Antiproton Beam Extraction for the ALPHA Antihydrogen Experiment CERN Summer Student Project Report

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The ALPHA experiment produces antihydrogen atoms by slowly merging cold plasmas of positrons and antiprotons inside a nested Penning-Malmberg trap. During each experimental cycle, antiprotons are delivered from the the CERN Antiproton Decelerator (AD) in a dedicated Penning trap known as the Catching Trap (CT) before being transferred to other parts of the apparatus. There is around 20% loss of antiprotons during particle transfers in each experimental cycle, this antiproton loss reduces the antihydrogen atoms produced during each cycle. In order to fix this, simulations were ran in a simplified section of the current ALPHA antiproton catching trap in order to produce short pulsed beams with narrow energy distributions. A relation of between the longitudinal emittance of the antiproton beam and the initial Penning trap voltages was studied, along with proper time constants, initial voltages and particle numbers for running future simulations.

1. INTRODUCTION

Based at the CERN Antiproton Decelerator (AD), ALPHA has been conducting experiments since 2005. ALPHA's primary goal since 2012 has been the spectroscopy of antihydrogen to a precision matching that of normal hydrogen[1] .ALPHA's most recent achievements characterization of the 1S–2S transition in antihydrogen[2] and the observation of the 1S–2P Lyman-α transition in antihydrogen[3].

ALPHA's source of antiprotons is the Antiproton Decelerator(AD), which delivers 100-200ns long bunches of approximately 3×10^7 antiprotons at 5.3MeV every 100s. The antiprotons are further slowed by allowing them to pass through an aluminium foil. Antiprotons are stored in a special high-voltage Penning trap called the Catching Trap (CT)[1]. The catching trap is compromised of 11 electrodes, the first and ninth electrodes are designed to support high voltages to catch the antiproton bunches. In order to catch the antiprotons, the ninth electrode, referred to as HVB, is given a voltage of 4kV and 50ns after the antiprotons arrive, the first electrode, referred to as HVA is given the same voltage. This results in around 10^5 captured antiprotons from the initial AD output.[1] It is important to note that for the purposes of this project we already have the antiprotons stored in the catching trap. Section 3 describes a simplified version of the current ALPHA antiproton catching trap, table 1 refers to the electrodes represents the voltages of the simulated trap, wich is shown on figure 4.

Antiproton bunches are extracted from the CT at an energy of 50eV, with a wide energy spread on the order of 1.5eV[1]. This spread expands the bunches longitudinally when the antiprotons are transferred to different parts of the apparatus. When the aniprotons arrive at the antihydrogen mixing traps, the bunch is spread in the time domain, resulting in a loss of 20% of the antiprotons as they cannont be captured and are lost. Given that the experiments will soon be turned back online, it is important to run simulations with the objective to reduce this loss of antiprotons in preparation for the end of the current shutdown in 2021 . This project aims to optimise the Penning trap voltages used to extract antiprotons from the catching trap, to create short pulsed beams with very narrow energy distributions.

To run the simulation a software capable of producing complex electric fields and calculating the trajectory of charged particles inside of them is required as the electric potential inside a real Penning trap is created by applying voltages to cylindrical electrodes. This means that the electric potential can only be found by solving the Laplace equation with complex boundary conditions, this solution too complex to do analytically. This is why the SIMION[5] software is used for the simulations.

From 2021, ALPHA will receive more antiprotons after ELENA (Extra Low ENergy Antiproton Ring). Elena is capable of slow down the antiprotons to 0.1MeV, aditionally the density of the beams will be improved, resulting in an increase of the efficiency with which the experiments can capture the antiprotons in their traps by a factor of 10 to 100. [4]

2. THEORY

2.1. Penning Traps

Penning-Malmberg traps are used to capture charged particles. These devices are constructed from hollow, cylindrical electrodes, a voltage is applied to the electrode configuration to generate an electric field, while a homogeneous magnetic field points in the same direction as the axis of the trap. The electric field prevents the atoms inside the trap to escape axially, while the magnetic field prevents a radial escape.[6]

The electrodes have a hollow cylinder shape, with voltages around the 100V mark. While the catching trap has electrodes that reach up to 5kV[1], the electrode voltages are limited to ±100V to prevent electrical discharge between neighbouring electrodes as this would cause damage to the trap and degrade the vacuum inside of the experiment. The magnetic field necessary to trap the particles varies from 0.1T to 10T.

