CERN Summer School project report

Study of the azimuthal correlation function of prompt D mesons with charged particles using the Von Mises function for fitting the data

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Abstract

The angular correlations of heavy-flavor and charged particles in high-energy proton–proton collisions can be used for probing the charm-quark production and fragmentation. A study of the the azimuthal correlation function of prompt D mesons with charged particles fitted with the Von Mises function in pp collisions performed at ALICE detector at the center-of-mass energy of $\sqrt{s} = 13$ TeV is reported. The results are compared with the ones obtained using a Generalised Gaussian + Gaussian fitting function.
Finally, the Von Mises function has been used for fitting the azimuthal correlation function of prompt D mesons with charged particles also in pp and p-Pb collisions at $\sqrt{s} = 5.02$ TeV.

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1 Introduction

The fundamental properties of Quantum Chromodynamics (QCD) in hadrons collisions can be investigated through heavy-flavour quarks [2]. Their production, occurring in proton-proton collisions, provides a testing reference frame for the Quark-Gluon Plasma (QGP) in heavy-ion collisions.
In fact, in heavy-ion collisions, a deconfined state of matter of free quarks and gluons, the QGP, is created. Heavy-quarks can be used as a probe for studying this state of matter. While heavy-quarks pass through the medium, they interact with the QGP: these interactions make the heavy-quarks lose their energy and change their original directions.
These modifications change the angular correlation function between heavy-flavour hadrons and their associated charged particles compared to the angular correlation function in proton-proton collisions, used as a reference.
Thus, the angular correlations of heavy-flavor particles and their associated charged particles in high-energy proton–proton collisions can be used for studying heavy quarks production mechanism, and their fragmentation and hadronization processes.

Specifically, the interest of this work focuses on the study of the azimuthal correlation function between prompt D mesons and charged particles obtained from pp collisions performed at ALICE detector at center-of-mass energy of $\sqrt{s} = 13$ TeV. The "prompt" D mesons are those produced from charm-quark fragmentation and from the decay of excited charm-quark resonances.
This study has been already performed in [1], where the azimuthal correlation function was fitted using a function made up of a Gaussian and a Generalised Gaussian, whilist, in this work, a different fitting function, called Von Mises function, has been used for fitting the data.

A comparison between the two fitting functions is reported in order to see which one fits the best the data.

Finally, a similar study on the fitting functions is performed also for the data gathered from pp and p-Pb collisions at $\sqrt{s} = 5.02$ TeV [3] and the results are compared to the ones for pp collision at $\sqrt{s} =$ 13 TeV.

2 Azimuthal correlation function

The typical structure of the azimuthal correlation function features two peaks: a near-side (NS) peak, centered at $(\Delta\phi, \Delta\eta) = (0, 0)$, and an away-side (AS) peak, centered at $\Delta\phi = \pi$ and elongating over a wide range of $\Delta\eta$. $\Delta\phi$ represents the azimuthal angle difference $\Delta\phi = \phi_{\mathrm{assoc}} - \phi_{\mathrm{D}}$ and $\Delta\eta$ is the pseudorapidity difference $\Delta\eta = \eta_{\mathrm{assoc}} - \eta_{\mathrm{D}}$. Right below the two peaks there is a flat contribution, the baseline, that will be subtracted from the correlation function since it is related to underlying events which are uncorrelated to the charm fragmentation.

In this work, the azimuthal correlation function of D mesons with associated charged particles obtained from pp collisions performed at ALICE detector at center-of-mass energy of $\sqrt{s} = 13$ TeV is studied. This azimuthal correlation function has been computed for five D-meson $\mathrm{p_T}$ intervals $3 < {p_\mathrm{T}}^{\mathrm{D}} < 5$ GeV/c, $5 < {p_\mathrm{T}}^{\mathrm{D}} < 8$ GeV/c, $8 < {p_\mathrm{T}}^{\mathrm{D}} < 16$ GeV/c, $16 < {p_\mathrm{T}}^{\mathrm{D}} < 24$ GeV/c, $24 < {p_\mathrm{T}}^{\mathrm{D}} < 36$ GeV/c, and for the following associated particles $\mathrm{p_T}$ intervals $0.3 < {p_\mathrm{T}}^{\mathrm{assoc}} < 1$ GeV/c, ${p_\mathrm{T}}^{\mathrm{assoc}} > 0.3$ GeV/c, $1 < {p_\mathrm{T}}^{\mathrm{assoc}} < 2$ GeV/c, ${p_\mathrm{T}}^{\mathrm{assoc}} > 1$ GeV/c, $2 < {p_\mathrm{T}}^{\mathrm{assoc}} < 3$ GeV/c, ${p_\mathrm{T}}^{\mathrm{assoc}} > 2$ GeV/c, ${p_\mathrm{T}}^{\mathrm{assoc}} > 3$ GeV/c.

3 Fit of the azimuthal correlation function

3.1 Generalised Gaussian + Gaussian fitting function

The azimuthal correlation function was initially fitted using the following function:

$$f(\Delta\phi) = a + \frac{Y_{\mathrm{NS}}\beta}{2\alpha\Gamma(\frac{1}{\beta})} e^{(-\frac{\Delta\phi}{\alpha})^{\beta}} + \frac{Y_{\mathrm{AS}}}{\sqrt{2\pi}\sigma_{\mathrm{AS}}} e^{-\frac{(\Delta\phi-\pi)^2}{2(\sigma_{\mathrm{AS}})^2}} \tag{1}$$

This function is composed of a constant term a, representing the baseline, a Generalised Gaussian, describing the near-side peak, and a Gaussian, describing the away-side peak.
The main parameters of this function are:

- β: it is the Generalised Gaussian parameter. It is related to the shape of the peak, so varying β it is possible to change the shape of the peak. For $\beta = 2$, the Generalised Gaussian is reduced to a regular Gaussian;
- Y_{NS} and Y_{AS}: they are respectively the near-side and away-side yields;
- α: it is related to the near-side variance; specifically, the near-side variance is given by the quantity $\alpha\sqrt{\Gamma(3/\beta)/\Gamma(1/\beta)}$. It describes the width of the near-side peak;
- σ_{AS}: it is the away-side variance and it describes the width of the away-side peak;
- $\Delta\phi$: it is the mean. In the near-side peak, the mean is fixed to $\Delta\phi = 0$, while in the away-side peak it is fixed to $\Delta\phi = \pi$.

This was the function used for fitting the data in [1].
In this work, the Generalised Gaussian + Gaussian fitting function has been used for performing a comparison with the Von Mises fitting function.

3.2 Von Mises fitting function

The innovation brought by this work is that both the correlation peaks are described by the same function, that is the Von Mises function.
Thus, the azimuthal correlation function has been fitted using the following function:

$$f = a + \frac{Y_{\mathrm{NS}}}{2\pi I_0(k_{\mathrm{NS}})} \exp(k_{\mathrm{NS}} \cos(x - \mu_{\mathrm{NS}})) + \frac{Y_{\mathrm{AS}}}{2\pi I_0(k_{\mathrm{AS}})} \exp(k_{\mathrm{AS}} \cos(x - \mu_{\mathrm{AS}})) \tag{2}$$

This function is composed of the constant contribution of the baseline and by two Von Mises functions, one for each peak. The main parameters of this function are:

- Y_{NS} and Y_{AS}: they are respectively the near-side and away-side yields;
- k_{NS} and k_{AS}: they are related to the variances of the near-side and away-side peaks and they describe the width of the peaks;
- μ_{NS} and μ_{AS}: they are the means. In the near-side peak, the mean is fixed to $\mu_{NS} = 0$, while in the away-side peak it is fixed to $\mu_{AS} = \pi$;
- I_0: it is the Bessel function.

