

Searching for $D^0 \rightarrow \pi^+ \pi^- \nu \bar{\nu}$ decays and measuring angular asymmetries in polarised $\Lambda_c^+ \rightarrow p \mu^+ \mu^-$ decays at FCC-ee

Tabea Hacheney^a, Lars Röhrig^{a,b}, Dominik Mitzel^a, Angelo Di Canto^c, Stéphane Monteil^b

^a*Fakultät Physik, Technische Universität Dortmund, Dortmund, Germany*

^b*Université Clermont Auvergne, CNRS/IN2P3, LPC, Clermont-Ferrand, France*

^c*Physics Department, Brookhaven National Laboratory, Upton NY, United States*

Abstract

This note presents sensitivity studies for the rare charm decays $D^0 \rightarrow \pi^+ \pi^- \nu \bar{\nu}$ and $\Lambda_c^+ \rightarrow p \mu^+ \mu^-$ at FCC-ee, when operated at the Z pole with 6×10^{12} produced Z bosons. The results indicate that an observation of the decay $D^0 \rightarrow \pi^+ \pi^- \nu \bar{\nu}$ with a significance of $5(3)\sigma$ could be possible for branching fractions at the level of $[1.8 - 2.1] \times 10^{-7}$ ($[2.9 - 3.4] \times 10^{-7}$). Angular asymmetries in $\Lambda_c^+ \rightarrow p \mu^+ \mu^-$ could be measured in decays of polarized Λ_c^+ baryons with a precision of 0.9%. The precision expected at FCC-ee allows for the parameter space of a large class of motivated BSM models to be constrained significantly.

Contents

1	Introduction	1
2	Detector model and simulated data	3
3	Search for $D^0 \rightarrow \pi^+ \pi^- \nu \bar{\nu}$ decays	5
3.1	Weighting of simulated signal candidates	5
3.2	Candidate reconstruction	6
3.3	Selection optimisation	9
3.4	Sensitivity estimation	12
4	Angular asymmetries in $\Lambda_c^+ \rightarrow p \mu^+ \mu^-$ decays	14
4.1	Candidate reconstruction and selection	14
4.2	Sensitivity estimation	15
5	Conclusion	17
	References	18
A	Supplemental Material Search for $D^0 \rightarrow \pi^+ \pi^- \nu \bar{\nu}$ decays	21
A.1	Preselection variable distributions	21
A.2	BDT variable distributions	26
A.3	Correlation plots	31
A.4	Separation and Variable Importance	36
A.5	$K_{S/L}^0$ variables	41
B	Supplemental Material Angular asymmetries in $\Lambda_c^+ \rightarrow p \mu^+ \mu^-$ decays	43

1 Introduction

After the discovery of the Higgs boson at CERN’s Large Hadron Collider (LHC) in 2012 [1, 2], one of the main goals of current and future experiments is to precisely determine its properties, which, if found to be inconsistent with the standard model (SM), could give indirect hints of physics beyond the SM (BSM) appearing at the TeV scale. To accomplish this task, the experimental high-energy physics community is considering the possibility to build a future electron-positron collider, such as the Future Circular Collider (FCC-ee) [3]. The potential of FCC-ee goes well beyond the study of the Higgs boson. When operated at collision energies close to the Z pole, it can also serve as a facility for precise measurements in the sector of flavour physics as addressed by several studies focusing on bottom-hadron decays [4–6]. Despite the large samples of approximately 7×10^{11} $Z \rightarrow c\bar{c}$ decays expected to be produced at FCC-ee, FCC-ee’s potential in the sector of charmed hadrons remains largely unexplored. The field of charm physics plays an important role in the research program of current flavour experiments, such as LHCb [7] and Belle II [8]. Of particular interest are rare-charm decays into final states with leptons, which offer the opportunity to search for BSM physics [9]. Contrary to strange and bottom rare processes, where loop-induced transitions occur through the exchange of up-type quarks (u, c, t), down-type quarks (d, s, b) are exchanged in charm transitions. On the one hand, this makes charm transitions very suppressed, requiring large samples of experimental data to be studied, and subject to large theory uncertainties arising from strong-interaction effects at low energies, which cannot be calculated with perturbative methods. On the other hand, it means that rare charm decays offer complementary sensitivity to those performed with kaons and bottom mesons, giving access to orthogonal interaction couplings that cannot be probed otherwise and making them an alternative avenue for BSM searches.

Important tools for BSM searches that are less affected by theory uncertainties in rare charm decays are null tests, i.e., observables that are negligibly small in the SM due to exact or approximate symmetries or parametric suppression. A first example is provided by $c \rightarrow u\nu\bar{\nu}$ transitions: their SM branching fractions are tiny and well beyond the experimental reach, so any observation at current facilities would clearly indicate BSM physics [10, 11]. Example BSM scenarios that can be tested in $c \rightarrow u\nu\bar{\nu}$ decays include new scalar or vector bosons, leptoquarks, or light dark-matter particles mimicking the dineutrino signature. These are all theoretically well-motivated extensions of the SM that can address some of its limitations. Moreover, as the size of their branching fractions depends on neutrino flavour, rare charm dineutrino modes also test BSM physics in a model-independent approach: assuming lepton universality or charged lepton flavour conservation, which are exact symmetries in the SM, model-independent upper limits on branching fractions of flavour-summed dineutrino modes can be derived using $SU(2)_L$ symmetry and existing bounds on the less suppressed charged-lepton modes ($c \rightarrow u\ell^+\ell^-$ with $\ell = \mu, e$) [12]. With no additional assumptions about charged lepton flavour symmetries, the upper limits typically reach a few $\times 10^{-5}$. The upper limits are approximately four times smaller when assuming charged lepton flavour conservation and almost 10 times smaller in the case of lepton universality [10–12].

Experimentally, there is one only measurement by the BESIII collaboration, setting an upper limit on the decay $D^0 \rightarrow \pi^0\nu\bar{\nu}$ at 2.1×10^{-4} at 90% confidence level (CL) [13]. No other limit on any of the $c \rightarrow u\nu\bar{\nu}$ branching fraction exists, partly because of the experimental challenge of identifying final states with two neutrinos that leave no signature in the experimental apparatus.

A second example involves measuring angular asymmetries in charged-lepton decays of charmed hadrons, such as $\Lambda_c^+ \rightarrow p\mu^+\mu^-$. The LHCb collaboration has measured the forward-backward asymmetry, A_{FB} , in these decays [14]. Contrarily to proton-proton collisions, where Λ_c^+ baryons are produced predominantly unpolarised, FCC-ee would give access to polarised Λ_c^+ baryons, providing access to a wider range of angular observables. Many of these observables

constitute null tests of the SM and offer increased sensitivity to potential contributions from BSM physics.

This note describes sensitivity studies for the search of $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ decays and for the measurement of angular asymmetries in $\Lambda_c^+ \rightarrow p\mu^+\mu^-$ decays at FCC-ee, using an assumed sample of 6×10^{12} produced Z bosons and the IDEA detector concept [15]. These decays have been chosen to highlight the unique potential of FCC-ee in the field of rare charm decays, and its complementarity with respect to current flavour-physics experiments. The decay $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ has been chosen as an example of $c \rightarrow u\nu\bar{\nu}$ transitions, since it has the experimental advantage of providing a displaced secondary D^0 decay vertex that can be reconstructed from the two oppositely charged pions. Sensitivity estimates for related decay modes and topologies, such as $D^0 \rightarrow K^+K^-\nu\bar{\nu}$, $D^+ \rightarrow \pi^+\nu\bar{\nu}$ or $\Lambda_c^+ \rightarrow p\nu\bar{\nu}$ [10], are left for future studies. We show that the experimental conditions of e^+e^- collisions, the significant boost of the produced charm quarks, and the hermetic acceptance of the IDEA detector [15] enable efficient reconstruction and selection of $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ decays. The decay $\Lambda_c^+ \rightarrow p\mu^+\mu^-$ has been chosen to illustrate the potential of studying $c \rightarrow u\ell^+\ell^-$ transitions in initially polarised baryon decays. The clean environment of an e^+e^- collider and the large production rate of Z bosons enables high-precision measurements of angular asymmetries, potentially reaching sensitivities below the one-percent level for observables that have not been measured at hadron colliders.

The note is structured as follows. First, section 2 outlines the simulated samples of data used in the analysis and the IDEA detector concept. The analysis strategy for the search of the $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ decay is presented in section 3, which comprises a candidate selection, estimation of expected backgrounds and determination of the expected sensitivity. The study of angular asymmetries in $\Lambda_c^+ \rightarrow p\mu^+\mu^-$ decays follows in section 4. Final concluding remarks are given in section 5.

The results of this note have been submitted for publication as part of a combined paper on phenomenology and experimental prospects together with G. Hiller and D. Suelmann (TU Dortmund) in Ref. [16], where also more details about the size of possible NP effects in the studied decays are described.

2 Detector model and simulated data

The event generator **PYTHIA8** [17] is used to simulate the e^+e^- collision, as well as the hadronisation of the outgoing quarks. Modelling of the beam size is taken into account according to the latest beam-size parameters [18]. For background studies, inclusive $Z \rightarrow q\bar{q}$ decay samples are generated privately with the production parameters of the so-called **winter2023** campaign using **EvtGen** [19] for modeling the decay of unstable particles. In **EvtGen**, we use the decay table provided by the Belle II collaboration [20]. With respect to the official FCC-ee samples of the **winter2023** campaign, which rely exclusively on **PYTHIA8**, our approach gives a more realistic generation of the physics processes by accounting also for very rare decays of b and c hadrons, which are relevant for the presented analyses. Signal $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ and $\Lambda_c^+ \rightarrow p\mu^+\mu^-$ decays are arising from charm mesons directly produced in the primary e^+e^- collision (no charm from b -hadron decays) and also generated using **EvtGen** assuming uniformly distributed final-state particles in the decay phase space. Only decays of one flavour (D^0 and Λ_c^+) are considered to avoid the occurrence of the signal decay in both hemispheres, which would bias the event selection. The following numbers of events have been generated:

$$\begin{aligned} N(Z \rightarrow b\bar{b}) &= 10^8, & N(Z \rightarrow c\bar{c}) &= 10^8, \\ N(Z \rightarrow u\bar{u}) &= 10^8, & N(Z \rightarrow d\bar{d}) &= 10^8, \\ N(Z \rightarrow s\bar{s}) &= 10^8, & N(Z \rightarrow \tau^+\tau^-) &= 10^8, \\ N(D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}) &= 10^6, & N(\Lambda_c^+ \rightarrow p\mu^+\mu^-) &= 10^6. \end{aligned}$$

The $Z \rightarrow \tau^+\tau^-$ sample has been included to account for 3-prong τ decays into hadrons, which have missing momentum in the final state and are topologically similar to $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ signal candidates.

The detector model is based on the IDEA detector concept [15]. It consists of silicon vertex detectors close to the beam pipe, a drift chamber for quasi-continuous tracking of charged particles, a 2 T solenoid, as well as a dual-readout calorimetry and muon chambers. This model is widely used and tested in the community to derive detector requirements for both flavour and Higgs physics, with flavour-physics studies resulting in the most stringent requirements in terms of vertexing, tracking, and calorimetry [6]. For fast simulation, parametric detector responses have been modelled using **DELPHES** [21], including the full event reconstruction into particle-flow objects. The design of the IDEA detector is still to be optimised and we use publicly available performance numbers. More information can be found in the FCC feasibility study report Ref. [22]. The output, the community-wide **EDM4HEP** [23] standard, has further been processed within the **FCCAnalyses** framework, based on **ROOT**'s **RDataFrame** format [24].

For identification of charged hadrons, we combine particle-hypothesis separation using information from the drift chamber (cluster counting and dE/dx measurements, see left of Figure 1 taken from the conceptual design report Ref. [3]) and the silicon wrapper, which provides time-of-flight measurements at a radial distance of 2 m from the beam line. Following Ref. [15], we compute the separation using time-of-flight information assuming a resolution of 100ps and assuming a flight distance corresponding to particles emitted at $\theta = 90^\circ$. As an example, the $K \rightarrow \pi$ separation (dN/dx , time of flight and their combination) is shown on the right of Figure 1. In the analyses presented in this note, we consider $\pi \rightarrow K$ and $K \rightarrow p$ separation. When studying final states with muons, information from the muons system will play a crucial role. In the following, we assume a 0.4% pion-to-muon misidentification probability, as assumed in previous studies of rare decays at FCC-ee [25].

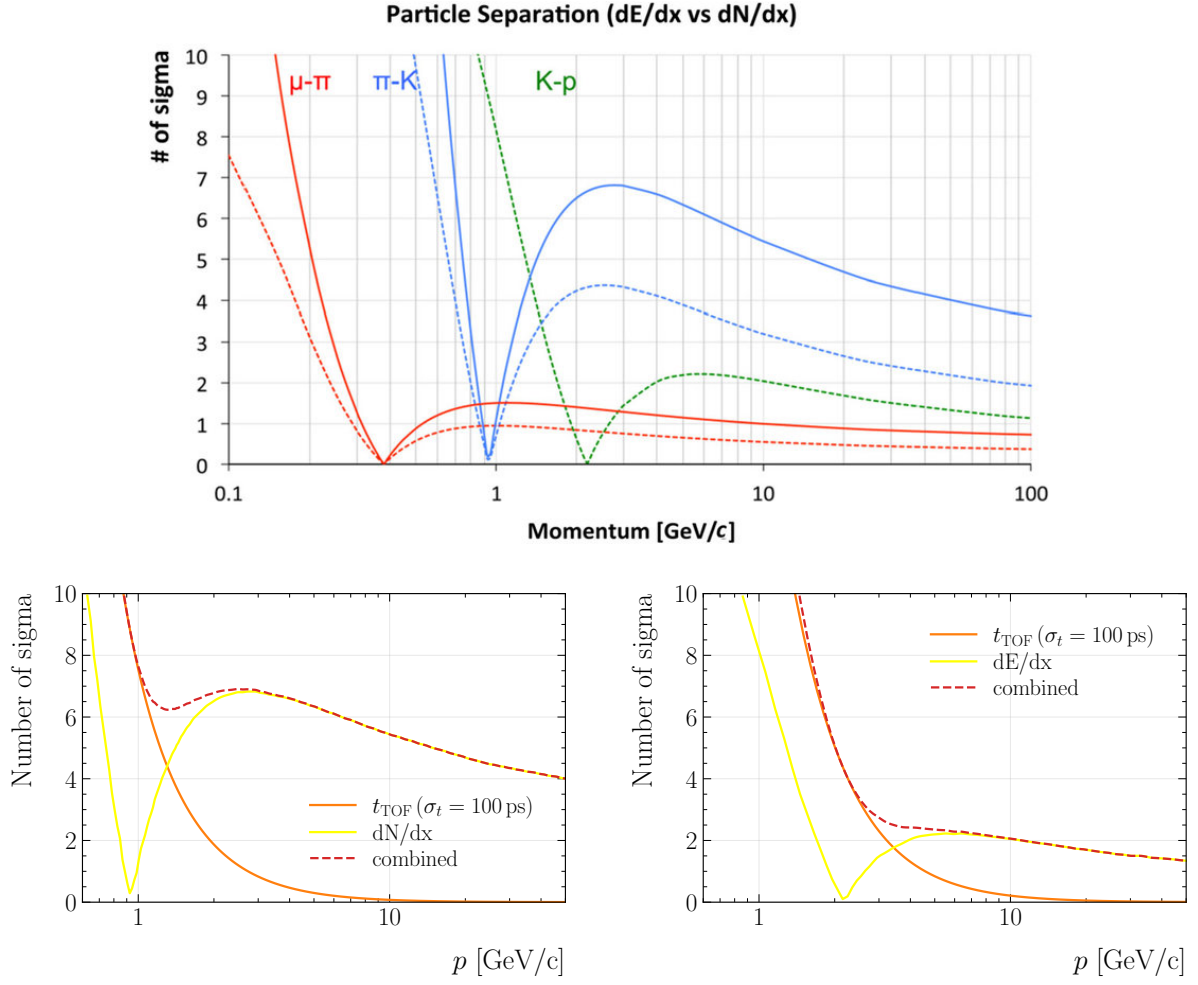


Figure 1: Energy-loss dE/dx measurements (dotted lines), expressed as separation power (in units of Gaussian standard deviations) between different particle species (top), reproduced from Ref. [3]. Combined separation of pions and kaons (lower left) and protons and kaons (lower right) using time-of-flight and dN/dx information.

3 Search for $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ decays

To estimate the FCC-ee sensitivity of the search for $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ decays, we develop a candidate reconstruction and selection strategy. This approach involves an initial cut-based preselection, followed by training and evaluating a Boosted Decision Tree (BDT). The selection performance might depend on the kinematics of the reconstructible final-state particles, which are affected by the underlying decay model, which remains unknown. To quantify the impact on the results of the phase-space decay model assumed in generation, we apply weights based on various hypothetical models to the simulated signal candidates, as detailed in the following section. Finally, we calculate the sensitivity using estimated efficiencies for both signal and background contributions.

3.1 Weighting of simulated signal candidates

Three alternative decay models, parametrized in terms of $q^2 = m^2(\nu\bar{\nu})$ and $p^2 = m^2(\pi^+\pi^-)$, have been provided by G. Hiller and D. Suelmann from TU Dortmund. The first model, referred to as **Dp**, uses the measured p^2 distribution of the decay $D^+ \rightarrow \pi^+\pi^-e^+\nu_e$ to infer the p^2 spectrum of the $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ decay. The other two models are derived from heavy hadron chiral perturbation theory (HHchiPT), including or excluding contributions from intermediate resonances in the dipion system, labelled as **HHchiPT_res** and **HHchiPT**, respectively. More details about the models can be found in Ref. [16]. Figure 2 compares the phase-space model used in the simulation (left) with the **Dp** model (right). Figure 3 shows the two HHchiPT-based models with (left) and without (right) resonances.

