



UNIVERSITÀ DI PISA
DOTTORATO DI RICERCA IN INGEGNERIA DELL'INFORMAZIONE

DIRECT DIGITISATION FOR POSITION MEASUREMENT OF CHARGED PARTICLE BEAMS: SAMPLING AND QUANTISATION EFFECTS ON RESOLUTION

DOCTORAL THESIS

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Pisa, April 2021

XXXIII



"Quanto più una lingua è ricca e vasta,
tanto ha bisogno di meno parole per esprimersi,
e viceversa quanto più è ristretta,
tanto più le conviene largheggiare in parole
per comporre un'espressione perfetta."

Giacomo Leopardi
Zibaldone di Pensieri, 1822,1

Acknowledgements

FIRST and foremost, I would like to thank my supervisors for their advise and support during my PhD study. Prof. Luca Fanucci has constantly encouraged my scientific growth, showing me the potential of my research path, especially when I was more doubtful. Andrea Boccardi has been an essential reference point: his expertise and knowledge, his mental clarity on the subject have been a fundamental guidance in my question making and answer attempting process.

I would like to express my sincere gratitude to the Università di Pisa and especially to the Department of Information Engineering for their always stimulating environment, that has been enriching and building me as engineer and researcher for more than nine years. In particular, I would like to thank Prof. Paolo Bruschi and Dott. Michele Dei for introducing me to the key concepts and the literature about the qualification of Analog to Digital Converters, when I was not even fully aware of the importance of it for my research.

I am extremely grateful to CERN, to the Beam Instrumentation group and to the Beam Position section for hosting me. I would like to thank in particular Manfred Wendt: his vast experience was fundamental for my approach to the particle accelerator world, he had an important role in the definition of my study, and, not to be forget, in the proofreading of my works. The guidance, helpfulness and advise of Dr. Rhodri Jones and Dr. Thibaut Lefevre were also a cornerstone of my PhD experience, therefore I am grateful to them. I would like to thank my first office mate, Dr. Jan Pospisil, for his work on the LHC beam extraction interlock BPM that is the foundation of this thesis, but especially for the always cherish time we spent together and for his bottomless knowledge of keyboard shortcuts. I am very grateful to Michele Bozzolan and Marek Gasior: their suggestions and lucid comments had a major constructive impact on my research. I would like to offer my special thanks to Manoel Barros Marin, for all the helpful advice on digital signal processing and on uncountable technical and not topics. I am also grateful to Michal Krupa and Tom Levens for definitely convincing me to start this adventure, and to all the other colleagues from the BP section and IQ section for making it special.

I would like to extend my sincere thanks to my friends in Pisa and my friends in

Geneva for doing, during these at times challenging years, what friends best do: supporting, sharing, laughing.

Finally, I would like to express my gratitude to my parents, my brother and my companion Giovanni. Without your endless understanding and encouragement it would have been tremendously hard to conduct this thesis.

Summary

WITH the technology improvements of Analog-to-Digital Converters (ADC), notable in terms of sampling rate and achievable resolution, the direct digitisation of beam pickup signals is of growing interest in the field of particle beam diagnostics for charged particle accelerators.

This thesis studies the possibility to acquire and measure the transverse position of a charged particle beam circulating in an accelerator, using an architecture based on minimum analogue signal conditioning of the Beam Position Monitor (BPM) electrodes signals, direct digital conversion, and measurement of the power of each beam bunch-related pulse signal in the digital domain. The aim is to analyse the impact of the characteristics of the sampling stage on the system resolution performance.

The effects of the distortion due to the sampling process, and of the finite resolution of the ADC, are discussed analytically in detail. The results are expressed in form of a general, analytic expression of the expected signal-to-noise ratio of the measurement for any pulse waveform as function of the characteristics of the analog-to-digital converter.

The presented model was applied to analyse the performance of the Large Hadron Collider beam extraction interlock BPM system, an example of system implementing direct-digitisation of BPM signals. A prototype setup was used to acquire beam data with two different ADCs, one with higher sampling rate, the other with higher resolution. The system was also replicated in a parametric simulation framework and the estimated performance from analytical prediction, simulation, and measurements are compared.

The analytic model of the effects of digitisation proves to be a valid tool to understand and quantify how the sampling stage limits the measurement resolution and how the choice of the ADC can be optimised.

Sommario

DATO lo sviluppo tecnologico dei convertitori analogico-digitali, osservabile nelle frequenze di campionamento e nelle risoluzioni che i nuovi componenti riescono a raggiungere, la digitalizzazione diretta degli impulsi generati dai sensori di fasci di particelle è una tecnica sempre più appetibile nel campo della progettazione di sistemi di misura per acceleratori di particelle cariche.

Questa tesi studia la possibilità di acquisire e misurare la posizione trasversale di fasci di particelle cariche all'interno di un acceleratore, usando un'architettura basata su: minimo condizionamento analogico del segnale del sensore di posizione del fascio, digitalizzazione diretta, e misura in digitale della potenza di ogni impulso legato a un singolo gruppo di particelle del fascio. Lo scopo è analizzare l'impatto delle caratteristiche del convertitore analogico-digitale nel definire le prestazioni del sistema in quanto risoluzione della misura.

Gli effetti della distorsione introdotta dal campionamento e della risoluzione limitata del campionatore sono discussi analiticamente in dettaglio. I risultati sono sintetizzati nell'espressione analitica del rapporto segnale-rumore della misura della potenza media di un generico impulso, in funzione delle caratteristiche del convertitore analogico-digitale.

Il modello presentato è stato impiegato nell'analisi delle prestazioni del prototipo del nuovo misuratore di posizione per la sicura estrazione del fascio di particelle nel Large Hadron Collider. Il sistema è un esempio pratico in cui viene effettuata la digitalizzazione diretta dei segnali del sensore di posizione. Il prototipo è stato usato per l'acquisizione di segnali generati dal fascio di particelle con due diversi convertitori analogico-digitali: uno ad elevata frequenza di campionamento, l'altro ad elevata risoluzione. L'intero sistema è stato descritto nei suoi elementi in un ambiente di simulazione numerica. Le predizioni analitiche e i risultati di simulazioni e misure sono discussi e comparati.

Il modello analitico degli effetti di campionamento e quantizzazione si dimostra un valido strumento sia per la comprensione e la stima dei limiti che la digitalizzazione pone alla risoluzione della misura, sia per l'ottimizzazione della scelta di un convertitore analogico-digitale per una specifica applicazione.

List of publications

International Journals

1. Degl’Innocenti, I., Boccardi, A., Fanucci, L., and Wendt, M. (2020). Analysis of sampling and noise error on the energy measurement of direct digitally acquired pulsed signals. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 984, 164571. Elsevier.

International Conferences/Workshops with Peer Review

1. Wendt, M., Barros Marin, M., Boccardi, A., Bogey, T., Degl’Innocenti, I., Kain, V., ... and Topaloudis, A. (2018, June). Upgrade of the CERN SPS Beam Position Measurement System. In *9th Int. Particle Accelerator Conf.(IPAC’18)*, Vancouver, BC, Canada, April 29-May 4, 2018 (pp. 2047-2050). JACOW Publishing, Geneva, Switzerland.
2. Degl’Innocenti, I., Fanucci, L., Albertone, J., Boccardi, A., Bogey, T., Guizan, C. M., ... and Wendt, M. (2018, December). Digital Acquisition Chain for the Upgrade of the CERN SPS Beam Position Monitor. In *2018 25th IEEE International Conference on Electronics, Circuits and Systems (ICECS)* (pp. 737-740). IEEE.
3. M. Wendt, M. Barros Marin, A. Boccardi, T.B. Bogey, I. Degl’Innocenti, and A. Topaloudis. (2019, September). Technology and First Beam Tests of the New CERN-SPS Beam Position System, in *8th International Beam Instrumentation Conference (IBIC’19)*, Malmö, Sweden, Sep. 2019, (pp. 655-659). JACoW Publishing, Geneva, Switzerland.
4. I. Degl’Innocenti, A. Boccardi, M. Wendt, and L. Fanucci. (2020, September) Direct Digitization and ADC Parameter Trade-off for Bunch-by-Bunch Signal Processing. In *9th Int. Beam Instrumentation Conf. (IBIC’20)*, Santos, Brazil, Sep. 2020, (pp. 288-294). JACoW Publishing, Geneva, Switzerland.

List of Abbreviations

A

AD	Antiproton Decelerator. 6
ADC	Analog to Digital Converter. 1–4, 8, 24–34, 36, 37, 48, 49, 52, 53, 56–59, 62, 64, 67, 70–74, 77, 80, 81, 83, 84, 86–91, 94, 97, 99, 100, 102–105
AWAKE	Advanced Proton Driven Plasma Wakefield Acceleration Experiment. 6

B

BNL	Brookhaven National Laboratory. 27
BPM	Beam Position Monitor. 3, 4, 7, 9–11, 15–17, 19, 20, 22–28, 30, 39, 67, 68, 77, 81, 85, 97, 102–105
BST	Beam Synchronous Timing. 71, 72

C

CERN	European Organization for Nuclear Research. 3–6, 71
CLEAR	CERN Linear Electron Accelerator for Research. 6

D

DAQ	Data AcQuisition. 71
DFT	Discrete Fourier Transform. 42, 44, 53, 61, 90
DMSPT	Delay-Multiplex Single Path Technology. 28

E

ELENA	Extra Low ENergy Antiproton ring. 6
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List of Abbreviations

ENOB Effective Number of Bit. 32–34, 55–57, 71, 72, 80, 89, 91, 97, 99, 103

F

FMC FPGA Mezzanine Card. 71, 72, 74

FPGA Field Programmable Gate Array. 25–27, 71

I

ISOLDE Isotope Separator On Line. 6

L

LEIR Low Energy Ion Ring. 6

LHC Large Hadron Collider. 3–5, 7, 12, 15–18, 20, 25–28, 53, 54, 58, 60, 61, 67, 69, 72–74, 83, 85, 87, 88, 97, 102–105

LSB Least Significant Bit. 30, 31, 48

M

MSB Most Significant Bit. 30

N

NCO Numerically Controlled Oscillator. 27

NSRRC National Synchrotron Radiation Research Center. 27

P

PDF Probability Density Function. 48, 56, 60, 64

PS Proton Synchrotron. 5, 24, 25, 27

PSB Proton Synchrotron Booster. 4, 25

R

RHIC Relativistic Heavy Ion Collider. 27

RMS Root Mean Square. 38, 73, 82, 87, 90, 94, 95, 105, 106

S

SINAD Signal to Noise and Distortion Ratio. 32–34, 103

SLS Swiss Light Source. 27

SNR Signal to Noise Ratio. 31, 32, 38, 52–54, 56–59, 62, 64, 91, 94, 100, 103, 105

SPS Super Proton Synchrotron. 5, 25, 73

SSRF Shanghai Synchrotron Radiation Facility. 27

STD Standard Deviation. 38, 105

V

VFC-HD VME FPGA Carrier. 71, 72
VME VERSAmodule Eurocard. 71

W

WBTN Wide Band Time Normalizer. 26

Notation

Symbols

$A_{X_k,N}$	coefficient of the real part of the phasor representing $\epsilon_{P_{T,\tau}}$. 45, 46, 52, 55, 90
A_{RMS}	Root mean square amplitude of the continuous pulse A. 94–97
BW	Finite bandwidth of a band-limited analog signal. 35, 36
$B_{X_k,N}$	coefficient of the imaginary part of the phasor representing $\epsilon_{P_{T,\tau}}$. 45, 46, 52, 55, 90
B_{RMS}	Root mean square amplitude of the continuous pulse B. 95, 96
C	button pick-up electrode capacitance to ground. 18, 19, 76
E	energy of the continuous pulse signal $x(t)$. 39, 40, 54
F_s	sampling rate. 33, 41, 42, 44, 59, 62
F	figure of merit of the performance of an ADC with respect to the dissipated power. 33
N_0	integer number defining the bandwidth limit of the periodic signal $x_T(t)$, such that the Fourier coefficients respect the following: $ X_i ^2 = 0 \quad \forall i : i > N_0$. 42, 44, 55, 59, 62
N_b	number of charges per bunch. 12, 13, 19, 20, 22, 60, 61, 75, 86, 94
N	number of samples of the signal $x(t)$ sampled at the sampling rate F_s over the window T . 41, 42, 44, 45, 48, 55, 56

P_T	averaged power of the periodic signal $x_T(t)$ over the period T . 40, 41, 44, 46, 48, 51–53, 94
P_{diss}	power dissipated by an ADC. 33
P	measurement of performance of an ADC. 33, 34, 37, 57, 58, 103
R_{Nyq}	Nyquist rate, equal to twice the bandwidth of a signal. 42, 44, 55, 59, 62, 64
R	beam pipe radius. 13, 15, 19, 22, 75, 76, 83, 84
T	window of observation of the continuous pulse signal $x(t)$. 39–42, 45, 54–56, 58, 61
V_{FSR}	full scale range voltage at the input of the ADC. 30, 31, 33, 58, 80, 86, 91
$X(f)$	Fourier transform of the continuous pulse signal $x(t)$. 39, 40
X_k	Fourier coefficients of the periodic signal $x_T(t)$. 40, 42, 54, 55, 59, 61
Z_0	pick-up characteristic impedance. 18, 20, 60, 61, 76
Δ	difference signal obtained by the difference of the signals of two opposing BPM pickup electrodes. 23, 24, 73, 104
Σ	sum signal obtained by the sum of the signals of two opposing BPM pickup electrodes. 23, 24, 73, 104
α	angular width of the pick-up electrode. 13–15, 18–20, 22, 60, 61, 75, 76, 83, 84
$\bar{A}_{\text{RMS},0}$	Measurement of the root mean square amplitude of the sampled pulse A subtracting the error mean value. 95, 96
$\bar{A}_{\text{RMS}}$	Measurement of the root mean square amplitude of the sampled pulse A. 94, 95
$\bar{B}_{\text{RMS},0}$	Measurement of the root mean square amplitude of the sampled pulse B subtracting the error mean value. 95, 96
$\bar{B}_{\text{RMS}}$	Measurement of the root mean square amplitude of the sampled pulse B. 95
$\bar{P}_T$	averaged power of the sampled sequence $\hat{x}_n$ of period T with additive noise ν . 41, 48, 51, 94
β_b	fraction of the lightspeed at which the particle beam moves. 20
β_s	fraction of the lightspeed at which the stripline signal propagates. 20

ℓ_c	cable length. 78
ℓ	pick-up electrode or stripline longitudinal length. 18–20, 22, 76
ϵ_q	quantisation error of an ADC. 31
$\epsilon_{P_{T,\tau}}$	averaged power error measurement caused by limited sampling rate with infinite resolution and sampling delay τ . 45–47, 51, 52, 59, 62–65, 90, 91, 94–96
η	averaged power error measurement due to limited resolution in the form of additive noise ν . 48–52, 56, 59, 62, 91, 94–96
$\hat{P}_T$	averaged power of the sampled sequence $\hat{x}_n$ of period T . 42, 44, 46, 48
$\hat{X}_k$	discrete Fourier transform coefficients of the sampled sequence $\hat{x}_n$. 42, 44
$\hat{x}_n$	sampled sequence of the continuous pulse signal $x(t)$ over the window T with infinite resolution. 42, 48, 51
μ_A	Mean value of the error of the measured root mean square amplitude of the sampled pulse A. 94–97
μ_B	Mean value of the error of the measured root mean square amplitude of the sampled pulse B. 95, 96
$\mu_{1,\epsilon}$	first moment of the error $\epsilon_{P_{T,\tau}}$. 46
$\mu_{2,\epsilon}$	second moment of the error $\epsilon_{P_{T,\tau}}$. 46
μ_ϵ	mean value of the error $\epsilon_{P_{T,\tau}}$. 46, 47, 94
μ_η	mean value of the error η . 49, 52, 56, 94–96
ν	random variable representing the additive noise on each ADC sample. 41, 47–50
σ_A^2	Variance of the error of the measured root mean square amplitude of the sampled pulse A. 95, 97
σ_b	standard deviation of the longitudinal Gaussian distribution of a bunch. 13, 19, 20, 22, 60, 61, 76
σ_{ϵ_q}	standard deviation of the quantisation noise of an ADC. 31, 80
σ_ϵ^2	variance of the error $\epsilon_{P_{T,\tau}}$. 46, 47, 52, 53, 55, 59, 62, 91, 95–97
σ_η^2	variance of the error η . 49–53, 56, 59, 62, 91, 95
σ_ν	standard deviation of the ADC noise random variable ν . 47–49, 52, 56, 57, 59, 91, 94

τ_ℓ	average propagation delay along the stripline length of the stripline signal and of the beam current. 20, 60, 61, 76
τ	random variable representing the sampling delay between the signal and the sampling clock. 41, 42, 44, 45, 47
B	number of bit used by the ADC to encode the samples. 30–32, 80
Q	elementary quantisation step of an ADC. 31
θ	beam angular displacement in cylindrical coordinates from the center of the beam pipe. 13, 14, 75
b_c	constant dependent on the cable electric and geometric properties. 78
c	speed of light (2.99792458×10^8 m/s). 12, 18–20, 76
$f_\epsilon(\epsilon)$	probability density function of the error $\epsilon_{P_{T,\tau}}$. 47
$f_\tau(t)$	probability density function of the uniformly distributed random variable τ . 45
i_A	portion of the induced wall current over the electrode A of angular width α . 13, 15
i_B	portion of the induced wall current over the electrode B of angular width α . 13, 15
i_b	beam current in the time domain. 12, 13, 15, 20
i_w	wall current induced by the beam current in the time domain. 12
j_w	density of the wall current induced by the beam current in the time domain. 13
q	charge of a single particle. 12, 13, 20, 60, 61, 76
r	beam radial displacement in cylindrical coordinates from the centre of the beam pipe. 13, 14, 75
s_{dB}	linearized pick-up sensitivity, given by the ratio between the electrode signal's amplitude variation in decibel and the displacement in millimeter. 86, 87
t_s	sampling period. 41, 42, 45
v_c	velocity of propagation along the cable. 78
v	particle velocity. 11, 12, 20
$x(t)$	continuous pulse signal in the time domain. 39, 40, 42, 51, 54

Notation

$x_T(t)$ periodic versions of period T of the continuous pulse signal $x(t)$. 39–42, 54

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CHAPTER 1

Introduction

Today, in most instrumentation or diagnostic systems the measured physical property is post-processed and presented in a digital format, regardless of the measured quantity, e.g. a velocity, a length dimension, an electric current, the strength of an electromagnetic field vector, etc. For many reasons – to be discussed – a quantised representation of the measured values in amplitude and time, i.e. its digital format, is in almost any case superior compared to an analog format, often a voltage, a current, or a frequency of an electrical signal. The key element converting the “analog world” into the digital domain is the *analog-to-digital converter* (ADC), an electronic component which *converts* a voltage signal presented as analog waveform into a sampled digital format, usually in form of binary data. The sampling rate and the resolution of today available ADCs are increasing, enabling to solve complex instrumentation problems, with new processing methods and acquisition architectures.

Typically, an instrumentation system can be divided in three main functional blocks (see Fig. 1.1). The first one is a sensor, or detector, interacting directly or indirectly with the physical system under observation. The sensor element converts the physical quantity to be characterised, or a related quantity, into an electric signal which has a property, e.g. the signal amplitude, that is connected to the physical quantity to be measured. This analog signal, generated by the sensor, is then conditioned, e.g. amplified, and appropriately processed (filtered, down-converted, linearised, calibrated, etc.) by a signal processing block, to extract the wanted information. Finally, the information is sent to an acquisition block, which makes the measured information available for display, storage, distribution and control operations. The sensor block usually generates analog signals, while the acquisition block manages digital information. Therefore, the signal processing block combines analogue conditioning and digital processing of the signal, with a digitisation stage in between to convert the information from one domain

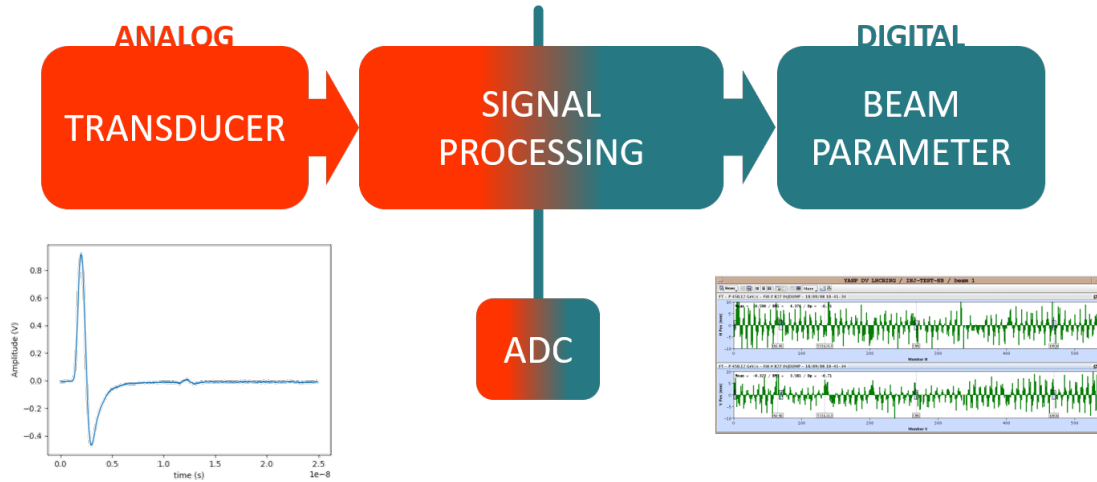


Figure 1.1: Functional blocks of an instrumentation system, which consists out of a sensor providing the analog signal, and the signal processing and acquisition elements to visualise the measured data, here the orbit of the proton beam circulating in the Large Hadron Collider at CERN, measured by its beam position monitoring system.

to the other.

In *direct digitisation* based instrumentation systems, the digitisation is performed at an early stage of the processing chain, using a minimum set of analog signal conditioning hardware.

Direct digitisation offers several advantages. The reduction of the number of the electronics components for analog conditioning means less uncontrolled parasitics, fewer drift effects and therefore less spread in board to board characteristics. Consequently, the system general performance is improved, e.g. better linearity and lower noise, and its robustness with respect to environmental conditions improves as well. Furthermore, direct digitisation facilitates the full use of the flexibility in the digital domain, e.g. the use of reprogrammable algorithms enabling a single hardware architecture to implement different instrument functions.

On the other hand, a direct digitisation based architecture imposes high demands on the digitisation stage. If the instrument processes high bandwidth signals and seeks high measurement repetition rate to discern physical events close in time, the converter must offer a sufficient high sampling rate. At the same time, the resolution of the measurement can be of critical importance, and in that case the converter needs to resolve small variations of the signal's amplitude, which on top might have large dynamic range.

Given the technology, there is a trade-off between the sampling rate an ADC can achieve and how noisy that ADC is (see Fig. 1.2). The designer of an instrument based on direct digitisation faces the question: what to prefer between high sampling rate and high resolution to digitise the analog signal of the instrument, and how that decision affects the quality of the measurement? How to quantify the impact of the ADC characteristics on the measurement performance?

Beam instrumentation for charged particle accelerators is among the applications that see an increased interest in direct digitisation based techniques. Beam diagnostic and instrumentation combine accelerator physics knowledge with electronics, mechanical and software engineering to design instruments to characterise and monitor the

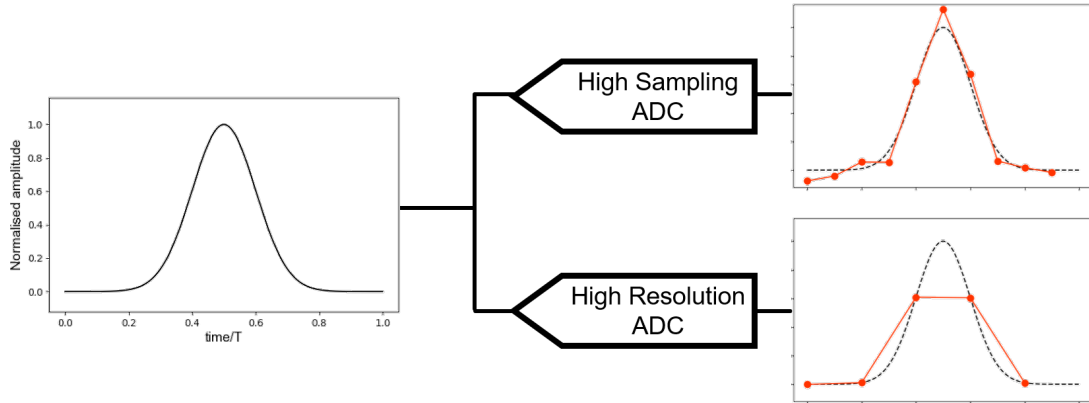


Figure 1.2: *Digital conversion trade-off between high sampling rate and high resolution.*

beam with the required accuracy and stability. The extreme requirements and operation conditions of these instruments demand a detailed study and a specific development of a tailored engineering solution.

This thesis studies the possibility to acquire and measure the transverse position of a charged particle beam circulating in an accelerator, utilising an architecture based on minimum analogue signal conditioning of the Beam Position Monitor (BPM) electrodes signals, direct digital conversion and measurement of the power of each beam bunch-related pulse signal in the digital domain. The aim is to analyse the trade-off, pitfalls and error sources of this type of architecture, especially related to the sampling stage. The outcome is the evaluation of the single-bunch, single-pass performance of the BPM system quantitatively, enabling the optimisation of the read-out system, particularly the selection of an ADC with an optimal sampling rate / resolution trade-off.

The research was led in the frame of the Doctoral Student program at the European Organization for Nuclear Research (CERN), in the Beams Department, Beam Instrumentation Group, Beam Position section. The work takes as reference of a charged particle accelerator the Large Hadron Collider (LHC) and the design of the LHC beam extraction interlock BPM system.

This first chapter gives a brief introduction to CERN accelerator complex and to Beam Instrumentation, necessary to contextualise the research and describe its goal and motivation.

The second chapter presents the general problem of beam position measurement and specifically in a hadron collider with the characteristics of the LHC. It provides the reader, who is new to the topic, with the basic knowledge of the typical signals to be acquired, the main physical concepts behind, the explanation of the fundamental keywords and the description of the main elements of the processing and acquisition electronics.

The third chapter describes the problem of analog-to-digital conversion and the design trade-off it imposes. It provides references to the state-of-the-art of those technologies, and how it can be modeled in a mathematical description. It also describes the process of the digitisation in a general form, and provides the present state-of-the-art on the studies of digitisation effects and limitations on under-sampled signal waveforms.

The fourth chapter discusses the analysis of the sampling and quantisation effects

for the specific case of the energy measurement of a direct digitally acquired pulse. Here, an analytical method is applied to examples of generic pulse waveforms.

The presented method is then applied in the fifth chapter to analyse the performance of the LHC beam extraction interlock BPM signal processing. This system is an example implementing direct-digitisation on BPM signals; a prototype setup was used to acquire beam data with two different ADCs. The system was also replicated in a parametric simulation framework and the estimated performance from analytical prediction, simulation, and measurements are compared.

Chapter six draws the conclusions, underlining the main results and the contribution of this work to the design and optimisation of signal processing and acquisition systems, with applications in the field of beam diagnostics.

1.1 The CERN accelerator complex

CERN [1] is the European Organization for Nuclear Research, located on the French-Swiss border, close to Geneva. It was founded in 1954 and counts 23 member states. It provides a complex of particle accelerators, detectors, and the necessary infrastructure to energise charged particles and observe their interactions for fundamental high energy physics research.

The particles are confined and directed within cylindrical, metallic tubes, often called beam pipes, which are evacuated to minimise unwanted collisions with residual gas molecules. As a beam of charged particles travels inside those vacuum pipes, it is energised, i.e. “accelerated”, by RF electric fields, and guided on a trajectory near the beam pipe center by magnetic fields, typically at a velocity near the speed of light.

We distinguish between linear and circular, or ring accelerators; in the latter ones particles are accelerated as they travel around the ring accelerator over many turns, they also can be *stored* in the accelerator ring for many hours at a given energy. A charged particle accelerator is designed for a specific range of beam energies, and while linear accelerators typically are used to accelerate the particles from rest to some higher energy, the ring accelerators cover higher energies. To maximise the energy of a particle beam, a *chain* of accelerators is utilised, with successive minimum injection and maximum extraction energies. The maximum energy a particle beam can reach in an accelerator depends on the characteristics of that machine, but also on the particle species, e.g. electron (positron), proton (anti-proton), or an ion beam of a given charge state.

A well known accelerator at CERN is the Large Hadron Collider (LHC), which is the last one in a chain of successive ring accelerators, linked together by transport-lines. The LHC is used for different types of collision experiments as the two counter-rotating beams are stored at a maximum beam energy of 7 tera electronvolts (TeV) for many hours. During those times the pre-accelerators of the LHC are also used for physics experiments or machine development.

Currently, two species of charged particles are utilised in the chain of accelerators towards the LHC: protons and lead ions (see Fig. 1.3). For protons, the Linac 4 is the new source, which started its operation in 2020. It is a linear accelerator which actually accelerates negative hydrogen ions (H^-) to an energy of 160 millions electronvolts (MeV). The ions are then stripped off their two electrons during the injection process in the first circular accelerator of the chain, the Proton Synchrotron Booster (PSB), which

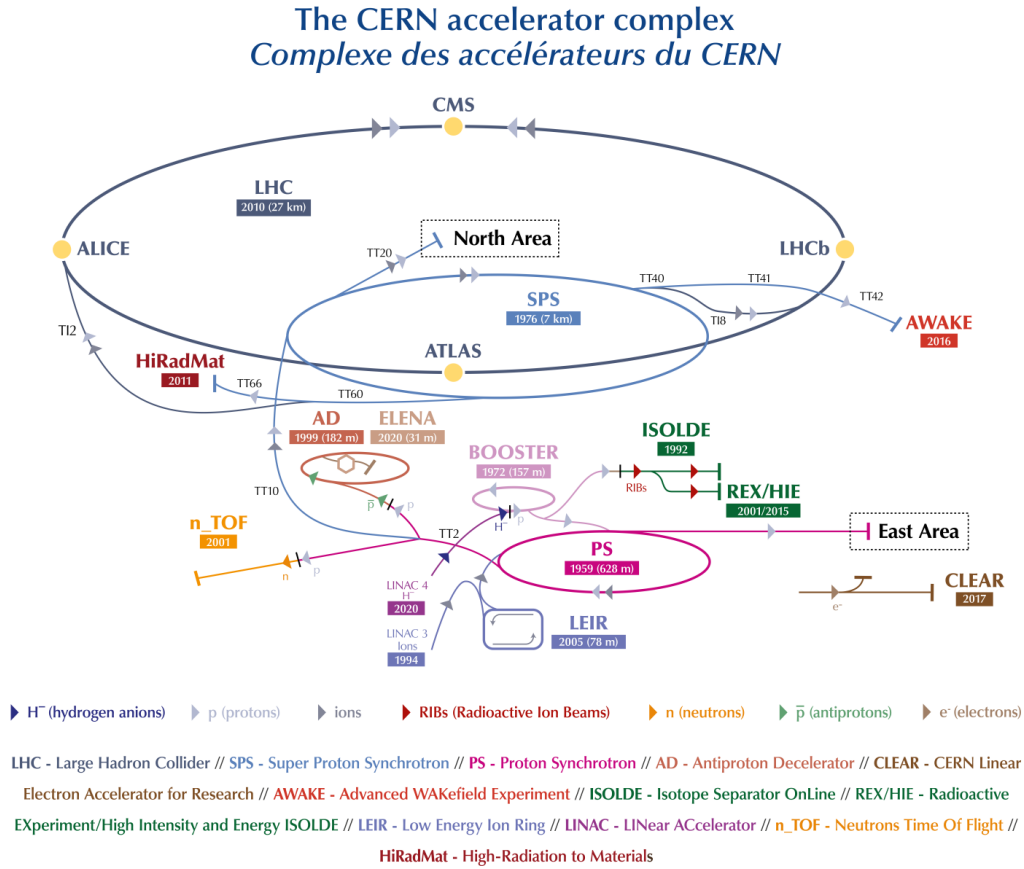


Figure 1.3: The CERN accelerator complex. Credits to CERN

is a stack of four independent ring accelerators; here the ions are converted to protons and accelerated to an energy of 1.4 giga electronvolts (GeV). The proton beam is then transferred to the Proton Synchrotron (PS), where its energy is boosted up to 25 GeV. The following circular accelerator is the Super Proton Synchrotron (SPS), the second largest of the complex with 6.7 kilometers (km) circumference; here the protons reach an energy of 450 GeV, before being injected into the last machine, the Large Hadron Collider (LHC).

A complicated scheme has to be followed to fill the two 27 km circumference LHC rings with protons, as the pre-accelerators are much smaller, thus can only fill a fraction of the LHC during a single injection. Once the filling of both LHC rings is completed, the counter-rotating proton beams are accelerated to a top energy of 6.5 TeV in each LHC ring, where they are stored for 10 hours or more to serve the LHC physics experiments. At dedicated interaction regions the otherwise in separate vacuum chambers circulating beams are focused and projected to each other in a combined beam pipe to reach a maximum particle collision rate at the vertex of a large scale experimental setup, the detector. The detector senses the collision, and follows the path and the characteristics of the fractured subatomic particles, a key element to help for a better understanding of the fundamental physics laws of nature.

The lead ions, after being extracted from the ion source and accelerated in the linear accelerator Linac 3 and then further boosted in the circular machine Low Energy Ion Ring (LEIR), are injected into the PS and hence follow the same path as the protons.

Other machines are also part of the CERN accelerator complex, expanding the scientific potential and possibilities at CERN: the Antiproton Decelerator (AD), to produce low energy antiprotons for antimatter studies, together with the soon to be fully operational Extra Low ENergy Antiproton ring, ELENA; the Isotope Separator On Line (ISOLDE) to produce radioactive nuclei; the CERN Linear Electron Accelerator for Research (CLEAR), a linear accelerator for general accelerator test and R&D; the Advanced Proton Driven Plasma Wakefield Acceleration Experiment (AWAKE), to investigate acceleration techniques based on the use of plasma wakefields driven by proton bunches.

1.2 Beam Diagnostics and Instrumentation

The observation of particle beams, and the measurement and monitoring of their parameters are fundamental to successfully operate, tune, optimise and improve particle accelerator machines.

The essential quantities to monitor and measure vary depending on the machine size and purpose, and on the accelerated particle species. An overview can be found in this lecture [2], below an – incomplete – list of some primary beam parameters which are mentioned in this work and used to characterise the particle beam in an accelerator.

- *Beam intensity*: the quantity of charges that are in the machine, a fundamental information to be monitored during beam operation, sometimes also referred to as *beam current*, which may lead to misunderstandings. The beam intensity allows to determine other beam characteristics, as the lifetime of the beam in a storage ring, or the injection / extraction efficiency of a machine.
- *Beam position*: the location of the particles in the vacuum chamber, specifically the transverse position of the *centre-of-charge* of the beam with respect to the centre of the pipe's cross-section. Usually an accelerator is equipped with many beam position measurement instruments distributed along the machine, which enable to characterise many other beam and machine parameters: the *beam orbit*, i.e. the averaged position of the beam trajectory along the accelerator; the operating point or *tune* in a ring accelerator, defined by the oscillations the particles undergo along their travel path; the *beam transfer function* and the *lattice function* of the beam optics, defined by the guide field magnets of the accelerator; and in combination with other instruments beam parameters such as *beam energy* and *chromaticity*. The beam position measurement system is the most powerful instrument in any accelerator, and is also used for trouble shooting and machine optimisation.
- *Beam profile*: the spacial distribution of the particles in the transverse and the longitudinal plane. It allows to determine the *beam emittance*, which is a key parameter to describe the quality of the particle beam. The measurement of the beam profile is fundamental to observe and study the dynamical processes of the beam during the acceleration cycle, from the injection to the collision.

Generally speaking, any beam measurement instrument relies on the physical interaction between the particle beam to be analysed and the beam detector. This interaction can be the response to the electromagnetic field generated by the moving charged particles, or it can be a nuclear interaction, e.g. scattering of the primary beam particles on targets or on residual gas; it can be an interaction with photons or other charged particles, either generated by the beam itself (synchrotron light), or by a dedicated target (laser wire, electron beam scanner).

The measurement scheme can in some cases perturb the beam: we speak about invasive, minimum invasive and non-invasive beam instruments. Therefore, the choice of the instrument's underlying physical principle naturally depends on the type of machine and particle species, but also on the operational conditions. As an example, diagnostics for the machine commissioning, employed to setup the accelerator, is not high demanding in terms of accuracy and can be invasive on the beam. Diagnostic for standard machine operation, instead, is used to precisely characterise the beam and optimise the accelerator performance, so it usually requires high accuracy and minimum beam disturbance. Another factor impacting the way the beam signals are processed, especially in circular machines, is the bandwidth of the instrument, which is directly related to the measurement integration time. This choice will determine the ability of the instrument to provide a measurement for single-passage of an assemble of particles (wideband), or an averaged measurement for the entire beam, perhaps on several passages, and eventually with a higher resolution (narrowband). Finally, other substantial design factors, conditioning the physical design of the processing architecture and the hardware choice, is whether the instrument is a singular entity or part of a widely distributed system, and if and how much of it is exposed to the unavoidable radiation in the accelerator tunnel caused by the operating machine.

1.3 Motivation

BPMs are fundamental for the day-to-day operation of any beam accelerator, so they must not alter or disturb the beam. Therefore, they usually rely on the interaction with the electromagnetic field of the charged particle beam to perform the measurement. The position signals generated at the BPM pickup are proportional to the beam current itself (as it will be described in 2.2), hence they are loaded with high potential of beam diagnostics information. BPMs are usually distributed systems; the LHC counts more than 1000 beam position monitors, distributed along the two 27 km long vacuum-pipes, located about 100 m underground, and parts of that system are exposed to ionising radiation.

These facts give particular challenges to the LHC beam position system in terms of reliability, complexity and costs, delivering thousands of signals that have to be processed and synchronised to the nanosecond time scale and delivered to the CERN Central Control (CCC) room.

On top of this, the performance of the accelerator is tied to the quality of the measurements of its diagnostics. The requirements for accuracy, resolution and stability are strict and must be met.

Direct digitisation techniques applied to beam instrumentation signals, such as beam position signals, have a huge potential to improve the performance of the measurement,

compared to traditional, more analog based signal processing techniques. They offer the chance to improve reproducibility, reliability and flexibility in the digital domain to process the information carried by the monitor source signals.

The achievable accuracy of a beam measurement based on direct digitisation signal processing techniques is given by the available resolution and sampling rate of state-of-the-art ADCs. Understanding how this limit affects the measurement is a key point for an optimised design.

CHAPTER 2

Beam Position Measurement

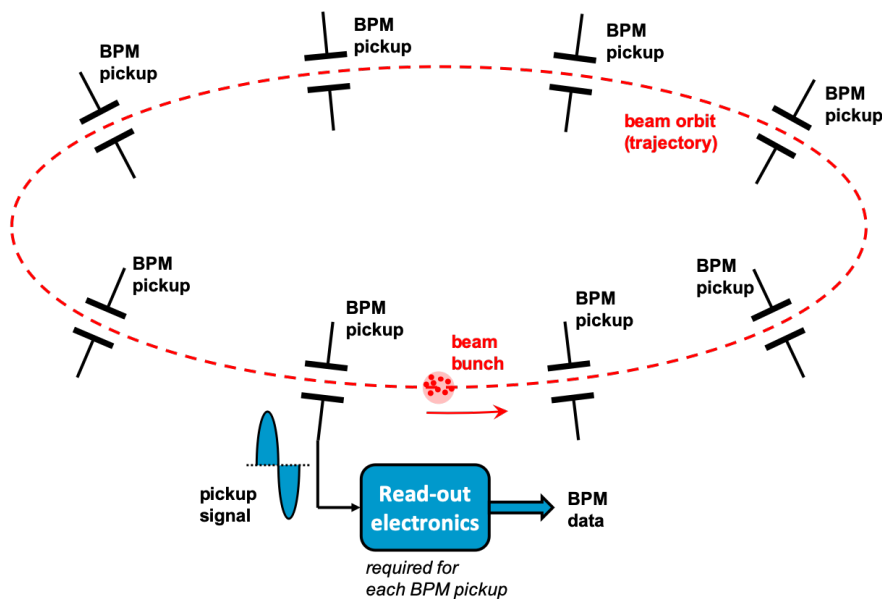


Figure 2.1: *Beam position monitors in a ring accelerator.*

The measurement of the beam position is fundamental in every particle accelerator and its implementation techniques are numerous. They differ depending on the type of accelerator – with the related beam parameters and formats – and the required performance of the measurement, and have evolved along with the available technologies.

The main components of a beam position monitor (BPM) are the transducers, also called BPM pickups, which convert the electromagnetic field related to the beam of

charged particles traveling inside the metallic vacuum chamber into electrical signals, and the read-out electronics to acquire and process those signals to calculate the beam position, which is further manipulated, stored and transmitted.

Figure 2.1 sketches a system of many BPMs located around a ring accelerator, each monitoring the passing beam. The beam typically consists out of one or several compact bunches, millimeter to centimeter in size, each being an assemble of many charged particles. As each BPM monitors the transverse beam position with respect to the beam pipe centre at its location, the position information of all BPMs in an accelerator allows to reconstruct the beam orbit (or beam trajectory) along the accelerator.

