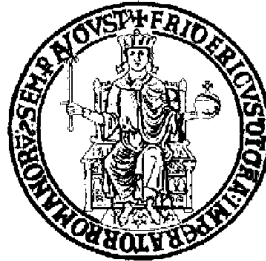


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Tesi di Laurea

*A new proposal for damping the resonances in CERN
accelerators, using HOM couplers*

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"The most exciting phrase to hear in science,
the one that heralds new discoveries, is not
"Eureka!" but "That's funny..."

Isaac Asimov

"L'espressione più eccitante da ascoltare nella scienza,
quella che annuncia le più grandi scoperte, non è
"Eureka" ma "Che strano..."

Isaac Asimov

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This acknowledgment is the result of number and number of changing and I don't know the reason, but I wrote the acknowledgment like the first thing, and now that is the last night useful for write my thesis, I'm here to rewrite it completely.

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CERN, Switzerland

Antonio Gilardi

Abstract

Parasitic resonances are a fundamental problem in the world of accelerators (there are also resonating cavities, but in this case the resonance is designed and is used to accelerate the beam).

These parasitic resonances must be reduced, and cut them down have multiple benefits.

The resonances studied in this thesis are the unwanted ones. Two enormous problems in accelerator physics are caused by these resonances: instability and overheating.

These problems will be explained in more details in the thesis.

Nowadays generally resonances in accelerator equipment are damped with ferrites. It can also be done by completely redesigning the equipment, but this is expensive and takes a long time and therefore damping with ferrites is preferred. However, when ferrites absorb the energy from the resonance, they become hot and start to outgas, which is detrimental to the vacuum. Also if the ferrite gets too hot and reaches the Curie temperature, it will no longer absorb the resonance and the beam will again be exposed to the resonance. In addition ferrites are fragile and easily break when attached to the equipment. In order to avoid the problems of the ferrites, we are now proposing a new method of damping the resonances. It uses a well-know technique from RF-cavities, where higher order modes (HOM) are removed with HOM-couplers. This technique have never been used before for other equipment than RF cavities, but we have now tested it with excellent results for the QUATTRO-TANK (QT) in SPS. The coupler for the QT is a coil, that couples to the magnetic fields in the resonance. The design is experimental, and in a later paper we will give a derivation of the theory for the coupling.

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Introduction

Particle accelerators are nowadays some of the most powerful instruments for scientists in different branches of science. One could think to the high energy physics that recently opened a new research path after the discovery at CERN of a particle compatible with the theorized Higgs boson, or the wide range of studies in synchrotron radiation facilities (biological, chemical, optical studies, etc.) or to the medical applications of the therapy. Particle accelerators are in continuous development in order to meet the always more challenging requirements in beam energy and quality.

Focusing on the high energy physics accelerators, in particular, the potential of new particle discoveries is strongly correlated with the intensity of the circulating particle beams and their size.

The CERN accelerator complex in Fig.1 is an example of the development of accelerators in order to push the frontiers of knowledge towards unexplored high energy boundaries. CERN, the European Organization for Nuclear Research, is currently the world's leading laboratory for particle physics. CERN mission is the fundamental research in physics pushing the frontiers of human knowledge. In support of that mission, CERN drives technology innovation, stimulates international collaboration and inspires a rising generation of scientists. CERN has its headquarters in Geneva.

In order to understand the challenges of accelerating particle beams in circular accelerators, we can virtually follow the process of acceleration of a particle beam from the source to the high energy of the CERN LHC, the Large Hadron Collider. Hydrogen atoms are taken from a bottle containing hydrogen from which protons are taken by stripping out the orbiting electrons.

The protons are then accelerated in the Linac 2 to the kinetic energy $T = 50\text{MeV}$. The extracted beam is injected in the PS (Proton Synchrotron) Booster, a piled-ring accelerator built in 1972 in order to provide high intensity beams up to 1.4GeV to the PS. The PS is one of the oldest machine at CERN, built in 1959, it was the first accelerator designed with strong focusing technique for acceleration. It accelerates particles up to 25GeV . Particles are then injected in the SPS (Super Proton Synchrotron) where a further acceleration brings them to 450GeV . The beam is then sent to the LHC through the transfer lines TI2 (clockwise) or TI8 (counter-clockwise). Particles circulate in opposite direction until the energy is increased and

CERN's Accelerator Complex

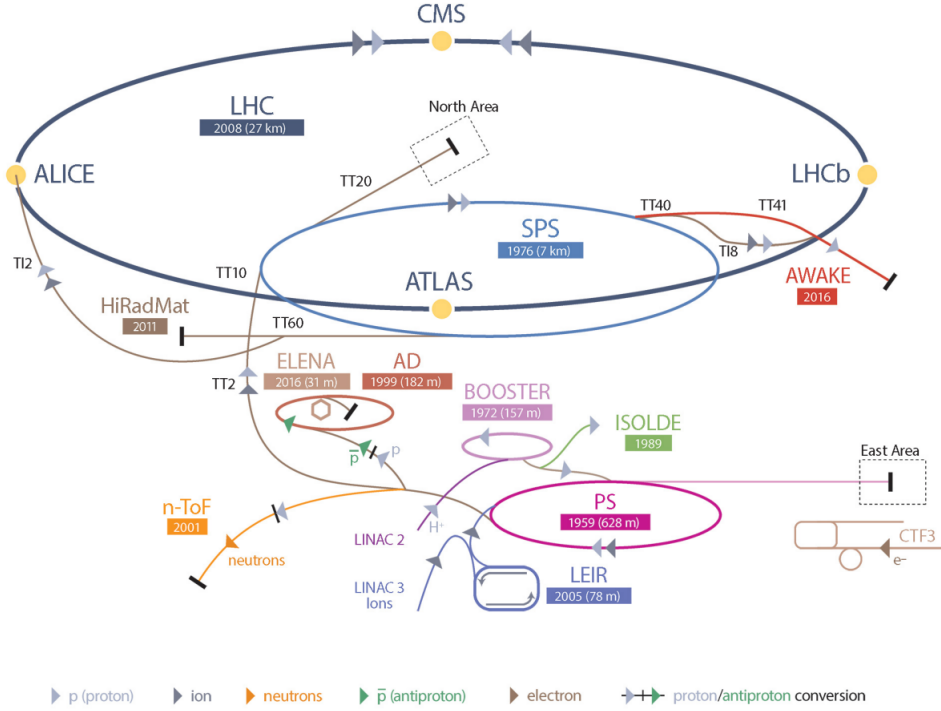


Figure 1 – Accelerators chain by Science and Technology Facilities Council.

the beams are brought into collision.

The amount and quality of beam collisions is crucial importance in order to provide sufficient data to the experiments. ATLAS and CMS, for example, during the first run of the LHC, were focused on the study of rare events associated with the decay of Higgs-like particles. Defined σ_p the cross-section of a particular event, the number of interactions per second $\frac{\delta R}{\delta t}$ is proportional to the luminosity $\mathcal{L}$, by the following formula:

$$\frac{\delta R}{\delta t} = \mathcal{L} \sigma_p \quad (1)$$

For n_b Gaussian bunches circulating at the revolution frequency f_{rev} with same horizontal, vertical and longitudinal beam sizes σ_x , σ_y and σ_b , and intensity per bunch N_b for the two colliding beams, the luminosity is given by:

$$\mathcal{L} = \frac{N_b^2 n_b f_{rev}}{4\pi \sigma_x \sigma_y} \quad (2)$$

As we can see from Eq.2, a high rates of events can be achieved either with small beam sizes or by increasing the number of circulating bunches or their intensity. The increase in beam intensity is often limited by beam instabilities due to the beam interaction with itself, the other beam, electrons or the accelerator devices. The machine beam coupling impedance,

for example, is a concept that allows to gather the electromagnetic field interactions with the beam itself and allows for useful stability, or instability, predictions. The knowledge of this parameter is therefore of great importance in order to correctly model and improve the machine performance.

This thesis has been subdivided into 6 Chapters and can be summarized as follows:

- in **Chapter 1** a set of basic informations on accelerator physics is provided, like the key concepts of wake functions and beam coupling impedance are explained;
- in **Chapter 2** the techniques used to measure the quality factor of a resonance are presented, focusing attention on two equipment, the Quattro-Tank (QT) and the Beam Gas Ionization monitor(BGI);
- in **Chapter 3** the techniques used to avoid or mitigate the resonance phenomenon are presented, highlighting the disadvantages and advantages of each individual method;
- in **Chapter 4** the new proposal to use HOM couplers to absorb a resonance is presented;
- **Chapter 5** where the measurement setups are produced (positioning of the HOM couplers, etc) and the characteristics that influence the measurement results are presented;
- in **Chapter 6** to conclude, I present and comment the measurement results obtained.

I hope that you will enjoy the reading.

CERN, Geneva, Switzerland 15 September 2017

A. G.

BACKGROUND **Part I**

1 Accelerator theory

This chapter defines general notions of single particle transverse and longitudinal motion in a circular accelerator and then focuses on the study of a specific type of collective effects called beam impedance deduced from the wake fields, in order to better understand simulations and results which will be presented in the following. This study of beam impedance is a relatively new subject in accelerator physics, which was introduced by A. Sessler and V. Vaccaro in 1967 [1]. Since there is a wide amount of literature on this topic, here the goal is not to give a complete perspective on it, rather to provide some basic notions, linked to the reference, to allow further and deeper analysis. Let us first consider a circular accelerator and define the coordinates that will be used.

1.1 Definition of the Coordinate System

In acceleration design, time is not used as independent variable; instead the longitudinal position s is used. The ideal trajectory is called design orbit. The design orbit can be a straight line (linac), a spiral (cyclotrons), or circular (really is succession of arcs and straight lines, synchrotron). The coordinates of a reference proton on an ideal circular design orbit of radius ρ is sketched in (Fig.1.1) [2]. The reference proton velocity is $v = \beta c$, where c is the speed of light and β is the relativistic factor.

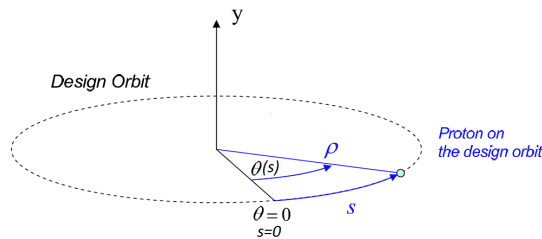


Figure 1.1 – Sketch of a proton on the ideal design orbit of a circular accelerator θ is the face advance.

The position of the proton is described by (s, x, y) , where $s = vt$ is the longitudinal coordinate, x is the horizontal and y is the vertical coordinate.

The coordinates of any other proton on a given trajectory are displayed on (Fig.1.2).

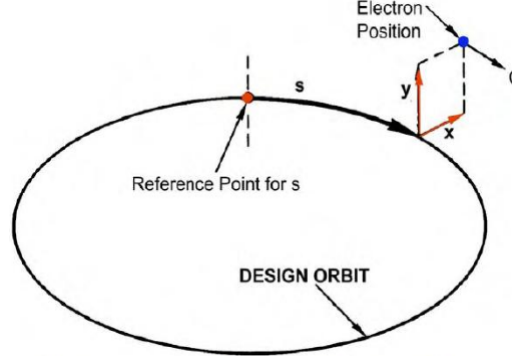


Figure 1.2 – Sketch of a proton on a trajectory different from the design orbit.

It is important to notice that the conventions classically used in accelerators impose that the horizontal position x is positive if the particle travels outward from the design orbit, the vertical position y is positive if the particle is above the design orbit, and the longitudinal position s is in the same direction as the velocity of the design particle. As a consequence, depending on the direction of rotation, the referential x, y, s may not be orthonormal (in the Fig.1.1 and Fig.1.2 we choose two different direction of the motion).

1.2 Single Particle Motion in a Synchrotron

The motion of charged particles forming a beam in an accelerator can be studied either individually or taking into account the electromagnetic interaction between them. In the former case, the beam is regarded as a collection of non-interacting particles and the forces acting on them, i.e. the driving terms in each particle's equations of motion, are fully prescribed by the accelerator design. The study of the single-particle dynamics is then complicated by all non-linear components of the applied electromagnetic fields. In practice, this description is sufficient as long as additional electromagnetic fields caused by the presence of the whole beam of particles are not strong enough to perturb significantly the motion imparted by the external fields.

For this reason the study of collective effects requires a prior understanding of the motion of single particles in a circular accelerator.

The motion of a single particle in a circular accelerator is driven by the Lorentz force $\vec{F}$.

$$\vec{F} = q(\vec{e} + \vec{v} \times \vec{b}) \quad (1.1)$$

where q is the charge of the particle ($q = e$ for the proton), $\vec{e}$ and $\vec{b}$ are the electric and magnetic fields at the location of the particle. The longitudinal and transverse motions of a

single particle are generally studied separately. In addition its preferred to treat the horizontal and vertical planes separately because the mathematics is easier. It is done by having a "linear" optics (basically bending magnets and quadrupoles magnets).

If we use higher order magnets (sextupoles or octupoles) the horizontal and vertical transverse motions are however coupled.

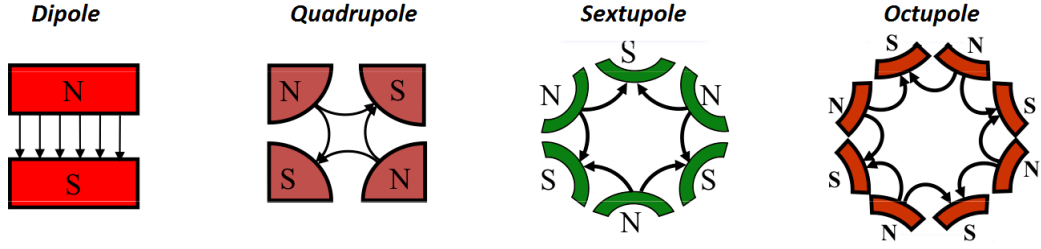


Figure 1.3 – Main magnets present in the LHC, with relative field. The dipole and quadrupole are linear magnets and lead to linear optics (horizontal and vertical plans are decoupled).

In the case of the CERN synchrotrons (PSB, PS, SPS, LEIR, AD and LHC), the protons have already been accelerated longitudinally before injection, and we can assume that their transverse velocity $v_{\perp}$ is much smaller than their longitudinal velocity v_s (in this reasoning we also use the symbol $v_{\parallel}$). As a consequence the total velocity is approximatively the longitudinal velocity ($v = \sqrt{v_x^2 + v_y^2 + v_s^2} \cong v_s$). We can assume that the longitudinal magnetic field $\vec{B}$, can be neglected, except in special cases (e.g. electron cooler and experimental magnets like CMS) more details about this experiment are given in the Appendix A.

In this case, the Lorentz Force $\vec{F}$ on a proton is decomposed in its longitudinal component F_s , and its transverse components F_x and F_y as

$$\begin{aligned} F_s &= eE_s F_x = e(E_x + v_s B_y) \\ F_y &= e(E_y + v_s B_x) \end{aligned} \quad (1.2)$$

If the beam is kicked (e.g. with a kicker magnet or quadrupole magnet) the beam start to oscillate sine-like motion. This is called betatron oscillation (Fig.1.4)

If we choose a point of observation along the ring, and start to observe the position of the beam in point the measured position follows a perfect sine wave (Fig.1.5).

This is caused because each turn gives a constant phase advance, this section of observation is also called Poincare' Section.

The sine-like motion a sine is given by the action of focusing and defocus by the quadrupole magnet. In reality the beam moves on straight lines between the magnets (see Fig.1.6). The focusing quadrupole bends the beam towards the design orbit while the defocussing quadrupole

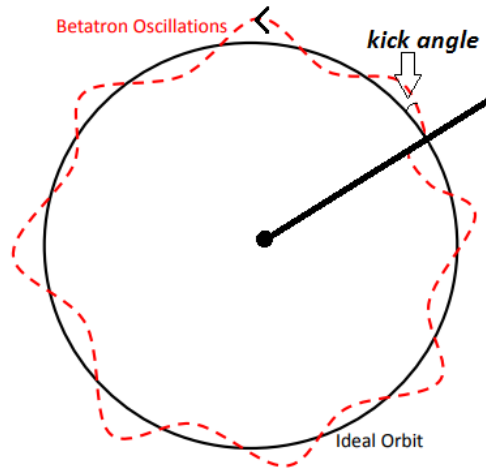


Figure 1.4 – Betatron oscillation around the design orbit. The beam moves anti-clockwise.

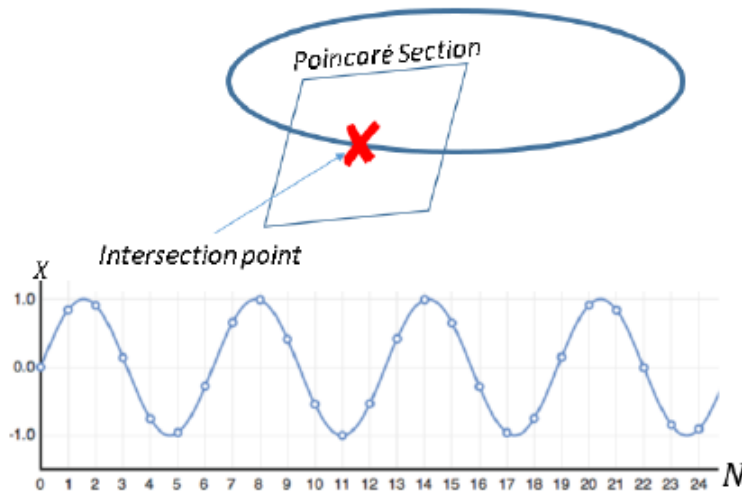


Figure 1.5 – Betatron oscillation in horizontal plane at the interaction point, x is the horizontal and N is the number of turns.

bend the beam away from the design orbit.

1.2.1 Phase space

In mathematics and physics, a phase space of a dynamical system is a space in which all possible states of a system are represented, with each possible state corresponding to one unique point in the phase space (an example of phase space is shown in Fig.1.7).

This topic is really important for a lot of area like: chemistry, astronomy, accelerators etc.

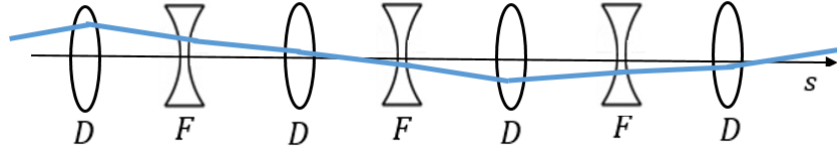


Figure 1.6 – Example of focusing and defocussing. The light blue line represents a beam trajectory.

For mechanical systems, the phase space usually consists of all possible values of position and momentum variables. Note the position and momentum are canonical variables, i.e. used in Hamiltonian systems, where by definition the energy is constant.

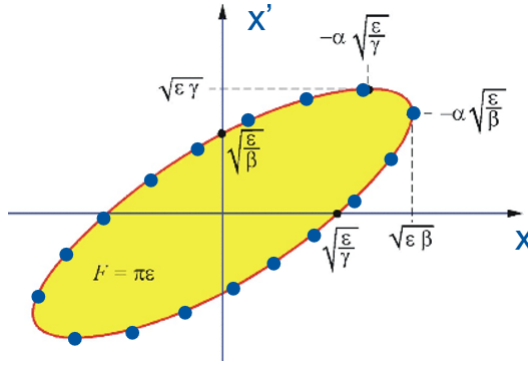


Figure 1.7 – Phase space in a linear accelerator. On the x-axis is the position of the particle and on the y-axis is the momentum. The yellow area is the emittance,

In linear accelerators the phase space is an ellipse. Defining $x'(s) = \frac{dx}{ds}$ it can be demonstrated that

$$\gamma(s)x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s) = \epsilon \quad (1.3)$$

where:

$$\begin{aligned} \alpha(s) &= -\frac{1}{2} \frac{d\beta(s)}{ds} \\ \gamma(s) &= \frac{1 + \alpha^2(s)}{\beta(s)} \end{aligned} \quad (1.4)$$

with ϵ is the area of the ellipse and $\beta(s)$ is the betatron function.

(α, β, γ) are the **Twiss parameters**. At a given position and emittance ϵ in the ring, they provide a complete description of the motion of the particles.

Emittance ϵ is a parameter of a charged particle beam and generally we want it to be as small as possible. A low-emittance particle beam is a beam where the particles are confined to a small space and have nearly the same momentum.

Liouville theorem explain why the phase advance is constant. The particle density in phase space is a constant if the particle move only under the influence of the forces for an external magnetic field. Phase space area is like an incompressible fluid, the implications are:

- The emittance is conserved when the beam is transported via a magnetic system. The phase space ellipse is distorted/stretched but its area is conserved (see Fig.1.8).

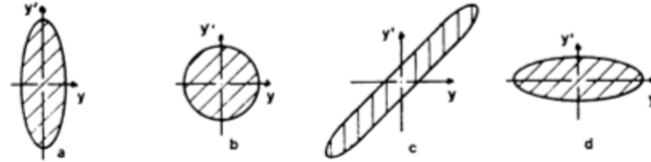


Figure 1.8 – The phase space changes shape but conserve the area.

- The emittance is not conserved if we accelerate.

In accelerators with electrons (e.g. LEP) the phase space is not constant because of synchrotron radiation (reduces phase space).

1.3 Wakefields and Impedance

When a bunch of charged particles passes through a chamber it leaves some electromagnetic fields behind it. This is called the wake field.

The term "Beam Impedance" is somewhat misleading. IS really important the kick given to the beam (via electromagnetic force) from preceding particles. Mathematically the wake functions describe the interaction of the beam with the surrounding environment.

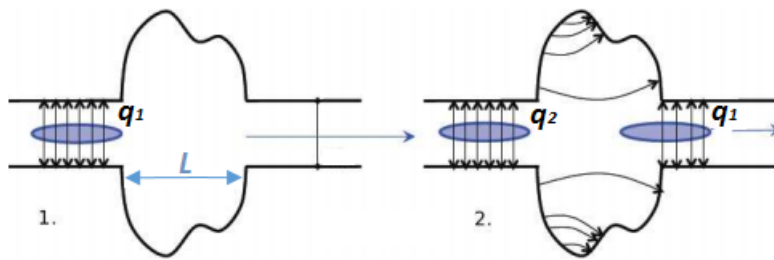


Figure 1.9 – Left side 1. the bunch q_1 passes through a discontinuity along a pipe. Right side 2. the created EM field stays in the cavity and interacts with the next bunch q_2 .

The electromagnetic (EM) problem is posed setting the Maxwell's equations with the beam as source term and boundary conditions given by the structure in which the beam propagates.

A wake function is the wake field of a single particle, and it is in time domain. The beam impedance is the Fourier Transformation of the wake function, and it is in frequency domain. As in the illustration of the wake field is generated by a particle q_1 going through a device of length L , leaving behind an oscillating field. A test particle q_2 behind q_1 , at distance z , will feel this wake field and get an impulse kick Fig.1.9 [3].

The integral of a force on the test particle q_2 over the device length, defines the wake function and its Fourier Transform is called the beam impedance of the device of length L .

As an introductory example we consider a bunch circulating in a storage ring and going through a passive cavity. When the bunch goes through the cavity it induces electromagnetic fields, Fig.1.10. These fields oscillate and decay slowly. In the next turn the bunch will find some field left, which will give a kick to the bunch, this can lead to an exponentially growing instability.

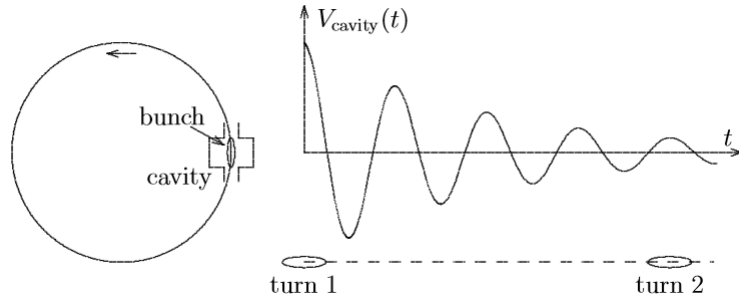


Figure 1.10 – Induced field acting on the bunch the next turn.

The single or multi-bunch behavior depends on length of the wake fields. Single bunch effects comes from short range wake field (e.g. resistive wall impedance).

The multi-bunch effects are due to the long range effects, for example a resonance with high Q value (e.g. narrow band resonator). The parameter that define if a resonator is narrow band broad band is the Q factor. We are going to explain more in details in section 1.5.1.

1.3.1 Longitudinal plane

The longitudinal wake function of an accelerator component is basically its Green function in the time domain [4] (i.e., its response to a pulse excitation) and is defined as follow:

$$W_{\parallel}(x_1, y_1, x_2, y_2, z) \left[\frac{V}{C} \right] = -\frac{1}{q_1 q_2} \int_0^L F_{\parallel}(x_1, y_1, s, x_2, y_2, z) ds \quad (1.5)$$

Is important to define the reference system between two bunch. According to Fig.1.11, the reference system is given with the two axis (s , and z) in opposition, so z is positive in the left side and s is positive in the right side.

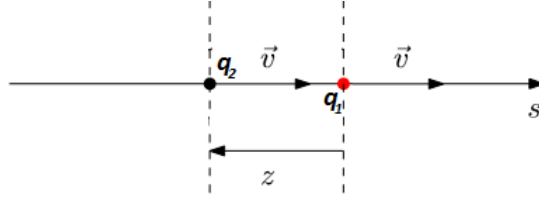


Figure 1.11 – The particle beam moves to the right. The source particle is in red, the test particle in black. It has to be recalled that z is positive from source to test particle.

It is worth noting that, z is the distance between the source and test particle and (x_1, y_1) define the transverse position of the source and (x_2, y_2) of the test particle with respect to the design orbit.

where $F_{\parallel}(x_1, y_1, s, x_2, y_2, z) = q_2 e_s(x_1, y_1, s, x_2, y_2, z)$ is the longitudinal component of Lorentz's force, $e_s(x_1, y_1, s, x_2, y_2, z)$ is the longitudinal component of the electric field induced by the source charge $q_1(x_1, y_1)$.

$W_{\parallel}(z)$ is discontinuous in $z = 0$, the field is infinite (test and source particle are on top of each other) and it vanishes for all $z < 0$ in case of ultra-relativistic drive particle (because the source particle cannot go faster of the speed of light).

The Fourier Transformation of the Wake function is the beam impedance.

The definition of wake function is very useful for macro particle models and simulations, because it can be used to describe the driving terms in the single particle equations of motion. Any 2D function can be decomposed into orthogonal higher functions (holomorphic decomposition) [5].

$$f(x, y) = a_0 + a_1 x + b_1 y + a_2 \left(\frac{x^2}{2} - \frac{y^2}{2} \right) + b_2 (xy) + a_3 \left(\frac{x^3}{3} - xy^2 \right) + b_3 \left(x^2 y - \frac{y^3}{3} \right) \dots etc. \quad (1.6)$$

The terms a_n are called "Normal" the terms b_n are called "Skew". The orthogonal functions are: $x, y, \left(\frac{x^2}{2} - \frac{y^2}{2} \right), xy, \dots$ etc.

An example of decomposition function is show in Fig. 1.12.

When we apply this decomposition on the longitudinal wake functions we obtain [6]:

$$W_{\parallel}(x_1, y_1, x_2, y_2, z) = C_{00} + (x_1 C_{01} + x_2 C_{10}) - (y_1 D_{01} + y_2 D_{10}) + (x_1^2 - y_1^2) C_{02} - 2x_1 y_1 D_{02} + x_1 (x_2 C_{11} - y_2 D_{11}) - y_1 (x_2 D_{11} + y_2 C_{11}) + (x_2^2 - y_2^2) C_{20} - 2x_2 y_2 D_{20} \quad (1.7)$$

This is the more generic expression of the wake functions using this series expansion.

1.3.2 Panofsky-Wenzel Theorem

The Panofsky-Wenzel (PW) Theorem describes the relationship between the longitudinal and the transverse wake fields produced by a beam as it travels through a device.

A particle, as it travels through a device and encounters a change in aperture, generates a wave excitation that can produce an integrated longitudinal and transverse momentum kick. The

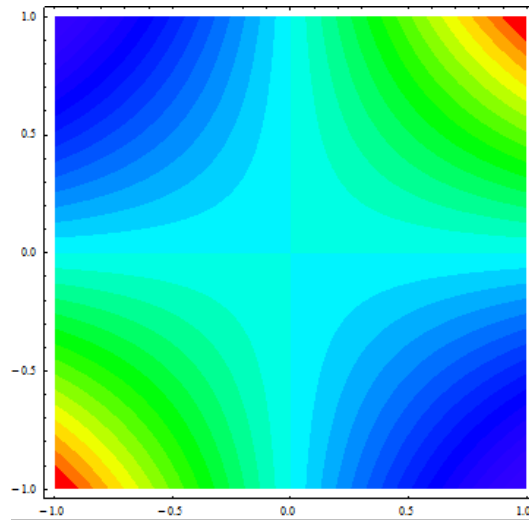


Figure 1.12 – An example of the decomposition function, skew quadrupole component (xy).

kick felt by the test particle depends on many factors:

- distance between driving and test particle;
- geometry of the chamber;
- longitudinal and transverse position of the charges respectively to the chamber;
- charge of the driving particle;
- speed of the driving particle.