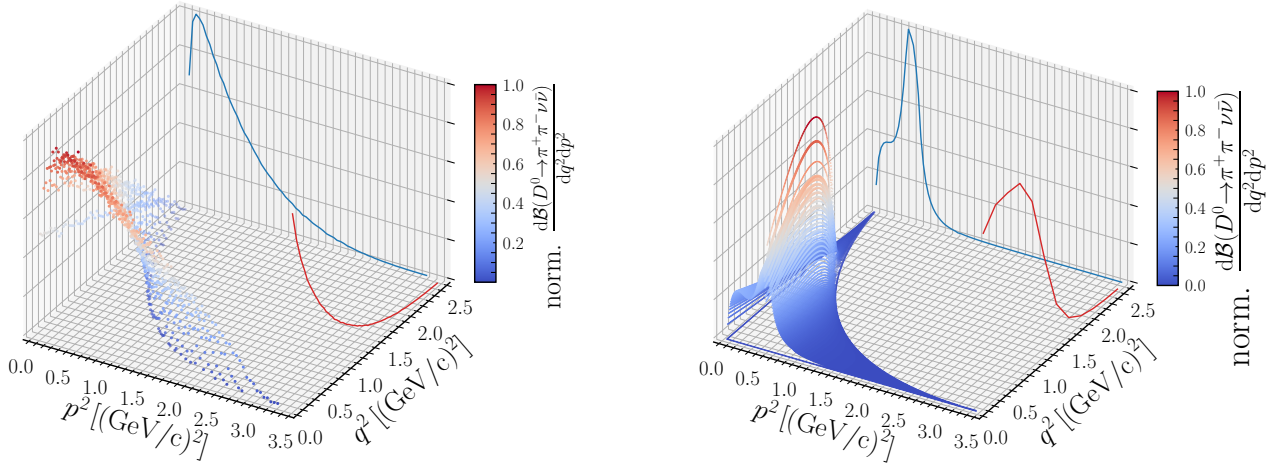


Figure 2: Normalised p^2 vs q^2 distribution of the simulated signal using phase space (left) and from the **Dp** theory model (right).

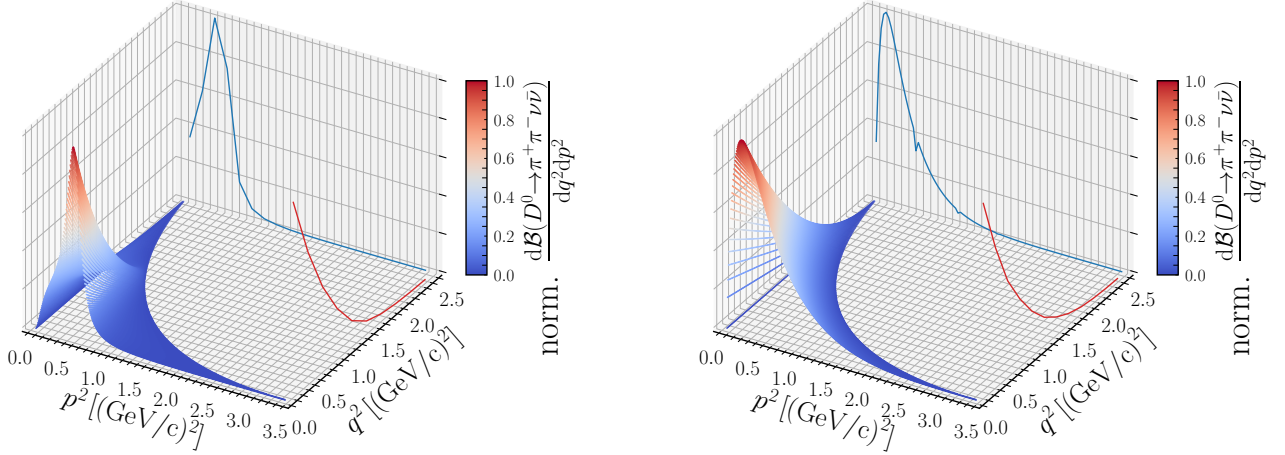


Figure 3: Normalised p^2 vs q^2 distribution from the HHchiPT_res (left) and HHchiPT (right) models.

For each alternative model, weights are obtained from the ratio between the two dimensional p^2 vs q^2 histograms of the alternative and phase-space model. The weights are then used to correct the generated signal decays on a per-event basis and evaluate the dependency of the result on the decay model. The entire analysis procedure is repeated for each alternative model (and the phase-space model), including the training of the BDT classifier and the optimisation of the selection requirements.

3.2 Candidate reconstruction

The reconstruction starts with the identification of tracks that originate from secondary vertices, well displaced from the primary e^+e^- interaction. The function `get_PrimaryTracks` from the `FCCAnalyses` library [26] identifies “primary” tracks by fitting them to a primary vertex (PV) which is constrained to be in the e^+e^- luminous region (beam spot). The constraint uses the sizes of the luminous region as uncertainties and the origin (0,0,0) as the reference centre. The actual fit, performed by the function `theVertexFit` [27], returns a χ^2 value computed as the sum of the χ_i^2 contributions of each track i , $\chi^2 = \sum_i \chi_i^2$. In an iterative procedure, tracks with a $\chi_i^2 > 25$ are removed from the fitted PV and identified as “secondary” tracks. The position of the PV is then determined and a vertex object is created with the function `VertexFitter_Tk` [26].

Secondary tracks are identified as pions and kaons using truth-level information. Per-track weights are then assigned to simulate realistic misidentification rates. For pions, a 100% identification probability is assumed by assigning a weight $w = 1$ to all true pions. Momentum-dependent weights emulating kaon-to-pion misidentification rates are shown in Figure 4. These are computed as

$$w(p) = 2 \times \int_{\sigma(p)}^{\infty} \mathcal{G}(x) dx, \quad (3.1)$$

where $\mathcal{G}$ is a Gaussian distribution with zero mean and unit width, and $\sigma(p)$ is the assumed momentum-dependent separation between different particle species using cluster counting and time-of-flight information shown in Figure 1. Correctly identified true pions and kaons misidentified as pions are embodied as pion candidates in the following. It is assumed that the particle-identification performance for a given track is independent from all other tracks. The weight assigned to D^0 candidates is then the product of the PID weights of the two pions. The misidentification of all other particle species is assumed to be negligibly small for this study.

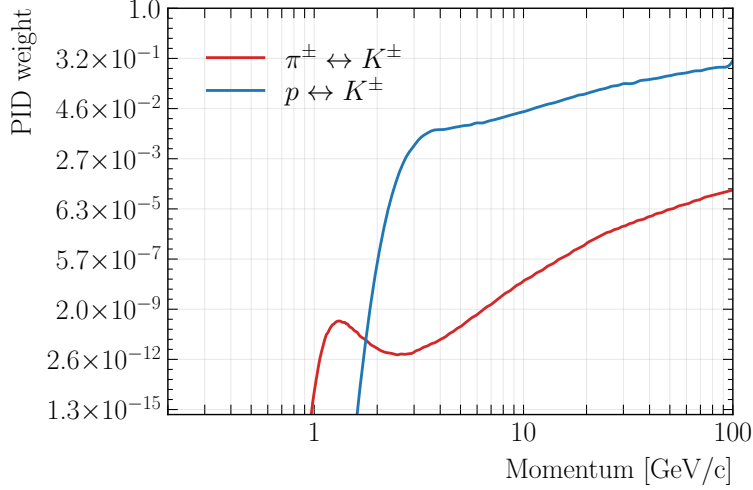


Figure 4: Assumed kaon $\rightarrow$ pion and kaon $\rightarrow$ proton misidentification probabilities as a function of momentum as obtained from the combined particle separation using information of the drift chamber and the time of flight.

To reconstruct candidate $\pi^+\pi^-$ secondary vertices from $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ decays, pairs of oppositely charged pion candidates are fitted to a common vertex. Candidate $\pi^+\pi^-$ vertices must have fits satisfying $\chi^2/\text{ndof} < 7.4$ and are excluded if, when combined with other charged particles in the event, form good vertices satisfying the same quality requirement. The selected vertices are considered D^0 signal candidates. The reconstruction efficiency is defined as the sum of the weights (PID weights for background and decay-model weights for signal) of all reconstructed D^0 candidates over the total number of initial events,

$$\frac{\sum_i w_{i,\text{rec}}}{N_{\text{initial events}}} . \quad (3.2)$$

For the signal dataset, this efficiency is found to be [58.5%, 57.9%, 58.0%, 57.8%] for the [PHSP, Dp, HHchiPT_res, HHchiPT] models. The average efficiency for background samples is 59.9%. The subsequent analysis is performed on a candidate-by-candidate basis, treating each candidate as an independent event, even if multiple candidates are reconstructed within the same event.

Non-reconstructed particles such as neutrinos appear as missing momentum in the event, which is simply calculated from momentum conservation via

$$\vec{p}_{\text{miss}} = - \sum_{i=0}^{n_{\text{particles}}} \vec{p}_i , \quad (3.3)$$

with the index i running over all particle-flow objects and $\vec{p}$ being their respective momentum vector. The presence of two neutrinos in the final state of the signal decay leads to a significant energy imbalance between the signal hemisphere and the opposite hemisphere. To quantify the energy imbalance, as well as other observables of the signal hemisphere, the thrust axis $\vec{t}$ is computed as implemented in the FCCAnalyses algorithms which uses the MIGRAD method of the Minuit minimizer [28] to maximize

$$\frac{\sum_i |\vec{p}_i \cdot \hat{t}|}{\sum_i |\vec{p}_i|} = \frac{\sum_i |p_{x_i} \cdot \hat{t}_x + p_{y_i} \cdot \hat{t}_y + p_{z_i} \cdot \hat{t}_z|}{\sum_i \sqrt{p_{x_i}^2 + p_{y_i}^2 + p_{z_i}^2}} , \quad (3.4)$$

where the index i denotes each reconstructed particle and $\hat{t}_n$ is the n^{th} component of the thrust vector normalized by the magnitude of $\vec{t}$. This axis is perpendicular to the plane separating

the two hemispheres. To make sure it always points towards the signal hemisphere, the thrust axis vector is transformed given the $\pi^+\pi^-$ candidate momentum $\vec{p}_{\pi^+\pi^-}$ via

$$\vec{p}_{\pi^+\pi^-} \cdot \vec{t} \begin{cases} > 0, & \vec{t} \rightarrow \vec{t} \\ < 0, & \vec{t} \rightarrow -\vec{t} \end{cases} \quad (3.5)$$

Variable	Requirement
Reconstructed energy of D^0 candidate	$> 1 \text{ GeV}$
IP($\pi^+\pi^-$)	$< 1 \text{ mm}$
Total number of reconstructed tracks in event	< 12
Total number of reconstructed particles in event	> 15
Number of reconstructed π^0 in event	< 60
Missing momentum of event p_{miss}	$> 1.2 \text{ GeV}$
Min. of the IP of the two π from D^0 candidate	$< 1 \text{ mm}$
Max. of the IP of the two π from D^0 candidate	$< 2 \text{ mm}$
Flight Distance of D^0 candidate	$< 10 \text{ mm}$
IP($\pi^+\pi^-$) - IP($\pi^+\pi^-p_{\text{miss}}$)	$> -0.5 \text{ mm}$
DIRA($\pi^+\pi^-$) - DIRA($\pi^+\pi^-p_{\text{miss}}$)	$> -0.5 \text{ rad}$
Opening angle α of π 's	$< 0.9 \text{ rad}$
score _{total, signal} ^b	< 0.62
score _{total, signal} ^{u/d}	< 0.2
score _{total, signal} ^g	< 0.2
score _{signal} ^c	> 0.01
Number of leptons in signal hemisphere	$== 0$
$m(\pi^+\pi^-)$	$< 1.8 \text{ GeV}$
$m(\pi^+\pi^-p_{\text{miss}})$	$< 3 \text{ GeV}$
m_{corr}	$> 0.6 \text{ GeV}$

Table 1: Requirements used in the preselection of the signal candidates. Signal and background distributions of each variable are shown in Appendix A.

The variables showing the highest discrimination power between signal and background are then used to apply a cut-based preselection suppressing the most evident background by about 99.4%. About [78.5%, 81.1%, 78.9%, 78.6%] of the reconstructed signal candidates for the [PHSP, Dp, HHchiPT_res, HHchiPT] decay model survive this preselection. The preselection requirements are presented in Table 1.

Here, the impact parameter (IP) is defined as the minimum distance of a particle trajectory to the reconstructed PV. The difference of the IP of the D^0 candidate with and without adding the missing-energy vector is referred to as IP($\pi^+\pi^-$) - IP($\pi^+\pi^-p_{\text{miss}}$). The requirement of the total number of reconstructed particles in the event removes the majority of background candidates from $Z \rightarrow \tau^+\tau^-$. The direction angle (DIRA) is the angle between the momentum vector of the signal candidate reconstructed with the two pions $\vec{p}_{\pi^+\pi^-}$ and the vector connecting the PV and the D^0 decay vertex. The response of a neural-network-based jet flavour tagger [29] is used to identify $c\bar{c}$ events. The tagger returns a probability, score^q, that a jet originates from a given parton flavour q ($q = u/d, s, c, b$ for quark jets or g for gluon jets). As the jet score is defined for each hemisphere, the score on the side consistent with the signal is

called $\text{score}_{\text{signal}}^q$, the score on the opposite side is called $\text{score}_{\text{non-signal}}^q$. The sum of the two scores is referred to as $\text{score}_{\text{all}}^q$. In addition, we compute the quantity $\text{score}_{\text{total}}^q$ for each flavour q as the product between the q score and the probabilities of not being of another flavour, $\text{score}_{\text{total,signal}}^q = \text{score}^q \cdot \prod_{k \neq q} (1 - \text{score}^k)$ in the signal hemisphere. The number of reconstructed π^0 is also used as selection variable, which is computed by summing the number of two photon combinations that meet certain criteria, such as an opening angle smaller than $\pi/6$ and an invariant mass in the range $[100, 160]$ MeV.

Further kinematic variables are introduced that partially account for the missing neutrinos in the final state and help to identify signal decays. The corrected mass of the D^0 meson (m_{corr}) is calculated as

$$m_{\text{corr}} = \sqrt{m^2(\pi^+\pi^-) + p_{\perp}^2} + p_{\perp}, \quad (3.6)$$

with $p_{\perp}$ being the component of the dipion momentum that is perpendicular to the D^0 flight direction, calculated from the measured decay and primary vertices as $\hat{e}_{\text{dir } D^0} = (\vec{v}_{D^0} - \vec{v}_{\text{PV}})/|\vec{v}_{D^0} - \vec{v}_{\text{PV}}|$. The magnitude of the momentum component of the D^0 candidate is approximated using the relation [30]

$$|\vec{p}(D^0)| = \frac{m_{D^0}}{m(\pi^+\pi^-)} \cdot (\vec{p}(\pi^+\pi^-) \cdot \hat{e}_{\text{dir } D^0}), \quad (3.7)$$

where m_{D^0} is the PDG value of the D^0 mass. This allows to compute q^2 as

$$q^2 = |\vec{p}(D^0) - \vec{p}(\pi^+\pi^-)|^2, \quad (3.8)$$

which is not used in this study, but could be used in future analyses to perform a q^2 binning.

Additional variables are computed to target background from $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ and $D^0 \rightarrow K_L^0 \pi^+ \pi^-$ decays. We reconstruct K_S^0 decays into two charged or neutral pions by requiring the dipion mass to be smaller than 750 MeV. The dipion opening angle must be smaller than 0.6 for the $(\pi^+\pi^-)$ candidates, and smaller than $\pi/6$ for the $(\pi^0\pi^0)$ candidates. We consider as K_L^0 candidate any neutral particle leaving energy deposits in the hadronic calorimeter, and no deposit in the electromagnetic calorimeter. Candidate $K_{S,L}^0$ mesons that are reconstructed within a cone around the direction of the dineutrino system, having a radius of $\sqrt{(\Delta\phi^2 + \Delta\eta^2)} = 0.2$, are combined with the dipion signal candidates to form candidate $D^0 \rightarrow K_{S,L} \pi^+ \pi^-$ decays. Here, $\Delta\phi$ and $\Delta\eta$ are differences in azimuthal angle and pseudorapidity between the dineutrino direction and the neutral kaon candidates, respectively. The dineutrino direction is obtained using the difference of the approximated D^0 momentum (Equation 3.7) and $\vec{p}(\pi^+\pi^-)$. The $K_{S,L}^0 \pi^+ \pi^-$ mass is used as discriminating variable. When more than one $K_{S,L}^0$ candidate are found within the cone, only the combination with mass closest to the known D^0 mass is considered. When no $K_{S,L}^0$ are found in the cone, a value of zero is assigned to the $K_{S,L}^0 \pi^+ \pi^-$ mass. Finally, the momentum imbalance of the signal candidate pions is also computed via

$$p_{\text{imb}}(\pi^+\pi^-) = \frac{|p(\pi^+)| - |p(\pi^-)|}{|p(\pi^+)| + |p(\pi^-)|}. \quad (3.9)$$

Plots showing the signal and background distributions of the variables used in the preselection can be found in Appendix A.

3.3 Selection optimisation

A Boosted Decision Tree (BDT) based on an Adaptive Boosting (AdaBoost) algorithm [31] is trained to separate signal and background candidates.¹

¹As most of the background after the preselection arises $Z \rightarrow c\bar{c}$ decays, a single BDT is found to be sufficient in contrast to an earlier version of this note [32] where BDTs were used in cascade.

The BDT is trained on the exclusively simulated $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ decays as signal, while the background sample consists of a merged dataset of all inclusive background samples described in section 2. The fractions of the different samples before the reconstruction of candidates reflect the natural proportions: 12.03% $Z \rightarrow c\bar{c}$, 15.12% $Z \rightarrow b\bar{b}$, 15.84% $Z \rightarrow s\bar{s}$, 11.17% $Z \rightarrow u\bar{u}$, 15.84% $Z \rightarrow d\bar{d}$, and 3.37% $Z \rightarrow \tau^+\tau^-$ [33]. The background dataset also includes events with misidentified K tracks with their corresponding PID weights. To validate the training results, the dataset is split into a training set (two-thirds) and a testing set (one-third).

