

Soft and Coulomb resummation:
squark and gluino production at the LHC

Wever, Christopher Stephen Paul

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Soft and Coulomb resummation: squark and gluino production at the LHC

Zachte en Coulomb hersommatie:
squark en gluino productie bij de LHC

(met een samenvatting in het Nederlands)

Proefschrift

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Christopher Stephen Paul Wever

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Promotor : Prof. dr. G. 't Hooft
Co-promotor: Dr. P. Falgari

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To Mai, Pai, Linda and Moises Curvas

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The study of nature at small scales is based on the principles of quantum mechanics that were put forward largely in the first quarter of the twentieth century by physicists such as Planck, Bohr, Heisenberg and Schroedinger among many others. Consider for example the uncertainty principle, complementarity, quantization or the principle of superposition. These were crucial to the understanding of early physical puzzles such as black-body radiation, the photoelectric effect and the double slit experiments. However, the mathematical basis for quantum mechanics set in the first quarter of the twentieth century lacked the ability to fundamentally describe the dynamical annihilation, creation and interaction of particles constantly taking place all around us or presently at high-energy particle colliders. Such a fundamental description required the concept of *fields*: particles and their corresponding forces are interpreted as interacting fields, naturally incorporating all of the principles named above. Field theoretical methods were first applied in physics in a classical sense by Newton in his theory of gravity. The inclusion of quantum mechanical principles in the classical field method was done in the second quarter of the twentieth century by theoretical physicists such as Dirac, Feynman, Schwinger among others. This quantization of classical fields led to the birth of *Quantum Field Theory (QFT)*.

The first successful QFT was arguably the $U(1)$ -abelian gauge theory *Quantum Electrodynamics (QED)*, which is the quantum mechanical counterpart of the classical electrodynamics theory of light by Maxwell. This laid the groundwork for the further study of non-abelian gauge-invariant QFT's by Yang and Mills in the fifties. After the proof of *renormalizability* of Yang-Mills gauge theories by 't Hooft and Veltman [1], the *electroweak (EW)* theory which semi-unified the weak and electromagnetic force was numerically set on equal footing as the successful theory of light. At the same time, Feynman's parton model and the Gell-Mann-Nishijima quark model were crucial for the further understanding of the inner structure of nuclei as well as for providing a framework for explaining the plethora of new particle discoveries. It was not until the discovery of *asymptotic freedom* in the seventies and its subsequent explanation of Bjorken-scaling observed in deep-inelastic scatterings that it was found that the partons, nowadays known as *quarks* and

gluons, were also subject to a fundamental QFT description called *Quantum Chromodynamics* (QCD).

The discovery of asymptotic freedom made perturbative calculations of higher-loop corrections at high energies possible in QCD. The calculated gauge-invariant observables such as *cross sections* and kinematic distributions could be expressed as a series expansion in the (squared) strong coupling constant $\alpha_s := g_s^2/(4\pi)$. One then uses the common terminology of *fixed order* N^k LO expansion to denote an expansion of the observables up to $\mathcal{O}(\alpha_s^k)$ -suppressed terms with respect to the tree level contribution. On the other hand, at low energies the coupling constant is thought to diverge when hitting the Landau pole at $\Lambda_{\text{QCD}} \sim \mathcal{O}(100 - 1000 \text{ MeV})$ because of the form of the QCD beta functions, which results in the breakdown of such a perturbative expansion in α_s . At energies close to Λ_{QCD} , one is forced to perform semi-classical calculations or use other nonperturbative methods (see e.g. [2]). In these approaches the perturbative expansion is implicitly reorganized and a subset of the expansion in α_s is summed. Such resummations are common in QFT and follow naturally when solving evolution equations such as the beta function for α_s or Schwinger-Dyson equations for Green's functions.

In collider physics there is also a type of perturbative breakdown which contrary to the above divergence resulting from hitting the Landau pole has instead a purely kinematical origin. If we imagine the production of a collection of heavy particles, the region in phase space where the produced particles have low kinetic energies in the centre of mass frame is called the *threshold region*. It has been known for some time that in this region the production cross section gets contributions from two classes of threshold enhanced corrections. The first are the so-called *soft logarithms* which are directly linked to infrared soft and collinear singularities and are of the form $\alpha_s^k \ln^l(\beta)$, where β roughly equals the non-relativistic velocity of the particles near threshold in the centre of mass frame [3]. The second type of corrections are so-called *Coulomb singularities* which appear from the exchange of non-relativistic virtual off-shell gluons between the produced heavy particles and are of the form $(\alpha_s/\beta)^m$ [4]. Since the eighties there has been much work on the study of these two classes of enhanced corrections and furthermore their resummation by deriving and calculating a factorized form of the partonic cross section. Most of these references can be found in [5]. The general idea in these papers is that by using the framework of *effective field theory* (EFT) the partonic cross section or kinematic distributions can be factorized, after which the corrections are resummed by solving the corresponding evolution equations for each factorized term. One then speaks of an N^k LL resummation where k specifies the class of enhanced terms.