We use the rigid bunch approximation for both drive and test particle.

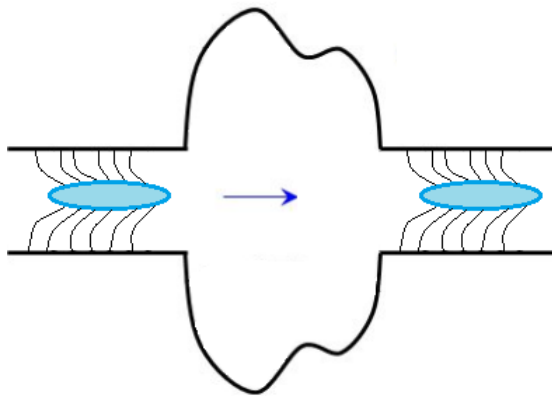


Figure 1.13 – Identical Wakefield before and after the cavity because of the rigid bunch approximation.

The rigid bunch approximation (see Fig. 1.14) consists of the following assumptions:

- the beam shape is rigid;
- the beam always moves with constant speed $\bar{v} = \beta c \bar{s}$,

where $\bar{s}$ is the unit vector in the direction of the propagation.

For a given structure, speed and charge of the driving particle, the kick that the test charge feels depends only on its position behind the driving particle. The fields are not static so they depend also on time.

However when the drive and test particle have passed the structure there is no any memory of the structure. This means, that the integral forces inside the structure must obey the same differential relations as the wake fields before the structure, the equations are:

$$\frac{\delta z}{\delta s} = -1 \qquad \frac{\delta z}{\delta t} = v \qquad (1.8)$$

This two equations plus the Maxwell equations give the Panofsky-Wenzel Theorem:

$$\nabla_{\perp} F_z(x, y, s, t) = - \frac{\delta \vec{F}_{\perp}(x, y, s, t)}{\delta z} \qquad (1.9)$$

with:

$$\nabla_{\perp} = \left(\frac{\delta}{\delta x}, \frac{\delta}{\delta y} \right) \qquad (1.10)$$

$$F_{\perp} = (F_x, F_y) \qquad (1.11)$$

Looking the wake function we have:

$$\nabla_{\perp} W_{\parallel}(x, y, s, \tau) = - \frac{\delta \vec{W}_{\perp}(x, y, s, \tau)}{\delta z} \qquad (1.12)$$

The formal derivation of this theorem is explained in detail in Appendix B. So we see, that the transverse wake function can be obtained differentiating the longitudinal wake function. Note, that (x, y) are the coordinates of the test particle.

1.3.3 Transverse plane

In an axisymmetric structure, or simply one with only a top-bottom or left-right symmetry, a source particle traveling on axis cannot induce net transverse forces on a witness particle also following on axis. We need to introduce a breaking of the symmetry to drive transverse effect, i.e. offset the source or the test particle. The transverse wake functions are defined as:

$$W_{x,y}(x_1, y_1, x_2, y_2, z)[V/C] = - \frac{1}{q_1 q_2} \int_0^L F_{x,y}(x_1, y_1, s, x_2, y_2, z) ds \qquad (1.13)$$

where $F_{x,y}(s, z) = [q(\vec{e}(x_1, y_1, s, x_2, y_2, z) + \vec{v} \times \vec{b}(x_1, y_1, s, x_2, y_2, z))]_{x,y}$ are the transverse components of Lorentz's force.

Similar to longitudinal plane we can expand the transverse wake function into a power series. Using the Panofsky-Wenzel Theorem we can also differentiate the longitudinal wake function, we obtain:

$$\begin{aligned} W_x(z) &= c_{10} + x_1 c_{11} + 2x_2 c_{20} - y_1 d_{11} - 2y_1 d_{20} \\ W_y(z) &= -d_{10} + x_1 d_{11} + 2x_2 d_{20} - y_1 c_{11} - 2y_1 c_{20} \end{aligned} \quad (1.14)$$

Note: the coefficient are not equal into the longitudinal and transverse case because to pass from one to the other there is a derivate to do 1.12.

For structures with symmetry both the horizontal (yz) and the vertical (xz) planes, the equations 1.14 looks like the same, but without the constant term.

$$\begin{aligned} W_x(z) &= W_x^{drive} x_1 + W_x^{test} x_2 \\ W_y(z) &= W_y^{drive} y_1 + W_y^{test} y_2 \end{aligned} \quad (1.15)$$

where $W_{x,y}^{drive}$ is the horizontal/vertical driving (also called dipolar) wake function and $W_{x,y}^{test}$ is the horizontal/vertical detuning (also called horizontal/vertical quadrupolar) wake function. For small offsets of both source and test particle, (1.15) is a very good approximation of the transverse wakes.

From the (1.15) we find the following relationship to link the driving and detuning contributions:

$$\begin{aligned} W_x^{drive}(z) \left[\frac{V}{mC} \right] &= \frac{W_x(z)}{x_1} \Big|_{x_2=0} \\ W_x^{test}(z) \left[\frac{V}{mC} \right] &= \frac{W_x(z)}{x_2} \Big|_{x_2=0} \\ W_y^{drive}(z) \left[\frac{V}{mC} \right] &= \frac{W_y(z)}{y_1} \Big|_{y_2=0} \\ W_y^{test}(z) \left[\frac{V}{mC} \right] &= \frac{W_y(z)}{y_2} \Big|_{y_2=0} \end{aligned} \quad (1.16)$$

Moreover, from Maxwell's equations and the definitions of wake functions can derive the general relation [6] (indeed, one can already see the relation from eq 1.14):

$$\frac{\partial W_x(z)}{\partial x_1} = -\frac{\partial W_y(z)}{\partial y_1} \quad (1.17)$$

from which it follows that:

$$W_x^{test}(z) = -W_y^{test}(z) \quad (1.18)$$

1.3.4 Beam Impedance

The beam impedance, which is generated by interaction between the beam and the surroundings environment, can lead to beam instability which can limit the intensity of an accelerator (e.g. Head-Tail instability is driven by the transverse beam impedance). The components seen by the beam are designed for a small impedance with the help of computer codes (CST, HFSS, etc) and of laboratory measurements on models.

The impedance is a function of the geometry and the electric properties of the beam surroundings and it is a frequency domain concept. It most often refers to beam moving an ultra-relativistic speed.

The definition of longitudinal beam impedance of the element under study is the Fourier Transform of the wake function:

$$Z_{\parallel}[\Omega] = \int_{-\infty}^{\infty} W_{\parallel}(z) e^{-\frac{j\omega z}{v}} \frac{dz}{v} \quad (1.19)$$

Here j is the imaginary unit and $\omega = 2\pi f$ is the angular frequency in $\frac{rad}{s}$.

Notice that $\frac{z}{v}$ is equivalent to τ (i.e. time), and we now see the normal structure of the Fourier Transform, .

Similarly to the longitudinal case, the transverse beam impedance of the element under study is defined as the Fourier Transform of the respective wake function:

$$\begin{aligned} Z_{\perp}^{drive}(\omega)[\Omega/m] &= j \int_{-\infty}^{\infty} W_{\perp}^{drive}(z) e^{-\frac{j\omega z}{v}} \frac{dz}{v} \\ Z_{\perp}^{test}(\omega)[\Omega/m] &= j \int_{-\infty}^{\infty} W_{\perp}^{test}(z) e^{-\frac{j\omega z}{v}} \frac{dz}{v} \end{aligned} \quad (1.20)$$

Analogously to its equivalent in the time domain the impedance in (1.20) can be expanded into a power series in the offsets of source and test particle:

$$\begin{aligned} Z_x &= Z_x^{drive} x_1 + Z_x^{test} x_2 \\ Z_y &= Z_y^{drive} y_1 + Z_y^{test} y_2 \end{aligned} \quad (1.21)$$

Eqs.1.21 are the horizontal and vertical component of the beam impedance and their dimension is $[\frac{\Omega}{m}]$.

It is important to underline that the longitudinal and transverse wakes, generated by the drive particle i.e. a response to a pulse excitation seen as a single particle: they are identified as wake functions.

If the source is a bunch of particles the resulting wake will be given by the convolution of the wake function by the charge density of the bunch $\lambda(z)$ and is called wake potential. Applying the convolution theorem, the beam impedance as defined in Eq.1.5 can be calculated as the

Fourier Transform of the wake potential divided by the Fourier Transform of the line density.

$$Z_{\parallel}(\omega) = \frac{V_{\parallel}(\omega)}{I_{\parallel}(\omega)} \quad (1.22)$$

Notice that $Z_{\parallel}$ is a frequency domain definition. There are different kinds of beam impedance, the most significant are:

1. Beam coupling impedance. This is generated by the induced image currents moving in the wall.
2. Direct space charge. Coulomb interaction between different particles inside the bunch.
3. Indirect space charge. Coulomb interaction between the particles in the bunch and the image beam charge.

1.4 Resistive Wall

A beam creates an EM field determined by the interaction with the vacuum chamber. From Maxwell's equations, there is a surface current on the beam pipe. Due to the finite impedance of the metal, the surface current is delayed with respect to the beam current (if the vacuum chamber would have been perfectly conductive then the surface current would have been no delay and the vacuum chamber would then act as perfect Faraday cage), see Fig.1.14.

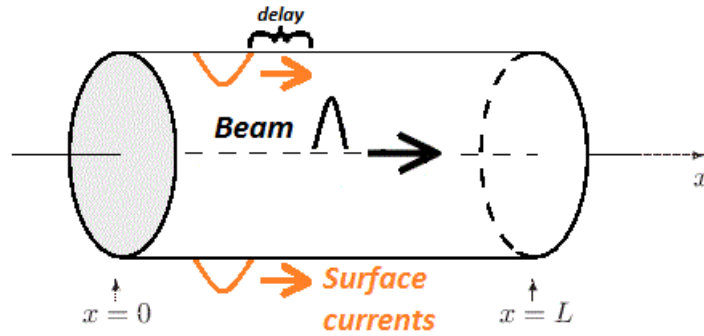


Figure 1.14 – Cross section of arbitrary device along an accelerator, in black the beam in orange the induced current.

The resistive wall impedance is a major concern for the LHC and has been widely studied over the years. Different formalisms have been introduced to treat it in different cases.

The classical theoretical formula for resistive wall impedance (see, e.g. [7]) are derived in a model of an infinitely long pipe. The longitudinal impedance of one turn of the accelerator, is

given by the following equation:

$$Z_{\parallel}(\omega) = [1 - \text{sign}(\omega)] \frac{L}{2\pi b} \sqrt{\frac{|\omega| Z_0}{2c\sigma_c}} \quad (1.23)$$

where ω is the frequency, b is the pipe radius, c is the speed of light, Z_0 is the impedance of free space, σ_c is the conductivity of the conductive layer and $L = 2\pi R$, is the full length of the accelerator ($L = 2\pi R$ for circular accelerator with R radius of the accelerator).

The $Z_{\parallel}(\omega)$ can be also called mode zero impedance $Z_{\parallel 0}(\omega)$.

Eq.1.23 is not valid for very high frequencies, because the electrons cannot move fast enough. In practice, one often has to deal with resistive inserts with a conductivity different from the rest of the pipe. When the conductivity of the pipe is an arbitrary function of the longitudinal coordinates the full derivation shown in [8].

An important relation is the Cauchy theorem, that link the real and the imaginary parts of the impedance by the Hilbert transforms [7].

$$\text{Re}Z_m^{\parallel}(\omega) = \frac{1}{\pi} P.V. \int_{-\infty}^{\infty} \frac{\text{Im}Z_m^{\parallel}(\omega')}{\omega' - \omega} d\omega' \quad (1.24)$$

$$\text{Im}Z_m^{\parallel}(\omega) = -\frac{1}{\pi} P.V. \int_{-\infty}^{\infty} \frac{\text{Re}Z_m^{\parallel}(\omega')}{\omega' - \omega} d\omega'$$

1.5 Resonance

Resonance occurs when a system is able to store and easily transfer energy between two or more different storage modes (such as kinetic energy or potential energy).

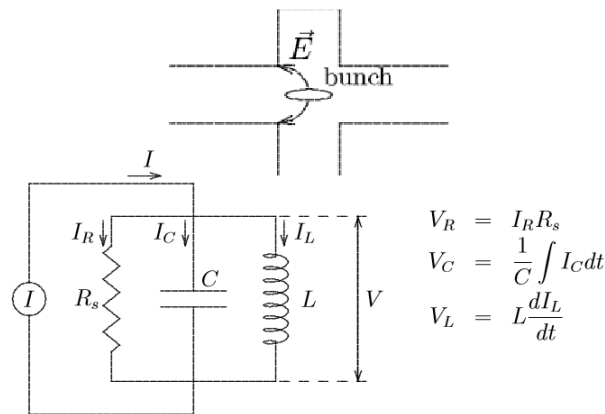


Figure 1.15 – Bunch pass through a cavity and after we modeling that cavity with RLF circuit. Cavity resembling an RLC-circuit.

Assuming the presence of one resonance inside a cavity. If the Fourier Transformation of the bunch shape contain the frequency of the resonance of the cavity then energy will be transferred to the field inside the cavity.

When a bunch pass a cavity, it creates oscillation fields inside the cavity.

A resonator naturally oscillates at a frequency, called its resonant frequency or eigen frequency. A cavity resonator is one in which waves exist in a hollow space inside the device. Cavity-like devices are objects which can cause coupled-bunch mode instabilities, because the induced fields oscillate for a relatively long time and have not degraded during the time interval between bunch passages.

Resonance also create heating of the component. The energy of resonance is dissipated in the cavity wall.

Such a cavity can be modeled by an RLC-circuit as shown in Fig.1.15.

The RLC-circuit has a shunt impedance R_s , an inductance L and a capacity C , see also Fig.1.15. In a real cavity these three parameters cannot easily be separated. For this reason we use some other related parameters which can be measured directly: the resonance frequency ω_r , the quality factor Q and the damping rate α :

$$\omega_r = \frac{1}{\sqrt{LC}} \quad (1.25)$$

$$Q = R_s \sqrt{\frac{C}{L}} \quad (1.26)$$

$$\alpha = \frac{\omega_r}{2Q} \quad (1.27)$$

The last parameter is not really interesting because can be evaluated from the other two. The way to evaluate this parameter will be show in the following chapter.

Please note, is possible model the cavity both with parallel RLC circuit or series RLC circuit. The main difference is the way to evaluate the Q-factor.

For the series circuit the expression is:

$$Q = R_s \sqrt{\frac{L}{C}} \quad (1.28)$$

In this thesis we are going to use the parallel RLC circuit like model of the resonance.

$\frac{R}{Q}$ is sometimes used as figure of merit. It is independent of the cavity material and it is only determined by the pure cavity geometry.

Once the calculated value of $\frac{R}{Q}$ is known, from simulations we can measure the Q-value of the cavity (will be also shown in chapter 2) and from this parameter we can evaluate the R_s .

A general idea, of how much the shape can influence the $\frac{R}{Q}$ and R_s is shown in Fig.1.16 [9].

The FM of an accelerating cavity should have a high $\frac{R}{Q}$ value, whereas higher modes with high

			
R/Q	higher	middle	lower
R _s	highest	middle	lowest

Figure 1.16 – Comparison of cavity geometries.

$\frac{R}{Q}$ values are dangerous to the beam stability. $\frac{R}{Q}$ itself can be calculated or measured directly by the perturbation method.

1.5.1 Quality factor

The quality factor (Q), of a resonant circuit is a measure of the “goodness” or quality of the resonant circuit. A higher value for this figure of merit corresponds to a more narrow bandwidth, which is desirable in some applications like the RF cavity and is necessary to avoid in other case like the parasitic resonance in the equipment.

In the context of resonators, there are two common definitions for Q , which aren’t necessarily equivalent. They become approximately equivalent as Q becomes larger, meaning the resonator becomes less damped. One of these definitions is the frequency-to-bandwidth ratio of the resonator:

$$Q = \frac{f_r}{\Delta f} \quad (1.29)$$

The other common equivalent definition for Q is the ratio of the energy stored in the oscillating resonator to the energy dissipated per cycle by damping processes:

$$Q(\omega) = 2\pi \frac{P_{stored}(\omega)}{P_{dissipated}(\omega)} \quad (1.30)$$

The $P_{dissipated}$ is the power loss in one cycle. In electrical systems, the stored energy is the sum of energies stored in lossless inductors and capacitors; the lost energy is the sum of the energies dissipated in resistors per cycle.

This formula is applicable to both series and parallel resonant circuits.

The Q factor determines the qualitative behavior of simple damped oscillators.

- A system with $Q < \frac{1}{2}$ is said to be over damped. Such a system does not oscillate at all, but when displaced from its equilibrium steady-state output it returns to it by exponential

decay, approaching the steady state value asymptotically. It has an impulse response that is the sum of two decaying exponential functions with different rates of decay. As the quality factor decreases the slower decay mode becomes stronger relative to the faster mode and dominates the system's response resulting in a slower system.

- A system with $Q > \frac{1}{2}$ is said to be underdamped. Underdamped systems combine oscillation at a specific frequency with a decay of the amplitude of the signal. Underdamped systems with a low quality factor (a little above $Q = \frac{1}{2}$) may oscillate only once or a few times before dying out. As the quality factor increases, the relative amount of damping decreases. A high-quality bell rings with a single pure tone for a very long time after being struck.
- A system with an intermediate quality factor $Q = \frac{1}{2}$ is said to be critically damped. Like an overdamped system, the output does not oscillate, and does not overshoot its steady-state output (i.e., it approaches a steady-state asymptote). Like an underdamped response, the output of such a system responds quickly to a unit step input. Critical damping results in the fastest response (approach to the final value) possible and especially without **overshoot**. Real system specifications usually allow some overshoot for a faster initial response or require a slower initial response to provide a safety margin against overshoot.

In accelerator physics when considering beam stability two types of resonance are treated: the narrow band resonance and the broad band resonance.

The first one is characterized by an high value of Q , the second one is characterized by a low value of Q . In Fig.1.17 three different resonances are shown (different values of Q).

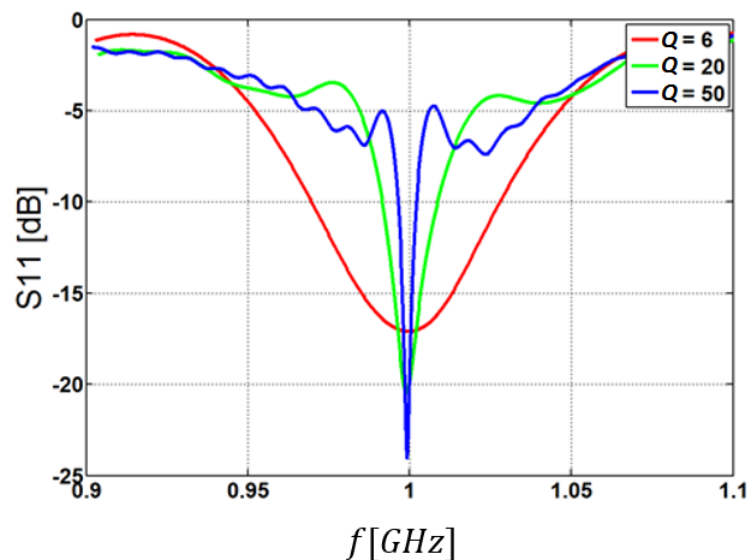


Figure 1.17 – The response of different parallel RLC-circuits with different Q -values.

Our particular interest in the Q factor comes from our interest in the beam stability and in the power loss; both of these topics are going to be show more in the detail int the next chapters. The link between the Q-factor and the instability comes from the fact that one of the main source of instability are given by the resonance.

While the link between the Q-factor and the power loss comes from the power loss not always proportional to the square of the number of particles per bunch. It depending on the shape of the impedance, it can be linear with the number of bunches (when the bunches are independent, i.e. for a sufficiently short-range wake-field — or broad-band impedance — which does not couple the consecutive bunches) or proportional to the square of the number of bunches (when the bunches are not independent, i.e. for a sufficiently long-range Wake field — or narrow-band impedance — which couples the consecutive bunches) [10].

1.6 Beam Instability

When a beam propagates in an accelerator, it interacts with both the external fields and the self-generated electromagnetic fields. If the latter are strong enough, the interplay between them and a perturbation in the beam distribution function can lead to an enhancement of the initial perturbation, resulting in what we call a beam instability.

Beam instabilities in particle accelerators have been studied and analyzed in detail since the late 1950s. The subject owes its relevance to the fact that the onset of instabilities usually determines the performance of an accelerator i.e. it limits the beam intensity.

The main source of instability, that can cause both, single or multi bunch instability, are:

- The resistive wall (see section 1.4).
- Resonance (that can be real cavity or accidental cavity see section 1.5).
- Electron cloud.

Synchrotron radiation from proton bunches in accelerator (i.e. LHC) creates photoelectrons at the beam screen wall. These photoelectrons are pulled toward the positively charged proton bunch. When they hit the opposite wall, they generate secondary electrons which can in turn be accelerated by the next bunch. Depending on several assumptions about surface reflectivity, photoelectrons and secondary electron yield, this mechanism can lead to the fast build-up of an electron cloud.

The main instability are:

A brief explanation of some instability is here following.

- The Robinson instability: In electron storage rings, radio frequency accelerator cavities are necessary to replenish the energy continually lost by the orbiting electrons to synchrotron radiation. The time dependence of the electric field in the cavity couples

	Transverse	Longitudinal
Single bunch	Transverse mode coupling instability (TMCI) Head tail instability	Negative mass instability Robinson instability Longitudinal microwave instability
Multi bunch	Coupled bunch modes	coupled bunch modes

Table 1.1 – A non exhaustive table of instabilities

with the energy dependence of the electron revolution period to produce 'synchrotron oscillations.

- Head Tail Instability: Single bunch effect of transverse wake fields generated by head of the bunch on it's own tail.
- Multi-bunch coupling instability: In this instability, the fields induced in a equipment resonance, generally caused by change in the aperture, and remain long enough to influence subsequent bunches.
- Longitudinal Microwave instability: This is a single bunch effect, driven by a broad-band impedance, which is caused by discontinuities in the beam pipe.

Cures of instabilities are:

- damping with a kicker;
- damping the resonance inside the equipment (with ferrite or with HOM couplers more detail are in the chapters 3 and 4);
- avoid resonance in the optic design (i.e. choosing the tunes in the optimal place);
- Landau Damping.

More detailed discussion about the instability in ref [11].

1.7 Power Loss

Let us consider a bunch of particles with line density $\lambda(z)$ ($\int \lambda(z) dz = eN_{bunch}$, where e is charge of the unity particle charge, N_{bunch} is the number of particles in the bunch to unity and the integral is over the bunch extension, Fig.1.18) passing once through an accelerator device generating a wake function $W_{\parallel}(z)$.

The total energy loss E_{loss} of the bunch can then be obtained by integrating the force $\overrightarrow{F(z)}$ over

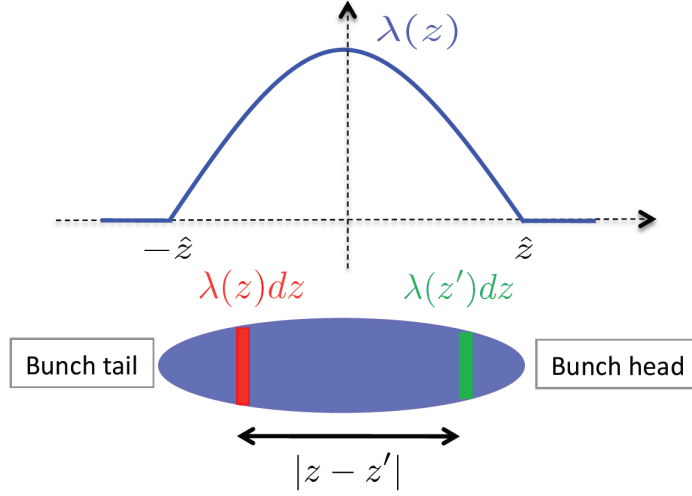


Figure 1.18 – Sketch of the bunch and line density. Source (in green) and test (in red) slices are also highlighted. The Wake field generated by the source slice at the position of the test slice is $W_{\parallel}(z)$

the full bunch extension:

$$E_{loss} = \int_{-\hat{z}}^{\hat{z}} F(z) dz \quad (1.31)$$

Where $\overrightarrow{F(z)}$ is the force give from the Lorentz force (see (1.2)) $\overrightarrow{F(z)} = \lambda(z)\Delta E(z)$.

$$E_{loss} = \int_{-\hat{z}}^{\hat{z}} \lambda(z)\Delta E(z) dz \quad (1.32)$$

The electric field, $\Delta E(z)$, on the test particle (of charge particle $\lambda(z)dz$) is the integral of the contributions from the wakes left behind by all the preceding charge ($\lambda(z')dz'$).

$$\Delta E(z) = \int_{-\hat{z}}^{\hat{z}} \lambda(z')W_{\parallel}(z - z') dz' \quad (1.33)$$

Combining the equation we obtain:

$$E_{loss} = \int_{-\hat{z}}^{\hat{z}} \lambda(z) \left(\int_{-\hat{z}}^{\hat{z}} \lambda(z')W_{\parallel}(z - z') dz' \right) dz \quad (1.34)$$

Transforming (1.34) into the frequency domain yields, using the Plancherel theorem and the convolution theorem we obtain:

$$\Delta E = \sqrt{2\pi} \int_{-\infty}^{\infty} |\bar{\lambda}(\omega)|^2 \text{Re}[Z_{\parallel}(\omega)] d\omega \quad (1.35)$$

where the Fourier Transformation is:

$$\Delta E = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{j\omega t} dt \quad (1.36)$$

The full derivation is shown in Appendix C.

The above formula is for a single bunch that does not interact with other bunches. Of course if we considering M bunches we have M times as much loss therefore:

$$P_{loss} \propto M \quad (1.37)$$

We have this linear behavior because we use the approximation of non interaction between the bunches, this approximation is valid for broad-band impedance (for this impedance the energy storage energy decrease very fast).

While if we have a narrow band impedance like a resonance at a resonance frequency, we have:

$$P_{loss} \propto M^2 \quad (1.38)$$

The energy loss of a bunches is normally given by the loss factor k , by the equation:

$$\Delta E = k q^2 \quad (1.39)$$

Where k is the loss factor.

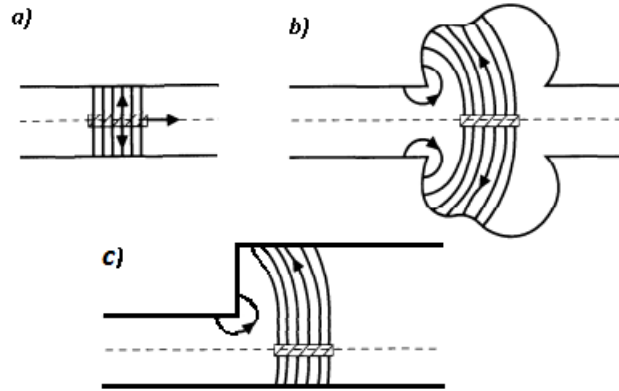


Figure 1.19 – The three different situations of the energy balance.

In the global energy balance, we have three different situations, the first is when the beam pass through a beam pipe in that case the energy lost comes from the resistive wall impedance Fig.1.19(a). The second is when the beam pass in a structure with an aperture changing

Fig.1.19(b) in that case, the energy lost by the source splits into: electromagnetic energy of modes that propagate down the beam chamber (above cut-off), which will be eventually lost on surrounding lossy materials, and electromagnetic energy of the modes that remain trapped in the device. In the second case two scenarios are possible:

1. this energy can be dissipated on lossy walls or through purposely designed HOM absorbers;
2. it keeps ringing without damping (perfect electric conducting (PEC like superconductivity cavity in LHC) walls), but can also be transferred to following particles (or the same over several turns) passing through the device, possibly feeding into an instability.

The third possibility is similar to the second. In this case Fig.1.19(c) we have a fast change in aperture (also called step change) the only difference between the second and the third case is that in the third case all the EM field is lost before get the resonance.

2 How to measure a resonance

In this chapter we are going to do a small overview over the main methods to measure the Q factor of a resonant cavity. The two main methods are the Bead-Pull measurement and the probe measurements. The probe measurement can be done both in transmission and in reflection. There is no absolute best way to measure the Q-factor but everything depends of the requirements. The best choice depends from many parameters (eg. precision, time and money spent) and for each method we are going to explain the advantages and the disadvantages. Before introducing these concepts, the definition of the scattering parameters of a Device Under Test (DUT) is introduced and we also introduce another kind of measurement, in order to do a preliminary test of the equipment, the wire measurement.