2.1 Principle of operation

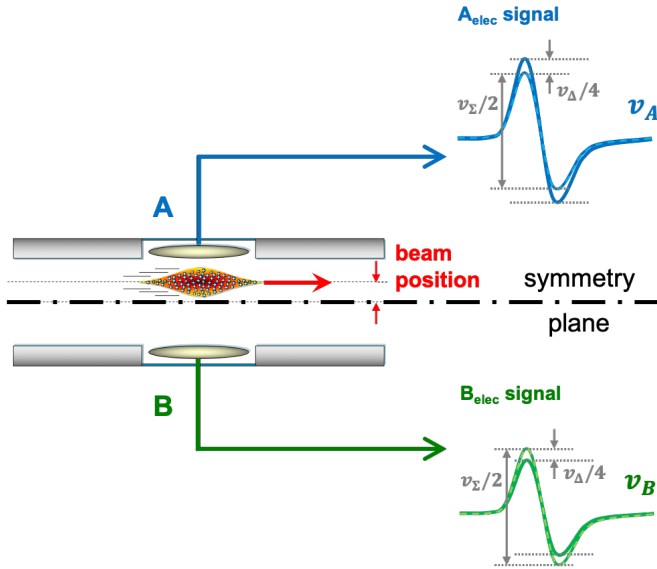


Figure 2.2: Principle of BPM operation.

Figure 2.2 shows the principle of operation of a BPM. The BPM pickup is indicated by two symmetrically arranged electrodes A and B , in a longitudinal section view of a cylindrical beam pipe. The passing beam bunch induces a pulse signal in each electrode; their voltage signals amplitude v_A and v_B are proportional to the bunch intensity and to the distance between beam and electrode, i.e. the transverse *beam position*. In this example the particle bunch is closer to electrode A , therefore $v_A > v_B$, as indicated by the time-domain pulse waveforms of the A_{elec} and the B_{elec} signals. The read-out electronics acquires and combines the A and B signals to extract the beam position information, often in the form:

$$\text{beam pos.} \propto \frac{v_{\Delta}}{v_{\Sigma}} = \frac{v_A - v_B}{v_A + v_B} \quad (2.1)$$

This *normalisation* is required because each electrode signal is proportional to the beam intensity, which provides a common mode contribution $v_{\Sigma}/2$ to both electrode signals.

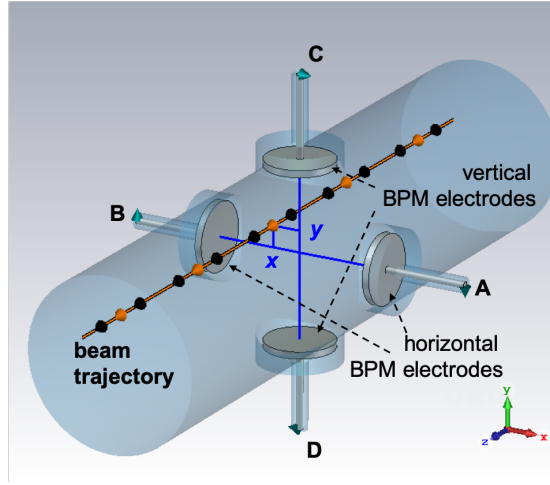


Figure 2.3: A BPM pickup for horizontal and vertical beam position measurements.

The Δ -signal $v_{\Delta} = v_A - v_B$ is proportional to both, the beam position *and* the beam intensity. The normalisation to the Σ -signal is required to ensure a beam intensity independent position information. In the case $v_A = v_B$ the Δ -signal, and therefore the beam position becomes zero, indicating the beam travels exactly in the centre of the beam pipe on the symmetry axis.

In other words, the beam position information is extracted by a symmetric detection system, monitoring and measuring the beam position as asymmetry originated by the two pickup signals. The concept is then expanded to both, the horizontal (x) and the vertical (y) plane, therefore usually the BPM pickup has two pairs of symmetrically arranged pickup electrodes. Fig. 2.3 shows a simplified example of an electrostatic, so-called “button”-style BPM pickup with four electrodes.

Typically the beam intensity related common-mode Σ -signal is large compared to the Δ -signal, which includes the position information imprinted as amplitude modulation on each electrode signal, but with opposite sign. The challenge of the read-out electronics lies in the detection of the small Δ -signal in presence of the large common-mode Σ -signal.

The optimal design choice or compromise towards a specific BPM pickup and processing electronics is dictated by the given input, i.e. the beam signal and formatting, the vacuum pipe aperture and available real estate, and by the desired output, i.e. the parameters to be measured with a specified accuracy, resolution and rate.

2.2 The input: the beam signal

A beam of charged particles travelling at a constant velocity v in the metallic beam pipe of a ring accelerator is equivalent to an average beam current, given by the total beam charge times the revolution frequency. As the average beam current depends on the revolution frequency, therefore on the size of the accelerator, it is more convenient to express the beam intensity in terms of number of charged particles or beam charge. The instantaneous beam current is defined as the time derivative of the beam charge, and reveals the longitudinal particle distribution of the beam bunches.

The moving charged particles are accompanied by an electromagnetic field, which

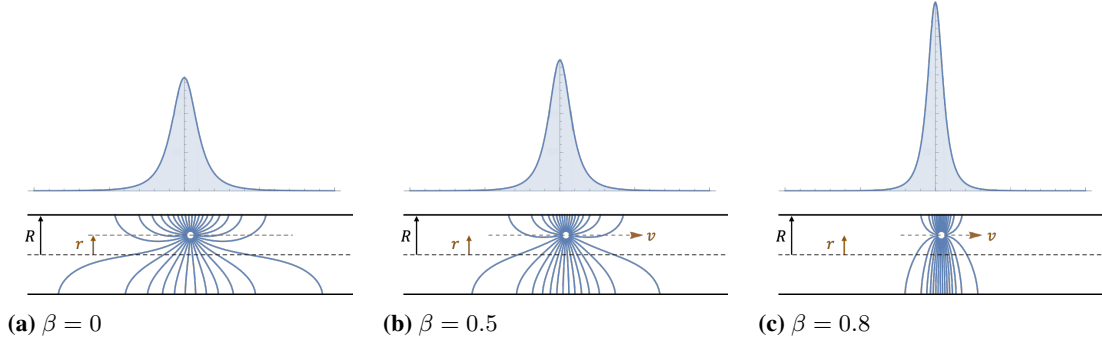


Figure 2.4: Electric field lines and wall current density for an off-center point charge moving at different velocities $\beta = v/c$.

generates compensating image charges of opposite sign distributed on the surface of the metallic beam pipe, leading to the so-called image or wall current [3]. Fig. 2.4 illustrates the electric field lines for a point charge q travelling at different constant velocities $\beta = v/c$ inside the beam pipe at a position offset r with respect to the centre of the beam pipe. It also shows the induced wall current distribution. At the limit $\beta \approx 1$ the electromagnetic field has no longitudinal components, it is a transverse electromagnetic field, and the wall current perfectly resembles the longitudinal particle distribution, see Fig. 2.5b.

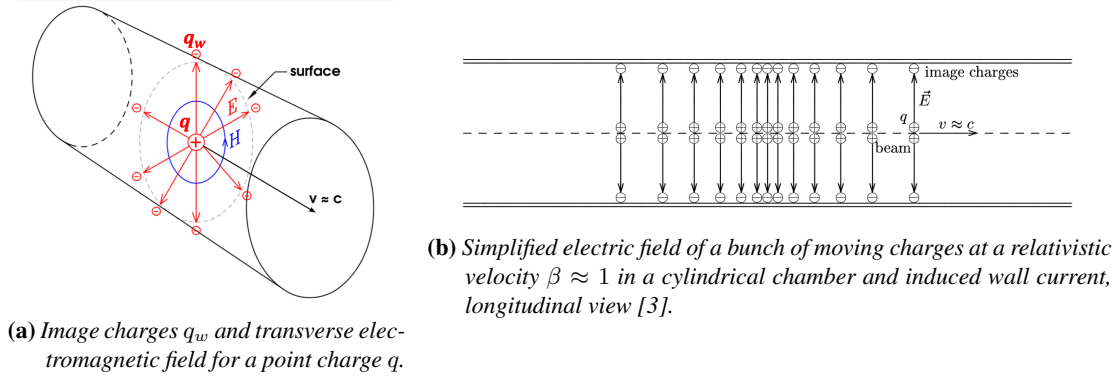


Figure 2.5: Beam particles in a vacuum chamber travelling at a relativistic velocity $\beta \approx 1$.

As mentioned, the particles are usually not uniformly or randomly distributed around the beam pipe circumference, but a *beam structure* is given by the acceleration scheme. For high energy ring accelerators, e.g. synchrotrons like the LHC, the longitudinal electric fields of radio-frequency (RF) powered cavities are used to accelerate the beam, i.e. to increase the beam energy. As result, the particles are grouped into bunches, based on the so-called longitudinal focusing concept [4].

The travelling bunch of particles is equivalent to a current $i_b(t)$, and induces a wall current $i_w(t)$ which, for $v \approx c$, has the same waveform, but opposite sign (see Fig. 2.5b).

The longitudinal distribution of the charges in a bunch, determining the instantaneous bunch current $i_b(t)$, can in most cases be approximated by a Gaussian distribution function. Considering a bunch of N_b particles of charge q , distributed in space-time

as a Gaussian with a standard deviation σ_b , describes the bunch instantaneous current as [5]:

$$i_b(t) = \frac{qN_b}{\sqrt{2\pi}\sigma_b} e^{\frac{-t^2}{2\sigma_b^2}} \quad (2.2)$$

2.2.1 Wall current and beam position characteristic

Consider a charged particle beam $i_b(t)$ of arbitrary longitudinal distribution, with all particles travelling at relativistic velocity exactly on the centre of a perfectly conducting beam pipe with circular cross-section of radius R . As Fig. 2.5a illustrates for a single point charge, the compensating image charges are uniformly distributed around the beam pipe circumference, leading to a wall current density:

$$j_w(R, t) = \frac{-i_b(t)}{2\pi R} \quad (2.3)$$

If the beam is displaced with respect to the centre of the circular, grounded, conducting pipe, the electromagnetic field interaction with the beam pipe changes and the density of the induced wall current varies accordingly (see Fig. 2.6). The solution of this 2-dimensional electrostatic problem can be found by applying the image current method, assuming the beam current as line charge, and by placing an image line charge of opposite sign such that the potential on the circle corresponding to the pipe is constant. Cylindrical coordinates $(\rho, \phi_w, z = 0)$ are assumed, the beam pipe circumference is described by the points $(\rho = R, \phi_w)$, and the beam is located at $(\rho = r, \phi_w = \theta)$. The solution reveals the wall current density distribution along the beam pipe circumference [5], i.e. in the cross-section:

$$j_w(R, r, \theta, \phi_w, t) = \frac{-i_b(t)}{2\pi R} \left[\frac{R^2 - r^2}{R^2 + r^2 - 2Rr \cos(\phi_w - \theta)} \right] \quad (2.4)$$

As indicated in the schematic cross-section view (Fig. 2.6), the wall current density is maximum where the beam has the closest distance to the beam pipe wall.

For the beam position measurement, a pair of pickup electrodes A and B is located symmetrically, each covering some fraction of the wall current density. Fig. 2.7 indicates two horizontal electrodes located at $\phi_w = 0$ and $\phi_w = \pi$, each covering an angular width of α on the beam pipe circumference. The amount of wall current on each electrode is given by:

$$i_{elec} = \int_{-\alpha/2}^{+\alpha/2} j_w(R, r, \theta, \phi_w, t) d\phi_w \quad (2.5)$$

i.e. the integrated wall current density on the pickup electrode as function of the beam position (r, θ) .

The two electrode currents i_A and i_B induced by a beam i_b displaced off the centre (r, θ) are calculated as:

$$i_A(R, \alpha, r, \theta, t) = \frac{-i_b(t)}{2\pi} 2 \arctan \left[\frac{R+r}{R-r} \tan \frac{\phi_w}{2} \right] \Bigg|_{-\theta-\alpha/2}^{-\theta+\alpha/2} \quad (2.6)$$

$$i_B(R, \alpha, r, \theta, t) = \frac{-i_b(t)}{2\pi} 2 \arctan \left[\frac{R+r}{R-r} \tan \frac{\phi_w}{2} \right] \Bigg|_{-\theta+\pi\alpha/2}^{-\theta+\pi+\alpha/2} \quad (2.7)$$

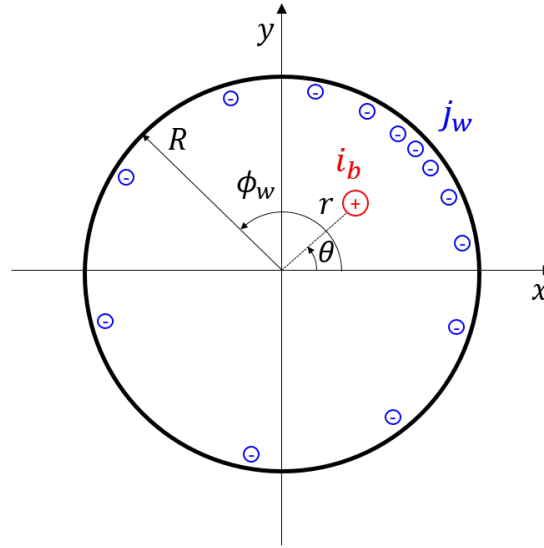


Figure 2.6: Graphic representation in cylindrical coordinates of the current density induced on the beam pipe wall by a displaced beam, cross-section view.

For small beam displacements, the resulting expressions can be linearised with good approximation at the first order. Considering the beam displacement now in Cartesian coordinates ($x = r \cos \theta$ and $y = r \sin \theta$), and only along the horizontal axis ($y = 0$)

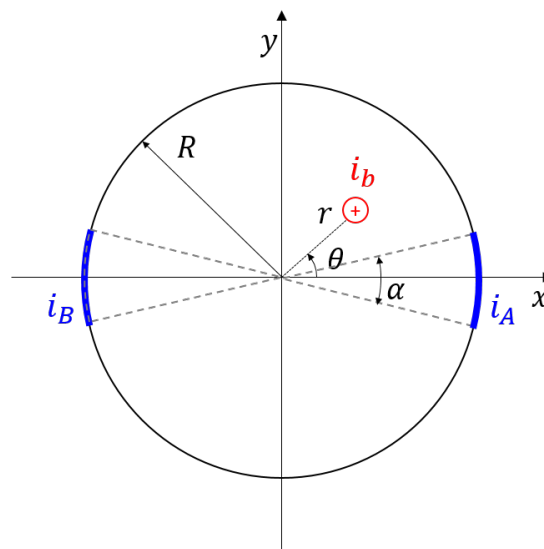


Figure 2.7: Graphic representation in cylindrical coordinates of the current density induced on the beam pipe wall by a displaced beam, transverse view, on two pipe sections of angular width α .

we find:

$$i_A(R, \alpha, x, t) \approx -i_b(t) \overbrace{\frac{1}{2\pi} \left[\alpha + 4 \sin \left(\frac{\alpha}{2} \right) \frac{x}{R} \right]}^{s_A(x)} \quad (2.8)$$

$$i_B(R, \alpha, x, t) \approx -i_b(t) \underbrace{\frac{1}{2\pi} \left[\alpha - 4 \sin \left(\frac{\alpha}{2} \right) \frac{x}{R} \right]}_{s_B(x)} \quad (2.9)$$

In both equations (2.8) and (2.9) is indicated the sensitivity functions s , i.e. an unit-less sensitivity function describing the transverse position characteristic of the BPM pickup electrode configuration as function of the beam position (x, y) , and for this approximation depends only on the horizontal beam displacement.

Finally we can compute the Δ over Σ ratio of the two electrode signals, observing that it is proportional to the displacement, normalised with respect to the beam intensity:

$$\text{hor. beam pos.} \propto \frac{\Delta}{\Sigma} = \frac{i_A - i_B}{i_A + i_B} \approx \frac{4}{\alpha} \sin \left(\frac{\alpha}{2} \right) \frac{x}{R} \quad (2.10)$$

This operation is among the most popular to extract the normalised position from the intensity dependent beam signals produced by a pair of opposite BPM electrodes.

2.3 The output: the position measurement specifications

The BPMs are one of the most powerful beam instrumentation and diagnostics system in the accelerator. The beam position is linked – directly or indirectly – to the monitoring and measurement of many beam parameters, and it is also used for trouble shooting and in feedback applications.

For any beam position measurement, i.e. the measurement of the position of the centre of charge of a bunched beam passing the BPM pickup with respect to the centre of the beam pipe, the measurement or integration time is an important variable, and usually linked to the beam formatting, e.g. bunch-to-bunch distance, length of a train of bunches, or the circumference of the ring accelerator. The requirements for the signal processing chain can vary largely, and typically there is a trade-off between the spatial resolution of the measurement and the time resolution, i.e. the measurement time.

Some typical measurement times for a ring accelerator, linked to the beam formatting, can be classified [6]:

- *Bunch-by-bunch*: each BPM is able to resolve the position of each beam bunch passing the pickup, enabling to monitor the beam position of individual bunches and its behaviour over many turns. The time distance between sequential bunches can be below 1 ns; for the LHC the nominal bunch spacing is 25 ns. The processing chain requires broadband electronics and fast digitisation, and a synchronisation scheme to label bunches and turns for all BPMs.
- *Turn-by-turn or trajectory*: the BPM resolves the beam position on a turn-by-turn basis, the measurement time is equal to the revolution time. This is the most fundamental and flexible operation mode, and is particularly important during the beam commissioning at the startup of a ring accelerator, when the beam is not yet

Chapter 2. Beam Position Measurement

stored, see Fig. 2.8. Turn-by-turn position data are used for several beam diagnostics applications, e.g the characterisation of the beam optics, the measurement of the machine *tune* and of the coupling between horizontal and vertical plane.

- *Closed orbit*: the position data of each BPM is averaged over many revolutions of the beam, typically some 100...1000 turns, requiring a stored beam. The measurement time is in the millisecond regime, such that beam oscillations are averaged out, and the closed orbit is measured with a high resolution. The processing scheme can make use of simple, narrowband electronics, reducing noise effects, which enables a beam position (orbit) measurement with a resolution in the sub- μm regime.

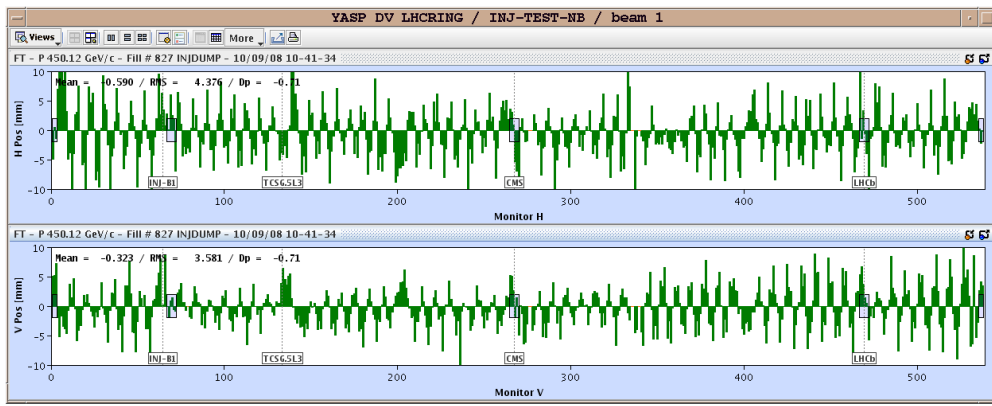


Figure 2.8: The beam position display as visible in the control room. Every green bar represents the beam position, here calculated over one turn for a specific BPM location along the machine; upper graph: hor. BPMs, lower graph: vert. BPMs. There are more than 500 BPMs per ring in the LHC. The display shows the first turn trajectory in the LHC, on September 10, 2008. Credits: The daily news of the LHC commissioning.

Depending on the application and requirements, some of the following measurements characteristics of a BPM system must be optimised more than others. The most important BPM measurement properties are [5–7]:

- *Position characteristic*: the law of the proportionality between the position signal strength and the beam displacement. Typically the BPM pickup has a non-linear position characteristic, see Eqs. (2.6) and (2.7). For large beam displacements the BPM pickup non-linearities have to be corrected, e.g. by an inverse polynomial fit function or a lookup table. The remaining non-linearities will limit the accuracy.
- *Accuracy*: the ability of a BPM to provide a calibrated, absolute information of the beam position, usually with respect to the centre of the beam pipe, typically reported in millimeter. It is limited by the contribution of mechanical misalignment and tolerances of the BPM pickup, calibration errors of the read-out electronics and/or the BPM pickup, and other unwanted effects like uncontrolled reflections and attenuation in the signal cables, electromagnetic interference, etc.
- *Resolution*: the ability of a BPM to detect small variations of the beam position. This BPM property is often more important than the accuracy; for a ring collider like the LHC measuring the relative position between two colliding beams and

small variations of this property is more valuable than knowing their absolute beam positions. The BPM resolution is mainly determined by the stochastic noise contribution of the input stage of the read-out electronics, but it can be improved substantially by averaging. The single-shot resolution corresponds to the standard deviation of each individual position measurement, e.g. for a single bunch or for a single turn, depending on the time resolution. The BPM resolution relates to the measurement time, and typically improves with the number of single-shot measurements for an averaged position measurement, e.g. over successive beam bunches and/or many turns.

- *Stability*: the ability of the BPM system to perform a precise beam position or orbit measurement, independent to changes of the input signal (beam intensity, beam structure, etc.), of the environment (temperature), and of the status of the read-out electronics (start-up conditions, aging effects, radiation effects, etc.). It is an important parameter, since it effectively limits the accuracy and the reproducibility of the system. It can be improved by an integrated online calibration system, but such actions are time and resource consuming for large, distributed systems like the BPMs of the LHC, and not always possible.
- *Bandwidth*: the frequency range over which the beam position is measured. It is limited by the combination of the bandwidth of the beam pickup and the analog section of the signal processing electronics, and should be matched to the beam signal spectrum. It defines minimum measurement time, and can be limited to achieve higher resolution.
- *Dynamic range*: the range of beam/bunch intensities (currents) that the BPM should cover to reliably measure the beam position. As discussed in 2.2, the level of the bunch signal is proportional to the number of charges, while the read-out electronics has to report the beam position independent of the actual beam intensity. If a change in the beam position is related – even partially – to a change of the beam/bunch intensity, the position measurement has no value.

2.4 The BPM Pickup

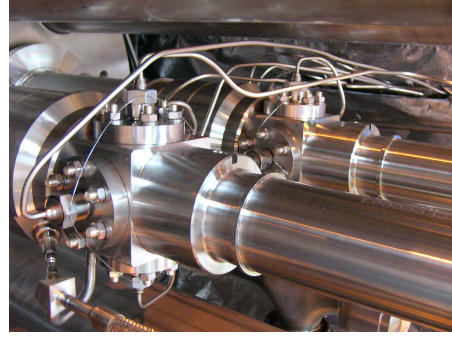
The pickup is the component of the BPM that converts the electromagnetic field of the beam, which is related to a beam signal (see section 2.2), into an electrical signal. There exist different types and geometries [8]; this thesis refers in particular to two of the most most popular pickups used in circular machines, which are the *button* and the *stripline* pickup [3]. They exploit different aspects of the electromagnetic interaction with the beam, and their response is characterised by the transfer impedance of the pickup electrode, i.e. the ratio of the electrode output voltage to the beam current.

2.4.1 The button pickup

The button pickup is most popular in ring accelerators, thanks to its compact and rigid design, and the relatively low production costs. It consists out of four symmetrically arranged coin-like electrodes, the “buttons”, see Fig. 2.3 and 2.9a, which couple to the electro-magnetic field of the beam. In practice the button electrode also fulfills the



(a) Photograph of a LHC button pickup feedthrough electrode.



(b) Photograph of the two LHC BPM pickups, one for each beam pipe of the collider. Each pickup is composed of two pairs of hor./vert. electrodes.

Figure 2.9: The LHC button BPM pickup.

function as vacuum feedthrough to pass the signal from the beam vacuum to the air side (Fig. 2.9b).

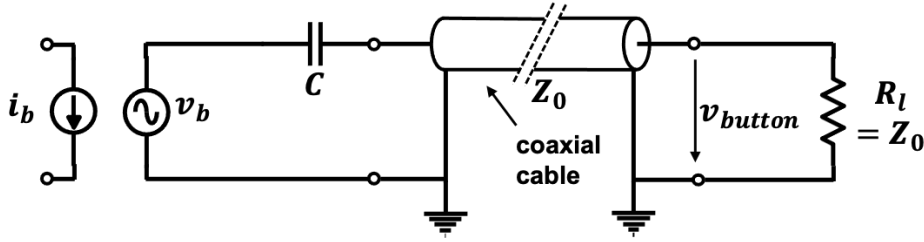


Figure 2.10: Equivalent circuit of a button electrode.

Each button electrode generates a high-pass filter like response to the induced image charge, with a signal magnitude proportional to the beam intensity and the beam-button distance, following the principle described in 2.2. The transfer impedance in the frequency domain follows as [9]:

$$Z(\omega) = \frac{V_{button}(\omega)}{I_b(\omega)} = \frac{\alpha}{2\pi} Z_0 \frac{\omega_1}{\omega_2} \frac{j\omega/\omega_1}{1 + j\omega/\omega_1} \quad (2.11)$$

with:

$$1/\text{time constant: } \omega_1 = \frac{1}{Z_0 C}; \quad 1/\text{transit time: } \omega_2 = \frac{c}{\ell}$$

where we assume an electrode with angular width α , length ℓ , capacitance C , and relativistic beam with velocity approximated by the speed of light c . Fig. 2.10 shows the equivalent circuit of the button electrode. As the button capacitance to ground C dominates the operation principle, the button BPM pickup is often referred as capacitive or electrostatic pickup. Examples of the button transfer impedance $|Z(\omega)|$ are plotted in Fig. 2.11, left.

The current flowing through the electrode in the capacitance is the time derivative of the induced image charge; the output voltage signal is proportional to the charge rate at low frequency, but it tends to the integrated current signal at high frequency. At high

frequencies, the transfer impedance of the button electrode is frequency independent [5]:

$$Z(\omega \gg \omega_1) = \frac{\alpha \ell}{2\pi c C} \quad (2.12)$$

Assuming the bunched beam current as a Gaussian particle distribution (2.2), the button pickup response in time follows as convolution of the bunch current signal with the inverse Fourier transform of the button transfer impedance (2.11), see the examples in Fig. 2.11, right.

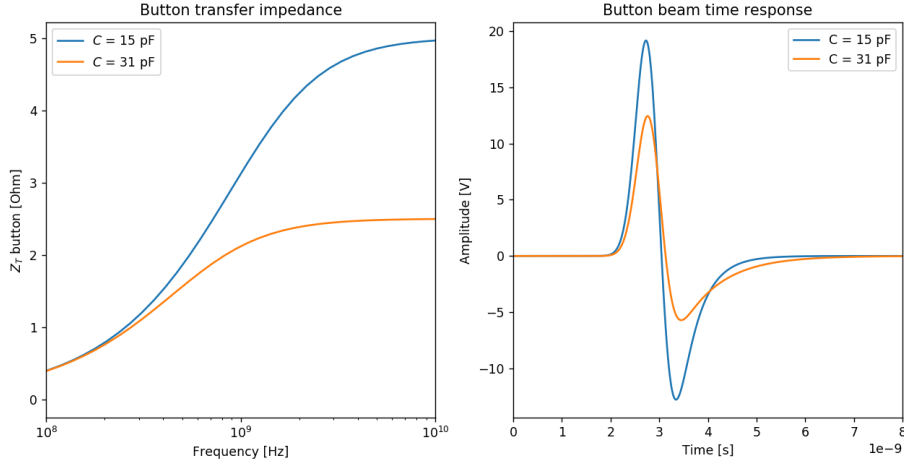
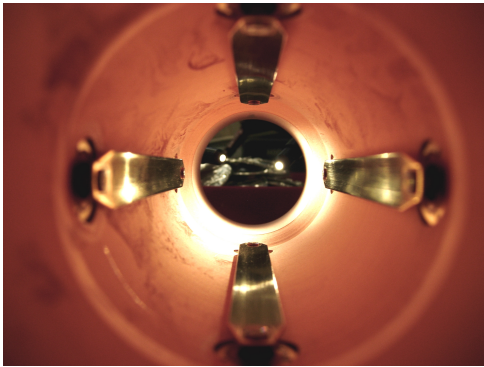
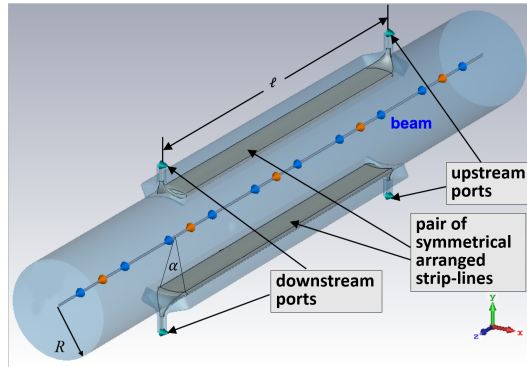


Figure 2.11: Transfer impedance of a button BPM in the frequency domain and its time response to a Gaussian bunch signal for two different capacitance values of the button electrode. The bunch length is $4\sigma_b = 1$ ns, the bunch intensity is $N_b = 1 \cdot 10^{11}$, the button length, i.e. its diameter, is $\ell = 24$ mm, the beam pipe radius is $R = 24.5$ mm and the beam is assumed relativistic.

2.4.2 The stripline pickup



(a) Photograph of four striplines in the mock-up of a BPM pipe.



(b) Schematic view of a stripline pickup (vertical plane only)

Figure 2.12: Representation of a stripline pickup.

If the length of the pickup electrode becomes comparable to the length of the bunch, signal propagation effects must be taken into account. In electromagnetic pickups, like

the directional coupler pickup, also referred to as *stripline* BPM pickup (see Fig. 2.12), signal propagation is part of the instrument design principle.

Each stripline electrode forms with the electrically grounded vacuum chamber a transmission line [5], with a defined characteristic impedance Z_0 . The stripline has two ports, referred as upstream (U) and downstream (D) port with respect to the beam direction. As the beam approaches the upstream port of the electrode, the wall current at the gap between the electrode and the beam pipe sees an impedance that is half the stripline characteristic impedance Z_0 , and induces a voltage signal at the gap proportional to the electrode's angular width α (Fig. 2.13, $t = 0$):

$$v_g(t) = \frac{Z_0}{2} \frac{\alpha}{2\pi} i_b(t) \quad (2.13)$$

This voltage excites the propagation of two waves, one traveling out the upstream port to the electronics, the other through the electrode towards the downstream port, on the side facing the beam pipe. Simultaneously the beam charge travels towards the downstream port, together with the image charge. At $t = \ell/c$ the beam and the image charge reach the downstream port, again inducing a voltage signal when passing the gap (Fig. 2.13, $t = \ell/c$). This second signal has opposite polarity with respect to the first one, and it also generates two waves, one moving out the downstream port, the other one traveling backwards to the upstream port. The net result at the two ports is the sum of two signals, how they combine depends on the ratio between the velocity of the signal on the stripline ($v_s = \beta_s c$) and the velocity of the beam ($v = \beta_b c$). The voltages at the two ports, $v_U(t)$ and $v_D(t)$, for a stripline of length ℓ and angular width α are then expressed as:

$$v_U(t) = \frac{Z_0}{2} \frac{\alpha}{2\pi} \left[i_b(t) - i_b \left(t - \frac{\ell}{\beta_b c} - \frac{\ell}{\beta_s c} \right) \right] \quad (2.14)$$

$$v_D(t) = \frac{Z_0}{2} \frac{\alpha}{2\pi} \left[i_b \left(t - \frac{\ell}{\beta_b c} \right) - i_b \left(t - \frac{\ell}{\beta_s c} \right) \right] \quad (2.15)$$

In the example of Fig. 2.13 the two propagation velocities are the same, $\beta_s = \beta_b$, therefore the net signal at the downstream port sums to ideally zero (annihilation). In this case the stripline has a directional characteristic, and this property is useful in colliders like the LHC, near the interaction points where both counter-rotating beams collide and there is the need to measure the position of both beams sharing the same pipe independently.

At the upstream port, the response to a beam bunch with Gaussian distribution, (2.2), can be expressed in a closed form:

$$v_U(t) = \frac{Z_0}{2} \frac{\alpha}{2\pi} \frac{qN_b}{\sqrt{2\pi}\sigma_b} \left\{ e^{-\frac{(t+\tau_\ell)^2}{2\sigma_b^2}} - e^{-\frac{(t-\tau_\ell)^2}{2\sigma_b^2}} \right\} \quad (2.16)$$

with:

$$\tau_\ell = \frac{\ell}{2c} \left(\frac{1}{\beta_b} + \frac{1}{\beta_s} \right)$$

Figure 2.14 shows the response in time and frequency domain of the stripline electrode to a beam bunch with Gaussian envelope function.

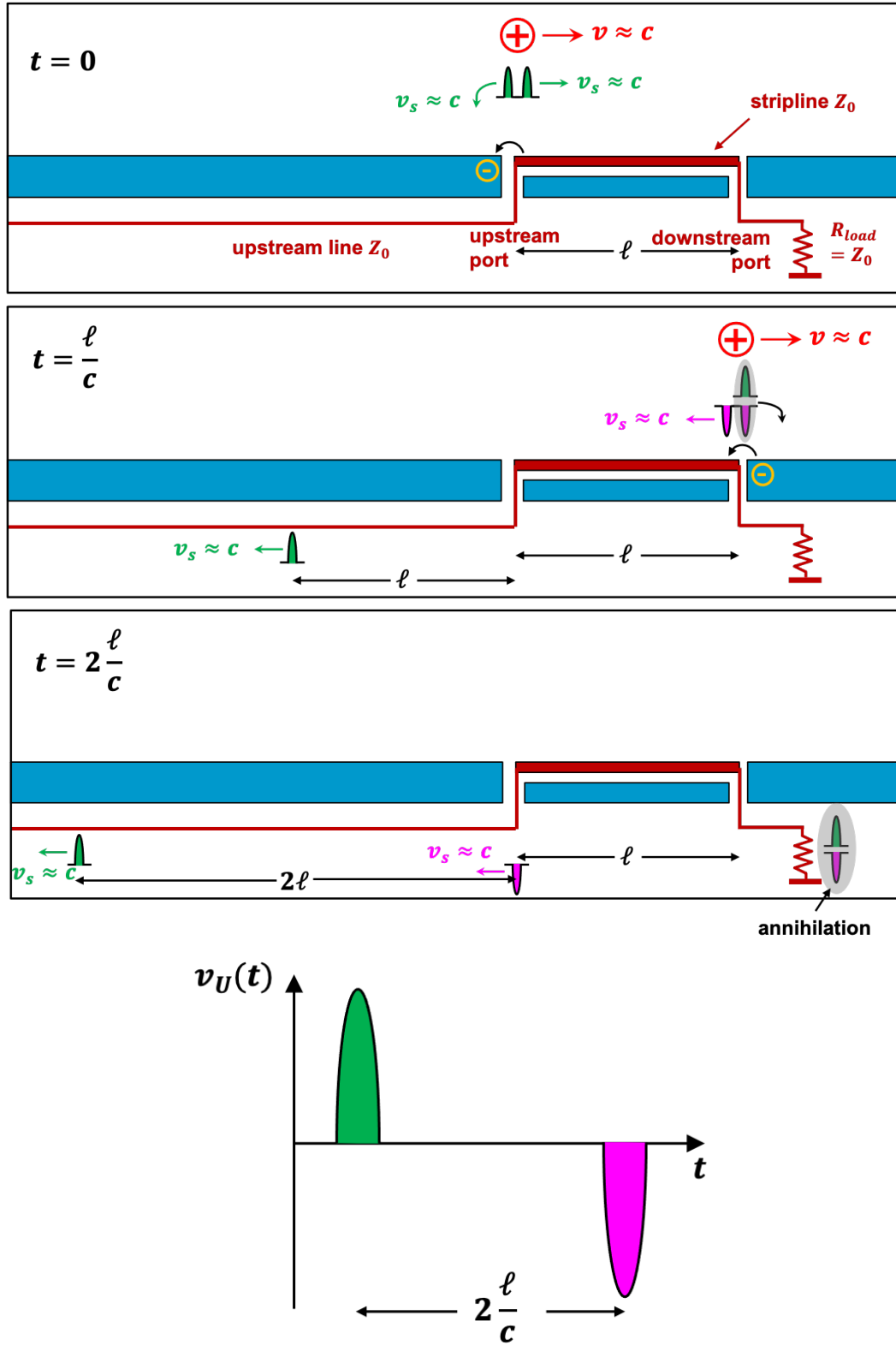


Figure 2.13: Operation principle of a stripline electrode.

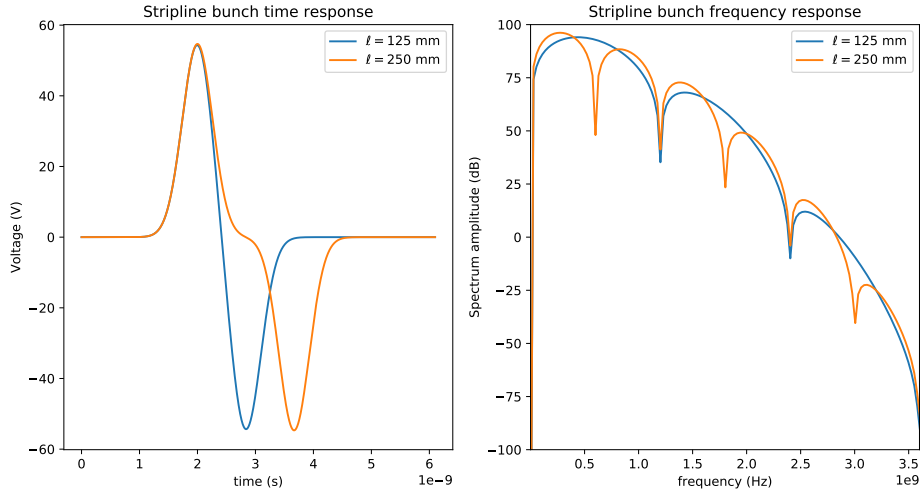


Figure 2.14: Time and frequency response of a stripline BPM for two different electrode lengths ℓ . The bunch length is $4\sigma_b = 1$ ns, the bunch intensity is $N_b = 1 \cdot 10^{11}$, the pipe radius is $R=44.5$ mm, the electrode width, approximately equal to αR , is 12 mm, and the beam is assumed relativistic.

2.5 BPM Signal Processing Electronics

The read-out electronic circuits processing the BPM pickup signals contribute to the performance and characteristics of the measurement. A comprehensive overview and comparison of analog BPM signal-processing schemes is given in [7], grouping them under the following three criteria: how the electrode signals are combined in the first stage of the signal processing chain; how the position information is normalised with respect to the beam intensity; what is the bandwidth of the signal processing electronics, thus the acquisition time. Most BPM signal processing systems can be classified with respect to which principle they apply in each of these categories.

Signal combination

The position information for the horizontal or vertical plane is extracted by comparing, i.e. *combining* the signals of opposite, symmetrically arranged pickup electrodes. The signal combination scheme defines how and at which point of the processing chain the signals are merged together.

In case of an *individual signal treatment* each electrode signal is continuously processed and acquired separately by dedicated electronics.

If the electrode signals are arranged to share the same electronics read-out channel we are talking of a *signal time-multiplexing* scheme. In the very early days of BPM signal processing, when electronics were expensive and unreliable, this was implemented by using a special single-shot analog storage oscilloscope (sometimes called “scan-converter”) and a complex coaxial relay, switching sequentially all electrodes of all BPM pickups to one or two channels of the read-out oscilloscope. Evidently, only one electrode signal is acquired per time, making it impossible to reach a high time resolution. Alternatively, the electrode signals are combined together into a single signal by an interleaving signal scheme, e.g. filling the interval between sequential bunches

with the bunch signal of the other electrode.

In both the above mentioned cases the position extraction is performed in the digital back-end electronics, where the entire signal content is still present. Another class of methods combines the signals in the analog domain to extract the position information before digitisation.

A popular analog signal processing scheme uses a 3 dB 4-port RF coupler, the so-called 180°-hybrid, to convert the two electrode signals into *sum and difference* (Σ and Δ) signals, necessary to perform the Δ/Σ normalisation (2.10). The limit of the scheme is the available bandwidth and precision of the hybrid, and the fact the the phase (sign) of the Δ -signal has to be preserved.

Also popular is the conversion of the two electrode signals from their amplitude ratio into a phase difference (sometimes referred as *AM-PM processing*) or time difference, e.g. by using a 3 dB 90°-hybrid (narrowband) or a time-delay network (wideband). The following signal processing typically uses analog comparators to become independent of the signal intensity and to acquire the position information hidden in the phase or time between two signals.

Normalisation

Normalisation with respect to the beam intensity is required to access the position information. It can be performed in the analog domain, producing a signal proportional to the beam displacement and independent on the beam intensity, e.g. as AM-PM processor. If no extra Σ -signal processing channel is implemented, the intensity information is lost, but the requirements on the digitisation stage in terms of analog bandwidth, sampling rate and dynamic range are somewhat relaxed.

The conversion from the amplitude ratio to a phase or time difference are two quasi passive normalisation techniques, utilising passive RF components to separate intensity and position information. Here, the relative phase or time between two output signals of the 90°-hybrid or delay-line network keeps the beam position information, while their amplitudes are the same and are proportional to the beam intensity. The following dual analog comparator and digital mixing stage generates a logic signal containing the position information, which can be transferred for integration and digitisation over long distances, e.g. by optical fibers.

