

Towards a cloverleaf type accelerator magnet

Thomas



TOWARDS A CLOVERLEAF TYPE ACCELERATOR MAGNET

Thomas Henricus Nes

TOWARDS A CLOVERLEAF TYPE ACCELERATOR MAGNET

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*Pour les enfants qui ont dû fuir leur foyer, qui m'ont
montré que même dans les moments les plus difficiles,
le rire nous unit tous, quelles que soient nos origines.*

Summary

The thesis delves into the development of a cloverleaf geometry based accelerator magnet made with high-temperature superconductor *ReBCO* tape. To this, various aspects are researched.

Two tapes are used in this work: tape from Shanghai Superconducting Technology (SST) for the pancake coils, and tape from SuperPower for a demonstrator cloverleaf coil. For predicting the performance of a superconducting magnet, detailed knowledge of the critical surface of the *ReBCO* tape used is of paramount importance. To this, the first part of the thesis revolves around the development of a fitting process for the critical surface of SST tape using publicly available measurement data. This approach of fitting, rather than simple interpolation, is essential for extrapolating to parameters beyond the established, measured, data range. The critical surface fit of the SST tape is subsequently compared with the critical current values provided by the manufacturer at 77 K, along with the nominal reported critical current density at 4.2 K and 20 T. Additionally, an interpolation method is employed for the SuperPower tape, based on measurement outcomes of a similar tape utilized in the EUCARD2 project. The critical current of the SST tape is measured at 77 K in self-field, aligning with the reported manufacturer's critical current of 400 A.

Among the various strategies aimed at enhancing the quench stability of *ReBCO* tape-based coils, the implementation of no-insulation technology stands out as a promising avenue. In-depth investigations are undertaken through a comparative study involving four single pancake coils, all identical in size. These coils, featuring inner and outer diameters of 52 mm and 102 mm respectively, are wound with 10 mm wide tape obtained from Shanghai Superconducting Technology. Two variations of no-insulation coils were fabricated: dry-wound with no interstitial material between the coil-windings, and the other solder-filled, where solder paste is added between the turns. Two different winding techniques were utilized: one employing a single tape, while the other adopts a twin-tape cable configuration with the superconducting layers facing each other. Crucial parameters such as time constants, inter-turn resistances, dynamic voltage behaviour, current distributions, magnetic field behaviour during charging and discharging processes, critical currents, and quench currents are meticulously derived and comprehensively analysed. These findings offer valuable insights into the quench behaviour and stability of no-insulation coils, serving as a basis for potential advancements in accelerator magnet applications and other high-field devices.

An essential aspect of the research pertains to the estimation of the effective time constant for non-insulated windings in accelerator type of magnets. The effective time con-

stant plays a critical role in determining the magnet's responsiveness, which needs to be closely correlated to the increasing momentum of particle bunches in an accelerator. To accurately estimate the effective time constant, an expanded model is proposed that accounts for mutual inductances between coil turns. This refined model provides a more precise estimation, enhancing the understanding of the temporal behaviour of non-insulated coils. Moreover, the study employs spectral analysis to unveil a set of eigenmodes, each characterized by a corresponding characteristic time constant. Among these eigenmodes, the most significant one dictates the overall decay time, which is crucial for the magnet's performance. This comprehensive analysis sheds light on the reasons why certain strategies, such as partial insulation or overcurrent, to reduce the effective time constant are more effective than others, opening up new avenues for optimization and improved performance of accelerator magnets.

A new innovative design approach is proposed for three-dimensional coil layouts and coil-end geometries, considering the geometrical constraints imposed by the large aspect ratio of *ReBCO* tape (width exceeding thickness). Through a systematic investigation, the research leads to novel design strategies that address various deformation modes of *ReBCO* tape and meet the requirements for effective coil-end design. The thin strip model, based on classical differential geometry, plays a pivotal role in modelling the large aspect ratio of *ReBCO* tape as a thin strip surface, yielding valuable insights for design optimization. To ensure the prevention of conductor degradation, new optimization criteria valid for three-dimensional geometries are introduced, emphasizing prevention of conductor creasing, minimization of overall bending energy, and avoidance of over-straining the conductor. These criteria guide the design process for two distinct 3D coil configurations: the helix, well-suited for *ReBCO* cables and solenoidal magnets, and the canted-cosine-theta, tailored for creating magnets with multipoles of any order. Moreover, a novel design method is proposed, employing Bézier splines for coil-end geometries, which substantially enhances design flexibility compared to conventional methods. Two examples of coil-end geometries generated with Bézier splines, namely the cloverleaf and the cosine-theta coil-end geometry, highlight the efficacy of this innovative design approach.

In a significant practical application, the design of a demonstrator cloverleaf-racetrack magnet is presented, exemplifying a candidate configuration for 20 T accelerator dipole magnets. This design synthesizes valuable insights gained from previous chapters, ensuring a coherent and optimized magnet system. The utilization of non-insulated coils for quench protection allows for transverse redistribution of coil current during quenching, enhancing operational safety and performance. A modelling-based comparison between dry-wound and soldered non-insulated coils is carried out, evaluating mechanical performance and time constant management. The findings indicate that the dry-wound coil exhibits a lower time constant and better mechanical stability, making it a more suitable choice for the cloverleaf-racetrack configuration, if no-insulation technology were to be utilized.

With the prospect of passing over the beam-pipe while minimizing hard-way bending, the cloverleaf configuration presents a practical solution. However, the thesis delves into addressing challenges associated with coil-end design and coil winding, cooling, and operational considerations. Through a detailed examination, an eight-turn cloverleaf coil has been wound using two 12 mm SuperPower tape conductors in parallel, with the *ReBCO* layers facing each other. The presence of 20 turns of 0.1 mm thick copper on both the inside

and outside of the coil serves as a stabilizer and facilitates current entry and exit points. Mechanically supporting the coil, the copper is fully soldered on the inside. The coil underwent rigorous testing, including cooling to 77 K and current supply at this temperature. Key parameters, such as critical current, n -value, effective time constants, current distribution, and magnetic field response, are thoroughly examined, providing valuable insights for future coil designs. Finally, a conceptual design of a 10 m long 20 T dipole magnet is presented.

Samenvatting

Dit proefschrift gaat over de ontwikkeling van een versnellermagneet op basis van een klaverbladgeometrie met hoge-temperatuur supergeleidende *ReBCO*-tape. Hiervoor worden verschillende aspecten onderzocht.

In dit werk worden twee tapes gebruikt: tape van Shanghai Superconducting Technology (SST) voor de pannenkoekspoelen en tape van SuperPower voor een demonstratie klaverbladspoel. Voor het voorspellen van de prestaties van een supergeleidende magneet is gedetailleerde kennis van het kritieke oppervlak van de gebruikte tape van het grootste belang. Daarom gaat het eerste deel van dit proefschrift over de ontwikkeling van een beschrijvingsmethode van het kritieke oppervlak van de SST tape met behulp van openbaar beschikbare meetgegevens. Deze aanpak, in plaats van eenvoudige interpolatie, is essentieel voor het extrapoleren naar parameters buiten het vastgestelde, gemeten bereik. De beschrijving van het kritieke oppervlak van de SST tape wordt vervolgens vergeleken met de kritieke stroomwaarden die door de fabrikant zijn opgegeven bij 77 K, samen met de nominale gerapporteerde kritieke stroomdichtheid bij 4.2 K en 20 T. Daarnaast wordt een interpolatiemethode gebruikt voor de SuperPower tape, gebaseerd op meetresultaten van een soortgelijke tape die gebruikt werd in het EUCARD2-project. De kritieke stroom van de SST tape is gemeten bij 77 K in eigenveld, wat overeenkomt met de gerapporteerde kritieke stroom van 400 A van de fabrikant.

Van de verschillende strategieën die gericht zijn op het verbeteren van de quenchstabiliteit van spoelen met tape op basis van *ReBCO*, is de toepassing van de geen-isolatietechnologie veelbelovend. Er is een diepgaand onderzoek uitgevoerd door middel van een vergelijkende studie met vier enkele pannenkoekspoelen, die allemaal dezelfde afmetingen hebben. Deze spoelen, met een binnen- en buitendiameter van respectievelijk 52 mm en 102 mm, zijn gewikkeld met 10 mm brede tape verkregen van Shanghai Superconducting Technology. Er werden twee varianten van isolatieloze spoelen gefabriceerd: drooggewikkeld zonder tussenmateriaal tussen de wikkelingen van de spoel, en de andere met soldeer gevuld, waarbij soldeer pasta tussen de wikkelingen wordt toegevoegd. Er werden twee verschillende wikkeltechnieken gebruikt: de ene maakt gebruik van een enkele tape, terwijl de andere een kabelconfiguratie met twee tapes gebruikt, met de supergeleidende lagen tegenover elkaar. Cruciale parameters zoals tijdconstanten, interwikkelingsweerstand, dynamisch gedrag van de spanning, stroomverdelingen, magnetisch veldgedrag tijdens oplaad- en ontladprocessen, kritieke stromen en quenchstromen zijn nauwkeurig afgeleid en uitgebreid geanalyseerd. Deze bevindingen bieden waardevolle inzichten in het quenchgedrag en de stabiliteit van geen-isolatiespoelen en dienen als basis voor mogelijke verbeteringen in toepassingen voor versneller magneten en andere

apparaten met een hoog magneetveld.

Een essentieel aspect van het onderzoek betreft de afschatting van de effectieve tijdconstante voor niet-geïsoleerde versneller-magneten. De effectieve tijdconstante speelt een cruciale rol bij het bepalen van de respons van de magneet, die nauw gecorreleerd moet zijn met het toenemende momentum van deeltjesbunches in een versneller. Om de effectieve tijdconstante nauwkeurig te kunnen afschatten, wordt een uitgebreid model voorgesteld dat rekening houdt met wederzijdse inducties tussen wikkelingen van spoelen. Dit verfijnde model levert een nauwkeurigere schatting op, waardoor het begrip van het temporele gedrag van niet-geïsoleerde spoelen wordt verbeterd. Bovendien maakt het onderzoek gebruik van spectrale analyse om een reeks eigenmodes te bepalen, elk gekenmerkt door een bijbehorende karakteristieke tijdconstante. Van deze eigenmodes dicteert de belangrijkste de totale vervaltijd, die cruciaal is voor de prestaties van de magneet. Deze uitgebreide analyse werpt licht op de redenen waarom bepaalde strategieën, zoals partiële isolatie of overstroom, om de effectieve tijdconstante te verlagen effectiever zijn dan andere, wat nieuwe wegen opent voor optimalisatie en verbeterde prestaties in versneller-magneten.

Nieuwe, innovatieve ontwerpbenaderingen zijn voorgesteld voor driedimensionale spoelconfiguraties en geometrieën van spoeleinden, rekening houdend met de geometrische beperkingen die worden opgelegd door de grote breedte-dikteverhouding van *ReBCO*-tape (breedte veel groter dan dikte). Door middel van systematisch onderzoek stelt het onderzoek nieuwe ontwerpstrategieën voor die verschillende vervormingsmodi van *ReBCO*-tape aanpakken en voldoen aan de vereisten voor een effectief spoeluiteinde-ontwerp. Het dunne strookmodel, gebaseerd op klassieke differentiaalmeetkunde, speelt een centrale rol in het modelleren van de grote breedte-dikteverhouding van *ReBCO*-tape als een dun strookoppervlak, wat waardevolle inzichten oplevert voor ontwerpoptimalisatie. Om ervoor te zorgen dat de geleider niet degradeert, zijn nieuwe optimalisatiecriteria geïntroduceerd die geldig zijn voor driedimensionale geometrieën, met de nadruk op het voorkomen van plooiën van de geleider, minimalisatie van de totale buigenergie en het vermijden van overbelasting van de geleider. Deze criteria geven richting aan het ontwerpproces voor twee verschillende 3D spoelconfiguraties: de helix, zeer geschikt voor *ReBCO*-kabels en solenoïde magneten, en de canted-cosine-theta, op maat gemaakt voor het maken van magneten met multipolen van elke orde. Bovendien is een nieuwe ontwerpmethode voorgesteld met Bézierkrommen voor geometrieën van spoeleinden, die de ontwerpflexibiliteit aanzienlijk vergroot ten opzichte van conventionele methoden. Twee voorbeelden van spoel-eindgeometrieën gegenereerd met Bézierkrommen, namelijk de klaverblad en de cosinus-theta spoel-eindgeometrie, benadrukken de efficiëntie van deze innovatieve methode.

In een belangrijke praktische toepassing is het ontwerp van een demonstratie klaverblad-racetrackmagneet gepresenteerd, dat een voorbeeld is van een kandidaatconfiguratie voor 20 T dipolen. Dit ontwerp is een synthese van waardevolle inzichten uit eerdere hoofdstukken en zorgt voor een coherent en geoptimaliseerd magneetsysteem. Het gebruik van niet-geïsoleerde spoelen voor quenchbeveiliging maakt een transversale herverdeling van de spoelstroom tijdens een quench mogelijk, wat de operationele veiligheid en prestaties verbetert. Er wordt een vergelijking op basis van modellen uitgevoerd tussen drooggewikkelde en soldeer-gevulde niet-geïsoleerde spoelen, waarbij de mechanische prestaties en het beheer van de tijdconstante worden geëvalueerd. De bevindingen

geven aan dat de drooggewikkelde spoel een lagere tijdconstante en betere mechanische stabiliteit vertoont, waardoor het een meer geschikte keuze is voor de klaverblad-racetrack configuratie, indien geen-isolatietechnologie zou worden toegepast.

Met het voordeel om over de bundelpijp te kunnen gaan en tegelijkertijd de harde buiging te minimaliseren, biedt de klaverbladconfiguratie een praktische oplossing. Het proefschrift gaat echter dieper in op de uitdagingen die gepaard gaan met het ontwerp van het spoeluiteinde en de wikkeling, koeling en operationele overwegingen van de spoel. Door middel van een gedetailleerd onderzoek wordt een klaverbladspoel met acht wikkelingen gewikkeld met twee tapes van 12 mm SuperPower tape geleiders parallel, met de *ReBCO*-lagen naar elkaar toegericht. De aanwezigheid van 20 wikkelingen van 0.1 mm dik koper aan zowel de binnen- als buitenkant van de spoel dient als stabilisator en vergemakkelijkt het in- en uitstromen van de stroom. Het koper ondersteunt de spoel mechanisch en is aan de binnenkant volledig gesoldeerd. De spoel heeft rigoureuze tests ondergaan, inclusief koeling tot 77 K en stroomtoevoer bij deze temperatuur. Belangrijke parameters, zoals kritieke stroom, n -waarde, effectieve tijdconstanten, stroomverdeling en magnetische veldrespons, zijn grondig onderzocht, wat waardevolle inzichten heeft opgeleverd voor toekomstige spoelontwerpen. Tot slot wordt een conceptueel ontwerp van een 10 m lange 20 T dipoolmagneet gepresenteerd.

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Chapter 1

Introduction to *ReBCO* particle accelerator magnets

Particle physics experiments at colliders provide a better understanding of how the physical world around us works. For the exploration of the energy frontier, it is desirable to have as much collision energy as possible, to have the largest mass range of particles produced in the collision. Currently, the highest collision energies are obtained in circular colliders, which require strong magnets to guide the particle beam. To generate these high magnetic fields, superconductors are essential, as they provide the high current densities needed. One of the most powerful superconductors is *ReBCO*, due to its high critical temperature, critical current density, and upper critical field. In this thesis, the possibility of using *ReBCO* material in high-field accelerator magnets is explored in a magnet configuration called the cloverleaf. This chapter provides an introduction to the thesis by giving the relevant background to particle colliders and superconductors, as well as an overview of *ReBCO* conductors and arguments to choose for a cloverleaf type accelerator magnet. The chapter ends with the scope of the thesis.

1.1 Particle colliders

‘What is the world around us made of?’ is a question that must have nagged at the homo sapiens since its emergence in Africa a few hundred thousand years ago. The ancient Greek and Indian philosophers already proposed that the universe is made of tiny particles that cannot be subdivided [1]. This school of philosophical thought is now called atomism, after the Greek word *ἄτομον* (atomon, ‘uncuttable’). It is now understood that their beliefs were partially correct, but faith has given it an ironic twist, as what is now known as the atom can be broken down into smaller particles. An atom consists of a nucleus, composed of protons and neutrons, and one or more electrons bound to the nucleus. Protons and neutrons can even be further subdivided into quarks and gluons that bind the quarks together [2]. Quarks and electrons belong to the elementary particle family, i.e. particles without an internal structure (etymologically, they should be called atoms).

In modern times, the role of the ancient philosophers on this topic has been taken over by physicists, who want to study the properties and interactions of elementary particles. The interactions can be studied by making the particles collide with each other. This is done at high energy, in order to overcome the repulsion forces of particles of the same charge and to increase the resolution or detail of the collisions. From particle-wave duality, it is known that particles also behave as waves and that each particle has an associated wavelength. Velocity and wavelength are inversely proportional to each other: the higher the velocity (or momentum) of the particle, the smaller its wavelength. More details are therefore visible when the particles collide at a high energy.

To attain high momenta, the particles must be accelerated in a complex known as a particle accelerator. For this, charged particles are the most suitable as they can be accelerated by electric fields in radio frequency (RF) cavities [3]. There are two basic types of particle accelerators: linear and circular. Linear accelerators propel particles along a straight line. Circular accelerators propel particles along a circular path. To steer the charged particles around the curved path of the circular collider, bending magnets are used. The main bending is achieved by dipole magnets. Focusing and corrections to the beam path and shape are performed by higher order magnets, e.g. quadrupoles and sextupoles. The advantage of a circular collider is that the particles get a kick each time they circle the complex and pass the RF cavity, allowing the particles to reach higher energies than in linear accelerators of a similar size. The fact that the particles circle the complex several times also means that there are many opportunities for collisions at the points where the particle beams are made to cross. To achieve the highest possible energy, circular accelerators are preferable for proton-proton colliders.

Currently, the largest and most powerful accelerator complex is the Large Hadron Collider (LHC) at CERN, which started operating in 2008 and is located on the border between France and Switzerland. Ionized hydrogen atoms are first accelerated in linear accelerators, where they are collected and bunched together. The proton beams are then sent to a chain of booster accelerators to increase their energy and intensity, before being injected into the Large Hadron Collider (LHC) in both directions and collided at the interaction points. An overview of the CERN accelerator complex is shown in Figure 1.1. The LHC was built in a circular tunnel with a circumference of about 27 km. In this tunnel, protons (from the hadron family) are accelerated to produce collisions. As of April 2022, these protons are accelerated to an energy of 6.8 TeV, giving a collision energy of 13.6 TeV. The

collision products are captured by detectors installed on the collider ring [4]. The LHC has achieved remarkable success by confirming many aspects of the standard model with enhanced accuracy and with the discovery of the Higgs Boson in 2012.

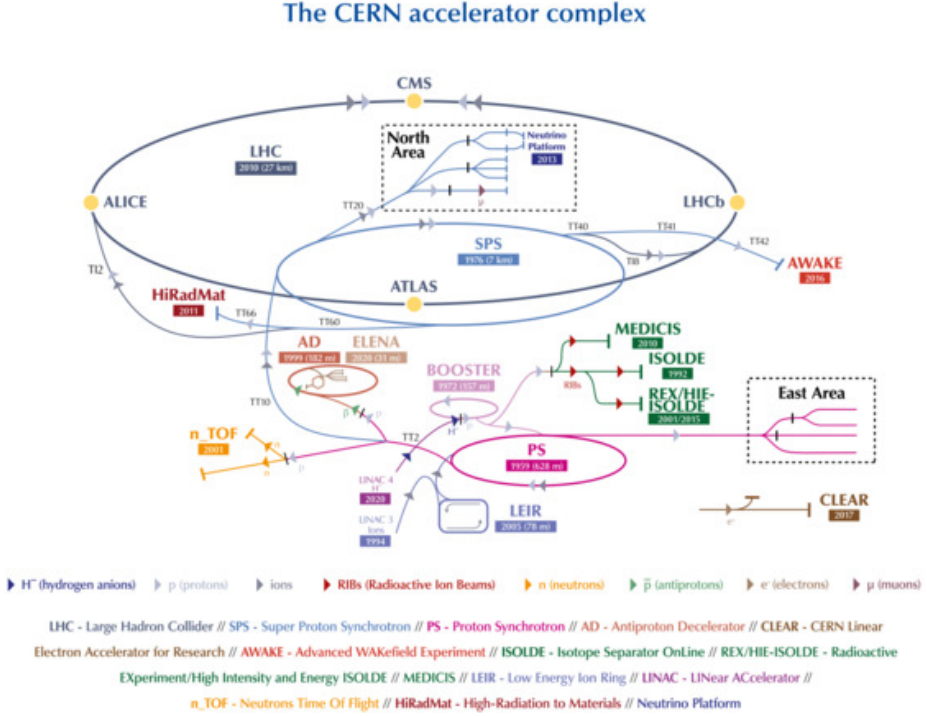


Figure 1.1: The CERN accelerator complex [5].

However, the LHC also observed that in detail there are few and very small discrepancies in the observations compared to the standard model. To go beyond the physics of the standard model, new accelerator complexes with higher proton-proton energies or high precision lepton-lepton (electron or muon) colliders are needed. As electrons and muons are elementary particles, lepton colliders are mostly suited for precision physics studies while the composite protons are most suitable as discovery machines for new particles.

Circular machines are envisioned for both proton-proton colliders and muon colliders. The maximum particle momentum p_{part} of a particle of unit charge in a circular collider depends on the radius of the accelerator R_{acc} and the strength of the bending magnets B_{acc} :

$$p_{\text{part}} \approx 0.3 B_{\text{acc}} R_{\text{acc}}, \quad (1.1)$$

where p_{part} is in units of GeV/c. There are then two approaches to increase the collision energy in a circular collider. One is to increase the bending radius, the other is to increase the magnetic field in the bending magnets.

CERN has proposed a new proton-proton collider called the Future Circular Collider (FCC) as a successor to the LHC, which aims to both increase the collider radius and magnetic field strength to reach higher collision energies. This program is outlined in the 2020 Update of the European Strategy for Particle Physics [6]. The collider is to be built in a 100 km long tunnel next to the existing LHC tunnels (see Figure 1.2). In the middle-long term, an electron-positron collider (FCC-ee) is foreseen as a first step. In the long term, a proton-proton collider (FCC-hh) is envisioned, which would operate with a collision energy of 100 TeV. This energy would allow for the discovery of new fundamental particles roughly of up to an order of magnitude higher in mass than can be produced with the LHC, and give insights on physical processes such as Higgs self-interaction [7]. Recently, another concept has gained momentum for a muon collider in the few TeV range [8].

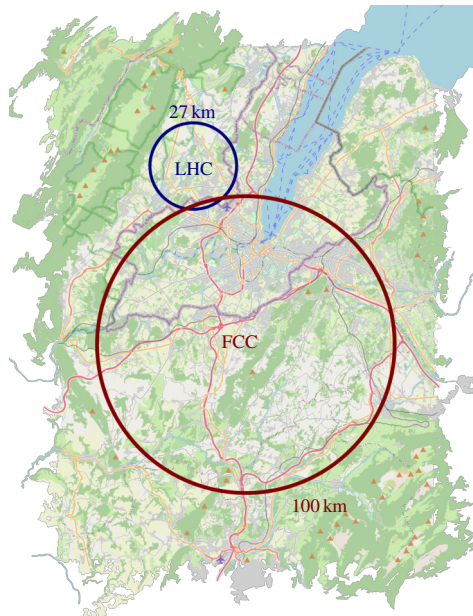


Figure 1.2: Locations of the Large Hadron Collider (LHC) and the proposed Future Circular Collider (FCC) in the greater Geneva area. The circumference of the LHC is 27 km, and the proposed circumference of the FCC is 100 km.

1.2 Practical superconductors for accelerator magnets

Discovered in 1911 by the Dutch physicist Heike Kamerlingh Onnes [9], superconductivity is a phenomenon characterised by the absence of electrical resistance and the expulsion of magnetic field (the Meissner-Ochsenfeld effect) in certain materials called superconductors. These effects only occur if the superconductor is operated at a sufficiently low combination of temperature T , current density J and magnetic field B . This region where superconductivity is present in the T , B , J space is bounded by the so-called critical surface, shown in Figure 1.3. If the superconductor is operated below the critical surface,

the conductor will be in a superconducting state, while above the critical surface, it will be in a normal state. The value of the current density at the critical surface is called the critical current J_c . The maximum temperature and magnetic field at which superconductivity occurs are called the critical temperature T_c and the upper critical magnetic field B_{c2} , respectively.

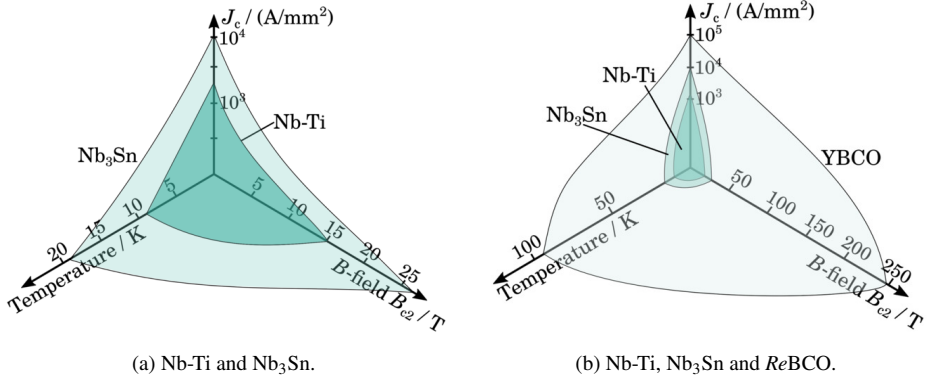


Figure 1.3: Illustration of the critical surface of technical superconductors. Image taken from S. Richter [10].

Superconductors are essential for the construction of high-field magnets, due to their high critical current densities. Superconducting cables are capable of carrying transport currents two to three orders of magnitude higher than normal conductors of similar size, for example copper or aluminium. Superconducting magnets can therefore be significantly more compact than normal conducting magnets.

Technical superconductors are so-called type-II superconductors. The group comprises alloys, such as Nb-Ti and Nb₃Sn, and complex ceramic oxides, such as BiSrCaCuO (BSCCO) and Rare-earth-BaCuO (*ReBCO*). In a type-II superconductor, magnetic field cannot penetrate the material below a lower critical field B_{c1} . If the magnetic field is increased above this critical value in the mT range, the magnetic field can enter the superconductor through so-called flux lines, each of which contain a single flux quantum $\Phi_0 = \frac{h}{2e}$. In a flux line, a shielding supercurrent circulates around a central core in which the material is in normal state. This mixed state with superconducting areas interspersed with flux lines is called the Abrikosov state. The existence of flux lines allows type-II superconductors to remain superconducting in high magnetic fields, which makes them indispensable as conductors for magnet applications. When the magnetic field is increased further, the vortex density increases, until the material can no longer contain the vortices at the upper critical field B_{c2} . Above this magnetic field strength, the whole material becomes normal conducting.

The transition from the superconducting state to the normal state is not infinitely sharp, but more gradual. This behaviour can be expressed in the form of a power law:

$$\frac{E}{E_c} = \left(\frac{I}{I_c} \right)^n, \quad (1.2)$$

where E is the electric field, E_c the electric field criterion above which the material is not

considered superconducting any more and n a power factor describing the sharpness of the transition. The tape critical current I_c is then defined as the current magnitude where the electric field is as large as the electric field criterion.

The most utilized superconducting material is Nb-Ti, which was discovered in 1962 [11]. It becomes superconducting at 9.2 K and has an upper critical field of 14.5 T. Nb-Ti is ductile material, making it practical to implement. It is therefore used in a large variety of applications, such as laboratory magnets for research, MRI scanners, and particle accelerators. The LHC comprises 1232 15 m long Nb-Ti dipole magnets, 392 Nb-Ti quadrupole magnets, each 5 m to 7 m long, three large Nb-Ti based detector magnet systems, and about 6000 small corrector magnets [12]. The current carrying performance of Nb-Ti increases profoundly at temperatures lower than the critical temperature. Therefore, to utilize this performance, the LHC operates at 1.9 K, which is achieved by cooling with superfluid helium. The nominal magnetic field for which the main dipoles of the LHC are designed for is 8.33 T, which gives a collision energy of 14 TeV [13].

Another niobium based compound that is used for superconducting magnets is Nb₃Sn, discovered in 1954 [14]. It becomes superconducting at 18.3 K and has a critical magnetic field of about 30 T. This makes Nb₃Sn suited for magnets between 9 T, the practical limit for Nb-Ti accelerator magnets, and 16 T, the practical limit of Nb₃Sn accelerator magnets due to the falling critical current density. Nb₃Sn is, however, a much more difficult material to work with than Nb-Ti, as the compound is very brittle. To use the material, it is normally necessary to wind the magnet first and then react the niobium and tin, so that the brittle compound is formed when the windings are already in place. The reaction is done at temperatures between 650 and 700 °C. This technique of winding first and then reacting is referred to as wind-and-react. In the context of FCC, it is considered to use Nb₃Sn magnets for the main dipoles, which have a magnetic field strength of 16 T [15]. Combined with a collider ring with a circumference of 100 km, this would allow for a collider with a centre of mass collision energy of 100 TeV.

To go beyond the 16 T practical limit of Nb₃Sn, so-called high temperature superconductors are required. These materials show remarkable differences with Nb-Ti and Nb₃Sn materials in terms of their physical properties. Figure 1.4 shows whole wire critical current densities for practical superconductors. Note that the lower limit considered for useful critical current density is about 300 A mm to 500 A mm⁻². The critical magnetic field of BSCCO and ReBCO is significantly higher than that of Nb-Ti and Nb₃Sn materials, and the current-carrying capacity does not decrease as quickly with increasing magnetic field. This makes magnets made with these materials a promising candidate for accelerator magnets, which are capable of reaching magnetic field strengths well beyond 20 T. Currently, no BSCCO or ReBCO type of dipole magnets are in use in accelerator complexes. However, research on their potential use continues in various projects, with Bi-2212 [16] and ReBCO [17, 18] being considered as potential candidates for the development of accelerator magnets.

The cuprate Bismuth Strontium Calcium Copper Oxide (BSCCO) was discovered to be a superconductor in 1988 [20]. Its generalized chemical formula is Bi₂Sr₂Ca _{$n-1$} Cu _{n} O_{2 $n+4+x$} . The variant Bi-2212 ($n = 2$) nowadays comes in the form of a round wire, although in the past flat tapes have also been investigated [21]. The variant Bi-2223 ($n = 3$) comes in the form of a flat tape [22]. Bi-2223 was the first tape material to be used for making practical superconducting wires, and therefore it is also referred to as the first-

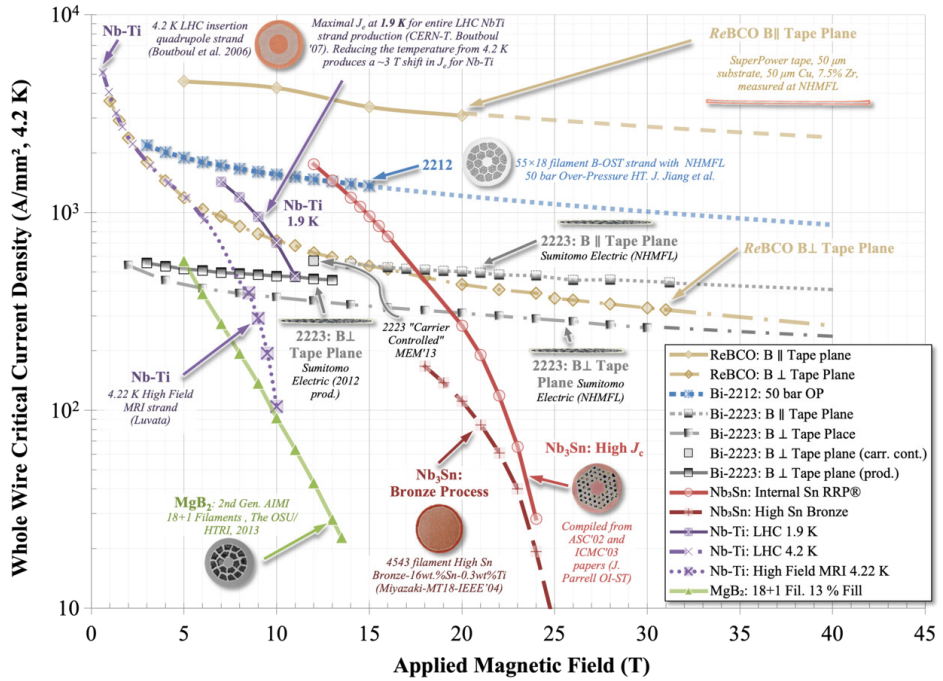


Figure 1.4: Whole wire critical current density of various superconducting materials, available in long lengths, as a function of the applied magnetic field. Adapted from a figure produced by the National High Magnetic Field Laboratory [19].

generation (1G) tape superconductor. Due to its higher critical current density and insensitivity to the direction of the background magnetic field, Bi-2212 is more frequently considered as a conductor in accelerator magnets, compared to Bi-2223 which has a lower critical current density that is also affected by the direction of the background magnetic field. Bi-2212 has a critical temperature of 95 K and upper critical field of around 200 T. The fabrication of Bi-2212 is however very challenging. To achieve a consistently high critical current density, the conductor must be heat-treated in a partial oxygen atmosphere at high temperatures up to 900 °C, in a partial oxygen atmosphere at total pressures between 20 bar and 100 bar [16]. After the heat treatment, the material is brittle. Another downside of BSCCO is that a silver matrix is needed to allow the penetration of oxygen during the heat treatment process. The price of the silver places a lower limit on the conductor cost [23].

Another promising candidate is the cuprate Rare-earth barium copper oxide (*Re*BCO). *Re*BCO was already discovered in 1986 [24], two years before BSCCO. However, due to early difficulties in the fabrication of tape out of *Re*BCO conductor, it took a little longer before they came commercially available. For this reason *Re*BCO based conductors are referred to as second-generation (2G) tape conductors. The chemical composition depends on the rare-earth element used, but for yttrium and gadolinium it can be written as $\text{ReBa}_2\text{Cu}_3\text{O}_{7-\delta}$. *Re*BCO coated conductors are available in the form of a flat tape, where

the *ReBCO* is deposited as a thin film on a carrier substrate, usually Hastelloy. *ReBCO* coated conductor has a critical temperature of 93 K and (theorized) critical field of some 250 T. Compared to BSCCO, the maximum obtainable critical current density is higher in *ReBCO*. However, *ReBCO* is an anisotropic material and the critical current density depends on the direction of the applied background magnetic field. In the worst magnetic field orientation, the critical current density of *ReBCO* is at the moment lower than that of BSCCO.

In this thesis, the focus is on *ReBCO* conductors for particle accelerator magnets due to their superior transport properties on a long-term compared to BSCCO, and because it does not require heat-treatment stages after winding like Bi-2212, simplifying the magnet fabrication process. However, the development of these conductors has not fully matured yet. In the following section, the characteristics of *ReBCO* are discussed in more detail.

1.3 *ReBCO* coated conductor

As stated in the previous section, *ReBCO* coated conductor is a good candidate for high-field magnet applications. In this section its micro-structural composition, tape architecture and electromagnetic and mechanical properties are discussed in more detail.

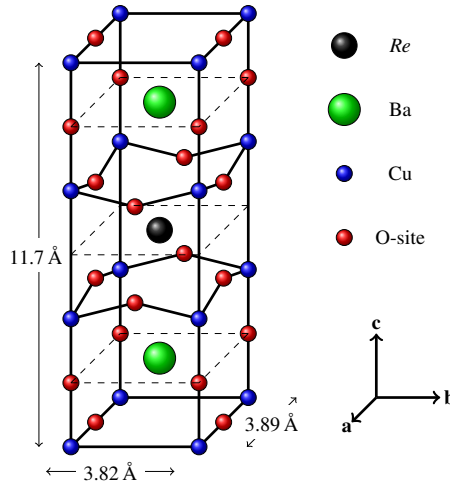
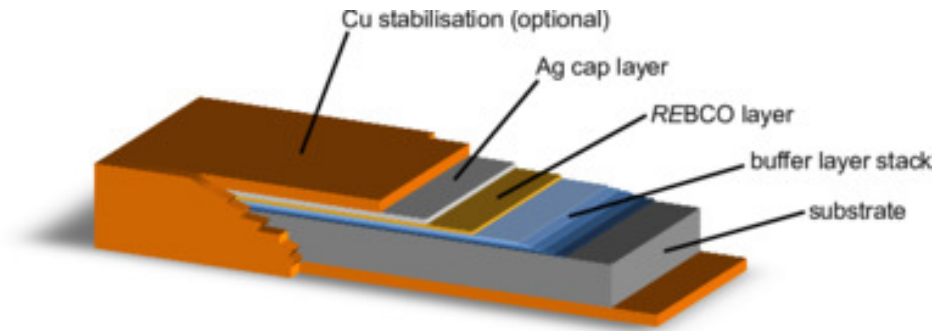
1.3.1 Crystal structure

The crystal structure of $ReBa_2C_3O_{7-\delta}$, as depicted in Figure 1.5, is an intricate variation of the perovskite structure [25]. The *ReBCO* unit cell is composed of a $ReCuO_3$ cube with neighbouring $BaCuO_3$ cubes on top and bottom. However, some oxygen sites are vacant. The oxygen atoms on the same horizontal plane as the *Re* atom are always unoccupied, which results in the oxygen atoms shifting slightly closer to the *Re* atom. The oxygen content is of influence to its superconducting properties. The optimum oxygen content is found at $\delta = 0.06$, as this gives the material its maximum critical temperature [26]. Superconductivity is due to the CuO_2 planes and CuO chains in the orthorhombic phase [25].

From the crystal structure, it can be seen that the material is anisotropic, which means that its electromagnetic properties are orientation dependent. In Section 1.3.3, these are discussed in more detail. Besides the crystal structure, the properties of *ReBCO* conductor are also dependent on the structure of the tape, which is discussed in the following section.

1.3.2 Tape composition and fabrication

Due to the weak link property of *ReBCO*, the alignment of the grains is a critical issue to obtain a high and uniform critical current [27]. *ReBCO* can be manufactured in both bulk [28] and thin film form. However, only thin films are used to create *ReBCO* based conductors. In Figure 1.6, a schematic of *ReBCO* coated conductor architecture is shown. Within the coated conductor configuration, a *ReBCO* film is deposited on a metal substrate with buffer layers, which allows for the precise control of grain alignment during the deposition process. As a result, the 1 μm to 3 μm thin *ReBCO* layer in the coated conductor exhibits exceptionally high average transport critical current densities due to the near-perfect grain alignment in the epitaxial layer.

Figure 1.5: Schematic of the $ReBCO$ crystal structure.Figure 1.6: Schematic of the $ReBCO$ conductor architecture [29].

The obvious way to control the grain-boundary problem is to texture the $ReBCO$ material biaxially. However, instead of texturing $ReBCO$ directly, Iijima et al. [30] showed that it is easier to texture the underlying material and deposit the $ReBCO$ layer epitaxially on it. Typically, Hastelloy is used for the substrate, as it is a strong and non-magnetic high-temperature resistant alloy with a similar thermal contraction coefficient as $ReBCO$.

Several methods are used to texture the template for the deposition of the $ReBCO$ layer [31]. The Ion Beam Assisted Deposition (IBAD) process involves using an argon ion gun to sputter a target material, such as magnesium oxide (MgO), on a polycrystalline substrate while an additional argon gun irradiates the growing oxide at a specific angle, resulting in the creation of a biaxially textured buffer layer on the substrate. Another technique, Inclined Substrate Deposition (ISD), involves positioning the substrate at a specific angle relative to the target material and irradiating it using a Pulsed Laser Deposition (PLD) or an electron gun. The Rolling Assisted Biaxially Textured Substrate (RABiTS) technique involves cold rolling and recrystallization of a metallic tape to form a biaxially textured

substrate, with a buffer layer added between the *ReBCO* layer and substrate to prevent chemical reactions and potential degradation of superconducting properties.

The *ReBCO* layer can be deposited on the prepared substrate through in-situ or ex-situ methods. In-situ methods, such as pulsed laser deposition (PLD), reactive co-evaporation (RCE), metal organic vapour deposition (MOCVD), and pulsed electron beam deposition, allow the *ReBCO* layer to be grown directly. Alternatively, an ex-situ method like metal organic deposition (MOD) can be used, where a precursor is deposited and then reacted to form the *ReBCO* layer [32]. Artificial pinning centres can be introduced to enhance the critical current density [33]. Subsequently, a silver layer is laid down on top of the *ReBCO* layer. This is needed to prevent a chemical reaction with the copper, with which the tape is coated in the final step of the production process. The copper helps cooling the conductor, facilitates connections and prevents damage from over-current conditions. It can be applied using methods such as electroplating or lamination.

1.3.3 Electromagnetic properties

The mean critical current density of *ReBCO* coated conductors is a crucial parameter for determining the performance in various applications. The performance and configuration of *ReBCO* coated conductors can vary greatly between different manufacturers [33]. The configuration of *ReBCO* tapes varies in several aspects, including tape width, type of stabilizer, type of substrate and its thickness, positioning of the stabilizer, and how artificial pinning centres are introduced. These factors all influence the engineering critical current density of the tape. Due to defects and inhomogeneities of the tape, the critical current can vary along the length of the tape, which can lead to weak spots in the application [34]. *ReBCO* tape performance can also be optimized for different temperature and magnetic field ranges [35].

One of the key characteristics of *ReBCO* coated conductor is the angular dependence of the critical current, which arises from the intrinsic anisotropy of the material. The critical current is highly dependent on the applied magnetic field, with the highest critical current generally observed when the magnetic field is aligned parallel to the tape and the lowest when the magnetic field is perpendicular to the tape. The angular dependence of the critical current density can be changed by skewed growth of the *ReBCO* layer or the introduction of artificial pinning centres. Artificial pinning centres can lead to secondary peaks in the angular spectrum [36] but can also reduce the magnetic field anisotropy [37]. As an example, the angular dependence of the critical current in Shanghai Superconducting Technology [38] tape is shown in Figure 1.7.

In this work, fitting relations for the critical surface of tape from two tape manufacturers used in the thesis are discussed. The scaling laws are compared to measurement results on tape of Shanghai Superconducting Technology to determine their accuracy and usability. The scaling laws are then used in the design of a cloverleaf type magnet, in order to estimate the performance of the magnet for various *ReBCO* tapes. The design considerations are discussed in the following section.

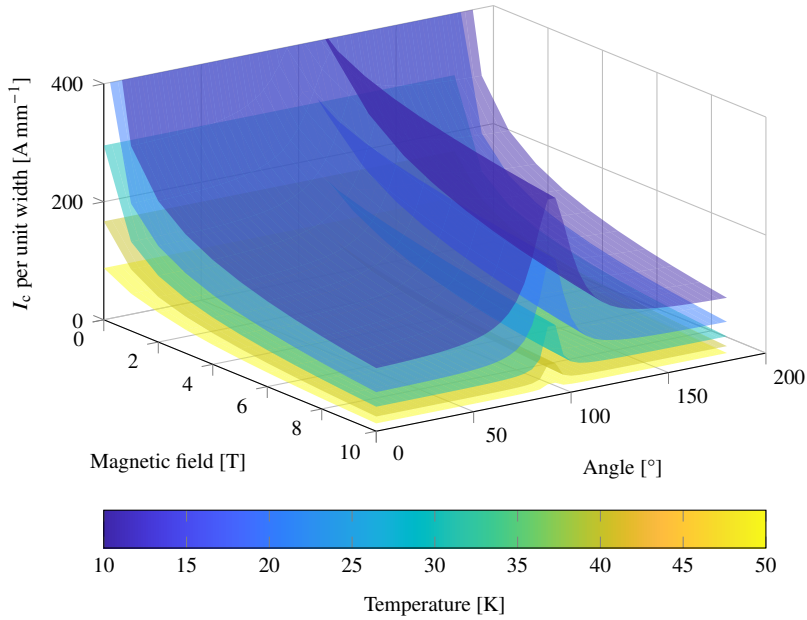


Figure 1.7: Critical current per unit width of Shanghai Superconducting Technology tape as function of magnetic field magnitude and angle, for different temperatures. The critical surface is the result of a fit presented in Chapter 2.

1.4 Considerations for a cloverleaf accelerator magnet

As discussed in Section 1.2, use of *ReBCO* and *BSCCO* conductors as materials are the only practical way to create accelerator magnets with magnetic fields over 16 T. The content of this thesis goes towards the development of a so-called cloverleaf-racetrack magnet based on *ReBCO* tape. The design of this magnet is described in Chapter 6.

To this design, some engineering choices are made to go from *ReBCO* tape to this cloverleaf-racetrack magnet. This section provides an overview of the possible configurations in cable type, coil layout and quench protection scheme, and the ones chosen for the cloverleaf-racetrack magnet. Briefly discussed are also mechanical arguments and magnetic field quality constraints that need to be included in the design and operation of such a magnet.

1.4.1 Cable configurations

A magnet that needs to be powered within a rather short time of some 30 to 60 minutes like an accelerator magnet cannot be made out of a single tape for various reasons. The magnetic field of a magnet is determined by its amp-turns: the larger the current or the number of turns, the higher the magnetic field. As the operating current is bounded by the critical surface, the most effective way is to increase the number of turns in the magnet. By using wires in parallel in a cable, the self-inductance of the magnet is kept low. A low self-

inductance is necessary to be able to ramp the magnet in 30 to 60 minutes, a requirement for accelerator magnets, and to ramp down the magnet faster in the case of a quench. It is also preferred that each strand within the cable has the same impedance, as this favours the current to be distributed more evenly among the strands and limits coupling currents. To this, transposition of the strands within the cable is required. Another issue for *ReBCO* is that it is difficult to fabricate it in long unit lengths without significant defects along the length. Therefore parallel current paths, i.e. using cables, is essential in the design of magnets.

Figure 1.8 shows an overview of possible cables made from *ReBCO* tape conductor. The most basic type of cable is a stack, where the tapes are placed parallel with respect to each other and features no transposition. The tape stack has the highest engineering current density out of all configurations, as it has the highest filling factor. A twisted stack is similar to a simply stacked cable, except for featuring a twist in the cable along its length to obtain a partial transposition [39]. The tape impedance of the ones in the centre of the stack are different to those towards the outside. One or several stacks of twisted conductors can be inserted into a former with grooves or a tube in order to secure the conductors in place and provide additional support. However, the support does lower the engineering current density significantly.

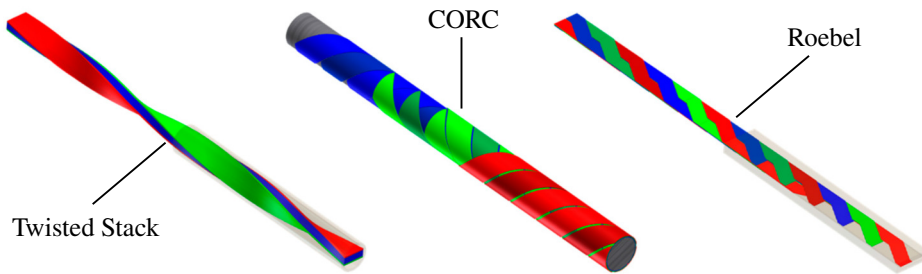


Figure 1.8: Illustration of three cable types made from *ReBCO* tape. Adaptation from Barth [40].

In a Roebel cable, typically the strands are punched from a tape into a zig-zag pattern, with the tapes subsequently meandering through the cable [41]. To create this cable, wide tapes are used, as a portion of the tape must be removed to assemble the Roebel pattern. The individual segments removed from the tape are called ‘meanders’. Using wider tapes and creating multiple meanders from a single tape can be more efficient, however, a significant amount of material is still lost during the process. Due to this, Roebel cable is a less efficient configuration. However, ongoing investigations are exploring the adoption of full-width tapes and in-plane bending for the construction of the Roebel configuration [42]. The advantage of the Roebel cable is that it is the only *ReBCO* cable configuration which is almost fully transposed. Like the stack configuration, the Roebel cable has a high engineering current density due to its high filling factor.

In a CORC (conductor on round core) cable (or wire), *ReBCO* tapes are wound helically around a cylindrical former [43]. A major advantage of this configuration is that the cable is round and quite flexible, so that it can be bent in any direction. In contrary to the other cable types, the cable performance does not depend on the magnetic field angle. Because of this helical nature, CORC cable requires some 90% more tape for the same

length of cable. As the cable is wound around a central core which does not carry any current, the mean current density of a CORC cable is the lowest of all configurations.

At the present stage in the development of *ReBCO* based accelerator magnets, no definitive choice has emerged for cable technology. In general, high engineering current densities are favoured, as it minimizes the amount of tape needed, and the need for transposition in *ReBCO* cables is not evident [44]. In this work, stacked tape cables are considered for the cloverleaf-racetrack magnet, as they require the least amount of tape and feature the highest engineering current density. A tape stack is much harder to bend over its thin sides (hard-way bending) than over its flat side (easy-way bending), of which the consequences are discussed in more detail in Chapter 5.

1.4.2 Dipole coil layout

After the cable layout, a next design choice to be made is the coil layout, as some coil designs favour different cable types over others. To generate the dipole magnetic field of an accelerator magnet, several coil configurations can be considered. Some coil geometries are shown in Figure 1.9.

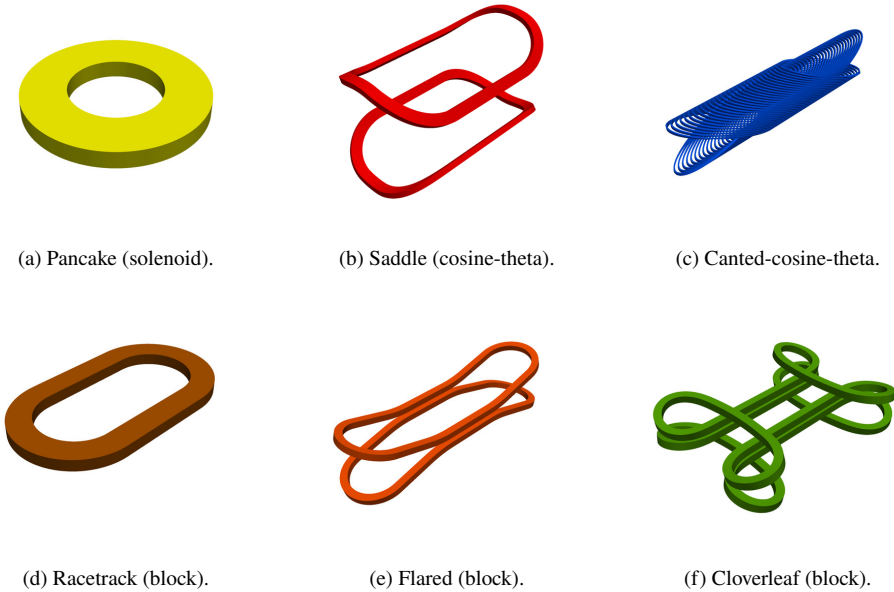


Figure 1.9: Collection of various coil types and coil-ends.

The geometrically simplest coil shape that can be made from *ReBCO* tape is a pancake coil (Figure 1.9a). In such a coil the tape is wound in a spiral from an inner radius to an outer radius. A stack of pancake coils forms a long solenoid. Pancake coils are generally not used in dipole magnets, as they produce a solenoidal field. However, due to their simplicity, they are ideal candidates to perform studies on coil behaviour.

Traditionally for accelerator magnets, the cosine-theta layout (Figure 1.9b) is used, which consists of a collection of coils placed around the aperture, so that the current density decreases as a cosine of the angle with respect to the mid-plane. Saddle coil configurations are used for the coil-ends to guide the conductor over the beam pipe. The geometric shape of a saddle coil is shown in Figure 1.9b. *ReBCO* cosine-theta coils are currently under construction for a dipole magnet made from Roebel cable at CEA [45].

Another option is a canted-cosine-theta (CCT) magnet (Figure 1.9c). This magnet consists of two nested solenoids that are tilted in opposite directions. By powering the coils in opposite polarity, the solenoidal component is cancelled and the dipole component remains. Currently, *ReBCO* CCT coils are being developed, where the coil is wound using CORC wires [46]. It is also possible to make n -th harmonic order CCT's by changing the shape of the helix by including n wiggles. The downside of a CCT is that significantly more conductor length is used compared to other configurations, as fundamentally a lot of conductor length is 'wasted' on creating two cancelling solenoidal fields. Although not the main topic of the thesis, some considerations on the geometry of CCT type magnets made with *ReBCO* conductor are made in Chapter 5.

A disadvantage of cosine-theta and canted-cosine-theta coils made of *ReBCO* conductor is that the magnetic field is not oriented optimally to the conductor surface. To achieve this, a block coil configuration can be used, where the tapes are oriented parallel to the magnetic field. In a normal block coil layout, the tapes are parallel everywhere to the aperture field. In an aligned block configuration the blocks are slightly canted to be locally parallel to the magnetic field. All block coils have similar topologies for the straight section, but the coil-ends can be different. In the framework of the R&D program EuCARD2, two aligned block dipole magnets called Feather M2.1-2 and M2.3-4 using Roebel cable were constructed and tested at CERN [17]. An illustration of the two coil-layouts with Nb-Ti/Nb₃Sn conductor is shown in Figure 1.10 for two different magnets: the LHC dipole and FRESKA2.

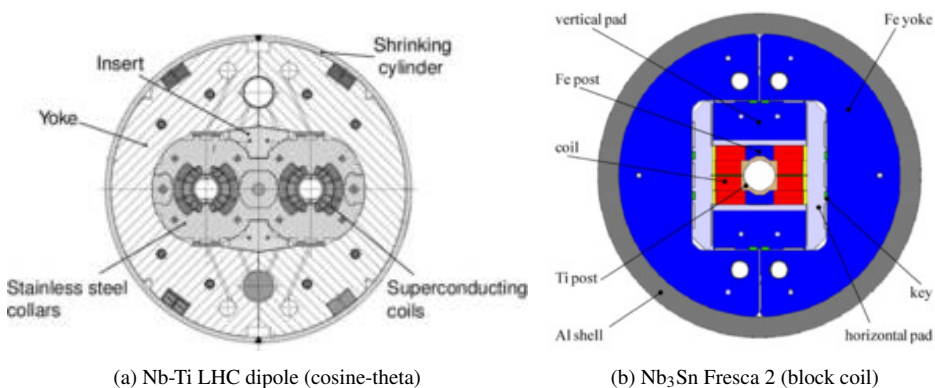


Figure 1.10: Cross-sections of a cosine-theta Nb-Ti coil and a Nb₃Sn block coil configuration [47].

A special type of block coil is the common coil, where two racetrack coils (Figure 1.9d) are placed left and right of two apertures running along both straight sections of the racetrack coils [48]. By powering the coils in opposite polarity, two opposing dipole fields

are created in the two apertures. This configuration has the advantage that the coils are two-dimensional, simplifying the winding and mechanical support structure of the coils and already has the dual aperture build in, which is needed in a circular collider such as the LHC.

An important aspect of the coil geometry is the coil-end. In an accelerator magnet, the aperture cannot be blocked by the coil, so the conductor has to be guided over the beam pipe. In a saddle coil, a saddle coil-end is used (Figure 1.9b). For block coils made with rectangular cable, this geometry is only possible when the tapes are aligned perpendicular to the aperture field. As this is the wrong magnetic field orientation for the *ReBCO* conductor due to its lower critical current, this configuration is a less attractive option for rectangular conductors such as stacked and Roebel. When the rectangular cable is placed parallel to the magnetic field in a block coil, different coil-end geometries are needed. The Feather coils use a so-called flared geometry to pass the conductor over the beam pipe (Figure 1.9e). This is possible as the Roebel cable allows for some hard-way bending, which is needed to create this geometry. Stack type of cables tolerate nearly no hard-way bending, and therefore this configuration is generally not possible. A saddle coil-end is possible when the tape is twisted at the coil-ends, but this has the downside that it requires a long length for the coil-end in order to perform the twist.

In this thesis, a cloverleaf geometry is proposed (Figure 1.9f). In this geometry, the amount of hard-way bending is minimized. In Chapter 5, the cloverleaf coil-end is discussed in detail. To enhance the magnitude of the magnetic field, racetrack coils are used to boost the field. For this reason it was chosen to name the developmental magnet the cloverleaf-racetrack magnet. In Figure 1.11, the coil layout of the cloverleaf-racetrack magnet is shown. The design of this magnet is discussed in Chapter 6. Note that this design concerns a short demonstrator magnet. For a full-size, accelerator compatible magnet, the magnet length should be increased to some 10 m. A conceptual design of the coil-system of such a magnet is also presented in Chapter 6.

1.4.3 Quench protection

One of the key problems in superconducting magnet design is to protect the magnet from damage caused by quenches. Due to local disturbances, a local section of the superconducting cable can transition into the normal state. Here, the current cannot flow through the superconductor and is forced through the resistive matrix, resulting in local heating. If the local cooling power is large enough, the normal zone is cooled away and the conductor recovers to the superconducting state. However, if the normal zone is large enough, the ohmic heating exceeds the cooling power and the normal zone starts to grow. This in turn leads to more heating, leading to a thermal run-away situation. A sudden thermal runaway is referred to as a quench. The amount of energy needed to create a spreading normal zone is the minimum quench energy (MQE). It is an indication of the stability of the magnet: the higher the MQE, the less chance that a disturbance can cause a quench, i.e. the magnet is more stable. *ReBCO* shows remarkable stability compared to Nb-Ti and Nb₃Sn conductors. It requires three orders of magnitude more energy to transition an HTS (high-temperature superconductor) material, such as *ReBCO* and BSCCO, to the normal state compared to an LTS (low temperature superconductor) material, such as Nb-Ti and Nb₃Sn.

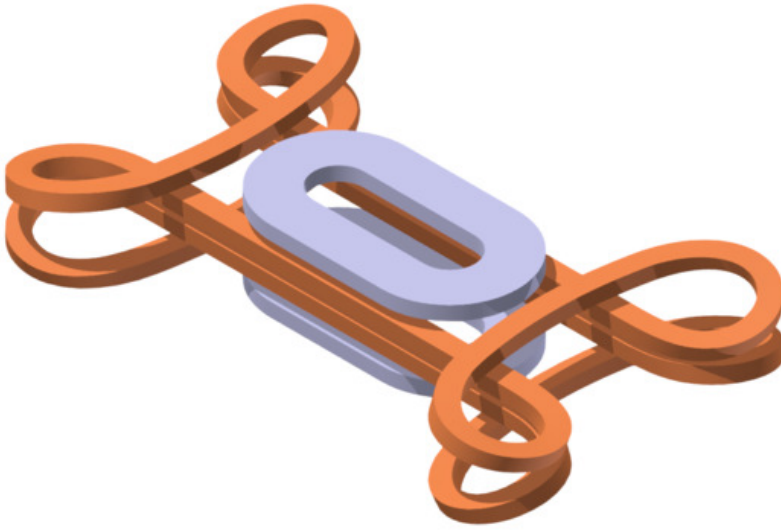


Figure 1.11: Coil layout of the short version cloverleaf-racetrack magnet presented in Chapter 6.

If the magnet is not protected against quenches, it can get catastrophically damaged due to burning or melting of the conductor and rapid expansion of the liquid helium into gas. The onset of a quench is monitored by measuring the voltage of the coil. A rising voltage indicates the presence of a growing normal zone, triggering the quench protection. Quench heaters can be used, which heat up the magnet, spreading the normal zone faster and therefore allowing for the energy to be dissipated over a larger volume.

For *ReBCO* magnets, protecting against quench is more challenging, as the normal zone spreads much more slowly. Combined with the high MQE, this makes it difficult to use quench heaters. An alternative to heaters is to use a coupling loss induced quench system (CLIQ) to heat up the magnet [49]. If ultimate performance is not required, the magnet can be operated at higher temperatures in helium gas flow for increased stability. Tests of the Feather M2.1-2 and M2.3-4 in helium gas flow showed that the onset of a quench is preceded by a drift in voltage and temperature several minutes before a quench, providing ample time to lower the current to prevent a quench [50]. The higher operational temperature also has as an additional benefit of reduced cooling costs. The disadvantage of this method is that it requires more conductor and larger magnets compared to operation at 4.2 K in liquid helium.

The aforementioned protection methods for *ReBCO* magnets are all active solutions for quench protection. Another solution is a passive quench protection by using non-insulated coils [51]. In a non-insulated coil, the turns are separated by finite electrical resistance, providing a bypass for current at hot spots, thus enhancing thermal stability and increasing the quench detection time. When there is a local normal zone, the current can flow through the finite resistance to the next turns, bypassing the defective spot. The principle behind this is shown in Figure 1.12. However, non-insulated coils display a different dynamic electromagnetic behaviour compared to insulated coils under normal charging and transi-

ent quench conditions. The disadvantage of using non-insulated coils is the significantly higher time constant compared to insulated coils, which is due to the presence of a non-superconducting current path, resulting in longer charging and discharging times. This (dis)charging time can be characterized by an effective time constant τ_{eff} , which depends in first approximation on the ratio of the self-inductance L and the transverse resistance R of the coils:

$$\tau_{\text{eff}} = \frac{L}{R}. \quad (1.3)$$

Lower transverse resistances and higher self-inductances lead to larger charging and discharging times. As a result, when designing the coil, it is necessary to consider a trade-off between stability and the effective time constant of the coil, based on the specific requirements set for the coil's application.

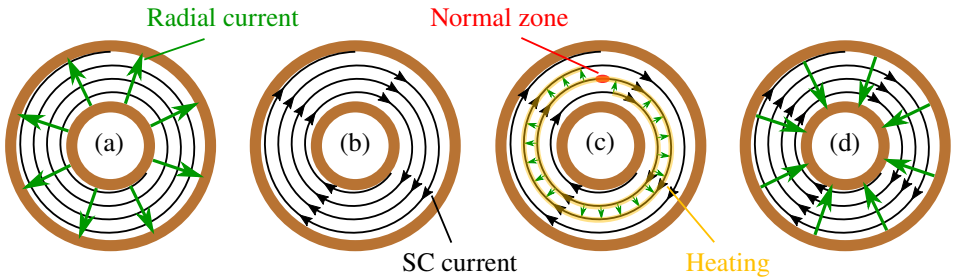


Figure 1.12: Expected current flow in single pancake coils in the following scenarios: (a) coil charging, (b) steady state operation, (c) presence of a normal zone, and (d) coil discharge.

In this thesis, the focus is on using no-insulation technology in particle accelerator magnets. To this, no-insulation coil technology is envisioned for the cloverleaf-racetrack magnet. Before winding a full-size cloverleaf-racetrack magnet, it is preferred to study non-insulated technology on a smaller scale. To this, four pancake coils with single or double tape windings and different inter-turn resistances were fabricated to study non-insulated coil technology for quench protection. To describe the electromagnetic behaviour of non-insulated coils, an equivalent network model is used as described in Chapter 4. This network model is validated with measurement results of the small pancake coils, and is then used to predict the charging time of the cloverleaf-racetrack magnet.

1.4.4 Mechanical constraints

A crucial aspect of designing a superconducting magnet is ensuring its mechanical integrity. In a superconducting magnet, the thermal contraction due to cool-down as well as the electromagnetic forces occurring during powering cause mechanical loads on the conductor. The allowable amount of stress and strain in a *ReBCO* tape or cable is limited, and must be taken into account when designing a *ReBCO* (accelerator) magnet.

The superconducting properties of *ReBCO* material deteriorate non-linearly as strain increases. The longitudinal strain can cause a decrease in critical current when below a certain strain limit. When the deformation of the *ReBCO* tape exceeds a critical strain, the

critical current deteriorates irreversibly, indicating that a portion of the superconducting material may have been damaged. Characteristic critical values for the longitudinal strain lie in the range of 0.5% to 0.7% [29]. In the same fashion, there is a limit on the maximum allowable transverse stress on the tape of some 600 MPa to 800 MPa [29]. For transverse tensile stress, critical values are reported ranging from 5 MPa to 110 MPa [52]. Besides longitudinal and transverse stresses and strains, shear stress is another important mechanical property for *ReBCO* tape in coils. Maximum reported shear stress values range from 1 MPa to 12 MPa at 77 K [53, 54]. For the majority of samples, the critical current decreases quickly as the shear stress approaches the maximum value. For all critical stresses and strains, they depend on operating conditions in temperature and magnetic field [55].

Current degradation occurs due to several modes of failure of a *ReBCO* tape. One such a mode is delamination, where the *ReBCO* layer gets separated from the substrate. Too high shear stress and transverse tensile stress have delamination as typical failure mode. The delamination can be caused by hoop stress [56], screening currents [57] and differences in thermal contraction between the conductor and the epoxy impregnation materials used in the coil [58]. Another failure mode is cracks in the *ReBCO* layer due to longitudinal strain and transverse compressive stress.