2.1 Wire Measurement

In the last section of this chapter we are going to show the two equipment where we are going to measure the Q-factor of some resonances: the Quattro-Tank and the Beam Gas Ionization monitor. In both equipment the frequency response was obtained by a wire measurement (see Fig.2.1).

The wire setup is used to find the position of the resonance frequencies. It cannot be used for the calculation of the Q-factor, because the wire introduces an additional loss of energy to the resonant cavity and a consequent depletion of the quality factor and it also modify the boundary conditions.

So, the wire measurements gives the resonance frequencies and that makes the probe measurements easier, because we will not be confused by other resonances generated by the probes.

The wire measurement setup is done by adding a wire in the middle of the equipment, the presence of the wire will shift the cut-off frequency to zero by introducing a TEM mode that is allowed to always propagate independent from the frequency (it works as a coaxial cable). The wire will therefore extract energy at all frequencies. The TEM methods allowed by the wire, looks like the Wakefield of the beam.

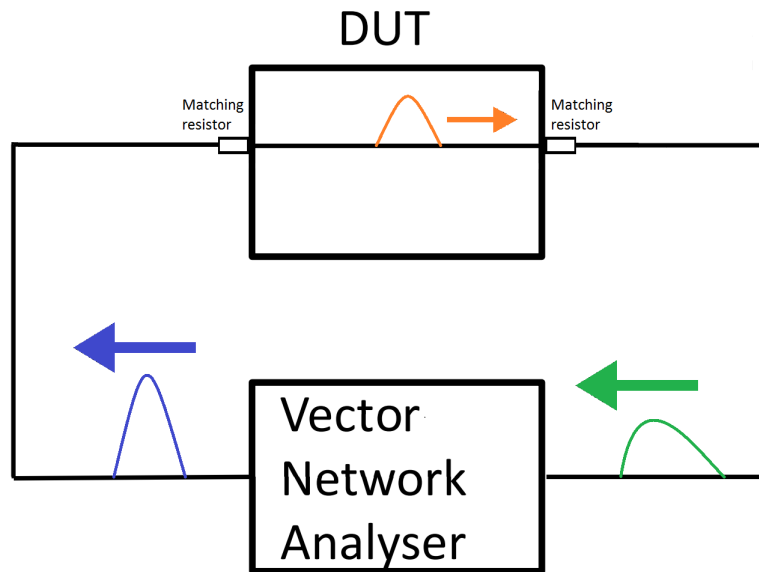


Figure 2.1 – An example of the wire measurement setup.

2.2 Scattering Parameters

Like the impedance or admittance matrix for an N-port network, the scattering matrix provides a complete description of the network as seen at its N ports. While the impedance and admittance matrices relate the total voltages and currents at the ports, the scattering matrix relates the voltage waves incident on the ports to those reflected from the ports (this waves express the power flow in and out of the equipment (in RF technology everything is express in term of power eg. dB)).

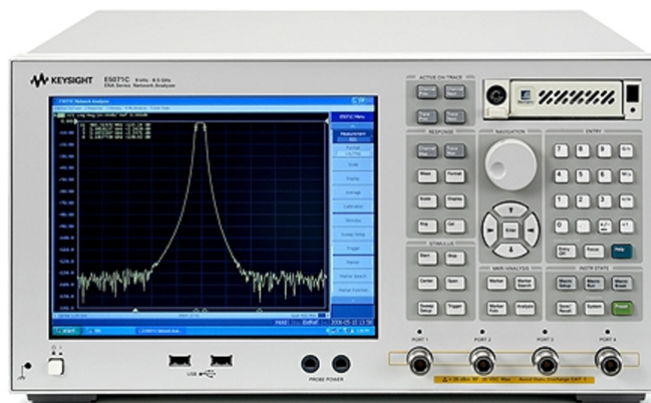


Figure 2.2 – Vector Network Analyzer (VNA) E5071C of the Keysight , used in the measurement in the thesis.

For some components and circuits, the scattering parameters can be calculated using network analysis techniques. Otherwise, the scattering parameters can be measured directly with a vector network analyzer; a photograph of a modern network analyzer is shown in Fig.2.2. Once the scattering parameters of the network are known, conversion to other matrix parameters can be performed, if needed [12].

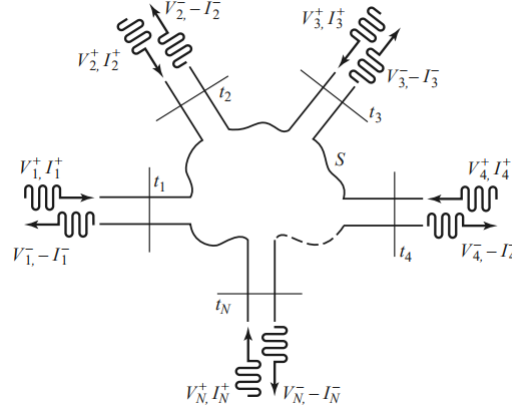


Figure 2.3 – An arbitrary N-port network.

Consider the N-port network shown in Fig.2.3, where V_n^+ is the amplitude of the voltage wave incident on port n and V_n^- is the amplitude of the voltage wave reflected from port n. The scattering matrix, or S matrix, is defined in relation to these incident and reflected voltage waves as:

$$\begin{pmatrix} V_1^- \\ V_2^- \\ \vdots \\ V_N^- \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & \dots & S_{1N} \\ S_{21} & & & \\ \vdots & & & \\ S_{N1} & \dots & & S_{NN} \end{pmatrix} \begin{pmatrix} V_1^+ \\ V_2^+ \\ \vdots \\ V_N^+ \end{pmatrix} \quad (2.1)$$

or directly in the following form:

$$[V^-] = [S][V^+] \quad (2.2)$$

In this case we assume the characteristic impedance is the same for all the channels of N-port network, otherwise we have to use $\frac{V}{\sqrt{Z}}$ where Z is the characteristic impedance. A specific element of the scattering matrix can be determined as

$$S_{ij} = \left. \frac{V_i^-}{V_j^+} \right|_{V_k^+ = 0 \text{ for } k \neq j} \quad (2.3)$$

In words, the (2.3) says that S_{ij} is found by driving port j with an incident wave of voltage V_j^+ and measuring the reflected wave amplitude V_i^- coming out of port i . The incident waves on all ports except the j th port are set to zero, which means that all ports should be terminated

in matched loads to avoid reflections.

Thus, S_{ii} is the reflection coefficient Γ_i seen looking into port i when all the other ports are terminated in matched loads, and S_{ij} is the transmission coefficient T_{ij} from port j to port i when all other ports are terminated in matched loads. A lossless network is one which does not dissipate any power. The sum of the incident powers at all ports is equal to the sum of the reflected powers at all ports (if the port are close). This implies that the S-parameter matrix is unitary, that is:

$$[S^H][S] = [I] \quad (2.4)$$

with $[S^H]$, is the conjugate transpose of $[S]$, and $[I]$, is the identity matrix and also the sum of the squares of the coefficients of each column must be 1, so this means also that the determinant of the $[S]$ matrix is 1 for a lossless network.

Another important feature of the scattering matrix is that it is symmetric. The symmetry is a consequences of the Lorentz reciprocity theorem, this is true for all passive network (from the practical point we can write $S_{12} = S_{21}$).

We are going to show three common examples in the accelerators physics.

Example 1: 3dB attenuator

The 3dB attenuator is a two port device show in Fig.2.4.

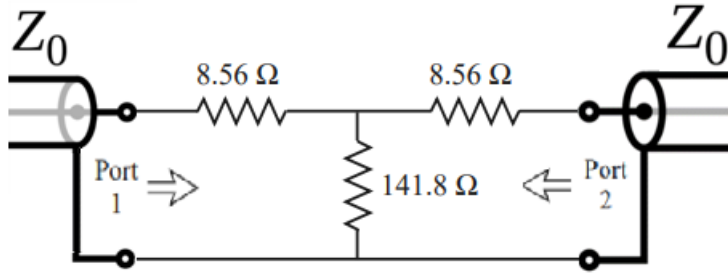


Figure 2.4 – Classic 2 port device, a matched 3 dB attenuator with a 50 characteristic impedance

From the (2.3), the S_{11} can be found as the reflection coefficient seen at port 1 when port 2 is terminated in a matched load ($Z_0 = 50\Omega$):

$$S_{11} = \frac{V_1^-}{V_1^+} \Big|_{V_2^+=0} = \Gamma^{(1)} \Big|_{V_2^+=0} = \frac{Z_{in}^{(1)} - Z_0}{Z_{in}^{(1)} + Z_0} \Big|_{Z_0 \text{ on port 2}} \quad (2.5)$$

with $Z_{in}^{(1)} = 8.56 + \frac{141.8(8.56+50)}{(141.8+8.56+50)} = 50\Omega$, so $S_{11} = 0$. Because of the symmetry of the circuit, $S_{22} = 0$.

We can find S_{21} by applying an incident wave at port 1, V_1^+ , and measuring the out coming

wave at port 2, V_2^- . This is equivalent to the transmission coefficient from port 1 to port 2:

$$S_{21} = \frac{V_2^-}{V_1^+} \Big|_{V_2^+ = 0} \quad (2.6)$$

From the fact that $S_{11} = S_{22} = 0$, we know that $V_1^- = 0$ when port 2 is terminated in $Z_0 = 50\Omega$, and that $V_2^+ = 0$. In this case we have that $V_1^+ = V_1$ and $V_2^- = V_2$. By applying a voltage V_1 at port 1 and using voltage division twice we find $V_2^- = V_2$ as the voltage across the 50Ω load resistor at port 2:

$$V_2^- = V_2 = V_1 \left(\frac{41.44}{41.44 + 8.56} \right) \left(\frac{50}{50 + 8.56} \right) = 0.707 V_1 \quad (2.7)$$

where $41.44 = 141.8 \left(\frac{58.56}{141.8 + 58.56} \right)$ is the resistance of the parallel combination of the 50 load and the 8.56 resistor with the 141.8Ω resistor. Thus, $S_{12} = S_{21} = 0.707$.

If the input power is $\frac{|V_1^+|^2}{2Z_0}$, then the output power is $\frac{|V_2^-|^2}{2Z_0} = \frac{|S_{21} V_1^+|^2}{2Z_0} = \frac{|S_{21}|^2}{2Z_0 |V_1^+|^2} = \frac{|V_1^+|^2}{4Z_0}$, which is one-half $-3dB$ of the input power.

Example 2: 180 degree hybrid coupler

Another common device used to measure the transverse impedance (Fig 2.5), is the hybrid coupler (Fig 2.6).

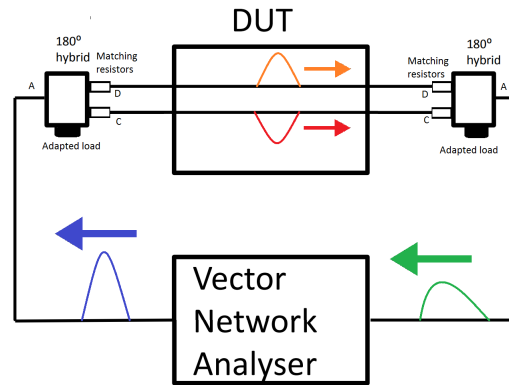


Figure 2.5 – Example of setup for the measurement of the transverse impedance.

The hybrid coupler is a four-port network, with a 180° phase shift between the two outputs. It also can be operated with outputs in phase.

A current pulse input at port A give a positive output pulse on port D and negative pulse output on port C.

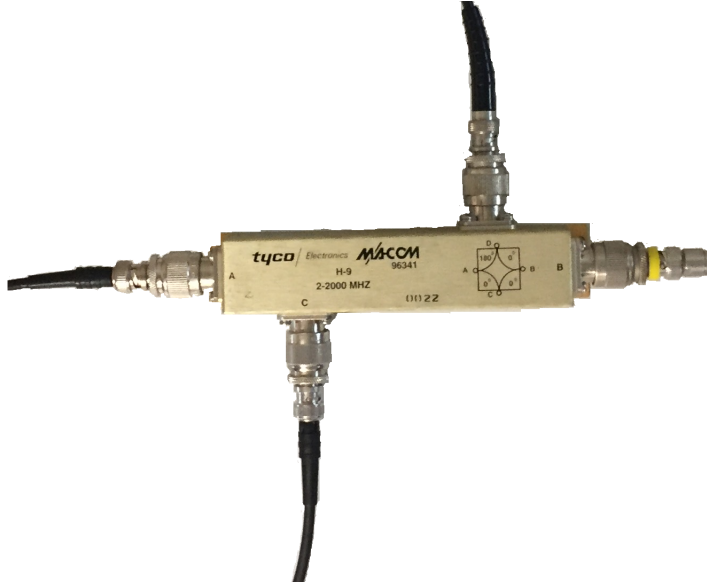


Figure 2.6 – Hybrid coupler used for transverse measure of impedance.

The S-matrix for this hybrid is given by:

$$\mathbf{S} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -1 \\ -1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix} \quad (2.8)$$

The hybrid coupler is not symmetric on its ports; choosing a different port as the input does not necessarily produce the same results. Please note that the $\text{Det}[\mathbf{S}] = 1$, it means the $[\mathbf{S}]$ is for ideal hybrid coupler.

Example 3: Unknown impedance

Another example of the utility of the S parameters is, if you have an unknown load. This type of calculation is used when doing a measurement of the longitudinal beam impedance. It is used to calculate the matching resistor (see Fig.2.7).

We can then calculate the transmission coefficient by the formula present in literature [13]:

$$S_{21,DUT} = \frac{b_2}{a_1} = \frac{\frac{V_2 + Z_0 I}{2\sqrt{Z_0}}}{\frac{V_1 + Z_0 I}{2\sqrt{Z_0}}} = \frac{V_2 + Z_0 I}{V_1 + Z_0 I} = \frac{V_2 + Z_0 I}{V_2 + Z_L I + Z_0 I} = \frac{Z_0 I + Z_0 I}{Z_0 I + Z_L I + Z_0 I} = \frac{2Z_0}{2Z_0 + Z_L} \quad (2.9)$$

$$S_{21,REF} = \frac{\frac{V_2 + Z_0 I}{2\sqrt{Z_0}}}{\frac{V_1 + Z_0 I}{2\sqrt{Z_0}}} = 1 \quad (2.10)$$

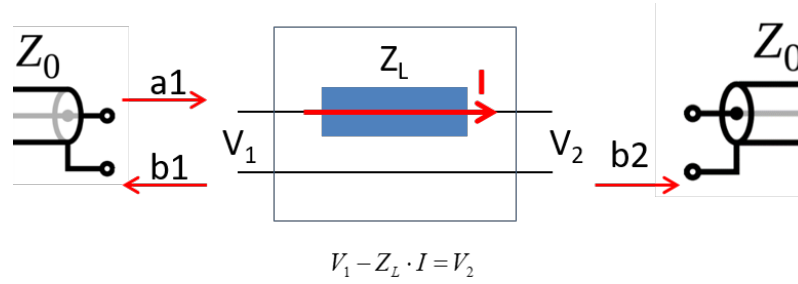


Figure 2.7 – Two port with unknown load in the middle.

The equation 2.10 is equal to 1 because Z_L is zero (i.e. not existing in the reference measurement). Now we can use the formula 2.9:

$$\frac{S_{21,DUT}}{S_{21,REF}} = \frac{2Z_0}{2Z_0 + Z_L} \quad (2.11)$$

that means:

$$\frac{Z_L}{Z_0} = 2 \frac{S_{21,REF}}{S_{21,DUT}} - 2 \quad (2.12)$$

2.3 Smith Chart

The Smith chart, shown in Fig.2.8, is a graphical aid that can be very useful for solving transmission line problems. Although there are a number of other impedance and reflection coefficient charts that can be used for such problems [14], the Smith chart is probably the best known and most widely used. It was developed in 1939 by P. Smith at the Bell Telephone Laboratories [15]. The reader might feel that, in this day of personal computers and computer-aided design (CAD) tools, graphical solutions have no place in modern engineering. The Smith chart, however, is more than just a graphical technique.

Besides being an integral part of much of the current CAD software and test equipment for microwave design, the Smith chart provides a useful way of visualizing transmission line phenomenon without the need for detailed numerical calculations.

The smith chart displays the reflection coefficient in magnitude and phase (polar) form as $\Gamma = |\Gamma|e^{j\theta}$.

Then the magnitude $|\Gamma|$ is plotted as a radius ($|\Gamma| \leq 1$) from the center of the chart, and the angle θ ($-180^\circ \leq \theta \leq 180^\circ$) is measured counterclockwise from the right-hand side of the horizontal diameter. Any passively realizable ($|\Gamma| \leq 1$) reflection coefficient can then be plotted as a unique point on the Smith chart.

The Smith chart have really two main utilities, the first is the possibility to find the matching impedance just moving on the Smith Chart, because depending of the movement, we modify the impedance view from the output. This modification can be modeling like an addition of an capacitor or an inductor in series or in parallel, the size of that component depend of the movement (see Fig.2.9).

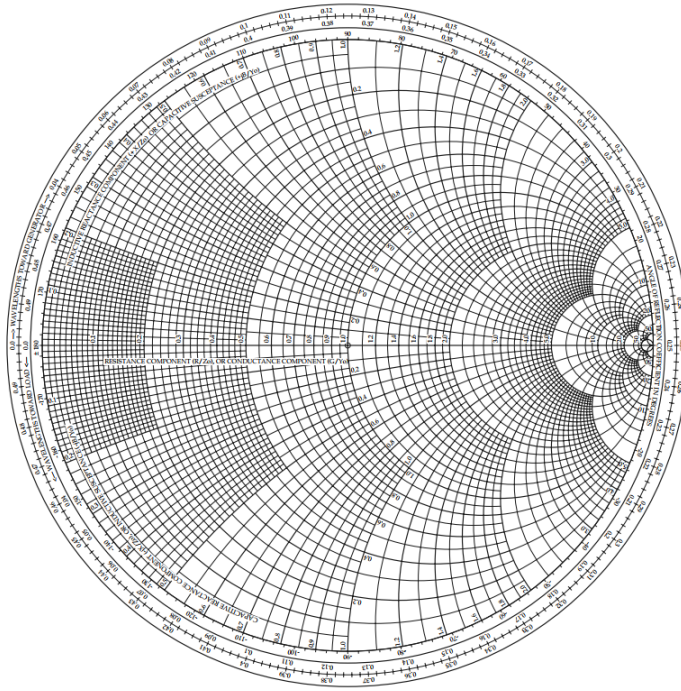


Figure 2.8 – The Smith chart.

The second utility is it can be used to convert from reflection coefficients to normalized impedances (or admittances) and vice versa by using the impedance (or admittance) circles printed on the chart (see Fig.2.10). When dealing with impedances on a Smith chart, normalized quantities are generally used, which we will denote by lowercase letters. The normalization constant is usually the characteristic impedance of the transmission line. Thus, $z = \frac{Z_L}{Z_0}$ represents the normalized version of the impedance Z .

If a lossless line of characteristic impedance Z_0 is terminated with a load impedance Z_L , the reflection coefficient at the load can be written as:

$$\Gamma = \frac{V_0^-}{V_0^+} \quad (2.13)$$

Where V_0^+ is the wave entering into the load Z_L and V_0^- is the reflected wave. From 2.25 we can write:

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (2.14)$$

If we use the normalized variable we obtain:

$$\Gamma = \frac{z_L - 1}{z_L + 1} \quad (2.15)$$

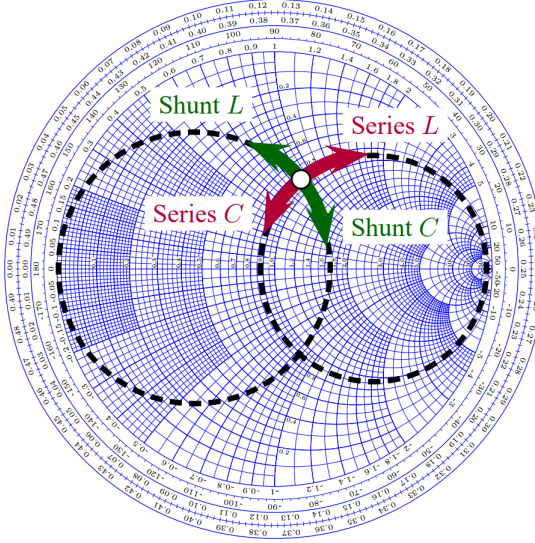


Figure 2.9 – Illustration of navigation in the Smith chart when adding lumped elements.

This relation can be solved for z_L in terms of Γ to give

$$z_L = \frac{1 + |\Gamma|e^{j\theta}}{1 - |\Gamma|e^{j\theta}} \quad (2.16)$$

This complex equation can be reduced to two real equations by writing Γ and z_L in terms of their real and imaginary parts $\Gamma = \Gamma_r + j\Gamma_i$, and $z_L = r_L + jx_L$, giving

$$r_L + jx_L = \frac{(1 + \Gamma_r) + j\Gamma_i}{(1 - \Gamma_r) - j\Gamma_i} \quad (2.17)$$

The real and imaginary parts of this equation can be separated by multiplying the numerator and denominator by the complex conjugate of the denominator to give

$$r_L = \frac{1 - \Gamma_r^2 + \Gamma_i^2}{(1 - \Gamma_r)^2 + \Gamma_i^2} \quad (2.18)$$

$$x_L = \frac{2\Gamma_i}{(1 - \Gamma_r)^2 + \Gamma_i^2} \quad (2.19)$$

Rearranging 2.18 gives

$$\left(\Gamma_r - \frac{r_L}{1 + r_L}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1 + r_L}\right)^2 \quad (2.20)$$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \left(\frac{1}{x_L}\right)^2 \quad (2.21)$$

which are seen to represent two families of circles in the Γ_r, Γ_i plane. Resistance circles are defined by (2.20) and reactance circles are defined by (2.21).

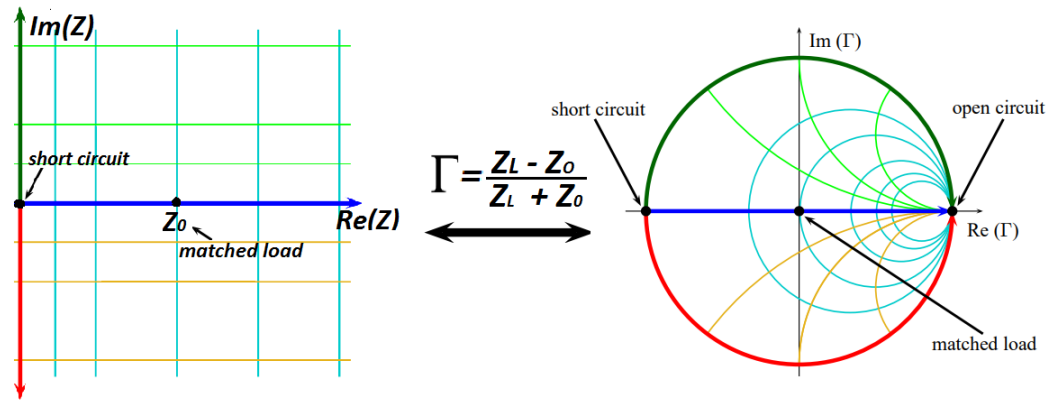


Figure 2.10 – Mapping of the admittance plane into the Γ plane.

The upper half of the chart is inductive, since it corresponds to the positive imaginary part of the impedance.

The lower half is capacitive as it is corresponding to the negative imaginary part of the impedance.

There are three important points in the chart:

1. Open circuit with $\Gamma = 1$, $z \rightarrow \infty$
2. Short circuit with $\Gamma = -1$, $z = 0$
3. Matched load with $\Gamma = 0$, $z = 1$

They all are located on the real axis at the beginning, the end, and the center of the circle (Fig.2.10 right side).

We can explain a bit more on this Fig.2.10, if we are moving on the light blue circle we are looking of impedance with real part constant, while if we are moving on the orange circle we are looking for impedance for imaginary part constant and negative at the end if we are moving on the green circle we are looking for impedance with imaginary part constant and positive (this two last circles are not full draw because doesn't matter what we have out of the circle of $|\Gamma| = 1$) [16].

2.4 The loaded Q-factor

When we talk about the Q-factor, a clarification is necessary. We have to distinguish the three different types of Q: the load Q (Q_L), the unloaded Q (Q_0) and the Q of the external world (Q_{ext}).

When a resonant circuit is connected to the external world, its losses will be affected. In this

case we represent the external world by a resistor, named R_{ext} . So, in order to obtain the Q_0 of the circuit is necessary considering both the Q_{ext} and the Q_L . In order to better explain this concept, let's make an example.

Considering an RLC parallel circuit, and modeling the connection with the external world with the resistance R_{ext} (see Fig.2.11).

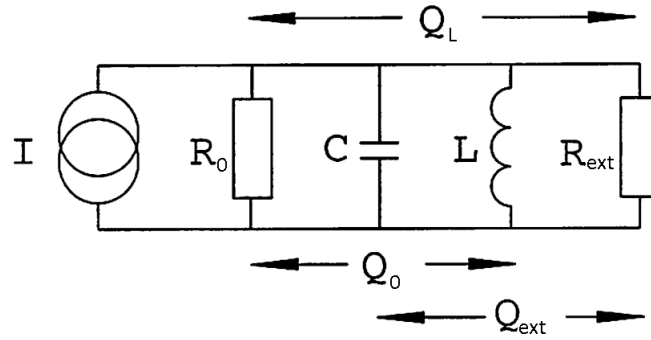


Figure 2.11 – The RLC parallel circuit that model the resonance (with the external world resistance).

When we measure the Q of the circuit unavoidably we measure the Q_L . Since we are interested in the real Q (Q_0) we have to measure also the Q_{ext} . The way to obtain directly the Q_0 is measure the circuit considering only R_0, C, L while to obtain Q_{ext} is necessary considering only R_{ext}, C, L [16].

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} \quad (2.22)$$

Since this two measurements are practically unfeasible, we use a parameter in order to obtain Q_0 . The parameter is named β , the coupling factor.

It is then possible to evaluate Q_0 by the formula:

$$Q_0 = Q_L(1 + \beta) \quad (2.23)$$

where β could be evaluate, by:

$$\beta = \frac{R_0}{R_{ext}} \quad (2.24)$$

This parameter expresses the coupling with the external world.

Is clear from (2.22) and (2.23) that there is a link between the Q_{ext} and β . To derivate this dependence, we can rewrite the (2.22) in this way:

$$Q_L = \frac{Q_0 Q_{ext}}{Q_0 + Q_{ext}} \quad (2.25)$$

Is possible rewrite also the (2.23) in this way:

$$Q_L = \frac{Q_0}{1 + \beta} \quad (2.26)$$

Using the last two (2.25) and (2.26) we obtain:

$$\frac{Q_0}{1 + \beta} = \frac{Q_0 Q_{ext}}{Q_0 + Q_{ext}} \quad (2.27)$$

and easily we can simplify and obtain:

$$\beta = \frac{Q_0}{Q_{ext}} \quad (2.28)$$

This result also find justification in the way to measure the β (see Eq.2.24) where Q_0 is represented by $Re[R_0]$ and Q_{ext} is represented by R_{ext} .

2.5 Bead-Pull measurement

The Bead-Pull measurement technique is typically used to derive the electric and magnetic fields in a cavity. It was developed in 1951 by J.C Slater [17]. The basic concept is to place a perturbing object (bead or needle for instance) through the longitudinal axis (beam-pipe axis) of the cavity.

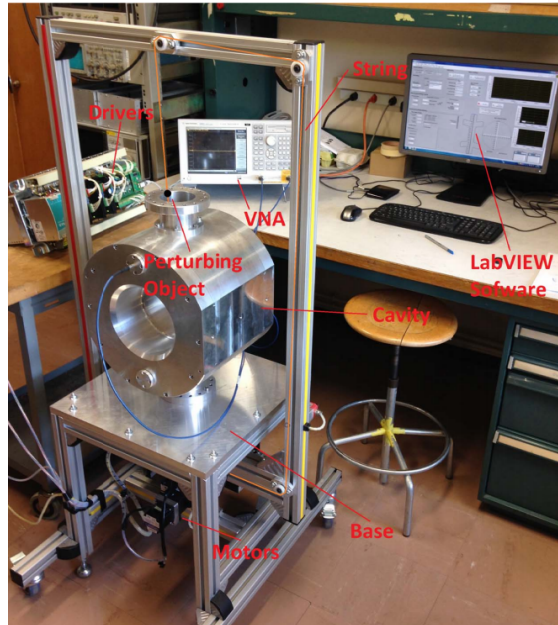


Figure 2.12 – Overview of the system to perform Bead-pull measurements.