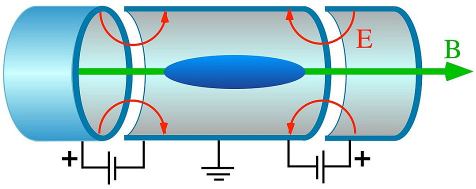


FIG. 1: Diagram of a Penning-Malmber trap, the magnetic field (green line) points in the direction of the trap axis.[7]

This configuration makes it possible to use several combinations of voltages to create complex potentials. Due to the ability to set every single parameter of the trap, Penning traps are useful when it comes to make a detailed analysis of the behaviour of different particles inside of it.

The electric potential and magnetic field inside the an ideal Penning trap are defined by the following equation, for a trap that has its axis on the z axis

$$\begin{cases} \phi(z,r) = V_R C(z^2 - \frac{r^2}{2}) \\ \vec{\mathbf{B}} = B_0 \hat{\mathbf{z}} \end{cases} \tag{1}$$

V_R represents the voltage at certain position, ϕ is the electric potential and C is a constant.

The motion of the particle inside of the trap is described in the following image

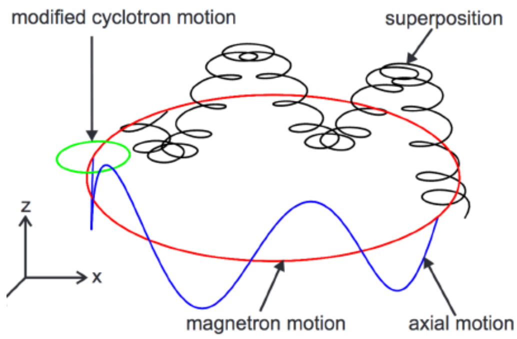


FIG. 2: Typical motion inside of a Penning-Malmberg trap[8]

Along the axis of the trap,the particle oscillates along the trap axis at the bounce frequency, defined by the equation:

$$\omega_x = \sqrt{2V_R C \frac{q}{m}} \tag{2}$$

q acts as the charge of the particle and m is the mass of said particle. The magnetic field causes charged particles to undergo a two-frecuency oscillation perpendicular to the trap axis with frecuencies:

$$\omega_\pm = \frac{\omega_a}{2} \pm \sqrt{(\frac{\omega_a}{2})^2 - \frac{\omega_x^2}{2}} \tag{3}$$

ω_+ corresponds to the fast component of the oscillation and is known as the modified cyclotron frequency, while ω_- is associated with the slow component and it gives the magnetron frequency. Lastly, the term ω_a describes the angular frecuency

$$\omega_a = \frac{qB}{m} \tag{4}$$

Where B is the magnitude of the magnetic field.

At ALPHA, antiprotons are released as beams by quickly changing the electric potential of the penning trap, this change takes approximately $1\mu s$. This voltage change is done to the fifth electrode and is described in equation 8. Changing only the fifth electrode prevents the particles from escaping in the opposite direction as the fourth electrode remains with the same voltage shown on table 1, this makes the escape of the antiprotons happen in the desired direction. The energy of the extracted beam is determined by the temperature of the plasma cloud, given that temperatures are around a few hundred Kelvin[1], the energy of the beam is usually 0.03eV.

