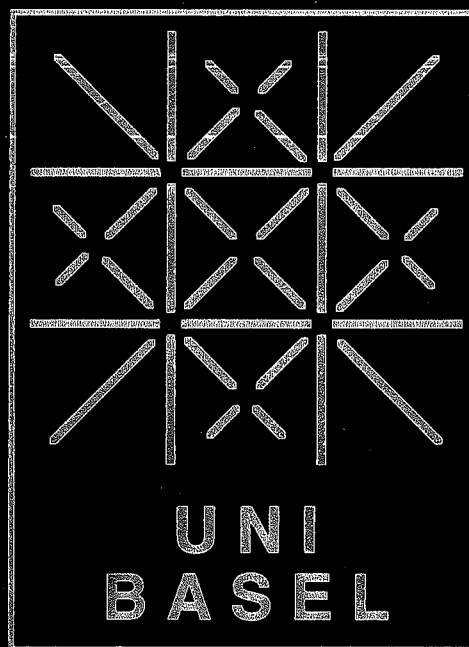




Measurements of the $K_L \rightarrow \pi^+ \pi^- \pi^0$
decay slope parameters

Fabian R. Leimgruber
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Thesis-1997-Leimgruber

L. Tard

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Measurements of the $K_L \rightarrow \pi^+ \pi^- \pi^0$
decay slope parameters

Inauguraldissertation

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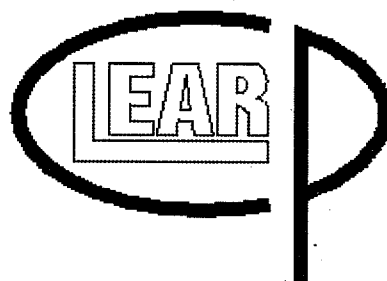
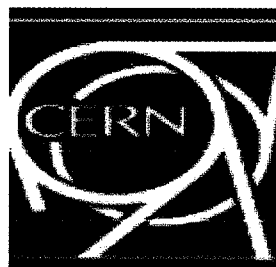
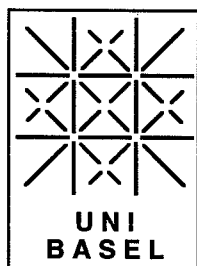
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Die richtige Methode der Philosophie wäre eigentlich die: Nichts zu sagen, als was sich sagen lässt, also Sätze der Naturwissenschaft - also etwas, was mit Philosophie nichts zu tun hat -, und dann immer, wenn ein anderer etwas Metaphysisches sagen wollte, ihm nachzuweisen, dass er gewissen Zeichen in seinen Sätzen keine Bedeutung gegeben hat. Diese Methode wäre für den anderen unbefriedigend - er hätte nicht das Gefühl, dass wir ihn Philosophie lehrten - aber *sie* wäre die einzig streng richtige.

Ludwig Wittgenstein

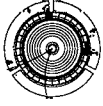
Tractatus logico-philosophicus

Abstract

The decay amplitudes for $K_L \rightarrow \pi^+\pi^-\pi^0$ are studied on the basis of a Dalitz-plot analysis of the complete selected $\pi^+\pi^-\pi^0$ data sample (508'032 events) of the CPLEAR experiment at CERN. The Dalitz-plot density is conventionally parameterized as $|M|^2 \propto 1 + gY + hY^2 + jX + kX^2 + fXY$, where X and Y are the Dalitz-plot variables. Our fit yields $g = 0.6823 \pm 0.0044 \text{ stat. } \pm 0.0044 \text{ sys.}$, $h = (6.13 \pm 0.38 \text{ stat. } \pm 1.45 \text{ sys.}) \cdot 10^{-2}$, $j = (0.10 \pm 0.24 \text{ stat. } \pm 0.30 \text{ sys.}) \cdot 10^{-2}$, $k = (1.06 \pm 0.17 \text{ stat. } \pm 0.24 \text{ sys.}) \cdot 10^{-2}$ and $f = (0.45 \pm 0.24 \text{ stat. } \pm 0.59 \text{ sys.}) \cdot 10^{-2}$.

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Chapter 1

Introduction

Since its discovery in 1946[1][2] the weak interaction has always been one of the most valuable sources for a deeper understanding of elementary particle interaction. Weak decays of hadrons containing heavy quarks are employed today in various ways to test the Standard Model and its parameters. In particular, this is the only direct way to measure the $\mathcal{CP}$ violating parameters.

$\mathcal{CP}$ violation was first discovered in the $K^0, \bar{K}^0$ system[3]. The fact that the mass matrix eigenstates of $K^0, \bar{K}^0$ mesons are different from their $\mathcal{CP}$ eigenstates leads to $\mathcal{CP}$ violation in the $K^0, \bar{K}^0$ mixing. Another $\mathcal{CP}$ breaking mechanism called direct $\mathcal{CP}$ violation or $\mathcal{CP}$ violation in the decay originates in the decay matrix elements with non-zero imaginary parts independent of phase conventions. Up to now no experimental proof exists for the latter effect.

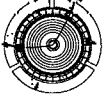
The neutral kaon system is the only system in which $\mathcal{CP}$ violation has so far been observed. Today, it belongs to the interesting questions of particle physics, whether $\mathcal{CP}$ violation effects can also be seen in the bottom meson $B^0, \bar{B}^0$ system[4]. In the bottom meson system direct $\mathcal{CP}$ violation effects are expected to be more evident. Another question is whether the $\mathcal{CP}$ violation as introduced by the δ phase of the Cabibbo-Kobayashi-Maskawa (CKM) model can explain the observations.

The CPLEAR experiment[5] at the Low Energy Antiproton Ring (LEAR) at CERN is designed to study $\mathcal{CP}$, $\mathcal{T}$ and $\mathcal{CPT}$ violation in the neutral kaon system. CPLEAR is a new way to measure $\mathcal{CP}$ violation in the neutral kaon system. The CPLEAR experiment uses initially well defined $K^0, \bar{K}^0$ states produced in $p\bar{p}$ annihilation at rest, instead of K_L or K_S beams like common experiments [6] [7] [8]. From all the $p\bar{p}$ annihilation channels the experiment selects the reactions:

$$(p\bar{p})_{\text{rest}} \longrightarrow \begin{cases} K^0 K^- \pi^+ \\ \bar{K}^0 K^+ \pi^- \end{cases}$$

The strangeness of the neutral kaon is tagged by identifying the charged kaon. The $\mathcal{CP}$ violating parameters are extracted from the time dependent asymmetries between the decay rates $R(K^0 \rightarrow f)$ and $R(\bar{K}^0 \rightarrow f)$ of a neutral kaon into the final state f of the form:

$$A_f(t) = \frac{R(\bar{K}^0 \rightarrow f) - R(K^0 \rightarrow f)}{R(\bar{K}^0 \rightarrow f) + R(K^0 \rightarrow f)}$$



This work focuses on the study of the Dalitz-plot slope parameters of the decay $K_L \rightarrow \pi^+\pi^-\pi^0$ done with the complete data sample of 508'032 selected events by the CPLEAR experiment after the final run in summer 1996. The Dalitz-plot density is conventionally parameterized[9] using $|M|^2 \propto 1 + gY + hY^2 + jX + kX^2 + fXY$, where X and Y are the Dalitz-plot variables. The slope parameters g, h, j, k and f allow the extraction of the decay amplitudes α, β, ξ and ζ of the neutral kaon to the different isospin eigenstates of the $\pi^+\pi^-\pi^0$ final state. The existing predictions of the decay amplitudes up to the sixth order of Chiral perturbative theories make a precise cross-check against the values derived in the fit to our experiment possible. Our results are also compared to the values of previous experiments. These two consistency checks of the CPLEAR data sample and the CPLEAR data selection criteria provide an important basis for the other measured parameters as e.g. $Re(\eta_{+-0})$ or $Im(\eta_{+-0})$ by the $\pi^+\pi^-\pi^0$ -group at CPLEAR.

The accuracy of today's experiments[10] allows us to measure the quadratic terms of the Dalitz-plot parameterization, i.e. the slope parameters h and k . This means that, within the framework of Chiral perturbative theory, higher order Lagrangian terms with higher derivatives must be taken into consideration in order to account for non-vanishing quadratic coefficients. At present calculations[11] of the decay amplitudes of $K_L \rightarrow \pi^+\pi^-\pi^0$ exist using Chiral perturbative theory up to $\mathcal{O}(p^6)$ in order to eliminate the discrepancy between theory and experiment. Yet, these calculations show some weak points. First, the standard approach by the vacuum-saturation method, i.e. by calculating the effective weak Hamiltonian in conjunction with the vacuum-insertion method, fails to explain the nonleptonic weak decays of charmed and strange mesons[12]. Second, the coupling constants of high order strong and weak Chiral Lagrangians are not very well known, since they must be empirically determined from various low-energy hadronic processes together with Zweig-rule arguments[13]. Our precise measurements of the decay amplitudes are an important test of the calculations of the Chiral perturbation theory and can serve as new input to the theory. In this paper we will establish the world best value for one of the decay amplitudes.

A detailed introduction to $\mathcal{CP}$ violation and the phenomenology of the neutral kaon system is given in chapter 2. The different decay channels are discussed with emphasis on the Dalitz-plot analysis of the $\pi^+\pi^-\pi^0$ channel.

In chapter 3 the CPLEAR detector and trigger environment is described. In particular, the development of a fast online trigger step based on artificial neural network algorithms is treated in detail. This trigger step is designed and tested on the CPLEAR experiment and its data.

Finally, chapter 4 is dedicated to the data analysis of the CPLEAR experiment for the decay channel $K^0, \bar{K}^0 \rightarrow \pi^+\pi^-\pi^0$. The fit procedure of the Dalitz-plot parameters uses an unbinned maximum likelihood method. The large data sample as well as the high order of the fitting polynomials require a special treatment of the fit, which is also discussed. Finally, after a careful study of the systematics, the results for the slope parameters g, h, j, k, f and the decay amplitudes $\alpha, \beta, \xi, \zeta$ of the $\pi^+\pi^-\pi^0$ channel are presented and compared to other work published so far.

Chapter 2

Phenomenology

2.1 CP symmetry and the Standard Model

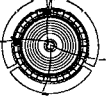
In the Standard Model several common discrete symmetries exist. These are: charge conjugation (C), parity (P), and time reversal (T). The free quark fields possess these symmetries separately, but interaction may break them. A special position is taken by the combination of all three transformations CPT . It is shown in the CPT -theorem or Lüders-Pauli-theorem[14][15] that, for any local Lorentz-invariant field theory in which the observables are represented by Hermitian operators, CPT invariance must hold. R. Jost showed[16][17] that even a weaker condition, the Weak Local Commutativity(WLC), is already sufficient to derive CPT invariance in a theory.

CPT invariance has important implications for the Standard Model. Assuming CPT invariance masses, total lifetimes, and gyro-magnetic ratios of a particle and its anti-particle should be equal, which is in complete agreement with the observations up to now[9]. Thus it is common sense to believe that the CPT symmetry is not broken in any interaction. However it should be mentioned that a tiny violation of CPT is conceivable in extensions of quantum field theory, such as quantum gravity[18] or string theory[19].

In the Standard Model P or C violation come in through the V-A or $\gamma_\mu(1-\gamma_5)$ structure of the charged weak current, whereas the origin of CP violation is still not clear. Many speculations have been proposed since 1964, the year of the discovery of CP violation in the neutral kaon system[3].

As an example I mention here a well-known theory by L. Wolfenstein[20], where CP violation is produced by a new kind of interaction. He calls this interaction 'very weak' or 'super weak'. The interaction has $\Delta S=2$ character and would only be observable in the $K^0, \bar{K}^0$ system. Another possibility of explaining CP violation is to assume the existence of additional $SU(2)$ complex scalar doublets, i.e. extra Higgs bosons. Since $W^\pm$ and Z^0 already have mass because of the original Higgs, the new scalar fields can not be eliminated by gauge transformation. This creates the possibility to introduce CP violation in many ways[21] [22].

Despite all these different models, there is a completely natural and straightforward way to explain CP violation within the frame of the minimal gauge symmetry $SU(2) \otimes U(1)$ of the Standard Model without adding new particles or



forces, other than those already appearing in the Standard Model. There, $\mathcal{CP}$ violation in the neutral kaon mass matrix is provided by the complex phase δ of the CKM (Cabibbo-Kobayashi-Maskawa) matrix in the Kobayashi-Maskawa version[23] of the Weinberg-Salam gauge theory $SU(2) \otimes U(1)$.

CP violation in the Standard Model

The Weinberg-Salam $SU(2) \otimes U(1)$ theory contains three left-handed doublets of quarks and leptons in the standard model. The three quark doublets are $(u, d')_L$, $(c, s')_L$ and $(t, b')_L$ and the gauge group eigenstates d'_L , s'_L and b'_L are linear combinations of the left-handed mass eigenstates d_L , s_L and b_L . The quark mass eigenstates $q(-1/3)_L$ are related to the gauge group eigenstates $q(-1/3)'_L$ according to the unitary transformation, described by the CKM unitary 3×3 quark mixing matrix V :

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \quad (2.1)$$

We get the (V-A) charged weak current for the free two-quark transition of the form $J_\mu = q_i(2/3)_L \gamma_\mu (1 - \gamma_5) V_{ij} q_j(-1/3)_L$, where $q_i(2/3) = (u, c, t)$ and $q_j(-1/3) = (d, s, b)$ for $i=1,2,3$. I.e. the component V_{ij} of the CKM matrix multiplied by $g/\sqrt{2}$ is the coupling constant of the q_i and the q_j quark field to the W-bosons. The relation between the coupling constant g and Fermi's coupling constant G_F is given by $g^2/2m_W^2 = 4G_F/\sqrt{2}$. To derive the complete $\Delta S=1$ weak Hamiltonian in the usual current-current form at the one loop level for quark transition we have to replace the single quark fields by a six-component field for each colour given as $\psi = (u, c, t, d, s, b)$ and $\bar{\psi} = (\bar{u}, \bar{c}, \bar{t}, \bar{d}, \bar{s}, \bar{b})$. The Hamiltonian is written as:

$$H = -\sqrt{1/2} G_F J_\mu^+ J^{-\mu} \quad (2.2)$$

where the weak current $J_\mu^\pm$ is now:

$$\begin{aligned} J_\mu^+ &= \bar{\psi} \gamma_\mu (1 - \gamma_5) U \psi \\ J_\mu^- &= \bar{\psi} \gamma_\mu (1 - \gamma_5) U^\dagger \psi \end{aligned} \quad (2.3)$$

Colour indices are suppressed here. The 6×6 matrix U has the form

$$U = \begin{pmatrix} 0 & V \\ 1 & 0 \end{pmatrix} \quad (2.4)$$

and V is the CKM matrix mentioned above.

Five out of nine free real parameters in the unitary CKM matrix can be absorbed by phase transformations. Three of the remaining four parameters can be regarded as Cabibbo-like rotation angles. As a consequence of the additional fourth parameter at least a few of the CKM matrix components can become complex. In other words, the transformation V can no longer be regarded as a pure rotation. The fourth parameter δ can now be interpreted as a CP-violating phase.

2.1 CP symmetry and the Standard Model



In the standard parameterization by Kobayashi and Maskawa[23] the matrix V depends on three CP-conserving rotation angles $\Theta_{1,2,3}$ and one phase δ :

$$V = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 - s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix} \quad (2.5)$$

where $c_1 = \cos(\Theta_1)$, $s_1 = \sin(\Theta_1)$ etc. The original Cabibbo-angle is $\Theta_1 = \Theta_C$, which is easy to see by decoupling the third quark family ($\Theta_2 = \Theta_3 = 0$).

The occurrence of CP violation introduces several conditions[24] governing the CKM matrix as well as the mentioned mass mixing matrices M , M' appearing in the Lagrangian of the Yukawa coupling of the fermions to the Higgs-field.

$$\mathcal{L} \propto (\bar{u} \quad \bar{c} \quad \bar{t}) M \begin{pmatrix} u \\ c \\ t \end{pmatrix} + (\bar{d} \quad \bar{s} \quad \bar{b}) V M' V^\dagger \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (2.6)$$

i.e. M and M' refer to charge $2/3$ respectively $-1/3$ quarks. Next we write down the commutator of M and M'

$$[M, M'] = iC \quad (2.7)$$

and get for the determinant of C

$$\det C = -2FF'J \quad (2.8)$$

$$\text{with : } F = (m_t - m_c)(m_t - m_u)(m_c - m_u)/m_t^3 \quad (2.9)$$

$$\text{and : } F' = (m_b - m_s)(m_b - m_d)(m_s - m_d)/m_b^3$$

$$\text{and : } J = \text{Im}(V_{11}V_{22}V_{12}^*V_{21}^*) \quad (2.10)$$

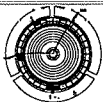
The factors m_t^3 and m_b^3 are introduced through the convenient normalization of M by the top quark mass and M' by the bottom quark mass. The determinant of C (2.8) becomes 0 if and only if there is CP invariance in the theory[24]. Furthermore, it can be shown that all CP violation effects in the Standard Model are proportional to J and that J is independent of phase-conventions. Therefore, in the CKM parameterization (2.5), with

$$J = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin(\delta) \quad (2.11)$$

CP violation occurs if the angles $\Theta_{1,2,3}$ and the phase δ are neither minimal nor maximal, i.e. $\Theta_{1,2,3} \neq (0, \pm\pi/2, \pm\pi, \dots)$ and $\delta \neq (0, \pm\pi, \pm2\pi, \dots)$, since otherwise J vanishes. Condition (2.10) includes the demand for complexity of the CKM matrix in the case of CP violation. In the Cabibbo model with two quark generations, the CKM matrix can always be chosen to be real. Therefore at least three quark generations are needed to explain CP violation this way. With F and F' (2.9) we find another important condition for CP violation, which is that all quark masses have to be different to get $\det C \neq 0$.

Beyond the Standard Model

Two major difficulties of the CKM description of CP violation in the Standard Model have to be mentioned, since they are still not very well understood.



Whilst CP violation is a necessary part of baryogenesis[25] in the early universe, it is believed that CP violation, as described by the Standard Model due to the phase δ , is too small to explain the observed cosmological baryon asymmetry[26]. The other difficulty arises due to the axial anomaly of the U(1) current and the nontrivial vacuum structure in QCD[27]. This introduces a new θ dependent term in the QCD Lagrangian, which violates both $\mathcal{P}$ and $\mathcal{T}$ invariance:

$$\theta \frac{\alpha_s}{4\pi} \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu} \quad (2.12)$$

where θ is the dimensionless vacuum angle of the QCD vacuum states. The additional CP violating term (eq. 2.12) in the QCD Lagrangian can be observed with precise measurements of the electric dipole momentum of the neutron. The upper limit given by such experiments is $|\theta| < 10^{-9}$ [9]. The search for the mechanism producing not necessarily vanishing value of θ is referred to as the strong CP problem. The theory so far seems to require an extreme fine-tuning to derive the correct value of θ , which is seemingly unnatural.

There are at present two main theories which offer a way out of the problems mentioned. Peccei and Quinn[28] suggest the introduction of an additional Higgs doublet in the electroweak Lagrangian. The two Higgs doublets acquire non-zero vacuum expectation values and the $U_{PQ}(1)$ symmetry is spontaneously broken. This creates a new additional pseudoscalar Goldstone boson, which is known as the axion. Although Peccei-Quinn $U_{PQ}(1)$ symmetry solves the strong CP problem new difficulties arise. For example the axion, with an expected mass $m_a \geq 180 \text{ keV}/c^2$, could not be discovered so far and not understood domain-wall structures at large scales of the universe[29] appear.

The other type of favoured models came up in the last years[29]. They create the possibility to trace back the two apparently different CP violation effects, the cosmological baryon asymmetry and the low energy K_L decay into two charged pions, to the same source of CP nonconservation. In these models the CP symmetry is spontaneously broken at the grand unification scale and then transmitted down to the low energy sector either by fermion mixing effects or by radiative corrections[29]. In both cases it is easy to construct realistic models to satisfy the constraint $0 < |\theta| < 10^{-9}$. In these theories the magnitude of the observed baryon asymmetry and the ϵ parameter, which is a measurement for the CP violation in the $K^0, \bar{K}^0$ mixing, are linked together, since they find their origin in the same source of CP nonconservation. This is the real beauty of these models.

Experimental limits for GUT

Today, two types of measurements of $\mathcal{CP}$ violating parameters can essentially be distinguished. The measurements of the electric dipole moment of the neutron has already been mentioned, where electric dipole moments of elementary particles are in general sensitive probes of CP violating effects. The current experimental upper limit of the electric dipole moment of the neutron is $|d_n| < 1.1 \times 10^{-25} \text{ ecm}$ (95% CL)[9]. But with the experimental value of ϵ'/ϵ from $K_L \rightarrow 2\pi$, every $\Delta S=1$ theory of CP violation, such as the CKM version of the Standard Model, leads to a $|d_n| \approx 10^{-34} \text{ ecm}$ [30]. Therefore an observation of a non-zero electric dipole momentum with today's technology would indicate a $\mathcal{CP}$ violation source other than the CKM δ phase.

2.1 CP symmetry and the Standard Model



Apart from the measurements of electric dipole moments, the most interesting observables for $\mathcal{CP}$ violation are provided by the weak decay of $K^0, \bar{K}^0$ and $B^0, \bar{B}^0$, where $\mathcal{CP}$ violation in the neutral B system has not so far been measured. In the neutral kaon system the only evidence for $\mathcal{CP}$ violation are the parameters η_{+-}, η_{00} and the semileptonic decay charge asymmetry for K_L . The current best values[9] are listed below and will be carefully introduced in the next section.

$$|\eta_{00}| = \left| \frac{A(K_L \rightarrow \pi^0 \pi^0)}{A(K_S \rightarrow \pi^0 \pi^0)} \right| = (2.275 \pm 0.019) \times 10^{-3} \quad (2.13a)$$

$$|\eta_{+-}| = \left| \frac{A(K_L \rightarrow \pi^+ \pi^-)}{A(K_S \rightarrow \pi^+ \pi^-)} \right| = (2.285 \pm 0.019) \times 10^{-3} \quad (2.13b)$$

$$\left| \frac{\eta_{00}}{\eta_{+-}} \right| = 0.9956 \pm 0.0023 \quad (2.13c)$$

$$\epsilon'/\epsilon \approx \text{Re}(\epsilon'/\epsilon) = \frac{1}{3} \left(1 - \left| \frac{\eta_{00}}{\eta_{+-}} \right| \right) = (1.5 \pm 0.8) \times 10^{-3} \quad (2.13d)$$

$$\phi_{+-} = 43.7 \pm 0.6 \quad (\text{phase of } \eta_{+-}) \quad (2.13e)$$

$$\phi_{00} = 43.5 \pm 1.0 \quad (\text{phase of } \eta_{00}) \quad (2.13f)$$

$$\begin{aligned} \gamma(e) &= \left(\frac{\Gamma(K_L \rightarrow \pi^- e^+ \nu) - \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu})}{\Gamma(K_L \rightarrow \pi^- e^+ \nu) + \Gamma(K_L \rightarrow \pi^+ e^- \bar{\nu})} \right) \\ &= (0.333 \pm 0.014)\% \end{aligned} \quad (2.13g)$$

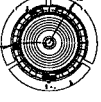
$$\begin{aligned} \gamma(\mu) &= \left(\frac{\Gamma(K_L \rightarrow \pi^- \mu^+ \nu) - \Gamma(K_L \rightarrow \pi^+ \mu^- \bar{\nu})}{\Gamma(K_L \rightarrow \pi^- \mu^+ \nu) + \Gamma(K_L \rightarrow \pi^+ \mu^- \bar{\nu})} \right) \\ &= (0.304 \pm 0.025)\% \end{aligned} \quad (2.13h)$$

$$\begin{aligned} \text{Re}(x) &= \text{Re} \left(\frac{A(\bar{K}^0 \rightarrow \pi^- \ell^+ \nu)}{A(K^0 \rightarrow \pi^- \ell^+ \nu)} \right) = \\ &= \text{Re} \left(\frac{A(\Delta S = -\Delta Q)}{A(\Delta S = \Delta Q)} \right) = 0.006 \pm 0.018 \end{aligned} \quad (2.13i)$$

$$\text{Im}(x) = -0.003 \pm 0.026 \quad (2.13j)$$

Problems, such as the cosmological baryon asymmetry and the strong CP problem, demand to be solved within the contents of Grand Unified Theories as extensions to our present Standard Model. Such an extension would be for example the Peccei-Quinn $U_{PQ}(1)$ symmetry. Only precise measurements of those CP violating observables which are accessible by experiments, are able to verify or reject the different existing models or at least to stem the incoming flood of new models. Up to now the neutral kaon system still allows the only unambiguous measurement of $\mathcal{CP}$ violating parameters, which shows the importance of their precise measurements.

This thesis focuses on the neutral kaon system, especially on the decay mode of the neutral kaon to $\pi^+ \pi^- \pi^0$. The determination of slope parameters and the decay amplitudes of $K_L \rightarrow \pi^+ \pi^- \pi^0$ allows an independent consistency check for the CPLEAR data sample and the CPLEAR data selection criteria by comparing the extracted values to theory and other experiments. Although we dispose of too few statistics to become sensitive to $\mathcal{CP}$ violation effects by using the Dalitz-plot analysis described later, this measurement of the slope parameters



creates an important basis for the remaining work done by the $\pi^+\pi^-\pi^0$ -group at CPLEAR, which measures e.g. the $\mathcal{CP}$ sensitive parameters $\text{Re}(\eta_{+-0})$ and $\text{Im}(\eta_{+-0})$. Also, our independent measurement of the complete set of slope parameters at an accuracy not reached so far can serve as test and new input to the Chiral perturbative theories, which provides the necessary conditions for a deeper understanding of the weak decays.

I like to end the introductory discussion by adding a quotation of the famous physicist M. Schwarzschild[31]: ‘If simple perfect laws uniquely rule the universe, should not pure thought be capable of uncovering this perfect set of laws without having to lean on the crutches of tediously assembled observations? True, the laws to be discovered may be perfect, but the human brain is not. Left on its own, it is prone to stray, as many past examples sadly prove. In fact, we have missed few chances to err until new data, freshly gleaned from nature, set us right again for the next steps.’

2.2 Phenomenology of the neutral kaon system

Discrete symmetries

In nature there are four pseudoscalar K mesons, namely the charged K^+ and K^- and the neutral K^0 and $\bar{K}^0$. They can be arranged into a complex two-component vector (K^+, K^0) which transforms as a doublet under isospin transformation, its isospin thus being $I=1/2$. Together with the complex conjugate doublet $(K^-, \bar{K}^0)$ they are each eigenvectors of the strangeness operator which possess by convention the eigenvalues:

$$\begin{aligned} \mathcal{S}|K^0\rangle &= |K^0\rangle & \mathcal{S}|\bar{K}^0\rangle &= -|\bar{K}^0\rangle \\ \mathcal{S}|K^+\rangle &= |K^+\rangle & \mathcal{S}|K^-\rangle &= -|K^-\rangle \end{aligned} \quad (2.14)$$

Under parity $\mathcal{P}$, charge conjugation $\mathcal{C}$ and time reversal $\mathcal{T}$ the pseudoscalar mesons show the transformations¹:

$$\begin{aligned} \mathcal{P}|K^0(\vec{p})\rangle &= e^{i\theta_P}|K^0(-\vec{p})\rangle & \mathcal{P}|\bar{K}^0(\vec{p})\rangle &= e^{-i\theta_P}|\bar{K}^0(-\vec{p})\rangle \\ \mathcal{C}|K^0(\vec{p})\rangle &= e^{i\theta_C}|\bar{K}^0(\vec{p})\rangle & \mathcal{C}|\bar{K}^0(\vec{p})\rangle &= e^{-i\theta_C}|K^0(\vec{p})\rangle \\ \mathcal{T}|K^0(-\vec{p})\rangle &= e^{i\theta_T}|\bar{K}^0(-\vec{p})\rangle & \mathcal{T}|\bar{K}^0(\vec{p})\rangle &= e^{-i\theta_T}|K^0(\vec{p})\rangle \end{aligned} \quad (2.15)$$

θ_P, θ_C and θ_T are arbitrary phases. Thus the combined transformation $\mathcal{CP}$ acts on the neutral meson states as follows:

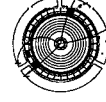
$$\mathcal{CP}|K^0(\vec{p})\rangle = e^{i\theta_{CP}}|\bar{K}^0(-\vec{p})\rangle \quad \mathcal{CP}|\bar{K}^0(\vec{p})\rangle = e^{-i\theta_{CP}}|K^0(-\vec{p})\rangle \quad (2.16)$$

The assumption that all commutations of the three operators $\mathcal{P}$, $\mathcal{C}$ and $\mathcal{T}$, i.e. the six operators $\mathcal{TCP}$, $\mathcal{TPC}$, ..., $\mathcal{PCT}$ commute equally with the Hamiltonian H puts a condition on the phases θ_P , θ_C and θ_T :

$$e^{i\theta_P} = e^{i\theta_C} e^{i\theta_T} = \pm 1 \quad (2.17)$$

¹It is worth mentioning that most of the formalism introduced in this and the next sections holds equally well to $K^0, \bar{K}^0, B^0, \bar{B}^0$ and $D^0, \bar{D}^0$, although I will only use the kaon state vectors in the equations and only the particularities of the kaon system are elucidated in this context.

2.2 Phenomenology of the neutral kaon system



Since the commutation relation above is required by the $\mathcal{CPT}$ theorem, it is a necessary condition for $\mathcal{CPT}$ conservation that, if $\mathcal{C}$, $\mathcal{P}$ or $\mathcal{T}$ is not conserved, at least one other must also not be conserved or all three not conserved separately. According to the conventional transformation properties of the quark fields under $\mathcal{C}$ and $\mathcal{P}$ in the Standard Model, conventionally the phase is chosen to be

$$\mathcal{CP}|K^0(\vec{p})\rangle = -|\bar{K}^0(-\vec{p})\rangle \quad \mathcal{CP}|\bar{K}^0(\vec{p})\rangle = -|K^0(-\vec{p})\rangle \quad (2.18)$$

and the $\mathcal{CP}$ eigenstates can be constructed as

$$\begin{aligned} |K_1\rangle &= \frac{1}{\sqrt{2}} [|K^0\rangle - |\bar{K}^0\rangle] & \mathcal{CP} &= 1 \\ |K_2\rangle &= \frac{1}{\sqrt{2}} [|K^0\rangle + |\bar{K}^0\rangle] & \mathcal{CP} &= -1 \end{aligned} \quad (2.19)$$

where the mass difference of $|K_1\rangle$ and $|K_2\rangle$ is induced by the strangeness violating component of the weak interaction. In the weak interaction isospin and strangeness are no longer conserved quantum numbers. Therefore K^0 and $\bar{K}^0$ do not have a definite lifetime and do not have definite mass. Instead, two independent linear combinations of the states $|K^0\rangle$ and $|\bar{K}^0\rangle$ exist, namely, $|K_S\rangle$ and $|K_L\rangle$, which are associated with the particles of definite and distinct mass m_S and m_L and the mean lifetimes Γ_S^{-1} and Γ_L^{-1} . If $\mathcal{CP}$ were an exact symmetry, $|K_S\rangle$ and $|K_L\rangle$ could be identified as $|K_1\rangle$ and $|K_2\rangle$.

