



CLIC – Note – 989

OPTICS DESIGN OF INTRABEAM SCATTERING DOMINATED DAMPING RINGS

Fanouria Antoniou (CERN)

Abstract

A e^+e^- linear collider, the Compact Linear Collider (CLIC) is under design at CERN, aiming to explore the terascale particle physics regime. The collider has been optimized at 3 TeV center of mass energy and targets a luminosity of $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$. In order to achieve this high luminosity, high intensity bunches with ultra low emittances, in all three planes, are required.

The generation of ultra low emittance is achieved in the Damping Rings (DR) complex of the collider. The large input beam emittances, especially the ones coming from the positron source, and the requirement of ultra low emittance production in a fast repetition time of 20 ms, imply that the beam damping is done in two stages. Thus, a main-damping ring (DR) and a predamping ring (PDR) are needed, for each particle species. The high bunch brightness gives rise to several collective effects, with Intra-beam scattering (IBS) being the main limitation to the ultra-low emittance. This thesis elaborates the lattice design and non-linear optimization of a positron pre-damping ring and the lattice optimization of a damping ring, under the influence of IBS. Several theoretical models, describing this effect, are discussed and compared for different lattices, while two multi-particle tracking algorithms are bench-marked with the theoretical models. Finally, IBS measurement results, at the Swiss Light Source (SLS) and the Cornell electron storage ring Test Accelerator (Cesr-TA), are presented and compared with theoretical predictions



Optics design of Intrabeam Scattering dominated damping rings

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for the degree of Doctor of Philosophy in Physics

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ΕΘΝΙΚΟ ΜΕΤΣΟΒΙΟ ΠΟΛΥΤΕΧΝΕΙΟ
ΣΧΟΛΗ ΕΦΑΡΜΟΣΜΕΝΩΝ ΜΑΘΗΜΑΤΙΚΩΝ
ΚΑΙ ΦΥΣΙΚΩΝ ΕΠΙΣΤΗΜΩΝ

Βέλτιστη οπτική σχεδίαση των δεσμών του
νέου γραμμικού επιταχυντή CLIC
λαμβάνοντας υπόψη τα φαινόμενα
ενδοσκέδασης των σωματιδίων της δέσμης

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ΑΘΗΝΑ, Ιανουάριος 2013



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ΑΘΗΝΑ, Ιανουάριος 2013

*To my husband and my son
To Iasonas, Nefeli, Sofia, Nikos and Andreas
for bringing light to our lifes*

*Στο σύζυγο μου και στο γιο μου
Στον Ιάσωνα, στη Νεφέλη, στη Σοφία, στο Νίκο και στον Αντρέα
που γεμίζουν με φως τη ζωή μας*

Abstract

A e^+/e^- linear collider, the Compact Linear Collider (CLIC) is under design at CERN, aiming to explore the terascale particle physics regime. The collider has been optimized at 3 TeV center of mass energy and targets a luminosity of $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$. In order to achieve this high luminosity, high intensity bunches with ultra low emittances, in all three planes, are required. The generation of ultra low emittance is achieved in the Damping Rings (DR) complex of the collider. The large input beam emittances, especially the ones coming from the positron source, and the requirement of ultra low emittance production in a fast repetition time of 20 ms, imply that the beam damping is done in two stages. Thus, a main-damping ring (DR) and a pre-damping ring (PDR) are needed, for each particle species. The high bunch brightness gives rise to several collective effects, with Intra-beam scattering (IBS) being the main limitation to the ultra-low emittance. This thesis elaborates the lattice design and non-linear optimization of a positron pre-damping ring and the lattice optimization of a damping ring, under the influence of IBS. Several theoretical models, describing this effect, are discussed and compared for different lattices, while two multi-particle tracking algorithms are bench-marked with the theoretical models. Finally, IBS measurement results, at the Swiss Light Source (SLS) and the Cornell electron storage ring Test Accelerator (Cesr-TA), are presented and compared with theoretical predictions.

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Περίληψη

Ένας γραμμικός συγκρουστήρας ηλεκτρονίων-ποζιτρονίων (e^+/e^-), ο Γραμμικός Συμπαγής Συγκρουστήρας (*Compact Linear Collider: CLIC*), είναι υπο μελέτη στο *CERN* με σκοπό τη μελέτη της φυσικής των υψηλών ενεργειών στο φάσμα των τερα-ηλεκτρονβολτ. Η βελτιστοποίηση της μελέτης έχει γίνει για ενέργεια κέντρου μάζας των 3 TeV και φωτεινότητα των $10^{34}\text{ cm}^{-2}\text{s}^{-1}$. Στο Σχήμα 1 παρουσιάζεται η σχηματική αναπαράσταση του συμπλέγματος του Συμπαγούς Γραμμικού Συγκρουστήρα.

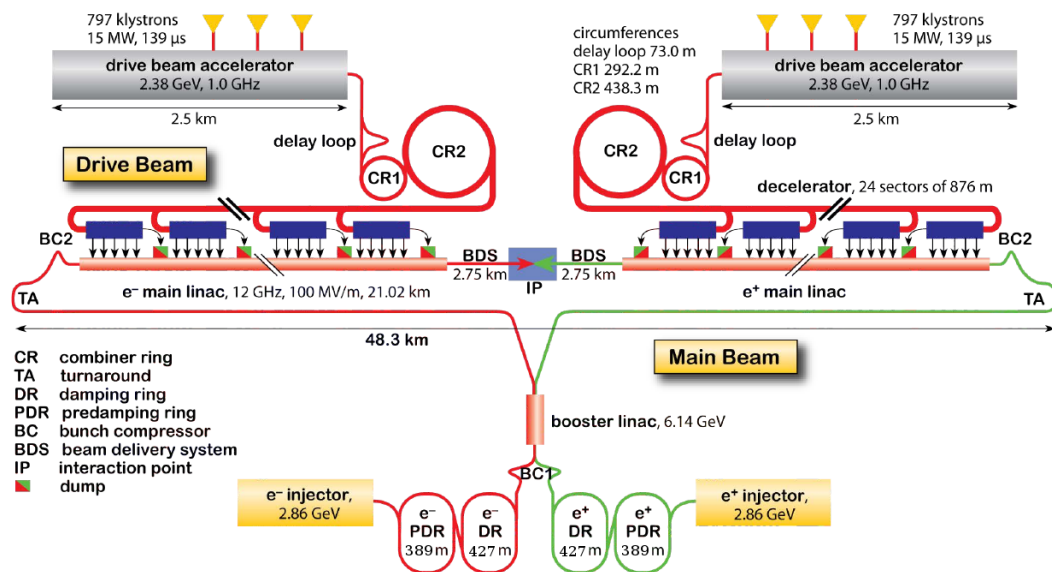


Figure 1: Σχηματική αναπαράσταση του συμπλέγματος του Συμπαγούς Γραμμικού Συγκρουστήρα.

Για την επίτευξη της μεγάλης φωτεινότητας, μεγάλης έντασης και μικρής εκπεμπικότητας πακέτα σωματιδίων της δέσμης είναι απαραίτητα. Ο σκοπός των δακτυλίων απόσβεσης είναι η παραγωγή αυτών των πολύ μικρής εκπεμπικότητας και μεγάλης έντασης πακέτων σωματιδίων. Οι προκλήσεις του σχεδιασμού τους καθορίζονται από τις παραμέτρους του επιταχυντή αλλά και των συστημάτων που προηγούνται και αυτών που ακολουθούν το σύστημα των δακτυλίων απόσβεσης. Η μεγάλη εκπεμπικότητα των δεσμών στην είσοδο των δακτυλίων απόσβεσης, ειδικά για τις δέσμες ποζιτρονίων, η απαίτηση πολύ μικρής εκπεμπικότητας στην έξοδο και ο ταχύς ρυθμός επανάληψης των 50 Hz , απαιτούν ότι η απόσβεση πρέπει να γίνει σε δύο στάδια, με ένα δακτύλιο προ-απόσβεσης (*PDR*), και ένα κύριο δακτύλιο απόσβεσης (*DR*) για κάθε είδος σωματιδίου. Επιπλέον, λόγω της εξαιρετικά χαμηλής εκπεμπικότητας και μεγάλης έντασης σωματιδίων της δέσμης, το φαινόμενο της ενδοδεσμικής σχέδασης γίνεται το κυρίαρχο φαινόμενο, περιορίζοντας την απόδοση των δακτυλίων. Η παρούσα διατριβή παρουσιάζει το σχεδιασμό και τη βελτιστοποίηση της οπτική του μαγνητικού πλέγματος των δακτυλίων προ-απόσβεσης και των κυρίως δακτυλίων απόσβεσης του *CLIC*, με παραμέτρους δέσμης για τις οποίες το φαινόμενο της ενδοδεσμικής σχέδασης καθορίζει την τελική εκπεμπικότητα της δέσμης στην έξοδο του συστήματος των δακτυλίων απόσβεσης.

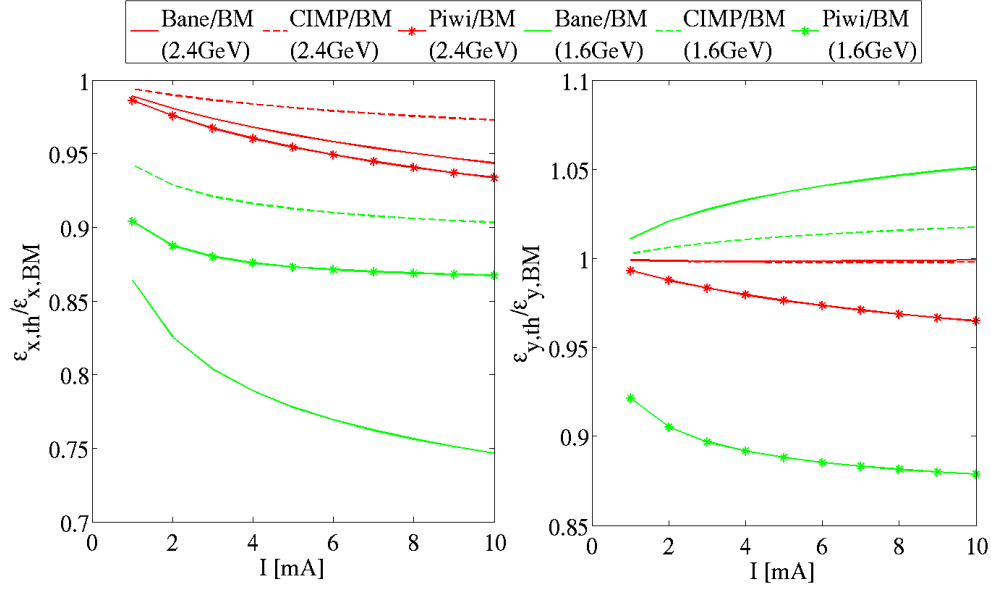


Figure 2: Λόγοι μεταξύ της οριζόντιας (αριστερά) και κάθετης (δεξιά) εκπεμπτικότητας όπως υπολογίστηκαν χρησιμοποιώντας τα μοντέλα της *IBS* ως προς τις τιμές που υπολογίστηκαν από το μοντέλο *BM* για την βασική ενέργεια (κόκκινο) και για χαμηλότερη ενέργεια (πράσινο) για διαφορετικά ρεύματα δέσμης.

Τα συμβατικά θεωρητικά μοντέλα που περιγράφουν το φαινόμενο της ενδοδεσμικής σκέδασης (*IBS*), *Piwiniski* και *Bjorken – Mtingwa*, έχουν μελετηθεί λεπτομερώς και επαλήθευτεί με πειραματικά δεδομένα μόνο για δέσμες αδρονίων. Σύγκριση μεταξύ αυτών των μοντέλων και δύο ευρέως χρησιμοποιούμενων προσεγγίσεων αυτών, *Bane* και *CIMP*, υπό την παρουσία της ακτινοβολίας συγχρότρου (*SR*) και της κβαντική διέγερσης (*QE*) παρουσιάζεται εδώ για τα πλέγματα τριών διαφορετικών επιταχυντικών διατάξεων: τους δακτυλίους απόσβεσης του *CLIC*, την ελβετική πηγή ακτινοβολίας συγχρότρου (*SLS*) και τον δακτύλιο αποθήκευσης ηλεκτρονίων του *Cornell (CESR – TA)*. Πολύ καλή συμφωνία έχει αποδειχθεί μεταξύ όλων των μοντέλων, αν το φαινόμενο της *IBS* είναι ασθενές. Ωστόσο, η απόκλιση μεγαλώνει όταν η επίδραση του φαινομένου στην τελική κατάσταση είναι ισχυρή, όπως φαίνεται στο Σχήμα 2. Δύο κωδικές προσομοίωσης πολλών σωματιδίων, *SIRE* και *IBStrack*, έχουν συγκριθεί με τα θεωρητικά μοντέλα, αποδεικνύοντας καλή συμφωνία, ειδικά με το φορμαλισμό *Piwiniski*. Για όλα τα θεωρητικά μοντέλα και τους κωδικές προσομοίωσης, η ίδια τάση στην εξέλιξη της εκπεμπτικότητας γύρω από τους δακτυλίους έχει υπολογισθεί και για τα τρία πλέγματα, ακόμη και όταν οι τελικές τιμές σταθερής κατάστασης αποκλίνουν, όπως φαίνεται στο Σχήμα 3.

Η κυψελίδα ελάχιστης εκπεμπτικότητας (*TME*) χρησιμοποιήθηκε ως η κύρια διάταξη για το σχεδιασμό των *PDR* και των *DR* του *CLIC*. Οι κυψελίδες *TME* μπορούν να παρέχουν την ελάχιστη δυνατή εκπεμπτικότητα, ενώ είναι οι πιο συμπαγείς. Αναλυτικές εκφράσεις για τις εντάσεις των τετραπολικών πεδίων και μια πλήρης παραμετροποίηση της *TME* έχει εξαχθεί, χρησιμοποιώντας βασικά επιχειρήματα γραμμικής οπτικής και την προσέγγιση λεπτών φακών. Επιπλέον, κριτήρια ευστάθειας έχουν εφαρμοστεί στις λύσεις, τόσο στο οριζόντιο όσο και στο κατακόρυφο επίπεδο. Ο

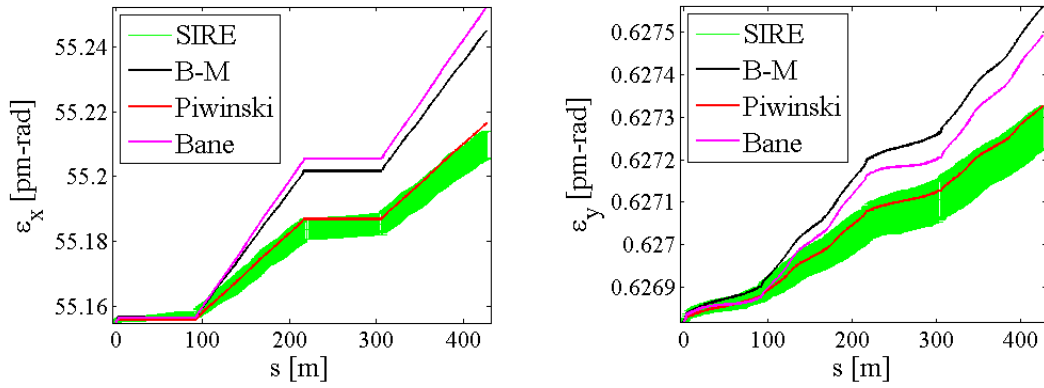


Figure 3: Σύγκριση της εξέλιξης της οριζόντιας (αριστερά) και κατακόρυφης (δεξιά) εκπεμπτικότητας σε μία στροφή των δακτυλίων απόσβεσης μεταξύ του κώδικα προσομοίωσης *SIRE* και των θεωρητικών μοντέλων *BM*, *P* και *Bane*.

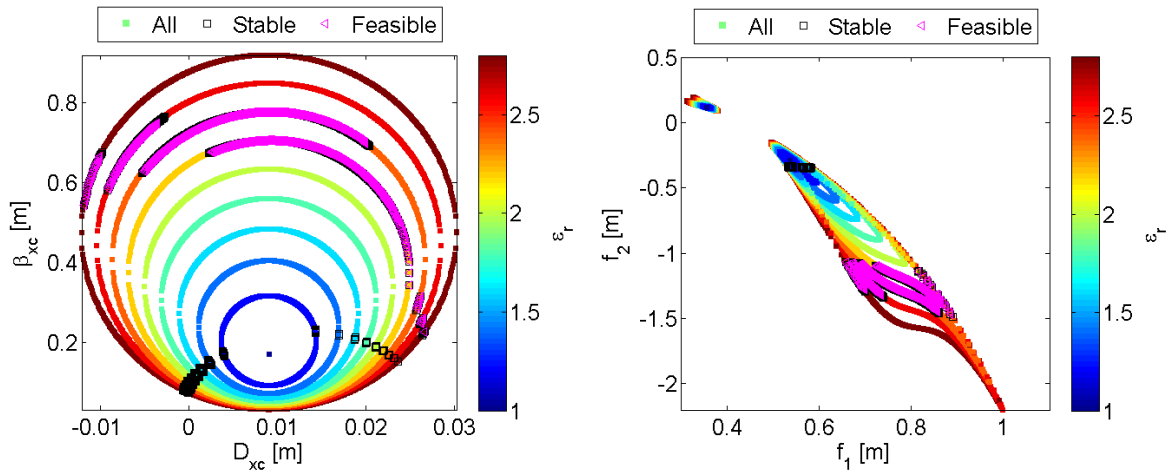


Figure 4: Παραμετροποίηση της οριζόντιας βήτα συνάρτησης β_{xc} και της διασποράς D_{xc} στο κέντρο του διπόλου (αριστερά) και των εστιακών αποστάσεων των τετραπόλων (δεξιά), με τον παράγοντα αποσυντονισμού της κυψελίδας ε_r . Οι ευσταθείς λύσεις παρουσιάζονται με τα μαύρα τετράγωνα ενώ οι ευσταθείς και τεχνολογικά εφικτές λύσεις με τα μοβ τρίγωνα.

πλήρης παραμετρικός χώρος των κυψελίδων, συμπεριλαμβανομένων των οπτικών και γεωμετρικών παραμέτρων, μπορούν στη συνέχεια να διερευνηθούν και η βελτιστοποίηση της διάταξης μπορεί να γίνει σύμφωνα με οποιαδήποτε σχεδιαστική απαίτηση. Στο όριο της απόλυτα ελάχιστης εκπεμπτικότητας, προσεκτική επιλογή των αποστάσεων μεταξύ των μαγνητών είναι σημαντική για τη ευστάθεια των λύσεων και για την ελαχιστοποίηση της χρωματικότητας. Ωστόσο, ακόμη και για τη βέλτιστη επιλογή, η χρωματικότητα είναι μεγάλη και στα 2 επίπεδα, αφού απαιτείται ισχυρή εστίαση για να επιτευχθεί η μικρή διασπορά στο μέσον του διπόλου. Για μεγαλύτερες τιμές εκπεμπτικότητας, που επιτυγχάνονται για χαμηλές προόδους φάσης, η ευστάθεια εξασφαλίζεται σχεδόν για κάθε επι-

λογή των μηκών των αποστάσεων και η χρωματικότητα μπορεί να μειωθεί σημαντικά. Στις περιοχές μικρών τιμών για τις προόδους φάσης, επιτυγχάνεται επίσης ελαχιστοποίηση του φαινομένου της *IBS* και του φαινομένου χωρικού φορτίου. Τέλος, η αναλυτική προσέγγιση επικυρώθηκε μέσα από μια σύγκριση με τα αποτελέσματα από τον κώδικα σχεδιασμού και προσομοίωσης επιταχυντικών διατάξεων *MADX*, όπως φαίνεται στο Σχήμα 5.

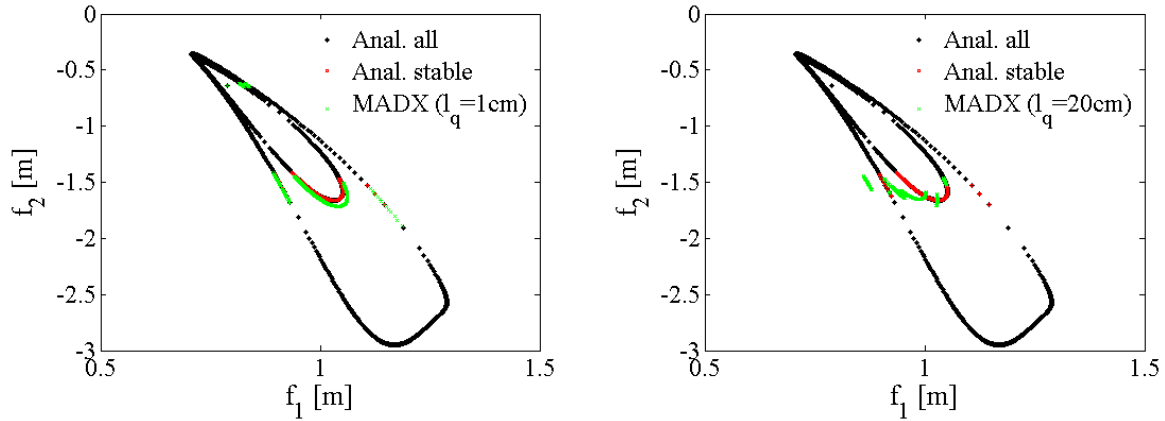


Figure 5: Σύγκριση μεταξύ της αναλυτικής προσέγγισης και των αποτελεσμάτων από τον κώδικα προσομοίωσης *MADX*. Οι αναλυτικές λύσεις παρουσιάζονται με μαύρο, οι λύσεις που ικανοποιούν τις συνθήκες ευστάθειας με κόκκινο, και τα αποτελέσματα από του κώδικα προσομοίωσης με πράσινο, για διαφορετικά μήκη τετραπόλων: $l_q = 1 \text{ cm}$ (αριστερά), $l_q = 10 \text{ cm}$ (δεξιά).

Το πρώτο στάδιο απόσβεσης των δεσμών ηλεκτρονίων και ποζιτρονίων γίνεται στους δακτυλίους προ-απόσβεσης *PDR*. Ιδιαίτερη έμφαση δόθηκε στο σχεδιασμό του δακτυλίου ποζιτρονίων, δεδομένου ότι οι παράμετροι της δέσμης ποζιτρονίων είναι πιο απαιτητικοί. Λόγω των μεγάλων τιμών εκπεμπτικότητας της δέσμης σε όλα τα επίπεδα, ο σχεδιασμός του πλέγματος των *PDR* επικεντρώθηκε στην βελτιστοποίηση της δυναμικής αποδοχής. Με βάση την αναλυτική παραμετροποίηση της κυψελίδας *TME* καθώς και επιχειρημάτων που αφορούν τα μη-γραμμικά φαινόμενα, ένας βέλτιστος αριθμός 17 κυψελίδων ανά τόξο επιλέχθηκε, συντονισμένες σε χαμηλές τιμές προόδου φάσης και στα 2 εγκάρσια επίπεδα με $\mu_x = 5/17$ και $\mu_y = 3/17$. Η επιλογή αυτή ελαχιστοποιεί τη διέγερση των μη γραμμικών συντονισμών, παρέχοντας ταυτόχρονα την απαιτούμενη εκπεμπτικότητα εξόδου. Ο ρυθμός επανάληψης των 50 Hz επιβάλλει την χρήση ειδικών μαγνητικών στοιχείων, των *damping wigglers*, προκειμένου να επιτευχθούν οι γρήγοροι χρόνοι απόσβεσης. Ένας μόνιμος μαγνήτης έντασης πεδίου των $B_w = 1,9 \text{ T}$, μήκους κύματος $\lambda_w = 30 \text{ cm}$ και απόσταση μεταξύ των πόλων $g = 41 \text{ mm}$ επιλέχθηκε, προκειμένου να χωρέσει εισερχόμενη δέσμη μεγάλης εκπεμπτικότητας και να παράγει την απαιτούμενη εκπομπτικότητα εξόδου, εντός του χρόνου επανάληψης των 20 ms . Συμπερασματικά, αυτή η διάταξη παρέχει επαρκή δυναμική αποδοχή για αποτελεσματική εισδοχή των μεγάλης εκπεμπτικότητας δεσμών και παράγει δέσμες εξόδου με παραμέτρους κατάλληλες για την εισαγωγή τους στο κυρίως δακτυλίο απόσβεσης. Η οπτική του δακτυλίου και η δυναμική περιοχή παρουσιάζονται στο Σχήμα 6.

Σε αντίθεση με τους δακτυλίους προ-απόσβεσης, *PDR*, ο σχεδιασμός των κυρίως δακτυλίων απόσβεσης εστιάζονται στην παραγωγή της εξαιρετικά χαμηλής εκπεμπτικότητας, για την οποία η

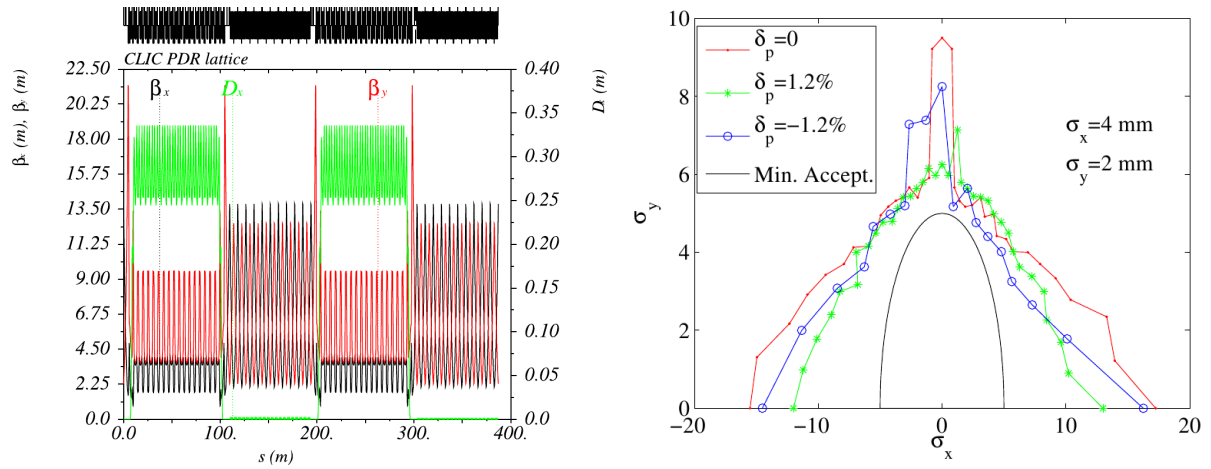


Figure 6: Αριστερά: Οπτικές συναρτήσεις των δακτυλίων προ-απόσβεσης του *CLIC*. Δεξιά: Η δυναμική αποδοχή των δακτυλίων προαπόσβεσης για τιμές διασπορά στην ορμή: $\delta p=0$ (κοκκίνο), 1.2% (πράσινο) και -1.2% (μπλέ).

επίδραση του φαινομένου της *IBS* γίνεται κυρίαρχη. Προηγούμενα στάδια του σχεδιασμού, αν και επιτύγχαναν τις απαιτούμενες παραμέτρους απόδοσης, είχαν κάποιους τεχνολογικούς και δυναμικούς περιορισμούς. Ειδικότερα, οι εκπεμπικότητες εξόδου κυριαρχούνταν από την επίδραση της *IBS* και του φαινομένου χωρικού φορτίου, ενώ η σταθερή φάση της κοιλότητας ραδιοσυχνοτήτων ήταν στη μη-γραμμική περιοχή λειτουργίας της κοιλότητας. Αριθμητικοί υπολογισμοί των τιμών εκπεμπικότητας εξόδου σε διαφορετικές ενέργειες, λαμβάνοντας υπόψη την επίδραση της *IBS*, μας οδήγησαν στην επιλογή μιας υψηλότερης ενέργειας λειτουργίας από ό,τι πριν, πηγαίνοντας από τα 2.424 GeV στα 2.86 GeV. Αυτό μείωσε την επίδραση της *IBS* κατά έναν παράγοντα 2, ενώ η εκπεμπικότητα εξόδου παρέμεινε η ίδια, όπως φαίνεται στο Σχήμα 7. Με βάση την αναλυτική παραμετροποίηση της κυψελίδας *TME*, η κυψελίδα τόξου των *DR* τροποποιήθηκε, επιλέγοντας χαμηλότερο πεδίο και επομένως μεγαλύτερο μήκος διπόλου και μειώνοντας την οριζόντια πρόοδο φάσης. Αυτό είχε ως αποτέλεσμα την αύξηση του συντελεστή συμπίεσης ορμής, και επομένως της διαμήκους εκπεμπικότητας της δέσμης, δίνοντας περιθώριο για βελτίωση της απόδοσης του συστήματος ραδιοσυχνοτήτων (*RF*). Η μείωση της προόδου φάσης οδήγησε στη μείωση των φαινομένων ενδοδεσμικής σκέδασης και χωρικού φορτίου, όπως φαίνεται στο Σχήμα 8.

Για την επίτευξη της εξαιρετικά χαμηλής εκπομπικότητας απαιτείται σε αυτή την περίπτωση η χρήση υπεραγωγών *damping wigglers*. Αριθμητικοί υπολογισμοί των τιμών εκπεμπικότητας εξόδου για διαφορετικές παραμέτρους των *damping wigglers*, έδειξε ότι υψηλό πεδίο και μικρό μήκος περιόδου είναι απαραίτητα για την επίτευξη των μικρών τιμών εκπεμπικότητας εξόδου. Από την άλλη πλευρά, η επίδραση της *IBS* σε αυτή την περιοχή παραμέτρων γίνεται εξαιρετικά ισχυρή. Για τη μείωση του φαινομένου σε συνδυασμό με την παραγωγή της απαιτούμενης εκπεμπικότητας εξόδου, τα υψηλότερα πεδία είναι ενδιαφέροντα, αλλά για μέτρια μήκη περιόδου, όπως φαίνεται στο Σχήμα 9. Τέλος, με βάση τα πιο πάνω, προτείνεται μια διάταξη μαγνητικού πλέγματος για τους κυρίως δακτυλίους απόσβεσης που επιτυγχάνει όλες τις απαιτούμενες παραμέτρους δέσμης στην έξοδο του.

Ο απώτερος στόχος για την κατανόηση της επίδρασης του φαινομένου της *IBS* υπό την παρουσία

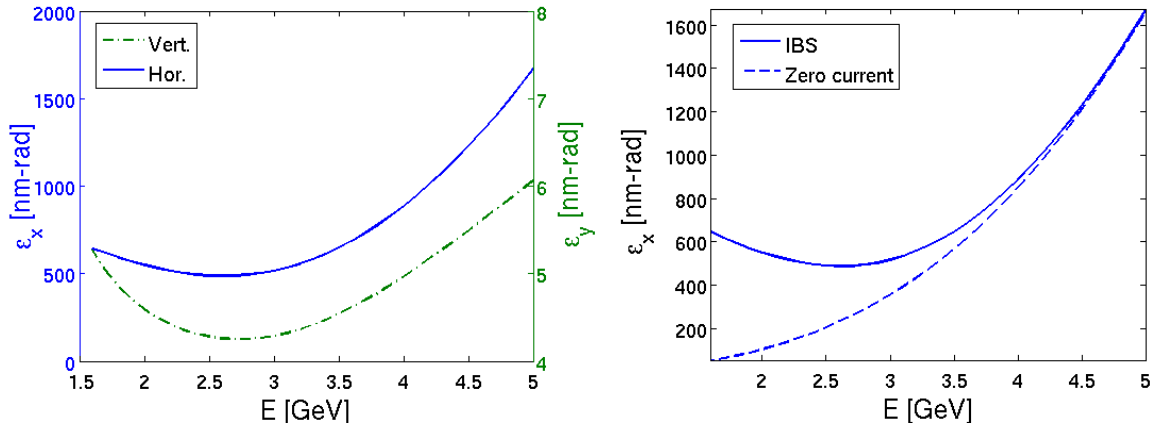


Figure 7: Αριστερά: Παραμετροποίηση της οριζόντιας (μπλε) και κάθετης (πράσινο) εκπεμπτικότητας εξόδου με την ενέργεια, λαμβάνοντας υπόψη το φαινόμενο της ενδοδεσμικής σκέδασης. Δεξιά: Σύγκριση της εκπεμπτικότητας εξόδου (συνεχής γραμμή) και της εκπεμπτικότητας μηδενικού ρεύματος (διακεκομμένη γραμμή) στο οριζόντιο επίπεδο συναρτήσει της ενέργειας.

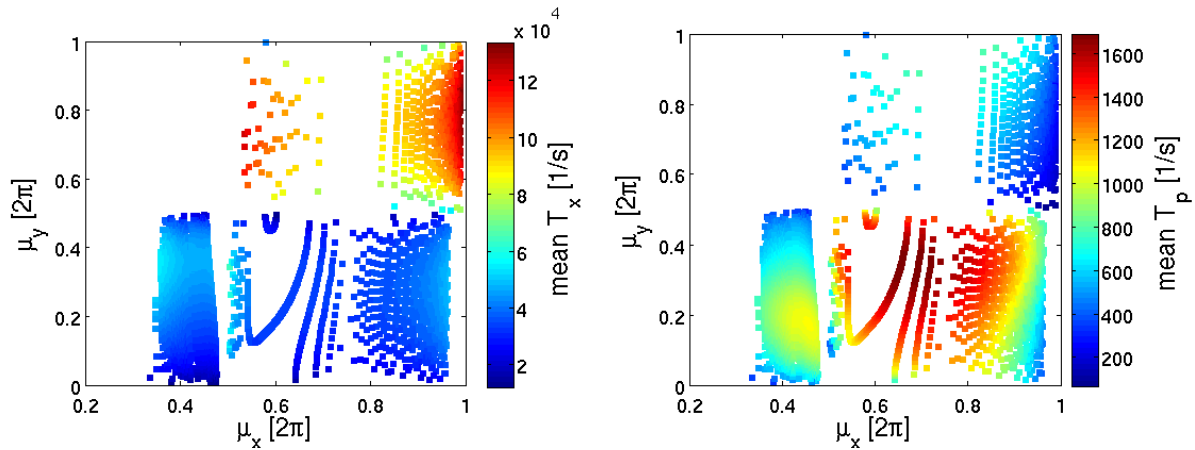


Figure 8: Παραμετροποίηση των μέσων ρυθμών αύξηση της εκπεμπτικότητας της δέσμης λόγω *IBS*, στο οριζόντιο (αριστερά) και διαμήκες (δεξιά) επίπεδο, με τις οριζόντιες και κάθετες προόδους φάσης της κυψελίδας ελάχιστης εκπεμπτικότητας, βάση της αναλυτικής προσέγγισης.

ακτινοβολίας συγχρότρου και του φαινομένου της χβαντικής διέγερσης, και ειδικά σε περιοχές όπου έχει μεγάλη επίδραση στις τελικές εκπεμπτικότητες της δέσμης, είναι η συγκριτική αξιολόγηση των θεωρητικών μοντέλων και των κωδίκων προσομοίωσης με πειραματικές μετρήσεις. Οι επιταχυντικές διατάξεις *SLS* και *CESR – TA* αποτελούν ιδανικές εγκαταστάσεις για μετρήσεις που αφορούν το φαινόμενο της *IBS*. Πειραματικά δεδομένα που συλλέχθηκαν και από τις δύο προαναφερθείσες διατάξεις έδειξαν ότι είναι απαραίτητος ο διαχωρισμός της *IBS* από άλλα φαινόμενα που εξαρτώνται επίσης από το ρεύμα της δέσμης και τα οποία επίσης οδηγούν στην αύξηση της εκπεμπτικότητας της δέσμης, έτσι ώστε να είναι δυνατή η συγκριτική αξιολόγηση των μοντέλων με τις μετρήσεις. Σε συνθήκες όπου η *IBS* είναι ασθενής, έχει αποδειχθεί καλή συμφωνία μεταξύ των θεωρητικών

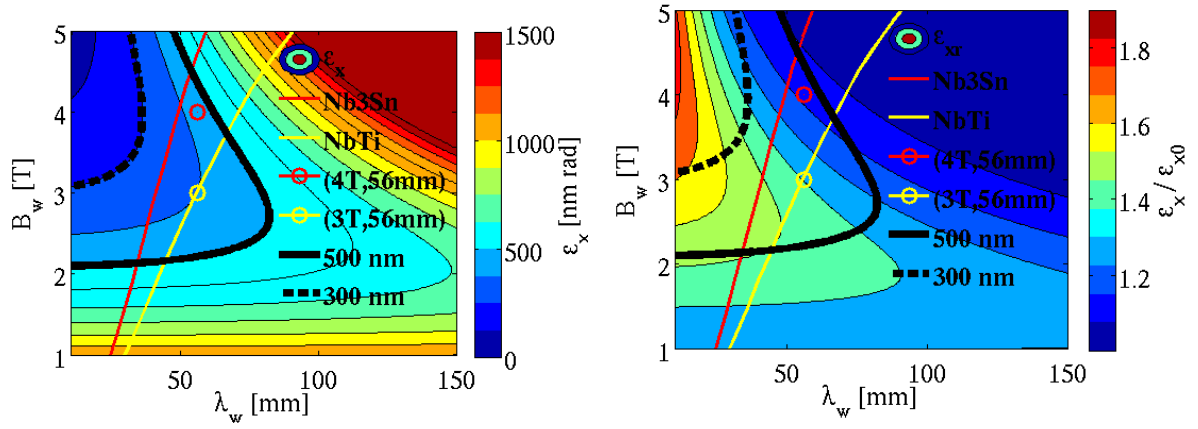


Figure 9: Εξάρτηση της εκπεμπτικότητας εξόδου (αριστερά) και του λόγου της με την εκπεμπτικότητα μηδενικού ρεύματος (δεξιά), συναρτήσει του πεδίου και της περιόδου των *wigglers*. Τα όρια για τις 2 τεχνολογίες υπεραγωγίων μαγνητών παρουσιάζονται με κόκκινο (*NbTi*) και κίτρινο (*Nb3Sn*). Οι μαύρες καμπύλες παρουσιάζουν τις περιπτώσεις εκπεμπτικότητας των 300 nm και 500 nm .

μοντέλων και των μετρήσεων. Στο Σχήμα 10 παρουσιάζονται μετρήσεις του μεγέθους της δέσμης στο οριζόντιο (αριστερά) και κάθετο (δεξιά) επίπεδο στην επιταχυντική διάταξη *SLS*. Οι θεωρητικές προβλέψεις για διαφορετικά μήκη πακέτων της δέσμης (μηδενικού ρεύματος) και διαφορετικές τιμές κάθετης εκπεμπτικότητας σε μηδενικό ρεύμα παρουσιάζονται με συνεχείς γραμμές. Περαιτέρω μετρήσεις σε συνθήκες όπου η *IBS* είναι ισχυρή, βρίσκονται σε εξέλιξη και στις 2 πειραματικές διατάξεις.

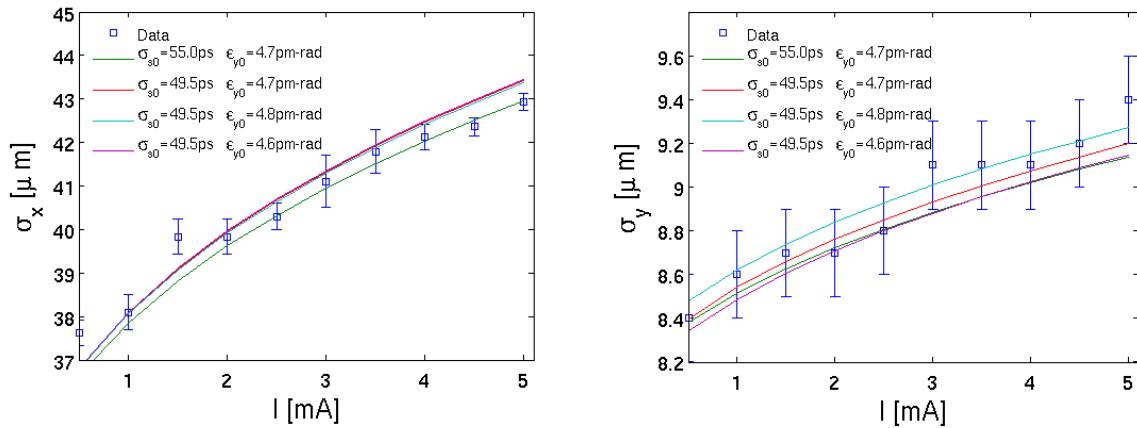


Figure 10: Μετρήσεις μεγέθους δέσμης στο οριζόντιο (αριστερά) και κάθετο (δεξιά) επίπεδο, για διαφορετικά ρεύματα δέσμης. Οι θεωρητικές προβλέψεις με βάση τα μοντέλα της *IBS*, για διαφορετικά μήκη πακέτων δέσμης και διαφορετικές κάθετες εκπεμπτικότητες, παρουσιάζονται με τις συνεχείς γραμμές.

Introduction

1.1 Physics potential of a future Linear Collider

The past few months have been very exciting for the particle physics community. The two high luminosity LHC experiments, ATLAS and CMS, announced the discovery of a Higgs-like boson with mass of ~ 125 GeV [1]. Even though the experimental access to Terascale physics is becoming reality with the LHC, the particle physics community worldwide has, in parallel, expressed a consensus that the results of the LHC will need to be complemented by experiments at a lepton collider in the tera-electron-volt (TeV) energy range. The required energy range and detailed physics requirements are expected to be defined from LHC results when substantial integrated luminosity has been accumulated at full LHC energy, tentatively by 2015-16 [2].

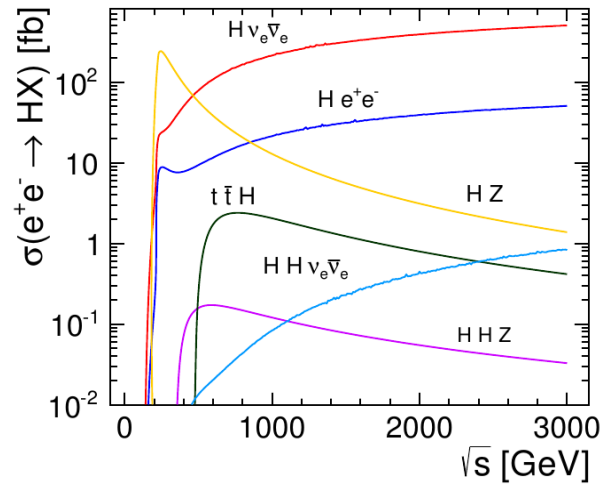


Figure 1.1: The cross-section for several different production modes of the Higgs boson in e^+e^- interactions for a 125 GeV Higgs boson mass, as a function of the center-of-mass energy of collisions [3].

The physics motivation for an e^+e^- linear collider (LC) has been studied in detail for more than 20 years. These studies have established the requirement for a LC as the next collider at the energy frontier. In a lepton collider the center-of-mass energy and initial-state polarizations are precisely known and can be adjusted, and backgrounds are many orders of magnitude lower than the QCD backgrounds that challenge hadron collider environments. Thus, the measurements and searches for new phenomena in lepton colliders are unbiased and comprehensive. These favorable experimental conditions will enable the LC to measure the properties of physics at the TeV scale with unprecedented precision and complementary to the LHC. Figure 1.1 illustrates

the cross-section for several different production modes of the Higgs boson in e^+e^- interactions, for a mass of 125 GeV, as a function of the collision energy at the center of mass [3].

Two options for a future e^+e^- LC have been developed, with different main linac acceleration schemes. The International Linear Collider (ILC) uses superconducting Radio-Frequency (RF) cavities, whereas the Compact Linear Collider (CLIC) uses a separate drive beam to provide the accelerating power to normal conducting (or copper) RF cavities. The ILC technology provides an option for a Higgs and top factory to be constructed on a relatively short timescale. It aims at colliding beam energy of 500 GeV, upgradeable to 1 TeV [4]. The Compact Linear Collider (CLIC) study is exploring the possibility of extending the energy range of linear colliders into the multi-TeV region by developing a novel technology of Two-Beam Acceleration (TBA), providing colliding beams up to 3 TeV. At different energy stages of CLIC, precision measurements of various observables of the Standard Model Higgs boson can be carried out.

In recent years, there has been extensive collaboration between ILC and CLIC physicists with the goal of realizing a Linear Collider (LC) as the next major accelerator facility at the high-energy frontier.

1.2 CLIC overview

Following preliminary physics studies based on an electron-positron collider in the multi-TeV energy range [5], the CLIC study is focused on the design of a linear collider with a center-of-mass collision energy of $E = 3$ TeV. The CLIC concept however, allows its construction to be staged, starting from a 500 GeV CLIC, without major modifications. Figure 1.2 illustrates the layout of the CLIC accelerator complex at 3 TeV [2].

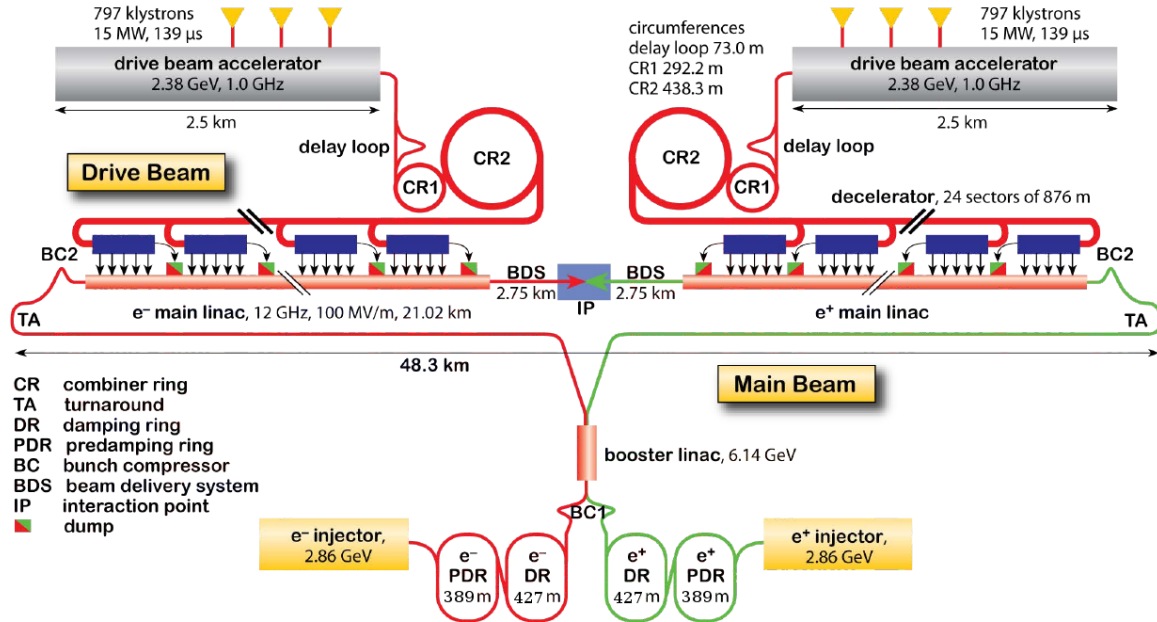


Figure 1.2: The CLIC layout at 3 TeV [2].

Due to the novel acceleration scheme of CLIC, the complex consists of two main areas: the Main (bottom of Fig. 1.2) and the Drive Beam (top of Fig. 1.2) part. The Main Beam is generated and pre-accelerated in the injector linacs and then enter the Damping Rings complex for reducing the phase space beam dimensions, i.e. the beam emittances. The aim is to produce beams of 500 nm and 5 nm emittances, normalized to the beam energy, in the horizontal and vertical planes respectively, at the exit of the injector complex. The small emittance beams are further accelerated in a common linac before being transported through the main tunnel to the turnarounds. After the turnarounds the acceleration of the Main Beam begins with an accelerating gradient of 100 MV/m. In the acceleration scheme of CLIC, the classical approach of the klystron powering of the RF cavities is replaced by the generation of a second “Drive Beam” and its compression and reversion into RF power close to the Main Beam accelerating structures. The Drive Beam pulses are generated in the two Main Linacs of the top part of the figure and then compressed in the Delay Loops and Combiner Rings (CR1 and CR2). They are then transported through the Main Linac tunnel to 24 individual turnarounds. Each Drive Beam segment is directed by pulsed Power Extraction and Transfer Structures (PETS), for the final RF power generation, into the accelerating structures of the Main Beams. The beams collide after a long Beam Delivery Section (BDS) (collimation, final focus) in one interaction point (IP) in the center of the complex.

This novel scheme is called the two beam acceleration scheme and it was proposed because the goal of the design of the CLIC collider is the high luminosity at a high energy at the lowest possible construction cost and power. Superconducting technology cannot achieve the very high acceleration gradient of 100 MV/m, thus would require a much longer linear accelerator.

The luminosity of a collider can be written as:

$$\mathcal{L} = H_D \frac{N^2}{4\pi\sigma_x\sigma_y} n_b f_r, \quad (1.1)$$

where:

H_D is a correction factor representing the combined effect of “hour-glass” (change of beta function in longitudinal direction over the collision region) and disruption enhancement (due to the attractive force that the two colliding bunches exert on each other).

N is the number of particles per bunch.

$\sigma_{x,y} = \sqrt{\frac{\varepsilon_{x,y}\beta_{x,y}}{\beta\gamma}}$ the horizontal (x) and vertical (y) r.m.s. beam sizes at the collision point.

$\gamma = E_b/E_0$ the beam energy normalized to the rest electron energy, $E_0=511$ keV.

$\varepsilon_{x,y}$ is the normalized horizontal (x) and vertical (y) emittances.

$\beta_{x,y}$ is the horizontal (x) and vertical (y) beta functions at the collision point.

n_{bt} the number of bunches per beam pulse.

f_r the repetition frequency of the beam pulses.

Table 1.1: CLIC main parameters for 500 GeV and 3 TeV.

Description [units]	500 GeV	3 TeV
Total (peak 1%) luminosity	2.3 (1.4) $\times 10^{34}$	5.9 (2.0) $\times 10^{34}$
Total site length [km]	13.0	48.4
Loaded accel. gradient [MV/m]	80	100
Main Linac RF frequency [GHz]		12
Beam power/beam [MW]	4.9	14
Bunch charge [$10^9 e^+/e^-$]	6.8	3.72
Bunch separation [ns]		0.5
Bunch length [μm]	72	44
Beam pulse duration [ns]	177	156
Repetition rate [Hz]		50
Hor./vert. norm. emitt. [$10^{-6}/10^{-9}$ m]	2.4/25	0.66/20
Hor./vert. IP beam size [nm]	202/2.3	40/1
Beamstrahlung photons/electron	1.3	2.2
Hadronic events/crossing at IP	0.3	3.2
Coherent pairs at IP	200	6.8×10^8

The main beam and main linac parameters for a luminosity of $\mathcal{L} = 2 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ at $E=3 \text{ TeV}$, are summarized in Table 1.1.

In order to achieve the high luminosity, high intensity bunches and very small transverse beam sizes or emittances are required.

1.3 The CLIC Damping Rings

The CLIC damping rings purpose is to produce the ultra-low emittance with high bunch charge necessary for the luminosity performance of the collider. The performance challenges of the damping rings (DRs) are driven by the main parameters of the collider and the requirements of the upstream and downstream systems, which are summarized in Table 1.2 [6]. The large input emittance, especially coming from the positron source, the requirement of ultra low output emittances and the high repetition rate of 50 Hz, requires that the beam damping is done in two stages, with a main-damping ring (DR) and a pre-damping ring (PDR) for each particle species. A careful lattice design and non-linear dynamics optimization is necessary for providing a solid PDR design, with large dynamic and momentum aperture, enabling the efficient digestion of the incoming beam. On the other hand, the ultra low output emittance and the collective effects associated with this, make the lattice design of the main DRs very challenging with respect to beam dynamics and the associated technology. The DR complex layout is shown in Figure 1.3.

1.4 Scope of the thesis

The subject of this thesis has two parts. It elaborates the design of the optics and optimization of the performance of the positron pre-damping ring of CLIC and the optimization of the optics and

Table 1.2: The required injection and extraction parameters of the DR complex.

Parameters	Injected		Extracted
	e^-	e^+	e^-/e^+
Bunch Population [10^9]	4.4	4.6	4.1
Bunch spacing [ns]	0.5/1	0.5/1	0.5
Bunches/train	312/156	312/156	312
Number of trains	2	2	1
Repetition rate [Hz]	50	50	50
Norm. horiz. emittance [nm·rad]	100×10^3	7×10^6	500
Norm. vert. emittance [nm·rad]	100×10^3	7×10^6	5
Norm. long. emittance [keV·m]	2.86	2288	6

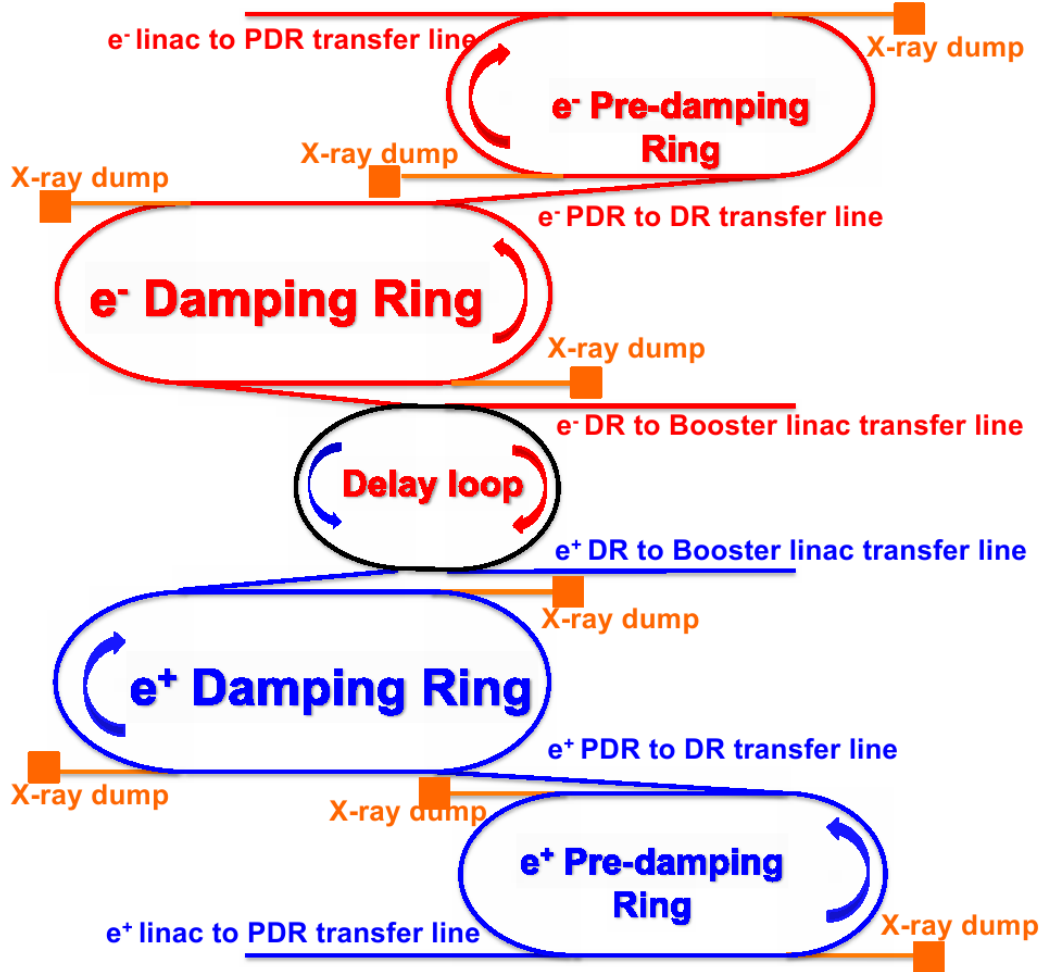


Figure 1.3: The damping ring complex layout.

the performance of the CLIC main damping rings, under the influence of the strong intrabeam scattering (IBS) effect. The work described in this PhD thesis was performed in the framework of the CLIC study group.

