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**Study of the $K^+ \rightarrow e^+ \nu_e \gamma$ decay
with the NA62 experiment**

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Abstract

The present thesis is focused on the measurement of the form factors of the $K^+ \rightarrow e^+ \nu_e \gamma$ decay ($K_{e2\gamma}$) with data collected by the NA62 experiment in 2007 during a preliminary phase of data taking, dedicated to a precision test of lepton flavour universality. For this phase, the NA62 collaboration used the pre-existing apparatus of the NA48/2 experiment. The measurement of the form factors is a test for the effective theories of strong interaction at low energies as the Chiral Perturbation Theory (ChPT) and the Light Front Quark Model (LFQM). The only kinematic configuration accessible to the analysis is the structure-dependent configuration in which the photon is emitted preferentially with positive helicity (SD⁺ term).

The measurement of the $K_{e2\gamma}$ form factors is obtained by analysing 11170 reconstructed $K_{e2\gamma}(\text{SD}^+)$ candidates with 4% background contamination, about ten times the statistics collected by earlier experiments. The final result of the analysis, assuming ChPT at $\mathcal{O}(p^6)$, is:

$$\begin{aligned} F_V(0) + F_A(0) &= 0.127 \pm 0.002_{\text{stat}} \pm 0.010_{\text{syst}} \\ \lambda &= 0.60 \pm 0.05_{\text{stat}} \pm 0.39_{\text{syst}} \end{aligned}$$

The LFQM prediction of the form factors was also tested but it was found to be completely inconsistent with data.

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Chapter 1

Physical motivations

1.1 Introduction

Since the first observation of the $\pi^+ \rightarrow e^+ \nu$ decay at CERN in 1958 [1, 2], there has been a considerable interest on the radiative decays of mesons. At that time, the measurement of the branching fraction:

$$R_\pi = \frac{\Gamma(\pi \rightarrow e \nu)}{\Gamma(\pi \rightarrow \mu \nu)} \quad (1.1)$$

gave one of the first experimental verifications of the V-A structure of the weak interactions proposed by Feynman and Gell-Mann [3] to explain the parity violation observed in nuclear β decay [4–6]. They postulated that weak interactions of leptons are generated by an equal mixture of vector and axial-vector currents [7–9]:

$$\mathcal{L}_{\text{weak}} = \frac{G_F}{\sqrt{2}} J_{\text{weak}}^\mu J_{\mu, \text{weak}}, \quad J_{\text{weak}}^\mu = J_V^\mu - J_A^\mu \quad (1.2)$$

where:

$$J_V^\mu = \sum_{\ell=e, \mu} \bar{\ell} \gamma^\mu \nu_\ell, \quad J_A^\mu = \sum_{\ell=e, \mu} \bar{\ell} \gamma^\mu \gamma_5 \nu_\ell \quad (1.3)$$

$G_F = 1.166 \cdot 10^{-5} \text{ GeV}^{-2}$ [10] being the Fermi constant, ℓ and ν_ℓ the lepton and neutrino fields respectively, and γ_μ , γ_5 the Dirac matrices [8, 9]. The observation of other leptonic processes such as $\pi \rightarrow \mu \nu$ and $\mu \rightarrow e \nu \bar{\nu}$, similar in strength and structure to β -decay, led to the postulate of “lepton universality”: all leptonic V-A currents are coupled to one another with equal strength given by G_F . In the same years, new measurements [11, 12] had become sufficiently accurate to justify a calculation of the effects of electromagnetic corrections. In 1959, the works of Kinoshita and Berman [13, 14] showed that radiative corrections can give large contributions to R_π , of the order of 10%; it was realized that the

evaluation of high order corrections were necessary to achieve better theoretical precision. Infinities encountered in the calculations of virtual corrections arising from Quantum Electro Dynamics (QED) required a redefinition of R_π in order to get rid of the infra-red divergences and obtain a finite value of the decay amplitude:

$$R_\pi = \frac{\Gamma(\pi \rightarrow e\nu + \pi \rightarrow e\nu\gamma)}{\Gamma(\pi \rightarrow \mu\nu + \pi \rightarrow \mu\nu\gamma)} \equiv \frac{\Gamma(\pi \rightarrow e\nu(\gamma))}{\Gamma(\pi \rightarrow \mu\nu(\gamma))} \quad (1.4)$$

where (γ) indicates that an extra real photon is present in the final state of the process. A similar quantity can be defined for kaons¹:

$$R_K = \frac{\Gamma(K \rightarrow e\nu(\gamma))}{\Gamma(K \rightarrow \mu\nu(\gamma))} \quad (1.5)$$

The Standard Model of particle physics (SM) [15–17] elucidates the origin of the V-A interaction extending weak interactions to the quarks sector. In the SM, the decay amplitude at tree level of the process $P^+ \rightarrow \ell^+ \nu_\ell$ (also called $P_{\ell 2}$, where $P = \pi, K$ denotes the pseudo-scalar meson and $\ell = e, \mu$) can be written as [8]:

$$\mathcal{M}^{(\text{tree})}(P \rightarrow \ell\nu) = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} f_P p^\mu \bar{u}_\ell \gamma_\mu (1 - \gamma_5) v_\nu \quad (1.6)$$

where u and v are the spinors of the outgoing particles, $V_{q_1 q_2}$ is the CKM element between the constituent quarks $q_1 q_2$ in P , p_μ is the four-momentum of the parent meson and the parameter f_P is the decay constant²; the experimental values of the CKM elements for the $u \rightarrow d$ and $u \rightarrow s$ transitions are respectively $|V_{ud}| = 0.974$ and $|V_{us}| = 0.225$ [10], while the measured values of f_P for pion and kaon decays are $f_\pi = 130.41 \pm 0.20$ MeV and $f_K = 156.1 \pm 0.8$ MeV [10]. By using momentum conservation and the Dirac equation, Eq. 1.6 can be rewritten in the form:

$$\mathcal{M}^{(\text{tree})}(P \rightarrow \ell\nu) = \frac{G_F}{\sqrt{2}} V_{q_1 q_2} m_\ell f_P \bar{u}_\ell (1 - \gamma_5) v_\nu \quad (1.7)$$

The amplitude is proportional to the lepton mass m_ℓ and vanishes for $m_\ell \rightarrow 0$. This can be interpreted as a physical consequence of the term $(1 - \gamma_5)$ which is the helicity projection operator for massless leptons; in weak interactions only left-handed massless

¹ R_K is defined to include Inner Bremsstrahlung radiation, ignoring Structure Dependent contributions (see Section 1.3).

²The decay constant f_P is defined through the matrix element of the hadronic axial current $J_A^{\mu;a}$ between the one-meson state and the vacuum: $\langle 0 | J_A^{\mu;a} | P^b \rangle = i \frac{f_P}{\sqrt{2}} p^\mu \delta^{ab}$, where a, b are the isospin indices and p_μ is the meson four-momentum [9].

	Technique	PID	$R_K(\times 10^{-5})$	$\delta R_K/R_K$
KLOE [18]	In-flight ($\phi \rightarrow KK$)	E/p and TOF	2.493 ± 0.031	1.3%
NA62 [19]	In-flight ($P_K = 75 \text{ GeV}/c$)	E/p	2.488 ± 0.010	0.4%
TREK [20]	Stopped K^+	TOF and C	Proposal	0.2%
PDG [10]			2.45 ± 0.11	5%
SM [21]			2.477 ± 0.001	0.04%

Table 1.1: A summary of the recent measurements of R_K ratio and the corresponding SM prediction.

	Technique	PID	$R_\pi(\times 10^{-4})$	$\delta R_\pi/R_\pi$
PIBETA [22]	Stopped π^+	E/p	1.2346 ± 0.0050	0.4%
Britton et al. [23]	Stopped π^+	$\pi \rightarrow \mu \rightarrow e$	1.2265 ± 0.0056	0.4%
PEN [24]	Stopped π^+	E/p	On-going	$< 0.05\%$
PIENU [25]	Stopped π^+	$\pi \rightarrow \mu \rightarrow e$	On-going	$< 0.1\%$
PDG [10]			1.230 ± 0.004	0.3%
SM [21]			1.2352 ± 0.0001	0.01%

Table 1.2: A summary of the recent measurements of R_π ratio and the corresponding SM prediction.

particles can be emitted by a vertex. If $m_e = 0$ only a positron with positive helicity and a neutrino with negative helicity are allowed but angular momentum conservation would prohibit the transition (“helicity suppression”). However, for electron and muon, both positive and negative helicity states are mixed by an amount proportional to the mass, resulting in non-zero decay rates. To lowest order, the decay width for the process $P \rightarrow \ell\nu$ is:

$$\Gamma^{(\text{tree})}(P \rightarrow \ell\nu) = \frac{G_F^2}{8\pi} |V_{q_1 q_2}|^2 f_P^2 m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right) \quad (1.8)$$

where M_P is the meson mass. The ratio of decay widths for $P \rightarrow e\nu$ and $P \rightarrow \mu\nu$, at tree level, is independent of f_P and $V_{q_1 q_2}$:

$$R_P^{(\text{tree})} = \frac{\Gamma(P \rightarrow e\nu)}{\Gamma(P \rightarrow \mu\nu)} = \frac{m_e^2 (M_P^2 - m_e^2)^2}{m_\mu^2 (M_P^2 - m_\mu^2)^2} \quad (1.9)$$

What makes R_P interesting is that hadronic uncertainties cancel to a very large extent in these ratios and that the electronic decay modes $K \rightarrow e\nu$ and $\pi \rightarrow e\nu$ are strongly suppressed in the SM. As a consequence, R_P is very sensitive to possible contributions due to New Physics (NP), in the sense that heavy new particles may contribute to the decay amplitudes on the same footing as SM particles, and its measurements give the best

experimental constraints on lepton universality so far. At present, the SM predictions for R_π and R_K are precise at level below 10^{-3} , one order of magnitude better the current experimental accuracy (Tab.1.1 and Tab. 1.2). In general, π beam intensities are much higher than K^+ intensities and the achievable experimental uncertainty is expected to be better for R_π measurements compared to R_K measurements.

1.2 Lepton Universality in Charged-Current Interactions

In the SM, the charged-current (CC) interactions are governed by an universal coupling g [8, 9, 26, 27]:

$$\mathcal{L}_{\text{CC}} = \frac{g}{2\sqrt{2}} \left(W_\mu^- \sum_\ell \bar{\ell} \gamma^\mu (1 - \gamma_5) \nu_\ell + W_\mu^- \sum_{i,j} \bar{u}_i \gamma^\mu (1 - \gamma_5) V_{ij} d_j \right) + \text{H.c.} \quad (1.10)$$

where $W_\mu^\pm$ is the $W^\pm$ boson field, $\ell = e, \mu, \tau$ is the lepton field, u_i and d_j are the “up-type” ($i = u, c, t$) and the “down-type” ($j = d, s, b$) quark fields, respectively, and V_{ij} is the CKM unitary matrix [8, 9]. One of the features of $\mathcal{L}_{\text{CC}}$ is the universality of the lepton couplings to the W boson (“lepton universality”), *i.e.* the fact that the coupling of the W to leptons does not depend on their flavour ($g_e = g_\mu = g_\tau = g$). Concerning the quark sector, the universality of the charged-current interaction is spoiled by the presence of the CKM matrix.

The lepton sector provides a clean environment to study the structure of the weak currents and the universality of their couplings to the gauge bosons. In the pure leptonic transitions, such as $\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau$, strong interactions are only present through small higher-order corrections. In processes, such as the semileptonic decay $\tau \rightarrow \pi \nu_\tau$ and the leptonic decay $\pi \rightarrow \mu \nu_\mu$, hadronization is present; however, by taking appropriate ratios of different transitions with identical hadronic components, the QCD effects cancel to a very good approximation.

1.2.1 Tests of lepton universality

1.2.1.1 $W \rightarrow \ell \bar{\nu}_\ell$ decays

Lepton universality can be tested by measuring the branching ratios of the $W \rightarrow \ell \bar{\nu}_\ell$ decays: according to the prediction of the SM, these decays should occur with the same rate, except for phase space corrections, for all generations of leptons.

The partial width of the decay, without the assumption of lepton universality, is

proportional to:

$$\Gamma(W \rightarrow \ell \bar{\nu}_\ell) \propto g_\ell^2 M_W \quad (1.11)$$

where M_W is the mass of the W boson. The effects of the fermion masses and the radiative corrections are negligible compared to the experimental precision of the branching ratio measurements and have been neglected. The latest experimental results on the ratios of the coupling constants g_μ/g_e and g_τ/g_e , are obtained by measurements done at LEP and Tevatron colliders [10]:

$$\left(\frac{g_\mu}{g_e}\right) = 0.983 \pm 0.018; \quad \left(\frac{g_\tau}{g_e}\right) = 1.047 \pm 0.023$$

In literature, also the following quantity is reported [28]:

$$R_{\tau\ell}^W = \frac{\Gamma(W \rightarrow \tau \bar{\nu}_\tau)}{[\Gamma(W \rightarrow e \bar{\nu}_e) + \Gamma(W \rightarrow \mu \bar{\nu}_\mu)]/2} = 1.055 \pm 0.023$$

g_τ/g_e (or, equivalently, $R_{\tau\ell}^W$) differs from the SM expectation by $\approx 2\sigma$.

1.2.1.2 Leptonic decays of τ and μ

An experimentally clean way to test lepton universality in charged-current interactions is through the leptonic decays of the τ , where, as in the μ decay, the interaction between initial and final state particles is mediated by a virtual W boson. The only difference with respect to the μ decay is that several final states are kinematically allowed: $\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau$, $\tau \rightarrow e \bar{\nu}_e \nu_\tau$, $\tau \rightarrow \nu_\tau \bar{u} d$ and $\tau \rightarrow \nu_\tau \bar{u} s$. Owing to the universality of the W-couplings in SM, all these decay modes have equal amplitudes (if final fermion masses and QCD interactions are neglected).

In general, the partial decay widths for a heavier lepton L decaying to a lighter lepton l are, neglecting neutrino masses and radiative corrections [8]:

$$\Gamma(L \rightarrow \nu_L l \bar{\nu}_l) \propto (g_L g_l)^2 m_L^5 \quad (1.12)$$

with m_L is the mass of the heavier lepton. Thus, the measurements of the branching ratios of the two leptonic τ decays can be used to determine the ratio g_μ/g_e , while the comparison of the decay $\tau \rightarrow e \bar{\nu}_e \nu_\tau$ with the decay $\mu \rightarrow e \bar{\nu}_e \nu_\mu$ allows for a measurement of g_τ/g_μ [26, 27]. The most precise measurements of these τ branching ratios come from e^+e^- colliders (LEP and PEP-II), in which the τ particles are produced in pairs. The

current experimental results on g_μ/g_e and g_τ/g_μ are [29]:

$$\left(\frac{g_\mu}{g_e}\right) = 1.0018 \pm 0.0014; \quad \left(\frac{g_\tau}{g_\mu}\right) = 1.0006 \pm 0.0021$$

in good agreement with the hypothesis of lepton universality. These results are about 10 times more precise than the results obtained with $W \rightarrow \ell \bar{\nu}_\ell$ decays.

1.2.1.3 $P \rightarrow \ell \bar{\nu}_\ell$ and $\tau \rightarrow \nu_\tau P$ decays and other tests

The ratio g_μ/g_e can be measured also through leptonic decays of mesons such as $P \rightarrow \ell \bar{\nu}_\ell$ ($P = \pi, K$), as described in Section 1.1 (Eq. 1.9). The effects of the strong interactions are contained in the constants f_P , which parameterize the hadronic matrix element of the corresponding weak current. Taking the ratios of widths (R_P) of the electron and muon modes, involving the same meson P , the dependence on these decay constants factors out. Therefore, those ratios can be predicted with an high accuracy. The latest measurements of R_π and R_K are reported respectively in Tab. 1.2 and Tab. 1.1. Using the theoretical predictions [21], which are one order of magnitude more precise than the current experimental accuracy, these results translate into:

$$\left(\frac{g_e}{g_\mu}\right)_{R_\pi} = 0.9979 \pm 0.0016; \quad \left(\frac{g_e}{g_\mu}\right)_{R_K} = 1.0022 \pm 0.0020$$

The experimental values are in excellent agreement with the SM expectations.

Similarly, τ decays partial widths to hadrons, $\tau \rightarrow h \nu_\tau$ ($h = \pi, K$), compared to the same hadron decay to muons, $h \rightarrow \mu \bar{\nu}_\mu$, measure the τ - μ universality [26, 29]. The current results for the ratio g_τ/g_μ for the pion and the kaon mode are:

$$\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9956 \pm 0.0031; \quad \left(\frac{g_\tau}{g_\mu}\right)_K = 0.9852 \pm 0.0072$$

in good agreement with the SM expectation $g_\tau/g_\mu = 1$.

Recently, other tests of lepton universality have been carried out in the b -flavoured mesons by the BaBar collaboration, such as the ratio of the decay widths of the $\Upsilon(1S)$, $R_{\tau\mu}(\Upsilon(1S)) = \frac{\Gamma(\Upsilon(1S) \rightarrow \tau^+ \tau^-)}{\Gamma(\Upsilon(1S) \rightarrow \mu^+ \mu^-)}$ sensitive to g_τ/g_μ , and the ratio $R_{\tau/\ell} = \frac{\Gamma(B \rightarrow D \tau \bar{\nu}_\tau)}{\Gamma(B \rightarrow D \ell \bar{\nu}_\ell)}$. The measurement of $R_{\tau\mu}(\Upsilon(1S))$ by BaBar [30] shows no deviation from the expected SM value, while the experimental result for $R_{\tau/\ell}$ [31] is significantly larger ($\approx 3\sigma$) than the SM value.

1.3 The $P \rightarrow \ell \nu \gamma$ radiative decay

The radiative process $P \rightarrow \ell \nu \gamma$ (also denoted $P_{\ell 2\gamma}$) plays a crucial role in the determination of R_P because it cancels out the infra-red divergences arising in radiative corrections. However, the $P \rightarrow \ell \nu \gamma$ decay deserves interest on its own being sensitive, in suitable kinematic configurations, to the hadron structure and strong dynamics at low energies. Radiative meson decays have been studied for the first time in the pion mode $\pi \rightarrow e \nu \gamma$ by Vaks and Ioffe in 1958 [32] and more in detail by Bludman and Young in 1960 [33]. These authors described the $\pi \rightarrow \ell \nu \gamma$ decay amplitude as the sum of two contributions related to distinct physical processes (Fig. 1.1):

$$\mathcal{M}(\pi \rightarrow \ell \nu \gamma) = \mathcal{M}_{\text{IB}} + \mathcal{M}_{\text{SD}}$$

$\mathcal{M}_{\text{IB}}$ is the “Inner Bremsstrahlung” (IB) amplitude: the hadron emits the lepton and the neutrino via the axial-vector current and the photon is radiated from the charged particle. It’s the “trivial” part of the process in the sense that effects of strong interactions are absent; it is calculated using the usual rules of QED and like $P \rightarrow \ell \nu$ decay it’s helicity suppressed. The IB energy spectrum diverges at $E_\gamma = 0$ and falls off very rapidly as the energy increases, so that IB photons are mostly of low energy (“soft photons”).

$\mathcal{M}_{\text{SD}}$ is the “Structure Dependent” (SD) amplitude in which strong interactions are acting. The SD amplitude describes the emission of photons from intermediate states generated by strong interactions (*e.g.* hadronic states and quarks) and the description of the complex mechanism involved at that level requires models.

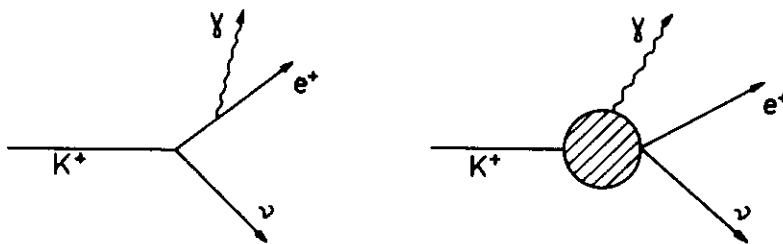


Figure 1.1: Diagrams contributing to the $K_{e2\gamma}$ decay amplitude (from [34]). (Left) Inner Bremsstrahlung (IB) process. (Right) Structure Dependent (SD) process. The vertex marked by the circle symbolizes the emission of photons from intermediate states generated by strong interactions.

The information about the hadronic structure is encoded in the form factors F_V and

F_A that parametrize the effects of strong interactions and are related to the vector and axial-vector part of the weak currents, respectively³; in general, the form factors are functions of the squared four-momentum of the lepton pair $\ell\nu$, $q^2 = (p_K - p_\gamma)^2$, where p_K and p_γ are the four-momenta of the kaon and the photon, respectively.

In 1961, Neville [36] drew attention to $K_{\ell 2\gamma}$ decays in which the pion is replaced by a K meson. Terms in the spectra proportional to lepton mass were retained in the computation so that the results were applicable to the muon modes $K_{\mu 2\gamma}$ and $\pi_{\mu 2\gamma}$. It's useful to compare the radiative decays of kaons and pions. The formulas for IB and SD rates can be derived qualitatively by dimensional arguments: in fact, the IB decay width have to be proportional to the the electron charge squared e^2 (electromagnetic coupling), G_F^2 (weak coupling), m_ℓ^2 (helicity suppression), $|V_{q_1 q_2}|^2$ (CKM element) and f_P^2 (describing the hadronic vertex), so that:

$$\Gamma(P \rightarrow \ell\nu\gamma)_{\text{IB}} \propto \alpha G_F^2 |V_{q_1 q_2}|^2 f_P^2 m_\ell^2 M_P \quad (1.13)$$

where $\alpha = e^2/4\pi \approx 1/137$ is the fine-structure constant [10]. On the other hand, the SD amplitude is not helicity suppressed and the hadronic structure is parametrized by form factors F_V and F_A (chosen to be dimensionless) so that, for dimensional reasons:

$$\Gamma(P \rightarrow \ell\nu\gamma)_{\text{SD}\pm} \propto \alpha G_F^2 |V_{q_1 q_2}|^2 (F_V \pm F_A)^2 M_P^5 \quad (1.14)$$

As a consequence, the absolute Bremsstrahlung rate is slightly smaller in kaons than in pions, while SD terms are about two order of magnitude more prominent in K decay than π decay⁴. A test of time reversal invariance (“T-invariance”) in $K_{\mu 2\gamma}$ decays was also suggested by Neville [36]. In fact, if T holds in processes with weak and electromagnetic interactions, the form factors must be real. If T invariance is broken, these form factors acquire an imaginary part which for maximal breaking could be of the same order of magnitude as the real part. Then T-odd terms proportional to $\text{Im}(F_V)$ and $\text{Im}(F_A)$ can occur in the formula of the differential decay rate. These contain the lepton ($\boldsymbol{\sigma}$) or photon ($\boldsymbol{\varepsilon}$) polarization vectors and are of the type:

$$\boldsymbol{\sigma} \cdot (\mathbf{P}_\gamma \times \mathbf{P}_\ell)$$

³In literature there are several definitions of form factors. In the present thesis we follow the definition given in [35], so that F_V and F_A are dimensionless.

⁴For the SD^+ term, using the current experimental values of the form factors [10], one obtains: $\Gamma(K \rightarrow \ell\nu\gamma)_{\text{SD}^+} / \Gamma(\pi \rightarrow \ell\nu\gamma)_{\text{SD}^+} \approx 300$.

or

$$\varepsilon \cdot (\mathbf{P}_\gamma \times \mathbf{P}_\ell)$$

where $\mathbf{P}_\ell$ and $\mathbf{P}_\gamma$ are the momentum vectors of the lepton and the photon, respectively. Thus, if the lepton has any net polarization P_T perpendicular to the plane defined by the lepton and photon directions, T invariance is violated in the process. The only measurement of the muon polarization in $K_{\mu 2\gamma}$ decays has been done by the E246 experiment at KEK in 2003, in which muons were stopped and polarization measured from the decay to positrons. The result $P_T = (-0.64 \pm 1.85) \cdot 10^{-2}$ is consistent with no T-violation in this decay [37].

Before the development of effective quantum field theories based on Quantum Chromodynamics (QCD), the computations involving strong interactions relied on general methods as dispersion relations, current algebra and sum rules. The main drawbacks of these old-fashioned calculations were that they did not give clear and accurate predictions and did not follow from a unique theoretical framework. The first attempts to evaluate pion and kaon form factors rested on the assumption that the radiative photon is emitted only from the resonance states with lower masses dominating over other intermediate states (“vector meson dominance” (VMD)). In $\pi_{\ell 2\gamma}$ decays, vector and axial form factors were supposed to be generated by the exchange of the lowest states with quantum numbers allowed by conservation rules (*e.g.* spin, parity and G-parity), the A_1 and ρ mesons, while in $K_{\ell 2\gamma}$ decays, they received the main contribution from K^* and K_A mesons, respectively⁵ (Fig. 1.2). The results of these calculations gave the form factors as a function of the coupling constants of the intermediate process. As an example, the $K_{\ell 2\gamma}$ vector form factor was usually written as [38]:

$$F_V(q^2) \propto \frac{g_{K^*} \cdot G_{K^*K\gamma}}{M_{K^*}^2 - q^2} \quad (1.15)$$

where $G_{K^*K\gamma}$ and g_{K^*} are the coupling constants of the $K^+ \rightarrow K^*\gamma$ and $K^* \rightarrow \ell\nu$ intermediate processes, respectively. Then, $SU(3)$ symmetry and sum rules were used to relate these constants to couplings measured from other known processes. Similar expressions hold for the $\pi_{\ell 2\gamma}$ decay. Various calculations of the $K_{\ell 2\gamma}$ vector form factors agreed to about 20% whereas estimates for the ratio $\gamma_K \equiv F_A(0)/F_V(0)$ yielded incoherent results.

During the 60’s and 70’s, form factors were estimated also in other theoretical frameworks. A set of models as the static and relativistic quark models [39] dealt from the

⁵Main proprieties: vector meson ρ ($J^{PC} = 1^{--}$, $m_\rho = 770$ MeV/ c^2); axial-vector meson A_1 now called a_1 ($J^{PC} = 1^{++}$, $m_{A_1} = 1230$ MeV/ c^2); vector meson K^* ($J^P = 1^-$, $m_{K^*} = 892$ MeV/ c^2); axial-vector meson K_A now called K_1 ($J^P = 1^+$, $m_{K_A} = 1272$ MeV/ c^2).

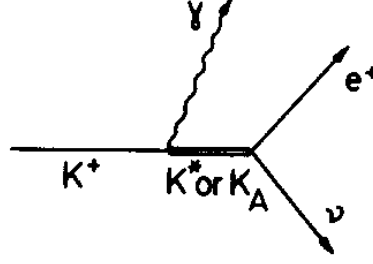


Figure 1.2: Diagram contributing to the $K_{e2\gamma}$ SD amplitude with K^* and K_A as intermediate states (from [34]).

beginning with the quark content of pions and kaons. A complete review of the different approaches to the computation of the form factors can be found in [39]. Despite the theoretical efforts, there were no major improvements on the predictions for strong interacting particles phenomena until the advent of effective field theories based on the symmetries of QCD. In the next sections, modern approaches to strong interactions at low energies, as Chiral Perturbation Theory and Light Front Quark Model will be discussed. Particular attention will be devoted to $K_{e2\gamma}$ decays and a detailed description of its kinematics will be presented.

1.4 The Chiral Perturbation Theory (ChPT)

The current theory of strong interactions is Quantum Chromodynamics (QCD), a theory formulated in terms of quarks and gluon fields built on the principle of gauge invariance with the color gauge group $SU(3)_c$. The large strong coupling constant at low energies (a consequence of the phenomenon of asymptotic freedom) prevents use of a perturbative approach, so there is no direct link between QCD's fundamental degrees of freedom and the relevant hadronic degrees of freedom as observed in the spectrum of mesons and baryons. An alternative approach for studying strong interactions at low energies is to build an effective theory that deals from the beginning with mesons and baryons [40]. The basic idea of an effective theory is the following [9, 41–45]: as long as one is only interested in a particular range of distances or energies, scales much bigger or much smaller than this range should not influence the description of the system in question too strongly. In particular, the dynamics at low energies is supposed to be independent on the details of the dynamics at high energies. As a result, low energy physics can be described using an effective Lagrangian that contains only the relevant degrees of freedom, ignoring the

heavy ones, and that shares the symmetries of the underlying theory. In order to build the effective theory for strong interactions, we have to investigate the symmetries of the QCD Lagrangian. For this purpose, we decompose the quark field into its chiral components:

$$q = \frac{1}{2} (1 - \gamma_5) q + \frac{1}{2} (1 + \gamma_5) q \equiv q_L + q_R \quad (1.16)$$

The QCD lagrangian is then:

$$\begin{aligned} \mathcal{L}_{\text{QCD}} &= \mathcal{L}_{\text{QCD}}^0 + \mathcal{L}_{\text{QCD}}^m + \dots, \\ \mathcal{L}_{\text{QCD}}^0 &= \frac{1}{2} \text{Tr} G_{\mu\nu}^a G^{a,\mu\nu} + i \bar{q}_L \not{D} q_L + i \bar{q}_R \not{D} q_R \\ \mathcal{L}_{\text{QCD}}^m &= \bar{q}_L \mathcal{M} q_R + \bar{q}_R \mathcal{M}^\dagger q_L \end{aligned} \quad (1.17)$$

where D_μ is the covariant derivative, $G_{\mu\nu}^a$ is the gluon field strength tensor, q collects the light quark flavours $q^T = (u, d, s)$ and $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$ is the quark mass matrix; the dots in the first line denote terms containing the heavier quark flavours. In the limit of vanishing quark masses, $\mathcal{L}_{\text{QCD}}$ displays the $U(3)_L \times U(3)_R$ *chiral symmetry*:

$$(q_R, q_L) \rightarrow (L q_L, R q_R) \quad (1.18)$$

where the matrices L and $R \in U(3)_{L,R}$. Above a typical hadronic mass scale $\Lambda_{\text{hadr}} \approx 1 \text{ GeV}/c^2$, there is a large number of states, both meson resonances and baryons. Only very few (pseudoscalar) states, however, are significantly lighter than this mass scale: in particular the pions ($M_\pi \approx 140 \text{ MeV}/c^2$), but also kaons ($M_K \approx 495 \text{ MeV}/c^2$) and the eta ($M_\eta \approx 550 \text{ MeV}/c^2$). The masses of the three light quarks are small compared to Λ_{hadr} ,

$$m_{u,d,s} \ll 1 \text{ GeV}/c^2 \approx \Lambda_{\text{hadr}} \quad (1.19)$$

The first assumption of Chiral Perturbation Theory [42, 46–49] is that we can perform a perturbative expansion around the chiral limit ($m_u = m_d = m_s = 0$). The chiral group can be decomposed according to:

$$U(3)_L \times U(3)_R = SU(3)_L \times SU(3)_R \times U(1)_V \times U(1)_A \quad (1.20)$$

where we have introduced the vector $V = L + R$ and the axial vector $A = L - R$ transformations. The Noether current associated with $U(1)_V$ is conserved in the SM, while the $U(1)_A$ current is broken by quantum effects ($U(1)_A$ anomaly [50, 51]). The $SU(3)_L \times SU(3)_R$ symmetry is broken by the mass term. There are two ways to realize

a global chiral symmetry: in the *Wigner mode*, where the symmetry is exact and act on fields by linear operators or in the *Goldstone mode*, where the symmetry is spontaneously broken [9] and realized by non-linear transformations [46, 48, 52, 53]. The Wigner mode realization of chiral symmetry would lead to a parity doubling in the hadron spectrum that is not observed: we find (approximate) $SU(3)_V$ multiplets and no parity doubling is observed. Several theoretical and numerical studies suggest that chiral symmetry is spontaneously broken and, in accordance with the observations of hadronic multiplets, the breaking pattern is [54]:

$$SU(3)_L \times SU(3)_R \rightarrow SU(3)_V \quad (1.21)$$

The axial charges commute with the Hamiltonian, but don't leave the ground state invariant. As a consequence, eight pseudoscalar Goldstone bosons appear. These can be identified with:

$$\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, \eta$$

The Goldstone particles have the important property that their interactions become arbitrarily weak at low momenta: it can be demonstrated [53] that in a theory with a spontaneously broken symmetry, the Goldstone fields appear in the Lagrangian with at least one derivative. As a result, the coupling of the Goldstone bosons vanish in the limit of vanishing momenta. In order to build an effective theory of the strong interactions at low energies, we must write the most general Lagrangian in terms of the Goldstone fields, invariant under chiral symmetry (in this chapter we deal only with the eight Goldstone bosons and we don't include heavier states such as baryons, in the effective Lagrangian). The general formalism for building effective Lagrangians for spontaneously broken symmetries was worked out by Callan, Coleman, Wess and Zumino [55, 56]. It's convenient to parametrize the Goldstone fields $\pi^a(x)$ as:

$$U(\pi) \equiv \gamma^2(\pi) \equiv \exp \left(i \frac{2\pi}{F} \right) \quad (1.22)$$

where:

$$\pi = \pi^a T^a = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \quad (1.23)$$

T^a , $a = 1 \dots 8$ are the Gell-Mann matrices, chosen as generators of the broken group and F is a dimensional constant (to be determined). Under the chiral group, π transforms

as [53]:

$$g_0 \cdot \gamma(\pi) = \gamma(\pi') \cdot h(\pi, g_0) \quad (1.24)$$

where $g_0 \in SU(3)_L \times SU(3)_R$ and $h \in SU(3)_V$. From the invariance of the chiral group under parity ($SU(3)_R \leftrightarrow SU(3)_L$) it follows that:

$$U(\pi) \rightarrow g_L U(\pi) g_R^\dagger \quad (1.25)$$

where $g_{L,R} \in SU(3)_{R,L}$. Once that the transformation rule of the Goldstone boson fields is known, one can proceed to write a Lagrangian in terms of the matrix U that is invariant under $SU(3)_L \times SU(3)_R$ chiral symmetry. The guiding principle to write a low-energy effective theory is to use the power of momenta p (or equivalently, the derivatives of the fields) of the Goldstone bosons as a parameter to order the importance of various possible terms and perform a perturbation theory. “Low energy” here refers to a scale well below Λ_{hadr} , *i.e.* an energy region where the Goldstone bosons are the only relevant degrees of freedom. ChPT relies on the fact that the more momenta of the particles involved in a process are low with respect to Λ_{hadr} , the less terms with higher derivatives in the Lagrangian contribute to the amplitude. Lorentz invariance dictates that Lagrangian terms can only come in even powers of derivatives, so that the natural perturbation parameter of ChPT is of the order of $p^2/\Lambda_{\text{hadr}}^2$. The most general Lagrangian invariant under (1.25) must be product of terms of the form⁶ $\text{Tr}(\partial_\mu U \partial^\mu U^\dagger \dots \partial_\nu U \partial^\nu U^\dagger)$. The only invariant term at $\mathcal{O}(p^2)$ order in the chiral expansion with two derivative is:

$$\mathcal{L}^{(2)} = \frac{F^2}{4} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) \quad (1.26)$$

Expanding U in powers of π , we have:

$$\mathcal{L}^{(2)} = \frac{1}{2} \text{Tr}(\partial_\mu \pi \partial^\mu \pi) + \frac{1}{6 F^2} \text{Tr}[\pi, \partial_\mu \pi]^2 + \dots \quad (1.27)$$

The constant F can be determined by the matrix element of the conserved axial current, associated to the $SU(3)_A$ symmetry, and Goldstone states: $\langle 0 | J_A^{\mu,a} | \pi^b \rangle = i F p^\mu \delta^{ab}$ so that F can be identified (in the chiral limit) as the pion decay constant $f_\pi/\sqrt{2}$, determined experimentally from $\pi \rightarrow \mu\nu$ decay rate. Chiral symmetry is explicitly broken by the quark mass matrix so that a breaking term must be added to Eq. 1.26:

$$\mathcal{L}_m = B \frac{F^2}{2} \text{Tr}(U^\dagger \mathcal{M} + \mathcal{M}^\dagger U) \quad (1.28)$$

⁶Terms such as $\text{Tr}(U U^\dagger \dots U U^\dagger)$ are constant ($U U^\dagger = 1$).

Expanding U to the second order in π , we find the mass terms of the Goldstone bosons, for example:

$$M_{\pi^\pm}^2 = B(m_u + m_d), \quad M_{K^\pm}^2 = B(m_u + m_s), \quad M_{K^0}^2 = B(m_d + m_s) \quad (1.29)$$

It can be demonstrated [46] that the free parameter B is related to the value of the matrix element of the quark-anti-quark pair $\langle 0 | q\bar{q} | 0 \rangle = -BF^2$. Finally, the full chiral Lagrangian at $\mathcal{O}(p^2)$ and at first order in the quark masses is:

$$\mathcal{L}^{(2)} = \frac{F^2}{4} \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + B \frac{F^2}{2} \text{Tr}(U^\dagger \mathcal{M} + \mathcal{M}^\dagger U) \quad (1.30)$$

The number of independent terms increases rapidly at higher orders. In fact, $\mathcal{L}^{(2)}$ contains 2 terms, the $\mathcal{O}(p^4)$ Lagrangian $\mathcal{L}^{(4)}$ contains 10 terms and the $\mathcal{O}(p^6)$ Lagrangian $\mathcal{L}^{(6)}$ 90 terms [57–59]. The constants associated to these high-order terms are called the “low-energy constants” (LECs) which are the effective coupling constants of ChPT and the analogue of the two quantities F and B . In order to account for the effects due to the $U(1)_A$ anomaly, additional terms have to be included in the chiral Lagrangian; these “anomalous” parts begin at $\mathcal{O}(p^4)$ with the Wess-Zumino-Witten term (WZW) and are responsible for processes as $\pi^0 \rightarrow \gamma\gamma$ and $K \rightarrow \pi\pi e\nu$ and in particular, at order $\mathcal{O}(p^6)$ they are the only terms contributing to the vector form factor F_V of the $K_{e2\gamma}$ decay. One of the main problem in ChPT is the occurrence of these LECs: if they are treated as arbitrary parameters, the predictive power of the theory is strongly reduced. Most of the $\mathcal{O}(p^4)$ couplings of the strong chiral Lagrangian are rather well known from low energy data (*i.e.* $\pi\pi$ scattering, K and π decays) and can be used as input in other different processes; however, at higher orders a theoretical estimation of LECs is needed. In principle, LECs are calculable, determined by physics at energies above the Λ_{hadr} scale that manifest itself only indirectly, through the values of the effective couplings. The matching between ChPT and the underlying theory (SM) is a very difficult task. In many cases one resorts to phenomenology to determine the values of the LECs; for instance, the bulk of values of LECs can be explained in the framework of VMD by contributions from the virtual exchange of higher-mass states, *e.g.* the ρ -meson ($M_\rho \approx 770 \text{ MeV}/c^2$) [60]. In addition to phenomenological models, the study of QCD in the limit of an infinite number of quark colors (large- N_c limit) [61–63] provide a useful framework for theoretical estimates of LECs. The resulting numerical estimates are in general in good agreement with recent QCD lattice computations [64].

1.5 The Light Front Quark Model (LFQM)

The foundations of light front (LF) quantization [65–68] date back to Dirac [65]. The basic idea behind the LF quantization relies on the fact that physics is independent from different parametrization of space-time, described by the coordinates x^μ . Usually, time $ct = x^0$ and space $\mathbf{x} = (x^1, x^2, x^3)$ are treated as if they were completely separate issues but in a relativistic theory, time and space are only different aspects of four-dimensional space-time. These concepts can be put more formally by conveniently introducing generalized coordinates $\tilde{x}^\nu = \tilde{x}^\nu(x^\mu)$. The dynamic of a physical system is described by the Hamiltonian H and can be expressed equivalently in terms of either x^μ or $\tilde{x}^\mu$. There is however the subtle point of the “initial conditions”. By matter of convenience, when one quantizes the system in a quantum theory, one sets the initial conditions at the same “initial time”. This is called the equal time (ET) quantization and defines a hypersphere in four-space, characterized by a fixed initial “time” $t = 0$. For example, given the wave functions of an n -electron atom at an initial time, $\Phi_n(0, \mathbf{x})$, one can use the equation of motions to evolve $\Phi_n(t, \mathbf{x})$ to later times t . Following Dirac [65], there are no more than three basically different but physically equivalent space-time parametrizations that cannot be mapped on each other by a Lorentz transformation. They differ by the hypersphere on which the fields are initialized, and correspondingly one has different “times”. Each of these space-time parametrizations $\tilde{x}^\mu$ has thus its own Hamiltonian, and correspondingly Dirac speaks of the three forms of Hamiltonian dynamics (Fig. 1.3): the ET form (called also instant form) is the familiar one, with its hypersphere given by $t = 0$, while in the light front form the initial hypersphere is a tangent plane to the light cone $z = ct$; in the point form the hypersphere has a shape of a hyperboloid. The LF coordinates are related to the usual ET coordinates by the formulae:

$$x^+ = \frac{x^0 + x^3}{\sqrt{2}} \quad (1.31)$$

$$x^- = \frac{x^0 - x^3}{\sqrt{2}} \quad (1.32)$$

$$\mathbf{x}_\perp = (x^1, x^2) \quad (1.33)$$

so that the LF initial condition reads $x^- = 0$. The same physical state may have very different wave functions in the ET and LF approaches because of different basis for expanding a state. Although the ET form is the conventional choice for quantizing field theory, the LF quantization has many practical advantages for QCD calculations.

Quantum field theories as QCD are relativistic many body formulations that neces-

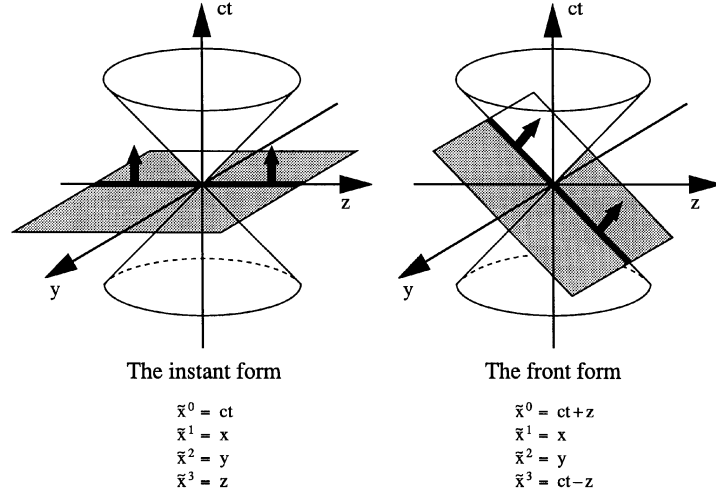


Figure 1.3: The instant (or equal time) and the light front parametrization of space-time [68].

sarily involve (anti-)particle creation and annihilation. Particle number is generally not conserved in a relativistic quantum field theory, thus, each state is represented as a sum over Fock states of arbitrary particle number. In QCD, an hadron is characterized by a set of state functions representing the probability amplitudes for finding different combinations of quarks and gluons in the hadron. For example, the state of a meson $|M\rangle$ can be written as:

$$|M\rangle = \sum_{q\bar{q}} \psi_{q\bar{q}} |q\bar{q}\rangle + \sum_{q\bar{q}g} \psi_{q\bar{q}g} |q\bar{q}g\rangle + \dots \quad (1.34)$$

where $\psi_{q\bar{q}}$, $\psi_{q\bar{q}g}$ are the probability amplitudes for a given Fock state and the ellipses denote terms with a higher number of particles. However, in QCD a conceptual difficulty concerning this many-particle picture appears. At low energies, hadrons, the bound states of QCD, are reasonably described in terms of two or three “constituent quarks” and thus as few-body systems. These effective quarks Q are collective excitations of the QCD field which can be described as one-particle states with an effective mass $M_Q \neq 0$. They are used as the basic degrees of freedom in the “Constituent Quark Model” (CQM) [69]. This model yields a reasonable mass spectroscopy of hadrons, but its foundations are not very well established theoretically. In fact, a non relativistic treatment of light hadrons is not justified and it is rather unclear how a few-particle picture can arise in a quantum field theory such as QCD.

In the Light Front Quark Model (LFQM) [68, 70–75] the idea of LF quantization is used to overcome theoretical contradictions of CQM arising in the canonical quantization

due to a few-body relativistic treatment of hadrons. The CQM description of hadrons suggests to truncate the Fock expansion by retaining only the lowest Fock states describing the constituent quarks, so that in the LFQM, the state of a meson with mass M and momentum $\mathbf{P}$ can be rewritten as:

$$|M(\mathbf{P})\rangle = \sum_{\lambda_1, \lambda_2} \int [dk_1][dk_2] 2(2\pi)^3 \delta^3(\mathbf{P} - \mathbf{k}_1 - \mathbf{k}_2) \times \Phi^{\lambda_1, \lambda_2}(x, \mathbf{k}_\perp) |\bar{Q}(\mathbf{k}_1, \lambda_1) Q(\mathbf{k}_2, \lambda_2)\rangle \quad (1.35)$$

where $[dk] \equiv \frac{dk^+ dk_\perp^2}{2(2\pi)^3}$ is the differential in the LF coordinates and $\Phi^{\lambda_1, \lambda_2}(x, \mathbf{k}_\perp)$ is the meson wave function in momentum space depending on the transverse LF component of the relative momentum of the quark pair $\mathbf{k}_\perp = (\mathbf{k}_1 - \mathbf{k}_2)_\perp$, on the fraction x of the momentum of the meson carried by one of the two quark and on the spins λ_1, λ_2 of Q and $\bar{Q}$, respectively. The meson wave function depends on the dynamics and can be found by solving the light front bound state equation [70–72].

1.6 The $K^+ \rightarrow e^+ \nu_e \gamma$ decay

1.6.1 Matrix element and kinematics

The matrix element for the generic $K^+ \rightarrow \ell^+ \nu_\ell \gamma$ decay ($\ell = e, \mu$) has the structure⁷:

$$\mathcal{M} = -ie \frac{G_F}{\sqrt{2}} V_{us}^* \varepsilon_\mu^* [F_K L^\mu - H^{\mu\nu} \ell_\nu] \quad (1.36)$$

with

$$\begin{aligned} L^\mu &= m_\ell \bar{u}(p_\nu)(1 + \gamma_5) \left(\frac{p_K^\mu}{p_K \cdot p_\gamma} - \frac{2p_\ell^\mu + \gamma^\mu}{2p_\ell \cdot p_\gamma} \right) v(p_\ell) \\ \ell^\mu &= \bar{u}(p_\nu) \gamma^\mu (1 - \gamma_5) v(p_\ell) \\ H^{\mu\nu} &\equiv \langle 0 | J_{\text{em}}^\mu J_{\text{weak}}^\nu | K \rangle = i \frac{F_V(q^2)}{M_K} \varepsilon^{\mu\nu\alpha\beta} (p_\gamma)_\alpha (p_K)_\beta - \frac{F_A(q^2)}{M_K} [(p_\gamma \cdot q) g^{\mu\nu} - q^\mu p_\gamma^\nu] \end{aligned}$$

Here p_K , p_ℓ , p_γ and p_ν are the four momenta of kaon, lepton, photon and neutrino, respectively, J_{em}^μ is the electromagnetic current, ε^μ is the photon polarization vector, V_{us} the CKM matrix element for the $s \rightarrow u$ transition and $q = p_K - p_\gamma$ is the four-momentum of the leptonic pair. The term proportional to L^μ describes the IB part of the amplitude $\mathcal{M}_{\text{IB}}$ and does not contain unknown quantities, it is determined by the amplitude of the non radiative decay $K^+ \rightarrow \ell^+ \nu$. The term proportional to the hadronic tensor $H^{\mu\nu}$ is called the structure dependent contribution $\mathcal{M}_{\text{SD}}$; effects of strong dynamics are parametrized by the form factors F_V and F_A related to the vector and axial part of the weak current J_{weak}^μ . The kinematics of $K_{\ell 2\gamma}$ decay can be described by two variables, traditionally defined in literature as [39]:

$$x = \frac{2p_K \cdot p_\gamma}{M_K^2} = \frac{2E_\gamma^*}{M_K} \quad y = \frac{2p_K \cdot p_\ell}{M_K^2} = \frac{2E_\ell^*}{M_K} \quad (1.37)$$

where E_ℓ^* and E_γ^* are the energies of the lepton and the photon in the kaon rest frame. The physical region for x and y is:

$$0 \leq x \leq 1 - r_\ell, \quad 1 - x + \frac{r_\ell}{1 - x} \leq y \leq 1 + r_\ell \quad (1.38)$$

with $r_\ell = m_\ell^2/M_K^2$. For $K_{e2\gamma}$ decay terms in the amplitude proportional to $r_e \approx 1.1 \cdot 10^{-6}$ can be neglected whereas in the muon mode $r_\mu \approx 4.6 \cdot 10^{-2}$ has to be retained throughout

⁷Same equations hold also for radiative pion decays $\pi_{\ell 2\gamma}$ with the M_K and F_K replaced by M_π and F_π .

the calculations. Some authors [39] use also the variables:

$$z = \frac{2E_\nu^*}{M_K}, \quad \lambda = \sin^2 \left(\frac{\theta_{\ell\gamma}^*}{2} \right) = \frac{x+y-1}{xy} \quad (1.39)$$

where E_ν^* is the neutrino energy, $\theta_{\ell\gamma}^*$ is the angle between the charged lepton and the photon in the kaon rest frame. Energy conservation implies the kinematic constraint $x + y + z = 2$ (Fig. 1.4). The relation between the momentum q^2 and x is given by:

$$q^2 = M_K^2(1 - x).$$

The SD part can be rewritten as the sum of two non interfering amplitudes $\mathcal{M}_{\text{SD}^\pm}$ describing respectively the emission of photons with positive (SD^+) and negative (SD^-) helicity; their corresponding interference terms with IB will be denoted $\text{INT}^\pm$. The $K_{\ell 2\gamma}$ event density over the Dalitz plot (x, y) is the differential decay width [76]:

$$\frac{d\Gamma}{dx dy} \equiv \rho(x, y) = \rho_{\text{IB}}(x, y) + \rho_{\text{SD}}(x, y) + \rho_{\text{INT}}(x, y) \quad (1.40)$$

$$\rho_{\text{IB}}(x, y) = 4r_\ell \left(\frac{F_K}{M_K} \right)^2 A_{\text{SD}} \times f_{\text{IB}}(x, y) \quad (1.41)$$

$$\rho_{\text{SD}}(x, y) = A_{\text{SD}} \times [(F_V + F_A)^2 f_{\text{SD}^+}(x, y) + (F_V - F_A)^2 f_{\text{SD}^-}(x, y)] \quad (1.42)$$

$$\rho_{\text{INT}}(x, y) = 4r_\ell \left(\frac{F_K}{M_K} \right) A_{\text{SD}} \times [(F_V + F_A) f_{\text{INT}^+}(x, y) + (F_V - F_A) f_{\text{INT}^-}(x, y)] \quad (1.43)$$

where

$$f_{\text{IB}}(x, y) = \left[\frac{1 - y + r_\ell}{x^2(x + y - 1 - r_\ell)} \right] \left[x^2 + 2(1 - x)(1 - r_\ell) - \frac{2xr_\ell(1 - r_\ell)}{x + y - 1 - r_\ell} \right] \quad (1.44)$$

$$f_{\text{SD}^+}(x, y) = (x + y - 1 - r_\ell) \cdot [(x + y - 1)(1 - x) - r_\ell] \quad (1.45)$$

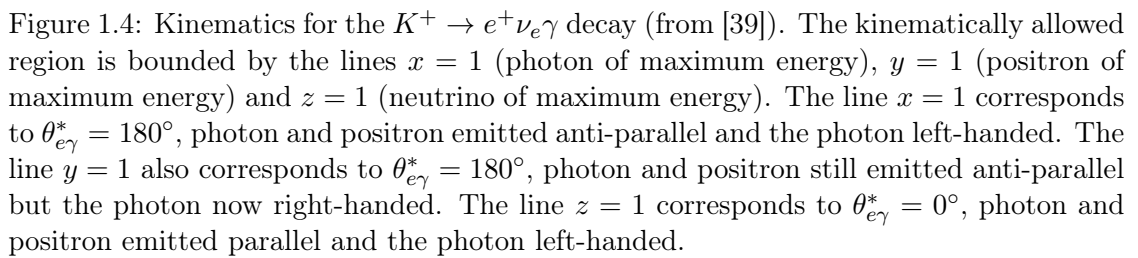
$$f_{\text{SD}^-}(x, y) = (1 - y + r_\ell) \cdot [(1 - x)(1 - y) + r_\ell] \quad (1.46)$$

$$f_{\text{INT}^+}(x, y) = \left[\frac{1 - y + r_\ell}{x(x + y - 1 - r_\ell)} \right] [(1 - x)(1 - x - y) + r_\ell] \quad (1.47)$$

$$f_{\text{INT}^-}(x, y) = \left[\frac{1 - y + r_\ell}{x(x + y - 1 - r_\ell)} \right] [x^2 - (1 - x)(1 - x - y) - r_\ell] \quad (1.48)$$

and

$$A_{\text{SD}} = \frac{M_K^5 \alpha G_F^2 |V_{us}|^2}{64\pi^2}$$



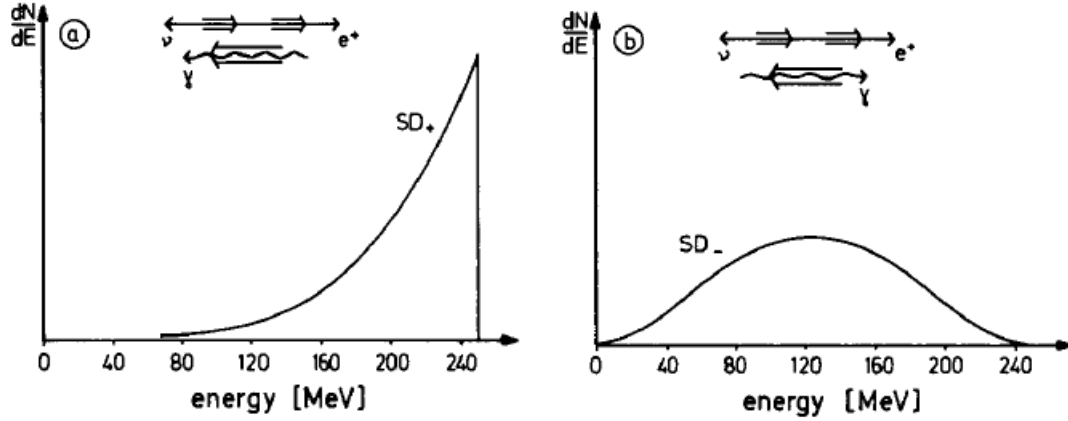


Figure 1.5: Sketch of the $K_{e2\gamma} E_e^*$ spectrum for favored event configuration (from [34]). (Left) SD^+ (Right) SD^- . The vertical scale is arbitrary.