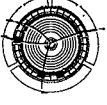
Two pion final states $\pi^+\pi^-$ and $\pi^0\pi^0$ are even under the $\mathcal{CP}$ operator irrespective of isospin considerations. The three neutral pion final state $\pi^0\pi^0\pi^0$ is $\mathcal{CP}$ odd. The final state $\pi^+\pi^-\pi^0$ can only be $\mathcal{CP}$ even if the $\pi^+\pi^-$ subsystem possesses an odd angular momentum. Since the decay amplitude of the $|K_1\rangle$ into three pion states with $\ell \neq 0$ is kinematically suppressed, we expect the $|K_1\rangle$ to decay mainly into two pion final states. In fact the experimental values are for the $|K_S\rangle$ decay a fraction $\Gamma_{\pi^+\pi^-}/\Gamma_{total} = 68.61 \pm 0.28\%$ and $\Gamma_{\pi^0\pi^0}/\Gamma_{total} = 31.39 \pm 0.28\%$ [9]. All other decay modes are negligible.

Considering the $|K_2\rangle$ decay modes it was in 1964, when Cronin et al.[3] observed that, besides the fully allowed decay channels $\Gamma_{\pi^0\pi^0\pi^0}/\Gamma_{total} = 21.12 \pm 0.27\%$ and $\Gamma_{\pi^+\pi^-\pi^0}/\Gamma_{total} = 12.56 \pm 0.20\%$, where the latter is predominantly an eigenstate of the $\mathcal{CP}$ operator with eigenvalue $+1$, there is a small but finite probability for the $\mathcal{CP}$ forbidden decay $K_L \rightarrow \pi^+\pi^-$. The branching ratio of $K_L \rightarrow \pi^+\pi^-$ is found to be $\Gamma_{\pi^+\pi^-}/\Gamma_{total} \simeq (2.067 \pm 0.035) \times 10^{-3}$. Therefore $|K_2\rangle$ can not be associated with $|K_L\rangle$ and the $\mathcal{CP}$ symmetry has to be slightly broken in nature.

Because of phase-space volume consideration K_L is expected to have a much smaller decay rate than K_S . This is indeed the case as experiments show and explains the name K_L for the long-lived meson and K_S for the short-lived meson.

CP violation in the mixing

In the following it is assumed that for the strong and electromagnetic interaction $\mathcal{C}$, $\mathcal{P}$ and $\mathcal{T}$ invariance holds separately. In general, the neutral mesons $K^0, \bar{K}^0$, $B^0, \bar{B}^0$ and $D^0, \bar{D}^0$, which are eigenstates of the strong and electromagnetic interaction, mix under the weak interaction via common decay channels. In the



case of neutral kaons the presence of the weak interaction induces strangeness changing decays $|\Delta S| = 1$ and they can oscillate into each other through $|\Delta S| = 2$ processes over e.g. the $\mathcal{C}$ even two pion final states:

$$|K^0\rangle \rightleftharpoons \pi\pi \rightleftharpoons |\bar{K}^0\rangle \quad (2.20)$$

Thus we can consider the neutral kaon state as a superposition of the flavour eigenstates

$$\Psi(t) = a(t)|K^0\rangle + b(t)|\bar{K}^0\rangle \quad (2.21)$$

By assuming the validity of the Wigner-Weisskopf[32][33] type of treatment, the time evolution of the neutral kaon system in its rest frame is described by the perturbation theory up to the second order of the weak interaction with the time-dependent Schrödinger equation

$$i\frac{\partial}{\partial t}\Psi(t) = \mathcal{H}\Psi(t) \quad (2.22)$$

where the effective Hamiltonian is $\mathcal{H} = H_0 + H_W$. H_0 is the Hamiltonian of the strong and electromagnetic interaction and H_W the Hamiltonian of the weak interaction. In this approximation $\mathcal{H}$ is represented by a non hermitian 2×2 mass matrix Λ , which can be decomposed into a hermitian mass matrix $\mathbf{M}$ and a hermitian decay matrix Γ :

$$\Lambda = \mathbf{M} - \frac{i}{2}\Gamma \quad (2.23)$$

Using perturbation theory[33][34] we get the matrix elements of $\mathbf{M}$ and Γ in the form

$$\begin{aligned} M_{ij} &= m_0\delta_{ij} + \langle i|\mathbf{H}_W|j\rangle + \mathcal{P} \sum_f \left(\frac{\langle i|\mathbf{H}_W|f\rangle\langle f|\mathbf{H}_W|j\rangle}{m_0 - E_f} \right) \\ \Gamma_{ij} &= 2\pi \sum_f \langle i|\mathbf{H}_W|f\rangle\langle f|\mathbf{H}_W|j\rangle \delta(m_0 - E_f) \end{aligned} \quad (2.24)$$

$\mathcal{P}$ stands for the Cauchy principal value and m_0 is the rest mass of the K^0 . The second term of M_{ij} denotes $\Delta S = 2$ transitions, whereas the Hamiltonian in the third term stands for $\Delta S = 1$ transitions. The Hamiltonians of Γ_{ij} both are $\Delta S = 1$ transitions. For M_{ij} the sum index f runs over all possible intermediate states, whereas for Γ_{ij} f denotes all final states. $K^0, \bar{K}^0$ are substituted by i, j . Due to the hermiticity of $\mathbf{M}$ and Γ we get

$$M_{11}, M_{22}, \Gamma_{11}, \Gamma_{22} \text{ real} \quad (2.25)$$

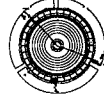
and

$$M_{12} = M_{21}^*, \Gamma_{12} = \Gamma_{21}^* \quad (2.26)$$

If we assume additionally $\mathcal{CPT}$ invariance, $H_W = (\mathcal{CPT})^{-1}H_W(\mathcal{CPT})$

$$M_{11} = M_{22}, \Gamma_{11} = \Gamma_{22} \quad (2.27)$$

2.2 Phenomenology of the neutral kaon system



and further for invariance under time-inversion $\mathcal{T}$, $H_W = (\mathcal{T})^{-1}H_W(\mathcal{T})$

$$M_{12} = M_{21}, \Gamma_{12} = \Gamma_{21} \quad \text{real} \quad (2.28)$$

which means that the mass eigenstates become equal to the $\mathcal{CP}$ eigenstates and Λ becomes hermitian.

Equation(2.23) can be solved and we get the time dependence of the real physical particles, which are the eigenstates $|K_S\rangle$ and $|K_L\rangle$ of the effective Hamiltonian $\mathcal{H}$

$$|\Psi(t)\rangle = a_S e^{-i\lambda_S t} |K_S\rangle + a_L e^{-i\lambda_L t} |K_L\rangle \quad (2.29)$$

where t is the time since production in the kaon rest frame and λ_S , λ_L the eigenvalues.

The K_S and K_L states show the exponential time dependence as required by the Wigner-Weisskopf approximation. Since the Hamiltonian $\mathcal{H}$ is not hermitian, its eigenvectors

$$\begin{aligned} |K_S\rangle &= \frac{1}{\sqrt{|p_S|^2 + |p_L|^2}} (p_S |K^0\rangle - q_S |\bar{K}^0\rangle) \\ |K_L\rangle &= \frac{1}{\sqrt{|p_S|^2 + |p_L|^2}} (p_L |K^0\rangle + q_S |\bar{K}^0\rangle) \end{aligned} \quad (2.30)$$

with

$$\frac{q_S}{p_S} = \frac{\Lambda_{21}}{\Lambda_{22} - \lambda_S} \quad \text{and} \quad \frac{q_L}{p_L} = \frac{\Lambda_{21}}{\Lambda_{22} - \lambda_L} \quad (2.31)$$

are not orthogonal, $\langle K_L | K_S \rangle \neq 0$, and the eigenvalues

$$\lambda_S = m_S - \frac{i}{2} \Gamma_S \quad (2.32a)$$

$$= \frac{1}{2} (\Lambda_{11} + \Lambda_{22}) - y$$

$$\lambda_L = m_L - \frac{i}{2} \Gamma_L \quad (2.32b)$$

$$= \frac{1}{2} (\Lambda_{11} + \Lambda_{22}) + y$$

with

$$y = \sqrt{\frac{1}{4} (\Lambda_{11} - \Lambda_{22})^2 + \Lambda_{21} \Lambda_{12}} \quad (2.33)$$

are complex. Their masses m_S and m_L and their mean lifetimes Γ_S^{-1} and Γ_L^{-1} are represented by the real and imaginary part of the eigenvalues λ_S and λ_L . With (2.32) we can write the differences of the masses and mean lifetimes of $|K_S\rangle$ and $|K_L\rangle$ as

$$\Delta m = m_L - m_S = 2\text{Re}(y) \quad (2.34a)$$

$$\Delta \Gamma = \Gamma_L - \Gamma_S = -4\text{Im}(y) \quad (2.34b)$$

It is an interesting experimental fact that for the neutral kaon system

$$\Delta \Gamma \simeq -2\Delta m \quad (2.35)$$



An alternative common notation is to define

$$p_{S,L} = \frac{1 - \epsilon_{S,L}}{\sqrt{2(1 + |\epsilon_{S,L}|)}}, \quad q_{S,L} = \frac{1 + \epsilon_{S,L}}{\sqrt{2(1 + |\epsilon_{S,L}|)}} \quad (2.36)$$

and ($|\epsilon_{S,L}| \ll 1$):

$$\frac{q_{S,L}}{p_{S,L}} = \frac{1 - \epsilon_{S,L}}{1 + \epsilon_{S,L}} \quad (2.37)$$

In the following we will use the fact that $|\epsilon_{S,L}| \ll 1$ and neglect terms which are proportional to the square of $\mathcal{T}$ or $\mathcal{CP}\mathcal{T}$ violating quantities. ϵ_S and ϵ_L can be taken as a measurement for the fraction of the $\mathcal{CP}$ -not-allowed $\mathcal{CP}$ eigenstate in $|K_S\rangle$ and $|K_L\rangle$, respectively.

$$|K_S\rangle = |K_1\rangle + \epsilon_S |K_2\rangle \quad (2.38a)$$

$$|K_L\rangle = |K_2\rangle + \epsilon_L |K_1\rangle \quad (2.38b)$$

In terms of ϵ_S and ϵ_L eq. 2.30 become

$$|K_S\rangle = \frac{1}{\sqrt{2}} \left((1 + \epsilon_S) |K^0\rangle - (1 - \epsilon_S) |\bar{K}^0\rangle \right) \quad (2.39a)$$

$$|K_L\rangle = \frac{1}{\sqrt{2}} \left((1 + \epsilon_L) |K^0\rangle + (1 - \epsilon_L) |\bar{K}^0\rangle \right) \quad (2.39b)$$

and the scalar product is

$$\langle K_S | K_L \rangle = \epsilon_S^* + \epsilon_L \quad (2.40)$$

Introducing a phase convention according to [35] ϵ_S and ϵ_L can be split into the two parameters ϵ and $\delta_{\mathcal{CP}\mathcal{T}}$

$$\epsilon_S = \epsilon_T + \delta_{\mathcal{CP}\mathcal{T}} \quad (2.41a)$$

$$\epsilon_L = \epsilon_T - \delta_{\mathcal{CP}\mathcal{T}} \quad (2.41b)$$

The formalism so far is not a consequence of the $\mathcal{CP}\mathcal{T}$ theorem. If we now assume $\mathcal{CP}\mathcal{T}$ invariance we get $q_S = q_L$ and $p_S = p_L$ and the ratio(2.31) can be written as

$$\left| \frac{q}{p} \right|^2 = \left| \frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}} \right| \quad (2.42)$$

Equation(2.42) is independent of phase-conventions and therefore physically meaningful. The validity of equation 2.43

$$\left| \frac{q}{p} \right| \neq 1 \iff \mathcal{CP} \text{ violation in the mixing} \quad (2.43)$$

would be a proof for $\mathcal{CP}$ violation in the mixing. Further under $\mathcal{CP}\mathcal{T}$ invariance we can write now equation 2.41 as

$$\delta_{\mathcal{CP}\mathcal{T}} = 0 \quad (2.44a)$$

$$\epsilon_S = \epsilon_L = \epsilon_T \quad (2.44b)$$

2.2 Phenomenology of the neutral kaon system



and

$$\langle K_S | K_L \rangle = 2\text{Re}(\epsilon) \quad (2.45)$$

where as under $\mathcal{T}$ invariance

$$\epsilon_T = 0 \quad (2.46a)$$

$$\epsilon_S = -\epsilon_L = \delta_{\mathcal{CP}\mathcal{T}} \quad (2.46b)$$

and

$$\langle K_S | K_L \rangle = 2\text{Im}(\delta_{\mathcal{CP}\mathcal{T}}) \quad (2.47)$$

Since $\mathcal{CP}\mathcal{T}$ noninvariance and/or $\mathcal{T}$ noninvariance implies $\mathcal{CP}$ noninvariance, no further parameters have to be introduced.

For the rest of this work, $\mathcal{CP}\mathcal{T}$ invariance is assumed, i.e. $\delta_{\mathcal{CP}\mathcal{T}}=0$.

Due to equation 2.29 the time evolution of initially pure K^0 and $\bar{K}^0$ can be written as

$$|\Psi(t)\rangle = \frac{1 - \epsilon_T}{\sqrt{2}} \left(e^{-i\lambda_S t} |K_S\rangle + e^{-i\lambda_L t} |K_L\rangle \right) \quad (2.48)$$

$$|\bar{\Psi}(t)\rangle = \frac{1 + \epsilon_T}{\sqrt{2}} \left(e^{-i\lambda_S t} |K_S\rangle - e^{-i\lambda_L t} |K_L\rangle \right) \quad (2.49)$$

As mentioned, initially pure K^0 and $\bar{K}^0$ states oscillate (eq. 2.20) over virtual intermediate states into each other. These strangeness oscillations with an oscillation frequency Δm are clearly visible before they are dumped away by the decay of the K_S component. Fig. 2.1 shows the strangeness fraction $\mathcal{S}(K^0(t))$ and the anti-strangeness fraction $\bar{\mathcal{S}}(K^0(t))$ of an initially pure K^0 beam. $\mathcal{S}(K^0(t))$ and $\bar{\mathcal{S}}(K^0(t))$ are defined as

$$\mathcal{S}(K^0, t) = \frac{\langle \bar{K}^0 | \Psi \rangle}{\langle \bar{K}^0 | \Psi \rangle + \langle K^0 | \bar{\Psi} \rangle} \quad (2.50)$$

$$\bar{\mathcal{S}}(K^0, t) = \frac{\langle K^0 | \bar{\Psi} \rangle}{\langle \bar{K}^0 | \Psi \rangle + \langle K^0 | \bar{\Psi} \rangle} \quad (2.51)$$

If we take the small value for ϵ derived from experiments into account, we can write

$$\mathcal{S}(K^0(t)) \simeq \frac{1}{2}(1 - \text{Re}(\epsilon)) \frac{1 - f(t)}{1 - 2\text{Re}(\epsilon)f(t)} \quad (2.52)$$

$$\bar{\mathcal{S}}(K^0(t)) \simeq \frac{1}{2}(1 + \text{Re}(\epsilon)) \frac{1 + f(t)}{1 + 2\text{Re}(\epsilon)f(t)} \quad (2.53)$$

with

$$f(t) = \frac{2e^{-\Delta\Gamma t} \cos(\Delta m t)}{e^{-\Gamma_S t} + e^{-\Gamma_L t}} \quad (2.54)$$

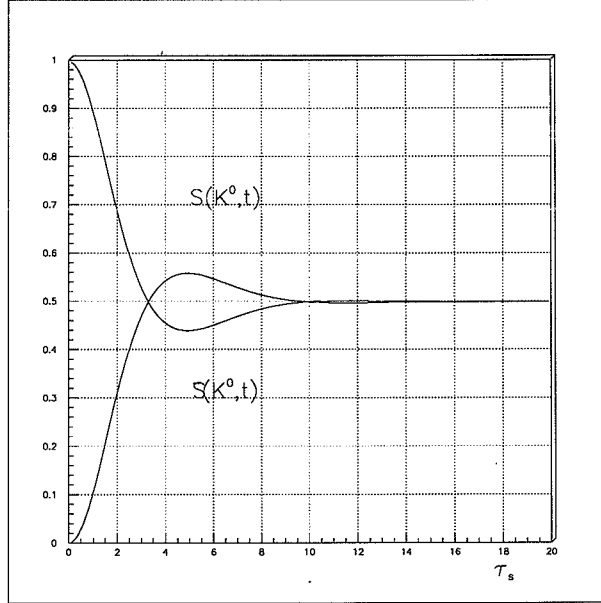
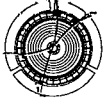


Figure 2.1: *Strangeness oscillations for a initially pure K^0 beam. The strangeness and anti-strangeness fractions $S(K^0(t))$ and $\bar{S}(K^0(t))$ are plotted against the decay time.*

Since $\mathcal{CP}$ is violated, $S(K^0(t))$ and $\bar{S}(K^0(t))$ do not oscillate around the value 0.5, but instead around $0.5 - \langle K_S | K_L \rangle$ for K^0 and $0.5 + \langle K_S | K_L \rangle$ for $\bar{K}^0$. Responsible for this is the fact that the transmission probability of K^0 to $\bar{K}^0$ and $\bar{K}^0$ to K^0 differ slightly as a result of the violated $\mathcal{T}$ symmetry. This fact is also known as violated microreversability.

CP violation in the decay

Two decay processes shall be related to each other by $\mathcal{CP}$ transformation. $|M\rangle$ and $|\bar{M}\rangle$ shall be the conjugated pseudoscalar meson states, and $|f\rangle$ and $|\bar{f}\rangle$ some $\mathcal{CP}$ conjugated final states both with the freedom of choosing a phase

$$\begin{aligned}\mathcal{CP}|M\rangle &= e^{i\varphi_M} |\bar{M}\rangle \\ \mathcal{CP}|f\rangle &= e^{i\varphi_f} |\bar{f}\rangle\end{aligned}\tag{2.55}$$

Applying a phase convention the $\mathcal{CP}$ conjugated decay amplitudes A and $\bar{A}$ are given as:

$$\begin{aligned}A &= \langle f | \mathcal{H} | M \rangle = \sum_i A_i e^{i\delta_i} e^{i\phi_i} \\ \bar{A} &= \langle \bar{f} | \mathcal{H} | \bar{M} \rangle = e^{i(\varphi_M - \varphi_f)} \sum_i A_i e^{i\delta_i} e^{-i\phi_i}\end{aligned}\tag{2.56}$$

A_i are the real partial amplitudes, which are described by different Feynman diagram contributions, and $\mathcal{H}$ is the effective Hamiltonian. There are two additional phases in the decay amplitudes. The weak phases ϕ_i are introduced by

2.2 Phenomenology of the neutral kaon system



the Lagrangian which violates $\mathcal{CP}$, whereas the strong phases δ_i from rescattering effects due to the strong interactions of the final states. Although the weak phases ϕ_i and the strong phases δ_i , which are convention dependent, appear one can show that the ratio

$$\left| \frac{\bar{A}}{A} \right| = \left| \frac{\sum_i A_i e^{i\delta_i} e^{i\phi_i}}{\sum_i A_i e^{i\delta_i} e^{-i\phi_i}} \right| \quad (2.57)$$

is phase convention independent and therefore physical meaningful. The condition

$$\left| \frac{\bar{A}}{A} \right| \neq 1 \iff \text{direct } \mathcal{CP} \text{ violation} \quad (2.58)$$

implies $\mathcal{CP}$ violation resulting from the interference of decay amplitudes leading to the same final states.

The processes $|K^\pm\rangle \rightarrow |\pi^\pm\pi^0\rangle$ may serve as example for the above. It is common to define the $\mathcal{CP}$ asymmetry

$$\mathcal{A}_{\pi^\pm\pi^0} = \frac{\Gamma(K^+ \rightarrow \pi^+\pi^0) - \Gamma(K^- \rightarrow \pi^-\pi^0)}{\Gamma(K^+ \rightarrow \pi^+\pi^0) + \Gamma(K^- \rightarrow \pi^-\pi^0)} = \frac{1 - |\bar{A}/A|^2}{1 + |\bar{A}/A|^2} \quad (2.59)$$

Therefore a value $\mathcal{A}_{\pi^\pm\pi^0} \neq 1$ would indicate $\mathcal{CP}$ violation in the decay for these processes.

The 2π channel

In the neutral meson decays, where the presence of mixing is unavoidable, the access to the $\mathcal{CP}$ violating parameters of the decay is not so straightforward. In this section the parameter ϵ' , which indicates $\mathcal{CP}$ violation in decay of the neutral kaons, is carefully introduced by discussing the two pionic final state $\pi^+\pi^-$ and $\pi^0\pi^0$.

Since the neutral kaons as well as the pions are spinless particles, the two pionic final state can occur just with $\mathcal{CP} = +1$, i.e. the observation of the processes $K_L \rightarrow \pi^+\pi^-$ and $K_L \rightarrow \pi^0\pi^0$ is a clear indication for $\mathcal{CP}$ violation either in the mixing with $\epsilon \neq 0$ or in the decay or in both. The isospin operator commutes with the Hamiltonian of the strong interaction and therefore simultaneous eigenstates exist for the different possible isospin values. The isospin decomposition of the two pionic final states into $I = 0$ and $I = 2$ states can be written as

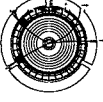
$$|\pi^+\pi^-\rangle = \sqrt{\frac{2}{3}}|\pi^+\pi^-, I=0\rangle + \sqrt{\frac{1}{3}}|\pi^+\pi^-, I=2\rangle \quad (2.60a)$$

$$|\pi^0\pi^0\rangle = \sqrt{\frac{1}{3}}|\pi^0\pi^0, I=0\rangle - \sqrt{\frac{2}{3}}|\pi^0\pi^0, I=2\rangle \quad (2.60b)$$

We define

$$\eta_{+-} = |\eta_{+-}|e^{i\Phi_{+-}} = \frac{\langle \pi^+\pi^- | \mathcal{H} | K_L \rangle}{\langle \pi^+\pi^- | \mathcal{H} | K_S \rangle} \quad (2.61a)$$

$$\eta_{00} = |\eta_{00}|e^{i\Phi_{00}} = \frac{\langle \pi^0\pi^0 | \mathcal{H} | K_L \rangle}{\langle \pi^0\pi^0 | \mathcal{H} | K_S \rangle} \quad (2.61b)$$



η_{+-} and η_{00} can be composed of the two isospin contributions $I = 0$ and $I = 2$ (eq. 2.60). To parameterize the $\mathcal{CP}$ violation in the isospin amplitude $I = 0$ and $I = 2$ we introduce the new parameters ϵ_0 and ϵ_2 as

$$\epsilon_0 = \frac{A(K_L \rightarrow \pi\pi, I=0)}{A(K_S \rightarrow \pi\pi, I=0)} \quad (2.62a)$$

$$\epsilon_2 = \frac{1}{\sqrt{2}} \frac{A(K_L \rightarrow \pi\pi, I=2)}{A(K_S \rightarrow \pi\pi, I=0)} \quad (2.62b)$$

Both decay channels are normalized to the $\pi\pi$ decay channel with $I=0$. The K_S channel with $I=0$ is due to the $\Delta I=1/2$ rule dominant. The $I=2$ decay channel with $\Delta I = 3/2$ is suppressed by the factor[36]

$$|\omega| = \left| \frac{A(K_S \rightarrow \pi\pi, I=2)}{A(K_S \rightarrow \pi\pi, I=0)} \right| \simeq 1/25 \quad (2.63)$$

which is a measure of the $\Delta I=1/2$ rule. With the eq. 2.60 we can write

$$\eta_{+-} = \frac{\epsilon_0 + \epsilon_2}{1 + \omega/\sqrt{2}} \quad (2.64a)$$

$$\eta_{00} = \frac{\epsilon_0 + 2\epsilon_2}{1 - \sqrt{2}\omega} \quad (2.64b)$$

A measure of the different strength of $\mathcal{CP}$ violation in the channels $I=0$ and $I=2$ is the new variable

$$\epsilon' = \frac{\omega}{\sqrt{2}} \left(\sqrt{2} \frac{\epsilon_2}{\omega} - \epsilon_0 \right) \quad (2.65)$$

with

$$\eta_{+-} = \epsilon_0 + \frac{\epsilon'}{1 + \omega/\sqrt{2}} \quad (2.66a)$$

$$\eta_{00} = \epsilon_0 - \frac{2\epsilon'}{1 - \sqrt{2}\omega} \quad (2.66b)$$

Applying the common Wu-Yang phase convention[37], the dominant decay amplitude $A(K_L \rightarrow \pi\pi, I=0)$ is chosen to be real ($\theta_0 = 0$) and therefore $\mathcal{CP}$ invariant. This is coterminous with fixing the relative phase of $|K^0\rangle$ and $|\bar{K}^0\rangle$ [38] to $\mathcal{CP}|K^0\rangle = |\bar{K}^0\rangle$ in contrast to eq. 2.18. In the Wu-Yang phase convention we get $\epsilon_0 = \epsilon$ and eq. 2.66 becomes

$$\eta_{+-} = \epsilon + \frac{\epsilon'}{1 + \omega/\sqrt{2}} \simeq \epsilon + \epsilon' \quad (2.67a)$$

$$\eta_{00} = \epsilon - \frac{2\epsilon'}{1 - \sqrt{2}\omega} \simeq \epsilon - 2\epsilon' \quad (2.67b)$$

2.34 and 2.40 become

$$\Delta m = 2\text{Re}(M_{12}) \quad (2.68a)$$

$$\Delta\Gamma = 2\text{Re}(\Gamma_{12}) \quad (2.68b)$$

$$\langle K_S | K_L \rangle = 2\text{Re}(\epsilon) \quad (2.68c)$$

2.2 Phenomenology of the neutral kaon system



with

$$\epsilon = -\frac{1}{2} \frac{\Lambda_{12} - \Lambda_{21}}{\Delta m - \frac{i}{2} \Delta \Gamma} \quad (2.69)$$

ϵ' can now be written explicitly as

$$\epsilon' = \frac{i}{\sqrt{2}} \frac{\text{Im}(A_2)}{A_0} e^{i\delta_2 - \delta_0} \quad (2.70)$$

Unlike the ϵ , which depends on the hermitian mass matrix M as well as on the decay matrix Γ , ϵ' depends only on the decay matrix Γ and thus is finally the searched measure for direct $\mathcal{CP}$ violation. As we will see ϵ' can also describe the rate difference of $K^0 \rightarrow \pi^+ \pi^-$ and $\bar{K}^0 \rightarrow \pi^+ \pi^-$. Finally combining the eq. 2.67 we derive

$$\text{Re}\left(\frac{\epsilon'}{\epsilon}\right) \simeq \frac{1}{3} \left(1 - \left|\frac{\eta_{00}}{\eta_{+-}}\right|\right) \quad (2.71)$$

For the experimental approach we define the decay rates in the neutral kaon system according to the Schroedinger equation(2.22) as

$$R_f(t) \sim |\langle f | T | \Psi(t) \rangle|^2 \quad (2.72a)$$

$$\bar{R}_f(t) \sim |\langle f | T | \bar{\Psi}(t) \rangle|^2 \quad (2.72b)$$

T is the transition operator in the specific final state $|f\rangle$. With [39] and eq. 2.67 and 2.68 the decay rates can be written as

$$R_f(t) \sim \frac{|A_S|^2}{1 + 2\text{Re}(\epsilon)} \left[e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} + 2|\eta_{+-}| e^{-\bar{\Gamma} t} \cos(\Delta m t - \Phi_{+-}) \right] \quad (2.73a)$$

$$\bar{R}_f(t) \sim \frac{|A_S|^2}{1 - 2\text{Re}(\epsilon)} \left[e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} - 2|\eta_{+-}| e^{-\bar{\Gamma} t} \cos(\Delta m t - \Phi_{+-}) \right] \quad (2.73b)$$

with $\bar{\Gamma}(\Gamma_L + \Gamma_S)/2$. Thus the decay rates decrease exponentially in time with Γ_S and Γ_L and possess an exponentially decreasing interference term of the frequency Δm . In the neutral kaon system these interference terms can be distinctively seen due to the relation 2.35. $\mathcal{CP}$ violation becomes apparent with this interference term, which is known as the Seghal-Wolfenstein theorem[40]:

For any possible nonleptonic decay mode of the K^0 meson the observation of an interference effect between K_L and K_S in the partial decay rate of this mode is a clear evidence for $\mathcal{CP}$ violation.

We define particle-antiparticle asymmetries as

$$\mathcal{A}_f(t) = \frac{\bar{R}_f(t) - R_f(t)}{\bar{R}_f(t) + R_f(t)} \quad (2.74)$$

With equation 2.73 2.74 takes the form

$$\mathcal{A}_{+-}(t) = 2\text{Re}(\epsilon) + \frac{2e^{-\bar{\Gamma} t} |\eta_{+-}| \cos(\Delta m t - \Phi_{+-})}{e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t}} \quad (2.75)$$

Under the assumption of $\mathcal{CP}$ invariance the asymmetry vanishes.