A widely used active technique for BPM signal normalisation is based on RF logarithmic amplifiers, sometimes called RF logarithmic envelope detectors. For small beam displacement, the logarithm of the ratio of the amplitude of the two electrode signals is proportional to the beam position [5]. Since the logarithm of the ratio of two signals is equal to the difference of the logarithm, the two electrode signals are processed with two, or preferably a dual logarithmic amplifier, which also acts as envelope detector, i.e. demodulator. The signal difference of the demodulated waveforms, i.e. the beam position signal, is then processed by a simple operational amplifier, which is followed by a digitisation stage. This processing scheme allows to cover a wide dynamic range, but is highly non-linear and involves a dynamic range compression.

Processing bandwidth

The bandwidth of the signal processing electronics, i.e. the lower and upper frequency limit, defines the portion of the pickup signal spectrum to be processed and therefore

determines the acquisition time, i.e. the response time to resolve a sudden change in the beam position in the BPM pickup.

Broadband systems usually try to preserve the bunch or turn structure information and are able to perform a bunch-by-bunch or turn-by-turn beam position measurement. The system bandwidth is typically in the 100 MHz regime. Low-pass filtering schemes are applied for noise reduction and/or signal pulse stretching, but the beam structure can still be resolved. The acquired signals are either the raw pickup electrode signals, or their pre-processed version in terms of Σ and Δ , amplitude-to-time signal, or logarithm value. The pre-processed analog signals are then digitised by ADCs with sufficient bandwidth and sampling rate, or, if direct digitisation is not possible, the peak signal amplitude is temporarily frozen by a track&hold circuit or the sampling point is selected with the help of an external trigger synchronised to the beam passage.

Narrowband systems utilise the periodic time structure of the bunched beam, circulating in the ring accelerator. The spectrum of the periodic BPM pickup signal has harmonics at the revolution frequency of the ring accelerator, at the frequency of the RF acceleration system, and at frequencies related to the beam format, e.g. bunch spacing. A band-pass filter centred over one or more of these harmonics is used to select a narrow area of the spectrum, reducing significantly the broadband noise. Furthermore, heterodyne downconversion and/or direct demodulation schemes are used on the Δ -signal modulated electrode signals to extract the position information. The demodulated analog signal has quasi-DC characteristics, therefore can utilise rather slow, high-resolution ADCs for the digitisation. This frequency-domain harmonic signal processing corresponds to averaging the position information over many bunches and revolutions. The information of individual bunches or turns is lost, but the resolution of the position measurement is high.

Figure 2.15 shows the response signal of a stripline electrode to a beam of equidistant spaced bunches in time and frequency domain. The direct bunch signal response of the coupler is shown, followed by the signals filtered by a low-pass filter – for a broadband processing system – and by a band-pass filter – for a narrowband processing system. From the broadband signal it is still possible to distinguish the individual bunches. The signal for narrowband electronics, however, requires the passage of multiple bunches before reaching a steady-state, the time-domain waveform is almost sinusoidal.

2.5.1 Examples of BPM signal processing systems used at CERN accelerators and elsewhere

The properties of the accelerator and the beams, and the requirements for the beam measurement system guide the instrument designer in the selection of the processing scheme from a wide variety of combinations and architectures.

Already at a single laboratory like CERN we find a variety of different BPM system implementations, because of the different characteristics of the machines and their beams in the accelerator complex, but also because of the available technology at the time of their design. Looking at the three largest synchrotrons at CERN, we notice how the different signal combination and normalisation techniques are applied, and what bandwidth was implemented for the signal processing.

In the PS a broadband system is used, based on Σ and Δ signal combination, fast

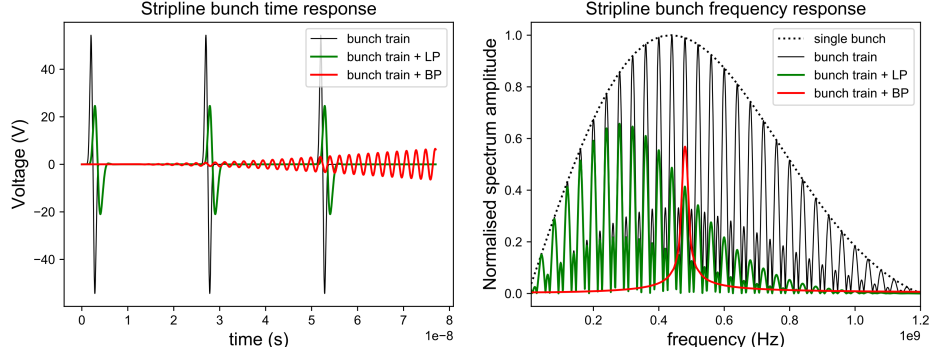


Figure 2.15: Numerical simulations in time and frequency domain, visualising the response of a stripline pickup to a beam of 25 ns spaced bunches, applying two different filtering schemes. The low-pass filter is a 4th order Bessel filter with a 3 dB cutoff frequency of 480 MHz. The band-pass filter is a 4th order Bessel filter centered at 480 MHz and with 12 MHz bandwidth.

digitisation and digital signal processing in a FPGA [10]. The sum and difference signals are digitised separately with 14-bit ADCs at a 125 MHz clock rate, and delivered to the FPGA (see Fig. 2.16). The processing is broadband, each single bunch can be resolved. The FPGA firmware integrates the sum and difference signal for the position calculation and generates the trigger events to synchronise the acquisition with the beam. The acquisition and signal processing modules are controlled and managed by a single board computer. It is worth to notice, the four rings of the CERN PS booster synchrotron (PSB) use an almost identical BPM system.

In the SPS a new BPM system is on the way to be commissioned, based on logarithmic normalisation [11–13]. The signals from the pickup electrodes are processed separately, and filtered with a band-pass filter centred at 200 MHz, making the processing narrowband. Instead of downconversion and demodulation based on a heterodyne scheme, the ringing signals are processed and demodulated by a logarithmic detector, followed by a low-pass filter. To cover the wide dynamic range of the beam signals in the SPS, three channels with different gain are acquired simultaneously with 12-bit ADCs at 40 millions samples per second (MSps), see Fig. 2.17. The difference of the logarithm signals is performed in the back-end electronics' FPGA, together with the event detection, followed by position, orbit and trajectory calculation.

The BPM system currently used in the LHC is, similar to that in the PS, a broadband BPM read-out system able to process each passing bunch circulating in the LHC, but it

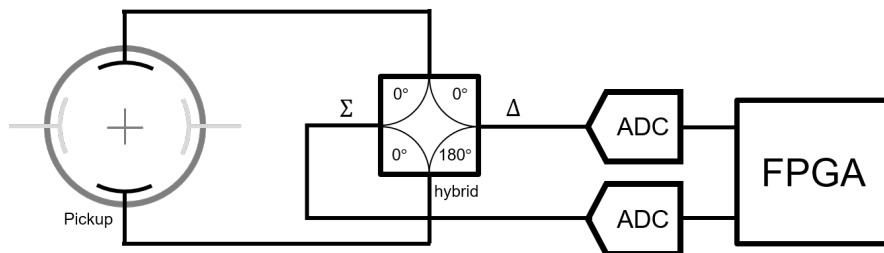


Figure 2.16: Simplified block schematics of the BPM hardware implementation of the PS digital beam trajectory and orbit system (TMS).

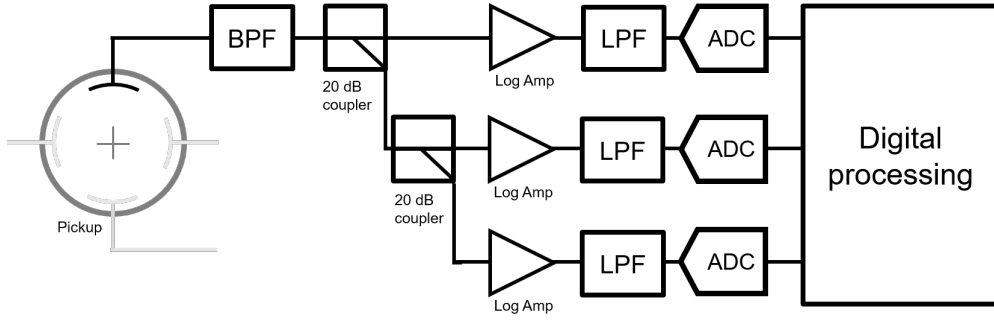


Figure 2.17: Simplified block schematics of the hardware implementation of the SPS logarithmic beam position system (ALPS), shown for one BPM electrode.

is based on the amplitude-to-time normalisation principle [14,15]. The analog front-end electronics is based on the Wide Band Time Normalizer (WBTN) which encodes the beam position information into a logic doublet pulse of constant amplitude, but varying time distance as function of the bunch position. The position is computed by measuring the time difference of the two pulse signals opportunely obtained as combination of a direct and a delayed signals of the pickup electrodes (see Fig. 2.18). The doublet pulse of each BPM channel is transmitted over long optical fibers to a radiation-free area. Here, the back-end electronics accommodate an integrator using the doublet pulse as start/stop command and a 10-bit ADC operating at 40 MSps to acquire the beam position of each passing beam bunch in the LHC, nominally spaced by 25 ns. The raw position data is acquired by a FPGA based electronics, for position, trajectory and orbit calculation. This system allows to cover a wide range of bunch intensities of >40 dB, and as it makes use of optical fibers for the signal transmission, it was the favorable option for large accelerators like the LHC. Nevertheless, the analog electronics is quite complex and maintenance intensive. Moreover, additional circuitry would be required to process the bunch intensity.

With the increasing availability of high frequency and high resolution ADCs on the market, the design of direct digitisation based BPM systems becomes attractive [16]. Those chips are especially developed for the telecommunications market, matching to some extent the need of modern BPM read-out electronics, based on individual signal treatment and narrowband frequency signal processing. The use of FPGAs allow the

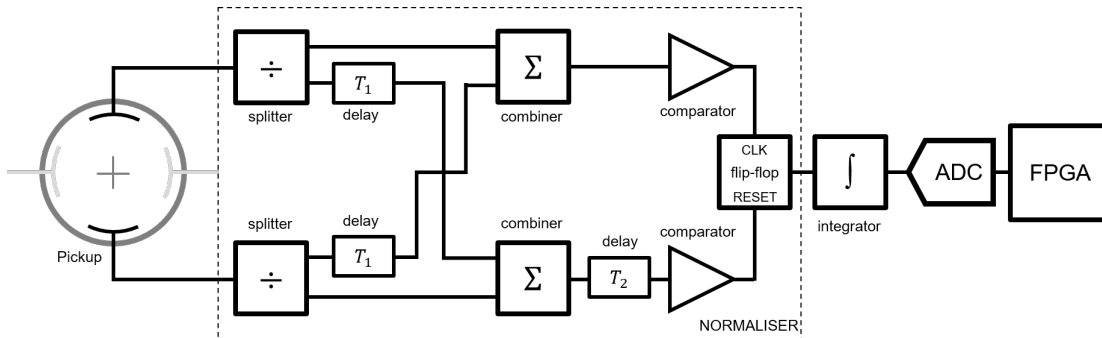


Figure 2.18: Simplified block schematics of the hardware implementation of the LHC wideband amplitude-to-time normaliser beam position system.

implementation of digital filters, with flexible bandwidth to fulfill the acquisition time and spacial resolution requirements of a BPM system.

There are many examples of BPM architectures exploiting the flexibility of digital signal processing, but still relying in their first stages on RF analog processing, e.g. implementations in light source accelerators like the Swiss Light Source (SLS) and the National Synchrotron Radiation Research Center (NSRRC) [17, 18]. Those are narrow-band BPM systems, with a first analog downconversion stage to bring a ~ 500 MHz RF signal to an about ~ 30 MHz IF signal, which then is digitised using high-resolution ADCs at rates below 50 MSps. A second downconversion is performed in the digital domain, with numerically controlled oscillators (NCO) and programmable digital low-pass filters to adapt bandwidth and resolution. The maximum time resolution allows to resolve turn-by-turn position information.

A similar approach, but completely digital, has been implemented in another synchrotron radiation machine, the Shanghai Synchrotron Radiation Facility (SSRF) [19], using the Software Defined Radio technology. A sharp band-pass filter is used on the BPM pickup signal to select the 500 MHz harmonic, but the downconversion is performed as part of the digitisation, directly undersampling the signal at an opportunely selected sampling rate. Further downconversion is performed in the digital domain in the FPGA. The performance of those implementations are comparable to the analogue narrowband approach, but without the need to implement the complicated analog downconverter. On the other hand, the requirements on the digitiser are more strict. The analog input bandwidth must be higher than 500 MHz and the sampling rate, even in undersampling conditions, is above 100 MHz.

There are also examples of wideband processing systems based on direct digitisation, in some way the PS system is one of them [10]. Again for the SSRF, a bunch-by-bunch digital BPM system was developed [20]: the four electrode signals are separately processed, each bunch pulse is sampled at the peak, acquired at a clock rate of approximately 500 MSps, synchronous to the RF acceleration frequency, with individual and precise adjustable phase-shifters. The signal samples are then processed in the FPGA based digital electronics. At Brookhaven National Laboratory (BNL) the stripline BPM signals of the Relativistic Heavy Ion Collider (RHIC) are processed by broadband electronics to acquire the position turn-by-turn of a selected bunch [21]. The signals pass through a 20 MHz low-pass filter and selectable switched gain stages; they are sampled precisely at the peak with a 16-bit digitiser and with the aid of track&holds and self-trigger circuitry. A digital signal processor subsystem and a FPGA are used for position computing and event decoding. For the CRYRING, a hadron accelerator for low energy heavy ions at GSI, it is possible to use slower ADCs for the pickup signal processing, but still obtaining more samples per bunch pulse signal, thanks to the long ion bunches. The BPM system developed at GSI [22] acquires simultaneously and separately the four electrode signals with 16-bit ADCs, sampling at a rate of 125 MSps. The sampling clock is not synchronised with the accelerator frequency, but the RF accelerator signal is used for data window selection. The sample sequence is processed in the FPGA to extract the beam position from several samples available per bunch, which enables to explore more advanced beam position estimation algorithms [23].

Among the wideband processing systems based on direct digitisation we can also list the prototype of the Interlock BPM system for the LHC [24]. Instead of process-

ing the electrodes signals separately, the signals of two opposing electrodes are time-multiplexed together into a single channel processing chain. Under the name of Delay-Multiplex Single Path Technology (DMSPT), this processing scheme, combining all the four electrodes, was first developed at DESY laboratory in 1986 and it was adopted in 2008 to upgrade the BPM system in the PETRA III pre-accelerator chain [25]. Each system measures the accelerated single bunch using a single channel processing chain. A self-triggered diode clipping peak-hold detector is used to sample the four peaks of the low-pass filtered combination of the four electrodes signal. In the case of the LHC Interlock BPM system, the choice of combining two electrode signals, together with the beam formats of the LHC and the need to keep the system independent, i.e. asynchronous to the acceleration RF, forces the selection of high sampling rate ADCs, in the order of >1 giga samples per second (GSps). This system and the choices of the sampling-stage are discussed extensively in chapter 5.

CHAPTER 3

Analog to digital conversion: definitions, limits and consequences

Analog-to-digital converters are the electric parts that translate an analog quantity, usually related to the physical world, into its encoded and discrete representation in the digital domain. Therefore, the ADC is a fundamental component of an instrumentation system that aims to perform digital signal processing on an analog signal generated by a physical phenomenon.

The characteristics of the ADC can be the performance-defining factor for an instrument based on direct digitisation. The advantages of direct digitisation are several (see the introduction in chapter 1), yet it poses strict requirements on the digitisation stage, in terms of resolution and achievable sampling rate.

The evolution of technology and architectures in the digital conversion field has been bringing to the market always better parts, making direct digitisation for beam instrumentation an increasingly more suitable option. However, a trade-off exists between the sampling rate and the resolution of the converter, making fundamental to understand how to analyse the impact of those features on the overall design.

This chapter firstly outlines the main definitions about digital conversion adopted in this work. Secondly, it gives some indications about the trade-off between sampling rate and resolution, how it can be modeled and its trend in the state-of-the-art of ADCs. Finally, this chapter provides some main principles of sampling theory and reference to recent works about undersampling and rate-distortion analysis.

3.1 Main definitions

An analog-to-digital converter converts an analog input into a digital output (see Fig. 3.1) [26].

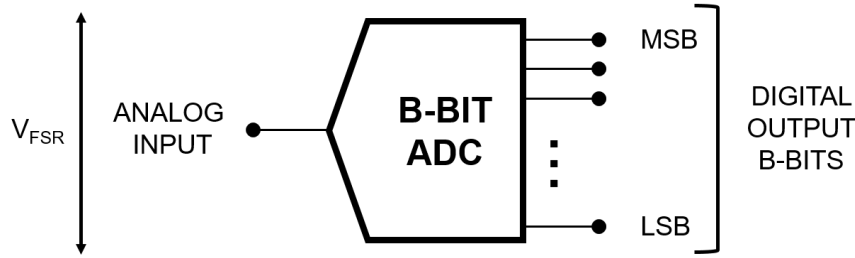


Figure 3.1: Analog-to-Digital Converter (ADC) input and output definition.

The analog input variable is an electrical signal, often a voltage or a current, assuming a value as function of time. This value, to be correctly converted, is normalised within the full scale range of the ADC input (the voltage V_{FSR} in Fig. 3.1, in case of input voltage).

The digital output is discrete in time and amplitude. At certain instants of time, the digital output assumes a voltage within a discrete set of values, as function of the value of the analog input. We limit the description to the case of *uniform sampling*, in which the time interval between the sampling instants is constant and it is the *sampling period*, i.e. the inverse of the *sampling rate* defined as number of samples per unit of time.

The output voltage is encoded by digital numbers, composed of a fixed number of *digits* (fixed number generalised as B in Fig. 3.1). We narrow here the description only to binary digital converters, in which case the code is binary code and each digit is a *bit* that can assume only two values: a low voltage value correspondent to a logical "0", and a high voltage value correspondent to a logical "1".

A group of B bits assigned to each digital output is generally defined a *word*. In particular, a group of eight bits is called *byte*. The single bits inside a word are defined by their position in the sequence, going from the *least significant bit* at the right-end (LSB) to the *most significant bit* at the left-end (MSB).

Every output word can be communicated in parallel on B lines, all at the same time (as in Fig. 3.1), or serially in a time sequence on one or more lines, abiding to a determined protocol to distinguish each single word and the order of the bits in it.

Each possible sequence of B bits in a word represents a specific portion of the analog input range. Hence, the input range is divided in 2^B uniform portions. How each sequence is assigned to each input portion depends on the implemented encoding system and on the ADC input-output transfer function.

The application of BPM readout often requires the digitisation of analog signals that can be positive and negative; in which case the encoding system uses a bipolar code to assign a word to each interval of the bipolar input range. One of the most popular codes is the *twos complement*, particularly convenient because it facilitates the implementation of mathematical operations. Its first definition by John von Neumann can be found in section 8.1 of its 1945 proposal for an electronic digital computer [27]. Fig. 3.2 shows the transfer function for an ideal bipolar 3-bit ADC with twos complement encoding. The bipolar analog input range is divided into $2^3 = 8$ intervals and the graph shows which digital word is assigned to the input, following a staircase characteristic.

The size of these uniform intervals is the elementary quantisation step and nominal

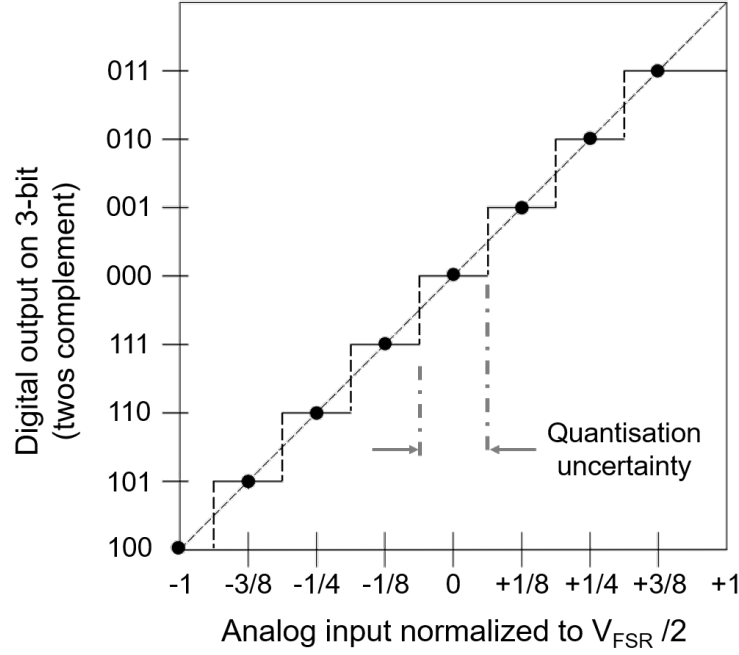


Figure 3.2: Transfer function for an ideal 3-bit ADC with twos complement digital output encoding.

resolution of the converter, equal to $V_{FSR} \cdot 2^{-B}$ and often referred to as LSB or Q. In an ideal transfer function, as the one shown in Fig. 3.2, the first transition takes place at $Q/2$ above the minimum value, and from there every Q. Hence, the maximum difference between an analog input and the value assigned to it is $Q/2$ and this error ranges between $-Q/2$ and $+Q/2$. It is called *quantisation error* and it is the source of *quantisation noise* (see Fig. 3.3).

The spectrum of quantisation noise was studied by W. R. Bennett [28], who also proposed to use the approximation of white noise, i.e. to assume same probability for all values of quantisation error, with good accuracy for most practical cases in which there is no correlation between the quantisation error and the digitised signal. With such approximation the quantisation error ϵ_q over one elementary quantisation step has zero mean value and varies uniformly between $-Q/2$ and $+Q/2$. Therefore, the error standard deviation σ_{ϵ_q} becomes:

$$\sigma_{\epsilon_q} = \sqrt{\int_{-Q/2}^{+Q/2} \epsilon_q^2 \frac{1}{Q} d\epsilon} = \frac{Q}{\sqrt{12}} = \frac{V_{FSR} \cdot 2^{-B}}{\sqrt{12}} \quad (3.1)$$

This expression is used to calculate the signal-to-noise ratio (SNR) of an ideal ADC with quantisation noise as only uncertainty source, when the input signal is a sine waveform having the input full scale range V_{FSR} as voltage peak-to-peak value. The calculated SNR in decibel results in the well-known linear formula as function of the number of bits B:

$$\text{SNR}_{dB} = 20 \log_{10} \left(\frac{V_{FSR}/(2\sqrt{2})}{V_{FSR} 2^{-B}/\sqrt{12}} \right) = 6.02 \cdot B + 1.76 \quad (3.2)$$

In a physical ADC there are additional uncertainty sources and flaws, impacting

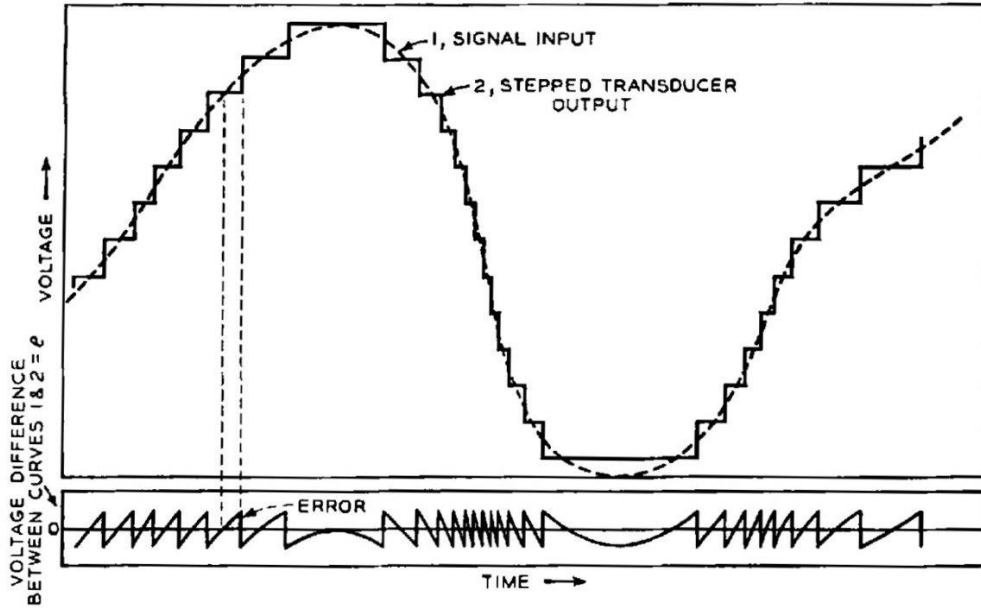


Figure 3.3: Representation of the quantised output and of the quantisation error with respect to an input analog signal, as function of time. The figure is from the paper of W. R. Bennett about quantisation noise [28].

the converter performance and defining its characteristics. Especially relevant for high speed applications are the dynamic characteristics of the ADC, measured by spectral analysis of the digital output.

A popular measurement is the *Signal to Noise and Distortion Ratio* (SINAD), defined as the ratio between the averaged power of the tonal input signal and the integrated power of all the other spectral components for frequencies above zero and up to half of the sampling rate. It is usually measured for tonal signals at different frequencies. It includes all components that constitute the overall noise and distortion, including quantisation noise, circuit noise (as thermal noise and capacitance noise) and aperture uncertainty (caused by the variation of the sampling interval).

The information given by the SINAD can also be expressed in the form of *Effective Number of Bits* (ENOB). Its definition comes from Eq. (3.2), and it expresses the number of bits of an equivalent ideal ADC whose quantisation noise has the same averaged power of the overall measured noise of the real ADC. Thus, the ADC's SINAD in decibel substitutes the SNR term in Eq. (3.2), obtaining:

$$\text{ENOB} = \frac{\text{SINAD}_{\text{dB}} - 1.76}{6.02} \quad (3.3)$$

Assuming that the total noise accounted for by the SINAD parameter can be characterised as Gaussian white noise, its standard deviation would be equal to the root mean square value of the quantisation noise of the ideal ADC with $B = \text{ENOB}$. Referring to Eq. (3.1) and replacing the number of bits B with the ENOB, the standard deviation of the overall Gaussian white-noise could be expressed as a function of the ADC full scale

range and resolution in ENOB.

$$\sigma_{\text{ENOB}} = \frac{V_{\text{FSR}}}{\sqrt{12}} 2^{-\text{ENOB}} \quad (3.4)$$

3.2 ADC state-of-the-art: sampling rate vs. resolution

There is a large variety of ADCs, differing for architecture and performance, covering the requirements of very different applications. Their performance and characteristics are described by a multitude of parameters, but there are three main quantities that are commonly compared: the ADC resolution (expressed for example by SINAD or ENOB), the maximum conversion rate F_s , and the dissipated power P_{diss} [29–31].

In his 1999 paper [29] Walden defined two figures of merit to compare the performances of different ADCs and analyse their evolution and trends. The first is the measurement of performance P , defined as the product of the effective number of quantisation levels times the maximum sampling rate:

$$P = 2^{\text{ENOB}} \cdot F_s \quad (3.5)$$

The second is the figure of merit F , that weights the efficiency with respect to the dissipated power.

$$F = \frac{2^{\text{ENOB}} \cdot F_s}{P_{\text{diss}}} \quad (3.6)$$

The inclusion of the power consumption is of great interest to analyse technology improvements, and meets the demand of those applications that highly value power consumption, e.g., portable applications. However, for high-performance instrumentation systems, like those developed for beam instrumentation, the dissipated power is not a primary requirement; thus, the figure of merit P , taking into account resolution and speed, better answers for our application needs.

In his work, Walden analysed the distribution of the resolution and speed characteristics of a group of commercial ADCs produced between 1978 and 1997. He observed how approximately one bit of resolution is lost for every doubling of the sampling rate, and that the aperture uncertainty is the main responsible mechanism for this limit. At the corners of the sampling rate range other factors further limit the performance: electronic noise limits the performance of slow high-resolution ADCs, whereas the maximum sampling rate achievable is restricted by the speed with which comparators are able to respond to small voltage variations. For most sampling rates the slope defined by a constant P describes quite accurately the envelope of the performance and its evolution with time.

In 2005 Bin Le et al. adjourned the survey and analysis work of Walden [30]. They observed that a constant P over a wide sampling rate range does not provide a faithful description of the population performance envelope. They registered an almost exponential growth of P starting in 1994, still not visible in Walden data. This improvement was due to the increased availability of fast converters with relatively high resolution, causing an unbalanced growth of P for the mid-high sampling rates.

A more recent survey from Keller, Murmann and Manoli [31] considers the data of converters until 2015. They confirmed the aperture uncertainty to be the main limiting

factor of the performance P . However, the performance P of the top designs has kept improving, indicatively doubling every four years, pushed by converters in the mid-high sampling rate range, that is in the order of the giga samples per second.

Murmann has been updating an online database [32], collecting the performances of the converters presented at the International Solid-State Circuit Conferences and at the Very Large-Scale Integration Symposia, starting from 1997 up to 2020.

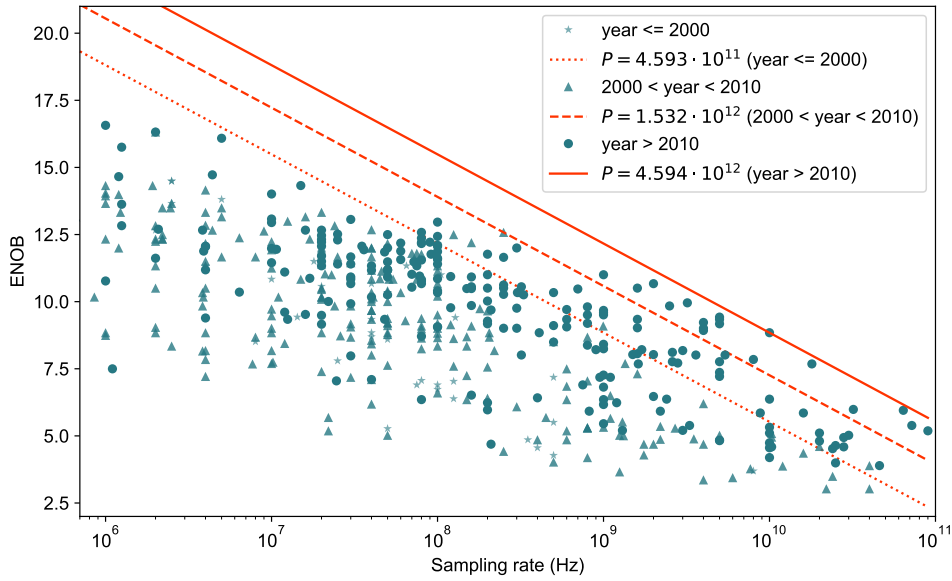


Figure 3.4: Resolution (ENOB) versus sampling rate in logarithmic scale for the ADCs presented between 1997 and 2020, as reported in Murmann’s database [32]. Only ADCs with sampling rate above 1 MHz are represented. The plot shows also the slopes defined by the best P before the year 2000, between 2000 and 2010, and between 2010 and 2020.

Fig. 3.4 shows the evolution of the ADCs’ performance. The ENOB is calculated as in Eq. (3.3), taking as input the SINAD values reported in the database [32] for input tones close to half of the maximum sampling rate. The ADCs are divided in three groups, as function of their year, whether they were presented before 2000, between 2000 and 2010, and between 2010 and 2020. For each of the three groups, the best P was found and the corresponding slopes are also plotted.

The evolution of P is evident in Fig. 3.4, with a gain of one order of magnitude in twenty years. The plot also shows how the improvement mainly regards high sampling rate converters, with frequency above one giga samples per second. In that range, the slope defined by the state-of-the-art P is an accurate description of the performance distribution envelope.

The trend in performances is promising for direct-digitisation based beam instrumentation applications. The bandwidth required to acquire directly the beam wideband pulses is increasingly more likely to be met without giving up too much on resolution.

However, the trade-off between sampling rate and resolution persists and it is interesting to understand, for a given pulse, where the optimum sampling point lies, along the state-of-the-art performance slope.

3.3 Sampling distortion

Several factors motivate the choice to save in sampling rate, e.g. the availability of higher resolution converters, the reduction of power consumption, and the simplification of acquisition and processing electronics.

Therefore, it is of high interest to define the lower boundary of the sampling rate for an input signal in order to fulfill a prefixed requirement.

Intuitively, the lower is the sampling rate, the more information is lost about the input signal. The loss of information is typically evaluated as the error of the reconstructed signal waveform based on the samples, with respect to the input signal. This error caused by sampling is referred to as *sampling distortion*.

Several works in the first half of the 20th century studied the relationship between the sampling rate and the distortion of the reconstructed waveform for bandlimited analog continuous signals, especially whether it exists a minimum sampling rate such that the input signal can be reconstructed without any distortion. Whittaker presented the first remarks from the mathematical point of view [33], and the problem applied to the communication field was solved on parallel by Kotelnikov [34] and by Nyquist and Shannon [35, 36]. Especially Shannon's publication [36] formalised in the communication field what is known as the Nyquist-Shannon sampling theorem. The theorem states that an analog signal whose Fourier spectrum is zero for frequencies higher than BW hertz, is fully described by its samples uniformly distanced by $(2BW)^{-1}$ seconds or less. Shannon named this time interval the *Nyquist interval* and the corresponding sampling rate is the *Nyquist rate*. In other words, a bandlimited signal sampled with infinite resolution and with rate equal to or higher its Nyquist rate, can be reconstructed from its sample with no distortion.

Shannon demonstrated the sampling theorem recurring to the definition of the Fourier transform of a continuous signal, and to the definition of the Fourier-series coefficients of a periodic signal. He observed that if a signal $x(t)$ has finite bandwidth BW , its Fourier spectrum $X(f)$ is different from zeros only at frequencies between $-BW$ and $+BW$, so the integral's extremes in the synthesis equation can be chosen accordingly (3.7). As a consequence, the signal's sample at the instant $n/(2BW)$ is also the n -th coefficient of the Fourier-series expansion of the signal spectrum $X(f)$, seen as a periodic signal with period $2BW$ (3.8).

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{j2\pi ft} df = \int_{-BW}^{+BW} X(f) e^{j2\pi ft} df \quad (3.7)$$

$$x\left(\frac{n}{2BW}\right) = \int_{-BW}^{+BW} X(f) e^{j2\pi f \frac{n}{2BW}} df \quad (3.8)$$

Hence, the samples of the time signal are enough to describe completely the analog signal's spectrum, thus the analog signal itself.

This conclusion can also be intuitively grasped looking at the spectrum of a sampled signal, when the sampled signal is modeled as the product between the continuous signal and a train of Dirac deltas distanced by the sampling period. The spectrum of the sampled sequence is the convolution of the input signal bandlimited spectrum with a comb of Dirac delta, and the result is a periodic spectrum composed of replicas of the bandlimited spectrum, spaced by the sampling frequency (see Fig. 3.5). If the sampling

rate is equal or higher than twice the signal bandwidth, there is no overlapping between the replicas in the spectrum. Ideally, low pass filtering the sampled signal is enough to reconstruct the input signal. If the sampling rate is lower than the Nyquist rate, there is overlapping between the replicas, also called *aliasing*, and it is source of distortion.

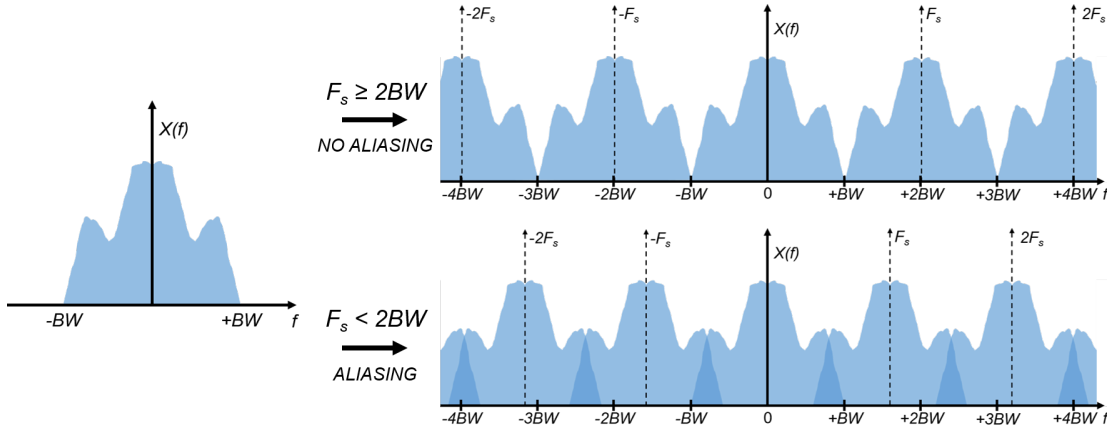


Figure 3.5: Schematic representation of an example of bandlimited Fourier spectrum $X(f)$, with bandwidth BW , in the frequency domain (left). On the right are represented how the replicas of the spectrum would combine in case of sampling rate equal to the Nyquist rate (top) and below the Nyquist rate (bottom).

One assumption of the Nyquist-Shannon sampling theorem is the infinite resolution of the sampler. As already discussed, there are several factors limiting the effective resolution of a physical analog-to-digital converter. Therefore, it is generally impossible to guarantee no distortion even sampling at frequencies above the Nyquist rate.

A linked problem is the one of specifying the minimum rate required to represent a source in presence of noise with a predefined maximum distortion; it is a classic subject of study in Information Theory, under the category of *rate-distortion theory*.

The first introduction and developments were given in 1948 by Shannon, who provided an expression for the minimum rate (as bits per second) to encode a bandlimited stochastic Gaussian source, guaranteeing a minimum target distortion [37]. An historical review of the results achieved in this field until 1999 can be found in this work [38]. Most of the literature analyses the source encoding problem for different kinds of stochastic sources, finding the minimum rate with the optimal encoding scheme. Regarding the impact of sampling distortion, there are studies quantifying the minimum mean squared error of the reconstructed waveform in case of sampling below the Nyquist rate and in presence of noise for a stationary process [39], for a Gaussian stationary process [40] and for a periodic Gaussian process [41]. In particular Kipnis et al. [40] added the dependence on the sampling rate and showed that the minimum distortion could be achieved with sampling rates lower than the Nyquist rate.

It is important to mention that the encoding scheme is considered in a very general way as an optimised linear function mapping the input signal in the digital space. The reference is not the classical ADC implementation, where sampling rate and samples resolution are both specified, and where the encoding scheme is usually scalar quantisation. In more recent works, Kipnis et al. [42, 43] merged the source coding and the sampling problem, considering that each sample of a stationary stochastic process

is encoded by scalar quantisation and the bitrate is limited, introducing a trade-off between sampling rate and bit-precision. They demonstrated that under this condition the optimal rate is for most cases below the Nyquist rate, since "some distortion due to sampling is preferred in order to increase the quantiser resolution".

These results encourage the research of a similar optimal solution in the more applied framework of an instrumentation system, with fundamental differences.

First, in the case of a Beam Position Monitor the distortion of the reconstructed bunch signal would not be of primary interest, the quantity to be retrieved with minimum error is the signal amplitude or its averaged power measurement.

Second, the input signal is not a stationary process, but a deterministic pulse signal; therefore, the relationship between the phase of the signal and the phase of the sampling clock is not to be neglected.

Finally, the trade-off linking sampling rate and bit resolution is not the one given by a fixed bit-rate; sampling rate and resolution would be dictated by the actual characteristics of commercial ADCs, or, for general observations, their trade-off could be modeled as function of the state-of-the-art measurement of performance P .

CHAPTER 4

Analysis of sampling effects on the average power measurement of direct digitally acquired pulse signals

In a beam position monitor the quantity to be calculated about each electrode beam bunch pulse typically is its amplitude. The reference algorithm for our study is the one called *Standard Deviation Method* (STD) in the article of A. Reiter and R. Singh [23], article that summarises several processing options for directly digitising BPM systems. This algorithm has the advantage of minimising measurement uncertainty while being less sensitive than others to differences in the sampling time of the signals from two electrodes. In this algorithm the amplitude information of each digitised waveform is calculated as the square root of the mean value of the square of the samples, where the mean value of the sequence is subtracted to each sample. For the pickups considered in this thesis, the beam induced signals do not have a continuous component, their mean value is zero. The resulting chosen algorithm is therefore a simplified version of the STD method, where the signal mean value is zero, and we call it *Root Mean Square Method* (RMS).

The RMS value is the square root of the average power of the sequence over its window of acquisition and it can be related to the average power and the energy of the continuous pulse, before digitisation.

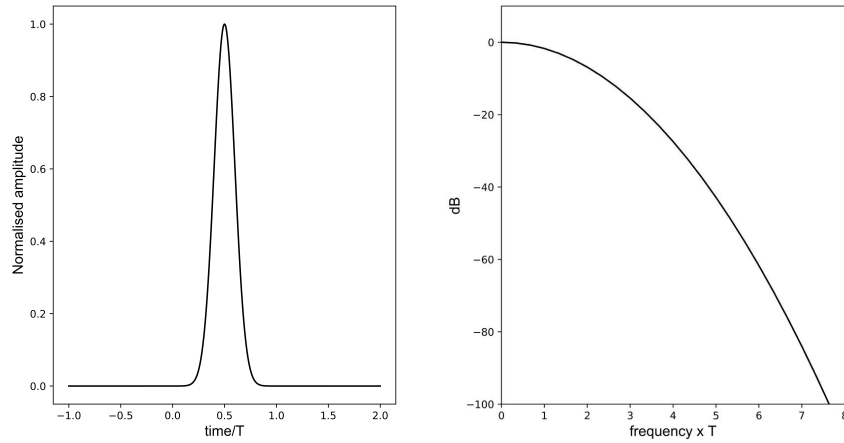
This chapter describes analytically the relationship between these two quantities: the average power of the continuous pulse and the average power of the sample sequence. In the first part it quantifies the error introduced by a limited and asynchronous sampling rate, when the converter has infinite resolution. In the second part, it assumes a converter with limited resolution. Finally the results are combined together in an analytical description of the SNR of the average power measurement. This analysis is

applied to some example pulses: a Gaussian pulse, the derivative of a Gaussian pulse, and the stripline pickup response to a single bunch.