2.1.1. *Rotating Wall Technique*

On realistic scenarios, plasma traps have asymmetries that expand the plasma. To compensate this expansion, electrical signals at a frequency f_{RW} drive sectors of an electrode surrounding an axial portion of the plasma. The electric field induces a dipole moment, which results in a torque. This torque mitigates the expansion of the plasma.[7]

2.2. Boltzmann Equation

In order to run the simulations with many particles it was necessary to develop a method in order to create random data for all of the particles.

The Boltzmann equation is an important result in statistical mechanics and is used to sample the initial positions and velocities of the particles from a distribution that reflects a trapped non-neutral plasma at thermal equilibrium

$$P(E) = Se^{-\frac{1}{k_b T}E} \tag{5}$$

Being[9]:

- **P(E)**: The probability of having an energy in a small range between E and $E + \delta E$.

- **S**: A scaling factor.
- $e^{-\frac{1}{k_b T}E}$: Known as the Boltzmann factor, this corresponds to the probability distribution. k_b represents the Boltzmann constant, T is the temperature measured in Kelvin and E represents the energy.

For the purposes of this experiment the scaling factor is chosen to that the probability of finding a particle with any energy of $E > 0$ is equal to one, figure 6 shows how does the Boltzmann factor behaves. By manipulating the parameters of the Boltzmann factor, it is possible to find the probability distribution for the position and kinetic energy of each of the particles in the simulation.

2.3. Phase Space

SIMION is capable of providing detailed information about the particle at certain time steps or positions, depending on what the user asks the program to do. While this provides the necessary data to see how does the particle act during the simulations, analysing the data might prove difficult. For this project, the data was taken at the start of the simulation and when the particle reached the end of the trap. The aim of the project is to reduce the spread of the beam. In this case, a visual representation of the data is preferred. To achieve said representation, the phase space is a valuable tool for analysing the data.

By selecting two variables, in this case the energy and the time of arrival, it is possible to analyse the data gathered from the simulations using a phase space. In a phase space a point P(E,t) represents the state of a particle at a given energy and time. The phase space corresponding to P(E,t) is called the longitudinal phase space. The complete path in the phase space constitutes the phase diagram.[10]

2.3.1. Twiss Parameters

In particle physics the Louville Theorem[10] is applied, this means that the area of the phase space, the emittance, is approximately conserved during beam transport. This makes the emittance a good measure of the spread of particles in the phase space.For this reason, the generated phase space usually is analysed fitting an ellipse to it, as seen in figure 3. This ellipse is defined by the following equation

$$\gamma x + 2\alpha x x\prime + \beta x\prime^2 = \epsilon \quad (6)$$

The parameters $\gamma, \alpha, \beta, \epsilon$ are known as the twiss parameters, the combination of these parameters determines the dimensions of the ellipse as shown in the following figure:

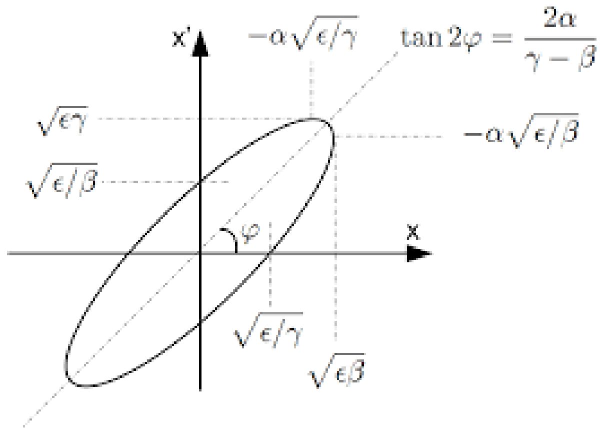


FIG. 3: Dimensions of the ellipse in terms of the twiss parameters

The term ϵ is known as the emittance, this parameter indicates the area of the phase space, divided by π[11]. For a fixed emittance, we have that

- The term β is related to the spread of particles in the time domain. This relation is given by the equation $\sqrt{\beta\epsilon}$, shown in figure 3
- γ corresponds to the spread of particles in the energy domain.
- Finally, α tells us how tilted the ellipse is going to be.

