



Evaluation of Longitudinal Single-Bunch Stability in the SPS and Bunch Optimisation for AWAKE

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Summary

We examine the longitudinal single-bunch stability of proton beams in the SPS using Matrix Equations for LOngitudinal beam DYnamics calculations (MELODY). We determine the minimum stable bunch length achievable at a bunch intensity of $N_p = 4.0 \times 10^{11}$, and focus on its application to AWAKE. Our calculations use the expected SPS operational limits following LS2. We find that the shortest bunch length obtainable without bunch rotation is $\tau_{4\sigma} = 0.99$ ns, which requires maximising the RF voltage amplitudes. Bunch rotation can reduce $\tau_{4\sigma}$ significantly: to 0.77 ns without controlled emittance blow-up and to 0.84 ns with blow-up. Optimisation of phase and voltage jumps can improve the symmetry of the bunch shape and can further decrease the bunch length to 0.71 ns and 0.78 ns, respectively. However, this option requires modifications of the 800 MHz Low Level RF system.

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1 Introduction

Due to their very small mass, electrons lose significant energy from bremsstrahlung radiation in a synchrotron. Thus, it is often preferable to accelerate them with linear accelerators. However, in order to reach very high energies, these accelerators need to be very long, which can become costly. The size required can be reduced if the accelerating voltages are increased. While current methods reach gradients on the order of MV/m, it is known that plasmas can sustain fields beyond the order of GV/m. The Advanced WAKEfield Experiment (AWAKE) [1], located at extraction of the Super Proton Synchrotron (SPS), is designed to test the capabilities of plasma wakefields to accelerate beams. If conclusive, this new technology could greatly reduce the size of linear accelerators needed to reach very high energies. The experiment requires high density single bunches to be injected into a plasma chamber at 400 GeV. This can be achieved by minimising proton bunch lengths and maximising bunch intensities.

Prior to the second long shutdown (LS2), the SPS could deliver single bunches with 2×10^{11} particles with a 4σ bunch length of 1.1 ns [2, 3], and more intense bunches had longer bunch lengths due to instabilities during the ramp. The SPS is equipped with a double-RF system with frequencies 200 MHz and 800 MHz. The main (200 MHz) RF system used a maximum voltage of ~ 7 MV during the AWAKE cycle, while the fourth-harmonic (800 MHz) RF system could provide a maximum voltage of ~ 2.5 MV [4]. Rearrangement of four existing cavities (with two spare sections) and RF power upgrades [5] during LS2 are expected to increase the maximum amplitude of the 200 MHz RF system to ~ 14 MV, while that of the 800 MHz system should remain at 2.5 MV. This will allow for much more freedom in the voltage settings during the beam acceleration cycle, leading to possible improvements in the beam quality. Moreover, the impedance reduction campaign [6] will be beneficial for beam stability.

We examine how to best take advantage of the new operational limits of the SPS to minimise the bunch length at high bunch intensities. We study three different scenarios: no bunch rotation on the SPS flat top, bunch rotation without controlled longitudinal emittance blow-up and bunch rotation with blow-up. In Section 2 we give a brief overview of the semi-analytical framework used for single-bunch stability calculations, implemented in the code MELODY (Matrix Equations for LOngitudinal beam DYnamics calculations) [7]. We also compare these semi-analytic results with macro-particle simulations done with the code BLonD (Beam Longitudinal Dynamics) [8]. In Section 3 we provide our results for the three different scenarios.

2 Longitudinal Single-Bunch Stability

2.1 Main Equations

The longitudinal motion of a particle in a synchrotron can be written in terms of the energy and phase deviations, ΔE and ϕ , of the particle relative to the synchronous particle. More precisely, if h is the harmonic number and ω_0 is the angular revolution frequency, then

the conjugate coordinates $\Delta E/(h\omega_0)$ and ϕ are related by

$$\dot{\phi} = \frac{h^2\omega_0^2\eta}{\beta^2 E_0} \left(\frac{\Delta E}{h\omega_0} \right), \quad (1)$$

$$\frac{d}{dt} \left(\frac{\Delta E}{h\omega_0} \right) = \frac{1}{2\pi h} [qV_t(\phi) - \delta E_0], \quad (2)$$

where β is the particle speed, $\eta = 1/\gamma_{\text{tr}}^2 - 1/\gamma^2$ is the slip factor, $\gamma^2 = 1/(1 - \beta^2)$, γ_{tr} is the transition Lorentz factor, E_0 is the synchronous particle energy, q is the particle charge and δE_0 is the energy gain per turn of the synchronous particle. These relations can be found in standard textbooks (e.g. [9]). The total voltage $V_t(\phi) = V_{\text{RF}}(\phi) + V_{\text{ind}}(\phi)$ includes contributions from both the RF system and the beam-induced fields. For the SPS, where the RF system is a double system with frequencies 200 MHz and 800 MHz and with controllable amplitudes V_{200} and V_{800} , the resulting RF voltage experienced by a particle with a phase ϕ (relative to the synchronous phase) is

$$V_{\text{RF}}(\phi) = V_{200} \left[\sin(\phi) + \frac{V_{800}}{V_{200}} \sin(4\phi + \Phi_{800}) \right], \quad (3)$$

where Φ_{800} is the phase offset between the two RF systems.