3.3 Possible advantages in using the Von Mises fitting function

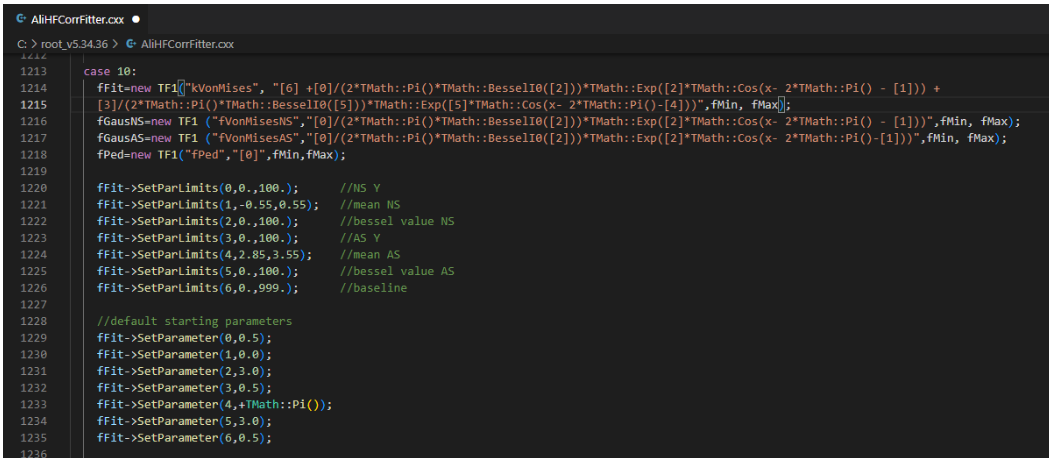
There are some reasons to believe that the Von Mises function might be a better candidate than the Generalised Gaussian + Gaussian for fitting the azimuthal correlation function. First of all, it has already been successfully used for other azimuthal correlation analyses (like heavy-flavor hadron decay electrons) [4]. In addition, it is more suitable for studying the azimuthal correlation function since it is a periodic function, differently from the Gaussian function, which is defined in the range [$-\infty$,$+\infty$]. Finally, the greatest advantage is that it does not need the β parameter of the Generalised Gaussian, which forces the function to fit the data well.

4 How to extract the results

In order to gather the results, several codes in C++ have been designed and compiled in ROOT. They are based on those that have been already used for other analyses, like [1], but some new features have been implemented for introducing the Von Mises as fitting function.

4.1 Implementation of the Von Mises function

The *AliHFCorrFitter.cxx* macro is the official macro where the fit classes are implemented. Nine cases were used for defining different fitting functions for the azimuthal correlation function. An additional case, *case 10*, has been added for including the Von Mises fitting function. Thus, the Von Mises functions, which describe the near-side and the away-side peaks, as well as the baseline, which is just a constant, have been defined and the parameters of the functions are defined and initialized to their default starting values.
In Figure 1, the core of the *case 10* is reported.

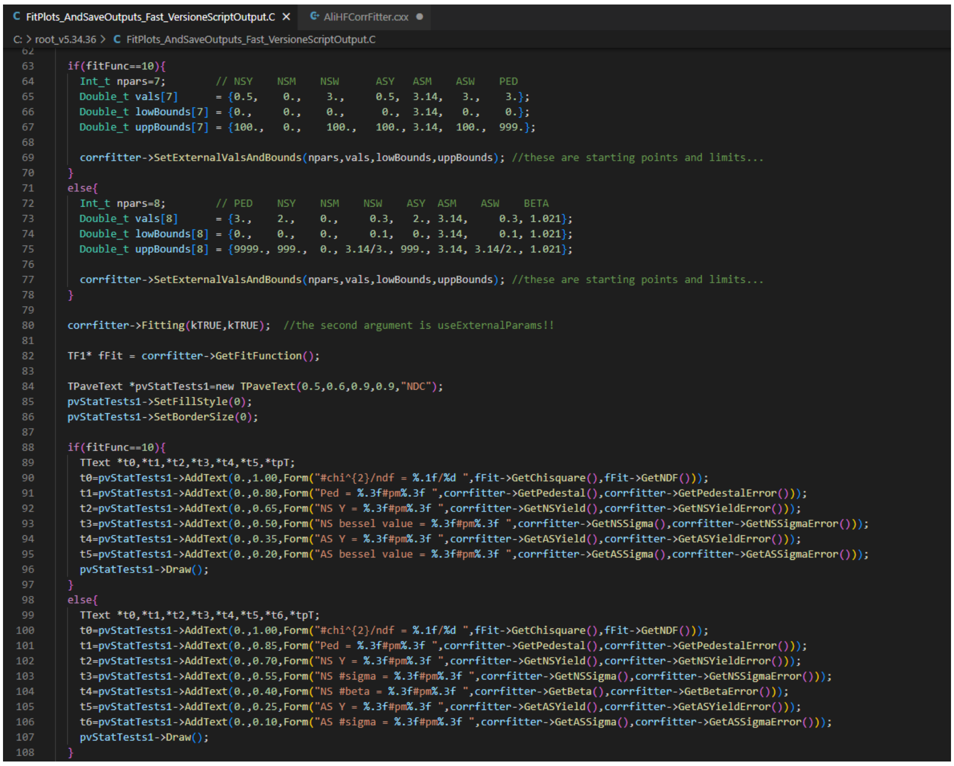



```
case 10:
  fFit=new TF1("kVonMises", "[6] +[0]/(2*TMath::Pi()*TMath::BesselI0([2]))*TMath::Exp([2]*TMath::Cos(x- 2*TMath::Pi() - [1])) +
  [3]/(2*TMath::Pi()*TMath::BesselI0([5]))*TMath::Exp([5]*TMath::Cos(x- 2*TMath::Pi()-[4]))",fMin, fMax);
  fGausNS=new TF1 ("fVonMisesNS","[0]/(2*TMath::Pi()*TMath::BesselI0([2]))*TMath::Exp([2]*TMath::Cos(x- 2*TMath::Pi() - [1]))",fMin, fMax);
  fGausAS=new TF1 ("fVonMisesAS","[0]/(2*TMath::Pi()*TMath::BesselI0([2]))*TMath::Exp([2]*TMath::Cos(x- 2*TMath::Pi()-[1]))",fMin, fMax);
  fPed=new TF1("fPed","[0]",fMin,fMax);

  fFit->SetParLimits(0,0.,100.);      //NS Y
  fFit->SetParLimits(1,-0.55,0.55);   //mean NS
  fFit->SetParLimits(2,0.,100.);      //bessel value NS
  fFit->SetParLimits(3,0.,100.);      //AS Y
  fFit->SetParLimits(4,2.85,3.55);    //mean AS
  fFit->SetParLimits(5,0.,100.);      //bessel value AS
  fFit->SetParLimits(6,0.,999.);      //baseline

  //default starting parameters
  fFit->SetParameter(0,0.5);
  fFit->SetParameter(1,0.0);
  fFit->SetParameter(2,3.0);
  fFit->SetParameter(3,0.5);
  fFit->SetParameter(4,+TMath::Pi());
  fFit->SetParameter(5,3.0);
  fFit->SetParameter(6,0.5);
```

Figure 1: Part of the code in *AliHFCorrFitter.cxx* where the Von Mises function is implemented