By integrating over the y variable (F_V and F_A depend on x), one obtains the differential decay spectrum:

$$\frac{d\Gamma}{dx} \equiv \rho_{IB}(x) + \rho_{SD}(x) + \rho_{INT}(x) \quad (1.49)$$

where

$$\rho_{IB}(x) = \frac{4r_\ell F_K^2}{M_K^2} \left[\frac{(x + r_\ell - 1)[x^2 + 4(1 - r_\ell)(1 - x)]}{1 - x} - \frac{x^2 + 2(1 - r_\ell)(1 - x + r_\ell)}{x} \ln \frac{r_\ell}{1 - x} \right] \quad (1.50)$$

$$\rho_{SD^+}(x) = (F_V + F_A)^2 x^3 \left[\frac{1 - x}{3} - \frac{r_\ell}{2} + \frac{r_\ell^3}{6(1 - x)^2} \right] \quad (1.51)$$

$$\rho_{SD^-}(x) = (F_V - F_A)^2 x^3 \left[\frac{1 - x}{3} - \frac{r_\ell}{2} + \frac{r_\ell^3}{6(1 - x)^2} \right] \quad (1.52)$$

$$\rho_{INT^+}(x) = \frac{4r_\ell}{M_K} F_K (F_V + F_A) x \left[\frac{1 - x}{2} - \frac{r_e^2}{2(1 - x)} + r_\ell \ln \frac{r_\ell}{1 - x} \right] \quad (1.53)$$

$$\rho_{INT^-}(x) = \frac{4r_\ell}{M_K} F_K (F_V - F_A) x \left[\frac{-1 + 3x}{2} + \frac{r_e^2 - 2xr_\ell}{2(1 - x)} + (x - r_\ell) \ln \frac{r_\ell}{1 - x} \right] \quad (1.54)$$

In $K^+ \rightarrow e^+ \nu_e \gamma$ decays, because of the smallness of the r_e , the structure dependent terms have different E_e^* spectra: in the massless limit (a good approximation for electrons) the neutrino and the positron are always emitted with negative and positive helicity respectively. Angular momentum conservation then requires the photon spin to be opposite to the positron spin. Thus a right-handed γ (SD^+ configuration) is emit-

ted preferentially anti-parallel to the e^+ , whereas a left-handed γ (SD^- configuration) is emitted preferentially anti-parallel to the ν . Consequently, the SD^+ positron spectrum is peaked at $E_{\text{max}}^* = M_K/2 = 0.247$ GeV, whereas the SD^- spectrum has its maximum at $E_{\text{max}}^*/2$ (Fig. 1.5).

The IB contribution becomes large and dominates over both SD and INT terms in the region at small x or small angle $\theta_{e\gamma}^*$ and it diverges at $x = 0$ (Fig. 1.6). In Fig. 1.7, the distribution of the various contributions to the $K_{e2\gamma}$ decay rate are shown over the Dalitz plot; it should be noted that the SD^- contribution is large only in the region where IB is large. This makes difficult a measurement of the $F_V - F_A$ form factor combination. In the study of radiative decays, one is interested to the portion of the Dalitz plot in which the SD decays dominates those occurring via IB. In $K_{e2\gamma}$ decays, for photon energies greater than $E_\gamma^* = 50$ MeV, the Bremsstrahlung rate is about 1% of the SD rate. Thus in this region, SD decays are much more probable than IB decays while the interference term INT, proportional to m_e , is small compared to SD and it may be neglected.

In $K_{\mu2\gamma}$ decays, the distribution of events on the Dalitz plot is very similar to the electronic mode. On the other hand, due to the fact that m_μ is no more negligible, interference terms $\text{INT}^\pm$ become important (Fig. 1.8).

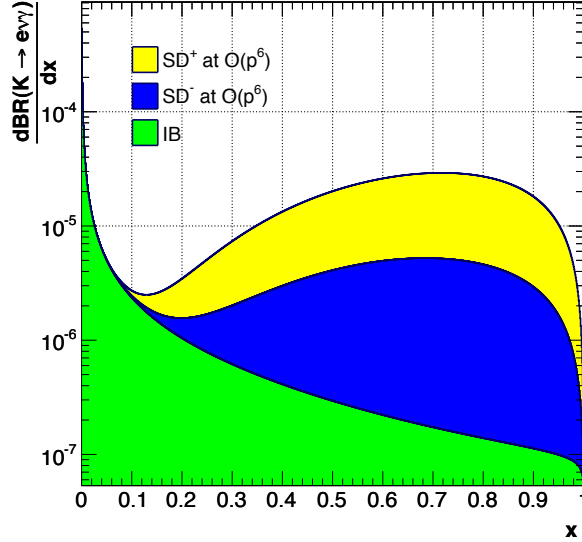


Figure 1.6: The contributions of SD (according to ChPT at $\mathcal{O}(p^6)$) and IB terms to the differential decay rate (logarithmic scale).

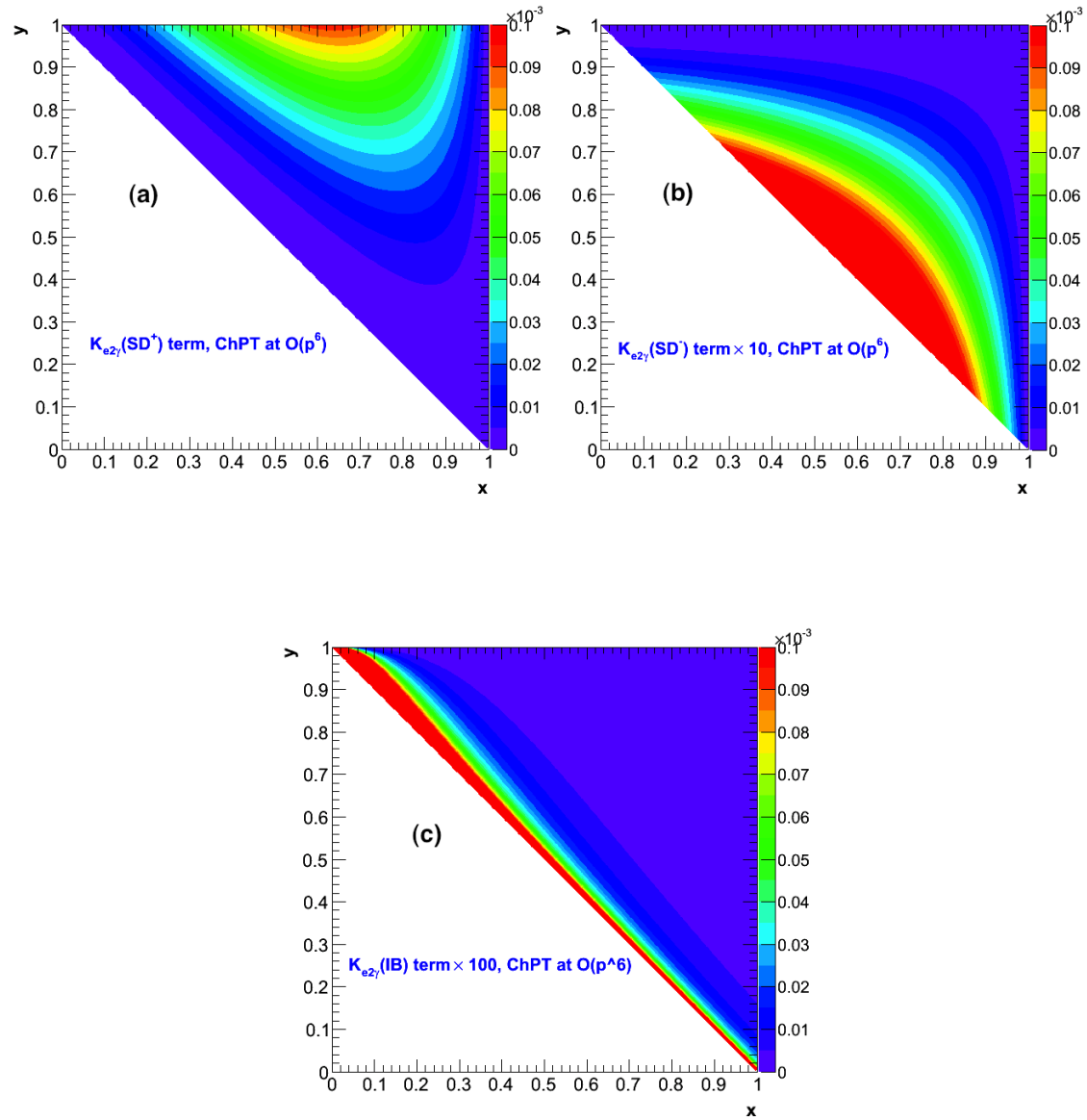


Figure 1.7: Theoretical SD and IB terms on the Dalitz plot (x, y) . (a) SD^+ . (b) SD^- . (c) IB. The SD terms are drawn according to ChPT at $O(p^6)$.

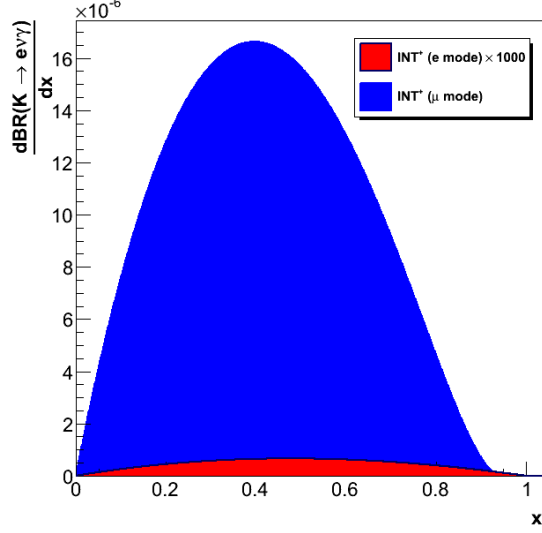


Figure 1.8: Differential branching ratio of INT^+ term for the muon and electron modes. Due to the $\mu - e$ mass difference, INT^+ decays in $K_{\mu 2\gamma}$ are about 10000 times more frequent than in $K_{e 2\gamma}$.

1.6.2 Theoretical estimation of form factors

ChPT result

At leading order $\mathcal{O}(p^2)$, ChPT gives:

$$F_V(q^2) = 0 \quad F_A(q^2) = 0$$

and the rate is entirely given by the IB contribution at this order. At the one-loop level $\mathcal{O}(p^4)$, one finds [76–78]:

$$F_V(q^2) = F_V(0) = \frac{\sqrt{2}M_K}{8\pi^2 F_\pi} = 0.096 \quad (1.55)$$

$$F_A(q^2) = F_A(0) = \frac{4\sqrt{2}M_K}{F_\pi}(L_9^r + L_{10}^r) = 0.042 \quad (1.56)$$

where $L_9^r = (6.9 \pm 0.7) \cdot 10^{-3}$ and $L_{10}^r = (-5.5 \pm 0.7) \cdot 10^{-3}$ are the low-energy couplings constants [77, 78]. At this order, the form factors do not exhibit any q^2 dependence. A non trivial dependence only occurs at the next order $\mathcal{O}(p^6)$ in which two-loop effects are taken into account. They were worked out first in [77] for the vector form factor and later confirmed in [78]. To this order, F_V gets contributions also from the anomalous sector and acquires a non-trivial dependence on q^2 which can be approximated by a

linear slope λ . The contributions of $\mathcal{O}(p^6)$ to F_A worked out in [78] involve two-loop graphs with insertion of the lowest-order ChPT Lagrangian $\mathcal{L}^{(2)}$, one-loop graphs with one insertion of $\mathcal{L}^{(4)}$, and tree-level graphs from $\mathcal{L}^{(6)}$. It was found that, at this level of approximation, F_A is essentially independent of q^2 with an uncertainty induced by the low-energy couplings at the percent level. The result for the form factors at $\mathcal{O}(p^6)$ can be summarized as:

$$F_V(q^2) = F_V(x) = F_V(0) \cdot [1 + \lambda(1 - x)] \quad (1.57)$$

$$F_A(q^2) = F_A(0) \quad (1.58)$$

where, according to [79]:

$$F_V(0) = 0.078 \quad (1.59)$$

$$F_A(0) = 0.034 \quad (1.60)$$

$$\lambda = 0.4 \quad (1.61)$$

Some of the values of the parameters entering in form factor calculations (*e.g.* F_π and masses) may be different from one study to another. However, the spread in the results due to a different set of parameters is less than 5% [78].

LFQM result

The study of $K_{e2\gamma}$ decay in the framework of LFQM was done in [35]. The general structure of the phenomenological light front meson wave function is based only on $q\bar{q}$ quark states (Section 1.5). It can be expressed by a quark u and an anti-quark $\bar{s}$ with total momentum $\mathbf{P}$:

$$|K(\mathbf{P})\rangle = \sum_{\lambda_1, \lambda_2} \int dk_1 dk_2 2(2\pi)^3 \delta^3(\mathbf{P} - \mathbf{k}_1 - \mathbf{k}_2) \times \Phi^{\lambda_1, \lambda_2}(x, \mathbf{k}_\perp) |\bar{s}(\mathbf{k}_1, \lambda_1) u(\mathbf{k}_2, \lambda_1)\rangle \quad (1.62)$$

where the same notations of Eq. 1.35 hold. The wave function $\Phi^{\lambda_1, \lambda_2}(x, \mathbf{k}_\perp)$ of the $u\bar{s}$ bound system can be found using the light front QCD bound state equation [70–72]. However, it has been shown that a reasonable approximation of the exact wave function is a Gaussian [35, 73–75]:

$$\Phi^{\lambda_1, \lambda_2}(x, \mathbf{k}_\perp) \propto [\bar{u}(k_1, \lambda_1) \gamma_5 v(k_2, \lambda_2)] \cdot \exp(-\mathbf{k}^2/2\omega_K) \quad (1.63)$$

where ω_K is a phenomenological parameter related to the physical size of the meson. The form factors resulting from the LFQM calculations, show a more complex q^2 dependence with respect to the linear behavior predicted by ChPT and they are given in an integral form in [35]. The LFQM prediction of the form factors at the point $q^2 = 0$ is [35]:

$$F_V(0) = 0.106 \quad (1.64)$$

$$F_A(0) = 0.036 \quad (1.65)$$

$$(1.66)$$

Comparison between models

The theoretical expectations of form factors within ChPT and LFQM frameworks are summarized in Tab. 1.3. It's worth to mention that the theoretical errors are not quoted in [35]; a recent estimate of the uncertainties in ChPT is given in [79]. They assign a theoretical uncertainties of 6% on $F_V(0)$ and 1% on $F_A(0)$, while the linear term λ is assigned a 33% of accuracy. The errors are obtained by looking at the spread in results obtained using different computations of the low-energy constants entering in the chiral Lagrangian (Tab. 1.4). Thus, a measurement of the $K_{e2\gamma}$ form factors with a precision better than the theoretical one, represents an important test for ChPT.

Model	$F_V(0)$	$F_A(0)$
ChPT at $\mathcal{O}(p^4)$	0.096	0.042
ChPT at $\mathcal{O}(p^6)$	0.078	0.034
LFQM	0.106	0.036

Table 1.3: The $K_{e2\gamma}$ form factors at $q^2 = 0$ in ChPT at $\mathcal{O}(p^4)$, $\mathcal{O}(p^6)$ and LFQM [35]

Form factor	ChPT estimated error
$F_V(0)$	0.078 ± 0.005
λ	0.3 ± 0.1
$F_V(0) + F_A(0)$	0.112 ± 0.005
$F_V(0) - F_A(0)$	0.044 ± 0.010

Table 1.4: Theoretical estimation of the form factors in the ChPT framework as reported in [79]. The error on $F_A(0)$ is estimated to be at the percent level while the uncertainty on $F_V(0) - F_A(0)$ is the difference between the central values at $\mathcal{O}(p^6)$ and $\mathcal{O}(p^4)$.

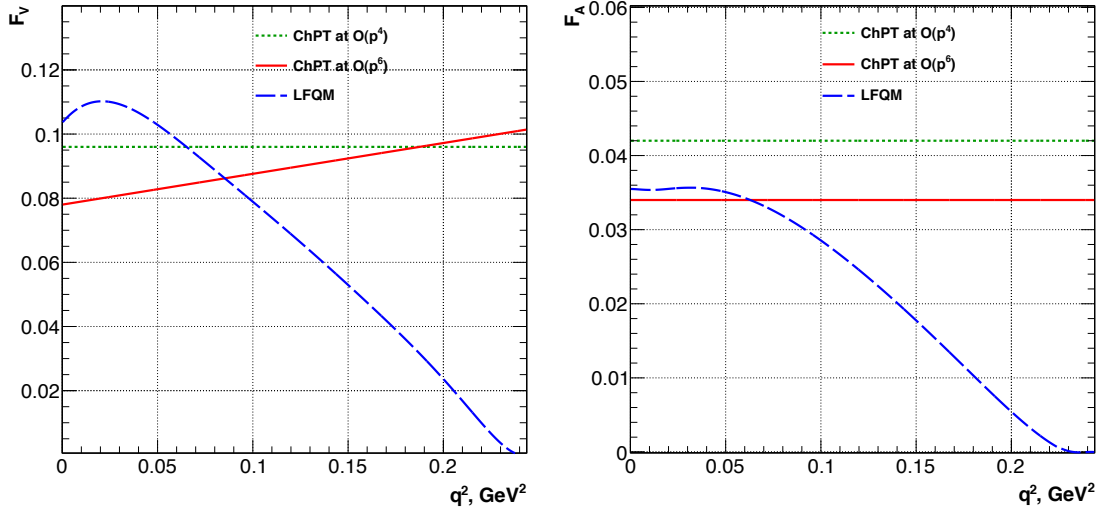


Figure 1.9: F_V (left) and F_A (right) as a function of the momentum squared q^2 .

The form factors as a function of the momentum are shown in Fig. 1.9 for the two models; the q^2 dependence predicted by LFQM is quite different with respect to the one in ChPT: both vector and axial form factors drop to zero at $q^2 = M_K^2$ (or $x = 1$), while in ChPT at $\mathcal{O}(p^6)$, F_A remains constant and F_V varies within 2% over the allowed kinematic region. The differential decay branching ratio as a function of x is plotted in Fig. 1.10: it can be seen that in the region $x < 0.7$ the decay branching ratio predicted in the LFQM is smaller than the one in the ChPT. On the other hand, in the region $x > 0.7$ the behavior is reversed. The theoretical results for the $K_{e2\gamma}$ branching ratio and form factors in ChPT at $\mathcal{O}(p^4)$, $\mathcal{O}(p^6)$ and LFQM are listed in Tab. 1.5. Due to the infra-red divergence affecting the IB amplitude at $E_\gamma = 0$, a lower cut on x must be applied. The SD^- branching ratio is about 10 times smaller than SD^+ ($\frac{BR_{SD^+}}{BR_{SD^-}} \approx \frac{(F_V + F_A)^2}{(F_V - F_A)^2}$) whereas IB and SD^- are of the same order in this region.

Model	IB	SD^+	SD^-	INT^+	INT^-	Total
ChPT at $\mathcal{O}(p^4)$	$0.69 \cdot 10^{-1}$	1.34	$1.93 \cdot 10^{-1}$	$6.43 \cdot 10^{-5}$	$-1.10 \cdot 10^{-3}$	1.60
ChPT at $\mathcal{O}(p^6)$	$0.69 \cdot 10^{-1}$	1.15	$2.58 \cdot 10^{-1}$	$6.22 \cdot 10^{-5}$	$-1.21 \cdot 10^{-3}$	1.47
LFQM	$0.69 \cdot 10^{-1}$	1.12	$2.59 \cdot 10^{-1}$	$4.33 \cdot 10^{-5}$	$-1.29 \cdot 10^{-3}$	1.44

Table 1.5: The decay branching ratio of $K_{e2\gamma}$ (in units of 10^{-5}) in ChPT and LFQM. The cut $x > 0.1$ is used.

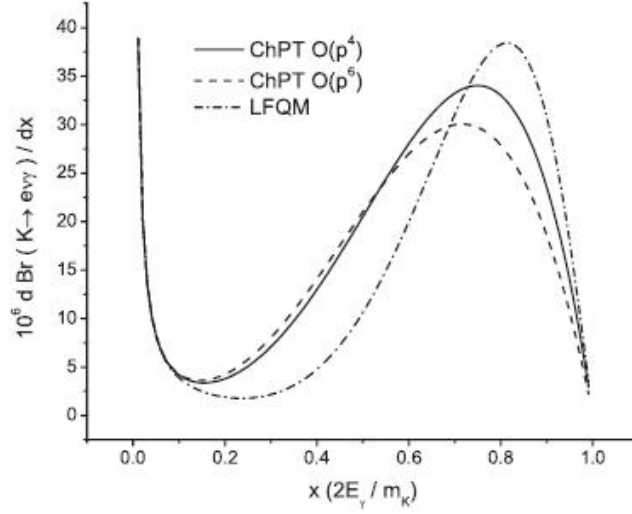


Figure 1.10: The differential decay branching ratio as a function of x (from [35])

1.7 Experimental status

Search for $K_{e2\gamma}$. Macek et al. 1970

The first experiment to study of $K \rightarrow e \nu \gamma$ decays was performed in 1970 [80] at the Princeton-Pennsylvania Accelerator (Fig. 1.11). A beam of K^+ , produced by the collision of the proton beam with a platinum target was transported at 500 MeV/ c to a moderator where it was stopped. A coincidence of three threshold gas Cerenkov counters assured the observation of a sample of electrons and a spectrometer measured the momentum; scintillation counters placed after the stopping region identified photons. The measurement of $K_{e2\gamma}$ events was made by searching for photons in a large angle coincidence with positrons in the momentum range above the maximum allowed in K_{e3} decay; in this kinematic region the main contribution to the decay rate comes from SD^+ and the background from K_{e3} decay is kinematically suppressed. No event with a positron with momentum greater than 234 MeV/ c was observed in coincidence with a photon; this result was used to set an upper limit on the $K_{e2\gamma}$ SD^+ branching ratio:

$$\text{BR}(SD^+) < 7.1 \cdot 10^{-5} \quad \text{at 90\% C.L.}$$

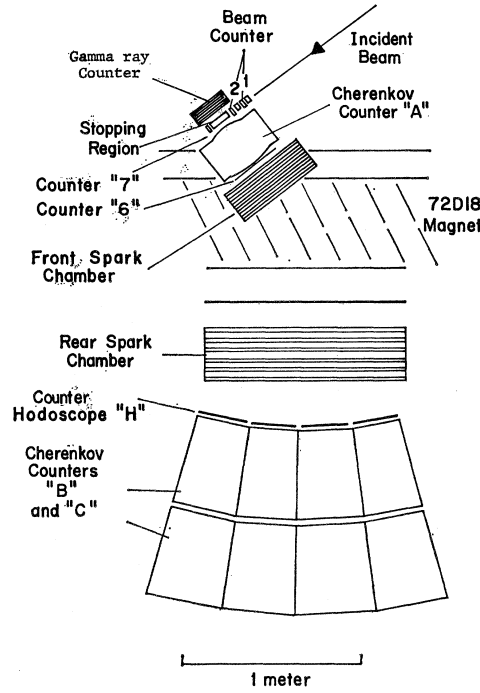


Figure 1.11: Top view of the apparatus of the Princeton-Pennsylvania experiment (from [80]).

First observation of $K_{e2\gamma}$ events at CERN. Heard et al. 1975

In 1975, the experiment of Heard et al. [81] at CERN-PS accelerator was able to observe for the first time $K_{e2\gamma}$ events (Fig. 1.12). Low energy K^+ were stopped in a lithium hydride (LiH) target. Kaons were identified by their high pulse in the scintillation counter hodoscope, while pions were vetoed in a plexiglass Cerenkov counter. The kaon track was recorded in proportional chambers. The momentum of the charged decay particle was measured in a magnetic spectrometer equipped with a multiwire proportional chamber and drift chambers. Charged particles coming from the target and passing the spectrometer were detected in scintillation counter hodoscopes, placed beside the target and behind the last drift chamber. A photon detector, consisting of three lead glass blocks, was placed beside the target, opposite to the spectrometer and was used to detect photons from the π^0 of K_{e3} decays and from $K_{e2\gamma}$ decays. The analysis was restricted to $K_{e2\gamma}$ decays with e^+ energies above the endpoint of K_{e3} spectrum and the kinematic region explored was $x > 0.4$, $y > 0.96$ and $\theta_{e\gamma}^* > 120^\circ$; thus, this experiment was sensitive only to the SD^+ term. At the end of the analysis, 56^{+8}_{-12} SD^+ events were identified. The measured values of the SD^+ branching ratio (normalized to K_{e2}) and form factors

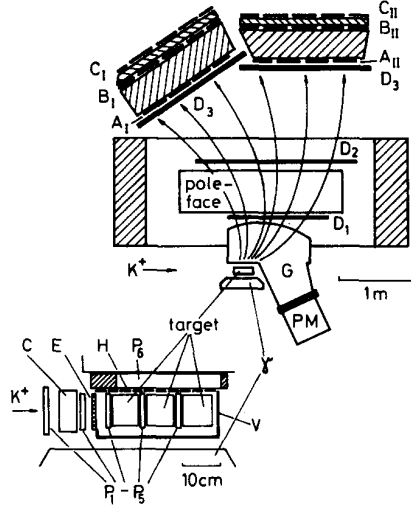


Figure 1.12: Experimental arrangement of the CERN experiment (from [81]). P1-P6: multiwire proportional chambers. D1-D3: drift chambers. C: plexiglass threshold Cerenkov counter. E, H, A, B, C: scintillation counter hodoscopes. G: gas Cerenkov counter. V: veto counters around the target. The bending magnet and typical tracks for $p = 240 \text{ MeV}/c$ are indicated.

(assuming no q^2 -dependence) were [81]⁸:

$$\frac{\text{BR}(\text{SD}^+)}{\text{BR}(K_{e2})} = 1.05^{+0.25}_{-0.30} \cdot 10^{-5}$$

$$F_V(0) + F_A(0) = 0.150^{+0.018}_{-0.023}$$

An upper limit at 90% confidence level was also set for the SD^- term:

$$\frac{\text{BR}(\text{SD}^+)}{\text{BR}(\text{SD}^-)} < 85$$

CERN. Heintze et al. 1979

In 1979, the same experiment at CERN collected further data on $K_{e2\gamma}$. Compared to the previous experiment [81], the apparatus was improved: the LiH stopping target was replaced by a scintillator target, mounted at an inclination of 30° to the beam axis, to identify kaons from their pulse height, NaI counters were placed behind the spectrometer to measure the energy of the charged decay particles. The analysis was performed in the

⁸ [81] quoted absolute value $|F_V + F_A| \sin \theta_C$, where θ_C is the Cabibbo angle.

kinematic region $x > 0.2$, $y > 0.95$ and $\theta_{e\gamma}^* > 140^\circ$. The observed momentum spectrum of electrons is shown in Fig. 1.13.

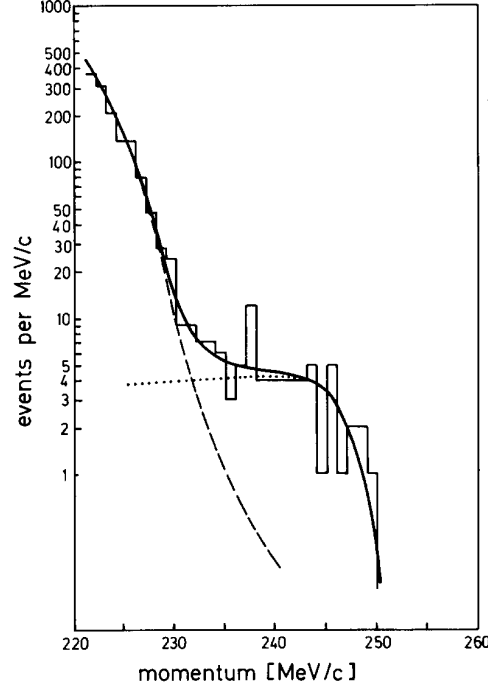


Figure 1.13: Observed momentum spectrum of electrons with a signal in the photon counter (from [34]). Dashed curve: calculated K_{e3} spectrum. Dotted curve: calculated SD^+ spectrum. Solid curve: sum of calculated K_{e3} and SD^+ spectra.

The results for the SD^+ branching ratio and form factors, obtained from 51 ± 3 SD^+ events, were [34]:

$$\frac{\text{BR}(SD^+)}{\text{BR}(K_{\mu 2})} = (2.33 \pm 0.42) \cdot 10^{-5}$$

$$F_V(0) + F_A(0) = 0.147 \pm 0.011$$

For the corresponding SD^- part an upper limit at 90% of confidence level was set by searching for a sample of electrons with energies $220 \text{ MeV} < E_e^* < 230 \text{ MeV}$ and with no photon in the backward direction:

$$\text{BR}(SD^-) < 1.6 \cdot 10^{-4}$$

This upper limit translates into a constraint on the ratio:

$$\frac{|F_V(0) + F_A(0)|}{|F_V(0) - F_A(0)|} < \sqrt{11}$$

The combined results of the experiments [34, 81] are [82]:

$$\text{BR}(\text{SD}^+) = (1.52 \pm 0.23) \cdot 10^{-5} \quad (1.67)$$

$$F_V(0) + F_A(0) = 0.148 \pm 0.010 \quad (1.68)$$

$$F_V(0) - F_A(0) < 0.49 \quad (90\% \text{ C.L.}) \quad (1.69)$$

with a large experimental error of about 15% on the branching ratio and 7% on the form factors.

KLOE at DAΦNE 2008

Recently the KLOE collaboration [18], using the DAΦNE collider, has published a new measurement of the $K_{e2\gamma}$ decay rate and form factors. DAΦNE is a e^+e^- collider operating at a total energy in the center of mass $E_{\text{CM}} \approx 1.2$ GeV; at this energy ϕ mesons are produced, essentially at rest, and decay into K^+K^- pairs. A sketch of the KLOE detector is shown in Fig. 1.14. During 2001-2005 KLOE collected an integrated luminosity of about 2.2 fb^{-1} , corresponding to about 3.3 billion of K^+K^- pairs. Kaons have a momentum of about 100 MeV/c. Observation of a $K^\pm$ tags the presence of a $K^\mp$ meson. Kaon production and decay are studied with the KLOE detector, consisting essentially of a drift chamber (DC) surrounded by an electromagnetic calorimeter (EMC). A superconducting coil provides a 0.52 T magnetic field. The measurement is based on the observation of 1484 $K_{e2\gamma}$ events combining both $K^\pm$, in the kinematic region $10 \text{ MeV} < E_\gamma^* < 250 \text{ MeV}$ and $E_e^* > 200 \text{ MeV}$ (or $x > 0.04$ and $y > 0.8$). In this analysis they didn't distinguish between $\text{SD}^\pm$ and IB components; the high-momentum part of the E_e spectrum is dominated by the SD^+ process while the SD^- and IB contributions account only for 2% and 1.3% to the total sample. The form factor parameters are obtained by fitting the measured x distribution with the theoretical differential decay rate in the framework of ChPT at $\mathcal{O}(p^6)$; this model predicts a dependence of the vector form factor on the momentum while the axial form factor F_A is assumed to be independent on x ; in the fit $F_V(x)$ was expanded as $F_V(x) = F_V(0) \cdot (1 + \lambda(1 - x))$. The small SD^- contribution did not allow a measurement of the related $F_V - F_A$ component, therefore this quantity was kept fixed at the expectation value of ChPT at $\mathcal{O}(p^4)$ while $F_V(0) + F_A(0)$ and λ were free parameters; due to the limited statistic, the E_γ spectrum was divided in 5 bins

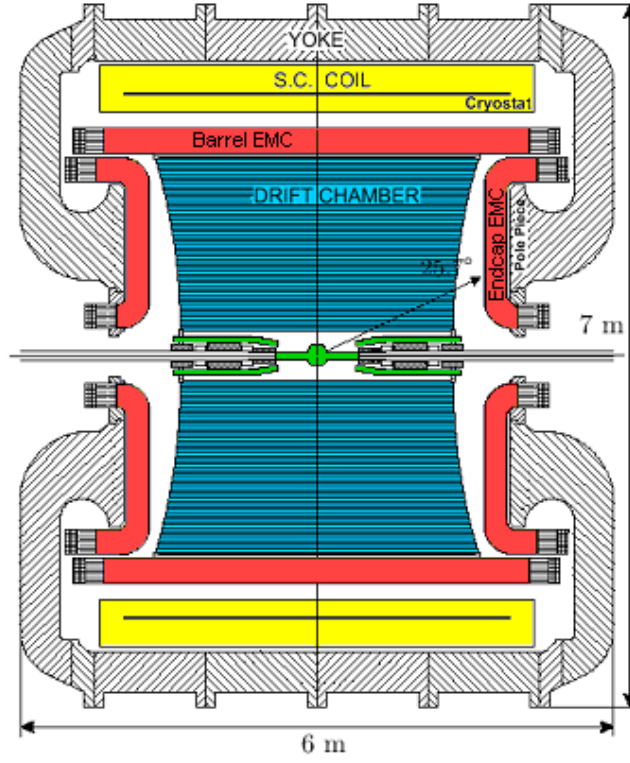


Figure 1.14: Experimental arrangement of the KLOE experiment at DAΦNE e^+e^- collider.

50 MeV width (Fig. 1.15). The final results are [18]:

$$F_V(0) + F_A(0) = 0.125 \pm 0.007_{\text{stat}} \pm 0.001_{\text{syst}} \quad (1.70)$$

$$\lambda = 0.38 \pm 0.20_{\text{stat}} \pm 0.02_{\text{syst}} \quad (1.71)$$

These results are in agreement with ChPT at $\mathcal{O}(p^6)$ and confirm at $\sim 2\sigma$ the presence of a slope in the vector form factor F_V . The $K_{e2\gamma}$ branching ratio is obtained by integrating the differential decay rate over E_γ^* ; KLOE quotes the value (normalized to $K_{\mu 2}$):

$$\frac{\text{BR}(K_{e2\gamma})}{\text{BR}(K_{\mu 2})} = (1.483 \pm 0.066_{\text{stat}} \pm 0.013_{\text{syst}}) \cdot 10^{-5}, \quad p_e > 200 \text{ MeV}/c \text{ and } 10 \text{ MeV} < E_\gamma^* < 250 \text{ MeV}$$

This result translates into:

$$\text{BR}(K_{e2\gamma}) = (9.4 \pm 0.4) \cdot 10^{-6}, \quad p_e > 200 \text{ MeV}/c \text{ and } 10 \text{ MeV} < E_\gamma^* < 250 \text{ MeV} \quad (1.72)$$

which is the value quoted by the Particle Data Group in 2012 [10].

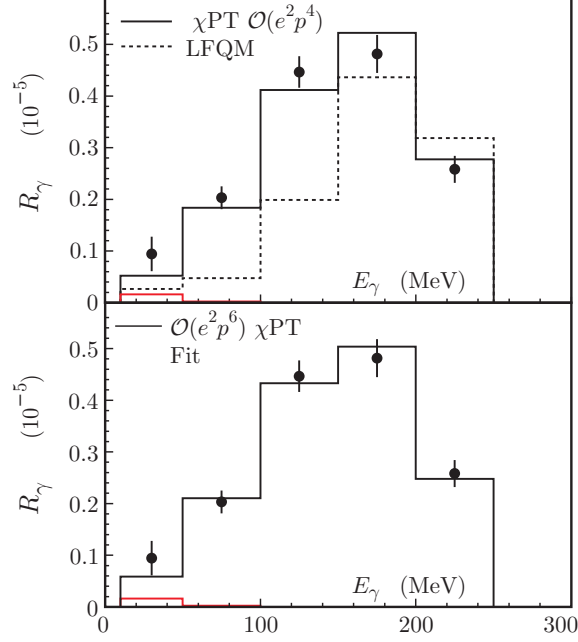


Figure 1.15: $\Delta R_\gamma = [1/\Gamma(K_{\mu 2})] \times [d\Gamma(K_{e 2\gamma})/dE_\gamma^*]$ vs E_γ^* (from [18]). On top data (black dots) are compared to ChPT predictions at $\mathcal{O}(p^4)$ and to the LFQM model. At the bottom data are fitted to ChPT at $\mathcal{O}(p^6)$. The IB contribution is shown (red line).

1.7.1 Form factors measurements in $K \rightarrow \mu\nu_\mu\gamma$ and $K \rightarrow \ell\nu_\ell e^+e^-$

In this section the most recent measurements of the form factors in $K_{\mu 2\gamma}$ and $K \rightarrow \ell\nu_\ell e^+e^-$ decays will be summarized and compared to the electron mode $K_{e 2\gamma}$. A complete list of past experimental results can be found in [10] and references therein. At present, the combination $F_V(0) - F_A(0)$ is only loosely constrained by $K_{e\nu\gamma}$ measurements [34] due to the suppression of the SD^- and INT terms with respect to SD^+ . However, in the $K_{\mu 2\gamma}$ decay, the INT term is comparable in size to the SD terms, making possible, in principle, to measure the sign as well as the magnitude of the form factor combination $F_V(0) - F_A(0)$. In 2000, the collaboration of the experiment E787 at the Brookhaven Alternating Gradient Synchrotron (AGS) studied $K_{\mu 2\gamma}$ decays and reported a value of $|F_V(0) + F_A(0)|$ at three-sigma level from the measurements in $K_{e 2\gamma}$ decay [83]:

$$F_V(0) + F_A(0) = 0.165 \pm 0.007_{\text{stat}} \pm 0.011_{\text{syst}}$$

For $F_V(0) - F_A(0)$, only a confidence interval was given. The best determination of $F_V(0) - F_A(0)$ comes from $K^+ \rightarrow \ell^+ \nu_\ell e^+ e^-$ [84] in which a virtual photon converts into $e^+ e^-$. In 2002, the experiment E865 at AGS reported a measurement of the branching ratio and form factors based on 410 $K^+ \rightarrow e^+ \nu_e e^+ e^-$ events and 2679 $K^+ \rightarrow \mu^+ \nu_\mu e^+ e^-$ events. The results of the form factors are obtained assuming a q^2 -dependence as predicted by vector meson dominance (Eq. 1.15) and combining electron and muon modes:

$$\begin{aligned} F_V(0) &= 0.112 \pm 0.018 \\ F_A(0) &= 0.035 \pm 0.019 \\ F_V(0) + F_A(0) &= 0.147 \pm 0.026 \\ F_V(0) - F_A(0) &= 0.077 \pm 0.028 \end{aligned}$$

Assuming constant form factors, they found⁹:

$$\begin{aligned} F_V(0) &= 0.131 \\ F_A(0) &= 0.034 \end{aligned}$$

Both results are consistent with ChPT predictions. Very recently, the ISTRA+ collaboration reported the results (based on the same data sample) of two independent analyses of the $K^- \rightarrow \mu^- \nu \gamma$ decay [85, 86]. They measured the sign and the value of $F_V - F_A$ assuming constant form factors at ChPT $\mathcal{O}(p^4)$ and observing the interference term between the IB and SD⁻ component in a sample of about 22000 $K^- \rightarrow \mu^- \nu \gamma$ events. The ISTRA+ results for the two analyses are [86]:

$$F_V(0) - F_A(0) = 0.126 \pm 0.027_{\text{stat}} \pm 0.043_{\text{syst}}$$

and [85]:

$$F_V(0) - F_A(0) = 0.21 \pm 0.04_{\text{stat}} \pm 0.03_{\text{syst}}$$

respectively, about 1.5 and 3 sigmas above the ChPT prediction [79]. At this stage it is clearly premature to claim a serious tension between data and theory.

⁹No errors are quoted in this case.

Chapter 2

The NA62 phase I experiment at CERN SPS

The NA62 experiment is located in the ECN3 hall in the North Area of the Super Proton Synchrotron (SPS) site at CERN. The experiment uses a decay-in-flight technique to identify the K^+ decay products that well match the characteristics of the CERN SPS. In 2007, the experiment had a preliminary phase of data taking, the “ R_K phase” or “phase I”, dedicated to a precision test of lepton flavour universality aiming to a measurement of R_K (Chapter 1) at level of 0.5%. For this phase, the NA62 collaboration used the pre-existing apparatus of the NA48/2 experiment. The data analyzed in the present thesis were collected during this period. After this preliminary phase, the NA62 collaboration plans a long-term program dedicated to the measurement of the branching ratio of the ultra rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ with a 10% accuracy, collecting $\mathcal{O}(100)$ events in two years of data taking with a signal to background ratio S/B ≈ 10 . For this second phase, most of the existing NA48/2 apparatus used in 2007 and 2008 has been dismantled and a new apparatus is under construction. A first physics run with the new detector is expected in 2014. In this chapter, the research program of the NA62 experiment, the SPS accelerator complex and the NA48/2 experimental layout used in 2007 and 2008 will be described, with particular attention for the sub-detectors used in the present analysis.

2.1 The physics program of the NA62 phase I

The aim of the NA62 experiment in phase I was the measurement of the R_K ratio with the unprecedented precision, $\delta R_K/R_K \approx 0.5\%$, in order to provide a stringent test of the SM [87]. For this preliminary phase, the NA62 collaboration used the existing NA48/2

experiment located at the CERN-SPS in the North Area with a beam line modified with respect to the old layout, to decrease the kaon momentum bite to about 2% (rms). The original aim of the NA48/2 experiment was the search of CP violation in the $K^+ - K^-$ system via the measurements of the decay asymmetries between positive and negative kaons in the $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ and $K^\pm \rightarrow \pi^\pm \pi^0 \pi^0$ decays [88–90]. The intense kaon flux needed for the measurements favoured also an intense physics program devoted to the study of rare kaon decays, clean environments for testing ChPT with an experimental accuracy comparable with theory. In most cases, the precision achieved by NA48/2 improved and superseded the existing world results. Recent NA48/2 results include the analyses of the flavour-changing neutral current decays $K^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ [91], the radiative decays $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ [92] and the semi-leptonic decay $K^\pm \rightarrow \pi^\pm \pi^- e^\pm \nu$ (also called K_{e4}), with statistics high enough (about 10^6 K_{e4} events) to allow to test the ChPT with an accuracy better than 1% [93–95]. The data for the NA62 experiment phase I have been collected during four months in 2007-2008. A first result of the R_K measurement based on a partial data sample of about 60000 $K^\pm \rightarrow e^\pm \nu$ candidates has been published in 2011 [96]. The final result with the full data sample (~ 150000 reconstructed $K^\pm \rightarrow e^\pm \nu$ events) has been released in 2013 [19]:

$$R_K = (2.488 \pm 0.010) \times 10^{-5}$$

in agreement with the Standard Model expectation (Chapter 1). As by-products of the R_K measurement, other analyses of rare decay channels, based on the same data set, are ongoing:

- $K^+ \rightarrow e^+ \nu \gamma$ ($K_{e2\gamma}$) decay, the subject of the present thesis. This radiative decay is a test bench for probing ChPT beyond the $\mathcal{O}(p^6)$ approximation.
- $K^+ \rightarrow \pi^+ \gamma \gamma$ decay. A test for ChPT at $\mathcal{O}(p^6)$. The analysis is an advanced stage, preliminary results can be found in [97].
- $K^+ \rightarrow \pi^0 e^+ \nu$ decay (called K_{e3}). A precise measurement of K_{e3} form factors allows the extraction of the CKM matrix element $|V_{us}|$.
- Search for an heavy sterile neutrino ν_h through the decay chain $K^+ \rightarrow \mu^+ \nu_h \rightarrow \mu^+ \nu_\mu \gamma$.

2.2 The Super Proton Synchrotron accelerator at CERN

Protons for the accelerator complex at CERN are produced in a plasma source, injected into a linear accelerator (Linac2) and accelerated to an energy of 50 MeV. An 80 m long beam transport carries the Linac2 beam to the first ring accelerator of the chain: the PS Booster (PSB). In this synchrotron the protons reach an energy of 1.4 GeV and the bunches from each ring can be recombined in various ways at the injection into the Proton Synchrotron (PS). The PS was the first synchrotron built at CERN and started being operative in 1959. It can accelerate protons up to an energy of about 25 GeV and is still used as injector to the higher energy machines, as, for example, the Super Proton Synchrotron (SPS). A schematic representation of the CERN accelerator complex is shown in Fig. 2.1. A detailed description of the CERN synchrotrons can be found in [98]. The SPS is operative since 1976 and is installed in an underground tunnel tangent to the CERN site, with a radius of 2.2 km. It was foreseen for fixed target experiments in the West and in the North Experimental Areas and is still actively used today. The SPS was also used as a proton-antiproton collider for the UA-1 and UA-2 experiments that observed for the first time the W and Z bosons. It served as an injector for the Large Electron Positron collider (LEP) and it is the last injector of the Large Hadron Collider (LHC). In addition it provides a neutrino beam to Gran Sasso and protons to the North Area. The maximum proton energy that can be reached at the SPS is 450 GeV.

2.3 The NA48/2 experimental setup for the NA62 phase I

The NA48/2 beam line and sub-detectors used in 2007 for the NA62 phase I data taking and relevant to the $K_{e2\gamma}$ analysis will be described in the following sections.

2.3.1 The beam line

The primary protons used for $K^\pm$ production are provided by the SPS accelerator. The $1.8 \cdot 10^{12}$ protons per pulse are delivered with a period of 16.8 s and a flat top of 4.8 s. After extraction the protons are transported along a 800 m long beam line and focused on a cylindrical (2 mm in diameter and 400 mm long) beryllium target, to produce several types of particles (mainly protons, neutrons, photons, pions, hyperon, kaons and electrons). All neutral particles are dumped along the zero degree direction in an absorber. The NA48/2 beam line, originally designed to minimize differences in acceptance between the K^+ and K^- , is capable to transport either simultaneous or single beams of positive and/or negative kaons. The charged particles produced in the target are split according to

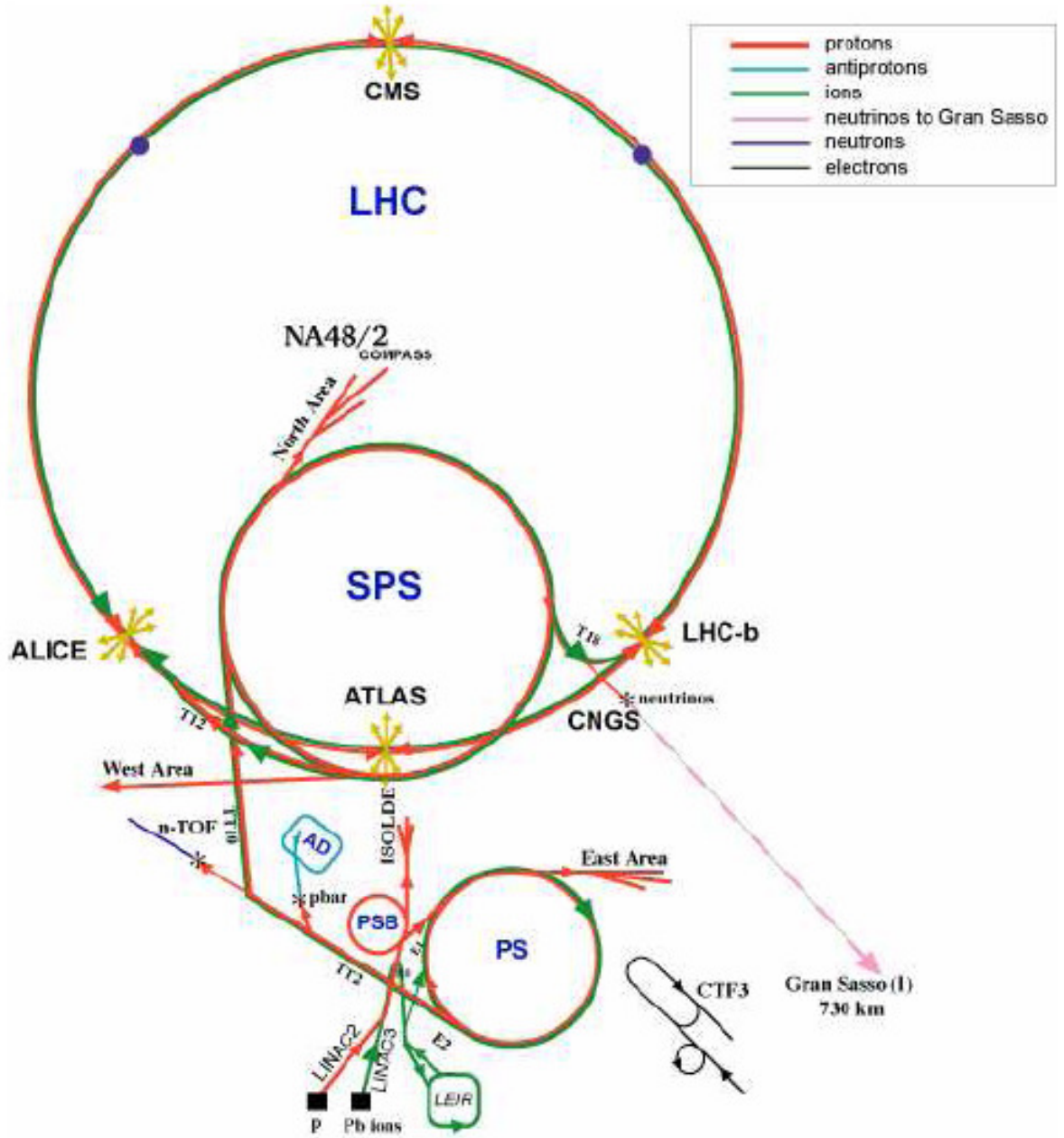


Figure 2.1: Schematics of the CERN accelerator complex. The NA48/2 experiment is located in the North Area.

their charge by the first two magnets in a four dipole achromat (“Front End Achromat”) and by momentum-defining slits incorporated into a 3.2 m thick copper/iron proton beam dump, which also provided a possibility of blocking either of the beams. This first set of magnets and collimators selects charged particles with a narrow momentum band around the nominal value of 74 GeV/ c with a spread of 1.4 GeV/ c (rms). The beam subsequently passes through acceptance-defining and cleaning collimators and a set of four quadrupoles of alternating polarity. In order to keep the beams within the beam pipe while traversing the apparatus, the beams are steered by a dipole (TRIM3) located upstream the entrance of the decay volume to compensate for the deflections by the downstream spectrometer magnet. These deflections were regularly reversed and optimized during the data taking. In addition, because the beam is accompanied by an intense flux of muons traveling outside the beam vacuum pipe, a system of toroids, named “muon scraper”, was installed upstream of the decay volume to suppress backgrounds associated with these “halo muons”. The kaon beams entered the decay fiducial volume with a transverse size of $\delta x = \delta y = 4$ mm (rms) and at angles of about ± 0.20 mrad with respect to the axis. The beam flux at the entrance of the decay volume was $2.5 \cdot 10^7$ particles per pulse, dominated by pions ($\pi^\pm$), with kaon ($K^\pm$) fractions of about 6%. The fraction of those beam kaons decaying in the vacuum tank at nominal momentum is 18%. The kaon beam spectrometer (KABES) used in the original NA48/2 set-up for track reconstruction of the kaon and for the measurement of its momentum was removed before 2007; for this reason the kaon momentum P_K is not measured directly event by event, but an average value is obtained using reconstructed $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ decays (Section 4.2.3). A schematic view of the NA48/2 beam line is illustrated in Fig. 2.2.

