on both R - and Φ -type sensors. On R -type sensors, the entire semi-circular sensor was divided into four 45° segments, sensor strips are concentric circular arcs, and routing lines that read out the signals collected by strips are perpendicular to the strips. The Φ -type sensor has two zones with inner and outer strips. The routing lines of the inner strips are orientated parallel to the outer strips. Strip pitches vary from $40\text{ }\mu\text{m}$ at the most inner region of sensors to $100\text{ }\mu\text{m}$ at the outer region of sensors. The layout of R - and Φ -type sensor strips and routing lines are shown in Fig. 7.8. The VELO ensured the excellent vertex reconstruction impact parameter (IP) resolution and decay time resolution achieved by LHCb.

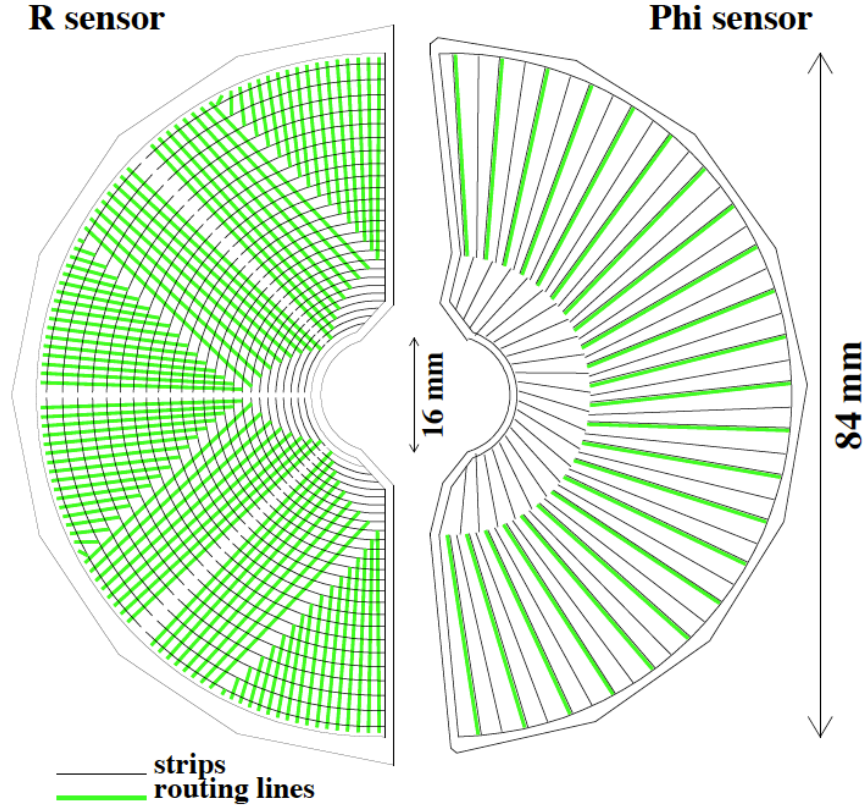


Figure 7.8: The layout of R - and Φ -type sensor strips and routing lines. Figure taken from Reference [6].

7.2.2 Tracking Stations

Together with the VELO, the tracking stations, TT and T1, T2, T3, compose the LHCb tracking system. TT and the other three tracking stations are separated by the magnet. The tracking stations are split into two projects based on their technology choices, with the TT and the Inner Tracker (IT) forming the Silicon Trackers (ST), where the IT constitutes the innermost regions of T1–T3. The Outer Tracker (OT) uses the technology of straw drift tubes, and makes up the remaining outer parts of T1–T3.

The Tracker Turicensis

The TT is a 157.2 cm wide and 132.4 cm high planar tracking station, which covers the full LHCb acceptance before the magnets. The TT uses a set of silicon micro-strip detectors, the sensors use p^+ implants on n -doped silicon substrates. The TT has four detection layers, in the first and last layers the sensors have vertical strips, and in the second and third layers the strips are

rotated by a stereo angle of -5° and $+5^\circ$, respectively. The strip pitches in the TT are $183\ \mu\text{m}$, and the thickness of the silicon layers is $500\ \mu\text{m}$. The TT has long read-out strips, the longest strip is $37\ \text{cm}$. The total active silicon area of TT is $8.4\ \text{m}^2$ and the total number of read-out channels is 143,360. The hit resolution of the TT was measured to be $52.6\ \mu\text{m}$ in 2011 and $53.4\ \mu\text{m}$ in 2012 [4]. The four layers of the TT are shown in Fig. 7.9.

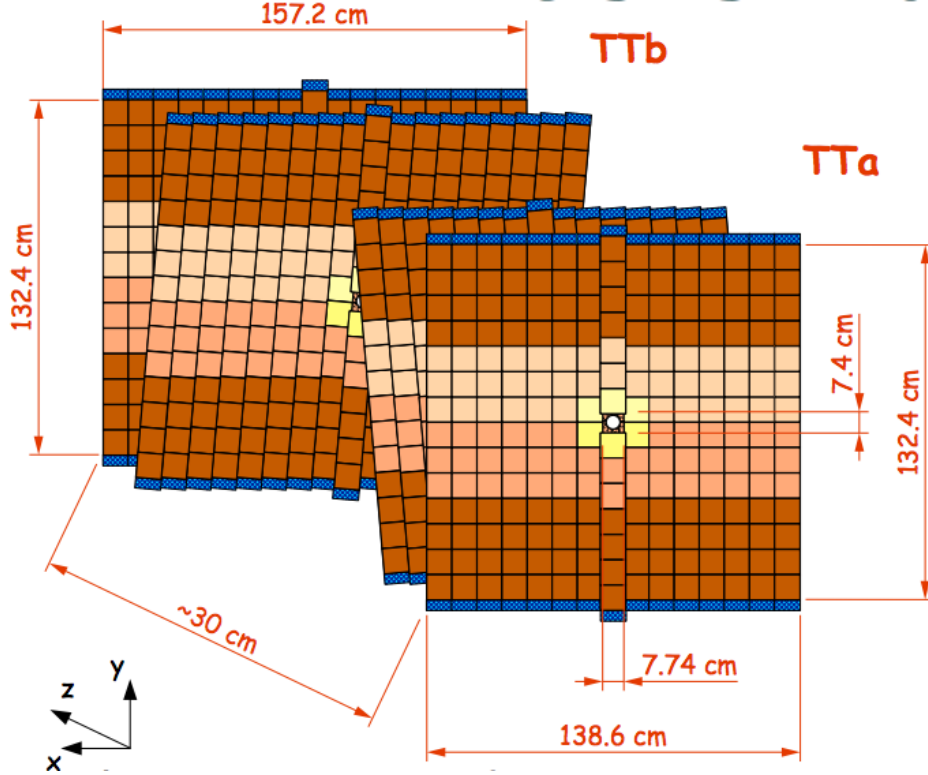


Figure 7.9: Layout of four layers of the TT. The second and third layers are rotated by $\pm 5^\circ$ with respect to the other layers.

The Inner Tracker

The IT covers the innermost regions of the three tracking stations, where the flux is highest in the tracking stations, and it has a finer hit resolution than the OT. At each station there are four detection boxes and in each box there are four layers of silicon. The second and third layers are rotated by $\pm 5^\circ$ with respect to the vertical first and fourth layers. Each of the four detection boxes is made of seven modules. The bottom and top boxes contain single sensor modules whereas the side boxes contain two-sensor modules. The layout of one IT station is illustrated in Fig. 7.10. The silicon sensors are single-sided sensors with a strip pitch of $198\ \mu\text{m}$. The thickness of the single-sensor modules is

320 μm , and of the two-sensor modules is 410 μm . There are in total 129,024 read-out channels for the IT and the total active silicon area is 4.2 m^2 . The hit resolution of the IT was measured to be 50.3 μm in 2011 and 54.9 μm in 2012 [4].

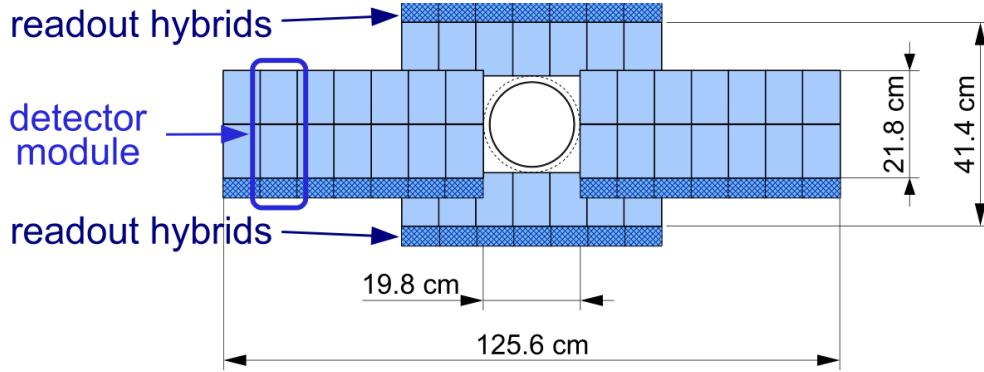


Figure 7.10: Layout of one IT station.

The Outer Tracker

The OT is used to provide precise tracking measurements of charged particles over a large active detector area. The OT uses the technique of straw tube drift-time detectors, each module contains two staggered layers of gas-tight drift-tubes of 4.9 mm inner diameter. The straw-tubes are filled with a mixture of 70% Argon, 28.5% CO_2 , and 1.5% O_2 gas. With this design, a drift time of less than 50 ns and a drift-coordinate resolution of 205 μm are obtained. However, the drift time in the OT is double the 25 ns minimum LHC bunch spacing time, the spillover effects between bunch crossings have become more significant in the run-2 operation of the LHC.

Just like the TT and IT, each of T1–T3 contains four layers of modules, where second and third layers are rotated by $\pm 5^\circ$ with respect to the vertical first and fourth layers. The total active area of the OT is $5971 \times 4850 \text{ mm}^2$. In total the OT has around 55,000 single straw-tube channels. The OT stations cover an acceptance of 300 and 250 mrad in the horizontal (bending) and vertical (non-bending) planes, respectively. The OT is illustrated in Fig. 7.11.

7.2.3 The dipole magnet

The LHCb dipole magnet is used to bend charged particle tracks, which allows the tracking system to determine the momentum of particles via the curvature of the trajectories in the magnetic field. The LHCb dipole magnet is a warm magnet (non-superconducting). The shape of the magnet is designed to cover the full acceptance of LHCb, and also ensures a good field uniformity along the

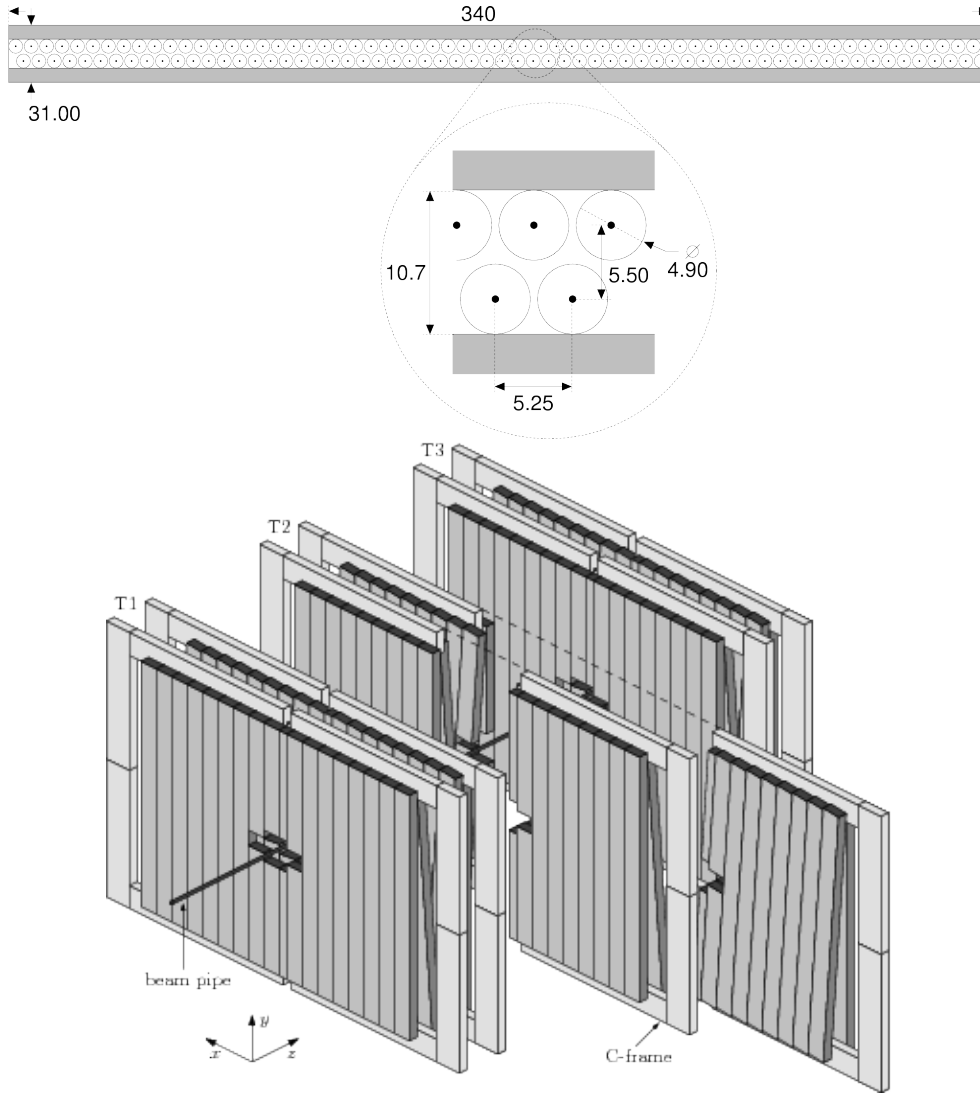


Figure 7.11: (top) Layout of OT straw tubes in one module. (bottom) Layout of three OT stations. Figure taken from [7].

transverse coordinate. This magnet provides an integrated field of 4 Tm for tracks passing through tracking stations, while the fiducial field inside RICH is below 2 mT.

7.2.4 Ring Imaging Cherenkov Detectors

The discrimination between charged pions, kaons and protons over a large range of momentum is mainly provided by two Ring Imaging Cherenkov detectors, RICH1 and RICH2, which are located upstream and downstream of the magnet.

The RICH detectors work by measuring emissions of Cherenkov radiation. Cherenkov radiation occurs when a charged particle passes through a certain medium (in LHCb RICH detectors, a dense gas) faster than light does, at which time the particle emits a cone of photons. In RICH1 the radiator is C_4F_{10} and in RICH2 the radiator is CF_4 . The photons are focussed by a combination of spherical and flat mirrors in the RICH detectors and then collected using hybrid photon detectors (HPDs). The photons strike the photo-cathode of the HPD and cause the emission of photo-electrons. The photo-electrons are accelerated by a 20 kV electric field and detected by a segmented silicon detector. The structure of the RICH detectors is shown in Fig.7.12.

The RICH detectors are placed at either side of the dipole magnet, and thus operate in the high magnetic field environment. The HPDs are surrounded by iron shields and placed in alloy cylinders with a high magnetic permeability. However, the residual field still can affect the images recorded by HPDs. Therefore, corrections studied with calibration runs are applied.

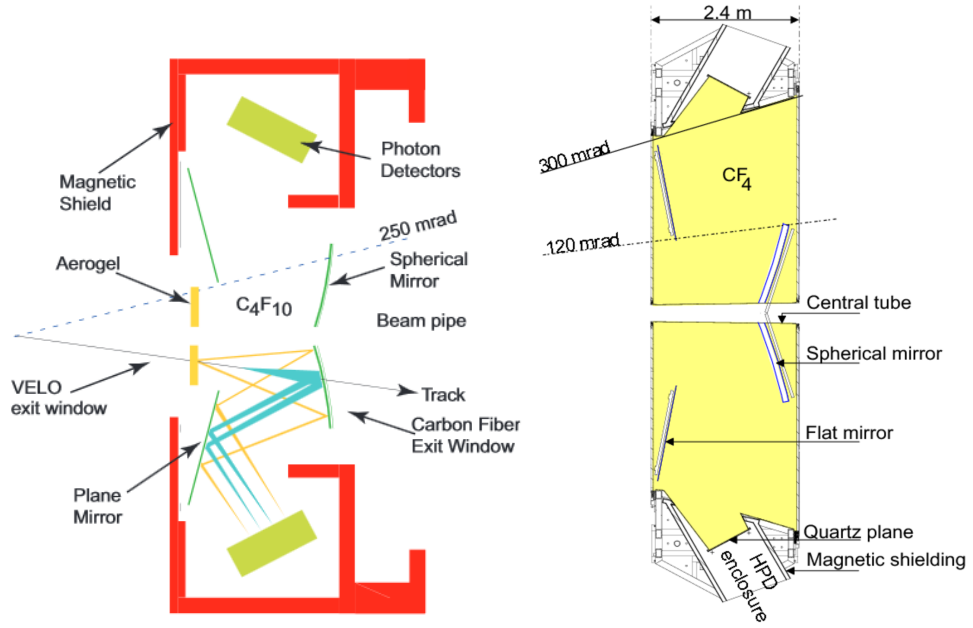


Figure 7.12: Schematic view of (left) RICH1 and (right) RICH2 detectors. Figure taken from [8].

The emission angle of the Cherenkov radiation θ_c can be calculated by

$$\cos \theta_c = \frac{c}{nv}, \quad (7.3)$$

where n is the refractive index, c is the speed of light in vacuum, and v is the speed of the particle. Having determined the momentum and direction of

particles from tracking detectors, v can be predicted for each hypothesis of the particle's types using the known masses. Therefore, different types of particles can be distinguished, as shown in Fig. 7.13.

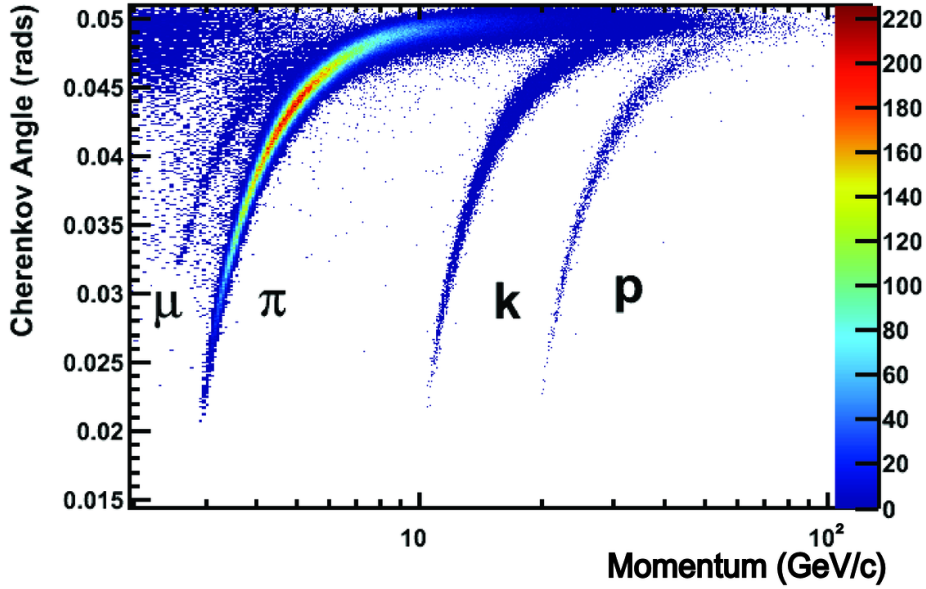


Figure 7.13: Reconstructed Cherenkov angle as a function of track momentum in C_4F_{10} radiator. The Cherenkov bands for muons, pions, kaons and protons are clearly separated and visible. Figure taken from [9].

RICH1 is responsible for low and intermediate momentum tracks, the momentum range that can be discriminated by RICH1 is approximately 2 – 40 GeV/c, while RICH2 is optimised for high momentum tracks, the momentum range covered by RICH2 is 15 – 100 GeV/c. RICH1 covers the full angular acceptance of LHCb, and RICH2 only covers an angular acceptance of 15 – 120 mrad, which is restricted to the region near the beam pipe. Due to the correlation between track momenta and polar angles, high momentum tracks are likely to be constrained in the small polar angle region, and in this region RICH2 has significant discriminatory power.

In order to assign a particle mass hypothesis to a track, an overall event log-likelihood algorithm is employed. Initially all particles that passed through RICH detectors are assumed to be pions. Using the mass of pions and the directions and momenta of tracks, the predicted Cherenkov rings are projected onto the HPD planes. By comparing the predicted pattern with the pattern observed by the HPDs a value of the log-likelihood is computed. Then, for each track, the mass hypothesis is changed to electron, muon, kaon, and proton and the value of the log-likelihood is recomputed. The hypothesis that gives

the largest increase in the log-likelihood is recorded and is set as the preferred value for the track. The difference in likelihood with respect to the pion is represented by delta log-likelihood (DLL) variables, for each track, one value is assigned for each (e, μ, K, p) hypothesis, and the pion DLL value is zero by definition. An example showing the kaon identification efficiency and pion misidentification rate measured on data as a function of track momentum can be found in Fig. 7.14.

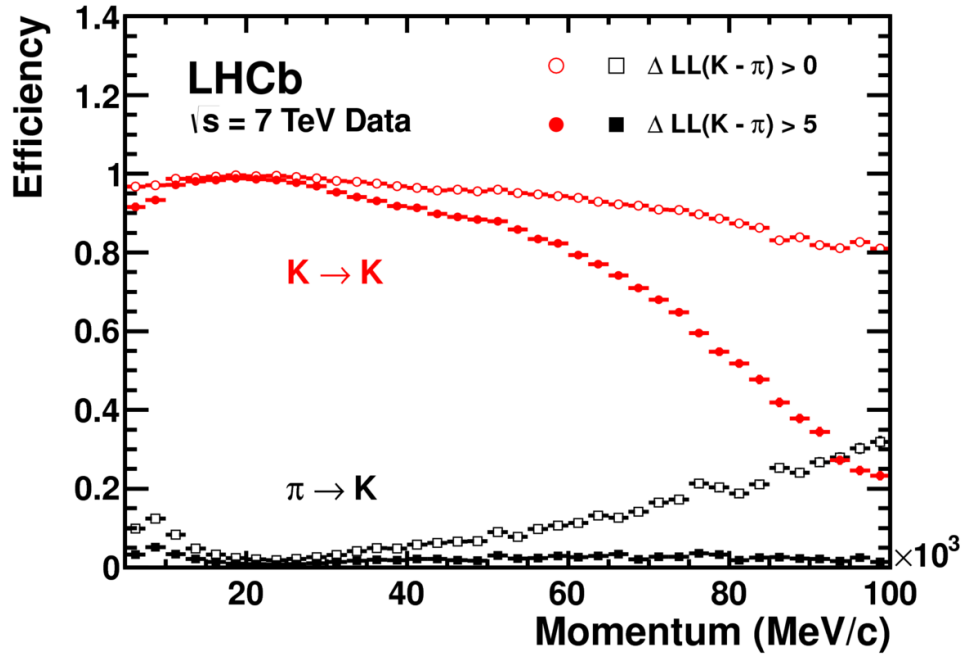


Figure 7.14: Kaon identification efficiency and pion misidentification rate as a function of track momentum with two different delta log-likelihood requirements. Figure taken from [9].

7.2.5 Calorimeter system

Given the precise momentum measurements provided by the tracking system, and the particle mass determined by using the RICH, the energy of charged particles can be calculated. Consequently the calorimeter system is often not used in many off-line analyses. The primary role of the calorimeter system in LHCb is in the first-level of triggering, due to the limited read-out bandwidth, not all collision events can be recorded. To record as many useful events as possible a hardware trigger (L0 trigger) relying on the calorimeter is necessary. In addition, the calorimeters are important for identifying and reconstructing

neutral particles, particularly photons, and for electrons. π^0 s, which decay into two photons, are used in this thesis in Chapter 10.

The calorimeter system can be divided into four components: a scintillator pad detector (SPD), that helps to determine whether the particle that is hitting the calorimeter system is charged or neutral; a pre-shower detector (PS), that provides discrimination between electrons and $\pi^\pm$, and between photons and neutral hadrons; an electromagnetic calorimeter (ECAL), that measures the deposited energy for electrons and photons; and a hadronic calorimeter (HCAL), that measures the energy of protons, neutrons and other hadrons.

The active layers of the SPD and PS detectors are very similar, they are all arrays of high granularity scintillator pads. Both the SPD and PS contain about 6000 pads. The cell granularity of the SPD and PS varies from 4×4 cm in the inner part of the detector, to 6×6 cm and 12×12 cm in the middle and outer parts. The cell granularity distribution can be found in Fig. 7.15. A 15 mm thick lead tile serves as the converter and is installed between the SPD and PS. Scintillation light in the SPD and PS is transmitted by wavelength-shifting fibre and collected by multi-anode photomultiplier tubes (MAPMTs).

The ECAL covers the LHCb acceptance from 25 mrad to 300 mrad in the horizontal (bending) plane and from 25 mrad to 250 mrad (non-bending) in the vertical plane. The ECAL contains scintillating tiles and lead plates, and scintillating light is detected by 10-stage photomultipliers. The cell granularity of the ECAL is the same as the SPD and PS, which facilitates the combined use of them in the γ/e separation. To achieve a more uniformly distributed hit density over the full area of the ECAL. The overall detector size is 7.76×6.30 m. The ECAL has an energy resolution of $\delta_E/E = (8.2\pm 0.4)\%\sqrt{E/GeV} \oplus (0.87\pm 0.07)\%$.

The HCAL is placed behind the ECAL, and is made of scintillating tiles and iron plates. The HCAL elements also have different sizes in the inner and outer regions, varying from 13×13 cm in the inner part of the detector to 26×26 cm in the outer part of the detector. The granularity distribution of the cells can be found in Fig. 7.15. Scintillating light is transmitted by wavelength-shifting fibers and collected by photomultipliers at the back side of the detector. The energy resolution of the HCAL is measured to be $\delta_E/E = (69\pm 50)\%\sqrt{E/GeV} \oplus (9\pm 2)\%$.

7.2.6 Muon system

The muon system is used in the first-level hardware trigger to select events that contain high transverse momentum muons in the decay and also provide muon information for offline analysis. Muons are final states of many b -hadron decays, therefore the precise reconstruction of muon tracks is essential in LHCb. The muon system is located at the downstream end of the LHCb detector. The muon system contains five stations (M1 - M5), with a size gradually increasing to cover a similar acceptance of ± 306 mrad in the horizontal plane

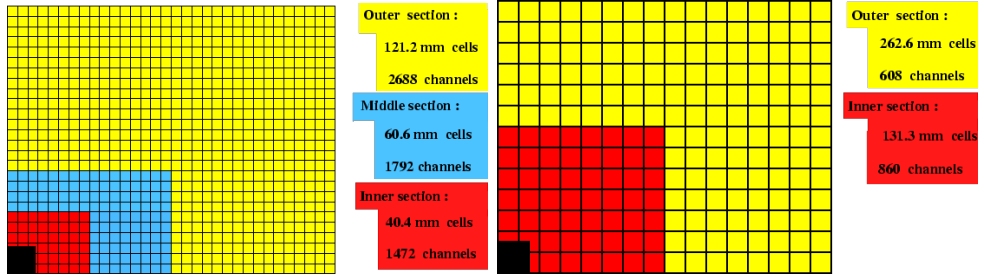


Figure 7.15: (left) The granularity of SPD/PS and ECAL. (right) The granularity of HCAL. One quarter of the detector front face is shown.

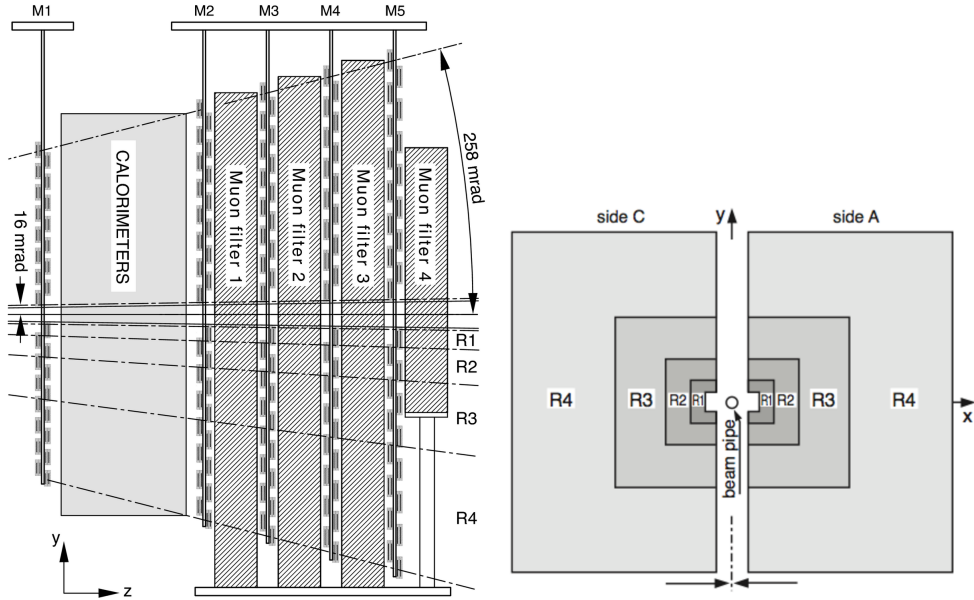


Figure 7.16: (left) Schematic side view of the muon stations. (right) Schematic front view of one muon station. Figure taken from [10] and [11].

and ± 258 mrad in the vertical plane. The total active area of the LHCb muon system is 435 m^2 across 1380 chambers. M1 is placed in front of the SPD/PS, M2 - M5 are placed behind the calorimeters (Figure 7.16). The M2 - M5 are separated from each other by 80 cm thick iron absorber plates to select penetrating muons. Each muon station can be divided into four regions (R1 - R4), the size of cells increases from inner region to outer region to achieve a similar total flux in each region. The spatial resolution in the first three muon stations (M1 - M3) are finer than the last two (M4, M5). M1 - M3 mainly contribute to the track momentum determination, providing a 20% estimate of the transverse momentum. M4 and M5 mainly contribute to the identification

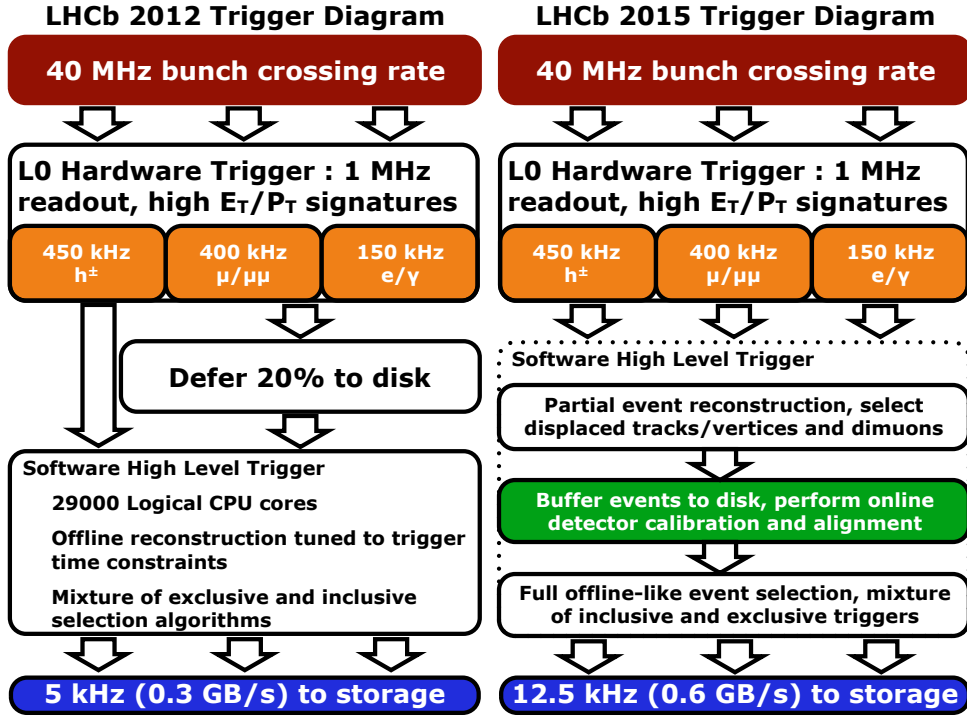


Figure 7.17: (left) The 2012 trigger data flow, including the manner in which the deferral of events is applied. (right) The run-2 trigger scheme showing the online detector alignment and calibration stage.

of penetrating muons. For all regions of the muon stations, except the innermost region (R1) of M1, Multi Wire Proportional Chambers (MWPC) are used to collect muon hits. In R1 of M1, where the flux is highest, triple-GEM (Gas Electron Multiplier) detectors are used as they have better properties against ageing in the presence of high fluxes. The layout of the muon regions is shown in Fig. 7.16.

7.2.7 Triggers

LHCb has three stages of trigger system, which filter most of the background events. Among the three stages of triggers, the first stage is a hardware trigger (L0). The bunch crossing frequency provided by the LHC can reach about 40 MHz, as discussed in the previous sections, the output rate of L0 is 1 MHz of events. L0 uses information from the calorimeter system and muon system, and is supplemented by the pile-up sensors in front of the VELO. L0 makes a selection of high E_T and p_T events.

On a positive L0 decision, the event data collected by all sub-detectors is sent to the high-level trigger (HLT) computing farm, where further selection by

the second and third stage of the triggers occurs. The HLT uses a combination of C++ algorithms running on 29,000 logical CPU cores. Starting from 2012, around 20% of data were deferred to disk, which allows the HLT farm to process these data during the periods that have no data taking. This technique increases the average CPU time for processing each event and hence more efficient event selection. There are two stages of the HLT, HLT1 and HLT2. HLT1 performs a partial reconstruction of events by requiring tracks to be reconstructed in the VELO and the tracking stations. At the HLT1 stage, pp collision vertices are formed, which allows the identification of long-lived particles by measuring their impact parameter (IP) with respect to the collision point. Events that do not have a track with significant IP within the VELO, or do not match the hits in muon stations are discarded. HLT1 reduces the 1 MHz of L0 output to around 30 kHz in run-1 or 80 kHz in run-2. Events that passed HLT1 are sent to HLT2, where a full event reconstruction is performed, and further reduces the read out rate to 5 kHz in run-1 and 12.5 kHz in run-2. Each layer of the HLT contains several selection algorithms, or trigger lines. These lines are optimised for different types of analyses. The trigger schemes in run-1 (2012) and run-2 (2015) are shown in Fig. 7.17.

8 Radiation damage in LHCb vertex locator

The study of particle physics with colliders requires the precise reconstruction of particle tracks. This is especially the case in heavy flavour physics which requires the measurement of the flight distance of the beauty or charm hadron. At the LHC this distance is typically in the millimetre range and thus the reconstruction of decay vertices very close to the collision point is required. At LHCb, this is performed with the VELO detector. The VELO is a silicon strip detector. Semiconductor detectors, primarily silicon detectors, are commonly used in the tracking systems of particle physics experiment, and are currently the best choice for achieving high precision. However due to the high cost of the semiconductor sensors, and the electronics to readout large numbers of channels, the usage of semiconductor detectors is usually limited to the region where the highest resolution is required, and the highest radiation fluence occurs, such as around the interaction point. The VELO is the closest detector to the interaction point amongst all the four major LHC detectors, and is exposed to the highest fluence during LHC operation. Hence continuous monitoring of the radiation damage effects is critical to ensure the performance of the VELO.

In this chapter, several studies on radiation damage effects in the VELO are discussed. A discussion of particle detection using silicon detectors is given in Sect. 8.1. The theory of radiation damage in silicon detectors is reviewed in Sect. 8.2. The VELO sensors are described in Sect. 8.3. Section. 8.4 to Sect. 8.7 describe the technique used in the VELO to monitor radiation damage effects and the impact of radiation damage on the VELO during LHC run-2 operation. The work reported here is based on the author's LHCb note [48]. Section 8.8 describes the efficiency of collecting charge in the VELO sensors and how it is affected by the routing lines on the VELO sensors. The studies described in this chapter are summarised in Sect. 8.9.

8.1 Particle detection with silicon detectors

Silicon is the second most abundant element in the Earth's crust after oxygen. In spite of these rich reserves, silicon usually exists in a variety of compounds,

such as silicon dioxide (SiO_2), and pure silicon crystals are rarely produced in nature. The atomic number of silicon is 14, a silicon atom has 14 electrons distributed in three shells with a (2, 8, 4) arrangement. The four electrons in the outer shell are valence electrons, a silicon atom can complete the outer shell by covalently bonding with four other silicon atoms to form a diamond lattice structure.

An electron from an isolated atom has to occupy one of a series of atomic orbitals with well defined discrete energy levels. However, in a molecule made of two atoms, the electrons orbitals overlap with each other. Due to the Pauli exclusion principle, no two electrons can have the same quantum numbers in a molecule, each atomic orbital will then be split into two molecular orbitals with a small difference in energy. Consider a crystal, with a large number of atoms, the energy level of the electrons will be split into a large number of orbitals and these orbitals are so close to each other that we consider them as a continuous band, this is known as the energy band.

At a temperature of absolute zero, the energy band which the highest energy electrons in the atoms occupy is called the valence band. At higher temperature, the electrons can populate a higher energy band, which is called the conduction band. An electron in the valence band is bound to its atom's lattice site, while an electron in the conduction band can be free to move in the lattice.

The energy difference between the valence band and the conduction band is called the band gap energy, E_g . The difference between conductors and insulators can be explained by the difference in electronic band structures of the material. Electrical conductivity in conductors is due to the presence of electrons in the conduction band and delocalized from the lattice. The valence band and conduction band in a conductor overlap, electrons can transfer from the valence band to the conduction band. On the contrary, in insulators, the energy gaps are usually large enough to prevent electrons in the valence band transferring to the conduction band due to thermal excitation. Semiconductors have a band gap that is smaller than that of an insulator, so that a significant fraction of electrons in the valence band can transfer to the conduction band due to thermal activity. The band gap energy of silicon is dependent on temperature, and is 1.12 eV at room temperature [49]. In the VELO at the operational temperature of about -10°C , an effective band gap energy of 1.21 eV is expected [50] and sensor current measurements have yielded a value of 1.16 eV [51]. The classification of metal, semiconductor, and insulator according to their band gap energy is shown in Fig. 8.1.

An electron in the conduction band is a negative charge carrier, the absence of an electron in the crystal lattice can also be considered to be a positive charge carrier, and is referred as a 'hole'. The kinematic behaviour of holes in an electromagnetic field is similar to electrons, but with an opposite charge.

The conductivity of a semiconductor is related to the number free charge carries, *i.e.* thermally excited electrons, and thus a function of temperature.

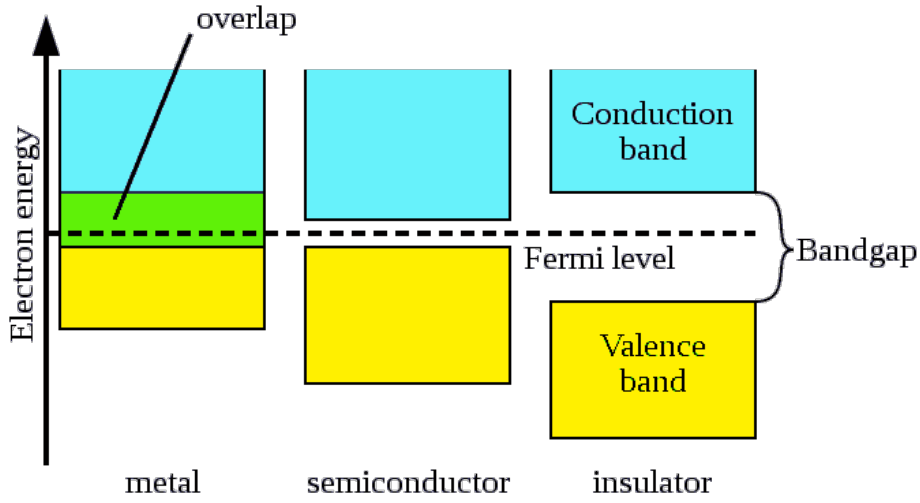


Figure 8.1: Band gap energies in metals, semiconductors and insulators. Conduction band and valence band are overlapped in metals, but have large band gap in insulators. Semiconductors have a band gap that is smaller than that of an insulator, therefore electrons in the valence band can transfer to the conduction band due to thermal activity.

It is typically several orders of magnitudes smaller than in conductors. In a pure silicon crystal lattice there are a very limited number of charge carriers that are free to move, which leads to a very low conductivity for pure silicon. However, the conductivity can be significantly increased by introducing some other element atoms in the lattice. The most commonly used dopant elements are group 15 elements (phosphorus or arsenic) and group 13 elements (boron or gallium). Group 15 elements have one additional electron than silicon in the outer shell, thus after covalently bonding with four other atoms in the lattice, the fifth electron is easily elevated to the conduction band and free to move in the lattice. The addition of these elements are called ‘donor’ dopants. Silicon doped with a donor dopant can form a *n*-type semiconductor. When a group 13 element is used as a dopant in the silicon lattice, the absence of an electron in the outer shell of these elements will lead to a ‘hole’ in the silicon lattice after the dopant has covalently bonded with four other atoms. These elements are called ‘acceptor’ dopants. Silicon doped with an acceptor dopant can form a *p*-type semiconductor.

A silicon diode is made with a combination of *p*-type and *n*-type silicon semiconductors. The boundary between *p*-type and *n*-type semiconductors is called a *p-n* junction, as shown in Fig. 8.2. The negative and positive charge carriers diffuse across the boundary and recombine together, forming

a region that is devoid of free charges. This region is called the depletion region. The dopant atoms are ionized and fixed in the lattice, therefore an electric field is established across the depletion region. This field has a built-in potential difference V_{bi} , which is opposite to the positive charge carrier diffusion direction and prevents the charges to diffuse further. If one side of the junction is significantly heavier doped than the other, the depletion region will extend further in the other side. This arrangement is commonly used in silicon detectors. A small reverse current, due to the thermal motion of excited electrons, flows across the junction and is known as the ‘leakage current’. This current can be reduced by decreasing the temperature. Silicon detectors are often operated at lower than room temperature to reduce the leakage current and to obtain a better signal to noise ratio. Reducing the operating temperature also assists with performance after irradiation (see below).

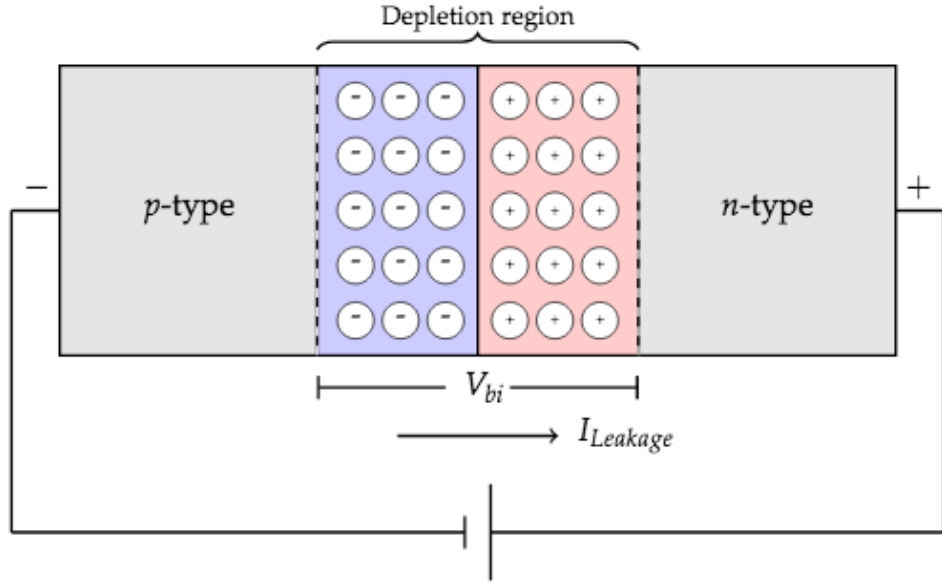


Figure 8.2: Illustration of p - n junction and depletion region. The negative and positive charge carriers diffuse across the boundary and form a depletion region.

When a charged particle transverses a silicon diode, it will interact with the atomic lattice to create electron-hole pairs and deposit energy along its path. On average a charged particle needs to deposit about 3.6 eV [52] to create an electron-hole pair. The mean energy loss of particles produced in LHCb in a silicon crystal lattice is about $388 \text{ eV}/\mu\text{m}$. For a silicon sensor with a $300 \mu\text{m}$ thickness (such as the VELO sensors) a charged particle passing through one layer of sensor perpendicularly will deposit a mean of 116 keV energy, or 32,300 electron-hole pairs. If the electron-hole pairs are created outside the depletion

region, in a region where there is no electric field, they will either recombine or diffuse via a random path. If the electron-hole pairs are created in the depletion region, due to a potential difference V_{bi} , the electrons and hole will move in opposite directions and can be collected at the edge of the silicon. This measured induced current is used in silicon detectors to identify the energy or position of the particle.

The depletion region can be increased if a reverse bias potential difference is applied on the p - n junction with the same direction as V_{bi} , *i.e.* a higher potential on the n -type side and a lower potential on the p -type side. In this case, free charges are pulled away from the edge of the depletion region by the applied field. The voltage required to deplete the entire silicon diode is called the ‘depletion voltage’, V_{dep} . The value of the depletion voltage is related to the difference in the concentrations of the ionized donor dopants and acceptor dopants or effective doping concentration, N_{eff} , by

$$V_{dep} + V_{bi} = \frac{q_0}{2\epsilon_r\epsilon_0}|N_{eff}|d^2, \quad (8.1)$$

where q_0 is the depletion charge, d is the depth of depletion region, $\epsilon_r\epsilon_0$ is the permittivity of silicon.

A typical silicon sensor thickness is several hundreds of micrometers thick and the depletion voltage is more than one order of magnitude higher than the built-in voltage. The efficiency of the silicon detector at collecting the deposited charge of a traversing particle is called the charge collection efficiency (CCE). To get the maximum charge collection efficiency the silicon diode needs to be fully depleted during operation.

There is a maximum reverse bias voltage that can be applied. At this breakdown voltage the current will rapidly increase with the reverse bias and the semiconductor then become electrically conductive, this phenomenon is called breakdown. Breakdown can potentially damage the silicon diode and hence the operation of silicon detectors close to the breakdown voltage is not recommended. If the depletion voltage of a silicon detector increases to a value beyond the breakdown voltage, the detector may have to be operated in a not fully depleted state and thus with a low CCE.

8.2 Radiation damage in silicon sensors

At the LHC a high fluence of high energy particles is created. These particles transverse the silicon detectors and not only do they ionize lattice atoms and create electron-hole pairs, but they also interact with lattice atoms via a non-ionising energy loss (NIEL) process and introduce defects in the lattice [12]. Figure 8.3 shows some types of defects. In these defects, nuclei are displaced from their lattice and both a vacancy in the lattice is created and the displaced atom forms an interstitial. These can be mobile in the lattice and a combination

of an interstitial and a vacancy in the lattice is called a Frenkel pair. To create this defect about 25 eV of energy is need to be transferred to the atom in the silicon lattice. If the displaced atom moves to the surface of silicon, a Schottky defect is created. Silicon atoms being “kicked-off” from the lattice can lead to the further removal of other atoms from their positions, which results in a dense cluster of defects. The majority of vacancies that are formed in this way can recombine with interstitials due to thermal motion. 60% of simple Frenkel pairs can be recombined, in a complex defect cluster, the ratio of recombined vacancies and interstitials can be up to 95%. The remaining defects can combine with each other, and form a more complex defect structure, for example some of the vacancies are combined together and formed a ‘divacancy’. Defects in the silicon lattice can change the property of silicon by changing the energy of the bands or introducing new energy levels in the band-gap. Electrons in the valence band can be excited to the conduction band, via this additional band-gap defect level, which lead to an increase of the device’s leakage current.

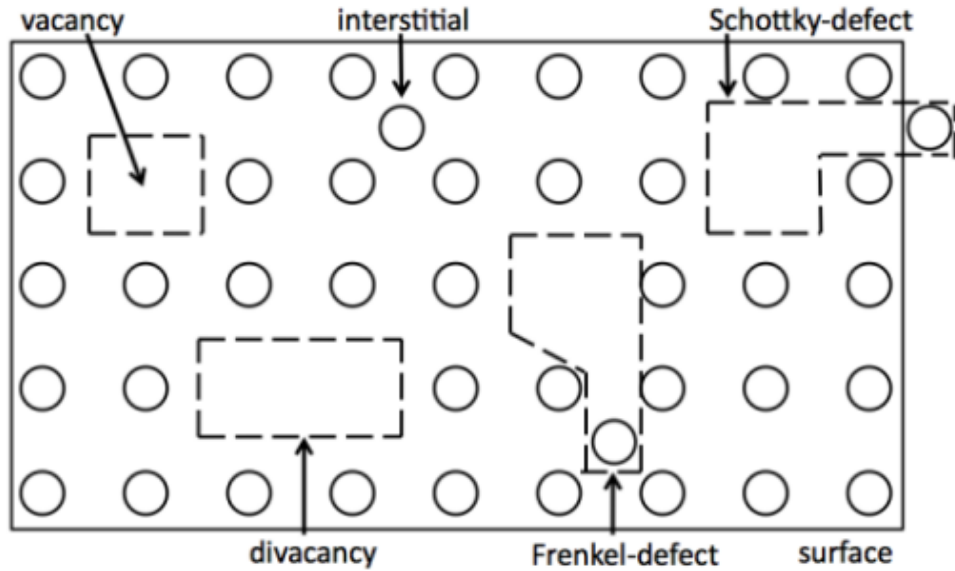


Figure 8.3: Illustration of radiation induced defects in a silicon lattice. Figure taken from [12].

The defects can also lead to an increase of the time that is needed to collect charge by a ‘charge trapping’ effect. A charge carrier can be trapped in the defect level before being emitted back to the conduction band and become less mobile. This effect can affect the total amount of charge collected in a readout period.

Radiation induced defects can behave similarly to donor or acceptor atoms, and lead to a change in the effective doping concentration in the silicon. When

the energy level of a donor (acceptor) defect is near the conduction (valence) band, the silicon atom is easily ionised and will introduce some additional free charge carriers. The depletion voltage will then be changed due to the present of these charge carriers.

In practice when n -type silicon is irradiated donor effects are removed and acceptor effects are introduced, consequently N_{eff} is found to decrease until the defect acceptor effect overweighs the initial donor effect, after this point an n -type silicon behaves in some sense like a p -type silicon. This point is called type-inversion. Figure. 8.4 shows the evolution of effective doping concentration in n -type silicon as a function of fluence.

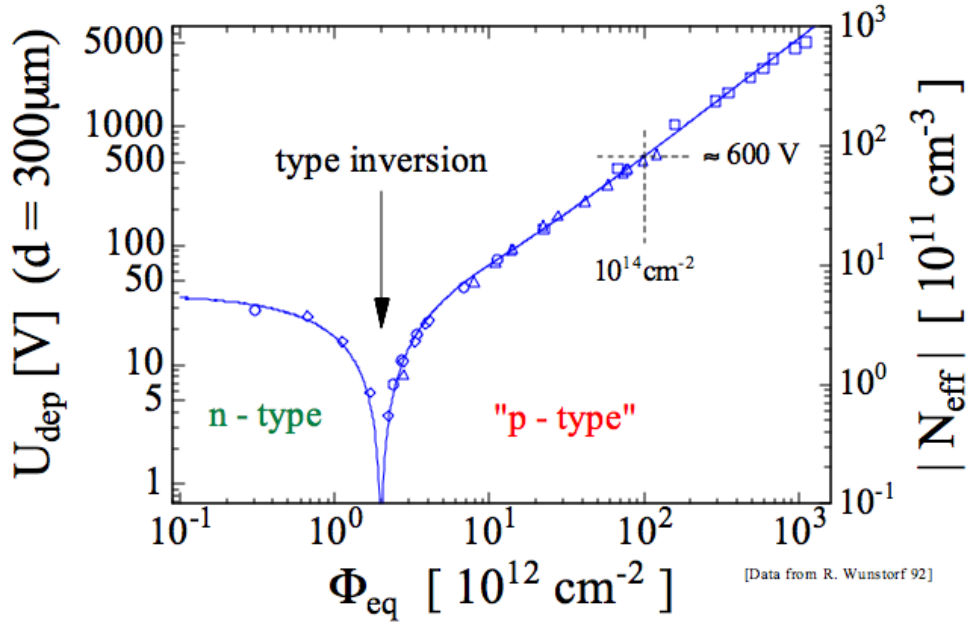


Figure 8.4: The evolution of effective doping concentration and depletion voltage for n -type silicon as a function of fluence. Figure taken from [13].

As discussed above, thermal motion can recombine some vacancies and interstitials and recover the defects. Increasing the temperature intentionally will accelerate this process. In the important case of an n -type silicon after type-inversion, the p -type N_{eff} is found to decrease with time [53]. This effect is called ‘annealing’. However, after a period of beneficial annealing, the N_{eff} is found to increase again. This is due to the remaining defects interacting with each other and forming more complex defects, and hence the annealing effect becomes damaging to the sensors. The VELO sensors are intended to be kept cooled, throughout operations and shutdowns, to reduce the annealing effect.

8.3 The VELO sensors

A brief overview of the VELO detector was given in Sect. 7.2.1. This section provides a more detailed description on the design and features of VELO sensors, necessary to understand the following sections of this chapter.

As shown in Fig. 7.6, the VELO detectors use two designs of silicon strip sensors. In R -type sensors the strips are arranged in sectors with concentric circles. In Φ -type sensors the strips are orientated in an approximately radial direction with a stereo angle of 20° in the inner region and -10° in the outer region. The choice of the stereo angle is driven by the purpose of improving the reconstruction efficiency and allowing ghost hits to be identified [54]. A VELO module is made of a Φ -type sensor and an R -type sensor which are attached back-to-back and mounted on a common support.

There are 2048 strips on each sensor. There is a narrow aluminium metal track running along the strips n^+ implant. This (AC coupled) metal track is known as the first metal layer, and is responsible for ‘collecting’ the deposited charges from the depletion region. The active strips are at a radial distance of 8.2 - 42 mm away from the beam axis. The width of the strips varies, from a minimum of $11\ \mu\text{m}$ in the innermost region, to a maximum of $38\ \mu\text{m}$ in the outermost region. The strip pitch describes the distance between the centre of two neighbouring strips, and it increases as a function of radius. For R -type sensors, the strips are distributed in four 45° sectors, each sector contains 512 strips. The strip pitches vary linearly from $42\ \mu\text{m}$ at the innermost radius to $102\ \mu\text{m}$ at the outermost radius. For Φ -type sensors, sensors are divided into two radius regions, with a boundary at 17 mm. The inner radius region contains 683 strips and in this region the pitch varies linearly from $38\ \mu\text{m}$ at the innermost radius to $78\ \mu\text{m}$ at 17 mm radius. The outer radius region contains 1365 strips, and the strip pitches vary linearly from $39\ \mu\text{m}$ at 17 mm radius to $97\ \mu\text{m}$ at the outermost radius. Some key properties of the VELO sensors are summarised in Table 8.1.

Table 8.1: Parameters of VELO sensors

Parameter	Value	
	R -sensors	Φ -sensors
Silicon thickness	300 μm	
Number of strips	2048	
Strip width	11 - 38 μm	
Routing line width	$\sim 11\ \mu\text{m}$	
Inner silicon edge to beam axis	7 mm	
Active strips to beam axis	8.2 - 42 mm	
Stereo angle	-	-10° - 20°
Strip pitch	42 - 102 μm	38 - 97 μm
Strip length	3.8 - 33.8 mm	5.9 - 24.9 mm

The strips on the R -type sensors and on the inner part of the Φ -type sensors do not extend to the outer edge of the VELO sensors, therefore to route out the charges collected by the first metal layer strips it is necessary to have routing lines to connect between the strips and the outer edge of the sensors where the readout electronics are located. The routing lines are approximately $11\ \mu\text{m}$ wide, and insulated from first metal layer by a $3.8\pm 0.3\ \mu\text{m}$ of SiO_2 . The routing lines are known as the second metal layer. For R -type sensors the routing lines are perpendicular to the sensor's strips, whereas for Φ -type sensors the routing lines of inner strips are parallel to the outer strips and overlaid behind every second strip. Photographs of sensor strips and routing lines of the R -type sensor are given in Fig. 8.5.

Most of the 84 sensors have an n^+ -on- n structure, which contains a highly-doped n^+ implant on an n -type bulk with a p -type backplane. A schematic cross-section of this configuration for an R -type sensor is shown in Fig. 8.6. Two of the sensors are of an n^+ -on- p type, with a similar structure but with p -type bulks. At the time of the detector construction this was a novel structure, it is now the baseline type for many LHC upgrades. All VELO sensors are enriched with oxygen to improve the radiation hardness. Oxygen can interact with interstitials and vacancies, and reduce the effect of radiation damage. The oxygen concentration is enhanced in the VELO sensors to $>1\times 10^{17}\ \text{cm}^{-3}$.

Signals are readout by 16 Beetle chips for each sensor, each Beetle chip can collect signals from 128 strips. The Beetle [55] is a dedicated ASIC designed for the VELO and silicon tracker of the LHC experiment. These Beetle chips store an analogue charge and, if the L0 trigger is fired, these analogue signals will be readout to a TELL1 boards and its Analogue-to-Digital Converter (ADC). The Beetle chips can accept a readout rate of up to 40 MHz, corresponding to the 25 ns LHC minimum beam bunch spacing. During LHC run-1 operation, with a 50 ns bunch spacing, the signals in the Beetle due to particles from two adjacent bunch crossing were well separated. However during run-2 operation, with a 25 ns bunch spacing, spill-over effects are introduced. The front-end pulse shape of a signal from the Beetle is shown in Fig. 8.7. The pulse from the signal in the previous bunch-crossing can spill over to the next readout clock cycle, which may introduce a fake 'signal' with a small ADC count value. However by looking at the ADC counts of the hits that are associated to a track the spill over effect can be removed.

Two formats of data can be output from the Beetle chips in the VELO: non-zero suppressed (NZS) data, and zero-suppressed (ZS) data. NZS data represents the complete data stream coming from the hardware with no thresholds or cuts applied, it is used only for calibration studies. ZS data consists of the information of VELO clusters, this is the normal output format of physics data. A cluster is defined as one or several adjacent strips with charge above a particular clustering threshold. Both types of data will be used in the studies described in this chapter.

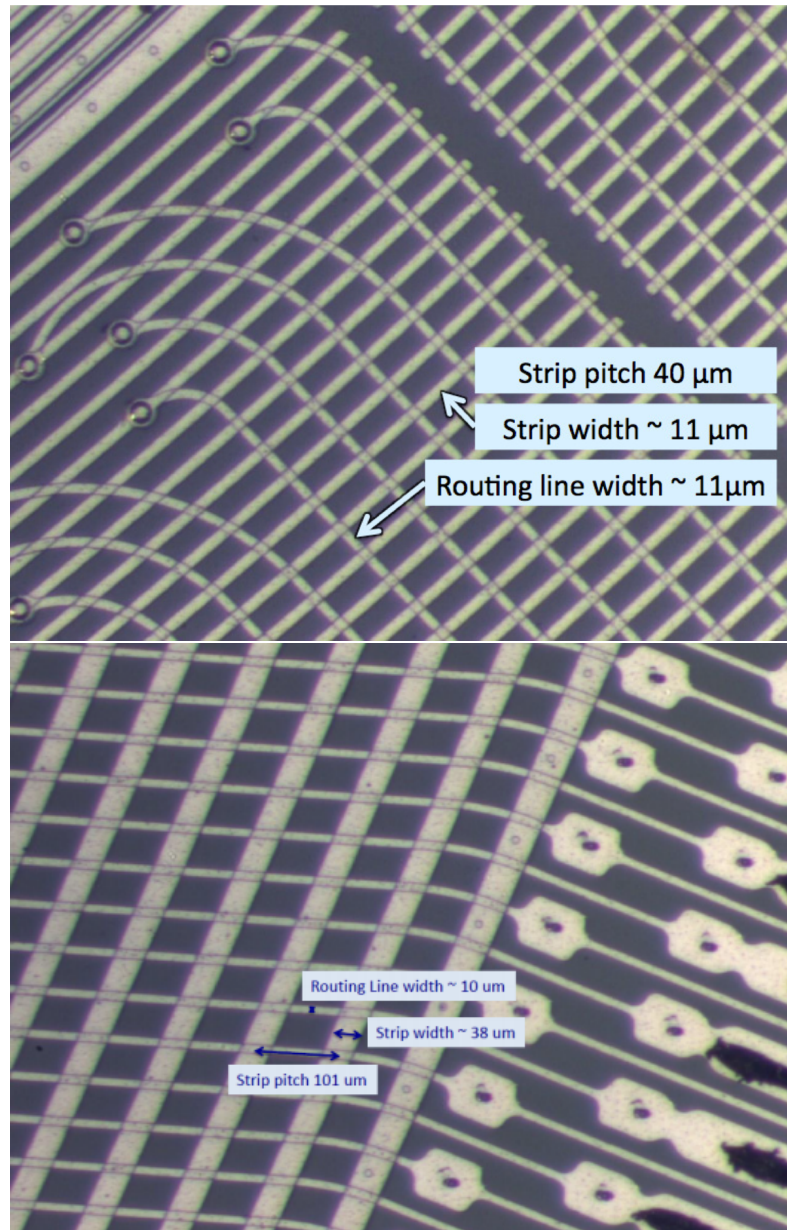


Figure 8.5: Photographs of the (top) innermost region and (bottom) outermost region of an *R*-type sensor. Each strip is connected by a routing line, signals are routed to the outer edge of sensors and collected by the readout electronics. Routing lines are orientated perpendicularly to the strips.

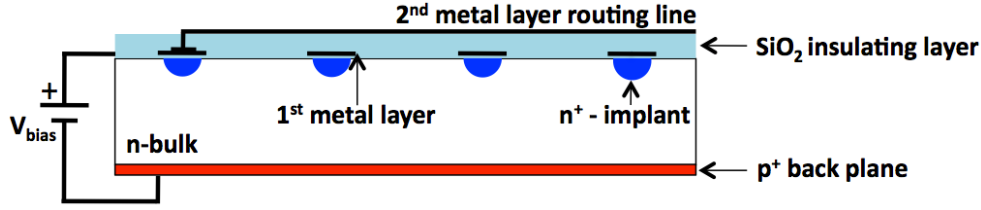


Figure 8.6: A schematic cross-section of a portion of an *R*-type sensor with n^+ -on-*p* structure. The first metal layer strips and n^+ implants run into and out of the page, while the second metal layer routing lines are perpendicular to strips and horizontally shown in the figure. A SiO₂ insulating layer separates the two metal layers. When operated at the depletion voltage or above the entire *n*-bulk region will be depleted.

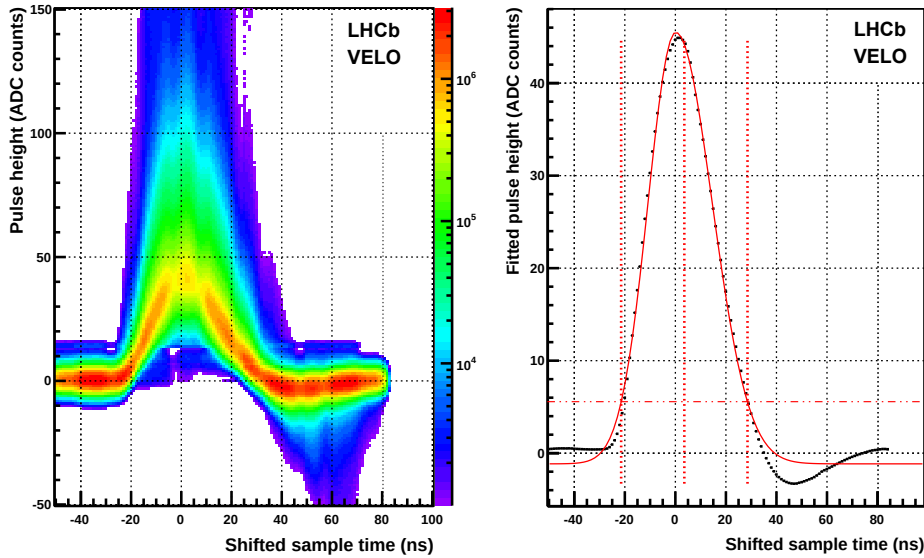


Figure 8.7: Pulse-shape obtained by combining data from all VELO sensors. (left) The ADC spectra versus time. (right) The most probable value for each time bin obtained from a fit of a Landau convolved with a Gaussian function. The fitted function also predicts the amount of pre-spill (25 ns before the chosen point) and the spillover (25 ns after), which are marked with red vertical dotted lines. Figure taken from [5].

8.4 Monitoring radiation damage of VELO sensors

There are several ways to monitor the health of the VELO sensors. The most commonly used methods are the IV scan and CCE scan, which are discussed

in this section.

8.4.1 IV scan

The study of the IV characteristics provides a method to track radiation damage using current (I) versus voltage (V) measurements of the VELO sensors. The IV scan consists of a voltage scans of all VELO sensors in 10V steps up to a certain voltage, with the maximum voltage step set to be higher than the nominal operational voltage. The IV scan is taken weekly and is taken between LHC fills, *i.e.* without beam and collisions, so that the currents measured in the IV scan do not have contributions from charges ionized by particle tracks produced in the collisions. The measured current has two main components, surface current and bulk leakage current. The surface currents can be due to surface contamination, or to the depletion region extending too close to the edge of the silicon sensor. It is expected to decrease with fluence and have only a weak dependence on temperature. The bulk leakage current is mainly due to the drift of charge carriers and is correlated with the radiation damage. Bulk leakage current is expected to increase with fluence and have an exponential temperature dependence.

Using the IV scan data the bulk leakage current can be monitored by plotting the measured current under a typical operational voltage (150 V in this case) versus time (see Fig. 8.8 top) or delivered luminosity (Fig. 8.8 bottom). On these plots it can be seen that sensors which started with a high surface current showed a reduction in current during the early stages of the detector operation. The bulk currents are seen to increase almost linearly with the delivered luminosity. The different slopes of the various sensors on these plots reflects the fluence received by these sensors. A significant decrease of the bulk leakage currents after the winter shutdown of the LHC at the end of 2011 and during the long shutdown of the LHC in 2013 and 2014 is clearly visible. This is due to beneficial annealing of the radiation damage.

8.4.2 Charge collection efficiency scan

As discussed in Sect. 8.1, the size of the depletion region in a silicon diode varies as a function of the applied bias voltage. The initial depletion voltage of the sensors was measured before the installation of the VELO detector by directly measuring the capacitance of each sensor at a number of bias voltages. The inverse of the capacitance squared increases until it reaches a plateau, the voltage at which the plateau is reached is then the depletion voltage. This method is known as the C-V method and is described in [56].

As a result of being exposed to radiation the depletion voltage of the VELO sensors is expected to evolve with exposed fluence and time. After the installation of the detector the direct measurement of the depletion voltage using the C-V method is no longer possible. Instead the measurement of the

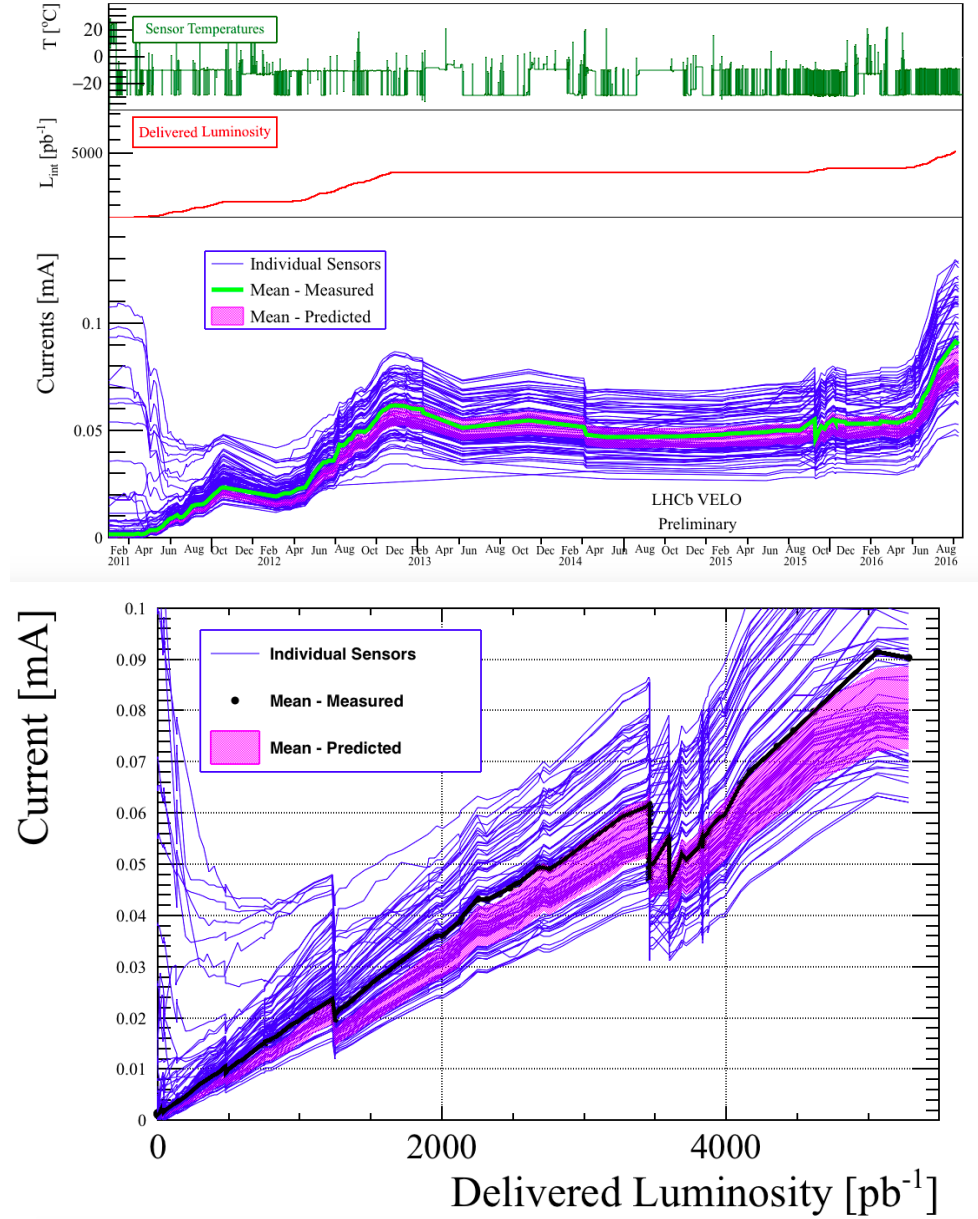


Figure 8.8: (top) Current measured from IV scans versus date and (bottom) delivered luminosity from the start of LHCb operations to September 2016. At the beginning of LHCb operations the surface current varies for the sensors, while after irradiation the bulk leakage current dominates the evolution of the trend.

ADC counts at different bias voltage is then used to determine the depletion voltage. This directly determines the relevant quantity for the performance of

the detector. The scan used for this study is called the CCE scan.

In a CCE scan the 42 modules are divided into five groups. The CCE scan is performed by taking one sensor in each group, known as the ‘test sensor’, and operating these at one of several voltages. The four modules either side of a test sensor remain at the nominal bias voltage. The test sensors are removed from the tracking algorithms, and the position of a particle track in a test sensor is determined by extrapolating the hit positions from the surrounding sensors, as shown in Fig. 8.9. This one-in-five pattern allows for the measurement of multiple sensors simultaneously. All five patterns are scanned over, one at a time, to ensure all sensors are measured.

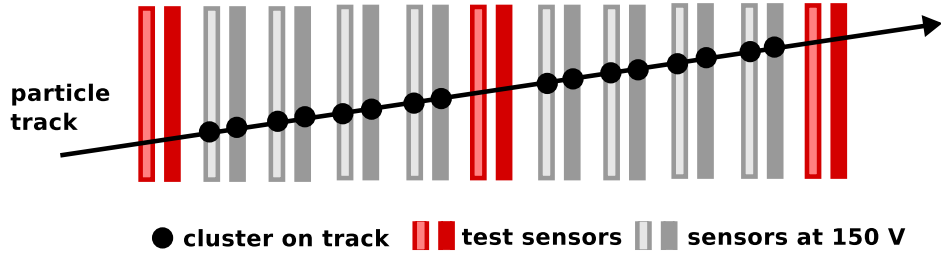


Figure 8.9: Schematic illustration of the method for determining the effective depletion voltage in a CCE scan. The test sensors are those for which the effective depletion voltage is being determined.

The total charge deposited in the five strips nearest to the extrapolated position on the test sensor is recorded. The collected charge is obtained for each sensor at each voltage. The ADC values are corrected for pedestal and common mode contributions [5]. The NZS data is used in this study, so that no zero-suppression cuts have been applied. The pedestal subtracted ADC distribution for these strips on the test sensor is then fitted with a Landau function convoluted with a Gaussian function, as shown in Fig. 8.10 (top) for several voltages. The most probable value (MPV) at each voltage is determined from the fit and plotted as a function of bias voltage, as shown in Fig. 8.10 (bottom). It can be seen that once a certain bias voltage is exceeded, the MPV of the ADC distribution reaches a plateau, indicating the sensor is fully depleted. The Effective depletion voltage (EDV) is then defined as the voltage at which a sensor would have an MPV of its ADC distribution that is 80% of the plateau value. This 80% value was chosen to give the best agreement between the laboratory determined depletion voltage and the EDV method from the first CCE scan, which occurred prior to any irradiation [14].

In addition to the study of the charge collection efficiency the CCE scan can also be used for studying the variation of the VELO cluster finding efficiency (CFE) with bias voltage. CFE is defined as the fraction of times a cluster is obtained at the track intercept on the test sensor.

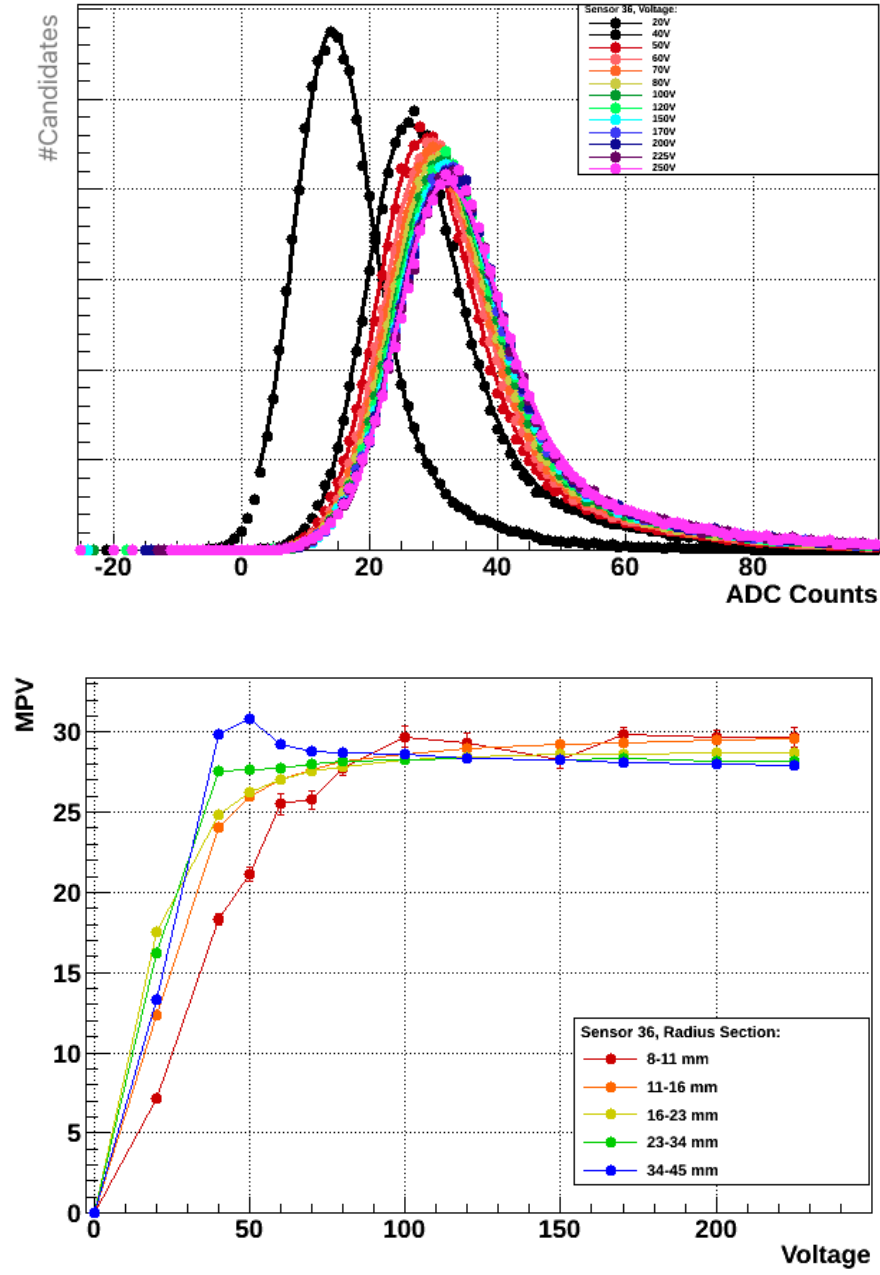


Figure 8.10: (top) Fits to the pedestal-subtracted ADC distributions at different bias voltages for an R sensor. (bottom) MPV of the ADC distribution as a function of bias voltage for an R sensor. The EDV is the voltage corresponding to 80% of the MPV of the ADC plateau value.

8.5 Determination of VELO sensor operation voltages for 2015 and 2016 operation

This section discussed the determination of the optimal operational voltages for the VELO sensors during the start of LHC Run 2. Figure 8.11 (top) shows the EDV versus fluence for all VELO sensors before the first long shutdown of the LHC in 2013. At this time the total delivered integrated luminosity was 3.4 fb^{-1} . As the fluence varies as a strong function of radius, the sensors are divided into five radial regions and a separate EDV measurement is plotted for each region of each sensor.

A number of features are clear from Fig. 8.11 (top). The EDV of the n^+ -on- n VELO sensors reaches a minimum, corresponding to type inversion [14], at around $20 \times 10^{12} \text{ MeV n}_{eq} \text{ cm}^{-2}$. In the n^+ -on- p sensors there is no minimum as type-inversion does not occur in these sensors. The innermost radial region of each sensor receives the most fluence, due its the closer proximity to the interaction point, and hence these points have the highest EDV. The EDV of the VELO sensors increases approximately linearly with fluence, after type inversion, as expected. The slope of EDV versus fluence is similar in the n^+ -on- p and n^+ -on- n sensors. Consequently, the absence of the drop in EDV, due to type inversion, in the n^+ -on- p sensors means the EDV at a given fluence is therefore higher than that of an n^+ -on- n sensor.

In both sensor types the sensor bias voltages should be increased as appropriate with accumulated luminosity to ensure the sensors remain operated above the EDV and the detector performance is optimised. Determining the EDV (or any other quantity) by radial region corresponds to an average across the region. The fluence received in the VELO follows an approximately $1/r^2$ dependence [5], where r is the radial distance from the centre of the beam pipe, and the radial regions are defined such that the fluence changes by a factor of two between regions. Hence the innermost part of each radial region receives a further $\sqrt{2}$ times the fluence than the mid-point of the region. Therefore the EDV of the inner radial edge of the most irradiated n^+ -on- n sensors is already approximately 140 V and this motivated an immediate increase in the sensor bias voltages for operations in run-2.

During the long shutdown of the LHC annealing could be expected to take place, although the amount was limited by keeping the VELO cooled. However, at the time of this analysis, the amount of annealing was not predictable due to a lack of data to tune the mathematical model. Therefore the prediction of the behaviour for the start of run-2 is based on a simple worst case assumption, *i.e.* assuming there is no annealing during LS1.

