

BBN Constraints on Universally Coupled Ultralight Dark Matter

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Abstract

Ultralight scalar dark matter (DM) can gain an interaction to all massive Standard Model (SM) particles through a universal coupling. Such a coupling can modify the properties of said SM particles through the generation of an effective mass. Flambaum and Stadnik [1] already exploited this effect in a similar model to constrain the DM phase space through its effect on Big Bang Nucleosynthesis (BBN). However there is also a back-reaction from SM to DM particles which modifies the DM mass. Due to the ultra low DM mass this effect, which has not been studied previously in the literature, can be substantial. By including this back-reaction in the DM evolution we derive new BBN constraints and find them to deviate significantly from those derived in previous work.

1 The Model

In this model we extend the Standard Model by including a real scalar field,

$$\mathcal{L}_\phi = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m_\phi^2 \phi^2. \quad (1.1)$$

This scalar field is then allowed to interact universally with the SM by perturbing the metric (or vielbeins) of the SM terms,

$$\begin{aligned} e\mathcal{L}_{SM} [e_a^\mu] &\rightarrow \\ \det(e_\mu^a(1+\alpha))\mathcal{L}_{SM} [e_a^\mu(1-\alpha)] & \quad (1.2) \\ \text{with } \alpha &= \pm \phi^2/\Lambda^2. \end{aligned}$$

We will soon expand this perturbation to see how such a coupling gives rise to a coupling to massive SM particles. Since we are dealing with curvature we include the relevant metric determinants and the complete Lagrangian is then

$$\begin{aligned} \mathcal{L}_{tot} &= \sqrt{-g} \frac{M_p^2}{2} R [g^{\mu\nu}] \\ &+ \det(e_\mu^a(1+\alpha))\mathcal{L}_{SM} [e_a^\mu(1-\alpha)] \\ &+ \sqrt{-g} \left(\frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m_\phi^2 \phi^2 \right). \quad (1.3) \end{aligned}$$

Because we in this work will be dealing with energies in the MeV range and below we can omit the substructure of the nucleons. We then only include electrons, protons and neutrons as the fermions and use an effective field theory for the SM Lagrangian:

$$\begin{aligned} \mathcal{L}_{SM} &= \mathcal{L}_{SM}^F + \mathcal{L}_{SM}^f + \mathcal{L}_{SM}^V \\ &= -\frac{F^2}{4} + \bar{f} (i\nabla + eQ\mathcal{A} - m_f) f - \frac{m_V^2}{2} V^2 \quad (1.4) \end{aligned}$$

where a sum over the fermions $f = e, n, p$ and the heavy vector bosons $V = W, Z$ is left implicit. The field strength tensor $F^{\mu\nu}$ is understood to include both the EM and weak gauge fields.

1.1 Expanding the coupling

To explore the consequences of this universal coupling we now seek to expand it in small α :

$$\begin{aligned} \det(e_\mu^a(1+\alpha))\mathcal{L}_{SM} [e_a^\mu(1-\alpha)] & \\ &\approx e\mathcal{L}_{SM} + \delta(e\mathcal{L}_{SM}) \\ &= e\mathcal{L}_{SM} + 4\alpha e\mathcal{L}_{SM} + e\delta\mathcal{L}_{SM} \quad (1.5) \end{aligned}$$

where we left implicit the dependence on the unperturbed vielbeins on the RHS by writing $\mathcal{L}_{SM} \equiv \mathcal{L}_{SM} [e_a^\mu]$. To evaluate the variation of

the determinant we used the standard identity $\delta e = e e_a^\mu \delta e_\mu^a = 4\alpha e$. We will now take the variations of the three components of our effective standard model Lagrangian.