2.3.2 The decay region

To avoid interactions of kaon decay products before detection the decay region is contained in an evacuated (at $\lesssim 10^{-4}$ mbar) cylindrical steel tank (called “blue tube”) of diameter 1.92 m, increasing to 2.4 m in its last 48 m. The total length is 113 m. The downstream part of the blue tube is closed by a thin (0.8 mm thick), convex hemispherical, 1.3 m radius Kevlar window. Its purpose is to separate the vacuum in the decay region from the helium at atmospheric pressure in the spectrometer. The thickness of the window corresponds to $3 \cdot 10^{-3}$ radiation lengths. Downstream the Kevlar windows the beam, containing mainly kaons and pions, continues in vacuum within a carbon pipe of 152 mm diameter and 1.2 mm thick. In the decay region a small residual magnetic field, mostly due to the Earth, is present; this field has been carefully measured in order to allow a posteriori correction on reconstructed tracks and in order to include the effect in Monte

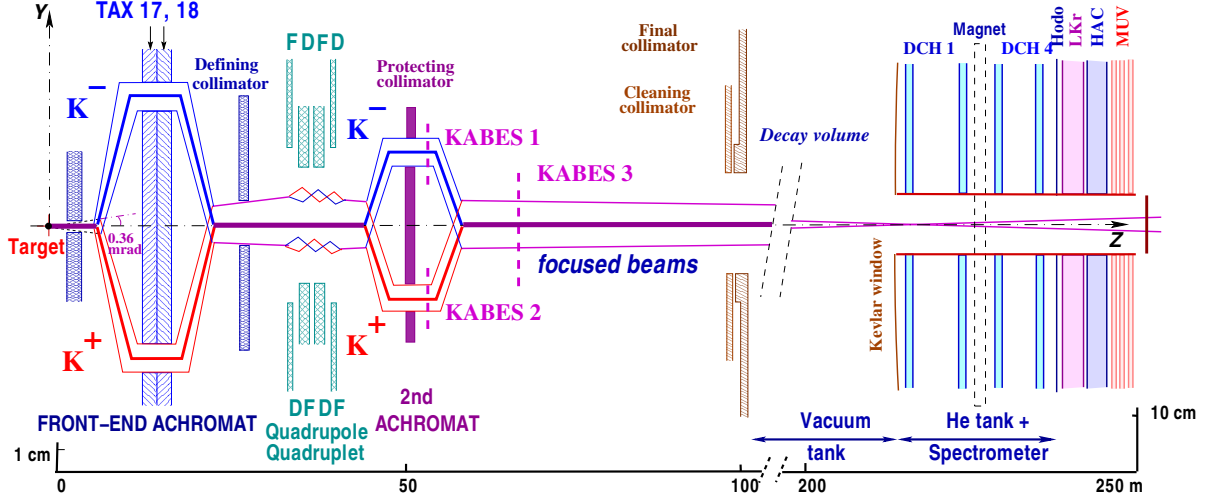


Figure 2.2: Vertical view of the NA48/2 simultaneous $K^\pm$ beams. The positive and negative beams are split and selected in momentum by using a system of magnets and collimators placed along the beam line.

Carlo simulations (Section 4.2.4).

2.3.3 Central detectors

The NA48/2 sub-detectors located downstream the decay volume and used in the present analysis will be described in the following sections. The position of all sub-detectors is given assuming as reference the kaon production target, while, at analysis level, the coordinate origin of the system is taken 120 m downstream the NA48/2 target (18 m from the last collimator), at the position where K_S were generated in the NA48 experiment. A schematic view of the central detectors is given in Fig. 2.3.

2.3.3.1 The magnetic spectrometer

The magnetic spectrometer is equipped with four drift chambers (DCH1-4), two upstream and two downstream of a dipole magnet (MNP33) placed in the center, providing an horizontal transverse momentum kick of $P_{\text{kick}}=0.265 \text{ GeV}/c$. The magnet polarity is frequently reversed in order to reduce the systematic due to the different detector acceptance for K^+ and K^- . The spectrometer is housed in a tank kept at atmospheric pressure and filled with pure helium to reduce multiple Coulomb scattering. The helium tank is delimited upstream by a $900 \mu\text{m}$ thick spherical window with a radius of 1.3 m

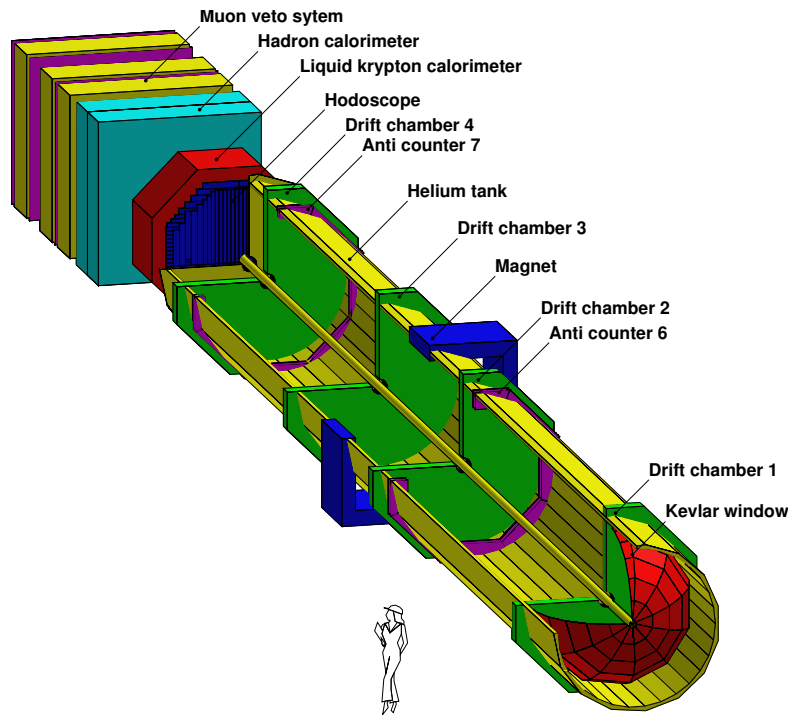


Figure 2.3: Schematics of the NA48/2 detector used for the preliminary phase of the NA62 experiment. Charged particle momentum measurement is performed by the magnetic spectrometer, time measurement by charged hodoscope. Photons are detected by the liquid Krypton calorimeter and the embedded neutral hodoscope

consisting of three layers of kevlar (~ 0.003 radiation lengths) and downstream by a 4 mm (~ 0.045 radiation lengths) thick aluminum window. The whole spectrometer has a length of about 24 m (Fig. 2.4). A charged particle traveling through a chamber ionizes the gas contained in it and the electrons that are freed in such a process drift, with known velocity, towards a sense wire. The position of the particle can be measured with a better resolution than the distance between the sense wires by exploiting the information given by the drift time. The drift chambers were designed to reach a detection efficiency as close as possible to 100% and a spatial resolution for the hits of about $100 \mu\text{m}$, with a fine granularity of the wires corresponding to drift distances of a few millimeters, necessary to stand the high particle flux (more than 1 MHz). Furthermore, a minimal amount of material was used in order to avoid energy loss for electromagnetic interacting particles. The geometry of a DCH plane shown in Fig. 2.5 (left) was chosen. A plane is composed by 256 gold-plated tungsten sense wires with a diameter of $22 \mu\text{m}$ and a spacing of 10 mm from each other. On each side of it, at a distance of 3 mm, gold-plated titanium-copper wires with a diameter of $120 \mu\text{m}$ and the same spacing as the sense wires generate the electric field. Each so-called “view” contains two planes of sense wires in order to resolve left-right ambiguities and to increase the space resolution of the reconstructed point (Fig. 2.5, right). The views are separated from each other by $22 \mu\text{m}$ thick mylar foils coated with graphite, which also serve to shape the electric field. A drift chamber contains four views oriented horizontally, vertically and along each of the two orthogonal 45° directions: $0^\circ(X, X')$, $90^\circ(Y, Y')$, $-45^\circ(U, U')$ and $+45^\circ(V, V')$. The shape of the chambers is octagonal with a width of 120 cm and a 160 mm diameter central hole for the beam pipe (Fig. 2.6). The redundancy in the position measurement given by the four views is used to compensate for the inefficiencies of the single views. From the hit positions in the two chambers before the magnet, the trajectory of the particles without deflection is computed and the position of the decay vertex can be extracted. The two chambers after the magnet give information on the deflection angle due to the magnetic field, which allows to measure the momentum. In principle one chamber after the magnet would be sufficient for this purpose, but the redundancy improves the detection and reconstruction efficiency. The resolution on the spatial position of the reconstructed hit is better than $100 \mu\text{m}$. The measured resolution on track momentum is:

$$\frac{\sigma(p)}{p} = 0.48\% \oplus 0.009\% \cdot p(\text{GeV}/c)$$

The first term is related to the multiple scattering in the helium tank and drift chambers, while the second one to the space point resolution of the chambers. The resolution of

track time measurement is about 700 ps.

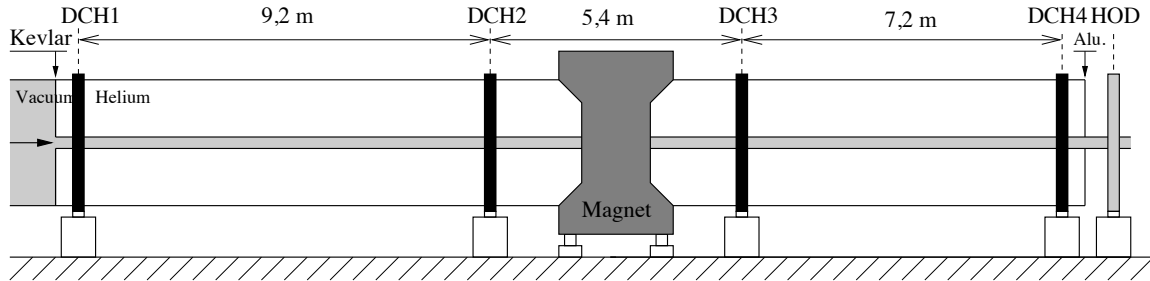


Figure 2.4: Schematic view of the magnetic spectrometer with four drift chambers and a dipole magnet.

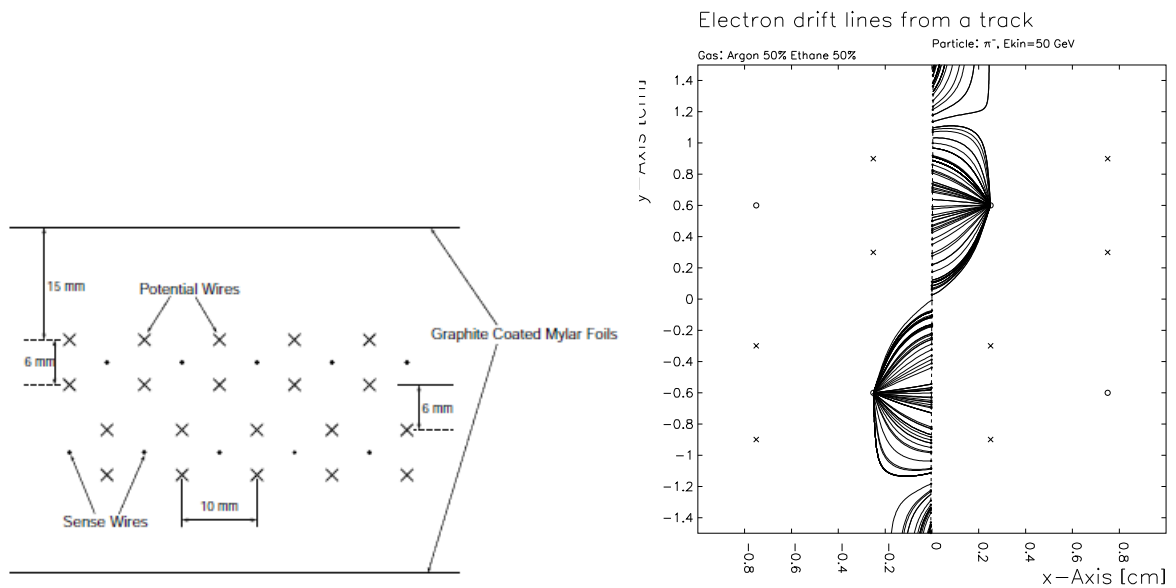


Figure 2.5: (Left) Internal structure of the drift chamber. Two staggered planes of sense wires compose a view. (Right) Signal induced by a 50 GeV/c pion going through a chamber. The staggered structure can solve the left-right ambiguity.

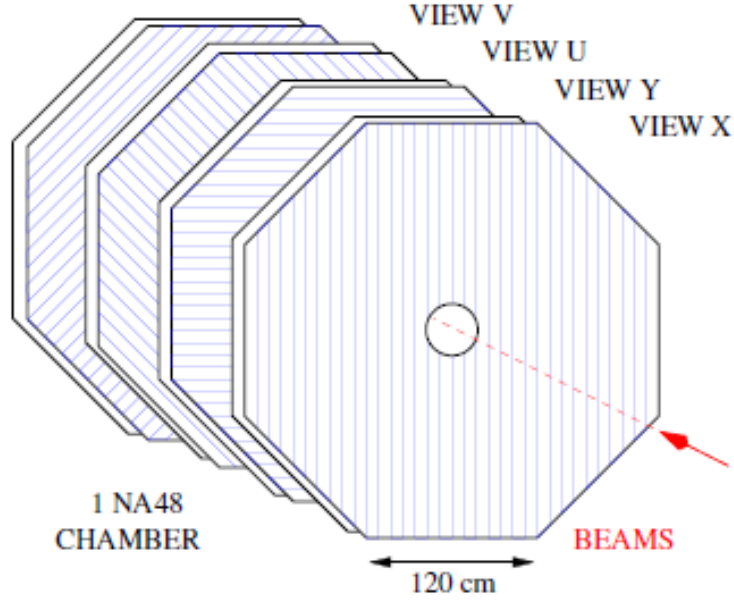


Figure 2.6: Orientation of the four views (X,Y,U,V) of a DCH.

2.3.3.2 The charged hodoscope (CHOD)

The charged hodoscope provides a fast trigger for charged events and is used as reference time detector both in the online and offline reconstructions. It is placed downstream the helium tank, in front of the LKr calorimeter. The hodoscope is composed by two plastic scintillator planes, one with vertical and one with horizontal slabs (Fig. 2.7). Each plane consists of 64 slabs of different length and width. The central hole, in which the beam pipe passes through, has a radius of 10.8 cm. The thickness (equal for each slab) is 2 cm, corresponding to 0.05 radiation lengths. The scintillation light produced at the passage of a charged particle is collected at the edge of a counter from a plexiglass fish-tail shaped light guide connected to a photo-multiplier. The horizontal plane is at 80 cm from the LKr, while the vertical plane is 75 cm in front of the horizontal one. The different timing related to the distance between the two planes is used to tag false coincidences due to the back-splash coming from the calorimeter surface, i.e. particles of the electromagnetic shower going back to the hodoscope. The CHOD time resolution is about 150 ps. Using logic conditions on the CHOD it is possible to define several trigger signals and easy and fast conditions at the first level of the trigger. For instance if a quadrant in each view is fired, a signal useful for many one track triggers is defined (Chapter 5).

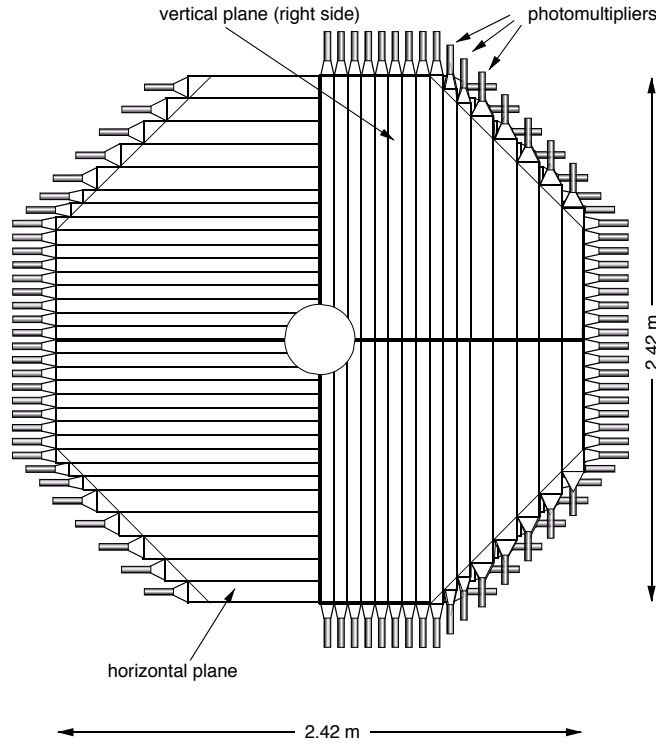


Figure 2.7: Sketch of the horizontal and vertical planes of the charged hodoscope.

2.3.3.3 The liquid Krypton (LKr) electromagnetic calorimeter

For the study of CP violation with the required precision, the NA48/2 experiment needed an electromagnetic calorimeter with good energy, position and time resolution, precise charge calibration, long-term stability and a fast read-out. In order to meet these requirements a liquid Krypton calorimeter (LKr) was built, an almost homogeneous ionization chamber with a tower structure geometry. A photon or an electron entering the active volume of the calorimeter produces an electromagnetic shower via repeated pair production and Bremsstrahlung processes until the energy of the particles is below the critical energy. The low energy charged particles in the shower can then ionize the Krypton atoms producing a certain numbers of electron-ion pairs, which is proportional to the deposited energy. The calorimeter works as an ionization chamber: all the electrons produced and accelerated towards the anode are collected before they can recombine, but the electrons do not get enough energy to produce secondary ionization. The choice of a liquefied noble gas is due to the good resolution and linearity response in energy, to the absence of aging problems and to the relative short radiation length which allows a

compact design without the need of heavy passive parts, typical of sampling calorimeters. The homogeneity ensures a good energy resolution, the granularity improves the spatial resolution and allows to separate showers close to each other. Radioactivity of the Krypton is negligible with respect to electronic noise and has therefore no effect on the resolution, but, due to the low boiling temperature of Krypton at 120 K, the whole detector has to be kept inside a cryostat where only temperature variations of few per mille are allowed, since the drift velocity of the electrons depends strongly on the temperature ($\Delta v_d/v_d \propto \Delta T/T$). Working at cryogenic temperature greatly increases the stability of the detector response and accuracy of its calibration. The LKr is octagonal, containing a circle of 128 cm radius, 127 cm thick, corresponding to about 27 radiation lengths¹, and has a hole at the centre of 9 cm radius for the beam pipe. The total active volume of about 7 m³ liquid Krypton is divided into 13248 cells (towers) by 18 mm wide, 40 μ m thick copper-beryllium ribbons at a distance of 1 cm from each other and with no longitudinal segmentation. The ribbons are used as electrodes to collect the ionization signal. A cell consists of a central anode, to which is applied a voltage of 3 kV, and two cathodes, one at each side, so that each cathode is in common between two cells. The separation between two cell layers is 2 mm. A small fraction of LKr cells (about 60) are marked as “dead cells” because of problems in the electronic chain (for example excessive noise or no calibration). The cells define a projective geometry of the calorimeter pointing at about 90 m in front of it, inside the decay region. This particular geometry was chosen in order to achieve the best possible accuracy in the measurement of the angle between the flight path of photons and the beam direction. To avoid response variations depending on the lateral distance between the shower core and the anode, the cells are not straight along the flight path of the original particles but follow a “zig-zag” shape or an “accordion geometry”, Figs. 2.8 (left) and 2.9. The readout of the LKr calorimeter works in a current -sensitive mode using the initial induced current, proportional to the ionization generated in the LKr by the shower and proportional to the energy of the incoming particle. The sampling and the digitization of the signal is performed every 25 ns (Fig. 2.10). A zero suppression is applied to the cells below a certain threshold, in this way the number of read cells is reduced to about 100 per shower. The presence of a threshold causes a non linear response of the calorimeter at low energy. In order to take into account this effect, an offline correction on the measured values of the cluster energies is applied (Section 4.2.4.1). The energy resolution of the calorimeter, measured using an electron beam of different momenta (15, 25, 50, 100 GeV/c) and 0.1% momentum bite,

¹A shower of 50 GeV deposits 95% of its energy in 24 radiation lengths.

can be parametrized as (Fig. 2.11):

$$\frac{\sigma(E)}{E} = \frac{3.2\%}{\sqrt{E(\text{GeV})}} \oplus \frac{9\%}{E(\text{GeV})} \oplus 0.42\%$$

where the first contribution is the Poisson term from stochastic fluctuations, the second is mainly due to electronic noise and Krypton radioactivity, and the last one comes from the non perfect inter-calibration of the cells. For a 20 GeV particle the energy resolution is about 1%.

Space resolution was measured by comparing the extrapolated track impact point of electrons from a special calibration run with the reconstructed center of energy deposition, yielding the following result:

$$\sigma_{x,y} = \left(\frac{0.42}{\sqrt{E(\text{GeV})}} \oplus 0.06 \right) \text{ cm}$$

where the first term is due to by the statistical fluctuation of the particles in the shower and the second one is due to the size of the cells. For a typical energy of 20 GeV, the space resolution is 1.1 mm in each coordinate. The time resolution on a single shower is at the level of 500 ps.

An example of the resolution performance of the calorimeter is given by the reconstruction of the distance D of the $\pi^0 \rightarrow \gamma\gamma$ longitudinal decay vertex to the LKr plane. Using only the energies E_i and the relative distance r_{12} of the two photons on the calorimeter face, one has (see also Eq. 4.2):

$$D = \frac{\sqrt{E_1 E_2 r_{12}^2}}{M_{\pi^0}}$$

where M_{π^0} is the nominal mass of the π^0 [10]. The resolution is found to be better than 1m at 70 m from the LKr. Moreover, thank to the high granularity, two photons are resolved if their relative distance is at least 2 cm.

2.3.3.4 The neutral hodoscope (NHOD)

The neutral hodoscope is made of 256 bundles of scintillating fibers immersed inside the LKr volume at the depth of 9.5 radiation lengths, where the shower of photons of 25 GeV reaches its maximum development (Fig. 2.12). The signal is read by 32 photomultipliers. The typical time resolution is at the level of 250 ps. The hodoscope, divided in 4 quadrants, provides an independent time measurement of the shower and is useful to measure the efficiency of other triggers.

Atomic number, Z	36
Mass number, A	84
Density at $-150\text{ }^{\circ}\text{C}$, ρ	2.4 g/cm^{-3}
Radiation length, X_0	4.7 cm
Moliere radius, R_M	4.7 cm
Energy to produce an e^- +ion pair	20.5 eV/pair
Hadronic interaction length, λ_h	60 cm
Drift velocity at 1 KV/cm, v_{drift}^e	$2.7\text{ mm}/\mu\text{s}$
Critical energy, E_c	21.5 MeV

Table 2.1: Liquid Krypton properties.

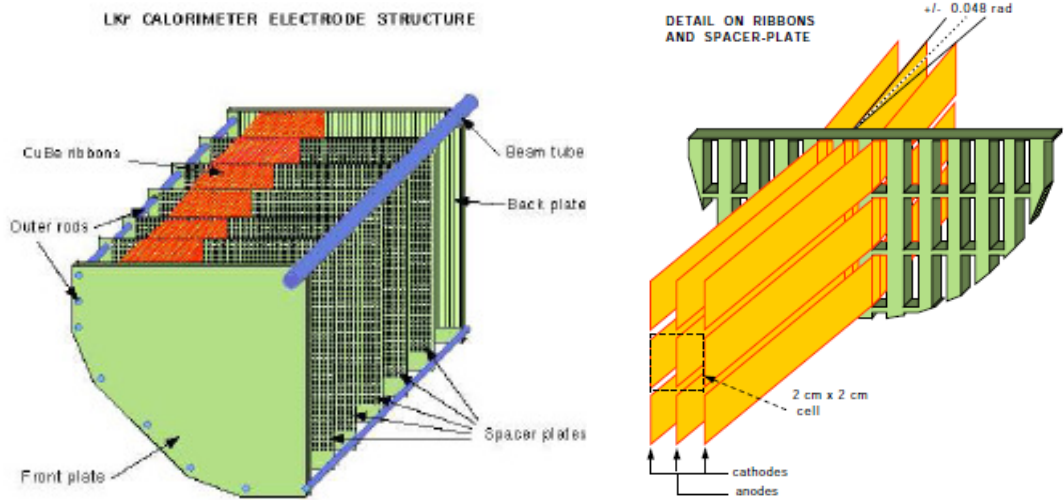


Figure 2.8: (Left) The LKr electrode structure. (Right) Detail of the LKr cell structure showing the “accordion geometry” structure of the ribbons.

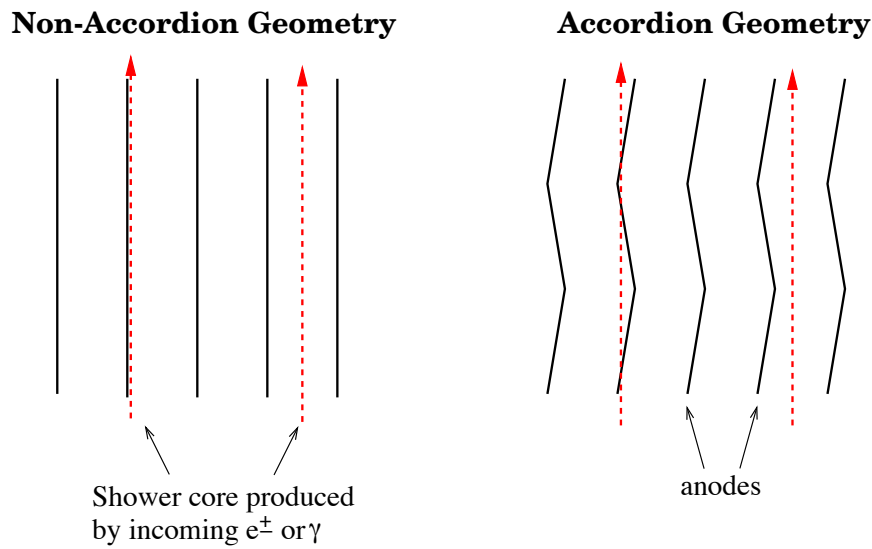


Figure 2.9: Calorimeters with straight electrodes show a marked dependence of the response on the transverse position of the shower in the cell. The accordion geometry is used to reduce this effect and to minimize inefficient ionization of the showers developing along the anode.

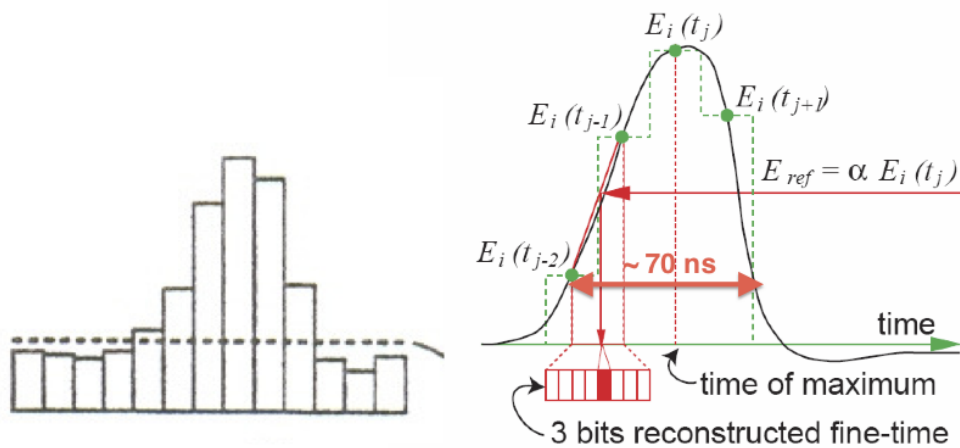


Figure 2.10: Sampling and digitization of the LKr calorimeter signal. The output signal has approximatively a Gaussian shape with 70 ns FWHM and a pulse height proportional to the initial current induced on the anode of a cell.

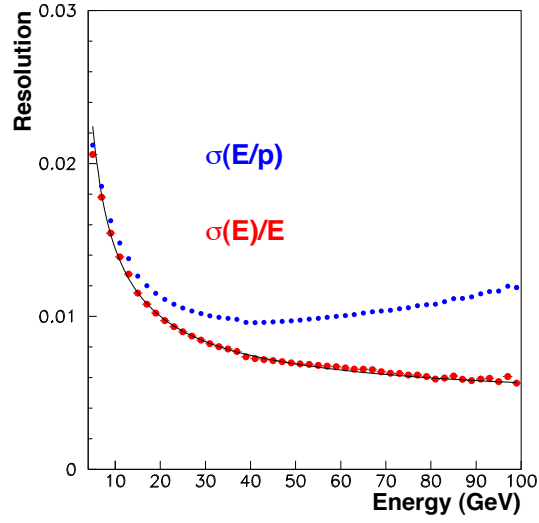


Figure 2.11: Resolution of the LKr calorimeter, calculated using an electron beam, as a function of the energy. The resolution of the E/p ratio between the energy as measured by the calorimeter and the momentum of the electron as measured by the magnetic spectrometer, is also shown.

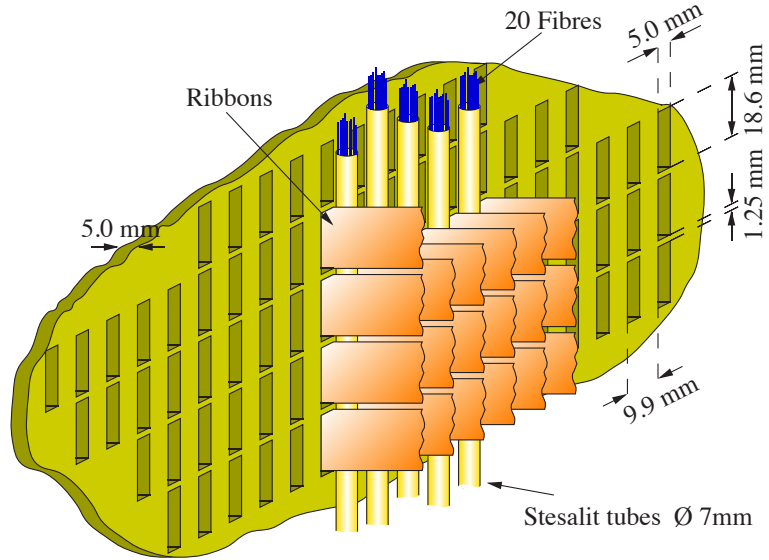


Figure 2.12: The fibers of the neutral hodoscope inside the LKr calorimeter.

2.3.4 The trigger system

The NA62 phase I trigger is a multilevel system, designed to avoid dead time with a rate of particles in the main detectors at the level of 1 MHz (Fig. 2.13). The full acquisition system is pipelined, synchronized by a 40 MHz clock. Circular memories 204.8 μ s deep store the informations from each detector every 25 ns. The Level 1 (L1) trigger system is subdivided in two parts: the charged (L1C) and the neutral (L1N) trigger. The two sub-systems are controlled by the Trigger Supervisor (TS) which collects all the information from the neutral and charged triggers and takes the final decision to read-out or discard the event.

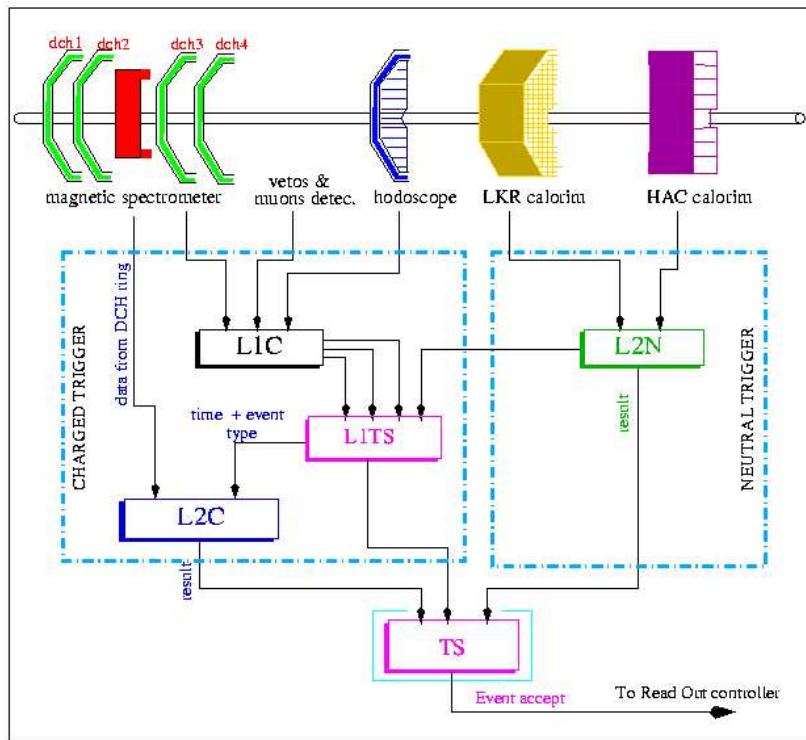


Figure 2.13: The NA48/2 trigger system. In 2007, the L2C trigger signal and the information coming from the muon veto detector (MUV) and the hadron calorimeter (HAC) have not been used in the trigger chain.

2.3.4.1 Charged trigger

The Charged Trigger of NA48/2 was originally divided in two levels in order to reduce the rate of events registered by the detectors, a pre-trigger based on a fast hardware system (L1C) and a second level based on a software processor farm (L2C). During the

data taking in 2007 only the L1C has been used. Each level of the trigger system had a reduction factor from input to output of the order of 10.

The goal of the L1C is to reduce the input rate to the second trigger level below 150 kHz. It is a fast hardware system collecting the information from the CHOD, NHOD and DCHs. The information of the muon veto and hadron calorimeter, used in the original NA48/2 L1C trigger chain, were not used in 2007. The CHOD provides a simple information on the hit pattern of the event; space coincidence and time consistency of hits between the horizontal and the vertical planes of the CHOD are used to identify different classes of events. The following logical signals are defined using CHOD information:

- Q1: a single coincidence; it tags events with at least a signal on each plane of the CHOD;
- Q2: two coincidences; it tags events that have at least two matching hits on each planes;

DCHs provide informations on the number of hits in the four views of the four chambers which are combined to produce triggers and select events with a different number of charged particles. A signal produced by DCHs and used in the analysis is 1TRK-LM signal. The 1TRK-LM trigger condition is based on the multiplicity of hits in the drift chambers, and it requires at least one hit in more than one view and less than 15 hits in any view in DCH1 and DCH2 and DCH4 (Chapter 5).

The NHOD triggers on a shower in the LKr; the signals sent to the L1C system provide a time measurement of the neutral events, independent of that obtained from the calorimeter; the NHOD provide the T0N signal, a loose coincidence between opposite halves of the NHOD, useful as minimum bias trigger for efficiency studies.

The L1C output signals are sent to the L1 Trigger Supervisor (L1TS) subsystem that combines them with the energy information from the LKr calorimeter. Here some loose selection based on the pattern and energy of the events are made. Before the event is sent to the Trigger Supervisor, a time alignment of the signals from the detectors is performed, a time-stamp is attached to the event, and a code that identifies the kind of event is produced.

2.3.4.2 Neutral trigger

The neutral trigger is completely independent from the charged trigger. It is a single level trigger which use only the information of the electromagnetic calorimeter. Several important quantities like the total energy, the center of gravity and the kaon vertex position are directly computed by dedicated hardware. In order to compute these quantities,

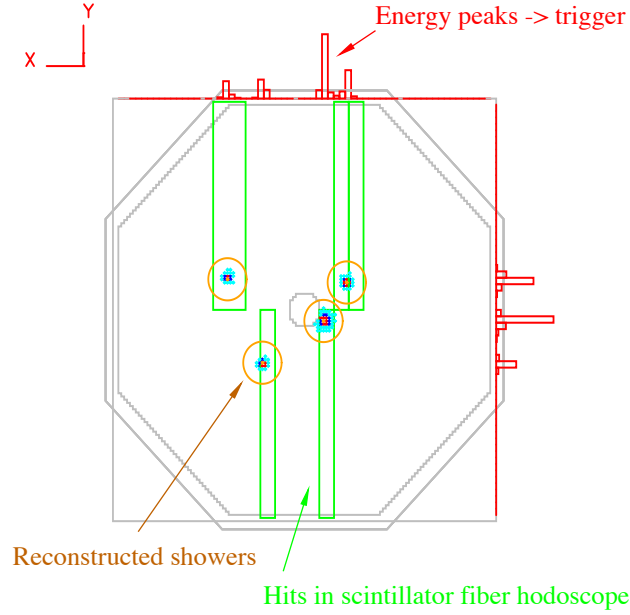


Figure 2.14: The event display of a typical $K^+ \rightarrow \pi^+ \pi^0 \pi^0$ event. The red histograms represent the energy projections in the x and y views and the green bars are the NHOD hits.

the energy information of the x and y views of the calorimeter are used (Fig. 2.14). The neutral trigger system receives in input the information on the energy of super cells, blocks of 2×8 single cells defined both in x and y views. The signals from super cells are digitized and filtered to suppress those below a certain threshold. Then the signals of up to 16 super-cells in a row or in a column are summed to build 64 horizontal and 64 vertical projections. A dedicated system computes, separately in the horizontal and vertical views, the total energy (m_0) and its first (m_1) and second (m_2) spatial moments and counts the number of peaks in each projection (n_{px} and n_{py}), defined as a local maximum in time and space with an energy above a certain threshold. The moments are calculated according to the formulae:

$$\begin{cases} m_{0x} = \sum_{i=1}^{64} E_i^x, & m_{0y} = \sum_{j=1}^{64} E_j^y \\ m_{1x} = \sum_{i=1}^{64} E_i^x (i - 32), & m_{1y} = \sum_{j=1}^{64} E_j^y (j - 32) \\ m_{2x} = \sum_{i=1}^{64} E_i^x (i - 32)^2, & m_{2y} = \sum_{j=1}^{64} E_j^y (j - 32)^2 \end{cases}$$

where E_i^x and E_j^y are the energies of the i -th horizontal projection and j -th vertical projection, respectively. A look-up table system combines the values computed for the

two views to obtain the physical quantities necessary for the trigger decision. The total deposited energy in the LKr (E_{TOT}), the distance of the energy-weighted average event position with respect to the LKr center (r_{cog}) and the distance of the longitudinal decay vertex to the LKr plane (D) are computed as:

$$E_{\text{TOT}} = \frac{m_{0x} + m_{0y}}{2}$$

$$r_{\text{cog}} = \frac{\sqrt{m_{1x}^2 + m_{1y}^2}}{E_{\text{TOT}}}$$

$$D = \frac{\sqrt{E_{\text{TOT}}(m_{2x} + m_{2y}) - m_{1x}^2 - m_{1y}^2}}{M_K}$$

where M_K is the nominal PDG kaon mass. The signals produced by the neutral trigger are:

- NT-PEAK: more than two LKr cluster peaks in at least one projection (x or y);
- NT-NOPEAK: an energy deposition in the LKr calorimeter greater than 30 GeV with a center of gravity smaller than 25 cm and a kaon decay position distance below 90 m;
- $E_{\text{LKr}}(10 \text{ GeV})$: the deposited energy in the electromagnetic calorimeter is greater than 10 GeV.

With these reconstructed quantities, the neutral trigger performs a selection and sends the information to the TS.

2.3.4.3 Trigger Supervisor

The L2 Charged and Neutral Trigger results are sent to the Trigger Supervisor (TS), a digital system which correlates the information coming from different trigger sources and takes the final decision on the selection of the event. It also provides a final trigger word and time-stamp. All trigger signals sent to the TS, coming from the Charged and Neutral Trigger, are also sent to external acquisition units, called Pattern Units (PU), for monitoring purposes and for trigger efficiency studies.

2.3.5 Data acquisition and data format

The NA62 data acquisition system (Fig. 2.15) is composed by 11 Sub-Detector PCs, 8 Event Builders PCs and one Control PC. Any given Sub-Detector PC stores the data

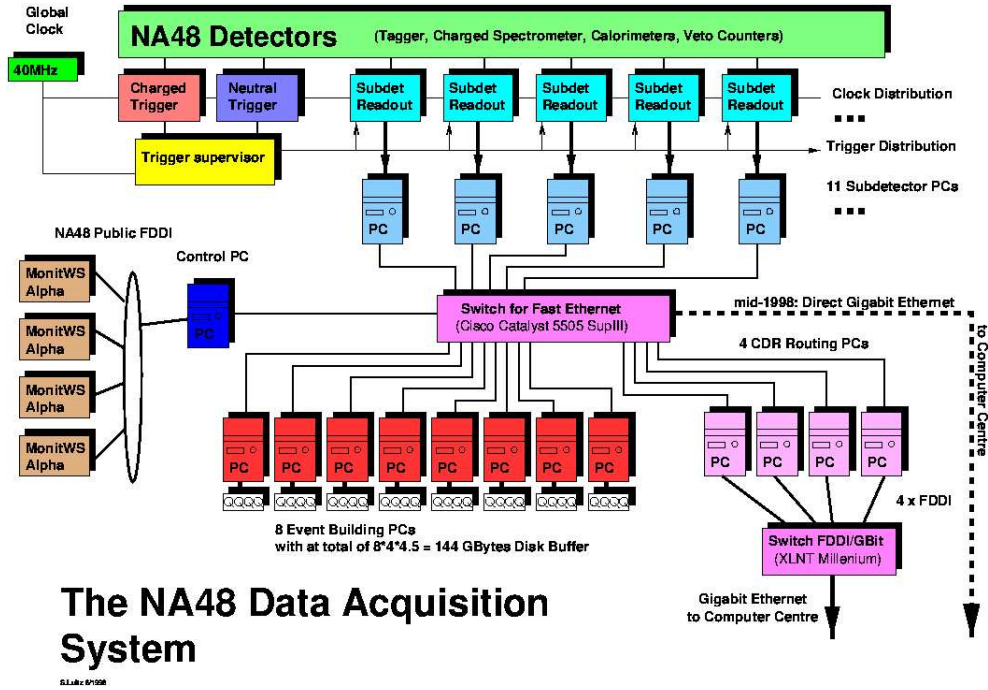


Figure 2.15: Schematic of the DAQ complex.

recorded by the connected detector during the SPS burst. Data are assembled, partitioned in eight blocks and sent to the Event Builder PCs. The Event Builder PCs arrange the data in complete raw events that are sent to ten disk-servers (average capacity about 10 TB) at the CERN computing center via a dedicated link. In case of an event number mismatch from different sub-detectors, the whole burst is discarded. In normal conditions the amount of data transferred from the experiment to the disks is around 3 TB/day. The raw data are processed by a software program, the level 3 trigger (L3), running on the CERN Batch System (LSF) that performs the event reconstruction, filters the data and creates different output streams. The program output are in COMpACT format, containing reconstructed information and physics quantities [99]. Both COMpACT and RAW files are recorded on tapes. The average time from data acquisition until the end of processing is about 30 minutes: this allows a quite fast feedback to check the data quality. The overall efficiency of the offline processing is slightly greater than 99%. The typical size on one RAW burst is about 500 MB while a typical COMpACT file is about 120 MB. RAW data are sometime reprocessed to apply the best recomputed calibration constants. During this process another compressed format, called SuperCOMpACT, is produced to reduce the total amount of data and to speed up the analysis time.

Run periods in 2007				
Period	Runs	Pb wall/Straws	Online K_{e2} count	K sign
1	20114 - 20203	Pb	10K	+/-
2	20209 - 20285	Pb	17K	+/-
3	20286 - 20324	Pb	10K	+/-
4	20332 - 20404	Pb	18K	+
4a	20385 - 20386	Pb	0.4K	+/-None
5	20410 - 20486	None	40K	+
6	20487 - 20531	None	8K	-/None
7	20611 - 20695	Straws	8K	+
8	21088 - 21120	None	N.A.	-

Table 2.2: A summary of the run periods in 2007. The online raw estimation of K_{e2} events gives an idea of the statistics accumulated in each period.

2.3.5.1 Level 3 trigger

The final stage of the online processing is the reconstruction of the events performed by the software L3 trigger level. Thanks to this level the data are rapidly available for monitoring purposes during the data taking. The typical time for the processing is about 10 min. During the 2007 runs, two main streams optimized for the R_K analysis, were produced by L3 trigger:

- K_{e2} stream: at least one track reconstructed by the spectrometer with momentum in the range $5 \text{ GeV}/c < p < 90 \text{ GeV}/c$ and the ratio between the track energy released in the calorimeter E and its momentum $E/p > 0.6$
- $K_{\mu2}$ stream: at least one track reconstructed in the magnetic spectrometer with momentum $p < 90 \text{ GeV}/c$

2.3.6 The 2007 data taking

Data were collected from June 23th until October 22th 2007, plus a week in September 2008, for a total of about 120 days. The data taking was subdivided in several periods (Tab. 2.2). Run periods differs concerning beam geometry and intensity, trigger conditions and on the presence of a lead wall between the two planes of the CHOD (Fig. 2.16). The lead wall was used to stop all charged particles except muons with the purpose of measuring the probability for a muon to be mis-identified as an electron. This measurement was essential for the R_K analysis to estimate the $K_{\mu2}$ contamination in the K_{e2} sample. The wall was present in the first part of the data taking (about 64 days).

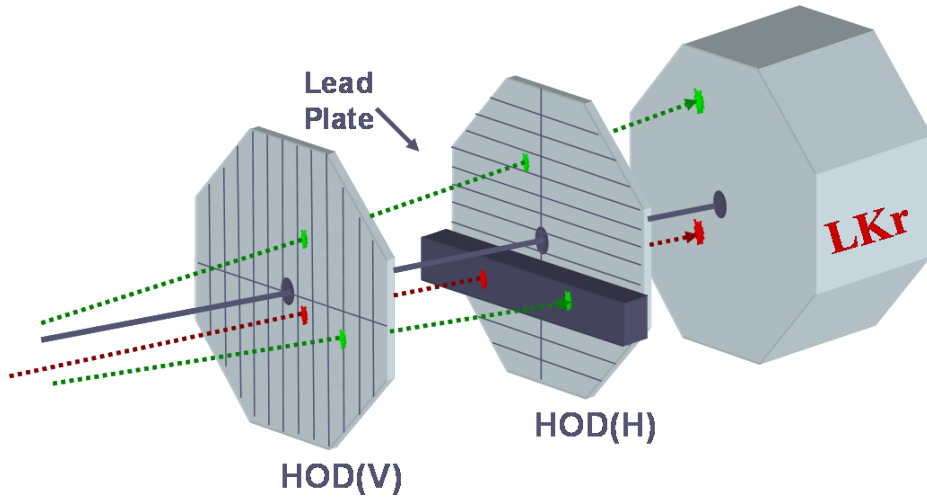


Figure 2.16: Sketch of the lead wall installed during some periods of the 2007 data taking to study the muon mis-identification probability for the R_K analysis.

During the data taking some special runs with different beams (muon beam, beam without kaons and K_L beam) were also collected in order to study some systematic effects on the R_K analysis and background due to other particles of the beam as muons and pions. Moreover, dedicated runs were scheduled to test the prototype of the straw chambers for the new NA62 experiment. During period 5 (P5), 53 days long, about 40% of the whole statistics was acquired and about $2 \cdot 10^9$ total triggers were collected, subdivided into 80551 good bursts. During this period only positive kaons were delivered by the beam line. For this analysis only data collected during P5 have been used, because of this period has the most stable data taking conditions. For the entire period the beam type and trigger conditions were not changed. On the contrary, different beam and trigger conditions were present during the data taking of the other periods. Another motivation for the use of P5 is the absence of the lead wall, which allows a full acceptance of the calorimeter, useful to increase as much as possible the number of signal candidates available for the $K_{e2\gamma}$ analysis.

Chapter 3

Strategy of the analysis

3.1 Goal of the analysis

As described in Chapter 1, the structure dependent amplitude of the $K_{e2\gamma}$ decay can be parameterized by the vector and axial-vector form factors $F_V(x)$ and $F_A(x)$, respectively. The evaluation of form factors requires a significant theoretical effort due to the non-perturbative behavior of the strong interactions at low energies. In this respect, ChPT provides a universal framework for treating in a perturbative way, leptonic, semi-leptonic and non-leptonic decays, including radiative modes. In particular, it gives a clear and unambiguous prediction of the $K_{e2\gamma}$ form factors in terms of a set of parameters obtained by fitting other decay data. Recent estimations in the framework of ChPT (Tab. 1.4) assign theoretical uncertainties of 6% on $F_V(0)$ and 1% on $F_A(0)$, while the linear slope λ is calculated with a 33% accuracy (at order $\mathcal{O}(p^6)$, F_V can be approximated by $F_V(x) = F_V(0) \cdot [1 + \lambda(1 - x)]$, Section 1.6.2). A measurement of the form factors with a precision better than the theoretical one represents an important test of ChPT. The data collected by the NA62 experiment in 2007 allows to measure $K_{e2\gamma}$ form factors with an unprecedented precision. The aim of the present analysis is to measure $K_{e2\gamma}$ form factors using a partial (40%) data sample, with a precision improved by a factor ≈ 3 with respect to the most recent result by the KLOE experiment [18]. In order to keep the background to a few percent level, the measurement is performed in a restricted kinematic region, where the SD^+ component dominates over SD^- and IB parts. In the following sections, the description of the fitting method will be discussed. The main sources of systematic errors, their estimations and the major issues limiting the precision of the measurement will be also addressed in the following chapters.