A simple solution for run-2 would be to increase the bias voltages of all sensors to the maximum immediately. However, there are a number of reasons for not doing so. Firstly, as indicated by Fig. 8.11 (bottom) the CFE of the sensors exhibits some voltage dependence. In the inner radial regions the CFE

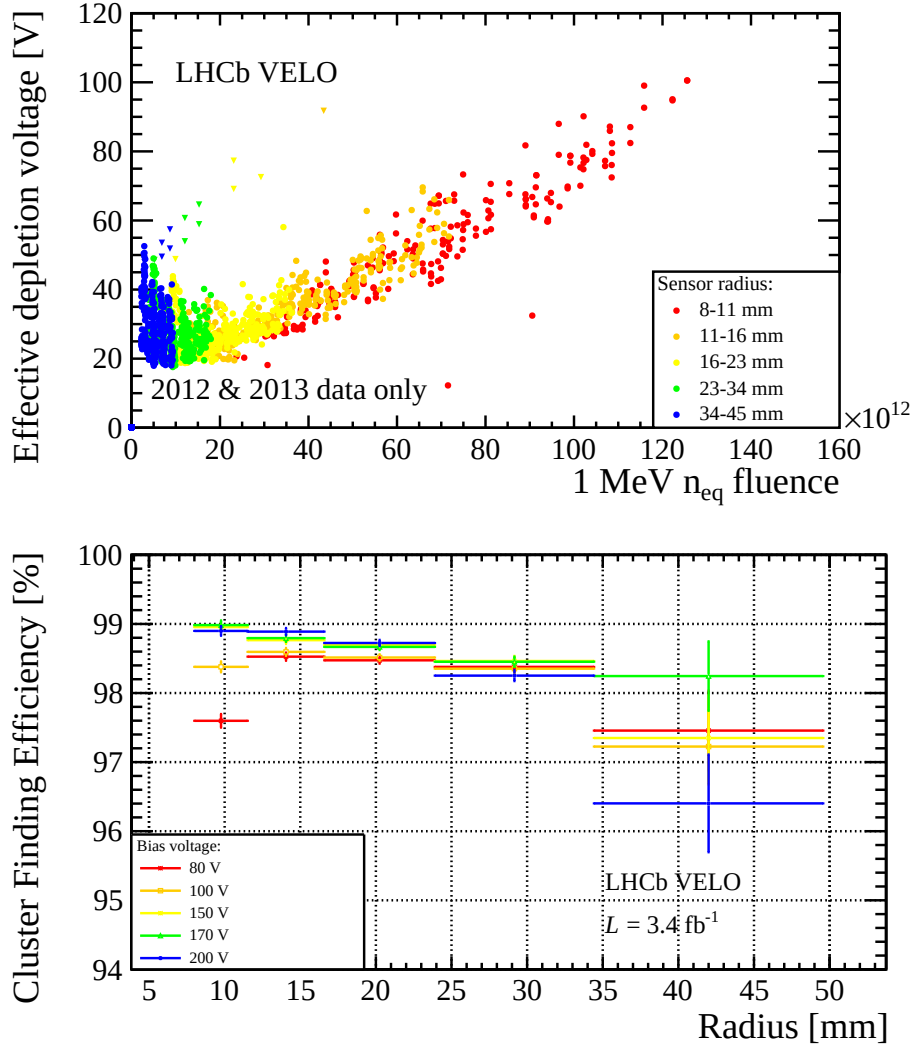


Figure 8.11: (top) EDV with fluence for all sensors before LHC LS1. Data from CCE scans during 2013 are included in the plot. The triangular points represent the n^+ -on- p sensors. (bottom) The dependence of the CFE on the sensor bias voltage for a downstream R -type sensor at the end of run-1.

increases with voltage, until the depletion voltage is reached. This is expected as for the CFE to be large the CCE must be at its maximum. However, in the outer radial regions the CFE decreases with voltage due to an unexpected second metal layer effect, which will be described in Sect. 8.8.1. Therefore in order to also optimise the CFE, the sensor bias voltages should only slightly exceed the EDV. Secondly, the cluster resolution has also been found to decrease with voltage when the sensors are operated above the EDV [57], due to a reduction

of charge sharing between strips. Lastly, the sensors had not been fully tested in operation close to their maximum design bias voltage by the time of the LHC long shut-down and indeed voltage trips had been observed during tests. These trips could permanently damage the sensors and must be avoided.

In order to determine a suitable operating voltage the EDV with fluence curve can be fitted and extrapolated to the expected fluence. The way this is performed depends somewhat on whether the sensors are n^+ -on- n or n^+ -on- p .

Method for n^+ -on- n sensors

For the n^+ -on- n sensors the extrapolation is performed with the following assumptions:

- The maximum expected values of the EDV are calculated based on the fluences at the inner radial edge of the sensors. As the change in fluence is approximately a factor of two across each radial region, the expected fluences for the inner edge of the sensors can be calculated by multiplying the fluences of the 8 – 11 mm radial regions by a factor of $\sqrt{2}$.
- Sizeable sensor-by-sensor variations on the EDVs are observed, as seen in the spread in the values at a given fluence in Fig. 8.11 (top). Focussing on the average EDV at a given fluence as the quantity of interest, the data are first binned in fluence and the uncertainties are calculated as the standard errors of the means for the different sensors within each bin.
- Only data from type-inverted sensors are considered for the extrapolation. Therefore a minimum fluence cut of $50 \times 10^{12} \text{ MeV n}_{eq} \text{ cm}^{-2}$ is applied.
- The centre-of-mass energy is taken as 14 TeV for all subsequent data taking¹. As the fluence is dependent on the centre-of-mass energy, the expected fluence is higher than that at the previous centre-of-mass energies of 7 and 8 TeV. Figure 8.12 shows the simulation predicted fluences in the inner regions of the sensors at 7 TeV (left) and 14 TeV (right). The maximum increase in the inner fluence with respect to the same sensor at 7 TeV is taken as a conservative estimate of the scaling and is found to be approximately 40% for the most downstream sensors.

Method for n^+ -on- p sensors

The extrapolation for the n^+ -on- p sensors is performed with similar assumptions to that described above. However, the minimum fluence cut is lowered to $10 \times 10^{12} \text{ MeV n}_{eq} \text{ cm}^{-2}$ as the sensors do not undergo type-inversion. In addition as there is only one n^+ -on- p sensor of each type (R or ϕ) the EDV

¹Whilst the centre-of-mass energy at the start of run-2 is 13 TeV, no fluence predictions are available for this energy, so 14 TeV predictions are used as a conservative estimate.

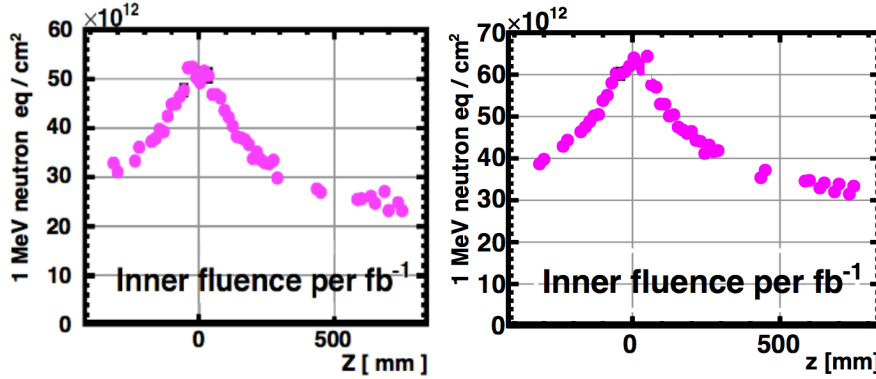


Figure 8.12: Variation of fluence with z-position of the sensor as determined from simulation at (left) 7 TeV and (right) 14 TeV. Figures from Ref. [14].

results are not averaged in bins of fluence. It should also be noted that due to limitations imposed by the VELO geometry there are no data points for the innermost radial region of the sensors. This in combination with the lower fluence at the upstream position of the n^+ -on- p sensors means that the range of fluences sampled is smaller than that from the n^+ -on- n sensors. However, the extrapolations are still performed assuming the fluence of the innermost radial region of the sensors multiplied by a factor of $\sqrt{2}$.

Recommendations for the start of run-2

Figures 8.13 and 8.14 show the results of a linear fit to the EDV of all n^+ -on- n and n^+ -on- p sensors. This assumes that the detectors are kept in a cold environment and undergo no appreciable annealing. Other parameterisations of the change in collected charge with fluence exist, such as that in Ref. [58], but the deviations from linear behaviour are typically only applicable at fluences greater than those relevant to the VELO.

Regular CCE scans must continue throughout run-2. To ensure the most consistent data, the bias voltages of the sensors should be set at the start of a year to exceed the value that is predicted for the EDV by the end of the operational year. A CCE scan should be taken every few months and at the start and end of each year's operation. For example, the first CCE scan in run-2 had no additional fluence with respect to the end of run-1, allowing to cross-check the behaviour after the long shut-down.

Conservative assumptions were made when planning for 2015 operation, 1 fb⁻¹ delivered luminosity was assumed for 2015 operation. In 2015 operation, the final delivered luminosity was 0.373 fb⁻¹. Assuming no annealing in the long shut-down of the LHC, and using the parameters of the linear fits shown in Figs. 8.13 and 8.14 the maximum expected EDV for n^+ -on- n sensors was determined to be 172 ± 12 V for R sensors and 164 ± 11 V for ϕ sensors by the

end of 2015. For n^+ -on- p sensors the expected EDVs were larger, at 261 ± 14 V for R sensors and 257 ± 17 V for ϕ sensors. It was therefore recommended that the bias voltages of the most irradiated n^+ -on- n sensors be increased to 200 V, and the bias voltages of the n^+ -on- p sensors be increased to 250 V for the start of run-2. For the n^+ -on- p sensors this is slightly lower than the expected EDV for these sensors, but was still a conservative estimate, given that some amount of beneficial annealing was already known to have occurred between the end of run-1 and the start of run-2.

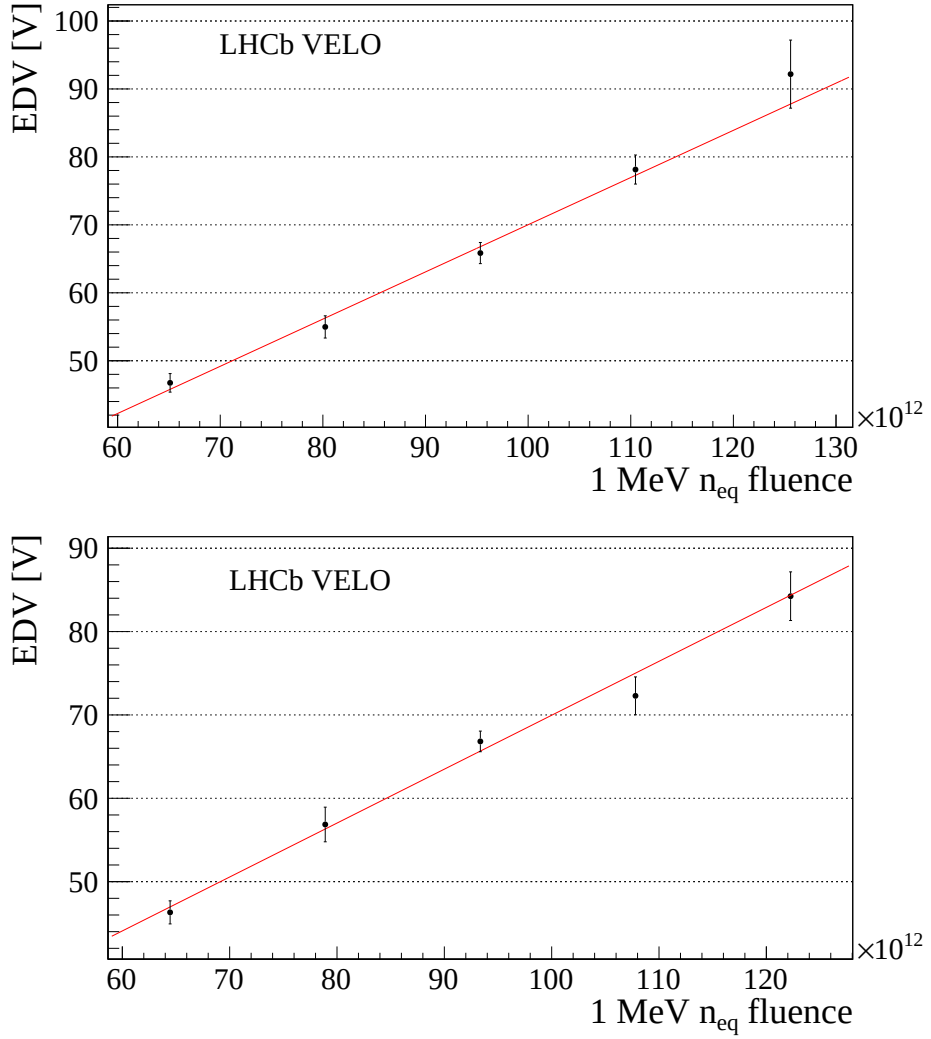


Figure 8.13: Linear fit to the EDV of the inner radius region of (top) R and (bottom) ϕ n^+ -on- n sensors. Fit based on January 2013 CCE scan data.

Figure 8.15 shows the expected distribution of EDVs for all sensors after an additional 1 fb^{-1} after run-1. As described previously, it is beneficial

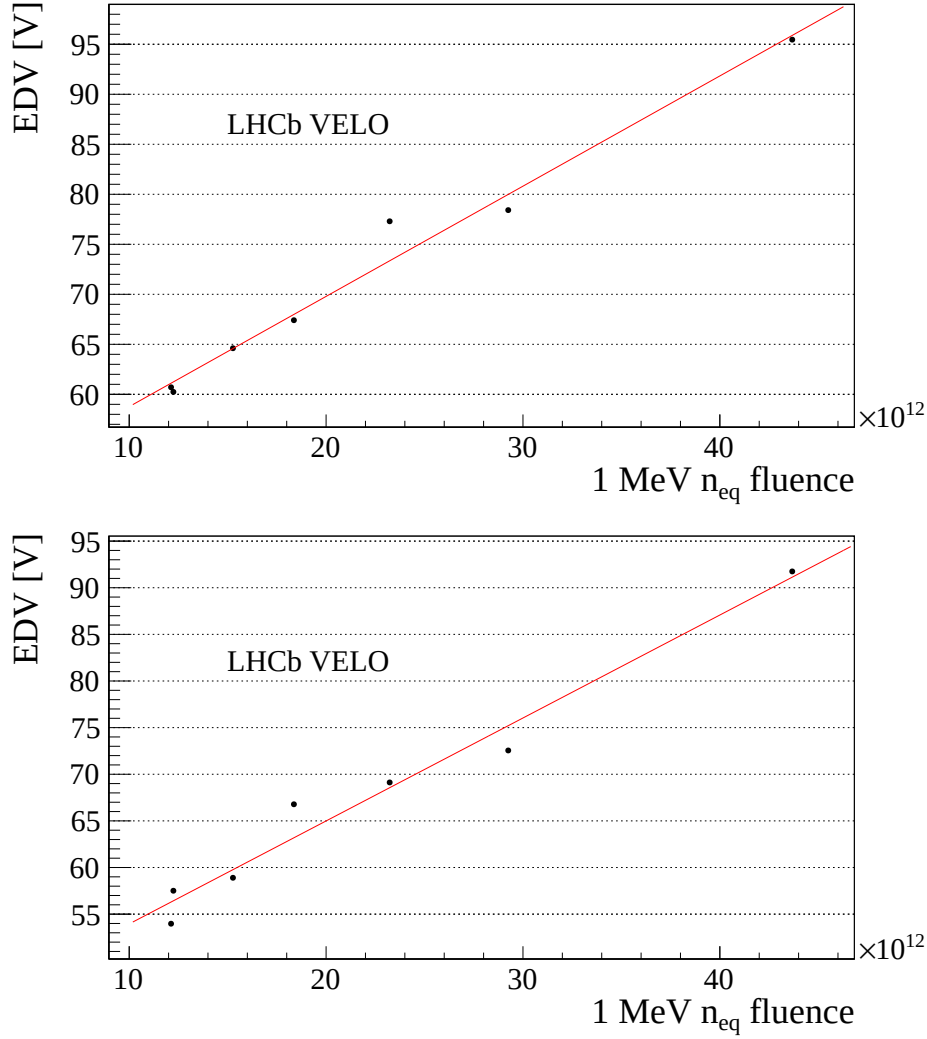


Figure 8.14: Linear fit to the EDV of the inner radius region of (top) R and (bottom) ϕ n^+ -on- p sensors. Fit based on January 2013 CCE scan data.

to minimise the increase in the bias voltages where possible. This can be achieved by dividing the sensors into groups, where the grouping is calculated based on the fluence. In Fig. 8.15, the n^+ -on- n sensors are divided into three groups according to their received fluence per fb^{-1} at 7 TeV, with the groups corresponding to < 20 , $20 - 30$ and > 30 $\text{MeV } n_{eq} \text{ cm}^{-2}$. A fourth group contains the n^+ -on- p sensors only. As the largest radiation dose is around the interaction point, the second group of n^+ -on- n sensors includes a set of sensors at either end of the VELO (stations 0–3 and 21–33). Within this scheme only one group of n^+ -on- n sensors needed to be operated at 200 V from the start of run-2, with the rest remaining at the nominal 150 V. The n^+ -on- p sensors

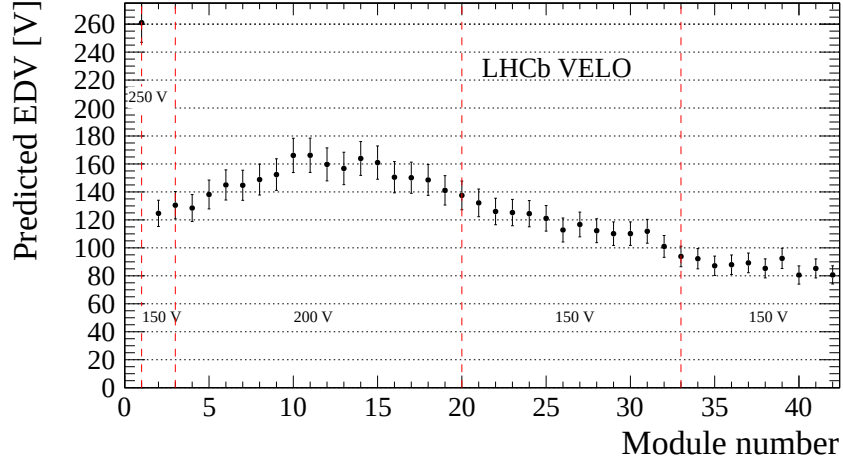


Figure 8.15: Predicted EDV with module number for all VELO sensors after a luminosity of 4.4 fb^{-1} grouped according to fluence at the sensor. Recommended operating voltages for the start of run-2 are overlaid. Each module contains one R and one ϕ sensor, and are numbered from upstream to downstream.

needed to be operated at 250 V as described previously.

After the start of run-2 operation, the analysis of the first CCE scan in run-2 showed a significant amount of beneficial annealing (see Fig. 8.16). Given that only 0.373 fb^{-1} was delivered during 2015 operation, the same operational bias voltage configuration as 2015 was decided to be used in 2016 operation.

8.6 Determination of VELO sensor operation voltage for 2017 operation

With data from several CCE scans collected in run-2, it is possible to tune a mathematical model to describe the EDV behaviour in a silicon sensor. The model that was used here is called the Hamburg model [13]. The Hamburg model can model the silicon sensor properties related to radiation damage effects, such as annealing and type inversion. The Hamburg model uses as its input chemical information (e.g. doping concentration) of the silicon diode, geometrical information (e.g. thickness of sensor), properties of the silicon lattice, and the historical or anticipated temperature and luminosity profiles. The model calculates the depletion voltage of the sensors. An example EDV versus fluence plot, overlaid with the Hamburg model, produced with data from the October 2016 CCE scan is shown in Fig. 8.17.

The Hamburg model describes the EDV in the high fluence region well, however, it does not fit the region near the type inversion point. The Hamburg model indicates that in the case of an ideal homogeneous silicon diode, the

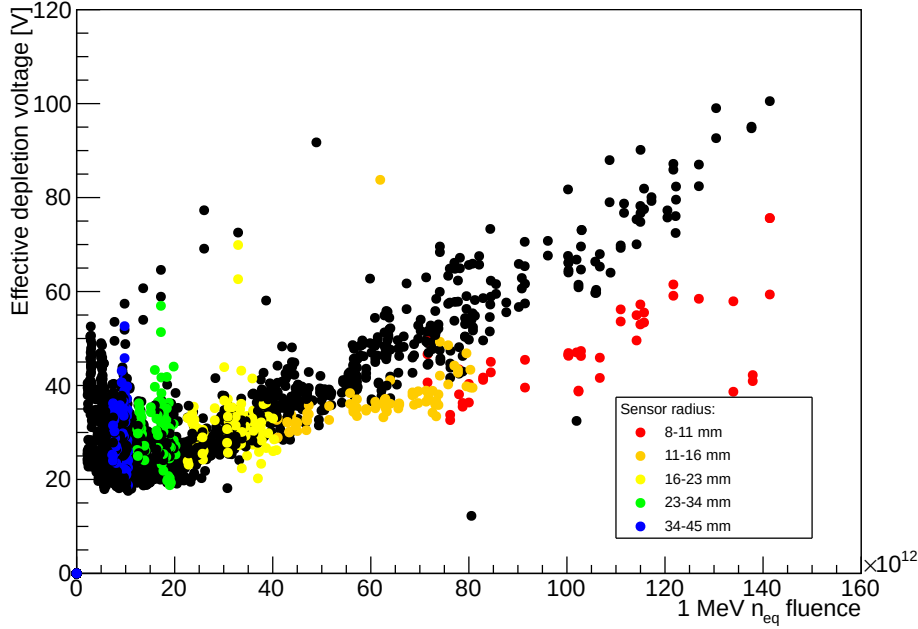


Figure 8.16: Comparison of the last EDV versus fluence plot from the 2012 and 2013 CCE scans (black dots) and the first CCE scan in run-2 (June 2015 scan, color dots). In the 2012 and 2013 scans the sensor EDVs show a consistent trend as function of versus fluence. However, in the June 2015 scan the sensor EDVs in the higher fluence region are lower than in previous scans, and the EDVs in the lower fluence region prior to type inversion are higher than in the previous scans. This is the consequence of beneficial annealing.

entire silicon can be fully depleted at the type inversion point, therefore no additional bias voltage needs to be applied to the silicon diode to drive the electron-hole pairs drift to the edge of silicon. However, in a real world detector, as the silicon volume is not infinitesimal, the fluctuation of hit positions and the complexity of defects make the silicon sensor not homogeneous. Consequently, the measured EDV of the sensors always has a non-zero value.

Using the CCE scan data the Hamburg model was adjusted to match the trend in the high fluence region. Using this adjusted Hamburg model, and given the history of luminosity (measured by van der Meer scans [59]) and temperature (measured with VELO temperature sensors), it is possible to simulate the evolution of the depletion voltage of each sensor as a function of time.

Figure 8.18 shows a result of the simulation of the depletion voltages of all n^+ -on- n type VELO sensors up to March 2017. From this plot, it is seen that the depletion voltage of the innermost part of the sensors (radial region of 8

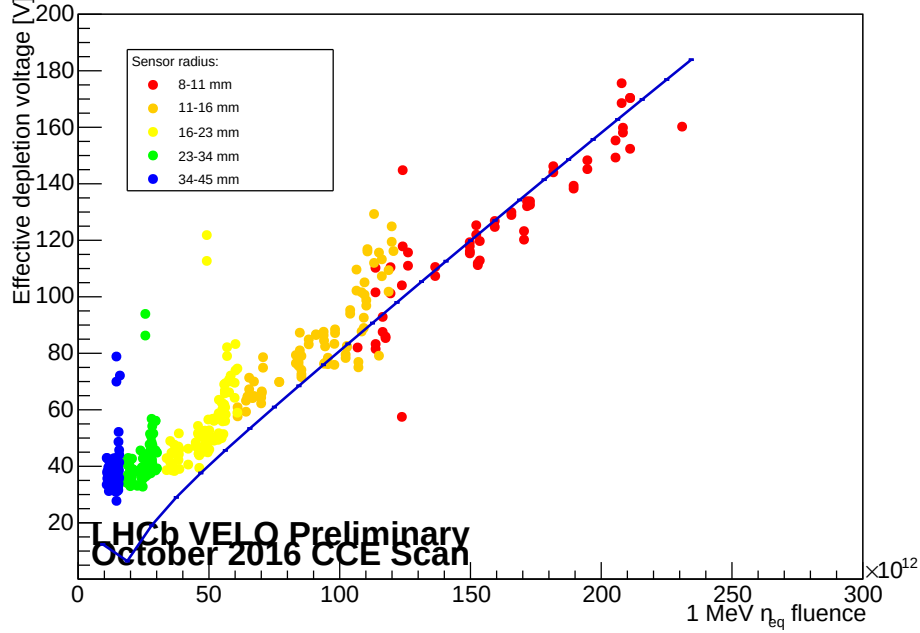


Figure 8.17: EDV versus fluence plot made with data from October 2016 CCE scan overlaid with Hamburg model simulation plot.

- 11 mm, red lines) drops very quickly at the beginning of LHCb operation, and the depletion voltage of the innermost part of all sensors reached the type inversion point during 2011 operation. Afterwards the depletion voltage in these parts increased during data taking. The outermost part of the sensors (blue lines) only reached type inversion in 2016. The decrease of depletion voltage, due to the beneficial annealing effect during shut-downs, is also clearly seen. In the long shut-down starting from 2013, the depletion voltages in the innermost parts dropped very quickly at the beginning of the shut-down, in this period interstitials and vacancies recombined. After middle of 2014, the depletion voltages in the innermost parts stopped dropping. At the start of 2015, depletion voltages even started slowly increasing. In this period more complex defects were formed, and the annealing effect was no longer beneficial. After the start of 2015 and 2016 operation, more interstitials and vacancies were introduced in the silicon sensors, therefore, during the 2015 and 2016 winter shut-down, the annealing effect became beneficial again.

8.6.1 Recommendations for 2017 and 2018 operation

At the end of 2016 operation, the EDV in the innermost part of the VELO sensors had already reached 200 V, therefore another increase of operation

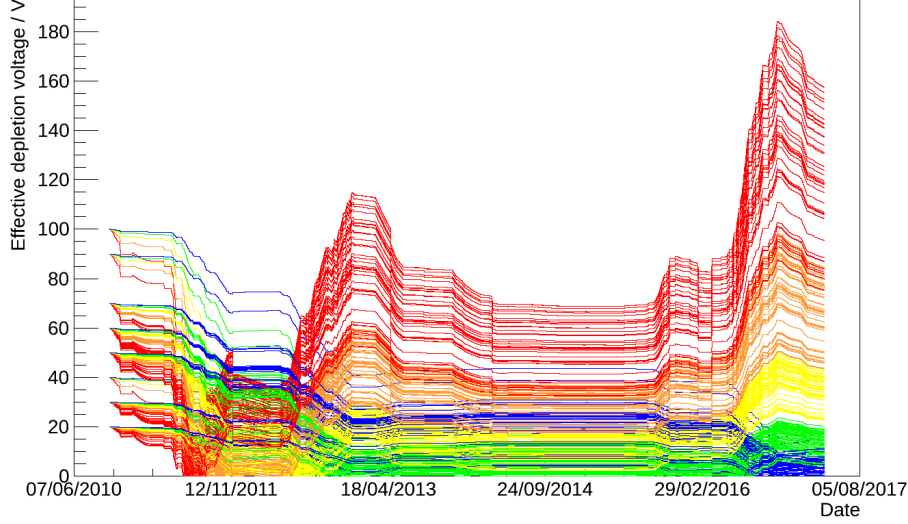


Figure 8.18: Evolution of depletion voltage of all sensors in five radial regions (red: 8 - 11 mm, orange: 11 - 16 mm, yellow: 16 - 23 mm, green: 23 - 34 mm, blue: 34 - 45 mm) with time using the Hamburg model.

voltage at the start of 2017 operational was necessary.

During 2016 operation, 1.91 fb^{-1} of luminosity was delivered to LHCb. Assuming that in 2017 and 2018 the same amount of luminosity can be delivered to LHCb, and the same temperature condition can be kept as 2016, the depletion voltage of the VELO sensors by the end of run-2 operation can be estimated. Considering the facts above, the evolution of depletion voltage of the innermost tips of the sensors is predicted and shown in Fig.8.19. It is indicating that if the LHC and LHCb is operating as expected, by the end 2017, there is no need to operate the VELO beyond 300 V, and by the end of run-2 operation, the maximum depletion voltage would be still below 400 V. The breakdown voltage of the VELO sensors is expected to be beyond 500 V, and all elements of the system are specified to 500V or beyond, therefore the VELO sensors are expected to remain fully depleted until the end of run-2 operations.

The further study of CFE shows that the second metal layer effect had become more stable with higher voltage, therefore to simplify the operation a decision to use a single voltage for all sensors was made. As a result of this study, it was recommended to operate all VELO sensors at 300 V in 2017. This choice satisfies the minimum depletion voltage requirement of the most irradiated sensor tips.

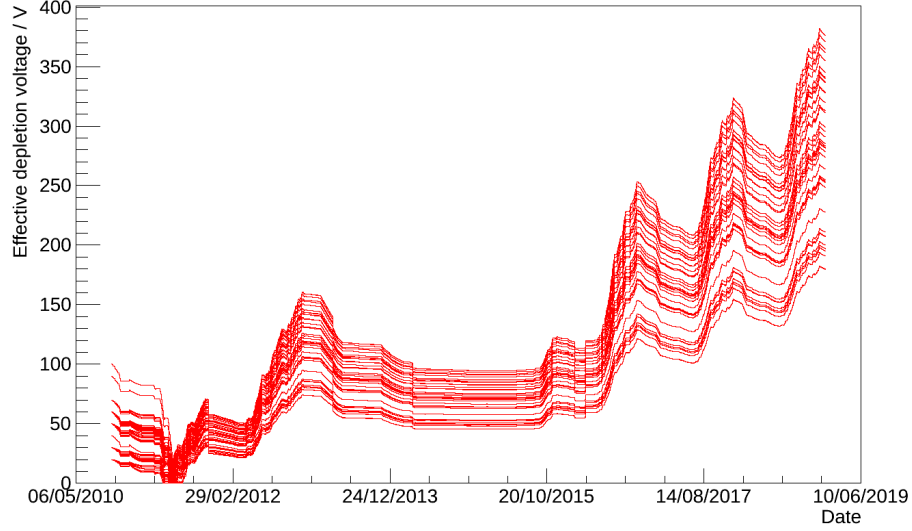


Figure 8.19: Prediction of depletion voltage of the innermost edge of all sensors till the end of 2018 operation. In this prediction, the temperature and luminosity in 2017 and 2018 is assumed to be the same as in 2016. The fluence exposed to the innermost edge is $\sqrt{2}$ times large than the average fluence in 8 - 11 mm radial region.

8.7 Optimisation of the CCE scan in run-2

At the end of run-1 the CCE scan had 13 test voltage steps, with steps at: 10 V, 20 V, 30 V, 40 V, 50 V, 60 V, 70 V, 80 V, 100 V, 120 V, 150 V, 170 V and 200 V. Due to constraints on the beam time taken to perform the scan this is likely to be the maximum number of steps. The number of steps is also agreed with the LHCb silicon tracker that performs similar scans simultaneously. Whilst it is possible to keep the total scan time constant and increase the number of steps whilst decreasing the time spent at each step, the statistics collected for each voltage may not be sufficient for every MPV fit to succeed. If several data points in Fig. 8.10 are missing, the fitted value of the EDV may become unreliable. This typically occurs when the fits to the MPV distributions also fail due to a lack of statistics. Poor quality fits (so called ‘failed fits’) are automatically excluded. With the configuration used in the January 2013 CCE scan there are very few failed fits, as indicated by the black triangles in Fig. 8.20 (top).

At the start of run-2 operations the VELO had a group of n^+ -on- n sensors operating at 200 V as described in Sect. 8.5. In CCE scans the maximum voltage step must remain beyond the predicted EDV such that the true EDV

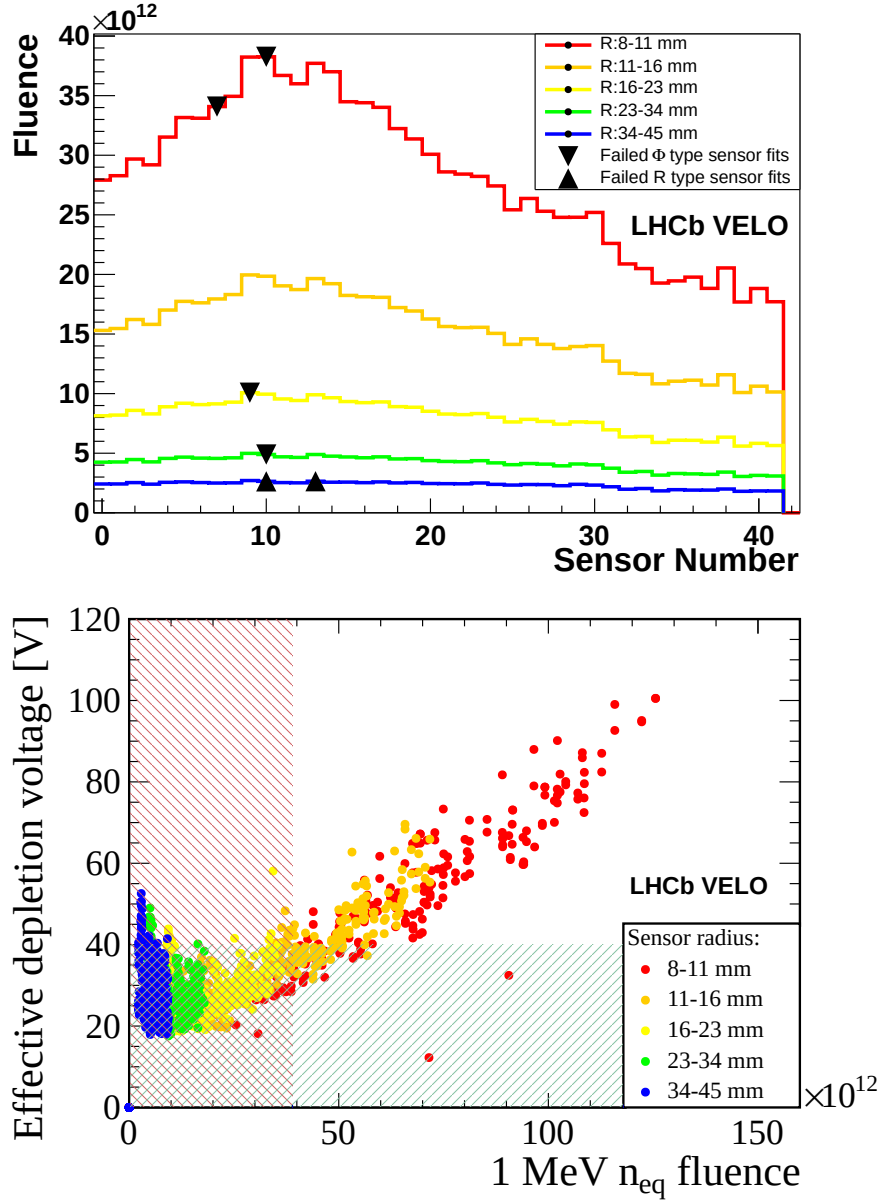


Figure 8.20: (top) Failed fits in the January 2013 CCE scan as function of sensor number for the different radial regions. The fluence values on the y-axis are based on 1 fb⁻¹ of delivered luminosity at 7 TeV, but increase proportionally for all sensors and in all radial regions. (bottom) Change in EDV with fluence for all sensors before LS1, with the excluded regions of fluence (vertical shade) and EDV (horizontal shade) described in the text.

can be determined². To ensure that the high voltage steps in the CCE scan at the start of run-2 were beyond the operational voltage, it was recommended that steps were added at 225 V and 250 V. As a result, two steps from the end of the run-1 CCE scan configuration needed to be removed. The CCE scan has to be performed with stable beams used normally for physics, hence the time taken by the scan must be minimised. Each CCE scan takes about one hour, and each step in a CCE scan collects 200,000 events. Each event contains 20 to 40 tracks and hence the total number of tracks collected is several million per step. Removing any steps will increase the number of failed EDV fits. A method for minimising the effect of removing steps is described below.

According to the LHC operations plan before the start of run-2, the maximum total luminosity delivered by the end of run-2 would be 10 fb^{-1} about three times larger than that collected by the end of run-1. As a result, the vertical shaded region in the left of Fig. 8.20 (bottom) will not receive a larger amount of fluence than that already received by the highest fluence sensors at the end of run-1. This region largely contains data from the outer radial regions of the sensors and also corresponds to the shaded horizontal regions in Fig. 8.21. As sensors in this fluence regime have already been closely studied using data from previous CCE scans, the data from sensors in this region is of limited interest in the future.

Therefore an increase in the number of failed fits in this low fluence region can be permitted without an effect on the quality of the results obtained from the data. Figure 8.21 shows the fits that fail when the January 2013 CCE scan analysis is repeated with a single step removed. The number of failed fits for all sensors and for sensors out of the low fluence region are listed in Table 8.2.

Removed step	# Failed fits	% Failed fits	# Failed fits outside of the shaded area	% Failed fits outside of the shaded area
10 V	10	4.8%	3	3.8%
20 V	10	4.8%	2	2.5%
30 V	12	5.7%	3	3.8%
40 V	12	5.7%	6	7.6%
50 V	13	6.2%	3	10.1%
60 V	9	4.3%	4	5.1%
70 V	7	3.3%	4	5.1%
80 V	13	6.2%	7	8.9%

Table 8.2: Numbers and percentages of failed fits when a single voltage step is removed from the January 2013 CCE scan analysis.

²Although the bias voltage of the n^+ -on- p sensors was 250 V additional steps to account for these sensors are not required, as the VELO geometry permits the EDV from being determined in the radial region where this bias voltage is relevant.

As well as increasing the number of failed fits removing voltage steps could also bias the results obtained from successful fits. Figure 8.22 shows the differences in EDV determined from the January 2013 CCE scan before and after a step is removed. Again, any biases in the shaded regions of Fig. 8.21 and Fig. 8.22 are of limited concern.

From Figs. 8.20 and 8.22 it can be seen that removing steps below 40 V will have a limited effect on the EDV determination for high fluence sensors. The effect of removing two steps below 40 V simultaneously was also studied, and it was found that all of the three possible combinations (10 V and 20 V, 10 V and 30 V, 20 V and 30 V) have a small impact on the results in the high fluence and high EDV region. Therefore to avoid removing two successive steps, it was recommended to remove the steps at 10 V and 30 V in the CCE scan at the start of run-2. The failed fits and differences in EDV resulting from removing both of these steps are shown in Fig. 8.23. The plots show good consistency with the nominal CCE scan configuration with only small changes in the results for high fluence sensors.

At the end of 2016 operation, The study of the CCE scan data at the end of 2016 operation showed that the bias voltage of the innermost part of the most irradiated VELO sensors has reached 200 V, therefore a further increase of operational bias voltage was implemented. In 2017 operation the bias voltage of all sensors has been increased to 300 V, hence a further increase of CCE step voltages was again needed. Two additional high voltage steps 280 V and 320 V (later changed to 300 V and 350 V for future CCE scans after the VELO operation team agreed to have 300 V as the operational bias voltage for all sensors) were added to the CCE scan configuration. A similar study to that described above showed that removing the steps at 50 V and 70 V have the smallest impact on the EDV distribution, Therefore, the 2017 CCE scan steps are 20 V, 40 V, 60 V, 80 V, 100 V, 120 V, 150 V, 170 V, 200 V, 225 V, 250 V, 280 V, 320 V (with the last two steps changed to 300 V and 350 V for the remaining CCE scans in 2017).

As the fluence delivered to the VELO sensors continues to increase throughout the remainder of run-2, the EDVs of n^+ -on- n sensors in the highest fluence group will exceed 300 V, and a further increase in the operation voltages of these sensors will be required. To ensure that there are still steps beyond the operational voltage additional high voltage steps will need to be added in future CCE scans. Whenever this is required the procedures described in this section can be repeated and the least important step from the current configuration can be removed. Table 8.3 summarises the CCE scans performed for the LHCb VELO so far.

³In November 2015 two CCE scans were performed with two different trigger configurations but similar delivered luminosities. Before this scan, events are proceed with L0 trigger and high level triggers during data taking. Due to the limited readout bandwidth, only about 10% of events that passed L0 trigger can be proceed with high lever triggers. Starting from this scan onwards, events passed L0 trigger are buffered in the disk to be proceed with high

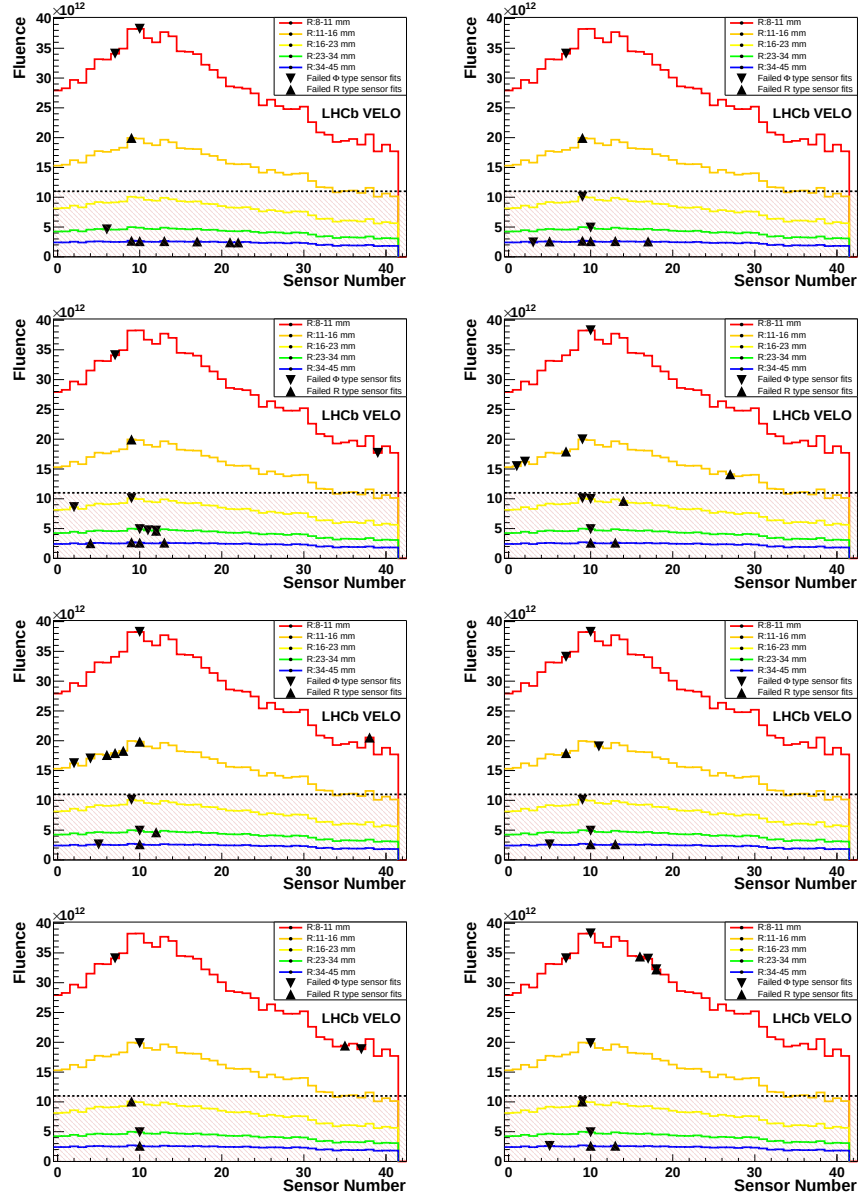


Figure 8.21: Failed fits when removing the steps at 10 V, 20 V, 30 V, 40 V, 50 V, 60 V, 70 V and 80 V. The shaded regions indicate sensors which will not receive more fluence by the end of run-2 than the highest fluence sensors at the end of run-1.

levels triggers offline. Therefore the statistics from this scan onwards are increased by a factor of 2.5 while the time taken is similar to previous scans.

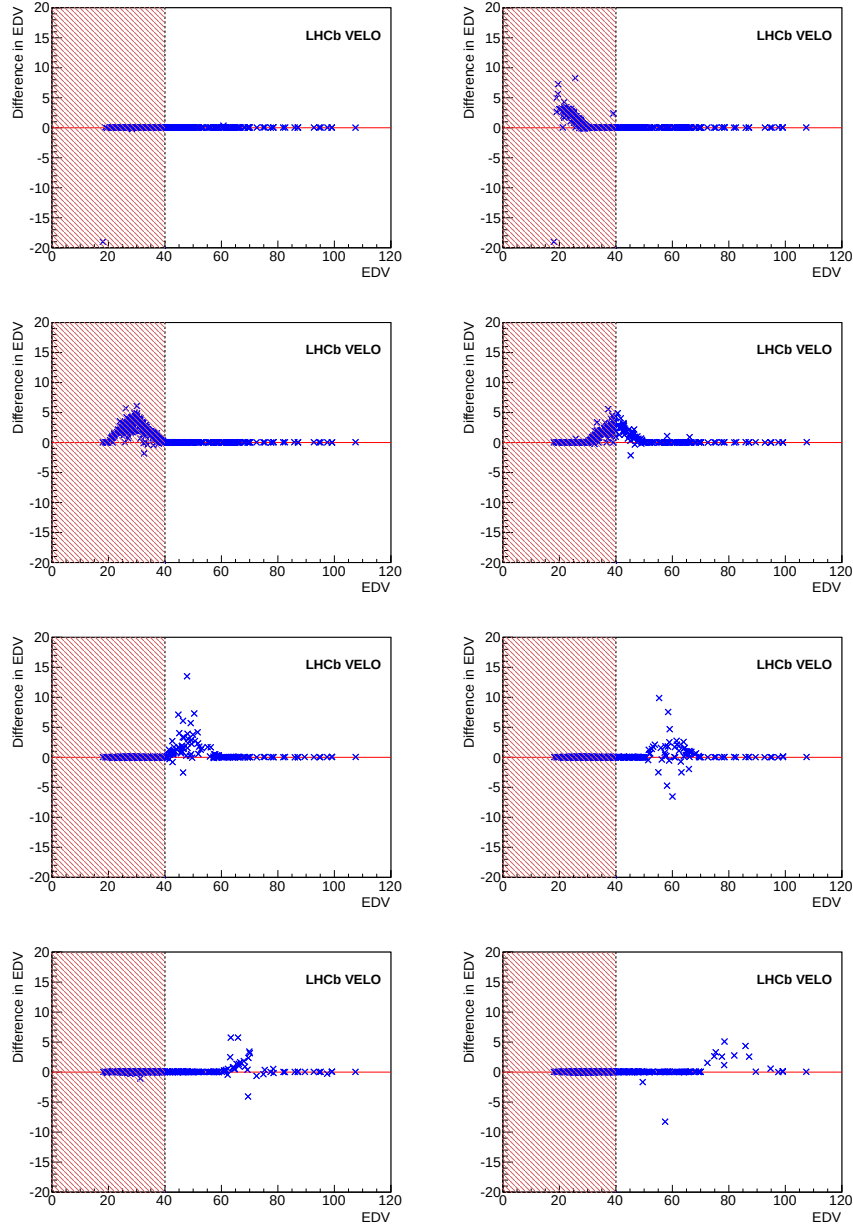


Figure 8.22: Differences in EDV before and after removing the steps at 10 V, 20 V, 30 V, 40 V, 50 V, 60 V, 70 V and 80 V. The shaded regions indicate EDVs below 40 V.

8.8 Study of cluster finding efficiency of VELO sensors

The reconstructed tracks in the LHCb VELO rely on the VELO sensor clusters that are found on the particle's trajectory. It is important that the CFE remains

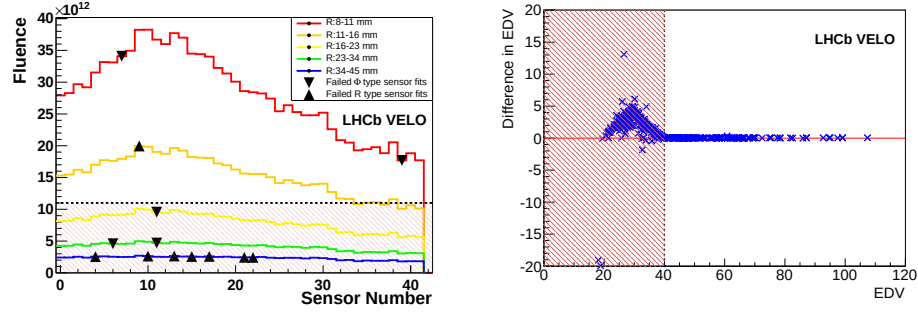


Figure 8.23: Failed fits and the difference in EDV when the steps at 10 V and 30 V are removed.

date	$\sqrt{s}$ [TeV]	Del. $\mathcal{L}$ [fb^{-1}]	Bias voltage steps
2010	7	0	0, 10, 30, 50, 70, 100, 150
April 2011	7	0.04	
July 2011		0.4	0, 10, 20, 30, 40, 50, 60, 70,
September 2011		0.8	80, 90, 100, 150
October 2011		1.2	
April 2012	8	1.2	0, 10, 20, 30, 40, 50, 60, 70, 80, 100, 120, 150
July 2012	8	1.9	
September 2012		2.7	10, 20, 30, 40, 50, 60, 70, 80,
January 2013		3.4	100, 120, 150, 170, 200
June 2015	13	3.4	
July 2015		3.4	
October 2015		3.7	
November 2015		3.8	20, 40, 50, 60, 70, 80, 100,
November 2015 ³		3.8	120, 150, 170, 200, 225, 250
April 2016		3.8	
August 2016		4.8	
October 2016		5.7	
June 2017	13	5.7	20, 40, 60, 80, 100, 120, 150, 170, 200, 225, 250, 280, 320

Table 8.3: List of CCE scans from 2009 to the start of 2017 operation. The determination of CCE scan steps are described in Sect. 8.7.

as high as possible. Before irradiation in LHCb the measured VELO sensors CFE was as high as 99.97% [6]. After irradiation the CFE is expected to drop with increasing fluence. The CFE can be plotted as a function of position on each sensor. Figure 8.24 shows the CFE on two sensors mounted on the same module measured at the end of 2016 operation. It is found that on downstream

modules the CFE is seen to have a very clear drop in the outer part of R -type sensors. Since the outer part of the sensors have a smaller fluence compared with the inner parts, this is a phenomena that was not expected and is not related to bulk radiation damage. This drop can be explained by charge loss to the second metal layer and will be described in Sect. 8.8.1.

CFE plots can also be used for spotting problematic sensors. Beside radiation damage of the silicon, the damage of electronics can also affect the performance of VELO sensors. For example, Fig. 8.25 (top) shows a low efficiency in a large area, this may be a problem related to sensor readout, and on Fig. 8.25 (bottom) two unresponsive strips are clearly seen. These problems have been reported to the VELO operation team and have been taken into account during operations.

Summaries of the average CFE for all R -type and Φ -type sensors studied with the October 2016 CCE scan data are shown in Figs. 8.27 and 8.26. For both types of sensors, there are visible drops in CFE at low radius due to the bulk radiation damage in that region, with the most damaged sensors being around the interaction region. By comparing plots produced at different bias voltages it can be seen that this drop can be partially recovered by increasing the operational voltage. For R -type sensors there is a wide spread in efficiencies measured at high radius with the most inefficient sensors being located in the downstream part of the VELO. The CFE in this high radius region is not improved by increasing the bias voltage, though neither is it harmed. For Φ -type sensors a small drop in efficiency at 17 mm is observed which is due to the boundary region between inner strips and outer strips.

8.8.1 Second metal layer effect of VELO sensors

The CFE of the sensors at the operational voltage can also be studied using standard physics collision data, allowing much larger statistics to be used than in the CCE scan data. With a large sample a closer look at the outer region of R -type VELO sensors becomes possible. Figure 8.28 shows a comparison between a CFE plot made with physics data and the second metal routing line pattern on an R -type sensor. It is clearly visible that on the R -type sensors the CFE is high where the routing lines are absent.

The data can also be visualised using two characteristic variables. The distance from the point where the extrapolated particle trajectory hits the sensor to the nearest sensor strip can be determined, known as the distance to strip. The distance from the point where the extrapolated particle trajectory hits the sensor to the nearest second metal routing line can also be calculated, known as the distance to RL. Figure 8.29 shows the CFE as function of the distance to strip and the distance to RL. The CFE is notably lower for particles that pass close to routing lines and far away from strips.

On the VELO sensors the first metal layer and second metal layer are separated by a layer of SiO_2 . After irradiation acceptors at the SiO_2 and silicon

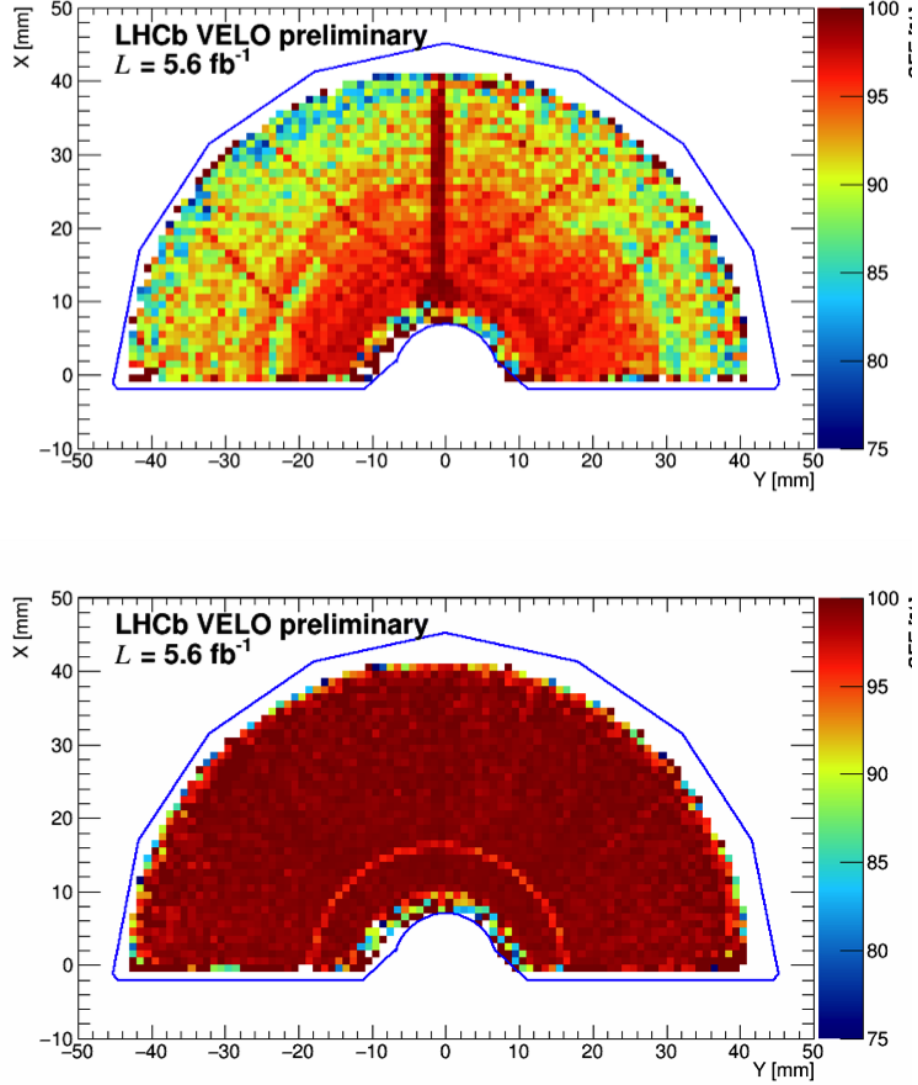


Figure 8.24: CFE as function of position for (top) sensor 36 (R -type sensor) and (bottom) sensor 100 (Φ -type sensor). These two sensors are on the same module. The CFE in the outer part of sensor 36 has a clear drop, while the CFE in sensor 100 does not. The plot is made with CCE scan data and the statistics are limited. Plots of all other sensors are shown in Appendix. A and Appendix. B.

interface are introduced which trap electrons in the interface layer. Then, due to the lower mobility of charge carriers after defects are introduced, for particles that are further away from strips the charges might be less able to move to the

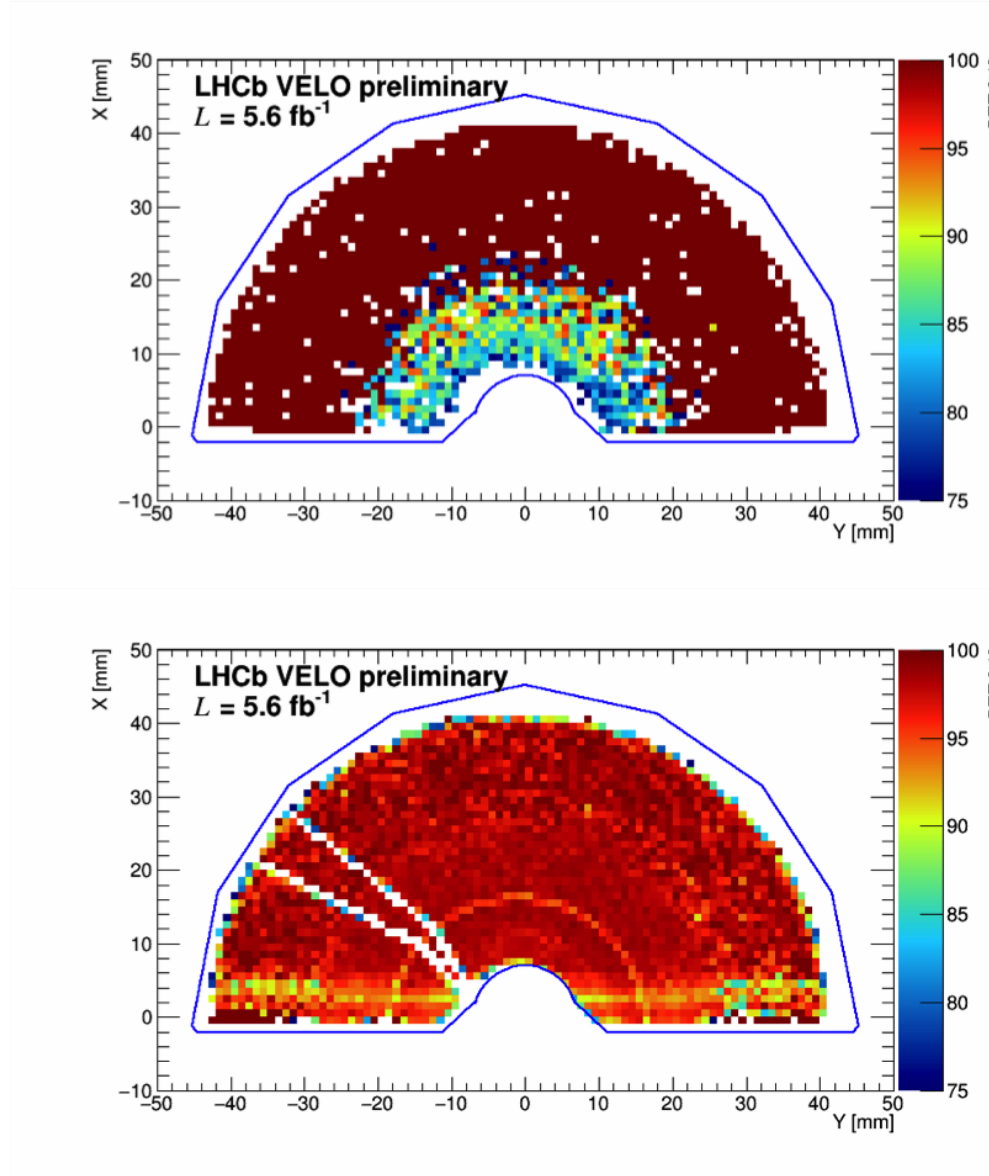


Figure 8.25: CFE as function of position for (top) sensor 76 (Φ -type sensor) and (bottom) sensor 97 (Φ -type sensor). Both sensors have some problems that may relate to their electronics readout chain. Plot made with October 2016 CCE scan data.

strips. The SiO_2 layer might also be less insulating after irradiation, some of the charges deposited in the silicon diode were then collected by the routing lines that connected to the inner part of sensor instead of the strips (see Fig. 8.30).

As a result of this effect charges are collected on the routing lines by particles passing through the outer region of the sensor. As these routing lines

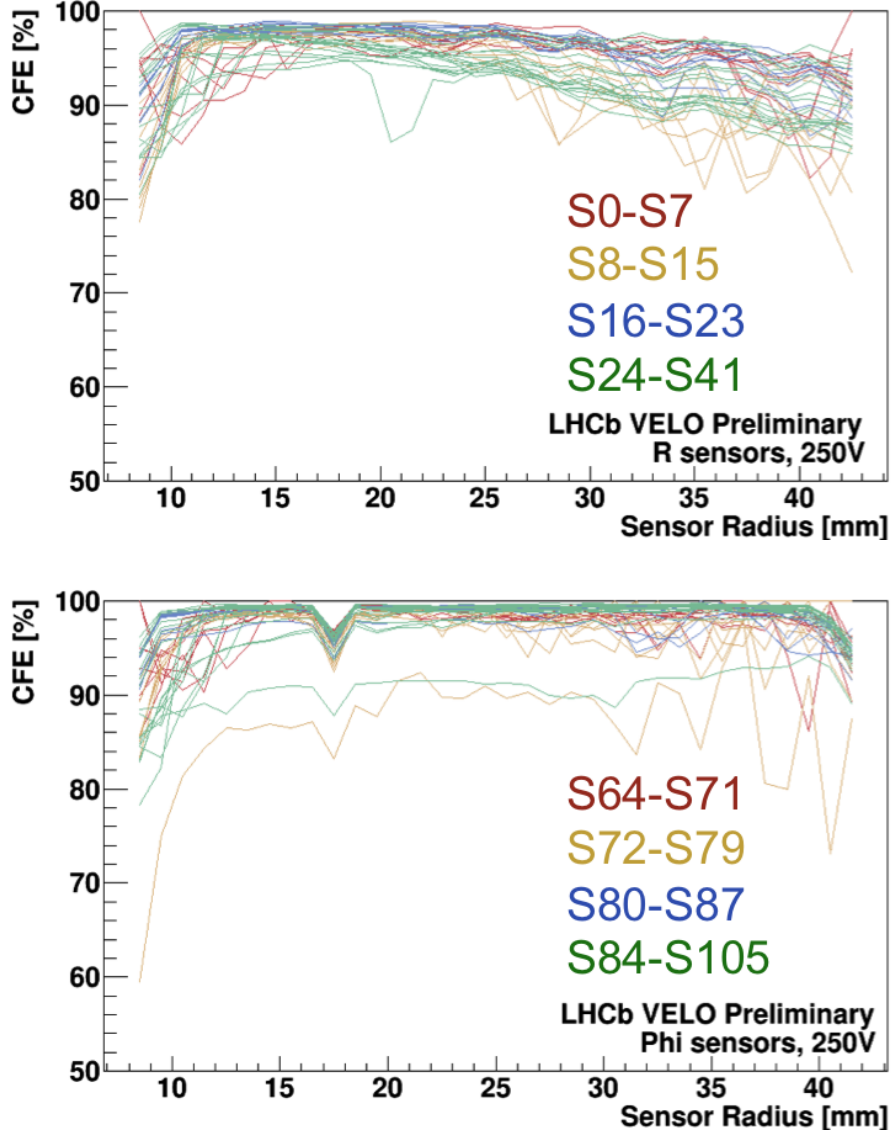


Figure 8.26: Summaries of CFE versus sensor radius for (top) R -type and (bottom) Φ -type sensors. The two curves with low efficiency on Φ -type sensors plots are the two problematic sensors shown in Fig. 8.25. Plots of each individual sensor are shown in Appendix. C and Appendix. D.

are connected to inner region strips, the signals are reconstructed as fake signals at inner strips. This is visible in the R -type sensor ADC distributions shown in Fig. 8.31 (top). However, if the ADC distribution is plotted for clusters that are associated with tracks these fake low ADC signals can be removed (see Fig. 8.31 (bottom)), and thus do not affect the tracking performance.

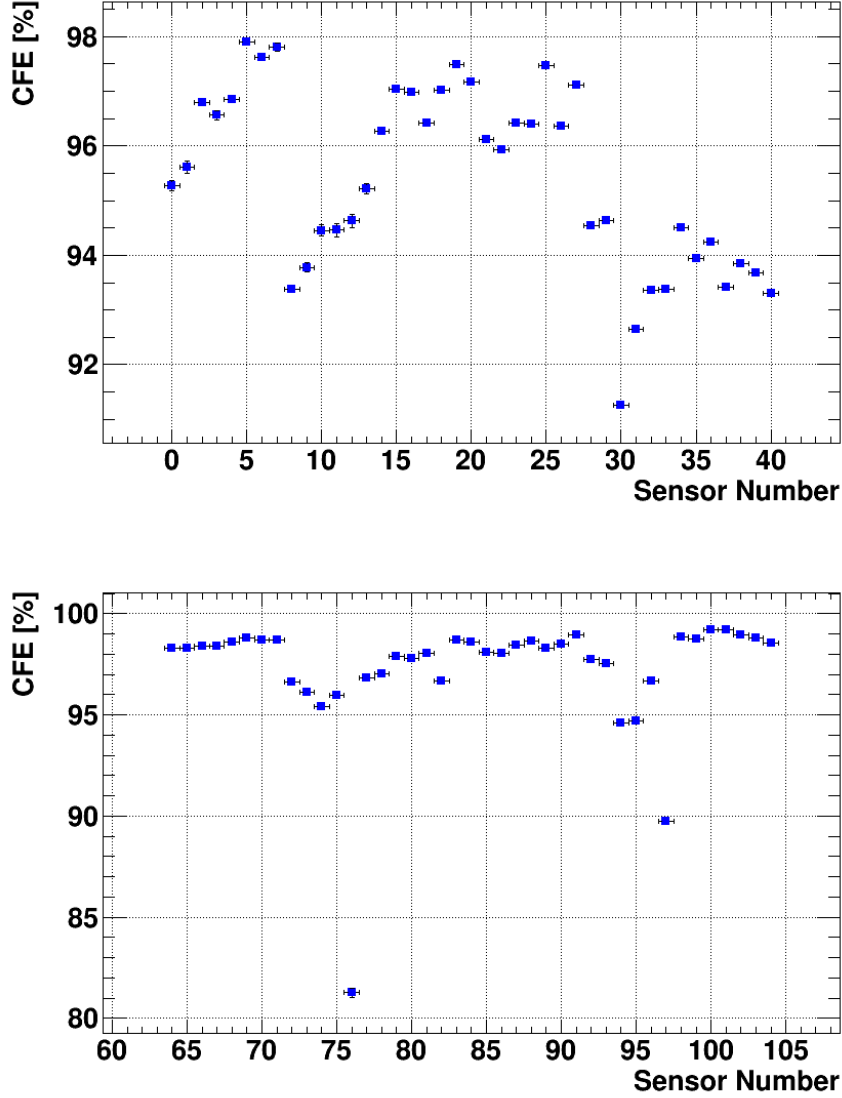


Figure 8.27: Summaries of CFE versus sensor number for R -type (top) and Φ -type (bottom) sensors. Plots are made with the October 2016 CCE scan data

8.8.2 Impact from the geometry of VELO detector

The different CFE patterns observed on the VELO sensors are related to the position of the sensor and the geometry of detector. Figure. 8.32 to Figure. 8.35 show the coverage of tracks in the sensor and the CFE as a function of pseudo-rapidity and sensor radius for four sensors in different regions of the VELO

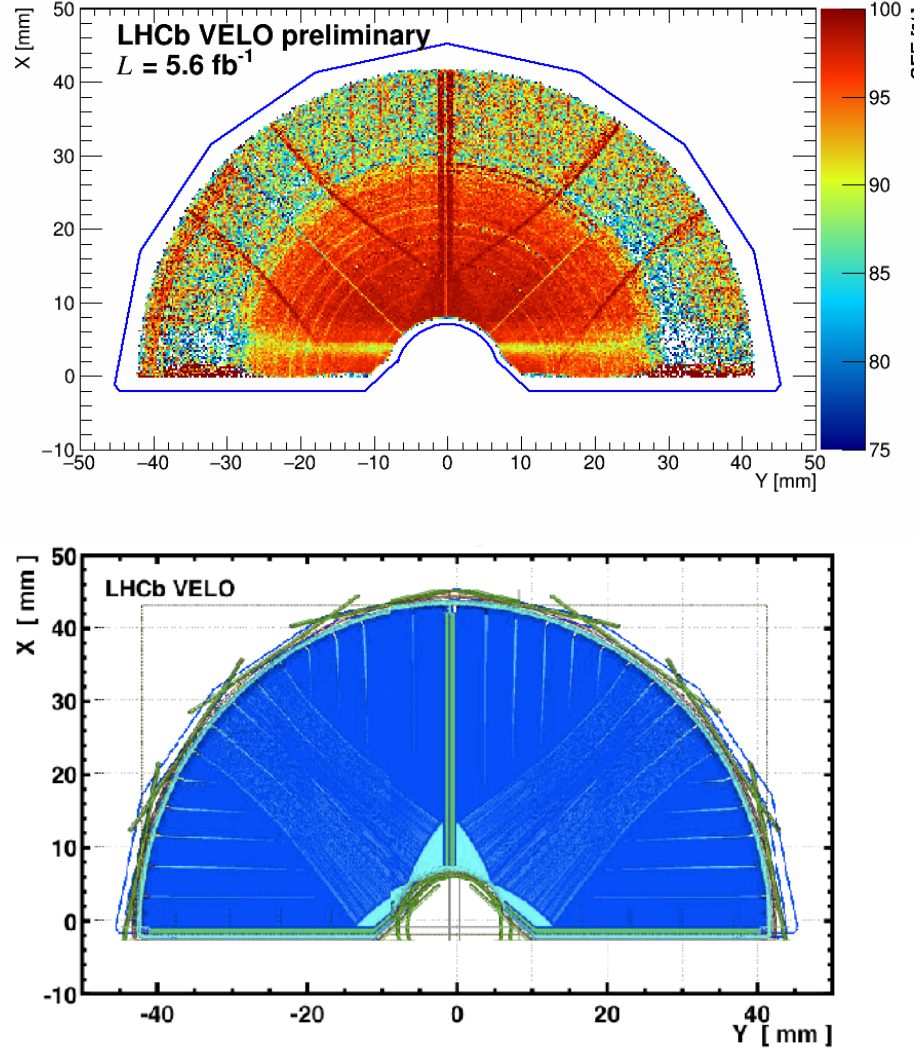


Figure 8.28: (top) CFE as function of position for sensor 32 (*R*-type sensor) made with physics data taken at the beginning of 2017 operation. (bottom) Routing line map on a *R*-type sensor. On this plot, dark blue indicates routing lines, and light blue is the area where routing lines are absent. Bottom plot is reproduced from Reference [6].

detector.

For sensors at the upstream (Fig. 8.32) and downstream ends of the detector (Fig. 8.35) tracks reaching the very inner part of the sensor cannot be reconstructed as this forms the edge of the geometrical acceptance of the detector. A low efficiency is measured with a large uncertainty in the inner region due to this lack of reconstructed tracks from the primary vertex and the presence

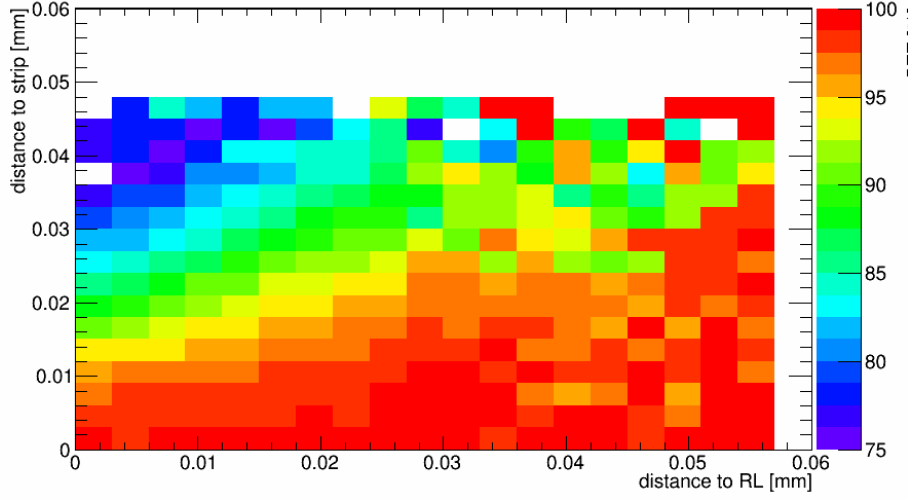


Figure 8.29: CFE as a function of distance to strip and distance to RL for sensor 36 (R -type sensor). This plot is made with October 2016 CCE scan data. The CFE remains high when the hit position of the particle track on the sensor is close to a strip. The CFE is low for regions that are further away from a strip and close to a routing line. Plots for all other R -type sensors can be found in Appendix. E.

of tracks from beam halo and secondary decays. The lower efficiency in the outer region of the sensors is mainly due to the second metal layer effect. For sensors near the interaction region (Fig. 8.33), tracks from the primary vertex reaching the very outer part of the sensor cannot be reconstructed. Hence the low efficiency observed in the outer radius region of these sensors is mainly due to a lack of tracks from the primary vertex and the presense of tracks from beam halo and secondary decays. In the inner part of this sensor the drop of efficiency is due to bulk radiation damage. For sensors in the forward region of the detector (Fig. 8.34) tracks from the primary vertex can be reconstructed across the full sensor. The low efficiency observed in the outer region of the sensor is mainly due to the second metal layer effect. The low efficiency in the very inner part of the sensors mainly arises from bulk radiation damage. The sample in this region has a low relative fraction of tracks from the beam halo tracks due to the geometry of the detector. The population and CFE as function of pseudo-rapidity and sensor radius plots for all other R -type sensors can be found in Appendix. F and Appendix. G.

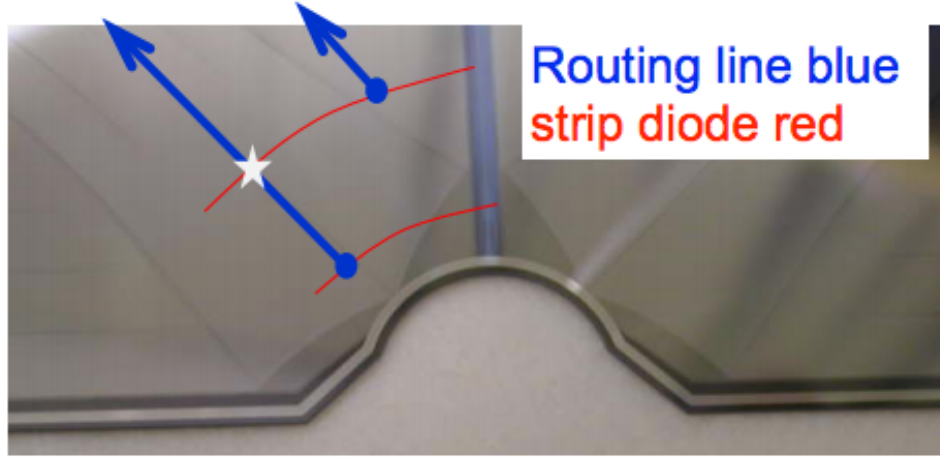


Figure 8.30: Illustration of second metal layer effect on a R -type sensor. If a track impacts at the star, signals should be collected by the red outer strip and routed out by the right-hand routing line. However a fraction of charges were picked up by the left-hand routing line which is close to the star, this is then reconstructed as a fake cluster on an inner region strip.

8.9 Conclusions

Changes in the EDV and CFE of VELO sensors have been studied using CCE scan data collected during run-2 operation. The configuration used for the collection of the CCE scan data has been optimised. The optimal operational bias voltages of the VELO sensors were determined using these studies. From the start of run-2 operation to the end of 2016 VELO sensors operated at a combination of 150 V, 200 V, and 250 V bias voltages, from the start of 2017 operation the VELO sensors were operated at 300 V.

The pattern of CFE observed in the detector was qualitatively understood to be due to charge loss to the second metal layer on the sensor and the geometry of the VELO detector. The study of how the VELO efficiency is affected by radiation damage is important for the operation of the detector and for ensuring the quality of the data for physics analysis. Physics analysis often also relies on Monte-carlo simulation data, and the second metal layer effect has been taken into account in LHCb simulations.

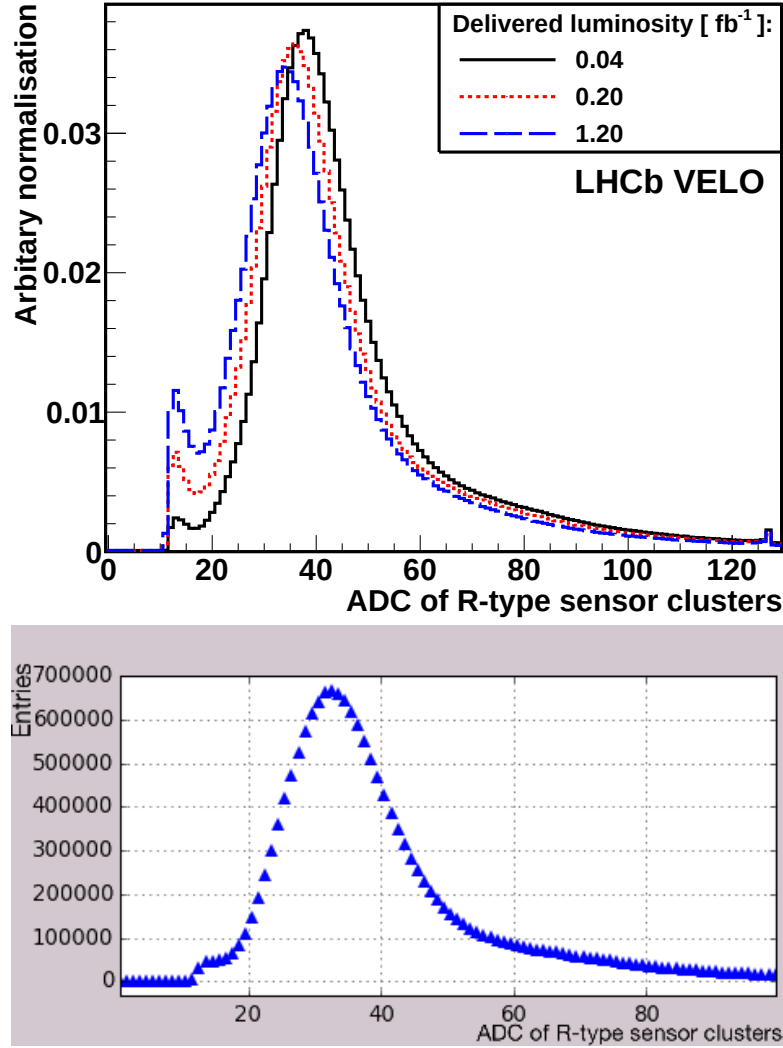


Figure 8.31: (top) ADC counts for clusters observed on R -type sensors for a range of delivered luminosity. After irradiation the peak at low-ADC counts due to the second metal layer effect is observed. This figure is reproduced from Reference [6]. (bottom) ADC counts for cluster on R -type sensors where the cluster is associated with a track. In this case the peak at low ADC is removed. This figure is taken with 2016 physics data from the LHCb VELO online data quality monitor.

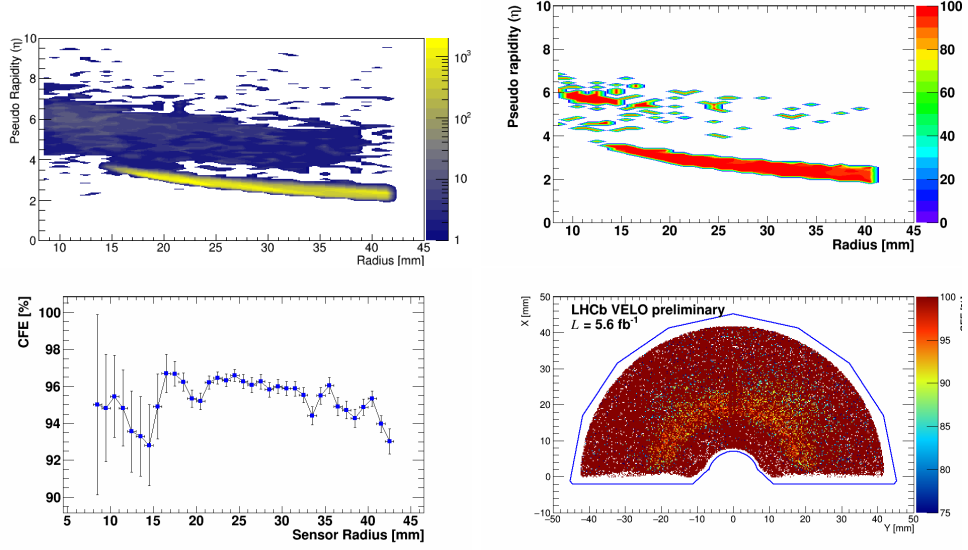


Figure 8.32: (top left) Population of tracks as a function of sensor radius and pseudo-rapidity. (top right) CFE as function of sensor radius and pseudo-rapidity. (bottom left) CFE as a function of sensor radius. (bottom right) CFE as a function of position on a sensor. These plots are made with an R -type sensor at the upstream end of the VELO detector (sensor number 1) using 2016 October CCE scan data.

