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ϕ meson production in proton-proton collisions
in the NA61/SHINE experiment
at CERN SPS

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PhD thesis prepared in the
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Kraków, 2016

Oświadczenie

Ja niżej podpisany Antoni Marcinek oświadczam, że przedłożona przeze mnie rozprawa doktorska pt. „ ϕ meson production in proton-proton collisions in the NA61/SHINE experiment at CERN SPS” jest oryginalna i przedstawia wyniki badań wykonanych przeze mnie osobiście, pod kierunkiem prof. dr. hab. Romana Płanety. Pracę napisałem samodzielnie.

Oświadczam, że moja rozprawa doktorska została opracowana zgodnie z Ustawą o prawie autorskim i prawach pokrewnych z dnia 4 lutego 1994 r. (Dziennik Ustaw 1994 nr 24 poz. 83 wraz z późniejszymi zmianami).

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Kraków, dnia 01.07.2016

Abstract

This thesis presents results on ϕ meson production in $p + p$ collisions at CERN SPS energies. They are derived from data collected by the NA61/SHINE experiment, by means of invariant mass spectra fits in $\phi \rightarrow K^+ K^-$ decay channel, using the so-called tag-and-probe method to remove bias due to inefficiency of kaon candidates selection with dE/dx .

These results include double differential spectra (first for ϕ mesons at CERN SPS energies) of rapidity y and transverse momentum p_T for beam momenta of 158 GeV/c and 80 GeV/c, as well as singly differential spectra of y or p_T for beam momentum of 40 GeV/c. Additionally, y spectra integrated over p_T were obtained from double differential spectra. Also total ϕ yields were determined by integration and extrapolation of y spectra and widths of these spectra along with yields at $y = 0$ were calculated from fits of these distributions with Gaussian functions.

Results were compared with world data on ϕ meson production in $p + p$ collisions showing consistency and superior accuracy. They served also as reference for Pb + Pb data on ϕ production, confirming and emphasizing earlier findings regarding phenomena in Pb + Pb. Finally, results were also compared with model predictions showing that none of considered models could describe properly all observables.

Streszczenie

Praca przedstawia wyniki dotyczące produkcji mezonów ϕ w zderzeniach $p + p$ przy energiach CERN SPS. Są one wyznaczone z danych zebranych przez eksperyment NA61/SHINE, przez dopasowania rozkładów masy niezmienniczej w kanale rozpadu $\phi \rightarrow K^+ K^-$, przy wykorzystaniu tzw. metody tag-and-probe w celu usunięcia strat sygnału ze względu na niewydajność selekcji kaonów przez dE/dx .

Wyniki obejmują podwójnie różniczkowe widma (pierwsze dla mezonów ϕ przy energiach CERN SPS) pospieszności y i pędu poprzecznego p_T dla pędów wiązki 158 GeV/c i 80 GeV/c oraz pojedynczo różniczkowe widma y lub p_T dla pędu wiązki 40 GeV/c. Dodatkowo, z rozkładów podwójnie różniczkowych, otrzymano widma y wyciąkowane po p_T . Wyznaczono również całkowite krotności ϕ przez całkowanie i ekstrapolację widm y , a także szerokości tych rozkładów i różniczkowe krotności w $y = 0$ z dopasowań tych widm funkcjami Gaussa.

Wyniki porównano z danymi światowymi na temat produkcji mezonów ϕ w zderzeniach $p + p$, pokazując ich zgodność i znacznie większą dokładność. Posłużyły one także jako dane referencyjne dla danych dotyczących produkcji ϕ w Pb + Pb, potwierdzając i wzmacniając wcześniejsze wnioski na temat zjawisk w Pb + Pb. W końcu, wyniki zostały również porównane z przewidywaniami modelowymi pokazując, że żaden z rozważanych modeli nie jest w stanie poprawnie opisać wszystkich obserwabli.

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Chapter 1

Introduction

1.1 Motivation

Motivation of study of ϕ meson production in proton-proton collisions is twofold. First, it is useful in itself to constrain hadron production models. Second, it may serve as reference for lead-lead measurements at the same energies to infer about strangeness-related phenomena in heavy ion collisions.

1.1.1 Constraints on hadron production models

The matter which surrounds us is built of atoms, which are in turn built of nuclei and electrons. Atomic nuclei are built of neutrons and protons, which are *hadrons* — particles built of quarks and gluons. Hadrons are divided into two groups: *mesons* having even number of valence quarks (in most cases two) and *baryons* with odd number of valence quarks (in most cases three). The *valence quarks* are those which contribute to quantum numbers of hadrons in contrast to *sea quarks* that are virtual — they are continuously created in quark-antiquark ($q\bar{q}$) pairs from gluons and instantly annihilated still within hadrons. The theory which describes how hadrons, and therefore the surrounding matter, are bound together by the so-called *strong interactions* of quarks and gluons, mediated by gluons, is quantum chromodynamics (QCD).

The problem of this theory is, that actually little can be directly calculated from its equations. The standard technique of quantum mechanics or quantum field theory, the perturbation theory, is applicable only in the high energy or high momentum transfer domain, where the strong interaction becomes relatively weak (what is called *asymptotic freedom* of QCD). In terms of experimental observables it corresponds to high transverse momenta — the so-called *hard* regime. However, the bulk of hadron production in nu-

Table 1.1: Properties of the ϕ meson relevant for the analysis. Numbers are taken from Ref. [7]. $\mathcal{BR}()$ stands for branching ratio for the decay channel in parentheses.

mass [MeV]	width [MeV]	$\mathcal{BR}(\phi \rightarrow K^+ K^-)$ [%]
1019.461 ± 0.019	4.266 ± 0.031	48.9 ± 0.5

clear collisions, as well as many other interesting dynamical phenomena of QCD (*e.g.* hadron masses, thermodynamics), belong to the *soft* regime — relate to low transverse momenta.

Where the perturbative QCD [1] is not applicable, calculations on discrete lattices of space-time points (lattice QCD [2]) are carried out. Unfortunately, this is still a method with limited use, as it requires tremendous computing powers and suffers from numerical instabilities. Therefore, in practice various phenomenological models are used to take care of the soft region. They can be roughly divided into two families: those that attempt a QCD-motivated microscopic description and those that try a macroscopic one, *e.g.* hydrodynamic or statistical-thermal models. All of them need to be fitted using numerous experimental observables. For the first kind the number of free parameters can be of the order of 100 [3, 4], while for the second family it is below 10 [5, 6]. It should be noted, however, that microscopic models attempt to offer complete description of events along with collision energy dependence, while thermal models focus on bulk properties like total multiplicities and spectra and need to be fitted separately for each reaction. Consequently macroscopic models serve rather as interpretation of experimental data within certain scenarios instead of being truly predictive.

From above it is clear, that to draw firm conclusions about phenomena associated with strongly interacting objects, it is paramount to gather as large and versatile experimental data base as possible. This thesis contributes to this data base with results on ϕ meson production in $p + p$ collisions.

The ϕ meson is the lightest bound state of s and $\bar{s}$ quarks. It has a mass close to the proton mass and decays predominantly into kaons (see Table 1.1). Because of its hidden strangeness composition (*i.e.* it contains strange quarks, but as a hadron it is strangeness-neutral), it is especially interesting for constraining models of hadron production. In a purely hadronic scenario it should be insensitive to strangeness-related effects. On the other hand, if partonic degrees of freedom are significant, then it could react more violently than singly-strange particles.

1.1.2 Reference for Pb + Pb collision data

One of fields connected to the soft regime of QCD is physics of *strongly interacting matter*, especially its thermodynamics. The term ‘matter’ implies large amount of hadrons or partons interacting with each other, as could be the case in neutron stars or in the Universe soon after the Big Bang. In the laboratory it is investigated by relativistic collisions of heavy ions.

Should a dense and hot hadronic or deconfined medium be created in such collisions, it is expected to manifest itself in various phenomena observed in the final state. However, to discern actual medium effects from some other, non-expected, but independent of the existence of the medium, it is necessary to make comparison with similar reactions in which medium is not created. For the case of CERN SPS and BNL RHIC energies it is feasible by comparison with $p + p$ reactions at the same energy. If the phenomenon observed in Pb + Pb data is also observed in $p + p$ or can be reproduced by scaling of $p + p$ data by the system size, then it is considered not to be associated with the medium.

Thus results on ϕ production presented in this thesis may serve as reference for Pb + Pb results measured at the same energies by the NA49 experiment [8], to infer about strangeness-related phenomena in that heavy ion reactions.

1.2 Phenomena associated with ϕ mesons

The first effect connected to ϕ mesons, relevant for this thesis, is strangeness suppression in $p + p$ collisions or equivalently enhancement in heavy ion reactions.

In terms of statistical-thermal models strangeness production in $p + p$ collisions suffers from the so-called *canonical suppression*. At the given temperature average amount of strangeness produced in small systems described by the canonical ensemble with exact strangeness conservation is lower than in large systems described by the grand canonical ensemble, where strangeness is conserved only on average [9, 10]. Furthermore, in addition to this volume effect, it turns out, that production of strange particles is undersaturated with respect to expectation for hadron gas in full equilibrium. This has been shown to depend on the size of the considered system and to be the strongest in $p + p$ collisions [5].

From the perspective of QCD-inspired reasoning, the suppression of ϕ production in $p + p$ reactions happens due to the Okubo-Zweig-Iizuka (OZI) rule. It states that processes with disconnected quark lines in the initial and

final state are suppressed. In other words it means, that a final state where all valence quarks of the initial state changed flavour, is unlikely. That is the case of $p + p$ where there are no strange quarks, while in the produced ϕ only such are present. In case of heavy ion collisions this limitation may be lifted as there may appear a lot of strange quarks in the produced medium.

As for the mechanism of ϕ production, different choices are possible. On one hand there is direct production. On the other, ϕ mesons may be created in coalescence of kaons. The latter can be verified experimentally. Assuming that kaon rapidity distributions are Gaussian with widths respectively σ_{K^+} and σ_{K^-} , the rapidity distribution of ϕ mesons is a product of kaons' distributions with a width σ_ϕ given by:

$$\frac{1}{\sigma_\phi^2} = \frac{1}{\sigma_{K^+}^2} + \frac{1}{\sigma_{K^-}^2}. \quad (1.1)$$

1.3 World measurements of ϕ production

Production of ϕ mesons has been measured in colliding systems ranging from $e^- + e^+$ to Pb + Pb and at energies from GSI SIS to CERN LHC accelerators. For this thesis it was decided that comparisons are meaningful only with experimental data for $p + p$ collisions at different energies [11–22] or Pb + Pb data at the same energies as those analysed here [8]. In this way only one parameter of the reaction changes in the comparison, with respect to results of this thesis.

For $p + p$ data at CERN SPS and ISR energies, measurements provide information on differential and total inclusive cross-sections [11–14]. The NA49 experiment has measured single differential spectra of rapidity or transverse momentum in $p + p$ collisions at the energy of 158 GeV [15], the same reaction as one of analysed in this thesis, thus allowing for direct consistency checks. For higher energies mainly midrapidity region is measured [17–21], with exception of double differential cross-sections in the forward region by LHCb experiment [22].

Since this thesis considers multiplicities, not cross-sections, the latter can be transformed into the former using tables of total (σ_{tot}) and elastic (σ_{el}) proton-proton cross-sections as a function of collision energy [23]:

$$n = \frac{\sigma}{\sigma_{\text{tot}} - \sigma_{\text{el}}}, \quad (1.2)$$

where n is (differential) multiplicity and σ the measured (differential) cross-section for ϕ production.

1.4 Structure of this thesis

First, in Chapter 2, the NA61/SHINE experiment at CERN SPS is described including its physics programme, detector system components and basics of the accelerator complex. The purpose of this, is to give the reader the experimental context in which this work is done.

Next, Chapter 3 discusses calibration of energy loss of ionizing particles in the gas of main detectors of the NA61/SHINE experiment. While this subject is not directly related to ϕ production, some known problems associated with it cause serious implications for the ϕ analysis methodology.

The latter is depicted in Chapter 4, which says in detail how the analysis was performed and why certain methods were selected. It is by far the largest part of the thesis and most probably that is the one useful for others working in future on the same subject.

Finally, Chapter 5 presents results of the work along with their discussion in the context of the motivation given above. These findings, along with most important points of the analysis methodology, are summarized and concluded in Chapter 6.

The thesis contains also several appendices. Appendix A gives definitions which are obvious for experienced readers, but may be necessary for beginners in the field. Appendices B and C show studies related to the methodology of the analysis, that were considered too technical or too detailed for the main body, but that may nevertheless be interesting for others doing similar work.

Chapter 2

The NA61/SHINE experiment

The NA61/SHINE is a *fixed target* experiment conducted in the North Area (hence *NA* in the name) of the CERN Super Proton Synchrotron (SPS) accelerator complex. It was proposed in November 2006, a pilot run was executed in 2007 and first physics run took place in 2009. After the latter, data taking was done each year. The NA61/SHINE Collaboration consists of about 150 physicists from almost 30 institutes. Before the CERN Long Shutdown 1 (LS1) in 2013, it has been the second largest active non-LHC experiment at CERN.

The experiment owes its name (SHINE — **SPS Heavy Ion and Neutrino Experiment**) to its rich physics programme, depicted in Section 2.1, which serves several physics communities. Such a programme is feasible thanks to large acceptance of the NA61/SHINE hadron spectrometer based on Time Projection Chambers, as well as variety of beam (Section 2.2) and target combinations which can be delivered and with which the experiment is capable of making measurements. The detector system of NA61/SHINE is described briefly in Section 2.3 and thoroughly in Ref. [24]. Main hardware and software components of NA61/SHINE were inherited from the NA49 experiment [25]. To satisfy the physics goals, several upgrades of the facility were performed. Also a major overhaul of software is in progress to assure long-term stability and maintainability.

2.1 Physics programme

NA61/SHINE gathers representatives of communities associated with *three* different fields of physics — heavy ion physics, neutrino physics and cosmic-ray physics. The common denominator among them is a need for *precise hadron production measurements* in the range of beam energies available

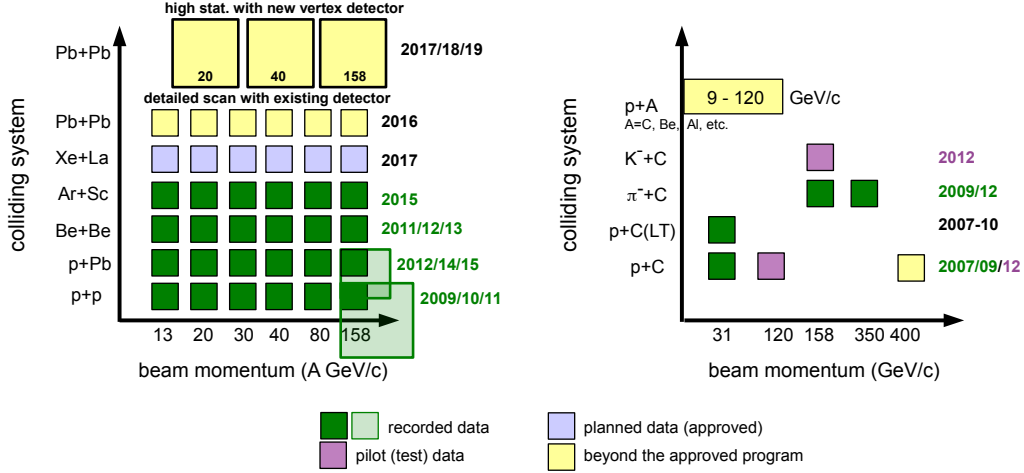


Figure 2.1: Schedule of finished and planned measurements of the NA61/SHINE experiment. The area of boxes visualizes size of data samples for given reaction. The smallest boxes correspond to about $3 \cdot 10^6$ events and the largest to about $50 \cdot 10^6$ events. Written are also years of data taking for the reactions. *Left:* Energy vs system size scan of the heavy ion programme together with large samples needed for high p_T physics ($p + p$, $p + \text{Pb}$) and rare processes ($\text{Pb} + \text{Pb}$). *Right:* Neutrino ($p + \text{C}$, $p + \text{A}$) and cosmic-rays ($\pi^- + \text{C}$, $K^- + \text{C}$) programmes. LT stands for Long Target, a replica of the T2K target. Exact reactions required by the extensions to the neutrino programme are not yet fully specified.

from the SPS accelerator (see Section 2.2). They do differ, however, in the exact set of required beam energies, as well as necessary colliding systems (projectile-target combinations) and physical observables of interest. Schedule of finished and planned measurements is summarized in Fig. 2.1.

Programme connected with heavy ion physics (*heavy ion programme*; left panel of Fig. 2.1) focuses on spectra, fluctuations and correlations and has itself several goals. The first are precision measurements of proton-proton and proton-lead collisions at various beam energies to serve as reference data to infer about nature of phenomena observed in heavy ion reactions measured by NA49 [26, 27]. These measurements include in particular study of high p_T particles production — energy dependence of the nuclear modification factor. Moreover, the experiment performs the first two-dimensional (collision energy versus size of colliding nuclei) scan in the history of relativistic heavy ion collisions. The scan aims at studying the properties of the onset of deconfinement discovered by NA49 and at search for the critical point of strongly interacting matter. The last goal is actually the most important one, as it has a potential for a significant discovery. The reason to perform the scan in size of colliding nuclei instead of dividing the data in collision

centrality classes, as is frequently done in heavy ion physics, is to reduce fluctuations of the size of colliding system. This is a must if one wants to look at fluctuation observables (needed for the critical point search), where physically interesting effects could be washed out by fluctuations in the entrance channel. Apart from above goals, the plethora of nuclear reactions measured in the scan provides comprehensive data base to constrain hadron production models. *Results presented in this thesis serve on one hand as a part of this data base and on the other, as a reference for Pb + Pb.*

More focused, in terms of goals, are neutrino and cosmic-ray programmes (right panel of Fig. 2.1 on the previous page). Both require measurements of hadron spectra in hadron-nucleus collisions. The first one is associated with the T2K long-baseline neutrino oscillation experiment at J-PARC laboratory in Japan. The neutrino beam of T2K is produced from decays of hadrons (mainly K and π mesons), which are in turn produced in interactions of 30 GeV proton beam with a long carbon target. Neutrino flux is measured close to its source, in the so-called near detector, and then again far away, in the so-called far detector. Based on measurement in the near detector and Monte Carlo modelling of the neutrino beam, a prediction is made for the neutrino flux in the far detector. This prediction is compared with the measurement to conclude about neutrino oscillations. Because both near and far detectors provide small geometrical acceptance of the neutrino beam and the beam itself has non-trivial shape due to its extended source, the result depends strongly on the beam MC. The MC modelling, in turn, relies on assumed K and π phase space distributions. These spectra are measured by the NA61/SHINE with the replica of the beam + target setup of T2K.

The cosmic-ray programme includes measurements required by the Pierre-Auger Observatory and KASCADE experiments. Cosmic rays of interest to those experiments have so much energy (*e.g.* 10^6 GeV), that they cause cascades of interactions in the atmosphere. These cascades, called extensive air showers, are measured at the ground level by vast arrays of detectors. To reconstruct the primary particle (cosmic ray) from plethora of tracks measured in these arrays, modelling of the shower is needed. During evolution of the cascade, most of collisions happen between pions produced in previous steps of evolution with nuclei of the atmosphere. Moreover, pions are the major source of muons, which are related to observables sensitive to mass composition of cosmic rays. This is why the modelling of extensive air showers relies heavily on modelling of hadron production in collisions of hadrons (especially pions) with light nuclei. Unfortunately, existing hadron production models are not consistent in description of such collisions. It is caused by the fact, that the data on such collisions is scarce, what prevents proper tuning of these models. The goal of the NA61/SHINE is to measure the relevant

reactions.

With a large part of the approved measurements performed, already some extensions to the programme are being discussed [28]. One is to include an energy scan with Pb + Pb collisions. With typical data taking campaign length of 40 days, it is easy to record 10 times more statistics than in the whole available NA49 data set. That would allow to decrease uncertainties of Pb + Pb results (both statistical and systematic due to detector upgrades), what would improve significantly both the search for the critical point and the study of the onset of deconfinement. Apart from that, gathering yet higher Pb + Pb statistics would allow to extend results on transverse-momentum spectra of identified hadrons from about 4.5 GeV/c up to about 7 GeV/c and together with a new vertex detector would open an opportunity to measure new observables. These include energy dependence of rare processes, in particular, production of D mesons and multi-strange hyperons. It should be stressed that this direct measurement of the open charm (D meson) would be the first one at SPS energies [29].

Another extension is associated with the neutrino program. Apart from the T2K there are several other ongoing and planned long-baseline neutrino oscillation experiments in the world — CERN-based LAGUNA-LBNO and Fermilab-based MINERνA, MINOS, NOνA and LBNE. The principle of these experiments is exactly the same as that of the T2K. Therefore, they share similar needs for hadron production measurements, although with different beam + target combinations, to improve their neutrino beam simulations. NA61/SHINE is a unique experiment in the world, which is capable of satisfying all these needs.

2.2 Beams

The physics programme described in the previous section is feasible thanks to several factors. One of them is that the SPS and the beamline in the North Area may deliver beams in a broad range of energies, including variety of projectile types. This section gives an overview of the subject; see Ref. [24] and references therein for a detailed description.

The layout of the CERN accelerator complex relevant for NA61/SHINE measurements is shown in Fig. 2.2 on the following page. Accelerator chains for proton and ion beams (in the figure the ion chain is exemplified with Pb ions) are slightly different. The protons leave their source already stripped from electrons, with energy of 750 keV. The lead ions are only partially stripped to the Pb29+ charge state and have energy of 2.5 keV/u. From sources, both beam types are injected to linear accelerators (LINACs). These

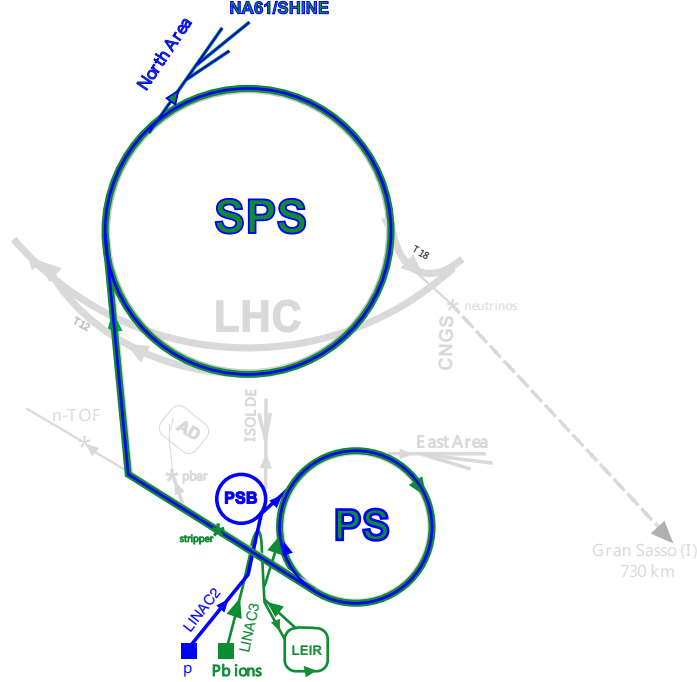


Figure 2.2: Schematic layout of the CERN accelerator complex relevant for NA61/SHINE measurements (top view, not to scale).

increase the beam energies to 50 MeV for protons and 4.2 MeV/u for lead. Exiting the LINAC3, the lead beam is stripped again by a thin ($0.3\mu\text{m}$) carbon foil and Pb^{54+} is selected by the subsequent spectrometer. From LINAC, the proton beam enters the PS booster (PSB), which accelerates it to 1.4 GeV, while the lead beam goes to Low Energy Ion Ring (LEIR), which increases its energy to 72 MeV/u. At this point two accelerator chains merge in the Proton Synchrotron (PS). The proton beam extracted from PS has momentum of 14 GeV/c, while the lead beam is accelerated to 5.9 GeV/u. At the exit of PS, the lead beam undergoes the last and complete stripping to Pb^{82+} traversing through a 1 mm thick aluminium foil. Next, beams are injected to the Super Proton Synchrotron (SPS) to be accelerated to 400 GeV/c in case of protons or 13 GeV/u to 160 GeV/u in case of ions. From the SPS, beams are extracted to the North Area. The typical *spill* (extraction) time is 10 seconds in which about 10^{13} protons or 10^8 lead ions are delivered. The extraction takes place once per the so-called *supercycle* of the SPS, which lasts about 30 to 60 seconds, in which several users (experiments) are served.

Until LS1, NA61/SHINE was mainly using *secondary beams* (there was

only a several hours long test period of low-intensity primary lead beam). The secondary beam, hadron or ion, is created by directing the primary SPS beam on a thick beryllium target located at the beginning of the North Area beamline, about 500 m upstream from the NA61/SHINE. In case of primary proton beam, various types of hadrons are produced, the majority of them singly charged, with broad momentum and angular distributions. This allows to provide to the experiment various beam energies (from about 10 GeV to 400 GeV) and projectile types using always the same 400 GeV/c proton beam in the SPS. In case of primary lead beam, the projectile is fragmented on beryllium, yielding secondary ions of various masses (A) and charges (Z). The fragments momentum per nucleon follows closely the one of the primary beam, with some (3 % to 5 % for light ions) smearing due to Fermi motion. Because of that, each energy setting of the secondary ion beam delivered to the experiment requires specific tuning of the primary beam in SPS, differently than in the primary proton beam case.

The secondary beam is transported to the experiment by the North Area beamline. It consists of a number of collimators, dipole and quadrupole magnets. It is responsible for selecting the momentum, controlling the intensity and the trajectory of the beam, as well as optimisation of abundance of the wanted projectile species. Its principle of operation is very similar to optical systems used *e.g.* in experiments in atomic physics. The role of lenses play quadrupole magnets which shape the beam. Dipole magnets act as prisms — they change beam direction and provide spatial separation between projectiles with different *magnetic rigidities* (momentum divided by charge, what effectively reduces to just the momentum in case of hadron beams). Collimators are used to select those particles that have wanted rigidities (momenta), to accept a window of angular distribution which optimizes the abundance of the wanted species and to attenuate the beam to the wanted overall intensity. The quality of the beam delivered to the experiment depends on both the physics of the beam production (initial distributions) and the transport in the beamline (*e.g.* accuracy of electrical current setting in magnets). Due to that, *e.g.* the high momentum hadron beams were easier to work with than those of low momentum.

Out of many beams used in the course of the experiment (see Fig. 2.1 on page 16), *this thesis is associated with proton beams used in 2009*. In the target vicinity, widths of these beams and widths of their angular distributions ranged from 2.5 mm to 5.5 mm and from 0.1 mrad to 0.9 mrad for beam momenta of 158 GeV/c and 20 GeV/c respectively [30].

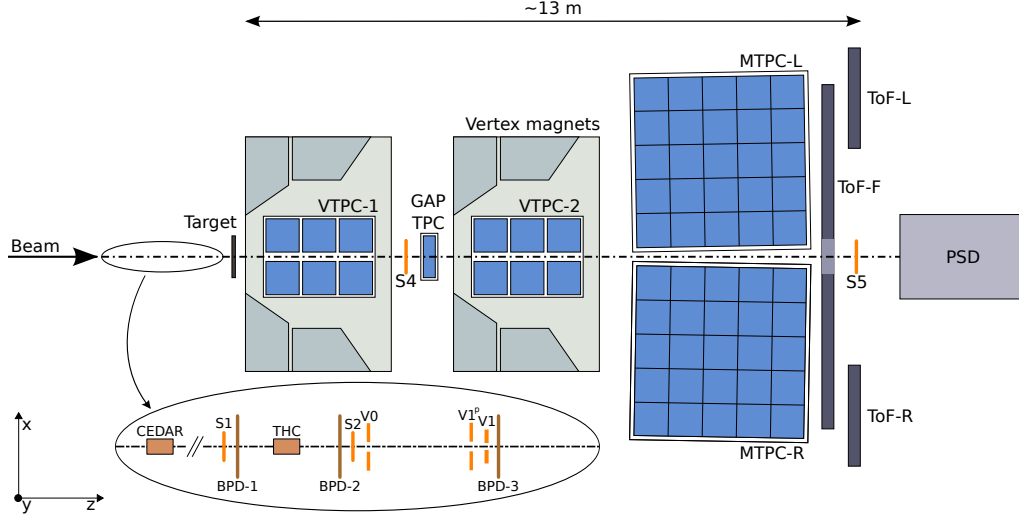


Figure 2.3: Schematic layout of the NA61/SHINE detector system (horizontal cut in the beam plane, not to scale). Also outlined are the coordinate system used in the experiment and beam detectors configuration used with secondary proton beams in 2009.

2.3 Detector components

Detector system of NA61/SHINE is depicted in Fig. 2.3 and described briefly below (see Ref. [24] for a detailed description). The figure shows also the coordinate system used in the experiment and consequently throughout this thesis, if not stated otherwise. Its origin is located in the centre of the second vertex magnet — the one surrounding the VTPC-2 in the picture. The z axis points *downstream* (relative to beam particles flux) along the nominal beam direction. The vertical y axis points upwards and the horizontal x axis towards the Jura mountains (left if one looks downstream).

2.3.1 Beam detectors and the trigger

As was outlined in Section 2.2, most of measurements were done with secondary beams. In case of hadron beams, of interest to this thesis, the beam which arrives at the experiment has well defined momentum. There are, however, few things that are not defined by the beamline (or defined not accurately enough) and need to be measured on the event by event basis.

One of them is projectile trajectory. It is assumed to be a straight line and it is determined by 3 multiwire proportional chambers called Beam Position Detectors (BPD-1,2,3 in Fig. 2.3). It serves three purposes. The first one is to constrain transverse position of the collision vertex, which cannot be assumed constant due to width of the secondary beam at the target, nor can

it be accurately determined using only TPC tracks extrapolated to the target due to low track multiplicities associated with hadron beams. The second purpose, related to the width of the beam angular distribution, is to know the angle at which the projectile hits the target. That angle defines the true longitudinal and transverse directions of the collision, rotated with respect to the coordinate system shown in Fig. 2.3 on the preceding page. Although possible rotations are not huge, they may yield significant systematic errors for certain ranges of phase space in certain accurate physical analyses (see section 4.8.5 ‘Beam divergence’ of Ref. [31]). The third purpose is on-line beam monitoring to be able to react to instabilities in the beamline during data taking.

Another value not defined by the beamline is the type of projectile. To identify hadrons delivered to the experiment, two Cherenkov detectors are used. The first one (CEDAR in Fig. 2.3 on the previous page) is a differential Cherenkov counter. It is filled with gaseous nitrogen for beam momenta lower than 60 GeV/c and helium for higher momenta. It is sensitive to a narrow range of opening angles of Cherenkov light cones. The Cherenkov angle is given by the equation $\cos\theta = 1/n\beta$, where n is refraction index which depends on the gas pressure and β is the velocity of the beam particle divided by the speed of light in vacuum. Therefore, for a given beam momentum one can set the gas pressure in CEDAR to make it sensitive to particles with wanted mass (*e.g.* protons). The second Cherenkov detector on the beam (THC in the Fig. 2.3 on the preceding page) is a threshold Cherenkov detector filled with carbon-dioxide, nitrogen or helium depending on the beam energy. The pressure is set to such a value, which makes the counter sensitive to all particles with masses lower than the wanted mass, which travel with high enough velocities to produce Cherenkov radiation, what allows to veto them. The two detectors provide purity of the identified proton beam on the level of 99 %.

The beam trigger, called T1, is formed by coincidence of two plastic scintillation counters (S1 and S2 in Fig. 2.3 on the previous page) with the CEDAR and anti-coincidence with the THC and veto scintillation counters (V0, V1, V1^p in Fig. 2.3 on the preceding page). The veto detectors are counters with holes through which the beam passes. Their role is to signal and remove events in which the beam particle travels too far from the nominal beam axis and may hit the target on the side or is accompanied by particles produced in interactions with the beamline equipment. The S1 counter serves also as a start detector for time-of-flight measurements.

The (minimum bias) interaction trigger, called T2, is formed by anti-coincidence of the beam trigger with the small scintillator placed downstream from the target (S4 in Fig. 2.3 on the previous page). So the event is consid-

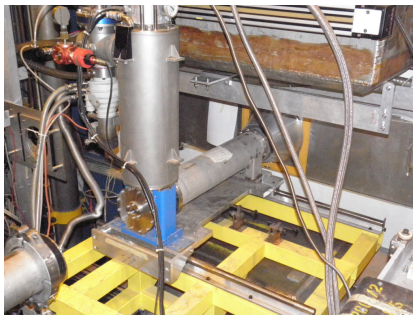


Figure 2.4: Liquid hydrogen target (LHT), with its cryogenic system, during installation in front of VTPC-1 for $p + p$ data taking. Veto scintillators and BPD-3 are going to be installed between LHT and a beam pipe (visible in the bottom left corner).

ered an inelastic interaction if there was a beam particle detected upstream from the target, but it did not appear downstream. There are however events, where an elastically scattered beam particle misses the S4, as well as events where products of an inelastic interaction hit S4. In the first case of the trigger inefficiency, an elastic event is incorrectly counted as an inelastic one. In the second case, an inelastic event is lost. That biases results of the analysis, so a correction is needed (see Section 4.5.4).

2.3.2 Targets

As was said, one of features that make the NA61/SHINE a multi-purpose hadron production measurement facility, is possibility to utilize many types of targets. Indeed, the data was already taken with various thin targets (carbon, lead, beryllium; 0.5 % to 4 % of nuclear interaction length λ_I) as well as a replica of the T2K carbon target (190 % λ_I) and a liquid hydrogen target. Usually, thickness of the target is chosen such as to minimize probability of secondary interactions in the target. When feasible, the target is also mounted in a helium atmosphere to reduce probability of interactions in the target proximity. Targets are installed in an open area in front of VTPC-1 (see Fig. 2.4). If the target is changed, beam detectors are rearranged in order to accommodate the target's holder and place the downstream veto counters and BPD-3 close to the target.

To measure proton-proton collisions, of interest to this thesis, a liquid hydrogen target (LHT), shown in Fig. 2.4, was used. It was 20.29 cm long (2.8 % of nuclear interaction length) and 3 cm in diameter. Its density was approximately $\rho_{\text{LH}} = 0.07 \text{ g/cm}^3$. Data were taken with the target filled with the liquid (the so-called *target inserted* configuration) and approximately 10 times less with the liquid removed (*target removed* configuration). In the

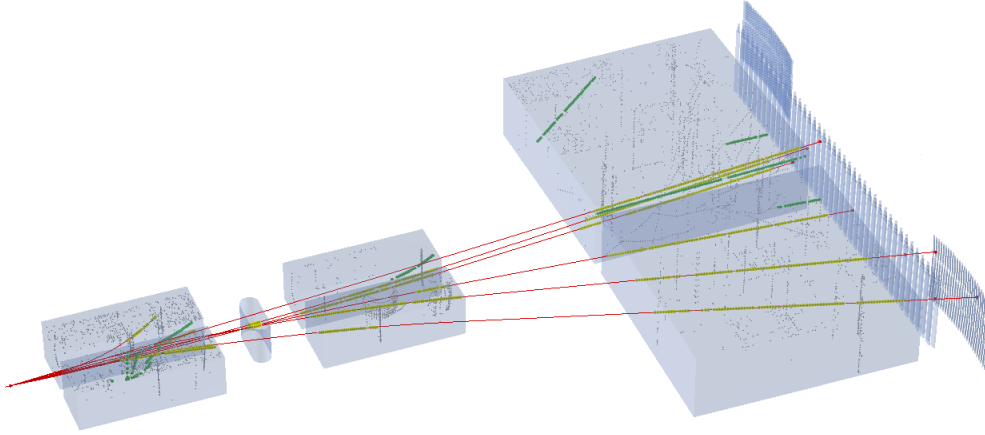


Figure 2.5: Example $p + p$ event at 158 GeV/c beam momentum. Red curves are main vertex tracks reconstructed using clusters depicted in yellow. Clusters considered as noise are shown in grey, while those in green correspond to tracks that do not originate from the main vertex.

latter case, the container was filled with gaseous hydrogen with density representing about 0.5 % of ρ_{LH} . Measurements in the target removed configuration were done in order to calculate a data-based correction for interactions with material surrounding the liquid hydrogen (see *e.g.* Ref. [30]). However, in case of ϕ meson analysis, which requires high statistics, target removed data and the correction were not used (see Section 4.5.2).

2.3.3 Time Projection Chambers

Main detectors of NA61/SHINE are four large volume Time Projection Chambers [32] (VTPC-1,2 and MTPC-L,R in Fig. 2.3 on page 21) inherited from the NA49 experiment. They are sensitive to charged particles detected as 3D tracks of energy deposits (*clusters*; see Fig. 2.5) in the gas filling the chambers. The energy deposits allow to measure indirectly mass of particles via the so-called dE/dx method (see Chapter 3). The Vertex TPCs (VTPC-1,2) are positioned inside two superconducting dipole magnets allowing to measure charges (actually signs of charge, but most of produced particles are singly-charged) and momenta associated with tracks, from their curvature and direction in which the tracks are bent. Magnetic field is set depending on beam energy to optimize the overall geometrical acceptance in the centre-of-mass frame. The Main TPCs (MTPC-L,R) add additional volume to improve dE/dx measurements. Apart from big TPCs, there is also a small GAP-TPC on the beamline between two VTPCs (see Fig. 2.3 on page 21), to cover a gap around the beam in the acceptance of big TPCs. Main pa-

Table 2.1: Main parameters of NA61/SHINE TPCs. Values on the left of ‘ $\times$ ’ symbols correspond to the transverse direction, while those on the right to the direction along the beam (neglecting rotation of MTPCs). The pad length in the VTPC-1 equals 16 mm only in the two upstream sectors. In the MTPCs, the 5 sectors closest to the beam have narrower pads and correspondingly more pads per padrow.

	VTPC-1	VTPC-2	MTPC-L/R	GAP-TPC
Horizontal sizes [cm ²]	200×250	200×250	390×390	81.5×30
Total height [cm]	98	98	180	70
Drift length [cm]	66.60	66.60	111.74	58.97
Drift velocity [cm/ μ s]	1.4	1.4	2.3	1.3
Drift field [V/cm]	195	195	170	173
Ar/CO ₂ proportions	90/10	90/10	95/5	90/10
# of sectors	2×3	2×3	5×5	1
# of padrows	72	72	90	7
# of pads/padrow	192	192	128(192)	96
# of pads/TPC	26 886	27 648	63 360	672
Pad sizes [mm ²]	$3.5 \times 28(16)$	3.5×28	$5.5(3.6) \times 40$	4×28

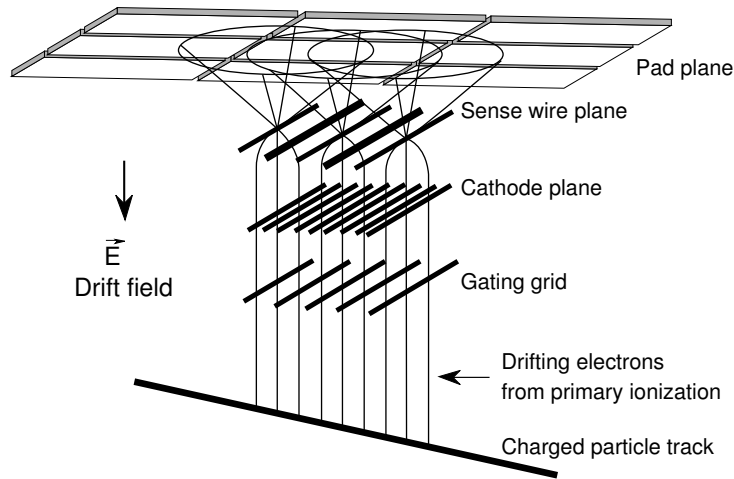


Figure 2.6: Schematic layout of the TPC readout chambers illustrating principle of operation of the NA61/SHINE TPCs. Picture taken from Ref. [25].

rameters of NA61/SHINE TPCs are given in Table 2.1 on the previous page, while their principle of operation is illustrated in Fig. 2.6 on the preceding page.

As could be read from Table 2.1 (‘Horizontal sizes’ $\times$ ‘Drift length’), the largest part of each TPC is its sensitive gas volume filled with Ar/CO₂ gas mixture and surrounded by a field cage. Charged particle which travels through the volume, ionizes atoms of the gas on its way. This process is called *primary ionization*. The field cage produces a uniform vertical electric field, which accelerates the electrons freed in the primary ionization. By collisions with the gas, the electrons soon reach an approximately constant drift velocity upwards (along the y axis of the NA61/SHINE coordinate system; see Fig. 2.6 on the previous page).

As was said, VTPCs are placed in a magnetic field. The nominal direction of this field is parallel to the electric field, however, due to the structure of magnets, some inhomogeneities of the field exist away from VTPC centres. That causes the drift velocity to diverge from the nominal vertical direction, so electrons from the primary ionization do not follow exactly vertical paths. This is called the $E \times B$ (pronounced ‘e-cross-b’) *effect*. A formula for the drift velocity $\vec{u}$ in the presence of electric and magnetic fields:

$$\vec{u} = -\frac{v_d}{E} \frac{1}{1 + \frac{v_d^2 B^2}{E^2}} \left(\vec{E} - \frac{v_d}{E} \vec{E} \times \vec{B} + \frac{v_d^2}{E^2} (\vec{E} \cdot \vec{B}) \vec{B} \right), \quad (2.1)$$

might be obtained from the *Langevin equation* of motion of a charged particle in electric and magnetic fields with friction coming from interactions with the gas, assuming that $\vec{u}$ does not change in time (see section 2.1 of Ref. [33]; Eq. (2.1) should be compared with equation (2.6) in Ref. [33] substituting v_d by $-e\tau E/m$ and introducing $\omega = eB/m$).

In Eq. (2.1), v_d is a drift velocity magnitude measured in absence of the magnetic field. It is measured on-line for each TPC by special detectors in the gas system of TPCs and then fine-tuned off-line in the calibration process using collision data. It depends on the magnitude of the electric field as well as gas mixture, temperature and pressure. As could be seen from Table 2.1 on the preceding page, gas mixture for MTPCs has slightly different proportions than the mixture for VTPCs and GAP-TPC. Larger argon content increases the drift velocity, what allows electrons to cover larger height of MTPCs in a similar time as that of VTPCs.

Both v_d and the electric field $\vec{E}$ are assumed to be constant in space within active volume of a given TPC. That is, however, not true for the magnetic field $\vec{B}$, so the drift velocity $\vec{u}$ depends on the point in space. Therefore $\vec{r}(t)$, position of primary ionization electrons drifting in TPCs, depending on

time t , is a (numerical) solution of a differential equation:

$$\vec{u}(\vec{r}) = \frac{d\vec{r}}{dt}. \quad (2.2)$$

It is some curve in VTPCs in a strong magnetic field or a straight line in MTPCs.

The drift, governed by Eq. (2.2), finishes in readout proportional chambers at the top of TPCs (Fig. 2.6 on page 25). Looking into Table 2.1 on page 25, one can see that each big TPC contains several such chambers, called *sectors*. Each chamber has three layers of wires — the gating grid, cathode plane and sense wires. The gating grid, when active, forms an electric field, which prevents drifting electrons and gas ions from entering the chamber. It opens for a short time during readout phase to allow electrons in and closes behind them to still block slow-going ions, that could disturb the electric field. The cathode plane separates the drift volume with homogeneous electric field from the amplification part. The latter is created by the sense wire plane, which consists of thin sense wires interleaved with thick grounded wires. Close to sense wires the electric field is so strong, that electrons accelerate enough to ionize the gas. New electrons again ionize the gas. This repeated secondary ionization leads to the so-called *gas amplification*, where the number of electrons increases by a factor of about 10^4 . It should be stressed that each sector has a separate high voltage supply, what leads to different amplification factors of sectors, behaving differently in time, what needs to be taken into account in dE/dx calibration (Section 3.2.2).

The amplified number of electrons collected on wires is enough to induce such an electric signal on the pad plane, which can be registered by the electronics. Each pad plane is divided into rows of pads (*padrows*) situated roughly along the x direction (MTPCs are rotated by about 1° with respect to the nominal beam direction). While padrows are parallel to each other, pads are tilted by variable angles to follow average track trajectories at the given point, to optimize tracking and dE/dx resolution. Each pad is connected to a separate electronics channel, giving about 180 000 channels in the full system. Part of electronics (motherboards responsible for initial processing of signals) was upgraded compared to NA49, providing ten times faster data acquisition — up to about 70 events per second.

For each pad, its position in space is known. Since electrons that come from vertically separated origins in the sensitive volume, travel through different drift distances, they arrive at different times. These times are registered with 200 ns sampling along with signal intensity. This allows to determine the position at which the primary ionization took place (where the track passed) from the solution of Eq. (2.2). Since signal from a single track is

distributed over several pads of a given padrow, in several time slices, to determine the primary ionization origin, a cluster is formed from signals on these pads and time slices. Its centroid defines the origin point. Clusters in a real experimental event are illustrated in Fig. 2.5 on page 24. Since each cluster corresponds to a different padrow, maximum number of clusters on track is defined by the total number of padrows that the track crosses. It should be noted, that clusters are artificial entities associated with NA61/SHINE reconstruction algorithms. Actual primary ionization, from a macroscopic point of view, happens continuously in space along the track.

2.3.4 Other components

The NA61/SHINE detector system consists of several detectors that were not mentioned until now, which however do not influence experimental results presented in this document. Here they are mentioned briefly for completeness:

- Time-of-Flight walls (ToF-L,R,F in Fig. 2.3 on page 21), which provide additional particle identification capabilities in phase space regions where dE/dx performs poorly due to overlap of Bethe-Bloch curves for different particles,
- Projectile Spectator Detector (PSD in Fig. 2.3), a hadron calorimeter needed for centrality selection in nucleus-nucleus collisions,
- Low Momentum Particle Detector (LMPD), a small TPC surrounding the target, used for centrality selection in hadron-nucleus interactions,
- A (beam nucleus mass) and Z (beam nucleus charge) detectors, necessary for measurements with secondary ion beams.