One can also use the energy $\mathcal{E}$ and phase ψ of the synchrotron oscillations to describe the longitudinal particle motion following the notations in Ref. [10]. These are related to the former conjugate variables by

$$\mathcal{E} = \frac{\dot{\phi}^2}{2\omega_{s0}^2} + U_t(\phi), \quad (4)$$

$$\psi = \text{sgn}(\eta\Delta E) \frac{\omega_s(\mathcal{E})}{\sqrt{2}\omega_{s0}} \int_{\phi_{\text{max}}(\mathcal{E})}^{\phi} \frac{d\phi'}{\sqrt{\mathcal{E} - U_t(\phi')}}, \quad (5)$$

where $U_t(\phi)$ is the total potential, $\omega_s(\mathcal{E})$ is the synchrotron frequency, $\phi_{\text{max}}(\mathcal{E})$ is the maximum phase of the particle with synchrotron energy $\mathcal{E} = U_t[\phi_{\text{max}}(\mathcal{E})]$, and

$$\omega_{s0}^2 = -\frac{h\omega_0^2\eta q V_{200} \cos \phi_{s0}}{2\pi\beta^2 E_0} \quad (6)$$

is the angular frequency of small synchrotron oscillations in a bare single-RF system, and $\phi_{s0} = \pi - \arcsin(\delta E_0/qV_{200})$ is the synchronous phase for the case when $\eta > 0$, $V_{800} = 0$, and intensity effects are neglected.

2.2 Perturbation Formalism

Consider the perturbation $\tilde{\mathcal{F}}$ to the stationary particle distribution function $\mathcal{F}$ in the longitudinal phase space. If this perturbation grows with time, then the beam is unstable. The initial time evolution of $\tilde{\mathcal{F}}$ is dictated by the linearised Vlasov equation (e.g. [11])

$$\frac{\partial \tilde{\mathcal{F}}}{\partial t} + \frac{d\mathcal{F}}{d\mathcal{E}} \frac{d\mathcal{E}}{dt} + \frac{d\psi}{dt} \frac{\partial \tilde{\mathcal{F}}}{\partial \psi} = 0. \quad (7)$$

This equation can be solved by expanding the perturbation $\tilde{\mathcal{F}}$ into the harmonics m of synchrotron motion with eigenfunctions $C_m(\mathcal{E})$ and $S_m(\mathcal{E})$ [12],

$$\tilde{\mathcal{F}}(\mathcal{E}, \psi, t) = e^{-i\Omega t} \sum_{m=1}^{\infty} [C_m(\mathcal{E}) \cos m\psi + S_m(\mathcal{E}) \sin m\psi], \quad (8)$$

leading to an eigenvalue problem [12]

$$\Omega^2 C_m(\mathcal{E}_n) = \sum_{n'=1}^{N_{\mathcal{E}}} \sum_{m'=1}^{\infty} M_{nmn'm'} C_{m'}(\mathcal{E}_{n'}), \quad (9)$$

where

$$M_{nmn'm'} = m^2 \omega_s^2(\mathcal{E}_n) \left[\delta_{nn'} \delta_{mm'} + \frac{4\omega_{s0}^2 \zeta \mathcal{E}_{\max}}{h\omega_s(\mathcal{E}_{n'}) N_{\mathcal{E}}} \frac{d\mathcal{F}(\mathcal{E}_n)}{d\mathcal{E}} \operatorname{Im} \left\{ \sum_{k=1}^{\infty} \frac{Z_k/k}{\operatorname{Im} Z/n} I_{mk}(\mathcal{E}_n) I_{m'k}^*(\mathcal{E}_{n'}) \right\} \right]. \quad (10)$$

Here δ_{ij} is the Kronecker delta, $\mathcal{E}_{\max}$ is the maximum synchrotron oscillation energy, $N_{\mathcal{E}}$ is the number of points used to discretise the energy, Z_k is the longitudinal impedance at frequency $k\omega_0$, and $\operatorname{Im} Z/n = 1$ ohm is the inductive wall impedance used here for impedance normalisation. More detailed derivations using variables $(\mathcal{E}, \psi)$ can be found in [10]. The intensity parameter, ζ , is a dimensionless parameter given by

$$\zeta = \frac{qh\omega_{\text{RF}} N_p \operatorname{Im} Z/n}{V_{200} \cos \phi_{s0}}, \quad (11)$$

where N_p is the bunch intensity and $\omega_{\text{RF}} = h\omega_0$ is the RF angular frequency. The function $I_{mk}(\mathcal{E})$ is defined as

$$I_{mk}(\mathcal{E}) = \frac{1}{\pi} \int_0^{\pi} d\psi e^{ik\phi(\mathcal{E}, \psi)/h} \cos m\psi. \quad (12)$$

2.3 Overview of the Code MELODY

The code Matrix Equations for LOngitudinal beam DYnamics calculations (MELODY) computes the stability of a bunch using the above framework. The stationary distribution including intensity effects (i.e. in the distorted potential well) is first calculated using an iterative procedure with an initial distribution that excludes intensity effects. The maximum number of azimuthal modes m is fixed to some finite value and the eigenvalues of the (reduced) matrix M in Eqs. (9) and (10) are then computed. The bunch is unstable whenever M has negative or complex eigenvalues [12].