Chapter 2 describes basic theoretical principles of beam dynamics in e^+e^- rings. The linear optics concept and the mathematical formulation, describing the single particle motion under the influence of electromagnetic forces, is introduced. Brief description of the synchrotron radiation theory is given, and the equilibrium beam properties due to synchrotron radiation and quantum excitation are defined. The effect of damping wigglers on beam dynamics is then discussed and the lattices used in low emittance rings design are presented. Basic concepts of non-linear beam dynamics are also introduced. Finally, a brief description of collective effects leading to current dependent emittance blow up, in high intensity and low emittance e^+/e^- circular accelerators, is presented.

Chapter 3, focuses on the theoretical description of the intrabeam scattering effect (IBS), which is the main limitation for achieving the ultra low emittance in the CLIC damping rings. In particular, the theoretical models of Piwinski and Bjorken-Mitingwa, their high energy approximations Bane and CIMP and two Monte-Carlo multiparticle tracking codes are discussed. The method of calculation of the final state emittances, taking into account intrabeam scattering, radiation damping and quantum excitation is also described.

The IBS theoretical models have been verified in the past for proton and ion machines, with good agreement between theoretical predictions and measurements, especially in the horizontal and longitudinal planes. However, the validity of the models has not been verified in lepton machines, where synchrotron radiation and quantum excitation are also present. A comparison between the theoretical models for three different lattices: the CLIC Damping Rings (DR), the Swiss Light Source (SLS) storage ring and the Cornell electron storage ring Test Accelerator (Cesr-TA), is presented in Chapter 4. The limitations of the theoretical models are also discussed. In order to understand further and overcome these limitations, Monte Carlo multiparticle tracking codes are very important. The bench-marking of two tracking codes with the theoretical models is also presented in this chapter.

Chapter 5 elaborates a detailed study and an analytical parameterization of the theoretical minimum emittance (TME) cells. The TME cells are instrumental magnet configurations for the low emittance lattice design, as they can provide the lowest possible emittance, in a compact cell. Using basic linear optics arguments, analytical expressions for the quadrupole strengths and a complete parameterization of the TME cell are derived, using thin lens approximation. In addition, stability criteria are applied, in both horizontal and vertical plane. All cell properties are then globally determined and the optimization procedure, following any design requirement, can be performed in a systematic way. Numerical examples are presented, to demonstrate the optimization procedure. This analytical approach is finally validated, through a comparison with the results from the Methodical Accelerator Design code MADX.

In Chapter 6, the linear lattice design and non-linear lattice optimization of the CLIC positron pre-damping ring (PDR) are studied. Due to the large beam emittance in all three planes, especially the one coming from the positron source, the lattice design of the PDR is focused on the momentum acceptance and the dynamic aperture optimization. Based on the analytical parameterization of the TME cell, the arc cell of the PDR is optimized in order to achieve

low natural chromaticity, and thus minimize the chromatic sextupole strengths. Based on the resonance free lattice concept, the cell phase advances are chosen to minimize both the possible resonance excitation and the amplitude dependent tune shift, while providing the required output emittance for efficient injection into the main Damping Rings (DR). The necessity of damping wigglers to achieve the fast damping, and the optimization of the wiggler FODO cell is also discussed. Finally, a design layout and the performance parameters of the ring are presented.

Chapter 7 is devoted to the optics and performance optimization of the CLIC main damping rings, under the influence of IBS. Other issues, like the Laslett space charge tune shift and the RF performance parameters are taken into account. Previous design stages of the CLIC main damping rings, although provided the required performance parameters, they suffered technological and beam dynamics limitations. An optimized design at a higher operational energy is proposed in this chapter, to reduce the IBS effect. Based on the analytical parameterization of the TME cell, the arc cell is modified to increase the momentum compaction factor, and thus the longitudinal emittance of the beam, giving room for improvement in the performance of the RF system. Optimal settings of phase advances are also defined, with respect to intrabeam scattering and space-charge tune shift. The necessity of damping wigglers to achieve the ultra-low emittance and their effect on the beam parameters are discussed. Parametric scans are then performed, studying the achievable emittances at different currents, under the influence of the IBS effect. Finally, a design layout and the performance parameters of the ring are presented.

The bench-marking of the IBS theoretical models and the tracking codes with experimental data is the ultimate goal for understanding the effect in IBS dominated regimes, in the presence of synchrotron radiation and quantum excitation. Chapter 8 presents IBS measurement results from the SLS storage ring at a low energy operation mode and measurement observations at CEsr-TA.

Finally, Chapter 9 summarizes the conclusions of this thesis.

Beam dynamics of e^+/e^- rings

2.1 Linear single particle beam optics

The motion of the particles in circular high energy accelerators, is controlled by electro-magnetic fields. In order to describe the motion of the particles in an accelerator, the understanding of their dynamics in the six-dimensional phase space is crucial. In the ideal case, of which particles interact only with the fields of the magnets around the accelerator and not with the fields of each other, the single particle dynamics can describe the whole system.

The magnetic focusing forces acting on the charged particles, keep their motion in the vicinity of the reference (ideal) orbit. For relativistic particles only magnetic fields are considered, as a static electric field cannot bend efficiently the trajectory of a relativistic particle. Dipoles are used to provide complete revolution of the particles around the ring, defining the closed orbit, while quadrupoles are used in order to focus the beam and force it to execute lateral oscillations around this closed orbit, the so-called betatron oscillations. While traveling around the ring, especially ultra-relativistic light particles as electrons and positrons, lose energy due to synchrotron radiation. This energy is recovered by a radio-frequency (RF) electric field. The particles in the bunch execute oscillations in longitudinal position and in energy, relative to an ideal reference (synchronous) particle in the center of the bunch, which are called synchrotron oscillations.

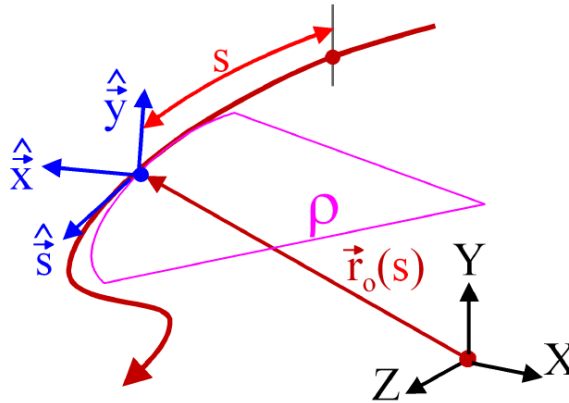


Figure 2.1: The Frenet-Serret curvilinear coordinate system.

In the presence of electromagnetic fields, charge particles undergo Lorentz forces:

$$\mathbf{F} = e(\mathbf{E} + \mathbf{u} \times \mathbf{B}), \quad (2.1)$$

where $\mathbf{u} = d\mathbf{r}/dt$ the velocity of the particles, e the charge and $\gamma = 1/\sqrt{1 - \mathbf{u}^2/c^2}$ the relativistic Lorentz factor. To describe the motion of the particles in the vicinity of the ideal orbit the Frenet-Serret coordinate system is used, which describes the kinematic properties of a particle moving along a continuous, differentiable curve in the three-dimensional Euclidean space [7]. Any point in the phase space can be expressed by $\mathbf{r} = \mathbf{r}_0 + x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$, where $\mathbf{r}_0(s)$ is the reference orbit and $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{s}}$ form the basis of the Frenet-Serret coordinate system, as shown in Figure 2.1.

In general, it is assumed that the magnetic field vector $\mathbf{B}$ is oriented perpendicular to the velocity vector $\mathbf{v}$, thus purely transverse field components are considered. The transverse components of the particles velocities for relativistic beams are small compared to the particle velocity u_z ($u_x \ll u_z$, $u_y \ll u_z$, $u_s \approx u_z$). Under these assumptions, from the equilibrium of the centrifugal force and the Lorentz force, the bending radius ρ_x , of a charged particle passing through the vertical homogeneous field B generated by a dipole magnet, is given by [7]:

$$\frac{1}{\rho_x}[\text{m}^{-1}] = 0.2998 \frac{|B[\text{T}]|}{\beta E[\text{GeV}]} \quad (2.2)$$

The deflection angle in a magnetic field is:

$$\theta = \int \frac{ds}{\rho_x}, \quad (2.3)$$

or for a uniform field of a dipole magnet of length l_d , is reduced to $\theta = l_d/\rho_x$. From Eq. (2.2), the momentum rigidity of the beam is defined as:

$$(B\rho_x) = \frac{\beta E[\text{GeV}]}{0.2998}, \quad (2.4)$$

which is a quantity defined only by the beam energy.

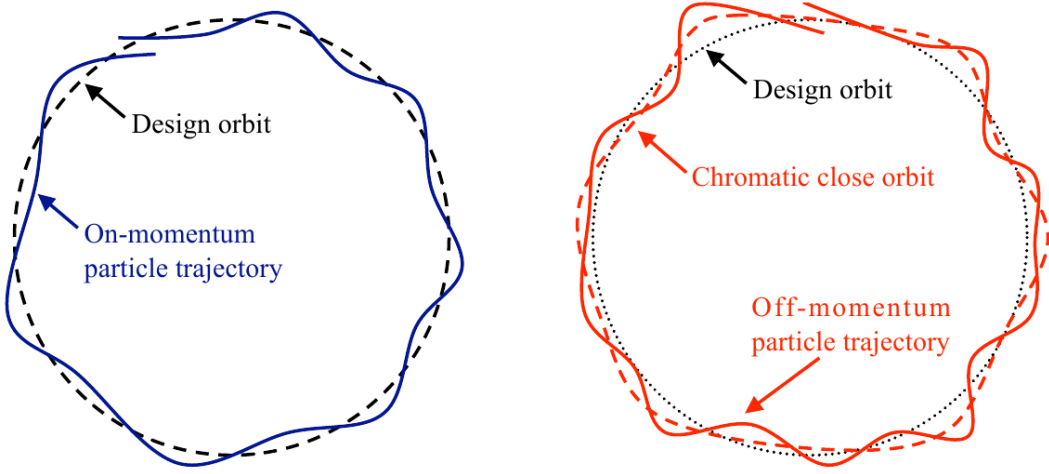
In order to keep the particle beam focused, and to generate specific beam properties at selected points around the accelerator, focusing forces are required. Focusing (or defocusing) forces are provided by quadrupole magnets, whose field is zero on the s axis, while it increases linearly with the distance from it:

$$B_y = gx, \quad B_x = gy, \quad \text{where} \quad g = \frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x}. \quad (2.5)$$

For a positively charged particle, the quadrupole with $\partial B_y/\partial x$ will provide horizontal focusing and vertical defocusing. This will be reversed, if the current direction or the particle charge is reversed. In beam dynamics, it is customary to define a focusing strength k , similar to the definition of the bending radius of Eq. (2.2):

$$k[\text{m}^{-1}] = 0.2998 \frac{g[\text{T/m}]}{\beta E[\text{GeV}]} \quad (2.6)$$

Note that $k > 0$ for horizontal focusing, while $k < 0$ for vertical focusing quadrupole.

Figure 2.2: Emittance ellipses change due to different $\delta p/p_0$ [9].

2.1.1 Transverse motion

The linear motion of particles in a circular machine, consisting only of dipole and quadrupole magnets, is described by Hill's equations [7, 8]:

$$\frac{d^2x}{ds^2} - K_1(s)x = \frac{1}{\rho_x(s)} \frac{\delta p}{p_0}, \quad (2.7)$$

$$\frac{d^2y}{ds^2} - K_1(s)y = 0. \quad (2.8)$$

The coefficient K_1 is the general focusing strength, which can be expressed, including the weak focusing from the dipole and the strong focusing from quadrupole magnets, in the general form:

$$K_1(s) = \frac{1}{\rho_{x,y}^2(s)} - \frac{1}{B\rho_x} \frac{\partial B_y(s)}{\partial x}, \quad (2.9)$$

where $\rho_x(s)$ is the bending radius of the element at position s . $K_1(s)$ and $\rho_x(s)$ are periodic functions of s , with a period at least equal to the circumference of the closed orbit of the machine. $\delta p/p_0$, is the relative momentum deviation of an off-momentum particle with momentum $p_0 \pm \delta p$ from the design (reference) momentum p_0 . It is interesting to notice, that Hill's equations are the ones of a harmonic oscillator with periodic coefficients.

The solution of Eqs. (2.7), (2.8) is not trivial if the whole accelerator is considered. On the other hand, it is rather easy if K_1 is considered piece-wise constant. The homogeneous equations (for $\delta p/p_0 = 0$) can then be expressed in matrix formulation (only for linear elements as drifts, dipoles and quadrupoles):

$$\begin{pmatrix} z(s) \\ z'(s) \end{pmatrix} = M(s|s_0) \begin{pmatrix} z(s_0) \\ z'(s_0) \end{pmatrix}, \quad (2.10)$$

where z refers to the horizontal x or vertical y plane.

Off-momentum particles are not oscillating around the design orbit, but around a chromatic closed orbit, as shown in Figure 2.2. For an off-momentum particle, the solution of the inhomogeneous Eq. (2.7) can be expressed as a linear superposition of the particular solution and the solution of the homogeneous equation (for $\delta p/p_0 = 0$):

$$x = x_\beta(s) + D(s) (\delta p/p_0), \quad (2.11)$$

where $D(s) (\delta p/p_0)$ the off-momentum closed orbit and $D(s)$ is called the dispersion function. The solution can then be expressed as:

$$\begin{pmatrix} D(s) \\ D'(s) \end{pmatrix} = M(s|s_0) \begin{pmatrix} D(s_0) \\ D'(s_0) \end{pmatrix} + \begin{pmatrix} d \\ d' \end{pmatrix}, \quad (2.12)$$

where d and d' is the dispersive part of the matrices.

The transfer matrices for a constant focusing function K are:

$$M(s|s_0) = \begin{cases} \begin{pmatrix} \cos(\sqrt{K}l_q) & \frac{1}{\sqrt{K}} \sin(\sqrt{K}l_q) \\ -\sqrt{K} \sin(\sqrt{K}l_q) & \cos(\sqrt{K}l_q) \end{pmatrix} & K > 0: \text{focusing quad}, \\ \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} & K=0: \text{drift space}, \\ \begin{pmatrix} \cosh(\sqrt{|K|}l_q) & \frac{1}{\sqrt{|K|}} \sinh(\sqrt{|K|}l_q) \\ \sqrt{|K|} \sinh(\sqrt{|K|}l_q) & \cosh(\sqrt{|K|}l_q) \end{pmatrix} & K < 0: \text{defocusing quad}. \end{cases} \quad (2.13)$$

In thin-lens approximation, where the quadrupole length $l_q \rightarrow 0$, the transfer matrix for a quadrupole reduces to:

$$M_{\text{quad}} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}, \quad (2.14)$$

where the focal length is given by $f = \lim_{l_q \rightarrow 0} \frac{1}{Kl_q}$.

In a similar way, the transfer matrix of a sector dipole, for which the particle trajectories enter and exit with perpendicular entrance and exit angles to the edge of the dipole field, is given by:

$$M_{\text{dip}} = \begin{pmatrix} \cos \theta & \rho_x \sin \theta \\ -\frac{1}{\rho_x} \sin \theta & \cos \theta \end{pmatrix}, \quad (2.15)$$

where $\theta = l_d/\rho_x$ the bending angle of the dipole, l_d the dipole length and ρ_x the bending radius. In the small-angle approximation, the transfer matrix reduces to:

$$M_{\text{dip}} = \begin{pmatrix} 1 & l_d \\ 0 & 1 \end{pmatrix}, \quad (2.16)$$

which is equivalent to that of a drift space.

The dispersive part of the matrices can be written in the form:

$$\begin{pmatrix} d \\ d' \end{pmatrix} = \begin{cases} \begin{pmatrix} \frac{1}{\rho_x K} (1 - \cos(\sqrt{K} l_q)) \\ \frac{1}{\rho_x \sqrt{K}} \sin(\sqrt{K} l_q) \end{pmatrix} & K > 0: \text{focusing quad,} \\ \begin{pmatrix} \frac{1}{\rho_x |K|} (-1 + \cosh(\sqrt{|K|} l_q)) \\ \frac{1}{\rho_x \sqrt{|K|}} \sinh(\sqrt{|K|} l_q) \end{pmatrix} & K < 0: \text{defocusing quad,} \\ \begin{pmatrix} \rho_x (1 - \cos \theta) \\ \sin \theta \end{pmatrix} & \text{sector dipole.} \end{cases} \quad (2.17)$$

In the small angle approximation the general transfer matrix with dispersion for a sector dipole is:

$$M = \begin{pmatrix} 1 & l_d & \frac{l_d \theta}{2} \\ 0 & 1 & \theta \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.18)$$

The transfer matrix for any intervals, made up of sub-intervals, is the product of the transfer matrices of the sub-intervals:

$$M(s_2|s_0) = M(s_2|s_1)M(s_1|s_0). \quad (2.19)$$

Using these matrices, the linear motion of particles can be tracked through the elements of the accelerator.

According to Floquet's theorem [8], the solutions to the homogeneous Hill's equations can be written in the form:

$$z(s) = A w(s) \cos(\phi_z(s) + \phi_0), \quad (2.20)$$

where $w(s) = w(s + C)$ and $\phi_z(s) = \phi_z(s + C)$ are periodic functions with the same period C , and $z = x$ or y . The phase ϕ_0 is determined by the initial conditions. Substituting Eq. (2.20) to Eq. (2.7) and (2.8), the betatron phase advance $\phi_z(s)$ and the betatron or twiss (Courant-Snyder parameters) functions $\alpha_z(s)$, $\beta_z(s)$ and $\gamma_z(s)$ can be defined, describing the motion of the particle with the maximum amplitude in the beam. The twiss functions are also periodic functions, with a period equal to the circumference of the machine C , and are related to each other and the betatron phase advance $\phi_z(s)$ by:

$$\begin{aligned} \alpha_{x,y}(s) &\equiv -\frac{1}{2} \beta'_{x,y}(s), \\ \gamma_{x,y}(s) &\equiv \frac{1 + \alpha_{x,y}^2(s)}{\beta_{x,y}(s)}, \\ \phi_{x,y}(s) &\equiv \int \frac{ds}{\beta_{x,y}(s)}. \end{aligned} \quad (2.21)$$

The horizontal and vertical tunes of a machine of circumference C are then defined by:

$$Q_{x,y} = \frac{1}{2\pi} \int_0^C \frac{ds}{\beta_{x,y}(s)}. \quad (2.22)$$

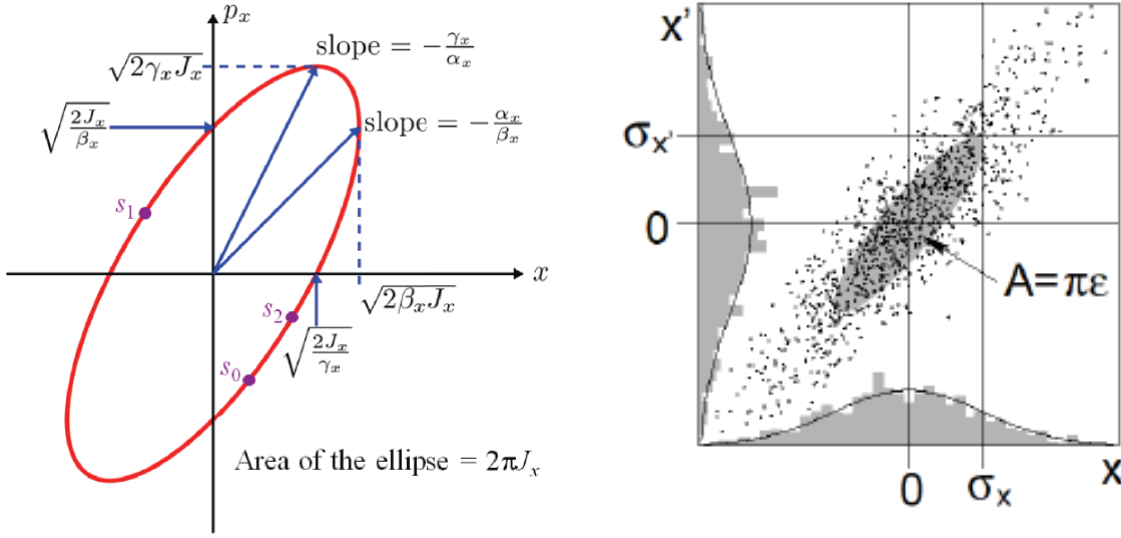


Figure 2.3: Left: The single particle emittance defined by the twiss parameters at a certain position in the lattice. Right: Statistical beam emittance depending on beam distribution [9].

Emittance

The motion of an on-momentum particle at any point of the lattice is described by the solution of Eq. 2.7, which, using the Floquet solutions of Eq. 2.20, can be written in the form [7]:

$$\begin{aligned} x(s) &= \sqrt{\varepsilon_x} \sqrt{\beta_x(s)} \cos(\phi_x(s) + \phi_0), \\ x'(s) &= -\frac{\sqrt{\varepsilon_x}}{\sqrt{\beta_x(s)}} [\sin(\phi_x(s) + \phi_0) + a_x(s) \cos(\phi_x(s) + \phi_0)], \end{aligned} \quad (2.23)$$

and they satisfy the equation of an ellipse:

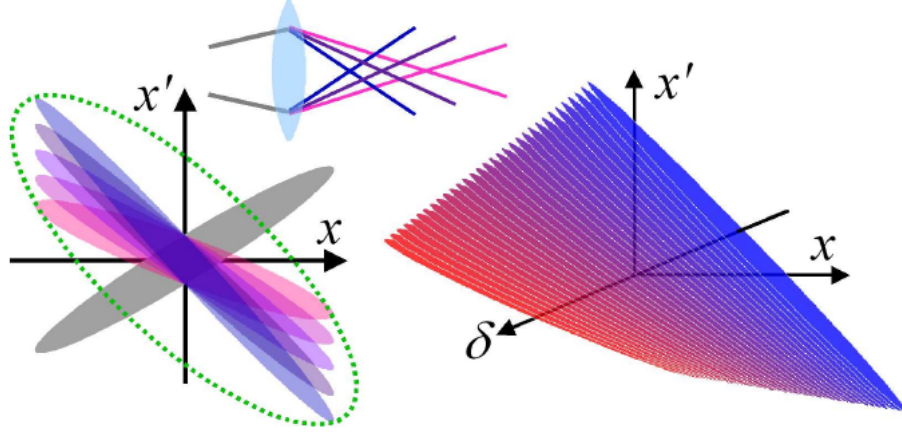
$$A_x = \gamma_x x^2 + 2\alpha_x x x' + \beta_x x'^2 \equiv \varepsilon_x. \quad (2.24)$$

This is an invariant expression, called the Courant-Snyder invariant, describing an ellipse with area $\pi \varepsilon_x$, where ε_x is called the natural or geometrical emittance. As it depends on the twiss parameters around the ring, it has a different shape at different positions, with an always constant area. In the case of acceleration, the quantity that is kept constant is $\beta \gamma \varepsilon_x$, called the normalized emittance. The geometrical meaning of the twiss functions and the emittance is shown in the left part of Figure 2.3.

The most general form for the transfer matrix M in one period, can then be expressed as:

$$M_{0 \rightarrow s} = \begin{pmatrix} \sqrt{\frac{\beta(s)}{\beta_0}} (\cos \phi + \alpha_0 \sin \phi) & \sqrt{\beta(s) \beta_0} \sin \phi \\ \frac{(\alpha_0 - \alpha(s)) \cos \phi - (1 + \alpha_0 \alpha(s)) \sin \phi}{\sqrt{\beta(s) \beta_0}} & \sqrt{\frac{\beta_0}{\beta(s)}} (\cos \phi - \alpha_0 \sin \phi) \end{pmatrix}, \quad (2.25)$$

where α , β and γ the twiss parameters and $\phi = \int_0^s \frac{ds}{\beta(s)}$ the phase advance between the positions

Figure 2.4: Emittance ellipses change due to different $\delta p/p_0$ [9].

$s = 0$ and s . The evolution of twiss functions is expressed through:

$$\begin{pmatrix} \beta_s \\ \alpha_s \\ \gamma_s \end{pmatrix} = \begin{pmatrix} M_{11}^2 & -2M_{11}M_{12} & M_{12}^2 \\ -M_{11}M_{21} & M_{11}M_{22} + M_{12}M_{21} & -M_{12}M_{22} \\ M_{21}^2 & -2M_{21}M_{22} & M_{22}^2 \end{pmatrix} \begin{pmatrix} \beta_{s0} \\ \alpha_{s0} \\ \gamma_{s0} \end{pmatrix}, \quad (2.26)$$

where M_{ij} the transfer elements of $M(s_1|s_0)$.

The action (J_x) - angle (ϕ_x) variables provide an alternative to the Frenet-Serret variables for describing the dynamics of a particle moving along a beam line. The advantage of action-angle variables is that the phase space motion is described by circles, thus the shape of the phase space is preserved around the machine. Under symplectic transport, the action of a particle is constant. The action-angle variables form a canonically conjugate pair. The action and the emittance are connected through $\varepsilon_x = 2J_x$ [7].

Considering a beam and not a single particle, the beam or statistical emittance is defined by a contour confining some fraction of particles, depending on the particle distribution (e.g. homogeneous, Gaussian, etc.). The statistical emittance is defined by: $\varepsilon_x = \langle A_x \rangle$. The statistical emittance is shown in the right part of Figure 2.3.

Particles with different $\delta p/p_0$, trace off-momentum orbits different than the ideal orbit, as shown in Figure 2.2. When they pass through the quadrupoles, particles with different energies get different focusing, as the focusing strength depends on the energy of the particles (see Eq. (2.6)). The effect of different betatron oscillations or tunes due to different momentum deviations is called chromaticity and is shown schematically in the top part of Figure 2.4. The beam ellipses are then differently transformed passing through the elements of the lattice, as shown in the bottom part of the figure.

In the non-zero momentum deviation case, there is another invariant of the lattice, called the dispersion invariant defined as:

$$\mathcal{H}_x = \gamma_x D_x^2 + 2\alpha_x D_x D'_x + \beta_x D_x'^2, \quad (2.27)$$

where D_x and D'_x the dispersion function and its derivative.

Similar expressions as the ones described above, are valid for the vertical plane too. In the ideal case, where the three planes are fully uncoupled, three independent two-dimensional beam emittances are defined. According to Liouville's theorem, under the influence of conservative forces, the particle beam volume in phase space stays constant [8].

Momentum compaction factor

Off-momentum particles on the dispersion orbit, travel in a different path length than on-momentum particles. The change of the path length with respect to the momentum spread is called momentum compaction factor [7]:

$$\alpha_p = \frac{\Delta C/C}{\delta p/p_0}. \quad (2.28)$$

The change in circumference for a particle with momentum $p_0 + \delta p$ is:

$$\Delta C = \oint D \frac{\delta p}{p_0} d\theta = \oint D \frac{\delta p}{p_0} \frac{ds}{\rho_x}, \quad (2.29)$$

and thus, the momentum compaction factor can be written as:

$$\alpha_p = \frac{1}{C} \oint \frac{D(s)}{\rho_x(s)} ds = \left\langle \frac{D(s)}{\rho_x(s)} \right\rangle. \quad (2.30)$$

The revolution frequency of a reference particle, is defined as:

$$f = \frac{u_z}{2\pi\rho_x} = \frac{\beta c}{C}. \quad (2.31)$$

The change in the frequency for an off-momentum particle can then be written as:

$$\frac{\delta f}{f} = \left(\frac{1}{\gamma^2} - \alpha_p \right) \frac{\delta p}{p_0}, \quad (2.32)$$

where the factor $(1/\gamma^2 - \alpha_p) \equiv \eta$ is called the slippage factor. For vanishing slippage factor, the transition energy is defined as:

$$\gamma_t = \frac{1}{\sqrt{\alpha_p}}. \quad (2.33)$$

Below the transition energy ($\eta < 0$), a higher momentum particle will have shorter revolution period than the reference one, and will arrive at a fixed location earlier than the reference particle. Above transition energy ($\eta > 0$) the opposite is true. At $\gamma = \gamma_t$ the revolution period is independent of the particle's momentum and all particles around the accelerator will travel with equal revolution frequencies.

2.1.2 Chromaticity

The particles in a beam distribution have different momenta, getting different focusing when passing through a quadrupole and thus, having different oscillation frequencies. The variation of

the oscillation frequency, or betatron tunes, with the relative momentum deviation defines the chromaticity:

$$\xi_{x,y} = \frac{\partial Q_{x,y}}{\partial \delta}. \quad (2.34)$$

The chromaticity caused only by the linear elements of the lattice, is called the natural chromaticity and is defined as [7]:

$$\begin{aligned} \xi_x &= \frac{\partial Q_x}{\partial \delta} = -\frac{1}{4\pi} \int_{s_0}^{s_0+C} \beta_x(s) K_1(s) ds, \\ \xi_y &= \frac{\partial Q_y}{\partial \delta} = \frac{1}{4\pi} \int_{s_0}^{s_0+C} \beta_y(s) K_1(s) ds. \end{aligned} \quad (2.35)$$

R.M.S. beam size

The beam size is defined as the half width of the beam particles distribution and is defined by the beam emittance, the value of the betatron function, the value of the dispersion function and the energy spread. It is thus a quantity that varies along the lattice [8]. For the case of a Gaussian beam distribution, which is very often the case for most particle beams, the contribution to the beam size from different sources add in quadratically, thus the horizontal x (or vertical y) beam size is defined as:

$$\sigma_{x,y}(s) = \sqrt{\varepsilon_{x,y} \beta_{x,y}(s) + D_{x,y}^2(s) \sigma_{p0}^2}. \quad (2.36)$$

It is important to notice, that a beam with a horizontal and vertical size of 1σ has a cross section of $2\sigma_x\sigma_y$ and includes only 46.59 % of the beam.

2.1.3 Longitudinal motion

The motion of particles in the longitudinal plane, for bunched beams, is dominated by the radio-frequency field applied [7]:

$$E_z = E_0 \sin(\omega_{rf}t + \phi_s), \quad (2.37)$$

where E_0 the amplitude of the field, ϕ_s is a phase factor and ω_{rf} is the angular frequency of the RF system. A particle, with revolution period T_0 and momentum p_0 , synchronized with the RF wave, is called a synchronous particle. Its angular revolution frequency will be $\omega_0 = \frac{2\pi}{T_0} = \omega_{rf}/h$, where h is an integer called the harmonic number. Under the influence of the electric field of the RF cavity, a synchronous particle gains in each turn an amount of energy:

$$\Delta E_s = eV_0 \sin(\phi_s), \quad (2.38)$$

where V_0 the amplitude of the RF voltage.

Non-synchronous particles, with phases $\phi = \phi_s \pm \delta\phi$, will gain a different amount of energy per turn, equal to $\Delta E_p = eV_0 \sin(\phi)$. Figure 2.5 shows schematically this effect, where the synchronous particle is denoted by P_1 (or P_2). In the case where the revolution frequency $f_0 = 1/T_0$, is higher for higher momentum particles (i.e. below transition, $\eta < 0$), the higher energy particle (N_1) will arrive earlier ($\phi_1 = \phi_s - \delta\phi$) than the synchronous one. Therefore, if $0 < \phi_s < \pi/2$, this particle will receive less energy gain from the RF voltage and will come closer to the synchronous particle in the next turn. In a similar way, the lower energy particle

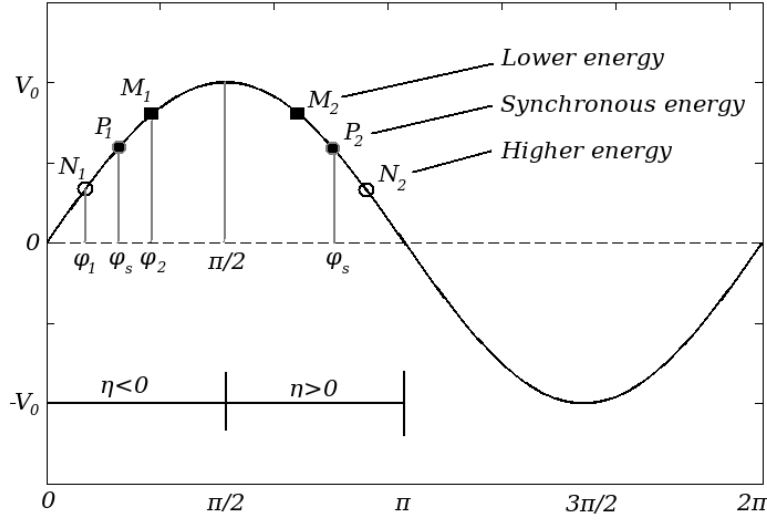


Figure 2.5: Schematic representation of phase stability of synchrotron motion [7].

(M_1) will arrive later ($\phi_2 = \phi_s + \delta\phi$) and gain more energy than the synchronous one. This phase focusing effect provides the phase stability of synchrotron motion. The opposite happens around point P_2 at $\pi - \phi_s$, i.e. M_2 and N_2 will further separate, thus P_2 is called an unstable point. Above transition ($\eta > 0$), where f_0 is lower for higher momentum particles, the stable point becomes the P_2 and the unstable the P_1 .

The energy gain per turn with respect to the energy gain of the synchronous particle is:

$$(\Delta E)_{\text{turn}} = \Delta E_p - \Delta E_s = eV_0(\sin \phi - \sin \phi_s). \quad (2.39)$$

Considering slow change of energy with respect to the revolution frequency, the equation of motion for the energy difference is:

$$\frac{d}{dt} \left(\frac{\Delta E}{\omega_0} \right) = \frac{1}{2\pi} eV_0(\sin \phi - \sin \phi_s), \quad (2.40)$$

or using the fractional off-momentum deviation:

$$\frac{\delta p}{p_0} = \frac{\omega_0}{\beta^2 E_0} \frac{\Delta E}{\omega_0} \Rightarrow \frac{d}{dt} \left(\frac{\delta p}{p_0} \right) = \frac{\omega_0}{2\pi \beta^2 E_0} eV_0 (\sin \phi - \sin \phi_s), \quad (2.41)$$

where E_0 is the energy of the synchronous particle and β the relativistic velocity. The time evolution of the phase angle variable is:

$$\frac{d\phi}{dt} = -h(\omega - \omega_0) = -h\Delta\omega. \quad (2.42)$$

Replacing the slippage factor from Eq. (2.32), the phase equation becomes:

$$\frac{d\phi}{dt} = h\omega_0\eta \frac{\delta p}{p_0} = \frac{h\omega_0^2\eta}{\beta^2 E_0} \left(\frac{\Delta E}{\omega_0} \right). \quad (2.43)$$

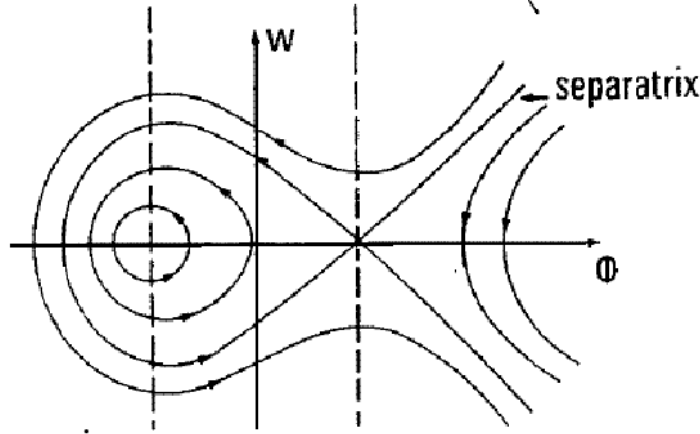


Figure 2.6: Trajectories of the particles in the longitudinal phase space. Red line indicates the separatrix. Trajectories outside the separatrix indicate unstable motion.

The pairs $(\phi, \Delta E/\omega_0)$ or equivalently $(\phi, \delta p/p_0)$ are pairs of conjugate phase space coordinates and Eqs (2.40) and (2.43) form the synchrotron equations of motion. Combining the energy and phase equations (2.40) and (2.43), a second order differential equation is obtained, which for the small angle oscillations is similar to the equation of the harmonic oscillator:

$$\frac{d^2}{dt^2}(\phi - \phi_s) = \frac{h\omega_0^2 e V_0 \eta_0 \cos \phi_s}{2\pi\beta^2 E_0}(\phi - \phi_s). \quad (2.44)$$

From this, the stability criterion of the synchrotron motion is:

$$\eta_0 \cos \phi_s < 0, \quad (2.45)$$

showing that, as was already discussed before, below transition energy, with $\eta_0 < 0$, the synchronous phase angle should be $0 < \phi_s < \pi/2$. Similarly, above transition, with $\eta_0 > 0$, the synchronous phase angle should be shifted to $\pi - \phi_s$. The angular synchrotron frequency is:

$$\omega_s = \omega_0 \sqrt{\frac{heV_0 |\eta_0 \cos \phi_s|}{2\pi\beta^2 E_0}}, \quad (2.46)$$

while the synchrotron tune, defined as the number of synchrotron oscillations per revolution is:

$$Q_s = \frac{\omega_s}{\omega_0} = \sqrt{\frac{heV_0 |\eta_0 \cos \phi_s|}{2\pi\beta^2 E_0}}. \quad (2.47)$$

In the longitudinal phase space, the motion is described by distorted circles in the vicinity of ϕ_s , which is called a stable fixed point, while for phases beyond $\pi - \phi_s$, which is called the unstable fixed point, the motion is unbounded as shown in Figure 2.6. The curve passing through $\pi - \phi_s$ is called the separatrix and the enclosed area the bucket. The height of the separatrix is called momentum acceptance and is defined by:

$$\left(\frac{\delta p}{p_0}\right)_{\max} = \mp \sqrt{\frac{eV_0}{\pi h \alpha_p E_0} (2 \cos \phi_s + (2\phi_s - \pi) \sin \phi_s)}, \quad (2.48)$$

where α_p the momentum compaction factor.

2.2 Synchrotron radiation damping and quantum excitation

When charged particles are accelerated in an electromagnetic field, they emit electromagnetic radiation. In the general case, the radiation power of an accelerated particle with charge e , moving with momentum $\mathbf{p} = m_0\mathbf{u}$, is given by Larmor's formula [10]:

$$P_s = \frac{e^2}{6\pi\epsilon_0 m_0^2 c^3} \left(\frac{d\mathbf{p}}{dt} \right)^2, \quad (2.49)$$

where $\epsilon_0 = 8.85419 \times 10^{-12} \text{ AsV}^{-1}\text{m}^{-1}$ is the permittivity of free space, m_0 the particle's rest mass and c the speed of light. This formula shows that the electromagnetic energy is emitted only when the particle's momentum changes, as a result of an applied force ($d\mathbf{p}/dt \neq 0$). For non-relativistic particles, this radiation is very weak and may be neglected. For relativistic particles, on the other hand, the radiation power can be written as:

$$P_s = \frac{e^2 c}{6\pi\epsilon_0 (m_0 c^2)^2} \left[\left(\frac{d\mathbf{p}}{d\tau} \right)^2 - \frac{1}{c^2} \left(\frac{dE}{d\tau} \right)^2 \right]. \quad (2.50)$$

In the case of linear acceleration this reduces to:

$$P_s = \frac{e^2 c \gamma^2}{6\pi\epsilon_0 (m_0 c^2)^2} \left(\frac{dp}{dt} \right)^2, \quad (2.51)$$

i.e. proportional to the accelerating gradient, which is negligible in the linear accelerators built today. For circular motion, where particles are bent perpendicular to their direction of motion, traveling in a circular path, the general radiation formula reduces to:

$$P_s = \frac{e^2 c}{6\pi\epsilon_0 (m_0 c^2)^2} \left[\left(\frac{d\mathbf{p}}{d\tau} \right)^2 - \frac{1}{c^2} \left(\frac{dE}{d\tau} \right)^2 \right]. \quad (2.52)$$

The change of momentum of a particle in a circular path, with bending radius R , can be written as $\frac{dp}{dt} = p\omega = p\frac{u}{R}$. The instantaneous radiation power from circular motion, called synchrotron radiation, is then given by Liénard's formula:

$$P_s = \frac{e^2 c}{6\pi\epsilon_0} \frac{1}{(m_0 c^2)^4} \frac{E^4}{\rho_x^2}, \quad (2.53)$$

where E is the energy of the particle, c the speed of light and ρ_x the local radius of curvature. The radiation power, thus, varies proportional to the fourth power of energy and inverse proportional to the fourth power of the rest mass m_0 . This radiation is important for electrons and positrons, while for protons, with high rest mass, it becomes important only for energies higher than 1 TeV [10].

The energy radiated, per revolution period, from a particle with energy E is:

$$U_0 = \oint P dt = \frac{C_\gamma}{2\pi} E^4 \oint \frac{1}{\rho_x^2} ds \equiv \frac{C_\gamma}{2\pi} E^4 I_2, \quad (2.54)$$

where β the relativistic factor, c the speed of light, ρ_x the local radius of curvature and $C_\gamma = \frac{4\pi}{3} \frac{r_0}{(mc^2)^3} = 8.846 \times 10^{-5} \text{ m/GeV}^3$ for electrons. I_2 is called the second radiation integral.

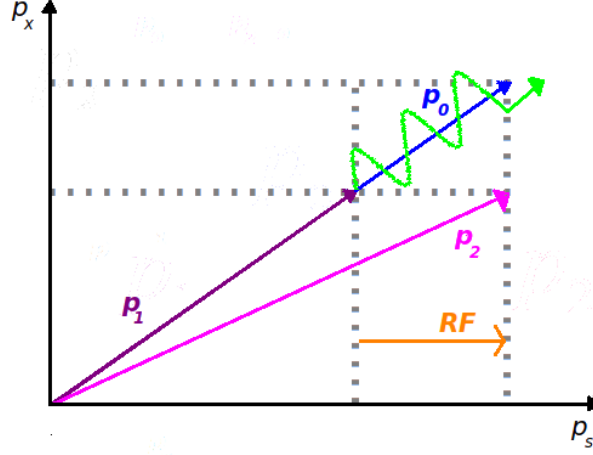


Figure 2.7: The synchrotron radiation damping mechanism [9].

The energy loss in the form of synchrotron radiation, results in a reduction of the particles' momenta in all three planes. As the RF field compensates the energy loss only in the longitudinal plane, this leads to steady reduction of the transverse betatron oscillation or to damping. A schematic representation of the effect is shown in Figure 2.7.

Betatron and synchrotron oscillation amplitudes are damped like: $A_i = A_{i0}e^{-\alpha_i t}$, where $i = x, y, s$. The α_i coefficients are called the damping increments, which are equivalent to the damping term of the harmonic oscillation with a frictional force. Taking into account the different, dispersive, orbit of off-momentum particles, the damping increments can be written in the form:

$$\begin{aligned}\alpha_z &= \frac{U_0}{2ET_0}(2 + \mathcal{D}) = \frac{U_0}{2ET_0}\mathcal{J}_s \quad \text{with} \quad \mathcal{J}_s = 2 + \mathcal{D}, \\ \alpha_y &= \frac{U_0}{2ET_0} = \frac{U_0}{2ET_0}\mathcal{J}_y \quad \text{with} \quad \mathcal{J}_y = 1, \\ \alpha_x &= \frac{U_0}{2ET_0}(1 - \mathcal{D}) = \frac{U_0}{2ET_0}\mathcal{J}_x \quad \text{with} \quad \mathcal{J}_x = 1 - \mathcal{D},\end{aligned}\tag{2.55}$$

with

$$\mathcal{D} = \frac{\oint \left[\frac{D}{\rho_x} \left(2k + \frac{1}{\rho_x^2} \right) \right]}{\oint \frac{ds}{\rho_x^2}},$$

where D the dispersion function, ρ_x the radius of curvature and k the quadrupole focusing strength. The $\mathcal{J}_x, \mathcal{J}_y, \mathcal{J}_s$, are called the damping constants.

The corresponding damping times, defined as $1/\alpha_i$, are given by:

$$\begin{aligned}\tau_x &= \frac{2E_0T_0}{\mathcal{J}_xU_0} = \frac{2E_0T_0}{(1 - I_4/I_2)U_0}, \\ \tau_y &= \frac{2E_0T_0}{\mathcal{J}_yU_0} = \frac{2E_0T_0}{U_0}, \\ \tau_s &= \frac{2E_0T_0}{\mathcal{J}_sU_0} = \frac{2E_0T_0}{(2 + I_4/I_2)U_0},\end{aligned}\tag{2.56}$$

where T_0 the revolution period and γ the relativistic energy factor. According to the Robinson Theorem, the sum of the three damping constants is invariant [7]:

$$\mathcal{J}_x + \mathcal{J}_y + \mathcal{J}_s = 4.$$

For machines with separated function magnets, $\mathcal{D} \ll 1$ and $\mathcal{J}_x = 1$, $\mathcal{J}_y = 1$ and $\mathcal{J}_s = 2$.

The beam parameters in a circular accelerator are modified due to synchrotron radiation, and they can be expressed through the radiation integrals. Some of them have been introduced earlier, while the following equations summarize all of them [11].

$$\begin{aligned}I_1 &= \oint \frac{D_x}{\rho_x} ds, & I_2 &= \oint \frac{1}{\rho_x^2} ds, & I_3 &= \oint \frac{1}{|\rho_x^3|} ds, \\ I_4 &= \oint \frac{D_x}{\rho_x} \left(\frac{1}{\rho_x^2} - 2K_1 \right) ds = \oint \frac{(1 - 2n)D_x}{\rho_x^3} ds, & I_5 &= \oint \frac{\mathcal{H}_x}{|\rho_x^3|} ds.\end{aligned}\tag{2.57}$$

The damping partition numbers are defined through the radiation integrals as:

$$\mathcal{J}_x = 1 - \frac{I_4}{I_2}, \quad \mathcal{J}_y = 1, \quad \mathcal{J}_s = 2 + \frac{I_4}{I_2}.\tag{2.58}$$

If radiation damping was the only effect, then after a while, the oscillation amplitude would reduce to zero and the beam would have a negligibly small emittance. However, photons are emitted with a distribution with an angular width $1/\gamma$, about the direction of motion of the electron. This leads to some vertical “recoil” that excites vertical betatron motion, resulting in a non-zero vertical emittance. Furthermore, if a particle emits a photon at a place with non-zero dispersion, it loses energy and instantly starts performing betatron oscillations about a different equilibrium orbit. This statistical nature of the emission of photons leads to a continuous increase of the betatron amplitudes and of the beam size. The balance between synchrotron radiation damping and quantum excitation, defines an equilibrium state of the beam distribution [11].

2.3 Equilibrium beam properties

In all six degrees of freedom, the equilibrium distribution is Gaussian with parameters defined by the damping times and the respective diffusion coefficients [8]. The emittance evolution with time is given by:

$$\varepsilon_x(t) = \varepsilon_{x,\text{inj}} e^{-2t/\tau_x}, \quad \varepsilon_y(t) = \varepsilon_{y,\text{inj}} e^{-2t/\tau_y}, \quad \sigma_p^2(t) = \sigma_{p,\text{inj}}^2 e^{-2t/\tau_p},\tag{2.59}$$

where the index inj refers to the injected beam emittances and energy spread and τ_x , τ_y , τ_p the horizontal, vertical and longitudinal damping times respectively.

Energy spread and bunch length

As the emission of photons is a statistical process and the RF cavity recovers only the average energy loss, there is a subsequent rms equilibrium energy spread in the beam [7]:

$$\sigma_{p0}^2 = \left(\frac{\delta E}{E} \right)^2 = C_q \gamma^2 \frac{\langle |1/\rho_x^3| \rangle}{\mathcal{J}_s \langle 1/\rho_x^2 \rangle} = C_q \gamma^2 \frac{I_3}{2I_2 + I_4}, \quad (2.60)$$

with $C_q = \frac{55}{32\sqrt{3}} \frac{\hbar c}{mc^2} = 3.8319 \times 10^{-19}$ m for electrons. For separated function magnets, it depends only on the particle energy and the bending radius, where for combined function magnets the partition number $\mathcal{J}_s$ can be modified accordingly to vary the energy spread.

In the case of circular electron accelerators the bunches are small compared to the bucket area, thus the small amplitude approximation is valid. The rms relative energy spread σ_p and rms bunch length σ_s [m] are then related through:

$$\sigma_{s0} = \sigma_{p0} C \sqrt{\frac{\alpha_p E}{2\pi h (eV_0^2 - U_0^2)^{1/2}}}, \quad (2.61)$$

where α_p is the momentum compaction factor, C the ring circumference, V_0 the amplitude of the RF voltage, h the harmonic number and U_0 the energy loss per turn.

Horizontal Emittance

The equilibrium horizontal beam emittance is defined as [7]:

$$\varepsilon_{x0} = C_q \gamma^2 \frac{\langle \mathcal{H}_x / |\rho_x^3| \rangle}{\mathcal{J}_x \langle 1/\rho_x^2 \rangle} = C_q \gamma^2 \frac{I_5}{I_2 - I_4}. \quad (2.62)$$

It scales as the square of the beam energy γ , and depends on the bending radius ρ_x and the dispersion invariant $\mathcal{H}_x$, defined in Eq. 2.27. It can be adjusted by appropriate choice of the optics functions along the bending magnets.

Vertical Emittance

Similarly to the horizontal emittance, the contribution of vertical dispersion to vertical emittance is [7]:

$$\varepsilon_{y0} = C_q \gamma^2 \frac{\langle \mathcal{H}_y / |\rho_y^3| \rangle}{\mathcal{J}_y \langle 1/\rho_y^2 \rangle}. \quad (2.63)$$

In order to minimize this effect, correction of the equilibrium orbit and the perturbation to the dispersion function is needed.

In the absence of vertical dispersion or coupling, $\mathcal{H}_y = 0$ and the vertical emittance is defined only by the opening angle of synchrotron radiation. The synchrotron radiation photons are emitted with an rms angle of $1/\gamma$, relative to the particle trajectory, affecting both the longitudinal and transverse momenta of the particle, defining a lower (the so-called quantum) limit of vertical emittance:

$$\varepsilon_{y0,\min} = \frac{13}{55} \frac{C_q}{\mathcal{J}_y} \frac{\oint \frac{\beta_y}{|\rho_y|^3} ds}{\oint \frac{1}{\rho_y^2} ds}. \quad (2.64)$$

However, this is negligible and the vertical emittance is mainly defined by vertical dispersion, due to alignment errors and by betatron coupling, if we don't take into account any current depended or collective effects. The contribution of vertical dispersion to vertical emittance is:

$$\varepsilon_{y0,d} \approx \mathcal{J}_s \langle \mathcal{H}_y \rangle \sigma_{p0}^2, \quad (2.65)$$

while the contribution of weak coupling to vertical emittance is:

$$\varepsilon_{y0,k} = k \varepsilon_{x0}, \quad (2.66)$$

where σ_{p0} is the rms relative momentum deviation and k the coupling factor.

In the presence of both vertical dispersion and betatron coupling, the vertical emittance is the sum of the above expressions:

$$\varepsilon_{y0} = \varepsilon_{y0,\min} + \varepsilon_{y0,d} + \varepsilon_{y0,k}. \quad (2.67)$$

The index 0 in all the equilibrium properties expressions, refers to the absence of any current depended effects, thus to the case of nearly zero current.

2.4 Effect of damping wigglers on beam dynamics

Fast damping for linear collider Damping Rings (DR) or specific radiation characteristics for light sources, cannot be obtained using only conventional bending magnets. Special insertion magnets are needed, called damping wigglers if they are high field or undulators if they are low field devices. They consist of a series of alternating magnet poles deflecting the beam periodically in opposite directions, enhancing the emission of synchrotron radiation.

The basic requirement of an insertion device is that, after passing through it, the beam should return to its nominal orbit, otherwise, a closed orbit distortion occurs around the machine. The presence of insertion devices may induce perturbations due to the magnetic field of the device, which results in linear optics distortion, tune shifts, resonance excitation and reduction of dynamic aperture. In addition, the radiation emitted by the beam in the insertion device changes the emittance and energy spread of the beam [12].

Using standard perturbation theory, the vertical tune shift and beta function distortions are estimated [7]:

$$\Delta\nu_y \approx \frac{\langle \beta_y \rangle L_{ID}}{8\pi\rho_x^2}, \quad (2.68)$$

$$\left(\frac{\Delta\beta_y}{\beta_y} \right)_{\max} \approx \frac{L_{ID} \langle \beta_y \rangle}{4\pi\rho_x^2 \sin(2\pi\nu_y)}, \quad (2.69)$$

where $\langle \beta_y \rangle$ the average lattice vertical beta function and L_{ID} the length of the insertion device. The insertion of damping wigglers along the particle beam path gives rise to fast damping and quantum excitation, resulting in different equilibrium states and damping times. For short wiggler poles, the magnetic field can be expressed by:

$$B_y(x, y=0, z) = B_0 \sin \frac{2\pi z}{\lambda_p}, \quad (2.70)$$

where B_0 the wiggler peak field and λ_p the wiggler period.

The radiation integrals can be generalized taking into account the contribution from the wigglers as:

$$I_i = I_{ia} + I_{iw}, \quad (2.71)$$

where the index i refers to the i^{th} integral and the indexes a and w to the contribution from the arcs and the wigglers respectively. Assuming wiggler magnets with sinusoidal field variation, the contribution to the radiation integrals can be written as:

$$\begin{aligned} I_{2w} &= \frac{L_{ID}}{2\rho_x + w^2}, & I_{3w} &= \frac{4}{3\pi} \frac{L_{ID}}{\rho_w^3}, & I_{4w} &= -\frac{3}{32\pi^2} \frac{\lambda_w^2}{\rho_w^4} L_{ID} \\ I_{5w} &= \frac{\lambda_w^4}{4\pi^4 \rho_w^5} \left[\frac{3}{5\pi} + \frac{3}{16} \right] \langle \gamma_x \rangle L_{ID} - \frac{9\lambda_w^3}{40\pi^4 \rho_w^5} \langle \alpha_x \rangle L_{ID} + \frac{\lambda_w^2}{15\pi^3 \rho_w^5} \langle \beta_x \rangle L_{ID}, \end{aligned} \quad (2.72)$$

where ρ_w the bending radius of the wiggler magnet and L_{ID} the wiggler length. The operator $\langle \dots \rangle$ denotes the average of the quantity over the wiggler length. For sinusoidal fields and considering $\langle \alpha_x \rangle$ very small through the wiggler length:

$$I_{5w} = \frac{\lambda_w^2}{15\pi^3 \rho_w^5} \langle \beta_x \rangle L_{ID}, \quad (2.73)$$

with the wiggler radius defined by:

$$\rho_w = \frac{(B\rho_x)}{B_w} \quad \text{or} \quad \rho_w[m] = \frac{0.0017\gamma}{B_w[T]}. \quad (2.74)$$

$(B\rho_x)$ is the standard energy depended magnetic rigidity.