Apart of ToF-L and ToF-R all of them are NA61/SHINE upgrades of the setup.

Yet another improvement with respect to the NA49 setup, but not a detector, are beam pipes filled with helium, installed in the gas volume of VTPCs to reduce the number of δ -electrons. That is required to significantly decrease event-by-event fluctuations of the track density in the TPCs and thus reduce systematic uncertainties of fluctuation measurements in nucleus-nucleus collisions [24].

Chapter 3

dE/dx calibration

As was outlined in Section 2.3.3, each TPC track is reconstructed from a collection of energy deposits (clusters) in the gas of a TPC. This chapter describes how from those deposits a single value, called dE/dx of the track, associated with the mass of the ionizing particle, is calculated. Also some problems regarding the calibration of the energy loss are discussed, which influence the analysis methodology (see Section 4.3.2 and Appendix C).

It has to be emphasized, that the purpose of this chapter is to give rough idea of the dE/dx topic, enough to understand its implications for the ϕ analysis, and not necessarily a deep understanding of the matter. Therefore, intentionally there are almost no equations that could distract the reader from the main subject of ϕ meson analysis.

Most of concepts considered here, were developed in the NA49 for the purpose of analysis of proton-proton collisions data. They are described in detail in Ref. [34]. For completeness it should be also mentioned, that in a high track density environment of heavy ion collisions, additional effects need to be taken into account. These are discussed in Ref. [35].

3.1 Energy loss of an ionizing particle

When an ionizing particle travels through the gas of a TPC and ionizes its atoms, it obviously loses (small) part of its energy. That loss, for a given track fragment, is a random variable governed by a highly-skewed Landau (or Landau-Vavilov) distribution [7]. This distribution is called a *straggling function*. The more collisions with atoms contribute to the observed loss, the more Gaussian-like the distribution becomes. When the loss is built from very few collisions, the distribution tends to Landau with undefined mean and variance.

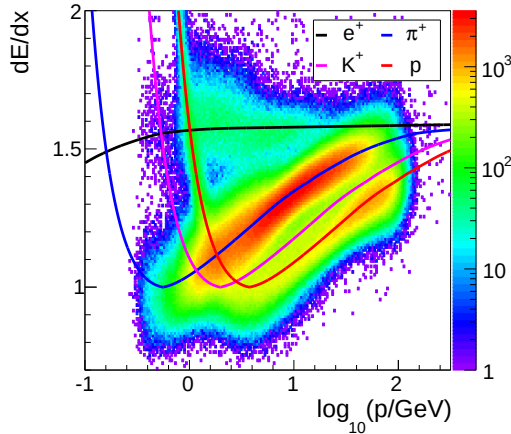


Figure 3.1: Bethe-Bloch functions overlaid on dE/dx distribution of positive tracks in 158 GeV data. It should be noted, that dE/dx in this picture is a truncated mean, which is not the same value as energy loss in Eq. (3.1), although their average values ought to behave similarly (see Section 3.3).

In the momentum range and for particle species measurable in the NA61/SHINE experiment, an average energy loss per unit length of path in the medium (mean of the straggling function) depends only on properties of the medium, the velocity of the particle and its electric charge:

$$\left\langle \frac{dE}{dx} \right\rangle = z^2 f_{\text{BB}}(\beta\gamma), \quad (3.1)$$

where β and γ are Lorentz factors of the particle (see Appendix A.2), electric charge z is equal to unity for most particles of interest and f_{BB} is the so-called *Bethe-Bloch function*. Intentionally no formula for that function is given here, as it is not well defined theoretically and there exist several parametrizations taking into account various effects [7]. A formula convenient for fits to data is given in Ref. [34].

Since $\beta\gamma = p/m$, where p is the momentum of the particle measured from track's curvature in magnetic field and m is its rest mass, the relation Eq. (3.1) says, how energy loss of a particle with known momentum is associated with its mass. Obviously, as the equation contains average loss, the energy loss measurement cannot in general uniquely determine particle's mass (and therefore species). Exemplary Bethe-Bloch functions for particles of different masses and known total momenta are shown in Fig. 3.1.

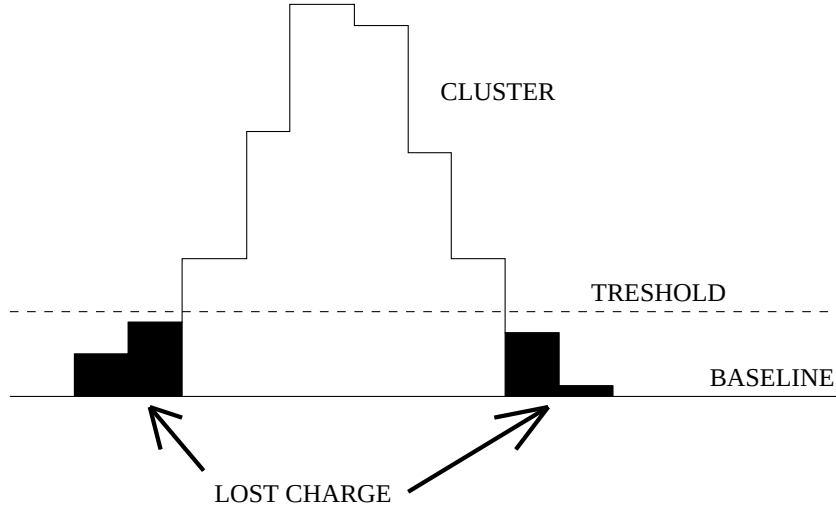


Figure 3.2: Projection of a two-dimensional cluster on pads or time slices axis. Vertical direction signifies signal level in readout electronics. The cluster loses part of its charge due to a threshold cut in the readout system (see the last paragraph of Section 3.2.1). Picture is taken from Ref. [36].

3.2 Corrections to cluster charges

At the end of Section 2.3.3, the concept of clusters in TPCs — two-dimensional signal distributions on pads and in time slices, was introduced. An illustration of such a cluster is shown in Fig. 3.2. From the point of view of tracking, the important feature of the cluster is its position in space calculated as a centroid of the structure in Fig. 3.2. However, for each cluster also its charge — a sum of signals in all time slices and on all pads belonging to the cluster, is computed. It is proportional to the energy deposited in the gas in terms of ionization, which is in turn proportional to the energy loss of the ionizing particle (not equal, as some energy is also lost for excitation of atoms and creation of δ -electrons). Therefore, the cluster charge could be considered as a sample of the energy loss measurement, governed by the straggling and Bethe-Bloch functions.

For the cluster charge to be useful in analysis, it has to be properly normalized and corrected for numerous detector effects. The purpose of these corrections is to assure, that for clusters coming from a certain particle species, at given total momentum, the straggling function is the same irrespective of place in the detector and point in time and that Eq. (3.1) on the previous page is obeyed.

Conceptually the simplest normalization is that to the unit length of path. The cluster charge is divided by the estimated track length that contributed

to the given cluster. The track length is normalized to the padrow width in the sector to which the cluster belongs, taking into account the $E \times B$ effect (projection of the track onto the pad plane is done along the curve given by Eq. (2.2) on page 27). Therefore, this normalization depends only on the track's angle with respect to the padrow.

From the point of view of the way in which corrections are determined, they can be grouped in two categories: those calculated from external information and those obtained using measured cluster charges themselves.

3.2.1 Corrections derived from external information

These corrections are computed once for a given setup (*e.g.* given gas mixture) and reused for all data sets collected with that setup. They are large, of the order of 10%. On top of them, data-driven, residual corrections are applied (Section 3.2.2).

The first one is the so-called *krypton calibration* of electronics channels. It is based on measurement of Kr decays in gas of TPCs, which have known energy spectrum. By comparison of peak positions in spectra for different channels, relative gain factors are calculated. It should be emphasised, that these measurements cannot be used to determine absolute gains, that would allow to equalize signals from different sectors. This is because TPCs operate at different conditions during Kr runs, than during normal data taking. This correction is slightly out of scope here, as it is applied before even clusters are formed during the reconstruction. But it affects dE/dx , so it has to be mentioned. It is described in detail in Ref. [37].

The second one is the pressure correction (see Ref. [34]). It takes care of most of time dependence in dE/dx . It is associated with variation of the gas gain with the density of the gas mixture. It is performed by dividing the cluster charge by a quadratic form in the measured atmospheric pressure. Parameters of this form were obtained for the NA49 experiment and might be slightly non-optimal for NA61/SHINE data. But this should be irrelevant in view of the time dependence calibration which is anyway performed for each data set (see below).

Finally, there is the correction for losses due to the threshold cut. That cut is done in the readout system of TPCs — after digitization only signals of at least 5 ADC units are recorded. This is done for the sake of noise and zero suppression. As can be seen from Fig. 3.2 on the previous page, this process removes tails of clusters. How much of the overall charge is lost, depends on the cluster's shape (width) and the original total charge. The shape, in turn, depends on the drift length (an electron cloud associated with the cluster widens during the drift because of diffusion) and track angles with respect

to pads. All these dependencies make the correction the most difficult one to compute. Details of the procedure are described in Ref. [36]. It requires simulation of cluster shapes as a function of all the mentioned parameters. The calculation is done once for a given gas mixture, so there is a single set of tables valid for all NA61/SHINE data (at least those collected until the time when this document was written). For completeness it should be also pointed out, that this correction was shown not to be necessary for the NA49 lead-lead data, for which cluster charge was obtained from a fit rather than from summing up signals [35].

3.2.2 Corrections inferred from measured charges

With exception of chip gains (see below), all these corrections are calculated separately for each collected reaction. Computations involve averaging of charges, which is done using truncated mean (see Section 3.3) to operate on similar quantities as those used later in the physics analysis. Corrections are evaluated separately for each sector, because each of them has a separate high voltage supply, with possibly different settings, and because there are construction differences between sectors. That leads to different amplification factors of sectors, behaving differently in time. Typically these corrections are smaller than those described in the previous section, with possible exception of sector constants.

The first one in this group is the time dependence correction. It is a residual smoothing correction on top of the pressure correction. First, for each sector, a time series of truncated mean charges is calculated. A single entry in the series is derived from a histogram of charge for the sector, after a predefined number of entries is accumulated. Therefore, the series have variable-size time bins. Second, by smoothing of that series, another one is obtained, with equally spaced, 10 min wide time bins. The smoothing is done by weighted averaging of entries in the first series in a time window of ± 15 min from the middle of the second series bin. The weight is a Gaussian function of a time distance of the given entry from the middle of the bin. The correction is then applied by linear interpolating the second series and dividing the cluster charge by the interpolated value.

Next is the y dependence correction (y is cluster's position along the drift direction). It is a residual correction on top of the correction for losses due to the threshold cut. First, for each sector, truncated mean charges are calculated in bins of y . Similarly to the time dependence correction, these mean values are derived from histograms, although here there is no limit on the number of entries. It is assumed, that while the mean charge may depend on y due to momentum distribution of tracks and particles

species composition, that dependence should be up/down symmetric (*i.e.* with respect to $y = 0$). Therefore, $|y|$ -dependent asymmetry has to be caused by losses during the drift. So the correction — asymmetry per meter of drift — is obtained as a slope of a line fitted to the truncated mean asymmetry parameter dependence on $|y|$. It is applied by summing up with linear part of the correction for losses due to the threshold cut [38].

The third correction inferred from measured charges are the so-called chip gains. They could be considered as taking care of residual effects not covered by the krypton calibration. For each chip (corresponding to a group of four pads), a histogram of charge ratios is built. The ratio is a cluster charge divided by the (non-truncated) mean of all cluster charges for a given track. The correction for a chip is an average of the related histogram. It is applied by dividing the cluster charge by that average ratio. Because calculation of chip gains requires large statistics, they are computed using big data sets and reused for smaller ones, collected close in time. Since charge ratios involve clusters from different sectors, determination of chip gains depends on sector constants (see below), and *vice versa*. That leads to an iterative procedure.

The last considered correction are the so-called sector constants. The goal of this correction is to equalize relative sector gains and to give proper scale to calibrated cluster charges — the so-called MIP (minimum ionizing particle) units instead of ADC units. Sector constants are derived by comparison of averaged truncated mean charges of pion tracklets in a given sector, in a narrow momentum range, with predictions of the Bethe-Bloch parametrization for pions. They are scaling parameters that align the average truncated means to the Bethe-Bloch. This obviously assumes, that the Bethe-Bloch parametrization is determined earlier. If not, then a procedure described in Ref. [34] needs to be utilized. A pion tracklet in the sector is identified using dE/dx (see Section 3.3) of the track to which the tracklet belongs, where the dE/dx is calculated with clusters from other sectors than the one under consideration. That makes the procedure of sector constants determination iterative. It usually requires up to four iterations. To apply the correction, the cluster charge expressed in ADC units is divided by the sector constant.

3.3 Track dE/dx

Having corrected cluster charges, the track dE/dx can be obtained. It is a truncated mean of charges of clusters that belong to the given track, corrected for a bias due to the number of clusters taken into account in the truncation.

The uncorrected mean is calculated using 50 % fraction of the lowest cluster charges on a track. Higher charges are discarded, hence the mean

is *truncated*. A special weighting scheme of entries is employed, to remove oscillatory behaviour of the mean depending on whether even or odd number of entries is taken [34]. The purpose of truncation is to improve resolution of dE/dx by effectively cutting off long tail of the straggling distribution.

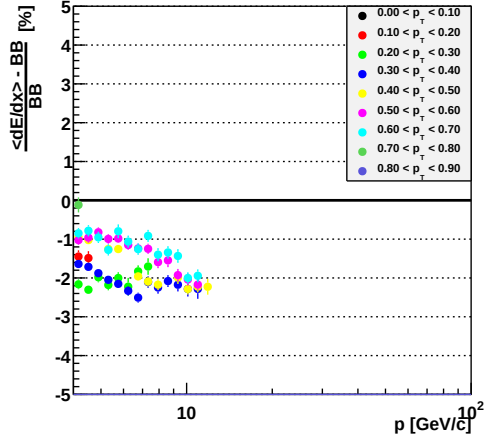
However, while the original straggling function has a well defined mean given by the Bethe-Bloch function, the mean of truncated mean values is not only systematically lower (quite obvious if we discard high charges), but is also dependent on the number of charges N taken into account in the truncation (number of all clusters on a track). It could be observed by sampling charges from some straggling distribution, computing truncated means for different values of N and averaging these means separately for each N . As a result we can see that the mean of truncated means increases with decreasing N , with a difference (bias) at $N \approx 10$ with respect to $N \approx 100$ on a level of 1 %. This bias is taken out by dividing the uncorrected truncated mean by a rational function of N . Parameters of this function can be obtained by fitting the dependence observed in the described toy MC study.

What is more, even with the above correction, the variance of the truncated mean still depends on N . That cannot be corrected, but can be simply parametrized — the variance turns out to be proportional to $\frac{1}{N}$. Because of that, fitting of dE/dx distributions requires quite complicated model function. The best currently known to the NA61/SHINE community is the one devised by Marco van Leeuwen [39]. It is a sum of asymmetric Gaussians (to account for remnants of asymmetry of the straggling function) taken over different values of N and different particle species contributing to the dE/dx spectrum, with $\sigma \propto \frac{1}{\sqrt{N}}$.

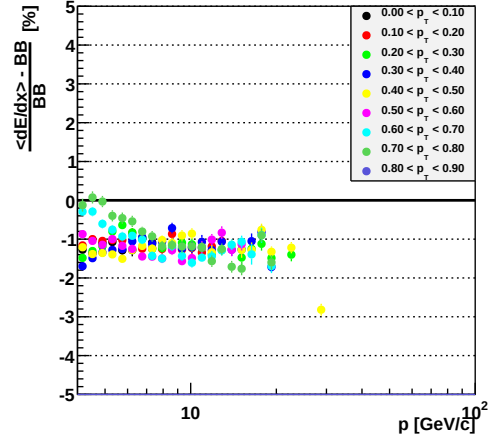
Finally, it should be noted, that dE/dx value associated with a track — a truncated mean — is different from the energy loss $\frac{dE}{dx}$ in Eq. (3.1) on page 30. The latter, proportional to energy deposited for ionization, should rather be identified with a single cluster charge. Consequently, due to mathematical properties of the truncation and the straggling function, mean track dE/dx does not follow the same Bethe-Bloch function as that in Eq. (3.1). However, it does follow a similar function, what is visible in Fig. 3.1. Because of the similarity, that one is also called a Bethe-Bloch formula.

3.4 Known problems

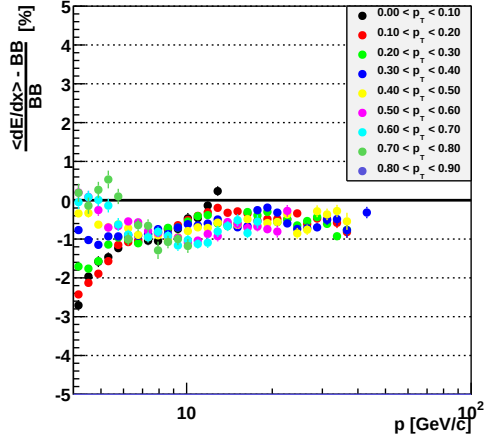
As a result of sector constants calibration, by construction, the truncated mean charge assigned to each track (Section 3.3), averaged over many tracks, should follow the Bethe-Bloch. To check if it is satisfied, a QA procedure has been introduced: after convergence of sector constants calibration, dE/dx fits



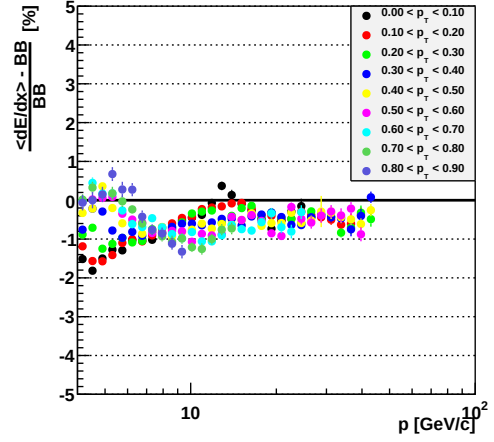
(a) $p + C$ @ 31 GeV/c



(b) $p + p$ @ 40 GeV/c



(c) $p + p$ @ 158 GeV/c



(d) $\pi^- + C$ @ 158 GeV/c

Figure 3.3: π^+ peak positions in dE/dx distributions from fits of Marco van Leeuwen method [39], compared with Bethe-Bloch values. Four reactions are shown to illustrate systematic problems.

are performed with Marco van Leeuwen method [39]. Pion peak positions resulting from fits are compared with the Bethe-Bloch function employed in the calibration. It turns out, that for all calibrated data sets, Bethe-Bloch overshoots the data on the level 0.5 % to 2 % depending on the reaction.

Example results of the QA are shown in Fig. 3.3 on the previous page for four reactions. Comparing Figs. 3.3a to 3.3d, one can see, that the bias increases with decreasing beam energy, while it is the same for different reactions at the same energy. There seems to be also some non-trivial dependence on the transverse momentum, which should be taken out by the y dependence calibration. Plots are only shown for positive pions, but the situations looks the same also for negatively charged particles. This systematic behaviour is repeated for all calibrated reactions.

The Bethe-Bloch function used in the calibration has been fitted once, to data coming from reactions at several different energies. Therefore, it would be understandable if for some energies an undershoot was seen and for some an overshoot. But it is not the case.

Unfortunately the reason for the observed problems is not known. There also was no manpower to solve them before the ϕ analysis was done, what influences the analysis methodology (see Section 4.3.2 and Appendix C).

Chapter 4

Analysis methodology

Before results of the analysis are presented and discussed, it has to be described how the analysis was done and what were the arguments for choices of the methods used.

The chapter starts with posing the analysis problem in Sections 4.1 and 4.2. Then the data set and cuts used for this analysis are defined in Section 4.3. In Section 4.4 method to extract raw yields from invariant mass spectra is discussed. It is followed by Section 4.5 which gives description of corrections to these raw yields, what defines the results of the analysis. Finally, Section 4.6 discusses various systematic effects giving rationale for choices of methods used in this analysis, as well as giving methods for estimation of systematic uncertainties.

Most of example plots shown further are derived from 158 GeV data. This is on one hand because of relatively high statistics available in this data set and on the other hand due to the fact, that for this energy a cross-check is possible with NA49 results [15].

4.1 Goal definition

The goal of the analysis is to give estimate of ϕ meson (see Table 1.1 on page 11) multiplicities, possibly differentially per units of transverse momentum p_T and/or centre-of-mass rapidity y (see Appendix A), in minimum bias proton-proton collisions for several beam energies measured in the NA61/SHINE experiment. The *multiplicity*, also called the *yield*, is the average number of particles of interest created per single inelastic event (collision).

It should be noted, that the choice of multiplicity instead of another popular value, the (differential) cross-section, is dictated by the fact that the whole study is done within heavy ion physics programme. In this commu-

nity, contrary to pure particle physics community, comparison with theory is frequently (but not solely) done using statistical thermodynamic models. Such models contain no notion of cross-section, only that of total number of particles produced and/or their momentum distributions. The yield might be converted to cross-section by multiplication by the total inelastic cross-section for the given reaction (provided it is measured elsewhere).

With its mean lifetime corresponding to travelling about 50 fm after creation, from the experimental point of view, ϕ meson decays *within* the main vertex of the interaction. Obviously, this makes it impossible to observe ϕ mesons directly in the experiment. However, an indirect measurement, using decay products and the so-called *invariant mass method*, is possible. The decay channel which provides measurement feasibility in NA61/SHINE, is the one into oppositely charged kaons (see Table 1.1 on page 11 for its branching ratio).

4.2 Invariant mass method

Due to energy and momentum conservation in the decay, the sum of four-momenta of observed decay products is equal to the four-momentum of the not observed mother particle. In case of $\phi \rightarrow K^+ K^-$ it might be written as

$$p_\phi^\mu = p_{K^+}^\mu + p_{K^-}^\mu . \quad (4.1)$$

In the experiment only momenta and charge signs of particles are measured. Therefore, to calculate the four-momentum, one has to *make assumption* about the mass of particle, which yields:

$$p_{K^\pm}^\mu = \left(\sqrt{p_\pm^2 + m_K^2}, \vec{p}_\pm \right) . \quad (4.2)$$

The magnitude of particle's four-momentum, invariant under Lorentz transformations, is just its rest mass (*invariant mass*). Using Eqs. (4.1) and (4.2), ϕ rest mass can be calculated from measured momenta and assumed masses according to

$$m_{\text{inv}} = \sqrt{p_\phi^\mu p_\mu^\phi} = \sqrt{(p_{K^+}^\mu + p_{K^-}^\mu)(p_\mu^{K^+} + p_\mu^{K^-})} . \quad (4.3)$$

This equation expresses *correlation of momenta* of decay products on which the invariant mass method is based.

Kaons from ϕ along with those from other sources, as well as other particles, are measured in TPCs. Since it is not known which tracks come from

Table 4.1: Millions of events recorded (*all*) in 2009 for 5 beam momentum values and those *selected* for analysis using cuts described in Section 4.3.2. Note, that due to statistics analysis was feasible only for beam momenta of 40 GeV to 158 GeV.

p_{beam} [GeV]	all [M]	selected [M]
158	3.5	1.3
80	4.5	1.3
40	5.2	1.6
31	3.1	0.8
20	1.3	0.2

ϕ decays, *all combinatoric choices of pairs* of oppositely charged tracks selected with certain criteria within an event are tried. For each pair in every event, m_{inv} is calculated from Eq. (4.3) on the previous page and filled into a histogram (*invariant mass spectrum*). Quantum mechanics predicts that fast decaying particles do not appear with a constant m_{inv} as stable particles do, but rather with various values of m_{inv} according to a certain probability distribution having resonant shape. The invariant mass method works by estimating the number of pairs in the spectrum which follow that distribution as opposed to those which follow the background distribution.

In the following, the symbol $m_{\text{inv}}(K^+, K^-)$ denotes an invariant mass associated with a pair of oppositely charged tracks, calculated with an assumption of kaon masses. Furthermore, the invariant mass spectra are binned into 100 bins from 990 MeV to 1090 MeV.

4.3 Data selection

4.3.1 The data — experimental and Monte Carlo

This analysis is done on the minimum bias proton-proton collision data collected by the NA61/SHINE experiment in 2009. Only the so-called *full target* or *target inserted* runs (see Section 2.3.2) are considered for reasons explained in Section 4.5.2. Event statistics are shown in Table 4.1. Reconstruction of the data was done within 14B026 production.

Initially the plan was to investigate all five available beam energies, but data sets associated with the lowest ones have too low statistics after cuts (see Fig. 4.1 on the next page). Finally only three data sets, for beam momenta of 40 GeV to 158 GeV, are used in the analysis.

For corrections described in Section 4.5 as well as for several systematic studies from Section 4.6, dedicated Monte Carlo (MC) production is used.

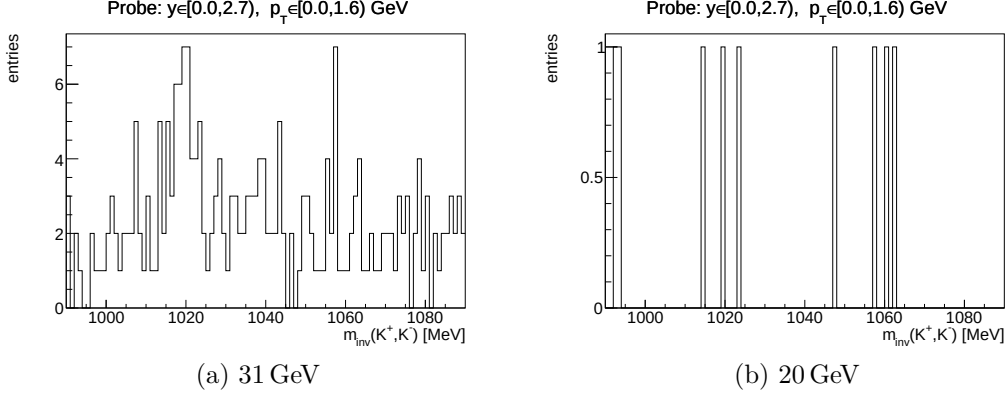


Figure 4.1: Invariant mass spectra in broad phase space for beam momenta of 31 and 20 GeV. It is clearly seen that for these energies analysis is not feasible. The meaning of ‘Probe’ is given in Section 4.4.3.

Samples for each energy contain 20M events. They are based on EPOS 1.99 model [40, 41] available within the CRMC 1.4 package [42]. Detector response is simulated using GEANT 3.21 package [43] and reconstruction is done with the same NA61/SHINE software version as for the experimental data. It should be noted, that EPOS code was slightly modified: ϕ natural width was changed to PDG value from the erroneous value of about 30 MeV, also branching ratio for $\phi \rightarrow K^+ K^-$ channel was set to 100 %, to increase usable statistics. The last change, due to small ϕ multiplicity compared to *e.g.* pions, has no significant effect on the overall event shape, so it does not deteriorate corrections quality. However, it does change the number of produced kaons enough to flaw comparisons of kaon results with EPOS model. This is why a dedicated MC production was done instead of including the changes in official NA61/SHINE MC productions.

As is written in Ref. [30], the choice of EPOS as primary event generator was dictated by the fact that other tested models performed worse in comparison with NA61/SHINE results on hadron production in hadron-hadron and hadron-nucleus interactions [44–47]. Also continuous support and documentation from the developers was taken into account. The GEANT 3.21 is utilized due to usage of legacy NA49 simulation software. An effort is currently under way [48, 49] to upgrade the simulation software to use the newer GEANT4 toolkit [50].

Table 4.2: Summary of event selection criteria. See text for a description of cuts. Thresholds for the ‘Not elastic’ cut are given respectively for 40 GeV and 80 GeV data. The cut is not applied for 158 GeV data.

name	value
BPD	beam track fitted & BPD3 good
Trigger	T2
Main vertex	ePrimaryFitZ
Fit quality	ePerfect
Vertex z	$\in (-590, -572)$ cm
Not elastic	< 35 GeV, < 74 GeV

4.3.2 Cuts

Most of event and track selection criteria (*cuts*) employed in this analysis are copied from other analyses of the same data set, especially from Ref. [30], to maintain self-consistency of NA61/SHINE results. Some of these cuts, however, are not used. On one hand they are specific to other analyses, as *e.g.* PID cut described later is specific to this analysis. On the other hand they reduce statistics too much, while being unnecessary. That is because accepting ϕ -related tracks with looser cuts and therefore possibly worse quality, only broadens the ϕ signal. That is taken into account in the fit of the invariant mass spectrum (see Section 4.4). Furthermore, some track pairs removed by these cuts contribute to the background in invariant mass spectra. However, since the contribution is small, deterioration of signal significance due to this, compared to deterioration due to overall statistics loss, is also small.

Both event and track cuts may cause biases — they remove inelastic events or ϕ -related tracks. This necessitates corrections. They are discussed in Section 4.5, apart from the correction for losses due to the PID cut. The latter is introduced at the end of this section and further explained in Section 4.4.3.

Event cuts

From Table 4.1 on page 40 one can read, that a lot of events were dropped from the analysis. Event selection criteria are divided into two groups — the *upstream cuts* (using information measured upstream from the target) and the *downstream cuts*. They are listed in Table 4.2 and discussed below.

The first group, independent of physics of investigated collisions, is applied only to the experimental data. It contains single element — the BPD

cut, which requires that the beam track is fitted and that signals from BPD3 (the one closest to the target) are used in the fit. This provides events with well reconstructed beam tracks, but also assures that the beam particle did not interact inelastically before the target vicinity. The latter is important, because NA61/SHINE trigger in 2009 allowed non-negligible amount of events in which such an unwanted interaction took place (this was improved in further years by including bigger veto counters).

The downstream cuts depend on investigated physics, therefore they are applied to both experimental and simulated data. The first of them selects events collected with the standard interaction trigger (called T2; see the end of Section 2.3.1). It requires special attention in simulation, since the triggering system as such does not exist in MC. The given event passes this cut if no track deposits signal in the volume of S4 counter.

Because the analysis is done on main vertex tracks, there are several cuts associated with the main vertex. The second downstream cut requires that the main vertex is reconstructed. It also demands that the vertex has x and y coordinates determined with Beam Position Detectors, while the z coordinate is fitted using TPC tracks (the so-called vertex type `ePrimaryFitZ`). Such a strategy provides best main vertex position estimation for the case of relatively low multiplicity proton-proton interactions in an elongated target. It is furthermore assured, that only cases with `ePerfect` fit quality flag are used, what means that the minimization package (`MINUIT`) did not run into any problems. Next, a tight cut is used for the main vertex longitudinal position — it has to be situated in the interval $(-590, -572)$ cm, well within the geometrical volume of the liquid hydrogen target (see Section 2.3.2). This requirement is further discussed in Section 4.5.2.

Finally, *events with a single, well measured positively charged track with absolute momentum close to the beam momentum are rejected* (event cut (v) in Ref. [30]). Such events are considered to be elastic events in which a beam proton is scattered into TPCs. After Ref. [30], the cut is applied for 40 GeV and 80 GeV data, but not for 158 GeV data. Track momentum thresholds for the two lower energies are listed in Table 4.2 on the preceding page. The *well measured* condition for the track is defined by similar cuts as those given in Table 4.3 on the next page (see the next section for description), without the PID cut and with a changed number-of-points cuts: $n_{\text{GTPC}} > 2$ instead of ($n_{\text{VTPC}} > 15$ or $n_{\text{GTPC}} > 4$).

Track cuts

Track selection criteria are listed in Table 4.3 on the following page. The first of them allows tracks which were successfully fitted to the main vertex.

Table 4.3: Summary of track selection criteria. See text for description of cuts.

name	value
Status	0
$ b_x , b_y $	$< 4 \text{ cm}$ and $< 2 \text{ cm}$
n_{all}	> 30
$n_{\text{VTPC}}, n_{\text{GTPC}}$	> 15 or > 4
PID	$< \pm 5 \%$ kaon Bethe-Bloch

It is analogical to the main vertex fit quality cut described above. Second condition involves components of a displacement vector from the main vertex to a point in which the xy plane at z of the main vertex is crossed by the extrapolated track. These components are called *track impact parameters* b_x, b_y and their magnitude (distance of the track from the main vertex) is required to be smaller than 4 cm and 2 cm respectively. Tracks with large values of $|b_x|$ or $|b_y|$, removed by the cut, are falsely associated with the main vertex, while truly stemming from weak decays, off-time interactions or secondary interactions. Next criterion requires that the track is reconstructed from more than 30 clusters in all TPCs. It assures reasonable dE/dx (see Section 3.3) resolution and nearly Gaussian dE/dx distributions. It also removes short, poorly measured tracks that could have survived fit status and impact parameter cuts. Moreover, it is demanded that the number of clusters reconstructed in magnetic field, attributed to the track, is larger than 15 for the measurement in VTPCs or more than 4 in GAP-TPC. This in turn assures reasonable momentum determination accuracy.

While above cuts provide well measured tracks, selection criterion having the most significant impact on the ϕ resonance analysis is the one responsible for particle identification (PID) of kaon candidates. It is illustrated in Fig. 4.2 on the next page. The variable used to differentiate kaons from other charged particles is dE/dx (see Section 3.3). Tracks are considered as kaons if their dE/dx value is within $\pm 5 \%$ of the kaon Bethe-Bloch value. Upper and lower limits of this cut are visualized as black curves in Fig. 4.2a. Importance of the cut is shown in Fig. 4.2b — without it the ϕ peak is invisible due to large background from pion combinatorics. As is obvious from Fig. 4.2a, the cut admits also particles other than kaons, *e.g.* pions and protons, which contribute to the background in an invariant mass spectrum. This is why the term *kaon candidates* is used to denote tracks allowed by the cut. Because some kaons are removed by the cut, also some number of ϕ mesons is lost. Therefore a correction of this loss is necessary. It is taken care of using a so-called *tag-and-probe* method described in Section 4.4.3.

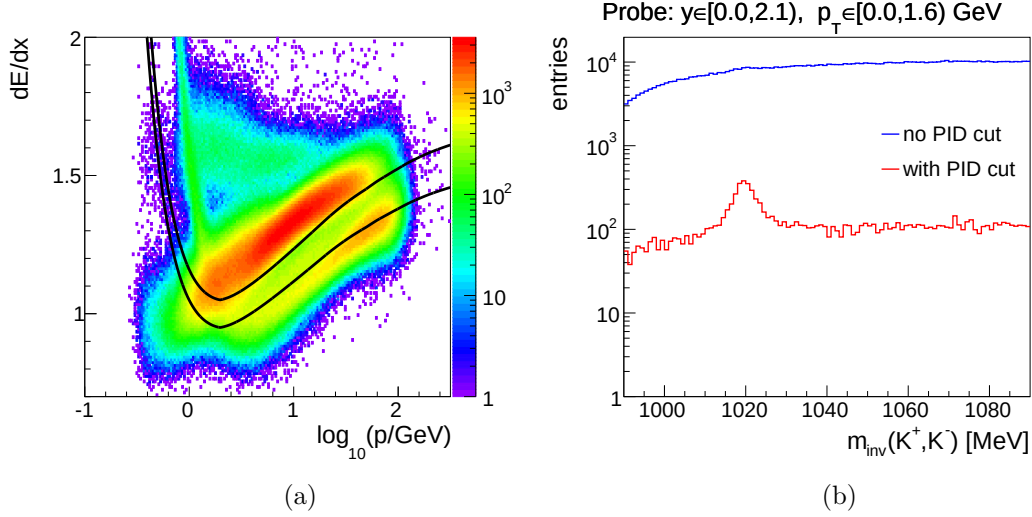


Figure 4.2: Illustration of the kaon candidate selection (PID) cut on the 158 GeV data. The kaon-rich band between black lines in picture (a) is accepted in the analysis. Structures associated with pions and protons are visible respectively above and below the band for $\log_{10}(p/\text{GeV}) > 0.5$ and the other way round for low values of $\log_{10}(p/\text{GeV})$. (b) shows background reduction due to the cut in the invariant mass spectrum in broad ϕ phase space, with the ϕ peak emerging when the cut is used.

This approach to kaon selection is slightly different from the one used traditionally. In NA49 [8, 15, 51] the band was also centred at the kaon Bethe-Bloch, but its size was defined with dE/dx resolution parameter $\sigma_{dE/dx}$ — momentum dependent Gaussian width of the fitted dE/dx distribution. A typical size was $\pm 1.5 \cdot \sigma_{dE/dx}$. Since dE/dx resolution is 4 % to 5 %, the cut used in this analysis is stricter than the traditional one. The main gain from the legacy approach is that the correction for losses due to the PID cut could be easily calculated. Assuming dE/dx distribution normalized to unity, an efficiency of the cut (ratio of accepted to all kaons) is just the integral of the distribution in the cut range. An efficiency for the pair of tracks is simply square of that value. The correction to ϕ yield is applied dividing by the pair efficiency.

The traditional approach relies heavily on the knowledge of dE/dx properties, associated with both the dE/dx distributions modelling and quality of dE/dx calibration. For example, if the assumption of Gaussian shape of dE/dx distribution is incorrect, then the correction is biased. Similar effect on the correction have flawed values of Gaussian parameters which come from fits to data. The method obviously requires, themselves non-trivial, dE/dx fits in total momentum bins and assumption of p_T invariance of dE/dx (or

2D binning in both p and p_T). That in turn necessitates a cut on track total momentum, because the fits are not possible for momenta where Bethe-Bloch curves for different particles cross. All in all, the traditional approach brings in quite a load of complications and possibilities for systematic biases. This is why, in view of an insufficient quality of dE/dx calibration, the method was changed in this analysis.

4.4 Signal extraction

Section 4.2 explained how invariant mass spectra are built from tracks selected according to criteria of Section 4.3.2. This section explains how ϕ yields are estimated from these spectra.

4.4.1 Phase space binning

As was written at the beginning of Section 4.1, the goal is to obtain the yields in bins of ϕ rapidity y and transverse momentum p_T . This requires to build a separate invariant mass spectrum for each (y, p_T) bin. The spectrum to be filled with an invariant mass of the given track pair (ϕ candidate), is chosen based on the candidate's y and p_T . These quantities are calculated (see Appendix A.2 for the formulas) using the candidate's four-momentum defined by Eqs. (4.1) and (4.2) on page 39. By convention, for each invariant mass spectrum shown in this document, the (y, p_T) bin is given in the plot's title (at the top) as ranges in both variables (see *e.g.* Figs. 4.2b and 4.4 on the previous page and on page 48).

In the course of the analysis several different types of binning are used. First, there is the so-called *binned phase space* or the *analysis binning*, which is the most differential (2D or 1D) binning considered for the given data set. It is illustrated by red lines in Fig. 4.3a on the next page for the case of a double differential analysis or in Figs. 4.3b and 4.3c for a single differential analysis. The term *1D binning* refers to a situation of binning in one of the two variables, while the second variable is limited (there is one bin in that variable). It is exemplified by red lines in Figs. 4.3b and 4.3c. The reason for the limit is to confine the analysis to the region of high acceptance. In the case of a double differential analysis, a 1D rapidity binning (Fig. 4.3b) is used for a certain purpose (see Section 4.4.4) with the transverse momentum limit such that the 1D binning covers the same region as the analysis 2D binning. Such limiting is called *broad*. Finally, associated with the latter, there is the so-called *broad* or *unbinned phase space*. It is used in both double and single differential analyses (see Section 4.4.4). In this case, shown

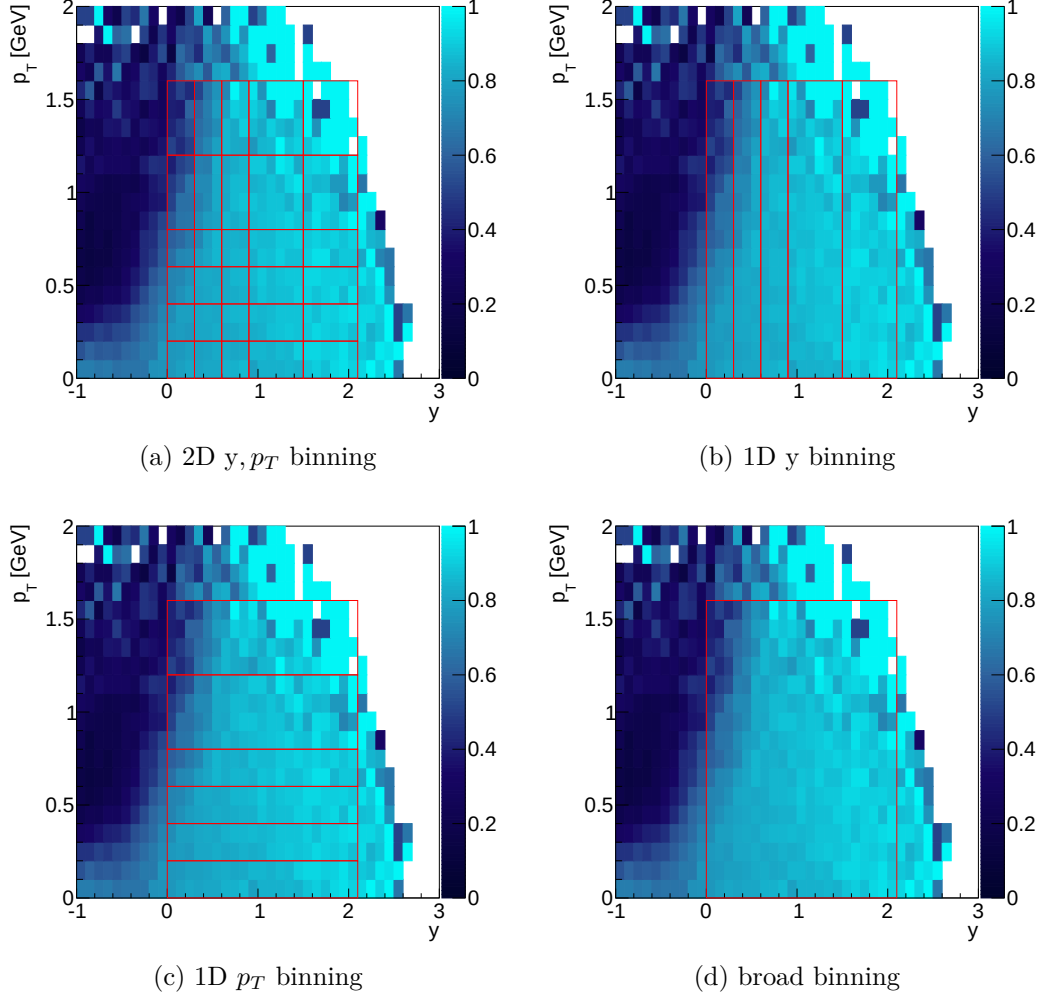


Figure 4.3: Illustration of binning types used in the analysis. Each binning type example is depicted as red lines. The *analysis binning* corresponds to the pane (a) for a double differential analysis or either of panes (b) and (c) for a single differential analysis. They are overlaid on ϕ registration probability plot for 158 GeV (see Section 4.5.4 for the method to calculate it). White regions in the plot correspond to bins where calculation of the probability was impossible due to insufficient statistics of generated tracks.

as red box in Fig. 4.3d, there is only a single broad bin, which covers the same two-dimensional region as the analysis (2D or 1D) binning. That could be opposed to the *total phase space* where the invariant mass spectrum is constructed without any limits on rapidity and transverse momentum.

For comparison with other experimental results, the results of this analysis are also presented in $(y, m_T - m_0)$ bins, where m_0 is the rest mass of ϕ . For technical reasons, however, the yields are extracted in (y, p_T) bins.

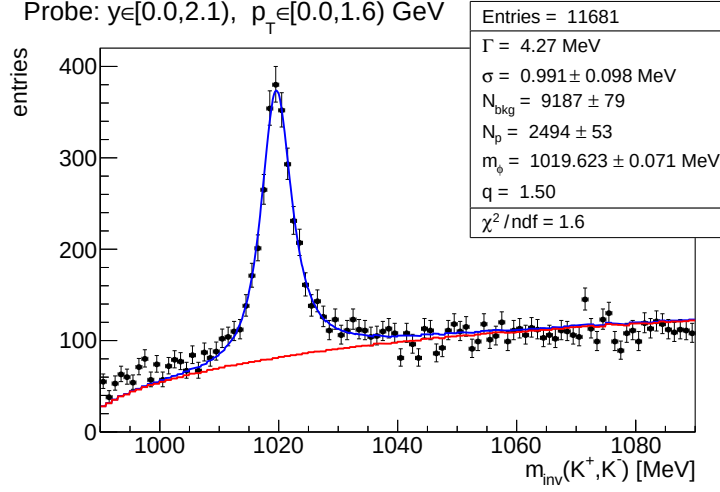


Figure 4.4: Example of a fitted invariant mass spectrum in the broad ϕ phase space for 158 GeV data. Signal shape parameters m_ϕ and σ resulting from this fit are used to constrain fits in binned ϕ phase space (see Section 4.4.4). Blue curve illustrates the fitting function defined by Eq. (4.7) on page 51, while red curve is the background component coming from the event mixing. See text for description of quoted parameters; the meaning of ‘Probe’ is given in Section 4.4.3.

The p_T bin edges are chosen such as to get desired bin edges in $m_T - m_0$, if m_T is calculated from p_T and m_0 , where m_0 is taken as given in PDG (Table 1.1 on page 11). That is formally incorrect, since m_0 should rather be the true rest mass of ϕ meson — the resonantly distributed m_{inv} . However, given the small width of ϕ , the error made in this way is rather insignificant. Looking at bin edges in Ref. [15], it seems that the same was done in the NA49 analysis.