This work reviews the way threshold enhanced QCD corrections to *heavy pair production* can be reorganized and resummed close to threshold. Because the corrections depend purely on the structure of QCD interactions, the resummed *partonic cross section* can be expressed in a largely model independent factorized form. This makes the resummation of the threshold enhanced QCD corrections possible for general gauge theories with gauge groups containing the $SU(3)$ colour group, which covers in particular all *Standard Model* (SM) extensions. This thesis

will focus on the application of the resummation methods to coloured *supersymmetry* (SUSY) production at the LHC. Under the assumption of R -parity, the processes that will be considered are *squark and gluino pair production* at the LHC. The reason for this is twofold. First, SUSY is a serious contender among many SM extensions because it introduces natural solutions to some unsolved physics problems as explained in Section 1.3. Secondly, since the above soft logarithm and Coulomb QCD corrections are enhanced near threshold, they are expected to be larger for the massive SUSY partners and in particular the squarks and gluinos which have the largest masses according to most phenomenologically viable models [6].

In the rest of this chapter we will address the relevant topics that appear in this thesis. Section 1.1 contains a discussion about the general setup of the heavy pair production of particles at hadron colliders, with emphasis on LHC collisions. In Section 1.2 we briefly consider EFT's and their use in deriving factorization and resummation formulas for the threshold enhanced terms. Section 1.3 contains a discussion of SUSY and its merits, where we specifically focus on the MSSM in the end. In Section 1.4 we consider the production of coloured SUSY particles at the LHC and discuss their LO and NLO production cross sections. Finally, in Section 1.5 we list the content and explain the outline of this thesis.

1.1 Heavy pair production in collider physics

In this section we consider the inclusive production cross section of a heavy pair of particles at hadron colliders. Anticipating the work in later chapters, throughout this section we will only discuss the specific case of the production of coloured SUSY pairs, i.e. squarks and gluinos, though everything that follows can be extended to the production of general heavy coloured particle pairs H, H' by simply replacing $\tilde{s} \rightarrow H, \tilde{s}' \rightarrow H'$ in the text and formulas below.

At hadron colliders, the dominant production channels for squarks $\tilde{q}$ and gluinos $\tilde{g}$ are pair-production processes of the form

$$N_1 N_2 \rightarrow \tilde{s} \tilde{s}' X, \quad (1.1)$$

where $N_{1,2}$ denote the incoming hadrons and $\tilde{s}, \tilde{s}'$ the two sparticles. The label X in (1.1) denotes the fact that we consider *inclusive* production cross sections and expresses any other number of final state particles. The total hadronic cross sections for the processes (1.1) can be obtained by convoluting the short-distance production *partonic cross sections* $\hat{\sigma}_{pp'}(\hat{s}, \mu_f)$ for the partonic processes

$$pp' \rightarrow \tilde{s} \tilde{s}' X, \quad p, p' \in \{q, \bar{q}, g\}, \quad (1.2)$$

with the parton luminosity functions $L_{pp'}(\tau, \mu)$:

$$\sigma_{N_1 N_2 \rightarrow \tilde{s} \tilde{s}' X}(s) = \int_0^1 d\tau \sum_{p, p' = q, \bar{q}, g} L_{pp'}(\tau, \mu_f) \hat{\sigma}_{pp'}(\tau s, \mu_f). \quad (1.3)$$

The factorization scale is denoted by μ_f and represents the scale which separates the short-distance hard processes (1.2) from the long-distance nonperturbative contributions to (1.1). The squared hadronic centre of mass energy of the hadrons N_1, N_2 is denoted by s , whereas $\hat{s} = \tau s$ specifies the squared partonic centre of mass energy of the two partons p, p' participating in the hard process (1.2). As a result, τ is varied inside the interval $[0, 1]$.