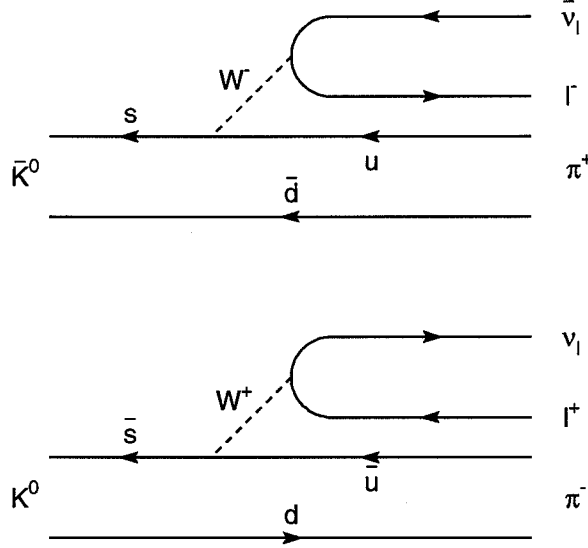


Figure 2.2: Feynman diagram contributions of the semileptonic decay channels

The semileptonic decays

Before passing to a careful discussion of the neutral kaon decay to the $\pi^+\pi^-\pi^0$ final state a section about the semileptonic decay channels is applied here to complete the different channels studied at the CPLEAR experiment. The semileptonic decay channels $\pi^+l^-\bar{\nu}_l$ and $\pi^-l^+\nu_l$ (fig. 2.2) deserve a special care and attention in the neutral kaon system, since semileptonic final states are not eigenstates of the $\mathcal{CP}$ operator. However, in these channels it is possible to observe the strangeness of the decaying kaon via the $\Delta S = \Delta Q$ rule,

$$\left. \begin{aligned} A_+ &= A(K^0 \rightarrow \pi^- l^+ \nu_l) \\ \bar{A}_- &= A(\bar{K}^0 \rightarrow \pi^+ l^- \bar{\nu}_l) \end{aligned} \right\} \quad \Delta S = \Delta Q \quad \text{allowed} \quad (2.76a)$$

$$\left. \begin{aligned} A_- &= A(K^0 \rightarrow \pi^+ l^- \bar{\nu}_l) \\ \bar{A}_+ &= A(\bar{K}^0 \rightarrow \pi^- l^+ \nu_l) \end{aligned} \right\} \quad \Delta S = -\Delta Q \quad \text{forbidden} \quad (2.76b)$$

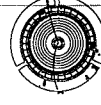
which allows $\Delta S = \Delta Q$ decays and forbids $\Delta S = -\Delta Q$ decays. Assuming that the $\Delta S = \Delta Q$ rule, is not violated it is possible to directly compare $|\langle K^0 | \mathcal{T} | \bar{K}^0 \rangle|^2$ with $|\langle \bar{K}^0 | \mathcal{T} | K^0 \rangle|^2$ or $|\langle K^0 | \mathcal{T} | K^0 \rangle|^2$ with $|\langle \bar{K}^0 | \mathcal{T} | \bar{K}^0 \rangle|^2$. In the first case we measure $\mathcal{T}$ noninvariance, in the second case we measure $\mathcal{CPT}$ noninvariance.

To consider a possible violation of the $\Delta S = \Delta Q$ rule we define the parameter x as

$$x = \frac{\int \int d\Omega A_+^* \bar{A}_+}{\int \int d\Omega |A_+|^2} = \frac{\int \int d\Omega A_-^* \bar{A}_-}{\int \int d\Omega |\bar{A}_-|^2} \quad (2.77)$$

where $\mathcal{CPT}$ invariance is assumed, i.e. $A_+ = -\bar{A}_-^*$ and $A_- = -\bar{A}_+^*$. The enumerator describes the forbidden $\Delta S = -\Delta Q$ decays, whereas the denominator

2.3 The decay $K_L, K_S \rightarrow \pi^+ \pi^- \pi^0$



describes the $\Delta S = \Delta Q$ decays. Applying the $\mathcal{CP}$ operator to the states we find as condition for $\mathcal{CP}$ invariance $A_+ = -\bar{A}_- e^{i\phi}$ and $A_- = -\bar{A}_+ e^{-i\phi}$ with ϕ being the relative phase between K^0 and $\bar{K}^0$. Direct $\mathcal{CP}$ violation induces a violation of the $\Delta S = \Delta Q$ rule through a value $\text{Im}(x) \neq 0$. A violation of the $\Delta S = \Delta Q$ rule independent of $\mathcal{CP}$ violation would be manifested in $\text{Re}(x) \neq 0$.

To obtain experimental access to the parameters above, we define the asymmetry $\mathcal{A}_{\Delta m}(t)$ as

$$\begin{aligned} \mathcal{A}_{\Delta m}(t) &= \frac{(R_-(t) + \bar{R}_+(t)) - (R_+(t) + \bar{R}_-(t))}{(R_-(t) + \bar{R}_+(t)) + (R_+(t) + \bar{R}_-(t))} \\ &= \frac{2e^{-\bar{\Gamma}t} \cos(\Delta mt)}{(1 - 2\text{Re}(x))e^{-\Gamma_L t} + (1 + 2\text{Re}(x))e^{-\Gamma_S t}} \end{aligned} \quad (2.78)$$

This asymmetry compares the $\Delta S = \Delta Q$ forbidden to the allowed semileptonic decays of K^0 and $\bar{K}^0$ and is sensitive to Δm and $\text{Re}(x)$.

The asymmetry $\mathcal{A}_x(t)$

$$\begin{aligned} \mathcal{A}_x(t) &= \frac{\bar{R}_-(t) + R_+(t)}{\bar{R}_-(t) + R_+(t)} \\ &= \frac{4\text{Im}(x)e^{-\bar{\Gamma}t} \sin(\Delta mt)}{(1 - 2\text{Re}(x))e^{-\Gamma_L t} + (1 + 2\text{Re}(x))e^{-\Gamma_S t} + 2e^{-\bar{\Gamma}t} \cos(\Delta mt)} \end{aligned} \quad (2.79)$$

compares the $\Delta S = \Delta Q$ allowed semileptonic decays of K^0 and $\bar{K}^0$ and is sensitive to $\text{Im}(x)$.

The asymmetry $\mathcal{A}_T(t)$ finally takes the form

$$\begin{aligned} \mathcal{A}_T(t) &= \frac{\bar{R}_-(t) + R_+(t)}{\bar{R}_-(t) + R_+(t)} \\ &= \frac{4\text{Re}(\epsilon)(e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2e^{-\bar{\Gamma}t} \cos(\Delta mt))}{(1 + 2\text{Re}(x))e^{-\Gamma_S t} + (1 - 2\text{Re}(x))e^{-\Gamma_L t} - 2e^{-\bar{\Gamma}t} \cos(\Delta mt)} \end{aligned} \quad (2.80)$$

which is sensitive to $\text{Re}(\epsilon)$. Therefore this asymmetry measures $\mathcal{T}$ violation in these channels.

2.3 The decay $K_L, K_S \rightarrow \pi^+ \pi^- \pi^0$

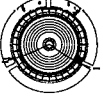
Considering the neutral kaon decay into the states $|\pi^0 \pi^0 \pi^0\rangle$ and $|\pi^+ \pi^- \pi^0\rangle$, it is in principle possible to choose the analogous formalism as in the 2π case. Just for completeness the $|\pi^0 \pi^0 \pi^0\rangle$ state is also introduced in this context. The $\mathcal{CP}$ operator acts on the states as follows

$$\mathcal{CP}|\pi^0 \pi^0 \pi^0\rangle = (-1)|\pi^0 \pi^0 \pi^0\rangle \quad (2.81a)$$

$$\mathcal{CP}|\pi^+ \pi^- \pi^0\rangle = (-1)^{\ell+1}|\pi^+ \pi^- \pi^0\rangle \quad (2.81b)$$

where as ℓ is the angular momentum of the $\pi^+ \pi^-$ subsystem. The isospin decomposition of the $|\pi^0 \pi^0 \pi^0\rangle$ state can be denoted as

$$|\pi^0 \pi^0 \pi^0\rangle = \sqrt{\frac{2}{5}}|\pi^0 \pi^0 \pi^0, I=3\rangle - \sqrt{\frac{3}{5}}|\pi^0 \pi^0 \pi^0, I=1\rangle \quad (2.82)$$



The decomposition is derived with regard to angular momentum conservation and bose symmetry by first coupling two of the pions to a state and then adding the third pion. For the state $|\pi^0\pi^0\pi^0\rangle$, which exists only with the $\mathcal{CP}$ eigenvalue (-1), the proof of the decay of $K_S \rightarrow |\pi^0\pi^0\pi^0\rangle$ would be a clear indication for $\mathcal{CP}$ violation.

As mentioned it is possible to use the same formalism as in the two pionic case. We define the parameter η_{000} , which is a measure for $\mathcal{CP}$ violation for $K^0, \bar{K}^0 \rightarrow \pi^0\pi^0\pi^0$, as

$$\eta_{000} = |\eta_{000}|e^{i\phi_{000}} = \frac{\langle \pi^0\pi^0\pi^0 | \mathcal{H} | K_S \rangle}{\langle \pi^0\pi^0\pi^0 | \mathcal{H} | K_L \rangle} = \frac{\int \int d\Omega A_{000}^S A_{000}^{L*}}{\int \int d\Omega |A_{000}^L|^2} \quad (2.83)$$

The decay rates of $K^0(\bar{K}^0) \rightarrow \pi^0\pi^0\pi^0$ are

$$R(t) \sim \frac{|A_S|}{1 + \langle K_S | K_L \rangle} \left(|\eta_{000}|^2 e^{-\Gamma_S t} + e^{-\Gamma_L t} + |\eta_{000}| e^{-\bar{\Gamma} t} \cos(\Delta m t + \phi_{000}) \right) \quad (2.84a)$$

$$\bar{R}(t) \sim \frac{|A_S|}{1 - \langle K_S | K_L \rangle} \left(|\eta_{000}|^2 e^{-\Gamma_S t} + e^{-\Gamma_L t} - |\eta_{000}| e^{-\bar{\Gamma} t} \cos(\Delta m t + \phi_{000}) \right) \quad (2.84b)$$

Thus the particle-antiparticle asymmetry of equation 2.74 takes for the $\pi^0\pi^0\pi^0$ channel the form

$$\mathcal{A}_{000} = 2\text{Re}(\epsilon) - \frac{2e^{-\bar{\Gamma} t} |\eta_{000}| \cos(\Delta m t + \phi_{000})}{|\eta_{000}|^2 e^{-\Gamma_S t} + e^{-\Gamma_L t}} \quad (2.85)$$

The interference will be observable mainly in the region of $0-8\tau_S$ as it is expected for $\mathcal{CP}$ violation in the K_S channel.

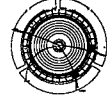
2.4 The isospin final states

In the $|\pi^+\pi^-\pi^0\rangle$ case the π^+ and the π^- are first coupled to a subsystem, which is later coupled to the remaining π^0 .

$$|\pi^+\pi^-\pi^0\rangle = (|1, 1\rangle \otimes |1, -1\rangle) \otimes |1, 0\rangle \quad (2.86)$$

where we used the notation $|I, I_z\rangle$. There are seven possible isospin states[39] with total isospin = 0,1,2 and 3. The notation is $|I, I_z\rangle_{I_{\pi^+\pi^-}}$. I_z is the third

2.4 The isospin final states



component of the isospin being 0 and $I_{\pi^+\pi^-}$ the isospin of the $\pi^+\pi^-$ subsystem.

$$|3,0\rangle_2 = \frac{1}{\sqrt{10}} \left(|\pi^+\pi^0\pi^- \rangle + |\pi^0\pi^+\pi^- \rangle + |\pi^+\pi^-\pi^0 \rangle + |\pi^-\pi^+\pi^0 \rangle + |\pi^0\pi^-\pi^+ \rangle + |\pi^-\pi^0\pi^+ \rangle \right) + \sqrt{\frac{2}{5}} |\pi^0\pi^0\pi^0 \rangle \quad (2.87a)$$

$$|2,0\rangle_2 = \frac{1}{2} \left(|\pi^+\pi^0\pi^- \rangle - |\pi^-\pi^0\pi^+ \rangle + |\pi^0\pi^+\pi^- \rangle + |\pi^0\pi^-\pi^+ \rangle \right) \quad (2.87b)$$

$$|2,0\rangle_1 = \frac{1}{\sqrt{12}} \left(|\pi^+\pi^0\pi^- \rangle - |\pi^0\pi^+\pi^- \rangle + |\pi^0\pi^-\pi^+ \rangle - |\pi^-\pi^0\pi^+ \rangle \right) + \frac{1}{\sqrt{3}} \left(|\pi^+\pi^-\pi^0 \rangle - |\pi^-\pi^+\pi^0 \rangle \right) \quad (2.87c)$$

$$|1,0\rangle_2 = \sqrt{\frac{3}{12}} \left(|\pi^+\pi^0\pi^- \rangle + |\pi^0\pi^+\pi^- \rangle + |\pi^0\pi^-\pi^+ \rangle + |\pi^-\pi^0\pi^+ \rangle \right) - \frac{1}{\sqrt{15}} \left(|\pi^+\pi^-\pi^0 \rangle + |\pi^-\pi^+\pi^0 \rangle \right) - \frac{2}{\sqrt{15}} |\pi^0\pi^0\pi^0 \rangle \quad (2.87d)$$

$$|1,0\rangle_1 = \frac{1}{2} \left(|\pi^+\pi^0\pi^- \rangle + |\pi^-\pi^0\pi^+ \rangle - |\pi^0\pi^+\pi^- \rangle - |\pi^0\pi^-\pi^+ \rangle \right) \quad (2.87e)$$

$$|1,0\rangle_0 = \frac{1}{\sqrt{3}} \left(|\pi^+\pi^-\pi^0 \rangle + |\pi^-\pi^+\pi^0 \rangle - |\pi^0\pi^0\pi^0 \rangle \right) \quad (2.87f)$$

$$|0,0\rangle_0 = \frac{1}{\sqrt{6}} \left(|\pi^+\pi^0\pi^- \rangle + |\pi^0\pi^-\pi^+ \rangle + |\pi^-\pi^+\pi^0 \rangle - |\pi^0\pi^+\pi^- \rangle - |\pi^+\pi^-\pi^0 \rangle - |\pi^-\pi^0\pi^+ \rangle \right) \quad (2.87g)$$

The states with total isospin $I=3$ and $I=0$ already have a totally symmetric or antisymmetric form. The $I=1$ states possess mixed symmetries in symmetric and antisymmetric form. The $I=2$ states can not be symmetrized alone or in combination. To derive the complete wave functions the unsymmetric states have to be symmetrized together with their orbital wave functions. The orbital wave functions take the form[39]

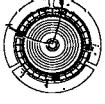
$$\phi_{mnp} = \vec{p}_1^{2m} \vec{p}_2^{2n} \vec{p}_1^{2p} f(\vec{p}_1^2 \vec{p}_2^2 \vec{p}_1^2) \quad (2.88)$$

in the rest frame of the $K^0, \bar{K}^0$. For m, n and p we get the relations

$$2m + 2n + 2p = \ell_A + \ell_B \quad (2.89)$$

ℓ_A denotes the relative angular momentum of the two pions in the subsystem and ℓ_B is the angular momentum of the remaining pion. Angular momentum conservation of the spinless particles implicates that $\ell_A + \ell_B = \ell_{\text{kaon}}$ and in the rest-frame of the kaon $\ell_A = \ell_B = \ell$.

The largest decay probabilities will be in decay states with the lowest orbital excitation. Therefore we first look at orbital wave functions with $\ell = 0$, which is in the lowest possibility the symmetric orbital wave function ϕ_{000} . $|3,0\rangle$ is



totally symmetric anyway and the $|1,0\rangle$ states can be symmetrized without vanishing.

$$|1,0\rangle_S = \frac{2}{3}|1,0\rangle_2 + \frac{\sqrt{5}}{3}|1,0\rangle_0 \quad (2.90a)$$

$$|1,0\rangle_M = \frac{\sqrt{5}}{3}|1,0\rangle_2 - \frac{2}{3}|1,0\rangle_0 \quad (2.90b)$$

The label S and M denote the states with symmetric and mixed symmetric form. Therefore we get three states with $\ell = 0$, $\mathcal{CP} = -1$ and in the first case with $\Delta I = 5/2$ and in the later cases $\Delta I = 1/2$.

$$|\pi^+\pi^-\pi^0\rangle_{\ell=0} = \frac{1}{\sqrt{12} \cdot m_K^2} (p_1^2 + p_2^2 - 2p_3^2)|1,0\rangle_M + \sqrt{\frac{2}{5}}|1,0\rangle_S + \sqrt{\frac{3}{5}}|3,0\rangle_2 \quad (2.91)$$

For the $\mathcal{CP} = 1$ states the symmetrization of the $I = 2$ states gives zero. Therefore there must be a non-zero orbital excitation. The lowest possibility is $m + n + p = 1$. With $\ell = 1$ we can form a fourth state by using the state $|2,0\rangle_1$

$$|\pi^+\pi^-\pi^0\rangle_{\ell=1} = \frac{1}{2m_K} (p_1^2 - p_2^2)|2,0\rangle_1 \quad (2.92)$$

The antisymmetric state $|0,0\rangle_1$ needs an antisymmetric orbital wave function to be symmetrized. The lowest angular momentum where this is possible is $\ell = 3$ with ϕ_{210} and m, n and p each being different from each other.

$$|\pi^+\pi^-\pi^0\rangle_{\ell=3} = \frac{1}{\sqrt{6}m_K^6} (p_1^4(p_2^2 - p_3^2) + p_2^4(p_3^2 - p_1^2) + p_3^4(p_1^2 - p_2^2))|0,0\rangle_1 \quad (2.93)$$

Recapitulating the results from the symmetrization of the complete wave functions, we get three states with angular momentum $\ell=0$ and $\mathcal{CP}=-1$ out of the seven $|\pi^+\pi^-\pi^0\rangle$ isospin states. Two of them possess isospin $I=1$ and one with $I=3$. One isospin state has an angular momentum $\ell=1$ and $\mathcal{CP}=1$ with $I=2$. Two states which possess $I=1$ and $I=2$ disappear due to cyclic permutation and the last isospin state has an angular momentum $\ell=3$ and $\mathcal{CP}=1$ with $I=0$. $\ell=3$ becomes necessary since for the antisymmetric state $|0,0\rangle_1$ needs an antisymmetric function ϕ_{mnp} .

$$\begin{array}{ll} \ell = 0 & \begin{cases} \mathcal{CP} = -1 & I = 3 \text{ with } \Delta I = 5/2 \\ \mathcal{CP} = -1 & I = 1 \text{ with } \Delta I = 1/2 \\ \mathcal{CP} = -1 & I = 1 \text{ with } \Delta I = 1/2 \text{ (mixed sym.)} \end{cases} \\ \ell = 1 & \mathcal{CP} = 1 \quad I = 2 \text{ with } \Delta I = 3/2 \\ \ell = 3 & \mathcal{CP} = 1 \quad I = 0 \text{ with } \Delta I = 1/2 \end{array} \quad (2.94)$$



2.5 The decay amplitudes

To write down the decay amplitudes in terms of kinematic variables, it is common to introduce the kinematic Lorentz-invariant variables X and Y

$$\begin{aligned} X &= \frac{s_2 - s_1}{m_{\pi^+}^2} \\ Y &= \frac{s_3 - s_0}{m_{\pi^+}^2} \end{aligned} \quad (2.95)$$

where the s_i are given by:

$$\begin{aligned} s_i &= (p_K - p_{\pi_i})_\mu (p_K - p_{\pi_i})^\mu \quad \text{for } i = 1, 2, 3 \\ s_0 &= \frac{1}{3}(s_1 + s_2 + s_3) \end{aligned} \quad (2.96)$$

p_K and p_{π_i} are the four-momenta of the kaon and the pions, respectively. The indices 1,2,3 correspond to π^+ , π^- and π^0 . For a given angular momentum ℓ and isospin I the decay amplitudes $\langle \pi^+ \pi^- \pi^0 | \mathcal{H} | K^0 \rangle$ and $\langle \pi^+ \pi^- \pi^0 | \mathcal{H} | \bar{K}^0 \rangle$ are given by[80]

$$\begin{aligned} A_{l,I} &= a_{l,I} e^{i\delta_I} \\ \bar{A}_{l,I} &= (-1)^{l+1} a_{l,I}^* e^{i\delta_I} \end{aligned} \quad (2.97)$$

The δ_I phase shifts arise due to the strong interaction of the final state and are considered to be small because of the small kinetic energies of the pions[41]. $a_{l,I}$ are the decay amplitude contributions of the weak interaction and depend on the kinematic variables of the system. The decay amplitudes for K_L and K_S are derived according to the eigenvalues of the effective Hamiltonian for the $\pi^+ \pi^- \pi^0$ final state. Therefore $\langle \pi^+ \pi^- \pi^0 | \mathcal{H} | K_L \rangle$ and $\langle \pi^+ \pi^- \pi^0 | \mathcal{H} | K_S \rangle$ can be written as

$$A_{l,I}^L = \frac{1}{\sqrt{2}} \left((1 + \epsilon_L) a_{l,I} - (-1)^{(l+1)} (1 - \epsilon_L) a_{l,I}^* \right) e^{i\delta_I} \quad (2.98a)$$

$$A_{l,I}^S = \frac{1}{\sqrt{2}} \left((1 + \epsilon_S) a_{l,I} + (-1)^{(l+1)} (1 - \epsilon_S) a_{l,I}^* \right) e^{i\delta_I} \quad (2.98b)$$

Due to the $\Delta I=1/2$ rule the state with $I=3$ of the isospin decomposition (eq. 2.87 a) can be neglected. Additionally, states with $\ell > 3$ can be neglected too, since the mass of the neutral kaons is only slightly larger than the mass of the three pions and they are kinematically suppressed:

$$A_{0,1}^L = \sqrt{2} a_{0,1} (1 - i\phi_1) e^{i\delta_1} \quad (2.99a)$$

$$A_{0,1M}^L = \frac{3}{2} \left(\frac{m_\pi}{m_K} \right)^3 a_{0,1M} (1 - i\phi_{1M}) e^{i\delta_{1M}} Y \quad (2.99b)$$

$$A_{1,2}^L = \frac{1}{\sqrt{2}} \left(\frac{m_\pi}{m_K} \right)^3 a_{1,2} (\epsilon_L + i\phi_2) e^{i\delta_2} X \quad (2.99c)$$

$$A_{0,1}^S = \sqrt{2} a_{0,1} (\epsilon_S + i\phi_1) e^{i\delta_1} \quad (2.99d)$$

$$A_{0,1M}^S = \frac{3}{2} \left(\frac{m_\pi}{m_K} \right)^3 a_{0,1M} (\epsilon_S + i\phi_{1M}) e^{i\delta_{1M}} Y \quad (2.99e)$$

$$A_{1,2}^S = \frac{1}{\sqrt{2}} \left(\frac{m_\pi}{m_K} \right)^3 a_{1,2} (1 - i\phi_2) e^{i\delta_2} X \quad (2.99f)$$



The phases ϕ_i are known to be small due to the small value of the fraction ϵ'/ϵ [9]. The first index in the amplitudes denotes the angular momentum ℓ and the second stands for the isospin I . There appear two states with the total Isospin equal to 1. The addition of 'M' denotes the states with mixed symmetry, i.e. their total wave function contains a symmetric and an antisymmetric part. Neglecting direct $\mathcal{CP}$ violation the decay amplitudes [11] are given by

$$A_S(X, Y) = \epsilon_S(\alpha e^{i\delta_1} - \beta Y e^{i\delta_{1M}}) \quad \mathcal{CP} = -1 \quad (2.100a)$$

$$A_S(X, Y) = \gamma X e^{i\delta_1} - \xi_{XY} X Y e^{i\delta_2} \quad \mathcal{CP} = +1 \quad (2.100b)$$

$$A_L(X, Y) = \alpha e^{i\delta_1} - \beta Y e^{i\delta_{1M}} + \xi(Y^2 - \frac{1}{3}X^2) + \zeta(Y^2 - \frac{1}{3}X^2) \quad \mathcal{CP} = -1 \quad (2.100c)$$

$$A_L(X, Y) = \epsilon_L \gamma X e^{i\delta_2} \quad \mathcal{CP} = +1 \quad (2.100d)$$

α , β and γ are proportional to the isospin decay amplitudes $a_{0,1}$, $a_{0,1M}$ and $a_{1,2}$, where ξ , ξ_{XY} and ζ are contributions due to higher angular momentum, isospin and final state interactions. $\mathcal{CP}$ violation may occur in the decay $K_S \rightarrow \pi^+ \pi^- \pi^0$ with the final state possessing $\mathcal{CP}$ eigenvalue (+1) and in the decay $K_L \rightarrow \pi^+ \pi^- \pi^0$ with the final state possessing $\mathcal{CP}$ eigenvalue (-1). As mentioned, the strong interaction phase shifts are assumed to be small[41] and therefore will be neglected here. The decay amplitudes become

$$A_S(X, Y) = \epsilon_S(\alpha - \beta Y) + \gamma X - \xi_{XY} X Y \quad (2.100a)$$

$$A_L(X, Y) = \alpha + \epsilon_L \gamma X - \beta Y + \zeta(Y^2 - \frac{1}{3}X^2) + \xi(Y^2 - \frac{1}{3}X^2) \quad (2.100b)$$

The integration over the whole phase space allows the observation of the interference between the $\mathcal{CP}$ conserving K_L decay amplitude and the $\mathcal{CP}$ violating K_S decay amplitude through the measurement of η_{+-0} .

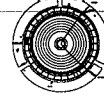
$$\eta_{+-0} = \frac{\int_{Y_{\min}}^{Y_{\max}} \int_{-X(Y_{\text{border}})}^{X(Y_{\text{border}})} dY dX A_S(X, Y) A_L^*(X, Y)}{\int_{Y_{\min}}^{Y_{\max}} \int_{-X_{\max}}^{X_{\max}} dY dX |A_L(X, Y)|^2} \quad (2.101)$$

In the integration the $\mathcal{CP}$ conserving K_S decay amplitude vanishes, since this amplitude is antisymmetric in X . The contribution of the $\mathcal{CP}$ violating part of K_L can be neglected and η_{+-0} becomes a measure for $\mathcal{CP}$ violation in the decay $K_S \rightarrow \pi^+ \pi^- \pi^0$.

The $\mathcal{CP}$ conserving K_S decay amplitude, which is antisymmetric in X , can only be observed by separating the events with $X > 0$ and $X < 0$. This can be achieved by defining the $\mathcal{CP}$ conserving parameter λ as

$$\lambda = \frac{\int_{Y_{\min}}^{Y_{\max}} \int_0^{X(Y_{\text{border}})} dY dX A_S(X, Y) A_L^*(X, Y)}{\int_{Y_{\min}}^{Y_{\max}} \int_{-X_{\max}}^{X_{\max}} dY dX |A_L(X, Y)|^2} \quad (2.102)$$

With λ it is also possible to measure the $\mathcal{CP}$ conserving K_S decay amplitude.



2.6 The asymmetry $\mathcal{A}_{\pi^+\pi^-\pi^0}$

α , β and γ are proportional to the three considered isospin states with $\ell \leq 1$. The contributions of higher angular momentum, isospin and final state interaction give the additional terms ζ and ξ proportional to X^2 and Y^2 for $A_{\pi^+\pi^-\pi^0}^L$ and ξ_{XY} proportional to XY for $A_{\pi^+\pi^-\pi^0}^S$. With the decay rates $K^0(\bar{K}^0) \rightarrow \pi^+\pi^-\pi^0$

$$\begin{aligned} R(X, Y, t) \sim & (1 - 2\text{Re}(\epsilon_S)|A_L(X, Y)|^2 e^{-\Gamma_L t} + \\ & (1 - 2\text{Re}(\epsilon_L)|A_S(X, Y)|^2 e^{-\Gamma_S t} - \\ & 2(\text{Re}(A_S(X, Y)A_L^*(X, Y)) \cos(\Delta mt) - \\ & \text{Im}(A_S(X, Y)A_L^*(X, Y)) \sin(\Delta mt)) e^{-\bar{\Gamma} t} \end{aligned} \quad (2.103a)$$

$$\begin{aligned} \bar{R}(X, Y, t) \sim & (1 + 2\text{Re}(\epsilon_S)|A_L(X, Y)|^2 e^{-\Gamma_L t} + \\ & (1 + 2\text{Re}(\epsilon_L)|A_S(X, Y)|^2 e^{-\Gamma_S t} + \\ & 2(\text{Re}(A_S(X, Y)A_L^*(X, Y)) \cos(\Delta mt) - \\ & \text{Im}(A_S(X, Y)A_L^*(X, Y)) \sin(\Delta mt)) e^{-\bar{\Gamma} t} \end{aligned} \quad (2.103b)$$

we obtain the particle-antiparticle asymmetry (eq. 2.74) for the $\pi^+\pi^-\pi^0$ final state.

$$\mathcal{A}(t) = 2\text{Re}(\epsilon_S) - 2(\text{Re}(q) \cos(\Delta mt) - \text{Im}(q) \sin(\Delta mt)) \quad (2.104)$$

with

$$q = \frac{\int \int dX dY A^S(X, Y) A^{L*}(X, Y)}{\int \int dX dY |A^L(X, Y)|^2}$$

Since it is used in later parts the complete asymmetry is written down here with all the X and Y terms. A term proportional to $\epsilon_L \gamma$ in the $A_L(X, Y)$ amplitude has been neglected.

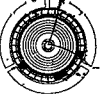
$$\begin{aligned} \mathcal{A}(X, Y, t) = & 2\text{Re}(\epsilon_S) - \\ & \frac{(\text{Re}(\kappa(X, Y)) \cos(\Delta mt) - \text{Im}(\kappa(X, Y)) \sin(\Delta mt)) e^{-\frac{1}{2}\Delta\Gamma t}}{1 + \iota(X, Y) e^{-\frac{1}{2}\Delta\Gamma t}} \end{aligned} \quad (2.105)$$

with κ defined as

$$\kappa(X, Y) = \frac{\epsilon_S - 2\epsilon_S \frac{\beta}{\alpha} Y + \frac{\gamma}{\alpha} X - (\frac{\beta\gamma}{\alpha^2} - \frac{\xi'}{\alpha}) XY}{1 - 2\frac{\beta}{\alpha} Y + (\frac{\beta}{\alpha})^2 Y^2 + 2\frac{\zeta}{\alpha} (Y^2 + \frac{1}{3}X^2) + 2\frac{\xi}{\alpha} (Y^2 - \frac{1}{3}X^2)}$$

and ι as

$$\iota(X, Y) = \frac{(\frac{\gamma}{\alpha})^2 X^2}{1 - 2\frac{\beta}{\alpha} Y + (\frac{\beta}{\alpha})^2 Y^2 + 2\frac{\zeta}{\alpha} (Y^2 + \frac{1}{3}X^2) + 2\frac{\xi}{\alpha} (Y^2 - \frac{1}{3}X^2)}$$



2.7 The slope parameters g, h, j, k and f

The Dalitz-plot density represents the time integrated rates of K^0 and $\bar{K}^0$ added together. By omitting shorter life times, i.e. only events above $2\tau_S$ are considered in the fit, the A_S contribution becomes negligible. Under this condition we derive

$$\int_{t_2}^{t_1} (R(X, Y, t) + \bar{R}(X, Y, t)) dt \sim |A_L(X, Y)|^2 \quad (2.106)$$

Then we can write with eq. 2.100

$$|A_L(X, Y)|^2 \sim 1 + gY + hY^2 + jX + kX^2 + fXY \quad (2.107)$$

where as usual second order terms are pooled in the new variables g, h, j, k and f , which are called slope parameters[42]. Higher order terms can be neglected in this context due to kinematic arguments. By convention eq. 2.107 is the standard parameterization of the Dalitz-plot density in the $K_L \rightarrow \pi^+\pi^-\pi^0$ analysis as it is also referred by the Particle Data Group[9]. The relations between the decay amplitudes and the slope parameters are given by

$$\begin{aligned} j &= 2\text{Re}(\epsilon_L) \frac{\gamma}{\alpha} & f &= -2\text{Re}(\epsilon_L) \frac{\beta\gamma}{\alpha^2} \\ g &= -2\frac{\beta}{\alpha} & k &= \frac{2}{3} \frac{\zeta - \xi}{\alpha} & h &= \frac{2}{3} \frac{\zeta + \xi}{\alpha} - \left(\frac{\beta}{\alpha} \right) \end{aligned} \quad (2.108)$$

Therefore in the first order terms $j \neq 0$ would indicate $\mathcal{CP}$ violation, where as $f \neq 0$ would stand for $\mathcal{CP}$ violation in the second order terms. g only has $\ell=0$ contributions, whereas j and f also have $\ell=1$ contribution. k and h possess beside the $\ell=0$ contributions, contributions of angular momenta $\ell > 1$.