Besides limits on the stress and strain on the conductor, another aspect to be considered for accelerator magnets is power cycling. The cycling can lead to fatigue damage in the conductor (and support structure). There are two categories of fatigue investigations on *ReBCO* conductors: high-cycle fatigue testing, where periodic dynamic alternating load stress/strain is applied to conductor samples, and static fatigue measurement, where for a long time a constant high force is exerted on the conductor samples. Studies on high cycle fatigue testing showed degradation of the conductor for large number of cycles if the strain was above a certain limit; below this limit over 200k cycles have been performed without degradation of the *ReBCO* conductor [59–62]. Static fatigue behaviour of conductors has not been extensively studied. In a study by De Leon et al. [63], it was found that the conductor showed no degradation in the applied stress level of 90% of the yield stress and after 100 hour. However, high applied stress levels resulted in significant drop in critical current, particularly when the stress was close to or equal to the yield stress.

For high-field *ReBCO* magnets, the mechanical design is especially challenging due to the very high electromagnetic forces. The direction and magnitude of the electromagnetic forces per unit volume in the coil winding pack are given by:

$$\mathbf{f} = \mathbf{J} \times \mathbf{B}_m, \quad (1.4)$$

where $\mathbf{f}$ is the body force density, $\mathbf{J}$ the local current density and $\mathbf{B}$ the local magnetic field. As the magnetic field scales linearly with the current (if no non-linear materials such as iron are present in the magnet), the electromagnetic forces scale quadratically with the magnetic field. This scaling relation makes that the mechanical design of high-field magnets is of paramount importance. Thus for high-field *ReBCO* coils, stresses due to thermal shrinkage as well as forces introduced by electromagnetic forces are to be considered in the design of the magnet. In Chapter 6, mechanical simulations of the cloverleaf-racetrack magnet are performed to show that the magnet stays within the mechanical limits of the *ReBCO* tape and the support structure.

1.4.5 Magnetic field quality

Another important aspect of an accelerator magnet is sufficiently good magnetic field quality in the aperture for circulating the beam. At an energy of 7 TeV per proton, the stored energy in the whole beam of the LHC is 360 MJ, equivalent to an intercity train going 220 km/h¹. The importance of a stable beam and hence good magnetic field quality cannot be understated.

The magnetic field in the aperture can be expressed as a sum of harmonics, where the first harmonic corresponds to the dipole field, the second harmonic to the quadrupole field etc. [64]. An illustration of the magnetic field harmonics is shown in Figure 1.13. Two types of harmonics exist, the skew harmonics denoted with A and the normal harmonics denoted with B. They are $90^\circ/n$ rotations of each other, with n the field harmonic number. By symmetric placement of the coils, the skew harmonics are cancelled out, so that only the normal harmonics need to be considered. An accelerator magnet is then designed to create a certain harmonic, for example a dipole magnet creates a dipole magnetic field in the aperture, a quadrupole a quadrupole field etc. The magnetic field created by the magnets contains errors caused by higher harmonics, which must be minimized for increased beam stability. Accelerator magnets are designed so that the normalized field error is less than 10^{-4} units of the main field, within the center 2/3 of the aperture area. The minimization is done by optimizing the position of the conductor, with respect to the aperture.

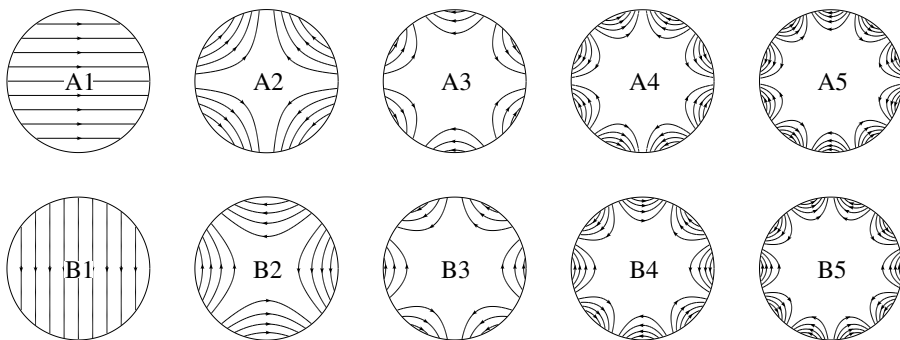


Figure 1.13: Illustration of the magnetic field lines of the first five harmonics. Top: skew harmonics A. Bottom: normal harmonics B.

In *ReBCO* conductor based magnets, the magnetic field quality is particularly difficult due to the width of the superconductor, acting as a large mono-filament. This causes large magnetization currents when a magnetic field is applied in the perpendicular direction [65]. By placing the conductor parallel to the magnetic field, some of this effect can be mitigated, but this does not prevent magnetic field quality errors due to local variations in the transport current, which can be attributed to defects in the *ReBCO* layer [66]. If non-isolated coil technology is used, there are even more possible current paths that can cause magnetic field errors.

¹ Assuming that the train weighs 200 tons.

To solve this problem, several strategies can be used, ranging from tape to coil level. The *ReBCO* layer in the tape can be cut into smaller filaments using a laser, a process called striation, to reduce magnetization currents [67]. Transposed cables such as Roebel, or partially transposed cables such as twisted stack and CORC, reduce the magnetization currents. Persistent *ReBCO* shim coils inserted in the aperture can be used to cancel individual harmonics [68]. Another solution for dipole magnets is to insert superconducting screens into the aperture [69]. The screens are aligned parallel to the magnetic field, so that higher order harmonics are cancelled out by the magnetization currents in the screens themselves.

In this work, the focus is not on magnetic field quality, as this is the last problem that needs to be solved in the development of a *ReBCO* tape based magnet. Some magnetic field calculations were performed on the cloverleaf-racetrack magnet to show that it is possible to optimize the field quality without taking into account second-order effects. The design of the magnet is made such that they can be cancelled by placing shim coils around the aperture, and rely on those to correct for the field quality.

1.5 Scope and objective of the study

The 2020 Update of the European Strategy for Particle Physics states, inter alia, that [6]: ‘*The particle physics community should ramp up its R&D effort focused on advanced accelerator technologies, in particular that for high-field superconducting magnets, including high-temperature superconductors.*’ The high magnetic field program at CERN follows the outlined strategy and ultimately aims to construct and cold-test a demonstrator accelerator magnet made with *ReBCO* conductor, which can operate stand-alone and can reach fields beyond 20 T with a usable aperture of at least 50 mm diameter within the decade [70].

This thesis is a product of this endeavour towards high-magnetic field (beyond 20 T) accelerator dipole magnets. Specifically, it focuses on the design and development of a non-insulated cloverleaf-type accelerator magnet. As depicted in Figure 1.14, the thesis is divided into five main chapters. In Chapter 2, an analysis of the critical current performance of Shanghai Superconducting Technology (SST) tape employed in the pancake coils and the SuperPower tape utilized in the cloverleaf coil is presented. Chapters 3 and 4 concern non-insulated coil technology. In Chapter 3, non-insulated pancake coils are discussed, which are study cases for the behaviour of the larger scale cloverleaf magnet. These coils are made with SST tape. Chapter 4 presents a model to describe the time-wise behaviour of the voltage of the coil. The last part of the thesis concerns the cloverleaf coil geometry and winding. Chapter 5 presents a way to model coil-geometries. Chapter 6 is dedicated to assessing the feasibility of the cloverleaf configuration in 20 T magnets. The lessons learnt from the pancake coils and the network model are there used to predict the charging behaviour of the cloverleaf-racetrack magnet. The thesis concludes with a summary of the results, and an outlook towards high-field accelerator magnets made with *ReBCO* conductor.

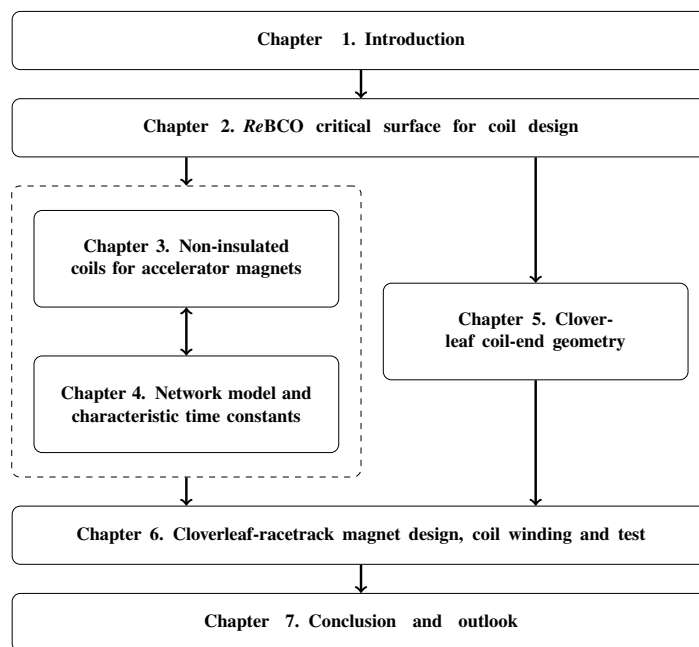


Figure 1.14: Flow diagram of the structure of the thesis chapters.

Chapter 2

***ReBCO* critical surface for coil design**

For designing and analysing superconducting coil performance, knowledge of the critical surface of the utilized tape is of paramount importance. In this chapter, the critical current performance of the Shanghai Superconducting Technology (SST) tape used in the pancake coils and the SuperPower tape used in the cloverleaf coil is discussed. A fit is made of the critical surface of SST tape, using publicly available measurement data from the University of Wellington. Fitting of the critical surface instead of merely interpolating the data is required in order to extrapolate to 4.2 K, as the available measurement data from Wellington is only present up to 15 K. The critical surface fit of the SST tape is compared with the critical current at 77 K as provided by the manufacturer and with the nominal reported critical current density at 4.2 K and 20 T, and found to be in agreement. For the SuperPower tape, an interpolation is made based on measurement results of a similar tape which was used in the EUCARD2 project. The critical current of the SST tape is measured at 77 K in self-field, and is in agreement with the reported critical current of 400 A by the manufacturer.

2.1 Introduction

In the last 20 years, there have been consistent improvements in the technology for making *ReBCO* coated conductors due to extensive research and development in the US, Europe, and Asia. To improve the quality of the *ReBCO* tape, the manufacturing process has been refined and several key parameters have been optimized, such as homogeneity, current density and magnetic field performance [33].

ReBCO tape conductors can be purchased from manufacturers such as Fujikura [71], Shanghai Superconducting Technology (SST) [38], SuNAM [72], Faraday Factory Japan [73], SuperPower [74], and THEVA [75]. Each of these tapes have different properties, such as critical surface [33] and mechanical properties [29], due to their specific manufacturing processes, which were discussed in Chapter 1.

Designing very high-field magnets of *ReBCO* coated conductors requires careful consideration of the unique $I_c(B, T, \alpha)$ characteristics of the tapes from the different manufacturers, where B is the magnetic field, T the temperature and α the magnetic field angle. However, the wide range of performance variability necessitates conducting extensive critical current measurements in the environment in which the magnet operates, which can mean magnetic fields up to 50 T at various orientations and temperatures between 4.2 K and T_c for each batch of conductors, which is a labour-intensive process. Scaling relations can minimize the amount of measurements needed on the tape in order to be able to design the magnet to its required specifications.

The development of *ReBCO* coated conductor is still in its early stages, and the shape of the critical surface can depend on several controlled and uncontrolled factors. The engineering current density can be controlled with the amount of stabilizer, substrate thickness, and with artificial pinning centres [33]. Due to manufacturing defects and inhomogeneities, the critical current can vary along the length of the tape [34, 76]. Consequently, fitting relations have to be regarded as approximate estimates for regimes where no data is available, offering a general reference for the critical surface characteristics rather than precise determinants.

Fitting of the critical surface has been performed by a couple of researchers. Senatore et al. [77] and Francis et al. [78] developed fitting relations for the critical current as a function of temperature and magnetic field. However, these fitting relations do not consider the angular dependence of the tape, which due to the anisotropic critical current density can be significant in the magnet design [66]. In the present study, fitting equations were employed that incorporate an angular-dependent scaling relation for the critical surface. This approach is based on the work of Fleiter and Ballarino [79], as their fitting relations account for temperature, magnetic field, and angular dependence, making it one of the few fitting approaches that incorporates all these factors.

In this thesis, two *ReBCO* tapes are used, SST tape for the pancake coils, which are discussed in Chapter 3, and SuperPower tape for the demonstrator cloverleaf coil, which is presented in Chapter 6. To predict the performance of the pancake coils, the critical current of the SST tape needs to be known in a temperature range between 4.2 K to 77 K and magnetic field up to 13 T with the angle ranging in the full 0° to 180° spectrum. In this chapter, a fit is presented of the critical surface of SST tape in order to cover this range. This fitting approach relies on the method from Fleiter [79] and utilizes publicly accessible measurement data on SST tape. The critical current performance of the tape

is experimentally investigated at 77 K in self-field to verify the nominal critical current performance as provided by the manufacturer.

The cloverleaf coil (see Section 6.4.3) employs tape supplied by SuperPower. However, no analogous fit is attempted for this tape variant, given that the cloverleaf coil solely underwent testing at a temperature of 77 K, and no characterization of a broader temperature range was required.

2.2 SST tape for the pancake coils

In this section, fitting relations for the critical surface of *ReBCO* coated conductors from SST are presented, which is the tape used for the small pancake coils that are described in Chapter 3. The critical surface properties of the SST conductor have been experimentally measured by the University of Wellington and are available in their public HTS database [80]. This data set was chosen to base the fit on, as the data can be readily accessed and encompasses the temperature range in the range from 15 K to 90 K, magnetic field in the 0 T to 7 T range, and full angular spectrum of 360°. For the pancake coils, two spools of different batches of SST tape were used. The main parameters defining the employed SST tape are listed in Table 2.1.

Table 2.1: Tape characteristics for the two tape lengths available from SST, as provided by the manufacturer.

Spool length [m]	552	456
Nominal J_e (4.2 K, 20 T) [$A\ mm^{-2}$]	600	600
Width [mm]	10	10
Tape thickness [μm]	95	95
Hastelloy substrate thickness [μm]	50	50
<i>ReBCO</i> layer thickness [μm]	1 - 3	1 - 3
Ag layer thickness [μm]	2	2
Cu layer thickness [μm]	10 - 40	10 - 40
Minimum I_c at 77 K [A]	382	346
Average I_c at 77 K [A]	413	384

2.2.1 Cross-section analysis

To gain insight in the winding of the pancake coils and performance characteristics during operation, a cross-section of the tape was made at CERN by Buchanan [81]. This study specifically targeted the uniformity of the copper layer thickness and substrate thickness of the tape.

Two samples from each spool were taken, leaving two cross-sections of each spool to be examined. The set cold mounted samples were ground with SiC papers from P80 down to P1200 and then polished first with 6 μm and then with 3 μm grain size with a water-based colloidal silica solution. The examination was carried out with a Keyence VHX-6000 digital microscope and thickness measurements taken at intervals of around 1.25 mm

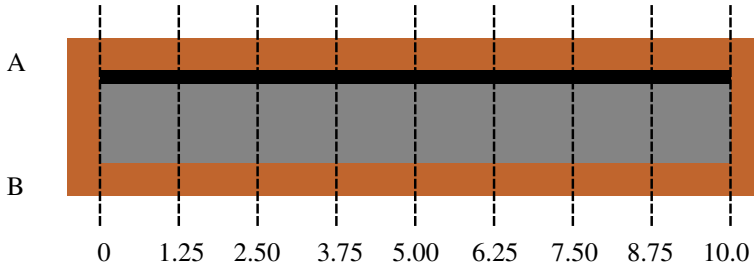


Figure 2.1: Positions in mm along the tape where the thickness has been measured.

along the length of the 10 mm substrate (see Figure 2.1). Measurements were taken of the copper thickness at each side of the tape and the substrate thickness. In Figure 2.2, two examples of cross-sections of the SST tape are shown.

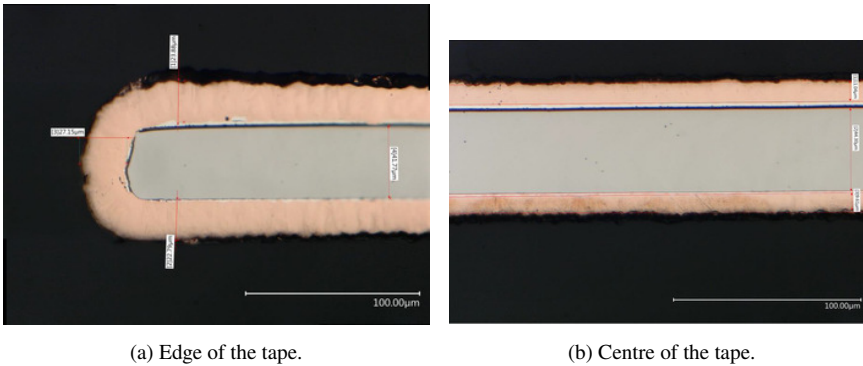


Figure 2.2: Cross-sections of the SST tape.

In Figure 2.3, the average thickness across the four samples of the copper on both sides of the tape and the substrate thickness are shown. The examinations showed that the copper thickness thins towards the centre of the substrate and then increases towards the edges. This effect is also referred to as dog-boning. The copper thickness ranges from 11 µm to 29 µm between both samples of the first spool and between 10 µm to 25 µm for the second spool. The substrate had a more constant thickness ranging between 42 µm to 44 µm for spool one and 42 µm to 45 µm for spool two, respectively. Unlike the copper, the substrate showed a pattern of increasing thickness towards the centre of the sample.

The dog-boning of the SST tape has implications for the contact resistance between two tapes with the broad sides in direct contact with each other, as is the case for the dry-wound non-insulated coils presented in Chapter 3. In this scenario, the contact will be near the edges of the tape, and there is a gap in the centre between the tapes. This reduces the effective contact area considerably, and the current can only pass over the edges of the tape.

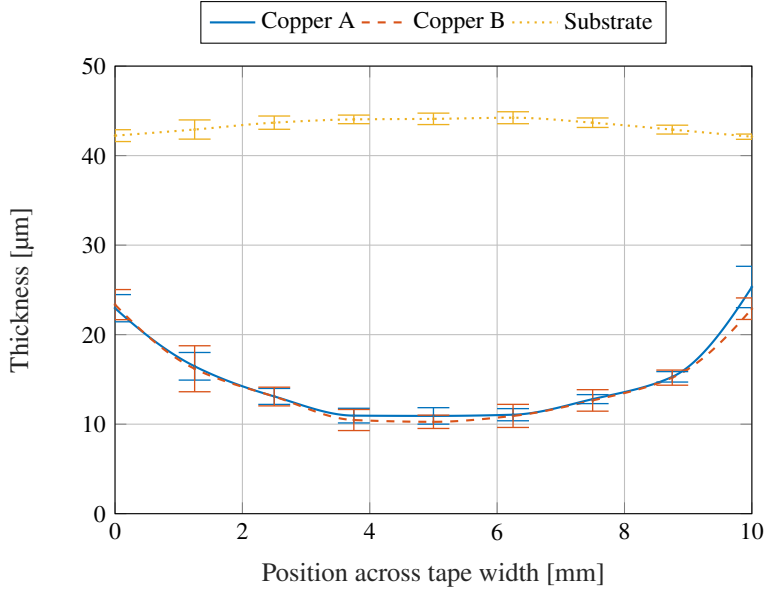


Figure 2.3: Measured average thickness across the four samples of the copper layer and the Hastelloy substrate of the SST tape along its width. Copper A lies on the *ReBCO* side of the tape and copper B on the Hastelloy side.

2.2.2 Critical surface fitting

In this section, the equations for the fitting relation are described. They are taken from Fleiter and Ballarino [79].

The scaled temperature is given by:

$$t = \frac{T}{T_{c0}}, \quad (2.1)$$

where T is the temperature and T_{c0} is the critical temperature at zero magnetic field. The anisotropic dependence of irreversibility magnetic fields, i.e. the magnetic field magnitude of the critical surface, of the *ab*-plane and *c*-axis of the *ReBCO* crystal structure (see Section 1.3.1) are given by:

$$B_{i,ab} = B_{i0,ab} [(1 - t^{n_1})^{n_2} + a(1 - t^n)], \quad (2.2)$$

and

$$B_{i,c} = B_{i0,c} (1 - t^n), \quad (2.3)$$

respectively, where $B_{i0,ab}$ and $B_{i0,c}$ are the magnitudes of the irreversibility magnetic field in the *ab*-plane and *c*-axis at zero temperature, and the constants a , n , n_1 and n_2 are fitting parameters. The reduced irreversibility magnetic fields are:

$$b_{i,ab} = \frac{B}{B_{i,ab}}, \quad b_{i,c} = \frac{B}{B_{i,c}}, \quad (2.4)$$

where B is the magnitude of the magnetic field the superconductor finds itself in, which is a contribution of the self-field and the applied magnetic field. The critical current of the tape with a magnetic field aligned to the ab -plane of the tape is:

$$J_{c,ab}(B, T) = \frac{\alpha_{ab}}{B} b_{i,ab}^{p_{ab}} (1 - b_{i,ab})^{q_{ab}} [(1 - t^{n_1})^{n_2} + a(1 - t^n)]^{\gamma_{ab}}, \quad (2.5)$$

and the critical current with the magnetic field aligned to the c -axis is:

$$J_{c,c}(B, T) = \frac{\alpha_c}{B} b_{i,c}^{p_c} (1 - b_{i,c})^{q_c} (1 - t^n)^{\gamma_c}, \quad (2.6)$$

where α , γ , p and q are fitting parameters whose indices refer to their corresponding planes. The full critical surface is described by combining Equations 2.5 and 2.6, and adding a anisotropy factor to describe the angular dependence:

$$J_c(B, T, \theta) = \min(J_{c,c}, J_{c,ab}) + \frac{\max(0, (J_{c,ab} - J_{c,c}))}{1 + \left(\frac{\theta - \theta_c}{g}\right)^\nu}, \quad (2.7)$$

where the min and the max functions take the lowest value and highest value of their two inputs, respectively. This addition is required to prevent strange behaviour when $J_{c,c}$ is higher than $J_{c,ab}$, which happens at high temperatures, as was pointed out by Van Nugteren [66]. The angle θ_c is the angle at which the field is maximum, and ν and g are fitting parameters, with:

$$g(B, T) = g_0 + g_1 e^{-g_2 B e^{g_3 T}}, \quad (2.8)$$

being the anisotropy factor, where g_0 , g_1 , g_2 and g_3 are fitting parameters. Note that the angle in Equation 2.7 is taken in radians, not degrees. In order to convert the critical current density into current carrying capacity of the tape, one can use:

$$I_c = J_{cpw} d = J_c d w, \quad (2.9)$$

where J_{cpw} is the critical current density per unit width, d the thickness of the superconducting layer and w the width of the tape.

A downside of the the fitting function is that it is not rotational symmetric, and only covers the angle range from 90° to 180° . The other angles need to be found by mirroring the curve in the 0° to 90° range, and copying this to the 180° to 360° range. In reality, the critical current can be asymmetric around the peak and be different when the tape is rotated by 180° [80], and this fitting relation does not incorporate this.

Using the aforementioned relations, a fit of the surface can be made. The measurement data of the SST tape has been provided by the University of Wellington [80].

First, the optimum magnetic field angle θ_c is determined by locating the angle at which the critical current is maximal from the measurement set. The critical current at this angle is then associated with $J_{c,ab}$. The angle $\theta_c + \pi/2$, i.e. a 90° rotation, is then associated with $J_{c,c}$. The fit is then valid on the interval $I = [\theta_c, \theta_c + \pi/2]$.

First, the critical temperature T_{c0} is determined by the point at which the critical current density drops to approximately zero from the critical current versus temperature data at zero magnetic field. The irreversibility magnetic fields $B_{i0,ab}$ and $B_{i0,c}$ are assumed to be 250 T and 140 T, respectively [82]. The factor n is also set to 1, in order to decouple

the critical currents $J_{c,ab}$ and $J_{c,c}$, as n is the only parameter common to the two. The thickness of the superconductor layer is assumed to be $2\text{ }\mu\text{m}$, which lies in the range of the manufacturer specified thickness of $1\text{ }\mu\text{m}$ to $3\text{ }\mu\text{m}$.

Then, a fit for $J_{c,ab}$ is made with Equation 2.5, by taking the data points of the critical current versus temperature at different magnetic fields at angle θ_c , and optimizing the parameters α_{ab} , γ_{ab} , ν , a , n_1 , n_2 , p_{ab} and q_{ab} such that the standard deviation σ is minimized. The standard deviation is calculated by:

$$\sigma = \sqrt{\left(\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2 \right)}, \quad (2.10)$$

where x_i is the measurement point, $\bar{x}$ the fit of the measurement data, N is the number of data points used to make the fit. A lower standard deviation means a more accurate fit, and thus the most optimal fit is found by minimizing the standard deviation. In a similar way, a fit is made for $J_{c,c}$ with Equation 2.6, where the parameters α_c , γ_c , p_c and q_c are determined by minimizing σ . The data points are the critical current versus temperature at different magnetic fields at a fixed angle of $\theta_c + \pi/2$. A plot of the fits for the SST tape for 90° and 180° is shown in Figure 2.4.

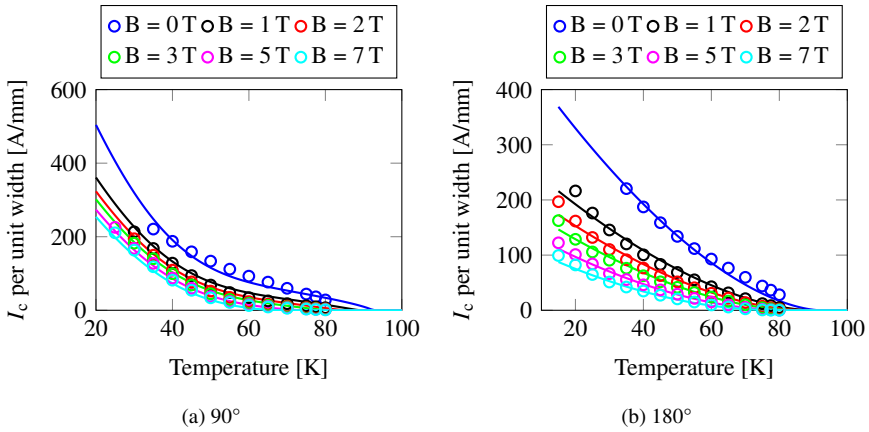


Figure 2.4: Temperature dependence of the critical current for various background magnetic fields applied parallel (a) and perpendicular (b) to the tape. In these graphs, the points are the measurement data [80], and the lines are the fits.

As the last step, the anisotropy factor is determined with Equation 2.7, where the parameters g_0 , g_1 , g_2 and g_3 are found by minimizing σ . The data points of the critical current versus angle at different temperatures and magnetic fields are used for this. A plot of the fits for 3 T and 7 T is shown in Figure 2.5.

With the obtained fitting parameters, it is now possible to describe the critical surface of the SST tape in full and extrapolate to 4.2 K and 13 T, needed for the pancake coils discussed in Chapter 3. Table 2.2 presents the fitting parameters and their corresponding values specifically for the SST tape. As an illustration of what the critical surface looks like, Figure 1.7 shows the critical surface obtained using the fitted parameter values. Using

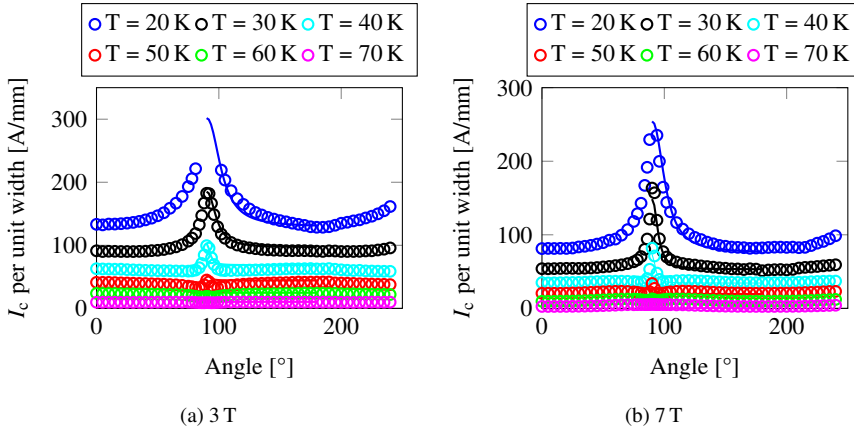


Figure 2.5: Dependence of the critical current per unit width to the applied magnetic field angle for various temperatures, and for (a) 3 T and (b) 7 T. The points are measurement data [80], and the lines are the fits.

the fit, a critical current density of 614 A mm^{-2} is predicted at 4.2 K and 20 T. This is close to the nominal critical current density of 600 A mm^{-2} as stated by the manufacturer.

Table 2.2: Fitting parameters to describe the critical surface and their calculated values for the SST tape.

Fitting parameter	Value	Unit	Fitting parameter	Value	Unit
α_{ab}	35	MATmm^{-2}	g_1	0.19	-
α_c	8.2	MATmm^{-2}	g_2	0.001	T^{-1}
γ_{ab}	1.6	-	g_3	0.018	K^{-1}
γ_c	2.46	-	n	1	-
θ_c	$\pi/2$	-	n_1	1.4	-
ν	2.22	-	n_2	6.9	-
a	0.1	-	p_{ab}	0.86	-
$B_{i0,ab}$	250	T	p_c	0.8	-
$B_{i0,c}$	140	T	q_{ab}	1.6	-
d	2	μm	q_c	9.7	-
g_0	0.05	-	T_{c0}	93	K

2.3 SuperPower tape for the cloverleaf coil

The cloverleaf coil presented in Chapter 6 was constructed using SuperPower tape. Table 2.3 provides the properties of the *ReBCO* tape employed in this coil. To predict the critical current of this coil at 77 K and self-field, measured data from KIT on SuperPower tape was utilized [83]. The measurements include the critical current as a function of magnetic field angle for the full 0° to 360° spectrum and background magnetic field up to 300 mT (spool

I.D.: 20170821). The tape measured at KIT was from a different batch than used in the cloverleaf coil, but have similar specifications as both batches were procured and used for the EUCARD2 project [84]. Figure 2.6 illustrates this dataset. To calculate intermediate points in magnetic field and angle, a spline interpolation method was applied for a smooth interpolation. As no extrapolated data outside of 77 K and above 300 mT was needed, no fit was required as for the SST tape. The data is used in Chapter 6 to estimate the critical current of the cloverleaf coil.

Table 2.3: Properties of the SuperPower *ReBCO* tape used for the cloverleaf coil.

Property	Value
Spool ID	20170713
Tape width	12.042 mm $\pm$ 5%
Tape thickness	0.0454 mm $\pm$ 10%
Silver overlay thickness	3.6 μ m $\pm$ 0.5 μ m
Hastelloy substrate thickness	0.03 mm
Copper stabiliser thickness	0.040 mm + 0.006 mm / -0.003 mm
I_c (77 K, self-field, minimum)	512 A
I_c (77 K, self-field, average)	525 A, 1.1% standard deviation
Estimated critical tensile stress	>625 MPa

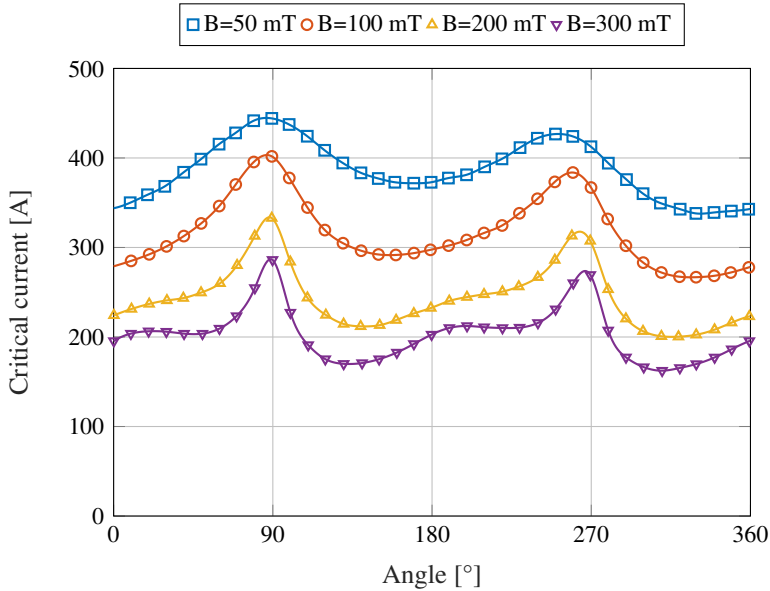


Figure 2.6: Critical current of the SuperPower tape at 77 K as a function of magnetic field angle for various background magnetic fields as measured by KIT [83].

2.4 Critical current measurements of SST tape at 77 K

To verify the critical current data provided by the manufacturer for the SST tape, measurements were conducted at a temperature of 77 K at the University of Twente. In total, ten samples of the SST tape were tested in liquid nitrogen at 77 K. This was performed to verify the provided critical current at 77 K by the manufacturer, as well as to investigate how to reduce the width of the tape for high-current measurements at 4.2 K.

In Figure 2.7, the resulting VI-curves near the chosen electric field criterion of $100 \mu\text{V m}^{-1}$ are shown. The voltage taps were placed 40 mm apart. The average measured critical current for spool 1 was 403 ± 6 A, and the measured n -factor is 34 ± 1 , and for spool 2 it was 380 ± 20 A and 33 ± 2 . The error in all cases is the standard deviation. The observed values are consistent with the specifications provided by the manufacturer for both spools.

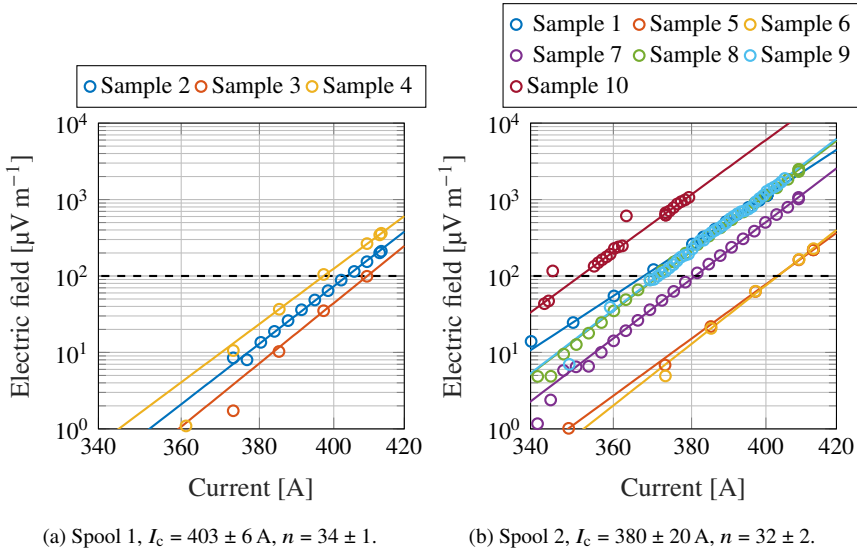


Figure 2.7: Measured VI-curves of the SST tape from both spools, indicating the critical current at 77 K, self-field.

2.5 Conclusion

The assessment of superconducting coil performance hinges significantly on a thorough understanding of the critical current properties of the utilized tape. In this chapter, attention was directed towards evaluating the performance of the SST tape used in pancake coils (Chapter 3), as well as the SuperPower tape in the cloverleaf coil (Chapter 6).

A cross-section analysis of the SST tape revealed that the tapes are dog-boned, where the copper thickness thins towards the substrate's center and increases towards the edges. The copper thickness ranged from $11 \mu\text{m}$ to $29 \mu\text{m}$ for the first spool to $9 \mu\text{m}$ to $25 \mu\text{m}$ for the second, while substrate thickness remained relatively consistent, varying between $42 \mu\text{m}$

to 44 μm and 42 μm to 45 μm for spools one and two, respectively. This phenomenon has implications for the effective contact resistance between coil turns in non-insulated coils, as is discussed in Chapter 3.

A fitting of the critical surface for the SST tape was performed utilising measurement data on SST tape performed by the University of Wellington. The critical current coming from the fit matched with the by the manufacturer stated critical current performance at 77 K and nominal critical current at 4.2 K in a 20 T background magnetic field. The critical current of the SST tape was measured as well at 77 K in self-field. The measurement results were in agreement with the provided data from the manufacturer, which stated an average critical current of 400 A at 77 K in self-field. Preferably, measurements at 4.2 K and in a magnetic field are performed in order to assess the critical current in application-relevant conditions. However, a measurement campaign at 4.2 K and in a background field is not part of this thesis, but recommended as a future research avenue. In the following chapter, the fit of the SST tape is used to predict the performance of pancake coils made from this tape under different temperature and magnetic field conditions.

For the SuperPower tape used in the cloverleaf coil presented in Chapter 6, an interpolation of the critical current was made using measured data from KIT of the critical current at 77 K. The dataset included critical current measurements across a full 0° to 360° spectrum of magnetic field angles and background magnetic fields up to 300 mT, with the sample sourced from a batch similar to that used in the EUCARD2 project. With this, the critical current performance of the cloverleaf coil is predicted in Chapter 6.

Chapter 3

Non-insulated coils for accelerator magnets

One approach to enhance the quench stability of *ReBCO* tape-based coils entails the adoption of no-insulation technology. To investigate its viability for accelerator magnet applications such as a cloverleaf type of magnet, a comparative study is conducted using four single *ReBCO* pancake coils that are identical in size. Two variations of no-insulation coils were fabricated using different techniques: one employing a single tape and the other utilizing a twin-tape cable with the superconducting layers facing each other. Of each conductor type, one coil is wound dry, i.e. with no interstitial material between turns present, and one coil is wound with solder present between the turns.

Comprehensive tests were conducted under various conditions, including tests in liquid nitrogen at 77 K in self-field, at temperatures ranging from 4.2 K to 70 K in self-field, and for a selected coil a test at 4.2 K in the presence of a background magnetic field of up to 7 T. The study is focussed on deriving several key parameters, including time constants and inter-turn resistances, voltage distributions across different sections of the coil in the radial direction, magnetic field behaviour during charging and discharging processes, as well as critical currents and quench currents of the coils.

The results show indications that at 77 K, the inner section of the coils exhibits the earliest onset of voltage generation as the current increases. This happens at currents significantly lower than the predicted coil critical current, thereby suggesting that the coils are damaged in this section. Furthermore, at a temperature of 10 K, the average effective time constant of the solder-filled coil was found to be 1600 times higher than that of the dry-wound coil. Notably, at all temperatures tested, the solder-filled coils demonstrated a higher quench current margin relative to the critical current compared to the dry-wound coils. This suggests that the solder-filled coils exhibit superior stability but larger time constants when compared to the dry-wound versions¹.

¹Part of this chapter has been published in T. H. Nes et al., "Effective Time Constants at 4.2 to 70 K in *ReBCO* Pancake Coils With Different Inter-Turn Resistances," in *IEEE Transactions on Applied Superconductivity*, vol. 32, no. 4, pp. 1-6, June 2022, Art no. 4600806, doi: 10.1109/TASC.2022.3148968.

3.1 Introduction

For magnets made from *ReBCO* conductors, one of the main problems is the protection of the magnet in the case of a quench (as discussed in Section 1.4.3). Two strategies can be discerned to manage this: actively by instrumenting the coil so that the onset of a quench is detected and proper action is taken, and passively by changing the coil's inter-turn resistance to safely survive the occurrence of a quench by creating a low resistive bypass of the quenched zone. Here, the passive strategy is investigated, by which the coil winding pack is made non-insulated by separating the turns with a finite resistance [51]. In a local normal zone, the current can bypass a defective spot or local dip in the tape critical current due to tape inhomogeneity through the finite resistance to the next turns [85].

The drawback of non-insulated coils is the much higher effective time constant compared to their insulated counterparts, due to the existence of a low resistance transverse current path, leading to longer charging and discharging times. Consequently, in the design of the coil, a trade-off has to be made between effective quench protection and the effective time constant of the coil, depending on the requirements set by the coil's application.

Various research groups have employed different no-insulation strategies. 'Dry-wound', where there is no insulation material between the turns and the copper tape stabilizer of adjacent *ReBCO* tapes are directly in contact [86]. 'Solder-filled', where solder is added between the turns [87–89]. 'Metal-insulated', where a metal strip is co-wound between the tapes [90–93]. Another variant is applying metal cladding, where a metallic coating is added to the *ReBCO* tape [94–97]. Also conductive epoxy has been tried as an interstitial material [98]. Studies directly comparing the different types of non-insulated coils are, however, less frequently conducted [99].

Here, a comparative study is conducted, involving coils of same size but featuring different inter-turn configurations. To achieve this objective, four dimensionally identical small pancake coils made of *ReBCO* were fabricated. The coils were manufactured using two distinct no-insulation technologies: dry-wound and solder-filled. Additionally, each of these configurations is implemented using both single and double tape set-ups. The insights gained from the pancake coil program serve as valuable foundations for the subsequent development of the cloverleaf magnet and reduced-turn demonstrator coil, discussed in Chapter 6. The key goals of the pancake coil development are listed below.

- *Development of winding technique.* The coil-pack of a *ReBCO* magnet is produced by winding one or several tapes around a mandrel. The idea of winding small pancake coils first is to provide insight in the technique of *ReBCO* coil winding, which in turn can be up-scaled to larger and more demanding applications.
- *Understanding the effects of different inter-turn resistances.* For future high-temperature superconducting high-field magnets constructed with *ReBCO* tape, it is proposed to use no- or partial-insulation strategies to deal with quench protection. In a non-insulated coil the turns are separated by a finite resistance, providing a bypass for the current around possible hot-spots greatly improving thermal behaviour and quench detection time. However, these coils show very different dynamic behaviour than their insulated counterparts. The pancake coils are used to study these effects on a small scale, before moving towards larger devices such as a cloverleaf coil.

- *Validation of simulation models.* Various models have been developed to predict the behaviour of *ReBCO* coils with different inter-turn resistances. In Chapter 4, a network model to describe the voltage behaviour during charging and discharging is presented and verified using the results of the pancake coils.

This chapter commences by presenting the design, fabrication, and assembly of the four pancake coils. The first coil concerns the dry-wound configuration, allowing for a preliminary experience with winding *ReBCO* tape. Subsequently, the solder-filled coil is wound using a novel process, where solder material is introduced between the turns during the winding process using a syringe. It is anticipated that the solder-filled coil will exhibit a lower inter-turn resistance [87]. Following this, the two double tape coils are wound.

Subsequently, comprehensive measurement results are presented. All coils underwent testing in a liquid nitrogen environment at 77 K. Both single tape coils were subjected to testing at various temperatures ranging from 10 K to 70 K within a variable flow cryostat. The cryostat enables precise control of the helium gas flow to achieve the desired coil operating temperature. Additionally, the single-tape dry-wound coil was tested in liquid helium at 4.2 K. Moreover, the double tape dry-wound coil was tested at 4.2 K in presence of a background magnetic field of up to 7 T.

3.2 Coil design and mechanical analysis

In this section, the pancake coil is discussed and mechanically analysed. The pancake coil consists of a winding pack and an inner and outer copper ring, which serve as the current entry and exit terminals. The main focus of the mechanical analysis is to ensure that no de-bonding occurs between the coil windings and their enclosing copper rings.

3.2.1 Coil geometry and current terminals

In Figure 3.1, a schematic of the pancake coil is shown. The main properties of the four coils are listed in Table 3.1. All parameters will be presented in more detail in the next sections.

The dimensions of all coils are the same in order to make a fair comparison between them. The inner diameter of the pancake is 52 mm and the outer diameter 102 mm. The thickness of the coil-pack is 25 mm. The thickness of the SST tape is 95 μm , allowing to wind 25 mm/95 $\mu\text{m} \approx 263$ turns. The total length of the tape is approximately:

$$L_t \approx \int_0^{263} 2\pi \cdot (26 \text{ mm} + x \cdot 0.095 \text{ mm}) dx \approx 64 \text{ m}. \quad (3.1)$$

Whereas the coil-pack configuration differs from coil to coil, the overall size remains the same. This means that the number of turns can vary depending on the thickness of the solder and the (partial) insulation between the tapes. When applying solder, the real amount of tape used will be less due to the finite thickness of the solder layer.

In order to facilitate the flow of current in and out of the *ReBCO* coil-pack, two copper rings, positioned on the inner and outer sides of the coil-pack, are employed. These rings measure 10 mm in height and 8 mm in thickness for the inner copper ring and 6 mm in

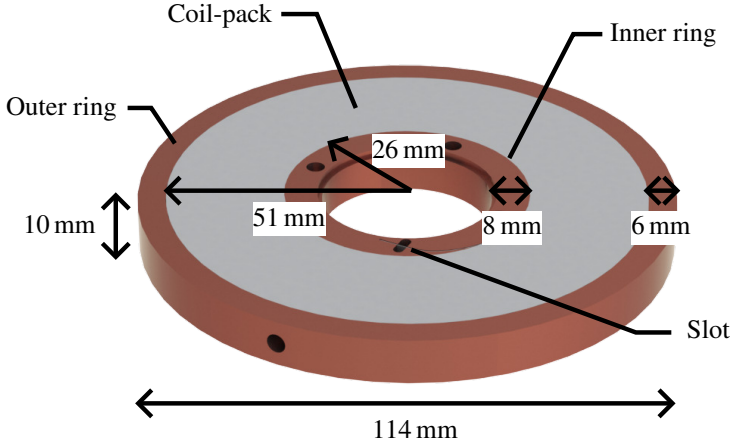


Figure 3.1: Schematic of the coil-pack and accompanying inner and outer copper terminal rings.

Table 3.1: Main properties of the four pancake coils.

	DW-1	S-1	DW-2	S-2
Number of turns	268	261	135	130
Number of tapes in parallel	1	1	2	2
Inner diameter [mm]	52.0	52.0	52.0	52.0
Outer diameter [mm]	102.1	102.0	102.3	100.4
Self-inductance [mH]	5.9	5.6	1.5	1.4
Estimated I_c at 77 K [A]	140 ± 20	140 ± 20	280 ± 40	280 ± 40
Estimated I_c at 4.2 K [kA]	2.1 ± 1.1	2.1 ± 1.1	4.2 ± 2.2	4.2 ± 2.2
Estimated I_c at 4.2 K, 7 T [kA]	1.3 ± 1.1	1.3 ± 1.1	2.6 ± 2.2	2.6 ± 2.2
B_T max. magnetic field on conductor [$T \text{ kA}^{-1}$]	8.4	8.0	4.2	4.1
B_T central magnetic field [$T \text{ kA}^{-1}$]	4.5	4.4	2.3	2.2

thickness for the outer copper ring. During the winding process, the first turn must be secured against the inner copper ring. To achieve this, a slot is created in the inner copper ring, allowing the tape to be inserted and held in place. The tape is fixed by puncturing it with a pin and inserting it in the slot. The presence of the pin prevents the tape from slipping out when tension is applied during the winding process. The first and the last turn are soldered to the copper terminals over their entire surfaces in order to reduce the electrical joint resistance.

3.2.2 Mechanical analysis of the coil

In order to prevent tape degradation and delamination of the coil windings in particular, a mechanical analysis is performed with basic mechanical simulations and calculations. A detailed mechanical analysis is beyond the scope of this thesis. During the cooling down process, the coil windings undergo less thermal contraction compared to the enclosing copper rings. As a result, the inner copper ring tends to loosen from the coil pack, while the outer ring shrinks over the coil pack. To ensure a robust electrical connection, it is crucial to prevent delamination between the inner copper ring and the coil pack. Therefore, the contact pressure between the inner copper ring and the coil windings is a parameter of interest.

In order to investigate and understand this behaviour more comprehensively, a finite element model of the small pancake coils has been developed using ANSYS [100]. In the model, a worst case scenario is assumed, where the coil-winding pack is cooled down to 4.2 K and powered with a current of 2 kA. It is assumed that there are 263 turns present in the coil-pack. For the material properties of the coil windings, the properties of the copper, Hastelloy, silver and *ReBCO* layer are averaged weighted by the thickness of the material in the tape. A Young's modulus of 110 GPa and a thermal expansion coefficient of $16 \mu\text{m/m/K}$ is assumed for the copper, and 170 GPa and $14 \mu\text{m/m/K}$ for the superconductor. It is also assumed that the coil windings are solid and the turns cannot slide relatively to each other. In Figure 3.2, an example of the contact pressure between the inner copper ring and the coil windings is shown. To determine the contact pressure, the average value of the contact pressure over the interface is taken.

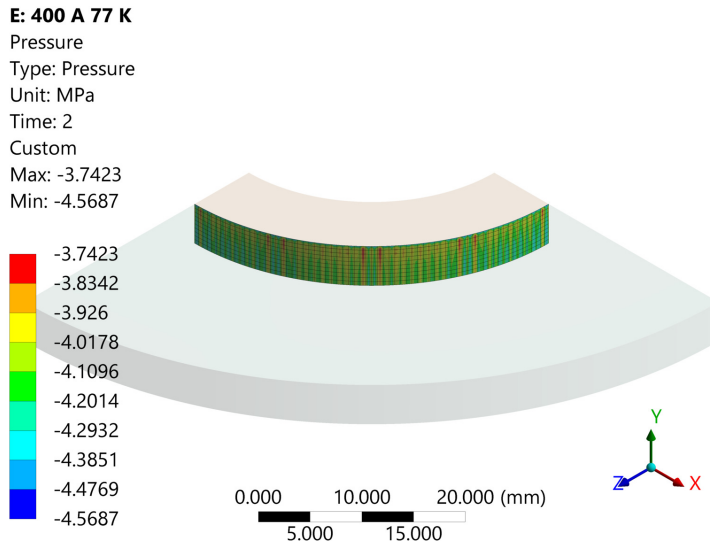


Figure 3.2: ANSYS model result showing the contact pressure between the inner copper ring and the coil-pack for 400 A and 77 K.

In the ANSYS model, the stress due to thermal contraction and powering is modelled,

but the pressure generated by the winding tension is ignored. The winding tension increases the contact pressure on the interface between the inner copper ring and the coil windings, and therefore needs to be taken into account. As a linear system is assumed, the stress due to the winding tension can be added to the results from the finite element model. Arp [101] derived the stress due to winding tension for an anisotropic material. He showed that the radial stress σ_r due to the winding tension can be calculated with:

$$\sigma_r(r) = \frac{C - D}{r}, \quad (3.2)$$

where

$$C = -(1 + B(kL + V))(r/a)^k I, \quad (3.3)$$

$$D = -(1 - B(kL - V))(a/r)^k I, \quad (3.4)$$

with

$$B = Y_{\text{mandrel}}, \quad (3.5)$$

$$R = 1/Y_r, \quad (3.6)$$

$$L = 1/Y_\theta, \quad (3.7)$$

$$V = \nu_{r\theta}/Y_\theta = \nu_{\theta r}/Y_r, \quad (3.8)$$

$$k = (R/L)^{1/2}, \quad (3.9)$$

$$I = \int_{r/a}^{b/a} \frac{a\sigma_w dx}{(1 + B(kL + V))x^k - (1 - B(kL - V))x^{-k}}, \quad (3.10)$$

where ν is Poisson's ratio, Y is Young's modulus of the coil-pack, with the subscripts r and θ indicating the coordinates in a cylindrical coordinate system, Y_{mandrel} is Young's modulus of the mandrel onto which the coil is wound, and σ_w is the stress on the tape due to the winding tension.

To simplify the set of equations, it is assumed that the material properties are isotropic, so that Poisson's ratios are all equal and the anisotropy factor k is equal to unity. With this, the winding stress on the inner radius can be readily calculated by setting $r = a$. For the given coil dimensions and winding on a copper core, this yields a pressure on the inner copper ring of 0.6 MPa.

In Figure 3.3, the stress on the contact interface between the inner copper ring and the coil-pack is shown, resulting from the finite element model with the stress due to the winding tension added to it. A positive stress means that they are pressed together, a negative stress means that they are being pulled apart. From the figure, it can be seen that even with a low winding tension of 10 N the inner terminal and the coil-pack are still pressed together. As the critical current of the Shanghai tape, which is used in the pancake coil, has an average of 413 A at 77 K in self-field, the coil critical current cannot be higher than this. In Section 3.5, the actual coil critical currents for the fabricated coils are predicted based on a model. Thus for the measurements at 77 K, this winding tension would already suffice to keep the coil together. However, a higher winding tension is still preferred for powering the coils with higher currents at lower temperatures.

Using the result of Figure 3.3, a suitable solder to make the connection between the inner ring and the coil-pack can be made. In Table 3.2, a list of solders with a melting

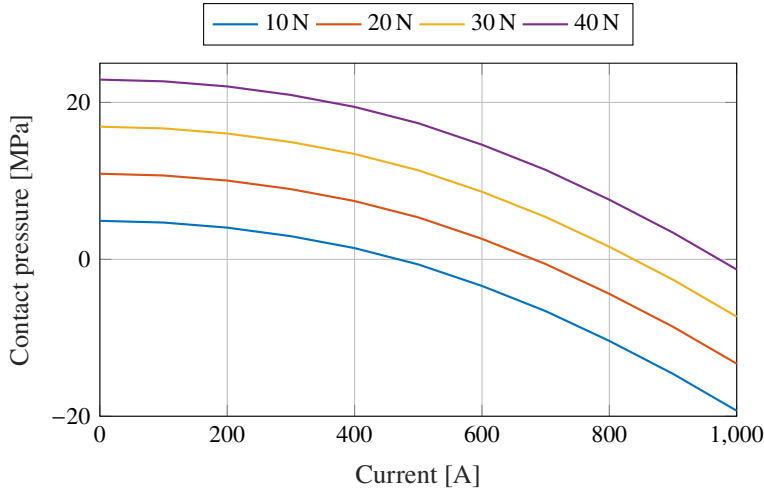


Figure 3.3: Contact pressure on the inner copper ring as function of coil current at 77 K in self-field, for different winding tensions ranging from 10 N to 40 N. Under positive stress conditions, the coil windings experience compression towards the inner copper ring. Conversely, under negative stress, the coil windings exhibit a tendency to detach from the inner copper ring, a phenomenon mitigated by the cohesive strength of the solder interfacing the inner copper ring with the coil windings.

temperature below 200 °C is shown. It is required that the temperature is lower than this temperature, as higher temperatures lead to degradation of the *ReBCO* [102–104].

From a technical perspective, Wood’s metal would be the most suitable solder, as it has a very low melting temperature of 58 °C and comparable tensile strength to tin-lead-silver solder of 42 MPa. However, Wood’s metal contains bismuth, which is highly toxic. Instead, it was chosen to use a $\text{Sn}_{62}\text{Pb}_{36}\text{Ag}_2$ based solder: Sirius 1DI. This solder has a high tensile strength compared to indium based solders, and can be procured with flux intermixed. A higher tensile strength prevents detachment of the tape from the inner copper ring. However, this does not prevent the possibility of delamination of the tape due to tensile strain. This solder also has the additional advantage that it can be applied using a syringe, which makes it very easy to apply to the tape surface. It also allows for a homogeneous distribution of the solder over the tape, allowing for a controlled thickness of the solder layer. More on the technique using a syringe is explained in Section 3.4, where the coil winding is discussed.

3.3 Coil enclosure

In this section, the design of the coil enclosure to provide structural support is discussed, outlining its overall layout and key features. Subsequently, the crucial aspect of clamping force is addressed to ensure proper electrical connections. Lastly, the instrumentation included in the structure is presented.

Table 3.2: Melting temperature and tensile strength of various solders [105].

Solder name	Melting temperature [°C]	Tensile strength at 293 K [MPa]
63Sn-37Pb	183	52
62Sn-36.65Pb-0.35Sb	183	-
62Sn-36Pb-2Ag (Sirius 1DI)	179	44
In	157	1.9
97In-3Ag	143	5.5
55.5Bi-44.5Pb (Cerrobased)	124	44
52In-48Sn	118	12
66.3In-33.7Bi	72	-
50Bi-26.7Pb-13.3Sn-10Cd (Cerrobend)	70	41
50Bi-25Pb-12.5Sn-12.5Cd (Wood's metal)	65 - 70	31
49Bi-18Pb-12Sn-21In	58	42

3.3.1 Design

For the coil enclosure, two designs were developed in collaboration with the superconducting magnet technology section of CERN. In the initial design, the coil was oriented vertically within the cryostat. However, to conduct tests involving a magnetic background field, a horizontal orientation for the coil was required. Consequently, the original design was modified to accommodate this new configuration. The two enclosures are illustrated in Figures 3.4a and 3.4b.

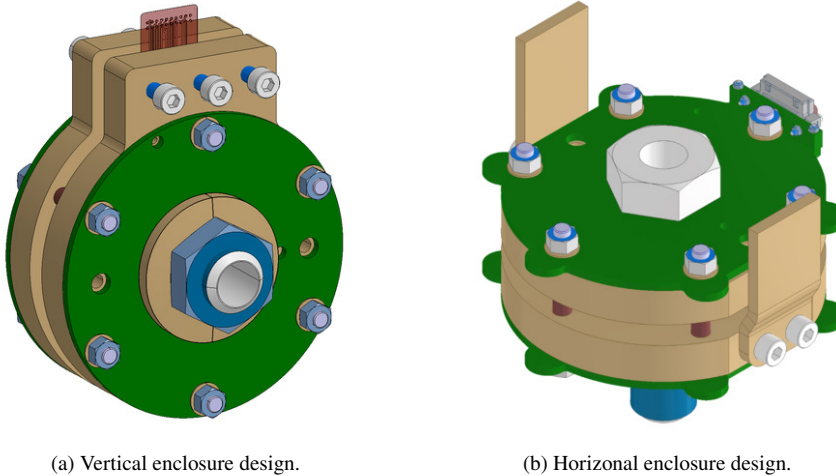


Figure 3.4: CAD renders of the two coil enclosure structures.

In Figure 3.5, a cross-section of the initial assembly design is shown. The flow of current through the pancake coil is visually depicted by red arrows. A current lead can be connected with the three bolts, which are attached to a copper plate. From the copper plate,

the current flows through an indium clamped connection to the outer copper ring of the annulus depicted in Figure 3.1. This copper ring is in contact with the outer radius surface of the coil-pack, enabling the current to enter. The current then continues through the coil-pack to the inner copper ring, where it can exit the coil-pack. From the inner copper ring, the current proceeds to the other copper plate via an indium clamped connection. Finally, it exits the assembly through the return current lead, which is connected in a similar manner to the other copper lead. The current direction can naturally be in the opposite direction, depending on the polarity of the power supply. To prevent short circuits, G11 plates are employed as insulation. Figure 3.6 provides an exploded view of the vertical enclosure, with labelling of all the individual components.

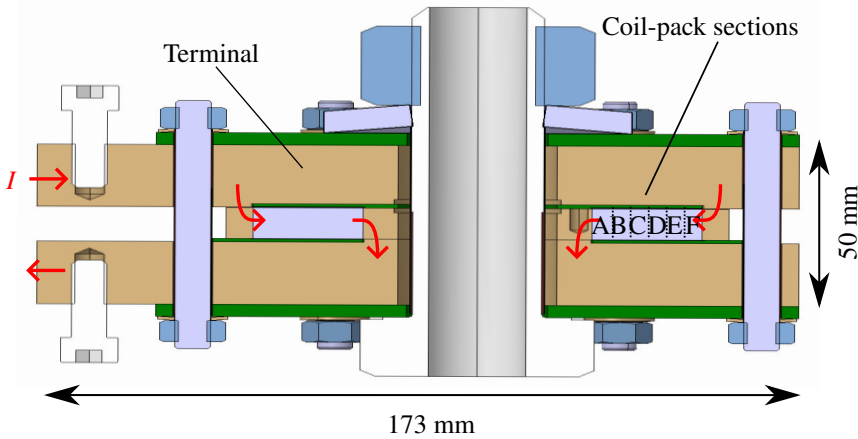


Figure 3.5: Cross-sectional view of the coil and its enclosure providing structural support. The red arrows indicate the flow of the current through the coil. Indicated as well are coil sections A to F, which corresponds to a sixth of the coil.

3.3.2 Instrumentation

For measurement purposes, the pancake coils are instrumented with voltage taps, Hall probes and pick-up coils to measure magnetic field, and temperature sensors.

The voltage is monitored over different sections A to F of the coil pack, with section A on the inside of the coil and section F on the outside of the coil (see Figure 3.5). One method of measuring the voltage across windings in a pancake coil is by the use of a so-called voltage trace. A rendering of the voltage trace can be seen in Figure 3.7a. The trace is a printed circuit board (PCB), in which printed contact points act as voltage taps and tracks are made to an extended connector base, where a connector can be attached to transfer the signals to the data acquisition system. The voltage trace is placed on top of the coil-pack, and the taps are pressed against the coil-pack by tightening the coil enclosure. To the trace, there are eight voltage taps lying on a line, on opposite sides of the surface of the coil-pack. Seven of these taps lay equidistant on the coil-pack, dividing the coil-pack in six sections. The eighth tap lies on the outer copper ring. Additionally, an inductive cancellation track is present in the trace. This track allows for measuring the voltage across

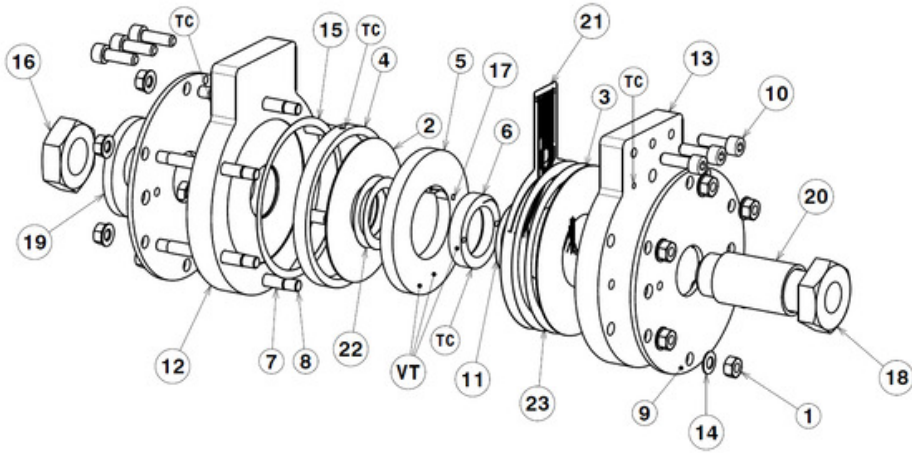


Figure 3.6: Explosion-view of the pancake coil and its encasing. (1) Stainless steel M8 nut, (2) G11 insulation plate, (3) G11 insulation plate, (4) Copper outer ring, (5) *ReBCO* coil-pack, (6) Copper inner ring, (7) Polyimide bolt insulation, (8) Stainless steel tie-rod, (9) G11 insulation disk, (10) Stainless steel M10 bolt, (11) G11 insulation ring, (12) Copper plate, (13) Copper plate, (14) Phosphor-bronze spring washer, (15) G11 insulation layer, (16) Stainless steel M30 nut, (17) Stainless steel pin, (18) Stainless steel M30 bolt, (19) M30 washer, (20) Polyimide insulation, (21) Trace, (22) Polyimide layers, VT = voltage tap, TC = temperature sensor.

the coil with cancellation of the inductive component of the signal if needed, as the track follows the coil-pack windings.

A combination of Hall probes and pick-up coils is used to measure the magnetic field along different axial positions in the pancake coil. These are placed on an insert, which is shown in Figure 3.7b. The sensor is designed to be inserted in the aperture of the pancake coil, allowing for magnetic field measurements. The design of the sensor was done by the magnetic measurements section of CERN.

The temperature is measured using carbon ceramic sensors on the inner copper ring and in one copper terminal plate. In each plate, there is space for a single temperature sensor, but for all measurements only one of the two was mounted. The location of the temperature sensors is indicated as well in Figure 3.6.

3.3.3 Clamping

To ensure a good electrical connection through the indium joints, the connection has to be firmly clamped. A contact pressure of 5 MPa was determined as sufficient to achieve a good electrical connection. This is achieved by tensioning the six M8 bolts passing through the assembly and the central M30 bolt. Spring washers are placed on the bolts to maintain the clamping pressure during the cool-down process.