For the field strength term we apply the corresponding variation $\delta g^{\mu\nu} = -2\alpha g^{\mu\nu}$ to get

$$\begin{aligned}\delta \mathcal{L}_{SM}^F &= -\frac{F_{\alpha\beta} F_{\lambda\rho}}{4} \delta \left(g^{\lambda\alpha} g^{\rho\beta} \right) \\ &= -4\alpha \mathcal{L}_{SM}^F.\end{aligned}\quad (1.6)$$

Notice that the variation of $\mathcal{L}_{SM}^F$ cancels identically with the determinant term in eq. 1.5 so that the net result is no effect on the field strength terms. The massive vector boson terms yields

$$\delta \mathcal{L}_{SM}^V = -\frac{1}{2} m_V^2 V_\mu V_\nu \delta g^{\mu\nu} = -2\alpha \mathcal{L}_{SM}^V, \quad (1.7)$$

which only has incomplete cancellation with eq. 1.5. We conclude that the universal coupling yields an interaction with massive vector bosons but not (at tree-level) with the massless photons.

The fermion term is complicated by the appearance of the spin connection in the covariant derivative. To absorb the extra terms introduced by the connection we perform a simultaneous field redefinition of the fermion fields:

$$f \rightarrow f \left(1 + \frac{3}{2}\alpha \right) \quad \text{where } f = n, p, e. \quad (1.8)$$

Note that while we simply have $\delta \bar{f} = 3/2\alpha \bar{f}$ we pick up an extra term from f because of the derivative acting right:

$$\begin{aligned}\bar{f} i \not{\nabla} \delta f &= \frac{3}{2} \bar{f} i \gamma^a e_a^\mu \nabla_\mu (\alpha f) \\ &= \frac{3}{2} \alpha \bar{f} i \not{\nabla} f + \frac{3}{2} \bar{f} i (\not{\nabla} \alpha) f\end{aligned}\quad (1.9)$$

The full variation is then

$$\begin{aligned}\delta \mathcal{L}_{SM}^f &= 3\alpha \mathcal{L}_{SM}^f + \frac{3}{2} \bar{f} i (\not{\nabla} \alpha) f \\ &\quad + \bar{f} (i \delta \not{\nabla} + e Q \delta \not{A}) f\end{aligned}\quad (1.10)$$

For the EM term we simply have

$$\delta \not{A} = \gamma^a \delta(e_a^\mu) A_\mu = -\alpha \not{A} \quad (1.11)$$

but the spin connection introduces extra terms for the derivative term:

$$\delta \not{\nabla} = -\alpha \not{\nabla} - \frac{1}{4} \delta \left(\omega_\mu^{ab} \right) \gamma_{ab} \quad (1.12)$$

where ω_μ^{ab} is the spin connection and γ_{ab} is the antisymmetric product $\gamma_{ab} = 1/2(\gamma_a \gamma_b - \gamma_b \gamma_a)$. By expressing the connection in terms of vielbeins and performing some manipulations involving index manipulation and gamma matrix anticommutation rules we arrive at

$$\delta \not{\nabla} = -\alpha \not{\nabla} - \frac{3}{2} \not{\nabla} \alpha, \quad (1.13)$$

such that the extra terms cancel and finally have

$$\delta \mathcal{L}_{SM}^f = 2\alpha \mathcal{L}_{SM}^f - \alpha m_f \bar{f} f. \quad (1.14)$$

We can apply the background EOM, which implies $\mathcal{L}_{SM}^f = 0$, to further simplify the perturbing terms. This yields

$$\delta \mathcal{L}_{SM}^f \approx -\alpha m_f \bar{f} f, \quad (1.15)$$

which we recognize as a DM-fermion coupling.

In summary the effective SM Lagrangian is then extended with the following DM-SM interaction terms:

$$\begin{aligned}\det(e_a^\mu(1+\alpha)) \mathcal{L}_{SM} [e_a^\mu(1-\alpha)] / e &\approx \\ \mathcal{L}_{SM} \mp \frac{\phi^2}{\Lambda^2} m_f \bar{f} f \pm \frac{\phi^2}{\Lambda^2} m_V^2 V^2\end{aligned}\quad (1.16)$$

We conclude that the DM couples universally to all massive particles as expected.