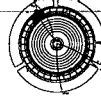
2.8 Regeneration

A pure $|K_L\rangle$ beam converts into a mixture of $|K_L\rangle$ and $|K_S\rangle$ as it passes through matter. This effect is called regeneration. We again denote $|\Psi\rangle$ to be the two-component neutral kaon state, as in equation 2.21. The change in $|\Psi\rangle$ with respect to its proper time τ as the beam propagates through matter has vacuum and nuclear components

$$\frac{\partial \Psi}{\partial \tau} = \frac{\partial \Psi}{\partial \tau} \Big|_{vac} + \frac{\partial \Psi}{\partial \tau} \Big|_{nuc} \quad (2.109)$$

Direct scattering of a $K_L(K_S)$ at a single nucleon in beam direction ($\Theta=0$) is expressed by the nuclear components. Cumulative scattering effects due to the whole medium are considered via the vacuum components. Remembering eq. 2.39 $|K^0\rangle$ and $|\bar{K}^0\rangle$ are the eigenstates of the strong and electromagnetic Hamiltonian and we get the two appropriate scattering amplitudes $f(\theta)$ and $\bar{f}(\theta)$. The nucleon components can be treated analogous to optical phenomena. The index of refraction n and $\bar{n}$ for $|K^0\rangle$ and $|\bar{K}^0\rangle$ can be expressed in terms of

2.8 Regeneration



the forward-scattering amplitudes $f(0)$ and $\bar{f}(0)$ as

$$n = 1 + \frac{2\pi N}{p_K} f(0) \quad (2.110a)$$

$$\bar{n} = 1 + \frac{2\pi N}{p_K} \bar{f}(0) \quad (2.110b)$$

where N is the number of scattering centres per unit volume and p_K the kaon momentum.

$$i \frac{\partial}{\partial \tau} \Psi(\tau) \Big|_{vac} = -\frac{2\pi N}{m_K} \begin{pmatrix} f(0) & 0 \\ 0 & \bar{f}(0) \end{pmatrix} \Psi \quad (2.111)$$

For the vacuum contribution the two mass components do not mix and the time evolution is given by

$$i \frac{\partial}{\partial \tau} \Psi(t) \Big|_{vac} = (M - \frac{i}{2}\Gamma) \Psi \quad (2.112)$$

Thus we get a modified effective Hamiltonian[43] for eq. 2.22

$$\mathcal{M}' = M' - \frac{i}{2}\Gamma' = M - \frac{i}{2}\Gamma - \frac{2\pi N}{m_K \gamma} \begin{pmatrix} f(0) & 0 \\ 0 & \bar{f}(0) \end{pmatrix} \quad (2.113)$$

The mixing of the mass components is specified by the regeneration parameter r , which is only dependent on the properties of the medium.

$$r = i\pi \frac{\Lambda_S N}{1/2 - i\Delta m \tau_S} \frac{f(0) - \bar{f}(0)}{p_K} \quad (2.114)$$

By adding together the contributions of all scattering centres in lowest order we derive the geometry-dependent regeneration parameter $\varrho(L)$, which is a measure for the regeneration power of a given medium[44].

$$\varrho(L) = i\pi \Lambda_S N G(L) \frac{f(0) - \bar{f}(0)}{p_K} \quad (2.115)$$

In terms of r we get:

$$\varrho(L) = r \left(1 - \exp(i\Delta\lambda L/\gamma v) \right) \quad (2.116)$$

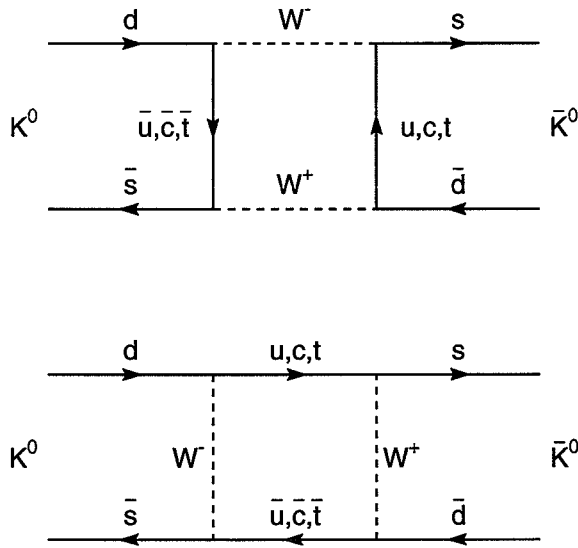
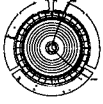
$L = \gamma v \tau_S$ is the thickness of the regenerator with the Lorentz transformation coefficient $\gamma = \sqrt{1 - (v/c)^2}$. Λ_S is the mean decay length of the K_S and $G(L)$ is a geometrical factor. We assume $\mathcal{CPT}$ invariance, but not $\mathcal{CP}$ invariance. Equation 2.113 with the modified $\mathcal{M}'$ can be solved. Its eigenvalues $\lambda'_{S/L}$ and eigenvectors $|K_S'\rangle$ and $|K_L'\rangle$ can be expressed in terms of the vacuum states $|K_S\rangle$ and $|K_L\rangle$. They are given by

$$|K_S'\rangle = |K_S\rangle - \varrho |K_L\rangle \quad (2.117a)$$

$$|K_L'\rangle = |K_L\rangle + \varrho |K_S\rangle \quad (2.117b)$$

and

$$\lambda'_{S/L} = \lambda_{S/L} - \frac{2\pi N}{m_{K^0}} (f(0) - \bar{f}(0)) \quad (2.118)$$


 Figure 2.3: box diagram contributions for $\Delta S=2$ processes

With this we can construct the decay rates and interference terms, which now include the regeneration effects. In our data analysis the regeneration corrections are evaluated event by event and added. As we will see the regeneration effects play a negligible role in the analysis of the $\pi^+\pi^-\pi^0$ channel.

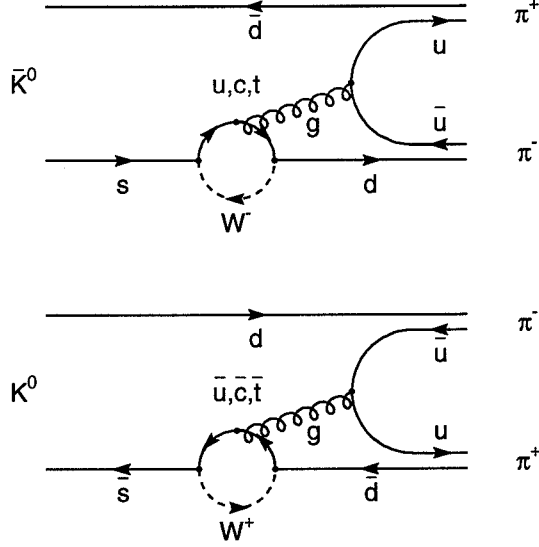
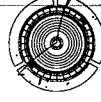
2.9 The CKM δ phase and the mixing parameter ϵ

To conclude the discussion of the phenomenology of the neutral kaon system the relation between the δ phase and the mixing parameter ϵ shall be given here. Yet to keep this section within an acceptable frame various assumptions have to be made to simplify the calculations.

We assume that the observed $\mathcal{CP}$ nonconservation arises solely from the CKM phase angle δ . In the six-quark scheme the charged hadronic current takes the form of eq. 2.3. To show how δ contributes to $\mathcal{CP}$ violation we introduce a parameterization of the CKM matrix[2.1] by Wolfenstein[45], which takes the observed smallness of $\mathcal{CP}$ violation into account.

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ \lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - (\rho + i\eta)) & A\lambda^2 & 1 - \frac{1}{2}\lambda^2 \end{pmatrix} + \mathcal{O}(\lambda^4) \quad (2.119)$$

$\lambda = V_{us} = 0.224$, $A \simeq 0.79$ and $\sqrt{\rho^2 + \eta^2} \simeq 0.36$. Only the off-diagonal elements V_{td} appear complex and $\mathcal{CP}$ violation occurs due to V_{td} coupling. But since appropriate decays are twice Cabibbo suppressed, the partial decay width


 Figure 2.4: Penguin diagrams contribution for $\Delta S=1$ processes

is small and measurements are difficult. In the standard CKM form the Wolfenstein parameters are given by

$$\lambda = s_1 \quad (2.120a)$$

$$A\lambda^2 = (s_2^2 + s_3^2 + 2s_2s_3 \cos(\delta))^{1/2} \quad (2.120b)$$

$$A^2\lambda^4\eta^2 = s_2s_3 \sin(\delta) \quad (2.120c)$$

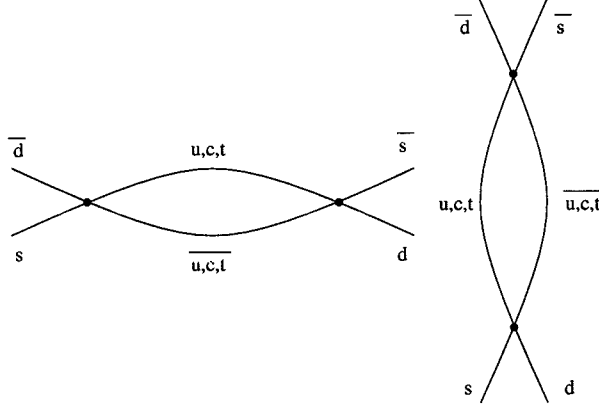
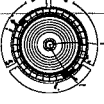
$$A\lambda^2(\rho^2 + \eta^2)^{1/2} = s_3 \quad (2.120d)$$

$$A\lambda^2((1-\rho)^2 + \eta^2)^{1/2} = s_2 \quad (2.120e)$$

In this parameterization J , as defined in eq. 2.11, becomes

$$J = A^2\lambda^6 \sin(\eta) \quad (2.121)$$

The neutral kaon decay can be described by Feynman graphs, as shown in fig. 2.3 and 2.4. The calculation of these diagrams will give us the relation between V_{td} and the parameters ϵ and ϵ' . $\mathcal{CP}$ violation in the mass matrix with $\epsilon \neq 0$ and $\Delta S = 2$ is described by the box diagrams (fig. 2.3), which are second order contributions to $K^0 \rightarrow \bar{K}^0$. Contributions to $\mathcal{CP}$ violation through direct $\mathcal{CP}$ violation with $\epsilon' \neq 0$ and $\Delta S=1$ are described by the penguin diagrams. Penguin diagrams require that $u\bar{u}$, $c\bar{c}$ or $t\bar{t}$ are created over gluons in the final states. Therefore penguin diagrams are strongly suppressed by the Zweig rule[46], which forbids final-state hadrons, whose quark lines are all created inside a diagram. The exact amount of suppression is difficult to estimate, since details of the gluon exchange are not understood. Limits for the suppression can be given by the experimental fact, that $\text{Im}(\Gamma_{12}) \ll \text{Im}(M_{12})$ due to the small fraction of ϵ'/ϵ [9].


 Figure 2.5: Four fermion diagram with $m_W \rightarrow \infty$

Due to the Zweig-rule suppression we neglect the contribution of penguin diagrams in a first approximation and assume that only ϵ contributes to $\mathcal{CP}$ violation according to eq. 2.69. Eq. 2.69 can be written as

$$\epsilon = -\frac{\text{Im}(M_{12})}{\Delta m - \frac{i}{2}\Delta\Gamma} \quad (2.122)$$

with $\Delta m = 2\text{Re}(\langle \bar{K}^0 | \mathcal{H} | K^0 \rangle)$. The $\mathcal{CP}$ violating mass term $\text{Im}(M_{12})$ is related to the $K^0 \rightarrow \bar{K}^0$ transition amplitude by

$$\text{Im}(M_{12}) = \text{Im}(\langle \bar{K}^0 | \mathcal{H} | K^0 \rangle) \quad (2.123)$$

The conventional calculations[47][50] of the amplitude $A(K^0 \rightarrow \bar{K}^0)$ begin with the determination of the free-quark transition amplitude

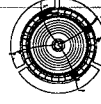
$$A(\bar{s}d \rightarrow s\bar{d}) = \quad (2.124)$$

$$|\langle \bar{K}^0 | (\bar{\psi}_s \gamma_\mu (1 - \gamma_5) \psi_d) (\bar{\psi}_s \gamma^\mu (1 - \gamma_5) \psi_d) | K^0 \rangle|^2 \quad (2.125)$$

in the Fermi-limit of point interaction. First order contributions to the amplitude tend to cancel[47] and we will only consider the second order contributions of the diagrams 2.2. In the Fermi-limit the box diagrams of fig. 2.3 becomes the four fermion diagrams of fig. 2.5. The amplitude of figure 2.5 takes the form of the current-current interaction (eq. 2.2). The coefficients of the transition amplitude can be parameterized by the quark masses and by the mixing angles, which characterize the matrix V (eq. 2.1). To compute the physical $K^0 - \bar{K}^0$ transition amplitude, the matrix element $\langle \bar{K}^0 | \mathcal{H} | K^0 \rangle$ must be estimated by formally inserting a complete set of intermediate states in all possible ways, which can be achieved by saturating the set with the contribution of only the vacuum state. This method is called vacuum-insertion calculation. Finally the following expression is found[47]

$$\text{Im}(M_{12})/\text{Re}(M_{12}) = 2s_2 c_2 s_3 \sin(\delta) P(\Theta_2, \eta) \quad (2.126)$$

2.9 The CKM δ phase and the mixing parameter ϵ



with

$$\eta = m_c^2/m_t^2 \quad (2.127)$$

and

$$P(\Theta_2, \eta) = \frac{s_2^2 \left(1 + \left(\frac{\eta}{1-\eta} \right) \ln(\eta) \right) - c_2^2 \left(\eta - \frac{\eta}{1-\eta} \ln(\eta) \right)}{c_1 c_3 \left(c_2^4 \eta + s_2^2 - 2s_2^2 c_2^2 \frac{\eta}{1-\eta} \ln(\eta) \right)} \quad (2.128)$$

$s_i = \sin(\Theta_i)$ and $c_i = \cos(\Theta_i)$ as defined in the CKM parameterization of eq. 2.5. Eq. 2.126, 2.122 and 2.68a give the desired relation between the mixing parameter ϵ from the neutral kaon system and the CKM δ phase.

$$\epsilon = \frac{s_2 c_2 s_3 \sin(\delta) P(\Theta_2, \eta)}{\left(\frac{i\Delta\Gamma}{\Delta m} - 2 \right)} \quad (2.129)$$

With eq. 2.129 we can calculate the theoretical predictions for ϵ . The present experimental values are

$$\Delta m = m_{K_L} - m_{K_S} = (3.510 \pm 0.018) \cdot 10^{-12} \text{MeV} \quad (2.130a)$$

$$\Gamma_{K_S} = (0.8927 \pm 0.0009) \cdot 10^{-10} s \quad (2.130b)$$

$$\Gamma_{K_L} = (5.17 \pm 0.04) \cdot 10^{-8} s \quad (2.130c)$$

$$m_c = 1.0 \text{ to } 1.6 \text{GeV} \quad (2.130d)$$

$$m_t = 180 \pm 12 \text{GeV} \quad (2.130e)$$

Θ_1 , Θ_2 and Θ_3 are established from measurements of the matrix elements of the CKM matrix. The present best values[9] are

$$\begin{pmatrix} 0.9745 \text{ to } 0.9757 & 0.219 \text{ to } 0.224 & 0.002 \text{ to } 0.005 \\ 0.218 \text{ to } 0.224 & 0.9736 \text{ to } 0.9736 & 0.036 \text{ to } 0.046 \\ 0.004 \text{ to } 0.014 & 0.034 \text{ to } 0.046 & 0.998 \text{ to } 0.9993 \end{pmatrix} \quad (2.131)$$

The three real CKM angles Θ_1 , Θ_2 and Θ_3 are

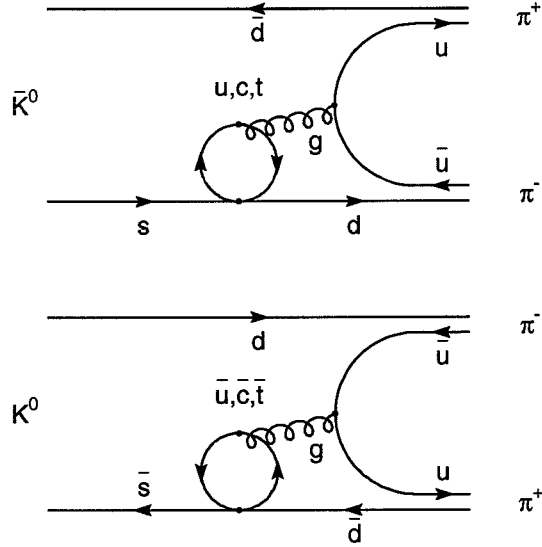
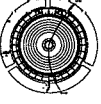
$$\Theta_1 = 12.6568^\circ \text{ to } 12.9669^\circ \quad (2.132a)$$

$$\Theta_2 = 0.9715^\circ \text{ to } 1.0223^\circ \quad (2.132b)$$

$$\Theta_3 = -0.9760^\circ \text{ to } -1.0223^\circ \quad (2.132c)$$

To derive a value for ϵ the CKM phase δ is chosen within its present permitted regions once within $0.34 < \delta < 0.73$ and once within $2.78 < \delta < 3.04$ [48]. In both cases it is possible to reproduce the correct order of magnitude for $\text{Re}(\epsilon) \simeq 10^{-3}$, which is required by the measurements. The limitations of the calculations above must be seen. The cancellation of the first order contributions are imperfect because of the t-c quark mass difference. Direct $\mathcal{CP}$ violation is neglected as well as diagrams of higher than the second order and the whole calculations are done in the Fermi-limit.

It is also possible to calculate the contributions of direct $\mathcal{CP}$ violation by considering Feynman graphs such as fig. 2.4. The general features of the calculation of direct $\mathcal{CP}$ violation are described by [49]. To simplify the calculations


 Figure 2.6: First order penguin diagrams with $m_W \rightarrow \infty$

we again use the Fermi-limit of point interaction. In the Fermi-limit the diagrams of fig. 2.4 become the diagrams of fig. 2.6. In addition, we assume the approximation $m_{c,t} \gg m_u$ and use the fact that $m_u \ll k \simeq m_{K^0}$ and $m_{c,t} \gg k$. Within these limits the desired ratio[49] is given as

$$|\epsilon'| \simeq s_2 c_2 s_3 \sin(\delta) \frac{\ln(m_t^2/k^2) - \ln(m_c^2/k^2)}{c_2^2 \ln(m_c^2/k^2) + s_2^2 \ln(m_t^2/k^2)} \quad (2.133)$$

where k is the gluon momentum transfer. In this approximation first order contributions other than those diagrams shown in fig. 2.6 again are neglected. In more precise calculation they are considered too, and we would get $\epsilon' = \epsilon_{Penguin} + \text{other first order diagrams}$. The diagrams of fig. 2.6 require that $c\bar{c}$ or $t\bar{t}$ annihilate into gluons in the final state. As mentioned such processes are suppressed by Zweig's rule. The suppression is caused by the dynamics of the strong interaction, which is not understood in detail. Theoretical estimation of the suppression reduce the amplitude by a factor 0.1 to 0.01.

There are calculations[50] which do not use the above approximations, i.e. they consider both, $\mathcal{CP}$ violation in the mixing and direct $\mathcal{CP}$ violation. Still the uncertainties concerning the different parameters are so large that it is not possible to derive firm values for the $\mathcal{CP}$ violating parameters ϵ and ϵ' . Nevertheless, the CKM model seems to yield approximatively the right magnitude of $\mathcal{CP}$ violation[47] or at least is not in contradiction with the experimental measurements.

Chapter 3

Fast neural network online trigger for CPLEAR

3.1 Introduction

The high luminosity of 10^6 Hz $\bar{p}$ and the large background are two major factors to be considered when designing the first level triggers for the CPLEAR experiment. High energy physics (HEP) experiments have to deal with analogous problems [51] providing an additional challenge to the study and development of first trigger steps for CPLEAR. The requirements for such a trigger are speed and selectivity. Trying to cope with the large luminosity soon revealed the advantage of highly parallel architectures. Trigger processors using artificial neural network algorithms (ANN) provide a possible approach to this problem [52].

The ANN-trigger stage presented here is based on a 3-layer feed-forward artificial neural network with two neurons in the hidden layer. This simple structure was chosen to keep the trigger decision time to a minimum. The network was trained using the error back propagation algorithm [52]. The purpose of this project was to realize the ANN proposed by G. Athanasiu et al. [52] as a very fast hardwired first trigger step and to test its performance with CPLEAR[5] data.

Feed-forward network architectures are well suited for hardware implementation, since they are certain to reach their decision within fixed time limits. This facilitates their integration into a sequential trigger hierarchy. The trigger was supposed to be upgradable to other experiments. In future HEP-experiments detectors are expected to become very complex and the number of signals to be processed will become larger as will the number of connections from the front-end to any other system [53]. With their simple structure feed-forward neural networks are well suited to minimize the number of connections, especially since they do not need any control mechanism to look for equilibrium conditions within them.

The error back propagation algorithm [54] was chosen to train the network, since it is simple to implement, faster than many other 'general' approaches, well tested in the field, and many efficient implementations exist. Additionally feed-forward networks trained with that algorithm are able to store a vast number of patterns far larger than their vector dimensionality [55].

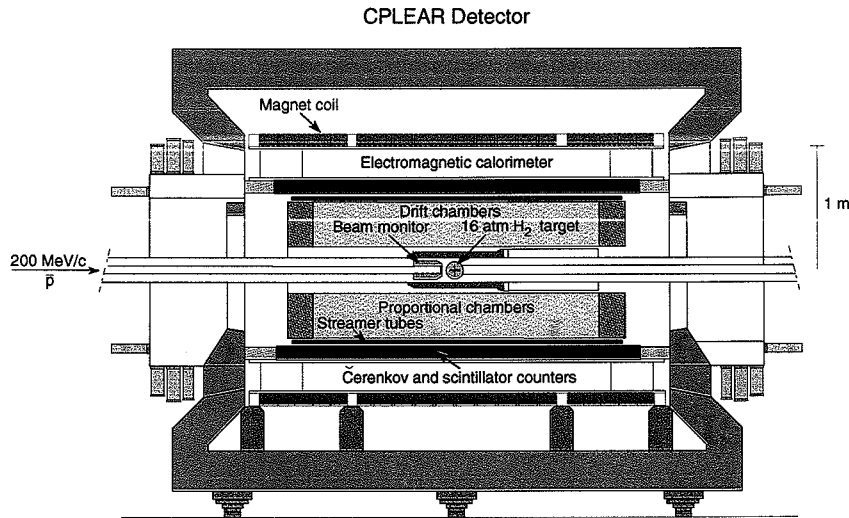
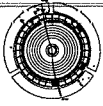


Figure 3.1: Layout of the CPLEAR detector

3.2 The CPLEAR detector

The cylindrical CPLEAR detector [56] is built up of several layers of wire chambers around a gaseous hydrogen target (fig. 3.1).

Starting from the target, i.e. the centre of the detector, there are first two layers of proportional chambers (PC), 6 layers of drift chambers (DC) and 2 layers of streamer tubes (ST), followed by a scintillator (S1) Čerenkov (C) scintillator(S2) sandwich [57]. A gas sampling calorimeter surrounds these tracking detectors. The whole detector is placed inside a solenoid providing a uniform magnetic field of 0.44 T along the beam axis. The solenoid possesses a length of 326 cm and a inner radius of 100 cm.

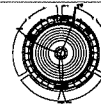
The beam axis also defines the coordinate system used: The origin of the coordinate system is chosen to be the centre of the target. The z-axis is equivalent to the beam axis with positive values pointing in the beam flight direction. The y-axis points upwards and the x-axis is given by the right handed coordinate system. In the following the various components are briefly described.

The antiprotons are cooled down to 200 MeV/c in the LEAR(Low Energy Antiproton Ring at CERN) and extracted with a intensity of about 1 MHz. The beam, after passing the beam-monitor and beam-counter, is stopped inside a spherical gaseous hydrogen target with a radius of 7 cm and a pressure of 15 atm. There the antiprotons and protons form protonium before their final annihilation.

The beam-monitor consists of two multiwire proportional chambers and is needed to adapt the beam profile and focus the beam position towards the centre of the target. The beam-counter consists of a 1 mm thick scintillator. It is situated close to the entrance window of the target and produces the start signal for the trigger electronic. The target walls are 0.8 mm thick kevlar and the entrance window is 0.12 mm thick mylar.

Different types of wire chambers are used as tracking system. First there are

3.3 The CPLEAR trigger



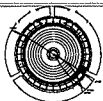
two layers of proportional chambers (PC1, PC2) at a radius of 9.7 cm and 12.7 cm followed by six layers of drift chambers (DC1,...,DC6) at a radius of 25.5 cm, 30.6 cm, 35.7 cm, 40.7 cm, 45.8 cm and 50.9 cm. The chambers are equipped with cathode strips for z readout. The proportional chambers as well as the drift chambers are used for momentum determination of the charged particles. Two layers of streamer tubes (ST1, ST2) at a radius of 58.2 cm and 60 cm allow the online z determination of the tracks. Additionally, using the information of the PCs a first classification of the tracks is done by the trigger. Tracks with at least one hit in one of the PCs are taken as primary tracks, as they are assumed to originate from the annihilation vertex. Tracks with no hit in a PC are taken to be secondary tracks, i.e. that they originate from the decay vertex of a neutral kaon. The efficiency of both PC layers is 99.9% and their transverse spatial resolution is $\approx 340\mu\text{m}$. The DCs show a slightly lower efficiency of 96% and a transverse spatial resolution of $\approx 300\mu\text{m}$. Each streamer tube has a central anode wire. Measuring the time difference of the propagation of the pulse along the wire to each end of the streamer tube allows to determine the z position of the track online with an uncertainty of about 1.4 cm. The efficiency of both ST is 98.2%. In 1994 the target was exchanged for a smaller one, which made it possible to place an additional proportional chamber PC0 at a radius of 1.8 cm in the detector. The information of PC0 mainly helps to suppress the pionic annihilation background, which minimizes the dead time for the detector.

The particle identification detectors (PIDs) allow us to distinguish between the charged pions and the charged kaons. The PIDs consist of a 3 cm thick scintillator (S1) at a radius of 63.8 cm, a 8 cm thick Cerenkov counter (C) at a radius of 69.4 cm and a second 1.5 cm thick scintillator (S2) at a radius of 74.2 cm. The Cerenkov medium is the liquid radiator C_6F_{14} (fluorinert FC72). PPO (2,5 Diphenyl Oxazole) is used as a wave length shifter to get a Cerenkov threshold at $\beta=0.84$, which is the maximum velocity of the kaons. Thus the Cerenkov detectors produce no light for kaons but they will produce light for the faster pions. A charged kaon possesses the signature $\text{S1}\overline{\text{C}}\text{S2}$, which means that the two scintillators gave a signal while the Cerenkov had no signal, whereas a charged pion carries the signature S1CS2 . To improve the particle identification the energy loss of the particles in the scintillators and the time of flight is also considered.

To detect photons an electromagnetic calorimeter consisting of 18 lead-streamer tube layers is used. It is placed at a radius of 77 cm to 99 cm. The wires are equipped with cathode strips at an angle of 30° and -30° to allow the determination of the position of the electromagnetic shower parallel to the wires. The resolution of the calorimeter is 4.7 mrad in the azimuthal angle and 8 mm in z direction. The energy resolution is $0.15/\sqrt{E(\text{GeV})}$. The efficiency for electrons above 150 MeV is measured to be 90%, while below 150 MeV the efficiency decreases significantly down to 50% at 50MeV.

3.3 The CPLEAR trigger

The CPLEAR trigger processors [58, 59] have to identify 2 or 4 track events and provide the location of the different tracks on the detector's hitmap. Two tracks of a good ('golden') event belong always to the $(K^\pm, \pi^\mp)$ pair originating from the annihilation point (primary vertex). Remaining tracks originate from



the decay of the K^0 (secondary vertex).

The signal is overlapped by a pionic background 200 times larger. The CPLEAR trigger reaches the desired reduction rate of the background by using a multi-level architecture (fig. 3.2). It has been custom designed in fast ECL electronics. The decision time of the different levels varies from 60ns up to 70 μ s. The first level of the CPLEAR trigger takes its decision within 60ns. All higher levels, e.g. calculating the transversal momentum, fine tracking, parameterization and kinematics of the tracks or particle identification (dE/dx , time of flight etc), have decision times higher than 0.5 μ s.

The task of the first trigger level, the Early Decision Logic (EDL), is to identify events containing a $K^\pm$ candidate and at least one other charged particle. A hit in both scintillators and the lack of a corresponding hit in the Čerenkov for a given sector identifies the charged kaon candidate. This is the PID signature S1 $\overline$ C52 mentioned earlier.

Slow pions faking a kaon pattern are rejected by a cut on the transverse momentum which demands $p_T > 270$ MeV/c. The p_T is calculated by using the informations of DC1 and DC6.

In the intermediate decision logic (IDL) a rough classification of the tracks is performed and the tracks are counted. At this level it is possible to distinguish between kaons, primary and secondary tracks. Kaons have to fulfil the PID signature S1 $\overline$ C52 and the p_T cut. Primary and secondary tracks have either hits in the PCs or no hits. The kaon candidates are also counted as primary tracks. An event is only passed to the the next trigger step if the following characteristics are fulfilled: The event has to possess at least one kaon candidate and the number of primary tracks has to be either 2 with a total number of tracks of at least 2, or 3 or 4 with a total number of tracks of at least 4.

The track follower (TF) completes the track reconstruction by assigning the fired wires to the different tracks. With the information of the steamer tubes the z direction of the track is calculated. The track is completely parameterized and the signature for a good event at this level becomes as follows: there has to be at least one primary pion with the charge opposite to the kaon. If there are two more tracks one of them has to possess the same charge as the kaon, which means that it descends from the neutral kaon decay.

Parallel to the track follower, the longitudinal and transversal components of the momenta are computed and a cut on the sum of the the momenta of the kaon and the associated primary pion is applied with $p_K + p_\pi > 750$ MeV/c.