3. NUMERICAL SIMULATIONS

3.1. Configuring the trap

The penning trap used in the final set of simulations consisted of 9 electrodes. Electrodes had a radius of 15mm and an axial length of 13mm. A 2mm gap was left between each of the electrodes. This configuration represents a simplified section of the current ALPHA antiproton catching trap.

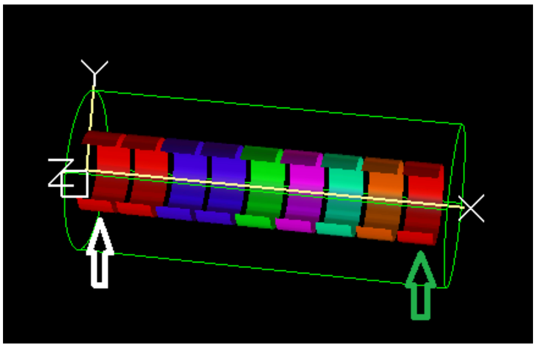


FIG. 4: Side view of the Penning trap. The first electrode is marked by the white arrow, while the ninth electrode is marked by the green arrow.

Before their extraction into the ALPHA beamline, antiprotons are held inside the catching trap using an asymmetric electrostatic potential well (Figure 5). Table 1 lists the voltages that were applied to the nine electrodes to produce this electric potential along the trap axis.

Antiprotons are stored in a potential well of 20.64846276V. Inside the potential, the antiprotons oscillate with a theoretical bounce frecuency of 4.3MHz. Once the voltage was changed using equation 8, this theoretical value changed to 1.89MHZ

Electrode	Initial Voltage[V]
1	0
2	-140
3	-140
4	100
5	100
6	-45
7	-40
8	0
9	0

TABLE I: Voltages applied to the nine Penning trap electrodes at the start of the simulation. The combination of voltages produced the on-axis electric potential shown on figure 5. In order to extract the particles as a beam, the voltage on the electrode 5 is changed during the simulation.

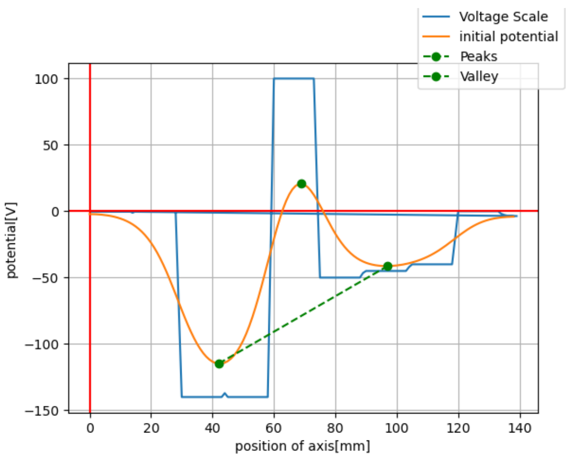


FIG. 5: Electric potential on the axis used to trap the antiprotons at the beginning of the reference simulation. The potential well is created using voltages given in table 1

In order to release the antiprotons from the potential well, the voltage of the fifth electrode is changed to -51V. Finally, the trap has a homogeneous magnetic field of 3T along the axis of the trap, in this case the x axis. The SIMION resolution was set to 1mm and the simulation was tasked to record particle data when the particles are initialized and when they reach the edge of the simulation.

3.2. First Simulation

The first simulation consisted of a single anti-proton that was placed in the fifth electrode, 69mm along the x axis in the simulation coordinate system shown in figure 4. The particle was given an energy of 0.03eV and left to oscillate for 10μs and then the trap would be turned off, meaning that all of the voltages would change immediately to 0V.By fitting a sine wave to the motion of the particle along the x axis, the axial bounce frequency of 4.00MHz was extracted.