4.1 Problem Definition

We define $x(t)$ as an analog, deterministic *pulse*, i.e. a signal with a rapid transient change in its amplitude from a baseline value to a higher or lower value, followed by a rapid return to the baseline in a limited time (see Fig. 4.1a). We consider the baseline value to be zero and we define $X(f)$ the continuous spectrum calculated by Fourier transform (see Fig. 4.1b):

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-j2\pi ft} dt \quad (4.1)$$



(a) Gaussian pulse signal $x(t)$ in the time domain. (b) Amplitude of the spectrum $X(f)$ of the pulse signal $x(t)$.

Figure 4.1: Example of a pulse signal in the time and in the frequency domain.

The signal's energy is the quantity E , and it can be calculated both in the time and in the frequency domain, as stated by the Parseval's theorem for the Fourier transform.

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{+\infty} |X(f)|^2 df \quad (4.2)$$

For time and energy limited signals, like those typical of BPM systems, the integral (4.2) converges to a finite value, and a window T can be defined such that outside this window $x(t) = 0$.

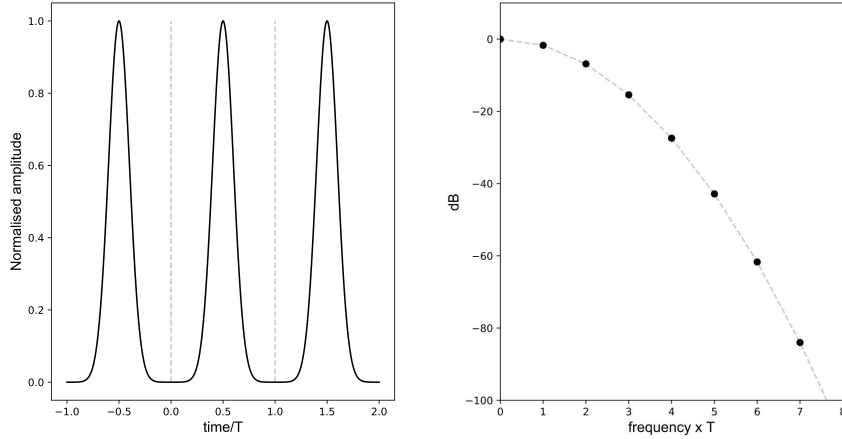
Those signals can be made periodic by adding replicas of $x(t)$ spaced by nT (see Fig. 4.2a), without altering the signal amplitude within the window T :

$$x_T(t) = \sum_{n=-\infty}^{+\infty} x(t - nT) \quad (4.3)$$

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The spectrum of $x_T(t)$ is discrete and defined by its Fourier coefficients X_k , proportional to the spectrum $X(f)$ sampled at multiples of $1/T$ (see Fig. 4.2b).

$$X_k = \frac{1}{T} \int_0^T x_T(t) e^{-j2\pi t \frac{k}{T}} dt = \frac{1}{T} X\left(\frac{k}{T}\right) \quad (4.4)$$



(a) Periodic pulse signal $x_T(t)$ in the time domain. (b) Amplitude of the Fourier coefficients X_k of the pulse signal $x_T(t)$.

Figure 4.2: Example of a periodic pulse signal in the time and in the frequency domain

The average power P_T of the periodic signal $x_T(t)$ over the period T is defined as follows:

$$P_T = \frac{1}{T} \int_0^T |x_T(t)|^2 dt = \frac{1}{T} \int_0^T |x(t)|^2 dt \quad (4.5)$$

Given the Parseval's theorem for the Fourier transform of periodic signals, the average power can also be expressed as function of the Fourier coefficients:

$$P_T = \sum_{k=-\infty}^{+\infty} |X_k|^2 \quad (4.6)$$

Since $x(t)$ only has significant power within the window T , the energy E , Eq. (4.2), and the average power P_T , Eq. (4.5), are proportional:

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_0^T |x(t)|^2 dt = T \cdot P_T \quad (4.7)$$

We have so defined the quantity E , that is a function only of the continuous time limited signal and can be related to its average power over an acquisition window of length T (4.5) or to the sum of the squared samples of its spectrum (4.6).

Since the power P_T is a limited quantity, the series $|X_k|^2$ (4.6) must converge. We can assume our pulse signal to be effectively bandlimited, with Fourier coefficients vanishing above the harmonic N_0 :

$$|X_i|^2 = 0 \quad \forall i : |i| > N_0 \quad (4.8)$$

which allows to rewrite (4.6) as:

$$P_T = \sum_{k=-N_0}^{+N_0} |X_k|^2 \quad (4.9)$$

The signal $x_T(t)$ is sampled with a sampling clock of rate F_s , resulting in a sampling period $t_s = 1/F_s$, and the phase of the sampling clock with respect to the signal is unknown, expressed by the random variable τ . The result is the sequence $\hat{x}_{n,\tau}$, with N samples over the observation period T :

$$\hat{x}_{n,\tau} = x_T(nt_s - \tau), \quad n = 0, 1, \dots, N-1 \quad (4.10)$$

Assuming the sampling performed by an analog to digital converter, the limited resolution of such converter can be modelled by the random variable ν_n to be added to the samples (see Fig. 4.3).

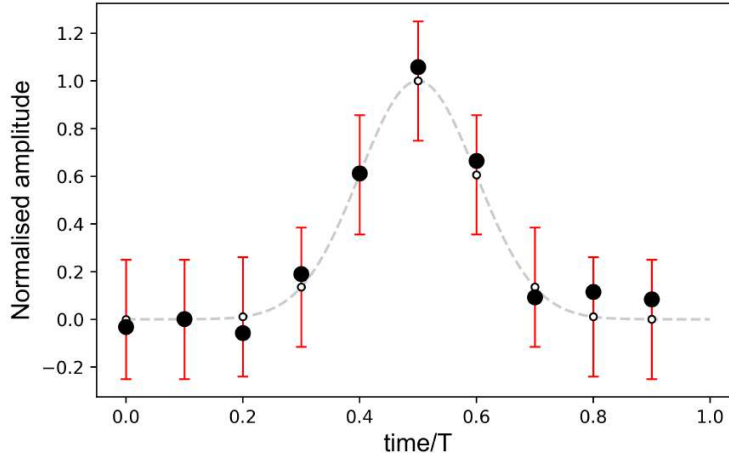


Figure 4.3: Sampled sequence, result of the digitisation of the example pulse $x_T(t)$ over the window T with limited sampling rate and limited resolution. The noise of each sample is described by the errorbars, with size equal to the standard deviation of the sample noise ν_n .

The average power of the sampled signal sequence over a period T is:

$$\bar{P}_T = \frac{1}{N} \sum_{n=0}^{N-1} (\hat{x}_{n,\tau} + \nu_n)^2 \quad (4.11)$$

In order to establish quantitatively the quality of $\bar{P}_T$ as estimator of P_T , the following questions are addressed respectively in sections 4.2 and 4.3:

1. What is the effect of the sampling assuming a limited rate and asynchronous clock?
2. What are the mean value and the variance of the error of the estimated power if the digitisation is performed with limited resolution?

4.2 Error due to a limited sampling rate

In the first part of this analysis we assume a digitisation with infinite resolution, and for ease of notation the sampling delay τ is omitted. For a certain sampling rate F_s , the acquisition window length T is set to be an integer multiple of the sampling period $t_s = 1/F_s$ that verifies the condition $x(t) = 0 \forall t > T$.

The discrete sequence $\hat{x}_n$ is the result of the sampling process, and $N = F_s T$ gives the integer number of samples within the window T . The associated Discrete Fourier Transform (DFT) coefficients are the sequence $\hat{X}_k$ of N complex coefficients, so defined:

$$\hat{X}_k = \frac{1}{N} \sum_{n=0}^{N-1} \hat{x}_n e^{-j2\pi k \frac{n}{N}} \quad (4.12)$$

The average power of the sampled signal sequence $\hat{x}_n$ of period T is then given as:

$$\hat{P}_T = \frac{1}{N} \sum_{n=0}^{N-1} \hat{x}_n^2 \quad (4.13)$$

Applying the Parseval's theorem for the Discrete Fourier Transform, the power estimator $\hat{P}_T$ can also be expressed as a function of the DFT coefficients $\hat{X}_k$ of the sampled sequence:

$$\hat{P}_T = \sum_{k=0}^{N-1} |\hat{X}_k|^2 \quad (4.14)$$

Since $\hat{x}_n$ is the result of the sampling of $x_T(t)$, its DFT coefficients $\hat{X}_k$ can be expressed as function of the Fourier coefficients X_k of the continuous periodic signal. As described in section 3.3, the spectrum of a sampled signal is the result of the sum of an infinite series of shifted replicas of the original spectrum (see Fig. 4.4). Hence, the coefficients $\hat{X}_k$ are composed of a series of overlapping replicas of the spectrum coefficients X_k , shifted by integer multiples of the sampling frequency F_s , in the form of the coefficient order N .

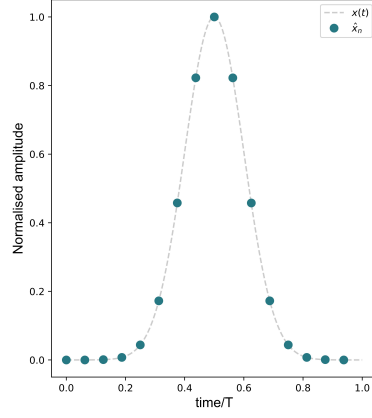
$$\hat{X}_k = \sum_{m=-\infty}^{+\infty} X_{k+mN}, \quad k = 0, 1, \dots, N-1 \quad (4.15)$$

With the assumption of a bandlimited signal (4.8), Nyquist-Shannon theorem [36] states that if the sampling rate is higher than twice the signal bandwidth, the original continuous signal can be reconstructed from its samples without loss of information. With our notation, this condition is translated into $F_s > 2N_0/T$, i.e. $N > 2N_0$. Such minimum sampling rate is the Nyquist rate (R_{Nyq}), above which the replicas composing the spectrum (see Eq. (4.15)) do not have any significant overlap causing distortion (see Fig. 4.4).

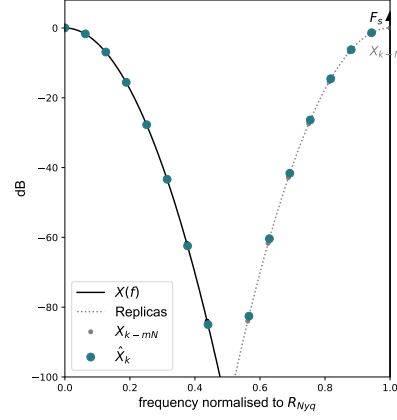
If the Nyquist-Shannon criterion is respected ($F_s > R_{Nyq}$, i.e. $N > 2N_0$) the DFT coefficients $\hat{X}_k$ expressed as in Eq. (4.15) can be simplified to:

$$\hat{X}_k = \begin{cases} X_k, & \text{if } k = 0, 1, \dots, \lfloor N/2 \rfloor \\ X_{k-N}, & \text{if } k = \lfloor N/2 \rfloor + 1, \dots, N-1 \end{cases}$$

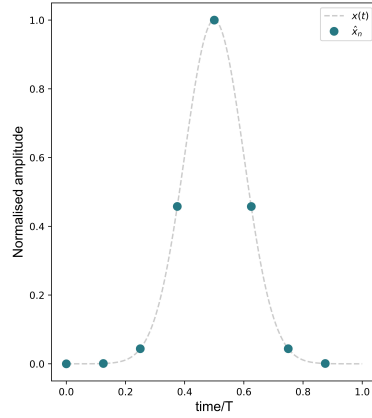
4.2. Error due to a limited sampling rate



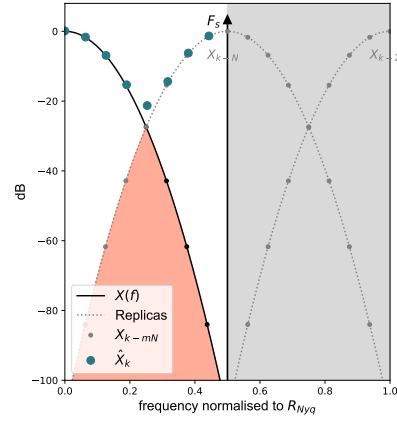
(a) Pulse sampled with $F_s = R_{Nyq}$.



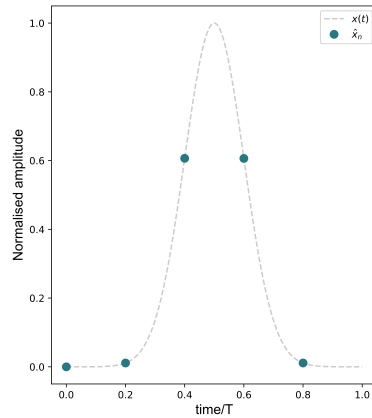
(b) DFT coefficients for $F_s = R_{Nyq}$.



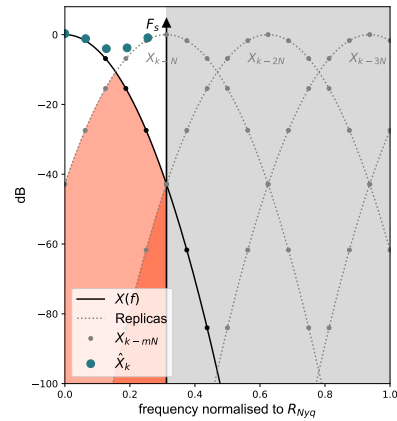
(c) Pulse sampled with $F_s = 0.5R_{Nyq}$.



(d) DFT coefficients for $F_s = 0.5R_{Nyq}$.



(e) Pulse sampled with $F_s = 0.3125R_{Nyq}$.



(f) DFT coefficients for $F_s = 0.3125R_{Nyq}$.

Figure 4.4: Aliasing of a Gaussian-shape pulse sampled with different sampling rates.

Chapter 4. Analysis of sampling effects on the average power measurement of direct digitally acquired pulse signals

Using the average power expression in the frequency domain (4.14) it follows that, under this condition and given Eq. (4.9), $\hat{P}_T$ is a perfect estimator of P_T :

$$\hat{P}_T = \sum_{k=0}^{N-1} |\hat{X}_k|^2 = \sum_{l=-\lfloor N/2 \rfloor}^{\lfloor N/2 \rfloor} |X_l|^2 = \sum_{l=-N_0}^{N_0} |X_l|^2 = P_T \quad (4.16)$$

where $l = k - \lfloor N/2 \rfloor$.

We can not say the same when the Nyquist-Shannon criterion is not respected by the chosen sampling rate ($F_s < R_{Nyq}$). An interesting case is when the sampling rate is below the Nyquist rate, but still above half of the Nyquist rate, i.e. $R_{Nyq}/2 < F_s < R_{Nyq}$. Given this condition only the first replica of the spectrum produces a significant overlap, and the coefficients $\hat{X}_k$ for $k = 0, 1, \dots, N-1$ can be expressed as:

$$\hat{X}_k = X_{k-N} + X_k \quad (4.17)$$

To calculate the resulting average power $\hat{P}_T$, we first write the square of the absolute value of the coefficients $\hat{X}_k$ for $k = 0, 1, \dots, N-1$, as a function of the Fourier coefficients $X_k = |X_k|e^{j\phi_k}$, indicated in their modulus and phase:

$$|\hat{X}_k|^2 = |X_k|^2 + |X_{k-N}|^2 + 2|X_k||X_{k-N}| \cdot \cos(\phi_k - \phi_{k-N}) \quad (4.18)$$

The average power of the sequence, $\hat{P}_T$, follows:

$$\hat{P}_T = \sum_{k=-N}^{N-1} |X_k|^2 + \sum_{k=0}^{N-1} 2|X_k||X_{k-N}| \cdot \cos(\phi_k - \phi_{k-N}) \quad (4.19)$$

Given the condition $F_s > R_{Nyq}/2$, so $N > N_0$, the first term of the sum is equivalent to the average power P_T of the continuous signal (see Eq. (4.9)), and therefore the second term expresses the error of the estimator $\hat{P}_T$ due to undersampling. It is possible to see that the error depends not only on the modulus of the spectrum, but also on its phase. We therefore expect the sampling phase variation to have an impact.

Consequently, we now take in consideration the phase relation between the sampling clock and the signal to sample, expressed by the time delay variable τ as defined in (4.10). The delay term appears also in the DFT coefficients of the sampled sequence, and here we look always at the case of sampling rate in the range $R_{Nyq}/2 < F_s < R_{Nyq}$:

$$\hat{X}_{k,\tau} = X_{k-N} \cdot e^{-j2\pi(k-N)\frac{\tau}{T}} + X_k \cdot e^{-j2\pi k\frac{\tau}{T}}, \quad k = 0, 1, \dots, N-1 \quad (4.20)$$

The square of the absolute value of the coefficients follows as:

$$|\hat{X}_{k,\tau}|^2 = |X_k|^2 + |X_{k-N}|^2 + 2|X_k||X_{k-N}| \cdot \cos\left(\phi_k - \phi_{k-N} - 2\pi N\frac{\tau}{T}\right) \quad (4.21)$$

The power estimator can be written including the dependency of the error due to the sampling delay τ and the equation (4.9):

$$\hat{P}_{T,\tau} = P_T + 2 \sum_{k=0}^{N-1} |X_k||X_{k-N}| \cdot \cos(\phi_k - \phi_{k-N} - 2\pi F_s \tau) \quad (4.22)$$

The average power measurement error appears explicitly and we define it as:

$$\epsilon_{P_{T,\tau}} \doteq \hat{P}_{T,\tau} - P_T = 2 \sum_{k=0}^{N-1} |X_k| |X_{k-N}| \cdot \cos(\phi_k - \phi_{k-N} - 2\pi F_s \tau) \quad (4.23)$$

Remembering that $\cos(a - b) = \cos a \cos b + \sin a \sin b$, the cosine term in the sum can be split:

$$\cos(\phi_k - \phi_{k-N} - 2\pi F_s \tau) = \cos(\phi_k - \phi_{k-N}) \cos(2\pi F_s \tau) + \sin(\phi_k - \phi_{k-N}) \sin(2\pi F_s \tau)$$

The average power measurement error term $\epsilon_{P_{T,\tau}}$, caused by sampling distortion, can then be rewritten as:

$$\begin{aligned} \epsilon_{P_{T,\tau}} = & 2 \cos(2\pi F_s \tau) \sum_{k=0}^{N-1} |X_k| |X_{k-N}| \cdot \cos(\phi_k - \phi_{k-N}) \\ & + 2 \sin(2\pi F_s \tau) \sum_{k=0}^{N-1} |X_k| |X_{k-N}| \cdot \sin(\phi_k - \phi_{k-N}) \end{aligned} \quad (4.24)$$

The arguments of the two series are now only function of the signal spectrum and of the sampling frequency, while the delay τ is out of the series. To get a more synthetic expression, the following coefficients are defined:

$$\begin{aligned} A_{X_k,N} & \doteq \sum_{k=0}^{N-1} |X_k| |X_{k-N}| \cdot \cos(\phi_k - \phi_{k-N}) \\ B_{X_k,N} & \doteq \sum_{k=0}^{N-1} |X_k| |X_{k-N}| \cdot \sin(\phi_k - \phi_{k-N}) \end{aligned} \quad (4.25)$$

and the error is finally written as:

$$\epsilon_{P_{T,\tau}} = A_{X_k,N} \cdot 2 \cos(2\pi F_s \tau) + B_{X_k,N} \cdot 2 \sin(2\pi F_s \tau) \quad (4.26)$$

The error appears to be the combination of two harmonics at the sampling rate frequency, with $\pi/2$ phase difference, modulated by the coefficients $A_{X_k,N}$ and $B_{X_k,N}$, which are functions of the input signal spectrum and of the sampling rate. To better understand the impact of the sampling delay, $\epsilon_{P_{T,\tau}}$ can also be seen as twice the projection of a phasor, whose real and imaginary part are respectively $A_{X_k,N}$ and $-B_{X_k,N}$, and that rotates with the change of the sampling delay (see Fig. 4.5):

$$\epsilon_{P_{T,\tau}} = 2 \Re \left((A_{X_k,N} - j B_{X_k,N}) e^{j 2\pi F_s \tau} \right) \quad (4.27)$$

As the sampling delay changes, the error grows and decreases, changing sign and keeping its absolute value below a maximum value defined by the coefficients $A_{X_k,N}$ and $B_{X_k,N}$. The variation of the sampling delay τ can be modeled with a random variable, described by a probability density function. If there is no form of synchronisation between the sampling clock and the pulse signal, we can assume the variable τ to be a continuous uniform random variable, defined in the interval between zero and one sampling period $t_s = T/N$, i.e. $\tau = \mathcal{U}[0, t_s]$. The sampling delay τ probability density function is:

$$f_\tau(t) = \begin{cases} \frac{1}{t_s} & 0 \leq t \leq t_s \\ 0 & \text{otherwise} \end{cases} \quad (4.28)$$

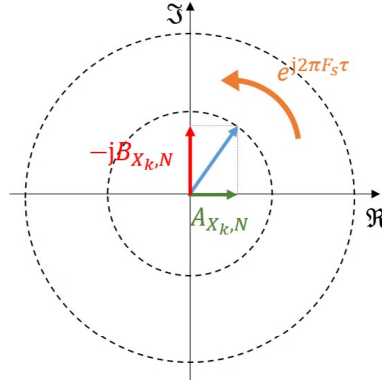


Figure 4.5: Complex phasor showing the power measurement error caused by sampling a pulse signal with a certain delay τ [44].

Being the error $\epsilon_{P_{T,\tau}}$ an analytical function of a random variable, it is possible to calculate its statistical properties, as the mean value, the variance and its own probability density function.

Starting from Eq. (4.26) we obtain its first and second moment, respectively $\mu_{1,\epsilon}$ (i.e. the mean value) and $\mu_{2,\epsilon}$.

$$\begin{aligned}
 \mu_{1,\epsilon} &= E\{\epsilon_{P_{T,\tau}}\} \\
 &= \int_{-\infty}^{+\infty} (A_{X_k,N} \cdot 2 \cos(2\pi F_s t) + B_{X_k,N} \cdot 2 \sin(2\pi F_s t)) \cdot f_\tau(t) dt \\
 &= 2A_{X_k,N} \cdot \int_0^{1/F_s} \cos(2\pi F_s t) \cdot F_s dt + 2B_{X_k,N} \cdot \int_0^{1/F_s} \sin(2\pi F_s t) \cdot F_s dt \\
 &= 0 + 0 \\
 &= 0
 \end{aligned} \tag{4.29}$$

$$\begin{aligned}
 \mu_{2,\epsilon} &= E\{\epsilon_{P_{T,\tau}}^2\} \\
 &= E\{(A_{X_k,N} \cdot 2 \cos(2\pi F_s t) + B_{X_k,N} \cdot 2 \sin(2\pi F_s t))^2\} \\
 &= E\{4A_{X_k,N}^2 \cdot \cos^2(2\pi F_s t)\} + E\{4B_{X_k,N}^2 \cdot \sin^2(2\pi F_s t)\} \\
 &\quad + E\{8A_{X_k,N}B_{X_k,N} \cos(2\pi F_s t) \sin(2\pi F_s t)\} \\
 &= E\{2A_{X_k,N}^2 \cdot (1 + \cos(4\pi F_s t))\} + E\{2B_{X_k,N}^2 \cdot (1 - \cos(4\pi F_s t))\} \\
 &\quad + E\{4A_{X_k,N}B_{X_k,N} \sin(4\pi F_s t)\} \\
 &= 2A_{X_k,N}^2 + 0 + 2B_{X_k,N}^2 + 0 + 0 \\
 &= 2(A_{X_k,N}^2 + B_{X_k,N}^2)
 \end{aligned} \tag{4.30}$$

The mean value μ_ϵ of the error, equal to the first moment $\mu_{1,\epsilon}$ (4.29), is found to be zero (4.31), so $\hat{P}_T$ is an unbiased estimator of P_T for a noiseless signal and a uniformly distributed sampling phase.

The variance σ_ϵ^2 is the difference between the second moment and the square of the first moment; the latter being zero, it is equal to the second moment $\mu_{2,\epsilon}$ (4.32); it is defined by the coefficients $A_{X_k,N}$ and $B_{X_k,N}$ and depends on both the signal spectrum

and the sampling rate.

$$\mu_\epsilon = 0 \quad (4.31)$$

$$\sigma_\epsilon^2 = 2 (A_{X_k,N}^2 + B_{X_k,N}^2) \quad (4.32)$$

The error probability density function provides further indications about the error behaviour, and, since $\epsilon_{P_T,\tau}$ is a transformation of τ and the fundamental theorem of random variable transformation can be applied, can be expressed as:

$$f_\epsilon(\epsilon) = \sum_i \frac{f_\tau(t_i)}{|\epsilon'(t_i)|} \quad (4.33)$$

with t_i being the set of the solutions to the equation $\epsilon(t) = \epsilon$:

$$A_{X_k,N} \cdot 2 \cos(2\pi F_s t) + B_{X_k,N} \cdot 2 \sin(2\pi F_s t) = \epsilon \quad (4.34)$$

The equation (4.34) has two solutions:

$$t_{1,2} = \frac{1}{F_s \pi} \arctan \left(\frac{4B_{X_k,N} \pm \sqrt{16A_{X_k,N}^2 + 16B_{X_k,N}^2 - 4\epsilon^2}}{2(2A_{X_k,N} + \epsilon)} \right) \quad (4.35)$$

Combining the power estimation error expression (4.26), the solutions described in the expression (4.35) and the probability density function of the uniformly distributed sampling delay (4.28) with the probability density function transformation formula (4.33), the result is the probability density function for the power estimation error (4.36), as a function of the signal, the sampling frequency and the sampling delay.

$$\begin{aligned} f_\epsilon(\epsilon) &= d(p_1(\epsilon))^{-1} + d(p_2(\epsilon))^{-1} \\ d(p) &\doteq |4\pi (B_{X_k,N} \cos(2 \arctan(p)) - A_{X_k,N} \sin(2 \arctan(p)))| \\ p_{1,2} &\doteq \frac{4B_{X_k,N} \pm \sqrt{16A_{X_k,N}^2 + 16B_{X_k,N}^2 - 4\epsilon^2}}{2(2A_{X_k,N} + \epsilon)} \end{aligned} \quad (4.36)$$

See Fig. 4.6 for the probability density function of the example Gaussian pulse, calculated following expression (4.36) and simulated numerically. It is of particular interest that the error probability is higher for large errors, within the limits given by the range of possible error values. This distribution must be taken into account when designing systems using asynchronously sampled data and the Nyquist criterion is not respected, especially in the case where the measurement is used for threshold based decisions. The standard deviation of the distribution is not a significant parameter for the area around the mean value where most of the outcome results fall, the maximum error value is more relevant.

4.3 Error due to a limited resolution

The analysis to this point assumed the sampler to have infinite resolution, but in real analog-to-digital converters each sample is affected by multiple non idealities, as the limited number of quantisation levels and the electronic noise. In this work a comprehensive additive white noise variable has been used to introduce those effects, defined by a zero-mean Gaussian variable ν_i , with variance σ_ν^2 .

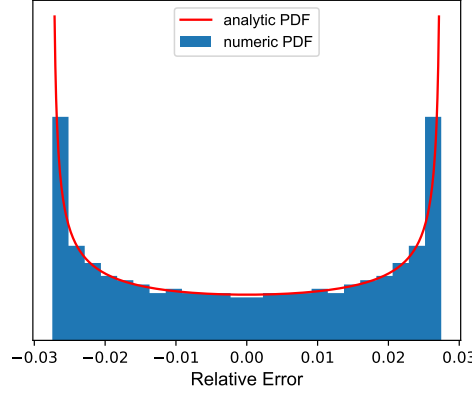


Figure 4.6: Probability Density Function (PDF) of the relative error of the average power measurement of a Gaussian pulse signal, when sampled at half of its Nyquist rate (analytical and numerical result) [44].

The additive white Gaussian noise on each sample is propagated in the average power estimator $\bar{P}_T$, when calculating the mean value of the squared samples of the sequence:

$$\bar{P}_T = \frac{1}{N} \sum_{n=0}^{N-1} (\hat{x}_n + \nu_n)^2 = \hat{P}_T + \frac{1}{N} \sum_{n=0}^{N-1} \nu_n^2 + \frac{1}{N} \sum_{n=0}^{N-1} 2\hat{x}_n \nu_n \quad (4.37)$$

The average power, comprehensive of noise, results to be equal to the average power affected by limited sampling rate with infinite resolution plus a new error term, defined as:

$$\eta(P_T, \sigma_\nu^2, N) \doteq \bar{P}_T - \hat{P}_T = \frac{1}{N} \sum_{n=0}^{N-1} \nu_n^2 + \frac{1}{N} \sum_{n=0}^{N-1} 2\hat{x}_n \nu_n \quad (4.38)$$

It should be noted that the error η appears to be a function of the sampled sequence $\hat{x}_n$, that is not deterministic being the sampling delay a random variable.

The mean value and the variance of the error η can be calculated. Here are listed the hypotheses on the signals:

1. The noise variable ν_i is a zero-mean Gaussian variable;
2. The observations of the noise variable ν_i are independent and identically distributed;
3. The signal samples $\hat{x}_i$ and the noise variable ν_i are independent.

Since the ADC noise variable includes the quantisation noise that depends on the signal, the third hypothesis is not always fully respected, but it is a valid approximation in most practical cases, when the signal amplitude is significantly higher than the LSB of the converter. The properties of the expectation ($E[\cdot]$), variance ($\text{Var}[\cdot]$) and covariance ($\text{Cov}[\cdot]$) operators that are used in the passages below, are the following:

1. If X and Y are independent random variables, the expectation of their product is the product of their expectations:

$$E[XY] = E[X] E[Y] \quad (4.39)$$

2. If X and Y are independent random variables, the variance of their product is:

$$\text{Var}[XY] = (\text{Var}[X] + \text{E}[X]^2) (\text{Var}[Y] + \text{E}[Y]^2) - \text{E}[X]^2 \text{E}[Y]^2 \quad (4.40)$$

3. The covariance of two random variables X and Y can be expressed as:

$$\text{Cov}[X, Y] = \text{E}[XY] - \text{E}[X] \text{E}[Y] \quad (4.41)$$

4. The variance of the sum of random variables X_i can be expressed as:

$$\text{Var}\left[\sum_{i=1}^N X_i\right] = \sum_{i=1}^N \text{Var}[X_i] + 2 \sum_{1 \leq i < j \leq N} \text{Cov}[X_i, X_j] \quad (4.42)$$

Making use of the linearity of the operator, the expectation of the error η , i.e. its mean value, is manipulated as follows, resulting in the sum of two terms:

$$\mu_\eta = \text{E}\left[\frac{1}{N} \sum_{n=0}^{N-1} \nu_n^2 + \frac{1}{N} \sum_{n=0}^{N-1} 2\hat{x}_n \nu_n\right] = \frac{1}{N} \sum_{n=0}^{N-1} \text{E}[\nu_n^2] + \frac{1}{N} \sum_{n=0}^{N-1} 2\text{E}[\hat{x}_n \nu_n] \quad (4.43)$$

Since the ADC noise variable ν_i is zero-mean (Hyp. 1), the expectation of its squared value is the noise variance σ_ν^2 . Then, being the sampled signal sequence $\hat{x}_i$ and the noise ν_i independent (Hyp. 3), the expectation of their product is the product of their expectations (property (4.39)), and the noise expectation is zero (Hyp. 1). So the mean value is found to be:

$$\mu_\eta = \sigma_\nu^2 \quad (4.44)$$

Also the variance σ_η^2 can be calculated exploiting the hypotheses and the operators' properties. In a first passage, applying property (4.42), we obtain:

$$\begin{aligned} \sigma_\eta^2 &= \text{Var}\left[\frac{1}{N} \sum_{n=0}^{N-1} (\nu_n^2 + 2\hat{x}_n \nu_n)\right] \\ &= \frac{1}{N^2} \sum_{n=0}^{N-1} \text{Var}[\nu_n^2 + 2\hat{x}_n \nu_n] \\ &\quad + \frac{2}{N^2} \sum_{0 \leq n < m \leq N-1} \text{Cov}[\nu_n^2 + 2\hat{x}_n \nu_n, \nu_m^2 + 2\hat{x}_m \nu_m] \end{aligned} \quad (4.45)$$

Each term of the sum in Eq. (4.45) can be treated separately to be simplified.

The argument of the first series in the sum, applying again variance property (4.42), becomes:

$$\text{Var}[\nu_n^2 + 2\hat{x}_n \nu_n] = \text{Var}[\nu_n^2] + \text{Var}[2\hat{x}_n \nu_n] + \text{Cov}[\nu_n^2, 2\hat{x}_n \nu_n] \quad (4.46)$$

The covariance term in (4.46) results to be 0, applying property (4.41) and under the hypothesis 1 on the noise variable (the odd moments of a zero-mean Gaussian variable are 0) and hypothesis 2 on the independence between noise and signal, so that property

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(4.39) can be applied:

$$\begin{aligned}
 \text{Cov} [\nu_n^2, 2\hat{x}_n\nu_n] &= \text{E} [2\hat{x}_n\nu_n^3] - \text{E} [\nu_n^2] \text{E} [2\hat{x}_n\nu_n] \\
 &= \text{E} [2\hat{x}_n] \text{E} [\nu_n^3] - 2\sigma_\nu^2 \text{E} [\hat{x}_n] \text{E} [\nu_n] \\
 &= 0 - 0 \\
 &= 0
 \end{aligned} \tag{4.47}$$

The argument of the second series in (4.45) can be simplified applying first property (4.41):

$$\begin{aligned}
 &\text{Cov} [\nu_n^2 + 2\hat{x}_n\nu_n, \nu_m^2 + 2\hat{x}_m\nu_m] = \\
 &= \text{E} [\nu_n^2\nu_m^2 + 2\hat{x}_n\nu_n\nu_m^2 + 2\hat{x}_m\nu_m\nu_n^2 + 4\hat{x}_n\nu_n\hat{x}_m\nu_m] \\
 &\quad - \text{E} [\nu_n^2 + 2\hat{x}_n\nu_n] \text{E} [\nu_m^2 + 2\hat{x}_m\nu_m]
 \end{aligned} \tag{4.48}$$

Then property (4.39) and the linearity of the expectation operator can be also applied, remembering hypotheses 2 and 3:

$$\begin{aligned}
 &\text{Cov} [\nu_n^2 + 2\hat{x}_n\nu_n, \nu_m^2 + 2\hat{x}_m\nu_m] = \\
 &= \text{E} [\nu_n^2] \text{E} [\nu_m^2] + \text{E} [2\hat{x}_n] \text{E} [\nu_n] \text{E} [\nu_m^2] \\
 &\quad + \text{E} [2\hat{x}_m] \text{E} [\nu_m] \text{E} [\nu_n^2] + \text{E} [4\hat{x}_n\hat{x}_m] \text{E} [\nu_n] \text{E} [\nu_m] \\
 &\quad - (\text{E} [\nu_n^2] + \text{E} [2\hat{x}_n] \text{E} [\nu_n]) (\text{E} [\nu_m^2] + \text{E} [2\hat{x}_m] \text{E} [\nu_m])
 \end{aligned} \tag{4.49}$$

Taking into account that the variable ν is zero-mean (Hyp. 1), the covariance is finally zero:

$$\begin{aligned}
 &\text{Cov} [\nu_n^2 + 2\hat{x}_n\nu_n, \nu_m^2 + 2\hat{x}_m\nu_m] = \\
 &= \text{E} [\nu_n^2] \text{E} [\nu_m^2] + 0 + 0 + 0 - (\text{E} [\nu_n^2] + 0) (\text{E} [\nu_m^2] + 0) \\
 &= 0
 \end{aligned} \tag{4.50}$$

The results in equations (4.46), (4.47) and (4.50) can be substituted in equation (4.45) to obtain the following expression of the error η variance σ_η^2 :

$$\sigma_\eta^2 = \frac{1}{N^2} \sum_{n=0}^{N-1} \text{Var} [\nu_n^2] + \frac{1}{N^2} \sum_{n=0}^{N-1} \text{Var} [2\hat{x}_n\nu_n] \tag{4.51}$$

Some further simplification can be applied. The argument of the first sum in (4.51), the variance of the squared variable ν , becomes:

$$\begin{aligned}
 \text{Var} [\nu_n^2] &= \text{E} [(\nu_n^2 - \sigma_\nu^2)^2] \\
 &= \text{E} [\nu_n^4 + \sigma_\nu^4 - 2\nu_n^2\sigma_\nu^2] \\
 &= \text{E} [\nu_n^4] + \sigma_\nu^4 - 2\text{E} [\nu_n^2] \sigma_\nu^2 \\
 &= 3\sigma_\nu^4 + \sigma_\nu^4 - 2\sigma_\nu^2\sigma_\nu^2 \\
 &= 2\sigma_\nu^4
 \end{aligned} \tag{4.52}$$

The argument of the second sum in (4.51), the variance of the product between the signal sample and the added noise, given hypotheses 3 and 1 and applying property (4.40),

4.4. Total error and Signal to Noise Ratio expression

can be rewritten as:

$$\begin{aligned}
 \text{Var} [2\hat{x}_n\nu_n] &= (\text{Var} [2\hat{x}_n] + \text{E} [2\hat{x}_n]^2) (\text{Var} [\nu_n] + \text{E} [\nu_n]^2) - \text{E} [2\hat{x}_n]^2 \text{E} [\nu_n]^2 \\
 &= (\text{Var} [2\hat{x}_n] + \text{E} [2\hat{x}_n]^2) (\sigma_\nu^2 + 0) - \text{E} [2\hat{x}_n]^2 \cdot 0 \\
 &= 4 (\text{Var} [\hat{x}_n] + \text{E} [\hat{x}_n]^2) \sigma_\nu^2
 \end{aligned} \tag{4.53}$$

In the expression (4.53) the expectation and the variance of the sampled sequence $\hat{x}_n$ appear. They are properties of the waveform $x(t)$, of the sampling rate and of each n , each sampling point in the waveform. A possible assumption is that $x(nt_s - \tau)$ might be linearised on the sampling interval around the value of the continuous signal in the middle of the sampling interval. Under this hypothesis, the expectation could be approximated by $x((n - 1/2)t_s)$ (4.54) and the variance would be proportional to the square of the derivative times the square of the sampling period (4.55).

$$\text{E} [\hat{x}_n] = \int_{-\infty}^{+\infty} x(nt_s - t) \cdot f_\tau(t) dt \approx x((n - 1/2)t_s) \tag{4.54}$$

$$\text{Var} [\hat{x}_n] = \int_{-\infty}^{+\infty} (x(nt_s - t) - \text{E} [\hat{x}_n])^2 \cdot f_\tau(t) dt \approx \left(\left. \frac{dx(t)}{dt} \right|_{(n-1/2)t_s} \right)^2 \frac{t_s^2}{12} \tag{4.55}$$

A second assumption, to simplify the sum, is that the variance could be neglected in the sum with the squared expectation. Under these assumptions, the variance σ_η^2 of the error η can finally be expressed and approximated as:

$$\begin{aligned}
 \sigma_\eta^2 &= \frac{2}{N} \sigma_\nu^4 + \frac{4}{N^2} \sigma_\nu^2 \sum_{n=0}^{N-1} (\text{Var} [\hat{x}_n] + \text{E} [\hat{x}_n]^2) \\
 &\simeq \frac{2}{N} \sigma_\nu^4 + \frac{4}{N^2} \sigma_\nu^2 \sum_{n=0}^{N-1} \left(\left(\left. \frac{dx(t)}{dt} \right|_{(n-1/2)t_s} \right)^2 \frac{t_s^2}{12} + x((n + 1/2)t_s)^2 \right) \\
 &\simeq \frac{2}{N} \sigma_\nu^4 + \frac{4}{N^2} \sigma_\nu^2 \sum_{n=0}^{N-1} x((n + 1/2)t_s)^2 \\
 &\simeq \frac{2}{N} \sigma_\nu^4 + \frac{4}{N} \sigma_\nu^2 P_T
 \end{aligned} \tag{4.56}$$

The accuracy of both assumptions increase with higher sampling rates: with a small sampling period the linearisation becomes a better approximation of the signal and also the approximated variance follows the square of the sampling period, becoming more negligible with respect to the signal expectation. Furthermore, also the approximation of the mean of the squared samples with the average power of the continuous signal (last step of Eq. (4.56)) is more accurate for high sampling rates.

4.4 Total error and Signal to Noise Ratio expression

Combining the results of the sections 4.2 and 4.3, the average power estimator $\bar{P}_T$ defined in Eq. (4.11) is found to be equal to the average power of the continuous pulse signal plus two error terms:

$$\bar{P}_T = P_T + \epsilon_{P_{T,\tau}} + \eta \tag{4.57}$$

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Both the errors are known in terms of their mean value and variance, so we can obtain a general description of the total error.

The mean value of the total error is equal to the sum of the mean of the two errors, so it is equal to the mean of the error caused by the limited resolution (see Eq. (4.44)), since the mean of the limited sampling rate error is zero (see Eq. (4.31)).