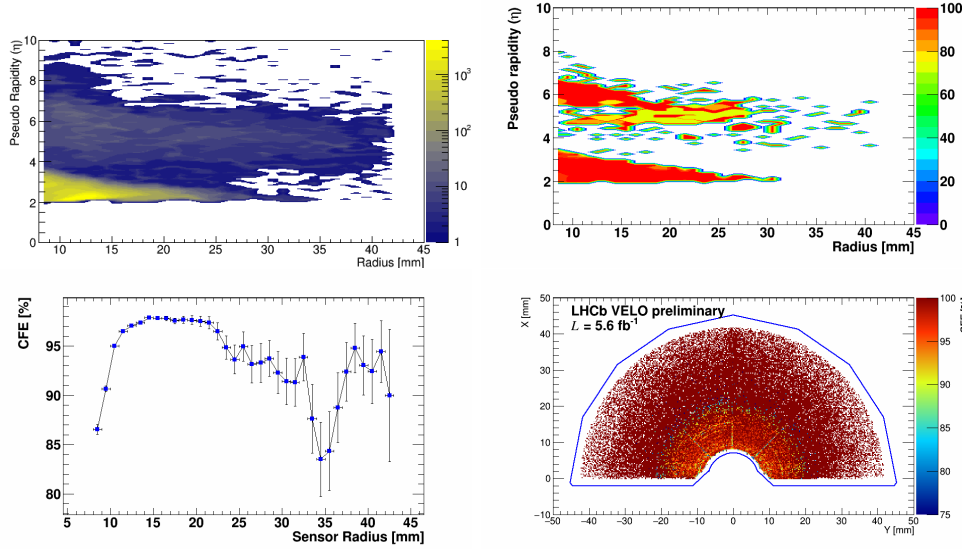


Figure 8.33: Same set of plots as in Fig. 8.32. These plots are for an R -type sensor near the pp interaction region (sensor number 10).

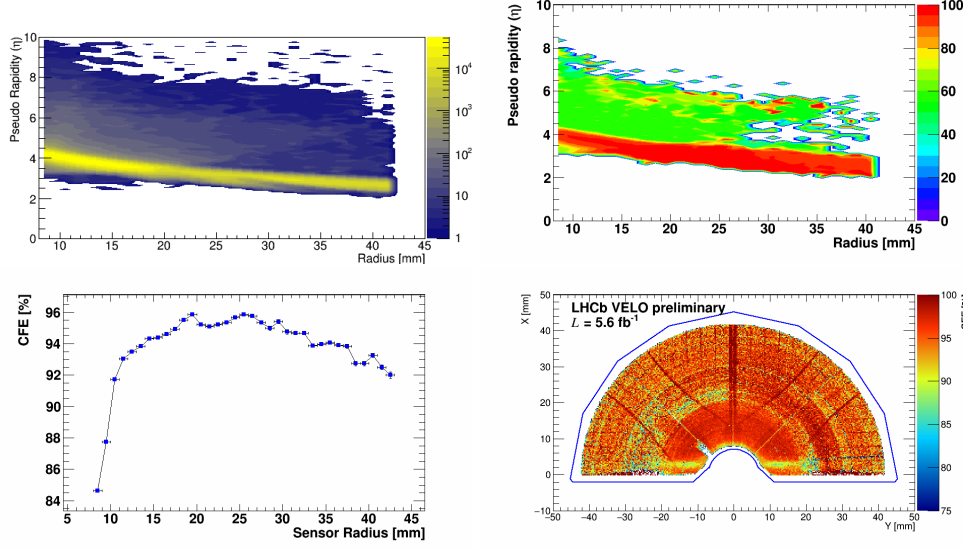


Figure 8.34: Same set of plots as in Fig. 8.32. These plots are for an R -type sensor in the forward region of the VELO (sensor number 31).

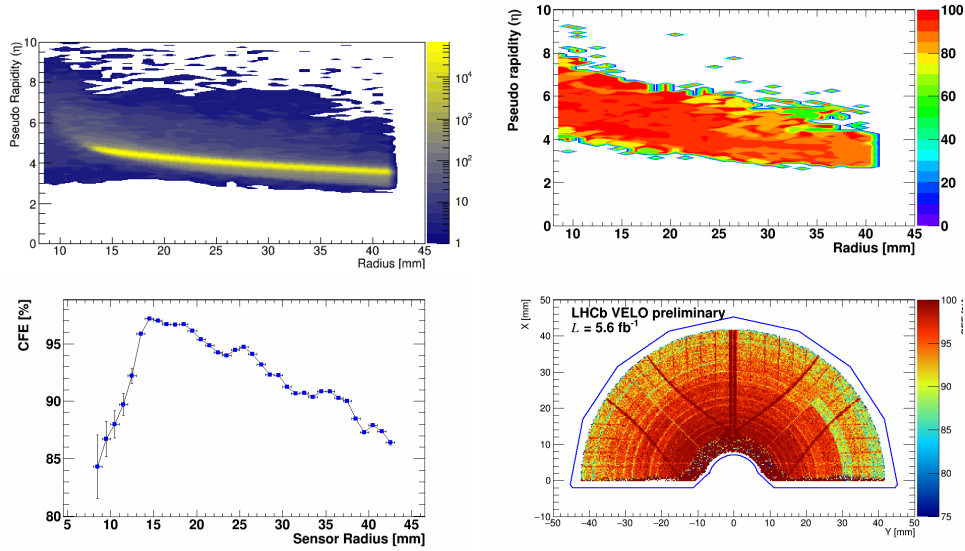


Figure 8.35: Same set of plots as in Fig. 8.32. These plots are for an R -type sensor in the very downstream interaction region of the VELO (sensor number 40).

9 Searches for CP violation in multi-body decays

The study of Charge-Parity (CP) violation allows for a sensitive probe of new physics from beyond the Standard Model of particle physics. CP violation is incorporated in the Standard Model of particle physics through a complex phase in the CKM matrix. Contributions from new particles, at mass scales that cannot be directly probed, can enhance the amount of CP violation observed.

This chapter is based on the publication in Reference [60], with only minor modifications. In this chapter, model-independent searches for direct CP violation, *i.e.* CP violation in decays, in multi-body final states are discussed. The rich phase-space of interfering resonances in multi-body decays provides excellent opportunities for CP violation measurements. Two techniques that have been used in the literature are discussed in Sect. 9.1. These are a binned χ^2 test and an unbinned technique based on the average weighted distance between events in phase-space known as the energy test. The energy test technique was applied for the first-time to CP violation searches in the analyses presented in Chapter 10 and Chapter 11. The performance and optimisation of these techniques is discussed. An approach to study CP violation in the presence of background events using unbinned techniques is presented in Sect. 9.5 through the extension of the energy test formalism. This is the first time that the treatment of background within the energy test has been considered for samples where the relative signal density throughout the phase-space being compared is not already known.

There are many possibilities for selecting the coordinates used for analysing CP violation in multi-body decays. In Sect. 9.6 the options are described and selections are made in the case relevant to the analysis in Chapter 10 and Chapter 11.

A novel method for analysing P -odd CP violation, which is accessible in any decay for which a parity violating observable can be defined such as decays to four pseudo-scalar particles, is presented in Sect. 9.7. This method is applicable for use with any two-sample comparison test. The first application of this technique is presented in Chapter 11.

The implementation of the energy test method in a computationally efficient

manner using GPUs is discussed in Sect. 9.8.

9.1 Techniques for multi-body CP violation searches

Model-independent searches for direct CP violation have been carried out using a number of different techniques. Tests have often been performed using a binned χ^2 approach to compare the relative density in the Dalitz plot [61] of a decay and its CP -conjugate sample (see for example [62] for three-body and [63] for a generalisation to the phase-space of four-body decays). This method is discussed further below. More recently unbinned techniques have also been applied. A technique known as the energy test has been applied (see [64] for three-body and [15] for four-body), again this is discussed below. A nearest neighbours approach has been used in Reference [65], albeit with a very small number of neighbours, and the angular moments of the cosine of the helicity angle of the studied particle have also been utilised [62] in three-body decays. A class of measurements based on triple products of momenta in four-body meson decays have also been performed [66]. The P -odd CP violation test proposed in Sect. 9.7 is related to these. P -odd CP violation can also be accessed in the baryon sector owing to significant P -parity violation even in two-body decays of baryons. This type of CP asymmetry can be measured by comparing P asymmetries for baryon and anti-baryon decays [67].

9.1.1 Binned χ^2

The simplest and most commonly applied technique used in the literature, here referred to as S_{CP} [62, 68], is a two-sample binned χ^2 test. The phase-space is divided into bins. The statistical significance of the difference in number of entries in the bin for the X and $\bar{X}$ samples is computed.

$$S_{CP}^i = \frac{N^i(X) - \alpha N^i(\bar{X})}{\sqrt{N^i(X) + \alpha^2 N^i(\bar{X})}}, \quad \alpha = \frac{N_{tot}(X)}{N_{tot}(\bar{X})}, \quad (9.1)$$

where $N^i(X)$ and $N^i(\bar{X})$ are the numbers of X and $\bar{X}$ candidates in the i^{th} bin, and the N^i values are sufficiently large that Gaussian uncertainties may be assumed. α is the ratio between the total yield of X and $\bar{X}$ events. The parameter α is introduced to account for global asymmetries which may occur due to production effects. The small correction to the significance term in the denominator varies in the literature [62, 63] where the form given here is recommended.

In the absence of local CP asymmetries the S_{CP}^i are distributed according to a Gaussian of unit width and zero mean. The χ^2 test value is computed from $\chi^2 = \sum (S_{CP}^i)^2$. The corresponding p -value for the compatibility of the observed data with the no CP violation hypothesis can be computed directly

from the observed χ^2 value and the number of degrees of freedom; here equal to number of bins–1. The test is straightforward to implement and requires only minimal computing resources.

The number, size and location of the bins need to be selected by the analysts. General advice for χ^2 comparison tests is that the number of bins must be sufficient not to miss local regions of asymmetry but limited to ensure sufficient numbers of entries in the bins to not affect the sensitivity.

The number of bins used in the method in the literature have varied significantly. The initial application and discussion of the method [62, 68] divided the Dalitz plane into $\mathcal{O}(10^3)$ bins. Applications of the method have also used a smaller number of bins, $\mathcal{O}(10)$ to $\mathcal{O}(10^2)$ [69, 63].

We recommend that the number of bins used in the method should be kept to a relatively small number. The use of bins enables significant effects to be localised with low pollution from regions of phase-space that contain little CP violation. However, only a relatively small number of bins are typically needed to achieve the optimal performance of the method, and beyond this point the introduction of additional bins degrades the sensitivity by increasing the degrees of freedom in the test and reducing the number of events in each bin. This is illustrated in Fig. 9.2, where the increased sensitivity of using a smaller number of bins is clearly observed. In this study simulation samples of 100,000 events were generated using the analysis package Laura++ [70] according to the model in Table. 9.1. The Dalitz plot of a sample generated with this model is shown in Fig. 9.1. This simple dimensionless model was originally introduced to test the power of different two-sample tests [71]. CP violation was introduced by changing the amplitude of the resonance with the largest fraction in that model. The conclusion that a relatively small number of bins should be used when applying this method has also been reached in studies of other models.

Table 9.1: A simple dimensionless resonance model of a three-body scalar decay $X \rightarrow abc$. The mass of mother particle X is in arbitrary unit.

Daughters	J^P	Mass	Width	Fit fraction
ab	0^+	0.3	0.025	6%
ab	2^+	0.6	0.05	2%
ac	1^-	0.4	0.04	18%
ac	0^+	0.7	0.1	43%
bc	1^-	0.35	0.01	10%
bc	0^+	0.75	0.02	17%
abc	non-resonant			1%

In a binned approach there is a clear trade-off between minimising the number of bins used and retaining sensitivity to the rich phase-space of interfering amplitudes in the decay. This is particularly true in the case of four-body

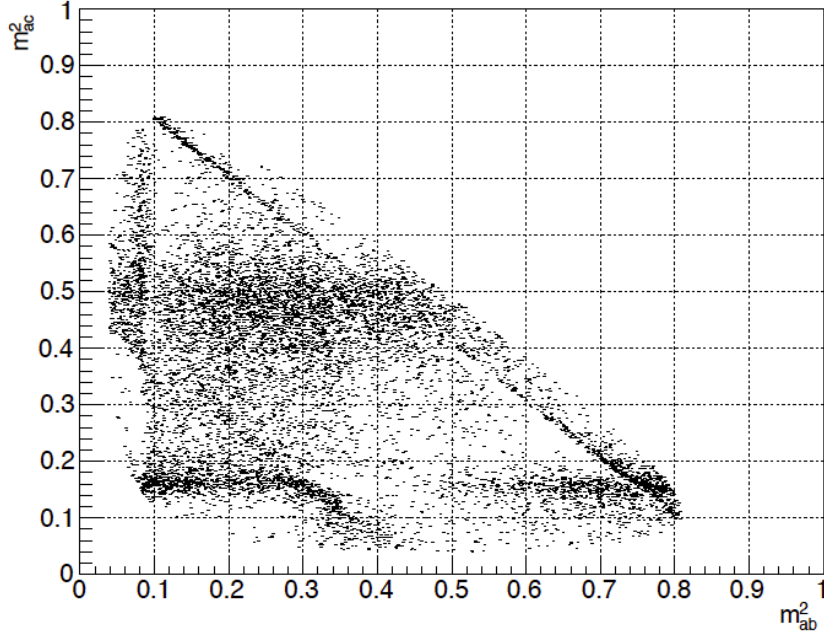


Figure 9.1: Dalitz plot of a sample generated with the model described in Table. 9.1.

decays where five coordinates are required to describe the phase-space (see Sect. 9.6).

Binned distributions of the CP asymmetry in the phase-space can also be used to test for CP violation. This technique was successfully applied in References [72, 73] to observe local CP asymmetries in $B^+ \rightarrow h^+ h^+ h^-$ decays with $h = \pi, K$. In this application of $\mathcal{O}(10^2)$ bins were used. The placement of the bins was physically motivated by the observed location of the resonances. The bin sizes varied across the plane to equalise the number of entries in each bin.

9.1.2 Energy test

A wide range of unbinned two-sample tests exist in the literature [74]. Well-known examples include the nearest neighbour approach, which has previously been applied to CP violation tests [65], and multi-dimensional extensions of the Kolmogorov-Smirnov test, commonly used in the astronomy literature [75]. The former has previously been shown to be relatively insensitive for this class of problem [76] and studies by the authors have concluded similarly for the latter.

The class of multi-variate tests based on distances between observables are of particular interest. A statistical method called the energy test was introduced

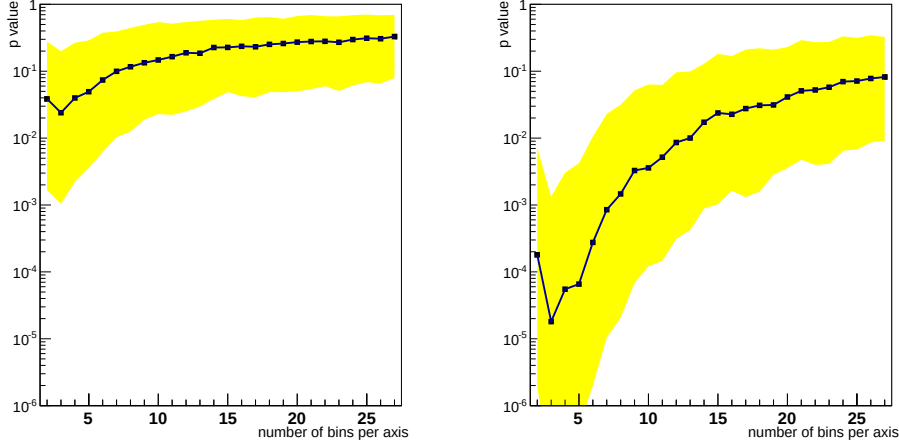


Figure 9.2: p -value versus number of bins in SCP method for simulation samples with CP violation introduced at the (left) 3% and (right) 5% level in one amplitude (see text). 1000 samples were generated and the mean p -values are shown by the points and the one-sigma confidence level range indicated by the yellow-band.

in References [77, 78]. Reference [71] suggests applying this method to Dalitz plot analyses and demonstrates the potential to obtain improved sensitivity to CP violation over the standard binned approach. The distribution of the test statistic is not known, and the permutation method is required to be applied in order to determine the significance of a result. This method was applied to data in the analysis of $D^0 \rightarrow \pi^+ \pi^- \pi^0$ and $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ presented in Chapter 10 and Chapter 11. The method is described below and used in the studies in this chapter. Many other tests of this class are known, including cases where the exact distribution is known under the null-hypothesis, and hence no permutations would be required (*e.g.* Cross-match statistic test [79]).

In the energy test method a test statistic, T , is used to correlate differences between the probability density functions (p.d.f.s) f and $\bar{f}$ in two samples. In the standard test the two samples are those of different flavours, particle and anti-particle. T is formed as

$$T = \frac{1}{2} \int \int (f(\vec{x}) - \bar{f}(\vec{x})) (f(\vec{x}') - \bar{f}(\vec{x}')) \times \psi(\Delta\vec{x}) d\vec{x} d\vec{x}'. \quad (9.2)$$

In practise, the T value can be calculated by approximating the integral by a sum over events which compares average distances of events in phase-space [78],

$$T = \sum_{i,j>i}^n \frac{\psi_{ij}}{n(n-1)} + \sum_{i,j>i}^{\bar{n}} \frac{\psi_{ij}}{\bar{n}(\bar{n}-1)} - \sum_{i,j}^{n,\bar{n}} \frac{\psi_{ij}}{n\bar{n}}, \quad (9.3)$$

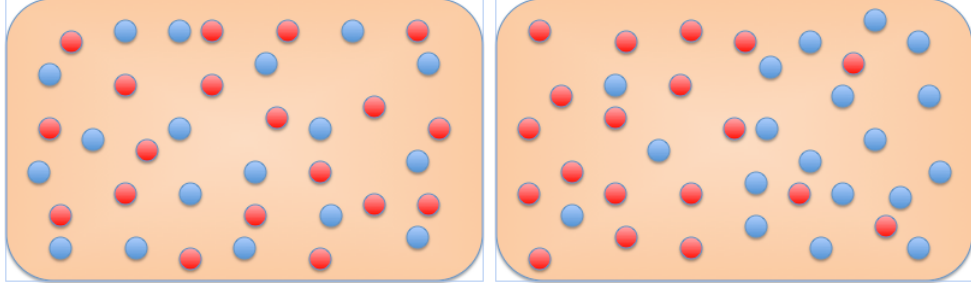


Figure 9.3: An illustration of an isolated system containing particles carrying opposite signed charges. If the distributions of particles are identical for the opposite charges (left), the electrical energy would be smallest and in the limit of a large number of particles would tend to zero. If the distributions of particles are different (right), the electrical energy would be larger than 0.

where ψ_{ij} is a distance function of pairs of events ij , belonging to two samples, and $\Delta\vec{x} = |\vec{x} - \vec{x}'|$ is the distance between two events in phase space. The first and second terms correspond to a weighted average distance of events within the n events of the first sample and the $\bar{n}$ events of the second sample, respectively. The third term measures the weighted average distance of events in one flavour sample to events of the opposite flavour sample. The normalisation factors in the denominator remove the impact of global asymmetries between the two samples.

The $\psi(\Delta\vec{x})$ function plays the role of translating the distance between events into the strength of their correlations, and it can be chosen as any monotonically decreasing function. The closer the events are, the stronger corrections between the events will be, therefore the larger impact on the T value will be. The original proposal suggested the use of $1/|\Delta\vec{x}|$ as a possible function, leading to the calculation of T values having the form of the calculation of the electrical potential energy of the system. Consider an isolated system which contains particles carrying two oppositely signed charges, if the distribution of both charges is identical, the electrical energy will be zero. If any difference in the distributions of opposite charges is observed the electrical energy would increase. An illustration of this kind of system is illustrated in Fig. 9.3. This property can be used in the CP violation searches where the system is considered to be in a multi-dimensional phase space and the opposite signed charges are considered to be decays from two opposite flavoured samples. The discussion above results in the method being named the energy test method.

In the analyses described in Chapter 10 and Chapter 11, Gaussian functions are used as $\psi(\Delta\vec{x})$ function:

$$\psi(\Delta\vec{x}) = e^{-\Delta\vec{x}^2/2\sigma^2}. \quad (9.4)$$

The characteristic width, in this case the σ , plays a role analogous to the

bin size in a binned χ^2 method, σ should be larger than the resolution of $\Delta\vec{x}$ but small enough not to dilute locally varying asymmetries.

If the distributions of events in both samples are identical the measured T value will fluctuate around a value close to zero; differences between these distributions increase the value of T . This is translated into a p -value under the hypothesis of CP symmetry by comparing the measured T value to a distribution of T values obtained from permutation samples. The permutation samples are constructed by randomly assigning events to either of the samples, thus simulating a situation without CP violation. The p -value for the no- CP -violation hypothesis is obtained as the fraction of permutation T values greater than the observed T value.

9.2 Fitting the T-value distribution

If large CP violation is observed, the observed T value is likely to lie outside the range of permutation T values. In this case the permutation T distribution can be fitted with a generalised-extreme-value (GEV) function, as demonstrated in References [77, 78] and used in References [64, 80]. The p -value from the fitted T distribution can be calculated as the fraction of the integral of the function above the observed T value. The GEV function is defined as

$$f(x; \mu, \sigma, \xi) = N \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{(-1/\xi)-1} \exp \left\{ - \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{-1/\xi} \right\}, \quad (9.5)$$

with normalisation N , location parameter μ , scale parameter σ , and shape parameter ξ . The function is exactly zero for $x > \mu - \sigma/\xi$ for $\xi > 0$, and for $x < \mu - \sigma/\xi$ for $\xi < 0$. The function has the cumulative distribution

$$F(x; \mu, \sigma, \xi) = \exp \left\{ - \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{-1/\xi} \right\}. \quad (9.6)$$

Examples of fits to the measured distribution T values in simulated events with a GEV function are shown in Fig. 9.4.

The default p -value extraction uses the counting method as long as three permutation T values are found to be larger than T value observed in data. Beyond that the p -value is extracted from integrating the fitted extreme value function. In the first case, the counting method, the uncertainty is simply extracted as a binomial error on the p -value. In the second case the uncertainty on the p -value is extracted by randomly resampling the fit parameters within their uncertainties, taking into account correlations, and by extracting p -values for each of these samplings. The spread of the p -value distribution then gives the uncertainty. This is either expressed by giving 68% confidence limits or, if the lower limit is zero, by giving 90% and 95% upper limits for the p -value as shown in Fig. 9.5. In practice, to extract a small p -value from this

GEV function requires the function extend to infinitely large on the right side, therefore requires ξ to be equal or larger than 0. We fix this shape parameter ξ to be 0 when there are less than 100 permutations or the fit result is less than 0 (Table.9.2).

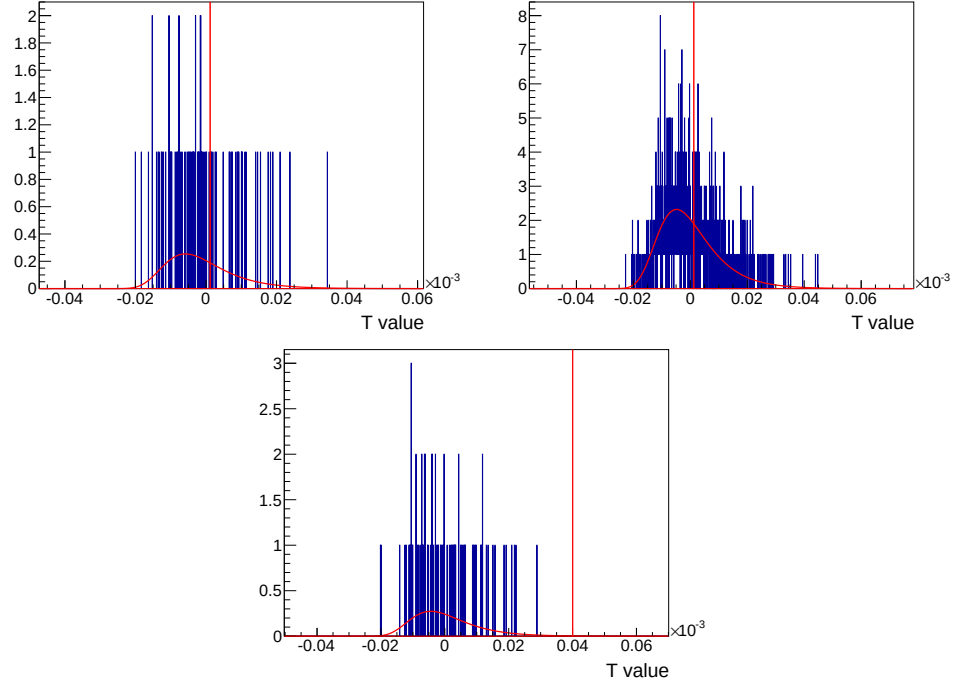


Figure 9.4: Examples of T values calculated from simulated data samples (red lines) and permutation T values (blue lines). The distributions of permutation T -values are fitted with GEV functions. In the plots 100 (top left), 800 (top right) and 100 (bottom) permutations are plotted. In all cases each sample uses 100,000 events generated with model described in Table. 9.1. The upper plots use a CP symmetric sample, the lower plot uses a CP violating sample, where CP violation was introduced by changing the amplitude of one resonance.

The number of permutations that are available is limited by the available computing time. For cross-checks as few as 30 permutations are sufficient provided that the p -value is not too small. More permutations can be added a-posteriori if needed without biasing the interpretation as they will only reduce the uncertainty on the p -value. For the nominal result a larger number of permutations should be run to obtain a sufficiently precise estimate of the p -value. The order of 1000 permutations should suffice in this case as this would allow a 3σ significance result to be obtained from the counting method.

Table 9.2: Fit parameters of the GEV function, for the fits to T vlaue distributions that are shown in Fig. 9.4. All fitted values are consistent with each other.

Fit parameter	Large p -value		Small p -value	
	100 permutations	800 permutations	100 permutations	100 permutations
N	6.90 ± 0.87	6.31 ± 0.28	7.43 ± 0.93	
μ	$-5.82 \pm 0.83 \times 10^{-6}$	$-4.85 \pm 0.32 \times 10^{-6}$	$-4.39 \pm 0.86 \times 10^{-6}$	
σ	$7.89 \pm 0.61 \times 10^{-6}$	$8.55 \pm 0.23 \times 10^{-6}$	$8.17 \pm 0.61 \times 10^{-6}$	
ξ	0 (fixed)	0 (fixed)	0 (fixed)	

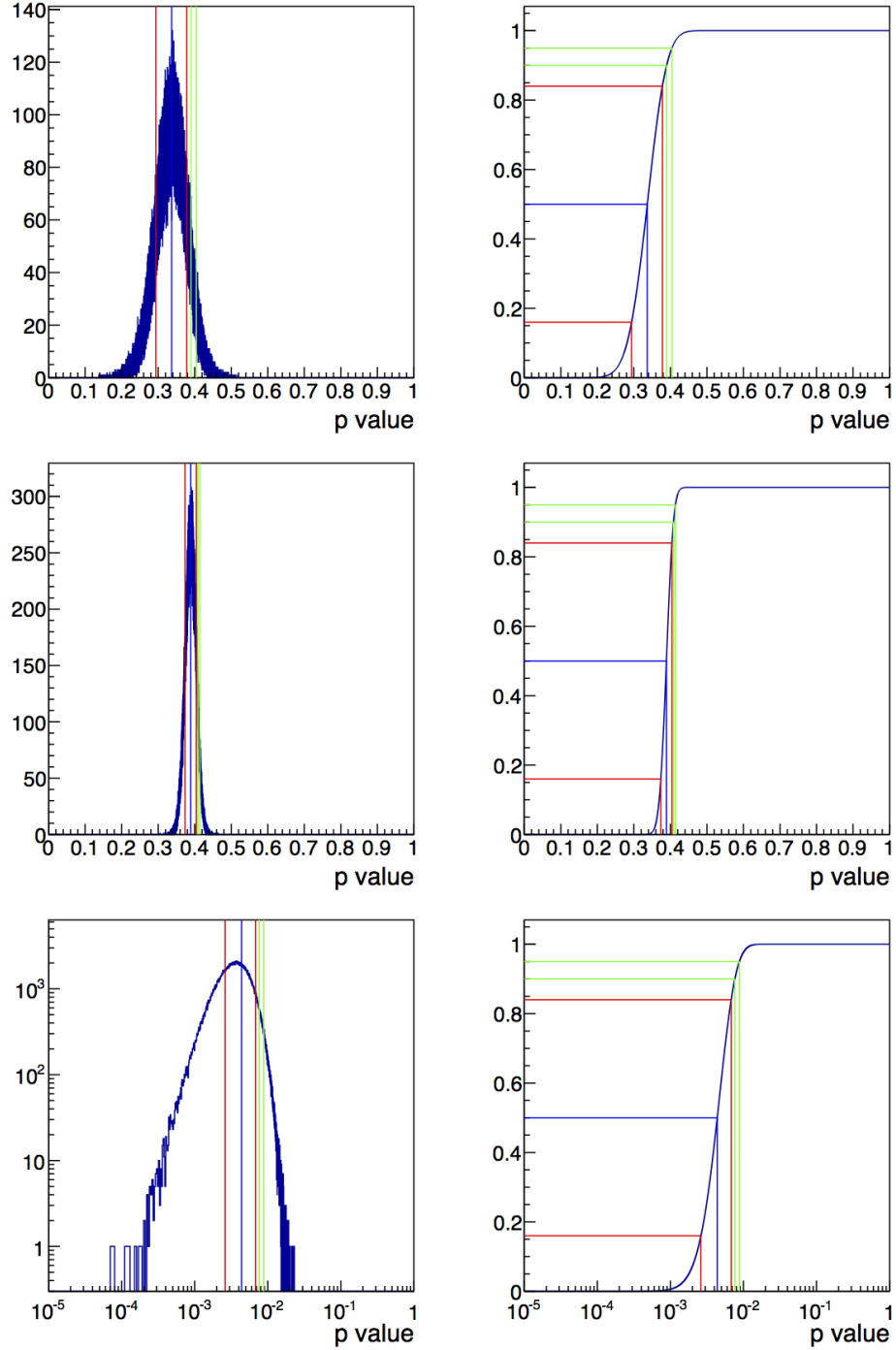


Figure 9.5: Examples of (left) p -value distribution and (right) cumulative p -value distribution from random sampling of the fit parameters of a GEV fit that used in Fig. 9.4. The top and middle plots show an example for a large p -value while the bottom plots show the scenario with a small p -value. The only difference between top and middle plots are in the top plots, the fit of GEV function is based on 100 permutations while in the middle plots, the fit is based on 800 permutations. The fit in the bottom plots is also based on 100 permutations. With more permutations, the fit of GEV function will be more accurate and therefore p -value will have smaller uncertainty. The lines indicate the central value (blue) and the 68% confidence level interval as well as the 90% and 95% confidence limits.

9.3 Sensitivity of energy test

The performance of the energy test method, based on one of the samples used in Fig. 9.2, is shown in Fig. 9.6. In this case CP violation has been introduced through a 3% change in the amplitude of resonance ($ac, 0^+$) in Table. 9.1. The sensitivity to the parameter σ is shown. The improved performance of this method over the S_{CP} method, that was discussed in Sect. 9.1.1, is seen by comparing the figures; indeed, a 5% amplitude asymmetry results in p -values from the energy test below 10^{-10} . Studies from the authors show this enhancement in sensitivity is common.

The sensitivity of the optimal σ to various parameters of a three-body decay model was studied. The value was found to be largely independent of the Dalitz plot structure or the width of the resonance in which CP violation was introduced. A variable σ set according to the density of points in the plot was also studied but not demonstrated to have any advantage over a fixed σ . As trivially expected, the optimal σ is dependent quadratically on the mother particle mass in a three-body decay.

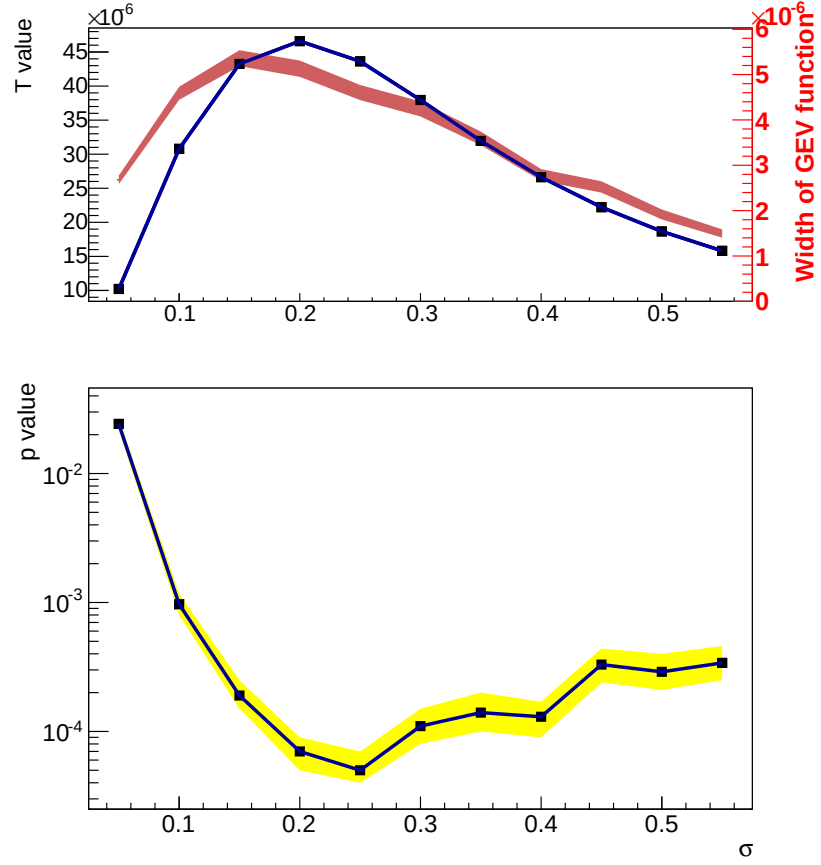


Figure 9.6: (top) T values and (bottom) p -values versus σ in the energy test method for simulation samples with CP violation introduced at the 3% level in one amplitude (see text). The p -values are evaluated using 100 permutation samples where their distribution is fitted with a GEV function. The width of the GEV function and its uncertainty is also shown on the top plot. The yellow band on the lower plot shows the uncertainty on the p -value accounting for the uncertainties in the GEV fit, including the correlation of the fitted parameters.

9.4 Visualisation of asymmetries

The technique described above produces a single p -value. In the case that evidence for CP violation is obtained it will be important to know which areas of the phase space are contributing. A full understanding of the CP violation would require a model-dependent analysis.

A visualization tool is suggested in this method to provide an intuitive guide to which regions of the phase space are contributing with larger CP asymmetries. Instead of summing all the interactions among events over the phase space, each single event is investigated separately. The contribution of a single positive event T_i and a single negative event $\bar{T}_i$ to the total T value are defined as

$$T_i = \frac{1}{2n(n-1)} \sum_{j \neq i}^n \psi(\Delta \vec{x}_{ij}) - \frac{1}{2n\bar{n}} \sum_j^{\bar{n}} \psi(\Delta \vec{x}_{ij}). \quad (9.7)$$

$$\bar{T}_i = \frac{1}{2\bar{n}(\bar{n}-1)} \sum_{j \neq i}^{\bar{n}} \psi(\Delta \vec{x}_{ij}) - \frac{1}{2n\bar{n}} \sum_j^n \psi(\Delta \vec{x}_{ij}). \quad (9.8)$$

For a specific event i the distances to all other events are grouped into those to events with the same flavour and those with the opposite flavour. The first term in Eqn. 9.7 is the sum over the distances of event i with all the other same flavour events, the second term is the sum over all events in the opposite flavour set. The same discussions applies for the corresponding Eqn. 9.8. It follows that $T = \sum_i T_i + \sum_i \bar{T}_i$.

Having obtained the T_i values¹ for all events, a permutation method is also used here to define the level of significance. The permutation values T_i are assumed to have a Gaussian distribution. For each permutation, the maximum and minimum T_i values are marked as T_i^{MAX} and T_i^{MIN} respectively, and these values would have distributions with many permutations. The tails of the T_i^{MAX} and T_i^{MIN} distributions are used to set the level of significance of T_i 's in the original sample (see Fig 9.7). This significance level can then be plotted on a standard Dalitz plot.

For the case of a resonance having an amplitude difference between the CP conjugate states, the region of significant asymmetry shows up as a band around the mass of the resonance on the plot. For the case of a resonance having a phase difference between the CP conjugate states, regions of positive and negative asymmetry appear around the resonance on the plot, indicating the phase shift. Examples of these visualisation plots are given later in the thesis in the analysis chapters in Fig. 10.15 and Fig. 11.23

¹All T_i and $\bar{T}_i$ values are denoted as T_i , unless otherwise specified.

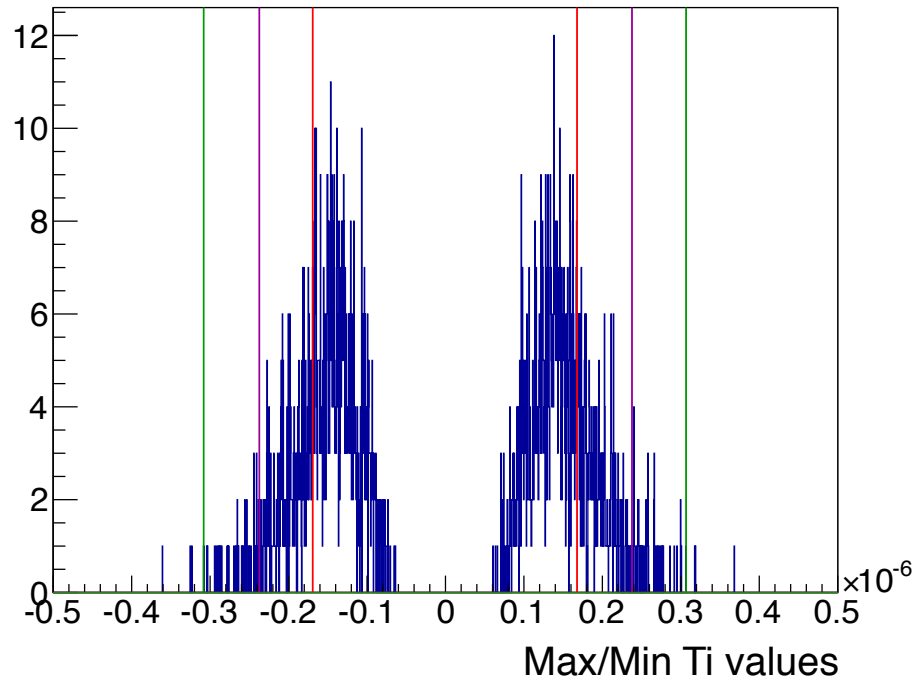


Figure 9.7: T_i^{MAX} and T_i^{MIN} distributions and the significance levels set by them. Red, magenta, and green lines are 1 σ , 2 σ and 3 σ respectively.

9.5 Presence of background

CP violation is expected to appear in relatively low rate decays where two amplitudes of similar magnitude interfere. The samples of signal events under study may be significantly polluted by additional background events from different physics processes that contribute at a similar rate to signal. Such pollution reduces the fraction of signal events (the purity) of the sample under study, and requires the use of additional techniques such that any significant effect can be associated with CP violation in the signal process.

This section describes an approach to treating the case of problematic backgrounds. In the analyses presented in this thesis in Chapter 10 and Chapter 11, the background fractions are relatively low and the backgrounds are found to be CP symmetric, and hence this procedure has not been applied.

The presence of background is problematic when studying local CP violation in the case that the purity is different in the two samples that are being compared. This can be generated, for example, through the presence of global production asymmetries between the two samples that are different for signal and background, even if no CP violation is present. Similarly problematic is the case where the background process itself exhibits CP violation. In these scenarios the local densities of events will differ between the two samples, since background populates the phase space differently to signal. For the binned S_{CP} test different yields will be found in the phase space bins (even after normalising for the overall yield), and a significant effect could be inferred. The effect of background on the S_{CP} test is typically removed by fitting the invariant mass distribution in each bin to determine the number of signal events in each bin, and then comparing these between the two samples. The energy test also exhibits similar complications. Differences in purity between the two samples are not reproduced when events are randomly assigned a flavour, since the purities and event densities will be consistent for the permuted samples; the densities of samples used in the permutation studies will not accurately model differences in density between the two ‘real’ data samples. Consequently background events can lead to significant T values and p -values being calculated even when the signal process exhibits no CP violation. It is therefore important to consider techniques to remove the effect of background from the energy test, so that any significant T value can be associated with CP violation in the studied decay. This section sets out such an approach.

The weighting of events to remove the effect of background was considered in Reference [71]. Such weights give the relative purity of each sample in the particular region of phase-space that the event is drawn from. Therefore, when T is calculated two weights are used for each pair of events, with one weight for each event. This allows each pair of events to be weighted according to the relative number of pairs of signal events expected to contribute to the calculation of T from such locations within the phase space, and allows the estimation of the T value that would be found if only signal were present.

However, this approach relies on knowledge of the signal density within the phase-space, which in many practical cases is not known.

An alternative method is suggested here to remove the bias introduced by background if additional representative samples of background events are available, for example from signal side-bands. This method will be equally applicable to other unbinned model independent two-sample tests but is discussed here for the energy test. These additional samples will be labelled here as ‘background samples’, as opposed to the ‘main samples’ which are being explicitly tested for differences in local event densities. The events in the background samples can be used to subtract off the effect of signal and background event pairs, and background and background event pairs that arise when considering the T value set out in Eq. 9.3 when background is present in the main samples. This can be achieved by altering the test statistic to

$$\begin{aligned}
T = & \frac{1}{2w(w-1)} \left(\sum_i^n \sum_{j \neq i}^n \psi_{ij} - \frac{2b}{b_s} \sum_i^n \sum_j^{b_s} \psi_{ij} + \frac{b(b+1)}{b_s(b_s-1)} \sum_i^{b_s} \sum_{j \neq i}^{b_s} \psi_{ij} \right) \\
& + \frac{1}{2\bar{w}(\bar{w}-1)} \left(\sum_i^{\bar{n}} \sum_{j \neq i}^{\bar{n}} \psi_{ij} - \frac{2\bar{b}}{\bar{b}_s} \sum_i^{\bar{n}} \sum_j^{\bar{b}_s} \psi_{ij} + \frac{\bar{b}(\bar{b}+1)}{\bar{b}_s(\bar{b}_s-1)} \sum_i^{\bar{b}_s} \sum_{j \neq i}^{\bar{b}_s} \psi_{ij} \right) \\
& - \frac{1}{w\bar{w}} \left(\sum_i^n \sum_j^{\bar{n}} \psi_{ij} - \frac{\bar{b}}{b_s} \sum_i^n \sum_j^{\bar{b}_s} \psi_{ij} - \frac{b}{b_s} \sum_i^{b_s} \sum_j^{\bar{n}} \psi_{ij} + \frac{b\bar{b}}{b_s\bar{b}_s} \sum_i^{b_s} \sum_j^{\bar{b}_s} \psi_{ij} \right),
\end{aligned} \tag{9.9}$$

where w and $\bar{w}$ are the number of signal events in the main samples, and b and $\bar{b}$ are the number of background events in the main samples, while b_s and $\bar{b}_s$ denote the number of background events in the background samples. The additional terms in comparison with Eq. 9.3 sum over pairs of events in the background and main samples or sum over pairs of events in the background samples, removing on average the effects of pairs of background and signal events and pairs of background events, when calculating T . The other notable difference with Eq. 9.3 is the inclusion here also of terms with $j < i$ when the sums are over the same sample. This is balanced with an additional factor of $\frac{1}{2}$, and is included for simplicity when considering all terms. Assuming that the density of events in the background samples reflects the density of background events in the main samples (up to an overall normalisation factor), this T value provides an unbiased estimate of the T value that would be calculated in the presence of signal alone.

An example of the effect of background on the energy test is shown in Fig. 9.8. Here, 4,000 signal events were generated (without CP violation) using the same model as in Reference [71], and assigned a flavour randomly, with a 50% chance of each flavour. Background events were generated assuming no variation in event density across the 3-body phase space, with 1,600 events contaminating the main samples. Again, the flavour was assigned randomly,

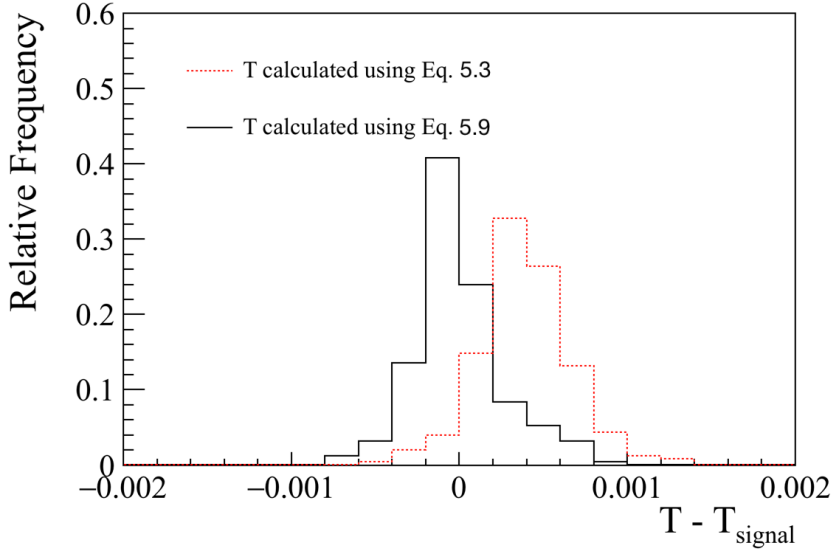


Figure 9.8: The T value relative to the T value found from only considering signal events (T_{signal}), when calculated using Eq. 9.3, where background is not removed, and Eq. 9.9, which subtracts (on average) the effect of background. The first case shows a clear bias; the presence of background can lead to small p -values when the permutation method is used to determine significance. The mean of the second distribution is consistent with zero, removing the bias associated with background.

though on average 75% of events were assigned one flavour and 25% the other. An additional 1,600 events (with the same 3:1 asymmetry in flavour) were used to create additional background samples that could be used to remove any bias introduced by background in the main samples. This was performed 250 times, and in each trial the energy test was calculated using a σ parameter of 0.25. Such a scenario can generate large T values if Eq. 9.3 is used to calculate the T value, and the permutation method yields significant p -values. However, if the background samples are also considered, and Eq. 9.9 is used to remove the effects of background on average, an unbiased estimate of the T value of signal events is recovered. This allows the use of the permutation method to estimate p -values (using the same method to remove background from the randomly permuted samples), and allows any significant effect to be associated clearly with the signal channel. Therefore, for the rest of this chapter, the case where background is present is neglected.

9.6 Coordinate selection

A decay of a pseudo-scalar particle M into n pseudo-scalar particles ($A, B, C...$) $M \rightarrow ABC...(n)$ can be described by n four-vectors $p_A^\mu, p_B^\mu, p_C^\mu, \dots$, and consequently $4n$ parameters. The known masses of the identified final state particles $A, B, C...$ remove n degrees of freedom. Energy (E) and momentum ($\mathbf{p}$) conservations remove an additional four degrees of freedom. The system can be rotated freely around all spatial axes, removing a further three degrees of freedom. Hence, $3n - 7$ degrees of freedom remain. Consequently a three-body decay phase-space is fully described by the two variables conventionally used in Dalitz plot analyses. The phase-space of a four-body decay can be fully described by five parameters. The selection of the variables used to describe the phase-space is discussed in this section.

9.6.1 Distances in Phase-Space

The S_{CP} method requires the division of the decay phase-space into bins. The energy test method relies on the distance between events in the phase-space. The result of model-independent two-sample comparison tests will typically depend on the distance metric, and the coordinates chosen. The choice of coordinates in three-body decay and four-body decay cases are described below.

9.6.2 Coordinates in three-body decays

The distance between two points in phase-space in a three-body decay is usually measured as the Euclidean distance in the Dalitz plot, in which all kinematic information of the decay is contained. In Reference [71] a standard Dalitz plot, using two of the squared invariant masses of pairs of final state particles as the axes, was used in the calculation of T value. However, there are three possible choices of these axes, the distance used in the T value calculation depends on the choice of the axes of the Dalitz plot.

The easiest way to picture the axis dependence is a pure phase-space distribution, which results in a flat Dalitz plot, and CP violation resulting in an enhancement of events in one corner and a deficit of events in another corner of phase space. The permutations will result in a flat distribution regardless of the axis choice and hence the T value distribution will remain the same. The nominal sample containing the CP violation will, however, have a contribution to observed T value, which is proportional to the distance between the two corners of phase space. This distance does depend on the axis choice and thus would the extracted p -value. The same argument applied to the so-called square Dalitz plot, which uses one two-body invariant mass and the associated helicity angle. This dependence is no fundamental flaw in the method as the interpretation via sensitivity studies to different CP violation scenarios, using

the same axis choice, would remain accurate. Nevertheless it is desirable to avoid this dependence.

This dependence can be removed by using all three invariant masses to determine the distance, d_{ij} , calculated as the length of the displacement vector $\Delta\vec{x}_{ij}$ in a 3D space. Using all three invariant masses does not add information, but it avoids an arbitrary choice that could impact the sensitivity of the method to different CP violation scenarios. In the case of a fixed parent particle mass all events lie in the same plane in this 3 D phase space, as shown in Figure 9.9. This method avoids an arbitrary choice of preferred axes, though no additional physical information is added by using the third coordinate.

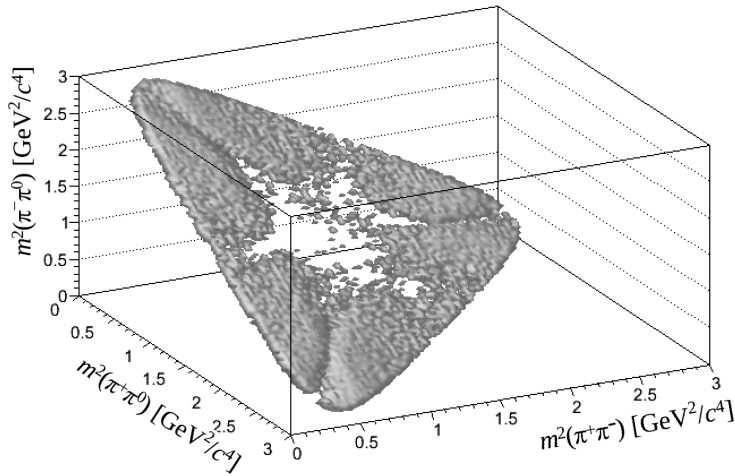


Figure 9.9: An example distribution of events in a simplified simulation Monte Carlo $D^0 \rightarrow \pi^+ \pi^- \pi^0$ decay sample in 3D Dalitz space. The three possible invariant masses squared of final state particle pairs are the axes. The distance between pairs of events in this 3D space is used in the computation of the T value (see text).

9.6.3 Coordinates in four-body decays

Four-body decays offer both challenges and opportunities with respect to three-body decays. A four-body decay phase-space is obviously more complicated and five coordinates are needed for its full description [81]. No unique choice of variables exists and depending on the decay dynamics and the purpose of the measurement one can try and optimize the set of coordinates.

For cascade-type decays proceeding via resonances in a three-body subsystem and followed by two-body decays (e.g. $D^0 \rightarrow a_1(1260)^+ (\rightarrow \rho^0(770) \pi^+) \pi^-$) a three-body invariant mass and a Dalitz-style distribution of the three-body decay would be a preferred option. For decays occurring through two two-body resonances, in particular ones with non-zero spins, (e.g. $D^0 \rightarrow \rho^0(770) \rho^0(770)$)

the natural choice is the so called transversity basis [82] comprising invariant masses of two-body subsystems, their helicity angles and the acoplanarity angle between decay planes of the two resonances. In some decays both decay types may contribute significantly (*e.g.* both of the examples given here contribute to $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ decays). The most general amplitude analysis that aims at a description of decay dynamics is performed in the phase-space constructed with the four-momenta of the final state particles and any coordinates are chosen only to illustrate the results.

The energy test is a statistical method comparing the events' distributions in phase-space (see Sect. 9.1). Therefore it is sensitive to the position of an event in phase-space and to the choice of coordinates building this phase-space. All choices are not equivalent in terms of the sensitivity of the analysis as it will change the distance between events in the phase-space.

There are many possible choices for the variables to describe the degrees of freedom of the phase-space. A natural set of invariants is $p_i p_j$, or equivalently s_{ij} defined as $s_{ij} = (p_i + p_j)^2 = m_i^2 + m_j^2 + 2(p_i p_j)$. There are $\frac{1}{2}n(n-1)$ such invariants; for four-body decays there are six of them.

Three-body mass invariants, s_{ijk} , can be made from linear combinations of the two-body invariants: $s_{ijk} = (p_i + p_j + p_k)^2 = s_{ij} + s_{ik} + s_{jk} - m_i^2 - m_j^2 - m_k^2$, so can be used interchangeably (for $n > 3$) for the s_{ij} . While they carry the same information as s_{ij} , the choice will change the distance measure between events used in the CP violation search. There are $\frac{1}{3!}n(n-1)(n-2)$ such invariants; there are four of them in four body decays.

Consequently there are ten mass invariants, six two-body and four three-body, which can be used to characterise a four-body decay. We consider a physically motivated choice of the coordinates. Invariant mass combinations corresponding to 'unphysical' doubly-charged meson resonances may be excluded. A further reduction in coordinates can be made by excluding mass combinations with small resonance contributions.

An additional complication in many decays of interest will be the presence of identical final state particles (*e.g.* $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$), which is analysed in Chapter 11. Identical particles of the same charge can be swapped; as a result in the example decay each event can be placed in four points in phase-space. The energy test is sensitive to such particle swapping as well. To get a unique output from the energy test, as well as to get optimal sensitivity, the order of the particles, *i.e.* the input variables of the energy test need to be determined.

Consider the experimentally interesting example of the singly Cabibbo suppressed $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ decay. The charge order of the particles in the D^0 decay $\pi_1 \pi_2 \pi_3 \pi_4$ is fixed to $\pi^+ \pi^- \pi^+ \pi^-$. For a $\bar{D}^0$ decay, the order of $\pi_1 \pi_2 \pi_3 \pi_4$ is the C -conjugated one: $\pi^- \pi^+ \pi^- \pi^+$. The invariant masses of all possible $\pi^+ \pi^-$ pairs are calculated and sorted for each event. Once the $\pi^+ \pi^-$ pair with the largest invariant mass is fixed to be $\pi_3 \pi_4$, the order of all four pions is fully determined. As only a small fraction of the $\rho(770)$ resonance, either produced directly from D^0 or through $a_1(1260)$ decays, contributes

Table 9.3: Coordinates used, and those excluded, in the measurement of $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ decays [15] and coordinates suggested for $D^0 \rightarrow K^+K^-\pi^+\pi^-$ decays. Excluding the mass marked in brackets would reduce to the minimal five coordinates that span the phase-space.

$D^0 \rightarrow \pi_1^+\pi_2^-\pi_3^+\pi_4^-$	Two-body masses	Three-body masses
Unphysical	m_{13}, m_{24}	
Physical	$m_{12}, m_{14}, m_{23}, m_{34}$	$m_{123}, m_{124}, m_{134}, m_{234}$
Selected	m_{12}, m_{14}, m_{23}	m_{123}, m_{124}
$D^0 \rightarrow K_1^+K_2^-\pi_3^+\pi_4^-$	Two-body masses	Three-body masses
Unphysical	m_{13}, m_{24}	
Physical	$m_{12}, m_{14}, m_{23}, m_{34}$	$m_{123}, m_{124}, m_{134}, m_{234}$
Selected	$m_{12}, m_{14}, m_{23}, m_{34}$	$m_{134}, (m_{234})$

to the largest $m(\pi^+\pi^-)$, the $\pi_3\pi_4$ combination is excluded [82]. Two-body masses except for the $m(\pi_3\pi_4)$ and three-body mass combinations that do not contain the $\pi_3\pi_4$ are kept. In that way we end up with exactly five invariant masses, which contain most of the dominant resonance contributions, as listed in Table 9.3. This choice has been adopted in the analysis presented in Chapter 11. Simulation studies comparing the performance of the test with these five coordinates and with the eight ‘physical’ mass combinations have been performed. No significant difference in sensitivity was obtained, though the optimal sigma is larger when using eight coordinates.

The other singly Cabibbo suppressed D^0 decay with charged long-lived hadrons in the final state is $D^0 \rightarrow K^+K^-\pi^+\pi^-$. In this case no significant three-body resonances containing K^+K^- are observed [83], and thus these masses can be excluded as coordinates. The other coordinates have significant contributions and thus for an energy test method of this final state a physically motivated choice would be to proceed with six coordinates, as shown in Table 9.3, or to reduce to five coordinates by further excluding the mass combination with the smallest resonance contribution (m_{234}).

9.7 Parity-even and Parity-odd CP Violation Tests

The choice of only invariant masses as coordinates, as discussed in the previous section, has a limitation. Invariant masses, of both two- and three-body systems, can be expressed through the double product of particle momenta ($\vec{p}_a \cdot \vec{p}_b$) and, as such, are even under the P -parity transformation (changing $\vec{p}$ into $-\vec{p}$). Thus, invariant masses allow only P -even CP asymmetries to be probed. This is also true of helicity angles.

In three-body decays only P -even amplitudes can be present and the conventional model-independent test of comparing particle and anti-particle samples is sufficient. In four-body decays P -odd amplitudes can be present and accessed with P -odd quantities. These are triple products of a general form $\vec{p}_a \cdot (\vec{p}_b \times \vec{p}_c)$; the acoplanarity angle between decay planes of two resonances is one example. There is a class of measurements based on the triple products, often called T -odd measurements², which probe the P -odd type of CP asymmetry only [84].

An asymmetry of this P -odd kind is induced by interferences between P -even and P -odd decay amplitudes, thus the sensitivity to CP violation depends on the P -odd amplitude contribution in the decay. However, it is proportional to the cosine of the strong-phase difference between the interfering partial waves [85] and thus will be enhanced where P -even CP asymmetry, proportional to sine of the strong-phase difference, lacks sensitivity.

Consider again the general four-body decay $M \rightarrow ABCD$ and its antiparticle equivalent $\bar{M}$. A triple product $C_T = \vec{p}_A \cdot (\vec{p}_B \times \vec{p}_C)$ is constructed for M decays. The $\vec{p}_A$, $\vec{p}_B$ and $\vec{p}_C$ are vector momenta of particles A, B, C in the M centre-of-mass frame. The corresponding triple product for the $\bar{M}$ decays is obtained by applying the CP parity transformation, $CP(C_T) = -C(C_T) = -\bar{C}_T$. The $\bar{C}_T$ is constructed with the anti-particles $\bar{A}, \bar{B}, \bar{C}$, being the C -conjugations of the ones entering C_T . The total sample may be divided into four subsamples according to the particle/antiparticle flavour and the triple product sign:

$$[I] D(C_T > 0), \quad [II] D(C_T < 0), \quad [III] \bar{D}(-\bar{C}_T > 0), \quad [IV] \bar{D}(-\bar{C}_T < 0). \quad (9.10)$$

The relationships between the samples under symmetry transformations is illustrated in Fig. 9.10. Samples [I] and [III] are related by CP transformation; and so are [II] and [IV]. There are thus two potential sample comparison tests for CP violation using the full data sample: comparing a sample consisting of data set [I] and [II] with a sample containing data [III] and [IV]; or comparing the combined [I]+[IV] with the combined [II]+[III]. Both tests span the full C_T space.

Consider the more familiar case of asymmetries, these may be measured in the C_T regions using the number of events populating the four samples in

²Triple products are also odd under T -parity, although time reversal is not what is typically measured.

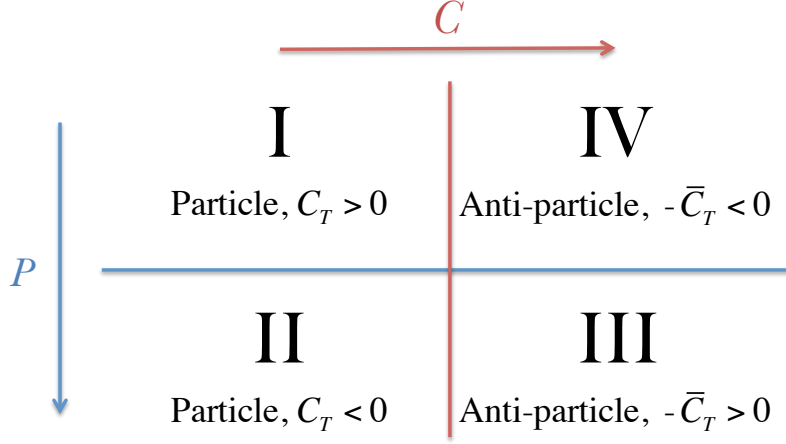


Figure 9.10: Symmetry transformation relationships of the four data samples used in the CP violation tests.

Eq. 9.10:

$$A_{CP}(C_T > 0) = \frac{N(I) - N(III)}{N(I) + N(III)}, \quad A_{CP}(C_T < 0) = \frac{N(II) - N(IV)}{N(II) + N(IV)}. \quad (9.11)$$

In the absence of CP Violation both of the asymmetries are expected to be compatible with zero.

CP asymmetries can be extracted from these samples that are P -even or P -odd (i.e. even and odd with respect to C_T) simply by adding or subtracting the asymmetries measured in the C_T regions:

$$A_{CP}^{P-even} = \frac{A_{CP}(C_T > 0) + A_{CP}(C_T < 0)}{2}, \quad A_{CP}^{P-odd} = \frac{A_{CP}(C_T > 0) - A_{CP}(C_T < 0)}{2}. \quad (9.12)$$

The A_{CP}^{P-even} test corresponds to integrating over C_T , and is equivalent (within normalisation) to the default sample comparison test in which particle (samples [I] and [II]) and anti-particle samples (samples [III] and [IV]) are compared in the phase-space built with invariant masses only. A binned or unbinned comparison of the phase-spaces of the decays is then performed using techniques such as those described in Sect. 9.1.

The A_{CP}^{P-odd} from Eq. 9.12 is equivalent to the quantity measured in the T -odd analyses. The asymmetry is typically measured integrated over the whole phase-space or asymmetries can be measured in the phase-space regions [84].

Unbinned model independent techniques do not allow for an asymmetry measurement. However, the P -odd CP asymmetry can be tested by comparing

the combined sample $I + IV$ with the combined sample $II + III$. This comparison may be performed in the same phase-space as the default P -even approach and allows the probing of the P -odd contribution into the CP asymmetry; the P -even contribution cancels out.

In the case of four-body meson decays, P -odd amplitudes can contribute only if the intermediate-resonance configuration is VV , VT or TT (V and T stand for vector and tensor meson respectively) and if both resonances have helicities of either ± 1 or ± 2 . For example, In the $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ decays there is one significant P -odd amplitude. It is the one describing perpendicular helicity ($A_\perp$) of the $D^0 \rightarrow \rho^0(770)\rho^0(770)$ decays. Alternatively, in the partial wave basis, it is the amplitude corresponding to the P -wave $D^0 \rightarrow \rho^0(770)\rho^0(770)$ decays, meaning relative orbital momentum of the two $\rho(770)$ mesons equal to 1. In such cases, the default approach may be extended to make a complementary test of the P -odd CP asymmetry.

A P -odd CP asymmetry is induced by interferences between P -even and P -odd decay amplitudes, thus the sensitivity to CP violation depends on the P -odd amplitude contribution in the decay. Consider a model made of two resonances, $\mathcal{A}_1 = a_1 e^{i\Phi_1}$ and $\mathcal{A}_2 = a_2 e^{i\Phi_2}$ where $\mathcal{A}_1$ is a P -odd amplitude and $\mathcal{A}_2$ is a P -even amplitude, then in samples sets $[I]$ and $[III]$, where $C_T(-\overline{C}_T) > 0$, we have the total amplitude is

$$\mathcal{A}_{I/III} = a_1 e^{i\Phi_1} + a_2 e^{i\Phi_2} \propto |a_1|^2 + |a_2|^2 + 2a_1 a_2 \cos(\Phi_1 - \Phi_2). \quad (9.13)$$

In sample sets $[II]$ and $[IV]$, where $C_T(-\overline{C}_T) < 0$, the total amplitude is

$$\mathcal{A}_{II/IV} = -a_1 e^{i\Phi_1} + a_2 e^{i\Phi_2} \propto |a_1|^2 + |a_2|^2 - 2a_1 a_2 \cos(\Phi_1 - \Phi_2). \quad (9.14)$$

If a CP asymmetry between particles and anti-particles exist in the amplitude of a a_1 resonance, *e.g.* in one flavour (samples sets $[I]$ and $[II]$) a_1 is replaced by a'_1 , then in a P -even test, by combining sample sets $[I]$ and $[II]$ or $[III]$ and $[IV]$, the contribution from the phase cancels,

$$\mathcal{A}_I + \mathcal{A}_{II} \propto 2 \times (|a'_1|^2 + |a_2|^2), \quad (9.15)$$

$$\mathcal{A}_{III} + \mathcal{A}_{IV} \propto 2 \times (|a_1|^2 + |a_2|^2), \quad (9.16)$$

in which case $\mathcal{A}_I + \mathcal{A}_{II} \neq \mathcal{A}_{III} + \mathcal{A}_{IV}$, hence the CP asymmetry effect can be observed. In the P -odd test, the contribution from the phase cannot be cancel out, we have

$$\mathcal{A}_I + \mathcal{A}_{IV} \propto [|a'_1|^2 + |a_2|^2 + 2a'_1 a_2 \cos(\Phi_1 - \Phi_2)] + [|a_1|^2 + |a_2|^2 - 2a_1 a_2 \cos(\Phi_1 - \Phi_2)], \quad (9.17)$$

$$\mathcal{A}_{II} + \mathcal{A}_{III} \propto [|a'_1|^2 + |a_2|^2 - 2a'_1 a_2 \cos(\Phi_1 - \Phi_2)] + [|a_1|^2 + |a_2|^2 + 2a_1 a_2 \cos(\Phi_1 - \Phi_2)], \quad (9.18)$$

hence the CP asymmetry effect $4a_2(a'_1 - a_1) \cos(\Phi_1 - \Phi_2)$ is suppressed by the phase difference.

If a CP asymmetry exists in the phase of a_1 resonance, *e.g.* in one flavour (samples sets $[I]$ and $[II]$) Φ is replaced by Φ' , then in the P -even test, by combining sample sets $[I]$ and $[II]$ or $[III]$ and $[IV]$, the contribution from the phase cancels,

$$\mathcal{A}_I + \mathcal{A}_{II} \propto 2 \times (|a_1|^2 + |a_2|^2), \quad (9.19)$$

$$\mathcal{A}_{III} + \mathcal{A}_{IV} \propto 2 \times (|a_1|^2 + |a_2|^2), \quad (9.20)$$

hence no CP asymmetry effect can be observed. In P -odd test, the contribution from the phase cannot be cancelled out, we have

$$\mathcal{A}_I + \mathcal{A}_{IV} \propto [|a_1|^2 + |a_2|^2 + 2a_1a_2 \cos(\Phi'_1 - \Phi_2)] + [|a_1|^2 + |a_2|^2 - 2a_1a_2 \cos(\Phi_1 - \Phi_2)], \quad (9.21)$$

$$\mathcal{A}_{II} + \mathcal{A}_{III} \propto [|a_1|^2 + |a_2|^2 - 2a_1a_2 \cos(\Phi'_1 - \Phi_2)] + [|a_1|^2 + |a_2|^2 + 2a_1a_2 \cos(\Phi_1 - \Phi_2)], \quad (9.22)$$

hence the CP asymmetry effect $4a_1a_2[\cos(\Phi'_1 - \Phi_2) - \cos(\Phi_1 - \Phi_2)]$ can be observed.

9.8 Implementation of unbinned techniques

The principal drawback of a number of unbinned statistical methods is the computational time required for large sample sizes. In methods, such as the energy test, that require the pairwise distance between all events in the sample to be calculated, the computational time grows quadratically with the sample size. Furthermore, in the energy test a significant number of permutations are required for the random comparison samples to get a sufficient precision on the probabilistic interpretation of the T value, for example to demonstrate that evidence ($> 3\sigma$) for CP violation was observed over one thousand permutations would be needed. In probing CP violation in b -hadron decays the computational constraints are typically not a limitation in these methods currently. However, in multi-body decay channels of interest in charm physics data samples of order one million events are available at the LHCb experiment. At this sample size one permutation for the energy tests requires around one day of CPU time on a typical computing node. Consequently the generation of 1000 permutations would require significant computational resources.

Even though modern multi-core CPUs have realised thread-level parallelism, the number of parallel threads is still very limited. This has been overcome in our studies by implementation on Graphical Processing Units (GPUs). A GPU is a specialised electronic circuit designed to rapidly manipulate and alter memory to accelerate the creation of images for output to a display. Their highly parallel structure makes them more efficient than CPUs for algorithms where large blocks of data are processed in parallel. Our implementation utilises the Compute Unified Device Architecture (CUDA) and Thrust library developed by NVIDIA [87]. The parallelisation is utilised for the calculation of the pairwise event distances. One permutation for one million events takes approximately 30 minutes to be computed on the two GPU systems utilised (NVIDIA M2070, NVIDIA K40c). Both manual and Grid submissions systems have been used. This implementation has made the energy test computationally feasible for the first time in CP violation searches with large data sets. The code of this implementation of the energy test, Manet, has been made available [88].

9.9 Conclusions

The performance and optimisation of two techniques for model-independent searches for direct CP violation in multi-body decays is discussed. It is shown that binned comparisons are best performed with a smaller number of bins than has previously been used in some of the literature. The potential advantages of unbinned techniques are discussed and demonstrated, and an approach to account for the presence of background is suggested. An implementation of the unbinned energy test technique is provided on GPUs, which renders this method feasible for the largest data sample sizes currently available at experiments.

Considerations in the choice, and symmetrisation, of the coordinates used to describe the phase-space of three and four-body decays are discussed, with specific examples given. A novel method for analysing P -odd CP violation in multi-body decays in unbinned model independent searches is presented.

10 CP violation in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ decays

This chapter is based on the publication in Reference [64] and the corresponding LHCb document [89], which described the searches of CP violation in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ decays with the energy test method. Section 10.1 gives the motivation for searching for CP violation in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ decay, Section 10.2 describes the dataset and signal selection. Sensitivity studies with the energy test are given in Sect. 10.3, and the sources of possible systematic uncertainties are explored in Sect. 10.4. The results of the study are given in Sect. 10.5.

10.1 Introduction

The $D^0 \rightarrow \pi^- \pi^+ \pi^0$ decay¹ proceeds via a singly-Cabibbo-suppressed $c \rightarrow u \bar{d} d$ transition. There are possible contributions from both tree-level and penguin amplitudes, thus providing sensitivity to particles beyond the Standard Model participating in the loops of the penguin amplitude. The interference of these amplitudes may give rise to CP violation, which may be present as an asymmetry between the various resonant contributions to the decay phase space.

The decay $D^0 \rightarrow \pi^- \pi^+ \pi^0$ is dominated by the the $\rho\pi$ resonances, which are the CP eigenstate $\rho^0 \pi^0$, and the flavour eigenstates $\rho^+ \pi^-$, $\rho^- \pi^+$, with the ρ meson decaying into a pair of pions [62]. According to the model used by BaBar, higher ρ resonances that contribute are $\rho(1450)$ and $\rho(1700)$. Furthermore, the following f_0 resonances contribute $f_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$, as well as $f_2(1270)$ and $\sigma(400)$.

The importance of singly-Cabibbo-suppressed decays has been pointed out by Grossman, Kagan, and Nir in 2006 [90]. In a more recent paper, Grossman, Kagan, and Zupan have pointed out the particular interest in the decay $D^0 \rightarrow \pi^- \pi^+ \pi^0$ [91]. They derive isospin relations of the different $\rho\pi$ amplitudes and discuss the possible input from CP violation measurements for identifying contributions from beyond the SM.

This analysis uses the energy test as tool to search for CP -violation, and it is the first application of the energy test in CP violation searches. A visualisation

¹charge conjugates are implied unless stated otherwise

method is also used that allows regions of the plot in which CP violation is observed to be identified as discussed in the previous chapter in Sect. 9.4.

The π^0 candidates are reconstructed using events where two separate photon-candidates are reconstructed and where only one merged energy cluster is observed in the electromagnetic calorimeter. Both samples contribute significantly to the precision of the analysis and provide coverage of complementary regions of the Dalitz plot. This analysis was the first LHCb publication on CP Violation using the π^0 .

The most precise previous search for CP violation in this decay has been performed by the BaBar collaboration [62]. It excludes CP -violating effects larger than a few percent in resonance amplitude changes. The BaBar measurement is based on about 82500 signal candidates with a purity of 98% as measured in a $\pm 1\sigma$ mass window from a dataset corresponding to a luminosity of 385 fb^{-1} . This measurement uses data sample of eight times the size of the BaBar sample, with a purity of 82-91% (see Sect. 10.2). The efficiency in this analysis has a significant variation across the Dalitz plane, but even in the region of lowest efficiency this measurement has significantly more statistics than the BaBar measurement.

10.2 Dataset and signal selection

The analysis is performed using the data collected by LHCb during 2012 operation. In this analysis, the two π^0 categories reconstructed in LHCb are exploited [92]. These are pions for which both final state photons are reconstructed separately (resolved pions), as well as pions that have higher momentum and thus a smaller opening angle of the two photons (merged pions). These π^0 mesons are detected in the calorimeter as one merged cluster which is further split into two subclusters based on the expected shape of the photon shower. The merged pions make up about 30% of the reconstructed π^0 mesons. Among the resolved π^0 mesons there are also candidates made of photons which, after interacting with detector material, have converted into an e^+e^- pair. They comprise about 55% of the resolved sample. The two π^0 samples provide coverage of complementary regions of the $D^0 \rightarrow \pi^-\pi^+\pi^0$ Dalitz plot and thus the use of both contributes significantly to the sensitivity of the analysis. In this section, the trigger, stripping and offline selection of $D^0 \rightarrow \pi^-\pi^+\pi^0$ events of both categories are discussed. The selection of the decay $D^0 \rightarrow K^-\pi^+\pi^0$ is used as a reference sample in this analysis is also discussed.

10.2.1 Trigger selection

The trigger in LHCb consists of a hardware stage, based on high- p_T signatures from the calorimeter and muon systems, followed by a two-level software stage, which applies partial event reconstruction. The software trigger at its first level requires at least one good quality track associated with a particle having high p_T and high χ_{IP}^2 , defined as the difference in χ^2 of the primary pp interaction vertex (PV) reconstructed with and without this particle.

A dedicated second-level trigger partially reconstructs $D^0 \rightarrow \pi^-\pi^+\pi^0$ candidates coming from $D^{*+} \rightarrow D^0\pi_s^+$ decays using only information from the charged particles. These requirements ensure the suppression of combinatorial background without distorting the acceptance in the decay phase space. The $\pi^-\pi^+$ pair is combined with a pion to form a D^{*+} candidate, which is accepted if it has p_T greater than 2.5 GeV/ c and a difference of invariant masses $m(\pi^-\pi^+\pi_s^+) - m(\pi^-\pi^+) < 285 \text{ MeV}/c^2$. All the charged-particle tracks used must have good quality, $p_T > 0.3 \text{ GeV}/c$ and $p > 3.0 \text{ GeV}/c$, while the charged pions from the D^0 decays must also have high χ_{IP}^2 . The $\pi^-\pi^+$ combination is required to form a good quality secondary vertex significantly displaced from the PV, while the pion that decays from D^{*+} must originate from the PV.

The inclusive D^{*+} trigger was introduced at the start of 2012. The data collected by the LHCb experiment in 2012 corresponds to an integrated luminosity of 2 fb^{-1} of pp collisions collected at a centre-of-mass energy of 8 TeV. The magnetic field polarity is reversed regularly during the data taking with

approximately half the data collected at each polarity to reduce the overall effect of any charge-dependent detection and reconstruction efficiency.

10.2.2 The LHCb centralised selection

The centralised selection of interesting events after the reconstruction at LHCb is called the stripping selection. The stripping selection that is used in this analysis is documented in Reference [93]. Table 10.1 shows the stripping selection criteria. The global event selection criteria remove only the events with high multiplicity, which would require excessive computing resources to reconstruct. The strategy is then to make as loose a selection as is possible on the individual particles, while requiring that one of the D^0 daughters is sufficiently hard to pass the high level trigger selection. Finally, the full D^{*+} candidate is required to have satisfy the requirements in dedicated inclusive D^{*+} stripping selections.

In this analysis, the stripping selection consists of the two algorithms (stripping lines), resolved and merged lines, for the two categories in which π^0 candidates are reconstructed. The resolved line involves π^0 for which both final state photons are reconstructed separately in the ECAL, and the merged line contains π^0 which are higher momentum so that an opening angle between the two photons is smaller and they are detected in the ECAL as one merged energy cluster. The reconstruction of these π^0 candidates is discussed in Reference [94].

Resolved π^0 candidates typically have momenta up to 50 GeV/c and p_T up to 5 GeV/c, while the p and p_T spectra for merged π^0 start around 10 GeV/c and 2 GeV/c respectively. The resolution of the merged π^0 invariant mass is significantly worse than that for the resolved π^0 , thus there is no related selection criteria applied in the stripping on the merged π^0 invariant mass.

10.2.3 Offline selection

The offline selection is split into a pre-selection, which follows the trigger selection, and a selection based on a multivariate analysis technique called a boosted decision tree (BDT) [95, 96]. All the D^{*+} candidates are required to pass both levels of the software trigger and stripping lines. In addition, the pre-selection requires more stringent kinematic criteria than those applied in the trigger and stripping lines; in particular $p_T > 0.5$ GeV/c is required for all the D^0 decay products to reduce the combinatorial background. For resolved π^0 mesons, the di-photon invariant mass has to be within 15 MeV/ c^2 of the known π^0 mass, this corresponds to about three times the $m(\gamma\gamma)$ resolution. The invariant mass of merged photons, due to its lower resolution, is required to be within the range of 75 – 195 MeV/ c^2 . The purity of the merged π^0 sample is nevertheless significantly higher with respect to the resolved π^0 , as it benefits from large transverse energy and much lower combinatorial background.

Table 10.1: Stripping selection criteria for the LHCb centralised selection algorithms (stripping lines) that used in this $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis.

Particle	Quantity	Selection criteria
GEC	number of long tracks	< 180
$\pi^\pm$	transverse momentum p_T ghost probability delta-log-likelihood $\Delta \log \mathcal{L}_{K-\pi}$	$> 500 \text{ MeV}/c$ < 0.35 < 0
π^+ or π^-	χ^2_{IP} w.r.t. best PV transverse momentum p_T	> 36 $> 1700 \text{ MeV}/c$
$\pi^\pm$ pair	χ^2 -distance of closest approach between two daughter particles $m_{\pi\pi}$ vertex χ^2/DoF flight distance χ^2	< 15 $< 1850 \text{ MeV}/c^2$ < 3 > 100
π^0	transverse momentum p_T	$> 500 \text{ MeV}/c$
π^0 (resolved only)	$ m_{\gamma\gamma} - 135 \text{ MeV}/c^2 $	$< 15 \text{ MeV}/c^2$
D^0	vertex χ^2/DoF transverse momentum p_T $ m_{\pi^+\pi^-\pi^0} - 1864.84 \text{ MeV}/c^2 $	< 20 $> 1400 \text{ MeV}/c$ $< 150 \text{ MeV}/c^2$
π_s	transverse momentum p_T ghost probability delta-log-likelihood $\Delta \log \mathcal{L}_{e-\pi}$ min. χ^2_{IP}	$> 300 \text{ MeV}/c$ < 0.35 < 5 < 9
D^{*+}	$m_{\pi^+\pi^-\pi^0\pi_s} - m_{\pi^+\pi^-\pi^0}$ χ^2 -distance of closest approach between two daughter particles vertex χ^2/DoF	$< 180 \text{ MeV}/c^2$ < 20 < 9

Cross-feed from $D^0 \rightarrow K^- \pi^+ \pi^0$ decays, with a kaon misidentified as a pion, is reduced with requirements on the $\pi^\pm$ particle identification based on the ring-imaging Cherenkov detectors.

