

Search for Charged Charmonium-like Z (Tetraquark) States with the CMS Experiment at LHC

A Thesis

Submitted to the
Tata Institute of Fundamental Research, Mumbai
for the degree of Doctor of Philosophy
in Physics

by

Nairit Sur

School of Natural Sciences



Mumbai

September, 2019

Final Version Submitted in January, 2021

To my parents

DECLARATION

This thesis is a presentation of my original research work. Wherever contributions of others are involved, every effort is made to indicate these clearly, with due reference to the literature, and acknowledgement of collaborative research and discussions.

The work was done under the joint supervision of Prof. Tariq Aziz and Prof. Gagan B. Mohanty at the Tata Institute of Fundamental Research, Mumbai.



Nairit Sur

In our capacity as joint supervisors of the candidate's thesis, we certify that the above statements are true to the best of our knowledge.



Prof. Tariq Aziz



Prof. Gagan B. Mohanty

Acknowledgements

ন হি কৃতমুপকারং সাধবো বিস্মরন্তি

na hi kṛtamupakāraṁ sādhave viśmaranti

The wise never forget any help received.

My generally enjoyable and occasionally arduous journey of PhD would not have come to a fruitful end without the constant support and guidance from my thesis advisors, Prof. Tariq Aziz and Prof. Gagan Mohanty. They have been my saviours on countless occasions - academic or otherwise. I am eternally grateful to them for the ideas, discussions, insights, and most importantly, for having confidence in me throughout this rather long adventure.

I consider myself extremely fortunate to be able to work in TIFR with its picturesque campus, inspiring academic environment, and smooth functionality in terms of administrative procedures. Special mention must be made of Prof. Sreerup Raychaudhuri for his words of encouragement and for guiding me through graduate school formalities.

I take this opportunity to thank CERN, where I spent some time during the period of my PhD, for being a gracious host and a truly global arena for collaborative research. I got the chance to make valuable academic and personal connections from all over the world, participate in many activities and enrich myself as a person. I'm indebted to my colleagues at CMS, especially the B-physics working group conveners for sharing valuable knowledge and experience. I learnt a lot while extensively collaborating with Alexis Pompili, Leonardo Cristella, Adriano Di Florio, and Vincenzo Mastrapasqua. In particular, I'd like to thank Alexis for inviting me to visit and work at INFN Bari, Italy. Besides being academically worthwhile, my visits to Bari allowed me to explore the exquisite cuisine, culture, and hospitality of the beautiful country of Italy.

This thesis would never have been possible without the presence of certain people in my life and here goes a feeble attempt to befittingly acknowledge them. I'm incredibly fortunate to have a super-supportive mother and father who have stood by me through all my life decisions. They have never doubted my abilities and cleared out a smooth path for me to pursue my aspirations.

I am indebted to my alma mater, South Point High School, Kolkata where I came across some amazing teachers who, among other things, instilled in me the love for physics. It is in South Point that I met two of my closest friends, Chitrak and Aritra, now physicists themselves, who have been constant sources of inspiration throughout my academic and personal life. I was lucky to share some time with each of them in TIFR. These two people have helped me sail through the most difficult times of my PhD life. The bonds that were forged during my school and college days have withstood the test of time. Samit, Iku, Bhatta, Aashish, Abhirup, Sumantra among others have never let my life become mundane or devoid of humour and positivity.

Graduate school at TIFR was a daunting but joyous affair thanks to my amazing seniors, especially Sayan, Shamashish, Saikat, Sanmay, Gourab, Nilay, Umesh, Rahul, Sumanta, Kolahal and his wife Tamali, Gouranga to name a few. My incredible luck in terms of finding the purest of friendships continued in TIFR as well. It has been an absolute delight to have in my life the Sinha trio - Atreyee, Ayesha and Ritam, who continue to provide the silly and the profound in the exact right doses. They form a part of the solid foundation based on which I can afford to lead an audacious lifestyle. I'm thankful to have had Debjyoti, Soureek, Shubhadeep, Khadiza, Rickmoy, Joe, Gope, Nilakash and Mangesh by my side to share the pains, puns, and pleasures of the PhD life. I wish to express my sincere gratitude to Prof. Arnab Bhattacharya and Surendra Kulkarni for inducting me into the TIFR Outreach team, thus allowing me to give back a little to society through science communication.

This section would be incomplete without the mention of my juniors Indranil, Sudip, Soumyajit, Soham, Sumeru, and Arkadipta who have been the "light of Eärendil's star" while I struggled to cut through the webs of multifarious complications which plagued the later days of my PhD life. I will forever cherish the passion we share for gastronomy, history, music, travelling, comic books, and pop-culture among many other things. The tea-time banters with Suman, Sandhya, Pallabi, Aravind, Uttiya, Mintu, Brij and other friends from my department have been therapeutic. I'd like to give a big shout-out to Sayani, Triparno, and Shilanjani for all the refreshing discussions about life, universe, and everything.

It has been an extraordinary privilege to be able to call Mumbai my home for the

last few years. It is a city that never sleeps, that is teeming with life at every corner, that exemplifies the true spirit of coexistence, that is honest, hardworking, ruthless, nonchalant but caring in its own unique way. It is safe and non-judgemental and with its wide variety of competent civic amenities, allows one to choose one's own brand of happiness. The relentless thrashing of waves on the Marine Drive tetrapods, the fierce sea-wind in the TIFR colonnade, the dimly lit buzz of Gokul and Sunlight, the colonial hangover in the by-lanes of Fort, the murals of Mondegar, the old school charm of the Parsi and Irani cafes, the ever-smiling waiters of New Martin, the din and bustle of Mohammad Ali road, the fight for desserts in Theobroma, Kala Ghoda Cafe and Five Spice, and the void left by some favourite establishments closing down have all played significant roles in calming my overworked mind and starting afresh with renewed vigour time and again.

In this day and age, a person cannot survive without machines and gadgets. I am immensely thankful to my Dell, and later, Macbook laptops for being my constant companions in this journey and delivering excellent performance under all circumstances. My trusty old Nikon has withstood severe use and abuse but never failed to lift my morale with top-notch results whenever I have turned to photography for much-needed distractions.

And finally, I must admit that I have, at times, used this thesis as an excuse to cover for my shortcomings as a person and failure to carry out responsibilities. I would like to acknowledge everyone who stood by me irrespective of that, and to all others who tried. Thank you.

Contents

Declaration	ii
Acknowledgements	iii
List of figures	x
List of tables	xiv
Introduction	1
1 Exotic hadron spectroscopy	5
1.1 Exotic charmonia and XYZ states	5
1.1.1 The XYZ mesons	6
1.2 The charged Z states	8
1.2.1 The $Z(4430)^\pm$	8
1.2.2 $Z_1(4050)$ and $Z_2(4250)$ states	10
1.2.3 Amplitude Analysis of the $Z(4430)^\pm$	10
1.2.4 $Z(4200)$ and $Z(4430)$ in $B^0 \rightarrow J/\psi K\pi$	13
1.2.5 The $Z(3900)$ state	15
2 The experimental setup	17
2.1 The LHC	17
2.1.1 The LHC parameters	19
2.1.2 Luminosity	20
2.2 Kinematic variables and coordinate system	21
2.3 The CMS detector	22
2.3.1 The CMS magnet	24
2.3.2 The inner tracker	25
2.3.3 The electromagnetic calorimeter	27

2.3.4	The hadron calorimeter	29
2.3.5	The muon system	31
2.4	The trigger system	33
2.4.1	B-physics triggers	36
2.5	The software framework	37
3	Reconstruction of $B^0 \rightarrow J/\psi K\pi$	39
3.1	Data sample, trigger and preselection	39
3.1.1	Analysis workflow and data sample	39
3.1.2	Event preselection	40
3.1.3	B^0 mass constrained kinematic fit	42
3.2	Final selection of B^0 candidates	45
3.2.1	Studies of reflection backgrounds	45
3.2.2	Multiplicity of B^0 candidates	47
3.3	Cut-based selection	49
4	Amplitude Analysis of $B^0 \rightarrow J/\psi K\pi$	53
4.1	Fit Formalism	53
4.2	Fit Validation	56
4.2.1	Implementation using GPU-based GooFit	56
4.2.2	Validation with pure K^* signal model	57
4.2.3	Inclusion and validation of $B^0 \rightarrow KZ (\rightarrow J/\psi\pi)$ in the signal model	62
4.2.4	LASS Parametrization	68
4.2.5	Sensitivity of the fitter to Z contributions	74
5	Amplitude analysis fits on data	79
5.1	Efficiency and background in the fit model	79
5.1.1	Efficiency correction	80
5.1.2	Background parametrization	81
5.1.3	Closure tests with efficiency and background included in the fit model	82
5.2	Fits on data	88
5.3	Future developments	93
	Summary	93

Appendix	97
A Lorentz invariant expression for the decay angle in a two-body decay . .	97
B Implementation of the angles for decays via Z	99
B.1 Calculation of θ_Z in the Z rest frame	99
B.2 Calculation of $\tilde{\theta}_{J/\psi}$ and $\tilde{\varphi}$ in the J/ψ rest frame	99
B.3 Calculation of α in the J/ψ rest frame	102
Publications and CMS internal notes	104
References	105

CONTENTS

List of Figures

1.1.1 The spectrum of charmonium and charmonium-like mesons.	7
1.2.1 $\pi^+\psi(2S)$ invariant mass distribution from Belle and BaBar	9
1.2.2 Belle $M_{\psi(2S)\pi}^2$ distribution showing a projection of 4D fit results with a $Z(4430)$	11
1.2.3 LHCb $M_{\psi(2S)\pi}^2$ distribution and Argand plot	12
1.2.4 LHCb background-subtracted and efficiency corrected $M_{\psi(2S)\pi}$ distribu- tion	13
1.2.5 Belle fit results with the $Z(4200) J^P = 1^+$	14
1.2.6 Belle $Z(4200)$ with $Z(4430)$ fit results and Argand plot	14
1.2.7 $J/\psi\pi$ invariant mass distributions from Belle and BESIII with promi- nent $Z(3900)$ peaks.	15
2.1.1 The CERN accelerator complex.	18
2.1.2 A schematic layout of the LHC.	19
2.1.3 Integrated luminosity for pp data delivered to and recorded by CMS . .	21
2.3.1 A sectional view of the CMS detector [38].	23
2.3.2 A transverse slice of the CMS detector	24
2.3.3 CMS magnetic field	25
2.3.4 A sketch of the tracker layout	26
2.3.5 Resolution of various track parameters for single muons	28
2.3.6 Global track reconstruction efficiency for muons and pions	29
2.3.7 Geometry of the HCAL	30
2.3.8 Geometry of the CMS muon system	31
2.4.1 The CMS trigger and data acquisition system	34
2.4.2 Comparison of the pure rates (in Hz) for the different physics analysis groups of CMS [50].	36
2.4.3 CMS dimuon mass distribution collected with various dimuon triggers .	37

3.1.1 Scatter plots of $(p_x^{\text{refit}} - p_x^{\text{orig}})/p_x^{\text{orig}}$ for different tracks	43
3.1.2 Scatter plots of $(p_y^{\text{refit}} - p_y^{\text{orig}})/p_y^{\text{orig}}$ for different tracks	44
3.1.3 Scatter plots of $(p_z^{\text{refit}} - p_z^{\text{orig}})/p_z^{\text{orig}}$ for different tracks	45
3.2.1 Sketch of a $B^0 \rightarrow J/\psi(\rightarrow \mu^+\mu^-)K\pi$ decay in a pp collision with the three important discriminating variables defined.	46
3.2.2 B_s^0 veto	48
3.3.1 Distribution of $m(J/\psi K\pi)$ for events with single B^0 candidate after the cut-based selection.	51
4.1.1 A sketch illustrating the definitions of the two independent angular variables, $\theta_{J/\psi}$ and φ , for the amplitude analysis of the $B^0 \rightarrow J/\psi K\pi$ decay.	54
4.2.1 Projection of $m_{K\pi}$ spectrum of the 4D dataset generated according to an ideal signal model with fit results	57
4.2.2 Projection of $m_{J/\psi\pi}, \cos\theta$, and φ of the 4D dataset generated according to an ideal signal model with fit results	59
4.2.3 Pull distributions of the amplitude parameters obtained from a fit to generated events with all the 10 K^* resonances.	60
4.2.4 Pull distributions of the phase parameters obtained from a fit to gener- ated events with all the 10 K^* resonances.	61
4.2.5 Projection on the $m_{K\pi}$ spectrum of the 4D dataset generated according to an ideal signal model with 10 K^* s and 2 Z s and fit results.	63
4.2.6 Projection on the $m_{J/\psi\pi}, \cos(\theta)$ and φ of the 4D dataset generated ac- cording to an ideal signal model with 10 K^* s and 2 Z s and fit results. . .	65
4.2.7 Pull distributions of the amplitude parameters obtained from a fit to generated events with all the 10 K^* plus 2 Z resonances.	66
4.2.8 Pull distributions of the phase parameters obtained from a fit to gener- ated events with all the 10 K^* plus 2 Z resonances.	67
4.2.9 Projection on the $m_{K\pi}$ spectrum of the 4D dataset generated according to an ideal signal model with LASS parametrization of low-mass $K\pi$ S-wave resonances with fit results	69
4.2.10 Projections of the $m_{J/\psi\pi}, \cos\theta$ and φ of the 4D dataset generated accord- ing to an ideal signal model with LASS parametrization of low-mass $K\pi$ S-wave resonances with fit results	71
4.2.11 Pull distributions of the amplitude parameters obtained from a fit to generated events with the LASS model.	72

4.2.12	Pull distributions of the phase parameters obtained from a fit to generated events with the LASS model.	73
4.2.13	Pull distributions of the amplitude parameters obtained when a dataset generated with ten K^* s is fitted with a [ten K^* s + $Z(4430)$ + $Z(4200)$] model.	74
4.2.14	Pull distributions of the phase parameters obtained when a dataset generated with ten K^* s is fitted with a [ten K^* s + $Z(4430)$ + $Z(4200)$] model.	75
4.2.15	Pull distributions of the amplitude parameters obtained when a dataset generated with all K^* s (with LASS) is fitted with an [all K^* s (LASS) + $Z(4430)$ + $Z(4200)$] model.	76
4.2.16	Pull distributions of the phase parameters obtained when a dataset generated all K^* s (with LASS) is fitted with an [all K^* s (LASS) + $Z(4430)$ + $Z(4200)$] model.	77
5.1.1	Relative reconstruction efficiency over the scatter plot of the two mass and two angular variables	81
5.1.2	Scatter plot of the mass and angular variables of the background candidates from the lower and upper sidebands	82
5.1.3	Projection on the $m_{K\pi}$ spectrum of the 4D dataset generated according to an ideal signal model with 10 K^* s and 2 Z s including efficiency and background parametrization and fit results.	83
5.1.4	Projection on the $m_{J/\psi\pi}$, $\cos(\theta)$ and φ of the 4D dataset generated according to an ideal signal model with 10 K^* s and 2 Z s including efficiency and background parametrization and fit results.	85
5.1.5	Pull distributions of the amplitude parameters obtained from a fit to generated events with all the 10 K^* plus 2 Z resonances including efficiency and background.	86
5.1.6	Pull distributions of the phase parameters obtained from a fit to generated events with all the 10 K^* plus 2 Z resonances including efficiency and background.	87
5.2.1	Projection on the $m_{K\pi}$ spectrum in data with ten K^* s with fit results . . .	89
5.2.2	Projection on the $m_{J/\psi\pi}$, $\cos\theta$, and φ in data with ten K^* s with fit results	90
5.2.3	Fit results to data in different $m_{K\pi}$ ranges with ten K^* s.	91
5.2.4	Fit results to data in different $m_{J/\psi\pi}$ ranges with ten K^* s.	91
A.1	Definition of the decay angle.	97
B.1	Z rest frame	100

LIST OF FIGURES

B.2	J/ψ rest frame	101
B.3	$\mu^+\mu^-$ decay in the J/ψ rest frame	101

List of Tables

2.1.1 Design parameters of LHC for proton-proton and lead-lead collisions [34].	20
3.1.1 Officially produced MC datasets used in this analysis. The first one consists of a signal sample generated according to the phase-space model.	40
3.1.2 Time evolution of the inclusive J/ψ trigger through the 2012 data taking period.	41
4.2.1 The magnitudes and phases of the helicity amplitudes obtained from a fit to the ideal signal model generated with ten K^* s.	58
4.2.2 The magnitudes and phases of the helicity amplitudes obtained from a fit to the ideal signal model generated with 10 K^* s and 2 Z s.	64
4.2.3 The magnitudes and phases of the helicity amplitudes obtained from a fit to the ideal signal model generated with only K^* s with LASS parametrization for low-mass $K\pi$ S-wave resonances.	70
5.1.1 The magnitudes and phases of the helicity amplitudes obtained from a fit to the ideal signal model generated with 10 K^* s and 2 Z s including efficiency and background parametrization.	84
5.2.1 Fitted values of $K_0^*(800)$ mass and width from a fit on data with five K^* s. NLL value after fit: 424522 , NLL value after fit with LASS: 426501	88
5.2.2 Fitted values of $K_0^*(800)$ mass and width from a fit on data with ten K^* s. NLL value after fit: 422975 , NLL value after fit with LASS: 424611	89
5.2.3 The magnitudes and phases of the helicity amplitudes obtained from a fit to the data in the model with ten K^* s.	92

Introduction

Observations and discoveries in physics, especially since the 1930s, have provided a deep understanding of the fundamental structure of matter. Matter and energy, as of today, are best understood in terms of the kinematics and interactions of a set of elementary particles which are the basic building blocks of our universe. Theories and experiments over the last century have narrowed down the laws governing the interactions of these elementary particles to a small set of fundamental laws. The standard model (SM) of particle physics is a gauge theory describing three of the fundamental laws of physics – weak, strong and electromagnetic interactions. It includes the Glashow-Salam-Weinberg electroweak model [1–3] and quantum chromodynamics (QCD), the theory of strong interaction introduced by Gross, Wilczek, and Politzer [4–7]. The SM has been able to predict or explain a huge number of phenomena observed in high energy physics experiments with great accuracy over the last few decades.

As the name suggests, QCD is the quantum field theoretical description of strong interaction between the quarks and gluons that make up composite hadrons which are observed experimentally. It is a non-abelian gauge theory with the gauge group $SU(3)$. The charge associated with this group is the colour charge, and the associated fundamental particles, the quarks and the gluons, are coloured objects. Two unique features of this theory are “colour confinement” which prevents occurrence of an isolated colour charge in nature, and “asymptotic freedom” which gives rise to a steady reduction in the strength of interaction between quarks and gluons as the energy scale of interactions increase (and the corresponding length scale decreases).

Beyond the conventional $qq'q''$ baryons or $q\bar{q}'$ mesons, QCD does not forbid the existence of more complex (colour singlet) bound states like mesons with more than two quarks, baryons with more than three quarks, or hybrid combinations of quarks and gluons. During the search for orbital and radial excitations of charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$) states, different experiments found evidence of many resonances which cannot be directly placed into the spectrum of heavy quarkonia. These “exotic” mesons are usually classified as XYZ states that decay into final states containing a

pair of D/B mesons or charmonium/bottomonium plus light mesons, despite being well above the open-charm or -bottom thresholds. The theoretical models proposed so far, viz. tetra- and penta-quark models, hadronic molecules, hadro-charmonium, quark-gluon hybrids, kinematically-induced threshold effects etc. have not yet been able to describe these quarkonium-like states in a uniform and self-consistent manner. Thus, the field of exotic spectroscopy needs to be driven forward by experimental observations of new states and a detailed study of the properties of existing states, like their quantum numbers.

Charged charmonium-like Z states are manifestly exotic and particularly interesting as candidates for compact tetraquark states with a possible quark content of $|c\bar{c}d\bar{u}\rangle$. Recent studies by Belle [8, 9] and LHCb Collaborations [10] have established the $Z(4430)^-$ state as a 1^+ resonance in the $B^0 \rightarrow (\psi(2S)\pi^-)K^+$ decay. Belle has also found evidence for $Z(4430)^-$ in the $B^0 \rightarrow (J/\psi\pi^-)K^+$ decay, along with a dominant new state, the $Z(4200)^-$ [11]. Very recently, LHCb demonstrated the potential contributions of exotic states to the $B^0 \rightarrow J/\psi K\pi$ signal via a model-independent moment analysis [12]. The $Z(3900)^-$ state was observed by both BESIII [13] and Belle [14] in the study of $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ near the $Y(4260)$ region. However, it was not reported in Belle's search in the $B^0 \rightarrow (J/\psi\pi^-)K^+$ channel.

The CMS experiment with its excellent muon identification and tracking systems, is suited for the study of final states containing J/ψ and $\psi(2S)$ that can be well reconstructed in their decays to a $\mu^+\mu^-$ pair. Using large data samples of such dimuon events with a relatively low threshold on the muon transverse momentum (p_T), CMS can perform measurements and searches for new states in the field of exotic quarkonium spectroscopy.

The thesis is structured as follows. Chapter 1 gives an overview of the field of exotic hadron spectroscopy with an emphasis on charged charmonium-like Z states. Chapter 2 comprises a description of the LHC accelerator complex followed by the CMS detector and its subsystems including the trigger system and software framework. The reconstruction and purification of the $B^0 \rightarrow J/\psi K\pi$ decay have been discussed in Chapter 3. It details the analysis workflow, the data samples and triggers used, studies of reflection backgrounds and application of kinematic fit. The amplitude analysis formalism has been described in Chapter 4 followed by the validation of its implementation in the GPU based GooFit. This chapter also contains the inclusion and validation of the fit models with the exotic Z contributions as well as the description and implementation of the LASS parametrization. Chapter 5 is devoted to the fits on real data starting with the efficiency correction and background parametrization

as well as their subsequent inclusion in the fitter. The fit performances on data under different conditions have been extensively discussed in this chapter followed by a summary of the analysis. Detailed calculations of some of the angular variables used in the analysis can be found in Appendices [A](#) and [B](#).

INTRODUCTION

Exotic hadron spectroscopy

1.1 Exotic charmonia and XYZ states

With the development of QCD over the last few decades, it is clear that beyond the standard three-quark baryons and quark-antiquark mesons, it is in principle possible to have other combinations of quark and/or gluon bound states. Such exotic multi-quark states have long been predicted in the light scalar sector [15, 16] but it has been a challenge to distinguish them from the densely populated conventional states.

Charmonium states consisting of a charm and anticharm quark ($c\bar{c}$) are powerful tools for studying the strong interaction. The high mass of the charm quark ($m_c \simeq 1.275$ GeV) implies that the constituent velocities in charmonium mesons are relatively small ($v \simeq 0.45c$) and relativistic effects can be treated as perturbative corrections to ordinary quantum mechanical calculations [17, 18]. This allows for a possibility to describe dynamical properties of the $c\bar{c}$ system in terms of nonrelativistic potential models, in which the functional form of the potential is chosen to reproduce the known asymptotic properties of the strong interaction with the free parameters fixed from experimental results. The large c -quark mass heavily suppresses the production of $c\bar{c}$ pairs via quark→hadron fragmentation processes at low energies. If a newly observed state is found to decay into final states containing a $c\bar{c}$ pair, these quarks must be present in the original particle. If the state only consists of c and $\bar{c}$ quarks, then the particle must be a charmonium and occupy one of the unassigned states (represented in *gray*) in the charmonium spectrum in Fig. 1.1.1. The limited number of unassigned charmonium states with masses below 4.5 GeV and the theoretical expectation

that most of the unassigned states will be relatively narrow and have nonoverlapping widths make the identification of nonstandard charmonium-like mesons less ambiguous than the light-quark sector. Similar argument is also valid for states decaying to a $b\bar{b}$ pair and the bottomonium ($b\bar{b}$) spectrum. The spectrum of experimentally established charmonium states is shown in yellow in Fig. 1.1.1.

Open-charm thresholds ($D\bar{D}$, $D^*\bar{D}^*$, ...) play a significant role in understanding the charmonium spectrum. Below them, the states are predicted to be narrow resonances slowly decaying into noncharmed mesons or dilepton pairs, whereas, above them, the states are expected to be broad resonances decaying rapidly into charmed mesons pairs. All expected states with mass below the $D\bar{D}$ threshold have been observed experimentally and found to have masses as well as other properties in agreement with potential model expectations. Thus, the charmonium system provides a clean environment with its well-predicted spectrum and distinct properties (zero-charge, zero-strangeness, constrained decay transitions) and is suitable for investigation of different types of exotic states viz. charged ($c\bar{c}u\bar{d}$), strange ($c\bar{c}d\bar{s}$), or both ($c\bar{c}u\bar{s}$).

1.1.1 The XYZ mesons

The XYZ mesons are the recently discovered resonance-like structures with hadronic final states that contain either a $c\bar{c}$, or a $b\bar{b}$ quark pair, with properties different from the expectations for any of the currently unassigned charmonium or bottomonium states. All the quantum numbers of each such state are still not experimentally determined. In Fig. 1.1.1, they are represented in purple and red having been arranged according to the best guesses of their J^{PC} quantum numbers. As far as their nomenclature is concerned, the neutral states with $J^{PC} = 1^{--}$ are denoted by Y , the charged (isospin = 1) ones are called Z , and the rest as X .

Starting with the observation of the $X(3872)$, the first of the XYZ mesons by Belle [20] in 2003, exotic hadron candidates have been discovered at a steady rate. Some of the other notable unexpected states are as follows:

- $X(3940)$ [21] and $X(4160)$ [22] observed in the invariant mass distributions of the $D\bar{D}^*$ and $D^*\bar{D}^*$ systems, respectively, recoiling from the J/ψ in $e^+e^- \rightarrow J/\psi D^{(*)}\bar{D}^*$ in Belle;
- $Y(4260)$ [23] and $Y(4360)$ [24] observed in the $\pi^+\pi^- J/\psi$ and $\pi^+\pi^-\psi(2S)$ mass spectra respectively, in the initial state radiation (ISR) processes $e^+e^- \rightarrow \gamma_{\text{ISR}}\pi^+\pi^- J/\psi(\psi(2S))$ in BaBar;

1.1. EXOTIC CHARMONIA AND XYZ STATES

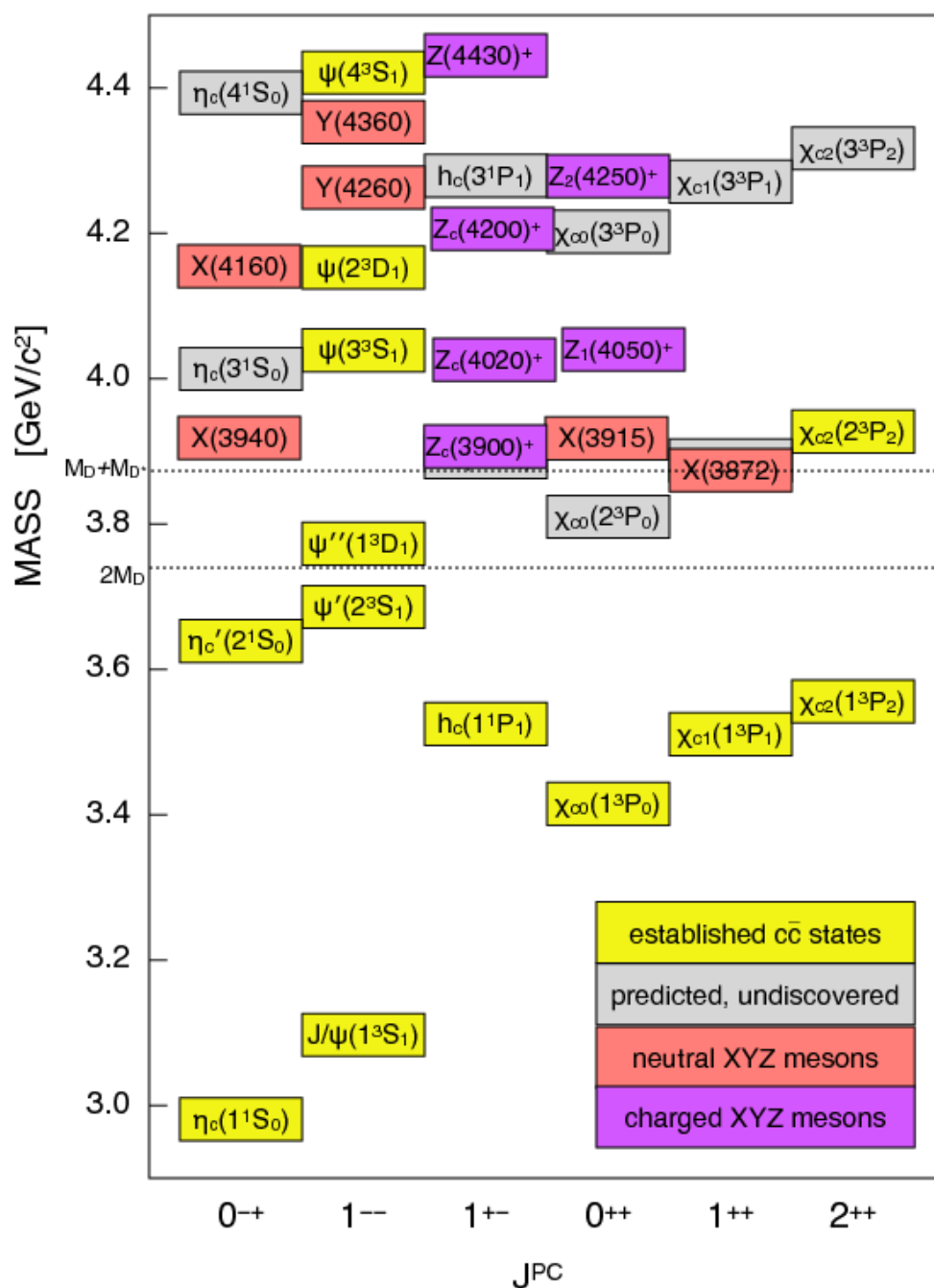


Figure 1.1.1: The spectrum of charmonium and charmonium-like mesons. Figure is taken from Ref. [19]

- a set of charged states : $Z(4430)$, $Z(4200)$, $Z_1(4050)$, $Z_2(4250)$ and $Z(3900)$, discussed in detail in Sec. 1.2.

Many of the experiments capable of discovering and studying the properties of these exotic states are in operation or being upgraded (e.g. LHCb, BESIII, CMS, ATLAS, JLab12, Belle II) or still in the development stage (e.g. PANDA).

The two main production processes of charmonium-like states in hadron colliders (like the LHC) are:

- prompt production: pp or $p\bar{p} \rightarrow (c\bar{c})^* + \text{others}$;
- nonprompt production in b-jets.

Establishing the existence of new states through both production processes would be ideal. However, for the prompt production, an inclusive search is needed which is experimentally challenging due to the high backgrounds and restricted trigger thresholds for low- p_T prompt muons in all LHC experiments except for LHCb. Searches in the nonprompt production modes are typically done through exclusive B-meson decays i.e., $B \rightarrow (c\bar{c})^* + \text{other}$ and are relatively easier to perform. The typical decay processes in which they can be searched for are:

- hadronic transition to a lighter $c\bar{c}$ meson through the emission of light hadrons ($\pi, \pi\pi, \rho, \phi$);
- electromagnetic transition to a lighter $c\bar{c}$ meson through the emission of photon.

Even though the former can be triggered on the dimuons arising from J/ψ and $\psi(2S)$, it is still difficult to reconstruct with minimal backgrounds in experiments such as CMS which does not have the capability of charged hadron identification. The latter scenario is challenging owing to the small reconstruction efficiency of low energy photons. The absence of charged hadron identification in a general purpose detector like CMS makes the reconstruction of the decay modes such as $B^0 \rightarrow J/\psi K^+\pi^-$ or $B^0 \rightarrow \psi(2S)K^+\pi^-$ considerably more challenging than LHCb or B-factory experiments like Belle. Extracting a sufficiently pure and statistically significant number of such decays in CMS is one of the key aspects of this thesis.

1.2 The charged Z states

1.2.1 The $Z(4430)^\pm$

The first charged charmonium-like state to be discovered was the $Z(4430)^\pm$ as a narrow structure in the $\psi(2S)\pi^\pm$ mass distribution in Belle [8] with a statistical significance of

1.2. THE CHARGED Z STATES

about 6.5σ . They looked into the $B^0 \rightarrow \psi(2S)K\pi^\pm$ decay channel where the $\psi(2S)$ decays into $\ell^+\ell^-$ or $\pi^+\pi^-J/\psi$ with $J/\psi \rightarrow \ell^+\ell^-$, the ℓ being an electron or a muon. Both charged and neutral kaons ($K_S^0 \rightarrow \pi^+\pi^-$) are used. In order to reduce the effect of dominant two-body resonance decays $B^0 \rightarrow K^*(892)\psi(2S)$ and $B^0 \rightarrow K_2^*(1430)\psi(2S)$, events with $K\pi$ invariant masses within ± 0.10 GeV of the $K^*(892)$ and $K_2^*(1430)$ nominal masses [25] were vetoed. The resulting prominent peak in the $M_{\psi(2S)\pi}$ distribution was fitted with a Breit-Wigner (BW) function as shown in Fig. 1.2.1 (left). The fitted values of the mass and width are 4433 ± 5 MeV and 45_{-18}^{+35} MeV, respectively. Since this state is electrically charged and decays into hidden-charm final states, it must have an exotic minimal four-quark ($c\bar{c}u\bar{d}$) substructure.

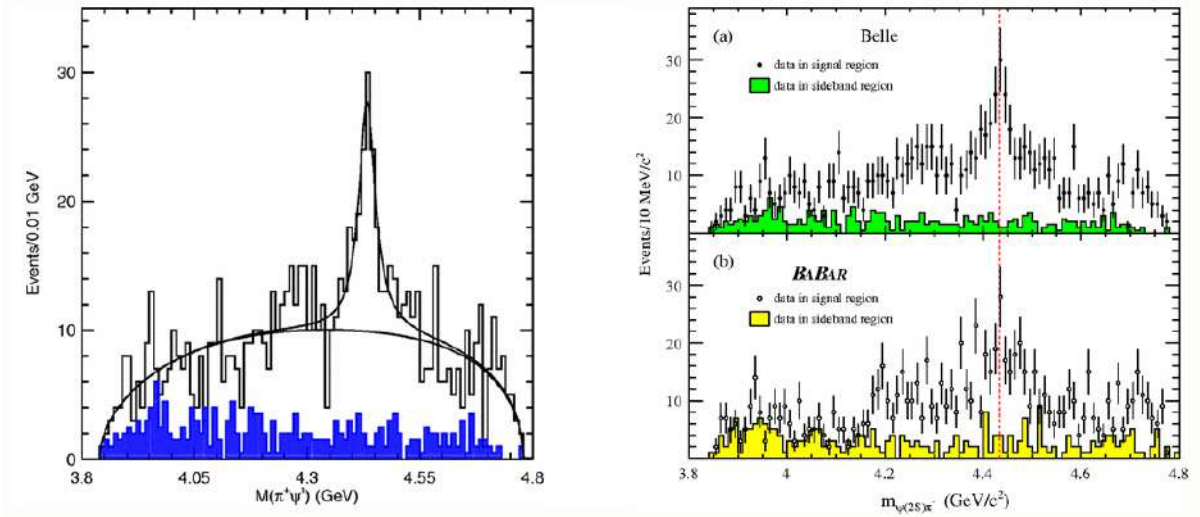


Figure 1.2.1: (left) $\pi^+\psi(2S)$ invariant mass distribution from Belle [8] after applying the K^* s veto as described in the text. Non- $\psi(2S)$ background estimated from the B^0 sidebands are represented by the shaded histogram while the curves represent the result of a fit that returned the mass and width values quoted in the text. (right) A direct comparison of Belle (right upper) and BaBar (right lower) data [26] under identical analysis conditions.

The BaBar experiment performed a similar analysis in some B -meson decay channels including $B^0 \rightarrow \psi(2S)K\pi$ as well as $B^0 \rightarrow J/\psi K\pi$ [26] but was unable to confirm the Belle result. An almost direct comparison of the Belle and BaBar results for the $\pi^+\psi(2S)$ mass distribution from $B^0 \rightarrow \psi(2S)K\pi$ decay is shown in Fig. 1.2.1, (right). The data sample used by BaBar for the comparative study is statistically compatible with that of Belle. To test whether reflections of the known $K\pi$ resonances and their

angular distributions are enough to explain the data, a model independent approach was adopted. Although the BaBar plot shows an excess of events in the same region as reported in Belle, their fit using Belle's mass and width values yielded a statistically marginal ($\sim 2\sigma$) $Z(4430) \rightarrow \psi(2S)\pi$ signal, thereby neither confirming nor rejecting Belle's result. BaBar's data in the $B^0 \rightarrow J/\psi K\pi$ channel was found to be consistent with the known $K\pi$ resonance reflections.

1.2.2 $Z_1(4050)$ and $Z_2(4250)$ states

The Belle Collaboration reported an observation of two new charged resonance-like states, $Z_1(4050)$ and $Z_2(4250)$, in the $\pi^+\chi_{c1}$ invariant mass spectrum in the exclusive decay of $\bar{B}^0 \rightarrow K^-\pi^+\chi_{c1}$ [27]. BaBar also searched for these states [28] in both $\bar{B}^0 \rightarrow K^-\pi^+\chi_{c1}$ and $B^+ \rightarrow K_S^0\pi^+\chi_{c1}$ processes where $\chi_{c1} \rightarrow J/\psi\gamma$. After background subtraction and efficiency correction, the $\pi^+\chi_{c1}$ mass distribution was modelled with the $K\pi$ mass distribution and the corresponding normalized $K\pi$ Legendre polynomial moments. It was found that there was no need for the inclusion of exotic resonant structures in the description of the $\pi^+\chi_{c1}$ mass distribution and the statistical significance for any additional resonance was found to be less than 2σ .

1.2.3 Amplitude Analysis of the $Z(4430)^\pm$

In order to address the possibility of $M_{\psi(2S)\pi}$ reflection peaks due to interference between different partial waves in the $K\pi$ resonance channels, Belle reanalysed their data with a 2D Dalitz plot fit with two kinematic variables, $M_{K\pi}$ and $M_{\psi(2S)\pi}$ [29]. It provided a strong $Z(4430)$ signal with a statistical significance of 6.4σ , albeit with a larger width ($\Gamma = 107^{+86+74}_{-43-56}$ MeV).

Belle followed up with a full 4D amplitude analysis of the $B^0 \rightarrow \psi(2S)K\pi$ [9] incorporating all four independent kinematic variables: $M_{K\pi}$, $M_{\psi(2S)\pi}$, the $\psi(2S)$ helicity angle θ , and the azimuthal angle between the decay planes, φ (discussed in Ch. 4). The amplitude is calculated in the helicity formalism following and isobar model approach, and is sensitive to angular correlations and interference effects. The mass and width obtained from this analysis are $M = 4485^{+22+28}_{-22-11}$ MeV and $\Gamma = 200^{+41+26}_{-46-35}$ MeV. Strong interference effects that are constructive below, and destructive above, the resonance mass are quite prominent from Fig. 1.2.2. This analysis favours a $J^P = 1^+$ assignment to the $Z(4430)$ state.