1.2 Effective mass

These interactions correspond to those considered by Flambaum and Stadnik [1]. As noted in [1] the interactions can be understood as an effective mass for the SM particles,

$$m_{f,V} \rightarrow m_{f,V} \left(1 \pm \frac{\phi^2}{\Lambda^2} \right). \quad (1.17)$$

However there is also a back-reaction on the DM which endows this with an effective mass,

$$m_\phi^2 \rightarrow m_{eff}^2 = m_\phi^2 \pm 2\Lambda^{-2} m_f \bar{f} f \mp 2\Lambda^{-2} m_V^2 V^2. \quad (1.18)$$

This DM effective mass modifies the evolution of ϕ and has not been studied in previous work[1]. In the remainder of this work we consider the DM field ϕ in the cosmological background in the period between today and BBN. We therefore interpret $m_f \bar{f} f$ as the mass density of fermions which in cosmology usually is denoted by baryonic matter ρ_b . We drop the contribution of heavy vector bosons because the density of them in the universe is vanishingly low. The effective mass picked up by the DM is then

$$m_{eff}^2 \approx m_\phi^2 \pm 2\Lambda^{-2}\rho_B \quad (1.19)$$

In the following we study how this effect modifies the DM evolution and how this impacts constraints from BBN.

2 Dark Matter Evolution

To evaluate the effect of DM on BBN we first need to scale the density of DM at present back to the time of BBN. To do so we first need to determine how ϕ^2 scales with a . Then we can apply this scaling to evolve ϕ^2 back to BBN.

2.1 The scaling of DM

To solve the evolution of ϕ we first write the Klein-Gordon equation in an expanding homogeneous spacetime:

$$\ddot{\phi} + 3H\dot{\phi} \pm 2\Lambda^{-2}\rho_B\phi + m_\phi^2\phi = 0 \quad (2.1)$$

We do not have a global solution to this equation. To approximate a solution we solve it in three distinct phases dominated respectively by:

- **Bare mass:** $+2\Lambda^{-2}\rho_B\phi$ dropped
- **Effective mass:** $+m_\phi^2\phi$ dropped
- **Hubble friction:** Both masses dropped

The solutions in two of the above phases can be found analytically. Taking Bessel functions in the high oscillation limit and translating time dependence to scale factor dependence we find the following scaling:

$$\text{BM: } \phi^2 \propto a^{-3} \quad \text{H: } \phi^2 \propto a^0 \quad (2.2)$$

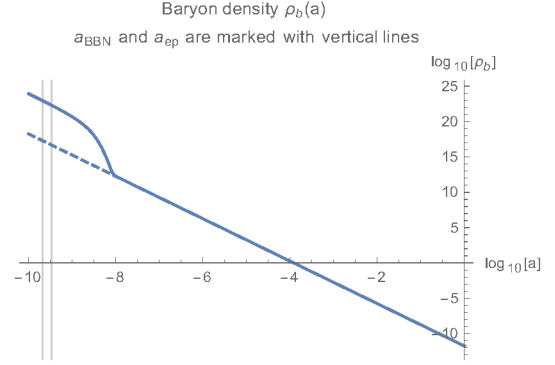


Figure 1: The evolution of ρ_b as a function of scale factor. Notice the strongly modified behaviour in the time near a_{ep} defined by $T = m_e$. Bare $\rho_b \propto a^{-3}$ evolution is displayed with a dashed line for reference.

The scaling in an effective mass dominated phase depends on the time dependence of the baryon density ρ_b . For the majority of the history we are concerned with this is $\rho_b \propto a^{-3}$ which dampens the DM evolution to $\phi^2 \propto a^{-3/2}$. However the baryon density is strongly modified by the presence of electron-positron pairs in the time before and immediately after $T = m_e$. When the baryon density is calculated with e/p pairs properly accounted for (detailed in appendix A) the evolution is as in figure 1. This deviation from $\rho_b \propto a^{-3}$ causes a deviation in the scaling of ϕ^2 which we can identify by solving the effective mass EOM for each scaling of ρ_b . As shown in figure 2 the dampening of the DM evolution is strongly amplified when the change in baryon density becomes steeper. Note that the above only applies in the case of a positive $+\phi^2/\Lambda^2$ coupling. In a model with negative coupling the DM becomes tachyonic in the EM phase.