The input variables capture the overall event topology, including hemisphere energy asymmetry, track multiplicity, and the response of the jet flavour tagger. They also include variables that characterize more directly the $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ decay, like the opening angle of the two pions or their IP. The input variables are selected to ensure minimal correlation within either the signal or background dataset while maximizing their discrimination power. The variables used in the BDT are chosen to have a maximum separation power between signal and background and a high variables importance in the training of the classifier. If multiple variables show high correlations, only the one with higher separation is retained. The variables are:

- c -jet score of signal side,
- sum of q -jet score of signal and non-signal side ($q \in [c, b, g]$),
- number of reconstructed π^0 in event,
- reconstructed energy of D^0 candidate,
- total number of charged tracks and total number of particles in event
- total number of leptons in event
- total reconstructed energy of charged tracks in event,
- reconstructed invariant mass of PV using primary tracks,
- number of primary tracks,
- missing momentum of event,
- total number of secondary vertices in the event
- momentum imbalance of two-pion system $p_{\text{imb}}(\pi^+\pi^-)$,
- corrected mass m_{corr} ,
- $m(K_S^0\pi^+\pi^-)$ and $m(K_L^0\pi^+\pi^-)$,
- flight distance of D^0 candidate,
- number of tracks (π^0 candidates) within a 0.5(0.4)-radius cone around the reconstructed D^0 momentum ($\vec{p}(\pi^+\pi^-)$),
- $\text{IP}(\pi^+\pi^-) - \text{IP}(\pi^+\pi^-p_{\text{miss}})$,
- difference of the mass of the two pion combination with and without adding the missing momentum, $m(\pi^+\pi^-p_{\text{miss}}) - m(\pi^+\pi^-)$
- $\log(1 - \cos(\alpha))$, where α is the angle between the two pion tracks,
- total reconstructed energy of the event,

- $\log(1 - \cos(\theta))$, where θ is the angle between the reconstructed signal-candidate momentum ($\vec{p}(\pi^+\pi^-)$) and the thrust axis
- energy asymmetry of hemispheres using the reconstructed energy of all particles.

Appendix A reports the separation power and importance of each variable, and their linear correlation coefficients. The distributions of the three BDT responses are shown in Figure 5. The area under the ROC curve of the classifier (AUC) is [0.979, 0.982, 0.983, 0.980] for the [PHSP, Dp, HHchiPT_res, HHchiPT] decay model, which displays a successful training.

The requirement on the BDT response is chosen by maximising the figure of merit (FOM) [34]

$$\text{FOM}_{n\sigma} = \frac{\epsilon_{\text{BDT},\text{sig}}}{\frac{n}{2} + \sqrt{B}}, \quad (3.10)$$

where B is the total expected background yield (as computed in the next section), $\epsilon_{\text{BDT},\text{sig}}$ is the signal efficiency determined as the ratio of the sum of the signal weights before ($N_{\text{w, preselection}} = \sum_i w_{i, \text{preselection}}$) and after ($N_{\text{BDT}} = \sum_i w_{i, \text{BDT}}$) the BDT requirement $\epsilon_{\text{BDT},\text{sig}} = \frac{N_{\text{BDT}}}{N_{\text{preselection}}}$. This metric is particularly useful when the absolute number of signal events is unknown, as it depends on the signal efficiency rather than the total number of signal events. The optimization is performed for $n = 3, 5$, corresponding to $3, 5\sigma$ levels of signal significance. The FOM for either level of significance yields similar BDT cut requirements. The evolution of the FOM as a function of the BDT requirement for the $n = 3$ case is shown in Figure 6. The BDT requirements that maximise the FOMs for the PHSP, [Dp, HHchiPT_res, HHchiPT] decay models are $> [0.208, 0.178, 0.216, 0.202]$. They result in background rejections of [99.1%, 98.8%, 99.5%, 99.2%] and signal efficiencies of [43.9%, 54.8%, 38.3%, 43.7%], respectively. Remaining background predominantly arise from $Z \rightarrow c\bar{c}$ decays with partially reconstructed or semi-leptonic charm decays.

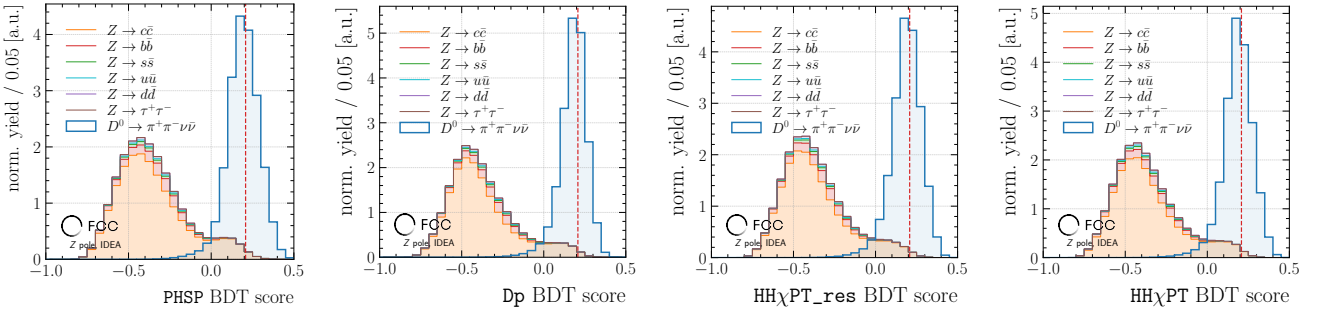


Figure 5: The response of the BDTs trained on (left to right) [PHSP, Dp, HHchiPT_res, HHchiPT] on all inclusive backgrounds and the signal. The backgrounds are normalized to their expected relative fractions after the preselection. The red dotted lines represent the applied BDT cuts.

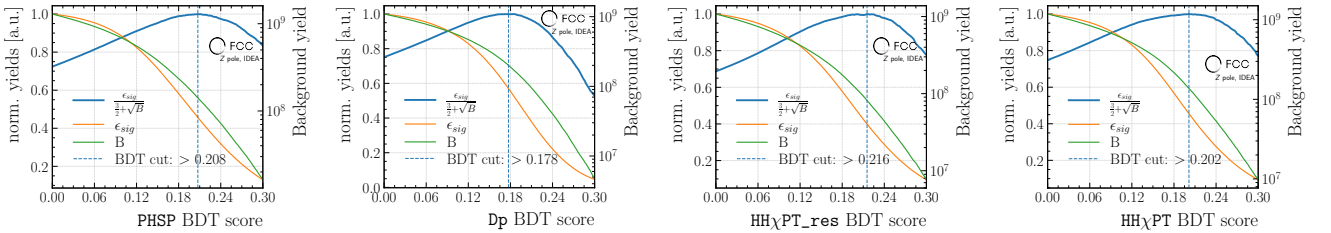


Figure 6: Signal efficiency (ϵ_{sig} , orange), normalized background yield (B , green), and $\text{FOM}_{3\sigma}$ (blue) as a function of the BDT cut for (from left to right) [PHSP, Dp, HHchiPT_res, HHchiPT] decay models.

300 A Kolmogorov–Smirnov test is performed to check for overtraining. The test compares the
 301 BDT score distributions of the training and test subsamples. The values found for the train and
 302 test sample for the [PHSP, Dp, HHchiPT_res, HHchiPT] decay models are [0.184, 0.249, 0.473,
 303 0.259] for the signal and [0.394, 0.787, 0.557, 0.224] for the background datasets respectively,
 304 proving no overtraining.

305 3.4 Sensitivity estimation

306 To evaluate the sensitivity, the statistical significance of the signal is approximated as

$$s = \frac{S}{\sqrt{S+B}}. \quad (3.11)$$

307 where S and B are the expected signal and background yields, respectively. The expected
 308 number of signal events S is calculated from the assumed signal branching fraction $\mathcal{B}(D^0 \rightarrow$
 309 $\pi^+\pi^-\nu\bar{\nu})$ as

$$S = 2 \times N_Z \times \mathcal{B}(Z \rightarrow c\bar{c}) \times f_{D^0} \times \mathcal{B}(D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}) \times \epsilon_{\text{total,sig}}, \quad (3.12)$$

310 where:

- 311 • The factor of 2 accounts for the production of both charm and anti-charm quarks per Z
 312 decay,
- 313 • $N_Z = 6 \times 10^{12}$ is the expected number of produced Z bosons,
- 314 • $\mathcal{B}(Z \rightarrow c\bar{c}) = 12.03\%$ is the branching fraction of the Z to a charm quark pair,
- 315 • $f_{D^0} = 0.59$ is the charm hadronization fraction to a D^0 meson [35],
- 316 • and $\epsilon_{\text{total,sig}}$ is the signal efficiency, which is the product of the reconstruction and selection
 317 efficiency.

318 The total expected number of background candidates B is given by

$$B = N_Z \left(\sum_{q=c,b,s,u,d,\tau} \mathcal{B}(Z \rightarrow q\bar{q}) \epsilon_{\text{total,q}} \right), \quad (3.13)$$

319 where:

- 320 • $\mathcal{B}(Z \rightarrow q\bar{q})$ are the branching fractions of the Z boson to various fermion final states,
- 321 • $\epsilon_{\text{total,q}}$ represents the total efficiency for each inclusive background category q , which is
 322 the product of the reconstruction and selection efficiency.

323 The efficiencies, after each analysis step, for all background samples and for signal decays are
 324 reported in Table 2. The significance as a function of the assumed branching fraction for each
 325 decay-model assumption is illustrated in Figure 7. The minimal branching fractions leading to
 326 significances of 3σ (5σ) are found to be $[2.1, 1.9, 1.8, 2.1] \times 10^{-7}$ ($[3.4, 3.2, 2.9, 3.4] \times 10^{-7}$) for the
 327 [PHSP, Dp, HHchiPT_res, HHchiPT] decay model. This is within the parameter space allowed by
 328 possible BSM scenarios, which, as a reminder, could lead to branching fractions at the level of
 329 few 10^{-6} depending on the assumptions about charged lepton flavour symmetries.

Bkg. process	Reco. [10^{-2}]	Preselection [10^{-3}]	BDT [10^{-3}]	Total
$Z \rightarrow c\bar{c}$	75.5	25.1	[10.0, 14.3, 5.8, 9.7]	$[1.9, 2.7, 1.1, 1.8] \times 10^{-4}$
$Z \rightarrow b\bar{b}$	84.2	1.7	[1.3, 1.4, 0.8, 1.5]	$[2.0, 2.1, 1.2, 2.1] \times 10^{-6}$
$Z \rightarrow u\bar{u}$	42.6	0.6	[1.1, 1.1, 0.3, 0.6]	$[2.7, 2.7, 8.6, 1.4] \times 10^{-7}$
$Z \rightarrow d\bar{d}$	44.9	0.6	[0.7, 1.1, 0.3, 1.0]	$[1.8, 2.8, 0.7, 2.7] \times 10^{-7}$
$Z \rightarrow s\bar{s}$	62.6	0.6	[0.8, 1.0, 0.4, 0.6]	$[2.8, 3.6, 1.3, 2.3] \times 10^{-7}$
$Z \rightarrow \tau^+\tau^-$	9.9	0.1	[23.3, 52.3, 17.4, 23.3]	$[1.9, 4.2, 1.4, 1.9] \times 10^{-7}$
Signal	Reco. [10^{-1}]	Preselection [10^{-1}]	BDT [10^{-1}]	Total [10^{-2}]
$D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$	[5.9, 5.8, 5.8, 5.8]	[7.8, 8.1, 7.9, 7.9]	[4.4, 5.5, 3.8, 4.4]	[20.2, 25.8, 17.5, 19.9]

Table 2: Efficiencies at different selection steps for background and signal samples and for [PHSP, Dp, HHchiPT_res, HHchiPT] decay models.

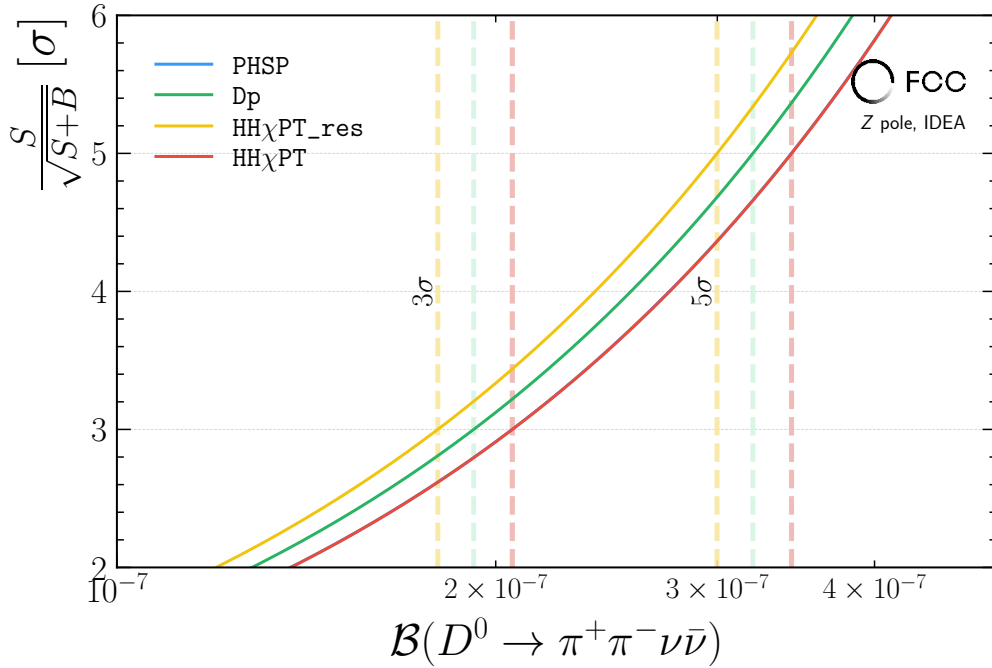


Figure 7: Sensitivity estimate as a function of the branching fraction of $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ for the [PHSP, Dp, HHchiPT_res, HHchiPT] decay model ([blue, green, yellow, red]) with the 3σ and 5σ thresholds. Note that the red and blue curves are almost identical.

4 Angular asymmetries in $\Lambda_c^+ \rightarrow p\mu^+\mu^-$ decays

To estimate the uncertainty on angular asymmetries in decays of polarized Λ_c^+ baryons to $p\mu^+\mu^-$ final states, a dedicated candidate reconstruction and cut-based selection is developed. Using the signal and background efficiencies, the sensitivity for a generic null-test asymmetry is determined, which serves as a proxy for future measurements of angular asymmetries.

4.1 Candidate reconstruction and selection

As done in section 3, we assume 100% identification probability for correctly identified particles, *i.e.* muons and protons. The background samples defined in section 2 are used with fractions of the different samples adjusted as described in subsection 3.3. Momentum-dependent weights emulate kaon $\leftrightarrow$ proton misidentification with probabilities as obtained from Figure 4. For this study a pion $\rightarrow$ muon misidentification of 0.4% per track is assumed as done in other FCC studies regarding rare semi-leptonic decays [25].

Muon candidates are required to have a transverse momentum larger than 300 MeV. Proton candidates are required to have transverse momentum larger than 1.5 GeV. Combinations of one proton candidate and two oppositely charged muon candidates are formed and their mass must be between 2.25 and 2.3 GeV. Each combination is fitted to a common secondary decay vertex, and the resulting vertex fit quality must satisfy $\chi^2/\text{ndof} < 5$. For signal, the efficiency of the reconstruction is found to be 89.6%. The average efficiency for the background samples is 6.7×10^{-5} . The subsequent analysis is performed on a candidate-by-candidate basis, treating each candidate as an independent event, even if multiple candidates are reconstructed within the same event. As the final state is fully reconstructed, a simple cut-based selection is sufficient to isolate the rare signal.

Variable	Requirement
$p_T(\mu)$	$> 300 \text{ MeV}$
$p_T(p)$	$> 4 \text{ GeV}$
$\text{score}_{\text{all}}^s$	< 0.9
$\text{score}_{\text{all}}^{u/d}$	< 0.9
$\text{score}_{\text{all}}^b$	< 1.2
$\sum_{q=s,u,d,b,g} \text{score}_{\text{all}}^q - \text{score}_{\text{all}}^c$	< 1.8
$\text{IP}(p\mu^+\mu^-)$	$< 0.26 \text{ mm}$
$\text{IP}(p)$	$> 2.5 \text{ } \mu\text{m}$
$m(p\mu^+\mu^-)$	$> 2.275 \text{ GeV}$
$m(p\mu^+\mu^-)$	$< 2.300 \text{ GeV}$
$E(p\mu^+\mu^-)$	$> 20 \text{ GeV}$
Flight distance of Λ_c^+ candidate	$< 2.8 \text{ mm}$
$\text{DIRA}(p\mu^+\mu^-)$	$< 2.8 \text{ rad}$
Number of π^0 within an angular distance $R \leq 0.4$	< 13
Number of reconstructed π^0 in event	< 40
χ^2/ndof of Λ_c^+ vertex	< 1.6
Number of particles in event	> 15

Table 3: Requirements used in the selection of the signal candidates. Signal and background distributions of each variable are shown in Appendix B.

The variables showing the highest discrimination power between signal and background are listed in Table 3, together with the requirements that are applied. The variables are defined in analogy to section 3. Plots showing the signal and background distributions of the variables used in the preselection can be found in Appendix B. The total reconstruction and selection efficiency is found to be 1.6×10^{-9} for the background and 41.79% for the signal. No further selection optimisation is required. Most of the remaining background is of combinatorial origin with one or two particles mis-identified.

4.2 Sensitivity estimation

To evaluate the sensitivity on the measurement of angular asymmetries, the simulated signal and background samples are scaled to the expected yields. To estimate the expected number of signal candidates S , we approximate the branching fraction of $\Lambda_c^+ \rightarrow p\mu^+\mu^-$ decays as the sum of the branching fractions, $\mathcal{B}(\Lambda_c^+ \rightarrow pV)$ with $V = \eta, \rho, \omega, \phi$ with subsequent decay of $V \rightarrow \mu^+\mu^-$, of the dominant intermediate hadronic resonances yielding a value of $\mathcal{B}(\Lambda_c^+ \rightarrow p\mu^+\mu^-) = 4.15 \times 10^{-7}$ [33]. In analogy to Equation 3.11, the total number of expected signal candidates is then

$$S = 2 \times N_Z \times \mathcal{B}(Z \rightarrow c\bar{c}) \times 0.06 \times \mathcal{B}(\Lambda_c^+ \rightarrow p\mu^+\mu^-) \times \epsilon_{\text{total,sig}}, \quad (4.1)$$

where:

- The factor of 2 accounts for the production of both charm and anti-charm quarks per Z decay,
- $N_Z = 6 \times 10^{12}$ is the expected number of produced Z bosons,
- $\mathcal{B}(Z \rightarrow c\bar{c}) = 12.03\%$ is the branching fraction of the Z to a charm quark pair,
- 0.06 is the charm hadronization fraction to a Λ_c^+ meson [35],
- and $\epsilon_{\text{total,sig}}$ is the signal efficiency, which is the product of the reconstruction and selection efficiency.