3.2 Fitting procedure

The measurement of the form factors is performed by fitting the binned x distribution of the SD^+ candidates, after background subtraction, to the expected binned x distribution given by theory. The predicted number of $K_{e2\gamma}(\text{SD}^+)$ decays in a given bin i of the x distribution (N_i^{th}) is computed according to the theoretical differential decay rate, with the normalization factor given by the kaon flux Φ_K :

$$N_i^{\text{th}} = \Phi_K \cdot d\text{BR}_i$$

where $d\text{BR}_i = \frac{d\text{BR}}{dx_i} \cdot \Delta x$ is the theoretical differential branching ratio evaluated at the bin centre and multiplied by the bin width Δx . In the framework of ChPT at $\mathcal{O}(p^6)$, $d\text{BR}_i$ depends on $F_V(0)$, $F_A(0)$ and λ . The flux Φ_K is defined as the total number of kaon decays in the fiducial decay region, integrated over the time period of the data taking, and depends on the number of protons impinging on the target. In principle, the flux can be considered as a free fit parameter and estimated together with form factors. However, the flux and the form factors result to be highly correlated, making the fit result unstable and very dependent on the initial value of the parameters. A method to avoid this problem, with the advantage of reducing the statistical error and the correlations between form factors, consists in measuring the flux directly from data using an independent decay mode (“normalization channel”); in this way, the measured flux can be used as an external parameter and fixed throughout the fit procedure. The normalization channel has to be chosen in such a way that:

- the statistical error on the flux is negligible; this requires to choose a decay channel with a high statistics (much higher than $K_{e2\gamma}(\text{SD}^+)$)
- the systematic effects on the flux are kept as low as possible. This can be achieved by considering a normalization channel with a final state similar to the signal one and triggered by the same trigger logic. In principle, this choice has the advantage of reducing the systematic error thanks to a partial cancellation of systematic effects common to signal and normalization

In the case of the $K_{e2\gamma}$ analysis, a natural choice for the normalization channel is the $K^+ \rightarrow \pi^0 e^+ \nu$ (K_{e3}) decay: its branching ratio is about 3300 more frequent than $K_{e2\gamma}$. The K_{e3} final state is similar to the $K_{e2\gamma}$ one and can be collected by the same trigger chain used for the signal, the only difference being an additional photon coming from π^0 decay. The analysis of the normalization channel will be discussed in Chap. 4.

The expected number of events in a given bin i of the x distribution is obtained from N_i^{th} by convoluting (folding) the theoretical spectrum with the detector resolution and acceptance given by a Monte Carlo of the signal. The effects of resolution and acceptance are encoded in a folding matrix $P_{i'j'}$ defined as the number of SD^+ Monte Carlo events¹ $N_{i'j'}^{\text{rec}}$ generated in bin i' and reconstructed in bin j' (Fig. 3.1) over the number of total events generated in bin i' :

$$P_{i'j'} = \frac{N_{i'j'}^{\text{rec}}}{N_{i'}^{\text{gen}}}$$

The matrix $P_{i'j'}$ is thus the conditional probability that an event will be found with measured value x in bin j' given that the true value was in bin i' . Primed indices refer to the “convolution bins”, while unprimed indices are used to denote the “fit bins”: in fact, the binning used for the convolution can be different with respect to the one used for the fit. $P_{i'j'}$ depends on the theoretical model implemented in the Monte Carlo; this dependence cancels out if the binning used in the computation of $P_{i'j'}$ is small enough that the resolution is approximately constant over the bin i' . The expected number of $K_{e2\gamma}(\text{SD}^+)$ events is then:

$$N_i^{\text{exp}} = \sum_{i' \in \text{bin } i} \left(\sum_{k'} N_{k'}^{\text{th}} \cdot P_{k'i'} \right) = \Phi_K \cdot \sum_{i' \in \text{bin } i} \left(\sum_{k'} d\text{BR}_{k'} \cdot P_{k'i'} \right) \quad (3.1)$$

where the sum in brackets extends over all the convolution bins, while the other runs only over the convolution bins inside the fit bin i . In order to compare the number of data events in each bin N_i^{data} with the expected number of SD^+ events, the total number of residual background events N_i^{B} in the i^{th} bin have to be estimated and subtracted from the number of selected SD^+ candidates $N_i = N_i^{\text{B}} + N_i^{\text{data}}$. The best estimate of the form factors compatible with the data is found by minimizing the χ^2 function [100]:

$$\chi^2 = \sum_{i=1}^{N_{\text{BIN}}} \frac{(N_i^{\text{data}} - N_i^{\text{exp}})^2}{\sigma_{N_i^{\text{data}}}^2 + \sigma_{N_i^{\text{exp}}}^2}$$

where N_{BIN} is the number of bins. The values of the form factors corresponding to the minimum of the χ^2 function are estimated numerically; the χ^2 minimization is performed by means of the computer program MINUIT [101]. The choice of the bin width used in the fit is dictated by the limited statistics of the SD^+ candidates (Chapter 4).

¹The signal Monte Carlo sample is generated assuming the form factors as measured by KLOE [18].

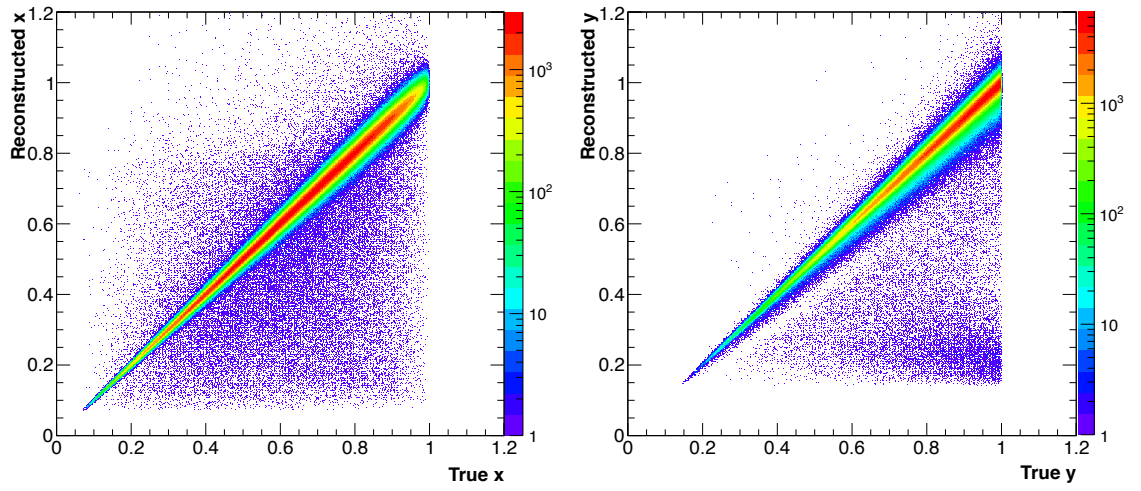


Figure 3.1: Distributions of the generated x versus the reconstructed x (left) and the generated y versus the reconstructed y (right) for SD^+ MC events passing the signal selection, except for the x , y and $M_{\text{miss}}^2(e\gamma)$ cuts. Events reconstructed with the radiative photon energy (positron momentum) lower than the true energy (momentum) are the most evident feature.

Chapter 4

$K^+ \rightarrow e^+ \nu_e \gamma$ (SD^+) event selection

The detectors involved in the $K_{e2\gamma}(\text{SD}^+)$ analysis are mainly the DCHs and the LKr electromagnetic (EM) calorimeter. For each detector a reconstruction program was developed which elaborates the information from the read-out into physical quantities such as the energy of an EM cluster in the calorimeter (*raw data*). The raw data format is not suitable for analysis purpose due to its large size. The COMPACT data format (Section 2.3.5), containing reconstructed information and physics quantities with a size reduced by a factor 5 with respect the raw format, is then obtained directly at the L3 trigger level, to allow a fast feedback and a first analysis to evaluate the detector performances. At this stage of the data processing, the detectors calibration constants (*e.g.* reference times, pedestals) are computed and a second version of COMPACT data is produced. However, even the COMPACT data set is large to handle for analysis purposes, so that the SuperCOMPACT (SC) format is developed starting from the COMPACT one without any selection, but reducing and digitally compressing the information available. In producing the SC format several routines are called, mainly regarding the calorimeter, in order to correct at best for all known effects. The SC format is then used to apply the selection and fully reconstruct the kinematic of the $K_{e2\gamma}(\text{SD}^+)$ event.

The first part of the $K_{e2\gamma}(\text{SD}^+)$ event reconstruction (Section 4.2.1) is dedicated to the selection of only one “good” DCH track (positron candidate), produced in the fiducial decay region and coming from the kaon decay vertex, and to the identification of the EM cluster associated to the energy deposit of the particle in the calorimeter (associated cluster), defined as the closest cluster to the track position at the LKr with a time consistency with the track. In the second part of the selection (Section 4.2.2), the cluster multiplicity in the LKr is considered by requiring only one “good” cluster not associated to the energy deposit of the track on the calorimeter (radiative photon candidate). In

the final part (Section 4.2.5), kinematic criteria are imposed to optimize the signal to background ratio. The reconstruction program of DCH and LKr and the $K_{e2\gamma}(\text{SD}^+)$ event selection will be described in the following sections.

4.1 Event reconstruction

4.1.1 DCH track reconstruction

The charged tracks are reconstructed using DCHs. The raw informations from DCHs are the hits on single wires. The hits are grouped into clusters if there are contiguous hits in both planes of one view (X,Y,U and V); only space consistency is required using the wire's number, while at this stage the hit times are not taken into account. Clusters are mainly made of two hits (one hit per plane). A cluster made of three hits can be generated when a particle is very close to a wire in a plane. In this case both wires in the corresponding staggered plane are used. If a plane is not fully efficient, some hit doublets are transformed into singlets (clusters made of one single hit).

Then *front segments* are built using all possible combinations of clusters from the corresponding views in DCH1 and DCH2. A front segment is a projection of the track in one of the four possible views.

The next step of the reconstruction algorithm is the definition of the *front tracks*, built using the front segments. If there are less than four front segments a front track could be defined by three or two segments adding, possibly, an additional cluster in DCH1 or 2. The extrapolated front tracks have to cross a virtual cylinder of 10 cm in diameter around the nominal beam axis, to exclude particles with trajectories external to the decay volume (Fig. 4.1).

Once a front track is computed, *space points* can be defined in DCH1 and DCH2 as their intersection with the front track. The full track can now be built matching the front tracks with the space points in DCH4. The timing information of the hits is then introduced to reject points not consistent in time. The hit times are used to compute the time and the “quality” of the track, defined as the fraction of hits in each DCH view close in time to the average time of the total sample; a good value of the quality parameter is close to unity. The physical parameters of the track (position, slope in x and y and momentum) are computed by a fitting algorithm using the information of each space point. Once the tracks are built, the clusters in DCH3 compatible with the extrapolated track position are added to the track and its physical parameters are recalculated.

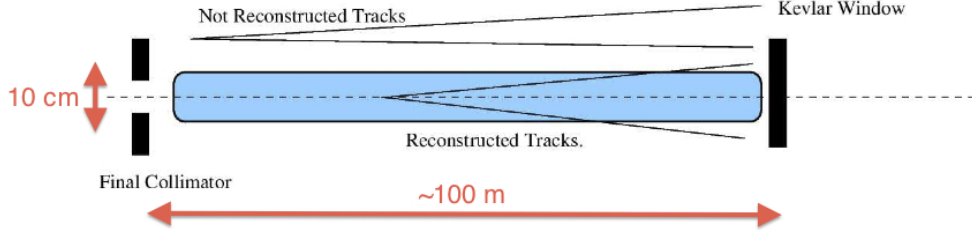


Figure 4.1: Example of tracks crossing and not crossing the virtual cylinder defined at the reconstruction level. The tracks outside the cylinder are excluded from the reconstruction.

4.1.2 LKr energy reconstruction

The LKr reconstruction program determines the energy, time, position and size of the clusters generated by particles hitting on the calorimeter. The first step is to look for cells with an energy greater than 0.25 GeV (*seeds*). The seeds are then ordered by decreasing energy and a loop is made to find clusters, defined when a seed has more energy than the 8 surrounding cells and satisfies:

$$E_{\text{seed}} > 0.18 + 1.8E_{\text{av}}$$

where E_{av} is the average energy of the 8 surrounding cells. Once a cluster is found, its position is estimated by calculating the energy-weighted average coordinates (barycenter) in this 3×3 box and a first order estimate of its energy is obtained from the central cell (Fig. 4.2). For each cell in the LKr a list of clusters within 11 cm from the cell is made. If there is only one cluster for which the cell time is ± 20 ns around the time of the cluster seed, the energy of the cell is added to that cluster. If there is more than one, the energy of the cell is shared among the clusters according to the formula

$$E_i = E_{\text{cell}} \frac{W_i}{\sum_i W_i}$$

where the weight W_i , representing the expected fraction of energy in the cell due to the i -th cluster, is estimated using the cluster energy, the time and the distance with a shower profile simulation (taken from a GEANT Monte-Carlo). After this step, the energy and the barycenter of the cluster are recomputed and corrected for known effects (mainly due to the overall energy scale and the energy loss for clusters at the edges of the calorimeter). Specific algorithms are applied if there are two clusters within 11 cm from each other: if

there is a cluster with very low energy (< 1.5 GeV) close (< 10 cm) to an high energy cluster, the low energy cluster is assumed to come from the same photon and the two clusters are joined (cluster merging). If a dead cell is among the 3×3 cells surrounding the cluster seed, the region used to measure energy and position is extended to 5×5 cells. In the case where the dead cell is adjacent to the seed, the energy and position are re-estimated by comparing the observed energy shape to the expected one for various assumed photon positions.

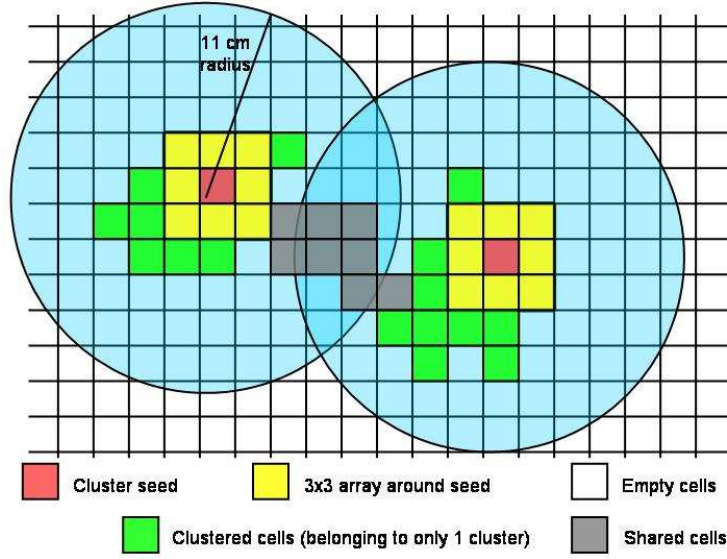


Figure 4.2: Example of cluster reconstruction in case of two close clusters. The energy of the overlapping cells is shared using a shower profile simulation taken from a Monte Carlo.

4.2 $K^+ \rightarrow e^+ \nu_e \gamma$ (SD^+) event selection

The selection of $K_{e2\gamma}(\text{SD}^+)$ events requires at least a track measured by the spectrometer with an associated energy deposit in the LKr calorimeter (positron) and at least another cluster not associated to any track (radiative photon). The cuts applied to the track and the clusters and the reconstruction of the $K_{e2\gamma}$ kinematic variables will be described in the following sections.

4.2.1 Track selection

The track selection aims at identifying a well reconstructed track from the signal, while being independent as much as possible on the presence of possible tracks from accidental beam-induced activity. Moreover, the track selection is useful for background rejection, as well as for restricting the signal sample to a given kinematic region and performing an accurate extraction of the form factors. The selection of the candidate lepton track is obtained in two steps:

- Exactly one “good” track is selected (the candidate lepton track). The definition of a “good” track is set as loose as possible to include with high efficiency tracks from kaon decays, while minimizing the probability to have an accidental track which is labelled as “good”. In SD^+ MC events, extra good tracks are originated by conversion of the radiative photon in the material before the spectrometer (the fraction of MC events with more than one good track is $\approx 0.5\%$), while in data, additional good tracks, not associated to kaon decays, are produced also by the interaction of the beam with the final collimator ($Z_{\text{coll}} \approx -1800$ cm) (Fig. 4.3). To estimate the number of accidental events with an extra track in time with the candidate lepton track, events with two good tracks were selected and the number of such events in which one of the two tracks passed the $K_{e2\gamma}$ selection were counted. The fraction of these events with respect to the total number of SD^+ candidates is about 1%.
- Once the candidate lepton track has been found, a tighter selection is applied in order to identify the selected track with the positron associated to the $K_{e2\gamma}$ decay.

Within the first step of the track selection a sample of “bad” tracks is identified for each event; a track is considered “bad” if it satisfies at least one of the following criteria (it is otherwise marked as “good”):

- $Z_{\text{vtx}} < -2000$ cm or $Z_{\text{vtx}} > 9000$ cm, where Z_{vtx} is the longitudinal decay vertex obtained by evaluating the Closest Distance of Approach (CDA) between the

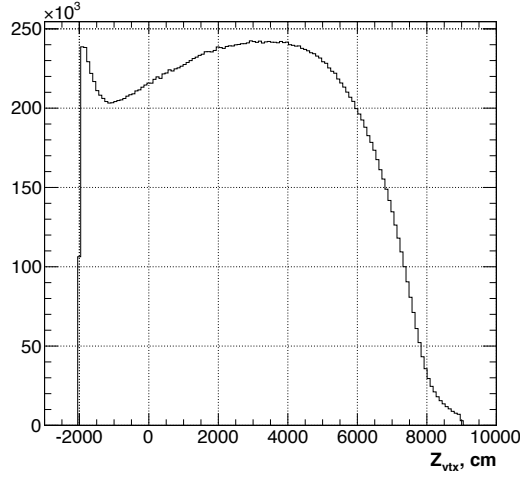


Figure 4.3: Data distribution of the longitudinal decay vertex position (Z_{vtx}), after the first step of the track selection, for events with exactly one good track. The peak around $Z_{\text{vtx}} = -1800$ cm is due to the interaction of the beam with the final collimator.

nominal kaon and track trajectories. These conditions, if satisfied, allow to exclude tracks coming from outside the fiducial decay region (the final collimator is at $Z_{\text{coll}} \approx -1800$ cm, while the Kevlar window closing off the upstream section of the helium tank of the spectrometer is at ≈ 9300 cm). The vertex reconstruction algorithm takes as input a point anywhere on the trajectory and a vector on it for both track and kaon: for the track, the position at DCH1 and its direction (before the magnet bending) are measured by the spectrometer, while for the kaon, the average slope and position are computed offline using a $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ ($K_{3\pi}$) decay sample (Section 4.2.3). The algorithm finds the points $\vec{P}_1$ and $\vec{P}_2$ along the two lines where the distance between them is minimum and it returns the vertex position $\vec{P}_{\text{vtx}} = \frac{\vec{P}_1 + \vec{P}_2}{2}$ and CDA.

- CDA > 10 cm. This cut, combined with the previous requirement on the longitudinal decay vertex position, allows to discard tracks not compatible with a kaon decay (Fig. 4.4, left plot).
- Track momentum $P_{\text{trk}} < 3$ GeV/ c or $P_{\text{trk}} > 75$ GeV/ c . This condition allows to suppress low-momentum tracks not well reconstructed by the DCH reconstruction algorithm and tracks not coming from kaon decays (the beam central momentum is 74 GeV/ c with a momentum spread of ± 1.4 GeV/ c)
- The track time measured by the DCH is outside ± 2 time slots around the slot

corresponding to the trigger signal (each time slot is 25 ns wide): $|\Delta t_{\text{trk-off}}| > 62.5$ ns, where $\Delta t_{\text{trk-off}}$ is the track time from which a constant offset was subtracted to synchronize the DCH time to the L2C trigger. This requirement allows to mark as “bad” accidental tracks not in time with the triggered event.

- The track is a “ghost track”. For any pair of tracks with momentum $3 \text{ GeV}/c < P < 75 \text{ GeV}/c$ and a relative distance $R < 0.5$ cm at DCH1 plane, the one with lower quality is marked as “ghost”. If qualities are equal, the one with larger $|y_4^a - y_4^b|$ is defined as ghost (y_4^b and y_4^a are the y track positions at DCH4 before and after bending, respectively). Ghost tracks are artifacts of the track reconstruction; the track reconstruction routine efficiency is improved by allowing the space points in DCH4 to be along the front track extrapolation in the y coordinate (the magnetic field in this direction is negligible), but it might happen that fake tracks are reconstructed in the proximity of the real one. These tracks are close to the real one (less than 1 cm) and have worse track quality. The ghost track rejection is done to exclude these fake tracks generated by ambiguities coming from hits with similar y coordinate in the DCH4.

An event is accepted only if exactly one good track has been found, which is considered the candidate lepton track. Within the second step of the track selection the lepton candidate must satisfy the following additional requirements:

- Positive charge $q > 0$.
- Quality > 0.7 (quality = fraction of hits in each DCH view near in time to the average time of the total sample).
- Track time (given by CHOD): $118 \text{ ns} < t_{\text{trk}} < 158 \text{ ns}$ (Fig. 4.4, right plot).
- Track impact points in the geometrical acceptance of the detectors:
 - Track radius at the DCH 1,2,4 planes: $12 \text{ cm} < R_{\text{DCH}} < 115 \text{ cm}$.
 - Track radius at the CHOD plane: $14 \text{ cm} < R_{\text{CHOD}} < 115 \text{ cm}$.
 - Distance to nearest dead cell at LKr $d > 2 \text{ cm}$.
 - Minimum track radial distance from the beam pipe at the LKr calorimeter $R_{\text{LKr}} > 15 \text{ cm}$. Cells with read-out problems are also excluded from the LKr acceptance (Fig. 4.5).

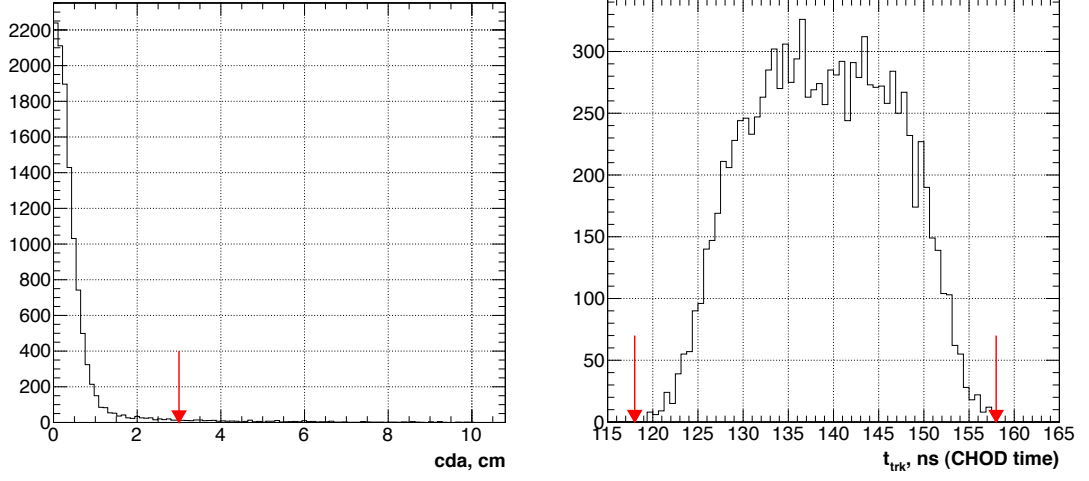


Figure 4.4: (Left) CDA distribution for data after the selection except the CDA cut. (Right) Track time distribution for data. The arrows represent the cuts applied.

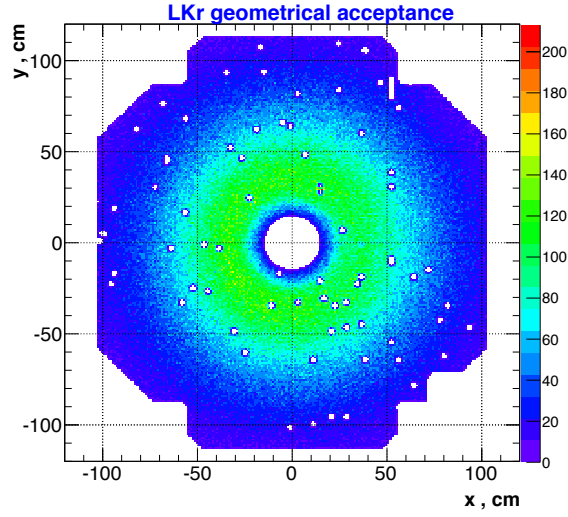


Figure 4.5: Track impact point on the LKr calorimeter. The dead cells are clearly visible.

- $-1600 \text{ cm} < Z_{\text{vtx}} < 9000 \text{ cm}$ (Fig. 4.6, left plot). The lower limit is chosen to reject events with a reconstructed longitudinal decay vertex too close to the final collimator plane $Z_{\text{coll}} \approx -1800 \text{ cm}$, taking into account resolution effects (the Z_{vtx} resolution is $\delta Z_{\text{vtx}} \approx \pm 2 \text{ m}$, Fig. 6.24): as shown in Fig. 4.3, the interaction of the beam with the material of the final collimator can produce charged particles not originated by kaon decays.
- $\text{CDA} < 3 \text{ cm}$ (Fig. 4.4, left plot)
- $10 \text{ GeV}/c < P_{\text{trk}} < 55 \text{ GeV}/c$ (Fig. 4.6, right plot). The lower limit on the track momentum, together with the lower cut on the photon energy, is chosen to ensure high efficiency of the $E_{\text{LKr}} > 10 \text{ GeV}$ trigger condition (Chapter 5) with more than 15 GeV energy deposit in the LKr. The upper value is chosen to minimize the $K^+ \rightarrow \pi^+ \pi^0$ ($K_{2\pi}$) background contamination (Fig. 4.7, right plot).
- The particle ID is based on the ratio E/p of track energy deposit in the calorimeter to its momentum measured by the spectrometer. The cluster associated to the track (associated cluster) is defined as the closest cluster to the track position at the LKr in a radius less than 1.5 cm, in time within 6 ns with the track (track time given by the CHOD) (Fig. 4.8) and with the pulse of the cluster cells correctly reconstructed. To identify the selected track as an electron a cut on E/p is made. For electrons the quantity E/p is expected to show a peak at $E/p = 1$ because all the energy has been fully released as electromagnetic shower. For charged pions, the E/p distribution is broader with respect to electrons and only a small fraction of events lies at $E/p > 0.9$ (mainly due to the “charge exchange” process $\pi^+ n \rightarrow \pi^0 p$ followed by $\pi^0 \rightarrow \gamma\gamma$). For muons that mostly interact via ionization and do not initiate a EM shower due their large mass ($m_\mu/m_e \approx 200$), the E/p distribution peaks at small values. Charged particles are identified as electrons if they satisfy $0.95 < E/p < 1.1$ (Fig. 4.9).

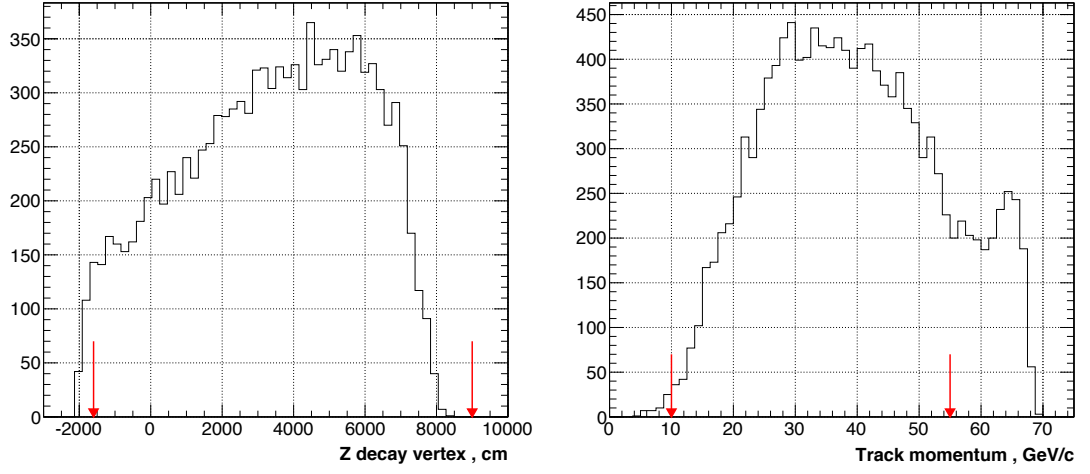


Figure 4.6: (Left) Z_{vtx} distribution for data after the selection, except the Z_{vtx} cut. (Right) Track momentum distribution in the laboratory frame for data after the selection except the momentum cut: the peak at 65 GeV/c is due to $K_{2\pi}$ background events (see Fig. 4.7). The arrows represent the cut applied.

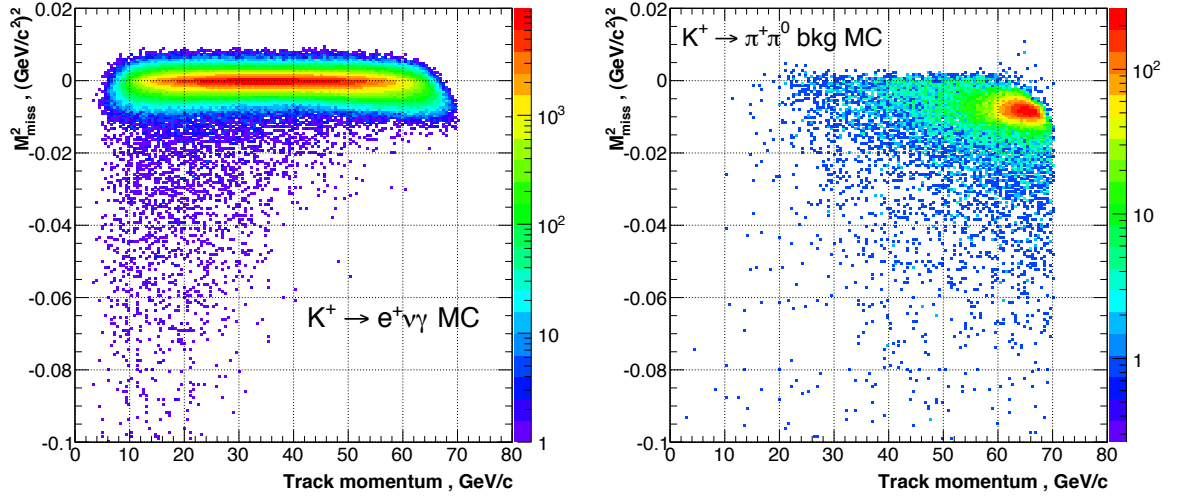


Figure 4.7: Distribution of the $K_{e2\gamma}(SD^+)$ missing mass squared (Section 4.2.5) versus track momentum for MC events. (Left) SD^+ events. (Right) $K_{2\pi}$ background events to SD^+ decay.

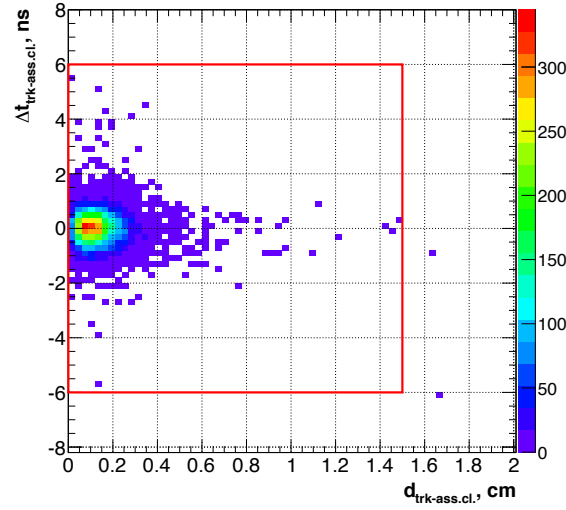


Figure 4.8: Distribution of the distance between the track position at LKr and its associate cluster ($d_{\text{trk-ass.cl.}}$) versus the difference between the track time (given by the CHOD) and its cluster time ($\Delta t_{\text{trk-ass.cl.}}$) for data events passing the SD^+ selection, except for the $d_{\text{trk-ass.cl.}}$ and $\Delta t_{\text{trk-ass.cl.}}$ cuts. The rectangle represents the cut applied in the analysis.

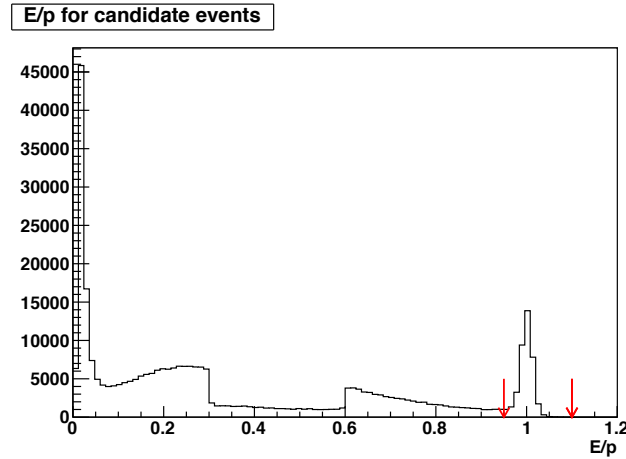


Figure 4.9: E/p distribution for data: the depletion of events in the region $0.3 < E/p < 0.6$ is due to a cut in the L3 trigger.

4.2.2 Photon selection

The cluster associated to the radiative photon from $K_{e2\gamma}$ (SD⁺) decay is selected requiring exactly one “good” cluster in the calorimeter. A good cluster is defined according to the following criteria:

- Not associated to the track (Section 4.2.1).
- Cluster energy $E > 5$ GeV. Together with the cut $P_{\text{trk}} > 10$ GeV/ c this condition ensures more than 15 GeV energy deposit in the LKr calorimeter and high efficiency of the $E_{\text{LKr}}(10 \text{ GeV})$ trigger condition (Chapter 5). The distribution of the energy of the cluster associated to the radiative photon is shown in Fig. 4.10.
- Inside the LKr acceptance (Fig. 4.5).
- In time within 6 ns with the track (track time given by the CHOD) (Fig. 4.11).
- Distance from the nearest dead cell greater than 2 cm.
- Cells of the cluster correctly reconstructed.
- Distance from any other cluster greater than 20 cm to minimize cluster overlapping and energy sharing effects. This cut rejects also the fraction of events ($\approx 0.7\%$) in which an additional photon is emitted by the positron via external Bremsstrahlung (EB) in the material crossed before the magnet of the spectrometer, and the photon associated to the $K_{e2\gamma}$ decay remains undetected because out of acceptance; in this case, the EB photon is selected as the candidate photon. The EB photon is emitted nearly collinear to the positron [10] so its cluster position is nearly the same of the undeflected track position on the LKr (Fig. 4.12).

The event is accepted only if exactly one good cluster has been found. The loss of signal acceptance due to this requirement is about 1.5% (SD⁺ MC events with extra good clusters are due to EB and photon conversion in the material before the spectrometer). Events with two good clusters are mainly due to $K^+ \rightarrow \pi^+ \pi^0$ and $K^+ \rightarrow \pi^0 e^+ \nu$ decays with both photons from π^0 decay detected in the calorimeter. The number of accidental events, in which an extra good cluster is in time with the candidate photon cluster (the two clusters not being associated to π^0 decays), is about 0.5% of the total number of SD⁺ candidates.

An additional check is done on extra clusters by vetoing events with additional clusters in time with the track ($|\Delta t_{\text{trk-cl}}| < 6$ ns) and with energy $E_{\text{veto}} > 2$ GeV. This requirement provides a photon veto to reduce the main backgrounds from $K^+ \rightarrow \pi^+ \pi^0$ and $K^+ \rightarrow$

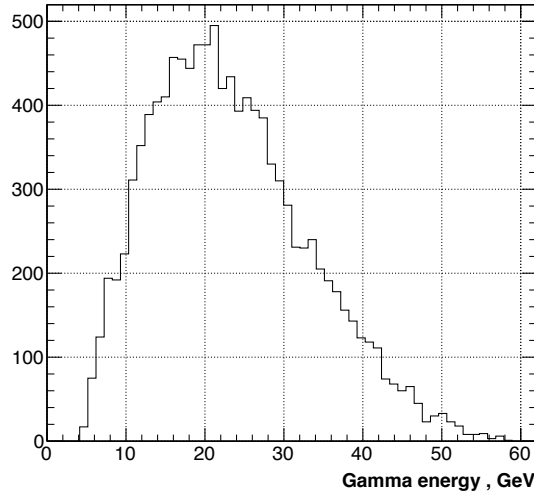


Figure 4.10: Photon energy distribution for data in the laboratory frame.

$\pi^0 e^+ \nu$ decays. The value of E_{veto} has been chosen to avoid effects due to the non-linear response of the LKr (Sections 2.3.3.3 and 4.2.4.1) maximizing the veto efficiency (*i.e.* for $E_{\text{veto}} = 3$ GeV the background increases by $\approx 7\%$ with respect to $E_{\text{veto}} = 2$ GeV).

The good cluster is assumed to be the photon candidate, on which further cuts are applied:

- If the selected photon loses energy by interacting with the material on the beam line (*e.g.* the DCH flanges) the corresponding reconstructed missing mass is shifted and contributes to the positive tail of the distribution. Such effect was studied with a control sample of K_{e3} decays. The data-MC comparison for the photon radius at the first DCH (Fig. 4.13) shows a discrepancy for photon radii < 12 cm, such distance being compatible with the actual size of the DCH flanges. To avoid the photon interactions with the DCHs flanges, a radial cut $R_{\text{DCH1}}(\gamma) > 12$ cm on the photon impact point at the DCH1 is imposed.

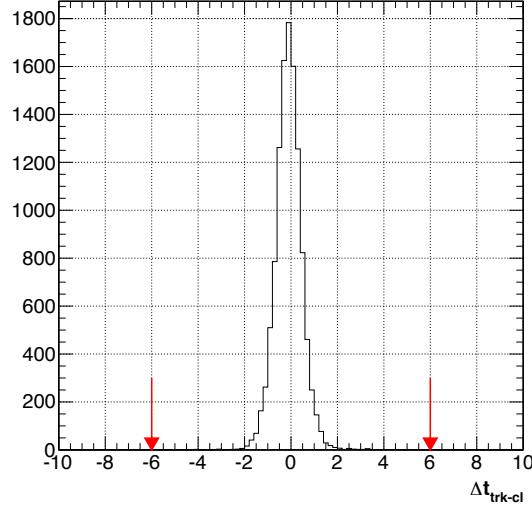


Figure 4.11: Track-candidate photon timing $\Delta t_{\text{trk-cl}}$ for data after the selection, except for the $\Delta t_{\text{trk-cl}}$ cut.

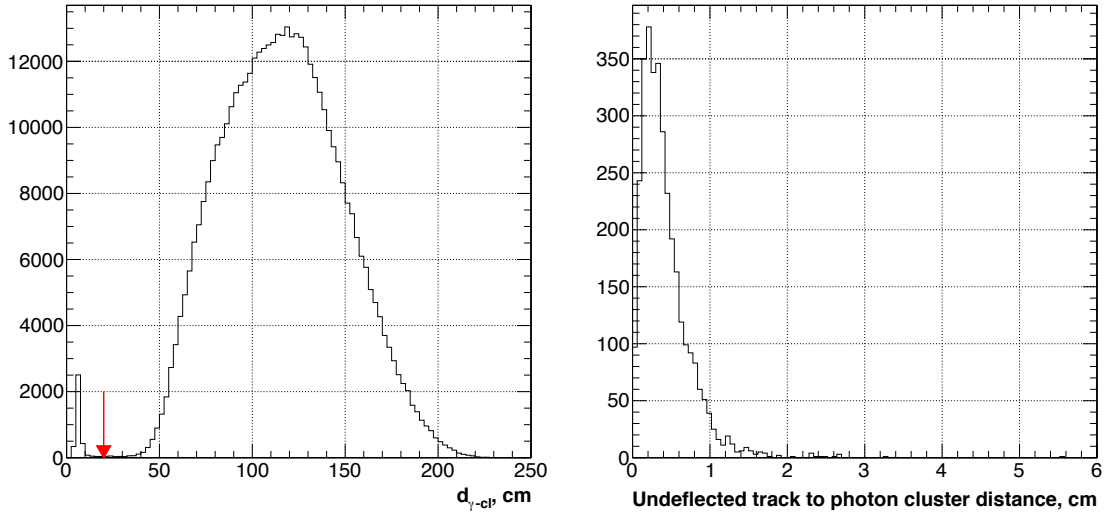


Figure 4.12: (Left) Distribution of the distance of the candidate photon cluster to any other cluster in the LKr calorimeter ($d_{\gamma\text{-cl}}$) for SD⁺ MC events. The peak below 10 cm is due to events in which the positron emits a photon via EB in the material crossed before the magnet of the spectrometer, and the photon from $K_{e2\gamma}$ decay remains undetected. (Right) Distribution of the distance between the undeflected track position at the LKr and the candidate photon cluster for $d_{\gamma\text{-cl}} < 10$ cm.

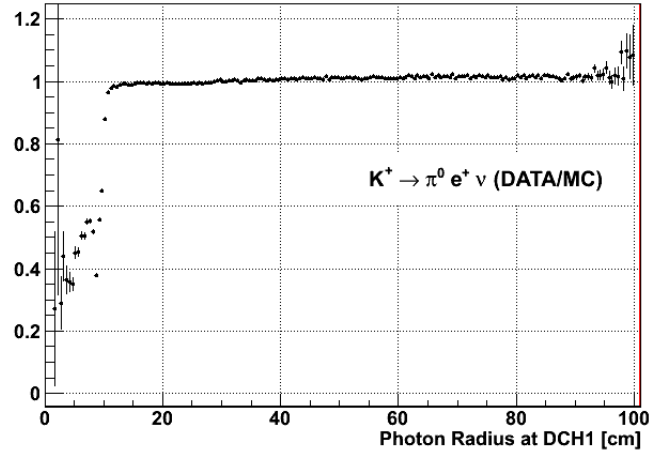


Figure 4.13: Photon radius at the DCH1: data-MC ratio for K_{e3} events.

4.2.3 Kaon momentum

In 2007 the beam line was optimized for charged kaons, which were delivered with a central momentum of 74 GeV/ c and a spread of 1.4 GeV/ c (rms). The kaon momentum vector $\mathbf{P}_K$ was not directly measured for each event but its average, measured with reconstructed $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ decays, has been used to compute the $K_{e2\gamma}(\text{SD}^+)$ kinematic variables (further details about the procedure can be found in [102, 103]). The collected data were divided into small time intervals (few hours each) with stable data taking conditions¹ and the distributions of all relevant quantities (central beam momentum P_K , transverse positions x_K , y_K at the $z=0$ cm plane and directions $(dx/dz)_K$, $(dy/dz)_K$) were fit in each time interval to extract the average reconstructed values, which were then written in a database. The same procedure has been applied to MC simulated events with one set of constants per run (all events are generated under same conditions within a given run). The average beam momentum, y_K position and $(dy/dz)_K$ slope varied slowly over time, in the ranges of about 0.1 GeV/ c , 1 mm and 10 μrad , respectively (Figs. 4.14-4.16), while the average x_K position and $(dx/dz)_K$ slope, correlated with the polarity of the TRIM3 dipole (Section 2.3.1), varied over time respectively from 0.2 cm to -0.8 cm and from -200 μrad to +230 μrad according to the TRIM3 current.

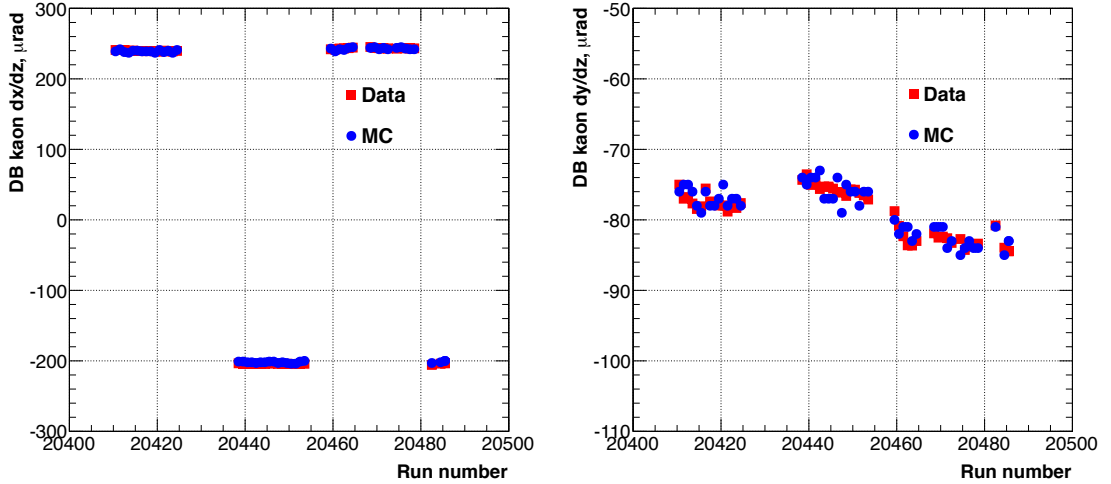


Figure 4.14: Average kaon trajectory slope $(dx/dz)_K$ (left) and $(dy/dz)_K$ (right) from database (P5) versus the run number for both data (red points) and MC (blue points). The TRIM3 dipole, placed before the final collimator, bends the beam particles along the x axis, so that $(dx/dz)_K$ varies following the inversion of the magnet polarity.

¹If the run conditions changed, the current sampling was ended and a new one started.

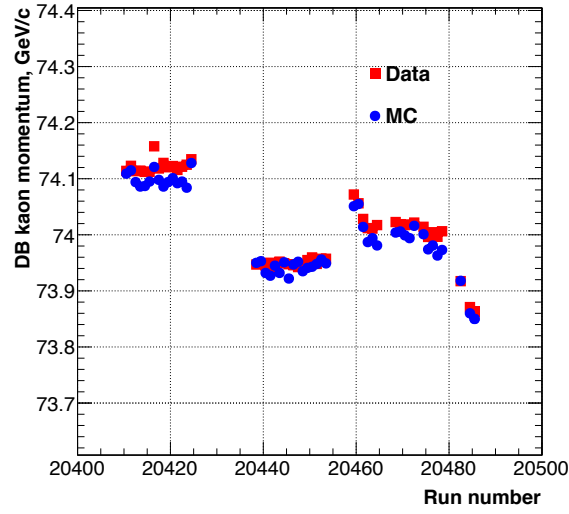


Figure 4.15: Average kaon momentum from database versus the run number (P5).

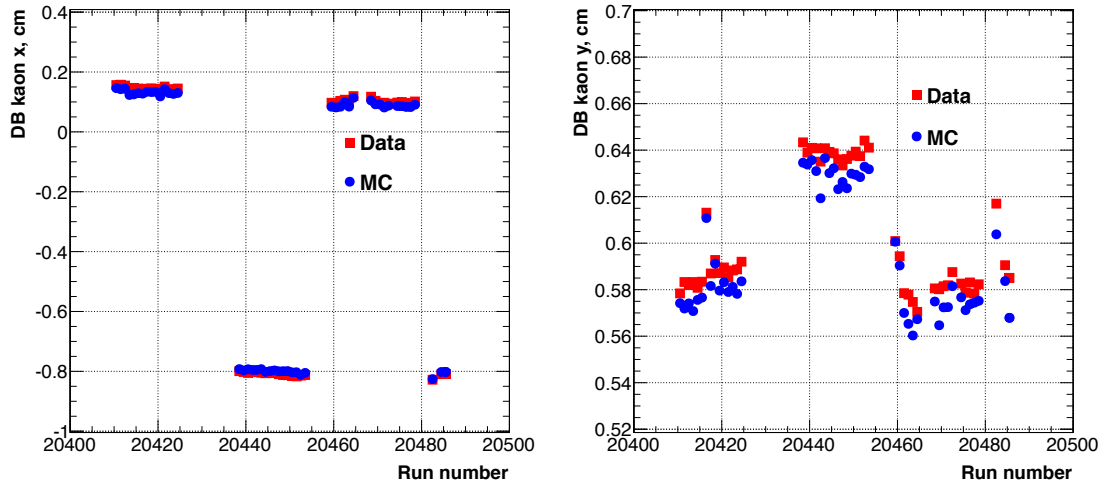


Figure 4.16: Average transverse kaon positions at $z = 0$ cm plane, x_K (left) and y_K (right) versus the number for data (P5).

4.2.4 Corrections applied to the events

Several kind of corrections are applied to the events in order to improve the resolution on the track momentum as well as on the energy and position of the LKr clusters. These corrections were evaluated for the R_K analysis [19, 96] and studied with dedicated runs and procedures. The main points are briefly summarized below.

4.2.4.1 LKr energy corrections

- A threshold applied to the calorimeter cells readout introduces a non-linear response between the deposited and measured energy which is not reproduced by the MC simulation. A correction is applied to the data to account for this non-linearity effect. Such correction is negligible for cluster energies above 10 GeV and it can be otherwise parameterized as a polynomial in the measured energy.
- In order to improve the resolution and uniformity of the LKr response and to decrease the electron ID inefficiency, the energy inter-calibration of the LKr cells was studied separately for each data taking period by using a sample of electrons from K_{e3} decays. A correction factor, dependent on the impact point of the track on the calorimeter, is applied to the energy of the clusters to equalize the LKr response and fix the E/p peak at 1 for electrons (Fig. 4.17).

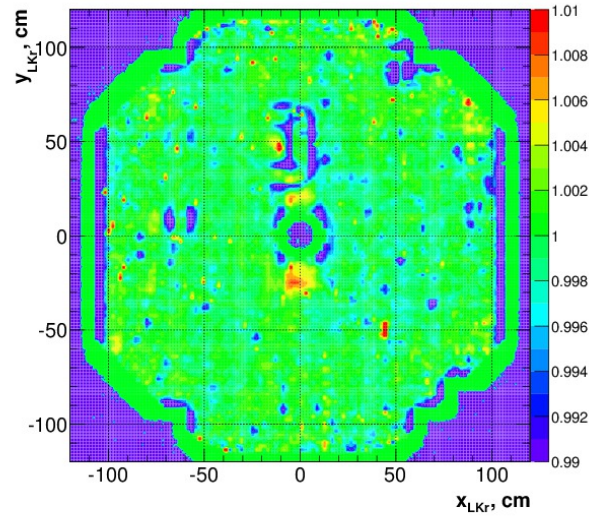


Figure 4.17: Energy recalibration factor for data as a function of the track impact points on the LKr front face.

4.2.4.2 Corrections to cluster position: projectivity and LKr to DCH alignment

The geometry of the LKr is such that the axis of the ionization cells points to a point P (projectivity point) at a distance $D_P = 10998$ cm in front of the LKr plane. A particle originating close to this point impinges on a cell parallel to the electrodes (Fig. 2.9) with the result that the reconstructed transverse coordinates of the cluster produced by the particle do not depend on the longitudinal position of the shower inside the calorimeter. However, if the decay vertex is far from the projectivity point, a small correction to the cluster position depending on the shower depth has to be applied (projectivity correction, Fig. 4.18). In addition, the cluster positions have to be corrected for a LKr to DCH residual misalignment. The complete correction to cluster positions, accounting for both projectivity and LKr to DCH misalignment, is applied to data and MC events and can be written as follows [104, 105]:

$$\begin{aligned} x_{\text{sh}} &= (x_0 + \delta x + \Theta \cdot y_0) \cdot \left(1 + \frac{d_{\text{sh}}}{D_P}\right) \\ y_{\text{sh}} &= (y_0 + \delta y - \Theta \cdot x_0) \cdot \left(1 + \frac{d_{\text{sh}}}{D_P}\right) \\ z_{\text{sh}} &= Z_{\text{LKr}} + d_{\text{sh}} \end{aligned}$$

where δx and δy are the LKr-DCH position shift values, Θ is the LKr-DCH rotation angle (small angle approximation is used), $d_{\text{sh}} = 16.5 + 4.3 \log(E(\text{GeV}))$ cm is the effective shower depth and Z_{LKr} is the longitudinal position of the LKr front plane; (x_0, y_0) are the un-corrected transverse cluster positions measured by the calorimeter at the Z_{LKr} plane, while $(x_{\text{sh}}, y_{\text{sh}})$ are the corrected cluster coordinates at the plane of the shower maximum $Z_{\text{LKr}} + d_{\text{sh}}$. For data, $\delta x = 0.136$ cm, $\delta y = 0.300$ cm and $\Theta = 0.87$ mrad, while for MC, $\delta x = -0.013$ cm, $\delta y = 0$ cm and $\Theta = 0$ mrad². The size of the correction to the transverse cluster positions is usually less than 0.5 cm, while the typical value of the shower maximum depth is about 30 cm.