The parton luminosity functions contain the information of the nonperturbative hadronic structures of $N_{1,2}$ and are expressed in terms of the parton density functions (**PDFs**) as

$$L_{pp'}(\tau, \mu) = \int_0^1 dx_1 dx_2 \delta(x_1 x_2 - \tau) f_{p/N_1}(x_1, \mu) f_{p'/N_2}(x_2, \mu). \quad (1.4)$$

The PDFs $f_{p/N}(x, \mu)$ can be interpreted as the probability of finding a parton p residing inside the hadron N , carrying a fraction x of the total momentum. They are found by using global fits to deep-inelastic scattering and probing experiments, typically at small enough values of the factorization scale μ_f . There are many experimental groups using different ansatz and data for fitting the PDFs, resulting in different PDF sets [7]. For example in this thesis the MSTW PDF sets [8] for the proton P are used, whose ansatz for the PDFs at $\mu_f = 1$ GeV are roughly:

$$f_{p/P}(x, 1 \text{ GeV}) = A_p x^{\eta_p} (1-x)^{\tilde{\eta}_p} (1 + b_p \sqrt{x} + c_p x). \quad (1.5)$$

These are then evolved to larger values of the factorization scale by use of the DGLAP renormalization equations:

$$\mu^2 \frac{d}{d\mu^2} f_{p/N}(x, \mu) = \int_x^1 \frac{dy}{y} \sum_{\tilde{p}} P_{p\tilde{p}}(x/y, \alpha_s(\mu)) f_{\tilde{p}/N}(y, \mu). \quad (1.6)$$

The splitting function $P_{p\tilde{p}}(z, \alpha_s)$ indicates at LO the probability of a parton $\tilde{p}$ splitting into a parton p with momentum fraction z and another parton and is expressed as an expansion in the (squared) strong coupling constant. Their precise forms are not needed in the thesis.

The lower bound in the convolution (1.3) is taken here as 0, though in the case of stable final states it is given by $\tau_0 := 4M^2/s$, with the average sparticle mass

$$M = \frac{M_{\tilde{s}} + M_{\tilde{s}'}}{2} \gg \Lambda_{\text{QCD}} \sim 1 \text{ GeV}. \quad (1.7)$$

The production *threshold* is $\sqrt{\hat{s}} = 2M$ or in other words $\tau = \tau_0$. The non-relativistic velocity β of the SUSY pair as seen in the centre of mass frame is small:

$$\beta := \sqrt{1 - \frac{4M^2}{\hat{s}}} \ll 1, \quad \text{close to threshold.} \quad (1.8)$$

As such, the total relativistic kinetic energy of the final states as observed from the centre of mass frame equals:

$$E := \sqrt{\hat{s}} - 2M \simeq M\beta^2, \quad \text{close to threshold.} \quad (1.9)$$

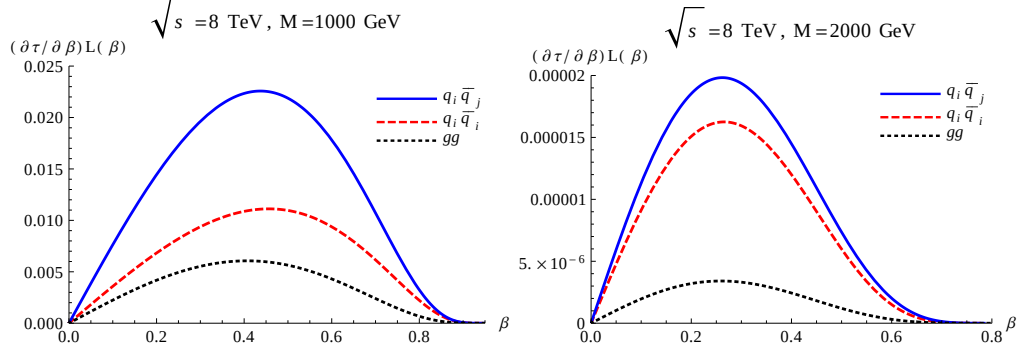


Figure 1.1: The parton luminosities for the sum over initial $p = q_i, p' = \bar{q}_j, i \neq j$ quarks (thick blue), diagonal $p = q_i, p' = \bar{q}_i$ quarks (dashed red) and initial gluons $p = p' = g$ (dotted black) for an average mass of $M = 1000$ GeV (left) and $M = 2000$ GeV (right) with $\sqrt{s} = 8$ TeV at the LHC.

The region in phase space where $E < 0$ is called the region *below threshold*, whereas $E > 0$ is called the region *above threshold*.