A shift of the resonant frequency is observed while the object travels the entire length of the cavity. This frequency shift is proportional to the square of the relative electric field $|\vec{E}|^2$ and magnetic field $|\vec{H}|^2$ at the position of the bead. This relation is shown by eq.2.29 (see the derivation from ref. [18] [19]).

$$\frac{\Delta f}{f_0} = \frac{\int \int \int_{\Delta V} \epsilon_0 E^2 dV - \int \int \int_{\Delta V} \mu H^2 dV}{4U} \quad (2.29)$$

U is the total stored energy, Δf is the frequency shift and f_0 is the initial resonant frequency. It is often easier to work in term of phase shift instead of frequency shift due to the difficulty to read it on the VNA, the correspondence is given by (2.30).

$$\frac{\Delta f}{f_0} = \frac{\tan[\phi(f_0)]}{2Q_L} \quad (2.30)$$

Where Q_L is the loaded Q factor and $\phi(f_0)$ is the phase shift of the transmission parameter S_{21} at f_0 .

An example of the measurement setup is show in Fig.2.12.

The advantage of this method is that it is extremely precise but the big disadvantage is that it is necessity to analyze the equipment alone in a lab and the need of moving the bead.

2.6 Probe measurements

The probe measurement can be carried out with 2 probes (Q_L measured in transmission) and with only one probe (Q_L measured in reflection). The two setups are shown in Fig.2.13.

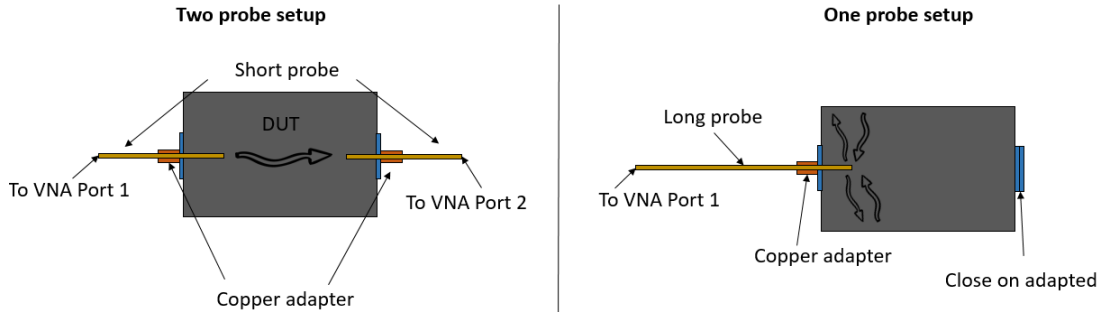


Figure 2.13 – The two probe measurement setup.

The electronic models are following shown in Fig.2.14 (made with ANSYS):

The coupling capacitor represents the connection to the resonance. Usually when the probe measurement is used the Q factor is evaluated using both, the reflection method and transmission method.

In the reflection measurement, we can identify the place of the resonance and in transmission

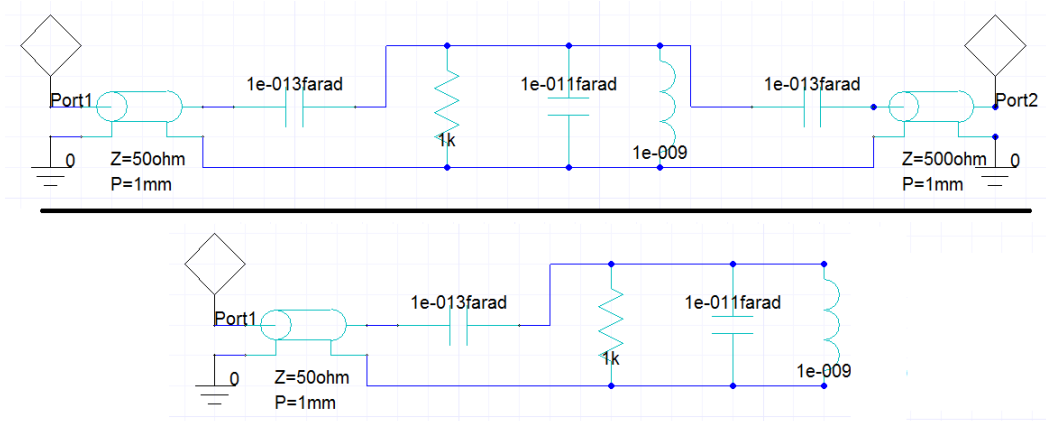


Figure 2.14 – The two electronic models, on the top the transmission model on the bottom the reflection.

obtain information about the Q-factor.

2.6.1 Transmission measurement

In the transmission measurement, the signal is sent through the structure and consequently the signal that reaches the second port is analysed.

In this type of measurement, we can assume $Q_L \approx Q_0$, since the β factor is usually close to zero because of the small coupling capacitor (the coupling capacitor is smaller because we are far from the resonance).

On this signal, the classic 3dB method can be used.

So to do it, we have to search the resonance frequency f_r and the frequencies f_1 and f_2 , where the peak is attenuated by 3dB.

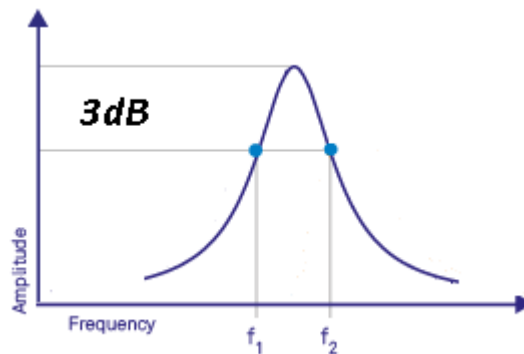


Figure 2.15 – The bandwidth of a damped oscillator is shown on a graph of energy versus frequency. The Q factor of the damped oscillator. The higher the Q, the narrower the peak is.

To that we have to look the frequency response as shown in Fig.2.15.

After the detection of these points, the Q is evaluated through the equation:

$$Q = \frac{f_r}{\Delta f} \quad (2.31)$$

with $\Delta f = f_2 - f_1$.

The disadvantage of this method is the necessity of short distance between the two probes. That is because the field sent by first probe have to pass some portion of free space first to reach the second probe. In this interval the signal is attenuated exponentially.

So, if the distance between the probes is too long, then the attenuation is too large, and the VNA is not sensitive enough to measure it.

The advantage of this method otherwise is the fast setup and the possibility of the real time measurement of the Q on the VNA.

2.6.2 Reflection measurement

If the measurement is carried out in reflection, the Smith chart should be used. In this case we have to distinguish the Q_L and the Q_0 .

To evaluate the Q_L is necessary the use of two lines at 45 degree. Those lines starts from the point $(-1, 0)$ (also called detuned short position) and intersect the circle (the resonance is a circle in the smith chart). These points are the equivalent of 3dB points (see Fig.5.3). The disadvantage of this method is the high probability of detecting parasites resonances

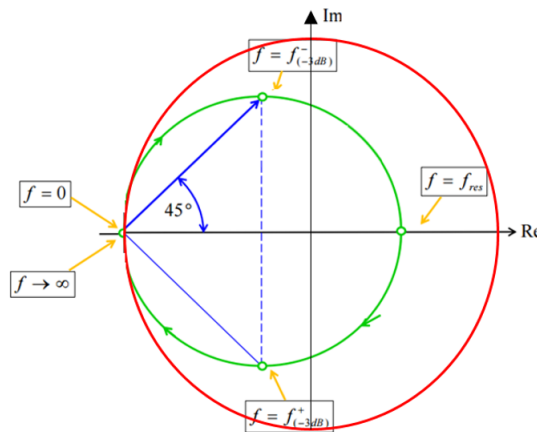


Figure 2.16 – Smith chart with the legend of the main point.

between the probe and the walls of the equipment and the necessity of post processing for the evaluation of the Q-factor.

But this measurement setup have the advantage it does not be limited by the length of the device, also the setup is really easy and, must importantly, that method is very precise because the probe is close to the resonance.

2.7 Example of measurements

Two different devices chosen to test the new way to attenuate a resonances with the HOM coupler, the Quattro-tank (QT) and the Beam Gas Ionization monitor (BGI) (more detail in the next chapter 4).

In this section I'm going to show a preliminary study on these equipment to show the resonances inside.

In the thesis, the experimental method will be applicate on the QT.

This measurement was done with the wire in order to find the position of the resonances.

2.7.1 Quattro-Tank

The QuattroTank (QT) (shown in Fig.2.17) is an SPS component that is used as a testbed for crystal collimation. The main idea is insert inside a crystal at given distance away from the beam (usually 3σ , where σ is the transverse beam size) and all particle that aren't into this space are going to deviate away in some collimator.

The QT has a relatively simple internal structure, and is more or less only constructed out of a single material.

When taking measurements of the QT, most of the structure takes the form of a coaxial cable. For these reasons, it is an ideal test structure for developing a new method to damp the resonances.

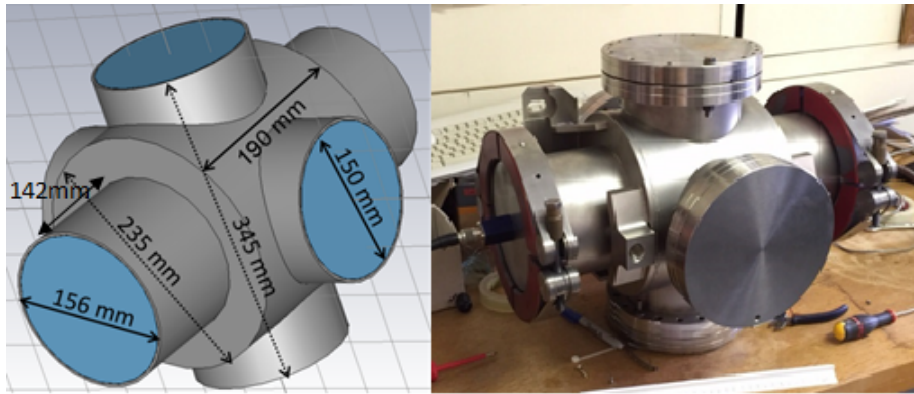


Figure 2.17 – On left the 3D model of the QT and on the right the real QT.

If we do a wire measurement of the equipment with our VNA (2.3) measure the transmission coefficient S_{12} we obtain:

Fig.2.18 shown clearly that we have 2 main resonances. The first is at $f_1 = 1.018GHz$ while the second one is at $f_2 = 1.40GHz$.

The Q-factor of f_2 will be show in the chapter 6.

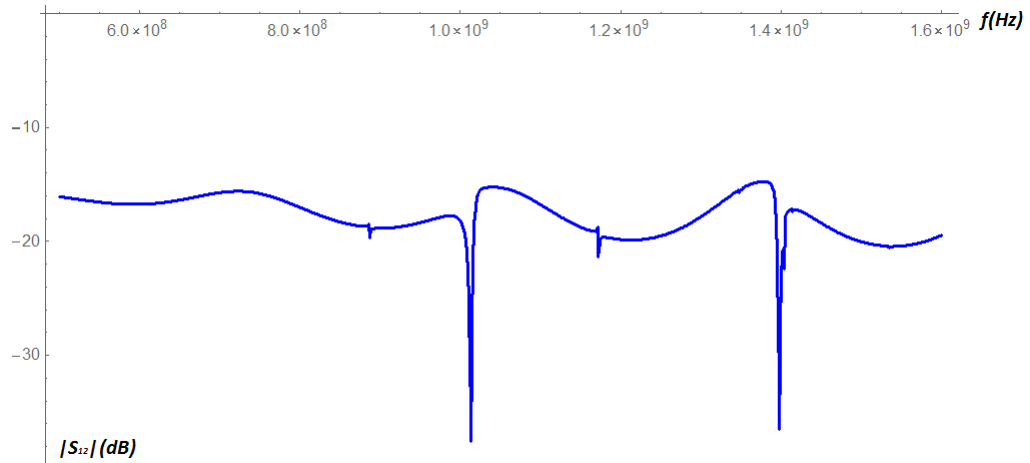


Figure 2.18 – Measurement of the frequency response obtained by wire measurement coefficient of the QT.

2.7.2 BGI monitor

The Beam Gas Ionization monitors (BGI) [20] operating in the LHC at CERN is a beam instrument design for beam size measurements. The beam profiles are obtained by collecting the electrons produced by the collision between the beam and the residual gas.

Fig.2.19 presents the picture of the BGI in the LHC.

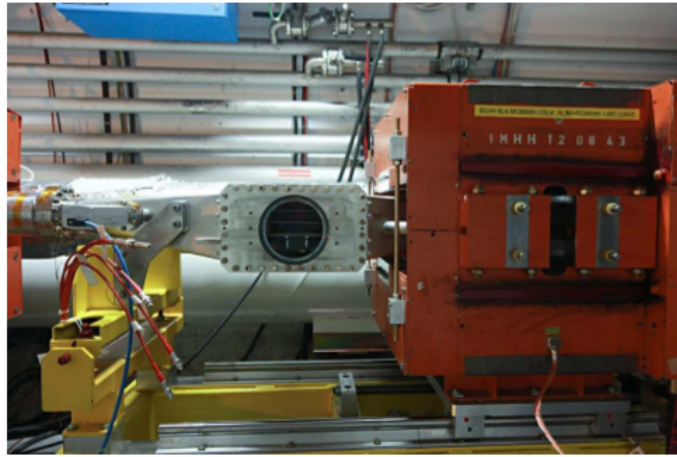


Figure 2.19 – BGI in LHC. The magnet is shifted to show the chamber and optical port.

A cut through the BGI ionization chamber (Fig.2.20) will be used for explaining the operating principle. We assume that the beam is passing in the z -direction, into the paper. As the chamber is filled with the low pressure (8 – 10 *mbar*) Neon gas, the circulating beam liberates the electrons and ions by the ionization process.

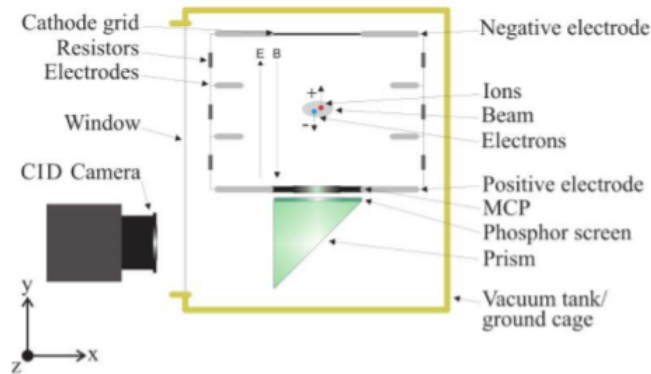


Figure 2.20 – Sketch of the operating principle.

The orange magnet seen on the Fig.2.19 is responsible for creating the constant magnetic field of $0.2T$ in the y -direction. The electrons with the momentum obtained in the ionization process and due to interactions with a beam are therefore forced to follow the spiral trajectory. It should prevent the electrons from getting the transverse spread in space. When the electrons reach the anode, they hit the Micro Channel Plate (MCP) used for multiplying the number of electrons. The MCP is a $0.5mm$ thin plate, usually made from the lead glass, where the electron multipliers (called channels) are densely spaced and oriented parallel to each other (see Fig.2.21). A high voltage is applied across the channels, so an incoming electron reaching a channel generate a cascade of electrons inside the channel. As the result, the electrons are multiplied with a gain of around $10^3 - 10^4$ which allows to obtain the clear signal.

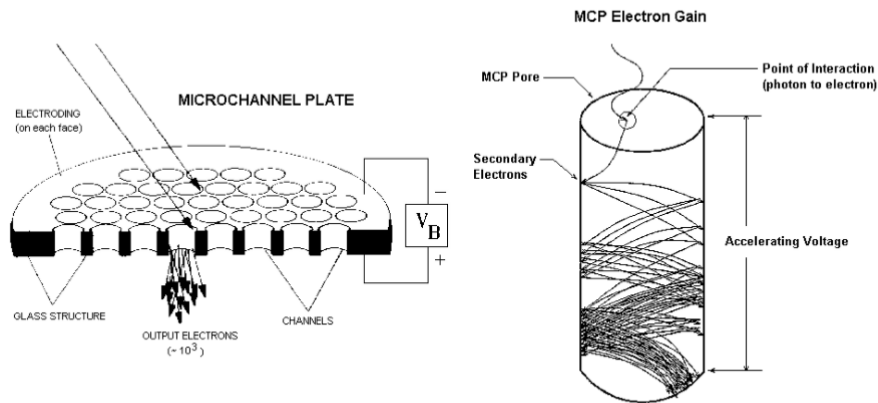


Figure 2.21 – The operating principle of MCP.

If we do a wire measurement of the equipment with our VNA (2.3) measure the transmission coefficient S_{12} we obtain:

Fig.2.22 shows an huge quantity of resonance. We already know that resonance like those are

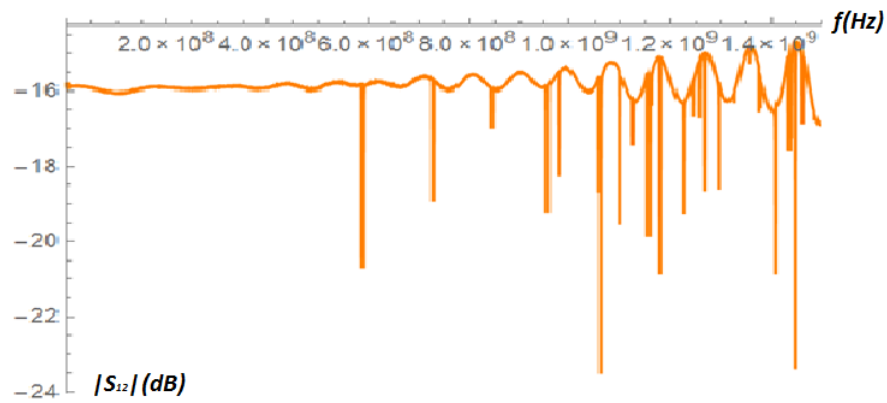


Figure 2.22 – Measurement of the frequency response obtained by wire measurement coefficient of the BGI.

a big problem for many reasons (for more detail look into the chapter 1).

This equipment will be study in the future.

Into the next chapter 3 we are going to show the classic ways to reduce this resonance.

3 Ways to mitigate a resonance

Due the effects of wake fields on both beam stability and machine equipment it is often necessary to consider reducing the beam impedance of different components in particle accelerators. There are a number of ways in which it is possible to reduce the impedance (that directly determining an reduction of problem with the resonance) depending on whether the impedance is primarily geometric or material dependent in nature and the most commonly used will be reviewed.

In this chapters it will often be said to reduce the incidence instead of resonance, this is because we can easily see resonance as an impedance at a well-defined frequency.

Different solutions from mechanical changes in the structures to damping materials placed to damp resonances are discussed [21]. Further references are given to provide more in depth knowledge as required. In particular the use of ferrite to damp cavity modes that may not be removed by redesign, due to either time or mechanical constraints, has recently become a product of intensive study due to the high temperatures seen in many devices that have ferrite placed in them. The ferrites may heat beyond their Curie temperature (the temperature at which a magnetic material starts to experience significant changes in magnetic properties, drastically affecting their use as either damping material or as electromagnet yokes) during regular operation leading to a deteriorating case for the machine impedance.

At the same of the ferrite, all solutions have problems that can not be overlooked, for this reason in the next chapter 4, we will show the new proposal that improves these issues.

3.1 Transition Pieces

Often it is necessary to have transitions in the beam pipe which can not be tapered, either due to space constraints or the operational requirements of the device containing the transition. This a common requirement in devices that require some mechanical degree of freedom (i.e. longitudinal or transverse movement is expected), such as bellows, or electrical isolation from the beam pipe, such as kicker magnets. For these devices it is often possible to use a transition piece, that is one or several pieces of conducting material to screen any transition. These may

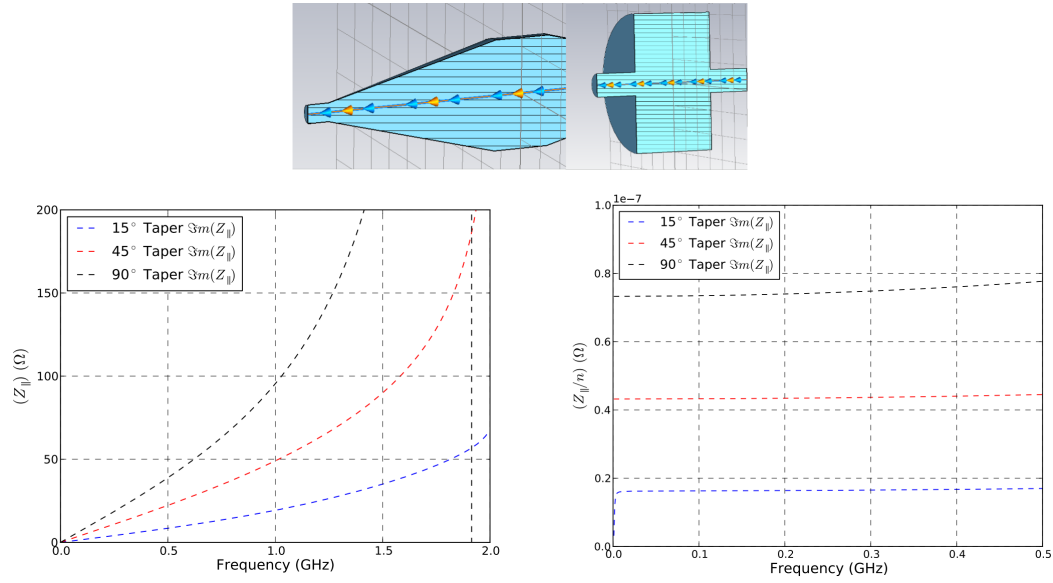


Figure 3.1 – An example pillbox structure with and without a tapered transition region, in this case with the taper of 15° . The resulting imaginary component of the longitudinal impedances are shown in (left), with $\frac{Z_{\parallel}}{n}$, where n is the number of bunch, is shown in (right), as these are the most significant for beam stability.

be rigid or movable as shown in Fig.3.2, often referred to as RF fingers.

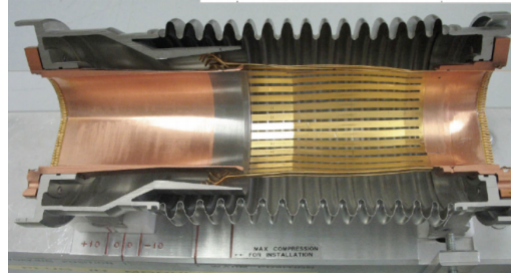


Figure 3.2 – Example of RF fingers (in this case for the PIMS (Plug In ModuleS) module, placed between cryo-modules in the LHC).

As an example of a cavity with and without RF fingers and a number of intermediary steps, see Fig.3.3, which illustrates the case of the VMTSA, a vacuum interconnect in the injection region of the LHC [22].

The VMTSA was screened by a long set of RF fingers, which functioned well when good surface contact was maintained between the fingers and the beam pipe. This method of impedance reduction is effective for a number of reasons. Firstly, it provides a short, good conducting path for the image currents to flow that does not make the cavity created by the transition visible to the beam, and, in the case of bellows, the image current does not have to follow to long contoured path of the bellows, such as shown in Fig.3.2. This serves to reduce the broadband

impedance increase and, by shielding the contours of the bellows, prevents an increase in the imaginary longitudinal impedance, due to the increased electrical length of the device. Secondly, by correctly designing the spacing in the transitions, it is possible to minimize field leakage to the surrounding cavities therefore decreasing the visibility of cavity resonances.

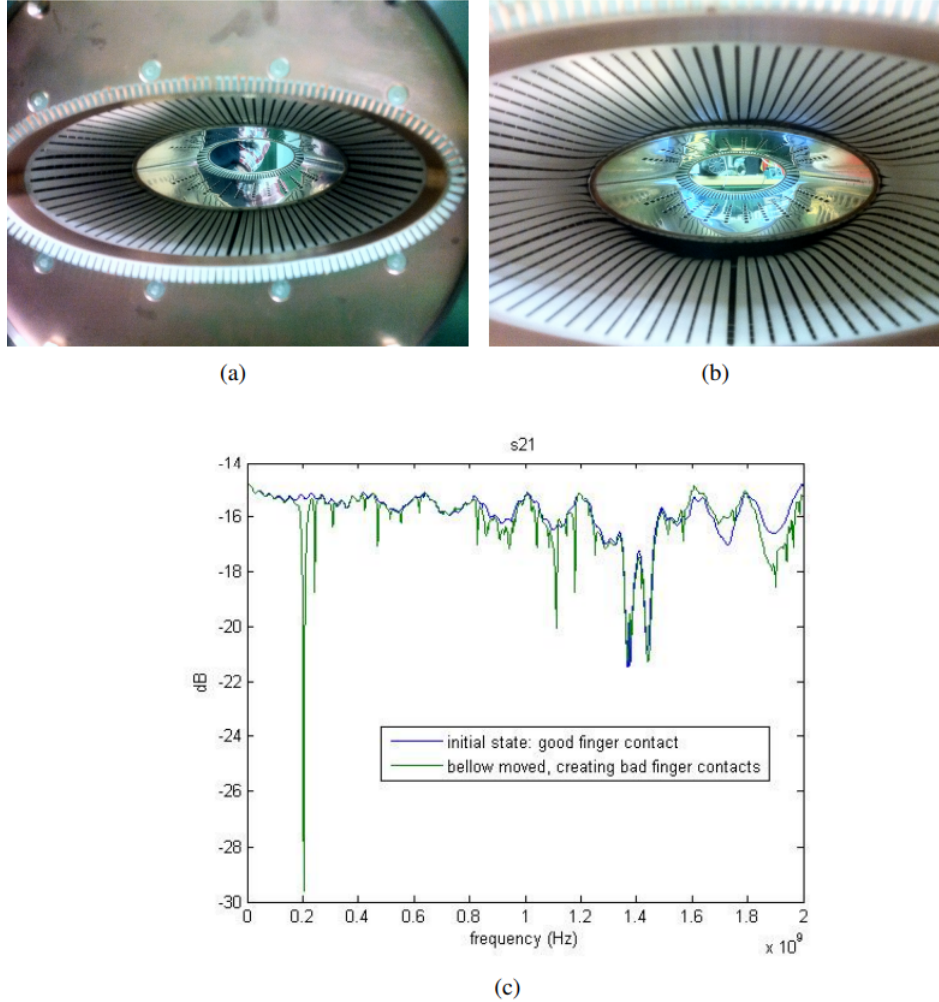


Figure 3.3 – The layout of the RF fingers in the VMTSA both in (a) the fully operational configuration and (b) when some RF fingers lose contact. (c) shows the transmission parameter S_{21} for the VMTSA module with and without good electrical contact between the fingers and the beam pipe as acquired by coaxial wire measurements.

3.2 Conductive Coatings

As was seen in section 1.4, a higher conductivity in the material seen by the beam in a particle accelerator results a lower beam coupling impedance. Typically this rule of thumb is followed in the design of particle accelerators, however the operational requirements of devices in the

machine often require that they dose not be made from a good conducting material. Examples of this include collimators (requiring high strength, mechanical stability and certain radiation properties), beam instrumentation and numerous other devices. It has been shown [22] that a thin layer of high conductivity material placed on the surface of a poorly conducting material can effectively screen the beam from interacting with the poorly conducting material for a large frequency range. This can be explained by considering the skin depth δ of a material:

$$\delta(\omega) = \sqrt{\frac{2}{\mu_r(\omega)\mu_0\sigma\omega}} \quad (3.1)$$

The skin depth can be thought of as the distance of penetration of the magnetic field into the material. It can thus be seen that for a good conducting material like copper ($\sigma_{cu} \approx 6 \times 10^7 Sm^{-1}$), for frequencies of the order of a hundred megahertz or above, a thickness of $10\mu m$ is larger than the skin depth at $100MHz$ ($\delta(100MHz) = 6\mu m$), thus effectively screening the layer below.

3.3 Serigraphy

For existing devices, limitations of both time and budget may require the use of retroactive solutions to reduce large beam impedances. Often these must be added to the original equipment, as the continued correct operation of the device requires minimal disruption to the geometry and surfaces of the device.

In this case, an innovative solution was found, the use of serigraphy.

This entailed the printing of a set of interleaved fingers (see Fig.3.4) made from a good conductor (silver), which form a good conductive path for the beam image currents: the ferrite dielectric provides a reasonable capacitive coupling between the interleaved fingers.

This serves to replace the broadband impedance typically associated with a ferrite dominated resistive wall impedance with a low broadband impedance, with strong resonant impedances due to the capacitive coupling and physical length of the fingers.

The results in the case of the SPS-MKE can be seen in Fig.3.4(c).

3.4 Damping materials (Ferrite)

For a number of devices it is unavoidable to have a cavity present in the structure.

In this case, it is necessary to find a way of reducing the beam impedance by altering the properties of resonances.