4.2.4.3 Relative DCH alignment: α and β corrections

The relative misalignment of the drift chambers and the mis-calibration of the magnetic field in the spectrometer can induce a bias in the measurement of the track momentum. The technique used to study this effect is to reconstruct $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ decays [106]. Let us suppose that a small misalignment is present along the x direction in the DCH4. Once

²Studies on LKr to DCH misalignment have been done also for MC events, revealing a small shift in the x position that has been taken into account in the correction.

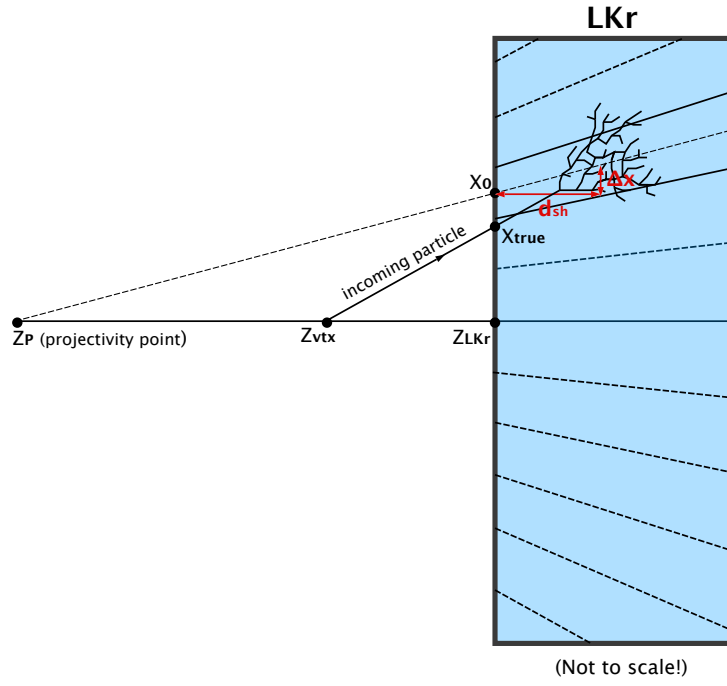


Figure 4.18: Projectivity correction to cluster position. The geometry of the LKr calorimeter is such that the cluster position measured by the LKr (indicated as X_0) of particles originating far from the projectivity point P differs from the true position X_{true} according to the shower depth. The cluster coordinates are then re-evaluated at the position of the shower depth d_{sh} : $X_{\text{sh}} = X_0 + \Delta X = X_0 + \frac{X_0}{D_P} \cdot d_{\text{sh}}$.

the direction of the spectrometer magnetic field and the direction of the DCH4 position shift are fixed, the measured angles of two “even charge pions” (the pions with positive charge) are, for instance, overestimated while the angle of the odd pion is underestimated. Due to this error, the measured momentum of the charged tracks is lower or higher (respectively) than the real one. As a consequence, we could observe that the average reconstructed invariant mass of the K^+ $\langle M(K^+) \rangle$ is below the nominal value $M(K)_{\text{PDG}}$:

$$\langle M(K^+) \rangle < M(K)_{\text{PDG}}$$

With the same argument, for K^- we would have:

$$\langle M(K^-) \rangle > M(K)_{\text{PDG}}$$

Moreover, the sign of the inequalities flips when the spectrometer magnetic field is inverted. This effect is observed on the data studying the reconstructed kaon mass as a function of the run number. An empirical correction is implemented to the measured pion momentum P_0 by requiring that the kaon mass is the same in K^+ and in K^- decay. The corrected track momentum P can be obtained using the formula [106, 107]:

$$P = P_0 \cdot (1 + q\alpha P_0)$$

where $q = \pm 1$ according to the pion charge and:

$$\alpha = -\text{sign}(B) \cdot \frac{\langle M(K^+) \rangle - \langle M(K^-) \rangle}{1.7476}$$

where B is the spectrometer magnetic field. The value in the denominator is an empirical number relating the positive and negative mass difference to the measured momentum difference [108]. During P5, only the positive charged kaon beam was delivered, so that for this period it was not possible to measure the α parameter by means of $K^\pm \rightarrow \pi^\pm \pi^+ \pi^-$ decays (however, α was measured for P1, P2, part of P3 and in the run 20486 between P5 and P6, where kaon beams of both signs were available). In order to correct for the misalignment also in the P5 data (the misalignment is not simulated so that $\alpha = 0$ for MC), the absolute value of the α parameter was assumed to be independent from the run number and chosen to improve the data to MC agreement of the dependence of the $K^+ \rightarrow \mu^+ \nu$ missing mass on the track momentum [109] (for data, the dependence is different for two magnet polarities, while for MC it is the same). The magnitude of the α correction for P5 was found to be consistent with the results obtained with P2 and P3

datasets, used as cross-check.

Another source of bias in the momentum measurement is due to the energy scale of the spectrometer and the fact that the magnitude of the magnetic field is not exactly equal for the two polarities $|B^+| \neq |B^-|$. This effect is the consequence of the relation between momentum and the bending field resulting in the relative error $\Delta P/P = \Delta B/B$. It can be demonstrated [106] that this uncertainty on the momentum translates into an uncertainty on the reconstructed kaon mass $\Delta M/M \approx 0.2\Delta B/B$ so that the average values of the distribution of the K^+ and K^- masses are not centered on the nominal kaon mass and they depend on the the sign of the magnetic field. The problem is fixed by defining the β parameter as:

$$\beta = -\frac{\langle M(K^\pm) \rangle - M(K)_{\text{PDG}}}{0.2 \cdot M(K)_{\text{PDG}}}$$

and rescaling the pion momentum by $(1 + \beta)$.

The track momentum P corrected for effects of misalignment and field mis-calibration can be obtained using the formula [106, 107]:

$$P = P_0 \cdot (1 + q \cdot \alpha \cdot P_0) \cdot (1 + \beta)$$

where q is the charge of the track. The absolute values of the α and β parameters are of the order of 10^{-5} and 10^{-3} respectively (Fig. 4.19). The typical value of the β -correction to the kaon momentum $\Delta P_K = P_K \cdot \beta \approx 75 \cdot 10^{-3} \text{ GeV}/c$ fully explains the $\approx 0.1 \text{ GeV}/c$ variation of the database average kaon momentum P_K^{DB} with respect to the magnet polarity (Fig. 4.20): in fact, kaon variables were measured and recorded in the database during the data taking concurrently with the α - β parameters, so that the β -correction to the average kaon momentum has been applied for data and MC only at the offline analysis level, according to the formula:

$$P_K^{\text{DB}} = P_K^{\text{DB}} \cdot (1 + \beta)$$

4.2.4.4 Blue field effect

A stray magnetic field is present in the decay region; it is called “blue” field after the colour of the steel tube in which the decay region is contained (Chapter 2). This field, whose main component is due to the Earth magnetic field, bends the trajectories of charged particles with respect to their initial direction. To correct for this effect, the charged tracks are traced backwards through the blue field to the decay vertex position and their

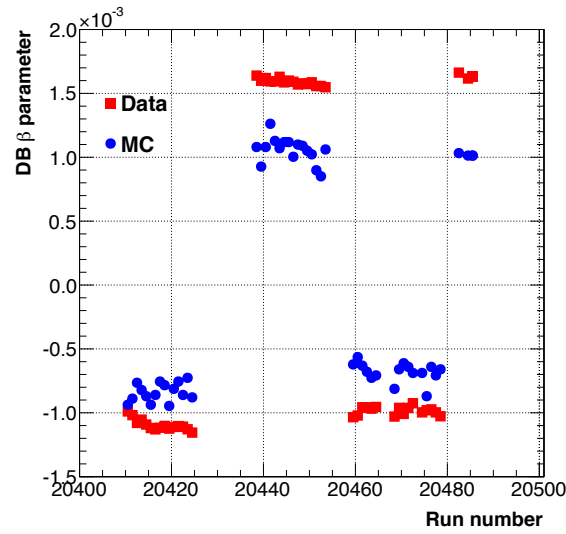


Figure 4.19: β -parameter for data as a function of the run number for P5. The α -parameter is not shown because for data α is assumed to be independent of the run number, while for MC is zero (see text).

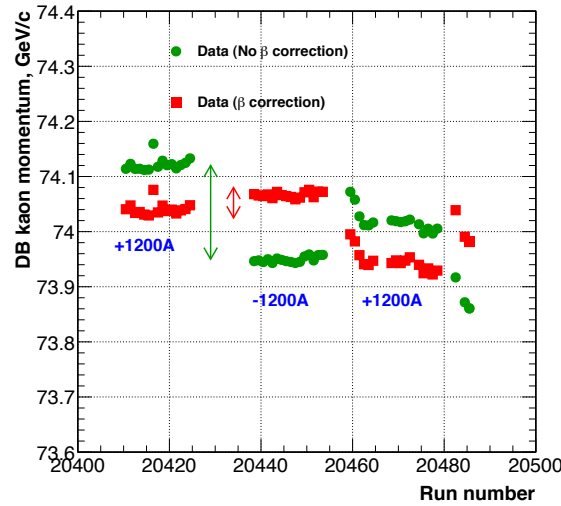


Figure 4.20: Kaon momentum from database for data with and without β correction: the variation of the mean momentum due to the inversion of the magnet current is reduced after applying the β correction.

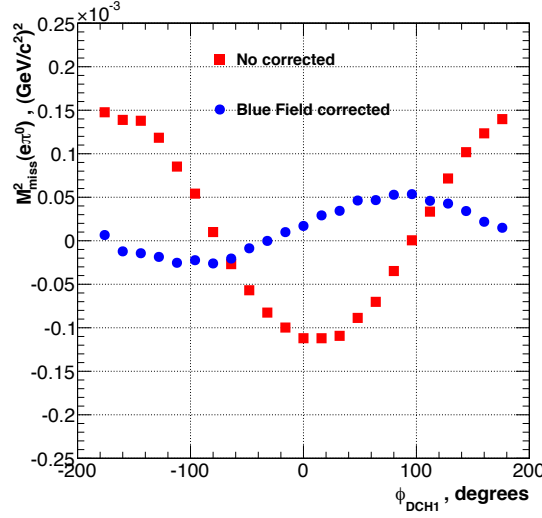


Figure 4.21: The missing mass squared $M_{\text{miss}}^2(e\pi^0)$ for K_{e3} MC events as a function of the angle ϕ_{DCH1} formed by the x axis of the DCH1 with the vector joining the impact point of the charged track on DCH1 and the centre of the detector.

directions are corrected according to the field map measured before the run (“blue field correction”). The effect of the residual magnetic field is shown in Fig. 4.21, where the missing mass squared for K_{e3} events $M_{\text{miss}}^2(e\pi^0)$ (Section 4.3) as a function of the angle ϕ_{DCH1} is plotted; ϕ_{DCH1} is defined as the angle formed by the x axis of the DCH1 with the vector joining the impact point of the charged track on DCH1 and the centre of the detector (by convention, $\phi_{DCH1} \in [-180^\circ, 180^\circ]$ with the zero at $y = 0$ and $x > 0$, while the anti-clockwise direction is taken as positive). In principle, $M_{\text{miss}}^2(e\pi^0)$ is independent of ϕ_{DCH1} but the residual magnetic field inside the blue tube induces a large modulation that biases the measurement of $M_{\text{miss}}^2(e\pi^0)$. This effect is strongly reduced after applying the blue field correction to data and MC events. Typical values of the correction for the slopes before the magnet are of the order of 10^{-2} mrad.

4.2.4.5 Correction for kaon momentum spectrum width

The kaon beam used in MC is simulated by using a package called TURTLE [110]. The relevant parameters of the beam line (*e.g.* the aperture of the slits of the defining collimators) have been tuned on a run-by-run basis in order to reproduce the momentum distribution of the beam. The MC reproduces with good accuracy the mean value of the data kaon momentum distribution as a function of the run number (Fig. 4.15) but it differs with respect to data in simulating the width of the P_K distribution. In particular,

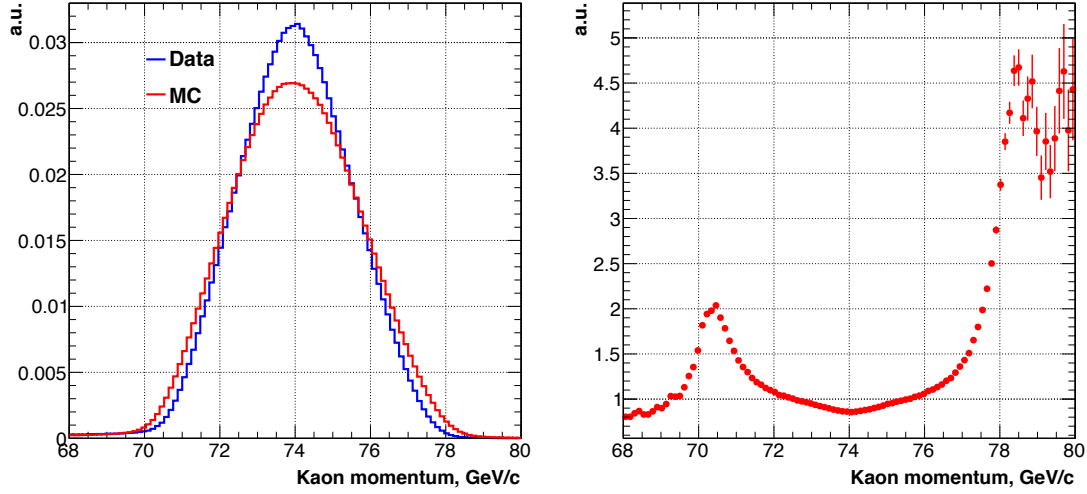


Figure 4.22: Kaon momentum distributions for data and MC (normalized to unit area) reconstructed using $K_{3\pi}$ events (left) and the corresponding data to MC ratio (right). The formula for the weight is chosen to flatten the parabolic-like shape visible in the data over MC distribution.

the kaon momentum width in the simulation is narrower than the one in data (Figs. 4.22-4.23). This discrepancy is due to imperfections in the simulation of the beam optics which cannot be easily fine-tuned by using TURTLE. To correct for this difference, the P_K spectra for data and MC, both reconstructed by using $K_{3\pi}$ events, have been compared to each other (Fig. 4.22) to evaluate a weight w which was applied event-by-event to the MC at the analysis level [102, 107, 109]:

$$w = 1 + \tilde{\alpha} \cdot (P_K^{\text{gen}} - 74 \text{ GeV}/c)^2$$

where P_K^{gen} is the generated kaon momentum and $\tilde{\alpha}$ is a time-dependent parameter in the range $-0.08 < \tilde{\alpha} < 0.08$ for P5. The correction strongly improves the data to MC agreement of the K_{e3} missing mass $M_{\text{miss}}^2(e\pi^0)$ resolution as a function of the track momentum (Fig. 4.24)

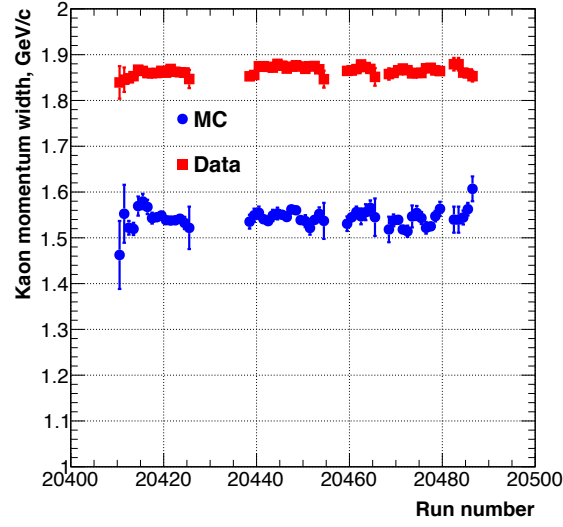


Figure 4.23: Width of the kaon momentum distribution for data and MC as a function of the run number.

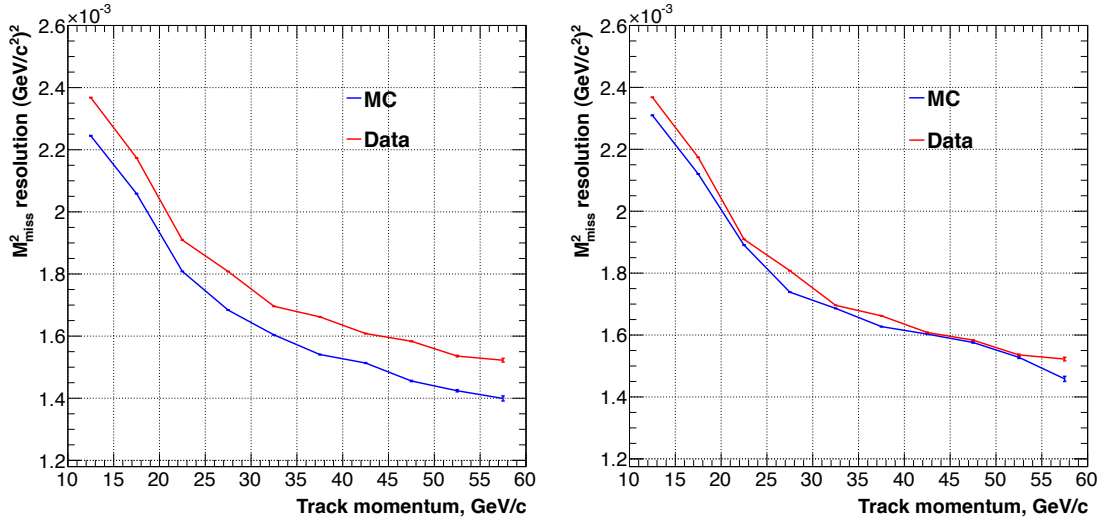


Figure 4.24: The resolution of the K_{e3} missing mass $M_{\text{miss}}^2(e\pi^0)$ as a function of the track momentum for data and MC, without (left) and with (right) the correction for kaon momentum spectrum width. The correction strongly improves the data to MC agreement.

4.2.5 $K_{e2\gamma}(\text{SD}^+)$ event reconstruction and definition of the signal region

The reconstruction of the $K_{e2\gamma}(\text{SD}^+)$ events is done using the information of the selected track and the cluster associated to the candidate photon. The positron four-momentum p_e is reconstructed by using the momentum and directions (blue field corrected) of the track measured by the spectrometer in the hypothesis of electron mass, while the photon four-momentum p_γ is computed using the energy and the position of the corresponding LKr cluster and the decay vertex position. The slopes of the photon trajectory $(dx/dz)_\gamma$ and $(dy/dz)_\gamma$ are reconstructed with the corrected position of the photon cluster $(x_{\text{cl}}, y_{\text{cl}}, z_{\text{cl}})$ and the position of the decay vertex $(x_{\text{vtx}}, y_{\text{vtx}}, z_{\text{vtx}})$:

$$(dx/dz)_\gamma = \frac{x_{\text{cl}} - x_{\text{vtx}}}{z_{\text{cl}} - z_{\text{vtx}}}, \quad (dy/dz)_\gamma = \frac{y_{\text{cl}} - y_{\text{vtx}}}{z_{\text{cl}} - z_{\text{vtx}}} \quad (4.1)$$

The average kaon momentum vector $\mathbf{P}_K$ is taken from the database. The $K_{e2\gamma}$ kinematic variables x and y are reconstructed using the formulae (Section 1.6):

$$x \left(= \frac{2E_\gamma^*}{M_K} \right) = \frac{2p_K \cdot p_\gamma}{M_K^2}, \quad y \left(= \frac{2E_e^*}{M_K} \right) = \frac{2p_K \cdot p_e}{M_K^2}$$

In the kaon rest frame the SD^+ events are mainly confined in a kinematic region of the Dalitz plot where the photon and the positron are emitted essentially anti-parallel (they have a relative angle $\theta_{e\gamma}^* > 120^\circ$ in more than 95% of the cases) with the positron energy spectrum peaking at $E^*(e) = \frac{M_K}{2} \cdot (1 + r_e)$ or $y = 1 + r_e$ ($r_e \equiv (\frac{m_e}{M_K})^2 = 1.07 \cdot 10^{-6}$). On the other hand, the IB and SD^- events are emitted preferentially at small angles $\theta_{e\gamma}^* \approx 0^\circ$ or equivalently, due to the relation $\sin^2 \left(\frac{\theta_{e\gamma}^*}{2} \right) = \frac{x + y - 1}{xy}$ (Eq. 1.39), are constrained in a kinematic region along the line $x + y = 1$: in the SD^- case, the left-handed photon is emitted with $\theta_{e\gamma}^* \approx 0^\circ$ while in the IB case, the photon has very low energy (the spectrum diverges at $E_\gamma^* = 0$) so that the positron and the neutrino are energetic ($x \approx 0, y, z \approx 1$) and $\theta_{e\gamma}^* \approx 0^\circ$.

The main backgrounds to $K_{e2\gamma}(\text{SD}^+)$ are due to the K_{e3} ($\text{BR}(K_{e3}) \approx 3000 \times \text{BR}(\text{SD}^+)$ [10]) and $K_{2\pi}$ ($\text{BR}(K_{2\pi}) \approx 14000 \times \text{BR}(\text{SD}^+)$ [10]) decays, when a photon from $\pi^0 \rightarrow \gamma\gamma$ remains undetected, and their Dalitz modes, in which the Dalitz decay $\pi^0 \rightarrow \gamma e^+ e^-$ (denoted π_D^0) occurs ($\text{BR}(\pi_D^0) = 1.17\%$ [10]). The kinematic endpoints of K_{e3} and $K_{2\pi}$ backgrounds in the y variable are (assuming the positron hypothesis for the track) $y^{\text{max}}(K_{2\pi}) \approx y^{\text{max}}(K_{e3}) = 0.925$. A cut $y > y^{\text{max}}(K_{e3})$ is necessary to select the SD^+ term and reject most of the background.

The following kinematic cuts define the SD^+ signal region:

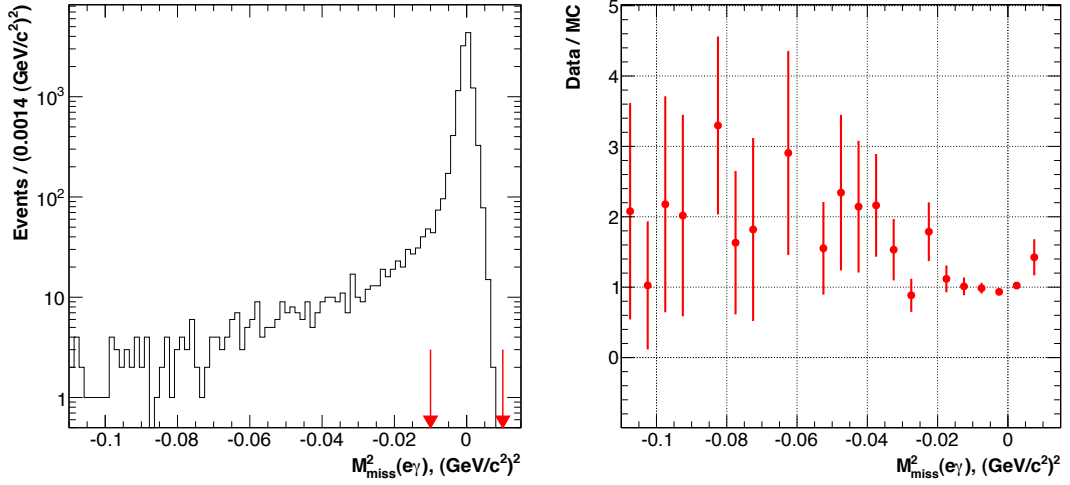


Figure 4.25: Squared missing mass distribution for data (logarithmic scale), with the complete event selection applied except for the $M_{\text{miss}}^2(e\gamma)$ cut (left), and the corresponding ratio between data and MC, inclusive of signal and background events (right). The plot is drawn using a larger binning with respect to the plot on the left.

- $|M_{\text{miss}}^2(e\gamma)| < 0.01 \text{ GeV}^2/c^4$, where $M_{\text{miss}}^2(e\gamma) = (p_K - p_e - p_\gamma)^2$ is the missing mass, which for the signal is equal to the neutrino mass $m_\nu \approx 0 \text{ GeV}/c^2$ (Fig. 4.25);
- $x > 0.2$ and $y > 0.95$. The lower limit in x is chosen to reduce the IB background that increases at low x , while the lower limit in y is chosen to minimize the main background component from K_{e3} (Fig. 4.27, left plot);

The number of $K_{e2\gamma}(\text{SD}^+)$ candidates at the end of the event selection is $N(\text{SD}^+) = 11170$. The study and the evaluation of the background will be described in Chapter 6.

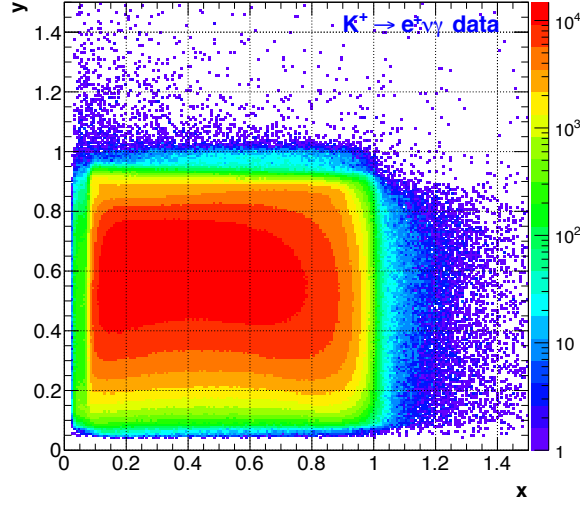


Figure 4.26: Distribution of the $K_{e2\gamma}$ sample events over the Dalitz plot without kinematic cuts (logarithmic colour scale). The selected events are mostly K_{e3} and $K_{2\pi}$ backgrounds to $K_{e2\gamma}$ (see Fig. 4.27). By loosening the cuts on x and y it's not possible to separate $\text{SD}^\pm$ and IB contributions. In this plot, in order to enlarge the accessible kinematic region up to the edges of the Dalitz plot, cuts on the track momentum and photon energy have been loosened to $3 \text{ GeV}/c < P_{\text{trk}} < 70 \text{ GeV}/c$ and $E_\gamma > 3 \text{ GeV}$, respectively.

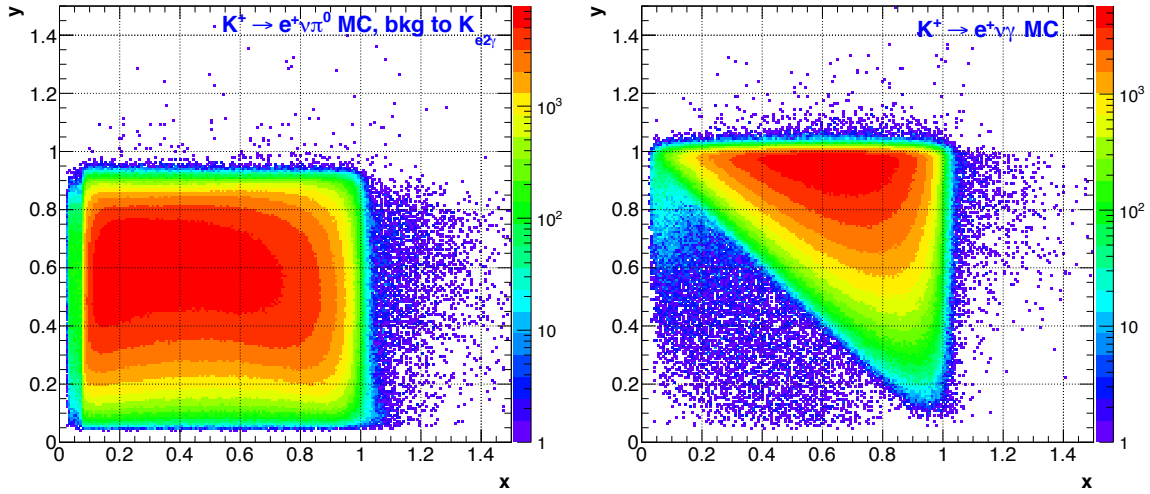


Figure 4.27: Distribution of x and y kinematic variables for K_{e3} (left, bkg) and $K_{e2\gamma}(\text{SD}^+)$ (right, signal) MC events, without kinematic cuts (logarithmic colour scale). The normalization is arbitrary.

4.3 Normalization channel

4.3.1 $K^+ \rightarrow \pi^0 e^+ \nu$ (K_{e3}) event selection

The kaon flux needed for the fit to the form factors and for the estimation of the background contamination to SD⁺ candidates (Sections 3.2 and 6.2) is evaluated by using the K_{e3} decay (followed by $\pi^0 \rightarrow \gamma\gamma$) as a normalization channel. This decay has a signature very similar to the signal (one charged track in the final state) and was acquired with the same trigger chain. It differs from the signal because of the presence of two photons from the $\pi^0 \rightarrow \gamma\gamma$ decay instead of a single one. Due to its signature and relatively high branching ratio ($\text{BR}(K_{e3}) = 5.07\%$, [10]), the K_{e3} decay is quite easy to select with an almost negligible background contamination. The K_{e3} events have been simulated according to the matrix element reported in [111]. The charged track selection is the same as for the signal event selection (Section 4.2.1), so that almost no systematic effects related to the difference between the signal and the normalization channel selections are introduced. The same photon selection reported in Section 4.2.2 is used to select two clusters in the LKr calorimeter which are generated by a $\pi^0 \rightarrow \gamma\gamma$ decay. A tight requirement on the time difference between the two photon clusters being within 3 ns ensures their time consistency (Fig 4.28, left plot). The π^0 is reconstructed using the cluster positions on the calorimeter and the distance D of the kaon decay vertex from the LKr front plane. The formula for the invariant mass of two photons is:

$$M_{\pi^0} = \sqrt{\left(\sum_{i=1}^2 E_i\right)^2 - \left(\sum_{i=1}^2 \mathbf{P}_i\right)^2} = \sqrt{2 E_1 E_2 (1 - \cos(\theta_{12}))}$$

where E_i , $\mathbf{P}_i$ are respectively the energies and the vector momenta of the photons and θ_{12} the angle between them. Because of typical values of θ_{12} are few mrad, the approximation $1 - \cos(\theta_{12}) \approx \theta_{12}^2/2 = (r_{12}/D)^2/2$, where r_{12} is the distance of the 2 clusters on the LKr surface, is valid to a good accuracy, and M_{π^0} can be rewritten as follows:

$$M_{\pi^0} = \frac{\sqrt{E_1 E_2 r_{12}^2}}{D} \quad (4.2)$$

The cut $125 \text{ MeV}/c^2 < M_{\pi^0} < 145 \text{ MeV}/c^2$ is then applied to the reconstructed π^0 mass (Fig 4.28, right plot).

The main contribution to the background is due to $K_{2\pi}$ decays, with the charged pion mis-identified as a positron. A kinematic separation between K_{e3} and $K_{2\pi}$ is achievable by constraining the missing transverse momentum $P_T = |\mathbf{P}_T|$ with respect to the kaon

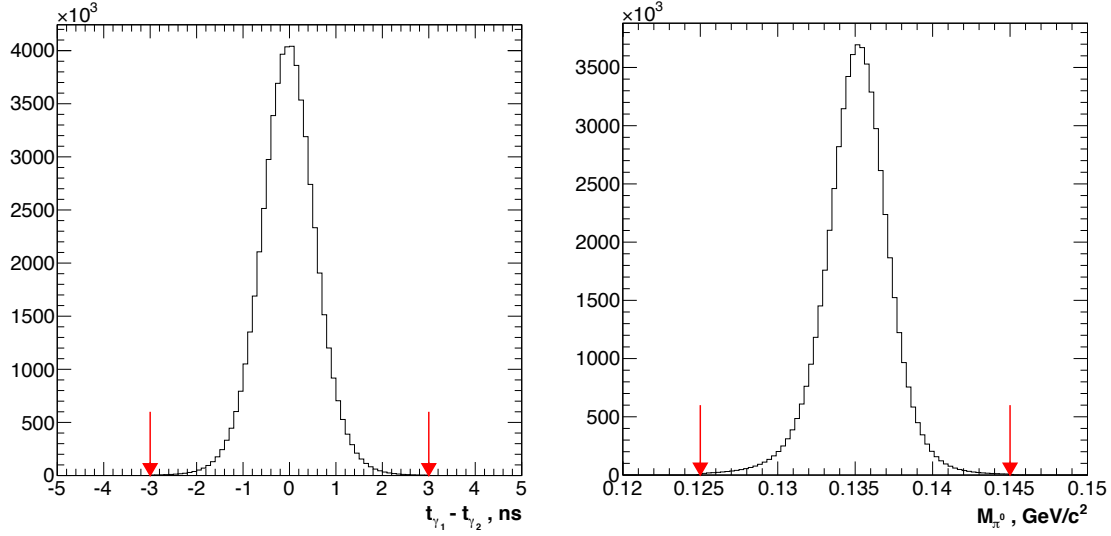


Figure 4.28: (Left) Distribution for data of the difference between the times at LKr of the two selected clusters associated to the $\pi^0 \rightarrow \gamma\gamma$ decay. (Right) Distribution of the reconstructed π^0 mass for data.

direction: in K_{e3} decays the neutrino is undetectable and produces a loss of transverse momentum, while in the $K_{2\pi}$ decay all the particles in the final state are detected. The kinematic separation of K_{e3} and $K_{2\pi}$ is illustrated in Fig. 4.29: the P_T spectrum for $K_{2\pi}$ events is mainly constrained at zero and is rejected by the cut $P_T > 0.02$ GeV/c. A further cut is applied to the reconstructed squared missing mass $|M_{\text{miss}}^2(e\pi^0)| = |(p_K - p_e - p_{\pi^0})^2| < 0.01$ GeV²/c⁴; the spectrum of this quantity is showed in Fig. 4.30.

A less significant source of background is due to K_{e3} decays followed by the $\pi^0 \rightarrow e^+e^-\gamma$ Dalitz decay: one of the two leptons from the Dalitz decay can be lost, the π^0 is reconstructed with the photon and the remaining lepton and the measured pion momentum is smaller than the true one. The contribution to the background coming from $K_{2\pi}$ decay followed by $\pi^+ \rightarrow e^+\nu$ and $K^+ \rightarrow \pi^+\pi^0\pi^0$ decay channel was checked to be negligible.

After the selection cuts applied, the K_{e3} is almost background free, with a background to signal ratio at the level of 10^{-4} . The number of K_{e3} candidates and the corresponding acceptance (Section 6.1.1 for the definition of acceptance) are reported in Tab. 4.1. Finally, the kaon flux, defined as the total number of K^+ decays in the fiducial volume $-2000 \text{ cm} < Z_{\text{vtx}} < 9000 \text{ cm}$, integrated over the runs of P5, can be measured by making

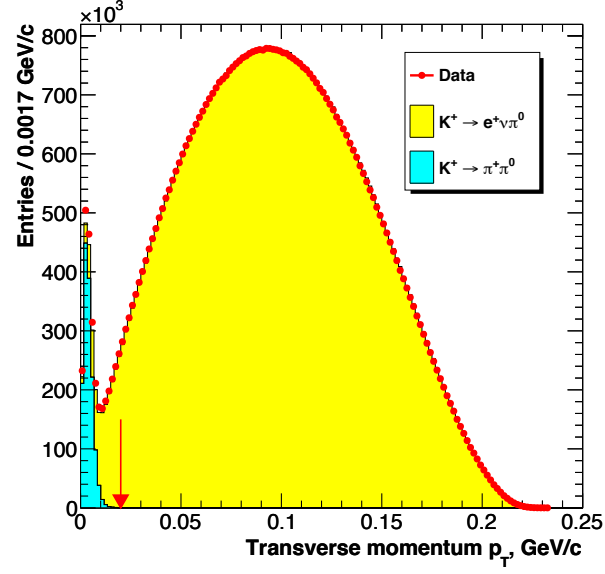


Figure 4.29: P_T distribution for K_{e3} candidates, stacked with the MC histograms of the expected signal and its main background $K_{2\pi}$.

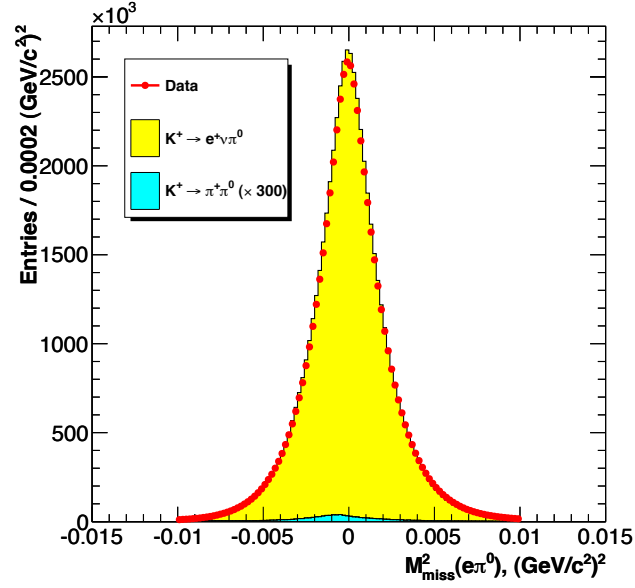


Figure 4.30: $M^2_{\text{miss}}(e\pi^0)$ distribution with full K_{e3} selection applied. After the P_T cut, the background due to $K_{2\pi}$ decays is strongly suppressed, with a background to signal ratio below the per mille level.

K_{e3} selection (normalization)	
Number of candidates, $N(K_{e3})$	$56.7 \cdot 10^6$
Acceptance, $\text{Acc}(K_{e3})$	$(11.264 \pm 0.002_{\text{stat}})\%$

Table 4.1: The number of K_{e3} candidates and the corresponding acceptance. The background to signal ratio is at the level of 10^{-4} .

use of the formula:

$$\Phi_K = \frac{N(K_{e3})}{\text{Acc}(K_{e3}) \cdot \varepsilon(K_{e3}) \cdot \text{BR}(K_{e3})} \quad (4.3)$$

where $\varepsilon(K_{e3})$ is the trigger efficiency for K_{e3} events (Chapter 5 and Eq. 5.3) and $\text{BR}(K_{e3})$ is the K_{e3} branching ratio taken from [10]. In Eq. 4.3, the background contamination has been neglected. The measured value of the flux for the P5 is then:

$$\Phi_K = (10.103 \pm 0.001_{\text{stat}} \pm 0.080_{\text{ext}}) \cdot 10^9 = (10.10 \pm 0.08) \cdot 10^9 \quad (4.4)$$

The external uncertainty is due to the error on the value of the K_{e3} branching ratio ($d\text{BR}(K_{e3})/\text{BR}(K_{e3}) \approx 1\%$ [10]).

The data-MC comparison and the identification and evaluation of the background to $K_{e2\gamma}(\text{SD}^+)$ will be described in detail in Chapter 6.

Chapter 5

Trigger efficiency and related issues

The trigger used for the acquisition of $K_{e2\gamma}$ events was originally implemented and optimized during the 2007 for K_{e2} events, the so called “main K_{e2} ” trigger. It consists of coincidences of signals in the two CHOD planes (Q1 signal), a loose multiplicity cut on the number of hits in the drift chambers (1TRK-LM signal) and an energy deposition in the LKr electromagnetic calorimeter of at least 10 GeV ($E_{\text{LKr}}(10 \text{ GeV})$ signal). A sketch of the trigger logic is shown in Fig. 5.1. A list of triggers available in P5, mainly used as control triggers to the K_{e2} and $K_{\mu2}$ signals, is shown in Tab. 5.1. The trigger efficiency was directly measured with a control data sample by means of Pattern Units (PU): digital registers which allows to retrieve¹ the trigger conditions satisfied during data acquisition every 25 ns. The bits corresponding to the relevant trigger sub-signals are:

- bit 1 in PU channel 6 for the $E_{\text{LKr}}(10 \text{ GeV})$ trigger;
- bit 9 in PU channel 4 for the 1-TRKLM trigger;
- bit 7 in PU channel 4 for the Q1 trigger.

The control data sample is selected with a control trigger signal provided by the neutral hodoscope (NHOD) which triggers on a shower in the LKr calorimeter. The NHOD trigger rate is higher compared with the triggers to be measured, and the control data sample has been downscaled by a factor of 150. The trigger efficiency for the condition i (*e.g.* Q1) is defined as:

$$\varepsilon_i = \frac{N_{\text{PU}(i)*\text{NHOD}}}{N_{\text{NHOD}}} \quad (5.1)$$

¹Each bit of the registered word corresponds to a trigger condition and is set to one when this is fulfilled.

where N_{NHOD} is the number of events in the control data sample and $N_{\text{PU}(i)*\text{NHOD}}$ is the number of such events in which the trigger condition i was found to be satisfied in the PU. The total trigger efficiency is measured by requiring the coincidence of the three conditions taken from the PU and the control trigger signal NHOD:

$$\varepsilon = \frac{N_{\text{PU}(Q1)*\text{PU}(1\text{TRK})*\text{PU}(\text{LKr})*\text{NHOD}}}{N_{\text{NHOD}}} \quad (5.2)$$

The measurement of the $K_{e2\gamma}$ trigger efficiency $\varepsilon(K_{e2\gamma})$ is limited by the statistically poor data sample of $K_{e2\gamma}$ candidates (about 11000 events) together with the large downscaling of the control trigger ($D=150$).

The strategy used to solve this issue and give an estimate of the $K_{e2\gamma}$ trigger efficiency is to measure the main K_{e2} efficiency for the normalization channel ($\varepsilon(K_{e3})$), which is similar to the signal, and to study each trigger component (Q1, $E_{\text{LKr}}(10 \text{ GeV})$ and 1TRK-LM) as a function of all the relevant variables of the analysis. The number of K_{e3} events triggered by NHOD is $\approx 3 \cdot 10^5$. The idea of the method is that if it can be demonstrated that the efficiency does not depend on any variable of the analysis, the assumption $\varepsilon(K_{e2\gamma}) = \varepsilon(K_{e3})$ can be legitimated. Whenever possible (see Section 5.3), the efficiency of each trigger component is measured using a control sample selected without making use of the information of the detectors involved in the corresponding trigger signal.

The measured efficiencies for the single trigger component and the main trigger are reported in the following sections.

Hex. code	Trigger name	Downscaling	Comments
0001	Q1	500	control trigger: 1TRK-LM efficiency
0004	NHOD	150	control trigger: 1TRK-LM, Q1, $E_{\text{LKr}}(10\text{GeV})$
0008	$Q1 * E_{\text{LKr}}(10\text{GeV})$	100	control trigger: 1TRK-LM
0200	$Q1 * 1\text{TRK-LM}$	150	main $K_{\mu 2}$
0400	$Q1 * 1\text{TRK-LM} * E_{\text{LKr}}(10\text{GeV})$	1	main K_{e2}

Table 5.1: A list of trigger signals available in P5. The triggers denoted as “main K_{e2} ” and “main $K_{\mu 2}$ ” are the triggers used for the R_K analysis implemented to collect K_{e2} and $K_{\mu 2}$ decays, respectively.

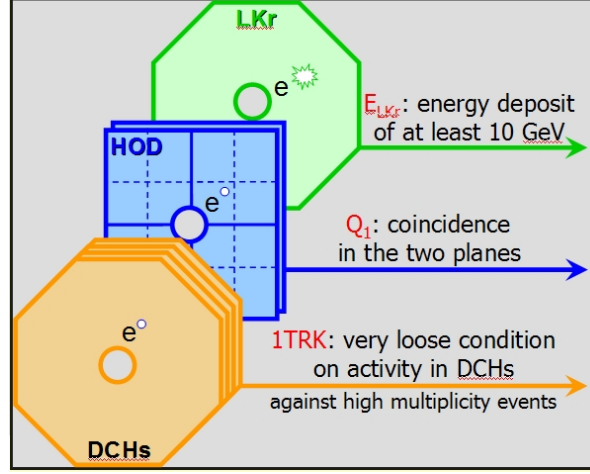


Figure 5.1: Sketch of the trigger logic of the main K_{e2} signal.

5.1 Q1 trigger efficiency

The Q1 signal is part of the main K_{e2} trigger, it requires at least one time coincidence between a signal from a vertical scintillator and a signal from a horizontal scintillator in the same hodoscope quadrant. The Q1 trigger efficiency is measured using the K_{e3} sample and the NHOD control trigger.

A source of Q1 inefficiency is geometrical: small “cracks” exist in the positions corresponding to the end of the horizontal and vertical scintillator bars. Fig. 5.2 shows the Q1 trigger inefficiency for K_{e3} events plotted as a function of the track impact point on the hodoscope $x - y$ front plane: the “cracks” generated by the gaps between the scintillators are visible. The Q1 trigger efficiency estimated by using K_{e3} events is:

$$\varepsilon_{Q1} = 0.99725 \pm 0.00009_{\text{stat}}$$

5.2 $E_{\text{LKr}}(10 \text{ GeV})$ trigger efficiency

The $E_{\text{LKr}}(10 \text{ GeV})$ trigger condition requires a minimum energy deposit in the LKr calorimeter. In order to study the corresponding efficiency, an independent control sample has been selected by requiring only a charged track inside the DCHs and LKr acceptances, with a momentum $3 \text{ GeV}/c < P_{\text{trk}} < 70 \text{ GeV}/c$ and with a cluster associated to it (defined as the cluster within 5 cm from the extrapolated track position at LKr and within 6 ns with respect to the track time), while no condition on the cluster multiplicity has been

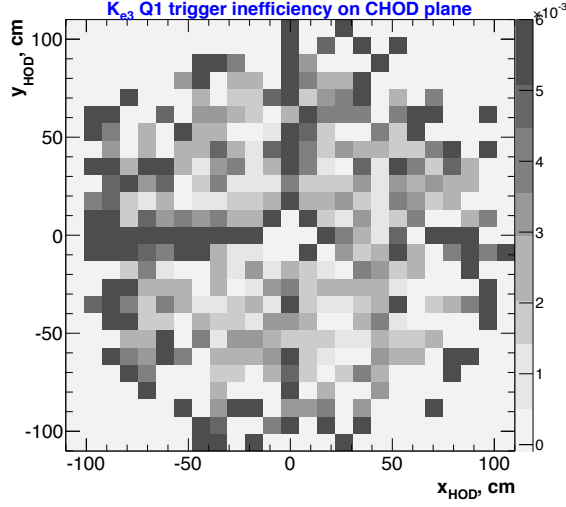


Figure 5.2: The Q1 inefficiency as a function of the track impact point on the CHOD, evaluated using K_{e3} events. The “cracks” generated by the gaps between the scintillators are visible.

applied; the control sample has been collected using the main $K_{\mu 2}$ trigger (Q1*1TRK-LM/150). The energy distribution of the associated cluster and the $E_{\text{LKr}}(10 \text{ GeV})$ trigger efficiency as a function of the total energy in the calorimeter are shown in Fig. 5.3: the efficiency becomes highly efficient and energy independent for energies greater than 15 GeV/c. The selection criteria for $K_{e2\gamma}$ and K_{e3} decays, described in Chapter 4, were chosen to guarantee an energy deposit in the LKr calorimeter above the 15 GeV energy threshold and avoid the inefficient region. The $E_{\text{LKr}}(10 \text{ GeV})$ trigger efficiency for K_{e3} events in bins of the sum of the track and the π^0 energy depositions at the LKr calorimeter is shown in Fig. 5.4²: the efficiency is very high and independent of the energy. The $E_{\text{LKr}}(10 \text{ GeV})$ trigger efficiency estimated by using K_{e3} events is:

$$\varepsilon_{E_{\text{LKr}}} = 0.99991 \pm 0.00002_{\text{stat}}$$

²The errors bars of the trigger efficiency plots are evaluated at the 68% of confidence level using the Binomial distribution and the Bayesian approach [100, 112].

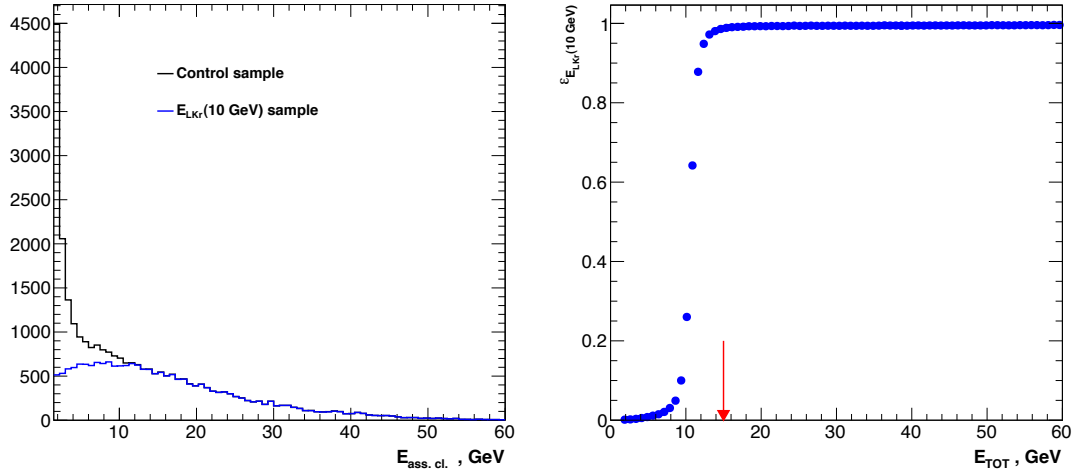


Figure 5.3: (Left) The distribution of the energy of the associated cluster for the control sample. (Right) The $E_{\text{LKr}}(10 \text{ GeV})$ trigger efficiency as a function of the total energy deposited in the calorimeter, measured with the control sample. The arrow indicates the cut used in the $K_{e2\gamma}$ selection.

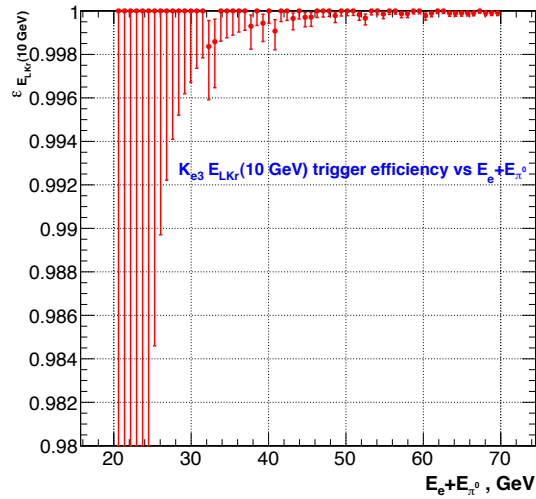


Figure 5.4: The $E_{\text{LKr}}(10 \text{ GeV})$ trigger efficiency, evaluated using K_{e3} events, as a function of the sum of the e^+ and π^0 energy deposits in the LKr calorimeter.