The change of the damping rate due to the wiggler is conventionally defined by the relative damping factor:

$$F_w = \frac{I_{2w}}{I_{2a}} = \frac{L_w B_w^2}{4\pi(B\rho_x)B_a} = \frac{1}{4\pi \cdot 0.0017[Tm]} \frac{L_w B_w^2}{\gamma B_a} \geq 0. \quad (2.75)$$

B_a is the magnetic field of the dipole magnets and the damping is dominated by the wigglers if $F_w > 1$. The total energy loss in the presence of wigglers is then defined as:

$$U_0 = U_{0a}(1 + F_w) = 3.548 \times 10^{-12} [MeV] \gamma^3 B_a [T] (1 + F_w), \quad (2.76)$$

where U_{0a} the energy loss from the dipole magnets. The damping partition number is defined as:

$$\mathcal{J}_x = \frac{\mathcal{J}_{xa} + F_w}{1 + F_w}, \quad (2.77)$$

where $\mathcal{J}_{xa}$ the contribution from the arc cells. The radiation damping times are:

$$\begin{aligned} \tau_x &= \frac{2E_0 T_0}{\mathcal{J}_x U_0} = \frac{3(B\rho_x)C}{2\pi r_0 c \gamma^3 B_a (\mathcal{J}_{xa} + F_w)} = E_2 \frac{C}{B_a \gamma^2 (\mathcal{J}_{xa} + F_w)}, \\ \tau_y &= \frac{2E_0 T_0}{\mathcal{J}_y U_0} = \frac{3(B\rho_x)C}{2\pi r_0 c \gamma^3 B_a (1 + F_w)} = E_2 \frac{C}{B_a \gamma^2 (1 + F_w)}, \\ \tau_s &= \frac{2E_0 T_0}{\mathcal{J}_s U_0} = \frac{3(B\rho_x)C}{2\pi r_0 c \gamma^3 B_a (3 - \mathcal{J}_{xa} + 2F_w)} = E_2 \frac{C}{B_a \gamma^2 (3 - \mathcal{J}_{xa} + 2F_w)}, \end{aligned} \quad (2.78)$$

where,

$$E_2 = \frac{3(B\rho_x)}{2\pi r_0 c \gamma} = \frac{3 \cdot 0.0017[Tm]}{2\pi r_0 c} = 960.13 \left[\frac{T \cdot sec}{m} \right]. \quad (2.79)$$

As the equilibrium emittance and energy spread and the momentum compaction factor are defined through the radiation integrals, these quantities considering a hard edge wiggler, for which the fields of the magnets are considered to begin and end exactly in the beginning and end of the magnets, can be expressed as [13]:

$$\varepsilon_{x0} \approx \frac{C_q \gamma^2}{12(\mathcal{J}_{xa} + F_w)} \left[\frac{\varepsilon_r \theta^3}{\sqrt{15}} + \frac{F_w |B_w|^3 \lambda_w^2 \langle \beta_x \rangle}{16(B\rho_x)^3} \right], \quad (2.80)$$

$$\sigma_{p0} \approx \gamma \sqrt{\frac{C_q I_3}{2I_2 + I_3}} = \gamma \left[\frac{C_q |B_a|}{(B\rho_x)} \frac{1 + F_w \frac{B_w}{B_a}}{3 - \mathcal{J}_{xa} + 2F_w} \right]^{1/2}, \quad (2.81)$$

$$\begin{aligned} \alpha_p &\approx \frac{3\pi}{2} \left(d \frac{4\sqrt{15}}{9} \right)^{2/3} \frac{(B\rho_x)(1 + F_w)^{2/3}}{C |B_a| \gamma^2} \\ &\times \left(\frac{\gamma \varepsilon_{x0}}{C_q} - \frac{|B_w|^3 \lambda_w^2 \langle \beta_x \rangle \gamma^3}{192(B\rho_x)^3} \frac{F_w}{\mathcal{J}_{xa} + F_w} \right)^{2/3} \times \frac{\sqrt{5} + \sqrt{\varepsilon_r^2 - 1}}{\varepsilon_r^{2/3}}. \end{aligned} \quad (2.82)$$

2.4.1 Wiggler modeling in MADX

As the wiggler does not exist as a standalone element in the design code MADX [14], a simplified model using alternating gradient dipoles, separated by drifts, is used.

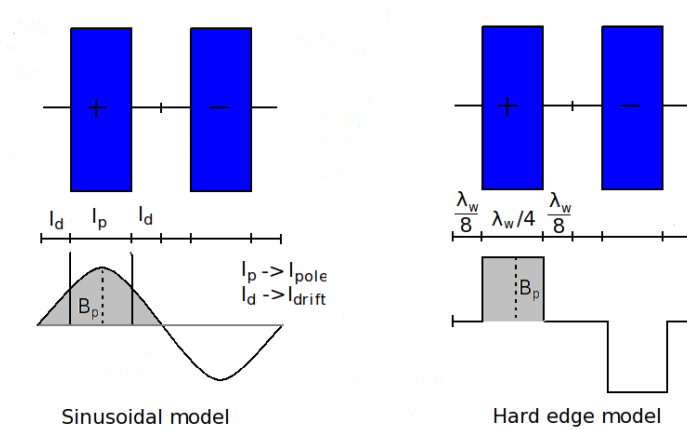


Figure 2.8: Wiggler modeling in MADX for a sinusoidal (left) and for a hard edge (right) form of the wiggler field.

In a first order approximation, the vertical field component of a wiggler has a sinusoidal form: $B_y = B_w \sin(z\pi/\lambda_w)$, where B_w the peak field and λ_w the wiggler period. This field distribution can be produced by a hard edge model or by a sinusoidal model, as shown in Figure 2.8. In the first case, the wiggler period λ_w consists of a drift space of length of $\lambda_w/8$, a magnetic pole with length of $\lambda_w/4$ producing positive vertical dipole field, a drift space of length of $\lambda_w/4$, a

magnetic pole with length of $\lambda_w/4$ producing negative vertical dipole field and a drift space of length of $\lambda_w/8$. In the second case, the wiggler period λ_w consists of a drift space of length of l_d , a magnetic pole with length of l_p producing positive vertical dipole field, two drift spaces of length of l_d each, a magnetic pole with length of l_p producing negative vertical dipole field and a drift space of length of l_d .

The integrated wiggler field in a wiggler pole with a sinusoidal field can be written as:

$$B_s = \int_0^{\lambda_w/2} B_w \sin(z\pi/\lambda_w) dz = \frac{B_w \lambda_w}{\pi}. \quad (2.83)$$

Considering a pole length l_p , a field B_p with an integrated field equivalent to the sinusoidal one, needs to be applied:

$$B_s = B_p \cdot l_p = \frac{B_w \lambda_w}{\pi} \implies B_p = \frac{B_w \lambda_w}{\pi l_p}, \quad (2.84)$$

where: $2l_d + l_p = \lambda_w/2 \implies l_p = \lambda_w/2 - 2l_d$. The length of the pole and the drifts need then to be matched in order to get the correct radiation integrals.

The procedure described above, is used to model sinusoidal field wigglers in madx. For a certain peak field and wiggler period, the wiggler pole length is chosen in order to get the required radiation integrals.

2.5 Low emittance lattices

The lattice design of e^+/e^- synchrotrons, used as light sources or damping rings, is focused on the minimization of the emittance. The equilibrium beam emittance, considering $\mathcal{J}_x \approx 1$, depends on the mean value of the lattice function $\mathcal{H}_x(s)$ in regions where $\rho_x \neq 0$, thus on the dipoles (see paragraph 2.3). Assuming a lattice where the optics functions are the same in all dipoles, the average value in all the ring is the same as in one dipole:

$$\langle \mathcal{H}_x \rangle = \frac{1}{L_d} \int_0^{L_d} \mathcal{H}_x(s) ds, \quad (2.85)$$

where L_d the length of the dipole magnet with constant bending radius ρ_x . The task of the ring emittance minimization is then reduced to the minimization of $\langle \mathcal{H}_x \rangle$ in the dipole. Knowing the twiss parameters at the entrance of the dipole, the function $\mathcal{H}_x$ can be determined at any point within the bending magnet, using the definitions of paragraph 2.1. Using the latest, the equilibrium emittance can be written in the form:

$$\varepsilon_{x0} = \mathcal{F}_{\text{lattice}} C_q \gamma^2 \theta^3, \quad (2.86)$$

where the scaling factor $\mathcal{F}_{\text{lattice}}$ depends on the lattice design.

There are two basic layouts in modern e^+/e^- rings: one imposes an achromat condition, i.e. $D_x = D'_x = 0$ in the entrance (or exit) of the bending magnet while the other impose a symmetry condition, i.e. both beta (β_x) and dispersion (D_x) functions have a minimum at the center of the bending magnets ($\alpha_x = D'_x = 0$) as shown in Figure 2.9 [15]. The first layout is widely used by light sources, where many dispersion free regions are required for the insertion of the synchrotron radiation beam lines. On the other hand, DR need only two long straight

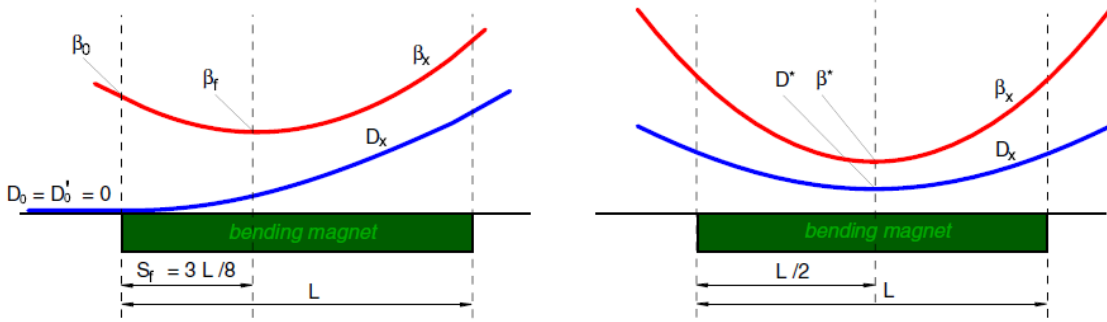


Figure 2.9: The two basic layouts for low emittance lattices. Left: the achromat condition. Right: the symmetry condition [15].

sections for the insertion devices (i.e. wiggler magnets, RF cavities and injection/extraction equipment), connected by two arc sections. This configuration is called “racetrack”, providing emittance minimization in a more compact ring.

2.5.1 The FODO cell

FODO cells have been used as the building blocks of high energy colliders and storage rings due to their simplicity. A FODO cell is usually configured as $\{\frac{1}{2} Q_F B Q_D B \frac{1}{2} Q_F\}$ where Q_F , Q_D focusing and defocusing quadrupoles and B a bending magnet.

The scaling factor for the FODO cell is:

$$\mathcal{F}_{FODO} = \frac{1 - \frac{3}{4} \sin^2(\phi_x/2)}{\sin^3(\phi_x/2) \cos(\phi_x/2)} \mathcal{J}_x^{-1}, \quad (2.87)$$

where $\mathcal{J}_x$ the horizontal damping constant and ϕ_x the horizontal phase advance. The factor $\mathcal{F}_{FODO}$ increase rapidly with the phase advance and has a minimum of about 1.2 at $\phi_x = 137^\circ$ [7].

2.5.2 The Double Bend Achromat cell

The Double bend achromat (DBA) or basic Chasman-Green cell, is the most common one in high brilliance light sources. It consists of two dipoles, one for producing and the second for suppressing dispersion, and either a focusing quad (basic scheme) or a triplet (extended scheme) between them. In the basic scheme, the quadrupole provides focusing only in one plane, making the structure rather inflexible, while the triplet in the extended scheme, restores focusing in both transverse planes and provides flexibility [7, 15].

Applying the achromat condition, the minimum emittance for the DBA cell is for:

$$\mathcal{F}_{DBA} = \frac{1}{4\sqrt{15}\mathcal{J}_x}, \quad (2.88)$$

with $\beta_{x0} = \sqrt{12/5}L$ and $\alpha_{x0} = \sqrt{15}$ at the entrance of the dipole. The resulting minimum emittance is smaller by a factor $4\sqrt{15} \approx 15.5$ from the FODO cell lattice.

2.5.3 The Theoretical Minimum Emittance cell

The emittance can be further minimized, if the achromat condition is not necessary. The theoretical minimum emittance can be achieved if the symmetry condition is satisfied. In this case only one dipole is needed and the structure is more compact [16].

The scaling factor for the theoretical minimum emittance is equal to:

$$\mathcal{F}_{TME} = \frac{1}{12\sqrt{15}\mathcal{J}_x}, \quad (2.89)$$

with $D_{xc} = \frac{\theta L}{24}$ and $\beta_{xc} = \frac{L}{2\sqrt{15}}$ at the center of the dipole. The minimum emittance in this case is a factor of 3 smaller than in the DBA lattice.

The very small emittance they can achieve and their compactness make the TME cells very interesting for the Damping Rings lattice design. For this reason, an analytical solution for the TME cells was developed in order to have a global understanding of all cell properties. This will be described in detail in Chapter 5.

The above characteristics are for ideal lattices. In reality, tuning to the minimum emittance requirements is not easy due to other constraints. In the case of the DBA, the beta function is not the optimal, while tuning the TME to the absolute minimum conditions is very difficult. Low emittance lattices in general are intrinsically high chromaticity lattices due to the strong quadrupole strengths needed for the low dispersion and beta functions. This leads to strongly nonlinear motion and limited dynamic aperture.

2.6 Dispersion suppression

If insertion devices, like wigglers or undulators, are placed in a dispersive region they enhance quantum excitation, leading to the enlargement of equilibrium emittances. For this reason, dispersion free regions are required, for the placement of such devices. In the case of achromats, the dispersion is zero in the two ends of the cell, thus no extra configuration is needed for suppressing dispersion. In the case of TME type cells, however, extra configuration is needed in the two ends of the arcs, for dispersion suppression.

The optical functions between the arcs and the straight sections are matched with the dispersion suppression - beta matching (DS-BM) sections. As dispersion only gets propagated by quadrupoles and is generated in dipoles, one dipole per dispersion suppressor is needed in order to suppress dispersion. For simplicity, the dipole is considered to have the same characteristics (ρ_x, l_d) , as the dipole of the arc cell. From the evolution of the dispersion in a dipole (see Eq. (2.18)):

$$\begin{aligned} \eta(s) &= \eta_0 + \eta'_0 s + s^2/2\rho_x, \\ \eta'(s) &= \eta'_0 + s/\rho_x, \end{aligned} \quad (2.90)$$

the conditions in the entrance of the dipole for zero dispersion and dispersion derivative in the exit of the dipole are:

$$\eta_0 = \frac{l_d^2}{2\rho_x}, \quad \eta'_0 = -\frac{l_d}{\rho_x}. \quad (2.91)$$

In order to match the dispersion and dispersion derivative, between the exit of the arc cell and the entrance of the DS dipole, to the required values for the dispersion suppression, two quadrupoles

are needed. At least four more quadrupoles are then required for matching the $(\beta_x, \alpha_x, \beta_y, \alpha_y)$ between the exit of the arc cell and the entrance of the long straight section. Thus a minimum number of six quadrupoles are necessary in the DS-BM cells. Extra quadrupoles may be added for flexibility.

2.7 Non-linear dynamics

Since the transverse dimensions of the beam are small compared to the radius of curvature of the particle trajectory, the magnetic field in the vicinity of the nominal trajectory can be expanded:

$$B_y(x, s, y) + iB_x(x, s, y) = (B\rho_x) \sum_n (i\alpha_n(s) + b_n(s)) (x + iy)^{n-1}, \quad (2.92)$$

with n the multipole order and $2n$ the number of poles in the magnet, i.e. $n=1$ indicates dipole, $n=2$ quadrupole, $n=3$ sextupole, etc. The b_n are the regular multipoles ($B_x = 0$ for $x = 0$) and α_n the skew multipoles, obtained through a rotation around the s -axis by $90^\circ/n$. Differentiating Eq. (2.92), the multipole magnet strength is derived:

$$b_n = \frac{1}{B\rho_x} \frac{1}{(n-1)!} \frac{\partial^{(n-1)} B_y(x, y)}{\partial x^{n-1}} \Big|_{y=0}, \quad (2.93)$$

where $(B\rho_x)$ is the magnetic rigidity, defined in Eq.(2.4). The pole-tip field of a regular magnet is then defined:

$$B_{\text{pt}} = (B\rho_x) b_n R^{n-1} = \frac{R^{n-1}}{(n-1)!} \frac{\partial^{(n-1)} B_y(x, y)}{\partial x^{n-1}} \Big|_{y=0}. \quad (2.94)$$

Non-linear dynamics, refers to the effect of non-linear elements (i.e. sextupoles, octupoles, etc.) on the dynamics of the beam. The non-linear elements are introduced into the lattice, either through the imperfections of the linear elements or as correction elements.

The introduction of non-linear elements into the lattice allows to compensate the natural chromaticity, as the closed orbit of off-momentum particles is displaced with respect to the reference orbit by $D(\delta p/p_0)$. Passing through a sextupole, an off-momentum particle, with initial coordinates $(x + D\delta, y)$, receives a kick:

$$\begin{aligned} x' &= - \left[D\delta x + \frac{1}{2} \left(D \left(\frac{\delta p}{p_0} \right) \right)^2 + \frac{1}{2} (x^2 - y^2) \right] K_2 l, \\ y' &= \left[D \left(\frac{\delta p}{p_0} \right) y + xy \right] K_2 l, \end{aligned} \quad (2.95)$$

where $K_2 = \frac{e}{p_0} \frac{\partial^2 B_y}{\partial x^2}$ is the normalized sextupole strength. The first order contribution to the chromaticity is given by:

$$\begin{aligned} \frac{\partial Q_x}{\partial \delta} &= \frac{1}{4\pi} \int_{s_0}^{s_0+C} \beta_x(s) K_2(s) D(s) ds, \\ \frac{\partial Q_y}{\partial \delta} &= -\frac{1}{4\pi} \int_{s_0}^{s_0+C} \beta_y(s) K_2(s) D(s) ds. \end{aligned} \quad (2.96)$$

Comparing Eqs. (2.35) and (2.96) general rules for efficient chromaticity correction can be extracted: the sextupoles with $K_2 > 0$ have to be placed at places with large β_x values and $\beta_y \ll \beta_x$, the sextupoles with $K_2 < 0$ at places with large β_y values and $\beta_x \ll \beta_y$ and all of them at positions where the dispersion function gets high values.

2.7.1 Hamiltonian treatment

In order to understand the effect of non-linear elements to the motion of the beam, the single particle Hamiltonian has to be studied.

Considering only dipoles, quadrupoles and sextupoles, the Hamiltonian can be written as [17, 18]:

$$H(s) = \underbrace{\frac{p_x^2 + p_y^2}{2(1 + \delta)}}_{\text{kinetic}} - \underbrace{\frac{b_1 x \delta}{2}}_{\text{dispersive}} + \underbrace{\frac{b_1^2 x^2}{2}}_{\text{focussing}} + \underbrace{\frac{b_2}{2}(x^2 - y^2)}_{H_2(s)} + \underbrace{\frac{b_3}{3}(x^3 - 3xy^2)}_{H_3(s)}. \quad (2.97)$$

The first term is the kinematic term, while b_1 , b_2 and b_3 are the dipole, quadrupole and sextupole strengths respectively, defined by Eq. (2.93) and $\delta \equiv \delta p/p_0$ is the particle's momentum deviation. The non-linear dynamics optimization, aims in finding sextupole configurations for which $H_2 + H_3$ becomes achromatic while the equations of motion remain linear.

The equations of motion are obtained from Hamilton's equations:

$$\begin{aligned} x' &\equiv \frac{dx}{ds} = \frac{\partial H}{\partial p_x} = \frac{p_x}{1 + \delta}, \\ p'_x &\equiv \frac{dp_x}{ds} = -\frac{\partial H}{\partial x} = b_1(s)\delta - (b_1^2(s) + b_2(s))x - b_3(s)(x^2 - y^2), \\ y' &\equiv \frac{dy}{ds} = \frac{\partial H}{\partial p_y} = \frac{p_y}{1 + \delta}, \\ p'_y &\equiv \frac{dp_y}{ds} = -\frac{\partial H}{\partial y} = b_2(s)y + 2b_3(s)xy. \end{aligned} \quad (2.98)$$

Substituting the linear equations of motion from Eqs. (2.23) to Eq. (2.97), the quadrupole/sextupole Hamiltonian can be represented by the sum over the different phase advances between the sextupoles of the lattice:

$$\int_{\text{cell}} [H_2(s) + H_3(s)] ds = \sum h_{jklmp}, \quad (2.99)$$

where:

$$h_{jklmp} \propto \sum_n^{N_{\text{sext}}} (b_3 l_s)_n \beta_{xn}^{\frac{j+k}{2}} \beta_{yn}^{\frac{l+m}{2}} D_n^p e^{i((j-k)\phi_{xn} + (l-m)\phi_{yn})} - \left[\sum_n^{N_{\text{quand}}} (b_2 l_s)_n \beta_{xn}^{\frac{j+k}{2}} \beta_{yn}^{\frac{l+m}{2}} e^{i((j-k)\phi_{xn} + (l-m)\phi_{yn})} \right]_{p \neq 0}. \quad (2.100)$$

The Hamiltonian modes are sums of complex vectors. Each vector corresponds to a sextupole, its length given by the sextupole's integrated strength l_s and its complex phase by the modal multiple of the betatron phases at its location, denoted by ϕ_{xn} and ϕ_{yn} . β_{xn} , β_{yn} and D_n , are the horizontal and vertical beta functions and dispersion at the sextupole's location, respectively.

Nine Hamiltonian modes are finally found. Two of them have no dependence in the betatron phase advance, corresponding to the linear chromaticities:

$$\begin{aligned} h_{11001} &= +J_x \delta \left[\sum_n^{N_{\text{sext}}} (2b_3 L)_n \beta_{xn} D_n - \sum_n^{N_{\text{quand}}} (b_2 L)_n \beta_{xn} \right], \\ h_{00111} &= -J_y \delta \left[\sum_n^{N_{\text{sext}}} (2b_3 L)_n \beta_{yn} D_n - \sum_n^{N_{\text{quand}}} (b_2 L)_n \beta_{yn} \right]. \end{aligned} \quad (2.101)$$

The other seven modes (and their complex conjugates) have non-zero phase arguments, thus they do not add up over many turns but they show a resonant structure: long term behavior is

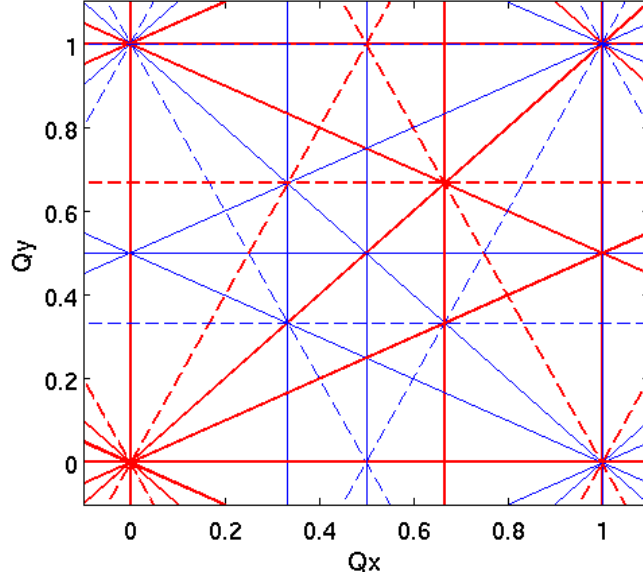


Figure 2.10: The tune diagram showing the resonance lines up to third order. In red, all the systematic resonances are shown while in blue the non-systematic ones. Solid lines correspond to resonances due to normal elements while dashed lines due to skew elements.

revealed by considering N repetitions of the lattice structure, i.e. many cells and many turns, and extrapolating to infinity:

$$\begin{aligned} |h_{jklmp}^\infty| &= \left| \sum_{n=0}^{\infty} h_{jklmp} e^{i((j-k)2\pi n Q_x^{\text{cell}} + (l-m)2\pi n Q_y^{\text{cell}})} \right| \\ &= \frac{|h_{jklmp}|}{2 \sin \pi [(j-k) Q_x^{\text{cell}} + (l-m) Q_y^{\text{cell}}]}, \end{aligned} \quad (2.102)$$

where $|h_{jklmp}|$ are called the resonance driving terms. From this, the resonance condition is:

$$(j-k)Q_x^{\text{cell}} + (l-m)Q_y^{\text{cell}} = n, \quad (2.103)$$

where, j, k, l, m, n are integers. Eq. (2.103) defines resonances, corresponding to forbidden lines in the Q_x, Q_y plane called a tune diagram. Working points close to these lines must be avoided, otherwise particles will be resonantly excited and get lost. Figure 2.10 shows the tune diagram with resonance lines up to third order. Red lines correspond to systematic resonances, for which the integer n is an integer multiple of the periodicity of the lattice, while blue lines correspond to non-systematic resonances. Solid lines correspond to regular multipoles, while dashed lines to skew multipoles.

High chromaticity would lead to the excitation of many resonances in the tune diagram, affecting the beam stability. Therefore, most storage rings operate with zero or slightly positive chromaticity.

Up to now, only ideal magnets were considered, for which no longitudinal variation of the fields is taken into account. In reality, however, there is a finite extend of the fields, called the

fringe fields, which can have an important impact on beam dynamics, and especially for rings of relatively small circumference but large acceptance [19]. In particular, the fringe field of a multipole of order n excites, in leading order expansion, $n + 2$ order fields, i.e dipole fringe fields excite quadrupole fields, sextupole fringe fields excite octupole fields etc [20]. In the non-linear dynamics optimization procedure, such fields has to be taken into account, as their contribution to the dynamic aperture may be important.

Summarizing, in order to correct chromaticity, sextupoles must be placed at high dispersion and well splitted beta functions regions, called the “chromatic” sextupoles. However, if the natural chromaticity is large, the sextupole strengths needed for the chromaticity correction are strong, introducing strong non-linearities and reducing the dynamic aperture (the maximum amplitude of stable betatron oscillations). This can be corrected by adding extra sextupoles even in dispersion-free regions, called “harmonic” sextupoles, which do not contribute to the chromaticity correction, but they are used for the minimization of the resonance driving modes. As the resonance driving terms depend on the phase advance between the sextupoles, a proper choice of the phase advances between the sextupoles can lead to the minimization of the driving terms. The effect of magnet fringe fields in the non-linear dynamics has to be taken into account, as their contribution to the dynamic aperture can be important.

2.8 Collective effects

Due to their charges, particles interact with Coulomb forces leading to intrabeam scattering or space charge effects. They also interact with their environment inducing charges and currents in the surrounding structures which create electromagnetic fields, called wake fields in space domain or impedances in frequency domain. Collective processes can have a significant contribution to beam dynamics, thus the understanding of all collective effects and the instabilities due to them is important for a successful accelerator design. In the next the effects of intrabeam scattering (IBS), Touschek scattering and space charge and the effect of potential well distortion are briefly described. Those are the main effects leading to current dependent emittance blow up in high intensity - low emittance e^+/e^- rings.

2.8.1 Intrabeam scattering

Intrabeam scattering (IBS) refers to the small angle multiple Coulomb scattering events between charged particles within accelerator beams, leading to the redistribution of the particle momenta and to the diffusion of the six-dimensional phase space. As this is the effect that this thesis elaborates, it will be discussed in detail in the next Chapter (see Chapter 3).

2.8.2 Touschek scattering

The elastic Coulomb collisions of particles in high intensity bunches, is called the Touschek effect [21]. The colliding particles exchange momentum, leading to a change in their oscillation amplitudes. If their amplitude exceeds the acceptance limits of the machine, the particles can get lost, leading to a reduction of the beam lifetime.

Considering a Gaussian particle distribution, the beam life time is given by:

$$\frac{1}{\tau} = -\frac{1}{N_b} \frac{dN_b}{dt} = \frac{r_c^2 c N_b}{8\pi \sigma_x \sigma_y \sigma_l} \frac{\lambda^3}{\gamma^2} D(\varepsilon), \quad (2.104)$$

$$D(\varepsilon) = \sqrt{\varepsilon} \left[-\frac{3}{2} e^{-\varepsilon} + \frac{\varepsilon}{2} \int_{\varepsilon}^{\infty} \frac{\ln u}{u} e^{-u} du + \frac{1}{2} (3\varepsilon - \varepsilon \ln \varepsilon + 2) \int_0^{\infty} \frac{e^{-u}}{u} du \right], \quad (2.105)$$

where r_c the classical particle radius, σ_x , σ_y , σ_l the standard values of the Gaussian bunch width, height and length respectively and $\lambda^{-1} = (\delta p/p_0)_{\max}$ the momentum acceptance parameter, assuming that is limited by the RF voltage. The argument ε of function D is:

$$\varepsilon = \left(\frac{(\delta p/p_0)_{\max}}{\gamma \sigma_l} \right), \quad (2.106)$$

with $\sigma_l = \frac{mc\gamma\sigma_x}{\beta_x}$. The Touschek effect is stronger at low energies and high bunch densities and the Touschek lifetime increases faster with the RF-acceptance than it decreases with the bunch length. It is an important effect in storage rings where the beam circulates for many hours.

2.8.3 Space charge

The space charge force refers to the force acting on one particle by the self field produced by the particle distribution [22]. For Gaussian bunches a linear approximation of the electric field can be derived (for small amplitudes). Using this linear approximation, the space-charge force appears as a linear defocusing force resulting a space charge tune shift, called the Laslett tune shift, given by:

$$\delta Q_{x,y} = -\frac{Nr_e}{(2\pi)^{3/2} \beta^2 \gamma^3 \sigma_s} \oint \frac{\beta_{x,y}}{\sigma_{x,y}(\sigma_x + \sigma_y)} ds, \quad (2.107)$$

where β the relativistic velocity, γ the relativistic energy, N the particles per bunch, σ_s the rms bunch length, r_e the electron radius, $\sigma_{x,y}$ the horizontal and vertical beam sizes and $\beta_{x,y}$ the horizontal and vertical beta functions. The integral is calculated around the ring circumference and depends both on the beam and lattice properties.

As each particle in the bunch experiences a different tune shift, space charge introduce a tune spread rather than tune shift called the incoherent tune shift and it is not possible to be compensated. This can move the beam onto a resonance, leading to emittance blow up or beam loss.

If in the above description, the effect of image charges of metallic and magnetic surfaces is taken into account, refers to the indirect space charge effect and is out of the scope of this thesis.

2.8.4 Potential well distortion and microwave instability

A given beam current in a beam pipe, induces fields produced in the vacuum chamber. In the time domain, those fields are called wake fields, while in the frequency domain they are called impedances. The bunch length depends on the interaction of the beam with the RF-field in the accelerating cavities and with any other field encountered in the ring. Potential well distortion refers to the influence of wake fields on the bunch length.

In addition to potential well bunch lengthening an increase of bunch current I_b can lead to longitudinal instabilities of a single bunch, mainly due to the high frequency part of the impedance. The energy spread of an electron bunch remains unchanged with increasing current up to the turbulent threshold, determined by sychrotron radiation. Above this threshold both bunch length and energy spread increase with current.

According to theory, the bunch lengthening factor σ_s/σ_{s0} as function of single bunch current is given, beyond the threshold for onset of turbulent bunch lengthening (TBL) (also referred to as microwave instability) by:

$$\sigma_s/\sigma_{s0} = \left(K \left| \frac{Z_{||}}{n} \right|_0^{bb} I_b \right)^A \quad \text{for } I_b > I_b^{\text{th}}, \quad (2.108)$$

with:

$$K = \frac{C}{(2\pi)^{3/2} \alpha_p (E/e) \sigma_p^2 \sigma_s}. \quad (2.109)$$

The exponent A for the TBL beyond the threshold is usually $A=1/3$, while the threshold I_{th} can be found for $\sigma_s/\sigma_{s0}=1$ in Eq. 2.108. C is the circumference of the ring, α_p the momentum compaction factor, E the energy, e the electron charge, σ_p the energy spread and σ_s the bunch length.

The impedance involved in the turbulent bunch lengthening effect, is the total longitudinal impedance of the ring, approximated by a resonator with a broad frequency spectrum, centered at the beam pipe cut-off frequency. Writing the above equation in a logarithmic form:

$$\ln x = A \ln \left(K \left| \frac{Z_{||}}{n} \right|_0^{bb} \right) + A \ln I_b = B + A \ln I_b, \quad (2.110)$$

the impedance term B and the exponent A can be evaluated by the slope and the intercept of a straight line respectively.

Theory of Intrabeam scattering

Intrabeam scattering (IBS) is a small angle multiple Coulomb scattering effect between charged particles within accelerator beams, leading to the redistribution of the particle momenta and to the diffusion of the six-dimensional phase space. It is an important effect in e^+/e^- damping rings and high intensity/low energy light sources [23], but also in high intensity hadron [24] and ion [25] circular machines. In this chapter, the standard IBS theories, their high energy approximations and two multi-particle codes, are summarized.

3.1 Introduction

The IBS theory for accelerators was first introduced by Piwinski [26] and extended by Martini [27] establishing a formulation called the standard Piwinski (P) method. Bjorken and Mtingwa (BM) [28] a few years latter described the effect using a different approach and taking into account the strong focusing effect. Bane modified later the standard Piwinski method in order to take into account the strong focusing effect, giving the so called Modified Piwinski (MP) method [29]. Several approximations were also developed over the years: in particular, the high energy approximations by Bane (Bane) [29] and the completely integrated modified Piwinski (CIMP) [30], which are valid under certain conditions. Those are some of the most widely used theoretical models for IBS calculations in lepton rings. Moreover, a method for fast numerical evaluation of the B-M integrals is described in [31] while in [32] the BM method is re-derived to deal with betatron coupling. A different approach, developed for hadron beams, based on a Boltzmann type integro-differential equation, taking into account betatron coupling, can be found in [33].

All theories and approximations calculate the horizontal (x), vertical (y) and longitudinal (p) growth rates and are defined as:

$$\frac{1}{T_p} = \frac{1}{\sigma_p} \frac{d\sigma_p}{dt}, \quad \frac{1}{T_x} = \frac{1}{\varepsilon_x^{1/2}} \frac{d\varepsilon_x^{1/2}}{dt}, \quad \frac{1}{T_y} = \frac{1}{\varepsilon_y^{1/2}} \frac{d\varepsilon_y^{1/2}}{dt}, \quad (3.1)$$

with

$$\frac{1}{T_i} = \langle f_i \rangle. \quad (3.2)$$

The functions f_i are integrals that have a complicated dependence on the beam properties as bunch charge and energy, the beam optics and the equilibrium rms horizontal, vertical emittances and energy spread defined by the radiation damping. This complicated dependence comes from the coupling of the three planes through the dispersion, which couples the betatron and synchrotron oscillations; a momentum change of a particle in a non zero dispersion region, leads to

a change in the betatron oscillations and it can occur all over the machine. The average of these functions all over the ring defines the growth rates.

The IBS effect on the beam emittance is calculated assuming a Gaussian distribution of the particles. While Gaussian distribution is the stationary solution of the Fokker-Planck equation for the particle distribution in the phase space, including radiation damping and quantum excitation [34], there is no evidence that in a strong IBS regime the distribution would remain Gaussian. It is then important to be able to calculate the combined effect of IBS, radiation damping and quantum excitation regardless of the distribution of the particles. To this end, two multi-particle Monte Carlo codes have been developed, capable of such calculations; the Software for IBS and Radiation Effects (SIRE) [35], and the IBStrack [36] implemented also in the collective effects simulation tool CMAD [37, 38].

3.2 The standard Piwinski formalism

In the standard Piwinski method [26], the Coulomb collisions of the particles are described by the classical Rutherford cross-section and the particles in phase space are considered to follow Gaussian distributions. The relative energy spread and transverse emittance growth times are then given by:

$$\frac{1}{T_p} = A \left\langle \frac{\sigma_h^2}{\sigma_p^2} f(\tilde{a}, \tilde{b}, \tilde{q}) \right\rangle, \quad (3.3)$$

$$\frac{1}{T_x} = A \left\langle f\left(\frac{1}{\tilde{a}}, \frac{\tilde{b}}{\tilde{a}}, \frac{\tilde{q}}{\tilde{a}}\right) + \frac{D_x^2 \sigma_h^2}{\beta_x \varepsilon_x} f(\tilde{a}, \tilde{b}, \tilde{q}) \right\rangle, \quad (3.4)$$

$$\frac{1}{T_y} = A \left\langle f\left(\frac{1}{\tilde{b}}, \frac{\tilde{a}}{\tilde{b}}, \frac{\tilde{q}}{\tilde{b}}\right) + \frac{D_y^2 \sigma_h^2}{\beta_y \varepsilon_y} f(\tilde{a}, \tilde{b}, \tilde{q}) \right\rangle. \quad (3.5)$$

The 6-dimensional invariant phase space volume of a bunched beam is given by:

$$A = \frac{c N r_0^2}{64 \pi^2 \beta^3 \gamma^4 \varepsilon_x \varepsilon_y \sigma_s \sigma_p}, \quad (3.6)$$

where r_0 is the classical particle radius, with $r_0 = 2.82 \times 10^{-15} \text{ m}$ for electrons or positrons and $1.53 \times 10^{-18} \text{ m}$ for protons. c is the speed of light, N the number of particles per bunch, β the particle velocity divided by c , γ the Lorentz energy factor and σ_s the rms bunch length. The factor d is the maximum impact parameter, usually considered to be the vertical beam size and:

$$\frac{1}{\sigma_h^2} = \frac{1}{\sigma_p^2} + \frac{D_x^2}{\beta_x \varepsilon_x} + \frac{D_y^2}{\beta_y \varepsilon_y}, \quad (3.7)$$

$$\tilde{a} = \frac{\sigma_h}{\gamma} \sqrt{\frac{\beta_x}{\varepsilon_x}}, \quad (3.8)$$

$$\tilde{b} = \frac{\sigma_h}{\gamma} \sqrt{\frac{\beta_y}{\varepsilon_y}}, \quad (3.9)$$

$$\tilde{q} = \sigma_h \beta \sqrt{\frac{2d}{r_0}}. \quad (3.10)$$

The general Piwinski scattering function f is defined as [26]:

$$f(\tilde{a}, \tilde{b}, \tilde{q}) = 2 \int_0^\infty \int_0^\pi \int_0^{2\pi} e^{-r[\cos^2 \theta + (\tilde{a}^2 \cos^2 \phi + \tilde{b}^2 \sin^2 \phi) \sin^2 \theta]} \ln(\tilde{q}^2 r) (1 - 3 \cos^2 \theta) \sin \theta d\phi d\theta dr, \quad (3.11)$$

where f satisfies the relations:

$$\begin{aligned} f(\tilde{a}, \tilde{b}, \tilde{q}) &= f(\tilde{b}, \tilde{a}, \tilde{q}), \\ f(\tilde{a}, \tilde{b}, \tilde{q}) + \frac{1}{\tilde{a}^2} f\left(\frac{1}{\tilde{a}}, \frac{\tilde{b}}{\tilde{a}}, \frac{\tilde{q}}{\tilde{a}}\right) + \frac{1}{\tilde{b}^2} f\left(\frac{1}{\tilde{b}}, \frac{\tilde{a}}{\tilde{b}}, \frac{\tilde{q}}{\tilde{b}}\right) &= 0. \end{aligned} \quad (3.12)$$

Evans and Zotter [39] wrote f in a single integral representation as:

$$f(\tilde{a}, \tilde{b}, \tilde{q}) = 8\pi \int_0^1 du \frac{(1 - 3u^2)}{PQ} \left\{ 2 \ln \left[\frac{\tilde{q}}{2} \left(\frac{1}{P} + \frac{1}{Q} \right) \right] - 0.577... \right\}, \quad (3.13)$$

where

$$P^2 = \tilde{a}^2 + (1 - \tilde{a}^2)u^2, \quad (3.14)$$

$$Q^2 = \tilde{b}^2 + (1 - \tilde{b}^2)u^2. \quad (3.15)$$

3.3 The modified Piwinski formalism

In order to take into account the twiss functions variations around the accelerator, Bane proposed the following replacements in the standard Piwinski theory:

$$\frac{D_{x,y}^2}{\beta_{x,y}} \longrightarrow \mathcal{H}_{x,y} = \frac{1}{\beta_{x,y}} \left[D_{x,y}^2 + \left(\beta_{x,y} D'_{x,y} - \frac{1}{2} \beta'_{x,y} D_{x,y} \right)^2 \right], \quad (3.16)$$

which means that σ_h , $\tilde{a}$, $\tilde{b}$ from the standard Piwinski formalism become σ_H , a , b with:

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_p^2} + \frac{\mathcal{H}_x}{\varepsilon_x} + \frac{\mathcal{H}_y}{\varepsilon_y}, \quad (3.17)$$

$$a = \frac{\sigma_H}{\gamma} \sqrt{\frac{\beta_x}{\varepsilon_x}}, \quad (3.18)$$

$$b = \frac{\sigma_H}{\gamma} \sqrt{\frac{\beta_y}{\varepsilon_y}}. \quad (3.19)$$

This representation is called the Modified Piwinski formalism.

3.4 The Bjorken-Mtingwa formalism

Bjorken and Mtingwa describe the effect using a different approach, namely relativistic "Golden Rule" for the transition rate due to a 2-body scattering process. The relative energy spread and transverse emittance growth times are given by:

$$\frac{1}{T_i} = 4\pi A(\log) \left\langle \int_0^\infty d\lambda \frac{\lambda^{1/2}}{[\det(L + \lambda I)]^{1/2}} \left\{ Tr L^{(i)} Tr \left(\frac{1}{L + \lambda I} \right) - 3 Tr L^{(i)} \left(\frac{1}{L + \lambda I} \right) \right\} \right\rangle, \quad (3.20)$$

where $(\log) \equiv \ln(r_{\max}/r_{\min})$ is the Coulomb logarithm that is the ratio of the maximum $r_{\max}$ to the minimum $r_{\min}$ impact parameter in the collision of two particles in the bunch. For typical flat beams, the $r_{\max}$ is taken to be equal to the vertical beam size σ_y , while $r_{\min}$ is taken to be $r_{\min} = r_0 \beta_x / (\gamma^2 \varepsilon_x)$. The Coulomb logarithm is then estimated by:

$$(\log) = f_{CL} \ln \left(\frac{\gamma^2 \varepsilon_x \sqrt{\beta_y \varepsilon_y}}{r_0 \beta_x} \right). \quad (3.21)$$

The auxiliary matrices are defined as:

$$L = L^{(p)} + L^{(x)} + L^{(y)}, \quad (3.22)$$

$$L^{(p)} = \frac{\gamma^2}{\sigma_p^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (3.23)$$

$$L^{(x)} = \frac{\beta_x}{\varepsilon_x} \begin{pmatrix} 1 & -\gamma \phi_x & 0 \\ -\gamma \phi_x & \gamma^2 \mathcal{H}_x / \beta_x & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (3.24)$$

$$L^{(y)} = \frac{\beta_y}{\varepsilon_y} \begin{pmatrix} 0 & 0 & 0 \\ 0 & \gamma^2 \mathcal{H}_y / \beta_y & -\gamma \phi_y \\ 1 & -\gamma \phi_y & 1 \end{pmatrix}, \quad (3.25)$$

where,

$$\phi_{x,y} \equiv D'_{x,y} - \frac{\beta'_{x,y} D_{x,y}}{2\beta_{x,y}}, \quad (3.26)$$

and

$$\mathcal{H}_{x,y} = \frac{1}{\beta_{x,y}} \left[D_{x,y}^2 + \left(\beta_{x,y} D'_{x,y} - \frac{1}{2} \beta'_{x,y} D_{x,y} \right)^2 \right]. \quad (3.27)$$

For Gaussian bunches, the factor $f_{CL} = 1$, however, it is not clear what is the effect of IBS on the final beam distribution. According to Raubenheimer [40], IBS populates the tails of the bunch distribution, leading to a reduction in the growth rates of the core emittance, which can be represented by a reduction in the factor f_{CL} to a value as low as 0.5.

3.5 The high energy approximation by Bane

Starting from the B-M solution, Bane derived a simplified model for IBS, valid for high energy beams in storage rings. Changing the integration variable to $\lambda' = \lambda\sigma_H^2/\gamma^2$ then:

$$(L + \lambda'I) = \frac{\gamma^2}{\sigma_H^2} \begin{pmatrix} a^2 + \lambda' & -a\zeta_x & 0 \\ -a\zeta_x & 1 + \lambda' & -b\zeta_y \\ 0 & -b\zeta_y & b^2 + \lambda' \end{pmatrix}, \quad (3.28)$$

with a, b, σ_H as given in paragraph 3.3 and $\zeta_{x,y} = \phi_{x,y}\sigma_H\sqrt{\frac{\beta_{x,y}}{\varepsilon_{x,y}}}$. The high energy assumptions of Bane are:

- The beam is cooler longitudinally than transversely, thus $a, b \ll 1$. Under this assumption, the second term in the braces of Eq. 3.20 is small, compared to the first and can be dropped.
- Drop off-diagonal terms, thus set $\zeta = 0$.

After these assumptions, the integrals are simplified and can be written in the form:

$$\frac{1}{T_p^p} \approx \frac{r_0^2 c N(\log)}{16\gamma^3 \varepsilon_x^{3/4} \varepsilon_y^{3/4} \sigma_s \sigma_p^3} \sigma_H g\left(\frac{a}{b}\right) (\beta_x \beta_y)^{-1/4}, \quad (3.29)$$

$$\frac{1}{T_p} = \left\langle \frac{1}{T_p^p} \right\rangle, \quad (3.30)$$

$$\frac{1}{T_{x,y}} = \frac{\sigma_p^2}{\varepsilon_{x,y}} \left\langle \frac{\mathcal{H}_{x,y}}{T_p^p} \right\rangle, \quad (3.31)$$

with

$$g(\alpha) = \frac{2\sqrt{\alpha}}{\pi} \int_0^\infty \frac{du}{\sqrt{1+u^2}\sqrt{\alpha^2+u^2}}. \quad (3.32)$$

The complicated integrals of B-M reduce like this to elliptic integrals with analytic solutions.

3.6 The Completely Integrated Modified Piwinski (CIMP) high energy approximation

Referring to relative terms for the size of the arguments of the Piwinski scattering function, it is shown in [41] that, whenever we have $f(\tilde{a}, \tilde{b}, \tilde{q}) = f(\text{large}, \text{small}, \text{large})$, the integrals in f can be explicitly computed. Using this and Eq. 3.12, Ref. [41] gives in the high energy limit:

$$f(\alpha, \omega, \delta) \approx \frac{-4\pi^{3/4} \ln \delta}{\alpha} g(\omega), \quad (3.33)$$

where,

$$g(\omega) = \sqrt{\pi/\omega} \left[P_{-1/2}^0 \left(\frac{\omega^2 + 1}{2\omega} \right) \pm \frac{3}{2} P_{-1/2}^{-1} \left(\frac{\omega^2 + 1}{2\omega} \right) \right], \quad (3.34)$$

where, $P_\nu^{-\mu}$ are the associated Legendre functions. The plus sign is valid for $\omega \geq 1$ and the minus one for $\omega \leq 1$. The type 3 associated Legendre functions were found to be the adequate

ones to use, giving the best approximation to the full integrals [30]. The growth rates for the high energy CIMP are then given by:

$$\frac{1}{T_p} \approx 2\pi^{3/2} A(\log) \left\langle \frac{\sigma_H^2}{\sigma_p^2} \left(\frac{g(b/a)}{a} + \frac{g(a/b)}{b} \right) \right\rangle, \quad (3.35)$$

$$\frac{1}{T_x} \approx 2\pi^{3/2} A(\log) \left\langle -ag\left(\frac{b}{a}\right) + \frac{\mathcal{H}_x \sigma_H^2}{\varepsilon_x} \left(\frac{g(b/a)}{a} + \frac{g(a/b)}{b} \right) \right\rangle, \quad (3.36)$$

$$\frac{1}{T_y} \approx 2\pi^{3/2} A(\log) \left\langle -bg\left(\frac{a}{b}\right) + \frac{\mathcal{H}_y \sigma_H^2}{\varepsilon_y} \left(\frac{g(b/a)}{a} + \frac{g(a/b)}{b} \right) \right\rangle, \quad (3.37)$$

where the factor $(\log)$ is defined as in Eq. 3.21.

3.7 The multi-particle tracking codes

The conventional IBS theories always assume Gaussian final distributions. However, this is not proven to be the case especially if IBS has a big impact on the final emittance. The assumption of Gaussian bunches does not permit investigation of interesting aspects of IBS, such as its impact during the damping process and its effect on the beam distribution. For this reason two multi-particle tracking codes have been recently developed, SIRE [35] and IBStrack [36]. Both algorithms are based in a Monte Carlo code called MOCAC, developed by Zenkevich et al [42, 43], which calculates the IBS effect for arbitrary distributions. SIRE and IBStrack include also Radiation Damping (RD) and Quantum Excitation (QE).

Even though based in the same algorithm they have one main difference: SIRE uses only the twiss functions around the ring in order to track the effect, calculating only the pure IBS effect without taking into account coupling or non-linear effects, while IBStrack uses the element-by-element composed one turn map taking into account the phase advance between the elements. Non-linear effects and coupling can then be included in the calculations. IBStrack has been implemented into the self-consistent, parallel processing code CMAD, which simulates the electron cloud build-up and instabilities [37]. IBStrack in CMAD can run in parallel processing mode, which drastically reduces the simulation time [38].

The existence of both codes is important for comparison. Both algorithms are under development and testing for different lattices and different particle species.

3.8 Equilibrium emittances due to IBS

Piwnski showed that, because of symmetry of $f(a, b, q)$ (see Eq. 3.12), there is a conservation of the following emittance dependent quantity [26]:

$$\varepsilon_l \left(\frac{1}{\gamma^2} - \left\langle \frac{D_x^2}{\beta_x^2} \right\rangle - \left\langle \frac{D_y^2}{\beta_y^2} \right\rangle \right) + \left\langle \frac{\varepsilon_x}{\beta_x} \right\rangle + \left\langle \frac{\varepsilon_y}{\beta_y} \right\rangle = \text{const}, \quad (3.38)$$

where, ε_l is the longitudinal emittance. Equation (3.38) shows that, below transition ($\gamma^2 < \gamma_{tr}^2 = 1/\alpha_p \approx 1/\langle D_x^2 \beta_x^2 \rangle$) the sum of the three (positive) invariants is limited, and an equilibrium

can exist. Above transition, ε_s , ε_x and ε_y can grow simultaneously and an equilibrium does not exist.

Even though e^+/e^- rings run normally above transition, where IBS leads to continuous emittance growth and equilibrium does not exist, synchrotron radiation damping counteracts the IBS growth, leading to new steady-state emittances.

From the growth rates, one can obtain the steady state properties, for which $\frac{d\varepsilon_x}{dt} = \frac{d\varepsilon_y}{dt} = \frac{d\sigma_p^2}{dt} = 0$:

$$\varepsilon_i = \frac{\varepsilon_{i0}}{1 - \tau_i/T_i}, \quad i = x, y \quad \text{and} \quad \sigma_p^2 = \frac{\sigma_{p0}^2}{1 - \tau_p/T_P}, \quad (3.39)$$

where ε_{x0} , ε_{y0} , σ_{p0} are the zero-current (without the effect of IBS) equilibrium horizontal and vertical emittances and rms energy spread. τ_x , τ_y and τ_p are the synchrotron radiation damping times. Equations 3.39 can be written in a differential form, describing the evolution of the horizontal, vertical emittance and energy spread with time:

$$\begin{aligned} \frac{d\varepsilon_x}{dt} &= -\frac{2}{\tau_x}(\varepsilon_x - \varepsilon_{x0}) + \frac{2\varepsilon_x}{T_x(\varepsilon_x, \varepsilon_y, \sigma_p)}, \\ \frac{d\varepsilon_y}{dt} &= -\frac{2}{\tau_y}(\varepsilon_y - \varepsilon_{y0}) + \frac{2\varepsilon_y}{T_y(\varepsilon_x, \varepsilon_y, \sigma_p)}, \\ \frac{d\sigma_p}{dt} &= -\frac{1}{\tau_p}(\sigma_p - \sigma_{p0}) + \frac{\sigma_p}{T_p(\varepsilon_x, \varepsilon_y, \sigma_p)}. \end{aligned} \quad (3.40)$$

Piwinski suggested to iterate numerically Eqs. 3.39 until a self-consistent solution is found. Another way is to solve numerically the three coupled differential equations 3.40, using small time iteration steps δt which are much smaller than the damping time and for which the emittances change adiabatically, in order to obtain the evolution of the beam emittances and the relative energy spread. The latest is the method that is used for the calculations presented in the next Chapters.

In order to solve Eqs. 3.40, it is important to identify the source of the equilibrium vertical emittance, at zero current. As discussed in Chapter 2 (see sec. 2.3), vertical dispersion due to vertical orbit errors, x - y coupling or a combination of the two can create a vertical emittance larger than its quantum limit. The vertical emittance for each case is given by Eqs. 2.65-2.67. If coupling is the main contributor to vertical emittance, the corresponding differential equation can be dropped and simply evaluate the vertical emittance through $\varepsilon_y = k\varepsilon_x$, with k the coupling factor. In the case where vertical dispersion is also important, the solution can be approximated by replacing the parameter ε_{y0} by the quantity [29]:

$$\varepsilon_y = k\varepsilon_x \left(1 - \frac{\tau_y}{T_y}\right) + \varepsilon_{y0d}, \quad (3.41)$$

where ε_{y0d} is the part of ε_{y0} due to dispersion only.

The practice of solving IBS equations assuming no vertical errors, which tends to result in near 0 or even negative vertical emittance growth, may describe a state that is unrealistic and unachievable.

3.9 Simulation tools

The theoretical models of B-M, P, Bane and CIMP have been implemented in Mathematica [44] and used for all IBS calculations in the next chapters.

The B-M formalism is implemented in the Methodical Accelerator Design (MAD-X) code as a module called IBS. A revision of the model was recently done, where bugs of the previous code were corrected and extra features added [14, 45].

A more detailed discussion about the IBS simulation methods can be found in Chapter 4.

Benchmarking of the IBS theoretical models with Monte-Carlo codes

The IBS theoretical models have been studied in detail and bench-marked with experimental data for hadron beams over the years [24,25]. In hadron machines, the IBS effect cause emittance dilution with time, limiting their performance. Lepton machines on the other hand, were operating until today in regimes where the IBS effect was negligible. Future linear collider Damping Rings, new generation light sources and B-factories, however, enter in regimes where the IBS effect can be predominant. It is thus important to study the IBS theories in the presence of synchrotron radiation and quantum excitation, benchmark the existing theoretical models and tracking codes with experimental data and identify their limitations.