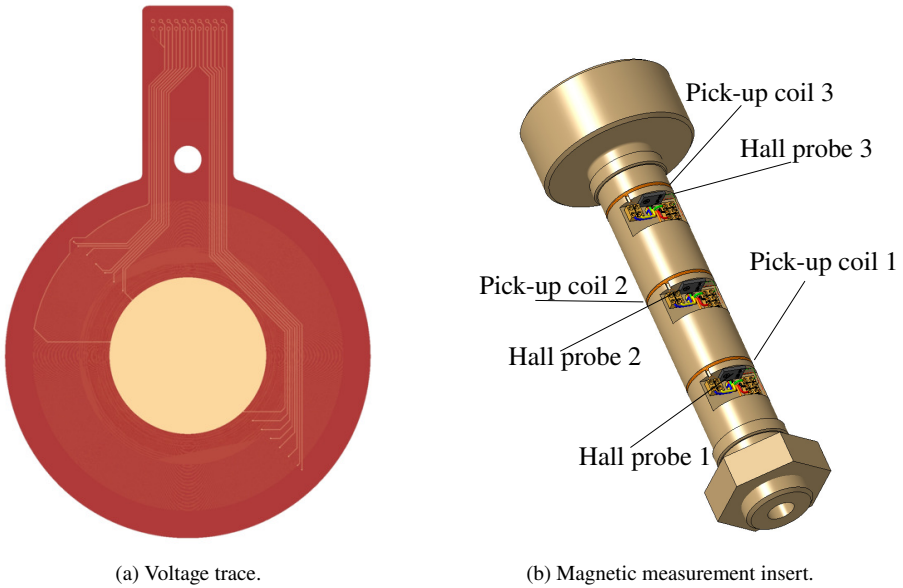


Figure 3.7: CAD renders of the the voltage trace and magnetic measurement insert.

3.4 Winding and fabrication of the NI coils

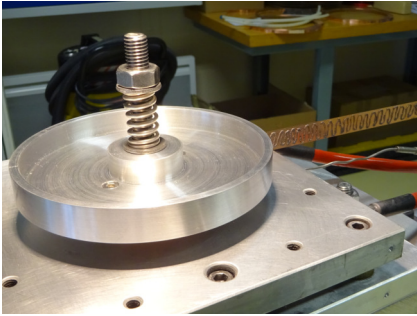
This section encompasses the method and techniques involved in winding the pancake coils. A preliminary phase was conducted, wherein trial coils were wound using copper tape. The ensuing segment delves into the insights gained from the coiling trials. The knowledge acquired serves as a guide for the subsequent winding of the *ReBCO* coils.

3.4.1 Coil winding trials

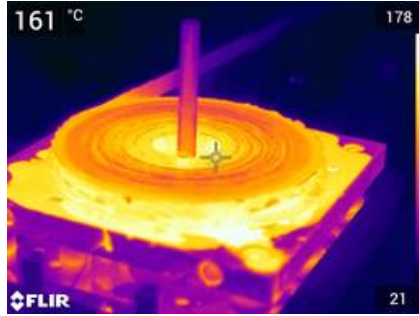
The winding trials started by winding a dry-wound coil in a horizontal orientation. This was done using 12 mm wide copper tape, relatively straightforward and no issues arose. After the first, it was decided to wind the first *ReBCO* coil in a horizontal orientation.

Winding trials with solder-filled coils were performed as well (see Figure 3.8a). Winding the solder-filled coils in a horizontal orientation proved to be more difficult. The first approach tried is to wind the coil with solder paste added to the tape, and after winding heat-up the whole coil-pack (see Figure 3.8b). However, when the solder melts, the coil de-spools due to the loss of tension in the coil-pack. It does so in slick-stick motion, leading to potentially dangerous conditions. It was decided to wind the coil hot, so that the coil cannot de-spool itself during the soldering process.

It was difficult to keep the top and bottom sides of the coil surface-defect free. The top surface of the solder-filled coils can remain surface defect-free if the coil is cleaned while the coil temperature is maintained at 176 °C. However, the surface situated on the bottom side of the coils frequently exhibits surface imperfections, such as surplus solder. The solder tends to go down due to gravity, and accumulates more on the bottom than at



(a) Winding of the coil in a horizontal orientation.

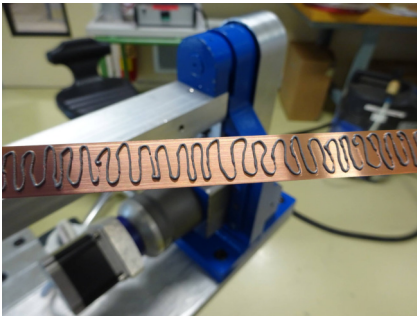


(b) Infra-red photo of the soldering of the coil.

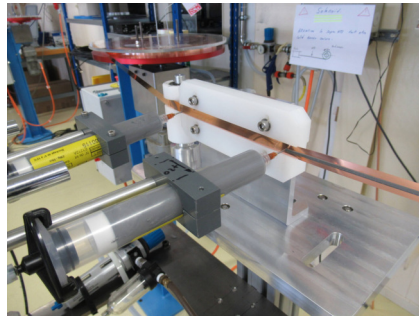
Figure 3.8: Horizontal winding and soldering of a trial coil.

the top, leading to surface defects. Therefore, for solder-based coils, it was decided to try an approach where the coils are wound in a vertical orientation.

During winding of the horizontal coils, it was found that the manual application of the solder on the tape leads to inconsistent distribution of the solder over the tape surface, as well as that it is very labour intensive (see Figure 3.9a). To remedy this, an automated system was developed. The set-up can be seen in Figure 3.9b. This worked by applying air pressure to the top end of the syringe, which forced the solder out from the top of the needle with a consistent rate of flow. For redundancy, two syringes are placed next to each other. In the case one fails, the other can take over. Solder is manually introduced to the portion of the tape that lacked solder.



(a) Manual solder deposition.

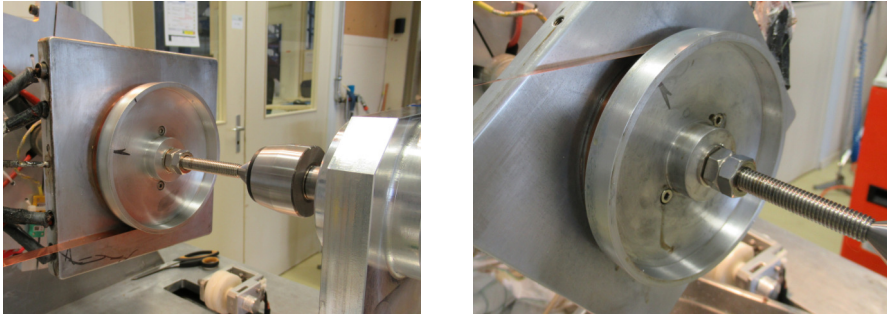


(b) Automatic solder deposition.

Figure 3.9: Two different methods to deposit solder paste on the tape.

In Figure 3.10, the winding of the coils in a vertical orientation is shown. There are two possible configurations: one where the tape enters the coil from the top, and one where the tape enters from the bottom of the coil-pack. It was found that the approach where the tapes enters from the bottom leads to better distribution of the solder than the overhand approach, from visible inspection from the top of the coil.

In addition to the position of entry of the tape, achieving optimal outcomes hinges on the positioning of the flanges on both sides of the coil. When the copper ring shares



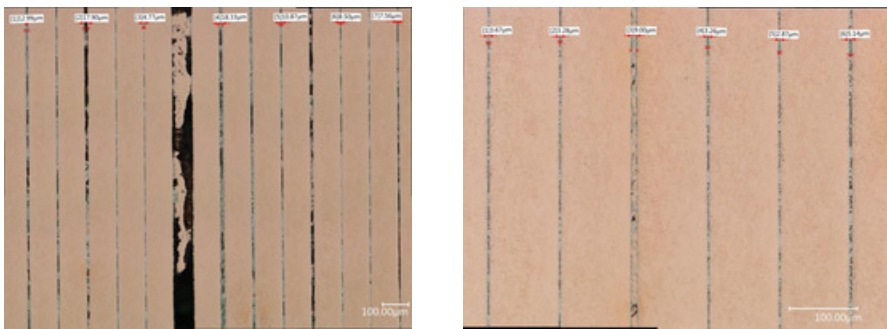
(a) Winding from the bottom.

(b) Winding from the top.

Figure 3.10: Two winding methods in a vertical orientation.

the same thickness as the copper tape, the absence of a gap between the flanges and the windings leads to challenges during the winding process. Consequently, this scenario also results in absence of solder on the coil's sides. Conversely, if the copper ring boasts a thickness exceeding that of the copper tape, a gap manifests between the flanges and the winding. This discrepancy culminates in an excess of solder on either side of the coil, coupled with a staggered arrangement of winding turns. If the internal and external flanges are not parallel, a gap appears between the flanges and the winding. This situation leads to an excess of solder on both sides of the coil, accompanied by a misalignment of the coil pack's turns, resulting in a staggered configuration.

The winding tension also plays a role in the distribution of the solder throughout the coil windings. Two solder-filled copper tape based dummy coils were wound with different winding tensions, one with a low tension of 10 N and another one with a higher tension of 33 N. From both coils, a cross-section was made and inspected by microscopy at CERN [106]. Two images of the cross-section of the two coils are shown in Figure 3.11.



(a) Coil wound with 10 N of winding tension.

(b) Coil wound with 33 N of winding tension.

Figure 3.11: Microscopy cross-section of two solder-filled coils wound with different winding tensions.

Throughout the coils, both samples show a significant difference in solder layer thickness, with the low-tension wound coil ranging from 3.7 μm to 85 μm and the high-tension

wound coil ranging from $1.3\text{ }\mu\text{m}$ to $30\text{ }\mu\text{m}$. On average, the high-tension wound coil had a thinner solder thickness of $5 \pm 2\text{ }\mu\text{m}$ with a smaller standard deviation compared to that of the low-tension wound coil, which had an average solder layer thickness of $10 \pm 5\text{ }\mu\text{m}$. Lack of bonding was present on both samples, however, more and substantial debonding was present on the low-tension wound coil. The largest gap in this coil had a width of $85\text{ }\mu\text{m}$, compared to the high-tension wound coil, which had a maximum gap of $30\text{ }\mu\text{m}$.

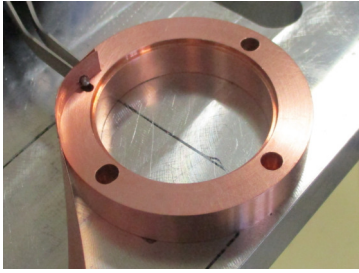
Cumulatively, the experiences with winding in both orientations shows that employing a vertical approach during winding yields superior soldering outcomes compared to the horizontal method. Consequently, the decision was taken to execute the winding of solder-filled coils in a vertical orientation and having the tape enter from the bottom, with a winding tension of 33 N.

3.4.2 *ReBCO* coil winding

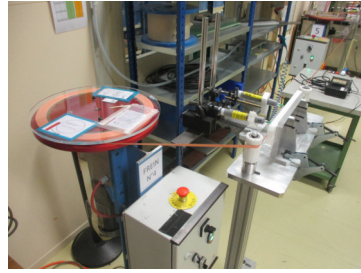
In Figure 3.12, some images are shown of the winding process of the *ReBCO* pancake coils. The coils are wound using SST tape, of which the properties were discussed in Section 2.2. The construction of all coils starts with the inner copper ring on which the superconductor tape is wound. The tape is fixed in a slot in the inner copper ring (see Figure 3.12a), which assures that it cannot be pulled out. For the single tape, one tape is placed in the slot, and for the double tape coils two tapes. Each tape is wound from a spool, which is sitting on top of a tensioning system (see Figure 3.12b). A winding tension of 33 N is used for all coils. The first turn is soldered to the inner copper ring using Sirius 1D solder paste. The coil turns are then wound around the inner copper ring, until an outer diameter of 102 mm is reached, measured by a calliper. For the single tape coils, the winding took about one hour, and for the double tape coils the winding took about half an hour.

For the solder-filled coil, Sirius 1D solder paste is added between the turns using a syringe, and the solder is pushed out at a continuous rate by pressurizing the top of the syringe with a pressure of 1.5 bar. For the solder-filled double-wound coil, solder is added to each tape on one side (see Figure 3.12c). The coil is clamped between a heating plate and a metal flange (see Figures 3.12d and 3.12e). By placing a droplet of solder on the aluminium disk, which is next to the coil, it is determined if the plate is hot enough for the solder to melt. For the single tape solder-filled coil, the heater is set to $190\text{ }^{\circ}\text{C}$ for the first 66 turns, then to $183\text{ }^{\circ}\text{C}$ up to 82 turns, then $176\text{ }^{\circ}\text{C}$ up to 99 turns, and the remaining turns are wound with the heater set to $174\text{ }^{\circ}\text{C}$. For the double wound coil, the whole coil is wound with a temperature setting of $194\text{ }^{\circ}\text{C}$, as the solder only melts at that temperature setting. As the temperature sensor is on the edge of the heating plate, it is expected that it is somewhat cooler there compared to the center of the heating plate where the coil is. The coil is wound warm in about one hour for the single tape coil and in 30 minutes for the double tape coil.

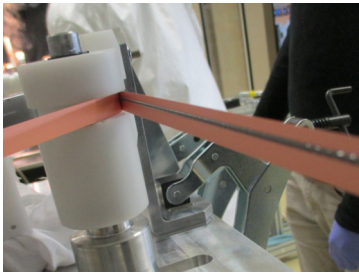
Figure 3.12f shows a coil after all the turns have been wound. Now, the tape sticking out of the coil still needs to be removed. In the case of the single tape dry-wound coil, the last turn was glued over a length of 20 mm using a cyanoacrylate adhesive. Subsequently, the tape is separated at this juncture by cutting it with a knife. For the dry-wound double tape coil, to the inside of the outer copper ring, Sirius 1D solder paste is applied to make the connection between the last turn of the coil and the outer copper ring. The whole coil



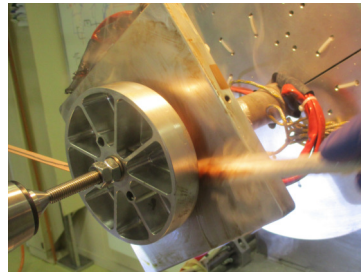
(a) Placing tapes in the slot.



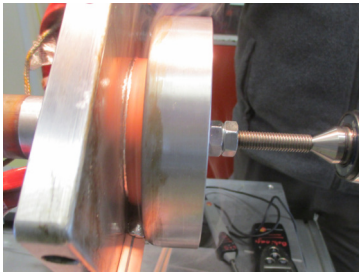
(b) Winding mandrel on top of tensioning system and soldering station.



(c) Joining two tapes in cable.



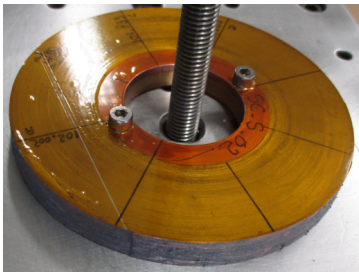
(d) Winding station side-view.



(e) Winding station front-view.



(f) Coil after winding.



(g) Coil prepared for shrink fitting.

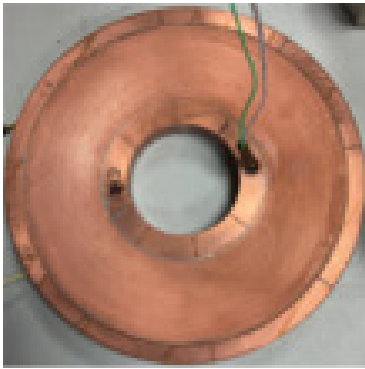


(h) Placing outer copper ring.

Figure 3.12: Winding and shrink fitting of the outer copper ring on the solder-filled double tape coil.

is heated up to 190 °C to solder the joint. For the solder-filled coils, a soldering iron is used to locally heat up the coil to melt the solder, and the tape is de-attached where the solder is molten, and then cut with a knife.

After the winding process is completed, the coil is removed from the winding station and prepared for the shrink-fitting of the outer copper ring. To all coils, Sirius 1D solder paste is applied to the outside of the coil. Both the top and bottom surfaces of the coil are covered in polyimide foil to protect their respective surfaces (see Figure 3.12g). An outer copper ring is fabricated, which has a outer radius 50 μm smaller than the measured outer dimension of the coil. The outer copper ring is heated up to 400 °C, and then is pulled over the coil-pack (see Figure 3.12h). The coil-pack remains at room temperature during the process. In Figure 3.13, all four wound coils in finished state are shown.



(a) Dry-wound single tape coil (DW-1).



(b) Solder-filled single tape coil (S-1).



(c) Dry-wound double tape coil (DW-2).



(d) Solder-filled double tape coil (S-2).

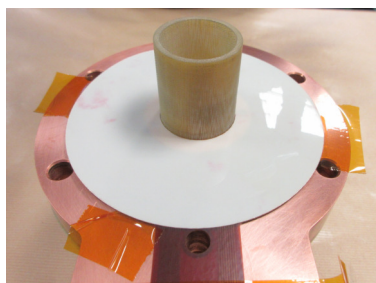
Figure 3.13: The four *ReBCO* pancake coils.

3.4.3 Coil structure assembly

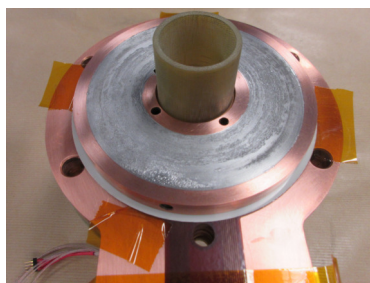
After coil winding, the coils are assembled within their housing, establishing an electrical connection between the copper terminal plates and the copper rings. Additionally, there is

the possibility of adding a trace plate, which is pressed onto the coil windings. For achieving a more homogeneous pressure distribution over the assembly, shimming is required. The shimming is accomplished using polyimide film and G11 glass fibre rings of various thicknesses and diameters.

To ensure proper load distribution, Fujifilm paper [107] is employed in the various assemblies. This paper is sensitive to pressure, and depending on the grade discolours if pressure is applied on the paper. Multiple iterations of assembly, with varying shimming thickness, were performed to optimize pressure distribution, guarantee a good electrical contact and to verify if the voltage trace makes contact with the coil-pack. In Figure 3.14, the assembly of the coil with Fujifilm paper is shown.



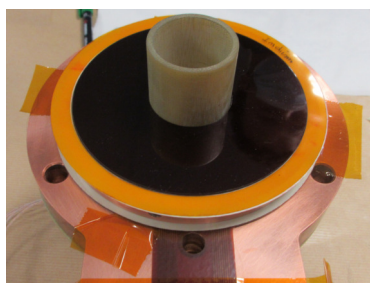
(a) Fujifilm paper over trace plate.



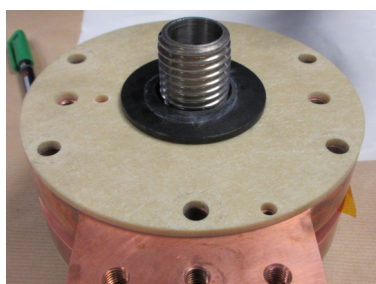
(b) Placing the coil.



(c) Fujifilm on top of the coil.



(d) Shimming using polyimide foils.



(e) Placing the top terminal.

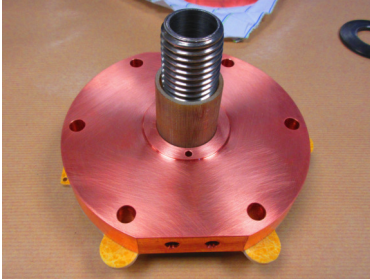


(f) Fujifilm result.

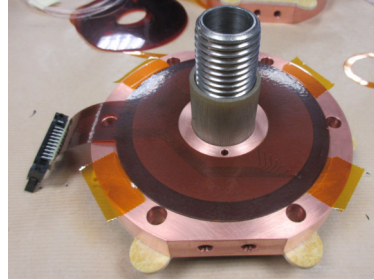
Figure 3.14: Images showing some of the steps in the use of Fujifilm to determine the required shimming thickness for optimal contact.

In order to enhance electrical contact, $50\text{ }\mu\text{m}$ thick indium rings are placed between each of the copper contacts between the copper rings and terminals, in order to reduce the contact resistance. The use of highly ductile indium ensures improved contact once the assembly is tightened. However, it is important to note that indium is extremely fragile. Consequently, during the various trials conducted prior to the final assembly, the two indium rings were replaced by polyimide rings of the same thickness and dimensions.

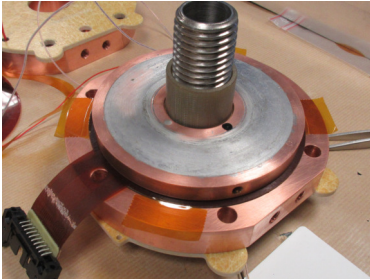
After the right amount of shimming was determined, the coil was assembled with the indium rings included. In Figure 3.15, some images of the assembly are shown.



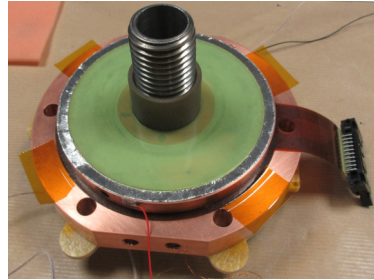
(a) G10 plate and copper terminal placed over the central bolt.



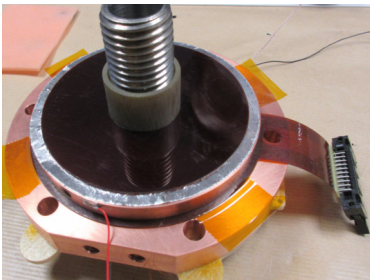
(b) Voltage trace added.



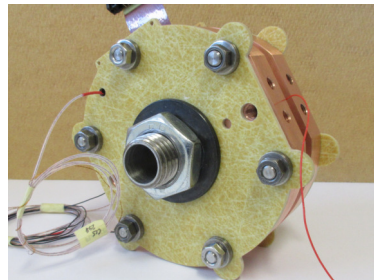
(c) Coil placed on top.



(d) G11 insulation and indium added.



(e) Shimming added.



(f) Final coil assembly.

Figure 3.15: Several images depicting some of the steps in the assembly of the solder-filled double tape coil for the horizontal mounting.

The electrical contact surfaces of the copper parts are deoxidized using ethyl alcohol and Scotch-Brite [108]. All the holes and edges through which the instrumentation cables pass are chamfered. The first copper end plate and the fibreglass insulation plate are carefully assembled, taking care to prevent the instrumentation wires from passing through the fibreglass plate. The assembly is secured using the M30 stainless steel bolt and nut. To aid in centring the parts, two M8 tie rods are placed opposite each other. The central fibreglass alignment tube is placed in the aperture.

The stacking process commences with the thinnest shims and concludes with the thickest G11 insulation plate to ensure proper positioning during assembly. The inner indium ring is gently brushed with Scotch-Brite to remove the oxygen layer and then degreased with a clean cloth and ethyl alcohol. The location of the hole for the instrumentation probes is marked on the washer and subsequently cut using a 5 mm diameter punch. The indium washer is then positioned. The wires from the instrumentation probes are threaded through the hole in the end plate, and the coil was placed on it. The outer indium ring is gently brushed with Scotch-Brite and degreased with a clean cloth and ethyl alcohol.

Similarly, to ensure the correct positioning of the washers in the copper end plate, the process begins with the thickest and ends with the thinnest polyimide washers. The copper end plate is carefully placed and its centring is verified. The fibreglass insulation plate is then fitted to the assembly. The central washer is added, and the M30 nut is hand-tightened. Six tie rods, washers, and M8 nuts are also fitted and hand-tightened. The M30 nut is tightened to 50 Nm. Six polyimide sheets are used to insulated the six outer bolts from the assembly, preventing a short circuit.

The nuts on the six M8 tie rods are gradually tightened in a crosswise pattern to a torque of 5 Nm. The M30 nut is tightened to 75 Nm. The gradual cross-tightening of the M8 nuts is repeated, to ensure the torque is still present. Then, the M30 nut is tightened to 100 Nm. The M8 nuts are finally tightened in a crosswise manner again. This concludes the assembly of the *ReBCO* coil in its casing.

3.5 Modelled coil electromagnetic properties

Knowing the dimensions and number of turns, the coils can be modelled for their relevant electromagnetic properties, such as expected coil critical current, self-inductance, and transfer function B_T , which tells how much magnetic field is generated in the aperture or on the conductor as function of the coil current. In Table 3.1, the main properties of the wound pancake coils were listed. The self-inductances and magnetic field calculations are performed by modelling the coils in FIELDS [109]. The coil is modelled as an annulus with a homogeneous current density. The self-inductances of the coil, the peak magnetic field on the conductor and magnetic field in the centre of the coil were calculated using FIELDS [109].

To estimate the coil critical current, the transfer function for the maximum magnetic field on the conductor is used. The coil critical current is the intersection of the critical surface with the load line, of which the transfer function is the slope (see Figure 3.16). The coil critical current can then be found from the following relation:

$$I_c = wJ_{cpw}(B_T I_c, T, \alpha), \quad (3.11)$$

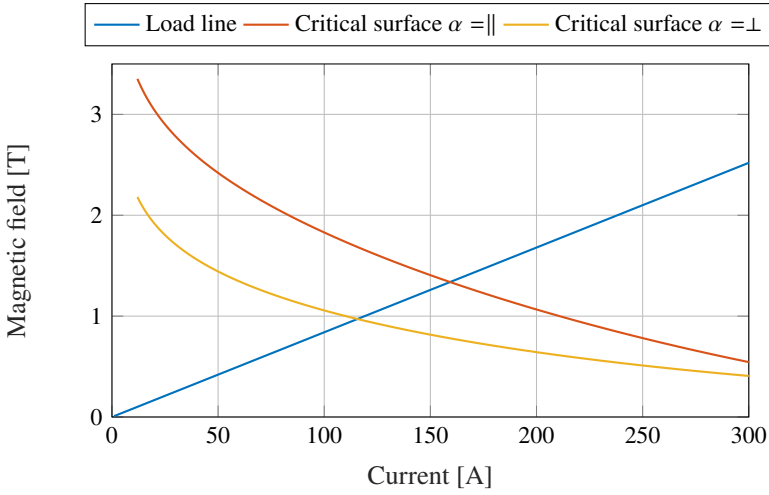


Figure 3.16: Magnetic field versus coil current. Load line of the single-tape dry-wound coil and the critical surface for and magnetic field angle of 90° (parallel to the tape) and of 180° (perpendicular to the tape). The intersection of the load line with the critical surface is the coil critical current.

where w is the width of the tape, and $J_{\text{cpw}}(B, T, \alpha)$ the critical current density per unit width, with $B_T I_c$ the transfer function times the critical current as input for the magnetic field B . Due to the anisotropy of the tape, the magnetic field angle is assumed to be either parallel or perpendicular to the tape to estimate an expected upper and lower bound to the coil critical current. In reality, the exact angle of the magnetic field with respect to the tape is a complicated function of the position along the tape width [110]. Solving Equation 3.11 numerically yields the coil critical current. For the critical surface of the tape, the fit of the SST tape presented in Chapter 2 is used.

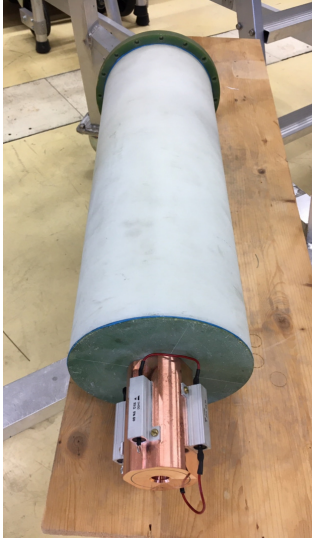
3.6 Measurement campaign

To study the behaviour of the pancake coils, various experiments were performed, in different operating conditions.

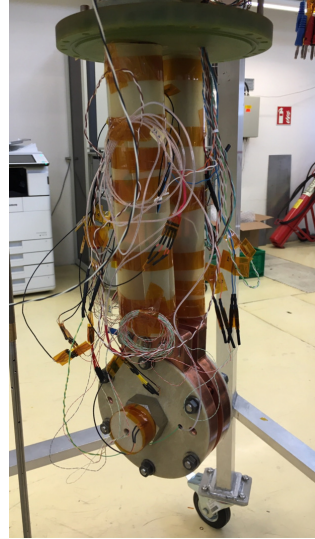
First, all coils were tested in liquid nitrogen at 77 K to determine their performance. The first three coils (DW-1, S-1, DW-2) were measured at CERN, the fourth (S-2) was measured at the University of Twente. For all coils except the dry-wound single tape coil, the voltage trace plate was used to measure across the individual coil sections.

The two single tape coils were tested in a variable temperature environment at the CERN Cryogenics laboratory (see Figure 3.17). To this, a variable flow cryostat was used, where the temperature can be set between 10 K and 70 K. At the bottom of the cryostat, there is an evaporator that causes the liquid helium to boil off, resulting in a flow of helium gas. This gas flow is directed through a flow conditioner, which carefully regulates the temperature of the gas to the desired level for the coil being tested. The dry-wound coil was also tested at 4.2 K in this set-up by directly emerging it in the helium fluid bath.

The double tape dry-wound coil was tested at the University of Twente at 4.2 K in



(a) Variable temperature G11 pot.



(b) Coil hanging on its current leads.

Figure 3.17: Variable flow measurement set-up at the CERN Cryogenics laboratory.

liquid helium in a background magnetic field up to 7 T (see Figure 3.18). In order to facilitate the characterization of the pancake coil in background magnetic fields of up to 7 T, an insert was designed and manufactured [111]. This insert integrated current leads, instrumentation wires, and mechanical support. The current leads are constructed using *ReBCO* tapes and vapour-cooled sectors, enabling a maximum transport current of 4 kA.

After the variable temperature campaign and the background magnetic field campaign, the coils were re-measured at the University of Twente in liquid nitrogen to determine if the coils were degraded. In Table 3.3, a list of measurements is presented in chronological order.

Table 3.3: Measurement sequence performed on the four pancake coils in chronological order.

	DW-1	S-1	DW-2	S-2
77 K, self-field	X	-	-	-
Variable temperature, self-field	X	X	-	-
77 K, self-field	X	X	X	X
4.2 K, 7 T	-	-	X	-
77 K, self-field	-	-	X	-

During the measurement campaigns, several parameters are studied.

- *Critical current and quench margin:* The critical current and quench current were determined by applying an electric current to the coil, varying from an initial zero value to the magnitude at which the coil experiences a quenching event. This upper limit of attainable current can surpass the critical current. In this scenario, the

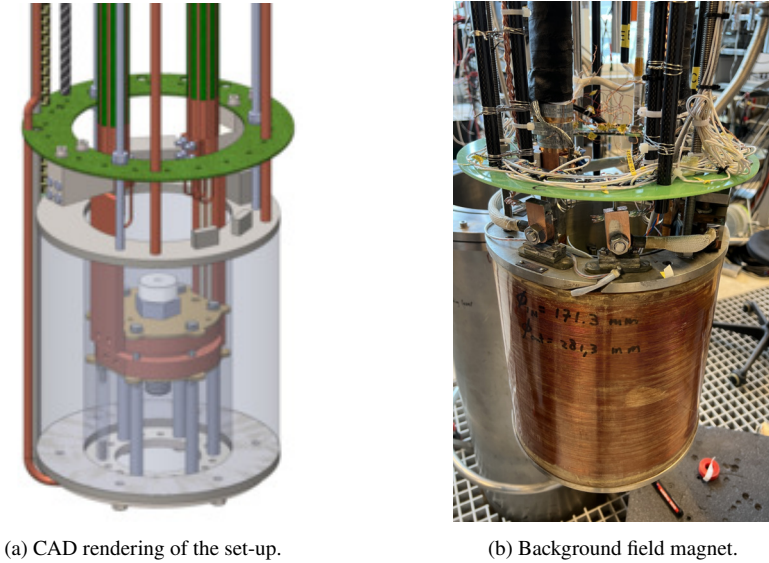


Figure 3.18: Overview of the background magnetic field measurement set-up at the University of Twente, showing a schematic in (a) and the real system in (b).

coil runs in saturated mode, where the critical current in the tape turns flows azimuthally and the excess current flows radially. Provided effective cooling mitigates the thermal consequences of this radial current, it will not incite quenching. Measurements of voltage were undertaken at distinct locations across coil winding sections, to ascertain whether the coil sustains superconductivity between these designated voltage points, or whether resistivity is exhibited. Concurrently, continuous monitoring of the coil's temperature was transpired, facilitating the determination of its operational temperature and the detection of localized thermal elevation.

- *Effective time constant:* The axial component of the magnetic field was measured inside the bore of the coil. Knowing the current going through the coil and the produced magnetic field, the radial and azimuthal current can be determined, since only the latter contributes to the magnetic field. The coil was discharged by using an open circuit to determine the discharging time.
- *Self-inductance:* The self-inductances of the coils were measured at 77 K by applying a linear ramp in current. The self-inductance is then determined using the relation $L = V/(dI/dt)$.
- *Transverse resistivity:* The transverse resistivity was determined from the effective time constant and the coil inductance, using the relation $R = L/\tau_{\text{eff}}$.

In the following section, relevant results of the measurement campaign are presented on these parameters of interest.

3.7 Test results of the four pancake coils

In this section, the results obtained from the measurement campaign are presented. As mentioned in the previous section, the critical current and quench margin, effective time constant and transverse resistance of the coils were investigated, for various operating conditions. All coils were tested at 77 K in liquid nitrogen. Additionally, the two single tape coils were tested in a temperature range of 4.2 K to 77 K in self-field, and the dry-wound double tape coil was tested at 4.2 K in a background magnetic field of up to 7 T.

3.7.1 Critical current and quench margin

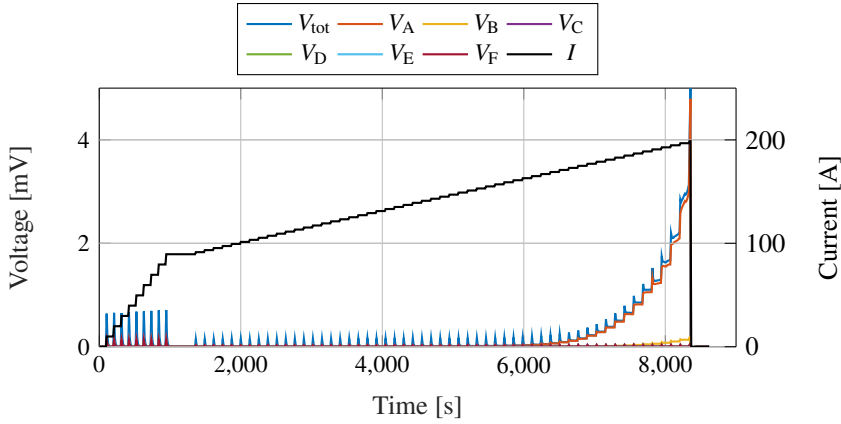
In Figure 3.19, an example of a critical current and quench current measurement is presented. The procedure involved a step-wise increment of current, as depicted in Figure 3.19a. The incremental elevation of current elicited a corresponding voltage response across the coil sections, also exemplified in Figure 3.19a. The resultant voltage-current curve is shown in Figure 3.19b. From this, it can be seen that the first coil section to generate a voltage is the inner section of the coil. This behaviour is typical for all four pancake coils. In all conducted measurements on the four coils, it was found that the innermost section consistently exhibited the earliest onset of voltage generation. The critical current of the coil is taken as the value where the measured voltage over the tape length in the coil section is higher than the electric field criterion. For the dry-wound coils, a criterion of $10 \mu\text{V m}^{-1}$ is taken, as the coils experienced quenches before the more typical criterion of $100 \mu\text{V m}^{-1}$ and for the solder-filled coils even a criterion of $1 \mu\text{V m}^{-1}$ is used, as a higher criterion would lead to an unrealistic high coil critical current due to the low transverse resistance.

In Table 3.4, the coil critical current of the pancake coils at 77 K is tabulated before and after the various measurement campaigns. It can be observed that the single-tape dry wound tape did not reduce in performance. However, the dry-wound double tape coil got damaged during the background magnetic field test. Remarkably, the critical currents for both dry-wound coils is lower than expected from the model by about 40% at 77 K.

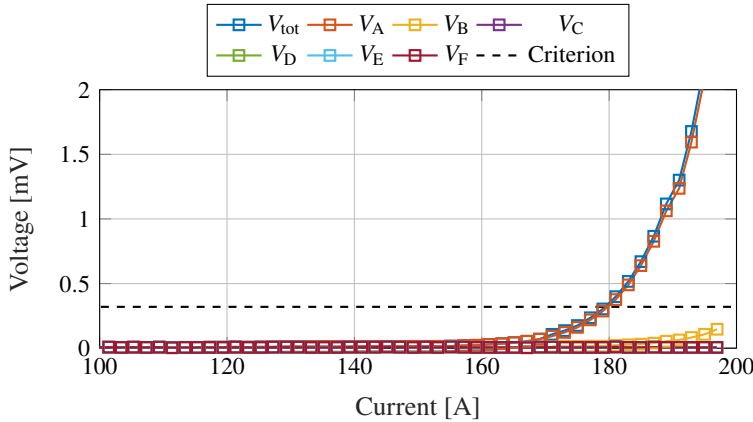
Table 3.4: Coil critical current I_c and quench current I_q of the pancake coils at 77 K after various measurement campaigns. The criterion for the dry-wound coils is $10 \mu\text{V m}^{-1}$ and for the soldered coils it is $1 \mu\text{V m}^{-1}$.

	DW-1	S-1	DW-2	S-2
I_c [A]	93 ± 5	-	180 ± 5	163 ± 7
I_c after variable T [A]	95 ± 4	40 ± 20	-	-
I_c after background [A]	-	-	11 ± 1	-
I_q [A]	118 ± 2	-	194 ± 3	>240
I_q after variable T [A]	116 ± 3	>400	-	-
I_q after background [A]	-	-	125 ± 5	-

The solder-filled single tape coil has a very low critical current compared to the other coils at 77 K. However, this was measured after the variable temperature measurement campaign, so it is possible that damage occurred during these measurements. The solder-



(a) Voltage across the entire coil, voltages across sections A to F, and supplied current as function of time.



(b) Voltage across the entire coil and sections A to F as function of applied current. The dashed line indicates the voltage criterion for the entire coil of $10 \mu\text{V m}^{-1}$.

Figure 3.19: Example of a critical current and quench current measurement on the dry-wound double tape coil. In (a), the temporal evolution of voltage and current is depicted, while (b) shows the resulting voltage-current relation.

filled double tape coil also has a lower critical current than its dry-wound counterpart. A possible reason for this, is that the criterion is rather arbitrarily chosen at the low value of $1 \mu\text{V m}^{-1}$, leading to a low coil critical current given the low n -value of ReBCO coils. Moreover, the length in the electric field criterion is rather undefined in the case of a non-insulated coil. In a single tape exposed to a uniform magnetic field and temperature, the length is clearly the distance between the voltage taps. However, in a non-insulated coil, the length can be the distance between the taps, or the length of the tape in the particular section in the coil windings.

The quench current is also lower than the expected critical current by some 16% for the dry-wound single tape coil and some 30% for the dry-wound double tape coil. This

does hint to potential damage to the coils, assuming the model is valid. Unwinding of the dry-wound coils could lead to clues if damage is present. The solder-filled coils did not experience quenches at 77 K. The coils may exhibit quenching under potentially higher applied currents. However, this aspect was not explored or investigated in the present study.

During the measurement campaign in liquid nitrogen, instabilities in the voltage behaviour were observed. An example of this behaviour is shown in Figure 3.20. Here, after an initial current ramp, the coil is operated at a current of 140 A, which is higher than the coil critical current. When the ramp is over, the coil voltage seems to settle to a constant value, which is expected in steady state conditions. However, at around $t \approx 280$ s, the coil voltage suddenly drops, and at $t \approx 400$ s, the coil voltage rises again. Typically, these instabilities were observed when the current was above the critical current of the coil, however, some instances of current redistributions were also observed when the coil operated below the critical current. A possible explanation for this behaviour is thermal instability. A detailed study of this behaviour is out of the scope of this study.

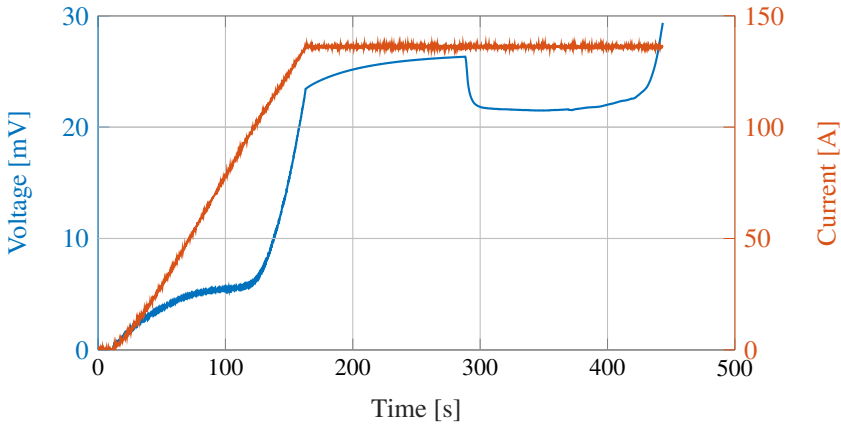


Figure 3.20: Voltage across the entire coil and supplied current as function of time for the dry-wound single tape coil (DW-1). A charging instability can be observed at $t \approx 280$ s.

When the coil is close to its critical current, some turns cannot take up additional current. The current will instead bypass the turn and flow transversely to its adjacent neighbouring turns. In this case, the turn does not contribute to the longitudinal magnetic field in the aperture. To demonstrate this effect, the axial component of the magnetic field in the aperture is plotted against current in Figure 3.21. When the measured magnetic field starts to deviate from the load line, some of the turns are saturated. From the figure, it can be seen that the solder-filled coils can still generate additional magnetic field when powered beyond their respective critical currents. However, they cannot provide more magnetic field than their dry-wound versions, even when running deep into over-current.

The reduction in magnetic field for the dry-wound single tape coil for high coil currents is possibly due to joule heating of the coil by the transverse currents, which lowers the critical surface of the coil due to the increased temperature, thereby lowering the performance of the coil.

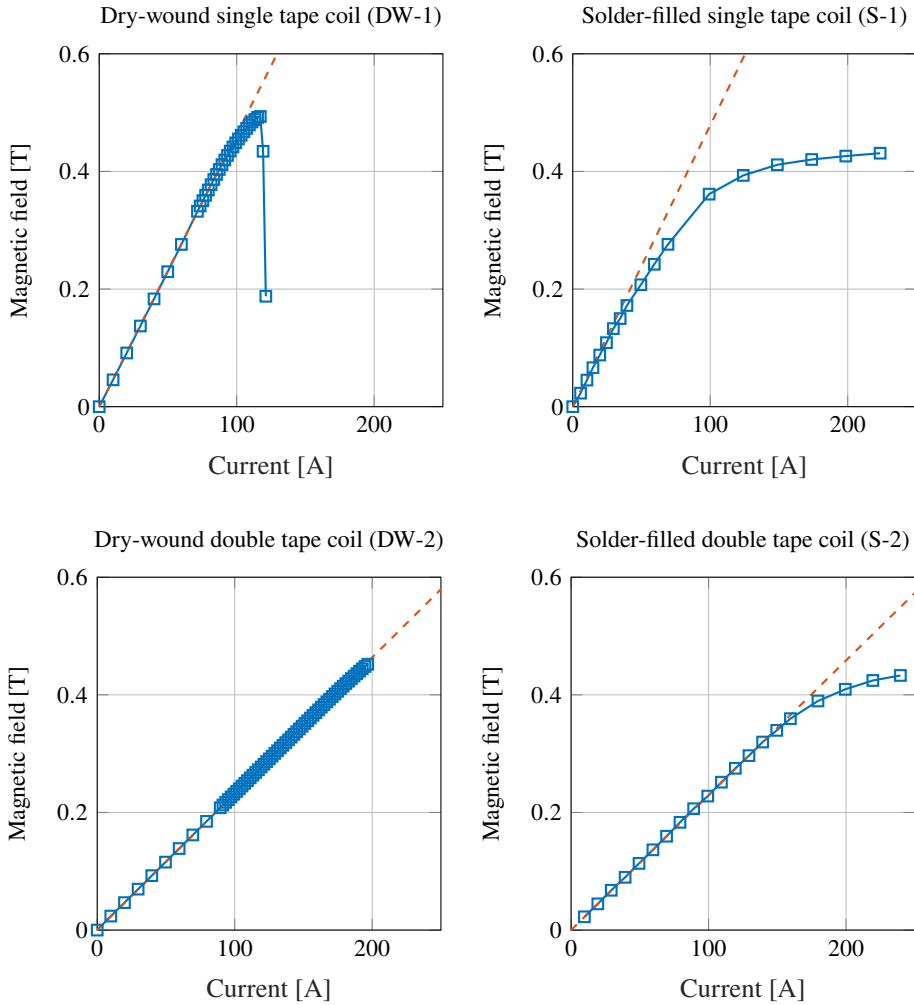


Figure 3.21: Axial component of the magnetic field in the aperture as function of supplied current for all four pancake coils at 77 K in self-field.

In Figure 3.22, the quench current of the dry-wound single tape coil is shown as function of temperature. The minimum and maximum expected coil critical currents are determined by taking the maximum magnetic field on the conductor and assuming that the magnetic field angle is either parallel or perpendicular to the tape. The latter is in order to give an lower and upper estimation to the coil critical current. From the figure, it can be seen that the quench current lies in this range. At 4.2 K, the quench current of the coil is lower than at 10 K. The reason for this is not quite understood, as a higher quench current is expected for a lower temperature. It is possible that the coil got locally damaged, however, the coil did not show any reduction in coil critical current after re-measuring at 77 K.

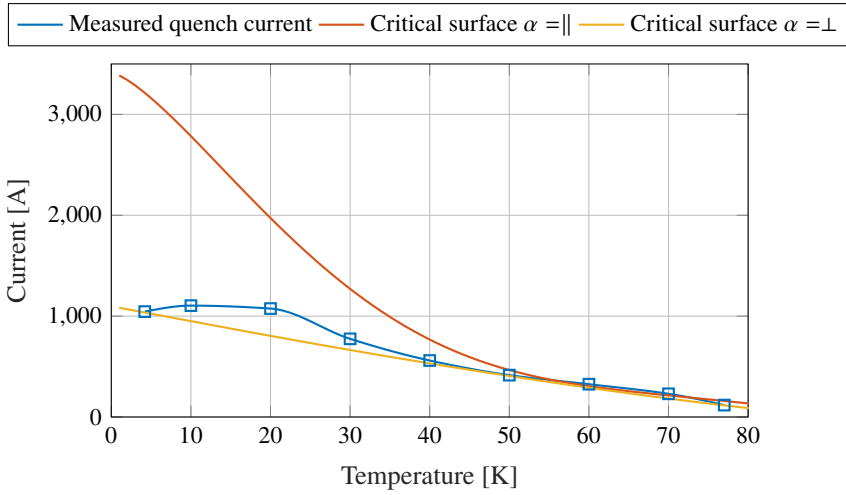


Figure 3.22: Quench current versus temperature for the dry-wound single tape coil (DW-1).

In the variable temperature measurement campaign, the quench and coil critical currents of the solder-filled single tape coil (S-1) were not investigated due to the large charging delay this coils exhibited. However, some statements regarding the coil can be made.

Figure 3.23 shows the voltage, magnetic field and temperature responses to a current step of 2.4 kA as a function of time. The measurement was performed on the single tape solder-filled coil (S-1) at 10 K. At this temperature, the expected coil critical current is 1.7 kA. When the coil is fully charged, the coil is thus expected to run in over-current.

At $t \approx 3160$ s, a self-recoverable transition can be observed, with a corresponding sharp drop in magnetic field. A possible explanation can be the following. Due to the lower self-inductance of the inner turns they charge first. As the outer turns fill up later, the magnetic field on the inner turns is increased, lowering the critical surface of the tape in these sections. If the turns were carrying more current than the critical current, this current now needs to be expelled radially, leading to a local increase in temperature, thereby lowering the critical current density even further. If the heat load is too big, this cascades into a quench.

The dry-wound double tape coil (DW-1) was tested at 4.2 K in a background magnetic field of 0 T to 7 T. In Figure 3.24, the voltage per unit tape length as function of current at 0 T is plotted. It can be seen that the inner section of the coil is damaged, as already from zero current there is an increase in voltage across the inner section. The total coil critical current is 750 A, whereas a coil critical current of 2.6 kA was expected. After electromagnetic cycling in background magnetic field, the coil degraded even further to 530 A (at $100 \mu\text{V m}^{-1}$ criterion), as can be seen in Figure 3.25.

Since the coil structure experiences the most extreme conditions when exposed to external magnetic field, additional calculations were made. Here, the mechanical model in Section 3.2.2 is used analyse at the contact pressure between in the inner ring and the coil-pack in the case of the coil operating at 4.2 K and in a background magnetic field up to 7 T, in order to investigate if the high Lorentz force eventually can play a role in the

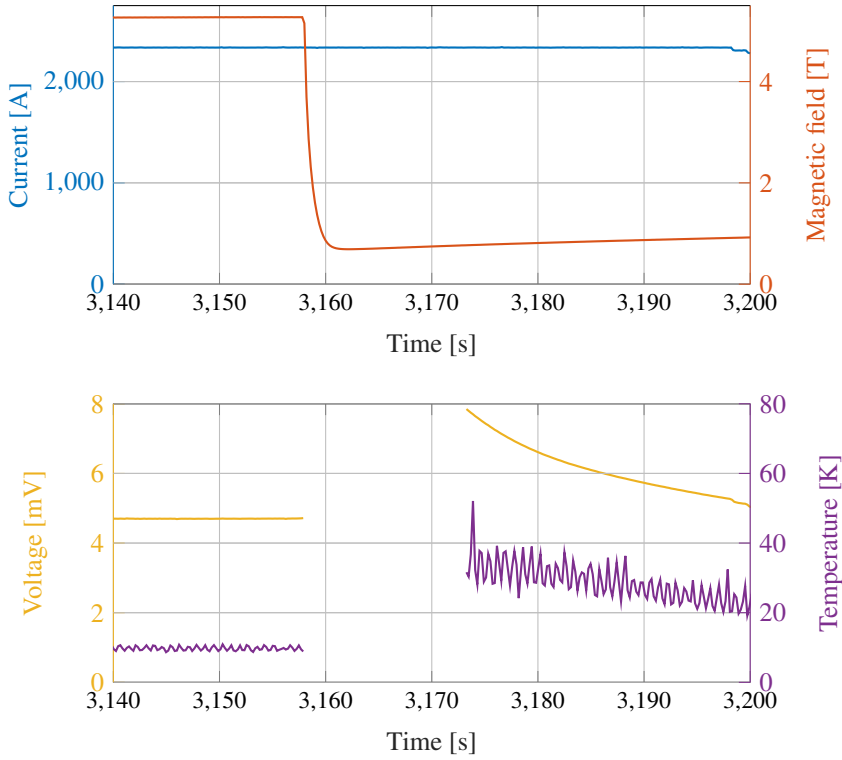


Figure 3.23: Top: current and magnetic field as function of time. Bottom: voltage and temperature as function of time. The figure depicts the self-recovery behaviour of the solder-filled single tape coil at 10 K in self-field.

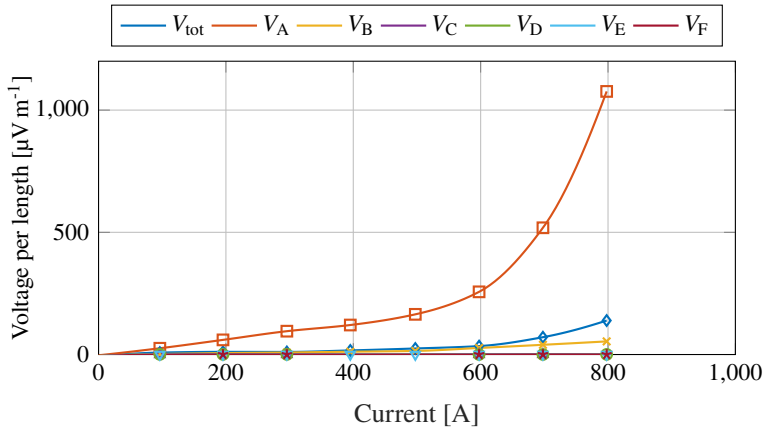


Figure 3.24: Voltage per unit tape length across the dry-wound double tape coil (DW-2) and coil sections A to F at 4.2 K in self-field.

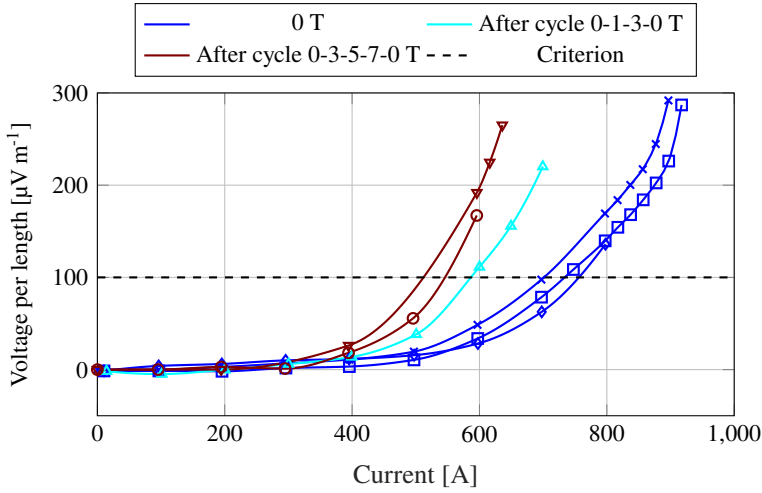


Figure 3.25: Voltage per unit tape length across the dry-wound double tape coil as function of supplied current at 4.2 K and self-field. The measurements were taken before and after cycling of the background magnetic field. Before the first cycle three measurements are shown, after the first cycle one measurement is shown, and after the second cycle three measurements are shown.

degradation of the coil in the inner sections.

In Figure 3.26, the result is shown. For an inner ring made from copper, it can be seen that at a coil current of about 350 A in a 7 T background magnetic field, the coil windings detach from the inner copper ring. As the first turn of the coil-pack is soldered onto the inner copper ring, two distinct modes of failure can occur. Either the tape can delaminate or rupture, or the solder can break. A possible strategy to mitigate this, is to use a titanium core instead of a copper one. In Figure 3.26, the result of this is shown as well. Now only at 1.5 kA, the coil windings detach. This is still well below the expected coil critical current of 2.6 kA. Therefore, for high magnetic field applications, making a joint by fixing it to an internal structure is not a good design. A possible solution to this is to wind a double pancake coil, where from a single conductor two pancake coils are wound which are stacked on top of each other. The layer jump between the two coils is then not to be attached to the inner structure, merely supported.

3.7.2 Effective charging time constant

The effective time constant is an indicator of the time it takes for the current to enter and exit the superconductor in a non-insulated coil. The voltage decay can be approximated well using a double exponential:

$$U \approx a_1 e^{t/\tau_{\text{eff}}} + a_2 e^{t/\tau_i}, \quad (3.12)$$

where a_1 and a_2 are constants depending on the initial condition and τ_{eff} and τ_i are the time constants, corresponding to the effective and initial decay response of the system. In the next chapter, this approximation for the discharging behaviour is justified.

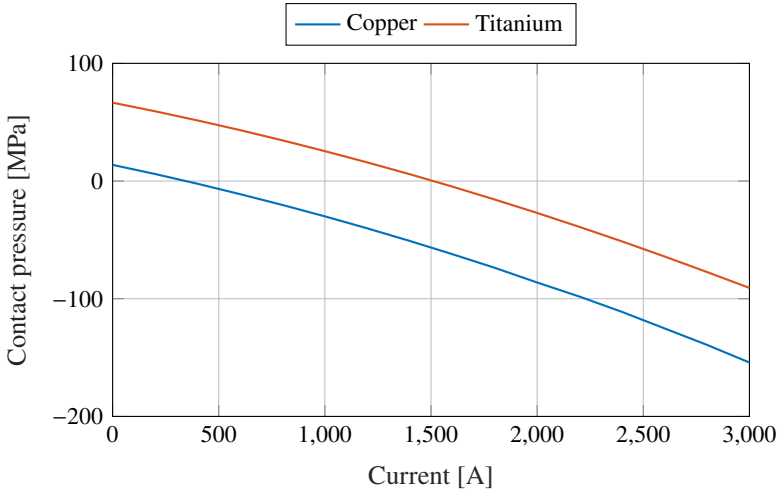


Figure 3.26: Contact pressure on the inner ring as function of coil current, for 4.2 K and in a background magnetic field of 7 T. A winding tension of 33 N is assumed. Shown are the cases for an inner ring made of copper and titanium.

The effective time constant determines the required charging and discharging times of the magnet. Here, it is determined using a current step-down. An example of a measurement to determine the effective time constant is shown in Figure 3.27. Here, the behaviour of the axial magnetic field in the aperture of the coil is related to a current-step as function of time. The axial magnetic field in the aperture does not increase and decrease immediately following a current step-up or step-down. This is due to the time required for the current to settle into the azimuthal path, which is responsible for generating the axial magnetic field along the aperture length. By fitting Equation 3.12 to the measured magnetic field decay, the effective time constant is determined.

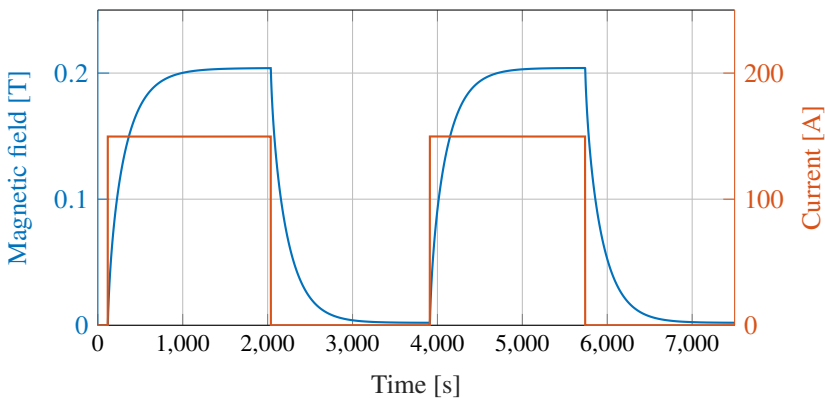


Figure 3.27: Axial magnetic field and coil current as function of time, when charging and discharging the solder-filled double tape coil (S-2) at 77 K in self-field.

In Figure 3.28, τ_{eff} for the dry-wound single tape coil (DW-1) is plotted as a function of measurement number, taken at different temperatures. For a healthy coil, one can anticipate a monotonically increasing effective time constant at lower temperatures, as resistance generally decreases with temperature reduction. However, in this case, an anomalous behaviour is observed, characterized by an abnormal jump in the effective time constant at a distinct event. It can be seen that at 30 K the effective time constant drops considerably. The measurements were repeated at 70 K, and the time constant was found a factor of 10 lower compared to the first measurements at this temperature, indicating an irreversible increase in the inter-turn-resistance. The measured coil critical current was the same for both measurements at 70 K.

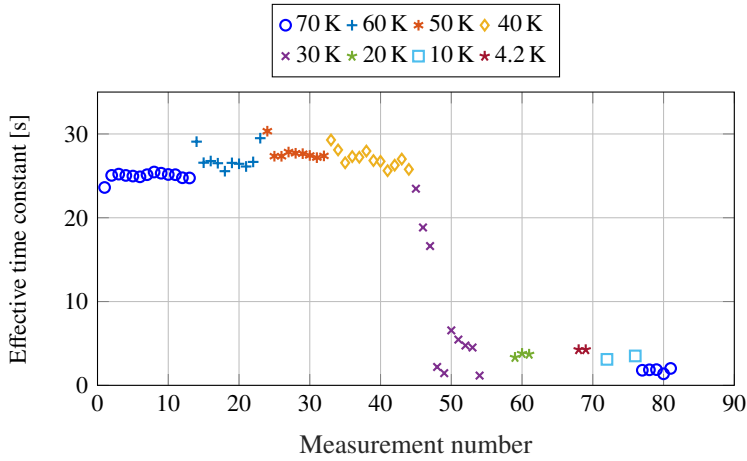
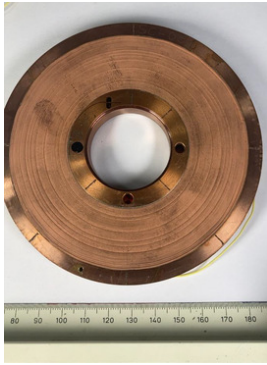


Figure 3.28: Sequential development of the effective time constant as function of measurement number for the dry-wound single tape coil (DW-1). The error from the double exponential fit is estimated to be 5%.

At present, the reason for the dramatic reduction in effective time constant is not well understood. A possible explanation is a changing contact resistance due to loosening of the turns. In Figure 3.29, an image of the dry-wound single tape coil and close-up of the coil windings are shown. It can be observed that there are gaps between the coil turns. Characteristic gap sizes between turns in the coil are estimated at $3\text{ }\mu\text{m}$ to $60\text{ }\mu\text{m}$, and the average gap size is $12 \pm 10\text{ }\mu\text{m}$, clearly demonstrating the effect of cool-down and excitation.

The loosening of turns, manifested as gaps between turns and lateral shifting of turns, can likely be attributed to the process of stacking and energizing non-ideally shaped turns (dog-boning), the flow of copper stabilizer in the tapes, and non-uniform friction between turns. These phenomena are induced by the Lorentz force, which varies locally.

When the turns of the coil exhibit reduced compression against one another, the resultant effect is a decrease in contact pressure, leading to a corresponding reduction in contact resistance. This phenomenon contributes to an increase of the effective time constant of the coil. Another possibility that can contribute is damage to the copper coating due to the radial currents. Unwinding of the coil and inspection of the tape can confirm this.



(a) Whole coil.



(b) Close-up.

Figure 3.29: Image of the dry-wound single tape coil (DW-1) after the variable temperature measurement campaign. Loose turns, i.e. gaps between adjacent turns, can be observed in the coil windings.

For the single-tape solder-filled coil (S-1), this irreversible reduction of the effective time constant is not observed. In Figure 3.30, τ_{eff} for the single-tape solder-filled coil is plotted as a function of temperature. Between 77 K and 10 K, the effective time constant becomes a factor of four larger. Between the dry-wound coil and the solder-filled coil, there is a factor of 40 difference at 70 K at the start of the measurement campaign, and a factor of 400 at the end of the measurement campaign. At 10 K the average time constant differs by a factor of 1600.

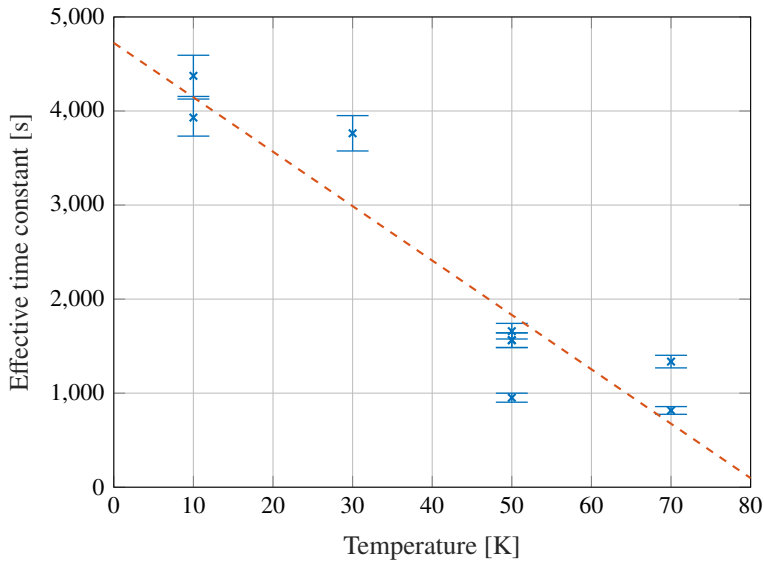


Figure 3.30: Effective time constant as function of temperature for the solder-filled single tape coil (S-1). The error from the double exponential fit is estimated to be 5%.

Not only is the effective time constant dependent on the temperature, but as well as on the applied background magnetic field. This is due to magneto-resistance, where the resistance of the material increases with magnetic field. In Figure 3.31, the effective time constant of the dry-wound double tape coil is plotted as function of background magnetic field. From the figure, it can be seen that the effective time constants drops as function of the applied background magnetic field. The decrease in time constant matches quite well that of the expectation of the effect of the magneto-resistance from copper [105].