In this work we perform calculations with five azimuthal modes ($m \leq 5$); in some cases convergence is reached when up to 10 modes are used. The energy space is sliced into $N_{\mathcal{E}} = 200$ points, and the frequency space is sliced into $N_k = 100000$ points (after interpolation of 200 points) with a maximum at $k_{\max} = 32h$ which covers the SPS impedance [13] up to 6.4 GHz with frequency resolution of ~ 64 kHz. We use a binomial stationary particle distribution function,

$$\mathcal{F}(\mathcal{E}) = \frac{1}{2\pi\omega_{s0} A_N} \left(1 - \frac{\mathcal{E}}{\mathcal{E}_{\max}} \right)^{\mu}, \quad (13)$$

where the normalisation constant is

$$A_N = \omega_{s0} \int_0^{\mathcal{E}_{\max}} \frac{d\mathcal{E}}{\omega_s(\mathcal{E})} \left(1 - \frac{\mathcal{E}}{\mathcal{E}_{\max}}\right)^\mu. \quad (14)$$

The corresponding line density can be written as

$$\lambda(\phi) = 2\omega_{s0} \int_{U(\phi)}^{\mathcal{E}_{\max}} d\mathcal{E} \frac{\mathcal{F}(\mathcal{E})}{\sqrt{2[\mathcal{E} - U_t(\phi)]}} = \frac{\sqrt{\mathcal{E}_{\max}} \Gamma(\mu + 1)}{\sqrt{2\pi} A_N \Gamma(\mu + 3/2)} \left[1 - \frac{U_t(\phi)}{\mathcal{E}_{\max}}\right]^{\mu+1/2}. \quad (15)$$

For $\mu \rightarrow \infty$ a bunch has a Gaussian line density and the corresponding bunch length $\tau_{4\sigma}$ is typically defined as four times the Root-Mean-Square bunch length σ , i.e. $\tau_{4\sigma} = 4\sigma$. The bunch length $\tau_{4\sigma}$ can be easily calculated from the Full-Width Half-Maximum (FWHM) bunch length τ_{FWHM} ,

$$\tau_{4\sigma} = \tau_{\text{FWHM}} \sqrt{2/\ln 2}. \quad (16)$$

In practice, SPS proton bunches are far from being Gaussian, but the best fit to measured bunch profiles corresponds to $\mu \approx 1.5$. For the sake of simplicity and comparison with measurements, however, we still use Eq. (16) as a definition of the bunch length for the rest of the present work. For completeness we also define the total emittance (in eV s) as

$$\epsilon = \oint d\phi \frac{\Delta E}{h\omega_0} = 2\sqrt{-\frac{V_{200} \cos \phi_{s0} q\beta^2 E_0}{\pi\eta\omega_0^2 h^3}} \int_{\phi_{\min}}^{\phi_{\max}} d\phi \sqrt{(\mathcal{E}_{\max} - U_t(\phi))}, \quad (17)$$

where $\phi_{\min}$ and $\phi_{\max}$ are the minimum and maximum phase of the particle with energy of synchrotron oscillation $\mathcal{E}_{\max}$.

The parameters of the bunches are inputted via the intensity parameter ζ in Eq. (11) and the maximum synchrotron oscillation energy $\mathcal{E}_{\max}$, which are dimensionless parameters and can be re-scaled to a given bunch intensity and emittance.

2.4 Comparison with the Code BLonD

We now compare stability calculations performed with MELODY with simulations done with the Beam Longitudinal Dynamics (BLonD) code [8]. We consider the case of the acceleration of beams to 450 GeV whose stability was studied in dedicated measurements operating with a single-RF system [14]. We compute the stability of bunches with bunch intensities ranging from $0.5 - 4.0 \times 10^{11}$ and 4σ bunch lengths ranging from 1.0 – 1.9 ns (approximately, for ϕ_{s0} furthest from π) at different times along the acceleration cycle using the corresponding impedance model (referred to as “present2018” in [13]). We focus in particular on the second half of the cycle, where the momentum is sufficiently high for space charge effects to become negligible [14].

We find that the majority of the instabilities during the second half of the cycle occur when the synchronous phase ϕ_{s0} is furthest from π . Moreover, we observe a good agreement between our results and simulations from BLonD. Figure 1 compares the stability map from BLonD with that from MELODY at 450 GeV flat top (end of the cycle) and during the acceleration ramp. At flat top, the same patterns in the stability are seen across the whole range, including the region of instability near the centre (bunch length of 1.5–1.9 ns),