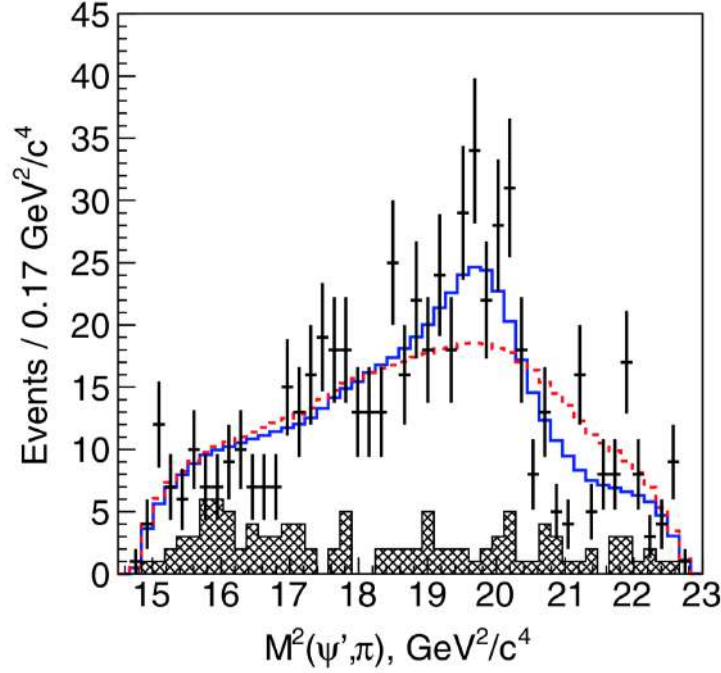


Figure 1.2.2: The data points with the error bars are the Belle $M_{\psi(2S)\pi}^2$ distribution with the K^* veto applied. The solid blue histogram shows a projection of the Belle 4D fit results with a $Z(4430) \rightarrow \psi(2S)\pi$ with $J^P = 1^+$. The hatched histogram is the $\psi(2S)$ sidebands. The dashed red curve shows the best fit results with no resonance in the $M_{\psi(2S)\pi}$ spectrum. Plot is taken from Ref. [9].

Confirmation of $Z(4430)$ by LHCb

LHCb did a complete 4D amplitude analysis of $B^0 \rightarrow \psi(2S)K\pi$, similar to Belle with a sample of more than 25K events and confirmed the existence of the $Z(4430)$ resonance with a significance of 13.9σ [10]. The measured mass and width $M = 4475 \pm 7_{-25}^{+15}$ MeV and $\Gamma = 172 \pm 13_{-34}^{+37}$ MeV are consistent with the values obtained from Belle but much more precise due to almost ten times larger statistics. This analysis favours the $J^P = 1^+$ assignment over any other possibility by 17.8σ . A comparison of the LHCb fit results with the data is shown in the left panel of Fig. 1.2.3, where strong interference effects, similar to those seen by Belle (Fig. 1.2.2), are clearly visible.

The large data sample allowed LHCb to avoid the BW assumption for the Z amplitude rather to directly measure the real and imaginary parts of the $1^+ \psi(2S)\pi$ amplitude as a function of the mass. The result is shown in the right panel of Fig. 1.2.3. The rapid phase variation near the resonance peak is a clear sign of a BW amplitude as in-

indicated by the nearly circular red curve superimposed on the plot. However, this does not demonstrate its resonant nature since this behaviour, by itself, does not necessarily rule out the possibility of such structures arising from rescattering [30].

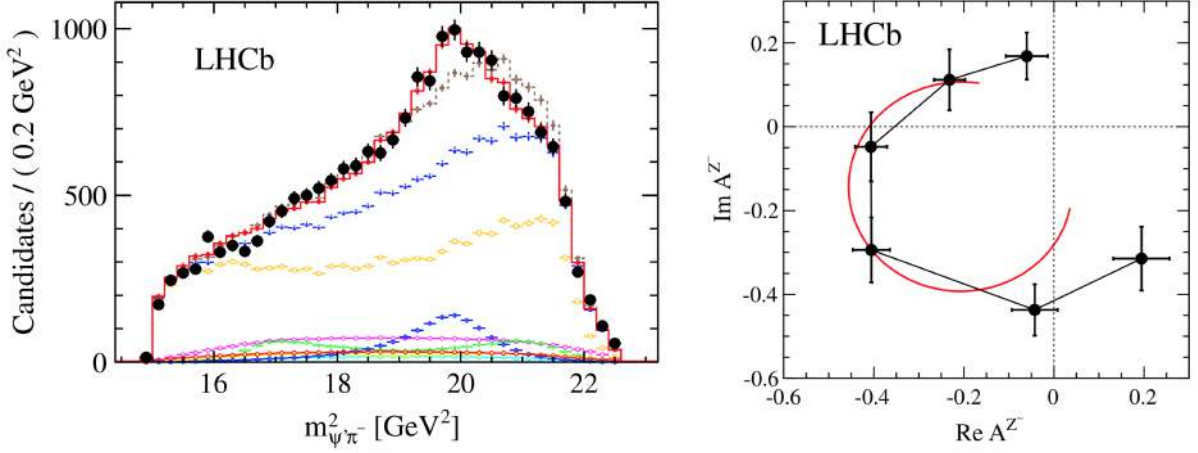


Figure 1.2.3: (left) The LHCb $M_{\psi(2S)\pi}^2$ distribution for all events (without any K^* veto), with projections from the 4D fits. The solid red histogram shows the fit with a $Z \rightarrow \psi(2S)\pi$ resonance term; the dashed brown histogram shows the fit with no resonance in the $M_{\psi(2S)\pi}$ channel. (right) The real (horizontal) and imaginary (vertical) parts of the $J^P = 1^+ Z \rightarrow \psi(2S)\pi$ amplitude for six mass bins spanning the 4430 MeV mass region. The red curve shows expectations for a BW resonance amplitude. Plots are taken from Ref. [10].

Model-independent confirmation of $Z(4430)$ by LHCb

A model-independent description of the $\psi(2S)\pi$ mass spectrum from $B^0 \rightarrow \psi(2S)K\pi$ has been obtained by LHCb using the $K\pi$ mass spectrum and angular distribution derived directly from data without the need of a theoretical description or modelling of resonance lineshapes and their interference effects [31]. It is found that the $\psi(2S)\pi$ mass distribution cannot be reproduced by reflections from the $K\pi$ channel by their model-independent approach that determines Legendre polynomial moments up to fourth order in $\cos \theta_{K^*}$ in bins of $K\pi$ mass, where θ_{K^*} is the $K\pi$ helicity angle, and reflects them into the $\psi(2S)\pi$ channel. A clear mismatch can be seen in the $Z(4430)$ mass region in Fig. 1.2.4, which shows a comparison of the $M_{\psi(2S)\pi}$ data with a fit that only uses reflections from the $\cos \theta_{K^*}$ moments, indicating the need for an exotic component to the spectrum.

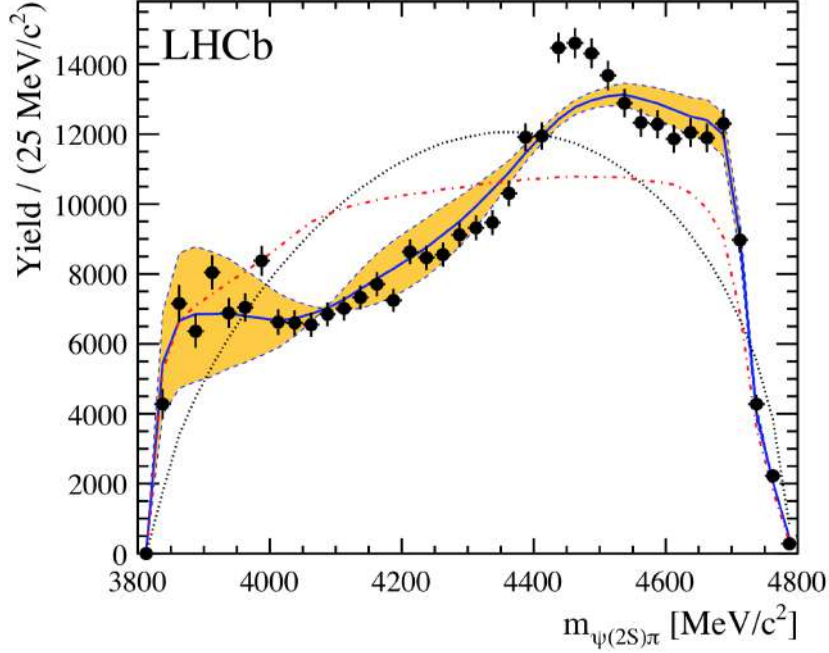


Figure 1.2.4: The data points in black show LHCb’s background-subtracted and efficiency corrected $M_{\psi(2S)\pi}$ distribution. The dotted black curve corresponds to MC distribution generated according to phase-space; in the dash-dotted red curve, the $M_{K\pi}$ is weighted to reproduce the experimental distribution. The solid blue curve shows the result of the fit using model-independent reflections from $\cos \theta_{K^*}$ moments up to fourth order. The golden shaded band indicates the range of uncertainties associated with the fit. The plot is taken from Ref. [31].

1.2.4 $Z(4200)$ and $Z(4430)$ in $B^0 \rightarrow J/\psi K \pi$

Belle performed a full 4D amplitude analysis on a nearly background free data sample of about 30K events from the $B^0 \rightarrow J/\psi K \pi$ decay channel and found a 6.2σ signal of a new resonance, the $Z(4200)$, in the $J/\psi \pi$ spectrum [11]. It is a broad resonance with mass and width $M = 4196^{+31+17}_{-29-13}$ MeV and $\Gamma = 370^{+70+70}_{-70-132}$ MeV, and a preferred quantum number assignment of $J^P = 1^+$. The channel also had a 4σ significant contribution from the $B^0 \rightarrow K Z(4430)$, $Z(4430) \rightarrow J/\psi \pi$. The fit results and their components are shown for different $K \pi$ slices in Fig. 1.2.5.

The fit results with and without the $Z(4430)$ (the $Z(4200)$ is already included in the model) are shown in the left and middle panels of Fig. 1.2.6. The rapid phase motion

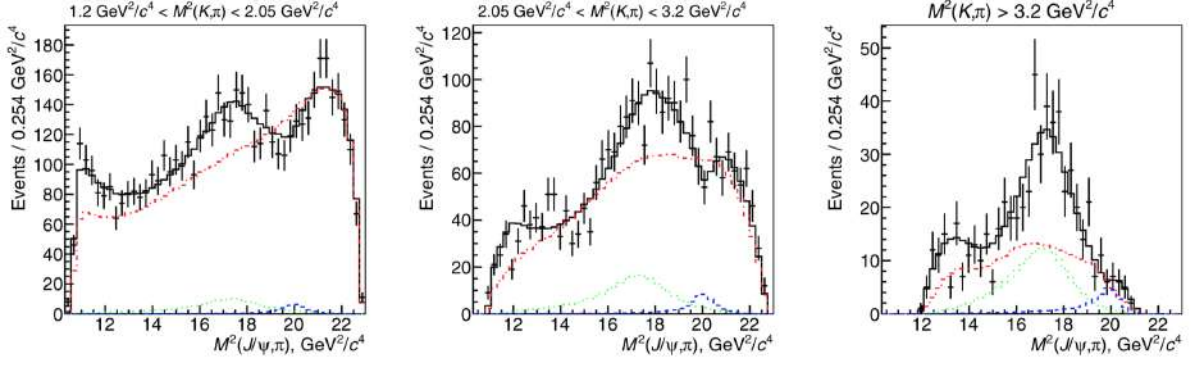


Figure 1.2.5: The fit results with the $Z(4200) J^P = 1^+$ in the default model. The points with error bars are data; the solid histograms are fit results, the blue dashed histograms are the $Z(4430)$ contributions, the green dotted histograms are the $Z(4200)$ contributions and the red dash-dotted histograms are contributions of all K^* resonances. The plots are taken from Ref. [11].

near 4200 MeV as seen in the right panel of Fig. 1.2.6 clearly indicates the BW nature of the peak.

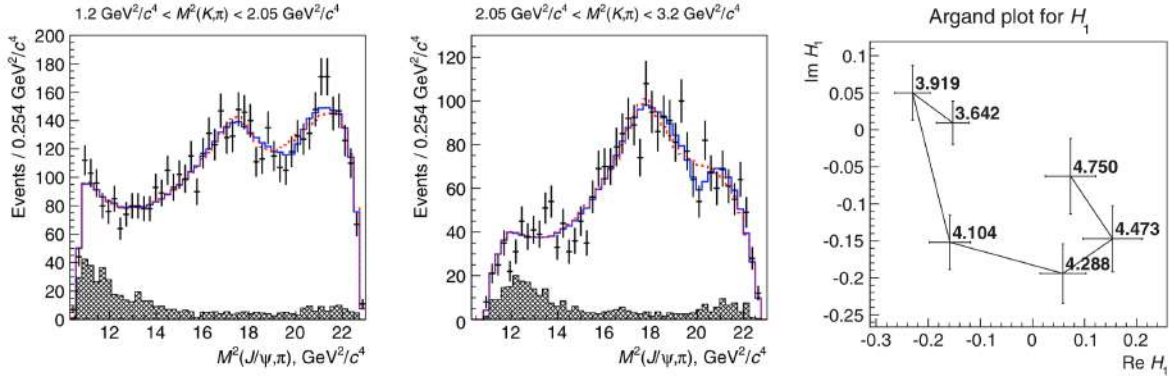


Figure 1.2.6: (left and middle) The fit results with (solid line) and without (dashed line) the $Z(4430)$ [the $Z(4200)$ included in the model]. The hatched histograms are the J/ψ sidebands. (right) The real (horizontal) and imaginary (vertical) parts of the $1^+ Z(4200) \rightarrow J/\psi\pi$ amplitude for six mass bins spanning the 4200 MeV mass region. The bin central mass values (in GeV) are shown near the points. The plots are taken from Ref. [11].

1.2.5 The $Z(3900)$ state

The $Z(3900)$ was seen as a resonance structure in the $J/\psi\pi$ mass distribution in the channel $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ at a center-of-mass energy of 4.260 GeV in BESIII [13]. A fit using a mass-independent-width BW function to represent the $J/\psi\pi$ mass peak yielded a mass and width of $M = 3899.0 \pm 6.1$ MeV and $\Gamma = 46 \pm 22$ MeV. The $Z(3900)$ was observed by Belle in isr data at the same time [14]. The $J/\psi\pi$ invariant mass distributions are shown in Fig. 1.2.7. The preferred quantum numbers are found out to be $J^P = 1^+$. Notably, no evidence of $Z(3900)$ was found in Belle's analysis of $B^0 \rightarrow J/\psi K\pi$ for floating mass values and any choice of J^P upto spin-2.

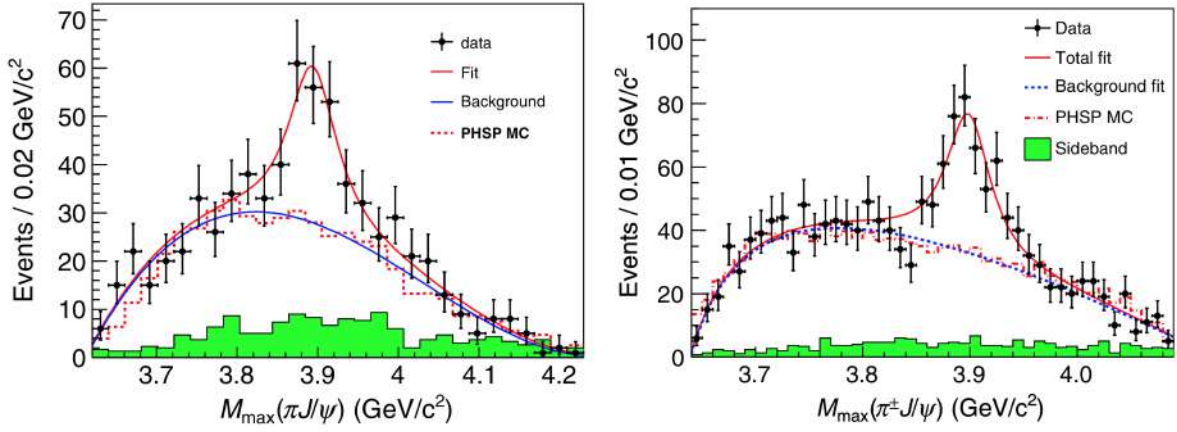


Figure 1.2.7: $J/\psi\pi$ invariant mass distributions from Belle (*left*) and BESIII (*right*) with prominent $Z(3900)$ peaks.

Focus of this thesis

This thesis focuses on the reconstruction and purification of $B^0 \rightarrow J/\psi K\pi$ events from proton-proton collisions at a centre-of-mass energy $\sqrt{s} = 8$ TeV collected by the CMS experiment in 2012 to search for exotic charged charmonium-like Z states. A full 4D amplitude analysis framework has been developed in the novel GPU-based GooFit to look for exotic contributions to the decay. Implementation and validation of this amplitude analysis framework are key aspects of this thesis.

CHAPTER 1. EXOTIC HADRON SPECTROSCOPY

The experimental setup

The European Organization for Nuclear Research, better known as **CERN** (acronym for *Conseil Européen pour la Recherche Nucléaire*), operates the largest particle physics laboratory in the world. Located on the Swiss-French border in Geneva, CERN runs particle accelerators and provides other infrastructures needed for research in high-energy physics, viz. powerful computing facilities primarily used to store and analyse data from experiments, as well as simulate events. A number of experimental facilities are constructed and operated at CERN through a massive international collaboration of about 2,500 scientific, technical, and administrative staff member as well as more than 12,000 visiting scientists, of which India is a prominent participant. The Compact Muon Solenoid (CMS) experiment is one of the two general purpose particle detectors at the Large Hadron Collider (LHC) of CERN. This chapter gives a general overview of the LHC followed by a somewhat detailed description of CMS.

2.1 The LHC

The LHC [32], one of the largest scientific instrument ever built, is a superconducting accelerator and collider of protons and heavy ions at TeV energy scales. It is built inside a 27 km long tunnel which used to house the erstwhile LEP (Large Electron-Positron Collider), at an average depth of 100 m below the surface. The LHC is the final element of the CERN accelerator complex (Fig. 2.1.1) that is a sequence of devices each of which accelerates particles up to a certain energy and injects the boosted particles into the next machine in the sequence to be accelerated to a higher energy.

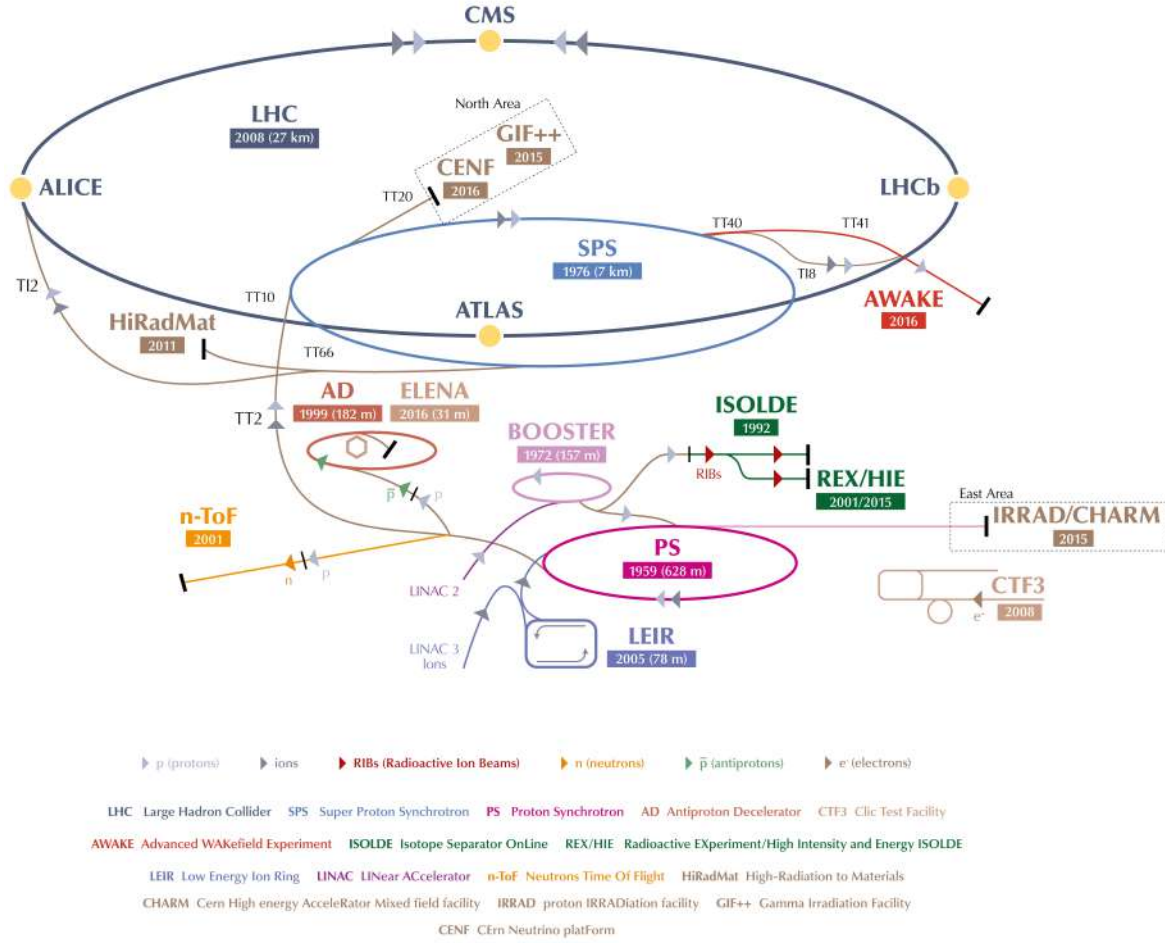


Figure 2.1.1: The CERN accelerator complex. The diagram is taken from Ref. [33].

The protons to be accelerated come from a bottle of hydrogen gas whose electrons are stripped off by applying an electric field. They are first injected into the LINAC2, where they are accelerated to 50 MeV. In the next step, they are pushed into the Proton Synchrotron Booster which accelerates the beam to 1.4 GeV and then into the Proton Synchrotron to be accelerated to 25 GeV. The protons are later transferred to the Super Proton Synchrotron and further accelerated up to 450 GeV, ready to be injected into the LHC. The protons flow through the two parallel beam pipes of the LHC in opposite directions and are accelerated to the target energy of 6.5 TeV in about 20 min.

The general layout of the LHC is shown in Fig. 2.1.2. The LHC is divided into eight arcs with the four interaction points (IPs) being located at the centers of the octants 1, 2, 5, and 8. The two general purpose experiments are located at diametrically opposite octants: the ATLAS experiment at IP 1 and the CMS experiment at IP 5. In 2010 and

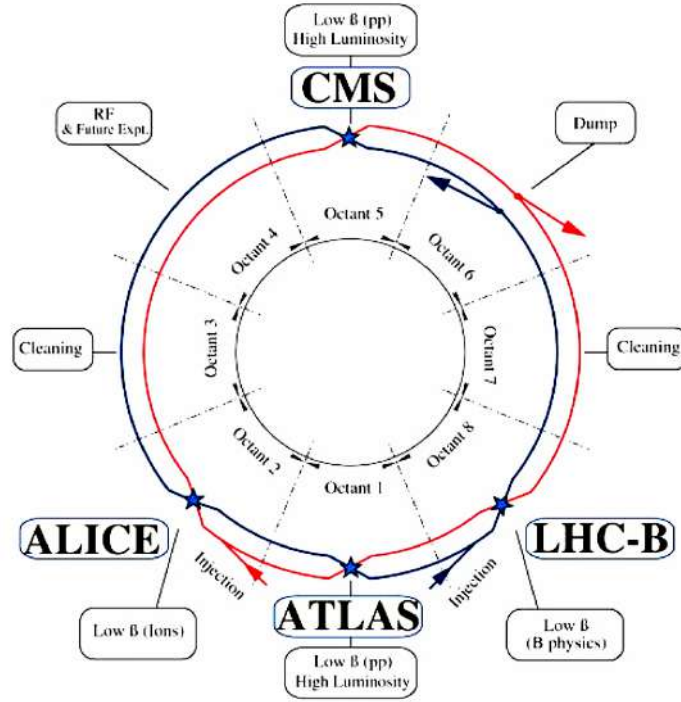


Figure 2.1.2: Schematic layout of the LHC. Two beams, shown in red and blue, are accelerated in opposite directions and made to cross each other at four interaction points where the four major experiments are built. The two general purpose detectors, CMS and ATLAS, are located diametrically opposite to each other in Point 5 and Point 1, respectively.

2011, the proton beams were made to collide at a center-of-mass energy $\sqrt{s} = 7$ TeV, i.e, each beam had an energy of 3.5 TeV. The collision energy was increased to 8 TeV in 2012 with a bunch spacing of 50 ns. This period of data taking was termed as Run-1. The LHC was shut down for two years for upgradation in spring 2013. The Run-2 collisions were started in June 2015 with $\sqrt{s} = 13$ TeV. The machine has again been shut down last december for further upgrades and will tentatively resume collisions in May 2021.

2.1.1 The LHC parameters

The LHC has 1232 superconducting dipoles with a peak magnetic field of 8.33 T and maintained at a temperature of 1.9°K. The current in main dipole is 11800 A and the total stored energy in the magnets is 11 GJ with an energy density of 500 kJ/m. At the interaction points, the beams cross each other at an angle of $285 \mu\text{rad}$. The LHC design

parameters for proton-proton and lead-lead collisions are listed in Table 2.1.1.

Table 2.1.1: Design parameters of LHC for proton-proton and lead-lead collisions [34].

			p-p	Pb-Pb
Energy/nucleon	E	TeV	7	2.76
Luminosity	$\mathcal{L}$	$\text{cm}^{-2}\text{s}^{-1}$	10^{34}	10^{27}
Luminosity lifetime	τ_L	hr	15	6
No. of bunches	n_b		2808	592
No. of particles/bunch	N_p		1.15×10^{11}	7.0×10^7
Bunch separation		ns	25	100
β -value at IP	β^*	m	0.55	0.5
RMS beam radius at IP	σ^*	μm	16.7	15.9

2.1.2 Luminosity

During the collisions, the number of events produced per unit time is proportional to the production cross section σ_{event} of that event, given as

$$N_{\text{event}} = \sigma_{\text{event}} \times \mathcal{L}, \quad (2.1.1)$$

where $\mathcal{L}$ is the instantaneous luminosity delivered by the LHC. It is a measure of the number of collisions per second, often given in units of $\text{cm}^{-2}\text{s}^{-1}$, and defined by:

$$\mathcal{L} = \frac{N_1 N_2 n_b f_{\text{rev}}}{A}. \quad (2.1.2)$$

Here, N_1 and N_2 are the number of particles in the two colliding bunches, n_b is the number of bunches in a beam, A is the transverse area of overlap of the two bunches, and f_{rev} is the revolution frequency of each bunch (designed to be 11245 Hz). For proton-proton collisions at the LHC, $N_1 = N_2 = N_p$. As the area of overlap transverse to the beam direction is difficult to measure in an accelerator, the transverse beam shape is assumed to follow a Gaussian distribution. Thus the instantaneous luminosity can be recast as

$$\mathcal{L} = \frac{N_p^2 n_b f_{\text{rev}} \gamma F}{4\pi \epsilon_n \beta^*}, \quad (2.1.3)$$

where γ is the relativistic Lorentz factor, ϵ_n is the normalised transverse beam emittance¹ (with a design value of $3.75 \mu\text{m}$), β^* is the betatron function at the IP, and F

¹ ϵ is a measure of the average spread of particle coordinates in position-and-momentum phase space

2.2. KINEMATIC VARIABLES AND COORDINATE SYSTEM

is the geometric reduction factor due to finite crossing angle at the IP. The integrated luminosity is the cumulative luminosity delivered by the machine over a total period of time T :

$$\mathcal{L}_{\text{int}} = \int_0^T \mathcal{L} dt. \quad (2.1.4)$$

To date, LHC has delivered 192.3 fb^{-1} (inverse-femtobarn²) of pp collision data for $\sqrt{s} = 7, 8$, and 13 TeV of which CMS has recorded 177.7 fb^{-1} as can be seen in the *right* panel of Fig. 2.1.3.

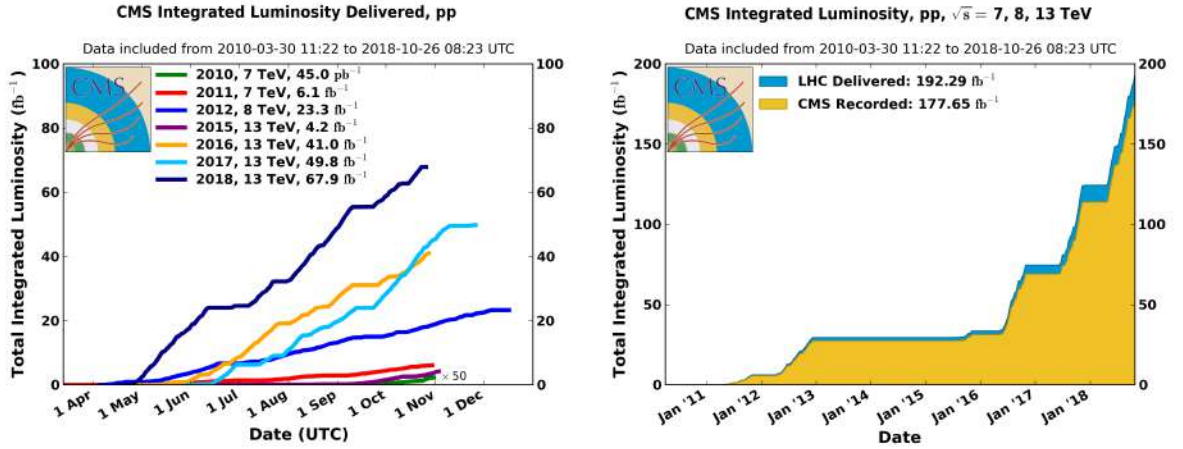


Figure 2.1.3: (*left*) Integrated luminosity for pp data versus day delivered to CMS during stable beams for pp collisions at nominal center-of-mass energy for 2010-2012 and 2015-2018. (*right*) Delivered and recorded luminosity cumulative over all years during stable beams for pp collisions at nominal center-of-mass energy in CMS [35].

2.2 Kinematic variables and coordinate system

The origin of the coordinate system adopted by CMS is the nominal collision point inside the detector with the y -axis pointing vertically upwards and x -axis pointing towards the center of the LHC ring, making the z -axis parallel to the counterclockwise proton beam. The azimuthal angle ϕ is measured from the x -axis in the x - y plane and the polar angle θ is measured with respect to the z -axis.

Since the proton is a composite particle, the exact energy of the colliding partons remains unknown and thus the total missing energy only in the plane transverse to the beam can be estimated. Moreover, the center-of-mass may be boosted along the

²1 b = $10^{-24} \text{ cm}^2 \implies 1 \text{ fb}^{-1} = 1 \times 10^{-39} \text{ cm}^2$

beam direction which requires the usage of kinematic variables invariant under such boosts. The momentum of a particle can be expressed in terms of two components, the longitudinal momentum p_z and the transverse momentum p_T , given by

$$p_T = \sqrt{p_x^2 + p_y^2}. \quad (2.2.1)$$

The rapidity, which is additive under Lorentz boosts along the z direction, is defined as

$$y = \frac{1}{2} \cdot \ln \left(\frac{E + p_z}{E - p_z} \right), \quad (2.2.2)$$

E being the energy of the particle. For highly relativistic particles ($p \gg m$), the rapidity can be expressed approximately in terms of pseudorapidity

$$\eta = -\ln \left(\tan \frac{\theta}{2} \right), \quad (2.2.3)$$

where θ is the same polar angle discussed earlier.

2.3 The CMS detector

The Compact Muon Solenoid (CMS) [36, 37] is a multipurpose detector designed to study a wide variety of phenomena produced in high energy particle collisions. It is located 100 m under the ground at IP5 of the LHC tunnel. It is a 28.7 m long, 14000 t cylindrical structure comprising a 21.6 m long main cylinder with a diameter of 15 m, and forward calorimeters towards both ends. The central barrel and the endcaps on the two ends ensure a nearly 4π hermetic coverage. A sectional view of the detector is presented in Fig. 2.3.1.

CMS has been designed keeping in mind the physics goals that can be achieved from the high energy pp (and also heavy ion) collisions at the LHC. One of the major considerations while designing the detector was the need for an accurate identification, reconstruction and measurement of momenta of muons and electrons. Another important goal was to precisely measure secondary vertices and impact parameters that are crucial for heavy flavour studies. A large number ($\sim 10^{11}$) of protons per bunch are squeezed into a narrow beam, and hence more than 20 inelastic collisions can happen per bunch crossing at the design luminosity. To minimise this deleterious pileup effect, high granularity detectors with low occupancy need to be used which in turn require not only huge number of detector channels but also an excellent synchronization among them. Also, a good time resolution is crucial to distinguish the interaction under study from those occurring in neighbouring bunch crossings due

2.3. THE CMS DETECTOR

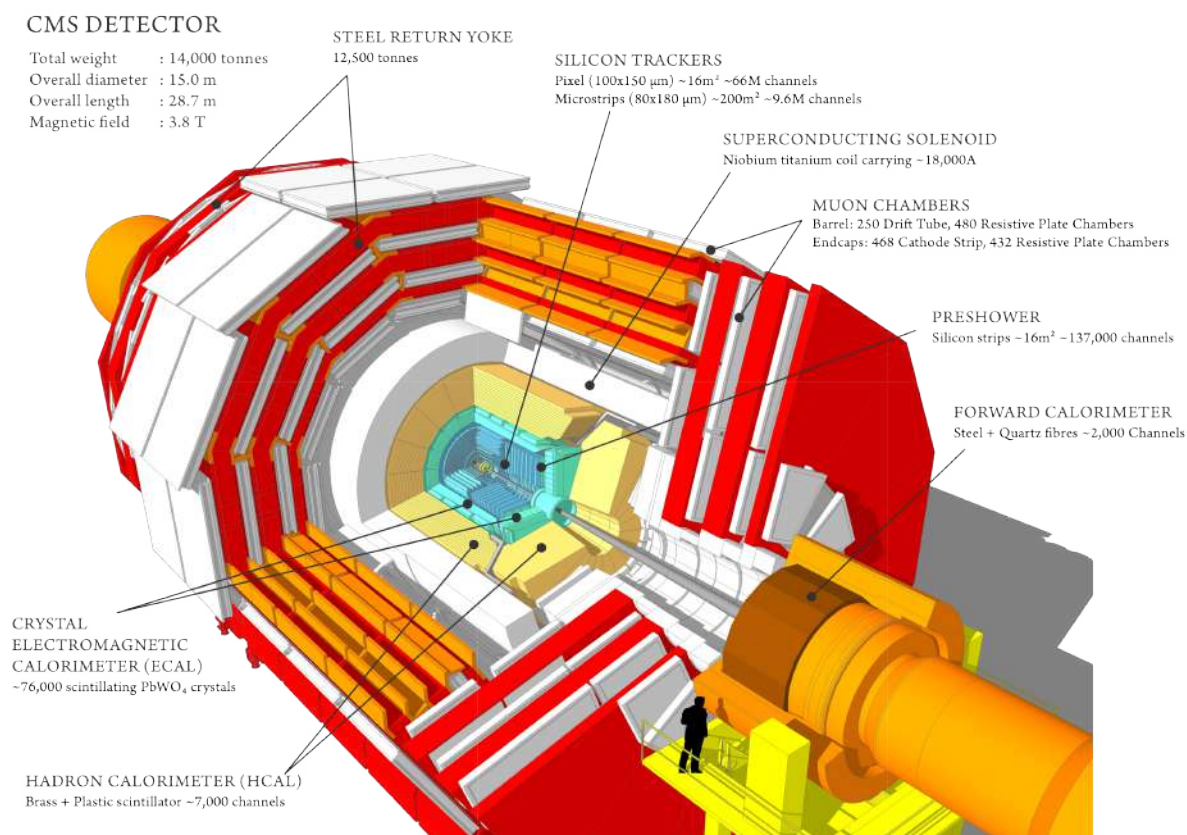


Figure 2.3.1: A sectional view of the CMS detector [38].

to a very high bunch crossing rate (40 MHz for pp collisions). A large flux of particles near the IP leads to high radiation levels, requiring the detectors and frontend electronics to be radiation hard. The salient features of the CMS detector are:

- a 4 T superconducting solenoid,
- a silicon based inner tracker (inside the solenoid),
- a homogeneous electromagnetic calorimeter (inside the solenoid),
- a sampling type hadron calorimeter (inside the solenoid, with a few additional layers outside),
- iron yokes outside the solenoid to absorb the return field (~ 2 T), and
- four muon stations integrated into the iron yokes.

A schematic diagram of a transverse slice of CMS with all the constituent subdetectors are shown in Fig. 2.3.2. The plot also demonstrates how different kinds of particles

originating from the collisions, viz. muons, electrons, neutral and charged hadrons, and photons interact with various subdetectors.

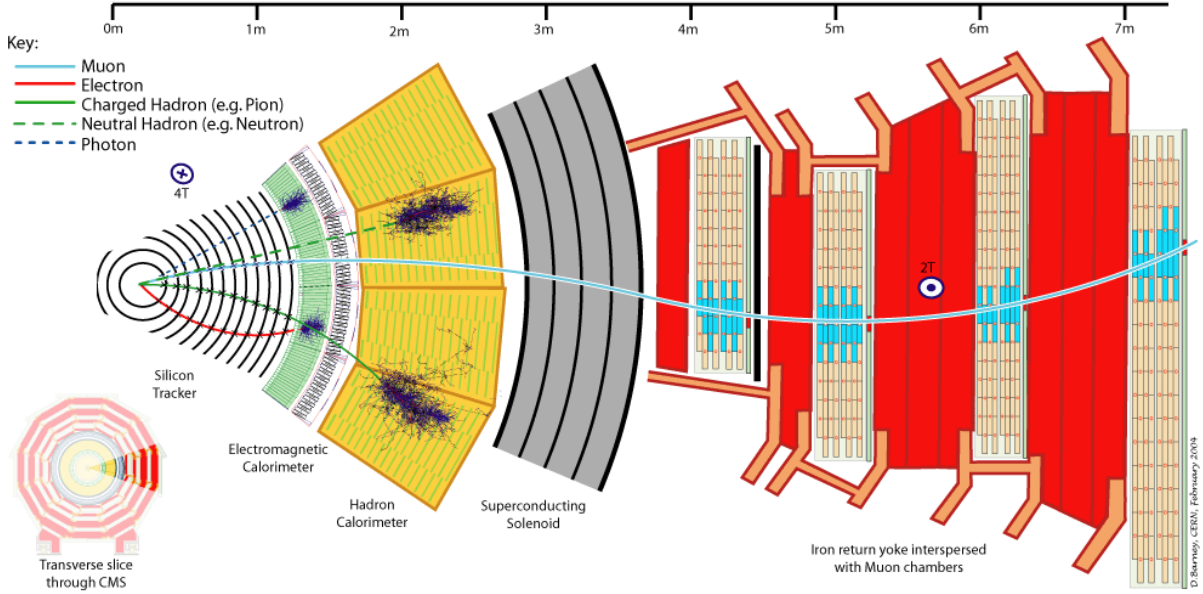


Figure 2.3.2: A transverse slice of the CMS detector showing how muons, electrons, neutral and charged hadrons, and photons interact with different subdetectors [39].