As seen before, the total expected number of background candidates B is given by

$$B = N_Z \left(\sum_{q=c,b,s,u,d,\tau} \mathcal{B}(Z \rightarrow q\bar{q}) \epsilon_{\text{total,q}} \right), \quad (4.2)$$

where:

- $\mathcal{B}(Z \rightarrow q\bar{q})$ are the branching fractions of the Z boson to various fermion final states,
- $\epsilon_{\text{total,q}}$ represents the total efficiency for each inclusive background category q , which is the product of the reconstruction and selection efficiency.

In total, we estimate an expected signal yield of 15 021 and 7 240 for the background.

Angular asymmetries can be measured as yield asymmetries of subsamples, which are defined by angular observables (see, for example, Ref. [14] for the measurement of the forward-backward asymmetry in $\Lambda_c^+ \rightarrow p\mu^+\mu^-$ decays). Thus, we estimate the sensitivity of the measurement by determining the uncertainty on a dummy asymmetry measured in a toy data set representing the expected signal and background yields, which is randomly divided in two halves to emulate a null test asymmetry. An extended maximum-likelihood fit to the mass

distributions of the two Λ_c^+ subsamples is performed, where the asymmetry is a shared parameter. A sum of two Gaussian functions with relative fraction f , shared mean and widths σ_1 and σ_2 is used to describe the signal and an exponential function to describe the background. The line shape parameters are determined in fits to the scaled simulated samples as shown in Figure 8. From the toy studies, we obtain an asymmetry with an uncertainty of 0.9%. The effective width of the signal component, defined as $\sigma_{\text{eff}} = \sqrt{f \cdot \sigma_1^2 + (1 - f) \cdot \sigma_2^2}$, is found to be $\sigma_{\text{eff}} = (2.0 \pm 0.1) \text{ MeV}$, where the uncertainty is the statistical uncertainty from the fit.

Enhanced sensitivity to new physics can be obtained by measuring the asymmetries in regions of dimuon mass. We therefore repeat the study making toy experiments generating the yields which are expected in two regions of dimuon mass centred around the ϕ -meson mass as defined in Ref. [14], defined as $\phi_{\text{low}} = [979.46, 1019.46] \text{ MeV}$ and $\phi_{\text{high}} = [1019.46, 1059.46] \text{ MeV}$. The yields in these two regions is estimated using the known branching fraction of $\Lambda_c^+ \rightarrow p\phi(\rightarrow \mu^+\mu^-)$ and the ratio of yield measured in Ref. [14]. The results are summarized in Table 4. No systematic uncertainties are evaluated in the scope of this study. However, for the measurements of this class of asymmetries, they are expected to be factored out and well below the level of the estimated statistical uncertainties.

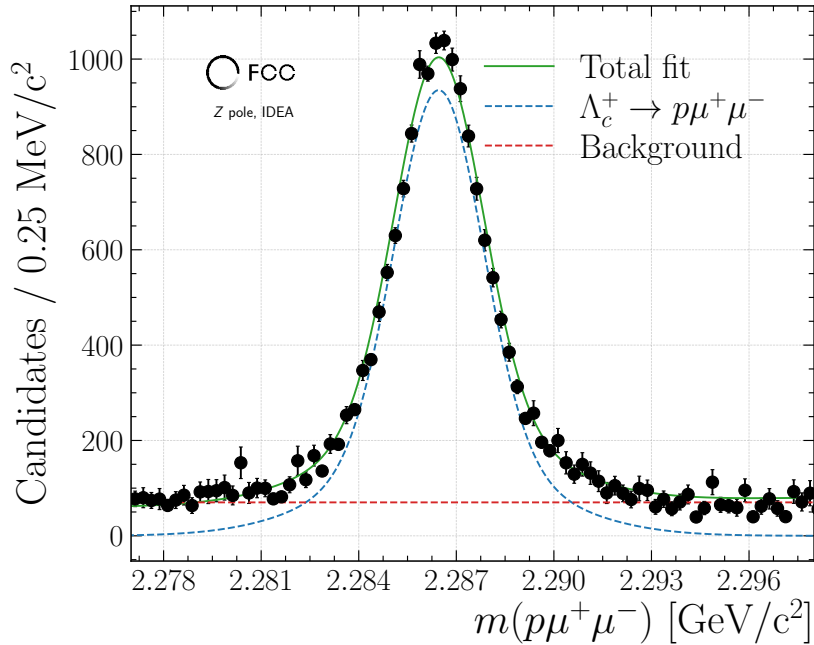


Figure 8: Mass distribution of selected simulated signal and background $\Lambda_c^+ \rightarrow p\mu^+\mu^-$ candidates scaled to 6×10^{12} Z decays. The projection of a fit is also shown with details given in the main text.

$m(\mu^+\mu^-)$	N_{exp}	σ_A
total	15 021	0.9%
ϕ	10 967	1.0%
ϕ_{low}	4 826	1.5%
ϕ_{high}	6 141	1.3%

Table 4: Expected signal yields and uncertainty on asymmetries in the total dimuon-mass range and in two regions around the intermediate ϕ meson.

5 Conclusion

In this note, a sensitivity study for the decay $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ is presented. While $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ decays are highly suppressed and approximate null tests of the SM, BSM contributions can result in branching fractions at the level of $\mathcal{O}(10^{-6})$. This study estimates that branching fractions at the level of $\mathcal{O}(10^{-7})$ are within reach at FCC-ee. These results can be extended to other charmed hadron decays into final states with two neutrinos, of which the $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ decay represents only one example.

Furthermore, the note describes a sensitivity study for the measurement of angular asymmetries in polarised $\Lambda_c^+ \rightarrow p\mu^+\mu^-$ decays, where a precision below the percent level can be reached. The decays of polarised Λ_c^+ baryons offer a variety of null-test measurements in the angular distributions of the final state particles where NP signatures can reach the percent level. These studies open the door to a new area in charm physics and the opportunity to look for new phenomena in processes that have not yet been explored experimentally, and highlight the complementary of FCC-ee with respect to existing flavour-physics experiments.

References

- [1] ATLAS Collaboration. Observation of a new particle in the search for the standard model higgs boson with the atlas detector at the lhc. *Physics Letters B*, 716(1):1–29, 2012.
- [2] CMS Collaboration. Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC. *Physics Letters B*, 716(1):30–61, 2012.
- [3] A. Abada, M. Abbrescia, S. S. Abdus Salam, I. Abdyukhanov, J. Abelleira Fernandez, A. Abramov, M. Aburaia, A. O. Acar, P. R. Adzic, P. Agrawal, and J. A. Aguilar-Saavedra. FCC-ee: The Lepton Collider. *The European Physical Journal Special Topics*, 228(2):261–623, 06 2019.
- [4] Yasmine Amhis, Matthew Kenzie, M  ril Reboud, and Aidan R. Wiederhold. Prospects for searches of $b \rightarrow s\nu\bar{\nu}$ decays at FCC-ee. *Journal of High Energy Physics*, 2024(1):144, 2024.
- [5] Xunwu Zuo, Marco Fedele, Cl  ment Helsens, Donal Hill, Syuhei Iguro, and Markus Klute. Prospects for B_c^+ and $B^+ \rightarrow \tau^+\nu_\tau$ at FCC-ee. *The European Physical Journal C*, 84(1):87, 2024.
- [6] Tristan Miralles and St  phane Monteil. Study of the feasibility of the observation of $B^0 \rightarrow K^*(892)\tau^+\tau^-$ at FCC-ee and related vertex detector performance requirements, September 2024. <https://doi.org/10.17181/d772d-egz40>.
- [7] LHCb Collaboration. The LHCb Physics Programme. In *34th Rencontres de Moriond: QCD and Hadronic interactions*, pages 409–414, Hanoi, 2001. The Gioi.
- [8] Belle II Collaboration. The belle ii physics book. *Progress of Theoretical and Experimental Physics*, 2019(12):123C01, 12 2019.
- [9] Hector Gisbert, Marcel Golz, and Dominik Stefan Mitzel. Theoretical and experimental status of rare charm decays. *Mod. Phys. Lett.*, A36(04):2130002, 2021.
- [10] Rigo Bause, Hector Gisbert, Marcel Golz, and Gudrun Hiller. Rare charm $c \rightarrow u\nu\bar{\nu}$ dineutrino null tests for e^+e^- machines. *Phys. Rev. D*, 103(1):015033, 2021.
- [11] Marcel Golz, Gudrun Hiller, and Tom Magorsch. Probing for new physics with rare charm baryon ($\Lambda_c, \Xi_c\Omega_c$) decays. *JHEP*, 09:208, 2021.
- [12] Rigo Bause, Hector Gisbert, Marcel Golz, and Gudrun Hiller. Lepton universality and lepton flavor conservation tests with dineutrino modes. *Eur. Phys. J. C*, 82(2):164, 2022.
- [13] M. Ablikim et al. Search for the decay $D^0 \rightarrow \pi^0\nu\bar{\nu}$. *Phys. Rev. D*, 105(7):L071102, 2022.
- [14] Roel Aaij et al. Search for resonance-enhanced CP and angular asymmetries in the $\Lambda_c^+ \rightarrow p\mu^+\mu^-$ decay at LHCb. *Phys. Rev. D*, 111(9):L091102, 2025.
- [15] The IDEA Study Group. The IDEA detector concept for FCC-ee. 2025. arXiv:2502.21223 [hep-ex].
- [16] Angelo Di Canto, Tabea Hacheney, Gudrun Hiller, Dominik Stefan Mitzel, St  phane Monteil, Lars R  hrig, and Dominik Suelmann. New opportunities for rare charm from $Z \rightarrow c\bar{c}$ decays. *arXiv preprint*, 2025. arXiv:2509.10447 [hep-ph].

- [17] Torbjörn Sjöstrand, Stefan Ask, Jesper R. Christiansen, Richard Corke, Nishita Desai, Philip Ilten, Stephen Mrenna, Stefan Prestel, Christine O. Rasmussen, and Peter Z. Skands. An introduction to pythia 8.2. *Computer Physics Communications*, 191:159–177, 2015.
- [18] FCC Collaboration. <https://hep-fcc.github.io/FCCeePhysicsPerformance/General/#common-event-samples>. Accessed: 2025-02-18.
- [19] David J. Lange. The evtgen particle decay simulation package. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 462(1):152–155, 2001. BEAUTY2000, Proceedings of the 7th Int. Conf. on B-Physics at Hadron Machines.
- [20] Belle II Collaboration. https://github.com/belle2/basf2/blob/main/decfiles/dec/DECAY_BELLE2.DEC. Accessed: 2025-02-18.
- [21] The DELPHES 3 Collaboration. Delphes 3: a modular framework for fast simulation of a generic collider experiment. *Journal of High Energy Physics*, 2014(2):57, 02 2014.
- [22] M. Benedikt et al. Future Circular Collider Feasibility Study Report: Volume 1, Physics, Experiments, Detectors. 2025.
- [23] Frank Gaede, Thomas Madlener, Placido Declara Fernandez, Gerardo Ganis, Benedikt Hegner, Clement Helsens, Andre Sailer, Graeme A. Stewart, and Valentin Voelkl. EDM4hep - a common event data model for HEP experiments. *PoS, ICHEP2022*:1237, 11 2022.
- [24] Rene Brun and Fons Rademakers. Root — an object oriented data analysis framework. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 389(1):81–86, 1997.
- [25] Alberto Lusiani, Guy Wilkinson, Jernej F. Kamenik, and Stephane Monteil. Prospects in Flavour Physics at the FCC, 2025. Contribution to the European Strategy for Particle Physics Update 2025–2026, <https://doi.org/10.17181/jnzpp-1fw39>.
- [26] FCC Collaboration. <https://github.com/HEP-FCC/FCCAnalyses/blob/master/analyzers/dataframe/src/VertexFitterSimple.cc>. Accessed: 2025-01-11.
- [27] Franco Bedeschi. A vertex fitting package. 2024. arXiv:2409.19326 [hep-ex].
- [28] M. Hatlo, L. Moneta, F. James, and V. Innocente. Developments of the MINUIT minimization package within ROOT. 2005.
- [29] Franco Bedeschi, Loukas Gouskos, and Michele Selvaggi. Jet flavour tagging for future colliders with fast simulation. *The European Physical Journal C*, 82(7), July 2022.
- [30] Roel Aaij et al. Measurement of the ratio of branching fractions $\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B}^0 \rightarrow D^{*+}\mu^-\bar{\nu}_\mu)$. *Phys. Rev. Lett.*, 115(11):111803, 2015. [Erratum: Phys.Rev.Lett. 115, 159901 (2015)].
- [31] Yoav Freund and Robert E Schapire. A decision-theoretic generalization of on-line learning and an application to boosting. *Journal of Computer and System Sciences*, 55(1):119–139, 1997.

- 493 [32] Tabea Hacheney, Lars Röhrig, Dominik Stefan Mitzel, Angelo Di Canto, and Stephane
494 Monteil. Sensitivity study of the search for the rare decay $D^0 \rightarrow \pi^+ \pi^- \nu \bar{\nu}$ at FCC-ee,
495 March 2025. <https://doi.org/10.17181/e3nw3-fx653>.
- 496 [33] S. Navas and others (Particle Data Group). Review of particle physics. *Phys. Rev. D*,
497 110:030001, 2024.
- 498 [34] Giovanni Punzi. Sensitivity of searches for new signals and its optimization. In *Statistical*
499 *Problems in Particle Physics, Astrophysics, and Cosmology (PHYSTAT 2003)*, 2003.
- 500 [35] Mykhailo Lisovyi, Andrii Verbytskyi, and Oleksandr Zenaiev. Combined analysis of charm-
501 quark fragmentation-fraction measurements. *The European Physical Journal C*, 76(7), July
502 2016.

503 **A Supplemental Material Search for $D^0 \rightarrow \pi^+\pi^-\nu\bar{\nu}$ decays**

504 The variables used in the preselection and BDT training are shown in subsection A.1 and
505 subsection A.2. The correlations of variables used in the BDT for the signal and background
506 are shown in subsection A.3. Additionally, the separation and variable importance of the
507 variables used in the BDT are shown in subsection A.4. All variable distributions shown in
508 subsection A.1 and subsection A.2 are exemplary done for the PHSP decay model. The variables
509 used to select π^0 , K_S^0 and K_L^0 candidates are presented in subsection A.5. All distributions are
510 normalized to unity.

511 **A.1 Preselection variable distributions**

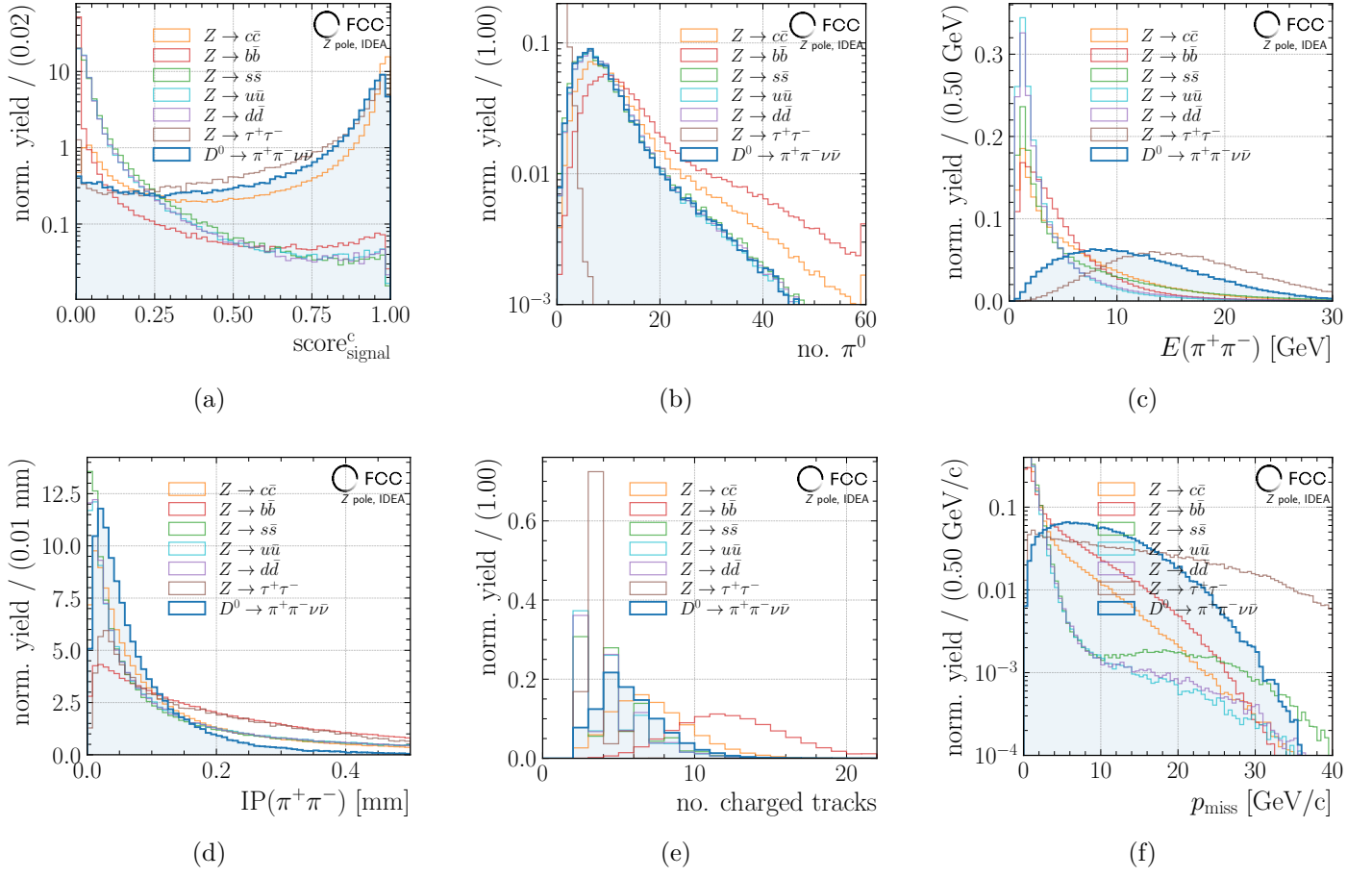


Figure 9: Variable distributions used in the **preselection**. (a) shows the jet flavour tagger response for the c-jet likelihood on the signal hemisphere, (b) the number of reconstructed π^0 candidates, (c) the reconstructed energy of the D^0 candidate, (d) the impact parameter of the D^0 candidate, (e) the total number of charged tracks in the event and (f) the missing energy of the event.