4.4.2 Single invariant mass spectrum

Once the invariant mass spectra in the analysis binning are constructed, they can be fitted to extract the ϕ yields. Before going to the tag-and-probe method which is finally used to obtain these yields, one should understand how a single invariant mass spectrum can be modelled. An example of such a histogram is shown as black points in Fig. 4.4. One can observe two easily distinguishable structures in the plot — the ϕ signal peak around 1020 MeV and a broad background.

Signal parametrization

The signal contribution is parametrized with a function which contains two components to take into account the natural shape of the resonance and the broadening due to the detector resolution.

The first component is described by a relativistic Breit-Wigner distribution of the same form as the one used by NA49 in Ref. [8]:

$$L(x; m_\phi, \Gamma) \propto \frac{x\Gamma_x(x)}{(x^2 - m_\phi^2)^2 + m_\phi^2\Gamma_x^2(x)}, \quad (4.4a)$$

with

$$\Gamma_x(x) = 2\Gamma \left(\frac{q(x)}{q(m_\phi)} \right)^3 \frac{q^2(m_\phi)}{q^2(x) + q^2(m_\phi)}, \quad (4.4b)$$

and

$$q(x) = \sqrt{\frac{1}{4}x^2 - m_K^2}, \quad (4.4c)$$

where m_ϕ is the peak position (expected to be equal within uncertainties to the mass of ϕ as quoted in PDG), Γ is the natural width of ϕ and m_K is the kaon mass. The choice of this parametrization is discussed in Section 4.6.3.

The second component is described by the q-Gaussian function:

$$G(x; \sigma, q) \propto \left[1 + (q-1) \frac{x^2}{2\sigma^2} \right]^{-\frac{1}{q-1}}, \quad (4.5)$$

where σ is a width parameter and q is a shape parameter. The q-Gaussian tends to Gaussian with standard deviation σ when $q \rightarrow 1$ and is equivalent to Lorentzian distribution when $q = 2$. The choice of this parametrization, in turn, is discussed in Section 4.6.2. As is stated in that section, the parameter q is never fitted to data, but is fixed using Monte Carlo. This is why it is dropped from the list of parameters in the following equations.

Finally the resonance peak model is given by a convolution:

$$V(x; m_\phi, \sigma, \Gamma) = L \otimes G = \int_{-\infty}^{+\infty} G(x'; \sigma) L(x - x'; m_\phi, \Gamma) dx'. \quad (4.6)$$

In practice it is not possible to fit at the same time both width parameters σ and Γ . Therefore the latter is fixed to the PDG value (Table 1.1 on page 11) and subsequently dropped from further equations, similarly to q .

Background from event mixing

The background contribution is modelled using the *event mixing method*. It is based on an assumption (discussed below) that the background is composed of uncorrelated track pairs. Since tracks in different events are obviously not correlated, one can obtain a background shape template (red curve in Fig. 4.4 on page 48), the so-called (invariant mass) *mixed-events spectrum*, combining into pairs tracks from one event with those from other events. Of course the considered events and kaon candidates are the same (they pass the same cuts) as those used to build the invariant mass spectrum under analysis (*same-events spectrum*). Also the mixed-events spectrum is built in the same ϕ phase space binning as the same-events one.

In this analysis event mixing is implemented by taking kaon candidates from the given event and combining them with candidates from 500 events previously processed in the loop. This number of events is arbitrary and should be chosen such, to obtain smooth mixed-events spectra, but not too huge due to performance reasons — the more events are used, the slower the processing and the smoother the mixed-events spectra.

As was mentioned, modelling of the background with the event mixing method fails in presence of correlated pairs in the background. In fact such pairs may occur due to several different reasons, *e.g.*:

- kinematic constraints due to energy-momentum conservation within the event,
- other resonances decaying into the same channel,
- other resonances decaying into another channel, when daughters are misidentified by the PID cut,
- correlations due to Coulomb or strong interactions,

Because of that, event mixing rarely works and it should be considered quite lucky that it seems to work in this case — looking at the Fig. 4.4 on page 48, one can see that there is no gross problem with the background description. However, there seem to be some small systematic deviations of data points from the fitting curve left and right of the resonance peak. They are further discussed in Section 4.6.1.

For completeness it should be said what are the other possible methods for the background treatment. First, there is a modification of the event mixing, which tries to deal with correlations in the background due to energy-momentum conservation. In this method only events from the

same multiplicity classes are mixed and then spectra for different multiplicities are averaged using multiplicity distribution to obtain the spectrum used to model the background. Such an approach was found to be paramount for $K^*(892)^0$ analysis in NA49 [52, 53]; it was also applied for such analysis in STAR [54] and ALICE [20]. In case of ϕ analysis in NA49 no difference was observed between the two approaches to the event mixing [51] and only the standard method was employed for published results [8, 15].

Another possible method for the background treatment is the so-called *like-sign mixing*. In this approach one combines into pairs tracks with the same sign instead of oppositely signed. Because the tracks come from the same event, this could take into account correlations due to kinematic constraints even better than the multiplicity mixing. This method was used by STAR [54] and ALICE [20]. It is also utilized in NA61/SHINE in the ρ^0 analysis [55]. It turned out that it cannot be applied here, though, because rarely two like-sign kaon candidates appear in an event.

Finally, various analytic expressions are used to parametrize the background (*e.g.* in LHCb [22] and ATLAS [21]) or at least the residual effects after subtraction of a mixed-events spectrum (*e.g.* in NA49 [8, 51] and ALICE [20]). While these functions may take into account problematic effects of correlated contributions to the background, they have some drawbacks as compared to the above procedures:

- the proper formula needs to be guessed without any physics motivated choices,
- there may appear a significant fitting range dependence of the result,
- there are usually 2 to 3 free parameters in the fit, instead of only 1.

Especially the last point is troublesome for the discussed analysis due to relatively low statistics of the fitted histograms (see Fig. 4.7 on page 57).

The fitting function

Having models for both the signal and the background, the function used to fit the invariant mass spectrum (blue curve in Fig. 4.4 on page 48) can be defined:

$$f(m_{\text{inv}}; N_{\text{p}}, N_{\text{bkg}}, m_{\phi}, \sigma) = N_{\text{p}}V(m_{\text{inv}}; m_{\phi}, \sigma) + N_{\text{bkg}}B(m_{\text{inv}}), \quad (4.7)$$

where $V(m_{\text{inv}}; m_{\phi}, \sigma)$ is given by Eq. (4.6) on page 49, $B(m_{\text{inv}})$ is the mixed-events spectrum and both are normalised in such a way that N_{p} and N_{bkg}

are respectively the number of signal and background pairs in the spectrum. That makes four parameters to be fitted to data.

It should be noted that this procedure is different from the one used in other quoted analyses that utilize event mixing. There the mixed-events spectrum was normalized in a certain way (which differs between the experiments) and subtracted from the analysed same-events spectrum. Such an approach effectively prevents use of likelihood fits due to possible negative values in bins; one should also pay attention to the calculation of bin content uncertainties. Since the likelihood, contrary to the χ^2 enforced by the subtraction, is a natural language to describe probability distributions (histograms), especially in case of low statistics, the common choice seems not to be the optimal one. Moreover, the chosen normalization, equivalent to the amount of background in the same-events spectrum, is arbitrary. That needs to be translated into contribution to the systematic uncertainty. In the method used here, this effect is automatically incorporated in the statistical uncertainty. With above in mind, there is no clear advantage in using subtraction of mixed-events spectrum to extract the signal. Subtraction, though, might still be a convenient way to visualize the signal in data, along with the fitted function, if the signal is much smaller than the background.

Furthermore, even more troubles arise in case of normalization scheme of NA49 ϕ analyses [8, 15, 51], employed after Ref. [56]. There the mixed-events spectrum is normalized to the number of entries of the same-events spectrum (in other approaches with subtraction the normalization region, in which spectra have the same integral, does not cover the resonance peak). Then, even if the mixed-events spectrum properly describes the background shape, artefacts appear in the subtracted spectrum. That requires subtracted spectrum shape template from Monte Carlo simulation of the procedure, as stated in the original paper [56] defining the method. Alternatively, as done in Refs. [8, 15], a term can be added to the function fitted to the subtracted spectrum, to take into account the residual effects.

4.4.3 Tag-and-probe method

After modelling of a single invariant mass spectrum is explained, a procedure to extract the ϕ yields corrected for losses due to kaon candidate identification inefficiency, can be described. The tag-and-probe method is used in LHCb analysis [22] for the same purpose and in a similar way as here, as well as to improve Monte Carlo PID correction in ATLAS [21]. The method requires two data samples (two invariant mass spectra; see Fig. 4.5 on the next page). For both of them the same events are selected. Also the same track cuts are applied, apart from the PID cut.

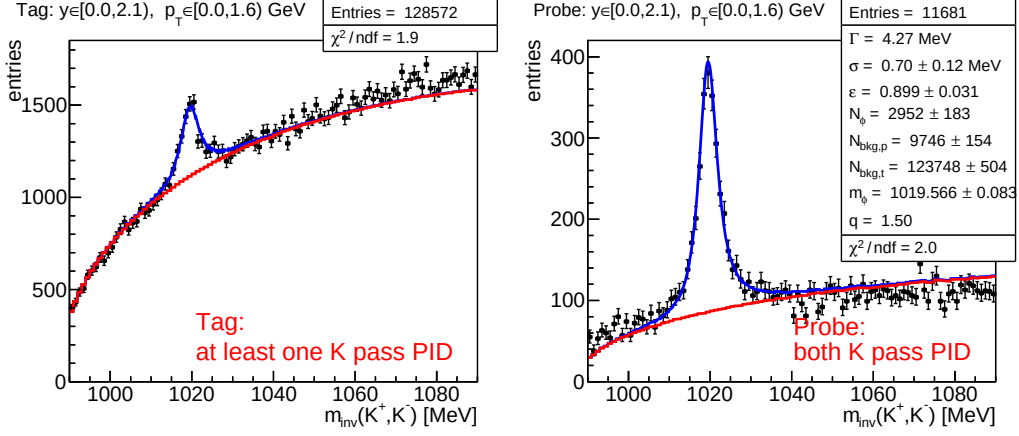


Figure 4.5: Illustration of a simultaneous tag-and-probe fit in broad ϕ phase space for 158 GeV data. Blue curves represent the fitting function defined by Eq. (4.9) on the following page, while red curves are background components coming from the event mixing. See text for description of quoted parameters.

The *probe* sample (right plot in Fig. 4.5) is built of track pairs where both tracks need to pass the PID condition for kaon candidate. It is the usual way of ϕ analysis, found *e.g.* in Refs. [8, 15]. Indeed that was implied in Section 4.3.2 while discussing cuts and these samples were shown until now as invariant mass spectra.

The *tag* sample (left plot in Fig. 4.5) is built of track pairs where at least one track passes the PID condition for kaon candidate. This is a looser pair cut than the one associated with the probe sample. This can be seen in Fig. 4.5 as much more pronounced background in the tag sample as compared to the probe sample. The huge increase in background yield is mostly due to pions which are about 10 times more abundant than kaons. The bump visible above the background estimation from event mixing, right from the ϕ peak, associated most probably with the $K^*(892)^0$, is discussed in Section 4.6.1.

In order to give expected signal yields in both spectra let's denote the efficiency of kaon selection (probability that the kaon is accepted by the PID cut) as ε and the total number of ϕ mesons contributing to the spectra as N_ϕ . It can be shown (see Appendix C.1), that the expected signal yield in the tag sample, as function of N_ϕ and ε , is

$$N_t(N_\phi, \varepsilon) = N_\phi \varepsilon (2 - \varepsilon), \quad (4.8a)$$

while that in the probe sample is

$$N_p(N_\phi, \varepsilon) = N_\phi \varepsilon^2. \quad (4.8b)$$

It should be emphasized that these formulas are valid only under the assumption that ε is constant for all kaons contributing to the signal in both spectra. That is not necessarily true in this analysis. Appendix C discusses what happens with the formulas in such a case. Systematic uncertainty of the estimated ϕ yield due to this effect is quantified in Section 4.6.5.

N_ϕ should be understood as the number of ϕ mesons, the daughters of which pass all track cuts not taking into account the PID cut. It means that the number should still be corrected for various effects other than PID (*e.g.* geometrical acceptance), to give an estimate of the number of ϕ mesons produced in the collisions. That issue is covered in Section 4.5.

Above allows to define the function (blue curves in Fig. 4.5 on the previous page) used to fit *simultaneously* both distributions:

$$f(m_{\text{inv}}) = \begin{cases} N_t(N_\phi, \varepsilon)V(m_{\text{inv}}; m_\phi, \sigma) + N_{\text{bkg},t}B_t(m_{\text{inv}}) & \text{for tag sample} \\ N_p(N_\phi, \varepsilon)V(m_{\text{inv}}; m_\phi, \sigma) + N_{\text{bkg},p}B_p(m_{\text{inv}}) & \text{for probe sample} \end{cases}, \quad (4.9)$$

where $V(m_{\text{inv}}; m_\phi, \sigma)$ is given by Eq. (4.6) on page 49, $B_t(m_{\text{inv}})$ and $B_p(m_{\text{inv}})$ are the mixed-events spectra for tag and probe samples respectively and the three are normalised in such a way that the terms N_t and N_p defined by Eqs. (4.8) on the previous page give the numbers of signal pairs respectively in the tag and probe spectra, while $N_{\text{bkg},t}$ and $N_{\text{bkg},p}$ numbers of background pairs in the respective histograms. That makes total of six parameters (N_ϕ , ε , $N_{\text{bkg},t}$, $N_{\text{bkg},p}$, m_ϕ , σ) to be fitted to data. Obviously only N_ϕ is the parameter of interest which allows to estimate the ϕ meson yield.

4.4.4 Fitting strategy

Due to limited statistics not all parameters mentioned above could be fitted in each analysis bin separately. Therefore, a three-step fitting strategy was developed. It was initially designed for a double differential analysis and this section is largely written in that context. Later it turned out that it also works fine for a single differential analysis. A small, necessary change in the second step for that case is described at the end of the respective fragment of this section.

All fits are extended binned log-likelihood fits (see *e.g.* Ref. [57]). The implementation utilizes the RooFit library [58] shipped with the ROOT framework [59]. It is chosen for its convenient support for

- building composite probability density functions (PDFs) including convolutions,
- automatic normalization of PDFs,

- wrapping histograms as PDFs (needed for mixed-events spectra),
- simultaneous fits (needed for tag-and-probe method),
- soft constraints on fit parameters.

All minimizations involved are performed with the MINUIT2 library [60] (also shipped with ROOT) and its MINIMIZE algorithm [61], which have been found to give satisfactory performance and stability.

In MINUIT2 there are two possible choices of algorithms for estimation of statistical uncertainties of fitted parameters — HESSE and MINOS [61, 62]. HESSE provides the so-called *parabolic errors*, which are symmetric. MINOS gives asymmetric uncertainties. The latter could have been preferable here, because in the tag-and-probe fit ε is limited in the $(0, 1)$ interval and values close to the upper limit occur quite frequently (see Fig. 4.7 on page 57). It was checked, however, that uncertainty estimates for N_ϕ differ at most only by 15 % between the two kinds (that is 15 % of about 20 % — a typical relative statistical uncertainty on N_ϕ). Furthermore, asymmetric uncertainties are less convenient to deal with in terms of interpretation (*e.g.* visual comparison of relative statistical and systematic uncertainties in dependence on p_T or y) than symmetric ones. Therefore it was decided to use uncertainties provided by the HESSE algorithm.

In the first step of the strategy, a probe sample in the broad phase space is fitted with function Eq. (4.7) on page 51 to obtain accurate values of signal shape parameters m_ϕ and σ on a high statistics histogram. This is illustrated in Fig. 4.4 on page 48. The fit is done in the full invariant mass spectrum range, *i.e.* 990 MeV to 1090 MeV. The values of m_ϕ and σ are then used to constrain fits in further steps — the parameters are fixed to these values. This is based on an assumption that signal shape parameters are the same or very close in different phase space bins. The assumption is verified with Monte Carlo in Section 4.6.2 and systematic uncertainties due to this are discussed in Section 4.6.4.

In the second step (for a double differential analysis), a 1D rapidity binning in the broad transverse momentum range is utilized (Fig. 4.3b on page 47). In each bin a simultaneous tag-and-probe fit is done with function Eq. (4.9) on page 54, with fixed signal shape parameters m_ϕ and σ . This is illustrated for one of rapidity bins in Fig. 4.6 on the following page. The fit is done in the range 990 MeV to 1060 MeV to minimize the impact of $K^*(892)^0$ reflection in the tag spectrum (see Section 4.6.1).

The goal of the second step is to calculate approximate values of PID efficiency parameter ε for all analysis p_T bins in the given y bin on a relatively high statistics histogram. The value ε_y of the ε parameter obtained for this

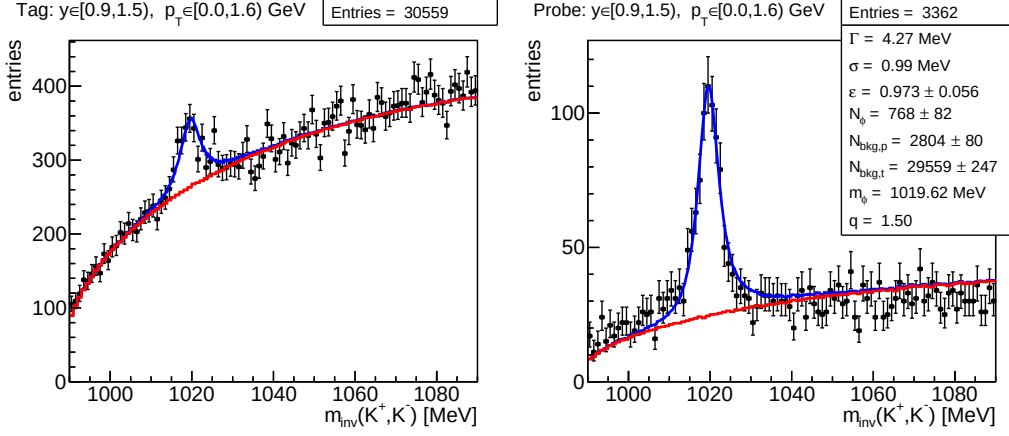


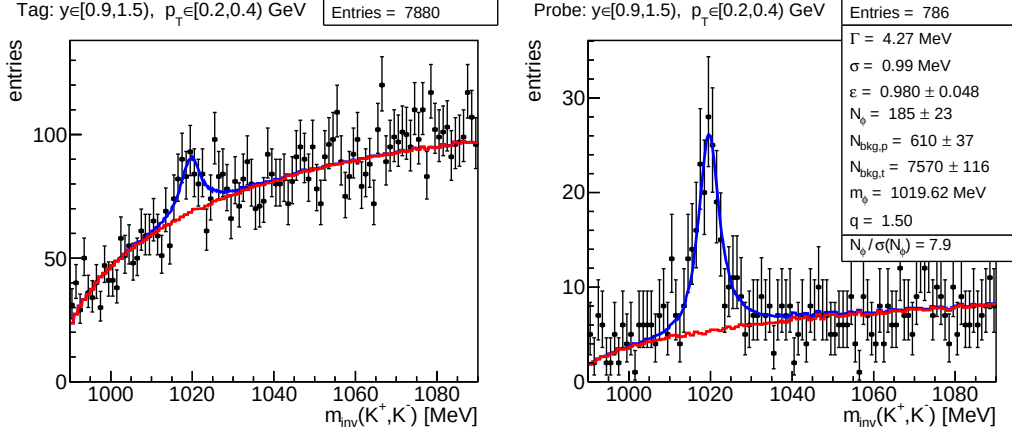
Figure 4.6: Example of a simultaneous tag-and-probe fit done in the second step of the fitting strategy, to constrain PID efficiency parameter ε in the next step. The spectra come from 158 GeV data. They are derived in one of the analysis y bins, in the broad p_T range. Signal shape parameters m_ϕ and σ are fixed to values obtained in the first step of the fitting strategy (Fig. 4.4 on page 48).

y bin is used to soft-constrain fits of the third step in all (y, p_T) bins for the same y . A Gaussian constraint in ε is applied with the mean equal to the ε_y and the standard deviation equal to the statistical uncertainty of ε_y from the fit.

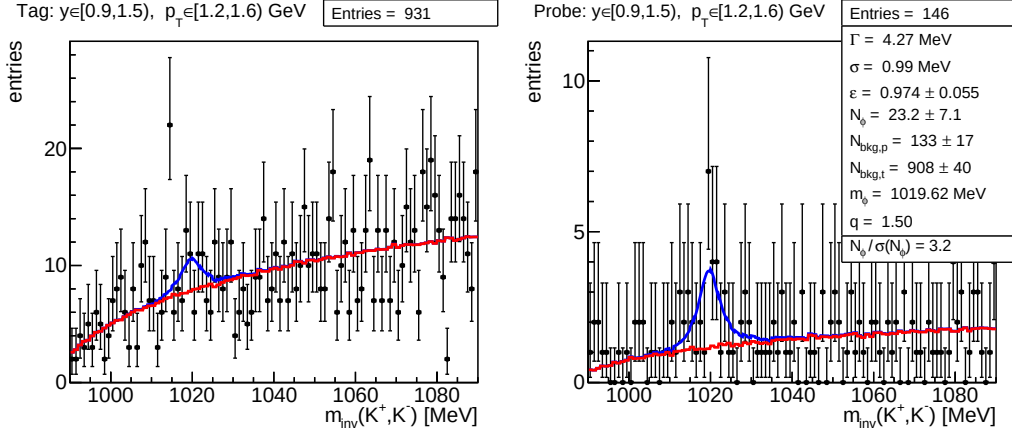
This step is based on an assumption that kaon identification efficiency changes between (y, p_T) bins, but should be similar (not equal — hence the soft constraint instead of fixing in the third step) for different p_T at the same y . That is supported by the fact that ε should primarily depend on the kaon total momentum p , while kaons from ϕ are clustered at low p_T , therefore have similar p_T and therefore y is roughly equivalent to p . Rationale for this is given in Appendix B. Systematic uncertainties arising due to such treatment are discussed in Section 4.6.5 along with a validation of the approach.

For the case of a single differential analysis, the change is that the constraint on ε is computed in the broad phase space instead of per 1D rapidity bin. That might initially seem unsound, as in the broad phase space, kaons of largely different total momenta are mixed, what makes the reasoning of Appendix B void in this case. However, the validation arguments of Section 4.6.5 hold. It means, that a small bias introduced by the constraint is less significant to the analysis than the stabilisation of fits.

Finally, in the third step, simultaneous tag-and-probe fits are done for all analysis (2D or 1D) bins separately to obtain the raw ϕ yields. Again function Eq. (4.9) on page 54 is employed with fixed signal shape parameters m_ϕ and σ , along with ε constrained according to the explanation above.



(a) a high statistics bin



(b) a low statistics bin

Figure 4.7: Examples of a simultaneous tag-and-probe fits done in the final step of the fitting strategy, to determine the raw ϕ yield. The spectra come from 158 GeV data. They are derived in two exemplary analysis bins to demonstrate the case of (a) a high statistics fit and (b) a low statistics one. The rapidity interval is the same as in Fig. 4.6 on the preceding page. Signal shape parameters m_ϕ and σ are fixed to values computed in the first step of the fitting strategy (Fig. 4.4 on page 48); ε is soft-constrained using results of the second step (Fig. 4.6 on the preceding page). Also signal significance parameters $N_\phi/\sigma(N_\phi)$ (inverse relative statistical uncertainties of N_ϕ) are shown.

Again the fits are done in the range 990 MeV to 1060 MeV.

Fitted invariant mass spectra in two example p_T bins, in the same y bin as Fig. 4.6 on the preceding page, are shown in Fig. 4.7. Panel (a) presents an example of a high statistics bin, while (b) displays the case of low statistics. At the bottom of statistics boxes on the right, the signal significance parameters $N_\phi/\sigma(N_\phi)$ (inverse relative statistical uncertainties

of N_ϕ) are given. It is visible that they are rather small. This is why such an elaborated fitting strategy was developed, which stabilizes the fits and trades some statistical uncertainty for the systematic uncertainty.

4.5 Corrections

4.5.1 Overview

In the previous section it was explained how the numbers N_ϕ of ϕ mesons contributing to the invariant mass spectra are obtained. Dividing these values for each analysis bin by the number of selected events N_{ev} , as well as by the transverse momentum bin width Δp_T and the rapidity bin width Δy , gives what is called the *normalized raw spectrum* of ϕ mesons. This is done for both double and single differential analyses, as by a convention of the NA49 single differential ϕ analysis, transverse momentum spectra are shown in a double differential normalization [8, 15].

As was already mentioned, these spectra need to be corrected for several known biases to become final experimental estimates of ϕ meson differential multiplicities in proton-proton collisions, the *corrected spectra*. Because the results of this analysis are compared with other results obtained with the same detector, it should be well understood which systematic effects are taken into account in this and the other analyses.

Already taken into account intrinsically by the extraction method is the bias due to kaon identification inefficiency, which needed an external correction in the NA49 ϕ analyses [15, 51]. Furthermore, as is explained in Section 4.5.2, this analysis, similarly to NA49 ϕ analyses, does not require a correction for off-target interactions. Such a correction was used in other NA61/SHINE analyses, *e.g.* the pion production analysis [30].

The simplest correction which is applied in this analysis as well as in the NA49, ALICE [20] and LHCb [22] ϕ analyses, but not in the main result of the ATLAS [21] ϕ analysis, is a correction associated with the branching ratio for the chosen decay channel. The raw spectrum is divided by the branching fraction taken from PDG [7]. Another semi-trivial corrections are a correction due to integration cut-off, described in Section 4.5.3 and a correction connected with unaccounted-for effects in the background, discussed in Section 4.6.1.

A more involved correction, described in Section 4.5.4, taking into account several different effects, is calculated with Monte Carlo simulation. The effects treated by this single correction are:

registration Efficiency of particle registration by the detector, which com-

bines purely geometrical acceptance (whether the particle hits the volume of the detector) with decays (the produced particle may not live long enough to leave a sufficiently long trace in the detector) and particle transport through the detector (secondary interactions).

reconstruction Combined track reconstruction efficiency (if a track which is registered in the above sense is reconstructed by the software and passes the track selection criteria) and bin migration due to track momentum resolution.

vertex Bias due to vertex cuts, which require at least that the main vertex is successfully fitted using TPC tracks and therefore favour high multiplicity events over low multiplicity ones (events with ϕ produced over those without).

trigger Trigger bias due to S4 counter vetoing inelastic events (see Section 2.3.1).

This approach is standard within NA61/SHINE analyses (*e.g.* pion production in proton-proton [30]). On the other hand the NA49 ϕ analyses to which comparison is performed, treated these issues differently. The **registration** efficiency was calculated with high statistics flat ϕ phase space Monte Carlo. The **reconstruction** efficiency was claimed [15] to be 100 % from simulation with embedding of MC tracks into real experimental events. The **trigger** bias was ignored without a comment, although it must have been present in the same way as in this analysis, due to the same definition of the interaction trigger (Section 2.3.1). Finally, a correction for the **vertex** bias is present in Ref. [51], but although it is mentioned in Ref. [15], it is not clear from the text if it was actually applied there. *Due to these last two points, a systematic difference between the results of this and NA49 analysis is expected, of the order of at least the **trigger** contribution to the overall MC correction.*

Having all the corrections listed above, the corrected double differential spectrum is defined as

$$\frac{d^2n}{dp_T dy} = \frac{N_\phi}{N_{\text{ev}} \Delta p_T \Delta y} \times \frac{c_\infty c_{\text{MC}} c_{\text{bkg}}}{\mathcal{BR}(\phi \rightarrow K^+ K^-)}, \quad (4.10)$$

where the first term is the normalized raw spectrum defined at the beginning of this section, c_∞ is the correction due to integration cut-off, c_{MC} is the Monte Carlo correction, c_{bkg} is the correction associated with unaccounted-for effects in the background and $\mathcal{BR}(\phi \rightarrow K^+ K^-)$ is given in Table 1.1 on page 11. Typical values of corrections are about 1.06 for c_∞ (see Section 4.5.3 for details), 1 to 3 for c_{MC} (Section 4.5.4) and 1.05 for c_{bkg} (Section 4.6.1).

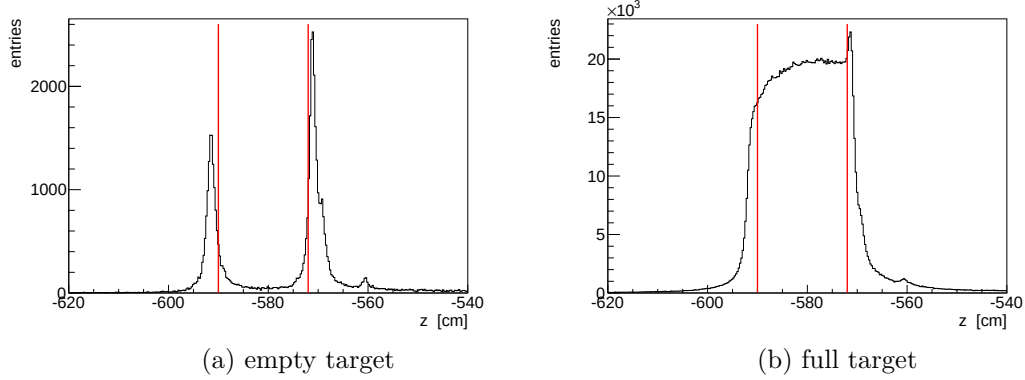


Figure 4.8: Distributions of the main vertex z coordinate for 158 GeV data, for the empty and full target configurations. In the empty target distribution, structures associated with interactions in windows of the LHT are pronounced. Vertical lines indicate limits of the tight main vertex z position cut employed in this analysis, while the full z range in each plot depicts the size of the loose cut utilized *e.g.* in Ref. [30].

4.5.2 Correction for off-target interactions

As was stated in Section 4.3.2, in this analysis a *tight* main vertex z position cut is used. Due to this, no correction for off-target interactions is needed. It is different from other NA61/SHINE analyses [30, 47, 63], where a *loose* cut, which accepts interactions in a broad vicinity of the target, is applied. Such a cut, in turn, allows significant amount of off-target interactions and therefore brings necessity of correction.

The first step to understanding of this difference in methods is the knowledge of the relation between the vertex cut and resulting biases (and corrections needed). This relation can be explained looking at the main vertex z position distributions for the liquid hydrogen target (LHT) used to collect the data being subject of this analysis. These distributions, for the empty and full target configurations, are shown in Fig. 4.8. The full z ranges in these plots correspond to the loose cut, while vertical lines indicate the tight cut limits.

From the distribution for the empty target sample it is visible that the tight cut removes most of interactions with windows of the LHT, which constitute the leading contribution to off-target interactions. Comparing the empty and full target distributions, one can see that what remains is negligible. This is why the correction for off-target interactions is unnecessary in this case. However, such a cut increases the **vertex** (see Section 4.5.1) contribution to the MC correction, favouring even more the high multiplicity events over low multiplicity ones. That is because the resolution of the

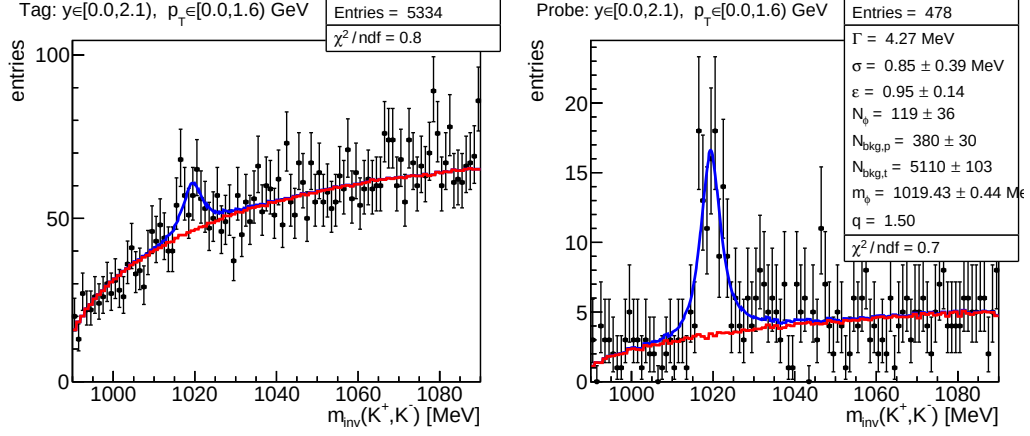


Figure 4.9: Empty target 158 GeV data: tag-and-probe invariant mass spectra in the broad phase space. The loose main vertex z position cut is used.

reconstructed main vertex z position is worse for low multiplicity events, so there is higher probability for such events, that the vertex leaks beyond the range of the cut. Another drawback of the tight cut is, that it degrades already modest statistics available for the analysis.

The loose cut accepts windows of the LHT, so obviously it has to be utilized in conjunction with the correction for off-target interactions. On the other hand, the **vertex** contribution to the MC correction is smaller than in the case of the tight cut, as the effect of the z position resolution is taken out. Since the cut is looser, it also gives higher usable statistics. So if only the off-target correction can be performed accurately, the approach reduces the overall uncertainty of the final result.

The usual data-based correction for off-target interactions [30, 47, 63] requires performing the same analysis on two data samples — with the target inserted and removed (full and empty LHT). The appropriately scaled target removed results are then subtracted from those for the target inserted sample to remove the contribution from off-target interactions.

Unfortunately in the case of ϕ analysis this procedure is not feasible. It is illustrated by Fig. 4.9, which shows invariant mass spectra in the broad phase space for 158 GeV empty target data. To apply the subtraction method, these should be divided into the order of 20 bins. Clearly available statistics are too low for such subdivision to be sound. *This is why the tight main vertex z position cut is employed.*

4.5.3 Correction due to integration cut-off

As is said in Section 4.4.3, the terms N_t and N_p in function Eq. (4.9) on page 54 are numbers of signal pairs respectively in the tag and probe spectra. It means that they correspond to integrals of the signal shape function Eq. (4.6) on page 49 within limits of the invariant mass histograms. Because N_t and N_p are proportional to N_ϕ , such meaning have also the fitted N_ϕ values.

But considered signal shape parametrizations have non-vanishing values beyond these limits. So N_ϕ do not estimate numbers of ϕ mesons produced in the measured inelastic collisions, but rather those produced with masses within limits of the invariant mass histograms.

Whether such a definition of the result is fine or not is arbitrary. On one hand, the integral is calculated in a range where the signal shape is in principle controlled by the data. On the other, whatever the true shape is, it has some tails outside the controllable range. In practice (see Section 4.6.3) the available statistics do not put significant constraints on the shape parametrization. Furthermore, the resonance models chosen are not blind guesses, but come from sound quantum-mechanical studies. So the fact that we are unable to validate them in the certain analysis, does not immediately mean that the analysis ought to be confined to the controlled area.

There seems, unfortunately, to be no common convention regarding this issue. In most papers [15, 20–22] it is not commented explicitly. Only one of NA49 papers clearly says what the integration limits are — from the threshold due to kaon masses to $m_\phi + 30\Gamma$ [8]. Where it is not commented, it may seem natural that the edges of invariant mass histograms are used. But if an analytically integrable parametrization is employed, such as Voigtian in LHCb [22] and ALICE [20] analyses, then the result as well may correspond to integration in an infinite range. Moreover, each analysis uses different histogram ranges, what causes ambiguity among cases where integrals are evaluated within these ranges.

Weighting the above considerations, *it was decided to define the result as corresponding to the integration in an infinite range*. Since signal shape parametrization used can only be integrated numerically, what implies integration in a finite range, a correction to N_ϕ is required. Because the shape parameters are fixed with the results of the first step of the fitting strategy, the correction needs to be calculated once in that step and its value reused for all analysis bins.

The idea is to determine the limit N_∞ of the integral N as the integration range goes to infinity, by extrapolation from a set of finite-range integration

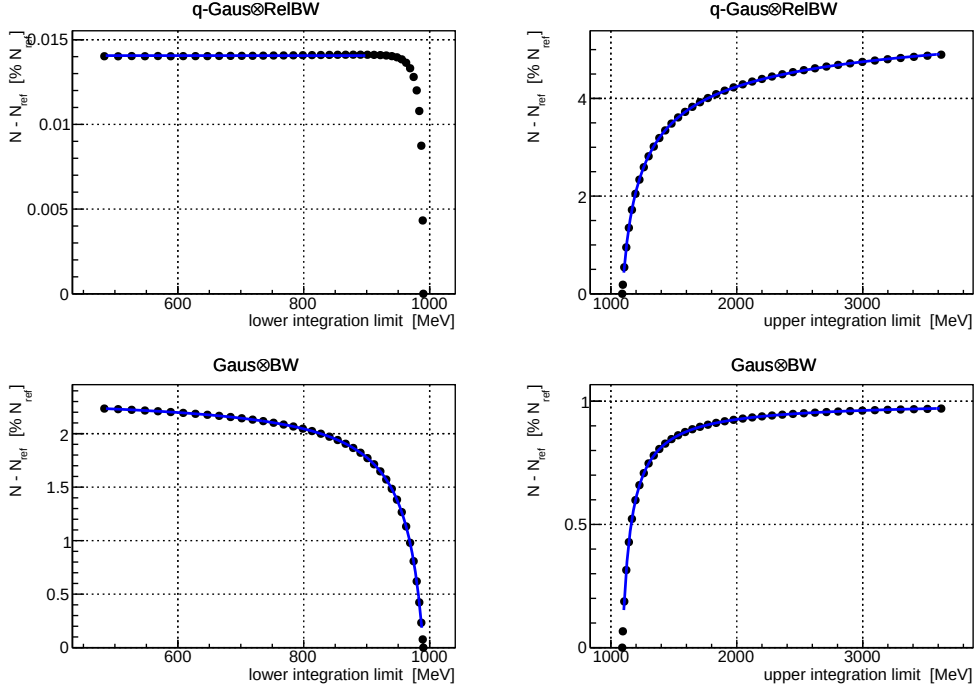


Figure 4.10: Signal shape integral values as function of the lower (*left*) and upper (*right*) integration limit, expressed as relative differences with respect to the integral using edges of the invariant mass histogram as limits. Two different signal shape parametrizations are shown: the one used in this analysis (*top*) and Voigtian (*bottom*); see Section 4.6.3. Blue curves are fits of function Eq. (4.12) on the next page. Signal shape parameters come from results of the first stage of the fitting strategy for 158 GeV data.

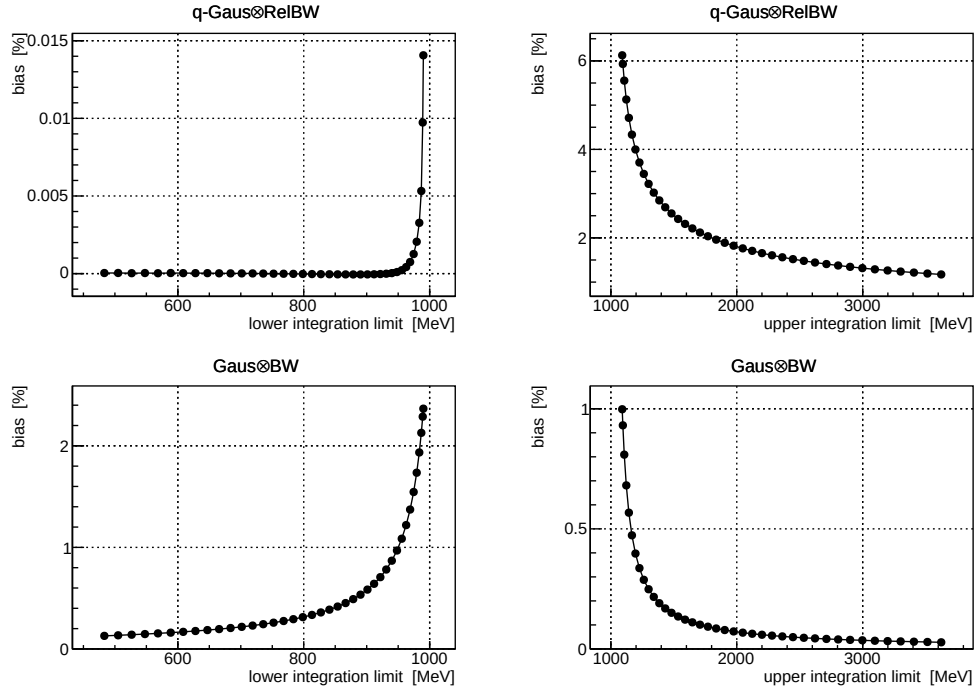


Figure 4.11: Bias of the signal shape integral as function of the lower (*left*) and upper (*right*) integration limit for two signal shape parametrizations. Bias values are derived from results of fits in Fig. 4.10. Points are connected with lines to guide the eye.

results. Two such sets are built: for the case when the lower integration limit is varied and the upper one is kept at the reference value (the upper edge of the m_{inv} histogram; see plots on the left-hand side of Fig. 4.10 on the preceding page), and the other way round (right-hand side plots).

In Fig. 4.10, for the sake of readability, differences between these integrals and the reference value N_{ref} , expressed as percentage of N_{ref} , are shown:

$$y = \frac{N - N_{\text{ref}}}{N_{\text{ref}}} 100\%, \quad (4.11)$$

where the reference value N_{ref} of the integral, proportional to N_ϕ , is an integral in the range of the invariant mass histogram.

By trial and error, it has been guessed, that points in the sets can be described by a function

$$f(x) = y_\infty - \frac{a}{|x - m_\phi|^b}, \quad (4.12)$$

with exception of the set in the top, left panel of Fig. 4.10, where a constant function is used. The latter is caused by the fact that the relativistic Breit-Wigner function vanishes for values of mass below the threshold for ϕ production. Cases of two distinct signal shape parametrizations are illustrated at the top and bottom of both Figs. 4.10 and 4.11 on the previous page to show robustness of formula Eq. (4.12).

From the fitted value of y_∞ parameter, a one-sided correction can be obtained. For example in case of the variation of the upper limit:

$$c_R = \frac{N_{\text{ref}} + N_R}{N_{\text{ref}}} = \frac{y_\infty}{100\%} + 1, \quad (4.13)$$

where N_R is the right tail contribution to N_∞ , *i.e.* an integral from the upper edge of the m_{inv} histogram to $+\infty$. Similarly, we get the

$$c_L = \frac{N_L + N_{\text{ref}}}{N_{\text{ref}}}, \quad (4.14)$$

where N_L is the left tail contribution to N_∞ .

Those one-sided corrections can be combined to obtain the wanted correction to N_ϕ :

$$\begin{aligned} c_\infty &= \frac{N_\infty}{N_{\text{ref}}} = \frac{N_L + N_{\text{ref}} + N_R}{N_{\text{ref}}} \\ &= \frac{N_L + N_{\text{ref}} + N_R + N_{\text{ref}} - N_{\text{ref}}}{N_{\text{ref}}} \\ &= c_L + c_R - 1. \end{aligned} \quad (4.15)$$

It should be noted that given the fact how well the points in Fig. 4.10 on page 63 are described by the fitted functions and given the small uncertainties from the fit of y_∞ parameters, the uncertainty of c_∞ is neglected. It means, that both N_ϕ and its statistical uncertainty are simply multiplied by c_∞ .

Furthermore, having one-sided corrections, it is possible to calculate bias of the finite-range integral in dependence on the range chosen. In the Fig. 4.11 on page 63 it is shown as

$$b = \frac{(N_{\text{ref}} + N_{\text{R}}) - N}{N} 100\%, \quad (4.16)$$

for the variation of the upper limit and analogously for the lower limit. With such a definition, it is visible that the bias of N_ϕ (integration in the m_{inv} histogram range) is about 6 % of N_ϕ value for the case of the signal shape parametrization used in this analysis (top plots in Figs. 4.10 and 4.11) and about 3 % for Voigtian (bottom plots).

Finally, it should also be added, that this correction determination method was validated against the Voigtian distribution, which is analytically integrable. The difference between the correction obtained with the described procedure and the ratio of the analytically calculated integral in infinite range to that in the m_{inv} histogram range, was found to be of the order 10^{-5} of the reference analytical value.

4.5.4 Monte Carlo correction

The last correction, which takes into account several different effects listed in Section 4.5.1, is calculated with Monte Carlo simulation data. Components of this simulation are described in Section 4.3.1.

Calculation

In order to determine this correction, first, two histograms are created:

$$n_{\text{gen}} = \frac{N_\phi^{\text{gen}}}{N_{\text{ev}}^{\text{gen}}} \quad \text{and} \quad n_{\text{sel}} = \frac{N_\phi^{\text{sel}}}{N_{\text{ev}}^{\text{sel}}}, \quad (4.17)$$

where $N_{\text{ev}}^{\text{gen}}$ and $N_{\text{ev}}^{\text{sel}}$ are numbers of events respectively generated and selected with the downstream cuts of Section 4.3.2, while N_ϕ^{gen} and N_ϕ^{sel} are numbers of respectively generated and selected reconstructed ϕ mesons in the given 2D momentum bin.

In N_ϕ^{gen} and N_ϕ^{sel} only those ϕ mesons are taken into account which decay into kaons. The selected reconstructed ϕ mesons are such, that come from

selected events and for which both daughters are reconstructed and both pass the same track cuts as are applied to experimental data (apart from the PID cut).

The n_{gen} is binned according to the generated momentum of ϕ mesons, while n_{sel} is binned according to the momentum obtained from reconstructed momenta of kaons. Thanks to this, the effect of bin migration due to track momentum resolution is included in the correction.

In case of a double differential analysis, the binning of both n_{gen} and n_{sel} is simply the same as the (2D) analysis binning for N_ϕ . In case of a single differential analysis, the correction (and consequently both n_{gen} and n_{sel}) still needs to be computed first in a 2D binning. That binning is constructed by combination of 1D analysis binnings in the rapidity and the transverse momentum. It is be illustrated by Fig. 4.3 on page 47: the combination of binnings in panels (b) and (c) yields the binning of panel (a).