2.3.1 The CMS magnet

The CMS detector is built around the superconducting magnet, which was designed to reach a 4 T magnetic field in a free bore of 6 m diameter and 12.5 m length with a stored energy of 2.6 GJ at full current. The high magnetic field is generated by the 4-layer winding made from an aluminium - stabilised reinforced NbTi conductors maintained at a temperature of 4.5°K. The magnet weighs 220 t and the ratio between the stored energy and cold mass is extremely high (11.6 KJ/kg). This ensures a high enough bending power (12Tm) within a relatively small volume which is essential for an unambiguous determination of the sign of muons and high momentum resolution of any charged particle. Values of the magnetic field in different sections of the detector at 3.8 T can be seen in Fig. 2.3.3.

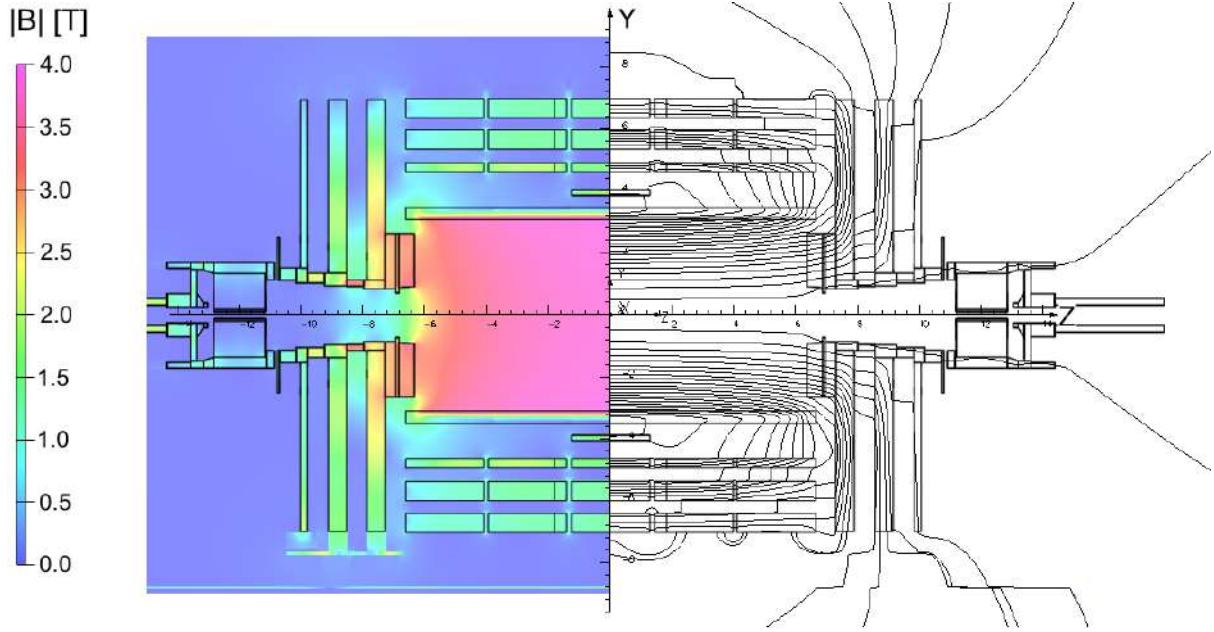


Figure 2.3.3: Value of $|B|$ (*left*) and field lines (*right*) on a longitudinal section of the CMS detector, at a central magnetic flux density of 3.8 T. Each field line represents a magnetic flux increment of 6 Wb. The plot is taken from Ref. [40].

2.3.2 The inner tracker

The silicon tracker is the subdetector closest to the beam pipe where the collisions occur. The aim of the CMS tracking system is to measure the trajectories of charged particles produced in the high energy collisions as well as to reconstruct secondary vertices. High instantaneous luminosity at the LHC results in thousands of particles crossing the tracking system per bunch crossing. This requires a high granularity detector technology with fast response and sufficient radiation hardness. Silicon crystals can be manufactured with high purity and is affordable for large scale production. Being a good thermal conductor, heat dissipation is faster in silicon. It also has a low threshold for electron-hole pair production and high hole mobility ensuring fast response. A sketch of the tracker layout (1/4 of the r - z view) is shown in Fig. 2.3.4.

The CMS tracker [41] has a cylindrical structure with an outer diameter of about 2.5 m and a total length of 5.8 m. It is immersed in the constant coaxial magnetic field of 3.8 T provided by the solenoid having its axis closely aligned with the LHC beam line. The tracker consists of two parts: a small silicon pixel detector followed by a large silicon microstrip system. In the central region, the pixel tracker is made of three coaxial barrel layers at radii between 4.4 and 10.2 cm whereas the strip tracker comprises

ten coaxial layers extending outwards to a radius of 110 cm. Both subdetectors are completed by endcaps on either end, each consisting of two disks in the pixel, and three small and nine large disks in the strip tracker. The endcaps extend the tracker acceptance of the tracker up to $|\eta| < 2.5$.

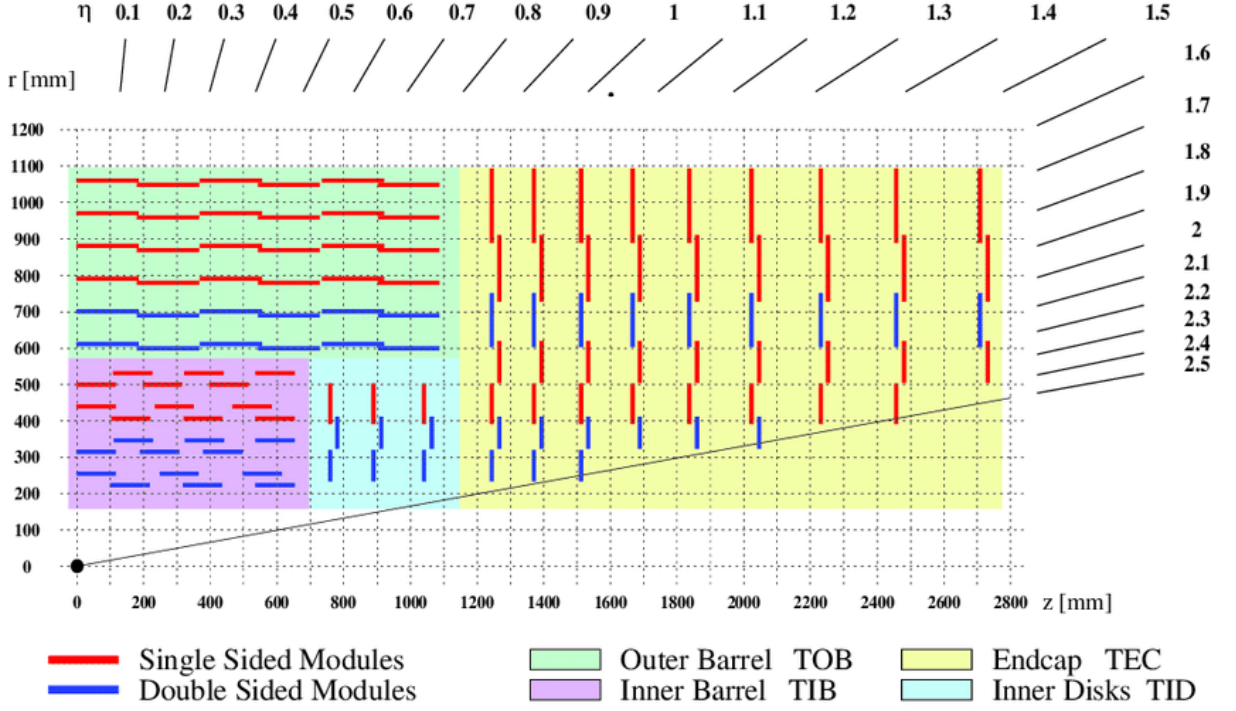


Figure 2.3.4: A sketch of the tracker layout (1/4 of the r - z view) (from [42])

Pixel detector

The pixel detector consists of cylindrical barrel layers at radii of 4.4, 7.3 and 10.2 cm, and two pairs of endcap disks at $z = \pm 34.5$ and ± 46.5 cm. It provides 3D position measurements of the hits arising from the interaction of charged particles with its sensors. The hit position resolution is approximately $10 \mu\text{m}$ in the transverse plane and $20\text{-}40 \mu\text{m}$ along the beam axis. In order to achieve the optimal vertexing resolution, a design with an “almost” square pixel shape of $100 \times 150 \mu\text{m}^2$ in both the (r, ϕ) and z coordinates has been adopted. The barrel is made of 768 pixel modules arranged into half-ladders of 4 identical modules. The endcap disks are assembled in a turbine-like geometry with blades rotated by 20° . In total, 1440 modules having 66 million pixels cover an area of about 1 m^2 .

Strip detector

The microstrip tracker has 15148 modules, which cover an active area of about 198 m² and have 9.3 million strips. It is composed of four subsystems. The Tracker Inner Barrel (TIB) and Disks (TID) cover $r < 55$ cm and $|z| < 118$ cm, and are composed of four barrel layers, supplemented by three disks at each end. These provide position measurements in $r\phi$ with a resolution of approximately 13-38 μm . The Tracker Outer Barrel (TOB) covers $r > 55$ cm and $|z| < 118$ cm, and consists of six barrel layers providing position measurements in $r\phi$ with a resolution of approximately 18-47 μm . The Tracker EndCaps (TEC) cover the region $124 < |z| < 282$ cm. Each TEC is composed of nine disks, each containing up to seven concentric rings of strip modules, yielding a range of resolutions similar to that of the TOB.

The modules in the TIB, TID and inner four TEC rings use 320 μm thick silicon, while those in the TOB and the outer three TEC rings use silicon of 500 μm thickness. In the barrel, the strips usually run parallel to the beam axis and have a pitch (the distance between two neighbouring strips) that varies from 80 μm in the inner TIB layers to 183 μm in the inner TOB layers. The endcap disks use wedge-shaped sensors with radial strips, whose pitch varies from 81 μm at small radii to 205 μm at large radii.

The modules in the innermost two layers of both the TIB and the TOB, as well as the modules in first and second rings of the TID, and first, second and fifth of the TEC, carry a second strip detector module, which is mounted back-to-back to the first one but rotated in the plane of the module by a “stereo” angle of 100 mrad. For the TIB, these stereo modules deliver 2D hit position, and lead to a single point resolution between 23 and 24 μm in the $r\phi$ plane and 230 μm along the z -axis. For the TOB, the stereo modules provide a single-point resolution ranging from 35-53 μm in the $r\phi$ plane and 530 μm along the z -axis.

The performance of the tracker is illustrated in Fig. 2.3.5, which shows the transverse momentum and impact parameter resolutions in the $r\phi$ and z planes for single muons with a p_T of 1, 10 and 100 GeV, as a function of pseudorapidity. Track reconstruction efficiency as a function of pseudorapidity for single muons and pions is shown in Fig. 2.3.6. The material inside the active volume of the tracker increases from $\approx 0.4X_0$ at $\eta = 0$ to around $1X_0$ at $|\eta| \approx 1.6$, before decreasing to $\approx 0.6X_0$ at $|\eta| = 2.5$.

2.3.3 The electromagnetic calorimeter

The Electromagnetic Calorimeter (ECAL) [43] is a hermetic, homogeneous calorimeter made of lead tungstate (PbWO_4) crystals and divided into two parts, barrel and end-

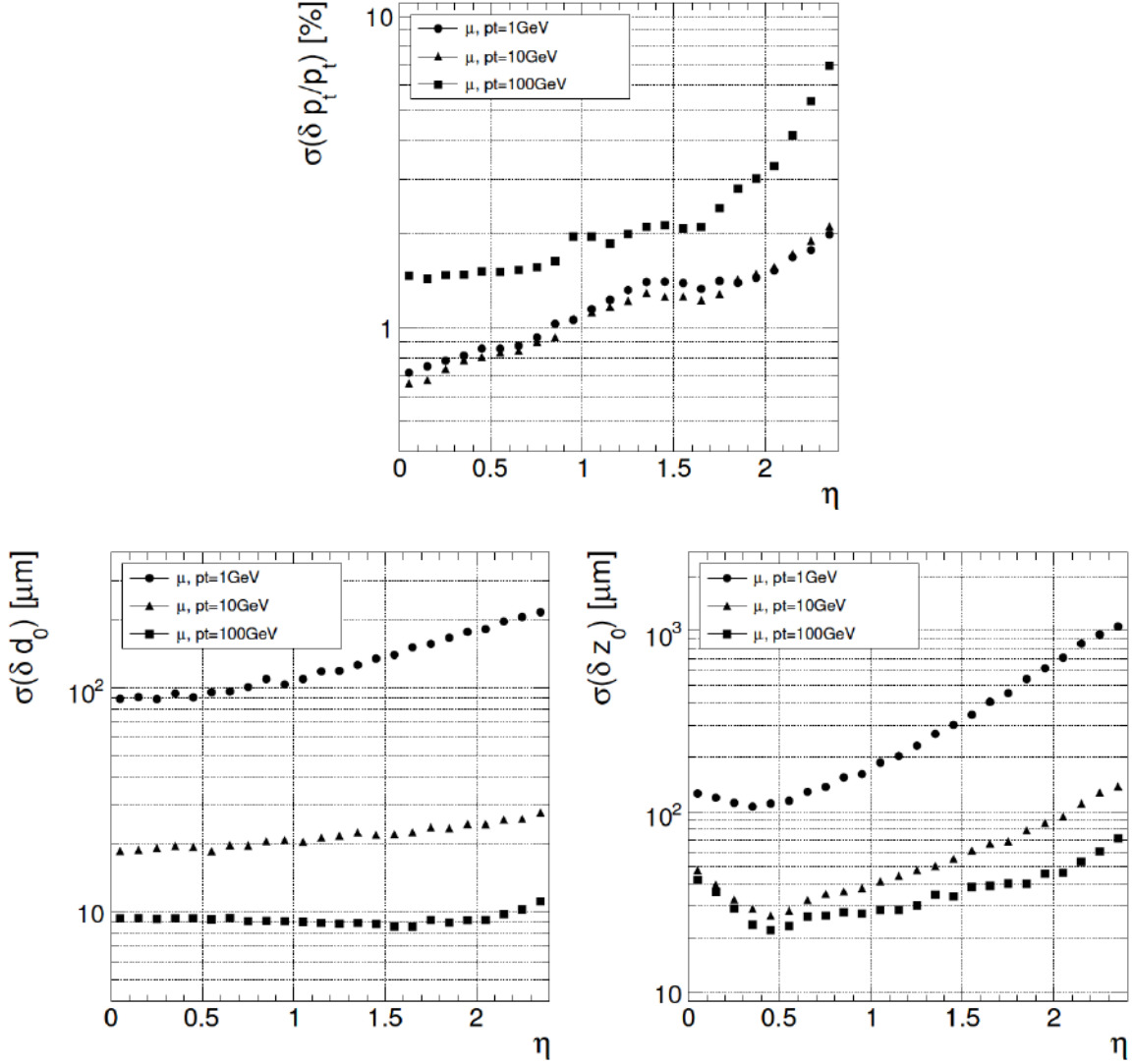


Figure 2.3.5: Resolution of various track parameters for single muons with p_T of 1, 10 and 100 GeV: (upper) p_T , (lower left) transverse, and (lower right) longitudinal impact parameters [41].

cap. The PbWO_4 scintillator have short radiation length ($X_0 = 0.89$ cm) and Moliere length (2.2 cm), are fast (80% of the light is emitted within 25 ns) and radiation hard (up to 10 Mrad). Its low light yield ($30\gamma/\text{MeV}$) is amplified by high gain photodetectors than can operate in a high magnetic field. Silicon avalanche photodiodes are used as photodetectors in the barrel and vacuum phototriodes in the endcaps. The choice of PbWO_4 crystals has thus allowed the design of a compact calorimeter inside the solenoid which is fast, has fine granularity, and is radiation hard. The barrel section consists of 61200 crystals and has an inner radius of 129 cm that covers the pseudora-

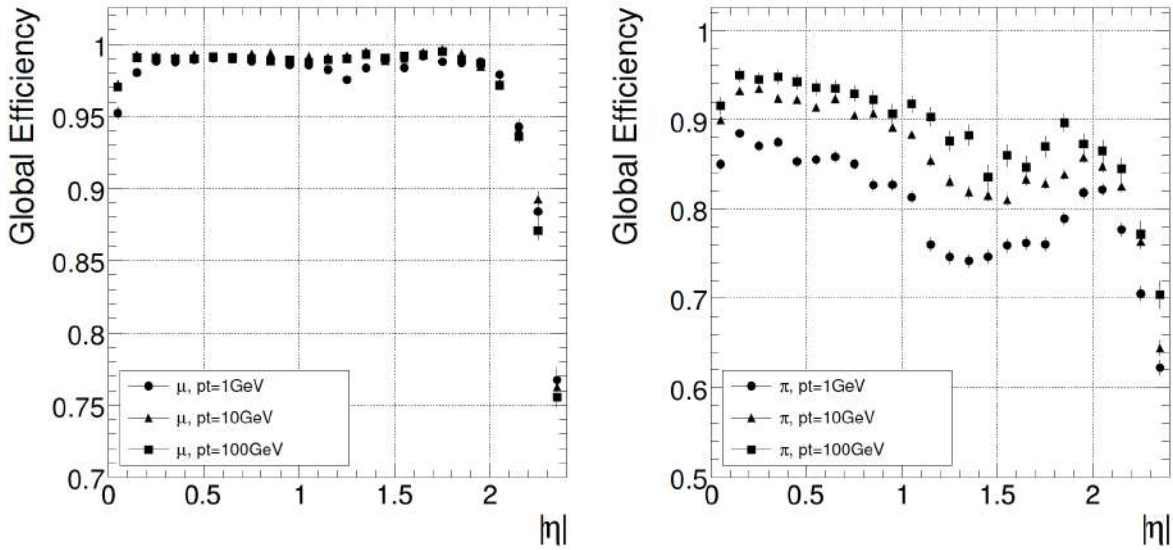


Figure 2.3.6: Global track reconstruction efficiency for muons (*left*) and pions (*right*) of p_T of 1, 10 and 100 GeV [41].

pidity interval of $0 < |\eta| < 1.479$. The crystals are quasi-projective (the axes are tilted at 3° with respect to the line from the nominal vertex position) and cover 0.0174 (i.e. 1°) in $\Delta\phi$ and $\Delta\eta$. The crystals have a front cross-section of $\approx 22 \times 22 \text{ mm}^2$ and a length of 230 mm, accounting for $25.8X_0$. The endcaps are made of 7324 crystals each and are located at a distance of 314 cm from the vertex on either end. They cover the pseudorapidity region $1.479 < |\eta| < 3.0$. The endcap crystals, like the barrel ones, off-point from the nominal vertex position, but are arranged in an x - y grid instead of an η - ϕ grid. They are all identical and have a front cross section of $28.6 \times 28.6 \text{ mm}^2$ and a length of 220 mm ($24.7X_0$).

2.3.4 The hadron calorimeter

The CMS hadron calorimeter (HCAL) is designed to estimate quark, gluon and neutrino directions and energies by measuring the energy and direction of particle jets and of the missing transverse energy flow.

The HCAL design maximizes material inside the solenoid in terms of interaction lengths. This is complemented by an additional layer of scintillators, referred to as the hadron outer (HO) detector, located just outside the solenoid. Brass has been chosen as the absorber material as it has a reasonably short interaction length and is nonmagnetic. Maximizing the amount of absorber inside the magnet requires minimizing the space for the active medium - the tile/fibre assembly. It consists of plastic scintilla-

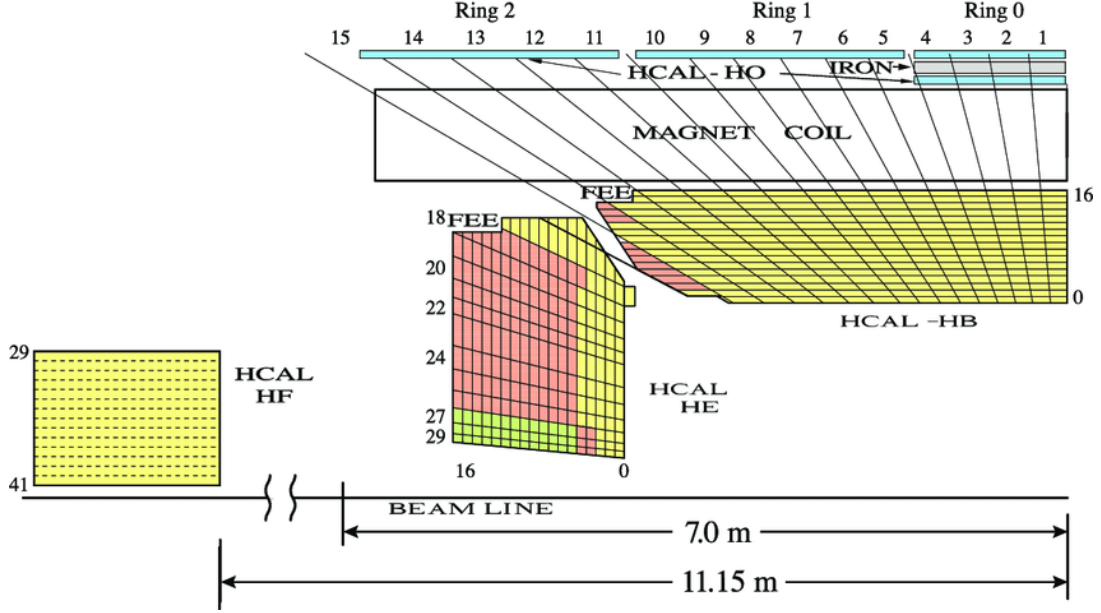


Figure 2.3.7: Geometry of the HCAL, for a quadrant of the CMS detector [44].

tor tiles read out with embedded wavelength-shifting (WLS) fibres. The WLS fibres are spliced to high-attenuation-length clear fibres outside the scintillator that carry the light to the readout system. The photodetection readout is based on multi-channel hybrid photodiodes. The hadron barrel (HB) part of HCAL covers the pseudorapidity region $|\eta| < 1.4$ with 2304 towers with a segmentation $\Delta\eta \times \Delta\phi = 0.087 \times 0.087$. The HB is read out as a single longitudinal sampling. There are 15 brass plates, each with a thickness of about 5 cm, plus 2 external stainless steel plates for mechanical strength.

The HO detector contains scintillators with a thickness of 10 mm, which lie outside the outer vacuum tank of the coil and cover the region $|\eta| < 1.26$. They sample the energy from penetrating hadron showers leaking through the rear of the calorimeters and so serve as a "tail-catcher" after the magnet coil. Each hadron endcap (HE) of HCAL consists of 14 η towers with 5° ϕ segmentation, covering the pseudorapidity region $1.3 < |\eta| < 3.0$. For the five outermost towers (at smaller η) the ϕ segmentation is 5° and the η segmentation is 0.087. For the eight innermost towers the ϕ segmentation is 10° , whilst the η segmentation varies from 0.09 to 0.35 at the highest η . The total number of HE towers is 2304.

The barrel and the endcap parts are complemented by a very forward calorimeter (HF), placed at ± 11.2 m from the interaction point, which extends the pseudorapidity range of the calorimetry up to $|\eta| < 5.2$. Due to high particle flux in the forward region, a radiation hard technology, using Cherenkov light in quartz fibres, was chosen

with steel as an absorber. The HF detector is also used as a real-time monitor for the luminosity on a bunch-by-bunch basis.

2.3.5 The muon system

The muon system placed outside the magnet provides an independent muon identification, robust trigger, and accurate measurement of momentum and charge, for muons with momenta ranging from a few GeV to a few TeV. It consists of four stations of gas-based detectors integrated into the iron flux return yoke so that both the 3.8 T magnetic field inside the solenoid and the 1.8 T average return field can be combin- ingly used for bending the muons. It is composed of three independent subdetectors. In the barrel ($|\eta| < 1.1$), where the track occupancy and residual magnetic field are low, drift tubes (DTs) are installed. In the endcaps, where the particle rate is higher and a large residual magnetic field is present, cathode strip chambers (CSCs) are used. The coverage of the DT and CSC systems goes up to $|\eta| < 2.4$. In the region $|\eta| < 1.8$, resistive plate chambers (RPCs) are also present. A schematic diagram of the muon system can be seen in Fig. 2.3.8.

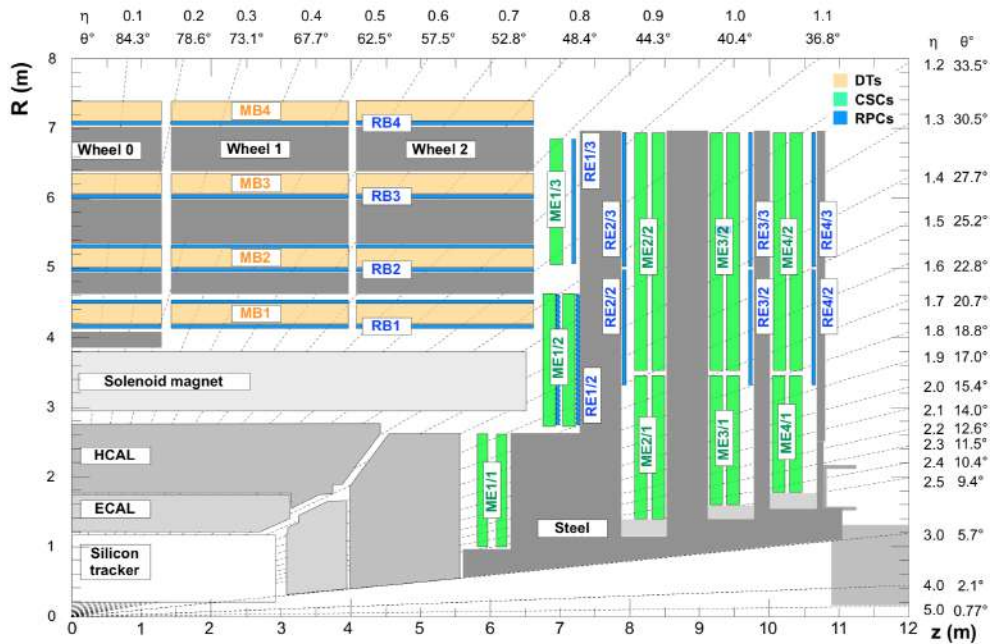


Figure 2.3.8: Geometry of the muon system, for a quadrant of the CMS detector [45].

RPCs are operated in avalanche mode to ensure good operation at high rates (up to 10 kHz/cm²), and provide a fast response with good time resolution but with a

coarser position resolution than the DTs or CSCs. They can therefore identify unambiguously the correct bunch crossing. The DTs or CSCs, and the RPCs operate within the level-1 trigger system, providing two independent and complementary sources of information. The combined system results in a robust, precise and flexible trigger device.

The DTs are aluminum tubes with a stainless steel anode wire in the middle. The segmentation of the barrel muon system along the beam direction follows the wheel structure of the yoke. There are four stations of DTs, interspersed among the layers of the iron yoke plates. The first three of them consist of 12 layers drift chambers each. The DT chambers in each station are further grouped into three superlayers, each containing four layers. The upper and lower superlayer measure the muon coordinates in the $r\phi$ bending plane while the middle one provide the muon coordinates along the z direction. The fourth station is composed of only two $r\phi$ measuring superlayers.

The Muon Endcap (ME) system comprises 468 CSCs in the two endcaps. Each CSC is trapezoidal in shape and consists of six gas gaps, each having a plane of radial cathode strips and a plane of anode wires running almost perpendicular to the strips. All CSCs except those in the third ring of the first endcap disk (ME1/3) are overlapped in ϕ to avoid gaps in the muon acceptance. The gas ionization and subsequent electron avalanche caused by a charged particle traversing each plane of the chamber produces a charge on the anode wire as well as an image charge on a group of cathode strips. The signal on the wires is fast and used at the Level-1 Trigger. However, it results in a coarser position resolution. A precise position measurement is made by determining the centre-of-gravity of the charge distribution induced on the cathode strips. Each CSC measures up to six space coordinates (r, ϕ, z) . The spatial resolution provided by each chamber from the strips is about $200 \mu\text{m}$ ($100 \mu\text{m}$ for ME1/1). The angular resolution in ϕ is of the order 10 mrad.

Muon system performance

Some representative figures of merit for the muon system performance [46] include:

- reconstructed hit spatial resolution $\approx 50 - 300 \mu\text{m}$;
- reconstructed hit efficiency $\approx 94 - 99\%$;
- segment timing resolution $< 3 \text{ ns}$;
- segment efficiency $\approx 97\%$;

- trigger bunch crossing identification $> 99\%$;
- trigger efficiency $> 90\%$;
- muon timing resolution ≈ 1.4 ns;
- muon reconstruction and identification efficiency $> 96\%$;
- muon isolation efficiency $> 95\%$;

The detector performance so far has remained within the design specifications and the muon reconstruction results are well reproduced by Monte Carlo simulation.

2.4 The trigger system

The LHC provides proton-proton collisions with a bunch crossing period of 25 ns which corresponds to a frequency of 40 MHz. At the designed luminosity, approximately 10^9 interactions occur per second resulting in a raw data rate of 40 TByte per second. But the peak storage rate for CMS is approximately 10 TByte per day (100 Hz). Therefore, the data volume must be reduced by a factor of 4×10^5 before the information can be written to archival media. Such a reduction is achieved by the CMS trigger and data acquisition system [47, 48] that ensures high data recording efficiency for a wide variety of physics objects and event topologies by applying selective online requirements.

CMS uses a two-level trigger design: the Level-1 (L1) trigger, built from custom hardware, which reduces the rate to a maximum of 100 kHz, and the High Level Trigger (HLT), running the reconstruction software on a processor farm, which performs higher level reconstruction and reduces the rate of events selected by the L1 trigger to about 400 Hz before the events are stored on disk. A schematic diagram of the trigger workflow in CMS is shown in Fig. 2.4.1.

Level-1 trigger

Due to the size of the LHC detectors and underground caverns, a minimum amount of transit time is required for signals to travel back and forth between the detector front-end electronics and the L1 trigger logic systems housed in the services cavern. The total time allocated for the transit and for reaching a decision to keep or discard data from a particular beam crossing is $3.2 \mu\text{s}$. During this time, the detector data are held in buffers while trigger data are collected from the frontend electronics to decide

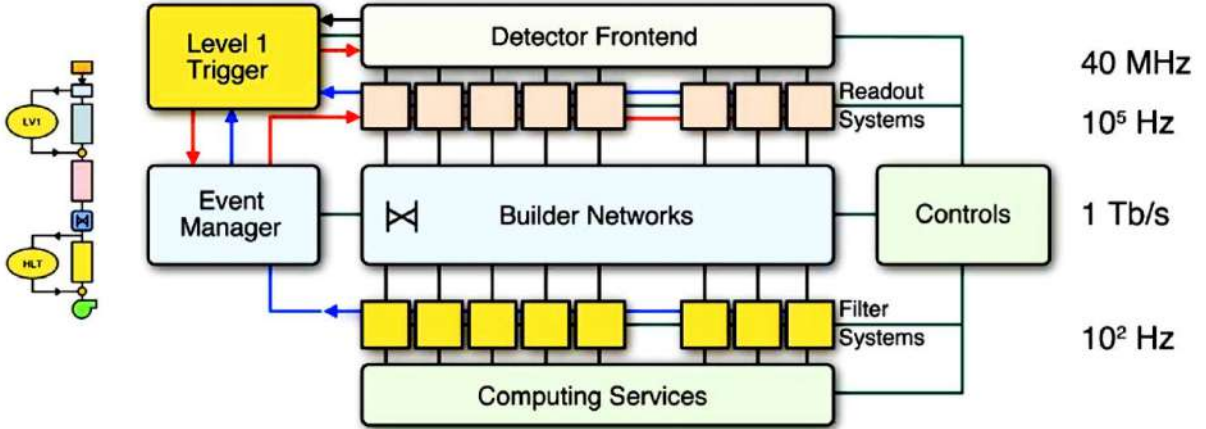


Figure 2.4.1: The CMS trigger and data acquisition system [49]. Data from a subset of the detector are first sent to the level-1 trigger for processing. Upon acceptance, the remaining data are read out to the High-level trigger for further processing.

which fraction of events are to be kept (typically 1 crossing in 1000) and the rest to be discarded. Of the total latency, the time allocated to L1 trigger calculations is less than $1 \mu\text{s}$.

The L1 trigger decisions are taken by custom hardware processors involving the calorimeters and muon systems, as well as some correlated information from both the systems. The coarsely segmented data from the calorimeters and muon systems are used to form the trigger “trigger primitive” objects and checked if they are above set E_T or p_T thresholds. These L1 seeds can have single object conditions or more complex structures (mixed, multiple-object) like $\Delta\eta$ between two objects and/or their invariant mass thresholds. For the triggering purposes, the calorimeters are subdivided into trigger towers. The trigger primitives are built by combining the HCAL readout towers or ECAL crystals. They also employ global sums of E_T and E_T^{miss} . In the muon systems, all three sub-detectors, namely DT, CSC and RPC take part in trigger. Each of them use their own trigger logic to deliver the muon trigger candidates which are then combined by the global muon trigger to achieve an improved momentum resolution and efficiency.

High-level trigger

All events satisfying the L1 triggers are transferred to a computer farm where the software based offline triggers (HLT) further filter the events using different reconstruction algorithms. Only events that pass at least one of the HLT conditions are kept for permanent storage thereby reducing the event rate from 100 kHz to 100 Hz, which is the accepted rate for storage. The multistage iterative algorithms of HLT perform reconstruction and selection steps of increasing complexity in order to discard events that lie within our current understanding, while simultaneously keeping a threshold for acceptance that is sufficiently low to allow all possible signatures of new physics to be recorded. The HLT codes are flexible enough to adapt to changes in data-taking conditions, like changes in luminosity or special conditions occurring during the CMS commissioning or dedicated LHC fills.

Each processor of the HLT farm can access the full raw L1 data and analyzes an event with a total processing time of ~ 1 s/event. Trigger objects are reconstructed only if they are needed for making the trigger decision. Fast algorithms help avoid unnecessary calculations by rejecting events as early as possible by using several stages of virtual trigger levels (L2 and L3) for selection and reconstruction. There is no limitation on the number of algorithms deployed or the number of virtual levels but the CPU time is limited. The L2 trigger sequence is used for the first HLT stage based on data from the calorimeters and muon systems, while the algorithms using the tracker data constitute the L3 sequence. The HLT algorithms process only parts of the detector information, which contain trigger objects found in the preceding trigger levels. Reconstruction is aborted in case the result cannot be altered by any further calculations. They provide online detector monitoring (specific trigger paths are designed for calibration and alignment) and are robust with respect to changes in alignment, calibration constants and pileup.

The HLT decision is taken as a logical “OR” of many independent trigger paths. Each path runs independently from the others (in parallel). The CMS software framework ensures that the same reconstruction block is not run twice and that all trigger paths are always run. The current HLT menu has ~ 600 independent paths. The HLT decision is used to split the events into online streams (for calibration and monitoring) and datasets (for further offline analysis) according to the needs of the different physics analysis groups (PAGs) e.g., MuOnia is the primary data set for b-physics (BPH) PAG. The PAGs are responsible for the development, maintenance and validation of their own HLT paths and datasets. The datasets are originally stored in the

CERN Tier0 of the worldwide LHC computing grid network and are eventually distributed around the entire network to be easily accessible by the members of the CMS collaboration all over the globe.

2.4.1 B-physics triggers

In contrast to other physics programs at CMS, the BPH PAG has different and peculiar trigger needs viz. low- p_T dimuons with opposite signs, displaced vertices, specific mass windows and so on. Hence the BPH trigger space is quite pure i.e, same events are rarely triggered by more than one trigger path (Fig. 2.4.2), and has a bandwidth quota of $\sim 10\%$ of the total.

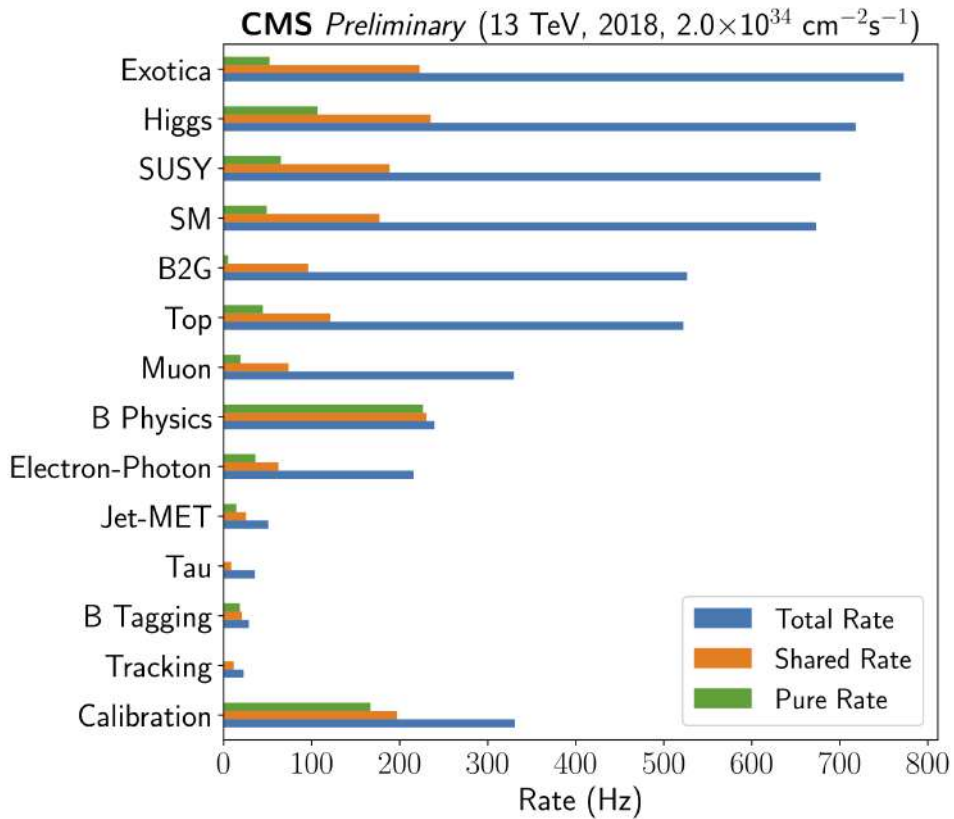


Figure 2.4.2: Comparison of the pure rates (in Hz) for the different physics analysis groups of CMS [50].

With ever increasing luminosity at the LHC, it is very difficult to keep a low p_T threshold for triggered muons due to hardware and computational limitations. The BPH PAG exploits correlations between trigger objects at the L1 level to keep the rate under control even with lower p_T thresholds. The η restrictions between muon objects

are already in place for many L1 seeds of BPH trigger paths and efforts are going on to implement invariant mass cuts at the L1 level. A graphical representation of the events collected by the physics triggers relevant for low mass dimuon physics can be seen in Fig. 2.4.3.