2.2 DM evolution

The DM evolution across an EM phase can be calculated by integrating the scaling as

$$\phi_{early}^2 = 10^{\int_{a_{late}}^{a_{early}} x \, d(\log_{10} a)} \phi_{late}^2, \quad (2.3)$$

and the DM evolution across a BM phase is

$$\phi_{early}^2 = \left(\frac{a_{late}}{a_{early}} \right)^{-3} \phi_{late}^2, \quad (2.4)$$

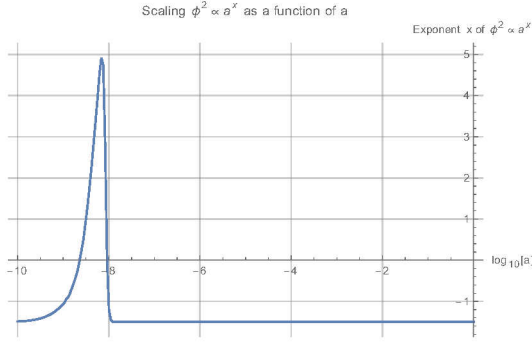


Figure 2: The scaling $\phi^2 \propto a^x$ in an EM dominated phase as a function of scale factor. As expected the scaling is $x = -3/2$ for most of the history. Around e/p annihilation the baryon effect is increased so the dilution of ϕ is not only slowed but actually reversed for a brief period. The deviation is discussed in appendix A.

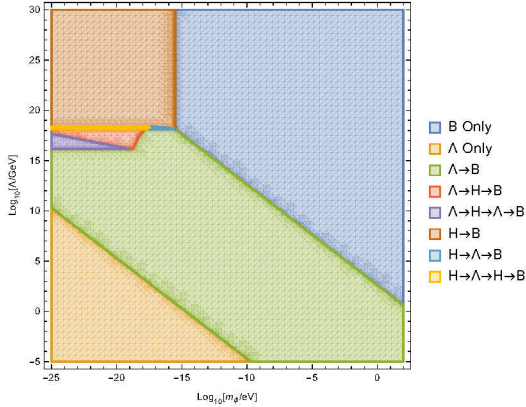


Figure 3: Map of the transition history of evolutions defined by various points in phase space.

while the DM density does not change across a Hubble friction phase. To evaluate the density $\rho_\phi = m_\phi^2 \phi^2$ at BBN all one has to do is identify which of the above phases the DM goes through and apply the above scaling over the transitions. To do so we map the transition history by using Mathematica to sample which prefactor in the EOM is dominant at various scale factors. This phase transition history map is shown in figure 3. Solving the transition times numerically one can then determine the correct evolution of ϕ^2 which is visualized in figure 4.

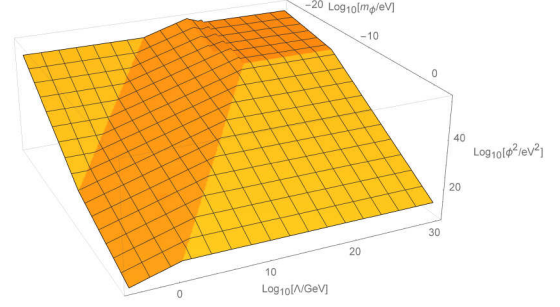


Figure 4: The density of dark matter $\phi^2 = \rho_\phi/m_\phi^2$ at BBN as evaluated using the scheme described in section 2.2.