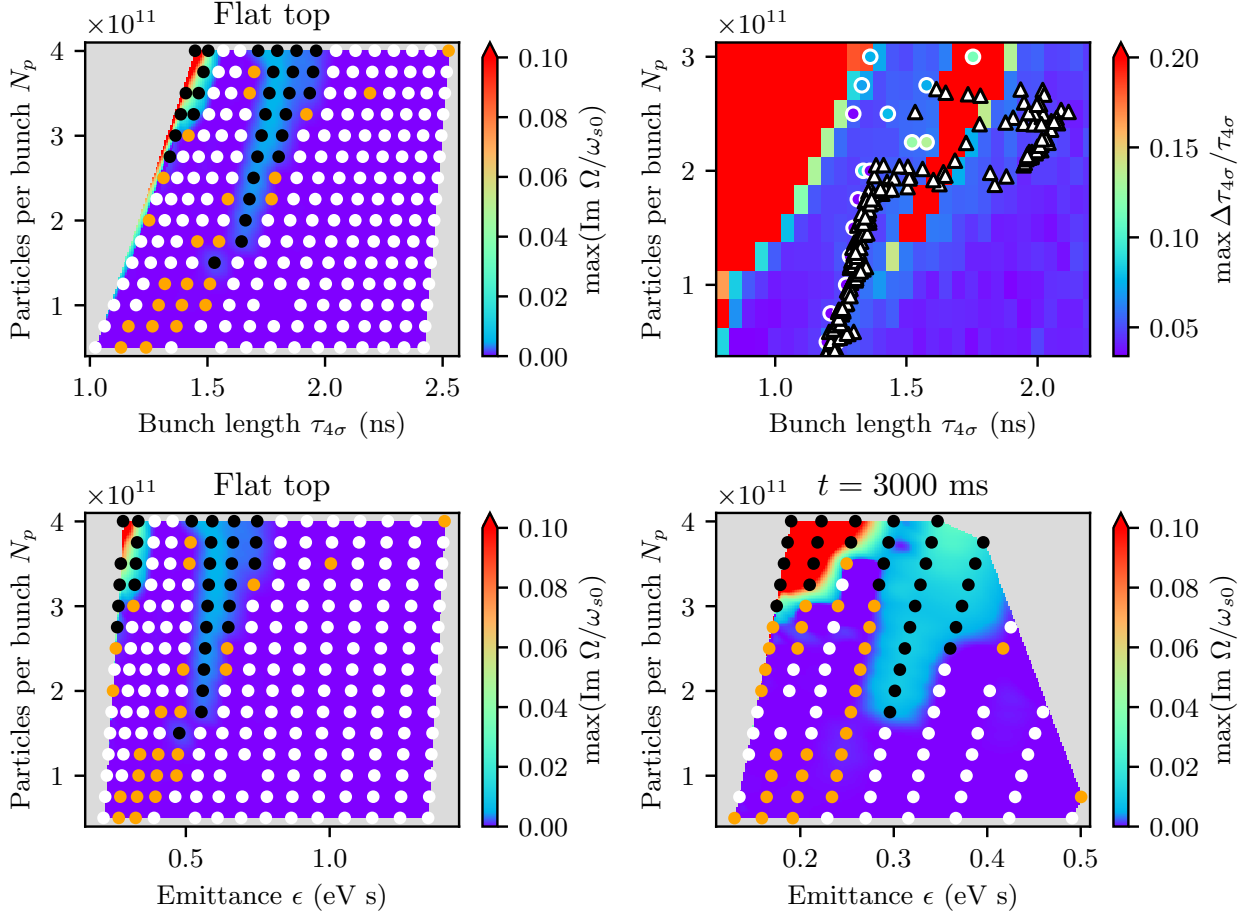


Figure 1: Calculations of longitudinal stability maps by MELODY compared with simulations by BLong and dedicated beam measurements. White circles are stable bunches, orange circles are bunches with maximum normalised growth rate $\text{Im } \Omega / \omega_{s0} < 0.002$ (essentially stable) and black circles are bunches with growth rate greater than $0.002\omega_{s0}$. The background colour corresponds to the interpolated growth rate. In the two plots at flat top (left), circles in the same relative position of the grid correspond to the same bunch. In the upper right plot, the background colour is the amplitude of bunch length oscillations obtained in simulations using BLong that indicates bunch stability at flat top; the circles are the simulation results during the energy ramp and the triangles are measurements.

with few discrepancies present. This unstable “island” was also observed in previous simulations [15]. We find that the mechanism of observed instabilities is the coupling of radial modes [16]. A direct comparison is more complicated during the ramp, as MELODY only computes stability at a given time in the cycle, while BLong simulates over a longer period of time. To get around this issue, we take advantage of the fact that a bunch’s emittance is constant during the acceleration cycle (in the absence of instabilities, strong noise, intra-beam scattering, etc.). For a particular bunch intensity and bunch length, we determine the corresponding emittance at flat top and look at its stability back in time. We again find a good agreement between MELODY and BLong. Figure 1 shows one example: a bunch with intensity 2×10^{11} and length 1.33 ns (unstable in BLong during the ramp) has an emittance

$\epsilon \approx 0.3$ eV s, which MELODY computes to be unstable at a time $t \approx 3000$ ms of the cycle.

Both codes are also in good agreement with experimental data [14] that was recently corrected taking into account a transfer function of the acquisition system. In particular we see that bunches with intensities larger than $\sim 2 \times 10^{11}$ suffer from instabilities during the ramp which result in uncontrolled emittance blow-up and larger bunch lengths at flat top. For smaller intensities, bunches are mostly found in regions calculated and simulated as stable by MELODY and BLonD, respectively.

3 Bunch-Length Minimisation

In this section we discuss three methods of obtaining very short bunches in the SPS under its expected operational capabilities following LS2. In particular, we assume that the maximum amplitude of the 200 MHz accelerating voltage, V_{200} , will be 13.5 MV, and the maximum amplitude of the 800 MHz accelerating voltage, V_{800} , will be 2.5 MV. Other important parameters of the SPS are provided in Table 1. We use the impedance model marked as “futurePostLS2” in [13]. We focus on the energy ramp in use for the AWAKE experiment, and we constrain the energy gain per turn $\delta E_0 = V_{200} \sin \phi_{s0}$ to be the same as used presently, so that the magnetic field strength as a function of time need not be changed. The injector of the SPS is the Proton Synchrotron (PS), so we only consider beams that can be produced by the PS and then fit in a 5-ns long SPS bucket; in particular, we limit the emittance at flat bottom to be less than approximately 0.4 eV s.