The D^0 candidates satisfying the above criteria and having invariant mass $m(\pi^- \pi^+ \pi^0)$ within 40(60) MeV/c^2 of the known D^0 mass are accepted in the resolved (merged) sample; this range corresponds to approximately four times the $m(\pi^- \pi^+ \pi^0)$ resolution. The D^{*+} candidates, formed with the D^0 and π_s^+ candidate, have their entire decay chain refitted requiring that the D^{*+} candidate originates from the corresponding PV, and π^0 and D^0 candidates have their nominal masses. Due to the mass difference between D^{*+} and D^0 is not too much larger than the mass of the pion coming from D^{*+} , the π_s^+ is called the slow pion. This improves the D^{*+} mass resolution and the resolution in the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ Dalitz plot, while a requirement put on the fit quality efficiently suppresses the background. This requirement also suppresses the

contribution from the D^{*+} mesons originating from long-lived b -hadrons. The remainder of this component is not affected by CP asymmetries in b -hadrons since the flavour tag is obtained from the D^{*+} meson.

This preliminary selection is followed by a BDT selection. Signal and background samples used to train the BDT are obtained by applying the *sPlot* technique [97] to a quarter of the real data. The *sWeights* for signal and background separation are determined from a fit to the distribution of the mass difference, $\Delta m \equiv m(\pi^- \pi^+ \pi^0 \pi_s^+) - m(\pi^- \pi^+ \pi^0)$, separately for the resolved and merged samples. The BDT uses the variables related to the kinematic and topological properties of the signal decays, as well as the π^0 quality. It is trained separately for the resolved and merged data categories. The most discriminating variables in the resolved sample are $p_T(\pi_s^+)$, $p_T(D^0)$ and $p_T(\pi^0)$, while in the merged sample these are $p_T(\pi_s^+)$, $p_T(D^0)$ and the $D^0 \chi_{\text{IP}}^2$. The optimal value of the BDT discriminant is determined by estimating the D^{*+} signal significance for various requirements on the BDT output. It retains approximately 75% (90%) of the resolved (merged) signal events while removing 90% (55%) of the background. Figure 10.1 shows the Δm distributions for the selected data set for events with resolved and merged π^0 candidates. The signal shapes, fitted in Fig. 10.1 with a sum of three Gaussian functions, significantly differ between both samples reflecting the different π^0 momentum resolutions. The lower momentum resolution of the merged π^0 mesons relative to the resolved π^0 mesons makes the core part of the merged signal distribution wider, while the low- p_T π^0 mesons contributing to the resolved signal enlarge its tail component. The background shape is fitted using a second-order polynomial multiplied by $\sqrt{1 - m_{\pi^+}/\Delta m}$.

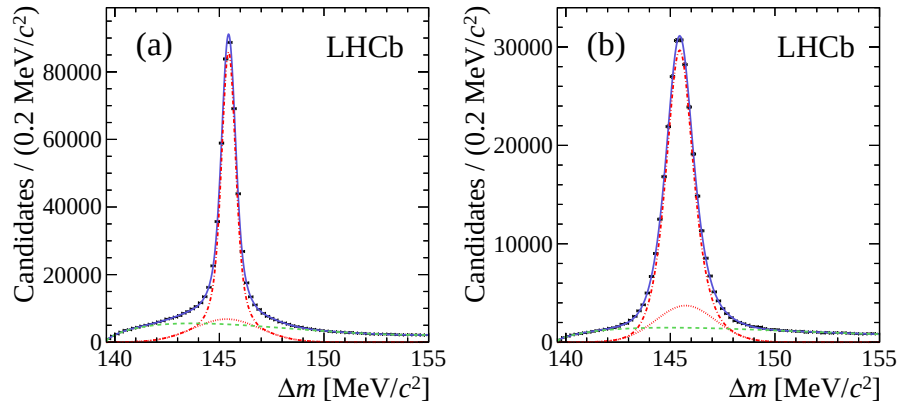


Figure 10.1: Distribution of Δm with fit overlaid for the selected data set with (a) resolved and (b) merged π^0 candidates in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis. The lines show the fit results for total signal (dot-dashed red), widest Gaussian signal component (dotted red), background (dashed green), and total (solid blue).

The final signal sample is selected requiring $|\Delta m - 145.4| < 1.8 \text{ MeV}/c^2$,

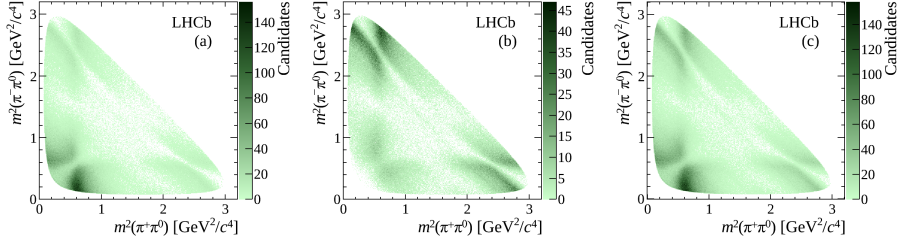


Figure 10.2: Dalitz plot of the (a) resolved, (b) merged and (c) combined $D^0 \rightarrow \pi^- \pi^+ \pi^0$ data sample. Enhanced event densities in the phase-space corners originate from the $\rho(770)$ resonances.

Table 10.2: Results of fit to the Δm distributions for the full data in the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis after BDT selection shown in Fig. 10.1. $\Delta m_{0\text{core}}$, $\Delta m_{0\text{mid}}$ and $\Delta m_{0\text{tail}}$ are the peak positions of the three Gaussians, σ_{core} , σ_{mid} and σ_{tail} are their widths, and f_{core} and f_{mid} are the signal fractions corresponding to the two narrow Gaussians.

Parameter	Resolved sample	Merged sample
Signal [10^3]	492.0 ± 2.2	272.6 ± 0.4
$\Delta m_{0\text{core}} [\text{MeV}/c^2]$	145.440 ± 0.001	145.440 ± 0.001
$\Delta m_{0\text{mid}} [\text{MeV}/c^2]$	$= \Delta m_{0\text{core}}$	$= \Delta m_{0\text{core}}$
$\Delta m_{0\text{tail}} [\text{MeV}/c^2]$	145.34 ± 0.01	145.73 ± 0.01
f_{core}	0.44 ± 0.02	0.208 ± 0.002
f_{mid}	0.28 ± 0.02	0.550 ± 0.002
$\sigma_{\text{core}} [\text{MeV}/c^2]$	0.295 ± 0.004	0.446 ± 0.003
$\sigma_{\text{mid}} [\text{MeV}/c^2]$	0.55 ± 0.02	0.752 ± 0.002
$\sigma_{\text{tail}} [\text{MeV}/c^2]$	1.58 ± 0.03	1.42 ± 0.01
Background [10^3]	297.7 ± 2.2	90.3 ± 0.3

which corresponds to roughly four times the effective Δm resolution. The effective resolution is similar for both resolved and merged π^0 samples when averaging the narrow and broad components of the peak. This gives 416×10^3 resolved and 247×10^3 merged signal candidates with a purity of 82% and 91%, respectively. The Dalitz plot of the final signal sample is shown in Fig. 10.2. The fit parameters of the signal sample are given in Table. 10.2. The smaller number of candidates in the low $m^2(\pi^+ \pi^-)$ region compared with the high $m^2(\pi^+ \pi^-)$ region is due to acceptance effects related to the π^0 reconstruction as discussed in Sect. 10.3.

10.2.4 Dalitz plot and mass projections for $D^0 \rightarrow \pi^- \pi^+ \pi^0$

The $D^0 \rightarrow \pi^- \pi^+ \pi^0$ Dalitz plot and its projections were studied using either the $|\Delta m - 145.4| < 1.8 \text{ MeV}/c^2$ signal region events with background estimated

from the $\Delta m > 150 \text{ MeV}/c^2$ sideband or using *sWeights* to obtain distributions for signal and background.

The Dalitz distributions, $m_{\pi^- \pi^0}^2$ versus $m_{\pi^+ \pi^0}^2$, are shown for the selected resolved and merged data samples in the plots in the left column of Fig. 10.3. A notation in which π^- denotes a pion with its charge opposite to mother D flavour is used, while π^+ is a pion of the charge the same as a D flavour. Therefore Dalitz distribution specified as $m_{\pi^- \pi^0}^2$ versus $m_{\pi^+ \pi^0}^2$, for $\bar{D}^0$ decays corresponds to $m_{\pi^+ \pi^0}^2$ versus $m_{\pi^- \pi^0}^2$.

The plots in the right column in Fig. 10.3 show the Dalitz plots for Δm sideband, $150 \text{ MeV}/c^2 < \Delta m < 155 \text{ MeV}/c^2$, which gives an approximation of the background in the Δm signal region. To obtain an equivalent number of background events as expected from the Δm fit in Fig. 10.1, the sideband distributions for the resolved sample have been prescaled by a factor of 1.3. Corresponding Dalitz plot projections, $m_{\pi^- \pi^0}$, $m_{\pi^+ \pi^0}$ and $m_{\pi^+ \pi^-}$, are shown in Fig. 10.4. Contributions from $\rho(770)^-$, $\rho(770)^+$ and $\rho(770)^0$ can be seen in the invariant-mass projections, while their two-blob helicity structures typical for longitudinally polarized vector particles are clearly seen in the Dalitz distribution. Other resonances contributing to the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ final state and listed in Table 10.4 are either too broad or have too small amplitudes to be visible by eye.

There are $\rho(770)$ contributions also in the background distribution, however their interference pattern visible in the signal-region Dalitz distribution is mostly missing and ρ^+ , ρ^- and ρ^0 contributions to the background are all similar despite their different amplitudes in the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ signal. This suggests that a significant part of the background comes from the combinatorial D^0 background in which a random ρ has been combined with a good or random pion. Some of the ρ 's are misreconstructed hence their shapes in mass projections are broadened while the corresponding Dalitz regions are significantly smeared. The latter is clearly visible in the bottom-left corner of the sideband Dalitz plot for the resolved sample (top right plot in Fig. 10.3) as a result of cross-feed between ρ^+ and ρ^- due to the swapped π^0 . An additional structure present in the background is a peak in $m_{\pi^+ \pi^-}$ at $1.3 \text{ GeV}/c^2$ (see Fig. 10.4). In the Dalitz plot for the sideband events (Fig. 10.3) this background appears as a band above the bottom left corner which is populated by one half of the $\rho(770)^+$ and $\rho(770)^-$ helicities. It suggests that this background is a cross-feed from the $\rho(770)^\pm$ reconstructed with a random π^0 of higher momentum (the band is orthogonal to the $m_{\pi^+ \pi^-}^2$ mass, so corresponds to the a π^0 of certain momentum). Corresponding reflections from the other $\rho(770)^\pm$ helicity parts would be from random π^0 of lower momentum, however any extra band in higher $m_{\pi^+ \pi^-}^2$ region is not clearly visible because of lower statistics in that part of the Dalitz plot.

Real D^0 s combined with a random pion that comes from D^{*+} (π_s) comprise a small contribution to the total background.

The Dalitz distributions and their projections are significantly different for

the resolved and merged samples as a consequence of the π^0 kinematics. Below a certain energy threshold π^0 mesons are reconstructed as resolved π^0 , while they form merged π^0 above this threshold. This threshold essentially forms a boundary on the $m_{\pi^+\pi^-}$ axis, which leads to the observation that events with resolved and merged π^0 cover two almost disjoint regions of phase space (see for example the efficiency plots in Fig. 10.16). In the analysis it is therefore important to merge the two samples in order to have information about the full phase space structure available simultaneously.

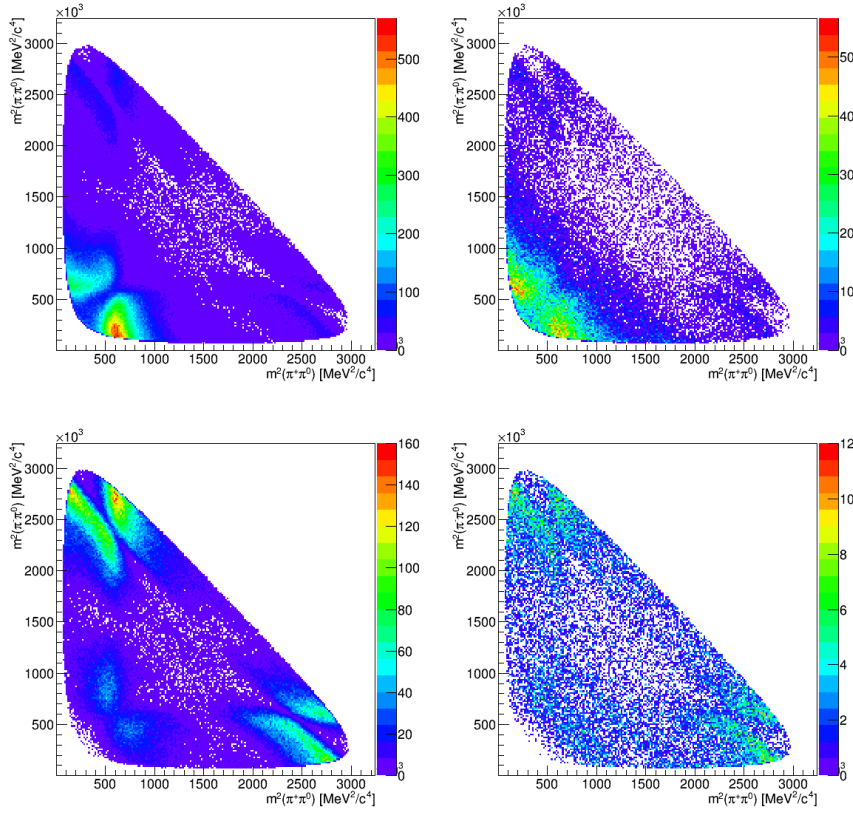


Figure 10.3: Dalitz plots $m^2_{\pi^-\pi^0}$ vs. $m^2_{\pi^+\pi^0}$ for (top row) resolved pions and (bottom row) merged pions samples for (left column) Δm signal region events and (right column) Δm sidebands in data for the $D^0 \rightarrow \pi^-\pi^+\pi^0$ analysis.

Based on the Δm fit in Fig. 10.1, *sWeights* are calculated and used to subtract background from both the Dalitz plot and mass projections. The *sWeighted* spectra are shown in Fig. 10.5 and Fig. 10.6. Unlike the spectra plotted for Δm signal and sideband regions, the *sWeighted* distributions are plotted in a full Δm region, thus the background contribution is significant.

Figure 10.7 shows that background distributions obtained from the Δm

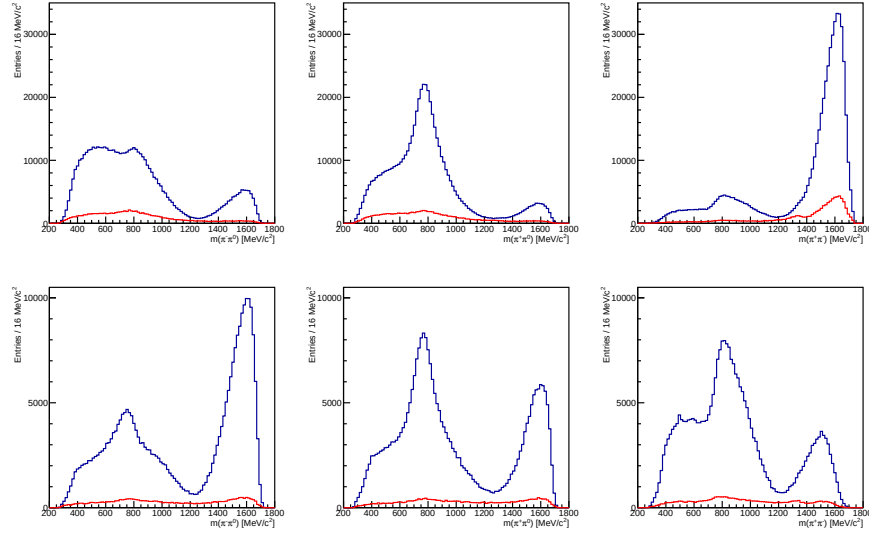


Figure 10.4: Dalitz plot projections (left column) $m_{\pi^-\pi^0}$, (middle column) $m_{\pi^+\pi^0}$, and (right column) $m_{\pi^+\pi^-}$ for (top row) resolved pions and (bottom row) merged pion samples in data for the $D^0 \rightarrow \pi^-\pi^+\pi^0$ analysis. Blue histogram is for Δm signal region events and red for Δm sideband region events.

sidebands and from the $sWeights$ are quite similar. Neither of them exhibits any visible CP asymmetry (see Figs. 10.28- 10.29 in Sec. 10.4.1). The $sWeighted$ background gives in general a better description of the background in the signal region than the sideband events which approximate background from a slightly different kinematic region. The statistics of the available Δm sidebands is also quite small. Therefore the study of the impact of asymmetries in the background (see Sect. 10.4.1) is performed with samples from a simplified simulation. In this simplified simulation, known as a ‘toy’ simulation, no modelling of the LHCb detector is performed. However, as described in Sect. 10.4.1, using both the energy test method and the binned method, a check is performed for asymmetries in the background estimated from the sidebands, and no significant asymmetry is obtained. A similar result is also obtained for the $sWeighted$ background tested with the binned method. Therefore, as concluded in Sect. 10.4.1, independent of the method used to analyse the backgrounds or its nature, no significant asymmetry is observed from the Dalitz plot. Consequently, the background can be neglected in the analysis.

Finally both resolved and merged data samples are combined and the resulting Dalitz plot and the mass projections are shown respectively in Fig. 10.8 and Fig. 10.9 for the Δm signal and sideband regions. Due to the lower overall reconstruction efficiency in the merged sample, the acceptance across the Dalitz

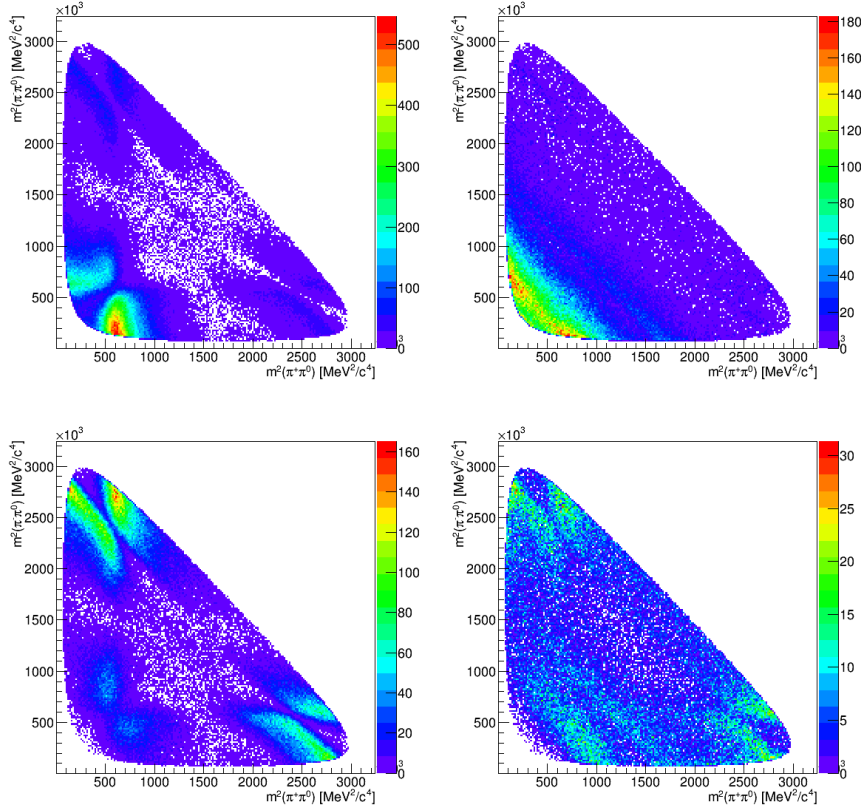


Figure 10.5: Dalitz plots $m_{\pi^-\pi^0}^2$ vs. $m_{\pi^+\pi^0}^2$ for (left column) *sWeighted* signal and (right column) *sWeighted* background for (top row) resolved and (bottom row) merged samples in data for the $D^0 \rightarrow \pi^-\pi^+\pi^0$ analysis.

plot for the combined sample is still lower in the region of small $m_{\pi^+\pi^-}$.

The Dalitz plots presented here are for the combined D^0 and $\bar{D}^0$ sample. The analysis method is applied to the Dalitz distributions for the combined merged and resolved sample (left plot in Fig. 10.8) in the Δm signal region but separated by D^0 flavour. The mass projections separated by D^0 flavour are shown in Fig. 10.10.

10.2.5 Selection of reference decay $D^0 \rightarrow K^-\pi^+\pi^0$

$D^0 \rightarrow K^-\pi^+\pi^0$ decays serve as a reference channel in this analysis. The reference channel is used to perform checks on whether various detection and production asymmetries, present in both signal and reference modes, have any impact on the CP violation search in $D^0 \rightarrow \pi^-\pi^+\pi^0$ decays. Being dominated by a Cabibbo-favoured $c \rightarrow s\bar{u}d$ diagram, with a small contribution of a doubly-

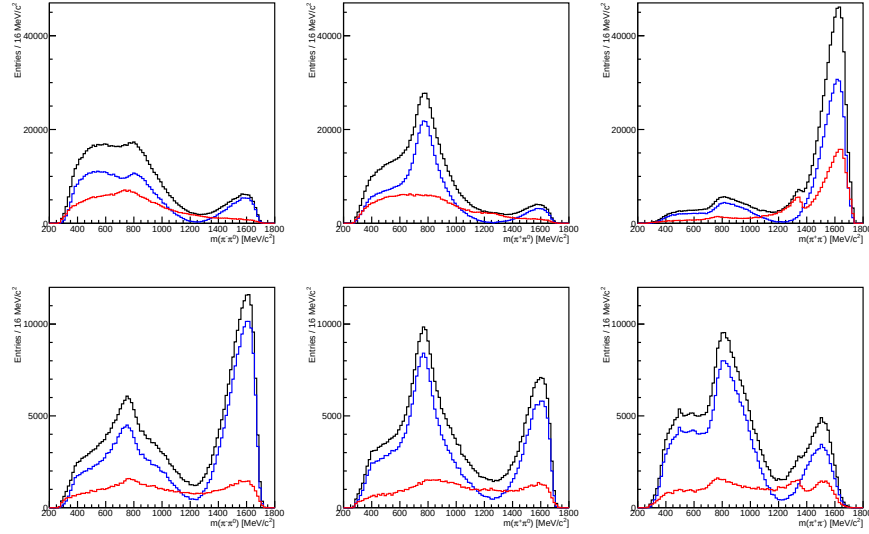


Figure 10.6: Dalitz plot projections (left column) $m_{\pi^-\pi^0}$, (middle column) $m_{\pi^+\pi^0}$, and (right column) $m_{\pi^+\pi^-}$ for (top row) resolved pion and (bottom row) merged pion samples in data for the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis. Blue histograms show $sWeighted$ signal, red ones are $sWeighted$ background, black histograms show total distributions. $sWeights$ are obtained from fits to Δm shown in Fig.10.1.

Cabibbo-suppressed $c \rightarrow d\bar{u}s$ amplitude, $D^0 \rightarrow K^- \pi^+ \pi^0$ is expected to be a CP violation free channel. Hence the application of the CP search to this channel is expected to yield a null result.

Although the $D^0 \rightarrow K^- \pi^+ \pi^0$ and $D^0 \rightarrow \pi^- \pi^+ \pi^0$ final states differ in some relevant features, the kinematics of their daughter particles are quite similar. The extent to which both channels are similar in terms of potential sources of non CPV-related asymmetries is discussed in Sect. 10.4.2. To get the relevant estimate of our sensitivity to these asymmetries, a total $D^0 \rightarrow K^- \pi^+ \pi^0$ sample selected from 2012 data is randomly divided into subsamples similar in size to the one of $D^0 \rightarrow \pi^- \pi^+ \pi^0$.

The stripping selection for $D^0 \rightarrow K^- \pi^+ \pi^0$ follows the one in Table 10.1 and includes additional selections related to the charged kaon: its ghost probability is required to be below 0.35, delta-log-likelihood $\Delta \log \mathcal{L}_{K-\pi} > 7$ and $p_T > 500 \text{ MeV}/c$. The stripping lines used here are dedicated $D^0 \rightarrow \pi^- \pi^+ \pi^0$ resolved and merged π^0 lines. Same pre-BDT selection criteria as for the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ were applied, employ the BDT weights as obtained for the resolved and merged signal samples and the same requirement on the BDT discriminators.

The selected $D^0 \rightarrow K^- \pi^+ \pi^0$ decay sample is about eight times larger than for the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ decay, and was divided into eight subsamples. The

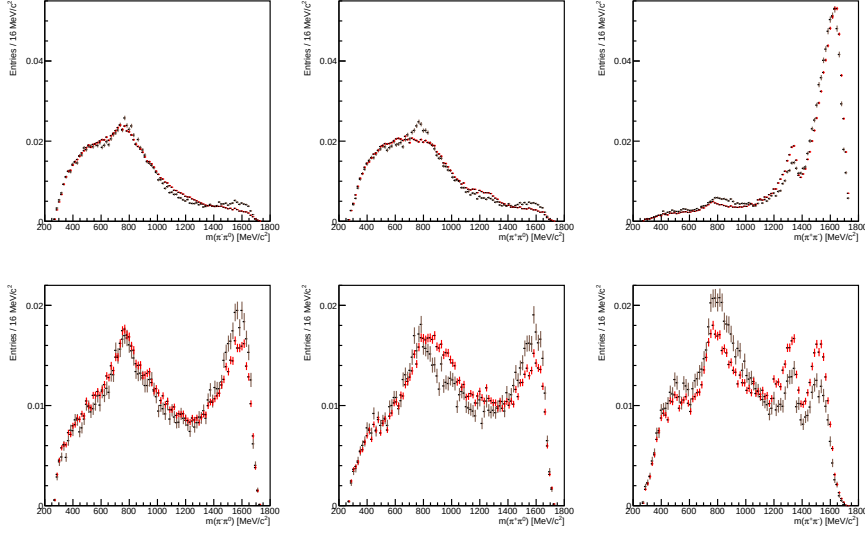


Figure 10.7: Comparison of background estimated from Δm sidebands (brown) and from the $sWeights$ (red) distributed on Dalitz plot projections (left column) $m_{\pi^-\pi^0}$, (middle column) $m_{\pi^+\pi^0}$, and (right column) $m_{\pi^+\pi^-}$ for (top row) resolved and (bottom row) merged samples in data for the $D^0 \rightarrow \pi^-\pi^+\pi^0$ analysis. The background distributions are normalized to unity.

events in the subsamples were chosen randomly. Below the properties of one of the eight $D^0 \rightarrow K^-\pi^+\pi^0$ subsamples are presented. Distributions of Δm for both $D^0 \rightarrow K^-\pi^+\pi^0$ data categories are shown in Fig. 10.11. A fit to the Δm distribution gives signal yields of 545×10^3 and 242×10^3 for the resolved and merged subsamples, respectively. The parametrisations used in the fit follow the ones used in the signal mode. The fit results are summarized in Table 10.3. The signal shapes are very similar to the $D^0 \rightarrow \pi^-\pi^+\pi^0$ ones.

Candidates within $|\Delta m - 145.4| < 1.8 \text{ MeV}/c^2$ are selected; inside one subsample this gives 431×10^3 resolved events and 227×10^3 merged events with purities of 93% and 97%, respectively. The combined $D^0 \rightarrow K^-\pi^+\pi^0$ subsample consists of 678×10^3 events as compared to 663×10^3 of $D^0 \rightarrow \pi^-\pi^+\pi^0$ events.

The Dalitz plot, $m_{K^-\pi^0}^2$ vs. $m_{\pi^+\pi^0}^2$, for the selected signal region events and for the Δm sideband region and for both resolved and merged subsamples combined, is shown in Fig. 10.12. Figure 10.13 shows the two-body invariant mass distributions. In addition to dominant $\rho(770)^+ \rightarrow \pi^+\pi^0$ component there are also $K^*(892)^-$ and $K^*(892)^0$ contributing to the $D^0 \rightarrow K^-\pi^+\pi^0$ final state.

The reference mode used is expected to give a good estimate of the asymmetry related to the detection of the charged pion from D^0 decays. As this asymmetry depends on the pion kinematics, it must be checked that the kin-

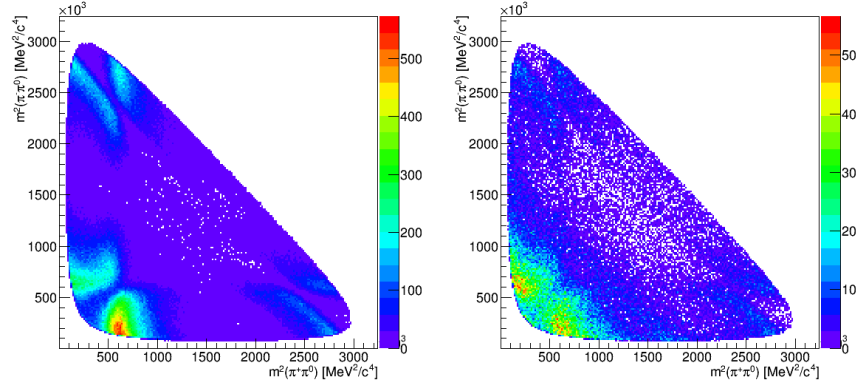


Figure 10.8: Dalitz plots $m_{\pi^- \pi^0}^2$ vs. $m_{\pi^+ \pi^0}^2$ for the combined data sample of $D^0 \rightarrow \pi^- \pi^+ \pi^0$ events for (left) Δm signal region events and (right) Δm sidebands region events.

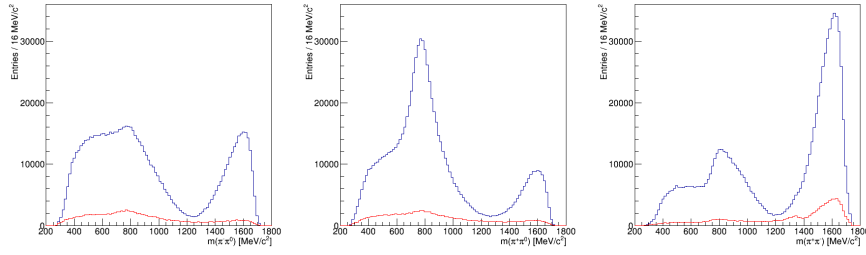


Figure 10.9: Dalitz plot projections (left) $m_{\pi^- \pi^0}$, (middle) $m_{\pi^+ \pi^0}$, and (right) $m_{\pi^+ \pi^-}$ for the combined data sample for the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis. Blue histogram is for Δm signal region events and red for Δm sidebands region events.

ematics of the π^+ from the signal channel and the reference channel are similar. Figure 10.14 shows the momentum and p_T distributions of the π^+ coming from the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ and $D^0 \rightarrow K^- \pi^+ \pi^0$ signal events. The spectra shows the momentum of the π^+ from the reference channel are slightly slower, but the p_T and p coverage and overall agreement are good enough to use $D^0 \rightarrow K^- \pi^+ \pi^0$ as a reference.

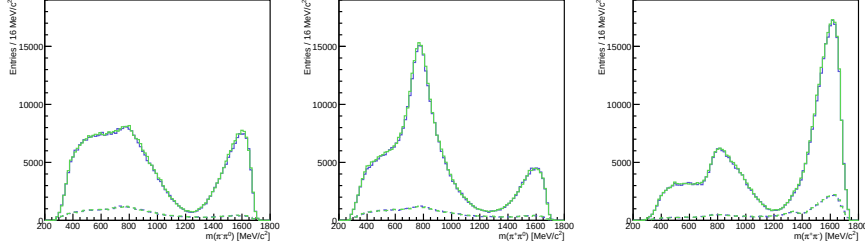


Figure 10.10: Comparison of mass projections for D^0 (blue) and $\bar{D}^0$ (green) for Δm signal region (solid line) and Δm sideband (dashed line) in data for the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis. The projections show (left) $m_{\pi^- \pi^0}$, (middle) $m_{\pi^+ \pi^0}$, and (right) $m_{\pi^+ \pi^-}$ for both the resolved and merged pion samples combined.

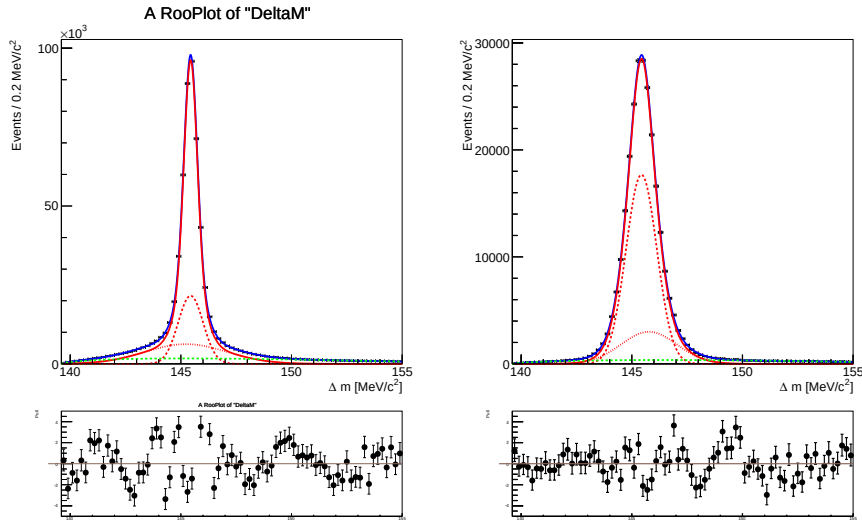


Figure 10.11: Δm distributions for $D^0 \rightarrow K^- \pi^+ \pi^0$ data after BDT selection for (left) resolved pion and (right) merged pion data samples. Lines show fit results for: total signal (solid red), *mid* signal component (dashed red), *tail* signal component (dotted red), background (green), total (blue).

Table 10.3: Results of fit to the Δm distributions for $D^0 \rightarrow K^- \pi^+ \pi^0$ shown in Fig. 10.11. $\Delta m_{0\text{core}}$, $\Delta m_{0\text{mid}}$ and $\Delta m_{0\text{tail}}$ are the peak positions of the three Gaussians, σ_{core} , σ_{mid} and σ_{tail} are their widths, and f_{core} and f_{mid} are the signal fractions corresponding to the two narrow Gaussians.

Parameter	Resolved sample	Merged sample
Signal [10^3]	544.8 ± 1.2	242.2 ± 0.6
$\Delta m_{0\text{core}} [\text{MeV}/c^2]$	145.440 ± 0.001	145.440 ± 0.002
$\Delta m_{0\text{mid}} [\text{MeV}/c^2]$	$= \Delta m_{0\text{core}}$	$= \Delta m_{0\text{core}}$
$\Delta m_{0\text{tail}} [\text{MeV}/c^2]$	145.24 ± 0.01	145.77 ± 0.02
f_{core}	0.45 ± 0.01	0.17 ± 0.02
f_{mid}	0.28 ± 0.01	0.64 ± 0.02
$\sigma_{\text{core}} [\text{MeV}/c^2]$	0.289 ± 0.003	0.41 ± 0.01
$\sigma_{\text{mid}} [\text{MeV}/c^2]$	0.56 ± 0.01	0.70 ± 0.01
$\sigma_{\text{tail}} [\text{MeV}/c^2]$	1.85 ± 0.01	1.26 ± 0.02
Background [10^3]	102.1 ± 2.2	22.0 ± 0.3

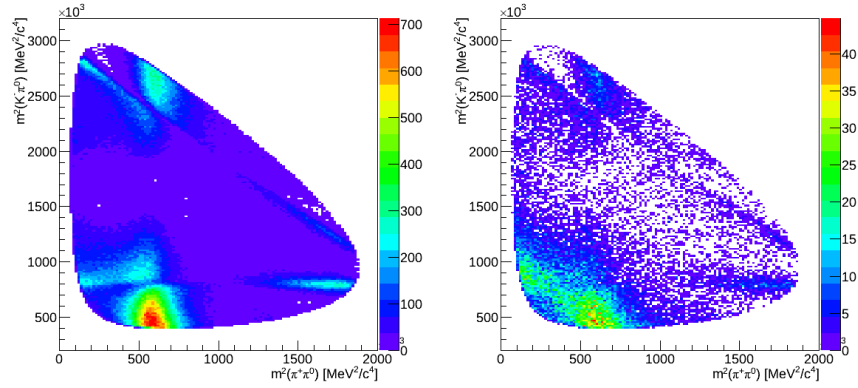


Figure 10.12: $D^0 \rightarrow K^- \pi^+ \pi^0$ analysis Dalitz plots $m_{K-\pi^0}^2$ vs. $m_{\pi^+\pi^0}^2$ for the combined data sample for (left) Δm signal region events and (right) Δm sidebands region events.

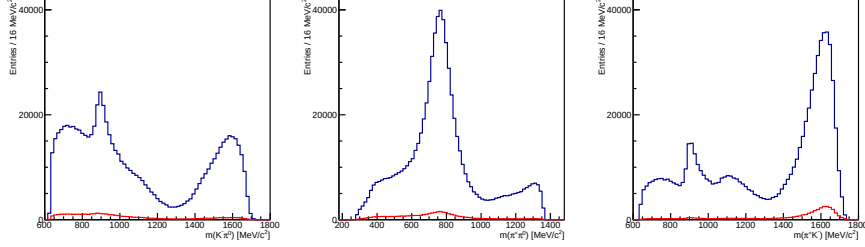


Figure 10.13: Projections of $D^0 \rightarrow K^-\pi^+\pi^0$ analysis selected data sample Dalitz plot: (left) $m_{K^-\pi^0}$, (middle) $m_{\pi^+\pi^0}$, and (right) $m_{\pi^+K^-}$ for both samples combined. Blue histogram is for Δm signal region events and red for Δm sidebands region events.

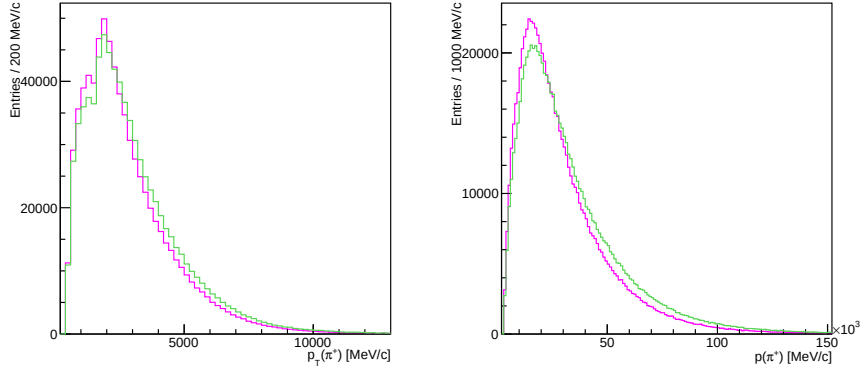


Figure 10.14: Distribution of (left) transverse momentum p_T and (right) momentum for the π^+ from $sWeighted D^0 \rightarrow \pi^-\pi^+\pi^0$ signal (green) and $sWeighted D^0 \rightarrow K^-\pi^+\pi^0$ signal data samples (magenta).

10.3 Sensitivity tests

A sensitivity study was performed in simulation to assess the level of CP violation beyond which an observation of CP violation could be anticipated. The energy test is implemented with the coordinates suggested in Sect. 9.6.2

10.3.1 Simulation Samples

The studies reported here on the sensitivity of the analysis are performed with a simple simulation, referred to here as a ‘toy’ simulation, which does not include a model of the LHCb detector. The realism of the toy simulation is improved by using standard LHCb simulation samples to parameterize the efficiency of the analysis across the Dalitz plane.

Samples of $D^0 \rightarrow \pi^+ \pi^- \pi^0$ decays generated using the Dalitz Plot analysis package Laura++ [70] are used in these studies. The Laura++ package includes a toy simulation sample generator which has been used in several analyses published by the BaBar collaboration. The simulation samples are constructed using the isobar model, where the total amplitude is the sum of a set of contributing amplitudes each with a complex coefficient:

$$A(\vec{x}) = a_{nr} e^{i\Phi_{nr}} + \sum a_r e^{i\Phi_r} A_r(\vec{x}). \quad (10.1)$$

Here $\vec{x} = (m_{\pi^+ \pi^0}^2, m_{\pi^- \pi^0}^2, m_{\pi^+ \pi^-}^2)$ represents the position in the Dalitz space as described in Sect. 9.6.2, $a_r e^{i\Phi_r}$ describes the complex amplitude of each resonance, and $a_{nr} e^{i\Phi_{nr}}$ describes the non-resonant contribution. $A_r(\vec{x})$ contains information about the decay dynamics. The resonance properties used to generate simulation samples are those measured in Reference [16] and are listed in Table 10.4.

Simulation samples are also generated including CP violation through a variation of this model. For example, the CP violation is introduced in the $\rho(770)^+$ resonance:

$$\frac{\Delta a_{\rho(770)^+}}{a_{\rho(770)^+}} = 1.02 \quad (10.2)$$

Hence, the magnitude in the D^0 decay samples are changed by 2%. The magnitude and phase in the $\bar{D}^0$ decay sample remain as in Table 10.4. The T value and visualisation result (see Sect. 9.4) of this toy simulation sample is shown in Fig. 10.15.

Table 10.4: List of resonances and their parameters used to generate $D^0 \rightarrow \pi^- \pi^+ \pi^0$ simulation samples [16].

Channel	Amplitude a_n	Phase $\phi_n(^{\circ})$	Fraction $f_n(\%)$
$\rho(770)^+$	0.823	0	67.8
$\rho(770)^0$	0.512	16.2	26.2
$\rho(770)^-$	0.588	-2.0	34.6
$\rho(1450)^+$	0.033	-146	0.11
$\rho(1450)^0$	0.055	10	0.30
$\rho(1450)^-$	0.134	16	1.79
$\rho(1700)^+$	0.202	-17	4.1
$\rho(1700)^0$	0.224	-17	5.0
$\rho(1700)^-$	0.179	-50	3.2
$f_0(400)$	0.091	8	0.82
$f_0(980)$	0.050	-59	0.25
$f_0(1370)$	0.061	156	0.37
$f_0(1500)$	0.062	12	0.39
$f_0(1710)$	0.056	51	0.71
$f_2(1270)$	0.115	-171	1.32
NonResonant	0.092	-11	0.84

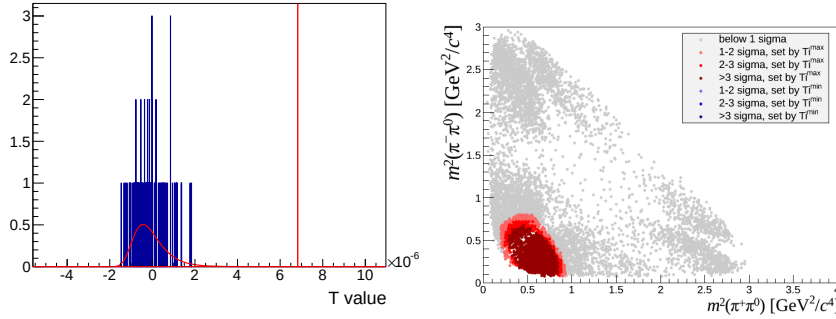


Figure 10.15: (left) T value result for a test sample (red line) and its permutations T values (histogram and fit). The percentage of permutation T values that are greater than the true data sample T value indicates the probability of the CP symmetric hypothesis. A GEV function is used to fit the T value distribution, this allows p values to be measured smaller than 1 over the number of permutations. (right) The visualization tool is applied to observe the regions which have an asymmetry on the Dalitz plot showing $m^2(\pi^- \pi^0)$ versus $m^2(\pi^+ \pi^0)$. The test sample is using toy simulation of the $D^0 \rightarrow \pi^+ \pi^- \pi^0$ decay with a 2% amplitude difference on $\rho(770)^+$. The colour on the visualization plot reveals the amplitude change around the $\rho(770)^+$ mass region.

10.3.2 Studies accounting for selection efficiencies and background effects

This section describes sensitivity studies on simulated data taking into account the phase-space bias from the selection. The selection efficiency is measured based on a full simulation sample comprising 5M $D^0 \rightarrow \pi^- \pi^+ \pi^0$ events generated with a flat phase-space distribution and the LHCb detector acceptance criterion. About 5000 events are retained after application of the stripping criteria, and about 1000 after adding the full offline selection. The efficiency is then defined as the number of reconstructed events divided by the number of generated events in a particular region of phase space. The efficiencies are determined simultaneously for merged and resolved π^0 candidates with an admixture that reflects that found in data.

The efficiency map on a 2D square Dalitz plot ($m_{\pi^+\pi^-}$ against cosine of helicity angle of $\pi^+\pi^-$ system) and the projection to the $m_{\pi^+\pi^-}$ are shown in Fig. 10.16. This parametrisation has been chosen as the largest efficiency variation is expected to be a function of $m_{\pi^+\pi^-}$ due to its correlation with the π^0 momentum. There is a clear efficiency dependence on $m_{\pi^+\pi^-}$ and no visible dependence on the helicity angle. The dependence is parametrised by a straight line. For the sensitivity studies conducted here, toy MC events are generated as described above and events are randomly discarded according to their efficiency as a function of $m_{\pi^+\pi^-}$ and assuming no dependence on the helicity angle.

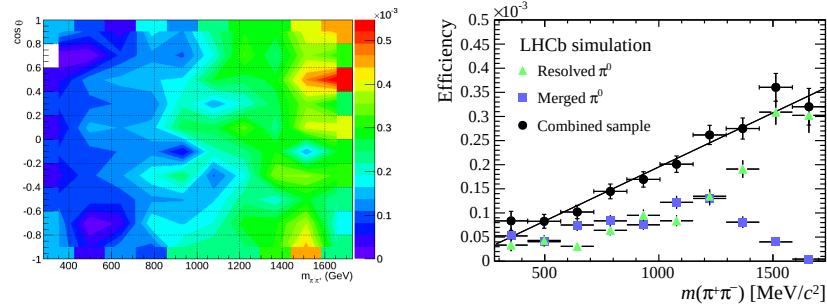


Figure 10.16: The efficiency map obtained with full LHCb simulation of $D^0 \rightarrow \pi^- \pi^+ \pi^0$ decay in (left) a square Dalitz plot ($m_{\pi^+\pi^-}$ against cosine of helicity angle) of the combined resolved and merged pion samples and (right) the corresponding projections on $m_{\pi^+\pi^-}$. The efficiency of the combined sample is fitted with a straight line.

After applying the efficiency map to toy MC data, the toy MC samples used in the sensitivity studies is obtained. A comparison of MC samples without and with the efficiency map applied is shown in Fig. 10.17. From the shape of the efficiency distribution it is obvious that the samples will have a reduced

Table 10.5: Summary of T values obtained from toy simulation samples of $D^0 \rightarrow \pi^- \pi^+ \pi^0$ with backgrounds generated with a range of CP violation scenarios applied to the $\rho(770)^+$ resonance. $\sigma = 0.2(\text{GeV}/c^2)^2$ is used in all tests.

Amplitude	Phase	T value	p -value (from fit)	p -value 90% CL
+1%	+0°	$+2.73 \times 10^{-7}$	$2.6^{+0.4}_{-0.4} \times 10^{-1}$	3.1×10^{-1}
+2%	+0°	4.51×10^{-6}	$1.8^{+1.9}_{-1.0} \times 10^{-4}$	4.4×10^{-4}
+0%	+0.5°	-3.41×10^{-7}	$6.5^{+0.4}_{-0.4} \times 10^{-1}$	7.0×10^{-1}
+0%	+1°	$+3.20 \times 10^{-6}$	$2.0^{+1.3}_{-0.8} \times 10^{-3}$	3.8×10^{-3}
+0%	+2°	1.02×10^{-5}	$7.6^{+32}_{-7.6} \times 10^{-10}$	6.1×10^{-9}

sensitivity in the region of the ρ^0 resonance.

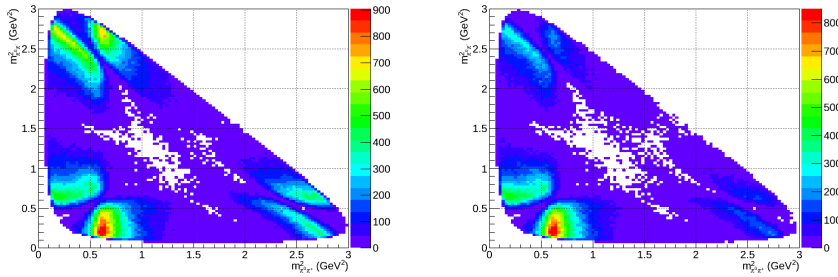


Figure 10.17: The Dalitz plots of the generated toy simulation sample for $D^0 \rightarrow \pi^- \pi^+ \pi^0$ (left) without and (right) with the efficiency map applied.

Example results for toy simulation samples are shown in Fig. 10.18 to Fig. 10.20. The T values and p values are summarized in Table 10.5 to Table 10.7. These samples are with background events added. These background events have been randomly generated reflecting the amount present in data and with shapes following those observed in Δm sidebands. The background shapes are the same for both flavours and the procedure is explained in more detail in Sect. 10.4.1. Judging by these results, the amount of background present in data does not lead to a significant change in sensitivity to CP violation.

10.3.3 Variation of metric parameter σ

The parameter σ defined in Eq. 9.4 which defines the rate at which the distance metric approaches 0 with increasing distance is set to $0.2(\text{GeV}/c^2)^2$ in the default analysis. It should be larger than the resolution as otherwise asymmetries are washed out on the scale of the metric. At the same time it should be small enough to resolve the regions where asymmetries lead to different densities.

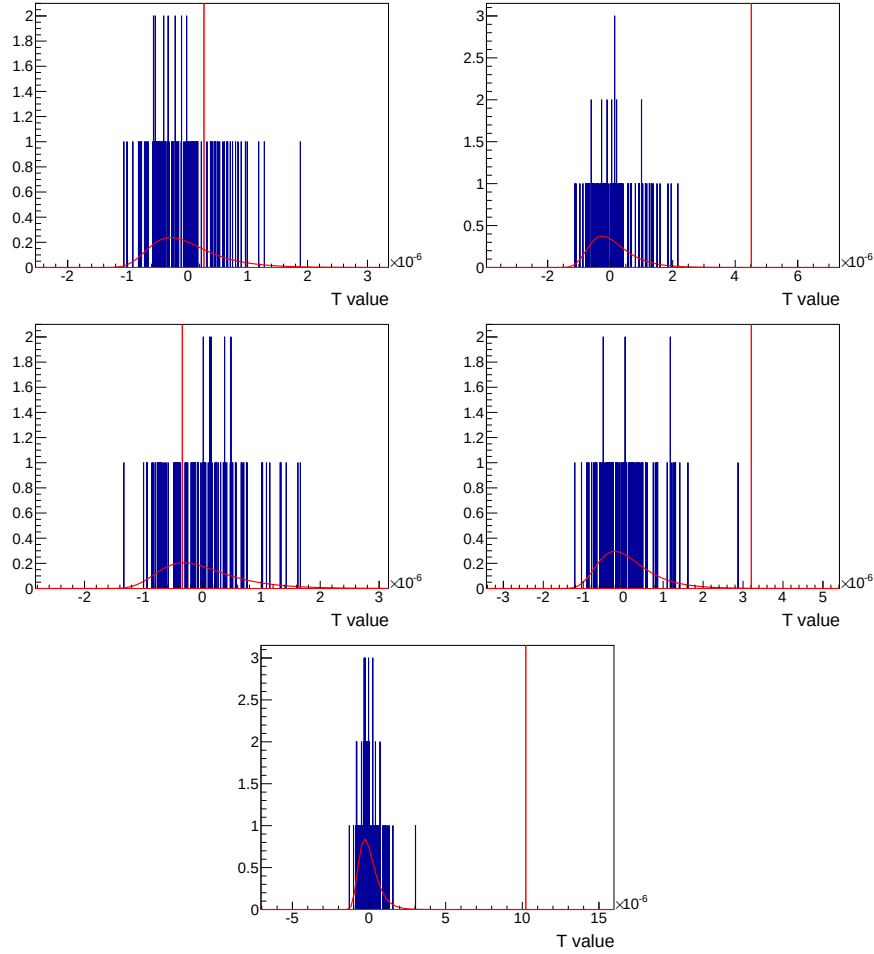


Figure 10.18: T value and permutation T values distribution for toy $D^0 \rightarrow \pi^- \pi^+ \pi^0$ simulation samples in $\rho^+(770)$ CP violation samples. From left to right, then from top to bottom, the changes in $\rho^+(770)$ resonance are: (1). Amplitude +1%, $\phi + 0^\circ$, (2). Amplitude +2%, $\phi + 0^\circ$, (3). Amplitude +0%, $\phi + 0.5^\circ$, (4). Amplitude +0%, $\phi + 1^\circ$, (5). Amplitude +0%, $\phi + 2^\circ$.

Within these two criteria the analysis should be rather insensitive to different values of σ .

For a given CP violation scenario, here the (+2%, +0°) sample is used, the dependence of the result on the choice of σ is tested for a range from $\sigma = 0.03(\text{GeV}/c^2)^2$ to $\sigma = 1.0(\text{GeV}/c^2)^2$. The lower end of this spectrum is in the range of the resolution and thus a deterioration of the efficiency can be expected. As the upper end of the test spectrum the σ is large compared to the overall extension of the phase space, hence some degradation in sensitivity may be expected. Table 10.8 to Table 10.11 lists the results, and Fig 10.21 to

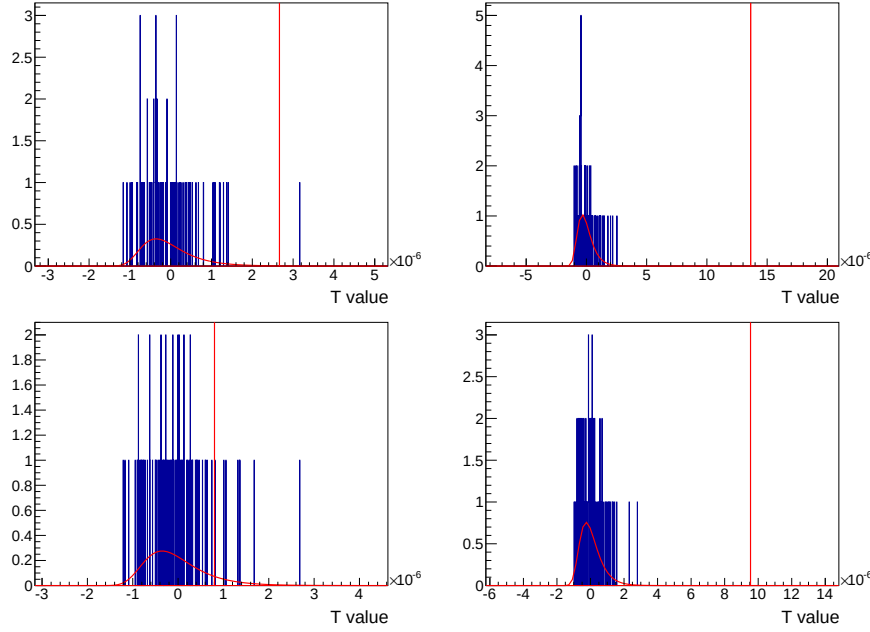


Figure 10.19: T value and permutation T values distribution for toy $D^0 \rightarrow \pi^- \pi^+ \pi^0$ simulation samples in $\rho^-(770)$ CP violation samples. From left to right, then from top to bottom, the changes in $\rho^-(770)$ resonance are: (1). Amplitude +2%, $\phi + 0^\circ$, (2). Amplitude +5%, $\phi + 0^\circ$, (3). Amplitude +0%, $\phi + 1^\circ$, (4). Amplitude +0%, $\phi + 2^\circ$.

Fig 10.24 are the corresponding T value distribution plots. Figure 10.25 and Figure 10.26 give an example of how the σ variation affects the visualisation plot. Indeed a deterioration of the sensitivity is visible at both ends of the spectrum with the optimal sensitivity to this CP violation scenario at approximately $\sigma = 0.2(\text{GeV}/c^2)^2$. The choice of the sigma value is somewhat analogous to the choice of the bin size in a binned analysis. The choice may affect the sensitivity of the analysis but it does not contribute a systematic uncertainty on the result.

Table 10.6: Summary of T values obtained from toy simulation samples in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis with backgrounds generated with a range of CP violation scenarios applied to the $\rho(770)^-$ resonance. $\sigma = 0.2(\text{GeV}/c^2)^2$ is used in all tests.

Amplitude	Phase	T value	p -value (from fit)	p -value 90% CL
+1%	+0°	$+8.84 \times 10^{-8}$	$5.1_{-0.7}^{+0.6} \times 10^{-1}$	5.8×10^{-1}
+2%	+0°	2.67×10^{-6}	$1.8_{-1.3}^{+1.6} \times 10^{-3}$	4.1×10^{-3}
+5%	+0°	1.36×10^{-5}	$2.4_{-2.4}^{+19} \times 10^{-12}$	3.6×10^{-11}
+0%	+0.5°	-2.26×10^{-7}	$6.1_{-0.6}^{+0.5} \times 10^{-1}$	6.8×10^{-1}
+0%	+1°	8.02×10^{-7}	$9.7_{-0.2}^{+0.3} \times 10^{-2}$	1.3×10^{-1}
+0%	+2°	9.54×10^{-6}	$4.1_{-4.1}^{+16} \times 10^{-9}$	3.0×10^{-8}

Table 10.7: Summary of T values obtained from toy simulation samples in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis with backgrounds generated with a range of CP violation scenarios applied to the $\rho(770)^0$ resonance. $\sigma = 0.2(\text{GeV}/c^2)^2$ is used in all tests.

Amplitude	Phase	T value	p -value (from fit)	p -value 90% CL
+1%	+0°	-8.60×10^{-7}	$9.7 \pm 0.1 \times 10^{-1}$	9.9×10^{-1}
+2%	+0°	2.07×10^{-7}	$3.3_{-0.4}^{+0.4} \times 10^{-1}$	3.8×10^{-1}
+5%	+0°	5.99×10^{-6}	$5.8_{-5.8}^{+17} \times 10^{-7}$	3.5×10^{-6}
+0%	+0.5°	-7.42×10^{-8}	$5.8_{-0.6}^{+0.6} \times 10^{-1}$	6.5×10^{-1}
+0%	+1°	-7.34×10^{-7}	$8.9_{-0.2}^{+0.2} \times 10^{-1}$	9.1×10^{-1}
+0%	+2°	-2.28×10^{-7}	$6.3_{-0.4}^{+0.4} \times 10^{-1}$	4.8×10^{-1}
+0%	+5°	9.86×10^{-6}	$2.5_{-2.5}^{+27} \times 10^{-9}$	4.4×10^{-8}

Table 10.8: Summary of T values obtained from toy simulation samples in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis generated with a CP violation of +2% applied to the amplitude of the $\rho(770)^+$ resonance for a range of value of the metric parameter σ .

$\sigma[(\text{GeV}/c^2)^2]$	T value	p -value (from fit)	p -value 90% CL
0.03	1.23×10^{-7}	$1.9_{-0.3}^{+0.3} \times 10^{-1}$	2.3×10^{-1}
0.06	5.53×10^{-7}	$5.7_{-1.6}^{+1.8} \times 10^{-2}$	8.2×10^{-2}
0.10	1.54×10^{-6}	$6.0_{-2.4}^{+3.3} \times 10^{-3}$	1.0×10^{-2}
0.20	4.51×10^{-6}	$1.8_{-1.0}^{+1.9} \times 10^{-4}$	4.4×10^{-4}
0.30	6.82×10^{-6}	$5.0_{-3.8}^{+8.8} \times 10^{-6}$	1.8×10^{-5}
0.40	8.23×10^{-6}	$1.6_{-1.2}^{+3.2} \times 10^{-6}$	6.3×10^{-6}
0.50	9.05×10^{-6}	$5.5_{-4.1}^{+9.6} \times 10^{-6}$	2.0×10^{-5}
1.00	9.83×10^{-6}	$2.5_{-2.9}^{+4.9} \times 10^{-6}$	9.9×10^{-6}

Table 10.9: Summary of T values obtained from toy simulation samples in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis generated with a CP violation of $+2^\circ$ applied to the phase of the $\rho(770)^+$ resonance for a range of value of the metric parameter σ .

$\sigma[(\text{GeV}/c^2)^2]$	T value	p -value (from fit)	p -value 90% CL
0.03	5.08×10^{-7}	$7.9^{+4.0}_{-3.0} \times 10^{-3}$	1.3×10^{-2}
0.06	1.78×10^{-6}	$1.7^{+1.5}_{-1.0} \times 10^{-4}$	3.9×10^{-4}
0.10	4.31×10^{-6}	$2.1^{+5.3}_{-1.6} \times 10^{-7}$	1.0×10^{-6}
0.20	1.02×10^{-5}	$7.6^{+32}_{-7.6} \times 10^{-10}$	6.1×10^{-9}
0.30	1.31×10^{-5}	$2.6^{+16}_{-2.6} \times 10^{-11}$	3.1×10^{-10}
0.40	1.37×10^{-5}	$1.2^{+8.0}_{-1.4} \times 10^{-10}$	1.5×10^{-9}
0.50	1.35×10^{-5}	$8.7^{+33}_{-8.7} \times 10^{-8}$	5.9×10^{-7}
1.00	1.12×10^{-5}	$1.1^{+2.8}_{-0.8} \times 10^{-6}$	5.1×10^{-6}

Table 10.10: Summary of T values obtained from toy simulation samples in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis generated with a CP violation of $+1^\circ$ applied to the phase of the $\rho(770)^+$ resonance for a range of value of the metric parameter σ .

$\sigma[(\text{GeV}/c^2)^2]$	T value	p -value (from fit)	p -value 90% CL
0.10	1.26×10^{-6}	$2.4^{+1.0}_{-0.8} \times 10^{-2}$	3.7×10^{-2}
0.20	3.20×10^{-6}	$2.0^{+1.3}_{-0.8} \times 10^{-3}$	3.8×10^{-3}
0.30	4.01×10^{-6}	$6.3^{+5.5}_{-3.3} \times 10^{-4}$	1.4×10^{-3}
0.40	4.22×10^{-6}	$9.4^{+7.7}_{-4.8} \times 10^{-4}$	2.0×10^{-3}
0.50	4.25×10^{-6}	$5.0^{+5.4}_{-2.9} \times 10^{-4}$	1.2×10^{-3}
1.00	3.50×10^{-6}	$2.1^{+1.9}_{-1.1} \times 10^{-3}$	4.7×10^{-3}

Table 10.11: Summary of T values obtained from toy simulation samples in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis generated with a CP violation of $+2\%$ applied to the amplitude of the $\rho(770)^-$ resonance for a range of value of the metric parameter σ .

$\sigma[(\text{GeV}/c^2)^2]$	T value	p -value (from fit)	p -value 90% CL
0.10	9.12×10^{-7}	$2.3^{+0.9}_{-0.7} \times 10^{-2}$	3.4×10^{-2}
0.20	2.67×10^{-6}	$1.8^{+1.6}_{-1.3} \times 10^{-3}$	4.1×10^{-3}
0.30	3.83×10^{-6}	$2.0^{+1.3}_{-0.9} \times 10^{-3}$	3.9×10^{-3}
0.40	4.71×10^{-6}	$4.6^{+5.0}_{-2.7} \times 10^{-4}$	1.2×10^{-3}
0.50	5.35×10^{-6}	$7.5^{+7.1}_{-4.1} \times 10^{-4}$	1.7×10^{-3}

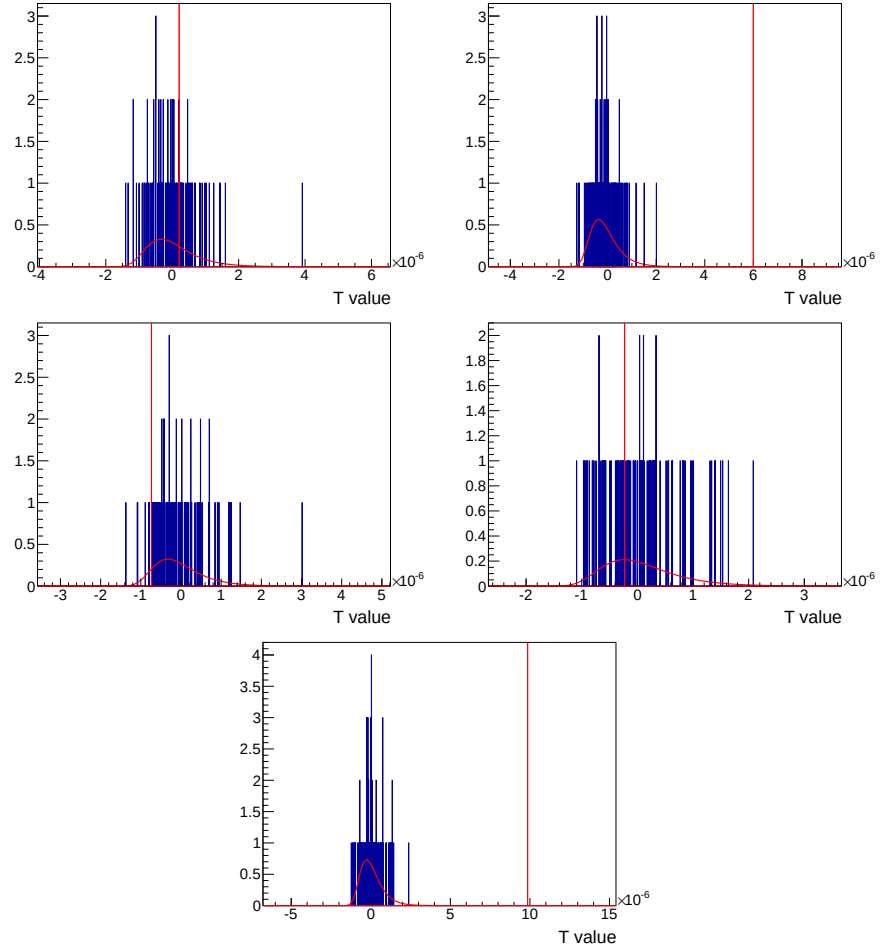


Figure 10.20: T value and permutation T values distribution for toy $D^0 \rightarrow \pi^- \pi^+ \pi^0$ simulation samples in $\rho^0(770)$ CP violation samples. From left to right, then from top to bottom, the changes in $\rho^0(770)$ resonance are: (1). Amplitude +2%, $\phi + 0^\circ$, (2). Amplitude +5%, $\phi + 0^\circ$, (3). Amplitude +0%, $\phi + 1^\circ$, (4). Amplitude +0%, $\phi + 2^\circ$, (5). Amplitude +0%, $\phi + 5^\circ$.

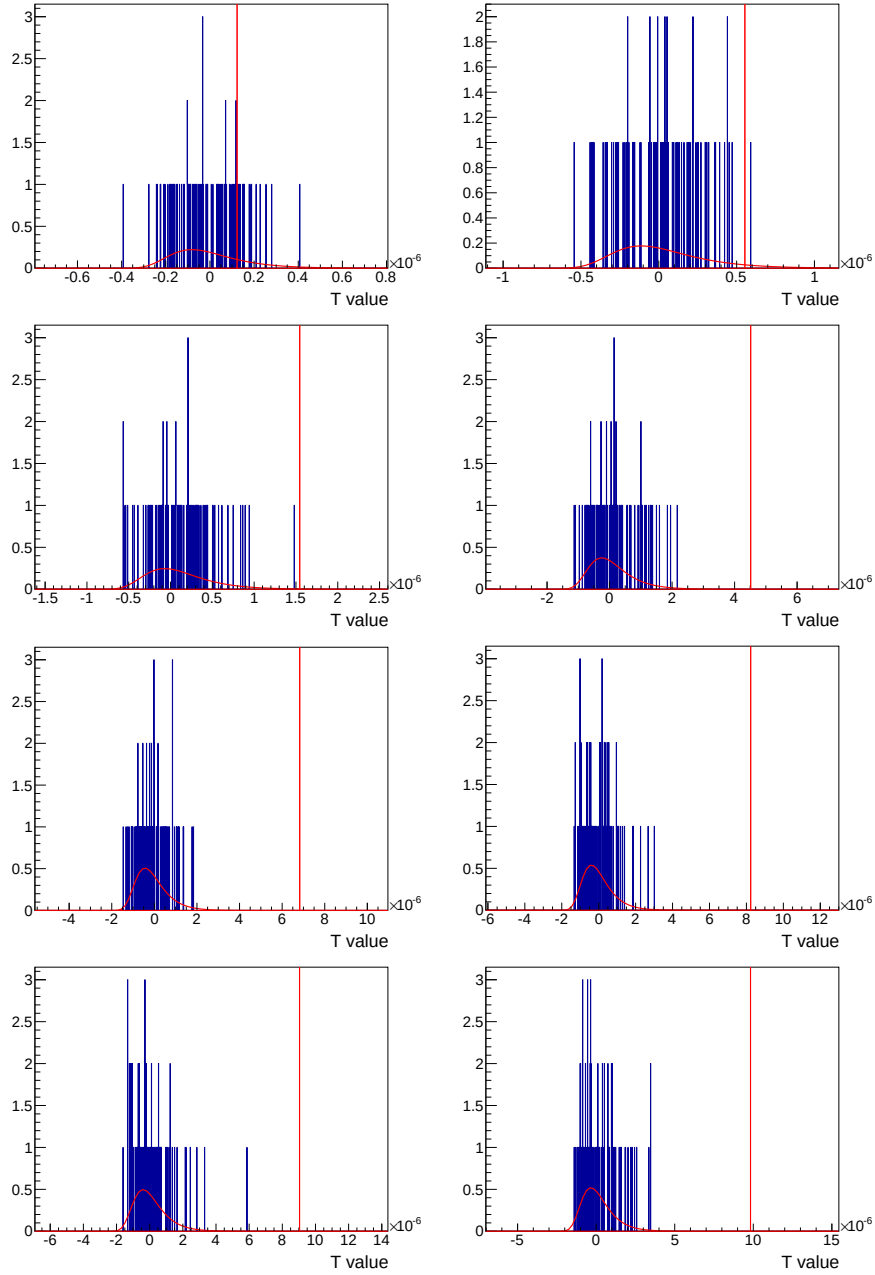


Figure 10.21: T value and permutation T values distribution for one toy $D^0 \rightarrow \pi^- \pi^+ \pi^0$ simulation sample (+2% in $\rho^+(770)$ amplitude) for a range of σ s. From left to right, then from top to bottom, σ are: (1). 0.03 $(\text{GeV}/c^2)^2$, (2) 0.06 $(\text{GeV}/c^2)^2$, (3) 0.1 $(\text{GeV}/c^2)^2$, (4) 0.2 $(\text{GeV}/c^2)^2$, (5) 0.3 $(\text{GeV}/c^2)^2$, (6) 0.4 $(\text{GeV}/c^2)^2$, (7) 0.5 $(\text{GeV}/c^2)^2$, (8) 1 $(\text{GeV}/c^2)^2$.

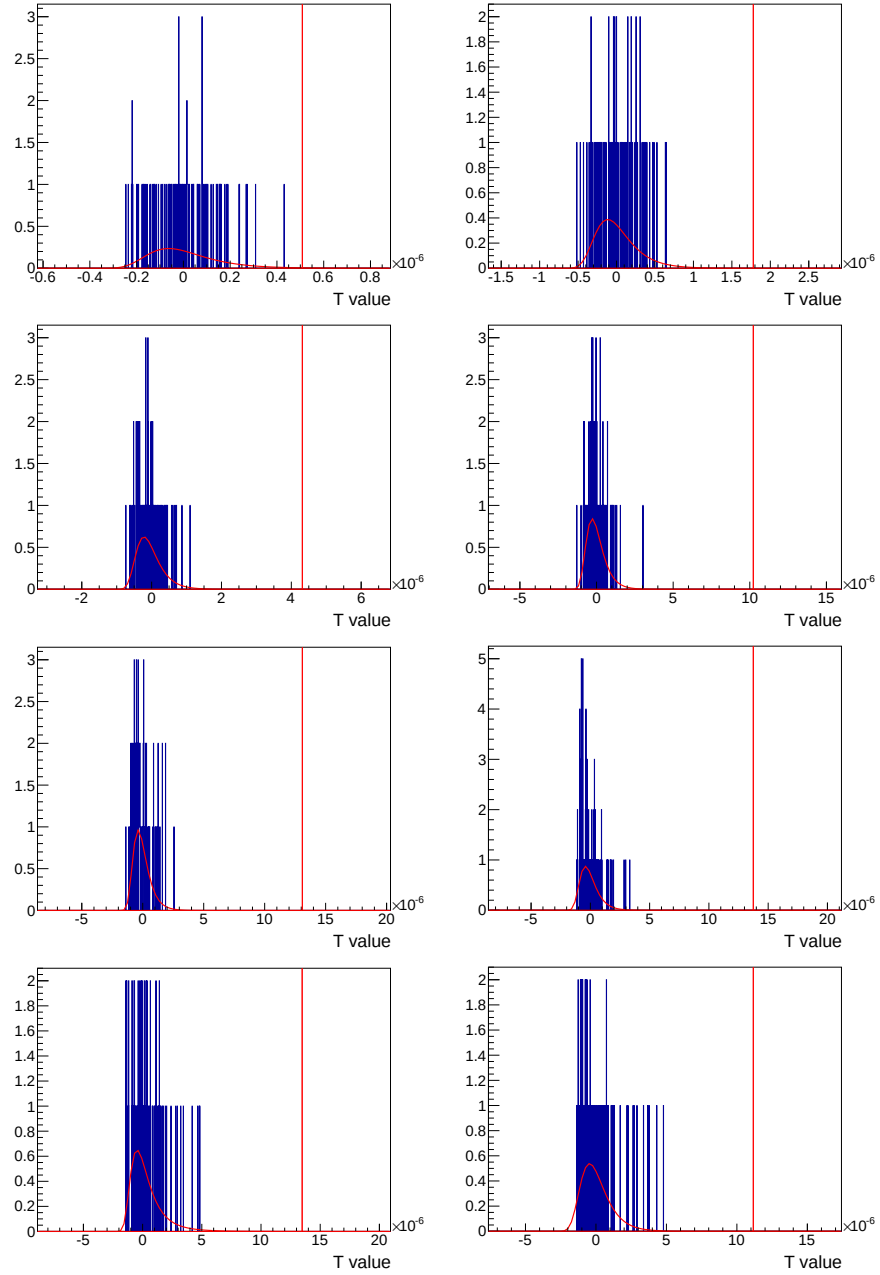


Figure 10.22: T value and permutation T values distribution for one toy $D^0 \rightarrow \pi^- \pi^+ \pi^0$ simulation sample ($+2^\circ$ in $\rho^+(770)$ phase) for a range of σ s. From left to right, then from top to bottom, σ are: (1). 0.03 (GeV/c^2)², (2) 0.06 (GeV/c^2)², (3) 0.1 (GeV/c^2)², (4) 0.2 (GeV/c^2)², (5) 0.3 (GeV/c^2)², (6) 0.4 (GeV/c^2)², (7) 0.5 (GeV/c^2)², (8) 1 (GeV/c^2)².

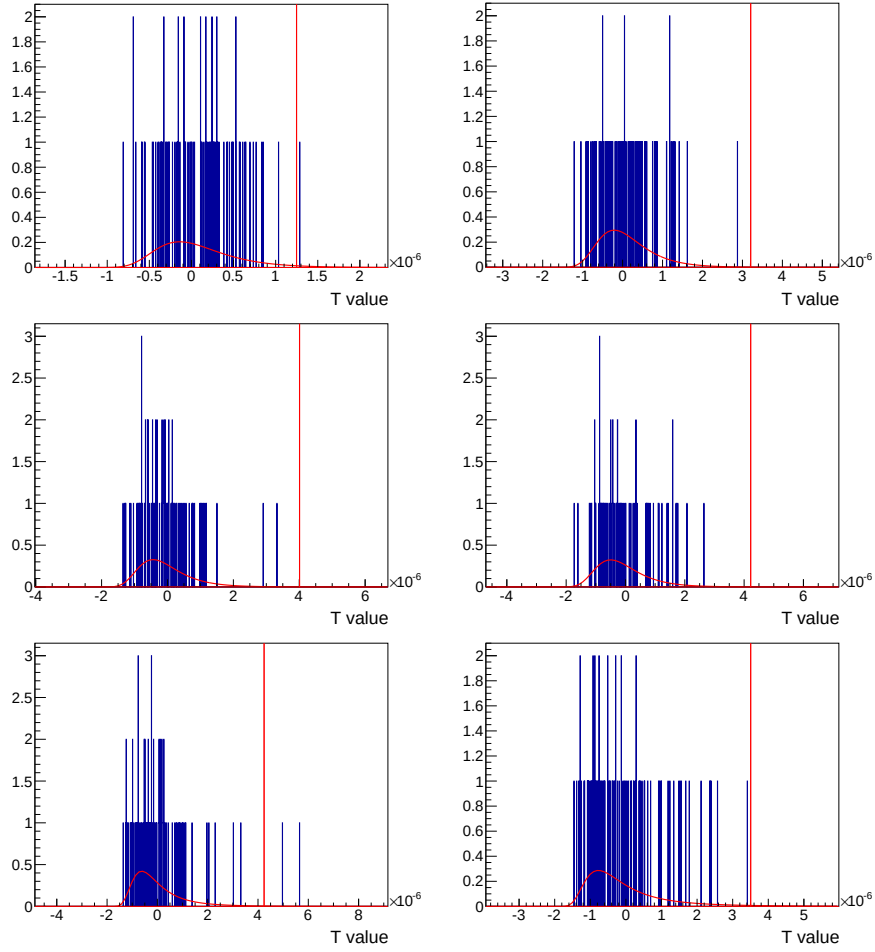


Figure 10.23: T value and permutation T values distribution for one toy $D^0 \rightarrow \pi^- \pi^+ \pi^0$ simulation sample ($+1^\circ$ in $\rho^+(770)$ phase) for a range of σ s. From left to right, then from top to bottom, σ are: (1) $0.1 \text{ (GeV}/c^2)^2$, (2) $0.2 \text{ (GeV}/c^2)^2$, (3) $0.3 \text{ (GeV}/c^2)^2$, (4) $0.4 \text{ (GeV}/c^2)^2$, (5) $0.5 \text{ (GeV}/c^2)^2$, (6) $1 \text{ (GeV}/c^2)^2$.

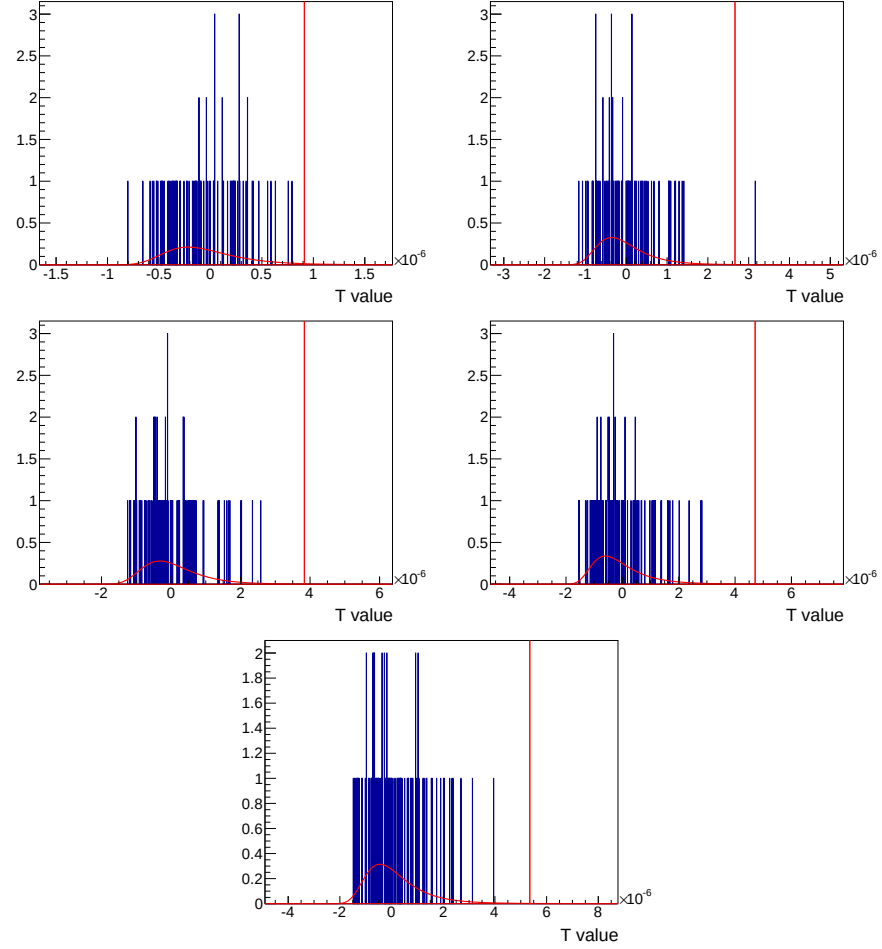


Figure 10.24: T value and permutation T values distribution for one toy $D^0 \rightarrow \pi^- \pi^+ \pi^0$ simulation sample (+2% in $\rho^-(770)$ amplitude) for a range of σ s. From left to right, then from top to bottom, σ are: (1) 0.1 (GeV/c²)², (2) 0.2 (GeV/c²)², (3) 0.3 (GeV/c²)², (4) 0.4 (GeV/c²)², (5) 0.5 (GeV/c²)².