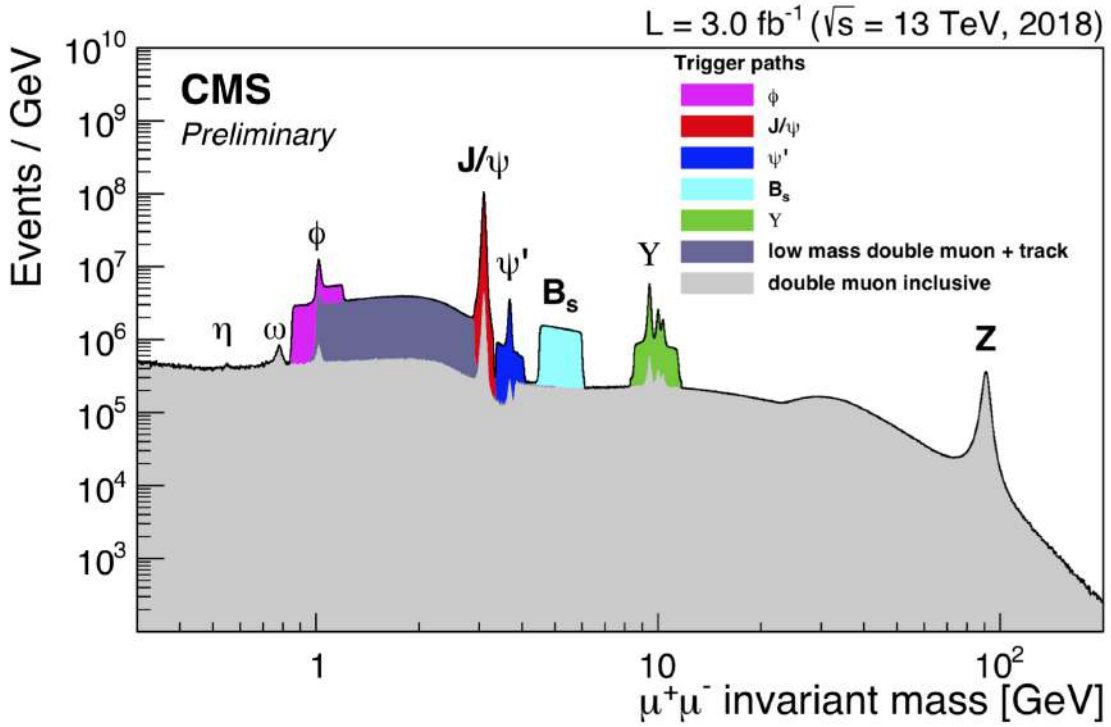


Figure 2.4.3: Dimuon mass distribution collected with various dimuon triggers at 13 TeV in 2018 with 3 fb⁻¹ of pp collision data. The coloured paths correspond to dedicated dimuon triggers with low p_T thresholds, in specific mass windows, while the light gray continuous distribution represents events collected with a dimuon trigger with high p_T thresholds [51].

2.5 The software framework

Collider experiments like CMS require robust and challenging softwares which are required to process and analyze the raw data collected by the readout electronics of various detector components. The CMS software framework, referred to as CMSSW [37, 52], is built around a Framework, Event Data Model (EDM), and services needed by simulation, calibration and alignment of the detector; reconstruction modules that

process event and analysis data; and the support of a distributed computing infrastructure. The main goal of the Framework and EDM is to facilitate the development and deployment of reconstruction and analysis software. The CMS EDM is centered around the concept of an Event. An Event is a C++ object container for all raw and reconstructed data related to a particular collision. During processing, data are passed on from one module to the next via the Event, and are accessed only through the Event. The CMSSW event processing model consists of one executable (`cmsRun`) and several plugin modules which are managed by the Framework. All the code needed in the event processing (calibration, reconstruction algorithms, etc.) are contained in the modules (C++ classes). The same executable is used for both detector and Monte Carlo simulations. The CMSSW framework facilitates the use of external software packages written in other languages, like event generation packages. The framework is also used for HLT processing.

Reconstruction of $B^0 \rightarrow J/\psi K \pi$

The reconstruction of events corresponding to the $B^0 \rightarrow J/\psi K \pi$ decay requires careful treatment of data using specific triggers and analysis techniques many of which are specific to b-physics analysis. This chapter discusses the data samples and triggers used, the analysis workflow including the cuts used for purification of data, studies of reflection backgrounds and application of kinematic fit.

3.1 Data sample, trigger and preselection

3.1.1 Analysis workflow and data sample

The data used for the analysis are a subset of all registered events produced in pp collisions at $\sqrt{s} = 8$ TeV in 2012 by the CMS experiment. The lower collision energy (as compared to 13 TeV from 2015 onwards) ensures a low p_T threshold for the triggers which is essential for this kind of analysis. The CMS reconstruction software version CMSSW 5_3_22 has been used to process these data. The dataset contains all dimuon triggers including the ones with low dimuon p_T thresholds and without any requirement on a displaced secondary vertex. The reconstruction is performed on all the events contained in runs and their luminosity sections certified by the CMS Data Quality Monitoring (DQM) standards. The DQM is performed online and offline, taking into account the status and behaviour of each detector subsystem, quality of reconstructed physics objects (muons, jets, electrons etc.) and higher level quantities (e.g., dilepton mass peaks), correlation between different physics objects or simple

kinematic distributions with more analysis oriented cuts. Only those luminosity sections (sub-section of a run during which the instantaneous luminosity is unchanging) that pass the stringent DQM conditions are certified to be used for subsequent physics analysis. The integrated luminosity is about 19.6 fb^{-1} for the 2012 data sample, which is divided into four sub-samples, listed below with their corresponding luminosity and run range:

- A (0.4 fb^{-1}) [190456 - 193621]
- B (4.3 fb^{-1}) [193833 - 196531]
- C (7.3 fb^{-1}) [198022 - 203742]
- D (7.6 fb^{-1}) [203777 - 208686]

The datasets of simulated Monte Carlo (MC) events used in this analysis are listed in Table 3.1.1. The sample in the first row is a signal MC sample generated according to the phase-space model with the purpose of the reconstruction efficiency evaluation. The same reconstruction workflow has been run over real and simulated samples, and it is referred as “default reconstruction” hereafter.

Table 3.1.1: Officially produced MC datasets used in this analysis. The first one consists of a signal sample generated according to the phase-space model.

MC type	Decay channel	# events
exclusive	$B^0 \rightarrow J/\psi K^+ \pi^-$	111×10^6
inclusive	$B^0 \rightarrow \psi(\rightarrow \mu^+ \mu^-) + X$	50.6×10^6
inclusive	$B_s^0 \rightarrow \psi(\rightarrow \mu^+ \mu^-) + X$	14.9×10^6
inclusive	$B^+ \rightarrow \psi(\rightarrow \mu^+ \mu^-) + X$	58.6×10^6
inclusive	$\Lambda_b \rightarrow \psi(\rightarrow \mu^+ \mu^-) + X$	2.3×10^6

3.1.2 Event preselection

The selection of $B^0 \rightarrow J/\psi K\pi$ events proceeds in multiple steps. Events are initially selected based on relaxed conditions followed by more stringent and sophisticated methods to attain high purity. The loose initial selection *aka* preselection implemented in the analyser is discussed in this section.

Inclusive J/ψ trigger

The trigger versions used in the analysis are listed in Table 3.1.2. Other than the com-

Table 3.1.2: Time evolution of the inclusive J/ψ trigger through the 2012 data taking period.

Menu	Trigger bit	Run range	L1 seed
5×10^{33}	HLT_Dimuon8_Jpsi_v3	190456 - 193621	L1_DoubleMu_5er_0er_HighQ_WdEta22
7×10^{33}	HLT_Dimuon8_Jpsi_v4	193833 - 194712	L1_DoubleMu0er_HighQ
7×10^{33}	HLT_Dimuon8_Jpsi_v5	194735 - 196531	L1_DoubleMu0er_HighQ
7×10^{33}	HLT_Dimuon8_Jpsi_v6	198022 - 198522	L1_DoubleMu0er_HighQ
8×10^{33}	HLT_Dimuon8_Jpsi_v7	203777 - 208686	L1_DoubleMu0er_HighQ

mon $p_T > 7.9$ GeV cut for the dimuon object, these trigger paths are characterised by:

1. a “pileup” protection through a distance of closest approach cut between the two muons to be less than 0.5 cm,
2. a cut on the vertex χ^2 probability at 0.5%, and
3. the dimuon invariant mass to lie in the window 2.8–3.5 GeV.

We have also the provision for the reconstruction of the $B^0 \rightarrow \psi(2S)K^+\pi^-$ signal for future studies. An inbuilt switch can select **either** of the inclusive J/ψ **or** $\psi(2S)$ trigger bits to be fired. The $\psi(2S)$ triggers are essentially similar to the J/ψ one as they share the same L1 seed differing only in the the mass window requirement (centred on the $\psi(2S)$ instead of J/ψ mass) and lower p_T threshold.

Building of B^0 candidates

For each event, we look for B^0 candidates first by reconstructing the J/ψ candidates for each pair of oppositely charged muons present in the event. The invariant mass of the J/ψ candidate, obtained by combining the four-momenta of two muons must lie within the mass window of $2.9 < m(\mu^+\mu^-) < 3.4$ GeV, and their vertex fit with the geometrical constraint of a common vertex must be valid (no further quality requirement at this stage). In the next step, for each J/ψ candidate, its combinations with any two additional, oppositely charged tracks (with $p_T > 0.35$ GeV) are investigated; these tracks must differ from the track of the muon candidates used to build the J/ψ candidate. Since the charged hadrons cannot be distinguished experimentally in CMS, the

tracks are assigned masses by hand. Both combinations (pion-kaon and kaon-pion) are taken into account and stored for further analysis. In principle, CMS can measure the ionization energy loss for charged tracks but this method of charged hadron identification is possible only for tracks with $p_T < 1$ GeV. This study was performed during the initial stages of our analysis but it was found that only few tracks with $p_T < 1$ GeV survive the final selection conditions for this method to be of any practical use.

B^0 candidates are obtained by combining the four-momenta of each set of J/ψ candidates and charged pion and kaon tracks with a common vertex. In order to enhance the mass resolution of the J/ψ candidate, a mass-constrained vertex fit is performed with the requirement for the dimuon invariant mass to be equal to the nominal J/ψ mass [25]. The B^0 candidate is retained for further processing if the constrained fit turns out to be valid. This is a loose requirement about the fact that the four tracks are approximately coming from a common (decay) vertex, thus discarding track pairs not geometrically compatible with the J/ψ candidate.

3.1.3 B^0 mass constrained kinematic fit

A four-track common vertex fit with the invariant mass being constrained to the nominal B^0 mass [25] is performed to fulfil kinematic boundary conditions of the three-body decay Dalitz plot. It was found that it is impossible to have the momenta of the refitted daughters by applying the B^0 mass constraint on all the daughters and simultaneously applying J/ψ mass constraint on the two muons within the standard kinematic fit frame framework of CMSSW. We compared the differences between the refitted momenta of B^0 daughters from the officially produced $B^0 \rightarrow J/\psi K^+ \pi^-$ phase-space MC sample, for the following cases :

- A) J/ψ mass constraint on the two muons and a four-track vertex fit (without B^0 mass constraint), and
- B) B^0 mass constraint on all four tracks (without J/ψ mass constraint on the muons).

A comparison is made with the original measured muons/tracks for events within the reconstructed B^0 mass window of 5.24–5.32 GeV. It can be seen from Figs. 3.1.1, 3.1.2, and 3.1.3 that the difference between the two cases are small and positively correlated.

It has been checked that there is no difference in terms of the B^0 candidate yields between the fits with and without the mass constraint. The B^0 mass constrained candidates are used only for the final amplitude analysis fit.

3.1. DATA SAMPLE, TRIGGER AND PRESELECTION

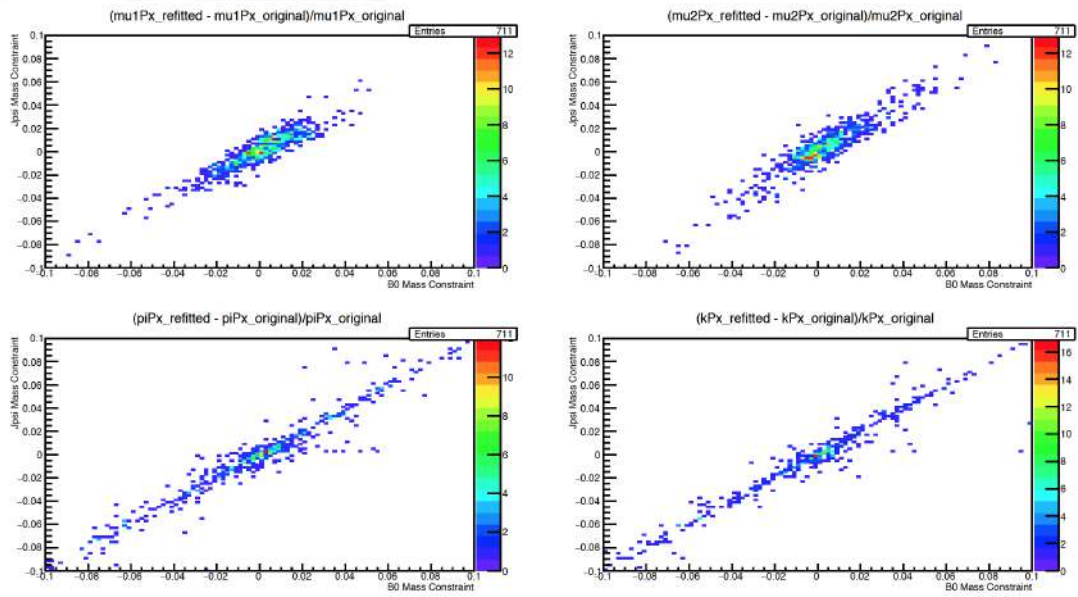


Figure 3.1.1: Scatter plots of $(p_x^{\text{refit}} - p_x^{\text{orig}})/p_x^{\text{orig}}$ for (top left) first muon track, (top right) second muon track, (bottom left) pion track, and (bottom right) kaon track. Here p_x is the x -component of the track momentum for case A and B of Section 3.1.3 plotted on the x - and y -axis, respectively.

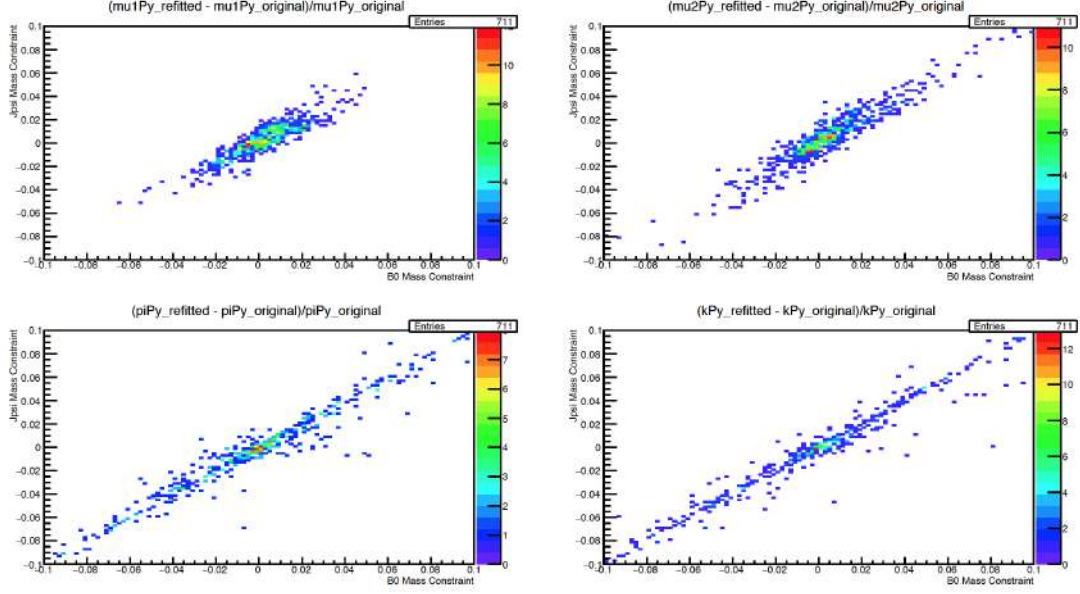


Figure 3.1.2: Scatter plots of $(p_y^{\text{refit}} - p_y^{\text{orig}})/p_y^{\text{orig}}$ for (top left) first muon track, (top right) second muon track, (bottom left) pion track, and (bottom right) kaon track. Here p_y is the y -component of the track momentum for case A and B of Section 3.1.3 plotted on the x - and y -axis, respectively.

Loose filtering of B^0 candidates

In order to keep the size of the output root files under control in presence of huge combinatorics, a preselection of the B^0 candidates has been implemented in the analysis. An event is stored in the output only if at least one preselected B^0 candidate is found in the event. The preselection requirements are:

- the invariant mass of the $J/\psi K\pi$ system should be $4.8 < m(\mu^+\mu^-K\pi) < 5.6$ GeV;
- all tracks must have at least four hits in the silicon tracker;
- the track fit χ^2/NDF must be < 7 (NDF is the number of degrees of freedom);
- the angular separation $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$ between the J/ψ candidate pseudo-momentum vector and the track momentum vector must be less than 1.5 for each track;
- $\Delta R(B^0, \text{track}) < 1.5$ for each track; and
- probability of the four-track vertex fit, $P_{\chi^2}(B^0 \text{ vtx}) > 0.5\%$.

3.2. FINAL SELECTION OF B^0 CANDIDATES

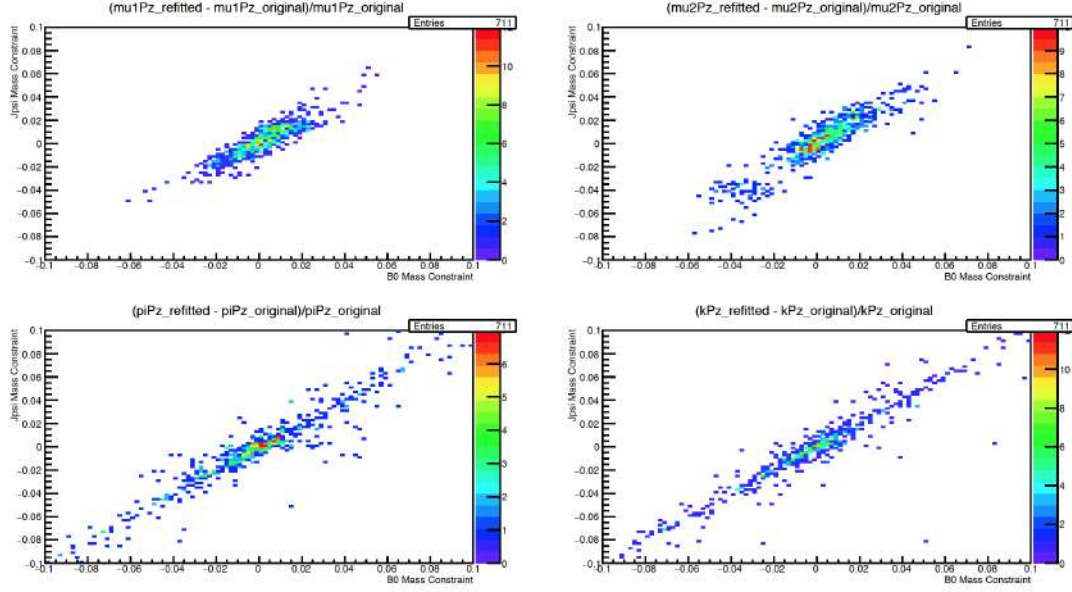


Figure 3.1.3: Scatter plots of $(p_z^{\text{refit}} - p_z^{\text{orig}})/p_z^{\text{orig}}$ for (top left) first muon track, (top right) second muon track, (bottom left) pion track, and (bottom right) kaon track. Here p_z is the z -component of the track momentum for case A and B of Section 3.1.3 plotted on the x - and y -axis, respectively.

Since the ΔR cut could bias the invariant mass distributions, a very loose value (1.5) has been used at this stage of the B^0 candidate selection.

3.2 Final selection of B^0 candidates

The output root files after preselection are merged and used for further analysis where the optimization and final selection of the B^0 candidates are performed. This is done with a ROOT macro run by means of PROOF-Lite to exploit parallel CPU computing capabilities.

To purify the B^0 signal, selection criteria are applied on three most important discriminating variables namely, the pointing angle α , the flight distance $B^0 c\tau$, and the reconstructed secondary B^0 decay vertex (Fig. 3.2.1).

3.2.1 Studies of reflection backgrounds

The $J/\psi \rightarrow \mu^+\mu^-$ provides a very clean signal with the background mainly coming from the two other tracks that form the B^0 . The absence of hadron identification may

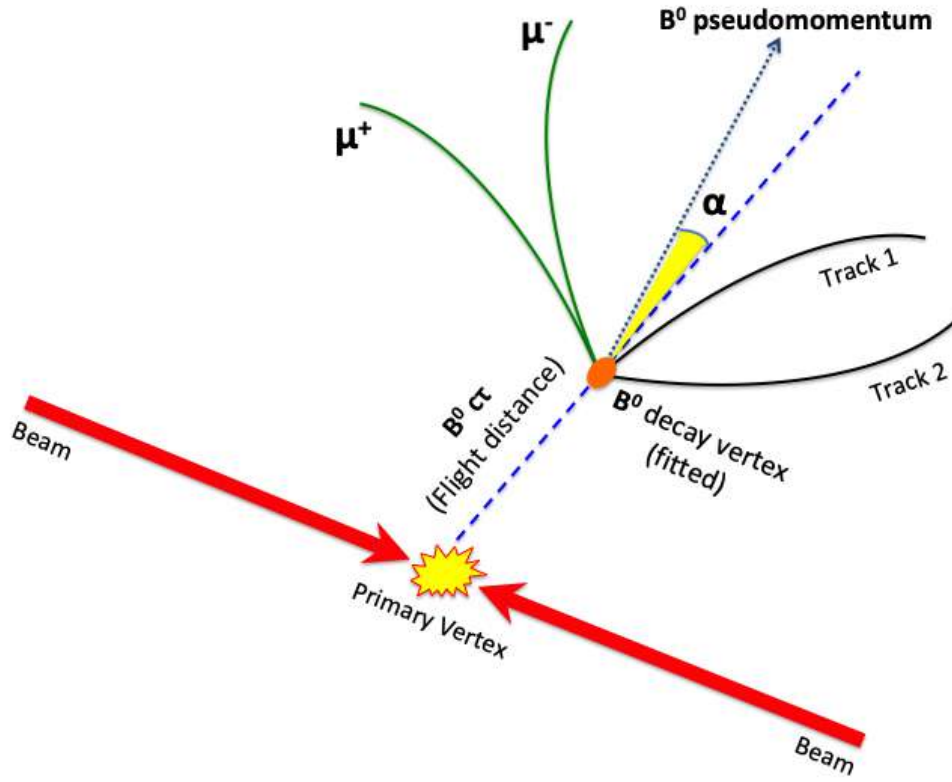


Figure 3.2.1: Sketch of a $B^0 \rightarrow J/\psi(\rightarrow \mu^+\mu^-)K\pi$ decay in a pp collision with the three important discriminating variables defined.

3.2. FINAL SELECTION OF B^0 CANDIDATES

produce distortions (hereafter called “reflections”) in the $J/\psi K\pi$ invariant mass spectrum due to possible physics background, on top of the huge combinatorial one. The major contamination is expected to arise from the misidentification of a charged pion with a kaon and vice versa, that would populate the higher and lower B^0 sideband, respectively. It is also possible to misidentify a charged pion or a kaon with a proton, but at a lower rate. For instance, signals of the following type could be expected: $\phi(1020), f_0(980) \rightarrow K^+ K^-$; $K_S^0, \rho^0(770), f_0(980) \rightarrow \pi^+ \pi^-$; and $\Lambda^0 \rightarrow p\pi, \Lambda^{0*} \rightarrow pK$. Two-track physics signals can be either uncorrelated or correlated with the J/ψ candidates. In the latter case, reflections can be associated, for instance, to $B_s^0 \rightarrow J/\psi\phi$, $B^0 \rightarrow J/\psi\rho$, and $\Lambda_b^0 \rightarrow J/\psi\Lambda^{0*}$.

The $B_s^0 \rightarrow J/\psi\phi$, where $J/\psi \rightarrow \mu^+ \mu^-$ and $\phi \rightarrow K^+ K^-$, contamination can be minimized by rejecting events if, after a $\pi \rightarrow K$ mass swap, the two-track invariant mass shows a peak at the nominal ϕ mass along with the $K^*(892) \rightarrow K\pi$ reflection, and at the same time the $\mu^+ \mu^-$ 2-track invariant mass falls within the B_s^0 mass range, as shown in Fig. 3.2.2. From a detailed MC study, other possible hadron misidentifications are found to have low impact on the $B^0 \rightarrow J/\psi K\pi$ peak.

3.2.2 Multiplicity of B^0 candidates

During the reconstruction of $B^0 \rightarrow J/\psi K\pi$ events, it was found that there could be more than one B^0 candidate within the mass window $5.15 < m(B^0) < 5.45$ GeV. As it is unlikely to have two real B^0 candidates characterised by the same decay in a given event, it is reasonable to assume that if there are more than one reconstructed B^0 candidates decaying into the $J/\psi K\pi$ final state, only one of them corresponds to the real candidate while the others are evidently fakes created in the reconstruction process.

Since the masses of charged tracks are assigned by hand, it may so happen that both combinations of [J/ψ , track 1(pion), track 2(kaon)] and [J/ψ , track 1(kaon), track 2(pion)] in an event passing the preselection have their invariant mass within the acceptable B^0 mass range of 5.15–5.45 GeV. Such cases are called *twins* which poses an inherent ambiguity in the selection of correct B^0 events. As the number of B^0 events without *twins* was found to be sufficiently large in data ($\sim 100K$), we decided to reject all events with *twins*. After the rejection of twins, the residual pion-kaon misidentification is found to be about 9% from MC studies.

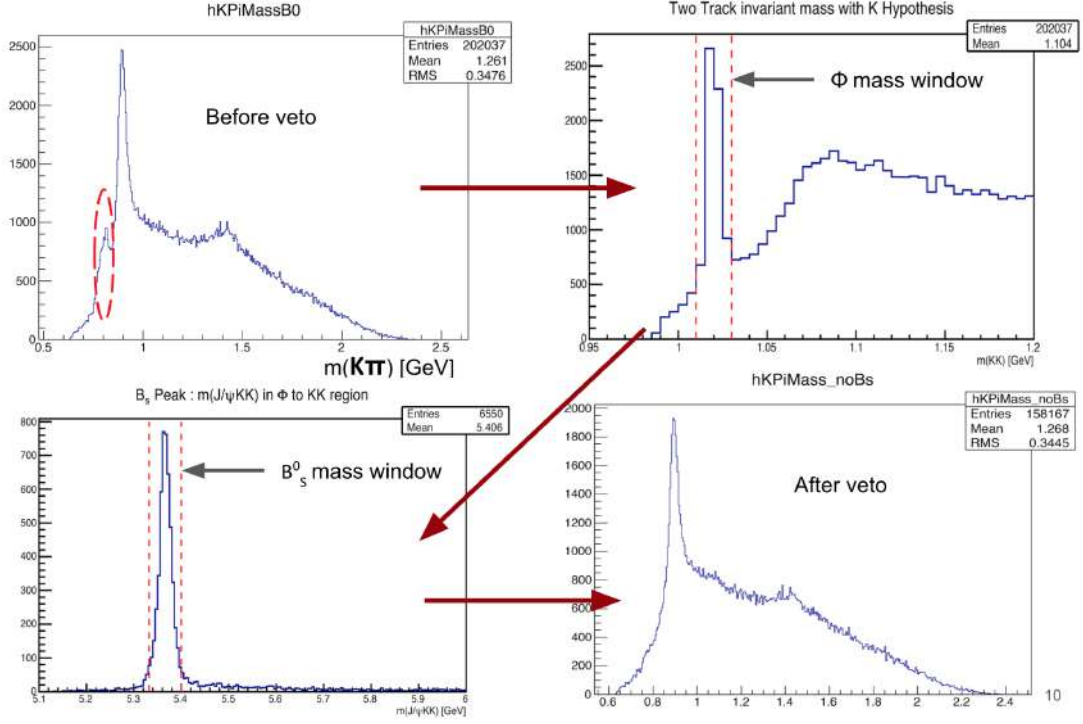


Figure 3.2.2: (top left) $m_{K\pi}$ distribution before veto with an anomalous peak (circled); (top right) two-track mass distribution after the $\pi \rightarrow K$ mass swap. The delimited peak is consistent with $\phi \rightarrow K^+ K^-$; (bottom left) $J/\psi K^+ K^-$ mass distribution with the $\phi \rightarrow K^+ K^-$ candidates, corresponding to the B_s^0 mass; and (bottom right) $m_{K\pi}$ distribution without any anomalous peak once the veto is applied.

3.3 Cut-based selection

It is found that the classic cut-based selection method yields a reasonably high signal purity. Selection criteria for each pion and kaon track are:

- The track fit χ^2/NDF must be less than 5.
- The track should have at least one hit in the pixel and 10 hits in the microstrip tracker.
- The angular separation ΔR between the J/ψ candidate pseudomomentum vector and the track momentum vector must be less than 1.0.
- Track p_T must be greater than 0.45 GeV and $|\eta|$ must be less than 2.4, as per the tracker acceptance.

Selection criteria for muons are:

- Each muon track fit χ^2/NDF must be less than 3.
- The muon track should have at least one hit in the pixel and five hits in the microstrip tracker.
- Longitudinal impact parameter should be less than 20.0 cm and transverse impact parameter should be less than 0.3 cm with respect to the primary vertex.
- The invariant mass of the dimuon system must lie within ± 0.12 GeV (1σ on either side) of the nominal J/ψ mass.
- Probability of the J/ψ vertex fit must be greater than 0.5%.
- Dimuon p_T must be greater than 5 GeV.
- For each muon, p_T must exceed 4 GeV if its $|\eta|$ value lies within 1.2, else it must be greater than 3.3 GeV.

The selection criteria used for the charged hadron tracks as well as muons are in fact the minimum quality conditions recommended by the CMS working groups responsible for the corresponding physics objects. No significant improvement in signal purity is observed by varying each cut. The cuts for the reconstructed B^0 candidates are chosen so as to maximize the B^0 signal purity. Selection criteria for the reconstructed B^0 candidates are:

- Probability of the B^0 vertex fit $P_{\chi^2}(B^0 \text{ vtx})$ must be greater than 0.9%.
- The cosine of the pointing angle, $\cos \alpha$, must be greater than 0.9985.
- $B^0 c\tau$ significance with respect to the primary vertex must be greater than 9.

Signal and sidebands

Figure 3.3.1 shows the mass distribution of the final selected B^0 candidates with the fit result overlaid. Since the distribution is asymmetric, the fit model uses a Johnson distribution [53] for signal and a fourth order Chebychev polynomial for background. The Johnson distribution is given by:

$$J(x; m, \sigma, \lambda, \delta) = \frac{\delta}{\sigma \sqrt{2\pi} \sqrt{1 + \left(\frac{x-m}{\sigma}\right)^2}} e^{-\frac{1}{2}[\lambda + \delta \sinh^{-1}\left(\frac{x-m}{\sigma}\right)]^2}.$$

Here, m and σ are the mean and width while λ and δ are the two shape parameters. The signal region is given by $[m - \sigma, m + \sigma]$, where m and σ are the fit values of the corresponding parameters in the Johnson distribution. The lower (left) and higher (right) sidebands are given by $[m - 4.5\sigma, m - 2.5\sigma]$ and $[m + 4.0\sigma, m + 6.0\sigma]$, respectively. A signal purity of 80% is reached within the $\pm 1\sigma$ window of the fitted B^0 peak.

3.3. CUT-BASED SELECTION

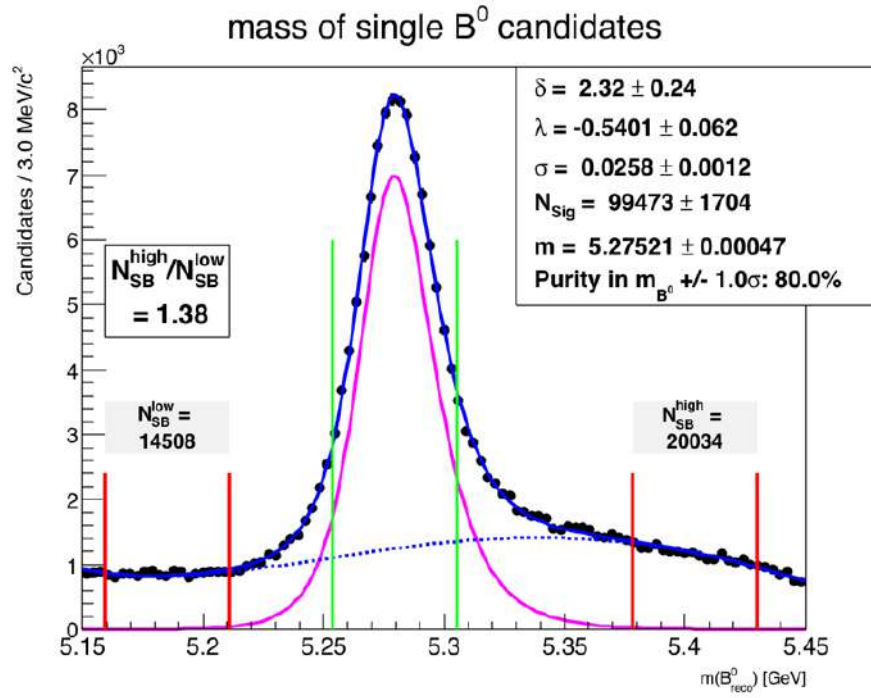


Figure 3.3.1: Distribution of $m(J/\psi K\pi)$ for events with single B^0 candidate after the cut-based selection. The solid blue curve represents the total fit function composed of a Johnson distribution (purple curve) and a fourth order Chebychev polynomial (dashed blue curve). The mass windows delimited by green (red) vertical lines display the signal region (sideband) as defined in the text.

Amplitude Analysis of $B^0 \rightarrow J/\psi K \pi$

A full 4D amplitude analysis framework [54] for the $B^0 \rightarrow J/\psi K \pi$ decay, developed based on helicity formalism, is discussed in this chapter followed by the validation of its implementation in the novel GPU based GooFit software framework. The inclusion and validation of the fit models with the exotic Z contributions as well as the description and implementation of the LASS parametrization are also addressed in detail in this chapter.

4.1 Fit Formalism

Three-body decays with an intermediate resonant state such as $P \rightarrow D_1 + D_{\text{res}}, D_{\text{res}} \rightarrow D_2 + D_3$ are generally analysed using a technique pioneered by Dalitz [55]. Here, P is the parent particle, D_1 is one of the daughters, D_{res} is the other daughter which is an intermediate resonance that in turn decays into the particles, D_2 and D_3 . A plot of $m_{D_1 D_2}^2$ vs. $m_{D_2 D_3}^2$ (2D distribution of invariant mass squared of any two daughter pairs), known as the Dalitz plot, shows a nonuniform distribution around the mass squared of the intermediate resonance, $m_{D_{\text{res}}}^2$. If one of the three daughters in the decay is a vector state, the traditional 2D Dalitz plot approach becomes insufficient as the angular variables are integrated over, leading to a loss of information about angular correlations among the decay products. The kinematics of the process $B^0 \rightarrow J/\psi K \pi$, $J/\psi \rightarrow \mu^+ \mu^-$ can be completely described by a four-dimensional variable space

$$\Phi \equiv (m_{K\pi}, m_{J/\psi\pi}, \theta_{J/\psi}, \varphi). \quad (4.1.1)$$

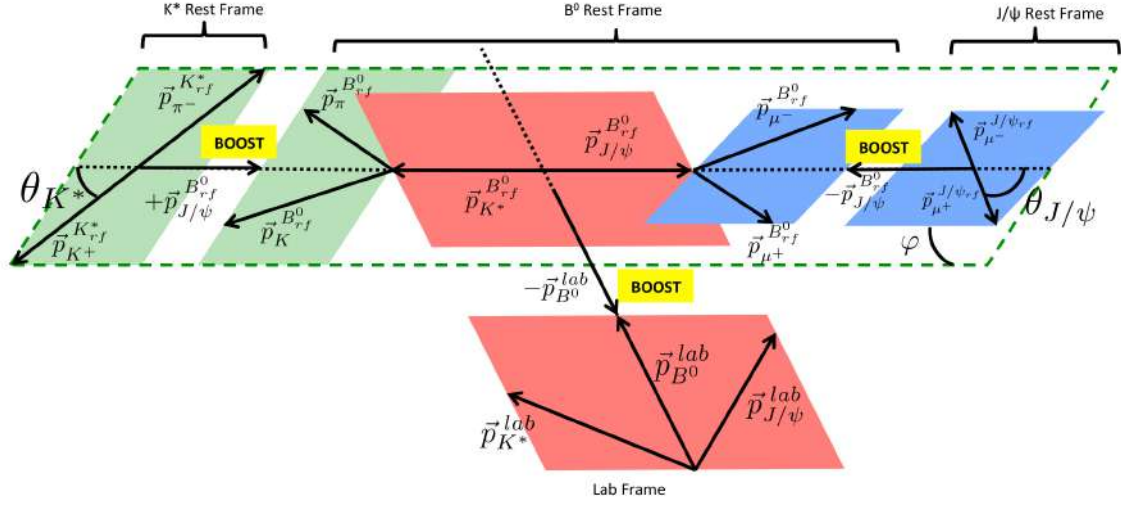


Figure 4.1.1: A sketch illustrating the definitions of the two independent angular variables, $\theta_{J/\psi}$ and φ , for the amplitude analysis of the $B^0 \rightarrow J/\psi K\pi$ decay.

The two angles, $\theta_{J/\psi}$ and φ are illustrated in Fig. 4.1.1.

The total decay amplitude of $B^0 \rightarrow J/\psi K\pi$ is represented by a coherent sum of the Breit-Wigner (BW) contributions for kinematically allowed intermediate resonant states. The angle-independent part of the decay amplitude for each resonance R is given by:

$$A^R(m_R^2) = \frac{F_B^{(L_B)} \left(\frac{p_B}{M_B} \right)^{L_B} F_R^{(L_R)} \left(\frac{p_R}{m_R} \right)^{L_R}}{M_R^2 - m_R^2 - iM_R\Gamma(m_R)}. \quad (4.1.2)$$

where the mass-dependent width of the resonance R is:

$$\Gamma(m_R) = \Gamma_0 \left(\frac{p_R}{p_{R0}} \right)^{2L_R+1} \left(\frac{M_R}{m_R} \right) F_R^2, \quad (4.1.3)$$

- m_R is the running invariant mass of the two daughters of the R resonance (for instance, for a K^* , $m_R = m_{K\pi}$);
- M_B is the B^0 meson mass;
- M_R is the nominal mass of the R resonance;
- L_B and L_R are the orbital angular momenta in the B^0 and R decay, respectively;
- p_B is the B^0 daughter momentum (i.e R momentum) in the B^0 rest frame;

4.1. FIT FORMALISM

- $F_B^{(L_B)}$ and $F_R^{(L_R)}$ are the Blatt-Weisskopf form factors [56] for B^0 and R decay, respectively, with the superscript denoting the orbital angular momentum of the (sub-)decay;
- Γ_0 is the nominal width of R ; and
- p_R and p_{R_0} are the momenta of R daughters in the former's rest frame, calculated from the running and pole mass of R , respectively.