4.1 Comparison results between theories and codes

The theoretical models of Bjorken-Mtingwa (BM), Piwinski (P), Bane and CIMP described in paragraphs 3.2-3.6, are applied and compared for three different rings: the CLIC Damping Rings (DR), the Swiss Light Source (SLS) and the Cornell electron storage ring Test Accelerator (Cesr-TA). The CLIC DR lattice is wiggler dominated, targeting ultra-low emittances in all three planes and small damping times, the SLS storage ring serves as a light source with ultra-low vertical emittance, while Cesr-TA used to be an e^+/e^- collider and now is used as a storage ring and test facility for DR R&D. Its emittance is dipole dominated, but wigglers are included to reduce further the emittance. Table 4.1, summarizes basic performance parameters of the three lattices.

Table 4.1: Basic equilibrium lattice parameters for the CLIC DR, the Cesr-TA and the SLS lattices.

	CLIC DR	Cesr-TA	SLS
En [GeV]	2.86	2.085	1.57
Circumference [m]	427.5	768.4	288
Hor. Emittance [nm-rad]	0.056	2.6	2.4
Vert. Emittance [pm-rad]	0.56	7.7	1
Bunch Length [mm]	1.6	9.2	1.98
Energy Spread	1e-3	8.1e-4	5.6e-4
Damping times ($\tau_x/\tau_y/\tau_l$) [ms]	2/2/1	56/56/28	30/30/15

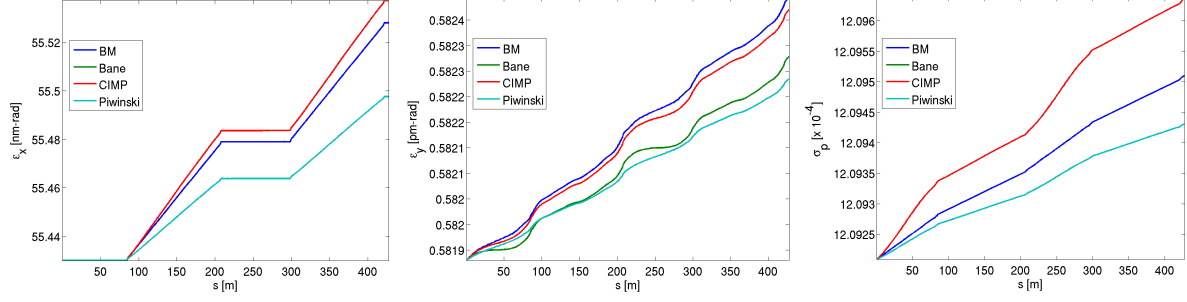


Figure 4.1: One turn comparison for the horizontal (left) and vertical (middle) emittance and energy spread (right) between the theoretical models BM (blue), P (light blue), Bane (green) and CIMP (red) for the CLIC DR lattice.

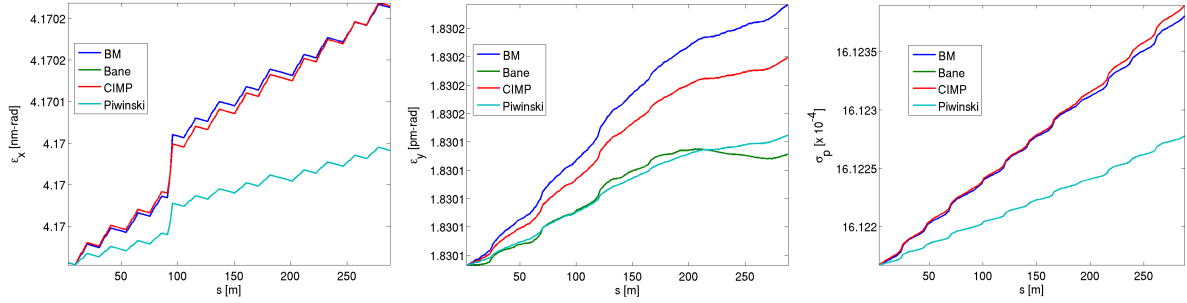


Figure 4.2: One turn comparison for the horizontal (left) and vertical (middle) emittance and energy spread (right) between the theoretical models BM (blue), P (light blue), Bane (green) and CIMP (red) for the the SLS lattice.

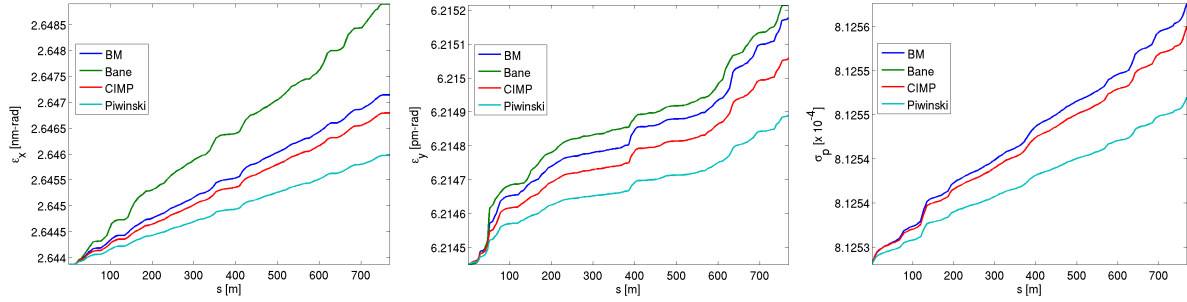


Figure 4.3: One turn comparison for the horizontal (left) and vertical (middle) emittance and energy spread (right) between the theoretical models BM (blue), P (light blue), Bane (green) and CIMP (red) for the Cesr-TA lattice.

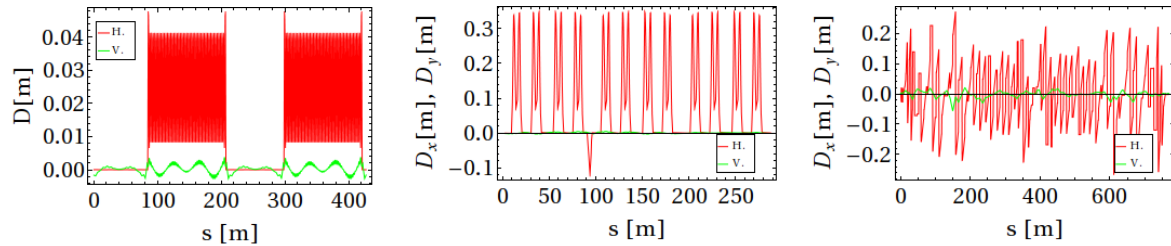


Figure 4.4: The horizontal (red) and vertical (green) dispersion for the CLIC DR (left), the SLS (middle) and the Cesr-TA (right) lattices.

For the IBS growth rate calculations, it is important that the IBS kicks are distributed over an adequate amount of points across the ring, such that the variation of the optics is taken into account and the areas where IBS is predominant are well represented. The starting emittance values are the zero current equilibrium ones. The steady state emittances, at least for the cases under study, do not depend on the starting point. In all cases, random quadrupole misalignments are applied to create the vertical dispersion, i.e. partly responsible for the non-zero value of vertical emittance.

Figures 4.1-4.3 show the evolution of the horizontal (left) and vertical (middle) emittances and energy spread (right) in one turn, for the CLIC DR (Figure 4.1), the SLS (Figure 4.2) and the Cesr-TA (Figure 4.3) lattices, as calculated by BM (blue), Piwinski (light blue), Bane (green) and CIMP (red) formalism. The horizontal (red) and vertical (green) dispersion for each lattice are shown on Figure 4.4. The dispersion plays a very important role in the redistribution of the phase space due to the IBS effect. This can be nicely demonstrated for the case of the CLIC DR (top plots of the figure). In the dispersion free regions (long straight sections) the IBS effect in the horizontal plane is almost zero. In the dispersive regions (arcs), on the other hand, the dispersion couples the horizontal and longitudinal planes and part of the IBS growth is transferred from the longitudinal to the horizontal plane: the slope of the growth in the longitudinal plane is reduced while in the horizontal one is increased. As the betatron coupling is considered zero and the vertical dispersion is very small, the vertical plane is uncoupled from the other two. Note that the trend of the emittance evolution is the same for all theoretical models and for all three examples, with Piwinski predicting always the smallest effect. In many cases the curve of Bane do not appear in the plots, as it overlaps with the curve of CIMP.

The evolution of the emittances until convergence (steady states), for the three lattices using the four theoretical models, is presented in Figure 4.5. The same color and order convention is used as in Figures 4.1-4.3. Table 4.2 summarizes the steady state horizontal geometrical emittances for the three lattices. The theoretical models of BM, Bane and CIMP are always in good agreement, while Piwinski underestimates the effect with respect to the other three, in most of the cases.

Table 4.2: Comparison between the steady state horizontal geometrical emittance calculated by BM, Piwinski, Bane and CIMP for the CLIC DR, the SLS and the Cesr-TA lattices. The bunch population and the energy is also given.

	ε_x (BM)	ε_x (Piw.)	ε_x (Bane)	ε_x (CIMP)	N [10^9]	E [GeV]
CLIC	95.4 [pm-rad]	85.8 [pm-rad]	98.1 [pm-rad]	97.2 [pm-rad]	4	2.86
SLS	3.6 [nm-rad]	3.3 [nm-rad]	3.7 [nm-rad]	3.2 [nm-rad]	30	1.57
Cesr-TA	9.6 [nm-rad]	7.7 [nm-rad]	11.0 [nm-rad]	9.0 [nm-rad]	32	2.085

Figure 4.6, shows a comparison between the theoretical models BM, P, Bane and CIMP with the multi-particle tracking code SIRE for the CLIC DR lattice [46]. As SIRE is a Monte-Carlo code, the tracking simulations were performed several times and the one standard deviation error-bars are also shown in the plots. The results from SIRE simulations are shown in green, from BM in black, from Piwinski in red and the ones from Bane in magenta. The classical formalism of Piwinski is in perfect agreement with the SIRE results, in all planes. This is not

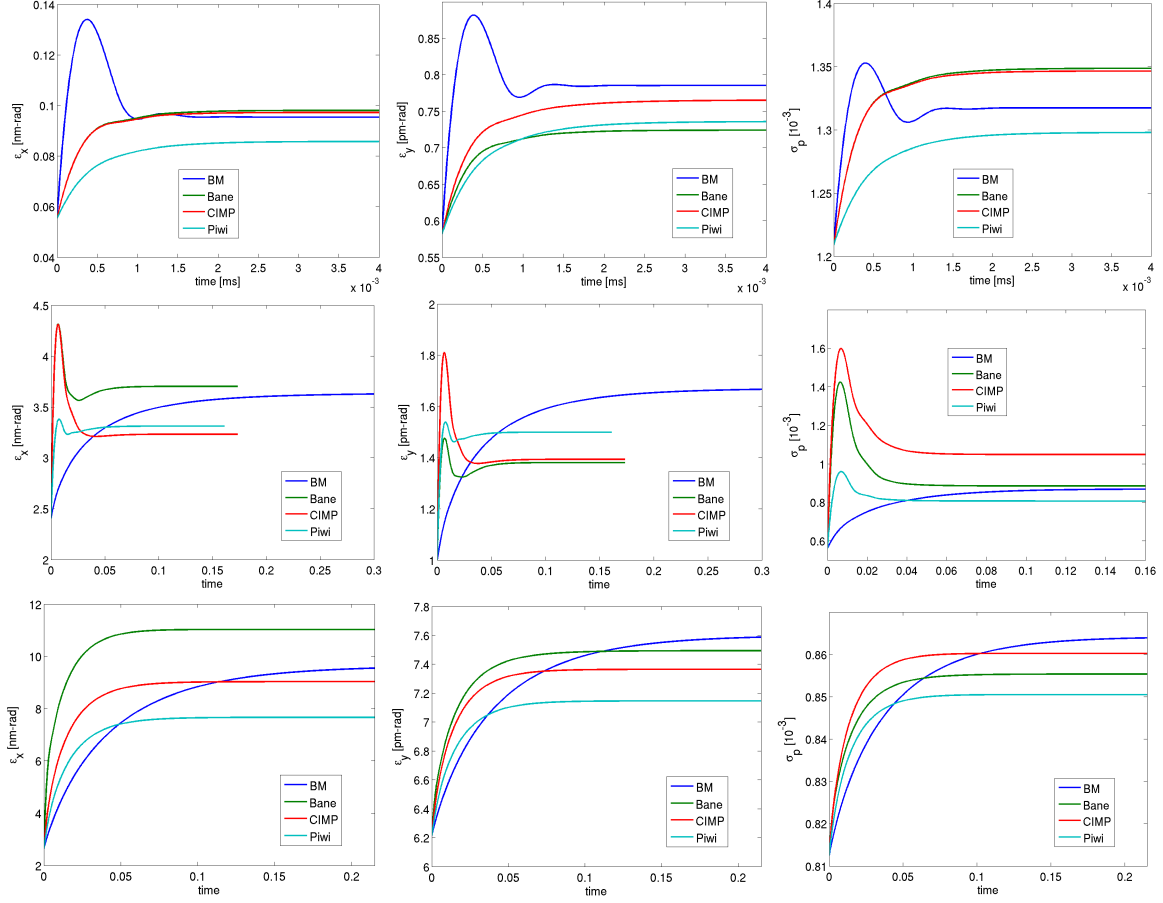


Figure 4.5: The emittance evolution in the horizontal (left) and vertical (middle) emittances and in energy spread (right) for the CLIC DR (top), the SLS (middle) and CEsr-TA (bottom) lattices.

a surprise, as both Piwinski formalism and the tracking codes use the Rutherford cross-section, to calculate the scattering probability in a solid angle. All theories and simulations seem to be in good agreement within 3σ in all planes.

The CLIC DR lattice was used for the bench-marking of the multi-particle tracking code CMAD-IBStrack [37] (green) with the theoretical models BM (black), Piwinski (red), Bane (magenta) and CIMP (blue). In this example, zero vertical dispersion is considered. The results are shown in Figure 4.7 for the horizontal plane (left). The CMAD-IBStrack was also bench-marked for the Super-B [47] collider storage ring (right part of the figure) where the standalone IBStrack code (blue), the IBStrack in CMAD (black) and the theoretical models of Piwinski (red) and Bane (green) are compared. In all cases the results give very good agreement between the theoretical models and the tracking code [38]. CMAD-IBStrack gives also the possibility to include betatron coupling and vertical dispersion.

In order to study the agreement of the theoretical models in different IBS regimes, the SLS lattice was used considering different bunch current values and different energies: the nominal 2.4 GeV (red) and the low 1.57 GeV (green) energy. All calculations are normalized to the ones

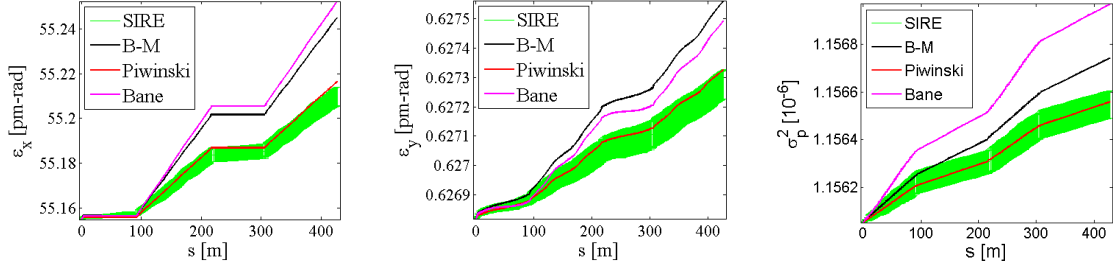


Figure 4.6: One turn comparison for the horizontal (left) and vertical (middle) emittance and energy spread(right) between the tracking code SIRE and the theoretical models BM, P and Bane for the CLIC DR lattice.

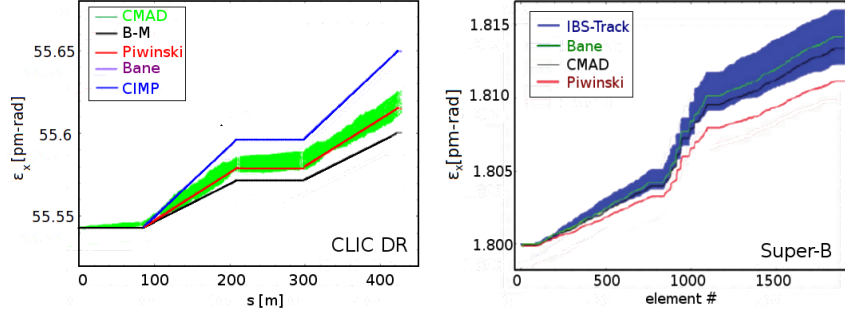


Figure 4.7: One turn comparison for the horizontal emittance between the tracking code CMAD-IBStrack and the theoretical models for the CLIC DR (left) and the Super-B lattices [38].

using the BM formalism. Figure 4.8 displays the results in the horizontal (left) and vertical (right) planes, for Bane (solid line), CIMP (dashed line) and Piwinski (starred line) formalisms. The agreement between the theoretical models seem to depend on the magnitude of the effect. For the nominal energy, where the effect is small, the agreement between the models is very good even at high currents, and always within less than 5% up to 10 mA for the horizontal and within 7% for the vertical plane. The longitudinal plane follows the trend of the horizontal one. For the low energy, the agreement of Bane, CIMP and BM is still very good in the horizontal plane, while Piwinski underestimates the effect with respect to the others. In the vertical plane BM, CIMP and Piwinski agree within less than 15% while Bane is diverging, especially at high currents. This is due to the fact that Bane's approximation deviates from the IBS theories for zero or very low vertical dispersion [41], which is the case of the SLS ring. A more detailed discussion about the Bane's approximation will be presented in the next paragraph (see section 4.1.1).

In Figure 4.9, results obtained using the tracking code CMAD-IBStrack described in [36] are compared to the analytical model predictions, for different values of the bunch population, assuming nominal parameters for the 2.4 GeV configuration of the SLS ring [48]. An initial distribution of 10^5 macro-particles has been generated at a particular location of the machine and the emittance evolution under the effect of IBS has been tracked for approximately 3 damping times, through a “realistic” model of the ring lattice. In all cases, at the end of the simulations the

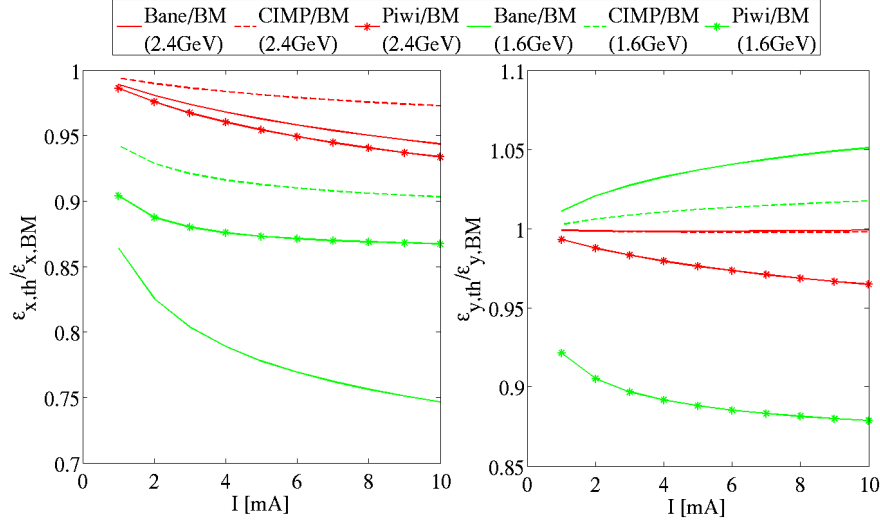


Figure 4.8: Horizontal (left) and vertical (right) emittance ratios for IBS models with respect to the BM method for nominal (red) and low energy (green) for different bunch currents.

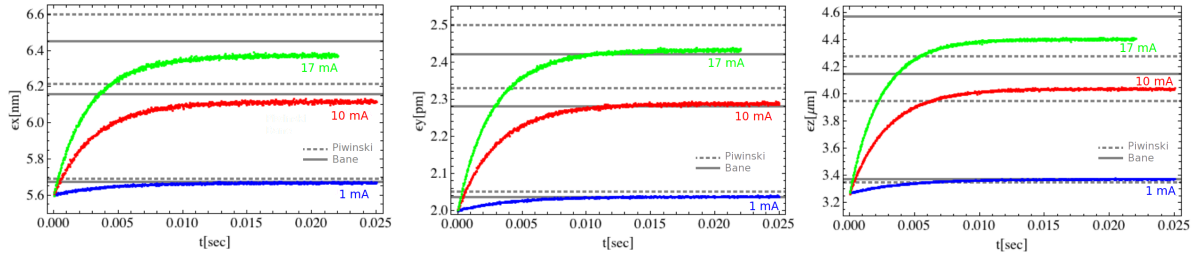


Figure 4.9: Evolution of horizontal (top, left), vertical (top, right), and longitudinal (bottom, left) emittances under the influence of IBS as obtained by the tracking code for different values of the bunch population: 6×10^9 (blue), 60×10^9 (red) and 100×10^9 (green). Horizontal lines represent the steady state values predicted by Piwinski (full) and Bane (dashed) models for the considered bunch populations [48].

emittances approach steady state values that are very close to those predicted by the theoretical models. In particular, the Piwinski model seems to provide the closest emittance prediction with respect to the tracking results. This is expected as the IBS kicks used in the tracking code are based in the Rutherford cross section also employed by the approach of Piwinski. The agreement between the theoretical models and the tracking codes is excellent at low current, where the IBS effect is less important, while the divergence between the results is getting larger for higher bunch currents. This is in agreement with what was observed in Figure 4.8.

4.1.1 Discussion on Bane's high energy approximation

In section 3.5, Bane's high energy approximation to the BM formalism was discussed. Bane's assumptions are not always valid, especially in the case of zero or very small vertical disper-

sion [41]. In this section, the validity of Bane's assumptions are discussed, for the CLIC DR, the SLS and the Cesr-TA lattices.

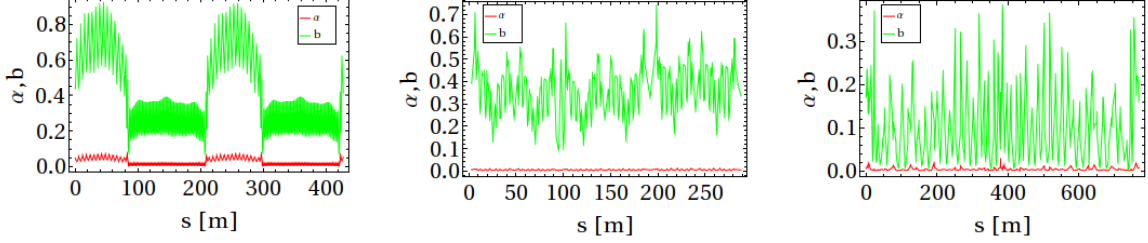


Figure 4.10: The α (red) and b (green) parameters around the CLIC DR (left), the SLS (middle) and the Cesr-TA (right) lattices.

Table 4.3: Validity of Bane's arguments for the CLIC DR, the SLS and the Cesr-TA lattices.

	CLIC	SLS	Cesr-TA
$\langle I_{1x} + I_{2x} \rangle / \langle I_{1x} \rangle$	1.14	1.17	1.014
$\langle I_{1y} + I_{2y} \rangle / \langle I_{1y} \rangle$	1.28	1.23	1.075
$\langle I_{1l} + I_{2l} \rangle / \langle I_{1l} \rangle$	1.25	1.18	1.068
D_x^{rms} [mm]	24.3	17.9	10.8
D_y^{rms} [mm]	1.4	1.6	10.5
$\langle \alpha \rangle$	0.027	4.83e-3	3.84e-3
$\langle b \rangle$	0.416	0.361	0.0992
$\langle \zeta_x \rangle$	0.015	0.015	0.105
$\langle \zeta_y \rangle$	-0.012	-0.041	7.17e-3

The first assumption of Bane ($\alpha, b \ll 1$) implies that, the second integral (I_2) of each growth time of the BM expressions (see Eq. (3.20)) can be ignored. The second assumption states that the non-diagonal elements of the matrices in Eq. (3.28) can be set to zero ($\zeta_x = 0$, $\zeta_y = 0$). Figure 4.10 shows the α (red) and b (green) parameters around the CLIC DR (left), the SLS (middle) and the Cesr-TA (right) lattices. Even though $\alpha \ll 1$ is always valid, this is not the case for b . For the CLIC DR and the SLS lattices, b can be close to 1 at certain locations around the rings. For the Cesr-TA lattice, the maximum value of b is around 0.5, however, in general, $b \ll 1$. Table 4.3 summarizes the values of Bane's assumptions for the different lattices. In particular, the mean values for the ratio of the sum of the two integrals over the first one, in the horizontal x , vertical y and longitudinal l planes, the mean values for α , b , ζ_x , ζ_y and the rms horizontal D_x and vertical D_y dispersion are given. The first assumption ($\alpha, b \ll 1$) is valid only in the case of the Cesr-TA lattice, where the rms vertical dispersion is one order of magnitude larger than for the CLIC DR and the SLS. This shows that Bane's formulation can underestimate the growth times, in the case of very small vertical dispersion. In our examples, the growth rates are underestimated by around 20 %. The second assumption ($\zeta_x = \zeta_y = 0$) is valid for all three lattices.

4.2 The Coulomb log factor

In the comparison between the theoretical models, Piwinski always seems to underestimate the IBS effect with respect to the other theoretical models. What diversifies Piwinski's method, is the different definition of the so called Coulomb factor. In BM, Bane and CIMP methods, the Coulomb factor is defined as:

$$(\log) = \ln \left(\frac{\gamma^2 \varepsilon_x \sqrt{\beta_y \varepsilon_y}}{r_0 \beta_x} \right). \quad (4.1)$$

In the Piwinski formalism on the other hand, the parameter d appears, i.e. the maximum impact parameter, and is normally taken as the vertical beam size. If the parameter d is chosen such that we have an effective Coulomb log factor which is the same as the $(\log)$ factor of Eq. (4.1), then Piwinski agrees with the other models. In the high energy limit, the Coulomb $(\log)$ for Piwinski can be written as [49]:

$$(\log) = \ln \left(\frac{d \sigma_H^2}{4 r_0 \alpha^2} \right). \quad (4.2)$$

Comparing the $(\log)$ factors of Eq. (4.1) and (4.2), then $d = 4 \sigma_y$.

4.3 IBS in MADX

The general formalism of Bjorken-Mtingwa following the approach of Conte and Martini [27], but including the effect of vertical dispersion was implemented in the Methodical Accelerator Design code MAD-X since 2006 [50]. This implementation included several bug fixes and the modified routine was crosschecked with the Mathematica implementation of the BM formalism for the CLIC DR and the SLS lattices [45]. The correct implementation is now available with the new release of MAD-X.

Table 4.4 compares the IBS growth rates computed by an old version of MAD-X (2.0), using the Conte-Martini formalism, and those from the new version of the code. While in the case of the ideal optics there is no difference, the vertical growth time becomes a factor 6 shorter when errors generating vertical dispersion are included. Figures 4.11 and 4.12 present the local IBS growth rates as a function of position around the CLIC DR and the SLS rings, when errors are included. The MAD-X results (red solid line) were cross-checked with the direct solution of the Bjorken-Mtingwa equations in Mathematica (blue dashed line), yielding a perfect agreement. More examples including, the LHC and the LHC upgrade optics are presented in [45].

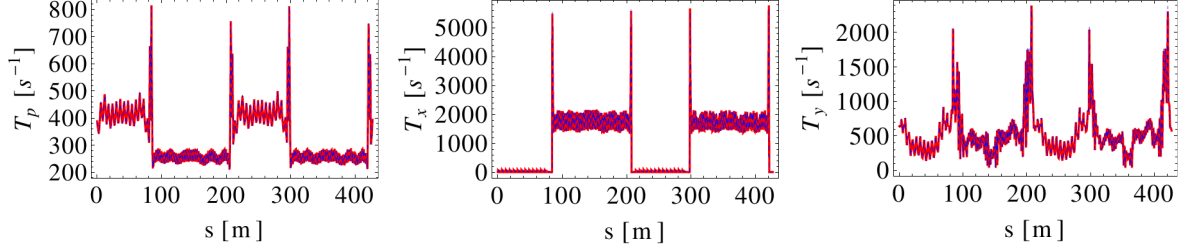


Figure 4.11: IBS growth rates comparison between the MADX ibs module results (blue) and the results from the BM implementation on Mathematica (red) for the CLIC DR lattice.

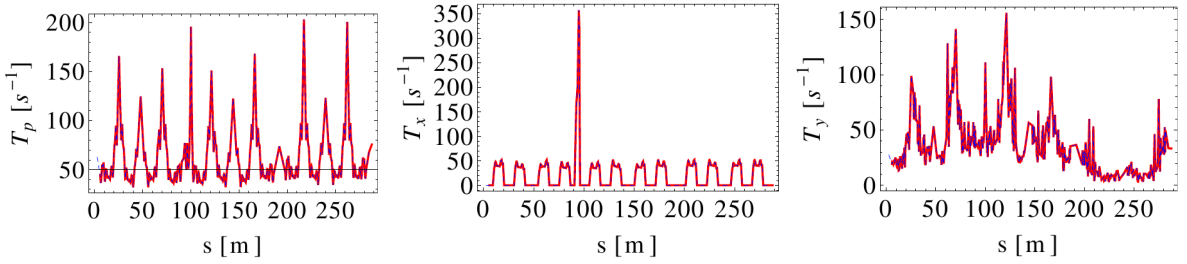


Figure 4.12: IBS growth rates comparison between the MADX ibs module results (blue) and the results from the BM implementation on Mathematica (red) for the SLS lattice.

Table 4.4: IBS growth rates in the CLIC damping ring computed with old (v. 2.0, compiled in 2004) and new (v. 5.01, from 2012) versions of MAD-X for the ideal optics and with random quadrupole roll angles.

	no errors		errors	
	old MAD-X	new MAD-X	old MAD-X	new MAD-X
T_l [ms]	3.043	2.989	3.043	3.071
T_x [ms]	0.871	0.855	0.871	0.880
T_y [ms]	5.577	5.477	5.577	1.426

Analytical parameterization of the TME cell

Under the influence of synchrotron radiation, the theoretical minimum emittance (TME) [51] is reached for specific optics conditions, including a unique high phase advance [15]. The strong focusing needed for accomplishing the TME conditions, results in cells with intrinsically high chromaticity. The chromatic sextupoles' strengths are enhanced by the low dispersion of the TME cell and reduce the Dynamic Aperture (DA). The ultimate target of a low emittance ring design is to build a compact ring, attaining a sufficiently low emittance with an adequately large DA, driven by geometrical aperture and injection requirements. The lattice design, however, is often based on numerical tools, whose optimization algorithms depend heavily on the initial conditions. Reaching the optimal solution necessitates several iterations, without necessarily having a global understanding of the interdependence between a series of optics parameters and knobs. Modern techniques, as the Multi-Objective Genetic Algorithms (MOGA) [52] or the Global Analysis of Stable Solutions (GLASS) [53] attempt to achieve a global optics optimization exploring numerically all possible solutions, within stability and performance requirements. Here, a different approach is followed, by obtaining an analytical solution for the quadrupole strengths and a complete parametrization of the TME cell, using thin lens approximation. In this way, all cell properties are globally determined and the optimization procedure, following any design requirements, can be performed in a systematic way.

5.1 Analytical solutions for the TME cells

A schematic layout of the TME cell is displayed in Fig. 5.1. It consists of one dipole D of length l_d and at least two families of quadrupoles Q1, Q2, as pictured. The quadrupole focal lengths are denoted by $f_1[m] = 1/(k_1 l_{q1})$ and $f_2[m] = 1/(k_2 l_{q2})$ and the drifts between the elements by s_1 , s_2 and s_3 . For simplicity, the center of consecutive dipoles is considered as the entrance and exit of the TME cell.

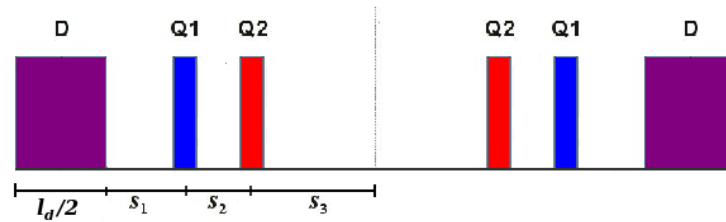


Figure 5.1: Schematic layout of the TME cell.

The horizontal emittance of the beam in an iso-magnetic ring:

$$\varepsilon_x = \frac{C_q \gamma^2}{\mathcal{J}_x \rho_x} \langle \mathcal{H}_x \rangle, \quad (5.1)$$

is determined by the average dispersion invariant in the dipoles, defined as $\mathcal{H}_x = \gamma_x D_x^2 + 2\alpha_x D_x D'_x + \beta_x D_x'^2$, where α_x , β_x , γ_x the twiss parameters and D_x and D'_x the dispersion and its derivative. The parameter $C_q = 3.84 \times 10^{-13}$ m, is the quantum fluctuation coefficient for the electron, γ the relativistic factor, $\mathcal{J}_x$ the damping partition number, and ρ_x the bending radius. The minimization of the dispersion invariant average, provides the conditions of β_x and D_x at the center of the dipole, for achieving the theoretical minimum emittance:

$$\beta_{xc}^{\min} = \frac{l_d}{2\sqrt{15}}, \quad D_{xc}^{\min} = \frac{\theta l_d}{24}, \quad (5.2)$$

where $\theta = \frac{l_d}{\rho_x} = \frac{2\pi}{N_d}$ is the bending angle for N_d dipoles in the ring. For a general TME cell, the geometrical emittance can be expressed as:

$$\varepsilon_x = \frac{C_q \gamma^2}{\mathcal{J}_x \rho_x} \left[\frac{1}{\beta_{xc}} \left(D_{xc}^2 - \frac{\theta D_{xc} l_d}{12} + \frac{\theta^2 l_d^2}{320} \right) + \frac{\theta^2 \beta_{xc}}{12} \right], \quad (5.3)$$

where D_{xc} and β_{xc} the dispersion and beta functions at the center of the dipole. Substituting the values of $D_{xc}^{\min}$ and $\beta_{xc}^{\min}$ in Eq. (5.3) with their TME expressions of Eq. (5.2), the emittance becomes $\varepsilon_{x\text{TME}} = \mathcal{F} C_q \gamma^3 \theta^3$. The scaling factor $\mathcal{F}$ for the TME lattice is $\mathcal{F} = \frac{1}{12\sqrt{15}\mathcal{J}_x}$ and the damping partition number $\mathcal{J}_x \approx 1$, in the case of isomagnetic, separated function dipoles [7]. Defining the ratios $\beta_r = \frac{\beta_{xc}}{\beta_{xc}^{\min}}$ and $D_r = \frac{D_{xc}}{D_{xc}^{\min}}$, it is useful to define the detuning factor:

$$\varepsilon_r = \frac{9 + 4\beta_r^2 + 5(D_r - 2)D_r}{8\beta_r}, \quad (5.4)$$

with $\varepsilon_x = \varepsilon_r \cdot \varepsilon_{x\text{TME}}$, which is an indication of how the emittance deviates from its theoretical minimum, for a given set of optics parameters at the center of the cell.

The beta β_{xc} and dispersion D_{xc} functions, at the dipole center, impose two independent optics constraints, and thus at least two quadrupole families are needed to meet them. The horizontal optics functions are fully controlled by these two pairs of quadrupoles, whereas, in the absence of additional knobs, the vertical plane optics is also uniquely defined. Calculating the transfer matrix for the half cell (see Appendix A), using thin lens approximation and for given β_{xc} and D_{xc} at the center of the dipole (or β_r and D_r), analytical expressions can be derived for the strengths of the quadrupoles:

$$\begin{aligned} f_1 &= \frac{s_2(4s_1 l_d + l_d^2 + 8D_{xc} \rho_x)}{(4s_1 l_d + l_d^2 + 8D_{xc} \rho_x) + 4s_2 l_d - 8D_s \rho_x} = \frac{l_d s_2 (12s_1 + l_d(D_r + 3))}{12l_d(s_1 + s_2) + l_d^2(D_r + 3) - 24D_s \rho_x}, \\ f_2 &= -\frac{8s_2 D_s \rho_x}{(4s_1 l_d + l_d^2 + 8D_{xc} \rho_x) - 8D_s \rho_x} = -\frac{24s_2 D_s \rho_x}{12l_d s_1 + l_d^2(D_r + 3) - 24D_s \rho_x}, \end{aligned} \quad (5.5)$$

which are parametrized with the drift lengths s_1 , s_2 , s_3 . The parameter D_s is the dispersion at the center of the cell (between two mirror symmetric quadrupoles) and is a function of the drift lengths, the optics functions at the dipole center and the bending characteristics:

$$D_s = \frac{A \pm \sqrt{A^2 + ABC}}{64B\rho_x^2}, \quad (5.6)$$

where:

$$\begin{aligned} A &= 8s_2\rho_x(l_d^4 + 64D_{xc}^2\rho_x^2 + 16l_d^2(\beta_{xc}^2 - D_{xc}\rho_x)) \\ &= \frac{8}{45}l_d^4s_2(12\beta_r^2 + 5(-3 + D_r)^2)\rho_x, \\ B &= l_d(2s_1l_d + l_d^2 + 8\beta_{xc}^2) - 8(2s_1 + l_d)D_{xc}\rho_x \\ &= \frac{1}{15}l_d^2(l_d(15 + 2\beta_r^2 - 5D_r) - 10s_1(-3 + D_r)), \\ C &= \frac{16s_3\rho_x(4s_1l_d + l_d^2 + 8D_{xc}\rho_x)}{s_2} = \frac{16l_ds_3(12s_1 + l_d(3 + D_r))\rho_x}{3s_2}. \end{aligned} \quad (5.7)$$

The calculation of D_s springs from the symmetry requirement in the middle of the cell, $\alpha_x = 0$. By applying the TME conditions in the middle of the dipole ($\alpha_x=0$, $D'_x=0$), the α_x function in the middle of the cell is of the form $\alpha_x = C_0 + C_1/D_s + C_2/D_s^2$, where, C_0 , C_1 and C_2 are functions of D_{xc} , β_{xc} , ρ_x , l_d , s_1 , s_2 and s_3 . The quadratic dependence of α_x to (D_s^{-1}) results the two solutions, with opposite sign in the second component, for D_s .

In the absolute minimum emittance limit, where $\beta_r = D_r = 1$, the parametric equations for the quadrupole strengths are reduced to:

$$\begin{aligned} f_1^{\text{TME}} &= \frac{(l_d + 3s_1)(3l_d + 5s_1)s_2}{(l_d + 3s_1)(3l_d + 5s_1) + (7l_d + 15s_1)s_2 \pm 2\sqrt{l_d(l_d + 3s_1)(3l_d + 5s_1) + l_d^2s_2^2}}, \\ f_2^{\text{TME}} &= \frac{2l_ds_2s_3}{l_d(s_2 + 2s_3) \pm \sqrt{l_d(l_d + 3s_1)(3l_d + 5s_1) + l_d^2s_2^2}}. \end{aligned} \quad (5.8)$$

It is interesting to study the behavior of those equations in the limits, where the drift spaces lengths are going to zero. Eqs (5.8) are then further reduced to:

$$\begin{aligned} f_1^{\text{TME}} &\xrightarrow{s_1 \rightarrow 0} \left(\frac{3l_ds_2}{3l_d + 7s_2 + 2\sqrt{s_2^2 + 3l_ds_3}}, \frac{3l_ds_2}{3l_d + 7s_2 - 2\sqrt{s_2^2 + 3l_ds_3}} \right), \\ f_1^{\text{TME}} &\xrightarrow{s_2 \rightarrow 0} (0, 0), \\ f_1^{\text{TME}} &\xrightarrow{s_3 \rightarrow 0} \left(\frac{(l_d + 3s_1)s_2}{l_d + 3(s_1 + s_2)}, \frac{(3l_d + 5s_1)s_2}{3l_d + 5(s_1 + s_2)} \right), \end{aligned} \quad (5.9)$$

and

$$\begin{aligned} f_2^{\text{TME}} &\xrightarrow{s_1 \rightarrow 0} \left(\frac{2s_2s_3}{s_2 + 2s_3 + \sqrt{s_2^2 + 3l_ds_3}}, \frac{2s_2s_3}{s_2 - 2s_3 - \sqrt{s_2^2 + 3l_ds_3}} \right), \\ f_2^{\text{TME}} &\xrightarrow{s_2 \rightarrow 0} (0, 0), \\ f_2^{\text{TME}} &\xrightarrow{s_3 \rightarrow 0} (0, 0). \end{aligned} \quad (5.10)$$

The two solutions in each case correspond to the two solutions for the D_s . In the limit where $s_2 \rightarrow 0$, both the focal lengths f_1 and f_2 go to zero too, thus the quadrupole strengths go to

infinity. This shows that a good separation of the two quadrupoles is necessary, in order to have a feasible TME cell. In the limit where s_3 goes to zero, f_2 goes to zero too, showing that the two mirror symmetric quadrupoles Q_2 cannot be merged, as the strength of the defocusing quadrupole will then go to infinity. f_1 , on the other hand, converges to a specific value. Finally, in the limit where $s_1 \rightarrow 0$, both f_1 and f_2 converge to specific values, depending on the dipole length and on the drift spaces lengths s_2 and s_3 . Thus, realistic solutions exist, even if the first quadrupole Q_1 is placed exactly after the dipole, without any space between them.

From Eq. (2.25), the horizontal and vertical phase advances of the cell can be defined through the trace of the cell transfer matrix as:

$$\cos \mu_{x,y} = \frac{M_{\text{cell},x,y}^{[1,1]} + M_{\text{cell},x,y}^{[2,2]}}{2}. \quad (5.11)$$

The horizontal phase advance can be written in a simple form as:

$$\cos \mu_x = \frac{(l_d^2 - 8D_{xc}\rho_x)^2 - 16l_d^2\beta_{xc}^2}{(l_d^2 - 8D_{xc}\rho_x)^2 + 16l_d^2\beta_{xc}^2} = \frac{5(D_r - 3)^2 - 12\beta_r^2}{5(D_r - 3)^2 + 12\beta_r^2}. \quad (5.12)$$

For $D_r = \beta_r = 1$, $\mu_x = \arccos(1/4) = 284.5^\circ$ independent on any cell parameter, which is a known property of the TME cells [15]. The expression for the vertical phase advance has a more complicated form:

$$\begin{aligned} \cos \mu_y = & 1 + (L_{12} + 2s_3)\left(\frac{1}{f_1} + \frac{1}{f_2}\right) + \frac{2L_1s_2 + 2s_2^2 + L_1s_3 + L_{12}s_3 + 2s_2s_3}{f_1f_2} \\ & + \frac{L_1s_2 + L_1s_3}{f_1^2} + \frac{L_{12}s_3}{f_2^2} + \frac{L_1s_2^2 + 2L_1s_2s_3}{f_1^2f_2} + \frac{L_1s_2s_3 + L_{12}s_2s_3}{f_1f_2^2} + \frac{L_1s_2^2s_3}{f_1^2f_2^2}, \end{aligned} \quad (5.13)$$

where $L_1 = l_d + 2s_1$ and $L_{12} = L_1 + 2(s_1 + s_2)$. In the limit of the absolute minimum emittance and of the drift spaces lengths going to zero, the $\cos \phi_y$ function goes to infinity. The vertical motion is thus unstable, if the cell is tuned to the absolute minimum emittance conditions, if $s_1 \rightarrow 0$ or $s_2 \rightarrow 0$ or $s_3 \rightarrow 0$.

Inverting Eq.(5.4) and solving with respect to β_r , the following expression is computed:

$$\beta_r = \varepsilon_r \pm \frac{1}{2}\sqrt{-9 + 4\varepsilon_r^2 - 5(-2 + D_r)D_r}. \quad (5.14)$$

The quadratic dependence on D_r of the argument in the square root in eq. (5.14), sets an upper and a lower limit for the dispersion at the center of the dipole, in order for β_r to be a real number:

$$1 - \frac{2\sqrt{-1 + \varepsilon_r^2}}{\sqrt{5}} \leq D_r \leq 1 + \frac{2\sqrt{-1 + \varepsilon_r^2}}{\sqrt{5}}. \quad (5.15)$$

The vertical beta function in the center of the dipole, has again a more complicated dependence on s_1 , s_2 , s_3 and l_d , of the form:

$$\beta_{y,c} = \left(\frac{(A_1A_2 - 48A_1l_dD_s\rho_x + s_{01}(24D_s\rho_x)^2)(A_1A_22s_3 - 48A_1l_ds_{23}D_s\rho_x + s_{01}s_{23}(24D_s\rho_x)^2)}{4(A_2A_3 - 48A_3l_dD_s\rho_x + (24D_s\rho_x)^2)(A_2A_32s_3 - 48A_3l_ds_{23}D_s\rho_x + s_{23}(24D_s\rho_x)^2)} \right)^{1/2}, \quad (5.16)$$

where:

$$\begin{aligned} A_1 &= 24s_1(s_1 + s_2) + l_d^2(3 + D_r) + l_d s_{12}(9 + D_r), \\ A_2 &= l_d^2(12s_1 + l_d(3 + D_r)), \\ A_3 &= 6s_{12} + l_d(3 + D_r), \end{aligned} \quad (5.17)$$

and $s_{01} = l_d + 2s_1$, $s_{23} = s_2 + 2s_3$, $s_{12} = 2s_1 + s_2$. The expression in the parenthesis should always be positive for real solutions, imposing further constraints on the drift lengths. Studying again the behavior of vertical beta function in the middle of the dipole, in the absolute minimum emittance limit and in the limits where the drift spaces lengths go to zero leads to complex solutions, showing again that the vertical plane is unstable if $s_1 \rightarrow 0$ or $s_2 \rightarrow 0$ or $s_3 \rightarrow 0$, for absolute minimum emittance conditions.

The momentum compaction factor, using the TME conditions ($D'_x = 0$ in the center of the dipole), can be written in the form:

$$\alpha_p = \left\langle \frac{D_x}{\rho_x} \right\rangle = \frac{1}{l_d} \int_0^{l_d} \frac{D_x(s)}{\rho_x} ds = \frac{7}{12} \theta^2 + \frac{2D_c}{\rho_x} = \frac{\theta^2}{12} (7 + D_r), \quad (5.18)$$

and depends only on the dipole characteristics and the dispersion in the middle of the dipole. The momentum compaction factor for the absolute minimum emittance ($D_r = 1$) becomes:

$$\alpha_p^{\text{TME}} = \frac{2\theta^2}{3}. \quad (5.19)$$

5.1.1 Optics stability and magnet constraints

The stability criterion for both horizontal and vertical planes is:

$$\text{Trace}(M_{x,y}) = 2 \cos \mu_{x,y} < 2, \quad (5.20)$$

where $M_{x,y}$ is the transfer matrix of the cell and $\mu_{x,y}$ are the horizontal and vertical phase advances per cell. The latest ensures the optics stability and can be used for constraining the cell characteristics (focal and drift lengths). However, even if satisfied, it does not necessarily guarantee technologically feasible magnet strengths. The feasibility of the quadrupoles and the chromatic sextupoles is ensured if the pole tip field is kept below a maximum value allowed by the chosen magnet technology. In addition, the radius of the magnets' aperture should be greater than a minimum value, defined by beam and lattice properties.

The quadrupole gradient (expressed in [T/m]) is defined as $g = k(B\rho_x)$, where k the quadrupole strength and $B\rho_x$ the magnetic rigidity. From the definition of the pole tip field: $B_q = R \frac{\partial B_y}{\partial x} |_{y=0} = Rg$, the gradient is $g = \frac{B_q}{R}$, where, R is the quadrupole aperture radius. Considering a circular beam pipe, the minimum required aperture radius in order to accept all the particles of the incoming beam, for a non-Gaussian beam distribution, is defined by the displacement of the particles with the maximum action in the beam, defined by an emittance ε_{max} and a momentum deviation $(\delta p/p_0)_{\text{max}}$ [54]:

$$R_{\text{min}} = \sqrt{2\beta\varepsilon_{\text{max}}} + \left(\frac{\delta p}{p_0}\right)_{\text{max}} \cdot D + d_{\text{co}}, \quad (5.21)$$

where β and D the beta and dispersion functions at this location, $(\delta p/p_0)$ the total energy spread of the beam and d_{co} a constant reflecting the tube thickness, mechanical tolerances and maximum orbit distortion. For a Gaussian beam distribution, Eq. (5.21) becomes: $R_{\min} = \sqrt{2\beta\varepsilon_{\max} + ((\frac{\delta p}{p_0})_{\max} \cdot D)^2 + d_{co}}$. The $R_{\min}$ can be computed for each element of the cell and takes its maximum value at the center of the quadrupoles, where the beta functions become maximum. The feasibility constraint for the quadrupole gradient or strength is then:

$$g \leq \frac{B_q^{\max}}{R_{\min}} \quad \text{or} \quad \frac{1}{fl_q} = k \leq \frac{1}{(B\rho_x)} \frac{B_q^{\max}}{R_{\min}}. \quad (5.22)$$

In a similar way, a feasibility constrained can also be set for the sextupole strengths. As already mentioned, the TME cells are intrinsically high chromaticity cells when targeting to their theoretical minimum emittance limit, as low dispersion and strong focusing are needed to achieve the ultra low emittance. The high chromaticity requires strong sextupoles for the chromaticity correction, reducing the dynamic aperture of the machine. The sextupoles used for the natural chromaticity correction are usually placed close to the quadrupoles, to large dispersion and beta function regions. In order to simplify the calculations, the sextupoles are considered to be placed on top of the quadrupoles, having the same lengths with them. The pole-tip field for the sextupoles is: $B_s = (B\rho_x)b_2R^2 = \frac{1}{2}R^2\frac{\partial^2 B_y}{\partial x^2}|_{y=0}$ and from this the sextupole gradient: $2(B\rho_x)b_2 = B_s/R^2$. As the sextupoles are set to cancel the chromaticity induced by the quadrupoles, the sextupole strengths can be calculated by:

$$\begin{aligned} \xi_x &= -\frac{1}{4\pi} \oint \beta_x [K_x(s) - S(s)D(s)] ds = 0, \\ \xi_y &= -\frac{1}{4\pi} \oint \beta_y [K_y(s) + S(s)D(s)] ds = 0, \end{aligned} \quad (5.23)$$

where $K_{x,y}$ the focusing and defocusing quadrupole strengths and $S = \frac{b_2}{(B\rho_x)}$ the sextupole strengths. Calculating the above expressions in all the cell, the expressions for the sextupole strengths that follows are:

$$\begin{aligned} S_1 &= -\frac{2\xi_y^q \pi \beta_{x,d} + 2\xi_x^q \pi \beta_{y,d}}{l_q \beta_{x,f} \beta_{y,d} D_{x,f} - l_q \beta_{x,d} \beta_{y,f} D_{x,f}}, \\ S_2 &= \frac{2\xi_y^q \pi \beta_{x,f} + 2\xi_x^q \pi \beta_{y,f}}{l_q \beta_{x,f} \beta_{y,d} D_{x,d} - l_q \beta_{x,d} \beta_{y,f} \eta_{x,d}}, \end{aligned} \quad (5.24)$$

where $\xi_{x,y}^q = -\frac{1}{4\pi} \oint \beta_{x,y} K_{x,y} ds$ and l_q the length of the quadrupoles. For simplicity, we consider all the quadrupoles to have the same length. In the expressions above, the index f denotes the values of the optics functions on the focusing quadrupoles while d the values on the defocusing quadrupoles. In order to have feasible solutions, these values need to satisfy the constrain:

$$S \leq \frac{2B_s^{\max}}{R_{\min}^2} \frac{1}{(B\rho_x)}. \quad (5.25)$$

Equations (5.5), (5.12), (5.14), (5.15), (5.22), (5.25) fully describe the TME cell. The parameter space of the cell, including geometrical and optical properties, can be determined giving the possibility to optimize the cell according to any design requirements.

5.2 Numerical Application

The analytical parameterization can be used to study the performance of any TME cell of interest. In the next, some numerical examples, applicable to the CLIC Pre-damping rings (PDR) lattice design, will be used to demonstrate the results. The operational energy of the CLIC Damping Rings complex of 2.86 GeV [55] and a dipole field of 1.2 T, are used. The required output normalized emittance from the CLIC PDR is $63 \mu\text{m-rad}$. Leaving also a blow up margin of 10 %, and using Eq. (2.86), at least 19 dipoles (or TME cells) of 1.2 T field and $\theta = 2\pi/N_d \approx 19^\circ$ bending angle, are needed. The example of a TME cell with 38 dipoles of 1.2 T bending field and $\theta \approx 9.5^\circ$ bending angle is also discussed. In order to set the feasibility constraints of the quadrupole and sextupole magnets, the maximum pole-tip field of the quadrupoles is set to $B_q^{\text{max}} = 1.1 \text{ T}$ and for the sextupoles $B_s^{\text{max}} = 0.8 \text{ T}$, which are typical values for normal-conducting magnets. Both quadrupole and sextupole lengths are set to $l_q = 0.3 \text{ m}$. Fixing those parameters, the free parameters left are the drift space lengths, s_1 , s_2 and s_3 , and the emittance ε_x , or the detuning factor ε_r . In the next, the parameterization with respect to drift spaces lengths and with respect to the emittance is treated separately.

5.2.1 Parametrization with the drift lengths

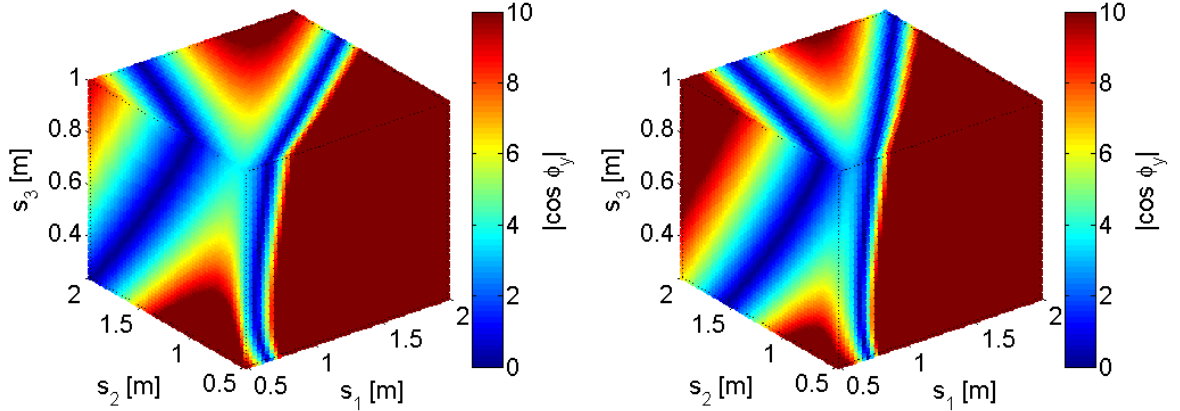


Figure 5.2: Numerical solution of the $|\cos \phi_y|$ function for the TME cell, for two different dipole bending angles, $\theta = 2\pi/19$ (left) and $\theta = 2\pi/38$ (right), when targeting their theoretical minimum emittance. Only the dark blue regions of solutions provide stability of the motion in the vertical plane.