5.3 1TRK-LM trigger efficiency

The 1TRK-LM trigger condition requires at least one hit in more than one view and less than 15 hits in any view in DCH1 and DCH2 and DCH4, allowing the suppression of events with high activity on the drift chambers. The 1TRK-LM trigger is highly efficient for K_{e3} events: Fig. 5.5 shows that it is independent of the energy of one of the photons from $\pi^0 \rightarrow \gamma\gamma$ decay (left plot) and of the track impact point on DCH4 (right plot). As further checks, the 1TRK-LM trigger efficiency for K_{e3} events has been studied also as a function of other variables of the analysis and no dependences have been observed (Fig. 5.6). The 1TRK-LM trigger efficiency estimated by using K_{e3} events is:

$$\varepsilon_{1\text{TRK-LM}}(K_{e3}) = 0.99869 \pm 0.00006_{\text{stat}}$$

The main drawback of the measurement of the 1TRK-LM efficiency using K_{e3} decays is that the reconstruction of K_{e3} events is based on the DCH information so that, in principle, a measure of 1TRK-LM efficiency can be affected by a bias. On the contrary, the efficiencies of the Q1 and $E_{\text{LKr}}(10 \text{ GeV})$ triggers have been measured using control samples independent of the information of the charged hodoscope and LKr, respectively (Sections 5.1 and 5.2). To overcome this issue, $K_{2\pi}$ decays were reconstructed without using the DCH information, exploiting the fact that all particles produced in the decay are detected. The $K_{2\pi}$ events were acquired by the $\text{Q1} * E_{\text{LKr}}(10 \text{ GeV})$ trigger (Tab. 5.1) and selected by requiring exactly three energy deposits in the calorimeter with energies greater than 5 GeV, separated by at least 20 cm from any other cluster and within $\pm 6 \text{ ns}$ from each other (Section 4.2.2). For each pair of clusters, the decay vertex position ($x_{\text{vtx}}, y_{\text{vtx}}, z_{\text{vtx}}$) was computed by making use of Eq. 4.2 (constraining the π^0 mass to its nominal value [10]) and the extrapolation of the database kaon position from $z = 0 \text{ cm}$ to z_{vtx} . For a given pairing, the π^0 and π^+ four-momenta p_{π^0} and p_{π^+} were computed using respectively Eq. 4.1 and the kinematic relation $p_{\pi^+} = p_K - p_{\pi^0}$ (the kaon four momentum p_K are taken from the database, Section 4.2.3). The combination most compatible with the hypothesis of the $K_{2\pi}$ decay is the one which minimizes the distance between the un-paired cluster and the expected position of the π^+ on the LKr surface ($d_{\pi^+-\text{cl}}$), computed using the momentum $\mathbf{P}_{\pi^+}$ corrected for the kick of the spectrometer. To ensure the purity of the sample, cuts were applied on the reconstructed kaon mass M_K and the pion energies in the kaon rest frame $E_{\pi^0}^*, E_{\pi^+}^*$ (Fig. 5.7). In addition, cuts on z_{vtx} and the timing between photon clusters were also made (Chapter 4). Due to the constraint $p_{\pi^+} = p_K - p_{\pi^0}$, no elliptic cut on the plane ($E_{\pi^+}^*, E_{\pi^0}^*$) was applied (for $K_{2\pi}$ events, $E_{\pi^0}^* = E_{\pi^+}^* = M_K/2$). After applying the selection, the number of $K_{2\pi}$ events reconstructed without using DCHs

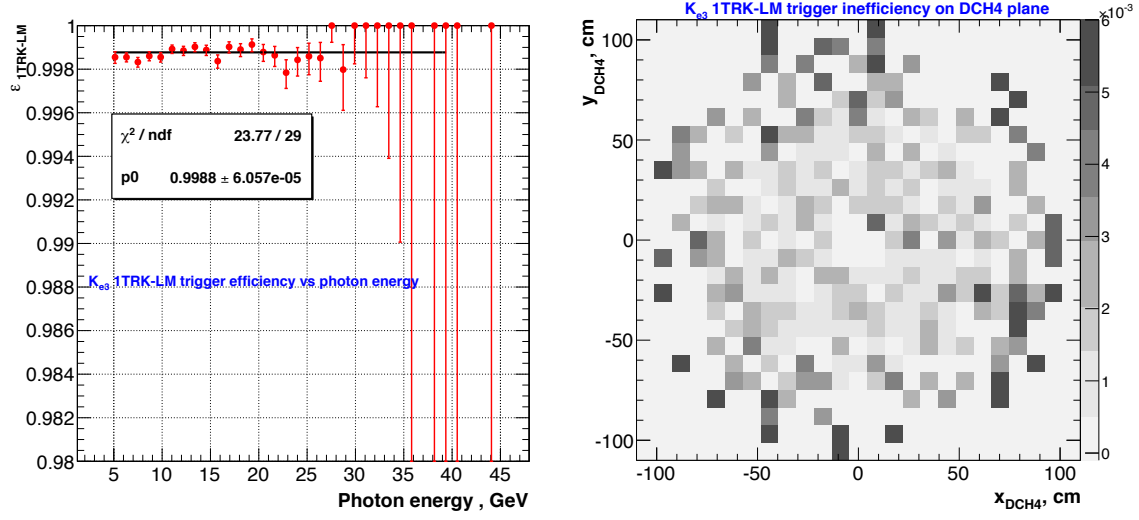


Figure 5.5: (Left) 1TRK-LM trigger efficiency $\varepsilon_{1\text{TRK-LM}}(K_{e3})$, evaluated using K_{e3} events, as a function of the energy of a photon from π^0 decay. (Right) 1TRK-LM inefficiency $1 - \varepsilon_{1\text{TRK-LM}}(K_{e3})$ as a function of the track impact point on DCH4.

information is about 10^6 . The 1TRK-LM trigger efficiency of the $K_{2\pi}$ control sample is then computed according to Eq. 5.1 and its value is found to be in good agreement with the one obtained with K_{e3} (Fig. 5.8 and Fig. 5.5).

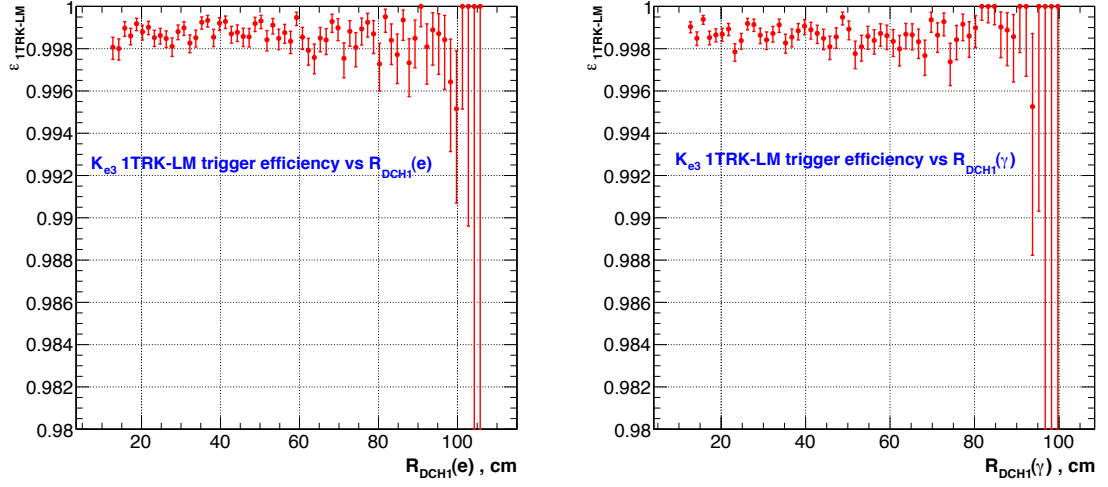


Figure 5.6: 1TRK-LM trigger efficiency evaluated using K_{e3} events as a function of the electron (left) and photon (right) radii at DCH1 plane (the radius of a particle at a given detector is defined as the distance of the impact point of the particle to the centre of the detector).

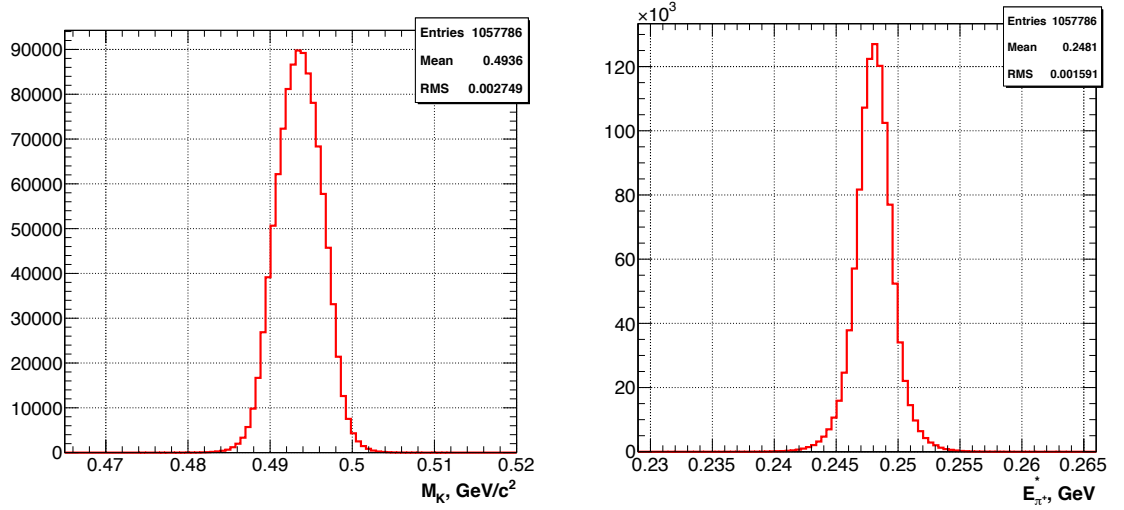


Figure 5.7: Distribution of the kaon mass (left) and the π^+ energy in kaon rest frame (right) for data events reconstructed without using DCHs information.

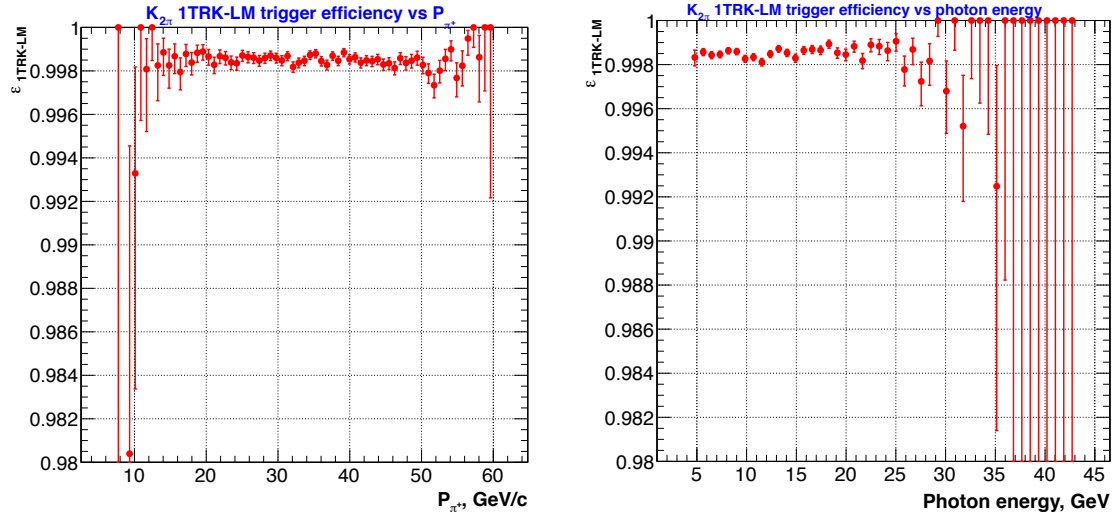


Figure 5.8: 1TRK-LM trigger efficiency evaluated using $K_{2\pi}$ data events, reconstructed without using DCHs information, as a function of the charged pion momentum (see text) (left) and the energy of a photon from π^0 decay (right). The measurement of the 1TRK-LM trigger efficiency obtained with the $K_{2\pi}$ sample is in agreement with the one obtained with the K_{e3} events.

5.4 Total trigger efficiency for K_{e3}

The total trigger efficiency for the K_{e3} selection $\varepsilon(K_{e3})$ is computed using Eq. 5.2. The total efficiency does not depend on the relevant variables of the analysis : Fig. 5.9 shows that $\varepsilon(K_{e3})$ does not exhibit any dependence on the reconstructed track momentum and the longitudinal decay vertex: this is confirmed by the constant fits applied in the plots. The trigger efficiency was also studied as a function of relevant variables in common with the $K_{e2\gamma}$ analysis, such as the energy of a photon from π^0 decay and the variables $x = 2E_{\pi^0}^*/M_K$ and $y = 2E_e^*/M_K$ that, in analogy with $K_{e2\gamma}$ decay, define the Dalitz plot of the K_{e3} decay: no dependence is observed (Fig. 5.10). A check on the stability of the trigger efficiency during the data taking of P5 was performed and showed no further dependence on the run number (Fig. 5.11). The total trigger efficiency evaluated using the K_{e3} selection is:

$$\varepsilon(K_{e3}) = 0.99473 \pm 0.00013_{\text{stat}} \quad (5.3)$$

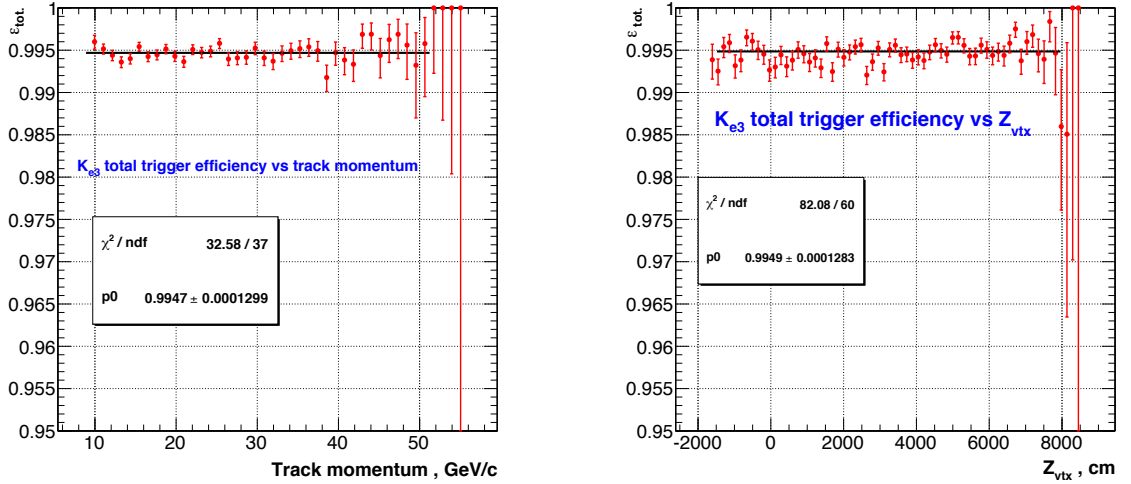


Figure 5.9: K_{e3} trigger efficiency plotted as a function of the reconstructed track momentum (left) and the reconstructed longitudinal decay vertex (right): no dependence is observed.

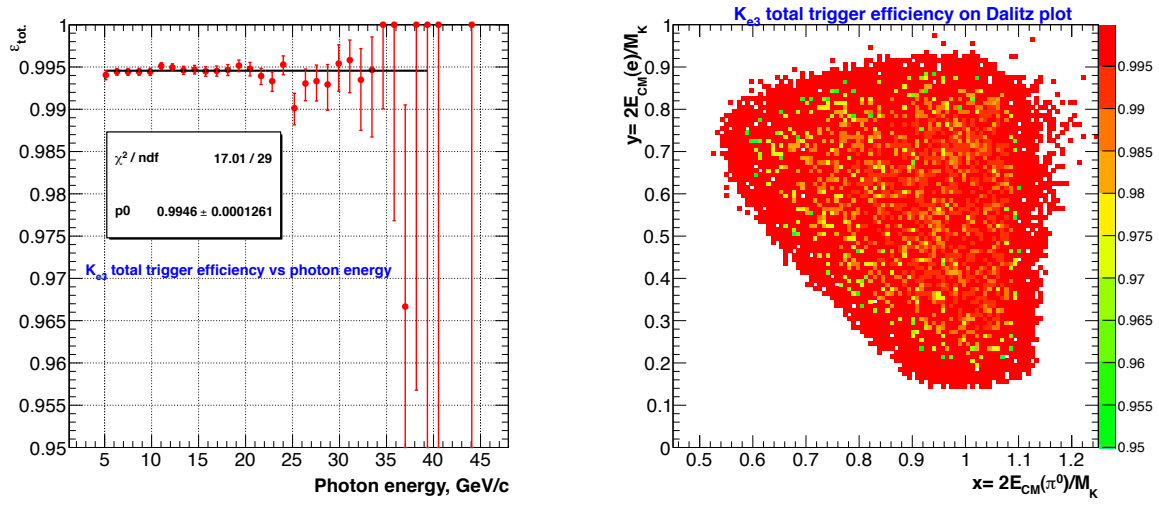


Figure 5.10: (Left) K_{e3} total trigger efficiency as a function of the energy of a photon from π^0 decay. (Right) The K_{e3} efficiency over the Dalitz plot, defined by the variables $x = 2E_{\pi^0}^*/M_K$ and $y = 2E_e^*/M_K$. The efficiency is independent of the photon energy and is flat over the whole K_{e3} Dalitz plot.

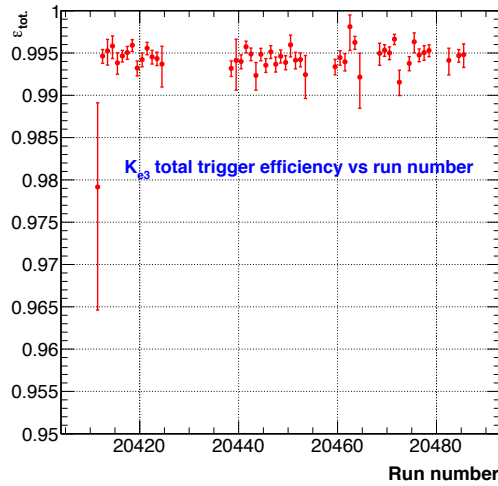


Figure 5.11: K_{e3} trigger efficiency as a function of the run number for the P5 sample.

5.5 Trigger efficiency for background to $K_{e2\gamma}$

The major limitation for a direct measurement of the $K_{e2\gamma}$ trigger efficiency is the low statistics in the $K_{e2\gamma}$ signal region. Nevertheless, the trigger efficiency for background events can be studied over a large range of the Dalitz plot using the $K_{e2\gamma}$ selection without kinematic cuts on the missing mass and x, y variables (“loose selection”); the resulting data sample is dominated by background events, mainly due to K_{e3} and $K_{2\pi}$ decays (see Chapter 4). Using this loose data sample, a source of inefficiency affecting the 1TRK-LM trigger component was found. In fact, the measurement of the 1TRK-LM trigger efficiency for background shows that the 1TRK-LM component is highly inefficient for events with photons out of the geometrical acceptance of the detector, as in the case of K_{e3} and $K_{2\pi}$ decays. These lost photons interact with the passive material of the detector (*e.g.* the beam pipe), showering and thus producing high multiplicity in the drift chambers; such events are most likely rejected by the 1TRK-LM trigger component. Moreover, due to the correlation between the energy and the direction of particles, a dependence of this inefficiency on the energy of the detected photon is observed; namely the inefficiency increases when the detected photon has low energy (the lost photon has most of the π^0 energy and is nearly parallel to the beam pipe). On the contrary, for the K_{e3} , all photons in the final state are detected and reconstructed and the 1TRK-LM trigger is indeed highly efficient (Fig. 5.5). Fig. 5.12 shows the 1TRK-LM trigger efficiency for background events selected with the loose selection, as a function of the photon energy (left) and on the Dalitz plot (right). The dependence of the inefficiency on the photon energy is clearly visible. The inefficiency increases up to 10% at low energies for the detected photon and is independent of the y variable. In order to evaluate the residual background to $K_{e2\gamma}$ over the whole Dalitz plot, the dependence of the trigger efficiency on the photon energy, not simulated by MC, must be taken into account by weighting the MC spectra for the 1TRK-LM trigger efficiency on a bin-by-bin basis; in principle, the 1TRK-LM inefficiency could be present also inside the signal region (in which the limited statistics make a direct estimate of the efficiency difficult) inducing a systematic effect on the measurement of the $K_{e2\gamma}$ form factors for which the shape of the distribution is important.

The trigger efficiency for K_{e3} and $K_{2\pi}$ backgrounds is measured directly from data: a pure sample of K_{e3} background events can be selected by looking at the events inside the control region $0.4 < y < 0.6$ below the kinematic endpoint of the $K_{2\pi}$ decay $y^{\max}(K_{2\pi}) = 0.925$ (Fig. 5.13, left), while a sample of $K_{2\pi}$ backgrounds events can be isolated by moving the E/p cut from $0.95 < E/p < 1.1$ (used to tag electrons) to $0.6 < E/p < 0.8$

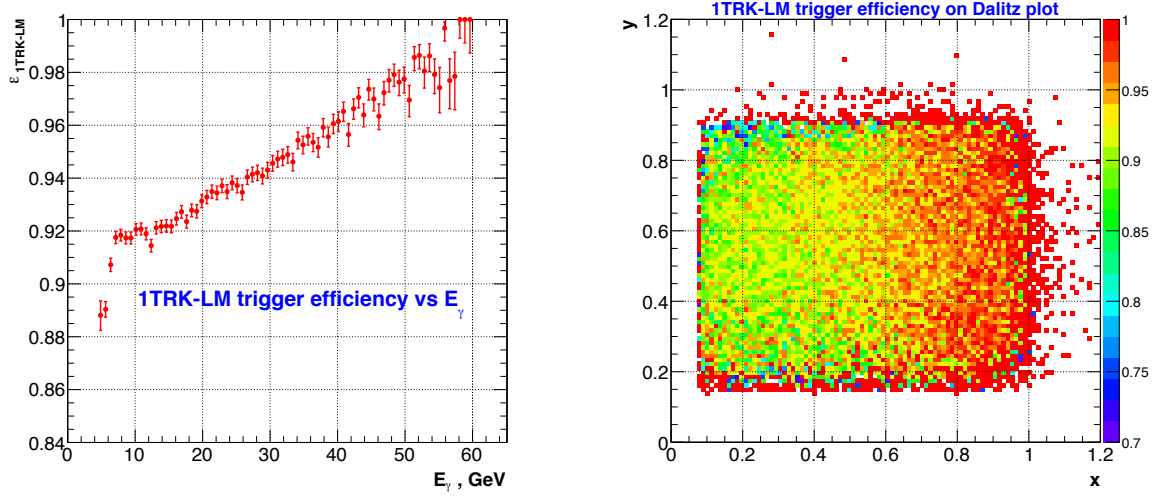


Figure 5.12: The 1TRK-LM trigger efficiency as a function of the photon energy in the laboratory frame (left) and on the Dalitz plot (right) computed using the $K_{e2\gamma}$ event selection with no kinematic cuts (missing mass, x and y). The sample is dominated by background events with lost photons (mainly K_{e3} and $K_{2\pi}$ decays) so that the efficiency showed in these plots is essentially the trigger efficiency of the background. The interactions of the lost photons with the material of the detector causes a trigger inefficiency up to 10%.

(see also Fig. 4.9) and selecting the events inside the control region $0.8 < y < 1$. (Fig. 5.13, right); the 1TRK-LM trigger efficiency is found to be independent of the track energy released in the LKr calorimeter as expected, so that a measurement of the efficiency in the range $0.6 < E/p < 0.8$ yields the same result as in the signal interval $0.95 < E/p < 1.1$. Figs. 5.13 show that at low energies ($E_\gamma < 20$ GeV) the 1TRK-LM trigger is more inefficient for the $K_{2\pi}$ background than for the K_{e3} background, while there is not appreciable difference at higher energies.

The trigger efficiencies for K_{e3} and $K_{2\pi}$ backgrounds will be used to correct the MC distributions and improve the data to MC agreement over the whole Dalitz plot (Chapter 6).

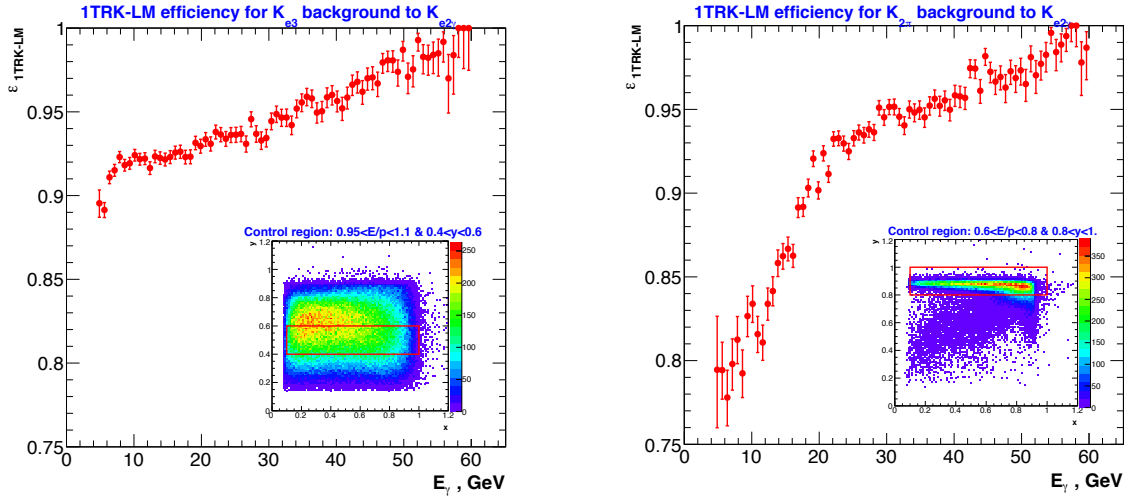


Figure 5.13: The 1TRK-LM trigger efficiency as a function of the photon energy in the laboratory frame. (Left) The 1TRK-LM efficiency for K_{e3} background events. The control region (indicated in the inset by a red rectangle) is defined by the cuts $0.95 < E/p < 1.1$ and $0.4 < y < 0.6$ and is used to compute the efficiency. (Right) The 1TRK-LM efficiency for $K_{2\pi}$ background events. In order to tag pions instead of electrons, the cut $0.6 < E/p < 0.8$ is applied.

5.6 Conclusions

The method for the measurement of the main K_{e2} trigger efficiency for $K_{e2\gamma}$ events is affected by the large downscaling ($D=150$) of the NHOD control trigger which limits the statistics available for trigger studies (about 100 $K_{e2\gamma}$ events). To overcome this issue, the trigger efficiency has been evaluated for the K_{e3} data sample, which is similar to the $K_{e2\gamma}$ signal. The efficiencies of the main K_{e2} trigger and its components have been studied as a function of several relevant variables of the analysis using independent control samples and no dependence with respect to any of these was observed. It was found that the main K_{e2} trigger inefficiency is dominated by the Q1 trigger component (coincidences of signals in the two planes of the charged hodoscope). In conclusion, it is reasonable to assume the trigger efficiency for the $K_{e2\gamma}$ signal events at the same level of the one measured for the normalization channel and make the assumption $\varepsilon_{\text{tot}}(K_{e3}) = \varepsilon_{\text{tot}}(K_{e2\gamma})$.

The trigger efficiency was studied also for background events using a loose $K_{e2\gamma}$ selection without kinematic cuts on the missing mass and x, y variables so that the data sample is dominated by background. The 1TRK-LM trigger efficiency for these events is affected by a severe inefficiency, not present in the K_{e3} sample, due to the interaction of lost photons with the passive material of the detector. The efficiency for background events, measured separately for K_{e3} and $K_{2\pi}$ backgrounds (inclusive of their Dalitz decay modes) using the E/p cut to tag pions and electrons, will be taken into account in the fitting procedure of the form factors to correct the MC spectra.

Chapter 6

Background evaluation and data-MC comparison

6.1 Monte Carlo simulations

In order to compare any theoretical prediction with the data, it is necessary to perform a Monte Carlo (MC) simulation of all the physical processes from the beam production to the signal read-out in the detector. In the simulation, single events are generated randomly, according to weights given by the expected probability distributions, such as the differential decay rates or the efficiencies and resolutions of the sub-detectors. The MC events have the same format as the data, but the “true values”, *i.e.* the four-vectors of the generated particles, their type and their production and decay positions, are known as well. This allows to apply the same event selection to data and MC, to measure the detector acceptance and resolution for any variable and to predict the distributions of signal and background events after the selection, to be compared with the data. The simulation program available for the NA48/2 experiment is called Charged Kaons Monte Carlo (CMC) and is based on GEANT3 [113]. CMC includes a detailed simulation of the apparatus, from the kaon generation on the beryllium target up to the output signals from detectors. In CMC, the four-momenta of the decay products are first generated in the kaon rest frame according to the phase space distribution then weighted with the proper matrix element and finally boosted to the laboratory frame. CMC can generate only one decay channel at a time, *e.g.* one kaon decay mode and, in case a π^0 is produced, one specific decay mode for it. The CMC simulation includes:

- a detailed description of detector geometry, including the misalignment of DCHs;

- interaction of radiation with matter (pair production and Bremsstrahlung);
- the implementation of radiative corrections using the PHOTOS package [114];
- multiple scattering in the helium between the drift chambers;
- possibility to generate a decay according to a user-defined matrix element;
- secondary production of particles (*e.g.* $\pi^\pm \rightarrow \mu^\pm \nu$ decay)
- effects of LKr dead cells and of DCH wire inefficiencies;
- simulation of some known detector effects, like the residual magnetic field in the decay volume;

The kaon beam is simulated using a package called TURTLE [110]. The relevant parameters of the beam line (*e.g.* the aperture of the slits of defining collimators) have been tuned in order to reproduce the momentum distribution of the beam. CMC was tuned on a run-by-run basis in order to simulate as much as possible the data taking conditions and reproduce time-dependent effects; for this reason, the generation of MC events was performed proportionally to the number of selected events in each run.

The complete simulation of an electromagnetic shower in the calorimeter with GEANT is very CPU-time consuming. In order to speed up the generation of millions of events, samples of electromagnetic showers for different particles and energies have been generated and the corresponding patterns of energy deposit in the LKr cells saved into “shower libraries”. CMC can read back from the libraries a shower chosen randomly for the necessary particle and energy and use the information of the saved pattern. Due to a poor knowledge of the interaction of hadrons with matter, shower development of pions inside the LKr is not well simulated by CMC, so that the background rejection of decays with pions in the final state (*e.g.* the $K_{2\pi}$ decay) based on the E/p cut (Section 4.2.1), has to be studied on data.

6.1.1 Acceptance computation

The acceptances of the $K_{e2\gamma}(\text{SD}^+)$ decay, the normalization channel and their backgrounds are computed as the ratios of the number of events passing the $K_{e2\gamma}(\text{SD}^+)$ selection criteria (N_{rec}) and the total number of generated events (N_{gen}). To speed up the CMC simulation, only kaon decays with a generated longitudinal decay vertex Z_{gen} in the range $-2000 \text{ cm} < Z_{\text{gen}} < 9000 \text{ cm}$ have been simulated; in fact, particles originating at $Z_{\text{gen}} < -2000 \text{ cm}$ or at $Z_{\text{gen}} > 9000 \text{ cm}$ are respectively dumped by the final collimator

($Z_{\text{coll}} \approx -1800$ cm) or lost inside the central hole of the DCHs. Therefore, in order to obtain the total number of generated events and compute the acceptance, the number of CMC generated events have to be divided by the probability for a kaon to decay within the range $-2000 \text{ cm} < Z_{\text{gen}} < 9000 \text{ cm}$. However, as can be seen from the Eqs. 4.3 and 6.2, these factors cancel out when calculating quantities as the number of residual background events. Then, the acceptance for a given decay channel (both geometric and relative to the event selection) can be conveniently defined as:

$$\text{Acc} = \frac{N_{\text{rec}}}{N_{\text{gen}}(-2000 \text{ cm} < Z_{\text{gen}} < 9000 \text{ cm})} \quad (6.1)$$

6.1.2 Simulation of $K_{e2\gamma}(\text{SD}^+)$ events

In CMC, the matrix element for $K_{e2\gamma}$ decay is implemented following [35, 76] with the possibility of selecting only single components ($\text{SD}^\pm$ and IB) and switching between different form factors (ChPT $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$, LFQM and as measured by KLOE (Eq. 1.70)). The LFQM form-factors (given in integral form in [35]) were pre-tabulated and approximated by polynomials, since the online computation was too time consuming. In the present analysis, the $K_{e2\gamma}(\text{SD}^+)$ events have been simulated according to form factors measured by the KLOE experiment. The generated and reconstructed Dalitz plot for SD^+ events are shown in Fig. 6.1.

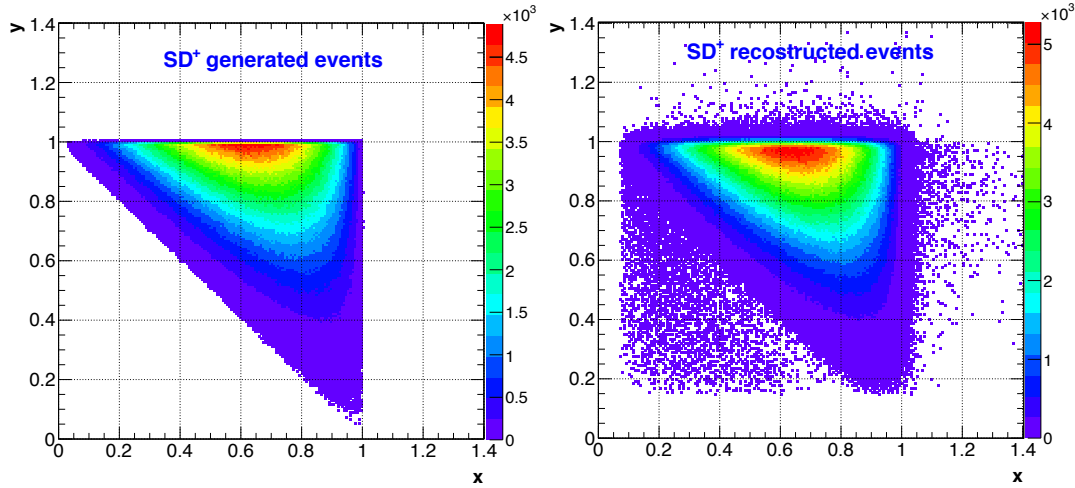


Figure 6.1: $K_{e2\gamma}(\text{SD}^+)$ events generated by CMC using form factors by the KLOE experiment (left) and after the reconstruction with no kinematic cuts applied (right). The color scale is arbitrary.

The $K_{e2\gamma}(\text{SD}^+)$ acceptance computed using CMC with KLOE form factors is shown in Tab. 6.1:

$K_{e2\gamma}(\text{SD}^+)$ selection	
Acceptance (KLOE form factors)	$0.06876 \pm 0.00004_{\text{stat}}$

Table 6.1: $K_{e2\gamma}(\text{SD}^+)$ acceptance

6.2 Background estimation

The main sources of background from kaon decays to $K_{e2\gamma}(\text{SD}^+)$ are due to K_{e3} and $K_{2\pi}$ decay modes whose final states are similar to $K_{e2\gamma}$. Their branching ratios are respectively ≈ 3000 and ≈ 14000 higher than that of the signal. The corresponding acceptances are listed in Tab. 6.2. The number of residual background events due to a kaon decay mode is evaluated using the formula:

$$N_{\text{B}}^{(i)} = \Phi_K \cdot \text{BR}^{(i)} \cdot \text{Acc}^{(i)} \quad (6.2)$$

where the index “ i ” denotes the decay channel contributing to the background, $\text{BR}^{(i)}$ its branching ratio, $\text{Acc}^{(i)}$ the corresponding “acceptance” (Eq. 6.1).

As discussed in Chapter 5, the 1TRK-LM trigger condition is highly inefficient for background events; in particular, the inefficiency strongly depends on the photon energy, or equivalently, on the Dalitz variables due to the interaction of the lost photons with the detector material (Chapter 5, Fig. 5.12). However, the trigger efficiency correction is essential for improving the data to MC comparison only outside the signal region, where the K_{e3} and $K_{2\pi}$ backgrounds overwhelm the signal. The effect of the trigger efficiency of the background on the measurement of the form factors will be discussed in Chapter 7. The estimate of the residual background for each single background source will be described in the following sections.

6.2.1 K_{e3} decay

A K_{e3} decay may lead to a $K_{e2\gamma}$ signature if one photon from the $\pi^0 \rightarrow \gamma\gamma$ decay is undetected (out of the calorimeter acceptance or in cells with read-out problems, Fig. 4.5) or if the π^0 decays in the Dalitz mode $\pi^0 \rightarrow e^+e^-\gamma$ (π_{D}^0) ($\text{BR} \sim 1.2\%$) and a e^+e^- pair is undetected. However, the track selection allows only one “good” track (Section 4.2.1) minimizing multi-track events and Dalitz decay modes. In fact, the background

Background to $K_{e2\gamma}(\text{SD}^+)$ from kaon decays		
Decay mode	BR [10]	Acceptance
$K^+ \rightarrow \pi^0 e^+ \nu (K_{e3})$	$(5.01 \pm 0.04)\%$	$(5.43 \pm 0.48) \cdot 10^{-7}$
$K^+ \rightarrow \pi^+ \pi^0 (K_{2\pi})$	$(20.42 \pm 0.08)\%$	$(8.01 \pm 0.19) \cdot 10^{-8}$
$K^+ \rightarrow \pi^0 \mu^+ \nu (K_{\mu3})$	$(3.35 \pm 0.03)\%$	$< 2 \cdot 10^{-8}$
$K^+ \rightarrow \pi^+ \pi^0 \pi^0$	$(1.76 \pm 0.02)\%$	$< 4 \cdot 10^{-8}$
$K^+ \rightarrow e^+ \nu \gamma (\text{IB}) (K_{e2\gamma}(\text{IB}))$	$(1.58 \pm 0.02) \cdot 10^{-5}$	$(7.52 \pm 0.29) \cdot 10^{-6}$
$K^+ \rightarrow e^+ \nu \gamma (\text{SD}^-) (K_{e2\gamma}(\text{SD}^-))$	$0.258 \cdot 10^{-5}$	$(3.13 \pm 0.03) \cdot 10^{-4}$

Table 6.2: Backgrounds from kaon decays to $K_{e2\gamma}(\text{SD}^+)$. The branching ratio and the acceptance relative to $K_{e2\gamma}(\text{SD}^+)$ selection are listed in the table. The acceptances of the K_{e3} and $K_{2\pi}$ backgrounds refers only to the $\pi^0 \rightarrow \gamma\gamma$ decay mode.

contribution from K_{e3} decay with subsequent Dalitz decay was studied using a dedicated MC simulation and was found to be negligible ($\approx 10^{-4}$). The distribution of the simulated K_{e3} background events over the Dalitz plot without cuts on x , y and $M_{\text{miss}}^2(e\gamma)$ is shown in Fig. 6.2.

6.2.2 $K_{2\pi}$ decay

Specific $K_{2\pi}$ decays, with and without the π_D^0 decay, have been simulated and analyzed with the signal event selection. A $K^+ \rightarrow \pi^+ \pi^0$ decay contributes to the background if one photon from the $\pi^0 \rightarrow \gamma\gamma$ decay is undetected and one of these two mechanisms happens:

- the charged pion is mis-identified as a positron because it releases a large fraction of its energy in the LKr calorimeter satisfying the electron-ID requirement (Section 4.2.1). This contribution has been addressed with the approach explained in Section. 6.2.3.
- the charged pion decays to a positron through the process $\pi^+ \rightarrow e^+ \nu_e$ ($\text{BR} \sim 10^{-4}$). This contribution has been specifically studied with a $K_{2\pi}$ MC sample in which the charged pion decays to a positron and was found to be negligible, with a background to signal ratio lower than 10^{-3} .

The $K_{2\pi}$ decay with a subsequent Dalitz decay may lead to a $K_{e2\gamma}$ signature in two ways:

- the π^+ is mis-identified as a positron and the $e^+ e^-$ pair is undetected;
- the charged pion and the electron are both undetected.

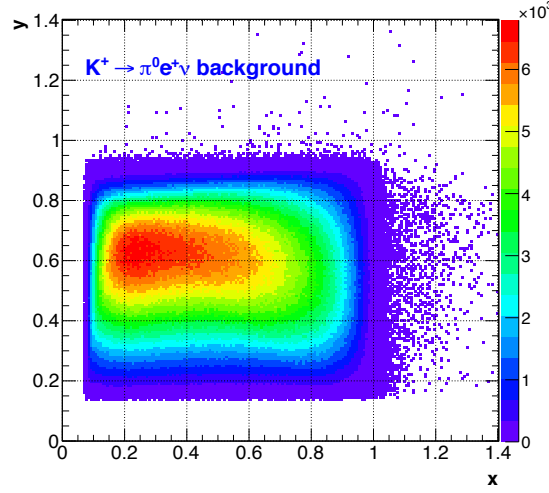


Figure 6.2: CMC simulation of K_{e3} background to $K_{e2\gamma}$ over the Dalitz plot. The color scale is arbitrary.

The former contribution was studied with a MC sample of $K_{2\pi}$ decays followed by $\pi^0 \rightarrow e^+e^-\gamma$ using the approach explained in Section 6.2.3, while the latter contribution was addressed using the same MC sample but assuming the positron hypothesis for the track (Section 4.2.1). Both contributions were found to be negligible at level of 10^{-4} .

6.2.3 Particle mis-identification probability $P(\pi \rightarrow e)$

The simulation of the energy released by the charged pion in the LKr calorimeter is inadequate for a particle ID based on the reconstructed ratio E/p , as described in Section 4.2.1, because the interaction and clusterization of hadrons inside the LKr calorimeter are difficult to reproduce with sufficient accuracy. To overcome this problem, the background contribution to SD^+ events due to the $K_{2\pi}$ decay (and its Dalitz mode) with the π^+ mis-identified as a positron, have been addressed with the following approach: for background MC samples with a charged pion in the final state, the cut $0.95 < E/p < 1.1$ related to the track energy deposit in the calorimeter, was released, while the mis-identification probability $P(\pi \rightarrow e)$ was measured directly from data using samples of $K_{2\pi}$ and $K_L \rightarrow \pi^\pm e^\mp \nu$ (K_{e3}^0) events [115]. $K_{2\pi}$ decays were selected by requiring one good track and two good clusters (Chapter 4), while, to reduce K_{e3} background, additional cuts were applied on the reconstructed kaon mass and the transverse momentum with respect to the kaon direction. K_{e3}^0 events were selected by requiring two good tracks with opposite charge and in time within 6 ns; to tag the electron, the condition $1.0 < E/p < 1.05$ was

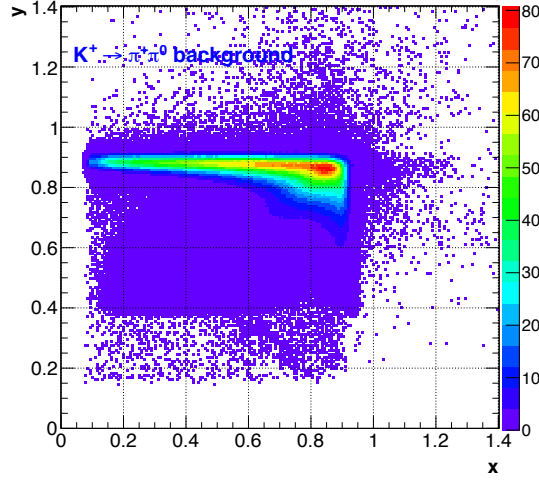


Figure 6.3: CMC simulation of $K_{2\pi}$ background to $K_{e2\gamma}$ over the Dalitz plot. The color scale is arbitrary.

required for one track. To increase the purity of the sample and reject other background channels such as $K_L \rightarrow \pi^\pm \pi^\mp \pi^0$, additional kinematic cuts were applied. MC events are then weighted with the appropriate measured values of the probability $P(\pi \rightarrow e)$ which is dependent on the pion momentum and has an average value of $\sim 4 \times 10^{-3}$ (Fig. 6.4). The distribution of $K_{2\pi}$ MC events on the Dalitz plot weighted for $P(\pi \rightarrow e)$ and without cuts on x , y and $M_{\text{miss}}^2(e\gamma)$ is shown in Fig. 6.3.

6.2.4 $K_{e2\gamma}(\text{SD}^-)$ and $K_{e2\gamma}(\text{IB})$ decays

The SD^- and IB components of the $K_{e2\gamma}$ decay have obviously the same final state of the signal but different distributions over the Dalitz plot, as shown in Fig. 6.5. A large fraction of SD^- events ($\approx 90\%$ of the SD^- MC events are at $y < 0.8$) is distributed in a kinematic region dominated by other backgrounds and not accessible to the analysis. The SD^- component is then considered as a background to the signal and is strongly reduced by the cut $y > 0.95$. The radiative photon emitted in the IB process is soft and its energy spectrum is different from the one emitted in the SD modes (Section 1.6); the IB background is then reduced to a negligible amount using a kinematic cut on the x variable.

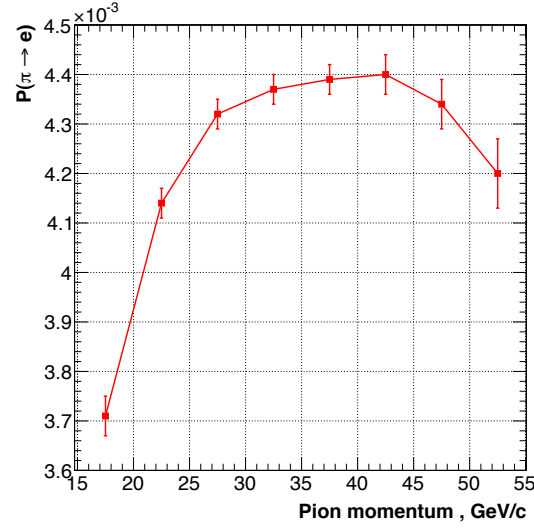


Figure 6.4: The pion mis-identification probability $P(\pi \rightarrow e)$ as a function of pion momentum. The values of the probability are the average of the measures obtained with $K_{2\pi}$ and K_{e3}^0 events. The error bars include statistical errors only.

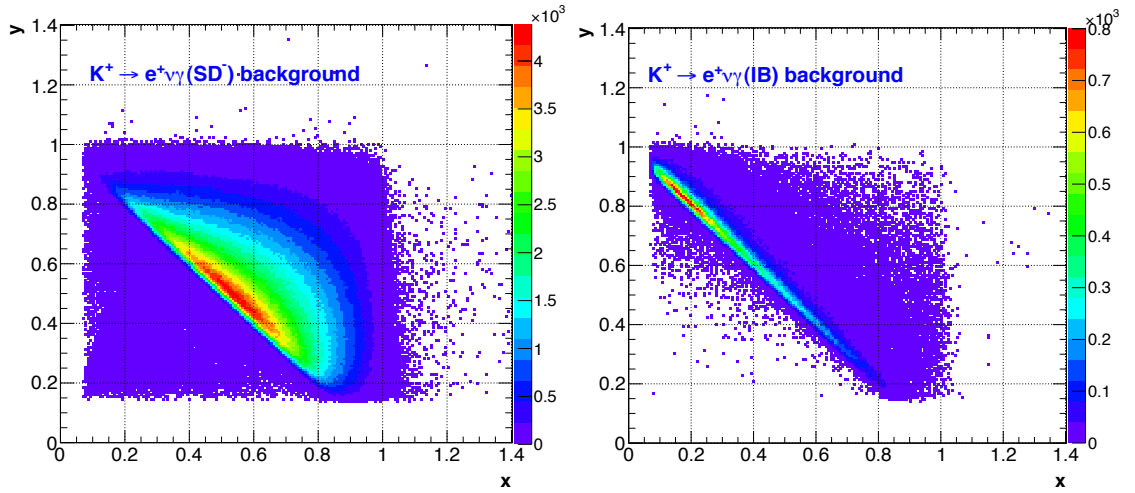


Figure 6.5: CMC simulation of $K_{e2\gamma}(\text{SD}^-)$ (left) and $K_{e2\gamma}(\text{IB})$ (right) backgrounds to $K_{e2\gamma}(\text{SD}^+)$ over the Dalitz plot, without cuts on x , y and $M_{\text{miss}}^2(e\gamma)$. The color scale is arbitrary.

6.2.5 Minor backgrounds

Another potential source of background is due to the $K^+ \rightarrow \pi^+ \pi^0 \pi^0$ decay ($\text{BR} \sim 1.8\%$), in which the charged pion is mis-identified as a positron and two photons from π^0 decays are undetected. However, this background is strongly reduced by vetoing on extra clusters in the calorimeter (Section 4.2.2), and its contribution can be neglected (after applying the selection, the background to signal ratio is about 10^{-6}).

Background channels with muons in the final state, such as the $K_{\mu 3}$ ($\text{BR} \sim 3\%$) and its Dalitz mode ($K_{\mu 3}$ followed by $\pi^0 \rightarrow e^+ e^- \gamma$), have also been considered. These channels may contribute to the background to $K_{e2\gamma}$ events if the charged muon is mis-identified as a positron and in one case a photon from the π^0 decay is undetected (absorbed before the LKr, or out of the calorimeter acceptance), while in the other case the π^0 decays in the Dalitz mode $\pi^0 \rightarrow e^+ e^- \gamma$ and the $e^+ e^-$ pair is undetected. A μ^+ can be mis-identified as a positron if:

- it completely releases its energy in the LKr calorimeter through a “catastrophic” Bremsstrahlung; this contribution is addressed with a similar approach to the one used for pions and explained in Sec. 6.2.3. In this case, the probability for a muon to be identified as a positron is $P(\mu \rightarrow e) \approx 10^{-6}$, three orders of magnitude lower than $P(\pi \rightarrow e)$ [116].
- it decays to a positron via the process $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$.

Both background contributions from $K_{\mu 3}$ and their Dalitz mode to $K_{e2\gamma}$ decay have been studied with specific MC samples, with and without the muon decaying to a positron. The background to signal ratio in the case of muon decay is lower than 10^{-3} , while in the case of muon-misidentification is $\lesssim 10^{-9}$ due to the very small value of $P(\mu \rightarrow e)$. In conclusion, backgrounds due to decay channels with muons in the final state were found to be negligible.

6.2.6 Accidental events

The contribution to the background due to accidental events was estimated by studying the time coincidence of the clusters associated to the positron and the photon with the hit of the track in the CHOD. Fig. 6.6 shows the distribution of the positron (Δt_{trk}) and photon (Δt_γ) clusters times, referred to the track time given by the CHOD, for data events passing all $K_{e2\gamma}$ criteria except for the timing cuts on Δt_{trk} , Δt_γ and the kinematic cuts on x , y and $M_{\text{miss}}^2(e\gamma)$. The main feature visible on the plot is a band populated by events in which the track and photon clusters are in time to each other but are out of time with respect to the CHOD hit. In addition, accidental events, in which the photon cluster is not in time with the track, populate a vertical band visible at $\Delta t_{\text{trk}} = 0$ ns. These events are strongly reduced after the cut on the missing mass. Their contribution inside the nominal time window, defined by $|\Delta t_{\text{trk}}| < 6$ ns and $|\Delta t_\gamma| < 6$ ns (Chapter 4), is estimated by extrapolation from the control regions of the Δt_γ distribution, $(-21:-15)$ ns and $(+15:+21)$ ns, to the nominal time window (Fig. 6.7). For $y > 0.8$, the number of reconstructed events is about $3 \cdot 10^6$, with only 2 events in the sidebands. In the signal region, no accidental events are found in the sidebands (Fig. 6.8), and only 7 events are outside a circle of radius 6 ns centered at $\Delta t_{\text{trk}} = \Delta t_\gamma = 0$ ns.

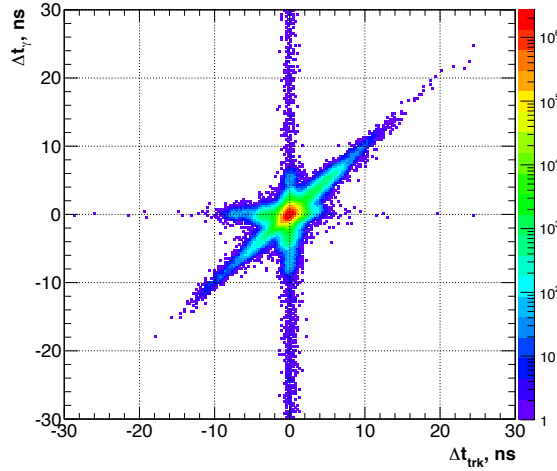


Figure 6.6: Data distribution of the positron (Δt_{trk}) and photon (Δt_γ) clusters times, referred to the track time given by the CHOD, for events passing the standard $K_{e2\gamma}$ selection, except for the Δt_{trk} , Δt_γ and kinematic cuts (logarithmic color scale). The oblique band is due to events in which the positron and photon clusters are in time to each other but are out of time with the hit of the track in the CHOD. Accidental events with the candidate photon cluster not in time with the track populate the vertical band at $\Delta t_{\text{trk}} = 0$ ns.