The need for a 2D binning in the latter case arises because shapes of the correction and of the ϕ spectrum are both non-trivial in (y, p_T) space. From that follows, that the correction calculated in a 1D binning could not come out unbiased without a special care (see the discussion of ϕ production model dependence of the correction further in this section and Section 4.6.7). One could observe, that the correction may have a non-trivial shape in the third, ignored dimension — the azimuthal angle φ . But, from the cylindrical symmetry of collisions, the ϕ spectrum is flat in φ and thus the correction is automatically properly averaged over φ .

Having the two histograms, they can be divided to get the 2D Monte Carlo correction:

$$c_{\text{MC}} = \frac{n_{\text{gen}}}{n_{\text{sel}}} . \quad (4.18)$$

In case of a single differential analysis, this correction is then averaged with the ϕ production model over the rapidity or the transverse momentum to obtain corrections in both 1D analysis binnings. That procedure is described in Section 4.6.7.

Statistical uncertainty

Determination of statistical uncertainty of c_{MC} due to finite statistics of MC data is not straightforward, because numbers from which c_{MC} is derived, are highly correlated. This can be understood when above formula is rewritten in the form

$$c_{\text{MC}} = \frac{\beta}{\alpha} , \quad (4.19)$$

with

$$\beta = \frac{N_{\text{ev}}^{\text{sel}}}{N_{\text{ev}}^{\text{gen}}} \quad \text{and} \quad \alpha = \frac{N_{\phi}^{\text{sel}}}{N_{\phi}^{\text{gen}}} . \quad (4.20)$$

Quantities β and α could be viewed as binomial proportions in processes of respectively event and ϕ meson selections. As is discussed *e.g.* by Brown *et al.* [64], the choice of good estimator of uncertainty (or equivalently confidence interval) for binomial proportion, is not trivial. The popular Wald interval, known also as Binomial formula, is defective — it has zero length (zero uncertainty) if number of successes k equals zero or equals number of tries n . Furthermore, its coverage probability, the probability that true value p of binomial proportion is contained in the interval, is systematically lower than the assumed confidence level [64]. This means, that the uncertainty is underestimated.

It should be noted that for the Monte Carlo correction, a perfect estimation of statistical uncertainty is not necessary. The idea is to have such statistics of MC, that the relative uncertainty of c_{MC} is much lower than that of N_{ϕ} . If this condition is not satisfied, simply more MC data should be produced. For this reason, the following requirements were put on estimator σ_p of statistical uncertainties of β and α :

- $\sigma_p \neq 0$ for $k = 0 \vee k = n$,
- it should not underestimate the uncertainty; overestimation is allowed, as in the worst case it causes demand for more MC data.

Based on above, it was decided to employ square root of variance of Bayesian posterior distribution of p with uniform prior, given by Ullrich and Xu [65]:

$$\sigma_p = \sqrt{\frac{(k+1)(k+2)}{(n+2)(n+3)} - \frac{(k+1)^2}{(n+2)^2}} . \quad (4.21)$$

Figure 4.12 on the following page illustrates properties of this estimator, named Ullrich, in comparison to the flawed Binomial formula. All calculations are done for number of tries $n = 100$, which is also the lowest allowed value for denominators in formulas for β and α . Phase space bins where this condition is not satisfied are discarded in correction breakdown plots shown further; obviously there is enough MC data produced for the condition to be satisfied for all bins of non-broken-down c_{MC} .

In the left panel of Fig. 4.12 on the next page, the unwanted behaviour of Binomial formula at $k = n$ is visible, which does not appear for the Ullrich estimator. Only a part of the plot close to $k = n$ is shown, as it is symmetric around $k/n = 0.5$.

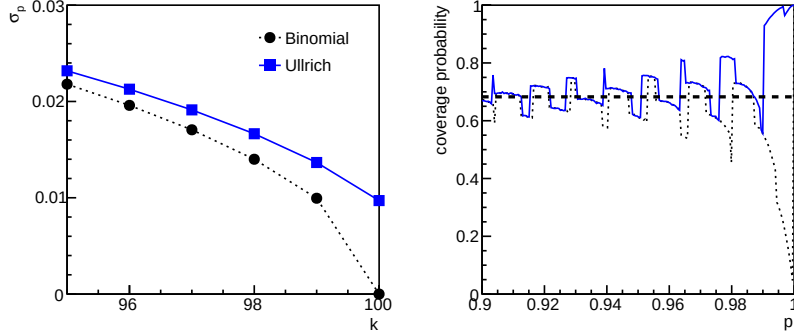


Figure 4.12: Comparison of uncertainty estimators σ_p for binomial proportion p , for number of tries $n = 100$. See text for the meaning of ‘Binomial’ and ‘Ullrich’. *Left*: values of the estimator if observed number of successes is k . *Right*: coverage probability of interval $k/n \pm \sigma_p$ as function of true value p of binomial proportion.

The right panel of Fig. 4.12 presents coverage probability for intervals associated with both estimators. These intervals are defined as $k/n \pm \sigma_p$. The horizontal line shows the confidence level equivalent to single standard deviation of Gaussian distribution, to which corresponds the usual meaning of statistical uncertainty. For both estimators characteristic oscillatory structures as in Ref. [64] are evident. It is visible, that Binomial formula underestimates the uncertainty. The chosen estimator’s coverage is on average closer to the expected value, overestimating the uncertainty for some values of p , especially close to 1. The plot is again symmetric around $p = 0.5$, so the same happens near $p = 0$.

With uncertainties of β and α calculated from Eq. (4.21) on the previous page, uncertainty of c_{MC} can be obtained by standard error propagation:

$$\sigma(c_{\text{MC}}) = \sqrt{\frac{\sigma^2(\beta)}{\alpha^2} + \frac{\beta^2 \sigma^2(\alpha)}{\alpha^4}}. \quad (4.22)$$

This obviously assumes that β and α are uncorrelated. It is not strictly true, because they are derived from the same MC events and selection of ϕ mesons for α involves selection of events, which defines β . But since very accurate uncertainty estimation is not necessary and there is no obvious, other way of $\sigma(c_{\text{MC}})$ calculation, this issue is ignored. For completeness, it should also be pointed out, that N_{ϕ}^{sel} contains small contribution of ϕ mesons from adjacent bins, that are not part of N_{ϕ}^{gen} . So the treatment of α as binomial proportion is also not strictly correct. But again this problem is deemed insignificant.

Finally, it is possible to compare computed uncertainties to those of uncorrected ϕ yield N_{ϕ} . This is done in Fig. 4.13 on the following page for

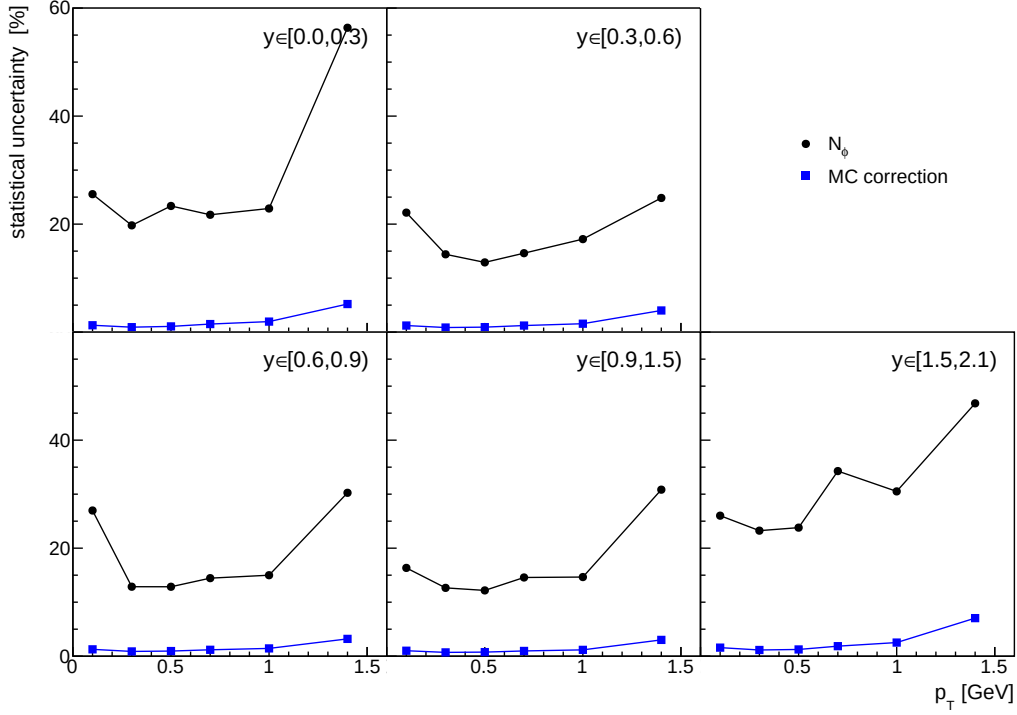


Figure 4.13: Comparison of relative statistical uncertainties of raw ϕ yield N_ϕ and Monte Carlo correction c_{MC} for 158 GeV data.

158 GeV data; pictures for other energies look qualitatively the same. It is clear, that relative uncertainty of the correction is about one order of magnitude smaller than that of N_ϕ . This shows that there is enough Monte Carlo data produced. Because of difference in magnitudes, when uncertainties are added in quadrature, MC correction uncertainty effectively has negligible contribution to the total statistical uncertainty of the corrected yield.

Model dependence

Monte Carlo correction depends on three models:

- ϕ production model,
- event model,
- detector model.

The ϕ *production model*, let's call it $g_\phi(\vec{p})$, corresponds to a generated ϕ momentum spectrum. If only MC correction, understood as a continuous

function of the momentum vector $c_{\text{MC}}(\vec{p})$, is not flat within a bin, its value c_{MC} for this bin depends on the ϕ production model:

$$c_{\text{MC}} = \frac{\int_{\vec{p} \in \text{bin}} g_{\phi}(\vec{p}) d\vec{p}}{\int_{\vec{p} \in \text{bin}} g_{\phi}(\vec{p}) \frac{1}{c_{\text{MC}}(\vec{p})} d\vec{p}}. \quad (4.23)$$

This equation corresponds to averaging of the efficiency (inverse of the correction). The reason why averaging is not applied directly to the correction is discussed in Section 4.6.7.

From Eq. (4.23) follows, that if the shape of $g_{\phi}(\vec{p})$ is the same as the shape of the true physical spectrum (the one that we are trying to measure), even with a non-constant $c_{\text{MC}}(\vec{p})$ within the bin, the result is unbiased. Otherwise, some bias occurs.

An obvious method to reduce the bias is therefore to choose small enough bins, what reduces variability of $c_{\text{MC}}(\vec{p})$ within a bin. This seems to be satisfied with the chosen 2D binning (see Figs. 4.14 and 4.15 on page 72 and on page 73), maybe with exception of higher p_T bins at mid rapidity. For these bins, however, other uncertainties are high enough to render this one insignificant. For example for the last p_T bin, second y bin in Fig. 4.15a on page 73 the change of correction is about 20 %. The bias is some fraction of this value, roughly proportional, but not equal to the ratio of slopes of generated MC and true physical spectra. Let it be even as high as a quarter, what yields 5 %. This should be compared (see Fig. 4.38b on page 111) to uncertainties on the level of 25 % (statistical) and 15 % (total systematic). Indeed, a toy Monte Carlo study presented in Section 4.6.7 suggests an even smaller bias, below 1 % (see Fig. 4.37 on page 109).

Another approach, necessary in case of a 1D analysis binning, is the one used in NA49 ϕ analysis [8]. There, correction is calculated in a fine 2D binning, where flatness condition is satisfied. Then values of the correction corresponding to the 1D analysis binning are obtained from Eq. (4.23), with $g_{\phi}(\vec{p})$ taken as a continuous phenomenological fit to the corrected ϕ spectrum. Since the latter requires the correction, the whole procedure is iterative with the uncorrected ϕ spectrum and finely binned correction on input and the corrected ϕ spectrum on output. Such algorithm, based on the NA49 one, actually applied for the single differential analysis of the 40 GeV data, is described in detail in Section 4.6.7.

The *event model* are distributions of all particles and correlations between them. Here not only shapes are important, but also multiplicities of particles. This is what governs the **vertex** and **trigger** contributions to

the correction (see Section 4.5.1 and further in this section). It should be noted, that the Monte Carlo correction is not sensitive to details of production of all particles, but rather to the overall properties of events defined by several most abundant particle species — in case of investigated reactions predominantly by pions. From this follows, that to reduce possible bias in the correction, model of the primary interaction should be chosen such, that it describes reasonably well the overall properties of reaction under study. As was mentioned in Section 4.3.1, that was shown to be true for EPOS.

The bias could theoretically be reduced further by factorization of the correction into an accurate dominant part that does not depend on the event model and a smaller part that depends. This is why a breakdown of the correction into various contributions is studied further.

At last, the *detector model* consists of the detector geometry, information about materials and models of interactions of ionizing particles with matter. While being fairly complicated entity, it is also quite reliable in comparison to previously discussed models. This is because it is based on physical knowledge of well studied phenomena (at least much better than proton-proton collisions at SPS energies). It is also verifiable in the context of the experiment and indeed was asserted by comparisons of certain distributions derived from experimental data with those from Monte Carlo, showing satisfactory agreement [30].

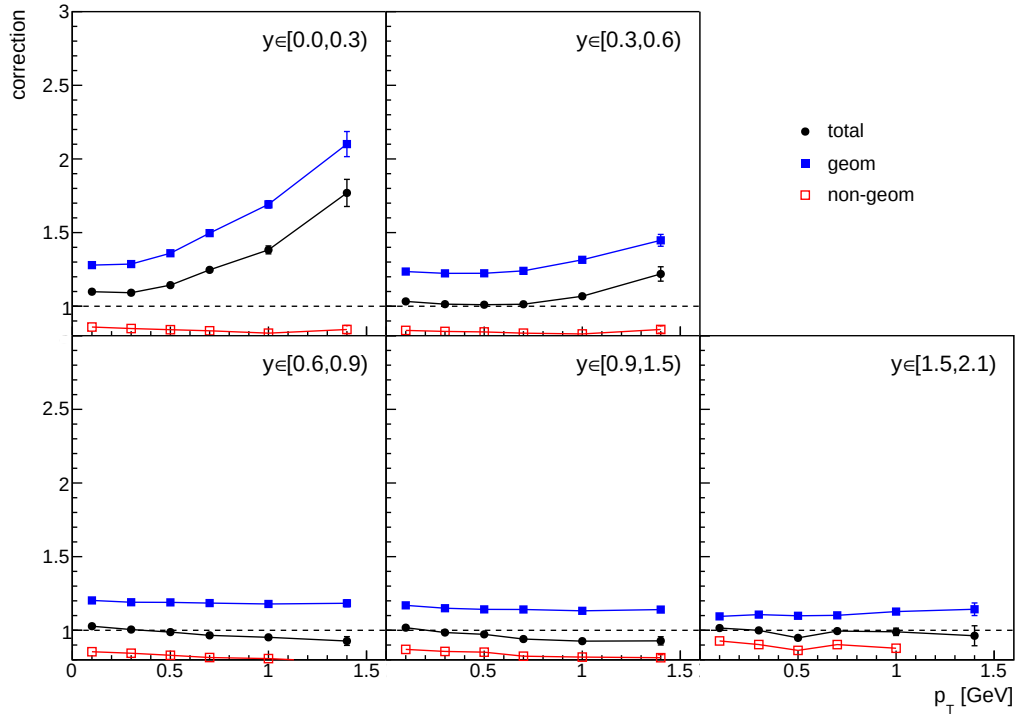
In spite of above arguments for lack of significant biases of the correction, obviously some systematic uncertainty due to the correction is to be expected. This is investigated in Section 4.6.6 by means of event and track cuts variations. Also for the case of a single differential analysis, the ϕ production model dependence may cause a non-negligible uncertainty, which is studied in Section 4.6.7.

Values and breakdown

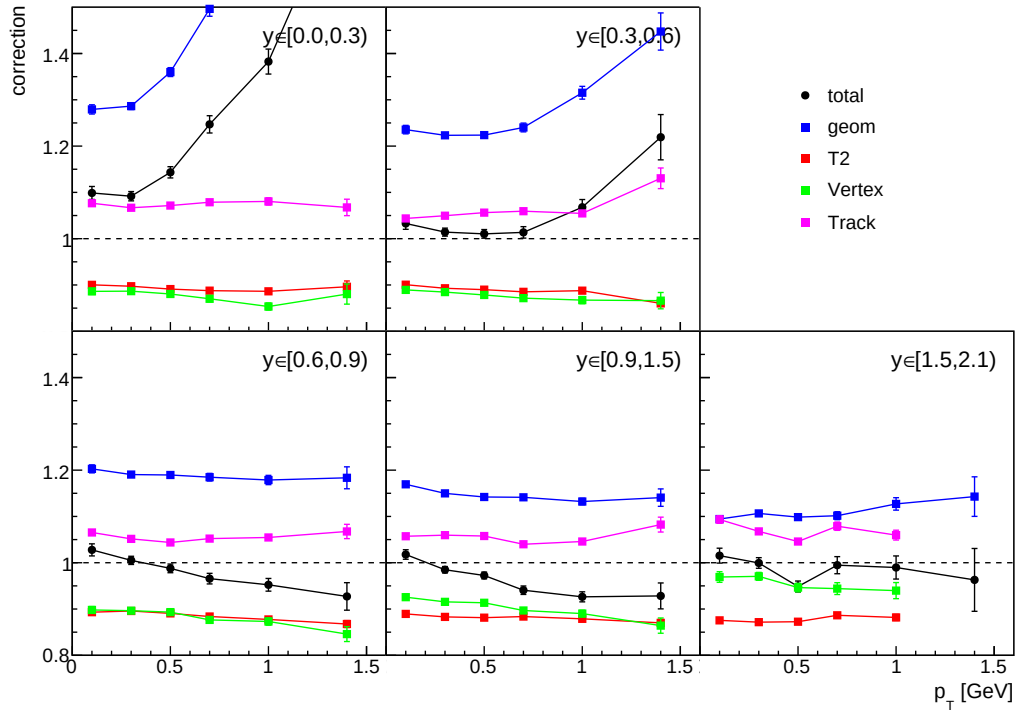
Values of c_{MC} for 158 GeV, 80 GeV and 40 GeV data are indicated with black circles (‘total’) in respectively Figs. 4.14 to 4.16 on pages 72–74. For most bins for two higher energies the correction is small (close to 1) and close to uniform. Only in mid rapidity bins there is large transverse momentum dependence and there are large values of the correction. It is not true for the lowest energy, where large values with large variability are observed over the whole phase space.

The breakdown into various contributions is accomplished by creation of histograms analogous to n_{gen} and n_{sel} , but with some cuts loosened or removed with respect to those that define n_{sel} .

Figures 4.14a, 4.15a and 4.16a show breakdown of the correction into two



(a)



(b)

Figure 4.14: Breakdown of the Monte Carlo correction for 158 GeV data as a function of the transverse momentum in each rapidity bin. Some points are missing due to the requirement of more than 100 entries in the denominator in efficiency calculations.

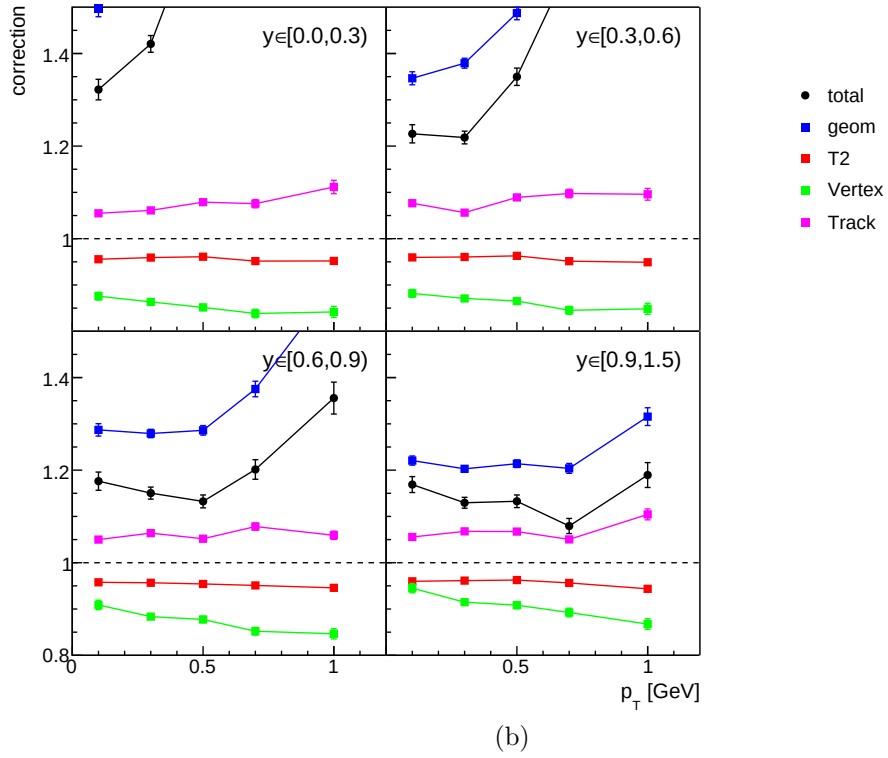
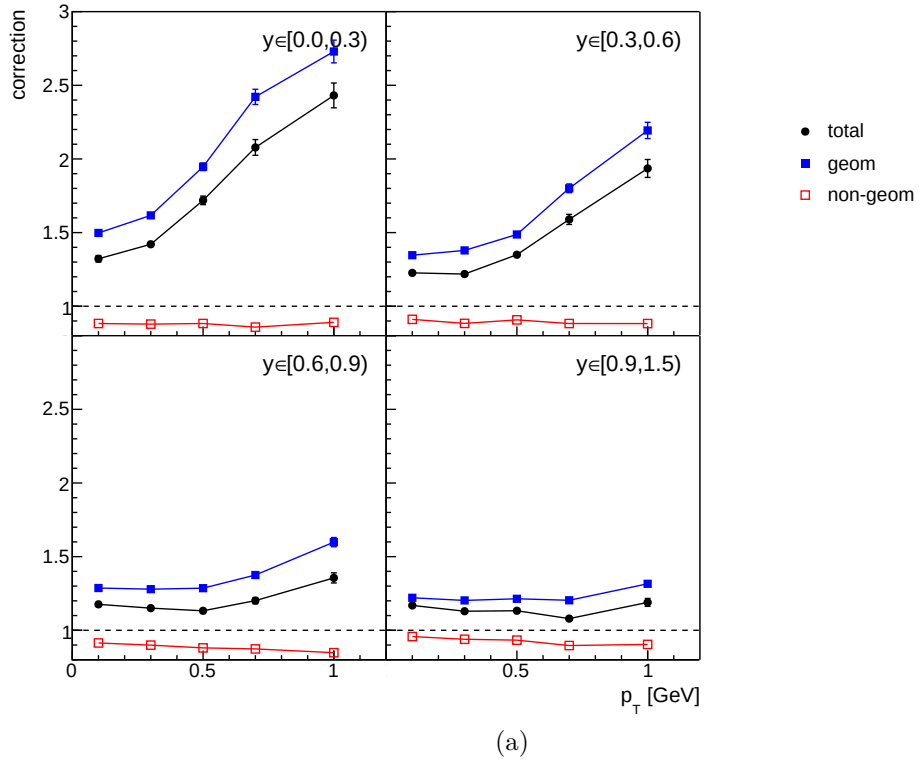
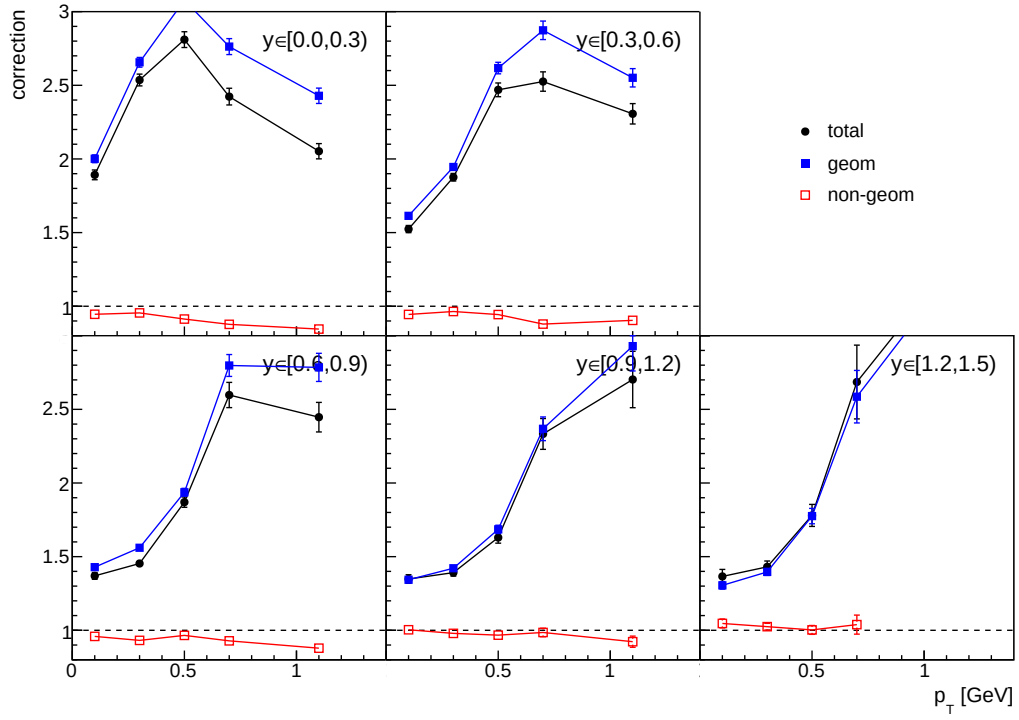
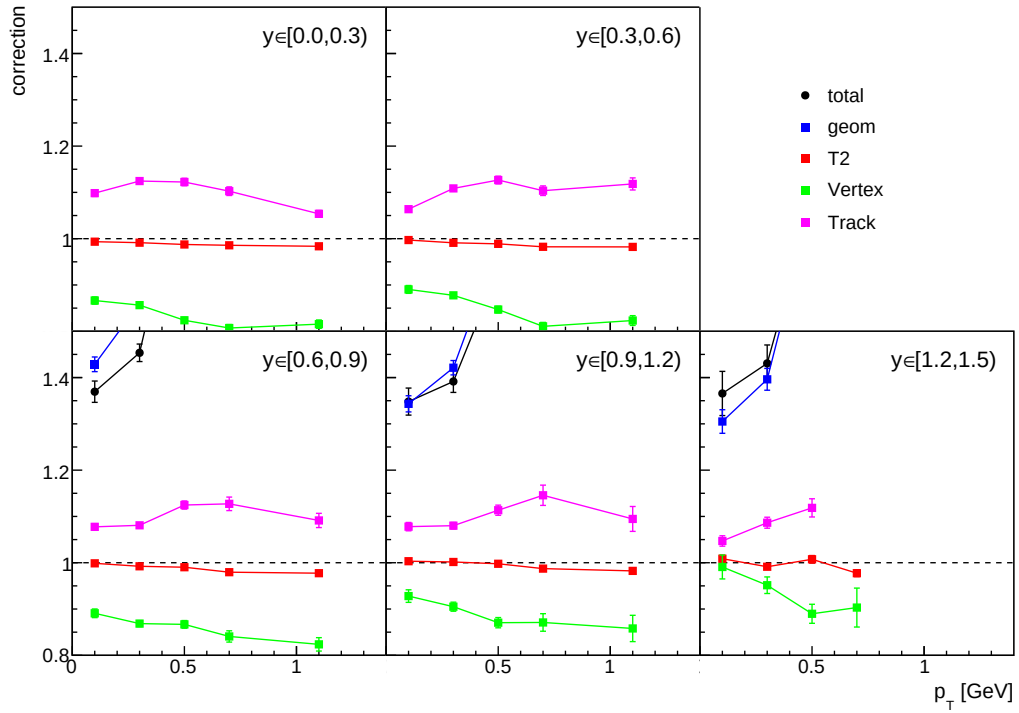


Figure 4.15: Breakdown of the Monte Carlo correction for 80 GeV data as a function of the transverse momentum in each rapidity bin.



(a)



(b)

Figure 4.16: Breakdown of the 2D Monte Carlo correction for 40 GeV data as a function of the transverse momentum in each rapidity bin. Some points are missing due to the requirement of more than 100 entries in the denominator in efficiency calculations.

main contributions when model dependency is considered. The first, ‘geom’, corresponds to the **registration** effect described in Section 4.5.1. Assuming that the binning is fine enough to avoid ϕ model dependence, it does not depend on physics under investigation, only on the detector model, mainly through purely geometrical acceptance. It is defined as

$$c_{\text{geom}} = \frac{n_{\text{gen}}}{n_{\text{reg}}}, \quad (4.24)$$

where n_{reg} is built using all generated events (the same as in case of n_{gen}) and generated ϕ mesons whose daughters are required to have more than 30 GEANT points in all TPCs and more than 15 GEANT points in VTPCs or more than 4 in GAP-TPC. It is a registered ϕ mesons spectrum. These number-of-points cuts are adaptations of analogous conditions on the reconstructed tracks level. Thanks to this, the **registration** (detector model-related) effect is separated from the **reconstruction** effect that governs what fraction of these deposits is actually reconstructed.

Inverse of c_{geom} is the *registration probability* plotted in Fig. 4.3 on page 47. There, a finer binning and in a broader phase space is employed, than the analysis binning utilized here.

This part of the overall correction is dominant in those bins where c_{MC} is significantly different from one. It is also responsible for most of transverse momentum dependence of c_{MC} . Comparing Figs. 4.14a, 4.15a and 4.16a on pages 72–74 it is evident, that there is significant dependence on energy of the reaction. It is not surprising, because with different energy of the reaction, different magnetic field strength is applied and the centre-of-mass system has different rapidity relative to the detector. This was also discussed by NA49 [8].

The second, ‘non-geom’, contribution depicted in Figs. 4.14a, 4.15a and 4.16a depends on the event model. It is almost uniform for all bins (so it is independent of the ϕ production model). It is defined as

$$c_{\text{non-geom}} = \frac{n_{\text{reg}}}{n_{\text{sel}}}, \quad (4.25)$$

so that

$$c_{\text{geom}} \cdot c_{\text{non-geom}} = c_{\text{MC}}. \quad (4.26)$$

These two components of the correction are natural candidates for the factorization mentioned earlier. The ‘geom’ part could in principle be calculated from high statistics, finely binned, flat phase space Monte Carlo (uniform $g_{\phi}(\vec{p})$ distribution). That would cover well corners of the analysis phase space, contrary to a realistic $g_{\phi}(\vec{p})$ which has high density in mid rapidity and

low transverse momentum regions. That was actually done in NA49 [8,15,51], but there the ‘non-geom’ part was ignored (see Section 4.5.1). For this analysis it was decided, however, not to be worth a rather large technical effort. Indeed, as was pointed out earlier, the ‘geom’ part is dominant only for some of bins in double differential analyses (not true for 40 GeV, where it is dominant over the whole phase space). What is more, in most of bins where it is dominant, the overall uncertainty is much higher than a possible bias due to large bins or a statistical uncertainty of the correction derived with relatively low statistics (that in turn is true for all energies). Still, this is something worth remembering if ever more accurate study would be performed with higher statistics data (possible with already collected 158 GeV data that are under calibration while this text is being written).

Another type of breakdown is illustrated in Figs. 4.14b, 4.15b and 4.16b on pages 72–74. Here different, event model dependent contributions are separated. First there is the ‘T2’ correction

$$c_{T2} = \frac{n_{\text{reg}}}{n_{T2}}, \quad (4.27)$$

where n_{T2} is created from all generated ϕ mesons that pass conditions for n_{reg} spectrum, but with additional event cut selecting only events with T2 trigger (no GEANT hits in S4 counter). It corrects for the loss of inelastic events vetoed by S4 (**trigger** effect listed in Section 4.5.1). It depends on the event multiplicity and topology — the probability that at least one charged particle in an event goes through such a trajectory that it hits S4 counter. An event that contains such a high momentum track is less likely to contain as well two kaons from ϕ . Therefore the uncorrected spectrum, which is a ratio of the number of ϕ mesons to the number of events, is overestimated due to the cut. So the correction needs to be smaller than one, what is indeed visible in Figs. 4.14b, 4.15b and 4.16b. It is expected to be larger (further from one) at high energies, since there are many high momentum, straight going tracks, and smaller (closer to one) at low energies. This holds true if one compares Figs. 4.14b, 4.15b and 4.16b.

Next, there is the ‘Vertex’ correction

$$c_{\text{Vertex}} = \frac{n_{T2}}{n_{\text{ver}}}, \quad (4.28)$$

where n_{ver} is built from all events and generated ϕ mesons that pass conditions for n_{T2} spectrum, but with additionally all vertex cuts applied. It corrects for the loss of low multiplicity events, which are more likely to have problems with vertex reconstruction than high multiplicity ones. Due to lower vertex z position resolution they are also more likely to leak beyond

the cut range. This is the **vertex** effect of Section 4.5.1. Similarly to c_{T2} it depends on the event model through the event multiplicity. To some extent it also depends on event topology, because it is easier to fit the vertex if tracks are going out from it at high angles to the beam track. Again, events that are removed by the cut are less likely to contain reconstructible ϕ mesons, which assure at least two charged tracks. So again the uncorrected spectrum comes out overestimated, what leads to a correction smaller than one, in agreement with the values calculated. In this case, however, the correction is expected to be smaller at high energies, where event multiplicities are high, and larger at low energies. It seems true if one compares Figs. 4.14b, 4.15b and 4.16b on pages 72–74.

Cuts utilized to isolate effects of c_{T2} and c_{Vertex} corrections are applied incrementally in a certain order depicted above. It should be noted that because they depend on the same properties of the event, a change in the sequence could result in assigning slightly different values to c_{T2} and c_{Vertex} in Figs. 4.14b, 4.15b and 4.16b. So the presented breakdown should rather be considered in a qualitative manner, to cross-check and validate the Monte Carlo correction, rather than as a definite quantitative assessment of various effects.

An interesting feature of these corrections is their flatness in ϕ phase space. The probable reason for this is that ϕ , due to its low multiplicity, relatively low mass and low width, consumes only a small portion of the available phase space in the event. As a consequence, other particles behave as if ϕ wasn't there at all.

Finally, the ‘Track’ correction,

$$c_{\text{Track}} = \frac{n_{\text{ver}}}{n_{\text{sel}}} , \quad (4.29)$$

separates the **reconstruction** effect listed in Section 4.5.1 through addition of all reconstructed tracks cuts and binning of n_{sel} according to the reconstructed momentum. It depends on reconstruction algorithms and negligibly on physics under investigation through the track density in the detector, which is small for considered reactions. It also depends on the magnetic field and detector geometry which define track momentum resolution. In this case no events are lost, but some ϕ mesons are, because of reconstruction efficiency and cuts. On the other hand, bin migration causes increase of ϕ mesons count in certain bins. But because bin sizes are large in comparison to the resolution, the effect is expected to be small. So the uncorrected spectrum is expected to be slightly underestimated and the correction higher than one, but rather small. This seems to be the case in Figs. 4.14b, 4.15b and 4.16b.

All in all, the calculated Monte Carlo correction seems to be sound.

4.6 Systematic studies and optimizations

Once values of the corrected ϕ spectra and their statistical uncertainties are defined in previous sections, it is necessary to estimate systematic uncertainties. This requires certain systematic studies where parameters of the analysis are varied or different methods are tested. As a result of these investigations, the algorithms were indeed optimized compared to what was done at the very beginning of the process (see Sections 4.6.3 and 4.6.7).

Before getting into details, it was considered that biases in this analysis may arise as a consequence of:

1. choice of integration range of signal parametrization curve to obtain the yield,
2. wrong choice of analytical parametrizations for resonance shape and detector resolution effect,
3. unaccounted-for effects in background description,
4. constraints used in fitting,
5. wrong assumptions associated with kaon selection efficiency,
6. improper Monte Carlo correction of losses due to cuts and of detector effects,
7. wrong ϕ production model assumption utilized in Monte Carlo correction averaging in case of a single differential analysis.

Investigations corresponding to the first three points and the last one led to refinements mentioned above. While most of these are discussed further in this section, the correction due to integration cut-off, connected to the first item, was already described in Section 4.5.3. Considerations associated with all but the first point, result in estimates of systematic uncertainties.

All discussed studies, apart from the one relevant only for a single differential analysis (Section 4.6.7), especially their certain important intermediate steps and details, are illustrated with examples for 158 GeV data. Nevertheless, the reasoning is valid for all energies and final systematic uncertainties are presented for all of them (Section 4.6.8).

Many times a weighted sample standard deviation σ of some fitted parameters x , is used. It is computed with the `gsl_stats_wsd` function, which yields [66]:

$$\sigma = \sqrt{\frac{\sum w_i}{(\sum w_i)^2 - \sum w_i^2} \sum w_i (x_i - \mu)^2} \quad \text{with} \quad \mu = \frac{\sum w_i x_i}{\sum w_i}. \quad (4.30)$$

As usually, weights w_i are taken equal to inverse values of squared statistical uncertainties of fitted parameters.

4.6.1 Background distortions

Looking at Fig. 4.17a on the next page, one can notice that the background seems to be underestimated for high m_{inv} in the tag sample and low m_{inv} in the probe sample, while being overestimated for high m_{inv} in the probe sample.

Several possible effects regarding background in ϕ meson analysis, which cannot be properly described by the event mixing method, were discussed at length in Ref. [51]. It was shown that no other resonance decaying into charged kaons is expected to distort the background. There was no clear conclusion given for possible reflections of other resonances decaying into different particles than a pair of charged kaons. That is to observe correlations stemming from pairs of tracks where at least one is not a kaon, but is misidentified as such. Also correlations of kaons due to Coulomb interaction were considered, that could be responsible for the underestimation at low m_{inv} in the probe sample. It was pointed out, that the Coulomb interaction alone cannot produce the observed effect, but strong interactions are not excluded. This is also mentioned in Ref. [8]. Finally, correlations associated with kinematic constraints and possible improvements to the event mixing method were investigated, but no significant change of background estimation was seen.

Because of lack of conclusions in the literature, it was decided to look into the problem using Monte Carlo. One technical difficulty that was encountered is lack of dE/dx simulation in NA61/SHINE MC. Because of that it is necessary to mock the PID cut for kaons in analysis of Monte Carlo data by randomly discarding prescribed fractions of reconstructed tracks that are matched to generated particles of certain species. To produce similar distortions as those seen in experimental data, 90 % of all charged kaons has to be allowed, 60 % of electrons and anti-electrons and 10 % of other species. The obtained tag-and-probe spectrum is shown in Fig. 4.17b on the following page.

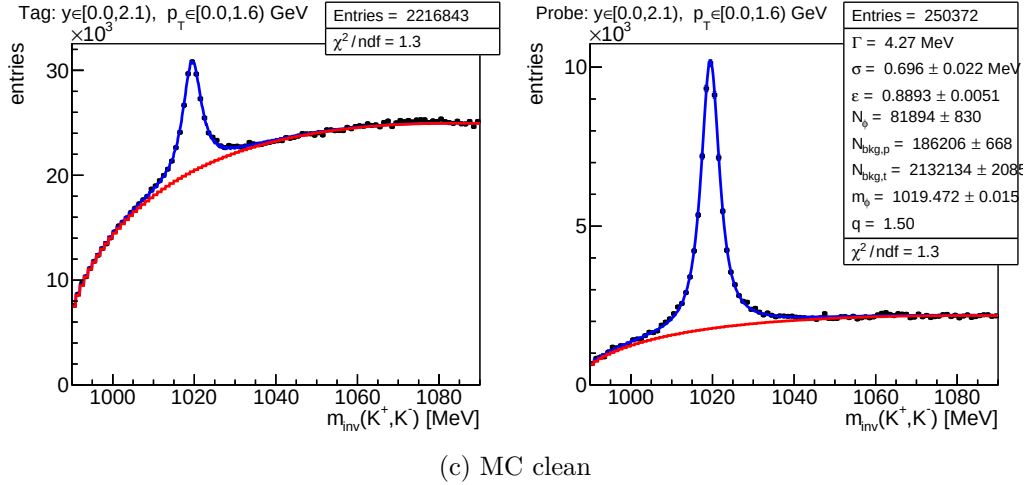
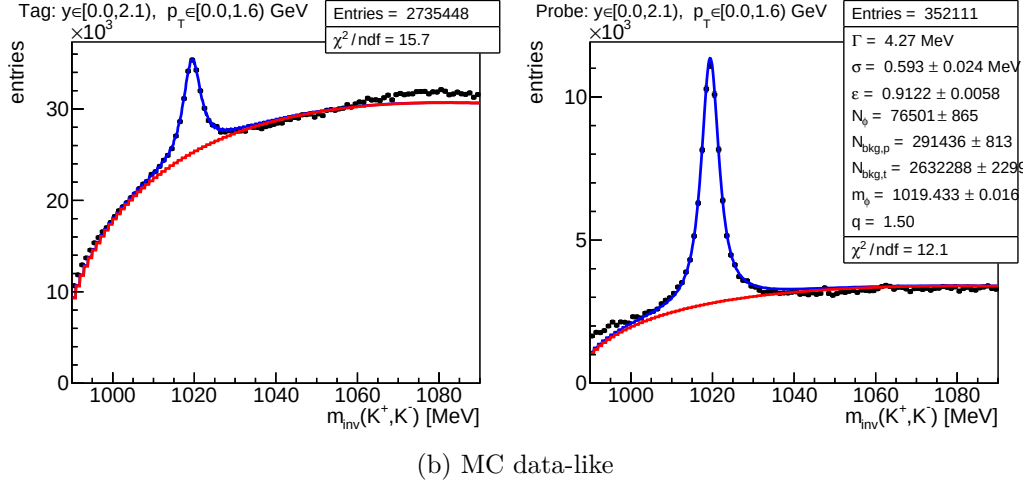
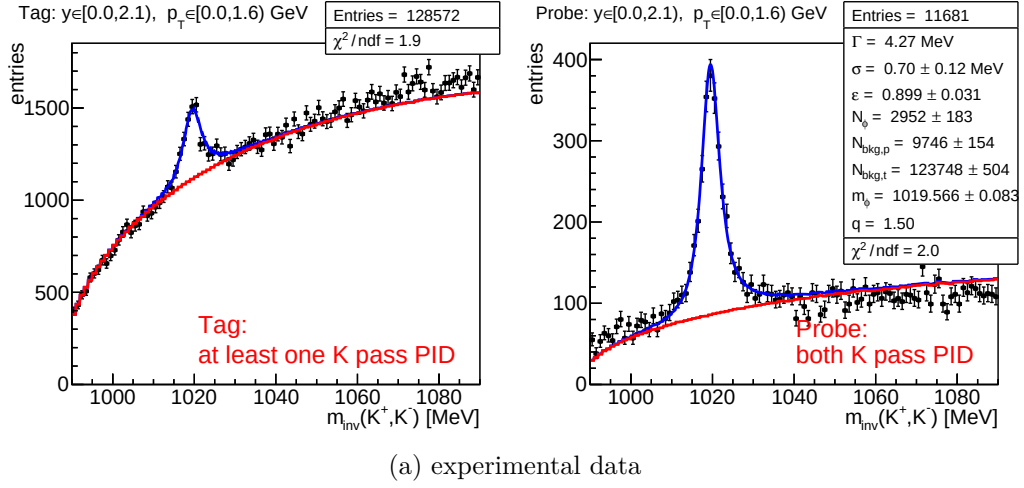


Figure 4.17: Fitted tag-and-probe spectra in broad phase space for 158 GeV experimental data and Monte Carlo with different cuts applied (see text).

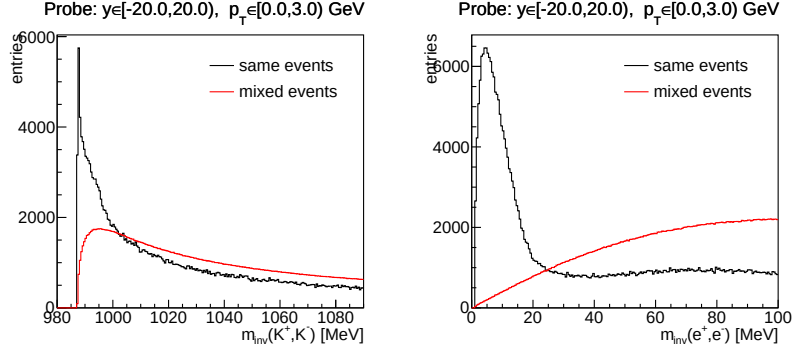


Figure 4.18: Invariant mass spectra for e^+e^- pairs in Monte Carlo with kaon mass hypothesis (*left*) and proper electron masses (*right*). Reconstructed tracks that match to generated electrons are used.

Required fractions were determined by trial and error, allowing and disallowing different species. In the course of these exercises it was discovered that the hill at high values of m_{inv} in the tag sample is related to misidentified pions from $K^*(892)^0$. The effect is much smaller in the probe sample, as both candidates need to pass the PID condition for kaon. The hill at low values of m_{inv} in the probe sample could only be acquired misidentifying large amount of electrons and anti-electrons. If no electrons and no daughters of $K^*(892)^0$ are allowed to invariant mass spectra, then no distortions appear. This is illustrated in Fig. 4.17c on the previous page. Such a clean sample yields an unbiased fit result.