The angle-dependent part of the amplitude is obtained using the helicity formalism [57]. For each K^* resonance, it is given by

$$A_{\lambda\xi}^{K^*}(\Phi) = H_{\lambda}^{K^*} A^{K^*}(m_{K\pi}^2) d_{\lambda 0}^{J(K^*)}(\theta_{K^*}) e^{i\lambda\varphi} d_{\lambda\xi}^1(\theta_{J/\psi}), \quad (4.1.4)$$

where, $A^{K^*}(m_{K\pi}^2)$, defined in Eq. (4.1.2), is explicitly rewritten for $R \equiv K^*$ and

- $J(K^*)$ is the spin of the considered K^* resonance;
- λ is the helicity of the J/ψ (the quantisation axis being parallel to the K^* momentum in the J/ψ rest frame). In general, λ can take the values $-1, 0$ and 1 . For K^* s with zero spin, only $\lambda = 0$ is allowed;
- ξ is the helicity of the dimuon system;
- $H_{\lambda}^{K^*}$ is the complex helicity amplitude for the decay via the intermediate K^* ;
- $d_{\lambda 0}^{J(K^*)}(\theta_{K^*})$ and $d_{\lambda\xi}^1(\theta_{J/\psi})$ are the Wigner small-d functions;
- θ_{K^*} is the K^* helicity angle, i.e. the angle between J/ψ and π momenta in the K^* rest frame or, equivalently, the angle between K momentum in K^* rest frame and the K^* momentum in the B^0 rest frame (Fig. 4.1.1);
- $\theta_{J/\psi}$ is the J/ψ helicity angle, i.e. the angle between μ^+ momentum in the J/ψ rest frame and J/ψ momentum in the B^0 rest frame (Fig. 4.1.1); and
- φ is the angle between the $J/\psi \rightarrow \mu^+\mu^-$ and $K^* \rightarrow K\pi$ decay planes (Fig. 4.1.1).

The signal density function, to be used in an Unbinned Maximum Likelihood (UML) fit, is obtained after appropriately summing over the helicity states and is given by

$$S(\Phi) = \sum_{\xi=1,-1} \left| \sum_{K^*} \sum_{\lambda=-1,0,1} A_{\lambda\xi}^{K^*} \right|^2. \quad (4.1.5)$$

The sum over K^* includes all kinematically allowed (up to $m_{K\pi} = 2.183$ GeV) resonance states: $K_0^*(800)$, $K^*(892)$, $K^*(1410)$, $K_0^*(1430)$, $K_2^*(1430)$, $K^*(1680)$, $K_3^*(1780)$, $K_0^*(1950)$, $K_2^*(1980)$, and $K_4^*(2045)$. As the above expression is sensitive only to the relative phases and amplitudes, we have the freedom to fix one phase and one amplitude in the fit. The helicity amplitude of the $K^*(892)$, the most dominant resonance, is chosen to be fixed, for $\lambda = 0$:

$$\left| H_0^{K^*(892)} \right| = 1 \quad , \quad \arg \left(H_0^{K^*(892)} \right) = 0. \quad (4.1.6)$$

The masses and widths of all the resonances are fixed to their world-average values [25].

4.2 Fit Validation

4.2.1 Implementation using GPU-based GooFit

The amplitude analysis model described in Section 4.1 requires a complex fit with four variables and a lot of free parameters. Before applying on real data, the fit (henceforth referred to as the AA fit) must be validated to check its consistency and capability of providing proper results. The validation is performed by generating, through a toy MC technique, a distribution according to the fit model and then checking, as a result of fitting to the same distribution, that the best estimates of parameters returned by the fit are consistent with their known input values.

The AA fit is implemented using the novel GPU based GooFit [58] because the standard RooFit tool designed to perform such UML fit was taking excessive processing time for the fits to converge. GooFit is an under-development open source data analysis tool, used in the HEP applications for parameters' estimation, which interfaces ROOT to the CUDA parallel computing platform on nVidia GPUs [59]. GPU-accelerated computing enhances application performances by offloading a sequence of elementary but computational-intensive operations to the GPU to be processed in parallel, while the remaining code still runs on the CPUs. GooFit acts as an interface between the MINUIT [60] minimization algorithm and the GPU, which allows any probability density function (PDF) to be evaluated in parallel over a huge number of data points. Fit parameters are estimated at each negative-log-likelihood (NLL) minimization step on the *host side* (CPU) while the PDF/NLL is evaluated on the *device side* (GPU). Since GooFit is still a limited open source tool, being developed by the users themselves according to their specific needs, significant sections needed for our AA fit

implementation have been either newly encoded or adapted starting from the existing classes and methods.

4.2.2 Validation with pure K^* signal model

A dataset of one million events is generated with the 10 K^* s mentioned in Section 4.1 with their masses and widths fixed to the nominal values. The helicity amplitude parameters for each of these resonances are fixed to the values obtained by Belle [8]. The $m_{K\pi}$ projection of the fit to the generated dataset is shown in Fig. 4.2.1 while the other projections are shown in Fig. 4.2.2. The consistency of the fitted values of the

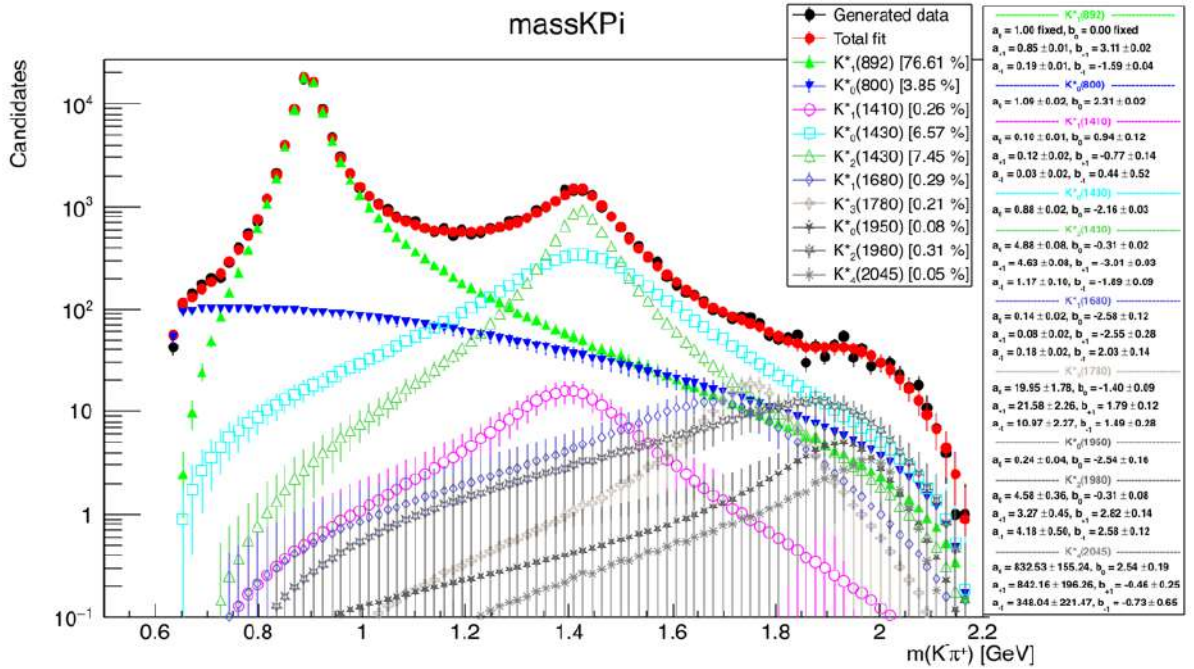


Figure 4.2.1: Projection of $m_{K\pi}$ spectrum of the 4D dataset generated according to an ideal signal model (black points with error bars). The fit result (red points with error bars) is superimposed along with the individual signal fit components corresponding to the different K^* s. The fitted values of the helicity amplitude parameters and fit fractions for each component are shown here as well as listed in Table 4.2.1.

free parameters is checked by comparing the pull distributions with their generated values as shown in Figs. 4.2.3 and 4.2.4.

Table 4.2.1: The magnitudes and phases of the helicity amplitudes obtained from a fit to the ideal signal model generated with ten K^* s.

Resonance	λ	$ H_\lambda $	$\arg H_\lambda$	Fit fraction
$K_0^*(800)$	0	1.09 ± 0.02	2.31 ± 0.02	3.85%
$K^*(892)$	+1	0.85 ± 0.01	3.11 ± 0.02	76.61%
	0	1.0 (fixed)	0.0 (fixed)	
	-1	0.19 ± 0.01	1.59 ± 0.04	
$K^*(1410)$	+1	0.12 ± 0.02	-0.77 ± 0.14	0.26%
	0	0.10 ± 0.01	0.94 ± 0.12	
	-1	0.03 ± 0.02	0.44 ± 0.52	
$K_0^*(1430)$	0	0.88 ± 0.02	-2.16 ± 0.03	6.57%
$K_2^*(1430)$	+1	4.63 ± 0.08	-3.01 ± 0.03	7.45%
	0	4.88 ± 0.08	-0.31 ± 0.02	
	-1	1.17 ± 0.10	-1.89 ± 0.09	
$K^*(1680)$	+1	0.08 ± 0.02	2.40 ± 0.14	0.29%
	0	0.14 ± 0.02	-2.58 ± 0.11	
	-1	0.18 ± 0.02	2.03 ± 0.14	
$K_3^*(1780)$	+1	21.58 ± 2.26	1.79 ± 0.12	0.21%
	0	19.95 ± 1.78	-1.40 ± 0.09	
	-1	10.97 ± 2.27	1.49 ± 0.28	
$K_0^*(1950)$	0	0.24 ± 0.04	-2.54 ± 0.16	0.08%
$K_2^*(1980)$	+1	6.43 ± 0.65	-0.20 ± 0.13	0.31%
	0	4.58 ± 0.36	-0.31 ± 0.08	
	-1	4.30 ± 0.73	-0.38 ± 0.19	
$K_4^*(2045)$	+1	842 ± 196	-0.46 ± 0.25	0.05%
	0	833 ± 155	2.54 ± 0.19	
	-1	348 ± 221	-0.73 ± 0.65	

4.2. FIT VALIDATION

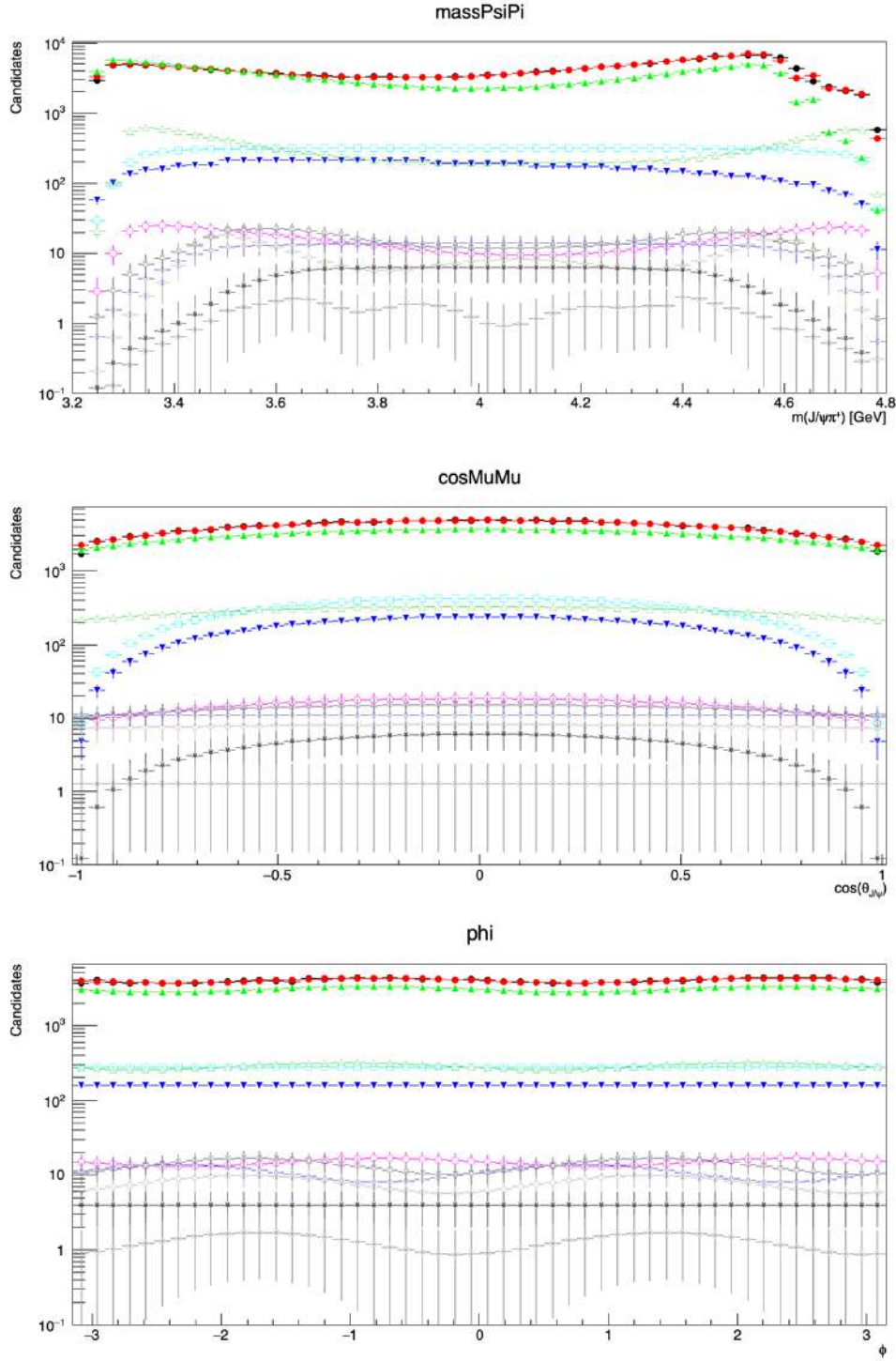


Figure 4.2.2: Projection of (from top to bottom) $m_{J/\psi\pi}$, $\cos(\theta)$, and φ of the 4D dataset generated according to an ideal signal model (black points with error bars). The fit result (red points with error bars) is superimposed along with the individual signal fit components corresponding to the different K^* s.

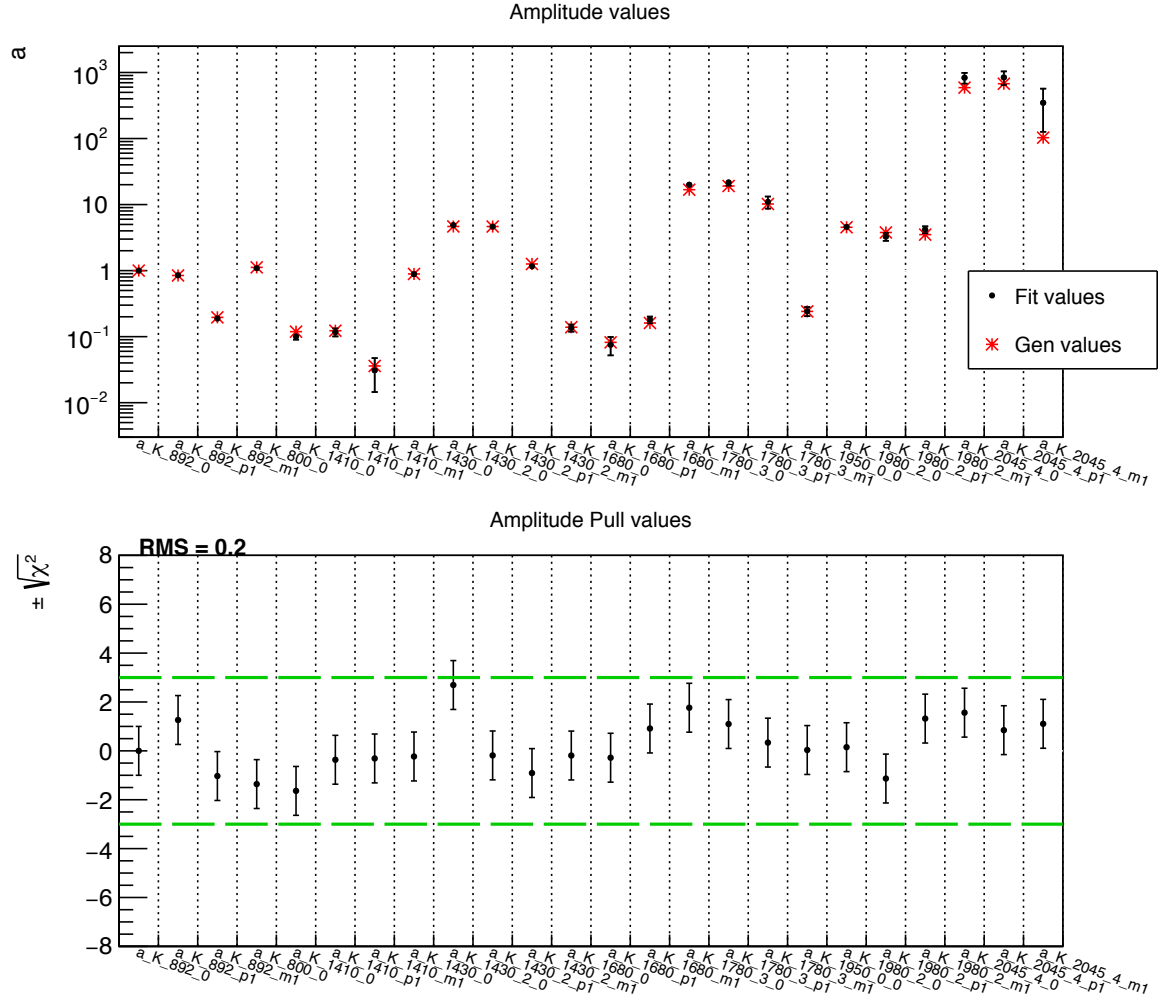


Figure 4.2.3: Comparison of generated and fitted values of the amplitude parameters (*above*) and the corresponding pull distribution (*below*) obtained from a fit to events generated with all the 10 K^* resonances.

4.2. FIT VALIDATION

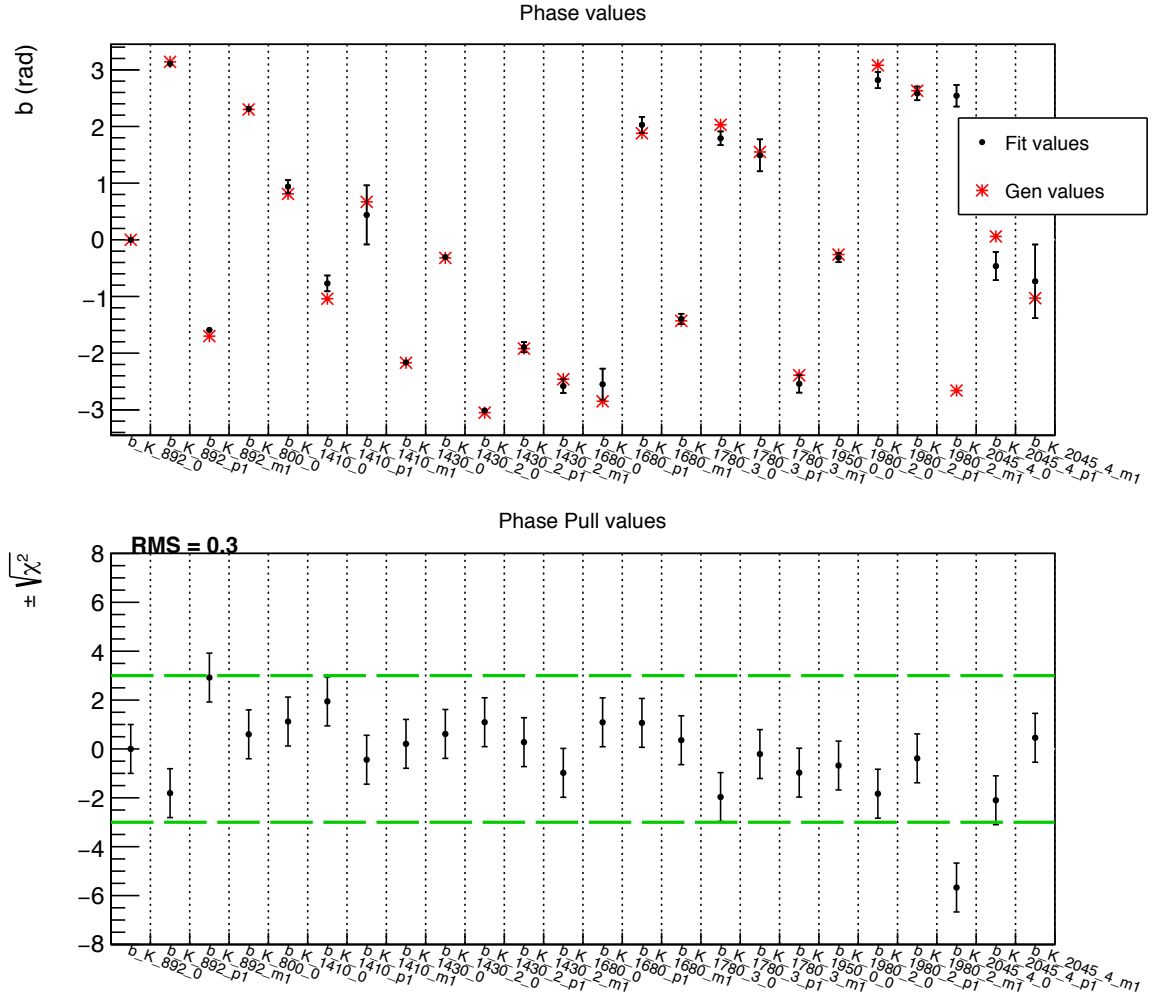


Figure 4.2.4: Comparison of generated and fitted values of the phase parameters (*above*) and the corresponding pull distribution (*below*) obtained from a fit to events generated with all the 10 K^* resonances.

4.2.3 Inclusion and validation of $B^0 \rightarrow KZ (\rightarrow J/\psi\pi)$ in the signal model

For the decay $B^0 \rightarrow KZ (\rightarrow J/\psi\pi)$, $J/\psi \rightarrow \mu^+\mu^-$ where Z can be $Z(4430)$ or $Z(4200)$ or any other exotic charged state, the angle-dependent amplitude is given as

$$A_{\lambda'\xi}^Z(\Phi) = H_{\lambda'}^Z A^Z \left(m_{J/\psi\pi^+}^2 \right) d_{0\lambda'}^{J(Z)}(\theta_Z) e^{i\lambda'\tilde{\varphi}} d_{\lambda'\xi}^1(\tilde{\theta}_{J/\psi}) e^{i\xi\alpha}, \quad (4.2.1)$$

where,

- $J(Z)$ is the spin of the Z resonance. We consider only 1^+ spin-parity of the Z s as per Belle result [8];
- λ' is the helicity of the J/ψ (quantisation axis parallel to the π momentum in the J/ψ rest frame);
- ξ is the helicity of the dimuon system;
- $H_{\lambda'}^Z$ is the complex helicity amplitude for the decay via the intermediate Z ;
- $d_{0\lambda'}^{J(Z)}(\theta_Z)$ and $d_{\lambda'\xi}^1(\tilde{\theta}_{J/\psi})$ are the Wigner small-d functions;
- θ_Z is the Z helicity angle, i.e. the angle between K and π momenta in the Z rest frame;
- $\tilde{\theta}_{J/\psi}$ is the J/ψ helicity angle, i.e. the angle between μ and π momenta in the J/ψ rest frame;
- $\tilde{\varphi}$ is the angle between the (μ^+, μ^-) and (K, π) planes in the J/ψ rest frame;
- α is the angle between the (μ^+, π) and $(\mu^+, K\pi)$ planes in the J/ψ rest frame.

The amplitudes for different λ' values are related by parity conservation:

$$H_{\lambda'}^Z = -P(Z)(-1)^{J(Z)} H_{-\lambda'}^Z. \quad (4.2.2)$$

After inclusion of the Z component, the signal density function of Eq. (4.1.5) becomes

$$S(\Phi) = \sum_{\xi=1,-1} \left| \sum_{K^*} \sum_{\lambda=-1,0,1} A_{\lambda\xi}^{K^*} + \sum_Z \sum_{\lambda'=-1,0,1} A_{\lambda'\xi}^Z \right|^2. \quad (4.2.3)$$

To validate the combined model, a dataset of one million events is generated with all 10 K^* s plus $Z(4200)$ and $Z(4430)$, and fitted. The mass, width and helicity amplitudes of the Z resonances are fixed to the values obtained by Belle [8]. The $m_{K\pi}$

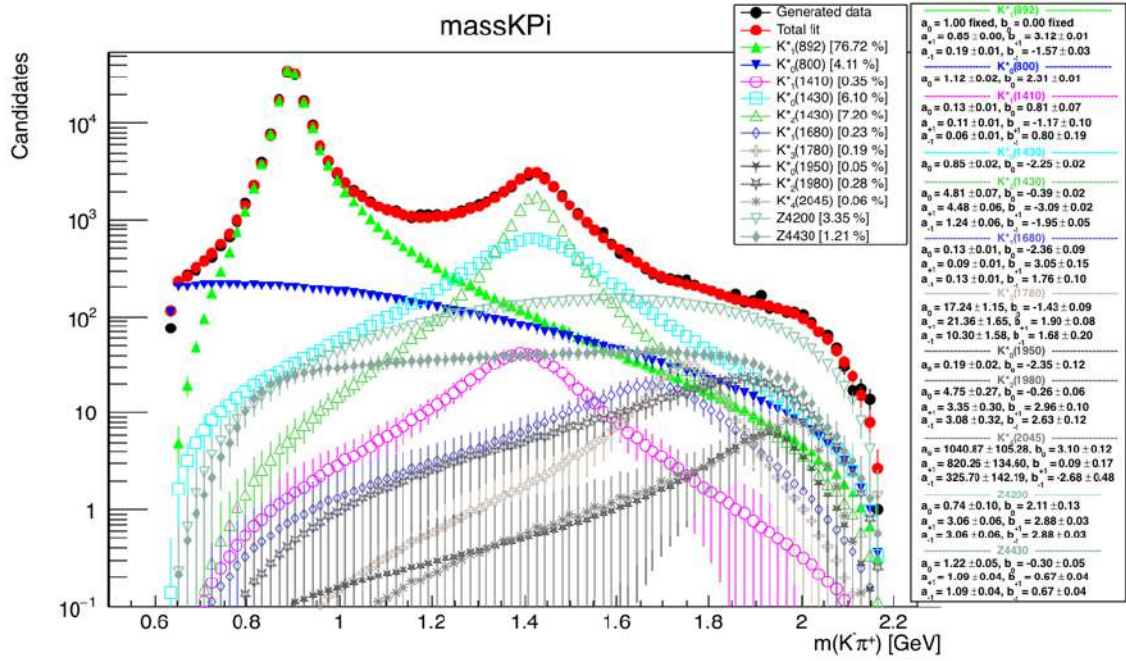


Figure 4.2.5: Projection on the $m_{K\pi}$ spectrum of the 4D dataset generated according to an ideal signal model (black points with error bars) with 10 K^* s and 2 Z s. The fit result (red points with error bars) is superimposed along with the individual signal fit components corresponding to the different K^* s and Z s. The fitted values of the helicity amplitude parameters and fit fractions for each component are shown here as well as listed in Table 4.2.2.

Table 4.2.2: The magnitudes and phases of the helicity amplitudes obtained from a fit to the ideal signal model generated with 10 K^* s and 2 Z s.

Resonance	λ	$ H_\lambda $	$\arg H_\lambda$	Fit fraction
$K_0^*(800)$	0	1.12 ± 0.02	2.31 ± 0.01	4.11%
$K^*(892)$	+1	0.85 ± 0.00	3.12 ± 0.01	76.72%
	0	1.0 (fixed)	0.0 (fixed)	
	-1	0.19 ± 0.01	-1.57 ± 0.03	
$K^*(1410)$	+1	0.13 ± 0.01	-1.17 ± 0.10	0.35%
	0	0.13 ± 0.01	0.81 ± 0.07	
	-1	0.06 ± 0.01	0.80 ± 0.19	
$K_0^*(1430)$	0	0.85 ± 0.02	-2.25 ± 0.02	6.57%
$K_2^*(1430)$	+1	4.48 ± 0.06	-3.09 ± 0.02	6.10%
	0	4.81 ± 0.07	-0.39 ± 0.02	
	-1	1.24 ± 0.06	-1.95 ± 0.05	
$K^*(1680)$	+1	0.13 ± 0.01	3.05 ± 0.15	0.23%
	0	0.13 ± 0.01	-2.36 ± 0.09	
	-1	0.13 ± 0.01	1.76 ± 0.10	
$K_3^*(1780)$	+1	21.36 ± 1.65	1.90 ± 0.08	0.19%
	0	17.24 ± 1.15	-1.43 ± 0.09	
	-1	10.30 ± 1.58	1.68 ± 0.20	
$K_0^*(1950)$	0	0.19 ± 0.02	-2.35 ± 0.12	0.05%
$K_2^*(1980)$	+1	3.35 ± 0.30	2.96 ± 0.10	0.28%
	0	4.75 ± 0.27	-0.26 ± 0.06	
	-1	3.08 ± 0.32	2.63 ± 0.12	
$K_4^*(2045)$	+1	820 ± 135	0.09 ± 0.17	0.06%
	0	1041 ± 105	3.10 ± 0.12	
	-1	326 ± 142	-2.68 ± 0.48	
$Z(4200)$	0	0.74 ± 0.10	2.11 ± 0.13	3.35%
	± 1	3.06 ± 0.06	2.88 ± 0.48	
$Z(4430)$	0	1.22 ± 0.05	-0.30 ± 0.05	1.21%
	± 1	1.09 ± 0.04	0.67 ± 0.04	

4.2. FIT VALIDATION

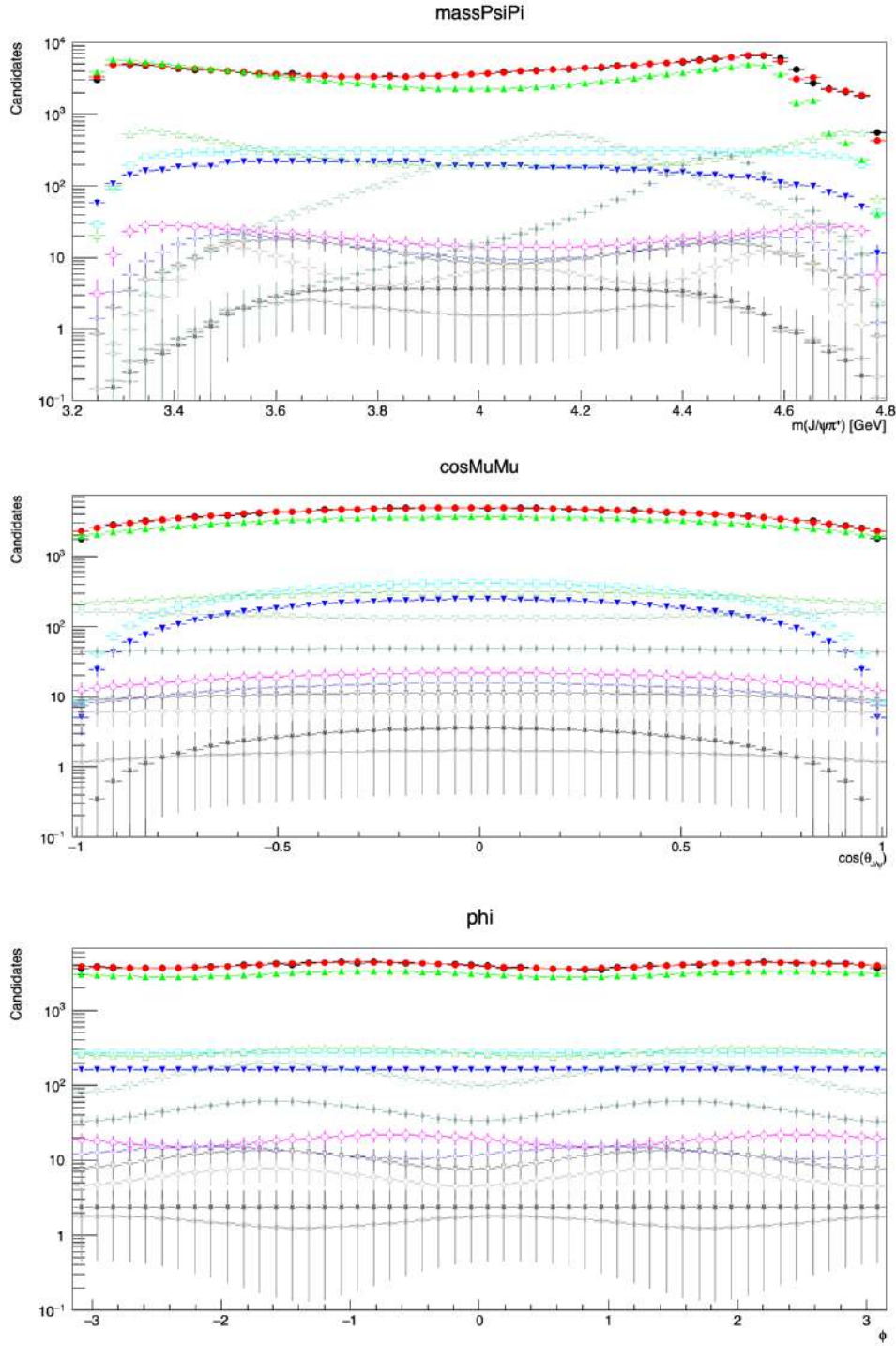


Figure 4.2.6: Projections of (from top to bottom) $m_{J/\psi\pi}$, $\cos\theta$ and φ of the 4D dataset generated according to an ideal signal model (black points with error bars). The fit result (red points with error bars) is superimposed along with the individual signal fit components corresponding to the different K^* s and Z s.

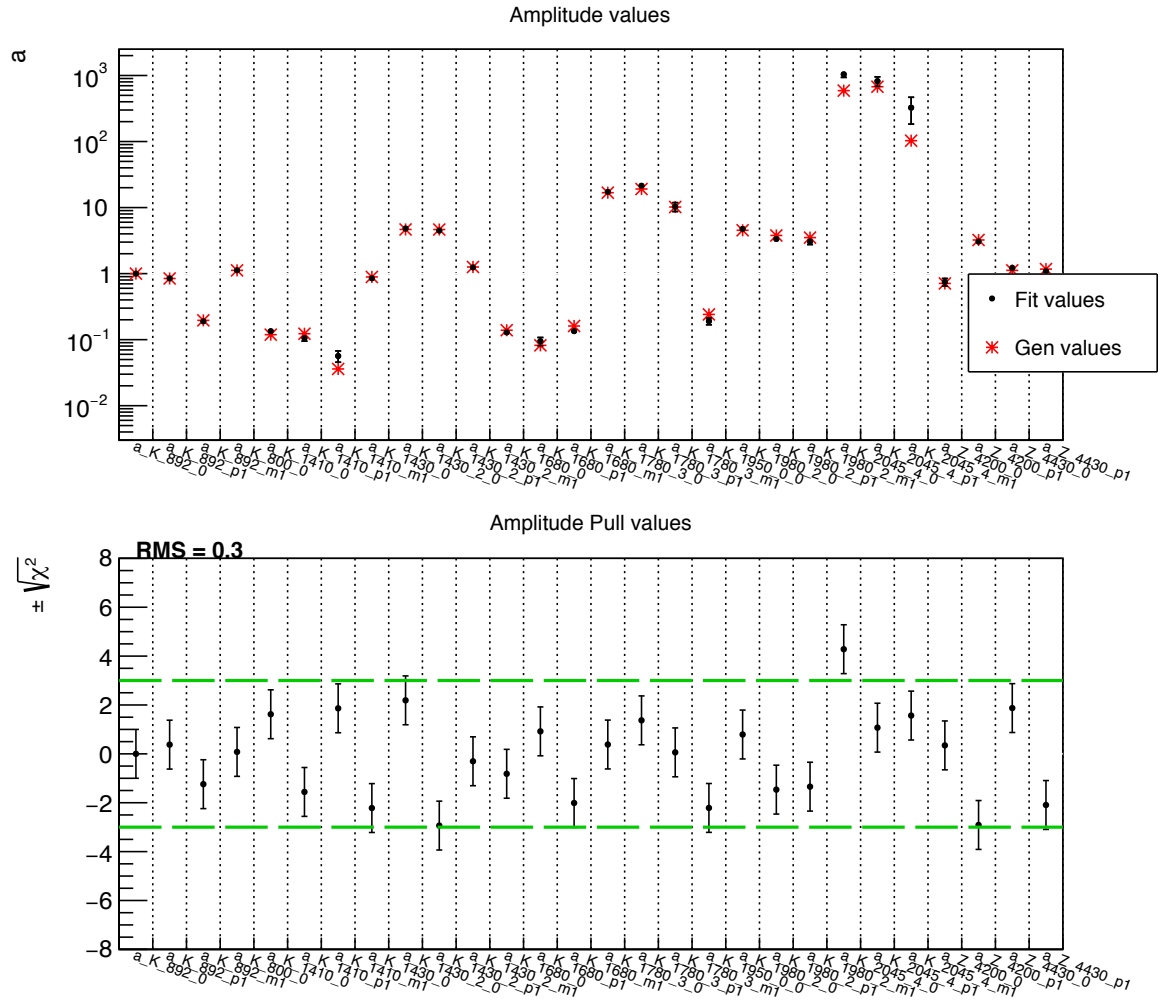


Figure 4.2.7: Comparison of generated and fitted values of the amplitude parameters (*above*) and the corresponding pull distribution (*below*) obtained from a fit to events generated with all the 10 K^* plus 2 Z resonances.