3 Effect on BBN

The neutron-proton ratio[2] is fixed at weak freeze-out according to the Boltzmann fraction

$$\frac{n_W}{p_W} = e^{-(m_n - m_p)/T_W} = e^{-m_{np}/T_W}, \quad (3.1)$$

where $T_W \approx 0.8$ MeV is the weak freeze-out temperature. Later almost all of the available neutrons will become locked up in helium-4 because of its high binding energy. However before the neutrons can participate in nuclear reactions to form He^4 the abundance is modified by neutron decay. This yields the neutron-proton ratio at BBN[2]:

$$\frac{n_{BBN}}{p_{BBN}} = \frac{n_W e^{-\Gamma_n t_{BBN}}}{p_W + n_W (1 - e^{-\Gamma_n t_{BBN}})}, \quad (3.2)$$

and the helium-4 fraction

$$Y = \frac{2}{1 + \frac{p_{BBN}}{n_{BBN}}} \quad (3.3)$$

where $n_{W/BBN}$ and $p_{W/BBN}$ refers to the neutron and proton densities at weak (n/p) freeze-out and BBN respectively. $\Gamma_n \approx 1/880s$ is the neutron decay rate and $t_{BBN} \approx 180s$ is the time of BBN[1]. The helium-4 fraction is predicted by standard BBN theory to be $Y_{theo} = 0.24709 \pm 0.00025$ [4] and measured to $Y_{exp} = 0.245 \pm 0.003$ [3]. Multiplying the uncertainties in quadrature these numbers gives the relative difference

$$\frac{\Delta Y}{Y} = \frac{Y_{exp} - Y_{theo}}{Y_{theo}} = -0.008458 \pm 0.012183 \quad (3.4)$$

This relative difference constraints how much the addition of dark matter is allowed (or required) to change the standard model prediction. Following the approach by Flambaum and Standnik[1] we extract the leading effect by expanding eq. 3.2 in small $\Gamma_n t_{BBN}$ and taking the variation of all masses:

$$\frac{\Delta Y}{Y} \approx \frac{\Delta(n/p)_W}{(n/p)_W} - \Gamma_n t_{BBN} \frac{\Delta \Gamma_n}{\Gamma_n} \quad (3.5)$$

where we assumed that t_{BBN} does not change as a result of DM. This assumption justified because we expect the timing of BBN to be set by strong forces which are unaffected by DM.

By taking the temperature of weak freeze-out to be $T_W = am_W^{4/3} \sin^{4/3}(\theta_W)/\alpha^{2/3} M_{planck}^{1/3} \approx 0.75 \text{ MeV}$ [2] we can carry through the variation of 3.1. Here a is a numerical constant, m_W is the mass of the W-boson, α is the fine structure constant and θ_W is the Weinberg angle. The variation of eq. 3.1 yields

$$\begin{aligned} \frac{\Delta(n/p)_W}{(n/p)_W} &= \frac{4}{3} \frac{m_{np}}{T_W} \frac{1 - \sin^2(\theta_W)}{\sin^2(\theta_W)} \frac{\Delta m_Z}{m_Z} \\ &+ \frac{4}{3} \frac{m_{np}}{T_W} \left[1 - \frac{1 - \sin^2(\theta_W)}{\sin^2(\theta_W)} \right] \frac{\Delta m_W}{m_W} \\ &- \frac{m_{np}}{T_W} \frac{\Delta m_{np}}{m_{np}}. \end{aligned} \quad (3.6)$$

With the prefactors written out numerically,

$$\frac{\Delta(n/p)_W}{(n/p)_W} \approx 8.04 \frac{\Delta m_Z}{m_Z} - 5.74 \frac{\Delta m_W}{m_W} - 1.72 \frac{\Delta m_{np}}{m_{np}}, \quad (3.7)$$

it is apparent the largest effect on the freeze-out neutron-proton ratio comes from the change of the Z mass which modifies the weak coupling and hence the freeze-out temperature.