We determine the minimum stable bunch length achievable in the SPS for a bunch intensity $N_p \approx 4 \times 10^{11}$, which is the maximum achievable in the PS. We consider a bunch to be stable when the maximum growth rate $\max(\text{Im } \Omega / \omega_{s0})$ as calculated by MELODY is less than 0.002, as such low growth rates are not sufficient to significantly affect the bunch in the relevant timescales. In double-RF systems, we use bunch-shortening mode (BSM) in order to obtain the shortest bunch lengths, i.e. the phase offset is $\Phi_{800} = \pi - 4\phi_{s0}$.

3.1 Without Bunch Rotation

We first consider the case without bunch rotation. Our analysis includes results for a double-RF system with several voltage ratios, with particular focus on $V_{800}/V_{200} = 0.1$ and 0.18%. (Note that $V_{800}^{\max}/V_{200}^{\max} \approx 0.18$.) For each voltage ratio, we perform calculations for about twelve different synchronous phases ϕ_{s0} , and then plot stability maps at twenty different times during the acceleration cycle. We take space charge impedance into account until its magnitude drops below 0.5Ω (half of its magnitude at flat bottom). We find that the space charges do not alter the minimum stable bunch length, but do slightly change, at early times, the range of voltages that can produce a beam with this bunch length. In general, we find that the minimum stable bunch length and emittance decrease with increasing V_{200} and voltage ratio, respectively.

We observe that the minimum stable 4σ bunch length at intensity $N_p = 4.0 \times 10^{11}$ is $\tau_{4\sigma} \approx 0.99$ ns; such a bunch has a longitudinal emittance of $\epsilon = 0.31$ eV s, as shown in Figure 2. The upper bound on the emittance is set at flat top; it is possible to keep a beam with lower emittance (and thus which would have a shorter bunch length) stable for most

Table 1: Main properties of the SPS and of the beam at flat bottom and flat top.

Machine Parameters	
Circumference	6911.554 m
Transition Lorentz factor γ_{tr}	17.951 (Q20 optics)
Main harmonic number h	4620
<i>Main RF system</i>	
Frequency $f_{rf,200}$ at flat top	200.394 MHz
Maximum voltage V_{200}	13.5 MV (expected)
<i>Fourth-harmonic RF system</i>	
Frequency $f_{rf,800}$ at flat top	801.578 MHz
Maximum voltage V_{800}	2.5 MV (expected)
Beam Parameters	
<i>Flat bottom</i>	
Energy E_0	25.937 GeV
Momentum p	25.92 GeV/c
Slip factor η	1.795×10^{-3}
<i>Flat top</i>	
Energy E_0	400.001 GeV
Momentum p	400 GeV/c
Slip factor η	3.098×10^{-3}

of the ramp. Deviations in the phase Φ_{800} between the two RF systems do not significantly change the bunch length: a deviation of -30° decreases the bunch length by ~ 35 ps, while a deviation of $+30^\circ$ increases the bunch length by ~ 20 ps. The emittance however, does change more noticeably: up to -0.2 eV s at a -30° deviation. The minimum bunch length is obtained by maximising both the main and fourth-harmonic RF system: $V_{200} = 13.5$ MV, $V_{800}/V_{200} = 0.18$. One can see from Figure 2 that the minimum bunch length tends to increase with bunch intensity, which is the expected result.

Since there is usually a shot-by-shot variation of bunch parameters at injection, we consider a spread of about 10% in the intensity and in the initial emittance in the range from 0.3 eV s to 0.4 eV s. Given that the emittance of a bunch is constant during the ramp, the stability of a bunch with the above characteristics can be tracked through time, and the parameters required to keep the bunch stable can be determined. In this way, we obtain a range of ‘allowed’ voltages as a function of time, as well as their corresponding synchronous phase, which is constrained by $\delta E_0 = V_{200} \sin \phi_{s0}$. These are shown in Figure 3. We find that a voltage ratio of 10% is sufficient to ensure stability of the bunch for the first half of the ramp; however, during the second half the voltage ratio must be increased, and up to 18% at later times. Note that if the bunch characteristics lie beyond the range scanned with