To calculate the variance of the total error, the relationship between the two errors must be considered. For high sampling rates it is reasonable to assume that the dependence of the limited resolution error on the changes in the sampling phase is negligible, so that the noise error variance can be approximated as in (4.56). Still, we can not state that the two variables are independent.

The variance of their sum is the sum of their variances, plus twice their covariance. Given property (4.41) about the covariance, and being the expectation of the error $\epsilon_{P_{T,\tau}}$ zero (4.31), the covariance of the two errors is equal to the expectation of their product, that is found to be zero:

$$\begin{aligned} E[\epsilon_{P_{T,\tau}} \cdot \eta] &= E_\epsilon[\epsilon_{P_{T,\tau}} \cdot E_{\eta|\epsilon}[\eta]] \\ &= \sigma_\nu^2 \cdot E_\epsilon[\epsilon_{P_{T,\tau}}] \\ &= 0 \end{aligned} \quad (4.58)$$

Therefore, the total error variance is the sum of the two individual variances.

A meaningful way to synthesise and represent the information collected so far, is in the form of a Signal to Noise Ratio (SNR). The SNR is a useful parameter in the analysis and design of signal processing systems, giving a measure of the signal quality. In the case here under study the signal is the average power of the continuous signal, i.e. P_T , while the noise is the combination of the errors caused by the limits of the converter, in its sampling rate and resolution. Knowing how the SNR of the measurement varies as a function of the ADC features, allows to optimise the choice of the converter, on top of analyzing how the single parameters affect the measurement. The SNR expression is calculated as:

$$\begin{aligned} SNR_{dB} &= 10 \log_{10} \left(\frac{P_T^2}{\sigma_\epsilon^2 + \sigma_\eta^2 + \mu_\eta^2} \right) \\ &= 10 \log_{10} \left(\frac{P_T^2}{2(A_{X_k,N}^2 + B_{X_k,N}^2) + \left(\frac{2}{N} + 1\right) \sigma_\nu^4 + 4\frac{\sigma_\nu^2}{N} P_T} \right) \end{aligned} \quad (4.59)$$

with $A_{X_k,N}$ and $B_{X_k,N}$ as defined in (4.25).

In the expression (4.59), differently from the one presented in [44], at the denominator is the total error power and not the total error variance. The difference is between considering the contribute of the squared mean value in addition to the total variance, or not. With the error power, we are taking into account the mean value of the error, so the measurement offset. The expression considering only the variation of the error, i.e. with the error variance at the denominator of the SNR, is also here reported (4.60). Which one of the two expressions to use is a matter of what we are interested in to

measure and qualify.

$$\begin{aligned}
 SNR_{dB} &= 10 \log_{10} \left(\frac{P_T^2}{\sigma_\epsilon^2 + \sigma_\eta^2} \right) \\
 &= 10 \log_{10} \left(\frac{P_T^2}{2(A_{X_k,N}^2 + B_{X_k,N}^2) + \frac{2}{N}\sigma_\nu^4 + 4\frac{\sigma_\nu^2}{N}P_T} \right) \quad (4.60)
 \end{aligned}$$

4.5 Application to example waveforms

The analytical results described in sections 4.2, 4.3 and 4.4 have been applied to example waveforms of pulse signals. The waveforms selected are: a Gaussian pulse, the derivative of a Gaussian pulse, and the response of a LHC stripline to the passage of a particle bunch, as analytically modeled expression and as oscilloscope measurement.

The Gaussian pulse and its derivative are chosen as generic signals that fit several practical cases. For example, in beam diagnostic, a beam current transformer signal can be approximated with a Gaussian waveform. The derivative of a Gaussian pulse is instead used to approximate the response of differential systems, as button and stripline pickups. The LHC stripline offers a specific signal example, that we can model analytically, but of which we also have actual measurements available.

The application of the analysis described above to these waveforms allows to estimate what would be the SNR of the average power measurement after direct digitisation, and how it would vary changing the sampling rate and the resolution of the digitisation.

The selected signals and the digitisation parameters respect, to a good extent, the hypotheses adopted in the model development, namely:

- the signal has finite energy and is limited in time;
- the sampling frequency is in the range $R_{Nyq}/2 < F_s < R_{Nyq}$ and therefore only the aliasing of the first replica is relevant;
- the sampling phase is a uniformly distributed random variable;
- the noise is described by an independent and identically distributed, zero-mean, Gaussian variable.

The Gaussian pulse is treated fully analytically. Differently, the Gaussian signal derivative and the LHC stripline signals analyses are based on the numerically computed DFT coefficients, starting from an oversampled discrete sequence of the signal in time.

Another difference in the analysis application is about the physical magnitudes involved. The Gaussian pulse and its derivative, not being necessarily linked to a physical phenomenon, are generally treated as normalised signals in amplitude, time and frequency. This is not the case for the LHC stripline signal, for which real amplitude values, and time and frequency coordinates are used. This allows to consider the characteristics of existing ADCs as reference to describe the relationship between the sampling rate and the resolution.

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The final outcome for all the presented examples is the representation of the expected SNR of the average power measurement as a function of the sampling rate and the converter resolution.

For all the examples the analytical or 'quasi-analytical' results have been compared with numerical simulations. In the numerical simulations the variation of the error of the average power measurement was calculated out of the sequence obtained after sampling the input signal with a uniformly distributed sampling delay, and adding Gaussian white noise on each sample.

4.5.1 Gaussian waveform

A Gaussian waveform with standard deviation σ and its maximum amplitude in the center of time window T is given by:

$$x(t) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(t-T/2)^2}{2\sigma^2}} \quad (4.61)$$

With $\sigma = 0.01T$, the pulse width with respect to the window size is comparable to the one of a bunch pulse with respect to its bunch-to-bunch distance (see the LHC stripline example 4.5.3) and the signal can be considered to be zero outside this window.

The Fourier transformation of the expression (4.61) is calculated analytically and it is found to be:

$$X(f) = e^{-2\pi^2\sigma^2 f^2} e^{-j2\pi f T/2} \quad (4.62)$$

Both expressions, the signal in time (4.61) and the modulus of its spectrum (4.62), are represented in Fig. 4.7.

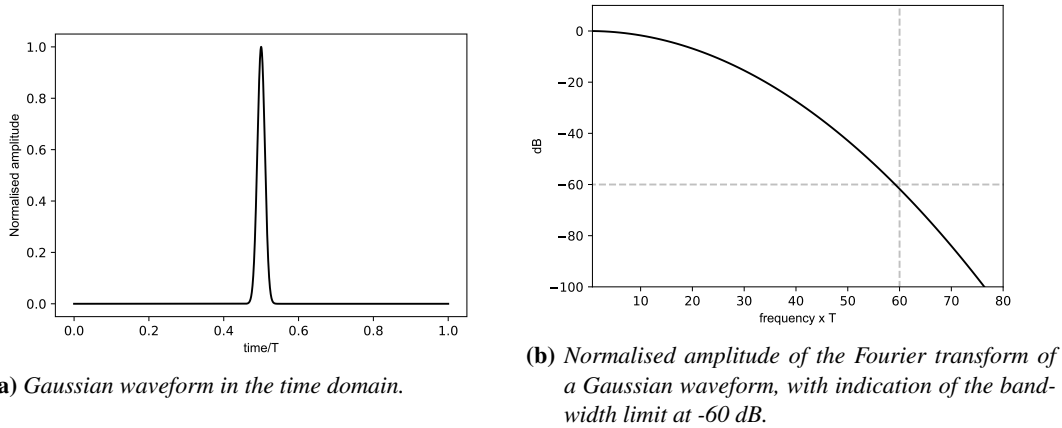


Figure 4.7: Example of a Gaussian waveform in the time and in the frequency domain [44].

The signal $x(t)$ has finite energy E , defined by:

$$E = \int_{-\infty}^{+\infty} x(t)^2 dt = \frac{1}{2\sqrt{\pi\sigma^2}} \quad (4.63)$$

We define the periodic signal $x_T(t)$ over the window T as in Eq. (4.3). Its Fourier transform coefficients X_k are expressed as in Eq. (4.4), extracted as samples of the

Fourier spectrum (4.62) at multiples of $1/T$:

$$X_k = \frac{1}{T} X\left(\frac{k}{T}\right) = \frac{1}{T} e^{-2\pi^2 \frac{\sigma^2}{T^2} k^2} e^{-j\pi k} \quad (4.64)$$

In order to identify the range of interesting sampling rates, we must in first instance define the signal's Nyquist rate R_{Nyq} , therefore its bandwidth limit in the form of the coefficient number N_0 defined in Eq. (4.8). Being the the Fourier transform of a Gaussian pulse a Gaussian function itself, there is no N_0 that perfectly satisfies the condition about the spectral density being equal to zero (Eq. (4.8)) and $|X(f)|^2 \neq 0 \forall f$. Therefore, it is necessary to define a threshold below which the spectrum amplitude can be considered negligible. In a real system a reference would be provided by the electronics noise level. For this example we set as threshold $|X_k|_{dB} \leq -60$ dB, for a spectrum normalised with respect to its maximum value (see Fig. 4.7b), limit that is compatible with a digitiser with a noise level equivalent to about 10 Effective Number of Bits (ENOB) as from the state of the art for multi GSps converters. The bandwidth condition is verified for $k \geq 60$, resulting in $N_0 = 60$ and the Nyquist rate $R_{Nyq} = 2N_0/T = 120/T$.

Once that the spectrum coefficients X_k are known, together with the Nyquist rate R_{Nyq} , it is possible to calculate the error caused by an asynchronous sampling rate in the frequency range $R_{Nyq}/2 < F_s < R_{Nyq}$.

The calculation of the coefficients $A_{X_k,N}$ and $B_{X_k,N}$, defined in (4.25), is straightforward:

$$A_{X_k,N} = (-1)^N \frac{1}{T^2} e^{-2\pi^2 \frac{\sigma^2}{T^2} N^2} \sum_{k=0}^{N-1} e^{4\pi^2 \frac{\sigma^2}{T^2} k(N-k)} \quad (4.65)$$

$$B_{X_k,N} = 0$$

and the variance of the error due to asynchronous sampling with infinite resolution and uniformly distributed sampling delay follows:

$$\sigma_\epsilon^2 = 2A_{X_k,N}^2 = 2 \left[\frac{1}{T^2} e^{-2\pi^2 \frac{\sigma^2}{T^2} N^2} \sum_{k=0}^{N-1} e^{4\pi^2 \frac{\sigma^2}{T^2} k(N-k)} \right]^2 \quad (4.66)$$

Also the probability density function (4.36) can be calculated and it is found to be:

$$f_\epsilon(\epsilon) = \left| 2\pi A_{X_k,N} \sin \left(2 \arctan \left(\sqrt{\frac{2A_{X_k,N} - \epsilon}{2A_{X_k,N} + \epsilon}} \right) \right) \right|^{-1} \quad (4.67)$$

with $A_{X_k,N}$ as in Eq. (4.65).

The expression (4.67) allows to estimate the behaviour of the average power measurement error when the pulse is sampled at a specified sampling rate, so with a specific number of samples N per window T , and with uniformly distributed sampling delay between each repetition of the measurement. The measurement was recreated in a numeric environment, with the aim to obtain a test array with acquisitions of the undersampled Gaussian pulse, where the delay of the first sample is uniformly distributed over a sample period. The distribution density of the numerically obtained relative error and the analytic expression (4.67) are compared for the same pulse and different sampling rates in Fig. 4.8.

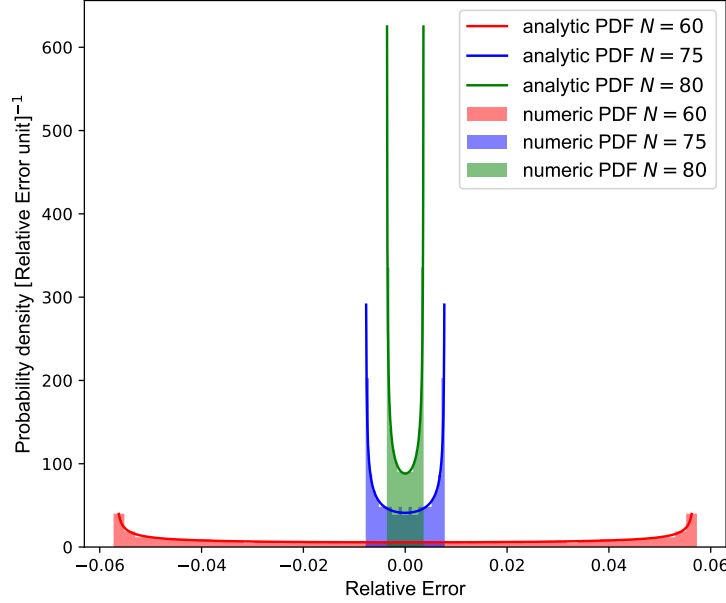


Figure 4.8: *Probability Density Function (PDF) of the relative error of the average power measurement of a Gaussian pulse signal, when sampled with infinite resolution at three different sampling rates between half of its Nyquist rate (equivalent to $N = 60$) and its Nyquist rate (equivalent to $N = 120$). Analytic and numeric distributions are compared.*

The results of the analysis, as from Fig. 4.8, confirm how the size of the maximum error decreases with higher sampling rates, and that larger errors have higher probability.

The limited resolution of the converter is included in the analysis, modeling the noise on each sample as a zero-mean Gaussian random variable, described by its variance σ_ν^2 . The error on the average power measurement, η , has mean value $\mu_\eta = \sigma_\nu^2$ (4.31) and variance σ_η^2 dependent on the noise on the single sample, the number of samples N acquired per window T , and the average power of the continuous pulse signal itself, that for this Gaussian example is a function of the pulse's variance σ^2 :

$$\sigma_\eta^2 \simeq \frac{2}{N}\sigma_\nu^4 + \frac{4}{N}\sigma_\nu^2 P_T = \frac{2}{N}\sigma_\nu^4 + \frac{2}{N}\sigma_\nu^2 \frac{T}{\sqrt{\pi}\sigma^2} \quad (4.68)$$

Finally, the SNR of the average power measurement of the Gaussian pulse can be represented for each sampling rate between half of the Nyquist rate and the Nyquist rate, and for a defined resolution of the sampler. Figure 4.9 shows the result for the Gaussian waveform (4.61) as SNR vs. F_s/R_{Nyq} (Eq. (4.60)), with expression (4.66) describing the contribution of sampling distortion to the error, and with two values of σ_ν reflecting the contribution of the resolution limit of the ADC to the error. In one case the standard deviation σ_ν is set to a value of $-70 \text{ dB} \equiv 3.16 \cdot 10^{-4}$ with respect to the full-scale-range (maximum) input signal value of the ADC, and equivalent to an 11.33 ENOB converter. In the other case the limited resolution contribution is doubled, with σ_ν set to a value of $-64 \text{ dB} \equiv 6.3 \cdot 10^{-4}$, and equivalent to an 10.33 ENOB converter.

The results from the analytical computation are observed to be confirmed by numerical analysis.

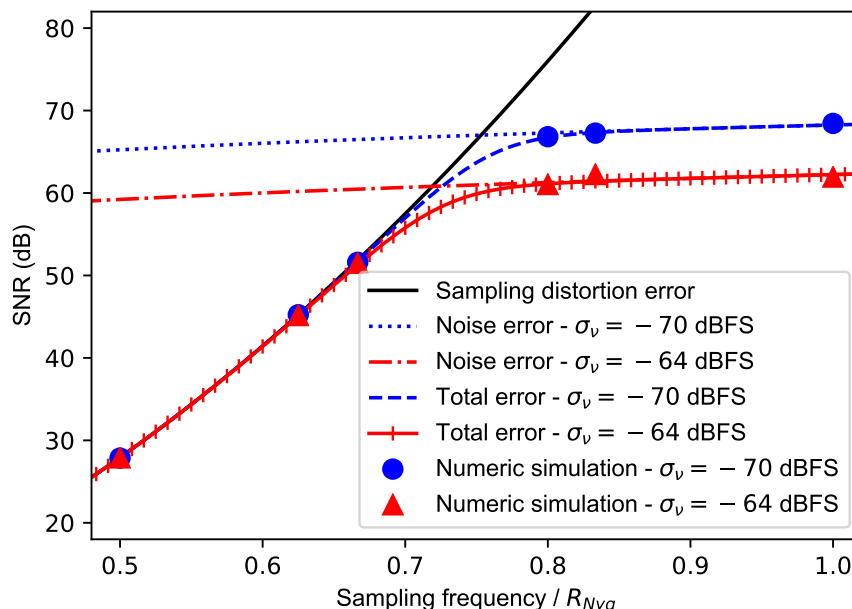


Figure 4.9: SNR as function of the normalised sampling rate with respect to the Nyquist rate for a Gaussian waveform [44].

In figure 4.9 are shown, together with the resulting SNR, the individual error contribution of the sampling distortion, and of the limited resolution. The sampling distortion error, as evaluated for a converter with infinite resolution, is the major contributor to the total error at lower sampling rates, and therefore limits the SNR. The error caused by the limited resolution, i.e. the noise on each single sample, here evaluated for two different standard deviations, decreases with higher sampling rates as the noise is averaged on more samples; nevertheless it is for high sampling rates that it becomes the limiting factor of the SNR.

The figure can therefore be split in three regions: at low sampling frequencies the achievable SNR is dominated by the limited sampling rate; at high sampling frequencies the SNR is limited by the ADC resolution; between these two regions a rather narrow transition area, the location of which is mostly determined by the equivalent ADC noise.

In the first part of this example we have ignored the relation between the maximum sampling rate of an ADC and its expected resolution: as discussed in detail in 3.2 there is a trade-off between these two parameters depending on the state of the art.

A way to reflect this relation in the SNR is to assume for the ADC a constant measurement of performance P , as defined in (3.5), making in this way the single sample noise variance σ_v^2 a function of the sampling rate. This results in an increased sampler's noise, expressed by a reduced ENOB, at the increase of the sampling rate.

Referring to the definition of ENOB (see Eq. (3.3)), we can express the standard

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deviation of the noise of the ADC as in Eq. (3.4):

$$\sigma_\nu = \frac{V_{FSR}}{\sqrt{12}} 2^{-\text{ENOB}} \quad (4.69)$$

where V_{FSR} is the full-scale range of the ADC input signal. For our example V_{FSR} was set to twice the maximum absolute value of the input signal, to take into account also the case of bipolar waveforms. By combining P definition (3.5) and Eq. (4.69), fixing the window of observation $T=24$ ns, within the bunch-to-bunch distance in the LHC, the variance of the noise of the ADC as a function of the sampling rate is found to be:

$$\sigma_\nu^2 = \frac{V_{FSR}^2 F_s^2}{12 P^2} \quad (4.70)$$

The result of the analysis using as constant performance P the maximum value for the converters reported in Murmann's ADC survey [32], is shown in figure 4.10.

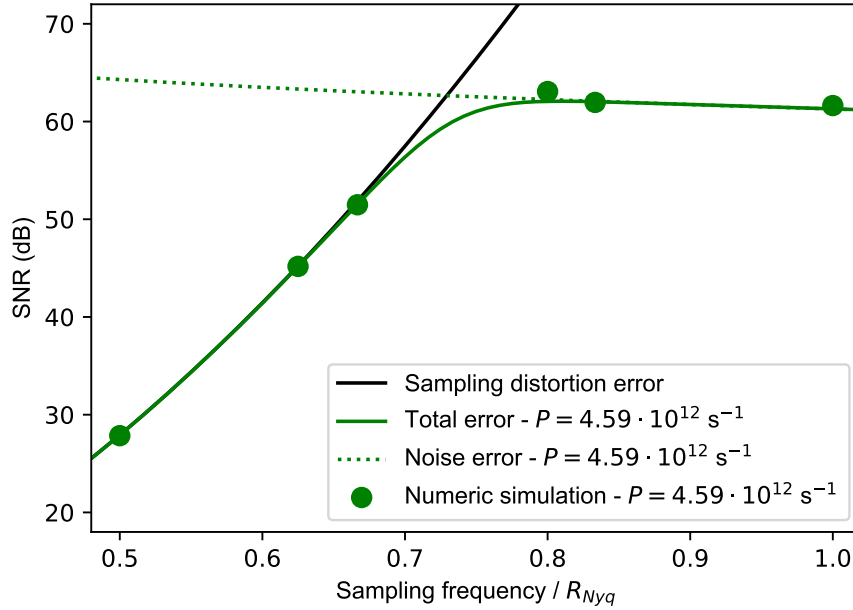


Figure 4.10: SNR as function of the normalised sampling rate with respect to the Nyquist rate for a Gaussian pulse and an ADC with constant performance $P = 4.59 \cdot 10^{12} \text{ s}^{-1}$ for all sampling rates.

The SNR curve in Fig. 4.10 does not represent the results achievable with a single converter; instead it traces the envelope of the possible performances that could be reached at each sampling rate with a state-of-the-art converter used at the peak of its capability.

It can be noted in Fig. 4.10 that, taking into account a fixed performance P that is linked to the current state of the art in the converters technology, the error due to the limited resolution does not decrease anymore with the increase of sampling rate: the deterioration of the ADC resolution weights more than the the higher number of samples available. This trend also indicates clearly that there is no obvious gain in looking for the fastest sampling option. The optimum sampling point sits in the transition between

the region dominated by distortion and the region dominated by the limited resolution. The example also shows that this optimum sampling point can be at sampling rates below the Nyquist Rate.

4.5.2 Derivative of a Gaussian waveform

The analysis can be applied to a bipolar pulse signal, in this case the derivative of the Gaussian pulse studied in 4.5.1, that also closely resembles the type of signal produced by many electromagnetic pickups in particle accelerators. The signal waveform in the time domain, and the amplitude of its Fourier spectrum are shown in Fig. 4.11.

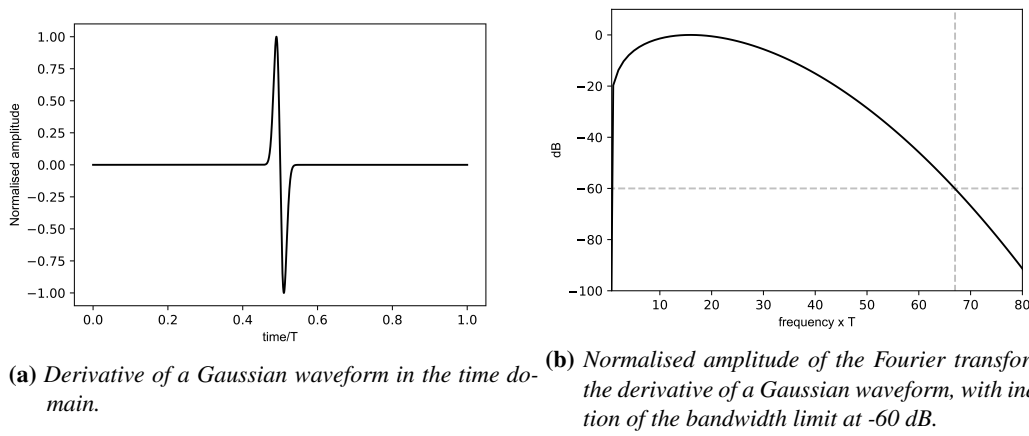


Figure 4.11: Example of derivative of a Gaussian waveform in the time and in the frequency domain [44].

The spectrum in the frequency domain has been calculated numerically out of a oversampled numeric representation of the signal, and it is available as the sequence of complex coefficients X_k .

The bandwidth condition (4.8) is verified for $k \geq 67$, resulting in $N_0 = 67$ and the Nyquist rate is defined as $R_{Nyq} = 2N_0/T = 134/T$.

All the information needed to estimate the error $\epsilon_{P_{T,\tau}}$ induced by sampling distortion are available. The probability density function of the error $\epsilon_{P_{T,\tau}}$ relative to the average power can be computed following expression (4.36). The results obtained for three different sampling rates are reported in Fig. 4.12, and it can be observed that the analytic expression of the function and the distribution obtained by numeric simulations are in close agreement. In this case as for the Gaussian pulse, the maximum error size decreases with higher sampling rates and larger errors are more likely to occur.

The variance σ_ϵ^2 of the error $\epsilon_{P_{T,\tau}}$ as function of the signal spectrum and of the sampling rate in the range $0.5 \cdot R_{Nyq} < F_s < R_{Nyq}$ can be computed through Eq. (4.32). The variance σ_η^2 of the error η as function of the ADC noise variance σ_ν^2 , the sampling rate and the pulse average power can be computed through Eq. (4.56).

The variance of the total error of the average power measurement is then known, and the expression of the SNR can be computed as a function of the sampling rate (see Eq. (4.60)), having the converter resolution as a parameter. Two ADCs with different resolution variance σ_ν^2 are considered and their performances are compared in Fig. 4.13.

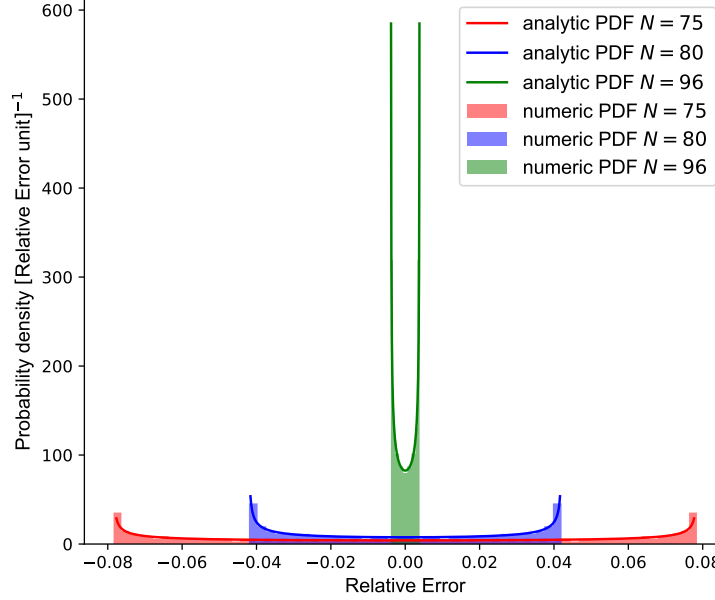


Figure 4.12: *Probability Density Function (PDF) of the relative error of the average power measurement of the derivative of a Gaussian pulse signal, when sampled with infinite resolution at three different sampling rates between half of its Nyquist rate (equivalent to $N = 67$) and its Nyquist rate (equivalent to $N = 135$). Analytic and numeric distributions are compared.*

A good agreement with the values obtained through numerical analysis can be noted also in this case. This confirms the validity of this methodology also for the analysis of bipolar signals and in the case in which the signal spectrum is not given as an analytic function.

4.5.3 Large Hadron Collider stripline response signal to a particle bunch

As example of an actual beam instrumentation signal we take the response of a LHC stripline pickup to a proton bunch. The general concept behind an electromagnetic pickup as the stripline is treated in section 2.4.2.

At the passage of a bunch of accelerated charged particles the upstream port of the stripline delivers a bipolar pulse output signal, that can be analytically modeled, as proposed in [5]. Consider a relativistic beam, i.e. with a velocity close to the speed of light, which has a Gaussian shaped longitudinal bunch profile (see Eq. (2.2)), and is passing by the BPM at the centre, that is along the line of transverse symmetry. The upstream port of each stripline electrode will generate a voltage pulse given by Eq. (2.16), here reported for reference:

$$V_U(t) = \frac{Z_0}{2} \frac{\alpha}{2\pi} \frac{qN_b}{\sqrt{2\pi}\sigma_b} \left\{ e^{\frac{-(t+\tau_\ell)^2}{2\sigma_b^2}} - e^{\frac{-(t-\tau_\ell)^2}{2\sigma_b^2}} \right\}$$

The definition of each parameter in Eq. (2.16) and their value for this example are listed in Table 4.1.

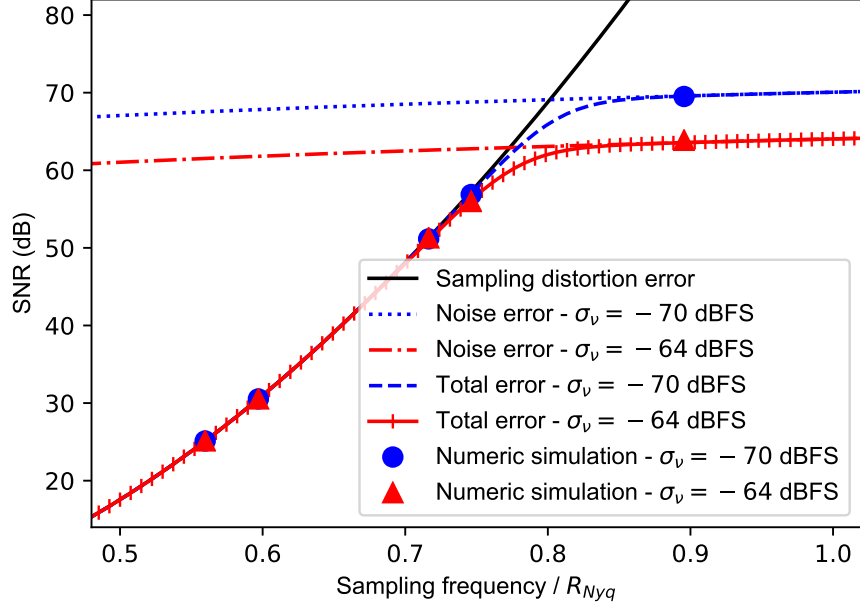


Figure 4.13: SNR as function of the normalised sampling rate with respect to the Nyquist rate for the derivative of a Gaussian waveform [44].

Symbol	Definition	Value
Z_0	Port impedance	50 Ω
τ_ℓ	Propagation delay	417 ps
α	Electrode azimuthal width	0.27 rad
q	Elementary charge	$1.602 \cdot 10^{-19}$ C
N_b	Bunch intensity	10^{10}
σ_b	Bunch length	0.25 ns

Table 4.1: Definition and value of the parameters of the LHC stripline BPM expression

Together with this analytic expression this example illustrates also the analysis of an actual measurement of a stripline response to a proton bunch in the LHC. The pickup signal was acquired by a broadband, fast sampling (60 GS/s) oscilloscope. The window of observation is $T = 24$ ns, and it is contained within the bunch-to-bunch distance of about 25 ns in the LHC.

The analytic expression (2.16) with parameters defined in Table 4.1, and the measured waveform normalised to the same bunch intensity $N_b = 10^{10}$ are shown in Fig. 4.14, left plot. The right plot of Fig. 4.14 shows the amplitude of their spectral coefficients X_k , the result of a numeric DFT, normalised to their peak value.

The measured signal and the analytic signal show close agreement in their spectral power density for low frequencies, but they start to diverge significantly at frequencies above 1.2 GHz. The simple Gaussian model of the beam current cannot approximate with great detail the complexity of the longitudinal particle dynamics of the LHC. Likewise for the ideal description of the electromagnetic coupler, which does not take into account any loss, dispersion or imperfection. Furthermore, the stripline is located in the accelerator tunnel, in an area not accessible when the machine is on; therefore the

Chapter 4. Analysis of sampling effects on the average power measurement of direct digitally acquired pulse signals

electrodes' signals are acquired through long coaxial cables, about 70 m long in this case.

The spectral amplitude plot shows also the bandwidth limit at -60 dB with respect to the peak. For the analytic signal the bandwidth limit is found at 2.21 GHz ($N_0=53$), which results in a Nyquist rate $R_{Nyq}=4.42$ GHz.

For the measured signal it is not straightforward to define such a threshold: even at high frequencies the spectral content can be significantly above 60 dB. But at 2.21 GHz can be set a bandwidth limit with threshold -40 dB with respect to the magnitude peak. The range of frequencies for which the average power measurement error analysis is performed is therefore $2.21 < F_s < 4.42$ GHz for both signals, corresponding to $R_{Nyq}/2 < F_s < R_{Nyq}$.

The condition of the spectrum amplitude being negligible after the bandwidth limit is poorly respected for the measured signal. The assumption of having only the first replica of the spectrum impacting the performances is not accurate, and we expect to see an effect of this in the results.

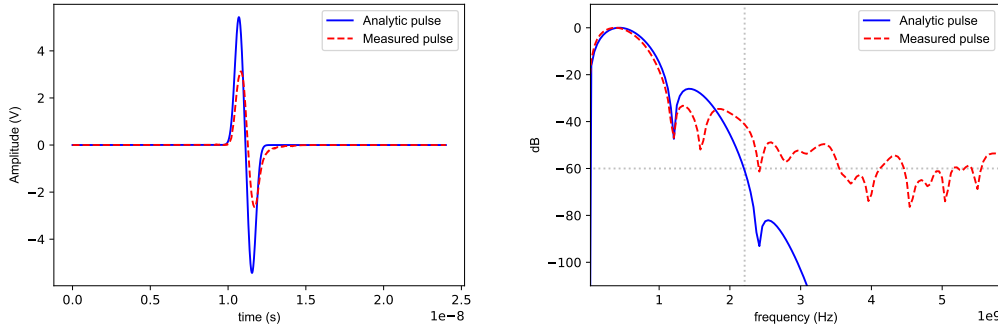


Figure 4.14: Analytic expression and measurement of LHC stripline signal, time signal and spectrum amplitude.

The error $\epsilon_{P_{T,\tau}}$ induced by sampling distortion with uniformly distributed sampling delay can be estimated in its variance σ_ϵ^2 (4.32) and its probability density function can be computed following expression (4.36). Fig. 4.15a shows the probability density function of the average power measurement of the analytic signal for two different sampling rates, comparing the analytic expression of the function and the distribution obtained by numeric simulations, with close agreement. Fig. 4.15b shows the same for the measured signal and we observe that the analytic estimation of the distribution, as expected, does not match accurately the numeric simulation results.

Taking also into account the limited resolution of the ADC, the variance σ_η^2 of the error η is calculated for both signals, leading to the comprehensive expression of the SNR of the average power measurement. As for the second part of the Gaussian signal analysis, the analysis was performed for an ADC with constant measurement of performance $P = 4.59 \cdot 10^{12} \text{ s}^{-1}$, i.e. with single sample noise variance function of the sampling rate (4.70). The analytic result is shown in Fig. 4.16, along with the numeric results, for both signals. The analytic signal analysis shows close agreement between the numeric simulation and the analytic result. The limited resolution of the ADC dominates the SNR for sampling rates higher than 3.5 GHz. As expected, distortion effects dominate the SNR of the measurement of the measured pulse signal, within the range

4.5. Application to example waveforms

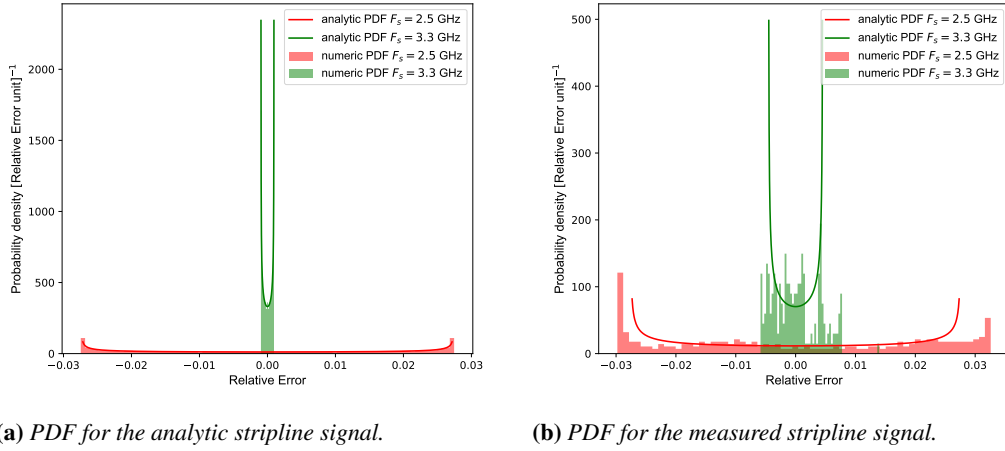


Figure 4.15: Probability density functions of the error $\epsilon_{P_{T,\tau}}$ for different sampling rates for the analytic stripline signal and the measured stripline signal.

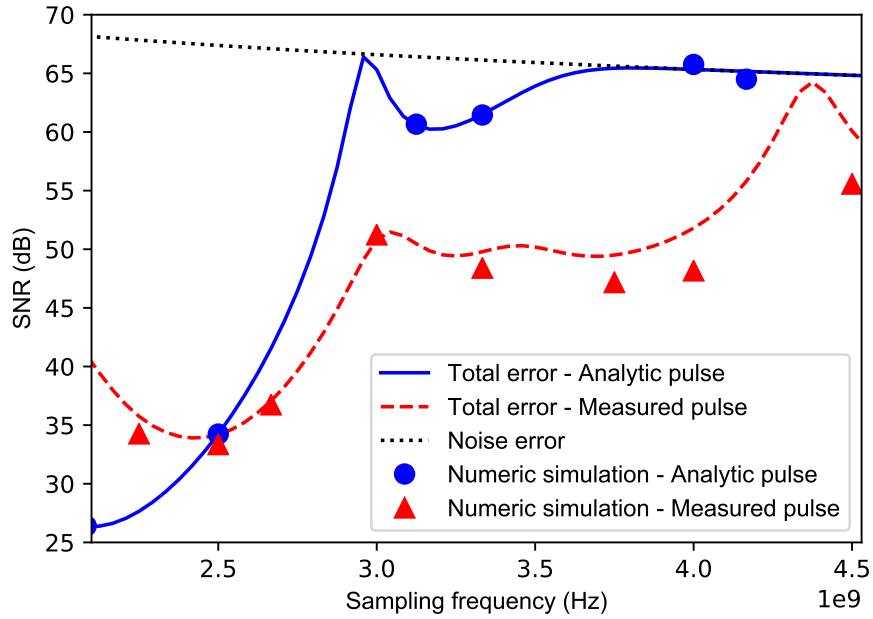


Figure 4.16: SNR as function of the sampling rate for the analytic expression and the measured signal of a stripline pickup, ADC with constant measurement of performance $P = 4.59 \cdot 10^{12} \text{ s}^{-1}$.

of sampling rates analysed. The plot also shows how the analytic prediction underestimates the distortion caused by replicas beyond the first order with respect to the first replica, and that the error is accentuated for higher sampling rates.

It is instructive to repeat the analysis flow on two modified versions of the same signals.

The analytic expression of the stripline response to a proton bunch is convoluted with the analytic pulse response of a model of a coaxial cable, fitted on the characteris-

Chapter 4. Analysis of sampling effects on the average power measurement of direct digitally acquired pulse signals

tics of the cable used in the actual stripline measurement (details about the cable model are reported in section 5.3). As a result, the analytic pulse signal is a closer representation of the measured pulse signal in the time domain and in the frequency domain, for low frequencies especially.

The measured signal has significant spectral content at high frequencies that can be reduced, for the benefit of the measurement and of the applicability of the analysis method. A sharp 8th order low-pass Bessel filter, with cut-off frequency set at 2.21 GHz, is applied numerically.

The two signals so modified are shown in Fig. 4.17, both in the time and frequency domain. The considered range of sampling rates for the analysis is kept the same, with $R_{Nyq}=4.42$ GHz. The probability density function of the error $\epsilon_{P_{T,\tau}}$ is computed analytically and numerically, with results shown in Fig. 4.18. For the modified analytic signal, the analytic PDF is still an accurate description of the error distribution. The dispersion of the cable has reduced the spectral content at high frequencies, causing a reduction of the maximum error size for the same sampling rates with respect to the signal before the cable. The PDF of the low-pass filtered measured signal presents an improved agreement between analytic predictions and numerical results, showing that the signal so modified respects better the hypotheses of the model.

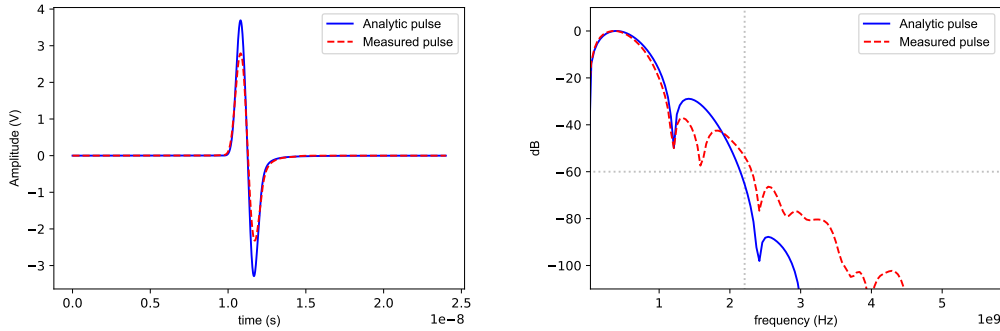


Figure 4.17: Analytic expression with cable and low-pass filtered measurement of LHC stripline signal, time signal and spectrum amplitude

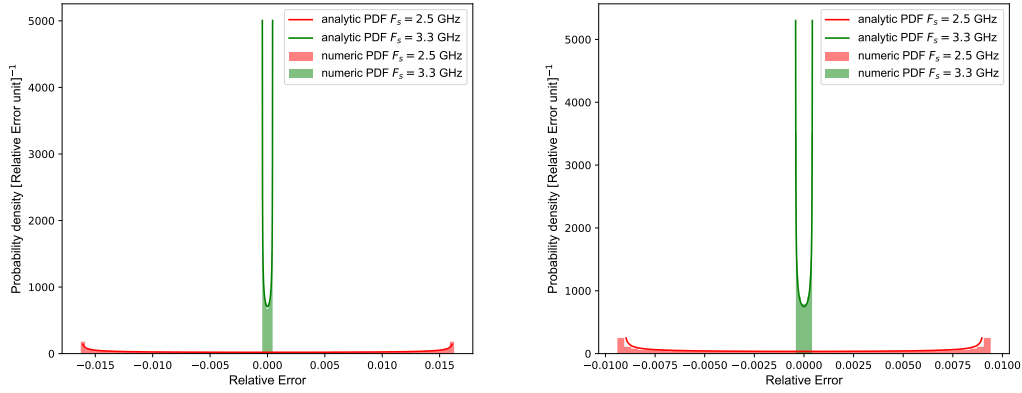
Finally, the general SNR function is computed analytically and numerically, with the same assumptions on the ADC resolution and performance. The results are plotted in Fig. 4.19. The improvement for the analytic signal, seen in the distortion error PDF in Fig. 4.18a, is confirmed in the SNR, for the sampling rates between 3 and 3.5 GHz. The limited resolution effect still dominates at sampling rates above 3.5 GHz.