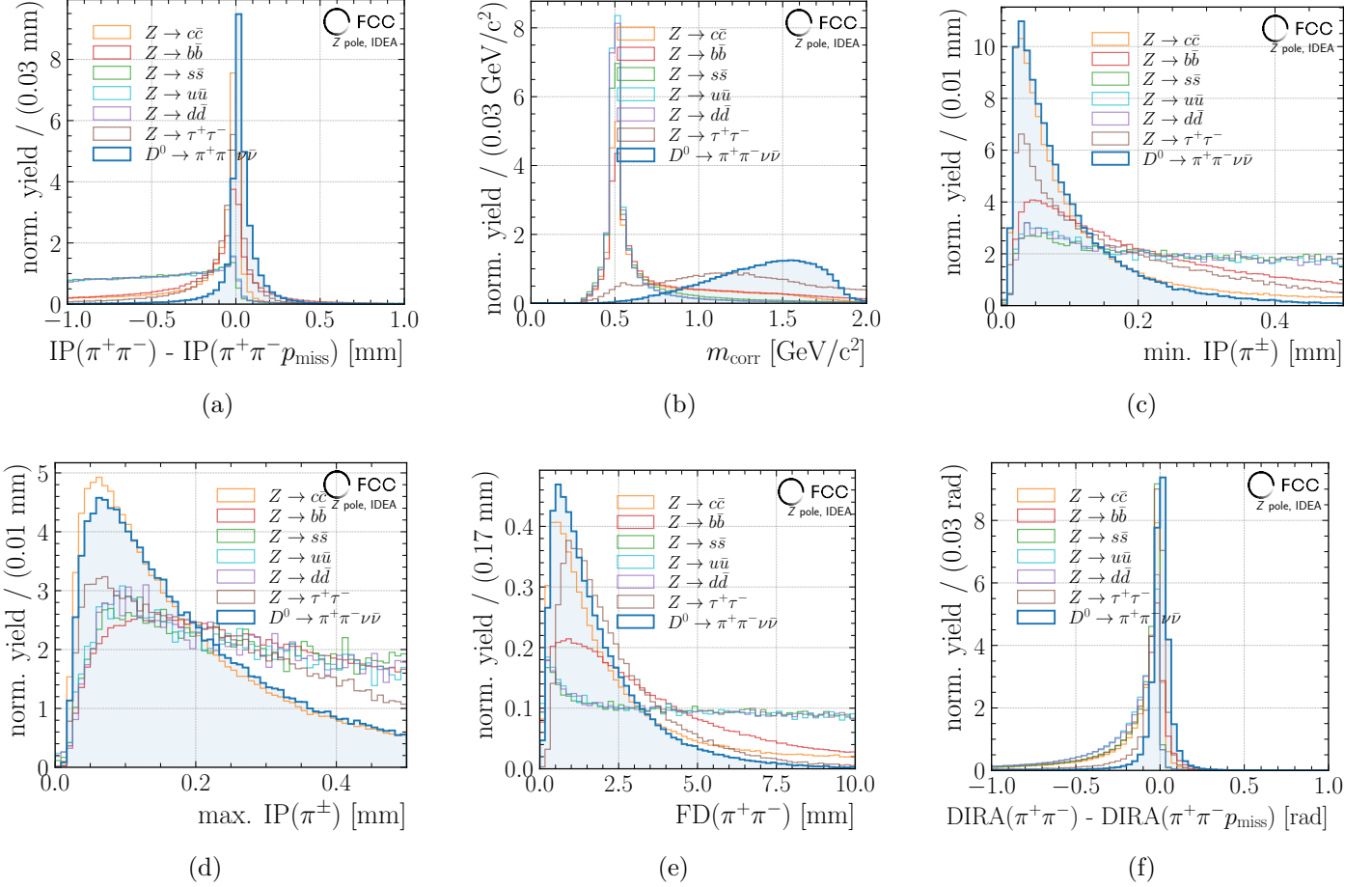


Figure 10: Variable distributions used in the [preselection](#). (a) shows the difference of the impact parameter between reconstructed D^0 using two pions and using two pions together with the missing momentum, (b) the corrected D^0 mass m_{corr} , (c) and (d) the minimum and maximum impact parameter of either pion used for the reconstruction of the D^0 candidate, (e) the flight distance of the D^0 candidate and (f) the difference of the direction angle between reconstructed D^0 using two pions and using two pions together with the missing momentum.

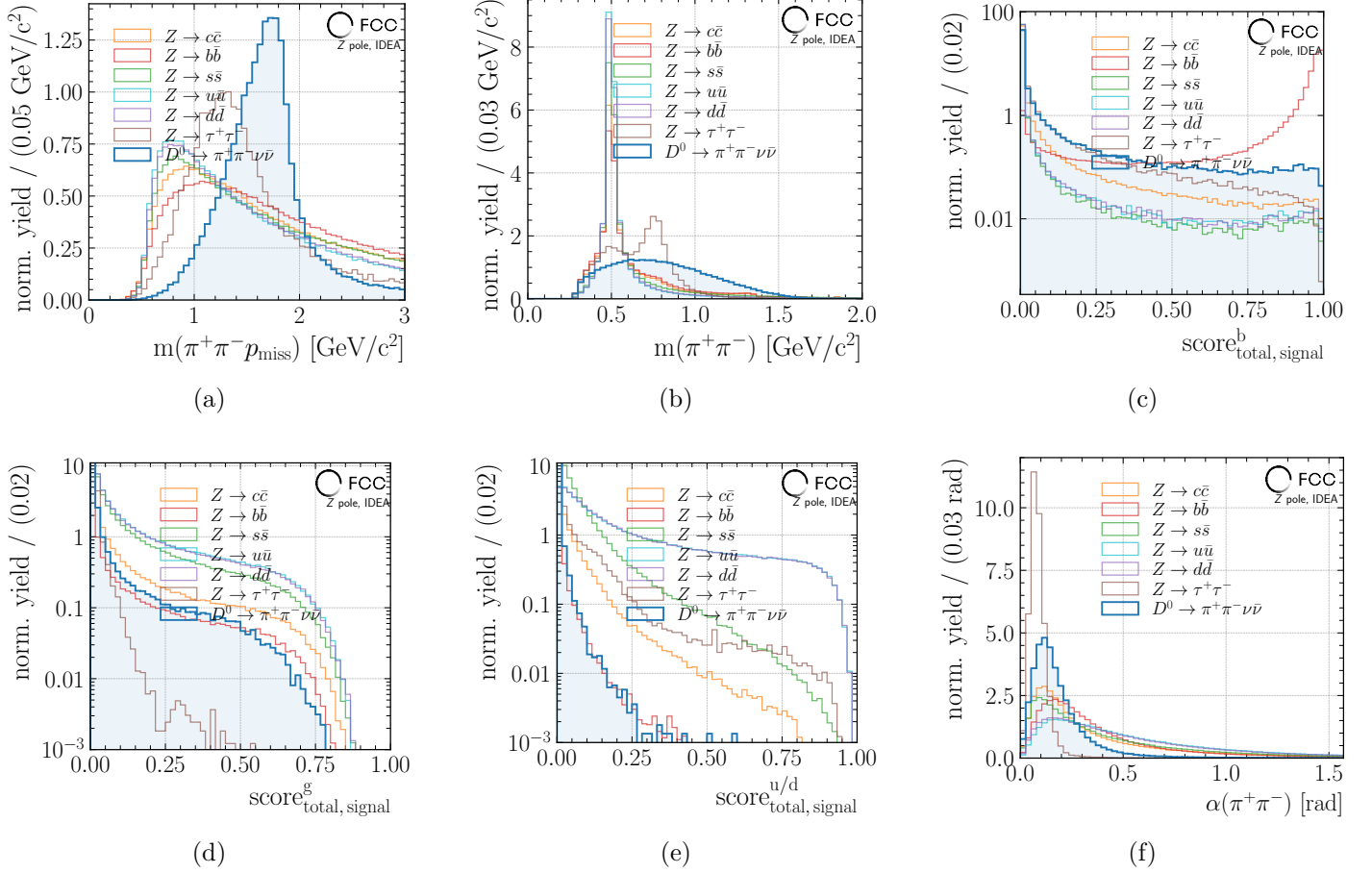


Figure 11: Variable distributions used in [preselection](#). (a) shows the difference of the reconstructed invariant mass using two pions and using two pions together with the missing momentum, (b) the reconstructed invariant D^0 mass, (c), (d) and (e) the multiplied chances of jet being specified flavour. (f) depicts the opening angle of the two pions used for the signal reconstruction.

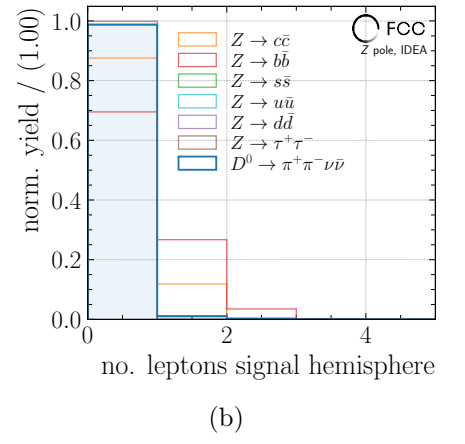
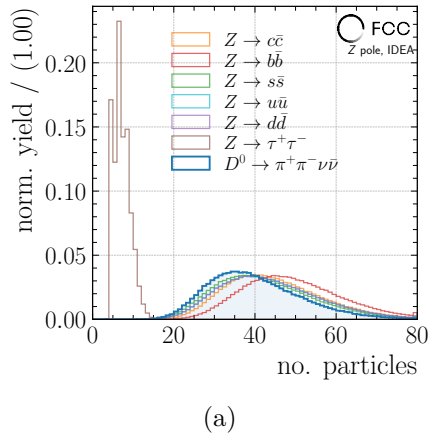


Figure 12: Variable distributions used in [preselection](#). (a) shows the number of particles and (b) the number of leptons in the signal-hemisphere.

A.2 BDT variable distributions

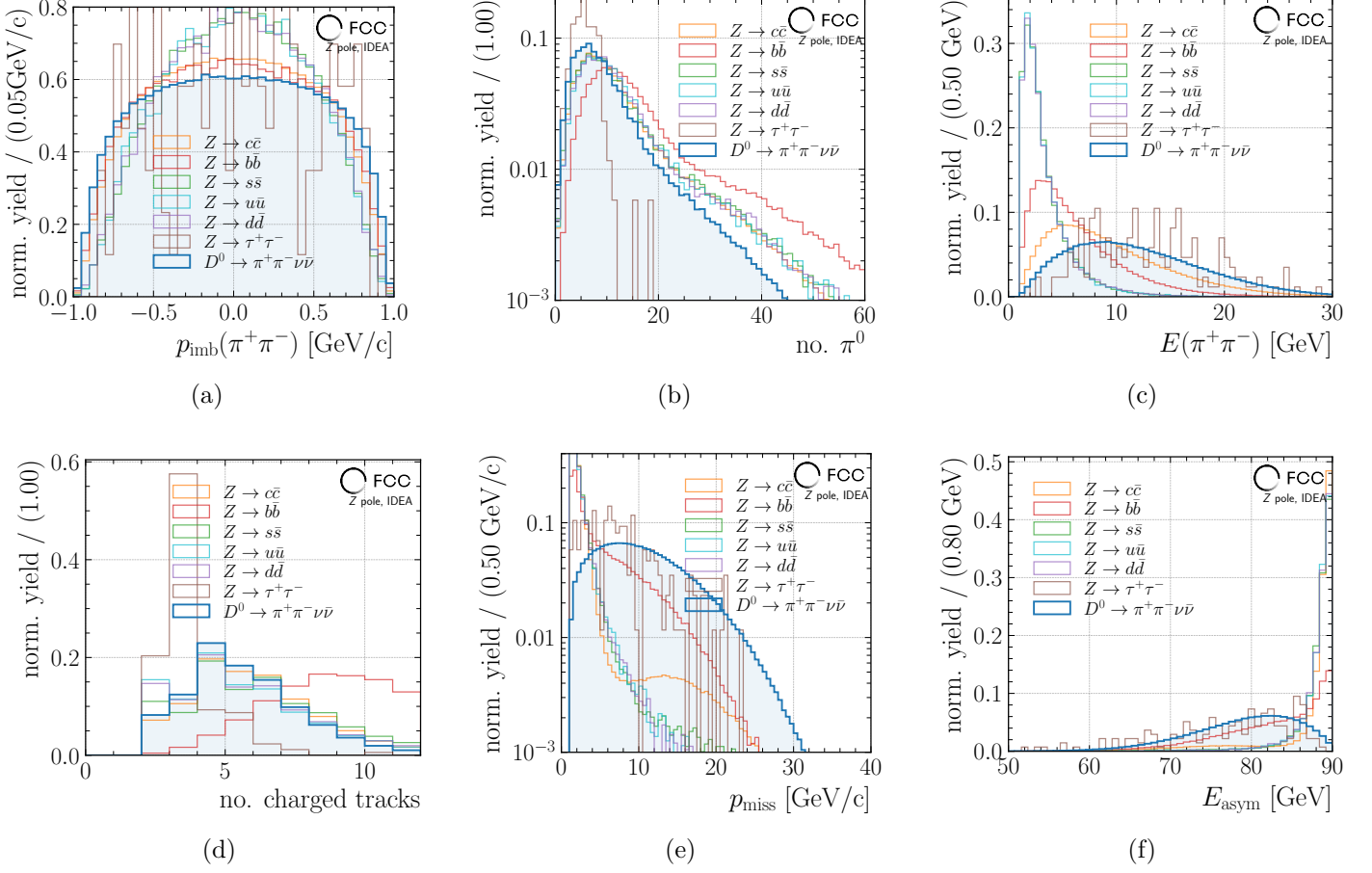


Figure 13: Variable distributions used in the BDT. (a) the number of reconstructed π^0 candidates around D^0 candidate, (b) the number of reconstructed π^0 candidates in the event, (c) the reconstructed energy of the D^0 candidate, (d) the the total number of charged tracks in the event, (e) the missing momentum of the event and (f) the total reconstructed energy of the event.

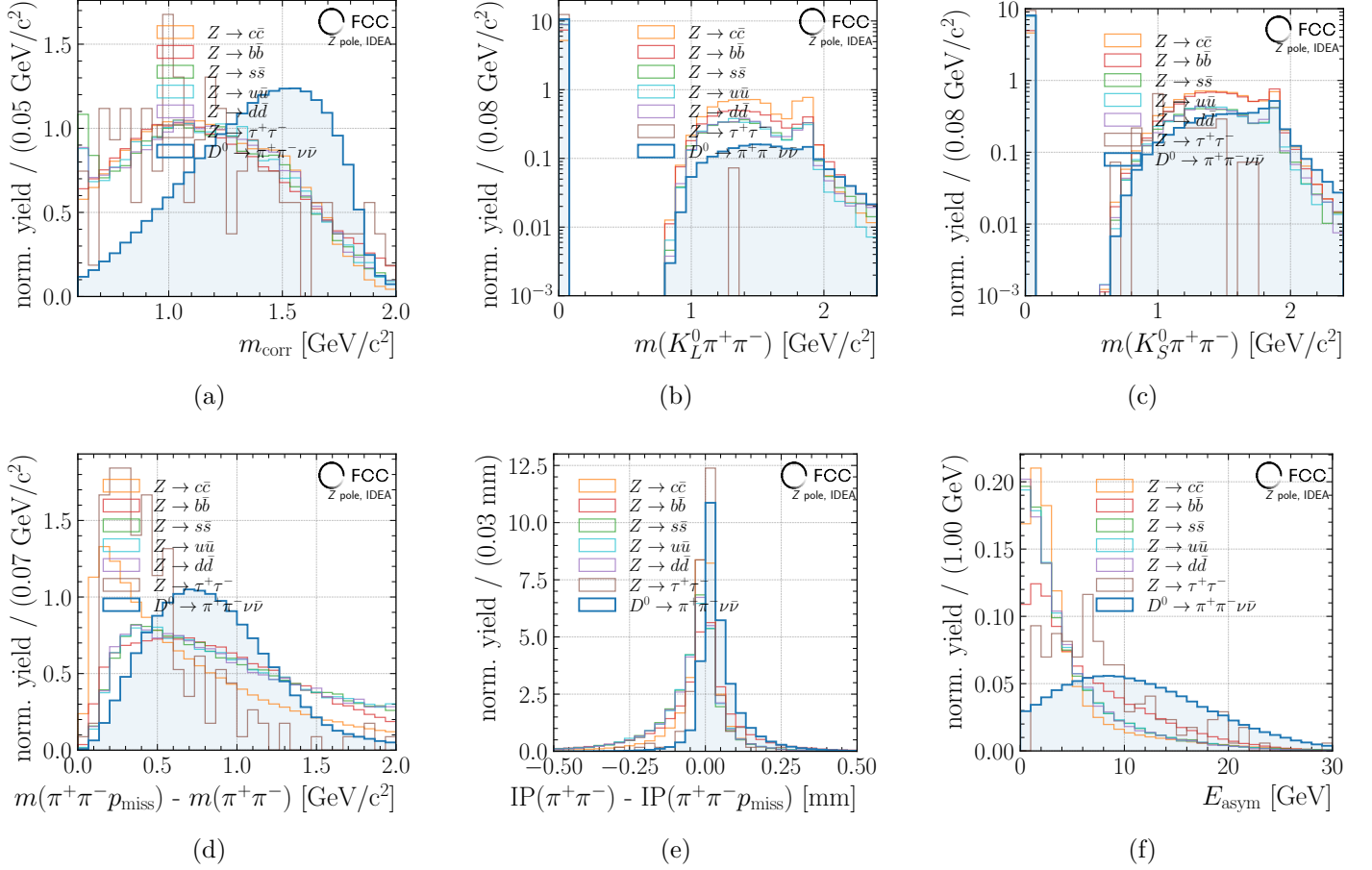


Figure 14: Variable distributions used in the BDT. (a) shows the corrected D^0 mass m_{corr} , (b) the reconstructed invariant mass of the signal candidate and the best K_L^0 candidate, (c) the reconstructed invariant mass of the signal candidate and the best K_S^0 candidate, (d) the difference of the reconstructed invariant mass using two pions and using two pions together with the missing momentum, (e) the difference of the impact parameter using two pions and using two pions together with the missing momentum and (f) depicts the energy asymmetry of particles between the signal and non-signal hemisphere.