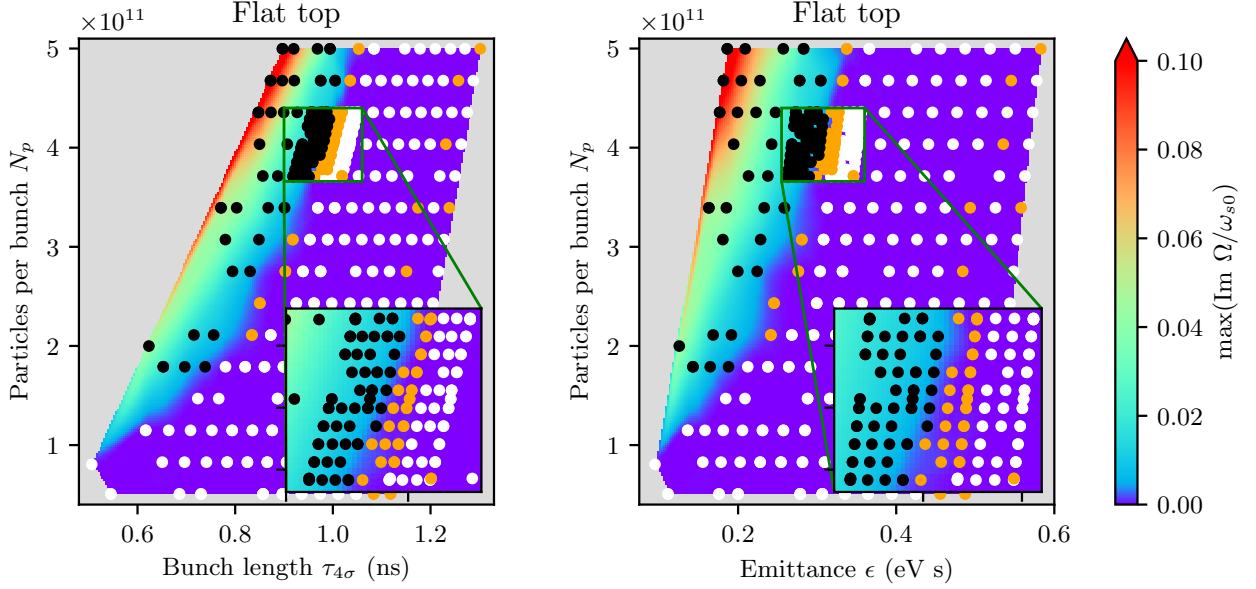


Figure 2: Longitudinal stability of single bunches at flat top ($V_{200} = 13.5$ MV, $V_{800}/V_{200} = 0.18$) with the expected SPS operational capabilities following LS2. The colours of the background and circles have the same meaning as in Figure 1. The minimum stable 4σ bunch length at $N_p = 4.0 \times 10^{11}$ particles per bunch is 0.99 ns, corresponding to an emittance $\epsilon = 0.31$ eV s.

MELODY, we deem it to be unstable, even if extrapolation suggests it should be stable. Thus, the range of allowed voltages is likely wider than presented in Figure 3; in particular, the lower bound can probably be reduced non-negligibly, and the upper bound can likely be smoothed. We have tested a voltage programme, selected from the range given in Figure 3, with BLonD, and results show that the bunch remains stable throughout the cycle.

3.2 With Bunch Rotation

3.2.1 Without Emittance Blow-up

We now consider a bunch of emittance $\epsilon \leq 0.4$ eV s and intensity $N_p = 4.0 \times 10^{11}$, and study the minimum stable bunch length achievable with bunch rotation at flat top. Note that such bunches can be achieved at flat top using a voltage programme that lies near the middle of the bounds given in Figure 3. Bunch rotation works by very rapidly increasing the voltage from a low value to a high value (voltage jump), causing the bunch to rotate in the longitudinal phase space. The bunch can then be extracted when its longitudinal length is shortest.

Bunch rotation is most effective in reducing the bunch length when the initial voltage and emittance are as low as possible. An analysis of the longitudinal stability of bunches with the above characteristics at voltage ratios ranging from 0% to 30% suggests that larger voltage ratios are preferable when the emittance is below 0.4 eV s (see Figure 4). At a ratio of 30%, a bunch with emittance 0.30 eV s can be maintained stable down to $V_{200} \approx 4.8$ MV. At low voltage ratios (up to $\sim 3\%$), bunches can be stable at a lower voltage, but this cannot be reached stably as one would need to ‘cross’ an unstable region. Calculations of bunch