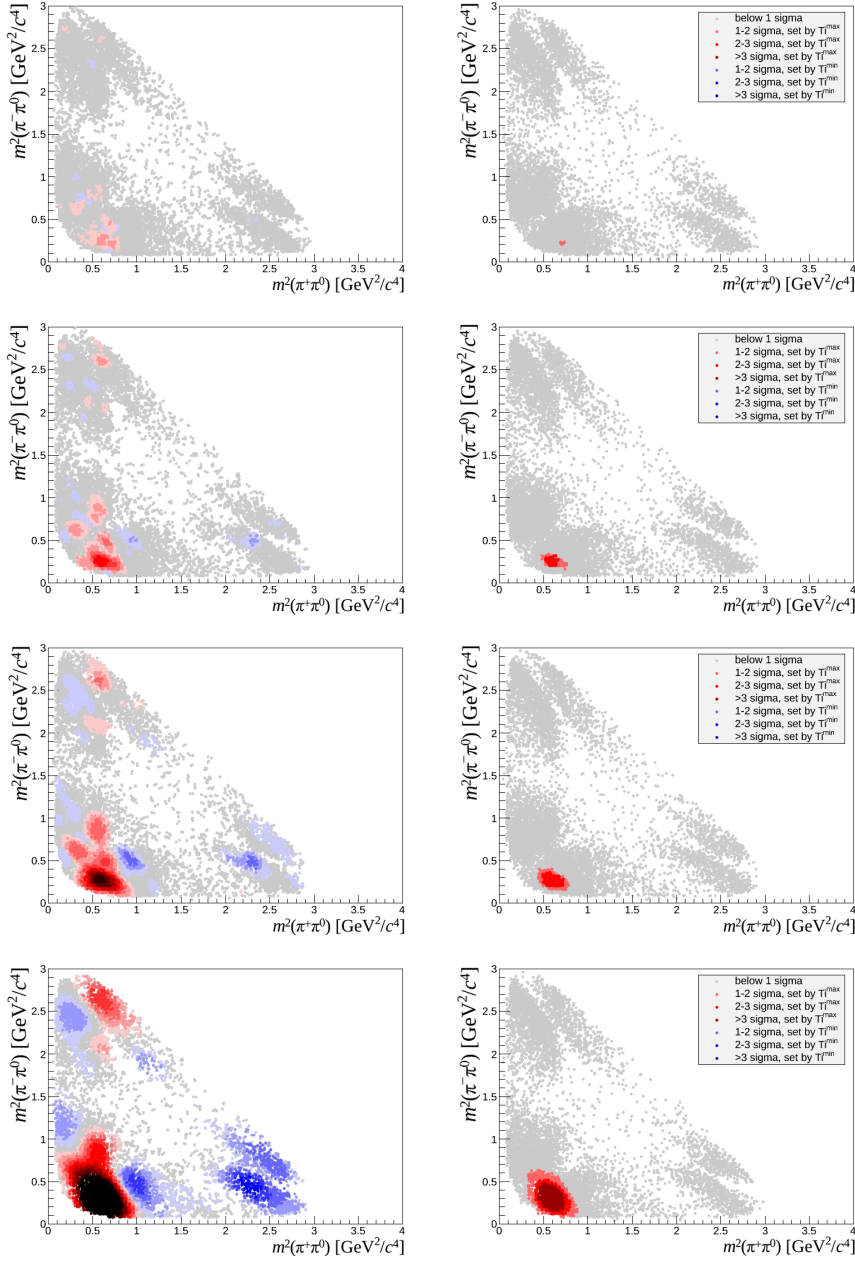


Figure 10.25: $D^0 \rightarrow \pi^- \pi^+ \pi^0$ T_i value distributions on Dalitz plot (left column) and the significance level plots (right column) for the same simulation sample (+2% in $\rho^+(770)$ amplitude) with a range of σ s. The Dalitz plot is $m_{\pi^- \pi^0}^2$ v.s. $m_{\pi^+ \pi^0}^2$. The $\rho^+(770)$ resonance is the vertical band around $m_{\pi^+ \pi^0}^2 = 0.6 \text{ (GeV}/c^2)^2$. With smaller σ , a finer structure could be seen, but with lower statistics a worse sensitivity is expected. With larger σ the asymmetric effect is amplified by averaging a larger region and gain some sensitivity but the structure information is lost. From top to bottom, σ are: (1) $0.03 \text{ (GeV}/c^2)^2$, (2) $0.06 \text{ (GeV}/c^2)^2$, (3) $0.1 \text{ (GeV}/c^2)^2$, (4) $0.2 \text{ (GeV}/c^2)^2$.

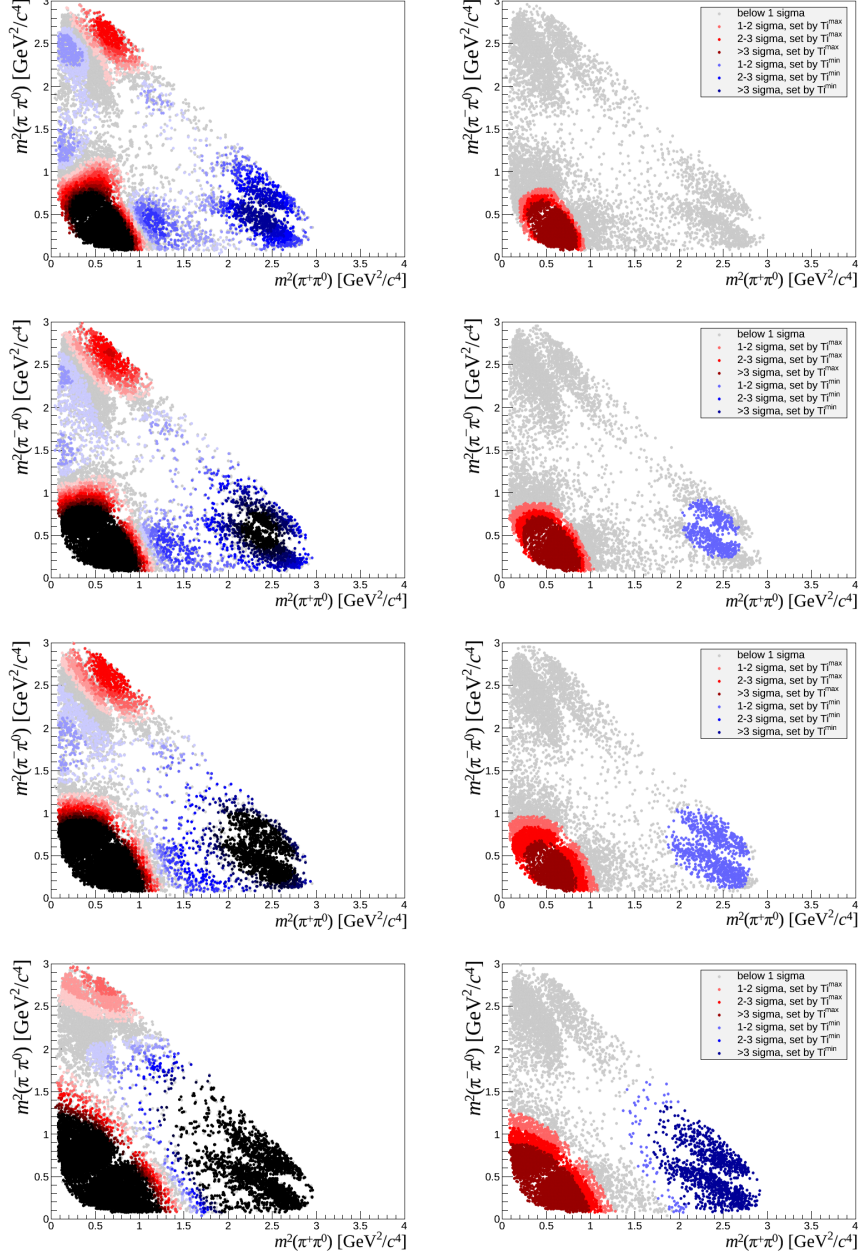


Figure 10.26: $D^0 \rightarrow \pi^- \pi^+ \pi^0$ T_i value distributions on Dalitz plot (left column) and the significance level plots (right column) for the same simulation sample as Fig. 10.25 with a range of σ s. From top to bottom, σ are: (1) $0.3 \text{ (GeV}/c^2)^2$, (2) $0.4 \text{ (GeV}/c^2)^2$, (3) $0.5 \text{ (GeV}/c^2)^2$, (4) $1 \text{ (GeV}/c^2)^2$.

10.4 Systematic uncertainties

In general, there are several sources of asymmetries that can affect the measurement presented here. As the method is only sensitive to local asymmetries in phase space, rather than global asymmetries, it is sufficient to concentrate on these local effects.

Detection asymmetries of the daughter particles can induce local asymmetries if they vary with the particle's kinematics. The impact of this can be estimated through the analysis of the decay mode $K^-\pi^+\pi^0$, which is expected to be subject to much larger asymmetries as the kaon detection asymmetry is not cancelled and therefore contributes at first order. This is discussed in more detail in Sec. 10.4.2.

Production asymmetries are known to vary with the particle's kinematics. Through a correlation of the particle subject to these asymmetries, here the D^{*+} , and the position in phase space, a local asymmetry may be induced. Figure 10.27 shows the kinematic distribution of the D^0 (which is highly correlated to the D^{*+}) for different regions of phase space. Some variation, in particular in the transverse momentum distribution is visible. The impact of this can be tested with a control mode as discussed in Sec. 10.4.2.

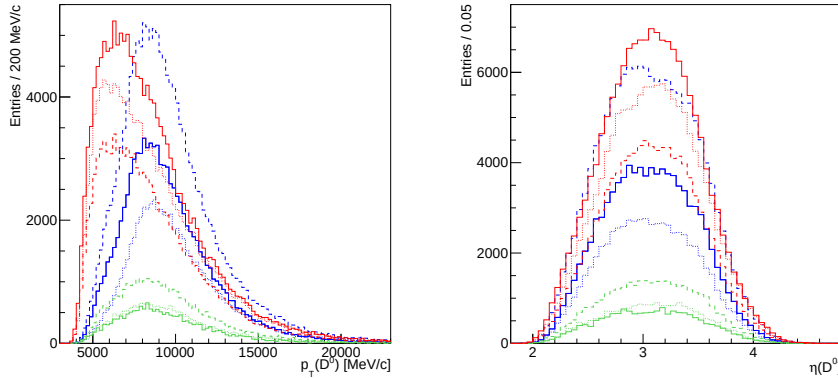


Figure 10.27: Kinematics of D^0 in regions in $m_{\pi^+\pi^-}^2$ and the helicity angle $\cos\theta_{\pi^+\pi^-}$. (left) D^0 momentum and (right) D^0 η distribution are shown. Histograms of the same colour show the spectra for different $\cos\theta_{\pi^+\pi^-}$ bins within the same $m_{\pi^+\pi^-}^2$ region: blue histograms are for $m_{\pi^+\pi^-}^2 < 1 \times 10^6$ (MeV/c²)², green ones for $1 \times 10^6 < m_{\pi^+\pi^-}^2 < 2 \times 10^6$ (MeV/c²)², red ones for $m_{\pi^+\pi^-}^2 > 2 \times 10^6$ (MeV/c²)². Histograms of the same style show spectra from the same $\cos\theta_{\pi^+\pi^-}$ bin: solid histograms are for $\cos\theta_{\pi^+\pi^-} < -0.33$, dotted for $-0.33 < \cos\theta_{\pi^+\pi^-} < +0.33$ and dashed for $\cos\theta_{\pi^+\pi^-} > +0.33$.

10.4.1 Background asymmetry

Although the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ sample in the Δm signal region has a high purity, asymmetries in the contributing background components could affect the CP Violation search. Any background asymmetry which is uniform across the Dalitz distribution (*i.e.* a global asymmetry) has no impact in the energy test method, however Dalitz-dependent (*i.e.* local asymmetries) can bias the measurement. In principle weightings to account for the background contribution can be introduced into the energy test method (see Sect. 9.5), however, if there are no significant asymmetries in the background it can be simply neglected.

Within this section the study of the background asymmetry in the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ signal sample is performed. A test on its impact using data based toy samples and comparing the measurement when using the background for D^0 and $\bar{D}^0$ candidates separately to that using a flavour-averaged background estimate is also performed.

The background asymmetry can be estimated from either the Δm sidebands or from *sWeighted* background distributions. Both options are checked below and they give consistent results. The toy samples from the probability density function (PDF) modelled with finely binned Dalitz distributions for *sWeighted* background are produced, the normalization of the PDFs is adjusted to the expected background yield in the Δm signal region.

Splitting the background for D^0 and $\bar{D}^0$ and finding their distributions in the three Dalitz variables, shown in Fig. 10.28, they are shown to be very consistent. The global asymmetry in the background is defined as $A_B = \frac{B_{D^0} - B_{\bar{D}^0}}{B_{D^0} + B_{\bar{D}^0}}$, where B_{D^0} and $B_{\bar{D}^0}$ are respectively Dalitz-integrated background yields for D^0 and $\bar{D}^0$. With the background yields obtained from a sum of the background *sWeights* a global asymmetry of $(-0.66 \pm 0.21)\%$ is obtained.

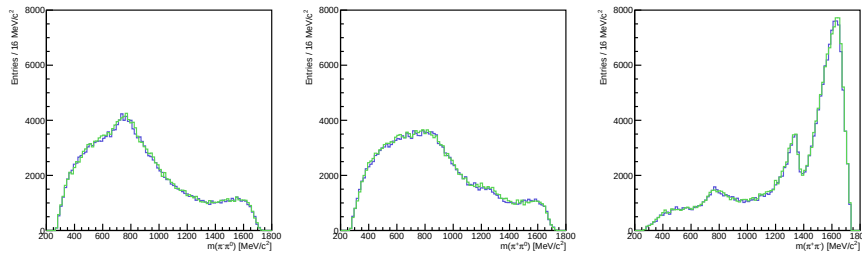


Figure 10.28: Comparison of *sWeighted* backgrounds for D^0 (blue) and $\bar{D}^0$ (green) in the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ sample data. The projections show (left) $m_{\pi^- \pi^0}$, (middle) $m_{\pi^+ \pi^0}$, and (right) $m_{\pi^- \pi^+}$ for both the resolved pion and merged pion samples combined.

The background estimated from the Δm sideband ($150 \text{ MeV}/c^2 < \Delta m < 155 \text{ MeV}/c^2$) has a global asymmetry of $(-0.68 \pm 0.34)\%$, calculated from

the number of events in the sideband region and their Poisson errors. The background from sidebands reveals no local asymmetry in any of the Dalitz projections (Fig. 10.29). The global background asymmetry can receive contributions from several sources. The background may use a D^0 where an incorrect particle, usually the π^0 , has been included, this is then combined with the correct π_s to make the D^{*+} , tagging the D^0 with the correct flavour. In this case the D^0 production asymmetry will contribute to the global background asymmetry. Another alternative is that the global background asymmetry can arise from a detection asymmetry of the π_s , this can occur even if the π_s is a random track, *i.e.* not the true π_s from the D^{*+} decay.

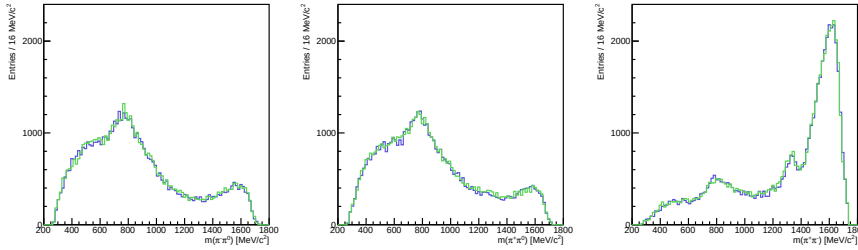


Figure 10.29: Comparison of background estimated from Δm sidebands for D^0 (blue) and $\bar{D}^0$ (green) in the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis data. The projections show (left) $m_{\pi^- \pi^0}$, (middle) $m_{\pi^+ \pi^0}$, and (right) $m_{\pi^+ \pi^-}$ for both the resolved and merged samples combined.

As a cross-check, the energy test for the sideband region is applied to the events selected from the $m(D^0)$ sideband region. The distribution of T value for 600 permutations is shown in Fig. 10.30; the measured p -value of 0.34 is consistent with no CP violation in the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ background.

The binned S_{CP} method was applied to the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ background. This allows a comparison with results from the energy test. The $\pi^- \pi^+ \pi^0$ Dalitz plot is divided into 8×8 uniform bins and only bins containing more than 50 entries are considered. When applying the S_{CP} test to the sideband events the $N_{D^0}^i$ and $N_{\bar{D}^0}^i$ from counting D^0 and $\bar{D}^0$ candidates are calculated in i^{th} bin, and use their Poisson errors, $\sigma_i = \sqrt{N^i}$. For the *sWeighted* background N^i are calculated from a sum of background *sWeights*, while σ_i^2 is a sum of the squared *sWeights*. Distributions of S_{CP}^i in bins of the $\pi^- \pi^+ \pi^0$ Dalitz plot are shown in Fig. 10.31, p -values are summarized in Table 10.12.

To check the impact of the background variation over the Dalitz space, two sets of toy simulation samples are produced, both including signal and background components. In the first toy set both the signal and background are based on combined D^0 and $\bar{D}^0$ Dalitz distributions (flavour-averaged estimate), in the second set the signal is flavour-averaged while the background is based

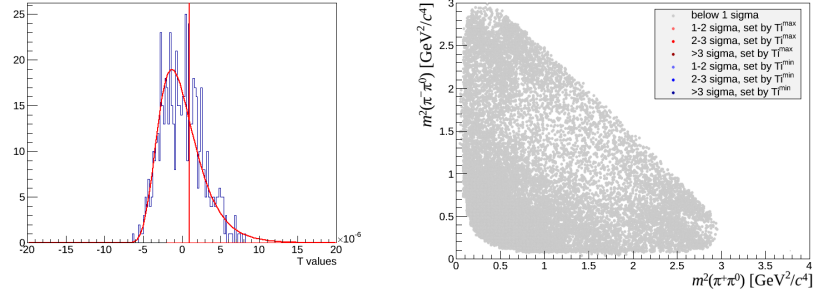


Figure 10.30: Results from applying the Energy test to the $m(D^0)$ sideband region events in the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis. (left) Distribution of T value obtained for 600 permutations. Measured T value is 9.27×10^{-7} and p -value=0.34. (right) A visualisation plot of the background tests.

Table 10.12: Results from applying the S_{CP} method to the $\pi^- \pi^+ \pi^0$ background

Sample	α [%]	χ^2	ndf	p -value
Δm sideband	98.7 ± 0.7	41.8	42	0.48
$sWeighted$ background	98.7 ± 0.4	49.2	42	0.21

on D^0 and $\bar{D}^0$ candidates separately. Results of the energy tests applied to these toy samples are presented in Table 10.13. Out of 10 experiments tested with 60 permutations, only one set with flavour-separated background gives a small p -value, the others are similar for both experiments. The p -value distributions are shown in Fig. 10.32 giving probabilities of 0.21 and 0.80 of being in agreement with the expected distribution. Therefore it can be stated that background does not bias our measurement and can be safely ignored.

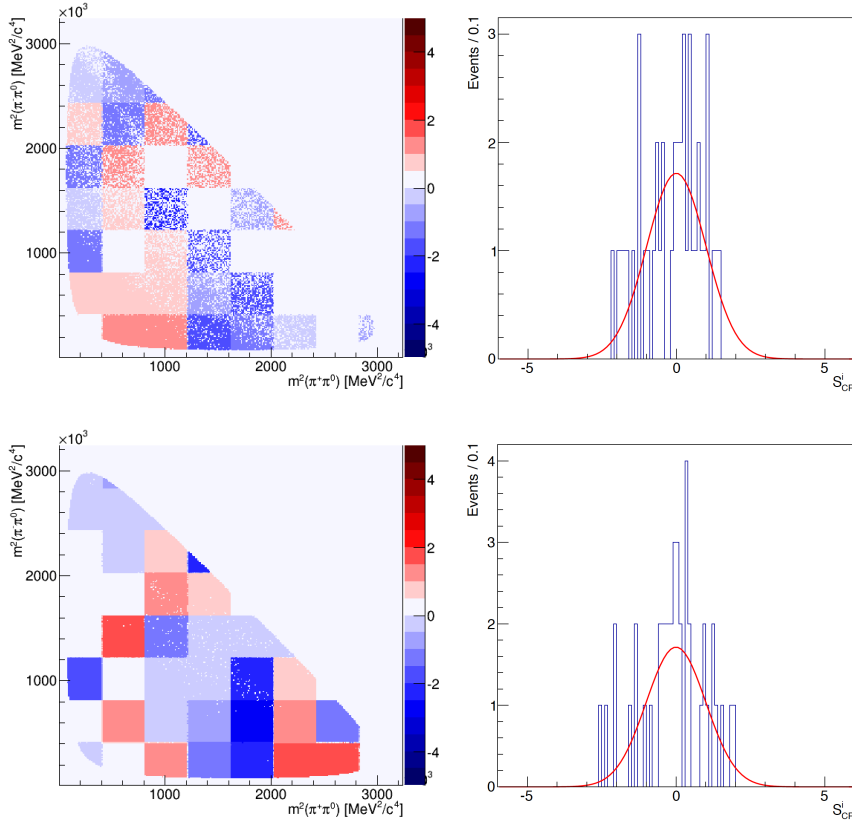


Figure 10.31: (left column) S_{CP}^i in bins of $\pi^-\pi^+\pi^0$ Dalitz plot for the background events. (right column) S_{CP}^i distribution compared with a Gaussian distribution expected for no- CP violation hypothesis. Distributions are for the events (top row) selected from the Δm sideband region, and for (bottom row) s Weighted background.

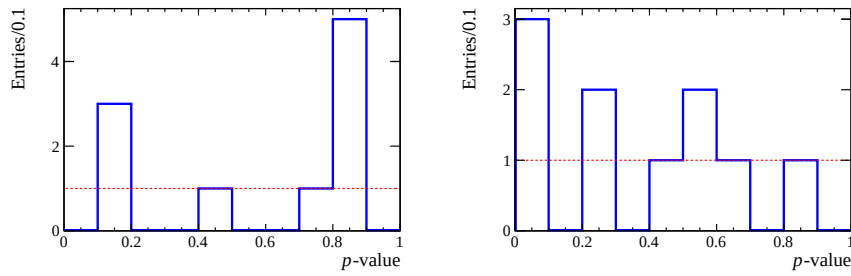


Figure 10.32: p -value distributions for (left) symmetric and (right) asymmetric toy MC samples as described in the text and listed in Tab. 10.13. The red line indicates the expected distribution and the probability of the distributions being in agreement with the line is 0.21 and 0.80 for the two sets, respectively.

Table 10.13: Results from applying the energy test for toys with flavour-averaged and flavour-separated background. Both samples include flavour-averaged signal component. A graphical representation of the p -values is given in Fig. 10.32.

Toy	Flavour averaged		Flavour separated	
	T value [10^{-7}]	p -value	T value [10^{-7}]	p -value
1	-4.43	0.87	-0.80	0.43
2	+0.86	0.42	-0.95	0.63
3	-1.25	0.72	+0.87	0.25
4	-4.11	0.88	-1.58	0.55
5	-4.28	0.88	+10.9	0.02
6	+2.50	0.15	-3.61	0.87
7	+2.25	0.17	+15.5	0.003
8	-3.34	0.85	+6.60	0.02
9	-3.28	0.80	+2.29	0.27
10	+4.96	0.15	-0.16	0.50

10.4.2 Cross-check using a control mode

The analysis method has been applied to a sample of $D^0 \rightarrow K^- \pi^+ \pi^0$ decay events with a size approximately the same as the final sample of decays into $\pi^- \pi^+ \pi^0$ decays as detailed in Sect. 10.2.5.

In comparison to the signal decay into $\pi^- \pi^+ \pi^0$ this control mode involves a $K^\mp \pi^\pm$ pair, which introduces a first order global asymmetry due to the average $K^\mp \pi^\pm$ detection asymmetry. More importantly though, the dependence of the $K^\mp \pi^\pm$ detection asymmetry on the particle kinematics leads to a phase-space dependent asymmetry.

Figure 10.12 shows the signal and Δm sideband Dalitz distributions. Figures 10.33 and 10.34 show the raw asymmetries as a function of Dalitz variables for signal and sideband region events, respectively.

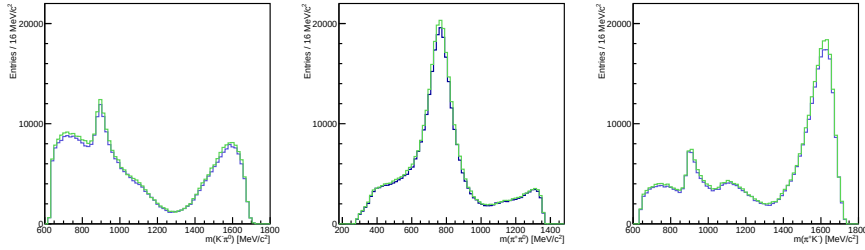


Figure 10.33: Comparison of $D^0 \rightarrow K^- \pi^+ \pi^0$ signals for D^0 (blue) and $\bar{D}^0$ (green) on Dalitz plot projections for Δm signal region events. The projections are (left) $m_{K^- \pi^0}$, (middle) $m_{\pi^+ \pi^0}$, and (right) $m_{\pi^+ K^-}$ for both the resolved and merged samples combined.

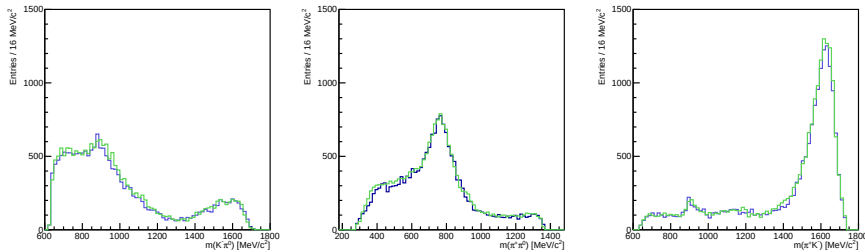


Figure 10.34: Comparison of $D^0 \rightarrow K^- \pi^+ \pi^0$ background for D^0 (blue) and $\bar{D}^0$ (green) on Dalitz plot projections for Δm sideband events. The projections are (left) $m_{K^- \pi^0}$, (middle) $m_{\pi^+ \pi^0}$, and (right) $m_{\pi^+ K^-}$ for both the resolved and merged samples combined.

The analysis method was applied to eight $K^- \pi^+ \pi^0$ subsamples, each is of

Table 10.14: Results from applying the energy test to the $K^- \pi^+ \pi^0$ control samples.

Sample	T value [10^{-7}]	p -value (counting)	p -value (fit)
1	+10.5	0.07 ± 0.03	0.07 ± 0.02
2	+1.9	0.39 ± 0.05	0.36 ± 0.05
3	+3.2	0.30 ± 0.05	0.25 ± 0.04
4	-5.1	0.74 ± 0.04	0.75 ± 0.03
5	-0.3	0.46 ± 0.05	$0.45^{+0.04}_{-0.05}$
6	+13.9	0.03 ± 0.02	0.02 ± 0.01
7	-3.6	0.67 ± 0.05	$0.69^{+0.03}_{-0.04}$
8	+4.25	0.21 ± 0.04	0.20 ± 0.04

Table 10.15: Results from applying the energy test to the $K^- \pi^+ \pi^0$ sample split according to the magnet polarity.

Sample	T value	p -value (fit)	90% CL
Magnet Up	2.46×10^{-7}	$0.08^{+0.04}_{-0.03}$	0.14
Magnet Down	9.99×10^{-8}	0.15 ± 0.06	0.24

similar size to the sample studied in the $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis. Distributions of T values and visualisations for all the $K^- \pi^+ \pi^0$ samples are shown in Fig. 10.35, the measured T values and p -values are summarized in Table 10.14 and in Fig. 10.36, which shows good agreement with the expected flat distribution in the absence of local asymmetries.

The total $K^- \pi^+ \pi^0$ sample is further split according to the magnet polarity. The results are presented in Table 10.15 and Fig. 10.37 and indicate no detector-related asymmetries.

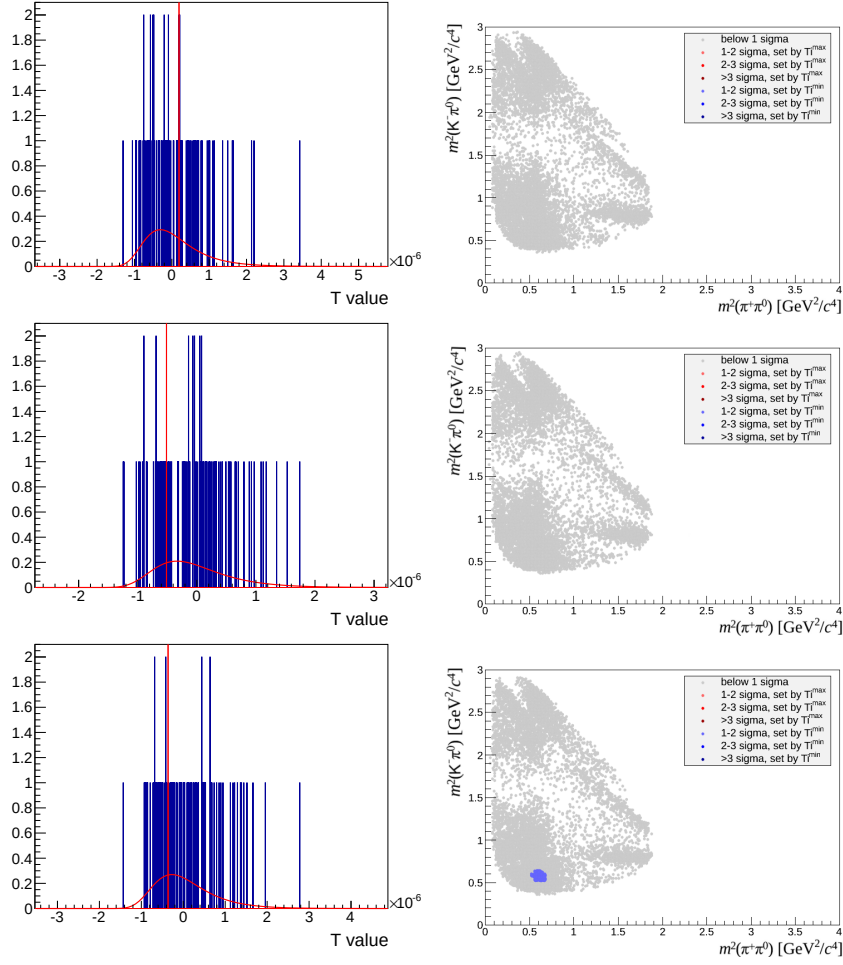


Figure 10.35: (left) T values of the permutation test of $K^-\pi^+\pi^0$ control mode for subsamples 2, 4 and 7. The nominal T value is shown as a thicker vertical line. (right) The corresponding visualisation on the Dalitz plot $m_{K^-\pi^0}^2$ vs $m_{\pi^+\pi^0}^2$.

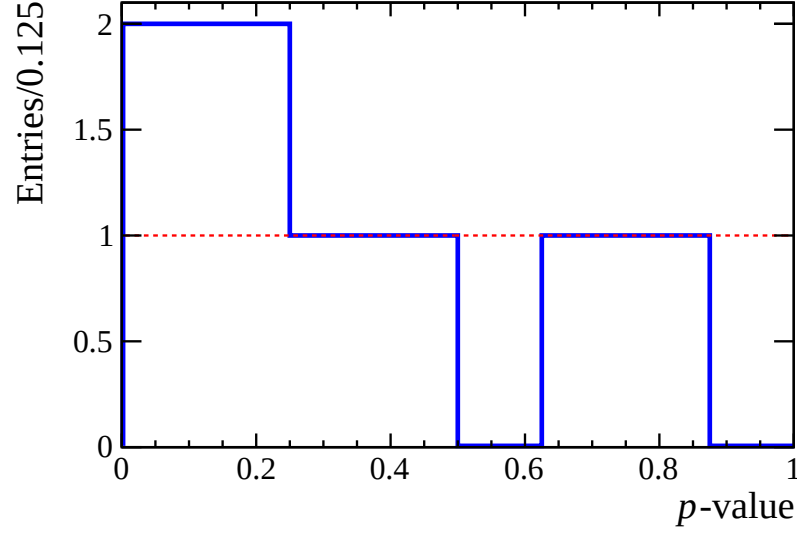


Figure 10.36: p -values from applying the energy test to the $K^- \pi^+ \pi^0$ control samples. Red line indicate the expected distribution.

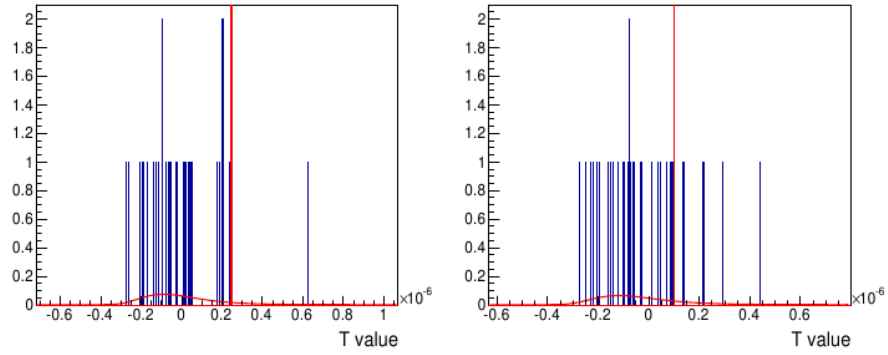


Figure 10.37: Distribution of the permutation T value for the $D^0 \rightarrow K^- \pi^+ \pi^0$ data selected with (left) magnet up and (right) magnet down polarities. The nominal T value is indicated with a vertical red line.

Table 10.16: Results from applying the S_{CP} method to the $K^-\pi^+\pi^0$ control samples

Sample	α [%]	χ^2	ndf	p -value	p -value ($sWeights$)
1	96.6 ± 0.2	50.4	66	0.92	0.87
2	96.3 ± 0.2	68.8	66	0.38	0.41
3	96.4 ± 0.2	79.1	66	0.13	0.08
4	96.3 ± 0.2	85.8	66	0.05	0.04
5	96.6 ± 0.2	63.3	66	0.57	0.56
6	96.1 ± 0.2	64.3	66	0.53	0.46
7	96.4 ± 0.2	72.9	66	0.26	0.20
8	96.5 ± 0.2	50.3	66	0.92	0.87

The results from energy test with results from the binned S_{CP} method are compared. The $K^-\pi^+\pi^0$ Dalitz plot is divided into 8×12 uniform bins; only bins containing more than 50 entries are considered. To compare directly with results from the energy test the $N_{D^0}^i$ and $N_{\bar{D}^0}^i$ are got from counting D^0 and $\bar{D}^0$ candidates in i th bin of the Dalitz distribution for the Δm signal region and without subtracting the background. As a cross-check the S_{CP}^i using $N_{D^0}^i$ and $N_{\bar{D}^0}^i$ are also calculated from a sum of signal $sWeights$. Distributions of S_{CP}^i in bins of the $K^-\pi^+\pi^0$ Dalitz plot are shown in Fig. 10.38 for the samples giving the smallest p -value from either energy test or the S_{CP} method (samples 2, 4 and 7). The corresponding p -values are summarized in Table 10.16. The distribution of p -values agrees with the expectation of no- CP violation case in this control mode, as shown in Fig.10.39.

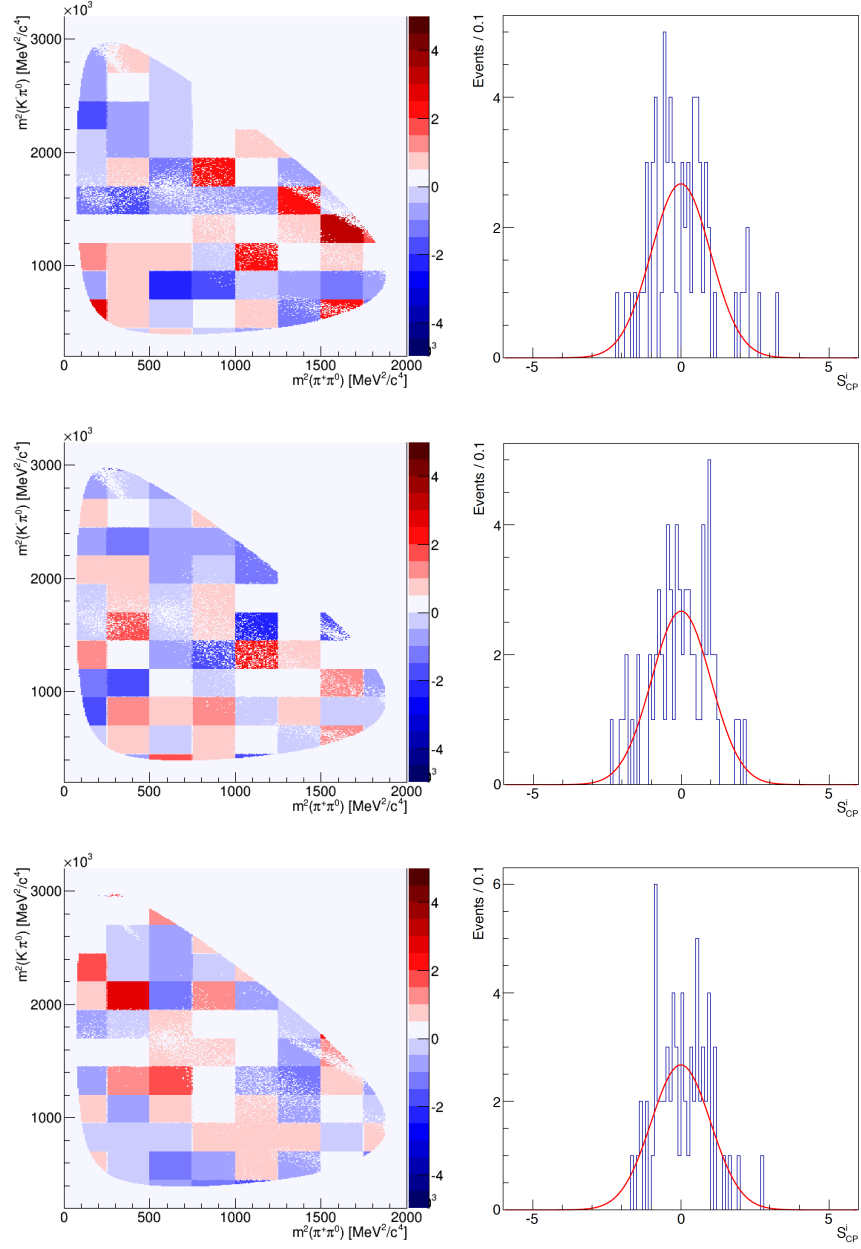


Figure 10.38: (left) S_{CP}^i in bins of $D^0 \rightarrow K^- \pi^+ \pi^0$ Dalitz plot for subsamples 2, 4 and 7. (right) The corresponding S_{CP}^i distribution compared with a Gaussian distribution expected for no- CP violation hypothesis.

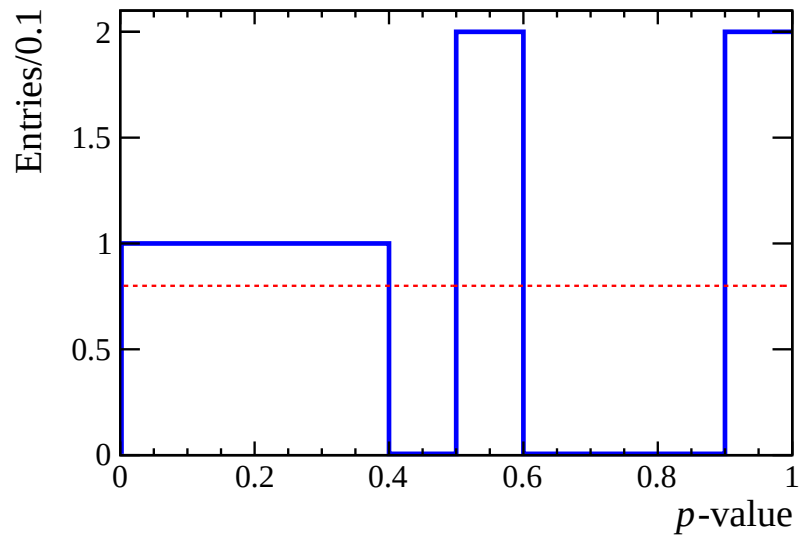


Figure 10.39: p -values from applying the S_{CP} method to the $K^-\pi^+\pi^0$ control samples. The red line indicates the expected distribution.

10.4.3 Tracking and detection asymmetries

Tracking and detection asymmetries may depend on the particle kinematics and can therefore lead to effective asymmetries that vary as a function of position in phase space. The check using the control mode (see Sec. 10.4.2, which would be influenced by the same issues) already shows that the method is not sensitive to these effects at the current level of precision. However, further tests are carried out to confirm this conclusion.

Detection asymmetries of slow pions

The LHCb detector itself is intended to be left-right symmetric, but some differences in detection efficiency will occur. The magnetic field bends the trajectory of charged particles and breaks the left-right symmetry. For the $D^0 \rightarrow \pi^+ \pi^- \pi^0$ decay, due to the self-conjugated final states, this effect does not matter to first order. However, as the D^0 is produced by the $D^{*+} \rightarrow D^0 \pi^+_s$ decay process, this effect can be significant for the π^+_s . A detection asymmetry of the slow pions can occur.

This effect has been previously studied in Reference [98], The data analysed uses data sets from both magnet polarities, but the cancellation between any asymmetries in these two data samples is not relied on. With some regions of the detector only being reached by the π^+ (π^-) depending on the magnet polarity. Here the fiducial selections applied in that study are used

$$\begin{aligned} |p_x| &\leq \alpha(p_z - p_0) \\ \alpha &= 0.317 \\ p_0 &= 2400 \text{ MeV}/c, \end{aligned} \tag{10.3}$$

and

$$\begin{aligned} p_1 - \beta_1 p_z &< |p_x| < p_2 + \beta_2 p_z \\ p_1 &= 418 \text{ MeV}/c \\ p_2 &= 497 \text{ MeV}/c \\ \beta_1 &= 0.01397 \\ \beta_2 &= 0.01605, \end{aligned} \tag{10.4}$$

only for $|p_y/p_z| < 0.02$ regions.

Figure 10.40 shows the slow pion kinematics in different regions of phase space. It is clearly visible that an asymmetry, which varies with the slow pion kinematics, can generate local asymmetries in the Dalitz plot.

The effect of a potential asymmetry due to the slow pion is assessed by applying the fiducial selections described above. They are expected to change significantly the induced asymmetry. Prior to unblinding, this test is carried out on the $K^- \pi^+ \pi^0$ control sample. Figure 10.41 shows the result, which yields a p -value of 50% thus confirming that fiducial selections do not change the local asymmetries present in phase space at a significant level.

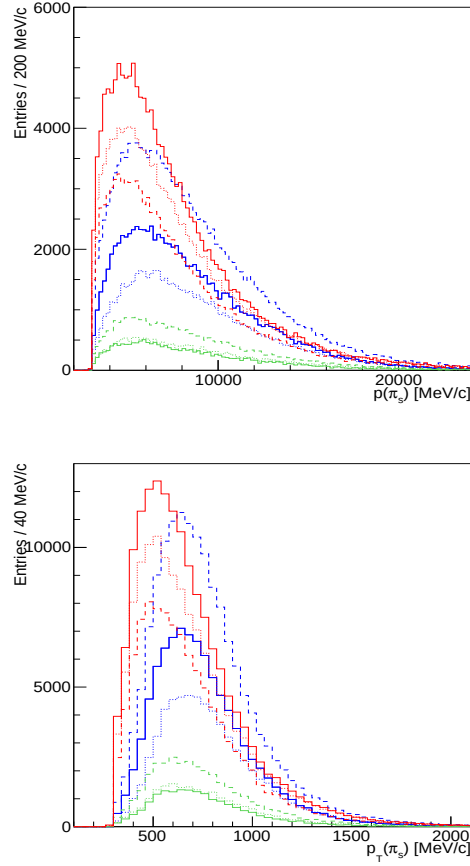


Figure 10.40: Kinematics of slow pion for different bins of $D^0 \rightarrow \pi^- \pi^+ \pi^0$ Dalitz plot. (top) π_s momentum and (bottom) the π_s p_T distributions for regions in $m_{\pi^+ \pi^-}^2$ and the helicity angle $\cos \theta_{\pi^+ \pi^-}$ are shown. Histograms of the same colour show the spectra for different $\cos \theta_{\pi^+ \pi^-}$ bins within the same $m_{\pi^+ \pi^-}^2$ region: blue histograms are for $m_{\pi^+ \pi^-}^2 < 1 \times 10^6$ (MeV/c²)², green ones for $1 \times 10^6 < m_{\pi^+ \pi^-}^2 < 2 \times 10^6$ (MeV/c²)², red ones for $m_{\pi^+ \pi^-}^2 > 2 \times 10^6$ (MeV/c²)². Histograms of the same style show spectra from the same $\cos \theta_{\pi^+ \pi^-}$ bin: solid histograms are for $\cos \theta_{\pi^+ \pi^-} < -0.33$, dotted for $-0.33 < \cos \theta_{\pi^+ \pi^-} < +0.33$ and dashed for $\cos \theta_{\pi^+ \pi^-} > +0.33$

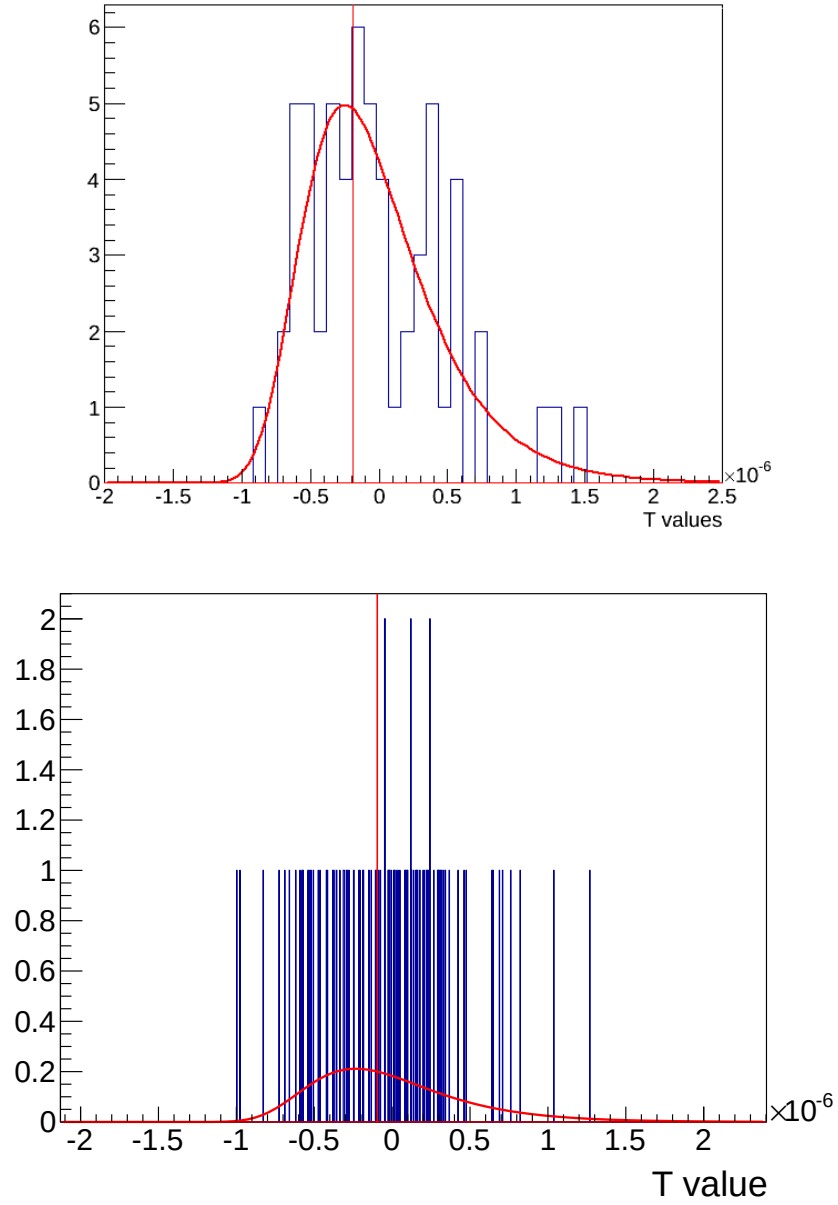


Figure 10.41: (top) T value distribution after applying selections that removed the asymmetries in the acceptance caused by the magnetic polarity to a sample of $D^0 \rightarrow K^- \pi^+ \pi^0$ decays. (bottom) T value distribution of the vetoed events.

10.5 Results

The application of the energy test to all selected $D^0 \rightarrow \pi^- \pi^+ \pi^0$ candidates using a metric parameter of $\sigma = 0.3 \text{ GeV}^2/c^4$ yields $T = 1.84 \times 10^{-6}$. The permutation T value distribution is shown in Fig. 10.42 (a). By counting the fraction of permutations with a T value above the nominal T value in the data, a p -value of $(2.6 \pm 0.5) \times 10^{-2}$ is extracted. Alternatively, extrapolation from a fit to the GEV function gives a p -value of $(2.1 \pm 0.3) \times 10^{-2}$. The significance levels of the T_i values are shown in Fig. 10.42 (b). This visualisation plot does not show any strong region of asymmetry. A small phase-space region dominated by the ρ^+ resonance contains candidates with a local positive asymmetry exceeding 1σ significance. Varying the metric parameter results in the p -values listed in Table 10.17; all the p -values are at the 10^{-2} level.

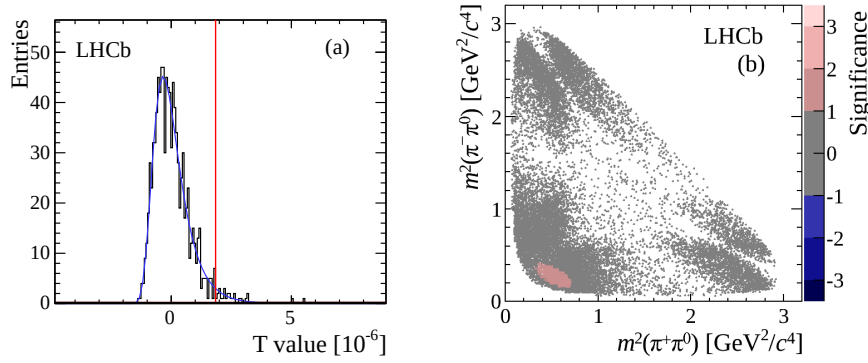


Figure 10.42: $D^0 \rightarrow \pi^- \pi^+ \pi^0$ analysis results. (a) Permutation T value distribution showing the fit function and the measured T value as a red line. (b) Visualisation of local asymmetry significances. The positive (negative) asymmetry significance is set for the D^0 candidates having positive (negative) contribution to the measured T value, respectively.

The data sample has been split according to various criteria to test the stability of the results. Analyses of sub-samples with opposite magnet polarity, with different trigger configurations, and with fiducial selection requirements

Table 10.17: Results for various metric parameter values. The p -values are obtained with the counting method.

σ [GeV^2/c^4]	p -value
0.2	$(4.6 \pm 0.6) \times 10^{-2}$
0.3	$(2.6 \pm 0.5) \times 10^{-2}$
0.4	$(1.7 \pm 0.4) \times 10^{-2}$
0.5	$(2.1 \pm 0.5) \times 10^{-2}$

removing areas of high local asymmetry of the tagging pion from the D^{*+} decay all show good consistency of the results.

In summary, a search for time-integrated CP violation in the Cabibbo-suppressed decay $D^0 \rightarrow \pi^- \pi^+ \pi^0$ is performed using a novel unbinned model-independent technique. The analysis has the best sensitivity from a single experiment to CP violation in this decay. The data are found to be consistent with the hypothesis of CP symmetry with a p -value of $(2.6 \pm 0.5)\%$.

11 CP violation in $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decays

This chapter is based on the publication in Reference [80], and the corresponding LHCb document [99], which describes the search for CP violation in the $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decays with the energy test method. Section 11.1 gives the motivation for searching for CP violation in $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decay, Section 11.2 describes the dataset and signal selection. The sensitivity test with the energy test are given in Sect. 11.3, the sources of possible systematic uncertainties are explored in Sect. 11.4. The results of the study are given in Sect. 11.5.

11.1 Introduction

The decay $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ (charge-conjugate decays are implied unless stated otherwise) proceeds via a singly Cabibbo-suppressed $c \rightarrow du\bar{d}$ transition with an admixture from a $c \rightarrow ug$ gluonic-penguin transition. Within the Standard Model (SM), these amplitudes have different weak phases, and the interference between them may give rise to a violation of the charge-parity (CP) symmetry in the decay. Another necessary condition for this direct CP violation to occur is interference of at least two amplitudes with different strong phases. The strong phase differences are known to be sizeable in charm decays and can enter through the resonances that abundantly contribute to the four-body final states. The sensitivity to CP violation is usually best for decays where the strong phases between interfering resonances have large differences. A rich spectrum of resonances contributes to the decay $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$, which according to the amplitude analysis performed by the CLEO collaboration [86], is dominated by the amplitudes for $D^0 \rightarrow a_1(1260)^+ \pi^-$ with $a_1(1260)^+ \rightarrow \rho^0(770) \pi^+$ and for $D^0 \rightarrow \rho^0(770) \rho^0(770)$.

In the SM, violation of the CP symmetry in the charm sector is expected at or below the $\mathcal{O}(10^{-3})$ level [90]. Contributions from particles that are proposed to exist in extensions of the SM may participate in higher-order loop contributions (penguin diagrams) and enhance the level of CP violation. Multibody decays, such as $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$, allow the CP asymmetries to be

probed across the phase space of the decay, and these local CP asymmetries may be larger than global CP asymmetries.

The analysis of $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ is primarily sensitive to direct CP violation. In addition to this direct CP violation, the time-integrated CP asymmetry in $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ decays can also receive an indirect contribution arising from either D^0 - $\bar{D}^0$ mixing or interference between direct decays and decays following mixing. While direct CP asymmetry depends on the decay mode, indirect CP violation is expected to be universal within the SM. The time-dependent measurements of $D^0 \rightarrow \pi^- \pi^+$, $K^- K^+$ decays constrain the indirect CP asymmetry below the $\mathcal{O}(10^{-3})$ level [100], which is beyond the sensitivity of this analysis.

Previously, the most sensitive search for CP violation in the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ decay was performed by the LHCb collaboration with data collected in 2011 [101]. A binned χ^2 technique (S_{CP}) was used to exclude CP violating effects at the 10% level. Four-body decays require five independent variables to fully represent the phase space (see Sect. 9.6.3). Consequently, binned analyses will have a trade-off between minimising the number of bins, in order to maximise the number of events per bin, and retaining sensitivity to the interference between all contributing resonances. The measurement presented here includes data collected in 2012, resulting in a signal sample that is about three times larger than in the previous LHCb analysis. The method exploited here, known as the energy test, is an unbinned technique, which is advantageous in the analysis of multi-body decays.

The energy test was applied for the first time to search for CP violation in decays of $D^0 \rightarrow \pi^- \pi^+ \pi^0$ [64], as presented in previous chapter. Here its first application to four-body decays is presented. The energy test is used to assess the compatibility of the observed data with CP symmetry. It is sensitive to local CP violation in the phase space and not to global asymmetries, which may also arise from different production cross-sections of D^0 and $\bar{D}^0$ mesons at a proton-proton collider. The method identifies the phase space regions in which CP violation is observed. Being model-independent, this method neither identifies which amplitudes contribute to the observed asymmetry nor measures the actual asymmetry. Consequently, a model-dependent amplitude analysis of D^0 and $\bar{D}^0$ decays would be required if evidence for a non-zero CP asymmetry is obtained.

The analysis presented here probes separately both P -even and P -odd CP asymmetries. The P -even test is performed through the comparison of the distribution of events in the D^0 and $\bar{D}^0$ phase spaces, characterised by squared invariant masses. Additionally characterising the events using a triple product of final-state particle momenta [85, 102, 60], as introduced in Sect. 9.7, gives sensitivity to P -odd amplitudes, and thus allows the first test for P -odd CP asymmetries in an unbinned model-independent technique.

11.2 Signal selection

The analysis is performed using the data collected during 2011 and 2012 at LHCb with integrated luminosities of approximately 1 fb^{-1} and 2 fb^{-1} respectively. The trigger, stripping and offline selection of the data are discussed in this section.

11.2.1 Trigger selection

The online event selection is performed by a trigger. As introduced in Sect.7.2.7, the trigger consists of a hardware stage, based on high- p_T signatures from the calorimeter and muon systems, followed by a two-level software stage. Events are required to pass both hardware and software trigger levels. The software trigger at its first level applies partial event reconstruction. It requires at least one good quality track associated with a particle having high p_T and not originating from a PV.

A second-level software trigger, optimised for four-body hadronic charm decays, fully reconstructs $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ candidates coming from $D^{*+} \rightarrow D^0 \pi_s^+$ decays. The charge of the pion coming from D^{*+} (slow pion, π_s) tags the flavour of the D mesons at production, which is needed as $\pi^+ \pi^- \pi^+ \pi^-$ is a self-conjugate final state accessible to both D^0 and $\bar{D}^0$ decays. The trigger selection ensures the suppression of combinatorial background while minimising the distortion of the acceptance in the phase space of the decay. The trigger requires a four-track secondary vertex with all tracks being of good quality and passing minimum momentum and transverse momentum requirements. The pions from the candidate D^0 decay are required to have a large impact parameter significance (χ_{IP}^2) with respect to all PVs, where χ_{IP}^2 is defined as the difference in the vertex-fit χ^2 of a PV reconstructed with and without the considered track. A part of the data collected in 2011 was taken with a different second-level trigger selection. In this selection only $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ candidates are reconstructed while the D^{*+} reconstruction is performed only during the offline selection. For a part of the data collected in 2012, additional events were selected in the trigger from the partial reconstruction of $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ candidates arising from $D^{*+} \rightarrow D^0 \pi_s^+$ decays, using only information from one $\pi^+ \pi^-$ pair and a slow pion.

11.2.2 The LHCb centralised selection

In this analysis, a dedicate LHCb centralised selection algorithm (stripping line) was used to select signal. Table. 11.1 summarizes the stripping selection criteria.

Table 11.1: Stripping selection criteria for the LHCb centralised selection algorithm (stripping line) that used in this $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis.

Particle	Quantity	Selection criteria
π^+ or π^-	transverse momentum p_T	$> 350 \text{ MeV}/c$
	momentum p	$> 3 \text{ GeV}/c$
	have RICH information	true
	delta-log-likelihood $\Delta \log \mathcal{L}_{K-\pi}$	< 3
	maximum track χ^2/DoF	< 4
	impact parameter χ_{IP}^2 w.r.t. best PV	> 3
D^0 before vertex fit	closest approach between two daughter particles	$< 0.12 \text{ mm}$
	transverse momentum p_T	$> 3 \text{ GeV}/c$
	$ m_{D^0} - 1864.84 \text{ MeV}/c^2 $	$< 68 \text{ MeV}/c^2$
	impact parameter χ_{IP}^2 for at least one daughter	> 30
D^0 after vertex fit	vertex χ^2/DoF	< 10
	flight distance χ^2/DoF	> 48
	impact parameter χ_{IP}^2	< 30
	$ m_{D^0} - 1864.84 \text{ MeV}/c^2 $	$< 65 \text{ MeV}/c^2$
	cosine of angle subtended by momentum and flight direction vectors of D^0	> 0.9998
D^{*+} before vertex fit	$m_{D^{*+}} - m_{D^0}$	$> 137.4 \text{ MeV}/c^2$
	$m_{D^{*+}} - m_{D^0}$	$< 159.4 \text{ MeV}/c^2$
	transverse momentum p_T	$> 3 \text{ GeV}/c$
	closest approach between D^0 and π_s	$< 0.22 \text{ mm}$
D^{*+} after vertex fit	vertex χ^2/DoF	< 20
	$m_{D^{*+}} - m_{D^0}$	$> 137.9 \text{ MeV}/c^2$
	$m_{D^{*+}} - m_{D^0}$	$< 158.4 \text{ MeV}/c^2$
π_s	transverse momentum p_T	$> 120 \text{ MeV}/c$

11.2.3 Offline selection

Refit before selection

In the offline selection some additional selection criteria are applied. Prior to these the events in the data sample are refitted to constrain the D^0 candidate and the slow pion to the same primary vertex, *i.e.* to require the D^* decay vertex to coincide with the primary vertex. The χ^2 of the refit is required to be less than 35. The four momentum of final state particles are calculated with a constrained D mass and constrained D^* vertex. These refit constraints significantly improves the resolution of the D^* mass - D^0 mass (Δm) distribution.

Removing detector asymmetries

The LHCb detector is intended to be left-right symmetric, but some differences in the detection efficiency can occur. To reduce the asymmetries that are introduced by different efficiencies of the left side and right side of the LHCb detector, the magnet up and magnet down samples are merged. In 2011, around 30% more luminosity was collected with the magnet in the down polarity configuration. Therefore the cancellation for magnetic polarity in 2011 sample is not complete. And for 2012 data, given that magnetic up and down samples have same size, the cancellation for magnetic polarity is almost complete. In the combined sample, there are about 9% more candidates with the magnetic field down than with the field up.

As discussed in Sect. 10.4.3, asymmetries can also arise due to acceptance effects. Charged particles, especially slow particles are bent when passing through the magnetic field, and for some slow pion momentum range, it is possible that one charge is bent out of the LHCb detector acceptance while the opposite charge is still within the acceptance.

We can observe this asymmetry in data by comparing the kinematics of positive and negative charged pions. Figure 11.1 shows the asymmetry between D^0 and $\bar{D}^0$ candidates as a function of the p and p_x of the slow pion in magnet up and magnet down data.

Asymmetries can also arise due to slow pions being bent back to the beam pipe. The pions can miss a tracking station by traversing the beam pipe. When this happens the pions are not fully reconstructed resulting in a beam pipe shadow in the p_x versus p_y plot, as shown in Fig. 11.2 .

Here, the fiducial selections applied in that study is used

$$\begin{aligned} |p_x| &\leq \alpha(p_z - p_0) \\ \alpha &= 0.317 \\ p_0 &= 2400 \text{ MeV}/c \end{aligned} \tag{11.1}$$

and

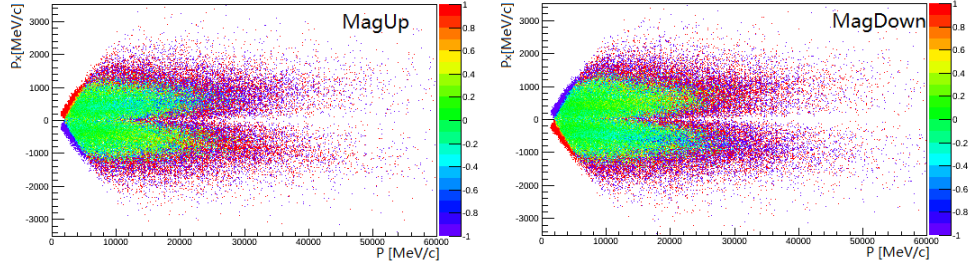


Figure 11.1: Asymmetry between D^0 and $\bar{D}^0$ candidates as a function of the p and p_x of the slow pion in (left) magnetic field up and (right) magnetic field down data samples for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis.

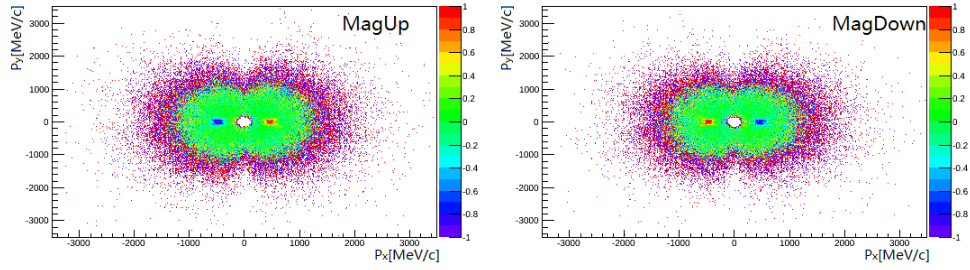


Figure 11.2: Asymmetry between D^0 and $\bar{D}^0$ candidates as a function of the p_x and p_y of the slow pion in (left) magnetic field up and (right) magnetic field down data samples for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis.

$$\begin{aligned}
 p_1 - \beta_1 p_z &< |p_x| < p_2 + \beta_2 p_z \\
 p_1 &= 418 \text{ MeV}/c \\
 p_2 &= 497 \text{ MeV}/c \\
 \beta_1 &= 0.01397 \\
 \beta_2 &= 0.01605
 \end{aligned} \tag{11.2}$$

only for $|p_y/p_z| < 0.02$ regions.

After applying these fiducial selection, the regions of largest asymmetry are removed, this removes about 15% of events, as shown in Fig. 11.3.

K_S^0 candidates

The presence of background from $D^0 \rightarrow K_S^0 \pi^+ \pi^-$, with $K_S^0 \rightarrow \pi^+ \pi^-$ is considered. By plotting the invariant mass distribution of the slowest and second

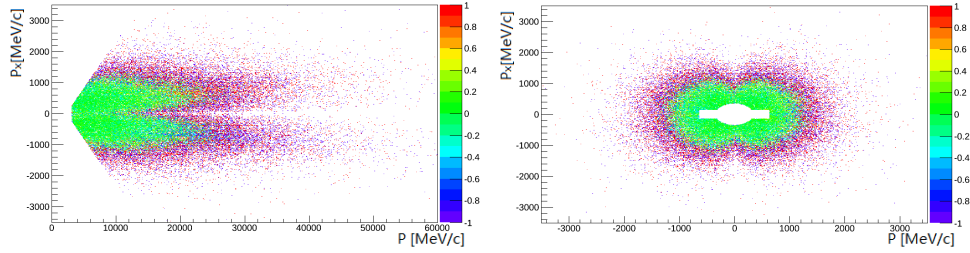


Figure 11.3: Asymmetries between D^0 and $\bar{D}^0$ candidates as a function of (left) the p versus p_x phase space and (right) the p_x versus p_y phase space of the slow pions after fiducial selections.

slowest $\pi^+\pi^-$ pairs, peaks around the K_S^0 mass ($497.611 \text{ MeV}/c^2$) region are observed (as shown in the first two figure in Fig. 11.7).

Fits with a Gaussian function are performed around the K_S^0 mass (Fig. 11.4, Table. 11.2), and this gives 9,270 K_S^0 candidates from the combined 2011 and 2012 sample. A dedicated study on the time integrated CP violation searches on $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ has been performed by CDF with 3.5×10^5 events, and did not observe any asymmetric effect [104]. In the energy test, the effect of background could be ignored if the background is small and symmetric for both flavours. Therefore, these K_S^0 candidates will be considered as background and will not be removed.

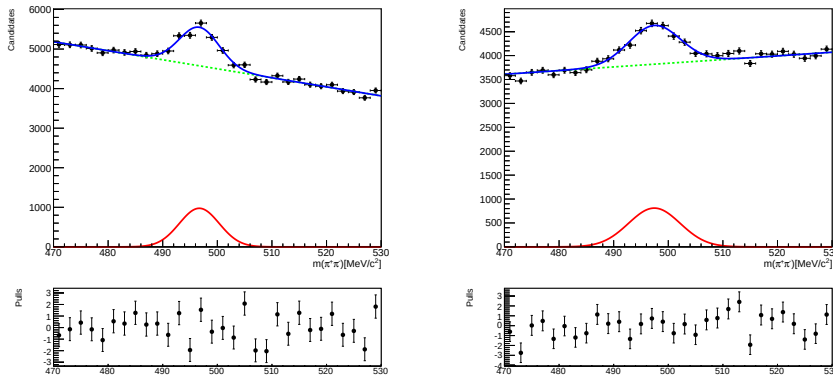


Figure 11.4: The Gaussian fit around K_S^0 mass for (left) slowest $\pi^+\pi^-$ pairs and (right) the second slowest $\pi^+\pi^-$ pairs in the selected $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ events. In the fit plots, the red line is the fit to the $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ candidates, the green line is the fit to the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ signals, blue line is the fit to combined distribution.

Table 11.2: Gaussian function fit it parameters of K_S^0 peaks in the selected $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ sample.

Parameter	Slowest $\pi^+ \pi^-$	Second slowest $\pi^+ \pi^-$
$\#K_S^0$	4464 ± 273	4806 ± 306
$m_{K_S^0} [\text{MeV}/c^2]$	496.70 ± 0.21	497.40 ± 0.26
$\sigma [\text{MeV}/c^2]$	3.65 ± 0.23	4.73 ± 0.32

Additional selections

Besides the selections described above, some more criteria are applied in the selection. The refit selection has been mentioned above, the selections on the slow pion increase the signal purity and the $m(D^0)$ and Δm selections select candidates that fall in the signal window in these variables.

$$\begin{aligned}
 D^0 \text{ refit } \chi^2 &< 35 \\
 D^{*+} \text{ refit } \chi^2 &< 35 \\
 p(\pi_{\text{slow}}) &> 3000 \text{ MeV}/c \\
 p_T(\pi_{\text{slow}}) &> 350 \text{ MeV}/c \\
 1852.03 \text{ MeV}/c^2 &< m(D^0) < 1881.73 \text{ MeV}/c^2 \\
 144.985 \text{ MeV}/c^2 &< \Delta m < 145.895 \text{ MeV}/c^2.
 \end{aligned} \tag{11.3}$$

In the selections above, the D^0 mass and Δm windows are tuned with fits and the range selected corresponds to the central value of their highest Gaussian functions plus/minus 1.8σ .

After selection, 319,948 candidates have been selected in the 2011 sample, with a purity of 96.9%, and 720,510 candidates in the 2012 sample, with a purity of 96.0%. The plots of Δm after D^0 mass selections and D^0 mass after Δm selections are shown in Fig. 11.5 and Fig. 11.6 respectively. Fit results are reported in Table. 11.3 and Table. 11.4. The Δm distribution is fitted with a triple Gaussian and the background with a polynomial. The D^0 mass distribution is fitted with a double Gaussian. As suggested in Sect. 9.6.3, the invariant mass of all four $\pi^+ \pi^-$ pairs are sorted and the corresponding distributions are shown in Fig. 11.7.

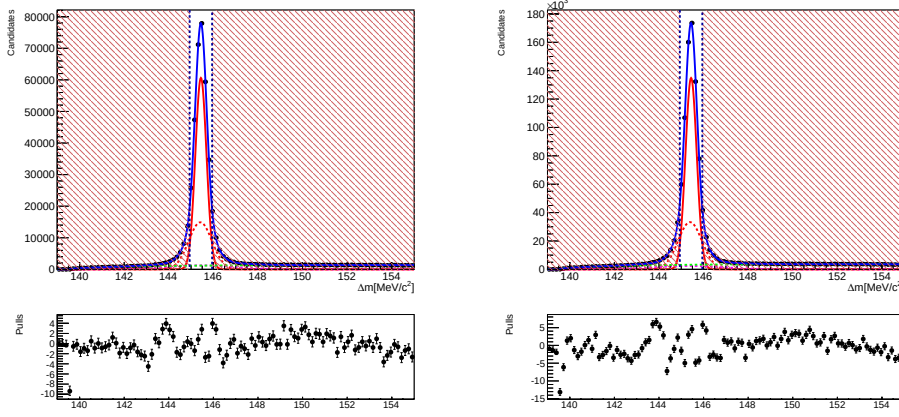


Figure 11.5: Δm distribution of 2011 (2012) data for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis after selection in the left (right) plot. Lines show fit results for: *core* signal component (solid red), *mid* signal component (dotted red), *tail* signal component (dotted purple), background (green), total (blue). Shaded areas are rejected in the final selection.

Table 11.3: Results of fit to the Δm distributions for selected 2011 and 2012 data for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis. $\Delta m_{0_{core}}$, $\Delta m_{0_{mid}}$, and $\Delta m_{0_{tail}}$ are the peak position of the three Gaussians, σ_{core} , σ_{mid} , and σ_{tail} are their widths, f_{core} and f_{mid} are the signal fractions of the two narrow Gaussians.

Parameter	2011 sample	2012 sample
Signal [10^3]	387.2 ± 0.6	869.5 ± 1.0
$\Delta m_{0_{core}} [\text{MeV}/c^2]$	145.441 ± 0.001	145.441 ± 0.001
$\Delta m_{0_{mid}} [\text{MeV}/c^2]$	145.415 ± 0.003	145.391 ± 0.002
$\Delta m_{0_{tail}} [\text{MeV}/c^2]$	145.46 ± 0.02	145.355 ± 0.014
f_{core}	0.66 ± 0.03	0.66 ± 0.015
f_{mid}	0.26 ± 0.03	0.27 ± 0.02
$\sigma_{core} [\text{MeV}/c^2]$	0.248 ± 0.003	0.252 ± 0.002
$\sigma_{mid} [\text{MeV}/c^2]$	0.519 ± 0.03	0.532 ± 0.02
$\sigma_{tail} [\text{MeV}/c^2]$	1.50 ± 0.004	1.60 ± 0.002
Background [10^3]	109.9 ± 0.5	299.7 ± 0.7

Table 11.4: Results of fit to the $m(D^0)$ distributions for selected 2011 and 2012 data for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis. $m(D)_{0_{mid}}$, and $m(D)_{0_{tail}}$ are the peak position of the two Gaussians, σ_{mid} , and σ_{tail} are their widths, f_{mid} is the signal fractions of the narrow Gaussian.

Parameter	2011 sample	2012 sample
Signal [10^3]	360.3 ± 0.4	811.4 ± 0.4
$m(D)_{0_{mid}}$ [MeV/ c^2]	1866.96 ± 0.12	1866.75 ± 0.02
$m(D)_{0_{tail}}$ [MeV/ c^2]	1863.5 ± 3.3	1863.7 ± 0.2
f_{mid}	0.81 ± 0.01	0.81 ± 0.01
σ_{mid} [MeV/ c^2]	8.51 ± 0.17	8.63 ± 0.02
σ_{tail} [MeV/ c^2]	15.4 ± 0.7	16.1 ± 0.4
Background [10^3]	26.5 ± 0.4	93.7 ± 3.3

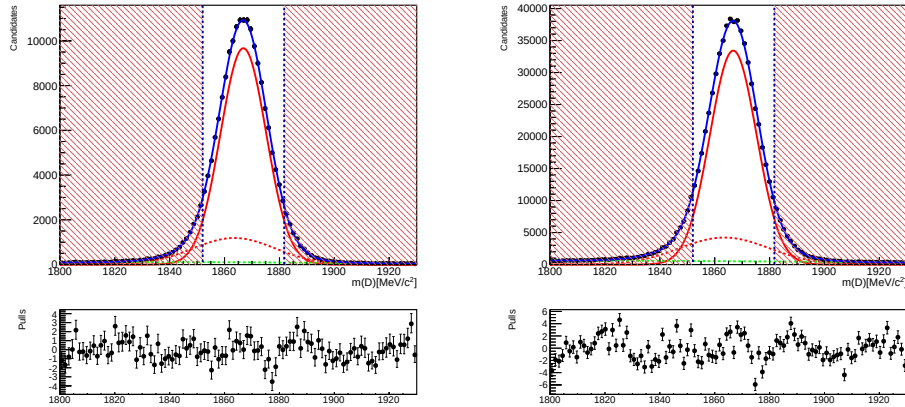


Figure 11.6: $m(D^0)$ distribution of 2011(2012) data for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis are shown in the left (right) plot. Lines show fit result for two Gaussian components (red), background (green), total (blue). Shaded areas are rejected in the final selection.

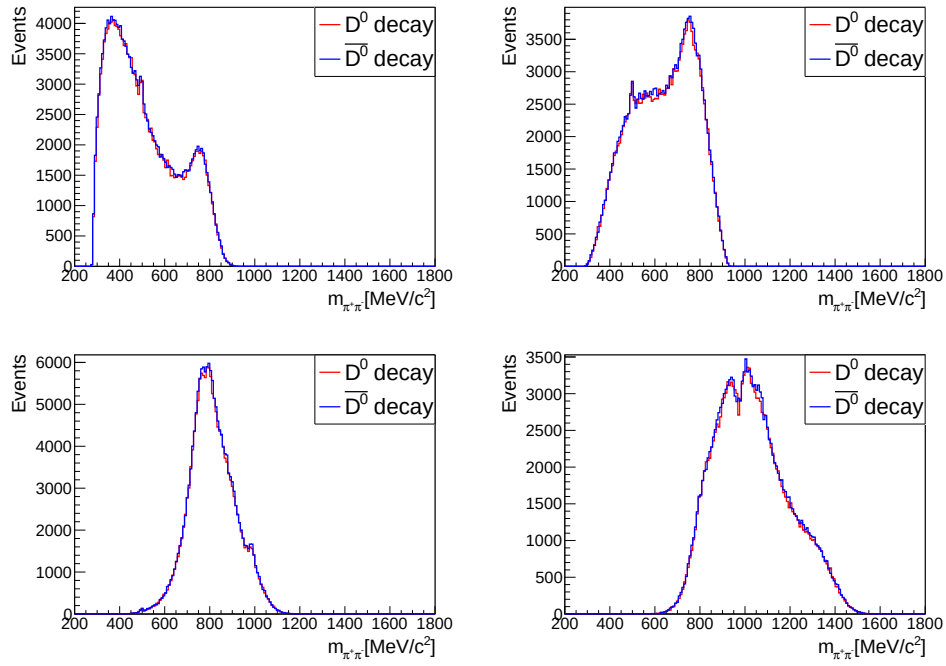


Figure 11.7: The four possible $\pi^+\pi^-$ invariant masses in the $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ data, ordered from top left to bottom right as the invariant mass of the pairing increases.

11.2.4 Background events

Background events are tested with the energy test to see if they introduce any asymmetry (see Sect. 11.4.1). Background events are obtained from a side band region in the $m(D^0) - \Delta m$ space. Events in a ring region from $\sim 4\sigma_{\text{midS}}$ to $\sim 9\sigma_{\text{midS}}$ away from the peak are taken into account. Signal events where final state radiation is emitted are removed in the selected background sample by rejecting events that peak in Δm and have low values for $m(D^0)$ (see Fig. 11.8).

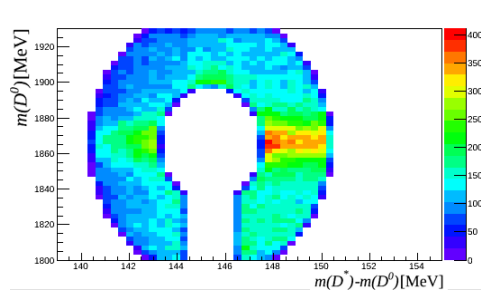


Figure 11.8: Background events for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis are taken in a ring region in the $m(D^0) - \Delta m$ space 4 to 9 σ s away from signal peak, signal events where final state radiation has been emitted are removed from this sample by cut off band regions with 2σ s half-width.

Combining the 2011 and 2012 samples, a total of 597,478 background candidates are obtained. From the fits applied in the signal peak region, about 33,329 background events in the combined signal peak region are expected. Hence, the background sample is divided into 17 sub-samples so that the background sample sizes are similar to that of the signal sample. Each background sample has 35,146 events.

11.2.5 Selection of reference decay $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$

$D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ decays serve as a reference channel with which checks are performed on whether various detection and production asymmetries, present in both signal and reference modes, have any impact on the CP violation search in $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decays. Being dominated by a Cabibbo favoured $c \rightarrow s\bar{u}d$ diagram, with a small contribution of a doubly Cabibbo suppressed $c \rightarrow d\bar{u}s$ amplitude, $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ is expected to have negligible CP violation.

Although the $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ and $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ final states are not identical, the kinematics of their daughter particles are quite similar. When testing the energy test method with the reference channel the kaon is treated in the same way as a pion, we refer to the pion or kaon generically as ‘ h ’. The kaon is known to have a larger detection asymmetry than the pion [25]. The extent to which both channels are similar in terms of the CP violation-unrelated asymmetries is discussed in Sect. 11.4.

The selection of the $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ reference channel is very similar to $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$. The stripping selection follows the one described in Sec. 11.2.2, the only difference in the stripping selection compared with the data channel is that in the reference channel kaon PIDK is required to be greater than 0 while for all pions the PIDK is required to be less than 3. Dedicated triggers for $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ are used for the 2011 and 2012 samples. The same offline selection is applied as for the $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$.

After selection, 3,740,127 candidates are obtained in the 2011 sample and 6,617,218 candidates in the 2012 sample. To get the relevant estimate of our sensitivity to these asymmetries, the total $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ sample is randomly divided into 10 subsamples, each subsample has about 1.04M candidates, which is similar in size to the number of events for the $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ analysis.