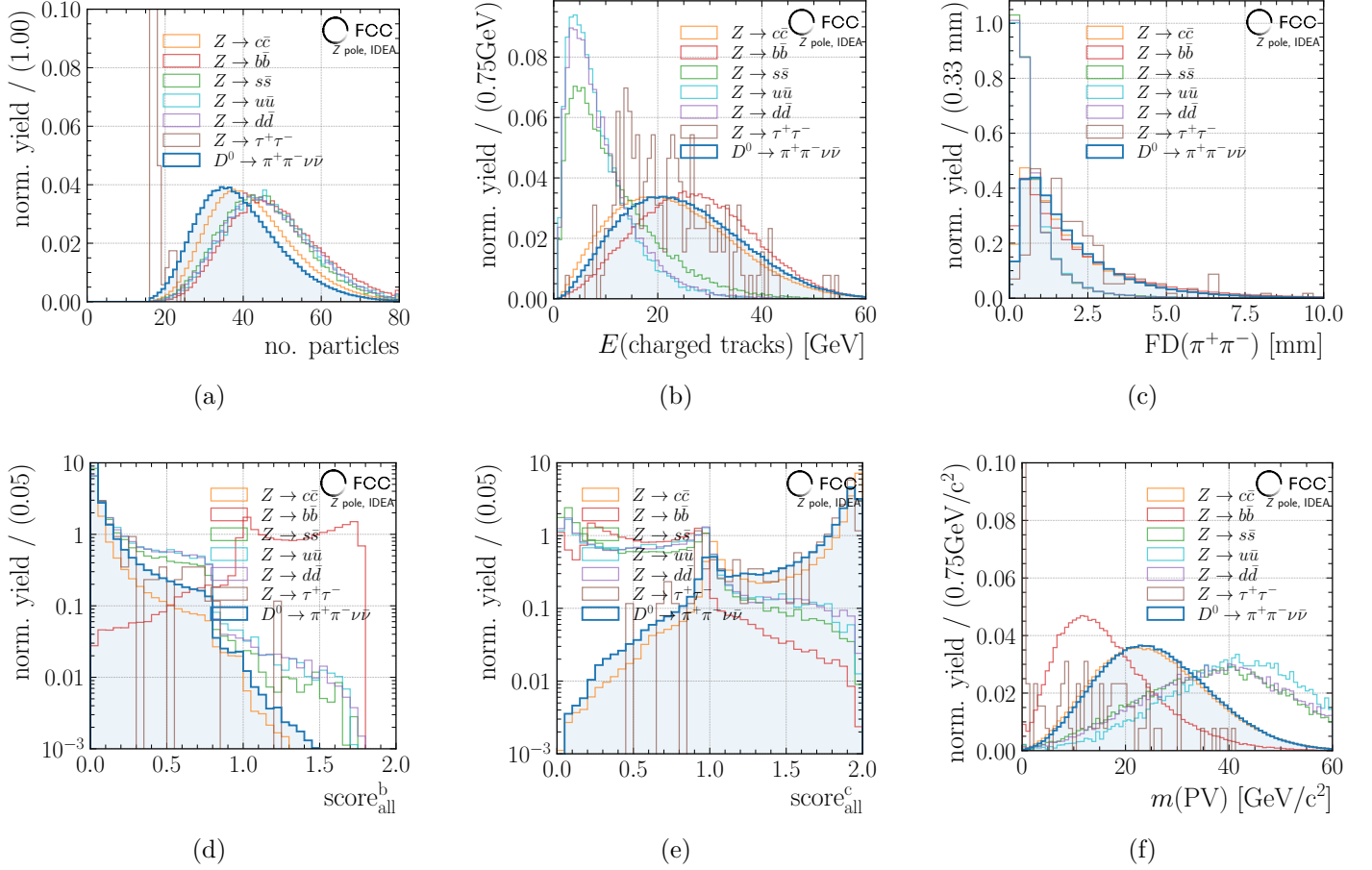


Figure 15: Variable distributions used in the **BDT**. (a) shows the total number of reconstructed particles, (b) the energy of all charged tracks, (c) the flight distance of the signal candidate, (d) signal + non-signal side b-jet score, (e) signal + non-signal side b-jet score, (f) the reconstructed mass of the primary vertex using tracks classified as primary tracks.

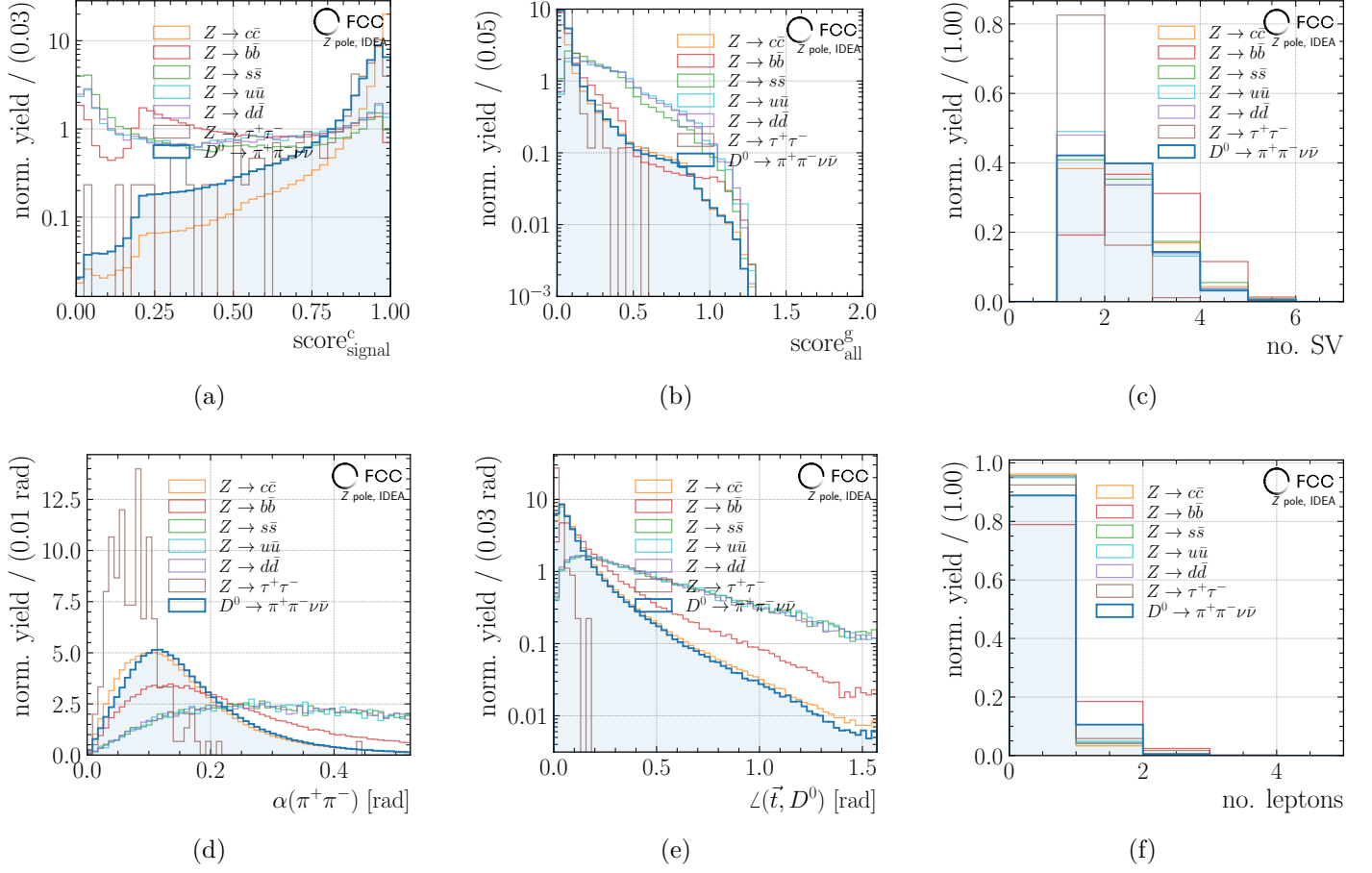


Figure 16: Variable distributions used in the **BDT**. (a) shows the c-jet score of the signal hemisphere, (b) the signal + non-signal g-jet score, (c) the number of secondary vertices, (d) the opening angles of the pions used to reconstructed the signal candidate, (e) the angle between the signal candidate and the thrust axis and (f) the number of leptons in the event.

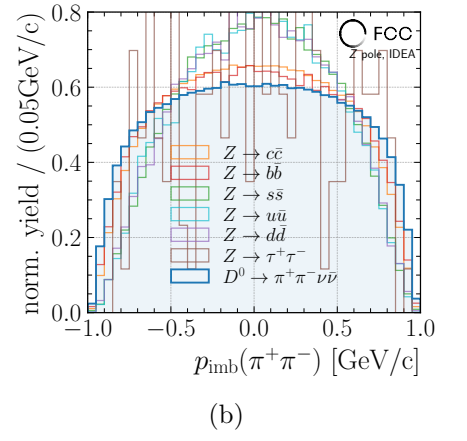
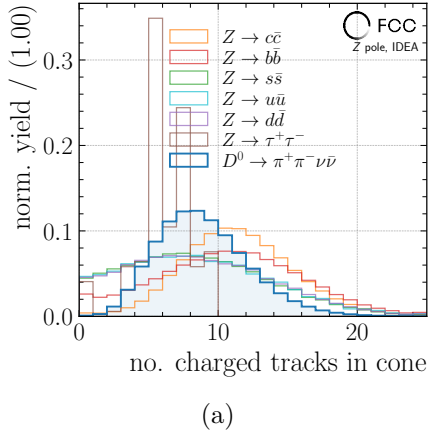


Figure 17: Variable distributions used in the **BDT**. (a) shows the number of charged tracks around the D^0 candidate and (b) the number of reconstructed π^0 candidates around the signal candidate.

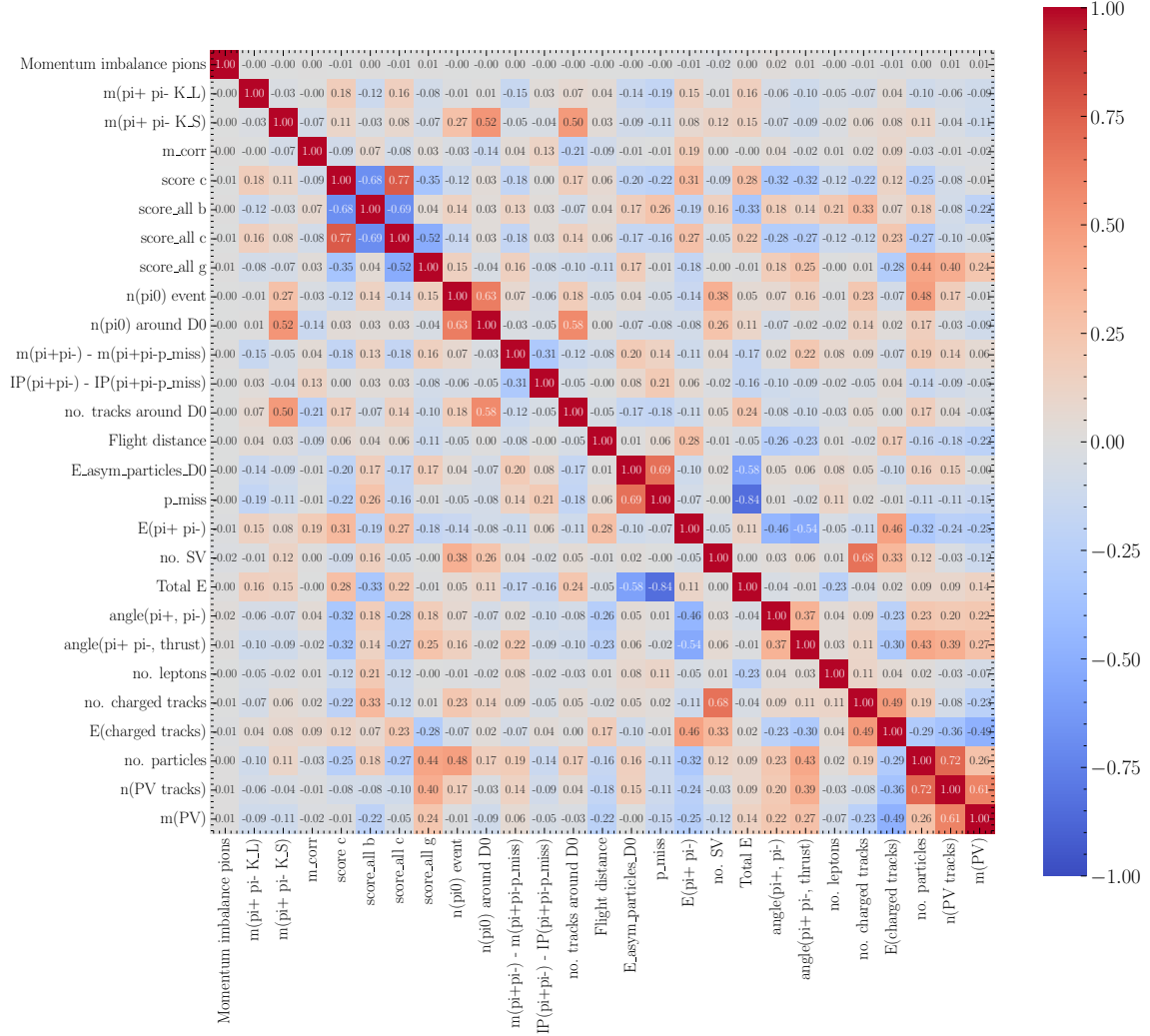


Figure 18: Correlation of variables used in the training of the BDT for the background.

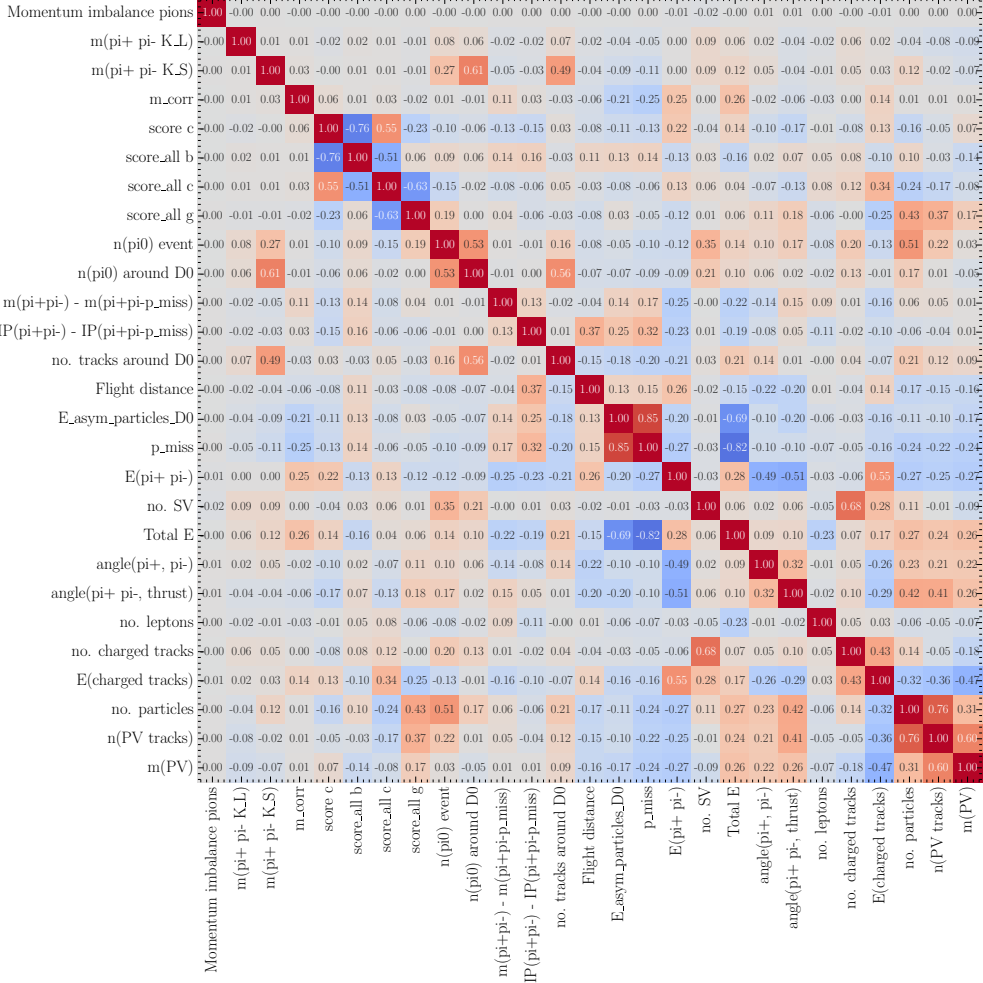


Figure 19: Correlation of variables used in the training of the BDT for the signal and PHSP theory assumption.