Often, it is only the peak value of the impedance attributable to a resonance that is of concern from a beam stability/beam induced heating point of view.

If we consider the defining properties of a resonant impedance, f_r , $\frac{R}{Q}$ and Q , there are a

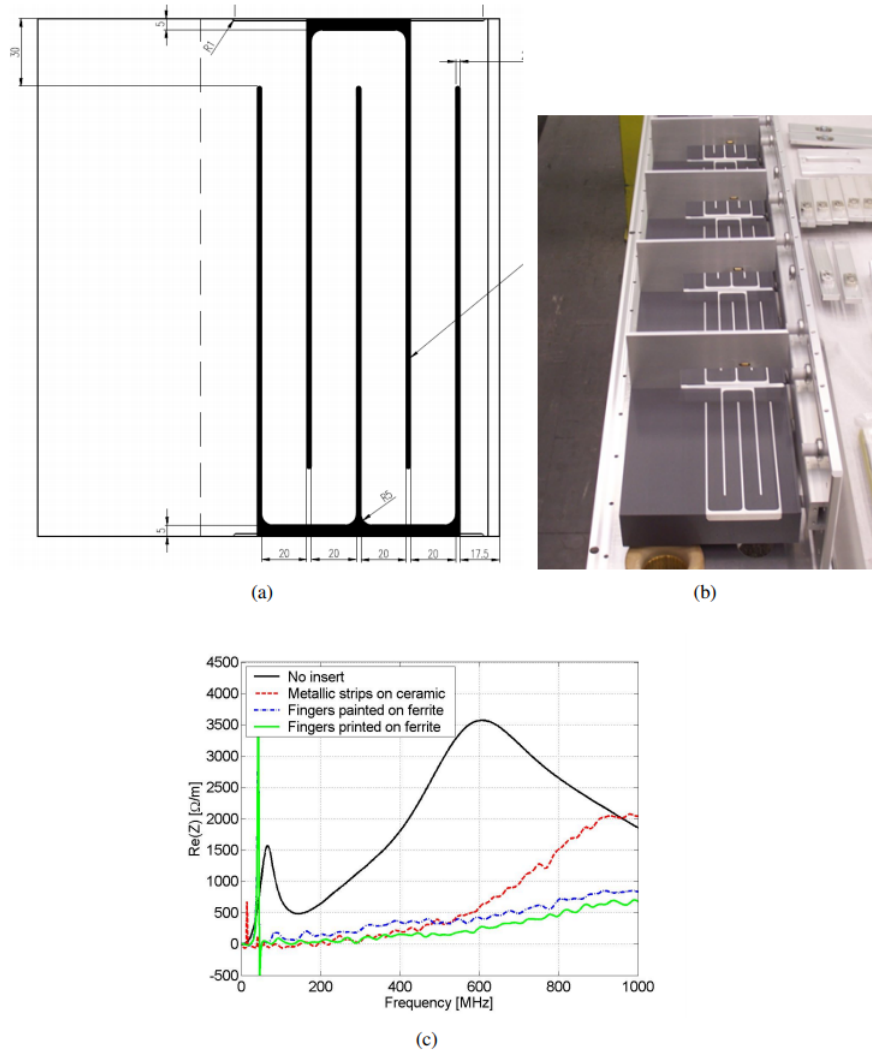


Figure 3.4 – An example of serigraphy in the SPS Extraction Kicker Magnets (SPS-MKE). The layout of the interleaved fingers is shown in (a) and the actual serigraphed ferrites in (b). A comparison of the measured real component of the longitudinal beam coupling impedance with and without the serigraphy is shown in (c).

number of properties that should be noted in changing them. Both f_r and $\frac{R}{Q}$ are strongly determined by the geometry of the structure, and thus cannot be significantly modified without possibly necessitating a modification of the device, which may hinder the intended operation. Thus one approach to use is to alter the Q of the resonance.

A well known method of altering the Q of a resonant cavity is to add a dispersive or ferritic material to the cavity volume, [23] that is a material that has complex permittivity or permeability, given by $\epsilon_r = \epsilon' - j\epsilon'' = \epsilon'(1 - j \tan \delta_\epsilon)$ and $\mu_r = \mu' - j\mu'' = \mu'(1 - j \tan \delta_\mu)$, respectively, where $\tan \delta_\epsilon = \frac{\epsilon''}{\epsilon'}$ and $\tan \delta_\mu = \frac{\mu''}{\mu'}$.

An example of the complex relative permeability of some sample ferrite damping materials is



Figure 3.5 – Some example of ferrite, of different shape, that can be used.

shown in Fig.3.6.

The addition of this family of materials to the cavity has the following effect on the cavity resonances:

1. The resonant frequency of any resonance ω_r is reduced by the inclusion of the dispersive material. This can be understood either by considering:
 - the $\frac{\mu'}{\epsilon'}$ increasing the effective electrical volume of the cavity by it's inclusion, thereby increasing the effective dimensions of the resonant cavity.
 - the RLC equivalent circuit of a cavity resonance, the inclusion of the dispersive material increases either or both of (depending on the properties of the material) the inductance and capacitance of the cavity, causing the resonant frequency to decrease as $\omega_r = \frac{1}{\sqrt{LC}}$.
2. The $\frac{R}{Q}$ of the cavity experiences little change. There is a slight modification; either an increase that can be attributed to the increased stored energy in the cavity mode due to $\epsilon' \neq 1$ or a decrease due to the rearranging of the field patterns (caused by the inclusion of the dispersive material) decreasing the stored energy.
3. The Q of the resonance is drastically reduced. This is due to the strong change in the damping time of the resonance due to the addition of the damping material. In terms of cavity properties, this can be thought of as the losses in the cavity increasing more rapidly than the stored energy in the cavity. In terms of the RLC circuit representation $Q = R_s \sqrt{\frac{C}{L}}$, thus is it can be seen that the inductance must increase more rapidly than the capacitance, or the capacitance decrease more rapidly than the inductance for the drastic reduction in the quality factor to occur.

As can be seen, the resulting effect is to drastically reduce the Q of a cavity resonance, and then by the relation $R_s = Q \sqrt{\frac{L}{C}}$ it can be seen that the shunt impedance will decrease proportionally to Q . This reduces the peak value of the resonance R_s , but broadens the width of

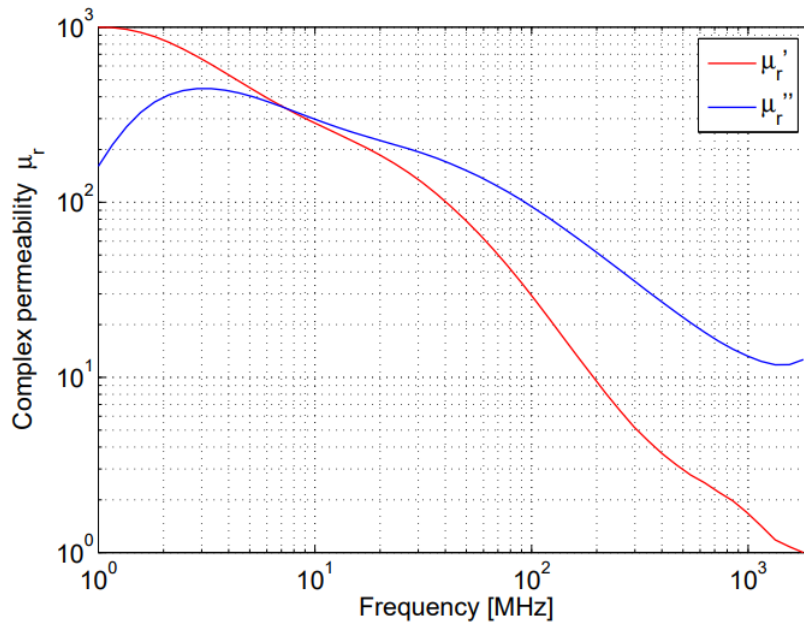


Figure 3.6 – The complex permeability of a sample ferrite of the sort used for damping. In this case 8C11. In usage a high Curie temperature (T_c) ferrite is recommended, such as TT2-111R.

the resonance peak. This indicates two effects of using damping materials as an impedance reduction technique; effects dependent on the shunt impedance R_s are suppressed, however effects dependent on the broadband behavior may suffer negatively as a result. Due to the strong frequency dependent nature of many impedance-driven instability mechanisms and beam-induced heating, these negative side effects rarely outweigh the benefits of using damping material in a cavity if necessary.

The placement of the damping material within the cavity is key to determining how effective the damping will be. This requires knowledge of the field patterns in the cavity modes of the structure. To effectively damp the cavity modes, the damping material must be placed in a position where it strongly interacts with the field associated with it's damping, normally the region of strongest fields.

For dielectric materials, this would be a region of strong electric fields and for ferritic materials a region of strong magnetic fields. In addition, if possible, the materials should not directly be seen by the beam due to the high resistive wall type impedance associated with these types of materials. Requirements for mechanical support often subsequently necessitate that these materials placed by the wall of the vacuum tank of the cavity also. The position should be choose in function of where is convenient to mount the damping materials, even away from the location of the cavity mode if it can be shown that the induced fields will be absorbed by the damping material.

In Appendix D, there are some examples of the main ferrites used (see also Fig.3.5).

3.4.1 Brief introduction of the magnetic materials

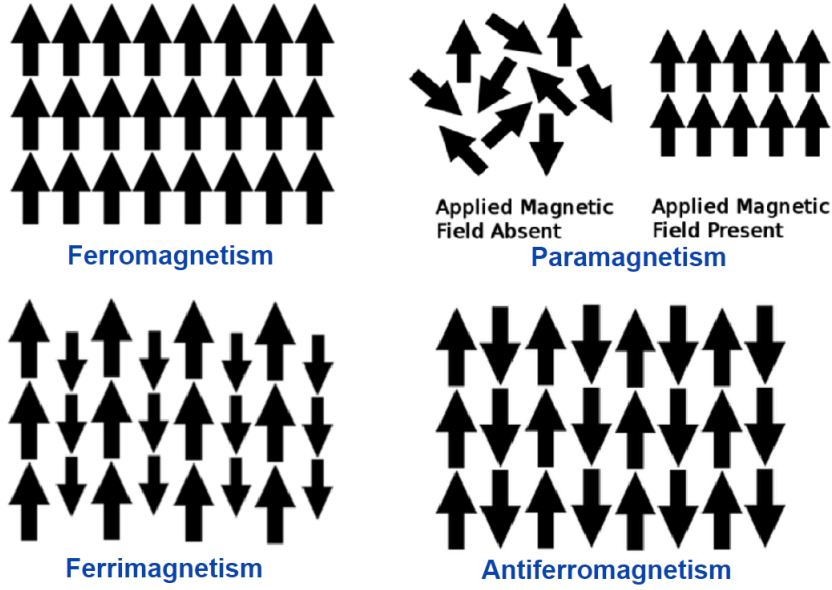


Figure 3.7 – Orientations of magnetic moments in materials

The force of magnetism is determined by the magnetic moment, a dipole moment within an atom which originates from the angular momentum and spin of electrons (see Fig.3.7). Materials have different structures of intrinsic magnetic moments that depend on temperature; the Curie temperature is the critical point at which a material's intrinsic magnetic moments change direction.

Permanent magnetism is caused by the alignment of magnetic moments and induced magnetism is created when disordered magnetic moments are forced to align in an applied magnetic field. Higher temperatures make magnets weaker, as spontaneous magnetism only occurs below the Curie temperature. Magnetic susceptibility above the Curie Temperature can be calculated from the Curie–Weiss law (3.3), which is derived from Curie's law (3.2).

$$\chi = \frac{M}{H} = \frac{M\mu_o}{B} = \frac{C}{T} \quad (3.2)$$

χ is the magnetic susceptibility (take into account the influence of an applied magnetic field on a material), M the magnetic moments per unit volume, H the macroscopic magnetic field, B the magnetic field, C the material-specific Curie constant.

The Curie–Weiss law is then derived from Curie's law to be:

$$\chi = \frac{C}{T - T_C} \quad (3.3)$$

where $T_C = \frac{C\lambda}{\mu_o}$, with λ that is the Weiss molecular field constant (for full derivation see [24]). The Curie temperature is named after Pierre Curie, who showed that magnetism was lost at a critical temperature.

In analogy to ferromagnetic and paramagnetic materials, the Curie temperature can also be used to describe the phase transition between ferroelectricity and paraelectricity.

- The Ferromagnetism:
the moments are ordered and of the same magnitude in the absence of an applied magnetic field.
- The Paramagnetism:
the moments are disordered in the absence of an applied magnetic field and ordered in the presence of an applied magnetic field.
- The Ferrimagnetism:
the moments are aligned oppositely and have different magnitudes due to being made up of two different ions. This is in the absence of an applied magnetic field.
- The Antiferromagnetism:
The moments are aligned oppositely and have the same magnitudes. This is in the absence of an applied magnetic field.

3.5 Drawback of the different methods

Ideally, the best vacuum chamber for an accelerator, is a uniform cross-section for the whole accelerator and with zero resistance.

All the methods shown in this chapter have issues that does not allow us to use in specific cases.

Cost is the most common issue for all: the transition pieces, the conductive coating and the serigraphy.

All three methods need physical modification of the component, which is costly. Cost is maybe the most important issue from a practical point of view.

Now we are going to entry more in the detail to better explain the issues for each methods.

3.5.1 Drawback of the transition pieces

This method is really sensitive to mechanical failure.

For example, when the connection was disrupted in the VMTSA (Fig.3.3(b)), the real component of the longitudinal beam coupling impedance increased drastically at around 200 MHz, shown by the large decrease in the transmission parameter S_{21} in the case of spring failure, causing further mechanical failure.

The breaking can be easily created via mechanical stress due to the weak pressure exerted by the affixing spring (this kind of failure is more common when the equipment is cold).

3.5.2 Drawback of the conductive coatings

The issue of this method is the dependence on of the frequency of interest.

Because the frequency range of concern is dependent on the machine (bunch shape): for beam-induced, the concern is at high frequencies (high depending on the machine, to a couple of gigahertz for proton machines, to tens of gigahertz for electron machines) where thin coatings are effective, whereas for coupled bunch instabilities, low frequencies are a concern, necessitating thicker conductive coatings to be effective. It is possible to use thicker coatings (on the order of millimeters) for machines that require a very broad frequency range screened. All this phenomenon comes from the skin depth effect (see Eq.3.1).

3.5.3 Drawback of the serigraphy

Serigraphy is only used for kickers. The issue of this method is the fact that often is not possible to design all equipment with serigraphy because some of the components are movable components (this is the case for a wire scanner or a collimator).

This is also a critical point for the transition pieces and for the coating.

3.5.4 Drawback of the damping materials

The placement of ferrite in a cavity does not always produce a significant reduction in the Q of a resonance.

The reason for this behavior is linked with the heating up of the ferrite. Because, when the ferrite "absorb" the resonance, its heats up and him reach the Curie Temperature, and it change for each type of ferrite.

Other two issues of the ferrite are: first, the strict range of action in frequency. It means that is necessary find the right ferrite in order to avoid a specific resonance. The second issue is the high fragility of the ferrite in the moment of mounting on the equipment (it is really easy to break a ferrite block while screwing it on the equipment).

PROPOSAL **Part II**

4 A new way to damp the resonances inside an accelerator

In this chapter I will introduce a new way to damp the resonance.

The idea is to use the HOM couplers (they will be explained in the next section 4.2) in order to catch the magnetic field that remains trapped into the equipment.

The HOM couplers that will use are not the same that already are used in the LHC. To absorb the resonance field is possible to choose if work with electric or magnetic field.

I choose to work with the magnetic field, because to catch the electric I would need an antenna inside the equipment and this antenna would need to have a specific length according to the frequencies it couples to, instead the magnetic field can be caught with coupler.

The basic idea is the magnetic field that interacts with the couplers will create an induced current and that current will be dissipated outside of the equipment.

4.1 Introduction of the problem

In order to solve this problem is necessary split it in four part:

1. Make a collection of different design of HOM couplers, to be tested for their efficiency.
2. Setup a measurement to judge the efficiency of the HOM couplers.
3. Find a possible positions of the HOM couplers.
4. Make scripts to find the resonant frequency and to measure the Q-value.

4.2 HOM Couplers

The Higher-order Mode (HOM) couplers are based on the idea of they interact with the magnetic or the electrical field (depend on the configuration used), which characterizes the resonance and dissipate that power outward (see Fig.4.1).

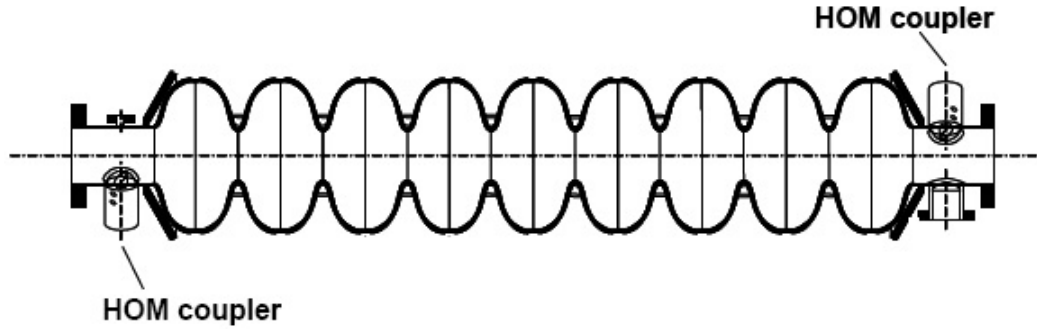


Figure 4.1 – An example of the HOM couplers inside the LHC acceleration cavities.

This actual method is already used in the LHC in order to remove the parasitic resonances inside the LHC acceleration cavities.

The necessity of HOM couplers inside a cavity is to minimize, as much as possible, the resonances that are not used to accelerate the beam, but which will cause beam instability (this concept is called beam loading).

Instabilities caused by real cavities are really dangerous because the Q-factor is really high (in order of 10^5).

In this thesis, I'm going to show a new method to use HOM couplers, in order not to damp a resonance at a specific frequency however to damp a much more large band of frequency (all modes couplers). This modification allows to implement couplers not only inside a cavity but also in a generic equipment. In order to install these HOM couplers inside a general equipment the future design should be smaller and easier. There are basically two methods to couple to the electromagnetic field of a cavity; probe coupling or loop coupling (see Fig. 4.2).

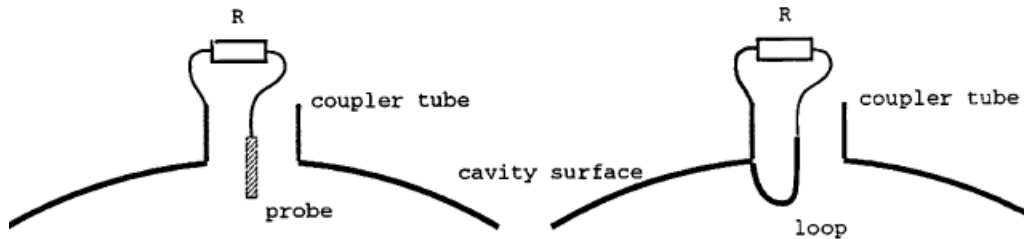


Figure 4.2 – Probe and loop coupling.

In order to understand how to maximize the extracted HOM-power for a given stored energy W_m , we first consider a non-resonant coupler and then modify the equations for the resonant coupler (more detailed discussion is provided in [25]).

The following is a brief explanation from F. Gerigk in how to extract energy from a resonance (the full derivation can be found in [26]).

4.2.1 Non-resonant coupling

With L as self-inductance of the loop and C as fringe field capacitance of the probe type, we can describe the two coupling ports with lumped elements.

According to the Thevenin Theorem we obtain the equivalent generator circuits of Fig. 4.3.

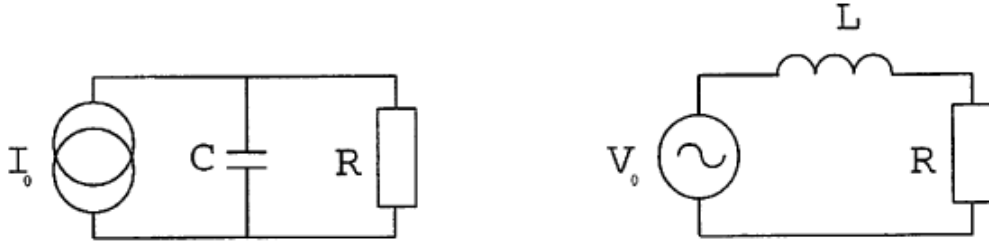


Figure 4.3 – Lumped circuit models of probe and loop coupling.

The current I_0 for the probe case is the displacement current which ends on the probe's surface. The voltage V_0 is induced by the magnetic field that penetrates the loop. We connect these generators to the terminating resistance R or to a transmission line with the characteristic impedance $Z_t = R$. Furthermore, Z is defined as the impedance seen by the current source and Y as the admittance seen by the voltage source. Then, the power extracted from the mode is given by:

$$P_{probe} = \frac{1}{2} I_0^2 \operatorname{Re}(Z) = \frac{1}{2} I_0^2 \frac{1}{Y_t} \frac{Y_t^2}{Y_t^2 + (\omega C)^2} \quad (4.1)$$

$$P_{loop} = \frac{1}{2} V_0^2 \operatorname{Re} Y = \frac{1}{2} V_0^2 \frac{1}{Z_t} \frac{Z_t^2}{Z_t^2 + (\omega L)^2} \quad (4.2)$$

Obviously, there are only two ways to increase the extracted HOM-power for this arrangement. The first one is to rise the induced current I_0 or the induced voltage V_0 , respectively. As the voltage depends on the amount of magnetic flux traversing the loop, we could widen the loop or regarding the probe-case, we could increase the probe's surface in order to get a higher current I_0 .

The following equations demonstrate that the available gain of these measures is strongly limited (the (4.1) and the (4.2) are sufficiently exact with the assumption that the wavelengths of the regarded electromagnetic field are large in comparison to probe and loop dimensions $\lambda \gg l$). This is certainly the case since λ (GHz) = 30cm.

$$I_0 = j\omega CV \quad (4.3)$$

$$V_0 = j\omega LI \quad (4.4)$$

Increasing the loop diameter results in a bigger loop inductance but lowers the real part of the admittance Y (see (4.2)). Therefore the gain, obtained by rising the induced voltage is partly lost. Trying to use this method for the probe results in the same problem.

The second possibility to increase the extracted HOM-power is to decrease Z_t for the loop or to reduce Y_t for the probe without changing the coupling elements (the maximum values for the real part of the impedance or the admittance is achieved for: $Y_t = \omega C$ and for $Z_t = \omega L$).

As the range for practical resistance values of the attached transmission line is limited and as the reactive elements still restrict the extractable HOM-power, this method's success is limited. However, a decisive change of the seen impedance can be attained at the price of some more sophistication.

4.2.2 Resonant coupling

The basic idea for resonant coupling is to reduce the loss of voltage (current), caused by the reactive loop or probe elements. For example, the inductance (capacitance) of the loop (probe) can be compensated by a series capacitor C_c (parallel coil L_c), at least at a certain frequency. With this simple compensation the (4.1) and the (4.2) become:

$$P_{probe_c} = \frac{1}{2} I_0^2 \frac{1}{Y_t} \frac{Y_t^2}{Y_t^2 + \left(\omega C - \frac{1}{\omega L_c} \right)^2} \quad (4.5)$$

$$P_{loop_c} = \frac{1}{2} V_0^2 \frac{1}{Z_t} \frac{Z_t^2}{Z_t^2 + \left(\omega L - \frac{1}{\omega C_c} \right)^2} \quad (4.6)$$

Together with the compensating reactive elements we have built the simplest form of a series (parallel) resonant circuit (see Fig. 4.4).



Figure 4.4 – Lumped circuit models of probe and loop coupling.

At resonance ($\omega = \omega_0$), the impedance (admittance) of both circuits becomes purely resistive assuming that the approaching transmission line resistance is real. The resonant frequency is:

$$\omega_0 = \frac{1}{\sqrt{LC_c}} \quad (4.7)$$

$$\omega_0 = \frac{1}{\sqrt{CL_c}} \quad (4.8)$$

In order to judge the effectiveness of this compensation, we compare the extracted power by dividing the (4.5) and the (4.6) through the (4.1) and the (4.2). This gives an improvement-factor of:

$$g_{probe_c} = \frac{P_{probe_c}}{P_{probe}} = 1 + \left(\frac{\omega C}{Y_t} \right)^2 \quad (4.9)$$

$$g_{loop_c} = \frac{P_{loop_c}}{P_{loop}} = 1 + \left(\frac{\omega L}{Z_t} \right)^2 \quad (4.10)$$

As ideal compensation is only obtained for a single frequency, the bandwidth Δf of the compensating network has to be examined.

The bandwidth can be calculated using the definition of Q for a resonant circuit.

$$Q_{probe} = \frac{f_r}{\Delta f} = \frac{\omega_0 W}{P_L} = \frac{\omega_0 C}{Y_t} \quad (4.11)$$

$$Q_{loop} = \frac{f_r}{\Delta f} = \frac{\omega_0 W}{P_L} = \frac{\omega_0 L}{Z_t} \quad (4.12)$$

From this equation we obtain:

$$\Delta f = \frac{Y_t}{2\pi C} \quad (4.13)$$

$$\Delta f = \frac{Z_t}{2\pi L} \quad (4.14)$$

In our idea of HOM coupler, we will not use a resonant circuit, but only an inductive component that can concatenate the magnetic field. This choice is dictated by the need not to be too selective in frequency. In this case we will talk about a coupler for all modes.

In general, any resonance will try to escape the confinement and therefore will have fields extending to the beam pipe. It should therefore be possible (for most resonances) to couple to them at the edge of the beam pipe.

5 Realization

In this chapter I will show the solution (i.e. the new method with the HOM couplers) found to solve the problem of damping resonances (shown into the chapter 4). Also I will analyze the main parameters that affect the resonance mitigation technique.

In order to find the best combination of parameters, a technique based on "trial and error" method was used, so the values obtained for the parameters are empirical (to get a model that takes on enough details into account longer time is needed).

In the next chapter (6), all the results will be reported.

5.1 Find the resonance frequency

Past works (ref.[27]) used interpolation to find an unknown frequency. This technique is not very precise. With certain test frequencies, we got errors in the order of few %.

Now I will introduce a much improved model, this model is based on the use of a windowing operation in order to filter out the noise floor (Nuttall window) is used. Then a Fourier Transformation of the signal is done and the three coefficients, in the frequency spectrum, closest to the frequency with the maximum amplitude ([28]).

Finally We can now find the frequency that minimizes the difference between the three Fourier coefficients in the measured spectrum and the three Fourier coefficients for the unknown

frequency. The Fourier coefficients are:

$$\begin{aligned} C_0 &= \frac{1}{N} \sum_{n=0}^{N-1} NuttSine(n) e^{-j \frac{2\pi}{T} f_{max0}} \\ C_1 &= \frac{1}{N} \sum_{n=0}^{N-1} NuttSine(n) e^{-j \frac{2\pi}{T} f_{max1}} \\ C_2 &= \frac{1}{N} \sum_{n=0}^{N-1} NuttSine(n) e^{-j \frac{2\pi}{T} f_{max2}} \end{aligned} \quad (5.1)$$

$$\begin{aligned} C_{u0} &= \frac{1}{N} \sum_{n=0}^{N-1} Nutt \cdot Sine(n) e^{-j \frac{2\pi}{T} f_{max0}} \\ C_{u1} &= \frac{1}{N} \sum_{n=0}^{N-1} Nutt \cdot Sine(n) e^{-j \frac{2\pi}{T} f_{max1}} \\ C_{u2} &= \frac{1}{N} \sum_{n=0}^{N-1} Nutt \cdot Sine(n) e^{-j \frac{2\pi}{T} f_{max2}} \end{aligned}$$

The first three Fourier coefficients (including windowing) are the coefficients around the frequency in the measurement. The last three Fourier coefficients are the coefficients for the unknown frequency.

Fig.5.1 shows an example of applying the Nuttall window (basically multiplying $f(n) \cdot w(n)$):

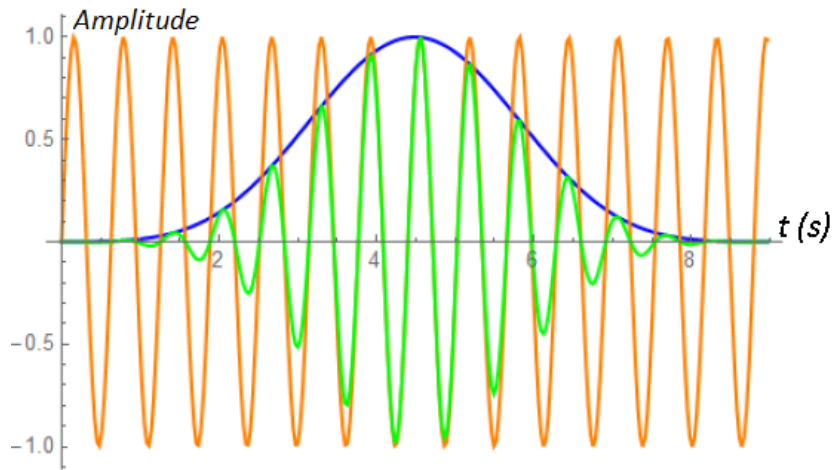


Figure 5.1 – Nuttall windows in blue, sine in orange and Nuttall window multiplied with the sine in green (in time).