The $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ sample has a 1.8% global asymmetry between D^0 and $\bar{D}^0$ decay: after selection 5,085,744 D^0 candidates and 5,271,601 $\bar{D}^0$ candidates are expected. The momentum and transverse momentum distributions of the D^0 from the reference channel are shown in Fig. 11.9.

Below the properties of the $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ sample are presented. Distributions of $\Delta(m)$ for the $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ data are shown in Fig. 11.10. A fit to the $\Delta(m)$ distribution is applied. The parametrisations used in the fit follow the ones used in the signal mode. The fit results are summarized in Table 11.5. The signal shapes are very similar to the $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$. Candidates within $|\Delta(m) - 145.4| < 0.5 \text{ MeV}/c^2$ are selected; this gives purities of 99%.

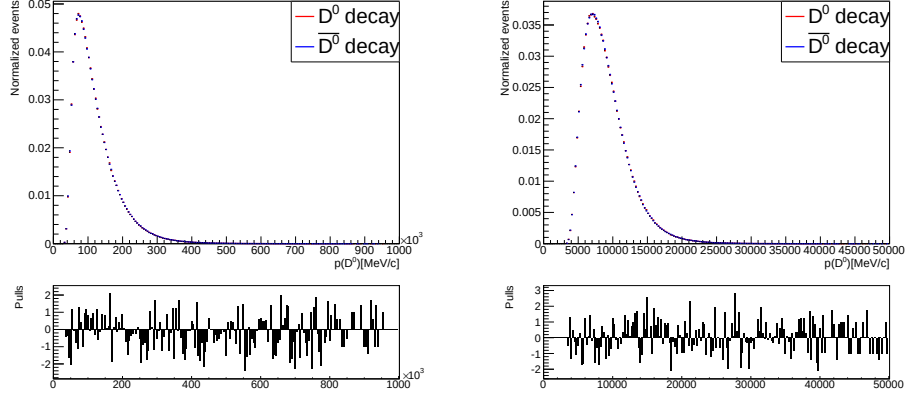


Figure 11.9: (left) Momentum and (right) transverse momentum of D^0 in the reference channel $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$.

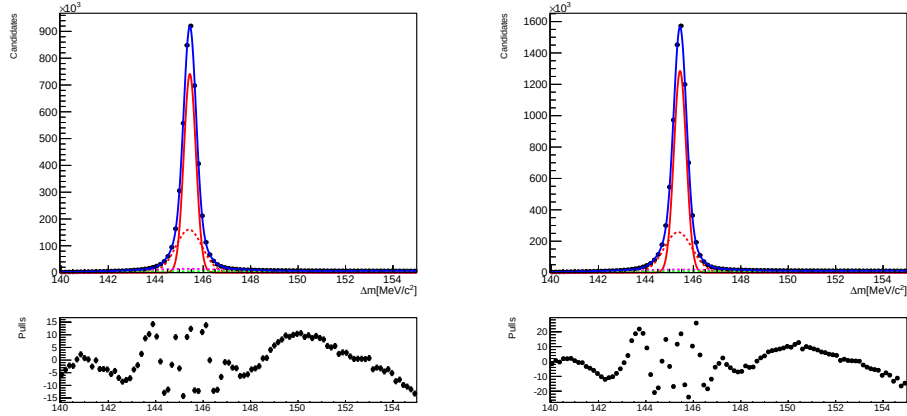


Figure 11.10: Δm distribution for $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ decay after selection for (left) 2011 and (right) 2012 data samples. Lines show fit results for: *core* signal component (solid red), *mid* signal component (dotted red), *tail* signal component (dotted purple), background (green), total (blue).

Table 11.5: Results of fit to the Δm distributions for the selected $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$ events shown in Fig. 11.10. $\Delta m_{0_{core}}$, $\Delta m_{0_{mid}}$, and $\Delta m_{0_{tail}}$ are the peak position of the three Gaussians, σ_{core} , σ_{mid} , and σ_{tail} are their widths, f_{core} and f_{mid} are the signal fractions of the two narrow Gaussians.

Parameter	2011 sample	2012 sample
Signal [10^6]	4.760 ± 0.005	8.201 ± 0.004
$\Delta m_{0_{core}} [\text{MeV}/c^2]$	145.440 ± 0.001	145.441 ± 0.001
$\Delta m_{0_{mid}} [\text{MeV}/c^2]$	145.401 ± 0.001	145.367 ± 0.001
$\Delta m_{0_{tail}} [\text{MeV}/c^2]$	145.43 ± 0.02	145.326 ± 0.005
f_{core}	0.6722 ± 0.0005	0.673 ± 0.002
f_{mid}	0.245 ± 0.001	0.249 ± 0.002
$\sigma_{core} [\text{MeV}/c^2]$	0.2530 ± 0.0001	0.2573 ± 0.0003
$\sigma_{mid} [\text{MeV}/c^2]$	0.560 ± 0.001	0.565 ± 0.003
$\sigma_{tail} [\text{MeV}/c^2]$	1.94 ± 0.03	1.81 ± 0.02
Background [10^3]	449.9 ± 0.9	840 ± 3

11.3 Sensitivity tests

The aim of this section is to describe the approximate sensitivity of the analysis to CP violation. This is illustrated by considering the effect of introducing CP violating effects in the primary resonances in the decay in a simplified toy simulation. The toy simulations are made more realistic by accounting for the primary efficiency variations across the phase-space in LHCb. The efficiency effects are parametrised using standard simulation (Gaudi GEANT based) ‘full’ simulation of LHCb. The energy test is implemented with the coordinates suggested in Sect. 9.6.3.

11.3.1 Simulation samples

The studies reported here on the sensitivity of the analysis are performed with a simple simulation, referred to here as a toy simulation, which does not include a model of the LHCb detector. In addition standard LHCb simulation ‘full’ samples are used to parameterise the efficiency of the analysis across the phase space.

Toy samples of $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ decays generated using the amplitude analysis package MINT are used in these studies. The MINT package includes a toy simulation sample generator which has been used in several LHCb four body decay analyses. These simulation samples are constructed using the isobar model, where the total amplitude is the sum of a set of contributing amplitudes each with a complex coefficient as described in Eq. 10.1.

The resonance properties used to generate Monte Carlo samples are a preliminary version of those measured in an amplitude analyses using CLEO-c data [86]. The properties of this model are listed in Table 11.6.

Toy simulation samples are also generated including CP violation through a variation of this model. For example, the CP violation is introduced in the $a_1(1260)^+ \rightarrow \rho^0(\pi^+ \pi^-) \pi^+$ (S-wave) decay mode:

$$\Delta a_{a_1(1260)^+ \rightarrow \rho^0(\pi^+ \pi^-) \pi^+ (S\text{-wave})} = 0.02 a_{a_1(1260)^+ \rightarrow \rho^0(\pi^+ \pi^-) \pi^+ (S\text{-wave})} \quad (11.4)$$

Hence, the magnitude in the D^0 decay samples are changed by 2%. The magnitude and phase in the $\bar{D}^0$ decay sample remain as in Table 11.6.

11.3.2 Studies accounting for selection efficiencies

This section describes sensitivity studies on toy simulated data taking into account the phase-space bias from the event selection using full simulation. The selection efficiency is measured based on a full simulated sample comprising 1M $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ events generated with a flat phase-space distribution and the the LHCb detector acceptance criterion. About 5000 events are retained after application of the stripping criteria and the full offline selection. Toy simulation samples are generated by MINT in the D^0 central of mass frame,

Table 11.6: List of CLEO-c model resonances and their parameters used to generate $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ simulation samples.

Component	Real part	Imaginary part	Amplitude	Phase	Fit fraction
$a_1^+(1260) \rightarrow \rho^0 \pi^+, S\text{-wave}$	1	0	1	0	0.382143
$a_1^+(1260) \rightarrow \rho^0 \pi^+, D\text{-wave}$	-0.0740463	0.287045	0.296442	104.465	0.032499
$a_1^+(1260) \rightarrow \sigma_1 \pi^+$	0.42593	0.444951	0.615953	46.2512	0.136431
$\rho^0 \rho^0, S\text{-wave}$	-0.0947318	0.174353	0.198427	118.517	0.014811
$\rho^0 \rho^0, P\text{-wave}$	-0.0994544	-0.380995	0.393762	-104.63	0.0640868
$\rho^0 \rho^0, D\text{-wave}$	-0.429147	0.224486	0.484315	152.386	0.0849663
$a_1^+(1640) \rightarrow \rho^0 \pi^+$	0.0426438	0.65117	0.652565	86.2532	0.15632
$a_1^+(1640) \rightarrow \rho^0 \pi^+$	0.19059	-0.176632	0.259853	-42.8233	0.0249813
$\rho^0 \sigma$	0.310192	-0.255412	0.401814	-39.468	0.055988
$f_0(1370) \sigma$	0.39547	0.33263	0.516758	40.0672	0.0934817
$f_0(1370) \rho^0$	-0.182548	-0.50309	0.535185	-109.943	0.101117
$f_2(1270) f_2(1270)$	0.0321765	-0.295171	0.29692	-96.2212	0.036311
NonRes	0.370854	0.156622	0.402571	22.8957	0.0512767

therefore all phase space events used in this efficiency study are boosted to the D^0 COM frame, the efficiency is then defined as the number of reconstructed events divided by the number of generated events in a particular region of phase space.

It is known that detected particles have to have enough energy to pass through the detector, the efficiency drops for pions in the low momentum region is expected. In this study, the four final state pions are sorted by their momentum, and then the efficiency is plotted of the 1st and 2nd slowest pions in the full simulation, which is shown in Fig. 11.11. A clear drop of efficiency in the slowest pion momentum is seen and parametrised by a $p_1 - e^{p_2 x + p_3}$ function. The fitted parameters for all events and for π^+ slowest and π^- slowest events separately can be found in Table. 11.7. These results are consistent with each other and there is no clear difference on the acceptance between π^+ and π^- .

Table 11.7: Fit parameters in the slowest pion momentum distribution for all events and for π^+ slowest and π^- slowest from full simulation.

parameter	All events	π^+ slowest	π^- slowest
p_1	0.0601 ± 0.0036	0.0572 ± 0.0042	0.0606 ± 0.0046
p_2	-0.0071 ± 0.0015	-0.0082 ± 0.0021	-0.0073 ± 0.0019
p_3	-2.805 ± 0.072	-2.740 ± 0.101	-2.767 ± 0.106

On the second slowest pion efficiency plot, a drop in the low momentum region is also seen. To understand the drop in the 2nd slowest pion momentum distribution, a plot of the efficiency of slowest pions versus 2nd slowest pion is made (see Fig. 11.12). From the plot, the drop of efficiency in 2nd slowest pion low momentum region is shown to be due to the shape of phase space and efficiency drop of the slowest pion.

For the sensitivity studies conducted here, toy simulation events are then generated as described above and events are randomly discarded according to the efficiency parametrisation extracted from full simulation as a function of slowest pion momentum and assuming no dependence on the other variables.

After applying the efficiency maps to toy simulation data, the toy simulation samples used in the sensitivity studies are obtained.

11.3.3 Sensitivity study with default metric parameter σ

The parameter σ defined in Eq. 9.4, which defines the rate at which the distance metric approaches 0 with increasing distance, is set to $0.5(\text{GeV}/c^2)^2$ in the default analysis. A range of CP violating scenarios are then tested using simulation samples to extract the expected sensitivity of the energy test method for the data sample. Table 11.8 and 11.9 summarize the scenarios that were tested, and the plots for T values of each scenarios are given in Fig. 11.13, 11.14, 11.15, and 11.16

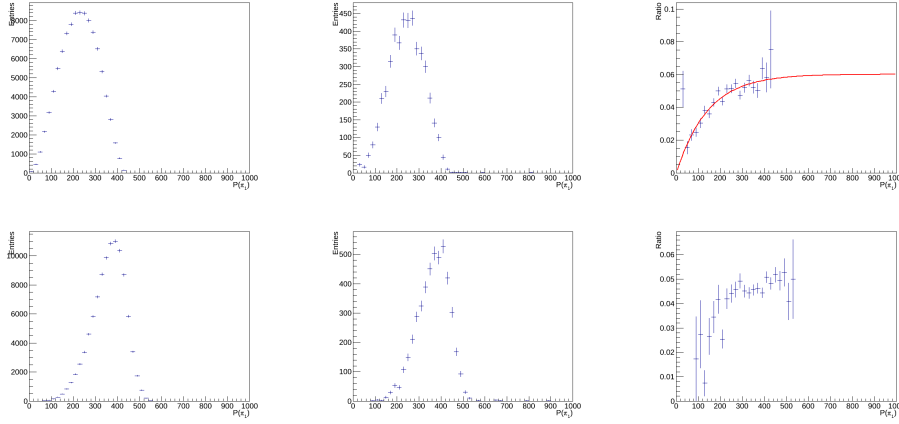


Figure 11.11: The left, middle, and right columns indicate the distributions of momentum of (top row) the slowest pion and (bottom row) 2nd slowest pion from generator level, reconstruction level, and the efficiency between them. These plots are made from LHCb full simulation of $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$. The slowest pion efficiency distribution is fitted with an exponential function, and is shown with red line.

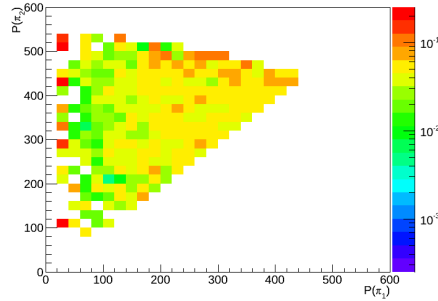


Figure 11.12: Acceptance of slowest pions versus 2nd slowest pion in LHCb full simulation events for $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$.

If a scenario gives a p -value less than 0.003 (*i.e.* more than 3 sigma in a Gaussian distribution), it is considered to be sensitive to this scenario. From these tests, it can be learned that with $\sigma = 0.5(\text{GeV}/c^2)^2$, the energy test is sensitive to a 3 degree phase asymmetry or 5% amplitude asymmetry on $D^0 \rightarrow (a_1^+ \rightarrow \rho^0 \pi^+) \pi^-$ decay mode, 4 degree phase asymmetry or 5% amplitude asymmetry on $D \rightarrow \rho \rho(D - \text{wave})$ decay mode. These scenarios are used to tune the metric parameter σ , which is described in Sec. 11.3.4.

Table 11.8: T values and p -value for $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decays simulation samples with phase and amplitude asymmetries in $D^0 \rightarrow (a_1 \rightarrow \rho\pi)\pi$.

asymmetries in $D^0 \rightarrow (a_1 \rightarrow \rho\pi)\pi$	T_0 value	p -value (fit)
a_1 Δ phase 2° , Δ Amp 0	1.77×10^{-6}	$0.015^{+0.0094}_{-0.0068}$
a_1 Δ phase 3° , Δ Amp 0	5.93×10^{-6}	$1.20^{+3.6}_{-1.20} \times 10^{-6}$
a_1 Δ phase 4° , Δ Amp 0	1.20×10^{-5}	$1.04^{+7.8}_{-1.04} \times 10^{-10}$
a_1 Δ phase 5° , Δ Amp 0	1.94×10^{-5}	$9.78^{+333}_{-9.78} \times 10^{-17}$
a_1 Δ phase 0, Δ Amp 2%	-7.22×10^{-7}	$0.931^{+0.019}_{-0.022}$
a_1 Δ phase 0, Δ Amp 3%	2.11×10^{-7}	$0.206^{+0.046}_{-0.046}$
a_1 Δ phase 0, Δ Amp 4%	1.52×10^{-6}	$0.014^{+0.0083}_{-0.0061}$
a_1 Δ phase 0, Δ Amp 5%	3.03×10^{-6}	$2.6^{+3.4}_{-1.7} \times 10^{-4}$

Table 11.9: T values and p -value for $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decays simulation samples with phase and amplitude asymmetries in $D \rightarrow \rho\rho(D - wave)$.

asymmetries in $D \rightarrow \rho\rho(D - wave)$	T_0 value	p -value (fit)
$\rho\rho(D)$ Δ phase 2° , Δ Amp 0	9.464×10^{-8}	$0.372^{+0.049}_{-0.052}$
$\rho\rho(D)$ Δ phase 3° , Δ Amp 0	2.052×10^{-6}	$0.010^{+0.0064}_{-0.0047}$
$\rho\rho(D)$ Δ phase 4° , Δ Amp 0	4.666×10^{-6}	$9.63^{+24}_{-7.2} \times 10^{-6}$
$\rho\rho(D)$ Δ phase 5° , Δ Amp 0	7.565×10^{-6}	$3.87^{+23}_{-3.87} \times 10^{-9}$
$\rho\rho(D)$ Δ phase 0, Δ Amp 2%	-7.931×10^{-7}	$0.961^{+0.013}_{-0.016}$
$\rho\rho(D)$ Δ phase 0, Δ Amp 3%	-9.234×10^{-8}	$0.578^{+0.045}_{-0.050}$
$\rho\rho(D)$ Δ phase 0, Δ Amp 4%	1.060×10^{-6}	$0.072^{+0.026}_{-0.0022}$
$\rho\rho(D)$ Δ phase 0, Δ Amp 5%	2.443×10^{-6}	$0.0038^{+0.0029}_{-0.0019}$

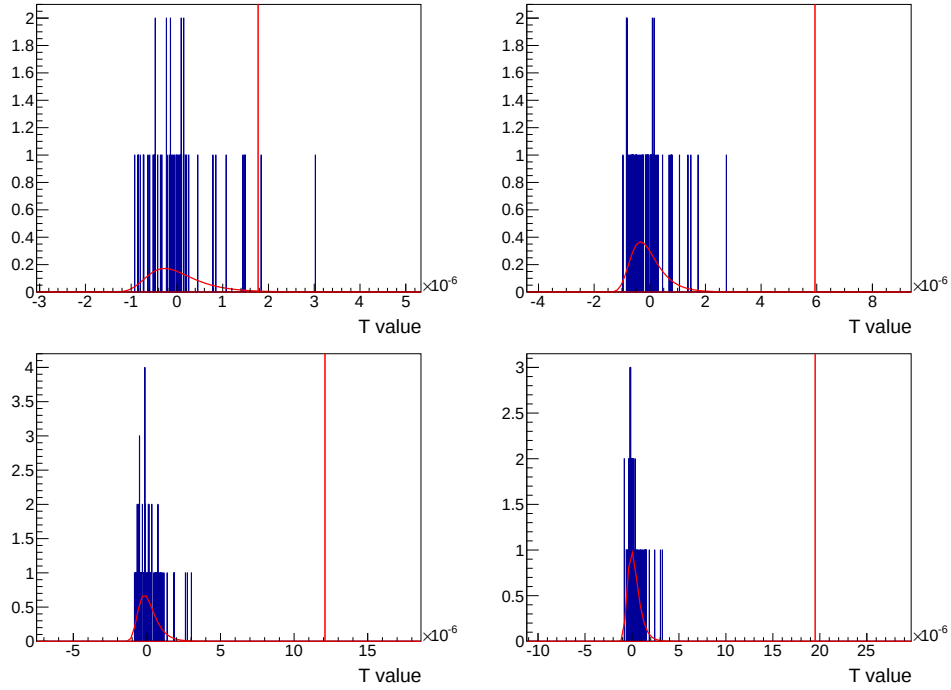


Figure 11.13: T value distributions of $D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$ decays simulation samples with (top left) 2 degree, (top right) 3 degree, (bottom left) 4 degree, and (bottom right) 5 degree phase difference in $D^0 \rightarrow (a_1 \rightarrow \rho \pi(S - wave)) \pi$ mode. Blue lines are T values from permutations, red line is the nominal T value calculated from the CP asymmetric sample.

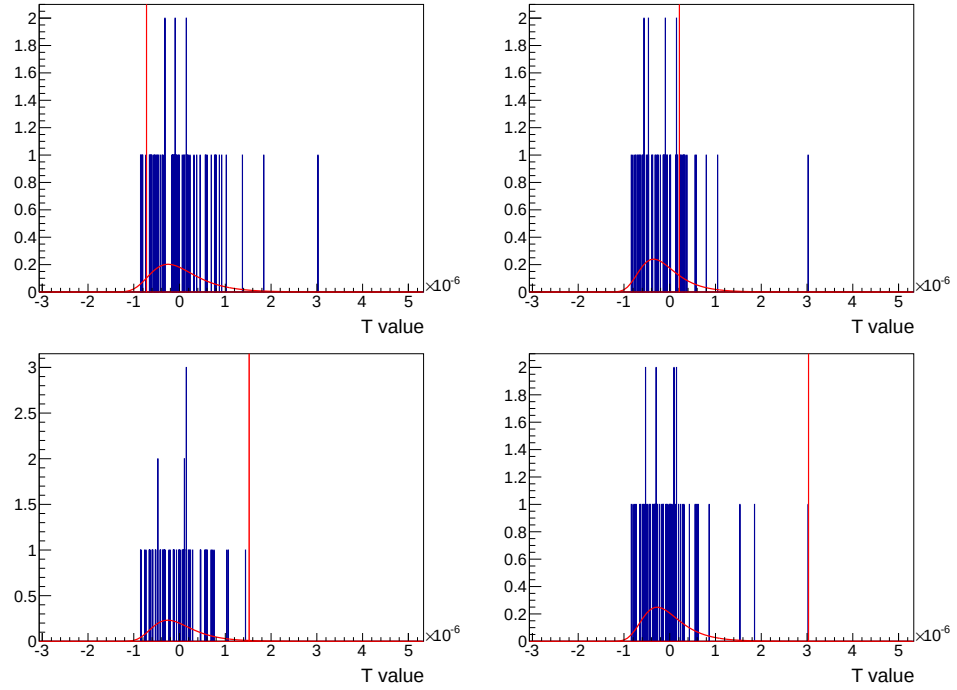


Figure 11.14: T value distributions of $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decays simulation samples with (top left) 2%, (top right) 3%, (bottom left) 4%, and (bottom right) 5% amplitude difference in $D^0 \rightarrow (a_1 \rightarrow \rho \pi(S - wave)) \pi$ mode. Blue lines are T values from permutations, red line is the nominal T value calculated from the CP asymmetric sample.

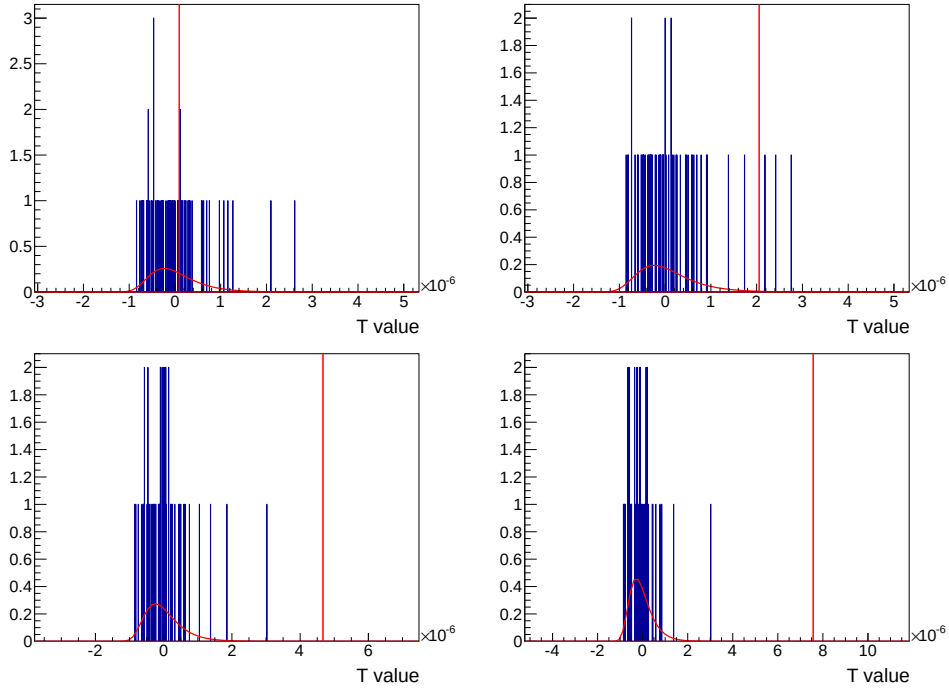


Figure 11.15: T value distributions of $D^0 \rightarrow \pi^- \pi^+ \pi^+ \pi^-$ decays simulation samples with (top left) 2 degree, (top right) 3 degree, (bottom left) 4 degree, and (bottom right) 5 degree phase difference in $D \rightarrow \rho \rho(D\text{-}wave)$ mode. Blue lines are T values from permutations, red line is the nominal T value calculated from the CP asymmetric sample.

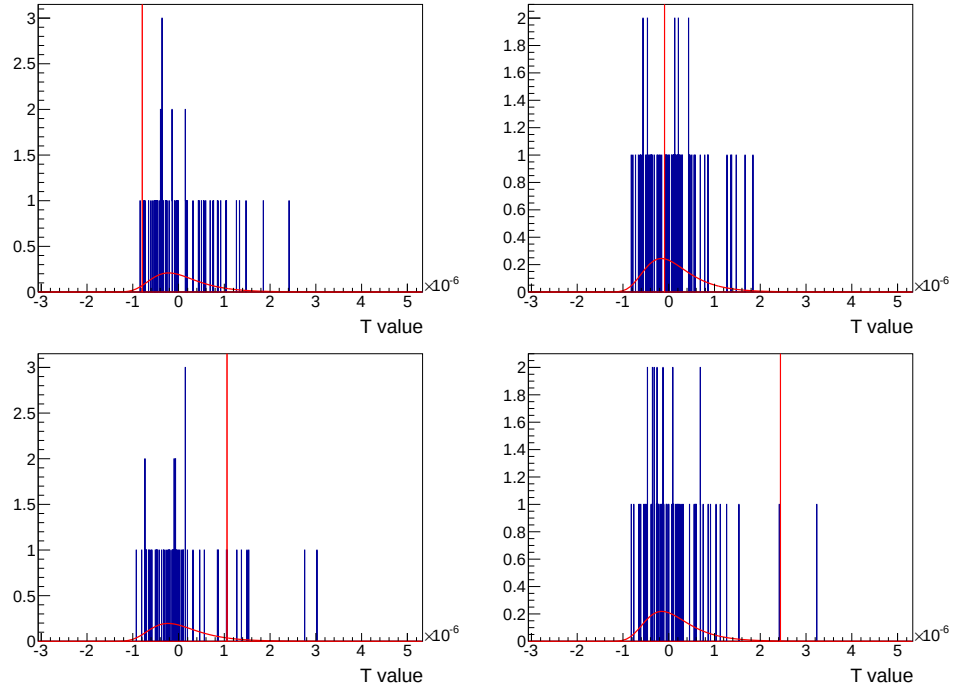


Figure 11.16: T value distributions of $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decays simulation samples with (top left) 2%, (top right) 3%, (bottom left) 4%, and (bottom right) 5% amplitude difference in $D \rightarrow \rho\rho(D - wave)$ mode. Blue lines are T values from permutations, red line is the nominal T value calculated from the CP asymmetric sample.

11.3.4 Variation of metric parameter σ

As discussed in Sect. 10.3.3, the metric parameter in the energy test needs to be tuned. For a given CP violation scenario, the dependence of the result on the choice of σ is tested for a range from $\sigma = 0.2(\text{GeV}/c^2)^2$ to $\sigma = 1.1(\text{GeV}/c^2)^2$. The lower end of this spectrum is in the range of the resolution and thus a deterioration of the efficiency can be expected. At the upper end of the range that is tested the σ value is large compared to the overall extension of the phase space and some degradation in sensitivity may be expected. Table 11.10 gives the results for a CP violation scenario with $+3^\circ$ phase asymmetry applied on the $a_1^+ \rightarrow \rho^0 \pi^+$ resonance, and Fig. 11.17 and Fig. 11.18 are the corresponding T value distribution plots. Results from other scenarios could be found in Appendix. H. Fig. 11.19 show the T value and corresponding p -value for specific CP violating scenarios as a function of the metric parameter σ . The optimal σ is found to be dependent on the specific CP violating scenario, however there is a broad agreement on the range of optimum values. Based on the range of optimum values obtained a default $\sigma = 0.5(\text{GeV}/c^2)^2$ is chosen, with results also given for other sigma values.

Table 11.10: Summary of T values obtained from toy simulation samples of $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ decays generated with a CP violation of $+3^\circ$ applied on the $a_1^+ \rightarrow \rho^0 \pi^+$ resonance for a range of values of the metric parameter σ .

$\sigma [\text{GeV}^2/c^4]$	T value	p -value
0.2	3.89×10^{-8}	$0.32724^{+0.07238}_{-0.07586}$
0.3	2.45×10^{-6}	$1.093^{+4.10}_{-1.09} \times 10^{-5}$
0.4	4.52×10^{-6}	$4.61^{+20.7}_{-4.61} \times 10^{-6}$
0.5	5.93×10^{-6}	$1.20^{+3.6}_{-1.20} \times 10^{-6}$
0.6	6.73×10^{-6}	$4.33^{+52}_{-4.33} \times 10^{-9}$
0.7	7.04×10^{-6}	$1.62^{+13}_{-1.62} \times 10^{-7}$
0.8	7.01×10^{-6}	$1.41^{+23}_{-1.41} \times 10^{-9}$
0.9	6.78×10^{-6}	$1.51^{+26}_{-1.51} \times 10^{-10}$
1.0	6.43×10^{-6}	$3.85^{+54}_{-3.85} \times 10^{-9}$
1.1	6.02×10^{-6}	$1.46^{+31}_{-1.46} \times 10^{-10}$

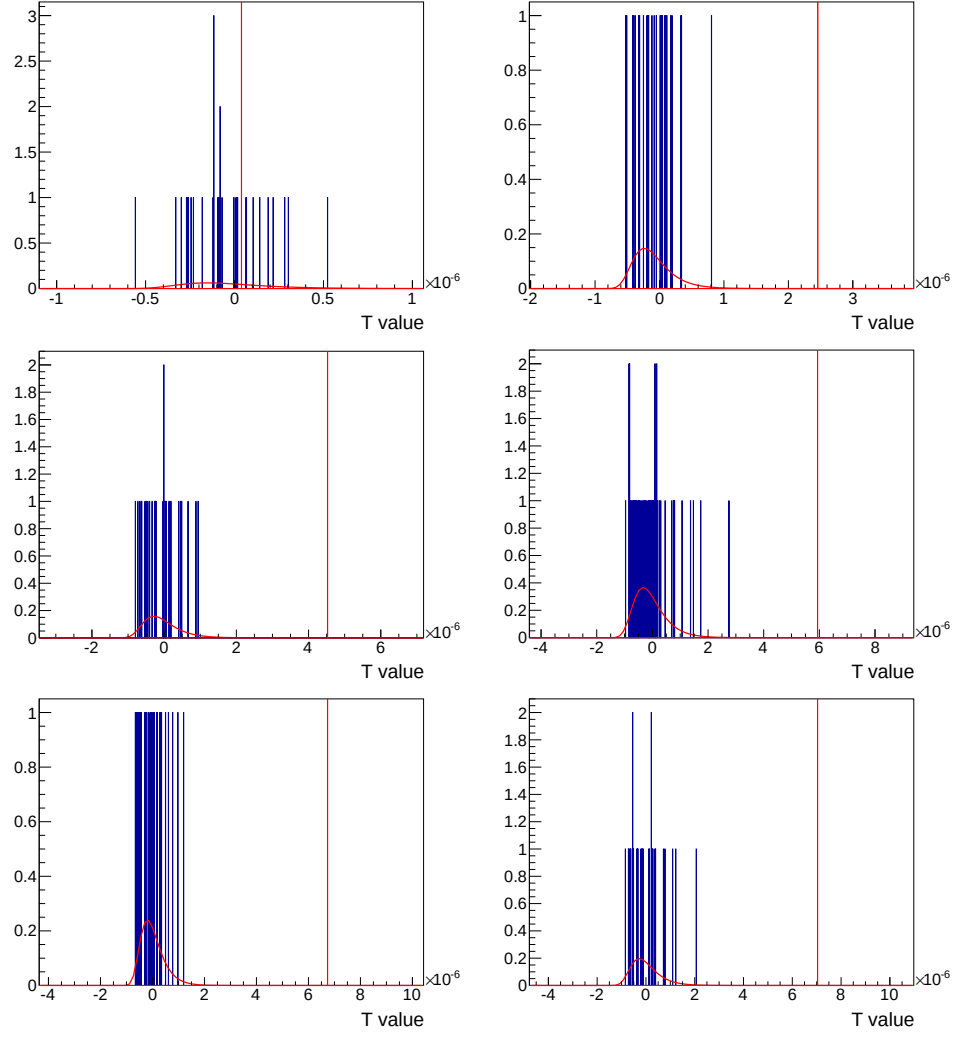


Figure 11.17: T value and permutation T values distribution for one toy simulation sample ($+3^\circ$ phase asymmetry applied on the $a_1^+ \rightarrow \rho^0 \pi^+$) for a range of σ s. From left to right, then from top to bottom, σ are (1) $0.2(\text{GeV}/c^2)^2$, (2) $0.3(\text{GeV}/c^2)^2$, (3) $0.4(\text{GeV}/c^2)^2$, (4) $0.5(\text{GeV}/c^2)^2$, (5) $0.6(\text{GeV}/c^2)^2$, (6) $0.7(\text{GeV}/c^2)^2$.

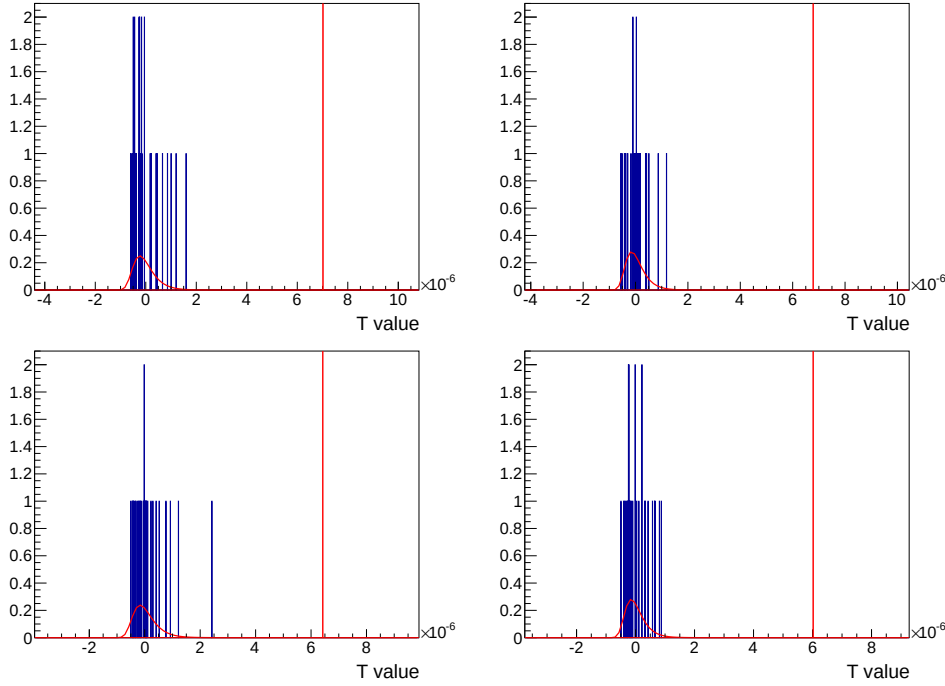


Figure 11.18: T value and permutation T values distribution for one toy simulation sample ($+3^\circ$ phase asymmetry applied on the $a_1^+ \rightarrow \rho^0 \pi^+$) for a range of σ s. From left to right, then from top to bottom, σ are (1) $0.8(\text{GeV}/c^2)^2$, (2) $0.9(\text{GeV}/c^2)^2$, (3) $1.0(\text{GeV}/c^2)^2$, (4) $1.1(\text{GeV}/c^2)^2$.

11.3.5 P -odd tests

As discussed in Sect. 9.7, in four-body decays a parity-odd amplitude can be accessed via dividing and recombining samples according to both the particle flavour and the triple products of the daughter particle momenta. This section describes the implementation in the $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decay.

To exploit the sensitivity of P -odd tests, simulation samples are generated with some phase and amplitude asymmetries in the P -wave $D^0 \rightarrow \rho^0(770)\rho^0(770)$ decays. First, the standard test for P -even CP violation is performed on these samples. The only way to get P -even CP violation from an asymmetry introduced into a P -odd amplitude is through its interference with itself i.e. the resonance intensity as in Eq. 11.5¹.

$$|a_{\rho\rho}|^2 = a_{\rho\rho} \times a_{\rho\rho}^* = |a_{\rho\rho}|e^{i\Phi_{\rho\rho}} \times |a_{\rho\rho}|e^{-i\Phi_{\rho\rho}}. \quad (11.5)$$

As the imaginary part cancels out in this case, it is expected that the P -even test will not be sensitive to an asymmetric P -odd amplitude. However, a

¹ P -odd amplitude multiplied with P -odd amplitude gives P -even and there is only single P -odd amplitude contributing here

P -odd CP violation effect will arise from the interference between this P -odd amplitude and any P -even amplitude.

The simulated $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decay samples are divided into four samples as suggested in Sect. 9.7, and by recombining the samples, both P -even and P -odd tests are performed. The test results of several scenarios are listed in Table. 11.11 and follow the expected pattern of sensitivity. An important conclusion from this study is that both P -even and P -odd tests should be performed to ensure that a full search for CP violation is made, this has often not be the case in previous studies.

11.3.6 Visualise the asymmetric region

The phase space of a four-body pseudo-scalar decay has five dimensions, therefore it is not easy to have an intuitive visualisation. To get the idea of which region in the phase space contribute to any CP violation signal, two visualisation options are explored.

The first option is to use a method called parallel coordinates. In this method ‘Dalitz like’ variables, i.e. invariant masses, are plotted with five dimensional parallel coordinates. An example is given in Fig. 11.20. This is a 2 D visualisation tool that is used to reveal 5 D information, consequently the drawbacks of parallel coordinates are very clear. The plots are not easy to interpret. It is not easy to see the population of events if the number of events is above a certain value, the entire graph may then covered by the lines that connect coordinates. Furthermore, the relationships between coordinates which are not plotted next to each are not straightforward to understand. These drawbacks can be reduced, but not removed, through the use of semi-transparent curves to display events and a wise choice of the order of coordinates to facilitate the understanding of the observed patterns. In $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ we know that the dominating decay modes are $D^0 \rightarrow (a_1 \rightarrow \rho^0 \pi) \pi$ and $D^0 \rightarrow \rho^0 \rho^0$. Hence, the lines connecting the ρ mass regions between $m(\pi^1 \pi^2)$ and $m(\pi^3 \pi^4)$ or between $m(\pi^1 \pi^4)$ and $m(\pi^2 \pi^3)$ can be used to illustrate the $D^0 \rightarrow \rho^0 \rho^0$ decay. Lines connecting the a_1 mass region on $m(\pi^1 \pi^2 \pi^3)$ coordinate and ρ mass region on $m(\pi^1 \pi^2)$ or $m(\pi^2 \pi^3)$ coordinate can be used to illustrate the $D^0 \rightarrow (a_1 \rightarrow \rho^0 \pi) \pi$ decay. Thus the order of $m(\pi^1 \pi^4)$, $m(\pi^2 \pi^3)$, $m(\pi^1 \pi^2 \pi^3)$, $m(\pi^1 \pi^2)$, $m(\pi^3 \pi^4)$ is chosen. An example of simulated $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ decays with the coordinates choice described above is shown in Fig. 11.20. Examples of parallel coordinate plots of simulation samples with only $D^0 \rightarrow (a_1 \rightarrow \rho^0 \pi) \pi$ (S-wave) and $D^0 \rightarrow \rho^0 \rho^0$ (D-wave) decays are shown in Fig. 11.21 and Fig. 11.22.

Table 11.11: T values and p -values for $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ decay simulation samples with phase and amplitude asymmetries in $D^0 \rightarrow \rho\rho$ ($P - wave$). Results from both P -even (default) and P -odd tests are listed. Yellow regions are scenarios where some level of sensitivity can be anticipated.

asymmetries in $D^0 \rightarrow \rho^0 \rho^0$ (P-wave)	P -even tests		P -odd tests	
	T_0 value	p -value (fit)	T_0 value	p -value (fit)
$\rho\rho(P)$ Δ phase 2° , Δ Amp 0	-3.91×10^{-7}	$0.73^{+0.02}_{-0.02}$	7.54×10^{-7}	$0.11^{+0.02}_{-0.02}$
$\rho\rho(P)$ Δ phase 4° , Δ Amp 0	1.91×10^{-7}	$0.30^{+0.03}_{-0.03}$	6.04×10^{-6}	$1.95^{+0.06}_{-1.95} \times 10^{-4}$
$\rho\rho(P)$ Δ phase 10° , Δ Amp 0	-3.85×10^{-7}	$0.73^{+0.02}_{-0.03}$	4.32×10^{-5}	$< 10^{-10}$
$\rho\rho(P)$ Δ phase 0° , Δ Amp 2	7.25×10^{-7}	$0.12^{+0.02}_{-0.02}$	-1.93×10^{-7}	$0.57^{+0.03}_{-0.03}$
$\rho\rho(P)$ Δ phase 0° , Δ Amp 4	2.31×10^{-6}	$3.02^{+1.2}_{-0.9} \times 10^{-3}$	2.13×10^{-7}	$0.41^{+0.03}_{-0.03}$
$\rho\rho(P)$ Δ phase 0° , Δ Amp 10	2.24×10^{-5}	$< 10^{-10}$	4.01×10^{-7}	$0.20^{+0.03}_{-0.03}$

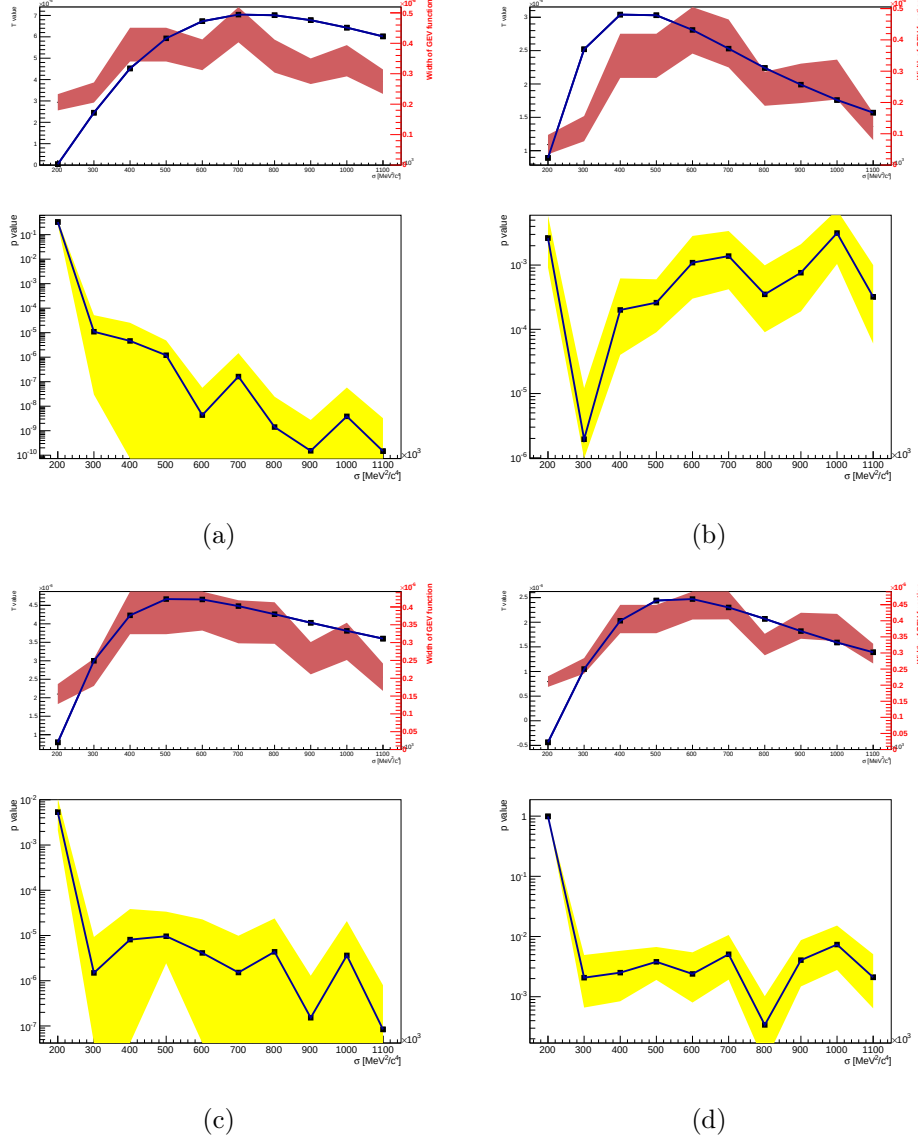


Figure 11.19: T value, p -value, and width of GEV function versus metric parameter σ for toy simulation $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ samples with (a) 3° phase asymmetry and (b) 5% amplitude asymmetry on $a_1^+ \rightarrow \rho^0 \pi^+$, (c) 4° phase asymmetry and (d) 5% amplitude asymmetry on $D^0 \rightarrow \rho \rho$. Yellow bands are the uncertainties of the p -values. T value has a maximum value for each CP asymmetry scenario, but the metric parameter σ that gives the maximum value varies.

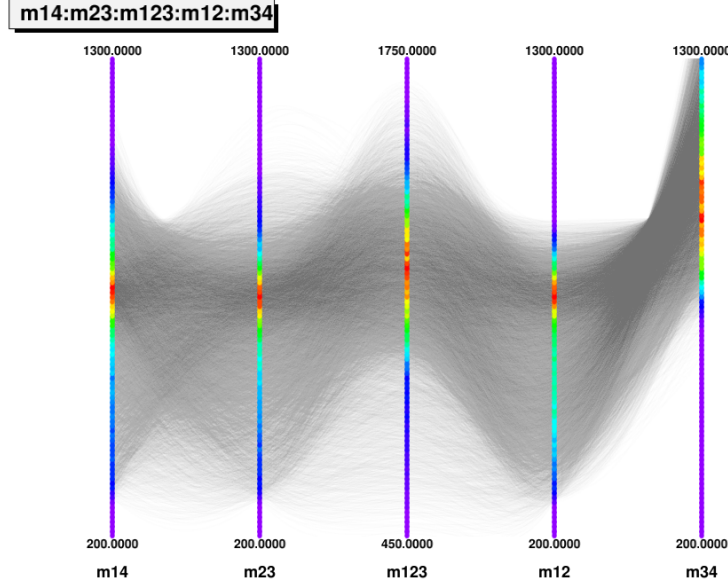


Figure 11.20: An example of a parallel coordinate display of simulated $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ decays. Each decay event is denoted with a curve connecting five axes. On these plots, the left two and the right two axes have neutral charge, the middle axis have positive charge. The left two axes are coordinates that could be a $\rho\rho$ pair, and the right two axes could be the other combination of a $\rho\rho$ pair. The middle axes are positive three body combination coordinates, which could be an a_1 resonance with the possibility to a rho that would appear on either the left or the right axis next to it. All ρ or a_1 events are lying in the middle region of their corresponding axes. Curves connecting the middle part of the left two axes or the right two axes indicate a $D^0 \rightarrow \rho\rho$ decay. Since the order of pions have been sorted, small amount of ρ s on $m(\pi^3\pi^4)$, *i.e.* the 5th axis on the plot is expected. Therefore most of $D^0 \rightarrow \rho\rho$ are seen on the left two axes. Curves connecting the middle region of the 3rd axes and the middle region of either 2nd or 4th axes indicate a $a_1 \rightarrow \rho\pi$ decay.

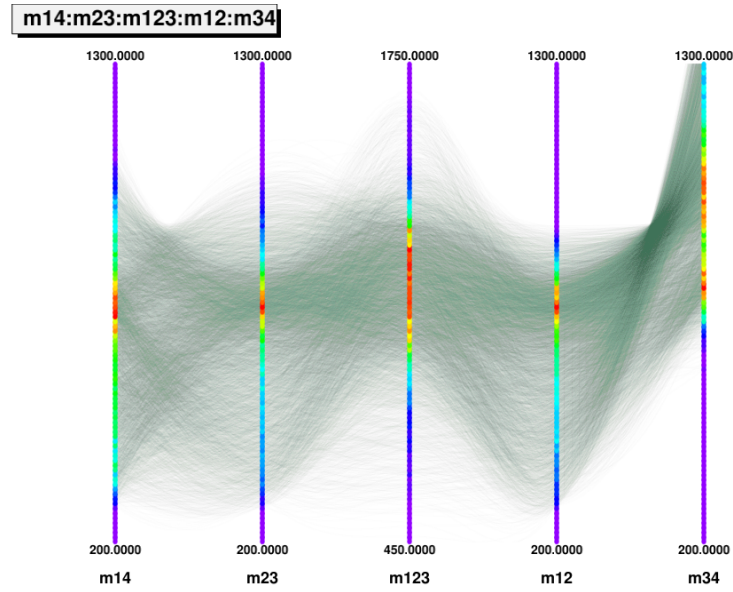


Figure 11.21: Parallel coordinate plots of simulated $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ samples with only the $D^0 \rightarrow (a_1^+ \rightarrow \rho^0 \pi^+) \pi^-$ (S-wave) decay. The a_1^+ resonance is shown in the middle region of the third axis (m_{123}), and there should be lines that connecting a_1^+ to the middle region of either the second (m_{23}) or the fourth (m_{12}) axis, indicating the $a_1^+ \rightarrow \rho^0 \pi^+$ decay, the a_1^+ is made from the combination of $\pi^1 \pi^2 \pi^3$, and the ρ^0 could be either the combination of $\pi^2 \pi^3$ or $\pi^1 \pi^2$. There should not be lines connecting the middle regions of the first (m_{14}) and the second (m_{23}) axes, as there will be no $\rho\rho$ component in this sample.

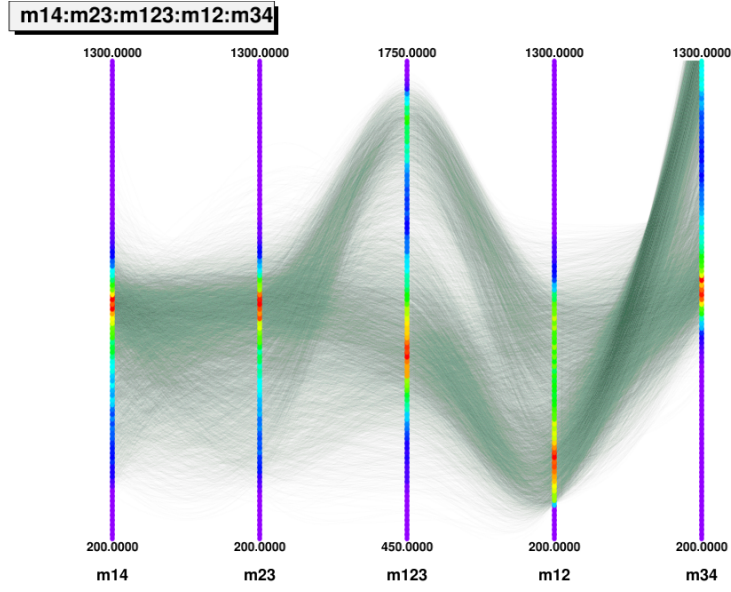


Figure 11.22: Parallel coordinate plots of simulated $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ with only the $D^0 \rightarrow \rho^0 \rho^0$ (D-wave) decay. The $\rho\rho$ resonances are made from two $\pi^+ \pi^-$ combinations, i.e., $\pi^1 \pi^4 + \pi^2 \pi^3$, or $\pi^1 \pi^2 + \pi^3 \pi^4$. Since $\pi^3 \pi^4$ is defined to be the $\pi^+ \pi^-$ pair that gives the largest invariant mass, and the fraction of $m(\pi^3 \pi^4)$ in the region of $\rho(770)$ mass is very small, the majority of the $\rho\rho$ resonances will be $\pi^1 \pi^4 + \pi^2 \pi^3$. Therefore, there should be horizontal lines connect the middle regions of the first (m_{14}) and the second (m_{23}) axes, which are clearly visible. There should not be lines that connecting the middle region of the third axis to either middle regions of either the second or the fourth axes, as there will be no $a_1^+ \rightarrow \rho^0 \pi^+$ component in this sample.

The second option is to use only 1 D or 2 D projections. 1D projections onto two-body and three-body invariant masses are shown in Fig. 11.23. In this figure both P -even and P -odd CP violation tests are visualised. The CP violation is introduced as a phase difference in either $a_1(1260)^+$ or P-wave $\rho^0(770)$ amplitudes. The shapes of the regions with significant asymmetry that are visible in the one-dimensional projections in Fig. 11.23 cannot be easily interpreted in terms of the amplitudes and phase differences of the contributing resonances; the two-dimensional spectra are not easy to understand either. However, repeating this exercise for different scenarios of CP violation, the observed features are found to be sufficiently distinguishable to help identify the origin of any CP asymmetry that is seen in the data.

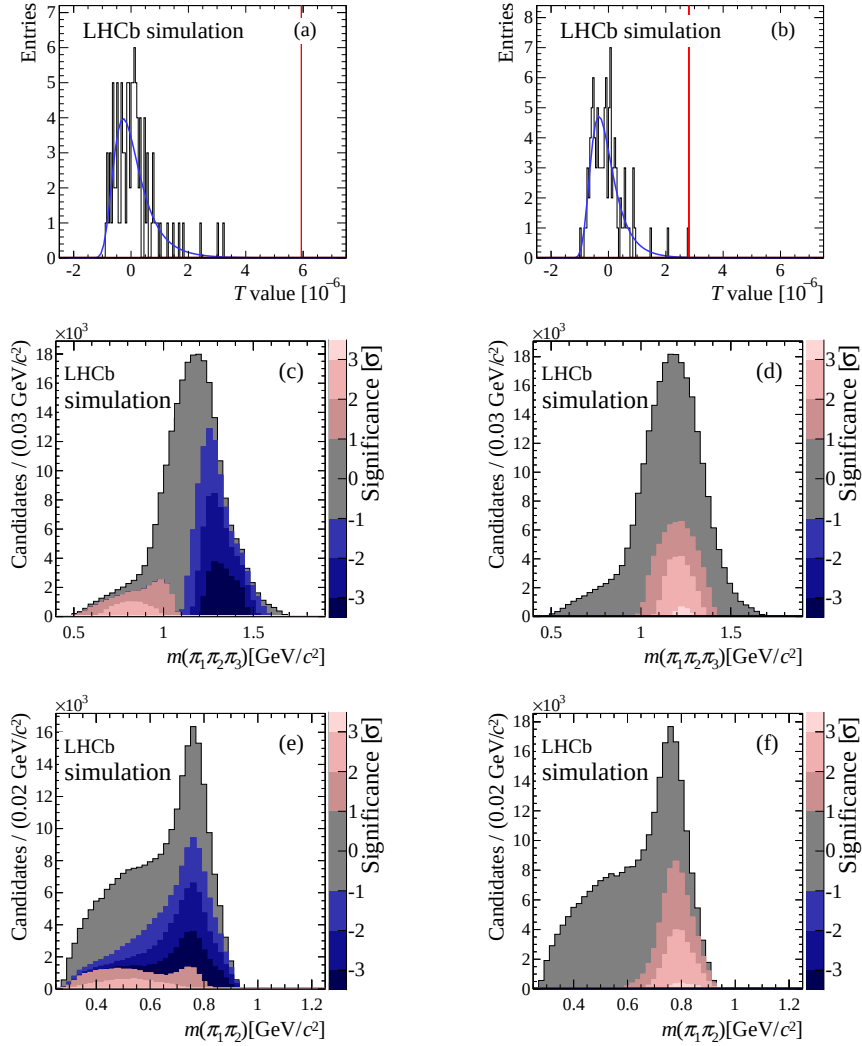


Figure 11.23: (a,b) Distribution of permutation T -values fitted with a GEV function for the simulated $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ sample and showing the measured T -value as a vertical line, and (c,d,e,f) local asymmetry significances. Left column plots are for a P -even CP violation test with a 3° phase CP violation introduced in the $a_1(1260)^+$ resonance (see text), projected onto the (c) $m(\pi_1\pi_2\pi_3)$ and (e) $m(\pi_1\pi_2)$ axes. Right column plots are for a P -odd CP violation test with 3° phase CP violation introduced in the P-wave $\rho^0(770)\rho^0(770)$ resonance projected onto the same axes. In plots (c,d,e,f) the grey area corresponds to candidates with a contribution to the T -value of less than one standard deviation. The pink (blue) area corresponds to candidates with a positive (negative) contribution to the T -value. Light, medium or dark shades of pink and blue correspond to between one and two, two and three, and more than three standard-deviation contributions, respectively.

11.4 Systematic uncertainties

In general, there are several sources of asymmetries that can affect the measurement presented here. As the method is only sensitive to local asymmetries in phase space, rather than global asymmetries, it is sufficient to concentrate on these local effects.

Detection asymmetries of the daughter particles can introduce local asymmetries if they vary with the particle kinematics. The impact of this can be estimated through the analysis of the decay mode $D^0 \rightarrow K^- \pi^+ \pi^- \pi^+$, which is expected to have much larger asymmetries as the kaon detection asymmetry is not cancelled and therefore contribute at first order. This is discussed in detail in Sec. 11.4.2.

Production asymmetries are known to vary with the particle's kinematics. The impact of this can be tested with a control mode as discussed in Sec. 11.4.2.

11.4.1 Background asymmetries

Although the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ sample in the (Δm) -(D mass) signal peak region is very clean, there still can be some asymmetry in the contributing background component which may affect the CP violation search. Any background asymmetries which are uniform across the phase space distribution have no impact in the energy test method, however, local asymmetries can bias the measurement. In principle weightings to account for the background contribution can be introduced into the energy test method (as discussed in Sect. 9.5), however, if there are no significant asymmetries in the background it can be simply neglected.

Within this section a study of the background asymmetry using the P -even method in the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ signal sample with the sideband background which is described in Sec. 11.2.4 is presented.

The background sample has a global asymmetry of $(1.34 \pm 0.14)\%$, calculated from the number of events in the sideband sample with their Poisson errors. The background from the sidebands reveals no local asymmetry in any of the ‘Dalitz-like’ variables (invariant mass squares, Fig. 11.24 and Fig. 11.25). The global asymmetry can receive contributions from the D^0 production asymmetry and the detection asymmetry of the slow pion.

The energy test was applied to all 17 sideband sub-samples, the distribution of T values for 50 permutations each is shown in Fig. 11.26. The T values and p -values for each sub-samples are listed in Table. 11.12. The p -value distributions are shown in Fig. 11.27, which shows good agreement with the expected flat distribution. Background events are random combination of tracks, therefore no difference between P -even and P -odd tests is expected. Hence, we can state that the background does not bias our measurement and can be safely ignored.

Table 11.12: Results from applying energy test for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ sideband sub-samples. A graphical representation of p -values is given in Fig. 11.27

Sample number	$\sigma = 0.5(\text{GeV}/c^2)^2$		$\sigma = 0.3(\text{GeV}/c^2)^2$		$\sigma = 0.7(\text{GeV}/c^2)^2$	
	T value	p -value	T value	p -value	T value	p -value
1	3.01×10^{-5}	$0.02^{+0.01}_{-0.01}$	1.48×10^{-5}	$0.08^{+0.03}_{-0.02}$	3.02×10^{-5}	$0.04^{+0.02}_{-0.01}$
2	-2.95×10^{-6}	$0.59^{+0.05}_{-0.06}$	-1.92×10^{-6}	$0.47^{+0.06}_{-0.06}$	-3.22×10^{-6}	$0.44^{+0.06}_{-0.06}$
3	-1.41×10^{-5}	$0.90^{+0.02}_{-0.03}$	-1.54×10^{-6}	$0.49^{+0.06}_{-0.06}$	-1.56×10^{-5}	$0.81^{+0.04}_{-0.04}$
4	1.01×10^{-5}	$0.25^{+0.05}_{-0.05}$	2.21×10^{-5}	$0.04^{+0.02}_{-0.01}$	2.98×10^{-6}	$0.34^{+0.06}_{-0.06}$
5	1.50×10^{-6}	$0.41^{+0.06}_{-0.06}$	-3.91×10^{-6}	$0.62^{+0.05}_{-0.06}$	7.51×10^{-6}	$0.19^{+0.05}_{-0.05}$
6	-1.26×10^{-5}	$0.86^{+0.03}_{-0.03}$	-9.54×10^{-6}	$0.80^{+0.04}_{-0.04}$	-1.10×10^{-5}	$0.66^{+0.05}_{-0.05}$
7	-7.82×10^{-6}	$0.73^{+0.04}_{-0.05}$	-2.86×10^{-7}	$0.52^{+0.06}_{-0.06}$	$-1/00 \times 10^{-5}$	$0.70^{+0.05}_{-0.05}$
8	3.21×10^{-7}	$0.40^{+0.06}_{-0.06}$	2.30×10^{-6}	$0.33^{+0.06}_{-0.06}$	-2.88×10^{-6}	$0.47^{+0.06}_{-0.06}$
9	-2.24×10^{-5}	$0.96^{+0.01}_{-0.01}$	-1.27×10^{-5}	$0.91^{+0.02}_{-0.03}$	-2.32×10^{-5}	$0.96^{+0.01}_{-0.02}$
10	-1.28×10^{-5}	$0.81^{+0.03}_{-0.04}$	-6.71×10^{-6}	$0.69^{+0.04}_{-0.05}$	-1.23×10^{-5}	$0.87^{+0.03}_{-0.04}$
11	9.00×10^{-6}	$0.12^{+0.04}_{-0.03}$	1.00×10^{-5}	$0.22^{+0.05}_{-0.05}$	1.24×10^{-5}	$0.23^{+0.05}_{-0.05}$
12	4.31×10^{-5}	$0.02^{+0.01}_{-0.01}$	1.94×10^{-5}	$0.04^{+0.02}_{-0.01}$	5.68×10^{-5}	$0.01^{+0.01}_{-0.01}$
13	-1.10×10^{-6}	$0.42^{+0.06}_{-0.06}$	2.66×10^{-6}	$0.29^{+0.06}_{-0.06}$	-3.22×10^{-6}	$0.47^{+0.06}_{-0.06}$
14	-3.27×10^{-6}	$0.59^{+0.05}_{-0.06}$	8.33×10^{-6}	$0.23^{+0.05}_{-0.05}$	-7.85×10^{-6}	$0.70^{+0.05}_{-0.05}$
15	2.24×10^{-5}	$0.11^{+0.04}_{-0.03}$	1.03×10^{-5}	$0.16^{+0.05}_{-0.04}$	2.80×10^{-5}	$0.08^{+0.03}_{-0.03}$
16	-2.09×10^{-6}	$0.50^{+0.06}_{-0.06}$	-7.15×10^{-6}	$0.82^{+0.04}_{-0.05}$	5.18×10^{-6}	$0.39^{+0.06}_{-0.06}$
17	3.85×10^{-6}	$0.39^{+0.06}_{-0.06}$	1.33×10^{-5}	$0.12^{+0.03}_{-0.03}$	-2.52×10^{-6}	$0.41^{+0.06}_{-0.06}$

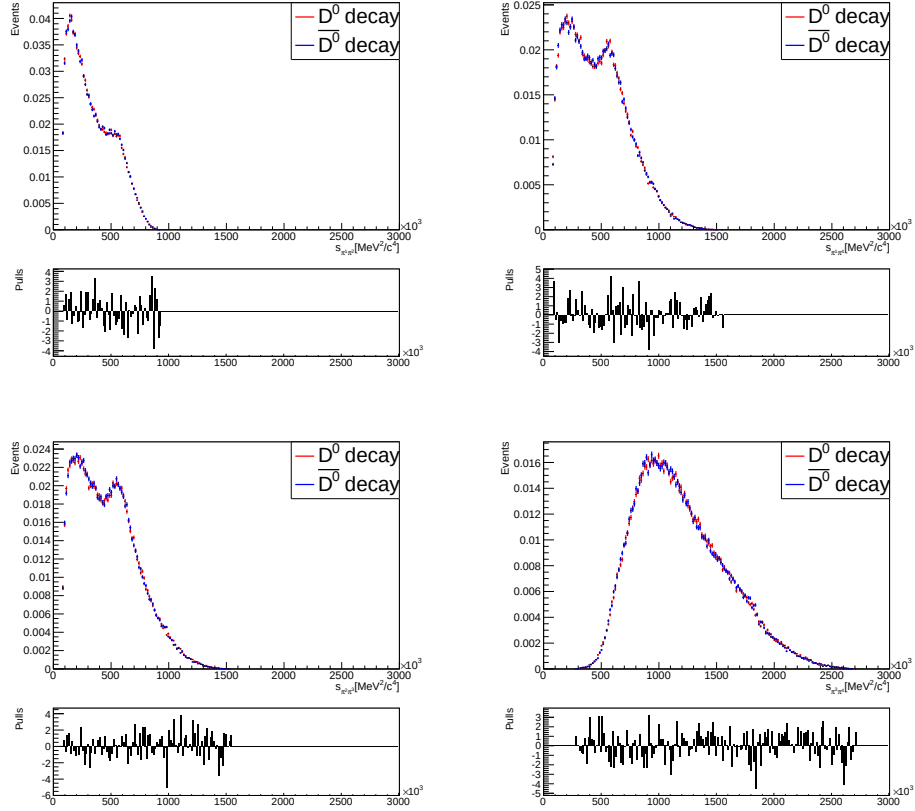


Figure 11.24: Comparison of background from Δm - D mass sideband for D^0 (red) and $\bar{D}^0$ (blue) for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ analysis. From left to right, then from top to bottom, the projections show $s(\pi^1 \pi^2)$, $s(\pi^1 \pi^4)$, $s(\pi^2 \pi^3)$, and $s(\pi^3 \pi^4)$. D^0 and $\bar{D}^0$ distributions are normalized to same number of events.

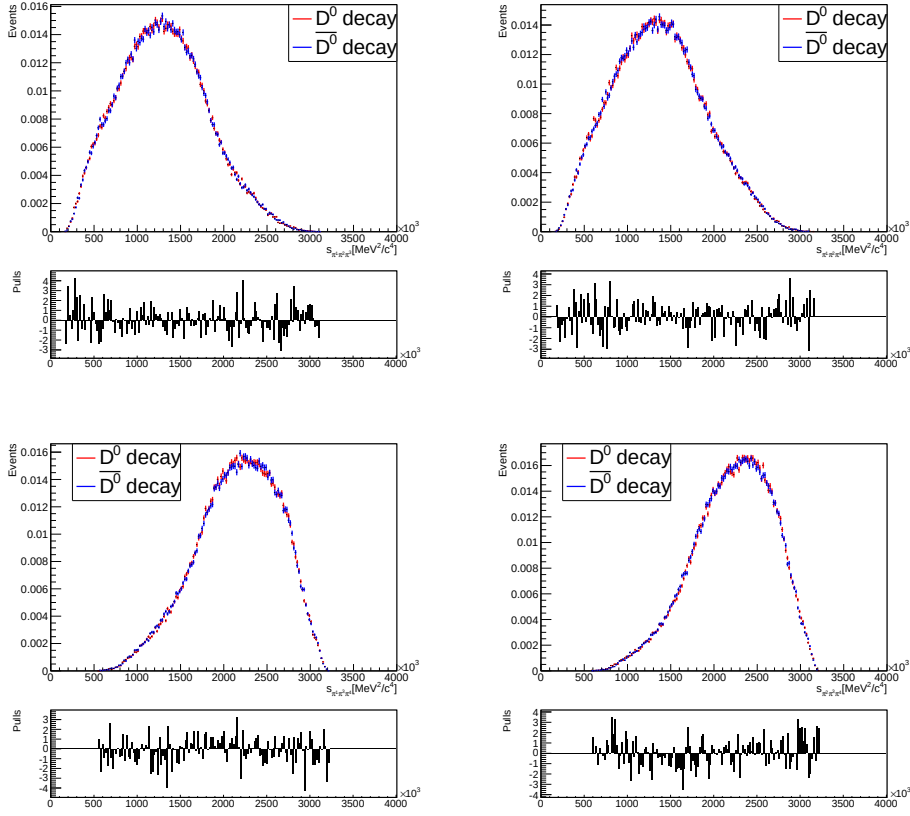


Figure 11.25: Comparison of background from Δm - D mass sideband for D^0 (red) and $\bar{D}^0$ (blue) for the $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ analysis. From left to right, then from top to bottom, the projections show $s(\pi^1\pi^2\pi^3)$, $s(\pi^1\pi^2\pi^4)$, $s(\pi^1\pi^3\pi^4)$, and $s(\pi^2\pi^3\pi^4)$. D^0 and $\bar{D}^0$ distributions are normalized to same number of events.

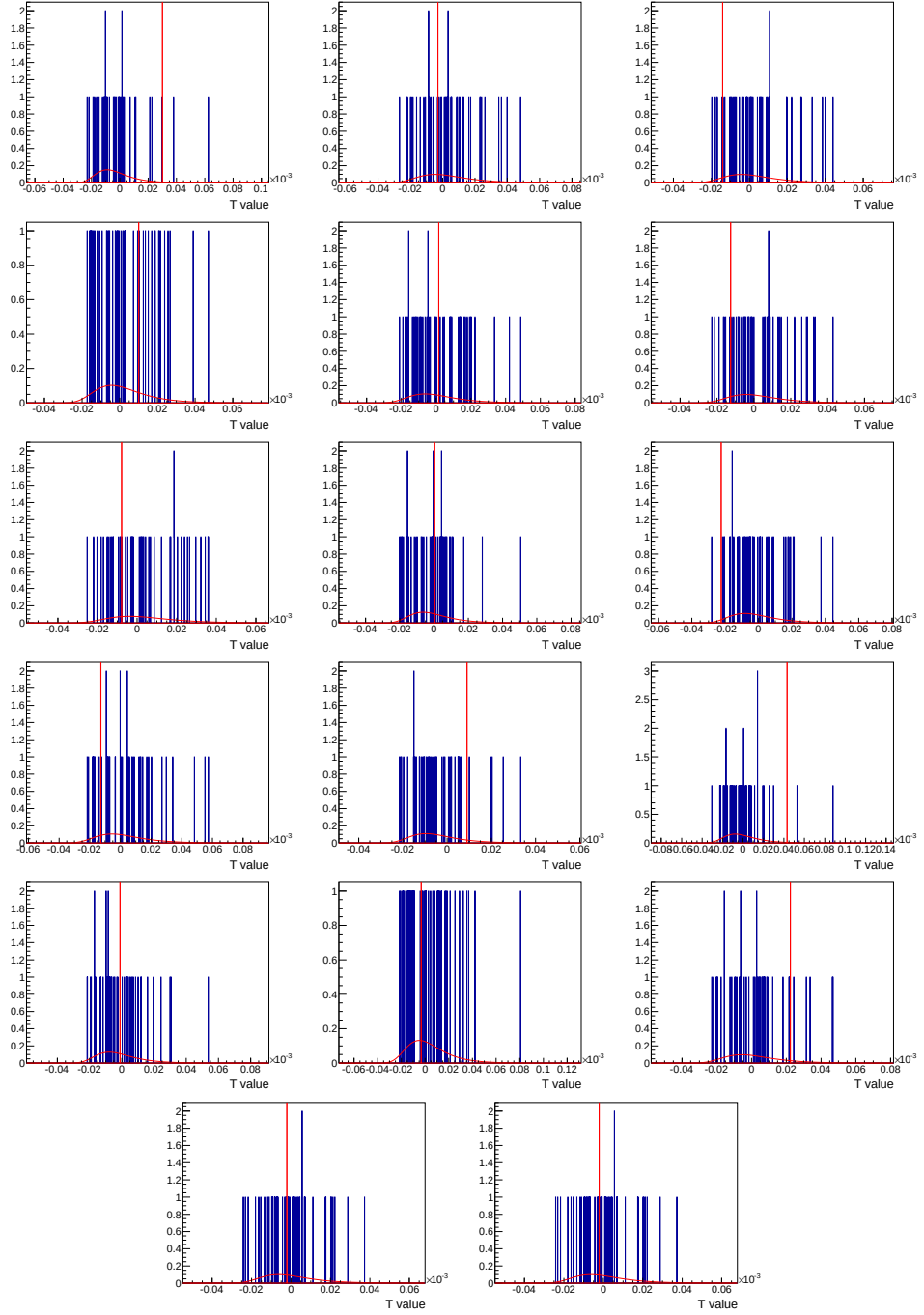


Figure 11.26: Results from applying energy test for the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ sideband sub-samples. Distribution of T values are obtained for 50 permutations each, the metric parameter used in these tests is $\sigma = 0.5(\text{GeV}/c^2)^2$. Measured T values are listed in Table. 11.12.

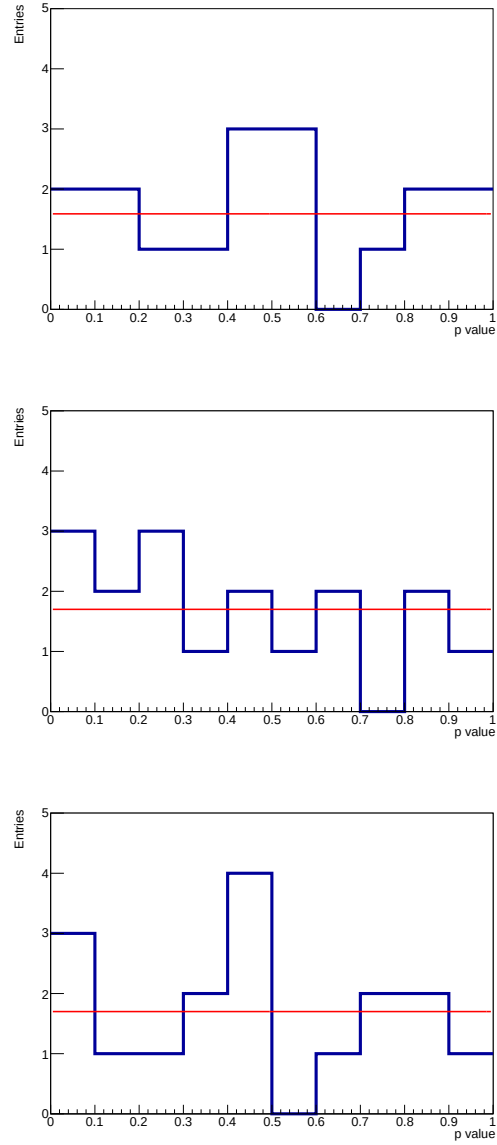


Figure 11.27: p -value distributions for $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ sideband background sub-samples that listed in Table. 11.12. From top to bottom the metric parameter σ s are $0.5 (\text{GeV}/c^2)^2$, $0.3 (\text{GeV}/c^2)^2$, and $0.7 (\text{GeV}/c^2)^2$. The line indicate the expected distribution.

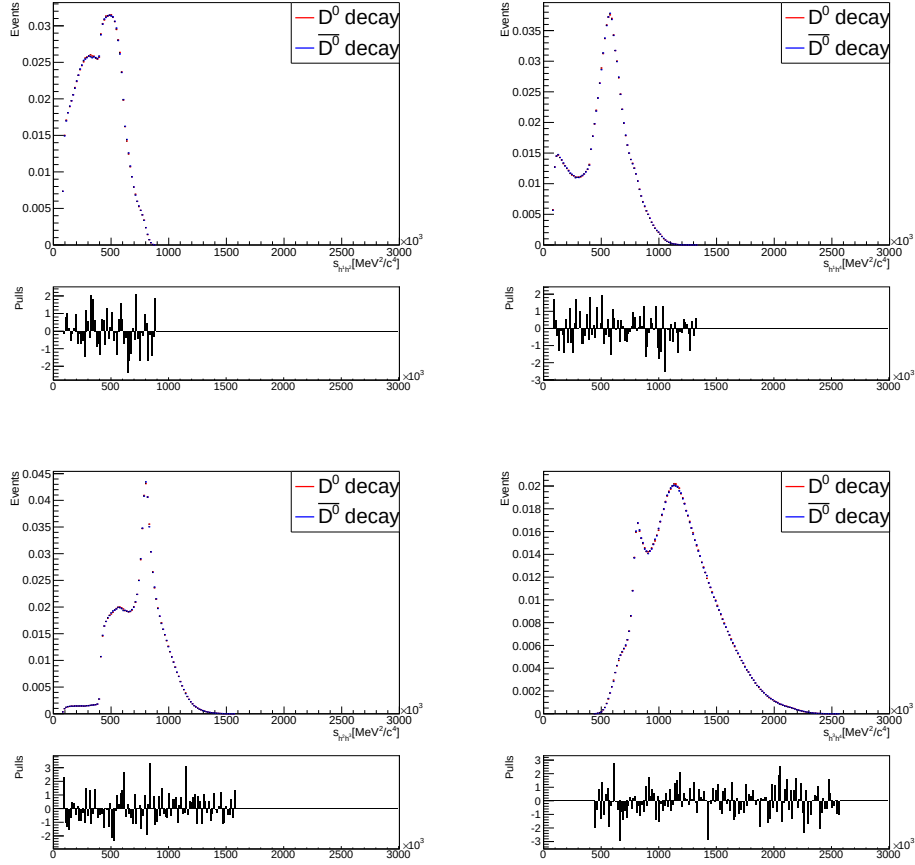


Figure 11.28: Comparison of reference sample from Δm - D mass for D^0 (red) and $\bar{D}^0$ (blue). From left to right, then from top to bottom, the projections show $s(\pi^1\pi^2)$, $s(\pi^1\pi^4)$, $s(\pi^2\pi^3)$, and $s(\pi^3\pi^4)$. D^0 and $\bar{D}^0$ distributions are normalized to same number of events.

11.4.2 Cross-check using a control mode

The P -even analysis method has been applied to samples of $D \rightarrow K^- \pi^+ \pi^- \pi^+$ with a size approximately the same as the final sample of $D \rightarrow \pi^- \pi^+ \pi^- \pi^+$ data. In comparing to the signal decay into $\pi^+ \pi^- \pi^- \pi^+$, this control mode involves a $K^\mp \pi^\pm$ pair, which introduces a first order global asymmetry. Figure 11.28 and 11.29 shows the normalized distribution of ‘Dalitz-like’ variables for selected signals, and no indication of asymmetries as function of these variables is seen.

The energy test was applied to 10 reference channel sub-samples. The T values and p -values for each of the sub-samples are listed in Table. 11.13. The p -value distributions are shown in Fig. 11.30, T values from reference

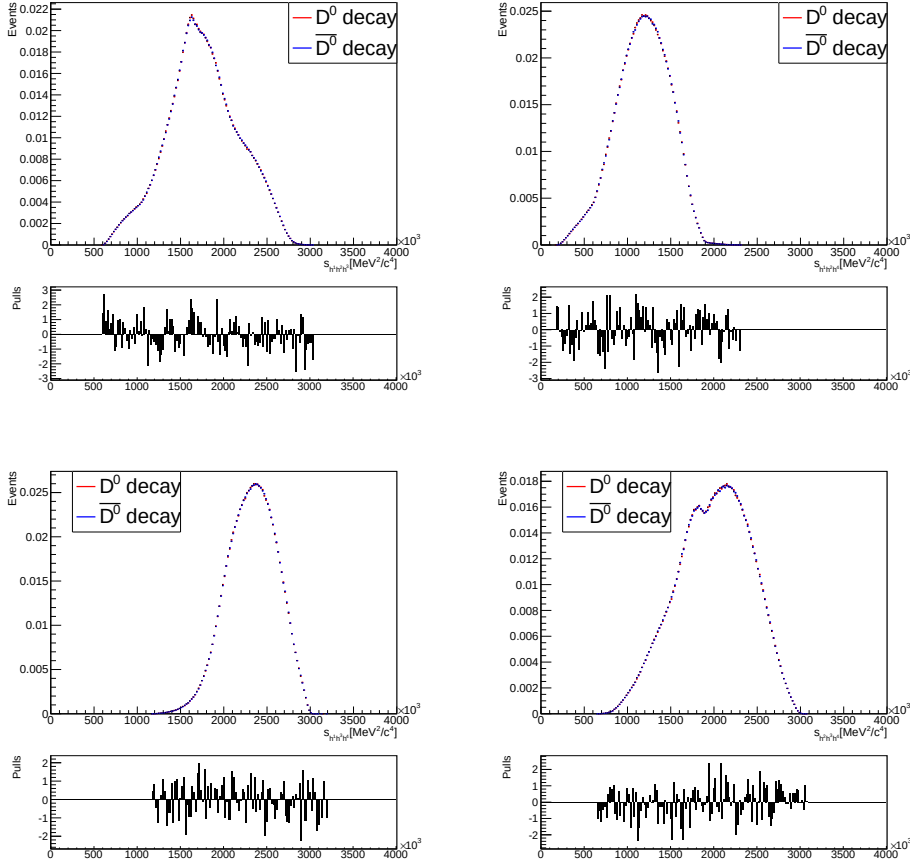


Figure 11.29: Comparison of reference sample from Δm - D mass for D^0 (red) and $\bar{D}^0$ (blue). From left to right, then from top to bottom, the projections show $s(\pi^1\pi^2\pi^3)$, $s(\pi^1\pi^2\pi^4)$, $s(\pi^1\pi^3\pi^4)$, and $s(\pi^2\pi^3\pi^4)$. D^0 and $\bar{D}^0$ distributions are normalized to same number of events.

sub-samples as well as their permutation T values are shown in Fig. 11.31, which shows good agreement with the expected flat distribution in the absence of global asymmetries. Since the P -odd test will mix the flavours and hence dilute the detection and production asymmetries, the study of the reference sample with the P -even test can represent the worst case of asymmetries that are not related to CP violation. Therefore it can be stated that the detection and production asymmetries do not bias our measurement and can be safely ignored.