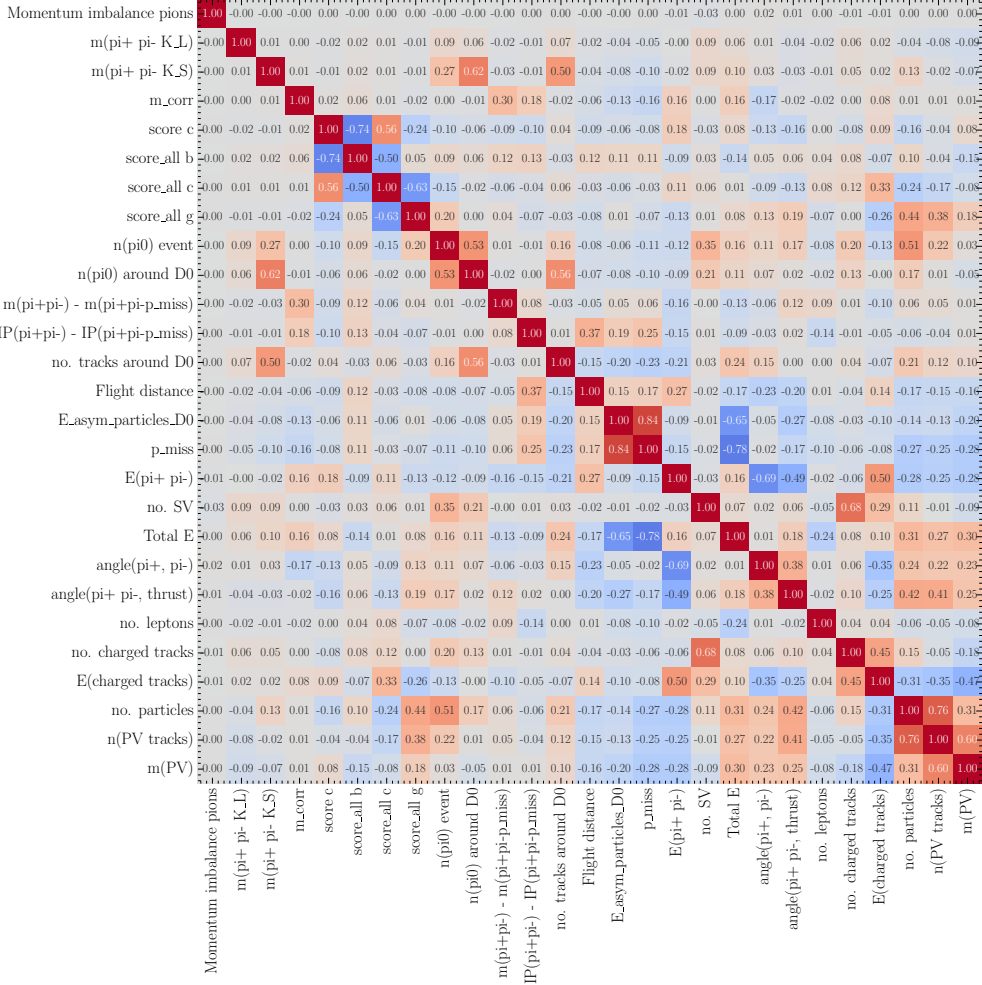


Figure 20: Correlation of variables used in the training of the BDT for the signal and D_p theory assumption.

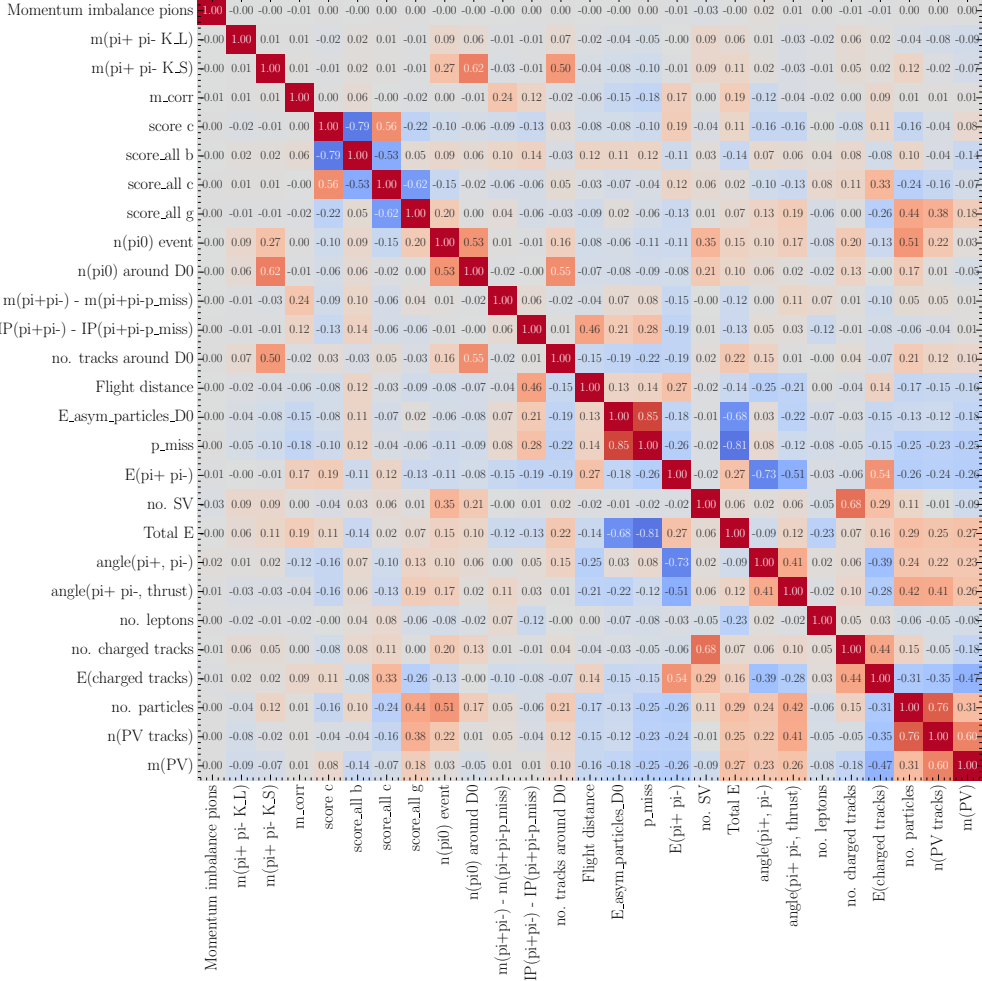


Figure 21: Correlation of variables used in the training of the BDT for the signal and `HHchiPT_res` theory assumption.

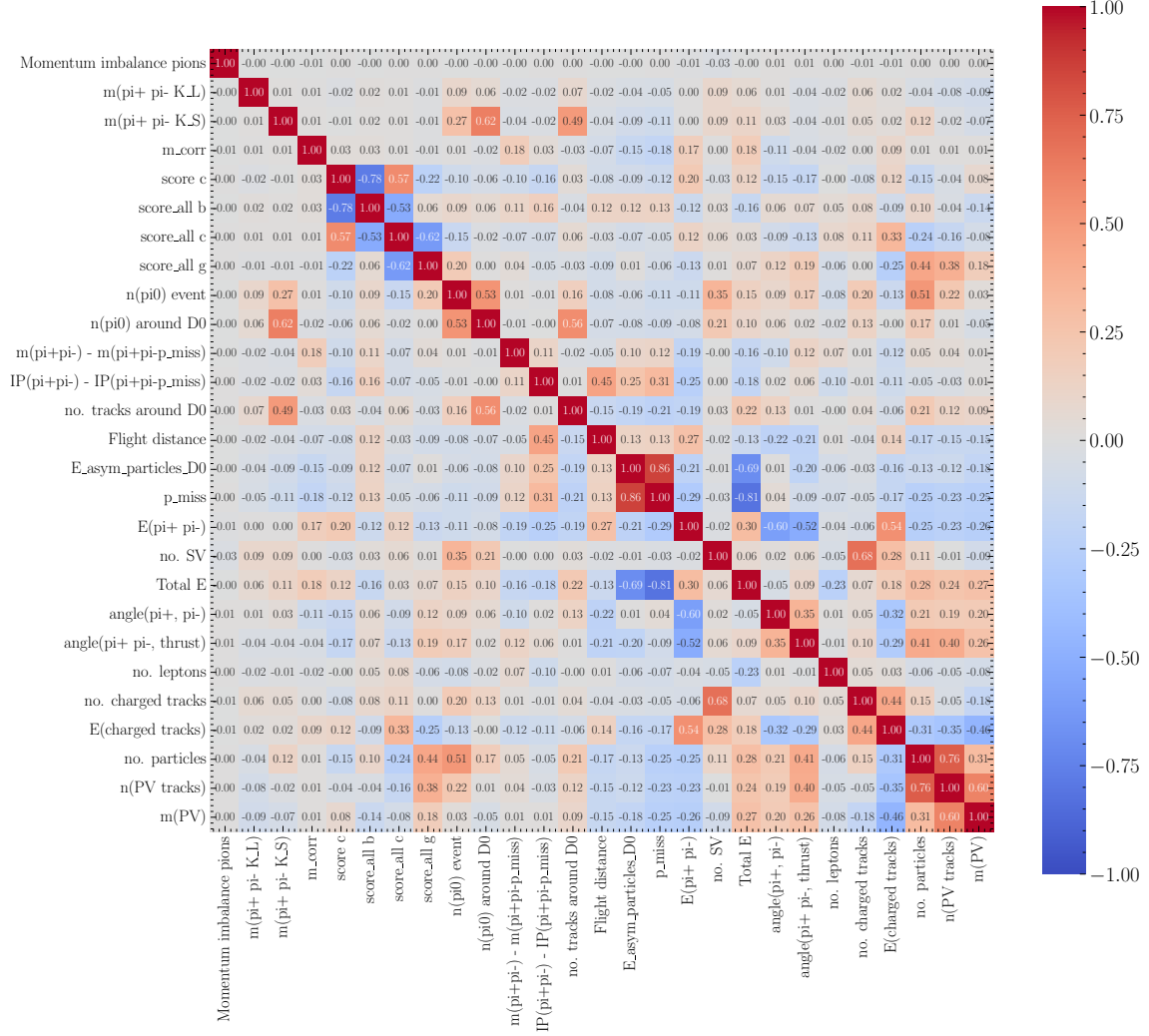


Figure 22: Correlation of variables used in the training of the BDT for the signal and HHchiPT theory assumption.

A.4 Separation and Variable Importance

Variable	Separation	Variable Importance
Total E	6.160×10^{-1}	1.222×10^{-1}
p_{miss}	5.975×10^{-1}	8.291×10^{-2}
E_{asym} particles	3.191×10^{-1}	2.374×10^{-2}
$m(\pi^+\pi^-K_L^0)$	1.876×10^{-1}	5.441×10^{-2}
$\text{IP}(\pi^+\pi^-) - \text{IP}(\pi^+\pi^-p_{\text{miss}})$	1.656×10^{-1}	3.970×10^{-2}
$m(\pi^+\pi^-) - m(\pi^+\pi^-p_{\text{miss}})$	1.377×10^{-1}	4.302×10^{-2}
$\text{score}_{\text{signal}}^c$	1.165×10^{-1}	3.324×10^{-2}
tracks around D^0	1.147×10^{-1}	3.345×10^{-2}
m_{corr}	9.674×10^{-2}	4.089×10^{-2}
$\text{score}_{\text{all}}^c$	8.833×10^{-2}	4.188×10^{-2}
$m(\pi^+\pi^-K_S^0)$	7.938×10^{-2}	2.892×10^{-2}
$E(\pi^+\pi^-)$	7.720×10^{-2}	6.793×10^{-2}
π^0 around D^0	7.196×10^{-2}	2.165×10^{-2}
$\text{score}_{\text{all}}^b$	6.855×10^{-2}	3.829×10^{-2}
$\text{score}_{\text{all}}^g$	3.137×10^{-2}	1.897×10^{-2}
no. particles	3.096×10^{-2}	2.093×10^{-2}
$n(\pi^0)$ event	2.887×10^{-2}	1.940×10^{-2}
no. tracks in event	1.918×10^{-2}	1.862×10^{-2}
no. leptons in event	1.202×10^{-2}	1.052×10^{-2}
$E(\text{tracks})$	1.166×10^{-2}	2.332×10^{-2}
$\angle(D^0, \text{thrust})$	1.070×10^{-2}	3.694×10^{-2}
$\angle(\pi^+, \pi^-)$	1.030×10^{-2}	5.479×10^{-2}
FD	7.985×10^{-3}	3.662×10^{-2}
$m(\text{PV})$	7.127×10^{-3}	2.705×10^{-2}
no. sec. vertices	6.389×10^{-3}	1.184×10^{-2}
pion momentum imbalance	4.791×10^{-3}	2.750×10^{-2}
no. PV tracks	2.178×10^{-3}	2.124×10^{-2}

Table 5: BDT variables sorted by their separation power along with their corresponding variable importance for the PHSP theory ansatz.

Variable	Separation	Variable Importance
Total E	6.678×10^{-1}	1.316×10^{-1}
p_{miss}	6.540×10^{-1}	7.499×10^{-2}
E_{asym} particles	3.741×10^{-1}	2.339×10^{-2}
$m(\pi^+\pi^-K_L^0)$	1.793×10^{-1}	4.088×10^{-2}
$\text{IP}(\pi^+\pi^-) - \text{IP}(\pi^+\pi^-p_{\text{miss}})$	1.691×10^{-1}	4.088×10^{-2}
$m(\pi^+\pi^-) - m(\pi^+\pi^-p_{\text{miss}})$	1.612×10^{-1}	4.640×10^{-2}
$\text{score}_{\text{signal}}^c$	1.359×10^{-1}	3.532×10^{-2}
tracks around D^0	1.138×10^{-1}	3.334×10^{-2}
$\text{score}_{\text{all}}^c$	9.900×10^{-2}	3.787×10^{-2}
$\text{score}_{\text{all}}^b$	7.511×10^{-2}	3.637×10^{-2}
π^0 around D^0	7.152×10^{-2}	2.309×10^{-2}
$m(\pi^+\pi^-K_S^0)$	6.899×10^{-2}	2.709×10^{-2}
$E(\pi^+\pi^-)$	5.348×10^{-2}	6.839×10^{-2}
m_{corr}	4.869×10^{-2}	4.617×10^{-2}
$\text{score}_{\text{all}}^g$	3.529×10^{-2}	2.106×10^{-2}
no. particles	3.100×10^{-2}	2.015×10^{-2}
$n(\pi^0)$ event	2.907×10^{-2}	1.604×10^{-2}
no. tracks in event	1.921×10^{-2}	2.114×10^{-2}
$\angle(\pi^+, \pi^-)$	1.669×10^{-2}	6.208×10^{-2}
no. leptons in event	1.457×10^{-2}	9.334×10^{-3}
FD	9.208×10^{-3}	4.468×10^{-2}
$E(\text{tracks})$	7.643×10^{-3}	2.220×10^{-2}
$m(\text{PV})$	7.142×10^{-3}	2.773×10^{-2}
$\angle(D^0, \text{thrust})$	6.995×10^{-3}	3.346×10^{-2}
no. sec. vertices	6.567×10^{-3}	1.179×10^{-2}
pion momentum imbalance	6.017×10^{-3}	2.126×10^{-2}
no. PV tracks	2.196×10^{-3}	2.327×10^{-2}

Table 6: BDT variables sorted by their separation power along with their corresponding variable importance for the Dp theory ansatz.

Variable	Separation	Variable Importance
Total E	6.321×10^{-1}	1.277×10^{-1}
p_{miss}	6.174×10^{-1}	8.228×10^{-2}
E_{asym} particles	3.413×10^{-1}	2.493×10^{-2}
$\text{IP}(\pi^+\pi^-) - \text{IP}(\pi^+\pi^-p_{\text{miss}})$	2.426×10^{-1}	4.591×10^{-2}
$m(\pi^+\pi^-) - m(\pi^+\pi^-p_{\text{miss}})$	2.043×10^{-1}	3.834×10^{-2}
$m(\pi^+\pi^-K_L^0)$	1.819×10^{-1}	4.059×10^{-2}
$\text{score}_{\text{signal}}^c$	1.279×10^{-1}	3.556×10^{-2}
tracks around D^0	1.141×10^{-1}	3.259×10^{-2}
m_{corr}	1.139×10^{-1}	4.870×10^{-2}
$\text{score}_{\text{all}}^c$	9.567×10^{-2}	3.798×10^{-2}
$\text{score}_{\text{all}}^b$	8.592×10^{-2}	4.134×10^{-2}
π^0 around D^0	7.201×10^{-2}	2.135×10^{-2}
$m(\pi^+\pi^-K_S^0)$	7.122×10^{-2}	2.961×10^{-2}
$E(\pi^+\pi^-)$	5.802×10^{-2}	6.685×10^{-2}
no. particles	3.115×10^{-2}	1.848×10^{-2}
$n(\pi^0)$ event	2.933×10^{-2}	1.770×10^{-2}
$\text{score}_{\text{all}}^g$	2.722×10^{-2}	2.095×10^{-2}
no. tracks in event	1.923×10^{-2}	1.742×10^{-2}
$\angle(\pi^+, \pi^-)$	1.615×10^{-2}	5.865×10^{-2}
no. leptons in event	1.294×10^{-2}	8.905×10^{-3}
FD	8.654×10^{-3}	4.008×10^{-2}
pion momentum imbalance	8.081×10^{-3}	2.225×10^{-2}
$E(\text{tracks})$	7.843×10^{-3}	2.426×10^{-2}
$m(\text{PV})$	7.489×10^{-3}	2.716×10^{-2}
$\angle(D^0, \text{thrust})$	7.155×10^{-3}	3.678×10^{-2}
no. sec. vertices	6.429×10^{-3}	1.118×10^{-2}
no. PV tracks	2.264×10^{-3}	2.246×10^{-2}

Table 7: BDT variables sorted by their separation power along with their corresponding variable importance for the $\text{HH}\chi\text{PT_res}$ theory ansatz.