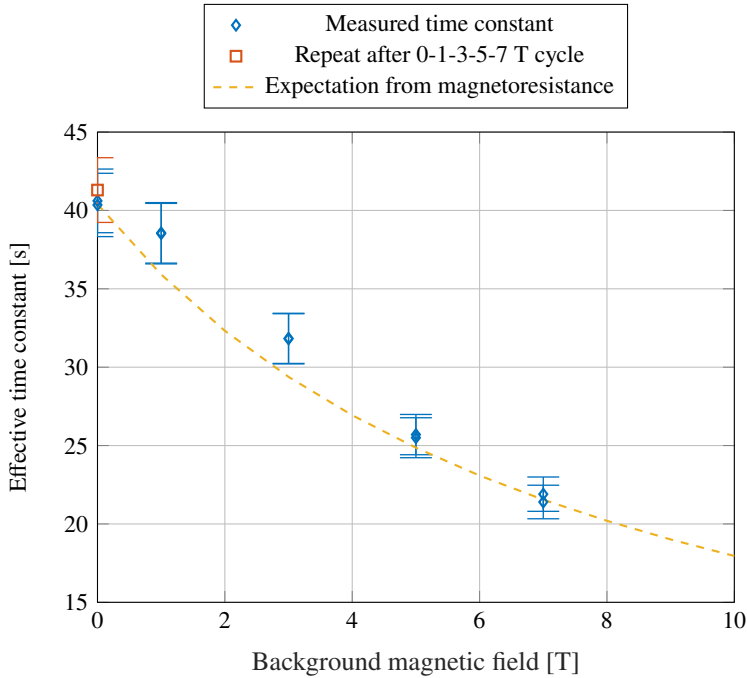


Figure 3.31: Effective time constant as function of background magnetic field for the dry-wound double tape coil (DW-2). The error from the double exponential fit is estimated to be 5%

As with the effective time constant of the single-tape dry-wound coil, the effective time constant of the double-tape dry-wound coil changed irreversibly during the measurement campaign, as can be seen in Figure 3.32. In the first eleven measurements, the effective time constant was determined. These are the points corresponding to Figure 3.31. Here, the effective time constant returned to its original value. From the twelfth point onwards, the coil critical current and quench current were measured. For these measurements, higher currents were supplied to the coil, and thus higher Lorentz force was present. Here, the effective time constant changed irreversibly. After the measurement campaign, loose turns were observed in the coil, similarly to the dry-wound single tape coil.

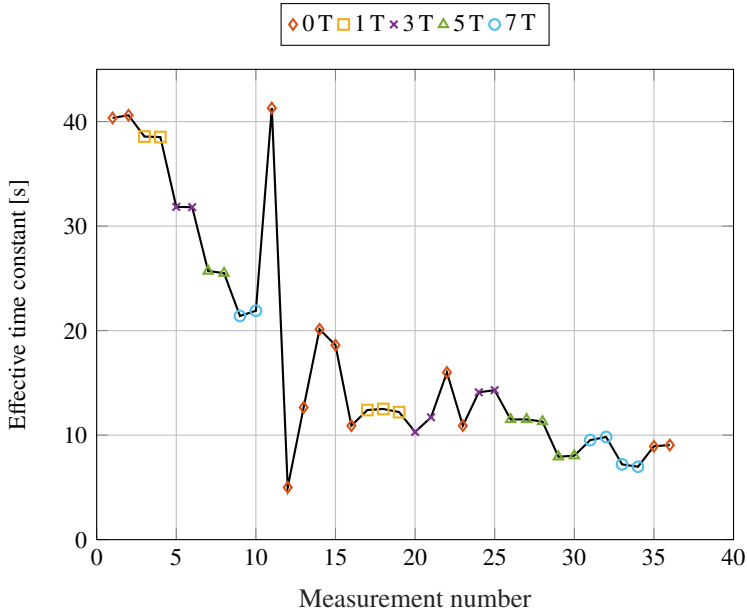


Figure 3.32: Effective time constant as function of measurement number for the double tape dry-wound coil (DW-2) at 4.2 K. The error from the double exponential fit is estimated to be 5%.

3.7.3 Self-inductance

The self-inductances of the pancake coils were measured by supplying a linear ramp in current to the coils. Due to the non-insulated nature of the coils, it takes some time for the voltage to settle to a steady state value. This settling time is related to the effective time constant as well, as is shown in Chapter 4. In Figure 3.33, an example of a self-inductance measurement is shown for the dry-wound single tape coil (DW-1). In Table 3.5, the measured self-inductances of the coils are listed. It can be seen that the self-inductance of the double-wound coils is reduced by a factor of four, which is expected from co-winding two tapes in parallel [112].

Table 3.5: Measured self-inductances of the pancake coils at 77 K.

	DW-1	S-1	DW-2	S-2
Self-inductance [mH]	5.3 ± 0.2	5.3 ± 0.2	1.4 ± 0.1	1.3 ± 0.1

3.7.4 Effective transverse resistivity

The effective transverse resistance of the coils windings is of importance, as it determines the charging time of the non-insulated coil and can be tuned by adding or changing the conductive material between the turns. For a first order estimation, the effective resistance

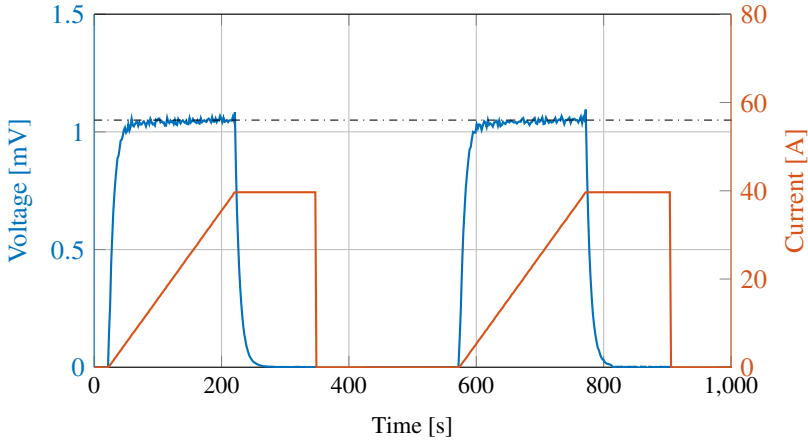


Figure 3.33: Voltage across the entire coil and supplied current as function of time. Example of a self-inductance measurement on the dry-wound single tape coil (DW-1). The current was ramped with a steady rate of 0.2 A s^{-1} . The dotted line indicates the voltage level at which the coil settles, which was 1.1 mV .

of the coil can be estimated from the relation:

$$R = \frac{L}{\tau_{\text{eff}}}, \quad (3.13)$$

where L is the coil self-inductance and τ_{eff} the measured effective time constant. The resistivity ρ_h of the coil can then be determined using the relation (see Chapter 4 for a derivation):

$$\rho_h = \frac{2\pi h R}{\ln\left(\frac{r_{\text{out}}}{r_{\text{in}}}\right)}. \quad (3.14)$$

Instead of transverse resistance, contact resistance can be used as well. The inter-turn contact resistance of the coil can be estimated using the following relation [113]:

$$R_k = \frac{2\pi h r_{\text{avg}} R}{n n_{\text{pt}}}, \quad (3.15)$$

where r_{avg} is the average radius of the coil, n the number of turns and n_{pt} the number of tapes in the cable.

In Table 3.6, the coil resistance, resistivities and contact resistances of the four pancake coils at 77 K and 4.2 K are tabulated. For the coil self-inductance, the measured self-inductance values listed in Table 3.5 are used. The resistivity of the dry-wound coils is close to the resistivity of copper, which is $20 \mu\Omega \text{ m}$ at 77 K [105]. The contact resistance of the dry-wound coils is similar to other measurements performed at 77 K on non-insulated coils made with SST tape [114].

Table 3.6: Resistance R , resistivity ρ_h and contact resistance R_k of the pancake coils at 4.2 K and 77 K. The estimated error is about 10% in the numbers stated.

	DW-1	S-1	DW-2	S-2
$R_{77\text{ K}}$ [$\mu\Omega$]	240	7.2	230	5.9
$R_{4.2\text{ K}}$ [$\mu\Omega$]	-	-	34	-
$\rho_{h,77\text{ K}}$ [$\mu\Omega\text{ m}$]	23	0.67	21	0.54
$\rho_{h,4.2\text{ K}}$ [$\mu\Omega\text{ m}$]	-	-	3.2	-
$R_{k,77\text{ K}}$ [$\mu\Omega\text{ cm}^2$]	22	0.67	21	0.52
$R_{k,4.2\text{ K}}$ [$\mu\Omega\text{ cm}^2$]	-	-	3.1	-

3.8 Conclusion

A comparative study of four single pancake coils, wound with 10 mm wide *ReBCO* tape obtained from Shanghai Superconducting Technology, was performed in order to investigate the potential of no-insulation coil winding technology for enhancing the quench performance of *ReBCO* tape-based coils in accelerator magnet applications. The study encompassed coils of identical size, featuring inner and outer diameters of 52 mm and 102 mm, respectively.

Two distinct variants of the no-insulation coils were fabricated, employing differing techniques: one utilising a single tape, and the other employing a double tape conductor with superconducting layers facing each other. Within each conductor type, one coil was wound without any interstitial material (dry), and another was wound with $\text{Sn}_{62}\text{Pb}_{36}\text{Ag}_2$ solder present between the turns.

A comprehensive array of measurements was undertaken. Each coil underwent tests at 77 K in a liquid nitrogen environment. Furthermore, the two single tape coils were measured within a temperature range of 4.2 K to 77 K in self-field. Additionally, the dry-wound double tape coil was subjected to measurements at 4.2 K in the presence of a background magnetic field ranging from 0 T to 7 T. The study focussed on deriving critical parameters, including time constants, inter-turn resistances, radial voltage distributions, magnetic field behaviour during both charge and discharge cycles, as well as critical and quench currents.

All coils did not reach their expected coil critical current at 77 K. A possible reason is that the criterion employed is too arbitrary, as the length used in the criterion is rather undefined. The initial manifestation of voltage generation was consistently observed within the innermost section of the coils as the current increased at a temperature of 77 K. The effective turn-to-turn resistance of the solder-filled coils is about a factor of 40 smaller than the turn-to-turn resistance of the dry-wound coils.

The dry-wound single tape coil experienced a not-expected drastic, irreversible reduction of a factor of 10 in effective time constant during the variable temperature measurement campaign. A possible reason for this is loosening of the turns in the coil due to high Lorentz forces, thereby effectively increasing the turn-to-turn resistance. This loosening of turns, i.e. gaps appearing between turns and or side-wards shifting of turns can plausibly be the consequence of stacking non-ideally shaped turns (dog-boning), flow of copper stabilizer in the tapes and non-uniform friction between turns, all caused by the Lorentz force acting on the turns and varying locally.

As expected, consequently, the solder-filled single tape coil did not experience this irreversible change in effective time constant. At 70 K the solder-filled single tape coil exhibited an average time constant approximately 40 times higher than that of the dry-wound coil and at 10 K about 1600 times higher. The dry-wound single tape coil did not show any sign of degradation during the measurement campaign.

Importantly, across all temperatures tested, the solder-filled coils demonstrated a higher quench current margin relative to the critical current compared to their dry-wound counterparts. This underscores the superior stability of the solder-filled coils, albeit with the trade-off of longer time constants in comparison to the dry-wound variants.

The dry-wound double tape coil did not reach its expected critical current at 4.2 K in self-field. Moreover, the coil showed signs of degradation when tested in background magnetic field. Additionally, the effective time constant decreased during the measurement campaign, and as with the dry-wound single tape coil, loose turns were observed.

For accelerator magnets, the magnet should be charged in a predictable manner in 20 minutes to 30 minutes. In the following chapter, the results of the pancake coils are compared to a network model. This can then be used to predict the charging behaviour of large scale applications, such as the cloverleaf-racetrack magnet presented in Chapter 6.

Chapter 4

Network model and characteristic time constants of non-insulated coils

The effective time constant of a non-insulated accelerator magnet is of critical importance to the function of the machine, as the magnet needs to be ramped in proportion with the increasing momentum of the particle bunches. It is desired that the time constant is as low as possible to increase the responsiveness of the magnet. Several strategies are explored to reduce the effective time constant of a non-insulated coil. To this, a model is needed to estimate the effective time constant. Typically, a lumped model can be used, which consists of a parallel connection of a resistor and an inductor. However, the model does not include mutual inductances. Here, an expansion of this model is presented by modelling each turn as a resistance and self-inductance connected in parallel, thereby taking the mutual inductances between the turns into account. This allows for a more accurate estimation of the effective time constant. The model is validated with measurement results of the small pancake coils in three scenarios: powering with constant current, powering with a linear ramp in current, and alternating current. Using spectral analysis, it is shown that the temporal behaviour of a non-insulated coil is governed by a set of eigenmodes each having a corresponding characteristic time constant, the largest of which is the effective time constant and determines the overall decay time. These eigenmodes allow for a physical interpretation as to why certain strategies to reduce the effective time constant are more effective than others. The last part of the chapter is devoted to explore several strategies to decrease the effective time constant and analyse their effectiveness with the use of the network model. These strategies are: locally increasing the inter-turn resistance, winding multiple tapes in parallel and applying an overshoot current¹.

¹Part of this chapter has been published in T. H. Nes et al., "Effective Time Constants at 4.2 to 70 K in ReBCO Pancake Coils With Different Inter-Turn Resistances," in IEEE Transactions on Applied Superconductivity, vol. 32, no. 4, pp. 1-6, June 2022, Art no. 4600806, doi: 10.1109/TASC.2022.3148968.

4.1 Introduction

The time-dependent behaviour is of major interest for non-insulated coils, as different applications have different requirements for the charging time of the coils. For a particle accelerator magnet, the shortest possible charging time is desired, whilst still retaining the quench protection properties of a non-insulated coil. A short charging time is required to have a responsive magnet. In a non-insulated coil, the charging time depends on the transverse resistance and self-inductance of the coil, with a higher transverse turn-to-turn resistance decreasing the charging time and a higher self-inductance increasing it [51].

Several methods can be employed to reduce the charging time, targeting either the transverse resistance or the self-inductance. The most obvious solution is to insert a (highly) conductive material between the turns to increase the resistance, while still allowing current to flow radially to bypass a local hot spot. The self-inductance can be modified by changing the size of the coil and the relative position of the coils in a multi-coil system, or by using a cable e.g. multiple tapes in parallel [112]. A third way to reduce the charging time to some extent is to apply an overshoot method, where an initially higher current is applied to force more current into the superconductor, and when the desired field level is reached, the current is lowered to the nominal operating level [115].

To make a prediction on the effectiveness of these methods, a simulation model is needed. There are two approaches that can be taken to model non-insulated coils: by finite element modelling (FEM) or by an electrical network model. Various network models have been used to simulate the voltage behaviour of non-insulated coils. The most basic one is a lumped electrical network model, first proposed by Hahn et al. [51, 86]. He used it to model the voltage behaviour of a non-insulated *ReBCO* pancake coil when a current is applied from a current source. The basic idea of the lumped model is that the current flowing through a non-insulated coil can choose two different paths. One path is transverse and passes through the finite resistance between the turns from one side of the coil to the other. The other path is through the superconductor. The model only takes electrical effects into account, other physical effects such as thermal and the superconducting to normal state transition are not included in this model. Another aspect not taken into account in the lumped network model is that a non-insulated coil consist of multiple turns, all of which have their own transverse resistivity and self-inductance, with mutual inductances between the turns. Furthermore, in a multi-coil configuration such as a stack of pancake coils or the cloverleaf-racetrack magnet (see Chapter 6), each coil may have different transverse resistances and self-inductances. A more advanced network model is required to include all these effects. Bhattarai et al. [116] used a network model for simulating a stack of thirteen non-insulated double pancake coils. To model redistribution of local currents, distributed models are used, where the coil windings are divided into several subsections [85, 115, 117, 118]. Such large distributed network models require solvers for a system of differential equations for each node to obtain the temporal behaviour of a non-isolated coil. This is a practical approach, but hides the physical effects in the output of the solver as it is unclear how the system is solved exactly, and can be time-consuming if non-linear effects are included.

Wang et al. [85, 119, 120] coupled a finite element method (FEM) to a network model. They used the network model to calculate the current redistribution in a non-insulated coil, and then imported the results into the FEM to calculate the electro-magnetic field

distribution inside the non-insulation coil and the magnetization loss [121]. Mataira et al. [122] established a 2D FEM based on a homogenization assumption to study the transient characteristics of non-insulated coils, which gives satisfactory results in a reasonable computational time. Another 2D study was performed by Duan et al. [123], who investigated temperature rises and current distributions in non-insulated coils. Three-dimensional FEM is also used to study non-insulated coils. Wang et al. [124] constructed a three-dimensional model based on the H-formulation to calculate the magnetization loss of a non-insulated coil for application in a maglev train, showing good agreement with experimental results. Zhao et al. [125] used a 3D FEM that allowed for detailed simulation of the transport current behaviour of non-insulated coils. Although finite element modelling has started to become more frequently used due to its good stability and accuracy, they are computationally expensive and most suitable to axially symmetric situations, as they can be performed in 2D. Although the pancake coils presented in Chapter 3 are axis symmetric, the cloverleaf-racetrack magnet presented in Chapter 6 is not. Therefore, in this thesis FEM is not considered to study non-insulated coils.

The models used are either very basic (lumped network model), or rather expansive (distributed network models and FEM). Currently, there is no middle ground between these two approaches. In this chapter, a different approach is taken by for the first time analytically solving a distributed network model of a non-insulated coil, so that the physics phenomena can be discerned and attributed. The analytical solution is linear, and can therefore be computed fast. The disadvantage of the analytical approach is that only solutions exist for a linear system, and therefore cannot include non-linear physics effects such as the superconducting to normal phase transition, as well as temperature effects. Having an analytical solution for the network model allows the solution to be split into so-called eigenmodes, each of which has a characteristic time constant. This technique is called spectral decomposition, and has already been applied to determine the characteristic time constants of current redistributions in superconducting cables [126]. Here, it will be applied for the first time to the study of the behaviour of current and voltage as a function of time in non-insulated coils.

The chapter starts with a description of the network model with a derivation of a general solution of the model. Next, an estimation of the transverse resistance and self-inductance of each node of the distributed network model is presented. In Section 4.4, the analytical solution to the network model is compared to measured results of the time-dependent behaviour of the voltage over the whole coil and the coil segments, for powering cases such as a current step, a linear current ramp and alternating current. Then, the spectral decomposition to determine the eigenmodes and characteristic time constants for non-insulated coils is presented. In the last section, methods are explored to reduce the charging time, such as insulating a single turn, intermittent insulations and current overshooting.

4.2 Network model

The most basic model for simulating non-insulated coils is the lumped model. Its equivalent network is shown in Figure 4.1a. In the lumped model, two parallel paths for the current are present, an azimuthal current path I_{sc} corresponding to the superconductor with a self-inductance L , and a radial current path I_R with a radial resistance R .

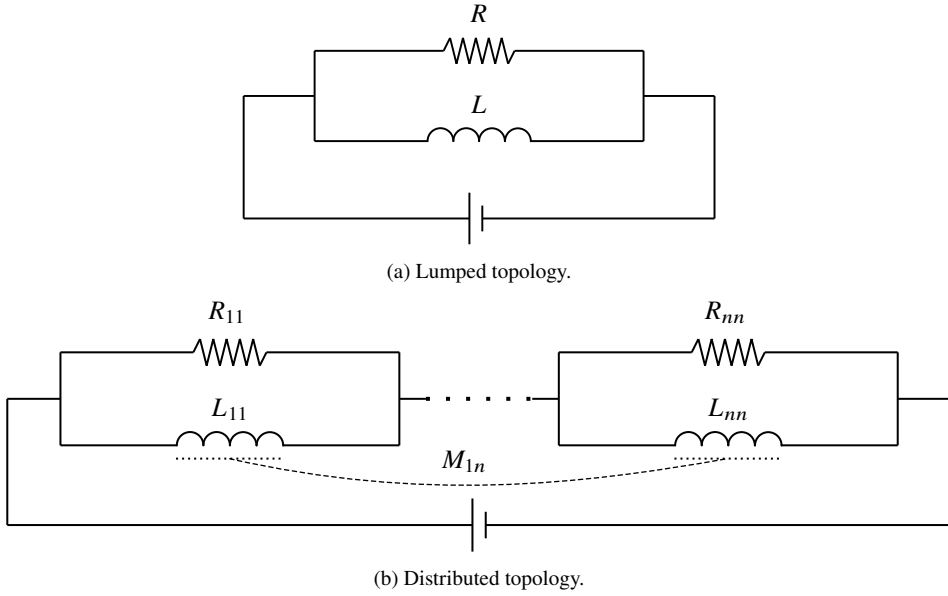


Figure 4.1: Two equivalent network models, with lumped and distributed topologies. These network topologies can be used to model non-insulated coils.

Here, a distributed network is proposed, based on an expansion of the lumped model. Instead of modelling the whole coil by a single resistor and a single inductor, each individual turn now consists of a resistor and an inductor, i.e. each turn has the topology of the lumped model (see Figure 4.1b). The system has n nodes, which corresponds to n turns in the coil, or n different single turn coils. One path corresponds to the radial path, and consists of a resistor, just as in the lumped model. The other path corresponds to the current flowing in the superconductor. It consists of a self-inductance, as in the lumped model, but also includes mutual inductances with the other turns.

The time-dependent behaviour of the currents and voltages within the topology of the distributed model can be solved analytically. So far, solving such a topology analytically has not been done for non-insulated coil geometries. However, a similar problem from a network topology point of view has been solved in the context of current redistributions in a Rutherford cable [126]. In the following, an analytical solution for non-insulated coils is presented.

In each node, the current must be conserved. In matrix form, this can be written as follows:

$$\mathbf{I}_{\text{tot}}(t) = \mathbf{I}_{\text{sc}}(t) + \mathbf{I}_{\text{R}}(t), \quad (4.1)$$

where the bold font indicates that it is a column vector. Each entry in the vector corresponds to a node. The voltage over the inductive part is then:

$$\mathbf{U}(t) = \tilde{\mathbf{L}} \frac{d\mathbf{I}_{\text{sc}}(t)}{dt}, \quad (4.2)$$

where $\tilde{\mathbf{L}}$ is the inductance matrix, which is of the form:

$$\tilde{\mathbf{L}} = \begin{pmatrix} L_1 & M_{12} & \dots & M_{1n} \\ M_{21} & L_2 & \dots & M_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ M_{n1} & M_{n2} & \dots & L_n \end{pmatrix}. \quad (4.3)$$

The inductance matrix can be calculated numerically, analytically for specific geometries, or using look-up tables [127]. The voltage over the resistive part is

$$\mathbf{U}(t) = \tilde{\mathbf{R}}\mathbf{I}_R(t) = \tilde{\mathbf{R}}(\mathbf{I}_{\text{tot}}(t) - \mathbf{I}_{\text{sc}}(t)), \quad (4.4)$$

where $\tilde{\mathbf{R}}$ is the resistance matrix:

$$\tilde{\mathbf{R}} = \begin{pmatrix} R_1 & 0 & \dots & 0 \\ 0 & R_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & R_n \end{pmatrix}. \quad (4.5)$$

The values of the resistance matrix can be determined by using first-order approaches based on the coil geometry, and refined using experimentally obtained data. Refinement is necessary because the radial resistance is influenced by the interstitial material [51], the winding tension [128] and the transverse resistance of the *ReBCO* tape [97].

By differentiating Equation 4.4 and using Equation 4.2, the voltage behaviour of each node can be described as follows:

$$\frac{d\mathbf{U}(t)}{dt} + \tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1}\mathbf{U}(t) = \tilde{\mathbf{R}}\frac{d\mathbf{I}_{\text{tot}}}{dt}. \quad (4.6)$$

The solution to this differential equation is given by the *variation of parameters* formula:

$$\mathbf{U}(t) = e^{-t\tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1}}\mathbf{c} + e^{-t\tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1}}\int_{t_0}^t e^{\xi\tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1}}\tilde{\mathbf{R}}\frac{d\mathbf{I}_{\text{tot}}}{d\xi}d\xi, \quad (4.7)$$

where $\mathbf{c} = (c_1, c_2, \dots, c_n)^T$ is a column vector with constant entries. In Equation 4.7, a matrix exponential is present, which is defined as:

$$e^{\tilde{\mathbf{X}}} = \sum_{k=0}^{\infty} \frac{1}{k!} \tilde{\mathbf{X}}^k. \quad (4.8)$$

For the lumped case, a similar expression can be obtained in terms of scalars:

$$U(t) = c_1 e^{-tR/L} + e^{-tR/L} \int_{t_0}^t \frac{dI_{\text{tot}}(\xi)}{d\xi} R e^{\xi R/L} d\xi. \quad (4.9)$$

The voltage behaviour of a non-insulating coil is then fully described by inserting the appropriate conditions in Equation 4.7. In the following section, it is discussed how to determine the values for the terms in the resistance and inductance matrices in the case of pancake coils.

4.3 Pancake coil self-inductance and resistance

The four pancakes considered were presented in Chapter 3. They are two dry wound pancakes and two solder-filled pancakes, of each type there is one wound with a single tape and one with two tapes in parallel in face-to-face configuration. The properties of the coils, such as resistances and self-inductances, were presented in Table 3.1.

4.3.1 Resistance

In the lumped case, the whole coil can be seen as an annulus of height h and inner and outer radii r_{in} and r_{out} , respectively (see Figure 4.2). Assuming an averaged homogeneous resistivity ρ_h , the differential resistance of an annulus where the current flows radially is then given by:

$$dR = \frac{\rho_h}{A} dr = \frac{\rho_h}{2\pi h r} dr. \quad (4.10)$$

Integrating the previous expression yields the total resistance in the radial direction:

$$R = \int_{r_{in}}^{r_{out}} \frac{\rho_h}{2\pi h r} dr = \frac{\rho_h}{2\pi h} \ln\left(\frac{r_{out}}{r_{in}}\right). \quad (4.11)$$

In practice, the resistance of the coil can be determined from measurements, as was performed by various researches [113, 129]. At room temperature, the resistance can be determined by Ohm's law. When the coil is in the superconducting state, the resistance cannot be determined in this way, because the current does flows azimuthally along the turns in steady-state operation.

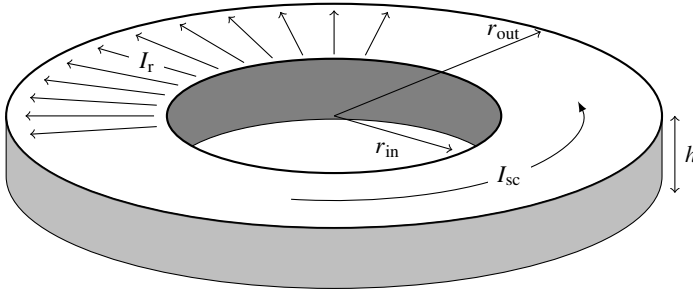


Figure 4.2: Schematic of a small pancake coil modelled as an annulus of inner radius r_{in} , outer radius r_{out} and height h . The normal current I_r flows radially through the annulus, and the superconducting current I_{sc} flows azimuthally through the annulus.

By rearrangement of the Equation 4.11, a homogenized resistivity ρ_h can be determined. This average resistivity can then be used as an input to estimate the transverse resistance per turn. Typically, investigators used an averaged radius of the turn to determine the radial resistivity [119, 130, 131]. A different approach is taken here, which takes into account the annular geometry of the turn. Assuming that each turn is a circle, the resistance per turn R_k can be determined by changing the integration constants in Equation 4.11 so

that they correspond to the inner and outer radii of the turn itself. This yields:

$$R_k = \frac{\rho_h}{2\pi h} \ln\left(\frac{b}{a}\right), \quad (4.12)$$

with inside radius:

$$a(k) = r_{\text{in}} + (r_{\text{out}} - r_{\text{in}})(k - 1)/n, \quad (4.13)$$

and outside radius:

$$b(k) = r_{\text{in}} + (r_{\text{out}} - r_{\text{in}})k/n, \quad (4.14)$$

where n is the total number of turns and k the turn number. The inner and outer radii of the turn are expressed in terms of the inner and outer radii of the coil and the number of turns. Thus, the thickness of the inter-turn layer (or gap size) is taken into account as well. The total resistance of the pancake is the sum of all the elements:

$$R = \sum_{k=1}^n R_k = \text{tr}(\tilde{\mathbf{R}}), \quad (4.15)$$

where the trace of the matrix $\tilde{\mathbf{R}}$ is the sum of all its diagonal elements.

It is assumed here that the resistivity is the same for each turn. It is true that the material properties are the same for each turn, but they are only one factor contributing to the resistivity. Another factor is the contact resistance between the turns, as the current must flow through the various contacts between a turn and its adjacent turns (or terminal at the first and last turn). This contact resistance depends among others on the contact pressure between the turns, as well as the local shape of the tape making the turns, and can reduce the radial resistance up to a factor of six at 200 MPa [132]. Throughout the coil, this contact pressure is different, due to the winding tension of each turn accumulating and maximum in the inner turns. It is therefore expected that the homogenized resistivity is lower on the inside of the coil due to the locally higher contact pressure.

Another assumption that has been made is that each turn is considered a circle. However, in a pancake coil, each turn is more like a flat spiral, where the turns increase in radius along their length. This changes the transverse resistance over each turn somewhat, as the radius of the turn becomes slightly larger during the turn. A spiral can be parametrized as:

$$r = r_{\text{in}} + c\theta, \quad (4.16)$$

where θ is the rotation along the spiral. Each turn corresponds to a rotation of 2π of the spiral. The term $c/(2\pi)$ is the difference in radius between each subsequent turn and c can therefore be expressed as follows:

$$c = \frac{b - a}{2\pi} = \frac{r_{\text{out}} - r_{\text{in}}}{2\pi n}. \quad (4.17)$$

The length of one turn is given by:

$$L_t = \int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta. \quad (4.18)$$

The area as a function of radius is then given by:

$$A = h \int_0^{2\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta = 2\pi h \sqrt{r^2 + c^2}. \quad (4.19)$$

The differential resistance is then given by:

$$dR = \frac{\rho_h}{A} dr = \frac{\rho_h}{2\pi h \sqrt{r^2 + c^2}} dr. \quad (4.20)$$

Integrating the previous expression from a to b yields the resistance over the k -th turn:

$$R_k = \int_a^b \frac{\rho_h}{2\pi h \sqrt{r^2 + c^2}} dr = \frac{\rho_h}{2\pi h} \ln \left(\frac{b + \sqrt{b^2 + c^2}}{a + \sqrt{a^2 + c^2}} \right). \quad (4.21)$$

For the small pancake coils with $r_{\text{in}} = 26$ mm and $r_{\text{out}} = 51$ mm, the difference between R_k for the circle (Equation 4.11) and for the spiral (Equation 4.21) is that the spiral approximation gives a resistance of 0.6% less for the first turn. For subsequent turns, the difference will be even smaller, a reason to neglect this effect for simplicity.

4.3.2 Self-inductance

The self-inductance of the coils can be calculated by assuming that each turn k is an annulus of height h and inner radius $a(k)$ and outer radius $b(k)$, with a homogeneous distribution of the current. The self-inductance is most easily calculated numerically using computer programs such as COMSOL [133], OPERA [134] or FIELDS [109]. Figure 4.3 shows the calculated self-inductances for the pancake coils that were investigated (see Section 3.7.3).

The total self-inductance for the lumped case is the self-inductance of an annulus of height h and inner radius r_{in} and outer radius r_{out} , which is equivalent to the sum of all matrix elements for the distributed case, i.e.:

$$L = \sum_{i,j} L_{ij}. \quad (4.22)$$

It was chosen to approximate the geometry as a circle and not a spiral, as it is easier and quicker to calculate the self-inductance of closed loops, as it avoids the problem of the existence of fringe fields at the beginning and end of each turn.

4.4 Modelling voltage behaviour

The voltage behaviour of a non-insulated coil can be modelled using the appropriate boundary conditions in Equation 4.7. In this section, the model is compared with measured data of the pancake coils for three cases: current step, linear ramp and alternating current.

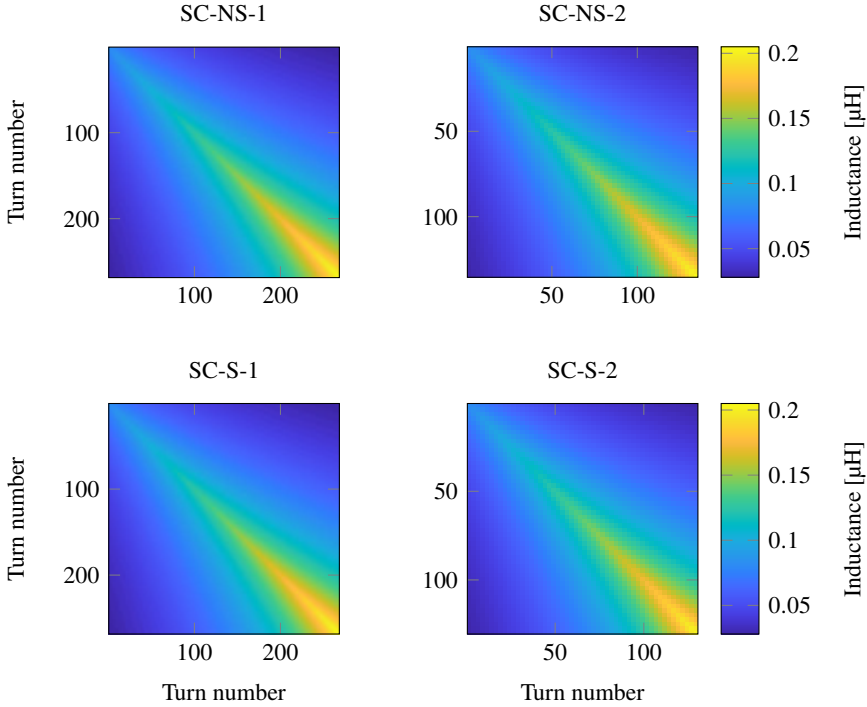


Figure 4.3: Inductance matrices of the four pancake coils that were investigated, shown as a colour plot to indicate the magnitude of the self- and mutual inductances.

4.4.1 Current step

A step-up or step-down in current being sent through a non-insulated coil is the purest method to determine the time-dependent behaviour of a non-insulated coil, as it excludes electromagnetic effects due to variations in currents over time. In a current step, the current is zero for $t < 0$ and has a constant value for $t > 0$. In practice, a current step up can never be perfect as no power supply is capable of sending current through the coil in an infinitely small time step. A step down can be achieved by turning off the power supply or with a circuit breaker.

After a current step up, the voltage in each node equals $\mathbf{U}_0 = \tilde{\mathbf{R}}\mathbf{I}$ at $t = 0$ and this serves as a boundary condition in Equation 4.7. As $\frac{d\mathbf{I}_{\text{tot}}}{dt} = 0$, the second term in Equation 4.7 is zero, and the solution to the equation is:

$$\mathbf{U}(t) = e^{-t\tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1}}\mathbf{U}_0. \quad (4.23)$$

In terms of current, the transverse flowing current is then:

$$\mathbf{I}_R(t) = \tilde{\mathbf{R}}^{-1}\mathbf{U}(t) = e^{-t\tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1}}\mathbf{I}_{\text{tot}}, \quad (4.24)$$

and the superconducting current is

$$\mathbf{I}_{\text{sc}}(t) = \left(\tilde{\mathbf{I}} - e^{-t\tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1}}\right)\mathbf{I}_{\text{tot}}, \quad (4.25)$$

with $\tilde{\mathbf{I}}$ the identity matrix.

Figure 4.4 shows the simulated response to a current step of 300 A of the dry-wound single tape coil (DW-1). The chosen value of the current is merely for illustration purposes, but must be under the critical current of the coil, as the model is not valid for situations where in some turns the critical current is exceeded. At $t = 0$, the current flows radially throughout the coil. At later times, the transverse current decreases. This is due to the fact that the current settles in the superconductor. The radial current decreases most rapidly near the inner radius of the coil winding pack. It takes longer for the current to stabilise near the outer sections of the coil. The comparison with the measured data is made in Figure 4.5. Here, the voltage response of sections A to C of the solder-filled coil is shown when a current step-up and step-down of 300 A is applied at 50 K. Each segment corresponds to one-sixth of the radius of the coil, with section A being on the inside, as presented in Figure 3.5. Based on the critical current fits of the SST-tape (see Chapter 2), at this temperature the critical current is expected to be 450 A. The resistivity is found by finding the best fit to the measurement data, and is $\rho_h = 280 \text{ n}\Omega \text{ m}$. For segments B and C, the network model shows good agreement with the measured data. For segment A however, the measured data is not in good agreement with the simulation. As this section is closest to the copper rings, the heating of the inner copper ring may generate a voltage over the first turns. To model this accurately, a more advanced network model is required, which include effects such as joule heating, thermal propagation and transitions from the superconducting to the normal state.

4.4.2 Linear ramp

Accelerator magnets are typically not stepped up in current, but ramped up so that the increasing magnetic field matches the momentum of the particle bunches. The current profile of the ramp has a non-trivial shape to reduce magnetic field errors and snap-back recovery rates [135]. To understand what is happening inside a non-insulated coil during ramping, the simpler case of linear ramping is considered, where the coils are ramped with a constant rate of current.

When the coils are ramped at a constant rate I_{ramp} , the change in total current through each turn is constant:

$$\frac{d\mathbf{I}_{\text{tot}}}{dt} = I_{\text{ramp}}\mathbf{e}, \quad (4.26)$$

where $\mathbf{e} = (1, 1, \dots, 1)^T$ is a vector with ones in all its entries. If the system is ramped from zero voltage, i.e. $\mathbf{U}_0 = 0$, the solution to Equation 4.7 is:

$$\mathbf{U}(t) = I_{\text{ramp}}(\tilde{\mathbf{I}} - e^{-t\tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1}})\tilde{\mathbf{L}}\mathbf{e}. \quad (4.27)$$

For $t \gg 0$, the voltage in each turn stabilizes at a constant value:

$$\mathbf{U}(t \gg 0) = I_{\text{ramp}}\tilde{\mathbf{L}}\mathbf{e}. \quad (4.28)$$

At large times, the current from the power supply is balanced by the current building up in the superconductor, leaving a constant net voltage. This result allows for an estimation of the self-inductance. By charging the coil with a linear current ramp rate and measuring

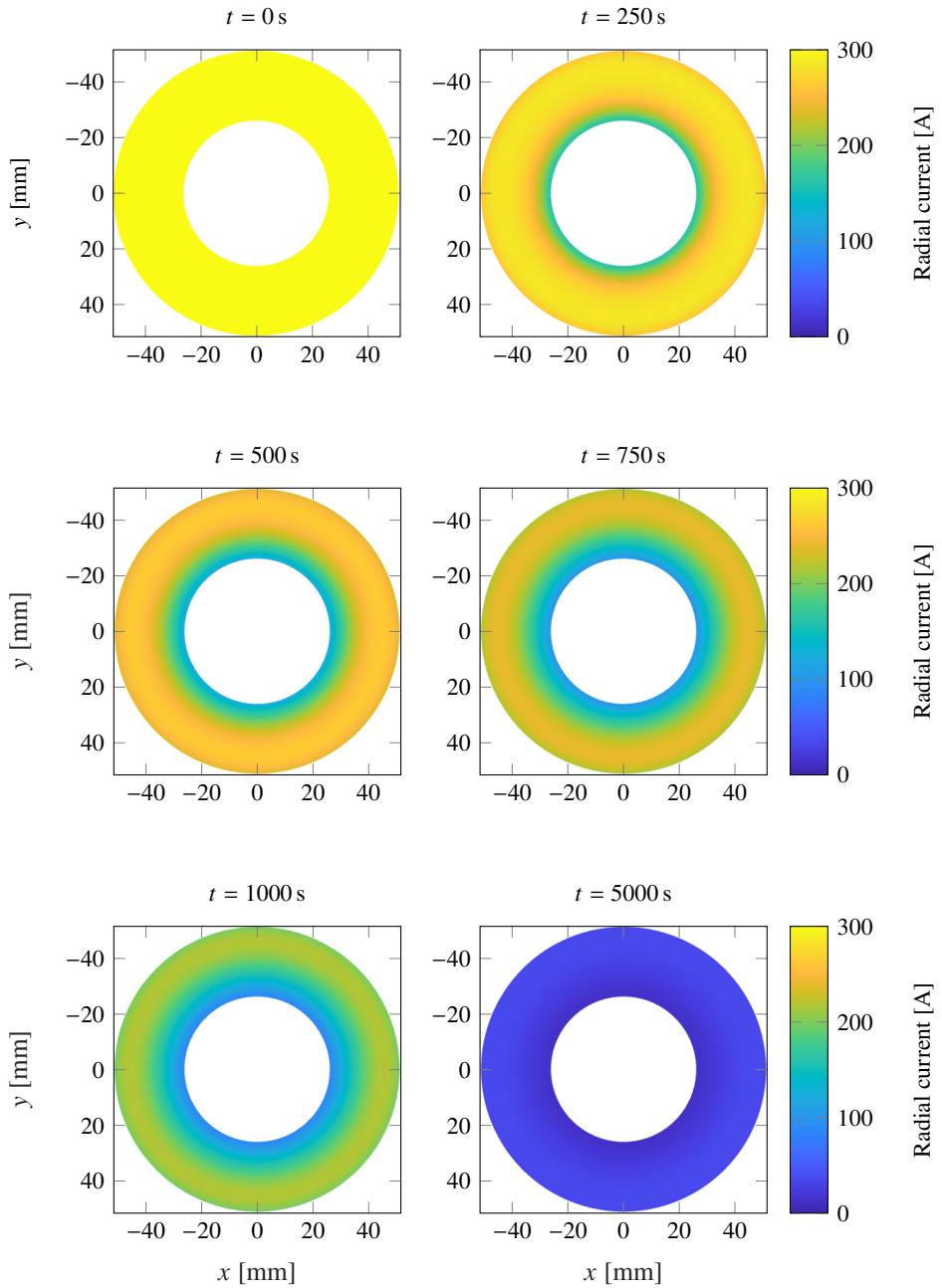


Figure 4.4: Modelled radial current decay for coil S-1, after applying a current step of 300 A at 50 K.

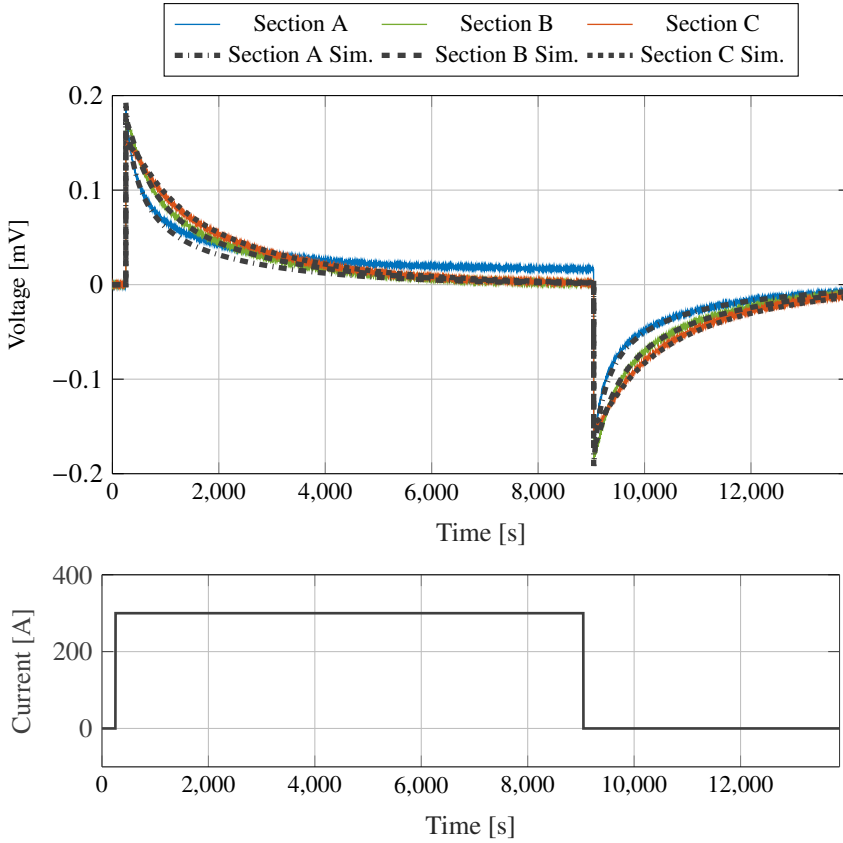


Figure 4.5: Time-dependent behaviour of the voltage across the first three segments of a sixth of coil S-1 in response to a step-up and step-down in current of 300 A. The measurement was performed at 50 K. The measurement data are compared with the simulated data, and it shows good agreement for segments B and C, but less for A.

the voltage once the system stabilizes, the self-inductance can be estimated using Equation 4.28. The ramp rate needs to be sufficiently low so that during the ramp the turns do not reach the critical current before the voltage has reached a constant level.

Figure 4.6 shows a comparison of the Equation 4.27 with measured results of dry-wound single tape coil (DW-1) at 77 K when the coil is energized with a constant ramp rate of 0.2 A s^{-1} . From the figure, it can be seen that the prediction of Equation 4.28 that the voltage reaches a constant value for time greater than zero is correct. The measured voltage is about 9% lower than estimated, which means that the self-inductance is lower than predicted by the model. At $t \approx 440 \text{ s}$, the voltage starts to deviate from the constant value. This is because the coil becomes oversaturated with current. Some of the turns cannot carry any additional current in the superconductor and there the current flows radially, generating voltage.

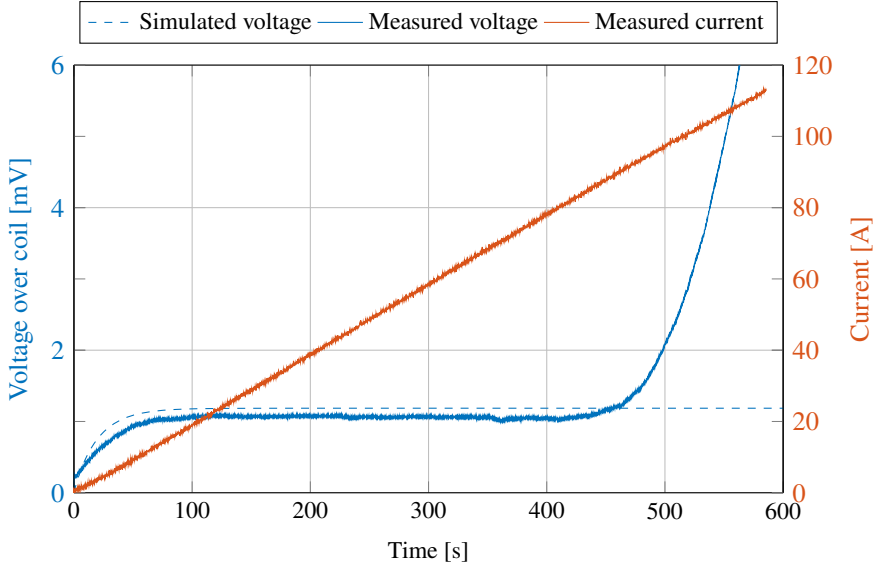


Figure 4.6: Coil voltage versus time. Modelled response to a linear ramp of 0.2 A s^{-1} compared with measured data from dry-wound single tape coil (DW-1), at 77 K. The average resistivity assumed of the coil is $32 \mu\Omega \text{ m}$.

4.4.3 Alternating current

For an accelerator magnet, application of an alternating current is not relevant. However, applying an alternating current to a non-insulated coil can be used to validate the model.

The current provided by the power supply is given by:

$$\mathbf{I}_{\text{tot}} = A_I \sin(\omega t) \mathbf{e}, \quad (4.29)$$

where A_I is the amplitude of the applied current and ω the angular frequency of the current. Plugging this into Equation 4.7 yields:

$$\mathbf{U}(t) = e^{-t\tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1}} \mathbf{c} + e^{-t\tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1}} \int_0^t A_I \omega \cos(\omega \xi) e^{\xi\tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1}} d\xi \tilde{\mathbf{R}} \mathbf{e}. \quad (4.30)$$

Working out the integral and using as a boundary condition that the initial voltage is zero everywhere, i.e. $\mathbf{U}_0 = 0$, results the following relation for the voltage over each turn:

$$\mathbf{U}(t) = \omega(\omega \sin(\omega t) \tilde{\mathbf{I}} + \cos(\omega t) \tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1} - e^{-t\tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1}})((\tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1})^2 + A_I \omega^2 \tilde{\mathbf{I}})^{-1} \tilde{\mathbf{R}} \mathbf{e}. \quad (4.31)$$

This rather complicated expression can be simplified if the lumped case (Equation 4.9) is considered. The voltage over the coil is then expressed as:

$$U(t) = \frac{A_I L R \omega}{L^2 \omega^2 + R^2} (L \omega \sin(\omega t) + R \cos(\omega t) - R e^{-tR/L}). \quad (4.32)$$

In an AC setting, a parameter of interest is the power loss in the coil. For $t \gg 0$, the exponential decay part can be omitted and the instantaneous power is:

$$P(t) = U(t)I_{\text{tot}}(t) = \frac{A^2 L R \omega}{L^2 \omega^2 + R^2} (L \omega \sin^2(\omega t) + \frac{R}{2} \sin(2\omega t)). \quad (4.33)$$

The energy loss per cycle is then:

$$E_{\text{cycle}}(\omega) = \int_0^{2\pi/\omega} P(t) dt = \frac{A^2 L^2 R \pi \omega}{L^2 \omega^2 + R^2}. \quad (4.34)$$

The average power loss is:

$$P_{\text{avg}}(\omega) = \frac{E_{\text{cycle}}}{2\pi/\omega} = \frac{A^2 L^2 R \omega^2}{2(L^2 \omega^2 + R^2)}. \quad (4.35)$$

In the edge case for low frequencies ($\omega = 0$), the loss is zero. For high frequencies ($\omega \rightarrow \infty$), the average loss is:

$$\lim_{\omega \rightarrow \infty} P_{\text{avg}} = \frac{A^2 R}{2}. \quad (4.36)$$

The critical frequency is the frequency at which losses become dominant. To define the critical frequency, the inversion point of Equation 4.35 is taken. This means that the second derivative of Equation 4.35 is equal to zero, i.e.:

$$\frac{d^2 P_{\text{avg}}}{d\omega^2} = 0. \quad (4.37)$$

Solving this equation for ω yields the critical frequency:

$$\omega_c = \frac{R}{\sqrt{3}L}. \quad (4.38)$$

The average power loss at the critical frequency can be found by plugging Equation 4.38 back into Equation 4.35, yielding:

$$P_c = \frac{A^2 R}{8}. \quad (4.39)$$

Having established an expression for the power loss for the lumped case, a similar expression can be found for the distributed model case. The total power loss becomes:

$$P_{\text{tot}}(t) = \mathbf{I}^T(t) \mathbf{U}(t) = A^2 \omega \mathbf{e}^T (\omega \sin^2(\omega t) \tilde{\mathbf{I}} + \frac{1}{2} \sin(2\omega t) \tilde{\mathbf{R}} \tilde{\mathbf{L}}^{-1}) ((\tilde{\mathbf{R}} \tilde{\mathbf{L}}^{-1})^2 + \omega^2 \tilde{\mathbf{I}})^{-1} \tilde{\mathbf{R}} \mathbf{e}. \quad (4.40)$$

The energy loss per cycle is expressed as:

$$E_{\text{cycle}}(\omega) = \int_0^{2\pi/\omega} P_{\text{tot}}(t) dt = A^2 \pi \omega \mathbf{e}^T ((\tilde{\mathbf{R}} \tilde{\mathbf{L}}^{-1})^2 + \omega^2 \tilde{\mathbf{I}})^{-1} \tilde{\mathbf{R}} \mathbf{e}, \quad (4.41)$$

resulting in an average loss of:

$$P_{\text{avg}}(\omega) = \frac{E_{\text{cycle}}}{2\pi/\omega} = \frac{A^2 \omega^2}{2} \mathbf{e}^T ((\tilde{\mathbf{R}} \tilde{\mathbf{L}}^{-1})^2 + \omega^2 \tilde{\mathbf{I}})^{-1} \tilde{\mathbf{R}} \mathbf{e}. \quad (4.42)$$

In the limit of high frequencies, the average power loss gives the same result as for the lumped case:

$$\lim_{\omega \rightarrow \infty} P_{\text{avg}}(\omega) = \frac{A^2 R}{2}, \quad (4.43)$$

with $R = \text{tr}(\tilde{\mathbf{R}})$ (Equation 4.15). For the distributed case, it is not possible to find a critical frequency in the same manner as Equation 4.37 due to the presence of vectors and matrices in Equation 4.42.

4.5 Spectral analysis

Spectral decomposition allows Equation 4.23 to be rewritten as a sum of exponentials, which each have an associated time constant. This is useful because it shows how the settling of the current into the superconductor behaves as a function of time. The spectral decomposition is achieved by rewriting Equation 4.23 in terms of its eigenvalues and eigenvectors:

$$\mathbf{U}(t) = \tilde{\mathbf{H}} e^{-t\tilde{\mathbf{\Lambda}}} \tilde{\mathbf{H}}^{-1} \mathbf{U}_0, \quad (4.44)$$

where $\tilde{\mathbf{\Lambda}}$ is the eigenvalue matrix, which has the eigenvalues λ of $\tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1}$ on its diagonal:

$$\tilde{\mathbf{\Lambda}} = \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_n \end{pmatrix}. \quad (4.45)$$

The matrix $\tilde{\mathbf{H}}$ is the eigenvector matrix, and has on its columns the eigenvectors $\boldsymbol{\eta}$ of $\tilde{\mathbf{R}}\tilde{\mathbf{L}}^{-1}$:

$$\tilde{\mathbf{H}} = \begin{pmatrix} \uparrow & \uparrow & \uparrow & \uparrow \\ \boldsymbol{\eta}_1 & \boldsymbol{\eta}_2 & \dots & \boldsymbol{\eta}_n \\ \downarrow & \downarrow & \downarrow & \downarrow \end{pmatrix}. \quad (4.46)$$

The numerical values of the eigenvalues and eigenvectors are almost impossible to find analytically due to the large number of nodes, so it is more efficient to solve them numerically.

The order of the eigenvalues and eigenvectors in the columns can be chosen arbitrarily, as long as an eigenvalue appears with its associated eigenvector in the same columns in $\tilde{\mathbf{\Lambda}}$ and $\tilde{\mathbf{H}}$, respectively. For example, when λ_1 is switched with λ_2 in $\tilde{\mathbf{\Lambda}}$, $\boldsymbol{\eta}_1$ and $\boldsymbol{\eta}_2$ must be switched as well in $\tilde{\mathbf{H}}$ for Equation 4.44 to still hold. Here, the order is chosen so that λ_1 is the smallest eigenvalue, and $\lambda_1 < \lambda_2 < \dots < \lambda_n$. The order is chosen in this way because of the relationship between the eigenvalues and the characteristic time constants of the coil, being:

$$\tau_k = \frac{1}{\lambda_k}. \quad (4.47)$$

Since the first eigenvalue is the smallest, the time constant $\tau_1 = 1/\lambda_1$ is the largest of all time constants. The time constant matrix is then the inverse of the eigenvalue matrix:

$\tilde{\mathbf{A}}^{-1} = \tilde{\mathbf{\Theta}}$:

$$\tilde{\mathbf{\Theta}} = \begin{pmatrix} \tau_1 & 0 & \dots & 0 \\ 0 & \tau_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \tau_n \end{pmatrix}, \quad (4.48)$$

and Equation 4.44 becomes:

$$\mathbf{U}(t) = \tilde{\mathbf{H}} e^{-t \tilde{\mathbf{\Theta}}^{-1}} \tilde{\mathbf{H}}^{-1} \mathbf{U}_0. \quad (4.49)$$

Rewriting this equation as a sum yields:

$$\mathbf{U}(t) = \sum_{k=1}^n c_k e^{-t/\tau_k} \boldsymbol{\eta}_k, \quad (4.50)$$

with $\mathbf{c} = \tilde{\mathbf{H}}^{-1} \mathbf{U}_0$, where $\mathbf{c}$ is the vector containing the coefficients c_k .

Equation 4.50 is a key formula for understanding the time-dependent behaviour of non-insulated coils. It states that the voltage decay behaviour of a coil is the sum of eigenmodes $c_k \boldsymbol{\eta}_k$, which each decay exponentially with a characteristic time constant τ_k . In total, there are as many eigenmodes as there are nodes, which in the model are equivalent to turns. Each eigenmode corresponds to a linear combination of turns, of which the contribution of each turn in the eigenmode is given by $\boldsymbol{\eta}$. However, not all eigenmodes contribute equally to the voltage decay.

As illustration, the first six eigenmodes of the dry-wound single tape coil (DW-1) are shown in Figure 4.7, assuming a total transverse resistance of $270 \mu\Omega$ and an inductance matrix given by Figure 4.3. They are a snapshot of the transverse current redistribution at $t = 0$ after applying a current step of 100 A. As can be seen from this plot, the amplitude of each eigenmode becomes smaller. For the first eigenmode the amplitude range is to 40 A to 120 A, but for the sixth eigenmode it is already reduced to -3 A to 3 A. The characteristic time constant of the first eigenmode τ_1 is 26 s, and that of the sixth eigenmode τ_6 is 0.68 s. This means that the transverse current of the sixth eigenmode decays much faster than that of the first eigenmode. The combination of the smaller current amplitude and the smaller time constant means that the higher eigenmodes contribute less and less to the voltage decay.

In Figure 4.8, the current for the first six eigenmodes is plotted as a function of the radius of the coil. An interesting point is the following. For each individual eigenmode, current conservation does not need to hold. This allows the transverse current in eigenmode 1 to be, at some points, greater than the original 100 A. This does not imply that the local transverse current is greater than the initial 100 A, which would not be physical. In a way, this ‘excess’ current is compensated by the other eigenmodes. For example, eigenmode 2 has a negative transverse current towards the outer radius of the coil. Adding up all the eigenmodes, the total transverse current is found, which is a transverse current of 100 A throughout the coil (which is shown in the top-left figure in Figure 4.4). This must be the case, because the total current needs to be conserved.

In Figure 4.9, the characteristic time constants of the dry-wound single tape coil (DW-1) are plotted as a function of eigenmode number. The most dominant time constant is

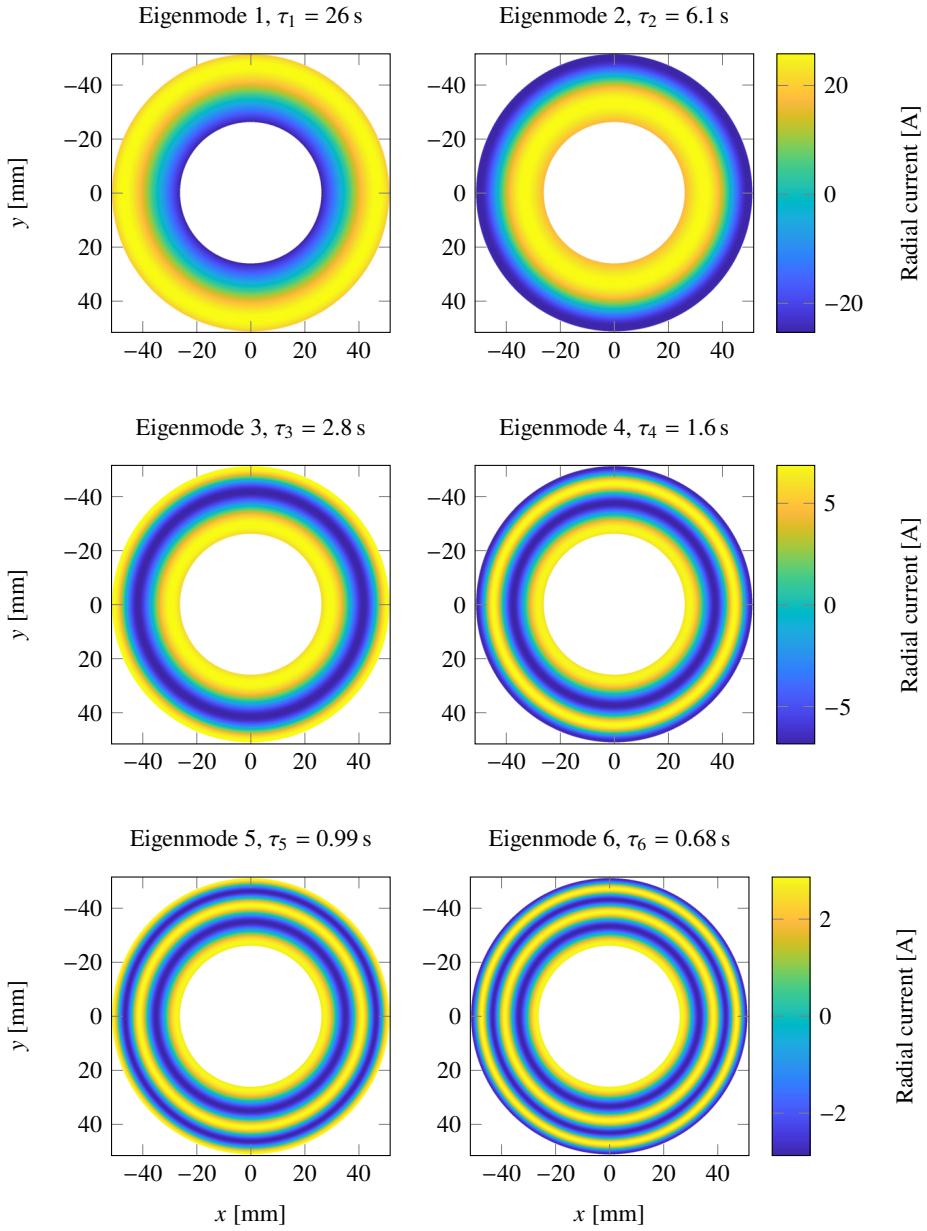


Figure 4.7: Current distribution of the first six eigenmodes of the dry-wound single tape coil (DW-1), with their corresponding time constants for a temperature of 50 K. Each sub-figure shows the transverse current distribution of an eigenmode at $t = 0$ for a current step of 100 A. The color scaling is different on each row to provide a better contrast of the current distribution.

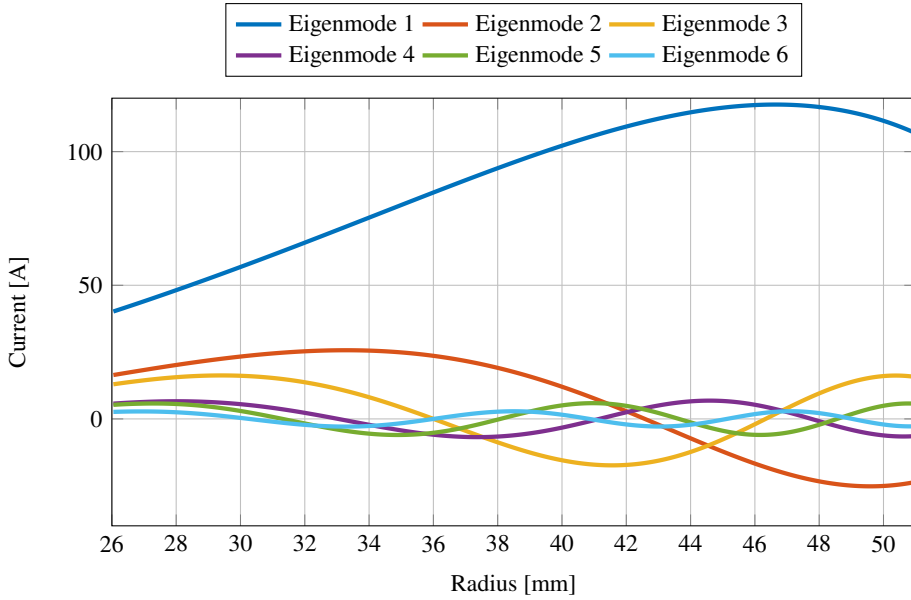


Figure 4.8: Current distribution of the first six eigenmodes of the dry-wound single tape coil (DW-1) plotted against coil radius at $t = 0$ for a current step of 100 A.

the first one. This time constant will be referred to as the *effective* time constant, as it has the greatest influence on the overall behaviour of the voltage decay. It is not necessary to have so many characteristic time constants, and this can be simplified for large t . For large t , only the largest time constants still contribute to the voltage. Using fitting functions, a double exponent can be matched well to the data, and Equation 4.50 can be approximated by:

$$U(t) \approx c_1 e^{-t/\tau_1} \eta_1 + c_2 e^{-t/\tau_2} \eta_2. \quad (4.51)$$

From the spectral analysis, it can be seen that this is approximately correct when all the higher modes have decayed sufficiently, i.e. when $t \gg \tau_3$.

The spectral eigendecomposition of the solution allows for a physical understanding which turns are responsible for the decay time of a non-insulated coil. In the next section, strategies to reduce the time constant are explored using the distributed network model introduced in Section 4.2 and analysed on their effectiveness using the spectral decomposition method.