4.2 Production of the histograms

The histograms that plot the fitting function are defined in *FitPlotsAndSaveOutputsFastVersione-ScriptOutput.C*. In Figure 2, the part of the code that produces the histograms for plotting the fitting function is shown. In particular, from line 63 to line 78, the range in which the parameters are let to vary is defined. It is possible to notice that, as already stated before, in the Von Mises case, only seven parameters are necessary, two of which - the near-side and the away-side means - are fixed, while in the other cases, including the one of the Generalised Gaussian + Gaussian, there is an additional parameter. In the following lines, the histograms are produced.




```
if(fitFunc==10){
  Int_t npars=7;        // NSY    NSM    NSW      ASY   ASM    ASW    PED
  Double_t vals[7]      = {0.5,    0.,    3.,      0.5,  3.14,   3.,    3.};
  Double_t lowBounds[7] = {0.,     0.,    0.,       0.,  3.14,   0.,    0.};
  Double_t uppBounds[7] = {100.,   0.,    100.,   100., 3.14,  100.,  999.};

  corrfitter->SetExternalValsAndBounds(npars,vals,lowBounds,uppBounds); //these are starting points and limits...
}
else{
  Int_t npars=8;        // PED    NSY    NSM    NSW    ASY  ASM    ASW    BETA
  Double_t vals[8]      = {3.,    2.,    0.,     0.3,   2., 3.14,     0.3, 1.021};
  Double_t lowBounds[8] = {0.,    0.,    0.,     0.1,   0., 3.14,     0.1, 1.021};
  Double_t uppBounds[8] = {9999., 999.,  0., 3.14/3., 999., 3.14, 3.14/2., 1.021};

  corrfitter->SetExternalValsAndBounds(npars,vals,lowBounds,uppBounds); //these are starting points and limits...
}

corrfitter->Fitting(kTRUE,kTRUE);  //the second argument is useExternalParams!!

TF1* fFit = corrfitter->GetFitFunction();

TPaveText *pvStatTests1=new TPaveText(0.5,0.6,0.9,0.9,"NDC");
pvStatTests1->SetFillStyle(0);
pvStatTests1->SetBorderSize(0);

if(fitFunc==10){
  TText *t0,*t1,*t2,*t3,*t4,*t5,*tpT;
  t0=pvStatTests1->AddText(0.,1.00,Form("#chi^{2}/ndf = %.1f/%d ",fFit->GetChisquare(),fFit->GetNDF()));
  t1=pvStatTests1->AddText(0.,0.80,Form("Ped = %.3f#pm%.3f ",corrfitter->GetPedestal(),corrfitter->GetPedestalError()));
  t2=pvStatTests1->AddText(0.,0.65,Form("NS Y = %.3f#pm%.3f ",corrfitter->GetNSYield(),corrfitter->GetNSYieldError()));
  t3=pvStatTests1->AddText(0.,0.50,Form("NS bessel value = %.3f#pm%.3f ",corrfitter->GetNSSigma(),corrfitter->GetNSSigmaError()));
  t4=pvStatTests1->AddText(0.,0.35,Form("AS Y = %.3f#pm%.3f ",corrfitter->GetASYield(),corrfitter->GetASYieldError()));
  t5=pvStatTests1->AddText(0.,0.20,Form("AS bessel value = %.3f#pm%.3f ",corrfitter->GetASSigma(),corrfitter->GetASSigmaError()));
  pvStatTests1->Draw();
}
else{
  TText *t0,*t1,*t2,*t3,*t4,*t5,*t6,*tpT;
  t0=pvStatTests1->AddText(0.,1.00,Form("#chi^{2}/ndf = %.1f/%d ",fFit->GetChisquare(),fFit->GetNDF()));
  t1=pvStatTests1->AddText(0.,0.85,Form("Ped = %.3f#pm%.3f ",corrfitter->GetPedestal(),corrfitter->GetPedestalError()));
  t2=pvStatTests1->AddText(0.,0.70,Form("NS Y = %.3f#pm%.3f ",corrfitter->GetNSYield(),corrfitter->GetNSYieldError()));
  t3=pvStatTests1->AddText(0.,0.55,Form("NS #sigma = %.3f#pm%.3f ",corrfitter->GetNSSigma(),corrfitter->GetNSSigmaError()));
  t4=pvStatTests1->AddText(0.,0.40,Form("NS #beta = %.3f#pm%.3f ",corrfitter->GetBeta(),corrfitter->GetBetaError()));
  t5=pvStatTests1->AddText(0.,0.25,Form("AS Y = %.3f#pm%.3f ",corrfitter->GetASYield(),corrfitter->GetASYieldError()));
  t6=pvStatTests1->AddText(0.,0.10,Form("AS #sigma = %.3f#pm%.3f ",corrfitter->GetASSigma(),corrfitter->GetASSigmaError()));
  pvStatTests1->Draw();
}
```

Figure 2: Part of the code in *FitPlotsAndSaveOutputsFastVersioneScriptOutput.C* where the histograms that plot the fitting function are produced

The values of the parameters and their statistical errors are taken from *AliHFCorrFitter.h*, as shown in Figure 3.

AliHFCorrFitter.h

C: > root_v5.34.36 > AliHFCorrFitter.h

```
Double_t GetNSI0(){return TMath::BesselI0(fFit->GetParameter("NS bessel value"));}
Double_t GetNSI1(){return TMath::BesselI1(fFit->GetParameter("NS bessel value"));}
Double_t GetASI0(){return TMath::BesselI0(fFit->GetParameter("AS bessel value"));}
Double_t GetASI1(){return TMath::BesselI1(fFit->GetParameter("AS bessel value"));}
Double_t GetNSK(){return fFit->GetParameter("NS bessel value");}
Double_t GetNSKerror(){return fFit->GetParError(fFit->GetParNumber("NS bessel value"));}
Double_t GetASK(){return fFit->GetParameter("AS bessel value");}
Double_t GetASKerror(){return fFit->GetParError(fFit->GetParNumber("AS bessel value"));}
Double_t GetNSSigma(){
  if(fTypeOfFitfunc==kConstThreeGausPeriodicity){
    return TMath::Sqrt(fFit->GetParameter("fract 1g")*fFit->GetParameter("NS #sigma 1g")*fFit->GetParameter("NS #sigma 1g")+
    (1.-fFit->GetParameter("fract 1g"))*fFit->GetParameter("NS #sigma 2g")*fFit->GetParameter("NS #sigma 2g"));
  }
  else if(fTypeOfFitfunc==kVonMises){
    return TMath::Sqrt(-2.*TMath::Log(GetNSI1()/GetNSI0()));
  }
  else return fFit->GetParameter("NS #sigma");
}
Double_t GetASSigma(){
  if(fTypeOfFitfunc==kConstThreeGausPeriodicityAS){
    return TMath::Sqrt(fFit->GetParameter("fract 1g")*fFit->GetParameter("AS #sigma 1g")*fFit->GetParameter("AS #sigma 1g")+
    (1.-fFit->GetParameter("fract 1g"))*fFit->GetParameter("AS #sigma 2g")*fFit->GetParameter("AS #sigma 2g"));
  }
  else if(fTypeOfFitfunc==kVonMises){
    return TMath::Sqrt(-2.*TMath::Log(GetASI1()/GetASI0()));
  }
  else return fFit->GetParameter("AS #sigma");
}
Double_t GetNSYield(){return fFit->GetParameter("NS Y");}
Double_t GetASYield(){return fFit->GetParameter("AS Y");}
Double_t GetBeta(){return fFit->GetParameter(7);}
Double_t GetPedestal(){return fBaseline;}
Double_t Getv2hadron(){return fFit->GetParameter("v_{2} hadron");}
Double_t Getv2Dmeson(){return fFit->GetParameter("v_{2} D meson");}
Double_t GetNSSigmaError(){
  if(fTypeOfFitfunc==kConstThreeGausPeriodicity){
    return TMath::Sqrt(fFit->GetParameter("fract 1g")*fFit->GetParError(fFit->GetParNumber("NS #sigma 1g"))*fFit->GetParError(fFit->GetParNumber("NS #sigma 1g"))+
    (1.-fFit->GetParameter("fract 1g"))*fFit->GetParError(fFit->GetParNumber("NS #sigma 2g"))*fFit->GetParError(fFit->GetParNumber("NS #sigma 2g")));
  }
  else if(fTypeOfFitfunc==kVonMises){
    return abs((1./GetNSSigma())*(GetNSI1()/GetNSI0()-GetNSI0()/GetNSI1()+1./GetNSK()))*GetNSKerror();
  }
  else return fFit->GetParError(fFit->GetParNumber("NS #sigma"));
}
Double_t GetASSigmaError(){
  if(fTypeOfFitfunc==kConstThreeGausPeriodicityAS){
    return TMath::Sqrt(fFit->GetParameter("fract 1g")*fFit->GetParError(fFit->GetParNumber("AS #sigma 1g"))*fFit->GetParError(fFit->GetParNumber("AS #sigma 1g"))+
    (1.-fFit->GetParameter("fract 1g"))*fFit->GetParError(fFit->GetParNumber("AS #sigma 2g"))*fFit->GetParError(fFit->GetParNumber("AS #sigma 2g")));
  }
  else if(fTypeOfFitfunc==kVonMises){
    return abs((1./GetASSigma())*(GetASI1()/GetASI0()-GetASI0()/GetASI1()+1./GetASK()))*GetASKerror();
  }
  else return fFit->GetParError(fFit->GetParNumber("AS #sigma"));
}
Double_t GetNSYieldError(){return fFit->GetParError(fFit->GetParNumber("NS Y"));}
Double_t GetASYieldError(){return fFit->GetParError(fFit->GetParNumber("AS Y"));}
Double_t GetBetaError(){return fFit->GetParError(7);}
Double_t GetPedestalError(){return fErrbaseline;}
Double_t Getv2hadronError(){return fFit->GetParError(fFit->GetParNumber("v_{2} hadron"));}
Double_t Getv2DmesonError(){return fFit->GetParError(fFit->GetParNumber("v_{2} D meson"));}
Double_t GetBinCountingYields(Double_t &nsYieldErr,Double_t &asYield,Double_t &asYieldErr){
  nsYieldErr=fEnsybc;
  asYield=fAsybc;
  asYieldErr=fEasybc;
  return fNsybc;
}
```