At first, a constant emittance is considered and seek for the drift spaces lengths satisfying the stability constraints in both horizontal and vertical planes. By construction, the horizontal plane is always stable, thus the stability constrain comes mainly from the vertical plane. The $\cos \phi_y$ function defined in Eq. (5.13) was calculated numerically, for all combinations of s_1 , s_2 and s_3 , for $s_1 \in (0.5, 2) \text{ m}$, $s_2 \in (0.5, 2) \text{ m}$ and $s_3 \in (0.25, 1) \text{ m}$. The solutions for two different TME cells with bending angles of $\theta = 2\pi/19$ (left) and $\theta = 2\pi/38$ (right), corresponding to dipole lengths $l_d = 2.6 \text{ m}$ and 1.3 m and equilibrium emittances of $\varepsilon_x^{\text{TME}} = 52 \mu\text{m-rad}$ and $6.5 \mu\text{m-rad}$

respectively, are shown in Figure 5.2. Only solutions for which $\cos \phi_y \in (-1, 1)$ guarantee the stability of the motion. The absolute value of the $\cos \phi_y$ function is plotted here. The function $|\cos \phi_y|$ approaches the $(0, 1)$ regime differently from the right (large s_1 , small s_2) and left part (small s_1 , large s_2) of the plot, and only a very small fraction of the solutions lie in this interval, in two different regions of the plot (dark blue).

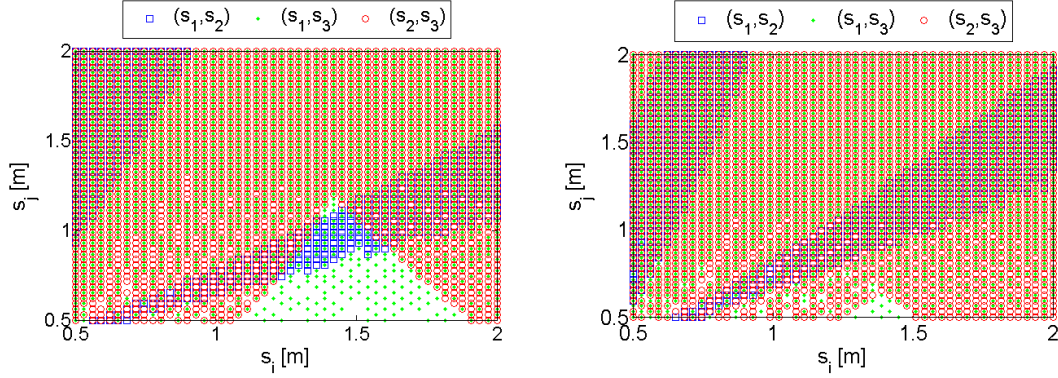


Figure 5.3: Combinations of s_1 , s_2 and s_3 satisfying the stability requirements for a TME cell with $\theta = 2\pi/19$ and for $\theta = 2\pi/38$.

Figure 5.3 shows the projection of the stable solutions in the planes (s_1, s_2) (blue squares), (s_1, s_3) (green dots) and (s_2, s_3) (red circles). For both cases under study, for $\theta = 2\pi/19$ (left) and $\theta = 2\pi/38$ (right), almost all combinations of (s_1, s_3) and (s_2, s_3) provide stability, except a small triangular part, which gets even smaller in the $\theta = 2\pi/38$ case in the (s_2, s_3) plane. On the other hand, only a small fraction of the (s_1, s_2) combinations, satisfy the stability requirements. This shows that the system is not sensitive to the choice of s_3 , with respect to stability, while a careful choice of a set of s_1 and s_2 is very important.

The combinations of drift lengths satisfying the stability requirements are then applied to Eqs. (5.5)-(5.16), for the calculation of all cell properties. Figure 5.4 shows the quadrupole focal lengths (top), f_1 (left), f_2 (middle) and the vertical phase advance μ_y (top, right), parametrized with the drift spaces lengths, s_1 , s_2 , s_3 . On the bottom part of the figure, the projection to the (s_1, s_2) plane, color-coded with the f_1 (left) and f_2 (right), is also presented.

In a similar way, Figure 5.5 shows the horizontal (left) and vertical (middle) chromaticities ξ_x and ξ_y respectively, and the beta function $\beta_{y,qd}$ at the defocusing quadrupole (right), for the different combinations of the drift lengths (top) and the projection to the (s_1, s_2) plane (bottom). The two manifold of stable solutions for the (s_1, s_2, s_3) triplets are clearly distinguished, with respect to the quadrupole focal lengths (especially for the defocusing quad), the horizontal and vertical chromaticities, and the vertical phase advance. The small s_1 - large s_2 region corresponds to low vertical phase advances, weaker quadrupole strengths and smaller chromaticities. The small s_2 region is characterized by large phase advances, strong quadrupole focal lengths (especially the vertical one) and large chromaticities. If the feasibility constraints are applied to these solutions, the latest region is rejected. In the low chromaticity region, a layer of large vertical chromaticity solutions appears, which is associated to large vertical beta functions at the

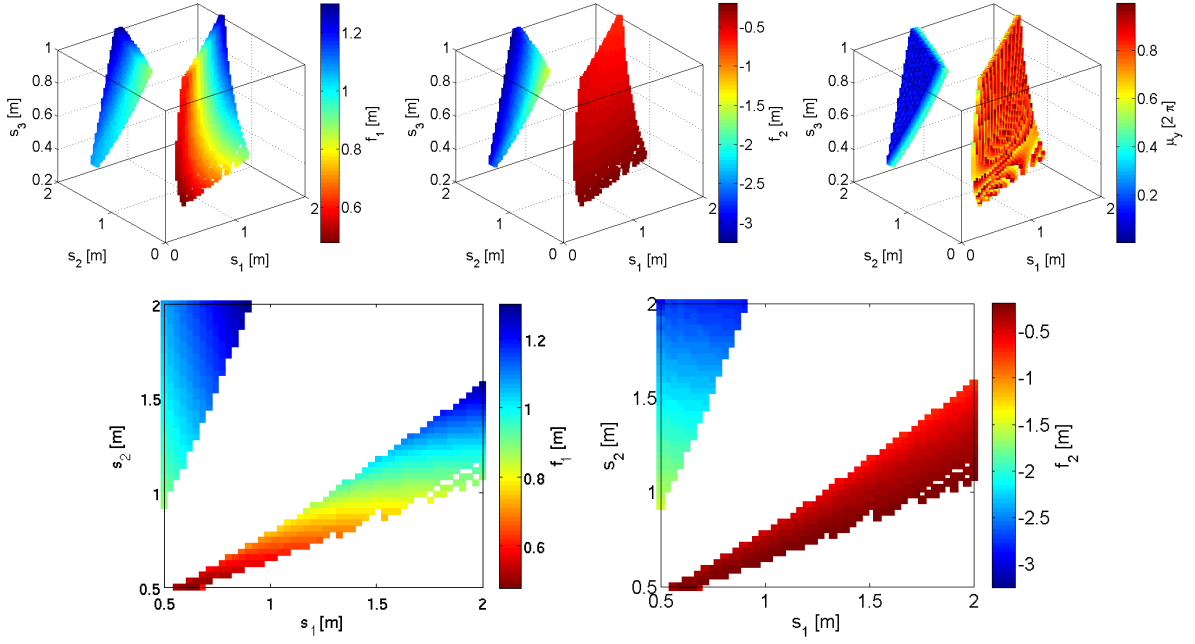


Figure 5.4: Top: Parameterization of the quadrupole focal lengths, f_1 (left) and f_2 (middle) and the vertical phase advance, μ_y (right), with the drift spaces lengths, s_1 , s_2 , s_3 , providing stable motion. Bottom: The projection to the (s_1, s_2) plane.

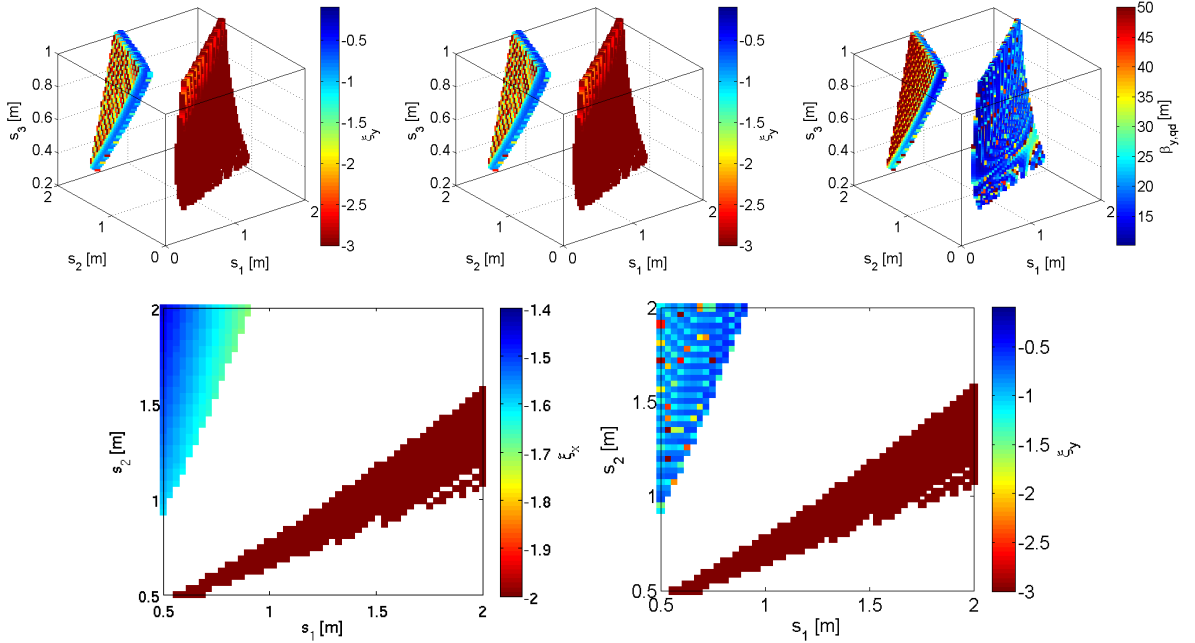


Figure 5.5: Top: Parameterization of the horizontal, ξ_x (left) and vertical, ξ_y (middle) chromaticities and the beta function at the defocusing quadrupole (right), with the drift spaces lengths, s_1 , s_2 , s_3 , providing stable motion. Bottom: The projection to the (s_1, s_2) plane, color-coded with the horizontal (left) and vertical (right) chromaticities.

defocusing quadrupole. Finally, we can conclude that proper choice of the drift spaces lengths triplet (s_1, s_2, s_3) can assure the stability of the motion, and lead to the minimization of the quadrupole strengths (maximum focal lengths) and to the minimization of the cell chromaticities in both planes, achieving always the same minimum emittance.

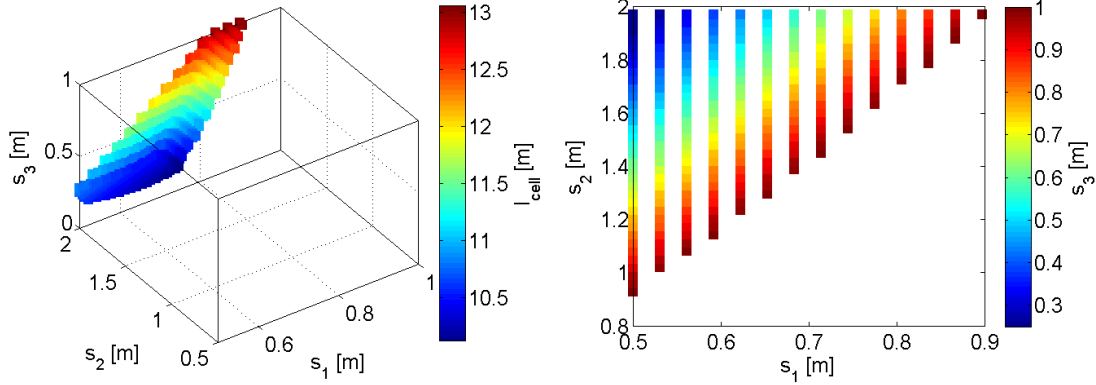


Figure 5.6: Left: The (s_1, s_2, s_3) combinations that provide the lowest chromaticity, $\xi_{x,y} \leq -2$ in both planes, colorcoded with the total length of the cell. Right: Projection of the low chromaticity solutions to the (s_1, s_2) plane, colorcoded with s_3 .

For the CLIC PDR, low chromaticity solutions are of interest as the lattice design is focused on the dynamic aperture optimization. The cell length, on the other hand, is preferred to stay as compact as possible, to minimize the circumference of the ring. Figure 5.6 shows the (s_1, s_2, s_3) for which the absolute chromaticity in both planes is less than 2, colorcoded with the total cell length l_{cell} (left), where $l_{\text{cell}} = l_d + 2(s_1 + s_2 + s_3)$. In the right part of the figure, the projection of the solutions to the (s_1, s_2) plane is shown, colorcoded with s_3 . For the absolute minimum emittance case, in order to keep the chromaticity in low levels and the cell length as compact as possible, small values of s_1 and s_3 are needed, ($s_1 \leq 0.65$ m, $s_3 \leq 0.5$ m), and large values of s_2 ($s_2 \geq 1.5$ m). However, even the minimum possible chromaticity of this cell is quite large ($\xi_x \sim -1.4$).

5.2.2 Parametrization with the emittance

Having the drift lengths fixed, Eq. (5.5) combined with Eqs. (5.14) and (5.15) are studied numerically, for different detuning factors ε_r . In this example, the dipole bending angle is set to $\theta = 2\pi/38$ and the drift lengths to $s_1=0.9$ m, $s_2=0.6$ m and $s_3=0.5$ m.

In order to achieve the absolute minimum emittance, only one pair of initial optics functions (D_{xc}, β_{xc}) exists [15]. However, relaxing this requirement and detuning the cell to higher emittance values ($\varepsilon_r > 1$), several pairs of (D_{xc}, β_{xc}) , lying in elliptical curves, can achieve the same emittance, as shown by equation (5.3). Figure 5.7 (left) shows the solutions of (D_{xc}, β_{xc}) color-coded with the detuning factor ε_r . Even though, by definition, all solutions are stable in the horizontal plane, only a small fraction of them satisfy the stability (black squares) and feasibility (magenta triangles) criteria. The parametrization of the focusing strengths with the emittance is displayed in Fig. 5.7 (right), with the same color-convention as before. The f_1, f_2 pairs,

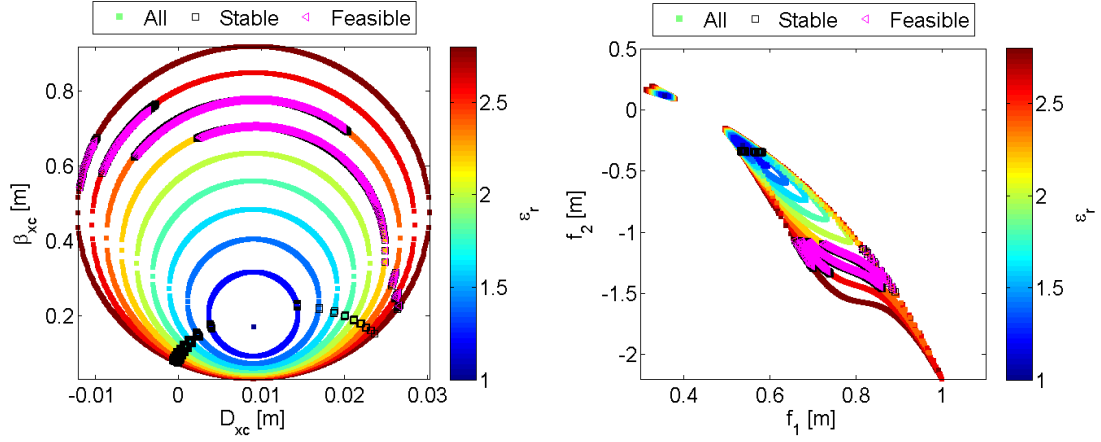


Figure 5.7: Parameterization of the horizontal beta β_{xc} and dispersion D_{xc} in the center of the dipole (left) and the quadrupole focal lengths (right), with the cell detuning factor ε_r . The stable solutions are indicated with the black squares, while the stable and feasible ones with the magenda triangles.

for the same emittance, lie in distorted ellipses, which get more distorted, while moving to high detuning factors. In order to tune the cell to the very low emittance, strong quadrupole strengths are needed and only one combination of (f_1, f_2) can tune the cell to the absolute emittance minimum. Moving away from the minimum emittance regime, the quadrupole strengths are relaxed for detuning factors greater than 2. In the upper left corner of the plot, solutions with both f_1 and f_2 positive, cannot provide stability as they always provide defocusing in the vertical plane. Those solutions correspond to negative dispersion in the middle of the cell ((-) sign in the second component of Eq. (5.6)). It is interesting to notice, that, changing the values of f_1 and/or f_2 by a small amount, the system remains stable if tuned in the relaxed ε_r regime but can easily get unstable if tuned in the absolute minimum emittance regime.

Figure 5.8 shows the parameterization of the detuning factor (top, left), the horizontal beta (top, middle) and dispersion (top, left) in the middle of the dipole, and the horizontal (bottom, left) and vertical (bottom, right) chromaticities, with the horizontal and vertical phase advances of the cell, μ_x and μ_y respectively. As was discussed earlier, there is only one pair of $(\mu_{x,\text{TME}}, \mu_{y,\text{TME}})$ that can achieve the theoretical minimum emittance. In the horizontal plane there is a unique phase advance, for $\mu_{x,\text{TME}} = 284.5^\circ$ independent on any cell characteristics, while in the vertical plane the $\mu_{y,\text{TME}}$ depends on the cell geometry and the dipole characteristics. Towards small phase advances, in both horizontal and vertical planes, the horizontal beta and dispersion functions in the center of the dipole and the detuning factor of the cell get high values. In the high horizontal phase advance regime, both the beta and dispersion functions get very small values, where the detuning factor gets also small. High detuning factor solutions exist also for very large horizontal phase advances. The latest correspond to the lower part of the ellipses of Figure 5.7 (left). Towards the minimum emittance (small detuning factors), the chromaticity values become large, while staying to values of $\mu_{x,y}/2\pi < 0.5$, the chromaticities are kept to low levels ($|\xi_{x,y}| < 0.5$). It is interesting to notice, that the high detuning factor solutions

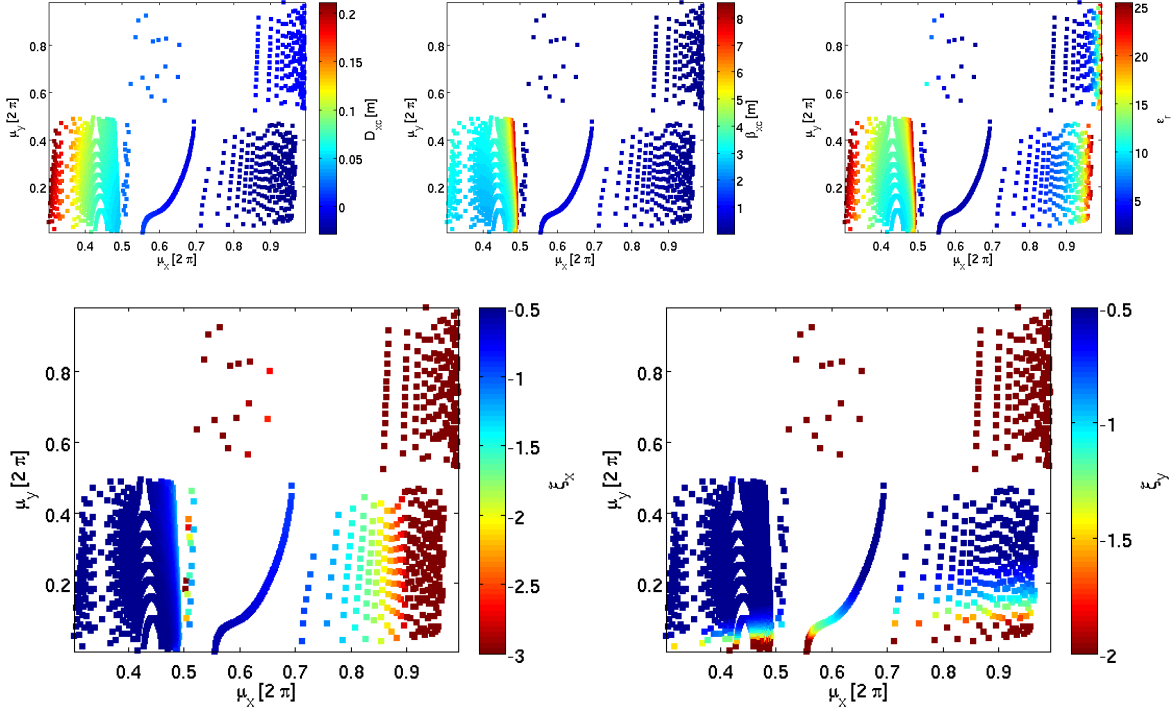


Figure 5.8: Top: Parameterization of the horizontal dispersion (left) and beta (middle) functions and the cell detuning factor (right) with the horizontal and vertical phase advances. Bottom: The dependence of the horizontal (left) and vertical (right) chromaticities on the horizontal and vertical phase advances.

at large horizontal phase advances, produce large chromaticities as they correspond to minimum dispersion and beta functions in the center of the dipole, which require strong focusing.

For the optimization of the CLIC PDR lattice design, low chromaticity solutions are of interest, as the design is focused on the dynamic aperture optimization. In section 5.2.1, it was demonstrated that careful choice of the drift spaces lengths is important for the chromaticity minimization, when the cell is tuned to the absolute minimum emittance. However, in this limit the chromaticity becomes very high, even for the optimal case of chromaticity. If low chromaticity is the goal of the design, it is thus preferable to choose a cell that can achieve an absolute minimum emittance much lower than the requirement of the design, and detune it to low phase advance values and large detuning factors, in order to minimize the chromaticity, staying always within the emittance requirements of the design.

In the previous section, the optimization of a cell, tuned to the conditions of the absolute minimum emittance, with respect to the drift spaces lengths, was described. Here, the example of a detuned cell is considered, for a dipole bending angle of $\theta = 2\pi/38$, corresponding to a minimum emittance of $6.5 \mu\text{m-rad}$ and for a detuning factor of $\epsilon_r = 10$. The emittance that this cell achieves, even if detuned by a factor of 10, is still within the requirements of the PDR design. Repeating the same procedure described in section 5.2.1, for a detuning factor of 10, we see that almost every choice of (s_1, s_2, s_3) triplet assures the stability of the motion. Figure 5.9

shows only the solutions for low chromaticity, in both horizontal and vertical planes ($|\xi_{x,y}| < 2$). The parameterization of the horizontal chromaticity (left) and the horizontal dispersion in the middle of the dipole (right) with the (s_1, s_2, s_3) triplets is presented in the top part of the figure, while the projection to the (s_1, s_2) plane, color-coded with the horizontal chromaticity, on the bottom. There is a clear correlation between the D_{xc} and the horizontal chromaticity ξ_x . High chromaticity values correspond to negative or small dispersion in the middle of the dipole, as low dispersion values require strong focusing by the quadrupoles. Higher dispersion values in the middle of the dipole, correspond to smaller chromaticity values of the cell.

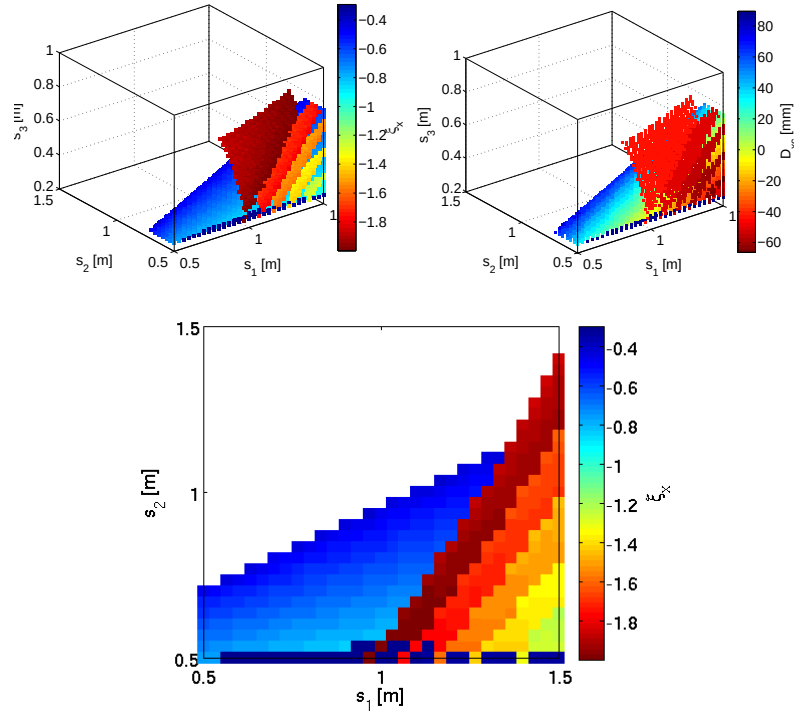


Figure 5.9: Top: Parameterization of the horizontal chromaticity (left) and the horizontal dispersion in the middle of the dipole (right), for a detuned cell ($\varepsilon_r=10$), with the drift spaces lengths, s_1 , s_2 and s_3 . Bottom: Projection of the low chromaticity solutions in the (s_1, s_2) plane, color-coded with the horizontal chromaticity.

As a conclusion, in order to achieve the ultra-low emittance, tuning of the cell in high horizontal and vertical phase advances is required. The absolute minimum emittance can be achieved only by a unique horizontal phase advance, $\mu_x = 287.5^\circ$ independent on any cell characteristics and by only one vertical phase advance, depending on the cell geometrical and dipole characteristics. In this regime, careful choice of the drift spaces lengths is required, in order to assure the stability of the motion in both horizontal and vertical planes. Small divergence from the (s_1, s_2, s_3) that guarantee stability, for the absolute minimum emittance, can lead to unstable motion. Careful choice of the (s_1, s_2, s_3) can also minimize the chromaticity, however, even in the optimal case, the chromaticity levels are high in the absolute minimum emittance regime. On the other hand, operating the cell in low phase advances (both horizontal and vertical),

even though larger emittances than the absolute minimum one are achieved, stability is assured almost for any combination of (s_1, s_2, s_3) triplets, and the chromaticity of the cell is much lower. Proper choice of (s_1, s_2, s_3) can minimize the chromaticity in both planes. In the case of the CLIC PDR, the lattice design is focused on the dynamic aperture optimization, thus low chromaticities are necessary. Due to non-linear optimization arguments that will be discussed in Chapter 6 (see sec. 6.3), a dipole bending angle of $\theta = 2\pi/38$ was chosen. As was demonstrated above, for this bending angle and for a detuning factor of 10, the triplet $(s_1, s_2, s_3)=(0.9, 0.6, 0.5)$ can achieve low chromaticities, in both horizontal and vertical planes. In the same way, the analytical parameterization described in this chapter, can be used for the optimization of the cell according to any requirement of the design

5.3 Comparison with MADX

The results of the analytical solution were compared with the results from the numerical simulation code MADX [14] for the thin and thick lens cases. The three plots of Fig. 5.10 show this comparison for three different values of the quadrupole lengths, $l_q = 1, 10$ and 20 cm. The three curves of each plot in Fig. 5.10, represent three different detuning factors, $\varepsilon_r = 1, 1.5$ and 2 . In black, all cases of the analytical solution are shown, in red the solutions satisfying the stability criterion while in green the MADX solutions. The agreement for the thin lens is excellent, demonstrating the validity of our analytical calculations. What is very interesting is the fact that, in the thick lens case, the agreement is still very good. The analytical solution can be a very good approximation of the simulation results and can be very helpful for the lattice optimization and understanding. This way the optimal dipole characteristics, the geometrical characteristics of the cell and the interesting phase advances can be defined. It can also be very useful, for the definition of initial conditions to be used for the lattice design using numerical tools, whose optimization algorithms depend heavily on the initial conditions.

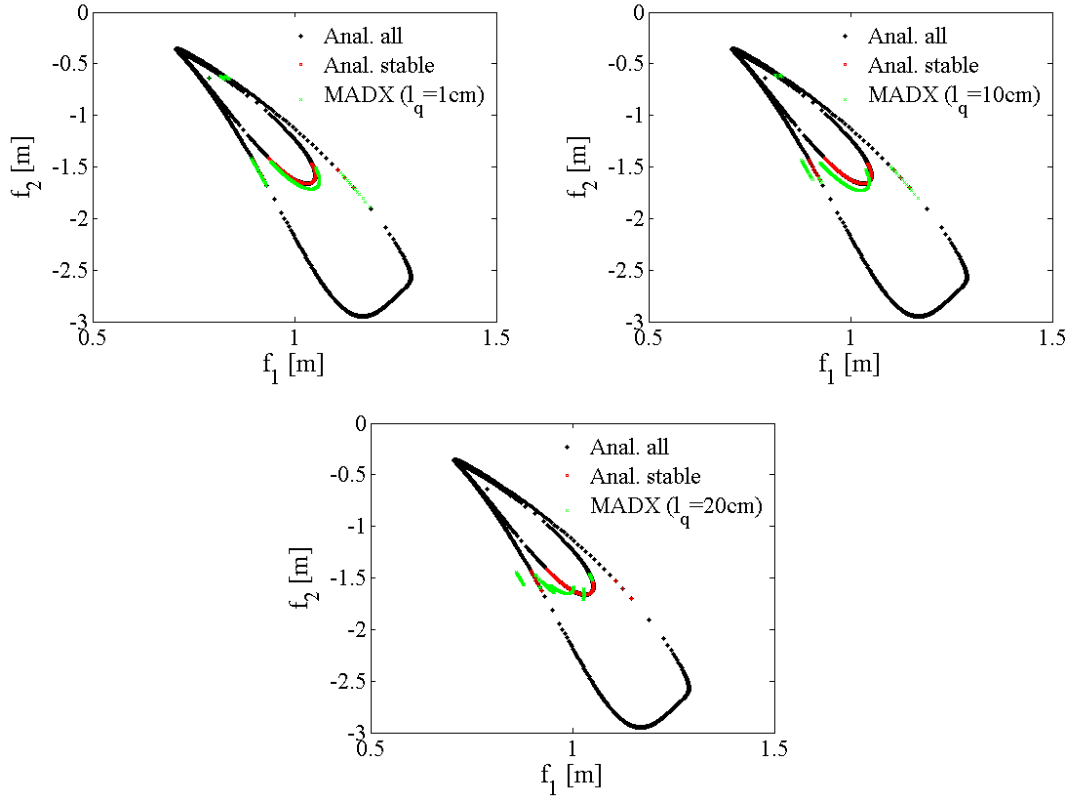


Figure 5.10: Comparison between the analytical solution, and the MADX results. In black are shown all the analytical solutions, in red the solutions satisfying the stability requirements and in green the results from MADX for different quadrupole lengths: $l_q = 1$ cm (top, left), $l_q = 10$ cm (top, middle) and $l_q = 20$ cm (bottom).

Conceptual design of the CLIC

Pre-damping Rings

The CLIC damping rings complex consists of a pre-damping ring and a main damping ring for each particle species. The damping of the beam needs to be done in two stages, as a damping of the emittance by 4 order of magnitudes has to be done within the repetition frequency of 50 Hz, of the collider. The first step of damping is performed in the pre-damping rings (PDR), where a ring with large dynamic and momentum acceptance has to accept the large beam, especially the one coming from the positron source, and damp it down by two order of magnitudes, in order to fit to the acceptance of the main damping rings. In this chapter, the linear lattice design and non-linear optimization of a positron pre-damping ring, achieving the requirements of the design and presenting an adequate dynamic aperture, is presented.

6.1 Introduction

The CLIC Pre-damping rings provide the first stage of damping of the e^+/e^- beams of the collider. They have to accommodate a 2.86 GeV beam with a large input emittance of 7 mm-rad, especially for the one coming from the positron source [56], and damp it down to a normalized emittance of 63 $\mu\text{m-rad}$ for injection into the main DR. The required input and output parameters are given in Table 6.1 [6].

Table 6.1: Parameters before the injection to the pre-damping rings and before the injection to the main damping rings.

Parameters	Injected		Extracted
	e^-	e^+	e^-/e^+
Bunch Population [10^9]	4.7	6.4	4.4
Bunch spacing [ns]	0.5/1	0.5/1	0.5
Bunches/train	312/156	312/156	312/156
Number of trains	1/2	1/2	1/2
Repetition rate [Hz]	50	50	50
Norm. horiz. emittance [μ m-rad]	100	7×10^3	63
Norm. vert. emittance [μ m-rad]	100	7×10^3	1.5
Norm. long. emittance [keV-m]	2.86	2288	143

Due to the characteristics of the injected beam and the requirements for the extracted beam both an electron and a positron rings are needed. Without the positron ring the injected normalized emittance of 7 mm-rad and rms energy spread of around 4% will not fit into the DR.

The electron injected emittance, although 2 order of magnitudes lower, is still 1 order of magnitude higher than the DR input emittance and 5 orders of magnitude higher than the extracted emittance of around 5 nm-rad. The electron beam coming from the low energy linac injected directly to the DR, may not encounter aperture restrictions. However, without an electron PDR and considering the damping time of around 2 ms in the DR, the large linac emittance will need at least 16.5 ms to reach equilibrium (without the effect of IBS), reaching almost the repetition time of 20 ms. Due to the more difficult characteristics of the positron beam, emphasis is given to the design of the positron pre-damping ring.

Unlike the DR, the PDR lattice design is not driven by the emittance requirements. The large energy spread and the large beam size of the injected beam, especially the one coming from the positron source, impose the requirements of large momentum acceptance and dynamic aperture. Thus the PDR lattice design is focused on the dynamic aperture optimization, providing a large enough momentum acceptance and the required output emittance.

6.2 Linear lattice design

The PDRs were chosen to have a racetrack configuration with two arc sections and two long straight sections (LSS), as a racetrack shape is the most compact one if only 2 dispersion free regions are required, which is valid for the case of the CLIC PDRs. The arc sections are composed by TME cells, being the most compact low emittance cells. On the other hand, the LSS are composed by FODO cells filled with damping wigglers. The damping wigglers are necessary to achieve the low emittance in a fast damping time, in order to fit into the 50Hz repetition rate of the collider.

6.2.1 Arc optimization

The large input emittance and the large momentum spread, especially for the case of the positron ring, impose the requirement of large dynamic aperture (DA) and momentum acceptance. In this respect, the optimization of the cell is focused on the DA and momentum acceptance optimization. In the low emittance lattices, the main limitation of the DA comes from the non-linear effects induced by the strong sextupole strengths, which are introduced in the lattice for the chromaticity correction. The strong sextupoles are necessary, as strong focusing is needed to achieve the low dispersion and the very low emittance, resulting high chromaticities. Based on the analytical parameterization of the TME cells described in Chapter 5, low horizontal and vertical phase advances per cell are needed ($\mu_x, \mu_y < 0.5$) for low cell chromaticity. Those solutions correspond to large detuning factors and relaxed dispersion values in the middle of the dipole. Following the procedure described in Chapter 5, an iteration between drift scan and emittance scan, for a dipole field of $B_d=1.2$ T and a dipole bending angle of $\theta = 2\pi/38$, lead to the choice $(l_1, l_2, l_3) = (0.9, 0.6, 0.5)$ m and a total cell length of 5.31 m.

The final choice of phase advances, was based on the resonance free lattice concept [57] and is ($\mu_x=0.2941, \mu_y=0.1765$). A detailed description of the non-linear optimization and the phase advance choice is given in section 6.3.

The optical functions of the TME cell are shown in Figure 6.1, where the horizontal (black) and vertical (red) beta functions and the horizontal dispersion (green) along the cell are depicted.

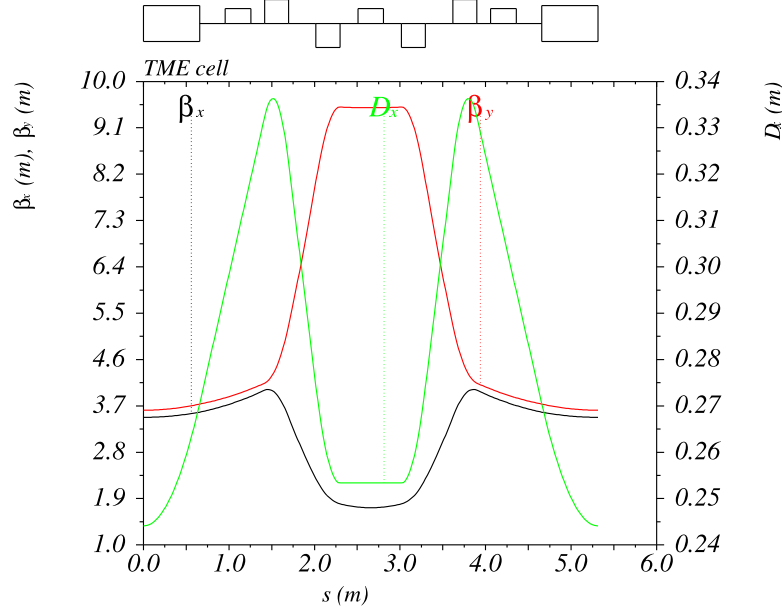


Figure 6.1: The optical functions of the TME arc cell of the PDR.

The top part of the figure, shows a schematic representation of the PDR TME cell. The entrance and the exit of the cell is the center of the dipole magnets, in the same way as in Figure 5.1. The two doublets of quadrupoles, with the focusing quadrupoles looking up and the defocusing ones looking down, are located where the horizontal and vertical beta functions have their maximum values respectively. Two families of sextupoles are also shown, one consisting of two sextupoles, placed between the dipole magnet and the first quadrupole and in the mirror symmetric position (looking up), and the other placed at the symmetry point of the cell (looking down), between the two mirror symmetric defocusing quads.

6.2.2 Straight section

Using Eq. (2.59) for the emittance evolution with time, at least 5 damping times are needed to reach a vertical equilibrium emittance of $1.5 \mu\text{m-rad}$, if the injected emittance is 7 mm-rad . Considering that the damping of the beam should be done within the repetition time of 20 ms , a damping time of less than 4 ms is needed. Taking into account the dipole characteristics of the TME cell and using Eq. (2.56), the damping time is $\tau_x \approx 10 \text{ ms}$, a factor of 2.5 times larger than the required one.

Figure 6.2 (left) shows the dependence of the horizontal damping time on the dipole field. Even for the maximum field of 1.7 T , for a normal conducting dipole, the damping time is reduced to 7 ms , which is still larger than required. Figure 6.2 (right) shows the horizontal damping time, if damping wiggler magnets are also included, parameterized with the wiggler total length L_w and peak field B_w . Using high wiggler peak field and/or large total wiggler length, small damping times can be achieved. In order to keep the ring circumference as compact as possible, large wiggler peak field is preferred, with an upper limit at 2 T to stay in the normal conducting regime for the wiggler technology. As a conclusion, the use of damping wigglers is necessary for

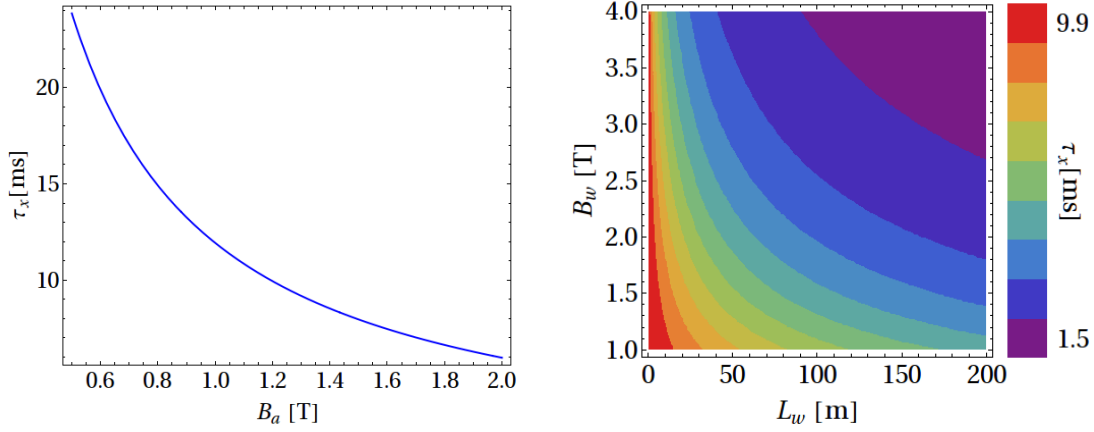


Figure 6.2: Left: Horizontal damping time with respect to dipole field. Right: Parameterization of the horizontal damping time with the wiggler total length and peak field.

the fast damping of the emittance by 2 orders of magnitude, in the repetition rate of 20 ms.

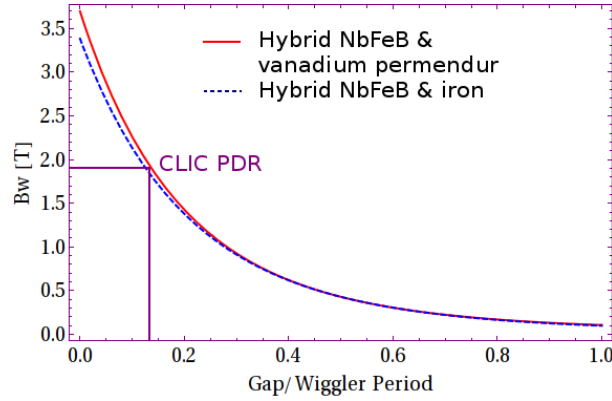


Figure 6.3: Peak field versus gap/period approximated by Halbach equation for a Hybrid NdFeB & vanadium permendur and a Hybrid NdFeB & iron wiggler.

According to Halbach [58], the peak magnetic field B_w , is related with the gap g and wiggler period λ_w , according to the fit:

$$B_w = a \exp\left[b \frac{g}{\lambda_w} + c \left(\frac{g}{\lambda_w}\right)^2\right],$$

where both B_w and a are expressed in units of Tesla, while b and c are dimensionless. These parameters depend on the wiggler performance and the materials used in the magnet construction. The coefficients a , b , c have been computed by Elleaume [59] for different materials, using a 3D magneto-static code. Figure 6.3 shows the peak field versus gap/period, approximated by Halbach equation, for two different materials, Hybrid NbFeB & vanadium permendur (red) and Hybrid NbFeB & iron (blue). Those materials are used in the Hybrid permanent magnet wigglers and they can achieve peak fields higher than 1.7 T, which is the limit for the nominal

electromagnets or pure permanent magnets. In the case of the CLIC PDR a wiggler gap of 41 mm is needed, in order to fit the large incoming positron beam. For a peak field of 1.9 T the gap/period has to be 0.13, corresponding to a wiggler period of 30 cm.

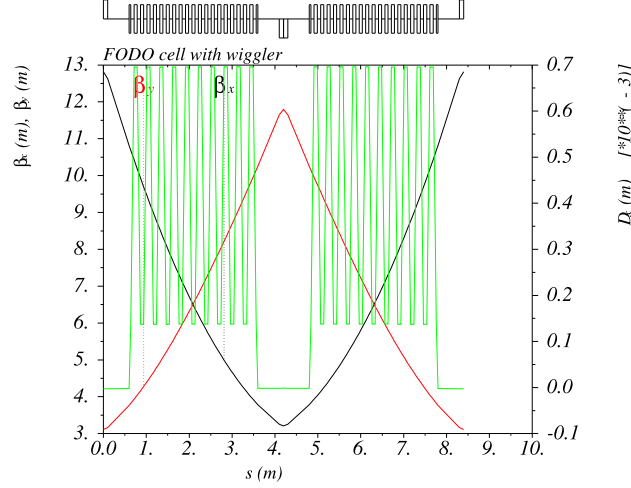


Figure 6.4: The optical functions of the FODO cell of the straight section, filled with damping wigglers.

The damping wigglers need to be placed in a dispersion free region, as the existence of dispersion in a wiggler region can lead to emittance blow up (see Chapter 2, sec. 2.4). Thus, the wiggler magnets are placed in the FODO cells of the dispersion free long straight sections (LSS). Two wigglers of 3 m each are placed in every FODO cell, of phase advances close to 90° . Each straight section consists of 13 FODO cells and the total number of wigglers is 52 with a total wiggler length of 108 m. This total number of wigglers is necessary to achieve the required output emittance.

As the wiggler does not exist as a standalone element in the design code MADX, a simplified model using alternating gradient dipoles separated by drifts is used, as described in section 2.4.1. The optical functions of the FODO cell is shown in Figure 6.4. The horizontal beta function is shown in black while the vertical one in red. The horizontal dispersion (green) along the wigglers is not zero, as the wigglers consist of opposite polarity dipoles, which generate and cancel dispersion. However, the dispersion created is very small ($D_x^{\max}=0.7$ mm) and does not have a negative impact on the final emittance. On the top part of the figure, the configuration of the FODO cell is depicted. The entrance and exit of the cell is the middle of the focusing quadrupole. A defocusing quadrupole is placed in the symmetry point of the cell. The wiggler elements are placed in the drift spaces between the quadrupoles.

A detailed description for the CLIC PDR and main DR wiggler design can be found in [60].

6.2.3 Dispersion suppression and matching section

The optical functions between the arcs and the straight sections are matched with the dispersion suppression - beta matching (DS-BM) sections, as was discussed in Chapter 2, sec. 2.6. For

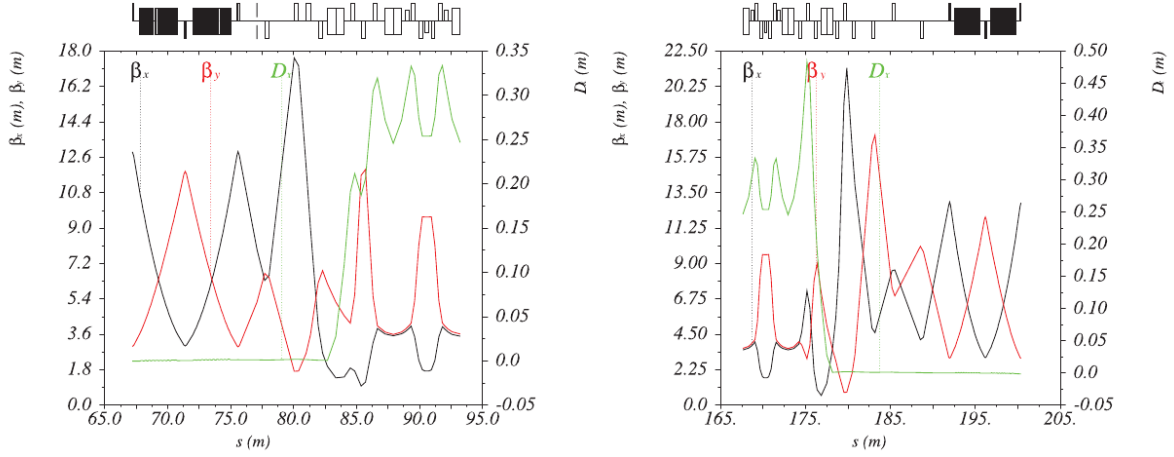


Figure 6.5: Optical functions of a TME cell, the two dispersion suppressor-beta matching sections and the FODO cell. Space is reserved for RF cavities in the DS-BM section in the left while for injection/extraction elements in the DS-BM section in the right plot.

the PDR lattice, two different DS-BM sections are used, upstream and downstream of each wiggler section, with space reserved for the injection/extraction elements upstream and the RF cavities downstream the wiggler section. The optical functions of the two dispersion suppressors, combined with the TME cell and the FODO cell optics, are shown in Figure 6.5. In the first DS-BM cell (left) seven quadrupoles are used, where the extra quadrupole is used for flexibility. In the second DS-BM cell (right) nine quadrupoles are used, as more space needs to be reserved for the injection - extraction elements.

6.2.4 RF cavities

Two RF options are under consideration for the CLIC DR complex. A 2 GHz RF system, in order to achieve the bunch spacing of 0.5 ns, presents a big technological challenge. In this respect, it was decided to consider a 1 GHz RF system and two bunch trains with 1 ns bunch spacing. This system is more conventional and an extrapolation from existing designs is possible. Nevertheless, the trains have to be recombined in a delay loop downstream the DRs [61]. A detailed description of the RF system design can be found in [62].

For large momentum acceptance (see Eq. (2.48)), large RF voltage and/or small momentum compaction factor are needed. Figure 6.6 shows the parameterization of the momentum compaction factor with the horizontal and vertical phase advances of the TME cell, based on the analytical parameterization. A small momentum compaction factor can be achieved, when the cell is tuned to high horizontal phase advances. In this regime, however, as was discussed in Chapter 5 (see sec. 5.2.2), the chromaticities get maximized, in both horizontal and vertical planes. That would require strong sextupole strengths for the chromaticity correction, leading to a small dynamic aperture. A compromise solution is thus used, with moderate α_p and large RF voltage. An RF voltage of 10 MV for the 2 GHz option can achieve an RF acceptance of $(\frac{\delta p}{p_0})_{\max} = \pm 1.2\%$, while for the 1 GHz option $(\frac{\delta p}{p_0})_{\max} = \pm 1.7\%$. Even for the $\pm 1.2\%$ acceptance, the e^+/e^- yield of the positron source is 0.453 which is within the limits, considering the

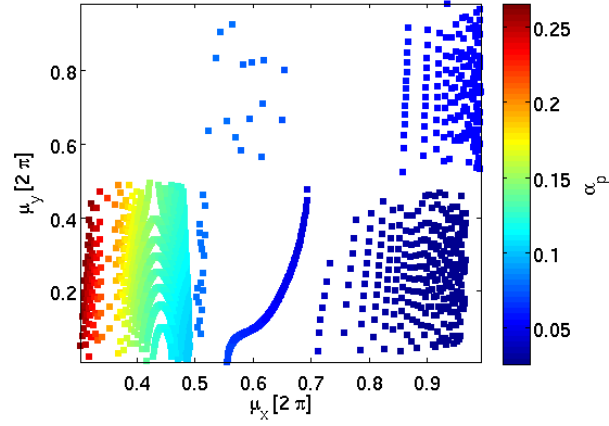


Figure 6.6: Parameterization of the momentum compaction factor, with the horizontal and vertical phase advances of the TME cell, based on the analytical parameterization.

target survivability of the positron source [63].

Four RF stations of 3 m each are required in order to provide the 10 MV voltage. Space is reserved in the dispersion free region downstream the wiggler sections.

6.3 Non linear optimization

The inclusion of sextupoles in the lattice introduces non-linear terms in the Hamiltonian of the system, as discussed in Chapter 2 (see section 2.7.1). Substituting into the sextupole Hamiltonian of Eq. (2.98), the equations of motion [18]:

$$\begin{aligned} x &= \sqrt{2J_x\beta_x(s)} \cos \phi_x(s) + D_x(s)\delta p/p_0, \\ y &= \sqrt{2J_y\beta_y(s)} \cos \phi_y(s), \end{aligned} \quad (6.1)$$

and integrating around the ring, the Hamiltonian can be represented by a sum over the betatron phase advances at the sextupoles' location:

$$\begin{aligned} \int_{\text{cell}} H_3(s)ds &= \sum h_{\text{ijklmp}}, \quad \text{with} \\ h_{\text{ijklmp}} &\propto \sum_n^{N_{\text{sext}}} (b_3L)_n \beta_{xn}^{\frac{j+k}{2}} \beta_{yn}^{\frac{l+m}{2}} D_n^p e^{i[(j-k)\mu_{xn} + (l-m)\mu_{yn}]}. \end{aligned} \quad (6.2)$$

Due to the phase arguments, the Hamiltonian modes h_{ijklmp} present a resonance behavior over many turns. Considering a sequence of identical cells, the sum can be easily calculated. The optics functions, β_x and β_y , have identical values at homologous places inside each cell and the phase advances in the $(p+1)^{\text{th}}$ cell are given by: $\mu_{x,p+1}(s) = \mu_{x,0}(s) + p\mu_{x,c}$ and $\mu_{y,p+1}(s) = \mu_{y,0}(s) + p\mu_{y,c}$, if the longitudinal coordinate s has its origin at homologous places inside each cell. $\mu_{x,c}$ and $\mu_{y,c}$ are the phase advances per cell in both planes and they are constants. Making

a change of variable: $s \rightarrow s + pl_{\text{cell}}$, where l_{cell} is the cell length, the contribution of the $(p+1)^{\text{th}}$ cell to the sum, defined by Eq. (6.2), can then be written as the product of an integral, which is the same for all cells, and a phase term depending only on the index p and the phase advances per cell:

$$e^{ip(n_x\mu_{x,c} + n_y\mu_{y,c})} \int_{\text{cell}} \beta_x^{\frac{n_x}{2}} \beta_y^{\frac{n_y}{2}} e^{i(n_x\mu_{x,0} + n_y\mu_{y,0})} ds. \quad (6.3)$$

The driving term of a resonance associated with the ensemble of N_c cells vanishes, if the resonance amplification factor is zero [57]:

$$\left| \sum_{p=0}^{N_c-1} e^{ip(n_x\mu_{x,c} + n_y\mu_{y,c})} \right| = \sqrt{\frac{1 - \cos[N_c(n_x\mu_{x,c} + n_y\mu_{y,c})]}{1 - \cos(n_x\mu_{x,c} + n_y\mu_{y,c})}} = 0. \quad (6.4)$$

This is achieved if: $N_c(n_x\mu_{x,c} + n_y\mu_{y,c}) = 2k\pi$, provided the denominator of Eq. (6.3) is non zero, i.e.: $n_x\mu_{x,c} + n_y\mu_{y,c} \neq 2k'\pi$, with k and k' are any integers. From this, a part of a circular accelerator will not contribute to the excitation of any non-linear resonances, except of those defined by $n_x\mu_x + n_y\mu_y = 2k_3\pi$, if the phase advances per cell satisfy the conditions: $N_c\mu_x = 2k_1\pi$ and $N_c\mu_y = 2k_2\pi$, where k_1 , k_2 and k_3 are any integers. Prime numbers for N_c , which in our case is the number of TME cells per arc, are interesting, as there are less resonances satisfying both diophantine conditions simultaneously.

The nonlinear optimization of the CLIC PDR lattice was based on the resonance free lattice concept, described above. From Eq. (5.3) and using a dipole field of $B_d=1.2$ T, at least 19 dipoles are needed in order to achieve the required output emittance. From this, convenient numbers of N_c (number of dipoles per arc) are 11, 13 and 17, which means 26, 30 and 38 dipoles in the ring respectively, including the dispersion suppressors' dipole. Following the results from the analytical parameterization of the TME cells, small horizontal and vertical phase advances and large detuning factors are favorable, for low cell chromaticity. The largest number of cells is better for increasing the detuning factor between the required and the minimum emittance and the cancellation of a larger number of resonance driving terms. Finally, the option of $N_c = 38$ was followed.

In first order perturbation theory, the inclusion of a sextupole field in the Hamiltonian of the system, excites only third order resonances. However, strong sextupoles are usually needed to correct chromaticity, which in higher order perturbation theory, can excite higher order resonances [18]. In the case of the CLIC PDR, strong sextupole fields are introduced in the Hamiltonian of the system, for the chromaticity correction. Magnet fringe fields are also taken into account, where the dominant quadrupole fringe field terms are octupole-like, as discussed in Chapter 2 (see sec. 2.7).