Low-pass filtering had a significant effect on the SNR of the measured signal, with the expected performances in line with those of the analytic pulse signal. Moreover, the analytic SNR expression replicates accurately the numeric simulation results.

This practical example demonstrates that the model helps to estimate how the signal spectrum, the ADC sampling rate, and the ADC resolution individually impact the average power measurement. Also, together with the other examples, it indicates that the optimum of the measurement can occur at sampling rates lower than the Nyquist rate.

Furthermore, this example shows how the model can be easily implemented in a numeric environment and on signals whose analytic expression is unknown, such as

4.5. Application to example waveforms



(a) PDF for the analytic stripline signal with cable. (b) PDF for the measured stripline signal, LP filtered.

Figure 4.18: Probability density functions of the error $\epsilon_{P_{T,\tau}}$ for different sampling rates for the modified analytic stripline signal and measured stripline signal.

the measured stripline pulse signal.

Lastly, the example illustrates how the model accuracy is limited in case of a spectrum that does not decrease significantly in power with increasing frequencies. In this case, indeed, the first replica of the spectrum is not strongly predominant with respect to the others in determining the distortion, and one of the main assumptions of the model is disproved. Nevertheless, most systems comprise an anti-aliasing filter to reduce the spectral content at high frequencies before digitisation, and this model can be

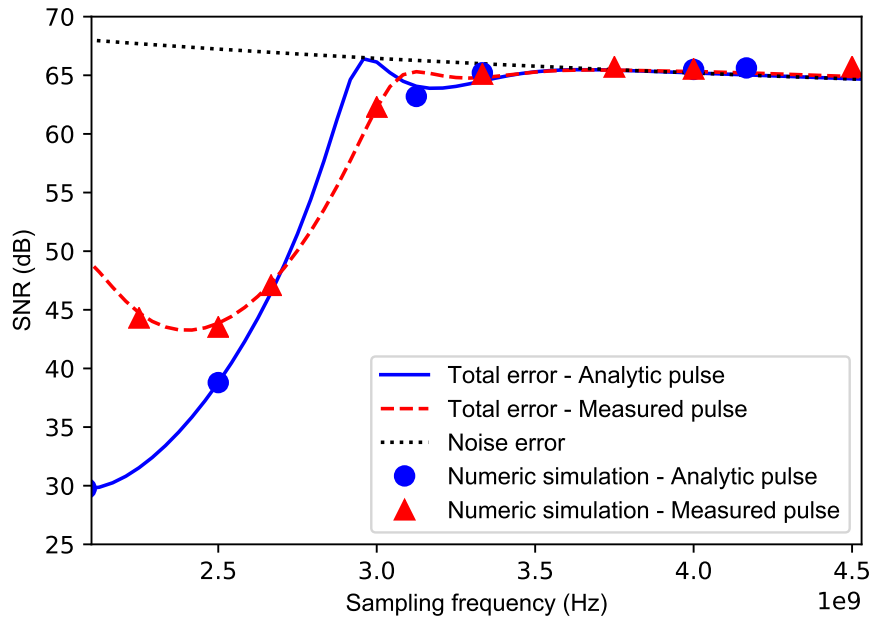


Figure 4.19: SNR as function of the sampling rate for the modified analytic expression and measured signal of a stripline pickup, ADC with constant measurement of performance $P = 4.59 \cdot 10^{12} \text{ s}^{-1}$.

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of assistance in the definition of such a filter.

CHAPTER 5

Study of the effect of digitisation in the LHC beam extraction interlock BPM prototype

The upgrade of the processing electronics of the beam position system for the beam extraction interlock of the Large Hadron Collider is under development. The new design falls in the category of direct digitisation based systems: every pickup bunch pulse is digitised after minimum analog conditioning. A first prototype of the analog front-end has been used to collect beam data with two different ADC mezzanine boards. The converters differ for sampling rate and resolution. Therefore, the system architecture and the collected data offer an interesting analysis case, with the possibility to apply the analytic method presented in chapter 4 for a practical system [45].

The LHC beam extraction interlock BPM system is part of the machine protection system that guarantees a safe beam extraction. When beam operations are over, or the beam quality, or the machine safety are compromised, the beam trajectory is deviated for extraction into the beam dump line by a kicker magnet. In the dump line, the beam is diluted and absorbed in a dedicated system (see Fig. 5.1a for a picture of the LHC beam dump). There is one beam dump line per beam of the LHC. The beam extraction interlock beam position system is composed of 16 monitors that measure continuously the beam position before and after the beam dump kickers (see Fig. 5.1b). Redundancy is required to guarantee the high-reliability required for machine protection systems.

In order to guarantee a safe extraction, the BPM Interlock system monitors that the beam position before and after the kicker magnet does not overcome a safety threshold for more than a predefined number of bunches during an observation window of a programmable number of turns. The threshold is such that the modified angular deviation given to the trajectory in case of extraction still allows the correct absorption of the beam in the foreseen structures (see Fig. 5.2). The system therefore prevents the beam from reaching a state where it can not be extracted safely.

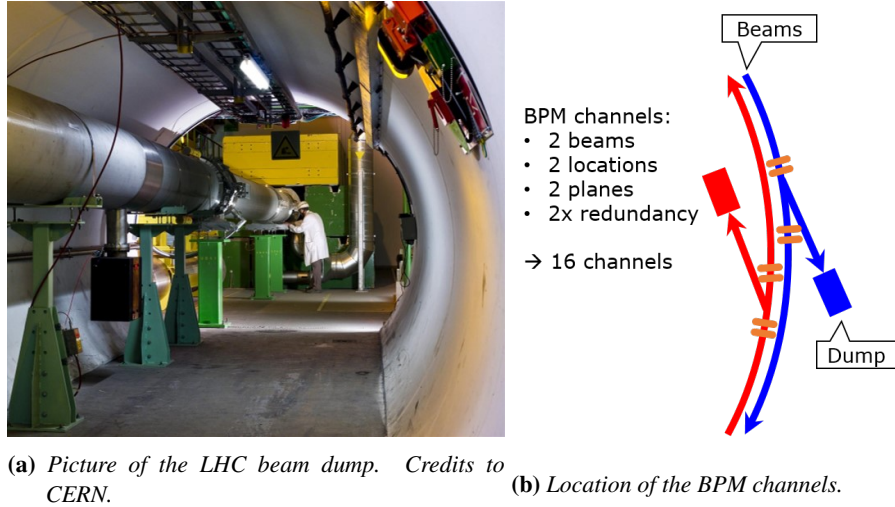


Figure 5.1: LHC Beam Dump and location of the Interlock BPM channels

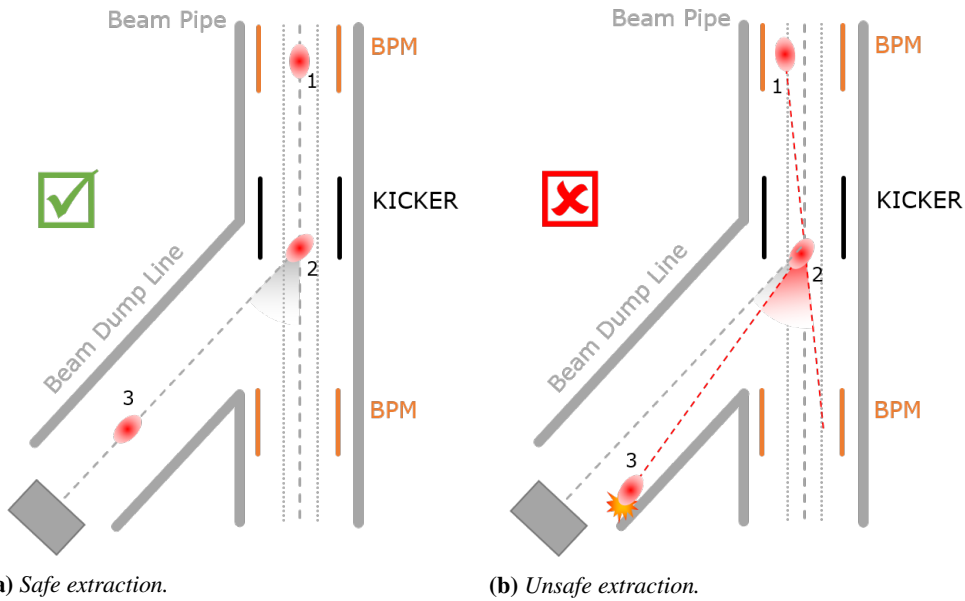


Figure 5.2: Schematic principle of the position measurement for protection of the beam dump line in case of beam extraction.

5.1 The LHC beam extraction interlock BPM prototype

The Interlock application demands high reliability and robustness. The new system is being designed to be as simple as possible and to be independent of other systems and infrastructures, even at the cost of performance. The architecture of the prototype system for the read-out of a single BPM plane is shown in Fig. 5.3 [24]. The inputs are the signals generated by the beam on two opposite stripline pickup electrodes like those described in section 2.4.2.

The main concepts behind the system architecture are:

- *Single processing chain:* the two signals of a single plane are time-multiplexed to

5.1. The LHC beam extraction interlock BPM prototype

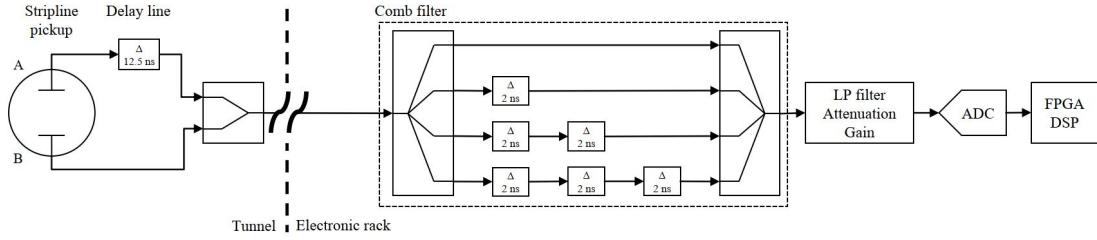


Figure 5.3: The LHC interlock BPM signal processing scheme [24].

form a new single one, and therefore they share the same processing and acquisition path. The effects on the beam position measurement caused by asymmetries or drifts in the electronics are in this way mitigated.

- *Direct digitisation:* the single channel, after minimal analog conditioning, is directly digitised at high sampling rate. This scheme should provide more flexibility on the repetition rate of the measurement, on top of the benefits of direct digitisation discussed in chapter 1.
- *No gain switching:* the system should be able to cover all the beam cases without changing settings during the fill. Therefore, variable gain stages are avoided, at the expense of the measurement resolution.
- *Independent from the accelerator RF clock distribution:* to further minimise possible points of failures, the acquisition system is independent from the RF distribution system, resulting in an asynchronous sampling.

5.1.1 Analog signal processing

The analog front-end is composed of two functional blocks. First, a 12.5 ns delay transmission line delays one of the two pickup signals, and a power combiner time-multiplexes the two signals into a single read-out channel (see Fig. 5.4). The delay of 12.5 ns is half of the bunch-to-bunch time distance in a standard LHC fill.

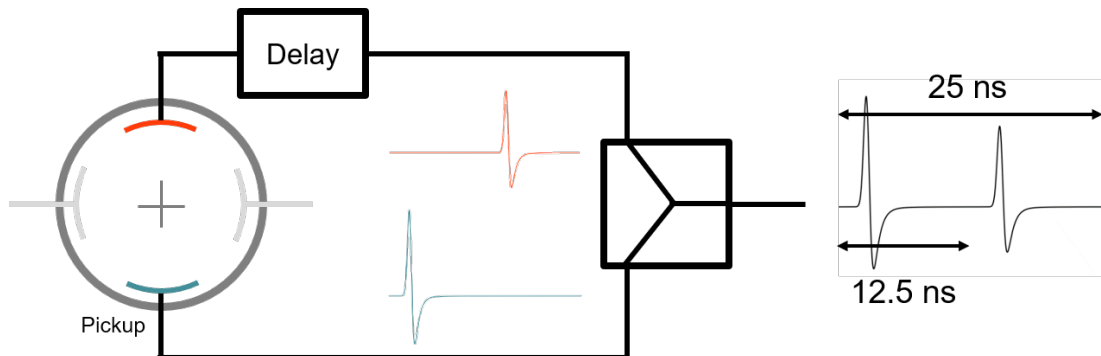


Figure 5.4: Time-multiplexing of the BPM electrode signals.

Second, a band-pass comb filter creates 4 delayed replicas of the signal and combines them. The replicas are delayed with respect to the first one by 2, 4 and 6 ns, resulting in two 8 ns sine-wave like burst signals spaced by 12.5 ns, being the pulse

Chapter 5. Study of the effect of digitisation in the LHC beam extraction interlock BPM prototype

signal from each electrode ~ 2 ns long. The filter is based on delay lines and power combiners, and its centre-frequency is 500 MHz, corresponding to the peak of the response of the stripline. Fig. 5.5 shows the simulation of the filter response to a stripline bunch signal, both in time and frequency domain. In Fig. 5.6 the physical realisation of the analogue processing prototype is pictured.

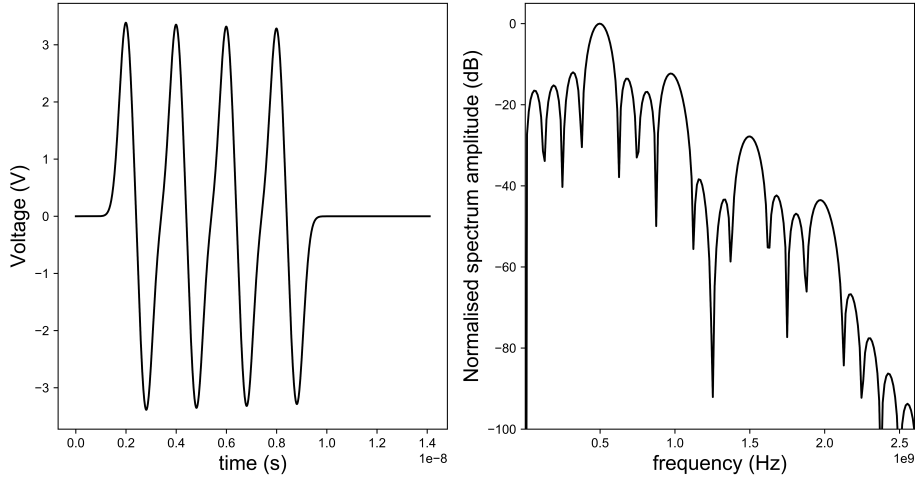


Figure 5.5: Simulation of the comb-filter response to a stripline bunch pulse, in the time and frequency domain.

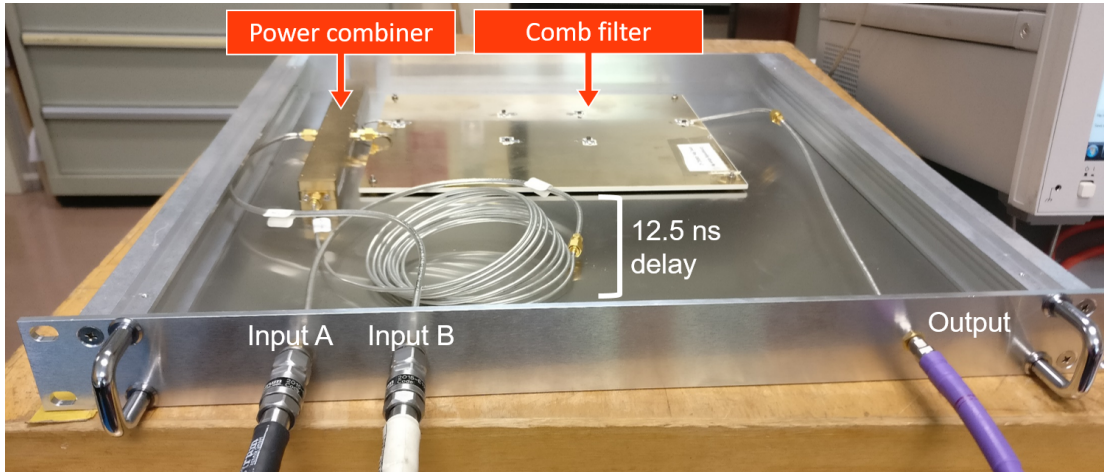


Figure 5.6: Picture of the prototype of the analog front-end.

An anti-aliasing filter can be added as part of the digitisation stage. Since variable gain stages are avoided, only the highest level signals of a full profile of the whole input range of the ADC.

Details about the design of the analog front-end prototype can be found in the Master thesis of Oskar Björkqvist [46].

5.1.2 Digital signal processing

The digital Data Acquisition system (DAQ) is based on an ADC mezzanine card plugged on a Field Programmable Gate Array (FPGA) based carrier board. The mezzanine card performs the digitisation, while the carrier board is responsible for acquisition control, data processing and communication.

The carrier board employed is the general purpose VME FMC Carrier (VFC-HD) board, developed by the Beam Instrumentation group at CERN [47, 48]. The board complies with the VME64x standard [49, 50], and it is to be used in VME crates for power, control and communication. Among the main features, it is relevant to note that the board hosts an Intel Arria V GX FPGA (5AGXMB1G4F40C4N) and it has a slot for one High Pin Count (HPC) FPGA Mezzanine Card (FMC) [51]. There are six optical transceivers' slots, of which one is dedicated to the reception of the optically transmitted Beam Synchronous Timing (BST), a stream of encoded messages distributing the timing of the machine and the triggers for specific events occurring during the fill [52]. The VFC-HD with its main components is pictured in Fig. 5.7.

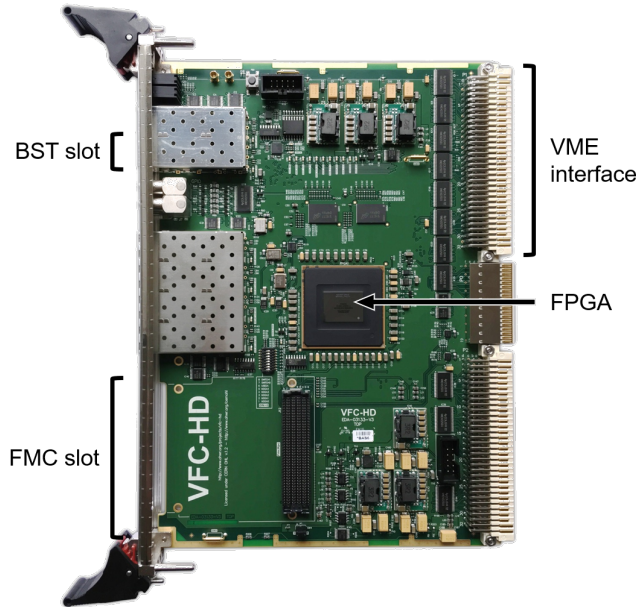


Figure 5.7: *The VFC-HD carrier board.*

The FMC standard connector makes possible the use of commercial mezzanines for the digitisation stage. Given the signal spectrum, the interest was for ADCs with sampling rate beyond 2.5 GSps. An upper limit on the sampling rate is imposed by the FPGA of the VFC-HD, whose transceivers' maximum speed is 6.5536 giga bits per second (Gbps). Two mezzanines with different ADCs were tested, one with higher sampling rate and the other with better resolution.

The first is the Vadatech FMC225 [53] (see Fig. 5.8a), that utilises the TI ADC12J4000 ADC, providing 12-bit conversion at rates of up to 4.0 GSps [54]. The converter's datasheet declares 8.8 ENOB and the connection to the VFC-HD allows a maximum sampling rate of 3.2 GSps.

The second is the HTG Dual 14-bit ADC (3 GSps)/ Dual 16-bit DAC (12.6 GSps) FMC Module [55] (see Fig. 5.8b), that has the 14-bit AD9208 ADC [56], able to reach up to 3 GSps sampling rate. The chip has higher resolution, the datasheet declares 9.6 ENOB, but on the VFC-HD can be used only up to a sampling rate of 2.6 Gsps.

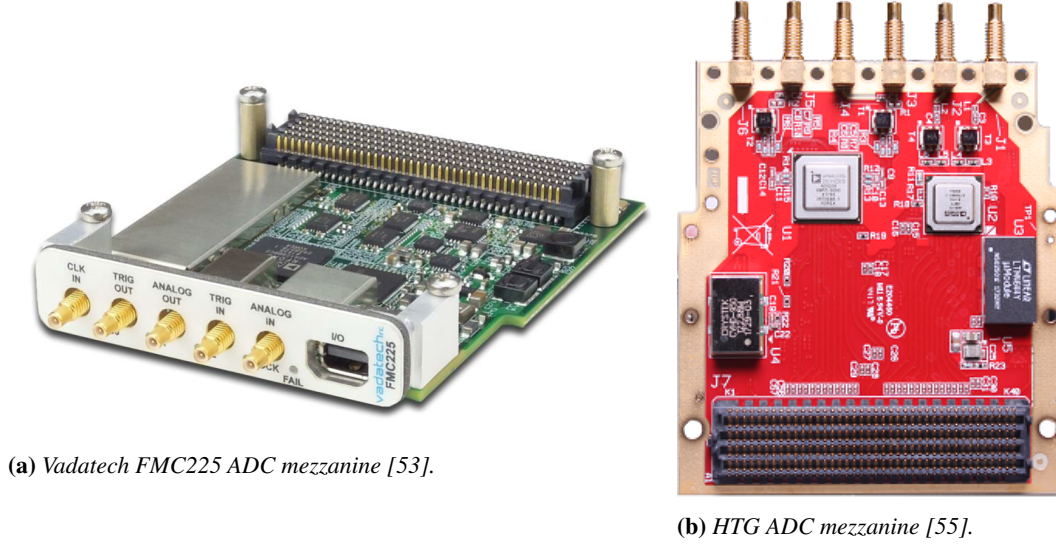


Figure 5.8: Pictures of the two tested ADC mezzanine cards.

5.2 Measurement set-ups

The prototype was used to acquire actual beam signals during run 2 of the LHC. This section describes a selection of three representative measurement set-ups and their differences. All the set-ups here presented used a single electrode split in two as signal source. Hence, the acquisition was decoupled from the actual beam movement. With the use of attenuators in different points of the acquisition chain (see Fig. 5.9) it was possible to emulate beam displacements and changes in beam intensity in a controlled way.

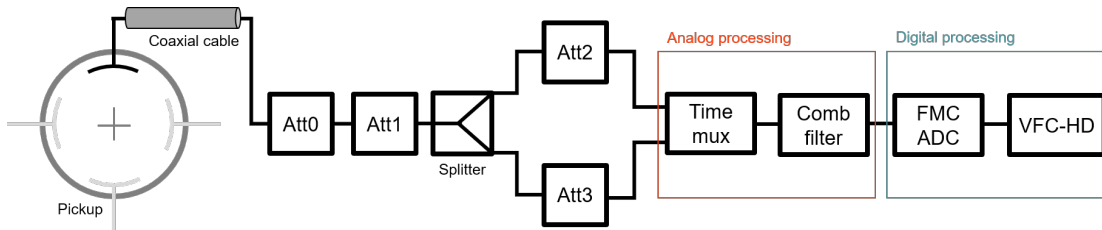


Figure 5.9: Schematic representation of the measurement set-up.

The set-ups differ for the pickup type and location used as signal source, and for the ADC mezzanine used for digitisation. The firmware of the carrier board implemented oscilloscope-like acquisition, in combination with the possibility to trigger the acquisition from BST events, and to segment the acquisition in periodic windows of defined duration and synchronised with the revolution frequency of the beam. Thus it was pos-

sible to observe and save the signals of specific groups of bunches over an extended number of turns, optimising the acquisition to the memory depth and bandwidth.

The processing, including the accurate detection of bunch margins required for the position calculation, was performed off-line. For the position calculation the selected algorithm was the calculation of the amplitude of each electrode pulse as RMS value of the samples [24], followed by the implementation of the Δ/Σ normalisation (see chapter 2 and Eq. (2.10)).

5.2.1 Measurement Set-up A: button pickup with high-sampling rate ADC

The first measurement campaign was performed on a SPS' button pickup the 16th of June 2017. The measurements were performed during LHC's filling cycles. The signal source was the up electrode of the vertical plane of the button pickup, connected through a circa 100 m coaxial cable. The ADC mezzanine was the Vadatech FMC225. The values of the attenuator used to emulate changes in beam intensity (*Att0* in Fig. 5.9) were 0, -3 dB and -6 dB. One attenuator before the splitter (*Att1* in Fig. 5.9) and the two attenuators after the splitter (*Att2* and *Att3* in Fig. 5.9) are used as an ensemble in order to emulate changes in beam position, maintaining the emulated beam intensity constant. Table 5.1 summarises the values of the attenuators used to emulate three beam displacements, quantified as the variation of the signal intensity in dB on the first electrode. Since the change in signal level was not induced by actual beam displacement, the value in dB is more interesting and can be translated in equivalent millimeters scaling it with the pickup sensitivity.

Displacement (dB)	Att1 [dB]	Att2 [dB]	Att3 [dB]
central (0)	0	-4	-4
positive (+1)	-2	-1	-3
negative (-1)	-2	-3	-1

Table 5.1: Attenuators' value settings for beam displacement emulation in measurement set-up A

5.2.2 Measurement Set-up B: stripline pickup with high-sampling rate ADC

Another set of measurements was performed on an actual LHC stripline pickup, on September 7th 2018. The measurements were performed at flat top, i.e. when the beam had reached its top energy. The signal source was the up electrode of the vertical plane of a stripline pickup connected through a coaxial cable about 70 m long. The ADC mezzanine was the Vadatech FMC225. The set of attenuator's values to emulate changes in beam intensity (*Att0* in Fig. 5.9) were 0 and -20 dB. For the emulation of beam displacement, Table 5.2 summarises the attenuators' values and the corresponding displacements in dB.

Displacement (dB)	Att1 [dB]	Att2 [dB]	Att3 [dB]
central (0)	-2	0	0
positive (+2)	0	0	-4
negative (-2)	0	-4	0

Table 5.2: Attenuators' value settings for beam displacement emulation in measurement set-up B and C

5.2.3 Measurement Set-up C: stripline pickup with high-resolution ADC

The last set of measurements here presented was performed, on October 26th 2018, on the same LHC stripline pickup of Set-Up B, and also in this case the measurements were performed at flat top. The signal source was for this set of measurements the down electrode of the vertical plane, also connected through a coaxial cable about 70 m long. The ADC mezzanine was the HTG Dual 14-bit ADC / Dual 16-bit DAC FMC. The set of attenuator's values to emulate changes in beam intensity ($Att0$ in Fig. 5.9) were 0, -10 dB and -20 dB. For the emulation of beam displacement, the set of attenuators was the same of the measurement set-up B, reported in Table 5.2.

5.3 Parametric simulation framework

A simulation framework was designed in order to better understand the dependences of the whole system performance on each of its components.

The framework is modular: the system is emulated as a chain of elements, and each of them is separately modeled.

The framework is parametric: each module has its own parameters and the whole system is configured at a system level to define which elements and in which order are part of it.

The modules are organised in four macro-groups: the pickup modules, the analog conditioning modules, the analog to digital converter modules and the digital signal processing modules. The structure is represented in Fig. 5.10.

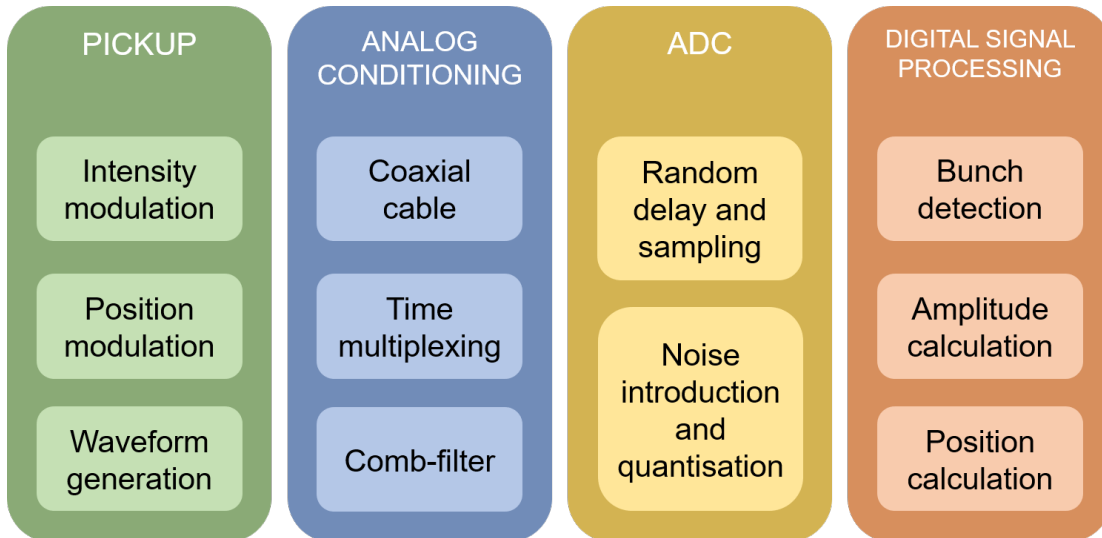


Figure 5.10: Main modules of the simulation framework and their organisation in macro-groups.

At a higher level, the beam test group of functions defines the system configuration and generates parametric beam tests, inserting sources of randomness in the digitisation stage. The parameters define the elements of the chains and their characteristics. The inputs are the beam's position and intensity. The output is the distribution of the error of the position measurement, characterised by its mean value and variance.

The language used is Python [57].

5.3.1 Pickup modules

The pickup modules simulate the waveforms generated by the four electrodes of a pickup, in response to the passage of a particle bunch. The kind of pickup and its geometry are the parametric information. The bunch intensity and transverse position are the input information.

The pickup electrode signal is described as the product of three elements: the bunch intensity N_b , the position modulation function $s(x, y)$, and the signal waveform $w_{PU}(t)$ defined by the bunch shape and by the pickup's impulse response.

$$V_{PU}(N_b, x, y, t) = N_b \cdot s(x, y) \cdot w_{PU}(t) \quad (5.1)$$

Intensity and position modulation

The bunch intensity N_b , the number of charged particles first defined in section 2.2, is directly an input information.

The position modulation function $s(x, y)$ varies for each of the four electrode signals. It is based on the solution of the 2-dimensional problem presented in section 2.2, resulting in the expressions (2.6) and (2.7) of the current induced on two opposite electrodes by a beam displaced off the centre. Referring to the same nomenclature introduced in section 2.2, α is the angular width of the pickup electrode and R the beam pipe radius; both are parameters of the simulation. The bunch position is in Cartesian coordinates (x, y) , corresponding in cylindrical coordinates to $(r = \sqrt{x^2 + y^2}, \theta = \arctan(y/x))$; the pickup electrodes are placed as represented in Fig. 5.11.

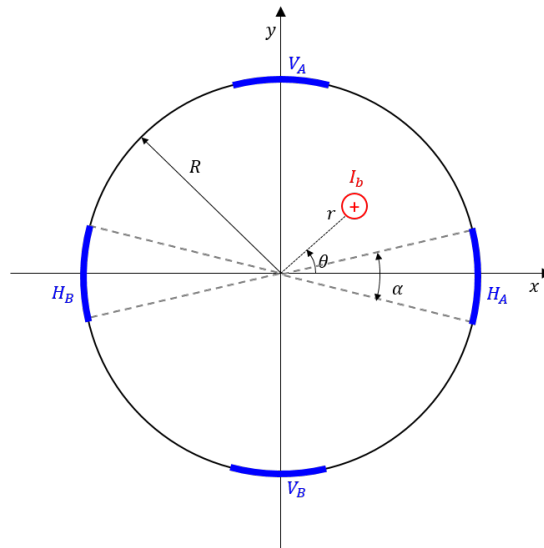


Figure 5.11: Reference system in Cartesian coordinates for the definition of position modulation function the $s(x, y)$ for the four electrodes, the two horizontal H_A and H_B and the two vertical V_A and V_B .

The position modulation function $s(x, y)$ for the positive horizontal electrode is:

$$s_{H_A}(x, y) = \frac{1}{\pi} \arctan \left[\frac{R + \sqrt{x^2 + y^2}}{R - \sqrt{x^2 + y^2}} \tan \left(\frac{-\arctan(y/x) + \alpha/2}{2} \right) \right] - \frac{1}{\pi} \arctan \left[\frac{R + \sqrt{x^2 + y^2}}{R - \sqrt{x^2 + y^2}} \tan \left(\frac{-\arctan(y/x) - \alpha/2}{2} \right) \right] \quad (5.2)$$

Given the symmetry of the system, the functions for the other electrodes are derived from (5.2):

$$s_{H_B}(x, y) = s_{H_A}(-x, y) \quad (5.3)$$

$$s_{V_A}(x, y) = s_{H_A}(y, -x) \quad (5.4)$$

$$s_{V_B}(x, y) = s_{H_A}(-y, x) \quad (5.5)$$

It is also possible to configure the module so to implement the linearised version of the function, in which the non-linearity of the pickup is neglected and the horizontal plane electrodes are only affected by horizontal displacements and vice versa for the vertical plane (See Eq. (2.8) and (2.9)). The linear monodimensional position modulation function $s_{\text{lin}}(x)$ for the positive horizontal electrode becomes:

$$s_{H_A, \text{lin}}(x) = \frac{1}{\pi} \left(\frac{\alpha}{2} + 2 \sin \left(\frac{\alpha}{2} \right) \frac{x}{R} \right) \quad (5.6)$$

For the other electrodes, expressions (5.3), (5.4) and (5.5) stay valid, neglecting the second variable.

Waveform generation

To define the waveform as function of time $w_{PU}(t)$, the bunch is assumed Gaussian with standard deviation σ_b (see Eq. (2.2)) and two parametric models of the pickup are implemented.

The first is the stripline pickup model, introduced in section 2.4.2. The parameters are: the characteristic impedance of the transmission line Z_0 ; the stripline length ℓ , that defines the average propagation delay τ_ℓ (see Eq. (2.4.2)), where the fraction of the lightspeed at which the particle beam moves and the stripline signal propagates are both assumed equal to 1. The function, the same all four electrodes, follows:

$$w_{PU, SL}(t) = \frac{Z_0}{2} \frac{q}{\sqrt{2\pi}\sigma_b} \left\{ e^{\frac{-(t+\ell/c)^2}{2\sigma_b^2}} - e^{\frac{-(t-\ell/c)^2}{2\sigma_b^2}} \right\} \quad (5.7)$$

where q and c are respectively the single particle charge and the constant speed of light.

The second pickup model produces the waveform of the button pickup, first introduced in section 2.4.1. The module uses the transfer function of the button (see Eq. (2.11)), with parameters the geometric and electric properties of the button electrode (its capacitance C and length ℓ , and its characteristic impedance Z_0). The waveform $w_{PU, B}(t)$ is the result of the convolution of the relativistic Gaussian bunch current with the inverse Fourier transformation of the button impedance transfer function:

$$w_{PU, B}(t) = \mathcal{F}^{-1} \left\{ Z_0 \frac{\ell}{c} \frac{j\omega}{1 + j\omega Z_0 C} \right\} * \frac{q}{\sqrt{2\pi}\sigma_b} e^{\frac{-t^2}{2\sigma_b^2}} \quad (5.8)$$

The linear filter function from the signal processing library of the Python-based ecosystem SciPy [58] is used to obtain the time response to a Gaussian bunch.

The waveforms produced by both models were compared with the results of electromagnetic field simulations. The used solver is CST Studio Suite, that provides the Wakefield solver to specifically calculate the three-dimensional fields around a particle beam [59]. Fig. 5.12 show the 3D models of the stripline BPM and of the button BPM. The simulated waveforms were used to calibrate the amplitude of the analytic waveforms. The comparison results for a bunch of 10^{10} charges are shown in Fig. 5.13.

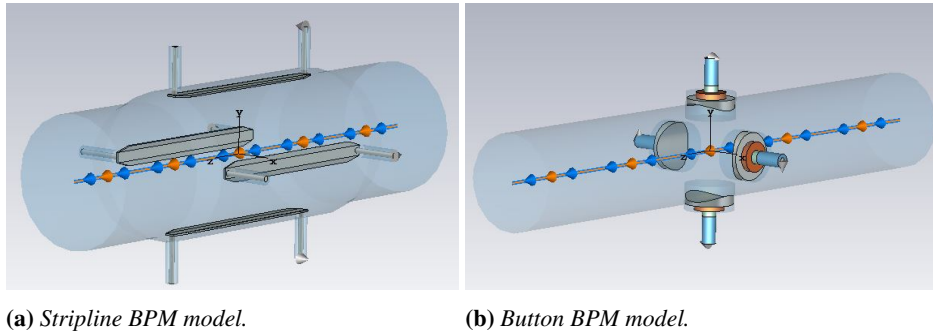


Figure 5.12: Three-dimensional models of the BPM pickup in the CST Studio Suite simulation environment,

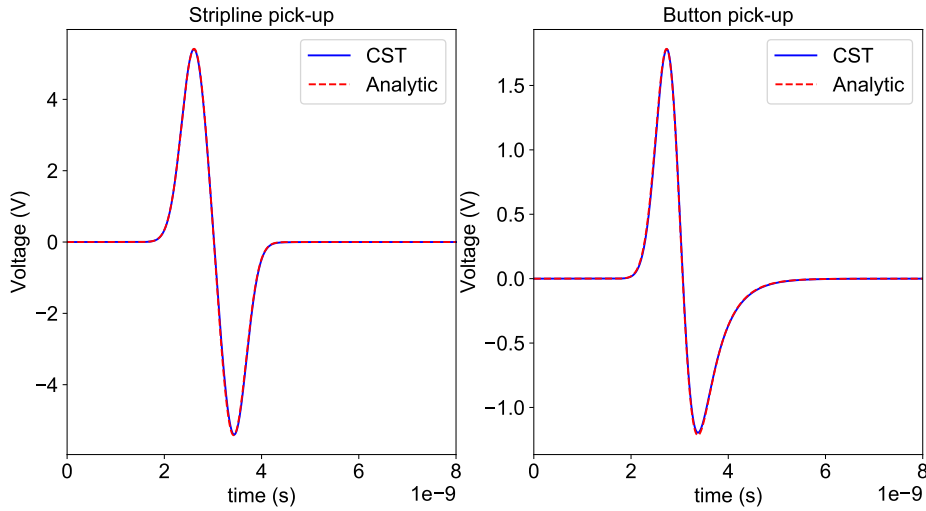


Figure 5.13: Comparison between the waveform generated by the analytic model and the waveform result of the electromagnetic solver simulation, for the stripline pickup and the button pickup, expressed in voltage amplitude versus time.

5.3.2 Analog conditioning modules

All the processing stages between the pickup and the ADC find place in the group of the analog conditioning modules.

Coaxial cable

The coaxial cable that connects the pickup to the processing electronics, often tens of meters away, is a necessary element of the acquisition chain and it has an effect on the signal shape. The simulation framework implements it by convolution of the signal with the simplified analytic expression of the cable impulse response, assuming negligible the leakage current between the inner and the outer conductors.

The cable transfer function in the frequency domain is expressed as [60]:

$$H_{\text{cable}}(f) = e^{-b_c \ell_c \sqrt{j4\pi f}} \cdot e^{-j2\pi f \ell_c / v_c} \quad (5.9)$$

where ℓ_c is the cable length, v_c is the velocity of propagation along the cable, and b_c is a constant depending on the cable electric properties and on the dimensions of its transverse section. They are all parameters of the simulation module; usually the cable length is known, the velocity of propagation is provided by the cable's datasheet and the constant b_c is the result of fitting the transfer function model on the measured transfer function of the cable, when available, or on the transfer function provided by the datasheet. As example, Fig. 5.14 shows the transfer function of the cable used in the measurement set-up B and C; the measured magnitude and phase of the transfer function are compared with the analytic model.

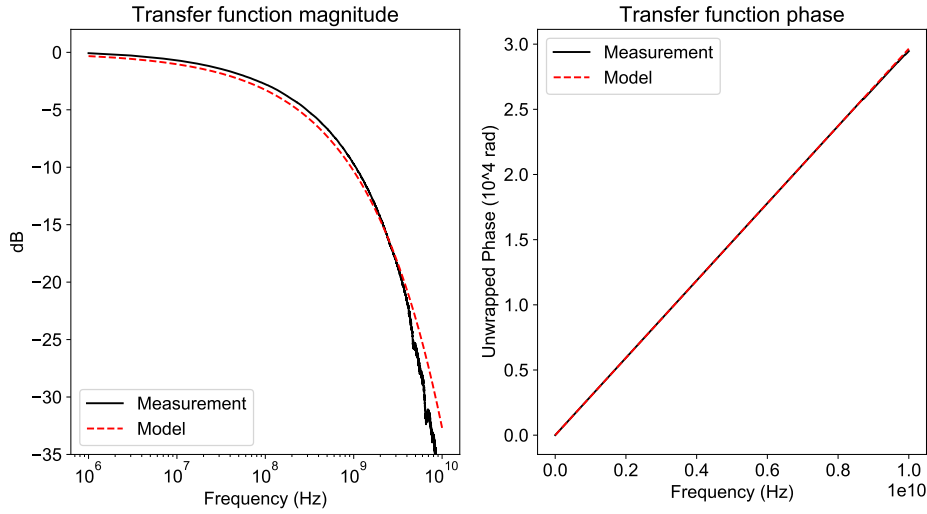


Figure 5.14: Magnitude and unwrapped phase versus frequency of the transfer function of the cable connected to the stripline used in set-up B and C, as result of the measurement and of the analytic model.