Figure 3: The values of the parameters and of their errors, shown in the histograms, are taken from this part of the code in *AliHFCorrFitter.h*

4.3 Plot of the fitting functions

The plot of the fitting function is obtained using the following commands in the ROOT terminal:

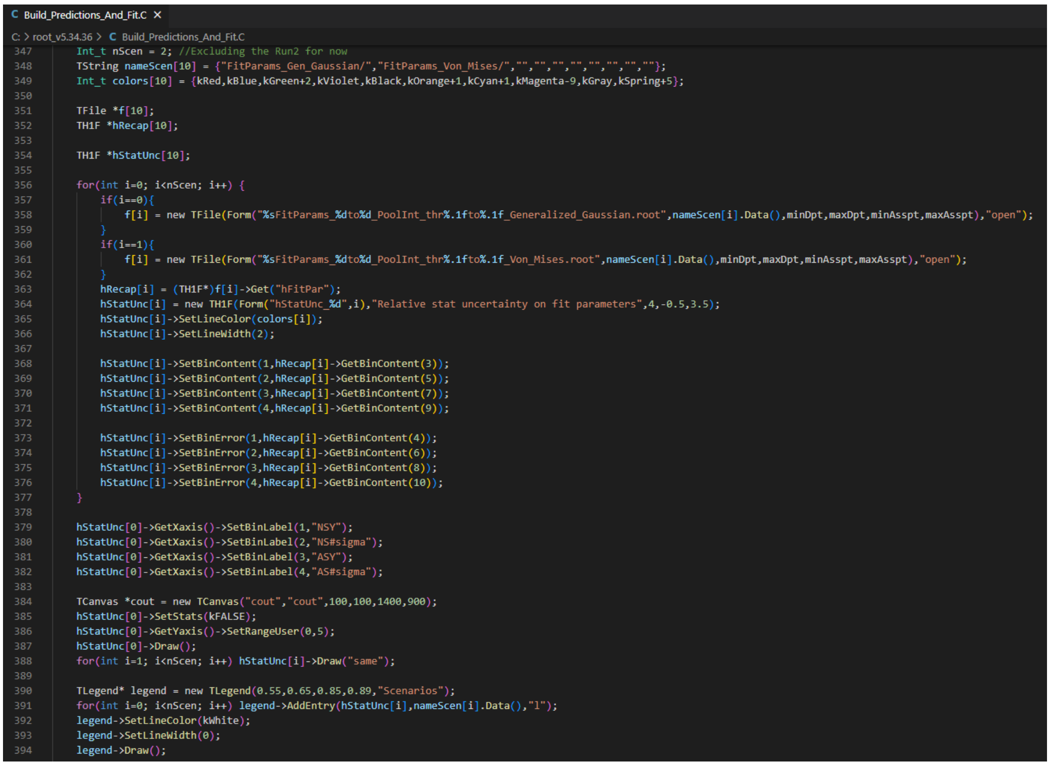
```
.L FitPlotsAndSaveOutputsFastVersioneScriptOutput.C

FitPlotsAndSaveOutputsFast(10,5,3,kTRUE,0.25*TMath::Pi(),0.5*TMath::Pi(),2,99,24,36)
```

The first line compiles the macro *FitPlotsAndSaveOutputsFastVersioneScriptOutput.C*, while the second line sets the arguments of the macro, specifically: the first argument (**10**) represents the case, so *case 10* is the Von Mises fitting function, while *case 7* is the Generalised Gaussian + Gaussian fitting function, the baseline fixed at average of the points in the transverse region $\mid (\pi/4 - \pi/2) \mid$ is implemented with the second argument of the macro set to **5**, the near-side mean fixed to zero and the away-side mean fixed to π is implemented with the third argument set to **3**, and the last four values, **(2, 99, 24, 36)** represents the first two the ${p_T}^{assoc}$ range and the last two the ${p_T}^{D}$ range.

A histogram of the fitted azimuthal correlation function and of the parameters of the fitting function in the chosen bin is created and stored.

4.4 Comparison of the fit parameters

The comparison of the parameters of the Von Mises and of the Generalised Gaussian + Gaussian functions is performed in *BuildPredictionsAndFit.C*, as reported in Figure 4.



```
Build_Predictions_And_Fit.C
C: > root_v5.34.36 > C Build_Predictions_And_Fit.C
    Int_t nScen = 2; //Excluding the Run2 for now
    TString nameScen[10] = {"FitParams_Gen_Gaussian/","FitParams_Von_Mises/","","","","","","","",""};
    Int_t colors[10] = {kRed,kBlue,kGreen+2,kViolet,kBlack,kOrange+1,kCyan+1,kMagenta-9,kGray,kSpring+5};

    TFile *f[10];
    TH1F *hRecap[10];

    TH1F *hStatUnc[10];

    for(int i=0; i<nScen; i++) {
        if(i==0){
            f[i] = new TFile(Form("%sFitParams_%dto%d_PoolInt_thr%.1fto%.1f_Generalized_Gaussian.root",nameScen[i].Data(),minDpt,maxDpt,minAsspt,maxAsspt),"open");
        }
        if(i==1){
            f[i] = new TFile(Form("%sFitParams_%dto%d_PoolInt_thr%.1fto%.1f_Von_Mises.root",nameScen[i].Data(),minDpt,maxDpt,minAsspt,maxAsspt),"open");
        }
        hRecap[i] = (TH1F*)f[i]->Get("hFitPar");
        hStatUnc[i] = new TH1F(Form("hStatUnc_%d",i),"Relative stat uncertainty on fit parameters",4,-0.5,3.5);
        hStatUnc[i]->SetLineColor(colors[i]);
        hStatUnc[i]->SetLineWidth(2);

        hStatUnc[i]->SetBinContent(1,hRecap[i]->GetBinContent(3));
        hStatUnc[i]->SetBinContent(2,hRecap[i]->GetBinContent(5));
        hStatUnc[i]->SetBinContent(3,hRecap[i]->GetBinContent(7));
        hStatUnc[i]->SetBinContent(4,hRecap[i]->GetBinContent(9));

        hStatUnc[i]->SetBinError(1,hRecap[i]->GetBinContent(4));
        hStatUnc[i]->SetBinError(2,hRecap[i]->GetBinContent(6));
        hStatUnc[i]->SetBinError(3,hRecap[i]->GetBinContent(8));
        hStatUnc[i]->SetBinError(4,hRecap[i]->GetBinContent(10));
    }

    hStatUnc[0]->GetXaxis()->SetBinLabel(1,"NSY");
    hStatUnc[0]->GetXaxis()->SetBinLabel(2,"NS#sigma");
    hStatUnc[0]->GetXaxis()->SetBinLabel(3,"ASY");
    hStatUnc[0]->GetXaxis()->SetBinLabel(4,"AS#sigma");

    TCanvas *cout = new TCanvas("cout","cout",100,100,1400,900);
    hStatUnc[0]->SetStats(kFALSE);
    hStatUnc[0]->GetYaxis()->SetRangeUser(0,5);
    hStatUnc[0]->Draw();
    for(int i=1; i<nScen; i++) hStatUnc[i]->Draw("same");

    TLegend* legend = new TLegend(0.55,0.65,0.85,0.89,"Scenarios");
    for(int i=0; i<nScen; i++) legend->AddEntry(hStatUnc[i],nameScen[i].Data(),"l");
    legend->SetLineColor(kWhite);
    legend->SetLineWidth(0);
    legend->Draw();
```