4.6 Reducing the charging time

For an accelerator magnet, it is of interest to reduce the charging time, which is due to the delay of the current settling into the superconducting path. There are three possible ways to reduce the charging time. The first method is to increase the turn-to-turn resistance, either locally or throughout, so that the current is forced into the superconductor. Another possibility is to decrease the self-inductance of the superconducting path, so that

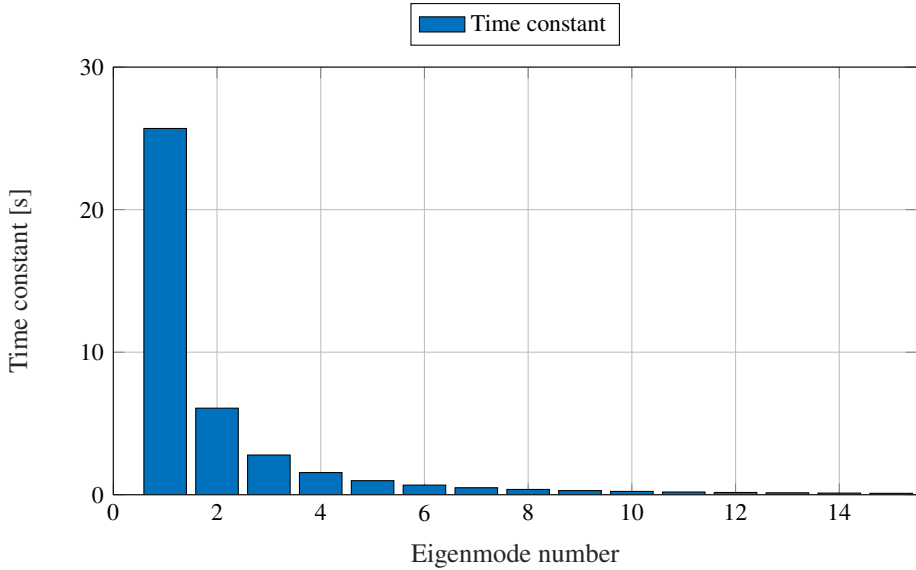


Figure 4.9: The first fifteen characteristic time constants of the dry-wound single tape coil (DW-1) as a function of eigenmode number for a temperature of 50 K. The first time constant is the most dominating, and is referred to as the effective time constant.

the current can go easier into the superconducting path. Lastly, an overshoot method can be applied, where an excess current is applied so that more current settles in the superconductor in a shorter time. In this section, the effectiveness of these methods is investigated using the network model and the first eigenmode found from the spectral analysis.

4.6.1 Changing the inter-turn resistance

In order to reduce the effective time constant, the most obvious solution is to change the transverse resistance throughout the coil. If the transverse resistance is doubled at every turn, all the characteristic time constants are halved. This can be easily seen by replacing $\bar{\mathbf{R}}$ in Equation 4.23 by $2\bar{\mathbf{R}}$, and going through the same steps from Equation 4.44 to Equation 4.49, which yields a new time constant matrix of $\frac{1}{2}\bar{\Theta}$.

However, it is sometimes not possible to increase the transverse resistance everywhere as this has a negative impact on the quench protection properties of non-insulated coils. Another strategy to consider is to increase the inter-turn resistance between two turns of the coil-pack, effectively dividing the coil into two segments. At the location of the added resistance, the current is forced into the superconductor, as it cannot flow as easily radially through the resistance. Figure 4.10 shows the effect on the effective time constant of the dry-wound single tape coil (DW-1). The most effective location of this resistance is around turn 234 of the coil i.e. at 87% of the coil thickness. However, the time constant is reduced by 0.6% only. Therefore, splitting the coil in two segments is not an effective option for reducing the time constant.

From a physics point of view, this can be explained by the current distribution of the

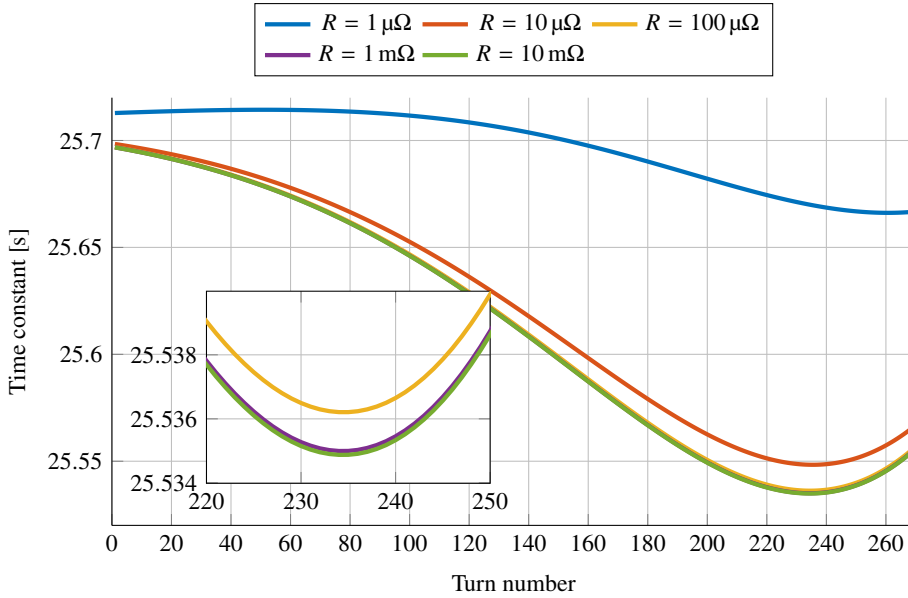


Figure 4.10: Time constant versus position in the coil windings i.e. turn number. The plot shows the influence on the time constant when increasing the inter-turn resistance between two turns.

first eigenmode (see Figures 4.7 and 4.8). In the first eigenmode, the outer turns contribute almost thrice as much as the inner turns, because the current magnitude of the first eigenmode is higher at the outside than at the inside. It is most efficient to place a resistance near the outer turns to decrease the time constant. However, as all turns contribute to the first eigenmode, the effective time constant is not significantly reduced by placing a single resistance. Thus, adding a single resistance between two turns is not sufficient to reduce the charging delay.

To significantly reduce the time constant, it can be considered to add a resistor between every k -th turn, so that more turns are influenced. The effect of this on the effective time constant of the dry-wound single tape coil (DW-1) is shown in Figure 4.11. The effective time constant drops to zero for high resistance at every turn, which is due to the fact that every turn is basically insulated now and there is no possibility for the current to flow anywhere radially through the coil windings. For high resistances, by adding a resistance every k -th turn, the effective time constant is reduced by $1/k$. For example, adding a resistance every other turn reduces the effective time constant by about $1/2$ (50%), and adding a resistance every five turns reduces the effective time constant by about $1/5$ (20%). Therefore, to decrease the effective time constant by increasing the resistance, it is needed to increase it throughout the coil windings and not at individual locations.

4.6.2 Changing the self-inductance

In addition to the resistance, the time constants also depend on the self-inductance. For a fixed coil geometry, the only way to change the self-inductance is to wind several tapes

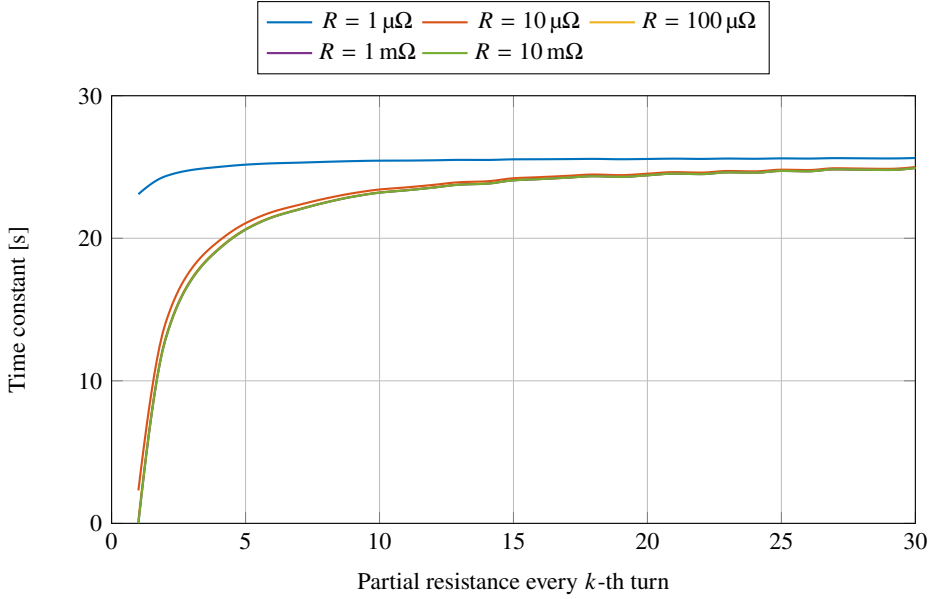


Figure 4.11: Time constant versus placement of a resistance every k -th turn. The plot shows the influence on the effective time constant by placement of a resistance between every k -th turn, ranging from $1 \mu\Omega$ to $10 \text{ m}\Omega$.

in parallel. The total self-inductance of the coil is inversely proportional to the number of tapes in parallel squared. This can be easily seen by considering that the total electromagnetic energy of the coil, which is:

$$U = \frac{1}{2}LI^2, \quad (4.52)$$

remains the same for the same magnetic field if the coil is wound with single or with parallel tapes. As the amount of ampere-turns also remains the same ($nI = \text{constant}$) to create the same magnetic field, the self-inductance of the coil with n_{pt} tapes in parallel L_{pt} is related to the self-inductance of a geometrically identical coil wound with single tape (L_{st}) by:

$$L_{\text{pt}} = \frac{L_{\text{st}}}{n_{\text{pt}}^2}. \quad (4.53)$$

For two tapes in parallel, the total self-inductance compared to a coil wound with single tape is divided by four, for three tapes in parallel, the self-inductance is divided by nine, etc. In matrix notation, it works a little bit differently than for doubling the resistance: one cannot replace $\tilde{\mathbf{L}}$ by $\frac{1}{4}\tilde{\mathbf{L}}$ in Equation 4.23. Instead, it is the size of the induction matrix that gets quartered if two tapes are in parallel (see Figure 4.3). The entries of the resistance matrix are found by replacing R_k by $R_{2k-1} + R_{2k}$. In this way, each node corresponds to two superconducting tapes in parallel. The disadvantage of adding several conductors in parallel is that the heat in-leak is higher, because the operating current must also be multiplied by the number of parallel windings to obtain the same magnetic field strength.

4.6.3 Overshoot method

To reduce the charging delay of a non-insulated coil, active current control can be applied. This was performed by Kim et al. [136], who applied this method to a non-insulated magnet consisting a stack of 13 non-insulated double pancake coils. The results demonstrated a reduction of the charging delay by over 2000 times. Also, a fully linear $\mathbf{B}(t)$ ramp was obtained with the control. This is relevant for accelerator magnets of which the magnetic field needs to be tunable so that the required field profile over time is achieved.

A less sophisticated version of active current control is an overshoot method [115]. In an overshoot, the current is ramped up to a higher current than is required to reach a given magnetic field. Once this magnetic field is reached, the current is reduced to the nominal current. This way, advantage is taken of the fact that more current settles in the superconductor when an initial higher current is provided to the non-insulated coil. To demonstrate the effect of an overshoot current on the time constants, an overshoot experiment with non-insulated coil S-1 has been performed. In Figure 4.12, an overshoot of 760 A applied to coil S-1 at 50 K is shown. In this way, the charging time is reduced by a factor of five.

The behaviour of the radial current is more dynamic than in the case of a single step. When the current is lowered to 300 A in a step down, some sections show a positive voltage and others negative voltages. This means that in some sections the radial current is flowing in the opposite direction to the other sections. From the eigenmodes shown in Fig 4.7, this can be physically interpreted as that the higher order eigenmodes have current distributions which are negative in some locations. With each step in current, all the eigenmodes are excited. For a single step, this cannot not lead to reverse currents in the coil, but for multiple current steps, the cumulative sum of all the eigenmodes leads to the existence of currents flowing in the opposite direction to the current supplied by the power supply in the coil. The characteristic time for these current redistributions to die out is the effective time constant, which follows from Equation 4.23.

The overshoot method can thus reduce the charging time. However, this comes with current redistributions within the coil. This would also be the case when an active control system is used. Magnetic field compensation techniques are then required to compensate for the field errors caused by these redistribution currents.

4.7 Conclusion

A new network model has been described to model the time-dependent behaviour of a non-insulated coil. In the network model, each individual turn is described with a node, of which its topology consists of an inductor and a resistor connected in parallel. The solution of this network can be solved analytically in the case that the current is below the critical current in each individual turn.

An analytical solution of the voltage and current behaviour in each individual turn has been derived for currents steps, linear ramp and alternating current. The solution can then be further decomposed using spectral decomposition. This shows that the voltage behaviour of a non-insulated coil is governed by a set of characteristic time constants. The largest of these time constants is the effective time constant, which dominantly determines the overall decay time of the voltage. This is an improvement from previous models, which

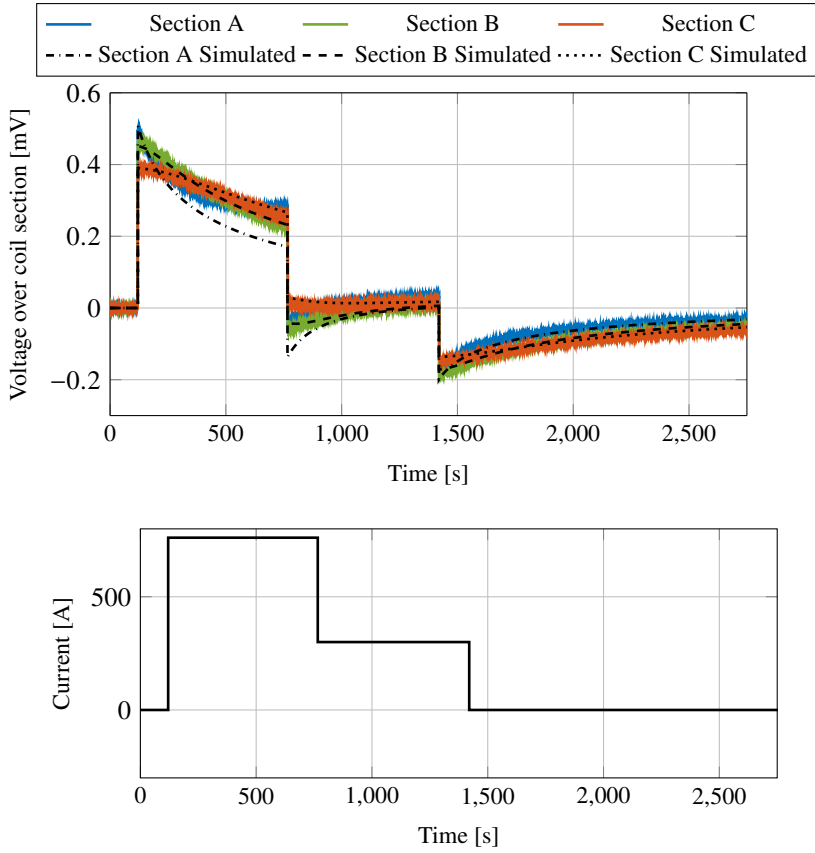


Figure 4.12: Voltage across coil section and current as function of time of the solder-filled single tape coil (S-1). The measured results are compared with simulated values in an overshoot scenario.

only described a single time constant. The behaviour of the voltage can be fitted by a double exponential, which is approximately equal to the analytical solution if $t \gg \tau_3$.

Using the network model and spectral analysis, several strategies for reducing the charging time have been discussed. Insulating a single turn does not lead to a significant reduction in the effective time constant. Alternately, adding a high resistance between each k -th turn can reduce the effective time constant by a factor of up to $1/k$. By winding n tapes in parallel, the charging delay gets reduced by a factor $1/n^2$. The overshoot method can be used to reach a magnetic field level faster, but within the coil there are always current redistributions, of which the decay time is governed by the effective time constant. In Chapter 6, the method to calculate time constants presented is used to estimate the charging delay of the cloverleaf-racetrack magnet.

Chapter 5

Cloverleaf coil-end geometry

ReBCO coated conductor always comes in the form of a flat tape, which has a large aspect ratio, that is, its width is much larger than its thickness. This poses design challenges to accelerator magnets using *ReBCO* tape, as the aspect ratio puts constraints on how the tape can be bent in three dimensions. In this chapter, a new design approach to three-dimensional coil layouts and coil-end geometries with tape conductor is proposed, which considers the tape's geometrical limitations. First, the different deformation modes of *ReBCO* tape and the requirements for coil-end design are addressed. The large aspect ratio of *ReBCO* tape allows it to be modelled as a thin strip surface, whose properties are well known from classical differential geometry. An overview of the thin strip model is outlined in this chapter. To prevent conductor degradation, new optimization criteria valid for three-dimensional geometries are presented, which are prevention of conductor creasing, minimization of overall bending energy, and prevention of over-straining the conductor. This will be applied to two 3D coil designs called the helix, which is used in *ReBCO* cables and solenoidal magnets, and the canted-cosine-theta, which can be used to create magnets with multipoles of any order. For the design of the coil-ends, a new design method using Bézier splines is proposed, which allows for much greater design flexibility than previous methods. Two examples of coil-end geometries generated with Bézier splines are presented: the cloverleaf and cosine-theta¹.

¹Part of this chapter has been published in T. H. Nes et al 2022 Supercond. Sci. Technol. 35 105011.

5.1 Introduction

Generally, bending of the conductor is required in the construction of coils. In magnets with an aperture, the free bore cannot be obstructed by the coil-end. In this case, one can use common coil type of magnets, which are double racetrack coils covering the two magnet apertures [137], or use 3D coil configurations to let the conductor pass over the aperture, such as cloverleaf or saddle coils in a cosine-theta dipole [138] and the MQXF quadrupole [139]. With tape conductors, a 3D configuration creates additional challenges, as thin tapes are very difficult to bend over their thin edges, which is referred to as hard-way or edge-wise bending [140]. This issue must be taken into account when designing the coil geometry. In this chapter, a mathematical description of the geometry of coils made with *ReBCO* tape conductor is presented, by taking into account its geometric constraints and bending modes. An image with the different bending modes of *ReBCO* is shown in Figure 5.1.

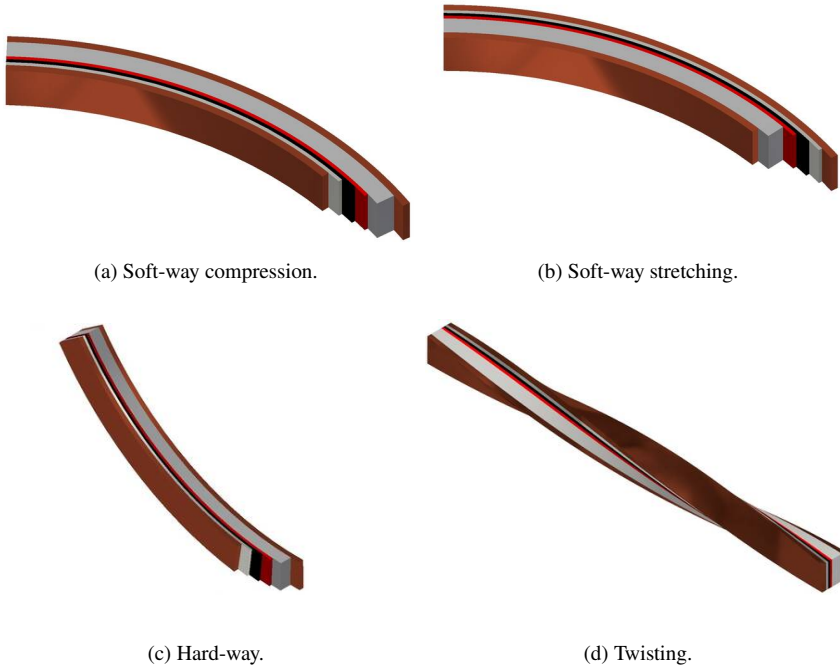


Figure 5.1: Deformation modes of *ReBCO* tape: soft-way bending with the *ReBCO* in compression (a) and stretch (b), hard-way bending (c) and twisting (d) which can be applied to *ReBCO* tape.

For this purpose, differential geometry is applied in the coil design. Previously, authors described the conductor geometry of rectangular conductors, in this case, Rutherford cables, using a mathematical model, referred to as the thin strip model [141, 142]. The assumption made in this model is that the conductor can be described as a thin strip surface with zero thickness (infinitely thin) and that the conductor is not bent over its thin edges. The model is also applicable to tape conductor, and here it will be used to describe the tape geometry.

For tape conductors, there are, however, additional requirements. In rectangular cables, some edge-wise bending is still possible due to their finite thickness. Tape conductor is much thinner than rectangular cable (around 0.1 mm for tape [29], 1.2 mm to 1.5 mm for common Nb₃Sn Rutherford cable [143]), and thus can tolerate much less hard-way bending. Therefore, a different minimization of energy is needed to describe the natural shape of the tape. Another aspect is that overbending of the tape can lead to critical current degradation. In *Re*BCO tapes, critical current degradation occurs due to excessive compressive or tensile strain [29]. The strain distribution in *Re*BCO tapes has been determined by Wang et al. for the helical and canted-cosine-theta (CCT) dipole and quadrupole configurations [121]. However, no general relation applicable to any geometry has been investigated yet. Here, a general approach is shown, which is applicable to any configuration that can be described by a parametrizable curve of which the curvature and torsion can be determined.

To put the optimization criteria of the coil design into context, they will be used in two geometries frequently used in superconducting coil and cable design called helix and CCT. Helical *Re*BCO windings are used to generate a solenoidal magnetic field in layer-wound solenoids [144, 145], to generate an alternating magnetic field in helical undulators to produce strong radiation in a narrow wavelength range [146], and in superconducting cables such as conductor on round wire (CORC) [147] or high temperature superconductor (HTS) power cables [148]. CCT is a winding configuration that can generate multipole magnetic fields for particle beam steering applications. It was originally proposed in 1969 [149] but has recently become increasingly attractive for various magnet applications, such as dipole magnets for particle accelerators [150], magnets for MRI [151], as well as higher-order magnets, such as quadrupole [152] and combined function magnets [153].

An important issue in the geometric design of the coils is the shape of the coil-end. In an accelerator magnet, the coil-end goes over the particle beam pipe. Previous design methods of the coil-end used polynomial expressions in the description of the shape [141]. However, the polynomial expression requires additional twisting to match the straight section with the coil-end, which leads to hard-way bending. Here, it will be shown that by using Bézier curves instead of polynomials, this mismatch can be avoided. To demonstrate the versatility of Bézier curves, two examples of accelerator magnet coil-end geometries generated with Bézier splines are shown: a cloverleaf coil-end and a cosine-theta coil-end.

5.2 Thin strip model

In order to model the surface of a *Re*BCO tape, it can be approximated as a strip of zero thickness. This allows surface to be described by the language of differential geometry. This is a rich mathematical discipline that can be used to describe many complex topologies [154], and is the basic frame work of physical theories such as Einstein's theory of General Relativity [155]. This section will outline the relevant theory of differential geometry, which is needed to model thin strip surfaces which are subjected to bending and twisting.

5.2.1 Space curves

The basis of modelling strips geometrically starts from a space curve. This is a curve that follows the tangential direction of the strip. This section describes the relevant mathematical properties of space curves.

A space curve is a curve that ‘lives’ in a three-dimensional Euclidean space $\mathbb{R}^3$, as opposed to a plane curve, which lives in $\mathbb{R}^2$. It is a very general form of curve. For the purpose of modelling strip surfaces, only real differentiable curves are of interest. A parametrized differentiable curve can be defined as a differentiable map $\phi : I \rightarrow \mathbb{R}^3$ of an open interval $I = (a, b) \subset \mathbb{R}$ into $\mathbb{R}^3$ [141, 156]. The mapping of a space curve is illustrated in Figure 5.2.

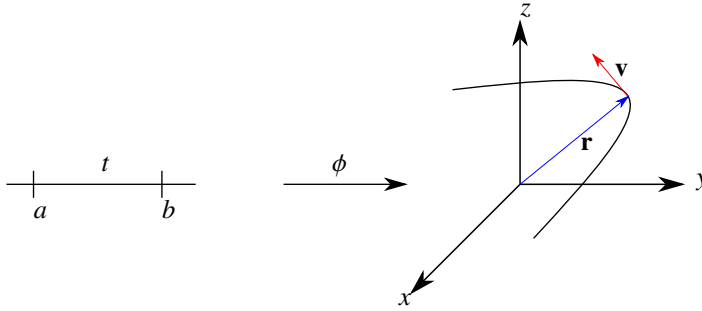


Figure 5.2: Mapping of a space curve $\phi : I \rightarrow \mathbb{R}^3$, adaptation from Russenschuck [141].

The space curve is an image of ϕ . The image set $\phi(I) \subset \mathbb{R}^3$ is called the trace of ϕ . The parametrized curve, which is a map, must be distinguished from its trace, which is a subset of $\mathbb{R}^3$. Curves that are parametrized differently, but have the same trace, are considered to be different. A curve is oriented by the direction it traces from a to b . It is positively oriented if it is traced from a to b , and negatively oriented if it is traced from b to a .

The trace of a space curve can be given by the locus of the position vector $\mathbf{r}(t)$, which is a smooth function of the real parameter t with a parameter space $I = [a, b] \subset \mathbb{R}$. Introducing a Cartesian coordinate system, $\mathbf{r}(t)$ can be written as:

$$\mathbf{r}(t) = x(t)\mathbf{e}_x + y(t)\mathbf{e}_y + z(t)\mathbf{e}_z. \quad (5.1)$$

If t changes in some interval I , the tangent of the space curve is the derivative $d\mathbf{r}/dt$ at $\mathbf{r}(t)$, which is:

$$\frac{d\mathbf{r}(t)}{dt} = \frac{dx}{dt}\mathbf{e}_x + \frac{dy}{dt}\mathbf{e}_y + \frac{dz}{dt}\mathbf{e}_z. \quad (5.2)$$

If the parameter t is interpreted as the time, then the tangent vector can be seen as the velocity $\mathbf{v}(t) = \dot{\mathbf{r}}(t)$, where the dot refers to the t -derivative. The tangent vector $\mathbf{T}(t)$ is the unit vector that is tangent to a curve (or surface) at any given point. It is defined as the unit vector in the direction of $\mathbf{v}(t)$:

$$\mathbf{T}(t) = \frac{\mathbf{v}(t)}{v(t)}, \quad (5.3)$$

with $v(t) = |\mathbf{v}(t)|$. A regular curve is defined as a curve which is differentiable and has a nonzero velocity for all $t \in I$. The curve is said to be of unit velocity if $v(t) = 1 \forall t$.

Going up to higher order derivatives, the acceleration is given by the second derivative of the curve: $\mathbf{a}(t) = \ddot{\mathbf{r}}(t)$, and the change of rate of the acceleration, which is sometimes referred to as jerk, is the third derivative of the curve with respect to time: $\dot{\mathbf{a}}(t) = \dddot{\mathbf{r}}(t)$.

The smoothness of a space curve is determined by the amount of continuous derivatives it possesses for all points along the curve. A space curve $\mathbf{r}(t)$ is considered to be of class C^n , if $\frac{d^n \mathbf{r}}{dt^n}$ exists and is continuous on I , where derivatives at the end-points are taken to be one sided derivatives (that is, at $t = 0$ from the right, and at $t = 1$ from the left).

A curve is said to be closed if $\mathbf{r}(a) = \mathbf{r}(b)$ on a closed interval $I = [a, b] \subset \mathbb{R}$. A closed curve is also referred to as a loop. A curve or loop is said to be simple if it does not intersect itself, that is, if $t_1, t_2 \in [a, b]$, $t_1 \neq t_2$, then $\phi(t_1) \neq \phi(t_2)$. Since *ReBCO* tape cannot intersect itself, only simple curves and loops can be considered.

Given $t_0 \in I$, the arc length of a space curve traced out by $\mathbf{r}$ from the point t_0 , is by definition:

$$s(t) = \int_{t_0}^t |\mathbf{v}(t)| dt. \quad (5.4)$$

Since differentiable curves are assumed ($v(t) \neq 0$), the arc length s is a differentiable function of t and $ds/dt = v(t)$. One can switch between parametrization with parameter t and arc-length parameter s using the following relation:

$$\frac{d}{ds} = \frac{d}{ds} \frac{dt}{dt} = \frac{dt}{ds} \frac{d}{dt} = \frac{1}{v} \frac{d}{dt}. \quad (5.5)$$

In the case of the tangent vector, it can be expressed in terms of arc-length as:

$$\mathbf{T}(s) = \frac{1}{v} \frac{d\mathbf{r}(t)}{dt} = \frac{d\mathbf{r}(s)}{ds}. \quad (5.6)$$

If the curve is of unit velocity, the curve is said to be arc-length parametrized with respect to parameter t , since $ds = dt$, and one can thus write the distance traced by the curve as:

$$s = \int_{t_0}^t dt = t - t_0. \quad (5.7)$$

Using unit-speed curves is advantageous in certain derivations, since it removes the velocity from the equations. In principle it is always possible to generate a unit-speed curve from a non-unit speed curve with the same trace by normalizing the velocity in Equation 5.4 and integrate it. However, it is often hard or impossible to find the anti-derivative, which makes the procedure of producing a unit-speed parametrization of a curve rather difficult in general. The whole theory of differential geometry applied to unit-speed curves is more theoretical than applicable in practice. Therefore, where possible, the general case of non-unit speed curves is considered in the following sections.

5.2.2 Frenet-Serret frame

A space curve only describes a line on the surface of the strip following the tangential direction. The conductor also has a width, and a normal perpendicular to the tape surface.

This section describes how to assign a frame to the space curve, which consists of the tangential vector $\mathbf{T}$, normal vector $\mathbf{N}$ and binormal vector $\mathbf{B}$ (see Figure 5.3). These vectors can then later be used to assign a normal direction pointing perpendicular to the tape surface and a binormal direction pointing in the direction of the tape width.

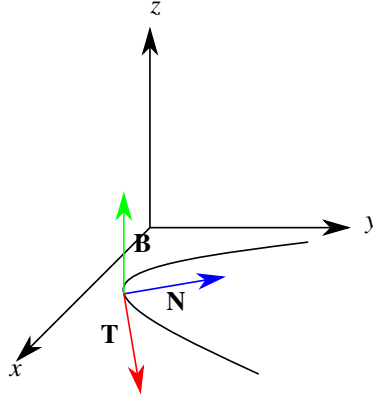


Figure 5.3: Space curve in the xy -plane with the Frenet-Serret frame.

Consider a regular arc-length parametrized curve. Since the tangent vector is a unit vector, it is normal to its s -derivative. This can be seen by considering the relation $\mathbf{T} \cdot \mathbf{T} = 1$, and differentiating this relation which gives $\mathbf{T} \cdot \mathbf{T}' = 0$. The prime refers to taking the s -derivative. By normalizing this vector, one can define the unit principle normal vector $\mathbf{N}$ as:

$$\mathbf{N} = \frac{\mathbf{T}'}{|\mathbf{T}'|}. \quad (5.8)$$

The absolute value of the vector $\mathbf{T}'$:

$$\kappa = |\mathbf{T}'|, \quad (5.9)$$

is called the curvature of the curve. One can interpret the curvature as the failure of the curve being a straight line. The reciprocal of the curvature $\rho = 1/\kappa$ is called the radius of curvature of the curve at the point $\mathbf{r}(s)$.

The vector:

$$\mathbf{B} = \mathbf{T} \times \mathbf{N}, \quad (5.10)$$

is called the unit binormal vector. Taking the s -derivative gives:

$$\mathbf{B}' = (\mathbf{T} \times \mathbf{N})' = \mathbf{T}' \times \mathbf{N} + \mathbf{T} \times \mathbf{N}' = \kappa \mathbf{N} \times \mathbf{N} + \mathbf{T} \times \mathbf{N}' = \mathbf{T} \times \mathbf{N}'. \quad (5.11)$$

As $\mathbf{B}$ is a unit vector, it is again normal to its s -derivative, and since it is also orthogonal to $\mathbf{T}$ (see Equation 5.11), a similar relation to Equation 5.8 holds:

$$\mathbf{N} = \frac{\mathbf{B}'}{|\mathbf{B}'|}. \quad (5.12)$$

The absolute value of the vector $\mathbf{B}'$:

$$\tau = -|\mathbf{B}'|, \quad (5.13)$$

is called the torsion of the curve. One can interpret the torsion a measurement of the failure of the curve to remain in-plane [157]. The minus-sign is there so that a positive torsion means an upward motion with respect to the plane. The reciprocal of the curvature $\sigma = 1/\tau$ is called the radius of torsion of the curve at the point $\mathbf{r}(s)$.

The vectors $\mathbf{T}$, $\mathbf{N}$ and $\mathbf{B}$, collectively called the Frenet-Serret frame [158, 159], form an orthonormal basis spanning $\mathbb{R}^3$ (see Figure 5.3).

Commonly, the plane spanned by $\mathbf{T}$ and $\mathbf{N}$ is referred to as the osculating plane, the span of $\mathbf{N}$ and $\mathbf{B}$ the normal plane and the span of $\mathbf{T}$ and $\mathbf{B}$ the rectifying plane (see Figure 5.4).

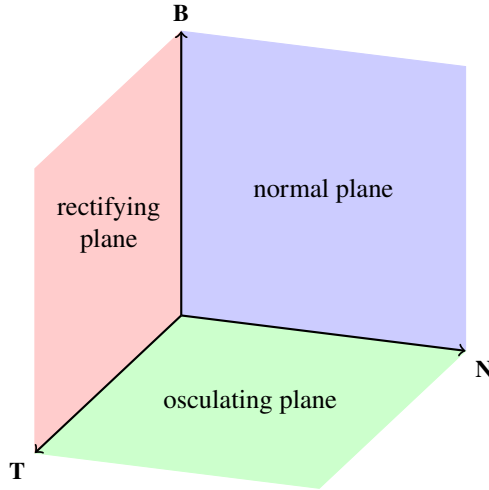


Figure 5.4: Nomenclature of the planes in the Frenet-Serret frame.

The relations between the Frenet-Serret frame at s and $s + ds$ are given by the Frenet-Serret equations:

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N}, \quad \frac{d\mathbf{N}}{ds} = \tau\mathbf{B} - \kappa\mathbf{T}, \quad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N}, \quad (5.14)$$

and in matrix form:

$$\begin{pmatrix} \mathbf{T}' \\ \mathbf{N}' \\ \mathbf{B}' \end{pmatrix} = \begin{pmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{pmatrix}. \quad (5.15)$$

The collection of $\mathbf{T}$, $\mathbf{N}$, $\mathbf{B}$, κ and τ is called the Frenet-Serret apparatus.

In terms of a space curve with parameter t , one can show that the Frenet-Serret frame is given by [141]:

$$\mathbf{T} = \frac{\mathbf{v}(t)}{|\mathbf{v}(t)|}, \quad \mathbf{N} = \mathbf{B} \times \mathbf{T}, \quad \mathbf{B} = \frac{\mathbf{v}(t) \times \mathbf{a}(t)}{|\mathbf{v}(t) \times \mathbf{a}(t)|}, \quad (5.16)$$

and the curvature and torsion are given by:

$$\kappa = \frac{|\mathbf{v}(t) \times \mathbf{a}(t)|}{|\mathbf{v}(t)|^3}, \quad \tau = \frac{(\mathbf{v}(t) \times \mathbf{a}(t)) \cdot \dot{\mathbf{a}}(t)}{|\mathbf{v}(t) \times \mathbf{a}(t)|^2}. \quad (5.17)$$

In matrix form the Frenet-Serret equations are:

$$\begin{pmatrix} \dot{\mathbf{T}} \\ \dot{\mathbf{N}} \\ \dot{\mathbf{B}} \end{pmatrix} = \begin{pmatrix} 0 & \nu\kappa & 0 \\ -\nu\kappa & 0 & \nu\tau \\ 0 & -\nu\tau & 0 \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{pmatrix}, \quad (5.18)$$

where an extra factor of ν is picked up from the chain rule (see Equation 5.5).

The fundamental theorem of curves states that if $\kappa \geq 0$ and τ are continuously differentiable over an open interval $I = (a, b) \subset \mathbb{R}$, there exists one unique curve, determined except for orientation and position in space, where κ and τ are its curvature and torsion respectively [160]. Two curves C_1 and C_2 with identical curvature and torsion but different locations in $\mathbb{R}^3$ are considered congruent and can be mapped onto each other by an rigid motion of $\mathbb{R}^3$. Thus, the uniqueness of the curve described by κ and τ implied by the fundamental theorem of curves means that all properties of the curve can be described in terms of κ and τ .

For modelling of the strip surface, the fundamental theorem of space curves is very useful. It guarantees that if the tape surface can be described by a single space curve, that the properties of curvature and torsion of the conductor surface are unique, and can be fully derived from the path vector $\mathbf{r}(t)$ of the space curve.

Rotation of space curves

Since the matrix in Equation 5.18 is skew symmetric ($\mathbf{A}^T = -\mathbf{A}$), the Frenet-Serret equations can be written as a cross product:

$$\begin{pmatrix} \dot{\mathbf{T}} \\ \dot{\mathbf{N}} \\ \dot{\mathbf{B}} \end{pmatrix} = \begin{pmatrix} 0 & \nu\kappa & 0 \\ -\nu\kappa & 0 & \nu\tau \\ 0 & -\nu\tau & 0 \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \nu\tau \\ 0 \\ \nu\kappa \end{pmatrix} \times \begin{pmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{pmatrix}. \quad (5.19)$$

The vector in front of the cross product in Equation 5.19 is the angular velocity vector of the Frenet-Serret frame [161]. Its direction determines the moving frame's momentary axis of motion (its centre) and its magnitude the angular speed. The angular velocity vector can thus be written as:

$$\boldsymbol{\omega} = \nu\tau\mathbf{T} + \nu\kappa\mathbf{B}. \quad (5.20)$$

The angular velocity vector is also known as the Darboux vector. The normalized Darboux vector is notated here as:

$$\hat{\mathbf{D}} = \frac{\tau\mathbf{T} + \kappa\mathbf{B}}{\sqrt{\kappa^2 + \tau^2}}. \quad (5.21)$$

For unit-speed curves, it gives another way to interpret the curvature κ and the torsion τ . Curvature is the measure of the rotation of the Frenet-Serret frame for unit-speed curves about the binormal unit vector, whereas torsion is the measure of the rotation of the Frenet-Serret frame for unit-speed curves about the tangent unit vector for unit speed curves [162]. For non-unit speed curves, an extra factor of ν is included in the rotation (see Figure 5.5).

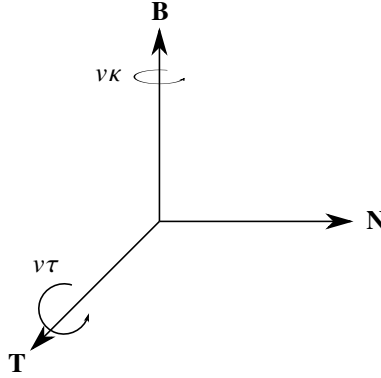


Figure 5.5: Rotation of the Frenet-Serret frame.

5.2.3 Differential geometry of strip surfaces

When modelling *ReBCO* tape as a thin strip, two fundamental properties have to be taken into account. One is that the width of the tape is fixed. The other is that due to its aspect ratio it is difficult to bend *ReBCO* tape over the thin side of the tape, the hard-way bend direction. A geometric model of the conductor must take these properties into account. Here, it is discussed how to model a thin strip using space curves and the Frenet-Serret frame. The thickness of the tape, as well as possible defects are neglected. These are discussed in Section 5.3.

By moving a line (called a generator) with variable length $g(t)$ and direction $\mathbf{g}(t)$ along a curve, one can trace out the surface $\mathbf{S}$ of a strip. A surface like this is called a ruled surface. It can be written down parametrically as:

$$\mathbf{S}(t, g) = \mathbf{c}(t) + g(t)\hat{\mathbf{g}}(t), \quad (5.22)$$

where $\mathbf{c}(t)$ is called the directrix (the base curve) of the representation, $g(t)$ the length of the generator and $\hat{\mathbf{g}}(t)$ the direction of the generator [157].

A developable surface is a special kind of ruled surface. It is a surface which can be created by transforming a plane, for instance by folding and bending, but not by stretching or compression. Conversely, a developable surface can be flattened on a plane without stretching or compressing it. Such a transformation is said to be isometric. Developable surfaces in $\mathbb{R}^3$ are always ruled surfaces, but not vice versa [157]. The properties of developable surfaces are also the properties of *ReBCO* tape, and hence developable surfaces can be used to model *ReBCO* tape.

As with space curves, strips also have an orthonormal frame associated with it. The tangent vector of the strip $\mathbf{t}$ points in the direction of the base curve, and the normal vector $\mathbf{n}$ is directed normal to the surface. The binormal vector $\mathbf{b}$ is orthonormal to both these vectors. The set $\{\mathbf{t}, \mathbf{n}, \mathbf{b}\}$ forms the frame for strips. This frame is called the Darboux frame [141]. Note that the Darboux frame does not necessarily equal the Frenet-Serret frame, but differs by a rotation α around the tangent vector. In the case $\alpha = 0$, the normal vector $\mathbf{n}$ and the binormal vector $\mathbf{b}$ of the strip are equal to the normal vector $\mathbf{N}$ and the binormal vector of the curve $\mathbf{b}$, respectively. In that case, the strip is free of any hard-way

bending and is called geodesic [156]. Note that the tangential vector of the strip $\mathbf{t}$ and of the curve $\mathbf{T}$ are always equal.

Darboux frame

Whereas the Frenet-Serret frame is used to study curves, the Darboux frame is used to study curves in a surface. Both are orthonormal frames where the tangent vector points in the direction of the curve. An additional condition for a curve in a surface is that the normal vector points normal to the surface. In order to accomplish this, the Frenet-Serret frame can be rotated with an angle α around the tangent vector:

$$\begin{pmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{pmatrix}. \quad (5.23)$$

Taking the derivative of the previous equation, and using its inverse and the Frenet-Serret equations, yields:

$$\begin{pmatrix} \mathbf{t}' \\ \mathbf{n}' \\ \mathbf{b}' \end{pmatrix} = \begin{pmatrix} 0 & \kappa \cos \alpha & -\kappa \sin \alpha \\ -\kappa \cos \alpha & 0 & \tau + \alpha' \\ \kappa \sin \alpha & -(\tau + \alpha') & 0 \end{pmatrix} \begin{pmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{pmatrix}. \quad (5.24)$$

Taking

$$\tau_r = \tau + \alpha', \quad \kappa_g = \kappa \sin \alpha, \quad \kappa_n = \kappa \cos \alpha, \quad (5.25)$$

the Darboux equations for strips are:

$$\frac{d\mathbf{t}}{ds} = \kappa_n \mathbf{n} - \kappa_g \mathbf{b}, \quad \frac{d\mathbf{n}}{ds} = \tau_r \mathbf{b} - \kappa_n \mathbf{t}, \quad \frac{d\mathbf{b}}{ds} = \kappa_g \mathbf{t} - \tau_r \mathbf{n}, \quad (5.26)$$

where τ_r the geodesic torsion κ_g is the geodesic curvature, and κ_n the normal curvature. The set $\{\mathbf{t}, \mathbf{n}, \mathbf{b}\}$ forms the Darboux frame with the tangent, normal and binormal unit vector of the strip (see Figure 5.6).

For a curve parametrized in t , the Darboux equations are:

$$\begin{pmatrix} \dot{\mathbf{t}} \\ \dot{\mathbf{n}} \\ \dot{\mathbf{b}} \end{pmatrix} = v \begin{pmatrix} 0 & \kappa_n & -\kappa_g \\ -\kappa_n & 0 & \tau_r \\ \kappa_g & -\tau_r & 0 \end{pmatrix} \begin{pmatrix} \mathbf{t} \\ \mathbf{n} \\ \mathbf{b} \end{pmatrix}. \quad (5.27)$$

The following equations connect the Frenet-Serret apparatus with the Darboux apparatus [141]:

$$\tau = \tau_r - \alpha', \quad (5.28)$$

$$\kappa = \sqrt{\kappa_n^2 + \kappa_g^2}, \quad (5.29)$$

$$\mathbf{T} = \mathbf{t}, \quad (5.30)$$

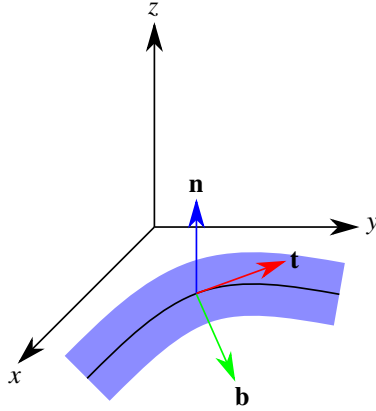


Figure 5.6: Non-geodesic strip in the xy -plane with the Darboux frame of the strip.

$$\mathbf{N} = \frac{1}{\kappa} \mathbf{T}' = \frac{\kappa_n \mathbf{n} - \kappa_g \mathbf{b}}{\sqrt{\kappa_n^2 + \kappa_g^2}}, \quad (5.31)$$

$$\mathbf{B} = \mathbf{T} \times \mathbf{N} = \frac{\kappa_n \mathbf{n} + \kappa_g \mathbf{b}}{\sqrt{\kappa_n^2 + \kappa_g^2}}. \quad (5.32)$$

In the case of a *ReBCO* strip, it takes much more energy to bend it in the so called hard-way direction, which is bending it around the normal direction of the strip than it is to bend in the soft-way direction, that is around the binormal vector of the strip. This is due to the large aspect ratio of *ReBCO* tape. Thus, for a physical strip, the geodesic curvature has to be zero, i.e. $\kappa_g = 0$. A strip with vanishing geodesic curvature is called geodesic. For a geodesic strip the normal and binormal vectors of the base curve and the strip are equal:

$$\mathbf{N} = \mathbf{n}, \quad \mathbf{B} = \mathbf{b}, \quad (5.33)$$

and the Darboux apparatus of the strips is equal to the Frenet-Serret apparatus of the base curve.

Rotation of strip surfaces

Similar to Equation 5.19, the Darboux equations can be written as:

$$\begin{pmatrix} \dot{\mathbf{T}} \\ \dot{\mathbf{N}} \\ \dot{\mathbf{B}} \end{pmatrix} = \begin{pmatrix} 0 & \nu \kappa_n & -\nu \kappa_g \\ -\nu \kappa_n & 0 & \nu \tau_r \\ \nu \kappa_g & -\nu \tau_r & 0 \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{pmatrix} = \begin{pmatrix} \nu \tau_r \\ \nu \kappa_g \\ \nu \kappa_n \end{pmatrix} \times \begin{pmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{pmatrix}. \quad (5.34)$$

The angular velocity vector is then:

$$\boldsymbol{\omega} = \nu \tau_r \mathbf{t} + \nu \kappa_g \mathbf{n} + \nu \kappa_n \mathbf{b}. \quad (5.35)$$

From this, one can interpret τ_r , κ_g and κ_n as the differential twist angles around $\mathbf{t}$, $\mathbf{n}$ and $\mathbf{b}$ respectively (see Figure 5.7).

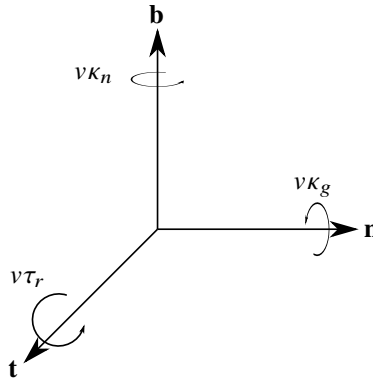


Figure 5.7: Rotation of the Darboux frame.

Note that the geodesic torsion came about by twisting the Frenet-Serret frame around the tangent vector. Since this leads to geodesic curvature, *ReBCO* tape cannot be twisted like this. However, *ReBCO* tape can be twisted in such a way that it adheres to the Frenet-Serret equations.

Principal and Gauss curvature

When dealing with surfaces, often reference is made to its Gauss curvature (for a rigorous description, see Do Carmo [156]). Given a point p on a surface S , one can draw an infinite number of curves C lying on S through p , coming from all directions. Depending on the orientation, the curves can have different normal curvatures. The maximum normal curvature κ_1 and the minimal normal curvature κ_2 of this set of normal curvatures are called the principal curvatures. The product of the two is called the Gaussian curvature:

$$K = \kappa_1 \kappa_2. \quad (5.36)$$

Three different surfaces with negative, zero and positive Gauss curvature are presented in Figure 5.8.

When a surface is transformed isometrically, its Gauss curvature is conserved everywhere. This property is known as Gauss's Theorema Egregium. A developable surface has zero Gauss curvature everywhere on the surface. Thus, a perfectly flat strip has zero Gaussian curvature, and when it is transformed isometrically, it will still have zero Gaussian curvature. Note that the converse that non-isometric transformations will lead to a change of Gaussian curvature does not hold. There exist non-isometric transformations which preserve the Gaussian curvature, for instance stretching a flat strip in the tangential direction.

When modelling *ReBCO* tape as a strip, it is assumed that the Gauss curvature of the *ReBCO* tape is zero everywhere. In practice the surface of a *ReBCO* tape is not perfectly flat, due to local variations in thickness of the copper coating of *ReBCO*, as well as dog boning effects, and hence, its Gaussian curvature is non-zero locally.

Closely related to Gauss's Theorem Egregium is the following statement. For a ruled

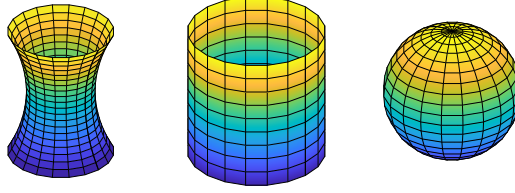


Figure 5.8: Example of Gauss curvature of three objects, the hyperboloid on the left has a negative Gaussian curvature, the cylinder in the middle has zero Gaussian curvature, and the sphere on the right has positive Gauss curvature.

surface $\mathbf{S}$ described by Equation 5.22 to be a developable surface, the following relation must hold [157, 162]:

$$\det(\dot{\mathbf{c}}, \mathbf{g}, \dot{\mathbf{g}}) = 0. \quad (5.37)$$

This relation will be used to find an expression for the generators of the surface.

Developable strips

Since for a geodesic strip the Frenet-Serret Frame is equal to the Darboux frame, the generator lies in the plane spanned by $\mathbf{T}$ and $\mathbf{B}$, i.e. a linear combination of $\mathbf{T}$ and $\mathbf{B}$:

$$\mathbf{g}(t) = \zeta \mathbf{T} + \xi \mathbf{B}, \quad (5.38)$$

Differentiating yields:

$$\dot{\mathbf{g}}(t) = \dot{\zeta} \mathbf{T} + \zeta \dot{\mathbf{T}} + \dot{\xi} \mathbf{B} + \xi \dot{\mathbf{B}} \quad (5.39)$$

$$= \dot{\zeta} \mathbf{T} + \zeta \kappa \mathbf{N} + \dot{\xi} \mathbf{B} - \xi \tau \mathbf{N}. \quad (5.40)$$

From Equation 5.37, the surface is developable when the following determinant is zero:

$$\det \begin{pmatrix} 1 & \zeta & \dot{\zeta} \\ 0 & 0 & \zeta \kappa - \xi \tau \\ 0 & \xi & \dot{\xi} \end{pmatrix} = -\xi(\zeta \kappa - \xi \tau) = 0. \quad (5.41)$$

Note that this statement is non other than stating that the Gauss curvature vanishes everywhere [162]. The determinant is zero when $\zeta = c\tau$ and $\xi = c\kappa$, where c is a constant. The generator is then:

$$\mathbf{g}(t) = c(\tau \mathbf{T} + \kappa \mathbf{B}). \quad (5.42)$$

The part between brackets is exactly the Darboux vector. Normalizing this vector yields the direction of the generator:

$$\hat{\mathbf{g}}(t) = \hat{\mathbf{D}}(t). \quad (5.43)$$

The direction of the generator is thus given by the direction of the Darboux vector. If the strip is developed from a base curve $\mathbf{r}(t)$ and generator direction $\hat{\mathbf{D}}(t)$, the parametric representation is:

$$\mathbf{S}(t, g) = \mathbf{r}(t) + g(t)\hat{\mathbf{D}}(t). \quad (5.44)$$

The generator length is not constant, but must have a specific length such that the width of tape remains the same everywhere along the curve. The magnitude of the generator length can be determined by comparing the two congruent triangles in Figure 5.9. From this, it follows that:

$$g(t) = w\sqrt{1 + \eta^2}, \quad (5.45)$$

with

$$\eta = \frac{\tau}{\kappa}. \quad (5.46)$$

A strip surface, which can be described by Equation 5.44, is a developable surface.

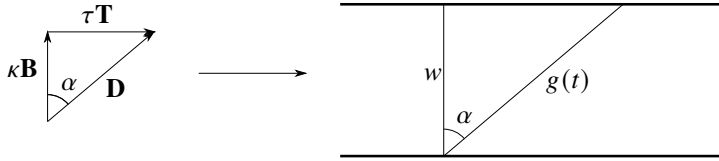


Figure 5.9: Determination of the generator length. On the left is the triangle formed by the Darboux vector and its Frenet-Serret components. On the right is drawn the tape surface with tape width w and generator length $g(t)$.

In order to make a drawing of the surface, one must draw the edges of the surface. This can be done by drawing the generators and connecting the top and the bottom of successive generators, which will form the perimeter $\mathbf{P}(t)$ of the tape:

$$\mathbf{P}(t) = \mathbf{r}(t) \pm \frac{1}{2}g(t)\hat{\mathbf{D}}(t). \quad (5.47)$$

Under isometric transformation, the length of the perimeter will not change. This fact is referred to as the *constant perimeter condition* [163]. By comparing the lengths of the two perimeters, it can be determined if the strip is geodesic. For a geodesic strip, that is, a strip without hard-way bending, the perimeters must be of equal length. The geometry of ReBCO tape can thus be modelled as a developable (geodesic) strip which adheres to the constant perimeter condition.

The development of a strip surface can be seen in the discrete setting depicted in Figure 5.10, where a strip is developed using a finite number of generators. From this it can

be seen that the generators can be interpreted as the lines over which the surface is folded. This naturally connects with the geometric fact that the generators are lying in the direction of the Darboux vector, which describes the rotation of the curve. The larger the number of generators used, the smoother the developed strip appears.

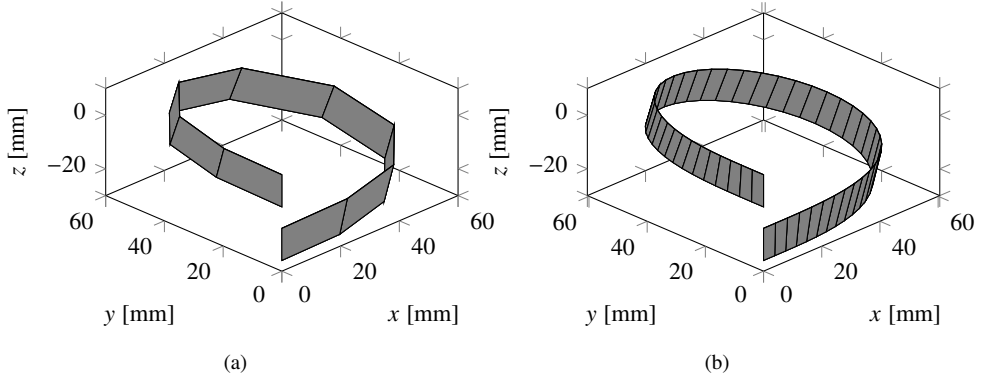


Figure 5.10: Strip development using (a) 10 generators, and (b) 50 generators.

For a real physical strip, such as *ReBCO*, the strip is transformed continuously, and not in a discrete setting. This is the same as saying that a real strip is bent over an infinite number of generators. When using a reflective strip, one can discern the generators by the reflections of the light on the strip (see Figure 5.11). Naturally, when modelling, it is impossible to use an infinite number of generators, so a discrete number must be chosen such that the modelled tape is sufficiently smooth in order to make models and drawings within the required accuracy for electromagnetic field computations and CAD modelling.

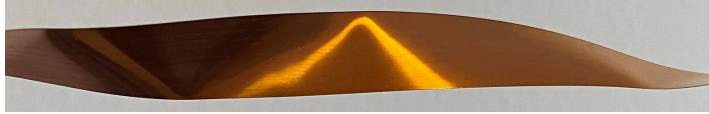


Figure 5.11: Bending of a copper strip. In the reflection of the light and shadows on the copper strip, one can trace out the generators.

5.2.4 Summary of the thin strip model

The steps to model *ReBCO* tape geometries can be summarized as follows:

- The first step is to generate a base curve $\mathbf{r}(t)$ which follows the path of the strip;
- Then, its derivatives $\mathbf{v}$, $\mathbf{a}$, and $\dot{\mathbf{a}}$ are calculated;
- From this, the Frenet-Serret apparatus $\{\mathbf{T}, \mathbf{N}, \mathbf{B}, \kappa, \tau\}$ (Equations 5.16 and 5.17) is calculated;
- Then the normalized Darboux vector $\hat{\mathbf{D}}(t) = (\tau\mathbf{T} + \kappa\mathbf{B})/\sqrt{\kappa^2 + \tau^2}$ and the generator length $g(t) = w\sqrt{1 + \eta^2}$ with $\eta = \tau/\kappa$ are calculated;

- The two perimeters of the surface can then be fully described with $\mathbf{P}(t) = \mathbf{r}(t) \pm \frac{1}{2}g(t)\hat{\mathbf{D}}(t)$.

The steps outlined above will give a strip surface conforming to the given base curve which does not possess any hard-way bending and has a fixed width, which are two requirements imposed by the geometry of *ReBCO* tape.

5.3 Optimization criteria for tape conductor

So far, the *ReBCO* tape has been modelled as an infinitely thin strip. In reality, the tape itself has a finite thickness of $50\mu\text{m}$ to $150\mu\text{m}$, and the coil winding pack has a finite thickness as well. In this section, the validity of the thin strip model for a thick *ReBCO* tape is discussed, and investigated if the thin strip model can be applied to model the geometry of the full coil winding pack.

In practice, the *ReBCO* tape is never perfectly flat due to dog-boning, where there is more copper on the corners on the tape due to the electroplating process. This effect is illustrated in Figure 5.12.



Figure 5.12: Illustration of a practical *ReBCO* tape with geometrical defects such as surface roughness and dog-boning.

5.3.1 Minimum bending energy state

The natural shape of a twisted and bent conductor is the one where its bending energy is minimized. Therefore, an expression of the bending energy is needed such that it can be minimized. For the case of a non-geodesic strip, Wunderlich [164] derived a mathematical expression stating that the total bending energy U can be expressed as a function of normal curvature and torsion:

$$U = \frac{D_w}{2} \int_0^L g(\kappa, \eta, \eta') ds, \quad (5.48)$$

with flexible rigidity

$$D = \frac{Yd^3}{12(1 - \nu^2)}, \quad (5.49)$$

where Y is Young's modulus and ν Poisson's ratio, and:

$$g(\kappa, \eta, \eta') = \kappa^2 (1 + \eta^2)^2 \frac{1}{w\eta'} \ln \left(\frac{2 + w\eta'}{2 - w\eta'} \right). \quad (5.50)$$

Equation 5.48 states that the bending energy is proportional to the thickness of the tape cubed. If one was to bend a *ReBCO* tape of 12 mm wide and 0.1 mm thick over its wide side, it would cost $(12/0.1)^3 = 1.7 \cdot 10^6$ times more energy to bend compared to bending over its thin side.

5.3.2 Conductor creasing

In terms of rulers, a local high density of generators means a high bending energy density, and a low generator density a corresponding low bending energy density. Therefore, when two rulers are close to crossing, there is locally an area with a high local bending energy [165]. This can lead to mechanically weak spots in the *ReBCO*. Also, the generator lines must not cross, as this would imply a locally infinite bending energy density and a real physical object cannot be deformed in such a way. With a real physical conductor, trying so would lead instead to creases, damaging it in the process. Thus, when designing a coil geometry, a condition must be made so that in the model the generator lines do not cross. The case where the generators cross is also referred to as edge of regression [141, 142]. Therefore, there is a need of an expression stating when the generators cross each other, presented next.

To find the conditions in which this occurs, the following case can be considered (see Figure 5.13). Two generators at $\mathbf{r}(t)$ and $\mathbf{r}(t + \Delta t)$, are separated by a distance Δs . From the point $\mathbf{r}(t)$, the perpendicular distance to the perimeter is $w/2$, and the length of the generator at $\mathbf{r}(t)$ to one edge of the strip is $\frac{1}{2}w\sqrt{1 + \eta^2}$. By Pythagoras, the projection 2 of the generators on the baseline is then $w\eta/2$. For the generators not to overlap, the following relation must hold:

$$\Delta s + \frac{w}{2}\eta(t) > \frac{w}{2}\eta(t + \Delta t). \quad (5.51)$$

As $\Delta s = \nu\Delta t$, this can be rewritten as:

$$\nu > \frac{w}{2} \frac{(\eta(t + \Delta t) - \eta(t))}{\Delta t}. \quad (5.52)$$

Writing this expression in differential notation yields the required condition to prevent generator crossing:

$$\frac{w}{2\nu} |\dot{\eta}| < 1. \quad (5.53)$$

The absolute value is taken here to take into consideration the case that the derivative of η is negative, which is the mirrored version of the presented case. The term $w|\dot{\eta}|/(2\nu)$ will be referred to as the regression. From Equation 5.53, it can be seen that in the case of a very wide strip ($w \rightarrow \infty$), that η must go to zero to satisfy the relation. This means that it is difficult to twist a strip with a large width without tearing it apart.

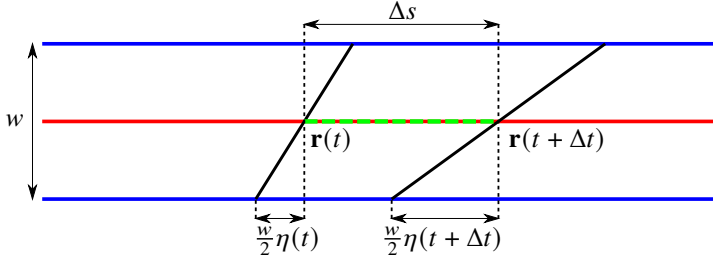


Figure 5.13: Determination of edge of regression. Two generators are located at positions $\mathbf{r}(t)$ and $\mathbf{r}(t + \Delta t)$, with a distance of Δs between them. If no edge of regression is to occur, the generators must not overlap.

If the baseline is chosen at the perimeter of the strip, the condition changes to $w|\dot{\eta}|/v < 1$, because now the full width of the strip must be considered, not the half-width. This has the consequence that the derivative of η can only be half as big compared to a baseline chosen in the centre of the strip for the generators not to cross. Therefore, it is best to choose the baseline in the centre of the strip instead of the perimeter, as this maximizes the allowable $\dot{\eta}$.

5.3.3 Strain in the *ReBCO* layer

Besides high local bending energy, *ReBCO* can also be degraded by overstraining. For the two specific cases of the helical and CCT dipole and quadrupole configurations, the strain distribution in *ReBCO* tapes has been determined by Wang et al. [121]. Here, a general expression for the strain in the *ReBCO* layer itself under arbitrary bending and twisting is derived. The most common form of bending is planar bending, i.e. bending along the width of the tape without any torsion present. From Figure 5.14, it can be derived that the strain ϵ is a function of distance from the neutral axis and the local curvature of the neutral axis [166]:

$$\epsilon = \frac{y}{R}, \quad (5.54)$$

where y is the distance from the neutral axis, and R is the radius of curvature. When the *ReBCO* layer is not on the neutral axis, the layer is subjected to strain, which can lead to a degradation of the critical current of the tape.

For a tape that has besides curvature also torsion, one can use the following reasoning to derive the strain. The generators can be interpreted as the folding lines around which the strip is folded; locally each generator can be seen as if the strip is bent around a cylinder. Using the angular velocity relation $v_{\perp} = \omega \cdot R$, the bending radius can be written as

$$R = \frac{v_{\perp}}{\omega} = \frac{v \cos(\alpha)}{v \sqrt{\kappa^2 + \tau^2}}. \quad (5.55)$$

Using that $\cos(\alpha) = \kappa / \sqrt{\kappa^2 + \tau^2}$ (see Figure 5.9) yields the generalized bending radius:

$$R = \frac{\kappa}{\kappa^2 + \tau^2}. \quad (5.56)$$

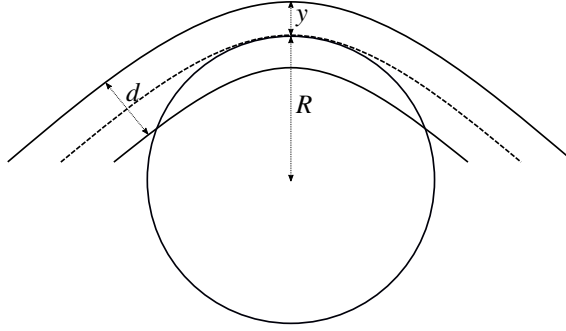


Figure 5.14: Schematic of the bending of a strip with thickness d . The neutral axis is indicated with the dashed line in the middle of the tape, and the distance from the neutral axis is indicated with y . The bending of the tape can be thought of a local bending over a circle with bending radius R .