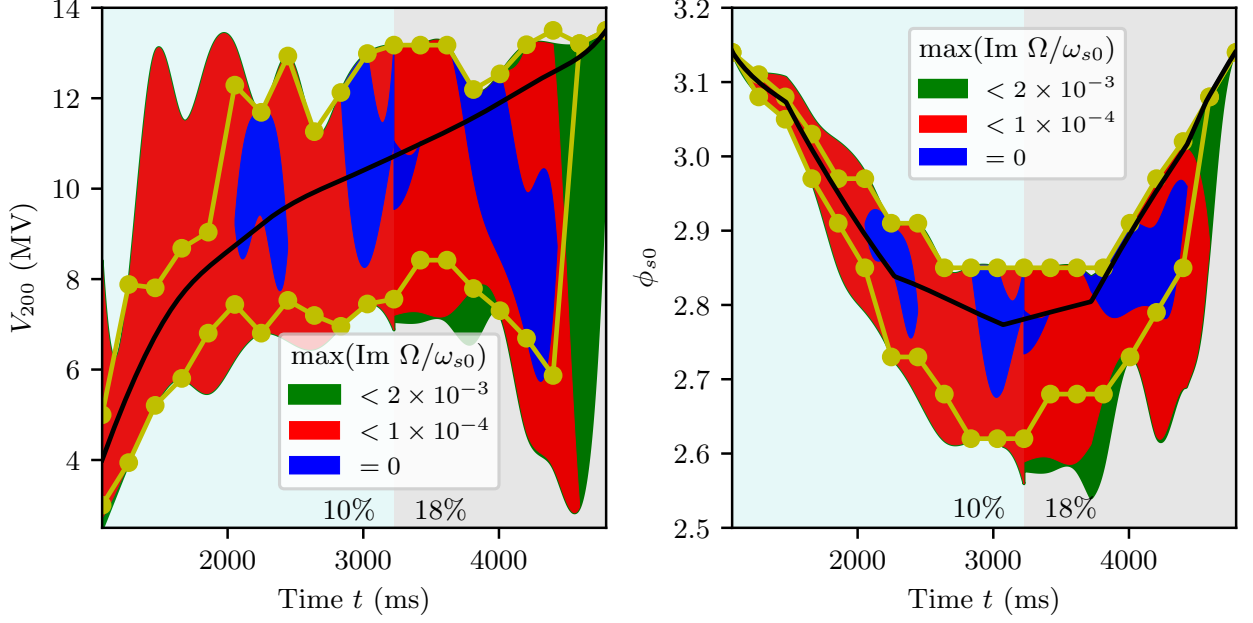


Figure 3: Range of voltages (left) and corresponding synchronous phase (right) that keep bunches with $N_p = 4.0 \times 10^{11}$ particles and emittance $\epsilon = 0.31$ eV s stable throughout the energy ramp. The ranges are calculated by interpolation of stability maps (such as those in Figure 2, for example), but no extrapolation is used, so the true ranges are likely slightly wider than shown here. The horizontal axis corresponds to the time in the cycle for the AWAKE experiment (injection is at 1075 ms). Blue regions are stable, while red and green regions are very nearly stable, with maximum normalised growth rates $\max(\text{Im } \Omega/\omega_{s0}) < 10^{-4}$ and 0.002, respectively. The boundaries of these regions should be taken as approximative. Yellow points are the ranges calculated without interpolation for parameters $N_p = (4.0 \pm 0.4) \times 10^{11}$ and $\epsilon = 0.33 \pm 0.02$ eV s. The black line is a sample cycle programme to produce the shortest bunch with $N_p = 4 \times 10^{11}$.

rotation show that a bunch with $\epsilon = 0.30$ eV s can be shortened to a 4σ bunch length $\tau_{4\sigma} \approx 0.77$ ns. This bunch length can be theoretically decreased to 0.71 ns if phase and voltage jumps are used in both 200 and 800 MHz RF systems. This option, however, will require a firmware modification of the 800 MHz Low Level RF system. These jumps can also help to reduce the asymmetry of the bunch profile caused by the potential well distortion. We find an optimal bunch shape if the phase jump is done with an initial phase $\phi_{s0} \approx 3.02$. The V_{200} and V_{800} voltage jumps should be done simultaneously and extraction should be performed about 25 turns after this in order to obtain the smallest bunch length. Depending on the quality of the bunch rotation programme used in practice, this method can lead to an improvement of 22 – 28% on the case without bunch rotation.

3.2.2 With Emittance Blow-up

The emittance of a bunch can be increased beyond the maximum achievable at injection using blow-up during the cycle. Blow-up inevitably leads to some particles obtaining a large trajectory in phase space compared with the bulk of the bunch. To remove those, one could