For the calculation of the resonance driving terms, the ptc-normal module of the MADX code is used, taking into account dipole and quadrupole fringe fields. The calculations are performed for different phase advances of the TME cell, while the resonance driving terms are calculated, for all the lattice.

Figure 6.7 shows the dependence of the third order resonance driving terms, for which $(j - k)\mu_x + (l - m)\mu_y = n$ and $|j - k| + |l - m| = 3$, on the horizontal and vertical phase advances of the TME cell. Blue regions correspond to small resonance excitation, while red regions indicate maximum excitation. Comparing the 5 Hamiltonian modes, the (2,1,0,0) mode is almost

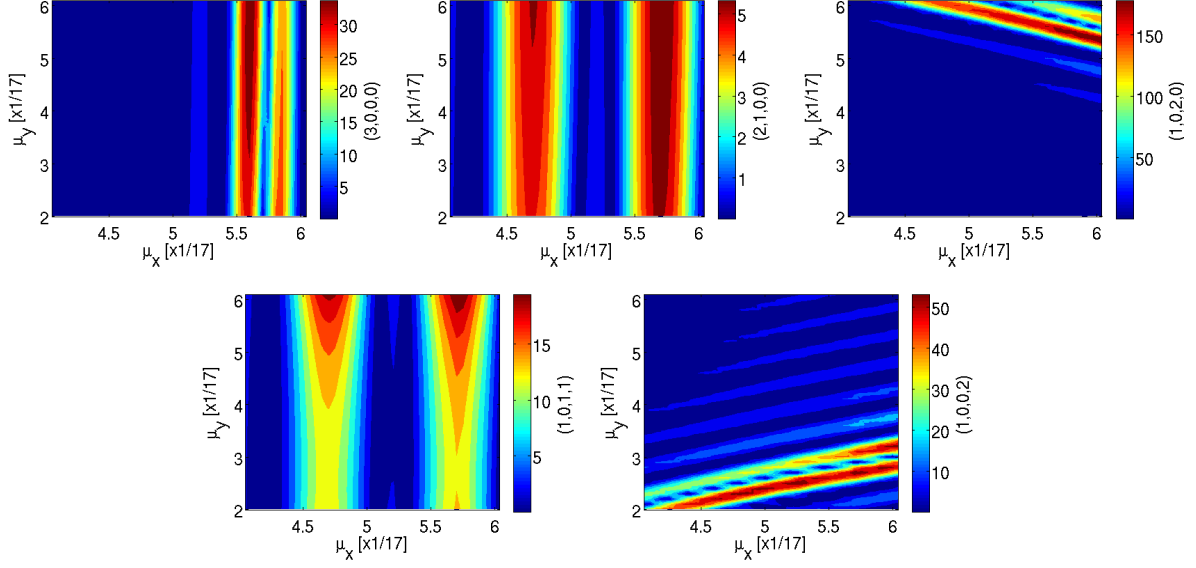


Figure 6.7: Horizontal and vertical phase advances of the PDR TME cell, parameterized with the third order Hamiltonian amplitudes.

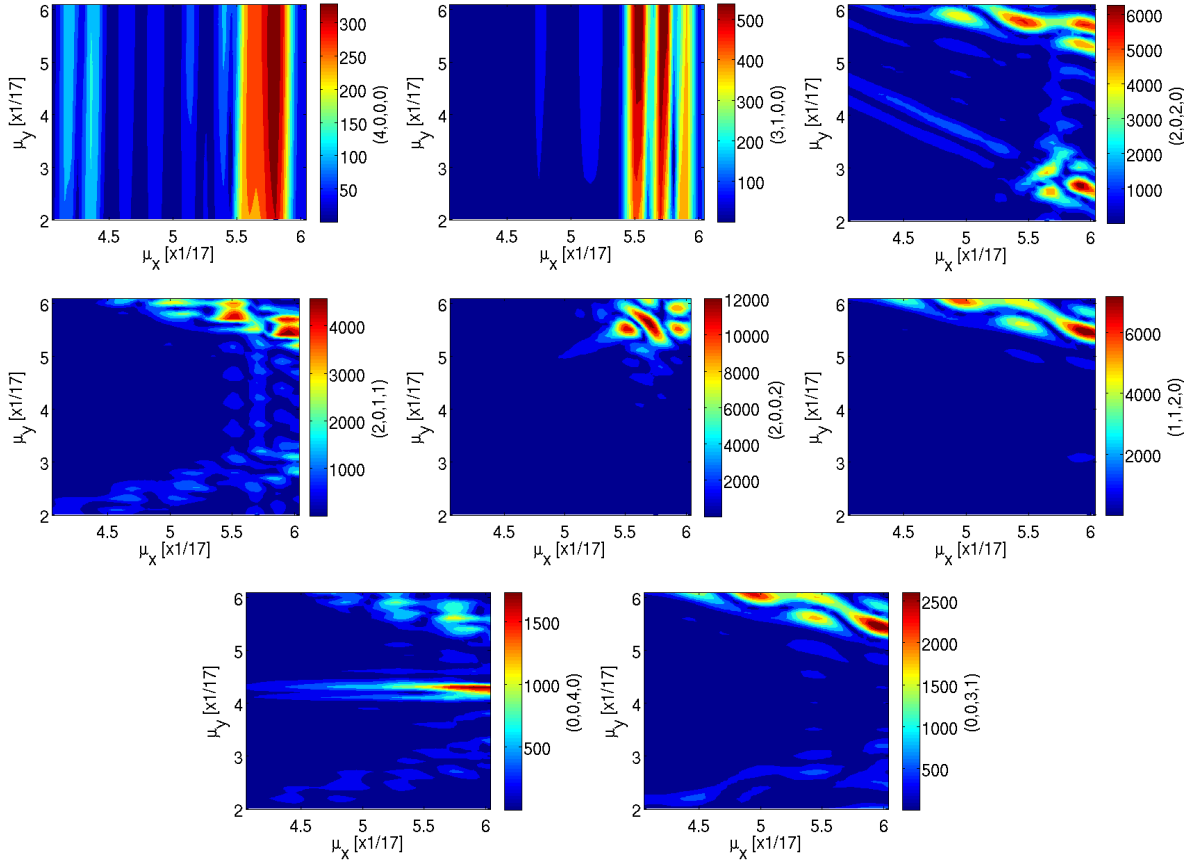


Figure 6.8: Horizontal and vertical phase advances of the PDR TME cell parameterized with the fourth order Hamiltonian amplitudes.

suppressed (a factor of 4-6 smaller than the other modes) and the (1,0,1,1) mode is weakly excited. The non-linear coupling term (1,0,2,0) is excited at high horizontal or high vertical phase advances. The horizontal mode (3,0,0,0) is also excited at high horizontal phase advances. In all cases, minimum excitation is observed, for integer multipoles of $1/17$.

Due to the fact that strong sextupoles are introduced in the PDR lattice, for the chromaticity correction, higher order resonances should also be considered. Figure 6.8 shows the dependence of the fourth order resonance driving terms, for which $|j - k| + |l - m| = 4$, on the horizontal and vertical phase advances of the TME cell. Maximum excitation is observed for the non-linear coupling terms (2,0,0,2), (2,0,2,0), (1,1,2,0) and (2,0,1,1), especially at the high horizontal or high vertical phase advance limit of the scan. The horizontal modes (4,0,0,0) and (3,1,0,0) are weakly excited with respect to the other modes. The vertical modes (0,0,4,0) and (0,0,3,1) are also excited, in the high horizontal phase advance limit for the first and in the high vertical phase advance limit for the second. All resonance driving terms are suppressed, for phase advances that are integer multipoles of $1/17$, as expected.

Here, the resonance driving terms are presented and discussed only to demonstrate the proof of principle of the resonance free lattice concept. In a further non-linear optimization of the lattice, especially when high-order magnet errors are included, additional families of sextupoles, in non-dispersive areas, can be used for the minimization of the resonance driving terms which limit the dynamic aperture.

Another quantity that has to be taken into account, is the amplitude dependent tune shift $\delta q_{x,y}/\delta J_{x,y}$. If the system is linear, the motion of the particles, with any initial conditions, will always give the same betatron frequency or tune. On the other hand, for a non-linear system, particles with different initial conditions will oscillate with different tunes and, in particular, particles at large amplitudes will have the largest deviation from the nominal tune, as they are more sensitive to the non-linearities of the system. From first order perturbation theory, the tune shift can be represented by [20]:

$$\begin{pmatrix} \delta q_x \\ \delta q_y \end{pmatrix} = \begin{pmatrix} \alpha_{hh} & \alpha_{hv} \\ \alpha_{vh} & \alpha_{vv} \end{pmatrix} \begin{pmatrix} 2J_x \\ 2J_y \end{pmatrix}, \quad (6.5)$$

where, α_{ij} are called the normalized anharmonicities and they describe the variation of the tune at different amplitudes (or action).

Figure 6.9 shows the dependence of the horizontal (top, left) and vertical (top, right) detuning with amplitude, δq_x and δq_y respectively, on the horizontal and vertical phase advances. The bottom plots show the parameterization of the factor $\delta q = \sqrt{\delta q_x^2 + \delta q_y^2}$ (left) and the fifth radiation integral I_5 (right), which is an equivalent to the horizontal emittance, with the horizontal and vertical phase advances. The amplitude dependent tune shift gets larger for large phase advances, while the emittance follows the opposite behavior. For this reason a compromise solution is chosen, where the horizontal emittance is reached for a small (but not minimum) detuning with amplitude. The optimal solution was chosen to be $\mu_x = 5/17$ and $\mu_y = 3/17$. With this choice, a compromise is achieved, for exciting the smallest number of resonances and achieve a rather small amplitude detuning and chromaticity, staying within the output emittance requirements of the design.

However, numerology shows that for this choice of phase advances, the non-linear fifth order coupling resonance driving terms are excited, for $|j - k| = 1$ and $|l - m| = 4$. In this case,

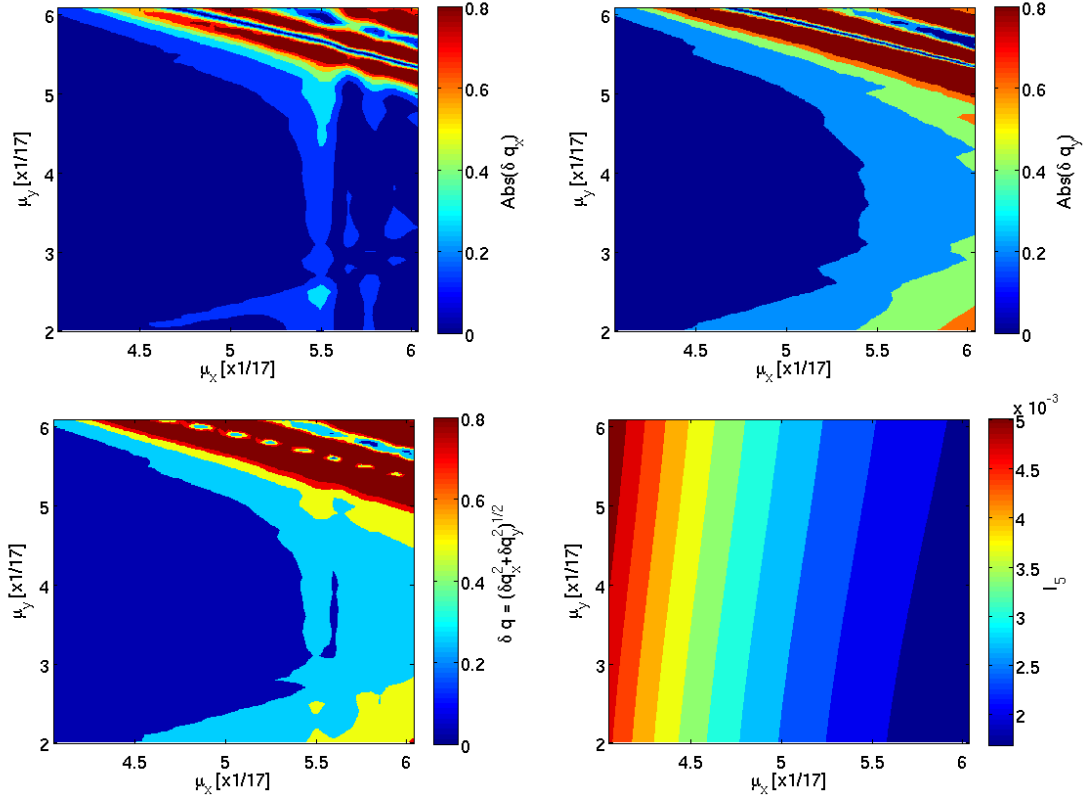


Figure 6.9: Top: Parameterization of the horizontal (top, left) and vertical (top, right) amplitude dependent tune shift, with the horizontal and vertical phase advances of the TME cell. Bottom: Parameterization of the square root of the quadratic sum of the horizontal and vertical amplitude dependent tune shifts (left) and of the fifth radiation integral (right), with the horizontal and vertical phase advances of the TME cell.

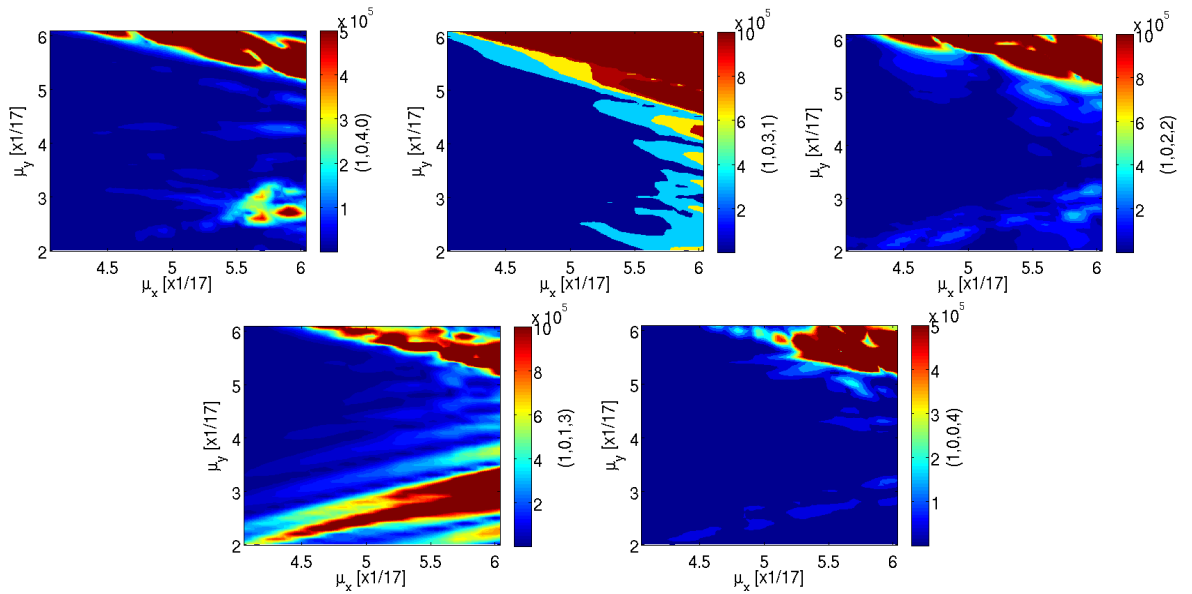


Figure 6.10: The fifth order resonance driving terms for which $|j - k| = 1$ and $|l - m| = 4$.

$\mu_x + 4\mu_y = 5/17 + 4 \times 5/17 = 1$. The five modes are presented in Figure 6.10, with the (1,0,1,3) mode being the dominant for $\mu_x = 5/17$ and $\mu_y = 3/17$. The other terms get excited for higher vertical phase advances.

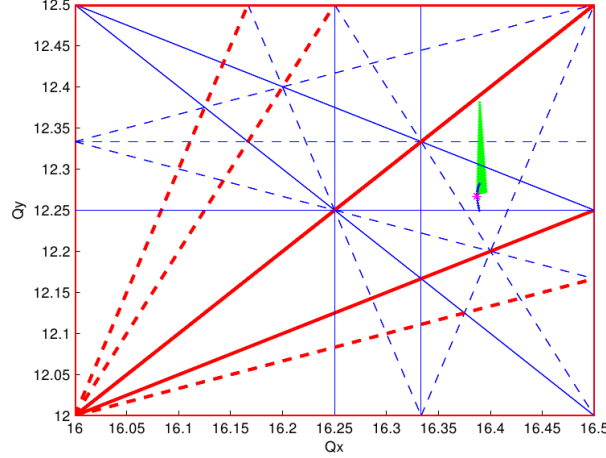


Figure 6.11: The working point in tune space for $\delta p/p$ from -1.2% to 1.2% (blue) and the first order tune shift with amplitude up to 6σ (green). The on-momentum working point is (16.39, 12.27).

For the chromaticity correction, four families of sextupoles are used. In order to minimize the sextupole strengths, and for efficient chromaticity correction, the sextupoles should be placed at places with large β_x and $\beta_y \ll \beta_x$, for $K > 0$, and with large β_y and $\beta_x \ll \beta_y$, for $K < 0$ (see sec. 2.1.2). A set of sextupoles are located before the focusing quadrupoles of the TME cells and a set of sextupoles after the defocusing ones. The same set-up is followed for the two other families of sextupoles, which are placed in the half TME cells of the dispersion suppressors. As those sextupoles are not placed in dispersive areas, they do not contribute to the chromaticity correction, but they can be used for further non-linear optimization of the lattice.

The change in the particles betatron frequencies, due to the non-linearities of the accelerator, can lead to the crossing of resonance lines in the tune diagram, defined by Eq. (2.103). This results in beam emittance blow up or in beam loss, thus, a careful choice of the betatron tunes of the linear lattice is very important for the beam quality and the beam life time. In the CLIC PDR lattice, the betatron tunes are controlled by the quadrupoles of the long straight section FODO cells. Figure 6.11 shows the working point in tune space for momentum deviations $\delta p/p_0$ from -1.2% to 1.2% (blue) and the first order tune shift with amplitude (green) up to $6 \sigma_{x,y}$. The on-momentum working point of the linear lattice is $(Q_x, Q_y) = (16.39, 12.26)$.

6.3.1 Dynamic aperture

The Dynamic aperture (DA) is defined as the maximum phase-space amplitude within which particles do not get lost as a consequence of single-particle effects [11]. The DA has to be at least equal or larger than the minimum beam transverse acceptance, $R_{\min}$. The beam coming from the positron source is not expected to be Gaussian, and the distribution in the storage ring

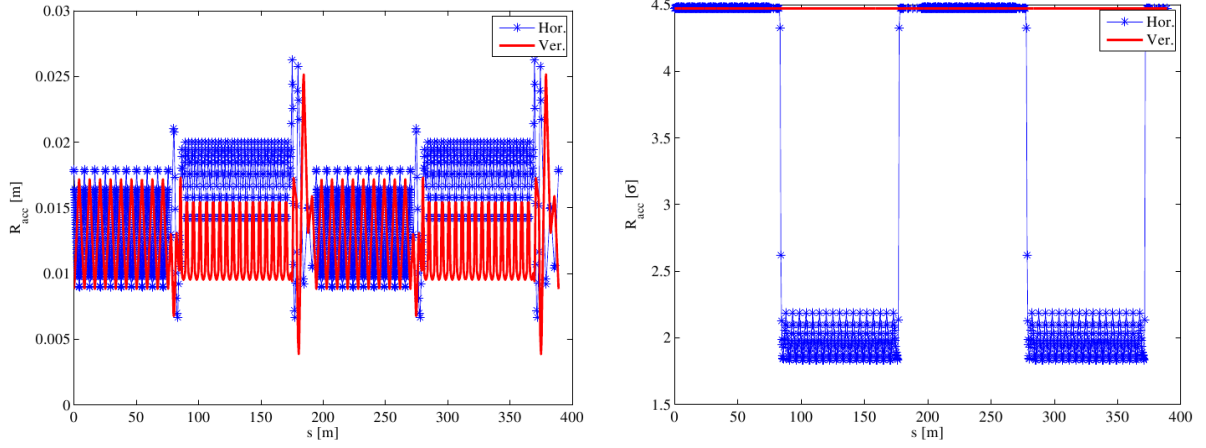


Figure 6.12: The required acceptance around the PDR in order to fit the positron beam in units meters (left) and in units of beam sizes (right).

is not modified, until the beam is damped close to equilibrium. For this reason, the minimum transverse acceptance is defined in terms of a maximum emittance $\varepsilon_{\max}$ of the particles with the maximum betatron action in the beam, and of a maximum relative momentum deviation $(\delta p/p_0)_{\max}$ [54]:

$$R_{\min} = \sqrt{2\beta\varepsilon_{\max}} + D(\delta p/p_0)_{\max}. \quad (6.6)$$

The incoming beam to the CLIC PDR is a round beam with same horizontal and vertical rms emittances of $\varepsilon_{x,y}^{\text{rms}} = 7$ mm-rad where, 99.9 % of the particles are inside a maximum emittance of $\varepsilon_{\max} = 10\varepsilon_{x,y}^{\text{rms}}$ and with maximum $(\delta p/p_0)_{\max} = 3\%$. Applying this to Eq. 6.6, the minimum acceptance can be calculated around the ring and is shown in Figure 6.12, in units of [m] (left) and in units of beam sizes $[\sigma]$ (right). A minimum DA of $4.5\sigma_{x,y}$ is required, in both horizontal (blue) and vertical (red) planes, in order to fit the large non-Gaussian beam coming from the positron source.

The DA of the ring was computed with numerical particle tracking, over 1000 turns, with the ptc module of MADX [14]. For lepton machines, if a particle survives for 1000 turns, its amplitude will have decreased, due to radiation damping, and is likely to survive over all the store time of the beam. Thus, for electron machines, the DA can be well approximated by the aperture within which particles survive for $\sim 15\%\tau_{\text{damp}}$ [11].

Figure 6.13 shows the initial positions of particles that survived over 1000 turns, normalized to the horizontal and vertical beam sizes, at the point of calculation ($\sigma_x = 4$ mm, $\sigma_y = 2$ mm). The results for $\delta p/p_0 = 0\%$ are shown in red, for $\delta p/p_0 = 1.2\%$ in green and for $\delta p/p_0 = -1\%$ in blue. The minimum acceptance is shown in black. For these calculations the magnet fringe fields are taken into account, while any magnet error effects are neglected. An adequate but tight dynamic aperture is demonstrated, following an optimization procedure based on the resonance free lattice concept, however, more optimization steps will be required when magnet errors and the effect of wigglers will be included.

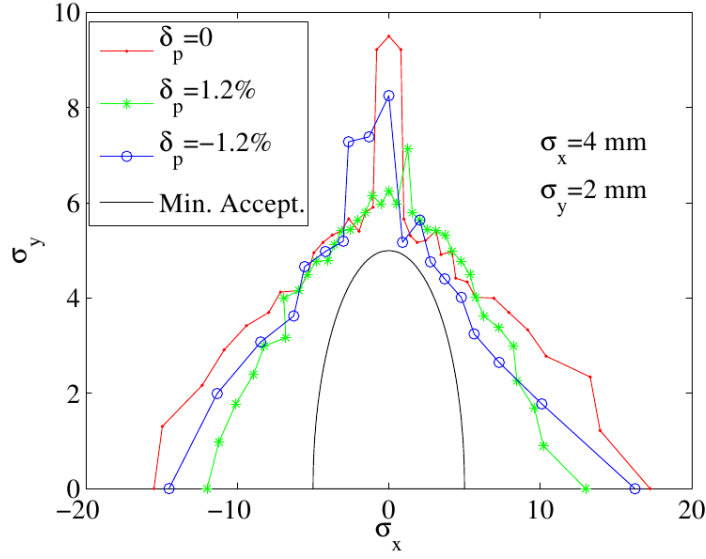


Figure 6.13: The on and off momentum Dynamic Aperture of the PDR for $\delta_p = 0$ (red), 1.2 % (green) and -1.2 % (blue).

6.3.2 Frequency maps

The frequency map analysis (FMA) examines the dynamics in frequency space rather than configuration space. Regular or quasi-regular periodic motion is a fixed point in frequency space characterized by a frequency or tune value. Irregular trajectories exhibit diffusion in frequency space, with the tunes changing in time. The mapping of configuration space (x & y) to frequency space (Q_x & Q_y) will be regular for regular motion and irregular for chaotic motion. Numerical integration of the equations of motion, for a set of initial conditions (x, y, x', y') and computation of the frequencies as a function of time (or turn number), constructs the map from the space of initial conditions to frequency or tune space, over a finite time span T [64]. An indication of how much the frequency is changing with time, is measured through the diffusion coefficient, defined by:

$$\mathcal{D} = \log \sqrt{(Q_{x1} - Q_{x2})^2 + (Q_{y1} - Q_{y2})^2} \quad (6.7)$$

where the index 1 refers to a certain number of turns, while, the index 2 to a consecutive same amount of turns. Large negative values of $\mathcal{D}$ denote long term stability while values of $\mathcal{D}$ close to zero denote chaotic motion [64].

Tracking of particles with different initial conditions for 1024 turns, was performed with MADX-PTC [65]. The ideal lattice including sextupoles and fringe fields is used, while no magnet errors are taken into account. The frequency map analysis was performed with the Numerical Analysis of Fundamental Frequencies (NAFF) algorithm [64].

Figure 6.14 (left) shows the initial positions of particles survived over 1024 turns, color-coded with the diffusion coefficient of Eq. (6.7), for on-momentum particles with $\delta p/p_0 = 0\%$ (top) and for off-momentum particles with $\delta p/p_0 = 1.2\%$ (middle) and $\delta p/p_0 = -1.2\%$ (bottom). The particle positions in the horizontal and vertical axis are expressed in units of horizontal and vertical

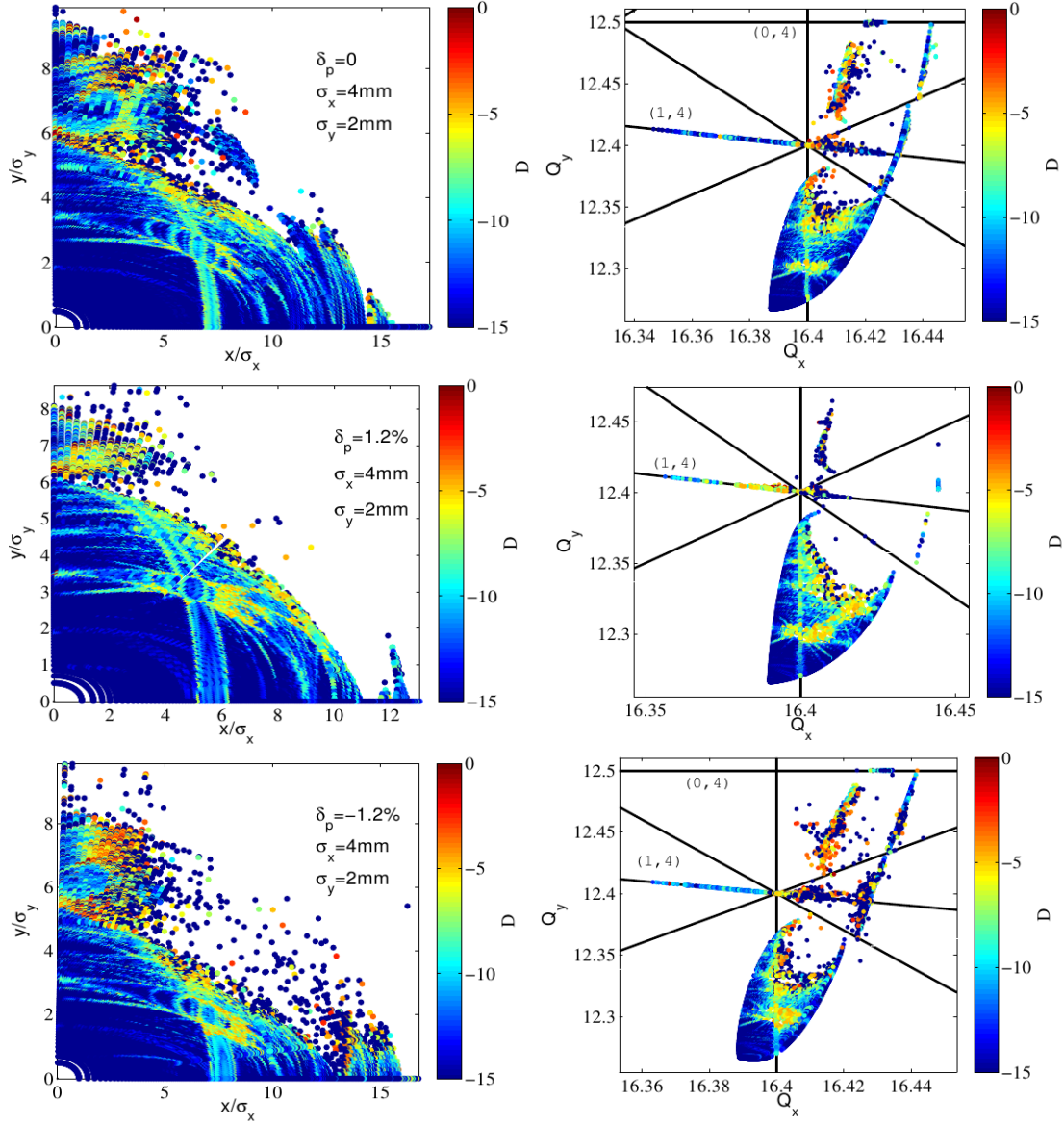


Figure 6.14: Diffusion maps (left) and frequency maps (right) for $\delta p/p=0$ (top), 1.2% (middle) and -1.2% (bottom).

beam sizes at the point of calculation, where $\sigma_x=4$ mm and $\sigma_x=2$ mm. The frequencies of the particles are presented in the right plots, called frequency maps. The color indicates the regularity of the orbits. Blue regions indicate very regular motion, while dark-red region indicate chaotic motion. The absence of dots means that the particles were lost. Resonance lines in the frequency maps are shown as distorted areas, while the colors allow to relate the resonant features observed, to regions of the physical space [64]. From the frequency maps it is observed that the tune is crossing the (1,4) resonance, which is not eliminated by the TME phase advance choice ($\mu_x = 5/17$, $\mu_y = 3/17$) as shown in Figure 6.10. This seems to be the main limitation of the DA.

The shape of the frequency maps, especially at high amplitudes, does not have the triangular shape expected by the linear dependence of the tune shift to the action, and they appear to be folded. This occurs when terms of higher order in the Hamiltonian become dominant over the quadratic terms as the amplitude increases [64]. This behavior occurs due to the suppression of the lower order resonances, following the resonance free lattice concept, which gives rise to higher order terms. Even though folded maps may lead to potentially very unstable designs, in our case this is not taken into account for the moment, as the folding of the map appears at high amplitudes, beyond the DA aperture limit.

6.4 Main magnet parameters and physical aperture

Table 6.2 summarizes the magnets' parameters of the PDR lattice. There are 38 main dipoles in one family, among which, 4 are located in the DS-BM section, for the dispersion suppression. There are 198 quadrupoles of three different types (i.e., lengths) for the arc, LSS and DS-BM sections. Their pole tip field is around 1 T. There are 110 sextupoles from which the 102 (2 families) are located in the arc and the remaining in the DS-BM section, with a pole-tip field of 0.8 T. All the magnets apart from the dipoles and wigglers have a circular aperture with 30 mm radius from the center to the magnet pole. The main bending magnets and wigglers have an elliptical aperture with vertical gaps of 30 and 41 mm, respectively.

Table 6.2: List of magnetic parameters for the CLIC PDRs.

Type	Location	Length [m]	Number	Families	Pole tip field [T]	Full aperture H/V [mm]
Dipoles	Arc	1.31	34	1	1.2	60/30
	DS-BM		4			
Quadrupoles	Arc	0.28	128	2	1.0	60/60
	LSS	0.20	36	2		
	DS-BM	0.35	32	16		
Sextupoles	Arc	0.3	68+34	2	0.5	60/60
	DS-BM		8	2		
Wigglers	LSS	3.00	36	1	1.9	60/41

The aperture radius, R [mm], for each element, is determined according to eq. 6.6. As shown on the right part of Figure 6.12, the required acceptance for both planes is everywhere below

Table 6.3: Basic PDR lattice parameters.

Parameters, Symbol [Unit]	Value (2 GHz)	Value (1 GHz)
Energy, E_n [GeV]	2.86	2.86
Circumference, C [m]	389.15	389.15
Bunches per train, N_b	312	156
Bunch population [10^9]	4.5	4.5
Bunch spacing, τ_b [ns]	0.5	1
Basic cell type	TME	TME
Number of dipoles, N_d	38	38
Dipole Field, B_a [T]	1.2	1.2
Tunes (hor./ver./sync.), $(Q_x/Q_y/Q_s)$	16.39/12.26/0.071	16.39/12.26/0.071
Nat. chromaticity (hor./vert.), (ξ_x/ξ_y)	-18.99/-22.85	-18.99/-22.85
Norm. Hor. Emit., $\gamma\varepsilon_0$ [mm·mrad]	53.89	53.89
Damping times, $(\tau_x/\tau_y/\tau_\varepsilon)$ [ms]	2.68/2.68/1.34	2.68/2.68/1.34
Mom. Compaction Factor, α_c [10^{-3}]	3.72	3.72
Energy loss/turn [MeV]	2.75	2.75
RF Voltage, V_{rf} [MV]	10	10
RF acceptance, ε_{rf} [%]	1.2	1.7
RF frequency, f_{rf} [GHz]	2	1
Harmonic number, h	2596	1298
Equil. energy spread (rms), σ_δ [%]	0.1	0.1
Equil. bunch length (rms), σ_s [mm]	3.2	4.6
Number of wigglers, N_{wig}	36	36
Wiggler peak field, B_w [T]	1.9	1.9
Wiggler length, L_{wig} [m]	3	3
Wiggler period, λ_w [cm]	30	30

30 mm which is the minimum magnet half gap for all magnets, apart the vertical wiggler. In that case it is 20.5 mm. In that area however, the vertical acceptance requirement is below 20 mm. Even though at present two identical PDRs are considered, the required acceptance for the electron PDR is smaller and thus could lead to lower magnet gaps, smaller magnets sizes and reduced cost and power consumption.

6.5 Layout and design parameters

The layout of the ring is shown in Figure 6.15 and the full optics of the ring in Figure 6.16. The performance parameters are summarized in Table 6.3 for both the 1 and 2 GHz options. The present design achieves the base line requirements for the output parameters and an adequate DA, although one could improved the DA by a further working point analysis. A necessary final step of the non-linear optimization, is the inclusion of non-linear errors in the main magnets and wigglers. The current design is the baseline design included in the CLIC Conceptual Design Report (CDR) [55].

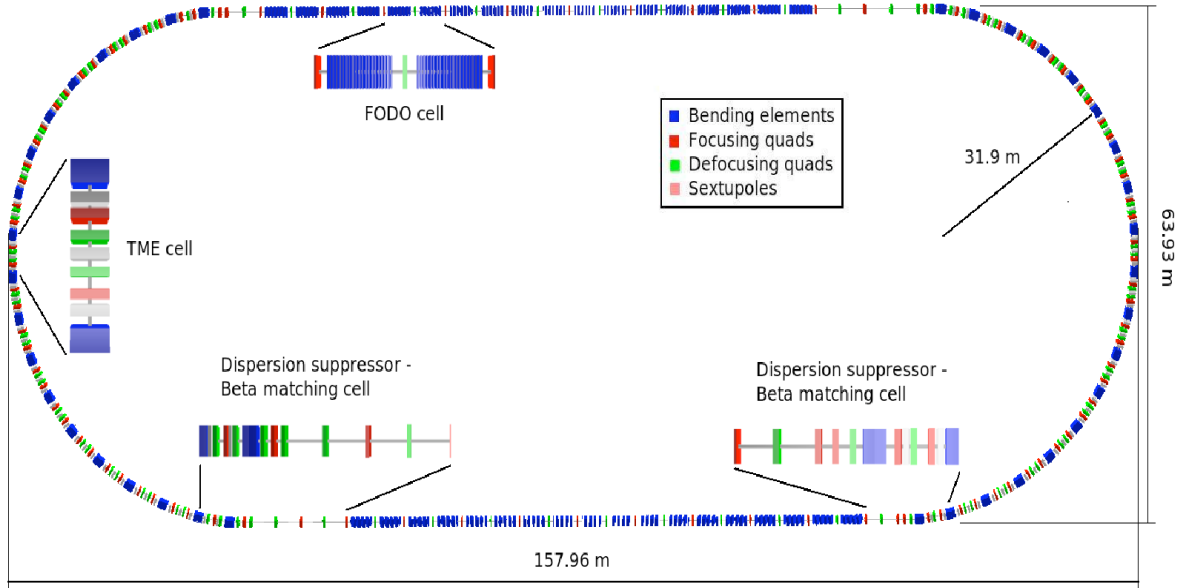


Figure 6.15: Schematic layout of the CLIC PDR.

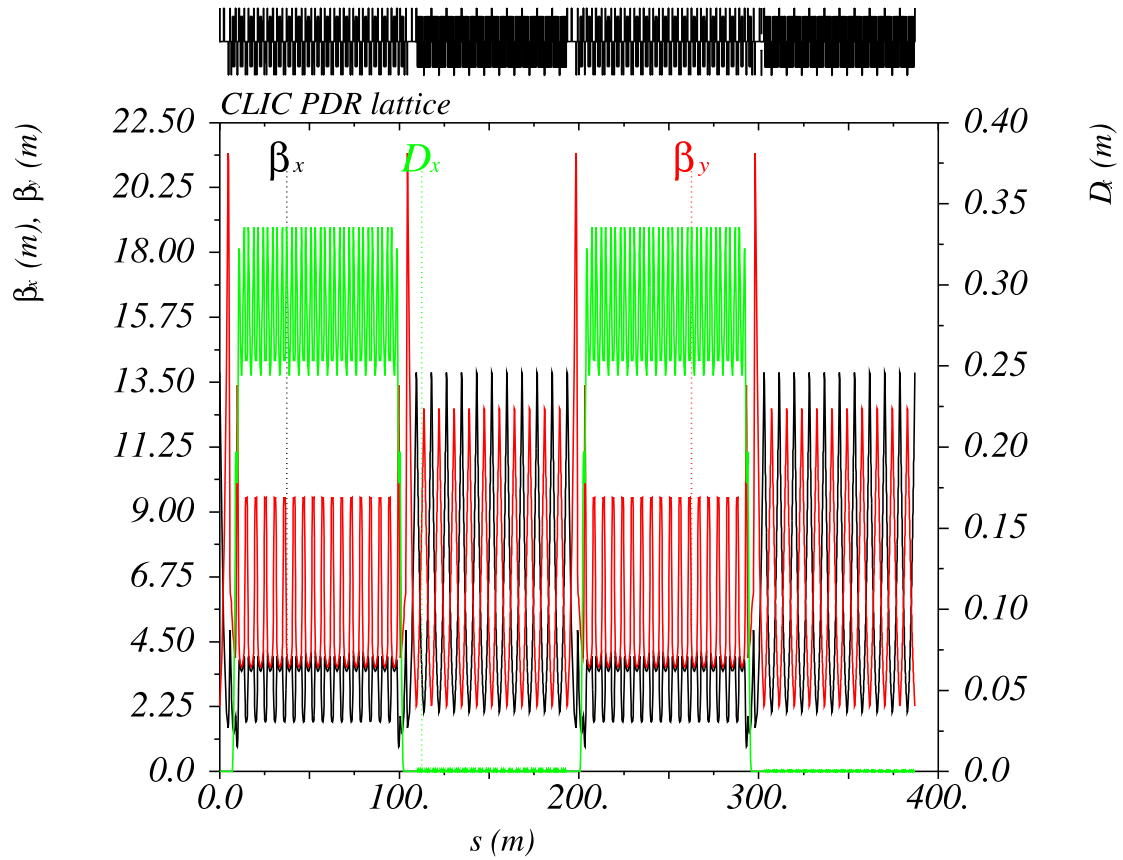


Figure 6.16: Optics functions of the full PDR.

Conceptual design of the CLIC Main Damping Rings

The role of the CLIC main damping rings (DR) is to provide the final stage of damping, in order to produce the required ultra-low emittance at a high bunch intensity and in a fast repetition time, following the luminosity requirements of the collider. The high bunch density and the large brightness of the beam, trigger a number of collective effects, with intrabeam scattering being the main limitation to the ultra-low emittance. In this Chapter, an optimized lattice design, with reduced intrabeam scattering emittance blow up and space charge tune shift is proposed, achieving the requirements of the collider and an adequate dynamic aperture. The possibility and limitations of using this design in low energy stages of CLIC is also discussed.

7.1 Introduction

The parameter requirements for the CLIC DR are summarized in Table 7.1. The bunch population of 4.1×10^9 particles, allows a 10 % margin with respect to the one required at the IP, taking into account transfer losses in the downstream systems. This high bunch intensity has to be delivered with ultra low horizontal and vertical emittances of 500 nm·rad and 5 nm·rad normalized to the beam energy, leaving also some blow-up margin. These low emittances, although unprecedented, are rapidly approached by modern light sources in operation (red rhombus) or in the construction phase (blue circles) [66], as shown in Figure 7.1. What indeed diversifies the required beam characteristics in the DRs is the very small longitudinal normalized emittance of 6 keV·m, which is imposed by the bunch compression requirements of the RTML (Ring To Main Linac), after the main DRs [67].

Table 7.1: Parameters before the injection and at the extraction of the main damping rings.

Parameters	Injected	Extracted
Bunch Population [10^9]	4.4	4.1
Bunch spacing [ns]	0.5/1	0.5/1
Bunches/train	312/15	312/156
Number of trains	1/2	1/2
Repetition rate [Hz]	50	50
Norm. horiz. emittance [nm·rad]	63 000	500
Norm. vert. emittance [nm·rad]	1500	5
Norm. long. emittance [keV·m]	143	6

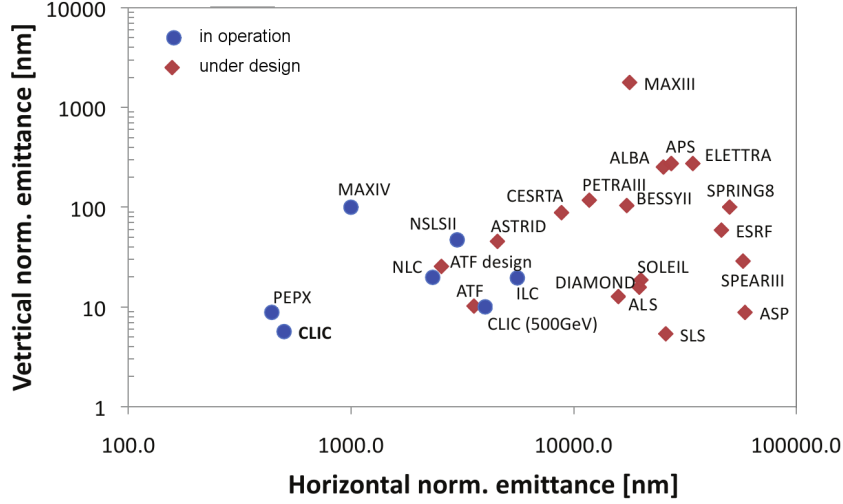


Figure 7.1: Horizontal versus vertical normalized emittance for low emittance rings in operation (red) and under design (blue)

The increased beam density, triggers a number of single bunch collective effects including Intra-beam Scattering (IBS), space-charge, Coherent Synchrotron Radiation (CSR) and Transverse Mode Coupling Instabilities (TMCI). Two stream phenomena such as ions or electron cloud build up can be amplified by the short bunch spacing of only 0.5 ns, making the vacuum technology and the photon absorption scheme quite demanding. The short bunch spacing is also creating a high peak current as seen by the RF system, which needs a very challenging low level system to cope with the beam loading transients. In addition, a high frequency (2 GHz) pulsed RF power source is not technologically available. Finally, the small emittance has to be extracted in a very stable and reproducible way imposing tight tolerances in the kicker system stability.

7.2 Previous Design stages

An original design of the CLIC damping rings is described in [68]. The lattice has a racetrack shape with 2 arcs, consisting of low-emittance TME cells and two long straight sections with FODO structure, to accommodate the damping wigglers, RF cavities, injection and extraction equipment. The repetition rate is at 50 Hz, while the energy was chosen to be 2.42 GeV [68].

Even though, this original design could demonstrate the parameters required by the performance of the linear collider in a very compact lattice (the circumference of the ring was only about 365 m), it had also some serious drawbacks. The lack of space between the magnetic elements to accommodate other accelerator components, serious problems with the evacuation of the high radiation power from damping wigglers and strong gradient of quadrupoles and sextupoles which can hardly be achieved in the frame of the normal-conducting magnet technology, provided an ideal but non-realistic solution. Another serious drawback is the very strong IBS effect which increase the emittance by a factor of 5.4.

Alternative solutions for the design were then proposed (V04 and V06) [69] with a larger ring circumference, achieving the same performance with more realistic technical parameters (V04).

A step further was the replacement of the nominal bending magnets with combined function dipoles (V06). The low emittance condition requires that only the horizontal betatron function should reach the minimum in the center of the bending magnets while the vertical one can be large. The defocusing gradient allows the increase of the vertical beta function in the center of the dipole, reducing like this the IBS growth rates and the final IBS emittance growth. Figure 7.2 shows the IBS growth rates (increments) for a nominal TME cell (left) and a modified TME cell with vertical gradient (right). The IBS growth factor in the second case is reduced, while the resulting output emittance is almost the same [69].

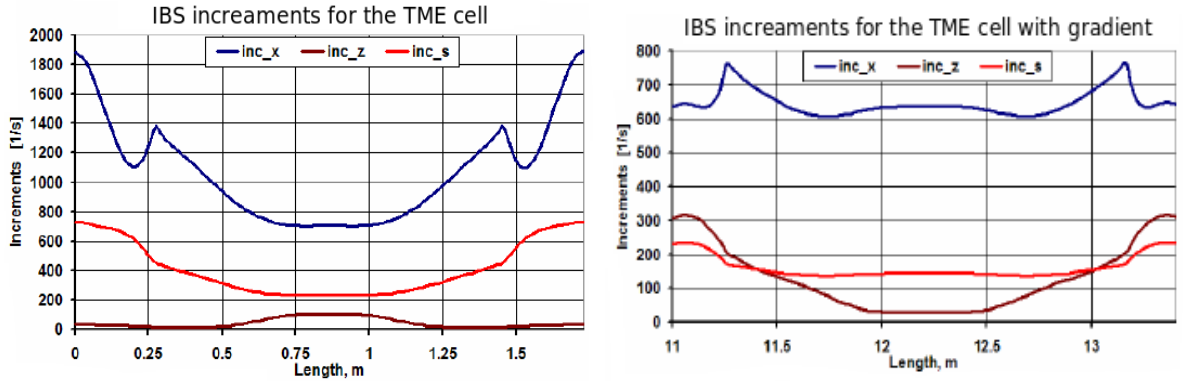


Figure 7.2: The IBS increments for V04 (left) and V06 (right) [69].

Table 7.2 summarizes the performance parameters of those 3 designs.

7.3 DR optimization

The two proposed optimized designs described above, even though achieving the performance parameters with realistic technical characteristics, are still strongly dominated by the IBS effect. The IBS growth factor in the best case is 2.9 (V06) while the circumference of the rings is long, increasing the cost of the DRs. In this chapter, an optimized design is proposed focused in the reduction of the IBS effect, but also taking into account the effect of space charge and the performance of the RF system.

7.3.1 Energy optimization

In the CLIC baseline, a polarized electron and an un-polarized positron beams are considered. Polarization is required for spin-dependent measurements at the collision point [70]. The requirement for polarized positron beams is relaxed due to technical difficulties of the positron source [71]. Therefore the ring energy should be chosen so that the spin tune is a half integer, to stay away from the strong integer spin resonances. This constrains the ring energy to:

$$\alpha\gamma = n + \frac{1}{2} \quad (7.1)$$

Table 7.2: Performance parameters of previous design stages of the CLIC Damping rings.

Parameters	Original	V04	V06
General			
Energy [GeV]	2.424	2.424	2.424
Circumference [m]	365.21	534.96	493.05
Longitudinal emittance [eV-m]	2713	2711	2716
Energy loss/turn [MeV]	3.86	3.98	3.98
RF voltage [MV]	4.38	4.35	4.60
RF harmonic (h)	2435	3569	3289
Energy Acceptance	0.0149	0.0128	0.0163
Natural chromaticity x/y	-102.6/-135.5	-186/-118	-148.8/-79.0
Momentum compaction factor	8.02×10^{-5}	4.56×10^{-5}	6.44×10^{-5}
Damping times x/y/s [ms]	1.5/1.5/0.8	2.2/2.2/1.1	1.99/2.0/1.01
Number of arc cells/wigglers	100/76	100/76	100/76
Phase advance per arc cell x/y	0.581/0.248	0.524/0.183	0.442/0.045
Dipole focusing strength $K_1[m^{-2}]$	0	0	-1.1
Dipole length [m]/field [T]	0.545/0.93	0.4/1.27	0.4/1.27
Without the IBS			
Normalized Hor. emittance [nm-rad]	86.3	121.4	148.0
Energy spread	1.12×10^{-3}	1.12×10^{-3}	1.12×10^{-3}
Bunch Length [mm]	0.919	0.848	0.951
Longitudinal Emittance [eV-m]	2493	2299	2584
With the IBS			
Bunch population	$4.1 \cdot 10^9$	$4.1 \cdot 10^9$	$4.1 \cdot 10^9$
Normalized Hor. emittance [nm-rad]	465	466	436
Emittance growth factor	5.4	3.8	2.9
Energy spread	1.57×10^{-3}	1.65×10^{-3}	1.56×10^{-3}
Bunch Length [mm]	1.3	1.25	1.32
Longitudinal Emittance [eV-m]	5000	5000	5000

where, $\alpha = 1.16 \times 10^{-3}$ is the anomalous magnetic moment of the electron (or positron) and n is an integer. The initial choice of energy was chosen for $n=5$ corresponding to the energy of 2.424 GeV.

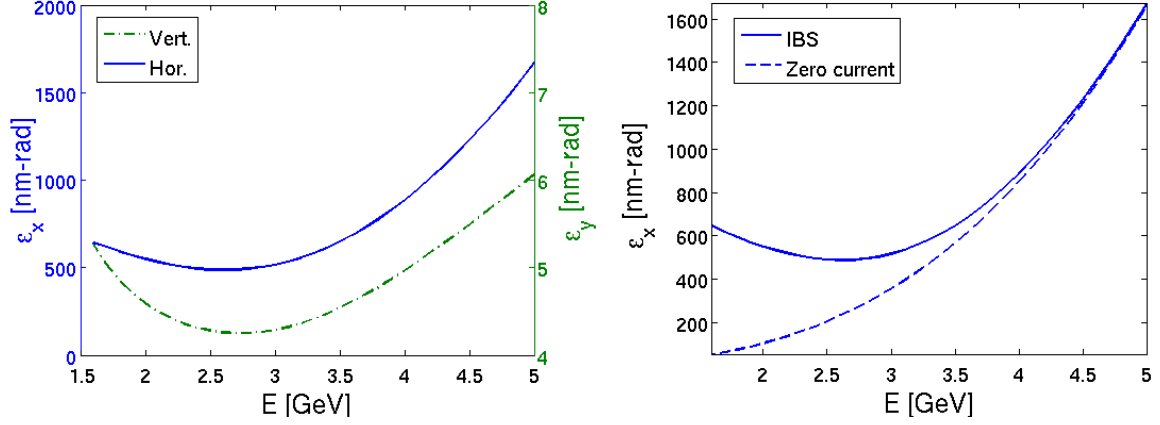


Figure 7.3: Scaling of the steady-state horizontal (blue) and vertical (green, dashed-dotted) emittances with energy, taking into account the effect of IBS (left) and comparison between the scaling of the steady-state (solid line) and zero current (dashed line) horizontal emittance with energy (right).

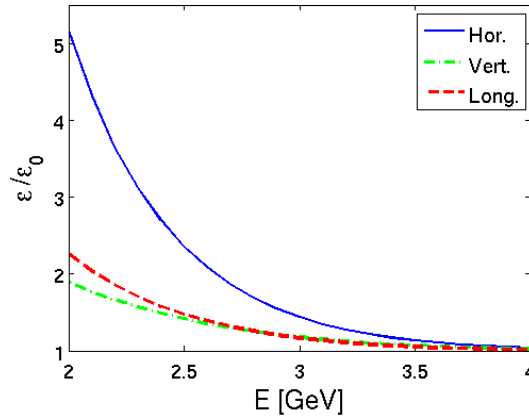


Figure 7.4: Dependence of the IBS effect (ratio between steady state and zero current emittances) on the energy, in the horizontal (blue), vertical (green, dashed-dotted) and longitudinal (red, dashed) planes.

Even though version V06 is focused on the IBS optimization, the output emittances are still strongly dominated by the IBS effect with an IBS emittance growth factor of 2.9. As the IBS growth rates but also the zero current equilibrium emittances depend on the energy of the beam, the scaling of the output emittance with the energy, taking into account the IBS effect, was evaluated. Figure 7.3 (left), shows the dependence of the steady state-state horizontal (blue,

solid) and vertical (green, dashed-dotted) emittances with the energy. In the right part of the figure, a comparison between the scaling of the steady-state (solid) and zero current (dashed) horizontal emittances with energy is presented. Figure 7.4 shows the scaling of the IBS effect, thus the ratio between the steady-state and zero current emittance, in the horizontal (blue, solid), vertical (green, dashed-dotted) and longitudinal (red, dashed) planes, with energy. A broad minimum is observed around 2.6 GeV for both horizontal and vertical emittances, while the IBS effect becomes weaker with the increase of energy. Although higher energies may be also interesting for reducing further collective effects, the output emittance is strongly increased due to the domination of quantum excitation. In this respect, it was decided to increase the DR complex energy from 2.424 GeV to 2.86 GeV. The new energy is close to a steady state emittance minimum but also reduces the IBS impact by a factor of 2 [46]. The absolute values of the steady-state emittances shown in the plots are slightly higher than the values quoted in the performance parameter tables, as for the scaling calculations the CIMP approximation was used, in order to reduce the computational time, while in the tables the full Piwinski formalism is used.

Reducing the IBS growth factor by a factor of 2, the requirement for the zero current horizontal emittance is also relaxed by a factor of 2. The number of wiggler magnets can then be reduced, reducing also the circumference of the ring. A number of 52 wiggler magnets (instead of 76) is sufficient to achieve the required output emittance. The optimization of the wiggler FODO cell is discussed in section 7.3.3.

7.3.2 Optimization of the arc TME cell

The intermediate design with increased energy and reduced number of wigglers, has three main weaknesses:

- The IBS effect, even though decreased with the increase of the energy to $\varepsilon_{x,IBS}/\varepsilon_{x0} \approx 1.5$, is still appreciable and dominates the output emittances and the performance of the DRs.
- The linear coherent space charge (or Laslett) tune shift, defined as (see section 2.8.3):

$$\delta Q_{x,y} = -\frac{N_b r_e}{(2\pi)^{3/2} \gamma^3 \sigma_s} \oint \frac{\beta_{x,y}}{\sigma_{x,y}(\sigma_x + \sigma_y)} ds,$$

is large in the vertical plane (~ 0.2), when the beam reaches steady state, and can lead to emittance growth and particle loss through resonance crossing.