Fig.5.2 shows the frequency spectrum of the signals shown in Fig. 5.1:

The algorithm will find the frequency that best match the three highest amplitude in the

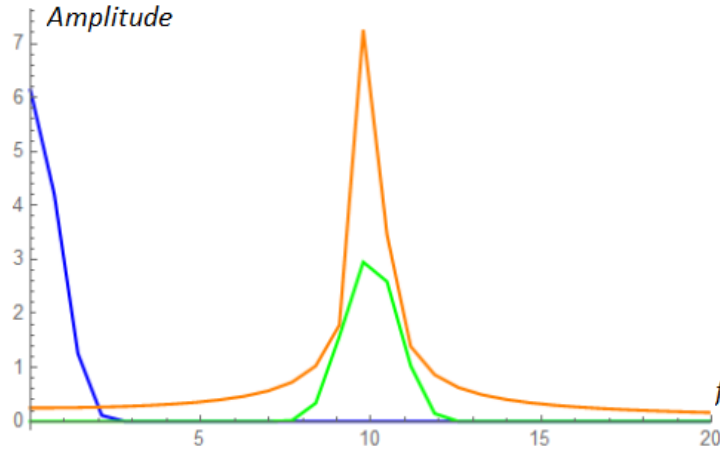


Figure 5.2 – Nuttall windows in blue, sine in orange and Nuttall window multiplied with the sine in green (in frequency $w_u = 10$).

measured frequency spectrum. The algorithm will minimize the following expression:

$$\begin{aligned} & ((R[C_0] - R[C_{u0}])^2 + (I[C_0] - I[C_{u0}])^2 + (R[C_1] - R[C_{u1}])^2 + \\ & + (I[C_1] - I[C_{u1}])^2 + (R[C_2] - R[C_{u2}])^2 + (I[C_2] - I[C_{u2}])^2 \end{aligned} \quad (5.2)$$

In order to minimize the above quantity as a function of w_u (unknown frequency) the initial frequency is chosen as the frequency of the highest peak.

Since our interest was the measurement of the Q-factor, we decided to test the algorithm in the presence of white Gaussian noise, and two separate fake peaks (for more detail see [29]), now the errors are in the order of part per million.

The full code is present in Appendix E

5.2 Q-factor measurement

I developed a script in order to evaluate the Q factor of a resonance measured with the VNA with the method of measuring in reflection.

This script repeat exactly what was described in the section 2.6.2 for the measurement in reflection. This script automatize the calculation of the Q-factor. The idea is to implement that script directly on the VNA in the near future. The script makes the calculation of the Q-factor much more precise and much faster. The main problem in the realization of this script was that the data is discrete and has a phase delay.

We can see that Fig.5.3 is really different from the Fig.2.16. We need to move the circle in the right place and to interpolate (using a quadratic interpolation) the data in order to obtain the desired shape of the circle. The result after the processing is shown in Fig.5.4:

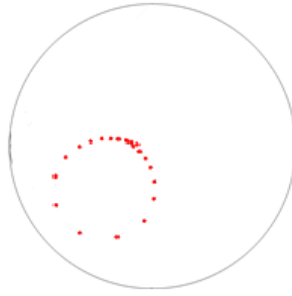


Figure 5.3 – This is the raw data taken from the VNA in a smith chart domain.

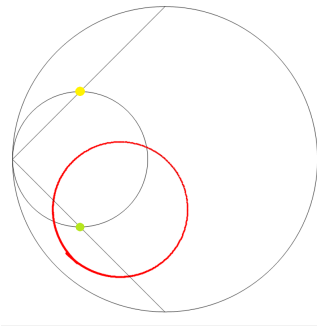


Figure 5.4 – This is the aspect of the data output of the script.

In Fig.5.3 we have the red "circle" that is the measurement data and after a processing, we can see in Fig. 5.4 we have the gray circle that is the interpolated data after the removing of the phase delay.

After that we are going to evaluate the the intersection point between the line a 45° degree and the circle (see green and yellow points in Fig.5.4).

The script is attached in the Appendix F.

5.3 Couplers

The most important feature is the design of the HOM couplers.

In order to find the best design, a large number of different types of inductors were tasted in different positions.

We chosen to test three different diameters (1 *cm*, 1,4 *cm* and 1,8 *cm*), three different lengths (3 *cm*, 6 *cm* and 7 *cm*) and three different numbers of turns (low numbers, medium numbers and high numbers).

In order to choose the best resistances to optimize the power to bring to the outside, potentiometers were used instead of resistors.

5.3.1 Positions

The main parameter to fix is the position of the HOM coupler, because the coupler should be able to strongly interact with the magnetic field trapped in the resonance. In order to be sure of the right positions, we did EM simulations with CST of the equipment. In particular I'm going to show the result of the QT (see section 2.7.1). The EM simulation is shown in Fig.5.5.

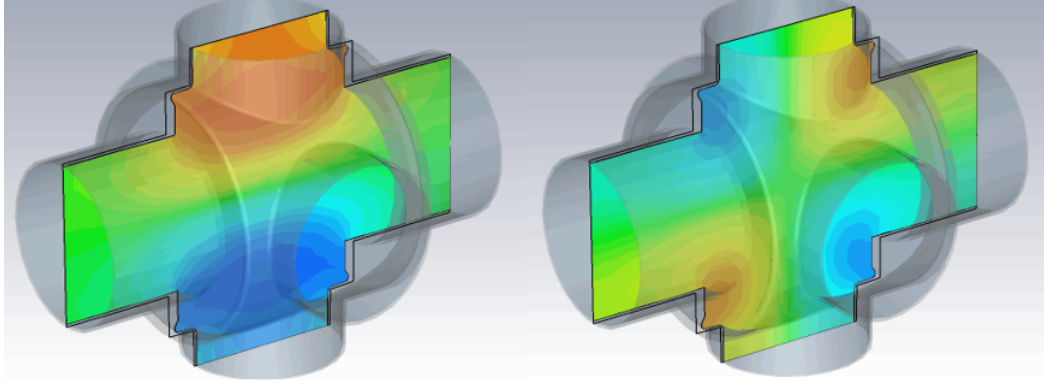


Figure 5.5 – The EM simulation of the QT, obtained with CST program. With the red color we show the highest value of the intensity of the magnetic field.

In this simulation we can see two different resonances, the first at 1.018GHz on the left side and the second at 1.40GHz on the right side.

These frequencies fit perfectly with the frequencies measurement done with the wire to find the resonant frequency in Fig.2.18.

The simulation shows where the magnetic field reaches the maximum value, and this allow us to physically fix the position of the coupler on this place. Two different positions were found and here the magnetic field can enter into the loops and generate the current that will be dissipate outside on the potentiometers.

An illustration of the situation is shown in Fig.5.6.

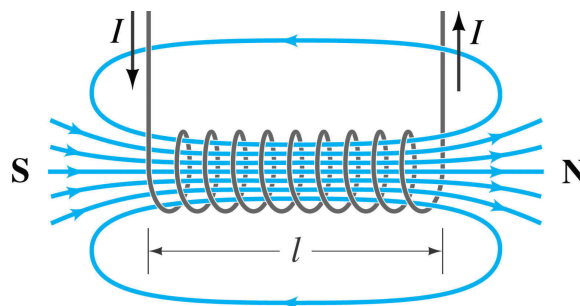


Figure 5.6 – The loop placed orthogonally to the beam passage.

In order to damp the first resonance (at 1.08GHz , located on the top of the equipment) a piece of plastic (used as a holder for the coupler) was attached to the top cover of the structure in

order to fix the height and the position inside the equipment. A picture of the setup is shown in Fig.5.7.

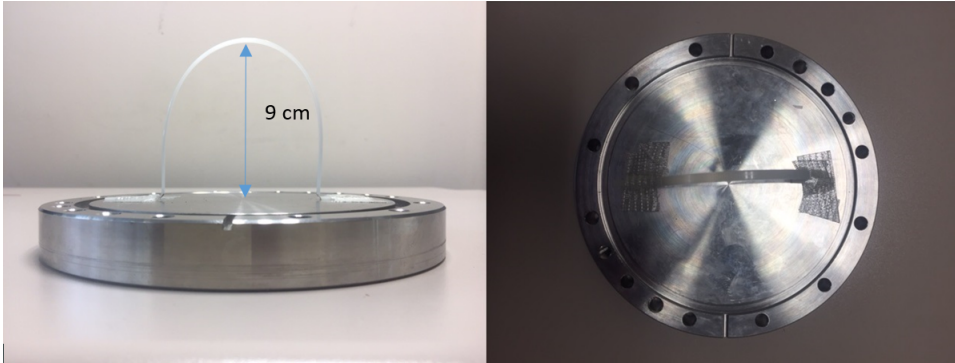


Figure 5.7 – A lateral and top view of the top cover of the QT with the plastic piece to fix the height and the position of the coupler.

The height of the position of the couplers is 9 cm . For the second resonance (at 1.40 GHz located on corner close to the side transition of the equipment) the coupler was directly attached close to the equipment with scotch tape.

After the positioning of the holders for the couplers, a measurement of the S_{12} was done in order to check that the insertion of these holders, only had negligible effect on the field. The measurement is shown in Fig.5.8.

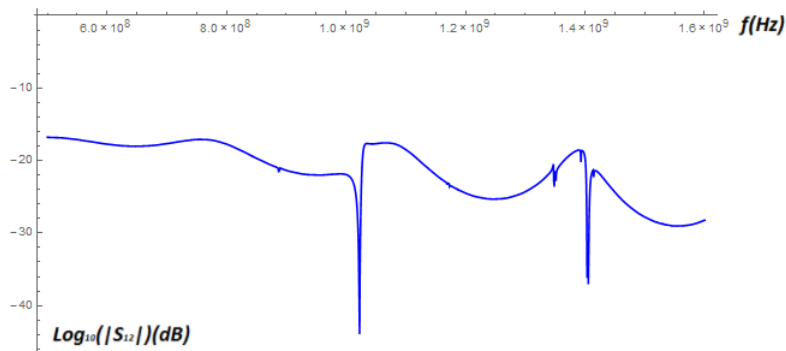


Figure 5.8 – A measurement of the S_{12} parameter of the QT after the insertion of the things for fix the couplers.

For a comparison between the Fig.2.18 and Fig.5.8 is clear that the insertion of these holders only introduce a negligible modification of the field.

5.3.2 Lengths

Three different lengths of the couplers were tested. The first, the shorter one, is 3 cm , the medium length is 6 cm and the big one is 7 cm .

5.3.3 Diameters

Also three different diameters were tested 1cm, 1,4cm and 1,8cm.

In the Fig.5.9 the 3 different cylinders used to made the couplers of the three different diameters are shown.

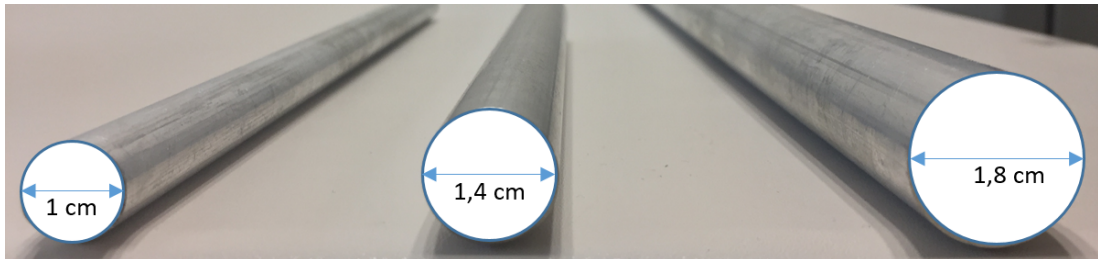


Figure 5.9 – The three different diameters used for realize the couplers.

5.3.4 Number of turns

Three different numbers of turns were used. Three different configurations in terms of numbers of turn were tested, the first with low numbers, the second with medium numbers and the last with high numbers. In the Fig.5.10 a view of the full set of couplers that were tested, is reported.

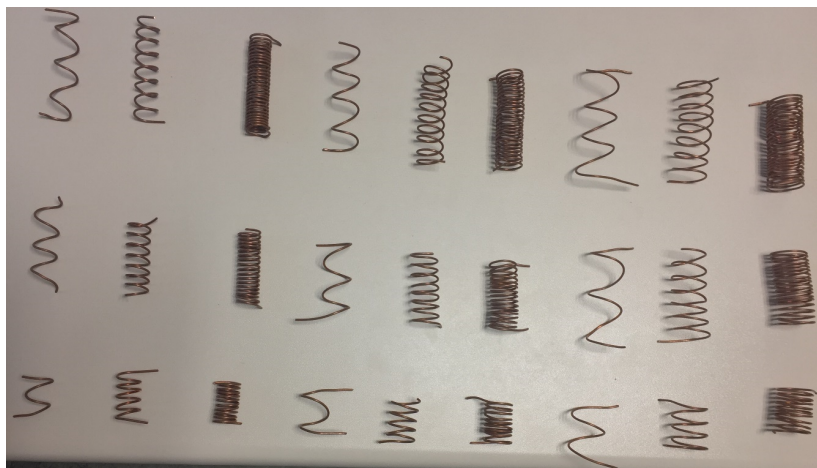


Figure 5.10 – The full set of couplers under test.

In the Fig.5.10 is easy to identify the three types of loop for each parameter.

5.3.5 Value of the resistors

As already mentioned above, instead of a fixed resistor, we preferred to use a potentiometer to be able to continuously vary the resistance value.

The potentiometer used is a commercial model of the *Bourns*, the model number is: 3339P – 1 – 102, the maximum value of resistance is $1\text{ k}\Omega$ and the tolerance is 10% [30].

5.3.6 Positions of the wires

Another important characteristic of the couplers are the that connect the loops to the potentiometer, they should be twisted. This is done to avoid to avoid interference from electromagnetic field that could be otherwise had space between them.



Figure 5.11 – The twisted wire to connect the potentiometer and couplers.

EXPERIMENTAL RESULT

Part III

6 Results

In this chapter, I'm going to show the setup and the results obtained using the methods shown in the previous chapters. All the results are for the QT, in the future everything will be measure also for the BGI in order to solve the problem of heating in the BGI. Already preliminary study has been done.

6.1 Experimental setup

In the QT campaign of measurements, you can identify different process steps. The first step concerns an identification of which coupler provides the best result, this identification will be done measuring all the couplers in the same position (exactly in the middle to interact with the first resonance) and evaluating the attenuation of the resonance.



Figure 6.1 – An overview of the measurement setup.

Chapter 6. Results

After identifying the suitable coupler, we proceed to the positioning of two of them in the positions where we consider the magnetic field to be strongest, see section 5.3 (in particular Fig.5.5).

In the measurements station (Fig.6.1), the network analyzer connected to the QT with the couplers inside.

Note also that we have a multimeter in order to check which value of resistance we are using on the potentiometers (in order to do that, is it necessary to desoldering the potentiometers from the couplers). An example of the couplers is show in Fig.6.2.

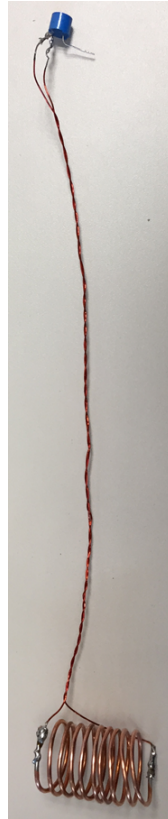


Figure 6.2 – An example of coupler with large diameter, short length and high number of turns.

To identify the effect of the HOM coupler, we use a wire measurement, this technique is only allowed to do a qualitatively study of the shape of the resonance, more details are explained in section 2.1. The wire used is shown in Fig.6.3.



Figure 6.3 – The wire used for the wire measurement in order to find the best coupler. Note the SUCOBoxes at both ends.

We emphasize the presence of matching resistors and the SUCOBoxes, the blue box at both ends of the equipment. The SUCOBoxes allow to connect coaxial cable to QT and back to VNA,

a SUCOBox is used, the SUCOBox allows coaxial cable and wire to be connected in QT. The characteristic resistance of the QT is 344Ω , coax cable is 50Ω , therefore SUCOBox must have a matching resistor of 294Ω .

After the physical setup, we proceed to the measurement of the quality factor. This step involves the removal of the wire to proceed with the measurement in reflection using a probe (The probe used is the shown in Fig.6.4).



Figure 6.4 – The probe used in the prob method to measure the Q-factor.

The result obtained from this measurement will then be used by the script for Q-measurements effectively to study the effect of the HOM couplers (using the script shown in Appendix F).

6.2 Experimental results

These are the results of the wire measurements, which is used to identify the best couplers for the first resonance (the one in the middle) of the QT, so all the coupler are tested in the middle of the QT. In order to have a first idea of which of the couplers should be used, the reduction, in dB, of the magnitude of the resonance was measured (this parameter was chosen because of the good indicator of the change of the quality factor). The results are reported in the following Tab.6.1. The best result is obtain, for the tester 9, which is characterized by a diameter of $1,8cm$ (the biggest) a length of $3cm$ (the shorter one) and a high number of turns. The coupler is shown in Fig.6.5.

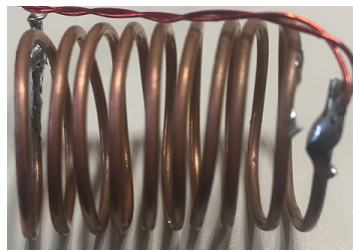


Figure 6.5 – The best coupler for the QT at $1,08GHz$.

After that identification, another identical coupler was done in order to put two of that in the right place (see Fig.5.5) to absorb both the resonance. The result obtained are shown in the Fig.6.6. In the Fig.6.6, we can see the resonance before the coupler (in blue) and after the addition of the coupler.

The result is attenuation of $17,55dB$ for the first one and an attenuation of $13,92dB$ for the second one.

Now using that result we can evaluate the Q-factor before and after the using of the couplers.

#	Diameter (cm)	Length (cm)	Intensity of turns	Attenuation (dB)
1	1	3	LOW	0,88
2	1	3	MEDIUM	3,13
3	1	3	HIGH	1,02
4	1,4	3	LOW	1,69
5	1,4	3	MEDIUM	1,68
6	1,4	3	HIGH	3,95
7	1,8	3	LOW	7,99
8	1,8	3	MEDIUM	7,43
9	1,8	3	HIGH	18,55
10	1	6	LOW	-0,36
11	1	6	MEDIUM	2,63
12	1	6	HIGH	3,76
13	1,4	6	LOW	1,40
14	1,4	6	MEDIUM	6,53
15	1,4	6	HIGH	5,83
16	1,8	6	LOW	7,78
17	1,8	6	MEDIUM	4,77
18	1,8	6	HIGH	3,44
19	1	7	LOW	8,98
20	1	7	MEDIUM	11,80
21	1	7	HIGH	7,37
22	1,4	7	LOW	13
23	1,4	7	MEDIUM	11,03
24	1,4	7	HIGH	8,42
25	1,8	7	LOW	8,28
26	1,8	7	MEDIUM	9,78
27	1,8	7	HIGH	1,39

Table 6.1 – Table of results of measurement for identify which coupler provides the best result.

That study was done for the second resonance, at 1,40GHz (see Fig.6.7).

The numerical result are shown in Tab.6.2

No coupler	With coupler
2282.14	848.07

Table 6.2 – The results in terms of Q-factor before and after the HOM couplers.

As a last step I compare our results with the damping obtained with ferrites (the current most widely used method, see section 3.4).

The result obtained by inserting ferrite blocks is shown in the Fig.6.8.

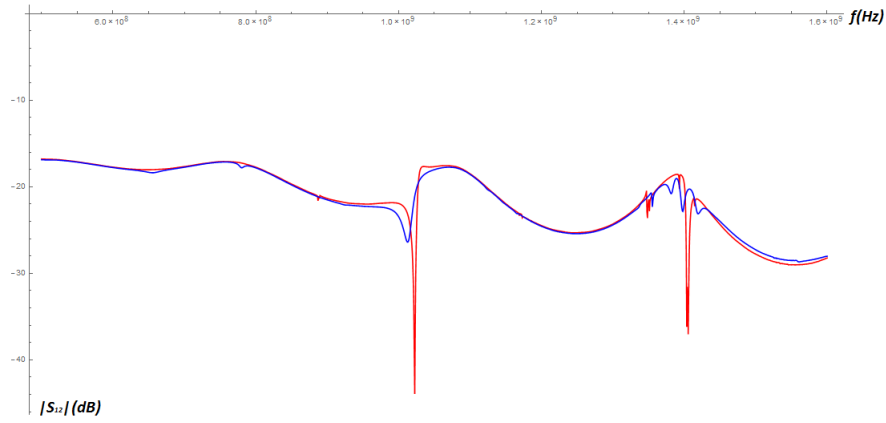


Figure 6.6 – The result using two couplers, in red is the measure without the couplers in blue is with.

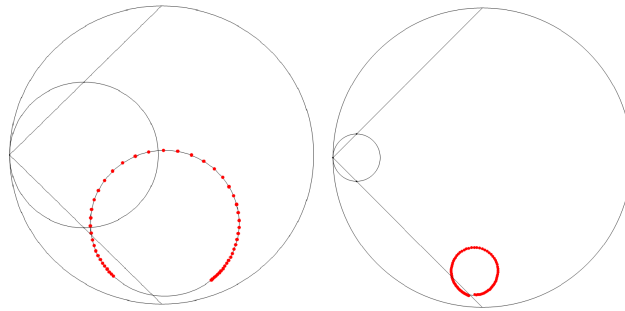


Figure 6.7 – How the resonance looks like in the script, before (left) and after (right) the HOM couplers.

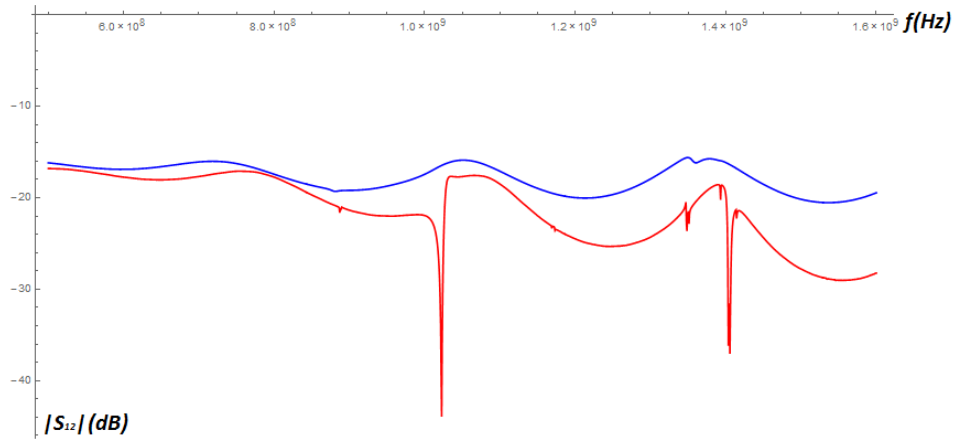


Figure 6.8 – Damping result obtained by the ferrite blocks.

All the consideration about the results, will be present in the conclusion (6.2).

Conclusion

The test results are very promising, the damping method using HOM couplers works very well in the case of the QT.

When comparing HOM couplers to ferrite (see the Fig 6.6 and the Fig.6.8), we can see that still the ferrite works better, but the damping obtained with the HOM couplers is not far off the damping with ferrites.

It is important to remark that this is an experimental project and for that reason the results presented are just to give a validation of the method. More work is needed to make the method more robust. There are still issues with the tolerance of the measurements done, because everything was done in order to obtain the best result in terms of repeatability and accuracy but was difficult to have strict control over some parameters, like the lengths and the intensity of turns of the couplers, because all the items were done manually.

The scope of this work, as said in the introduction, is to propose a new idea that needs to be studied further.

This study should be carried out taking into account the electromagnetic field produced by a particle that moves at the speed of the light and how this field can interact with couplers.

The idea of trying several different couplers was done in order to test the method. The strengths of this method are many, including:

- **Economic:**
this method does not require a redesign of the equipment, it does not involve expensive modifications such as coating, and the price of a coupler is negligible compared to the average equipment used in CERN's laboratories;
- **Ease of installation:**
after an EM simulation it is easy to understand where the coupler should be placed;
- **Physically robust:**
the couplers are not brittle like the ferrite blocks and are also insensitive to temperature;
- **All mode coupler:**
the use of a magnetic coupler that works for all frequency frees us from the frequency dependence of the ferrites.

Conclusion

During the QT measurement campaign, a serious drawback was found. The effect of the coupler is really sensitive (if the coupler is touched the damping effect change).

This sensitivity made the repeatability of experiments difficult. However, this problem is common for all circuit working at high frequencies.

It is important to underline that the promising result has been obtained with the HOM couplers is research type, and when the technique will be better developed, hopefully excellent results will be reach in the future.

In the future, a measurement campaign is planned on the BGI, which is overheating due to the resonances inside it.

A Appendix: Introduction to the CERN experiments

Six detectors have been constructed at the LHC, located underground in large caverns excavated at the LHC's intersection points. Two of them, the ATLAS experiment and the Compact Muon Solenoid (CMS), are large, general purpose particle detectors (see Fig.A.1). A Large Ion Collider Experiment (ALICE) and LHCb have more specific roles and the last two TOTEM and LHCf are very much smaller and are for very specialized research. ATLAS is one of two

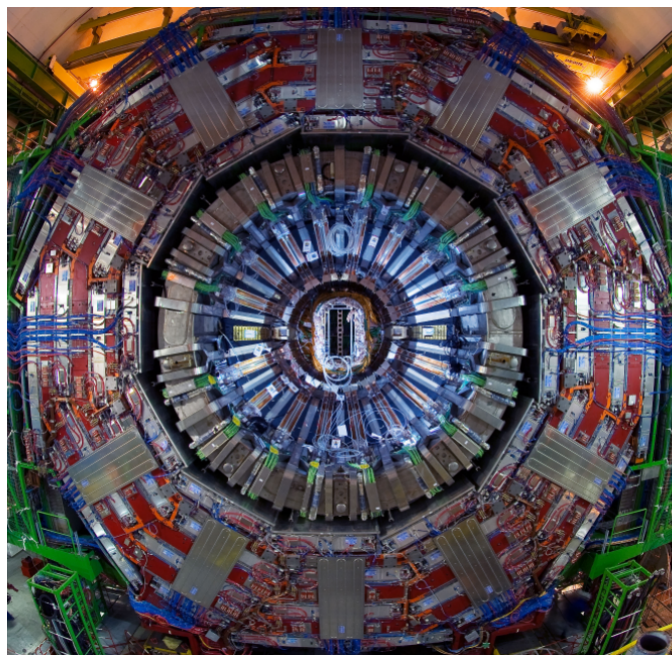


Figure A.1 – The CERN Compact Muon Solenoid detector (CMS).

so-called general purpose detectors. ATLAS will be used to look for signs of new physics, including the origins of mass and extra dimensions. CMS is the other general purpose detector by which the Higgs boson has been discovered and looks for the origin of the nature of the dark matter. ALICE will study a "liquid" form of matter called quark-gluon plasma that existed

Appendix A. Appendix: Introduction to the CERN experiments

shortly after the Big Bang. Equal amounts of matter and anti-matter were created in the Big Bang. LHCb will try to investigate what happened to the "missing" anti-matter.

B Appendix: Panofsky-Wenzel Theorem

B.1 Basic approximations

To prove the theorem, we need to look at how the fields are and how they act on the test charge. So, we consider a driving charge followed by a test charge that enters in a chamber and creates the wake fields.

Let's go to refresh the two basic approximations used in order to simplify the mathematical description of wake functions, the rigid bunch and the impulse approximations [7]. In the rigid bunch approximation, the beam traversing through the vacuum chamber is assumed to be not affected by its discontinuities. Looking at Fig.B.1, is the distance of the source particle along the vacuum chamber axis, from an arbitrary reference point. Let the source particle be at $s = \beta ct$ and the following the test particle at $s = z + \beta ct$, with $z > 0$ to indicate that the witness stays behind the source. Being the bunch rigid, both z and βc do not change after traversing the discontinuity, even if synchrotron motion is still allowed.