Variable	Separation	Variable Importance
Total E	6.324×10^{-1}	1.245×10^{-1}
p_{miss}	6.182×10^{-1}	8.635×10^{-2}
E_{asym} particles	3.436×10^{-1}	2.399×10^{-2}
$\text{IP}(\pi^+\pi^-) - \text{IP}(\pi^+\pi^-p_{\text{miss}})$	2.369×10^{-1}	4.302×10^{-2}
$m(\pi^+\pi^-) - m(\pi^+\pi^-p_{\text{miss}})$	1.947×10^{-1}	3.799×10^{-2}
$m(\pi^+\pi^-K_L^0)$	1.810×10^{-1}	4.641×10^{-2}
$\text{score}_{\text{signal}}^c$	1.221×10^{-1}	3.858×10^{-2}
tracks around D^0	1.142×10^{-1}	3.320×10^{-2}
m_{corr}	9.919×10^{-2}	5.009×10^{-2}
$\text{score}_{\text{all}}^c$	9.250×10^{-2}	3.978×10^{-2}
$\text{score}_{\text{all}}^b$	8.567×10^{-2}	3.873×10^{-2}
π^0 around D^0	7.247×10^{-2}	2.357×10^{-2}
$m(\pi^+\pi^-K_S^0)$	7.023×10^{-2}	2.804×10^{-2}
$E(\pi^+\pi^-)$	4.605×10^{-2}	6.150×10^{-2}
no. particles	3.115×10^{-2}	1.793×10^{-2}
$n(\pi^0)$ event	2.922×10^{-2}	2.102×10^{-2}
$\text{score}_{\text{all}}^g$	2.453×10^{-2}	2.037×10^{-2}
no. tracks in event	1.936×10^{-2}	1.968×10^{-2}
$\angle(\pi^+, \pi^-)$	1.675×10^{-2}	4.598×10^{-2}
no. leptons in event	1.310×10^{-2}	1.070×10^{-2}
FD	7.866×10^{-3}	4.175×10^{-2}
$m(\text{PV})$	7.467×10^{-3}	2.931×10^{-2}
no. sec. vertices	6.381×10^{-3}	1.164×10^{-2}
$\angle(D^0, \text{thrust})$	5.630×10^{-3}	3.948×10^{-2}
$E(\text{tracks})$	5.551×10^{-3}	2.669×10^{-2}
no. PV tracks	2.257×10^{-3}	2.249×10^{-2}
pion momentum imbalance	2.126×10^{-3}	1.717×10^{-2}

Table 8: BDT variables sorted by their separation power along with their corresponding variable importance for the $\text{HH}\chi\text{PT}$ theory ansatz.

A.5 $K_{S/L}^0$ variables

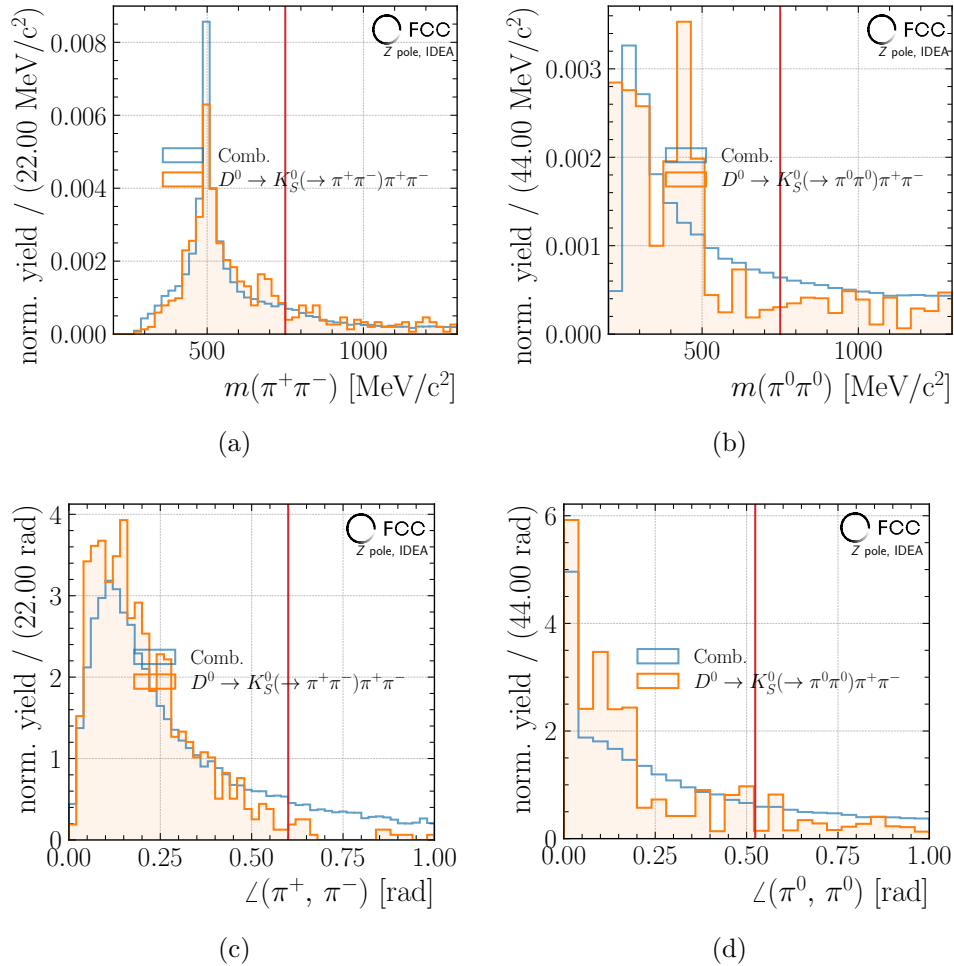
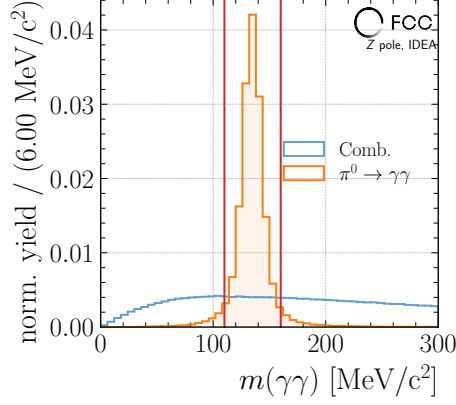
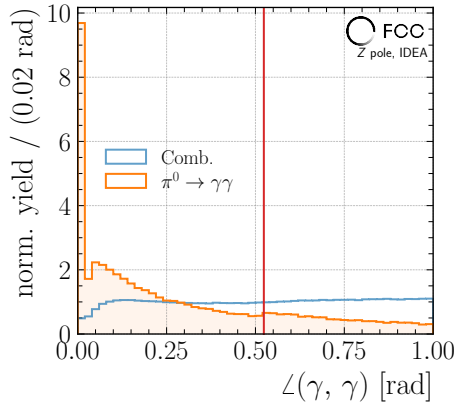


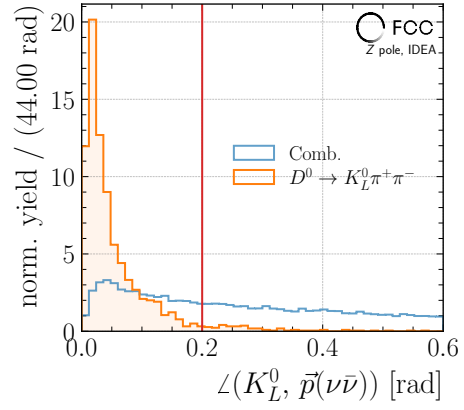
Figure 23: Variables used to select K_S^0 and K_L^0 candidates originating from $D^0 \rightarrow K_{S/L}^0 \pi^+ \pi^-$ decays, with the red lines illustrating the chosen cut value. (a) and (b) show the invariant mass of the two charged and neutral pions used to reconstruct K_S^0 candidates, with (c) and (d) showing the respective opening angles.



(a)



(b)



(c)

Figure 24: Variables used to select K_S^0 and K_L^0 candidates originating from $D^0 \rightarrow K_{S/L}^0 \pi^+ \pi^-$ decays, with the red lines illustrating the chosen cut value. (a) and (b) depict the invariant diphoton mass and the opening angle, used to select neutral pion candidates. (c) shows the angular distance between the K_L^0 candidate and the computed momentum vector of the dineutrino system.

B Supplemental Material Angular asymmetries in $\Lambda_c^+ \rightarrow p\mu^+\mu^-$ decays

This section shows the variables used in the reconstruction and selection of $\Lambda_c^+ \rightarrow p\mu^+\mu^-$ candidates. The signal and each background type are normalised separately so that their distributions integrate to one.

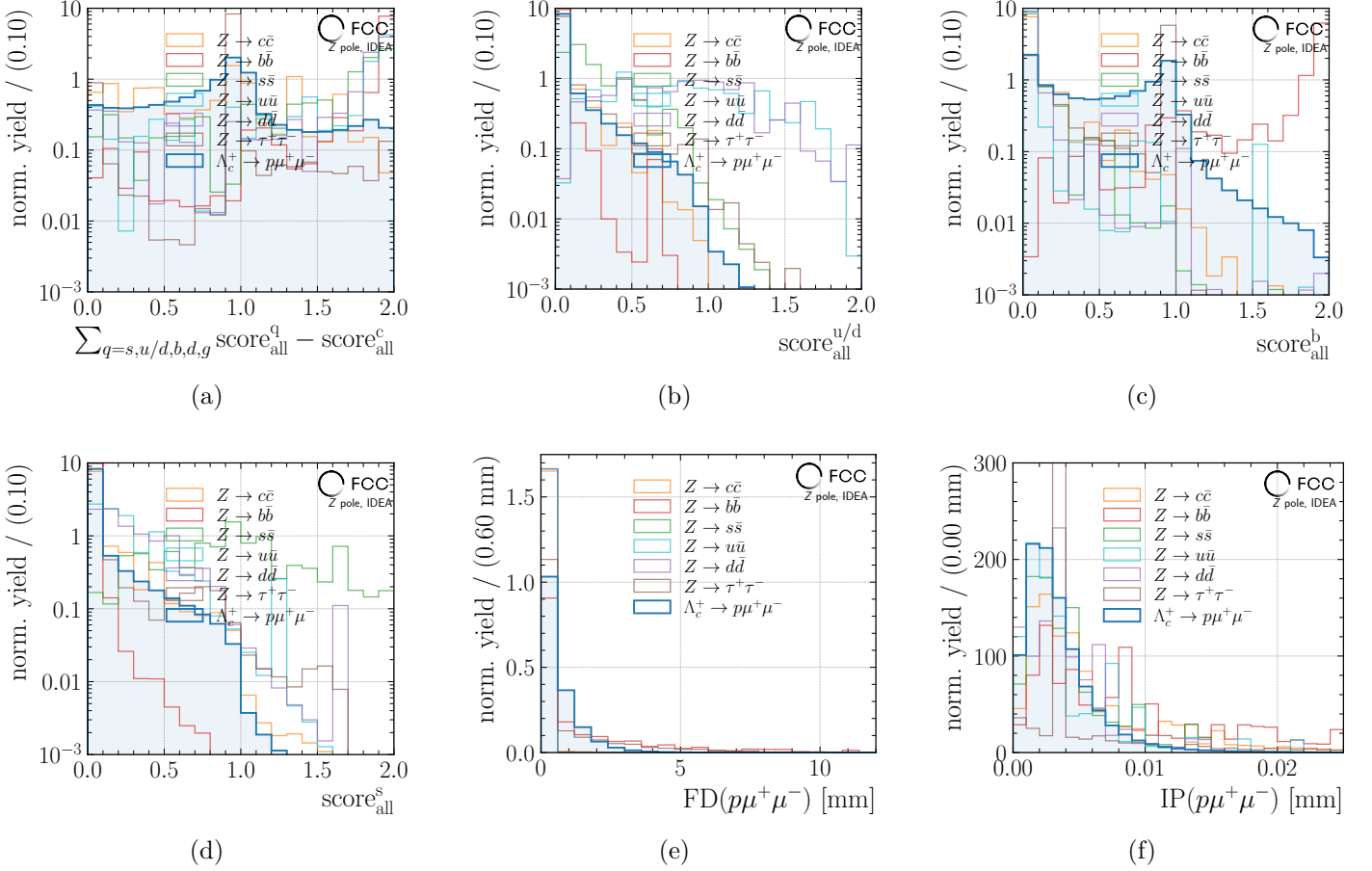
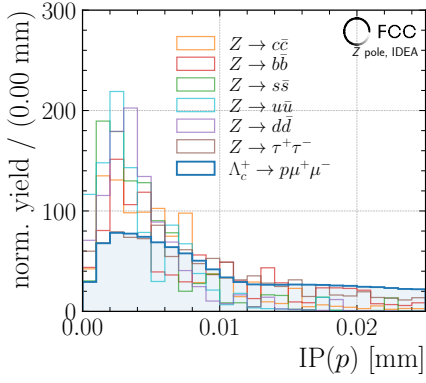
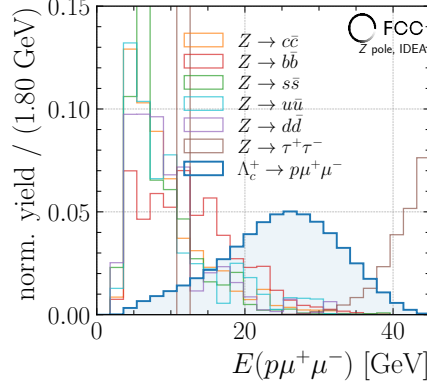


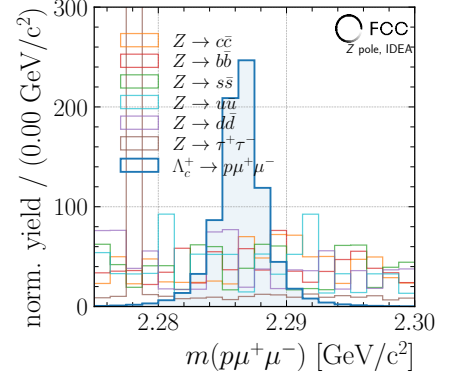
Figure 25: Variable distributions used in the selection: (a) shows the likelihood that the two jets do not originate from c-jets, (b) the likelihood of the jets being q-jets, (c) the likelihood of the jets being b-jets, (d) the likelihood of the jets being s-jets, (e) the Flight Distance of the Λ_c^+ (f) the Impact Parameter of the Λ_c^+ .



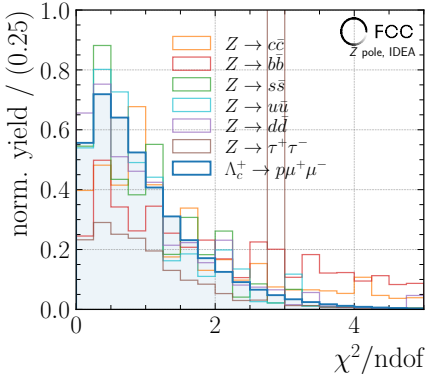
(a)



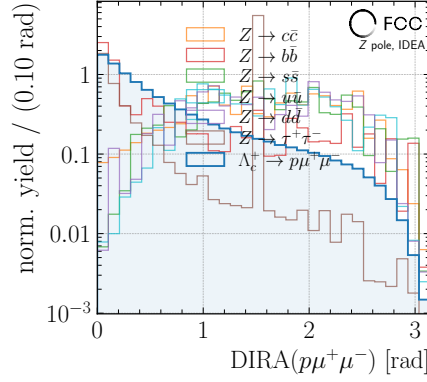
(b)



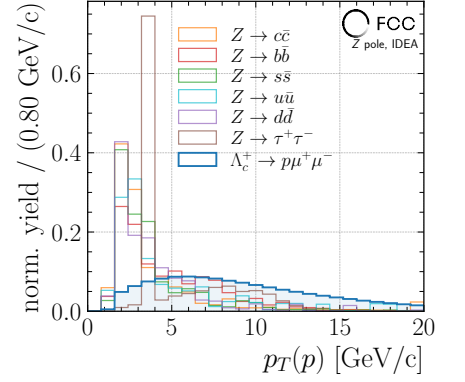
(c)



(d)



(e)



(f)

Figure 26: Variable distributions used in the selection: (a) shows the Impact Parameter of the proton, (b) the energy of the Λ_c^+ , (c) the mass of the Λ_c^+ (d) the χ^2/ndof value of the Λ_c^+ vertex (e) the direction angle of the Λ_c^+ , and (f) transverse momentum of the proton..

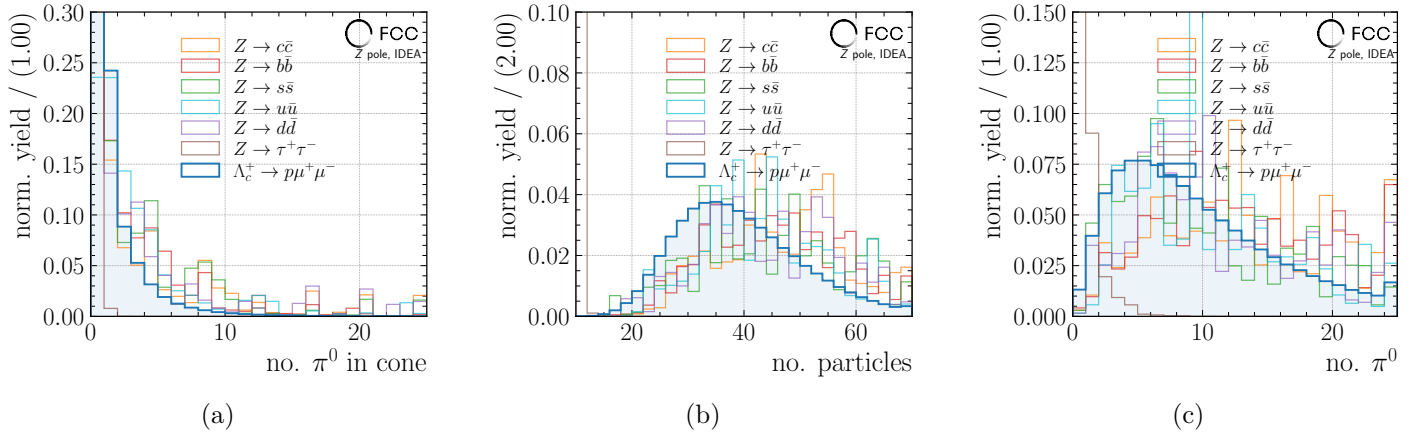


Figure 27: Variable distributions used in the selection:s (a) shows the number of π^0 around the Λ_c^+ candidate, (b) the number of reconstructed particles and (c) the number of π^0 in the event.