If an event has only two primary tracks and no further secondary tracks, the HWP2.5(HardWired Processors) trigger process is started, which counts the number of showers in the calorimeter. HWP2.5 is a special trigger step for decays of neutral kaons into exclusive neutral pions.

Finally, in the last trigger step HWP2 the kaon hypothesis is confirmed by using dE/dx information from S1 and the amount of Čerenkov-light. A momentum cut $p_K > 350$ is applied on the charged kaon. In addition the S1 information is used to compare the time of flight difference of the kaon and the associated primary pion with opposite charge to the expected value.

3.3 The CPLEAR trigger

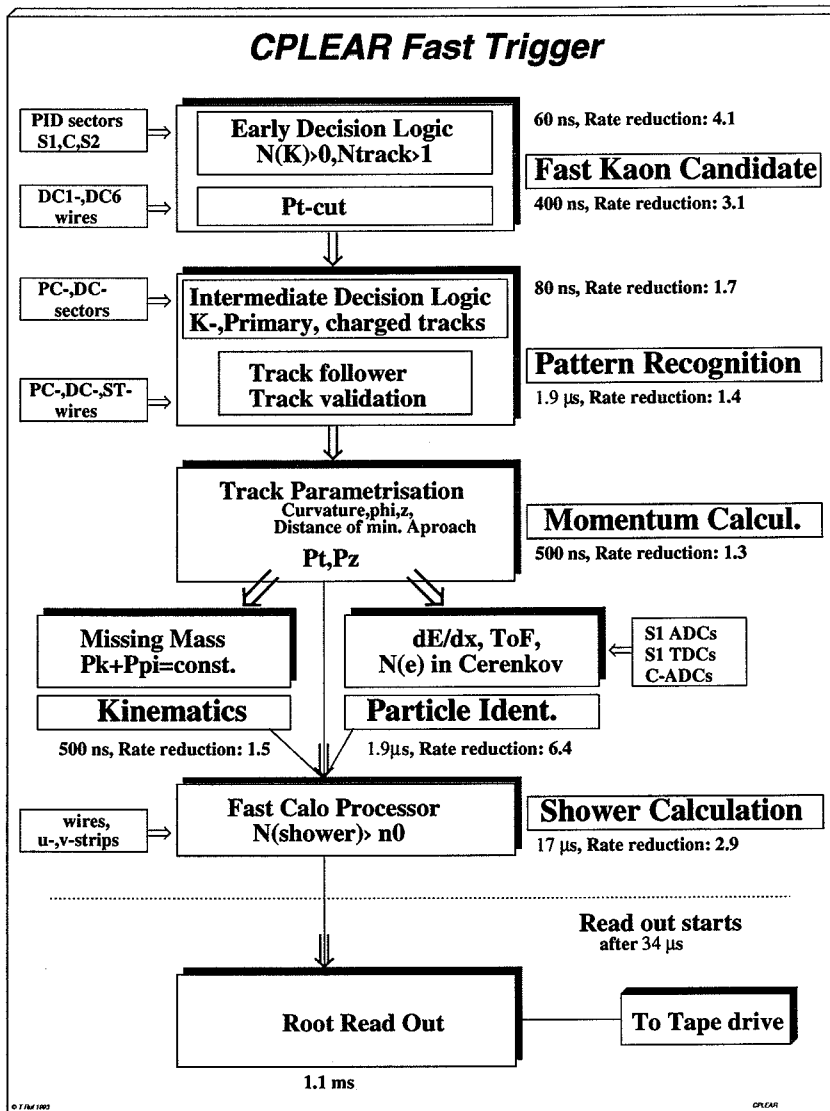
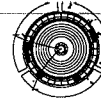


Figure 3.2: Layout of the CPLEAR trigger

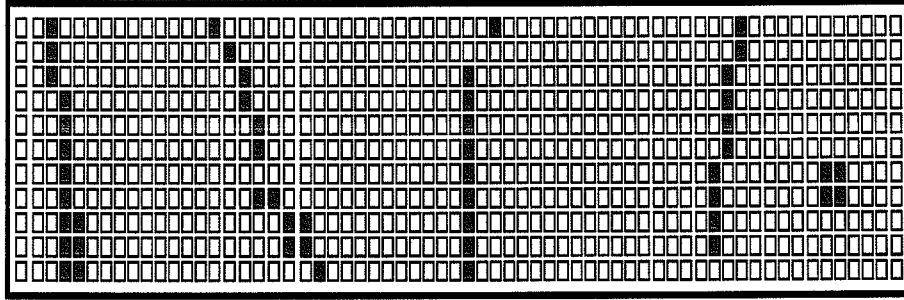
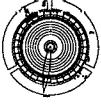


Figure 3.3: Typical CPLEAR event hitmap.

3.4 The artificial neural network

There are 55 input nodes to the ANN, i.e. a 11×5 matrix, corresponding to a 5 column hit-map of the CPLEAR detector. The 11 rows correspond to the number of detector layers, which are considered by the neural network. The 5 rows cover about 12% of the whole 2π detector angle, which is divided into 64 angular sectors. A typical event hitmap is shown in fig. 3.3. The learning set consisted of 400 events. The hitmaps were hand-selected by human experts, thus defining which object should be identified by the neural network as tracks. Imperfection of any existing pattern recognition algorithm was therefore avoided.

Since in a hardware implementation of an artificial neural network(ANN) one layer corresponds to one execution step (memory cycle), it was the intention when designing the ANN to keep the number of the layers as low as possible. Even the coordination of a reference frame costs time and should be avoided if possible. Therefore the simplest neural network configuration, the perceptron with just an input and an output layer, was tested first.

The input-output neurons of the system were chosen as a binary threshold unit. I.e. each input and output neuron computes the weighted sum of its inputs. If the sum is above or below a certain threshold or bias, the output T_i^μ becomes 1 or -1.

$$T_i^\mu = \Theta \left(\sum_j w_{ij} n_j - b_i \right) \quad (3.1)$$

where a step function or Heaviside function is used as transfer function $\Theta(x)$

$$\Theta(x) = \begin{cases} 1 & \text{if } x \geq 0, \\ -1 & \text{otherwise.} \end{cases} \quad (3.2)$$

T_i^μ is the output of the neuron i for the input pattern μ . The weight w_{ij} represents the strength of the synapse connecting neuron j to neuron i and n_j is the j input to the neuron. The cell specific parameter b_i is the threshold or bias value for neuron i . In terms of our network -1 and 1 in the input layer stand for no hit or hit and -1 and 1 in the output neuron stand for track or no track. The weights themselves were allowed to take any real value, initialized

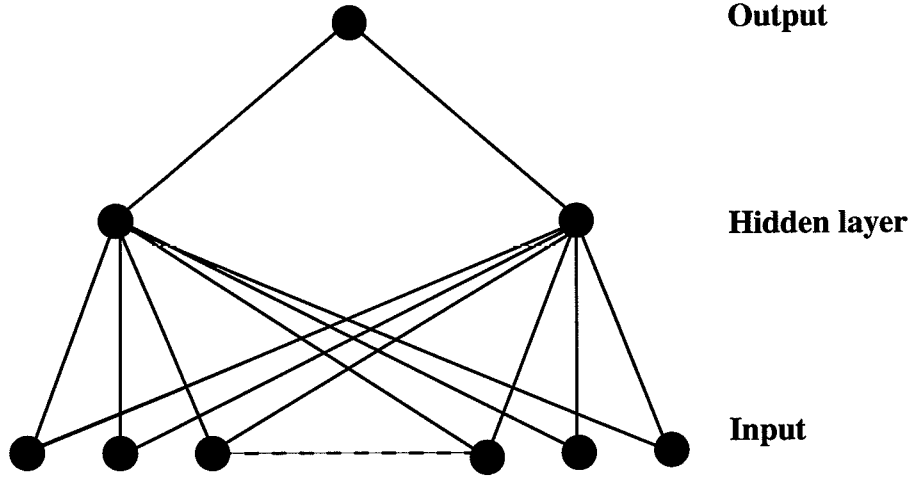
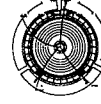


Figure 3.4: The three layered feed-forward structure of the neural network.

by random numbers in the range of -1 and 1. The adaption of the weights of the single layer perceptron during the learning phase was done according to

$$\Delta W_i = \epsilon \sum_{\mu} (T^{\mu} - O^{\mu}) x_i^{\mu} \quad (3.3)$$

where x_i^{μ} are the input hits of the example hitmap μ , $O^{\mu} = \sum_{\mu} W_i^{\mu} x_i^{\mu} - b$ and the constant b the arbitrary bias of the output neuron. T^{μ} is the expected answer of the network and ϵ is a constant, which determines the learning speed. The time evolution of the net during the learning phase is given by the cost function

$$C = \sum (T^{\mu} - O^{\mu})^2 \quad (3.4)$$

The purpose of the learning algorithm is now to minimize the cost function C . The performance[52] of the trained two layer network was 10% inefficiency of track detection and 3% probability of false track identification. Inefficiency of track detection is the probability that a real track is not detected by the neural network. The probability of false track identification is the probability that the neural network finds a track where there is no real one.

To get a better performance a three layered feed-forward neural network was introduced, with two hidden nodes in the hidden layer, a single output neuron and the 11×5 input neurons. The whole net has full local connectivity between the layers (fig. 3.4). Standard error back propagation [54] was used as training algorithm. The cost function is again defined by eq. 3.4 and the output O^{μ} for the test sample μ is now given as

$$O^{\mu} = \Theta \left(\sum_j w_j f(w'_{ij} x_i^{\mu}) \right) \quad (3.5)$$

The weights from the hidden nodes to the output node are w_j and the weights from the input neurons to the hidden nodes are w'_{ij} . The indices i, j run over the

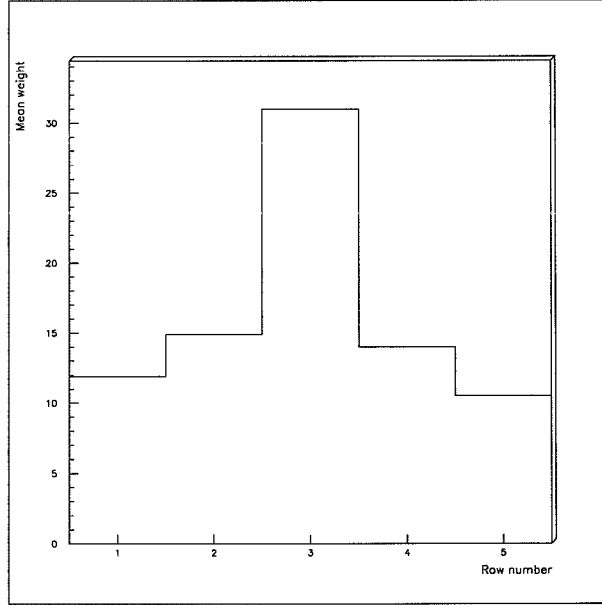
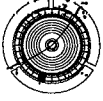


Figure 3.5: The on-cell distribution of the hidden node 1.

number of input and of hidden neurons. The response function f of the hidden neurons is chosen to be the typical sigmoid function

$$f(x) = \tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad (3.6)$$

Each event in the learning sample, which is presented to the neural network, consists of 4 tracks. As mentioned, such an event (fig. 3.3) accords with the chosen CPLEAR detector segmentation input for 64 equivalent ANNs, i.e. for the learning phase we get 60 non-track examples and 4 track examples with one event. This implies that most of the times the weights were pulled in a region where they force the system to respond there was no track. To compensate for that particular distribution of examples the learning constant ϵ was replaced by the expression

$$\epsilon \equiv \left(1 + \frac{T^\mu - O^\mu}{|T^\mu - O^\mu|}\right) \epsilon_r + \left(1 - \frac{T^\mu - O^\mu}{|T^\mu - O^\mu|}\right) \epsilon_l \quad (3.7)$$

Therefore with ϵ_r and ϵ_l we get different learning constants for the cases, where the network answer should be 1 and those where it should be -1. In addition, a temperature factor β was introduced to the argument $\sum_i W_i x_i^\mu$ of the response function f , which is now given as $\beta \sum_i W_i x_i^\mu$. The temperature factor is able to accelerate the learning by allowing easier transition between classes of misclassified examples. The weights derived from the learning phase are real numbers. In view of the later hardware implementation they are appropriately stretched and truncated to the closest integer numbers. The net with the integer weights was compared to the original net with the real weight. No deterioration of the system's performance could be observed.

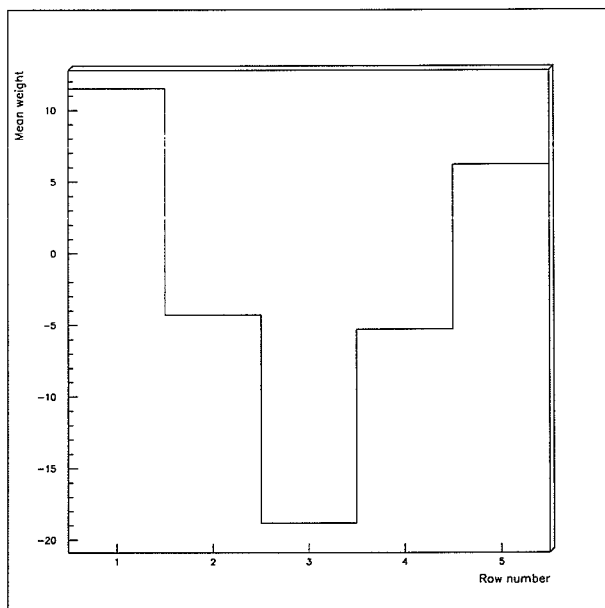
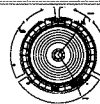


Figure 3.6: *The off-cell distribution of the hidden node 2.*

With the final performance of this neural network there was no observed inefficiency of track detection with the test sample of 400 CPLEAR events used as learning sample. False track identification was found to have a probability of 6%. The examples where the neural network identified a track incorrectly, can be subdivided into two categories. They are either noisy patterns which the network considered as tracks, or patterns of strongly bent tracks, such that adjacent networks recognised the track too. As we will see below topological constraints, as a special cluster treatment of the output neurons, achieve a further reduction of the later effect.

It is interesting to observe the behaviour of the final weights from the input to the hidden layer of the neural network. They show a structure described in fig. 3.5 and 3.6, which represents the averages over the 11 weights for each of the 5 columns plotted for each node. The two hidden nodes perform obviously complementary functions. The first hidden neuron serves as a so-called on-cell filter, whereas the second serves as an off-cell filter. The on-cell nodes manage to gather scattered hits, lying close together, to form a single track out of them, while the off-cell nodes pull apart slightly displaced hits to make well separated tracks out of them. The strong influence of the off-cell decision is origin of the high multiple counts of tracks mentioned. The off-cells drastically increase the discriminating power of the neural network.

On-cell and off-cell structures are actually also found in the first processing layer of neurons in the retina of mammals (ganglion cells). This structure[60] is responsible for an important data reduction and facilitates feature extraction, such as line recognition, before the transmission of the signal to the back of the brain for further processing. It is encouraging that the proposed algorithm con-

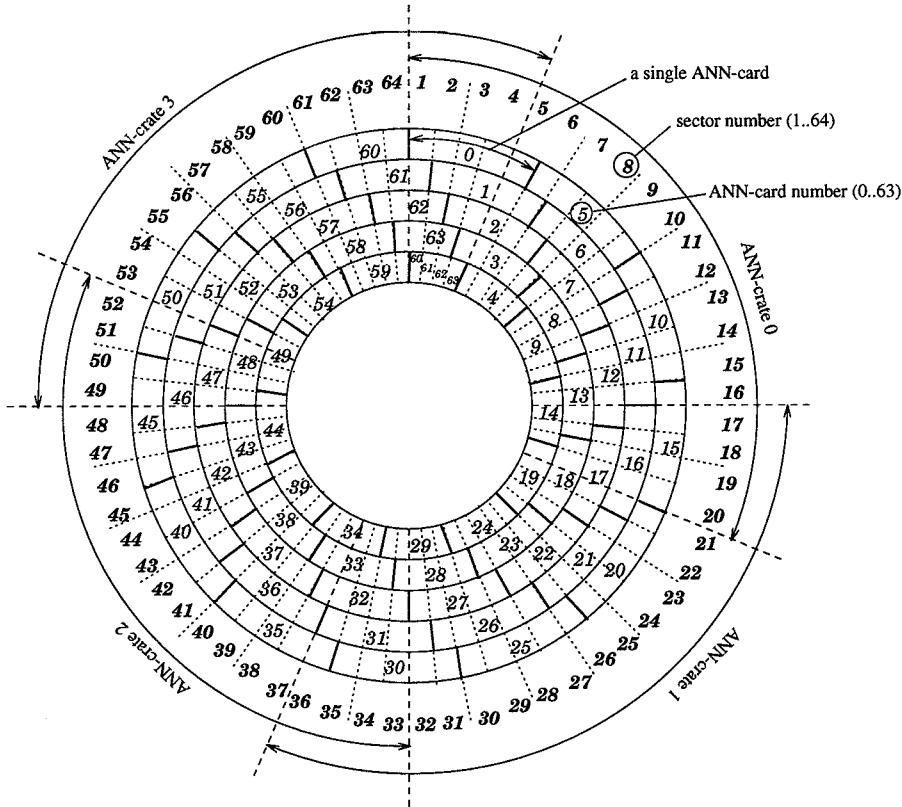
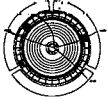


Figure 3.7: Sector segmentation of the detector by overlapping ANN-cards.

verged intrinsically to a solution that is observed in nature by natural selection too.

3.5 Hardware implementation

The purpose of the ANN-trigger presented here is to determine the number and location of the tracks in a event.

Eleven of the detector layers are used for the ANN trigger as input for track identification: PC1, PC2, DC1...DC6, ST1, ST2 and S1. For practical purposes the read-out of the detector is segmented into 64 angular sectors (1..64). 5 adjacent sectors are processed by one ANN-card. 64 ANN-cards, each one shifted by 1 segment with respect to the previous one, are needed in total to process the full detector. 16 such ANN-cards are packed in a CP-VME crate [61]. Therefore to cover the whole detector 4 such crates are required.

A single ANN-card is fed by 5 sectors with 11 layers, i.e. there are 5x11 input bits, starting with the detector layer PC1 and ending with S1. A set ANN decision-bit means that there was a track found in the central of the 5 sectors. For example ANN-card 15 has the sectors 16 to 20 as input and is assigned to tracks centred at sector 18 (see fig. 3.7).

3.5 Hardware implementation

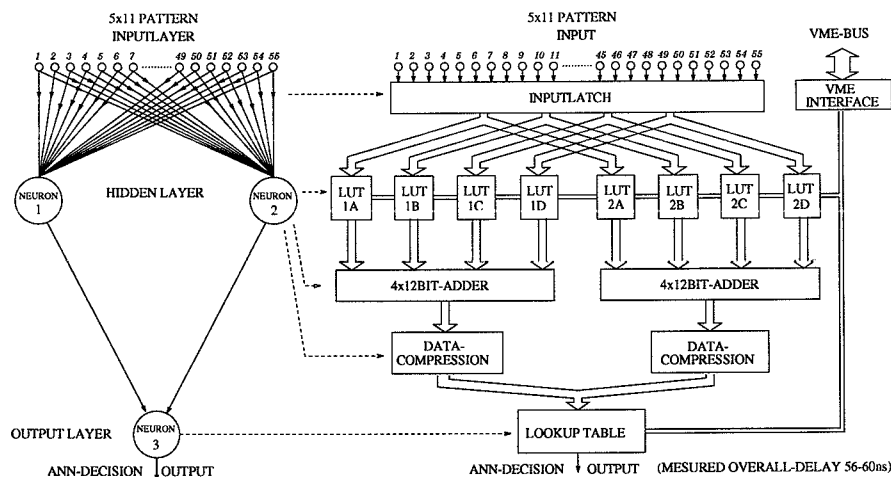


Figure 3.8: Hardware realization of the ANN-trigger card. For details see text.

One ANN-card represents one independent artificial neural network, processing its own part of the detector. All ANN cards execute precisely the same algorithm, i.e. they all have the same weight. The relation between the hardware and the applied neural network structure is presented in fig. 3.8 in a simplified block-diagram. If a track is recognized in a 55 bit input pattern, the corresponding ANN-card fires (i.e. the ANN-decision output goes high).

Complicated arithmetic operations could not be executed within the desired time scale. To keep the decision time short the nonlinear part of the arithmetic as well as the multiplications with the weights are therefore stored in look-up tables (LUT), i.e. in fast SRAM chips. LUT 1A,...,1D and LUT 2A,...,2D hold the sum of the input pattern already multiplied with the corresponding weights at the input addresses (fig. 3.8). To keep the size of the look-up tables reasonable, the inputs for hidden neuron 1 and hidden neuron 2 are distributed into four LUTs. Their output is added and compressed to 8 bits. The nonlinear part of the activation function for neuron 1 and 2, the summation of their output and the activation function for neuron 3 are included in LUT 3. All LUTs are loaded over a VME-bus, which is also used for testing the ANN-card logic.

High-speed additions are executed by standard fast TTL chips. However it is surprising that specially adapted FPGA-chips (Field Programmable Gate Arrays) were not considerably faster[62]. Other high speed adders, which were able to handle 12- or 16bit words, were mostly delivered together with multipliers or arithmetic units at a prohibitive price and were therefore not considered further.

It is important to note that the structure of the artificial neural network obtained on the PCB-board (fig. 3.8) is the most general. Fig. 3.9 presents a view of the complete hardware concept with the 64 ANN-cards divided into 4 crates. One of the advantages of this architecture is that the overall delay is obviously independent from the detector size.

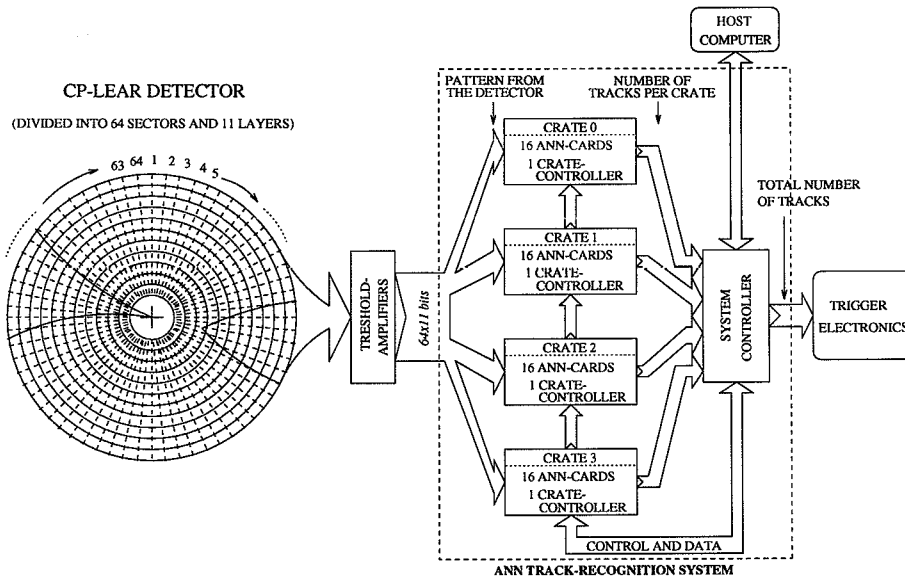
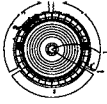


Figure 3.9: Hardware overview of the complete ANN-trigger stage. The schematic view of the sectors of the detector (containing 4 tracks) is on the left, signal distribution into the crates is on the right.

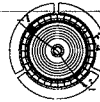
3.6 Performance of the ANN cards

Eighteen ANN-cards have been built and completely tested, i.e. a full crate (16 cards), which covers a quarter of the CPLEAR detector. These 18 ANN-cards underwent 32 different hardware tests [63] to ensure that they were working properly. Most tests were made with a PC connected via a VME-bus to the ANN-cards. The maximum overall time delay for each of the eighteen ANN-cards was measured using a pattern generator and a fast digital oscilloscope. For all 18 ANN-cards the measured power consumption in the working mode is in the range of 30W-34W. The maximum overall time delay of these 18 ANN-cards varies less than 10% from the fastest (56ns) to the slowest (60ns). Most of the ANN-cards have a maximum overall decision time of 59ns [61].

The maximum decision time of a complete ANN-crate is less than 75ns. The decision time of a crate increases due to the signal buffering and distribution on the crate. The detector data have to be present for a minimum of 10ns before the common data-driven ANN-clock is activated. The decision time is measured from the clock signal to the latest ANN-decision of the crate. The total power consumption of a single full ANN-crate is about 700W.

3.7 The ANN trigger

The two most interesting aspects of the ANN-cards' performance are: to what degree does the ANN-trigger decision with limited precision compare to the mathematical neural network model by G. Athanasiu et al.[52] and how well does the ANN-trigger perform in comparison to other tracking methods.



A software simulation of the ANN-trigger hardware was developed in an object oriented language in order to ensure a one to one representation of the hardware modules in the simulation. This enables the direct comparison of the mathematical artificial neural network model to the actual ANN-trigger card. Despite the data compression in the actual hardware, the ANN-trigger decision differs in less than 0.06% of all events from the decision of its model. Looking at these events, it was observed that the ANN-trigger tends to accept an event more easily than the ANN model.

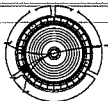
Concerning the comparison with other tracking methods, the ANN-trigger was compared with the present trigger and the offline pattern recognition of the CPLEAR experiment. The offline pattern recognition provides reliable decisions and is therefore used as reference standard. However its dedicated algorithms can never compete with the first level triggers discussed earlier in terms of execution time. On the other hand the sequential CPLEAR trigger is regarded as an example of a conventional well performing trigger setup.

The ANN-trigger starts evaluating an event as soon as the EDL identifies a good event candidate, and it completely identifies and locates the tracks. The number of tracks is obtained by counting the clusters of fired ANN-cards for the event. A cluster is defined as one or more consecutive ANN-cards, which have set their ANN-decision bit. Whenever a cluster consists of one or two fired ANN-cards, it is counted as one track, whereas clusters consisting of three or more fired ANN-cards are counted as two tracks. For comparison the corresponding pattern recognition of the CPLEAR experiment, the IDL (Intermediate Decision Logic), needs about $0.4\mu\text{s}$. In order to compare the quality of the ANN decisions with the decisions of the CPLEAR trigger in the next session, we have to use the more complete information of the third trigger level of the CPLEAR experiment (called 'track follower'). At this level fine tracking is already completed; this gives the number of tracks for a specific event.

3.8 Track recognition

For the evaluation of the system's performance on track recognition, Monte Carlo data from the CPLEAR experiment were used. From Monte Carlo data it is exactly known which hitmaps originate from which processes, and thus which hits belong to each track. This allows us to compare the different trigger methods with the 'ideal' recognition. Since at least three hits of a track are required for fitting the track parameters, only Monte Carlo tracks producing at least three hits in the hitmap of the detector were considered.

In this section we discuss recognition of single tracks. The performance in track recognition of the ANN-trigger has been evaluated and compared to two other pattern recognition procedures, namely the present equivalent CPLEAR trigger and the offline procedures to recognize tracks. The performance is described by the following criteria:



a) *The inefficiency of track detection*, i.e. the probability that a real track (a Monte Carlo track with at least 3 hits in the hitmap) is not detected by the system (table 3.1).

trigger system	inefficiency
CPLEAR trigger system	0.136
CPLEAR offline pattern recognition	0.035
ANN-card trigger	0.063

Table 3.1: Inefficiency of simple track recognition.

b) *The false track identification*, i.e. the probability that the system finds a track (from hits, which belong to real tracks) where there isn't any in reality (table 3.2).

trigger system	false track identification
CPLEAR trigger system	0.025
CPLEAR offline pattern recognition	0.012
ANN-card trigger	0.010

Table 3.2: False identification of single tracks.

A comparison of the performances shows that the ANN-trigger identifies tracks almost as efficiently as the sophisticated offline pattern recognition.

3.9 Event identification

In following we want to correlate the track identification strength with the overall density of hits in a event. The track identification system has to recognize 2 or 4 tracks in the event (good event). There are three relevant performance criteria:

a) *The inefficiency of event detection*, which is the probability that a good event is not detected by the system (table 3.3).

trigger system	2 track events	4 track events
CPLEAR trigger system	0.218	0.544
CPLEAR offline pattern recognition	0.083	0.217
ANN-card trigger	0.144	0.296

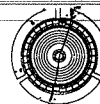
Table 3.3: Inefficiency of detecting 2 or 4 tracks in a sample of 2 or 4 track events respectively.

b) *The false event identification*, which is the probability that the system falsely finds a good event where none actually exists (table 3.4).

trigger system	2 track events	4 track events
CPLEAR trigger system	0.393	0.331
CPLEAR offline pattern recognition	0.207	0.225
ANN-card trigger	0.327	0.344

Table 3.4: False identification of 2 or 4 tracks in a sample of 2 or 4 track events respectively.

3.10 Future of the ANN trigger



c) The signal enhancement over the background, in a mixture of good events and background (table 3.5).

trigger system	2 track events	4 track events
CPLEAR trigger system	6.055	2.708
CPLEAR offline pattern recognition	15.004	4.614
ANN-card trigger	8.086	3.188

Table 3.5: Signal enhancement for 2 track events in a sample of 20.3% 2 track events and background, and for 4 track events in a sample of 42.7% 4 track events and background.

The performance figures for the ANN-trigger are in general significantly superior to those of the equivalent CPLEAR trigger stages. The offline selection is always better, with one exception, namely the loss of efficiency in event detection when going from 2 to 4 tracks. In this case the offline selection performs similarly to the CPLEAR trigger, while the ANN-trigger has the lowest loss and thereby demonstrates its capabilities of handling high hit densities.

The performance in selecting good events deteriorates, when passing from 2 to 4 tracks, by a factor 2 to 3 for all 3 recognition systems, reflecting the better separation of 2 tracks in the hitmap.

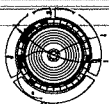
The performance of *false event identification* depends only moderately on the number of tracks. The false event identification is the only case where the ANN performs slightly worse than the other two systems in 4 track events. The reason is that the ANN-trigger is conceived so as to reject as few good event candidates as possible.

3.10 Future of the ANN trigger

Despite the many published proposals and NIM papers the CPLEAR ANN-card as a neural network based online trigger step, is still[64] the only system, which really got over the test phase and is included as a fixed part in a trigger system. Due to its open architecture it could be easily adapted in the year 1996 from the CPLEAR environment to the existing L3 trigger system[65, 66]. From June 1997 it will contribute to the online data selection of the L3 experiment. The ANN trigger is also projected to be used in the multilevel trigger of the DIRAC experiment[67][68].