- The RF stable phase, defined as (see section 2.1.3):

$$\phi_s = \arcsin\left(\frac{U_0}{V_0}\right) \approx 70^\circ,$$

is close to the crest of the sinusoidal field, where the field change with respect to phases is non-linear. This can lead to a non-linear bucket with reduced momentum acceptance (see Eq. (2.48)).

Considering the optics of the ring unchanged, for the same energy and target emittances, only the zero current bunch length can be increased in order to reduce the space charge tune

shift. On the other hand, for the reduction of the RF stable phase larger RF voltage is required, which leads to bunch length reduction, being in contradiction with the space charge tune shift decrease. In order to achieve a general optimization a re-tuning of the lattice is needed, as for the above considerations the optics of the ring is considered the same, while, optimizing the optics can overcome the above contradiction. For the moment, the wiggler working point remains the same as for the original design ($B_w=2.5$ T, $\lambda_w=5$ mm).

As was discussed in Chapter 2 (sec. 2.4), taking into account the wiggler contribution to the radiation integrals, the zero current equilibrium longitudinal emittance is defined as the product of the equilibrium energy spread and bunch length: $\varepsilon_l = \sigma_{p0}\sigma_{s0}$, where:

$$\sigma_{p0} = \gamma \left(\frac{B(C_q(1 + F_w \frac{B_w}{B}))}{(B\rho)(3 - (1 + F_w)\mathcal{J}_{x0} + 3F_w)} \right)^{1/2}, \quad (7.2)$$

$$\sigma_{s0} = \sigma_{p0} C \left(\frac{\alpha_p E}{2\pi h(V_0^2 - U_0^2)} \right)^{1/2}, \quad (7.3)$$

with $F_w = \frac{L_w B_w^2}{4\pi(B\rho)B}$.

The horizontal equilibrium emittance is also defined as:

$$\varepsilon_{x0} = \frac{C_q \gamma^3}{12(\mathcal{J}_x + F_w)} \left(\frac{\varepsilon_r \theta^3}{\sqrt{15}} + \frac{F_w B_w^3 \lambda_w^2 \beta_{xw}}{16(B\rho)^3} \right), \quad (7.4)$$

and the energy loss per turn as:

$$U_0 = C_\gamma \frac{E^4}{l_d} \theta \left(1 + \frac{L_w B_w^2}{4\pi(B\rho)B} \right). \quad (7.5)$$

Equation (7.3) shows that, in order to increase the bunch length, the momentum compaction

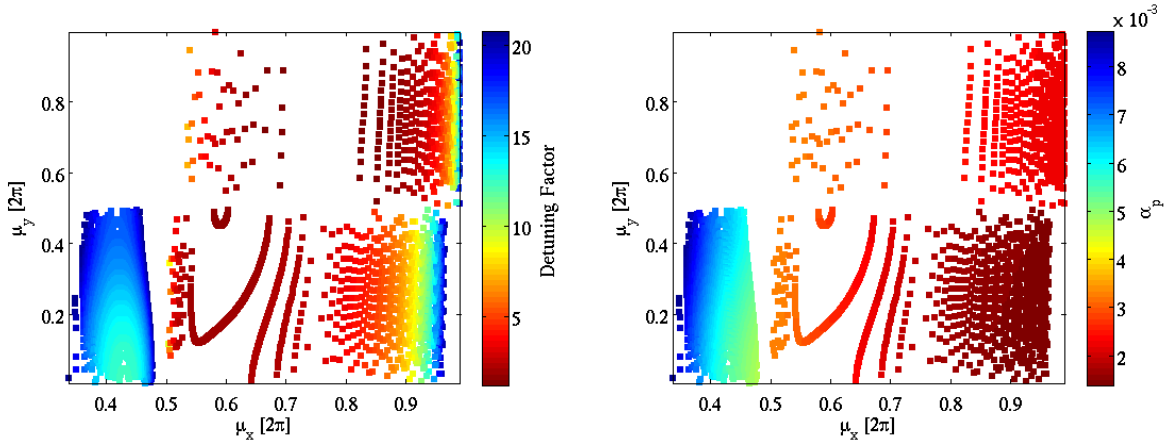


Figure 7.5: Parameterization of the detuning factor (left) and the momentum compaction factor (right), with the horizontal and vertical phase advances of the TME cell, based on the analytical parameterization.

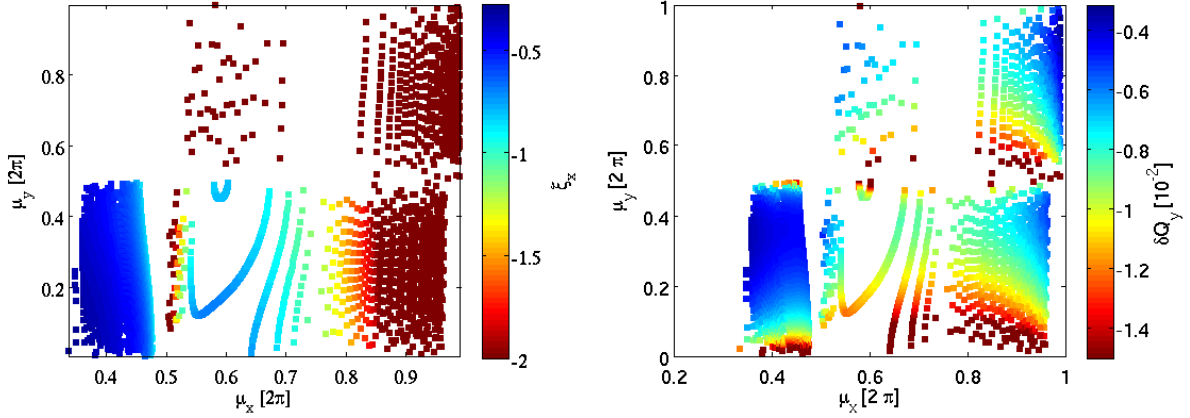


Figure 7.6: Parameterization of horizontal chromaticity (left) and the Laslett tune shift (right), with the horizontal and vertical phase advances of the TME cell, based on the analytical parameterization.

factor of the lattice has to be increased. Based on the analytical solution for the TME cell (see Chap. 5), the parameterization of the cell detuning factor (left) and the momentum compaction factor (right), is presented in Figure 7.5. The emittance, in this case, is defined only by the dipoles of the TME cell as the wiggler contribution to synchrotron radiation is not taken into account. For all solutions presented here, the same dipole length is considered. A smaller horizontal phase advance would increase the momentum compaction factor and thus the bunch length, while it has a very weak dependence on the vertical phase advance. At the same time, the horizontal chromaticity and the Laslett tune shift are also decreased, as shown in Figure 7.6. Those observations strengthen the initial statement, that a retuning of the lattice is necessary in order to overcome the weaknesses of the previous design, and the contradictions from a change in the RF voltage without changing the lattice parameters.

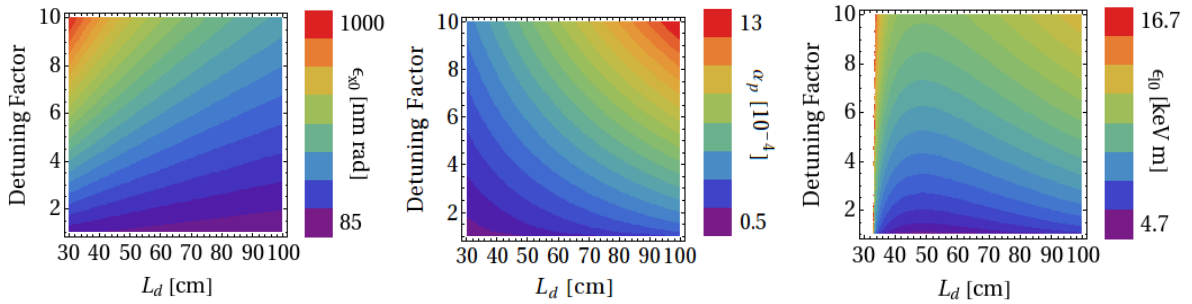


Figure 7.7: Equilibrium horizontal emittance (left) and momentum compaction factor (right) dependence on the dipole length and the cell detuning factor.

Figure 7.7 shows the parameterization of the horizontal emittance (left), the momentum compaction factor (middle) and the longitudinal emittance (right) with the dipole length and the cell detuning factor, based on Eqs. (7.2)-(7.5). Even though moving to larger detuning

factors, the output horizontal emittance can remain unchanged, if the dipole length is also increased (or the dipole field decreased). This is due to the fact, that the relative damping factor F_w , being inversely proportional to the dipole field (see Eq. (2.75)), gets increased. The output emittance then gets more wiggler dominated and less dipole dominated, compensating the increase of the detuning factor. At the same time the momentum compaction factor and the longitudinal emittance are increased.

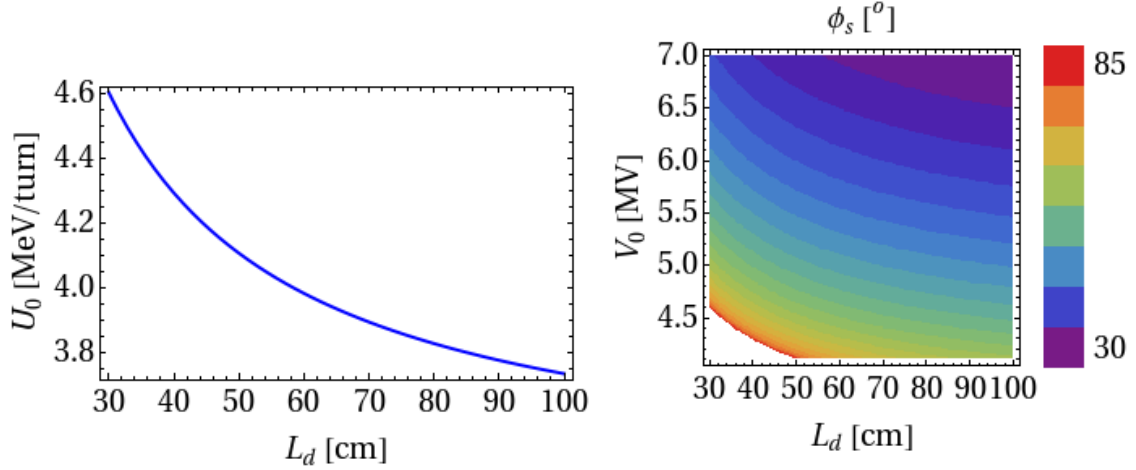


Figure 7.8: Left: Energy loss per turn dependence on the dipole length. Right: Parameterization of the RF stable phase with the dipole length and the maximum RF voltage.

Figure 7.8 shows the dependence of the energy loss per turn on the dipole length (left) and the parameterization of the RF stable phase, with the dipole length and the maximum RF voltage (right). Increasing the dipole length the energy loss per turn is reduced, which is the correct direction for our optimization procedure. Increasing the longitudinal emittance through the increase of the momentum compaction factor and decreasing the energy loss per turn by increasing the dipole length gives room to the RF voltage to be increased, reducing the same time the RF stable phase.

As discussed in Chapter 5, there are several pairs of (β_{xc}, D_{xc}) in the center of the dipole, that can achieve the same emittance, lying in elliptical curves. The different pairs of solutions define different optics functions along the cell, corresponding to different phase advances, while only a small fraction of them provide stable motion in both horizontal and vertical planes. The IBS growth rates, which will in the end define the IBS emittance growth, depend on the optics of the lattice. It is thus interesting to study the dependence of the IBS growth rates on the different optics options of the cell. Bane's IBS modeling was applied in all possible solutions that can achieve the same emittance and the IBS growth rates were calculated for the zero current equilibrium emittance, where the IBS effect is expected to be maximum. Figure 7.9 shows some examples of different optics options (top), that achieve the same emittance or detuning factor (in this example $\varepsilon_r=6$). In particular, the horizontal beta function (left) and dispersion (right) are presented. The respective horizontal (left) and longitudinal (right) IBS growth rates calculated along the cell, are shown in the bottom plots of the figure. A careful choice of optics is very important for the minimization of the IBS effect. In particular, for the examples presented here,

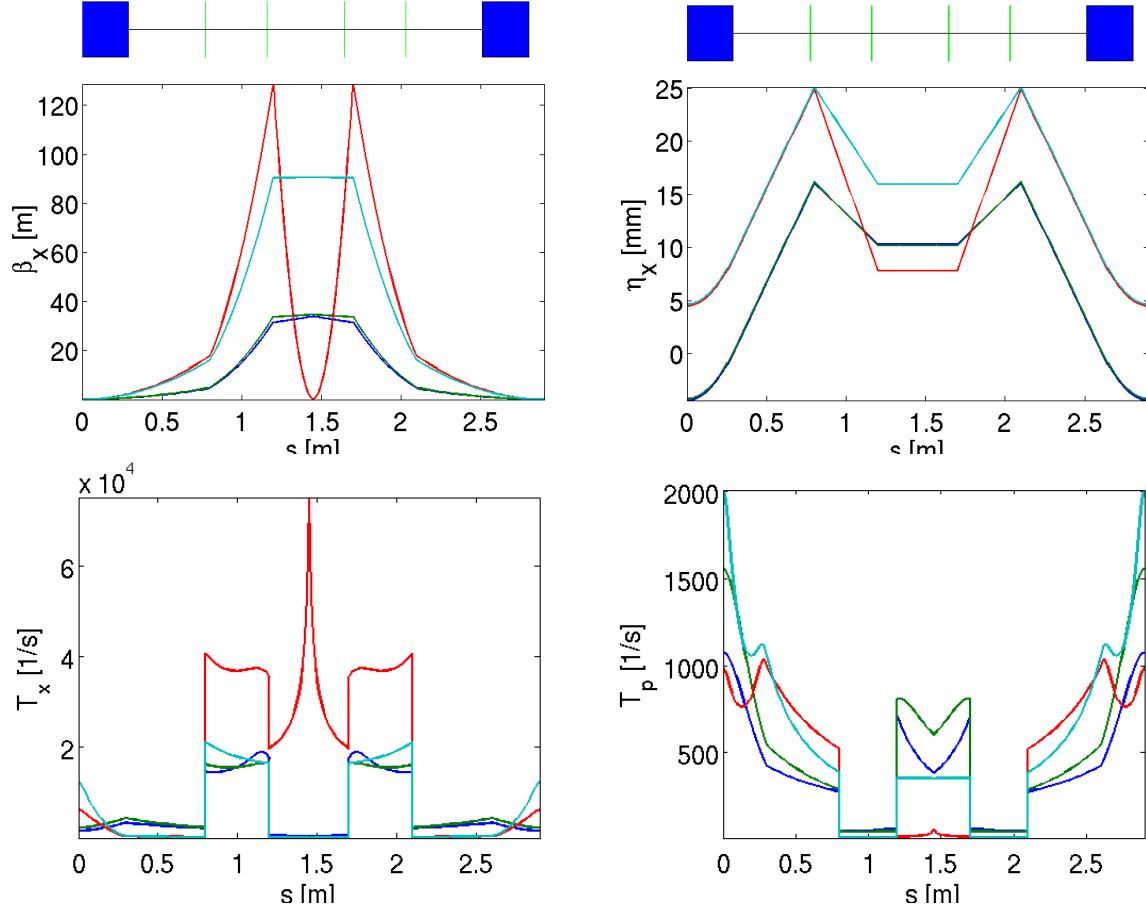


Figure 7.9: The different optics cases for a TME cell for the same detuning factor (top) and the respective horizontal (bottom, left) and longitudinal (bottom, right) IBS growth rates based on the analytical parameterization of the TME cells.

locations with small horizontal dispersion (absolute value) and beta functions, thus small beam sizes, present maximum horizontal growth rates. This is very clear in the solution presented in red, where maximum growth rate is observed in the middle of the cell, where the beam size is minimum. The effect in the longitudinal plane for this case is minimized. Smoother optics, without locations of ultra small beam sizes, like for the other 3 options (blue, light blue and green), present smoother behavior with respect to the IBS growth rates, which is preferable, as the mean value of the growth rate along the cell will in the end define the magnitude of the effect. For these calculations, zero vertical dispersion is considered for which Bane's formalism computes zero vertical growth rates (see Eq. (3.31)). In the limit of zero or very small vertical dispersion Bane's assumptions are not valid, as discussed in Chapter 4 (see sec. 4.1.1), however, here the effect is studied more qualitative than quantitative, thus Bane's formalism is adequate.

Scanning in several detuning factor options (from 1 to 25), the mean IBS growth rates in the horizontal and vertical planes can be calculated. Figure 7.10 shows the parameterization of the IBS growth rates with the horizontal and vertical phase advances. All the calculations were

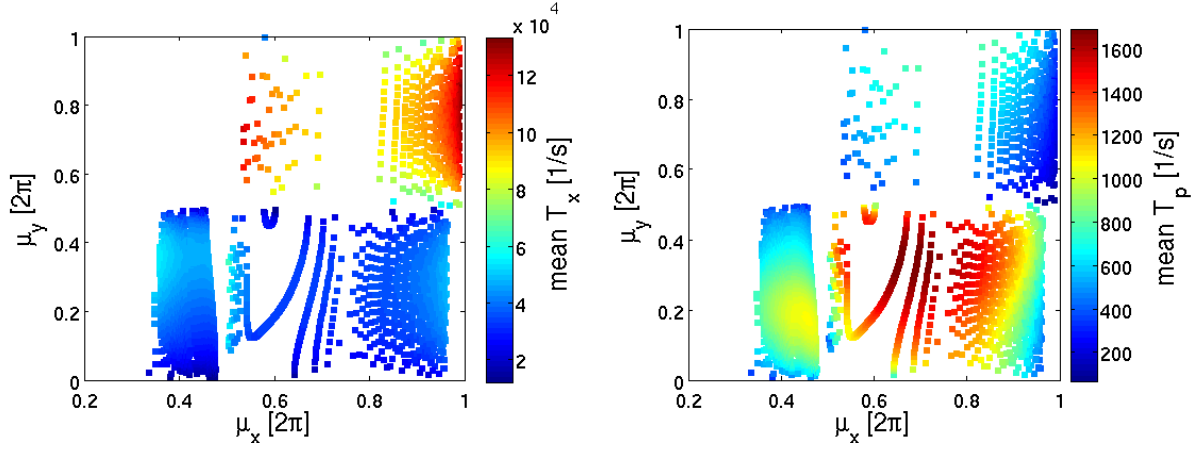


Figure 7.10: Parameterization of the mean IBS horizontal (left) and longitudinal (right) growth rates with the horizontal and vertical phase advances of the TME cell, based on the analytical parameterization.

done for the equilibrium transverse emittances, energy spread and bunch length of the CLIC DR lattice, where the effect is expected to be maximum. For low horizontal and vertical phase advances the mean growth rates are optimal, as both of them are in the low growth rate regime, even though not minimized simultaneously. More specifically, a good compromise for the horizontal phase advance is to be around 0.4, while the vertical phase advance can be used in order to find the optimal solution, staying always below 0.5. In this low horizontal and vertical phase advances regime, the horizontal and vertical chromaticities and the Laslett tune shift are minimized while the momentum compaction factor is maximized. Other regions of phase advances are also interesting, for each growth rate independently. Large μ_x and large μ_y values produce the minimum longitudinal growth rate, however the horizontal one gets maximized. Large μ_x and small μ_y minimize both growth rates, however, the Laslett tune shift gets maximized.

Summarizing the above, in order to reduce the space charge tune shift and the RF stable phase simultaneously, staying within the emittance requirements of the design, the TME arc cell needs to be detuned to a lower horizontal phase advance and the dipole length and the RF voltage need to be increased, if the output emittance is wiggler and not dipole dominated. At the same time, low horizontal and vertical phase advances are favorable for the reduction of the IBS effect, which is the main limitation to the ultra low emittance of the CLIC damping rings.

After several iterations, it was found that the horizontal phase advance of the TME cell has to be reduced to 0.408 from 0.452. The dipole length has to be increased to 0.58 m from 0.43 m and the RF voltage to 5.1 MV from 4.5 MV of the previous design. The optical functions of the TME cell are shown in Figure 7.11, where the horizontal (black) and vertical (red) beta functions and the horizontal dispersion (green) are depicted. On the top part of the figure, a schematic layout of the cell is presented. The entrance and exit of the cell is the middle of the dipole. Two doublets of quadrupoles are used, while in the symmetry point of the cell, between the two mirror symmetric defocusing quadrupoles, and between the dipole and the focusing quadrupoles, the sextupoles of the cell are placed. Unlike the TME cell of the pre-damping rings, the vertical

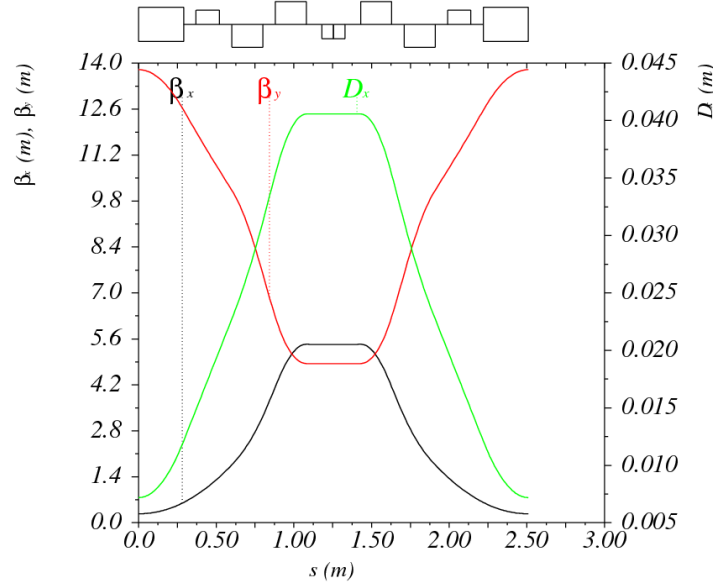


Figure 7.11: Optical functions of the arc TME cell.

beta function of this cell is reversed, due to the defocusing gradient of the combined function dipole.

For a dipole field of 1 T and a length of 0.58 m, the lattice functions to achieve the minimum emittance are $\beta_{x,\min} = 7.49$ cm and $D_{x,\min} = 15$ mm. The minimum emittance that this lattice can achieve, taking into account only the arcs, is 359.3 nm-rad normalized. However, in practice these values for the lattice functions are difficult to be reached and in the regime close to the minimum emittance the chromaticity is very high due to the strong focusing required for the very low dispersion.

Figure 7.12, shows the dependence of the horizontal damping time on the dipole length (left) for the nominal wiggler characteristics (blue) and without the wiggler contribution (red). A dipole dominated ring with a dipole length of $l_d=0.58$ m, would achieve a damping time of the order of 10 ms, which is very large compared to the repetition time of 20 ms. An injected beam of $\varepsilon_x=63\mu\text{m-rad}$ normalized emittance, would need at least 5 damping times to reach equilibrium, meaning that a maximum damping time of $\tau_x=4$ ms is required, without taking into account the IBS effect. Even a 30 cm long dipole, would achieve a damping time of 6 ms, which is still larger than the required one. A wiggler dominated ring, on the other hand, can achieve a much faster damping. In the case of dipole dominated ring, the damping time strongly depends on the dipole length (or field), while for a wiggler dominated ring, it is almost independent. The dependence of the horizontal damping time on the wiggler peak field and total length, is shown in the right part of the plot. The same arguments are valid for the vertical and longitudinal damping times, as well.

The above discussion leads to the conclusion that, in order to achieve the very low emittance in a fast damping time, imposed by the repetition frequency, keeping the chromaticity in low values and the ring circumference small, the use of wiggler magnets is mandatory.

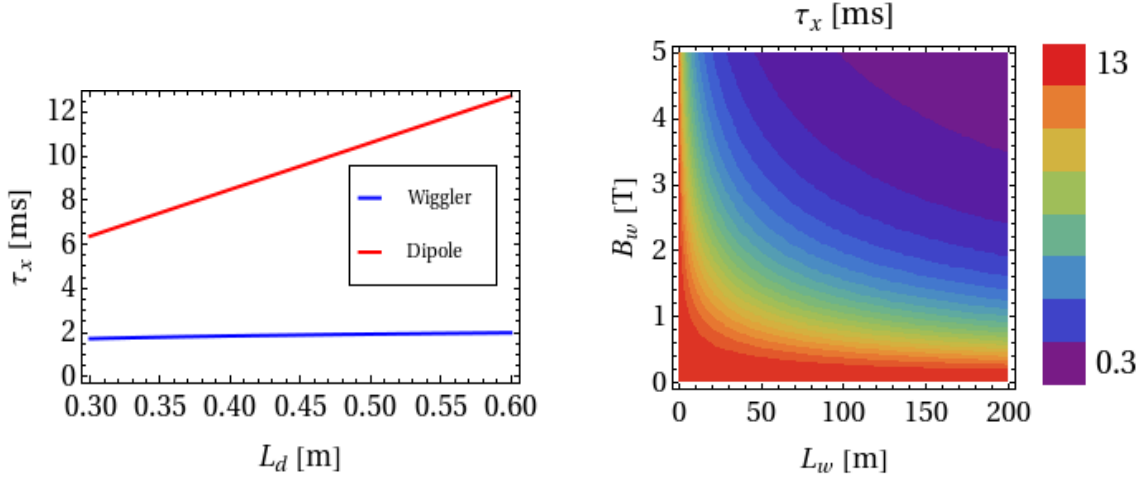


Figure 7.12: Damping time dependence on the dipole length (left) for the nominal wiggler parameters (blue) and without any wiggler contribution (red) and on the wiggler peak field and total length (right).

7.3.3 Optimization of the wiggler FODO cell

Figure 7.13 shows the parameterization of the horizontal normalized emittance with the wiggler peak field and period (left) and the mean beta function and total wiggler length (right). The horizontal emittance becomes minimum for large wiggler peak fields and small wiggler periods, for fixed total length of wigglers and dipole characteristics. Keeping the wiggler peak field and period fixed, the horizontal emittance becomes minimum for large total wiggler length and small mean beta function at the wigglers. The energy spread, on the other hand, becomes larger for large peak fields and longer wiggler length as shown in Figure 7.14. The energy loss per turn and the RF stable phase get smaller for smaller wiggler peak fields and wiggler total length, while the emittance gets larger, as shown in Figure 7.15. In all these plots, the IBS effect is not taken into account.

In order to explore the dependence of the output horizontal emittance on the wiggler characteristics with respect to IBS, a simulation was performed where the IBS effect on the emittance is computed, varying the wiggler peak field and period, while keeping the final vertical and longitudinal emittances fixed. The effect of the different wiggler working points in the optics is also not taken into account. The results are shown in Fig. 7.16. The left plots are color-coded with the horizontal steady state emittance, while the right ones with the ratio between the steady-state and the zero-current one. The highest field and the shortest period is indeed necessary for reaching the smallest emittance possible. On the other hand, the effect of IBS in that case becomes extremely strong. For reducing the blow-up due to IBS, still the highest fields are interesting but for moderate period lengths.

The maximum achievable flux density strength B_w with various wiggler types, as a function of the gap to period length ratio g/λ_w and the pole field B_p is [72]:

$$B_w = \frac{B_p}{\cosh\left(\pi \frac{g}{\lambda_w}\right)} \quad (7.6)$$

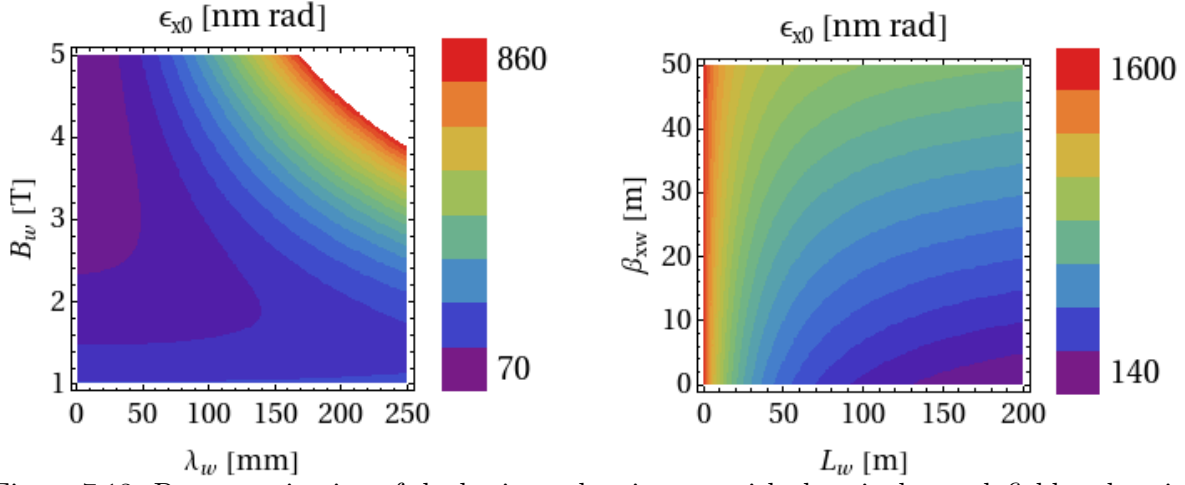


Figure 7.13: Parameterization of the horizontal emittance with the wiggler peak field and period (left) and the mean beta function and the total wiggler length (right).

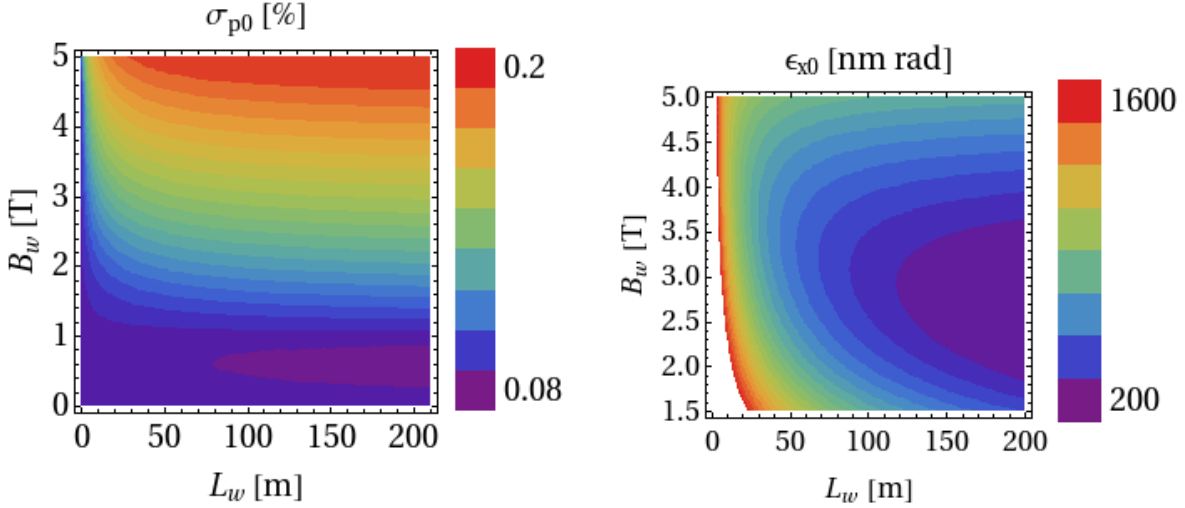


Figure 7.14: Parameterization of the energy spread with the wiggler peak field and the total wiggler length.

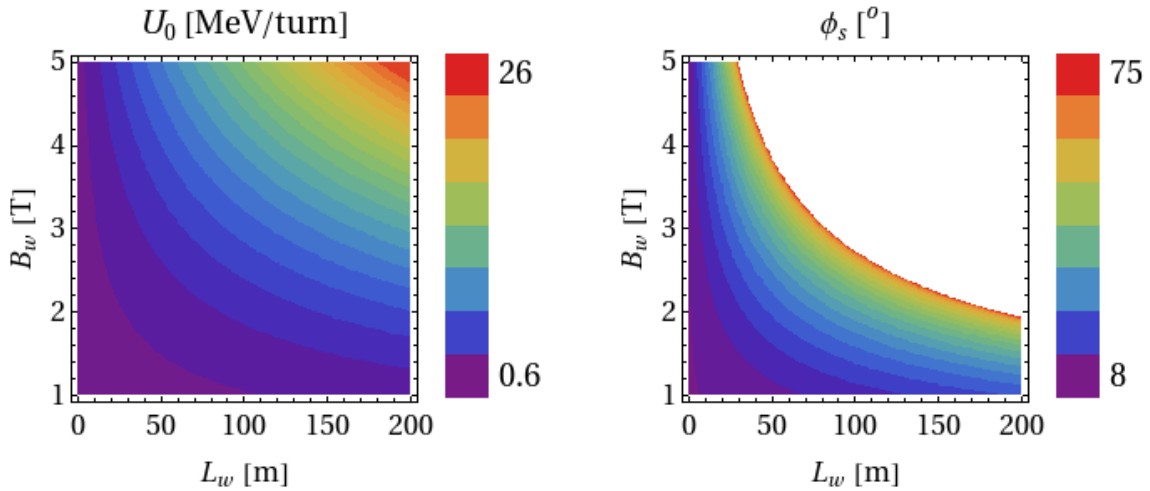


Figure 7.15: Parameterization of the energy loss per turn (left) and the RF stable phase (right) with the wiggler peak field and total length.

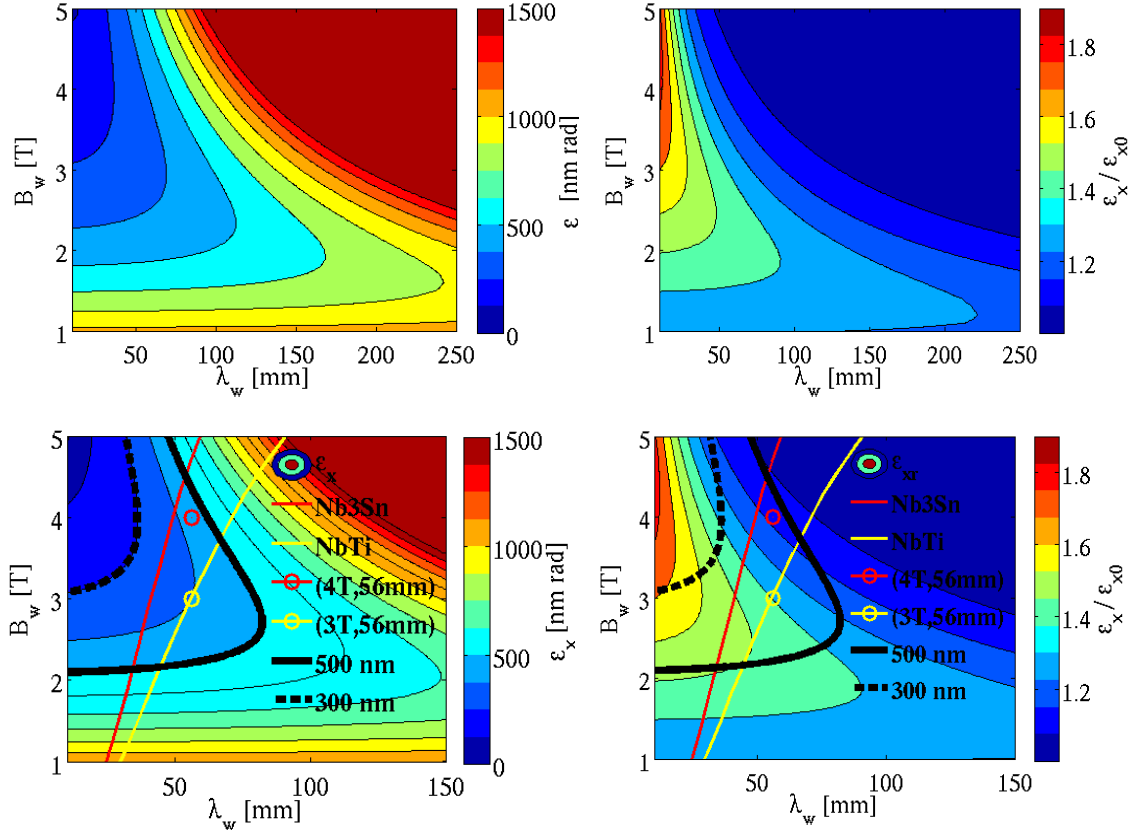


Figure 7.16: Top: Dependence of the steady state emittance (left) and its ratio with the equilibrium emittance (right) as a function of the wiggler peak field and period. Bottom: The limits for the two superconducting technologies are shown in red (NbTi) and yellow (Nb3Sn) and the 300 nm and 500 nm contours in black.

Hybrid-permanent wiggler magnets are not able to reach the required large magnetic flux densities. A theoretical upper limit of hybrid-permanent wiggler magnets is the magnetic saturation induction of iron ($B = 2.15$ T). Superconducting wigglers are able to achieve pole fields considerably higher than the magnetic saturation induction of iron. For this, the development of short-period superconducting wiggler magnets with a high magnetic flux density strength B_w and small gap g is required [60]. The limits for the two superconducting technologies, NbTi (yellow) and Nb3Sn (red) are shown in the bottom plots of Fig. 7.16 [60, 72]. The 300 nm and 500 nm contours are also indicated in black. Only wigglers with a period length λ_w of less than 80 mm and with a magnetic flux density amplitude B_w larger 2.2-2.5 T fulfill the requirements of the CLIC damping rings. For period lengths λ_w of 50 to 80 mm and magnetic flux densities B_w of 2.8-4.5 T, the effect of intrabeam scattering (Figure 7.16 (bottom, right)) can be minimized.

In each DR, it is foreseen to install 52 wigglers of peak field $B_w = 2.5$ T and 50 mm period, based on NbTi technology. A short prototype with these characteristics was developed and measured at Budker Institute achieving the field requirements. Another mock-up with more challenging design (2.8 T field, with 40 mm period) wound with Nb3Sn wire is also under testing

at CERN [72].

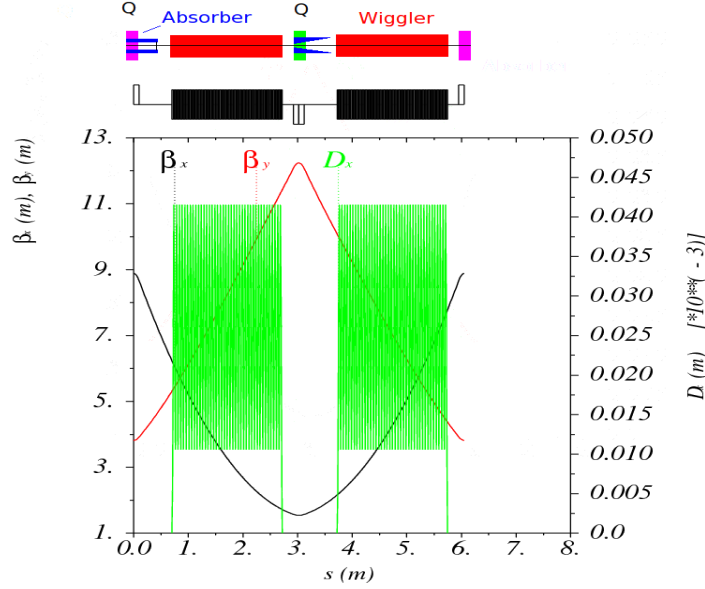


Figure 7.17: Optical functions for the FODO cell filled with superconducting damping wigglers.

The long straight sections have a FODO structure in order to accommodate the damping wigglers. There are 13 FODO cells per straight section with two wigglers per cell, thus 52 wigglers of 2.5 T peak field, 5 cm period and 13 cm full gap in total. In order to handle the intense synchrotron radiation, space is reserved for the placement of a radiation absorber downstream every wiggler, as sketched on the top part of Figure 7.17. The absorbers must reliably intercept the upstream radiation and prevent the heating of the vacuum chamber in the superconducting wigglers. Besides the absorber, the beam position monitor and the steering magnet are located in this reserved space [55].

Further emittance minimization can be made by optimizing the lattice functions in the wiggler. For a FODO cell, the minimum emittance is reached for horizontal phase advance $\mu_x \approx 0.31$ and for vertical phase tending to zero. The vertical phase advance can then be set as low as possible using other criteria. For example, for $\mu_y \approx 0.12$ the chromaticity is minimized. Another possible choice is $\mu_y \approx 0.25$ corresponding to minimum vertical beta and thus, maximum vertical acceptance [69]. The optical functions for this option is plotted in Figure 7.17, where the horizontal (black) and vertical (red) beta functions and the horizontal dispersion (green) are depicted. On the top part of the figure, a schematic layout of the TME cell and of the absorption scheme, are presented. The entrance and the exit of the cell are the middle of the focusing quadrupole, while a defocusing quadrupole is shown in the middle of the cell. In the space between the two quadrupoles, 2 damping wigglers of 2 m each are placed.

7.3.4 Dispersion suppression optimization

The lattice functions between the arcs and the straight sections are matched from the dispersion suppressors - beta matching sections, in the same way as discussed in Chapter 2 (see section 2.6).

The first part is a half TME cell, with different quadrupole strengths. This is done in order to use

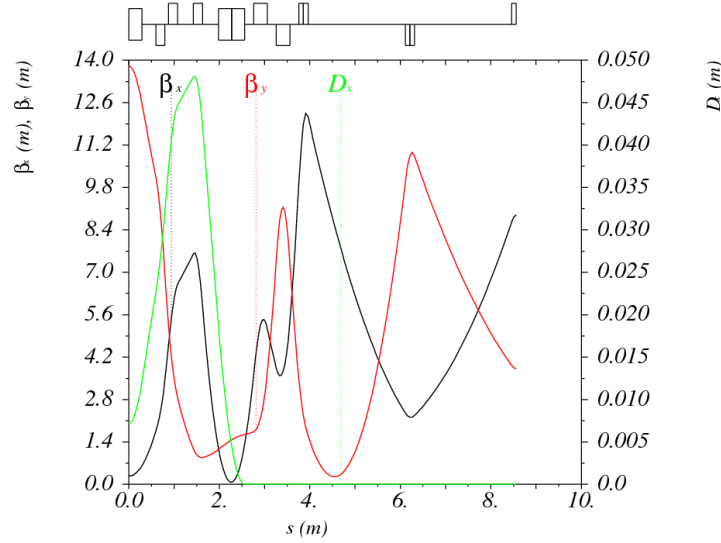


Figure 7.18: The optical functions of the dispersion-suppressor - beta matching section.

these two quads as knobs and minimize the length of the suppressor. A dipole is then used for the suppression of the dispersion and four more quads for the matching of all the optics functions. Space is reserved in the dispersion free region for injection/extraction and RF cavities. The optical functions of the dispersion suppressor are presented in the right part of Figure 7.18.

7.3.5 Magnets

Table 3.7 summarizes the main magnets' parameters of the main DRs. There are 100 main dipoles in one family, with same field, among which, 4 are located in the DS-BM section, for the dispersion suppression. There are 458 quadrupoles of two different types (0.2 and 0.31 m long). Their pole tip field is around 1 T. There are 282, 0.15 m long sextupoles in two families with a pole-tip field of 0.8 T. Finally, there are 52, 2 m super-conducting wigglers. All the magnets, apart from the dipoles and wigglers, have a circular aperture with 20 mm diameter. The main bending magnets and wigglers have an elliptical aperture, with vertical gaps of 20 and 13 mm, respectively. The geometrical acceptance of the whole ring is quite comfortable, considering the fact that the injected beam emittances are quite small, especially in the vertical plane.

The minimum aperture radius of the magnets (column R [mm]), is defined through the geometrical acceptance of the ring, as defined in Eq. (6.6). Figure 7.19, shows the minimum geometrical acceptance of the main DRs, in units of beam sizes (left) and meters (right). From this, a minimum dynamic aperture of 4 beam sizes is required, in both horizontal and vertical planes, in order to fit the Gaussian beam of $\varepsilon_x = 54 \mu\text{m-rad}$, coming from the pre-damping ring.

7.3.6 Dynamic aperture

The ptc-track module of MADX [14] was used to perform a 1000-turns tracking, in order to determine the dynamic aperture (DA), including chromatic sextupoles and fringe fields. The

Table 7.3: List of magnetic parameters for the CLIC DRs.

Type	Location	Length [m]	Number	Families	Pole tip field [T]	Full aperture H/V [mm]
Dipoles	Arc DS-BM	0.58	96 4	1	0.97	80/20
Quadrupoles	Arc	0.20	376	2	1.0	20/20
	LSS	0.20	28+26	2		
	DS-BM	0.20	24	12		
	DS-BM	0.31	4	2		
Sextupoles	Arc	0.15	188+94	2	0.5	20/20
Wigglers	LSS	2.00	52	1	2.5	80/13

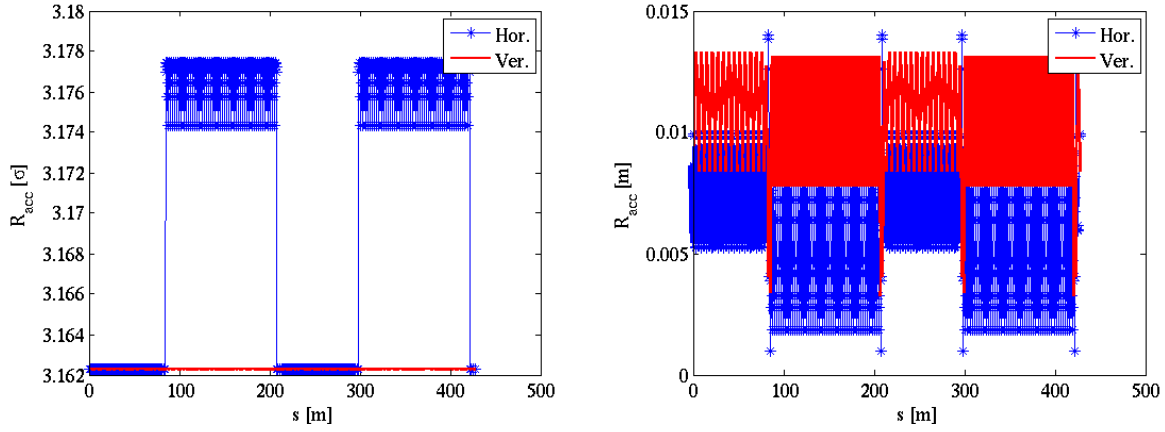


Figure 7.19: The required acceptance around the DRs in order to fit the positron beam in units of beam sizes (left) and in meters (right).

result is presented in Figure 7.20. A very large DA, translated to around ± 5 mm, is observed in both planes for on-momentum particles. A more detailed DA and non-linear optimization is ongoing including working point optimization, inclusion of magnet imperfections and the wiggler effect [73].

7.3.7 Longitudinal and RF parameters

Parameters relevant to the design of the DR RF system are presented in Table 7.4. The very high peak and average current corresponding to the full train of 312 bunches spaced by 0.5 ns, presents a big challenge due to the transient beam loading, especially for a 2 GHz RF system. In this respect, it was decided to consider two bunch trains with 1 ns bunch spacing which reduces significantly the beam loading. An RF system with frequency of 1 GHz is more conventional and an extrapolation from existing designs is possible. Nevertheless, the trains have to be recombined in a delay loop downstream the DRs with an RF deflector [61]. A detailed description of the RF

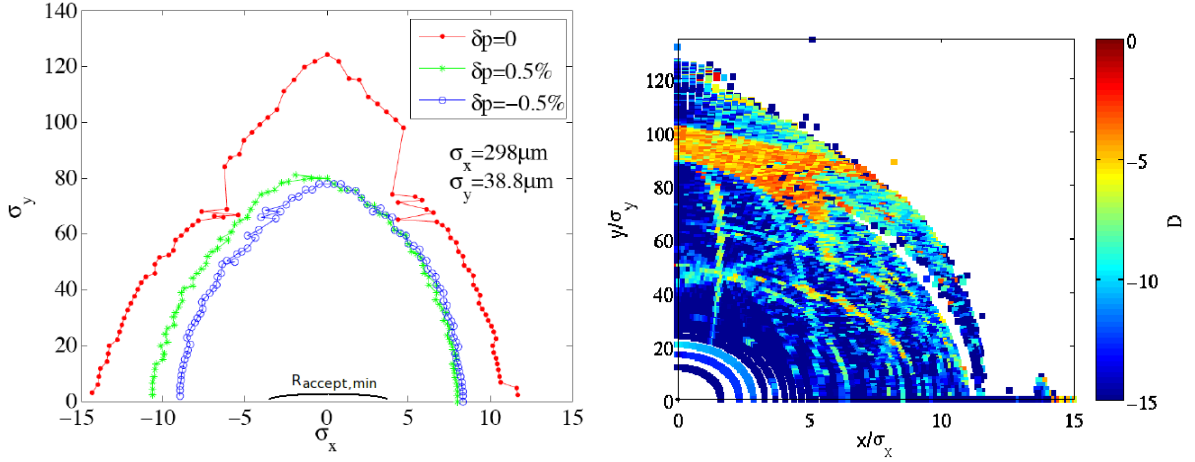


Figure 7.20: The on and off momentum Dynamic Aperture.

system design can be found in [62].

Table 7.4: CLIC DR RF parameters.

Parameter	DR @ 1 GHz	DR @ 2 GHz
Circumference [m]		427.5
Energy [GeV]		2.86
Mom. compaction factor		$1.3 \cdot 10^{-4}$
Energy loss per turn [MeV]		3.98
Energy spread (r.m.s.) [%]		0.1
Bunch length (r.m.s.) [mm]	1.6	1.
Longitudinal emittance [keV-m]	5.3	6.0
RF voltage [MV]	5.1	4.5
RF stationary phase [°]	62	51
Peak/Average current [A]	0.66/0.15	1.3/0.15
Peak/Average power [MW]	2.8/0.6	5.5/0.6

This choice has a positive impact in both the PDRs and the main DRs. Doubling the bunch spacing halves the harmonic number, increasing the momentum acceptance. The extraction kicker rise time becomes shorter but it is still long enough (560 ns). The 2-train structure may require two separate extraction kicker systems or one kicker with longer flat top (1 μ s) [74]. The beam loading is significantly reduced, as the larger bunch spacing reduces peak current and power by a factor of 2. Several beam dynamics issues are also eased due to double bunch spacing. The e-cloud production and instability is reduced while the fast ion instability will be less pronounced by doubling the critical mass above which particles get trapped [75]. The reduced number of bunches per train reduces the central ion density, the induced tune-shift and the rise time of the instability is getting doubled, thus relaxing the feedback system requirements. Finally, a bunch-by-bunch feedback system is more conventional at 1 than at 2 GHz [55].

7.4 Deliverable emittances from the CLIC DR

In the previous paragraphs, the impact of the IBS effect on the output emittances of the CLIC DR was discussed in detail. The transverse emittances delivered by the damping rings affect the luminosity of the collider, while the extracted longitudinal emittance, affect the performance of the downstream systems, where the bunch needs to be further compressed. It has been demonstrated that for the nominal current, for the 3 TeV option, the current design can achieve the required output parameters. However, in the lower energy CLIC options, higher bunch currents are required in order to achieve the high luminosity. For example, in the 500 GeV CLIC, the number of particles per bunch at collision should be $N_p = 6.8 \times 10^9$ which is around 85% higher than for the 3 TeV CLIC, where $N_p = 3.7 \times 10^9$ [2].

In order to study the impact of the different bunch currents on the output emittances, a scaling of the latest with respect to bunch current is performed. The optics of the ring is considered unchanged. For all the IBS calculations that will be presented in the next the CIMP formalism is used, as the computational time for this method is much faster than the original IBS formulations of Piwinski and Bjorken-Mtingwa. Bane's method, even though faster than CIMP, is not used as the approximations of this method are not valid for the case of the CLIC DR (see Chapter 4).

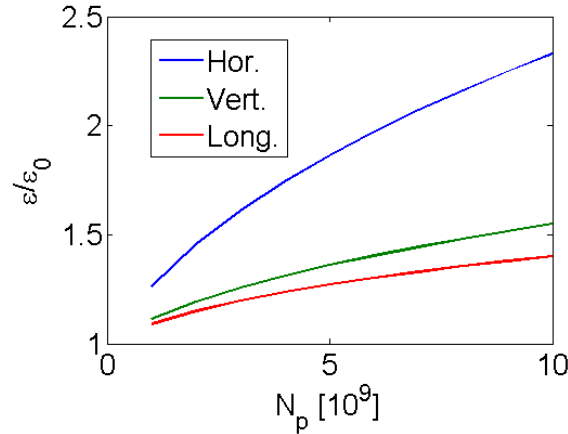


Figure 7.21: Parameterization of the IBS effect with bunch charge, in the horizontal (blue), vertical (green) and longitudinal (red) planes.

Figure 7.21 (left) shows the parameterization of the IBS effect with bunch charge in the horizontal (blue), vertical (green) and longitudinal (red) planes. The horizontal emittance has a stronger dependence on the bunch charge than the vertical and longitudinal emittances. In particular, for a bunch charge of 9×10^9 particles, the IBS growth factor in the horizontal plane is 2.1, in the vertical 1.6 and in the longitudinal 1.3.

The zero current equilibrium emittances, are an input to the IBS growth rate calculations. The horizontal emittance is defined by the lattice, and is considered always the same. The vertical emittance, produced by residual vertical dispersion and betatron coupling due to magnet alignment errors, on the other hand, can be controlled by an efficient orbit correction scheme,

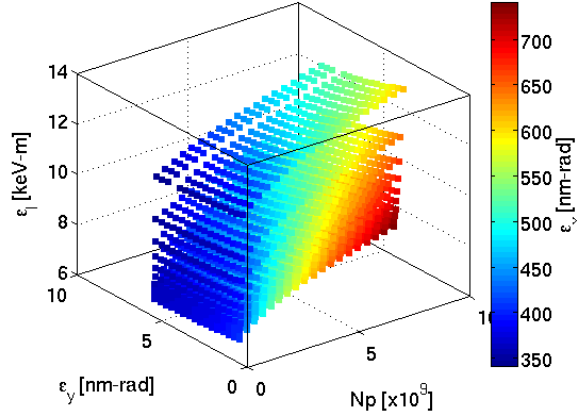


Figure 7.22: Parameterization of the horizontal emittance with the vertical and longitudinal emittances and bunch charge.

while the longitudinal one can be controlled by the RF voltage, provided that enough voltage is available from the RF system. In Figure 7.22, the output horizontal emittance is parameterized with the longitudinal and vertical emittances and with the bunch charge. For these calculations, different zero current beam parameters are considered. The horizontal emittance is much more sensitive to the longitudinal than to the vertical one, due to the coupling between the two planes through the horizontal dispersion. In the high current regime, the longitudinal emittance has to be doubled to keep the horizontal emittance at 500 nm-rad.