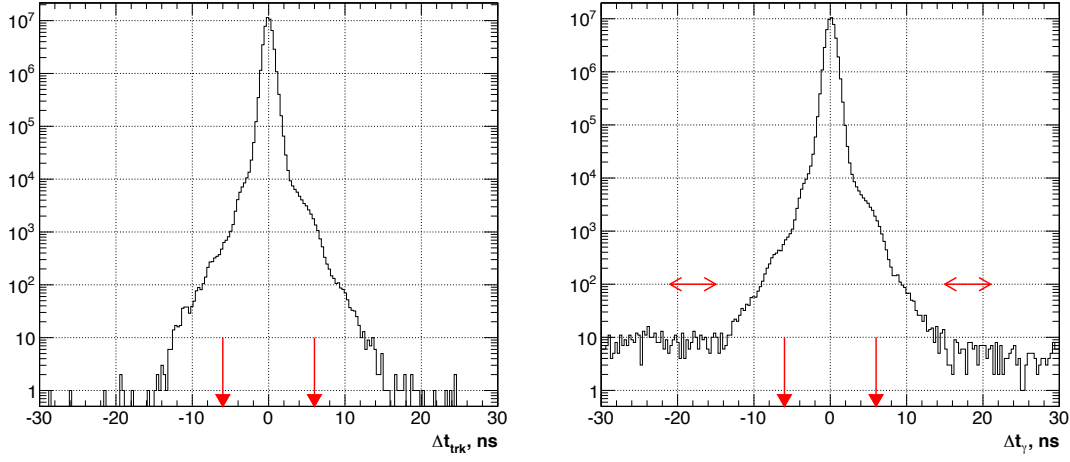


Figure 6.7: Data distributions of the positron (Δt_{trk} , left) and the photon (Δt_{γ} , right) clusters times, referred to the CHOD time, for events passing the standard $K_{e2\gamma}$ selection, except for the Δt_{trk} , Δt_{γ} and kinematic cuts (logarithmic scale). The vertical arrows indicate the nominal time window, while the horizontal arrows the control regions used for estimating the number of accidental events.

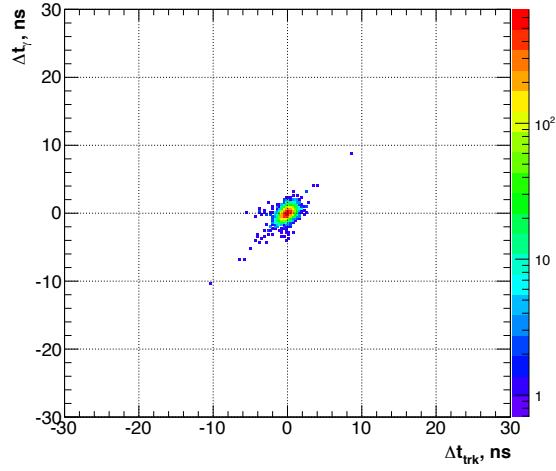


Figure 6.8: Data distribution of the positron (Δt_{trk}) and photon (Δt_{γ}) cluster times, referred to the track time given by the CHOD, for events passing all the $K_{e2\gamma}$ criteria, except for the Δt_{trk} and Δt_{γ} cuts (logarithmic color scale). After applying the $y > 0.95$ cut, the number of accidental events becomes negligible.

Total SD^+ candidates	11170
Acc(SD^+)	0.06876(4)
Background to $K_{e2\gamma}(SD^+)$ from kaon decays	
$N_B^{(K_{e3})}$	278 ± 24
$N_B^{(K_{2\pi})}$	166 ± 4
$N_B^{(SD^-)}$	8.2 ± 0.3

Table 6.3: Summary of the most relevant backgrounds to $K_{e2\gamma}(SD^+)$. The errors associated to the residual backgrounds are the total errors computed by error propagation of the terms in Eq. 6.2. Background contributions with a signal to background ratio $\lesssim 10^{-4}$ are not listed.

6.2.7 Background summary

The expected number of residual events for the main background components are shown in Tab. 6.3. From this table it can be inferred that the dominant background component is due to K_{e3} with a background to signal ratio $B/S(K_{e3}) \approx 2.6\%$, while the contribution due to the $K_{2\pi}$ decay is $B/S(K_{2\pi}) \approx 1.6\%$. This amount goes up to $B/S(K_{e3}) \approx 6.3\%$ and $B/S(K_{2\pi}) \approx 2.2\%$ when loosening the y cut from 0.95 to 0.94, because the resolution of the detector smears the K_{e3} and $K_{2\pi}$ endpoints from the kinematic limit $y_{\max} \approx 0.925$ up to the signal region, defined by the cut $y > 0.95$. On the other hand, the gain in signal acceptance is about 18%. Although a tight cut on y decreases the amount of residual background, it also causes a reduction of the signal region available for the measurement of the form factors. However, the approach followed in the present analysis is to keep the contribution of the residual background as low as possible, maximizing the purity of the sample.

The main backgrounds as a function of the track momentum, the photon energy and the y variable are shown in Figs. 6.9-6.12; the $K_{2\pi}$ background component, due to its kinematics, increases at high track momentum (Fig. 4.7), while both K_{e3} and $K_{2\pi}$ backgrounds grow rapidly just below the signal region.

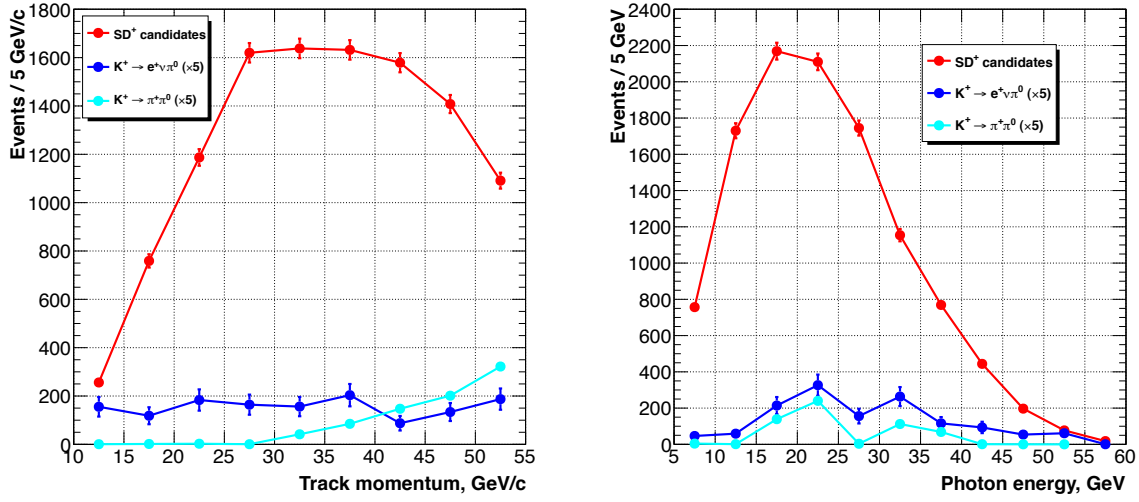


Figure 6.9: Residual background compared to SD⁺ candidates after the selection as a function of track momentum (left) and photon energy (right) (laboratory frame).

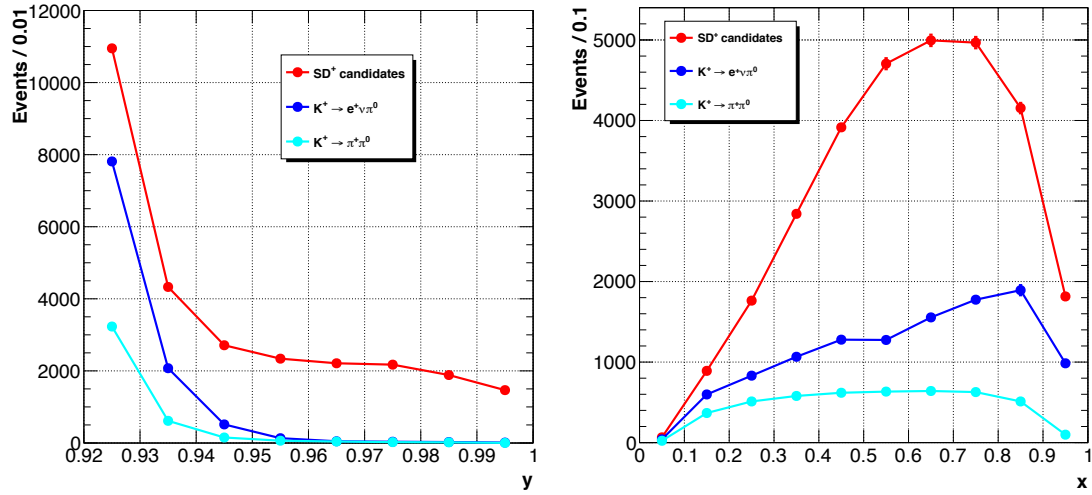


Figure 6.10: Residual background compared to the SD⁺ candidates as a function of the y (left) and x (right) variables, after applying the SD⁺ criteria with the looser cut $y > 0.92$. The signal region is at $y > 0.95$.

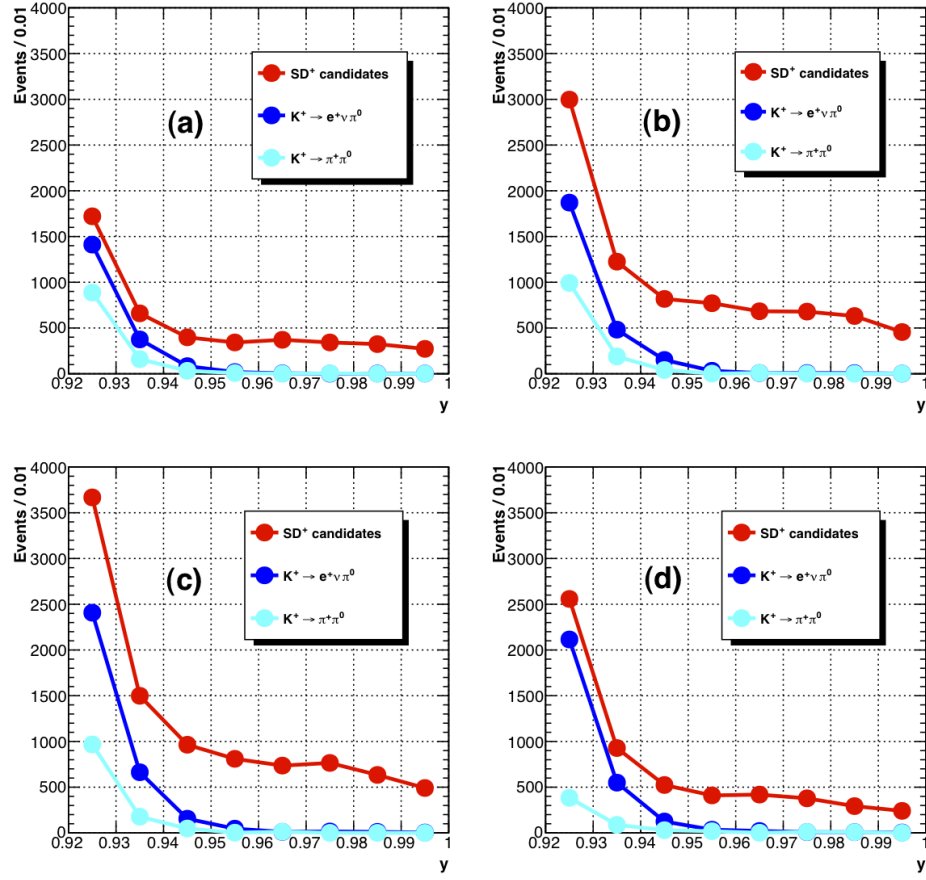


Figure 6.11: Residual background compared to the SD^+ candidates as a function of the y variable, after applying the SD^+ criteria with the looser cut $y > 0.92$, for different x intervals: (a) $0.2 < x < 0.4$, (b) $0.4 < x < 0.6$, (c) $0.6 < x < 0.8$, (d) $0.8 < x < 1.0$.

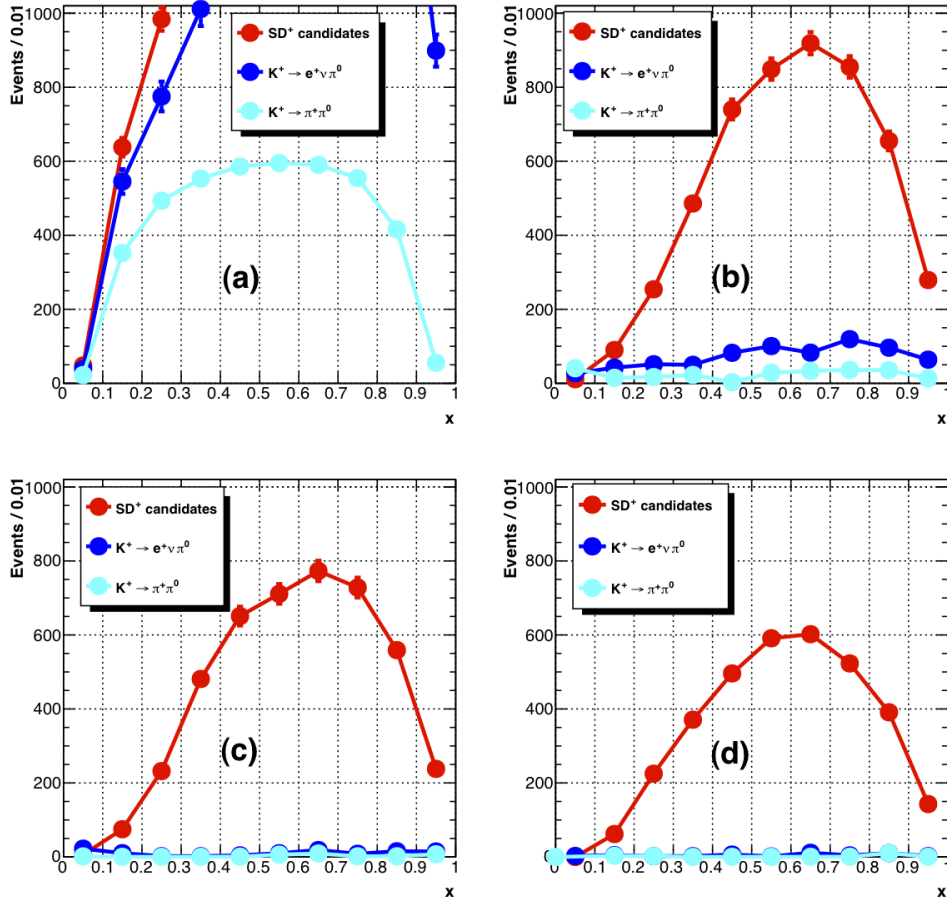


Figure 6.12: Residual background compared to the SD^+ candidates as a function of the x variable, after applying the SD^+ criteria with the looser cut $y > 0.92$, for different y intervals: (a) $0.92 < y < 0.94$, (b) $0.94 < y < 0.96$, (c) $0.96 < y < 0.98$, (d) $0.98 < y < 1.0$.

6.3 Data to MC comparison

6.3.1 Study of the Dalitz plot and related issues

The data to MC agreement over the whole Dalitz plot is essential to have the background under control also inside the SD^+ signal region. An imperfect description of the data/MC in the side bands could propagate inside the signal region and bias the background estimation. In order to compare data to MC expectations on the entire Dalitz plot, the cuts on x , y and missing mass were removed (the selected events are then mostly background due to K_{e3} and $K_{2\pi}$ decays) and MC distributions were corrected for the corresponding trigger efficiencies and normalized to data according to the formula:

$$N_{\text{B}}^{(i)} = \Phi_K \cdot \text{BR}^{(i)} \cdot \sum_{x,y} \text{Acc}^{(i)}(x,y) \cdot \varepsilon_{\text{trg}}^{(i)}(x,y) \quad (6.3)$$

where the same notation of Eq. 6.2 is used; $\text{Acc}^{(i)}(x,y)$ and $\varepsilon_{\text{trg}}^{(i)}(x,y)$ are the acceptance and the trigger efficiency calculated in a two-dimensional bin centered at x,y , while the sum extends over the whole Dalitz plot. The trigger efficiencies of K_{e3} and $K_{2\pi}$ backgrounds have been evaluated separately as described in Chapter 5.

The data to MC comparison over the Dalitz plot is shown in Fig. 6.13. In the background-enriched region $y \lesssim 0.8$, the MC is in agreement to data within 10%. For $y \gtrsim 0.9$, where the signal dominates with respect to background, the agreement is poorer than inside the background-enriched side-band $y < 0.8$: the data/MC disagreement in the signal region is due to the values of form factors (x -dependent) implemented in the simulation of SD^+ events; the best values of form factors compatible with data will be presented in Chapter 7. Outside the $K_{e2\gamma}$ kinematic limits $x,y > 1$, the MC fails to reproduce data (Fig. 6.13). This region contains non-physical events from the tails of x and y distributions, originated by detector resolutions. As shown in Fig. 6.21, the main contribution to the x and y resolution is given by the photon energy and track momentum (and their slopes), respectively. A good simulation of the resolutions is then important to have these tails under control (Section 6.4).

6.3.1.1 Normalization cross-check

A fit was done over the Dalitz plot to extract the overall normalization (common to signal and backgrounds) and to cross check the flux measurement obtained with K_{e3} . The number of data events was compared to the background expectation only outside the signal region where the background dominates over SD^+ events. In the fitting procedure

Fit summary over the Dalitz plot	
Number of data events	$69.6 \cdot 10^6$
Fitting region	$0.2 < x, y < 0.8$
Φ_K from fit	$(10.073 \pm 0.006_{\text{stat}}) \cdot 10^9$
χ^2/ndf	$1411.34/1319 = 1.07$

Table 6.4: Summary of the results of the fit to the flux over the Dalitz plot.

the branching ratio of each background source from kaon decays was fixed at the values given in Tab. 6.2 while the flux was left as free parameter. The flux is found by minimizing the χ^2 :

$$\chi^2 = \sum_{ij} \frac{(N_{ij} - N_{B,ij})^2}{\sigma_{N_{ij}}^2 + \sigma_{B,ij}^2} \quad (6.4)$$

where N_{ij} is the number of data in the bin ij of the Dalitz plot passing the loose selection, $N_{B,ij}$ is the total number of background events evaluated according to Eq. 6.3, $\sigma_{N_{ij}} = \sqrt{N_{ij}}$ is the error on data and $\sigma_{N_{B,ij}}$ is the error on the total background, computed by adding in quadrature the errors on acceptance and trigger efficiency. The result of the fit is shown in Tab. 6.4. The fitted flux is in good agreement with the value measured with K_{e3} (Eq. 4.4). The distribution of the χ^2 over the Dalitz plot is shown in Fig. 6.14.

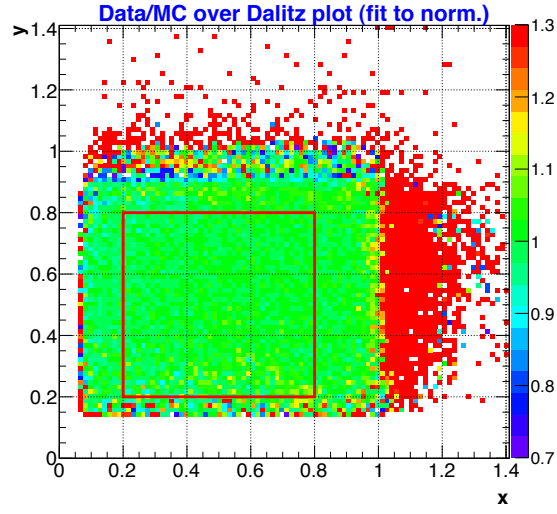


Figure 6.13: Data to MC comparison over the Dalitz plot (without kinematic cuts). The rectangle indicates the fit region used to extract the normalization (flux) to cross-check the flux measurement obtained with K_{e3} decay. The background distributions given by CMC are corrected for the trigger efficiency as described in Eq. 6.3 and Chapter 5. The data/MC disagreement is evident above the kinematic limits $x, y > 1$.

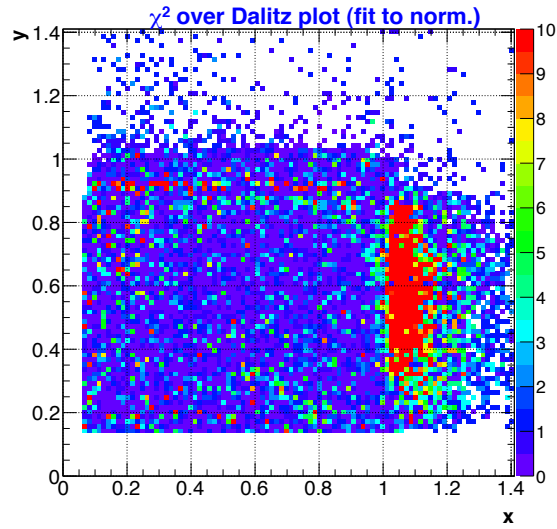


Figure 6.14: Fit χ^2 distribution over the Dalitz plot

6.3.2 Other data/MC comparisons

Figs. 6.15-6.19 show data/MC comparisons for some relevant variables of the $K_{e2\gamma}$ analysis after the full selection. The data to MC ratios of these variables are fitted to a constant and no appreciable deviation of the simulation from data was found, with the exception of the $M_{\text{miss}}^2(e\gamma)$ and x distributions, the latter depending on the values of the form factors used in the signal simulation. Fig. 6.19 shows the data to MC agreement of the missing mass in a wider $M_{\text{miss}}^2(e\gamma)$ range: the distribution has a long tail at negative values of $M_{\text{miss}}^2(e\gamma)$ extending well below the lower cut of the signal region $|M_{\text{miss}}^2(e\gamma)| < 0.01$ (GeV/c²)². From the conservation of four-momentum, the $K_{e2\gamma}$ missing mass squared for K_{e3} background events can be written as¹:

$$M_{\text{miss}}^2(e\gamma_{\text{seen}}) \equiv (p_K - p_e - p_{\gamma,\text{seen}})^2 = (p_{\gamma,\text{lost}} + p_\nu)^2 \geq 0 \quad (6.5)$$

where γ_{seen} and γ_{lost} denote the two photons from π^0 decay detected and rejected by the $K_{e2\gamma}$ selection, respectively and $p_{\gamma,\text{seen}}$ and $p_{\gamma,\text{lost}}$ their corresponding four-momenta. Eq. 6.5 implies that background events due to K_{e3} decays are distributed at positive values of the missing mass squared, while the tail at negative values are forbidden by kinematics and can arise only due to resolution effects. For the other main background component due to $K_{2\pi}$ decay a similar argument holds. Figs. 6.19-6.20 show that the resolution tail of the missing mass distribution is not correctly described by CMC: in fact, for $M_{\text{miss}}^2(e\gamma) \lesssim -0.02$ (GeV/c²)², the number of MC events is about 2 times smaller than the number of data events. This deficit is mostly outside of the signal region and is, at least in part, due to imperfections in the simulations of the detector resolution and the multiple scattering of the track at large angles. The inadequate MC description of events which are outside the kinematic limits (see also the data/MC comparison over the Dalitz plot for $x, y > 1$, Fig. 6.13) is a source of bias in the estimation of the residual background. As described in Chapter 7, this is an important issue of the analysis which limits the accuracy on the measurement of the $K_{e2\gamma}$ form factors.

¹ $(p_{\gamma,\text{lost}} + p_\nu)^2 = 2p_{\gamma,\text{lost}}p_\nu = 4E_{\gamma,\text{lost}}E_\nu \sin^2 \theta/2$, where $E_{\gamma,\text{lost}}$ and E_ν are respectively the energies of the neutrino and the lost photon and θ the angle between them.

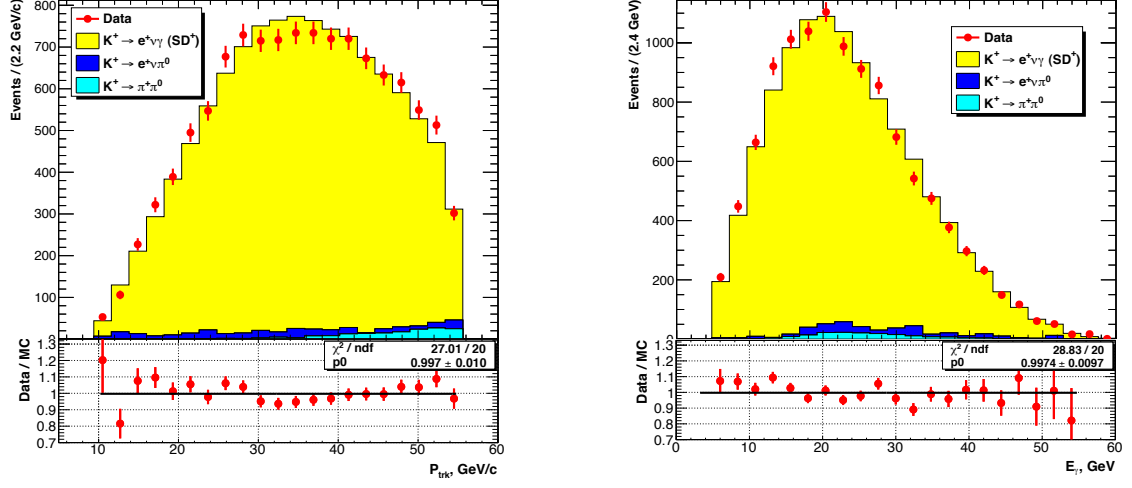


Figure 6.15: Data to MC comparison for $K_{e2\gamma}(SD^+)$ selection. Track momentum (left) and photon energy (right) in the laboratory frame.

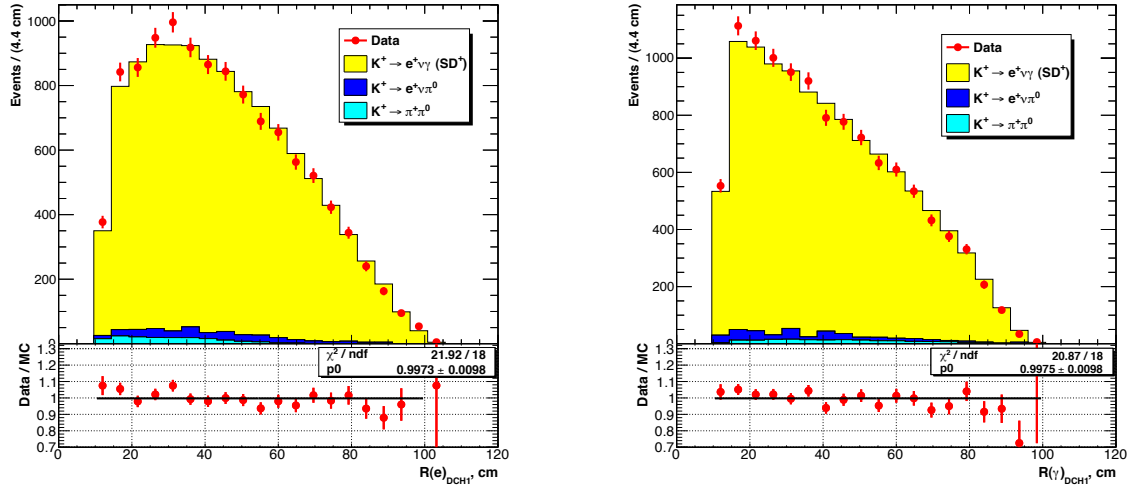


Figure 6.16: Data to MC comparison for $K_{e2\gamma}(SD^+)$ selection. Distances of the track (left) and photon (right) impact points to the DCH1 centre.

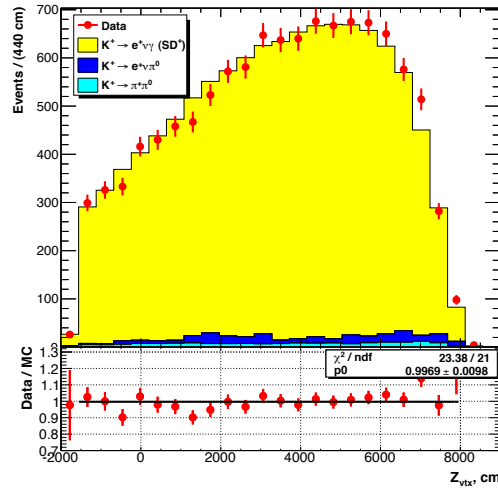


Figure 6.17: Data to MC comparison for $K_{e2\gamma}(SD^+)$ selection. Longitudinal decay vertex.

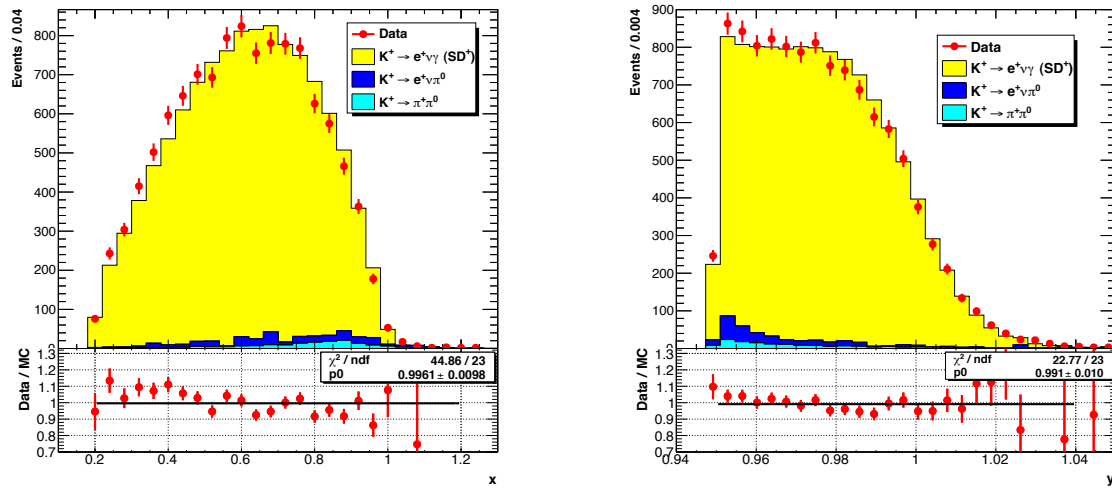


Figure 6.18: Data to MC comparison for $K_{e2\gamma}(SD^+)$ selection. Kinematic variables x (left) and y (right). The data/MC ratio for the x variable is incompatible with a constant; it strongly depends on the values of the form factors used in the signal simulation (in the present analysis the KLOE values have been used).

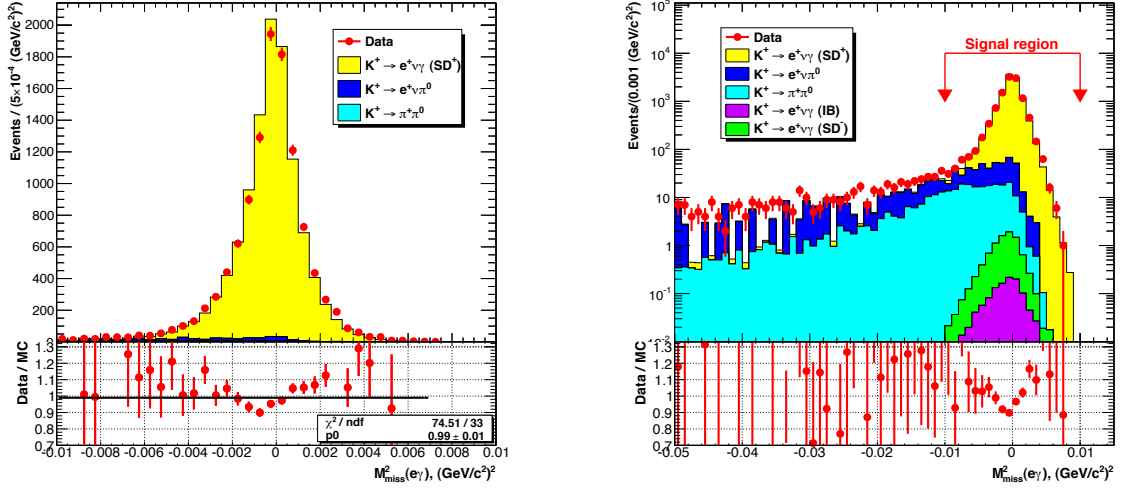


Figure 6.19: Reconstructed squared missing mass $M_{\text{miss}}^2(e\gamma)$ distributions of $K_{e2\gamma}(SD^+)$ candidates compared with the sum of normalized estimated signal and background components. Linear scale (left) and logarithmic scale in a wider $M_{\text{miss}}^2(e\gamma)$ range (right). The deficit of reconstructed MC events in the region below $0.01 \text{ GeV}^2/c^4$ (right) is mostly outside of the signal region and is, at least in part, due to resolution effects dominated by non Gaussian tails.

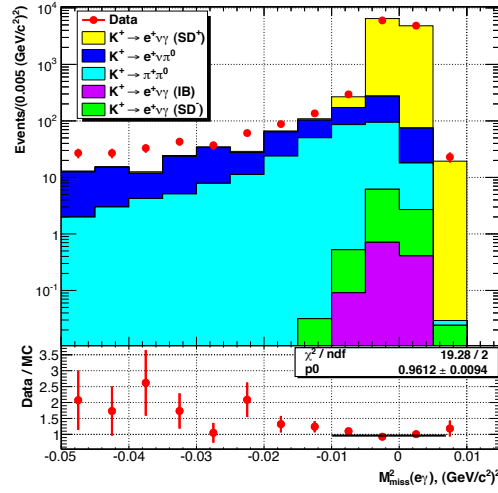


Figure 6.20: Reconstructed squared missing mass distribution, drawn with a binning 5 times larger than in Fig. 6.19 (right plot), to spot the data/MC disagreement in the tails of the distribution. For $M_{\text{miss}}^2(e\gamma) \lesssim -0.02 \text{ (GeV/c}^2\text{)}^2$, the number of events predicted by MC is about 2 times smaller than the number of data events.

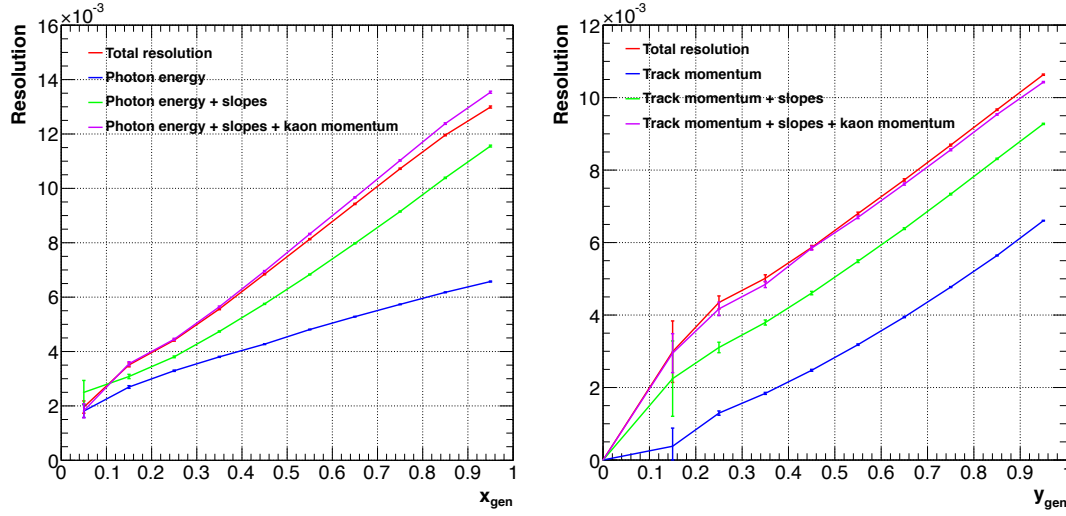


Figure 6.21: Contributions to the x (left) and y (right) resolutions. The main contributions to the resolutions on x and y are mainly due to the track and photon resolutions, respectively, while the contribution due to the kaon quantities (momentum and direction) is negligible.

6.4 Resolution studies on SD^+ events

Detector resolutions play a relevant role in the $K_{e2\gamma}$ analysis. The main effect of the resolution is to smear distributions to such an extent that background events are pushed inside the signal region above their kinematic end-point. The main contributions to the resolutions of the Dalitz variables are shown in Fig. 6.21: the y resolution is dominated by the resolution on the track variables (momentum and slopes) introduced by the spectrometer, while the x resolution is due to the measurements of energy and position of the photon, with a negligible contribution due to the kaon variables. Distributions of the difference between reconstructed and generated quantities for several variables are shown in Figs. 6.22, 6.23, 6.24. A partial check on the CMC performance in the simulation of the detector resolutions was done by comparing the width of the $M_{\text{miss}}^2(e\pi^0)$ distribution for data and CMC in bins of track momentum and π^0 energy (Fig. 6.25). The K_{e3} sample was chosen because it has low-background contamination ($B/S \approx 10^{-4}$) which cannot bias the data-MC comparison. For each bin, the resolution was measured by fitting the $M_{\text{miss}}^2(e\pi^0)$ distribution with a Gaussian. As shown in Fig. 6.25, the simulation reproduces data with an accuracy better than 2% over the all range of momentum and energy.

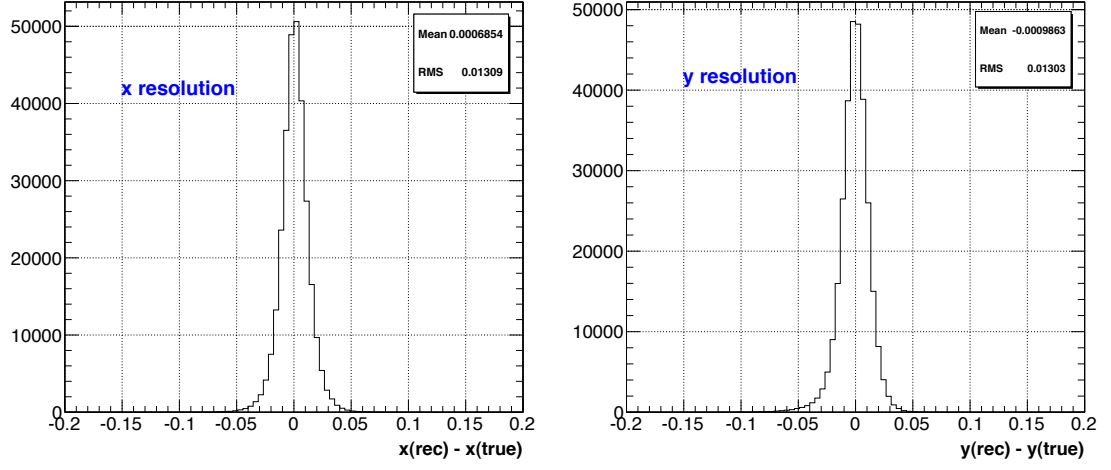


Figure 6.22: Resolution of the Dalitz plot variables x (left) and y (right), obtained from MC simulation.

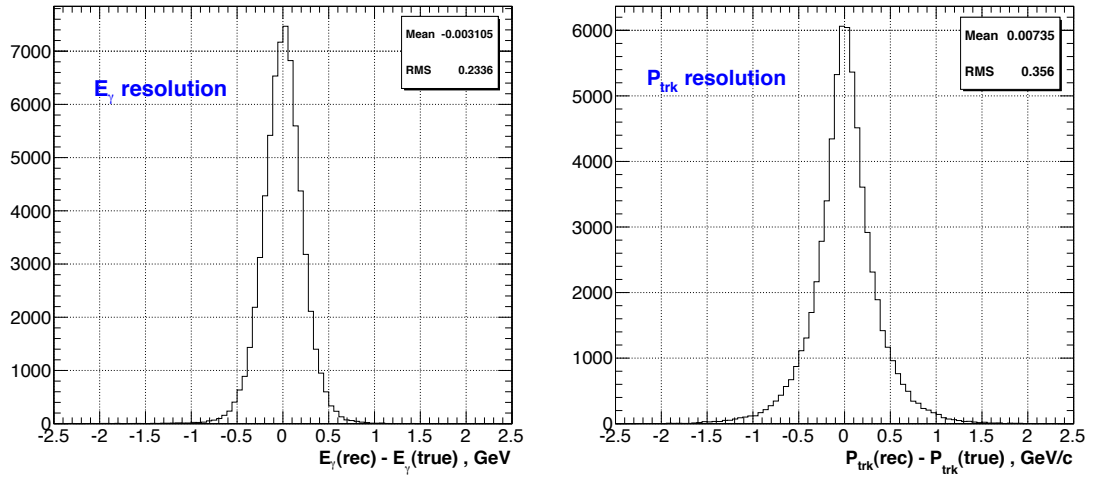


Figure 6.23: Resolution of the track momentum (left) and the photon energy (right), obtained from MC simulation.

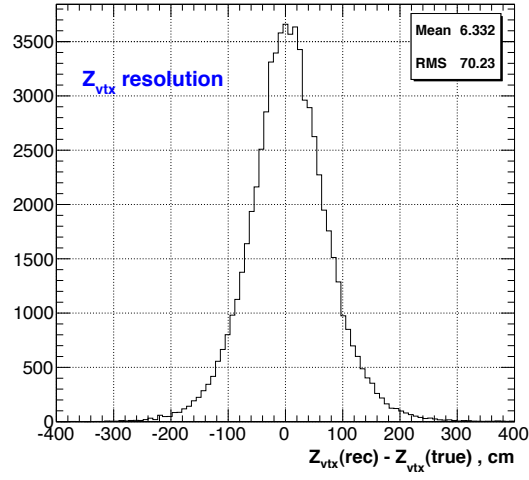


Figure 6.24: Resolution of the longitudinal decay vertex, obtained from MC simulation.

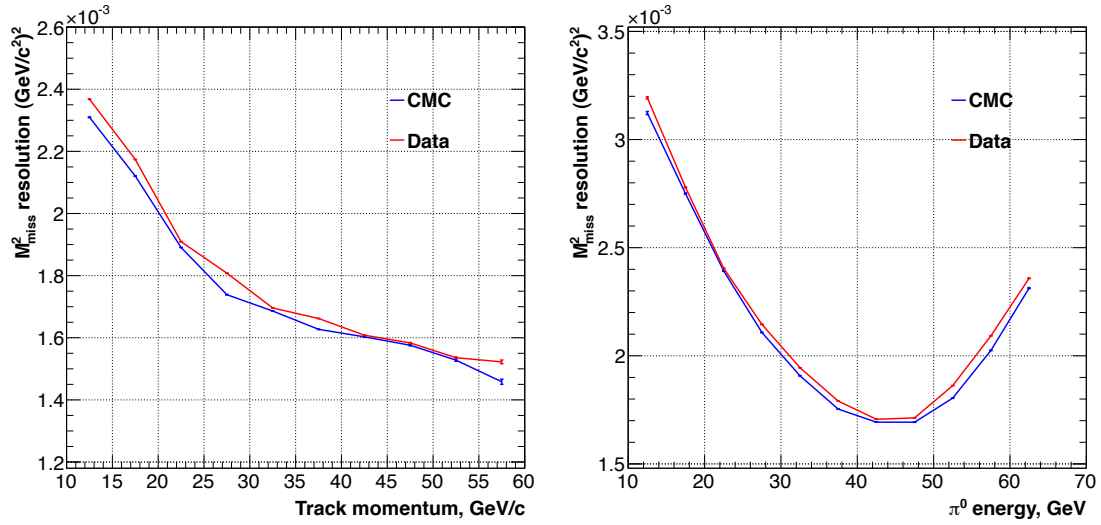


Figure 6.25: Data to MC comparison of the resolution of the K_{e3} missing mass squared $M_{\text{miss}}^2(e\pi^0)$ as a function of the reconstructed track momentum (left) and π^0 energy (right). The CMC reproduces the data with good accuracy over all momentum and energy ranges.

Chapter 7

Study of the systematic effects

The uncertainty on the form factors returned by the fit routine is only the estimation of the statistical error [100,101]. All the other sources of uncertainty due to the experimental method have to be taken into account in the systematic error. The origins of systematics can be originated by some instrumental effect, by wrong tuning of MC simulation (*e.g.* the simulation of the kaon beam and resolution of the detectors) or by unaccounted background sources. The aim of the systematic studies is to estimate the stability of the measurement under changes in the selection cuts, the precision of background subtraction and the dependence of results on the experimental conditions. In fact, if the value of the form factors depends on the applied selection cuts, this means that there is something out of control in the analysis that have to be corrected and/or included as systematic error. In particular, as discussed in Chapter 6, the agreement between data and MC simulation can in principle introduce a systematic uncertainty in the background estimation. On this purpose, all the effects to which the measurement is expected to be sensitive were considered.

7.1 Fit to the x distribution

After the selection of the SD^+ candidates and the subtraction of the residual background, a first measurement of the $K_{e2\gamma}$ form factors can be attempted. Although systematic errors are not yet estimated, quantitative conclusions on the agreement of data to the theoretical models (ChPT and LFQM) can be given. The fit to the x distribution has been done according to the procedure described in Chapter 3. The results of the fits are described in the following sections.

7.1.1 Data fit to LFQM

The LFQM features a complicated dependence of F_V and F_A on x (Fig. 1.9). In this framework, the $K_{e2\gamma}$ form factors are expressed in integral form and cannot be easily parametrized as in the case of ChPT at $\mathcal{O}(p^6)$ in which they are fully described by $F_V(0)$, $F_A(0)$ and λ (Section 1.6.2). The LFQM has been tested by making use of a polynomial approximation for the form factors (Section 6.1.2); for this reason there are no free fit parameters to compare to the LFQM prediction. The agreement with data is given only by the χ^2 obtained by overlaying the expected distribution (LFQM signal and background) with data. Fig. 7.1 shows the overlay (not a fit) considering only the LFQM signal and the main backgrounds K_{e3} and $K_{2\pi}$. The expected distribution disagrees with data, in particular at low values of x (or equivalently at high values of $q^2 = M_K^2(1-x)$) where the LFQM form factors differs substantially from the ChPT prediction (Fig. 1.9). The disagreement with data is quantified by the value of the $\chi^2/\text{dof} \approx 350$.

In conclusion, the LFQM is not able to reproduce the observed x -distribution of SD^+ events in the kinematic range $0.2 < x < 1$ and $y > 0.95$. More inputs from theory are needed to improve the agreement with data and a description of the form factors in terms of few physical parameters would be useful to perform a fit to the theoretical model.

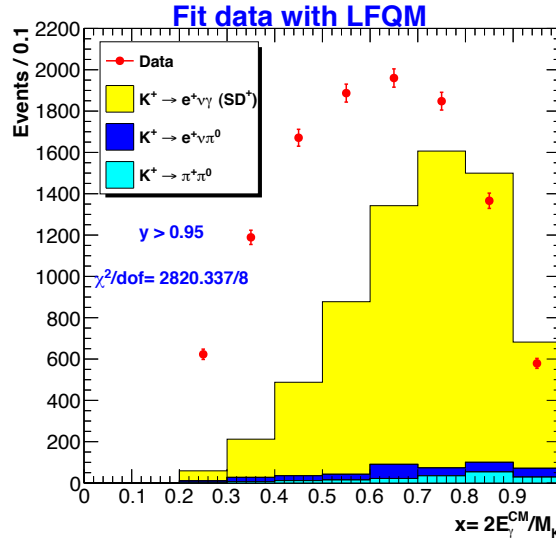


Figure 7.1: Overlay (not a fit) of the LFQM prediction and the residual background with data. Only the main backgrounds K_{e3} and $K_{2\pi}$ are drawn in this plot. The data are clearly in disagreement with the prediction of the LFQM, as the overlay gives $\chi^2/\text{dof} \approx 350$. The error bars refer only to the statistical uncertainty.

7.1.2 Data fit to ChPT

In the framework of ChPT, the $K_{e2\gamma}$ form factors have a rather simple form. At one-loop order $\mathcal{O}(p^4)$, ChPT predicts constant form factors $F_V(x) = F_V(0)$ and $F_A(x) = F_A(0)$ (Section 1.6.2). The agreement of the $\mathcal{O}(p^4)$ approximation to data was tested by fitting the combination $F_V(x) + F_A(x) = F_V(0) + F_A(0)$. Fig. 7.2 shows that ChPT fits data better than LFQM already at one-loop order, but in most of the bins the expected number of events differs from data up to 20%. The fit gives $F_V(0) + F_A(0) = 0.147 \pm 0.001_{\text{stat}}$ with $\chi^2/\text{dof} \approx 24$, while ChPT at $\mathcal{O}(p^4)$ predicts $F_V(0) + F_A(0) = 0.138$ (no errors are quoted at one-loop order, Tab. 1.3). At two-loop approximation $\mathcal{O}(p^6)$, the vector form factor is no more x -independent, but it varies linearly as $F_V(x) = F_V(0) \cdot [1 + \lambda(1 - x)]$ with the slope parameter $\lambda = 0.3 \pm 0.1$, while F_A remains constant (Section 1.6.2). In particular, the $\mathcal{O}(p^6)$ contribution enhances events at low x values with respect to the $\mathcal{O}(p^4)$ case, giving an explanation for the behavior of the fit residuals shown in Fig. 7.2 (right). The fit to ChPT at $\mathcal{O}(p^6)$ was done following the procedure used in the KLOE analysis (Section 1.7), in which $F_V(0) - F_A(0)$ was fixed at the expectation of ChPT at $\mathcal{O}(p^4)$, while $F_V(0) + F_A(0)$ and λ were free parameters. In fact, in the KLOE analysis, the small contribution from SD^- transitions to the selected events ($\approx 2\%$) did not allow a fit to the related $F_V - F_A$ component, while in the present analysis the SD^- component is only 0.1% of the total selected events and was subtracted as background to SD^+ . The result of the fit to ChPT at $\mathcal{O}(p^6)$ is summarized in Tab. 7.1. Fig. 7.3 shows that the agreement of the fit with data is improved with respect to the one-loop order, as the fit gives $\chi^2/\text{dof} \approx 0.89$ with a statistical correlation between parameters ρ compatible with the one reported by the KLOE experiment ($\rho_{\text{KLOE}} = -0.93\%$). In Fig. 7.4 the confidence level (CL) contours for the fitted parameters are compared to the result reported by the KLOE. The form factors measured at this stage of the analysis, with only the statistical error included in the result, are incompatible with the KLOE experiment at more than 3σ . The effect of the systematics on the measurement of $F_V(0) + F_A(0)$ and λ and the relative systematic errors will be discussed in the following sections.

Fit to ChPT at $\mathcal{O}(p^6)$	
Parameter	Fit result
$F_V(0) + F_A(0)$	$0.127 \pm 0.002_{\text{stat}}$
λ	$0.60 \pm 0.05_{\text{stat}}$
χ^2/dof	5.35/6
ρ	-92.6%

Table 7.1: Summary of the fit to SD^+ events using the ChPT at $\mathcal{O}(p^6)$. At this stage of the analysis, only the statistical error is included in the result.

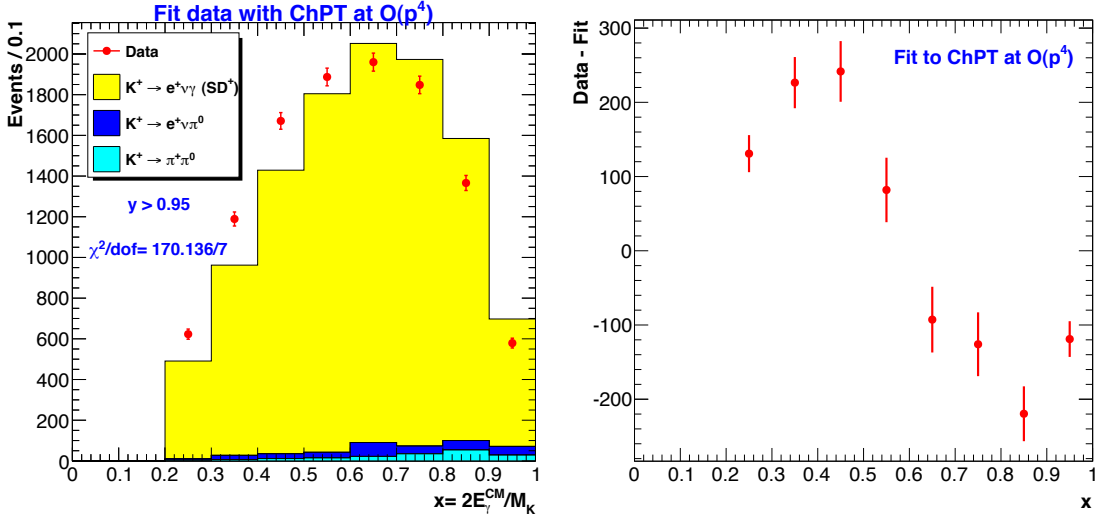


Figure 7.2: Fit to ChPT at $\mathcal{O}(p^4)$ with $F_V(0) + F_A(0)$ as free parameter. (Left) Data are compared to the fitted distribution (only the main backgrounds K_{e3} and $K_{2\pi}$ are drawn). (Right) Residuals of the fit. The ChPT at $\mathcal{O}(p^4)$ gives a better agreement to data with respect to LFQM but it is not able to describe the x spectrum with a sufficient accuracy. In both plots, the error bars refer only to the statistical uncertainty.