L3 is one of the four experiments running on the LEP electron-positron collider at CERN. It is a general purpose detector designed for both precision measurements and rare new phenomena searches. The original trigger system of L3 was optimized for selecting events when data were taken around the Z^0 peak. Today LEP is being upgraded to run above the W^+W^- pair production threshold. At these energies a considerable effort will be put into searches for new processes which are not accommodated for the time being within the present trigger decision. Therefore a new trigger stage was looked for to select the new types of events and reject the different background sources. This new trigger stage was designed on the basis of the CPLEAR ANN-cards. For a detailed description see [69].

The L3 trigger will be the first time that a neural network based online trigger step is used as a fixed part of a trigger system and it will therefore be



very interesting to await the long time experience of the L3 collaboration with the neural network trigger in a 'real' world environment. It will help to show the potential of this very fast and powerful technology as a reasonable tool for fast online tracking in high energy experiments.

Chapter 4

$\pi^+\pi^-\pi^0$ Dalitz-plot analysis

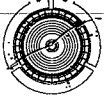
4.1 Introduction

This chapter is devoted to the study of the decay amplitudes of $K_L \rightarrow \pi^+\pi^-\pi^0$. The motivation for precise measurements of the amplitudes has various sources, even though a direct measurement of $\mathcal{CP}$ violating parameters is not possible with the analysis presented here because of too few statistics. An important motivation is the fact that the measurements can serve as a consistency check of the $\pi^+\pi^-\pi^0$ data sample and of the data selection criteria which are used for the $\pi^+\pi^-\pi^0$ analysis. Tabulated values of previous experiments[10] and theoretical prediction by calculation of the Chiral perturbation theory[11] exist up to the 6th order for the slope parameters as well as the decay amplitudes. Another motivation comes from the calculation of the Chiral perturbation theory itself. The precise measurements of the amplitudes are a test of the calculations of the Chiral perturbation theory and can serve as new input to these calculations.

In the analysis presented here the decay amplitudes and the slope parameters are extracted by an unbinned maximum likelihood analysis of the Dalitz-plot distribution of the decay. This kind of analysis reconsiders the entire data sample in each iteration step without losing any information. The use of such most complete methods for our large data sample of 508'032 events and the large parameter space is only made possible by the enormous development in the computer sector in the recent years.

In the first section the data selection criteria of the $\pi^+\pi^-\pi^0$ group are presented, which provided the data sample used for the analysis in this work. Since the Dalitz-plot distribution for $K^0, \bar{K}^0 \rightarrow \pi^+\pi^-\pi^0$ of the CPLEAR experiment is affected by large acceptance effects, the acceptance has first to be eliminated in order to parameterize the data by fitting the physical variables. In the case of a sufficiently complicated acceptance it can be rather difficult to find an analytic form to describe it. Therefore in the CPLEAR-experiment a Monte Carlo simulation representing the acceptance is used. In the following the parameterization of the acceptance and the background is presented.

Using the same fit engine and taking the parameterization of the background and acceptance as input, the Dalitz-plot slope parameters were fitted. Then a careful discussion of the systematics follows. The values derived by this method for the slope parameters and the decay amplitudes are comparable or better



than the present world averages. To conclude this work the comparison with the predictions of the Chiral perturbation theory up to the 6th order and with the values of earlier experiments is made and discussed.

4.2 $\pi^+\pi^-\pi^0$ data selection

In this section the offline data selection criteria and the applied cuts are discussed. The $\pi^+\pi^-\pi^0$ data selection can be divided into mainly three steps with sequential cuts. The three steps are referenced[70] as the general 3π filter, the super 3π filter and the hyper 3π selection of $K^0, \bar{K}^0 \rightarrow \pi^+\pi^-\pi^0$. It should be noticed that in the first step some of the cuts used online by the trigger are repeated offline, using the offline calculated quantities which provide better resolution and ensures the minimal requirements for a good events used already by the trigger.

4.2.1 The 3pi filter

This filter consists of a 4 track filter which requires that only events are accepted with four reconstructible charged tracks. Each track must show at least 3 DCs hits and two of them must have a 1 PC hit, i.e. they must be primary tracks. All tracks must have at least 2 pieces of information in the z direction of the detector, which is either two hits in the cathode strips or one hit in a cathode strip and one hit in a streamer tube.

The two primary tracks must possess opposite charges and one of the tracks has to show the kaon signature $S1\bar{C}S2$ in the same cell of the PIDs. Its momentum must be $300 \text{ MeV}/c < p_{K^\pm} < 850 \text{ MeV}/c$. For the associated primary pion (1 PC hit) a momentum of $75 \text{ MeV}/c < p_{\pi^\mp} < 750 \text{ MeV}/c$ is required.

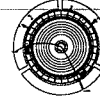
The secondary particles must be of opposite charges and their invariant mass m_{inv} must fulfil the condition $m_{\pi^+\pi^-\pi^0} \leq 600 \text{ MeV}/c^2$ under the assumption that $\pi^+\pi^-$ are the two secondary tracks and the total missing momentum belongs to the neutral pion. This cut suppresses background coming from $K_S \rightarrow \pi^+\pi^-$ and to a lower degree background from the semileptonic K_L decays.

To avoid badly reconstructed vertices, the separation between the primary and the secondary vertex in the transversal plane must be greater than three times its standard uncertainty. To avoid annihilation background the lower limit for the separation of the two vertices is set to 1.2 cm.

4.2.2 The super3pi filter

The super3pi analysis contains additional conditions on the primary tracks as well as on the secondary tracks. The two primary tracks must form a reconstructed vertex that lies completely inside the target. To ensure this condition the primary vertex radius is required to have $r < 2 \text{ cm}$ in the transverse plane and $|z| < 7 \text{ cm}$ in its position along the beam axis. Additionally, the energy loss dE/dx of the kaon candidate in S1 may not deviate by more than 3 times the standard uncertainty from the expectation and its momentum must now fulfil the slightly stronger condition of $300 \text{ MeV}/c < p_{K^\pm} < 800 \text{ MeV}/c$. Also, the primary pion undergoes a stronger momentum cut with $100 \text{ MeV}/c < p_{\pi^\mp} < 680 \text{ MeV}/c$.

4.2 $\pi^+\pi^-\pi^0$ data selection



For the secondary tracks the opening angle α_s between the tracks at the secondary vertex has to be larger than 8° and less than 172° in the transverse plane. This corresponds to the condition $|\cos(\alpha_s)| < 0.99$. The lower limit cut is applied to reject secondary tracks from γ conversion. The upper limit cut is used to reject secondary tracks faked by primary tracks which are backscattered from the outer detectors.

The π^0 hypothesis is supported by series of cuts, which are applied to the plane spanned by the total missing momentum p_{miss} and the total missing energy E_{miss} . The missing energy is calculated from the missing momentum at the secondary vertex. The cuts demand, that $p_{\text{miss}} > 40 \text{ MeV}/c$, $p_{\text{miss}} > 150 - E_{\text{miss}}$ and $p_{\text{miss}} > 0.5 \cdot (E_{\text{miss}} - 120)$. These conditions restrict the kinematic allowed region of the π^0 at the secondary vertex.

In addition at least one neutral shower and not more than three neutral showers must be observed in the calorimeter. A shower is regarded to be a neutral shower, if it cannot be associated with a charged track.

Finally a constraint kinematic fit is applied to the two primary tracks, demanding that the missing mass must correspond to the rest mass of the neutral kaon with a probability higher than 5 % (1C fit).

4.2.3 The hyper3 π selection

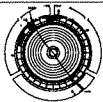
The third filter level, which is called hyper3 π filter, mainly contains constraint fits. There are also some cuts which have been already applied earlier, but which are now repeated after verifying the association of the $K^\pm - \pi^\mp$ and the $\pi^+ - \pi^-$. These cuts will no longer be listed from now on.

To verify the two possible associations $K^\pm - \pi^\mp$ and $\pi^+ - \pi^-$ both vertices must be well defined and may not show chamber hits before the vertex. The z position is verified. The two vertices must be separated in the transversal plane by at least 1.2 cm to suppress annihilation background.

A further cut is applied in the $\vec{p}_K - \vec{p}_\pi$ plane of the primary tracks to reject background events $p\bar{p} \rightarrow K^\mp \pi^\pm \pi^0 K^0(\bar{K}^0)$. This type of background has the same particles in the final state as the signal events, but the correlation between the momentum of the charged kaon and the momentum of the charged pion at the annihilation vertex is different. The primary pions and the kaon of golden events satisfy the relations $p_\pi < 26 p_K/21 + 180$, $p_\pi < -0.856 p_K + 1171$ and $p_\pi < -5 p_K/6 + 650$.

The vertex of the secondary tracks must now lie within $|z| < 75 \text{ cm}$, i.e. it must lie within the volume of the tracking chambers. Stricter cuts on the kinematic information in the $E_{\text{miss}} - p_{\text{miss}}$ plane demand, that $p_{\text{miss}} > 40 \text{ MeV}/c$, $p_{\text{miss}} > 150 - E_{\text{miss}}$, $p_{\text{miss}} > 1.143 E_{\text{miss}} - 285.75$ and $p_{\text{miss}} > 1.250 E_{\text{miss}} + 200$. A cut on the invariant mass of the two charged secondary pions, $m_{\text{inv}}^{2\pi_{\text{sec}}} > 285 \text{ MeV}/c^2$ rejects further events, where the charged secondary pions are faked by an electron-positron pair from a γ -conversion. Badly reconstructed events are rejected by requesting $m_{\text{inv}}^{2\pi_{\text{sec}}} < 400 \text{ MeV}/c^2$.

The background is further suppressed by several constraint fits, using the $K^0(\bar{K}^0) \rightarrow \pi^+\pi^-\pi^0$ hypothesis with a probability of more than 5%. The missing mass of the primary vertex has to correspond to the neutral kaon mass. The total missing mass must be the π^0 mass and the primary and secondary vertices must be aligned in the transversal direction with the missing momentum dedicated



to the neutral kaon. The information of the neutral showers has two cuts. The neutral showers must show $|z| < 150$ cm and the number of neutral showers may not be less than 1 and not more than 3.

Further it is demanded that the Dalitz-plot variables X and Y of the event lie inside the kinematic boundaries. If there are more than 2 primary tracks, the assignment of the primary tracks is not always clear. Therefore it is required that only one $K^\pm\pi^\mp$ combination survives the data selection, i.e. the assignment of tracks to the primary and secondary vertices must be unambiguous. The two cuts just mentioned do not only help to reject background, but also yield momenta and vertices with an improved resolution.

It is important for the analysis that consistency is achieved between the cuts applied by the trigger and the cuts applied offline. This problem is called 'matching'. For this an additional cut demands that all offline track assignments must agree with the online trigger assignment.

4.2.4 Final selection

In the analysis some of the cuts undergo stronger conditions a last time, e.g. the momentum of the $p_{K^\pm}$ must now lie between $350 \text{ MeV}/c < p_{K^\pm} < 850 \text{ MeV}/c$.

The background events $p\bar{p} \rightarrow K^\mp\pi^\pm\pi^0K^0(\bar{K}^0)$, where the primary pion is mistakenly assigned to the secondary vertex, is further reduced by a constraint fit. This constraint fit requires the $\pi^+\pi^-$ invariant mass to be the neutral kaon mass, using the primary pion which is assigned to the wrong track and the secondary pion with its opposite charge. To keep as many of the good events as possible this cut is only applied if the angle of the traverse plane between the momentum of the neutral kaon and the vertex separation is less than 90° where the kaon momentum is calculated from the secondary pions which is not correctly assigned. An event is rejected if the secondary vertex of the wrong combination is inside the radius of PC2 and the constraint fit yields a probability of more than 0.01.

There remains some background $p\bar{p} \rightarrow K^+K^-\pi^+\pi^-$, where either a track is badly reconstructed or a charged kaon undergoes decay shortly after its production. This background is minimized by rejecting events where a three track vertex is possible and where the missing momentum is $p_{\text{miss}} < 600 \cdot p_m \text{ MeV}/c$. p_m is the probability that a vertex with three or four tracks exists. Time of flight considerations help to fortify this cut.

A last source of background are events at short decay times, where a K_S decays into $\pi^0\pi^0$ and one of the π^0 undergoes a Dalitz decay or a γ conversion into a e^+e^- -pair. Events are rejected if the invariant mass and the time of flight of the secondaries agrees with the $\pi^0\pi^0$ -hypothesis.

CPLEAR took data from 1992 till 1996. After the last run period in August 1996, a total of 508'032 events survived the $\pi^+\pi^-\pi^0$ selection criteria (fig. 4.1). This complete sample of 508'032 events is the basis for the Dalitz-plot analysis of this work. From Monte Carlo studies the acceptance for $K^0(\bar{K}^0) \rightarrow \pi^+\pi^-\pi^0$ is derived to be $(2.464 \pm 0.0006) \cdot 10^{-4}$. A large part of the loss comes from K_L decaying outside the volume of the tracking chambers. Another large factor is the efficiency of the trigger and the offline data selection.

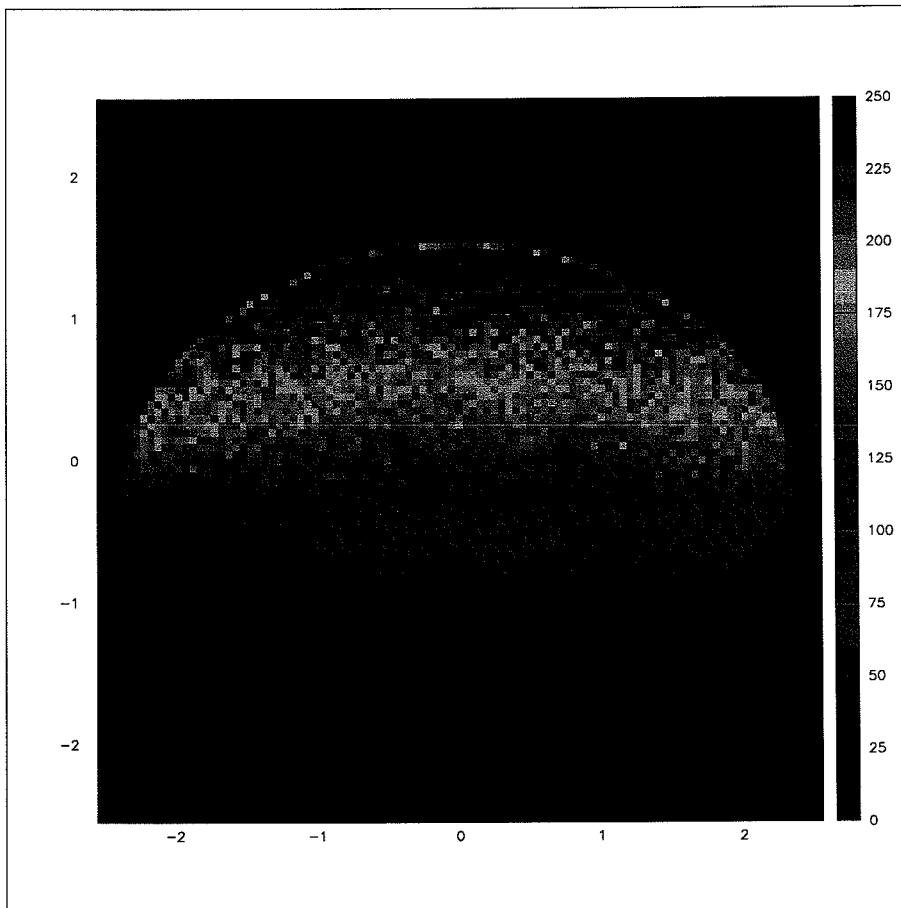
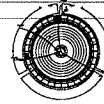
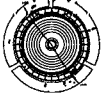


Figure 4.1: Dalitz-plot distribution of the decay $K_L \rightarrow \pi^+\pi^-\pi^0$ of the complete CPLEAR data sample of 508'032 selected events (1992-1996)



4.3 Normalization

K^0 and $\bar{K}^0$ are produced with the same rates $R(t)$ and $\bar{R}(t)$ at the $p\bar{p}$ -annihilation vertex. Because of the different strong interaction of the s quark of the K^+ and the $\bar{s}$ quark of the K^- in the PIDs, the detector efficiency for $K^+(\bar{K}^0)$ and $K^-(K^0)$ is not the same. Thus this effect has to be corrected for the data. The measured rate $R_{\text{mes}}(t)$ and $\bar{R}_{\text{mes}}(t)$ are given as

$$R_{\text{mes}}(t) = \kappa(t) \frac{1}{\alpha} R(t) + \sum_l B_l(t) \quad (4.1a)$$

$$\bar{R}_{\text{mes}}(t) = \kappa(t) \bar{R}(t) + \sum_l \bar{B}_l(t) \quad (4.1b)$$

$R(t)$ and $\bar{R}(t)$ are the K^0 and $\bar{K}^0$ rates and $B_l(t)$ and $\bar{B}_l(t)$ the various background contributions. $\kappa(t)$ denotes the detection efficiency for K^0 , $\bar{K}^0$ and α is the normalization factor accounting for the difference in detection efficiency between K^0 and $\bar{K}^0$.

The normalization factor α is a function of the kinematic variables of the primary tracks. It is assumed, that α is completely determined by these variables and has no dependence on the decay location of the neutral kaon in the detector. α can be calculated from the decay time regions where the rate difference of $K^0, \bar{K}^0$ due to other effects is negligible, which is true in the $\pi^+\pi^-\pi^0$ channel above $6 \tau_s$. Because of the better statistics the value of $\alpha_{\pi^+\pi^-}$ is taken instead of $\alpha_{\pi^+\pi^-\pi^0}$. $\alpha_{\pi^+\pi^-}$ is calculated from the decay $K^0, \bar{K}^0 \rightarrow \pi^+\pi^-$ at decay times below $4 \tau_s$,

The exact way of calculating α can be looked up in [71]. This method is called LUT (Look-Up Table) method. In these look-up tables the normalization is corrected event by event. To keep the correction small, only the deviations from the average normalization is corrected this way. The averaged normalization value α_0 is found to be

$$\alpha_0^{\text{data}} = 1.1268 \pm 0.0023 \quad \text{data} \quad (4.2a)$$

$$\alpha_0^{\pi^+\pi^-\pi^0} = 1.1227 \pm 0.0016 \quad \pi^+\pi^-\pi^0 \text{ MC acceptance} \quad (4.2b)$$

$$\alpha_0^{\text{elec.bg}} = 1.1223 \pm 0.0119 \quad \pi^\pm e^\mp \nu_e \text{ MC background} \quad (4.2c)$$

$$\alpha_0^{\mu \text{ bg}} = 1.1209 \pm 0.0176 \quad \pi^\pm \mu^\mp \nu_\mu \text{ MC background} \quad (4.2d)$$

4.4 Regeneration

Regeneration effects provide a further difference between K^0 and $\bar{K}^0$. To correct for this difference the effect on the neutral kaon is calculated event by event[44]. Unlike the 2π channel the effect of the corrections on the measured parameters in the $\pi^+\pi^-\pi^0$ channels was found to be extremely small. The corrected rate divided by the uncorrected rate of $K^0, \bar{K}^0$ versus the decay time is shown in fig. 4.2. For the Dalitz-plot analysis presented here it will turn out that regeneration effects are completely negligible.

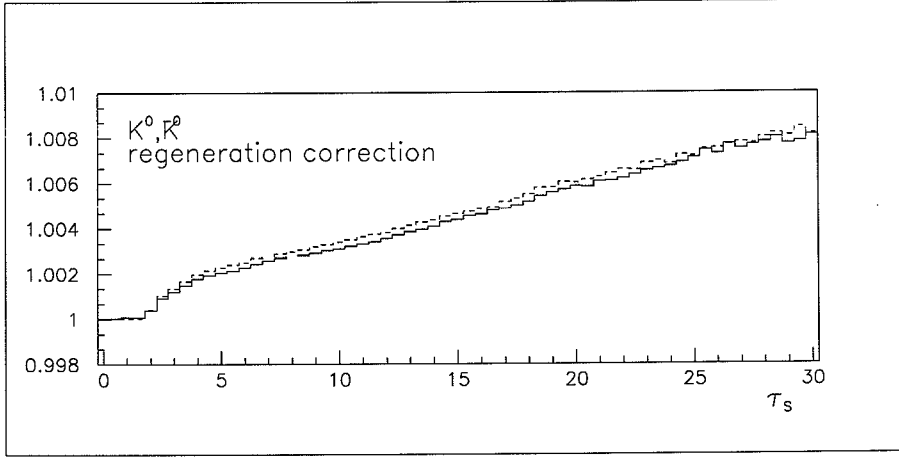
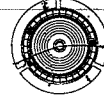


Figure 4.2: The corrected rate divided by the uncorrected rate of $K^0, \bar{K}^0$ versus the decay time. The dashed line shows the rate for K^0 and the solid line for $\bar{K}^0$.

4.5 Acceptance and background parameterization

The acceptance and the semileptonic background generated by Monte Carlo can be described by a polynomial of high order. In general, interpolation by high-order polynomials tends to follow the data too closely, while low-order interpolation may miss essential information. Therefore the χ^2 -value of the fit in dependence of the order of the fitting polynomial was minimized. The χ^2 -value becomes acceptable for polynomials of the order 10. Fit polynomials of even higher order brought no significant improvement.

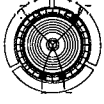
The events were generated uniformly over the whole phase space. The acceptance is obtained using CPGEANT[72]. To calculate the likelihood function of the acceptance a sample of about 2'000'000 events was used. The initial Monte Carlo sample is fitted in $2\tau_s$ -interval steps be sensitive to a possible dependence of the acceptance of the different τ_s -interval.

The likelihood function

The Lorentz-invariant variables X and Y for the Dalitz-plot are defined according to eq. 2.95. For a random sample of m events with phase space coordinates (X_i, Y_i) the likelihood function[73, 74] is defined as the product of the probability densities.

$$L_m(\vartheta_1, \vartheta_2, \dots, \vartheta_k) = \prod_{i=1}^m P(X_i, Y_i, \vartheta_1, \dots, \vartheta_k) \quad (4.3)$$

where $\vec{\vartheta}$ is a chosen set of parameters describing the acceptance. As discussed, the acceptance is parameterized by a polynomial $\wp$ with the coefficients $\vec{\vartheta}$ in the



Dalitz-plot variables X and Y . P can be explicitly written as:

$$P(X_i, Y_i, \vec{\vartheta}) = \frac{\wp(X_i, Y_i, \vec{\vartheta})}{\int \int \wp(X, Y, \vec{\vartheta}) dX dY} \quad (4.4)$$

The method of maximum likelihood estimates the k unknown parameters $\vec{\vartheta}$ by maximizing the likelihood function L_m . Since probabilities and densities are never negative and due to the monotony of the logarithm the negative logarithm of the likelihood function

$$l_m(\vec{\vartheta}) = -\ln(L_m(\vec{\vartheta})) \quad (4.5)$$

is often minimized instead of maximizing $L_m(\vec{\vartheta})$. A minimum of the log-likelihood function can be found by solving the following system of equations:

$$\frac{\partial l_m(\vec{\vartheta})}{\partial \vartheta_j} = 0 \quad \text{with } j = 1, \dots, k \quad (4.6)$$

The Hessian matrix $\mathcal{H}(\vec{\vartheta})$ is the second partial derivative matrix of the log-likelihood function $l_m(\vec{\vartheta})$.

$$\mathcal{H}_{k\ell} = \frac{\partial^2 l_m}{\partial \vartheta_k \partial \vartheta_\ell} \quad (4.7)$$

The matrix $\mathcal{H}(\vec{\vartheta})$ is positive definite at the location $\vec{\vartheta}_i$, if there is at least a local minimum. The diagonal elements of the inverse Hessian matrix $\mathcal{C}_{k\ell} \equiv [\mathcal{H}]_{k\ell}^{-1}$ are the variances.

$$\sigma^2(\vartheta_j) = \mathcal{C}_{jj} \quad (4.8)$$

In the CPLEAR data as well as in the Monte Carlo simulation the events are weighted. Including the weights ω_i in the likelihood function, we get the following probability distribution:

$$P(X_i, Y_i, \vec{\vartheta}) = \left(\frac{\wp(X_i, Y_i, \vec{\vartheta})}{\int \int \wp(X, Y, \vec{\vartheta}) dX dY} \right)^{\omega_i} \quad (4.9)$$

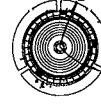
For the log-likelihood function $l_m(\vec{\vartheta})$ we get:

$$l_m(\vec{\vartheta}) = \sum_{i=1}^m \left(\omega_i \ln(P(X_i, Y_i, \vec{\vartheta})) \right) \quad (4.10)$$

4.6 Bent-off problems of the Dalitz plot boundaries

By fitting the acceptance by a polynomial of the order 10 it gets difficult to control the fit polynomials at the boundaries of the Dalitz-plot due to the contribution of high order terms. This problem is known as so-called bent-off problem. If it is possible to orthogonalize the polynomials over the considered region,

4.6 Bent-off problems of the Dalitz plot boundaries



the high order contributions will become controllable. Orthogonal polynomials have, besides other very nice properties, the advantage that an improvement of the approximation through addition of terms of higher order does not affect the values of the parameters from lower degrees. Above all, the higher orders do not worsen the fit-result of the lower orders.

In order to determine the orthogonal polynomials, the boundaries of the Dalitz-plot have to be known, so that the scalar products of the orthogonal polynomials can be calculated. The boundaries are given by:

$$X(Y) = \pm \sqrt{Y^2 - 4 \frac{s_0}{m_{\pi^\pm}^2} Y - \left(\frac{s_0}{m_{\pi^\pm}^2} - 4 \right) \left(5 \frac{s_0}{m_{\pi^\pm}^2} - 4 \right) + V \left(1 - \frac{4}{Y + \frac{s_0}{m_{\pi^\pm}^2}} \right)}$$

$$\text{with : } V = \frac{(m_K^2 - m_{\pi^0}^2)^2}{m_{\pi^\pm}^4} \quad (4.11)$$

For the minimum and maximum of Y we get:

$$Y_{min} = -\frac{(m_K^2 - 10m_{\pi^\pm}^2 + m_{\pi^0}^2)}{3m_{\pi^\pm}^2} = 4 - \frac{s_0}{m_{\pi^\pm}^2} \quad (4.12)$$

$$Y_{max} = \frac{2}{3} \left(\frac{m_K^2 - 3m_K m_{\pi^0} + m_{\pi^0}^2}{m_{\pi^\pm}^2} \right)$$

$$= 2 \left(\frac{s_0}{m_{\pi^\pm}^2} - 1 \right) - \sqrt{\left(3 \frac{s_0}{m_{\pi^\pm}^2} - 2 \right)^2 - V} \quad (4.13)$$

where s_0 is given by eq. 2.95.

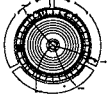
Orthogonalization over the Dalitz-plot

The coefficients of the orthogonal polynomials for a certain region are often simpler to calculate by the recursive formulas of Morris Weisfeld[75] instead of the better known Gram-Smith algorithm. Yet, it is possible to show that the Weisfeld formulas can be traced back to Gram-Smith orthogonalization, which is described in appendix A of this work. Additionally, the coefficients of the polynomials derived by either method are equivalent, which follows directly from the derivation of the Weisfeld formulas.

For the Weisfeld orthogonalization we define $\phi_j = X^{j_1} \cdot Y^{j_2}$ as a set of functions and $\sigma(j)$ as an order on Φ . The exact definitions are given in appendix A. The ϕ_i 's in the order σ use the indexing j_1 and j_2 , with $j_1 = n-k$ and $j_2 = k$. The elements ψ_j of the orthogonal base Ψ are constructed as follows:

$$\psi_{(0,0)} = \psi_0 = 1 \quad (4.14)$$

For $j > 0$ we define subsets of J with $J_{d,k} = \{j \mid \sigma(j) = d\}$, $k = 1, 2$ and $d = 0, 1, \dots$ with $d \leq n$. We choose $k=1$ if $j_2=0$ otherwise $k=2$. n is the highest existing order in Φ .



All $j \in J_{d,k}$ and $j \in J_{d,k-1}$ are uniquely determined by d and k . Further we define $\forall j > 0$ and $k=1$: $\hat{j} = (j_1 - 1, j_2)$ and for $k=2$: $\hat{j} = (j_1, j_2 - 1)$. As the scalar product for two functions f and g we define

$$(f, g) = \iint_{\text{Dalitz}} f g \, dX dY \quad (4.15)$$

If the two functions are orthogonal, the scalar product becomes 0. In this case we can write the ψ_j for $j > 0$ as:

$$\psi_j = X^{2-k} Y^{k-1} \cdot \psi_{\hat{j}} - \sum_m \alpha_j^m \psi_m \quad (4.16)$$

where the sum goes over all $m < j$ such that $\sigma(j) - \sigma(m) \leq 2$ and:

$$\alpha_j^m = \frac{(X^{2-k} Y^{k-1} \cdot \psi_{\hat{j}}, \psi_m)}{(\psi_m, \psi_m)} \quad (4.17)$$

Since we can write each ψ_j as

$$\psi_j = \phi_j + \sum_{i=1}^{i < j} \beta_j^i \phi_i \quad (4.18)$$

we have $(\psi_j, \psi_j) \neq 0$ due to the linear independence of the ϕ_i . The mapping $j \rightarrow \psi_j$ induces an order in Ψ . The choice of the α_j^m ensures the orthogonality of the set $\{\psi_n | n \in J, n \leq j \text{ and } \sigma(j) - \sigma(n) \leq 2\}$. For proof of this see [75].

As an example we will present the first three terms, i.e. ψ_0 , ψ_1 and ψ_2 , of the polynomial over the Dalitz-plot in the variables X and Y :

$$\psi_{(0,0)} = 1 \quad (4.19)$$

$$\psi_{(1,0)} = X - \frac{(1, X)}{(1, 1)} \quad (4.20)$$

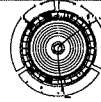
$$\psi_{(0,1)} = Y - a_{(1,0)}^{(0,1)} \cdot X - a_{(0,0)}^{(0,1)} \quad (4.21)$$

$$\text{with : } a_{(1,0)}^{(0,1)} = \frac{\left(Y, X - \frac{(X,1)}{(1,1)}\right)}{\left(X - \frac{(X,1)}{(1,1)}, X - \frac{(X,1)}{(1,1)}\right)}$$

$$\text{and : } a_{(0,0)}^{(0,1)} = \frac{(Y, 1)}{(1, 1)} - \frac{\frac{(X,1)}{(1,1)} \cdot \left(Y, X - \frac{(X,1)}{(1,1)}\right)}{\left(X - \frac{(X,1)}{(1,1)}, X - \frac{(X,1)}{(1,1)}\right)}$$

4.7 The fit procedure

The fit polynomial for the Dalitz-plot fit of the Monte Carlo data was chosen to be of the order ten. Due to the large amount of data and the high order of the fitting polynomial it is not at all trivial to achieve a satisfactory minimization



of the log-likelihood function. The usual minimization methods, e.g. gradient methods, failed during minimizing the log-likelihood function either by getting stuck in a local minimum or by never converging, even though the start-values for the parameters were chosen carefully.