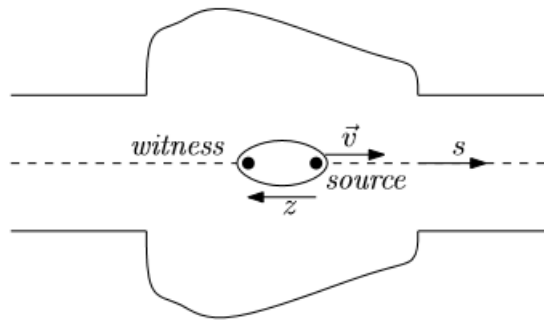


Figure B.1 – The rigid bunch approximation. Both distance between source and test particles, z , and particles velocity, βc , do not change during vacuum chamber discontinuities traversal.

B.2 Derivation of the Panofsky-Wenzel theorem

Let the Maxwell equations be rewritten for the test particle at (x, y, s, t) , with z constant and $s = z + \beta ct$, where β is the velocity factor $\beta = \frac{v}{c}$ and v is the speed of the particle.

$$\begin{aligned}
 \nabla \cdot \vec{E} &= \frac{\rho}{\epsilon_0} \\
 \nabla \times \vec{B} &= \frac{1}{c^2} \frac{\delta \vec{E}}{\delta t} + \beta c \rho \vec{s} \\
 \nabla \cdot \vec{B} &= 0 \\
 \nabla \times \vec{E} &= -\frac{\delta \vec{B}}{\delta t}
 \end{aligned} \tag{B.1}$$

where $\vec{s}$ the unit vector of the s direction.

Given the Lorentz force definition in Eq.1.1, the Panofsky-Wenzel theorem arise quite naturally from (B.1).

$$\begin{aligned}
 \nabla \times \vec{F} &= \nabla \times q(\vec{E} + \vec{v} \times \vec{B}) = q \left(-\frac{\delta \vec{B}}{\delta t} + \beta c \nabla \times \vec{s} \times \vec{B} \right) = \\
 &= q \left(-\frac{\delta \vec{B}}{\delta t} + \beta c [\vec{s}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{s}) + (\vec{B} \cdot \nabla) \vec{s} - (\vec{s} \cdot \nabla) \vec{B}] \right) = \\
 &= q \left(-\frac{\delta \vec{B}}{\delta t} + \beta c [\vec{s}(0) - \vec{B}(0) + B_z \frac{\delta \vec{s}}{\delta z} - (\vec{s}' \cdot \nabla) \vec{B}] \right) = q \left(-\frac{\delta \vec{B}}{\delta t} - \beta c \frac{\delta \vec{B}}{\delta z} \right)
 \end{aligned} \tag{B.2}$$

Let's look at the components of this equation:

$$\begin{aligned}
 \frac{\delta F_z}{\delta y} \bar{x} - \frac{\delta F_y}{\delta z} \bar{x} &= q \left(-\frac{\delta B_x}{\delta t} - \beta c \frac{\delta B_x}{\delta z} \right) \bar{x} = \\
 \frac{\delta F_x}{\delta z} \bar{y} - \frac{\delta F_z}{\delta x} \bar{y} &= q \left(-\frac{\delta B_y}{\delta t} - \beta c \frac{\delta B_y}{\delta z} \right) \bar{y} = \\
 \frac{\delta F_y}{\delta x} \bar{z} - \frac{\delta F_x}{\delta y} \bar{z} &= q \left(-\frac{\delta B_z}{\delta t} - \beta c \frac{\delta B_z}{\delta z} \right) \bar{z} =
 \end{aligned} \tag{B.3}$$

We'll take only the first equations, we obtain:

$$\begin{aligned}
 \frac{\delta F_y}{\delta z} \bar{x} &= \frac{\delta F_z}{\delta y} \bar{x} - q \left(-\frac{\delta B_x}{\delta t} - \beta c \frac{\delta B_x}{\delta z} \right) \bar{x} = \\
 \frac{\delta F_x}{\delta z} \bar{y} &= \frac{\delta F_z}{\delta x} \bar{y} + q \left(-\frac{\delta B_y}{\delta t} - \beta c \frac{\delta B_y}{\delta z} \right) \bar{y} =
 \end{aligned} \tag{B.4}$$

Now adding that equations, we obtain:

$$\frac{\delta F_x}{\delta z} \bar{y} + \frac{\delta F_y}{\delta z} \bar{x} = \frac{\delta F_z}{\delta y} \bar{x} + \frac{\delta F_z}{\delta x} \bar{y} + q \left(-\frac{\delta B_y}{\delta t} - \beta c \frac{\delta B_y}{\delta z} \right) \bar{y} + q \left(-\frac{\delta B_x}{\delta t} - \beta c \frac{\delta B_x}{\delta z} \right) \bar{x} \quad (\text{B.5})$$

Using the ∇ notation:

$$\nabla_{\perp} F_z = \frac{\delta \vec{F}_{\perp}}{\delta z} - q \left(-\frac{\delta B_y}{\delta t} \bar{y} - \beta c \frac{\delta B_y}{\delta z} \bar{y} + \frac{\delta B_x}{\delta t} \bar{x} + \beta c \frac{\delta B_x}{\delta z} \bar{x} \right) \quad (\text{B.6})$$

This equation is true for every particle that goes through some electromagnetic fields. It has still nothing linked to the accelerator or to the driving bunch. As we can see it expresses the relation between the longitudinal and transverse components of the force but it does not look so simply.

To simplify it a bit more we have to take in account the distance between the driving charge and the test particle. In other words we have to say that the fields are generated by the driving charge. We started the proof setting the distance. This means that this equation is true for a fixed distance s_0 . Let's consider a driving charge that at a time t is in a certain position νt . Behind it, at a distance s there is the test charge in the position (x, y, z) . This means that the position of the test particle is:

$$z = \nu t - s \quad (\text{B.7})$$

We expressed the distance as a space interval but we can actually use also the time. We write:

$$z = \nu(t - \tau) \quad (\text{B.8})$$

It is important to highlight that t and τ are both time but express different concepts: one is linked to the position of the particles, the other one is linked to the time delay in between them. On the other side s and τ express the same concept but in two different domain, respectively space domain and time domain. This means that we can use s or τ without any great change in the equations. We will choose to use. So defining z and τ as:

$$z = \nu(t - \tau) \quad \tau = \frac{\nu t - z}{\nu} \quad (\text{B.9})$$

We can redefine the fields as:

$$\vec{E}(x, y, z, \tau) \quad \vec{B}(x, y, z, \tau) \quad (\text{B.10})$$

These seem exactly the same fields we used before but there is a main difference: now the z and τ are linked together by s . This is what allow us to simplify the relation between the longitudinal and transverse components of the Lorentz force.

It is worth to emphasize that this is true only if the fields are created by the driving charge. Otherwise the fields depend on s .

Let's take the same example of before. We have a driving charge that at a time t is in a certain

Appendix B. Appendix: Panofsky-Wenzel Theorem

position. Behind it, at a distance s there is the test charge in the position (x, y, z) . Let's suppose to shrink the distance, taking s' . This is exactly same of shrinking the time interval τ into τ' . We can analyse this situation from two points of view as shown in the pictures.

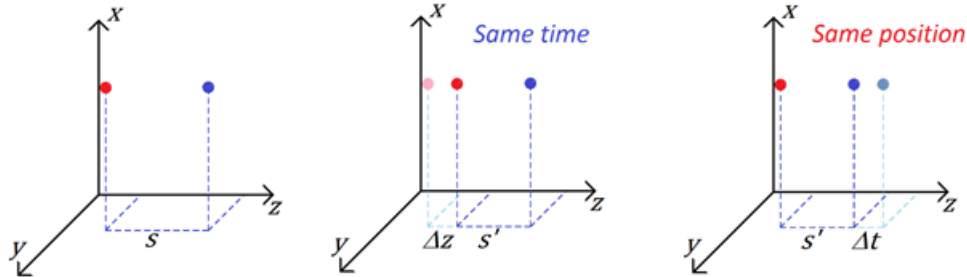


Figure B.2 – The possibility situations (same time and same position).

- Same time

In this case we are in the same instant of time. The driving particle is in the same position as before (because there is no change in its velocity) but given the shortened distance the test particle is in a position $z' < z$. The fields that it feels are:

$$\vec{E}(x, y, z', \tau) \quad \vec{B}(x, y, z', \tau) \quad (\text{B.11})$$

With z' equal to:

$$z' = z + (-\Delta s) \quad (\text{B.12})$$

Where we used Δs because the variation is positive considering positive the left side for z axes. Bringing the z on the other side we obtain:

$$z' - z = \Delta s \quad (\text{B.13})$$

$$-\Delta z = \Delta s \quad (\text{B.14})$$

- Same position

In this case the test particle is in the same position of before. Given the shortened distance this means that less time is passed, so $t' < t$. The fields that the test charge feels are

$$\vec{E}(x, y, z, \tau') \quad \vec{B}(x, y, z, \tau') \quad (\text{B.15})$$

With t' equal to

$$t' = t - \left(\frac{-\Delta s}{v} \right) \quad (\text{B.16})$$

Same considerations about $-\Delta s$ of before are applied. Bringing t on the other side we

B.2. Derivation of the Panofsky-Wenzel theorem

have:

$$t' - t = -\frac{\Delta s}{v} \quad (\text{B.17})$$

$$\Delta t = \frac{\Delta s}{v} \quad (\text{B.18})$$

From this considerations we have obtained to important relations:

$$\frac{\delta s}{\delta z} = -1 \quad (\text{B.19})$$

$$\frac{\delta s}{\delta t} = v \quad (\text{B.20})$$

Now looking again at the relation B.6, Where now the fields and the force depends also on τ (that is the same as s):

$$\vec{F}(x, y, z, \tau) \quad \vec{E}(x, y, z, \tau) \quad \vec{B}(x, y, z, \tau) \quad (\text{B.21})$$

We can change the partial derivatives:

$$\nabla_{\perp} F_z = \frac{\delta s}{\delta z} \frac{\delta \vec{F}_{\perp}}{\delta z} - q \left(-\frac{\delta s}{\delta z} \frac{\delta B_y}{\delta t} \bar{y} - \beta c \frac{\delta s}{\delta z} \frac{\delta B_y}{\delta z} \bar{y} + \frac{\delta s}{\delta z} \frac{\delta B_x}{\delta t} \bar{x} + \beta c \frac{\delta s}{\delta z} \frac{\delta B_x}{\delta z} \bar{x} \right) \quad (\text{B.22})$$

And therefore:

$$\nabla_{\perp} F_z = \frac{\delta \vec{F}_{\perp}}{\delta s} - q \left(\beta c \frac{\delta B_y}{\delta s} \bar{y} - \beta c \frac{\delta B_y}{\delta s} \bar{y} + \beta c \frac{\delta B_x}{\delta s} \bar{x} + \beta c \frac{\delta B_x}{\delta s} \bar{x} \right) \quad (\text{B.23})$$

Where we have used $v = \beta c$. The terms in the bracket cancel and we obtain:

$$\nabla_{\perp} F_z(x, y, z, t) = \frac{\delta \vec{F}_{\perp}}{\delta s} \quad (\text{B.24})$$

C Appendix: Power Loss derivation

Following is attached the full derivation of the power loss using the Wolfram Mathematica.

```
Clear[λ, ΔE, Wake, Fλ, FΔE, FWake, z, zp, w, ΔEnergy]
```

Functions in z domain (i.e. time)

```
a = 2 π;
Wake[z_] = Cos[z * 3.] + 1;
λ[z_] = { 0                                z < -a
         PDF[NormalDistribution[0, 1.5], z]  -a ≤ z ≤ a;
         0                                a < z
ΔE[z_] = Chop[ExpToTrig[Convolve[Wake[zp], λ[zp], zp, z]] // FullSimplify];
```

Functions in frequency domain (i.e w)

The Fourier transform of a function $f(t)$ is by default defined to be $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$.

```
FWake[w] = FullSimplify[Chop[FourierTransform[Wake[x], x, w]]];
Fλ[w] = FullSimplify[Chop[FourierTransform[λ[z], z, w]]];
FΔE[w] = FullSimplify[Chop[FourierTransform[ΔE[x], x, w]]];
```

ENERGY calculation

```
ΔEnergy = ∫-aa λ[z] * ΔE[z] dz // Chop
0.999944
```

Integrating from infinite to infinite gives the same result, because the charge distribution is limited to a window

```
ΔEnergy = ∫-∞∞ λ[z] * ΔE[z] dz // Chop
0.999944
```

Using Plancherel_theorem (i.e the integral form of the Parseval theorem)

In its common form:

$$\int_{-\infty}^{\infty} f(x) \overline{g(x)} dx = \int_{-\infty}^{\infty} \hat{f}(\xi) \overline{\hat{g}(\xi)} d\xi,$$

where

$$\int_{-\infty}^{\infty} f(x) e^{-2\pi i \xi x} dx = \hat{f}(\xi).$$

`Hyperlink["https://en.wikipedia.org/wiki/Plancherel_theorem"]`

`https://en.wikipedia.org/wiki/Plancherel_theorem`

$$\Delta \text{Energy} = \int_{-\infty}^{\infty} \text{Conjugate}[F\lambda[w]] * F\Delta E[w] dw // \text{Chop}$$

0.999944

Using the convolution theorem. since ΔE is a convolution of Wake and λ

$$\mathcal{F}\{f * g\} = \mathcal{F}\{f\} \cdot \mathcal{F}\{g\}$$

`Hyperlink["https://en.wikipedia.org/wiki/Convolution_theorem"]`

`https://en.wikipedia.org/wiki/Convolution_theorem`

$$\Delta \text{Energy} = \int_{-\infty}^{\infty} \text{Conjugate}[F\lambda[w]] * \sqrt{2\pi} * F\text{Wake}[w] * F\lambda[w] dw // \text{Chop}$$

0.999944

Combining: $\text{Conjugate}[F\lambda[w]] * F\lambda[w] = \text{Abs}[F\lambda[w]]^2$

$$\Delta \text{Energy} = \int_{-\infty}^{\infty} \text{Abs}[F\lambda[w]]^2 * \sqrt{2\pi} * F\text{Wake}[w] dw // \text{Chop}$$

0.999944

$$\Delta \text{Energy} = \sqrt{2\pi} * \int_{-\infty}^{\infty} \text{Abs}[F\lambda[w]]^2 * F\text{Wake}[w] dw // \text{Chop}$$

0.999944

D Appendix: Ferrites

There are a number of ferrites used for the damping of cavity modes in accelerator equipment, some of the details of which are given here.

For the use of ferrite as a damping material, the important material properties to consider are the complex permeability $\mu_r = \mu' - j\mu''$, which determines the degree of damping of cavity modes, and the Curie Temperature (TC) which determines to which temperature the ferrite will remain effective as a damping material, important due to the significant power loss that may occur in the ferrite.

Here we give examples of three ferrites commonly used for damping of cavity modes, and one additional ferrite that shows promise for use due to a high TC.

An additional figure of merit is the penetration depth of the ferrite. This figure determines how far the magnetic field penetrates into the ferrite, thus giving a guide as to how much ferrite is necessary to effectively damp any cavity modes. This is a frequency dependent value, given by the skin depth, we can write the frequency depending in the following way:

$$\delta(f) = \frac{c}{2\pi f} \frac{1}{\sqrt{1 - \epsilon_r(f)\mu_r(f)}} \quad (\text{D.1})$$

D.1 TT2-111R

TT2-111R is a ferrite made by Transtech.

It is a variant of the TT2-111 ferrite with a slight conductivity to reduce the build up of electrostatic charge. TT2-111R is useful in as it has a very high Curie Temperature $T_c \approx 375^\circ\text{C}$.

ϵ'	12.5	
Conductivity	10^{-1}	Sm^{-1}
T_c	375°	C

Table D.1 – A selection of the physical properties of TT2-111R

Appendix D. Appendix: Ferrites

Some material properties are shown in Tab.D.1. The complex permeability is shown in Fig.D.1(a) and the penetration depth in Fig.D.1(b).

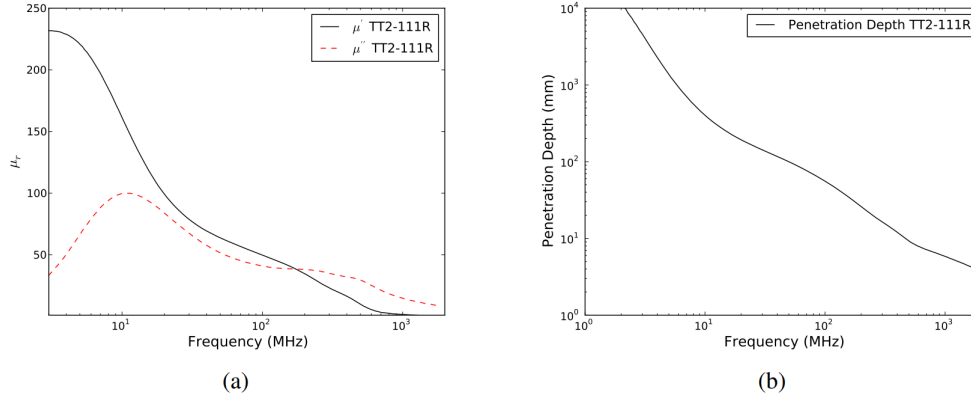


Figure D.1 – The permeability (a) and penetration depth (b) for TT2-111R.

D.2 4S60

4S60 is a ferrite made by Ferroxcube. It has a slight conductivity. Some material properties are shown in Tab.D.2.

ϵ'	10	
Conductivity	10^{-5}	Sm^{-1}
Tc	100°	C

Table D.2 – A selection of the physical properties of 4S60

The complex permeability is shown in Fig.D.2(a) and the penetration depth in Fig.D.2(b).

D.3 4A4

4A4 is a NiZn ferrite made by Transtech. It has a mild conductivity to reduce electrostatic buildup.

It is commonly used to damp cavity modes due to its good vacuum performance. Some material properties are shown in Tab.D.3.

The complex permeability is shown in Fig.D.3(a) and the penetration depth in Fig.D.3(b).

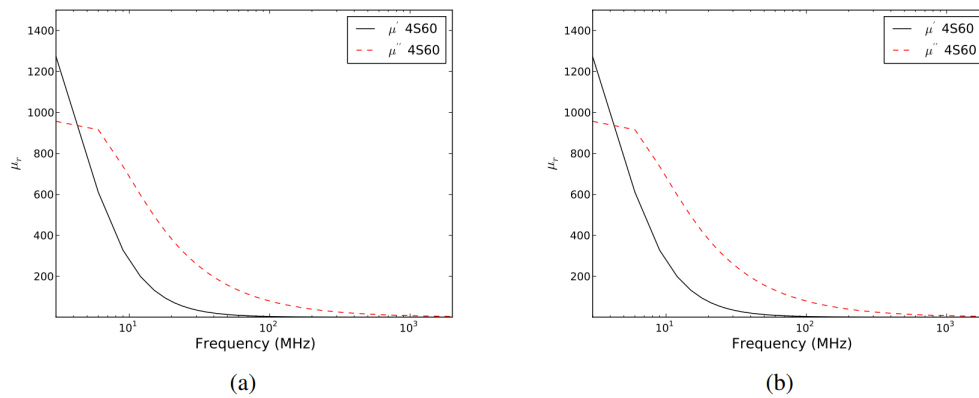


Figure D.2 – The permeability (a) and penetration depth (b) for 4S60.

ϵ'	10	
Conductivity	10^{-6}	Sm^{-1}
T_c	375°	C

Table D.3 – A selection of the physical properties of 4S60

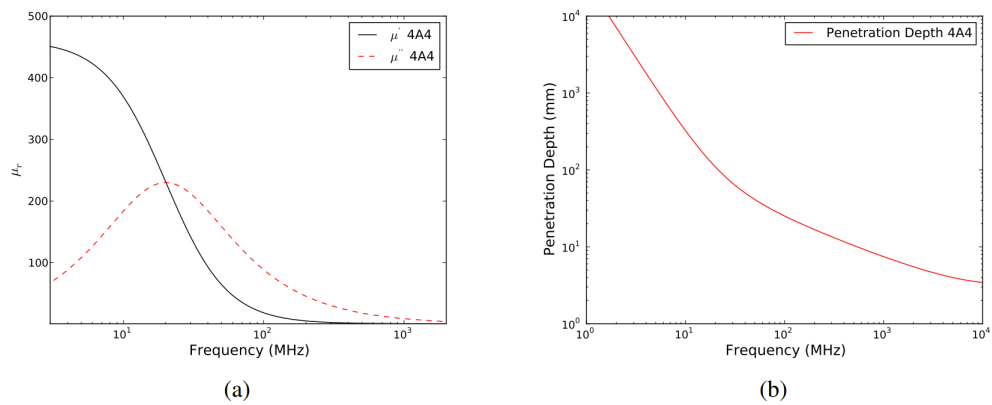


Figure D.3 – The permeability (a) and penetration depth (b) for 4A4.

D.4 4E2

4E2 is a NiZn ferrite made by Ferroxcube. It has a mild conductivity to reduce electrostatic buildup, and holds promise as a ferrite for use for damping cavity modes where the power loss to the ferrite might be expected to cause it to heat significantly due to its high Curie Temperature $T_c \geq 400^\circ C$. Some material properties are shown in Tab.D.4.

The complex permeability is shown in Fig.D.4(a) and the penetration depth in Fig.D.4(b).

ϵ'	10	
Conductivity	10^{-5}	Sm^{-1}
Tc	375°	C

Table D.4 – A selection of the physical properties of 4E2

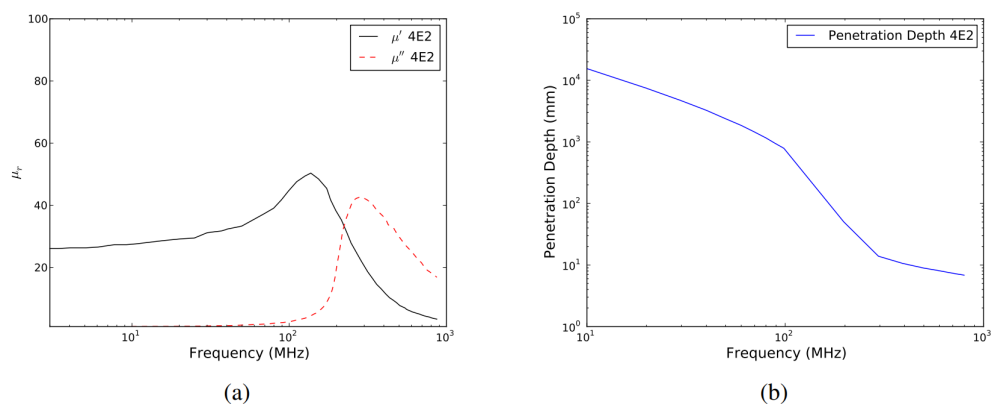


Figure D.4 – The permeability (a) and penetration depth (b) for 4E2.

E Appendix: Interpolated FFT

Following is attached the script where the much improved model, to find the resonance frequency, is tested with a white Gaussian noise, the script was done in Wolfram Mathematica.

```
In[1088]:= NN = 300.;
```

```
T = 9.;
```

```
dt = T / NN;
```

```
w0 = 25;
```

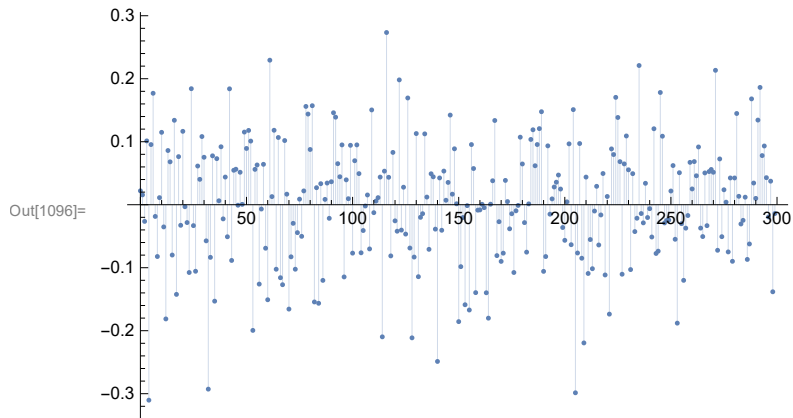
```
In[1092]:= df = 1 * 2  $\pi$  / T;
```

```
In[1093]:=  $\mathcal{P}$  = WhiteNoiseProcess[0.1];
```

```
noise = Normal[RandomFunction[ $\mathcal{P}$ , {0, IntegerPart[NN - 1]}]];
```

```
NOISE = Table[noise[[1]][[n]][[2]], {n, 1, NN}];
```

```
ListPlot[noise, Filling  $\rightarrow$  Axis]
```



```
In[1097]:=
```

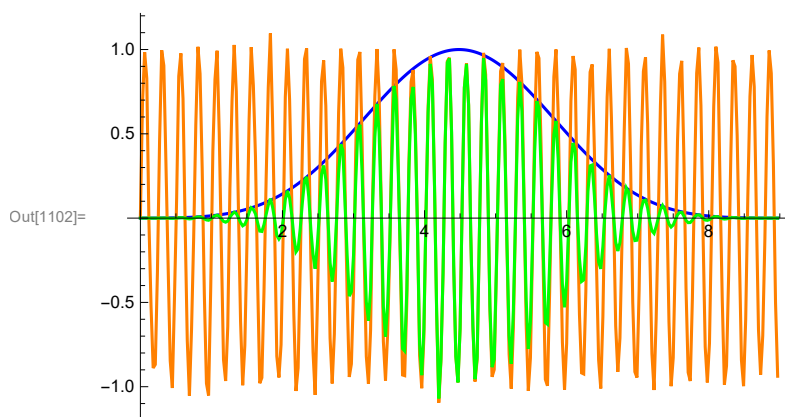
```
In[1098]:= Table[HannWindow[x], {x, -0.5, 0.5, 1 / (NN - 1)}];
```

```
In[1099]:= NuttW = Table[{i * dt, NuttallWindow[i * dt / ((NN - 1) * dt) - 0.5]}, {i, 0, NN - 1}];
```

```
Sine = Table[{i * dt, (Sin[w0 * i * dt] + 0.5 * NOISE[[i + 1]])}, {i, 0, NN - 1}];
```

```
NuttWSine = Table[{i * dt, NuttallWindow[i * dt / ((NN - 1) * dt) - 0.5] *  
(Sin[w0 * i * dt] + 0.5 * NOISE[[i + 1]])}, {i, 0, NN - 1}];
```

```
In[1102]:= ListPlot[{NuttW, Sine, NuttWSine},  
Joined  $\rightarrow$  True, PlotStyle  $\rightarrow$  {Blue, Orange, Green}]
```



```
In[1103]:= NuttWFourier = Abs[Fourier[NuttW[[1 ;; 2]]]];
```

```
SineFourier = Abs[Fourier[Sine[[1 ;; 2]]]];
```

```
NuttWSineFourier = Abs[Fourier[NuttWSine[[1 ;; 2]]]];
```

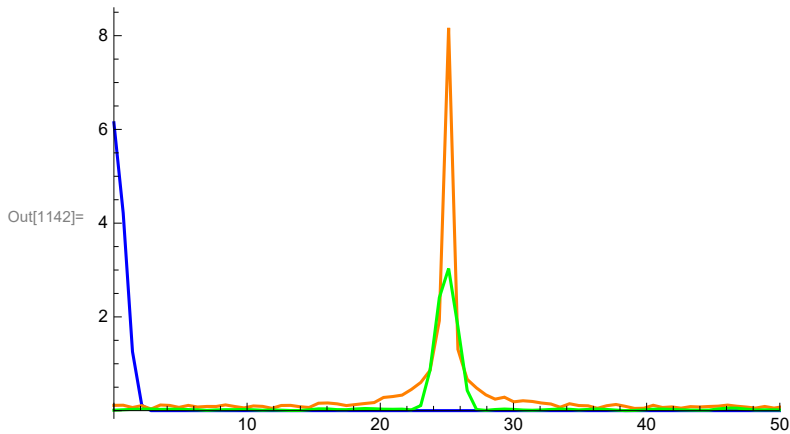
```

In[1106]:= wspectNutt = Table[{i * 2  $\pi$  / T, NuttWFourier[[i + 1]]}, {i, 0, NN - 1}];
wspectSine = Table[{i * 2  $\pi$  / T, SineFourier[[i + 1]]}, {i, 0, NN - 1}];

In[1108]:= wspectNuttSine = Table[{i * 2  $\pi$  / T, NuttWSineFourier[[i + 1]]}, {i, 0, NN - 1}];

In[1142]:= ListPlot[{wspectNutt, wspectSine, wspectNuttSine}, Joined  $\rightarrow$  True,
  PlotRange  $\rightarrow$  {{0, 50}, All}, PlotStyle  $\rightarrow$  {Blue, Orange, Green}]