Table 11.13: Results from applying energy test for the reference channel sub-samples A graphical representation of p -values is given in Fig. 11.30.

Sample number	$\sigma = 0.5(\text{GeV}/c^2)^2$		$\sigma = 0.3(\text{GeV}/c^2)^2$		$\sigma = 0.7(\text{GeV}/c^2)^2$	
	T value	p -value	T value	p -value	T value	p -value
1	-5.26×10^{-7}	$0.87^{+0.02}_{-0.03}$	-4.08×10^{-7}	$0.72^{+0.04}_{-0.05}$	-3.45×10^{-7}	$0.84^{+0.03}_{-0.04}$
2	-2.93×10^{-7}	$0.68^{+0.04}_{-0.03}$	2.30×10^{-8}	$0.32^{+0.06}_{-0.06}$	-3.57×10^{-7}	$0.76^{+0.04}_{-0.05}$
3	-4.39×10^{-8}	$0.44^{+0.05}_{-0.07}$	2.85×10^{-7}	$0.16^{+0.05}_{-0.04}$	-2.06×10^{-7}	$0.69^{+0.05}_{-0.05}$
4	1.93×10^{-7}	$0.26^{+0.05}_{-0.10}$	-1.14×10^{-7}	$0.46^{+0.06}_{-0.06}$	1.75×10^{-7}	$0.27^{+0.06}_{-0.06}$
5	-3.24×10^{-7}	$0.71^{+0.03}_{-0.04}$	2.79×10^{-8}	$0.43^{+0.06}_{-0.06}$	-3.39×10^{-7}	$0.76^{+0.04}_{-0.05}$
6	7.18×10^{-7}	$0.06^{+0.03}_{-0.06}$	7.88×10^{-7}	$0.06^{+0.03}_{-0.02}$	5.12×10^{-7}	$0.13^{+0.06}_{-0.06}$
7	6.72×10^{-7}	$0.07^{+0.03}_{-0.07}$	3.20×10^{-7}	$0.24^{+0.06}_{-0.06}$	6.45×10^{-7}	$0.09^{+0.05}_{-0.04}$
8	-2.09×10^{-7}	$0.61^{+0.04}_{-0.05}$	-3.55×10^{-7}	$0.80^{+0.03}_{-0.04}$	-2.94×10^{-8}	$0.34^{+0.09}_{-0.10}$
9	9.88×10^{-7}	$0.03^{+0.02}_{-0.03}$	5.48×10^{-7}	$0.09^{+0.03}_{-0.03}$	7.83×10^{-7}	$0.08^{+0.03}_{-0.03}$
10	-4.04×10^{-7}	$0.79^{+0.03}_{-0.03}$	-5.88×10^{-7}	$0.96^{+0.02}_{-0.03}$	-1.47×-7	$0.38^{+0.07}_{-0.09}$

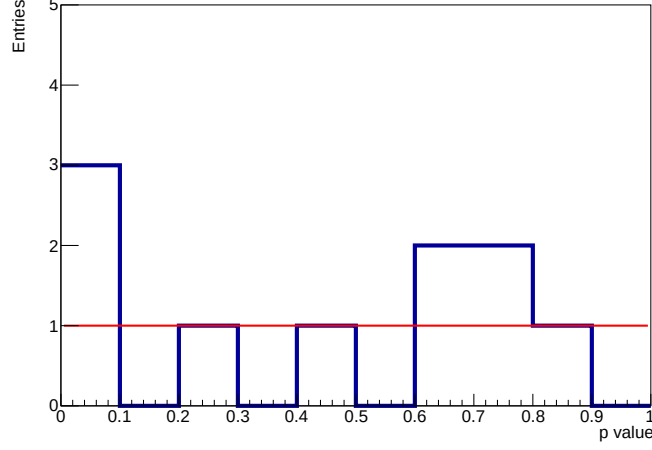


Figure 11.30: p -value distributions for reference channel sub-samples that listed in Table. 11.13. The metric parameter used in these tests is $\sigma = 0.5(\text{GeV}/c^2)^2$. The line indicate the expected distribution.

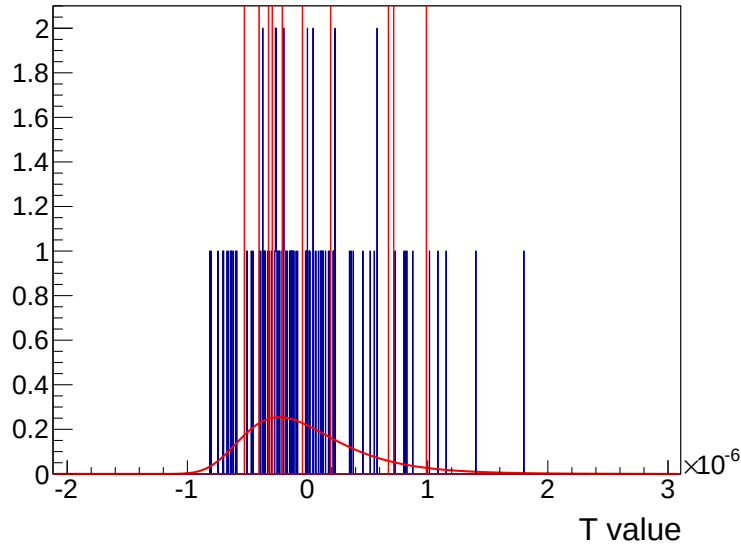


Figure 11.31: T value distributions for reference channel sub-samples that listed in Table. 11.13 (red lines), and permutation T value distributions generated from reference sample (blue lines). The metric parameter used in these tests is $\sigma = 0.5(\text{GeV}/c^2)^2$. The distribution of permutation T values are fitted with a GEV function (red curve), p -values are calculated from this GEV function.

11.5 Results

The application of the energy test to all selected $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ candidates using an effective radius of $\delta = 0.5 \text{ GeV}^2/c^4$ yields $T = 1.10 \times 10^{-6}$ in the P -even CP violation test and $T = 2.11 \times 10^{-6}$ in the P -odd CP violation test. The permutation T value distributions are shown in Fig. 11.32 (a,b). By counting the fraction of permutations with a T value above the nominal T value in the data, a p -value of $(4.6 \pm 0.5)\%$ is obtained in the P -even CP violation test and $(0.6 \pm 0.2)\%$ in the P -odd CP violation test. The central value of the P -odd result would correspond to being at least 2.7 standard deviations from the mean of a normal distribution. The significance levels of the T_i values are shown in Fig. 11.32 (c,e) projected onto the $m(\pi_1 \pi_2 \pi_3)$ axis, and Fig. 11.32 (d,f) projected onto the $m(\pi_1 \pi_2)$ axis. In the P -even test, a small phase space region contains candidates with a local negative asymmetry exceeding 1σ significance. Furthermore, in the P -odd test, candidates with a local positive asymmetry exceeding 2σ significance are seen in a small phase-space region dominated by the ρ^0 resonance, which can be compared with the corresponding plots in Fig. 11.23. Varying the effective radius results in the p -values listed in Table 11.14. The central values of the P -odd results for $\delta = 0.3 \text{ GeV}^2/c^4$ correspond to more than three standard deviations from the mean of a normal distribution but the significance falls below this level when considering their uncertainties.

In summary, a search for time-integrated CP violation in the Cabibbo-suppressed decay $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ is performed using a novel unbinned model-independent technique. This is the first application of the energy test to four-body decays and extends the approach to allow P -odd CP violation searches. This analysis has the best sensitivity from a single experiment to P -even CP violation and is the first test for P -odd CP violation in this decay. The data are found to be marginally consistent with the hypothesis of CP symmetry at the current level of precision.

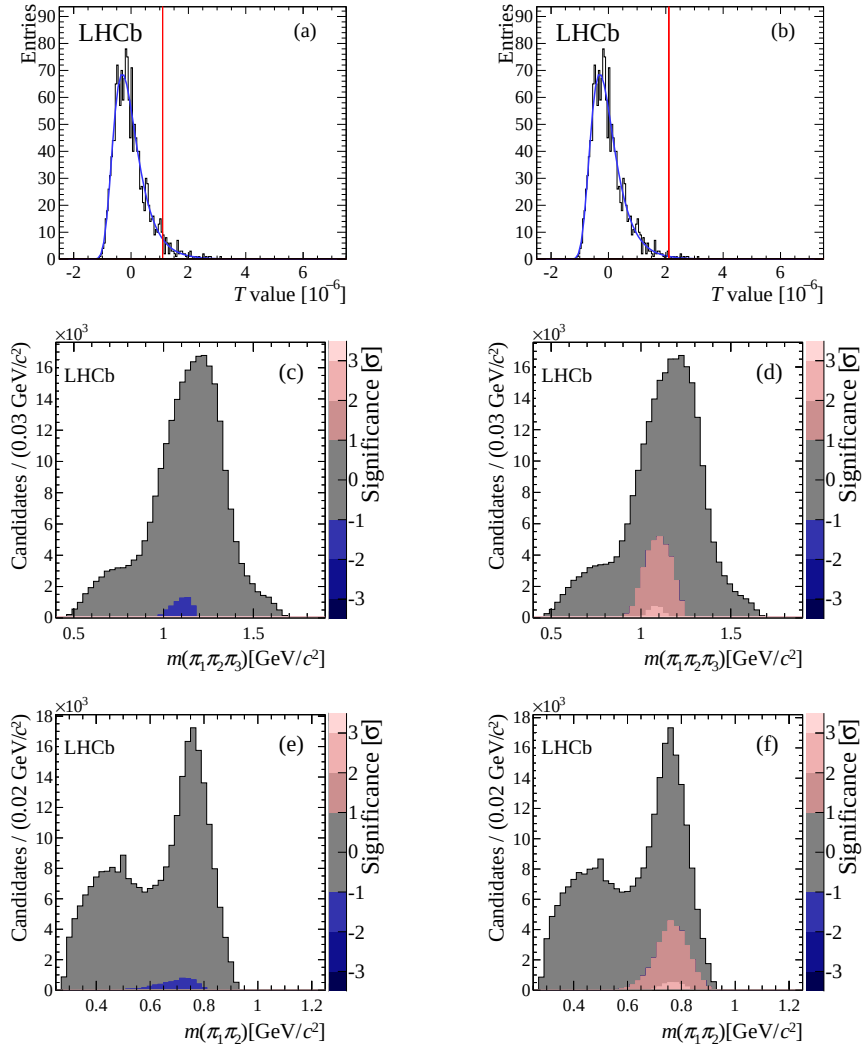


Figure 11.32: (a,b) Distribution of permutation T -values fitted with a GEV function and showing the T -value of the $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ data tests as a vertical line, and (c,d,e,f) local asymmetry significances. Left column plots are for the P -even CP violation test, projected onto the (c) $m(\pi_1 \pi_2 \pi_3)$ and (e) $m(\pi_1 \pi_2)$ axes. Right column plots are for the P -odd CP violation test projected onto the same axes. In plots (c,d,e,f) the grey area correspond to candidates with a contribution to the T -value of less than one standard deviation. In the P -even CP violation test the positive (negative) asymmetry significance is set for the D^0 candidates having positive (negative) contribution to the measured T value. In the P -odd CP violation test the positive (negative) asymmetry significance is set for sample $I + IV$ having positive (negative) contribution to the measured T value (see Sect. 9.7). The pink (blue) area corresponds to candidates with a positive (negative) contribution to the T -value. Light, medium or dark shades of pink and blue correspond to between one and two, two and three, and more than three standard deviation contributions, respectively.

Table 11.14: Results for the P -even and P -odd CP violation tests for three different values of the effective radius δ . The p -values obtained with both the counting and GEV fitting methods are given (see text). The counting method is the default method.

δ [GeV ² /c ⁴]	p -value P -even		p -value P -odd	
	Counting	GEV fit	Counting	GEV fit
0.3	(0.88 ± 0.26)%	(0.78 ± 0.10)%	(0.24 ± 0.14)%	(0.28 ± 0.04)%
0.5	(4.6 ± 0.5)%	(4.8 ± 0.3)%	(0.63 ± 0.20)%	(0.34 ± 0.05)%
0.7	(16 ± 2)%	(17 ± 2)%	(0.83 ± 0.48)%	(0.52 ± 0.16)%

12 Summary and outlook

This thesis presented studies of radiation damage effects in the LHCb VELO sensors, and the determination of the operational bias voltages for the VELO sensors. The LHCb VELO detector remains highly efficient at collecting data during the run-2 operation of the LHC. Tracks continue to be reconstructed with high efficiency and low fake rate, although at the sensor level the effects of radiation damage can be clearly seen. Continuous monitoring of the radiation damage effects for the VELO sensors is essential for data quality and operation of the VELO and LHCb. For the remainder of the period of the LHC run-2 operation, the operational bias voltages of the VELO sensors need to be carefully determined by the CCE scan method described in Chapter 8, and the change of the VELO cluster finding efficiencies needs to be measured to adjust simulations. Indeed the failure to apply suitable simulation adjustments for the effects presented in this thesis, recently led to the issuing of errata for several LHCb papers [105, 106, 107]. The projections to the end of run-2 operation presented at the end of Chapter 8, suggest that a bias voltage of approx. 400 V will be required to maintain the detector at full depletion, assuming the system is kept cold. As the system was designed to run up to 500 V this should be achievable. One further consideration for the final year of LHCb operation will be whether to consider any period of annealing for the VELO detector. As shown in Chapter 8, a short period at higher temperature would give rise to beneficial annealing. A dedicated study of the advantages and disadvantages of such a treatment is recommended.

Techniques to study direct CP violation were discussed in Chapter 9. This demonstrated that the energy test can often be significantly more powerful than the binned S_{CP} test that is most commonly used in the literature. A novel unbinned parity-odd model-independent CP violation test was introduced. It was shown that the energy test was computationally feasible for the current large data sample sizes obtained at LHCb through the use of GPUs. Recent work in the Manchester group has shown that further speed-up can be achieved in both GPUs and CPUs through careful optimization of the code, and that the use of multi-threaded processes in CPUs can also be beneficial,

The energy test technique was applied in searches for CP violation in multi-body decays in Chapter 10 and Chapter 11. The study in Chapter 10 of $D^0 \rightarrow \pi^- \pi^+ \pi^0$ was the first application of the energy test. Studies using

π^0 s were typically thought to be best performed in the lower background environment of the B-factories. However, a sample eight times that of Belle was selected using only the 2012 data. Samples using π^0 s that were identified as two separate photons and as one merged cluster in the calorimeter were used. Combining these samples allowed good coverage of the full Dalitz plot. This was the first CP violation search performed using π^0 s at LHCb. The result obtained was consistent with the hypothesis of CP symmetry with a p -value of $(2.6 \pm 0.5)\%$. This is perfectly compatible with CP conservation. However, it is recommended that the analysis be repeated with the much larger data set now available at LHCb. The data sample analysed was 2 fb^{-1} at $\sqrt{s} = 8 \text{ TeV}$. The data sample available now, using the full statistics collected to date (2012-2016) is many times bigger. The increase in centre of mass energy to 13 TeV at run-2 led to an approximate doubling of the production cross-section [105], and improvements have occurred to the trigger that further increase the sample. The integrated luminosity collected for 2012-2016 is 4.1 fb^{-1} .

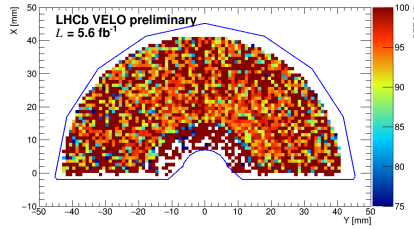
The study in Chapter 11 was the first application of the energy test in four-body decays and the first use of this with a parity-odd test. The study used the full run-1 data set. The results obtained were a p -value of $(4.6 \pm 0.5)\%$ in the parity-even case, and p -value of $(0.6 \pm 0.2)\%$ in the parity-odd case. The P -odd result, at nearly 3σ , is particularly intriguing. The group and I, are currently repeating the study using the full 2011-2016 data set, which is five times larger than that analysed in this thesis. Clearly this has the potential to be an important result for the field, and whether or not the hints seen in this thesis are confirmed it is of high importance that this study with the full data sample be performed.

The energy test technique can also be applied in many more decay channels at LHCb. A straightforward extension of the $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ analysis is to apply this to $D^0 \rightarrow K^- K^+ \pi^- \pi^+$. $D^0 \rightarrow K^- K^+ \pi^- \pi^+$ is the other singly-Cabibbo-suppressed four-body decay with only charged particles in the final state, and hence a natural target. This analysis is currently being pursued by myself and colleagues in the Manchester group. The techniques can also be applied in the B-sector. LHCb recently presented first evidence at the 3σ level of CP violation in baryons in the decay $\Lambda_b \rightarrow p \pi^- \pi^+ \pi^-$ [108], the additional power of the energy test over the binned technique used for this study may allow CP violation to be discovered in this channel. This is also being pursued in the Manchester group.

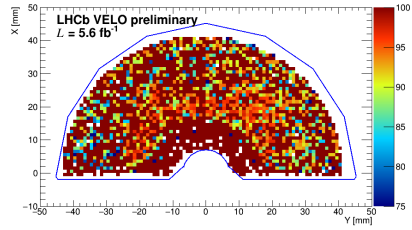
Appendices

A CFE as function of position on sensors 0 to 40 (R -sensors)

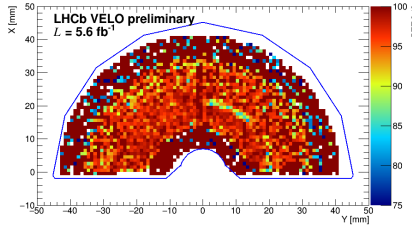
The following plots provide supplementary material to Chapter 8. These plots show the Cluster Finding Efficiency (CFE) for all VELO R -type sensors. the plots are made using the October 2016 Charge Collection Efficiency (CCE) scan data. The plots are made with the sensors at their operational bias voltage. The sensors number (S0-S40) is given below the figure.



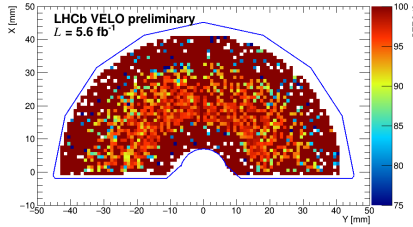
S0



S1

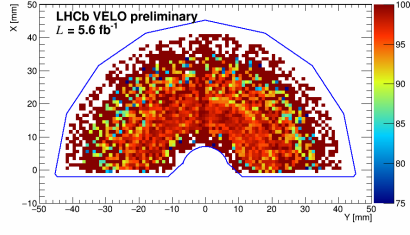


S2

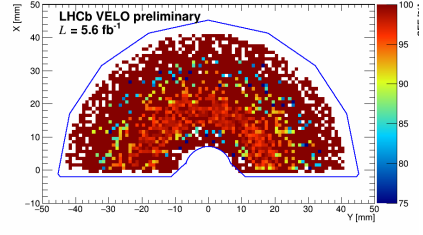


S3

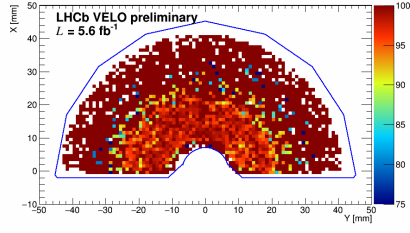
APPENDIX A. CFE AS FUNCTION OF POSITION ON SENSORS 0 TO
240 40 (R-SENSORS)



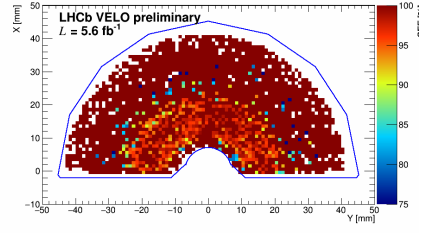
S4



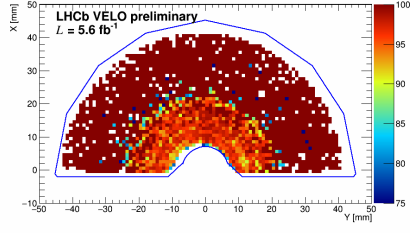
S5



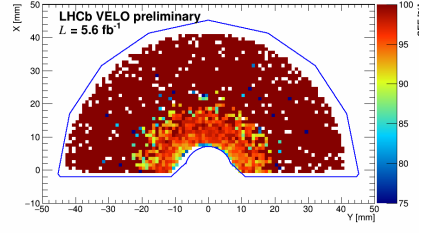
S6



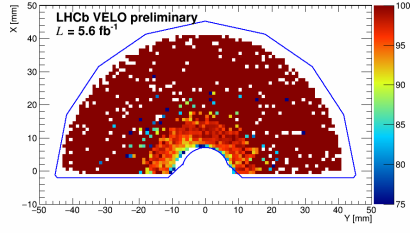
S7



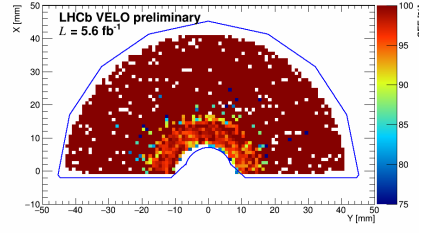
S8



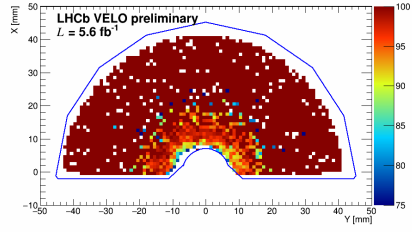
S9



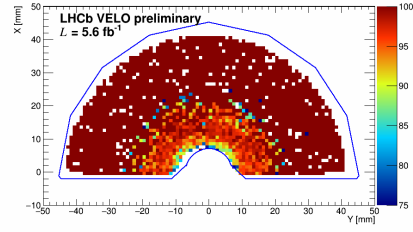
S10



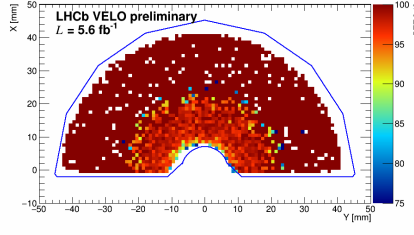
S11



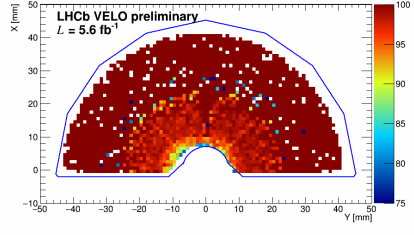
S12



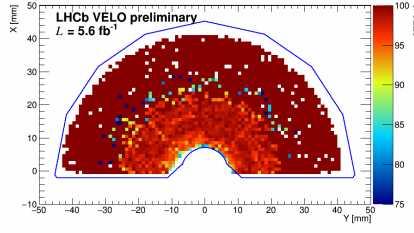
S13



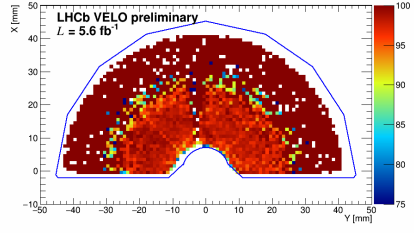
S14



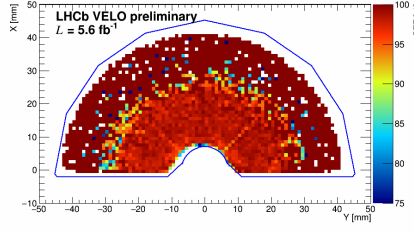
S15



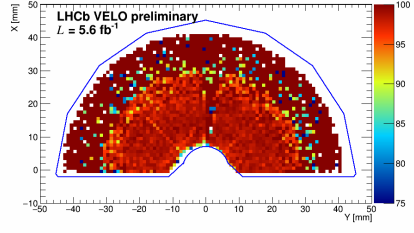
S16



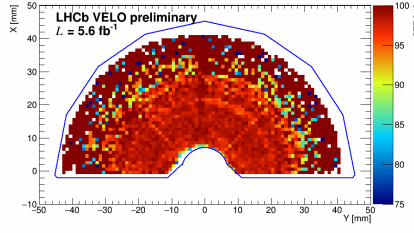
S17



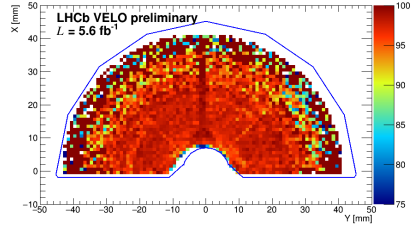
S18



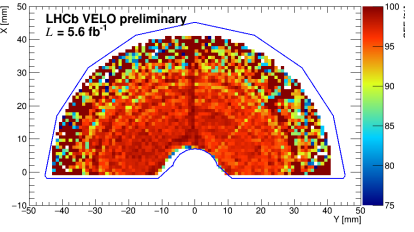
S19



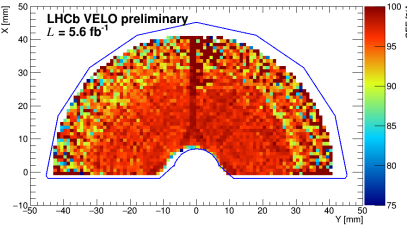
S20



S21

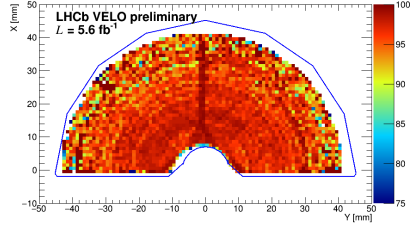


S22

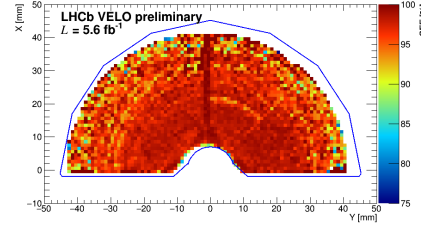


S23

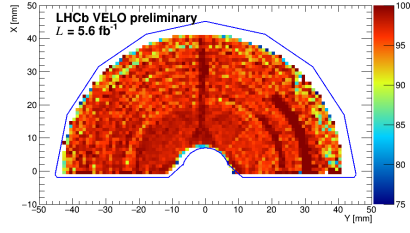
APPENDIX A. CFE AS FUNCTION OF POSITION ON SENSORS 0 TO
242 40 (R-SENSORS)



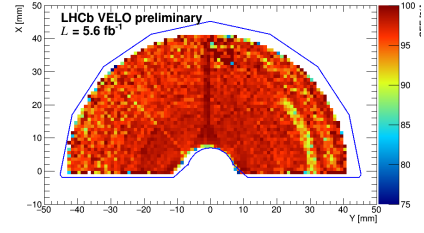
S24



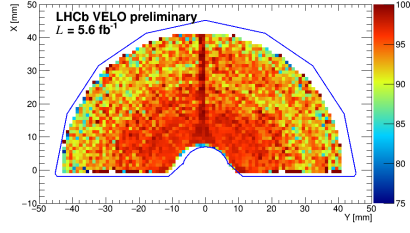
S25



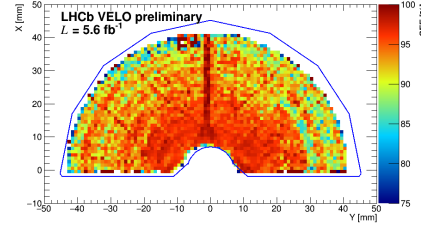
S26



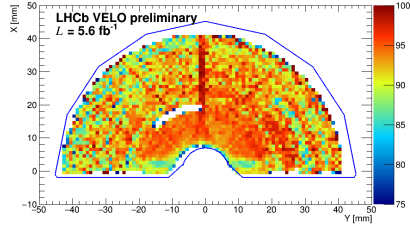
S27



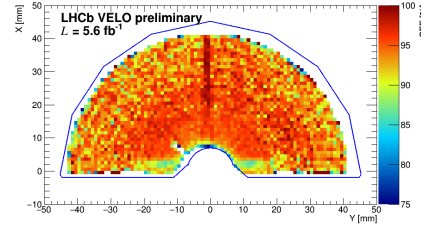
S28



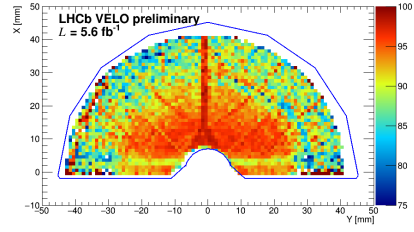
S29



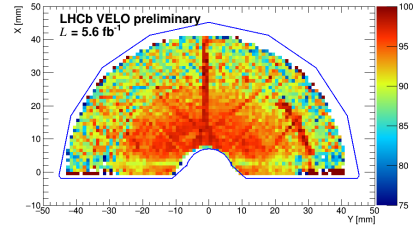
S30



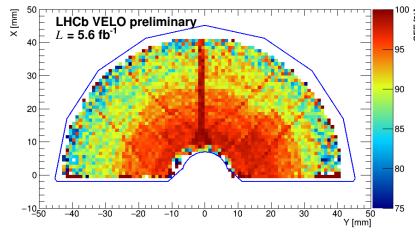
S31



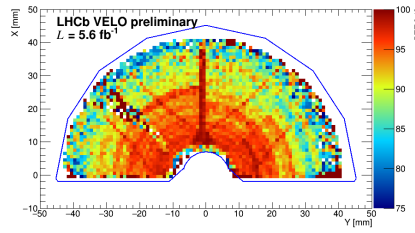
S32



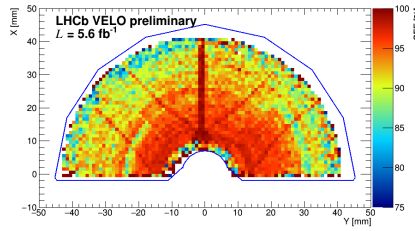
S33



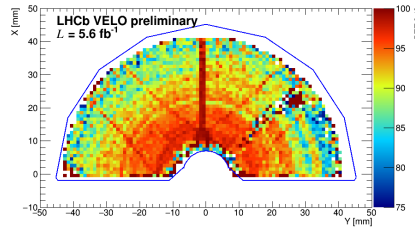
S34



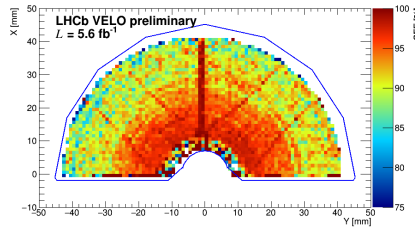
S35



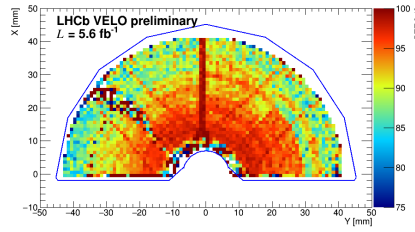
S36



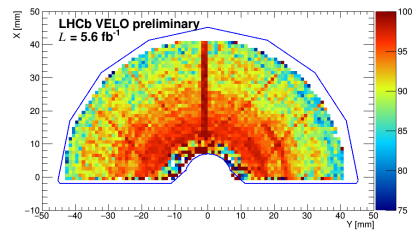
S37



S38



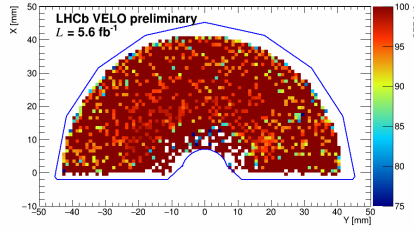
S39



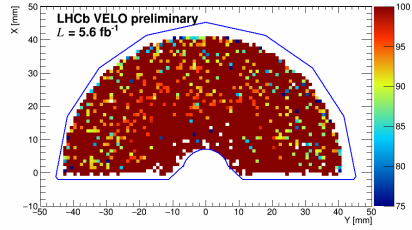
S40

B CFE as function of position on sensors 64 to 104 (Φ -sensors)

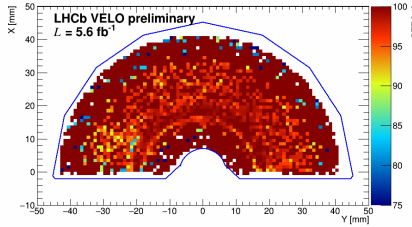
The following plots provide supplementary material to Chapter 8. These plots show the Cluster Finding Efficiency (CFE) for all VELO Φ -type sensors. the plots are made using the October 2016 Charge Collection Efficiency (CCE) scan data. The plots are made with the sensors at their operational bias voltage. The sensors number (S64-S104) is given below the figure.



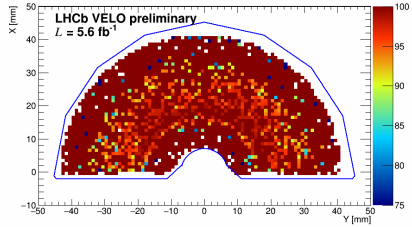
S64



S65

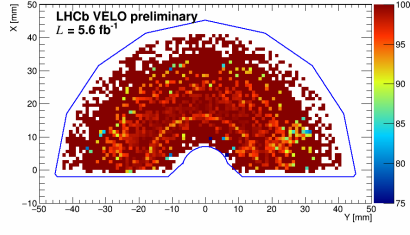


S66

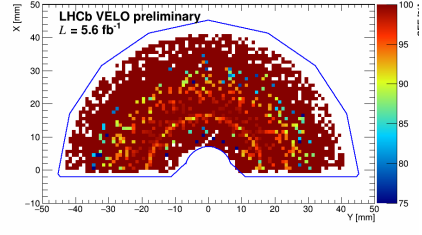


S67

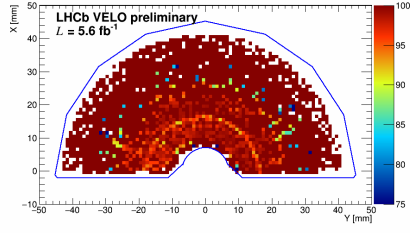
APPENDIX B. CFE AS FUNCTION OF POSITION ON SENSORS 64 TO
246 $(\Phi\text{-SENSORS})$



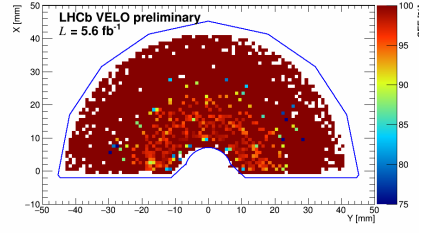
S68



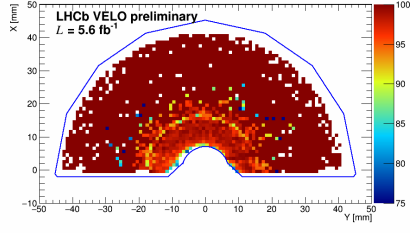
S69



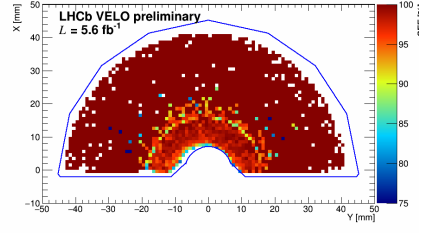
S70



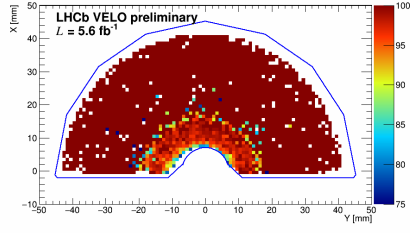
S71



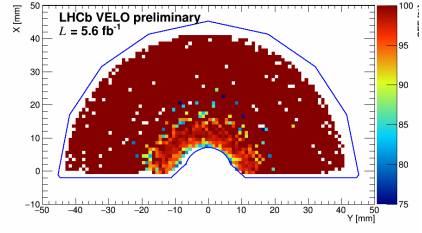
S72



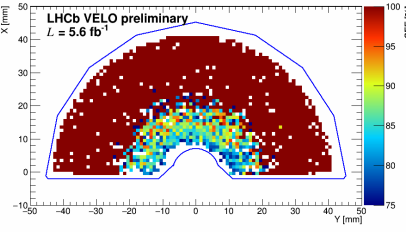
S73



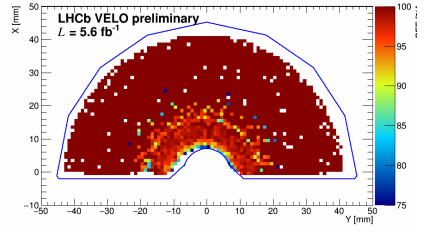
S74



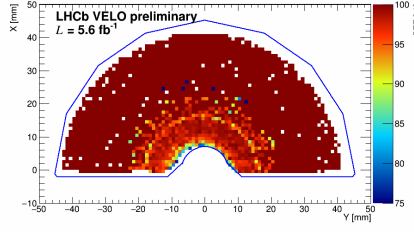
S75



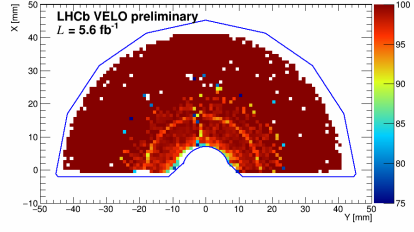
S76



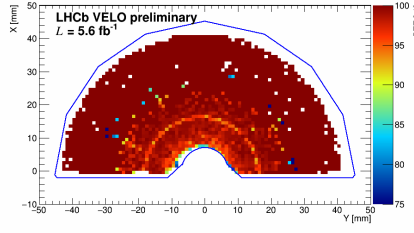
S77



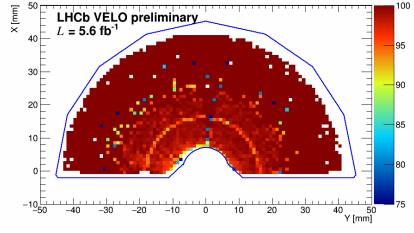
S78



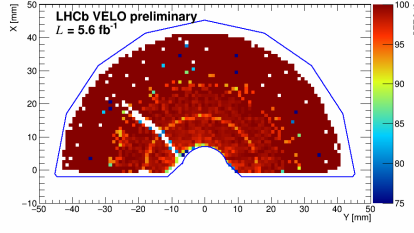
S79



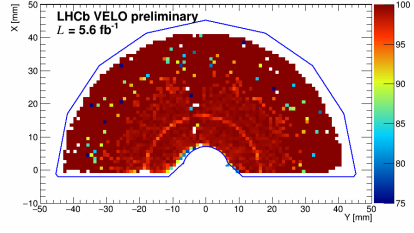
S80



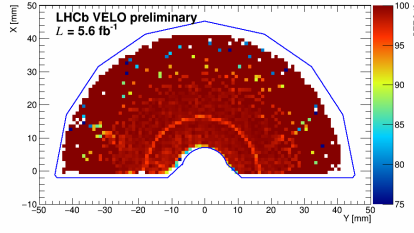
S81



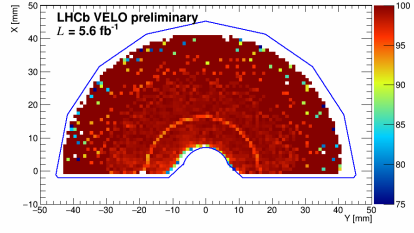
S82



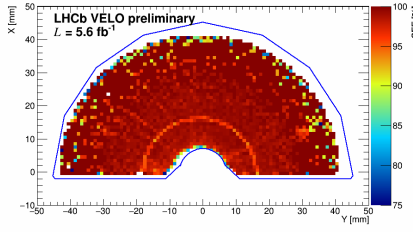
S83



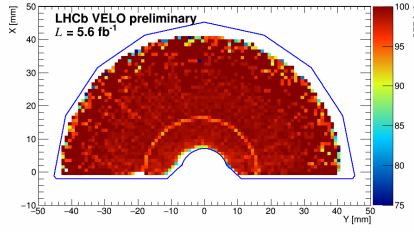
S84



S85

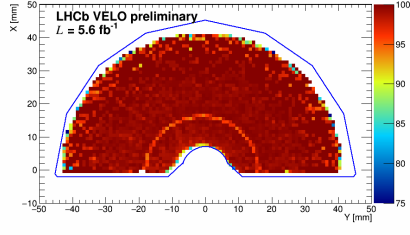


S86

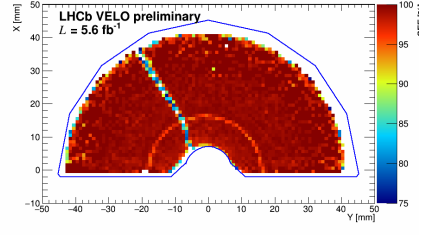


S87

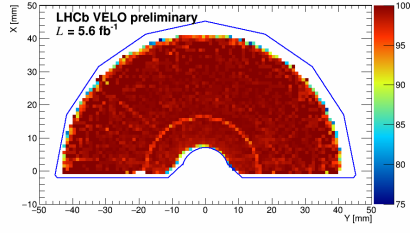
APPENDIX B. CFE AS FUNCTION OF POSITION ON SENSORS 64 TO
248 $(\Phi\text{-SENSORS})$



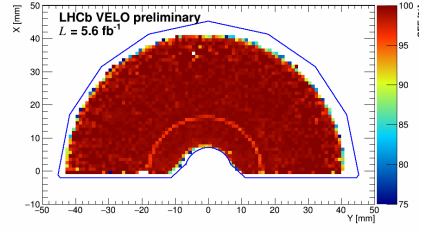
S88



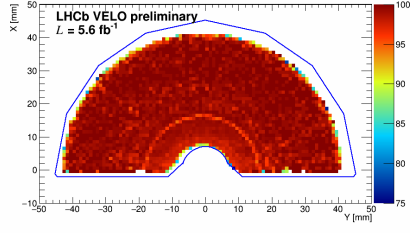
S89



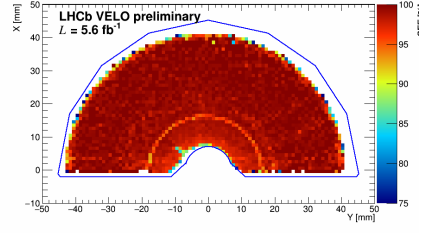
S90



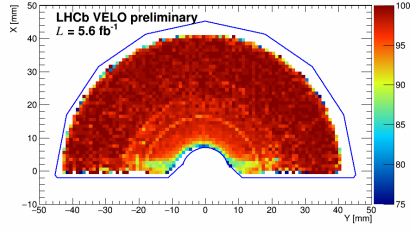
S91



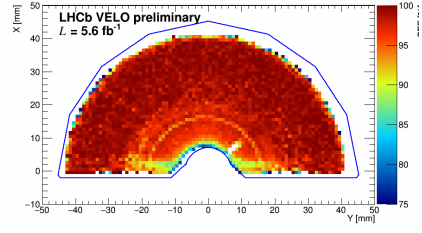
S92



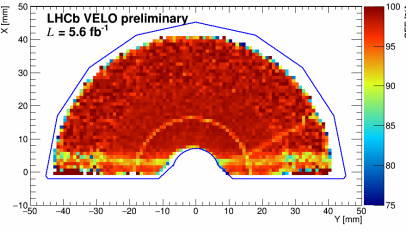
S93



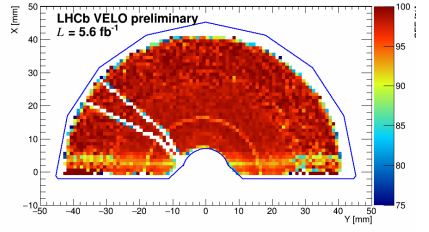
S94



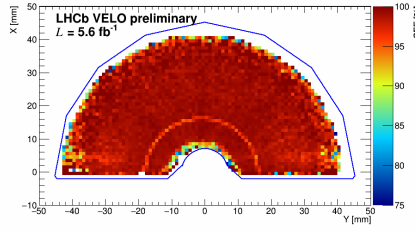
S95



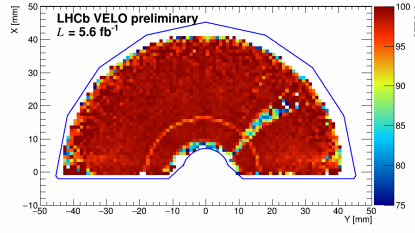
S96



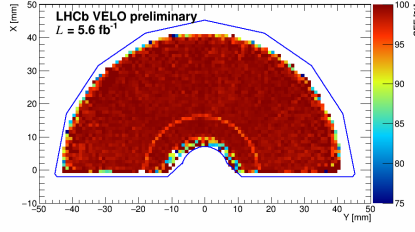
S97



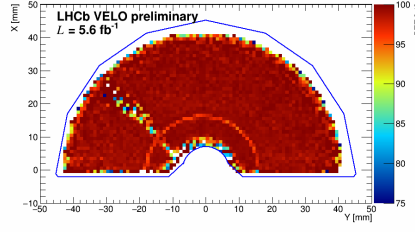
S98



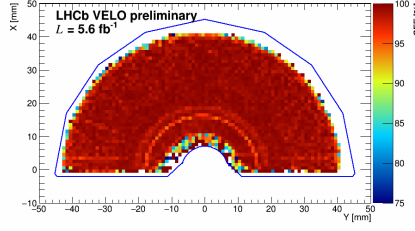
S99



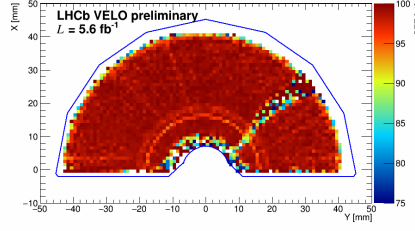
S100



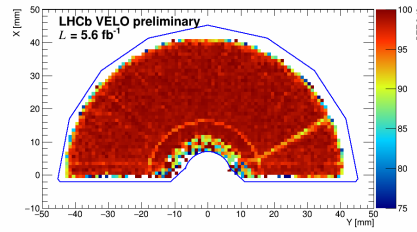
S101



S102



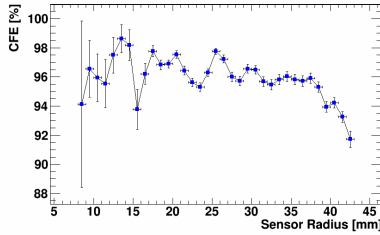
S103



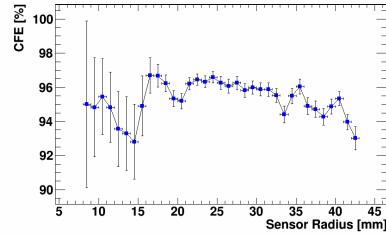
S104

C CFE versus sensor radius for sensor 0 to 40 (R -sensors)

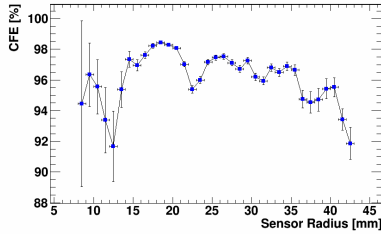
The following plots provide supplementary material to Chapter 8. These plots show the Cluster Finding Efficiency (CFE) for all VELO R -type sensors as a function of radius. The plots are made using the October 2016 Charge Collection Efficiency (CCE) scan data. The plots are made with the sensors at 250 V. The sensors number (S0-S40) is given below the figure.



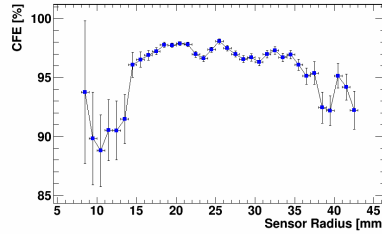
S0



S1

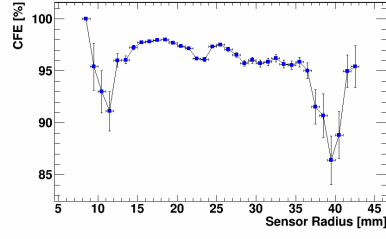


S2

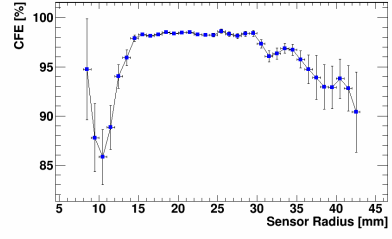


S3

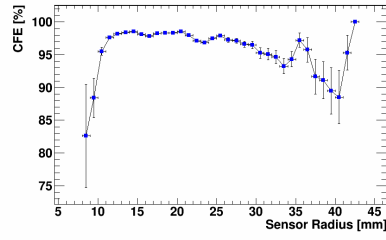
APPENDIX C. CFE VERSUS SENSOR RADIUS FOR SENSOR 0 TO 40
252 (R-SENSORS)



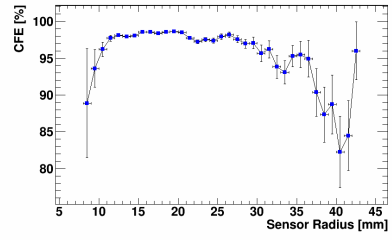
S4



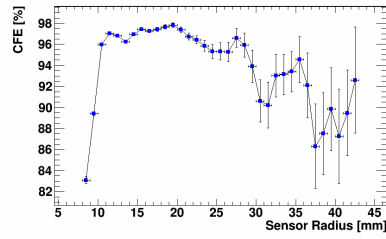
S5



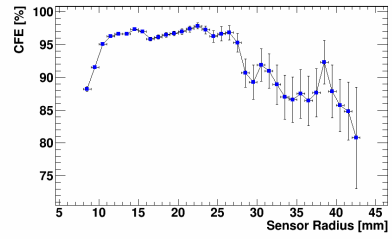
S6



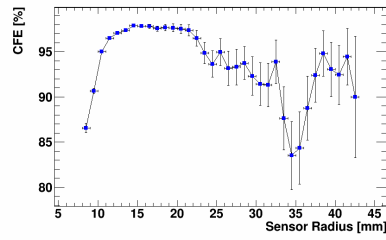
S7



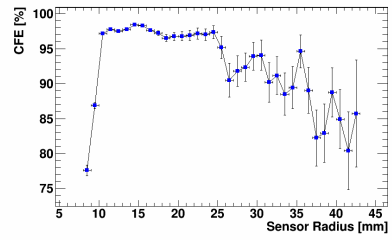
S8



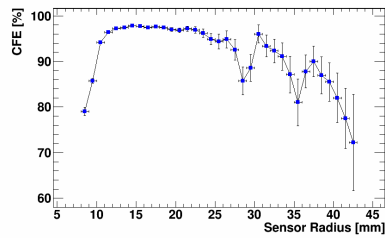
S9



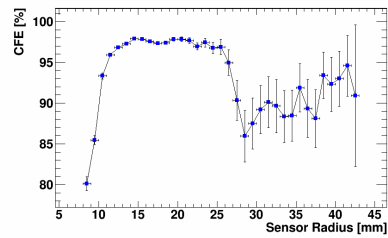
S10



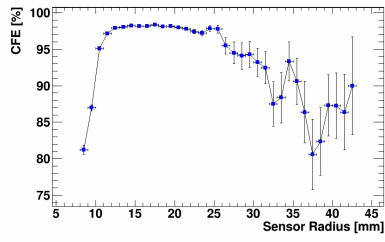
S11



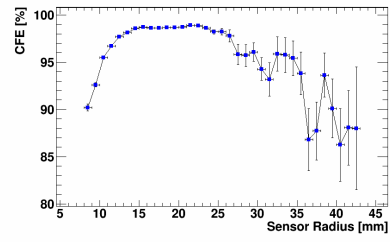
S12



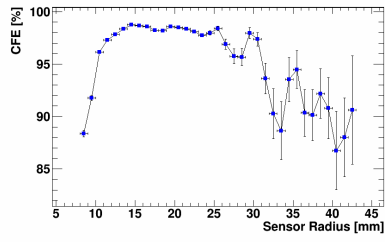
S13



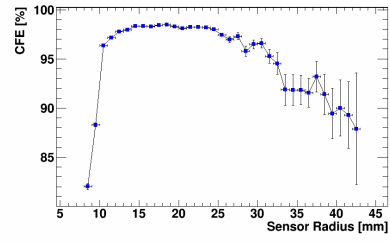
S14



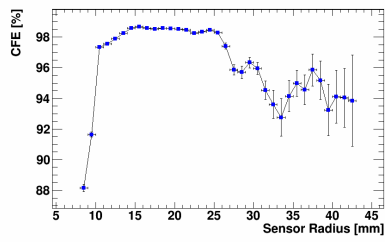
S15



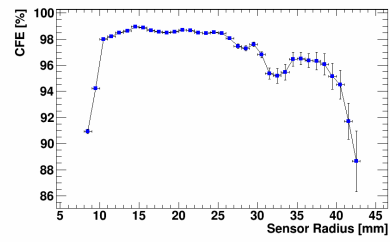
S16



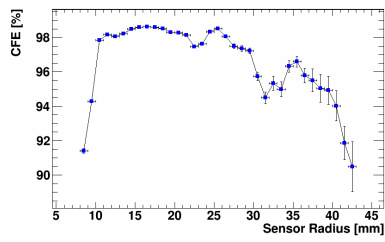
S17



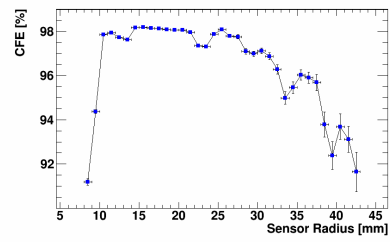
S18



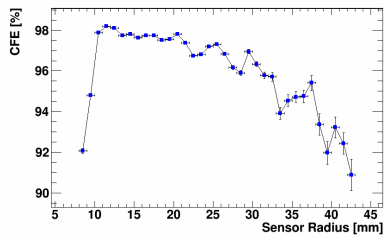
S19



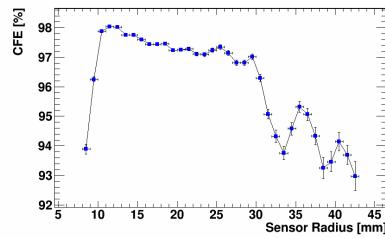
S20



S21

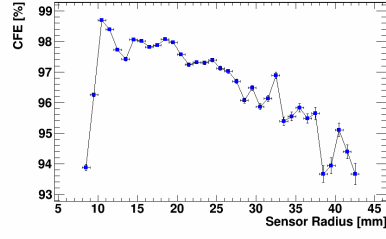


S22

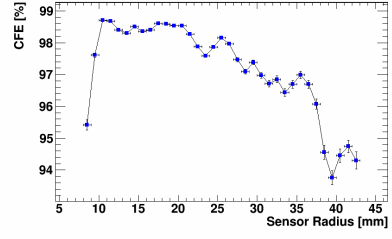


S23

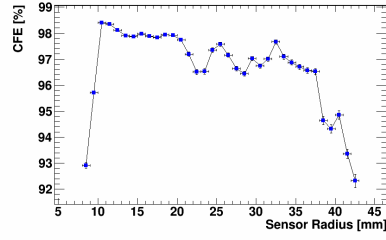
APPENDIX C. CFE VERSUS SENSOR RADIUS FOR SENSOR 0 TO 40
254 (R -SENSORS)



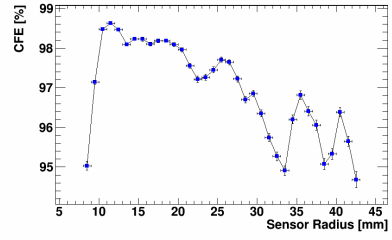
S24



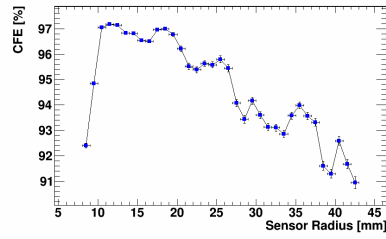
S25



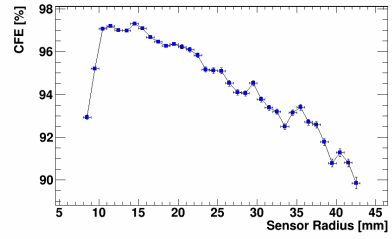
S26



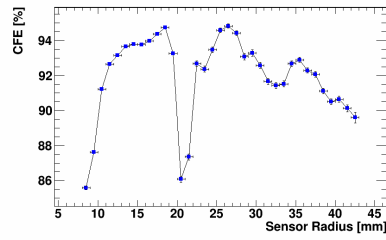
S27



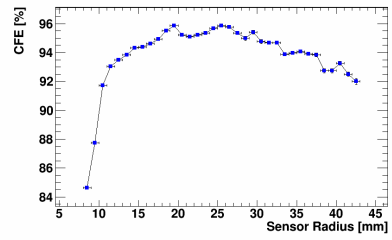
S28



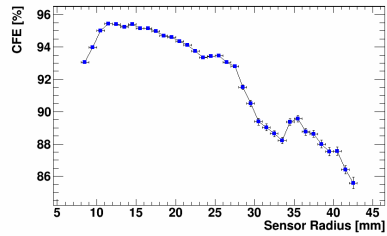
S29



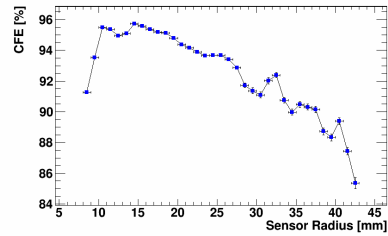
S30



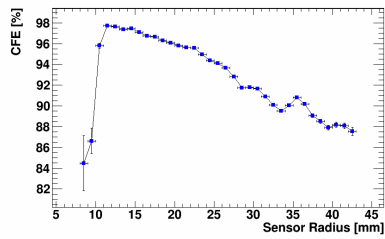
S31



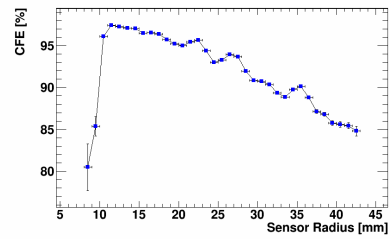
S32



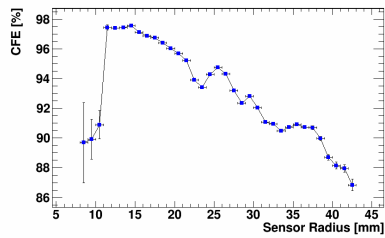
S33



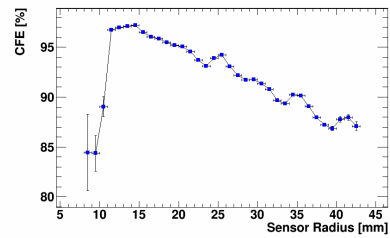
S34



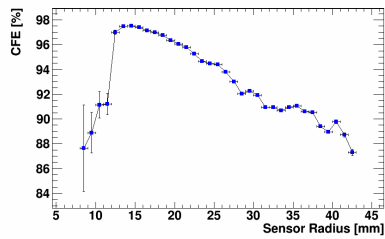
S35



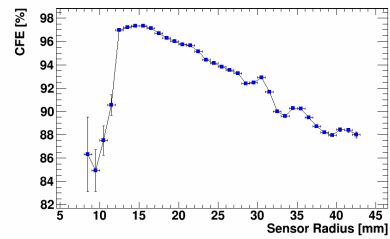
S36



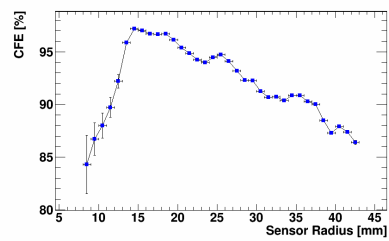
S37



S38



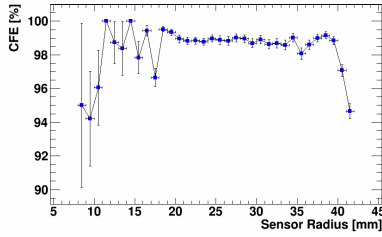
S39



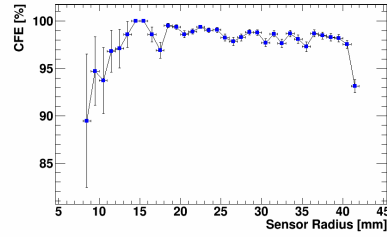
S40

D CFE versus sensor radius for sensor 64 to 104 (Φ -sensors)

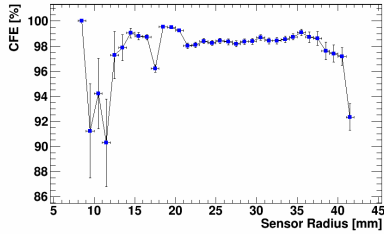
The following plots provide supplementary material to Chapter 8. These plots show the Cluster Finding Efficiency (CFE) for all VELO Φ -type sensors as a function of radius. The plots are made using the October 2016 Charge Collection Efficiency (CCE) scan data. The plots are made with the sensors at 250 V. The sensors number (S64-S104) is given below the figure.



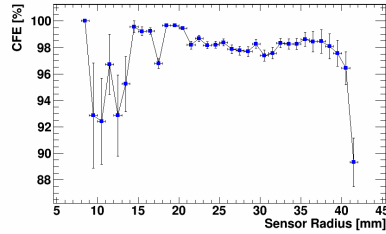
S64



S65

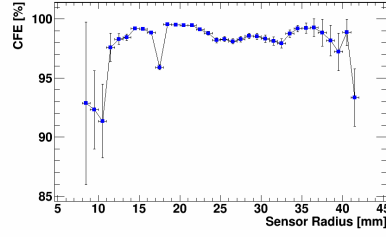


S66

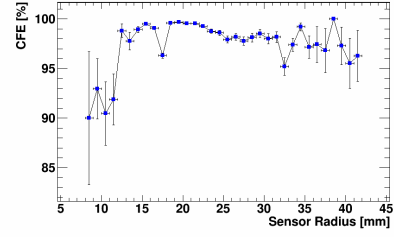


S67

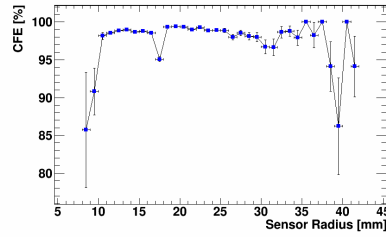
APPENDIX D. CFE VERSUS SENSOR RADIUS FOR SENSOR 64 TO 104
258 (Φ -SENSORS)



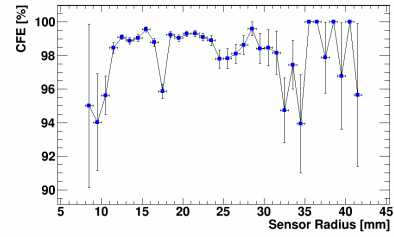
S68



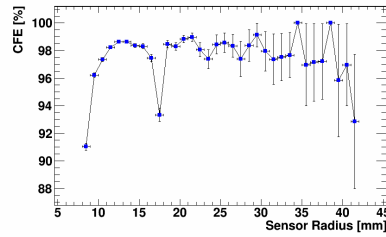
S69



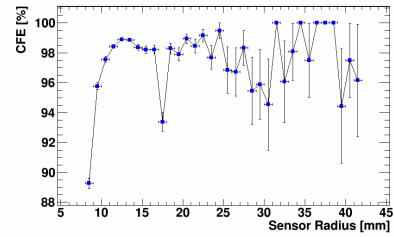
S70



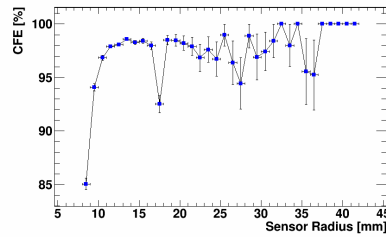
S71



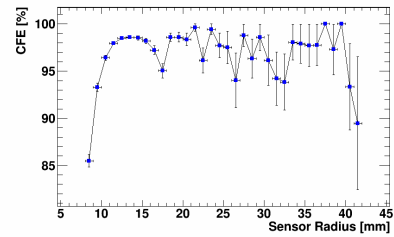
S72



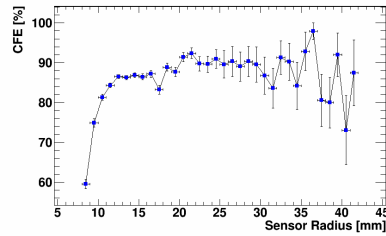
S73



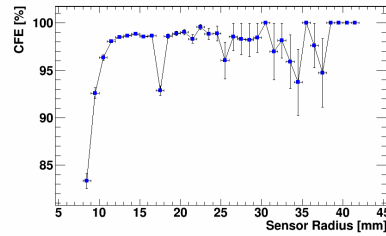
S74



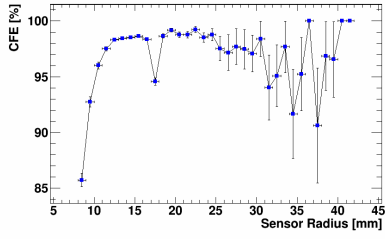
S75



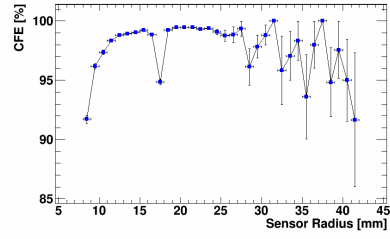
S76



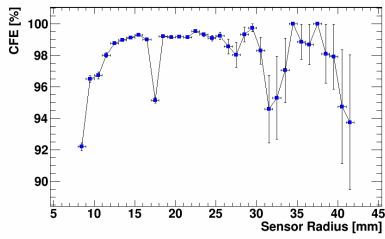
S77



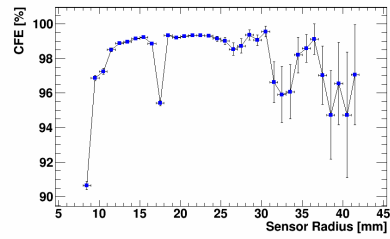
S78



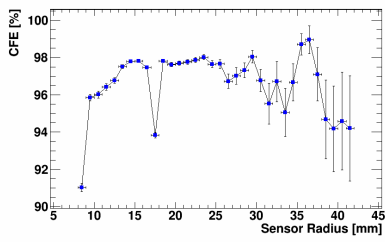
S79



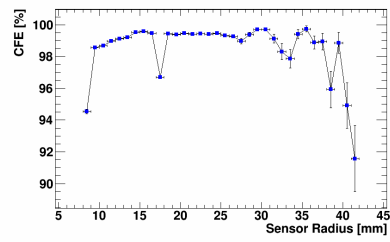
S80



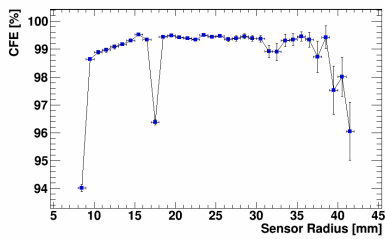
S81



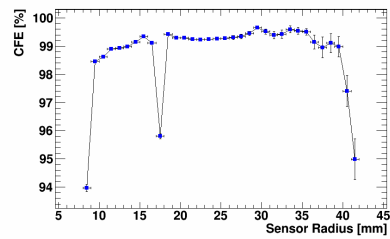
S82



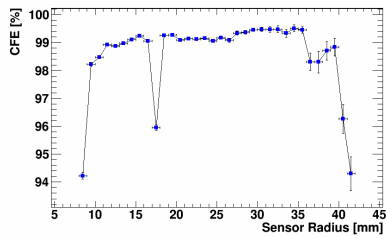
S83



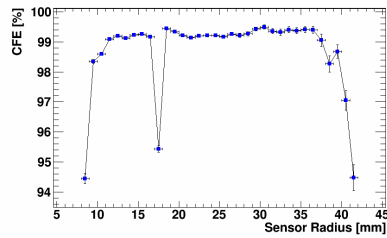
S84



S85

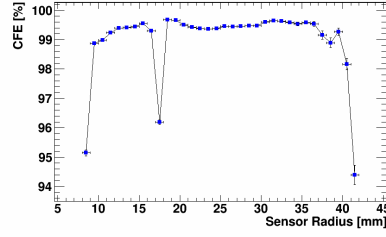


S86

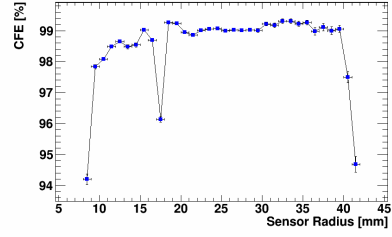


S87

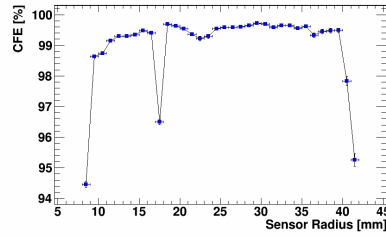
APPENDIX D. CFE VERSUS SENSOR RADIUS FOR SENSOR 64 TO 104
(Φ -SENSORS)



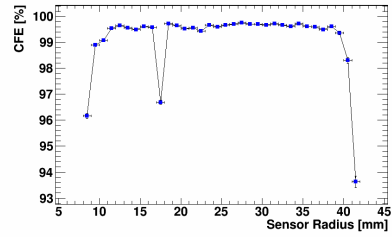
S88



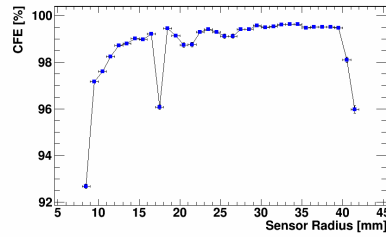
S89



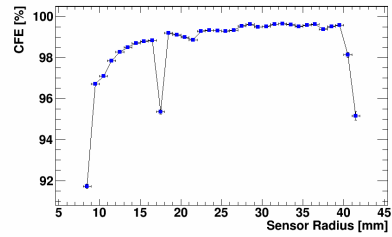
S90



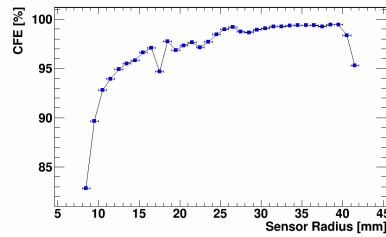
S91



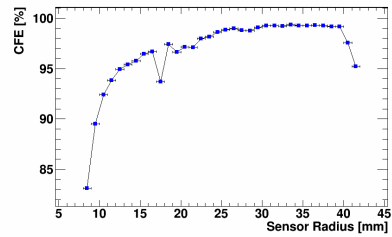
S92



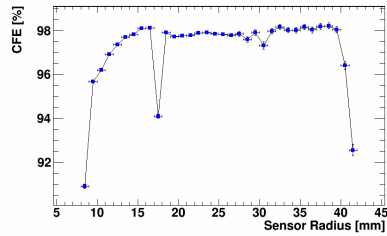
S93



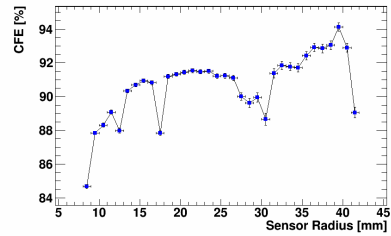
S94



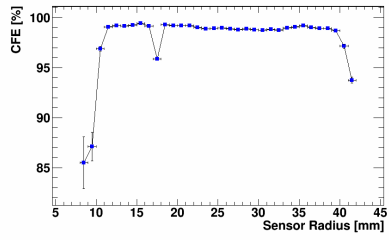
S95



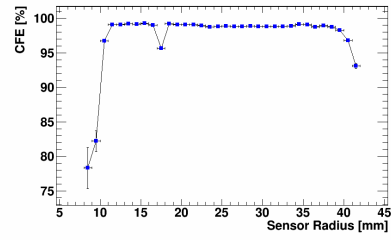
S96



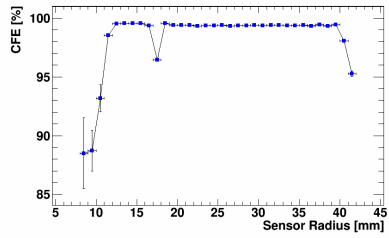
S97



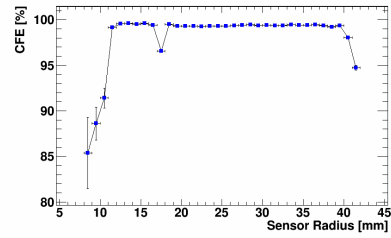
S98



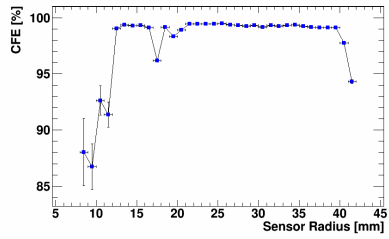
S99



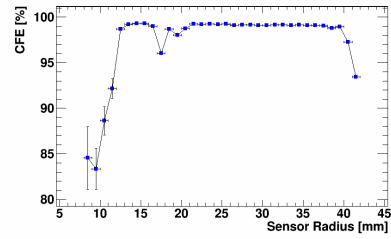
S100



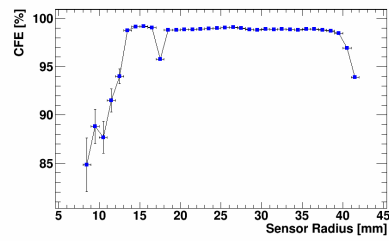
S101



S102



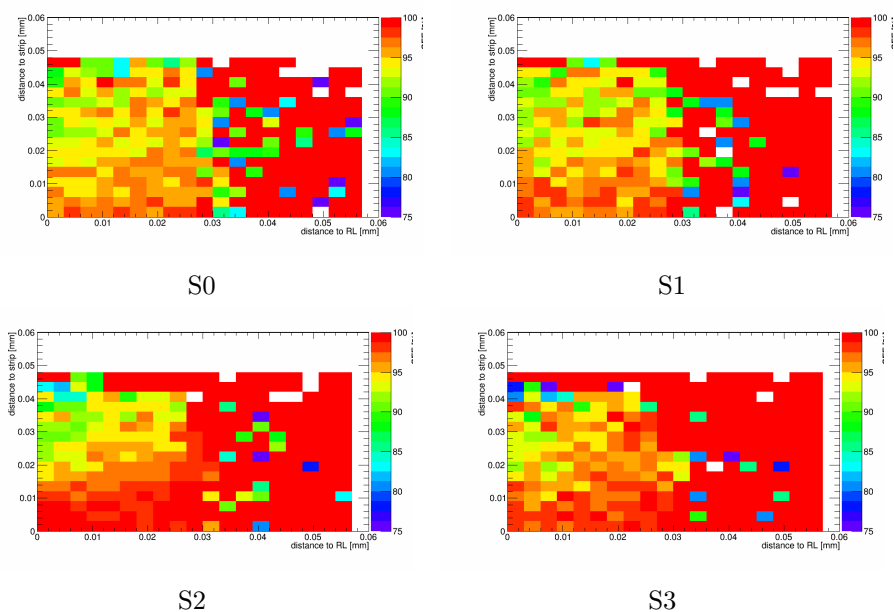
S103



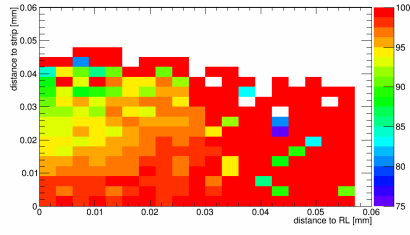
S104

E CFE as function of distances to routing lines and strips on sensors 0 to 40 (R -sensors)

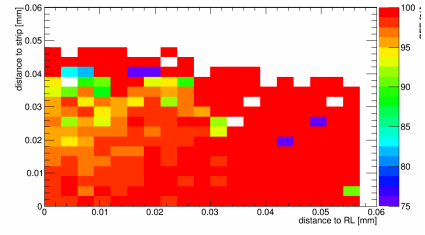
The following plots provide supplementary material to Chapter 8. These plots show the Cluster Finding Efficiency (CFE) for all VELO R -type sensors as a function of the distance of the hit to the strip and the distance of the strip to the routing line. The plots are made using the October 2016 Charge Collection Efficiency (CCE) scan data. The plots are made with the sensors at their operational bias voltage. The sensors number (S0-S40) is given below the figure.