The fact that e^+e^- pairs produce a similar correlated contribution in Monte Carlo as that observed in experimental data, is itself puzzling. Figure 4.18 shows invariant mass spectra if only reconstructed tracks that match to generated electrons are used. The *left* panel depicts the actual shape that contributes to the invariant mass spectrum in ϕ analysis when incorrect mass hypothesis is employed. It exhibits a Dirac delta-like peak at threshold for ϕ production. When the same pairs are plotted in the *right* panel with proper mass hypothesis, the shape looks more reasonable. The distribution vanishes at the threshold for e^+e^- pair production, in agreement with expectation. Mixed events spectra are also plotted to show that the structures really come from correlations.

These pictures are similar to those obtained for hadrons when background effects in analysis of correlations due to Coulomb interaction are studied [67]. It could be as well connected to kinematics of gamma conversion or Dalitz decays of neutral pions. However, any significant, electron-related effect in experimental data is not very probable, as the dE/dx cut for kaons should remove most of electrons, even though they do not obey the Bethe-Bloch

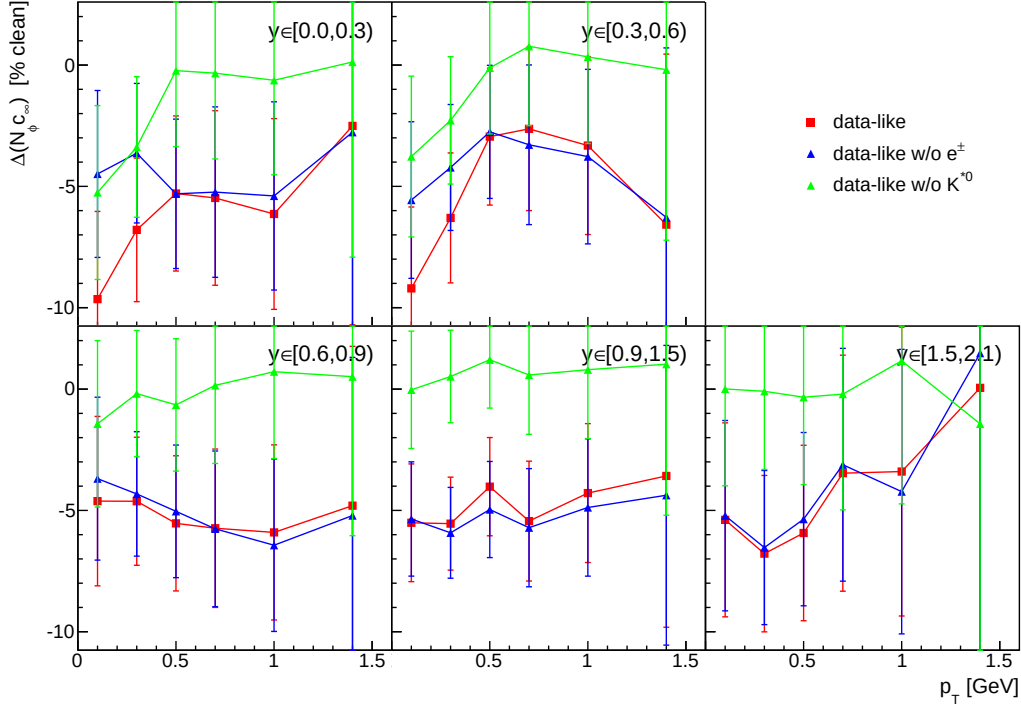


Figure 4.19: Comparison of biases of tag-and-probe fitted ϕ yields in Monte Carlo depending on background distortions included.

relation. So, maybe it is rather the mentioned correlation of kaons due to strong interactions, which probably is not included in EPOS. Whatever it is, it does not change the conclusion of this section, as for the Monte Carlo study it is only important that the shape is reproduced, not its source.

To determine the bias associated with background distortions, invariant mass spectra are fitted. For the resonance model the non-relativistic Breit-Wigner is used, because this is the parametrization employed in EPOS. Fitted yields are compared in Fig. 4.19 to those obtained for the unbiased clean sample (Fig. 4.17c on page 80). Apart from the data-like sample, also two additional ones are checked. First does not include distortion related to e^+e^- correlations, while the second does not include distortion due to $K^*(892)^0$.

This shows that the overall bias is up to -10% — the spectrum derived from experimental data is underestimated. Looking at the two additional samples, this comes mostly from $K^*(892)^0$. There is no indication that this effect could lead to overestimation of the yield. That would require setting a one-sided systematic uncertainty on the result. So it was decided, that it is better to apply a 5% correction and a double-sided systematic uncertainty of another 5% . Since the biased spectrum is about 0.95 of the unbiased one,

inverting this, the correction is $c_{\text{bkg}} = 1.05$, as was stated in Section 4.5.1. The validity of the correction and the systematic uncertainty estimate relies only on how well the shapes visible in the experimental data are reproduced in this Monte Carlo study. No significant differences in distortions between experimental data samples for different energies are observed, so the same values are applied to results for all of them.

One could also think about improving background model in the fit, to get rid of the bias. That could be done by using some analytic expressions. This was already discussed in Section 4.4.2 listing arguments against that idea. Another possibility is to use shape templates from this Monte Carlo study for correlated effects in the background. But, similarly to the previous option, it also increases the number of parameters, what leads to destabilization of fits. Still, this one looks as a plausible choice if data samples of higher statistics were to be analysed.

4.6.2 Resolution model study

One can recall, that the fitting strategy is defined such, that signal shape parameters m_ϕ and σ are fitted once to the probe sample in the broad phase space. Then they are fixed to these values in further fits (Section 4.4.4). This is based on an assumption that signal shape parameters are the same or very close in different phase space bins. But there is no *a priori* physical argument that this assumption should hold exactly.

Using Monte Carlo it is possible to verify it. In MC the true, Breit-Wigner distributed mass $m_{\text{inv}}^{\text{gen}}$ of ϕ meson is known. So one can subtract it from the reconstructed mass $m_{\text{inv}}^{\text{rec}}$ for each pair to enter the invariant mass spectrum, to get a distribution related only to the detector resolution effect, not the resonant shape. Doing this in the analysis binning allows to see how signal shape parameters actually behave. In the end, comparison of these parameters to those obtained in a fit to broad phase space spectrum is necessary. So, instead of using an actual difference of reconstructed and simulated masses, a histogram of

$$m_{\text{inv}} = m_{\text{inv}}^{\text{rec}} - m_{\text{inv}}^{\text{gen}} + m_{\text{EPOS}} \quad (4.31)$$

is created, where m_{EPOS} is the centre of Breit-Wigner function in EPOS. This does not change the shape of the distribution, but assures that the scale of variation in the location parameter is correct.

An example of such a histogram is presented in Fig. 4.20 on the next page. In all three panels the distribution, depicted with black dots, is exactly the same. Initially the Gaussian model (*left* panel) was chosen for fitting.

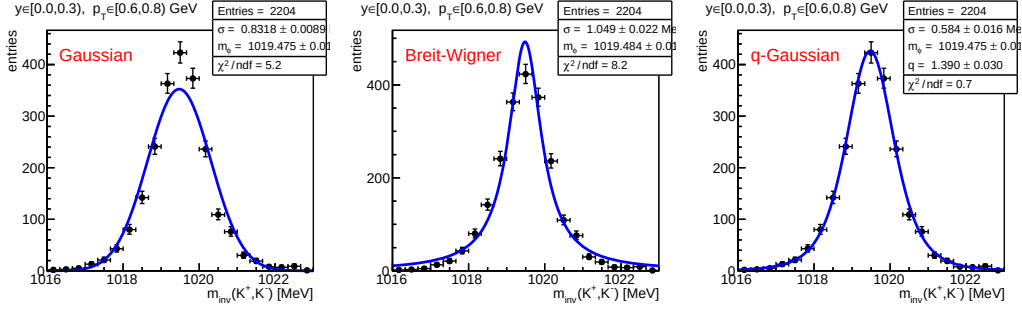


Figure 4.20: Comparison of fits with different models to the same distribution of variable defined by Eq. (4.31) on the preceding page, for one of analysis bins for 158 GeV MC data.

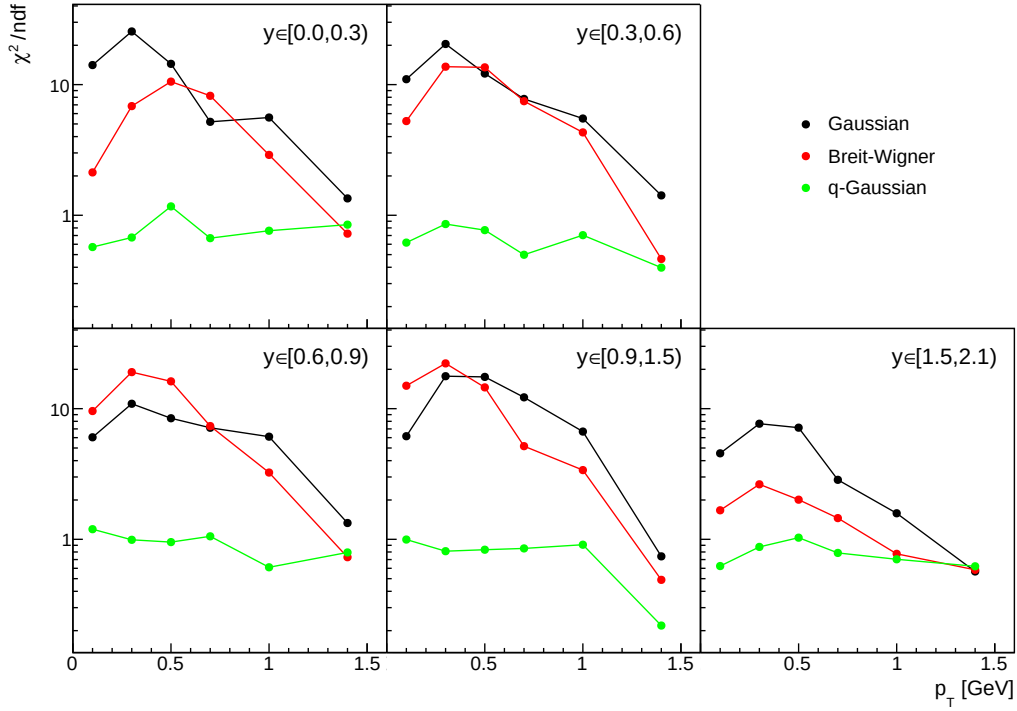


Figure 4.21: Systematic comparison of goodness-of-fit parameter for tested resolution models. Convergence at high p_T values happens due to insufficient statistics.

This was dictated by the fact, that in all studied cases of four different experiments [8, 15, 20–22] that parametrization was used. But, as is seen in Fig. 4.20, Gaussian does not fit properly the histogram.

This should not be too surprising, because there is no physical reason why the detector resolution effect should be distributed according to Gaussian. It is chosen for its simplicity and the fact that due to the central limit theorem we expect it to be applicable if there are many concurring random effects

Table 4.4: Example standard deviations of parameters in phase space for 158 GeV MC data.

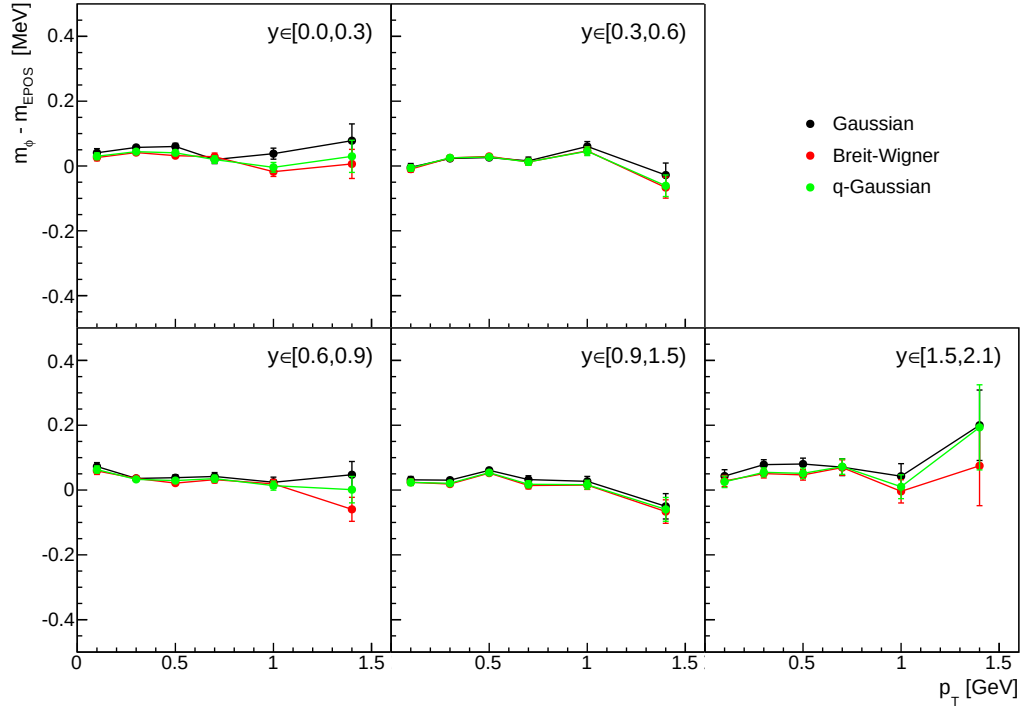
parameter	w.s.d. [% x_{broad}]		
	Gaussian	Breit-Wigner	q-Gaussian
m_ϕ	0.0021	0.0019	0.0018
σ	6.9	9.2	8.8
q			5.7

contributing to the resolution. But most probably nobody ever counted how many effects actually contribute. What is more, that could rather be applicable for single track resolution. In case of the invariant mass, its resolution comes from resolution of momentum measurements for both tracks and a non-linear relation between both momenta and the invariant mass. That alone breaks assumptions of the central limit theorem.

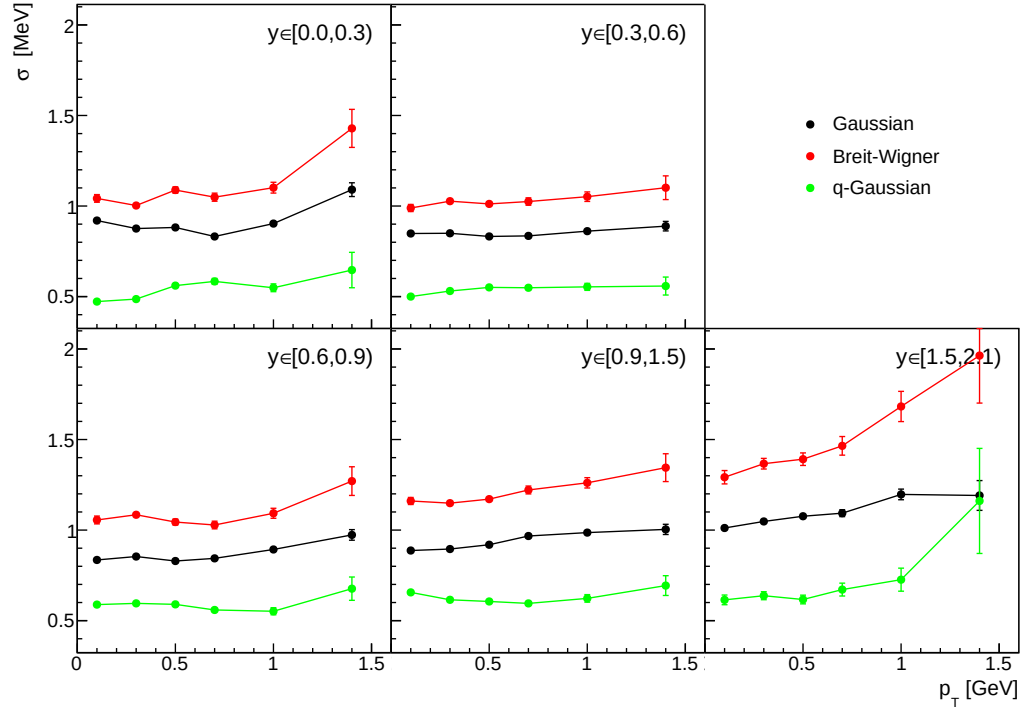
In such a situation, a better model had to be found, obviously a one more ‘peaked’. The second attempt became the Lorentz function (Breit-Wigner, *central* panel in Fig. 4.20 on the previous page). That too was not successful, being too ‘peaked’. Finally, the q-Gaussian (*right* panel, Eq. (4.5) on page 49) was tried, which has the q parameter, which steers the ‘peakedness’. That turned out to be fine. Figure 4.21 on the previous page shows that q-Gaussian is clearly favoured over two other tested models irrespectively of phase space location.

All models have a location parameter m_ϕ and a width parameter σ ; as was stated, q-Gaussian has also a shape parameter q . Their stability in phase space for 158 GeV MC data is presented in Figs. 4.22 and 4.23 on the following page and on page 87. For each of stability plots, weighted sample standard deviation (w.s.d.) values are computed and compared to parameters’ values x_{broad} from fits to broad phase space spectrum. Example results for 158 GeV are given in Table 4.4. Comparing these stability measures, one sees that there are no dramatic differences between models (that criterion could also influence the choice of model for fits to experimental). Values from the last column (q-Gaussian) are used in Section 4.6.4 to estimate systematic uncertainties due to constraints on signal shape parameters.

As a result of this study, the choice of detector resolution model in fits to invariant mass distributions for experimental data, was changed from the initially picked Gaussian parametrization to q-Gaussian. Because of the background distortion at low values of m_{inv} in the probe sample (Section 4.6.1), the q parameter, however, cannot be fitted along with other signal shape



(a)



(b)

Figure 4.22: Stability in phase space of (a) the location parameter and (b) the width parameter for 158 GeV MC.

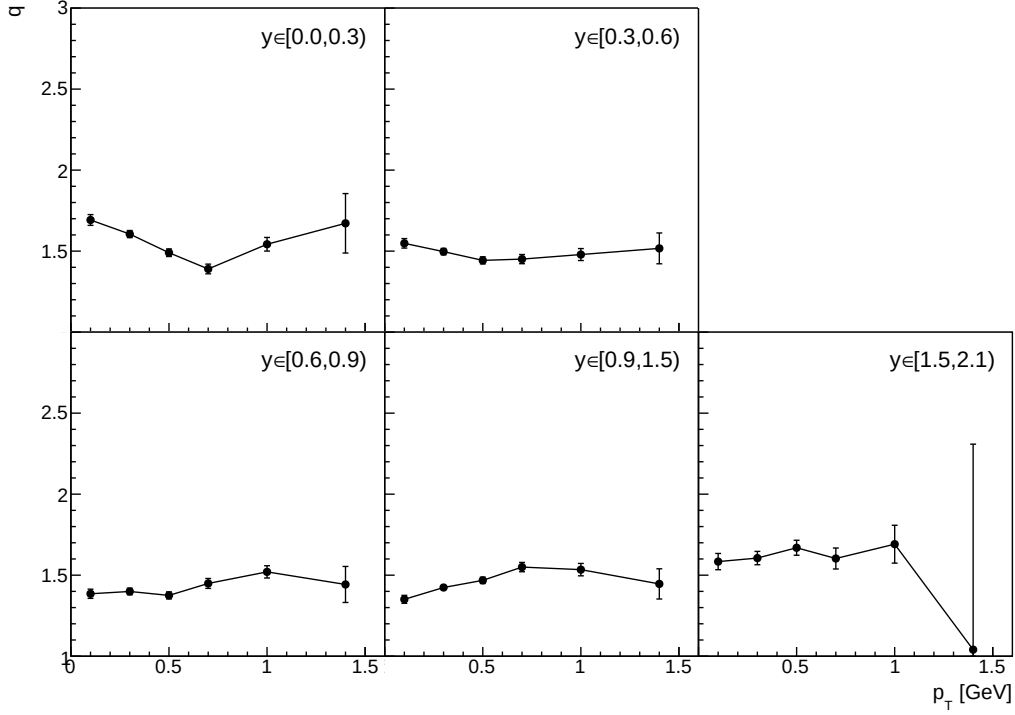


Figure 4.23: Stability of the q-Gaussian's shape parameter in phase space for 158 GeV MC.

parameters. This is because q-Gaussian is such an elastic parametrization that it can adapt to the distortion, reinterpreting it as a signal contribution. That yields about twice larger N_ϕ values and seems to be utterly wrong. Therefore, q is fixed to the value of 1.5, which is about the average of points in Fig. 4.23.

It is expected, that q depends in the first order on overall properties of the detector and the relation between the invariant mass and track momenta. Therefore, the Monte Carlo q value, unlike σ , should be valid in the first order for the experimental data. Higher order discrepancies are covered by the systematic uncertainty which takes into account variations of constraints on fit parameters.

4.6.3 Signal parametrization discussion

After issues regarding background description in invariant mass distributions and the effect of the detector resolution are explained, it is possible to conclude the topic of the signal component parametrization that is used in this analysis.

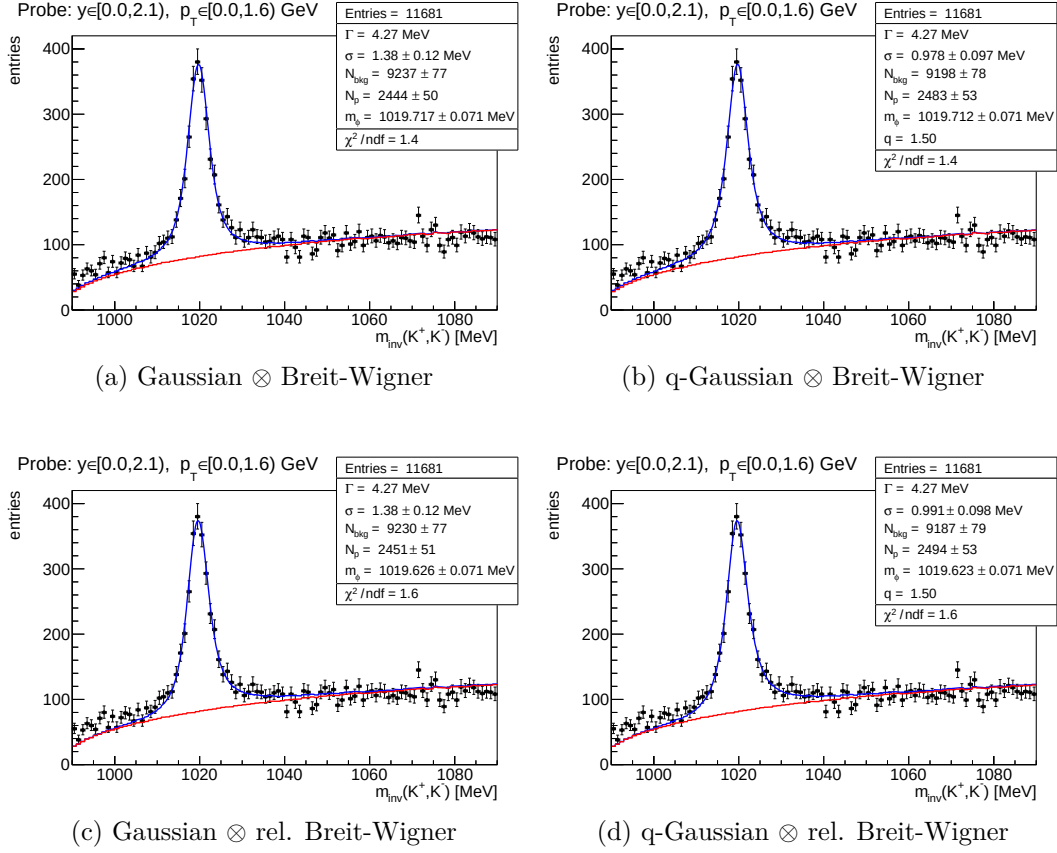


Figure 4.24: Comparison of fits with different parametrizations to the invariant mass spectrum in broad phase space, for 158 GeV data.

Initially Voigtian, a convolution of Gaussian and non-relativistic Breit-Wigner, was employed due to its technical convenience (availability in software libraries) and the fact that it was utilized in several other analyses of ϕ production. But, based on the Monte Carlo study of the resolution effect, it was decided to change Gaussian to q-Gaussian. Then it was realised, that non-relativistic Breit-Wigner yields couple of percent of the total signal integral in a region below the threshold for two kaons production. That is clearly non-physical, so the resonance component parametrization was changed to the relativistic Breit-Wigner. The latter goes to zero as the invariant mass goes to the threshold value.

Comparison of fits with different combinations of the detector resolution effect component and the natural shape of the resonance component are shown in Fig. 4.24. Clearly, no large effect is visible with the available statistics. One can notice a slight difference of χ^2/ndf values, in favour of

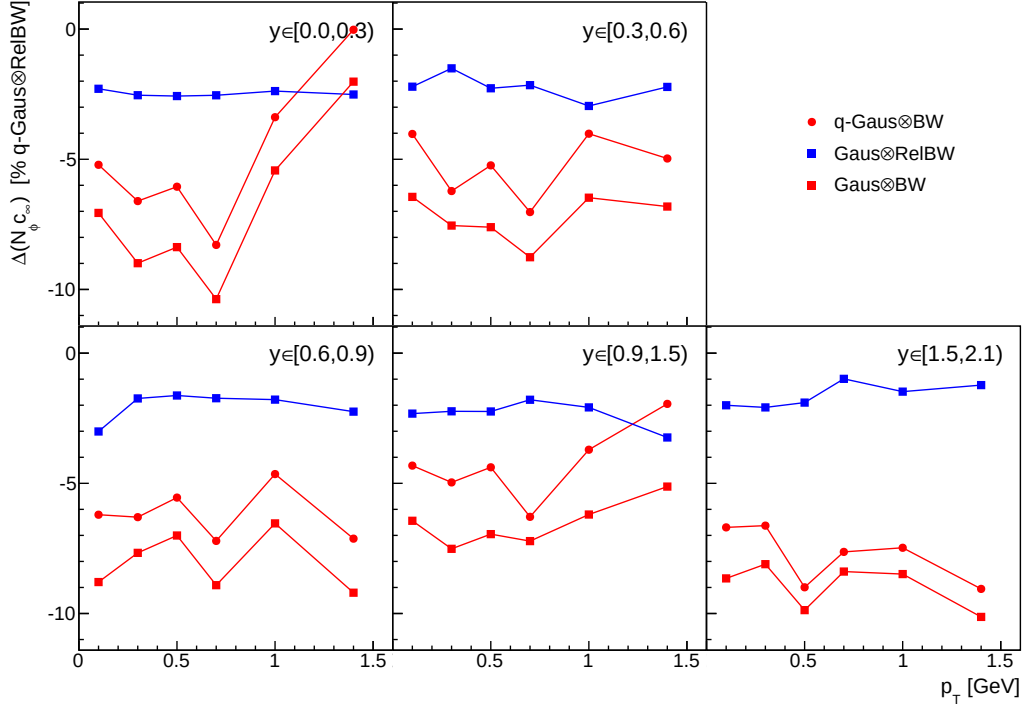


Figure 4.25: Changes of fitted infinite-domain yields obtained with different signal parametrizations, relative to the one used in this analysis. Fits are done on 158 GeV data. Uncertainties from fits are not shown to reduce clutter.

non-relativistic Breit-Wigner. But that comes from the fact that relativistic Breit-Wigner falls fast to zero to the left of the peak and the problem with background description in that region becomes more pronounced. So it should not be considered as an argument to use the non-relativistic version.

Cleaner differences are seen in the fitted yields in Fig. 4.25, which are given in the infinite domain using the correction described in Section 4.5.3. According to this comparison, Voigtian may lead to up to 10% bias of the yield. The difference between q-Gaussian and Gaussian resolution model is on average about 2%, while that between the relativistic and the non-relativistic resonance model is on average about 5%.

Such comparisons, if done, can be used to estimate systematic uncertainty associated with the choice of model of signal component in the invariant mass spectrum. It is done *e.g.* in Ref. [21], where instead of q-Gaussian, the Crystal-Ball model is checked and the same as here, two versions of Breit-Wigner are compared. Similar numbers are derived there as the ones above, and incorporated into the systematic uncertainty. Such an approach is valid, however, only if tested parametrizations are equivalent in the sense,

that none of them is preferred based on goodness-of-fit nor some prior, sound knowledge. The latter is not necessarily true in this case, due to non-physical sub-threshold production for non-relativistic Breit-Wigner and knowledge against Gaussian from the Monte Carlo study.

While relativistic Breit-Wigner Eqs. (4.4) on page 49 is taken from Ref. [8], it originally comes from a paper written in 1964 [68]. To be precise, it is a combination of Ref. [68] equations (4), (A.1) and (A.7). Out of these, (4) is the main relativistic Breit-Wigner formula, corresponding to Eqs. (4.4a) here, which has connection to the propagator of an unstable particle. It is not clear where (A.1) comes from. That equation, together with an empirical expression (A.7), defines energy dependence of the width of the resonance — Eqs. (4.4b) here. This term is actually responsible for the fact that the distribution falls to zero at the threshold, what is the reason why the relativistic Breit-Wigner is preferred over the non-relativistic one. But the article shows, that the subject, especially the energy dependence of the width, is not well defined on theoretical grounds. Indeed, a much newer paper from 2005 [69] expresses similar notion, actually questioning the main, propagator-related formula. It says nothing about width’s energy dependence. But that may be associated with the fact, that it is focused mainly on the Z boson, for which the resonance curve is located far from the threshold.

All in all, it seems that some systematic uncertainty connected with ambiguities in resonance theory needs to be assigned. Assuming that the non-relativistic parametrization is wrong and the chosen relativistic one is better, and having no other plausible choice from theory, it was decided to use a value of 3%. It is slightly higher than half of the average difference between two tested parametrizations.

4.6.4 Systematic uncertainties due to constraints on signal shape parameters

Using results of the Monte Carlo study of Section 4.6.2 it is possible to estimate systematic uncertainties due to constraints imposed on signal shape parameters according to the established fitting strategy (Section 4.4.4).

This is done in the following way for each of parameters and each of variation factors:

1. Reference values of all parameters are taken from the first step of the fitting strategy.
2. The parameter in question is varied (increased or decreased) by a given factor. This defines a new set of signal shape parameters with all but the given one at the reference (unvaried) values.

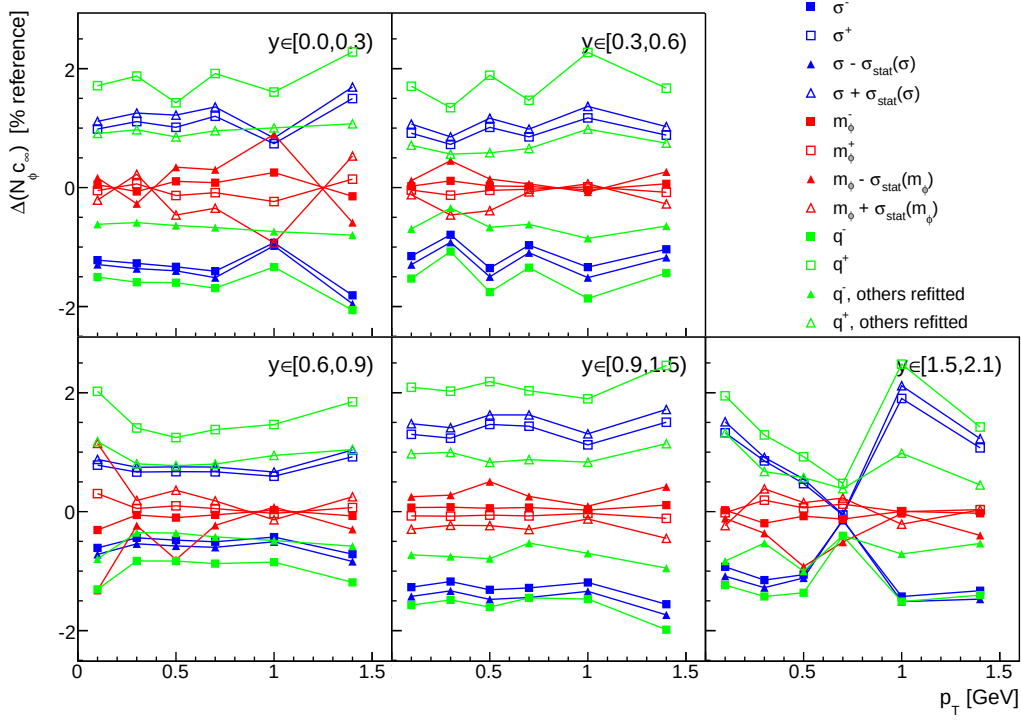


Figure 4.26: Changes of fitted infinite-domain yields obtained with varied constrained parameters, relative to the yield with reference, unvaried parameter values. Fits are done on 158 GeV data. Uncertainties from fits are not shown to reduce clutter.

3. Fits of the second and the third step of the strategy are done with the new set of fixed parameters.

Results of these fits for 158 GeV data are presented in Fig. 4.26. Superscripts ‘+’ and ‘−’ denote that the given parameter was respectively increased or decreased by the standard deviation of the parameter as comes from the study of Section 4.6.2. For 158 GeV data in Fig. 4.26, the values from Table 4.4 on page 85 are utilized. Next, in case of the q parameter, additionally an alternative approach is used, indicated by ‘others refitted’. The q parameter, which is normally kept constant when m_ϕ and σ are fitted in the first step, is varied before the first step of the strategy. That influences fitted m_ϕ and σ values. Last, denoted by $\pm\sigma_{\text{stat}}$, parameters m_ϕ and σ are also varied using their statistical uncertainties coming from the reference fit, as these uncertainties should also be propagated somehow to the final uncertainty of the result.

Because of correlations between parameters, a change in one parameter may give almost the same yield, but a different value of another parameter, as compared to the reference case. That is especially well seen for the alternative

approach to q variation, where a change in σ lowers the change in the yield if compared to the situation when q is varied and other parameters are kept constant. Also it is not clear how the systematic uncertainty should be estimated bin-by-bin, especially for the central p_T bin of the last y bin, where some aberrant behaviour is observed. Therefore, it was decided to assign for this case a bin-independent value of 2 % as an estimate of the systematic uncertainty due to constraints.

4.6.5 Tag-and-probe systematics

Next uncertainty to estimate is the one associated with biases of particle identification by dE/dx , which should be taken out by the tag-and-probe method. It is also an opportunity to validate the second step of the fitting strategy (Section 4.4.4). Known sources of systematic error in the tag-and-probe method include:

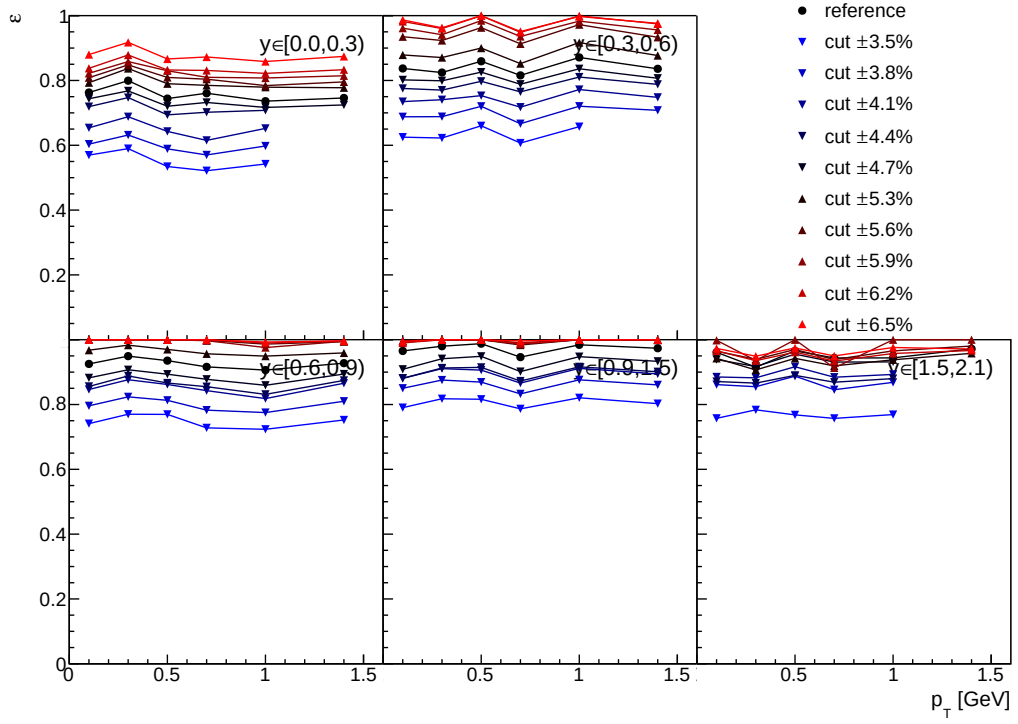
- non-constant value of PID efficiency ε within analysis bins (some theoretical considerations regarding this issue are presented in Appendix C),
- constraints on ε imposed according to the fitting strategy, if ε is non-constant between p_T bins for a given y bin.

These and other, unknown effects are studied by variation of the dE/dx window size around kaon Bethe-Bloch curve. The total range of the variation is $\pm 30\%$ of the reference window size, the same as the one used to estimate systematic uncertainty in Ref. [51]. The whole exercise is done twice, for two fitting strategies:

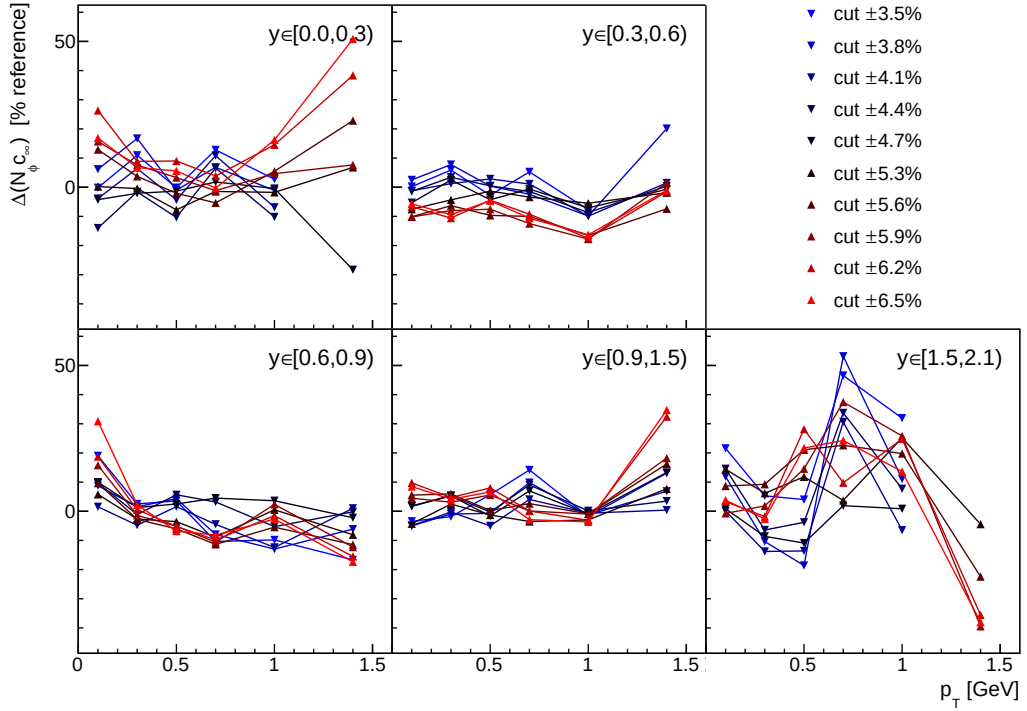
constrained The default strategy of the analysis, described in Section 4.4.4, where ε value in final fits, in 2D bins, is soft-constrained to the value fitted in a whole 1D rapidity bin. Results of fits for all variants are presented in Fig. 4.27 on the next page.

free There are no constraints on ε other than it has to be in the range $[0, 1]$. Results of fits for all variants are presented in Fig. 4.28 on page 94.

It is visible in Fig. 4.27a, that apart from one of variants in the last rapidity bin, ε changes monotonically with the window size. This agrees with the expectation. However, a different behaviour is observed in the **free** strategy, Fig. 4.28a. This unexpected result occurs probably due to instability of the fit with more free parameters. This is an argument in favour of validity of the chosen fitting strategy.

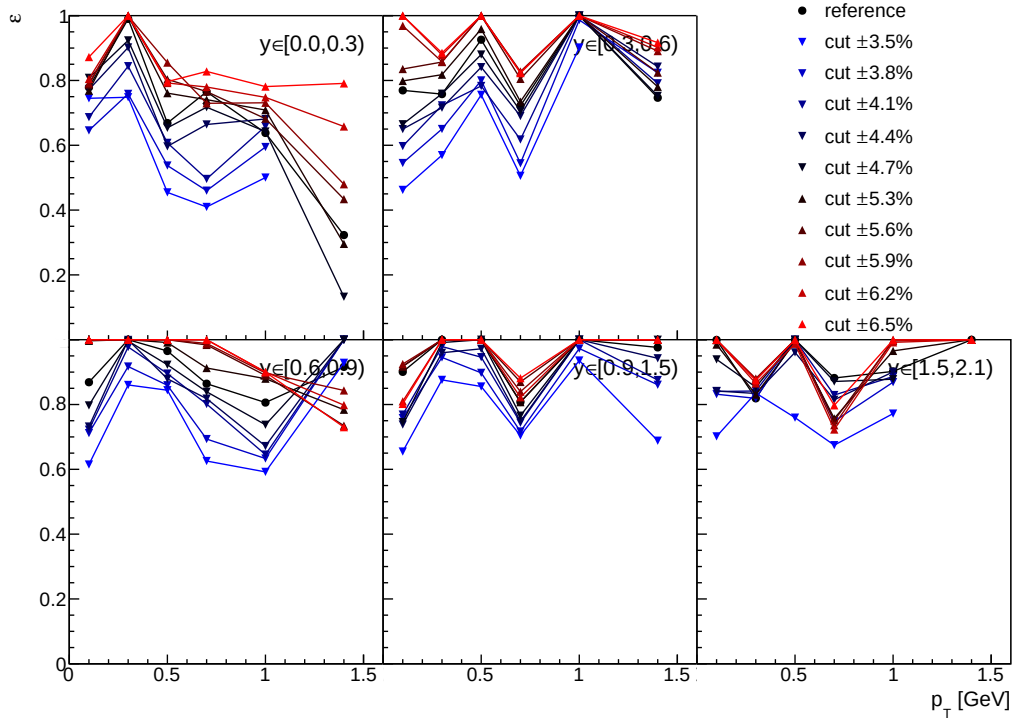


(a) fitted PID efficiency

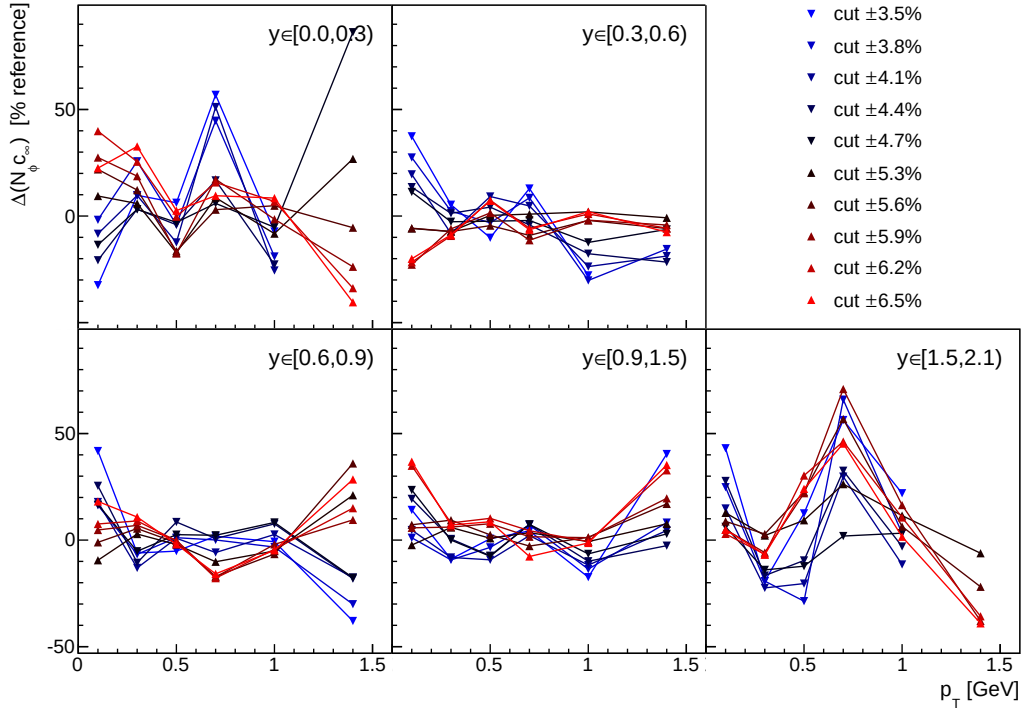


(b) relative changes of fitted infinite-domain yields

Figure 4.27: **Constrained** strategy fit results on 158 GeV data. Reference size of the dE/dx cut window is $\pm 5\%$ of the kaon Bethe-Bloch value (Section 4.3.2). Uncertainties from fits are not shown to reduce clutter.



(a) fitted PID efficiency



(b) relative changes of fitted infinite-domain yields

Figure 4.28: **Free** strategy fit results on 158 GeV data. Reference size of the dE/dx cut window is $\pm 5\%$ of the kaon Bethe-Bloch value (Section 4.3.2). Uncertainties from fits are not shown to reduce clutter.