Note that for zero torsion the generalized bending radius becomes $R = 1/\kappa$, the planar bending case. Filling this into Equation 5.54 yields the relation between the strain and the curvature and torsion:

$$\epsilon = y \frac{\kappa^2 + \tau^2}{\kappa}. \quad (5.57)$$

Van der Laan showed that the strain is anisotropic in the ab -plane [167]. It is therefore of importance to distinguish the strain along the length of the tape and the strain along the width of the tape. In terms of curvature and torsion, the strain along the tangential direction is:

$$\epsilon_T = \epsilon \cos(\alpha) = y \sqrt{\kappa^2 + \tau^2}, \quad (5.58)$$

and the strain along the binormal direction is:

$$\epsilon_B = \epsilon \sin(\alpha) = y \frac{\tau}{\kappa} \sqrt{\kappa^2 + \tau^2}. \quad (5.59)$$

Comparison with measured data of the anisotropic strain dependence of the *Re*BCO conductor in combination with Equations 5.58 and 5.59 can then be used to optimize the coil design.

Of interest is that if the *Re*BCO layer is on the neutral axis, the strain it experienced will be minimized the most. One possibility to achieve this is by soldering a copper strip onto the *Re*BCO conductor [168]. Another way to shift the *Re*BCO layer towards the neutral axis is to decrease the substrate thickness [169]. Symmetric Tape Round (STAR) wires are 1.6 mm to 1.9 mm in diameter and are made from winding 2.5 mm wide *Re*BCO tapes on a 1.02 mm diameter copper former. This small bending radius of the STAR wire is achieved by using tape with a specific architecture where the thickness of the copper stabilizer is matched to the thickness of the Hastelloy substrate, such that the *Re*BCO layer is placed close to the neutral axis [170].

Placing the *Re*BCO layer on the neutral axis does not mean that the tape can then be bent with an infinitely small radius. The *Re*BCO layer has a finite thickness, and when the middle of the layer lies on the neutral axis, the edges of the *Re*BCO layer are not, and thus experience strain under bending that can lead to degradation of the tape.

5.4 Extension to coils and conductor with a finite thickness

The thin strip model eliminates all hard-way bending from the surface of the strip. In this section it is explored if the addition of a finite thickness to the strip leads to a hard-way bend component.

Consider a curve $\mathbf{r}(t)$ to be the neutral axis of the tape. One can create a parallel curve $\mathbf{r}^*(t)$, which is offset by a distance λ in the normal direction (see Figure 5.15). Since the

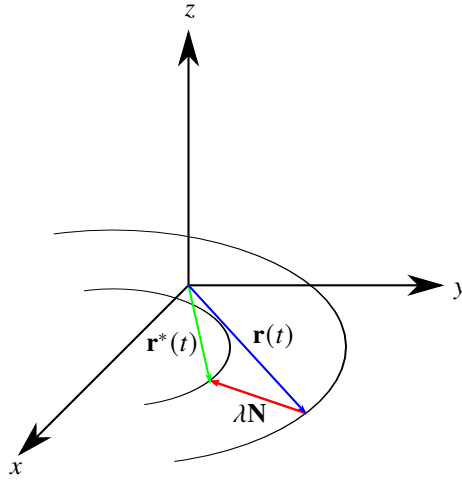


Figure 5.15: Drawing of a parallel curve $\mathbf{r}^*$ by offsetting the curve $\mathbf{r}$ in the normal direction $\mathbf{N}$.

surface of the tape can be developed by applying the Frenet-Serret equations to a base curve, the base curve of the parallel curve $\mathbf{r}^*(t)$ must then have the same normal vector $\mathbf{N}$ as the original curve $\mathbf{r}(t)$ for there to be no gap between the broad sides of the two tapes.

The parallel curve can be parametrized as:

$$\mathbf{r}^*(t) = \mathbf{r}(t) + \lambda \mathbf{N}(t). \quad (5.60)$$

Differentiating with respect to t yields:

$$\frac{d\mathbf{r}^*(t)}{dt} = \frac{d\mathbf{r}(t)}{dt} + \lambda \frac{d\mathbf{N}(t)}{dt}, \quad (5.61)$$

which is in terms of velocity:

$$\begin{aligned} \mathbf{v}^*(t) &= \mathbf{T}v + \lambda v(-\kappa_n \mathbf{T} + \tau \mathbf{B}) \\ &= v((1 - \lambda \kappa_n) \mathbf{T} + \lambda \tau \mathbf{B}). \end{aligned} \quad (5.62)$$

The magnitude of the velocity of the parallel curve is:

$$v^* = v \sqrt{(1 - \lambda \kappa)^2 + \lambda^2 \tau^2}. \quad (5.63)$$

The unit tangent vector of the parallel frame is:

$$\mathbf{t}^* = \frac{\mathbf{v}^*}{v^*} = \frac{(1 - \lambda\kappa)\mathbf{T} + \lambda\tau\mathbf{B}}{\sqrt{(1 - \lambda\kappa_n)^2 + \lambda^2\tau^2}}. \quad (5.64)$$

The normal vector of the parallel strip must be equal to the normal vector of the base curve:

$$\mathbf{n}^* = \mathbf{N}. \quad (5.65)$$

The unit binormal vector is then:

$$\mathbf{b}^* = \mathbf{t}^* \times \mathbf{n}^* = \frac{(1 - \lambda\kappa_n)\mathbf{B} - \lambda\tau\mathbf{T}}{\sqrt{(1 - \lambda\kappa_n)^2 + \lambda^2\tau^2}}. \quad (5.66)$$

The set $\{\mathbf{t}^*, \mathbf{n}^*, \mathbf{b}^*\}$ forms the Darboux frame of the parallel curve. In matrix form:

$$\begin{pmatrix} \mathbf{t}^* \\ \mathbf{n}^* \\ \mathbf{b}^* \end{pmatrix} = \begin{pmatrix} \frac{1 - \lambda\kappa_n}{\sqrt{(1 - \lambda\kappa)^2 + \lambda^2\tau^2}} & 0 & \frac{\lambda\tau}{\sqrt{(1 - \lambda\kappa)^2 + \lambda^2\tau^2}} \\ 0 & 1 & 0 \\ -\frac{\lambda\tau}{\sqrt{(1 - \lambda\kappa)^2 + \lambda^2\tau^2}} & 0 & \frac{1 - \lambda\kappa_n}{\sqrt{(1 - \lambda\kappa)^2 + \lambda^2\tau^2}} \end{pmatrix} \begin{pmatrix} \mathbf{T} \\ \mathbf{N} \\ \mathbf{B} \end{pmatrix}. \quad (5.67)$$

Taking the s -derivative yields:

$$\begin{pmatrix} \mathbf{t}^* \\ \mathbf{n}^* \\ \mathbf{b}^* \end{pmatrix}' = \begin{pmatrix} 0 & \frac{\kappa + \lambda(\kappa^2 + \tau^2)}{\sqrt{(1 - \lambda\kappa)^2 + \lambda^2\tau^2}} & \frac{\lambda(\lambda\kappa'\tau + (1 - \lambda\kappa)\tau')}{(1 - \lambda\kappa)^2 + \lambda^2\tau^2} \\ -\frac{\kappa + \lambda(\kappa^2 + \tau^2)}{\sqrt{(1 - \lambda\kappa)^2 + \lambda^2\tau^2}} & 0 & \frac{\tau}{\sqrt{(1 - \lambda\kappa)^2 + \lambda^2\tau^2}} \\ -\frac{\lambda(\lambda\kappa'\tau + (1 - \lambda\kappa)\tau')}{(1 - \lambda\kappa)^2 + \lambda^2\tau^2} & -\frac{\tau}{\sqrt{(1 - \lambda\kappa)^2 + \lambda^2\tau^2}} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{t}^* \\ \mathbf{n}^* \\ \mathbf{b}^* \end{pmatrix}. \quad (5.68)$$

This gives the rotation of the Darboux frame between s and $s + \Delta s$, but it is more useful to have an expression between s^* and $s^* + \Delta s^*$. Using:

$$\frac{d}{ds} = \frac{d}{ds} \frac{ds^*}{ds} \frac{dt}{dt} = \frac{d}{ds} \frac{v^*}{v} = \frac{d}{ds^*} \sqrt{(1 - \lambda\kappa)^2 + \lambda^2\tau^2}, \quad (5.69)$$

yields:

$$\frac{d}{ds^*} \begin{pmatrix} \mathbf{t}^* \\ \mathbf{n}^* \\ \mathbf{b}^* \end{pmatrix} = \begin{pmatrix} 0 & \frac{\kappa + \lambda(\kappa^2 + \tau^2)}{(1 - \lambda\kappa)^2 + \lambda^2\tau^2} & \frac{\lambda(\lambda\kappa'\tau + (1 - \lambda\kappa)\tau')}{((1 - \lambda\kappa)^2 + \lambda^2\tau^2)^{3/2}} \\ -\frac{\kappa + \lambda(\kappa^2 + \tau^2)}{(1 - \lambda\kappa)^2 + \lambda^2\tau^2} & 0 & \frac{\tau}{(1 - \lambda\kappa)^2 + \lambda^2\tau^2} \\ -\frac{\lambda(\lambda\kappa'\tau + (1 - \lambda\kappa)\tau')}{((1 - \lambda\kappa)^2 + \lambda^2\tau^2)^{3/2}} & -\frac{\tau}{(1 - \lambda\kappa)^2 + \lambda^2\tau^2} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{t}^* \\ \mathbf{n}^* \\ \mathbf{b}^* \end{pmatrix}. \quad (5.70)$$

In terms of parameter t and comparison with Equation 5.27 gives:

$$\begin{pmatrix} \dot{\mathbf{t}}^* \\ \dot{\mathbf{n}}^* \\ \dot{\mathbf{b}}^* \end{pmatrix} = v^* \begin{pmatrix} 0 & \kappa_n^* & -\kappa_g^* \\ -\kappa_n^* & 0 & \tau_r^* \\ \kappa_g^* & -\tau_r^* & 0 \end{pmatrix} \begin{pmatrix} \mathbf{t}^* \\ \mathbf{n}^* \\ \mathbf{b}^* \end{pmatrix}, \quad (5.71)$$

where the parallel relative torsion, geodesic curvature and normal curvature are:

$$\tau_r^* = \frac{\tau}{(1 - \lambda\kappa)^2 + \lambda^2\tau^2}, \quad (5.72)$$

$$\kappa_g^* = -\frac{\lambda(\lambda\dot{\kappa}\tau + (1 - \lambda\kappa)\dot{\tau})}{v((1 - \lambda\kappa)^2 + \lambda^2\tau^2)^{3/2}}, \quad (5.73)$$

$$\kappa_n^* = \frac{\kappa + \lambda(\kappa^2 + \tau^2)}{(1 - \lambda\kappa)^2 + \lambda^2\tau^2}, \quad (5.74)$$

respectively.

Equation 5.73 implies that for a general curve, some hard-way bending is present in the tape away from the neutral axis. Of interest are the geometries where no hard-way bending is present. In this case κ_g^* must be zero. This is when the numerator in Equation 5.73 equals zero. Omitting the case where $\lambda = 0$ (the curves coincide), the following relation must hold:

$$\lambda\dot{\kappa}\tau + (1 - \lambda\kappa)\dot{\tau} = 0. \quad (5.75)$$

The set of curves that satisfy this equation have common principle normals and are called Bertrand curves [156, 157]. One solution is that of planar curves ($\tau = 0$). Another solution are circular helices, which have constant torsion and curvature ($\dot{\kappa} = \dot{\tau} = 0$). The last possible solution is a skew circle, which has a constant curvature of $\kappa_c = 1/\lambda$ and non-zero torsion. This solution is not relevant for coil design, since it only holds for one particular λ .

For a strip of finite thickness, Equation 5.73 implies that under twisting, some hard-way bending is present. This means that if the *ReBCO* layer is offset from the neutral axis, it will be subjected to hard-way bending. The magnitude of this hard-way bending depends on the magnitude of the curvature and twist, and the first derivatives, and the thickness of the strip in a non-linear fashion. The thin strip approximation is thus only valid for thin strips which are not subjected to large changes in curvature and torsion. Also the derivation of the strain presented in Section 5.3.3 is not valid in this case, as it did not take into account strain due to hard-way bending.

Equation 5.73 also has consequences for coil-ends made out of *ReBCO* tape, or rectangular conductor, which is limited in the hard-way bending direction for that matter. The only hard-way free coil-ends made with rectangular conductor where the broad sides do not have a gap between them (excluding geometric defects) are made out of planar curves. An example of a planar coil-end is a racetrack coil. This is, of course, a very trivial result. More interesting is the fact that there exist no twisted coil-ends, which are hard-way bend free and where the broad sides of the tape are in full contact. When the second layer is wound on top of the first one, it cannot follow the exact shape. In the case of an accelerator dipole magnet, torsion is needed in the coil-ends to go over the particle beam pipe, thus no dipole coil-end can be designed which has the broad sides of the tapes in full contact.

5.4.1 Modelling of the tape thickness and stack

In this section it is discussed how to model the tape with thickness and the full coil winding pack from a strip surface. Whereas a section of a strip surface can be modelled with 4 nodes, the volume of the *ReBCO* tape can be modelled with 8 nodes. The location of the nodes $\mathbf{V}(t)$ is given by:

$$\mathbf{V}(t) = \mathbf{P}(t) \pm \mathbf{O}(t), \quad (5.76)$$

where $\mathbf{P}(t)$ is the perimeter (Equation 5.47) of the conductor and the vector $\mathbf{O}(t)$ an offset vector to give the tape its required thickness. As the tape is thin, it is natural to choose the normal vector $\mathbf{N}(t)$ of the surface as the offset vector. The location of the nodes can then be described with:

$$\mathbf{V}(t) = \mathbf{r}(t) \pm \frac{1}{2}g(t)\hat{\mathbf{D}}(t) \pm \frac{d}{2}\mathbf{N}(t), \quad (5.77)$$

where $\mathbf{r}(t)$ is the base curve, $g(t)$ the generator length, $\hat{\mathbf{D}}$ the normalized Darboux vector and d the thickness of the tape. In Figure 5.16, it is shown how to create the thickness of the coil-pack from a strip surface.



Figure 5.16: Drawing of the full coil-pack from the first turn, which is modelled as an infinitely thin strip. On the left a strip is shown, where an offset vector $\mathbf{O}(t)$ is depicted by the green arrows. It points in the direction of the coil-pack thickness. In the figure on the right, the coil-winding pack is shown.

Since the adjacent *ReBCO* tapes cannot be placed on top of the previous tape in the normal direction without hard-way bending, the exact shape of the coil-pack when wound cannot be calculated analytically, but has to be determined by experiment or numerical inquiries. The most precise results can be obtained by modelling each turn individually, but this is quite laborious. A less intensive method is to define some average offset vector $\mathbf{O}(t)$ that points in the direction in which the tapes stack, but ignores the gap between the individual tapes. A practical way to find this offset vector is by winding a coil-pack on a former where the shape of the first turn is created using Bézier splines. By measuring the shape of the coil-pack, the offset vector can then be determined.

5.5 Applications to coil and cable design

In this section, two case studies are presented of geometrical shapes for which *ReBCO* tape can be used, the helix and the canted-cosine-theta configuration.

5.5.1 Helix

Helical *ReBCO* windings are used to generate solenoidal magnetic fields in layer-wound solenoids [144, 145], to generate an alternating magnetic field in helical undulators to

produce strong radiation in a narrow wavelength range [146], and in cables such as CORC [171] and HTS power cables [172–174]. In Figure 5.17, a helix is shown.

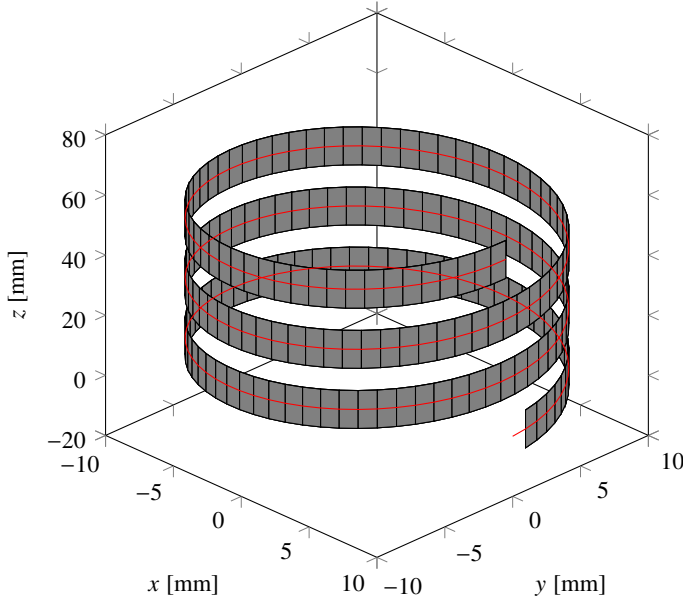


Figure 5.17: Three turns of a 12 mm wide strip wound in a helical shape with a radius $R = 10$ mm and pitch $p = 20$ mm.

A helical curve can be parametrized as:

$$\mathbf{r} = R \cos(t)\mathbf{e}_x + R \sin(t)\mathbf{e}_y + \frac{p}{2\pi}t\mathbf{e}_z, \quad (5.78)$$

where R is the radius of the cylinder around which the tape is wound and p is the pitch length. Setting $q = p/2\pi$, the Frenet-Serret apparatus can then be found using Equation 5.16:

$$\mathbf{T} = \frac{\mathbf{v}}{v} = \frac{-R \sin(t)\mathbf{e}_x + R \cos(t)\mathbf{e}_y + q\mathbf{e}_z}{\sqrt{R^2 + q^2}}, \quad (5.79)$$

$$\mathbf{B} = \frac{\mathbf{v} \times \mathbf{a}}{|\mathbf{v} \times \mathbf{a}|} = \frac{q \sin(t)\mathbf{e}_x - q \cos(t)\mathbf{e}_y + R\mathbf{e}_z}{\sqrt{R^2 + q^2}}, \quad (5.80)$$

$$\mathbf{N} = \mathbf{B} \times \mathbf{T} = -\cos(t)\mathbf{e}_x - \sin(t)\mathbf{e}_y, \quad (5.81)$$

$$\kappa = \frac{|\mathbf{v} \times \mathbf{a}|}{v^3} = \frac{R}{q^2 + R^2}, \quad (5.82)$$

$$\tau = \frac{\mathbf{v} \times \mathbf{a} \cdot \dot{\mathbf{a}}}{|\mathbf{v} \times \mathbf{a}|^2} = \frac{q}{q^2 + R^2}. \quad (5.83)$$

The angular velocity vector can be found using Equation 5.20:

$$\boldsymbol{\omega} = \nu \kappa \mathbf{B} + \nu \tau \mathbf{T} = \frac{\nu}{\sqrt{q^2 + R^2}} \mathbf{e}_z = \mathbf{e}_z. \quad (5.84)$$

The generators of the helix are thus all pointing in the z -direction, that is along the length of the surface of the cylinder. In the case of the helical parametrization of Equation 5.78, the magnitude of the angular velocity vector is unity.

Strain in helices

To find the maximal strain in the *ReBCO* layer, Equation 5.57 can be used. This yields:

$$\epsilon = y \frac{\kappa^2 + \tau^2}{\kappa} = \frac{y}{R}. \quad (5.85)$$

The maximum strain is thus the same as for a tape wound around a cylinder with radius R . The tangential (axial) component of the strain experienced by the *ReBCO* layer is then:

$$\epsilon_{\mathbf{T}} = y \sqrt{\kappa^2 + \tau^2} = \frac{y}{\sqrt{q^2 + R^2}}, \quad (5.86)$$

and the binormal (transverse) component:

$$\epsilon_{\mathbf{B}} = y \tau \frac{\sqrt{\kappa^2 + \tau^2}}{\kappa} = \frac{y}{R} \frac{q}{\sqrt{q^2 + R^2}}. \quad (5.87)$$

This is equivalent to the transverse and axial strains found by Wang [121], who expressed the strain in terms of the winding angle θ . The twitch pitch thus influences the relative amount of axial and transverse strain experienced by the *ReBCO* layer. The maximum amount of strain remains inversely proportional to the radius R of the cylinder around it is wound.

Another method to find the axial strain in helical windings is to look at the change of length of the helical path:

$$\epsilon_{\Delta L} = \frac{L_2 - L_1}{L_1}, \quad (5.88)$$

where L_1 is the original length of the tape and L_2 the length after deformation [43, 175]. Taking $L_1 = 2\pi\sqrt{R^2 + q^2}$ and $L_2 = 2\pi\sqrt{(R + y)^2 + q^2}$, the length of one revolution of a helix with radius R and $R + y$ respectively, one finds for the strain:

$$\epsilon_{\Delta L} = \frac{L_2 - L_1}{L_1} = \sqrt{\frac{(R + y)^2 + q^2}{R^2 + q^2}} - 1. \quad (5.89)$$

Series expansion of this equation for $y \rightarrow \infty$ yields:

$$\epsilon_{\Delta L} \approx \frac{y}{\sqrt{q^2 + R^2}}. \quad (5.90)$$

For large y , Equations 5.86 and 5.89 are equivalent. In the case of *ReBCO* however, the distance from the neutral axis is generally very small, i.e. $y \ll 1$, and thus Equation 5.89 is not valid.

Parallel tapes in a helical configuration

Here, the equations for the parallel frame are verified with the helical geometry. When winding multiple layers of a helix, the successive layers are placed on the outside of each other. Thus, the the stacks grows in the negative normal direction. For the parallel frame it is expected that for a tape placed a tape thickness d away Equations 5.78 to 5.84 are still valid, with the radius R substituted by $R + d$. Substituting $\lambda = -d$ into Equation 5.64 and the values for κ and τ yields:

$$\mathbf{T}^* = \frac{-(R + d) \sin(t) \mathbf{e}_x + (R + d) \cos(t) \mathbf{e}_y + q \mathbf{e}_z}{\sqrt{(R + d)^2 + q^2}}. \quad (5.91)$$

In the same manner one can find an expression for the parallel binormal vector using Equation 5.66:

$$\mathbf{B}^* = \frac{q \sin(t) \mathbf{e}_x - q \cos(t) \mathbf{e}_y + (R + d) \mathbf{e}_z}{\sqrt{(R + d)^2 + q^2}}. \quad (5.92)$$

The parallel torsion and curvature can be found using Equations 5.72 and 5.74:

$$\tau^* = \frac{q}{q^2 + (R + d)^2}, \quad (5.93)$$

$$\kappa^* = \frac{R + d}{q^2 + (R + d)^2}. \quad (5.94)$$

Note that all the equations for the parallel frame are offset by d in radius, as expected.

5.5.2 Canted-cosine-theta

The canted-cosine-theta (CCT) is a winding configuration that can generate multi-pole magnetic fields for particle beam steering applications. It was originally proposed by Meyer and Flasck in 1969 [149], but has recently been gaining traction for various magnet applications, such as dipole magnets for particle accelerators [176–178], medical magnets [151, 179] and electrical machines [180, 181], as well as higher order magnets, such as quadrupoles [152] and combined function magnets [153].

The base curve of a CCT coil is given by [182]:

$$\mathbf{r}(t) = R \cos(t) \mathbf{e}_x + R \sin(t) \mathbf{e}_y + \left(\frac{R \cot(\alpha)}{n} \sin(nt) + qt \right) \mathbf{e}_z, \quad (5.95)$$

where R is the radius of the cylinder around the CCT is wound, α the tilt angle, q the pitch, and n the order of the CCT ($n = 1$ is a dipole, $n = 2$ is a quadrupole etc). The pitch q can be written as:

$$q = \frac{d_1 + d_2}{2\pi \sin(\alpha)}, \quad (5.96)$$

where d_1 is the width of the tape and d_2 the gap between the two tapes in a CCT winding.

The curvature and torsion of a CCT winding can be found using Equation 5.17, and are:

$$\kappa = \frac{R\sqrt{R^2 + (q + R \cot(\alpha) \cos(nt))^2 + n^2 R^2 \cot(\alpha)^2 \sin(nt)^2}}{(R^2 + (q + R \cot(\alpha) \cos(nt))^2)^{3/2}}, \quad (5.97)$$

and:

$$\tau = \frac{q + (1 - n^2) \cot(\alpha) R \cos(nt)}{R^2 + (q + R \cot(\alpha) \cos(nt))^2 + n^2 R^2 \cot(\alpha)^2 \sin(nt)^2}, \quad (5.98)$$

respectively.

Following the procedure summarized in Section 5.2.4, the strip surface of a CCT winding where the centre of the strip follows the base curve given by Equation 5.95 can be developed. The developed strip surface is shown in Figure 5.18, where for the first four multi-poles five single tape turns are shown.

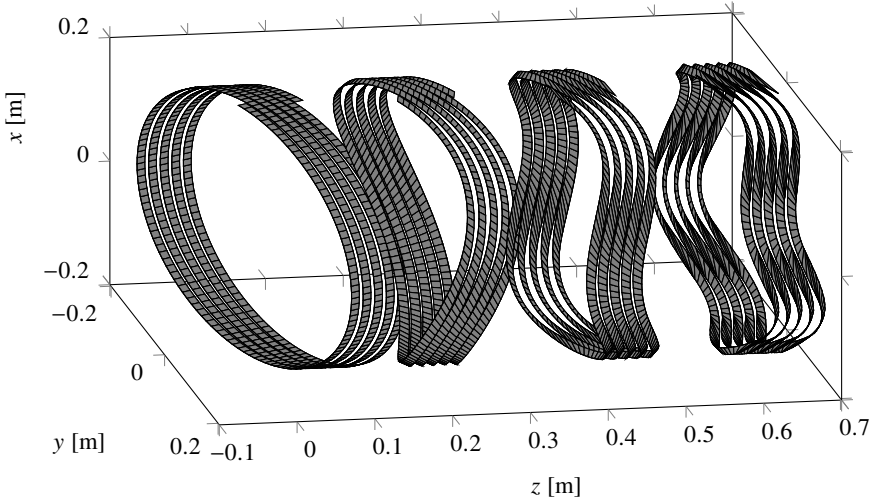


Figure 5.18: From left to right, five single tape turns of a CCT dipole, quadrupole, sextupole and octopole are shown, with a radius $R = 200$ mm, angle $\alpha = 70^\circ$, strip width $d_1 = 12$ mm and a gap between the tapes $d_2 = 2$ mm.

When designing a CCT configuration with rectangular conductor like *ReBCO*, it is of importance that the edge of regression condition is not violated, or buckling of the tape and a deviation from the imposed base curve is possible, leading to possible magnetic field errors. Using that $v(t) = \sqrt{R^2 + (q + \cot(\alpha) \cos(nt))^2}$, one can verify with the edge of regression condition $d_1 |j| / (2v) < 1$ if the designed configuration is possible. As an example of this, three versions of sextupole CCT windings using tape with a width of 12 mm are shown in Figure 5.19. They have a radius of 100 mm, a gap between the tapes of 1 mm, and have angles α of 20° , 40° and 60° degrees, respectively. In Figure 5.19a, it can be seen that for the case $\alpha = 20^\circ$ and $\alpha = 40^\circ$, some artefacts in the form of spikes in

the modelling occur. This is due to the generators overlapping each other at these points. From Figure 5.19b, it can be seen that for these angles the regression exceeds 1 at those points. It is physically not possible to create a winding with these parameters. If one desires to wind a 12 mm sextupole with a 1 mm gap at these angles, one can increase the radius R , which lowers the regression. For $\alpha = 60^\circ$, the regression remains below 1, and the generators do not overlap.

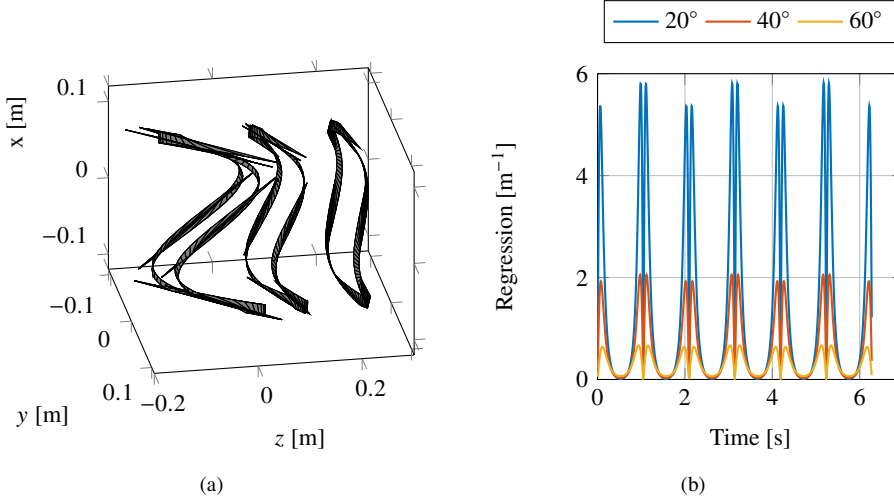


Figure 5.19: In (a), from left to right three CCT sextupole turns with angles α of 20° , 40° and 60° . with radius $R = 100$ mm, width $d_1 = 12$ mm, gap between turns $d_2 = 1$ mm. In (b), the regression versus time is plotted. For the 20° and 40° cases, edge of regression is violated.

An expansion to the straight CCT is a curved version. To model the shape with rectangular conductor, it is necessary to know the exact base curve in order to use the thin strip model. For a curved CCT, one can apply a rotation of the the base curve of a straight CCT with a radius r_1 around the origin, which yields:

$$\mathbf{r}(t) = \begin{pmatrix} (r_1 + R \cos(t)) \cos(qt) - \frac{1}{n} R \cot(\alpha) \sin(nt) \sin(qt) \\ R \sin(t) \\ (r_1 + R \cos(t)) \sin(qt) + \frac{1}{n} R \cot(\alpha) \sin(nt) \cos(qt) \end{pmatrix}, \quad (5.99)$$

where r_1 is the radius of curvature of the curved CCT. In Figure 5.20, three examples of curved CCT coils made with a single geodesic tape are shown.

5.6 Coil-end geometries for *ReBCO* accelerator magnets

In order to make accurate drawings of the coil-ends, a geometric description is needed of the *ReBCO* tape when it is bent and twisted. A single turn of the coil-end can be modelled as a thin strip, whose geometrical properties are described in the previous sections.

In magnet design, the geodesic strip description has been used extensively to model magnet coil-ends using rectangular superconducting cable [141, 142]. The aspect ratio

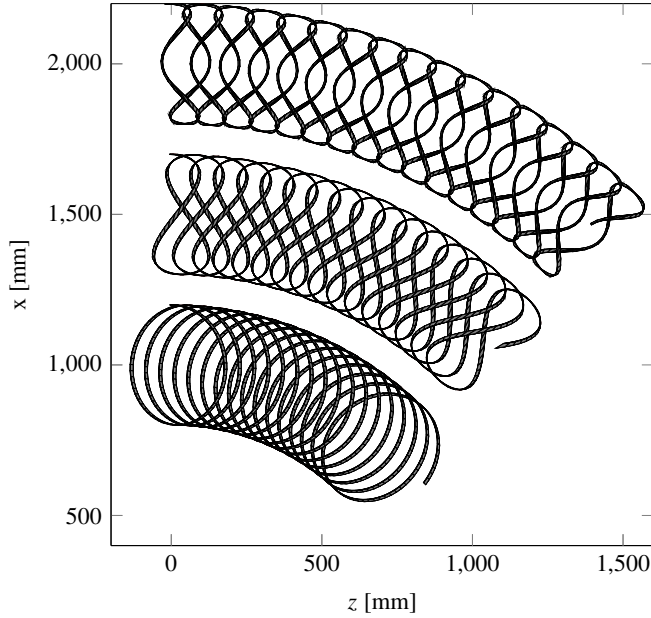


Figure 5.20: Top-down view of a dipole, quadrupole and sextupole curved CCT coil.

of the cable is smaller than that of a single *ReBCO* strip, which allows it more leeway in bending it in the hard-way direction. This extra leeway allows for the possibility to add additional twist in the design, in order for the coil-ends to line up with the straight section of the magnet [141]. This does lead to hard-way bending of the cable, and for a single *ReBCO* strip this hard-way bending is much more difficult to perform. Therefore, new designing methods for *ReBCO* coil-ends based on winding single or double stacked tapes, i.e. cables with a large aspect ratio, are necessary in order to accommodate for this behaviour.

In this section, a method is outlined to model coil-end geometries that are fully hard-way bend free. For this, Bézier functions are used, and first an overview of Bézier functions are given, which are then used to model coil-ends with the appropriate boundary conditions.

5.6.1 Bézier curves

A Bézier curve is a type of parametric curve, defined by a set of control points $\mathbf{P}_0$ through $\mathbf{P}_n$ in $\mathbb{R}^3$, where n is called its order or degree ($n = 1$ for linear, 2 for quadratic, etc.). The Bézier curve is a function of t , defined on the interval $I = [0, 1]$, and can be explicitly expressed as:

$$\mathbf{B}(t) = \sum_{k=0}^n b_{n,k}(t) \mathbf{P}_k, \quad (5.100)$$

where $b_{n,k}$ are the so called Bernstein polynomials, defined by:

$$b_{n,k}(t) = \binom{n}{k} (1-t)^{n-k} t^k, \quad (5.101)$$

with:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}. \quad (5.102)$$

The points $\mathbf{P}_k$ are the control points of the Bézier curve. A useful property of Bézier curves is that complicated shapes can be easily drawn out by the placement of the control points. In Figure 5.21, an example of a Bézier curve with the control points is shown.

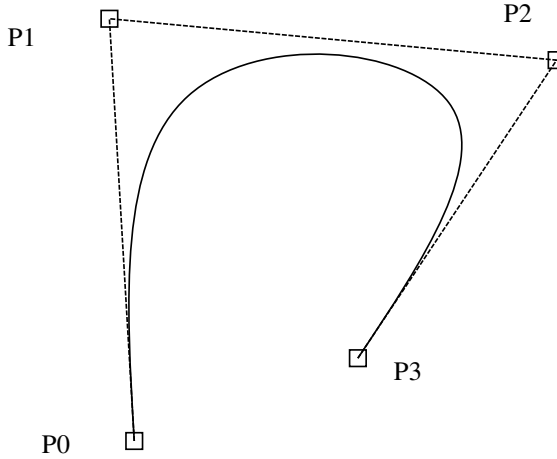


Figure 5.21: Bezier spline with four control points.

Another useful property is that its derivatives can be calculated analytically everywhere, even for geometrically complicated curves. They can be calculated using Casteljau's algorithm, which is described in Appendix A. In Table 5.1, the expression for the derivatives at $t = 0$ is shown.

Table 5.1: Derivatives of a Bézier curve $\mathbf{r}(t)$ at $t = 0$, calculated using the method outlined in Appendix A. For the derivatives at $t = 1$, all control points $\mathbf{P}_i$ get substituted with $\mathbf{P}_{n-i}$.

	Startpoint
$\mathbf{r}(0)$	$\mathbf{P}_0$
$\mathbf{v}(0)$	$n(\mathbf{P}_1 - \mathbf{P}_0)$
$\mathbf{a}(0)$	$n(n-1)(\mathbf{P}_2 - 2\mathbf{P}_1 + \mathbf{P}_0)$
$\dot{\mathbf{a}}(0)$	$n(n-1)(n-2)(\mathbf{P}_3 - 3\mathbf{P}_2 + 3\mathbf{P}_1 - \mathbf{P}_0)$
$\ddot{\mathbf{a}}(0)$	$n(n-1)(n-2)(n-3)(\mathbf{P}_4 - 4\mathbf{P}_3 + 6\mathbf{P}_2 - 4\mathbf{P}_1 + \mathbf{P}_0)$
$\ddot{\mathbf{a}}(0)$	$n(n-1)(n-2)(n-3)(n-4)(\mathbf{P}_5 - 5\mathbf{P}_4 + 10\mathbf{P}_3 - 10\mathbf{P}_2 + 5\mathbf{P}_1 - \mathbf{P}_0)$

5.6.2 Coil-end development with Bézier splines

One can draw a single turn of the coil-end as follows. The first step is to draw a three-dimensional curve $\mathbf{r}(t)$ that defines the shape of the coil-end, using a Bézier spline. In Figure 5.22, the steps for drawing a single turn of a coil-end is shown.

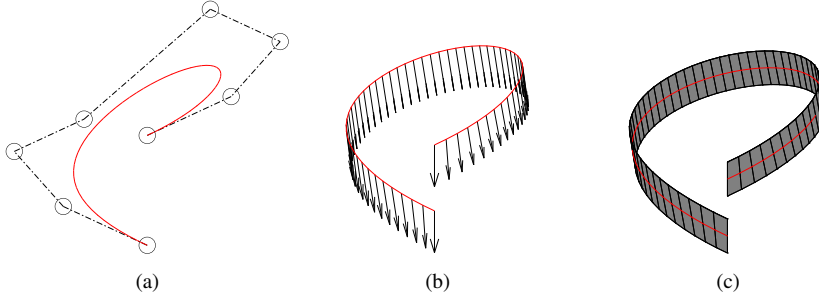


Figure 5.22: Steps taken to draw the coil-pack from a given base curve. In (a), the base curve is drawn using a Bézier curve. Then in (b), the Darboux vector field derived from the Frenet-Serret apparatus is applied to the base curve. And in (c), the strip surface is drawn using the Darboux vector.

The next step is to develop a strip surface free of hard-way bending which uses this base curve. In order to do this, first the Frenet-Serret apparatus is applied to each point of the base curve, and the Darboux vectors are calculated from this apparatus. With the Darboux vector, a strip surface can be developed on the base curve which is hard-way bend free. This surface is unique, there are no other surfaces which can be created using the base curve which are also hard-way bend free.

The coil-ends generally meet a straight section of the magnet, which has no curvature and torsion. These requirements can be fulfilled with the correct placement of the control points of the Bézier spline, as will be outlined next.

5.6.3 Boundary conditions

The coil-end commonly joins two straight sections with each other. In the straight section, the curvature and the torsion of the strip are zero. The curvature and torsion of the coil-end must be matched with the straight section for a continuous transition, and thus must be zero as well. The transition can have various degrees of smoothness, which means matching derivatives of the curvature and the torsion in the straight section and the coil-end. The transition smoothness is of order n if the derivatives $\frac{d^n \kappa}{dt^n}$ and $\frac{d^n \tau}{dt^n}$ are matching in the straight section and in the coil-end at the place where they join. Since all derivatives of the curvature and torsion in the straight section are zero, they must be zero in the beginning and end of the coil-end up to order n to have a transition smoothness of order n .

Curvature boundary conditions

When the coil-end joins a straight section, the beginning and end of the coil-end must have zero curvature. This implies:

$$\kappa = \frac{|\mathbf{v} \times \mathbf{a}|}{|\mathbf{v}|^3} = 0. \quad (5.103)$$

Since $v(t) \neq 0 \forall t$, this means that $|\mathbf{v} \times \mathbf{a}| = 0$. This condition can be fulfilled using a useful property of Bézier splines. In Table 5.1, the derivatives at the begin and end point of a Bézier curve as function of the control points of the curve are tabulated. From this, it can be seen that for the start of the curve the first derivative depends on the first two control points of the Bézier curve, the second derivative on the first three control points etc. For the end of the curve, it is the opposite. The first derivative depends on the last two control points, the second derivative depends on the last three control points etc. The key point to notice here is that they do not depend on all the other control points. This provides a way to adhere to the boundary conditions. For instance, the condition $|\mathbf{v} \times \mathbf{a}| = 0$ can be fulfilled by placing the first three control points on a line. This has the effect that the velocity and acceleration are pointing in the same direction, and hence their cross product equals zero. The other control points can then be used to form the rest of the curve, without influencing the curvature at the start of the curve. If a smoothness of the curvature of order $n = 1$ is required, the first derivative of the curvature has to be zero as well:

$$\frac{d\kappa}{dt} = \frac{(\mathbf{v} \times \mathbf{a}) \cdot (\mathbf{v} \times \dot{\mathbf{a}})}{|\mathbf{v} \times \mathbf{a}| |\mathbf{v}|^3} - \frac{3|\mathbf{v} \times \mathbf{a}| \mathbf{v} \cdot \mathbf{a}}{|\mathbf{v}|^5} = 0. \quad (5.104)$$

Since $|\mathbf{v} \times \mathbf{a}| = 0$, the second term is zero, but the first term is indeterminate. The indeterminate part of the term can be solved using L'Hôpital's rule, which states:

$$\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}. \quad (5.105)$$

Application of L'Hôpital's rule in the limit $t \rightarrow 0$ yields:

$$\begin{aligned} \lim_{t \rightarrow 0} \frac{d\kappa}{dt} &= \lim_{t \rightarrow 0} \frac{\frac{d}{dt}(\mathbf{v} \times \mathbf{a}) \cdot (\mathbf{v} \times \dot{\mathbf{a}})}{\frac{d}{dt} |\mathbf{v} \times \mathbf{a}|} \\ &= \lim_{t \rightarrow 0} \frac{(\mathbf{v} \times \dot{\mathbf{a}})(\mathbf{v} \times \dot{\mathbf{a}}) + (\mathbf{v} \times \mathbf{a})(\mathbf{a} \times \dot{\mathbf{a}}) + (\mathbf{v} \times \mathbf{a})(\mathbf{v} \times \ddot{\mathbf{a}})}{\frac{(\mathbf{v} \times \mathbf{a}) \cdot (\mathbf{v} \times \dot{\mathbf{a}})}{|\mathbf{v} \times \mathbf{a}|}}. \end{aligned} \quad (5.106)$$

The denominator is exactly the same indeterminate term on which L'Hôpital was applied. Making use of a substitution: $f(t) = (\mathbf{v} \times \mathbf{a}) \cdot (\mathbf{v} \times \dot{\mathbf{a}})$, $g(t) = |\mathbf{v} \times \mathbf{a}|$ and $h(t) = (\mathbf{v} \times \dot{\mathbf{a}})(\mathbf{v} \times \dot{\mathbf{a}}) + (\mathbf{v} \times \mathbf{a})(\mathbf{a} \times \dot{\mathbf{a}}) + (\mathbf{v} \times \mathbf{a})(\mathbf{v} \times \ddot{\mathbf{a}})$, Equation 5.106 can be rewritten as:

$$\lim_{t \rightarrow 0} \frac{f(t)}{g(t)} = \lim_{t \rightarrow 0} \frac{g(t)h(t)}{f(t)}. \quad (5.107)$$

Since the limit on the left side is the same as the limit on the right side it must hold that:

$$\frac{f(0)}{g(0)} = \frac{g(0)h(0)}{f(0)}, \quad (5.108)$$

provided that the limit exists. Rearranging and substituting into Equation 5.107 yields:

$$\lim_{t \rightarrow 0} \frac{f(t)}{g(t)} = \sqrt{h(0)}. \quad (5.109)$$

Since $|\mathbf{v} \times \mathbf{a}| = 0$, $h(0) = (\mathbf{v} \times \dot{\mathbf{a}}) \cdot (\mathbf{v} \times \dot{\mathbf{a}})$. The derivative of the curvature at the start point is then:

$$\lim_{t \rightarrow 0} \frac{d\kappa}{dt} = \frac{|\mathbf{v} \times \dot{\mathbf{a}}|}{|\mathbf{v}|^3}. \quad (5.110)$$

This can be achieved if $|\mathbf{v} \times \dot{\mathbf{a}}| = 0$, which can be achieved if the first four control points are on a line. This process can be continued for higher order smoothness. For the opposite end of the coil-end, where $t = 1$, one applies the limit $t \rightarrow 1$, which results that the last four control points need to be on a line for the first derivative of the curvature to match the straight section.

Torsion boundary conditions

The requirement $|\mathbf{v} \times \mathbf{a}| = 0$ gives an indeterminate form for the torsion:

$$\tau = \frac{(\mathbf{v} \times \mathbf{a}) \cdot \dot{\mathbf{a}}}{|\mathbf{v} \times \mathbf{a}|^2} = \frac{0}{0}. \quad (5.111)$$

One thus requires that the torsion in the limit of $t \rightarrow 0$ and $t \rightarrow 1$, where $|\mathbf{v} \times \mathbf{a}| = 0$, goes to zero. Applying L'Hôpital's rule twice on Equation 5.111 in the limit $t \rightarrow 0$ yields:

$$\lim_{t \rightarrow 0} \tau = \frac{(\mathbf{v} \times \dot{\mathbf{a}}) \cdot \ddot{\mathbf{a}}}{2|\mathbf{v} \times \dot{\mathbf{a}}|^2}. \quad (5.112)$$

Thus, in the limit $|\mathbf{v} \times \mathbf{a}| \rightarrow 0$, $\tau = 0$ when the nominator in Equation 5.112 equals zero, and the denominator is non-zero. The nominator is either zero when $\ddot{\mathbf{a}}$ is in-plane with $\mathbf{v}$ and $\dot{\mathbf{a}}$. From Table 5.1, it can be seen that this requirement can be fulfilled by placing $\mathbf{P}_4$ in the same plane as the plane formed by $\mathbf{P}_0$, $\mathbf{P}_1$ and $\mathbf{P}_2$, and $\mathbf{P}_3$. In the limit $t \rightarrow 1$, it follows by symmetry that the last 5 control points must be in-plane.

In the case that $|\mathbf{v} \times \dot{\mathbf{a}}| = 0$, there is another indeterminate form:

$$\tau = \frac{(\mathbf{v} \times \dot{\mathbf{a}}) \cdot \ddot{\mathbf{a}}}{2|\mathbf{v} \times \dot{\mathbf{a}}|^2} = \frac{0}{0}. \quad (5.113)$$

In this case, the limit $t \rightarrow 0$ where $|\mathbf{v} \times \dot{\mathbf{a}}| = 0$ and $|\mathbf{v} \times \mathbf{a}| = 0$ needs to be considered. Applying L'Hôpital four times on Equation 5.111 yields:

$$\lim_{t \rightarrow 0} \tau = \frac{(\mathbf{v} \times \ddot{\mathbf{a}}) \cdot \ddot{\mathbf{a}}}{3|\mathbf{v} \times \ddot{\mathbf{a}}|^2} \quad |\mathbf{v} \times \mathbf{a}| = 0, |\mathbf{v} \times \dot{\mathbf{a}}| = 0. \quad (5.114)$$

This is zero when $\ddot{\mathbf{a}}$ is in plane with $\mathbf{v}$ and $\dot{\mathbf{a}}$. Note that the nominator can be zero as well be setting $\mathbf{v} \times \ddot{\mathbf{a}} = 0$, but this gives another intermediate form, since the denominator is then zero as well, and can be evaluated by application of L'Hôpital's rule.

Edge of regression boundary conditions

The procedure described in the previous section does not guarantee no edge of regression. One case of edge of regression happens when the generator length $l = w\sqrt{1 + \eta^2}$ goes to infinity, due to $\kappa = 0$ and $\tau = 0$. Since the edges of the tape strip are formed by the generator, the generators must be pointing in the binormal direction of the tape at its beginning and end. In that case $l = w$, which means that $\eta = 0$ and thus the following relationship must hold:

$$\eta = \frac{|\mathbf{v}|^3 (\mathbf{v} \times \mathbf{a}) \cdot \dot{\mathbf{a}}}{|\mathbf{v} \times \mathbf{a}|^3}. \quad (5.115)$$

Since $|\mathbf{v} \times \mathbf{a}| = 0$ for the curvature to be zero at the ends, this yields an indeterminate form. Using L'Hôpital's rule thrice yields:

$$\lim_{t \rightarrow 0} \eta = |\mathbf{v}|^3 \frac{\mathbf{a} \times \dot{\mathbf{a}} \cdot \ddot{\mathbf{a}} + 2\mathbf{v} \times \dot{\mathbf{a}} \cdot \ddot{\mathbf{a}}}{6|\mathbf{v} \times \dot{\mathbf{a}}|^3}. \quad (5.116)$$

Therefore, the strip is only developable if $\ddot{\mathbf{a}}$ is in plane with $\mathbf{v}$ and $\dot{\mathbf{a}}$, or as seen from the perspective of the control points, $\mathbf{P}_5$ must be in plane with the $\mathbf{P}_0 \dots \mathbf{P}_4$.

Control point boundary conditions

In order to gain a more intuitive understanding of the boundary conditions, the effect of the control points on the curvature κ , torsion τ and ratio between the torsion and curvature $\eta = \tau/\kappa$ are plotted in Figure 5.23. On the left, a cloverleaf ear is developed using the Bézier method. The strip starts at the origin and ends displaced by -20 mm in the z -direction. On the right, the curvature, torsion and ratio between the torsion and curvature are plotted.

In Figure 5.23a, the Bézier curve is created using 16 control points. The first 4 control points are placed on a single line, the other 4 are placed in the osculating plane of the base curve formed by the tangential and normal vector at the starting point. The end points are placed in the same manner, but offset by -20 mm, rotated by 90° and mirrored in the xz -plane. The curvature, torsion and ratio are all zero at the beginning and end point, as is required for a physical strip.

In order to gain a understanding how the control points influence the strip, some control points are removed in the succeeding plots. Figure 5.23c, one of the control points in the oscillating plane is removed for both ends. The curvature and torsion are still zero at the end points, but their ratio is not any more. One can notice this by looking at the generators at the end points, which are not making a right angle with the base curve any more. In Figure 5.23e, one of the 4 points on the line is removed for both ends. It can be seen that now the ratio goes to zero again, but the smoothness of the curvature at the end points has decreased by one order, i.e. the derivative of the curvature is not zero any more at the end points. Another point in the oscillating plane has been removed in Figure 5.23g for both ends, and the ratio η is not zero any more at the end points. Next in Figure 5.23i, another point in the oscillating plane is removed, and the torsion now is a finite value at the end points, and since the curvature is still zero, the ratio shoots off to infinity. Lastly, in Figure 5.23k, a point on the initial line of control points is removed. Now the curvature at the end-points is not zero any more.

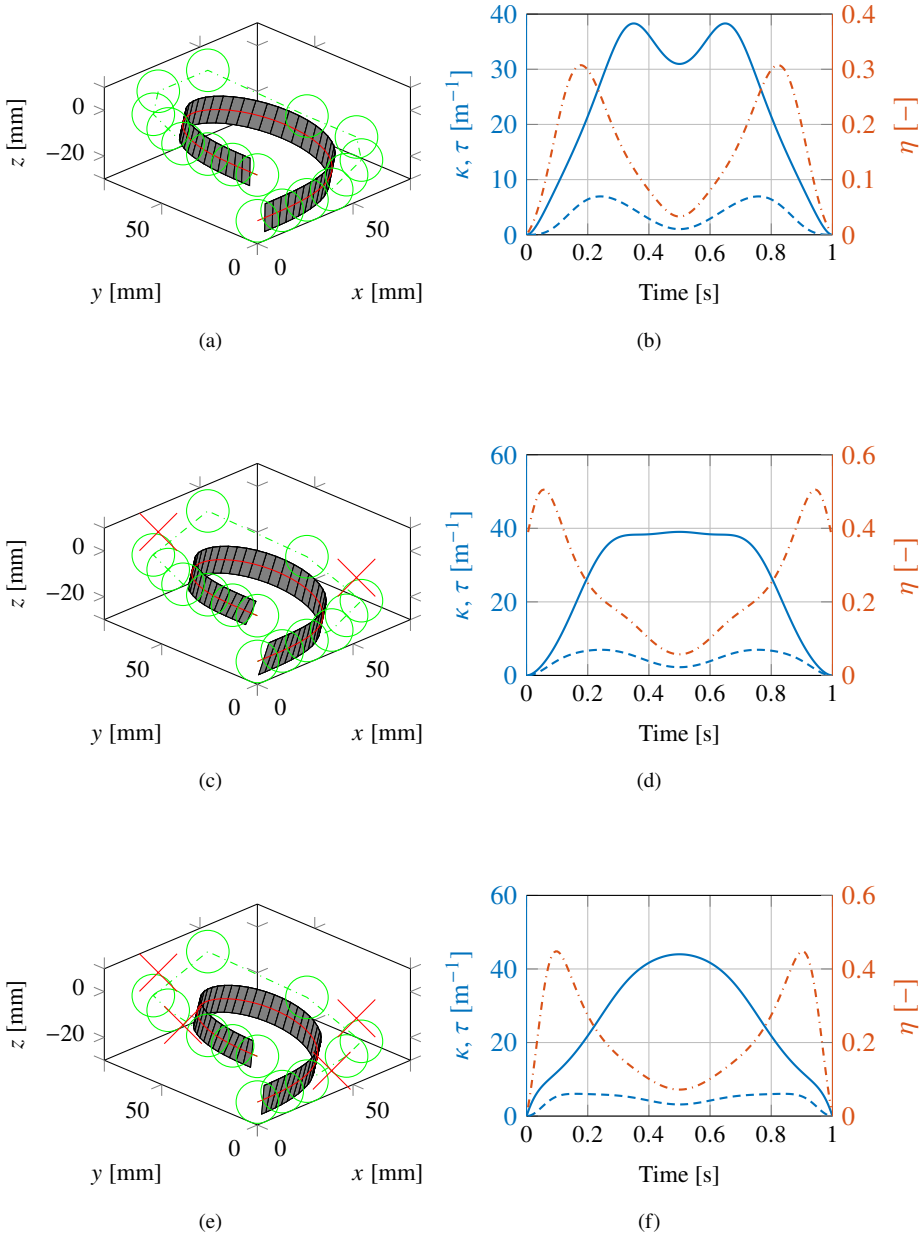


Figure 5.23: In the left figure, a turn of a cloverleaf ear is shown. In the right figure, the magnitude of the curvature (solid blue) and torsion (dashed blue) are plotted on the left y-axis, and the magnitude of η (dashdotted red) is plotted on the right y-axis. The green circles are control points, the red crosses are removed control points.

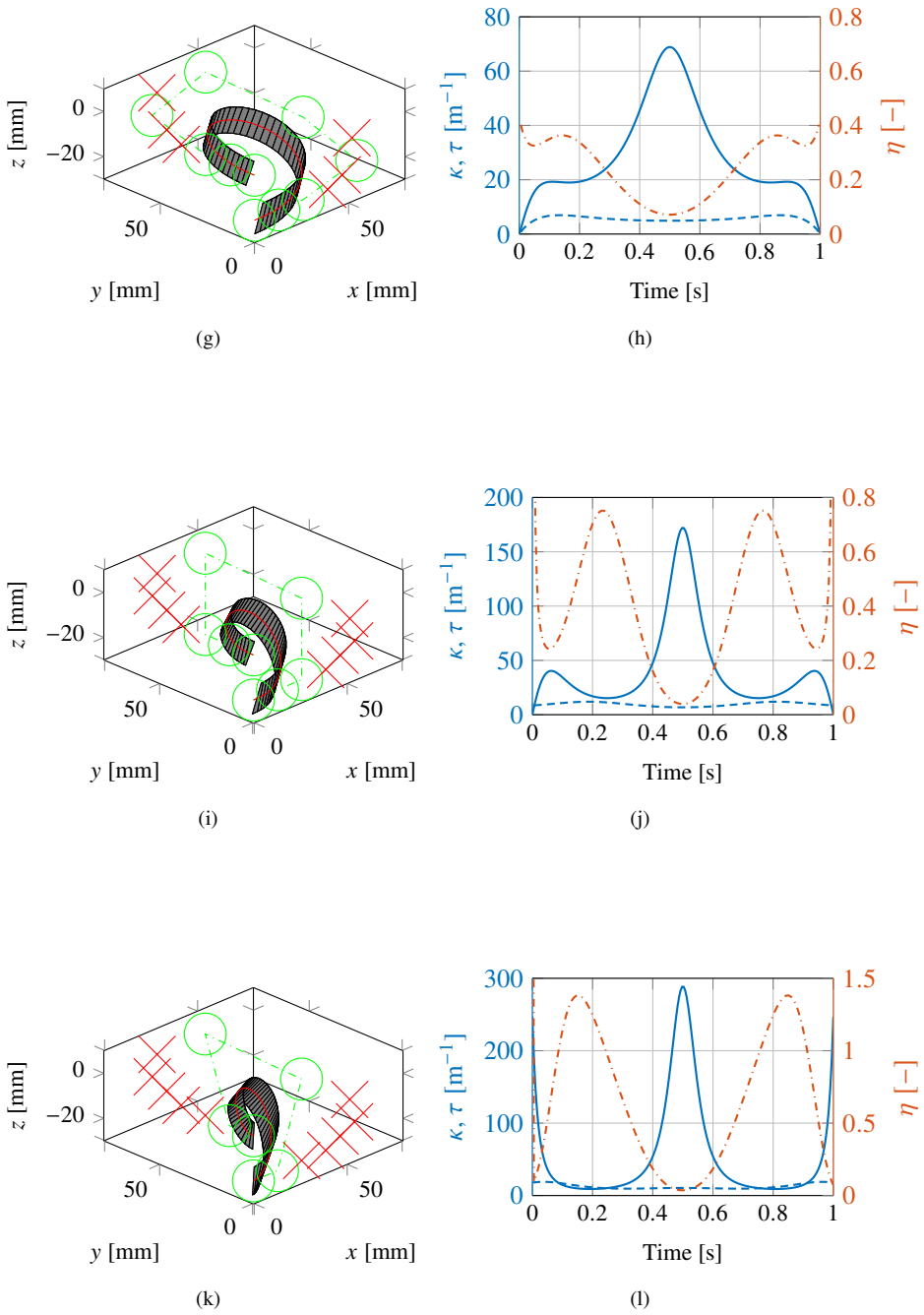


Figure 5.23: Continued.

5.6.4 Coil-end geometries made with Bézier splines

In this section, two examples are presented of accelerator magnet coil-ends that can be made using the Bézier method: a cloverleaf coil-end and a cosine-theta coil-end.

Cloverleaf coil-end

The cloverleaf geometry was first proposed by Gupta to eliminate the need for reverse bends for single aperture magnets [183]. The shape is named after the cloverleaf interchange which is used to connect two crossing roads or motorways. The advantage of the cloverleaf configuration to other configurations is that the tapes are aligned parallel to the field, which is the good field angle for the tape. The disadvantage of the cloverleaf configuration is that the ears are wide, making it less suitable as an insert magnet.

Here, the cloverleaf shape that is presented in Figures 5.10, 5.16 and 5.23 is discussed. In Chapter 6, the cloverleaf shape is discussed in a practical setting. Here it is discussed theoretically.

Using Equation 5.56, the bending radius of the cloverleaf can be determined, which yields a bending radius of 25.7 mm. Using Equation 5.73, it can be estimated that the minimal hard-way bend radius in the *ReBCO* layer is 145 m. Thus, the amount of soft-way bending is three orders of magnitude larger compared to the amount of hard-way bending in this cloverleaf configuration. This shows that it was reasonable to neglect the hard-way bending aspect for this particular geometry.

From a geometric perspective, the anticipation is for a gap to occur between the *ReBCO* tapes in a coil-end where there is locally torsion. The geodesic curvature came about by rotating the frame about the tangent vector. It can thus be assumed that the next tape layer rotates around the tangent vector such that the geodesic curvature is cancelled out. The rotation angle can be found by combining Equations 5.25 and 5.29:

$$\sin \theta = \sqrt{\frac{(k_g^*)^2}{(k_g^*)^2 + (k_n^*)^2}}. \quad (5.117)$$

Figure 5.24a shows a schematic of two tapes. The left tape is the first tape, and the right tape is the parallel tape, which rotates around the tangent vector, which is out of the paper. The gap d between the tapes is then given by:

$$d = w \tan \theta. \quad (5.118)$$

The assumption made here is that the base curve does not move when the parallel tape twists, in reality this does not need to be the case. Substituting Equation 5.117 into this yields:

$$d = \frac{w(k_g^*)^2}{\sqrt{2(k_n^*k_g^*)^2 + (k_n^*)^4}}. \quad (5.119)$$

For the cloverleaf configuration in Figure 5.23a, the estimated gap is plotted in Figure 5.24b. From this it can be seen that the gap between the tapes for this cloverleaf configuration is of the order of 10 nm to 100 nm.

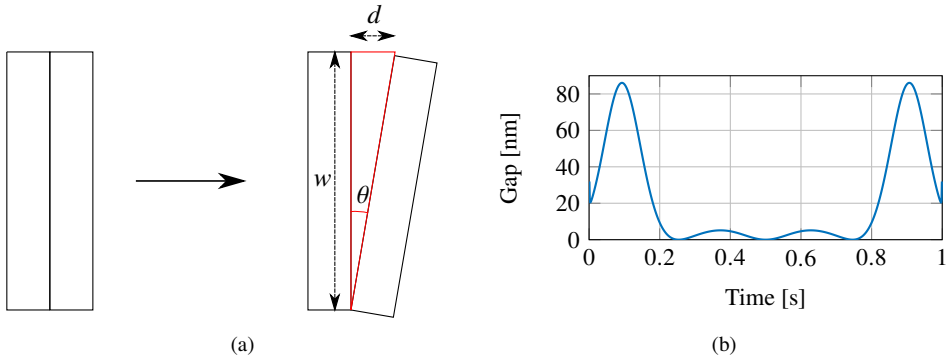


Figure 5.24: Estimation of the gap in the cloverleaf configuration between two adjacent tapes. In (a), a sketch of how the second tape is rotated in order to cancel the hard-way bending is shown. In (b), a plot of the estimated gap in nm between successive turns in the cloverleaf configuration is shown.

For the cloverleaf coil-pack, the following offset vector was used:

$$\mathbf{O}(t) = \mathbf{N}_{xy}(t) + \mathbf{N}(t), \quad (5.120)$$

where $\mathbf{N}$ is the normal vector to the base curve and $\mathbf{N}_{xy}$ the projection of the normal vector in the xy -plane, which is the plane normal to the tape at the start and end points of the ear. In Figure 5.16, the procedure of drawing the coil-pack thickness was shown for a cloverleaf ear.

Cosine-theta coil-end

The cosine-theta configuration has been used frequently in accelerator magnet design, mostly with Nb-Ti and Nb₃Sn. *ReBCO* tape has been used for a cosine-theta coil using 12 mm tape wrapped with Kapton CI insulation [184] and a cosine-theta magnet made with Roebel cable [45].

Bézier functions can also be applied to model a cosine-theta coil-end. In Figure 5.25a, a single turn of the coil-end of a cosine-theta magnet is presented. The turn lies on top of a cylindrical mandrel, and its end points are offset by 10°. The goal of designing a cosine-theta turn is to have one edge in perfect contact with the cylinder, without having hard-way bending. In Figure 5.25b, the lift-off from the cylinder is plotted as a function of distance. The maximum lift-off is in the order of 100 μm, which is the same order of magnitude as the thickness of the tape. For the shape presented here the minimal generalized bending radius is 18.1 mm, and the minimal hard-way bending radius is 81.7 m.

One downside of this configuration is that the magnetic field direction is perpendicular with respect to the tape, i.e. in the magnetic field angle direction where the critical current is lowest. This lowers the performance of the magnet compared to a configuration where the tapes are parallel with respect to the magnetic field.

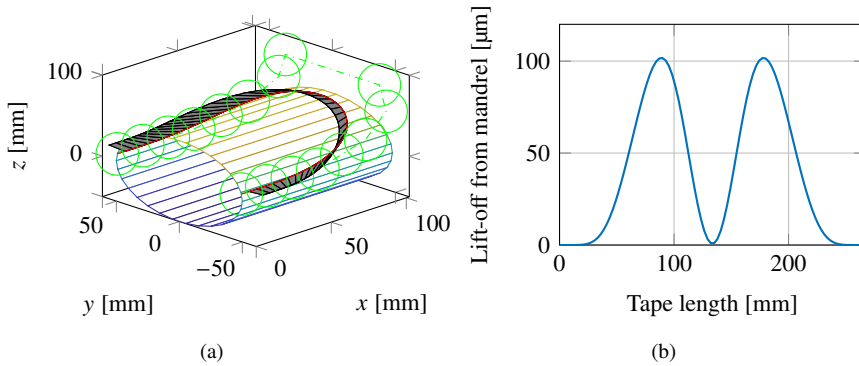


Figure 5.25: Overview of a cosine-theta coil-end designed using the Bézier method. In (a), a plot of a cosine-theta winding is shown. The winding is wound over a cylindrical mandrel. The green circles are the control points of the spline. In (b), a plot of the lift-off of the mandrel, i.e. the gap between the tape and the cylindrical mandrel for the created cosine-theta configuration is shown.

5.7 Conclusion

In this chapter, it was shown how to create coil geometries for tape conductors, such as *ReBCO* coated conductor. This is achieved by the thin strip model, which assumes that the strip is infinitely thin and creates the strip conductor surface by tracing out generators along a base curve. This minimizes the hard-way bending in the strip. The thin strip model is only valid for thin conductors. It was shown mathematically that in stacked-tape cables, hard-way bending is induced when the cables are stacked without a gap in between the turns.

Minimization of the hard-way bending is only one aspect that must be taken into account in the design of coils with *ReBCO* tape conductor. It was shown that for the most mathematically optimal shape, not only the hard-way bending component must be minimized with the thin strip model, but the overall bending energy due to the curvature and torsion has to be minimized as well. This creates geometries with a more natural shape, which is relevant in the design of the coil-end. A mathematical expression for the edge of regression requirement was derived, which states that a coil design is limited by the width of the tape and the ratio of change in curvature and torsion along the conductor length. Expressions were determined for the strain along the length and width of the conductor for generally bent and twisted conductor geometries.