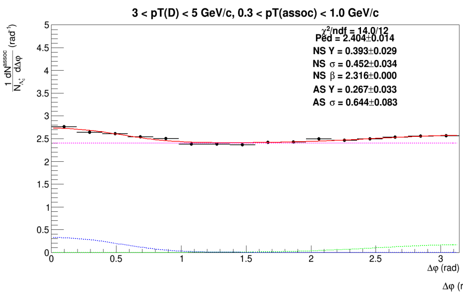
Figure 4: Part of the code in *BuildPredictionsAndFit.C* where the comparison of the fit parameters between the Generalised Gaussian + Gaussian and the Von Mises functions is performed

5 Results

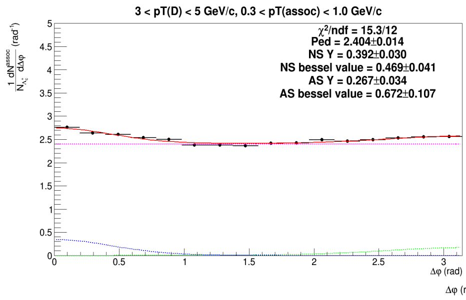
5.1 Comparison between the Generalised Gaussian + Gaussian and the Von Mises fitting functions for pp collisions at $\sqrt{s} = 13$ TeV

In order to compare the Generalised Gaussian + Gaussian and the Von Mises fitting functions, the azimuthal correlation function of prompt D mesons with associated particles for pp collisions at $\sqrt{s} = 13$ TeV has been fitted using the two functions and the systematic uncertainties evaluated accordingly. Then, a parameter comparison of the two fitting functions has been performed.

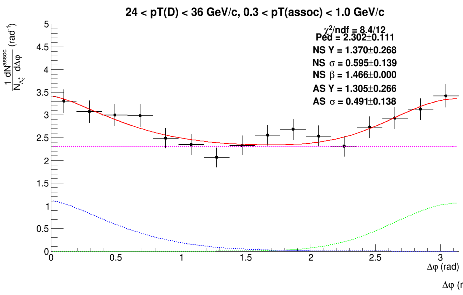
In Figure 5, Figure 6 and Figure 7, the results for the $\mathrm{p_T}^{\mathrm{assoc}}$ intervals $0.3 < p_{\mathrm{T}}^{\mathrm{assoc}} < 1$ GeV/c, $1 < p_{\mathrm{T}}^{\mathrm{assoc}} < 2$ GeV/c, $p_{\mathrm{T}}^{\mathrm{assoc}} > 3$ GeV/c for the $\mathrm{p_T}^{\mathrm{D}}$ ranges $3 < p_{\mathrm{T}}^{\mathrm{D}} < 5$ GeV/c and $24 < p_{\mathrm{T}}^{\mathrm{D}} < 36$ GeV/c are reported.



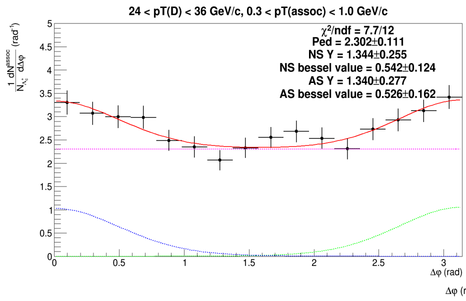
(a) Generalised Gaussian + Gaussian fitting function for the $p_T{}^D$ interval $3 < p_T{}^D < 5$ GeV/c



(b) Von Mises fitting function for the $p_T{}^D$ interval $3 < p_T{}^D < 5$ GeV/c

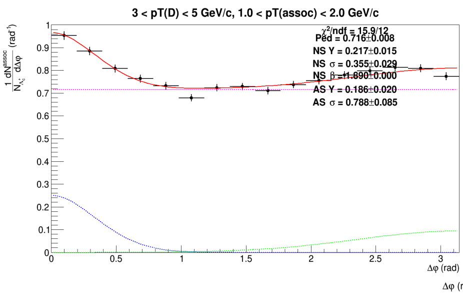


(c) Generalised Gaussian + Gaussian fitting function for the $p_T{}^D$ interval $24 < p_T{}^D < 36$ GeV/c

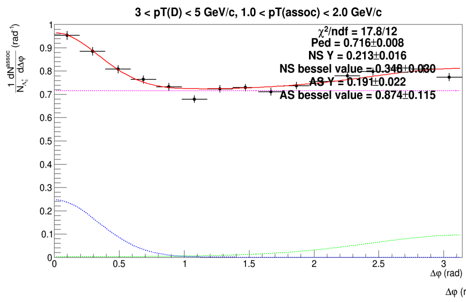


(d) Von Mises fitting function for the $p_T{}^D$ interval $24 < p_T{}^D < 36$ GeV/c

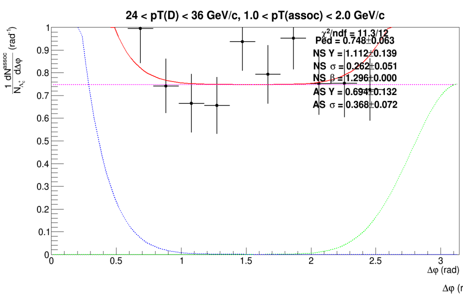
Figure 5: Azimuthal correlation functions of D mesons with associated particles in pp collisions at $\sqrt{s}$ = 13 TeV in the $p_T{}^{assoc}$ range $0.3 < p_T{}^{assoc} < 1$ GeV/c.