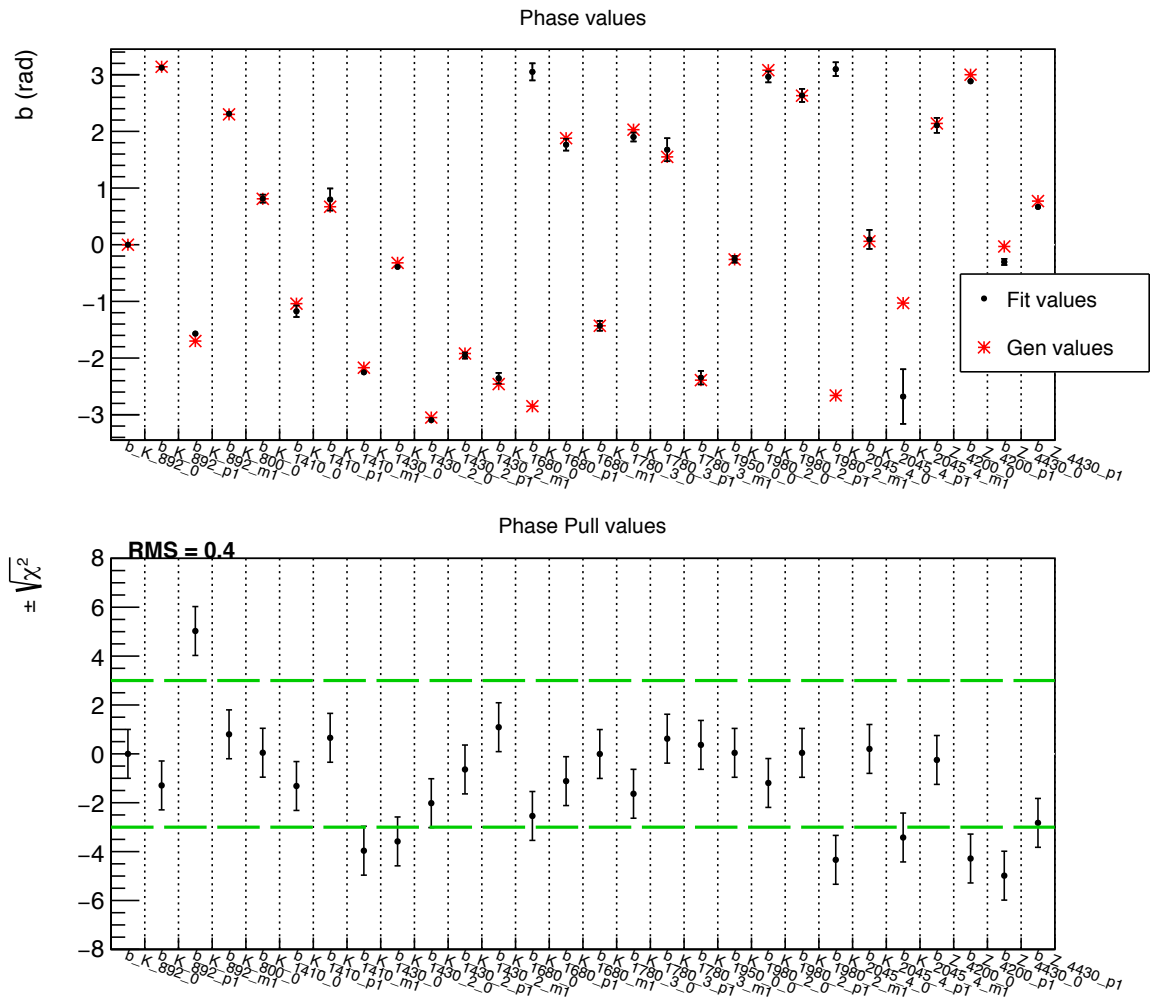


Figure 4.2.8: Comparison of generated and fitted values of the phase parameters (*above*) and the corresponding pull distribution (*below*) obtained from a fit to events generated with all the 10 K^* plus 2 Z resonances.

projection of the fit to this generated dataset is shown in Figs. 4.2.5 and 4.2.6. The consistency of the fitted values of free parameters are again checked by comparing the pull distributions with their generated values as shown in Figs. 4.2.7 and 4.2.8.

For the charge conjugate decay $\bar{B}^0 \rightarrow J/\psi K\pi$, the particles in the definitions of the angular variables change to the corresponding antiparticles ($K^+ \rightarrow K^-$, $\pi^- \rightarrow \pi^+$, and $\mu^\pm \rightarrow \mu^\mp$). If the parity transformation is applied, then the helicity angles do not change while the azimuthal angles change sign. Thus, the signal density for the conjugate decay is given by Eq. (4.2.3) with the opposite sign of the azimuthal angles ($\varphi \rightarrow -\varphi$, $\tilde{\varphi} \rightarrow -\tilde{\varphi}$ and $\alpha \rightarrow -\alpha$). The fitter is designed to automatically take into account the charge conjugate decays in the dataset and switch the variables internally.

4.2.4 LASS Parametrization

Generally, P and D waves states are considered to be well described by narrow resonance approximations. For the $K\pi$ system, the low mass S-wave $K_0^*(800)$ appears as a broad peak calling for a more careful treatment. The LASS experiment at SLAC used an effective range expansion to model the low-energy behaviour of the $K\pi$ S-wave [61]. We use a similar parametrization where the angle-independent part of the amplitude is a nonresonant contribution interfering with the $K_0^*(1430)$ BW:

$$A_{\text{LASS}} = \frac{m_{K\pi}}{q_{K\pi}} \sin \theta_B e^{i\theta_B} + 2e^{2i\theta_B} \frac{\left(m_{K_0^*(1430)}^2 / q_{K_0^*(1430)} \right) \Gamma_{K_0^*(1430)}}{M_{K_0^*(1430)}^2 - m_{K\pi}^2 - iM_{K_0^*(1430)} \Gamma(m_{K\pi})}, \quad (4.2.4)$$

with

$$\cot \theta_B = \frac{1}{a} + \frac{1}{2} b q_{K\pi} \quad \text{and,} \quad a = 1.95 \text{ GeV}^{-1}, \quad b = 1.76 \text{ GeV}^{-1}, \quad (4.2.5)$$

where

- $m_{K\pi}$ is the running mass of the $K\pi$ system;
- $q_{K\pi}$ is the momentum of either of the K^* daughters in the K^* rest frame;
- $\Gamma(m_{K\pi})$ is the running resonance width.

Therefore, the signal density with the LASS parametrization for the low mass $K\pi$ S-wave becomes

$$S(\Phi) = \sum_{\xi=1,-1} \left| H_0^{\text{LASS}} A_{0\xi}^{\text{LASS}} + \sum_{K^{*\prime}} \sum_{\lambda=-1,0,1} A_{\lambda\xi}^{K^{*\prime}} \right|^2. \quad (4.2.6)$$

4.2. FIT VALIDATION

Here the sum over K^* includes all K^* s except $K_0^*(800)$ and $K_0^*(1430)$ for the K^* -only model. For validation, a dataset is generated with the model with LASS (amplitude = 0.33, phase = 1.36 rad). These values are obtained by fitting a non-LASS generated dataset with a LASS model, keeping all other helicity amplitudes fixed. The $m_{K\pi}$ projection of the fit to this generated dataset is shown in Figs. 4.2.9 and 4.2.10.

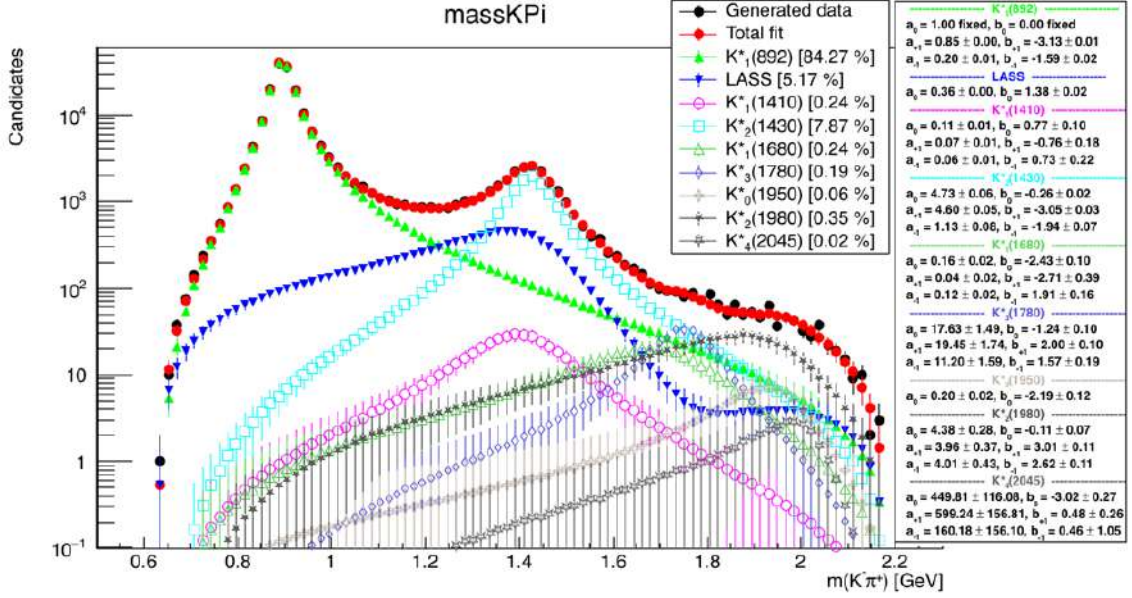


Figure 4.2.9: Projection on the $m_{K\pi}$ spectrum of the 4D dataset generated according to an ideal signal model (black points with error bars) with LASS parametrization of low-mass $K\pi$ S-wave resonances. The fit result (red points with error bars) is superimposed along with the individual signal fit components corresponding to the different K^* s and the LASS component. The fitted values of the helicity amplitude parameters and fit fractions for each component are shown here as well as listed in Table 4.2.3.

The consistency of the fitted values of the free parameters is checked by plotting the pull distributions with respect to their generated values, as shown in Figs. 4.2.11 and 4.2.12.

Table 4.2.3: The magnitudes and phases of the helicity amplitudes obtained from a fit to the ideal signal model generated with only K^* s with LASS parametrization for low-mass $K\pi$ S-wave resonances.

Resonance	λ	$ H_\lambda $	$\arg H_\lambda$	Fit fraction
LASS	0	0.36 ± 0.00	1.38 ± 0.02	5.17%
$K^*(892)$	+1	0.85 ± 0.00	-3.13 ± 0.01	84.27%
	0	1.0 (fixed)	0.0 (fixed)	
	-1	0.20 ± 0.01	-1.59 ± 0.02	
$K^*(1410)$	+1	0.07 ± 0.01	-0.76 ± 0.18	0.24%
	0	0.11 ± 0.01	0.77 ± 0.10	
	-1	0.06 ± 0.01	0.73 ± 0.22	
$K_2^*(1430)$	+1	4.60 ± 0.05	-3.05 ± 0.03	7.87%
	0	4.73 ± 0.06	-0.26 ± 0.02	
	-1	1.13 ± 0.08	-1.94 ± 0.07	
$K^*(1680)$	+1	0.08 ± 0.02	2.40 ± 0.14	0.24%
	0	0.16 ± 0.02	-2.43 ± 0.10	
	-1	0.12 ± 0.02	1.91 ± 0.16	
$K_3^*(1780)$	+1	19.45 ± 1.74	2.00 ± 0.10	0.19%
	0	17.63 ± 1.49	-1.24 ± 0.10	
	-1	11.20 ± 1.59	1.57 ± 0.19	
$K_0^*(1950)$	0	0.20 ± 0.02	-2.19 ± 0.12	0.06%
$K_2^*(1980)$	+1	3.96 ± 0.37	3.01 ± 0.11	0.35%
	0	4.38 ± 0.28	-0.11 ± 0.07	
	-1	4.01 ± 0.43	2.62 ± 0.11	
$K_4^*(2045)$	+1	599 ± 157	0.48 ± 0.26	0.02%
	0	450 ± 116	-3.02 ± 0.27	
	-1	160 ± 156	0.46 ± 1.05	

4.2. FIT VALIDATION

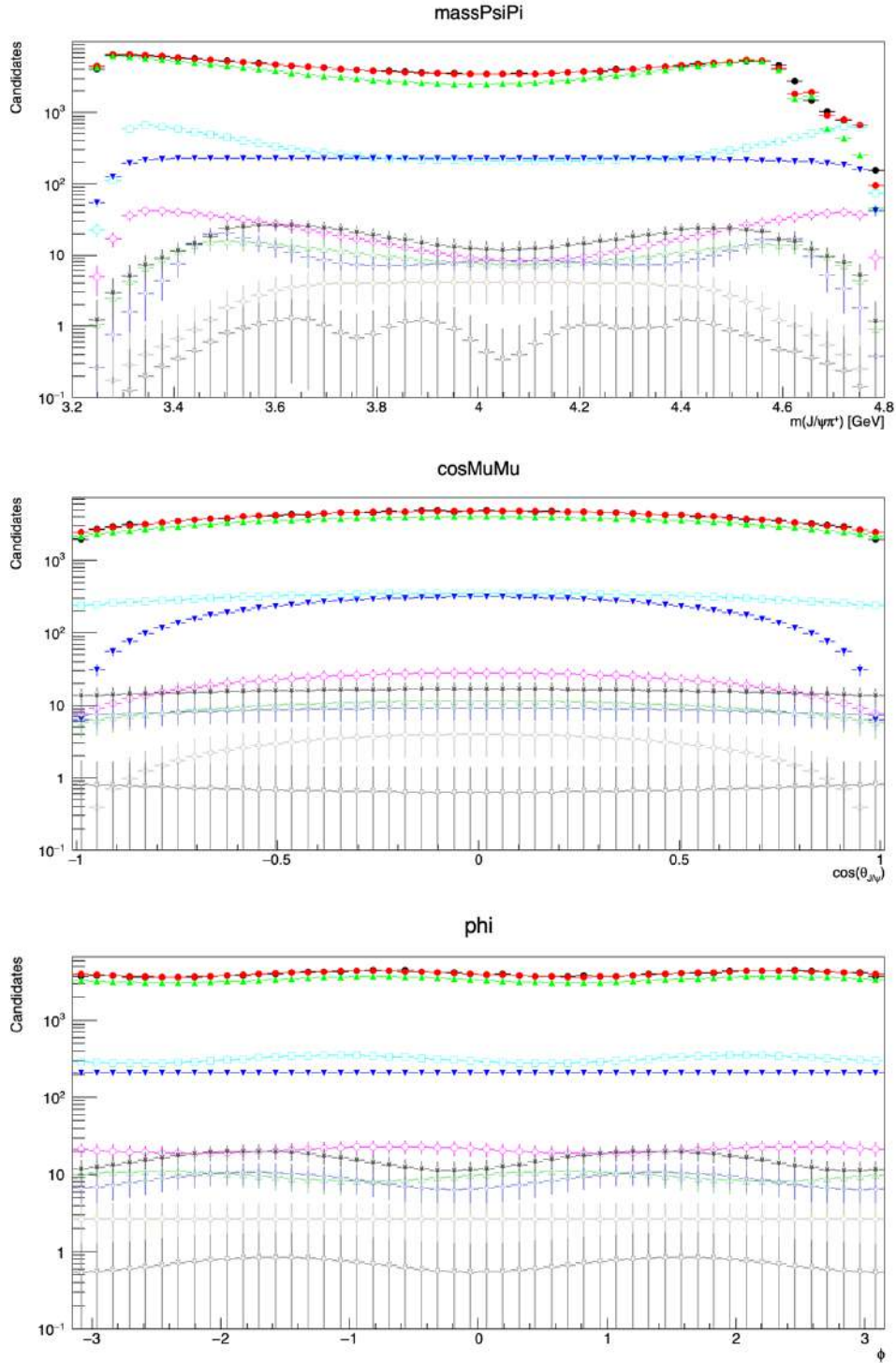


Figure 4.2.10: Projections of (from top to bottom) $m_{J/\psi\pi}$, $\cos(\theta)$ and φ of the 4D dataset generated according to an ideal signal model (black points with error bars). The fit result (red points with error bars) is superimposed along with the individual signal fit components corresponding to the different K^* s and the LASS component.

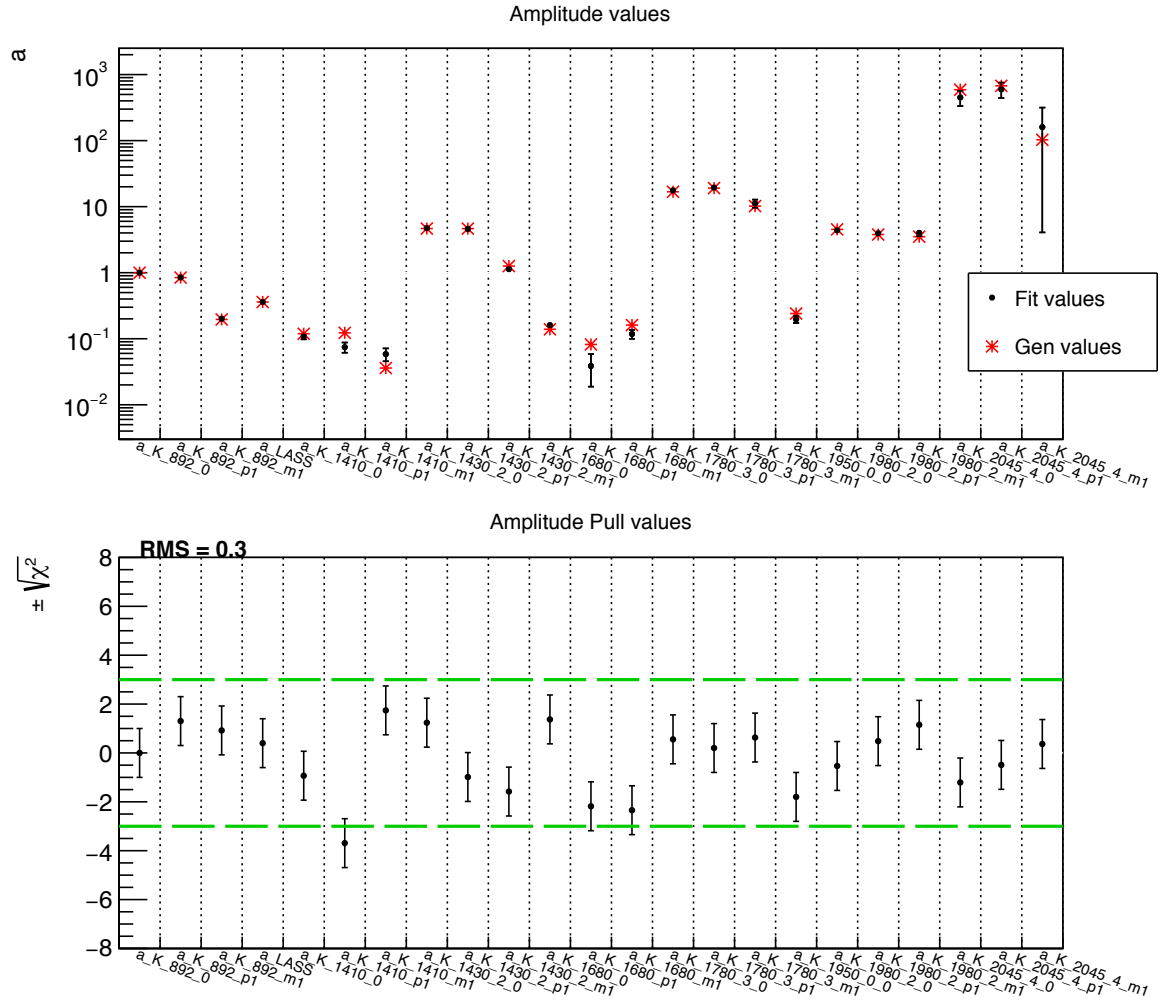


Figure 4.2.11: Comparison of generated and fitted values of the amplitude parameters (*above*) and the corresponding pull distribution (*below*) obtained from a fit to events generated with the LASS model.

4.2. FIT VALIDATION

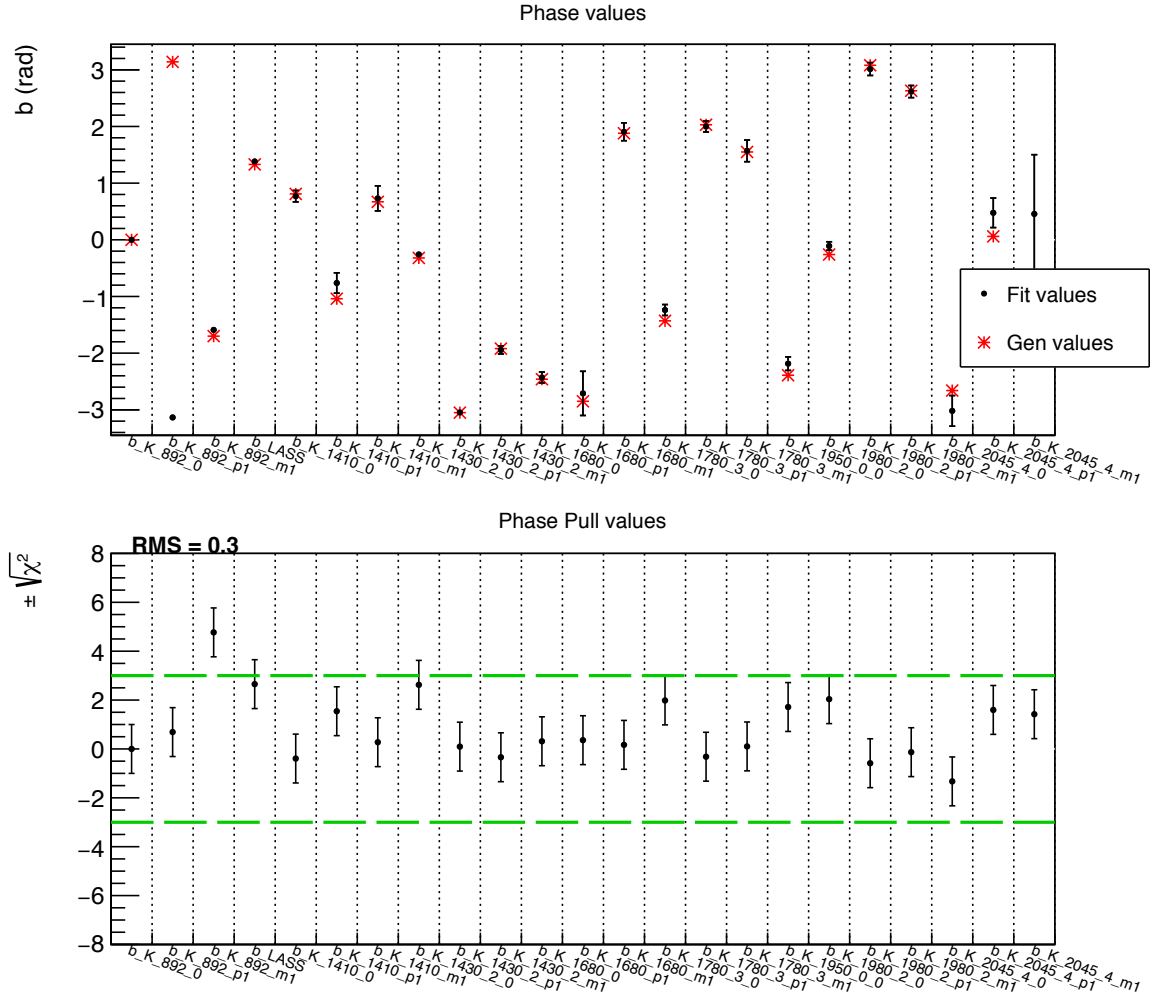


Figure 4.2.12: Comparison of generated and fitted values of the phase parameters (*above*) and the corresponding pull distribution (*below*) obtained from a fit to events generated with the LASS model.

4.2.5 Sensitivity of the fitter to Z contributions

The contribution of each Z state is expected to be of the order of a few percent from the Belle and LHCb results. An MC dataset was generated with the ten K^* s and fitted with a [ten K^* s + $Z(4430)$ + $Z(4200)$] model. The fit fraction for both $Z(4430)$ and $Z(4200)$ were found to be 0.01%. From Figs. 4.2.13 and 4.2.14, it can be seen that the fitted

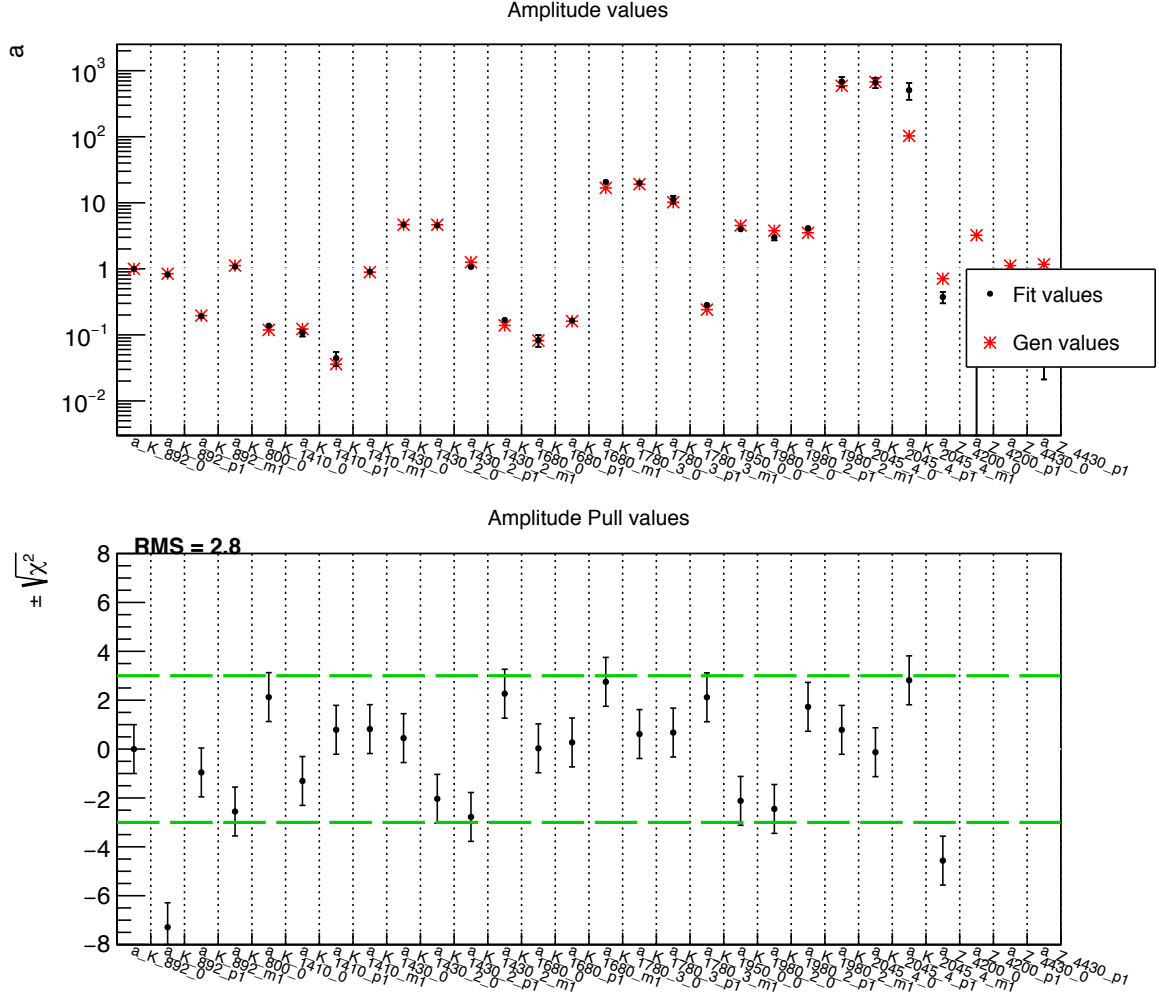


Figure 4.2.13: Comparison of generated and fitted values of the amplitude parameters (*above*) and the corresponding pull distribution (*below*) obtained when a dataset generated with ten K^* s is fitted with a [ten K^* s + $Z(4430)$ + $Z(4200)$] model.

helicity amplitude values for the K^* s are close to their generated values signifying that the contribution of the Z s are indeed consistent with zero.

Similarly, another MC dataset was generated with all K^* s with LASS lineshape and fitted with an [all K^* s (with LASS) + $Z(4430)$ + $Z(4200)$] model. The fit fractions

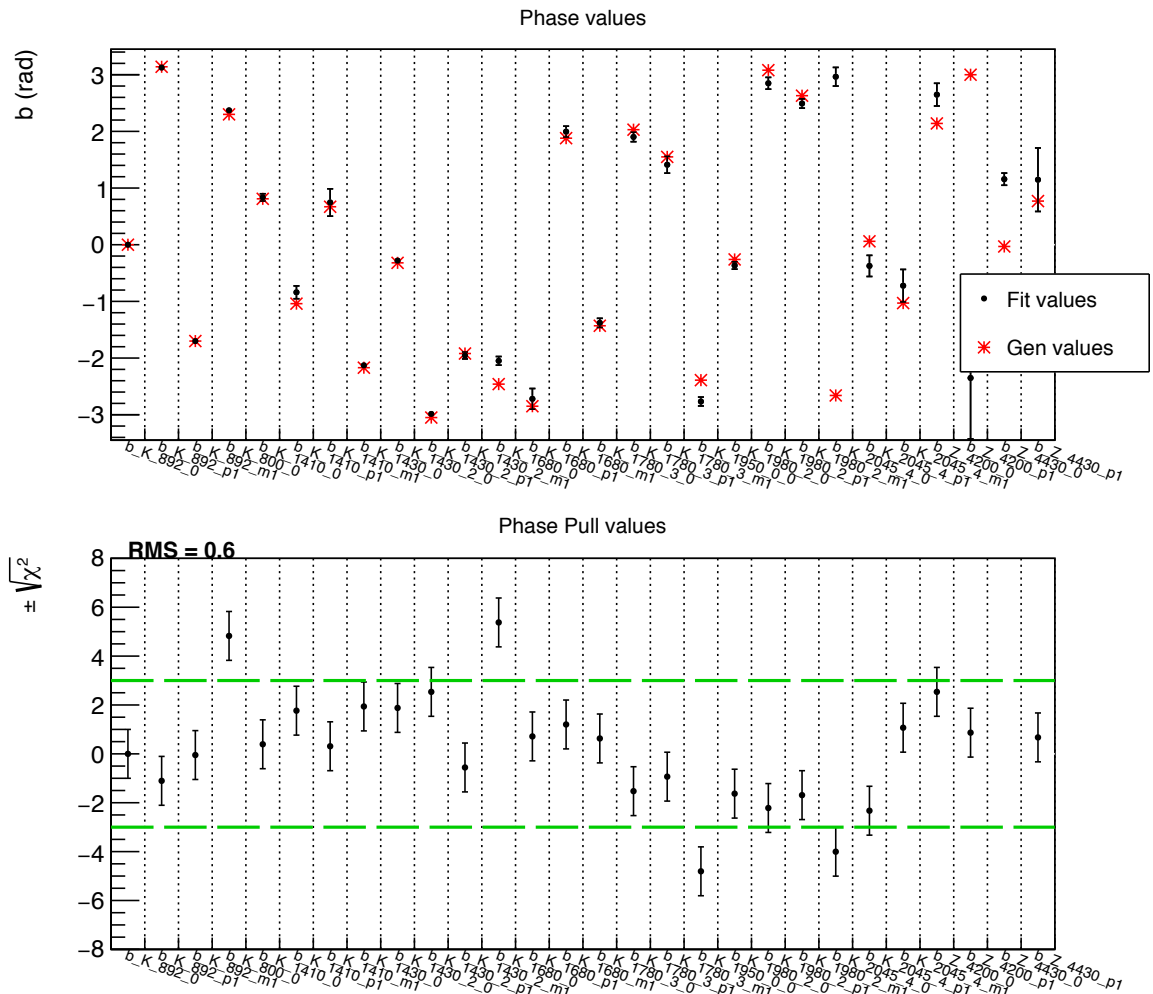


Figure 4.2.14: Comparison of generated and fitted values of the phase parameters (*above*) and the corresponding pull distribution (*below*) obtained when a dataset generated with ten K^* s is fitted with a $[\text{ten } K^*s + Z(4430) + Z(4200)]$ model.

for $Z(4430)$ and $Z(4200)$ were found to be 0.003% and 0.002%, respectively. From

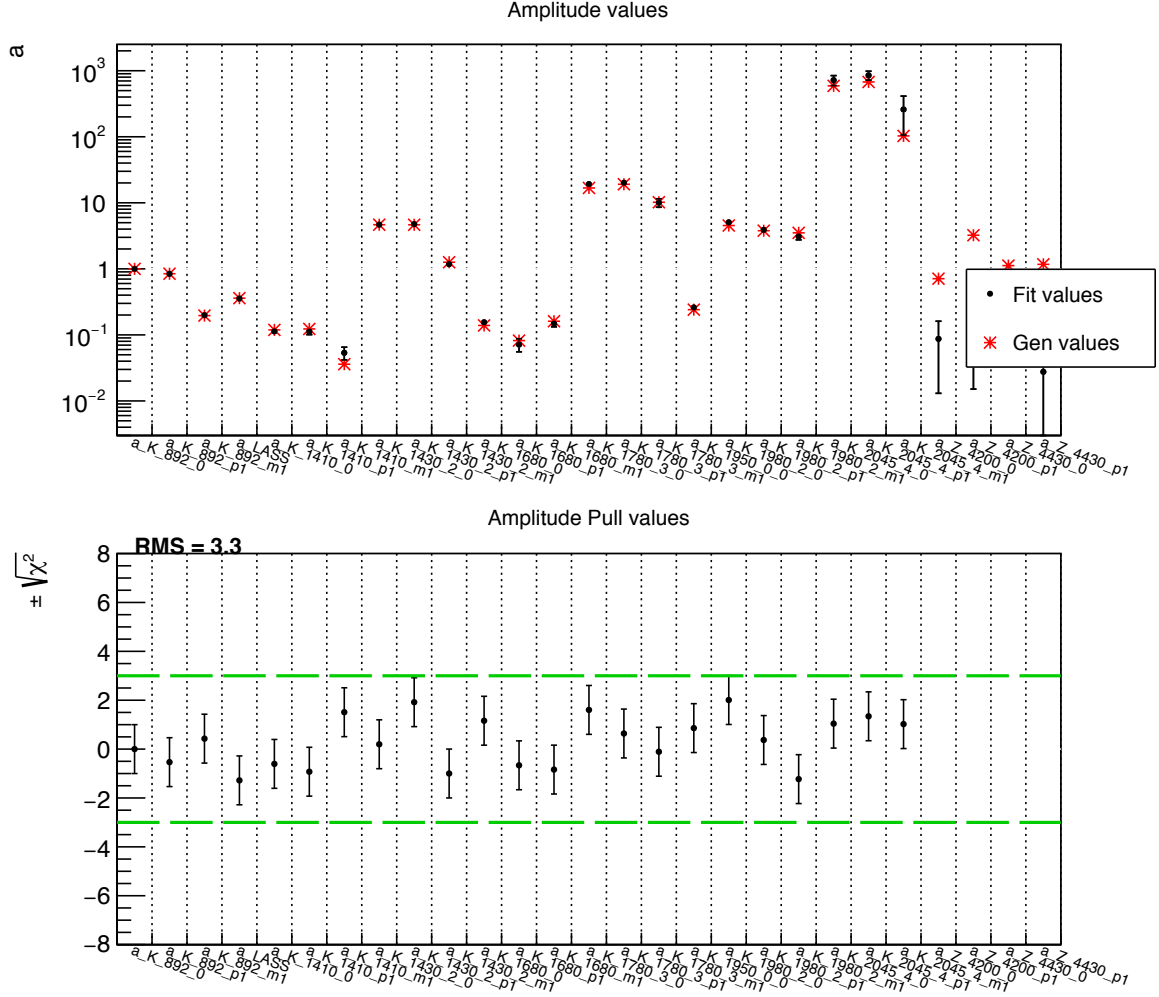


Figure 4.2.15: Comparison of generated and fitted values of the amplitude parameters (*above*) and the corresponding pull distribution (*below*) obtained when a dataset generated with all K^* s (with LASS) is fitted with an [all K^* s (LASS) + $Z(4430)$ + $Z(4200)$] model.

Figs. 4.2.15 and 4.2.16, it can be seen that the fitted helicity amplitude values for the K^* s are close to their generated values signifying that the contribution of the Z s are indeed consistent with zero.

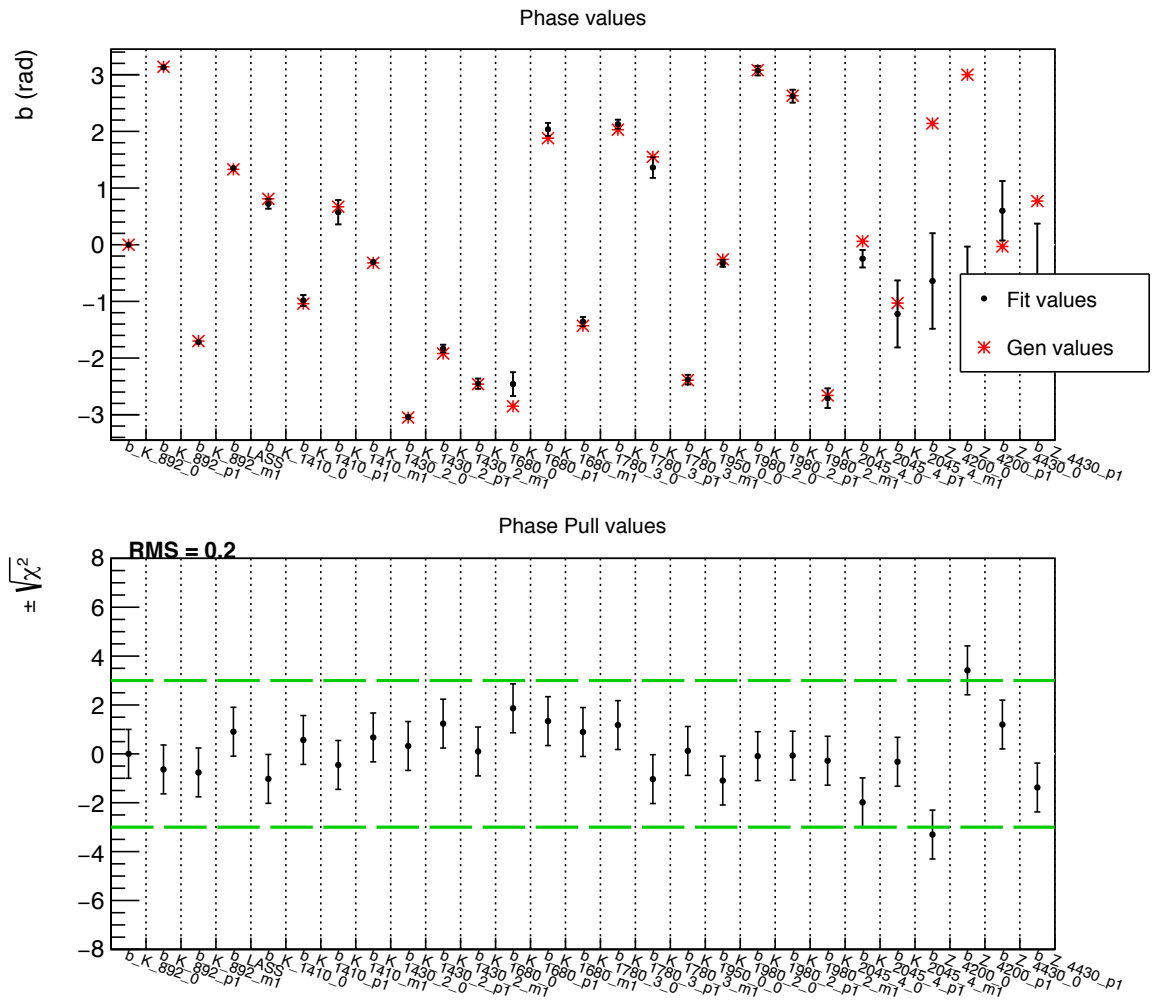


Figure 4.2.16: Comparison of generated and fitted values of the phase parameters (*above*) and the corresponding pull distribution (*below*) obtained when a dataset generated with all K^* s (with LASS) is fitted with an [all K^* s (LASS) + $Z(4430)$ + $Z(4200)$] model.

Amplitude analysis fits on data

Results of the fits of the amplitude analysis framework on real data are presented in this chapter. The latter also includes a discussion on the detector efficiency corrections derived from phase space MC events, and parametrization of the background from B^0 sidebands in data. The fit performances on data under different conditions are extensively discussed and a summary of the analysis is provided.