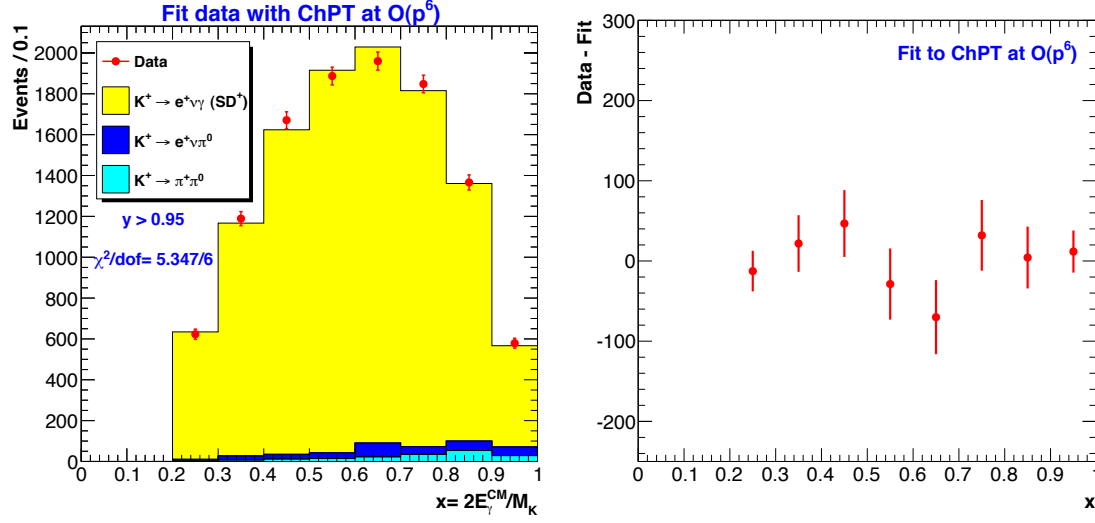


Figure 7.3: Fit to ChPT at $\mathcal{O}(p^6)$ with $F_V(0) + F_A(0)$ and λ as free parameters with $F_V(0) - F_A(0)$ fixed at the expectation of ChPT at $\mathcal{O}(p^4)$. (Left) Data are compared to the fitted distribution (only the main backgrounds K_{e3} and $K_{2\pi}$ are drawn). (Right) Residuals of the fit. The ChPT at $\mathcal{O}(p^6)$ gives a better agreement to data with respect to LFQM and ChPT at $\mathcal{O}(p^4)$. In both plots, the error bars refer only to the statistical uncertainty.

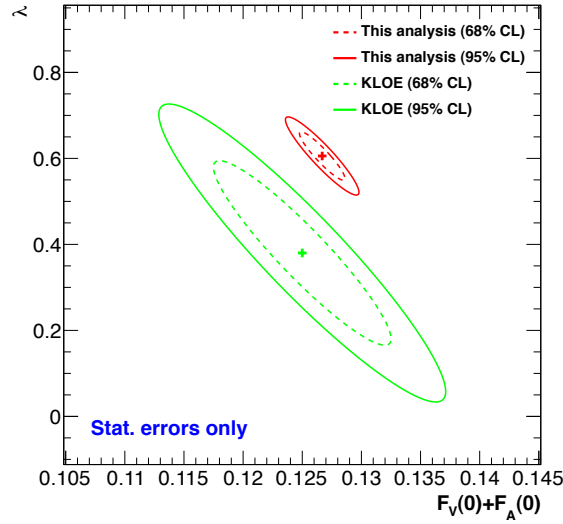


Figure 7.4: Confidence level contours for the form factors measurement obtained in the present analysis (statistical error only) and by the KLOE experiment. The crosses refer to the central values. Without considering systematics, the result of the present analysis shows a $\approx 3\sigma$ discrepancy with respect to the KLOE measurement.

7.2 Fit robustness

To test the fit procedure, an independent SD^+ MC sample (pseudo-data) was generated according to the form factor parametrization given by ChPT at $\mathcal{O}(p^6)$ but with values of the form factors ($F_V(0)$, $F_A(0)$ and λ) similar to the ones obtained from the fit to data. The pseudo-data sample contains about 10000 events (similar to the statistics of the SD^+ candidates) and was used in place of data in the fit procedure. In the test, the background is assumed to be correctly subtracted so that no background contamination has been added to the pseudo-data; in addition, the overall normalization is supposed to be known exactly. The result of the fit to pseudo-data is shown in Fig. 7.5 and Tab. 7.2. The fit retrieves the value of the true parameter $F_V(0) + F_A(0)$ within the statistical error, while λ is consistent with the generated value only at 2σ ; however, this discrepancy is a purely statistical effect: using a pseudo-data sample with more statistics (about 10^5 events), the fitted values of the form factors agree with the true values to better than 1%. The value of the correlation between $F_V(0) + F_A(0)$ and λ is similar to the one found by fitting the data (Tab. 7.1). In conclusion, the above results demonstrated the reliability of the fitting procedure for a measurement of the $K_{e2\gamma}$ form factors, assuming the background subtraction is under control and the normalization is correctly measured.

Fit to the pseudo-data		
Pseudo-data statistics	10440	
Form factor parameter	True value	Fitted value
$F_V(0) + F_A(0)$	0.1125	$0.110 \pm 0.002_{\text{stat}}$
λ	0.60	$0.68 \pm 0.04_{\text{stat}}$
χ^2/dof	0.37	
ρ	-92.3%	

Table 7.2: The result of the fit to the pseudo-data. The generated values of the form factors are $F_V(0) = 0.1$, $F_A(0) = 0.0125$ and $\lambda = 0.6$. The statistical error is similar to the one obtained by fitting data.

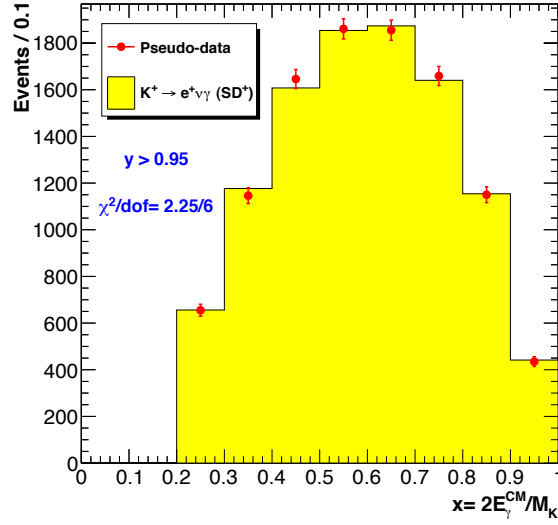


Figure 7.5: Fit to pseudo-data.

7.3 Systematic errors related to cuts variation

The results obtained for the $K_{e2\gamma}$ form factors should not depend on the choice of the selection cuts. The study of the stability of the measurement with respect to cuts variation is essential to spot some unaccounted systematic effect. The stability of the results was studied by varying one selection cut at time and re-evaluating the form factors. The measurements with the nominal and varied cut are obtained by using a common data sample so that the statistical uncertainties of the two values are correlated. In general, the two measurements can share other correlated uncertainties (*i.e.* the error on the normalization (flux) and other systematic errors not related to the cut variation). A positive (negative) correlation between errors implies that if one of the two measurements fluctuated with respect the true value, the other always fluctuated in the same (opposite) direction, hiding a possible systematic effect. In order to decide if the measurement obtained with the varied value of the cut is compatible with the measurement at the nominal value, the so-called un-correlated errors have to be quoted for the former:

$$\sigma_{\text{uncorr}}(\text{var}) = \sqrt{|\sigma^2(\text{nom}) - \sigma^2(\text{var})|} \quad (7.1)$$

where $\sigma(\text{nom})$ and $\sigma(\text{var})$ are the total errors of the measurements with the nominal and varied cut, respectively. If the absolute value of the difference between the results obtained with the varied cut and the nominal cut is bigger than the uncorrelated error,

a systematic error is associated to the cut; in formulae, this condition can be written as:

$$|FF(\text{nom}) - FF(\text{var})| > \sigma_{\text{uncorr}}(\text{var}) \quad (7.2)$$

where $FF(\text{nom})$ and $FF(\text{var})$ are the form factors measured at the nominal and varied cut, respectively. The quoted value of the systematic error is the largest value of the $|FF(\text{nom}) - FF(\text{var})|$ among the values obtained by varying the cut, provided the condition in Eq. 7.2 holds. If the variation is within the uncorrelated error, no systematic error is assigned to the cut.

In Tab. 7.3 the list of the cuts is reported. The variation range of each cut was chosen as a trade-off between the point above/below which the results become stable if a systematic trend is spotted, and the change in statistics with respect the nominal cut (a statistical uncertainty on the measurement with the varied cut much larger than the one with the nominal cut can hide or enhance a systematic effect).

List of the varied cuts		
Cut	Nominal value	Variation range
Track momentum (min), $P_{\text{trk}}^{(\text{min})}$	10 GeV/ c	5-30 GeV/ c
Track momentum (max), $P_{\text{trk}}^{(\text{max})}$	55 GeV/ c	65-40 GeV/ c
Photon energy (min), $E_{\gamma}^{(\text{min})}$	5 GeV	2-9.5 GeV
Longitudinal decay vertex (min), $Z_{\text{vtx}}^{(\text{min})}$	-1600 cm	-2000-1600 cm
Longitudinal decay vertex (max), $Z_{\text{vtx}}^{(\text{max})}$	9000 cm	9000-5500 cm
Track radius @ DCH1 (min), $R_{\text{DCH1}}(e)^{(\text{min})}$	12 cm	7-47 cm
Photon radius @ DCH1 (min), $R_{\text{DCH1}}(\gamma)^{(\text{min})}$	12 cm	7-47 cm
Energy of the extra clusters veto, E_{veto}	2 GeV	1-5 GeV
cda (max), $\text{cda}^{(\text{max})}$	3 cm	4.5-1 cm
Missing mass (min), $M_{\text{miss}}^2(e\gamma)^{(\text{min})}$	-0.01 (GeV/ c^2) ²	-0.03 - -0.002 (GeV/ c^2) ²
Missing mass (max), $M_{\text{miss}}^2(e\gamma)^{(\text{max})}$	+0.01 (GeV/ c^2) ²	+0.01 - +0.002 (GeV/ c^2) ²
x (min), $x^{(\text{min})}$	0.2	0.0-0.40
y (min), $y^{(\text{min})}$	0.95	0.92-0.97

Table 7.3: List of the varied cuts and their variation ranges. Each cut was varied above and below its nominal value, except for the $M_{\text{miss}}^2(e\gamma)^{(\text{max})}$ and $Z_{\text{vtx}}^{(\text{max})}$ cuts for which there are not data events above their nominal values.

The measured values of the form factors as a function of the cuts are shown in Figs. 7.6 - 7.15, while the systematic error assigned to each cut is reported in Tab. 7.4. The

form factors measurement is found to be unstable with respect to several cuts. According to Eq. 7.2, large systematic errors have to be assigned to the measurement of the form factors due to the poor stability on the cuts on the track momentum, longitudinal vertex position, photon radius at the DCH1 and y variable. The systematic trends visible in Fig. 7.6, in which the stability of the form factors with respect to the $P_{\text{trk}}^{(\text{min})}$ is plotted, points to a wrong subtraction of the background due to the $K_{2\pi}$ decay: in fact, the instability becomes important at $P_{\text{trk}}^{(\text{min})} \gtrsim 25 \text{ GeV}/c$, where the $K_{2\pi}$ contamination is no more negligible (as shown in Fig. 6.9, it increases at high momenta). A possible underestimation of the $K_{2\pi}$ component might also explain the instability with respect to the y variable (Fig. 7.14) as the background increases steeply below the signal region $y > 0.95$ (Fig. 6.10).

Summary of the systematics related to cuts variation		
Cut	Associated systematic error	
	$\delta[F_V(0) + F_A(0)] \times 10^3$	$\delta\lambda \times 10^2$
$P_{\text{trk}}^{(\text{min})}$	4.4	11.2
$P_{\text{trk}}^{(\text{max})}$	2.0	10.7
$E_{\gamma}^{(\text{min})}$	0.4	1.6
$Z_{\text{vtx}}^{(\text{min})}$	2.7	7.1
$Z_{\text{vtx}}^{(\text{max})}$	2.6	5.9
$R_{\text{DCH1}}(e)^{(\text{min})}$	2.5	2.4
$R_{\text{DCH1}}(\gamma)^{(\text{min})}$	5.1	12.3
E_{veto}	0.5	1.1
$\text{cda}^{(\text{max})}$	1.0	1.0
$M_{\text{miss}}^2(e\gamma)^{(\text{min})}$	1.2	5.5
$M_{\text{miss}}^2(e\gamma)^{(\text{max})}$	0.7	4.5
$x^{(\text{min})}$	0.3	1.3
$y^{(\text{min})}$	4.7	31.0

Table 7.4: Summary of the systematics (absolute values) associated to the $F_V(0) + F_A(0)$ and λ measurements and related to cuts variation.

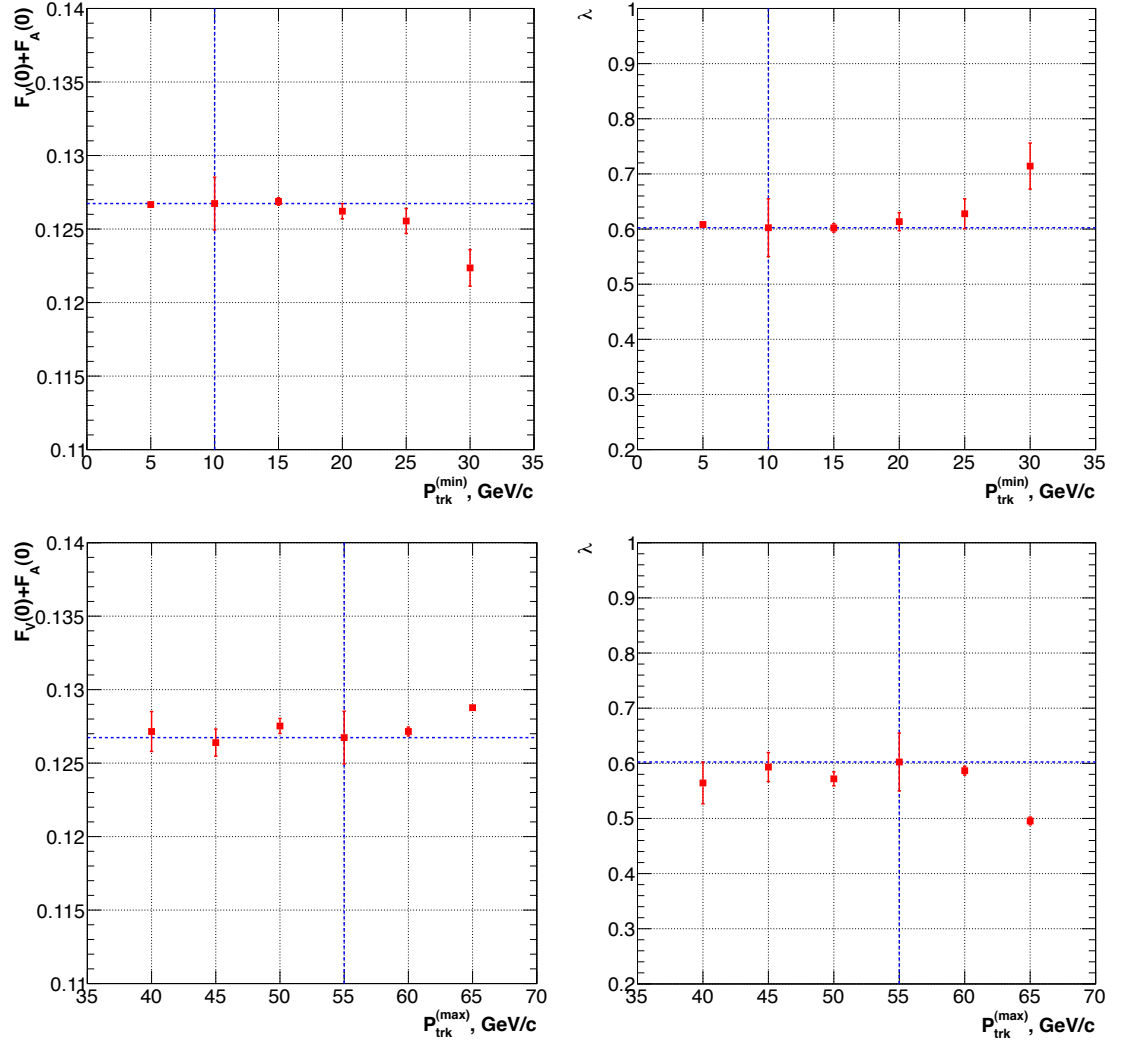


Figure 7.6: Form factor variation as a function of $P_{\text{trk}}^{(\text{min})}$ (upper plots) and $P_{\text{trk}}^{(\text{max})}$ (lower plots). The nominal values ($P_{\text{trk}}^{(\text{min})} = 10$ GeV/c and $P_{\text{trk}}^{(\text{max})} = 55$ GeV/c) are reported with the statistical error, while the uncorrelated error is reported for the varied cut. The horizontal dashed line indicates the value of the measurement at the nominal value of the cut (indicated by the vertical dashed line).

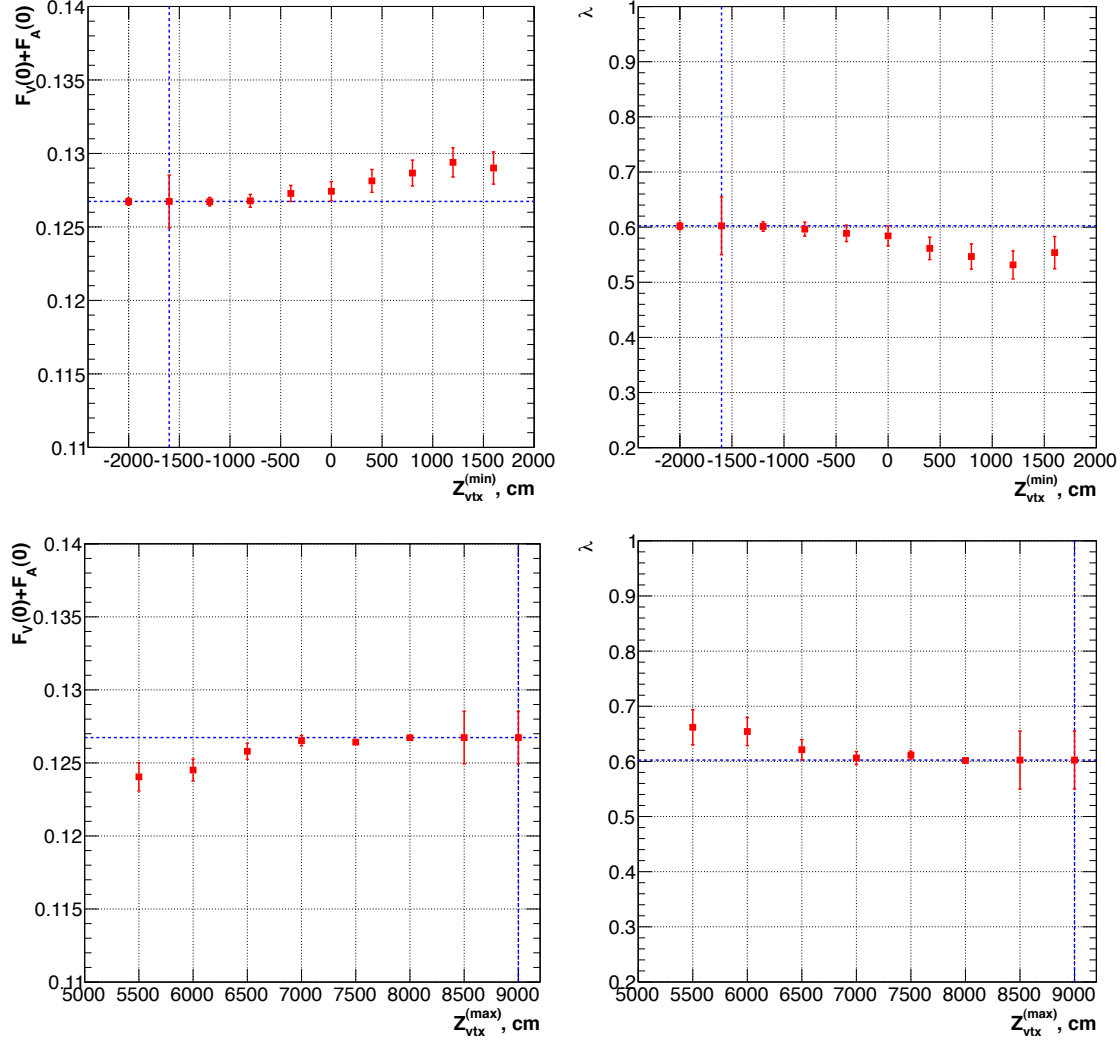


Figure 7.7: Form factor variation as a function of $Z_{\text{vtx}}^{(\text{min})}$ (upper plots) and $Z_{\text{vtx}}^{(\text{max})}$ (lower plots). The nominal values ($Z_{\text{vtx}}^{(\text{min})} = -1600$ cm and $Z_{\text{trk}}^{(\text{max})} = 9000$ cm) are reported with the statistical error, while the uncorrelated error is reported for the varied cut. The horizontal dashed line indicates the value of the measurement at the nominal value of the cut (indicated by the vertical dashed line). The $Z_{\text{vtx}}^{(\text{max})}$ cut was not varied above its nominal value because there are no events at $Z_{\text{vtx}}^{(\text{max})} > 9000$ cm (Fig. 4.6).

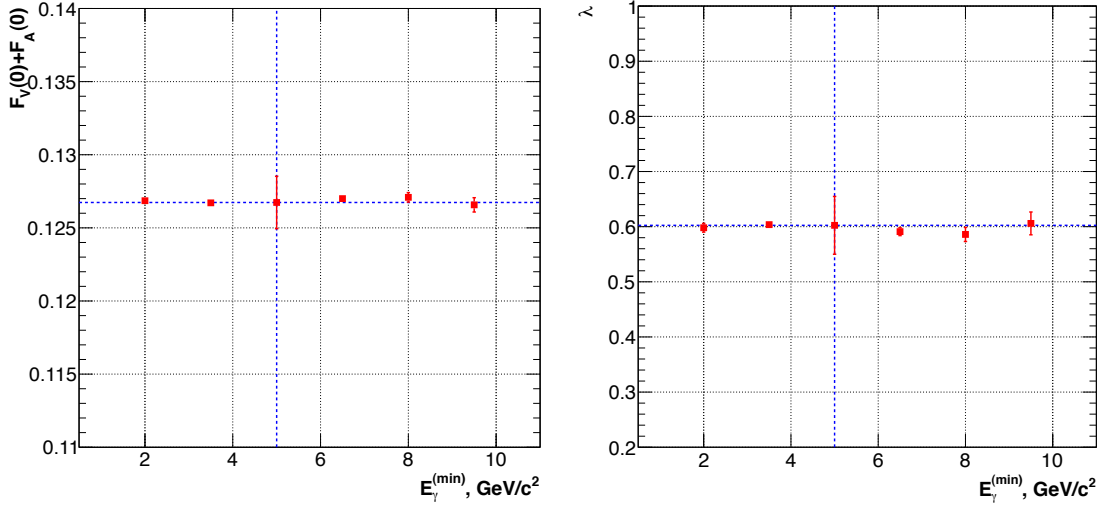


Figure 7.8: Form factor variation as a function of $E_\gamma^{(\min)}$. The nominal value ($E_\gamma^{(\min)} = 5 \text{ GeV}$) is reported with the statistical error, while the uncorrelated error is reported for the varied cut. The horizontal dashed line indicates the value of the measurement at the nominal value of the cut (indicated by the vertical dashed line).

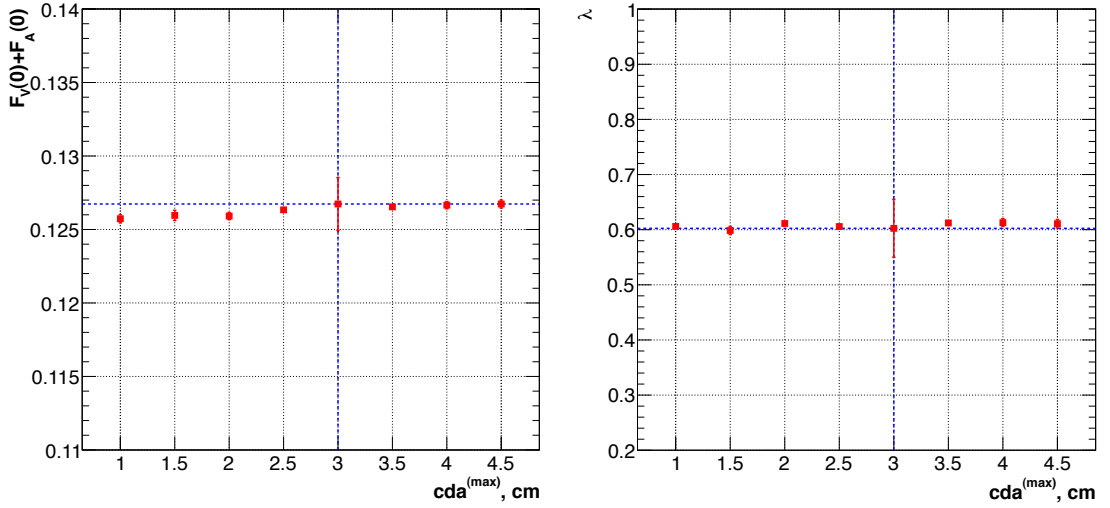


Figure 7.9: Form factor variation as a function of $cda^{(\max)}$. The nominal value ($cda^{(\max)} = 3 \text{ cm}$) is reported with the statistical error, while the uncorrelated error is reported for the varied cut. The horizontal dashed line indicates the value of the measurement at the nominal value of the cut (indicated by the vertical dashed line).

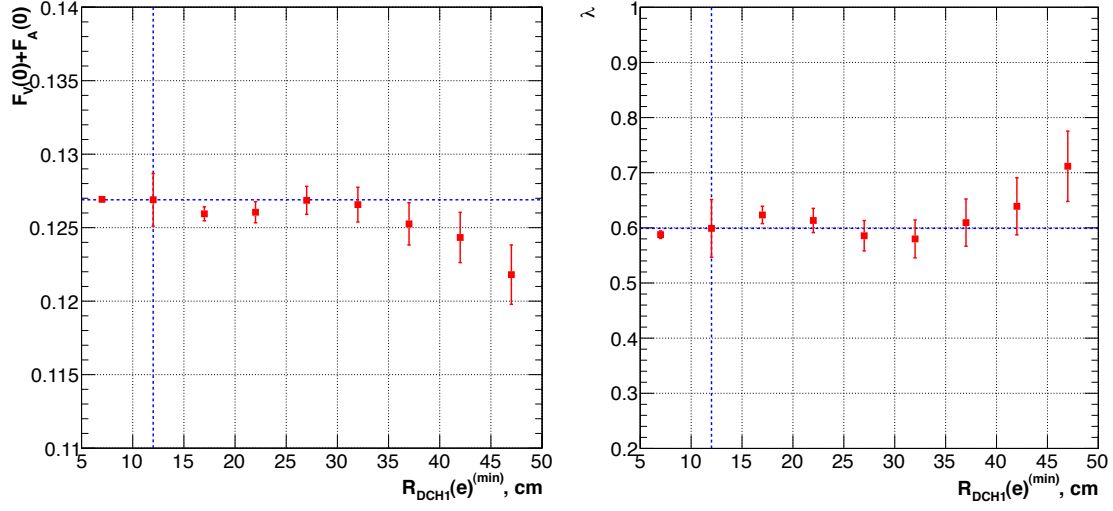


Figure 7.10: Form factor variation as a function of $R_{DCH1}(e)^{(min)}$. The nominal value ($R_{DCH1}(e)^{(min)} = 12$ cm) is reported with the statistical error, while the uncorrelated error is reported for the varied cut. The horizontal dashed line indicates the value of the measurement at the nominal value of the cut (indicated by the vertical dashed line).

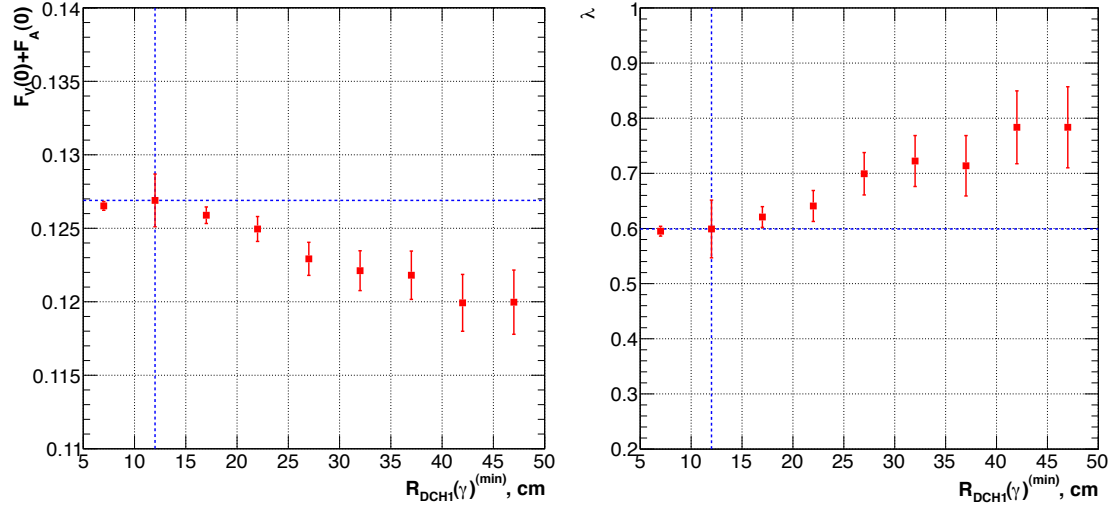


Figure 7.11: Form factor variation as a function of $R_{DCH1}(\gamma)^{(min)}$. The nominal value ($R_{DCH1}(\gamma)^{(min)} = 12$ cm) is reported with the statistical error, while the uncorrelated error is reported for the varied cut. The horizontal dashed line indicates the value of the measurement at the nominal value of the cut (indicated by the vertical dashed line).

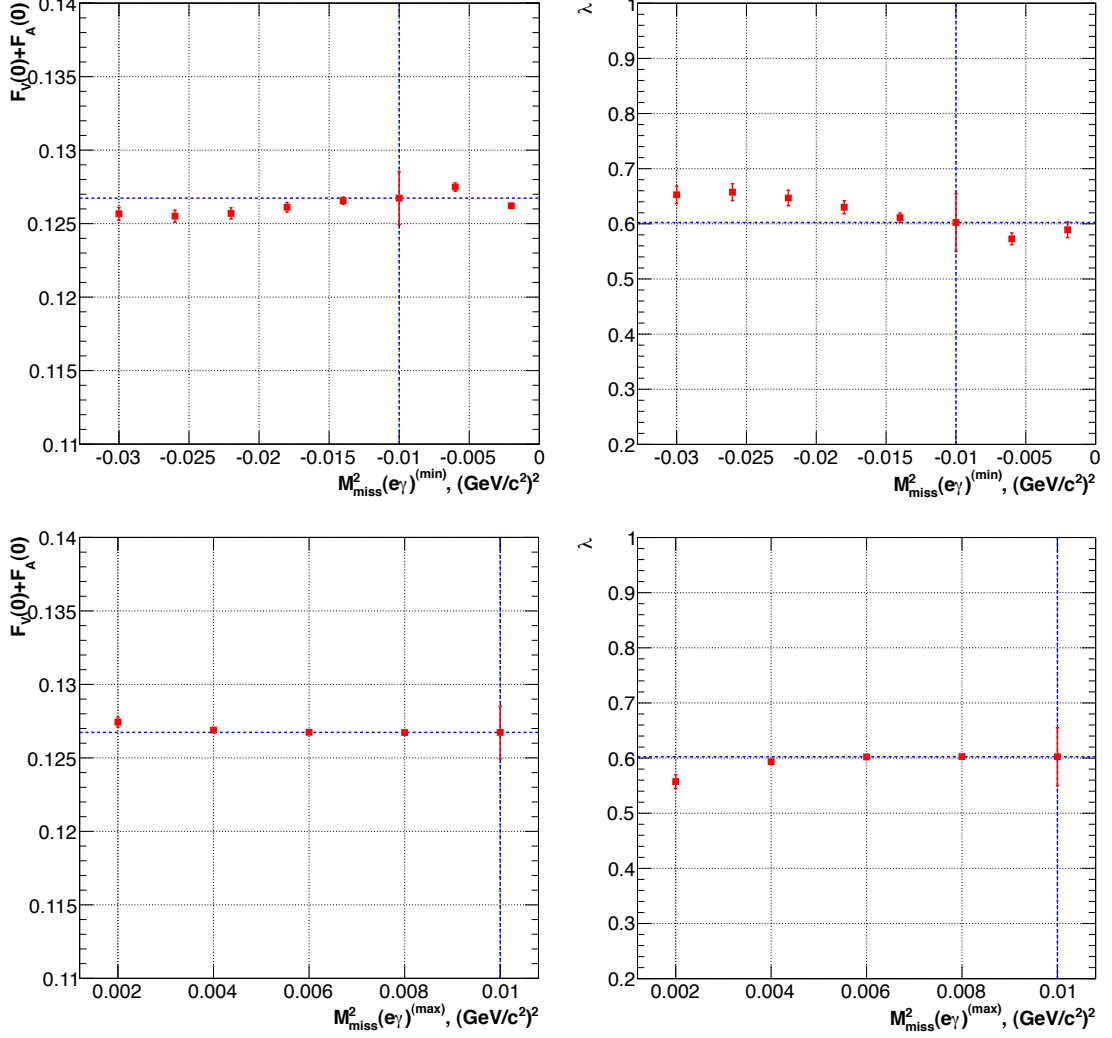


Figure 7.12: Form factor variation as a function of $M_{\text{miss}}^2(e\gamma)^{(\text{min})}$ (lower plots) and $M_{\text{miss}}^2(e\gamma)^{(\text{max})}$ (upper plots). The nominal values ($M_{\text{miss}}^2(e\gamma)^{(\text{min})} = -0.01$ $(\text{GeV}/c^2)^2$ and $M_{\text{miss}}^2(e\gamma)^{(\text{max})} = +0.01$ $(\text{GeV}/c^2)^2$) are reported with the statistical error, while the uncorrelated error is reported for the varied cut. The horizontal dashed line indicates the value of the measurement at the nominal value of the cut (indicated by the vertical dashed line). The $M_{\text{miss}}^2(e\gamma)^{(\text{max})}$ cut was not varied above its nominal value because there are no events at $M_{\text{miss}}^2(e\gamma) > 0.01$ $(\text{GeV}/c^2)^2$ (Fig. 6.19).

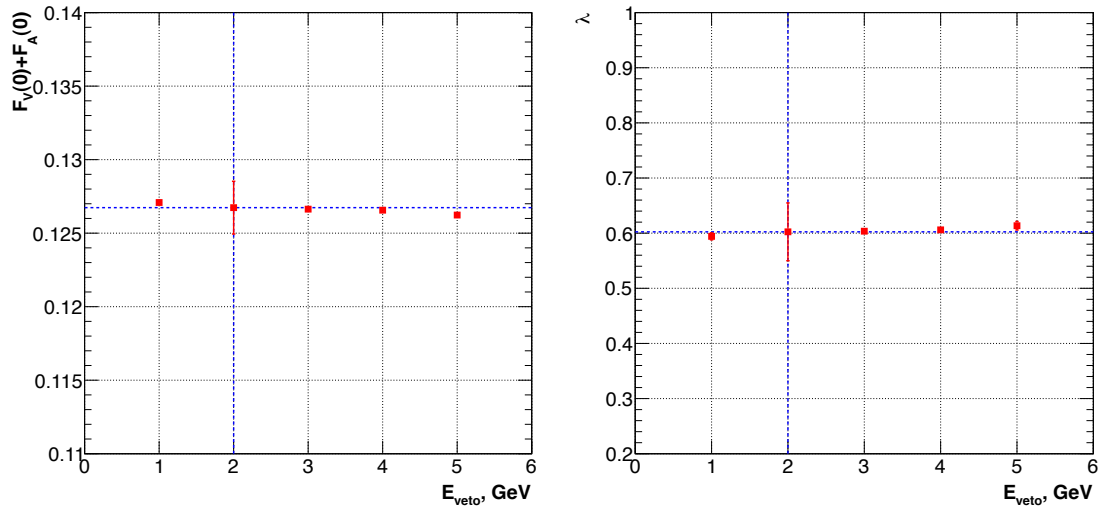


Figure 7.13: Form factor variation as a function of E_{veto} . The nominal value ($E_{\text{veto}} = 2$ GeV) is reported with the statistical error, while the uncorrelated error is reported for the varied cut. The horizontal dashed line indicates the value of the measurement at the nominal value of the cut (indicated by the vertical dashed line).

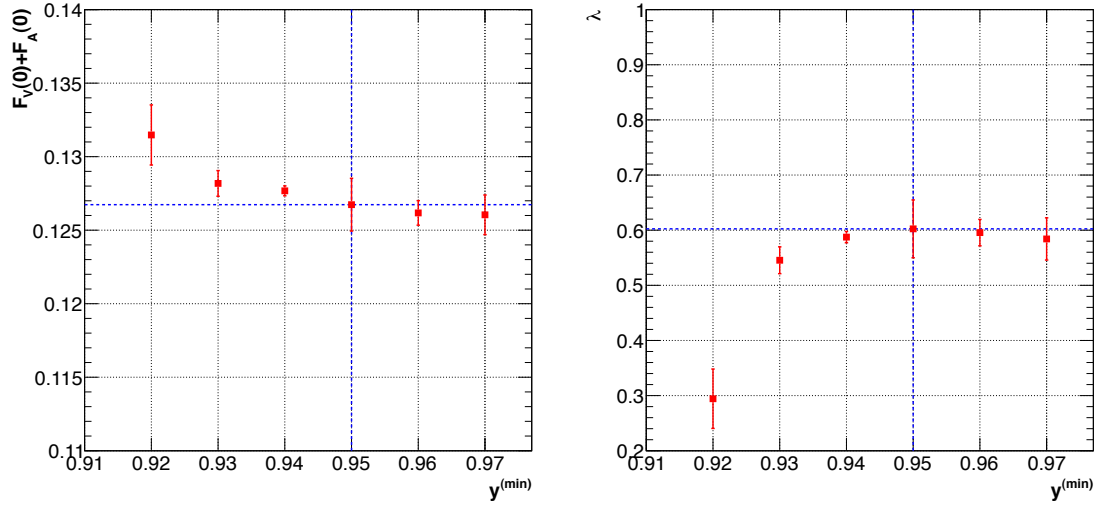


Figure 7.14: Form factor variation as a function of $y^{(\text{min})}$. The nominal value ($y^{(\text{min})} = 0.95$) is reported with the statistical error, while the uncorrelated error is reported for the varied cut. The horizontal dashed line indicates the value of the measurement at the nominal value of the cut (indicated by the vertical dashed line).

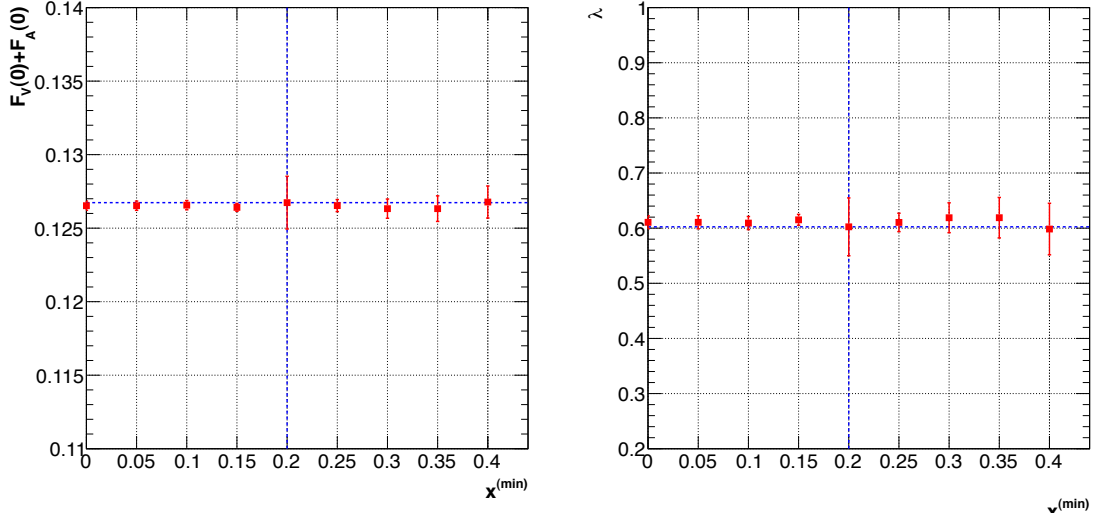


Figure 7.15: Form factor variation as a function of $x^{(\min)}$. The nominal value ($x^{(\min)} = 0.2$) is reported with the statistical error, while the uncorrelated error is reported for the varied cut. The horizontal dashed line indicates the value of the measurement at the nominal value of the cut (indicated by the vertical dashed line).

7.4 Systematics related to kaon variables from database

The experimental statistical precisions of the average kaon momentum, directions and positions (at $z=0$ cm plane) are respectively $\delta P_K \approx 50$ MeV/ c , $\delta(dx/dz)_K \approx \delta(dy/dz)_K \approx 1$ μ rad and $\delta x_K \approx \delta y_K \approx 0.01$ mm [103, 117], while their time-averaged values are $P_K \approx 74$ GeV/ c , $|x|_K, |y|_K \lesssim 1$ cm, $|dx/dz|_K \approx 200$ μ rad and $|dy/dz|_K \approx 80$ μ rad, with a variation over time of about 0.1 GeV/ c , 1 mm and 10 μ rad, respectively (Section 4.2.3). In order to assess the sensitivity of the form factor measurement with respect to the kaon variables, each variable was shifted in the MC samples by several times its corresponding statistical error. The variations of the measured form factors due to the shifts of the values in the database are reported in Tab. 7.5. The uncertainties on the form factors are negligible for shifts of the order of the statistical precision of the kaon variables.

Summary of the systematics related to kaon variables			
Variable	Shift	$\delta[F_V(0) + F_A(0)] \times 10^3$	$\delta\lambda \times 10^2$
Kaon momentum	1 GeV/ c	0.7	1.3
Kaon slope dx/dz	10 μ rad	0.1	0.4
Kaon slope dy/dz	10 μ rad	0.1	0.4
Kaon x position at $z=0$ cm plane	1 cm	0.2	0.1
Kaon y position at $z=0$ cm plane	1 cm	0.2	0.1

Table 7.5: Summary of the variations (absolute values) of the form factors with respect to the shift of the average values of the kaon variables.

7.5 Systematics related to trigger efficiency

The evaluation of the trigger efficiency for SD^+ and background events in the signal region is made difficult by the large downscaling factor of the control trigger (Chapter 5). In particular, the uncertainty of the efficiency becomes relevant for background events with lost photons, due to the high inefficiency (up to 10%) introduced by the 1TRK-LM trigger condition. An estimate of the systematic error on the form factors related to the trigger efficiency is then an important issue of the analysis.

The efficiency of each trigger component was measured using K_{e3} events and independent control samples and no appreciable dependence of the efficiency on the relevant variables of the analysis was found. In particular, the 1TRK-LM trigger efficiency was evaluated using K_{e3} and $K_{2\pi}$ decays, reconstructed respectively with and without the information of DCHs; both measurements are compatible to each other and show that, if all decay particles are detected (except for neutrino), the 1TRK-LM trigger is highly efficient ($\varepsilon_{1\text{TRK-LM}} \approx 99.8\%$).

To understand if the trigger efficiencies of signal and its background play some role in the measurement of the form factors, the expected SD^+ distribution (Eq. 3.1) has been corrected for the measured K_{e3} trigger efficiency ($\varepsilon(K_{e3}) = 99.47\%$) which is x -independent. The difference with respect to the result obtained assuming 100% trigger efficiency for the signal is quoted as systematic error.

In the same way, the MC distributions of K_{e3} and $K_{2\pi}$ backgrounds have been corrected for the trigger efficiency as a function of the x Dalitz variable (measured outside the signal region, Fig. 5.12) and the difference with respect to the result obtained assuming 100% trigger efficiency for the background is assessed as systematic uncertainty. The variations of the measured form factors due to the uncertainty on the trigger efficiency are reported in Tab. 7.6.

Summary of the systematic related to the trigger efficiency		
	$\delta[F_V(0) + F_A(0)] \times 10^3$	$\delta\lambda \times 10^2$
Signal trigger efficiency	0.3	< 0.1
Background trigger efficiency	0.2	0.2

Table 7.6: Summary of the systematic (absolute values) associated to $F_V(0) + F_A(0)$ and λ measurements and related to trigger efficiency.

7.6 Summary of the systematics and comment on the result

In Tab. 7.7 the contribution of all the evaluated systematic errors is reported. The 1% relative uncertainty on the kaon flux from the limited precision on $\text{BR}(K_{e3})$ translates into an uncertainties on the form factors and is included in the total error. The major systematic uncertainties, which limit the accuracy of the form factor measurement, are related to the stability with respect to cuts variation. Some unaccounted systematic effects and/or an inadequate precision in the simulation of the signal and background events can be the reasons for a wrong estimation of the background and could be the sources of the observed instability of the result. Even if some systematic estimation was conservative and possible correlations of systematics effect are ignored (*e.g.* the sum of the errors related to the stability with respect to the track momentum and the longitudinal decay vertex cuts is overestimated due to the correlation between the two variables), the form factor measurement obtained in the present analysis is however not solid enough and not competitive with the precision obtained by the KLOE experiment (Fig. 7.16).

In order to reach a better precision on the form factors and keep the systematic uncertainty at level of the statistical error, other issues not addressed in the present work could be studied in a future development of the analysis. Examples are the role of multiple scattering of the track by large angles on the tails of the track slope distributions (the main backgrounds contribute via the resolution tails) and the under/over estimation of the track momentum by reconstruction. Additional checks can be done to improve the data to MC agreement outside the kinematic limits $x, y > 1$, where the MC fails to describe data (Fig. 6.13). Finally, the estimation of the background due to the $K_{2\pi}$ and $K_{\mu 3}$ decays contributing via a photon from a π^0 decay merging with the cluster associated to the track, rather than by mis-identification. Understanding these effects might help to keep the background under control and reduce the systematic uncertainty.

Summary of the systematic uncertainties		
Cut	Associated systematic	
	$\delta[F_V(0) + F_A(0)] \times 10^3$	$\delta\lambda \times 10^2$
$P_{\text{trk}}^{(\text{min})}$	4.4	11.2
$P_{\text{trk}}^{(\text{max})}$	2.0	10.7
$E_\gamma^{(\text{min})}$	0.4	1.6
$Z_{\text{vtx}}^{(\text{min})}$	2.7	7.1
$Z_{\text{vtx}}^{(\text{max})}$	2.6	5.9
$R_{\text{DCH1}}(e)^{(\text{min})}$	2.5	2.4
$R_{\text{DCH1}}(\gamma)^{(\text{min})}$	5.1	12.3
E_{veto}	0.5	1.1
$\text{cda}^{(\text{max})}$	1.0	1.0
$M_{\text{miss}}^2(e\gamma)^{(\text{min})}$	1.2	5.5
$M_{\text{miss}}^2(e\gamma)^{(\text{max})}$	0.7	4.5
$x^{(\text{min})}$	0.3	1.3
$y^{(\text{min})}$	4.7	31.0
Kaon variables	< 0.1	< 0.1
Trigger efficiency	0.4	0.2
External input	0.5	< 0.1
Total	9.8	38.8

Table 7.7: Summary of the systematics (absolute values) associated to $F_V(0) + F_A(0)$ and λ measurements.

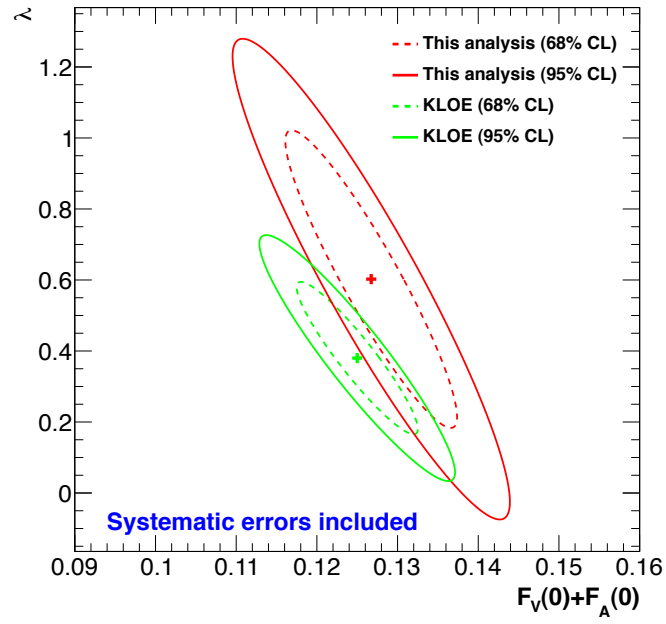


Figure 7.16: Confidence level contours for the form factors measurement obtained in the present analysis (statistical and systematic error) and by the KLOE experiment. The crosses refer to the central values.

Chapter 8

Conclusions

This thesis was devoted to the measurement of the form factors of the charged kaon decay $K^+ \rightarrow e^+ \nu \gamma$ ($K_{e2\gamma}$) with the statistics collected by the NA62 experiment in its first phase during the 2007 data taking. The $K_{e2\gamma}$ decay, in suitable kinematic configurations, is sensitive to the hadron structure and the strong dynamics at low energy. All the information about this structure dependent (SD) configuration is encoded in the form factors F_V and F_A , related respectively to the vector and axial part of the weak current. A measurement of the form factors is then a test for the effective theories of strong interaction at low energies as the ChPT and the LFQM.

The present work focused on the analysis of the SD^+ term with the photon emitted preferentially with positive helicity, which is the only kinematic configuration accessible to the analysis with a low background to signal ratio at level of 4%. The number of SD^+ candidates collected after the selection is $N(SD^+) = 11170$, about ten times the statistics obtained by the KLOE experiment. In principle, with the collected statistics, a precision on the measurement of the form factors improved by a factor ≈ 3 with respect to the result obtained by the KLOE experiment could be in reach. However, large systematic effects not adequately accounted for in the analysis, prevented the achievement of the proposed goal. The final result of the analysis on the measurement of the $K_{e2\gamma}$ form factors, assuming ChPT at $\mathcal{O}(p^6)$, is:

$$F_V(0) + F_A(0) = 0.127 \pm 0.002_{\text{stat}} \pm 0.010_{\text{syst}}$$
$$\lambda = 0.60 \pm 0.05_{\text{stat}} \pm 0.39_{\text{syst}}$$

which corresponds to a precision respectively of $\approx 8\%$ on $F_V(0) + F_A(0)$ and $\approx 70\%$ on λ . The LFQM prediction for the form factors was also tested but it was found to be completely inconsistent with data.

Future developments of the analysis include a deeper study on the effects of multiple scattering and of the limited precision of the simulation of the resolution on the background. Addressing these issues could help to reduce the systematics related to cuts variation and make the measurement of the form factors more solid.

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