The software measured the positions along the x, y and z axis, the total time of flight, its angles, the potential in that specific time step and the kinetic energy.

Once the error percentage was deemed acceptable and the motion of the anti-proton was similar to the expected motion, the data was used to generate more complex simulations.

3.3. Generating data

In order to perform realistic simulations of the antiproton plasma, the simulations included many particles. The initial positions were found by creating an array covering the length of the trap and solving the Boltzmann equation (equation 5) to determine the probability of finding a particle in a certain position. The temperature of the beam was set at $10^4 K$, which equates to an average energy of 0.8eV, the kinetic energy was assigned by creating a "pool" of acceptable kinetic energies and once again solving the Boltzmann equation to determine the probability of finding a particle with a certain kinetic energy.Since the radial size of the antiproton cloud can be controlled using the rotating wall technique, we assume that particles are normally distributed around the trap axis with a typical radial size of 0.4 mm. This size is consistent with experimental measurements made at ALPHA during 2018. When solving the Boltzmann equation to determine the initial position of the particles, only the axial position was accounted for. This is because, given the small radial size of the antiproton cloud (0.4mm), it is possible to assume that all particles remain close to the simulation axis.

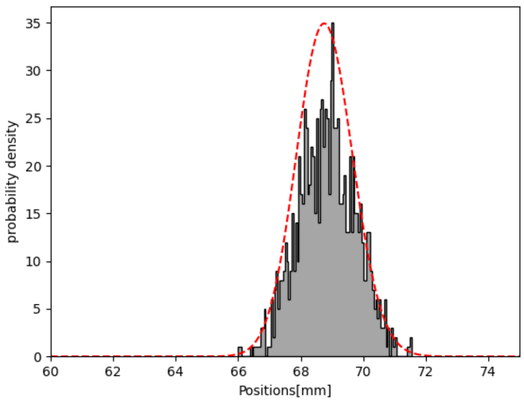


FIG. 6: Representation of the distribution of positions for 10^3 particles. The grey histogram represents the probability that a particle would be found in a certain position, while the dashed line gives a general outline of the distribution of the probabilities, which matches the Gaussian distribution[9]. The figure shows that particles are more likely to be located at the center of the trap and the probability decreases for positions located in sections far from the center.

3.4. Simulation Parameters

For these simulations, the anti-protons were left to oscillate for 4μs, this time is long enough for the particle to complete around 10 bounce oscillations, while the voltage of the fifth electrode was changed as a function of time, according to the following shape function

$$S_f(t_) = \frac{1}{e^{\frac{t_o - t_f}{t_c}} + 1} \tag{7}$$

With the variables being:

- S_f: The shape function
- t_o: Time when the electrode voltage is switched
- t_f: Time of flight
- t_c: Time constant

The voltage applied to the fifth Penning trap electrode is then defined as

$$V_5(t) = V_0(1 - S_f) + V_f S_f \tag{8}$$

In this case V_0 and V_f represent the initial and final voltages, respectively.

The last equation is of extreme importance. If the voltage switch happened immediately the antiprotons would find themselves with a sudden increase of their kinetic energy and they might have enough energy to escape the 100V wall set by the fourth electrode. By reducing the voltage periodically, the area of oscillation is reduced and the antiprotons are pushed to the edge of the potential well, ensuring that the antiprotons are released in the desired direction.

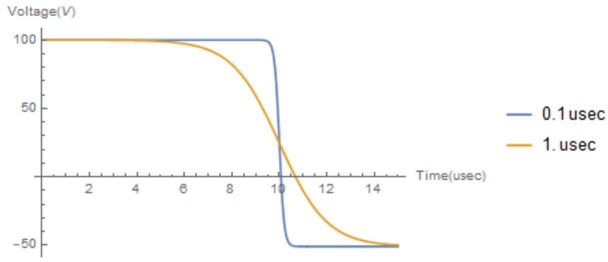


FIG. 7: Voltage behavior. Equation 8 is plotted as a function of time, the yellow line is the function with a time constant of $1\mu s$, while the blue line has a time constant of $0.1\mu s$. The blue line represents the ideal behavior of equation 8, as the voltage needs to be at -51V at the exact time mark. In the case of a high time constant, the voltage does not reach the desired voltage at the necessary time.