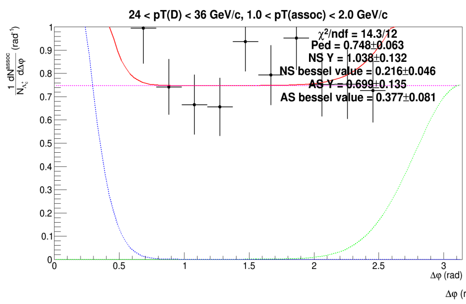
(a) Generalised Gaussian + Gaussian fitting function for the $p_T{}^D$ interval $3 < p_T{}^D < 5$ GeV/c



(b) Von Mises fitting function for the $p_T{}^D$ interval $3 < p_T{}^D < 5$ GeV/c

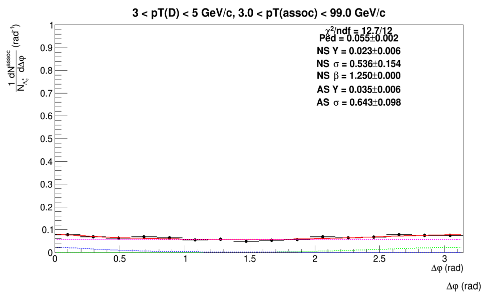


(c) Generalised Gaussian + Gaussian fitting function for the $p_T{}^D$ interval $24 < p_T{}^D < 36$ GeV/c

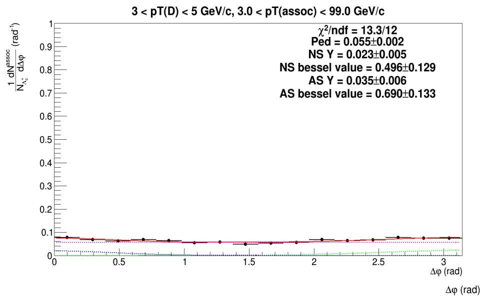


(d) Von Mises fitting function for the $p_T{}^D$ interval $24 < p_T{}^D < 36$ GeV/c

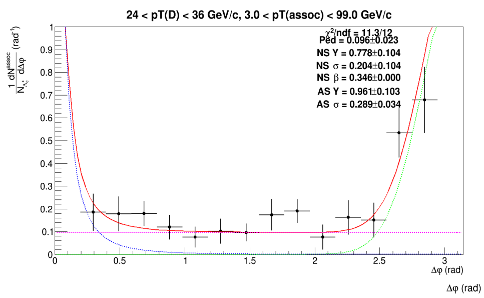
Figure 6: Azimuthal correlation functions of D mesons with associated particles in pp collisions at $\sqrt{s}$ = 13 TeV in the $p_T{}^{assoc}$ range $1 < p_T{}^{assoc} < 2$ GeV/c.



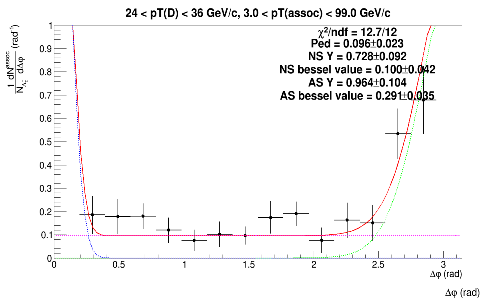
(a) Generalised Gaussian + Gaussian fitting function for the $p_T{}^D$ interval $3 < p_T{}^D < 5$ GeV/c



(b) Von Mises fitting function for the $p_T{}^D$ interval $3 < p_T{}^D < 5$ GeV/c



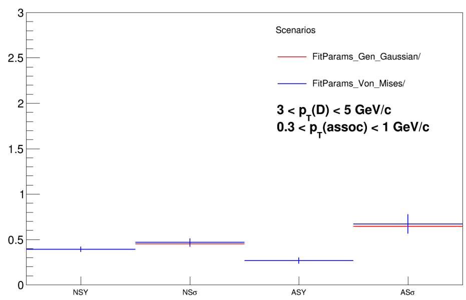
(c) Generalised Gaussian + Gaussian fitting function for the $p_T{}^D$ interval $24 < p_T{}^D < 36$ GeV/c



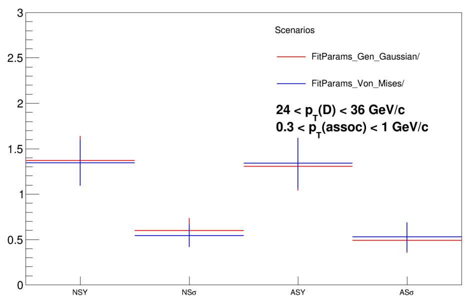
(d) Von Mises fitting function for the $p_T{}^D$ interval $24 < p_T{}^D < 36$ GeV/c

Figure 7: Azimuthal correlation functions of D mesons with associated particles in pp collisions at $\sqrt{s}$ = 13 TeV in the $p_T{}^{assoc}$ range $p_T{}^{assoc} > 3$ GeV/c.

In order to make a more quantitative comparison, the fit parameters of the Generalised Gaussian + Gaussian and of the Von Mises have been compared one by one. Specifically, the compared parameters are the yields and the widths of the near-side and away-side peaks. In Figure 8, Figure 9 and Figure 10, the parameters comparison for the $p_T{}^{assoc}$ intervals $0.3 < p_T{}^{assoc} < 1$ GeV/c, $1 < p_T{}^{assoc} < 2$ GeV/c, $p_T{}^{assoc} > 3$ GeV/c for the $p_T{}^D$ ranges $3 < p_T{}^D < 5$ GeV/c and $24 < p_T{}^D < 36$ GeV/c are reported.

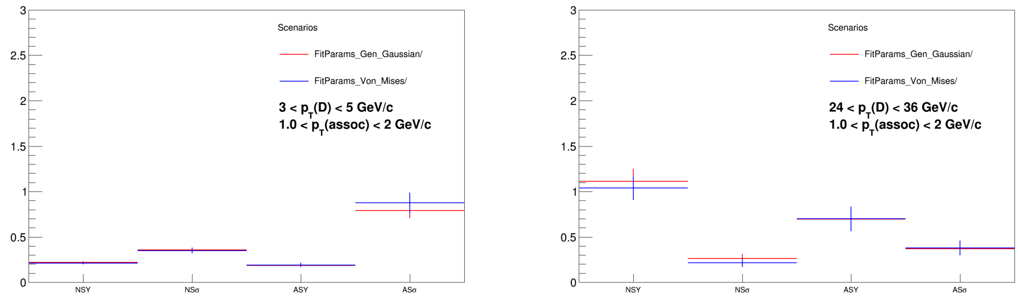


(a) Fit parameters comparison for the $p_T{}^D$ interval $3 < p_T{}^D < 5$ GeV/c



(b) Fit parameters comparison for the $p_T{}^D$ interval $24 < p_T{}^D < 36$ GeV/c

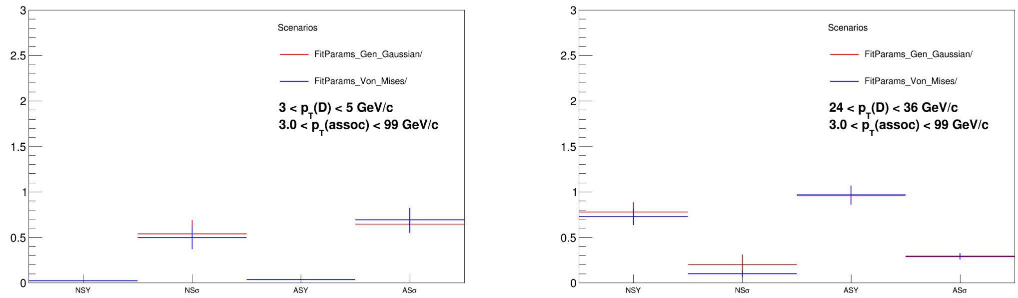
Figure 8: Comparison of the parameters of the azimuthal correlation function fitted with the Generalised Gaussian + Gaussian function (red) and with the Von Mises function (blue) in the $p_T{}^{assoc}$ range $0.3 < p_T{}^{assoc} < 1$ GeV/c.



(a) Fit parameters comparison for the $\mathrm{p_T}^{\mathrm{D}}$ interval $3 < p_{\mathrm{T}}{}^{\mathrm{D}} < 5$ GeV/c

(b) Fit parameters comparison for the $\mathrm{p_T}^{\mathrm{D}}$ interval $24 < p_{\mathrm{T}}{}^{\mathrm{D}} < 36$ GeV/c

Figure 9: Comparison of the parameters of the azimuthal correlation function fitted with the Generalised Gaussian + Gaussian function (red) and with the Von Mises function (blue) in the $\mathrm{p_T}^{\mathrm{assoc}}$ range $1 < p_{\mathrm{T}}{}^{\mathrm{assoc}} < 2$ GeV/c.