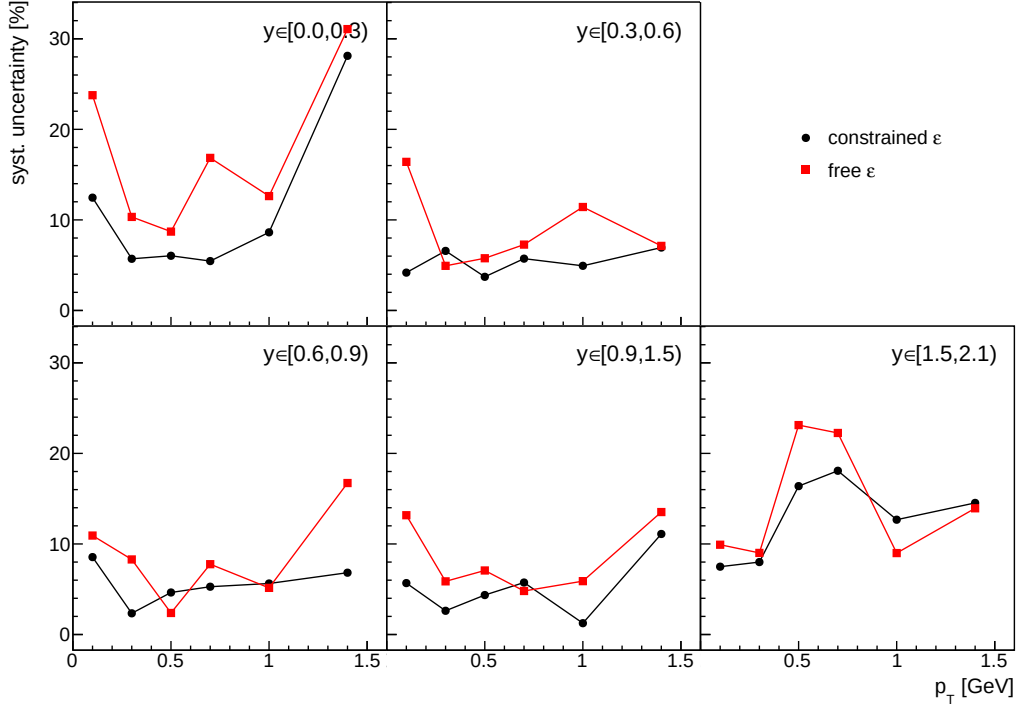


Figure 4.29: Tag-and-probe method systematic uncertainties for 158 GeV data.

Figure 4.27b on page 93 presents changes of fitted infinite-domain yields, relative to those obtained with the reference cut value, for the default strategy. If there was no systematic error, all points would cluster at zero difference value. Analogous results are shown in Fig. 4.28b on the preceding page for the **free** strategy. It is evident that the spread of points is larger than in Fig. 4.27b.

For both cases weighted standard deviations of relative yield changes are used as bin-by-bin estimates of systematic uncertainties. These are plotted in Fig. 4.29. Obviously the chosen strategy gives on average better systematic accuracy. This, together with the better behaviour of ε and also smaller statistical uncertainties from more stable fits, clearly favours the chosen strategy over the **free** one.

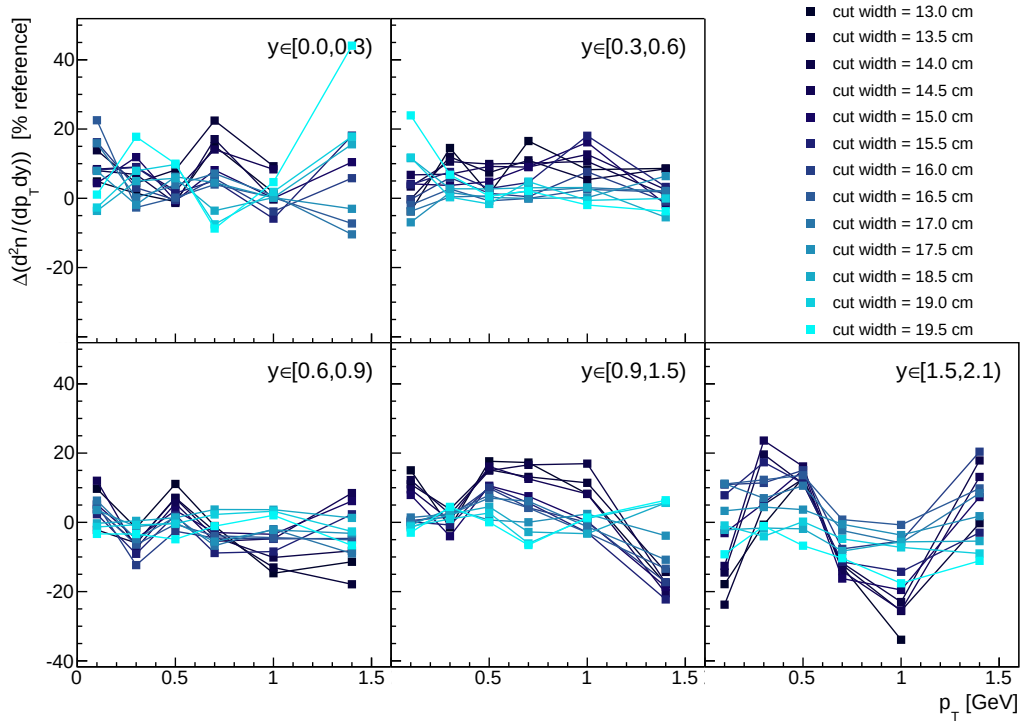
4.6.6 Systematic uncertainties related to event & track quality cuts

Finally, systematic uncertainties related to event and track cuts can be estimated by variations of cuts' values. These uncertainties are caused by improper Monte Carlo corrections of losses due to varied cuts and of detec-

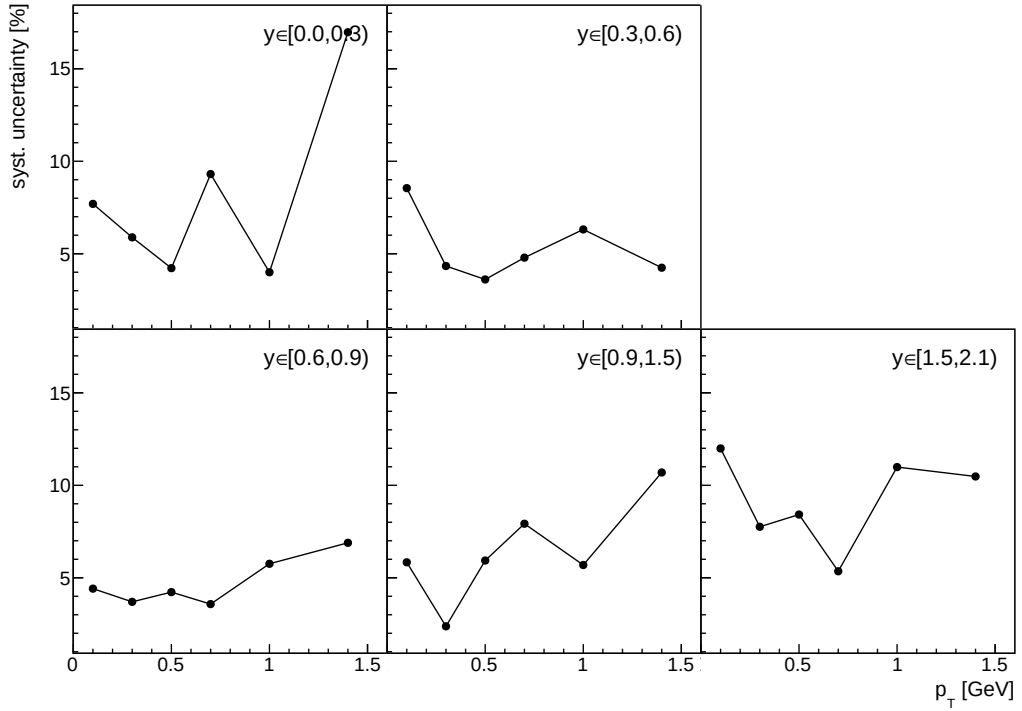
tor effects. Therefore, they could also be called ‘systematic uncertainties of the Monte Carlo correction’. For each cut variant a full analysis is repeated on both experimental data to get the uncorrected yields and on the Monte Carlo to derive correction values.

Among event cuts, the one on the main vertex z position needs to be scanned. The reference cut width is 18 cm (see Table 4.2 on page 42). Variation is done in a range 13 cm to 19.5 cm, what is not symmetric around the reference value, but it does not make sense to cover windows of the target (see Fig. 4.8 on page 60) with the varied cut. Figure 4.30a on the following page presents changes of corrected spectra, relative to those obtained with the reference cut value. Similarly to the previous section, if there was no systematic error, all points would cluster at zero difference value. Weighted standard deviations of relative corrected yield changes are used as estimates of bin-by-bin systematic uncertainties shown in Fig. 4.30b. It should be noted, that these uncertainties are not only connected with a bias of the MC correction, but also with a genuine bias of the experimental result due to residual contribution of events from windows of the target (this is why above it is said that coverage of windows should be minimised).

Among track selection criteria, number-of-points cuts are scanned and also a variant is tested in which the cut on track impact parameters b_x , b_y is not applied. Reference values of scanned cuts are given in Table 4.3 on page 44. Ranges of scans are motivated by a test in Ref. [30], where cut values were decreased to lower limits of scan ranges and that single variant was compared to the reference result. Both cuts on the number of all points in TPCs and the cut on the number of VTPC points are varied simultaneously. The criterion on the number of GAP-TPC points is not altered. Figure 4.31a on page 98 presents changes of corrected spectra, relative to those obtained with reference cut values. Again weighted standard deviations of relative corrected yield changes are used as estimates of bin-by-bin systematic uncertainties. These are shown in Fig. 4.31b on page 98. It is evident that in most bins they are negligible.

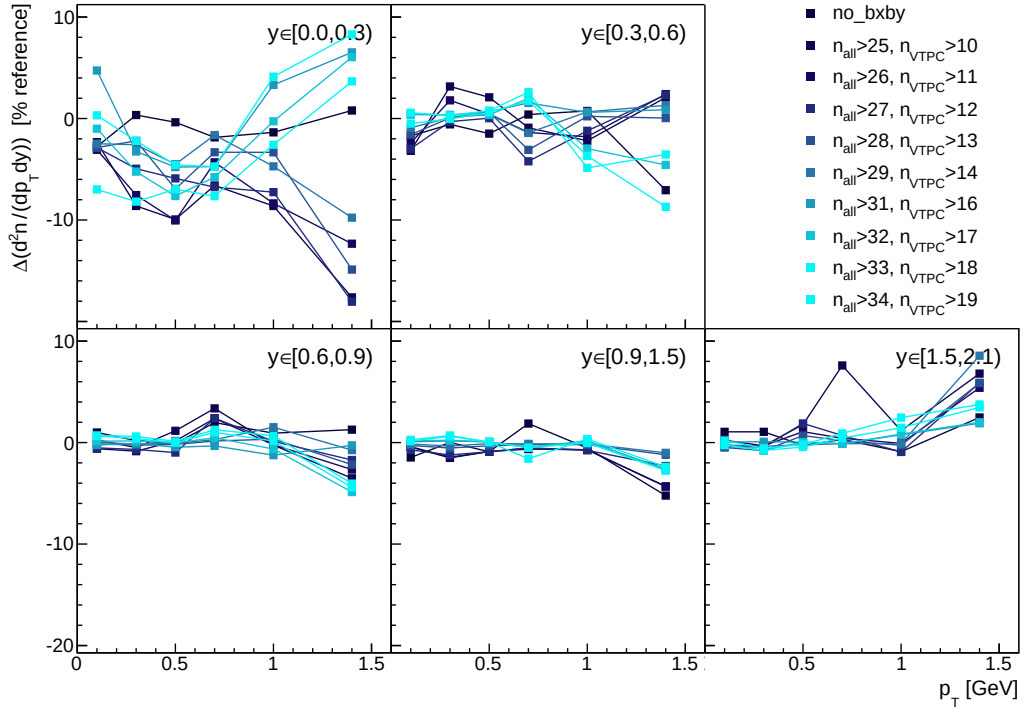


(a) relative changes of corrected spectra

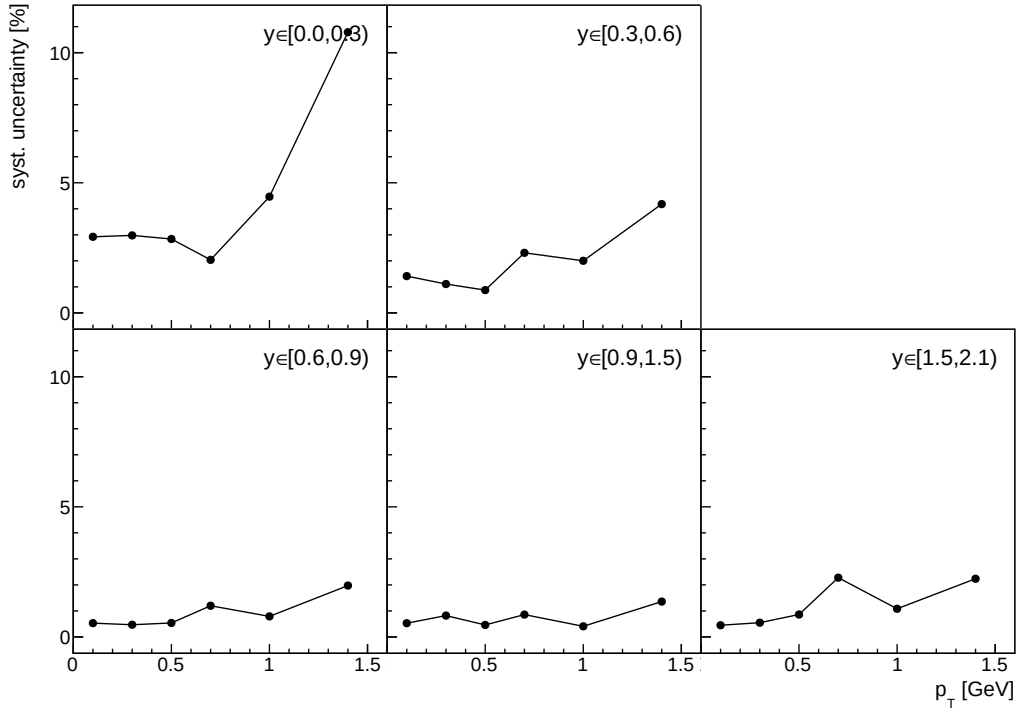


(b) systematic uncertainties

Figure 4.30: Illustration of estimation method of systematic uncertainties associated with the main vertex z position cut for 158 GeV data. Statistical uncertainties of corrected spectra are not shown in (a) to reduce clutter.



(a) relative changes of corrected spectra



(b) systematic uncertainties

Figure 4.31: Illustration of estimation method of systematic uncertainties associated with track cuts for 158 GeV data. Statistical uncertainties of corrected spectra are not shown in (a) to reduce clutter.

4.6.7 Monte Carlo correction averaging

As was written in Section 4.5.4, in case of a single differential analysis, Monte Carlo correction still has to be calculated in a 2D binning (Fig. 4.32a on the following page). But in order to apply it to the raw (1D) spectrum, its projection in the 1D analysis binning (Fig. 4.32b) is needed.

Algorithm of averaging

This projection is obtained by averaging of the 2D correction using a method based on the one developed for the NA49 ϕ analysis to calculate the average acceptance [8]. The overall procedure looks like that:

1. The transverse momentum spectrum is fitted with a thermally motivated formula $f(p_T; T)$ given by Eq. (A.1) on page 134, which has a single shape parameter T (the effective temperature).
2. The rapidity spectrum is fitted with a Gaussian $g(y; \sigma_y)$ centred at $y = 0$, which also has a single shape parameter σ_y (the standard deviation of a Gaussian).
3. A ϕ production model *ansatz* $h(y, p_T)$ is constructed using results of above fits.
4. In the spirit of Eq. (4.23) on page 70 integrals of $h(y, p_T)$ are used as weights in the averaging of the 2D correction within 1D bins.
5. 1D corrections are applied to the transverse momentum and the rapidity spectra yielding corrected spectra.

After the last point, corrected spectra are injected in the first two points in the place of uncorrected ones and all steps are repeated. So the algorithm is iterative and finishes when T and σ_y spectral parameters no longer change.

To understand the procedure of averaging with a given ϕ production model *ansatz*, let's focus on the correction for the 1D rapidity binning (the reasoning for transverse momentum binning is analogous with exchange of y and p_T). In that case the averaging has to be done over p_T bins for each y bin. So for a given y bin, let c_i be a value of the 2D correction for a p_T bin i . Moreover, let's denote the region in (y, p_T) plane to which that bin corresponds (blue tile in Fig. 4.32a on the next page) as S_i . Then assuming that Eq. (4.23) on page 70 is correct (that will be shown later), the average

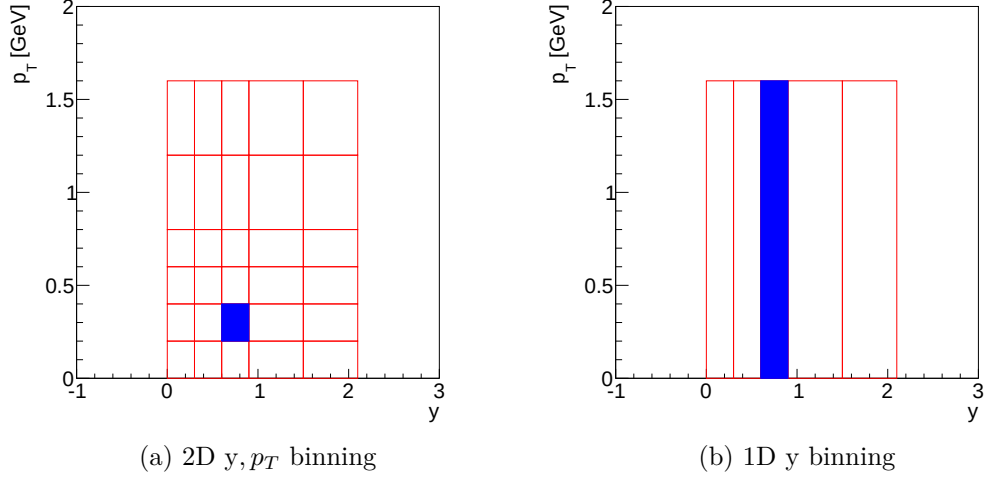


Figure 4.32: Monte Carlo correction needs to be projected from the binning in panel (a) to that in panel (b). Blue tile in (a) illustrates one of regions S_i (see text) for the third rapidity bin, while the blue tile in (b) corresponds to the region S (see text) for that rapidity bin.

correction for the considered y bin is given by:

$$\bar{c} = \frac{\int_S h(y, p_T) dy dp_T}{\int_S h(y, p_T) \epsilon(y, p_T) dy dp_T} = \frac{\sum_{S_i} \int h(y, p_T) dy dp_T}{\sum_{S_i} \frac{1}{c_i} \int h(y, p_T) dy dp_T} = \frac{\sum w_i}{\sum w_i / c_i}, \quad (4.32)$$

with

$$w_i = \int_{S_i} h(y, p_T) dy dp_T, \quad (4.33)$$

where $S = \bigcup S_i$ is the region in (y, p_T) plane corresponding to the considered bin of the 1D rapidity binning (blue tile in Fig. 4.32b) and the efficiency $\epsilon(y, p_T) = 1/c_i$ for $(y, p_T) \in S_i$. Sums run over p_T bins (all tiles in Fig. 4.32a that belong to the blue tile in Fig. 4.32b). The first equality in Eq. (4.32) is equivalent to Eq. (4.23) on page 70. The second one comes from dividing S into regions S_i where $\epsilon(y, p_T)$ is constant, substitution of that constant and use of integration properties.

From above follows, that the statistical uncertainty of the projected correction for the considered y bin can be calculated as

$$\sigma(\bar{c}) = \frac{\sum w_i}{\left(\sum w_i / c_i\right)^2} \sqrt{\sum w_i^2 \sigma_i^2 / c_i^4}, \quad (4.34)$$

where σ_i is an uncertainty of c_i .

To prove that indeed Eq. (4.23) on page 70 is correct, it is necessary to utilize the definition of the Monte Carlo correction given by Eq. (4.18) on page 66. The correction pertaining to the region S is a ratio of numbers of all generated (n_{gen}) and selected (n_{sel}) particles with momenta within that bin. The integral w_i gives the number of generated particles within region S_i . Thus

$$n_{\text{gen}} = \sum w_i. \quad (4.35)$$

Furthermore, if the correction c_i is constant within region S_i , the expected number of selected particles within region S_i is, by definition of c_i , just w_i/c_i . Consequently

$$n_{\text{sel}} = \sum w_i/c_i. \quad (4.36)$$

This proves, that the right-hand side of Eq. (4.32) on the preceding page indeed yields the projected correction. Therefore selecting infinitesimal regions S_i , where the assumption of constancy of c_i is fulfilled, and extrapolating the reasoning to three-dimensional momentum space, one gets Eq. (4.23) on page 70.

Obviously a term $w_i c_i$, that would arise if one tried to apply averaging directly to the correction, is meaningless within above reasoning based on the definition of the Monte Carlo correction given by Eq. (4.18) on page 66. Indeed within the toy Monte Carlo study described below, the bias caused by such an erroneous approach is of the order of 10 % to 20 %.

Choice of ϕ production model *ansatz*

Initially the same *ansatz* was used as in the NA49 analysis [8], that is the ‘factorized’ one:

$$h_{\text{factorized}}(y, p_T; T, \sigma_y) = f(p_T; T)g(y; \sigma_y). \quad (4.37)$$

However, there is no strong physical argument that such a simple model is sound. Indeed, the dependence of T and σ_y spectral parameters on y and p_T in Fig. 5.2 on page 119, for a double differential analysis of experimental data, hints against it.

The simplest test that can be done to check how significant that issue might be, is to take c_{MC} directly obtained in 1D binning. That corresponds to averaging with the model residing in EPOS. This model is not expected to be very well describing the reality (this why the whole analysis is done in the first place), but it is somehow physically motivated and it does not factorize. So it can be considered as a realistic worst-case scenario. Its comparison with the ‘factorized’ *ansatz* is shown as green squares in Fig. 4.36a on page 107.

Table 4.5: Spectral parameters in single differential analysis of 158 GeV and 40 GeV experimental and EPOS data. For experimental data first uncertainty is statistical, second is systematic. For EPOS statistical uncertainty is negligible compared to data uncertainty.

p_{beam} [GeV]	T_0 [MeV]		$\sigma_{y,0}$	
	data	EPOS	data	EPOS
158	$154 \pm 9 \pm 5$	112	$0.953 \pm 0.062 \pm 0.038$	1.044
40	$129 \pm 14 \pm 8$	99	$0.752 \pm 0.064 \pm 0.043$	0.645

While nothing spectacular happens for the transverse momentum binning, the difference for the rapidity binning reaches 13 %, what is rather significant. This is why it was decided to test models other than the simple ‘factorized’ one.

For an *ansatz* to be useful, it has to be related somehow to $f(p_T; T)$ and $g(y; \sigma_y)$ as these convey the only information that can be inferred about the model from a single differential analysis of experimental data. Therefore, it seems natural to construct the *ansatz* in the form

$$h(y, p_T) = f(p_T; T(y))g(y; \sigma_y(p_T)). \quad (4.38)$$

So what is left is to invent how T and σ_y should depend on y and p_T .

In order to do it, one can utilize the information from double differential analysis of 158 GeV experimental data shown in Fig. 5.2 on page 119. It may also prove useful to get similar plots from EPOS. It turns out, that σ_y has similar values in 158 GeV data and in EPOS. But T in data is systematically larger than in EPOS, although it does decrease with similar rate as a function of rapidity.

These values can then be compared to those obtained from single differential analysis of both data and EPOS (Table 4.5). The latter quantities are those, which are calculated in each iteration of the averaging algorithm by fits of $f(p_T; T)$ and $g(y; \sigma_y)$. In addition to the already observed differences between data and EPOS for 158 GeV, the single differential analysis reveals that the temperature is also higher for 40 GeV data than in EPOS.

Figure 4.33 on the next page shows results of a double differential analysis scaled by those coming from single differential analysis. It is obvious that for 158 GeV the scaled dependence of spectral parameters on y and p_T is very similar between the experimental data and EPOS. Noticing that single differential temperatures compare similarly to EPOS for both energies, *let’s make an assumption* that the scaled dependence of spectral parameters on y and p_T is also similar between EPOS and data for 40 GeV. Then EPOS

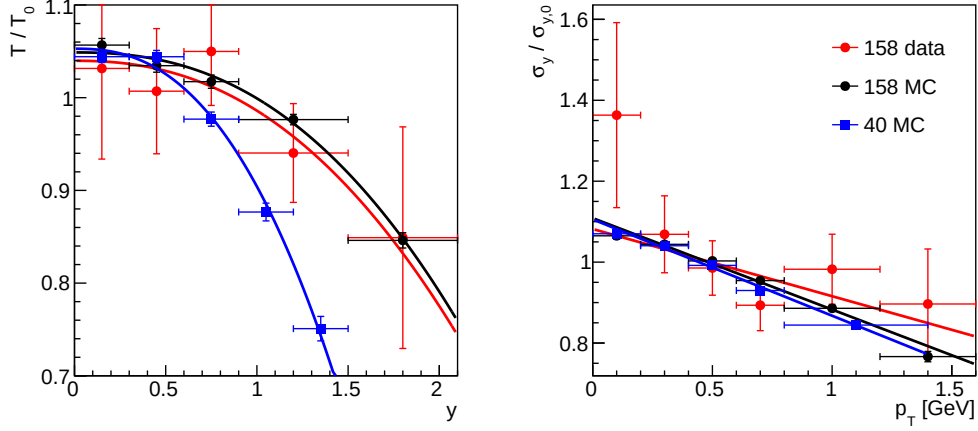


Figure 4.33: Dependence of scaled spectral parameters on the rapidity and transverse momentum for 158 GeV experimental and EPOS data, as well as 40 GeV EPOS data. Values of T and σ_y come from fits in double differential analysis (for 158 GeV data, are the same as in Fig. 5.2a on page 119). Values of T_0 and $\sigma_{y,0}$ come from fits in single differential analysis (Table 4.5 on the previous page). Curves are fits of formulas Eq. (4.41).

dependence may be used to construct $T(y)$ and $\sigma_y(p_T)$ to be utilized in the ϕ production model *ansatz*.

Denoting EPOS-related histograms visualized in Fig. 4.33 as

$$H_T^{\text{MC}}(y) = \left[\frac{T(y)}{T_0} \right]^{\text{MC}} \quad \text{and} \quad H_{\sigma_y}^{\text{MC}}(p_T) = \left[\frac{\sigma_y(p_T)}{\sigma_{y,0}} \right]^{\text{MC}}, \quad (4.39)$$

one obtains the ‘histograms’ *ansatz*:

$$\begin{aligned} h_{\text{histograms}}(y, p_T; T_0, \sigma_{y,0}, H_T^{\text{MC}}(y), H_{\sigma_y}^{\text{MC}}(p_T)) \\ = f(p_T; T_0 \cdot H_T^{\text{MC}}(y)) g(y; \sigma_{y,0} \cdot H_{\sigma_y}^{\text{MC}}(p_T)). \end{aligned} \quad (4.40)$$

It is built in the course of the averaging algorithm from histograms $H_T^{\text{MC}}(y)$ and $H_{\sigma_y}^{\text{MC}}(p_T)$ derived from analysis of MC generated spectra, while T_0 and $\sigma_{y,0}$ parameters come from fits to data in the first two steps of the algorithm.

Instead of using directly $H_T^{\text{MC}}(y)$ and $H_{\sigma_y}^{\text{MC}}(p_T)$, one can also try to parametrize them to get continuous $T(y)$ and $\sigma_y(p_T)$. Formulas that give satisfactory fit quality are

$$f_T(y; a, b, c) = a - b \cdot y^c \quad \text{and} \quad f_{\sigma_y}(p_T; d, e) = d - e \cdot p_T. \quad (4.41)$$

They are depicted as solid curves in Fig. 4.33. Employing them, one gets the

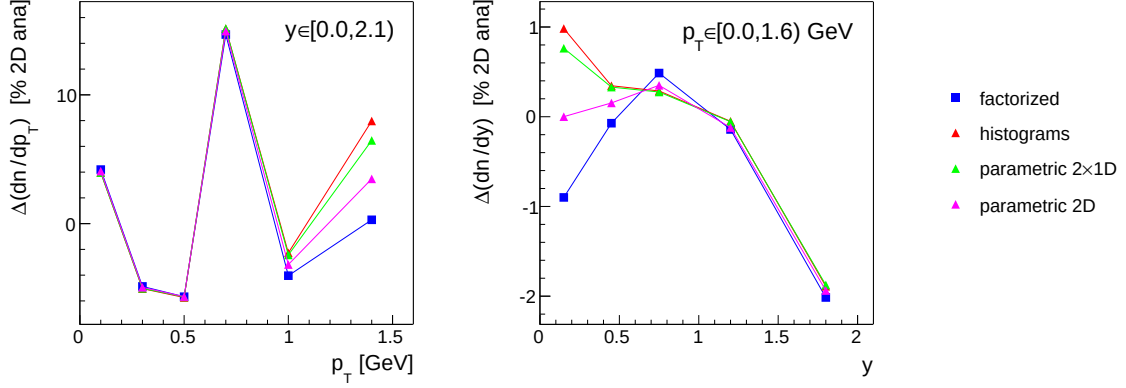


Figure 4.34: Bias of single differential spectra from a single differential analysis of 158 GeV data, with respect to those obtained by summation of results of the double differential analysis. Four kinds of ϕ production model *ansatz*, employed in correction averaging, are compared. No extrapolations to unmeasured parts of phase space were done, so the reference rapidity spectrum is not exactly the same as the one in Fig. 5.3a on page 120.

‘parametric’ *ansatz*:

$$h_{\text{parametric}}(y, p_T; T_0, \sigma_{y,0}, a, b, c, d, e) = f(p_T; T_0 \cdot f_T(y; a, b, c)) g(y; \sigma_{y,0} \cdot f_{\sigma_y}(p_T; d, e)). \quad (4.42)$$

In case of fitting $H_T^{\text{MC}}(y)$ and $H_{\sigma_y}^{\text{MC}}(p_T)$ histograms, it is called ‘parametric 2×1D’.

Yet further improvement might be done if instead of fitting histograms $H_T^{\text{MC}}(y)$ and $H_{\sigma_y}^{\text{MC}}(p_T)$, parameters a, b, c, d, e in Eq. (4.42) are obtained by fitting the generated 2D MC spectrum with another ‘parametric’ model, where parameters T_0 and $\sigma_{y,0}$ are calculated earlier (these are denominators in Eq. (4.39) on the previous page) and are not varied in the fit. A ‘parametric’ *ansatz* in which parameters a, b, c, d, e are computed in such an alternative way is called ‘parametric 2D’.

In order to decide which of four proposed kinds of ϕ production model *ansatz* should be used in the analysis of 40 GeV data, a validation method of the averaging procedure was devised. The validation is done against results of the double differential analysis of 158 GeV data. Single differential spectra are computed by summation of double differential ones, without any extrapolations to unmeasured regions of phase space. Then a single differential analysis of 158 GeV data is performed for each tested *ansatz*. Resulting spectra are compared in Fig. 4.34 to the reference ones from double differential analysis.

It is evident, that for some bins, especially in the transverse momentum spectrum, all variants of single differential analysis give similar results, significantly different from the reference ones. This effect is attributed to PID efficiency for kaons which is incorrectly estimated in the tag-and-probe method. That happens because, especially for the 1D p_T binning, all possible kaon total momenta get mixed within each analysis bin. It should be noted, that the size of the observed bias is in agreement with values estimated for the tag-and-probe systematic uncertainty.

These results could be compared to bias obtained in Fig. 4.35a on the next page for each *ansatz* variant within the toy MC study described below. It is striking, that differences between these variants are almost equal between the two studies (vertical displacement of points in Fig. 4.34 on the preceding page is very similar to that in Fig. 4.35a on the next page). From this comparison it is concluded, that

1. the toy MC study is sound, and
2. the ‘parametric 2D’ *ansatz* should be chosen to improve the averaging procedure with respect to the NA49 one.

Toy MC study

In order to validate the correction averaging method and to estimate the possible bias caused by a wrong assumption regarding the ϕ production model, a toy Monte Carlo study of the averaging procedure has been implemented.

It is performed separately for each considered energy, within the same binnings as those used in the analysis. The continuous 2D efficiency is obtained by interpolating the actual 2D correction for the given data set and inverting the value. Two toy models are employed: ‘data model’ to mock the experimental data and ‘MC model’ to mock the MC generated data. Both models are of the ‘parametric’ *ansatz* form. They share a, b, c, d, e parameters, which are fitted to the actual MC generated data. From that fit, also T_0^{MC} and $\sigma_{y,0}^{\text{MC}}$ parameters are derived completing the parameter list of the ‘MC model’. Moreover, the actual experimental data are utilized to determine T_0^{data} and $\sigma_{y,0}^{\text{data}}$ parameters to be fed into the ‘data model’ along with the common a, b, c, d, e parameters.

Once both models are prepared, large random samples are drawn from them. The one built with the ‘MC model’ is used to calculate the 2D correction to be averaged. It also serves as the MC generated spectrum for the purpose of computation of $H_T^{\text{MC}}(y)$ and $H_{\sigma_y}^{\text{MC}}(p_T)$ histograms, as well as a, b, c, d, e parameters when correction averaging procedures are exercised. The sample drawn from the ‘data model’ provides the reference ‘generated’

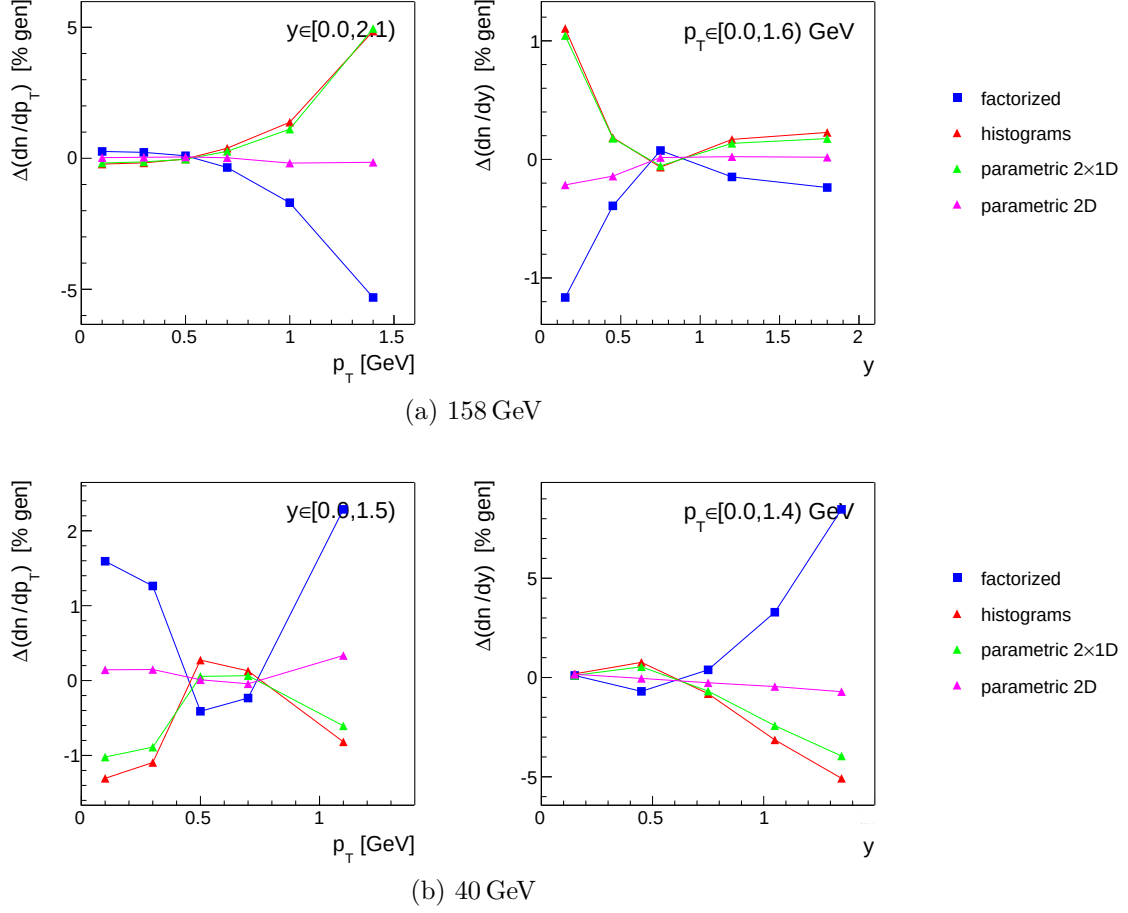


Figure 4.35: Bias of single differential spectra reconstructed within the toy MC study for (a) 158 GeV and (b) 40 GeV, with respect to generated spectra. Four kinds of ϕ production model *ansatz*, employed in correction averaging, are compared.

spectra denoted as ‘gen’, as well as the spectra ‘selected’ with the continuous 2D efficiency. They play a role of measured raw spectra, which constitute an input for the averaging procedure together with the 2D correction calculated with the ‘MC model’ sample.

Having the samples, correction averaging procedures can be exercised for all studied kinds of ϕ production model *ansatz*, to produce corrected single differential spectra. These are compared to the ‘generated’ ones in Fig. 4.35 for 158 GeV and 40 GeV data. Looking at these pictures, it seems that the employed correction averaging method is valid — there are no huge biases that could indicate errors in the implementation. Obviously there are differences between various kinds of ϕ production model *ansatz*. What is surprising, is that the ‘histograms’ and ‘parametric 2×1D’ cases give bias of

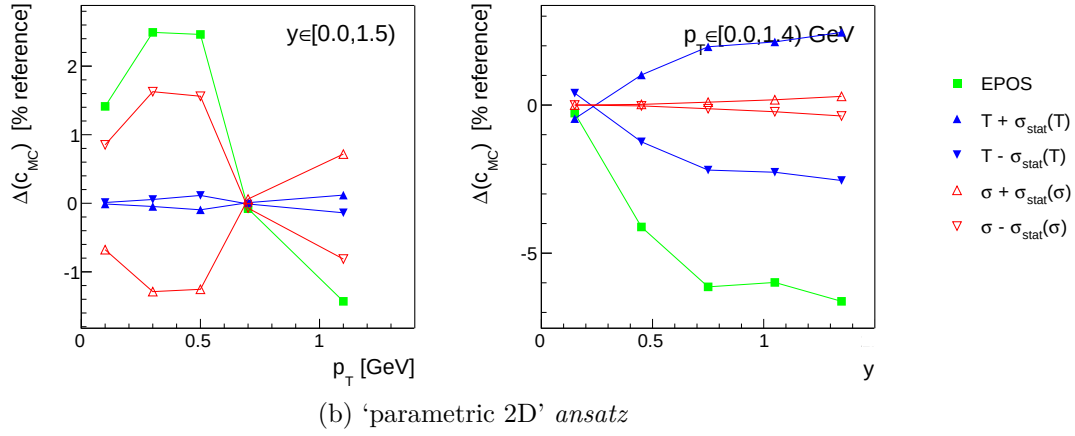
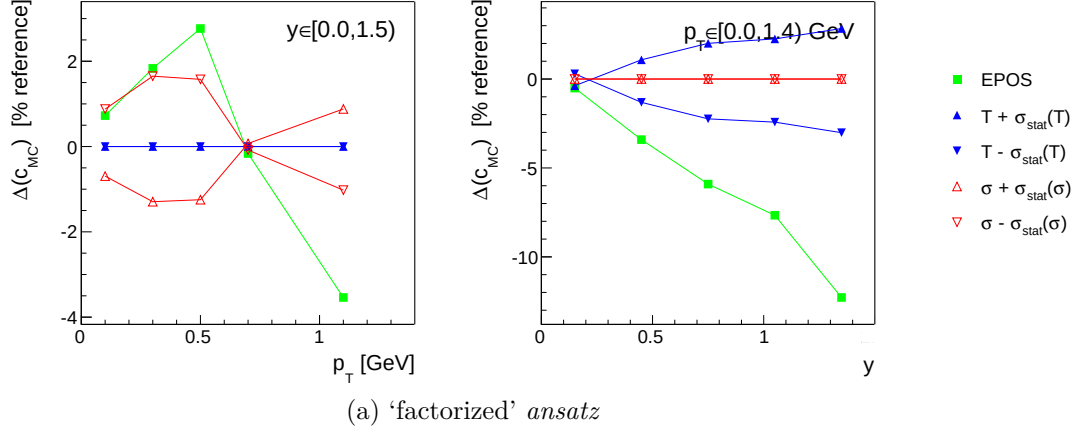


Figure 4.36: Changes of 1D MC corrections for 40 GeV data, obtained with alternative settings for the model used in averaging, relative to the reference ones coming from averaging with an iteratively optimized (a) ‘factorized’ and (b) ‘parametric 2D’ model. ‘EPOS’ denotes a case in which c_{MC} is directly obtained in 1D binning, what corresponds to averaging with the model residing in EPOS.

the same size, but opposite sign as the ‘factorized’ one. Only the ‘parametric 2D’ *ansatz* seems to be a clear improvement. What is more, the size of bias and its bin-dependence for the ‘factorized’ *ansatz* for 40 GeV data indicates, that indeed it is worth to make the improvement.

Systematic uncertainty due to averaging

Results of the toy MC study already suggest, that the reasonable estimate of the systematic uncertainty due to correction averaging in single differential analysis, is of the order of few percent.

To take into account uncertainties of spectral parameters fitted in the first two steps of the averaging algorithm, 1D corrections are recomputed with ϕ production model in which these parameters are shifted up or down by their fit uncertainties. This is intended to be a repetition of the NA49 procedure of Ref. [8], which states that the uncertainty was calculated by *determining the range of acceptance values allowed by the errors in T and σ_y* .

Outcome of this exercise is shown in Fig. 4.36 on the previous page for the 40 GeV data. It is presented as changes of 1D corrections, induced by shifts of spectral parameters, relative to the reference ones coming from the ordinary averaging process. Two kinds of ϕ production model *ansatz* are compared: the initially used ‘factorized’ *ansatz* and the improved one, the ‘parametric 2D’ *ansatz*. It is visible, that relative differences of 1D corrections are not sensitive to a change of the *ansatz*.

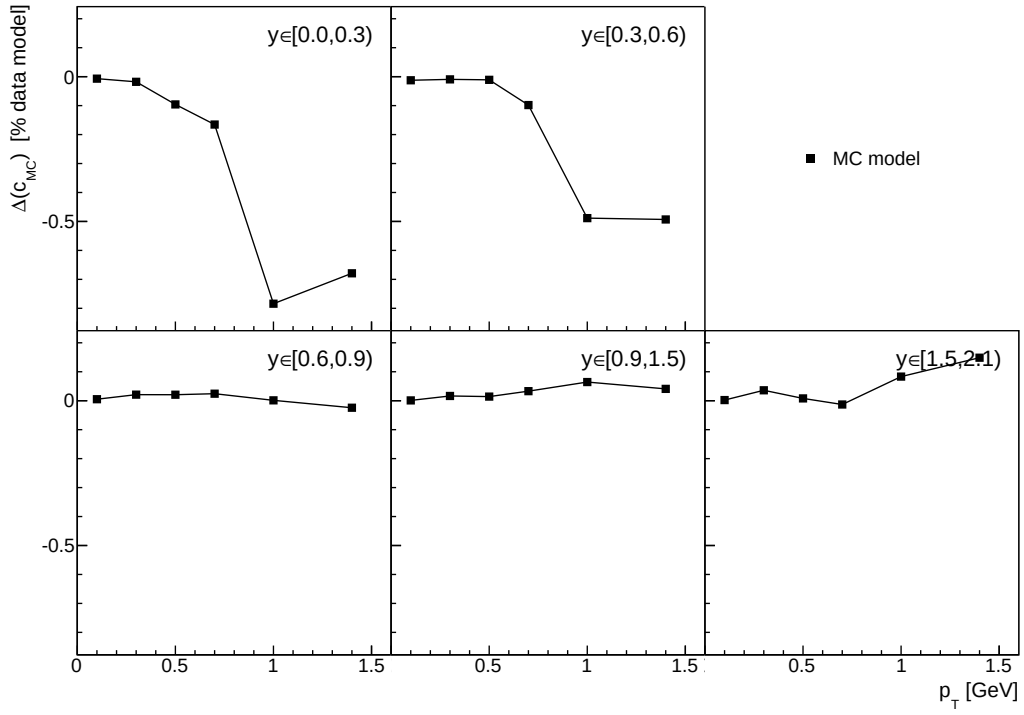
As was already mentioned, Fig. 4.36 illustrates also a realistic worst-case scenario associated with the model choice: when c_{MC} is directly obtained in 1D binning, what corresponds to averaging with the model residing in EPOS. Comparing both panels of the figure, one may notice, that the relative difference is slightly smaller (closer to one) for the improved *ansatz*. That is expected, because the improvement uses the spectrum generated by EPOS. Still, even in that case, a sizeable difference (similar in magnitude to the toy MC bias for the ‘factorized’ *ansatz*) occurs. It means that indeed the discussed, involved procedure of averaging is necessary.

Comparing Fig. 4.36 and Fig. 4.35b on page 106, it seems reasonable to assign a *bin-independent* systematic uncertainty of 3 % for 40 GeV single differential analysis, due to Monte Carlo correction averaging.