5.1 Efficiency and background in the fit model

The full Amplitude Analysis (AA) fit developed and configured in the present work is an UML fit in the 4D variables space

$$\Phi \equiv (m_{K\pi}, m_{J/\psi\pi}, \theta_{J/\psi}, \varphi) \equiv \vec{x}. \quad (5.1.1)$$

It is performed by minimising the NLL value of a 4D PDF $f(\vec{x}; \vec{\Theta})$ with respect to the parameter set $\vec{\Theta}$. To accurately model real data, variations of the reconstruction efficiency as well as background contribution over the 4D variable space must be taken into account. Thus, the 4D-model f can be expressed as a combination of two contributions: a signal PDF model $s(\vec{x}, \vec{\Theta})$ and a background PDF model $b(\vec{x}, \vec{\Theta})$, given by

$$f(\vec{x}, \vec{\Theta}) = \frac{n_s \cdot s(\vec{x}; \vec{\Theta}) + n_b b(\vec{x}; \vec{\Theta})}{n_s + n_b}, \quad (5.1.2)$$

where n_s and n_b are the numbers of signal and background candidates, respectively. The $s(\vec{x}; \vec{\Theta})$ and $b(\vec{x}; \vec{\Theta})$ used to describe the selected and reconstructed events implic-

itly include effects of the detector acceptance, and the reconstruction efficiency due to the selection criteria applied in the offline analysis.

The relation between the effective and the theoretical signal model is given by

$$s(\vec{x}; \vec{\Theta}) = \frac{\epsilon(\vec{x})}{\epsilon_s} S(\vec{x}; \vec{\Theta}), \quad (5.1.3)$$

where $\epsilon(\vec{x})$ is the efficiency model and ϵ_s is the overall signal efficiency, defined as

$$\epsilon_s = \int_{\Omega} \epsilon(\vec{x}) S(\vec{x}; \vec{\Theta}) d\vec{x}. \quad (5.1.4)$$

Here, Ω is the total phase-space of the 4D decay. The effective signal model $s(\vec{x}; \vec{\Theta})$ is, by construction, a normalised PDF [see Eqs. (5.1.3) and (5.1.4)]. The theoretical signal model $S(\vec{x}; \vec{\Theta})$ can be expressed in the context of the isobar model using the helicity formalism as done in Eqs. (4.1.5) and (4.2.3).

Since the effective background shape in this study is obtained from data using sidebands of the reconstructed B^0 signal [discussed in section 5.1.2], the background PDF $b(\vec{x}; \vec{\Theta})$ can be directly used in Eq. (5.1.2) without any need for efficiency correction. Thus, Eq. (5.1.2) can be rewritten as

$$f(\vec{x}; \vec{\Theta}) = p \frac{\epsilon(\vec{x})}{\epsilon_s} S(\vec{x}; \vec{\Theta}) + (1 - p) b(\vec{x}; \vec{\Theta}), \quad (5.1.5)$$

where, the signal *purity* p is given by

$$p = \frac{n_s}{n_s + n_b}. \quad (5.1.6)$$

5.1.1 Efficiency correction

The efficiency over the 4D variable space is calculated by using 111 million $B^0 \rightarrow J/\psi K\pi$ MC events generated with a uniform (phase space) distribution over the Dalitz plot. Events are then passed through the full detector simulation chain and subject to all the selection requirements applied on the real data as discussed in chapter 3. Since a full amplitude analysis is sensitive to the relative fluctuations of the efficiency shape, the *relative* efficiency is computed by dividing the absolute efficiency by its average value over the phase-space. The generated MC sample has finite size, so a binned relative efficiency is calculated. Due to limited statistics of the generated sample, most of the bins over the 4D variable space would be scarcely populated or empty. Thus, the overall efficiency is expressed as a combination of two 2D efficiencies over the mass variables, and angular variables, respectively:

$$\epsilon_r^{\text{MC}}(\Phi) = \epsilon_r^{\text{MC}}(m_{K\pi}, m_{J/\psi\pi}) \times \epsilon_r^{\text{MC}}(\theta_{J/\psi}, \varphi). \quad (5.1.7)$$

5.1. EFFICIENCY AND BACKGROUND IN THE FIT MODEL

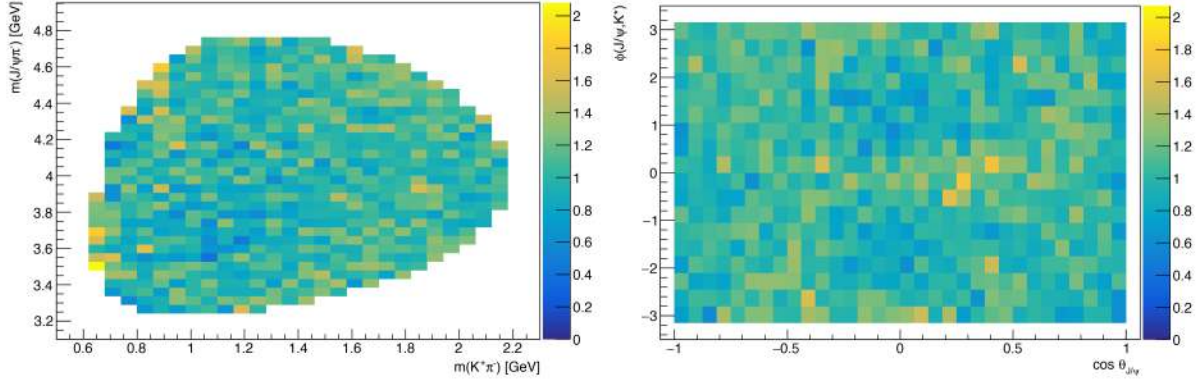


Figure 5.1.1: Relative reconstruction efficiency over the scatter plot of the two mass variables (*left*) and of the two angular variables (*right*).

Many techniques have been tried to include the relative efficiency in the PDF model of the AA fit: as maps, through a smooth interpolation in 2D, as high degree 2D polynomials, with and without constraints, with and without mirroring etc. In the end, a multidimensional Gaussian kernel estimation PDF is used to model the relative efficiency and a bicubic spline interpolation method to estimate the related systematic uncertainty.

5.1.2 Background parametrization

The background density function $b(\Phi)$ is determined with the B^0 mass sidebands as defined in Fig. 3.3.1. This is generally valid for the combinatorial background. Even in presence of contributions from the non-combinatorial background, directly or through reflections, this assumption is more or less valid as the overall level of background is rather low (20%) in this analysis. Note that from chapter 3 the purity, p value is taken as 0.8 that can be used in Eq. (5.1.5). Due to limited statistics, the $b(\Phi)$ is expressed as a combination of two 2D distributions over the Dalitz plot, and angular variables, respectively:

$$b(\Phi) = b_{\text{masses}}(m_{K\pi}, m_{J/\psi\pi}) \times b_{\text{angles}}(\theta_{J/\psi}, \varphi). \quad (5.1.8)$$

Similar to efficiency, the background is also modelled with a multidimensional Gaussian kernel estimation PDF.

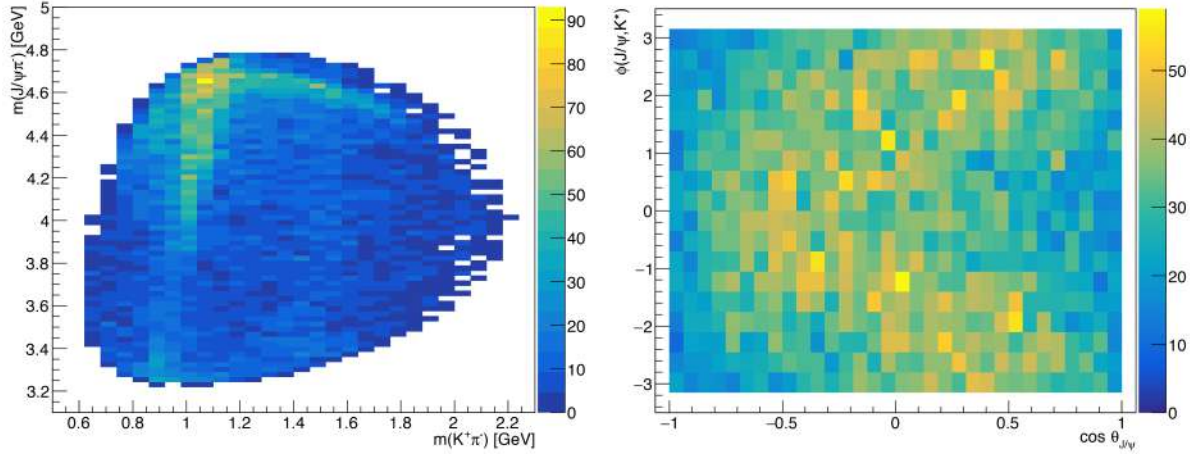


Figure 5.1.2: Scatter plot of (*left*) mass and (*right*) angular variables of the background candidates from the lower and upper sidebands. The sidebands are defined in Fig. 3.3.1.

5.1.3 Closure tests with efficiency and background included in the fit model

MC events were generated and fitted with all 10 K^* s plus $Z(4200)$ and $Z(4430)$ where the events are re-weighted according to the reconstruction efficiency and with 20% background contribution added. The mass, width and helicity amplitudes of the Z resonances are fixed to the values obtained by Belle [8]. The $m_{K\pi}$ projection of the fit to this generated dataset is shown in Figs. 5.1.3 and 5.1.4.

The consistency of the fitted values of free parameters are again checked by comparing the pull distributions with their generated values as shown in Figs. 5.1.5 and 5.1.6.

5.1. EFFICIENCY AND BACKGROUND IN THE FIT MODEL

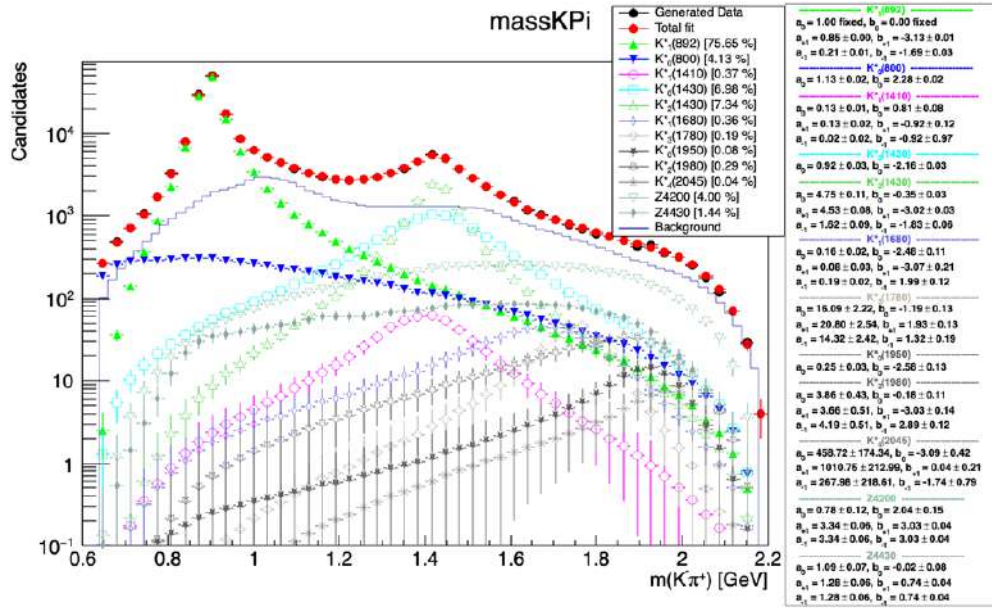


Figure 5.1.3: Projection on the $m_{K\pi}$ spectrum of the 4D dataset generated according to an ideal signal model (black points with error bars) with 10 K^* s and 2 Z s including efficiency and background parametrization. The fit result (red points with error bars) is superimposed along with the individual signal fit components corresponding to the different K^* s and Z s and background (solid blue histogram). The fitted values of the helicity amplitude parameters and fit fractions for each component are shown here as well as listed in Table 5.1.1.

Table 5.1.1: The magnitudes and phases of the helicity amplitudes obtained from a fit to the ideal signal model generated with 10 K^* s and 2 Z s including efficiency and background parametrization.

Resonance	λ	$ H_\lambda $	$\arg H_\lambda$	Fit fraction
$K_0^*(800)$	0	1.13 ± 0.02	2.28 ± 0.02	4.13%
$K^*(892)$	+1	0.85 ± 0.00	-3.13 ± 0.01	75.65%
	0	1.0 (fixed)	0.0 (fixed)	
	-1	0.21 ± 0.01	-1.69 ± 0.03	
$K^*(1410)$	+1	0.13 ± 0.02	-0.92 ± 0.12	0.37%
	0	0.13 ± 0.01	0.81 ± 0.08	
	-1	0.02 ± 0.02	-0.92 ± 0.97	
$K_0^*(1430)$	0	0.92 ± 0.03	-2.16 ± 0.03	6.98%
$K_2^*(1430)$	+1	4.53 ± 0.08	-3.02 ± 0.03	7.34%
	0	4.75 ± 0.11	-0.35 ± 0.03	
	-1	1.62 ± 0.09	-1.83 ± 0.06	
$K^*(1680)$	+1	0.08 ± 0.03	-3.07 ± 0.21	0.36%
	0	0.16 ± 0.02	-2.48 ± 0.11	
	-1	0.19 ± 0.02	1.99 ± 0.12	
$K_3^*(1780)$	+1	20.80 ± 2.54	1.93 ± 0.13	0.19%
	0	16.09 ± 2.22	-1.19 ± 0.13	
	-1	14.32 ± 2.42	1.32 ± 0.19	
$K_0^*(1950)$	0	0.25 ± 0.03	-2.58 ± 0.13	0.08%
$K_2^*(1980)$	+1	3.66 ± 0.51	-3.03 ± 0.14	0.29%
	0	3.86 ± 0.43	-0.18 ± 0.11	
	-1	4.19 ± 0.51	2.89 ± 0.12	
$K_4^*(2045)$	+1	1011 ± 213	0.04 ± 0.21	0.04%
	0	459 ± 174	-3.09 ± 0.42	
	-1	268 ± 217	-1.74 ± 0.79	
$Z(4200)$	0	0.78 ± 0.12	2.04 ± 0.15	4.00%
	± 1	3.34 ± 0.06	3.03 ± 0.04	
$Z(4430)$	0	1.09 ± 0.07	-0.02 ± 0.08	1.44%
	± 1	1.28 ± 0.06	0.74 ± 0.04	

5.1. EFFICIENCY AND BACKGROUND IN THE FIT MODEL

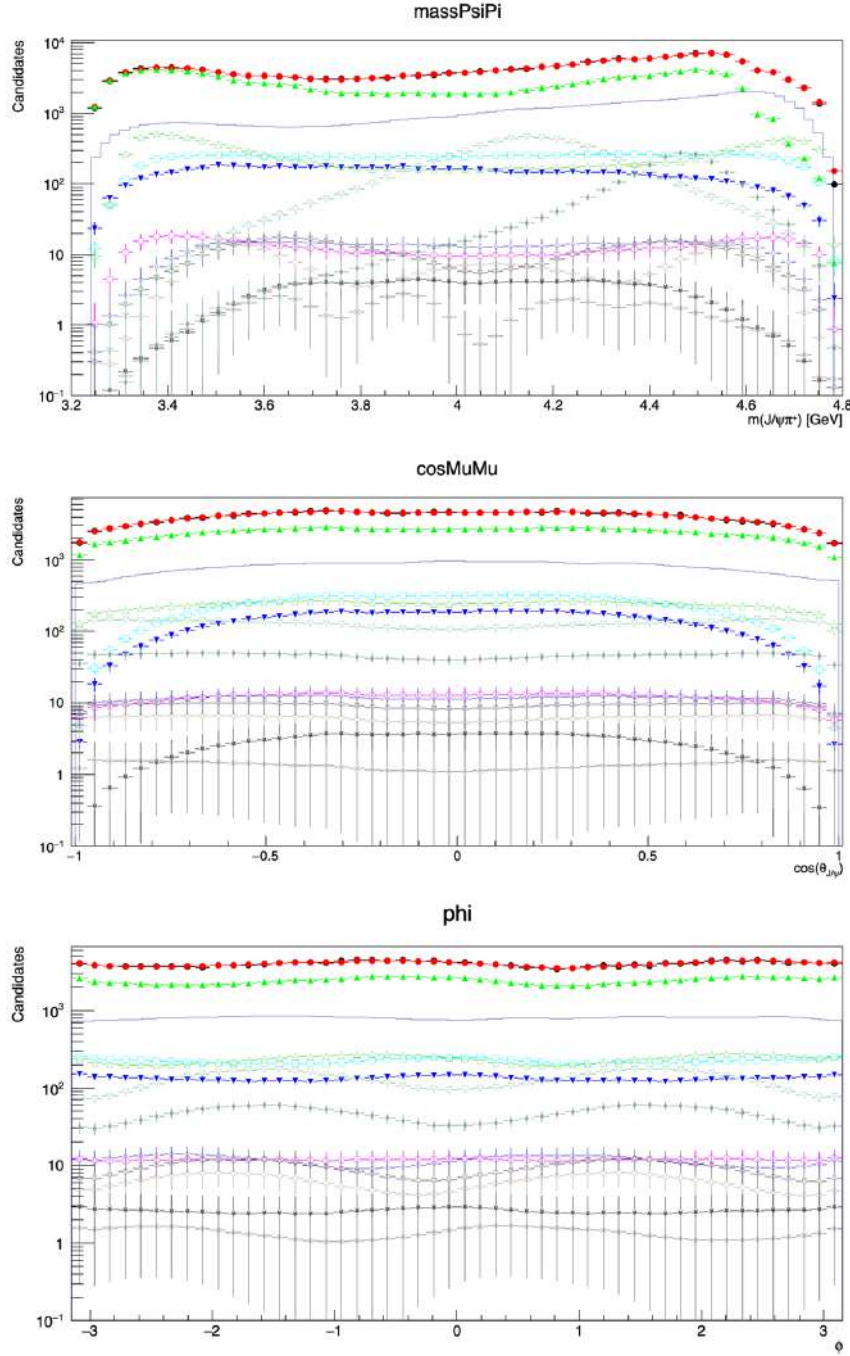


Figure 5.1.4: Projections of (from top to bottom) $m_{J/\psi\pi}$, $\cos\theta$ and φ of the 4D dataset generated according to an ideal signal model (black points with error bars) with 10 K^* s and 2 Z s including efficiency and background parametrization. The fit result (red points with error bars) is superimposed along with the individual signal fit components corresponding to the different K^* s and Z s as well as background (solid blue histogram).

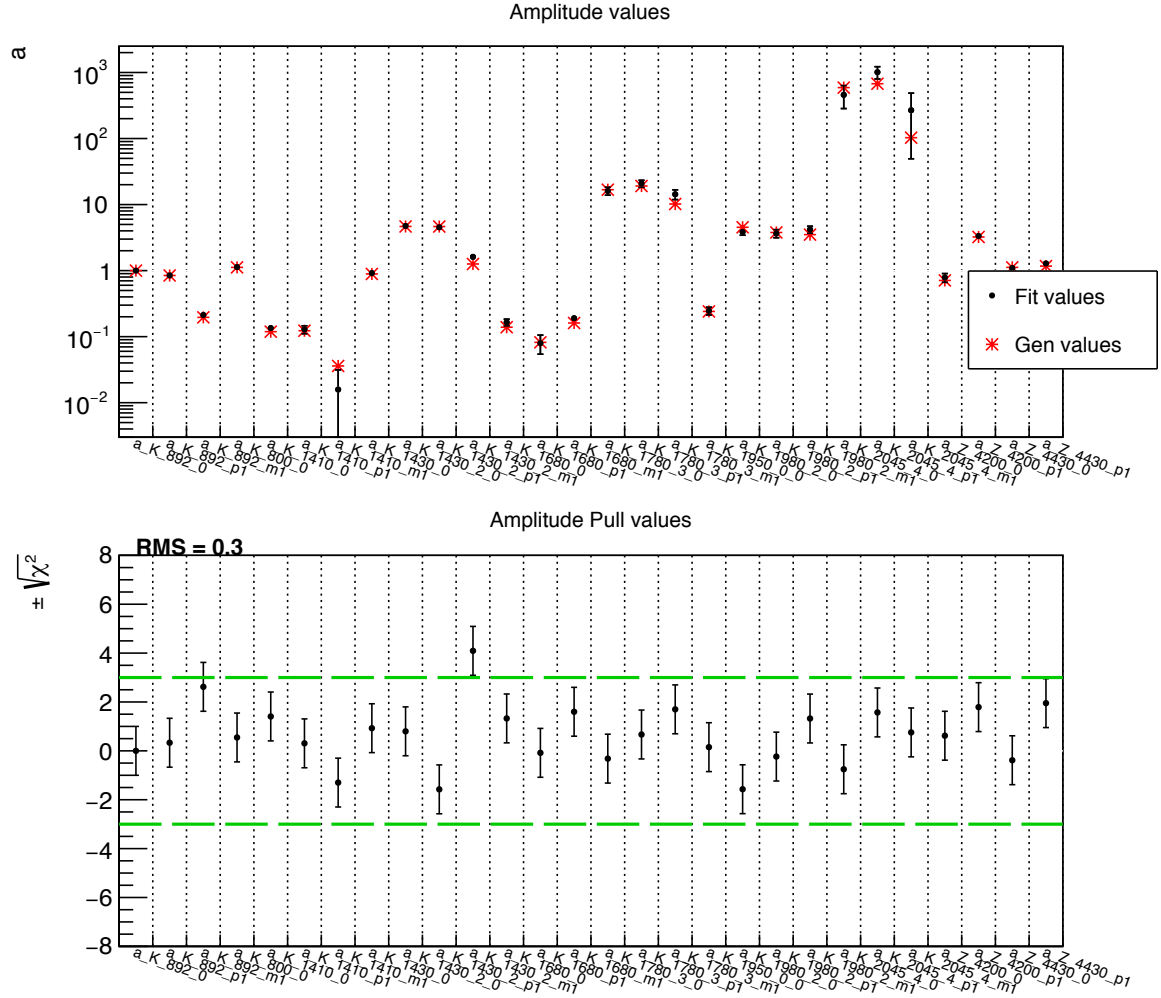


Figure 5.1.5: Comparison of generated and fitted values of the amplitude parameters (*above*) and the corresponding pull distribution (*below*) obtained from a fit to events generated with all the 10 K^* plus 2 Z resonances including efficiency and background.

5.1. EFFICIENCY AND BACKGROUND IN THE FIT MODEL

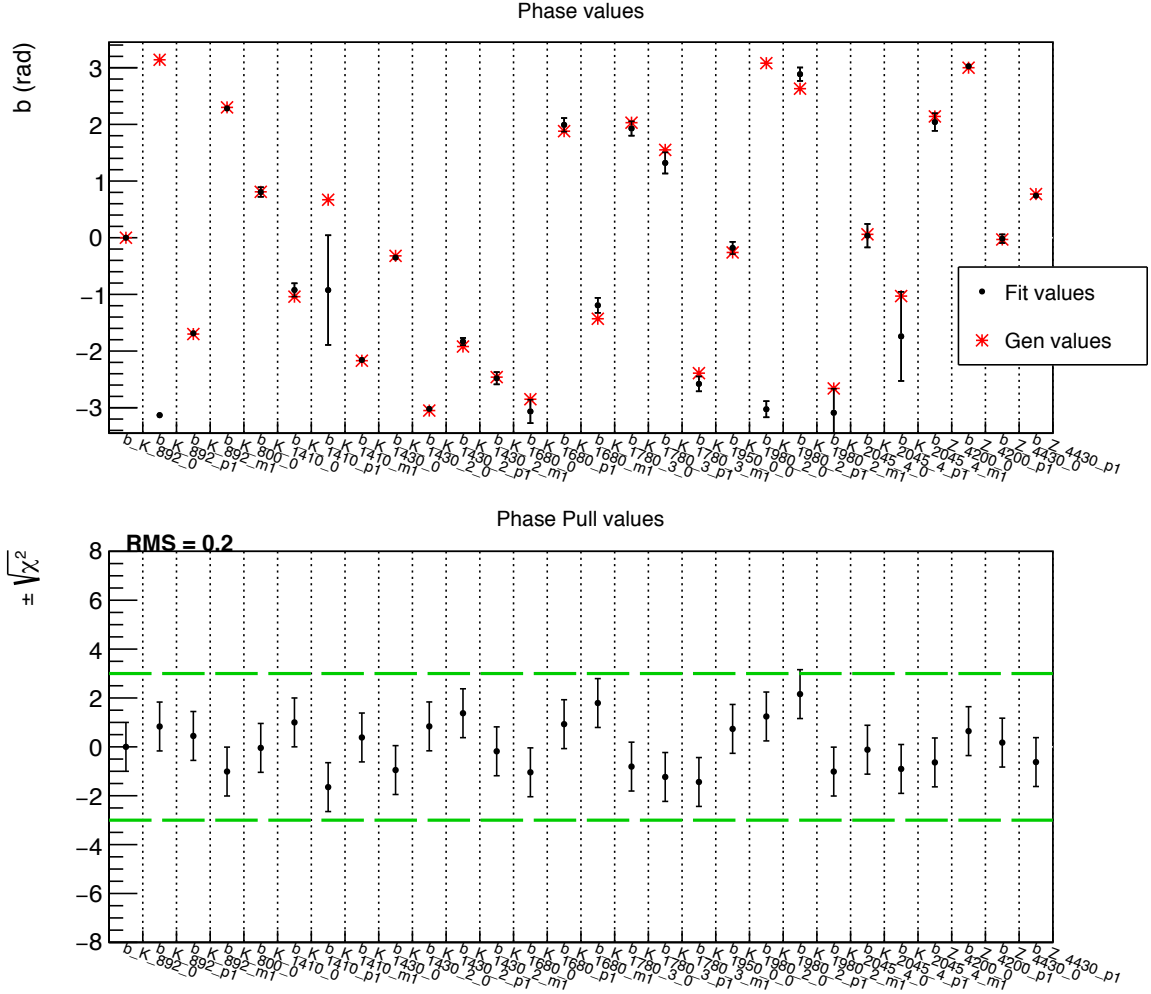


Figure 5.1.6: Comparison of generated and fitted values of the phase parameters (*above*) and the corresponding pull distribution (*below*) obtained from a fit to events generated with all the 10 K^* plus 2 Z resonances including efficiency and background.

5.2 Fits on data

The fit to data is performed in a sequence of steps described below:

- The data are first fitted with the five most dominant K^* s: $K_0^*(800)$, $K^*(892)$, $K_0^*(1430)$, $K_2^*(1430)$, and $K_2^*(1980)$. Since the low mass S-wave $K_0^*(800)$ is not well measured, its mass and width are allowed to float in the fit (Table 5.2.1).
- The fit with five K^* s is repeated with the alternative LASS parametrization for the S-wave system [$K_0^*(800)$ and $K_0^*(1430)$].
- The fit is performed again with all the ten kinematically allowed K^* s. Also in this case, the mass and width of $K_0^*(800)$ is allowed to float (Table 5.2.2).
- The same fit with ten K^* s is also repeated with the LASS lineshape.

The value of the NLL itself is taken as a measure of goodness of fit. We show that the fit yields a significantly lower NLL value (hence a better fit) when fitted with all the ten K^* s (NLL value: 422975) compared to that with only five K^* s (NLL value: 424522). It can also be seen that the BW assumption with floating mass and width is a better choice than LASS for describing the contribution of $K_0^*(800)$ and $K_0^*(1430)$ to the overall fit (NLL value with LASS for ten K^* s: 424611).

The $m_{K\pi}$ projection of the fit to data with all the ten K^* s model is shown in Fig. 5.2.1 while the other projections are shown in Fig. 5.2.2.

The fit results with the ten K^* model are presented in four mass ranges in $m_{K\pi}$ ($0.50 < m_{K\pi} < 1.10$ GeV, $1.10 < m_{K\pi} < 1.43$ GeV, $1.43 < m_{K\pi} < 1.78$ GeV, and $1.78 < m_{K\pi} < 2.30$ GeV) and three mass ranges in $m_{J/\psi\pi}$ ($3.00 < m_{J/\psi\pi} < 4.00$ GeV, $4.00 < m_{J/\psi\pi} < 4.36$ GeV, and $4.36 < m_{J/\psi\pi} < 5.00$ GeV) as can be seen in Figs. 5.2.3 and 5.2.4.

The amplitude magnitudes and phases in the model with ten K^* s are listed in Table 5.2.3.

Table 5.2.1: Fitted values of $K_0^*(800)$ mass and width from a fit on data with five K^* s. NLL value after fit: **424522**, NLL value after fit with LASS: **426501**.

Variable	Fitted value	Error	Lower limit	Upper limit
$K_0^*(800)$ mass [GeV]	0.879	0.005	0.6	1.0
$K_0^*(800)$ width [GeV]	0.461	0.014	0.3	0.7

Table 5.2.2: Fitted values of $K_0^*(800)$ mass and width from a fit on data with ten K^* s. NLL value after fit: **422975**, NLL value after fit with LASS: **424611**.

Variable	Fitted value	Error	Lower limit	Upper limit
$K_0^*(800)$ mass [GeV]	0.877	0.005	0.6	1.0
$K_0^*(800)$ width [GeV]	0.452	0.016	0.3	0.7

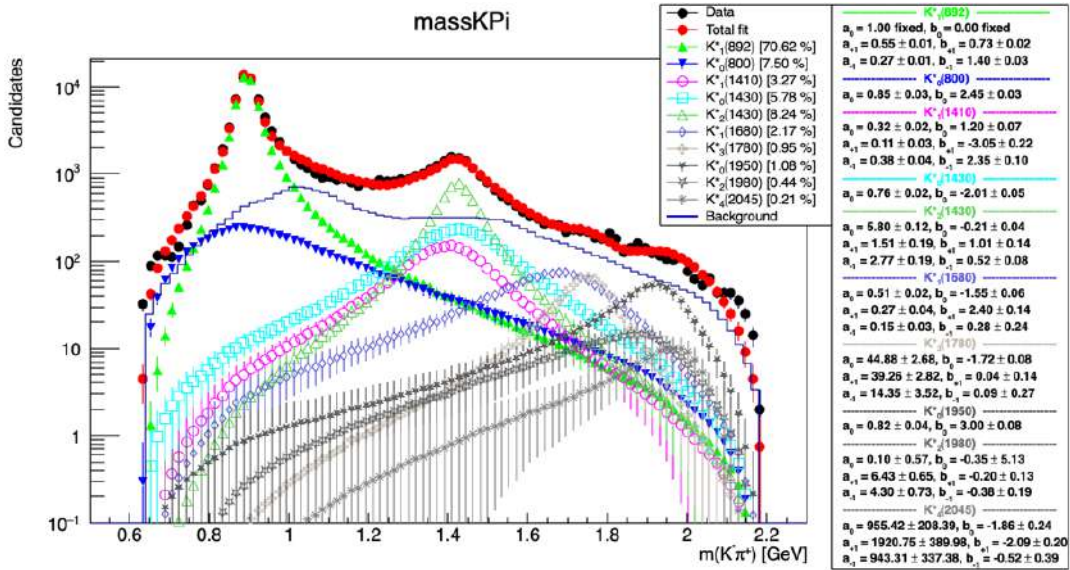


Figure 5.2.1: Projection on the $m_{K\pi}$ spectrum in data (black points with error bars) with ten K^* s. The fit result (red points with error bars) is superimposed along with the individual signal fit components corresponding to the different K^* s and background (solid blue histogram). The fitted values of helicity amplitude parameters and fit fractions of each component are also shown.

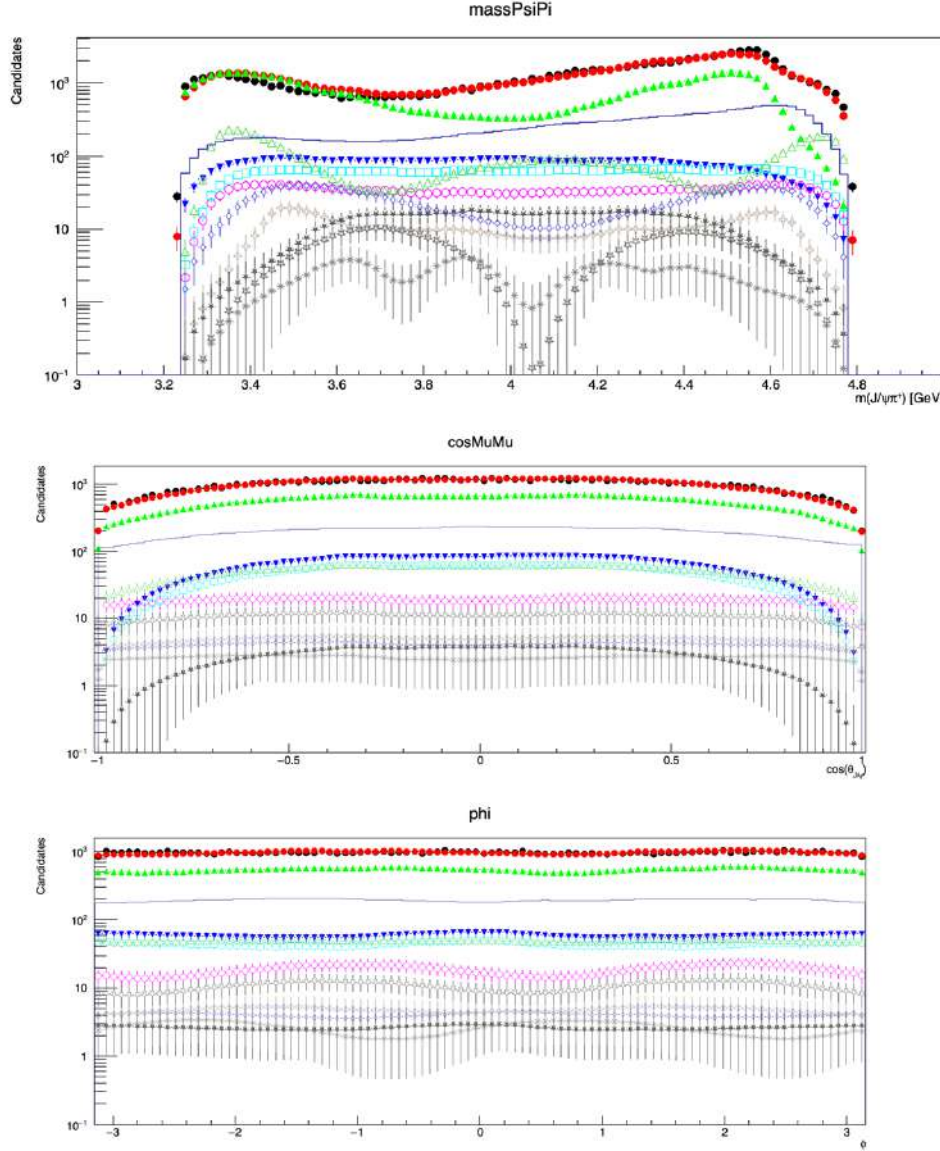


Figure 5.2.2: Projection of (from top to bottom) $m_{J/\psi\pi}$, $\cos\theta$, and φ after fitting the data with all the ten K^* s (black points with error bars). The fit result (red points with error bars) is superimposed along with the individual signal fit components corresponding to the different K^* s and background (solid blue histogram).

5.2. FITS ON DATA

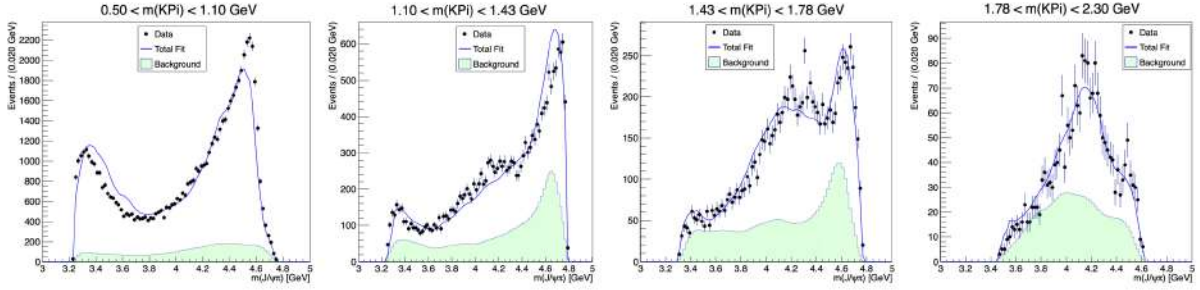


Figure 5.2.3: Fit results to data in different $m_{K\pi}$ ranges with ten K^* s. The solid blue curves are the fit result, the black points with error bars are the data, and the shaded regions represent the background.

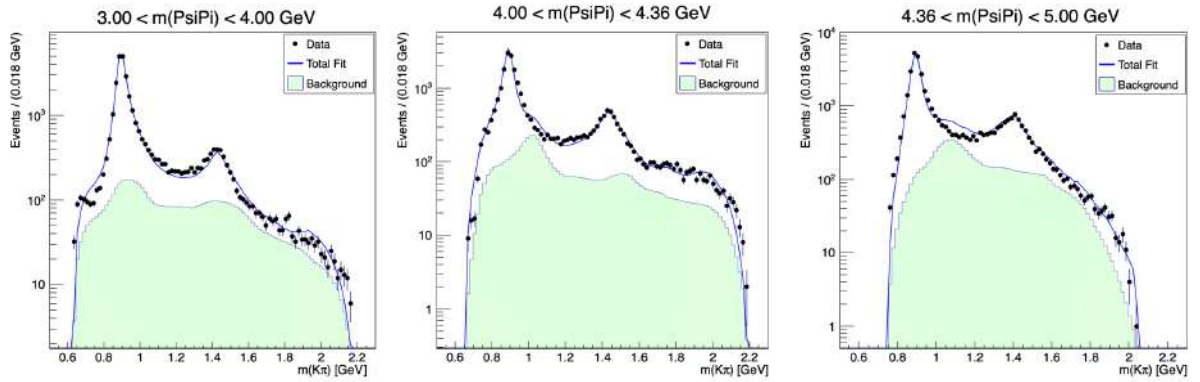


Figure 5.2.4: Fit results to data in different $m_{J/\psi\pi}$ ranges with ten K^* s. The solid blue curves are the fit result, the black points with error bars are the data, and the shaded regions represent the background.

Table 5.2.3: The magnitudes and phases of the helicity amplitudes obtained from a fit to the data in the model with ten K^* s.