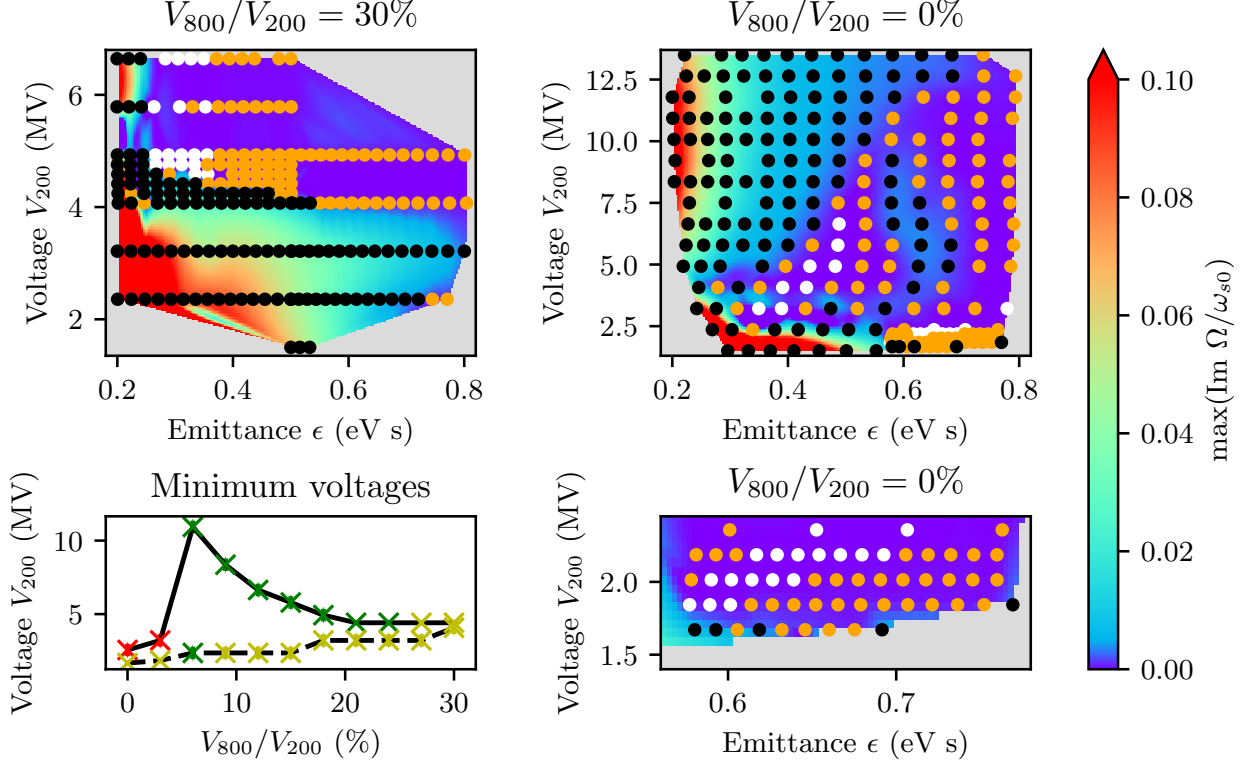


Figure 4: Longitudinal stability of single bunches as calculated by MELODY at voltage ratios $V_{800}/V_{200} = 0.3$ and 0 . The colours of the circles and background have the same meaning as in Figure 1. The lower left plot shows the minimum voltage at which a bunch can be stable as a function of voltage ratio, given upper bounds in emittance of 0.4 eV s (solid line) and 0.8 eV s (dashed line). Red points are unattainable from flat top as a region of instability must be crossed to get to this voltage. Yellow points are almost stable, with maximum normalised growth rate $\max(\text{Im } \Omega/\omega_{s0}) < 0.002$, while green points are stable. The lower right plot is a zoomed-in section of the upper right plot.

perform blow-up during the energy ramp rather than at flat top, so that they can be lost naturally through the cycle.

We again consider bunches with intensity $N_p = 4.0 \times 10^{11}$, and we set an upper bound on the emittance at 0.8 eV s. Calculations with MELODY show that bunches can be maintained stable at these large emittances for a wide range of voltages V_{200} . A similar analysis as in Section 3.2.1 suggests that the minimum voltage that can be reached stably is $V_{200} \approx 1.7$ MV, with the fourth-harmonic RF system turned off ($V_{800}/V_{200} = 0$). The smallest stable emittance at that voltage is $\epsilon \approx 0.62$ eV s (see Figure 4). In order to avoid instabilities while decreasing the voltage from flat top, one must first reduce V_{200} to about 4 MV with a voltage ratio of 18% , and then decrease V_{200} to 1.7 MV while V_{800} is gradually turned off.

Bunch rotation calculations show that given the above voltage settings, a bunch with $N_p = 4.0 \times 10^{11}$ protons and emittance 0.62 eV s can be shortened to $\tau_{4\sigma} \approx 0.84$ ns, if no phase or voltage jump optimisation is done. As in Section 3.2.1, optimisation improves the symmetry of the bunch shape and slightly reduces the bunch length. We find an optimal

phase jump with initial phase $\phi_{s0} \approx 2.691$ carried out simultaneously to the V_{200} jump, followed by a V_{800} jump 17 turns later. Extraction should be done about 32 turns after the jump in V_{200} . This procedure gives a minimum bunch length $\tau_{4\sigma} \approx 0.78$ ns according to calculations, which is marginally longer than that achievable with smaller emittances and larger voltages.

4 Conclusion

We have studied the longitudinal stability of single bunches in the SPS under different conditions with the code Matrix Equations for LOngitudinal beam DYnamics calculations (MELODY). We have observed good agreement between these results and simulations performed with Beam Longitudinal Dynamics (BLonD), with few discrepancies. Results are also in good agreement with experimental measurements. This suggests that using five azimuthal modes in the calculations is sufficient to obtain accurate results.

Our main objective was to determine the minimum stable bunch length achievable in the SPS with a bunch intensity of $N_p \approx 4.0 \times 10^{11}$, for use in the AWAKE experiment. We found that the minimum 4σ bunch length obtainable without bunch rotation is $\tau_{4\sigma} = 0.99$ ns. This requires both the main and fourth-harmonic RF systems to be set to their maximum amplitudes, and these to be phased in bunch-shortening mode. We have also determined the range of voltages that will keep bunches of this length stable throughout the acceleration cycle; a voltage ratio of 10% is sufficient for the first half of the ramp, but for later times a ratio of 18% is required. We have run a sample voltage programme with BLonD, which found the bunch to be stable through the cycle, in agreement with MELODY calculations.

Bunch lengths can be reduced non-negligibly with bunch rotation in the longitudinal phase space. A bunch length of $\tau_{4\sigma} = 0.77$ ns is achievable at intensity $N_p = 4.0 \times 10^{11}$ without requiring controlled longitudinal emittance blow-up by using a bunch with emittance $\epsilon = 0.30$ eV s and an initial voltage $V_{200} = 4.8$ MV at ratio 30%. This is a reduction of 22% compared with the minimum length obtained without bunch rotation. Controlled emittance blow-up makes it possible to achieve a very low voltage (1.7 MV) at a higher emittance (0.62 eV s). However, this does not lower the bunch length, but instead yields similar results ($\tau_{4\sigma} = 0.84$ ns) to without blow-up. Jumps in the phase and the voltage in both RF systems can theoretically decrease the bunch length to 0.71 ns without blow-up and to 0.78 ns with blow-up. In both cases, these jumps also reduce the asymmetry of the bunch profiles caused by the potential well distortion. However, this option requires modifications of the 800 MHz Low Level RF system.

One of the most important steps that remains to be done is, of course, experimental tests. Since the calculations show that stability can be maintained for a wide range of RF voltages, further optimisation can be performed to minimise RF power requirements and to avoid potential limitations in the transverse plane that are not covered in the scope of the present work. These experiments will be carried out after the current long shutdown, and should help to optimise operation as well as further test the accuracy of MELODY, BLonD and other beam dynamics codes.

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