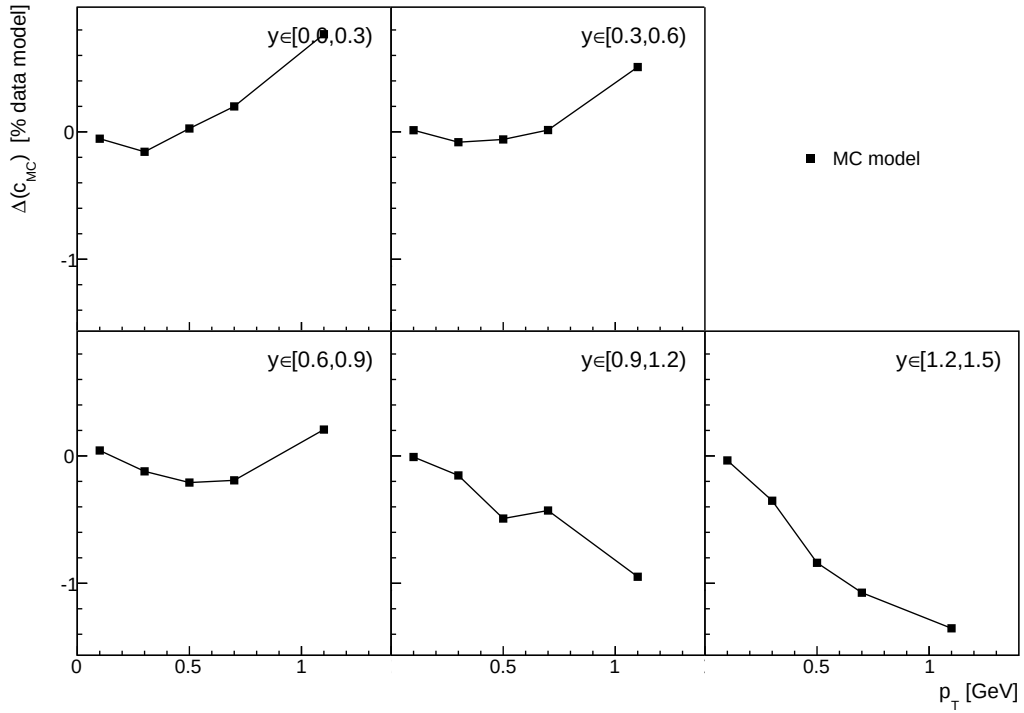
Bias of 2D c_{MC} due to Epos ϕ production model

Finally, as a side effect of the toy MC study, it is possible to make an estimate of the bias of the 2D correction due to an improper ϕ production model in EPOS. To achieve this, the correction calculated with the ‘data model’, which mocks the true correction, is compared to the one derived from the ‘MC model’, which mocks EPOS. Results of the comparison are presented in Fig. 4.37 on the next page for 158 GeV and 40 GeV data, in terms of differences between the wrong (‘MC model’) and the proper (‘data model’) corrections given as percentage of the proper one.

It is visible, that the bias is negligible as expected for 158 GeV, because the correction does not vary much inside a 2D bin (see Fig. 4.14a on page 72). The effect is larger for 40 GeV, because in that case the correction is much larger and also varies a lot (see Fig. 4.16a on page 74). But still the bias is negligible. For 80 GeV the magnitude of the effect is between the two



(a) 158 GeV



(b) 40 GeV

Figure 4.37: A toy MC estimate of the bias of the 2D correction due to an improper ϕ production model in EPOS for 158 GeV and 40 GeV data. For 80 GeV the magnitude of the effect is between the two shown cases. See the text for definitions of the ‘data model’ and the ‘MC model’.

Table 4.6: Bin-independent systematic uncertainties. ‘Total’ is calculated by adding contributions in quadrature.

Source	uncertainty value [%]		
	158 GeV	80 GeV	40 GeV
branching ratio	1	1	1
fitting constraints	2	3	4
theory	3	3	3
background	5	5	5
correction averaging	—	—	3
Total	6	7	8

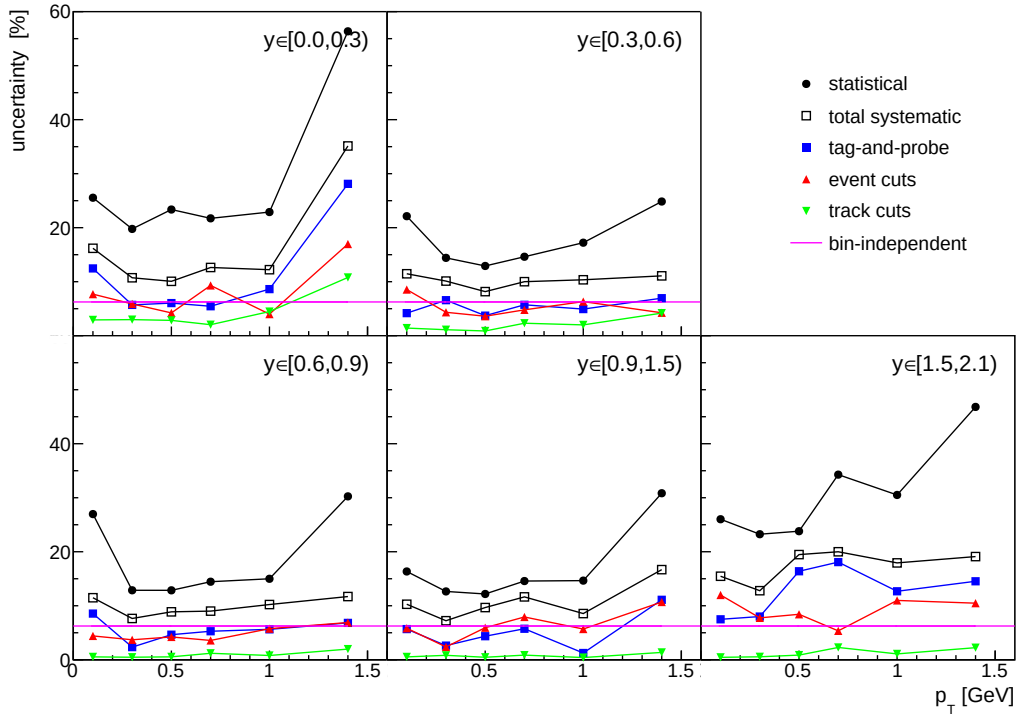
presented cases and therefore respective pictures are not shown for brevity.

4.6.8 Summary of uncertainties

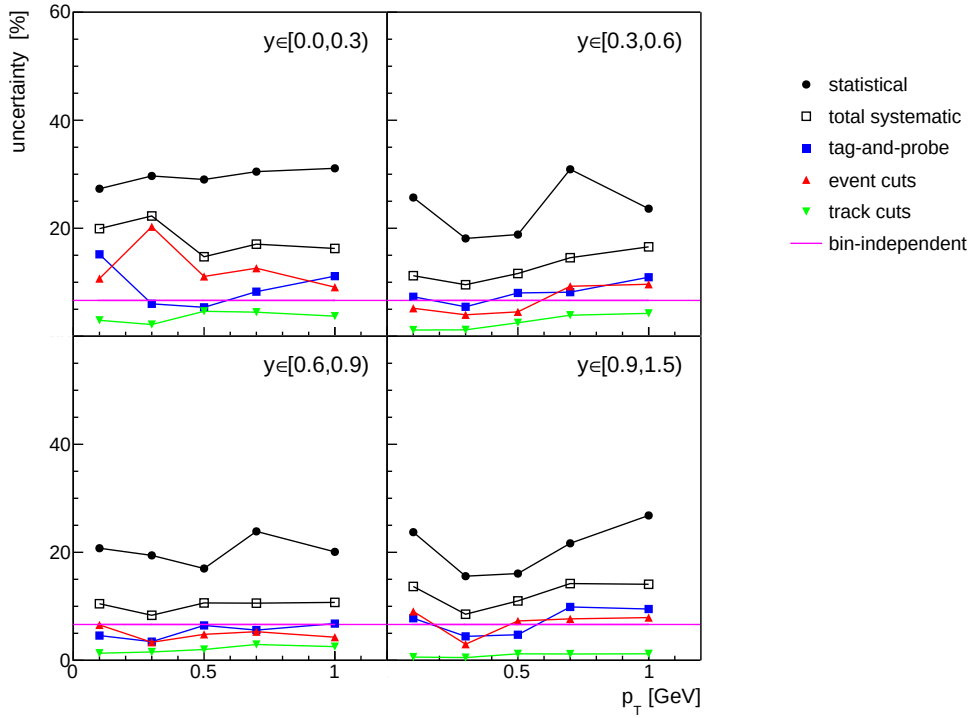
Figures 4.38 and 4.39 on the following page and on page 112 present statistical and systematic uncertainties for all analysed energies. Components of the bin-independent contribution to the systematic uncertainty are listed in Table 4.6.

It is evident, that the statistical uncertainty dominates in all bins. The total systematic uncertainty is calculated by adding contributions in quadrature. Because of that, effectively only the highest uncertainties have visible impact on the total systematic uncertainty. These are the tag-and-probe contribution (Section 4.6.5), event cuts-related uncertainty (Section 4.6.6) and the one associated with unaccounted-for issues in the background description (Section 4.6.1), which constitutes the major part of the bin-independent uncertainty. In several bins also the track cuts-related one (Section 4.6.6) is comparable to the leading ones. Conversely, the uncertainties due to constraints on signal shape parameters (Section 4.6.4) and the one related to the resonance theory (Section 4.6.3) are always insignificant in comparison to the others. Similar is the uncertainty due to Monte Carlo correction averaging in a single differential analysis (Section 4.6.7). The uncertainty of the branching ratio (Table 1.1 on page 11) taken from PDG [7] is completely negligible.

One possible bias/systematic uncertainty stays unfortunately unaccounted for — the effect of kaon decays in the detector. Kaons decay mostly into muons. The Monte Carlo correction takes into account only kaon-matched tracks, so it ignores contributions from decay products of kaons. These are



(a) 158 GeV



(b) 80 GeV

Figure 4.38: Comparison of statistical and systematic uncertainties for the double differential analysis of 158 GeV and 80 GeV data. Total systematic uncertainty is calculated by adding contributions in quadrature.

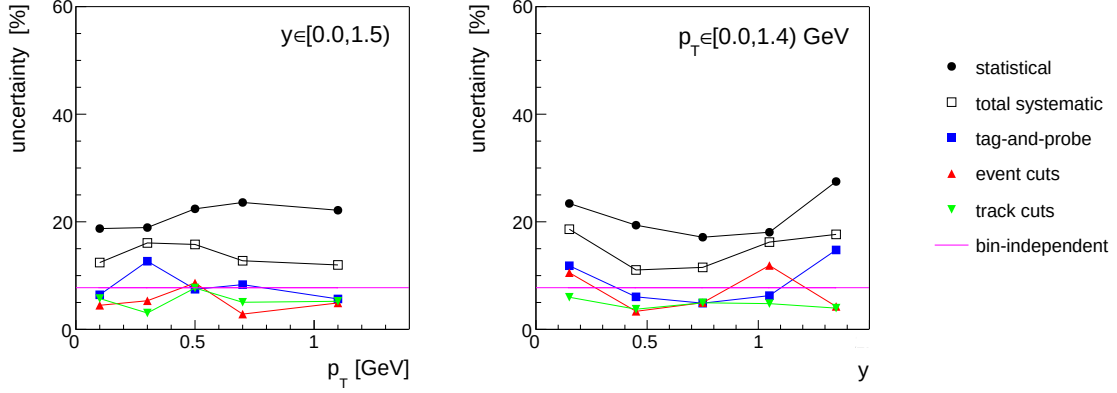


Figure 4.39: Comparison of statistical and systematic uncertainties for the single differential analysis of 40 GeV data. Total systematic uncertainty is calculated by adding contributions in quadrature.

partially taken into account in the data, if the muon has similar momentum to the kaon and therefore correlation associated with ϕ is not dissolved. Depending on the dE/dx value such tracks contribute to the probe sample (if dE/dx is kaon-like) or appear in the tag sample and are treated as PID inefficiency. So the MC correction overestimates the losses due to the registration effect (Section 4.5.1).

Some effort to estimate the influence of kaon decays was undertaken in Ref. [51], but the reasoning there does not match the case of tag-and-probe analysis. Based on that source, the bias could be of the order of several percent, but it is hard to judge how this could be extrapolated to tag-and-probe analysis. Still, if the estimate was not much higher, then the effect would be anyway smaller than the accuracy of this analysis. To correctly take the issue into account, would require proper simulation of dE/dx and performing full tag-and-probe fitting or at least counting of reconstructed selected ϕ mesons based on a dE/dx cut in Monte Carlo, not on the matching to kaons. This is something to be considered in future, more accurate analyses of higher-statistics data sets.

Chapter 5

Results and their discussion

After describing analysis methods, it is finally possible to present and discuss the results. It should be repeated that all elements of reasoning described in Chapter 4, although mostly illustrated with examples for 158 GeV data, are valid for all energies. This is why for lower energies only final results are shown.

This chapter starts with some more methodology-related issues in Section 5.1 stating how from primary results of the analysis, some further ones are derived. The reason why that is not a part of Chapter 4, is because that chapter is focused on primary results, while the derived ones belong rather to an ‘interpretation’ part, *i.e.* here. Because double and single differential analyses differ in primary results, they are presented separately, respectively in Sections 5.2 and 5.3. Discussion of derived results as reference for Pb + Pb data is done in Section 5.4, while comparison to world data and models is given in Section 5.5.

5.1 Methods for derived results

5.1.1 Primary vs derived results

Primary results are these which require experimental data and related Monte Carlo to be calculated. In the context of this work, they are double or single differential spectra of ϕ mesons. *Derived results* in contrast need only the primary ones as prerequisites. These are spectral parameters, single differential spectra from summation and extrapolation of double differential ones and the total yield. Therefore, they are more an interpretation, which could be done by anyone having just the tabularized primary results.

5.1.2 Spectral functions and parameters

Transverse momentum spectra are fitted with a thermally motivated formula

$$f(p_T) = \frac{A_f}{T(T + m_{T,\max}) \exp\left(-\frac{m_{T,\max}}{T}\right)} \times p_T \exp\left(-\frac{m_T}{T}\right), \quad (5.1)$$

where $m_{T,\max}$ is a transverse mass calculated for $p_{T,\max}$ — maximum transverse momentum covered by the analysis binning.

The purpose of this is twofold. On one hand, it gives the effective temperature parameter T , interesting for phenomenology of collisions. On the other, it allows for extrapolation to unmeasured, large transverse momenta, *i.e.* beyond $p_{T,\max}$. Normalization factor in the denominator assures, that A_f parameter gives the integral of the unmeasured tail:

$$\int_{p_{T,\max}}^{+\infty} f(p_T) dp_T = A_f. \quad (5.2)$$

That will prove necessary for obtaining rapidity spectra. (Section 5.1.3).

For analogous reasons rapidity spectra are fitted with a Gaussian

$$g(y) = \frac{A_g}{\sqrt{\frac{\pi}{2}}\sigma_y \left[1 - \operatorname{erf}\left(\frac{y_{\max}}{\sqrt{2}\sigma_y}\right)\right]} \times \exp\left(-\frac{y^2}{2\sigma_y^2}\right), \quad (5.3)$$

where $y_{\max}$ is the maximum rapidity covered by the analysis binning. Again σ_y , the width parameter, is interesting for phenomenology, while A_g parameter is the integral of the unmeasured tail:

$$\int_{y_{\max}}^{+\infty} g(y) dy = A_g. \quad (5.4)$$

The latter is needed for determination of the total yield. The fits are also repeated with a differently normalized Gaussian

$$g'(y) = \frac{dn}{dy}(y=0) \times \exp\left(-\frac{y^2}{2\sigma_y^2}\right), \quad (5.5)$$

to obtain midrapidity yields $\frac{dn}{dy}(y=0)$, where the width parameter is exactly the same as for $g(y)$.

Additionally, rapidity spectra are fitted with a sum of two identical Gaussian functions of width σ_0 symmetrically displaced by y_0 with respect to $y = 0$:

$$g_2(y) = A \times \left[\exp\left(-\frac{(y - y_0)^2}{2\sigma_0^2}\right) + \exp\left(-\frac{(y + y_0)^2}{2\sigma_0^2}\right) \right]. \quad (5.6)$$

It is done to maintain consistency with other analyses of similar data sets. For example in charged hadrons analyses of the same data as analysed here, such double Gaussian is said to give better description than a single Gaussian [30, 70]. However, in ϕ analysis of lead-lead collisions in NA49, it is not concluded which of the two alternatives better fits the data [8].

From y_0 and σ_0 parameters above, it is possible to compute double Gaussian's width parameter

$$\sigma'_y = \sqrt{y_0^2 + \sigma_0^2}, \quad (5.7)$$

which might be compared to σ_y for a single Gaussian.

Finally, transverse mass spectra, similarly to transverse momentum spectra, are fitted with a thermally motivated formula

$$\frac{d^2n}{m_T dm_T dy} \propto \exp\left(-\frac{m_T}{T}\right), \quad (5.8)$$

to get the effective temperature parameter T .

All these functions are fitted by means of least-squares fits [57]. In χ^2 calculation, the spectrum value for a bin is compared with the integral of the fitted function within that bin, divided by the bin width.¹ Only statistical uncertainties of bin contents are used.

The above procedure yields spectral parameters with statistical uncertainties as given by HESSE algorithm [62] of MINUIT2 [60]. To obtain their systematic uncertainties, a method from Ref. [8] was adapted:

1. Spectrum with statistical and systematic uncertainties summed is built following

$$\sigma_{\text{tot}, i} = \sigma_{\text{stat}, i} + \sigma_{\text{syst}, i}, \quad (5.9)$$

where $\sigma_{\text{stat}, i}$ denotes the statistical uncertainty associated with bin i of the considered spectrum, $\sigma_{\text{syst}, i}$ the systematic one and $\sigma_{\text{tot}, i}$ the total uncertainty assigned to the bin.

2. It is fitted according to above prescriptions for determination of spectral parameters with statistical uncertainties, yielding a 'total uncertainty' $\sigma_{\text{tot}}(\alpha)$ of a spectral parameter α .
3. Systematic uncertainty of a spectral parameter α is calculated as

$$\sigma_{\text{syst}}(\alpha) = \sigma_{\text{tot}}(\alpha) - \sigma_{\text{stat}}(\alpha), \quad (5.10)$$

where $\sigma_{\text{stat}}(\alpha)$ is a statistical uncertainty of the parameter α computed earlier in a fit which takes into account only statistical uncertainties in the spectrum.

¹The 'I' option of the `TH1::Fit()` method of the ROOT framework

5.1.3 Summation and extrapolation of spectra

To get a rapidity spectrum from a double differential spectrum of p_T and y , it is necessary to integrate the double differential spectrum over p_T for each y bin and add an estimated contribution from the unmeasured tail. Similarly, to determine the total ϕ yield $\langle\phi\rangle$, a rapidity spectrum needs to be integrated and extrapolated.

This is done by means of summation of the measured spectrum and use of fitted extrapolation parameters:

$$Y = Y_{\text{meas}} + \frac{A_\beta}{I_\beta} Y_{\text{meas}} , \quad (5.11)$$

with

$$Y_{\text{meas}} = \sum_i y_i \Delta x_i \quad \text{and} \quad I_\beta = \int_0^{x_{\text{max}}} \beta(x) dx , \quad (5.12)$$

where x stands for p_T or y , summation of spectrum values y_i multiplied by bin widths Δx_i runs over all measured x bins and the function $\beta(x)$ is one of functions $f(p_T)$ or $g(y)$ (Eqs. (5.1) and (5.3) on page 114) with respective tail integral parameter A_β .

Extrapolation contribution to the statistical uncertainty of the integrated yield stems from A_β parameter's fit uncertainty $\sigma_{\text{stat}}(A_\beta)$:

$$\sigma_{\text{stat}}(Y) = \sqrt{\sum_i (\sigma_{\text{stat}, i} \Delta x_i)^2 + \left(\frac{\sigma_{\text{stat}}(A_\beta)}{I_\beta} Y_{\text{meas}} \right)^2} , \quad (5.13)$$

while contribution to the systematic uncertainty is taken as half of the fitted tail integral to take into account arbitrariness of choice of the $\beta(x)$ function:

$$\sigma_{\text{syst}}(Y) = \sqrt{\sum_i (\sigma_{\text{syst}, i} \Delta x_i)^2 + \left(\frac{A_\beta/2}{I_\beta} Y_{\text{meas}} \right)^2} . \quad (5.14)$$

Above, bin-by-bin uncertainties $\sigma_{\text{stat}, i}$ and $\sigma_{\text{syst}, i}$ of the spectrum are denoted the same as in Eq. (5.9) on the previous page.

Additionally, only for calculation of the total ϕ yield, the results of above formulas are doubled. This is because measurements are done only for non-negative values of rapidity and from symmetry of proton-proton reactions, the rapidity spectrum is symmetric with respect to $y = 0$.

5.2 Double differential analysis

Double differential analysis was performed for 158 GeV and 80 GeV data. It should be emphasized, that this is the first double differential analysis of ϕ

production in proton-proton collisions at CERN SPS energies. These are at the same time the first ϕ production results ever for 80 GeV.

5.2.1 Analysis binnings

The only parameter of the analysis that was not discussed until now because of its strict dependence on the considered data sets, is the choice of analysis binnings.

The 2D binning chosen for 158 GeV data is motivated by the 1D binnings in p_T and laboratory rapidity y_{lab} used by NA49 in Ref. [15]. No (re)optimization attempt was undertaken, only bin edges were adapted to the centre-of-mass rapidity and one more bin in both dimensions was added.

In case of 80 GeV, almost the same binning was used — only the last p_T and the last y bin of 158 GeV analysis binning were removed due to insufficient statistics.

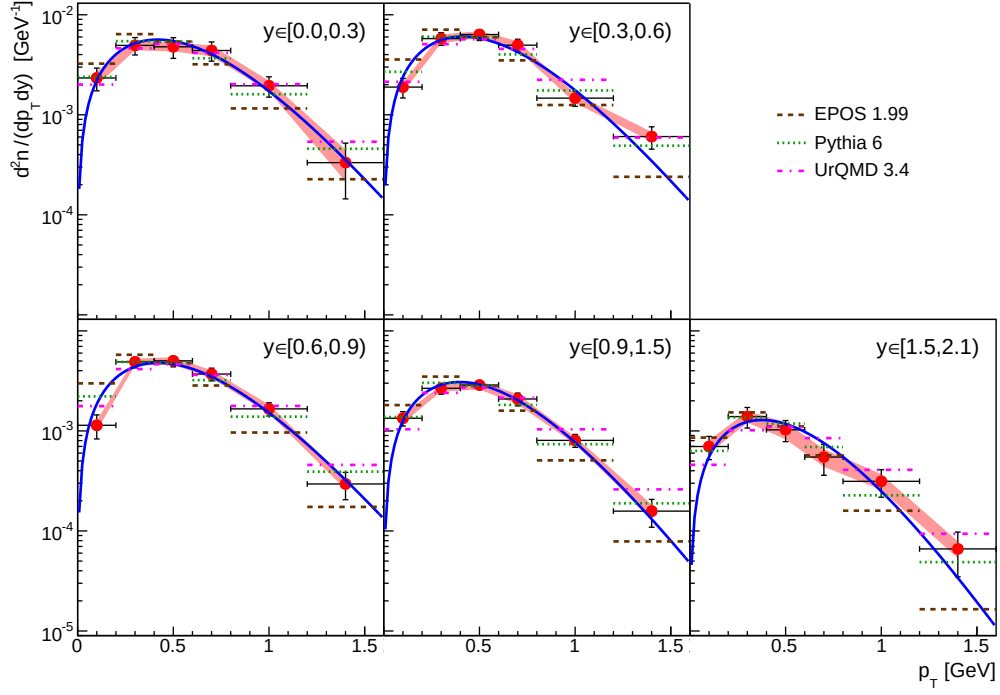
5.2.2 Double differential spectra

Double differential spectra of ϕ mesons are presented in Fig. 5.1 on the following page for both energies as transverse momentum spectra for each rapidity bin.

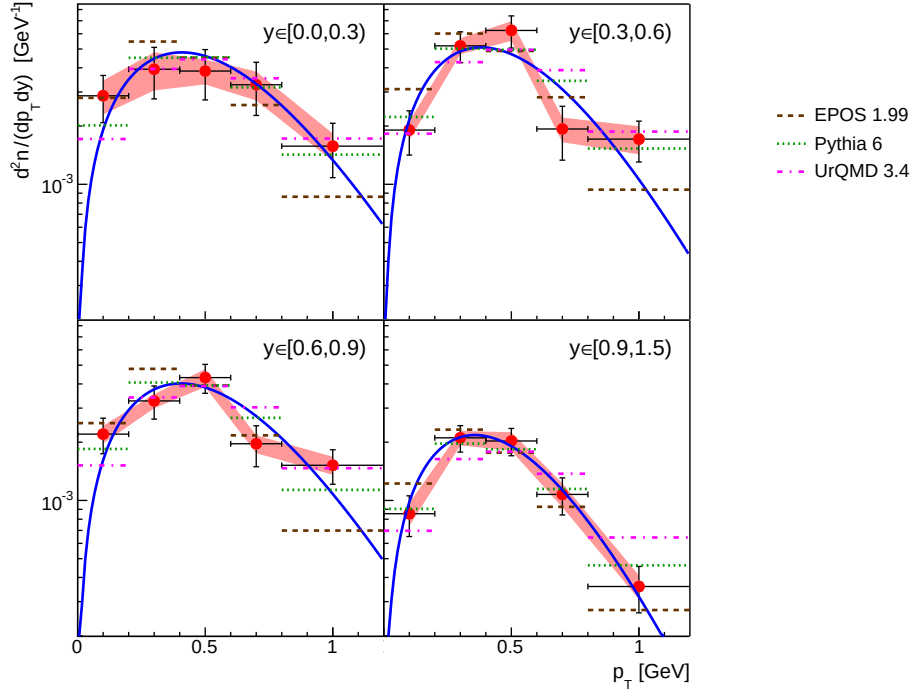
Thermally motivated functions $f(p_T)$ are fitted to estimate the unmeasured tail contributions to rapidity spectra. For 158 GeV these contributions are below 1 % for all rapidity bins, while for 80 GeV they are of the order of 1 % to 4 %. It is visible that $f(p_T)$ describes well the spectra for all rapidity bins.

From fits of $f(p_T)$ also effective temperature parameters are derived as a function of rapidity. Furthermore, if double differential spectra are drawn as rapidity spectra for each transverse momentum bin, they can be fitted with Gaussian functions $g(y)$. From these fits, in turn, Gaussian widths are obtained as a function of transverse momentum. Resulting dependence of spectral parameters on y and p_T is presented for both energies in Fig. 5.2 on page 119.

At least for 158 GeV, it seems that indeed there is some systematic behaviour, namely that both T and σ_y decrease with respectively y and p_T . It is in agreement with rapidity dependence of T parameter for pions in the same reaction (Fig. 19 of Ref. [30]). It also shows, that the NA49 assumption of T and σ_y being constant in phase space, used for correction averaging (Section 4.6.7), is incorrect. Unfortunately, there are no comparative results on p_T dependence of σ_y . The accuracy of spectral parameters systematics for

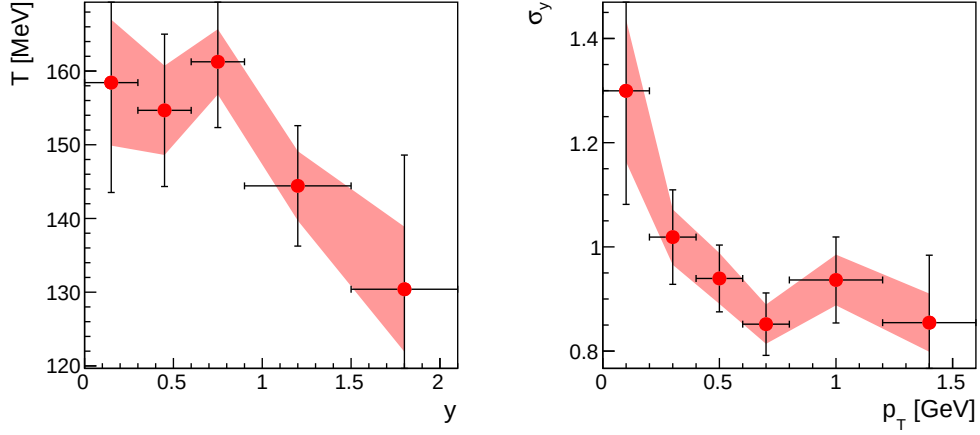


(a) 158 GeV

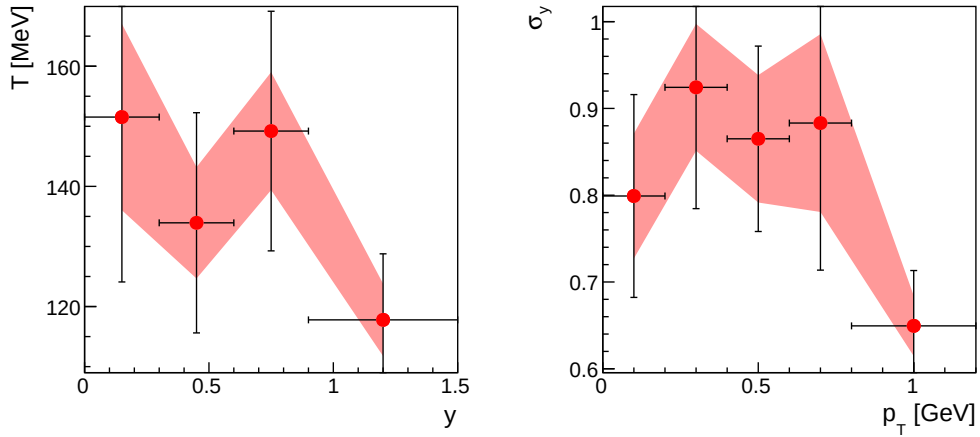


(b) 80 GeV

Figure 5.1: Transverse momentum spectra in rapidity bins for 158 GeV and 80 GeV data with statistical (vertical lines) and systematic (red bands) uncertainties. Horizontal lines give p_T bin sizes. Curves are fits of function Eq. (5.1) on page 114. Regarding models see Section 5.5.2.



(a) 158 GeV



(b) 80 GeV

Figure 5.2: Dependence of spectral parameters on the rapidity and transverse momentum for 158 GeV and 80 GeV data with statistical (vertical lines) and systematic (red bands) uncertainties. Horizontal lines illustrate y and p_T binning.

80 GeV is insufficient to draw any strong conclusions, but it looks like similar trends develop, as those at the top energy.

5.2.3 Rapidity spectra

From summation and extrapolation of double differential spectra according to methods introduced in Section 5.1.3, rapidity spectra arise. They are shown in Fig. 5.3 on the following page for both energies, along with single

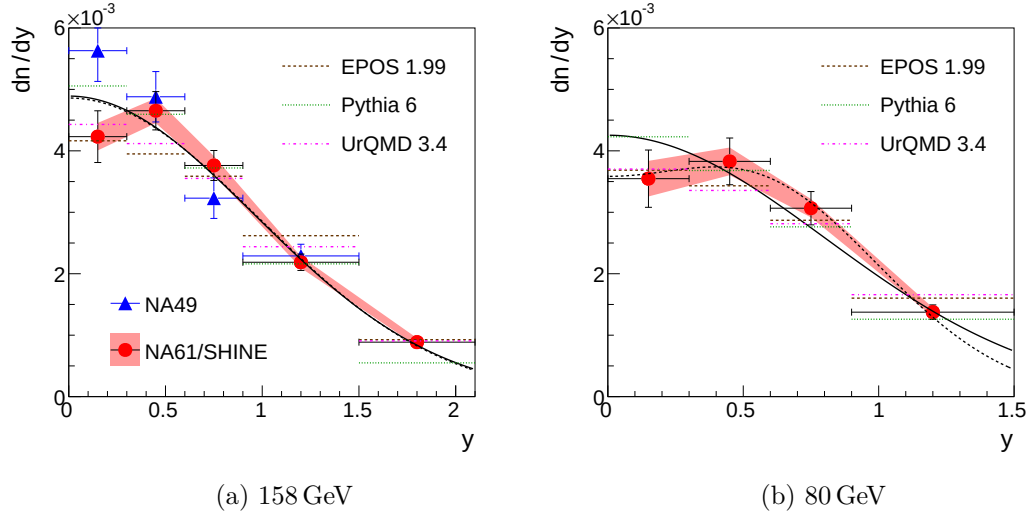


Figure 5.3: Rapidity spectra for 158 GeV and 80 GeV data with statistical (vertical lines) and systematic (red bands) uncertainties. Horizontal lines give y bin sizes. NA49 points come from Ref. [15]. Solid curves are Gaussian fits (Eq. (5.3) on page 114), while dashed ones are double Gaussian fits (Eq. (5.6) on page 114). Regarding models see Section 5.5.2.

and double Gaussian fits. Parameters derived from these fits are listed in Table 5.1 on the next page.

One can see, that single Gaussian describes well the spectra. Indeed, for 158 GeV no matter what starting parameters are chosen, double Gaussian always converges to a single Gaussian. For 80 GeV, on the other hand, the double Gaussian better follows the points. However, there are only four of them and it may well be accidental. Especially that for an even lower energy, although in a single differential analysis, again there is no significant difference between single and double Gaussian (Fig. 5.6b on page 124). Therefore, it is concluded that contrary to charged hadrons spectra in the same reactions [30, 70], there is no systematic indication that the double Gaussian is a better model of a rapidity spectrum than a single Gaussian. Consequently, double Gaussian's fitted parameters do not enter further discussions.

For 158 GeV comparative results of a single differential NA49 analysis exist [15]. They are presented in Fig. 5.3a as blue triangles. No systematic uncertainties for them are quoted in Ref. [15]. An overall consistency is seen, although the NA49 spectrum is expected to be overestimated due to lack of the trigger bias correction and probable lack of the correction for losses because of vertex cuts. That expectation is satisfied for all but the third point. What might seem surprising, is that statistical uncertainties are comparable, despite much less events collected by NA49 (event numbers suggest

Table 5.1: Parameters deduced from rapidity distributions for all analysed beam momenta (see Fig. 5.6b on page 124 for 40 GeV spectrum). The first uncertainty is statistical, the second one systematic.

p_{beam} [GeV]	σ_y	$\langle\phi\rangle$ [10^{-3}]	$\frac{dn}{dy}(y=0)$ [10^{-3}]
158	$0.958 \pm 0.036 \pm 0.022$	$11.60 \pm 0.41 \pm 0.29$	$4.89 \pm 0.24 \pm 0.14$
80	$0.802 \pm 0.046 \pm 0.028$	$8.43 \pm 0.44 \pm 0.36$	$4.25 \pm 0.32 \pm 0.19$
40	$0.752 \pm 0.064 \pm 0.043$	$5.34 \pm 0.52 \pm 0.39$	$2.87 \pm 0.38 \pm 0.26$

2 to 3 times larger relative uncertainties in the NA49 spectrum than in this analysis). It is attributed to the use of tag-and-probe method, which, due to more parameters, produces larger relative statistical uncertainties, reducing systematic uncertainty associated with identification of kaon candidates.

By summation, extrapolation and doubling of rapidity spectra, total yields $\langle\phi\rangle$ are calculated. They are quoted in Table 5.1. The unmeasured tail contributions to $\langle\phi\rangle$ from single Gaussian $g(y)$ fits are about 3 % for 158 GeV and 7 % for 80 GeV. The total yield corresponding to 158 GeV can be compared to that obtained by NA49: $(12.0 \pm 1.5) \times 10^{-3}$ [15]. It is evident, that the two values agree within uncertainties. However, the NA49 one is more than two times less accurate, mainly because of smaller rapidity coverage and therefore large contribution to the uncertainty from extrapolation to full phase space.

Another comparison with NA49 result is possible for σ_y parameter, estimated in Ref. [15] to be 0.89 ± 0.06 . Again the outcome of this analysis (Table 5.1, 158 GeV entry) and the NA49 one are compatible within uncertainties.

5.2.4 Transverse mass spectra at midrapidity

Figure 5.4 on the next page presents transverse mass spectra at midrapidity for both energies. They are fitted with thermally motivated functions Eq. (5.8) on page 115 to determine effective temperature parameters T , which are quoted in Table 5.2 on the next page.

As one may notice, these spectra are not new, independent results. They are indeed just a different representation of double differential spectra in Fig. 5.1 on page 118, for the first rapidity bin. They are derived in a binning, which gives sound bin edges in $m_T - m_0$ variable, which is not the case for the previously shown double differential spectra of y and p_T . The purpose of including here this alternative representation, is for convenience of those who would wish to compare them with analogous spectra for other hadrons

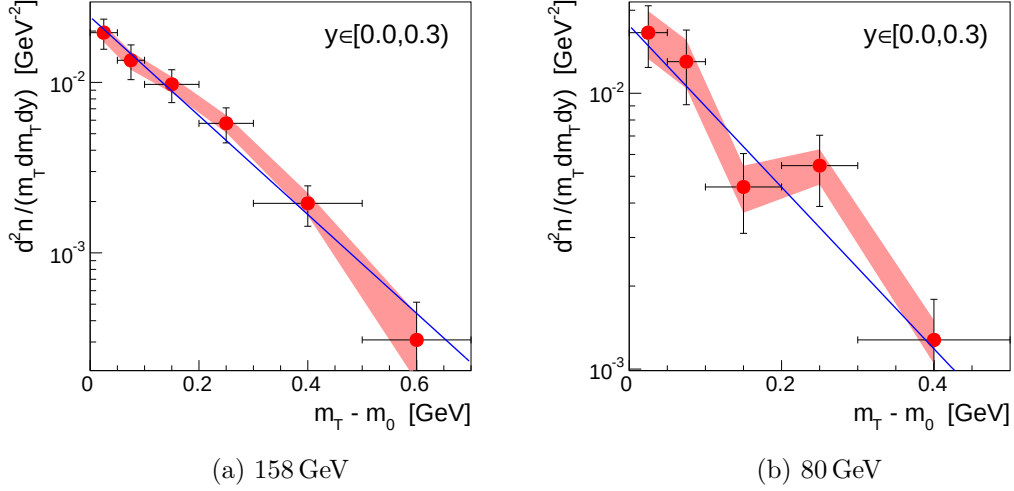


Figure 5.4: Transverse mass spectra at midrapidity for 158 GeV and 80 GeV data with statistical (vertical lines) and systematic (red bands) uncertainties. Horizontal lines give m_T bin sizes. Rest mass m_0 of ϕ is listed in Table 1.1 on page 11. Blue lines are fits of function Eq. (5.8) on page 115.

Table 5.2: Effective temperature parameters from fits of function Eq. (5.8) on page 115 to transverse mass spectra at midrapidity for 158 GeV and 80 GeV data. The first uncertainty is statistical, the second one systematic.

p_{beam} [GeV]	T [MeV]
158	$150 \pm 14 \pm 8$
80	$148 \pm 30 \pm 17$

in the same reactions (like *e.g.* Figure 10.5 in Ref. [70]).

5.2.5 Comparison with NA49 transverse mass spectrum

As was written in Section 5.2.1, the 2D binning for 158 GeV data is motivated, but not exactly the same as the NA49 binning of Ref. [15] (even disregarding that NA49 used two 1D binnings). Therefore, in order to compare results of this analysis to the NA49 transverse mass spectrum, it was necessary to prepare them in an NA49-like 2D binning (obviously again this is not a new, independent result, but rather a different representation of the one in Section 5.2.2). Then they had to be averaged over rapidity to get a transverse mass spectrum in broad rapidity range. The latter is shown in Fig. 5.5

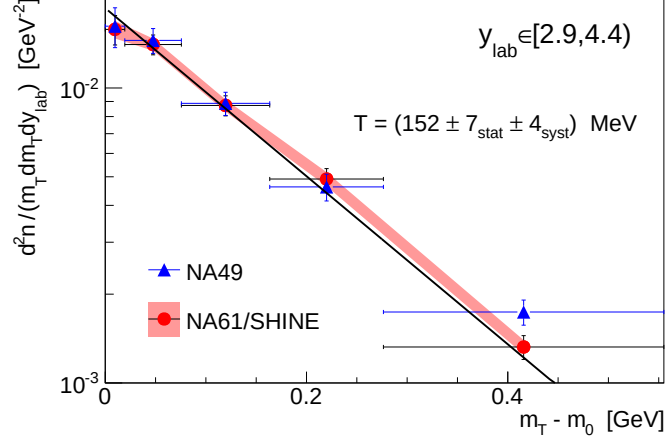


Figure 5.5: Transverse mass spectrum in broad rapidity range for 158 GeV data with with statistical (vertical lines) and systematic (red bands) uncertainties, from averaging of a double differential spectrum in NA49-like binning. Horizontal lines give m_T bin sizes. NA49 points come from Ref. [15]. Black line is a fit of function Eq. (5.8) on page 115 with its effective temperature parameter T value also quoted in the plot.

together with the NA49 m_T spectrum of Ref. [15].

It is visible, that the two spectra are consistent. The largest deviation appears for the highest m_T , but it is only about 2 statistical uncertainties of the NA49 point.

The spectrum coming from this analysis is fitted with a thermally motivated function Eq. (5.8) on page 115 to determine effective temperature parameter T , which is quoted in the plot. The latter can be compared to that obtained by NA49: (169 ± 17) MeV [15]. Again, compatibility is observed, however the level of agreement within uncertainties is lower than in case of the total yield and σ_y parameter (deviation expressed in the number of uncertainties is larger).

5.3 Single differential analysis

Single differential analysis was done for 40 GeV data due to insufficient statistics to perform a double differential analysis. It needs to be emphasized, that these are the first ϕ production results at this energy.

Both 1D binnings, in transverse momentum and in rapidity, are motivated by 2D binnings utilized in double differential analyses of higher energies. Only slight changes in high p_T and high y bins were made to adapt to available statistics. Spectra of p_T and y presented in Fig. 5.6 on the following page

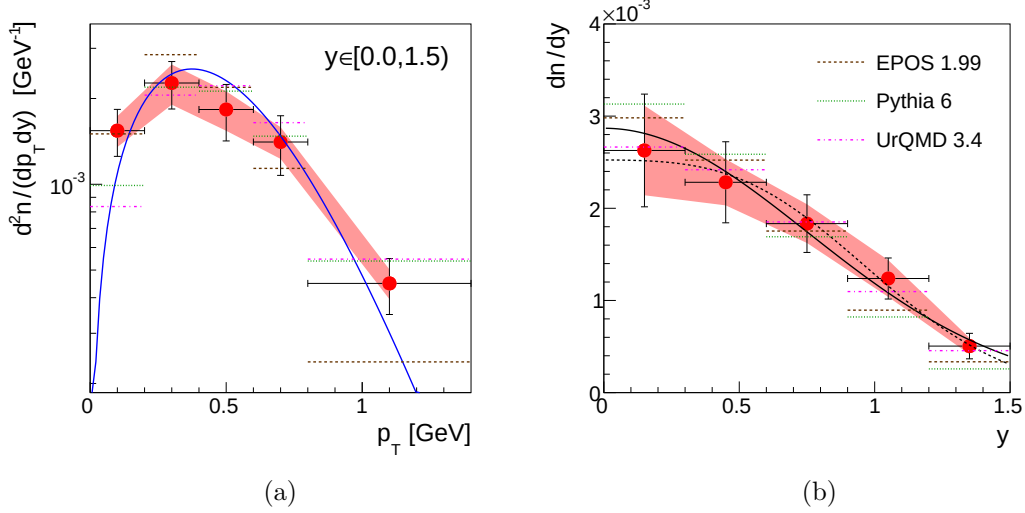


Figure 5.6: Transverse momentum spectrum in broad rapidity range and rapidity spectrum for 40 GeV data with statistical (vertical lines) and systematic (red bands) uncertainties. Horizontal lines give respectively p_T and y bin sizes. Solid blue curve in panel (a) is a fit of function Eq. (5.1) on page 114. Solid black curve in panel (b) is as Gaussian fit (Eq. (5.3) on page 114), while dashed one is a double Gaussian fit (Eq. (5.6) on page 114). Regarding models see Section 5.5.2.

are derived simultaneously by means of an iterative method described in Section 4.6.7.

Similarly to analogous results of the double differential analysis, the transverse momentum spectrum is fitted with function $f(p_T)$ to estimate the unmeasured tail contribution. It turns out to be below 1 %, thanks to the broad range of p_T covered. The fit also yields the effective temperature parameter $T = (129 \pm 14 \pm 8) \text{ MeV}$, which is smaller than average T values for the two higher energies (Fig. 5.2 on page 119).

The rapidity spectrum is formally corrected for the above p_T tail contribution in analogy to double differential analysis results, although effectively the correction is negligible. The spectrum is fitted with a single Gaussian $g(y)$ to allow for estimation of the y tail contribution to the total yield. It is about 5 %. The resulting total yield is given in Table 5.1 on page 121 along with the width and midrapidity yield parameters.

The issue of the double Gaussian is already discussed in Section 5.2.3.

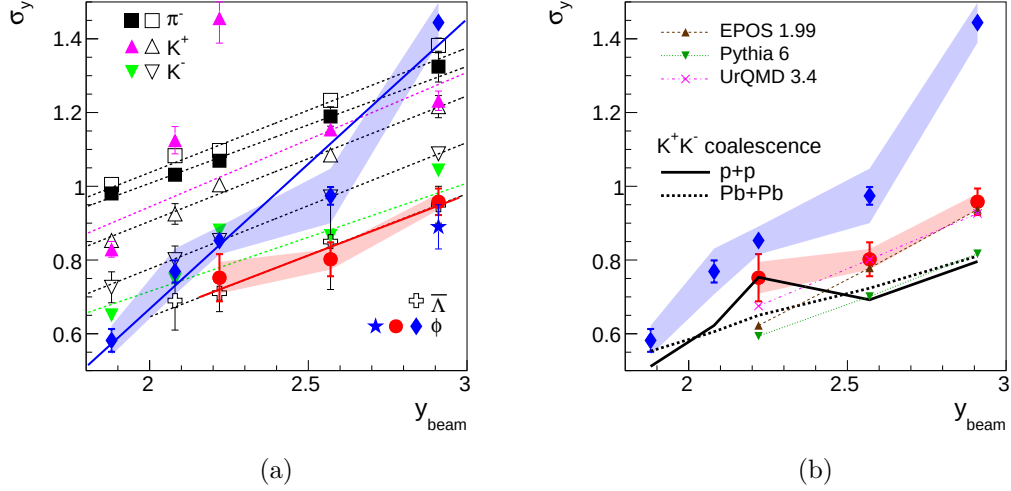


Figure 5.7: (a) Widths of rapidity distributions of various particles in $p + p$ (full symbols) and central Pb + Pb collisions (open symbols, apart from full diamonds for ϕ) as a function of beam rapidity. Full red circles are results of this analysis, the star is $p + p$ NA49 measurement [15], other $p + p$ points come from NA61/SHINE [30, 70], while Pb + Pb ones from NA49 [8, 26, 71, 72]. Lines are fitted to points to guide the eye. (b) Comparison of widths for ϕ mesons with expectations from kaon coalescence (see text) and models (see Section 5.5.2).