(a) Fit parameters comparison for the $\mathrm{p_T}^{\mathrm{D}}$ interval $3 < p_{\mathrm{T}}{}^{\mathrm{D}} < 5$ GeV/c

(b) Fit parameters comparison for the $\mathrm{p_T}^{\mathrm{D}}$ interval $24 < p_{\mathrm{T}}{}^{\mathrm{D}} < 36$ GeV/c

Figure 10: Comparison of the parameters of the azimuthal correlation function fitted with the Generalised Gaussian + Gaussian function (red) and with the Von Mises function (blue) in the $\mathrm{p_T}^{\mathrm{assoc}}$ range $p_{\mathrm{T}}{}^{\mathrm{assoc}} > 3$ GeV/c.

The results for the azimuthal correlation function of D mesons with associated particles for pp collisions at $\sqrt{s} = 13$ TeV has a similar trend when fitted with both the Generalised Gaussian + Gaussian and the Von Mises functions. In particular, it shows an increase of both the near-side and of the away-side yields increasing $\mathrm{p_T}^{\mathrm{D}}$ in all the $\mathrm{p_T}^{\mathrm{assoc}}$ intervals. Also, the near-side and the away-side widths slowly decrease when increasing $\mathrm{p_T}^{\mathrm{D}}$ in each $\mathrm{p_T}^{\mathrm{assoc}}$ interval; this peaks narrowing is caused by the increased boost of fragmenting partons at higher $\mathrm{p_T}^{\mathrm{D}}$.

The comparison of the fit parameters shows an overall agreement within statistical uncertainties of the parameters of the Von Mises and of the Generalised Gaussian + Gaussian fitting functions.
In addition to that, the values of χ^2/ndf are very similar in both the fits, where χ^2/ndf is a parameter that represents how well the calculated data describe the observed data. For $\chi^2/\mathrm{ndf} = 1$, the calculated data fit perfectly the observed ones.

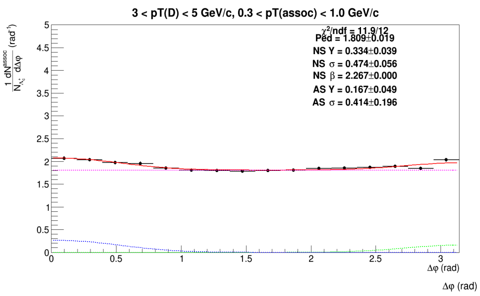
In the end, it is possible to state that the Von Mises function is a good candidate for fitting the azimuthal correlation function since it gives results comparable with those of the Generalised Gaussian + Gaussian fitting function and, most importantly, the Von Mises is way simpler thanks to the absence of the β parameter, thus it has one less parameter than the Generalised Gaussian + Gaussian. In fact, the β parameter not only is very hard to calculate - it has to be fixed to Monte Carlo value, inducing a slight, but unwanted, model dependence - but it forces the function to fit the data, losing some physics behind the processes. The main advantage of the Von Mises function is that it does not need to be forced for fitting the data well.

5.2 Comparison between the Generalised Gaussian + Gaussian and the Von Mises fitting functions for pp and p-Pb collisions at $\sqrt{s}$ = 5.02 TeV

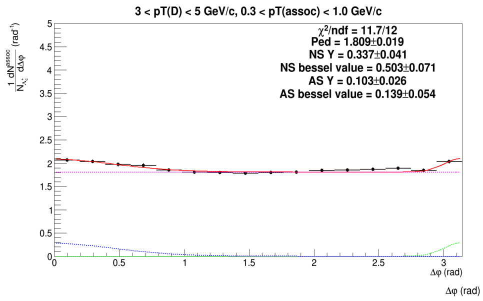
A similar data analysis has been performed on the azimuthal correlation function of D mesons with associated particles for pp and p-Pb collisions at $\sqrt{s}$ = 5.02 TeV. These data have been already fitted using the Generalised Gaussian + Gaussian in [3].

In this work, instead, the azimuthal correlation function has been fitted using both the Generalised Gaussian + Gaussian and the Von Mises functions, then the results from the fit and from the comparison of the parameters have been studied and compared to those of the pp collisions at $\sqrt{s}$ = 13 TeV.

Figure 11 and Figure 12 show the results of the azimuthal correlation distribution with the superimposed fit functions for pp collisions at $\sqrt{s}$ = 5.02 TeV in the p_T^{assoc} intervals $0.3 < p_T^{assoc} < 1$ GeV/c and $1 < p_T^{assoc} < 2$ GeV/c for the p_T^D range $3 < p_T^D < 5$ GeV/c, while Figure 13 shows comparison of the fit parameters in the same intervals of p_T.

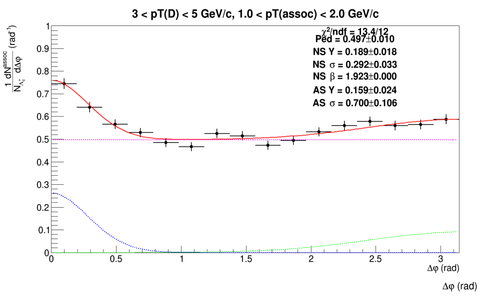


(a) Generalised Gaussian + Gaussian fitting function for the p_T^D interval $3 < p_T^D < 5$ GeV/c

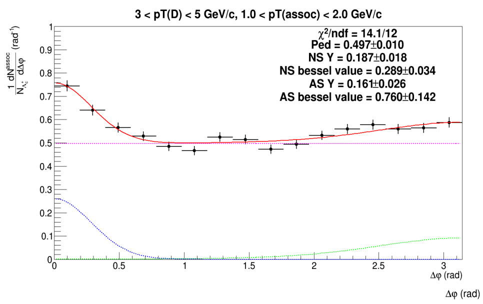


(b) Von Mises fitting function for the p_T^D interval $3 < p_T^D < 5$ GeV/c

Figure 11: Azimuthal correlation functions of D mesons with associated particles for pp collisions at $\sqrt{s}$ = 5.02 TeV in the p_T^{assoc} range $0.3 < p_T^{assoc} < 1$ GeV/c.

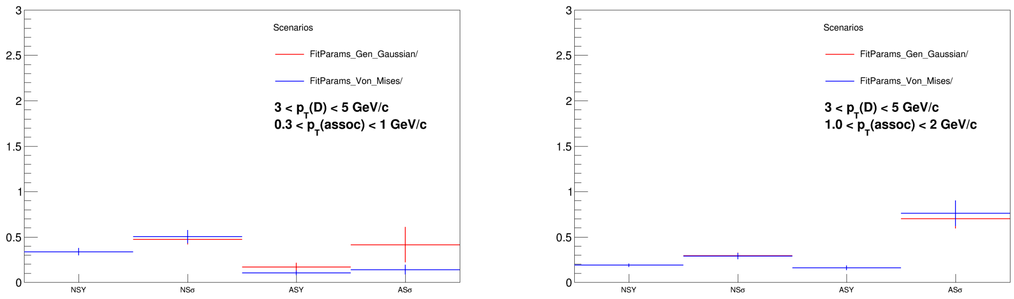


(a) Generalised Gaussian + Gaussian fitting function for the p_T^D interval $3 < p_T^D < 5$ GeV/c



(b) Von Mises fitting function for the p_T^D interval $3 < p_T^D < 5$ GeV/c

Figure 12: Azimuthal correlation functions of D mesons with associated particles for pp collisions at $\sqrt{s}$ = 5 TeV in the p_T^{assoc} range $1 < p_T^{assoc} < 2$ GeV/c.