```



```

In[1110]:= A1 = wspectNuttSine[[1 ;; IntegerPart[NN/2]]];
S1 = Sort[A1, #1[[2]] < #2[[2]] &]
Am0 = S1[[ (NN/2) - 2]][[2]]; fm0 = S1[[ (NN/2) - 2]][[1]];
Am1 = S1[[ (NN/2) ]][[2]]; fm1 = S1[[ (NN/2) ]][[1]];
Am2 = S1[[ (NN/2) - 1]][[2]]; fm2 = S1[[ (NN/2) - 1]][[1]];
data = {{fm0, Am0}, {fm1, Am1}, {fm2, Am2}};

```

```

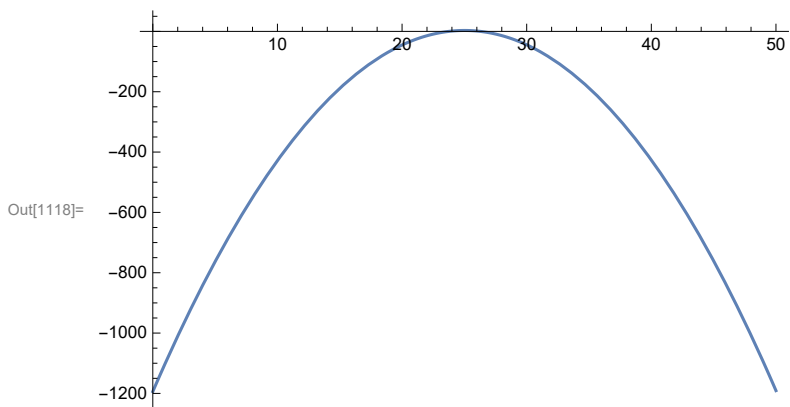
In[1116]:=

```

```

In[1117]:= parabola = Fit[data, {1, x, x^2}, x];
Plot[parabola, {x, 0, 50}]

```



```

In[1119]:= F = FindMaximum[parabola, {x, 0}]

```

```

Out[1119]= {3.05771, {x  $\rightarrow$  25.0164}}

```

```

In[1120]:=

```

```

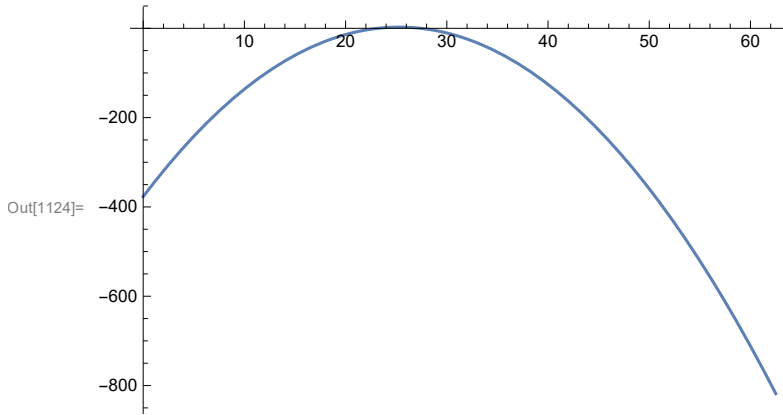
In[1121]:= wmax = fm1 + (Am2 - Am0) / (2 * (2 * Am1 - Am2 - Am0));

```

In[1122]:=

In[1123]:= **Ip1** = **NonlinearModelFit**[**data**, **a * (w - wmax) ^ 2 + h**, {**a**, **h**}, **w**];

In[1124]:= **Plot**[**Ip1**[**w**], {**w**, 0, 2.5 * **w0**}]



In[1125]:=

In[1126]:= **Q** = **FindMaximum**[**Ip1**[**w**], {**w**, **x** /. **F**[[2]]}]

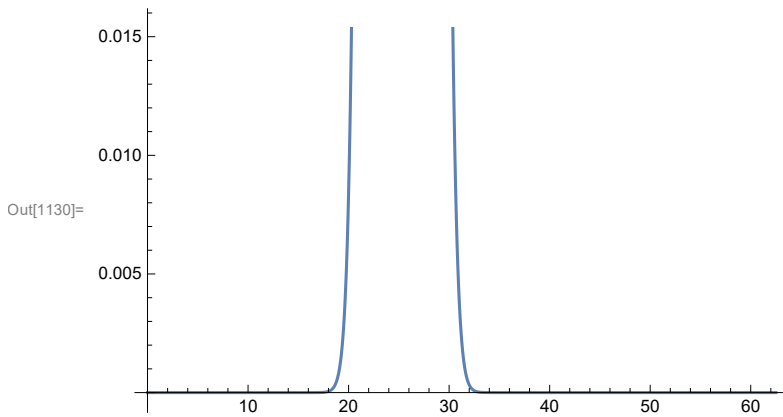
Out[1126]= {2.61957, {w → 25.2994}}

In[1127]:=

In[1128]:= **wmax** = **fm1** + **Log**[**Am2** / **Am0**] / (2 * (**Log**[**Am1** ^ 2 / (**Am2** * **Am0**)]));

In[1129]:= **Ip2** = **NonlinearModelFit**[**data**, {**Exp**[**a * (w - wmax) ^ 2 + h**]}, {**a**, **h**}, **w**];

In[1130]:= **Plot**[**Ip2**[**w**], {**w**, 0, 2.5 * **w0**}]



In[1131]:= **R** = **FindMaximum**[**Ip2**[**w**], {**w**, **x** /. **F**[[2]]}]

Out[1131]= {2.588, {w → 25.3297}}

In[1132]:=

In[1133]:= **C0** = $\left(\frac{1}{\text{NN}}\right) * \sum_{n=0}^{\text{NN}-1} \text{NuttWSine}[[n+1, 2]] * \text{Exp}[-i * ((\text{fm2} / \text{df}) * 2 * \pi / \text{T}) * n * \text{dt}];$

In[1134]:= **C1** = $\left(\frac{1}{\text{NN}}\right) * \sum_{n=0}^{\text{NN}-1} \text{NuttWSine}[[n+1, 2]] * \text{Exp}[-i * ((\text{fm1} / \text{df}) * 2 * \pi / \text{T}) * n * \text{dt}];$

```
In[1135]:= C2 =  $\left(\frac{1}{NN}\right) * \sum_{n=0}^{NN-1} \text{NuttWSine}[n+1, 2] * \text{Exp}\left[-i * \left(\frac{fm0}{df}\right) * 2 * \pi / T * n * dt\right];$ 
```

```
In[1136]:=
```

```
In[1137]:= C0U =  $\left(\frac{1}{NN}\right) * \sum_{n=0}^{NN-1} \text{NuttallWindow}[n * dt / ((NN - 1) * dt) - 0.5] * \text{Sin}[wU * n * dt] * \text{Exp}\left[-i * \left(\frac{fm2}{df}\right) * 2 * \pi / T * n * dt\right];$ 
```

```
In[1138]:= C1U =  $\left(\frac{1}{NN}\right) * \sum_{n=0}^{NN-1} \text{NuttallWindow}[n * dt / ((NN - 1) * dt) - 0.5] * \text{Sin}[wU * n * dt] * \text{Exp}\left[-i * \left(\frac{fm1}{df}\right) * 2 * \pi / T * n * dt\right];$ 
```

```
In[1139]:= C2U =  $\left(\frac{1}{NN}\right) * \sum_{n=0}^{NN-1} \text{NuttallWindow}[n * dt / ((NN - 1) * dt) - 0.5] * \text{Sin}[wU * n * dt] * \text{Exp}\left[-i * \left(\frac{fm0}{df}\right) * 2 * \pi / T * n * dt\right];$ 
```

```
In[1140]:= Res = FindRoot[ $\left(\left(\text{Re}[C0] - \text{Re}[C0U]\right)^2 + \left(\text{Im}[C0] - \text{Im}[C0U]\right)^2\right) + \left(\left(\text{Re}[C1] - \text{Re}[C1U]\right)^2 + \left(\text{Im}[C1] - \text{Im}[C1U]\right)^2\right) + \left(\left(\text{Re}[C2] - \text{Re}[C2U]\right)^2 + \left(\text{Im}[C2] - \text{Im}[C2U]\right)^2\right), \{wU, fm1\}]$ 
```

```
Out[1140]= {wU → 25.0005}
```


F Appendix: Q-factor script

Following is attached the script made in Wolfram Mathematica in order to evaluate the Q-factor in reflection.

x,PX (Select {x})

```
In[1]:= Clear[X]
      X[y_] := {y[[1]]}
      PX[y_] := {y[[2]]};
```

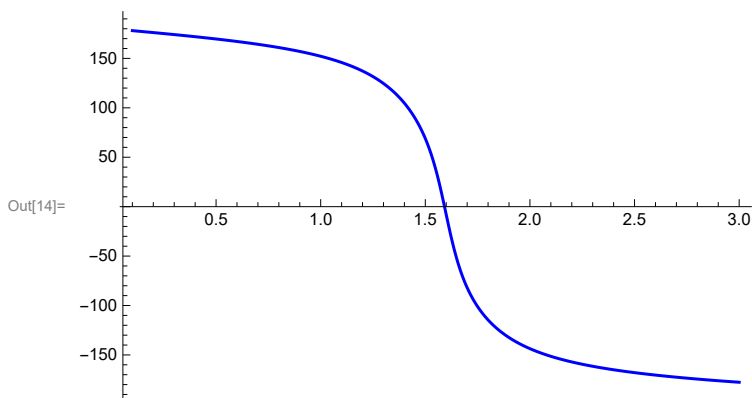
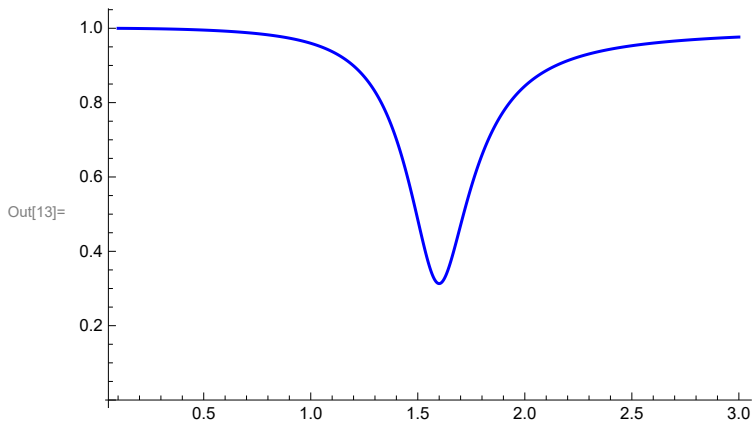
Data

```
In[4]:= S22ProbeTwo5 =
      Take[Import["\\\\cern.ch\\dfs\\Users\\a\\agilardi\\Desktop\\Export1.csv",
      "DateStringFormat" → {"1", "2", "3"}], 10 001];
      frequency = Table[S22ProbeTwo5[[n, 1]], {n, 2, 10 001}];
      S22ProbeTwo5Abs = Table[S22ProbeTwo5[[n, 2]], {n, 2, 10 001}];
      S22ProbeTwo5Phi = Table[S22ProbeTwo5[[n, 3]], {n, 2, 10 001}];
      S225 = S22ProbeTwo5Abs * Exp[i * (S22ProbeTwo5Phi) *  $\frac{\pi}{180}$ ];
```

```

In[9]:= S22InterpR5 = Interpolation[Table[Re[S225[[n]]], {n, 1, Length[frequency]}]];
(*S22InterpR5=Interpolation[
  Table[{frequency[[n]],Re[S225[[n]]}],{n,1,Length[frequency]}]];*)
S22InterpI5 = Interpolation[Table[Im[S225[[n]]], {n, 1, Length[frequency]}]];
(*S22InterpI5=Interpolation[
  Table[{frequency[[n]],Im[S225[[n]]}],{n,1,Length[frequency]}]];*)
freqInterp = Interpolation[Table[frequency[[n]], {n, 1, Length[frequency]}]];
fpoint = Table[Point[{n, S22ProbeTwo5Abs[[n]]}], {n, 1, Length[frequency]}];
ListLinePlot[
  {Table[{frequency[[n]], S22ProbeTwo5Abs[[n]]}, {n, 1, Length[frequency]}]},
  AxesOrigin -> Automatic, PlotRange -> {All, All}, PlotStyle -> {Blue}]
ListLinePlot[
  {Table[{frequency[[n]], S22ProbeTwo5Phi[[n]]}, {n, 1, Length[frequency]}]},
  AxesOrigin -> Automatic, PlotRange -> {All, All}, PlotStyle -> {Blue}]

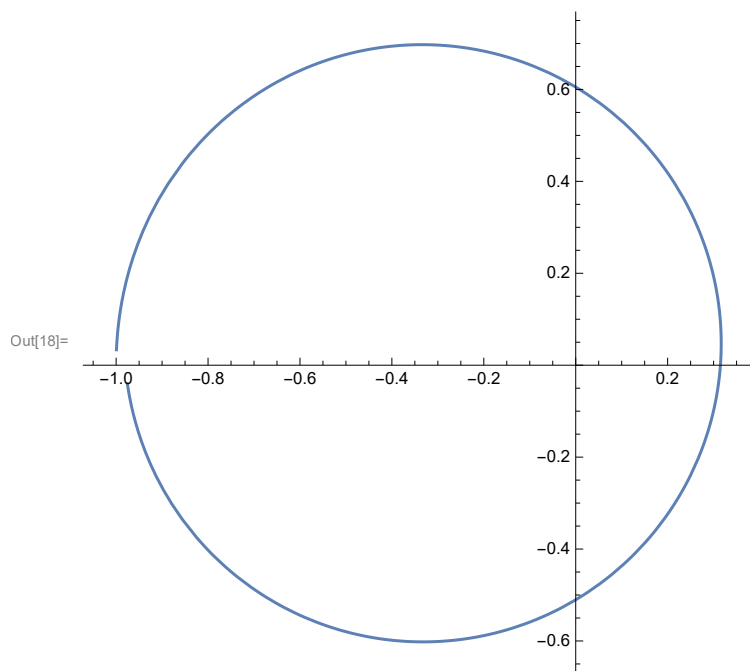
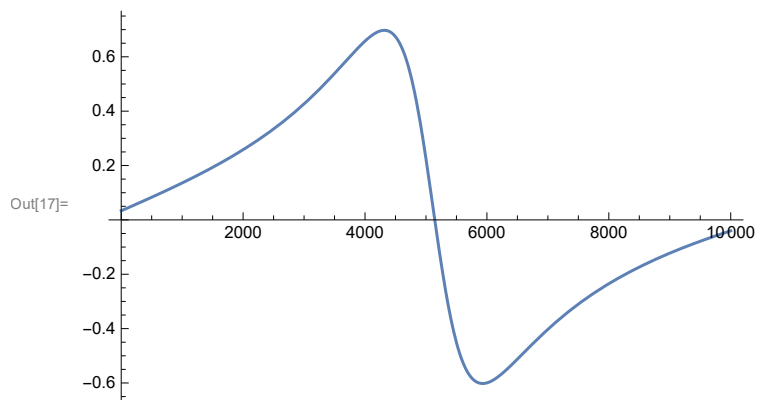
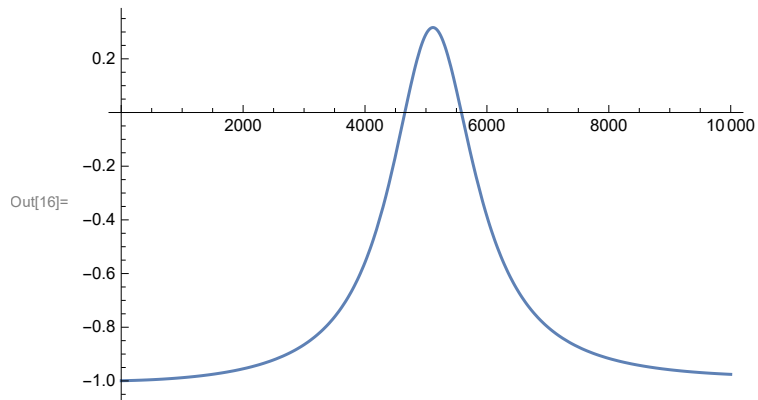
```



```

In[15]:= Island = Table[{Re[S225[[n]]], Im[S225[[n]]]}, {n, 1, Length[frequency]}];
Plot[S22InterpR5[x], {x, 1, Length[frequency]}, PlotRange -> {All, All}]
Plot[S22InterpI5[x], {x, 1, Length[frequency]}, PlotRange -> {All, All}]
ParametricPlot[{S22InterpR5[x], S22InterpI5[x]},
  {x, 1, Length[frequency]}, PlotRange -> {All, All}]

```



Finde Frequency

```
In[19]:= Parte = Table[S22ProbeTwo5Abs[[n]], {n, 1, Length[frequency]};
      x0 = Min[Parte]
      Parte[[1]]
      n0 = Position[Parte, x0]
      f0 = freqInterp[n0]

Out[20]= 0.313245

Out[21]= 0.999841

Out[22]= {{5173}}

Out[23]= {{1.60003}}
```

TwissIsland

```
In[24]:= TwissIsland[tempisl_] := Module[{theoutput, g, islandX, f1, a, b, r},
      Clear[a, b, r, x, y, f1];
      g[y_] := {y[[1]], y[[2]], 0};
      islandX = g/@tempisl;
      f1 = FindFit[islandX, (x - a)^2 + (y - b)^2 - r^2, {a, b, r}, {x, y}];
      a = a /. f1;
      b = b /. f1;
      r = Abs[r /. f1];
      theoutput = {a, b, r};
      theoutput];
```

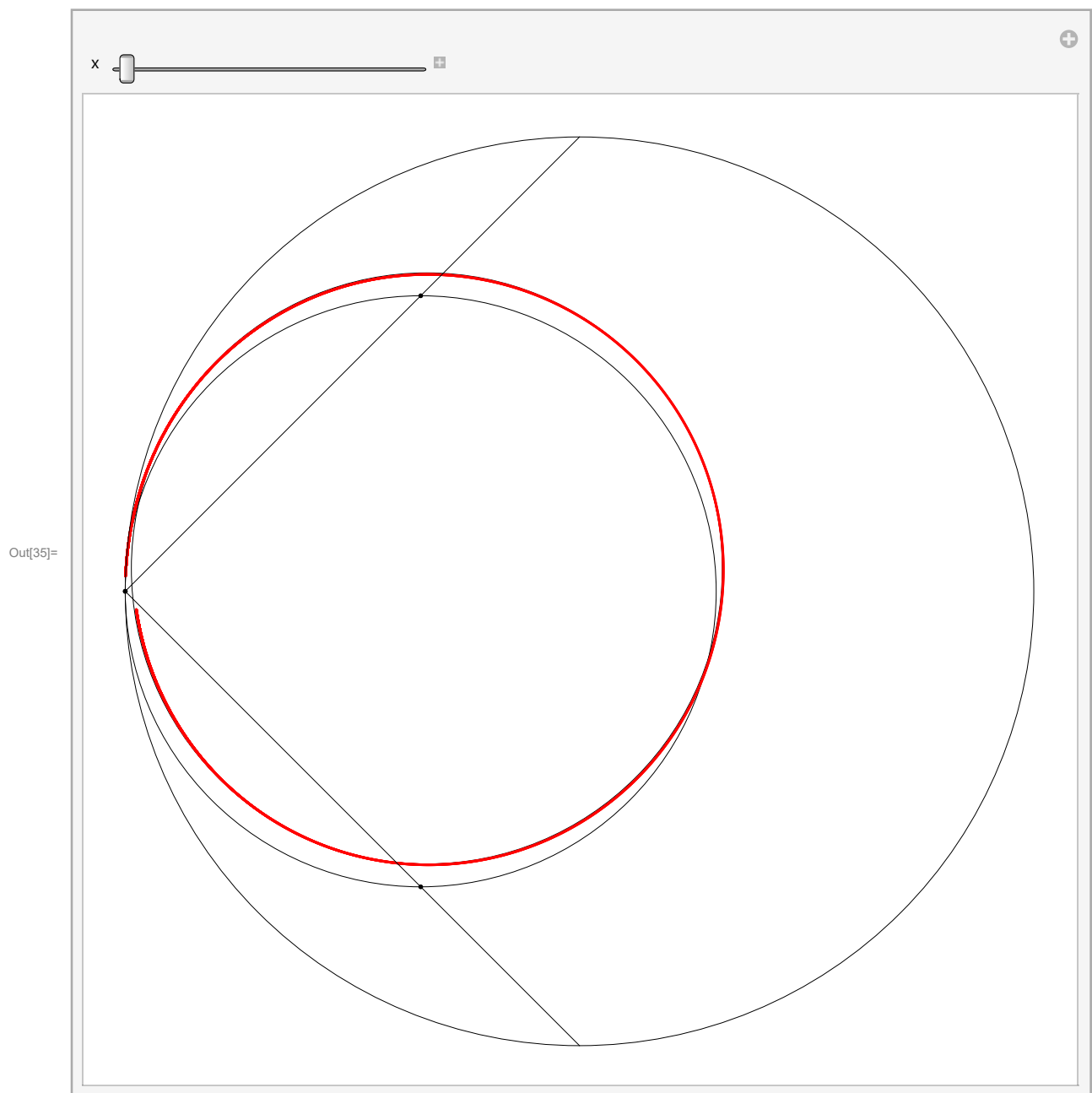
Calculate the OpticsParameters at (0,0) with TwissIsland

```
In[25]:= Clear[a, b, r];
      {a, b, r} = TwissIsland[Island]

Out[26]= {-0.336119, 0.0504013, 0.650429}
```

Display

```
In[27]:= P0 = Graphics[Circle[{a, b}, r]];
P1 = Graphics[Circle[{-1 + Abs[r], 0}, r]];
P2 = ListPlot[Table[Island[i]], {i, 1, Length[Island]}], PlotStyle -> Red];
P3 = Graphics[Circle[{0, 0}, 1]];
P4 = Graphics[Line[{{-1, 0}, {0, 1}}]];
P5 = Graphics[Line[{{-1, 0}, {0, -1}}]];
(*P6=Graphics[Circle[{1-Abs[r], 0}, r]];*)
PP1 = Graphics[Point[{-1 + r, r}]];
PP2 = Graphics[Point[{-1 + r, -r}]];
Manipulate[Show[Graphics[Point[{x, x + 1}]],
  Show[Graphics[Point[{x, -x - 1}]]], P0, P1, P2, P3, P4, P5, PP1, PP2,
  PlotRange -> All, ImageSize -> 600, AxesOrigin -> {0, 0}], {x, -1, 0}]
```



Post Evaluation

```
In[36]:= diameter = 2 * r;
Interp = NSolve[{y == x + 1, (x + 1 - Abs[r])^2 + (y)^2 == r^2}, {x, y}]
x1 = x /. Interp[[1]]
y1 = x1 + 1
y2 = -y1
```

```
Out[37]= {{x -> -0.349571, y -> 0.650429}, {x -> -1., y -> 0.}}
```

```
Out[38]= -0.349571
```

```
Out[39]= 0.650429
```

```
Out[40]= -0.650429
```

```
In[41]:= radius = Sqrt[a^2 + b^2];
alpha = ArcSin[b / radius]
(alpha * 180) /
  pi
phi = 180 - (alpha * 180) /
  pi
```

```
Out[42]= 0.148842
```

```
Out[43]= 8.528
```

```
Out[44]= 171.472
```

```
In[45]:= off = 1 - radius - r
```

```
Out[45]= 0.00969445
```

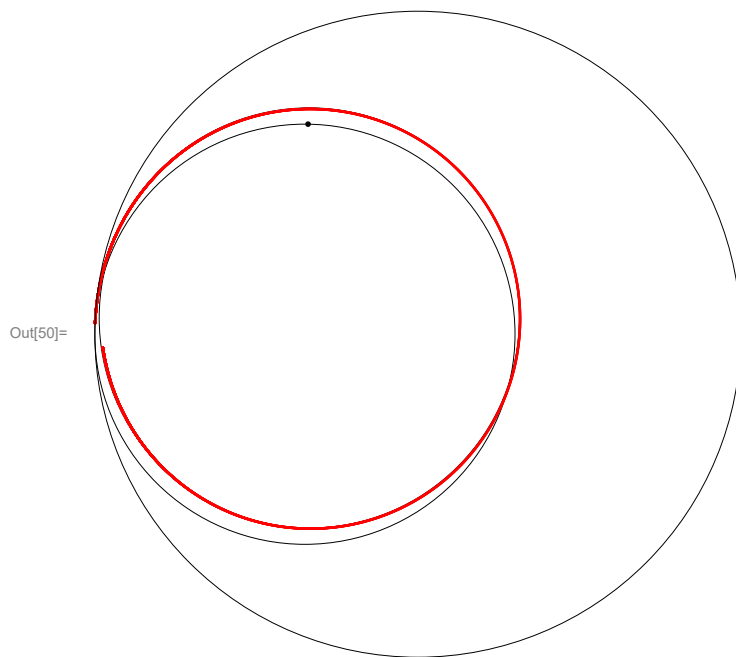
fb1 - fb2

```

In[46]:= Movimento1 = -1 + (r + off) + i r;
Movimento1Phi = Movimento1 * Exp[-i * (phi) * pi / 180.];
PlotMovimento1 = Graphics[Point[{Re[Movimento1], Im[Movimento1]}]];
PlotMovimento1Phi = Graphics[Point[{Re[Movimento1Phi], Im[Movimento1Phi]}]];
Show[P0, P1, P2, P3, PlotMovimento1, PlotRange -> All, AxesOrigin -> {0, 0}]

```

Out[47]= 0.432573 - 0.592836 i



```

In[51]:= R1 = FindMinimum[(S22InterpR5[n] - Re[Movimento1Phi])^2 +
(S22InterpI5[n] - Im[Movimento1Phi])^2, {n, n0}]
fb1 = freqInterp[R1[[2, 1, 2]]]

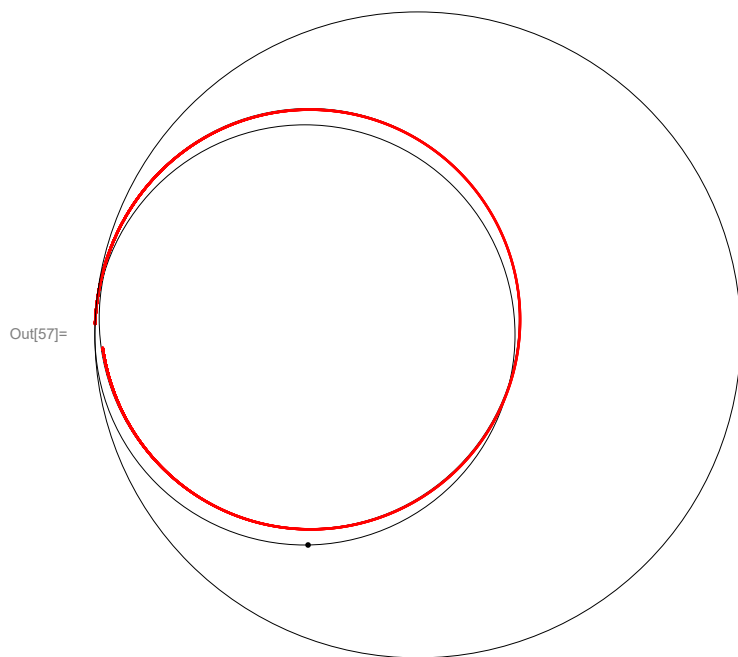
```

Out[51]= {0.121332, {n -> {{5413.85}}}}

Out[52]= {{1.66988}}

```
In[53]:= Movimento2 = -1 + r + off - i r;
Movimento2Phi = Movimento2 * Exp[-i * (phi) * pi / 180.];
PlotMovimento2 = Graphics[Point[{Re[Movimento2], Im[Movimento2]}]];
PlotMovimento2Phi = Graphics[Point[{Re[Movimento2Phi], Im[Movimento2Phi]}]];
Show[P0, P1, P2, P3, PlotMovimento2, PlotRange -> All, AxesOrigin -> {0, 0}]
```

Out[54]= 0.239665 + 0.693638 i



```
In[58]:= R2 = FindMinimum[(S22InterpR5[n] - Re[Movimento2Phi])^2 +
(S22InterpI5[n] - Im[Movimento2Phi])^2, {n, n0}]
fb2 = freqInterp[R2[[2, 1, 2]]]
```

Out[58]= {0.0457775, {n -> {{4747.69}}}}

Out[59]= {{1.47668}}

Q1

```
In[60]:= Q1 = f0 / Abs[fb2 - fb1]
```

Out[60]= {{8.28147}}

β

```
y1 = Solve[x - 50 / x + 50 == (-1 + 2 * r + off), x]
```

```
In[62]:=  $\beta = \mathbf{y1}[[1, 1, 2]] / 50$ 
```

```
Out[62]= 1.90087
```

Q0

```
In[63]:=  $Q0 = (1 + \beta) * Q1$ 
```

```
Out[63]= {{24.0235}}
```


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