The following parameters were modified during each simulation

- Particle count: Simulations were run for ranges of 1000 to 10000 particles
- Initial voltage on the fifth electrode: A voltage range from 75V to 125V in increments of 5V. Some fine tuning around the 90V to 110V in 1V increments was also performed.
- Time constant: Different time constants were tested ranging from 0.01μs to 1μs. It is important to note that the time constant is a property of the electrode power supplies and other hardware used at ALPHA. The simulations with different time constants were used to check how much does the time constant limits the capability to extract beams. Changing the time constant requires hardware development beyond the scope of this project.

It is important to note that any possible relativistic effects are neglected due to the low energies of the particles resulting in velocities slower than the speed of light. Given the number of antiprotons in the trap, the space charge interaction can also be neglected.

4. DATA ANALYSIS

4.1. Motion of the anti-proton

Along the axis of the trap (the x axis in this simulations), the antiproton had a sine wave behavior. By doing a superposition of the motion on the y and z axis, the result is what is expected for the magnetron motion of a particle inside of a penning trap. In regards of the bounce frequency, a theoretical value of 1.89MHz was calculated. Using the data fit method to obtain the actual frequency of the simulation a 1.95MHz frequency was measured, resulting in a 1.5% error, a reason for this error can be that the electric potential in SIMION is not exactly a quatratic function as equations 1 and 2 assume.

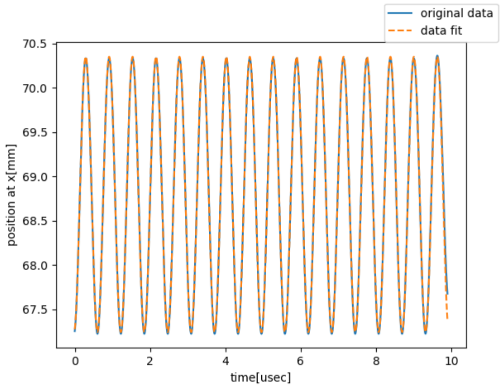


FIG. 8: Axial position of a single antiproton as a function of time, this motion was evaluated within the first $10\mu s$ of a SIMION simulation. After that time, the electrode voltage is altered so that the particle is no longer trapped. A sine wave has been fitted to the simulation data to extract the axial bounce frecuency ω_x (equation 2)

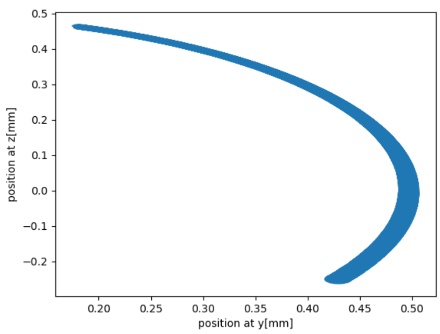
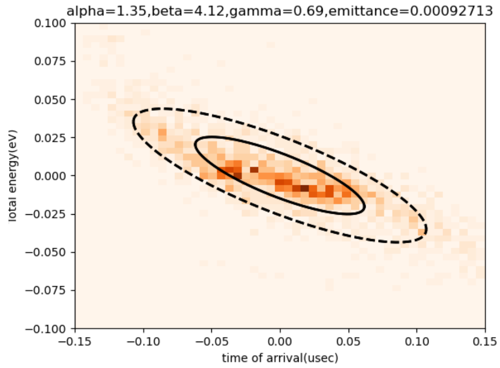
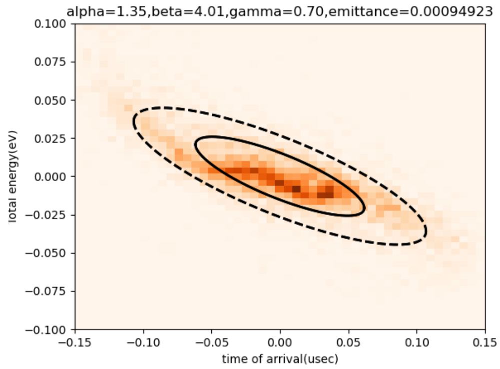