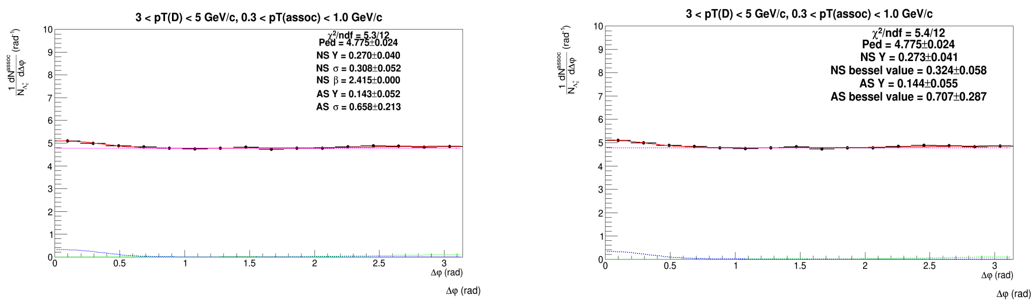


(a) Fit parameters comparison for the $\mathrm{p_T}^{\mathrm{assoc}}$ interval $0.3 < p_\mathrm{T}{}^{\mathrm{assoc}} < 1$ GeV/c

(b) Fit parameters comparison for the $\mathrm{p_T}^{\mathrm{assoc}}$ interval $1 < p_\mathrm{T}{}^{\mathrm{assoc}} < 2$ GeV/c

Figure 13: Comparison of the parameters of the azimuthal correlation function fitted with the Generalised Gaussian + Gaussian function (red) and with the Von Mises function (blue) in the $\mathrm{p_T}^{\mathrm{D}}$ range $3 < p_\mathrm{T}{}^{\mathrm{D}} < 5$ GeV/c for pp collisions at $\sqrt{s} = 5.02$ TeV.

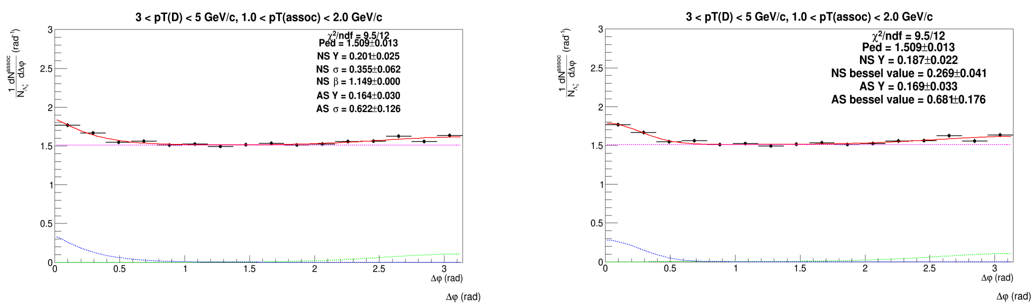
Figure 14 and Figure 15 show the results of the azimuthal correlation distribution with the superimposed fit functions for p-Pb collisions at $\sqrt{s} = 5.02$ TeV for the $\mathrm{p_T}^{\mathrm{assoc}}$ intervals $0.3 < p_\mathrm{T}{}^{\mathrm{assoc}} < 1$ GeV/c and $1 < p_\mathrm{T}{}^{\mathrm{assoc}} < 2$ GeV/c for the $\mathrm{p_T}^{\mathrm{D}}$ range $3 < p_\mathrm{T}{}^{\mathrm{D}} < 5$ GeV/c, while Figure 16 shows the results of the fit parameters comparison for the same intervals of $\mathrm{p_T}$.



(a) Generalised Gaussian + Gaussian fitting function for the $\mathrm{p_T}^{\mathrm{D}}$ interval $3 < p_\mathrm{T}{}^{\mathrm{D}} < 5$ GeV/c

(b) Von Mises fitting function for the $\mathrm{p_T}^{\mathrm{D}}$ interval $3 < p_\mathrm{T}{}^{\mathrm{D}} < 5$ GeV/c

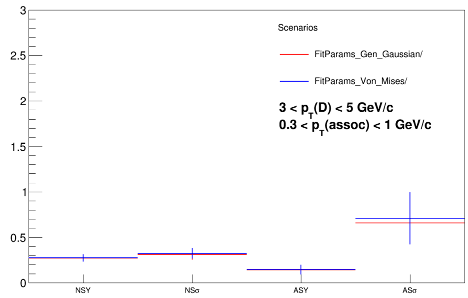
Figure 14: Azimuthal correlation functions of D mesons with associated particles for p-Pb collisions at $\sqrt{s} = 5.02$ TeV in the $\mathrm{p_T}^{\mathrm{assoc}}$ range $0.3 < p_\mathrm{T}{}^{\mathrm{assoc}} < 1$ GeV/c.



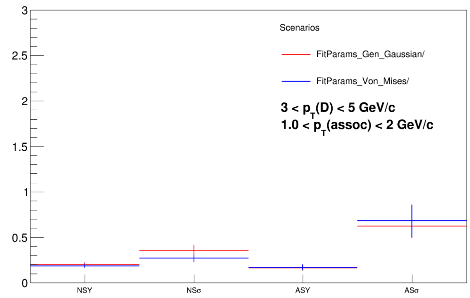
(a) Generalised Gaussian + Gaussian fitting function for the $\mathrm{p_T}^{\mathrm{D}}$ interval $3 < p_\mathrm{T}{}^{\mathrm{D}} < 5$ GeV/c

(b) Von Mises fitting function for the $\mathrm{p_T}^{\mathrm{D}}$ interval $3 < p_\mathrm{T}{}^{\mathrm{D}} < 5$ GeV/c

Figure 15: Azimuthal correlation functions of D mesons with associated particles for p-Pb collisions at $\sqrt{s} = 5.02$ TeV in the $\mathrm{p_T}^{\mathrm{assoc}}$ range $1 < p_\mathrm{T}{}^{\mathrm{assoc}} < 2$ GeV/c.



(a) Fit parameters comparison for the $p_T{}^{assoc}$ interval $0.3 < p_T{}^{assoc} < 1$ GeV/c



(b) Fit parameters comparison for the $p_T{}^{assoc}$ interval $1 < p_T{}^{assoc} < 2$ GeV/c

Figure 16: Comparison of the parameters of the azimuthal correlation function fitted with the Generalised Gaussian + Gaussian function (red) and with the Von Mises function (blue) in the $p_T{}^D$ range $3 < p_T{}^D < 5$ GeV/c for p-Pb collisions at $\sqrt{s} = 5.02$ TeV.

It is possible to see from the results that the near-side and away-side yields and the near-side width trends are compatible with those at $\sqrt{s} = 13$ TeV for pp collisions, while the away-side peak narrowing is not observed due to less precision in lower $\sqrt{s}$ collisions.

In addition to that, the Von Mises function gives results comparable to those of the Generalised Gaussian + Gaussian for both pp and p-Pb collisions, thus it can be said that the Von Mises function is a good candidate for fitting the azimuthal correlation function also for other collisions.

6 Conclusions

A study of the Von Mises function used for fitting the azimuthal correlation function of prompt D mesons with associated charged particles, measured in pp collisions at $\sqrt{s} = 13$ TeV with the ALICE detector, was presented.

The possible advantages of using the Von Mises function w.r.t the Generalised Gaussian + Gaussian function for fitting the azimuthal correlation function, also at lower-energy collisions, have been reported. Specifically, not only there is an overall agreement within the statistical uncertainties of the values obtained for the fit parameters between both the fitting functions, but also the Von Mises is a more suitable function for studying the azimuthal correlations thanks to its periodicity and, most importantly, to the absence of the β parameter, which forces the function to fit the data; thus, the Von Mises function has less parameters and less model dependence.

The study of the Von Mises fitting function might be relevant not only for understanding more deeply proton-proton collisions, but also in view of future heavy-ions collisions, like Pb-Pb collisions, that could be accessible during the Run 3 of LHC.

7 Acknowledgements

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