Resonance	λ	$ H_\lambda $	$\arg H_\lambda$	Fit fraction
$K_0^*(800)$	0	$(8.47 \pm 0.30) \times 10^{-1}$	2.45 ± 0.03	8.0%
$K^*(892)$	+1	$(5.45 \pm 0.10) \times 10^{-1}$	0.73 ± 0.02	69.7%
	0	1.0 (fixed)	0.0 (fixed)	
	-1	$(2.68 \pm 0.09) \times 10^{-1}$	1.39 ± 0.03	
$K^*(1410)$	+1	$(1.11 \pm 0.31) \times 10^{-1}$	-3.05 ± 0.22	2.1%
	0	$(3.24 \pm 0.16) \times 10^{-1}$	1.21 ± 0.07	
	-1	$(3.81 \pm 0.38) \times 10^{-1}$	2.35 ± 0.10	
$K_0^*(1430)$	0	$(7.65 \pm 0.25) \times 10^{-1}$	-2.01 ± 0.04	5.8%
$K_2^*(1430)$	+1	1.51 ± 0.19	1.01 ± 0.13	6.4%
	0	5.80 ± 0.12	-0.21 ± 0.03	
	-1	2.77 ± 0.19	0.52 ± 0.08	
$K^*(1680)$	+1	$(2.66 \pm 0.31) \times 10^{-1}$	2.40 ± 0.14	0.6%
	0	$(5.09 \pm 0.20) \times 10^{-1}$	-1.55 ± 0.06	
	-1	$(1.47 \pm 0.33) \times 10^{-1}$	0.28 ± 0.24	
$K_3^*(1780)$	+1	39.26 ± 2.82	0.03 ± 0.14	0.6%
	0	44.87 ± 3.68	-1.72 ± 0.08	
	-1	14.35 ± 3.52	0.09 ± 0.27	
$K_0^*(1950)$	0	$(8.21 \pm 0.39) \times 10^{-1}$	3.00 ± 0.08	0.4%
$K_2^*(1980)$	+1	6.43 ± 0.65	-0.20 ± 0.13	1.3%
	0	$(1.05 \pm 5.67) \times 10^{-1}$	-0.35 ± 5.13	
	-1	4.30 ± 0.73	-0.38 ± 0.19	
$K_4^*(2045)$	+1	1921 ± 390	-2.09 ± 0.20	0.3%
	0	955 ± 208	-1.85 ± 0.24	
	-1	943 ± 337	-0.52 ± 0.38	

5.3 Future developments

The validation and functionality tests for the AA fit have been carried out under many different conditions using generated events. The fitter can be used with models of varying complexity and has been found to be sensitive to small variations. Initial tests on data demonstrate that the general behaviour of the fitter adheres to expectations. Given that the purity of the sample is 80%, the background estimated from the sidebands plays a crucial role in the fits to data. Further investigations to pinpoint the exact nature of backgrounds can improve the fit results. Other than the alternative LASS parametrization for the spin-0 resonances and free masses and widths (with Gaussian constraints to their known values) of K resonances, other sources of systematic uncertainties viz. floating Blatt-Weisskopf r parameters (up to 5 GeV^{-1}), and the presence or absence of each K^* resonance are being studied. Further studies on the fit stability may help us improve the fit to data for a nominal model including the exotic Z contributions.

CHAPTER 5. AMPLITUDE ANALYSIS FITS ON DATA

Summary

Charged charmonium-like Z states are manifestly exotic and particularly interesting as candidates for compact tetraquark states with a possible quark content of $|c\bar{c}d\bar{u}\rangle$. Recent studies by Belle and LHCb Collaborations have established the $Z(4430)^-$ state as a 1^+ resonance in the $B^0 \rightarrow (\psi(2S)\pi^-)K^+$ decay. Belle has also found evidence for $Z(4430)^-$ in the $B^0 \rightarrow (J/\psi\pi^-)K^+$ decay, along with a dominant new state, the $Z(4200)^-$. Very recently, LHCb demonstrated the possibility of exotic contributions to the $B^0 \rightarrow J/\psi K\pi$ signal via a model-independent moment analysis. The $Z(3900)^-$ state was observed by both BESIII and Belle in their study of $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ near the $\Upsilon(4260)$ region. However, it was not reported in Belle's search in the $B^0 \rightarrow (J/\psi\pi^-)K^+$ channel.

This thesis embodies a thorough study for the decay channel $B^0 \rightarrow J/\psi K\pi$ based on 19.6 fb^{-1} of pp collision data collected in 2012 at $\sqrt{s} = 8 \text{ TeV}$ by CMS at the LHC. The signal events have been collected using a dimuon trigger corresponding to a J/ψ decay. The signal purification has been particularly challenging due to the absence of charged hadron identification capability in CMS unlike other experiments such as LHCb or Belle. The $B^0 \rightarrow J/\psi K\pi$ signal extraction and purification has been done on multicore clusters using the PROOF-Lite/ROOT parallel computing framework. Enormous combinatorial background has been eliminated through both kinematic and topological cut-based selections resulting in a signal purity of $\sim 80\%$. Keeping in mind that the purified signal will be used for a detailed amplitude analysis, mass constrained vertex fit has been applied to both the two-track J/ψ candidates and the four-track B^0 candidates to ensure that the final events lie within the kinematically allowed boundary. Reflection effects from physical backgrounds due to hadron misidentification have been eliminated through a detailed study of different Monte Carlo samples.

Using the helicity formalism, a four-dimensional amplitude analysis framework for unbinned maximum likelihood fit has been implemented. The fitting framework has been developed using the novel GPU based GooFit which is an underdevelopment open source data analysis tool, used in the HEP applications for parameters' es-

timization, which interfaces ROOT to the CUDA parallel computing platform on nVidia GPUs. The fit model has been validated by generating and fitting toy Monte Carlo samples under different conditions with the known K^* resonances. Since the low mass S-wave $K_0^*(800)$ is not satisfactorily described by a Breit-Wigner amplitude, the alternative LASS parametrization has been implemented on GooFit and thoroughly tested. Finally, the possible contribution of the exotic Z components has been calculated and incorporated within the fitter framework with enough robustness to allow for testing with any combination of J^{PC} values as well as without constraints).

The reconstruction efficiency over the whole of the 4D variable space has been obtained from officially generated phase space Monte Carlo samples and parametrized for the amplitude analysis fitter using a multidimensional Gaussian kernel estimation PDF. The background has been estimated from the B^0 sidebands and modelled similar to efficiency in the fitter.

The performance of the fitter on real data with only the known K^* models has been consistent. The fitter has proved to be sensitive to models with small contribution to the overall fit. It is seen that the NLL value reduces significantly when the model fitted is close to the actual one. The framework is sensitive enough to detect contribution of charged charmonium-like Z states, if at all present, with a high level of significance.

The work presented in this thesis demonstrates that a full-fledged state of the art GPU based amplitude analysis framework developed for three-body decays is performing according to expectations. It is hoped that this will considerably augment the capabilities of CMS in searches and measurements in the field of exotic charmonium spectroscopy.

Appendix

A Lorentz invariant expression for the decay angle in a two-body decay

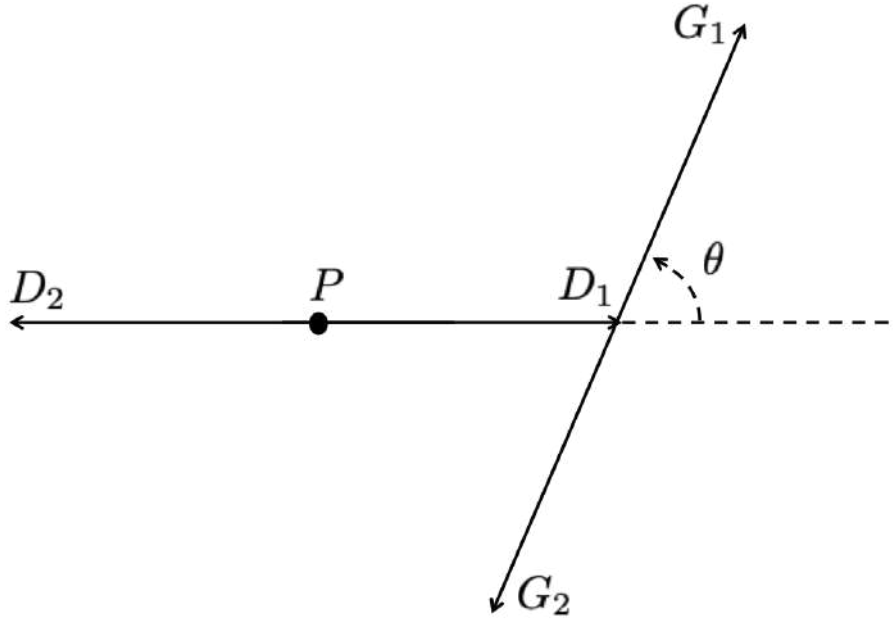


Figure A.1: Definition of the decay angle.

Consider a particle P decaying into two daughters D_1 and D_2 , where D_1 further decays into G_1 and G_2 (Fig. A.1). The decay angle θ is the angle between the flight direction of G_1 in the rest frame of D_1 , with respect to D_1 's flight direction in the rest frame of P . P , D_1 , and G_1 are the four-momenta of these particles in any frame of reference. The decay angle is computed (from Ref. [62]) using the (Lorentz invariant) expression

$$\cos \theta = \frac{(P \cdot G_1)M_{D_1}^2 - (P \cdot D_1)(D_1 \cdot G_1)}{\sqrt{[(P \cdot D_1)^2 - M_{D_1}^2 M_P^2][(D_1 \cdot G_1)^2 - M_{D_1}^2 M_{G_1}^2]}}, \quad (\text{A.1})$$

where, the M s denote the nominal masses of the corresponding particles. By exploiting relativistic kinematics, Eq. (A.1) can be expressed in terms of only the invariant masses. Considering a three-body decay, in the rest frame of P ,

$$P \cdot G_1 = E_P E_{G_1} - (\vec{p}_P \cdot \vec{p}_{G_1}) = M_P E_{G_1}, \quad (\text{A.2})$$

where, the E s denote the energies and $\vec{p}$ s denote the momentum vectors of the corresponding particles. It also holds true that $m_{D_2 G_2}^2 = M_P^2 + M_{G_1}^2 - 2M_P E_{G_1}$ where m s are the running invariant masses of the particles denoted in the subscripts combined. Thus,

$$P \cdot G_1 = M_P E_{G_1} = \frac{1}{2}[M_P^2 + M_{G_1}^2 - m_{D_2 G_2}^2]. \quad (\text{A.3})$$

Considering a two-body decay of the intermediate D_1 in its rest frame, ($\vec{p}_{D_1} = 0$),

$$D_1 \cdot G_1 = E_{D_1} E_{G_1} - (\vec{p}_{D_1} \cdot \vec{p}_{G_1}) = m_{G_1 G_2} E_{G_1}. \quad (\text{A.4})$$

By invoking Eq. (39.15) of the PDG, viz. $E_{G_1} = \frac{m_{G_1 G_2}^2 - M_{G_2}^2 + M_{G_1}^2}{2m_{G_1 G_2}}$, one gets

$$D_1 \cdot G_1 = m_{G_1 G_2} E_{G_1} = \frac{1}{2}[m_{G_1 G_2}^2 - M_{G_2}^2 + M_{G_1}^2]. \quad (\text{A.5})$$

Similarly, for the two-body decay of P in its rest frame, ($\vec{p}_P = 0$),

$$P \cdot D_1 = \frac{1}{2}[M_P^2 - M_{D_2}^2 + m_{G_1 G_2}^2]. \quad (\text{A.6})$$

So, Eq. (A.1) can be rewritten as

$$\cos \theta \equiv \frac{\text{Num}}{\text{Denom}}, \quad (\text{A.7})$$

where, after putting together the expressions obtained so far, one gets

$$\begin{aligned} \text{Num} &= \frac{m_{G_1 G_2}^2}{2}(M_P^2 + M_{G_1}^2 - m_{D_2 G_2}^2) \\ &\quad - \frac{1}{4}(M_P^2 - M_{D_2}^2 + m_{G_1 G_2}^2)(m_{G_1 G_2}^2 - M_{G_2}^2 + M_{G_1}^2) \end{aligned} \quad (\text{A.8})$$

and

$$\begin{aligned} \text{Denom} &= \sqrt{\left[\frac{1}{4}(M_P^2 - M_{D_2}^2 + m_{G_1 G_2}^2)^2 - m_{G_1 G_2}^2 M_P^2 \right]} \\ &\quad \times \sqrt{\left[\frac{1}{4}(m_{G_1 G_2}^2 - M_{G_2}^2 + M_{G_1}^2)^2 - m_{G_1 G_2}^2 M_{G_1}^2 \right]}. \end{aligned} \quad (\text{A.9})$$

B Implementation of the angles for decays via Z

In order to evaluate the signal density of the combined K^* plus Z model, the angles, θ_Z , $\tilde{\theta}_{J/\psi}$, $\tilde{\varphi}$, and α from Eq. (4.2.1) need to be calculated event by event in terms of the four variables, $m_{K\pi}^2$, $m_{J/\psi\pi}^2$, $\theta_{J/\psi}$, and φ in the correct rest frames.

B.1 Calculation of θ_Z in the Z rest frame

The Z helicity angle θ_Z in the Z rest frame is directly calculated from Eq. (A.1) by putting $P \equiv B^0$, $D_1 \equiv Z$, $D_2 \equiv K$, $G_1 \equiv J/\psi$, and $G_2 \equiv \pi$.

B.2 Calculation of $\tilde{\theta}_{J/\psi}$ and $\tilde{\varphi}$ in the J/ψ rest frame

In order to calculate $\tilde{\theta}_{J/\psi}$ and $\tilde{\varphi}$ in the J/ψ rest frame, one needs to know the four-momenta of μ^+ , μ^- , K , and π in the J/ψ rest frame in the decay $B^0 \rightarrow ZK$, $Z \rightarrow J/\psi\pi$, $J/\psi \rightarrow \mu^+\mu^-$.

Step 1: Z momentum in the B^0 rest frame

The magnitude of three-momentum of Z (or K) in the two-particle decay $B^0 \rightarrow ZK$ in the B^0 rest frame is given by

$$p^{Z(B^0)} = p^{K(B^0)} = \frac{\sqrt{M_{B^0}^4 - 2M_{B^0}^2 M_K^2 - 2M_{B^0}^2 M_Z^2 + M_K^4 - 2M_K^2 M_Z^2 + M_Z^4}}{2M_{B^0}}. \quad (\text{B.1})$$

Step 2: Kinematic variables in the Z rest frame

Magnitude of the K momentum in the Z rest frame, $p^{K(Z)}$, is obtained by applying boost along the Z momentum direction:

$$p^{K(Z)} = \frac{p^{K(B^0)} \sqrt{\frac{M_Z^2}{M_Z^2 + p^{K(B^0)}}} \left(M_Z^2 + p^{K(B^0)} - \sqrt{M_K^2 + p^{K(B^0)}} \sqrt{M_Z^2 + p^{K(B^0)}} \right)}{M_Z^2}. \quad (\text{B.2})$$

Since Z and K are back to back, the boosted momentum also acts along the same direction. Consider a coordinate system in the Z rest frame where the Z decay product π has its momentum along the $\hat{z}$ direction and the boosted K lies on the $\hat{x}\hat{z}$ plane making an angle θ_Z with the direction of π as shown in Fig. B.1. This θ_Z is nothing but the Z helicity angle as calculated in Section B.1.

The magnitude of three-momentum of J/ψ (or π) in the two-particle decay $Z \rightarrow J/\psi\pi$ in the Z rest frame is given by

$$p^{J/\psi(Z)} = p^{\pi(Z)} = \frac{\sqrt{M_\pi^4 - 2M_\pi^2 M_{J/\psi}^2 - 2M_\pi^2 M_Z^2 + M_{J/\psi}^4 - 2M_{J/\psi}^2 M_Z^2 + M_Z^4}}{2M_Z}. \quad (\text{B.3})$$

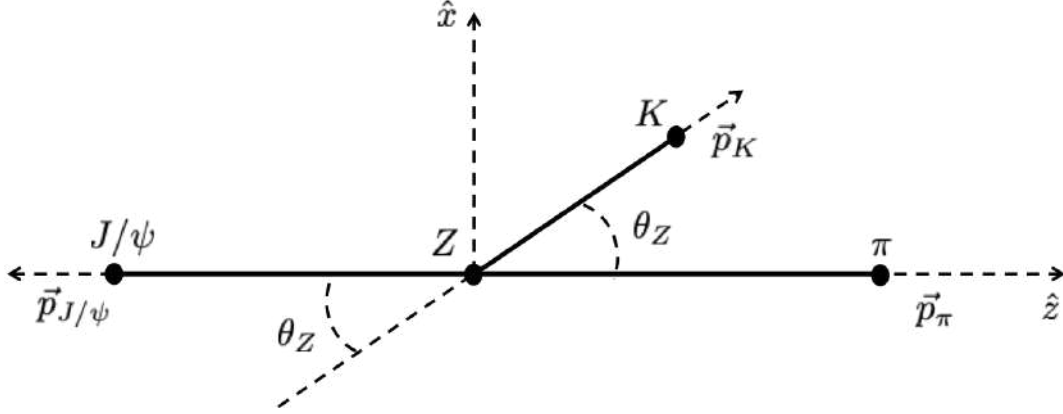


Figure B.1: Z rest frame where the coordinates have been chosen such that the Z decay product π has its momentum along the $\hat{z}$ direction, J/ψ momentum is along the $-\hat{z}$ direction and the boosted K lies on the $\hat{x}\hat{z}$ plane making an angle θ_Z with the direction of π .

Thus, the four-momenta of K , π , and J/ψ in the Z rest frame are given by

$$\begin{aligned} P_K &= \left(\sqrt{M_K^2 + (p^{K(Z)})^2}, p^{K(Z)} \sin \theta_Z, 0, p^{K(Z)} \cos \theta_Z \right), \\ P_\pi &= \left(\sqrt{M_\pi^2 + (p^{\pi(Z)})^2}, 0, 0, p^{\pi(Z)} \right), \\ P_{J/\psi} &= \left(\sqrt{M_{J/\psi}^2 + (p^{\pi(Z)})^2}, 0, 0, -p^{\pi(Z)} \right). \end{aligned} \quad (\text{B.4})$$

Step 3: Kinematic variables in the J/ψ rest frame

Since the K , π , and J/ψ four-momenta are completely determined in the Z rest frame, they can be easily calculated in the J/ψ rest frame by applying proper boost. The magnitude of three-momenta of muons in the J/ψ rest frame is given by

$$p_\mu = \frac{1}{2} \sqrt{M_{J/\psi}^2 - 4M_\mu^2}. \quad (\text{B.5})$$

From Fig. B.2, the muon four-momenta in the J/ψ rest frame can be written as

$$\begin{aligned} P_{\mu^+} &= \left(\sqrt{M^2 + p_\mu^2}, p_\mu \sin \tilde{\theta}_{J/\psi} \cos \varphi, -p_\mu \sin \tilde{\theta}_{J/\psi} \sin \varphi, -p_\mu \cos \tilde{\theta}_{J/\psi} \right), \\ P_{\mu^-} &= \left(\sqrt{M^2 + p_\mu^2}, -p_\mu \sin \tilde{\theta}_{J/\psi} \cos \varphi, p_\mu \sin \tilde{\theta}_{J/\psi} \sin \varphi, p_\mu \cos \tilde{\theta}_{J/\psi} \right). \end{aligned} \quad (\text{B.6})$$

$\tilde{\theta}_{J/\psi}$ cannot be measured directly, however it can be calculated in terms of the known quantities. On the other hand, $\theta_{J/\psi}$ is a known (measured) quantity that can also be written with the help of Eq. (A.1). From Fig. B.3 using Eq. (A.1),

B. IMPLEMENTATION OF THE ANGLES FOR DECAYS VIA Z

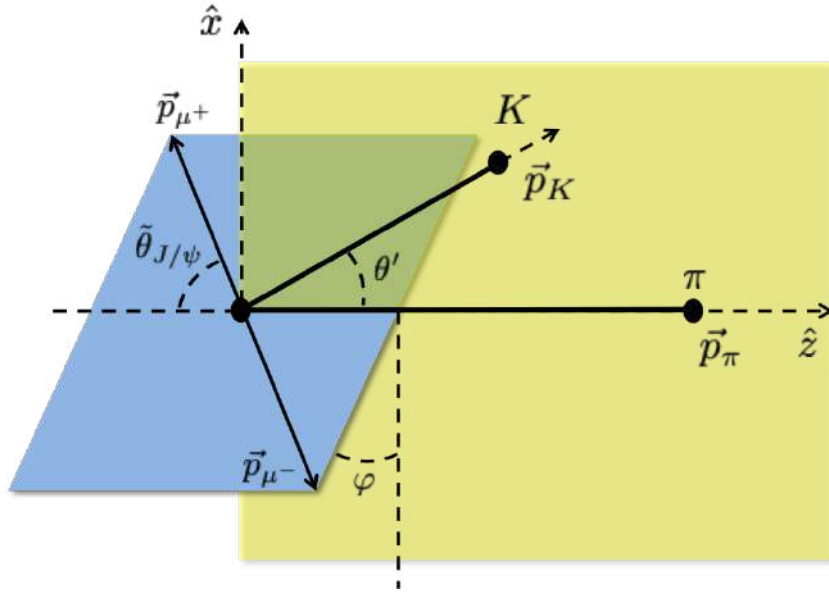


Figure B.2: J/ψ rest frame where the coordinates have been chosen such that the π momentum is along the $\hat{z}$ direction, the K momentum lies on the $\hat{x}\hat{z}$ plane making an angle θ' with the direction of π , and the J/ψ decay products μ^+ and μ^- lie on a plane making an angle φ with the $\hat{x}\hat{z}$ plane. μ^+ momentum makes an angle $\tilde{\theta}_{J/\psi}$ with the direction of π .

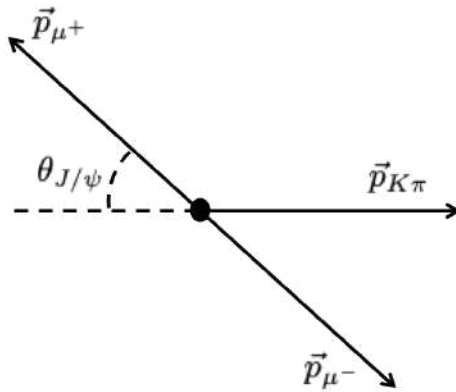


Figure B.3: $\mu^+\mu^-$ decay in the J/ψ rest frame in the process $B^0 \rightarrow ZK, Z \rightarrow J/\psi\pi, J/\psi \rightarrow \mu^+\mu^-$. Here, the angle $\theta_{J/\psi}$ made by $\vec{p}_{\mu^+}$ with the opposite direction of $K\pi$ pseudomomentum $\vec{p}_{K\pi}$ is a measured quantity and one of the four input variables of the total 4D PDF.

$$\cos \theta_{J/\psi} = \frac{1}{d} \left[\frac{1}{2} M_{J/\psi}^2 (M_{B^0}^2 - m_{K\pi\mu}^2 + M_\mu^2) - \frac{1}{4} M_{J/\psi}^2 (M_{B^0}^2 - m_{K\pi}^2 + M_{J/\psi}^2) \right], \quad (\text{B.7})$$

$$\text{where, } d = \sqrt{\left[\frac{1}{4} (M_{B^0}^2 - m_{K\pi}^2 + M_{J/\psi}^2)^2 - M_{B^0}^2 M_{J/\psi}^2 \right] \left[\frac{M_{J/\psi}^4}{4} - M_\mu^2 M_{J/\psi}^2 \right]}.$$

Here, $m_{K\pi}$ is the running mass of the $K\pi$ system and $m_{K\pi\mu}$ is the running mass of the $K\pi\mu^-$ system. Thus,

$$m_{K\pi\mu}^2 = \frac{1}{2} \left(-\frac{4 \cos \theta_{J/\psi} \times d}{M_{J/\psi}^2} + M_{B^0}^2 + m_{K\pi}^2 + 2M_\mu^2 - M_{J/\psi}^2 \right). \quad (\text{B.8})$$

$m_{K\pi\mu}^2$ can also be calculated from Fig. B.2 from the four-momenta of μ^- , K , and π in the J/ψ rest frame such that

$$\begin{aligned} m_{K\pi\mu}^2 &= (E_\mu + E_\pi + E_K)^2 - (-p_\mu \sin \tilde{\theta}_{J/\psi} \cos \varphi + p_{K\hat{x}})^2 \\ &\quad - (p_\mu \sin \tilde{\theta}_{J/\psi} \sin \varphi)^2 - (p_{\pi\hat{z}} + p_{K\hat{z}} + p_\mu \cos \tilde{\theta}_{J/\psi})^2. \end{aligned} \quad (\text{B.9})$$

Solving for $\tilde{\theta}_{J/\psi}$ in Eq. (B.9), we get

$$\begin{aligned} \cos \tilde{\theta}_{J/\psi} &= \frac{ab \pm \sqrt{c^2(-a^2 + b^2 + c^2)}}{b^2 + c^2}, \\ \sin \tilde{\theta}_{J/\psi} &= -\frac{ac^2 \mp b\sqrt{c^2(-a^2 + b^2 + c^2)}}{b^2c + c^3}, \end{aligned} \quad (\text{B.10})$$

where,

$$\begin{aligned} a &= (E_\mu + E_\pi + E_K)^2 - m_{K\pi\mu}^2 - (p_{K\hat{x}}^2 + p_{K\hat{z}}^2 + 2p_{K\hat{z}}p_{\pi\hat{z}} + p_\mu^2 + p_{\pi\hat{z}}^2), \\ b &= 2p_\mu(p_{K\hat{z}} + p_{\pi\hat{z}}), \\ c &= 2p_{K\hat{x}}p_\mu \cos \varphi. \end{aligned} \quad (\text{B.11})$$

Now, we know the four-momenta of μ^+ , μ^- , K , and π in the J/ψ rest frame and can easily calculate $\tilde{\varphi}$.

B.3 Calculation of α in the J/ψ rest frame

From Fig. 4.1.1, the magnitude of K or π momentum in the K^* rest frame can be calculated as

$$p^{K(K^*)} = p^{\pi(K^*)} = \frac{\sqrt{m_{K\pi}^4 - 2m_{K\pi}^2 M_K^2 - 2M_{K\pi}^2 M_\pi^2 + M_K^4 - 2M_K^2 M_\pi^2 + M_\pi^4}}{2m_{K\pi}}. \quad (\text{B.12})$$

B. IMPLEMENTATION OF THE ANGLES FOR DECAYS VIA Z

The angle θ_{K^*} is calculated from Eq. (A.1). The magnitude of J/ψ or K^* momentum in the B^0 rest frame is given by

$$p^{J/\psi(B^0)} = p^{K^*(B^0)} = \frac{\sqrt{M_{B^0}^4 - 2M_{B^0}^2 m_{K\pi}^2 - 2M_{B^0}^2 M_{J/\psi}^2 + m_{K\pi}^4 - 2m_{K\pi}^2 M_{J/\psi}^2 + M_{J/\psi}^4}}{2M_{B^0}}. \quad (\text{B.13})$$

The K and π four-momenta can be completely known in the J/ψ rest frame by boosting them first to the B^0 rest frame and then to J/ψ rest frame. The magnitude of muon momentum in the J/ψ frame is obtained from Eq. (B.5) and $\theta_{J/\psi}$ is known from measurement. Thus, α , the angle between the planes defined by (μ^+, π) and (μ^+, K^*) momenta can be directly calculated.

Publications and CMS internal notes

- **N. Sur**, L. Cristella, A. D. Florio, and V. Mastrapasqua. *A GPU based multidimensional amplitude analysis to search for tetraquark candidates*. [J Big Data 8, 16 \(2021\)](#)
- T. Aziz, L. Cristella, A. Di Florio, V. Mastrapasqua, G. Mohanty, A. Pompili, and **N. Sur**. “Amplitude Analysis of $B^0 \rightarrow J/\psi K\pi$ with 2012 data sample”. **CMS-AN-2016/236**

Not directly related to the thesis

- S. Dugad, M. Kaur, M. Mittal, **N. Sur**, B. Sutar, and S. Xie. “Saturation Study of HO with SiPM readout for the choice of Dynamic range and gain”. **CMS-DN-2015/015**
- A. Lanyov, I. Belotelov, S. Shmatov, D. Bourilkov, M. Schmitt, S. Stoynev, A. Everett, N. Neumeister, A. Svyatkovskiy, H. D. Yoo, J. Han, A. Bodek, S. E. Lee, G. N. Kim, D. H. Miller, M. Jones, D. Silvers, J. Rojo, **N. Sur**, and K. Mazumdar. “Measurement of the differential and double differential Drell-Yan cross section in proton-proton collisions at $\sqrt{s} = 7$ TeV in dimuon channel”. **CMS-AN-2012/063**, the results are published in [JHEP 12 \(2013\) 030](#).

Conference Proceedings

- **N. Sur**. “[Heavy-Flavor Spectroscopy and Exotic States at LHC](#)”. *16th Conference on Flavor Physics and CP Violation*. Springer International Publishing, 2019

References

- [1] S. L. Glashow. “Partial Symmetries of Weak Interactions”. *Nucl. Phys.* **22** (1961), 579–588.
- [2] S. Weinberg. “A Model of Leptons”. *Phys. Rev. Lett.* **19** (1967), 1264–1266.
- [3] A. Salam. “Weak and Electromagnetic Interactions”. *Conf. Proc.* C680519 (1968), 367–377.
- [4] D. J. Gross and F. Wilczek. “Ultraviolet Behavior of Nonabelian Gauge Theories”. *Phys. Rev. Lett.* **30** (1973), 1343–1346.
- [5] H. D. Politzer. “Reliable Perturbative Results for Strong Interactions?” *Phys. Rev. Lett.* **30** (1973), 1346–1349.
- [6] D. J. Gross and F. Wilczek. “Asymptotically Free Gauge Theories - I”. *Phys. Rev. D* **8** (1973), 3633–3652.
- [7] H. D. Politzer. “Asymptotic Freedom: An Approach to Strong Interactions”. *Phys. Rept.* **14** (1974), 129–180.
- [8] S.-K. Choi et al., Belle Collaboration. “Observation of a Resonancelike Structure in the $\pi^{+-}\psi'$ Mass Distribution in Exclusive $B \rightarrow K\pi^{+-}\psi'$ Decays”. *Phys. Rev. Lett.* **100** (2008), 142001.
- [9] K. Chilikin et al., Belle Collaboration. “Experimental constraints on the spin and parity of the $Z(4430)^{+}$ ”. *Phys. Rev. D* **88** (2013), 074026.
- [10] R. Aaij et al., LHCb Collaboration. “Observation of the Resonant Character of the $Z(4430)^{-}$ State”. *Phys. Rev. Lett.* **112** (2014), 222002.
- [11] K. Chilikin et al., Belle Collaboration. “Observation of a new charged charmoniumlike state in $\bar{B}^0 \rightarrow J/\psi K^{-}\pi^{+}$ decays”. *Phys. Rev. D* **90** (2014), 112009.
- [12] G. Cavallero (LHCb Collaboration). *Recent results on exotic hadrons at LHCb*. CERN-LHC Seminar, 2018.

- [13] M. Ablikim et al., BESIII Collaboration. “Observation of a Charged Charmoniumlike Structure in $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ at $\sqrt{s} = 4.26$ GeV”. *Phys. Rev. Lett.* **110** (2013), 252001.
- [14] Z. Q. Liu et al., Belle Collaboration. “Study of $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ and Observation of a Charged Charmoniumlike State at Belle”. *Phys. Rev. Lett.* **110** (2013), 252002.
- [15] R. L. Jaffe. “Multi-Quark Hadrons. 1. The Phenomenology of (2 Quark 2 anti-Quark) Mesons”. *Phys. Rev.* **D15** (1977), 267.
- [16] R. L. Jaffe. “ $Q^2\bar{Q}^2$ Resonances in the Baryon - anti-Baryon System”. *Phys. Rev.* **D17** (1978), 1444.
- [17] S. Godfrey and N. Isgur. “Mesons in a Relativized Quark Model with Chromodynamics”. *Phys. Rev.* **D32** (1985), 189–231.
- [18] T. Barnes, S. Godfrey, and E. S. Swanson. “Higher charmonia”. *Phys. Rev.* **D72** (2005), 054026.
- [19] S. L. Olsen. “XYZ Meson Spectroscopy”. *Proceedings, 53rd International Winter Meeting on Nuclear Physics (Bormio 2015): Bormio, Italy, January 26-30, 2015*. 2015.
- [20] S. K. Choi et al., Belle Collaboration. “Observation of a narrow charmonium - like state in exclusive $B^\pm \rightarrow K^\pm\pi^+\pi^- J/\psi$ decays”. *Phys. Rev. Lett.* **91** (2003), 262001.
- [21] K. Abe et al., Belle Collaboration. “Observation of a new charmonium state in double charmonium production in e^+e^- annihilation at $\sqrt{s} \approx 10.6$ GeV”. *Phys. Rev. Lett.* **98** (2007), 082001.
- [22] P. Pakhlov et al., Belle Collaboration. “Production of New Charmoniumlike States in $e^+e^- \rightarrow J/\psi D^{(*)}\bar{D}^{(*)}$ at $\sqrt{s} \approx 10.6$ GeV”. *Phys. Rev. Lett.* **100** (2008), 202001.
- [23] B. Aubert et al., BaBar Collaboration. “Observation of a broad structure in the $\pi^+\pi^- J/\psi$ mass spectrum around 4.26 GeV/ c^2 ”. *Phys. Rev. Lett.* **95** (2005), 142001.
- [24] B. Aubert et al., BaBar Collaboration. “Evidence of a broad structure at an invariant mass of 4.32 GeV/ c^2 in the reaction $e^+e^- \rightarrow \pi^+\pi^-\psi 2S$ measured at BaBar”. *Phys. Rev. Lett.* **98** (2007), 212001.
- [25] M. Tanabashi et al., Particle Data Group. “Review of Particle Physics”. *Phys. Rev.* **D 98** (2018), 030001.
- [26] B. Aubert et al., BaBar Collaboration. “Search for the $Z(4430)^-$ at BaBar”. *Phys. Rev.* **D79** (2009), 112001.

- [27] R. Mizuk et al., Belle Collaboration. “Observation of two resonancelike structures in the $\pi^+\chi_{c1}$ mass distribution in exclusive $\bar{B}^0 \rightarrow K^-\pi^+\chi_{c1}$ decays”. *Phys. Rev. D* **78** (2008), 072004.
- [28] J. P. Lees et al., BaBar Collaboration. “Search for the $Z_1(4050)^+$ and $Z_2(4250)^+$ states in $\bar{B}^0 \rightarrow \chi_{c1}K^-\pi^+$ and $B^+ \rightarrow \chi_{c1}K_S^0\pi^+$ ”. *Phys. Rev. D* **85** (2012), 052003.
- [29] R. Mizuk et al., Belle Collaboration. “Dalitz analysis of $B \rightarrow K\pi^+\psi'$ decays and the $Z(4430)^+$ ”. *Phys. Rev. D* **80** (2009), 031104.
- [30] P. Pakhlov and T. Uglov. “Charged charmonium-like $Z^+(4430)$ from rescattering in conventional B decays”. *Phys. Lett. B* **748** (2015), 183–186.
- [31] R. Aaij et al., LHCb Collaboration. “Model-independent confirmation of the $Z(4430)^-$ state”. *Phys. Rev. D* **92.11** (2015), 112009.
- [32] L. Evans and P. Bryant. “LHC Machine”. *JINST* **3** (2008), S08001.
- [33] E. Mobs. “The CERN accelerator complex. Complexe des accélérateurs du CERN” (2016). [General Photo](#).
- [34] O. S. Brüning et al. *LHC Design Report*. CERN Yellow Reports: Monographs. Geneva: CERN, 2004.
- [35] CMS Collaboration. *CMS Luminosity - Public Results*. Multi-year plots. 2019.
- [36] S. Chatrchyan et al., CMS Collaboration. “The CMS Experiment at the CERN LHC”. *JINST* **3** (2008), S08004.
- [37] G. L. Bayatian et al., CMS Collaboration. *CMS Physics: Technical Design Report Volume 1: Detector Performance and Software*. Geneva: CERN, 2006.
- [38] S. Chatrchyan et al., CMS Collaboration. “A New Boson with a Mass of 125 GeV Observed with the CMS Experiment at the Large Hadron Collider”. *Science* **338** (2012), 1569–1575.
- [39] S. R. Davis. “Interactive Slice of the CMS detector” ([Aug. 2016](#)).
- [40] S. Chatrchyan et al., CMS Collaboration. “Precise Mapping of the Magnetic Field in the CMS Barrel Yoke using Cosmic Rays”. *JINST* **5** (2010), T03021.
- [41] S. Chatrchyan et al., CMS Collaboration. “Description and performance of track and primary-vertex reconstruction with the CMS tracker”. *JINST* **9.10** (2014), P10009.
- [42] D. Abbaneo. *The Tracker Layout*. Geneva, 2002.

REFERENCES

- [43] [The CMS electromagnetic calorimeter project: Technical Design Report](#). Geneva: CERN, 1997.
- [44] S. Chatrchyan et al., CMS Collaboration. “Performance of the CMS Hadron Calorimeter with Cosmic Ray Muons and LHC Beam Data”. [JINST 5 \(2010\), T03012](#).
- [45] G. Abbiendi. “The CMS muon system in Run2: preparation, status and first results”. [PoS EPS-HEP2015 \(2015\), 237](#).
- [46] A. M. Sirunyan et al., CMS Collaboration. “Performance of the CMS muon detector and muon reconstruction with proton-proton collisions at $\sqrt{s} = 13$ TeV”. [JINST 13.06 \(2018\), P06015](#).
- [47] S. Dasu et al. “CMS. The TriDAS project. Technical design report, vol. 1: The trigger systems” (2000).
- [48] V. Khachatryan et al., CMS Collaboration. “The CMS trigger system”. [JINST \(2017\), P01020](#).
- [49] M. Jeitler, CMS Collaboration. “Upgrade of the trigger system of CMS”. [Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment 718 \(2013\). Proceedings of the 12th Pisa Meeting on Advanced Detectors, 11–15](#).
- [50] “Early 2018 High-Level Trigger rates” (2018). [CMS DP-2018/057](#).
- [51] “Heavy Flavour distributions from CMS with 2018 data at $\sqrt{s} = 13$ TeV” (2018). [CMS-DP-2018-036](#).
- [52] G. L. Bayatyan et al., CMS Collaboration. [CMS computing: Technical Design Report](#). Geneva: CERN, 2005.
- [53] N. L. Johnson. “Systems of Frequency Curves Generated by Methods of Translation”. [Biometrika 36.1-2 \(1949\), 149](#).
- [54] N. Sur et al. “A GPU based multidimensional amplitude analysis to search for tetraquark candidates”. [J Big Data 8 \(2021\), 16](#).
- [55] R. Dalitz. “On the analysis of τ -meson data and the nature of the τ -meson”. [Phil. Mag. Ser.7 44.357 \(1953\), 1068–1080](#).
- [56] J. M. Blatt and V. F. Weisskopf. *Theoretical Nuclear Physics*. Springer, New York, NY.
- [57] J. D. Richman. “An Experimenter’s Guide to the Helicity Formalism” (1984). [CALT-68-1148](#).

REFERENCES

- [58] R. Andreassen, B. T. Meadows, M. de Silva, M. D. Sokoloff and K. Tomko. “GooFit: A library for massively parallelising maximum-likelihood fits”. *Journal of Physics: Conference Series* 513.5 (2014), 052003.
- [59] J. Nickolls et al. “Scalable Parallel Programming with CUDA”. *ACM Queue* 6 (2008).
- [60] F. James and M. Roos. “Minuit – a system for function minimization and analysis of the parameter errors and correlations”. *Comput. Phys. Commun.* 10.6 (1975), 343.
- [61] D. Aston et al. “A Study of $K^-\pi^+$ Scattering in the Reaction $K^-p \rightarrow K^-\pi^+n$ at 11 GeV/c”. *Nucl. Phys. B* 296 (1988), 493.
- [62] A. Ryd et al. “EvtGen: A Monte Carlo Generator for B-Physics” (2005).