FIG. 9: Trajectory of a typical particle in the transverse (y, z) plane of the Penning trap

After performing simulations with different number of particles, it was decided that the simulation using 7000 particles provided the best balance in terms of Our ability to clearly resolve the distribution of particles in the longitudinal (t, E) beam phase space and time to perform it, about 45minutes.



(a) Sample of a 2k particles simulation



(b) Sample of a 7k particles simulation

FIG. 10: Two-dimensional histograms showing the longitudinal phase space of a simulated antiproton bunch at the end of the Penning trap. The histograms shown correspond to two identical simulations with a)2000 and b)7000 particles.

As figures 10.a and 10.b show, the less amount of particles represent a much less defined image. This can be noticed by the corners of each image, as the evidence of a filamentation effect is observed more easily when the amount of particles is higher, this filamentation is due to non linear forces acting in the simulation.

When it comes to the shape function behavior, the desired result is to have a voltage drop in an instant, for this, a time constant of $0.1\mu s$ presented the ideal behavior, higher or lower time constants left the shape function with a shape that prevented the voltage to finish the switch at the desired time.

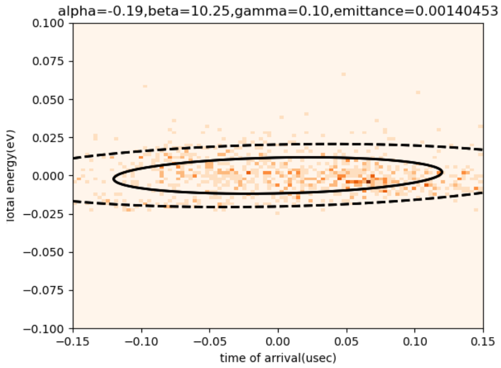


FIG. 11: Two-dimensional histogram showing the longitudinal phase space of an antiproton bunch at the end of the Penning trap. The simulation of the shown histogram was performed with a time constant of $0.1\mu s$ for the porpoises of determining how much does the time constant limit the capability of extracting beams. While the particles are not so spread in the energy domain, the time domain presents a spread that higher than the spread present in other histograms.

In order to analyze the data provided by the simulations, special consideration was taken with the emittance value. Figure 12 shows the longitudinal emittance as a function of the initial voltage applied to electrode 5.

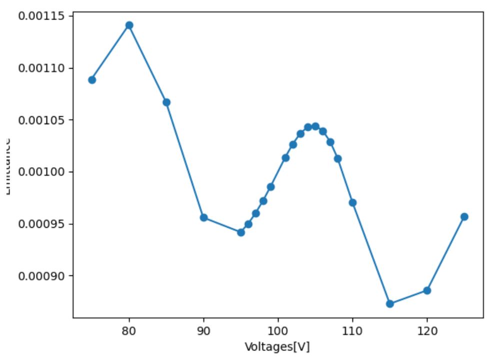


FIG. 12: Emittance summary plot. The emittance is plotted as a function of the initial voltage on electrode 5. A wave-like pattern is observed with the additional detail that higher initial voltages produce deeper potential wells.

A wave-like pattern is visible in the general behavior of the emittance, a "fine tuning" was performed around the 95V-110V in order to detail the behavior of this pattern, below the phase space plot corresponding the the minimum, half way point and maximum are shown. More investigation is needed to discover the motive behind this behaviour, one possible explanation can be that the phase of each particles bounce frequency is the same, resulting in the particles move in the same direction just before leaving the trap producing a smaller spread of velocities. This can be the result of simulation errors.