A method had to be found which is able to wander more freely among different minima without being easily fooled by local minima. Being caught by such minima mostly happens if the method urges fast convergence in order to save iterations. This is typical for cases where the gradient methods fail. Badly chosen start values, starting with too small a step size or decreasing the step size too fast results in an insufficient search of the parameter space. One possible way of evading this problem, still by using gradient methods, is by permanent repetition with different start values. This does not help in our case, since it only succeeds if the number of local minima is small compared to the number of start values.

Simulated annealing is a so-called 'slow cooling' method. One of its very nice features is that it provides a kind of inherent parallelism in its search for the best minima that other methods can not compete with. In other words, there is always a finite probability of leaving a local minima and finding a better one. This prevents the problems mentioned, but unfortunately at the expense of speed. As we will see, we choose our minimization strategy in such a way as to combine the advantage of the different methods.

4.7.1 Minimizing by simulated annealing

Unlike gradient minimization, which is a continuous method, simulated annealing [77] [78] is a discrete method. Therefore finding a minimum does not require calculating the derivatives.

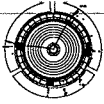
It can be shown that simulated annealing is a special case of genetic algorithm [79]. Its analogy in statistical mechanics are systems trying to find their lowest possible energy state, as e.g. in crystallization processes. This is achieved by cooling down the system. If the cooling too fast takes place, frustration-effects start to play an important role, i.e. the structure of the system is irregular and no minimal energy is reached. A slow and careful cooling can avoid these effects. In such slow cooling systems, nature normally is able to find a minimum energy state, which is at least very close to the global one.

If a physical system of particles is in thermodynamic equilibrium, the probability $P(T)$ to find a particle in an arbitrary energy state $E(\Lambda)$ with the free parameters λ_i , $i=1,2,\dots$ at a specific temperature T is given by the Boltzmann-distribution:

$$P(E) \propto e^{(-E/kT)} \quad (4.22)$$

If the temperature is lowered, states with lower energy become more probable. But even at low temperatures, a finite chance always exists, although perhaps a very small one, for a particle to attain a very high energy state. Therefore there is always a possibility that the system will escape a local energy minimum and find a better, more global one.

In analogy by simulating the slow cooling process it is possible to also find states with low 'energies' for non physical systems by choosing a certain state function, which describes the system. While looking for the minima, states with



lower energy are always accepted. States with higher energy are accepted only with the probability of the Boltzmann distribution, i.e. lowering T means that the number of minima to frequently visit is reduced till convergence is reached. One of the advantages of this algorithm is that the space of the function values is relatively well searched and local minima can easily be discarded by adapting the speed of decreasing the temperature T to the problem. Of course the slower the cooling the more iteration steps are needed to derive a certain convergence. The algorithm can be described as follows:

1. Find the state function in the relevant parameters for the system, which is in our case the log-likelihood function of the Dalitz-plot $l_m(\vec{\vartheta})$ for a certain parameter set $\vec{\vartheta}$. The energy of a given state is defined as the output of the log-likelihood function.
2. Choose a set of start parameters $\vec{\vartheta}$ and calculate the 'energy' of the initial state Υ_0 of the system. In our analysis the output of a least square fit of the Dalitz-plot performed with a rough binning is used as start parameter set. As we will see below this helps to speed up the fit procedure.
3. Choose the start value for the temperature T . This has to be done carefully since the algorithm depends heavily on T . Further, the criterion to decrease the temperature is defined. To decide where to decrease the temperature we use both the maximal number of iteration steps for calculating the state function and a fractional convergence criterion of the state function. If the criterion is reached in the fit, the temperature was decreased by 20%. This value was found to fit best to our problem. We tested other strategies as well, but they proved to be less efficient.
4. The next state Υ_{i+1} is generated according a transfer rule. This rule defines, how the next parameter set is calculated. We have chosen a stochastic transfer function, i.e. a logarithmical distributed random value varies a randomly chosen parameter of the state function. Logarithmic means that the probability to get a certain random value decreases logarithmically for bigger deviations from the start value.
5. Calculate the energy of the state. If the energy is lower than the energy of state Υ_i , the new parameters are always kept. Otherwise, the parameters are only kept with the Boltzmann probability at a certain temperature.
6. If T is below a chosen end value, the convergence of the program is stopped; otherwise the program returns to step 4.

4.7.2 Multi-step minimization strategy

We use a combined strategy for the minimization of the log-likelihood function. First a rough binned least square fit is performed on the Dalitz-plot distribution. The resulting parameter set serves as start values for the unbinned maximum likelihood fit by simulated annealing minimization. The main reason for this is, that simulated annealing minimization would be simply too slow if the start values for the parameters are not well chosen. In our fit the start temperature T had to be chosen quite high and the cooling process appropriately slow in order to get a reasonable minimum. The best start values for these parameters

4.7 The fit procedure

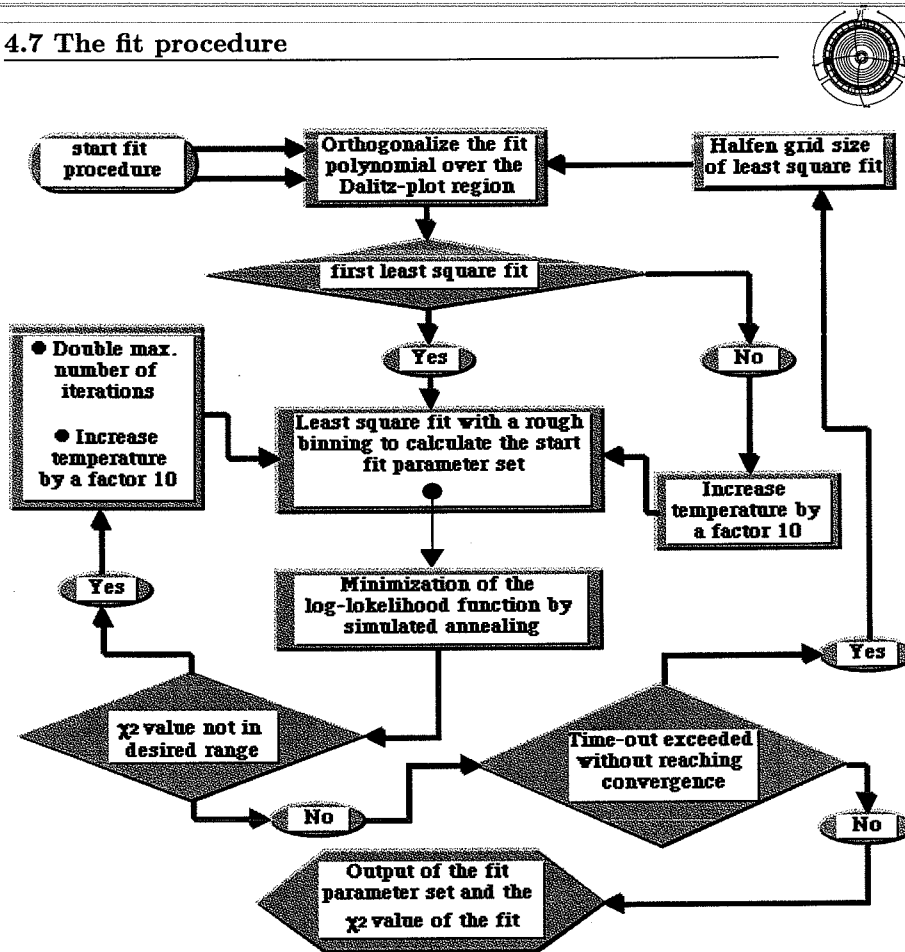


Figure 4.3: Overview of the two step minimization procedure used to fit the Dalitz-plot with high order polynomials.

to our problem were $T=10^8$ and the number of iteration steps $it = 100$ at a given temperature T . The fit procedure is given as:

1. The fit polynomial is orthogonalized over the Dalitz-plot region.
2. A normal least square fit together with a rough binning is performed to get a first estimation of the parameters.
3. The log-likelihood function $l_m(\vec{\vartheta})$ is minimized by the simulated annealing algorithm described above. The minimization starts with the resulting parameter set of step 2.
4. If the fit converges to slowly, i.e. a given time-out is exceeded the program returns to the step 2 by laying a thinner grid on the Dalitz-plot and decreases the start value for the temperature T in the step 3. The bins of the new grid were chosen to have the half width of the preceding size.
5. If the value of the χ^2 is not in the desired range the program returns to step 3 doubling the number of iterations to be performed at a specific value of the temperature T , which corresponds to a slower cooling of the system. In addition, the start temperature is increased by a factor of 10.

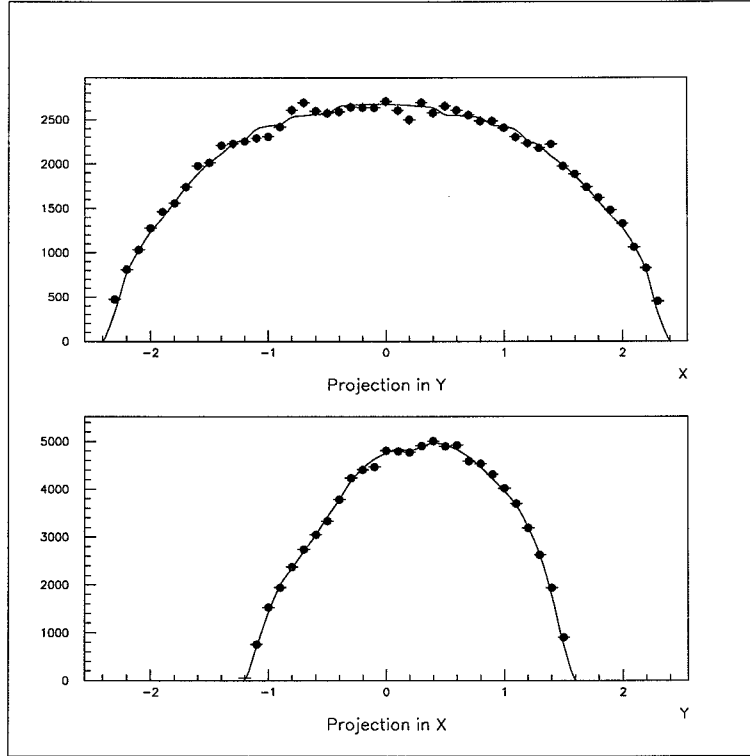
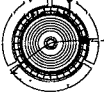


Figure 4.4: *Projection of the Dalitz-plot in the variables X and Y of the τ_s -interval $2-4\tau_s$.*

Despite the large number of fit-parameters and the large data sample we attained a good reliability in the convergence of the fit at a tolerable convergence speed by this two step minimization strategy. The maximal start temperature taken by the fit procedure was $T_{\max} = 10^{11}$ and the maximal iteration steps for one convergence level was $it_{\max} = 10^5$. An overview of the complete two step fit procedure is shown in fig. 4.3.

4.8 Results of the parameterization

In order to have a measure of how well the fits are performed we used the χ^2 -value of the fit as a quality function. This also allows us to compare the maximum likelihood fit results with the fit done by the least square method.

Beside the main goal of fitting the slope parameters and the decay amplitude it was regarded as an important step of this analysis to ensure the symmetry of the Dalitz-plot representation in the Dalitz-plot variable X . The Dalitz-plot is by definition (eq. 2.95, 2.96) chosen to be symmetric in X . A deviation of the symmetry in X produced by e.g. a different acceptance of π^+ and π^- in the detector would have a direct influence of the measured value of $Re(\eta)$. Since in the analysis presented here we disposed of too less statistics to directly measure CP violating parameters the symmetry test possesses a minor importance to

4.8 Results of the parameterization

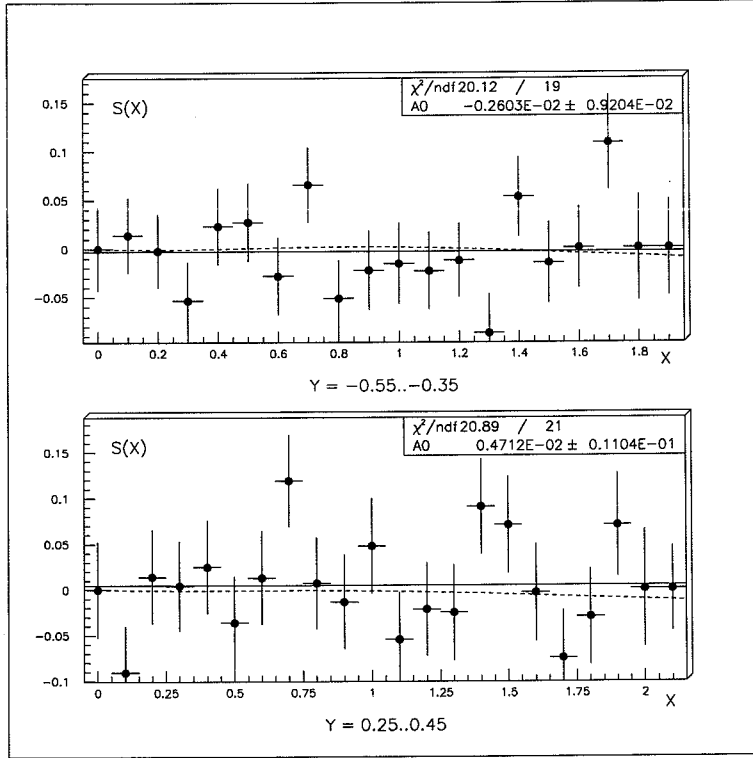
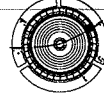
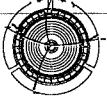


Figure 4.5: Two examples where the function $S(X)$ (dashed line), which is the normalized average deviation of the symmetry in X for a specific Y , is plotted against X for the interval $2-4 \tau_s$. The bold dots show the Monte Carlo $\pi^+\pi^-\pi^0$ acceptance together with their statistical errors. The solid line gives the fitted deviation of the Monte Carlo acceptance from 0.

this analysis. This is unlike to the the remaining work on the $\pi^+\pi^-\pi^0$ analysis which is sensitive to $\mathcal{CP}$ violating parameters by appropriate asymmetries. Since they use the same data sample the results of the symmetry test presented play an important role there.

The $\pi^+\pi^-\pi^0$ acceptance and the semileptonic background is fitted in $2\tau_s$ -interval to keep the sensitivity to an eventual dependence of the data on the different τ_s -intervals. The χ^2 -value of the maximum likelihood fit is stable around 1 ± 0.022 for all decay time intervals. That means that the fit also stays stable for small data samples, where the statistics are not as good. This is an important feature for Monte Carlo samples in high τ_s -intervals, in which only few statistics are available. Fig. 4.4 shows the projection of the Dalitz-plot fit in the X and the Y direction, respectively. The solid line is produced by the fitted polynomial. The fit proved to properly reproduce the data for all values of X and Y .

Comparing the results to the χ^2 -values of least square fits done in the same $2\tau_s$ -intervals as the maximum likelihood fits, the χ^2 of the least square fit displays a clear dependence on the statistics of the sample used. Unlike the maxi-



imum likelihood fit we observe a characteristic deterioration of the χ^2 -value due to statistics for the least square method in samples of the τ_s -intervals higher than $20\tau_s$. There the χ^2 differs up to ± 0.722 from 1.

4.8.1 Symmetry in X

By definition (eq. 2.95, 2.96), the Dalitz-plot distribution is symmetric in the Dalitz-plot variable X along Y . In order to test the symmetry in X of the Dalitz-plot distribution a function $S(X)$ is defined. $S(X)$ is the normalized average deviation of the symmetry in X for a specific Y and $\wp$ is the fitting polynomial function.

$$S(X) = \frac{\wp(X) - \wp(-X)}{\frac{1}{2}(\wp(X) + \wp(-X))} \quad (4.23)$$

The function $S(X)$ is plotted together with the propagated standard uncertainties introduced by the statistics of Monte Carlo simulation. The fig. 4.5 shows two arbitrary examples of the symmetry function $S(X)$ plotted with the data and their propagated standard uncertainties. In both plots the deviations from the symmetry of the Dalitz-plot are clearly compatible with the Monte Carlo standard uncertainties and can therefore be regarded as being of no further relevance.

Considering the terms of the fit polynomial which are odd in X , these terms are completely compatible with zero within their standard uncertainties for all orders and for all decay time intervals. This can be counted as another clear sign that the symmetry in X is really given in the Monte Carlo data sample. The parameterization of the acceptance and the background can be repeated by forcing the odd terms of the polynomial to zero. We will see later that in consequence the values of the slope parameters just change within their predicted systematic errors, which originate from the standard uncertainties of the parameterization of the acceptance and the background.

4.9 Determination of the slope parameters for $K_L \rightarrow \pi^+\pi^-\pi^0$

To extract the Dalitz-plot slope parameters for the decay $K_L \rightarrow \pi^+\pi^-\pi^0$ a fit in $2\tau_S$ slices of the Dalitz-plot distribution is performed, where the Dalitz-plot distribution is given as the time integrated rate of K^0 and $\bar{K}^0$ in a certain time interval and in the Dalitz-plot variable X and Y (eq. 2.95).

The lower and upper limits of the fit were chosen to be $2\tau_S$ and $24\tau_S$, respectively. With the lower limit some residual background, which is not well enough described by the Monte Carlo simulation, is cut. The upper limit is inducted by the given detector specifications and by the global χ^2 value of the fit, which reaches best values for this upper limit. Due to the lower limit of $2\tau_S$, the contribution of the A_S decay amplitude can be neglected. If in addition all direct $\mathcal{CP}$ violation effects are neglected, we can assume the validity of the expression 2.106. The parameterization of eq. 2.107 can then be used to fit the slope parameters.

4.9 Determination of the slope parameters for $K_L \rightarrow \pi^+ \pi^- \pi^0$

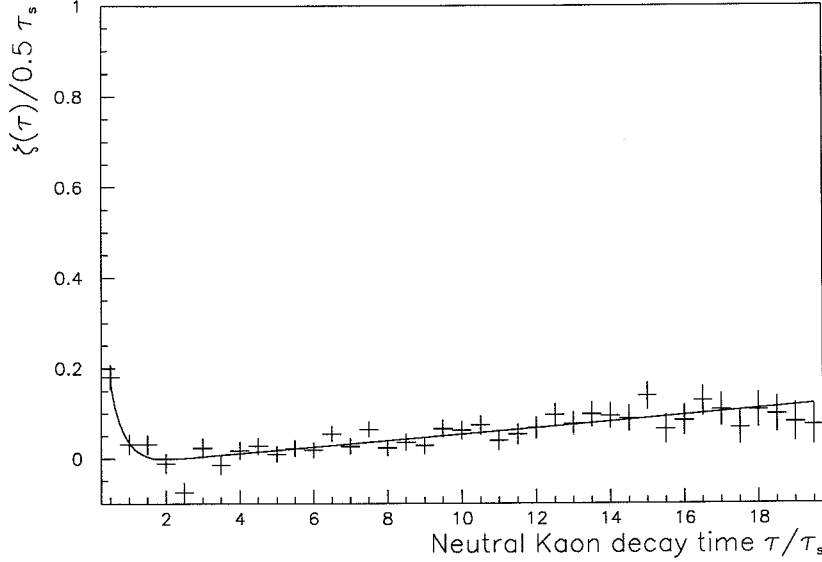
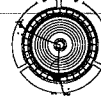


Figure 4.6: The background fraction $\zeta_B(t)$ (eq. 4.28) as a function of the decay time in τ_s units (CPLEAR collaboration[80]). In the fit presented in this work the residual background at short life times is cut by the lower fit-limit of $2\tau_s$.

Regeneration corrections are calculated event by event, which gives a additional weight to every event. The normalization due to the different acceptance of K^0 and $\bar{K}^0$ in the detector is also calculated for each event and corrected using Look-Up-Tables(LUT). The parameterization of the $\pi^+ \pi^- \pi^0$ acceptance and of the semileptonic background by Monte Carlo data is taken as input for the fit. Regarding the background the semileptonic contributions, which are $K_L \rightarrow \pi^\pm e^\mp \nu_e$ and $K_L \rightarrow \pi^\pm \mu^\mp \nu_\mu$, only have to be considered, since other sources are negligible above $2\tau_s$ (fig. 4.6). The agreement between data and Monte Carlo was already confirmed previously within 1% by other means[80].

There are three more parameters left free in the fit together with the 5 slope parameters. One is the constant parameter θ_3 of the parameterization of the Dalitz-plot, which is left free for the fit but is afterwards normalized to 1 by convention. Leaving the constant parameter free in the fit is a necessary condition of the likelihood method. The two other free parameters θ_1 and θ_2 are the fraction of the semileptonic background to the $\pi^+ \pi^- \pi^0$ acceptance and the fraction of the muonic background to the electronic background. $\theta_4.. \theta_8$ correspond to the slope parameter g, h, j, k and f , as they are introduced by eq. 2.107. Therefore we can write the likelihood function as:

$$L_m(\vec{\vartheta}) = \prod_{i=1}^m P(X_i, Y_i, \vec{\vartheta}) \quad (4.24)$$

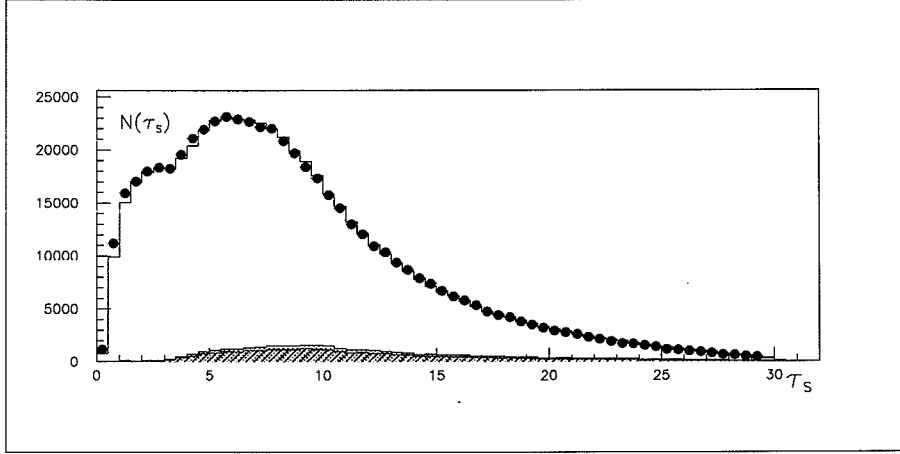
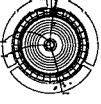


Figure 4.7: The decay time distribution of $K^0, \bar{K}^0 \rightarrow \pi^+\pi^-\pi^0$. The solid line shows the Monte Carlo $\pi^+\pi^-\pi^0$ and semileptonic background data weighted by the fractions ϑ_1 and ϑ_2 .

In contrast to eq. 4.4 the probability density P now consists of the two functions $\wp$ and α :

$$P(X_i, Y_i, \vec{\vartheta}) = \frac{\wp(\vartheta_1, \vartheta_2, X_i, Y_i) \cdot a(\vartheta_3.. \vartheta_8, X_i, Y_i)}{\int \int \wp(\vartheta_1, \vartheta_2, X_i, Y_i) \cdot a(\vartheta_3.. \vartheta_8, X_i, Y_i) dX dY}$$

with

$$\begin{aligned} \vartheta_1 &= \frac{\text{semileptonic bg}}{\pi^+\pi^-\pi^0} \\ \vartheta_2 &= \frac{\text{muonic bg}}{\text{electronic bg}} \end{aligned} \quad (4.25)$$

and

$\vartheta_3.. \vartheta_8$: the slope parameters

$\wp(\vartheta_1, \vartheta_2, X_i, Y_i)$ is the parameterization of the acceptance and the background for a certain X and Y value. ϑ_1 and ϑ_2 are two free fit parameters giving the ratio of the semileptonic background to the $\pi^+\pi^-\pi^0$ acceptance and the ratio of the muonic background to the electronic background (eq. 4.25). The shape of the acceptance and the background is taken from the previous fit and left unchanged at this level.

The same fit engine as for the parameterization of the acceptance and the background was used here. The fit yields

$$\begin{aligned} g &= 0.6831 \pm 0.0044 \\ h &= (6.17 \pm 0.38) \cdot 10^{-2} \\ j &= (0.99 \pm 2.44) \cdot 10^{-3} \\ k &= (1.04 \pm 0.17) \cdot 10^{-2} \\ f &= (3.65 \pm 2.38) \cdot 10^{-3} \end{aligned} \quad (4.26)$$

4.9 Determination of the slope parameters for $K_L \rightarrow \pi^+ \pi^- \pi^0$

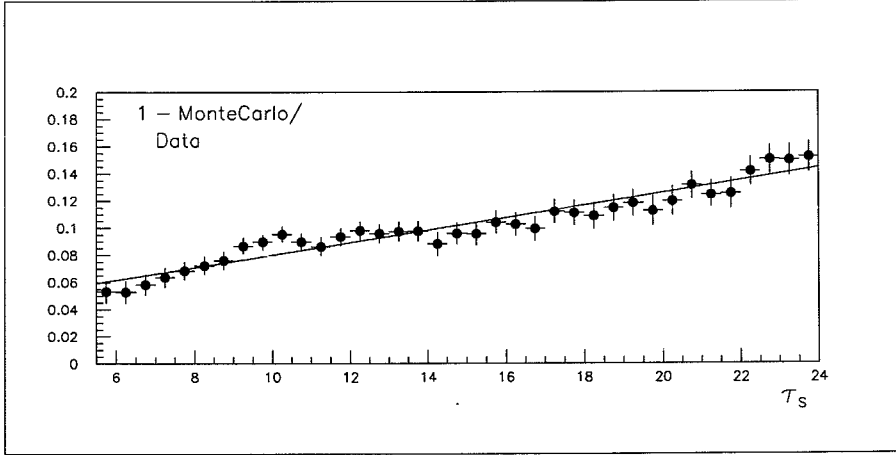
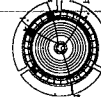


Figure 4.8: Fit of the total background fraction $\zeta_B(t)$ as a function of the decay time measured in units of $\tau_S/2$.

4.9.1 Total background fraction $\zeta_B(t)$

To determine the background we introduced the two parameters ϑ_1 and ϑ_2 above. The results of the fit are

$$\begin{aligned}\vartheta_1 &= (6.45 \pm 0.41) \cdot 10^{-2} \\ \vartheta_2 &= (76.04 \pm 0.45) \cdot 10^{-2}\end{aligned}\tag{4.27}$$

Fig. 4.7 shows the fitted decay time distribution for data and Monte Carlo events. The black dots are the data. The solid line shows the Monte Carlo $\pi^+ \pi^- \pi^0$ and semileptonic background data weighted by the fractions ϑ_1 and ϑ_2 . Fig. 4.7 shows clearly, how well the data sample is reproduced by the fit.

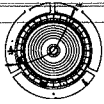
The parameters ϑ_1 and ϑ_2 can now be compared to the values derived independently by other members of the CPLEAR collaboration using completely different fit methods[80]. There the total background fraction $\zeta_B(t)$ is conventionally introduced as

$$\zeta_B(t) = 1 - \frac{N_{MC}(t)}{N_{data}(t)}\tag{4.28}$$

where $N_{MC}(t)$ is the fitted amount of $K^0, \bar{K}^0 \rightarrow \pi^+ \pi^- \pi^0$ Monte Carlo events in its relative normalization to $N_{data}(t)$ which is the amount of data at a certain decay time. A well fitting parameterization for $\zeta_B(t)$ is found by the expression

$$\zeta_B(t) = a + b \cdot t + e^{(c+d \cdot t)}\tag{4.29}$$

The exponential term describes the residual background at short life times and can be neglected in our fit. In fig. 4.8 the points with the error bars are the total background fraction, which is now fitted using the fit results of ϑ_1 and ϑ_2 . The plotted errors are purely statistical. According the total background fraction our fit is in good agreement with the existing results of the CPLEAR collaboration (fig. 4.6). It is worth noting that they derived their fit values using



neither the maximum likelihood method nor did they directly fit the Dalitz-plot distribution.

$$\zeta_B(t) = -(0.008 \pm 0.004) + (0.0061 \pm 0.0004) \cdot t \quad (4.30a)$$

$$\zeta_B(t) = -(0.004 \pm 0.009) + (0.0059 \pm 0.0007) \cdot t \quad (4.30b)$$

Eq. 4.30a gives the fit values for ζ_B according to [80], whereas eq. 4.30b gives the values derived by the above Dalitz-plot fit. The fact of this proper consistency between our fit and the results of the CPLEAR collaboration is a clear sign that the fit procedure works reliably and that we are able to control fitting correctly.

4.9.2 Self-consistency check by the Monte Carlo data

A lot of efforts was put on various tests of the fit to bolster confidence in the fit method and procedure and to prove that in spite of the large parameter space the fit works completely controllably in all parts and produces reliable results. An important part of these tests were extensive self-consistency checks on fit procedures.

The Monte Carlo data are produced by default with all slope parameters set to 0. The self-consistency of the fit was tested by splitting the $\pi^+\pi^-\pi^0$ acceptance sample randomly in two parts. The first half of the sample was used to parameterize the acceptance. The other half replaced the experimental data sample for the fit of the slope parameters. As expected the fitted values of the slope parameters are without exception compatible with 0 within their standard uncertainties.

The whole procedure can be repeated with the same two Monte Carlo samples, but we now weight the second data sample by any desired default slope parameter values which are not equal 0. We repeated the test several times not only by choosing the default slope parameter according to the expected values found in the PDG[9], but also by extending the parameter range far above these values. Again we proved that the fit behaves in perfect consistency with the expectations. In all cases which we tested, the fit reproduced the input slope parameter values properly within their statistical errors.

These self-consistency tests can be regarded as a further proof that the fit procedures are understood and produce reliable results.

4.9.3 Radiative corrections

Radiative corrections for $K_L \rightarrow \pi^+\pi^-\pi^0$ must be added to the Dalitz-plot distribution. Although these corrections show only small effects on most of the slope parameters, the radiative correction for the parameter g is about 18% of the statistical error and therefore not negligible. For the radiative correction for virtual photons and photons with a momentum less than 1 MeV a formalism by Neveu and Scherk[81] is used. All events were reweighted with this correction and the fit was repeated. For the inner Bremsstrahlung with photons possessing a momentum greater than 1 MeV a sample of $K_L \rightarrow \pi^+\pi^-\pi^0\gamma$ events was created and used as a background contribution according to Ferrari and Rosa-Clot[82].