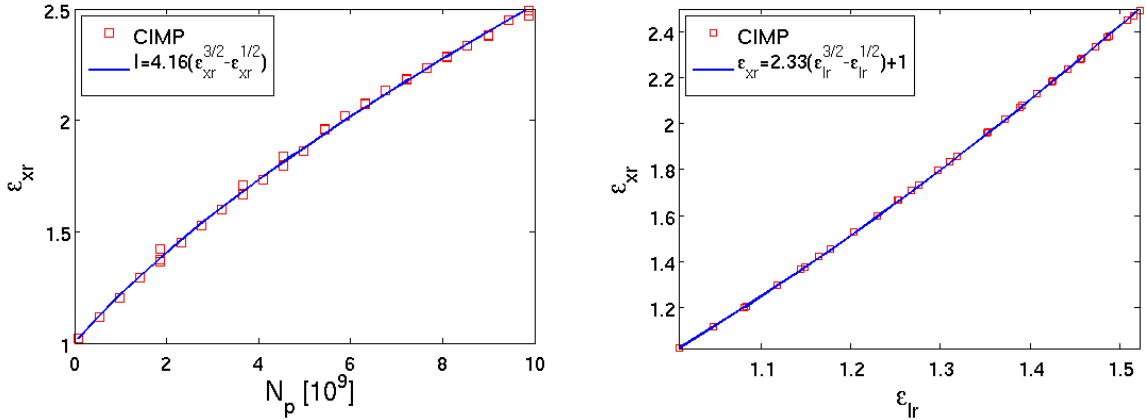


Figure 7.23: Scaling laws for the interdependence of the IBS effect in the horizontal and longitudinal plane (left) and for the dependence of the IBS effect in the horizontal plane on the bunch population (right).

Unlike the other two planes, the zero current horizontal emittance cannot change, if the lattice remains the same. It is thus interesting to study the dependence of the output horizontal emittance on the bunch charge, if the vertical and longitudinal emittances are kept constant to the nominal required values with $\varepsilon_y = 5 \pm 0.1$ pm-rad and $\varepsilon_l = 6 \pm 0.1$ keV-m respectively. This is

shown in the left part of Figure 7.23, while in the right the interdependence between the IBS effect in the horizontal and longitudinal planes is presented. In both cases, simple scaling laws are derived. In the first case, a scaling law similar to the one reported in [23] is derived:

$$I = b \left(\left(\frac{\varepsilon_x}{\varepsilon_{x0}} \right)^{n_1} - \left(\frac{\varepsilon_x}{\varepsilon_{x0}} \right)^{n_2} \right), \quad (7.7)$$

with $b=4.16$, $n_1=3/2$ and $n_2=1/2$. The b , n_1 and n_2 change if different output vertical and longitudinal emittances are considered. In the second case, the scaling law is:

$$\frac{\varepsilon_x}{\varepsilon_{x0}} = \alpha \left(\left(\frac{\varepsilon_l}{\varepsilon_{l0}} \right)^{3/2} - \left(\frac{\varepsilon_l}{\varepsilon_{l0}} \right)^{1/2} \right) + c, \quad (7.8)$$

where $\alpha=2.33$ and $c=1$. It is interesting to notice that this is valid for any RF voltage and any vertical emittance.

7.5 Ring layout and performance parameters

Table 7.5 summarizes the performance parameters of the DR for both RF options of 1 and 2 GHz. In the third column, the performance parameters of the previous design are also shown for comparison. For the IBS calculations presented in this table, the Piwinski formalism is used, as this has been traditionally the formalism used for all the design stages of the CLIC DR. The full optics of the ring is shown in Fig. 7.24, while the final layout in Fig. 7.25.

The proposed design achieves all the requirements imposed by the performance requirements of the collider. The IBS effect and the Lasslet space charge tune shift have been reduced and the performance parameters of the RF system have been optimized. Finally, it is considered as the baseline design and is included in the CLIC Conceptual Design Report (CDR) [55].

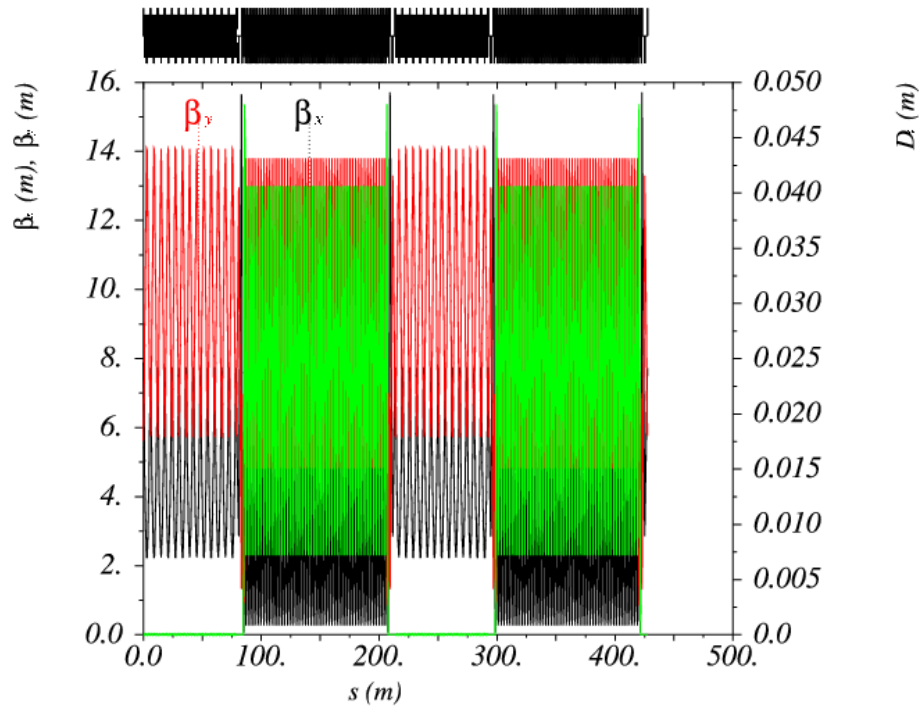


Figure 7.24: The optics functions of the CLIC DR.

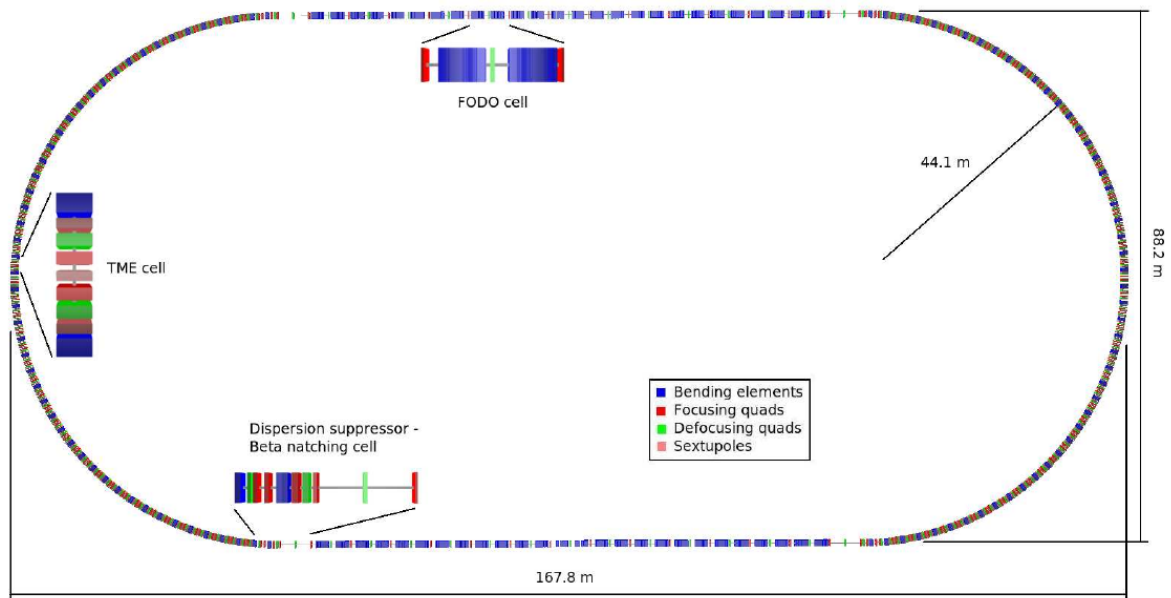


Figure 7.25: Damping ring layout.

Table 7.5: Performance parameters of the CLIC DR for the 1 GHz and 2 GHz options and the previous version (V06) of the DR lattice design.

Parameters	1 GHz	2 GHz	V06
General			
Energy [GeV]	2.86	2.86	2.424
Circumference [m]	427.5	427.5	493.05
Bunches per train	156	312	312
Energy loss/turn [MeV]	3.98	3.98	3.98
RF voltage [MV]	5.1	4.5	4.3
RF harmonic (h)	1425	2850	3287
RF stationary phase [$^\circ$]	51	62	67
Energy Acceptance [%]	1	2.5	0.98
Natural chromaticity x/y	-115/-85	-115/-85	-148.8/-79.0
Momentum compaction factor [10^{-4}]	1.27	1.27	0.644
Damping times x/y/s [ms]	2/2/1	2/2/1	2/2/1
Number of arc cells/wigglers	100/52	100/52	100/76
Phase advance per arc cell x/y	0.408/0.05	0.408/0.05	0.442/0.045
Dipole focusing strength $K_1[m^{-2}]$	-1.1	-1.1	-1.1
Dipole length [m]/field [T]	0.58/1.03	0.58/1.03	0.4/1.27
Without the IBS			
Normalized Hor. emittance [nm-rad]	312	312	148
Energy spread [10^{-3}]	1.2	1.3	1.12
Bunch Length [mm]	1.18	1.46	0.95
Longitudinal Emittance [keV-m]	5.01	4.39	2.58
With the IBS			
Bunch population [10^9]	4.1	4.1	4.1
Normalized Hor. emittance [nm-rad]	456	472	436
Normalized Vert. emittance [nm-rad]	4.8	4.8	5
$\varepsilon_{x,IBS}/\varepsilon_{x,0}$	1.44	1.5	2.9
Longitudinal Emittance [keV-m]	6	6	5
Space charge tune shift	-0.10	-0.11	-0.2

IBS measurements

In Chapter 3, the theoretical models of Piwinski and Bjorken-Mtingwa, their high energy approximations Bane and CIMP and two multi-particle tracking codes, describing the IBS effect, were discussed. A comparison between them was presented for three different lattices, showing that in the limit where the IBS effect is strong the divergence between the theoretical models and the codes is growing. The bench-marking of the theoretical models and the tracking codes with experimental data is the ultimate goal for understanding the effect in IBS dominated regimes, in the presence of synchrotron radiation and quantum excitation. First IBS measurements studies has been presented by the Accelerator Test Facility (ATF) at KEK [29], while recently similar studies are being performed at the Swiss Light Source (SLS) [48] and the Cornell Electron Storage Ring Test Accelerator (Cesr-TA) [76]. In this chapter, the measurement results from the SLS and observations at Cesr-TA, are presented and discussed.

8.1 IBS measurements at the SLS

The Swiss Light Source (SLS) storage ring is an ideal test facility for IBS experimental studies: a record vertical geometrical emittance of around 1 pm-rad at 2.4 GeV was recently achieved [77], but also the ring has the availability of emittance monitoring diagnostics and the ability to run at lower energies, where the IBS effect becomes stronger.

Since 2011 the Paul Scherrer Institut (PSI) is part of the EU-project TIARA (Test Infrastructure and Accelerator Research Area) collaboration [78]. The aim of one of the work packages (WP6) in this collaboration is to use the SLS as test facility for damping rings and future light source R&D. In this context, IBS studies is one subject of great interest.

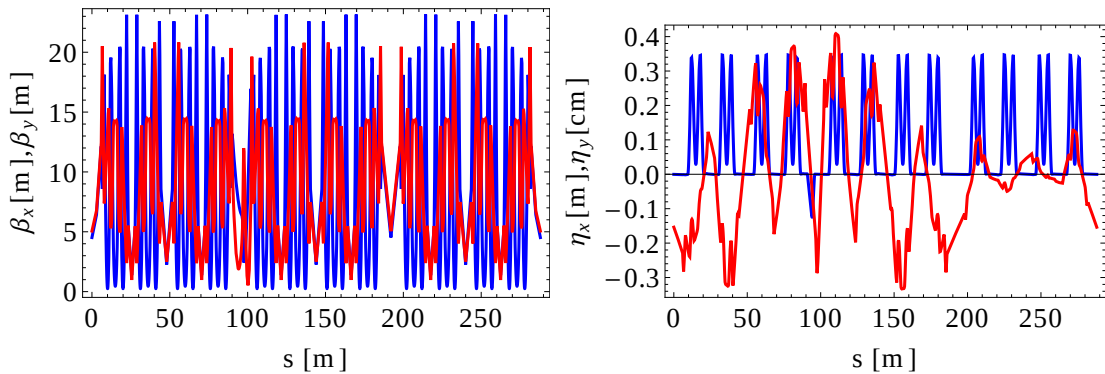


Figure 8.1: The twiss functions, beta (left) and dispersion (right), in the horizontal (blue) and vertical (red) planes around the SLS ring.

Table 8.1: Performance parameters of the SLS storage ring at nominal energy ($E_n=2.4$ GeV) and at lower energy ($E_n=1.57$ GeV).

Parameter [Unit]	SLS nominal	SLS low En.
E_n [GeV]	2.411	1.57
Circumference [m]	290	
Bunch current [mA]	0.5	0.1-5
Momentum Compaction Factor	6×10^{-4}	
Geom. Hor. Emittance [nm-rad]	5.6	2.4
Geom. Vert. Emittance [pm-rad]	1	1-10
Bunch Length [ps]	6	11.2-6.8
Energy Spread [%]	0.09	0.056
RF Voltage [MV]	2.1	0.6-2.1
Synchrotron frequency [kHz]	6.5	4.3-8.0

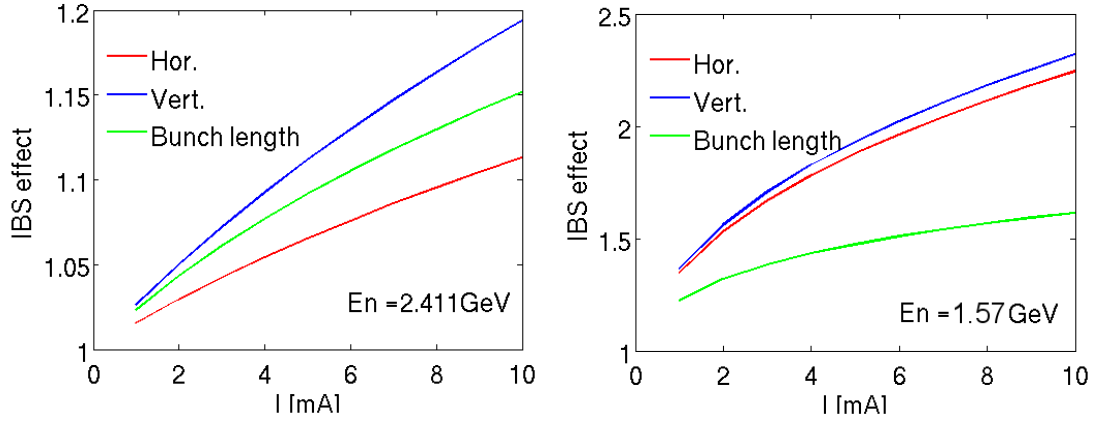


Figure 8.2: The IBS effect in the horizontal (red), vertical (green) and longitudinal (blue) plane at 2.4 GeV (left) and at 1.57 GeV (right).

Figure 8.1 shows the twiss functions around the SLS ring in the horizontal (blue) and vertical (red) planes. In order to produce the vertical dispersion and the 1 pm vertical emittance, random quadrupole rotations are applied. Table 8.1 summarizes the basic performance parameters of the SLS at nominal energy ($E_n = 2.4$ GeV) and at a lower operational energy ($E_n = 1.57$ GeV) [79]. Figure 8.2 shows the IBS effect, defined by the ratio of the steady state to zero current emittances, in the horizontal (red), vertical (blue) and longitudinal (green) planes, for different currents and different energies: at 2.4 GeV (left) and at 1.57 GeV (right). At nominal energy, the expected IBS effect is small even at high bunch currents in all three planes. On the other hand, in the low energy case the effect is appreciable, even at low currents, making this study option very attractive.

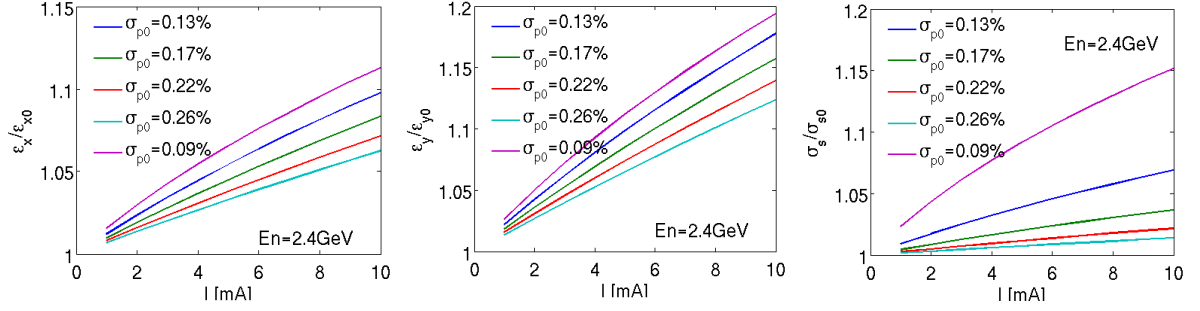


Figure 8.3: The IBS effect versus current in the horizontal (left) and vertical (middle) emittance and bunch length (right), for different equilibrium energy spread values at nominal energy.

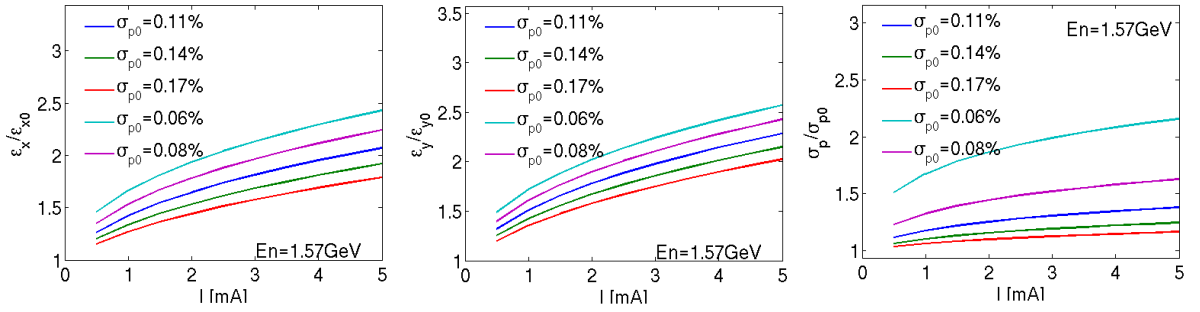


Figure 8.4: The IBS effect versus current in the horizontal (left) and vertical (middle) emittance and bunch length (right), for different equilibrium energy spread values at low energy (1.57 GeV).

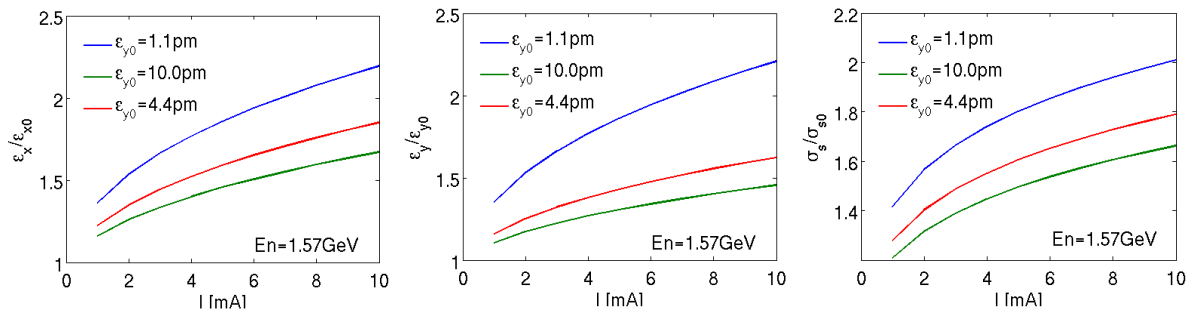


Figure 8.5: The IBS effect versus current in the horizontal (left) and vertical (middle) emittance and bunch length (right), for different equilibrium vertical emittance values at low energy (1.57 GeV).

8.1.1 IBS dependence on SLS parameters

For the IBS growth rates, and thus the steady state emittance calculations due to IBS, the equilibrium beam parameters are used as input. In the ideal case, the equilibrium beam states are defined only by synchrotron radiation. In the presence of other effects, however, and assuming that these effects are disentangle from scattering processes, new steady states are defined, and those will be the input to the IBS calculations. It is thus interesting to study the effect of different equilibrium or steady state beam parameters on the output steady states due to IBS.

At the SLS, the microwave instability (MI), also called turbulent bunch lengthening (TBL) effect, dominates the longitudinal phase space in the high intensity regime, leading to both bunch length and energy spread increase [80]. Assuming that this effect is disentangled with scattering processes, new steady states, different for each current, are defined in the longitudinal plane. Figures 8.3 and 8.4 show the IBS effect with respect to bunch current on the horizontal (left), vertical (middle) emittance and bunch length (right), at $E=1.57$ GeV and $E=2.4$ GeV respectively. The different color curves, correspond to different equilibrium energy spread σ_{p0} values. The emittance growth due to IBS is reduced, if the energy spread is increased. At nominal energy where the effect is small, increasing the energy spread makes the effect negligible, even at high currents. At low energy case, even for 3 times larger energy spread than the zero current one, the IBS effect is still significant.

At nominal energy, the SLS can achieve a vertical geometrical emittance of around 1 pm·rad. However, we cannot assume that this is the case for the lower energy or for high currents. Different ε_{y0} will have an impact on the IBS effect. Figure 8.5 presents the IBS growth in the horizontal (left) and vertical (middle) emittance and bunch length (right) for different zero current vertical emittance values, represented by different color curves. Even for large zero current vertical emittance values and relatively low currents, the IBS effect is predicted to be important.

8.1.2 Preparation for measurements at 1.6 GeV

The way to set up the low energy mode is to scale down the whole 2.4 GeV optics to 1.57 GeV and do the extraction from the booster to the storage ring by changing the extraction timing [79]. A brief description of the diagnostics is given in the next while a more detailed description can be found in [81].

Diagnostics

In the SLS storage ring, a dedicated beam-line is used to measure the transverse size of the electron beam. Utilizing radiation of the same dipole it is possible to simultaneously use two methods, X-rays for pinhole imaging and vis-UV π -polarized light method [82]. With the π -polarized method it is possible to measure very accurately small beam heights, however it is not the optimum way to measure the horizontal beam size. To have a precise measurement of the horizontal beam size a series of pinholes can be used [79].

To measure the bunch length, an Optronics Streak Camera installed in the diagnostic beam-line is used [83]. This Streak Camera enables to measure the bunch length of individual bunches with a precision better than 2 ps for currents of the order of 0.1 mA.

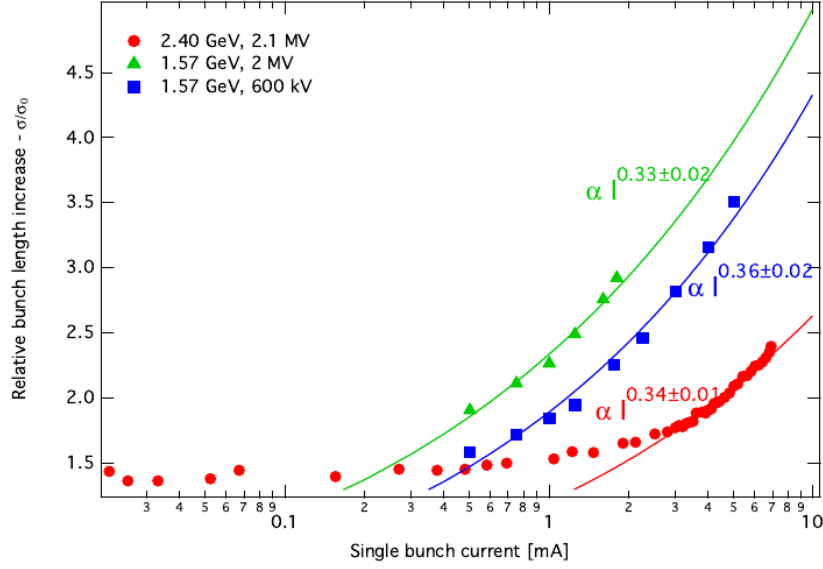


Figure 8.6: Measured bunch length as a function of single bunch current for 2.4 GeV ($V_{rf} = 2.1$ MV) and 1.57 GeV ($V_{rf} = 600$ kV and 2 MV). The lines are the fits to the turbulent regime and the exponents obtained in the fitting are shown beside each line [79].

8.1.3 Measurements at 1.6 GeV

Bunch length

In order to characterize the machine, a measurement of bunch length versus current was performed at the nominal mode of operation (2.4 GeV for a total accelerating voltage of 2.1 MV) and at lower energy (1.57 GeV) for two different cavity voltages ($V_{rf} = 600$ kV and 2 MV). As the purpose of this measurement is to have a model for the bunch length increase with current without the effect of IBS, the vertical emittance is kept large to stay outside the IBS regime.

Figure 8.6 shows the measurements and the fitted curves, corresponding to the I^α scaling for

Table 8.2: Turbulent bunch lengthening thresholds [79].

Energy [GeV]	Cavity voltage	I_{th} [mA]	$ Z/n $ [$m\Omega$]
2.41	2.1 MV	0.57 ± 0.02	432 ± 2
1.57	2.0 MV	0.08 ± 0.01	482 ± 1
1.57	600 kV	0.16 ± 0.02	417 ± 1

turbulent bunch lengthening (see Eq. (2.110)), for each case. From all three fits the scaling of $\alpha = 0.34 \pm 0.02$ on the bunch length with current is confirmed (already measured before [80]), verifying that turbulent effects are present. It is possible to extract the turbulent current threshold (I_{th}) and the longitudinal broad band impedance ($|Z/n|$) from each measurement through Eq. (2.110) and the results are summarized in Table 8.2. In all three measurements the impedance values for the longitudinal broad band impedance are in fair agreement with each

other and the values for the current threshold are very close to those predicted by the theory describing turbulent effects [79].

A first set of IBS measurements at the SLS storage ring tuned at low energy, were performed in May 2012. Due to technical problems of the pinhole camera both the horizontal and vertical beam sizes were measured with the synchrotron light beam size monitor. As this camera is optimized for the vertical beam size measurements, the absolute values of the horizontal one are not reliable, however, a relative trend can be measured. The machine was corrected at low current, achieving a vertical emittance of 3.4 pm-rad. Two sets of single bunch measurements for different RF voltage settings were taken, at 600 kV and 2 MV. The measurements were repeated for large zero current vertical emittance (at 50 pm-rad).

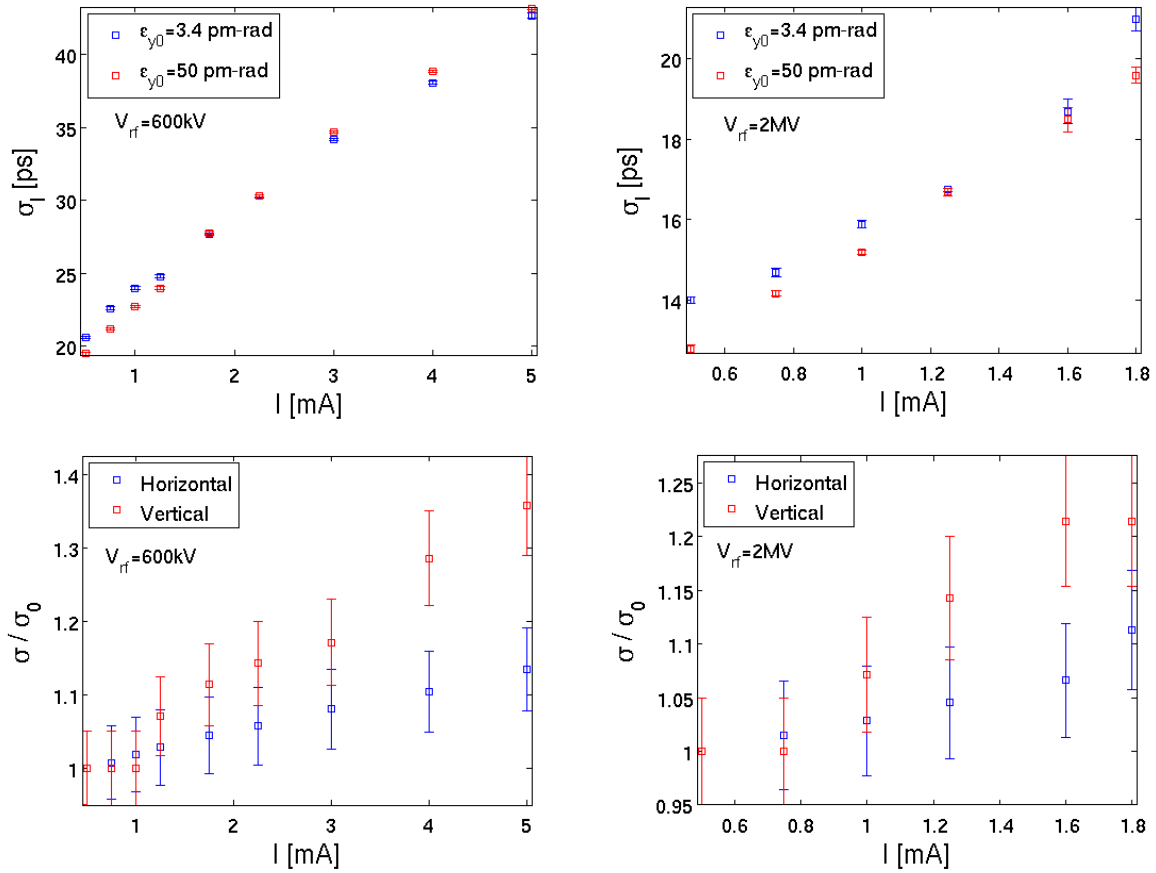


Figure 8.7: Left: Bunch length (top) and transverse beam size (bottom) data versus current for RF voltages of 600 kV (left) and 2 MV (right).

Figure 8.7 (top) shows the bunch length versus current at 600 kV (left) and at 2 MV (right). The blue curves correspond to the vertical emittance corrected to 3.4 pm-rad, while the red ones to the vertical emittance blown up to 50 pm-rad. The bunch length increase at high currents in the case of the 600 kV, seems to be dominated by TBL, as the blow up of the vertical emittance does not affect the bunch length. For the higher voltage, thus reduced bunch length, a larger traverse emittance blow up is observed for the low vertical emittance, which is an indication

of IBS. Figure 8.7 (bottom) shows the ratio of the horizontal (red) and vertical (blue) beam sizes with respect to the zero current values, for the two sets of measurements. A clear current dependent increase in both planes is observed, which gets larger if the bunch length is shorter (right plot). This is an indication of the presence of IBS.

Even though IBS is present, the longitudinal phase space is dominated by the MI effect. In the SLS storage ring, there is neither a measurement method nor a MI model for the energy spread. As the beam size monitors are placed in a dispersive area, the measured beam size includes also the contribution from the energy spread ($\sigma_x = \sqrt{\varepsilon_x \beta_x + (\sigma_p \eta_x)^2}$), which is changing mainly due to MI and less due to IBS. In order to compare the data with the theoretical predictions, it is important to develop a method to disentangle the two effects.

An other important parameter that has to be taken into account at this point, is the contribution from the third harmonic RF cavity. In the SLS, a passive 3rd harmonic RF cavity is installed, operating in bunch lengthening mode, serving in the increase of the beam lifetime dominated by the Touschek effect (see sec. 2.8.2). In addition, it introduces an increase of the incoherent synchrotron frequency spread inside the bunch, enhancing the effect of Landau damping¹, which allows the suppression of longitudinal instabilities in the beam [85]. The performance of the 3rd harmonic cavity depends on the total current of the machine. In the first set of measurements of May, the total current of the machine was low, and the third harmonic cavity was not excited and did not have any contribution to the longitudinal phase space motion.

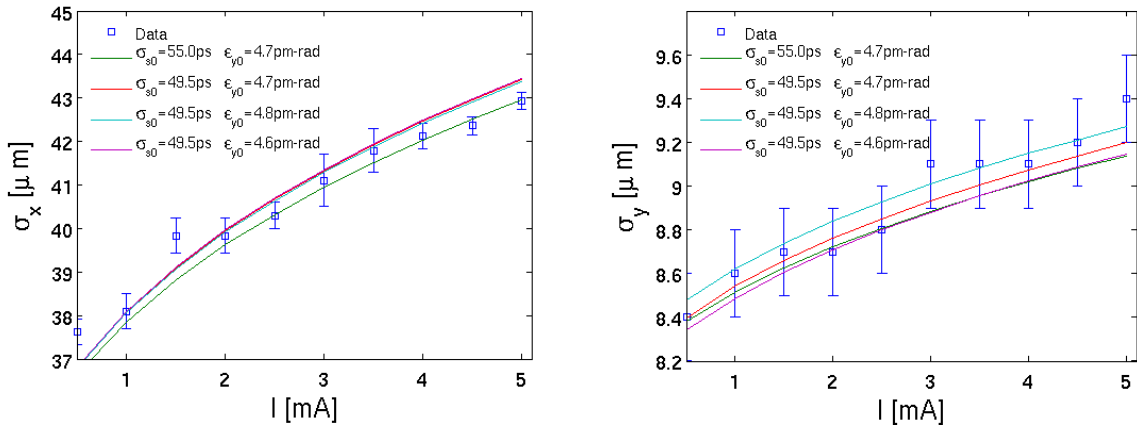


Figure 8.8: Horizontal (left) and vertical (right) beam size measurements for different bunch currents. The IBS predictions for different equilibrium bunch length and vertical emittance values are shown in solid lines.

A second set of measurements at low energy was performed in August 2012. At this time, all the instrumentation was working properly. In order to have the best performance of the pinhole camera, a minimum total current of the machine at 60-70 mA is required, single bunch measurements were not possible. To keep the total current of the machine constant, while changing the single bunch current, a different (random) filling pattern of the machine was used

¹Landau damping is an effect similar to the damping of the oscillation amplitudes in a system of coupled harmonic oscillators, if a spread in their frequencies is introduced [84].

for each measurement. The random filling pattern was found to be the optimal way to reduce the multi-bunch instabilities while increasing the bunch current. At a total current of 60-70 mA, the contribution of the 3rd harmonic cavity to the bunch length is appreciable. For this set of measurements, the bunch length is doubled as compared to the previous one. The MI instability in this regime is suppressed and the longitudinal phase space is dominated by the third harmonic cavity. Note that it is important to keep the total current of the machine constant in each measurement, to assure the same performance of the cavity for all measurements. Figure 8.8 shows the horizontal (left) and vertical (right) beam size measurements with current. The solid lines show the IBS predictions for three different assumptions for the zero current bunch length and vertical emittance. The IBS calculations were done using the CIMP formalism. The results are very promising as the measured data seem to follow the IBS predicted behavior, in the transverse planes. However, more measurements are required in order to define correctly the zero current emittances, which is an input to the IBS calculations, and explore the phase space in order to disentangle IBS from any other collective effects. Comparison with the tracking codes is also under development.

8.2 IBS observations at Cestr-TA

The Cornell electron storage ring - Test Accelerator (Cestr-TA) used to be an e^+/e^- collider and now is used as a storage ring and test facility for DR R&D. Cestr-TA is an ideal test facility for IBS measurements, having the ability to run at different energies, the capability of high single bunch current measurements and the availability of emittance monitoring diagnostics in all three planes. The purpose of a dedicated program for IBS measurements is to establish a model for low emittance beams at high single bunch currents. The results are summarized in [76] showing that, the measurement data follow the IBS theoretical predictions at low currents, up to 4-5 mA. At higher currents, however, a big divergence between measurements and theoretical predictions is observed, especially in the vertical plane, which was not understood.

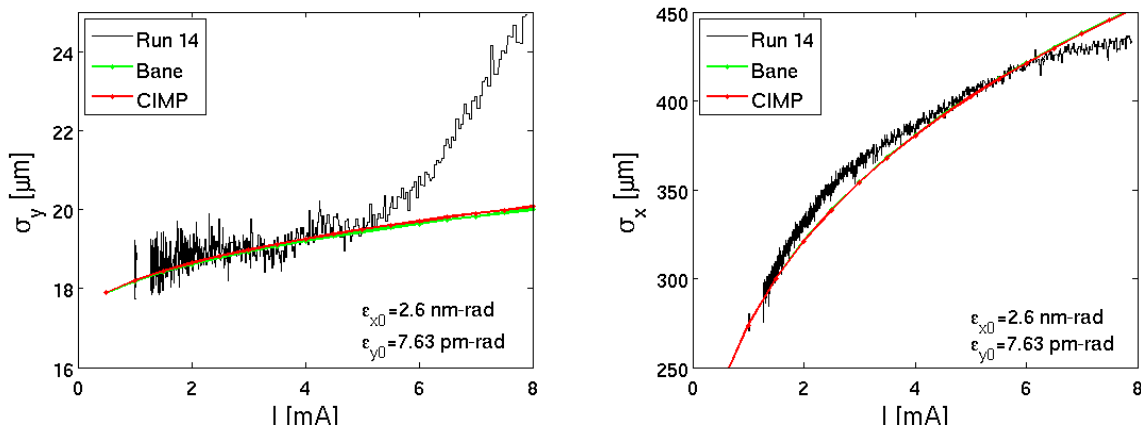


Figure 8.9: Vertical (left) and horizontal (right) beam size measurements at Cestr-TA, in comparison with the theoretical predictions using CIMP (red) and Bane (green) formalisms.

A set of data from the IBS measurements at Cesr-TA, obtained in November 2011, is presented here. Figure 8.10 shows the vertical (left) and horizontal (right) beam size measurements at different currents (blue) in comparison with the theoretical curves, using CIMP (red) and Bane (green) formalisms. The zero current horizontal emittance is $\varepsilon_{x0}=2.6$ nm, which is the theoretical one and is in agreement with the measurements at low current. A zero current vertical emittance of $\varepsilon_{y0}=7.63$ pm, was found to fit the data at low currents. It becomes obvious that at high currents ($I>5$ mA), a big divergence between the data and the theoretical predictions is observed, visible especially in the vertical plane. At the same time, a small reduction in the horizontal emittance is observed, which is again in disagreement with the IBS predictions. The IBS effect in the longitudinal plane is expected to be very small (of the order of 10% at 10 mA) and is becoming negligible if potential well distortion (PWD) bunch lengthening is also taken into account. This has been verified also by the measurement data, where small increase in bunch length has been observed, mainly due to PWD. In an attempt to find the sources of the unexpected beam size evolution with current, the effect of different betatron coupling at different currents is studied.

In the case of weak coupling, it is assumed that the betatron and dispersion functions do not change significantly, thus in the theoretical calculations always the same optics functions are considered. For the IBS calculations taking into account coupling, the procedure described in Chapter 3 (see sec. 3.8) is followed.

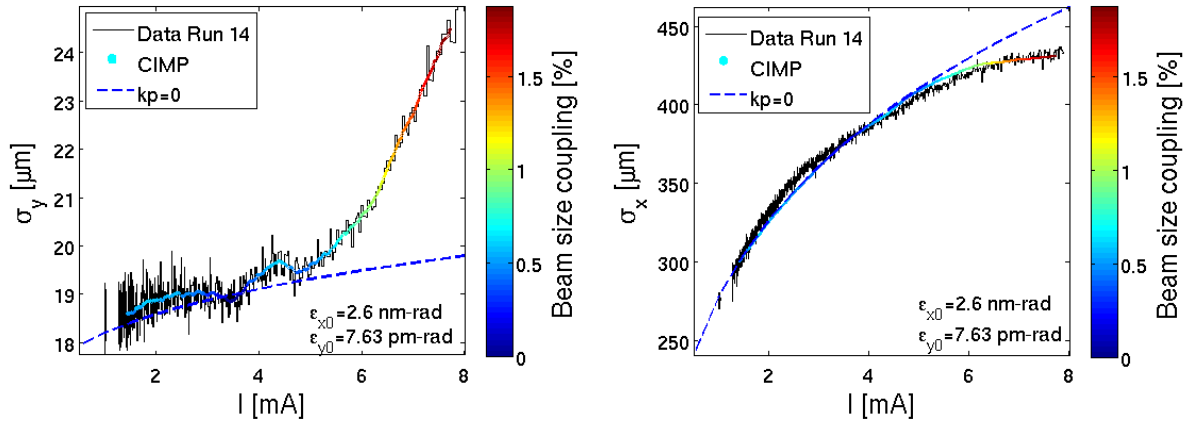


Figure 8.10: Comparison of vertical (left) and horizontal (right) beam size measurements with theoretical predictions for different betatron coupling coefficients.

In order to smooth the data points, a moving average was performed in the vertical beam size data. Figure 8.10 (left) shows the vertical beam size data at different currents (black), and the data after the moving average (colored). For each current, theoretical calculations were performed and a coupling coefficient was chosen, for which the theoretical calculation in the vertical plane is in agreement with the vertical beam size measurement, at the same current. The colorcode shows the value of this coupling coefficient. The respective calculations in the horizontal plane are then plotted on top of the horizontal beam size measurements (right part of Fig. 8.10). In both cases, the zero coupling IBS calculations at different currents are shown

with the blue dashed line. It is very interesting to notice, that the coupling coefficients chosen to fit the vertical plane, fit very well the horizontal beam size measurements too, without any further assumption or manipulation for the latest.

Figure 8.11 shows the betatron coupling coefficient with current, calculated with the above procedure. The coefficient for the initial data is shown in black and after the moving average in blue. At low currents, the betatron coupling seems to be stable with a mean value around 0.5 %. This corresponds to 0.07 % if translated to emittance coupling, which is consistent with the realistic machine coupling quoted in [76]. The scattered values at low current is due to the scattered data points in the vertical beam size measurement. With an on-set at 5 mA, a linear increase of coupling with current is observed.

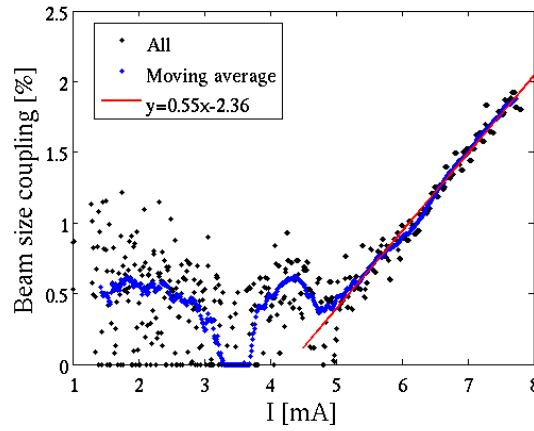


Figure 8.11: Coupling change with current, coming out from the fit procedure between the measurement data and the theoretical predictions.

The analysis presented above suggests that a possible source of the unexpected behavior of the measurement data, could be explained with a coupling change with current. This could come for example from vertical impedance kicks that change the orbit significantly. Vertical off-axis orbit passing through the sextupoles can introduce betatron coupling. In order to verify this, dedicated measurements are needed.

In summary, intrabeam scattering measurements are ongoing both at the SLS storage ring and at CEsr-TA. In both cases it is crucial to characterize the machines and develop methods to disentangle IBS from other single bunch effects, leading to current dependent emittance blow up. The results in the transverse plane are encouraging, following an IBS behavior especially in the regime where IBS is weak. More measurement results, with dedicated measurements for understanding all effects at high single bunch currents are necessary. Finally, benchmarking of the theoretical models and tracking codes with data are foreseen, especially for understanding the shape of the final beam distributions, due to the IBS effect.

Conclusions

A e^+/e^- linear collider, the Compact Linear Collider (CLIC) is under study at CERN, aiming to explore the terascale particle physics regime. The design study has been optimized at 3 TeV center of mass energy and targets a luminosity of $10^{34} \text{ cm}^{-2}\text{s}^{-1}$. In order to achieve this high luminosity, high intensity bunches with ultra low emittances, in all three planes, are required. The damping rings' purpose is to produce beams with the above specifications. Their design challenges are driven by the main parameters of the collider and the requirements of the upstream and downstream systems. The large input emittances, especially the one coming from the positron source, the ultra-low output emittance requirements and the fast repetition rate of 50 Hz, require that the damping should be done in two stages, with a pre-damping ring (PDR) and a main damping ring (DR) for each particle species. Furthermore, in the regime of ultra-low emittances with high bunch charge, intrabeam scattering becomes the predominant effect, limiting the performance of the DR. This thesis elaborated the optics design and optimization of the CLIC PDR and main DR, with beam parameters entering in a regime where intrabeam scattering (IBS) has a strong impact on the output emittances.

The conventional IBS theoretical models by Piwinski and Bjorken-Mtingwa have been studied in detail and verified over the years with experimental data only for hadron beams. A comparison between these models and two widely used high energy approximations of them, Bane and CIMP, in the presence of synchrotron radiation (SR) and quantum excitation (QE) has been presented here for three different lattices: the CLIC damping rings, the Swiss Light Source (SLS) storage ring and the Cornell electron storage ring test accelerator (Cesr-TA). A very good agreement has been demonstrated between all models, if the IBS effect is weak. However, the divergence grows when the impact of IBS to the final steady state is strong. Two multi-particle tracking codes, SIRE and IBStrack, have been bench-marked with the theoretical models, showing good agreement, especially with the Piwinski formalism. For all theoretical models and tracking codes, the same emittance evolution around the rings has been computed, for all three lattices, even when the final steady state values diverge.

The Theoretical minimum emittance (TME) cell was used as the main arc cell of both the PDR and the DR designs. TME cells can provide the minimum emittances, while being very compact. In this respect, analytical expressions for the quadrupole strengths and a complete parameterization of the TME cell has been derived, using basic linear optics arguments in thin lens approximation. In addition, stability criteria has been applied to the solutions, in both the horizontal and vertical plane. The full parameter space of the cell, including optical and geometrical parameters, can then be explored and the optimization of the cell can be done, according to any design requirement. In the absolute minimum emittance limit, careful choice of the drift lengths is important for the stability of the motion and for the chromaticity minimization. However, even for the optimal choice, the chromaticities are high in both planes, since strong focusing

is required to achieve the small dispersion in the middle of the dipole. On the other hand, for relaxed detuning factors, which are achieved for low phase advances, stability is assured almost for any choice of drift lengths and the chromaticities can be reduced significantly. In the low phase advance regime, minimization of the IBS growth rates and the Laslett space charge tune shift are also achieved, while the momentum compaction factor of the cell is large. Finally, the analytical approach was validated through a comparison with the results from the Methodical Accelerator Design code MADX.

The first stage of damping of the electron and positron beams is done in the PDR. Emphasis was given to the design of the positron ring, since the parameters of the positron beam are more demanding. A racetrack shape of the ring was chosen, as this gives the most compact design if only two dispersion free straight sections are needed. Due to the large beam emittances in all three planes, the lattice design of the PDR was focused on the momentum acceptance and the dynamic aperture optimization. Based on the analytical parameterization of the TME cell and the resonance free lattice concept, an optimal number of 17 cells per arc was chosen, tuned at low phase advances with $\mu_x=5/17$ and $\mu_y=3/17$. This choice minimized the excitation of non-linear resonances and the detuning with amplitude, while providing the required output emittance. The repetition rate of 50 Hz imposes the requirement of damping wigglers, in order to achieve the fast damping times. A permanent magnet wiggler of $B_w=1.9$ T peak field, $\lambda_w=30$ cm period length and $g = 41$ mm gap was chosen, in order to fit the large incoming beam and produce the required output emittance within the repetition time. In conclusion, this design provides adequate dynamic aperture and momentum acceptance for efficient injection of the large incoming beam and produces beams at extraction with the required parameters for injection to the main damping rings.

Unlike the PDR, the main damping rings design is focused in the generation of the ultra-low emittance, for which, the effect of IBS becomes predominant. Previous design stages, although provided the required performance parameters, suffered from technological and beam dynamics limitations. In particular, the output emittances were strongly dominated by the IBS effect and the Laslett space charge tune shift was large, while the stable phase was in the non-linear regime of the RF voltage. Numerical calculations of the output emittances at different energies, taking into account the effect of IBS, lead to the choice of a higher operational energy than before, from 2.424 GeV to 2.86 GeV. This reduced the IBS effect by a factor of 2, while the output emittances remained the same. Based on the analytical parameterization of the TME cell, the arc cell was modified, choosing lower field and thus longer length of the dipole and reducing the horizontal phase advance. This increased the momentum compaction factor, and thus the longitudinal emittance of the beam, giving room for improvement in the performance of the RF system. The reduction of the phase advances led to the reduction of the intrabeam scattering growth rates and Laslett space charge tune shift as well. The necessity of damping wigglers to achieve the ultra-low emittance and their effect on the beam parameters was discussed. Numerical calculations of the output emittances for different wiggler working points, showed that the highest wiggler peak field and the shortest period length are necessary for reaching the smallest possible emittances. On the other hand, the effect of IBS in that case becomes extremely strong. For reducing the blow-up due to IBS, still the highest fields are interesting but for moderate period lengths. As CLIC is planned to be build in different energy stages, high luminosity in lower energies

requires higher bunch charge. This will have an impact in the performance of the DR, as the IBS effect will be further enhanced. In this respect, parametric scans were performed, studying the achievable emittances at different currents, under the influence of the IBS effect. Scaling laws for the dependence of the horizontal emittance on the bunch charge and the interdependence between the IBS effect in the horizontal and longitudinal planes were derived. Finally, a design layout and the performance parameters of the ring were presented.

The ultimate goal for understanding the IBS effect in the presence of SR and QE, even in regimes where it has a big impact on the final emittances, is the bench-marking of the theoretical models and the tracking codes with measurements. SLS and CEsR-TA are ideal test facilities for IBS measurements. Data acquired in both machines showed that it is essential to disentangle IBS from other current dependent effects, leading to emittance blow up, for the bench-marking of the models with measurements. At weak IBS regimes, good agreement between theoretical models and measurements has been demonstrated. Measurement efforts are still in progress for the strong IBS regime in both machines. In particular, dedicated measurements for the development of an energy spread model at high currents is ongoing at the SLS and optics and coupling measurements at different currents are foreseen.

Analytical calculations for the TME cell

The transfer matrices for the quadrupoles, drifts and dipoles in thin lens approximation are defined as:

$$M_{quad} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}, \quad (A.1)$$

where for the focusing quad $f > 0$ while for the defocusing quad $f < 0$.

$$M_{drift} = \begin{pmatrix} 1 & l_{dr} \\ 0 & 1 \end{pmatrix} \quad (A.2)$$

$$M_{dip} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \quad (A.3)$$

The transfer matrix for the half cell is defined by the multiplication of the matrices of the elements:

$$M_{half} = M_{drift,s_3} \cdot M_{quad,f_2} \cdot M_{drift,s_2} \cdot M_{quad,f_1} \cdot M_{drift,s_1} \cdot M_{dip} \quad (A.4)$$

$$M_{x,half} = \begin{pmatrix} \frac{f_1(f_2-s_3)+s_2s_3-f_2(s_2+s_3)}{f_1f_2} & \frac{-(-s_2s_3+f_2(s_2+s_3))(2s_1+l_d)+f_1(-s_3(2s_1+2s_2+l_d)+f_2(2s_1+2s_2+2s_3+l_d))}{2f_1f_2} \\ -\frac{f_1+f_2-s_2}{f_1f_2} & \frac{f_1(2f_2-2s_1-2s_2-l_d)-(f_2-s_2)(2s_1+l_d)}{2f_1f_2} \end{pmatrix} \quad (A.5)$$

$$M_{y,half} = \begin{pmatrix} \frac{f_1(f_2+s_3)+s_2s_3+f_2(s_2+s_3)}{f_1f_2} & \frac{(s_2s_3+f_2(s_2+s_3))(2s_1+l_d)+f_1(s_3(2s_1+2s_2+l_d)+f_2(2s_1+2s_2+2s_3+l_d))}{2f_1f_2} \\ \frac{f_1+f_2+s_2}{f_1f_2} & \frac{f_1(2f_2+2s_1+2s_2+l_d)+(f_2+s_2)(2s_1+l_d)}{2f_1f_2} \end{pmatrix} \quad (A.6)$$

The dispersion transfer matrix in the horizontal plane is given in the same way, where the dispersion transfer matrices for each element are:

$$D_{quad} = \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (A.7)$$

$$D_{drift} = \begin{pmatrix} 1 & l_{dr} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (A.8)$$

$$D_{\text{dip}} = \begin{pmatrix} 1 & s & \frac{s^2}{2\rho} \\ 0 & 1 & \frac{s}{\rho} \\ 0 & 0 & 1 \end{pmatrix} \quad (\text{A.9})$$

and the half cell matrix is:

$$D_{x,\text{half}} = \begin{pmatrix} M_{x,\text{half}}^{[1,1]} & M_{x,\text{half}}^{[1,2]} & \frac{l_d((s_2 s_3 - f_2(s_2 + s_3))(4s_1 + l_d) + f_1(-s_3(4(s_1 + s_2) + l_d) + f_2(4(s_1 + s_2 + s_3) + l_d)))}{8f_2 f_2 \rho} \\ M_{x,\text{half}}^{[2,1]} & M_{x,\text{half}}^{[2,2]} & -\frac{l_d((f_2 - s_2)(4s_1 + l_d) + f_1(-4f_2 + 4(s_1 + s_2) + l_d))}{8f_1 f_2 \rho} \\ 0 & 0 & 1 \end{pmatrix} \quad (\text{A.10})$$

Because of the symmetry of the problem, the full cell matrix can be written in the form:

$$M_{\text{cell},x,y} = \begin{pmatrix} M_{\text{half}}^{[2,2]} & M_{\text{half}}^{[1,2]} \\ M_{\text{half}}^{[2,1]} & M_{\text{half}}^{[1,1]} \end{pmatrix} \cdot M_{x,y,\text{half}} \quad (\text{A.11})$$

The twiss transfer matrix is defined by the elements of the transfer matrix $M_{x,y}$ as:

$$B_{x,y} = \begin{pmatrix} M_{x,y}^{[1,1]2} & -2M_{x,y}^{[1,1]} M_{x,y}^{[1,2]} & M_{x,y}^{[1,2]2} \\ -M_{x,y}^{[1,1]} M_{x,y}^{[2,1]} & M_{x,y}^{[1,1]} M_{x,y}^{[2,2]} + M_{x,y}^{[1,2]} M_{x,y}^{[2,1]} & -M_{x,y}^{[1,2]} M_{x,y}^{[2,2]} \\ M_{x,y}^{[2,1]2} & -2M_{x,y}^{[2,1]} M_{x,y}^{[2,2]} & M_{x,y}^{[2,2]2} \end{pmatrix}, \quad (\text{A.12})$$

where x denotes the horizontal and y the vertical plane.

The optical functions at a point 1 knowing the optical functions at point 0 can then be defined as:

$$\{\alpha_1, \beta_1, \gamma_1\} = B_{x,y} \cdot \{\alpha_0, \beta_0, \gamma_0\} \quad (\text{A.13})$$

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