Two examples of applications, the geometry of helical and CCT windings made from tape conductor have been presented. General expressions were derived for the curvature and strain in a CCT, which can be used to give an estimation of the strain in the CCT turn. Combined with the critical strain of the used *ReBCO* tape in the windings, the coil design can be optimized. Edge of regression violation must be taken into account when designing a CCT. Not all CCT configurations are possible to create with tape conductor as it is impossible to bend the conductor in such a way that it follows the CCT base curve. Possible ways to alleviate this are using tape with a smaller width or larger tilt angles.

A method to develop coil-ends using Bézier splines has been outlined. It can be used

to create many diverse coil-end geometries that are fully hard-way bend free, and adhere to the boundary conditions imposed by the endpoints. By strategic placement of the control points, the straight section of the magnet can be met without the requirement of additional twist, and edge of regression violations can be prevented. The correct placement of these control points, as well as the minimum number of control points necessary, have been mathematically determined.

The method of Bézier splines in combination with the thin strip model is an improvement to previous methods using polynomials, which are less flexible in their geometric design and cannot meet the straight section without inducing extra hard-way bending.

To demonstrate the usability of Bézier splines, two accelerator dipole coil-end configurations made with Bézier splines were presented, the cloverleaf and the cosine-theta. Bézier splines in combination with the thin strip model are thus a powerful tool in the design of coil-end geometries with minimized the hard-way bending of *ReBCO* tape conductors.

Chapter 6

Cloverleaf magnet design, coil winding and test

In this chapter, a preliminary design of a demonstrator cloverleaf-racetrack magnet is presented, a candidate configuration for above 20 T accelerator dipole magnets. Design aspects considered are electromagnetic properties, a first order optimization of magnetic field quality and the mechanical support structure. For quench protection, it is looked at how the charging time of the coil would be if the coil were to be wound non-insulated. A modelling-based comparison between dry-wound and solder-filled non-insulated coils is made for mechanical performance as well as time constant management. It is still an open question if this is the right method to protect large scale accelerator magnets, due to the large charging times, magnetic field quality concerns and scalability of non-insulated coil technology to large scale coils. A dry-wound coil is momentarily considered more suitable for the cloverleaf-racetrack configuration due to a lower charging time constant and better mechanical stability.

The cloverleaf configuration provides a method of passing the coil over the beam-pipe while minimizing the amount of hard-way bending of the tape conductor. A major problem with a cloverleaf coil is how to make the ends of the coil without causing degradation due to coil winding, cooling, and operation. To address these issues, in this chapter the winding and testing is examined of a trial cloverleaf coil based on the dimensions of the demonstrator cloverleaf coil, but with a significantly reduced number of turns, being eight. This reduced-turns cloverleaf coil is wound with eight turns of two 12 mm SuperPower tape conductors in parallel, where the *ReBCO* layers face each other. The coil is cooled to 77 K and supplied with current at this temperature. The coil reached a critical current of 488 A, which is in the predicted range. Other aspects considered are *n*-value of the coil, effective time constants and magnetic field response, as well as the influence of the current terminal positioning on the current distribution within the coil. Lastly, a conceptual design of a 10 m long 20 T dipole magnet is presented¹.

¹Part of this chapter has been published in T. H. Nes et al., "Design of a Cloverleaf-Racetrack Dipole Demonstrator Magnet With Dual *ReBCO* Conductor," in *IEEE Transactions on Applied Superconductivity*, vol. 32, no. 6, pp. 1-5, Sept. 2022, Art no. 4002105, doi: 10.1109/TASC.2022.3155445.

6.1 Introduction

For most *ReBCO* tapes, the critical current is highest when the magnetic field is parallel to the broad side of the tape, and the lowest when the magnetic field is oriented perpendicular to the tape [77]. To maximize the critical current in the coils, it is thus of importance to align the tape parallel to the applied magnetic field. In accelerator magnet design, this is achieved by using a block coil configuration, where the tapes are oriented parallel to the dipole field in the aperture (see Chapter 1). The cloverleaf coil-end is a possible solution for guiding the conductor over the particle beam tube in a block-type coil layout. Using the differential geometry methods presented in Chapter 5, cloverleaf shapes can be obtained with minimal hard-way bending of the tape conductor.

Here, a preliminary design of a demonstrator cloverleaf-racetrack accelerator magnet is presented. The magnet is foreseen to be constructed and cold-tested at CERN and will act as a first stepping stone towards above 20 T accelerator magnets. In the first part of the chapter, the coil design of the magnet is discussed. The magnet consists of two poles each with a so-called cloverleaf coil and a racetrack coil. The quench protection method to be employed is still on debate. The appropriateness of employing no-insulation technology as the preferred method for safeguarding large-scale accelerator magnets remains an unresolved inquiry, primarily due to the protracted charging durations, qualms about magnetic field integrity, and the scalability of non-insulated coil technology to systems with coils of substantial size. It is briefly discussed what the consequences are for no-insulation technology for this cloverleaf-racetrack configuration with regards to the charging delay. Also, a first order optimization of the magnetic field quality is discussed. Then, the design of the magnet enclosure, which provides the mechanical support for the coils and incorporates the instrumentation and current leads, is presented. To show that the magnet can be operated within the allowable stress and strain levels, a mechanical simulation of the magnet assembly has been performed and is presented as well.

In the cloverleaf-ear design, the size and shape of the ears can be adjusted so that the conductor is not overly strained and aligns to the straight section smoothly. However, this only applies to an ideal coil geometry. In practice, the coil must be created by winding the tape into the desired shape. During this process, errors can occur in the geometry when the windings deviate from the prescribed geometry. In addition, the critical current of the cloverleaf coil must not be degraded by the winding process itself, which can occur due to damage to the tape during the winding process or overstraining of the conductor.

The first research on winding the cloverleaf configuration with *ReBCO* coated conductor was performed at Brookhaven National Laboratory in 2016 by Gupta et al. [185], and at CERN in 2017 by Van Nugteren [66]. Both winding tests were performed to demonstrate that it is possible to practically wind such a topology. At Brookhaven, 10 turns of single tape were wound on a glass-filled nylon winding mandrel, where the conductor was placed in a groove in the mandrel. The coil was tested in liquid nitrogen at 77 K in self-field and no degradation of the tape due to the winding process was observed. At CERN, the conductor used for the test was a stainless steel dummy Roebel cable. A total of five turns were wound. The test proved that it was possible to wind such a geometry, but it was also found that it is quite difficult to align the conductor with the winding mandrel during winding. Another cloverleaf coil was wound at the Brookhaven National Laboratory in 2021 [186]. They wound a Rutherford cable made of Nb_3Sn onto a titanium winding

mandrel. The winding was performed as part of a proof-of-principle for an 11 T Nb₃Sn block configuration coil, where the coil-ends are cloverleaf shaped.

To study the windability of the cloverleaf configuration, the manufacturing of a cloverleaf-shaped coil is examined in this chapter. This coil differs from the one wound at Brookhaven as it consists of two *ReBCO* tapes in parallel, its use of the optimized coil-end geometry presented in Chapter 5, and that the coil can be removed from the winding mandrel. First, a set of dummy coils is wound on different winding mandrels to gain experience with winding the cloverleaf shape. The next step is to wind a coil with *ReBCO* tape. This coil is made with two 12 mm wide SuperPower tape conductors in parallel, in order to study the feasibility of co-winding tapes in the cloverleaf geometry. The dimensions of the dummy coils and the *ReBCO* coil are the same as the coil used in the demonstrator cloverleaf-racetrack magnet design, except for the number of turns. Both the dummy coils and the *ReBCO* coil have 20 copper turns of 0.1 mm thickness inside and outside. The number of turns of the dummy coil and the *ReBCO* coil is reduced compared to the full-size coil, because the focus here is on the winding itself.

Using the properties of the reduced-turns coil, a prediction is made of the electromagnet performance of the coil. The reduced-turns coil is tested in liquid nitrogen at 77 K in self-field and results are compared to expected values for the critical current to determine if any degradation of the conductor has occurred. Other aspects investigated are the n -value of the coil, effective time constants and magnetic field response. In the final section of this chapter, a design and outlook towards a 10 m long 20 T dipole magnet is presented.

6.2 Cloverleaf-racetrack electromagnetic magnet design

The electromagnetic design of the demonstrator cloverleaf-racetrack magnet was performed using the RAT software package [187]. It has a central magnetic field of 9.4 T at 4.2 K, and is a first stepping stone towards above 20 T magnets, in order to test the concept on a smaller scale. The impetus for the dimensions used in the magnet design stemmed from the necessity for a ratio of aperture to length that is around five, to ensure accurate magnetic field measurements. Additionally, it was imperative to consider minimal bend radii to prevent overstraining of the conductor in the ears and to ensure compatibility with the dimensions of the cryostat. A conservative minimum radius of 27 mm was used in the ears, which is well above typical values for minimum bending radii of *ReBCO* tape [166]. The overarching objective was to establish a configuration featuring a substantial aperture that can function as a dedicated test station, facilitating the evaluation of the innovative *ReBCO* magnetic screen [69]. The design strategy involved a phased implementation, initially focusing on the fabrication of the cloverleaf component, followed by the subsequent incorporation of the racetrack coils.

In Figure 6.1, the layout of coils in the magnet is depicted. The magnet consists of two cloverleaf coils and two racetrack coils with a rectangular aperture. In Figure 6.2, a cross-section in the center of the coil-system is shown, with the magnetic field through the cross-section and magnetic field lines in the plane indicated.

The coils are wound using dual *ReBCO* conductors, where the superconducting layers are facing each other. This is so that the resistance between the two superconducting layers is minimized [143]. This can lead to better current sharing between the tapes in the case

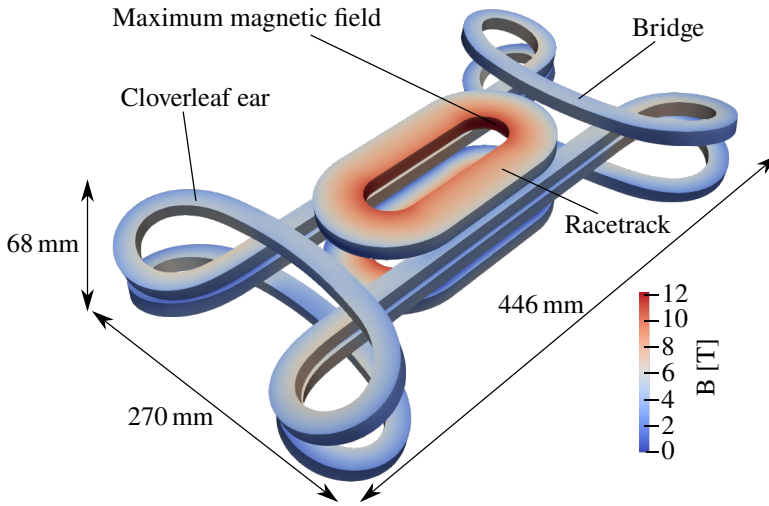


Figure 6.1: View of the coil-system layout in the magnet and its dimensions. It consists of two poles, with each a cloverleaf coil and a racetrack coil. Indicated in the figure is the magnetic field on the surface of the coil-packs for a current of 2 kA.

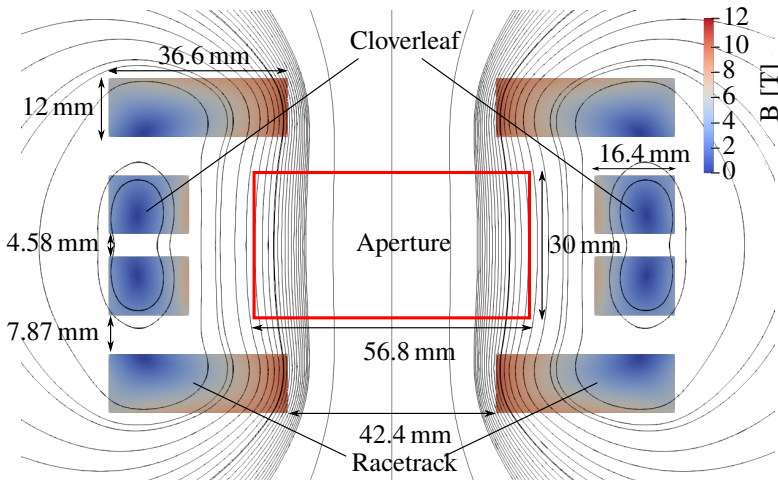


Figure 6.2: Cross-section of the coils through the mid-plane of the coil pack, with the magnetic field at 2 kA operating current. The main sizes of the coils and distances between coils are shown as well.

of a quench, and can improve the stability of the coils as well. There is no insulation in-between the tapes.

On the inside and outside surfaces of the *ReBCO* coil windings, 2 mm thick copper tape windings are present. These layers serve as a stabilizer as well as current entry and exits point along the first and the last turn of the coil windings. In Table 6.1, the most relevant coil parameters are listed.

Table 6.1: Main parameters of the cloverleaf-racetrack demonstrator magnet.

Parameter	Value
Operating current at 4.2 K	2 kA
Current density	833 A mm ⁻²
Aperture magnetic field	9.4 T
Maximum field on conductor	12.2 T
Tape thickness/width	0.1/12 mm
Number of tapes in cable	2
Length cable/tape per cloverleaf	143/286 m
Length cable/tape per racetrack	80/160 m
Total tape length	894 m
Number of cable turns cloverleaf	82
Number of cable turns racetrack	183

6.2.1 Charging time in the case of no-insulation coil windings

A possible way to deal with quench protection is to use no-insulation technology. This allows the current to bypass a local normal zone in the conductor by conduction in the radial direction [188]. A consequence of the use of no-insulation technology is the ramping time of the coils being significantly longer compared to insulated coils. For a system of non-insulated coils, the ramping time can be estimated by performing a spectral decomposition, as was discussed in Chapter 4. The ramping time is then defined as $t_{\text{ramp}} = 5\tau_{\text{max}}$, where τ_{max} is the largest eigenvalue of the time constant matrix. The inductance matrix, with self-inductances on the diagonal and mutual inductances off-diagonal, is shown in Table 6.2. The inductance matrix was calculated in RAT with the geometry of the coils as input and assuming a homogeneous current distribution within the coil windings.

Table 6.2: Inductance matrix with the self-inductances in mH of the cloverleaf (CL) and racetrack (RT) coils in poles 1 and 2 on the diagonal and mutual inductances between the coils on the off-diagonal entries. The self-inductance of the whole coil-system of 37 mH is the sum of all the listed inductances.

mH	CL 1	RT 1	CL 2	RT 2
CL 1	5.16	1.56	1.97	0.98
RT 1	1.56	5.05	0.98	1.10
CL 2	1.97	0.98	5.16	1.56
RT 2	0.98	1.10	1.56	5.05

For the purpose of calculating the ramping time, an average transverse resistivity between coil turns is used. From experiments with small pancake solenoids discussed in Chapter 3, an average radial resistivity of $3.4 \mu\Omega \text{ m}$ for a dry wound coil at 4.2 K was found. The resistance of the coil is then based on the averaged cross-sectional area of the coil windings along the length of the coil. The ramping times are then estimated to be some 300 minutes for the dry-wound coil and for a solder-filled coil it is even longer. Typical accelerator magnets require ramping times of some 30 minutes, which means that for

the demonstrator and a full-size magnet a higher transverse resistance is needed to keep the ramping time of the magnet at some 30 minutes. A possible way to achieve this is to add a specifically chosen resistive material in between the turns of the coil, or add more conductor in parallel in the cable to lower the inductance.

6.2.2 Magnetic field quality in the aperture

As mentioned in Chapter 1, the magnetic field quality of an accelerator magnet is of paramount importance for the operation of the machine. In an accelerator magnet, the magnetic field errors in the aperture are expressed as the Fourier series expansion of the radial magnetic field component at a given reference radius r_0 :

$$B_r(r, \phi) = \sum_{n=1}^{\infty} \left(\frac{r}{r_0} \right)^{n-1} (B_n(r_0) \sin(n\phi) + A_n(r_0) \cos(n\phi)), \quad (6.1)$$

where B_n and A_n are the normal and skew components respectively. This is for the 2D case. In three dimensions, the transverse magnetic field components can be given as function of the longitudinal position or integrated along the length of the magnet. The longitudinal component of the magnetic field is typically not considered for beam tracking, as the particles are moving in the same direction and the longitudinal component does only exert a negligible force on any off-axis particles in the beam. Typically, the harmonics are given in terms of units, which are defined as the ratio of the harmonic to the magnitude of the main component times 10^4 :

$$a_n = \frac{A_n}{B_1} \cdot 10^4, \quad b_n = \frac{B_n}{B_1} \cdot 10^4. \quad (6.2)$$

For the cloverleaf-racetrack demonstrator, the position of the conductor blocks was optimized for minimum integrated higher order magnetic field errors. This was achieved using RAT software [187]. The program does this in two steps. In the first step, the aperture size and thicknesses of the cloverleaf coil and the racetrack coils are variable. These are then optimized so that the required integrated magnetic field in the aperture is reached, and the quadrupole and sextupole components are minimized. The vertical positions of the coil and the current in the coil are fixed. In the second step, a correction to the coil dimensions is made based on the fact that a coil block consists of an integer number of cable turns. In this step, the aperture size and the distance between the cloverleaf coil from the mid-plane and the distance between the racetrack coil and the cloverleaf coil are variable. They are then optimized for the required integrated dipole field and minimized quadrupole and sextupole components. The number of turns and the current in the coil are fixed. The starting point of the cable was arbitrarily placed on the bridge for the cloverleaf coils and on the straight section for the racetrack coils.

The integrated magnetic field is 1.7 T m, and the minimum higher order magnetic field is 61 μ T m. In Figure 6.3, the magnitude of the harmonics is shown along the aperture length, and its integrated values along the aperture length are shown in Table 6.3. As the units of all higher order harmonics are smaller than one, the magnetic field quality is acceptable according to this first order calculation.

The optimization assumes a homogeneous current distribution in the conductor, which is difficult to achieve in *ReBCO* conductor due to the absence of fine filaments. Moreover,

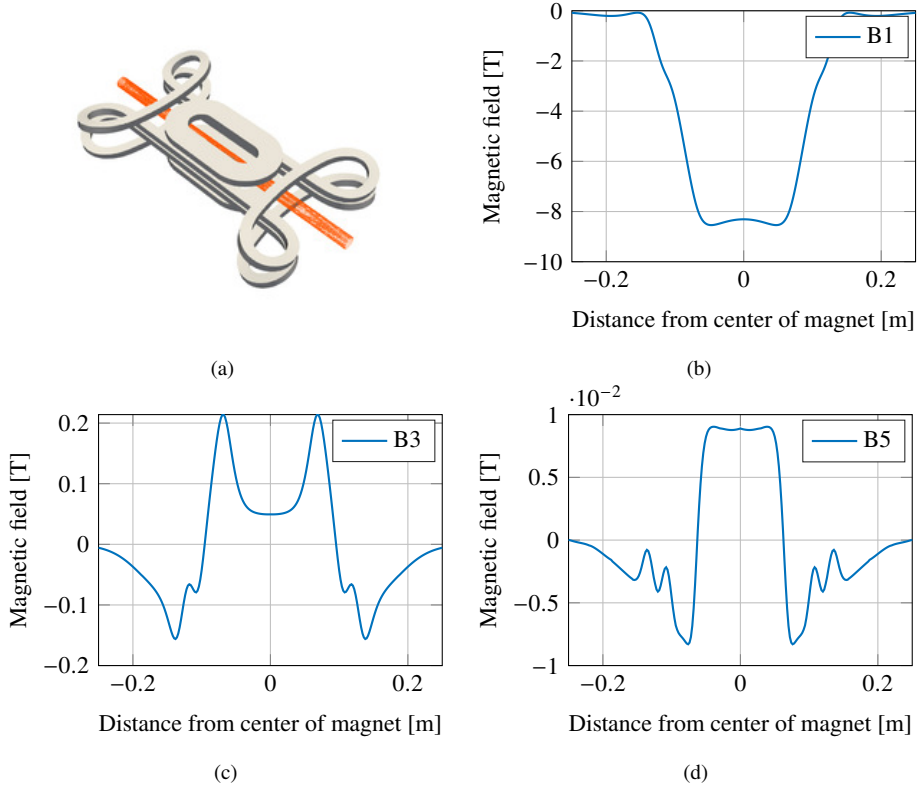


Figure 6.3: Calculation of the cylindrical harmonics along the aperture length of the magnet at a 17 mm reference radius (a). The dipole component is depicted in (b), and the higher order harmonics B3 and B5 are shown in (c) and (d). The origin is taken at the centre of the magnet.

Table 6.3: Magnetic field harmonics of the designed cloverleaf-racetrack dipole accelerator magnet.

Field multipole	A_n [Tm]	B_n [Tm]	a_n	b_n
1	$2.72 \cdot 10^{-9}$	-1.7	0.00	-10^4
2	$1.21 \cdot 10^{-6}$	$6.13 \cdot 10^{-5}$	0.01	0.36
3	$3.42 \cdot 10^{-9}$	$3.11 \cdot 10^{-9}$	0.00	0.00
4	$1.22 \cdot 10^{-6}$	$-1.27 \cdot 10^{-6}$	0.01	-0.01
5	$2.10 \cdot 10^{-9}$	$-1.61 \cdot 10^{-9}$	0.00	-0.00
6	$1.22 \cdot 10^{-6}$	$-8.35 \cdot 10^{-7}$	0.01	-0.00
7	$2.72 \cdot 10^{-9}$	$-9.96 \cdot 10^{-5}$	0.00	-0.59
8	$1.22 \cdot 10^{-6}$	$1.8 \cdot 10^{-9}$	0.01	0.00
9	$2.19 \cdot 10^{-9}$	$-4.91 \cdot 10^{-6}$	0.00	-0.03
10	$1.22 \cdot 10^{-6}$	$-5.42 \cdot 10^{-9}$	0.01	-0.00

in a non-insulated coil the current path is not well defined during ramping, leading to additional dynamic field effects. This makes it debatable if no-insulation technology can be used in accelerator magnets in the first place. A possibility to correct for these magnetic field quality errors, is by adding a *ReBCO* magnetic screen [69]. To accommodate these screens, the aperture was made rectangular.

6.3 Mechanical design

The presence of the cloverleaf ears requires a different mechanical support structure compared to usual cosine-theta coil configurations. To address this, a mechanical design of the cloverleaf-racetrack magnet was developed in collaboration with CERN's mechanical and materials engineering group and superconducting magnet technology section.

An overview of the support structure for the magnet is presented in Figure 6.4. The coils are situated within an aluminium alloy casing, in order to withstand the Lorentz force generated during coil operation. Aluminium alloy was chosen, as the material is easy to machine and still capable to take the mechanical forces for this cloverleaf-racetrack configuration. This will be demonstrated in the following section, where a mechanical model of the cloverleaf-racetrack configuration is discussed. All coils are surrounded by copper rings to act as a shunt for the inner and outer turns. To guide the instrumentation wires out of the coil, it is envisioned to use a voltage trace. To provide support for the bridge section of the cloverleaf coil, an aluminium alloy bridge support is incorporated.

Insulation between the coils and the former can be achieved through the use of a 3D-printed plastic [189] U-channel, which wraps around the coils. The plastic needs to be able to withstand operation at cryogenic temperatures. However, the appearance of cracks is not a critical issue, as long as they still allow for separation of the coil-pack to the former. To ensure a precise fit, the exact shape of the coils is envisioned to be determined via surface laser scanning, serving as input for the production of the U-channel. Additionally, a 3D-printed cap made from the same plastic material is utilized to insulate the top surface of the coils.

The two magnet poles are enclosed by an aluminium alloy shell, as depicted in Figure 6.5. This shell effectively holds the poles together, can provide pre-stress on the coil windings, and facilitates the mounting of the magnet assembly within a cryostat. Figure 6.6 provides a cross-sectional view of the complete magnet assembly.

The current leads are made of copper, and are attached to the copper rings in the straight section. To reduce the heat load, *ReBCO* tapes may be added to the surface of the copper. To lower the complexity of this first demonstrator, it was chosen to make the connections on the straight legs of the coils causing additional magnetic field errors. As is shown in the experimental results of the reduced-turns cloverleaf coil, this is not the best location, as this leads to unwanted current redistribution effects. A more suitable location would be on the bridge where the windings start. This is also better from a magnetic field quality point of view. The magnet is constructed in a modular way. The configuration allows the magnet to be partially cold-tested, first with just the cloverleaf coils, and later with the racetrack coils added. It also enables the replacement of a defect coil, interchanging coils made with different conductor, testing different no-insulation techniques or other conductor configurations.

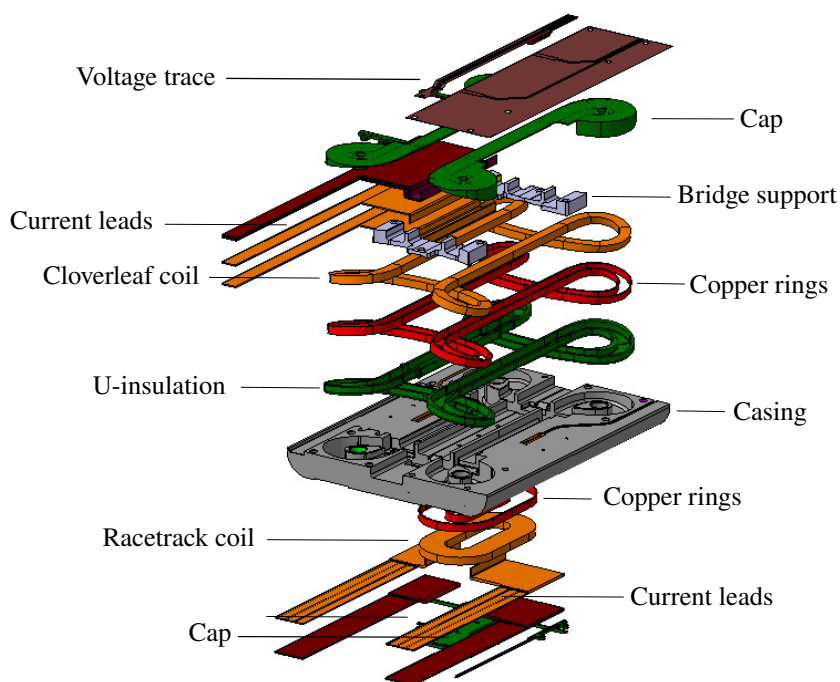


Figure 6.4: Exploded view of one magnet pole, where a cloverleaf coil and racetrack coil are sitting in an aluminium alloy encasing, electrically insulated by 3D-printed plastic situated between the encasing and the coils. The coils are surrounded by copper rings, which act as an electrical shunt. A voltage trace is foreseen to get the instrumentation wires out.

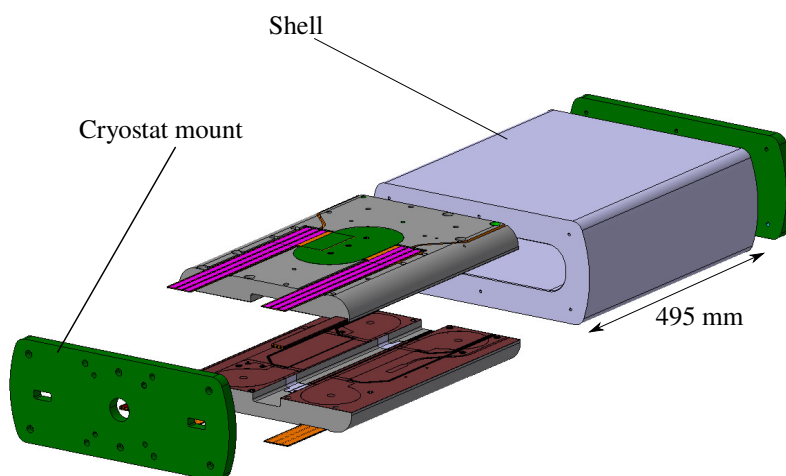


Figure 6.5: View of the two magnet poles enclosed by a tapered shell, to which the cryostat mount is attached.

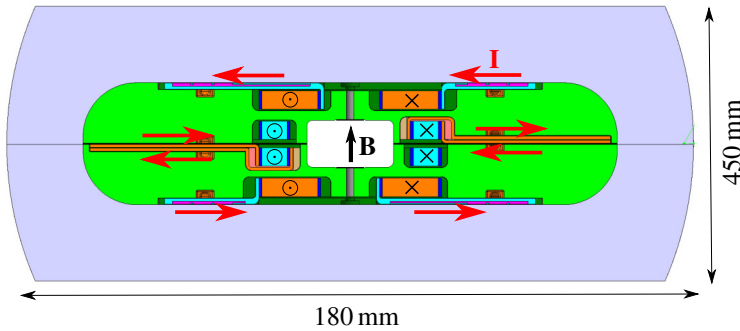


Figure 6.6: Cross-section of the magnet through the centre. The direction of the current and magnetic field is indicated in the picture. Outer dimensions of the magnet cold mass cross-section are indicated.

6.3.1 Mechanical modelling

In order to understand the mechanical performance of the magnet, and assess if any degradation of the tape occurs due to a combination of thermal contraction and Lorentz force when powering the coils, a mechanical model of the magnet is needed. One quarter of a magnet pole was modelled in ANSYS [100], where the magnet is cooled down to 4.2 K and is powered with a current of 2 kA.

As an input to the mechanical model, the Lorentz forces acting on the coil need to be included. They are calculated with the RAT software package for the cloverleaf-racetrack magnet when powered with a current of 2 kA. Assumed is a homogeneous current distribution in the coil windings and no movement of the conductor during powering. This yields a body force density up to 10 N mm^{-3} in the racetrack head.

In Figures 6.7 to Figure 6.9, the direction of the forces acting on elements of the coils of the cloverleaf-racetrack magnet are shown. In these figures, the focus is on the direction of the forces for illustration purposes. They are calculated by summing up the body force densities over a plane. From Figures 6.7 and 6.8, it can be seen that direction of the forces is mostly in the z -direction, i.e. the poles attract each other. In Figure 6.9, the direction of the forces projected in the xy -plane is shown. From this, it can be seen that the straight section and the bridge of the cloverleaf are pushed outward. This also holds for the ears of the cloverleaf. This leads to a change of direction of the forces in the xy -plane at the cross-over point of the cloverleaf.

In Figure 6.10, the local maximum equivalent stress and axial strain are plotted in the coil-winding pack for a friction coefficient of 0.15. In Figures 6.10a and c, the racetrack coil is split up in four different segments, where the turns can slide relatively to each other. It was chosen to split the coil in a few parts, as modelling all turns was computationally too intensive. Modelling all turns would correspond to the dry-wound case if no-insulation technology is utilized. The cases where the coil is split-up in two, four and eight parts are considered to model the dry-wound case. By extrapolation it can then be determined if the magnet is below critical stress and strain values of the *ReBCO* for the dry-wound case.

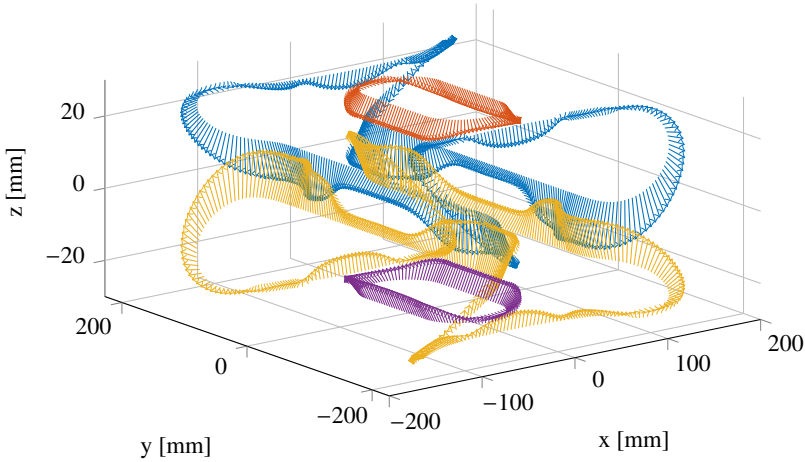


Figure 6.7: Direction of the Lorentz-forces acting on the coils of the cloverleaf-racetrack magnet.

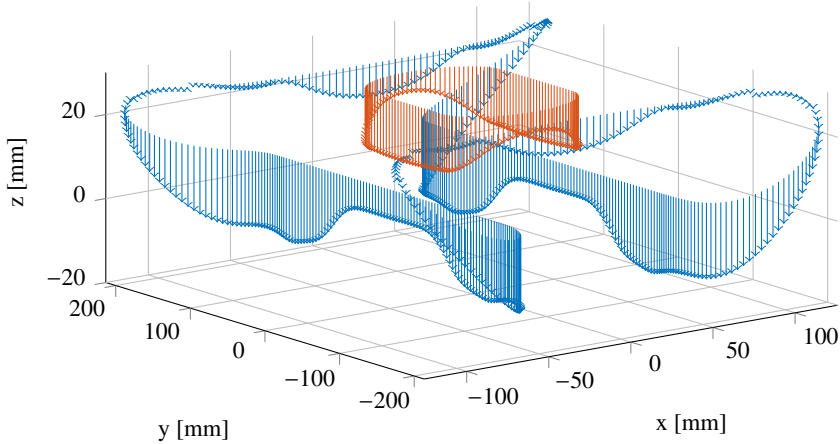


Figure 6.8: Averaged Lorentz-forces in the z -direction.

For the solder-filled case, both coils are modelled as a single block (see Figures 6.10b and d), where the turns cannot move relative to each other.

The examination of the equivalent stress plot reveals that the highest stress and strain values in the coils are concentrated within the inner region of the coil-end in the racetrack coil. To delve deeper into this observation, Figure 6.11 showcases the maximum equivalent stress and longitudinal strain as a function of the number of sections in the racetrack coil. The generated plots consider various friction coefficients to assess the impact of this parameter on the obtained results, as it is one of the significant unknowns in the mechanical modelling process. However, the investigation reveals that different friction coefficients do not yield substantially different outcomes.

For the solder-filled case, the stress approaches approximately 700 MPa, which is com-

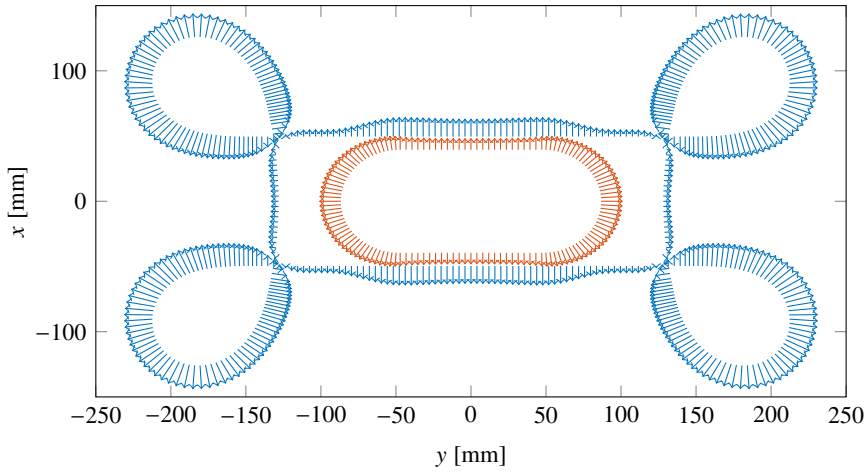


Figure 6.9: Projection of the averaged Lorentz force on the xy -plane.

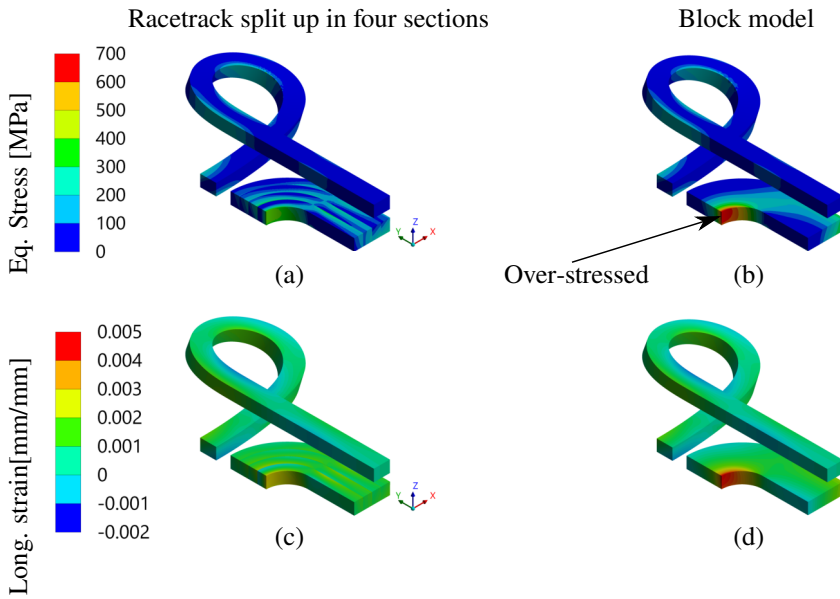


Figure 6.10: Simulation of the stress (a), (b) and strain (c), (d) in a quarter of the cloverleaf and racetrack coils, for a coil current of 2 kA. In (a) and (c), the racetrack is split in four different sections that can slide relative to each other. This is to simulate the individual turns in the coil windings. In (b) and (d), the racetrack coil is modelled as a block, which is representative for the solder-filled case. The maximum strain and stress are present on the inside of the racetrack coil-end.

parable to the maximum allowable stress that *ReBCO* tape can withstand, estimated to be 600 MPa to 800 MPa depending on the tape structure [29]. Consequently, the stress in the

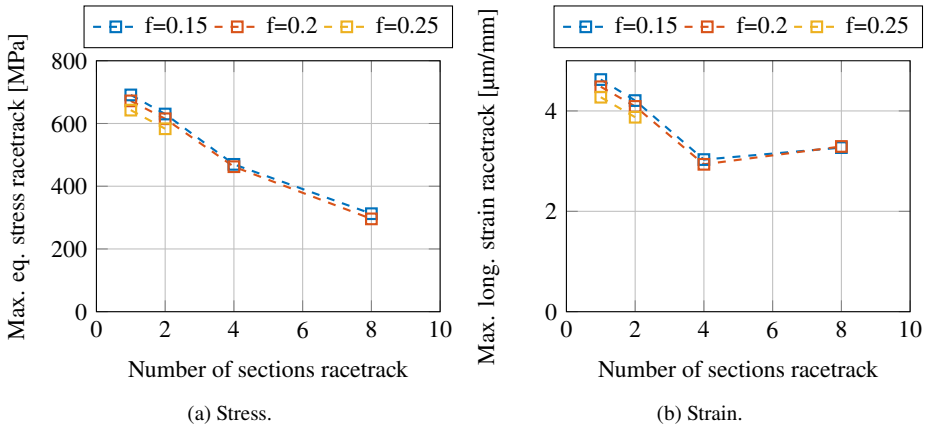


Figure 6.11: Maximum equivalent (Von-Mises) stress (a) and longitudinal strain (b) in the racetrack coil as function of the number of sections of said coil, for different friction coefficients f .

solder-filled case is right around the critical limit, posing a risk of irreversible damage to the tape. Conversely, the eight-way split racetrack coil remains below the critical stress limit, exhibiting a maximum stress of approximately 300 MPa. Extrapolating this trend, modelling each turn individually would yield even lower stress values. Therefore, according to the extrapolated finite element modelling results, the stress in the dry-wound coil remains well below the allowable stress. In the design of the cloverleaf-racetrack magnet, the solder-filled coils build up a large stress. However, this can possibly be mitigated by splitting up the racetrack coil-end into separate parts in order to reduce the stress.

The maximum longitudinal strain in the block racetrack coil is 0.46%, whereas in the eight-way split racetrack coil, it is 0.33%. The critical strain limit for *ReBCO* is reported to be between 0.5% and 0.7% [29]. Extrapolating to the dry-wound case, the predicted strain values are even lower. For the eighth-way partition, it can be seen that the longitudinal strain is slightly higher than for the four-way partition. This is due to a shift of location of the maximum strain from the inside of the racetrack to the interface between parts on the straight section. Consequently, in all cases, the strain remains below the limit, although the solder-filled racetrack coil offers little to no margin.

In contrast, the stress and strain levels in the cloverleaf coil remain significantly below these critical values. Therefore, from a mechanical perspective, the cloverleaf coil does not pose as critical of a concern as the racetrack coil. The most critical mechanical spot is identified as the inner bend of the racetrack coil. Consequently, in addition to meeting the time constant requirement, fully solder-filled coils are not suitable from a mechanical point of view in the current design.

The transverse stress and longitudinal strain are not the only two mechanical actors that can lead to damage to the *ReBCO* tape. Critical as well are transverse tensile force and shear forces occurring during cool-down and powering of the magnet, which can cause delamination. These two force components were not taken into account in the modelling yet.

The modelling assumed an ideal geometry of the conductor, however the shape of the

conductor used is never perfectly rectangular. In the following section, the practical issues of winding the cloverleaf coil with non-ideal tape geometry are discussed.

6.4 Winding of the cloverleaf coil

The geometry of the cloverleaf is unique and requires the development of a new winding method. The method presented here was initiated by Van Nugteren at CERN in 2017 [66]. To demonstrate the possibility of winding the cloverleaf geometry, five turns of stainless steel Roebel dummy cable were wound onto a glass-filled nylon winding mandrel. During the test it was found that it was quite challenging to align the conductor with the winding mandrel during the winding process. Here, an evolution of the winding procedure is presented, using a different shape of the winding mandrel, based on the differential geometry method discussed in Chapter 5. First, the experiences with the test windings are discussed. Then, a general overview of the winding procedure of a single turn is presented, before concluding with a description of the winding of the demonstrator cloverleaf coil consisting of eight turns of double *ReBCO* tape conductor. In Figure 6.12, the general coil layout is shown, with the ears of the coil A to D, straight sections A and B, and bridge sections A and B indicated.

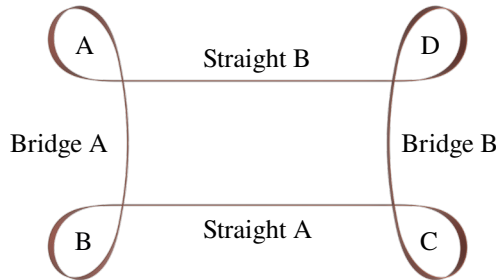


Figure 6.12: Render of the cloverleaf coil with the naming convention of sections of the coil indicated.

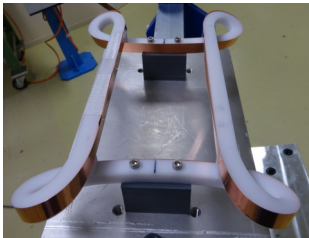
6.4.1 Development of the winding procedure

Before winding the *ReBCO* cloverleaf coil, several winding trials were carried out to investigate the winding procedure. A collection of images from the winding trials is shown in Figure 6.13. The first step was to design a new winding mandrel based on calculations using the differential geometry presented in Chapter 5. For this purpose, a cloverleaf ear was 3D-printed to test the geometry based on Bézier splines. This ear is shown in Figure 6.13a. It showed that a tape can be fitted perfectly onto the ear without any discernible gaps between the tape and the ear.

The geometry of the ear was then modified to accommodate a slightly curved bridge. The reason for the curved bridge is that it makes for a neater winding pack, as tape tends to become wavy when wound onto a perfectly straight section, as there is no force pushing the tape towards the winding mandrel. The first winding mandrel with this updated



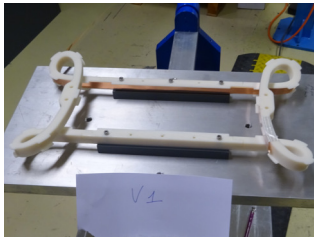
(a) First 3D-printed ear with stainless steel tape wrapped around.



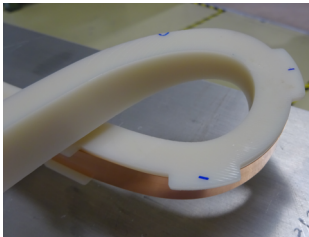
(b) First plastic winding mandrel with copper tape wound around it.



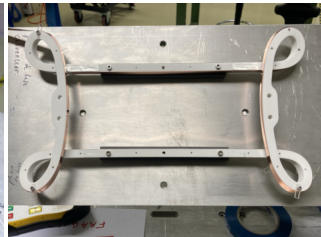
(c) Close-up of the ear.



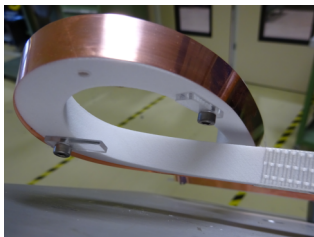
(d) Second plastic winding mandrel.



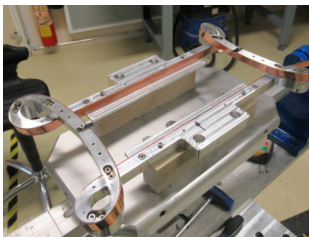
(e) Close-up of ear with guiding spacers.



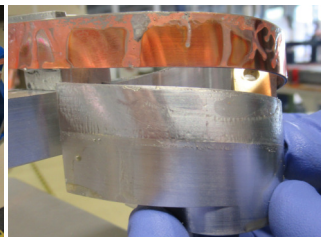
(f) Third winding mandrel.



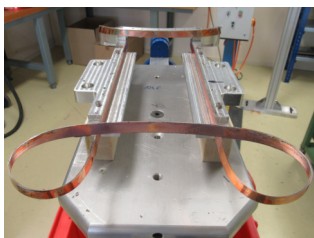
(g) Close-up of ear with plastic spacers to guide the tape.



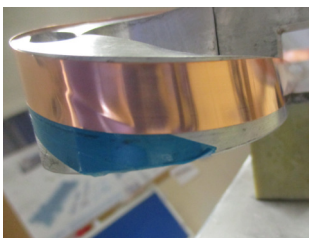
(h) Aluminium alloy winding mandrel.



(i) Removing the ear.



(j) Free floating cloverleaf shape.



(k) Buckling of the tape due to too high winding tension.



(l) Damage after copper tape slipped from the winding mandrel.

Figure 6.13: Images of winding trials performed for the cloverleaf coil.

geometry is shown in Figures 6.13b and 6.13c. This winding mandrel has almost the dimensions of the geometry of the cloverleaf-racetrack dipole presented in Section 6.2: only the straight section is a little shorter. As can be seen in Figure 6.13c, there is a discernible gap between the tape and the winding mandrel. At its widest, the gap is 2 mm. This shows that even though the cloverleaf design is completely free of bending in the hard-way bending direction, it does not guarantee a perfectly fitting geometry. A possible reason is that the bending energy of the tape must also be minimized to find the natural shape of the tape, as discussed in Chapter 5. In the end, it was decided to leave the shape like this, for time reasons. In practice, the gap is not a problem, as the first turn provides a good surface for the remaining turns to wind onto. However, for the cloverleaf-racetrack coil, it would be desirable to refine the design so that the winding mandrel provides a perfect fit. The design has to be modified to minimize the bending energy. Evidence that a good fit is possible is demonstrated by the ear of Figure 6.13a.

A second iteration of the winding mandrel was made. This version included spacers to guide the tape along the edges of the winding mandrel. However, the spacers caused difficulties in the ears. The tape would catch on them and crease, damaging it in the process. It was believed that this was due to the long length of the spacers. Therefore, a third 3D-printed plastic mandrel was made with holes where small spacers could be placed to guide the tape. This mandrel was also made out of several parts so that it can be removed after coil winding. However, the tape also tended to get stuck on these smaller spacers. It was therefore decided to remove the spacers altogether and manually push the tape into position.

Based on this latest design, an aluminium alloy winding mandrel was made by machining of the shape (Figure 6.13h). A metal version was required so that the coil or parts of the coil can be heated, allowing it to be soldered, if required. Once the coil or part of the coil has been soldered, the winding mandrel can be removed (see Figures 6.13i and 6.13j).

The winding tension is important. A too high winding tension leads to problems during the winding. Winding the copper strip with 32 N tension was doable in practice. Winding with 37 N tension was too much. With this winding tension, it was difficult to turn the winding mandrel, thereby making it more difficult to align the tape properly to the winding mandrel. Also, the tape would tend to buckle, as can be seen in Figure 6.13k. The difficulty in alignment also makes the tape prone to slipping off, as can be seen in Figure 6.13l. Some winding tests were carried out with copper-coated Hastelloy tapes. In this case, only a winding tension of 16 N could be used, as the stiffer tape made it more difficult to manoeuvre the winding mandrel.

6.4.2 Winding procedure for a single turn

For all winding tests, as well as for the winding of the demonstrator *ReBCO* cloverleaf coil, the same winding procedure was used. The tape is wound on a spool placed on top of the winding machine. This machine consists of a tensioner to exert a specific pull on the tape, which can provide a tension up to 37 N. This is the same machine used to wind the small pancake coils (see Chapter 3). The tape is then brought to the winding mandrel, onto which the cloverleaf-shaped coil was wound. Adhesive tape is used to attach the tape to the winding mandrel. From here, the winding process commences by rotating the winding mandrel on itself. The topology of the cloverleaf makes this operation a little more

complex than winding a flat coil, such as the small pancake coils presented in Chapter 3. Firstly, the three-dimensional geometry of the cloverleaf coil necessitates the ability of the winding mandrel to rotate in three axes. Secondly, due to the coil passing over itself in the cloverleaf topology, the winding process faces hindrances caused by the presence of the winding mandrel supports. This issue was resolved by employing two supports for the winding mandrel. If one support obstructs the winding path, it can be temporarily removed, while the other support ensures the coil remains adequately supported.

To illustrate the winding procedure, a series of images illustrating the winding of half a turn of a cloverleaf-shaped coil are presented in Figure 6.14. The image sequence begins at the point where ear B is almost completely wound and winding onto the straight section is about to commence (Figure 6.14a). The winding mandrel is held at an inclined angle of 30° with respect to the horizontal plane, and the beam supporting the winding mandrel is positioned 20° off-center from its neutral position.

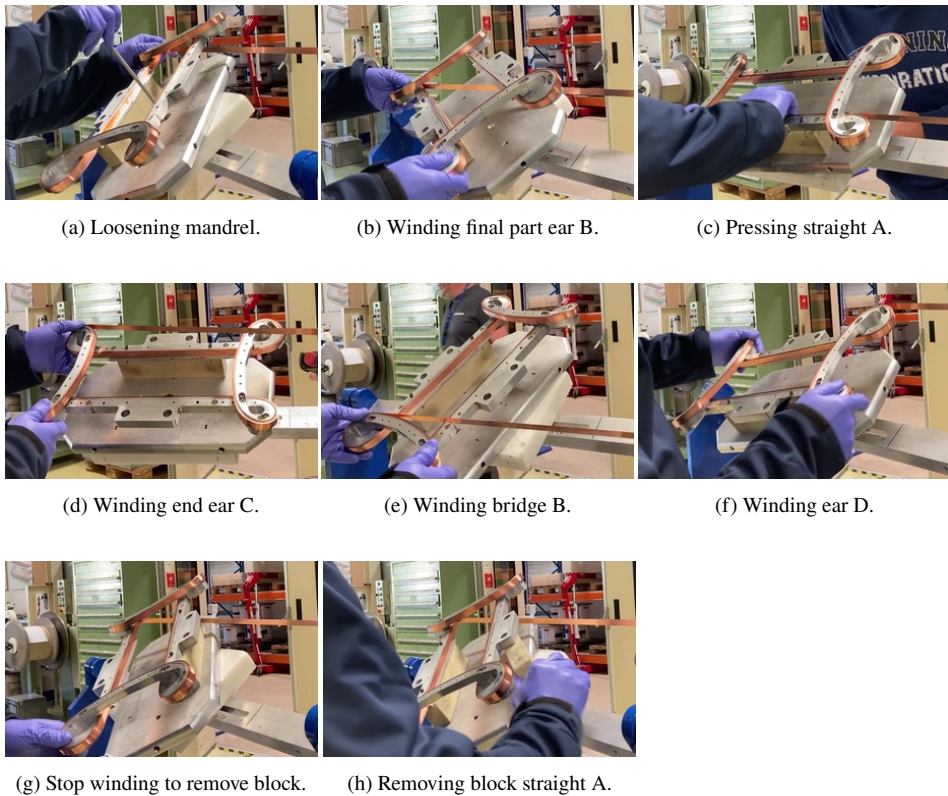


Figure 6.14: Sequential images of the winding of half a turn of a cloverleaf coil. In these pictures, a 0.1 mm thick copper strip is used to perform the winding.

By rotating the winding mandrel, the tape passes underneath (Figure 6.14b) and is carefully placed onto the straight section of the coil. Manual pressure is applied to push the tape onto the winding mandrel (Figure 6.14c). Subsequently, the tape is wound around the ear of the coil. This is achieved by simultaneous rotation of the winding mandrel and

the supporting beam. During the winding of the ear, the beam is rotated 40° in the opposite direction from its neutral position to ensure that the tape makes tangential contact with the winding mandrel. At the start of the ear winding, the tape is pressed against the straight section (Figure 6.14d). The tape is then wound onto the bridge section (Figure 6.14e), and then wound on ear D (Figure 6.14f).

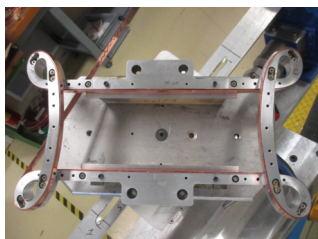
The rotation of the winding mandrel continues, and the supporting beam is returned to its initial position during the winding of the second ear. In Figure 6.14g, it can be seen that the block beneath the right-hand straight section obstructs the path of the tape. To address this, another block is placed beneath the opposing straight section, and the original block is removed (Figure 6.14h). Once this is completed, the block under the other straight section is removed, enabling the tape to pass underneath. This completes the cycle, and the procedure returns to the situation depicted in Figure 6.14a to continue the winding process.

6.4.3 Cloverleaf coil winding with *ReBCO*

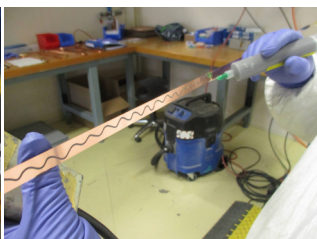
After the winding trials have shown that it is possible to wind the cloverleaf geometry, the next step was to wind a cloverleaf coil with *ReBCO* tape. Figure 6.15 shows the sequential winding stages of the *ReBCO* demonstrator coil. A spool containing a single strip of copper 12 mm wide and 0.1 mm thick is placed on the winding machine. This is the same tape used in the winding tests. The copper tape is attached using tape to the winding mandrel on ear A. Then twenty turns of copper are wound onto the winding mandrel using a winding tension of 32 N (Figure 6.15a). During the winding process, $\text{Sn}_{62}\text{Pb}_{36}\text{Ag}_2$ solder paste is added (Figure 6.15b). This paste was also used for the soldered pancake coils (see Chapter 3). But there it is used between the turns of the *ReBCO* tape. Here, it is only used in between the first twenty copper turns. In all the ears, the tapes are tightly stacked, although not all are perfectly aligned at the same height (Figure 6.15c). This discrepancy arises from the manual alignment of the tape, which is a drawback of the manual winding process.

The twentieth turn of copper tape in the winding process concludes at the same starting point where the winding has commenced. Subsequently, the coil and winding mandrel are detached from the three-axis winding bench and transferred to an aluminium alloy plate. This aluminium alloy plate is heated, causing the solder paste to melt. Once the plate is cooled, the solder solidifies, creating a sturdy inner structure of the coil. To achieve the heating, heaters were positioned inside the aluminium alloy plate and supplied with power. Additional aluminium alloy plates with embedded heating elements were placed on the bridge section. To facilitate the heating process, a heat gun is directed along each ear of the cloverleaf coil. It is worth noting that heating introduces a disadvantage, as the solder undergoes shrinkage, leading to the formation of gaps between the turns. This effect is particularly evident on the bridge and ears of the coil, as shown in Figure 6.15d and 6.15e. Despite the initial tight windings prior to soldering, gaps between the turns became noticeable after the heating process.

After heating, the coil and the winding mandrel are repositioned on the three-axis winding bench. The subsequent step involves winding the superconducting part of the coil, which consists of two *ReBCO* tapes wound in parallel to form a cable. The tape properties of the employed tape in the reduced-turns cloverleaf are listed in Table 2.3 in Chapter 2.



(a) Winding of the first turns of copper tape.



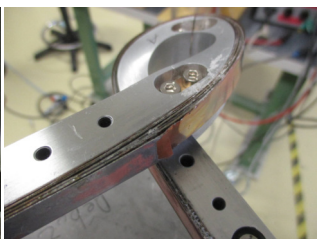
(b) Adding solder paste to the copper tape surface.



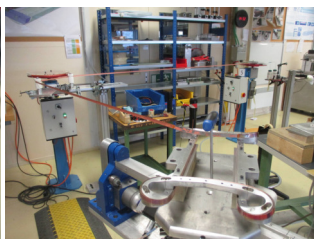
(c) Close-up of ear D after winding 20 turns of copper tape.



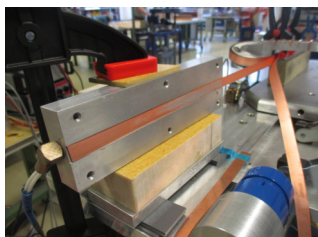
(d) Close-up of ear D after soldering.



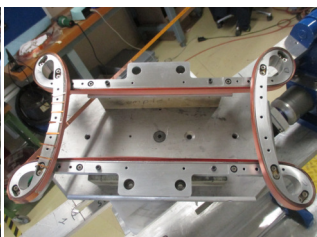
(e) Bridge section after solder repair.



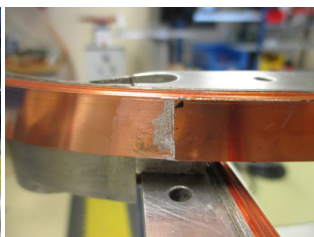
(f) Winding set-up of the *ReBCO* cable.



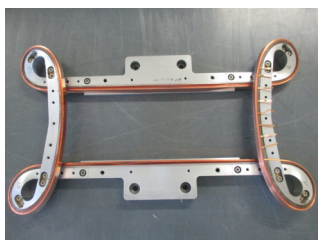
(g) Fabrication of the second *ReBCO* tape to copper tape joint.



(h) Winding of the last 20 copper tape turns.



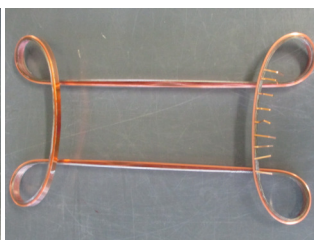
(i) Close-up of the joint after cutting off the remaining copper.



(j) Winding mandrel removed from bench.



(k) Removal of individual parts of the winding mandrel.



(l) Final resulting coil.

Figure 6.15: Sequential pictures of the winding of the *ReBCO* cloverleaf coil.

The two tapes are wound in a face-to-face configuration, with the superconducting layers facing each other. To create the joint, the end of the copper tape is inserted between the two superconducting tapes, and the same solder is applied between the copper tape and the superconducting tape. The joint is then placed within an aluminium block, which is heated to facilitate the melting of the solder paste.

The superconducting cable is subsequently wound onto the bridge section, with additional $\text{In}_{97}\text{Ag}_3$ solder applied to secure the cable and fill any gaps on the bridge. Two aluminium plates are placed on the bridge and heated to melt the solder paste. However, the repair did not succeed completely, and gaps were still visible (see Figure 6.15e). The superconducting cable is further wound onto the winding mandrel. Figure 6.15f presents the complete configuration of the winding mandrel on the three-axis machine, with the superconducting tapes originating from the spools on the winding machines.

A total of eight turns of ReBCO cable are wound, with a winding tension of 16 N and 37 N. The 16 N is used on the bridges and ear, and the 37 N is used on the straight sections. The lower tension allows the tape to be manoeuvred around the relatively complex three-dimensional ear geometry, and the higher tension on the straight section is intended to mitigate waviness there. This is done in combination with manual pushing on the conductor for alignment. However, despite these efforts, waviness persists on the straight sections. A possible cause is that pulling on the tape does not press it against the winding mandrel due to a lack of local curvature, allowing for waviness of the tape.

Next, a joint is made with the copper strip in a similar way to the first joint (Figure 6.15g). From there, another twenty turns of copper tape are wound (Figure 6.15h). The copper tape is then soldered to itself on bridge B. It is not soldered on bridge A, so that the other solder does not melt. It also means that straight A has one more layer of copper tape on the outside than straight B. Two blocks of aluminium are then used again to melt the solder paste. The result can be seen in Figure 6.15i. Next, the winding mandrel is removed from the three-axis machine and disassembled piece by piece to remove the coil from the winding mandrel (Figures 6.15j to 6.15l). Table 6.4 lists the main properties of the reduced-turns cloverleaf coil. In the following section, the measurement campaign and test results of the cloverleaf coil are discussed.

6.5 Performance of the reduced-turns cloverleaf coil

The test of the coil was conducted within a liquid nitrogen bath at a temperature of 77 K. The objective was twofold: firstly, to ascertain whether any degradation had taken place during the winding process, and secondly, to investigate no-insulation technology in a cloverleaf coil geometry. This section provides a detailed account of the simulated electromagnetic performance of the coil, the experimental set-up employed, and the resulting finding and conclusions.

6.5.1 Simulated coil performance

In order to estimate the critical current of the reduced-turns cloverleaf coil, its geometry has been modelled using RAT software and the maximum magnetic field on the conductor has been calculated. A conservative estimation method was employed, assuming that the coil

Table 6.4: Specification of the reduced-turns cloverleaf coil.

Property	Value
Overall length (ears included)	420 mm
Overall width (ears included)	240 mm
Overall height (ears included)	32 mm
Number of copper tapes inside section	20
Number of <i>ReBCO</i> tapes in cable	2
Number of <i>ReBCO</i> cable turns	8
Number of copper tapes outside section	20
Total length copper tape inside and outside	32 m
Length tape single turn	1.6 m
Length of superconducting cable	12.8 m
Total length <i>ReBCO</i> tape used	25.6 m
Estimated coil critical current	490 ± 50 A
Maximum magnetic field on conductor at coil I_c	0.28 T
Calculated self-inductance	58 μ H

attains its overall critical current when the critical current is locally reached at a specific point with the highest magnetic field. The properties of the critical surface of the tape used are described in Section 2.3.

Iterative calculations were performed to determine the critical current of the coil. The process involved adjusting the current and computing the corresponding critical current. If the calculated critical current exceeded the provided current, the current was increased in the model. This iterative procedure continued until the provided current and critical current were equal. The local magnetic field was assumed to be perpendicular or parallel to give a range of possible critical currents. This approach was used in Section 3.5 as well to estimate the critical current of the pancake coils. Using this approach, a predicted coil critical current of 490 ± 50 A was obtained, with a maximum magnetic field of 0.28 T at 490 A acting on the conductor.

6.5.2 Experimental set-up

The coil is fixed on a flat plate made out of G10 epoxy resin laminate by clamping it between blocks, which can be moved in slots in the plate and were fixed in position by tightening screws in the blocks from the underside. No outside support was required due to the low number of turns and thus low Lorentz force. The coil was clamped on the bridge sections and straight sections. The blocks on the bridge were made of G10 like the plate. On both straight sections, there were copper blocks to clamp them from one side and a G10 plate on the other side. Aluminium alloy U-shaped pieces were used for the clamping of the copper blocks and the epoxy resin plate by tightening a screw from the side of the aluminium alloy piece. The copper blocks are also used as current terminals.

The cloverleaf coil is instrumented with various devices. On bridge A and both straight sections, voltage taps are placed between each turn. Hall probes are placed at various places: in all cloverleaf ears, next to the conductor on bridge B and straight section B. A

picture of the cloverleaf coil, its mounting and instrumentation, is shown in Figure 6.16.