The cable impulse response can be calculated from (5.9) as [60]:

$$h_{\text{cable}}(t) = \frac{1}{t - \ell_c / v_c} \cdot \frac{b_c \ell_c}{\sqrt{2\pi(t - \ell_c / v_c)}} \cdot e^{-\frac{b_c^2 \ell_c^2}{2t - \ell_c / v_c}} \quad (5.10)$$

The function (5.10) is convoluted with the pickup signal to simulate the effect of a long coaxial cable (see Fig. 5.15).

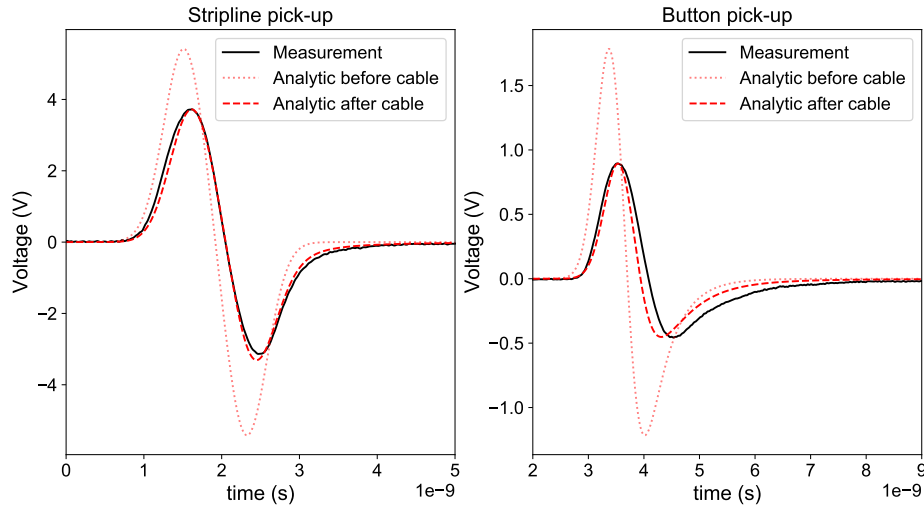


Figure 5.15: Analytic expression of the pickup response to a bunch signal before and after convolution with the cable pulse response, compared with the measurement of the signal of the pickup after the cable, acquired by oscilloscope. The plots show voltage amplitude versus time. Both the stripline case and the button case are represented, in each the cable model was adapted to the specific cable's properties.

Time-multiplexing

The combination of the two pickup electrode signals in a single signal is a straightforward logic function to implement. Its physical realisation can have some minor collateral effects on the signal quality and shape, but the interest of the study was focused on the effects of the digitisation part. Therefore, this module is implemented as an ideal logic function, with the possibility to assume an amplitude attenuation of the delayed signals.

The parameters of the module are: the multiplexing configuration (if one plane or the two planes are combined together and in which order); the delay between the combined signals and the attenuation of the delayed signals.

The signals are not cut, they are summed so that the tail of each pulse affects the following one, like in the actual implementation.

Comb-filter

The principle at the base of the comb-filter is similar to the time-multiplexing scheme. The main difference is that in the case of the comb-filter the input signals are attenuated replicas of the same signal. Also in this case the logic function is implemented as ideal, with the only addition of parametric attenuation. Other parameters are the number of replicas and the delay between each replica.

5.3.3 Analogue to Digital Converter modules

The digital conversion is divided in two logic functions: sampling and encoding. They are by design the sources of error and randomness in the simulation framework on the time axis and on the amplitude axis, respectively.

Sampling

The first module of the digital conversion implements the sampling function, selecting the samples of the signal according to the sampling rate and the sampling delay, given as parameter.

Since the framework is numeric, the input signal is itself a discrete sequence, therefore the sampling operation is not infinitely precise and the sampling rate emulated is an approximation of the required sampling rate. The smaller is the time step of the digital input sequence, the better is the sampling rate accuracy, at the expense of the processing time. As an example, with a time step of 1 ps, the error in the emulated sampling rate is: for a sampling rate of 2.6 GHz, about 2.6 MHz; for a sampling rate of 3.2 GHz, about 5.1 MHz. The discrepancy is in most cases negligible. Nevertheless, if the sampling rate must be emulated with the greatest accuracy, the time step can be set accordingly in the pickup module to be an integer multiple of the sampling period.

The source of randomness in the sampling module is the definition of the sampling delay. For each input signal the module generates multiple sequences of the same number of samples equally distanced in time; each sequence is a possible outcome of the pulse sampling. The sequences can differ for the position of the first sample, i.e. the start point, defined by the random sampling delay. The number of the output sequences is a parameter of the test.

The module implements two modes of operation: the synchronous mode and the asynchronous mode. In the synchronous mode the sampling delay is the same for each sequence of samples, emulating the case in which the ADC clock signal is perfectly synchronised with the start point of the signal to sample. The asynchronous mode, instead, emulates the case in which the ADC clock signal runs asynchronously with the input signal, so that the starting point for each sequence of samples is uniformly distributed in the interval of a sampling period.

Encoding

The encoding module receives the array of sampled signals generated by the sampling module as input. The parameters are the ADC input range in volts (V_{FSR}), the number of bits of the digital converter (B) and the effective number of bits (ENOB) describing the effective converter resolution.

As already introduced in chapter 3, the number of bits indicates the depth of the word used to encode each sample value, meaning that there is a minimum step between two distinguished values and that each input value is approximated by the closest multiple of this minimum voltage step.

On top of this error, known as quantisation error, the module introduces further uncertainty, in order to take into account the other noise sources of the converter. The information about the noise of the converter is provided by the ENOB value, that quantifies the number of bits of an equivalent converter with the same noise average power, but with only quantisation as noise source. The standard deviation of the quantisation noise of a converter with ENOB bits has been introduced in chapter 3 (see Eq. (3.4)), and used in chapter 4 (see Eq. (4.69)); it is here reported for reference:

$$\sigma_{\epsilon_q} = \frac{V_{FSR}}{\sqrt{12}} 2^{-ENOB} \quad (5.11)$$

The encoding module describes the ADC noise as a Gaussian random variable, with zero-mean and the expression (5.11) as standard deviation. To each sample in each sequence the module adds, before quantisation, a realisation of the noise random variable.

5.3.4 Digital Signal Processing modules

The digital signal processing modules emulate the main functions of the digital electronics that lead to the calculation of each bunch position, starting from the stream of samples provided by the ADC.

Bunch Detection

The first necessary step is the correct bunch detection. To increase the accuracy of the simulation of a system that, in most cases, deals with trains of bunches, the framework generates as input three bunches in a row, distanced by the parametric bunch-to-bunch distance.

The measurement is performed on the central bunch and the bunch detection module has as goal the selection of the samples of the central bunch and the areas of the signal belonging to each of the electrodes. The position of the optimal bunch selection window, window from here after, is not constant when the sampling is asynchronous with the signal, while the bunch-to-bunch distance and the distance between the combined signals are constant and known as parameters of the system. The relative time structure within each sequence is known, only the absolute time reference is to be found.

The optimal window is found by means of signal correlation: each sequence of three bunches signals, processed by the analog conditioning modules and digitised, is correlated with a master signal, generated from a pickup signal conditioned by the same processing chain. The correlation of two deterministic discrete real sequences $x[m]$ and $y[m]$, with M and N samples respectively, and $M \leq N$, is defined as:

$$(x \star y)[n] \doteq \sum_{m=0}^{M-1} x[m] \cdot y[m+n] \quad (5.12)$$

with $n = [0, \dots, N - M]$. In the bunch detection module, $x[m]$ is the master signal and $y[m]$ is any sequence generated by the previous stages of the framework.

As mentioned, the simulation framework generates sequences containing the response to three consecutive bunches, each with two electrode signals time-multiplexed, for a total of six electrode signals. The computation of the correlation function is repeated six times; each time the correlation function produces a maximum, corresponding to the beginning of the samples of an electrode. The area around this peak is muted before proceeding with the next correlation. This procedure produces six local maxima. Each maximum indicates the location in the sequence of the samples of an electrode (see Fig. 5.16). The first sample of the central bunch corresponds to the third peak of the correlation function. Starting from that reference, the samples of the two electrodes of the central bunch can be identified for each sequence (see Fig. 5.17).

Amplitude calculation

As described in section 2.2, the position information is modulated in the amplitude of the two electrodes' signals. In the Interlock BPM prototype the amplitude information

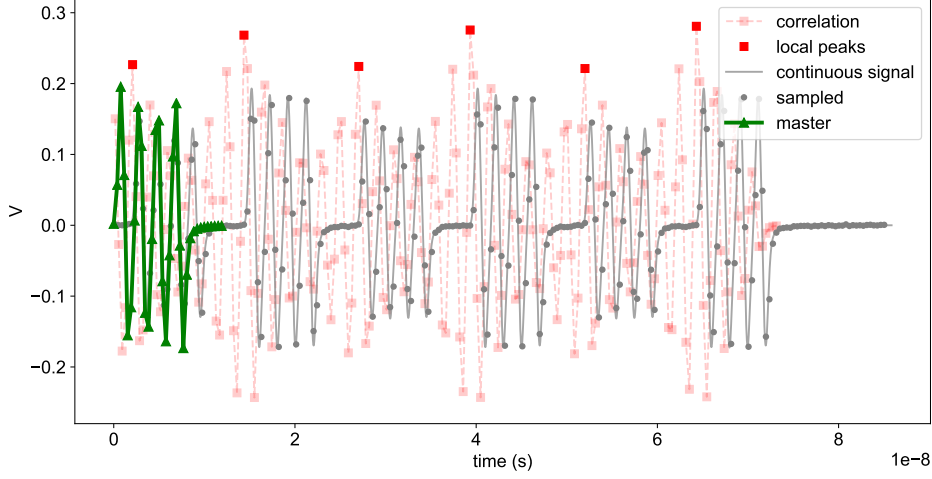


Figure 5.16: Example of correlation function between the master electrode signal and a sequence of samples resulting from the simulation of a train of three bunches processed by the Interlock BPM prototype. The six local peaks are evidenced.

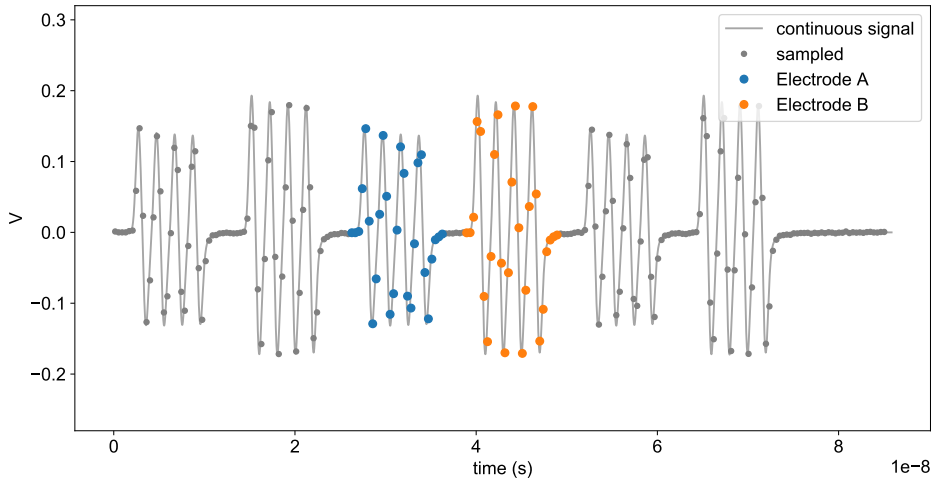


Figure 5.17: Identification of the two electrodes of the central bunch in the sequence.

of each electrode signal is calculated as the RMS value of its samples. The amplitude calculation module calculates the square root of the mean value of the squared samples of each electrode, those selected by the bunch detection module. If $x_{H_A,n}$ are the N samples of the pulse generated by the electrode H_A , the RMS value is defined as:

$$H_{A,\text{RMS}} = \sqrt{\frac{1}{N} \sum_{n=0}^{N-1} x_{H_A,n}^2} \quad (5.13)$$

For each bunch, the amplitude calculation module generates four RMS values ($H_{A,\text{RMS}}$, $H_{B,\text{RMS}}$, $V_{A,\text{RMS}}$, $V_{B,\text{RMS}}$) each proportional to the amplitude of the signal of one of the

four electrodes of the pickup.

Position calculation

Once that for each plane, horizontal and vertical, two values proportional to the two signals' amplitudes are extracted, the difference over sum normalisation formula (2.10) is applied. For the two planes are computed:

$$H_{\text{norm}} = \frac{H_{A,\text{RMS}} - H_{B,\text{RMS}}}{H_{A,\text{RMS}} + H_{B,\text{RMS}}} \quad (5.14)$$

$$V_{\text{norm}} = \frac{V_{A,\text{RMS}} - V_{B,\text{RMS}}}{V_{A,\text{RMS}} + V_{B,\text{RMS}}} \quad (5.15)$$

The position value in units of length is then calculated multiplying the two ratios by a coefficient, dependent on the pickup geometry, that is derived by the monodimensional and linearised position modulation functions (2.10). The coefficient is the same for both the horizontal and vertical plane, since the beam pipe has the same horizontal and vertical dimensions. Naming $\bar{H}$ and $\bar{V}$ the estimated horizontal and vertical position, respectively, they are computed as:

$$\bar{H} = H_{\text{norm}} \cdot \frac{R\alpha}{4 \sin \alpha/2} \quad (5.16)$$

$$\bar{V} = V_{\text{norm}} \cdot \frac{R\alpha}{4 \sin \alpha/2} \quad (5.17)$$

where α is the angular width of the pickup electrode and R is the radius of the beam pipe. These conversion formulas neglect the non linearity of the pickup, so they introduce a systematic error in the position estimation that becomes significant for great displacements. Obviously, this is not the case in the simulation framework if the pickup module directly uses the linear formula to describe the position modulation of the electrode signals.

The distribution of the error, defined as the difference between the input position used to generate the test array and the estimated position, is a measure of the expected performance of the system, for the defined beam conditions and acquisition configuration (see Fig. 5.18).

5.4 Beam measurement and simulation results

The measurements was performed on two different types of pickups: a SPS button pickup and a LHC stripline pickup. Two ADC mezzanines were tested (Vadatech mezzanine and HTG mezzanine), the first with higher sampling rate, the second with higher resolution. All the set-ups used the two electrodes per signal time-multiplexing scheme and the comb-filter with four 2 ns spaced repetitions. The output of the comb-filter was directly digitised. The beam raw data was analysed off-line to extract the position information and calculate the measurement error in terms of resolution. The prototype characteristics have been recreated in the simulation framework for the three different acquisition set-ups described with detail in section 5.2. The measurement analysis and the simulation results are here compared.

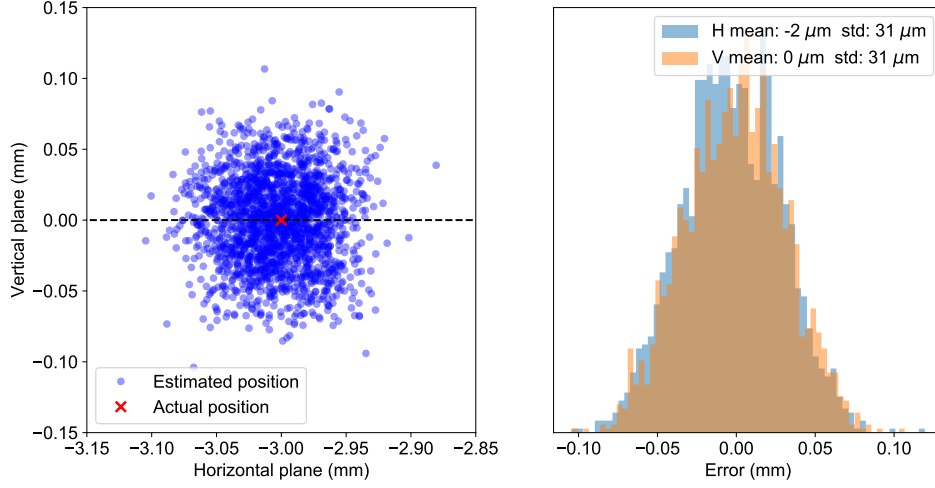


Figure 5.18: Example of the estimated position distribution calculated for a configuration replicating that of the measuring set-up C. For this implementation the simulation framework generated 2000 repetitions, for a bunch of $1 \cdot 10^{11}$ charges, central vertical position and -3 mm horizontal displacement. On the left a graphical representation of the position estimation for each repetition, on the right the histograms of the position error for the two planes, with in the legend the calculated mean values and the standard deviations.

The main set-up parameters implemented in the simulation framework are summarised in table 5.3.

Parameter	A	B	C
pickup	button	stripline	stripline
electrode azimuthal width (α)	0.98 rad	0.27 rad	0.27 rad
beam pipe radius (R)	24.5 mm	44.5 mm	44.5 mm
cable length	100 m	64 m	64 m
cable propagation velocity	81%	88%	88%
time multiplexing	2 electrodes	2 electrodes	2 electrodes
time multiplexing delay	11.5 ns	12.5 ns	12.5 ns
analog filter	comb	comb	comb
ADC rate	3.2 GSps	3.2 GSps	2.6 GSps
ADC FSR	0.725 Vpp	0.725 Vpp	2.04 Vpp
ADC bits	12	12	14
ADC ENOB	8.8	8.8	9.6

Table 5.3: Parameters settings to recreate the measurement set-up conditions in the simulation framework.

Figure 5.19 shows one bunch with positive offset acquired in the time domain with each of the three measurement set-ups, compared with the simulated signal before and after digitisation.

The acquisitions are decoupled from the beam motion, as explained in 5.2, and all changes in position and intensity are emulated and controlled by use of attenuators. The system must have enough margin in its input range to guarantee the absence of saturation, and therefore the intensity is scaled such that two thirds of the ADC full scale

5.4. Beam measurement and simulation results

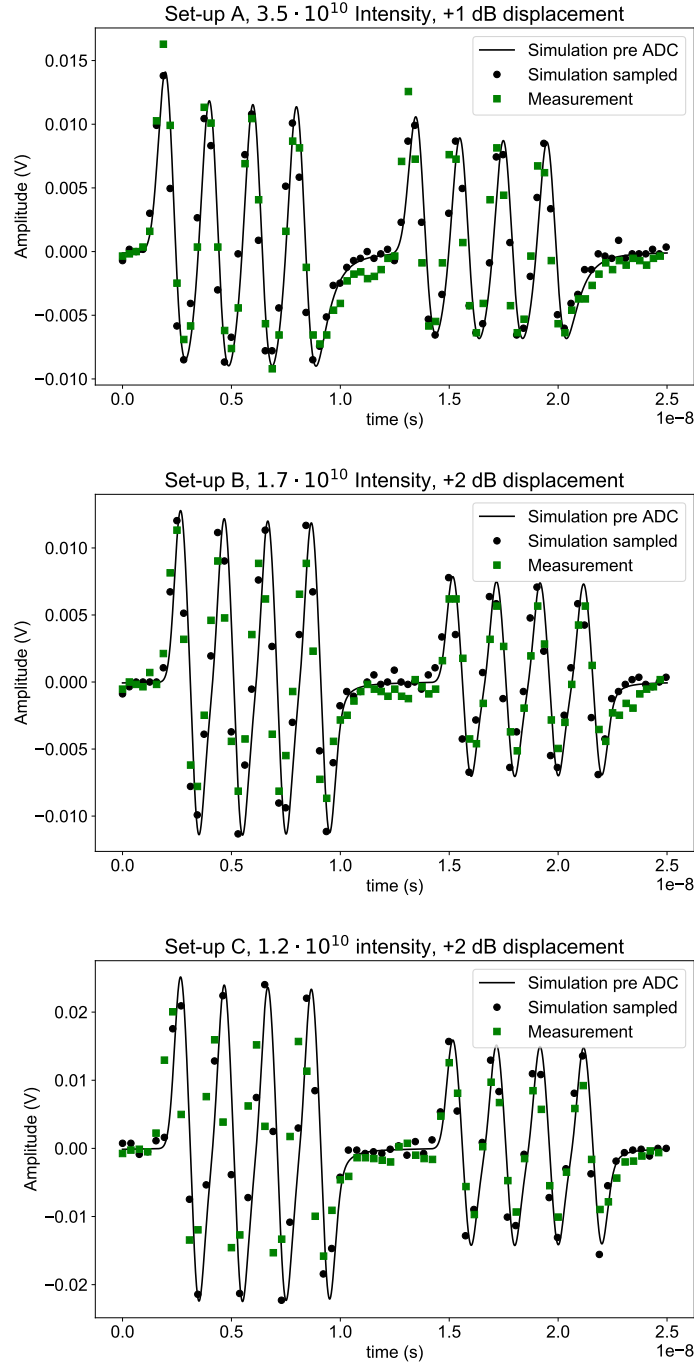


Figure 5.19: Acquisition with a prototype of the Interlock BPM electronics of a bunch from a SPS button pickup with the Vadatech mezzanine (top), and of a bunch from a LHC stripline with the Vadatech mezzanine (center) and the HTG mezzanine (bottom), compared with the correspondent simulation. The signals are expressed in volts at the ADC input versus time.

range could accommodate a bunch with $3 \cdot 10^{11}$ charges per bunch intensity and +7.5 mm displacement, maximum required intensity and measureable displacement. We define $A_{N_b,d}$ the peak of the amplitude of an electrode signal in response to a bunch of equiv-

alent charge number N_b to be established, and displacement d . With the given scaling $A_{3E11,7.5}$, later referred to as $A_{\max,7.5}$, is $0.67 \cdot V_{\text{FSR}}/2$. The equivalent displacement d , in millimeters, is determined by the attenuators and the pickup linearised sensitivity s_{dB} , i.e. the factor that converts beam displacement into pickup signal level variation in dB. For each bunch signal of measured peak amplitude $A_{N_b,d}$ on the electrode closest to the bunch, and known displacement d , the equivalent intensity, as particles per bunch, is calculated as follows:

$$N_b = 3 \cdot 10^{11} \frac{A_{N_b,0}}{A_{\max,0}} \quad (5.18)$$

where $A_{N_b,0}$ and $A_{\max,0}$ are respectively the peak of the signal's amplitude if the bunch had the same intensity but displacement equal to zero, and the peak of the signal's amplitude if the bunch had the maximum intensity $3 \cdot 10^{11}$ and displacement equal to zero. These two values are computed as:

$$A_{N_b,0} = A_{N_b,d} \cdot 10^{-s_{\text{dB}}|d|/20} \quad (5.19)$$

$$A_{\max,0} = A_{\max,7.5} \cdot 10^{-s_{\text{dB}}|7.5|/20} \quad (5.20)$$

For each acquisition set, defined as the data from a selected group of bunches acquired for several consecutive turns and with fixed emulated bunch intensity and position, a single bunch per turn was selected. The data of the selected bunch, i.e. its 25 ns window in the asynchronously acquired turn, is identified with the correlation method explained earlier in the chapter. Figure 5.20 shows as example the result of the selection of a single bunch over 1638 acquired turns, with the measurement set-up C. It can be noted the amplitude difference between the signal from the two electrodes, indicating here an emulated positive displacement.

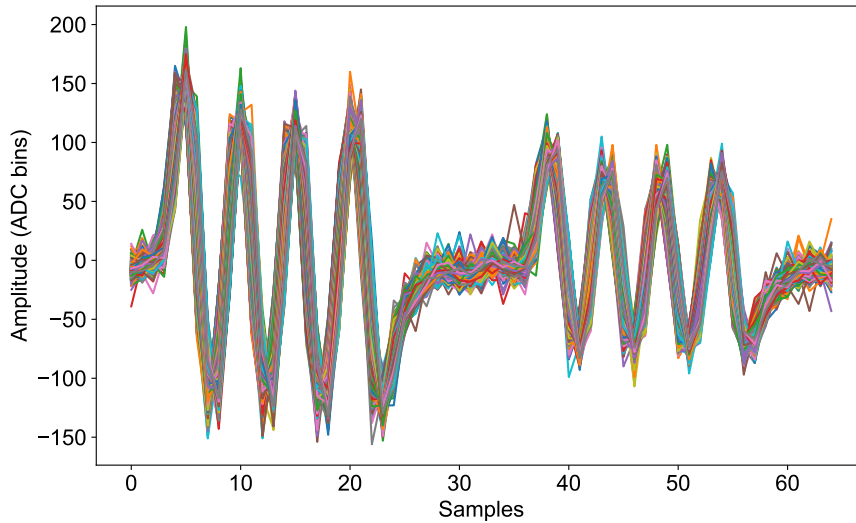


Figure 5.20: Acquisitions of the same bunch ($1.2 \cdot 10^{10}$ equivalent intensity, positive displacement) over 1638 turns, aligned via correlation. The acquisition is shown in ADC bins per ADC sample.

5.4. Beam measurement and simulation results

The 25 ns window of each acquisition was divided in two halves, one per electrode. The mean root square value of each electrode was computed (see Eq. (5.13)) and the position was evaluated as the ratio of the difference and the sum of the two RMS values (see Eq. (5.14)), multiplied by a factor dependent on the geometry of the specific pickup (see Eq. (5.16)). The parameters used are summarised in table 5.4. The pickup sensitivity used for the button measurements is the one of the LHC 24 mm diameter button. The SPS button was used as accessible button pickup, but the system is thought for the LHC machine. Since the displacement is emulated and not related to the actual beam position, this data is used as projection of the expected performance for an LHC button pickup.

Parameter	A	B	C
pickup sensitivity s_{dB}	0.6556 dB/mm	0.3892 dB/mm	0.3892 dB/mm
# turns per acquisition	2048	2048	1638
Intensity steps	0, -3 dB, -6 dB	0, -20 dB	0, -10 dB, -20 dB
Displacement steps	0, ± 1 dB	0, ± 2 dB	0, ± 2 dB

Table 5.4: Beam acquisition characteristics and parameters for intensity and position calculation

The result is, for each acquisition point, a set of calculated single-shot, single bunch beam positions. The standard deviation of each distribution gives an estimation of the expected one-sigma position resolution of the system in the various conditions.

Figures 5.21, 5.22 and 5.23 show the calculated standard deviation of the error as a function of the bunch intensity for the three measurement setups, together with the simulation results for a central bunch and 2000 repetitions. Each marker on the plots represents the standard deviation of the measurement error results for a specific acquisition settings, as for kind of pickup, ADC mezzanine, emulated bunch displacement and intensity. The simulations cover the intensity range between $3 \cdot 10^9$ and $3 \cdot 10^{11}$ charges per bunch.

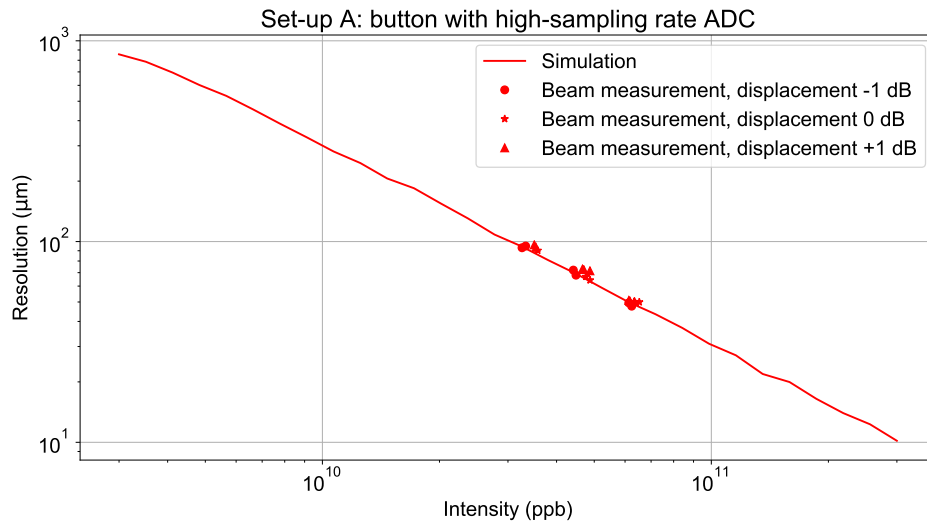


Figure 5.21: Measurements and simulation one-sigma position resolution results for the SPS button set-up with Vadatech mezzanine, as function of the bunch intensity.

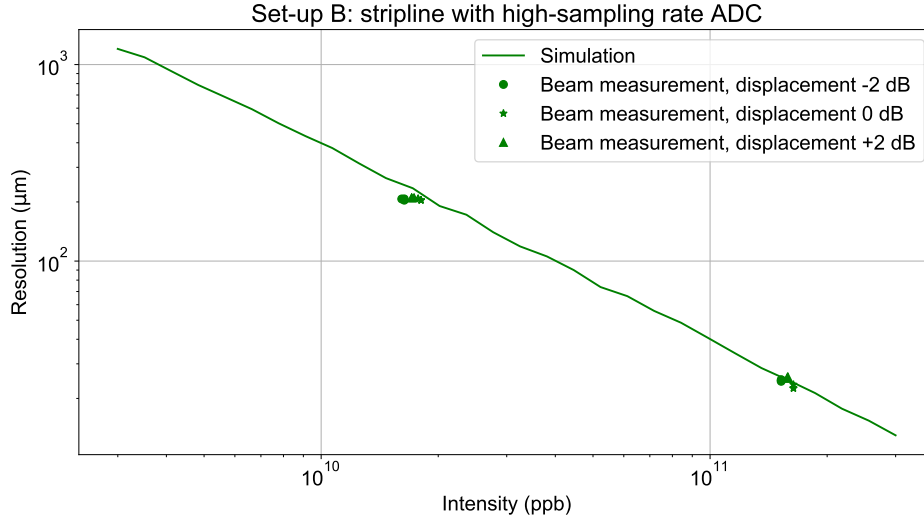


Figure 5.22: Measurements and simulation one-sigma position resolution results for the LHC stripline set-up with Vadatech mezzanine, as function of the bunch intensity.

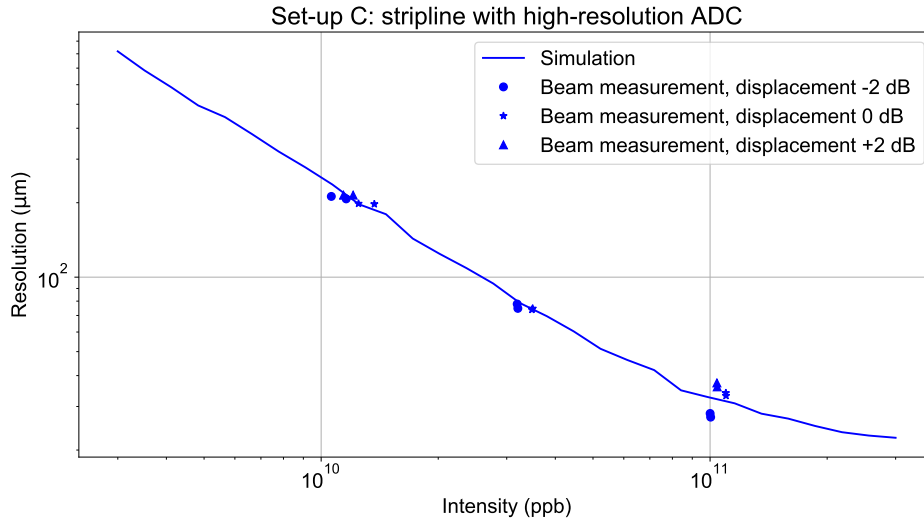


Figure 5.23: Measurements and simulation one-sigma position resolution results for the LHC stripline set-up with HTG mezzanine, as function of the bunch intensity.

The simulations show good agreement with the measurements, both in the signal generation and in the error estimation. Therefore, the simulated results can be used to interpolate the measurements and to better understand their trends.

As the beam signal level decreases linearly with the beam intensity, and since no gain adjustment was applied, the acquisition of low intensity bunches obviously does not use the full input range of the ADC. This is reflected in the degradation of the position resolution, indicating that for low intensity bunches the performance appears to be dominated by the ADC noise.

This dependence can be observed clearly when comparing the resolution results with the three different set-ups (see Fig. 5.24).

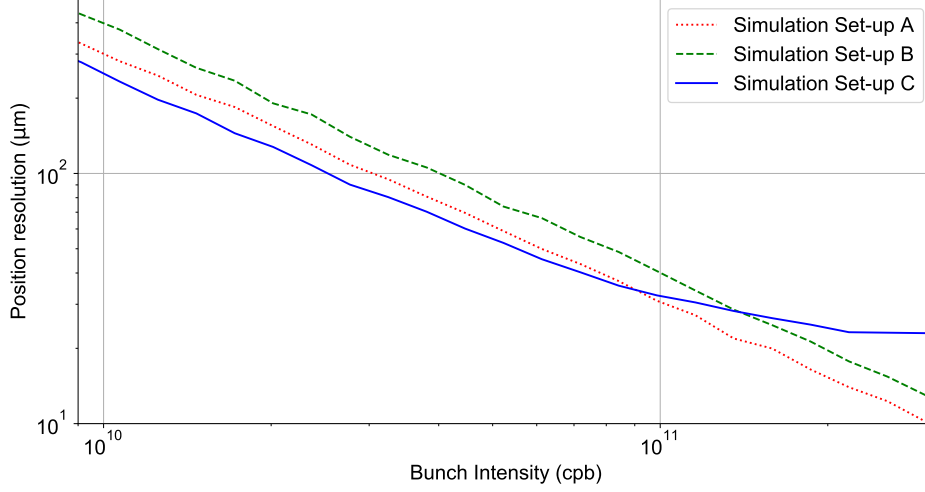


Figure 5.24: Simulation results for the position one-sigma resolution with respect to the bunch intensity for the three measurement set-ups.

Set-up B and C use the same input type, the stripline signal, but it is acquired with two different ADCs: the first with higher sampling rate (3.2 GSps with respect to 2.6 GSps), the second with higher resolution (9.6 ENOB with respect to 8.8 ENOB). The simulated one-sigma resolution of the position measurement of medium and low intensity bunches of set-up B, the one with the lower resolution ADC, is worse by a factor about 1.55, with respect to the other.

This result is consistent in trend and order of magnitude with the approximation that sees the ADC noise as the only source of uncertainty. In this case the position resolution would get better by a factor 2 for every ENOB of the converter, and the noise would improve by a factor $1/\sqrt{N}$, where N is the number of samples over which the ADC noise is averaged. Given two ADCs ($\text{ADC}_1(f_{s,1}, \text{ENOB}_1)$ and $\text{ADC}_2(f_{s,2}, \text{ENOB}_2)$), modeled by their sampling rate (f_s) and their resolution in ENOB, the ratio between the two resulting resolutions (σ_1 and σ_2) can be approximated by:

$$\frac{\sigma_1}{\sigma_2} \approx 2^{\text{ENOB}_2 - \text{ENOB}_1} \cdot \sqrt{\frac{f_{s,2}}{f_{s,1}}} \quad (5.21)$$

Using the specifications of the ADCs in the set-up B and in the set-up C (as reported in table 5.3) in the expression 5.21, the position resolution of the measurement with the Vadatech mezzanine is expected to be according to this approximation 1.57 times worse than with the HTG mezzanine.

This relationship between the two performances does not hold for high intensity bunches, with more than $\sim 5 \cdot 10^{10}$ charges per bunch. The measurements and, more clearly, the simulations, show how the position resolution improves at a constant rate increasing the signal level for the high sampling rate ADC (set-up A and B), and not for the high-resolution ADC with lower sampling rate (set-up C).

This aspect can be investigated with more detail applying to the signal at the input of the ADC the analytic model presented in chapter 4.

5.5 Application of the analytic model

The RMS calculation of each digitised electrode pulse from its samples is a fundamental part of the digital signal processing of the prototype described in the above sections. The RMS value is the square root of the average power; knowing how the average power measurement depends on the ADC characteristics is fundamental in understanding how the position measurement depends on it.

5.5.1 Computation of the SNR of the average power measurement

The model presented in chapter 4 estimates for a known pulse waveform, input of the ADC, the error of its average power measurement, as function of the sampling rate and of the converter resolution. It specifically takes into account the effect of the distortion introduced by sampling at sampling rates that are lower than the Nyquist rate.

The measurements and the simulations have as input signal of the ADC two pulse like waveform: they are both processed in the same way, with time multiplexing and comb-filtering, but one is generated by a button pickup (set-up A) and the other by a stripline pickup (set-up B and C). The two waveforms generated by the simulation framework and the amplitude of their spectra calculated by DFT, are represented in Fig. 5.25.

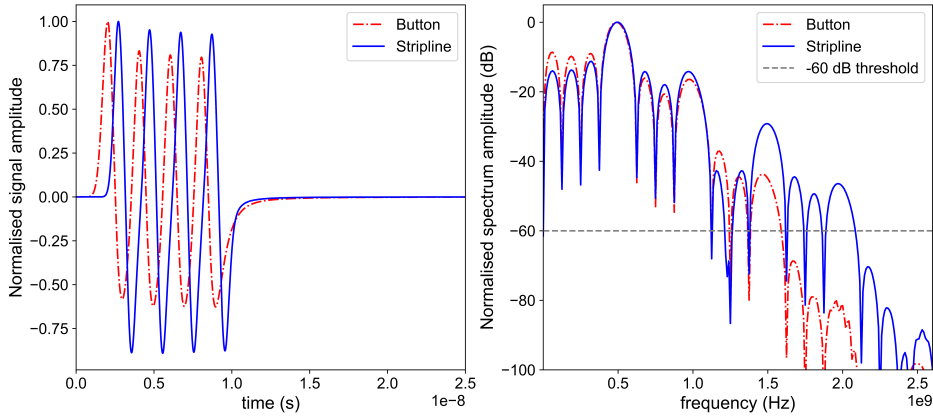


Figure 5.25: Single pulse waveforms at the input of the digitisation in the time domain (left) and in the frequency domain (right). They represent the signals generated by a bunch passing through a button pickup, and a stripline pickup, with the characteristics respectively of the measurement set-up A and B/C. The signals and the spectra are normalised with respect to their maximum value. In the plot of the frequency domain representation is marked the threshold line at -60 dB.

Fig. 5.25 shows, in the frequency domain plot, at which frequency each signal's spectrum module reaches -60 dB with respect to its own peak: about 1.6 GHz for the button signal, and about 2.1 GHz for the stripline signal. Therefore, the Nyquist rate defined relatively to the -60 dB threshold is 3.2 GSps for the button signal and 4.2 GSps for the stripline signal.

Following the procedure of section 4.2 to estimate the error $\epsilon_{P_{T,\tau}}$ of the average power measurement introduced by a limited and asynchronous sampling rate, the coefficients $A_{X_k,N}$ and $B_{X_k,N}$ (see Eq. (4.25)) are computed for both signals. The variance

of the error $\epsilon_{P_{T,\tau}}$, i.e. σ_ϵ^2 (4.32), is then known for each signal for sampling rates between half of the Nyquist rate and the Nyquist rate.

The contribution of the ADC noise to the error on the average power measurement is expressed by the error term η (see section 4.3), function of the standard deviation of the ADC noise on each sample (σ_ν) and of the signal spectrum. The variance of the error η , i.e. σ_η^2 , can be approximated by Eq. (4.56), function of the standard deviation σ_ν , of the sampling rate and of each signal averaged power.

The standard deviation σ_ν for each ADC is modeled as function of their ENOB and their input voltage full scale range V_{FSR} (see Eq. (4.69)). Remembering that for the system it was imposed that the maximum amplitude at the ADC input of a signal generated by a bunch with maximum charge intensity, i.e. $3 \cdot 10^{11}$ charges, and a displacement of 7.5 mm would be two thirds of the full scale, σ_ν can be expressed as Eq. (4.69), replacing V_{FSR} with $2A_{\text{max},7.5}/0.67$:

$$\sigma_\nu = \frac{2 \cdot A_{\text{max},7.5}}{0.67} \cdot \frac{2^{-\text{ENOB}}}{\sqrt{12}} \quad (5.22)$$

We have therefore all the data to calculate the SNR of the average power measurement with respect to the variance of the error (see expression (4.60)) for each of the three measurement set-ups, as function of the sampling rate.

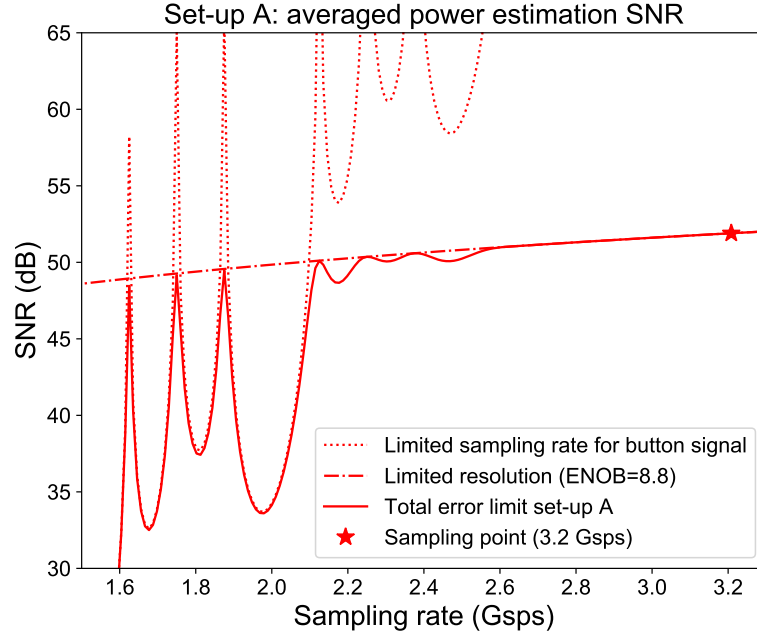
The SNR for the set-up A is represented in Fig. 5.26a, and the results for set-up B and C are shown together in Fig. 5.26b, since the measurements for those two were performed on the same pickup and for the same type of beam.