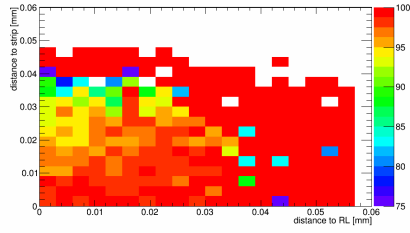
264 *APPENDIX E. CFE AS FUNCTION OF DISTANCES TO ROUTING
LINES AND STRIPS ON SENSORS 0 TO 40 (R-SENSORS)*



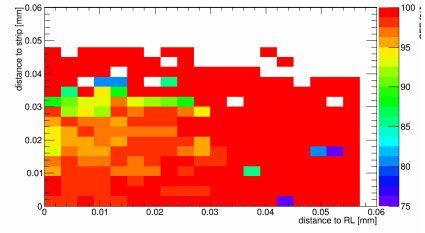
S4



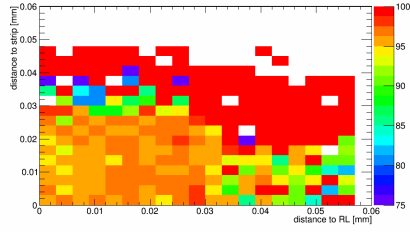
S5



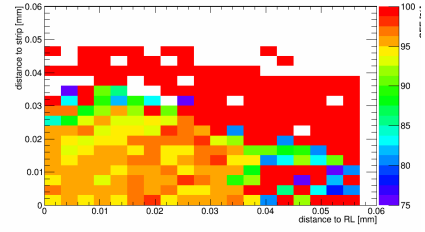
S6



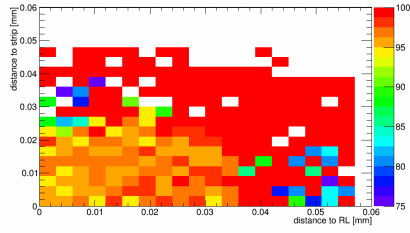
S7



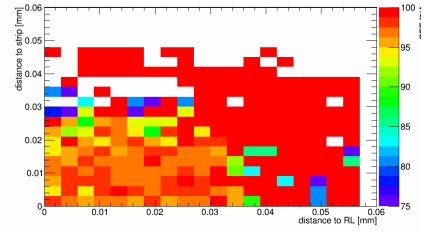
S8



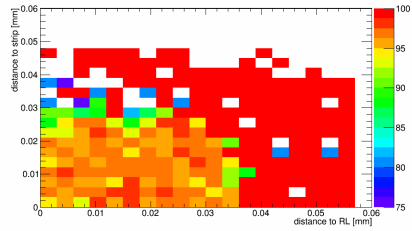
S9



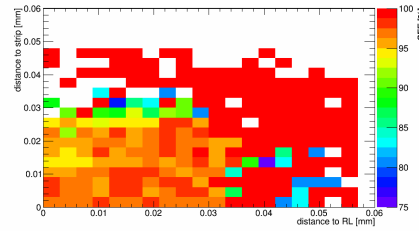
S10



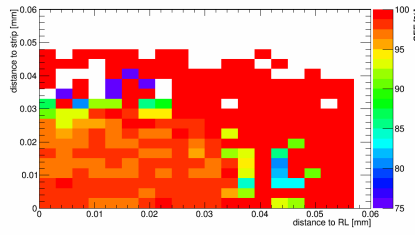
S11



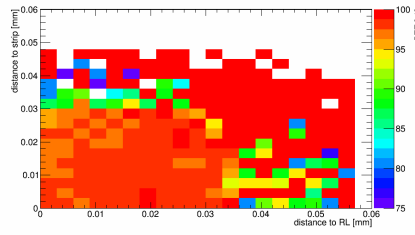
S12



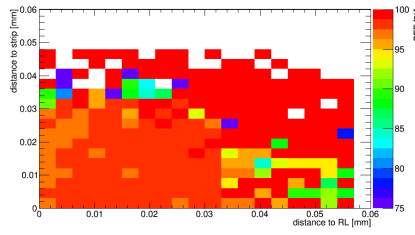
S13



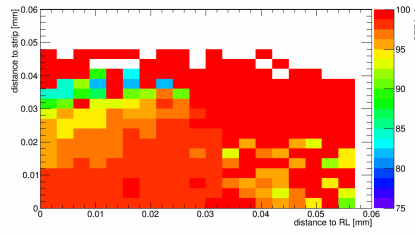
S14



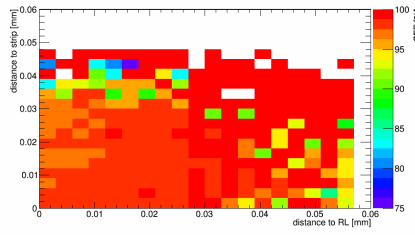
S15



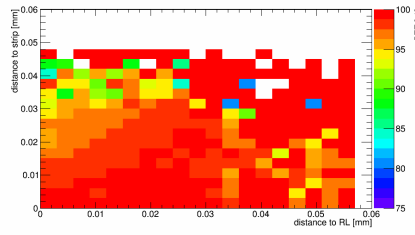
S16



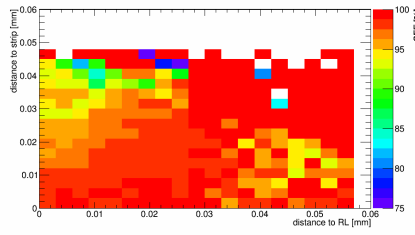
S17



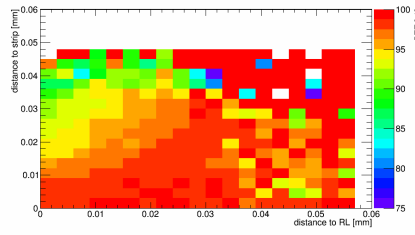
S18



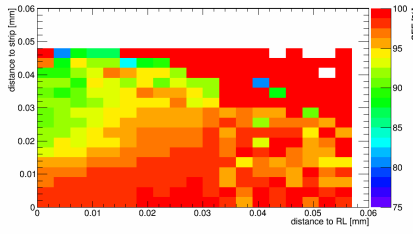
S19



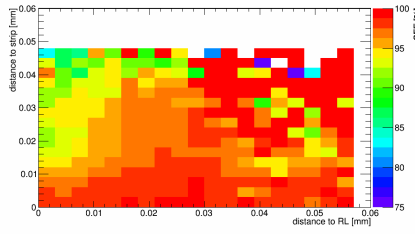
S20



S21

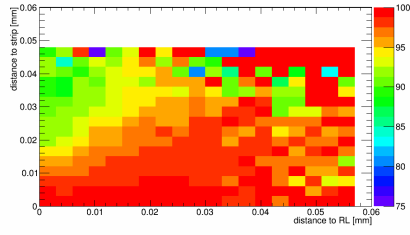


S22

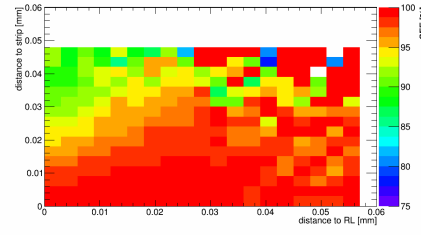


S23

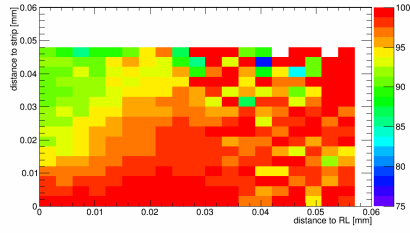
266 *APPENDIX E. CFE AS FUNCTION OF DISTANCES TO ROUTING
LINES AND STRIPS ON SENSORS 0 TO 40 (R-SENSORS)*



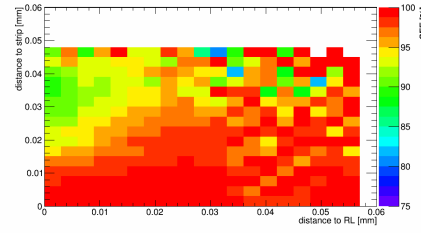
S24



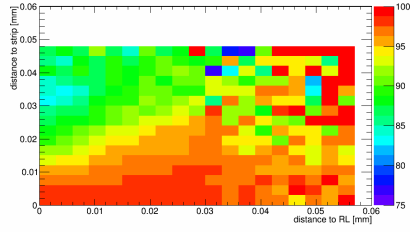
S25



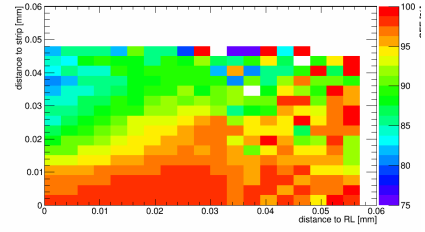
S26



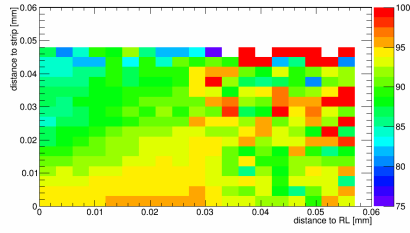
S27



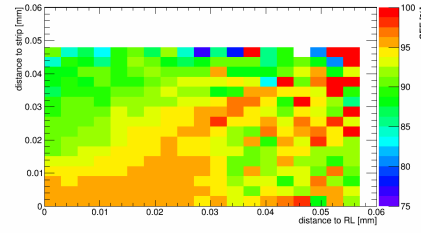
S28



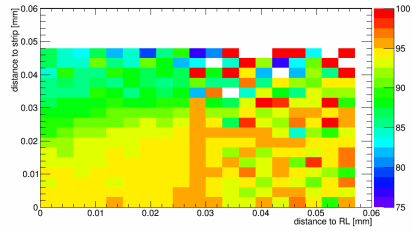
S29



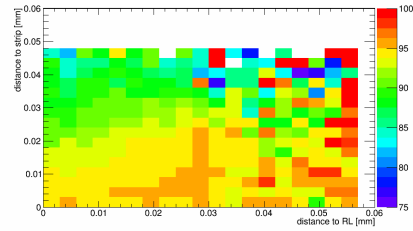
S30



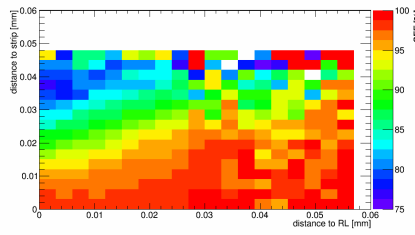
S31



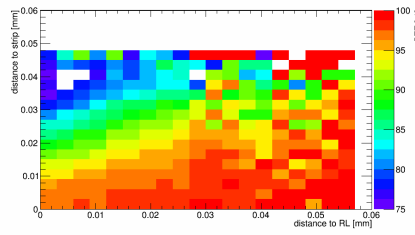
S32



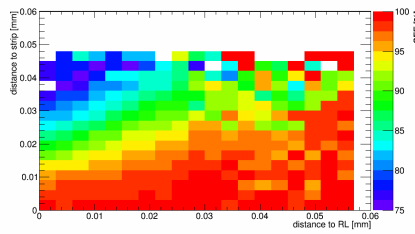
S33



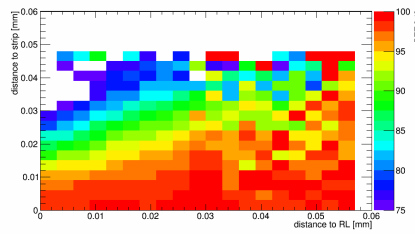
S34



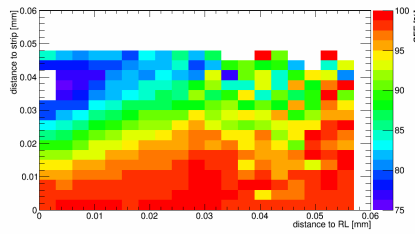
S35



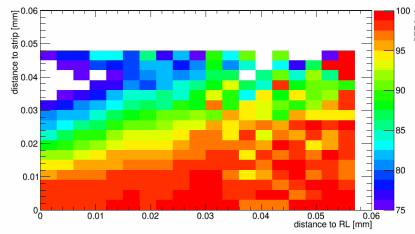
S36



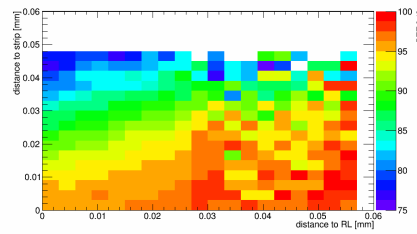
S37



S38



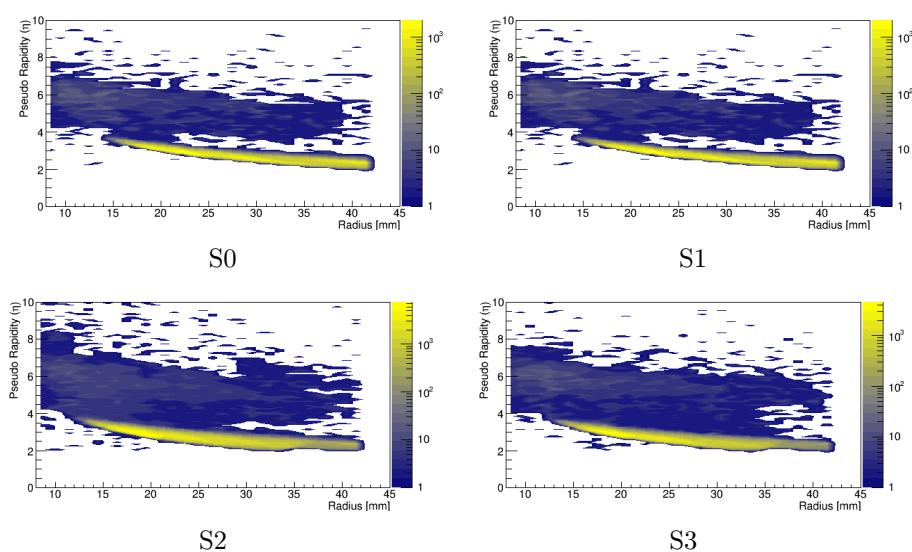
S39



S40

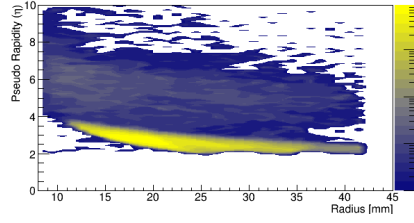
F Track population as function of pseudo-rapidity and sensor radius for sensor 0 to 40 (R -sensors)

The following plots provide supplementary material to Chapter 8. These plots show the population of hit positions of the tracks observed as a function of pseudo-rapidity and radius for all VELO R -type sensors. The plots are made using October 2016 Charge Collection Efficiency (CCE) scan data. The sensor number (S0-S40) is given below the figure.

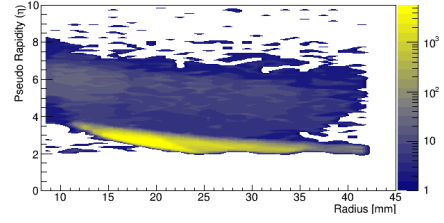


270

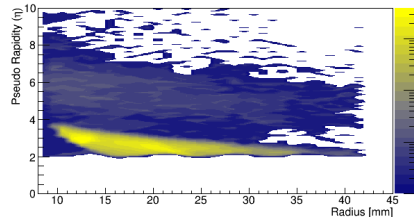
APPENDIX F. TRACK POPULATION AS FUNCTION OF
PSEUDO-RAPIDITY AND SENSOR RADIUS FOR SENSOR 0 TO 40
(R -SENSORS)



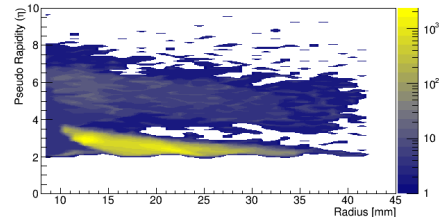
S4



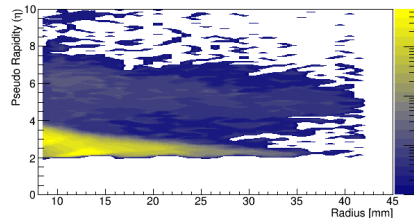
S5



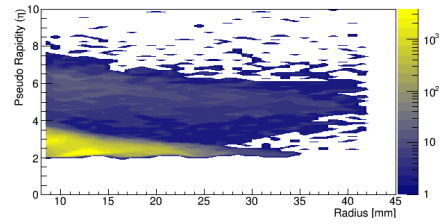
S6



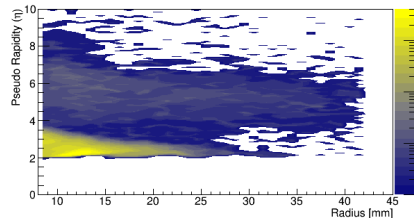
S7



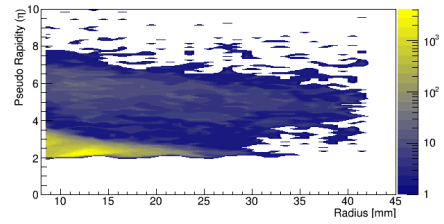
S8



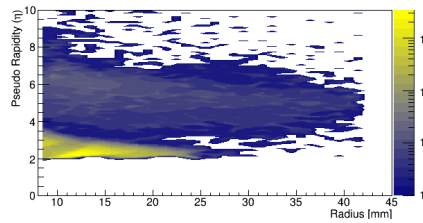
S9



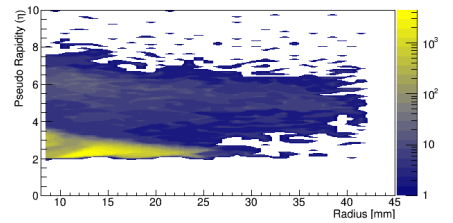
S10



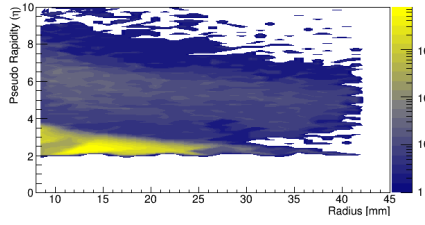
S11



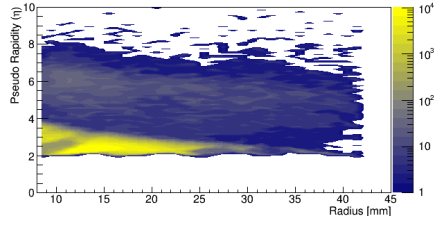
S12



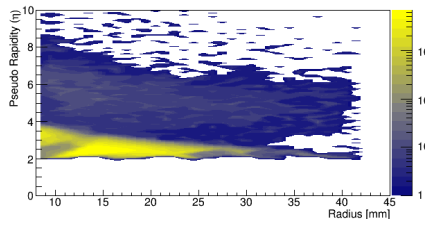
S13



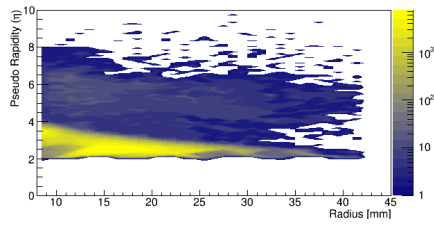
S14



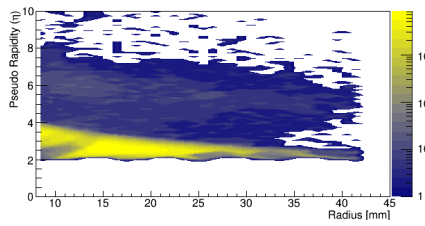
S15



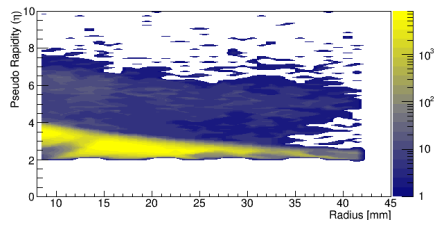
S16



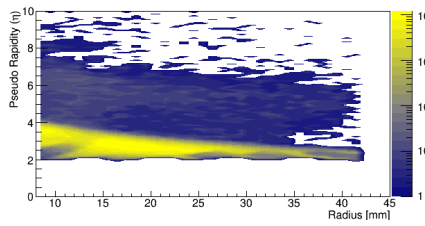
S17



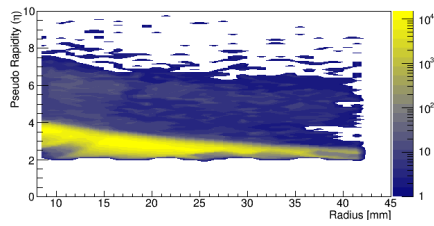
S18



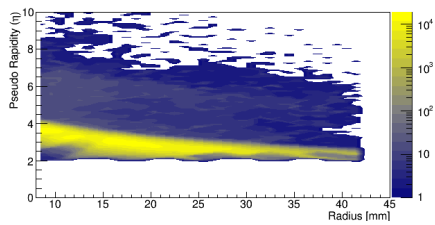
S19



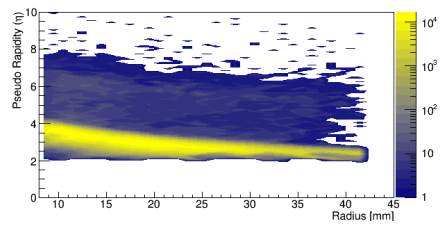
S20



S21



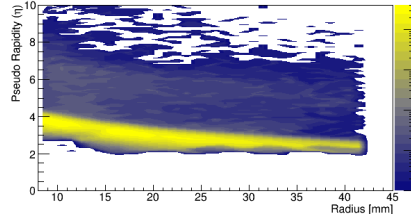
S22



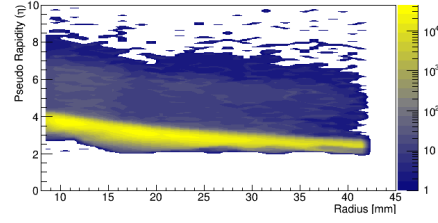
S23

272

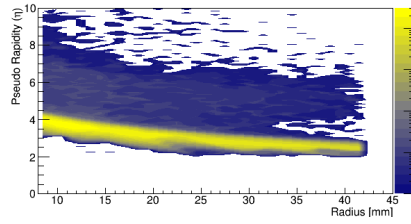
APPENDIX F. TRACK POPULATION AS FUNCTION OF
PSEUDO-RAPIDITY AND SENSOR RADIUS FOR SENSOR 0 TO 40
(R-SENSORS)



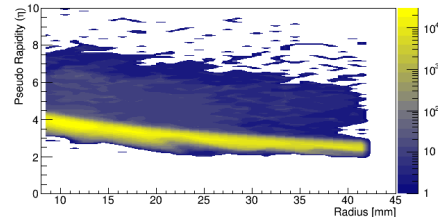
S24



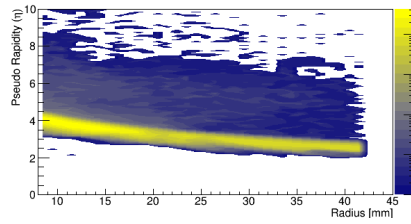
S25



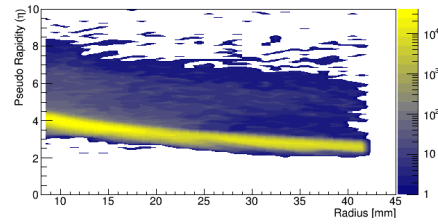
S26



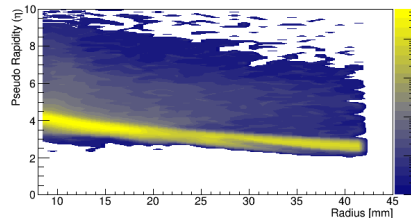
S27



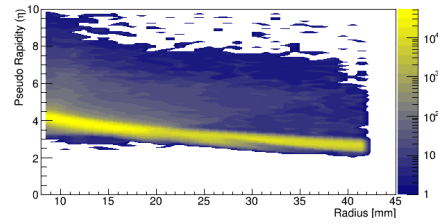
S28



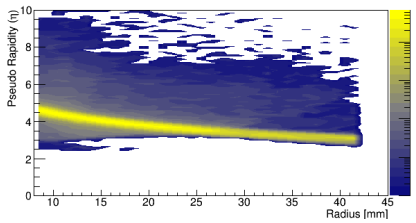
S29



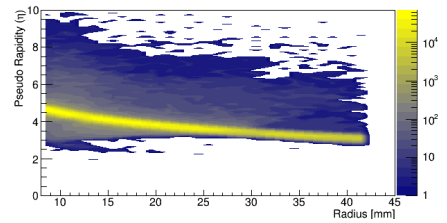
S30



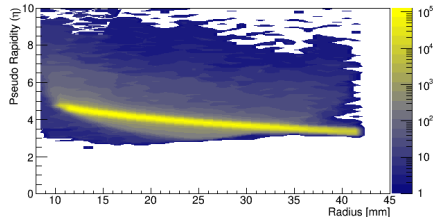
S31



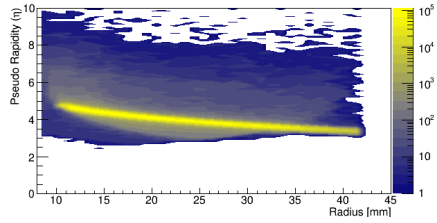
S32



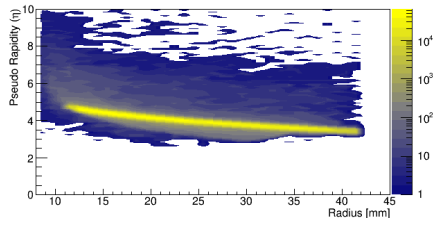
S33



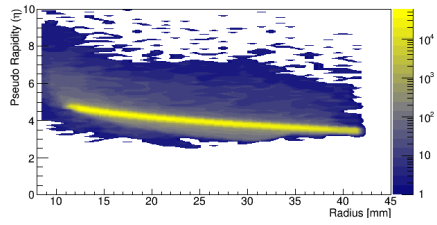
S34



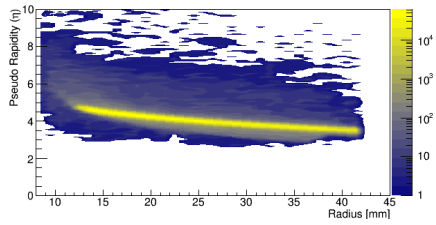
S35



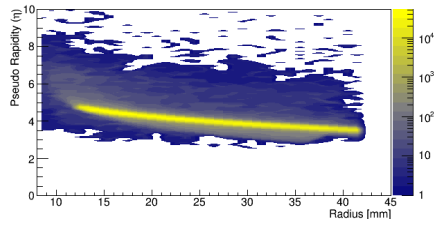
S36



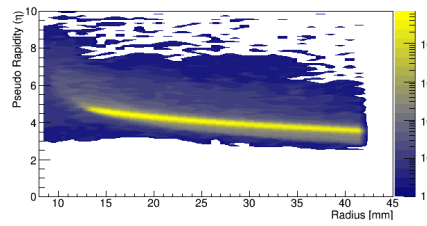
S37



S38



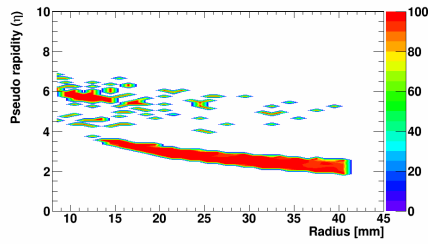
S39



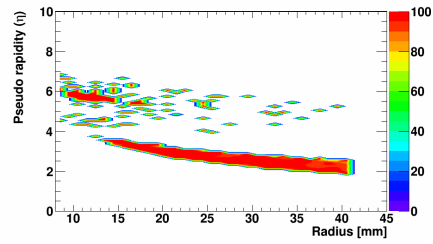
S40

G CFE as function of pseudo-rapidity and sensor radius for sensor 0 to 40 (R -sensors)

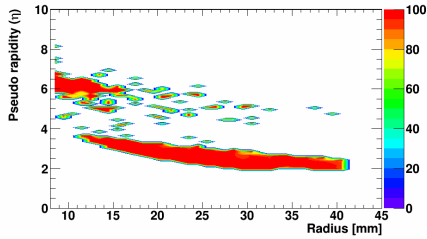
The following plots provide supplementary material to Chapter 8. These plots show the the Cluster Finding Efficiency (CFE) as a function of pseudo-rapidity and radius for all VELO R -type sensors. The plots are made using October 2016 Charge Collection Efficiency (CCE) scan data. The sensor number (S0-S40) is given below the figure.



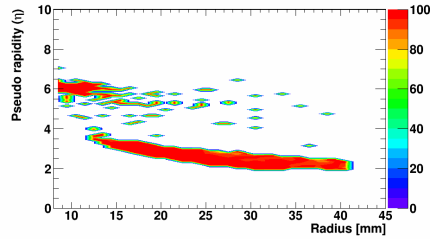
S0



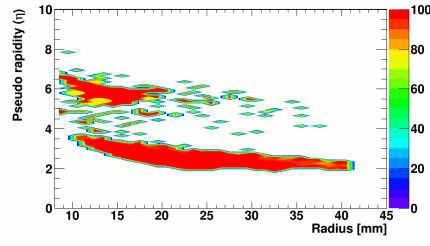
S1



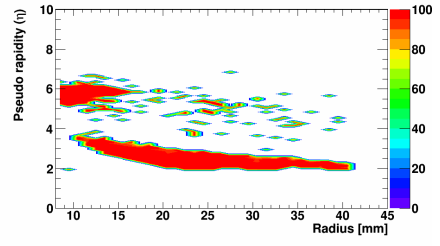
S2



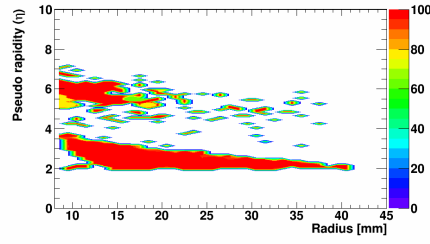
S3



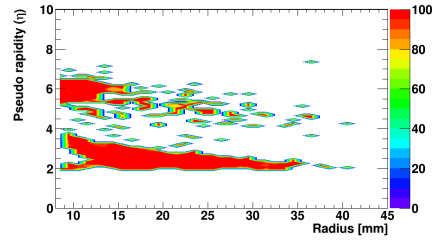
S4



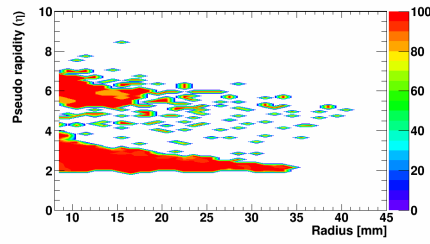
S5



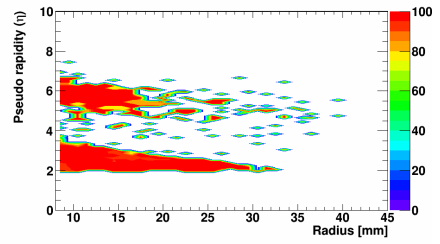
S6



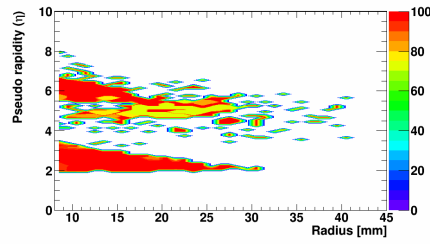
S7



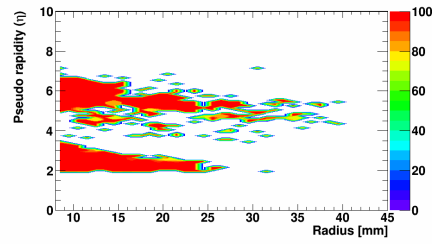
S8



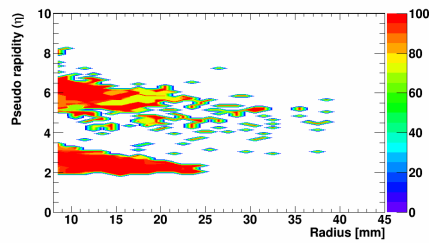
S9



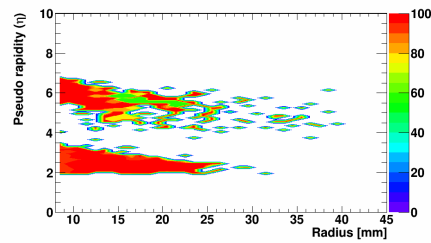
S10



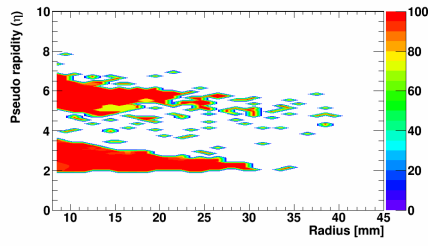
S11



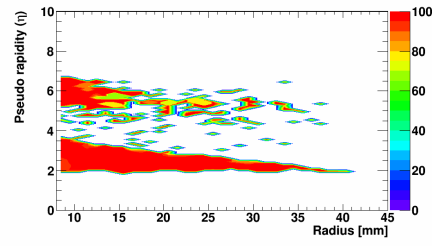
S12



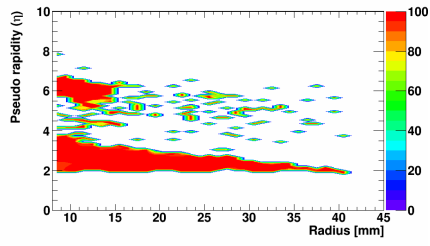
S13



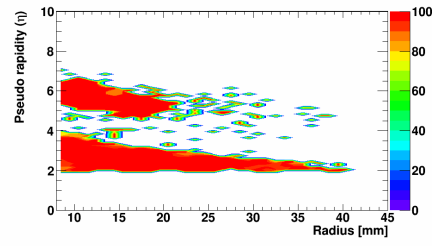
S14



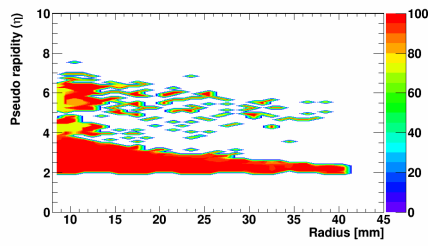
S15



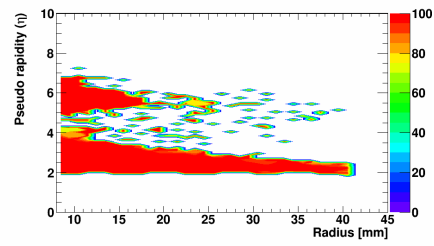
S16



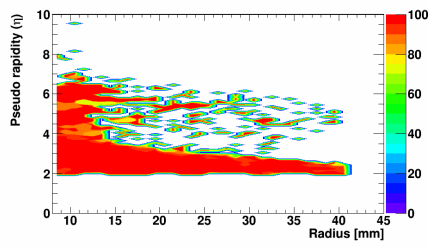
S17



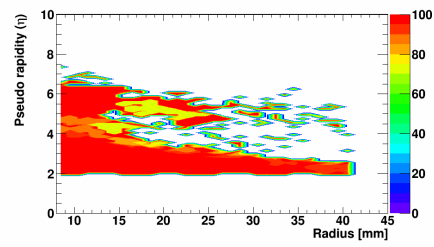
S18



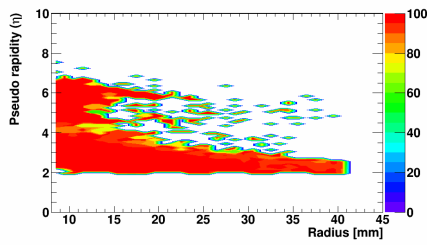
S19



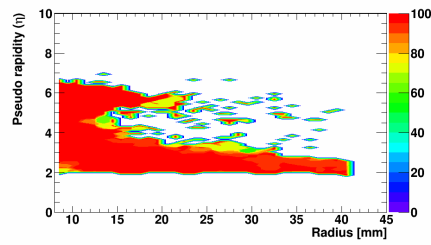
S20



S21

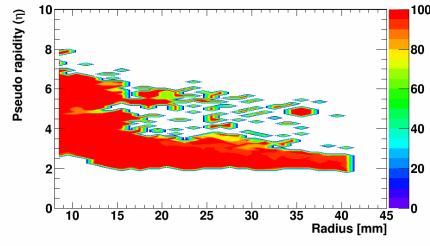


S22

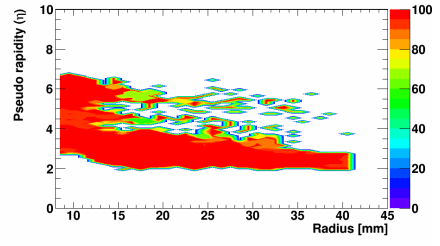


S23

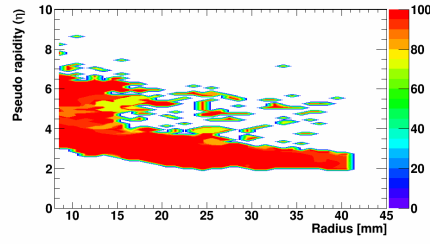
278 APPENDIX G. CFE AS FUNCTION OF PSEUDO-RAPIDITY AND
SENSOR RADIUS FOR SENSOR 0 TO 40 (R-SENSORS)



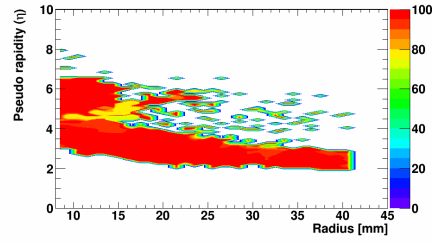
S24



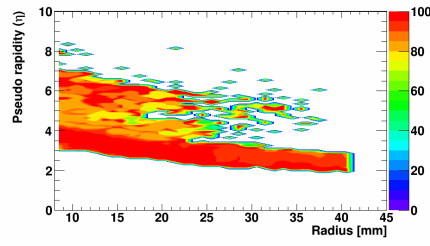
S25



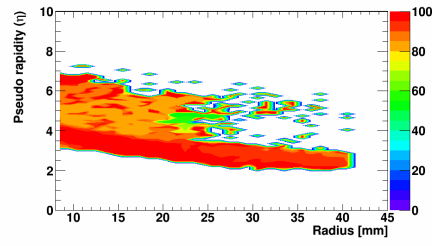
S26



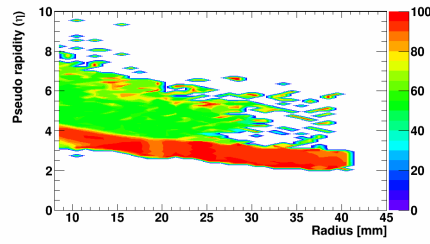
S27



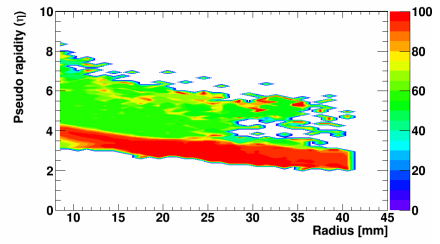
S28



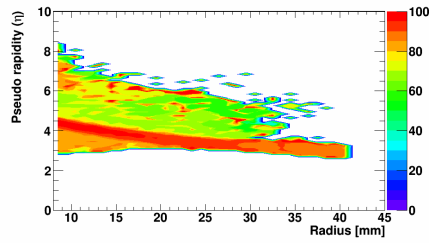
S29



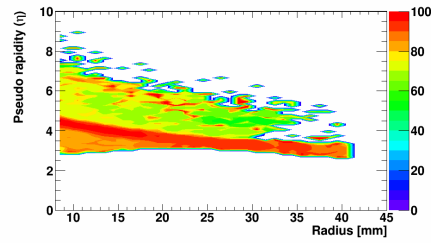
S30



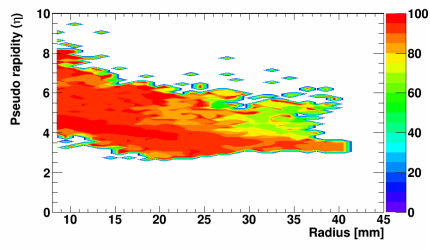
S31



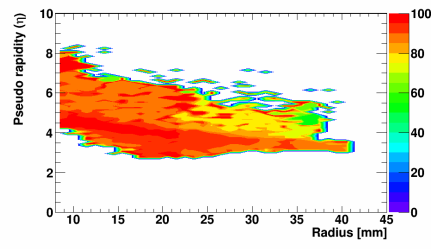
S32



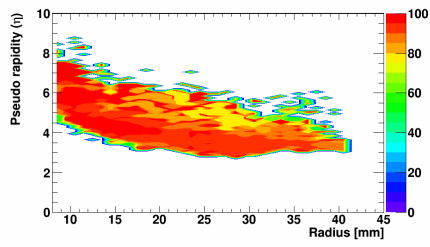
S33



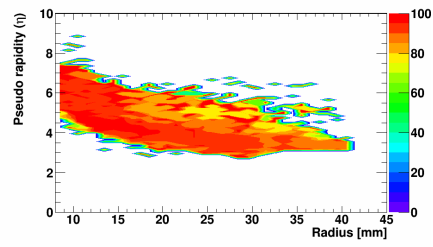
S34



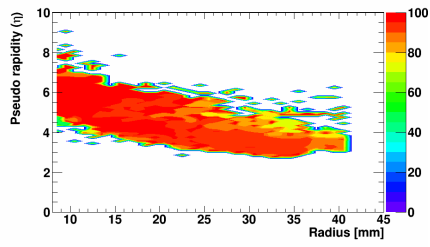
S35



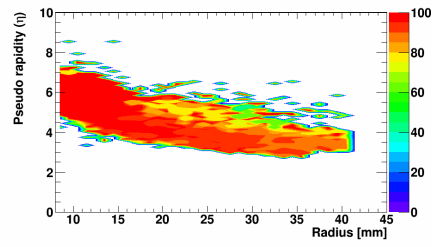
S36



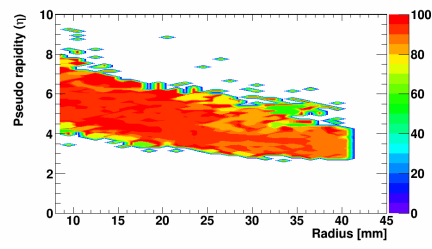
S37



S38



S39



S40

H Variation of metric parameter σ

σ [GeV ² /c ⁴]	T-value	p-value
0.2	8.92×10^{-7}	$2.63^{+3.26}_{-1.71} \times 10^{-3}$
0.3	2.52×10^{-6}	$1.95^{+1.0}_{-1.94} \times 10^{-6}$
0.4	3.04×10^{-6}	$2.0^{+4.2}_{-1.6} \times 10^{-4}$
0.5	3.03×10^{-6}	$2.6^{+3.4}_{-1.7} \times 10^{-4}$
0.6	2.81×10^{-6}	$1.09^{+1.75}_{-0.79} \times 10^{-3}$
0.7	2.53×10^{-6}	$1.38^{+2.01}_{-0.96} \times 10^{-3}$
0.8	2.24×10^{-6}	$3.5^{+6.4}_{-2.6} \times 10^{-4}$
0.9	1.99×10^{-6}	$7.6^{+13.5}_{-5.7} \times 10^{-4}$
1.0	1.76×10^{-6}	$3.15^{+4.06}_{-2.21} \times 10^{-3}$
1.1	1.57×10^{-6}	$3.2^{+6.8}_{-2.6} \times 10^{-4}$

Table H.1: Summary of T values obtained from toy simulation samples generated with a CP violation of 5% applied on the $a_1^+ \rightarrow \rho^0 \pi^+$ for a range of values of the metric parameter σ .

σ [GeV ² /c ⁴]	T-value	p-value
0.2	7.93×10^{-7}	$5.31^{+5.44}_{-3.17} \times 10^{-3}$
0.3	3.00×10^{-6}	$1.49^{+7.81}_{-1.49} \times 10^{-3}$
0.4	4.23×10^{-6}	$8.11^{+30.4}_{-8.11} \times 10^{-6}$
0.5	4.67×10^{-6}	$9.63^{+24}_{-7.2} \times 10^{-6}$
0.6	4.66×10^{-6}	$4.12^{+18.6}_{-4.12} \times 10^{-6}$
0.7	4.48×10^{-6}	$1.51^{+8.35}_{-1.51} \times 10^{-6}$
0.8	4.26×10^{-6}	$4.36^{+19.6}_{-4.36} \times 10^{-6}$
0.9	4.03×10^{-6}	$1.52^{+1.14}_{-1.52} \times 10^{-7}$
1.0	3.81×10^{-6}	$3.64^{+17.3}_{-3.64} \times 10^{-6}$
1.1	3.60×10^{-6}	$8.39^{+71.4}_{-8.39} \times 10^{-8}$

Table H.2: Summary of T values obtained from toy simulation samples generated with a CP violation of 4° applied on the $D^0 \rightarrow \rho\rho$ resonance for a range of values of the metric parameter σ .

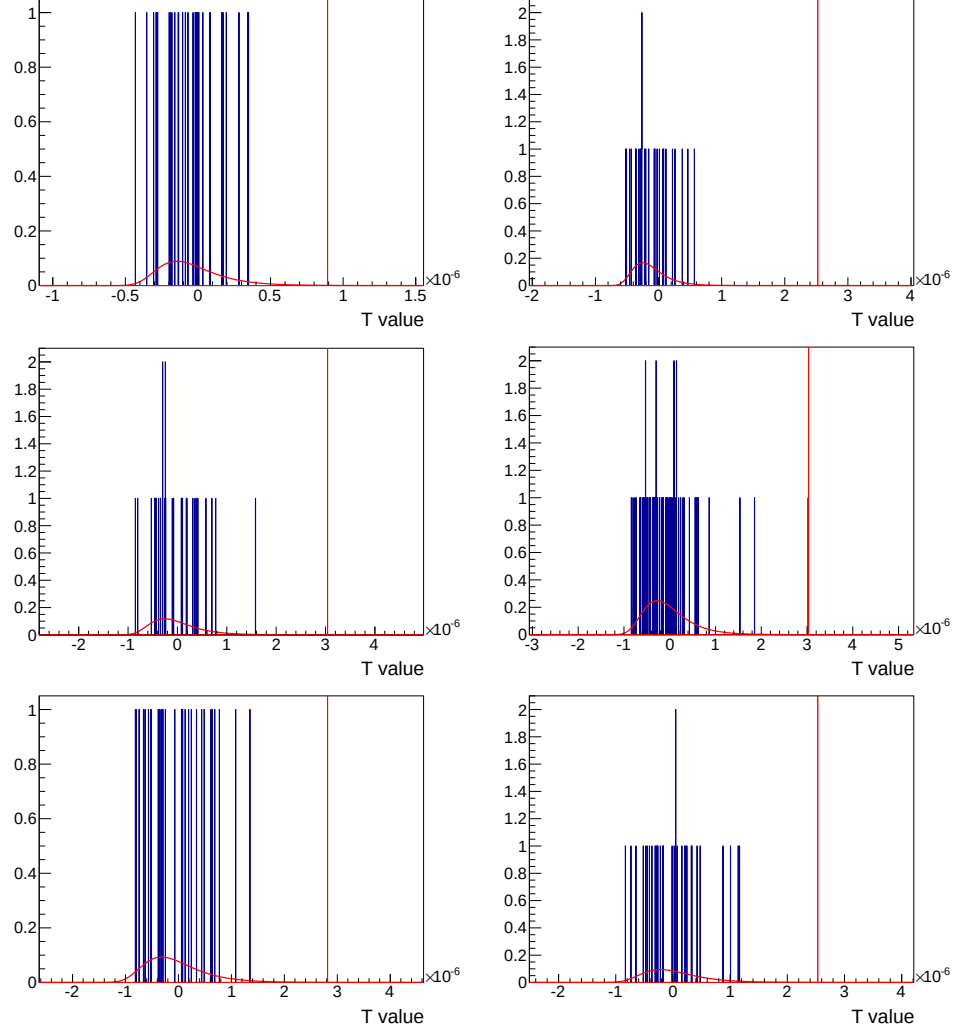


Figure H.1: T value and permutation T values distribution for one toy simulation sample (5% amplitude asymmetry applied on the $a_1^+ \rightarrow \rho^0 \pi^+$) in a range of σ s. From left to right, then from top to bottom, σ are (1) $0.2(\text{GeV}/c^2)^2$, (2) $0.3(\text{GeV}/c^2)^2$, (3) $0.4(\text{GeV}/c^2)^2$, (4) $0.5(\text{GeV}/c^2)^2$, (5) $0.6(\text{GeV}/c^2)^2$, (6) $0.7(\text{GeV}/c^2)^2$.

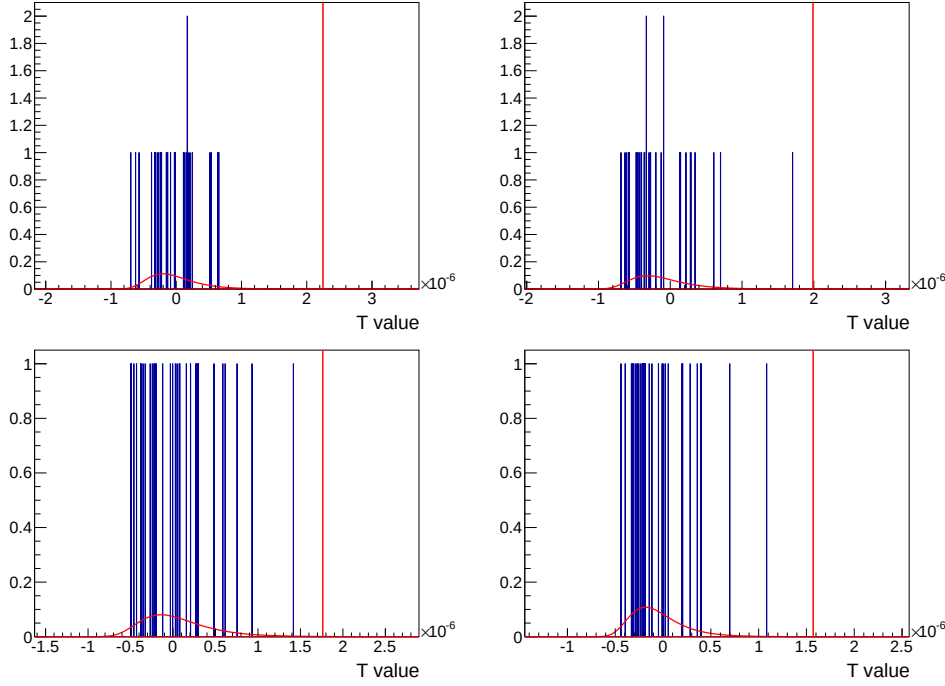


Figure H.2: T value and permutation T values distribution for one toy simulation sample (5% amplitude asymmetry applied on the $a_1^+ \rightarrow \rho^0 \pi^+$) in a range of σ s. From left to right, then from top to bottom, σ are (1) $0.8(\text{GeV}/c^2)^2$, (2) $0.9(\text{GeV}/c^2)^2$, (3) $1.0(\text{GeV}/c^2)^2$, (4) $1.1(\text{GeV}/c^2)^2$.

σ [GeV^2/c^4]	T-value	p-value
0.2	-4.38×10^{-7}	$0.999^{+0.0001}_{-0.00014}$
0.3	1.05×10^{-6}	$2.07^{+2.86}_{-1.41} \times 10^{-3}$
0.4	2.03×10^{-6}	$2.51^{+3.28}_{-1.67} \times 10^{-3}$
0.5	2.44×10^{-6}	$3.8^{+2.9}_{-1.9} \times 10^{-3}$
0.6	2.47×10^{-6}	$2.39^{+3.07}_{-1.59} \times 10^{-3}$
0.7	2.30×10^{-6}	$5.07^{+5.54}_{-3.15} \times 10^{-3}$
0.8	2.07×10^{-6}	$3.4^{+6.8}_{-2.6} \times 10^{-4}$
0.9	1.82×10^{-6}	$4.05^{+4.65}_{-2.57} \times 10^{-3}$
1.0	1.59×10^{-6}	$7.35^{+7.86}_{-4.57} \times 10^{-3}$
1.1	1.39×10^{-6}	$2.10^{+2.98}_{-1.46} \times 10^{-3}$

Table H.3: Summary of T values obtained from toy simulation samples generated with a CP violation of 5% applied on the $D^0 \rightarrow \rho \rho$ resonance for a range of values of the metric parameter σ .

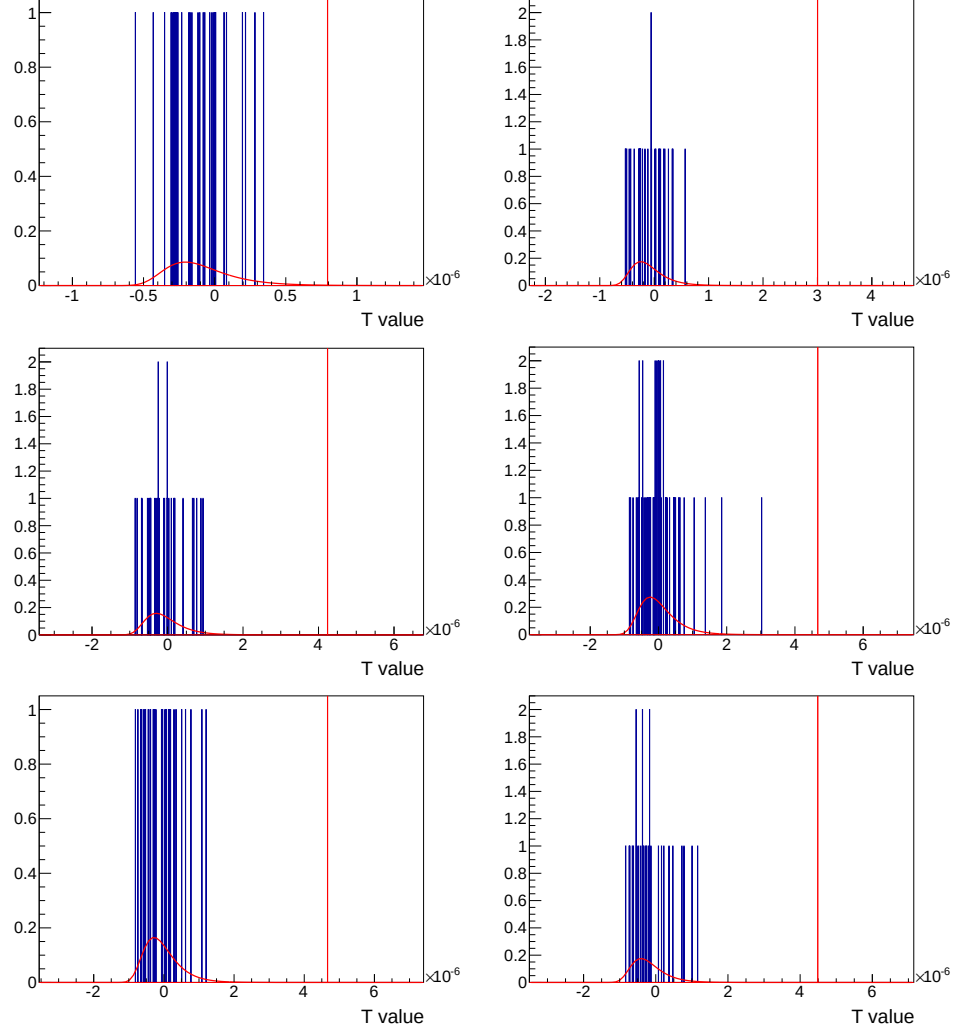


Figure H.3: T value and permutation T values distribution for one toy simulation sample ($+4^\circ$ phase asymmetry applied on the $D^0 \rightarrow \rho\rho$) in a range of σ s. From left to right, then from top to bottom, σ are (1) $0.2(\text{GeV}/c^2)^2$, (2) $0.3(\text{GeV}/c^2)^2$, (3) $0.4(\text{GeV}/c^2)^2$, (4) $0.5(\text{GeV}/c^2)^2$, (5) $0.6(\text{GeV}/c^2)^2$, (6) $0.7(\text{GeV}/c^2)^2$.

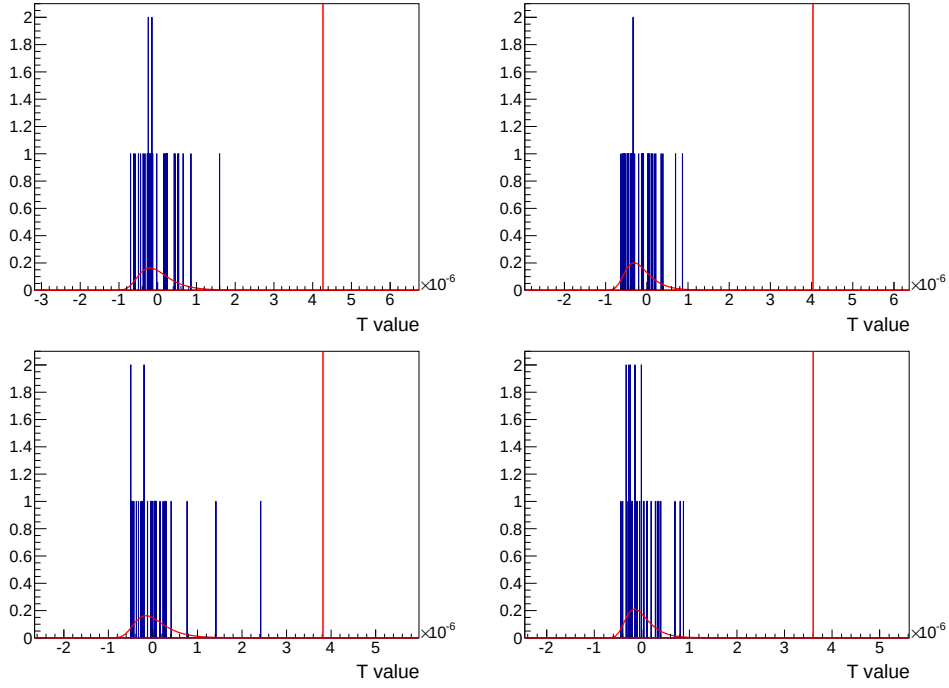


Figure H.4: T value and permutation T values distribution for one toy simulation sample ($+4^\circ$ phase asymmetry applied on the $D^0 \rightarrow \rho\rho$) in a range of σ s. From left to right, then from top to bottom, σ are (1) $0.8(\text{GeV}/c^2)^2$, (2) $0.9(\text{GeV}/c^2)^2$, (3) $1.0(\text{GeV}/c^2)^2$, (4) $1.1(\text{GeV}/c^2)^2$.

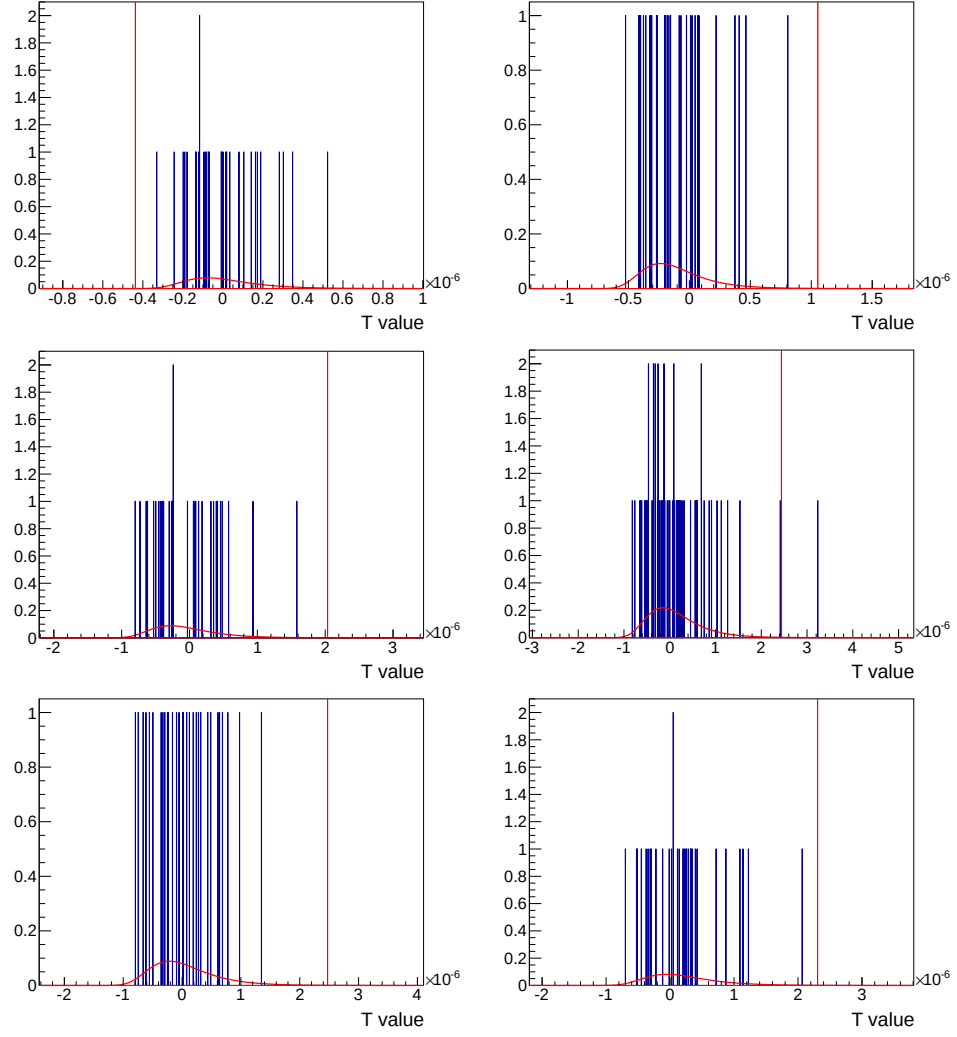


Figure H.5: T value and permutation T values distribution for one toy simulation sample (5% amplitude asymmetry applied on the $D^0 \rightarrow \rho\rho$) in a range of σ s. From left to right, then from top to bottom, σ are (1) $0.2(\text{GeV}/c^2)^2$, (2) $0.3(\text{GeV}/c^2)^2$, (3) $0.4(\text{GeV}/c^2)^2$, (4) $0.5(\text{GeV}/c^2)^2$, (5) $0.6(\text{GeV}/c^2)^2$, (6) $0.7(\text{GeV}/c^2)^2$.

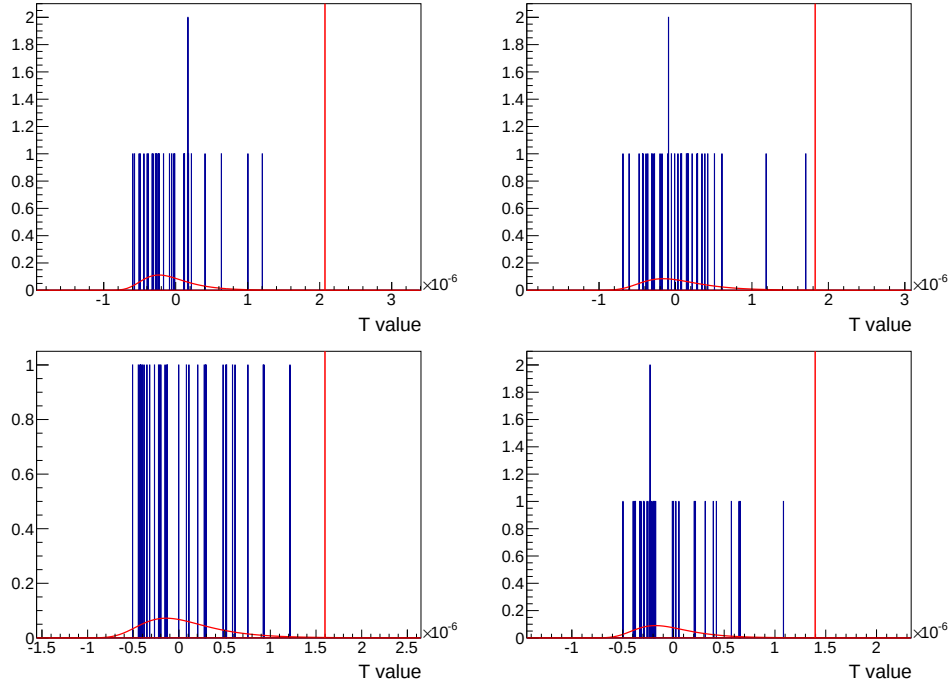


Figure H.6: T value and permutation T values distribution for one toy simulation sample (5% amplitude asymmetry applied on the $D^0 \rightarrow \rho\rho$) in a range of σ s. From left to right, then from top to bottom, σ are (1) $0.8 (\text{GeV}/c^2)^2$, (2) $0.9 (\text{GeV}/c^2)^2$, (3) $1.0 (\text{GeV}/c^2)^2$, (4) $1.1 (\text{GeV}/c^2)^2$.

References

- [1] Y. Amhis et al. Averages of b -hadron, c -hadron, and τ -lepton properties as of summer 2016. 2016.
- [2] Czeslaw Fluder, Enrique Blanco Viñuela, Juan Casas, P Dubert, P Gomes, Marco Pezzetti, A Tovar-Gonzalez, and L Zwalinski. The development of the control system for the cryogenics in the lhc tunnel. 01 2011.
- [3] Cinzia De Melis. The CERN accelerator complex. Complexe des accélérateurs du CERN. Jan 2016. General Photo.
- [4] Roel Aaij et al. LHCb Detector Performance. *Int. J. Mod. Phys.*, A30(07):1530022, 2015.
- [5] R. Aaij et al. Performance of the LHCb Vertex Locator. *JINST*, 9:09007, 2014.
- [6] A. Affolder et al. Radiation damage in the LHCb Vertex Locator. *JINST*, 8:P08002, 2013.
- [7] R Arink et al. Performance of the LHCb Outer Tracker. *JINST*, 9(01):P01002, 2014.
- [8] A. A. Alves Jr. et al. The LHCb detector at the LHC. *JINST*, 3:S08005, 2008.
- [9] M. Adinolfi et al. Performance of the LHCb RICH detector at the LHC. *Eur. Phys. J.*, C73:2431, 2013.
- [10] M. Anelli et al. Performance of the LHCb muon system with cosmic rays. *JINST*, 5:P10003, 2010.
- [11] F. Archilli et al. Performance of the Muon Identification at LHCb. *JINST*, 8:P10020, 2013.
- [12] *New Insulators, Devices and Radiation Effects*. North-Holland, 1999.
- [13] Michael Moll. *Radiation damage in silicon particle detectors: Microscopic defects and macroscopic properties*. PhD thesis, Hamburg U., 1999.

- [14] A. Affolder et al. Radiation damage in the LHCb vertex locator. *JINST*, 8:P08002, 2013.
- [15] Roel Aaij et al. Search for CP violation in the phase space of $D^0 \rightarrow \pi^+\pi^-\pi^+\pi^-$ decays. 2016.
- [16] Bernard Aubert et al. Measurement of CP Violation Parameters with a Dalitz Plot Analysis of $B^\pm \rightarrow D(\pi^+\pi^-\pi^0)K^\pm$. *Phys.Rev.Lett.*, 99:251801, 2007.
- [17] Sheldon L. Glashow. Partial-symmetries of weak interactions. *Nuclear Physics*, 22(4):579 – 588, 1961.
- [18] Steven Weinberg. A model of leptons. *Phys. Rev. Lett.*, 19:1264–1266, Nov 1967.
- [19] Abdus Salam. Weak and Electromagnetic Interactions. *Conf. Proc.*, C680519:367–377, 1968.
- [20] G. Aad et al. Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC . *Physics Letters B*, 716(1):1 – 29, 2012.
- [21] S. Chatrchyan et al. Observation of a new boson at a mass of 125 gev with the cms experiment at the lhc. *Physics Letters B*, 716(1):30 – 61, 2012.
- [22] Kenneth G. Wilson. Confinement of quarks. *Phys. Rev. D*, 10:2445–2459, Oct 1974.
- [23] David J. Gross and Frank Wilczek. Asymptotically free gauge theories. i. *Phys. Rev. D*, 8:3633–3652, Nov 1973.
- [24] David J. Gross and Frank Wilczek. Asymptotically free gauge theories. ii. *Phys. Rev. D*, 9:980–993, Feb 1974.
- [25] C. Patrignani et al. Review of particle physics. *Chin. Phys.*, C40:100001, 2016.
- [26] T. D. Lee and C. N. Yang. Question of parity conservation in weak interactions. *Phys. Rev.*, 104:254–258, Oct 1956.
- [27] C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson. Experimental test of parity conservation in beta decay. *Phys. Rev.*, 105:1413–1415, Feb 1957.
- [28] T. D. Lee, Reinhard Oehme, and C. N. Yang. Remarks on possible noninvariance under time reversal and charge conjugation. *Phys. Rev.*, 106:340–345, Apr 1957.

- [29] Richard L. Garwin, Leon M. Lederman, and Marcel Weinrich. Observations of the failure of conservation of parity and charge conjugation in meson decays: the magnetic moment of the free muon. *Phys. Rev.*, 105:1415–1417, Feb 1957.
- [30] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay. Evidence for the 2π decay of the K_2^0 meson. *Phys. Rev. Lett.*, 13:138–140, Jul 1964.
- [31] S. L. Glashow, J. Iliopoulos, and L. Maiani. Weak interactions with lepton-hadron symmetry. *Phys. Rev. D*, 2:1285–1292, Oct 1970.
- [32] J. J. Aubert, U. Becker, P. J. Biggs, J. Burger, M. Chen, G. Everhart, P. Goldhagen, J. Leong, T. McCorriston, T. G. Rhoades, M. Rohde, Samuel C. C. Ting, Sau Lan Wu, and Y. Y. Lee. Experimental observation of a heavy particle j . *Phys. Rev. Lett.*, 33:1404–1406, Dec 1974.
- [33] J. E. Augustin, A. M. Boyarski, M. Breidenbach, F. Bulos, J. T. Dakin, G. J. Feldman, G. E. Fischer, D. Fryberger, G. Hanson, B. Jean-Marie, R. R. Larsen, V. Lüth, H. L. Lynch, D. Lyon, C. C. Morehouse, J. M. Paterson, M. L. Perl, B. Richter, P. Rapidis, R. F. Schwitters, W. M. Tanenbaum, F. Vannucci, G. S. Abrams, D. Briggs, W. Chinowsky, C. E. Friedberg, G. Goldhaber, R. J. Hollebeek, J. A. Kadyk, B. Lulu, F. Pierre, G. H. Trilling, J. S. Whitaker, J. Wiss, and J. E. Zipse. Discovery of a narrow resonance in e^+e^- annihilation. *Phys. Rev. Lett.*, 33:1406–1408, Dec 1974.
- [34] Makoto Kobayashi and Toshihide Maskawa. Cp-violation in the renormalizable theory of weak interaction. *Progress of Theoretical Physics*, 49(2):652–657, 1973.
- [35] B. Aubert and others. Evidence for $D^0-\bar{d}^0$ mixing. *Phys. Rev. Lett.*, 98:211802, May 2007.
- [36] M. Starič et al. Evidence for $D^0-\bar{d}^0$ mixing. *Phys. Rev. Lett.*, 98:211803, May 2007.
- [37] G. Aad et al. The ATLAS Experiment at the CERN Large Hadron Collider. *JINST*, 3:S08003, 2008.
- [38] S. Chatrchyan et al. The CMS Experiment at the CERN LHC. *JINST*, 3:S08004, 2008.
- [39] K. Aamodt et al. The ALICE experiment at the CERN LHC. *JINST*, 3:S08002, 2008.
- [40] G. Anelli et al. The TOTEM experiment at the CERN Large Hadron Collider. *JINST*, 3:S08007, 2008.

- [41] O. Adriani et al. The LHCf detector at the CERN Large Hadron Collider. *JINST*, 3:S08006, 2008.
- [42] B. Acharya et al. The Physics Programme Of The MoEDAL Experiment At The LHC. *Int. J. Mod. Phys.*, A29:1430050, 2014.
- [43] R. Aaij et al. Measurement of the ckm angle γ from a combination of lhcb results. *Journal of High Energy Physics*, 2016(12):87, Dec 2016.
- [44] R. Aaij et al. Observation of $D^0-\overline{D}^0$ oscillations. *Phys. Rev. Lett.*, 110:101802, 2013.
- [45] R. Aaij et al. Observation of the doubly charmed baryon Ξ_{cc}^{++} . *Phys. Rev. Lett.*, 119:112001, Sep 2017.
- [46] Roel Aaij et al. Observation of J/ψ Resonances Consistent with Pentaquark States in $\Lambda_b^0 \rightarrow J/\psi K^- p$ Decays. *Phys. Rev. Lett.*, 115:072001, 2015.
- [47] Vardan Khachatryan et al. Observation of the rare $B_s^0 \rightarrow \mu^+ \mu^-$ decay from the combined analysis of CMS and LHCb data. *Nature*, 522:68–72, 2015.
- [48] Jon Harrison, Shanzhen Chen, and Chris Parkes. Determination of the operating voltages of the VELO and optimisation of the radiation damage scan data collection. Jul 2015.
- [49] Charles Kittel. *Introduction to Solid State Physics; 8th ed.* Wiley, Hoboken, NJ, 2005.
- [50] A Chilingarov. Generation current temperature scaling. Technical Report PH-EP-Tech-Note-2013-001, CERN, Geneva, Jan 2013.
- [51] Andrew Hickling, Tony Affolder, Kazu Akiba, Martin van Beuzekom, Paula Collins, Marco Gersabeck, Eddy Jans, Malcolm John, Thomas Latham, Alexander Leflat, and Chris Parkes. Use of IT (current vs temperature) scans to study radiation damage in the LHCb VELO. Technical Report LHCb-PUB-2011-021. CERN-LHCb-PUB-2011-021, CERN, Geneva, Oct 2011.
- [52] R. C. Alig and S. Bloom. Electron-hole-pair creation energies in semiconductors. *Phys. Rev. Lett.*, 35:1522–1525, Dec 1975.
- [53] K.V. Ravi. *Imperfections and Impurities in Semiconductor Silicon*. Wiley-Interscience publication. Wiley, 1981.
- [54] LHCb VELO (Vertex LOcator): Technical Design Report, 2001. LHCb-TDR-005.

- [55] S Lochner and M Schmelling. The Beetle Reference Manual - chip version 1.3, 1.4 and 1.5. Technical Report LHCb-2005-105. CERN-LHCb-2005-105, CERN, Geneva, Nov 2006.
- [56] P R Turner. VELO Module Production - Sensor Testing. Technical Report LHCb-2007-072. CERN-LHCb-2007-072, CERN, Geneva, Nov 2007. revised version submitted on 2007-11-06 12:07:30.
- [57] Adam Dane Webber. *Radiation damage studies in the LHCb VELO detector and measurement of the flavour-specific asymmetry in semileptonic B-decays*. PhD thesis, Manchester U., 2013. Presented 21 02 2013.
- [58] G. Kramberger et al. Charge collection studies on custom silicon detectors irradiated up to $1.6 \times 10^{17} n_{eq}/cm^2$. *Journal of Instrumentation*, 8(8):P08004, 2013.
- [59] Roel Aaij et al. Precision luminosity measurements at LHCb. *JINST*, 9(12):P12005, 2014.
- [60] Chris Parkes, Shanzhen Chen, Jolanta Brodzicka, Marco Gersabeck, Giulio Dujany, and William Barter. On model-independent searches for direct cp violation in multi-body decays. *Journal of Physics G: Nuclear and Particle Physics*, 44(8):085001, 2017.
- [61] R. H. Dalitz. On the analysis of tau-meson data and the nature of the tau-meson. *Phil. Mag.*, 44:1068–1080, 1953.
- [62] Bernard Aubert et al. Search for CP Violation in Neutral D Meson Cabibbo-suppressed Three-body Decays. *Phys.Rev.*, D78:051102, 2008.
- [63] R Aaij et al. Search for CP violation in the decay $D^+ \rightarrow \pi^- \pi^+ \pi^+$. *Phys.Lett.*, B728:585–595, 2014.
- [64] R. Aaij et al. Search for CP violation in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ decays with the energy test. *Phys. Lett.*, B740:158, 2015.
- [65] R. Aaij et al. Search for CP violation in the decay $D^+ \rightarrow \pi^- \pi^+ \pi^+$. *Phys. Lett.*, B728:585, 2014.
- [66] J. M. Link et al. Search for T violation in charm meson decays. *Phys. Lett.*, B622:239–248, 2005.
- [67] J. M. Link et al. Study of the decay asymmetry parameter and CP violation parameter in the $\Lambda \rightarrow \Lambda \pi^+$ decay. *Phys. Lett.*, B634:165–172, 2006.
- [68] I. Bediaga, I.I. Bigi, A. Gomes, G. Guerrer, J. Miranda, et al. On a CP anisotropy measurement in the Dalitz plot. *Phys.Rev.*, D80:096006, 2009.

- [69] R. Aaij et al. Search for CP violation in $D^+ \rightarrow K^- K^+ \pi^+$ decays. *Phys.Rev.*, D84:112008, 2011.
- [70] T. Latham, J. Back, and P. Harrison. Laura++ Dalitz plot fitting package. <https://laura.hepforge.org/>. [Accessed 10-June-2014].
- [71] Mike Williams. Observing CP violation in many-body decays. *Phys.Rev.*, D84:054015, 2011.
- [72] R Aaij et al. Measurement of CP violation in the phase space of $B^\pm \rightarrow K^\pm \pi^+ \pi^-$ and $B^\pm \rightarrow K^\pm K^+ K^-$ decays. *Phys.Rev.Lett.*, 111:101801, 2013.
- [73] Roel Aaij et al. Measurement of CP violation in the phase space of $B^\pm \rightarrow K^+ K^- \pi^\pm$ and $B^\pm \rightarrow \pi^+ \pi^- \pi^\pm$ decays. *Phys.Rev.Lett.*, 112(1):011801, 2014.
- [74] Frank C. Porter. Testing consistency of two histograms. 2008.
- [75] G Fasano and A. Franceschini. A multidimensional version of the Kolmogorov-Smirnov test. *Mon. Not. R. astr. Soc.*, 225:155–170, 1987.
- [76] Mike Williams. How good are your fits? Unbinned multivariate goodness-of-fit tests in high energy physics. *JINST*, 5:P09004, 2010.
- [77] B. Aslan and G. Zech. New test for the multivariate two-sample problem based on the concept of minimum energy. *J. Stat. Comput. Simul.*, 75:109–119, 2005.
- [78] B. Aslan and G. Zech. Statistical energy as a tool for binning-free, multivariate goodness-of-fit tests, two-sample comparison and unfolding. *Nucl. Instrum. Meth.*, A537:626 – 636, 2005.
- [79] P. R. Rosenbaum. An exact distribution-free test comparing two multivariate distributions based on adjacency. *J. R. Statist. Soc. B*, 67:515–530, 2005.
- [80] R. Aaij et al. Search for CP violation in the phase space of $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ decays. *Phys. Lett.*, B769:345, 2017.
- [81] Eero Byckling and K. Kajantie. *Particle Kinematics*. University of Jyvaskyla, Jyvaskyla, Finland, 1971.
- [82] J. M. Link et al. Study of the $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decay. *Phys. Rev.*, D75:052003, 2007.
- [83] M. Artuso et al. Amplitude analysis of $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$. *Phys. Rev.*, D85:122002, 2012.

- [84] R. Aaij et al. Search for CP violation using T -odd correlations in $D^0 \rightarrow K^+ K^- \pi^+ \pi^-$ decays. *JHEP*, 10:005, 2014.
- [85] German Valencia. Angular correlations in the decay $B \rightarrow VV$ and CP violation. *Phys. Rev. D*, 39:3339–3345, Jun 1989.
- [86] Philippe d’Argent, Jack Benton, Jeremy Dalseno, Evelina Gersabeck, Sam Harnew, Paras Naik, Claire Prouve, Jonas Rademacker, and Nicola Skidmore. Amplitude analysis of $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$ decays using CLEO-c data. In *8th International Workshop on Charm Physics (Charm 2016) Bologna, Italy, September 5-9, 2016*, 2016.
- [87] NVIDIA Corporation, *Thrust quick start guide*, DU-06716-001 (2014).
- [88] J. Brodzicka et al. Energy test implementation on GPUs. <https://www.hepforge.org/>. [Accessed 26-August-2016].
- [89] Jolanta Brodzicka, Shanzhen Chen, Marco Gersabeck, and Christopher Parkes. A model-independent un-binned search for CP violation in $D^0 \rightarrow \pi^- \pi^+ \pi^0$ decays. Mar 2014.
- [90] Yuval Grossman, Alexander L. Kagan, and Yosef Nir. New physics and CP violation in singly Cabibbo suppressed D decays. *Phys.Rev.*, D75:036008, 2007.
- [91] Yuval Grossman, Alexander L. Kagan, and Jure Zupan. Testing for new physics in singly Cabibbo suppressed D decays. *Phys.Rev.*, D85:114036, 2012.
- [92] R. Aaij et al. LHCb detector performance. *Int. J. Mod. Phys.*, A30:1530022, 2015.
- [93] Mika Vesterinen, Chris Parkes, and Kevin Maguire. Stripping lines for prompt tagged $D^0 \rightarrow hh\pi^0$ decays. Technical Report LHCb-INT-2013-049. CERN-LHCb-INT-2013-049, CERN, Geneva, Aug 2013.
- [94] O Deschamps, F P Machefert, M H Schune, G Pakhlova, and I Belyaev. Photon and neutral pion reconstruction. Technical Report LHCb-2003-091, CERN, Geneva, Sep 2003.
- [95] L. Breiman, J. H. Friedman, R. A. Olshen, and C. J. Stone. *Classification and regression trees*. Wadsworth international group, Belmont, California, USA, 1984.
- [96] Robert E. Schapire and Yoav Freund. A decision-theoretic generalization of on-line learning and an application to boosting. *J. Comput. Syst. Sci.*, 55:119, 1997.

- [97] Muriel Pivk and Francois R. Le Diberder. sPlot: A statistical tool to unfold data distributions. *Nucl.Instrum.Meth.*, A555:356–369, 2005.
- [98] Angelo Carbone, Vincenzo Vagnoni, Matthew Charles, and Guy Wilkinson. Search for time-integrated CP violation in $D^0 \rightarrow K^- K^+, \pi^- \pi^+$ in the 2011 data. Feb 2012. LHCb-ANA-2011-059, Linked to LHCb-ANA-2011-010.
- [99] Shanzhen Chen, Marco Gersabeck, Chris Parkes, and Jolanta Brodzicka. Energy test analysis of $D^0 \rightarrow \pi^+ \pi^- \pi^+ \pi^-$. Sep 2015.
- [100] Y. Amhis et al. Averages of b -hadron, c -hadron, and τ -lepton properties as of summer 2014. 2014. updated results and plots available at <http://www.slac.stanford.edu/xorg/hfag/>.
- [101] R. Aaij et al. Model-independent search for CP violation in $D^0 \rightarrow K^- K^+ \pi^+ \pi^-$ and $D^0 \rightarrow \pi^- \pi^+ \pi^- \pi^+$ decays. *Phys. Lett.*, B726:623, 2013.
- [102] Alakabha Datta and David London. Triple-product correlations in $B \rightarrow V_1 V_2$ decays and new physics. *Int. J. Mod. Phys.*, A19:2505–2544, 2004.
- [103] K. A. Olive et al. Review of particle physics. *Chin. Phys.*, C38:090001, 2014. and 2015 update.
- [104] T. Aaltonen et al. Measurement of CP-violation asymmetries in $D^0 \rightarrow K_S^0 \pi^+ \pi^-$. *Phys. Rev.*, D86:032007, 2012.
- [105] Roel Aaij et al. Measurements of prompt charm production cross-sections in pp collisions at $\sqrt{s} = 13$ TeV. *JHEP*, 03:159, 2016. [Erratum: JHEP05,074(2017)].
- [106] Roel Aaij et al. Measurement of the b -quark production cross-section in 7 and 13 TeV pp collisions. *Phys. Rev. Lett.*, 118(5):052002, 2017.
- [107] Roel Aaij et al. Measurement of forward J/ψ production cross-sections in pp collisions at $\sqrt{s} = 13$ TeV. *JHEP*, 10:172, 2015. [Erratum: JHEP05,063(2017)].
- [108] Roel Aaij et al. Measurement of matter-antimatter differences in beauty baryon decays. *Nature Phys.*, 13:391–396, 2017.