The decay rate can be approximated by: [1]

$$\Gamma_n = \frac{b\alpha^2 m_e^5}{m_W \sin^4(\theta_W)} P(x) \quad (3.8)$$

where $x = m_{np}/m_e$ and

$$\begin{aligned} P(x) &= \frac{1}{15} (2x^4 - 9x^2 - 8) \sqrt{x^2 - 1} \\ &+ x \ln \left(x + \sqrt{x^2 - 1} \right) \end{aligned} \quad (3.9)$$

Taking the variation of all masses as before we arrive at the fractional change in decay rate

$$\begin{aligned} \frac{\Delta \Gamma_n}{\Gamma_n} &= \frac{\partial P(x)/\partial x}{P(x)} \frac{\Delta m_{np}}{m_{np}} + \left(5 - \frac{\partial P(x)/\partial x}{P(x)} \right) \frac{\Delta m_e}{m_e} \\ &+ 4 \left(\frac{1 - \sin^2(\theta_W)}{\sin^2(\theta_W)} - 1 \right) \frac{\Delta m_W}{m_W} \\ &- 4 \frac{1 - \sin^2(\theta_W)}{\sin^2(\theta_W)} \frac{\Delta m_Z}{m_Z}. \end{aligned} \quad (3.10)$$

Writing again the prefactors numerically

$$\begin{aligned} \frac{\Delta \Gamma_n}{\Gamma_n} &\approx 6.54 \frac{\Delta m_{np}}{m_{np}} - 1.54 \frac{\Delta m_e}{m_e} \\ &+ 9.99 \frac{\Delta m_W}{m_W} - 13.99 \frac{\Delta m_Z}{m_Z} \end{aligned} \quad (3.11)$$

we conclude that the dominant contribution to the change in neutron decay is also the change in Z -mass.

Summing up all contributions and using that all masses change universally $\Delta m/m = \phi^2/\Lambda^2$ (eq. 1.17) we can express the fractional change in Helium-4 fraction according to eq. 3.5:

$$\frac{\Delta Y}{Y} = \left[\frac{1}{3} \frac{m_{np}}{T_W} - \Gamma_n t_{BBN} \right] \frac{\pm \phi^2}{\Lambda^2} \quad (3.12)$$

4 Helium-4 constraints

Combining the constraints on the Helium-4 fraction (eq. 3.4) with the DM effect (eq. 3.12) we arrive at a DM constraint:

$$\left[\frac{1}{3} \frac{m_{np}}{T_W} - \Gamma_n t_{BBN} \right] \frac{\pm \phi^2}{\Lambda^2} = -0.008458 \pm 0.012183 \quad (4.1)$$

Assuming a positive $+\phi^2/\Lambda^2$ coupling we can apply the time evolved $\phi^2(T_{BBN})$ from section 2.2. This yields a constraint on the DM phase space shown in figure 5 where it is also compared to the same constraint in a model which neglects the DM effective mass effect. It is apparent that the effective mass leads to a loosening of the constraints for DM with a mass of at least $\sim 10^{-18}$ eV. This happens because a part of the DM evolution is in an effective mass dominated phase with a slower

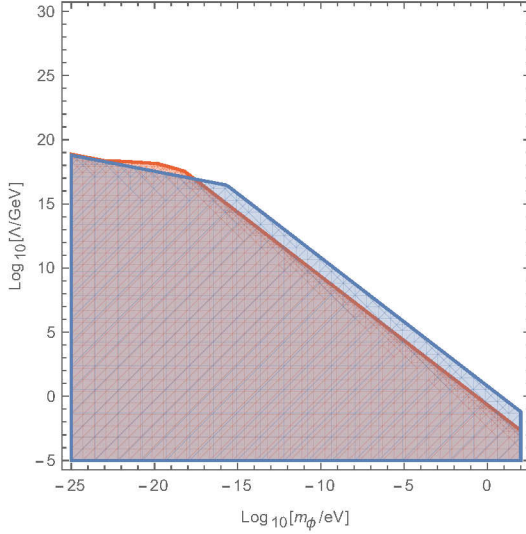


Figure 5: Comparison between the DM phase space constraint including DM effective mass in red vs. a model without effective mass in blue both in the case of a positive $+\phi^2/\Lambda^2$ coupling. The model in blue is similar to the results found by Flambaum and Stadnik[1].

evolution than in the bare mass dominated phase. Below $\sim 10^{-18}$ eV the constraint is amplified because the effective mass shields the DM from an otherwise frozen friction phase.