In Fig. 5.26, the global SNR labeled as *Total error limit*, and calculated as the expression (4.60), is plot together with the two partial SNR components, for which only the effect of one of the two error variances is considered at the time: the *Limited sampling rate* curve depending on σ_ϵ^2 and defined only by the input signal; the *Limited resolution* curve depending on σ_η^2 and defined by both the input signal and the ADC's ENOB. The limited resolution curves are plotted only up-to the maximum sampling rate of each ADC. The star markers in the plot indicate the sampling points at which each ADC operated in the measurement set-ups.

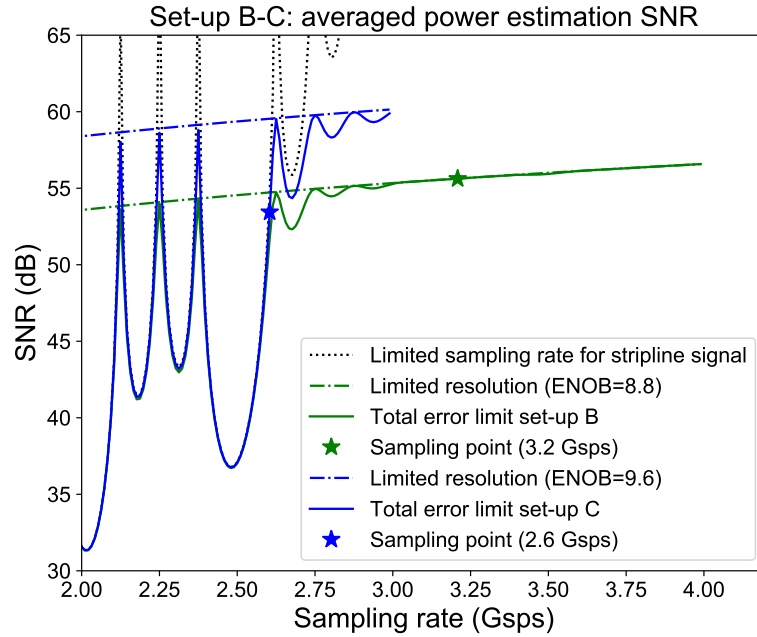
The division of the SNR in its components helps understanding for each of the set-up which characteristic of the ADC is mainly defining the quality of the measurement.

Set-up A, sampling at 3.2 GSps, is not limited by sampling distortion in its performance: the transition area between being dominated by the sampling distortion error and being dominated by the limited resolution of the converter is between 2.1 GSps and 2.6 GSps. The system would profit from the use of a slower and less noisy ADC.

The results are different for set-ups B and C, whose input comes from a LHC stripline. The high sampling rate ADC of set-up A is also used in set-up B. The spectrum of the stripline signal analysed is different from the button signal and in this case the sampling rate is close to the transition area between the two regimes dominated mostly by a single error source. The results of set-up C show how the SNR changes choosing a converter with better resolution. The limit imposed by the noise of the converter with 9.6 ENOB is significantly higher, moving the transition region towards higher sampling rates. Nevertheless, the higher resolution comes with a reduced sampling rate for the ADC of set-up C, that indeed operates at 2.6 GSps, and due to this the SNR is completely dominated by the sampling distortion effect.



(a)



(b)

Figure 5.26: SNR of the averaged power measurement as function of the sampling rate for the set-up A (a) where data was acquired from a SPS button, and set-up B and C (b), where data was acquired from an LHC stripline. The total SNR is plotted together with its partial components due to the limited sampling rate or the limited resolution. The star markers indicate the sampling rate at which the ADC operated during the measurements.

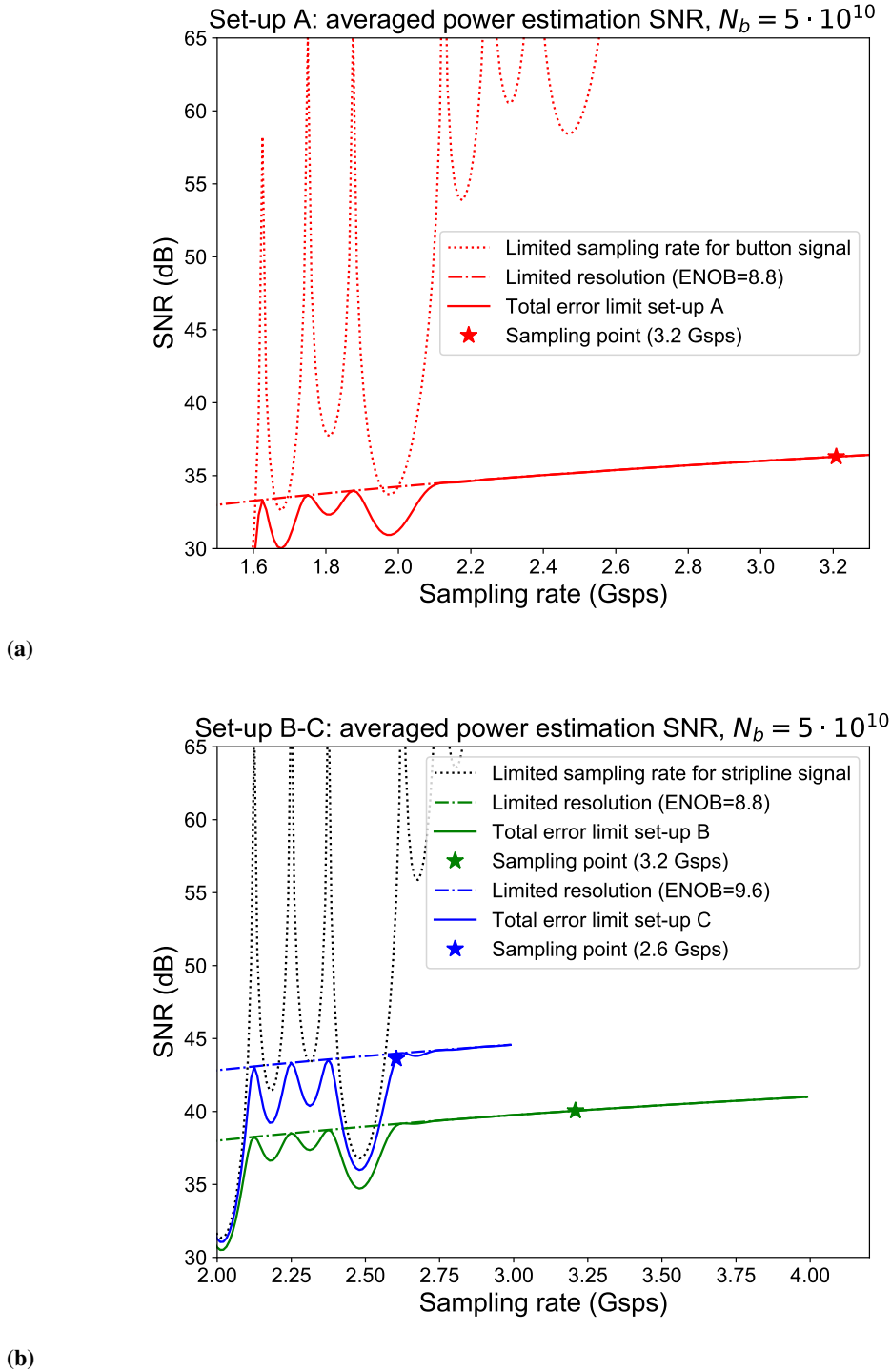


Figure 5.27: SNR of the average power measurement as function of the sampling rate for the set-up A (a) where data was acquired from a SPS button, and set-up B and C (b), where data was acquired from an LHC stripline. The ADC noise is computed as in Eq. (5.23) with bunch intensity six times lower than the maximum value ($5 \cdot 10^{10}$ charges per bunch with respect to $3 \cdot 10^{11}$ charges per bunch). The total SNR is plotted together with its partial components due to the limited sampling rate or the limited resolution. The star markers indicate the sampling rate at which the ADC was operated during the measurements.

For the same input signal, set-up B appears to perform better than set-up C, indicating that the higher sampling rate ADC is to be preferred.

The previous considerations relate to the case of bunches with maximum intensity. To describe the case in which the signal level decreases because of the decreasing number of charges N_b , we modulate the standard deviation of the ADC noise to grow with respect to the signal:

$$\sigma_\nu(N_b) = \frac{N_{b,\max}}{N_b} \frac{2 \cdot A_{\max,7.5}}{0.67} \cdot \frac{2^{-\text{ENOB}}}{\sqrt{12}} \quad (5.23)$$

The SNR is plotted again in Fig. 5.27 for the three measurement set-ups, but assuming the bunch intensity N_b to be six times smaller than the maximum intensity, i.e. $5 \cdot 10^{10}$ charges per bunch with respect to $3 \cdot 10^{11}$ charges per bunch. The standard deviation σ_ν is set accordingly.

It can be observed that the increased relative ADC noise, effect of the reduced beam intensity, impacts significantly more the performance of the studied cases, including set-up C. Fig. 5.27b shows how set-up C for lower bunch intensities allows to reach a better SNR than the one of set-up B, in opposition to what observed in Fig. 5.26b for the maximum intensity bunch. As already indicated by the measurements and by the simulations, for lower bunch intensities the ADC with better resolution, though with lower sampling rate, achieves a higher SNR.

5.5.2 Computation of the error propagation in the position formula

The model describes the error of the average power measurement; for the sake of comparison with the simulations and measurements, it is instructive to relate this information to the measurement of the single-bunch, single pass position resolution out of two electrodes A and B .

Let A_{RMS} be the RMS amplitude of pulse A , by definition equal to the square root of the average power P_T of the pulse. The measured RMS amplitude of the pulse, from here indicated by $\bar{A}_{\text{RMS}}$, is computed as the square root of the measured average power $\bar{P}_T$, including the measurement error:

$$\bar{A}_{\text{RMS}} = \sqrt{\bar{P}_T} = \sqrt{P_T + \epsilon_{P_T,\tau} + \eta} \quad (5.24)$$

The error of $\bar{A}_{\text{RMS}}$ is the following, and its expression can be linearised using the first order Taylor expansion of $\sqrt{1 + (\epsilon_{P_T,\tau} + \eta) / P_T}$:

$$\bar{A}_{\text{RMS}} - A_{\text{RMS}} = A_{\text{RMS}} \left(\sqrt{1 + \frac{\epsilon_{P_T,\tau} + \eta}{P_T}} - 1 \right) \simeq \frac{\epsilon_{P_T,\tau} + \eta}{2A_{\text{RMS}}} \quad (5.25)$$

The mean value of the error, i.e. μ_A , is the expectation ($\text{E}[\cdot]$) of Eq. (5.25):

$$\mu_A = \text{E} \left[\frac{\epsilon_{P_T,\tau} + \eta}{2A_{\text{RMS}}} \right] = \frac{\mu_\epsilon + \mu_\eta}{2A_{\text{RMS}}} = \frac{\sigma_\nu^2}{2A_{\text{RMS}}} \quad (5.26)$$

where the mean values μ_ϵ and μ_η are known from Eq. (4.31) and Eq. (4.44), respectively.

The variance of the error, i.e. σ_A^2 , is the variance ($\text{Var}[\cdot]$) of Eq. (5.25):

$$\sigma_A^2 = \text{Var} \left[\frac{\epsilon_{P_{T,\tau}} + \eta}{2A_{\text{RMS}}} \right] = \frac{\sigma_\epsilon^2 + \sigma_\eta^2}{4A_{\text{RMS}}^2} \quad (5.27)$$

where the variances σ_ϵ^2 and σ_η^2 are known from Eq. (4.32) and Eq. (4.56) respectively.

The second electrode's RMS amplitude is B_{RMS} , measured as $\bar{B}_{\text{RMS}}$, with error mean and variance as for the pulse A , but replacing A_{RMS} with B_{RMS} . The random error of the position measurement can be estimated by linear approximation of the propagation of the random errors of $\bar{A}_{\text{RMS}}$ and $\bar{B}_{\text{RMS}}$ in the position formula. To evaluate the error propagation, the RMS amplitudes are rewritten separating the mean value from the random component:

$$\bar{A}_{\text{RMS}} = \bar{A}_{\text{RMS},0} + \mu_A \quad (5.28)$$

$$\bar{B}_{\text{RMS}} = \bar{B}_{\text{RMS},0} + \mu_B \quad (5.29)$$

with:

$$\bar{A}_{\text{RMS},0} = A_{\text{RMS}} + \frac{\epsilon_{P_{T,\tau}A} + (\eta_A - \mu_\eta)}{2A_{\text{RMS}}} \quad (5.30)$$

$$\bar{B}_{\text{RMS},0} = B_{\text{RMS}} + \frac{\epsilon_{P_{T,\tau}B} + (\eta_B - \mu_\eta)}{2B_{\text{RMS}}} \quad (5.31)$$

To be noted, the errors $\epsilon_{P_{T,\tau}}$ and η are different for each pulse, while the mean value of the error η depends only on the digitiser resolution.

The position formula is:

$$\text{position} = k \cdot \frac{(\bar{A}_{\text{RMS},0} + \mu_A) - (\bar{B}_{\text{RMS},0} + \mu_B)}{\bar{A}_{\text{RMS},0} + \mu_A + \bar{B}_{\text{RMS},0} + \mu_B} \quad (5.32)$$

where k is the coefficient depending on the pickup geometry, as the one used in Eqs. (5.16) and (5.17).

The variance of a non-linear function $f(a, b)$ of two random variables with variances σ_a^2 and σ_b^2 and covariance σ_{ab} can be approximated by:

$$\sigma_f^2 = \left| \frac{\partial f}{\partial a} \right|^2 \sigma_a^2 + \left| \frac{\partial f}{\partial b} \right|^2 \sigma_b^2 + 2 \frac{\partial f}{\partial a} \frac{\partial f}{\partial b} \sigma_{ab} \quad (5.33)$$

For the position formula (5.32) we have the two partial derivatives:

$$\frac{\partial \text{position}}{\partial \bar{A}_{\text{RMS},0}} = k \cdot \frac{2(\bar{B}_{\text{RMS},0} + \mu_B)}{(\bar{A}_{\text{RMS},0} + \mu_A + \bar{B}_{\text{RMS},0} + \mu_B)^2} \quad (5.34)$$

$$\frac{\partial \text{position}}{\partial \bar{B}_{\text{RMS},0}} = k \cdot \frac{-2(\bar{A}_{\text{RMS},0} + \mu_A)}{(\bar{A}_{\text{RMS},0} + \mu_A + \bar{B}_{\text{RMS},0} + \mu_B)^2} \quad (5.35)$$

The covariance of $\bar{A}_{\text{RMS},0}$ and $\bar{B}_{\text{RMS},0}$ depends on the covariances of the errors $\epsilon_{P_{T,\tau}A}$, $\epsilon_{P_{T,\tau}B}$, η_A and η_B . It was already demonstrated that for a pulse the covariance of the error caused by limited sampling rate with the error caused by limited resolution is 0. Independence is also assumed between the error caused by the limited sampling rate of one pulse with respect to the error cause by the limited resolution of the other.

The errors caused by limited resolution in the two pulses are considered independent, as well. What is left is the covariance of the two errors $\epsilon_{P_{T,\tau}A}$ and $\epsilon_{P_{T,\tau}B}$ caused by limited sampling rate:

$$\begin{aligned} \text{Cov} [\bar{A}_{\text{RMS},0}, \bar{B}_{\text{RMS},0}] &= \text{E} [\bar{A}_{\text{RMS},0} \cdot \bar{B}_{\text{RMS},0}] - \text{E} [\bar{A}_{\text{RMS},0}] \text{E} [\bar{B}_{\text{RMS},0}] \\ &= \text{E} \left[\frac{\epsilon_{P_{T,\tau}A} + (\eta_A - \mu_\eta)}{2A_{\text{RMS}}} \cdot \frac{\epsilon_{P_{T,\tau}B} + (\eta_B - \mu_\eta)}{2B_{\text{RMS}}} \right] \\ &= \frac{\text{E} [\epsilon_{P_{T,\tau}B} \cdot \epsilon_{P_{T,\tau}A}]}{4A_{\text{RMS}}B_{\text{RMS}}} \end{aligned} \quad (5.36)$$

If there is a fixed delay between one pulse and the other, there is a fixed relation between the two pulses' sampling delays, hence a fixed relation between the two errors caused by limited sampling rate.

We look at the case of central beam, i.e. $A_{\text{RMS}} = B_{\text{RMS}}$ and $\mu_A = \mu_B$, and there are two extreme cases: one where the time-delay between the two pulses is an integer multiple of the sampling period, the other where the time-delay between the two pulses is an odd multiple of half the sampling period. In the first case the two errors are in phase, in the second they are in counter-phase (see Fig. 5.28).

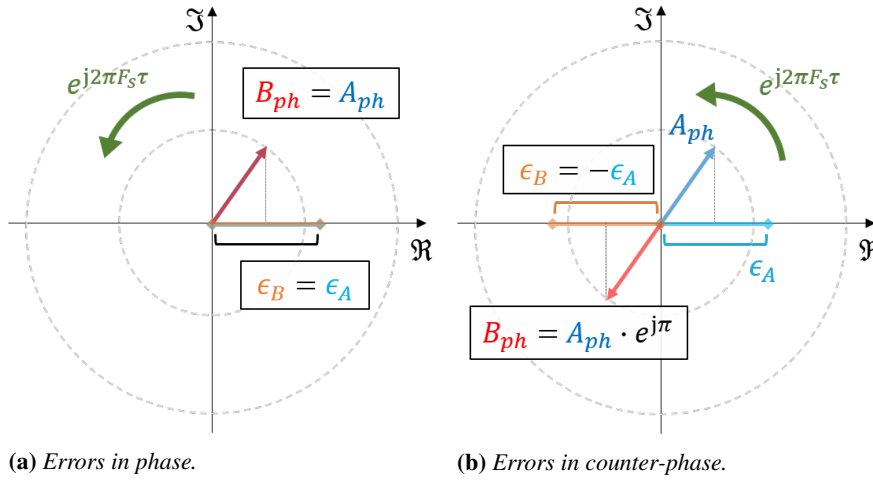


Figure 5.28: Relation between the errors caused by limited resolution for two pulses A and B , represented by their error phasor (see Eq. (4.27) and Fig. 4.5). The two extreme cases are represented, when the time-delay is an integer multiple of the sampling period (left), and when the time-delay between the two pulses is an odd multiple of half the sampling rate (right).

When the two errors are in phase, i.e. $\epsilon_{P_{T,\tau}B} = \epsilon_{P_{T,\tau}A}$, the covariance is positive and has its maximum value:

$$\text{Cov} [\bar{A}_{\text{RMS},0}, \bar{B}_{\text{RMS},0}]_{\text{max}} = + \frac{\sigma_\epsilon^2}{4A_{\text{RMS}}^2} \quad (5.37)$$

When the two errors are in counter-phase, i.e. $\epsilon_{P_{T,\tau}B} = -\epsilon_{P_{T,\tau}A}$, the covariance is negative and has its minimum value:

$$\text{Cov} [\bar{A}_{\text{RMS},0}, \bar{B}_{\text{RMS},0}]_{\text{min}} = - \frac{\sigma_\epsilon^2}{4A_{\text{RMS}}^2} \quad (5.38)$$

5.6. Discussion and comparison of the results

Combining all the results, the variance of the central position measurement in the two extreme cases is finally expressed as:

$$\sigma_{\text{position}}^2 = \frac{k^2}{4(A_{\text{RMS}} + \mu_A)^2} \left(2\sigma_A^2 \pm \frac{\sigma_\epsilon^2}{4A_{\text{RMS}}^2} \right) \quad (5.39)$$

where the contribution of the correlation can be added or subtracted, depending on the time-delay between the two pulses. If there is no relation between the sampling delay of the two pulses, the two errors are uncorrelated and the variance of the central position measurement is:

$$\sigma_{\text{position}}^2 = \frac{k^2 \sigma_A^2}{2(A_{\text{RMS}} + \mu_A)^2} \quad (5.40)$$

The dependency on the sampling rate and on the ADC noise are included in the variances σ_ϵ^2 , σ_A^2 and in the mean error μ_A .

5.6 Discussion and comparison of the results

The analytic estimation of the position resolution can be computed for each input signal, for any sampling rate in the range between half the Nyquist rate and the Nyquist rate of the input signal, and for any ADC of given ENOB. Changes in bunch intensity can be accounted for by varying the ADC noise as from Eq. (5.23).

Using this method, the expected position resolution for set-ups A, B and C has been calculated for beam intensities between $9 \cdot 10^9$ and $3 \cdot 10^{11}$ charges per bunch. The result is shown in Fig. 5.29. The estimated position resolution neglecting the covariance between the two electrodes' errors caused by limited sampling rate (5.40) is plotted as continuous line for the three measurement set-ups. The two boundary curves assuming correlation in the two extreme cases (5.39) are plotted as dotted lines and they differ significantly from the case with no correlation only for the measurement set-up C and for medium-high bunch intensities, that is when the error caused by the limiting sampling rate is significant. The simulation results and the beam measurement results are also plotted for reference and comparison.

The analytic model reproduces closely the simulation curves and the measurements: it describes for each measurement condition which ADC delivers better position resolution, and it provides an estimation of such resolution.

The application of the model to describe the effects of digitisation in a system like the LHC beam extraction interlock BPM prototype highlighted for each measurement set-up which characteristic of the ADC is limiting the performances. Set-up C is the only one whose performance is limited by sampling distortion, but only for high intensity bunches. Therefore, for this set-up and this bunch intensity conditions we expect to observe the effect of sampling distortion in the behaviour of the simulated and measured positions.

Fig. 5.30 compares the distribution of 2000 simulated position measurements emulating the conditions of the set-ups B and C, for the maximum bunch intensity. Set-up B results reach a better one-sigma resolution and have a Gaussian-like distribution around the mean value. The performance of set-up C are limited by sampling distortion and the distribution of the measured positions does not peak in its mean value, in accordance to what predicted for this type of cases (see Eq. (4.36) and its graphic representation for a

5.6. Discussion and comparison of the results

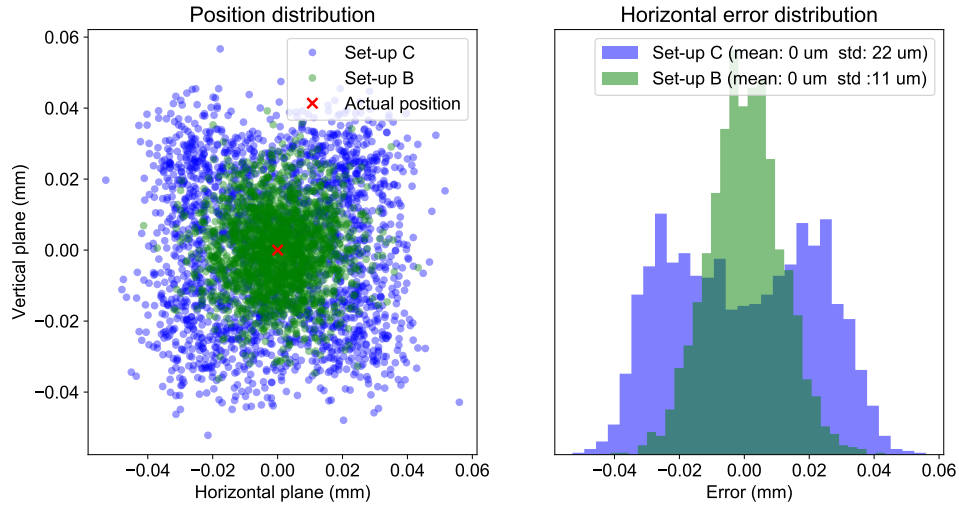


Figure 5.30: Results of the simulation of the beam measurement over 2000 maximum intensity bunches emulating the conditions of the measurement set-up B and C. On the left is represented the estimated position in the transverse plane for each measurement. On the right is reported in the form of histogram the result for the position measurement error in the horizontal plane.

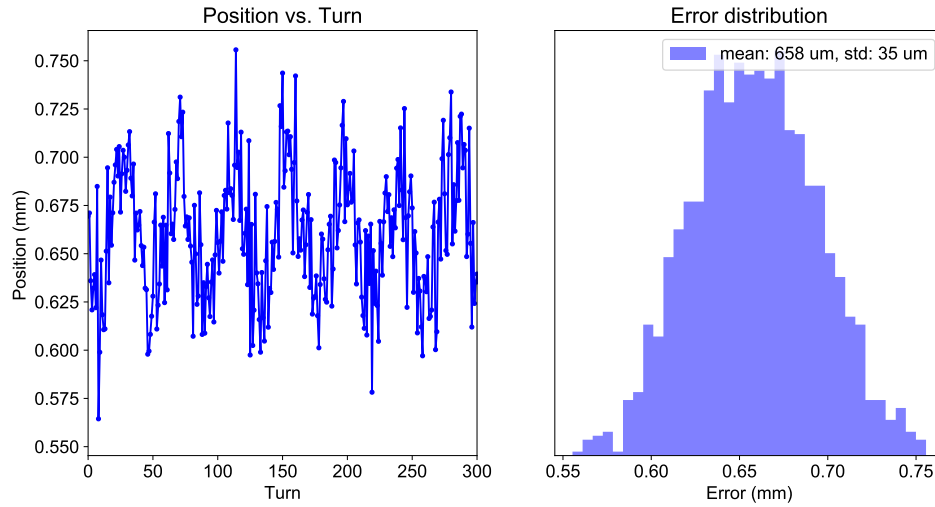


Figure 5.31: Measurement of the beam position with set-up C, central emulated position, equivalent bunch intensity of $1.1 \cdot 10^{11}$ charges per bunch. On the left is represented the evolution of the position of a single bunch over 300 consecutive turns. On the right the position measurement error distribution in the form of histogram.

tion caused by it, i.e. the sampling distortion error, leaving the ADC noise as the sole source of error; in a second moment, setting the ENOB to 14, minimising the ADC noise, but reintroducing the asynchronous sampling, thus the sampling distortion error. The results and the error distributions are shown in Fig. 5.32. Only the variation of the error is reported and not the mean value, that for the synchronous set-up would depend on the specific sampling phase selected. The two distributions have comparable

standard deviation, though with different shapes, and therefore contribute to the overall system performance with similar weight.

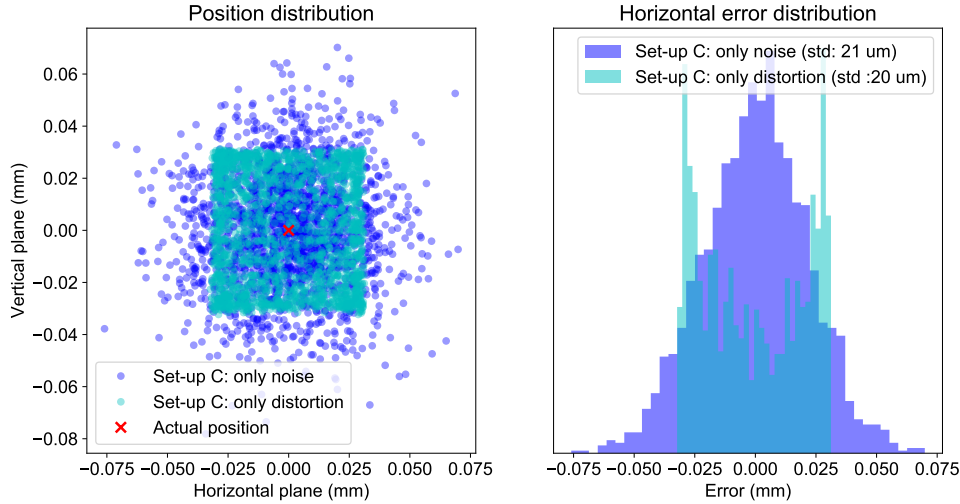


Figure 5.32: Results of a simulation for the measurements of a $1.1 \cdot 10^{11}$ charges per bunch beam in central position with two different configuration of set-up C. In one the measurement is synchronous, so that the only source of error is the ADC noise. In the other, the ADC's ENOB are set to 14, making the ADC noise negligible with respect to the sampling distortion effect. On the left is represented the distribution of the estimated position in the transverse plane. On the right the histogram of the measurement error for the horizontal position.

The concurrence of the two error sources is also predicted by the analytic model applied to the stripline pulse. Fig. 5.33 shows the SNR (4.60) of the average power measurement as function of the sampling rate, for the set-up C when the ADC noise is set such to emulate a bunch intensity of $1.1 \cdot 10^{11}$ charges (see Eq. (5.23)). The sampling rate of 2.6 GSps, used in the measurements, lies in between the sampling rate dominated error regime and the ADC resolution dominated error one.

An additional indication of the significance of sampling distortion effects in determining the resolution of beam measurements with the set-up C of bunch signals of intensity in the order of 10^{11} charges per bunch is given by the spread of the measured one-sigma resolution (see Fig. 5.29). For each acquisition emulating a beam displacement, different sets of attenuators were applied to the two analog front-end's inputs, altering the time-delay between the two pulses. Studying the behaviour of the sampling distortion error and how it propagates in the position formula, we have computed how the variation of the time-delay affects the position resolution and within which boundaries. Indeed, the measured values of one-sigma position resolution are spread approximately within the analytically calculated boundaries.

The analytic model proved to be a valid tool to analyse qualitatively, and also quantitatively, the expected performance of the measurement system under study, in dependence of the digitisation stage. It allows to compare different sampling schemes, giving an overview of how varying sampling rate and resolution of the converter would impact the overall measurement uncertainty and error behaviour.

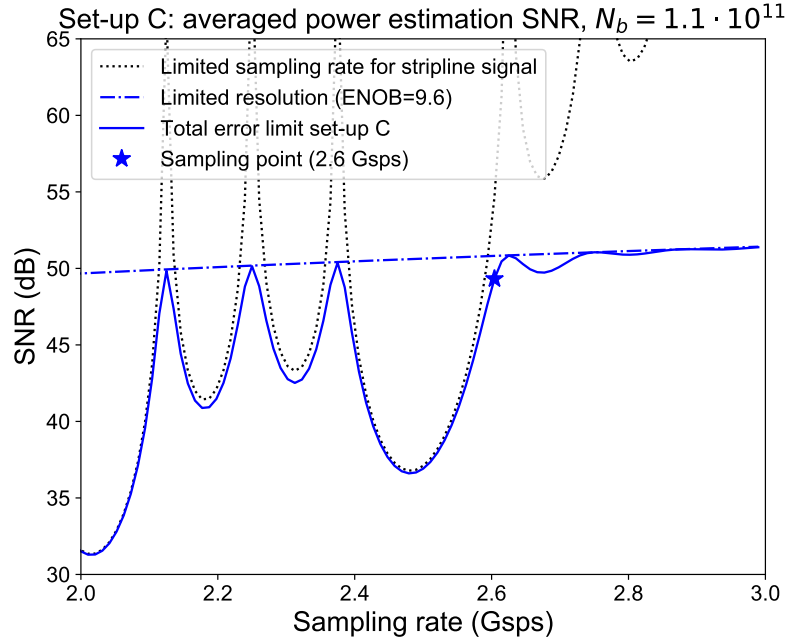


Figure 5.33: SNR of the average power measurement as function of the sampling rate for the measurement set-up C. The ADC noise is set according to Eq. (5.23) to emulate a bunch intensity of $1.1 \cdot 10^{11}$ charges per bunch. The total SNR is plotted together with the individual contributions of the limited sampling rate and of the limited resolution. The star marker indicates the sampling rate at which the ADC was operated during the actual measurements.

CHAPTER 6

Conclusions

This research aimed to study the impact of the ADC characteristics on the resolution of a direct-digitisation based beam position monitor system for charged particle accelerators, in particular the case of beam pickup signals that are digitised after minimum analog conditioning, evaluating them by a root-mean-square algorithm for bunch-by-bunch beam position measurements.

It was demonstrated how to model, for a given pulse signal, the performance of the average power measurement from noisy samples, as function of the sampling rate and the resolution of the converter. Furthermore, the analytic model proposed points to an optimal choice in terms of ADC characteristics for the average power measurement, given the trade-off for physical converters between sampling rate and ADC resolution. Such a model was applied to the specific example of the LHC beam extraction interlock BPM prototype electronics, where the calculation of the average power is a first step for the computation of the beam position by a root-mean-square algorithm. The analytic estimations of the position resolution were confirmed by beam measurements and simulations. The analytic results, in accordance with the beam measurements and the simulations, indicated between the sampling schemes which one is to be preferred to maximise the position resolution, depending on the measurement conditions.

Our specific problem to estimate the resolution of the beam position, measured by calculating the mean square values of two beam signals digitised with a specific sampling rate and converter resolution, was reduced to a more general one, i.e. the estimation of the error of the average power of a pulse, calculated from its measured samples. This generalisation allowed a simpler development of the model, and also increased the model applicability.

The analytic model aimed to describe two fundamental, while different aspects of the analog-to-digital conversion, *sampling* and *quantising*, and how they relate to the

average power measurement of a pulse signal. The goal was to obtain a simple and discrete description, therefore easily to be implemented in a numeric environment.

The first part of the problem was to evaluate the effect of sampling distortion, when the sampling rate is limited. By focusing on the average power measurement, instead of the more typical waveform reconstruction, I could make full use of the relationship between time and frequency domain, exploiting the Parseval's theorem. The class of pulse signals was chosen as generalisation of the single beam bunch signal, but also because it allows to treat the signal as periodic. The transition to a periodic signal is fundamental, since it enables to discretise the spectrum of the signal without loss of information. As main approximation I considered only the first replica of the spectrum to evaluate the effects of aliasing. Such approximation is valid only for signals whose spectrum amplitude decreases significantly with increasing frequency, trending to zero, not necessarily monotonically. This assumption is typically valid for physical signals of finite energy. A relevant and distinctive aspect of the model is the inclusion of the dependence on the sampling delay, i.e. the time or phase relation between the signal pulse and the sampling clock signal.

The limited sampling rate description was completed by also including the limited resolution of the conversion. The ADC noise was comprehensively modeled as additive white Gaussian noise, that propagates in the average power calculation. When referred to the measured SINAD or ENOB of an ADC, this noise variable is a cumulative description of the quantisation noise, of the aperture uncertainty and of the thermal noise of the chip.

The combination of the two effects, expressed as signal-to-noise ratio of the average power measurement, gives an immediate and practical tool to navigate through the ADC specification space. As a first outcome, the expression directly evaluates the expected performance among different sampling schemes, each defined by their sampling rate and resolution. From a practical point of view, it points, within a group of converters, to the one ADC which probably delivers the best measurement result. In addition, the expression allows to explore how the measurement performance varies for different values of the ADC sampling rate or resolution, especially indicating which one of these two ADC parameters determines the SNR limit. Aiming at optimisation, within the limits of the present state-of-the-art ADC technology, the trade-off between sampling rate and resolution was modeled with a constant figure of merit P , linking resolution and sampling rate. The SNR calculated for the best P indicates the envelope of the theoretical maximum performance that could be achieved with the present technology for a given sampling rate.

The analysis of the LHC beam extraction interlock BPM prototype offered a practical example of a beam position monitor system based on direct-digitisation to apply the model. The analog front-end prototype was used to acquire beam data from two BPM pickups using two different ADCs, one with higher sampling rate, the other with higher resolution. The results of the beam position measurements under controlled variation of the beam displacement (position) and intensity were analysed offline. For a more general analysis, including the trends beyond the experimental parameter space, the measurements were completed by a tailored simulation framework, replicating the system in all its elements.

The application of the analytic model to the ADC input pulse signals, processed

with the LHC beam extraction interlock BPM prototype, enabled the comparison of the expected performance of the average power measurement for both ADCs. The model discovered for various combinations of pickup signal, bunch intensity and type of ADC whether the performance was dominated by sampling distortion or by the ADC noise, coherently with the results observed from the measurements and the simulations. The error expression for the average power measurement was also extended to estimate the resolution of the position measurement by applying a Δ/Σ normalisation algorithm. Beam measurement results and simulations using the developed models also match well when analysing the beam bunch position resolution as function of the bunch intensity, demonstrating that the analytic approach is also valid to estimate the beam (bunch) position resolution.

These findings enable an optimised design of a real-world BPM system based on direct-digitisation techniques, applying an average power measurement algorithm to the digitised pulse signal, in terms of ADC sampling rate and resolution. Furthermore, the model allows to understand which ADC parameter dominates the performance limit, and if there is room for improvement. Finally, the model indicates the theoretical maximum performance to be expected by state-of-the-art ADCs, and which are the required sampling conditions.

From the application point of view the implications are vast. First of all, for a given system architecture it is straightforward to estimate if the maximum performance meets the minimum requirement, hence, if it is worth to investigate such architecture. If yes, the designer can search for suitable ADCs in the sampling rate regime of interest; and if several options are available, they can be directly compared based on their published specifications.

The purchase of ADC evaluation boards and their implementation into a BPM hardware test setup is time consuming and costly, also in terms of engineering manpower. The presented study stage can potentially reduce the number of design iterations and components to be tested, hence speeding up the design towards a successful prototype.

The development of new instrumentation systems for beam diagnostics can definitely profit from these results. The trend in the evolution of the characteristics of ADCs clearly shows the increasing availability of high-sampling rate converters with good resolution [30, 31]. Hence, direct digitisation of single bunch signals with only minimum analog conditioning and computation of their power or amplitude in the digital domain is becoming more feasible and convenient. In particle accelerators with rather low RF frequency, $\lesssim 50$ MHz, new designs for the measurement of the beam position and the beam current can already use this processing scheme. It allows to preserve the bunch structure in the digital domain and to implement some flexibility in the signal processing without modifying the hardware [6].

The LHC beam extraction interlock BPM, still in the prototype stage, is an example of such system [24], designed to acquire bunches with a repetition rate of 40 MHz. This type of architecture is also investigated for other beam position monitor systems at CERN, like the High-Luminosity LHC Interaction Region BPMs [61] and the LHC Consolidation BPM system [45]. There exist other beam instrumentation systems to monitor bunched beams in accelerators with lower RF frequencies [10, 22], and there are extensive studies about the optimisation of the algorithm to calculate the position from the direct digitised bunch signals [23].

Instead, this work focused on the optimisation of the sampling stage, applying a well known input signal, and a measurement based on the calculation of the average power. It is undeniable that, even if the ADC plays a determinant role defining the achievable performance, the optimisation of the overall design of a specific BPM system must consider additional aspects, not covered in this thesis.

As commonly known and also stressed in this work, the shape of the frequency spectrum of the signal to be digitised has a substantial impact on the overall performance, in so far as it alters the sampling distortion limit, hence, the balance between sampling rate and resolution in selecting the optimal ADC.

Also a change of the transfer function of the analog conditioning filter can potentially improve the measurement resolution, but how the transfer function can be changed depends on the specific system requirements, on the waveform of the input pulse and on the time-structure of the input pulse signal, i.e. the minimum bunch spacing.

Since the analytic model presented in this thesis explicitly relies on the expression of the spectrum of the ADC input signal, the transfer function of the analog anti-aliasing filter could be included into the model, widening the space of optimisation parameters. In this sense, this work can support the design of the overall system, facilitating a comprehensive optimisation study, which would be an interesting object for further research.

As specific example, in the LHC beam extraction interlock BPM system the addition of a low-pass filter in the acquisition chain would probably improve the performance of the measurement for high intensity bunches, as it would allow to use an ADC with higher resolution. Nevertheless, the filter needs to be carefully tailored, as it should not increase the interference between successive beam pulse signals.

Beside the optimisation of the most fundamental ADC parameters, some limitation of this work regards the choice of algorithm to evaluate the signal amplitude. The analytic model developed in this work covers two fundamental cases, i.e. the power estimation algorithm and the linked RMS algorithm. However, the results shown in this work may not be representative for other types of algorithms. Still, it should be noted, the power estimation algorithm, as well as the RMS algorithm are simple, robust and commonly adopted methods. When applied for signals without DC component, they are equivalent to the STD algorithm, and found to be the best performing to calculate the bunch position from asynchronously digitised pickup pulse signals, among the algorithms compared in this work [23].

Another interesting aspect for further studies is related to the observation of the SNR of the average power measurement as function of the sampling rate for a pulse signal similar to the one digitised in the LHC beam extraction interlock BPM. The pulse spectrum presents many lobes, and so does the SNR when limited by sampling distortion. This indicates the measurement error does not follow monotonically while reducing the sampling rate, and there might be a specific low sampling frequency with unexpected low sampling distortion error. Even though this would require a precise control of the sampling frequency and a constant input signal spectrum, it might be of interest and benefit to design a measurement set-up and obtain an experimental verification of this mechanism of undersampling of a wideband signal.

In summary, the analytical approach presented in this thesis proved to be effective

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in the description of the limitations that the analog-to-digital converter poses to the performance of a direct-digitisation based system that measures the RMS amplitude or the average power of pulse signals. Understanding and quantifying the limits of this kind of measuring scheme is one of the first steps for the successful implementation of an architecture that could offer many new processing opportunities, especially for beam instrumentation applications.

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