5.4 These results as reference for Pb + Pb

5.4.1 Width of rapidity spectra

Figure 5.7a shows widths of rapidity distributions of ϕ mesons and various other particles in $p + p$ and central Pb + Pb collisions as a function of beam rapidity in the centre-of-mass frame. Widths from this analysis as well as from NA49 analyses of $\bar{\Lambda}$ in Pb + Pb [72] and ϕ in $p + p$ [15] come from single Gaussian fits. Those for π^- [30] and K^+ , K^- [70]¹ in $p + p$, as well as for π^- , K^+ , K^- and ϕ in Pb + Pb [8, 26, 71] are associated with double Gaussian fits and are calculated according to Eq. (5.7) on page 115.

It is striking, that for almost all particles but the ϕ mesons in Pb + Pb, irrespectively of the colliding system, widths increase with the beam rapidity approximately linearly and with the same rate within the considered rapidity range. Some deviation from this trend happens also for kaons in $p + p$ at intermediate energies, but even for these, linear fits yield the common slopes.

¹Ref. [70] does not give numbers for widths of rapidity distributions, but the author graciously made his spectra accessible so that appropriate fits could have been done within this work.

The fact that it is not so for ϕ mesons in Pb + Pb was already observed in Ref. [8], but peculiarity of this result is emphasized with NA61/SHINE $p + p$ data. In particular, ϕ points from this analysis suggest (but not prove due to lack of the two lowest rapidity entries) that it is not the ϕ meson which is peculiar in itself. Rather, it is something specific to both together: the ϕ meson *and* the Pb + Pb system.

For example, Pb + Pb behaviour of σ_y is qualitatively consistent with rescattering of kaons from ϕ mesons decaying inside the fireball [8, 73]. Such kaons no longer contribute to the signal peak in the invariant mass spectrum. It is more likely to take place for slow kaons (*i.e.* those coming from low rapidity ϕ mesons), which travel longer through the fireball. Therefore, the ϕ rapidity spectrum gets depleted and this loss is the highest at midrapidity and decreases with increasing y values. So the spectrum becomes wider due to the rescattering. The effect should increase with the collision energy, because the higher the energy, the larger and denser the fireball.

Having widths for kaons, it is possible to verify the hypothesis of ϕ production through kaon coalescence. In order to do that, expected σ_y values for ϕ mesons are calculated with values for kaons from Eq. (1.1) on page 13 (in case of Pb + Pb it is an exact repetition of what was done in Ref. [8]). These expectations are illustrated as thick black lines for $p + p$ (solid) and Pb + Pb (dotted) in Fig. 5.7b on the preceding page. Only the lowest y_{beam} point for $p + p$, where the large deviation from the common trend for kaon σ_y occurs, agrees with the coalescence mechanism hypothesis. It should be noted, that the discrepancy between the coalescence expectation and the actual measurement is much lower for $p + p$ than for Pb + Pb points. This may suggest, that proportionally more ϕ mesons are produced through kaon coalescence in $p + p$ than in Pb + Pb collisions at high SPS energies. It would be very interesting to see what happens in $p + p$ collisions at the lowest SPS energies — whether there is an actual change of mechanism with collision energy or the observed coincidence with coalescence prediction is accidental.

5.4.2 Multiplicity ratios

Figure 5.8a on the next page presents ratios of total yields of ϕ mesons to mean total yields of pions in $p + p$ and central Pb + Pb [8] collisions as a function of energy per nucleon pair. Mean total yields for pions are calculated after Ref. [8] as

$$\langle \pi \rangle = \frac{3}{2}(\langle \pi^+ \rangle + \langle \pi^- \rangle). \quad (5.15)$$

It is visible that the ratio increases with energy for measured $p + p$ data.

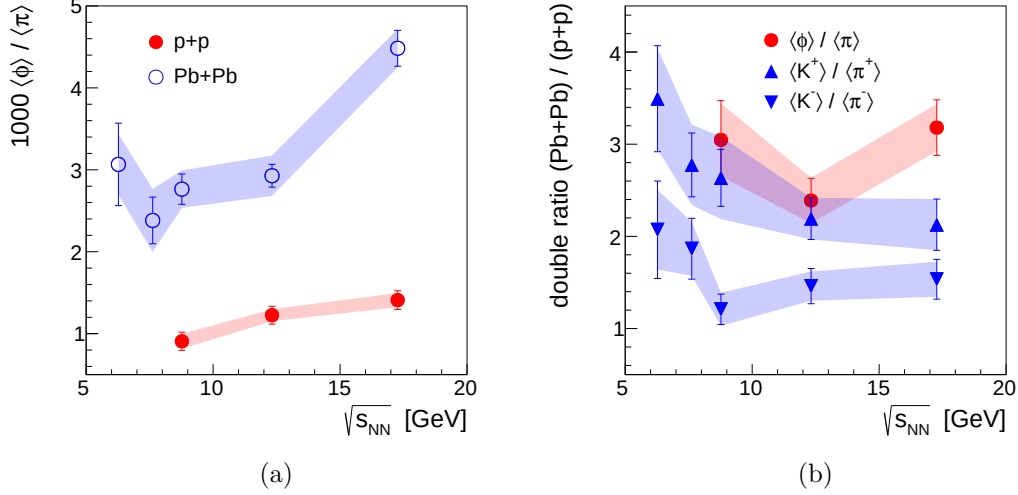


Figure 5.8: Energy dependence of (a) ratios of total yields of ϕ mesons to mean total yields for pions (Eq. (5.15) on the preceding page) in $p + p$ and $Pb + Pb$, (b) double ratios (see text). Full red circles correspond to results of this analysis, $Pb + Pb$ data come from NA49 [8, 26, 71], while $p + p$ kaon and pion data are taken from Ref. [70]. Possible correlations of uncertainties of yields within the same reaction are neglected, what may lead to slight overestimation of visualized uncertainties.

Results of this analysis confirm (not surprisingly) that the enhancement of ϕ production relative to pions in $Pb + Pb$ compared to $p + p$ collisions takes place also for lower SPS energies. Indeed, looking at *double ratios*:

$$\text{double ratio } (\langle \phi \rangle / \langle \pi \rangle) = \frac{(\langle \phi \rangle / \langle \pi \rangle)_{Pb+Pb}}{(\langle \phi \rangle / \langle \pi \rangle)_{p+p}}, \quad (5.16)$$

in Fig. 5.8b, one can see that ϕ production is enhanced roughly threefold for all 3 measured energies. That has already been observed in Ref. [8], but due to lack of $p + p$ data, a parametrization [74] of ϕ production cross-section was utilized as reference.

Enhancement in $Pb + Pb$ collisions expressed by double ratios is shown also for comparison for charged kaons relative to charged pions. It is systematically larger for ϕ mesons than for the other particles, being however comparable to that for positive kaons over positive pions. It contradicts the hypothesis that enhancement of ϕ production in $Pb + Pb$ is due to enhanced strangeness production in a partonic stage of the collision. It is, however, compatible with the kaon coalescence picture even in a purely hadronic scenario — ϕ production is proportional to kaon production, while kaons suffer canonical suppression [8].

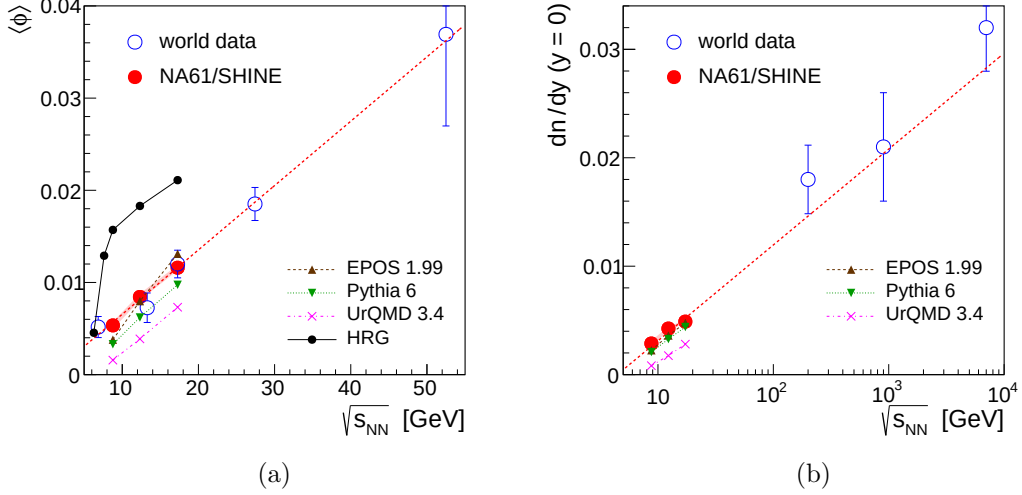


Figure 5.9: Energy dependence of (a) total yields and (b) midrapidity yields of ϕ mesons in $p + p$ collisions. World data on total yields come from Refs. [11–15], while on midrapidity yields come from Refs. [17, 19, 20]. Red dashed lines are fits to guide the eye (see text). Regarding models see Section 5.5.2.

5.5 Comparison with world data and models

5.5.1 World data

Figure 5.9 shows energy dependence of total and midrapidity yields of ϕ mesons produced in $p + p$ collisions. For CERN SPS and ISR energies total inclusive cross-sections are given in Refs. [11–15]. They are converted to multiplicities according to Eq. (1.2) on page 13. For BNL RHIC and CERN LHC the width of rapidity distributions and geometrical acceptance of detectors at colliders do not allow to measure total yields. Therefore midrapidity yields [17, 19, 20] are used for comparison. Wherever systematic uncertainties of world data are available, they are summed quadratically with statistical uncertainties for brevity of presentation.

Total yields dependence on $\sqrt{s_{NN}}$ is fitted with a red dashed line in Fig. 5.9a assuming proportionality between the total energy available for production and the number of produced ϕ mesons. All points, *i.e.* world and those from this analysis, are used in the fit. It is evident that in the presented energy range the proportionality assumption is valid and that results of this analysis are consistent with the world data.

Obviously, the new measurements are much more accurate (plotted uncertainties are much smaller than the symbol size) than the old ones. On

one hand, in all cases old experiments took smaller data samples. But on the other, for points at $\sqrt{s_{NN}} \approx 13 \text{ GeV}$ [12] and $\sqrt{s_{NN}} \approx 53 \text{ GeV}$ [13], large model-dependent extrapolations were done to calculate the total yield from measurements in a limited phase space, what resulted in big systematic uncertainties.

For midrapidity yields no simple choice of $\sqrt{s_{NN}}$ dependence exists. Red dashed line in Fig. 5.9b on the preceding page corresponds to a function

$$f(\sqrt{s_{NN}}) = a \log_{10}(\sqrt{s_{NN}}/b), \quad (5.17)$$

which is an unjustified guess, which happens to describe well the few points in the plot. Because there are no results on midrapidity yields at energies close to those considered in this thesis, the plot cannot be really used to make a statement on consistency of NA61/SHINE and world measurements. It can, however, serve to illustrate a difference between fixed target experiments with magnetic field perpendicular to the beam and collider experiments with magnetic field oriented along the beam.

In case of experiments with transverse magnetic field, including NA61/SHINE, uncertain extrapolations are necessary only to high p_T and high y values in ϕ mesons phase space. Since bulk of production occupies midrapidity and low p_T , the extrapolations and consequently systematic uncertainties are relatively small. Furthermore, these are predominantly necessitated by statistics, not the actual geometrical acceptance.

On the other hand for collider experiments with longitudinal magnetic field, low p_T kaons cannot be detected (there is usually also a cut on high p_T , but that one is not so significant). Since most kaons from ϕ mesons occupy low p_T , that necessitates large, ϕ production model-dependent extrapolations. That is a source of great systematic uncertainties and the reason why points in Fig. 5.9b on the previous page associated with modern experiments having huge amounts of data collected, have so large error bars.¹

5.5.2 Models

In the comparison three microscopic models are used — EPOS 1.99 [40, 41] from the CRMC 1.6.0 package [42] (it should be noted, that it is a newer version than the one utilized to calculate Monte Carlo corrections),

¹For that reason also, ATLAS decided to publish their results in a so-called *fiducial region* [21], which explicitly includes cuts in the kaon phase space. Such formulation doesn't allow to compare between experiments, but is perfectly suitable for verification of MC generators. Extrapolated results (the so-called *kinematic region*) were still shown to compare with ALICE [20], but seemed to be only an addition to the main product.

PYTHIA 6.4.28 [3] also shipped with the CRMC 1.6.0 and URQMD 3.4 [75, 76]. Apart from these, only in the context of total yields, the hadron resonance gas (HRG [6]) statistical model is employed.

For EPOS again, despite the newer version, ϕ width had to be adjusted to the PDG value from the erroneous value of about 30 MeV. In case of PYTHIA, the main Perugia 2011 tune 350 [4] is used. For URQMD elastic events are suppressed and ϕ decays are switched off (so that it appears in the standard output, what allows to propagate it further through the standard software chain).

The comparison with transverse momentum spectra in Figs. 5.1 and 5.6a on page 118 and on page 124 is performed in order to verify shapes of p_T distributions in considered models. Therefore, model results are normalized for each rapidity bin separately in such a way, that their integral in the presented p_T range is equal to that of the experimental spectrum in the same range. It is clear, that PYTHIA reproduces shapes of p_T spectra quite well, while URQMD produces slightly too hard distributions (*i.e.* decreasing slower with p_T than the experimental results and consequently having mean at higher p_T values) and EPOS yields visibly too soft spectra.

Shapes of rapidity distributions are verified in the same way in Figs. 5.3 and 5.6b on page 120 and on page 124. In this case both EPOS and URQMD give reasonable description of experimental data, while distributions in PYTHIA are more peaked at midrapidity. This is also visible in Fig. 5.7b on page 125, where widths of rapidity spectra are compared. Those for model calculations are extracted as standard deviations¹ from MC histograms finely binned in broad y range, without condition on p_T . While the rate of increase of σ_y with y_{beam} is similar for all models and the data, values for PYTHIA are significantly lower than those for the experimental measurements.

Finally, none of models is able to reproduce total nor midrapidity yields in Fig. 5.9 on page 128. Microscopic models mainly underestimate the yields, with URQMD being the worst, giving almost two times too low values. EPOS is exceptional, as it is compatible with experimental measurements for beam momentum of 80 GeV, while it overestimates the 158 GeV result and underestimates that for 40 GeV. In contrast, the HRG model overestimates the yields about two times, despite the fact that it is able to describe very well other particles in the same reactions [5, 6]. Still, all models but EPOS show correct rate of increase of yield values with $\sqrt{s_{NN}}$. The EPOS energy dependence seems to be too steep.

¹TH1::GetRMS() method of the ROOT framework

Chapter 6

Summary and conclusions

The goal of this analysis was to estimate differential multiplicities of ϕ mesons produced in proton-proton collisions at CERN SPS energies, from data collected by the NA61/SHINE experiment. It has been achieved for beam momenta of 40 GeV to 158 GeV; measurements for lower energies, which were also done by the experiment, yielded too small statistics in invariant mass spectra, rendering the analysis infeasible.

The analysis was done by means of invariant mass spectra fits in charged kaons decay channel. The so-called tag-and-probe method was adapted from LHC analyses. It allows to obtain ϕ multiplicities non-biased due to inefficiency of particle identification (PID) of kaon candidates, without strict conditions imposed on PID efficiency as a function of kaon momenta, nor any knowledge regarding modelling of PID variables distributions. It proved particularly useful in view of problems with quality of dE/dx calibration for the considered data sets. It should be noted, that this method is novel to the NA61/SHINE community.

For 158 GeV and 80 GeV the analysis was done double differentially yielding double differential spectra of rapidity and transverse momentum. From these, rapidity spectra, integrated over transverse momentum, were obtained. Also transverse mass spectra at midrapidity were derived as alternative representation of midrapidity double differential spectra. Statistics for 40 GeV allowed only for a single differential analysis resulting in single differential spectra of transverse momentum or rapidity.

Spectra were corrected for most significant known biases. Systematic uncertainties associated with various sources were estimated. Statistical uncertainties dominate over systematic ones for all energies and all phase space points.

For 158 GeV, previous single differential results exist, measured by the NA49 experiment [15]. An agreement was found between the latter and

the outcome of this analysis. Apart from clear improvement over NA49 results due to double differential analysis, also the phase space coverage was extended by one more bin in rapidity and one more bin in transverse momentum. This significantly reduced systematic uncertainty of the total yield compared to NA49.

NA61/SHINE results on ϕ production in $p + p$ collisions serve as reference for NA49 Pb + Pb data [8]. They emphasize the peculiarity of ϕ rapidity spectra widths dependence on beam rapidity in Pb + Pb. They also confirm earlier findings [8] regarding enhancement of ϕ production in Pb + Pb collisions and show that it is essentially independent of collision energy in the considered energy range.

Obtained ϕ rapidity spectrum width for 40 GeV is consistent with the hypothesis that ϕ mesons are predominantly produced through kaon coalescence at that energy in $p + p$ collisions. Values for higher energies suggest that although kaon coalescence cannot be a dominant mechanism of ϕ production at these energies, proportionally more ϕ mesons may be produced through coalescence in $p + p$ than at the same energies in Pb + Pb collisions.

Comparison with world data on total and midrapidity ϕ yields in $p + p$ collisions shows that results of this thesis are consistent with the world measurements and that their accuracy is superior to the others.

Outcome of this analysis is compared also to three microscopic models, EPOS 1.99 [40, 41] from the CRMC 1.6.0 package [42], PYTHIA 6.4.28 [3] and URQMD 3.4 [75, 76], as well as to a statistical thermal model HRG [6]. While each microscopic model is able to reproduce either the shape of transverse momentum or rapidity spectra, none can well describe both. Also none of models is able to reproduce total nor midrapidity yields, with discrepancies ranging from twice too low, to twice too high values. This shows, that outcome of this thesis ought to be valuable for tuning phenomenological models.

It has to be emphasized that these are the first results ever on ϕ production in proton-proton collisions at 80 GeV and 40 GeV. Furthermore, these are also the first ϕ meson double differential spectra from $p + p$ collisions at CERN SPS energies. Outcome of this analysis is planned to be published soon by the NA61/SHINE collaboration.

It may finally be noted, that analysis methods developed in the context of this thesis will serve to estimate ϕ production using yet higher statistics $p + p$ data at 158 GeV, collected by NA61/SHINE in 2010 and 2011, currently under calibration, as well as planned 350 GeV $p + p$ data.

Acknowledgements

First of all I wish to thank my supervisor, prof. dr hab. Roman Płaneta, for his support, patience, kindness and allowing me to be a member of the NA61/SHINE Jagiellonian University group.

I also want to thank other members of the Kraków group, especially prof. dr hab. Elżbieta Richter-Wąs and dr hab. Paweł Staszcz, for fruitful scientific discussions and constructive critics of what I were doing. I am particularly grateful to prof. dr hab. Elżbieta Richter-Wąs for bringing the tag-and-probe method to my attention, as well as for invaluable remarks concerning the methodology of scientific work in general. Although I felt it was too late to utilize the latter for this thesis, it will surely boost my efficiency in the future.

Next, I wish to express my gratitude to members of the NA61/SHINE collaboration, especially to Marek Gaździcki, Zoltan Fodor, Peter Seyboth, Tanja Susa and Michael Unger, for creating a friendly atmosphere in the collaboration and many valuable discussions and suggestions. I am greatly indebted to Szymon Puławski for running full Monte Carlo productions for my sole use and providing me his rapidity spectra.

This work would not have been finished without constant support and sacrifices of my family. To my wife Ewa, my parents and my parents-in-law: thank you so much!

This work was supported by the National Science Center of Poland (grant UMO-2012/04/M/ST2/00816) and the Foundation for Polish Science — MPD program, co-financed by the European Union within the European Regional Development Fund. I would like to thank prof. dr hab. Paweł Moskal, the coordinator of the MPD program at Jagiellonian University, for ever-kind and patient cooperation.

Appendix A

Basic definitions

A.1 Conventions used in this work

In order to reduce clutter in tables and pictures while giving or discussing results of the analysis (Chapters 4 and 5), a system of units is used in which:

- the velocity of light in vacuum $c = 1$, so that energy, momentum and mass are all expressed in electron volts (eV);
- Boltzmann constant $k_B = 1$, so that the effective temperature parameter T coming from fits to transverse momentum spectra with a thermally motivated formula:

$$\frac{dn}{dp_T} \propto p_T e^{-m_T/T}, \quad (\text{A.1})$$

is also expressed in electron volts (see Appendix A.2 for the meaning of variables above).

Furthermore, a right-handed Cartesian coordinate system is used, which is described at the beginning of Section 2.3. The *longitudinal* direction is the one along the beam line (along the z axis). The direction perpendicular to z axis is called the *transverse* direction and is denoted with subscript T .

Moreover, wherever the text mentions *data* it means the experimental data. The counterpart coming from a Monte Carlo simulation is either called *Monte Carlo data* or just *Monte Carlo* or *MC*.

Last, the symbol y indicates the rapidity (see Appendix A.2) in the centre-of-mass reference frame. It is different from the convention used initially in NA49 [15], where the same symbol denoted the rapidity in the laboratory frame. Here, if the latter is intended, it is indicated with y_{lab} . On the other hand, the total momentum in the laboratory frame is denoted simply as p , since the counterpart in the centre-of-mass frame is not used.

A.2 Kinematic variables

Relativistic ion collision experiments may be conducted with beam hitting a stationary target or with two beams hitting each other head-on. The collision may be symmetric with respect to masses of colliding objects or not. That gives experimenters couple of choices for the reference frame to use while presenting their results. Today usually the centre-of-mass frame is used, however historically other choices were utilized as well. All these frames are related to each other by Lorentz boosts along the beam.

The transformation of Cartesian coordinates of a momentum four-vector in such a boost is given by equations ($c = 1$, see Appendix A.1)

$$p'_x = p_x, \quad (\text{A.2a})$$

$$p'_y = p_y, \quad (\text{A.2b})$$

$$p'_z = \gamma(p_z - \beta E), \quad (\text{A.2c})$$

$$E' = \gamma(E - \beta p_z), \quad (\text{A.2d})$$

where the primed frame's velocity relative to the unprimed one's is β and the Lorentz factor $\gamma = 1/\sqrt{1 - \beta^2}$. Looking at these equations one can see, that *e.g.* the shape of differential yield expressed in these variables would transform non-trivially between the frames. Therefore, to facilitate comparison of results from different experiments, arises a need for kinematic variables which transform conveniently under longitudinal Lorentz boosts. Furthermore, it is reasonable to utilize the cylindrical symmetry of collisions.

The first convenient variable is the *transverse momentum*

$$p_T = \sqrt{p_x^2 + p_y^2}. \quad (\text{A.3})$$

From Eqs. (A.2a) and (A.2b) one sees that it is invariant under longitudinal boosts, so the same are observables being functions of this quantity. It is also interesting from the point of view of physics, because the entrance channel of the reaction by definition carries no transverse momentum, thus the p_T of outgoing particles is fully a collision product. Another useful variable, with similar properties to p_T , is the *transverse mass*

$$m_T = \sqrt{p_T^2 + m^2}, \quad (\text{A.4})$$

where m is a rest mass of the particle (in case of ϕ meson it is the value given in Table 1.1 on page 11).

Moreover, one can define the *rapidity* as

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}. \quad (\text{A.5})$$

It is associated with the longitudinal velocity v_z :

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} = \frac{1}{2} \ln \frac{\gamma m + \gamma m v_z}{\gamma m - \gamma m v_z} = \frac{1}{2} \ln \frac{1 + v_z}{1 - v_z} = \text{artanh } v_z, \quad (\text{A.6})$$

where γ is the Lorentz factor related to the total velocity of the particle (it has thus different meaning than γ in Eqs. (A.2) on the preceding page, which pertains to the relative velocity between two reference frames). Employing Eqs. (A.2c) and (A.2d) on the previous page, rapidity transforms according to

$$\begin{aligned} y' &= \frac{1}{2} \ln \frac{E' + p'_z}{E' - p'_z} = \frac{1}{2} \ln \frac{\gamma(E - \beta p_z) + \gamma(p_z - \beta E)}{\gamma(E - \beta p_z) - \gamma(p_z - \beta E)} = \\ &= \frac{1}{2} \ln \frac{E(1 - \beta) + p_z(1 - \beta)}{E(1 + \beta) - p_z(1 + \beta)} = \frac{1}{2} \ln \frac{(E + p_z)(1 - \beta)}{(E - p_z)(1 + \beta)} = \\ &= \frac{1}{2} \ln \frac{E + p_z}{E - p_z} + \frac{1}{2} \ln \frac{1 - \beta}{1 + \beta} = y - \text{artanh } \beta = y - y_b, \end{aligned} \quad (\text{A.7})$$

where y_b is a rapidity of the primed reference frame relative to the unprimed frame (the last equality comes from Eq. (A.6) and the meaning of β in Eqs. (A.2) on the previous page). So it transforms in the same way as the velocity in the Galilean transformation. Due to this property, shapes of particle distributions depending on rapidity are invariant under longitudinal boosts. They only undergo translation by y_b .

The last variable to describe the (four-)momentum of particles is the azimuthal angle φ in $p_x p_y$ plane. However, due to cylindrical symmetry of collisions, none of inclusive observables discussed in this work depend on it. So, to describe the transverse phenomena, the p_T is sufficient. It should be noted for completeness, that the azimuthal angle is useful in other analyses, with a different meaning — a relative angle between the particle's momentum and some other direction.

Appendix B

Phase space of kaons from ϕ

To investigate properties of kaons coming from ϕ decays, a toy Monte Carlo study was done. ϕ mesons were generated with uniform distributions of azimuthal angle, transverse momentum and rapidity within analysis bins (red rectangles in top plots in Fig. B.1 on the following page). Their masses were sampled from a non-relativistic Breit-Wigner distribution. For each of them a decay into K^+ and K^- was generated using the Raubold and Lynch method described in Ref. [77] and implemented in `TGenPhaseSpace` class of the ROOT framework [59].

Distributions of K^+ transverse momentum and rapidity for ϕ mesons generated in selected analysis bins are shown in top plots in Fig. B.1 on the next page. From symmetry of the decay K^- are distributed in the same way. Red rectangles signify ranges of p_T and y in which ϕ mesons were uniformly generated. Panels (a), (b), (d) and (e) of Fig. B.1 on the following page show results for corner bins of the analysis binning for 158 GeV, while (c) corresponds to one of middle bins and (f) illustrates the distribution for the full, unbinned analysis phase space.

It is visible, that kaons approximately follow ϕ rapidity distribution and they almost don't leak beyond ϕ rapidity range. On the other hand, their p_T distribution is squashed towards low p_T values with respect to the distribution of ϕ . Furthermore, comparing kaon registration probabilities from Fig. B.2 on page 139 (showing no significant differences for K^+ and K^-) with kaon distribution in Fig. B.1f on the following page, one can see that essentially all kaons considered in the analysis of 158 GeV data fall into measurable region.

The bottom plots in Fig. B.1 on the next page show distributions of (logarithm of) total laboratory momentum for K^+ . These should be compared with dE/dx vs $\log_{10}(p/\text{GeV})$ distribution (*e.g.* Fig. 4.2a on page 45). From Fig. B.1f one can learn, that for the analysis only the relativistic rise of the Bethe-Bloch curve, starting from about $\log_{10}(p/\text{GeV}) = 0.5$, is important.

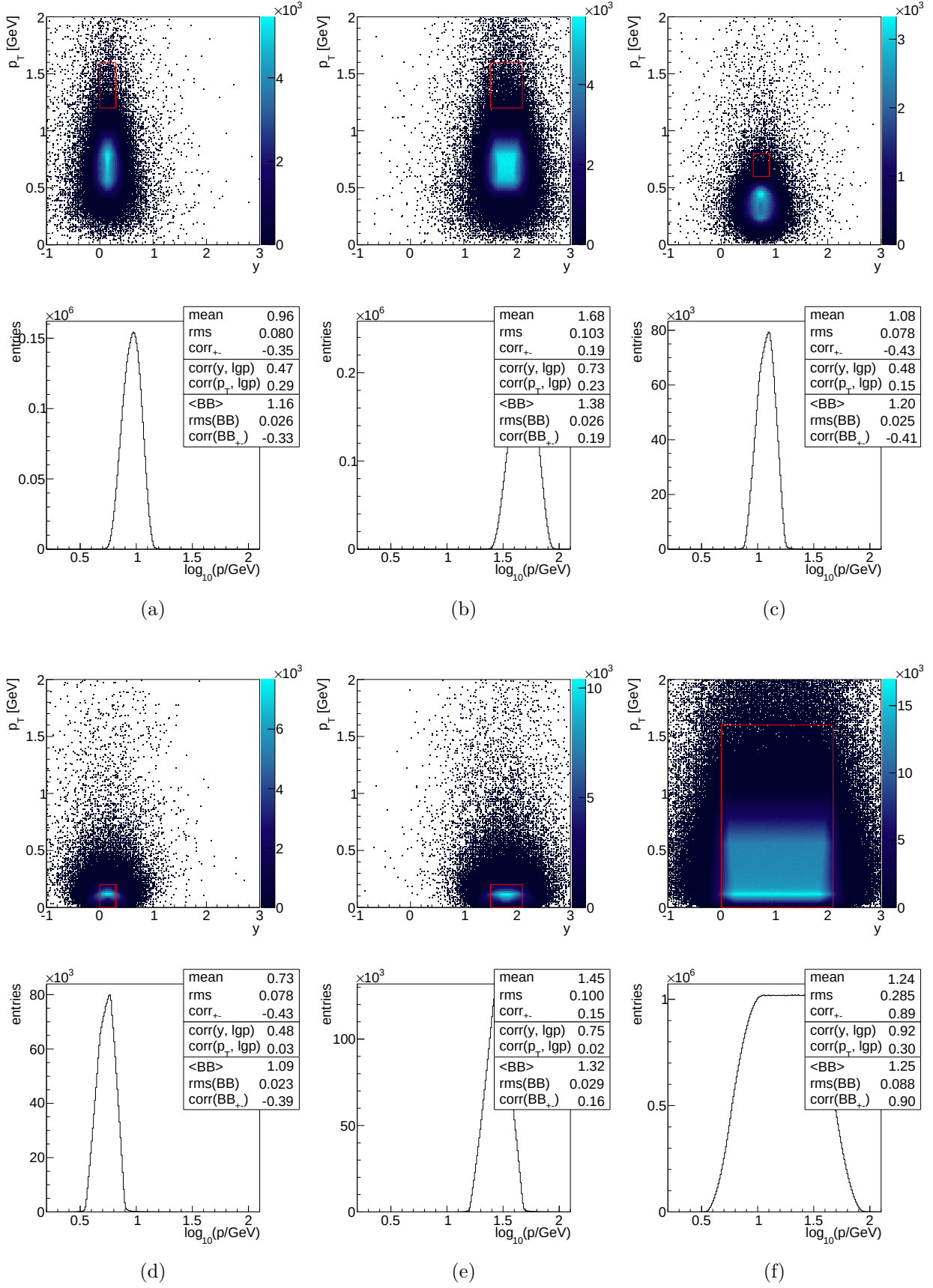


Figure B.1: Top plots in each panel show momenta distributions for K^+ mesons from ϕ decays, for ϕ mesons generated with uniform distributions in red rectangles. Bottom plots show corresponding total laboratory momentum distributions of K^+ together with some statistics (see text) derived from the MC sample.

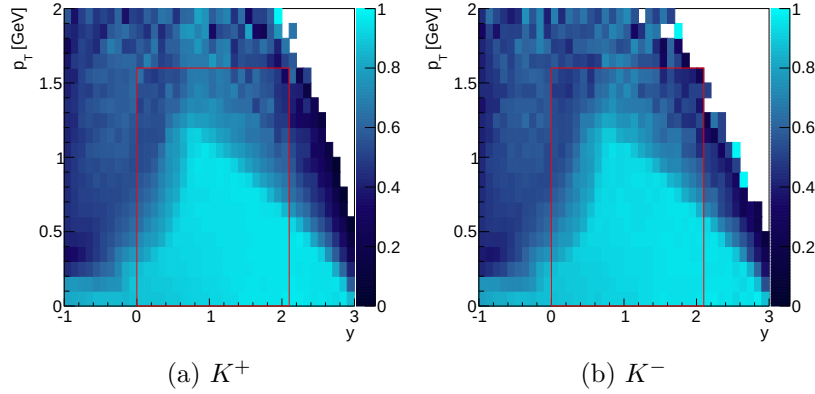


Figure B.2: Registration probability plots (see Section 4.5.4 for the method to calculate them) for K^+ and K^- for 158 GeV with overlaid broad analysis binning to guide the eye. To be compared with top plot of Fig. B.1f on the preceding page. White regions in the plot correspond to bins where calculation of the probability was impossible due to insufficient statistics of generated tracks.

Moreover, for each panel three boxes with statistics derived from the MC sample are shown to give arguments in favour of properties of kaon PID efficiency assumed in Section 4.4.4. The whole reasoning is based on an assumption that this efficiency depends primarily on the kaon total momentum and in a similar way to the Bethe-Bloch value (see also discussion in Appendix C.2).

The top boxes give mean and standard deviation (due to historical reasons denoted as rms) for $\log_{10}(p/\text{GeV})$ of K^+ , as well as the Pearson's correlation coefficient corr_{+-} of $\log_{10}(p/\text{GeV})$ for K^+ and K^- . Comparing mean and rms values for the corner bins, one can see that $\log_{10}(p/\text{GeV})$ differences between p_T bins at the same y are comparable to rms and smaller than differences between y bins at the same p_T . What's more, correlation coefficients $\text{corr}(y, \text{lgp})$ of ϕ rapidity and kaon $\log_{10}(p/\text{GeV})$ (middle boxes) are significantly higher than correlation coefficients $\text{corr}(p_T, \text{lgp})$ of ϕ transverse momentum and kaon $\log_{10}(p/\text{GeV})$. This is why the fitting strategy described in Section 4.4.4 uses 1D y bins in the broad p_T range to determine PID efficiency values utilized to constrain fits in the 2D analysis binning.

Last, the bottom boxes show mean values $\langle \text{BB} \rangle$ and standard deviations $\text{rms}(\text{BB})$ of K^+ Bethe-Bloch function values associated with K^+ momenta, as well as correlation coefficients $\text{corr}(\text{BB}_{+-})$ of Bethe-Bloch values for K^+ and K^- momenta. These are relevant for the issue of tag-and-probe bias due variable PID efficiency and therefore are discussed in Appendix C.3. Correlation coefficients $\text{corr}(\text{BB}_{+-})$ have very similar values to corr_{+-} because Bethe-Bloch curves are almost linear functions of $\log_{10}(p/\text{GeV})$ in the relativistic rise region.

Appendix C

Tag-and-probe with variable ε

Section 4.4.3 assumed that all kaons from ϕ are accepted by the PID cut with the same probability ε . It is interesting to learn what happens with the tag-and-probe method when ε is not constant. Especially, that it is likely in the case of this analysis due to how the PID cut is formulated and due to known problems in the dE/dx calibration (Section 3.4).

C.1 Derivation of formulas for N_t and N_p

First, let's see how formulas for expected signal yields in tag and probe samples are derived.

Let ε_+ and ε_- be probabilities that respectively the K^+ and the K^- from a ϕ decay are accepted by the PID cut. Furthermore, let ε_p and ε_t be probabilities that the ϕ decay contributes to the probe and to the tag sample, respectively. In case of the probe sample both kaons need to be accepted, so

$$\varepsilon_p = \varepsilon_+ \varepsilon_- . \quad (\text{C.1})$$

In case of the tag sample at least one of kaons needs to be accepted. The complement of such an event is that not K^+ nor K^- is accepted. The probability of this complementary event is therefore

$$1 - \varepsilon_t = (1 - \varepsilon_+)(1 - \varepsilon_-) = 1 + \varepsilon_+ \varepsilon_- - \varepsilon_+ - \varepsilon_- , \quad (\text{C.2})$$

so

$$\varepsilon_t = \varepsilon_+ + \varepsilon_- - \varepsilon_+ \varepsilon_- . \quad (\text{C.3})$$

Having N_ϕ decays and constant probabilities, the expected yields in both samples are given simply by products of N_ϕ and respective probabilities for

the samples:

$$N_p = N_\phi \varepsilon_p, \quad (\text{C.4a})$$

$$N_t = N_\phi \varepsilon_t. \quad (\text{C.4b})$$

If moreover $\varepsilon_+ = \varepsilon_- = \varepsilon$ as in Section 4.4.3, we get $\varepsilon_p = \varepsilon^2$ and $\varepsilon_t = 2\varepsilon - \varepsilon^2 = \varepsilon(2 - \varepsilon)$. Substituting these into Eqs. (C.4) gives exactly the Eqs. (4.8) on page 53 of Section 4.4.3.

If the probabilities for kaons are variable (are itself random variables), the expected yields are given by products of N_ϕ and expected values of the probabilities:

$$N_p = N_\phi \langle \varepsilon_p \rangle, \quad (\text{C.5a})$$

$$N_t = N_\phi \langle \varepsilon_t \rangle, \quad (\text{C.5b})$$

where

$$\langle \varepsilon_p \rangle = \langle \varepsilon_+ \varepsilon_- \rangle = \langle \varepsilon_+ \rangle \langle \varepsilon_- \rangle + \text{cov}(\varepsilon_+, \varepsilon_-) \quad (\text{C.5c})$$

and

$$\begin{aligned} \langle \varepsilon_t \rangle &= \langle \varepsilon_+ + \varepsilon_- - \varepsilon_+ \varepsilon_- \rangle = \langle \varepsilon_+ \rangle + \langle \varepsilon_- \rangle - \langle \varepsilon_+ \varepsilon_- \rangle \\ &= \langle \varepsilon_+ \rangle + \langle \varepsilon_- \rangle - \langle \varepsilon_+ \rangle \langle \varepsilon_- \rangle - \text{cov}(\varepsilon_+, \varepsilon_-). \end{aligned} \quad (\text{C.5d})$$

Above, the linearity property of expectations was used and the fact that $\text{cov}(X, Y) = \langle XY \rangle - \langle X \rangle \langle Y \rangle$. This makes N_p and N_t depend jointly, in the most general form, on four parameters: N_ϕ , $\langle \varepsilon_+ \rangle$, $\langle \varepsilon_- \rangle$ and $\text{cov}(\varepsilon_+, \varepsilon_-)$, contrary to the two parameters in Eqs. (4.8) on page 53 of Section 4.4.3.

C.2 ε_+ and ε_- distribution properties

Now let's consider properties of the joint probability distribution of ε_+ and ε_- described by the associated probability density function $g(\varepsilon_+, \varepsilon_-)$.

Probabilities ε_+ and ε_- depend on momenta of kaons through both the intrinsic properties of dE/dx and the formulation of the PID cut. From symmetry of the ϕ decay, the underlying joint probability distribution for the K^+ and K^- momenta $\vec{p}_+$ and $\vec{p}_-$, expressed by the probability density function $f(\vec{p}_+, \vec{p}_-)$, is symmetric in the two momenta. However, the positive and negative particles on average travel through different parts of the detector due to the magnetic field. If the dE/dx calibration is very bad and there are significant systematic discrepancies between the two parts of the detector, then $g(\varepsilon_+, \varepsilon_-)$ lacks the symmetry of $f(\vec{p}_+, \vec{p}_-)$.

But with a moderately good dE/dx calibration, when there is some total momentum dependent deviation from Bethe-Bloch, the same for positive and negative particles, as seems to be the case in the analysed data (see Section 3.4), the $g(\varepsilon_+, \varepsilon_-)$ is symmetric in ε_+ and ε_- . In this simpler, but still realistic and non-trivial situation, the expected values $\langle \varepsilon_+ \rangle = \langle \varepsilon_- \rangle = \mu$ and similarly variances (useful later in this discussion) $\text{var}(\varepsilon_+) = \text{var}(\varepsilon_-) = \sigma^2$.

Whether $g(\varepsilon_+, \varepsilon_-)$ follows the symmetry of $f(\vec{p}_+, \vec{p}_-)$ or not, ε_+ and ε_- are still correlated through the correlation of momenta given by Eq. (4.3) on page 39. Therefore their covariance, for brevity denoted c_{+-} in the following, does not vanish if only the probabilities are not constant.

C.3 Bias of the tag-and-probe method

Finally, it is possible to discuss a bias of tag-and-probe fit results due to variable PID efficiency. To be able to give some coarse estimation, let's consider the (simpler, yet realistic) situation when $g(\varepsilon_+, \varepsilon_-)$ is symmetric in ε_+ and ε_- .

Using results of the previous section, in this situation, Eqs. (C.5) on the previous page can be rewritten as

$$N_p = N_\phi (\mu^2 + c_{+-}), \quad (\text{C.6a})$$

$$N_t = N_\phi (2\mu - \mu^2 - c_{+-}). \quad (\text{C.6b})$$

Obviously it is not possible to use Eqs. (C.6) to improve the fit function Eq. (4.9) on page 54 for tag-and-probe method to get the unbiased N_ϕ , μ and c_{+-} parameters, as that would be equivalent to solving a system of two equations with three unknowns. The thing that could be done, is to calculate the relation between parameters resulting from the tag-and-probe fit and the true parameters that describe the process.

In terms of expected values, fit of function Eq. (4.9) on page 54 is equivalent to solving Eqs. (4.8) on page 53 for ε and N_ϕ . Let's denote the expected values of the fitted parameters as $\hat{\varepsilon}$ and $\hat{N}_\phi$. Solving Eqs. (4.8) on page 53 gives:

$$\hat{\varepsilon} = \frac{2N_p}{N_t + N_p}, \quad (\text{C.7a})$$

$$\hat{N}_\phi = \frac{(N_t + N_p)^2}{4N_p}. \quad (\text{C.7b})$$

Using Eqs. (C.6) on the previous page we get

$$\hat{\varepsilon} = \frac{2N_\phi(\mu^2 + c_{+-})}{N_\phi(2\mu - \mu^2 - c_{+-} + \mu^2 + c_{+-})} = \frac{2(\mu^2 + c_{+-})}{2\mu} = \mu + \frac{c_{+-}}{\mu} \quad (\text{C.8})$$

and

$$\hat{N}_\phi = \frac{(2\mu N_\phi)^2}{4N_\phi(\mu^2 + c_{+-})} = \frac{4N_\phi^2\mu^2}{4N_\phi(\mu^2 + c_{+-})} = N_\phi \frac{\mu^2}{\mu^2 + c_{+-}}. \quad (\text{C.9})$$

This allows to calculate the relative bias of the fitted ϕ yield:

$$\begin{aligned} \frac{\hat{N}_\phi - N_\phi}{N_\phi} &= \frac{\hat{N}_\phi}{N_\phi} - 1 = \frac{\mu^2}{\mu^2 + c_{+-}} - 1 = -\frac{c_{+-}}{\mu^2 + c_{+-}} \\ &= -\frac{1}{1 + \mu^2/c_{+-}} = -\frac{1}{1 + \frac{(\mu/\sigma)^2}{\rho_{+-}}}, \end{aligned} \quad (\text{C.10})$$

where σ^2 was introduced in the previous section and ρ_{+-} is the Pearson's correlation coefficient¹ for ε_+ and ε_- .

Assuming that PID efficiency depends on total momentum similarly to the Bethe-Bloch function, gives

$$\frac{\mu}{\sigma} \approx \frac{\langle \text{BB} \rangle}{\text{rms}(\text{BB})} \quad \text{and} \quad \rho_{+-} \approx \text{corr}(\text{BB}_{+-}), \quad (\text{C.11})$$

where symbols $\langle \text{BB} \rangle$, $\text{rms}(\text{BB})$ and $\text{corr}(\text{BB}_{+-})$ are defined at the end of Appendix B. Substituting values given in the bottom statistic box in the bottom panel of Fig. B.1f on page 138 (unbinned phase space) yields only $\frac{\hat{N}_\phi - N_\phi}{N_\phi} \approx -0.4\%$, while corresponding values for analysis bins are even smaller.

To estimate the worst case scenario, taking reasonable value of $\mu = 0.8$, relatively large $\sigma = 0.2$ and maximum $\rho_{+-} = 1$ yields in turn $\frac{\hat{N}_\phi - N_\phi}{N_\phi} \approx -5.9\%$. Taking $\rho_{+-} = -1$ — as can be seen in Fig. B.1 on page 138 at least for Bethe-Bloch values the anticorrelation case is also possible — yields $\frac{\hat{N}_\phi - N_\phi}{N_\phi} \approx 6.7\%$. So even this case does not give large bias. Indeed these numbers are similar to systematic uncertainties estimated in Section 4.6.5, which include also other effects.

All in all it seems that the analysis should not suffer from the bias due to variable PID efficiency.

¹Pearson's correlation coefficient is defined for two random variables X and Y as $\text{corr}(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{\text{var}(X) \text{var}(Y)}}$.

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