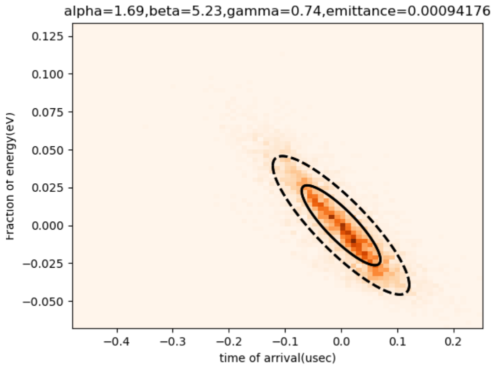


FIG. 13: Phase space at 95V

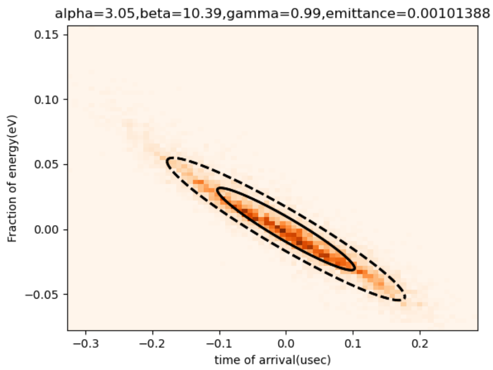


FIG. 14: Phase space at 101V

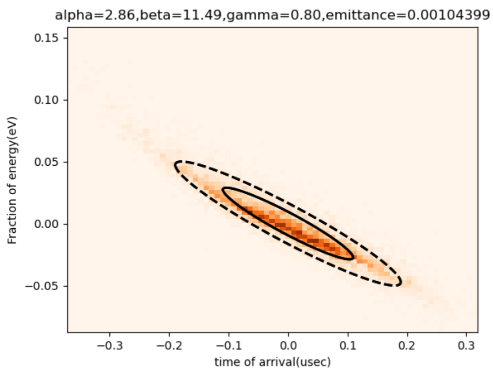


FIG. 15: Phase space at 105V

The results shown in the previous images are for the longitudinal emittance. The black solid line represents the ellipse fit to the phase space, while the dashed line represents the edge of the distribution at two standard deviations. For particles with a normal distribution this dashed line should enclose about 98% of the particles. For these simulations, the dashed line does not enclose most of the particles and has a lot of free space inside of it. The filamentation effect is responsible for these behaviour. The beam parameters have units of $\frac{\mu s}{eV}$, while the emittance has units of μseV

As it is shown, the filamentation effect appears to be more prevalent in the highest points of the wave. Starting from the lowest point, the 95V simulation, the phase space tends to rotate and stretch. It is worth noting that the area with the highest concentration remains at the same position in terms of the fraction of energy, while there is a small increment in the time of arrival, both for the late arriving particles and the early arriving particles. The most relevant movement is the previously mentioned filamentation.

5. CONCLUSIONS AND RECOMMENDATIONS

- A Penning trap with dimensions matching the ALPHA catching trap was constructed, this trap is able to simulate realistic electrostatic potentials.
- By solving the Boltzmann equation to obtain positions, energy and kinetic energy, random data for many particles was provided successfully. As a recommendation, doing a 2d interpolation for the potential might result in more accurate values as the radial contribution is taken into account.
- Optimal particle number and time constant value was successfully found for further simulations.
- An interesting behavior was found as the initial voltage of the fith electrode was changed. Further testing to discard any human error on this behavior is recommended.
- The simulations were ran at $10^4 K$, normal ALPHA temperatures are at 345K[1]. While the data from figure 12 depicts the general behavior of the emittance, the experiment should be re-run to verify the behavior at lower temperatures.

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