The evolution discussed in section 2.2 does not apply for DM with a negative $-\phi^2/\Lambda^2$ coupling. This is because the DM becomes tachyonic in the EM phase. However this can be used as a constraint in its own right since we do not expect the dark matter to go through an unstable tachyonic phase. This constraint is in fact stronger than the BBN constraint seen for positive coupling for most of the mass range. When one combines exclusion of all phase space containing tachyonic phases and the BBN constraint on the remaining phase space then one arrives at figure 6.

Interestingly the allowed phase space in the case of the negative coupling allows for a band where the DM stays non-tachyonic during cosmological evolution but still has a coupling strong enough to allow tachyonic behaviour inside extremely compact objects such as neutron stars. Taking a high estimate for neutron star density to be $\rho_{NS} = 6 \times 10^{17} \text{ kg/m}^3$ the allowed band is also displayed in figure 6.

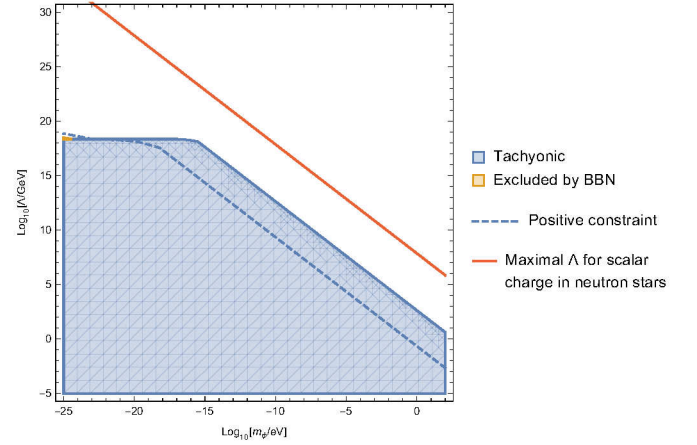


Figure 6: Phase space excluded in the model with negative $-\phi^2/\Lambda^2$ coupling as excluded tachyonic phases in blue and BBN in orange. The constraint for the positive coupling (fig. 5) is given by the blue line for comparison. The allowed band of phase space below the red line also allows for DM to become tachyonic inside neutron stars.

5 The Einstein and the Jordan pictures the Jordan frame

So far we have solved the model in the Einstein frame. In this frame the DM had interactions which allowed us to evaluate the DM evolution and its effect on the Helium-4 abundance. Alternatively we can absorb the interaction terms in a conformal rescaling of the metric, i.e. $g^{\mu\nu} \rightarrow g^{\mu\nu}(1 + 2\alpha)$. Such a transformation yields the Jordan frame Lagrangian:

$$\begin{aligned} \mathcal{L}_{tot} = & \sqrt{-\det(g_{\mu\nu}(1 - 2\alpha))} \frac{M_p^2}{2} R[g^{\mu\nu}(1 + 2\alpha)] \\ & e\mathcal{L}_{SM} + \sqrt{-\det(g_{\mu\nu}(1 - 2\alpha))} \\ & \times \left(\frac{g^{\mu\nu}(1 + 2\alpha)}{2} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m_\phi^2 \phi^2 \right) \end{aligned} \quad (5.1)$$

In this picture the DM becomes non-interacting with the SM at the price of a non-minimal coupling with gravity.