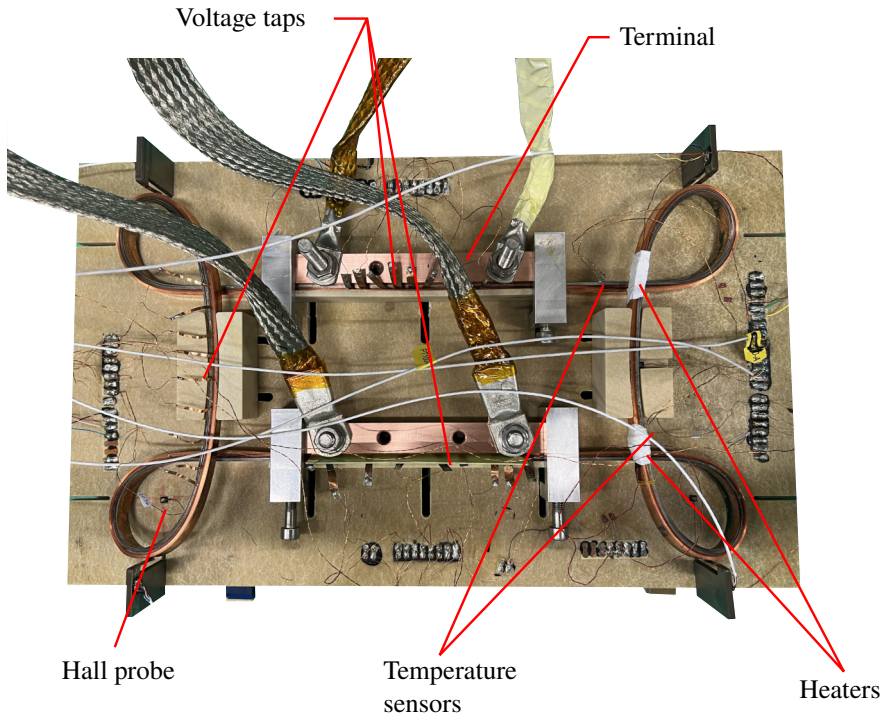


Figure 6.16: Picture of the cloverleaf mounted on a glass-reinforced epoxy resin flat plate, with current terminals and instrumentation indicated.

6.5.3 Measurement results

The reduced-turns cloverleaf coil was tested in an open bath of liquid nitrogen at 77 K in self-field. These conditions were chosen as they allow for a relatively easy assessment of the coil performance, and for observing if any degradation of the coil occurred during the winding process, cooling down and test. In Figure 6.17, the voltage across the coil as measured over the bridge section is plotted. The voltage criterion threshold of $10 \mu\text{V m}^{-1}$ to determine the coil critical current is indicated as well. From the figure, it can be seen that the critical current of the coil is $490 \pm 5 \text{ A}$, matching the predicted critical current. This indicates that the coil did not degrade during winding, showing that it is feasible to wind a cloverleaf geometry coil of this size.

It is explicitly noted here that strictly speaking the critical current and n -value of a piece of superconductor is well defined only in the case of uniform magnetic field and constant temperature between the voltage taps positioned on the conductor and using a generally accepted voltage criterion of $10 \mu\text{V m}^{-1}$ or $100 \mu\text{V m}^{-1}$. Obviously the uniform

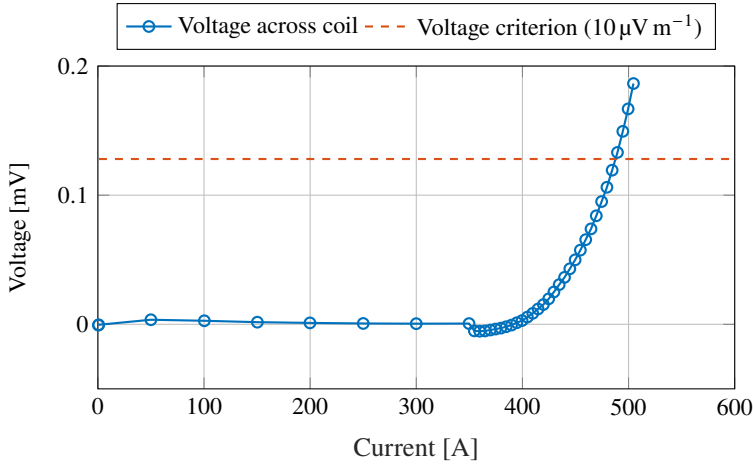


Figure 6.17: Voltage across the cloverleaf coil as function of current.

magnetic field condition is not fulfilled in the case of measuring the voltage across turns or across an entire coil. Nevertheless the terms critical current and associated n -value may be used provided the circumstances are clear, i.e. turn critical current / n -value or coil critical current / n -value. Also the use of a certain voltage per meter criterion is arbitrary. The relation to the real conductor critical current is not always obvious and conclusions must be formulated with care. Here, for the cloverleaf coil case, the measured coil and turn critical currents are used for showing a large spread in results depending on coil terminal positions and turns in the coil.

The current terminals are positioned at opposing sides of the straight sections, in order to study different current redistributions throughout the coil. The terminal configurations are denoted as configuration A and configuration B. Figure 6.18 depicts a schematic representation of these positions. The placement of the terminals relative to the straight section significantly influences the distribution of electrical current within the coil, owing to the winding direction and the initial position of the coil windings. Also shown are configuration C, which is the layout for the demonstrator cloverleaf, and configuration D represents a recommended adaptation to the design with respect to the terminal positioning. The advantage of configuration D is that the terminals are placed on a bridge section, thereby being less of a disturbance to the magnetic field quality on the straight section. It also ensures that all conductor length is used, and that there is no asymmetry in the current flow between the halves of the coil.

The winding starts at bridge A and progresses in a counter-clockwise direction along the coil. In configuration A, the first turn reaches proximity to the terminal after completing a quarter turn. Conversely, in configuration B, the first turn is in proximity to the terminal only after completing three quarters of a full turn. This pattern is also applicable to the last turn. Consequently, configuration A omits half a turn, while configuration B omits one and a half turn.

The disparities in terminal position between the two configurations manifest as variations in the current distribution within the coil. This can be seen in the VI-curves of the

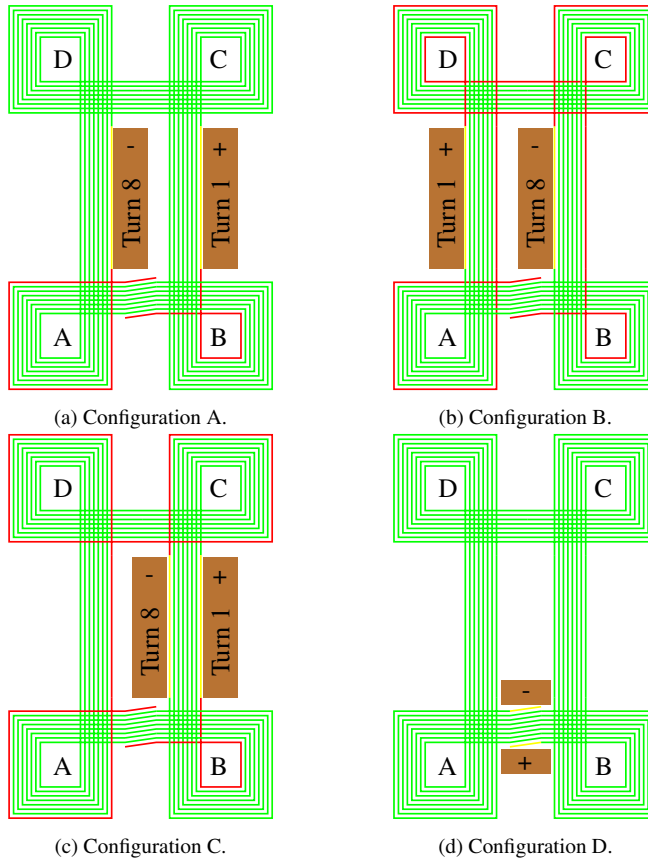


Figure 6.18: Four configurations with different terminal positions on the cloverleaf coil. The green color indicates the presence of current flow along the conductor, while the red color denotes the absence of current flow. The yellow color signifies the flow of current either from the terminal into the turn or vice versa. Configurations A and B were tested on the reduced-turns cloverleaf coil, configuration C is from the current design of the demonstrator cloverleaf coil, and configuration D is a recommended adaptation to the design regarding the terminal positions for the demonstrator cloverleaf coil.

coil, magnetic field response of the Hall probe and the effective time constant of the coil.

During the measurement campaign, different current levels are applied to the two configurations. In configuration A, the current is intentionally limited to prevent potential damage caused by over-current. This cautious approach is adopted to ensure that the coil would not quench and be damaged during the first measurements.

At first, configuration B was also tested at these lower current levels. After these tests were concluded, it was decided to push the coil to its limits. It was found that the coil could be operated and protected at these higher current levels. The results of the measurements with higher current levels are presented in this thesis. The higher current level allowed for a more comprehensive assessment of the coil's performance and ensured that its operating

limits were thoroughly examined. Configuration A was not tested again at these higher currents.

The VI-curves, representing the voltage-current characteristics, of each turn in the cloverleaf coil were analysed, as depicted in Figure 6.19. The curves were obtained from different sections of the coil windings and two terminal configurations.

For the results in configuration A, the current steps at relatively low currents are larger than at higher currents. Notably, a voltage drop in all the turns is observed immediately upon reducing the step size (Figures 6.19a, 6.19c and 6.19e). The settling time for the initial low current measurements is approximately 36 s, while for the subsequent measurements at higher currents, it is reduced to approximately 26 s. This indicates that the settling time itself is not a significant contributing factor to the observed voltage drop. If it were, a higher voltage can be expected, as less current has settled in the superconductor.

One possible explanation for this voltage drop is the influence of thermal effects. The magnitude of the voltage drop, in the order of microvolts, suggests that it can be attributed to thermal phenomena. In order to mitigate the voltage drops, a decision was made for configuration B to adopt small current steps even at relatively low currents. This modification was implemented with the aim of achieving more consistent VI-curves across the range of current levels.

The analysis of the VI-curves reveals significant variations between different measurement locations and terminal configurations in the cloverleaf coil. This indicates that the current distribution in the coil is not homogeneous. This leads to a certain degree of complexity in determining the apparent critical current and n -value per turn.

In Table 6.5, the critical current values for configuration B are presented for each turn, considering two electric field criteria: 10 and 100 $\mu\text{V m}^{-1}$. The chosen length for this analysis is 1.6 m. It can be observed that the apparent critical current can vary by up to 277 A per turn, emphasizing the substantial variability across the coil.

Table 6.5: Relation between the apparent turn critical current and both turn number and location, for criteria of 10 and 100 $\mu\text{V m}^{-1}$, for configuration B, within an uncertainty of ± 10 A.

Turn number	I_c at bridge A [A]	I_c at straight A [A]	I_c at straight B [A]
	10 100 $\mu\text{V m}^{-1}$	10 100 $\mu\text{V m}^{-1}$	10 100 $\mu\text{V m}^{-1}$
1	560 620	420 490	430 500
2	400 480	550 580	270 510
3	540 580	480 570	550 610
4	470 590	540 610	530 600
5	560 630	570 630	560 640
6	560 650	550 620	530 580
7	530 590	530 600	500 560
8	510 560	400 560	520 600
Full coil	470 580	470 580	450 570

The significant variation arises due to the challenges in interpreting the VI-curves accurately. The non-insulated nature of the coil, along with its relatively long and non-uniform geometry, makes it that the measured voltage difference is not solely indicative of the voltage difference over a particular turn, but rather merely reflects the voltage differ-

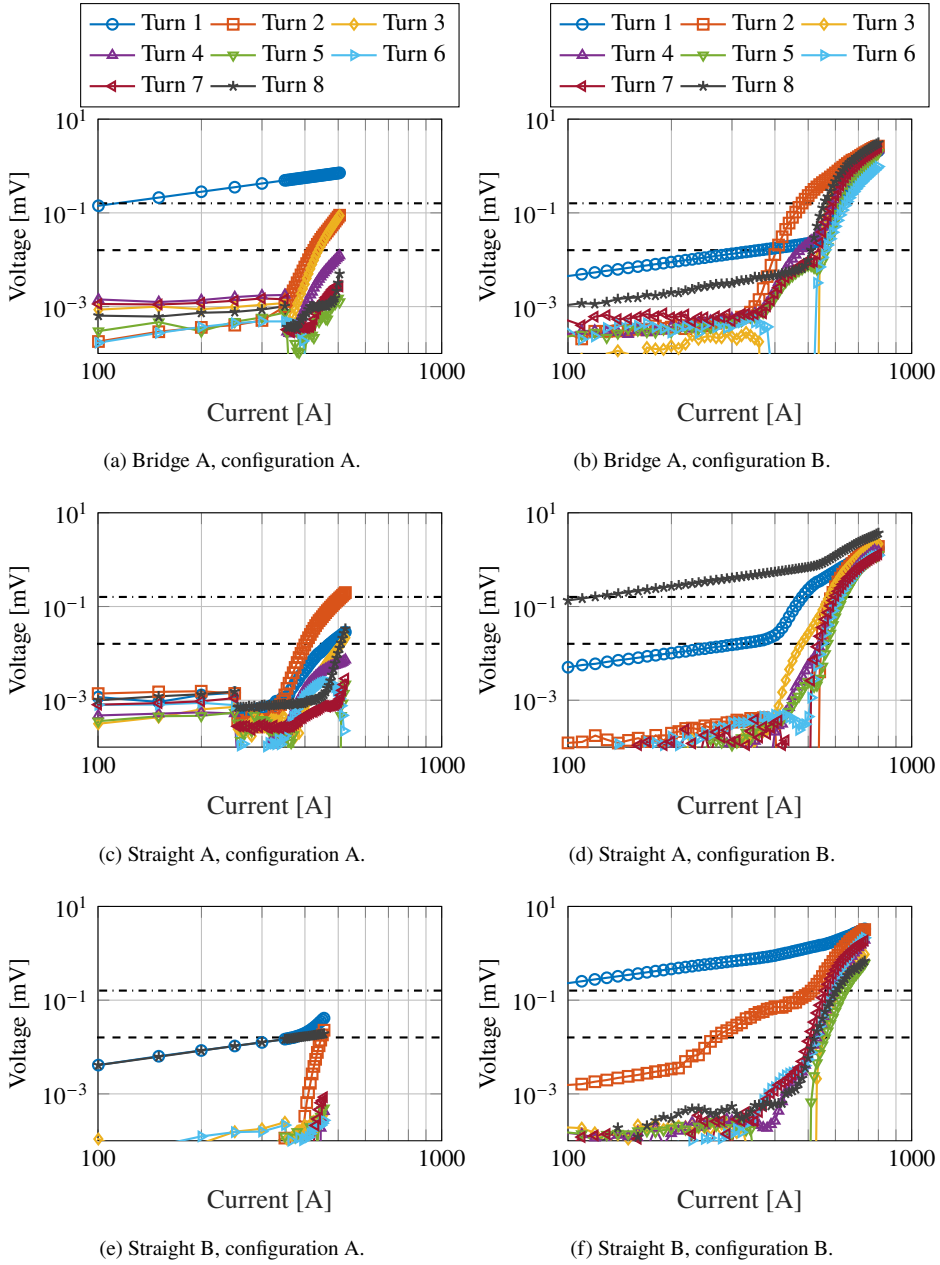


Figure 6.19: Measured VI-curves of each turn of the demonstrator cloverleaf coil in different sections of the coil and for the two different terminal configurations. The dashed line and dashdotted line indicate the voltage per length criteria of $10 \mu\text{V m}^{-1}$ and $100 \mu\text{V m}^{-1}$ respectively.

ence between the two voltage taps. As a result, Ohm's law, a fundamental assumption in

the voltage-current relation, is not applicable in this scenario.

The n -value is also subject to significant discrepancies across different turn numbers and locations, as demonstrated in Table 6.6. However, for the overall coil, the critical current values and n -values appear to exhibit greater consistency across the three locations.

Table 6.6: Relation between the n -value and both turn number and location, for criteria of 10 and $100 \mu\text{V m}^{-1}$, for configuration B, within an uncertainty of ± 5 .

Turn number	n -value at bridge A 10 $100 \mu\text{V m}^{-1}$	n -value at straight A 10 $100 \mu\text{V m}^{-1}$	n -value at straight B 10 $100 \mu\text{V m}^{-1}$
1	31 19	23 11	6 8
2	19 10	12 18	6 8
3	8 22	12 15	41 14
4	7 16	22 17	14 20
5	19 17	29 13	20 11
6	20 13	30 13	25 19
7	30 20	28 14	24 18
8	25 20	7 16	21 11
Full coil	10 13	11 13	9 13

The voltage-current relation was also determined across the terminals for the two configurations. The results are shown in Figure 6.20. The total resistance of the coil over the terminals in configuration A was $260 \pm 10 \mu\Omega$ and in configuration B it was $200 \pm 10 \mu\Omega$. By subtracting the linear component, the n -value can be determined, yielding 3.0 for configuration A and 3.2 for configuration B. The n -value is lower than measured across the full coil, which is expected, as now also the copper shunts inside and outside the coil winding pack are taken into account.

As discussed above, the large spread in measured critical current and n -factor across the individual turns stems from the fact that the magnetic field conditions over the voltage taps are not uniform. Moreover, in a non-insulated coil the current can flow between the turns all over the coil, leading to wildly different current distributions, and therefore measured critical currents and n -factors locally. Therefore, it is not very useful to define a local turn critical current and n -factor, as indicated from the wide spread in results. However across the full coil, the conditions are more uniform as there is a clear entry and exit point of the current at the terminals. This yields values of the critical current and n -factor that lie closer together when measured at different positions.

During the measurement campaign, two instances of quenching occurred in the coil. The first quench occurred at a current of 800 A, while the second quench occurred at 730 A. Following the second quench, the coil was successfully re-energized and reached a current of 800 A without experiencing further quenching.

Figure 6.21 displays the voltage-time relation for the two quench events. In the first quench, it was determined that turn 2 in the coil experienced the quench. However, in the second quench event, it is unclear in which specific turn in the coil the quench initiated. In an attempt to purposely trigger a quench, both heaters were activated, while the coil was subjected to a current of 800 A. However, in this particular scenario, the coil did not undergo a quench event.

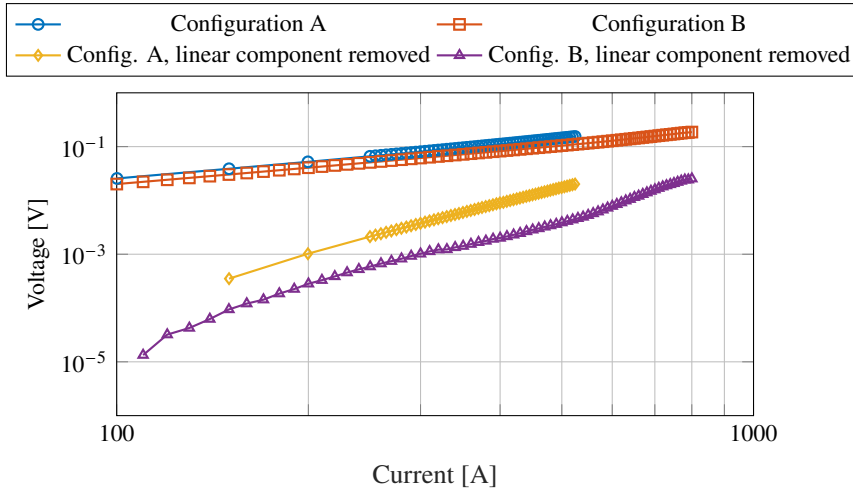


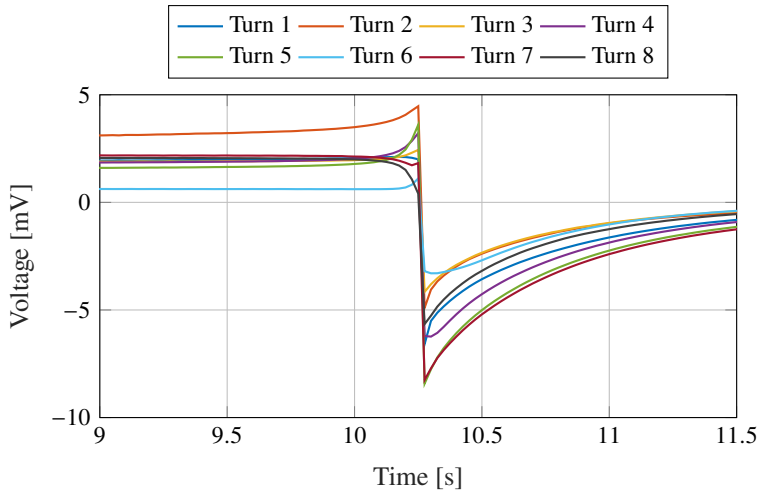
Figure 6.20: Voltage-current relation of the cloverleaf coil, measured across the terminals for the two terminal configurations.

The measured magnetic field demonstrates a clear dependence on the terminal configuration, primarily stemming from the dissimilarities in the current path between the two cases. Figure 6.22 illustrates the magnetic field measurements acquired near bridge B using a Hall probe, presenting the relation with current for the two terminal configurations.

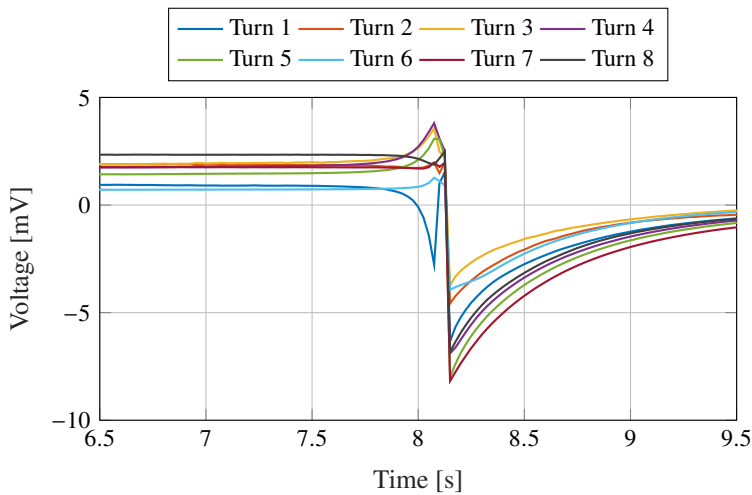
During each measurement, two thermal cycles were present. In configuration A, a distinct pattern emerges where, after each successive measurement within a thermal cycle, an offset magnetic field is evident at zero current. However, following the completion of a thermal cycle, this residual magnetic field disappears, returning to baseline levels. Similarly, in configuration B, the same effect is observed, but with a notable distinction. By powering the coil up to 800 A and subsequently deactivating the power supply, all residual magnetic fields are effectively eradicated without necessitating a full thermal cycle.

The linear part of the magnetic field curve displays distinct slopes for the two configurations. For the virgin curves, the slope for configuration A is $263 \pm 9 \mu\text{T A}^{-1}$ and for configuration B is $215 \pm 3 \mu\text{T A}^{-1}$. For the fully magnetized curves, the slope for configuration A is $236 \pm 1 \mu\text{T A}^{-1}$ and for configuration B is $198 \pm 1 \mu\text{T A}^{-1}$. Notably, the slope for configuration B is approximately 0.82 ± 0.04 times lower than that of configuration A. Considering that the current does not traverse the conductor at bridge B only through 6 out of the 8 turns, a reduction factor of 0.75 would be expected. Thus, the observed reduction in slope aligns well with the expectations based on the terminal configuration. This finding supports the plausibility of the terminal configuration as an explanatory factor for the observed variation in the magnetic field measurements. The differences in the current path caused by the placement of the terminals on opposing sides of the straight sections largely affect the magnetic field distribution near bridge B, leading to the discrepancy in slopes between the two configurations.

As with any non-insulated coil, this cloverleaf coil also has a significant time constant. To determine the effective time constant, the magnetic field response is examined by fitting



(a) Quench at 800 A.

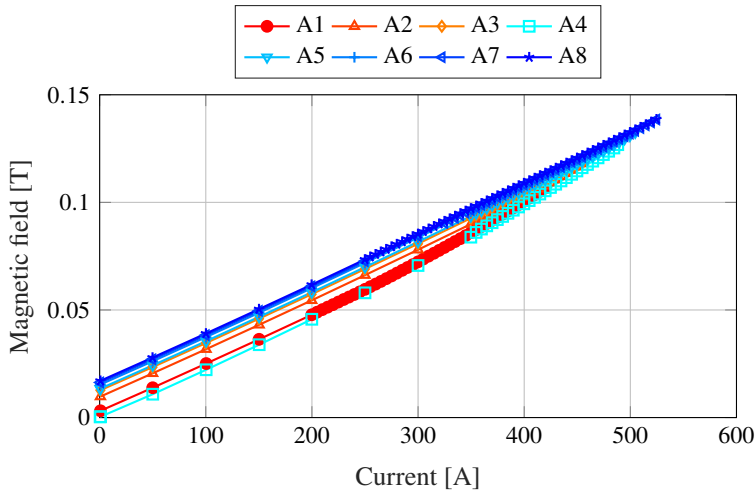


(b) Quench at 730 A.

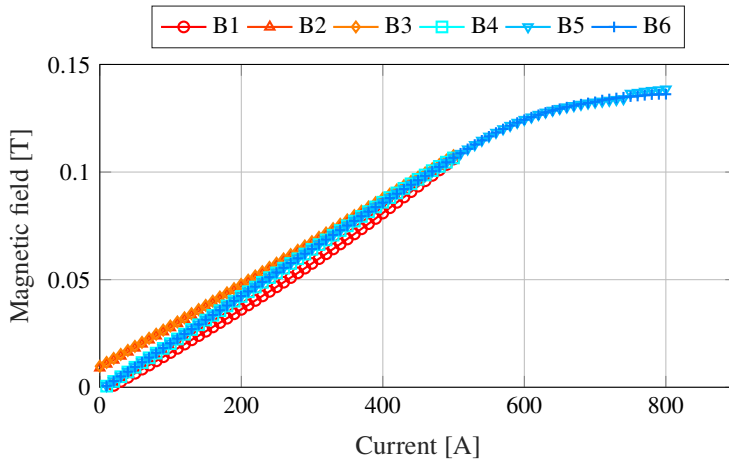
Figure 6.21: Voltage as function of time for each turn of the two quenches that occurred during the cloverleaf measurement campaign.

a single exponential decay to the data. Due to the small size of the system, this yielded more consistent results than a double exponential decay. For configuration A, the effective time constant is 2.6 ± 0.2 s for current steps in the 5 A to 30 A range.

For configuration B, the effective time constant was investigated as a function of current by analysing the response of the Hall probes. Figure 6.23a illustrates the magnetic field response recorded by a Hall probe placed in the centre of each of the four ears, in response to a step change in current of 50 A.



(a) Configuration A.

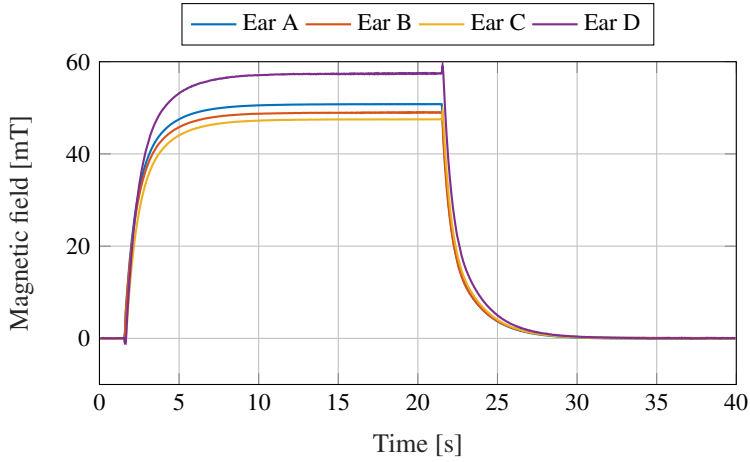


(b) Configuration B.

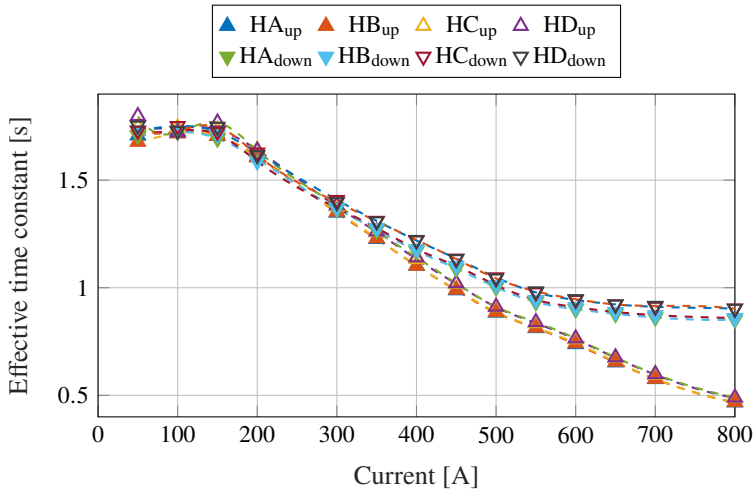
Figure 6.22: Measured magnetic field as function of current for two different terminal configurations.

By repeating this fitting for various magnitudes of current steps, the results are presented in Figure 6.23b. This graph displays the effective time constants corresponding to different current step sizes for each ear location. From this it can be seen that the effective time constant decreases with increasing current step size. Also around 400 A, the effective time constant of the cloverleaf coil for the step up starts to deviate to that of the step down. This is because less current can settle in the superconductor when I_c is reached.

Furthermore, it is evident that the effective time constants differ between ears A and B, and C and D, particularly during the step-down phase. One possible explanation for this discrepancy is the splitting of the normal flowing current, which can bifurcate and



(a) Magnetic field response to a current step of 50 A.



(b) Time constant as function of current.

Figure 6.23: Magnetic field response to a current step and effective constant versus current as for configuration B.

flow in either a clockwise or counter-clockwise direction between the two terminals. In configuration A, the current would flow opposite to the coil winding pack in ears A and B, and in the case of configuration B, the current would flow in the opposite direction compared to the coil winding pack in ears C and D.

To investigate this phenomenon more closely, a detailed examination of the magnetic field response immediately following a current step is necessary. Figure 6.24 presents the Hall probe response in the four ears for a step-up in current. During this time period, the oscillatory behaviour of the power supply can be observed as it adjusts to the requested

current level, taking approximately 15 ms to settle.

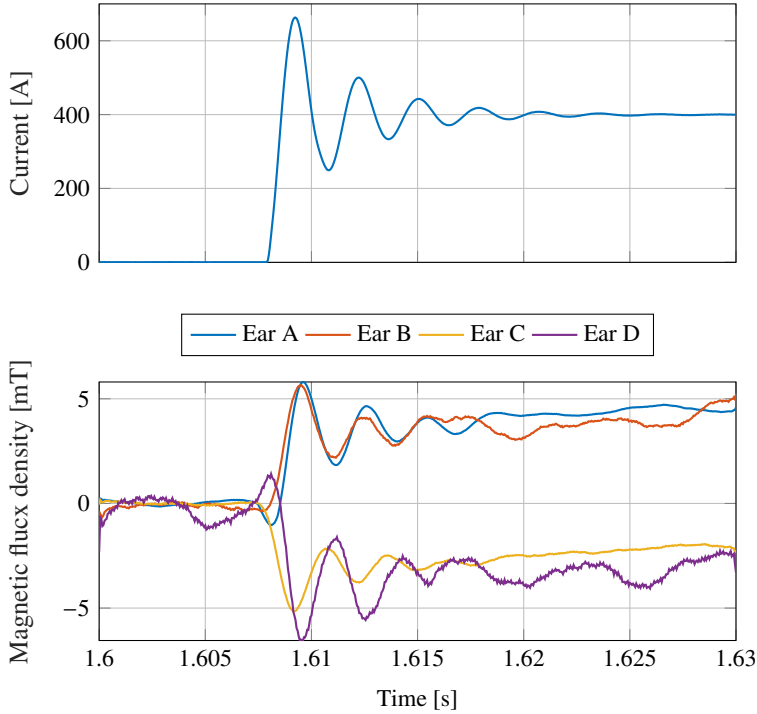


Figure 6.24: Hall probe response to a current step as a function of time in the ears A-D. In the top graph, the current is plotted. In the bottom graph, the Hall probe response is shown as function of time.

Of greater significance is the observation that the Hall probe responses of ears C and D exhibit negative values, with magnitudes opposite to those of ears A and B. This finding indicates that the current is divided between the two halves of the coil, flowing from one side to the other. The splitting of the current between the coil halves becomes evident in the opposite polarities observed in the Hall probe responses of ears A and B compared to ears C and D.

Using the effective time constant and the relation $\tau_{\text{eff}} = L/R$, an estimation of the mean transverse resistance between turns in the coil can be made. Taking the predicted self-inductance value of $58 \mu\text{H}$, the effective resistance is $64 \pm 10 \mu\Omega$ for $\tau_{\text{eff}} = 0.9 \text{ s}$. Using this, an extrapolation can be made what the time constant would be for the larger cloverleaf coil introduced in Section 6.2. This coil has ten times the number of turns compared to the small demonstrator coil. Assuming the transverse resistance is then ten times as large, and the inductance is 5.16 mH , this leads to a time constant of $\tau_{\text{eff}} = L/R = 8 \text{ s}$.

6.6 Outlook towards 20 T cloverleaf-type dipole magnets

A conceptual design of a 10 m long 20 T dipole is shown in Figure 6.25. The design is based on an early design by Van Nugteren [50]. It consists of two poles, each containing a double cloverleaf and double racetrack coil, making for a total of eight coils in the magnet. It uses a stacked tape cable of 12 mm wide and a critical current density of 700 A mm^{-2} . Typical non-Cu current densities of *ReBCO* tape can be over 2000 A mm^{-2} to 3000 A mm^{-2} at 18 T and 4.2 K [33]. The chosen current density is thus conservative. The total length of the cable is 5.9 km, and the length of the cable in the cloverleaf coil is 651 m and 812 m for the racetrack coil. These lengths can be bought commercially in a single piece. The cloverleaf coils each comprise 28 turns and the racetrack coils 40 turns, yielding a coil pack thickness of 56 mm for the cloverleaf coil and 80 mm for the racetrack coil, and giving an average length per turn of 23 m for the cloverleaf coil and 20 m for the racetrack coil. The total self-inductance of the magnet is 366 mH. Using an average resistivity of $3.0 \mu\Omega \text{ m}$, as found with the small coil measurements for the resistivity at 4.2 K and self-field, the transverse resistance of the coils would be $0.6 \mu\Omega$ for the cloverleaf coil and $1.0 \mu\Omega$ for the racetrack coil. The effective time constant is then taken from the spectral decomposition and yields 1000 hour. This is of course way too long for a charging time of an accelerator magnet. Thus for large scale magnets no-insulation technology is not feasible.

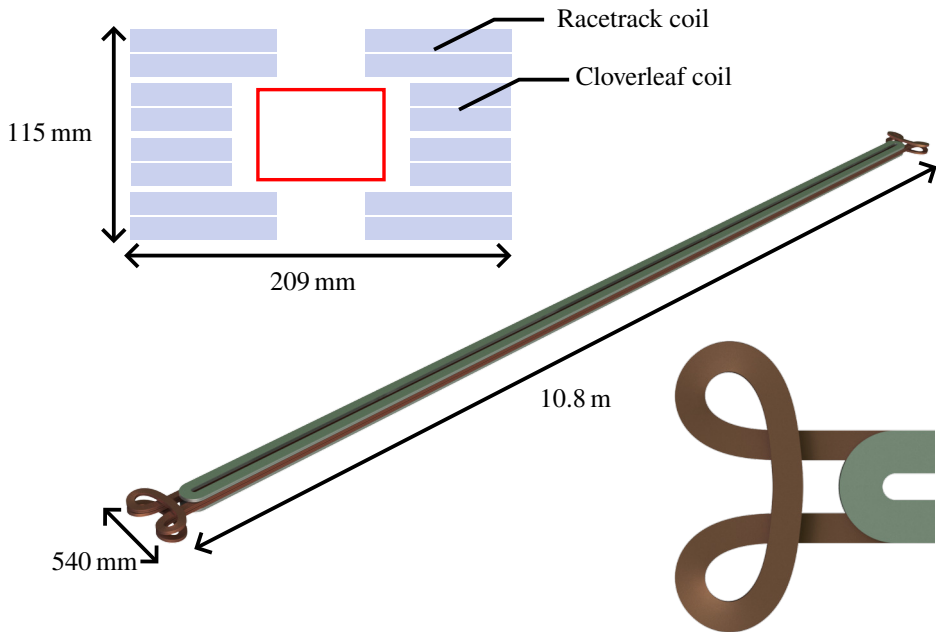


Figure 6.25: Rendering of a 10 m long 20 T dipole coil-system, made with two poles containing a double cloverleaf and double racetrack, making for 8 coils in total. In the top left, the cross-section of the coil-packs are shown. In the bottom right, the head of the magnet is shown.

A challenge for such a large magnet will be the winding of the coils. For the reduced-turns cloverleaf, the whole coil was turned and the spool containing the tape was stationary. For such a large scale magnet, this is rather impractical, as it requires the whole 10 m long magnet to turn in three dimensions. A more practical solution would be to move the spool around the coil and keep the coil stationary. Ideally, this has to be an automated process in order for good alignment of the tape. An important issue is that such a large magnet requires a high-current cable to keep the induction of the magnet sufficiently low. If the cable is a tape stack, issues occur at the head of the magnet, as the tapes on the outside of the cable need to be stretched and the tapes on the inside need to be compressed. To remedy this, the cable needs to be transposed, however, so far no *ReBCO* tape stack cables exist with a transposition length much smaller than the radius of curvature of the heads. A solution to this can be to co-wind multiple tapes in parallel, including the insulation layers and possibly mechanical support strips. Alternatively other cable types can be considered like Roebel or CORC-type cables.

Table 6.7: Main parameters of the proposed 20 T cloverleaf coil-system.

Property	Value
Aperture dimensions	70 mm × 50 mm
Cross-section dimensions	209 mm × 115 mm
Coil-system total length	10.8 m
Coil-system width (including ears)	540 mm
Cable thickness	2 mm
Engineering current density at 26 T	700 A mm ⁻²
Number of turns cloverleaf coil	28
Number of turns racetrack coil	40
Total cable length	6.9 km
Length of single racetrack coil	812 m
Length of single cloverleaf coil	651 m
Self-inductance	366 mH
Aperture magnetic field	20.6 T
Max. magnetic field on conductor	26.2 T

6.7 Conclusion

A novel design of a demonstrator cloverleaf-racetrack magnet has been introduced, intended as an initial step towards the realization of very high-field *ReBCO* tape based accelerator magnets within the scope of CERN's high magnetic field program. The magnet configuration comprises two poles, each comprising a racetrack coil and a cloverleaf coil. The utilization of the cloverleaf geometry facilitates smooth lifting of the tape conductor in the coil-ends to pass over the particle beam pipe.

In addition, a coil-enclosure design was presented, wherein the coils are enclosed within an aluminium alloy casing. Through mechanical modelling of the magnet assembly, it was demonstrated that dry-wound coils experience reduced mechanical stress compared

to solder-filled coils. This reduction in stress arises from the inherent ability of the turns within dry-wound coils to slide relative to each other.

No-insulation coil windings can probably not be used for full-size meters long accelerator magnets, as it is not sure if large scale and long dipole magnets can still be sufficiently protected with no-insulation technology. Long charging times and magnetic field quality requirements are also reasons not to use no-insulation technology. If no-insulation would be used, it is clear that dry-wound and solder-filled coils are not an option, as their charging time is too long. To fulfil the stringent requirement for the charging time of accelerator dipole magnets, at least partial insulation is required in the case no-insulation technology is used. For the cloverleaf-racetrack magnet, it was foreseen to use two tapes in parallel, but placing more conductors in parallel would lead to a lower charging time if required for large-scale dipoles. If the coil were to be wound non-insulated, multiple current paths are possible during charging. Therefore, an option is to use magnetic field screens in the aperture to correct for magnetic field errors.

From a coil winding perspective, it is feasible to create a cloverleaf coil geometry; however, it should be noted that the winding mandrel geometry used was not an ideal match. In order to address this issue, it is recommended to develop new mandrel geometries that consider the minimization of bending energy. By utilizing such optimized mandrel geometries, it is anticipated that the winding process can be significantly improved, leading to enhanced alignment of the tape on the mandrel. To achieve better alignment and consistency during the winding process, and to reduce costs as well, automation is strongly recommended. Automating the winding process would minimize human error and variability, resulting in improved tape placement accuracy and overall alignment. By employing automated techniques, such as robotic winding systems or advanced machinery, the precision and repeatability of the winding process can be significantly enhanced.

Experimental tests conducted on the cloverleaf coil immersed in a liquid nitrogen bath demonstrated that the coil did not exhibit signs of degradation during the conducted experiments. They also revealed a notable dependence of the coil's behaviour on the chosen terminal configuration. Furthermore, it was observed that the coil can be powered well beyond its coil and tape critical current, thereby highlighting the stability and robustness provided by non-insulated coils at 77 K. Based on the insights gained from these tests, a recommendation is put forth to relocate the terminals to the bridge sections. By doing so, the intention is to mitigate the occurrence of irregular current distributions throughout the coil. This repositioning of the terminals is expected to result in improved and more predictable current pathways within the coil windings.

Lastly, a conceptual design of a 10 m long 20 T dipole magnet coil-system was presented. The calculated effective time constant of some 1000 hours for charging the magnet renders no-insulation technology unfeasible for large-scale applications in accelerator magnets. Additionally, the winding process for such a large magnet poses significant challenges.

Chapter 7

Conclusion and outlook

In this thesis, comprehensive efforts are reported on the development of high-magnetic field (beyond 20 T) accelerator dipole magnets, based on *ReBCO* coated conductor. It is specifically focused on advancing a cloverleaf-type accelerator magnet, where it was investigated if non-insulated technology is a realistic option for quench protection of this type of magnet. To this, four *ReBCO* pancake coils were manufactured and tested as a first step, since pancake coils feature a relatively simple geometry. There after, a demonstrator cloverleaf coil was manufactured and tested to demonstrate the feasibility of such a coil.

Chapter 2 involved fitting and experimentally verifying the critical surface of SST *ReBCO* tape and a description of the critical surface of the SuperPower *ReBCO* tape, which were used for the pancake coils and cloverleaf coil, respectively.

Chapters 3 and 4 addressed non-insulated coil technology. In Chapter 3, the construction and testing of four non-insulated pancake coils made with SST tape was presented. The coils were tested in different environments: all at 77 K and in self-field, two in a variable temperature environment between 4.2 K and 70 K in self-field, and one at 4.2 K in a background magnetic field up to 7 T. In Chapter 4 a model was presented for describing the time-dependent behaviour of the voltage across a non-insulated coil while discharging. In the last two technical chapters of the thesis, the feasibility of a cloverleaf-type coil was explored.

In Chapter 5 a coil-geometry modelling approach was introduced, capable of creating a cloverleaf geometry.

In Chapter 6 a layout was presented for a demonstrator cloverleaf-racetrack magnet with 9.4 T in the aperture, as well as a coil-system of a 10 m long 20 T cloverleaf-racetrack accelerator magnet. Also presented in Chapter 6 was the coil winding, and testing at 77 K and self-field, of a demonstrator cloverleaf coil comprising eight turns of a cable with two *ReBCO* tapes in parallel.

In this final Chapter 7, a concise overview is provided, summarizing the key findings, discussions, and conclusions drawn from the preceding chapters. The significant contributions and key insights derived from the research conducted in the frame of this thesis are highlighted as achievements. Furthermore, the chapter also presents avenues for future research and potential applications stemming from the work presented.

7.1 Achievements

A list of major achievements realized from the thesis work is presented on a per chapter basis.

Chapter 2: ReBCO critical surface for coil design

- A fit was employed to assess the critical surface of SST tape. The measurement results aligned with the manufacturer's stated critical current of about 400 A at 77 K and nominal critical current of 600 A mm^{-2} at 4.2 K in a 20 T magnetic field. Self-field measurements confirmed the manufacturer's data. The fit was used in Chapter 3 to predict the critical current of the pancake coils.
- The tape critical surface of SuperPower tape used for the cloverleaf coil was presented, encompassing measured data at 77 K, a 360° magnetic field spectrum and up to 300 mT background field. An interpolation of this data was made in order to predict the performance of the demonstrator cloverleaf coil presented in Chapter 6.

Chapter 3: Non-insulated coils for accelerator magnets

- Four non-insulated pancake coils were manufactured at CERN, two dry-wound and two solder-filled, each with a single and dual-tape configuration. The coils were of identical size, with an inner diameter of 52 mm and out diameter of 102 mm, in order to make a fair comparison. All coils were tested at 77 K in self-field, and the dry-wound and soldered single tape coil were tested in a variable temperature environment between 4.2 K and 70 K. The dry-wound double-tape coil was tested at 4.2 K and in up to 7 T background field environment.
- The dry-wound coils had a higher critical current at 77 K in liquid nitrogen than their solder-filled equivalents. The solder-filled coils showed a higher margin between critical current and quench current. However, they showed a reduced critical current in the inner sections of the coil apparently due to damage introduced.
- The effective charging time constant of the solder-filled coils was about a factor of fifty higher for the soldered coils compared to the dry-wound coils at 77 K. The dry-wound coil showed a remarkable change in the effective time constant during the variable measurement campaign, which appeared irreversible. Consequently, its results can not be directly compared with the effective time constant of the solder-filled coil at lower temperatures.
- The resistivity of the dry-wound coils was $22 \pm 3 \mu\Omega \text{ m}$, and of the solder-filled coil $0.6 \pm 0.2 \mu\Omega \text{ m}$.
- It was shown that winding two tapes in parallel leads to a reduction of the self-inductance and the charging time by a factor of four, as was expected from straightforward calculations.
- The double tape dry-wound coil was tested in a background magnetic field in the range of 0 T to 7 T. It was found that the coil did not reach its predicted critical

current, and notably degraded over successive measurement cycles. The precise cause of the degradation has yet to be investigated and is beyond the scope of this thesis.

- In general, no-insulation coils can prevent quenches due to the enhanced stability of the coils by providing alternative current paths to by-pass a normal zone. However, this technology cannot prevent damage to the coils during quenches in all scenarios, as shown at 4.2 K and in a background magnetic field.

Chapter 4: Network model and characteristic time constants of non-insulated coils

- A new network model was introduced to characterize the time-dependent behaviour of (rather small) non-insulated coils. Through this model, each turn is represented as a node with an inductor and a resistor connected in parallel. Analytical solutions for voltage and current behaviour were derived for different powering scenarios, and spectral decomposition revealed characteristic time constants governing voltage behaviour. The largest of these time constants, the effective time constant, significantly influences the overall voltage decay time, an improvement over previous models that considered a single time constant. The model showed good agreement with measured results from the small pancake coils, thereby validating its use.
- The network model predicted that powering cycles of non-insulated coils, where the coil has not completely settled, can lead to situations where the currents are flowing in opposite transverse directions within the coil-pack. This corresponded well with observations during measurements performed on the pancake coils in the variable temperature measurement campaign.
- Various strategies for diminishing the charging time have been deliberated. Insulating an individual turn does not yield a noteworthy reduction in the effective time constant. Conversely, introducing high resistance between each k -th turn can result in a reduction of the effective time constant by a factor of up to $1/k$. Winding n tapes in parallel leads to a reduction in charging delay by a factor of $1/n^2$. While the overshoot method expedites the attainment of magnetic field, internal current redistributions persist within the coil, with the decay time governed by the effective time constant. These redistribution currents can potentially constrain good magnetic field quality.

Chapter 5: Cloverleaf coil-end geometry

- Accelerator coil geometries for thin tape conductors, such as *ReBCO* coated conductor, were developed using the thin strip model and Bézier splines. The thin strip model was used to minimize hard-way bending in the strip, while Bézier splines enabled flexible and optimal geometric designs for coil-ends, reducing hard-way bending. The method of Bézier splines, in conjunction with the thin strip model, provided improved geometric design capabilities compared to previous methods. Bézier splines were used to generate the cloverleaf shape, but can also be used to create other coil-ends, such as the cosine-theta coil-end.

- To ensure the prevention of conductor degradation, from a geometry point of view, new optimization criteria valid for three-dimensional geometries were introduced. These were prevention of conductor creasing, minimization of overall bending energy, and avoidance of over-straining the conductor. The optimization criteria can be used in coil and coil-end designs.
- It was shown mathematically that the thin strip model is only valid for thin conductors. In stacked-tape cables, hard-way bending is induced when the cables are stacked without a gap in between the turns.
- It was shown that it is not practical to build higher order multi-pole canted-cosine-theta magnets using rectangular conductor, as the conductor would crease due to the particular CCT geometry.

Chapter 6: Cloverleaf magnet design, coil winding and test

- A novel design of a 9 T demonstrator cloverleaf-racetrack magnet was introduced as a crucial step towards high-field *ReBCO* accelerator magnets in CERN's high magnetic field program.
- It was shown that it is possible to wind a cloverleaf coil geometry with two tapes in parallel without conductor degradation. The measured critical current matched the predicted critical current of the coil of 490 A. Furthermore, tests on the cloverleaf coil exhibited stable behaviour below the coil critical current and the potential for powering beyond the coil critical current.
- Correct positioning of the current entry and exit terminals on the cloverleaf coil is important for non-insulated coils, as they lead to different current paths, even leading to situations where the current is flowing initially in the opposite direction to the coil-winding direction.
- A coil-system for a 10 m long 20 T cloverleaf-racetrack coil-system was presented. It was shown that for large scale accelerator coils of several meters, the charging time for dry-wound non-insulated coils would be in the order of days, which is far too long for practical use.
- If no-insulation technology is to be used for large scale accelerator magnets, at least partial insulation is required, where an interstitial material with a specific resistance is added between the turns. This is needed in order to reduce the effective time constant of the magnet to a manageable level. However, it is yet unclear whether large scale magnet applications can be adequately protected in this way.

These findings collectively contribute to the understanding of non-insulated coil behaviour, geometric optimization, and the development of advanced accelerator magnet designs. In the following section, prospects are presented.

7.2 Prospects

A list with issues for further studying and recommendation towards high-field *ReBCO* dipoles is presented below.

- It is recommended to perform in-depth measurements of the critical current of the *ReBCO* tape used in application-relevant conditions at 4.2 K and in a background magnetic field. This is in order to verify the precision of the extrapolated fit and homogeneity of the tape, and to obtain a better prediction of the coil critical current of the pancake coils at these conditions.
- The dry-wound double tape coil exhibited degradation when tested in a background magnetic field, but the reason for this is unclear at the present stage. In order to understand this, it is recommended to unwind the coil and study the tape in order to understand what happened to the coils and the principle degradation mechanism present. Studies of the tape that can be performed are visible inspections of the tape for possible defects, and a measurement of the critical current of the tape along the length of the tape in order to pinpoint the location of the degradation.
- It is proposed to explore various mandrel geometries, which are designed with minimization of the overall bending energy, in contrast to the current practice of exclusively minimizing the hard-way bend component. It was found that for the cloverleaf mandrel, a substantial gap of up to 2 mm persisted between the superconducting tape and the mandrel, despite the fact that the hard-way bend component was minimized for this geometry. Minimizing the overall bending energy can possibly lead to better fitting cloverleaf mandrel geometries.
- It is unclear what the geometry of a turn comprising a thick stack of tapes is in a cloverleaf coil-end. The thin strip model is obviously not suitable to calculate precisely the optimal geometry of a thick coil-pack or a cable with a small width to thickness ratio. It is therefore proposed to wind a cloverleaf coil with many turns of a relatively thick stack of tape, and scan the resulting geometry of the coil. The measured geometry can then be used as an input in the model. This can be used to determine the magnetic field on the conductor and field quality in the aperture in detail, as well as for the geometry of the support structure of the coil. For consistent winding of the tape (stack), it is recommended to use automation for improved tape placement accuracy and alignment.
- A next step would be to wind a full-size coil of the cloverleaf-racetrack demonstrator. This magnet can be designed and constructed in a few steps. First a single cloverleaf coil can be wound and placed within the structure. This coil can be reduced in turn number, with the remaining coil-pack thickness filled with copper. After this a full-size cloverleaf coil can be fabricated and tested, afterwards joined by a second cloverleaf coil in the other magnet pole. Lastly, the racetrack coils can be added. If this magnet proves to be successful, one can undertake the next step towards 20 T dipole accelerator magnets.
- No-insulation technology is not a viable option for large scale accelerator dipole magnets, as the effective time constants are too long: in the order of many hours or

even days. Partial insulation can be used to reduce the time constant, however, this leads to diminishing the effect of the quench protection properties, whilst still having the additional drawback of having multiple current paths in the system, which is troublesome for the magnetic field quality of the magnet. Therefore, other smart and reliable methods need to be developed for quench protection of large scale *ReBCO* accelerator dipole magnets.

This concludes a four year development work towards the realization of a beyond 20 T accelerator magnet using the cloverleaf technology applied to *ReBCO* tape coil windings. Given the limited resources, successfully first, small, but considered essential steps were made to discover, and create solutions for the peculiarities in design, manufacturing and testing associated with the cloverleaf geometry. So far no hard show-stoppers were encountered and it is therefore recommended to undertake the next steps in realizing a full-size demonstrator.

Appendix A

Derivatives of Bézier Splines

A Bézier curve is given by:

$$\mathbf{B}(t) = \sum_{k=0}^n b_{n,k}(t) \mathbf{P}_k, \quad (\text{A.1})$$

where $b_{n,k}$ are the so called Bernstein polynomials, defined by:

$$b_{n,k}(t) = \binom{n}{k} (1-t)^{n-k} t^k, \quad (\text{A.2})$$

with:

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}. \quad (\text{A.3})$$

The derivatives of a Bézier curve can be found using the following strategy [190]. Since the control points are constant and thus independent of t , computing the derivative of $\mathbf{B}(t)$ reduces to the computation of the derivatives of the Bernstein polynomials $b_{n,i}(t)$. It can be shown that this yields:

$$\frac{d}{dt} b_{n,k}(t) = b'_{n,k}(t) = n(b_{n-1,k-1}(t) - b_{n-1,k}(t)). \quad (\text{A.4})$$

Computing the derivative of $\mathbf{B}(t)$ gives then:

$$\begin{aligned} \frac{d}{dt} \mathbf{B}(t) &= \dot{\mathbf{B}}(t) = \sum_{k=0}^{n-1} b_{n-1,k}(t) (n(\mathbf{P}_{k+1} - \mathbf{P}_k)) \\ &= \sum_{k=0}^{n-1} b_{n-1,k}(t) \mathbf{Q}_k, \end{aligned} \quad (\text{A.5})$$

where $\mathbf{Q}_k = n(\mathbf{P}_{k+1} - \mathbf{P}_k)$. Continuing this procedure for the second derivative gives:

$$\begin{aligned} \ddot{\mathbf{B}}(t) &= \sum_{k=0}^{n-2} b_{n-2,k}(t) ((n-1)(\mathbf{Q}_{k+1} - \mathbf{Q}_k)) \\ &= \sum_{k=0}^{n-2} b_{n-2,k}(t) (n(n-1)(\mathbf{P}_{k+2} - 2\mathbf{P}_{k+1} + \mathbf{P}_k)). \end{aligned} \quad (\text{A.6})$$

To create an expression for the m -th derivative of the Bézier curve, one can make use of finite difference. The point $\mathbf{D}_k^0$ is defined as the control point $\mathbf{P}_k$ for $0 \leq k \leq n$. Then the first level difference is defined as the difference of the previous level:

$$\begin{aligned}\mathbf{D}_0^1 &= \mathbf{D}_1^0 - \mathbf{D}_0^0 = \mathbf{P}_1 - \mathbf{P}_0 \\ \mathbf{D}_1^1 &= \mathbf{D}_2^0 - \mathbf{D}_1^0 = \mathbf{P}_2 - \mathbf{P}_1 \\ &\vdots \\ \mathbf{D}_{n-2}^1 &= \mathbf{D}_{n-1}^0 - \mathbf{D}_{n-2}^0 = \mathbf{P}_{n-1} - \mathbf{P}_{n-2} \\ \mathbf{D}_{n-1}^1 &= \mathbf{D}_n^0 - \mathbf{D}_{n-1}^0 = \mathbf{P}_n - \mathbf{P}_{n-1}.\end{aligned}\tag{A.7}$$

The first level difference has only n points, one less than the number of control points. The first level difference is defined as:

$$\mathbf{D}_k^1 = \mathbf{D}_{k+2}^0 - \mathbf{D}_k^0 \quad 0 \leq k \leq n-1.\tag{A.8}$$

The second level difference is defined to be the difference of the first level points:

$$\mathbf{D}_k^2 = \mathbf{D}_{k+2}^1 - \mathbf{D}_k^1 \quad 0 \leq k \leq n-2.\tag{A.9}$$

The second level has $n-1$ points, two less than the number of control points. Repeating the procedure, the m -th level can be defined as:

$$\mathbf{D}_k^m = \mathbf{D}_{k+2}^{m-1} - \mathbf{D}_k^{m-1} \quad 0 \leq k \leq n-m.\tag{A.10}$$

With this notation the higher order derivatives can be written down more concise. In this notation the first derivative becomes:

$$\begin{aligned}\dot{\mathbf{B}}(t) &= n \sum_{k=0}^{n-1} b_{n-1,k}(t)(\mathbf{P}_{k+1} - \mathbf{P}_k) \\ &= n \sum_{k=0}^{n-1} b_{n-1,k}(t)(\mathbf{D}_{k+1}^0 - \mathbf{D}_k^0) \\ &= n \sum_{k=0}^{n-1} b_{n-1,k}(t)\mathbf{D}_k^1,\end{aligned}\tag{A.11}$$

and the second:

$$\begin{aligned}\ddot{\mathbf{B}}(t) &= n(n-1) \sum_{k=0}^{n-2} b_{n-2,k}(t)(\mathbf{D}_{k+1}^1 - \mathbf{D}_k^1) \\ &= n(n-1) \sum_{k=0}^{n-2} b_{n-2,k}(t)\mathbf{D}_k^2,\end{aligned}\tag{A.12}$$

and the third:

$$\begin{aligned}\ddot{\mathbf{B}}(t) &= n(n-1)(n-2) \sum_{k=0}^{n-3} b_{n-3,k}(t)(\mathbf{D}_{k+1}^2 - \mathbf{D}_k^2) \\ &= n(n-1)(n-2) \sum_{k=0}^{n-3} b_{n-3,k}(t)\mathbf{D}_k^3.\end{aligned}\quad (\text{A.13})$$

Continuing this process, the m -th derivative can be written as:

$$\mathbf{B}^{[m]}(t) = n(n-1)(n-2) \dots (n-m+1) \sum_{k=0}^{n-m} b_{n-m,k}(t)\mathbf{D}_k^m. \quad (\text{A.14})$$

An interesting property for the beginning and end of the Bézier curve, where $t = 0$ and $t = 1$ respectively, is the following. Filling in $t = 0$ into Equation A.2 yields for all for all $k \neq 0$:

$$b_{n,k}(0) = \binom{n}{k}(1-0)^{n-k}0^k = 0. \quad (\text{A.15})$$

However, when $k = 0$:

$$b_{n,0}(0) = \binom{n}{0}(1-0)^{n-0}0^0 = 1. \quad (\text{A.16})$$

At the end-point $t = 1$. This yields for all for all $k \neq 0$:

$$b_{n,k}(1) = \binom{n}{k}(1-1)^{n-k}1^k = 0. \quad (\text{A.17})$$

However, when $n = k$:

$$b_{n,k}(1) = \binom{k}{k}(k-k)^01^k = 1. \quad (\text{A.18})$$

Thus $b_{n,k}(0) = 0$ for all $k \neq 0$ and $b_{n,k}(1) = 0$ for all $n - k \neq 0$, and the Bernstein polynomials are 1 when $k = 0$ and $n - k = 0$ and zero for all other k at the beginning and end. Looking at the first derivative of $\mathbf{B}$ (Equation A.5), it is seen that at $t = 0$:

$$\dot{\mathbf{B}}(0) = \mathbf{P}_1 - \mathbf{P}_0, \quad (\text{A.19})$$

$$\dot{\mathbf{B}}(1) = \mathbf{P}_{n-1} - \mathbf{P}_n. \quad (\text{A.20})$$

Similarly for the second derivative:

$$\ddot{\mathbf{B}}(0) = \mathbf{P}_2 - 2\mathbf{P}_1 + \mathbf{P}_0, \quad (\text{A.21})$$

$$\ddot{\mathbf{B}}(1) = \mathbf{P}_n - 2\mathbf{P}_{n-1} + \mathbf{P}_{n-2}. \quad (\text{A.22})$$

The first derivative is controlled by the last two control points, the second derivative by the last three control points. Looking at Equation A.14, it can be seen that this continues: at $t = 0$ the third derivative is controlled by the first four control points etc, and at $t = 1$, the third derivative by the last four control points etc. This is useful to adhere to the boundary conditions at the beginning and end of the curve.

Nomenclature

Matrices

$\tilde{\mathbf{A}}$	Eigenvalue matrix
$\tilde{\mathbf{\Theta}}$	Time constant matrix
$\tilde{\mathbf{H}}$	Eigenvector matrix
$\tilde{\mathbf{I}}$	Identity matrix
$\tilde{\mathbf{L}}$	Inductance matrix
$\tilde{\mathbf{R}}$	Resistance matrix
$\tilde{\mathbf{X}}$	Variable matrix

Scalars

α	Angle
α	Fitting parameter
α	Tilt angle
ϵ	Strain
η	Torsion to curvature ratio
γ	Fitting parameter
κ	Curvature
κ_g	Geodesic curvature
κ_n	Normal curvature
λ	Distance in normal direction
λ_k	Eigenvalue
ν	Fitting parameter
ν	Poisson's ratio

ω	Angular frequency
$\bar{x}$	Measurement fit
ϕ	Angle
ϕ	Map
ρ	Radius of curvature
ρ_h	Homogenized resistivity
σ	Radius of torsion
σ	Standard deviation
σ_w	Winding tension
σ_r	Radial stress
τ	Torsion
τ_{eff}	Effective time constant
τ_i	Initial decay time constant
τ_k	Characteristic time constant
τ_r	Relative torsion
θ	Angle
θ_c	Maximum magnetic field angle
ξ	Variable
ζ	Variable
A	Area
a	Constant
a	Fitting parameter

a	Turn inner radius	I_{sc}	Superconductor current
A_I	Current amplitude	I_{tot}	Total current
A_n	Skew component	J	Current density
a_n	Reduced skew component	J_{cpw}	Critical current per unit width
B	Magnetic field	J_c	Critical current density
b	Turn outer radius	J_e	Engineering current density
B_{acc}	Accelerator magnetic field strength	k	Turn number
B_c	Critical magnetic field	L	Inductance
B_i	Irreversibility field	L_{pt}	Inductance parallel tape coil
b_i	Reduced irreversibility field	L_{st}	Inductance single tape coil
B_n	Normal component	L_t	Length
b_n	Reduced normal component	M	Mutual inductance
c	Constant	N	Number of points
C^n	Class	n	n -factor
D	Flexible rigidity	n	Fitting parameter
d	Thickness	n	Number of turns
E	Electric field	n_1	Fitting parameter
E_{coll}	Collision energy	n_2	Fitting parameter
E_c	Electric field criterion	n_{pt}	Number of turns in parallel
g	Fitting parameter	p	Fitting parameter
g	Generator length	p	Twist pitch
h	Height	p_{part}	Particle momentum
I	Current	q	Fitting parameter
I	Interval	q	Twist pitch
I_c	Critical current	R	Bending radius
I_q	Quench current	R	Radius of curvature
I_{ramp}	Current ramp rate	R	Resistance
I_R	Radial current	r	Radius
		r_0	Reference radius
		R_{acc}	Accelerator radius

r_{avg}	Average radius	$\mathbf{B}_m$	Magnetic field
r_{in}	Inner radius	$\mathbf{c}$	Constant
R_k	Contact resistance	$\mathbf{c}$	Directrix
r_{out}	Outer radius	$\mathbf{D}$	Darboux vector
s	Length	$\mathbf{e}$	Unit vector
T	Temperature	$\mathbf{f}$	Body force density
t	Time	$\mathbf{g}$	Generator vector
t	Unit-less temperature	$\mathbf{I}_R$	Radial current
t_0	Initial time	$\mathbf{I}_{\text{sc}}$	Superconductor current
T_c	Critical temperature	$\mathbf{I}_{\text{tot}}$	Total current
U	Bending energy	$\mathbf{J}$	Current density
U	Voltage	$\mathbf{N}$	Normal vector in Frenet-Serret frame
v	Velocity	$\mathbf{n}$	Normal vector in Darboux frame
w	Width	$\mathbf{O}$	Offset vector
x_i	Measurement point	$\mathbf{P}$	Perimeter vector
Y	Young's modulus	$\mathbf{P}_k$	Control points Bézier curve
y	Distance from axis	$\mathbf{r}$	Position vector
Y_{mandrel}	Mandrel Young's modulus	$\mathbf{S}$	Surface
Vectors		$\mathbf{T}$	Tangent vector in Frenet-Serret frame
$\boldsymbol{\eta}$	Eigenvector	$\mathbf{t}$	Tangent vector in Darboux frame
$\boldsymbol{\omega}$	Angular velocity vector	U	Voltage
$\mathbf{a}$	Acceleration vector	U_0	Initial voltage
$\mathbf{B}$	Binormal vector in Frenet-Serret frame	$\mathbf{V}$	Volume
$\mathbf{B}$	Bézier curve	$\mathbf{v}$	Velocity vector
$\mathbf{b}$	Binormal vector in Darboux frame		

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