4.9 Determination of the slope parameters for $K_L \rightarrow \pi^+ \pi^- \pi^0$

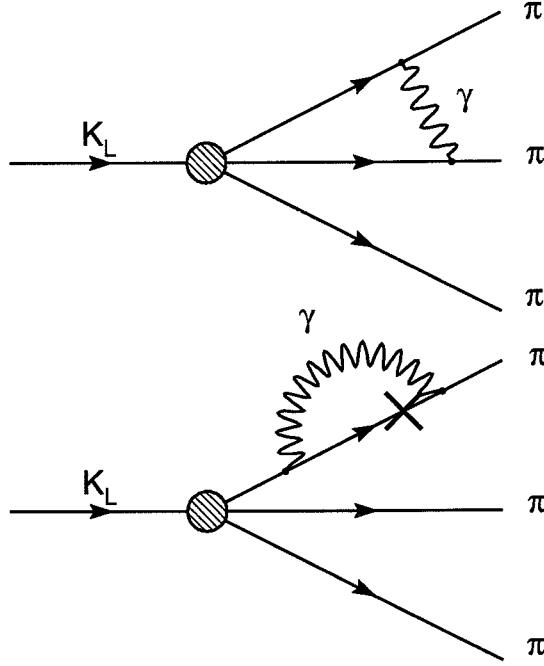
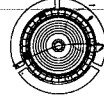


Figure 4.9: First order Feynman diagrams for radiative corrections of $K_L \rightarrow \pi^+ \pi^- \pi^0$.

Electromagnetic contribution by virtual photons

The contribution of virtual photons and photons with a momentum $< 1 \text{ MeV}/c$ [81] is applied by considering the Feynman diagrams of fig. 4.9. For Dalitz-plot regions, where the relative velocity between the pions is small, higher order Coulomb corrections become also important. But this occurs on a very small part of the Dalitz-plot, so that they become negligible[81].

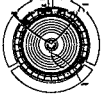
We define dw_0 to be the decay rate at a given point of the Dalitz-plot without electromagnetic corrections. The correction for dw_0 takes the form

$$dw = dw_0 \left((3\alpha/\pi) \ln(\Lambda/m_{\pi^+}) + a \cdot \ln(\epsilon/m_{\pi^+}) + \alpha\pi(1+v^2)/2v + (\alpha/\pi)C \right) \quad (4.31)$$

with

$$a = (\alpha/\pi) \left(-1 + \frac{1+v^2}{2v} \ln \left(\frac{1+v}{1-v} \right) \right) \quad (4.32)$$

Λ is the ultraviolet cut-off, ϵ is the upper limit for the energy of the unobserved photons emitted in the diagram and α is the Sommerfeld fine-structure constant. v is the velocity of the π^+ in the center of mass of the charged pions. C is a finite term of the form



$$C = \frac{1}{2v} \ln\left(\frac{1+v}{1-v}\right) + \frac{1+v^2}{2v} \left[\ln\left(\frac{1+v}{1-v}\right) + 2 \ln\left(\frac{1+v}{1-v}\right) \ln\left(\frac{1-v^2}{v^2}\right) + \right. \\ \left. f\left(\frac{2}{1-v}\right) - f\left(\frac{2}{1+v}\right) + 2f\left(\frac{1}{1+v}\right) - 2f\left(\frac{1}{1-v}\right) + \right. \\ \left. f\left(\frac{1-v}{1+v}\right) - f\left(\frac{1+v}{1-v}\right) \right] - 1/4 - 2 \ln(2) \quad (4.33)$$

The Spence function $f(x)$ is defined[83] as

$$f(z) = \int_0^z \frac{-\log|1-y|}{y} dy \quad (4.34)$$

The radiative corrections were calculated event by event. In our correction we used the values $\epsilon = 1$ MeV and $\Lambda = 1$ GeV. The correction weights depend on the kinetic energy T_{π^0} of the neutral pion in the rest frame of the K_L and reach a maximal deviation from unity of 9.9 % for maximal T_{π^0} .

The error in this electromagnetic correction can be estimated. There are different contributions: the strong interaction is not taken into account and there is a cut-off term $\ln(\Lambda/m_{\pi^+})$ which replaces a correctly evaluated term depending on the electromagnetic form factor of the pion. This would decrease the result by 0.6 % in absolute magnitude and introduces an uncertainty of about $\pm 5 \times 10^{-3}$ to the correction[81]. Feynman diagrams of higher order are neglected in the calculation and the correction cannot account for the $|\Delta I| = 3/2$ amplitude, which exists in the decay $K_L \rightarrow \pi^+\pi^-\pi^0$. The overall uncertainty amounts to 5.5 % of the electromagnetic correction.

Bremsstrahlung contribution to $K_L \rightarrow \pi^+\pi^-\pi^0\gamma$

The Dalitz-plot for the K_L decay can be fitted by the dependence of the decay amplitude A on the energy of the pions. A takes the form[82]

$$A = A_0 \left(1 + \frac{\lambda}{m_{K_L}} \left(\frac{3p_{K_L} p_{\pi^0}}{m_{K_L}} - m_{K_L} \right) \right) \quad (4.35)$$

A is the decay amplitude for the non radiative process. A_0 is a constant and the slope parameter is $\lambda = -2.10 \pm 0.18$ [82] for $K_L \rightarrow \pi^+\pi^-\pi^0$. The p 's are the four-momenta of the neutral kaon and the pions.

Using for $p_{K_L} \cdot p_{\pi^0}$ the relation

$$p_{K_L} \cdot p_{\pi^0} = \frac{m_{K_L}^2 + m_{\pi^0}^2 - 2m_{\pi^+}^2}{2} - p_{\pi^+} p_{\pi^-} \quad (4.36)$$

and applying perturbative theory we get the Bremsstrahlung amplitude M_μ to the first order of the electromagnetic coupling constant in the form[82]

$$\overline{M}_\mu = \sqrt{4\pi\alpha} \frac{A}{p_{\pi^+} k} (p_{\pi^+} - p_{\pi^-})_\mu \quad (4.37)$$

where $\overline{M}_\mu$ is given by those Feynman graphs in which the photon is emitted from an external line (fig. 4.10). The approximate amplitude $\overline{M}_\mu$ for Bremsstrahlung in the K_L -decay reproduces the true amplitude M_μ up to terms of the order

4.9 Determination of the slope parameters for $K_L \rightarrow \pi^+\pi^-\pi^0$

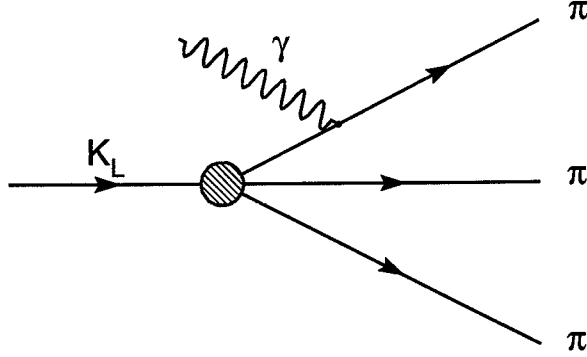
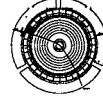


Figure 4.10: First order Feynman diagram for the Bremsstrahlung in $K_L \rightarrow \pi^+\pi^-\pi^0\gamma$.

$\mathcal{O}(k)$. α is the Sommerfeld fine-structure constant and k the energy of the emitted photon.

The emission probability of the photon with energy larger than k can be obtained by summing over the photon polarization and then integrating $\overline{M}_\mu^* \overline{M}^\mu$ over the relevant part of the phase space. With the integration we get the total rate for the emission of a photon with an energy larger than k . It is convenient to present the result of the integration by defining the rate $R_\gamma(k)$ for the radiative decay of $K_L \rightarrow \pi^+\pi^-\pi^0\gamma$ as

$$R_\gamma(k) = \frac{R_{\pi^+\pi^-\pi^0\gamma}(k)}{R_{\pi^+\pi^-\pi^0}} \quad (4.38)$$

where $R_{\pi^+\pi^-\pi^0\gamma}(k)$ is the rate for the emission of a γ -quantum with an energy larger than k and $R_{\pi^+\pi^-\pi^0}$ is the rate for $K_L \rightarrow \pi^+\pi^-\pi^0$. $R_\gamma(k)$ can be parameterized[82] as

$$R_\gamma(k) = \frac{a(k)\lambda^2 + b(k)\lambda + c(k)}{\alpha\lambda^2 + \beta\lambda + \gamma} \quad (4.39)$$

The values α , β and γ are independent of k and are given for the K_L -decay as

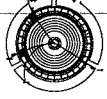
$$\alpha = 13.0 \quad (4.40a)$$

$$\beta = -65.0 \quad (4.40b)$$

$$\gamma = 1937.0 \quad (4.40c)$$

The values $a(k)$, $b(k)$ and $c(k)$ for various k are tabulated in [82].

Since we use $\overline{M}_\mu$ instead of the true amplitude M_μ we get an error in the values of $R_\gamma(k)$, which is dependent of k and the slope parameter λ . For a large λ , i.e. for a strong final interaction acting among the pions, the error becomes large too, whereas for a large k the corrections to M_γ are suppressed by the phase space factor. With $\lambda = -2.10 \pm 0.18$ for the K_L -decay the error amounts to 2.5 % of R_γ .



Result with radiative corrections

The fit was done twice, once by adding the radiative corrections and once without. The corrections due to Bremsstrahlung effects were considered by using them as an other background subtraction. We generated a sample of $K_L \rightarrow \pi^+\pi^-\pi^0\gamma$ Monte Carlo events and applied it to the acceptance fit as additional background.

The overall electromagnetic correction to the extracted values of the Dalitz-plot coefficients g , h , j , k and f is small but not negligible. The most significant effect is found for the slope parameter g . There the correction amounts to 18 % of the statistical error. The correction for h amounts to 11 %, for k to 12 % and for f to 38 % of their statistical errors. For the parameter j is the correction smaller than 10^{-5} . In table 4.1 and 4.2 the slope parameters are listed first by applying the electromagnetic corrections and then without applying them.

	value	stat.error
g	0.6831	± 0.0044
h	0.0617	± 0.0038
j	0.0010	± 0.0024
k	0.0104	± 0.0017
f	0.0037	± 0.0024

Table 4.1: Overview of the slope parameter values without applying the electromagnetic corrections.

	value	stat.error
g	0.6823	± 0.0044
h	0.0613	± 0.0038
j	0.0010	± 0.0024
k	0.0106	± 0.0017
f	0.0045	± 0.0024

Table 4.2: Overview of the slope parameters values by applying the electromagnetic corrections.

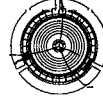
To estimate the additional uncertainty introduced into the values of the slope parameters by the uncertainty of the electromagnetic corrections, the fit procedure was repeated by varying the electromagnetic corrections within their standard uncertainties. The additional error was proved to be very small, i.e. $< 10^{-5}$ for all slope parameters, and therefore completely negligible.

4.9.4 Systematic errors

Various sources of systematic errors have been studied. The main contributors of systematic errors were found to be the finite X,Y -resolution of the detector and the parameterizations of the background and the acceptance. All systematic errors are discussed in the following paragraphs:

A systematic error occurs due to the standard uncertainties of the two fit parameters ϑ_1 and ϑ_2 , which give the background fraction and the fraction of

4.9 Determination of the slope parameters for $K_L \rightarrow \pi^+ \pi^- \pi^0$



muonic background to electronic background. The error was evaluated by varying the fractions within their 1σ uncertainty while redoing the fit. We derived

$$\begin{aligned}\Delta g &= \pm 1.5 \cdot 10^{-4} & \Delta h &= \pm 3.0 \cdot 10^{-5} \\ \Delta j &= \pm 7.7 \cdot 10^{-6} & \Delta k &= \pm 4.5 \cdot 10^{-5} \\ \Delta f &= \pm 2.1 \cdot 10^{-5}\end{aligned}\quad (4.41)$$

With the exception of the parameter g the error is for all parameters $< 10^{-4}$ and can be neglected.

The calculation of the electromagnetic corrections introduce another systematic error, since e.g. higher order Feynman graphs were not considered in the calculations. The standard uncertainties of the electromagnetic correction can be evaluated. The systematic error on the slope parameters was found by varying the electromagnetic corrections within their standard uncertainties. For all slope parameters this systematic error is extremely small and is therefore not listed here:

$$\Delta g, \Delta h, \Delta j, \Delta k, \Delta f < 10^{-5} \quad (4.42)$$

Another systematic error is originated by the normalization according to the LUT. The values of the averaged normalization factors α_0 (eq. 4.2) were varied within their standard uncertainties. We found

$$\begin{aligned}\Delta g &= \pm 3.0 \cdot 10^{-5} & \Delta h &= \pm 4.4 \cdot 10^{-5} \\ \Delta j &= \pm 3.2 \cdot 10^{-5} & \Delta k &= \pm 1.8 \cdot 10^{-5} \\ \Delta f &= \pm 1.5 \cdot 10^{-5}\end{aligned}\quad (4.43)$$

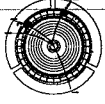
This systematic error can also be neglected for all slope parameters.

There is a further systematic error because of the finite X, Y -resolution of the detector. It should be noticed that this error was found not to be negligible, although it plays a minor role. To get the error the fit was repeated once using the true event information from the Monte Carlo simulation and once with the reconstructed values. We derived for the systematic errors

$$\begin{aligned}\Delta g &= \pm 4.7 \cdot 10^{-4} & \Delta h &= \pm 7.2 \cdot 10^{-4} \\ \Delta j &= \pm 6.1 \cdot 10^{-4} & \Delta k &= \pm 4.5 \cdot 10^{-4} \\ \Delta f &= \pm 1.0 \cdot 10^{-3}\end{aligned}\quad (4.44)$$

An important source of error in the uncertainty was introduced by the limited statistics for the parameterization of the acceptance and background, which e.g. for the acceptance was a sample of about 2'000'000 Monte Carlo events. There are uncertainties in each parameter fitted in the parameterization. The values for the systematic error were derived by error propagation of the standard uncertainties of the parameter fitted. The errors were

$$\begin{aligned}\Delta g &= \pm 1.5 \cdot 10^{-3} & \Delta h &= \pm 1.4 \cdot 10^{-4} \\ \Delta j &= \pm 2.2 \cdot 10^{-5} & \Delta k &= \pm 2.3 \cdot 10^{-5} \\ \Delta f &= \pm 7.9 \cdot 10^{-5}\end{aligned}\quad (4.45)$$



As described in a previous section, the data have to be corrected due to coherent regeneration. This difference is corrected by calculating the effect on the neutral kaons event by event. These calculations also adhere a systematic error. The fit was repeated by now omitting the regeneration corrections. The effect on the slope parameters is extremely small. For all slope parameters the systematic error contribution is

$$\Delta g, \Delta h, \Delta j, \Delta k, \Delta f < 10^{-5} \quad (4.46)$$

The main source of systematic error was found by applying a standard procedure of the Particle Data Group[9], which forces the global reduced χ^2 value over all τ_s intervals and all slope parameters to 1. To understand this error we have performed many tests on the data. All these tests seem to show that these systematic errors originate from the difficulty of the Monte Carlo simulation to produce absolute acceptances in the different decay time intervals. The hypothesis is confirmed by the fact that such difficulties are not observed when analysing asymmetries (eq. 2.74) where the Monte Carlo acceptance is cancelled. The systematic error was found to be

$$\begin{aligned} \Delta g &= \pm 4.4 \cdot 10^{-3} & \Delta h &= \pm 1.2 \cdot 10^{-2} \\ \Delta j &= \pm 3.6 \cdot 10^{-3} & \Delta k &= \pm 2.4 \cdot 10^{-3} \\ \Delta f &= \pm 5.9 \cdot 10^{-3} \end{aligned} \quad (4.47)$$

Test of the condition $\langle E_{CM}(\pi^+) \rangle = \langle E_{CM}(\pi^-) \rangle$

The assumption that the $\mathcal{CP}$ -allowed $K_S \rightarrow \pi^+\pi^-\pi^0$ decay amplitude cancels is only true if the acceptance of π^+ is equal to the acceptance of π^- for $\langle E_{CM}(\pi^+) \rangle = \langle E_{CM}(\pi^-) \rangle$, which means that the Dalitz-plot distribution of the $\pi^+\pi^-\pi^0$ acceptance must be symmetric in X . As mentioned, this symmetry possesses special importance not for the fit described in this work, but for the $\pi^+\pi^-\pi^0$ analysis using the asymmetries according to eq. 2.74.

With the analysis presented here it is now possible to perform another symmetry check on the data sample in addition to the previous one by using the normalized average deviation $S(X)$ (eq. 4.23) of the symmetry in X for a specific Y . This test can be regarded as a further independent verification of the validity of the assumption above for the remaining $\pi^+\pi^-\pi^0$ analysis.

For the symmetry check the fit was repeated by explicitly demanding the terms of the fit polynomial which are of odd powers of X to be 0. This way the parameterization of the acceptance and the background is forced to be completely symmetric in X . The fit yielded

$$\begin{aligned} g &= 0.6831 \pm 0.0045 & h &= (6.11 \pm 0.45) \cdot 10^{-2} \\ j &= (-1.8 \pm 2.44) \cdot 10^{-3} & k &= (1.04 \pm 0.18) \cdot 10^{-2} \\ f &= (2.70 \pm 2.77) \cdot 10^{-3} \end{aligned} \quad (4.48)$$

Since in the parameterization all the terms of odd powers in X were compatible with 0 within their standard uncertainties anyway, there is no additional contribution to the systematic error. Its contribution is already considered in

4.9 Determination of the slope parameters for $K_L \rightarrow \pi^+ \pi^- \pi^0$



the systematic error which is introduced by the Monte Carlo acceptance and background statistics.

The symmetry test shows that even if we force terms with the odd powers in X to 0 in the parameterization, the extracted values for the slope parameters vary only slightly and only within the systematic error which is introduced by the Monte Carlo statistics. In addition this test proves yet again that the fit procedure produces reliable results.

4.9.5 Results

Table 4.3 summarizes the fit results for all slope parameters with their statistic and systematic errors.

	para. value	stat.error	sys.error
g	0.6823	± 0.0044	± 0.0044
$h \cdot 10^{-2}$	6.13	± 0.38	± 1.45
$j \cdot 10^{-3}$	0.99	± 2.44	± 2.96
$k \cdot 10^{-2}$	1.06	± 0.17	± 0.24
$f \cdot 10^{-3}$	4.48	± 2.37	± 5.86

Table 4.3: Overview of the final results for the slope parameters.

The correlations between the slope parameters are all small. They are for all parameter < 0.075 and given in the normalized error-correlation matrix below

$$\begin{array}{c} j \\ g \\ k \\ f \\ h \end{array} \begin{pmatrix} j & g & k & f & h \\ 1 & & & & \\ -0.050 & 1 & & & \\ -0.017 & 0.003 & 1 & & \\ 0.052 & 0.066 & 0.005 & 1 & \\ 0.075 & 0.011 & 0.031 & 0.056 & 1 \end{pmatrix} \quad (4.49)$$

The same fit but by setting parameter f to 0, as it is common in literature, leaves the parameter values unchanged. Table 4.4 gives an overview to all systematic errors.

Systematic error	$g \cdot 10^{-3}$	$h \cdot 10^{-2}$	$j \cdot 10^{-3}$	$k \cdot 10^{-3}$	$f \cdot 10^{-3}$
time slice MC def.	± 3.8	± 1.4	± 2.8	± 2.3	± 5.6
accept. statistic	± 2.3	± 0.1	0.9	0.5	± 1.7
XY resolution	± 0.5	± 0.1	± 0.6	± 0.5	± 1.0
bg fractions	—	—	—	—	—
regeneration	—	—	—	—	—
normalization (LUT)	—	—	—	—	—
radiative correct.	—	—	—	—	—

Table 4.4: Overview of the systematic errors.

Where no value is given we got an error which is $< 10^{-5}$. All parameters are in good agreement with the values from PDG[9]. In addition, our fit result of the parameter g is better than the present world average by a factor of 2. For the rest of the parameters we derive larger errors than the best experiment[84] quoted by

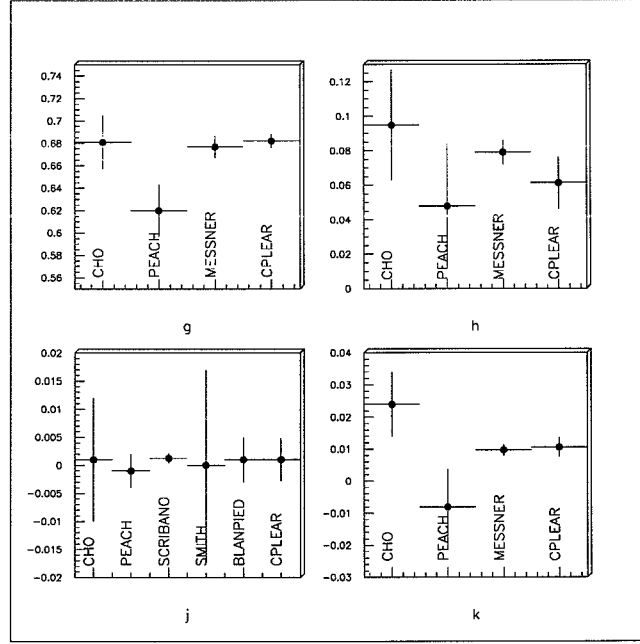
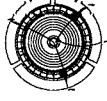


Figure 4.11: The fit results for the slope parameters in comparison with the experiments used by the PDG to fit their world average.

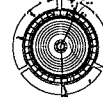
PGD, but we are still a factor of 3 to 10 better than the other experiments quoted [85, 86, 87]. Fig. 4.11 shows our values for the slope parameters in comparison with the values from the experiments used by the PDG to obtain their world averages. To obtain the world averages by including our measurements we repeated the fit performed by the Particle Data Group. The results are listed in table 4.5.

	World average	
	without CPLEAR	including CPLEAR
j	0.0011 ± 0.0008	0.0011 ± 0.0008
g	0.670 ± 0.014	0.675 ± 0.007
k	0.0098 ± 0.0018	0.0098 ± 0.0016
h	0.079 ± 0.007	0.077 ± 0.006
f	—	4.48 ± 6.32

Table 4.5: World average by PDG with and without our measurement.

With the relations 2.108 we can now express the slope parameters in the decay amplitudes β , ζ and ξ of $K_L \rightarrow \pi^+\pi^-\pi^0$. The decay amplitudes appear in ratios to the K_L decay amplitude α . Therefore α can not be determined using the slope parameters. We put α to the world average value according to [10]. $Re(\epsilon)$ is taken from the PDG too. The amplitude γ , which is also listed below, is not calculated from this analysis, but is derived from other work on the CPLEAR data sample.

4.9 Determination of the slope parameters for $K_L \rightarrow \pi^+\pi^-\pi^0$



The decay amplitudes can now be compared to their theoretical values using Chiral perturbation theory at $\mathcal{O}(p^6)$ [11]. Table 4.6 shows the decay amplitudes of the fit to our experiment, the world average and the theoretical values from perturbation theory at $\mathcal{O}(p^6)$.

	Chiral perturbation theory at $\mathcal{O}(p^6)$ (Kambor[34])	World average (Devlin[86])	CLEAR
α	84.2	84.3 ± 0.4	—
β	-28.1	-28.3 ± 0.6	-28.6 ± 0.2
γ	2.5	2.5 ± 0.4	2.6 ± 0.8
ξ	-1.4	(-1.6 ± 0.1)	-1.9 ± 0.1
ζ	-0.6	(-0.7 ± 0.1)	-0.5 ± 0.1

Table 4.6: Expected values for the decay amplitudes of $K_L \rightarrow \pi^+\pi^-\pi^0$ by using Chiral perturbation theory at $\mathcal{O}(p^6)$ [34], together with the world average according to [86] and with the fit to the experiment.

Our values are in good agreement with the theoretical prediction as well as with the values obtained by older experiments. The result for the decay amplitude β is better than the present world average by a factor of 3. The precise measurements are an important test to the calculations of the Chiral perturbation theory up to the order $\mathcal{O}(p^6)$ and can also serve as new input to these calculations.

Chapter 5

Conclusion

It has been demonstrated that the online first trigger step for real time tracking based on neural network algorithms[52] can be realized. The trigger step has proofed to be both fast and reliable during operation.

The inherent structure of the ANN trigger accounts for its easy adaptability to evolving experimental conditions or new trigger environments. The same hardware structure can be used with different weight sets to meet various requirements for input data. This also means that increased knowledge of event topologies within the detector can be met by a new set of properly adapted weights. The ANN trigger developed at CPLEAR is now used in other experiments' completely different trigger environments[69][68] which proofs the flexibility of this online trigger step.

We applied a unbinned maximum likelihood procedure for the Dalitz-plot analysis of the decay $K_L \rightarrow \pi^+\pi^-\pi^0$ of CPLEAR events. This is the most complete analysis to use, where no information is lost in any step of the analysis. With this method we extracted a new, independent set of slope parameters. For the slope parameter g and the decay amplitude β we established values which are better than their present world averages[9]. Our values improve the world average for the slope parameter g by a factor of 2 and for the decay amplitude by a factor of 3. Our precise measurements are an important test to the calculations of the Chiral perturbation theory up to the order $\mathcal{O}(6)$ [11] and can also serve as new input to these calculations.

Appendix A

Gram-Smidt orthogonalization

A.1 Orthogonality

For $D \subset \mathbb{R}^2$ let f and g be two mappings of $D \rightarrow \mathbb{R}$. A scalar product (f, g) shall be defined as $\int_D f g d\vec{x}$. Two functions f and g are called **orthogonal** if

$$(f, g) = 0 \quad \text{with } f \neq g \quad (\text{A.1})$$

We choose a set $\Phi = \{\phi_j | j = 1, \dots, k\}$ of ordered, linear independent, real-valued mappings of D . An **orthogonalization** of Φ is to find an ordered orthogonal base Ψ of real-valued mappings ψ_j for Φ , i.e. for each i , ϕ_i can be written as a finite linear combination of elements of the set $\{\psi_k | k \leq i\}$, where the ψ_k are pairwise orthogonal.

A.2 Orthogonalization of $\Phi = \{\phi | D \rightarrow \mathbb{R}, D \subset \mathbb{R}^2\}$

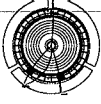
Φ shall be the set of functions $\phi_j = X^{j_1} \cdot Y^{j_2}$ with $\vec{j} = (j_1, j_2) \in J$, j_i are non-negative integers in the vector coordinates and X, Y being the coordinate variables of the Dalitz-plot. We introduce an order $\sigma(j) = j_1 + j_2$ in Φ by defining the order of J as:

i) $i < j$: if $\sigma(i) < \sigma(j)$
and

ii) $i < j$: if $\sigma(i) = \sigma(j)$ and $i_2 < j_2$

The set Φ of linearly independent functions ϕ_i can be orthogonalized with the aid of the Gram-Schmidt orthogonalization process[76]. The orthogonal basis of functions in X and Y over the Dalitz-plot region shall be $\Psi = \{\psi_{(n,k)}\}$ with n giving the degree of the polynomial and $k = (1, \dots, n)$. To construct the ψ_i we choose the basic approach

$$\psi_{(n,k)} = X^{n-k} Y^k + \sum_{i=0}^{k-1} a_{(n,i)}^{(n,k)} X^{n-i} Y^i + \sum_{\ell=0}^{n-1} \sum_{j=0}^{\ell} a_{(\ell,j)}^{(n,k)} X^{\ell-j} Y^j \quad (\text{A.2})$$



The indices of the equation are written in brackets and follow the introduced order σ naturally with $j_1 = n - k$ and $j_2 = k$. The first term corresponds to the next ϕ to be orthogonalized. The second terms are correlated to the ψ_i of the same order which are already orthogonalized. The third term is derived from all lower orders which are already orthogonalized. We choose the first ψ to be:

$$\psi_{(0,0)} = \psi_0 = 1 \quad (\text{A.3})$$

For the recursion formula we assume all polynomials up to $\psi_{(n,i)}$ to be known. For the $\psi_{(n,i+1)}$ we get the condition from the scalar product:

$$(\psi_{(n,i+1)}, \psi_{(m,\ell)}) = 0 \quad (\text{A.4})$$

$$\begin{aligned} (X^{n-k}Y^k, \psi_{(m,\ell)}) + \sum_{i=0}^{k-1} a_{(n,i)}^{(n,i+1)} (X^{n-i}Y^i, \psi_{(m,\ell)}) + \\ \sum_{\ell=0}^{n-1} \sum_{j=0}^{\ell} a_{(\ell,j)}^{(n,i+1)} (X^{\ell-j}Y^j, \psi_{(m,\ell)}) = 0 \end{aligned} \quad (\text{A.5})$$

Therefore the problem of orthogonalizing Φ is equivalent to solving the system of linear equation for each ψ_i :

$$M^{(n,i+1)} \vec{a}^{(n,i+1)} = \vec{V}^{(n,i+1)} \quad (\text{A.6})$$

$$\text{with : } M_{((n,k),(m,\ell))}^{(n,i+1)} = (X^{n-k}Y^k, \psi_{(m,\ell)}) \quad (\text{A.7})$$

$$\text{and : } V_{(m,\ell)}^{(n,i+1)} = - (X^{n-(i+1)}Y^{i+1}, \psi_{(m,\ell)}) \quad (\text{A.8})$$

Dank

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¹Der mich auf die Wichtigkeit der Existenz von Meerjungfrauen aufmerksam machte.

²Für Seine unermüdlichen Bemühungen, diese Arbeit stilistisch auf ein Niveau zu bringen, welches Seinen ästhetischen Ansprüchen auch nur halbwegs zugenügen vermag.

³Dem es nie zuviel wurde diese Arbeit zu unterstützen, indem er mich zu Forschungszwecken an die entlegensten und düstersten Winkel des Abendlandes begleitete. So behauptet er, dass er von einer dieser Forschungsexpeditionen noch heute in klaren Vollmondnächten die kalten Hände Zitas im Rücken spühre.

⁴Mein spezieller Dank gilt Seinem unglaublich wichtigen Beitrag: 'Wenn zufälligerweise die stabile Mannigfaltigkeit eines Fixpunktes mit der instabilen Mannigfaltigkeit dieses oder eines anderen Fixpunktes zusammenfällt, gehören diese nichttransversalen homoklinen bzw. heteroklinen Separatrizen auch zur invarianten Menge Λ , aber die Struktur der invarianten Menge bleibt immer noch relativ einfach'.

⁵Für die musikalische Untermalung der Arbeit.

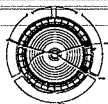
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Während meines Studiums an der Universität Basel habe ich Lehrveranstaltungen der folgenden Dozentinnen und Dozenten besucht: K. Alder, E. Allerton, G. Backenstoss, C. Bandle, E. Baumgartner, G. Baur, H. Burkhardt, H. Christen, J. Fünfschilling, H.-J. Güntherodt, H. Huber, H. Kraft, P. Oelhafen, H. Rudin, I. Sick, U. Steinlin, P. Talkner, G. A. Tammann, L. Tauscher, H. Thomas, D. Trautmann, C. Trefzger, C. Ullrich, R. Wagner, I. Zschokke.

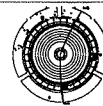
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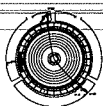


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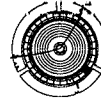


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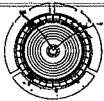


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