To go beyond Helium-4 and investigate the abundances of other light elements we will have to dive into the complex network of nuclear interactions. Traditionally this problem is handled

with numerical codes several of which are publicly available. In the Einstein frame our model involves continuously modified fermion masses and reaction rates which it would take a substantial effort to accommodate in a standard code. Instead of undertaking such an effort we make use of the Jordan frame where the standard model is left unchanged but evolving in a universe with modified expansion.

In the future development of this work we will seek to exploit the Jordan frame to calculate other abundances. To avoid calculating the expansion through the modified Friedman equations which follow eq. 5.1 it may be possible to use the evolution of ϕ as solved in the Einstein frame to translate the hubble parameter between frames. Furthermore the publicly available code AlterBBNv2[8] already contains a mechanism to modify the cosmological expansion. In the code this is achieved by the addition of an effective dark density which may be useful if the build-in parametrization proves to yield a good fit to our modified expansion. It seems likely to achieve BBN constraints based on more light elements in the near future.

6 Acknowledgements

I would like to thank Sergey Sibirikov and Tien-Tien Yu for the great help they have provided during this project. Without time and effort they have put aside this project would not have been possible.

A Baryon density and DM scaling in the EM phase

Before a detailed study of the baryon density can take place we need to properly account for reheating in the temperature evolution. To accomplish this we make use of tabulated data on relativistic degrees of freedom borrowed from the MicroOMEGAs package[5] to evaluate the realistic $T \propto g_{*s}^{-1/3} a^{-1}$ evolution. With this information

we can calculate the comoving baryon density[7]:

$$\rho_{b(com)}(a) = \rho_{b0} + \frac{4m_e}{2\pi^2} \frac{g_{*s}(a_0)}{g_{*s}(a)} T_0^3 \int_{m_e/T}^{\infty} \frac{\sqrt{x^2 - m_e^2/T^2}}{e^x + 1} x dx \quad (\text{A.1})$$

Performing this integral numerically with interpolated values for g_{*s} yields the curve seen in figure 1 after converting from comoving to physical density. In particular the corrected ρ_b evolution violates $\rho_b \propto a^{-3}$ while the comoving density is saturating with e/p pairs. This breaks the expected $\phi^2 \propto a^{-3/2}$ evolution discussed in section 2.1. This will need to be corrected as the scaling becomes time dependent.

To calculate the corrected scaling x of $\phi^2 \propto a^x$ we solve the OEM in EM phase:

$$\ddot{\phi} + \frac{3}{2t} \dot{\phi} + A t^{y/2} \phi = 0 \quad (\text{A.2})$$

where A is a constant and y is the scaling $\rho_b \propto a^y$. Solving this equation analytically yields expressions in terms of rapidly oscillating Bessel functions which can be taken in the high oscillation limit to cleanly isolate the scaling of $\phi \propto t^z$. Translating this to $\phi^2 \propto a^x$ where $x = 4z$ for various values of y we discover a simple linear dependence

$$x = -\frac{1}{2}y - 3, \quad (\text{A.3})$$

which is plotted in figure 7.

Knowing exact evolution of the baryon density as a function of scale factor and how the scaling of ϕ^2 depends on the scaling of ρ_b we can compute the scaling of ϕ^2 as a function of scale factor also. This yields the curve in figure 2.

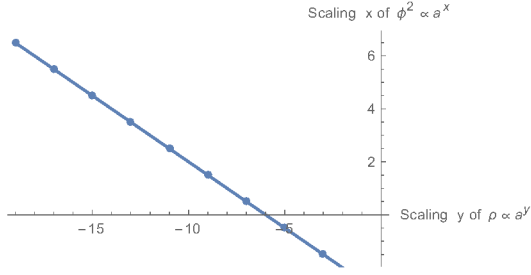


Figure 7: The scaling x of $\phi^2 \propto a^x$ as a function of the scaling y of $\rho_b \propto a^y$ as found by solving the EOM repeatedly for various values of y . The results of the individual solutions is showed by the points which are interpolated exactly by the linear relation $x = -\frac{1}{2}y - 3$ shown with the graph.

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