The hadronic cross section given by (1.3) can be expressed in terms of the non-relativistic velocity β by a variable substitution as:

$$\sigma_{N_1 N_2 \rightarrow \tilde{s} \tilde{s}' X}(s) = \int_0^{\beta_{\max}} d\beta \sum_{p, p' = q, \bar{q}, g} \left(\frac{\partial \tau}{\partial \beta} \right) L_{pp'}(\tau(\beta), \mu_f) \hat{\sigma}_{pp'}(\tau(\beta) s, \mu_f), \quad (1.10)$$

where typically $1 > \beta_{\max} = \sqrt{1 - 4M^2/s} \gtrsim 0.9$ at LHC energies and squark and gluino masses of order 1 TeV. As the average sparticle mass increases, the value of $\beta_{\max}$ decreases and the region close to threshold becomes relatively more important, which is further enhanced by the rise of the PDFs at small values of τ . The combination of these two effects can be seen in Figure 1.1, where we plot $\left(\frac{\partial \tau}{\partial \beta} \right) L_{pp'}(\beta)$ for initial quark-antiquark and gluon-gluon channels as a function of the non-relativistic velocity β . As the lower bounds on the sparticle masses become larger it thus becomes increasingly important to include the contributions of threshold enhanced corrections, which can be resummed by the use of EFT's as explained in the next section.

1.2 Effective field theories: factorization and resummation of threshold enhanced terms

The soft logarithms arise from an imperfect cancellation of the *infrared* (IR) soft and collinear singularities near threshold. To see this note that the Kinoshita-Lee-Nauenberg theorem [9, 10] states that QCD is infrared finite. IR pole cancellations takes place between those arising from

virtual Feynman loop integrals and those arising from phase space integrals of real (on-shell) soft gluons and collinear quarks or gluons. However, from Eq. (1.9) it follows that in the centre of mass frame near threshold, the kinetic energy of the final states labeled as X in (1.1) is restricted to $E_X \lesssim M\beta^2$. Accordingly, as the threshold is approached and $\beta \rightarrow 0$ the emission of outgoing on-shell real soft gluons is suppressed. This then also suppresses the delicate IR pole cancellations with the virtual Feynman loops, leading to singular terms in β . In Section 2.1 it will be shown that the singularities which appear as $\beta \rightarrow 0$ are of the form $\alpha_s^k \ln^l(\beta)$. These soft logarithms can be thus seen as arising from the emission of *soft gluons* with momenta $p \sim M\beta^2$. Furthermore, it will be shown that the above described Coulomb singularities $(\alpha_s/\beta)^m$ arise from the exchange of so-called *potential gluons* carrying momenta $p_0 \sim M\beta^2$, $\vec{p} \sim M\beta$. Near threshold, the relevant dynamical fields are thus the initial collinear partons, the outgoing heavy pair and the emitted soft and potential gluons. The momenta associated with these fields span a hierarchy of scales as $M \gg M\beta \gg M\beta^2$ close to threshold which makes heavy pair production a prime system for the application of the EFT framework [11].

Effective field theories are everywhere in physics. In fact, almost every theory in physics can be framed in the context of an EFT. Exceptions are candidates for the so-called *theory of everything*, with string theory still being the main contender for that title in our universe, or other *ultraviolet-* (UV) complete theories. However, even for these exceptions it does not make sense to use the full complete lagrangians when one is only interested in the dynamics at specific energy scales, where the use of their respective EFT's is much more powerful. Imagine a system which is characterized by a (set of) small Lorentz invariant parameters $\{x_i\}$. The general idea of EFT's is that any observable calculated with the lagrangian of the complete theory can be approximated by calculating the same observable with an effective theory whose effective lagrangian is an expansion in the (set of) small parameter(s) $\{x_i\}$ (see [12] and references therein).

The most common parameter(s) are those derived from a hierarchy of energy scales involved in most physical problems, in which case the parameter(s) $\{x_i\}$ is (are) the ratios of these scales. The effective lagrangian then only contains the relevant dynamical fields which are typically those with momenta that appear as poles in the Feynman diagrams of the complete theory when considering a specific energy scale E [13]. In the EFT framework the short-distance (higher-energy) degrees of freedom decouple from the long-distance (lower-energy) dynamical degrees of freedom in E [14]. This is a Lorentz invariant statement, though in practice it is more convenient to perform the calculations in some fixed Lorentz frame.¹ In this framework the cross section and other observables are also naturally factorized in a term containing the decoupled contributions and another term incorporating the dynamical fields. The decoupled contributions are found by *matching* to the full theory at corresponding scales resulting in *renormalization group* (RG) equations. Such factorizations are common and is the basis of the QCD factorization theorem expressed in Eq. (1.3) [15, 16].

¹For the production of heavy pairs one usually uses the *centre-of-mass* (c.o.m.) frame.

For heavy pair production close to threshold as described in Section 1.1 the small parameter is the non-relativistic velocity $\beta \ll 1$. The effective theory of the system near threshold is a combination of *soft collinear effective field theory* (SCET) [17, 18] describing the soft and collinear field interactions and the theory of *potential non-relativistic QCD* (pNRQCD) [19–21] describing the soft and potential field interactions. By using the EFT framework the *hard* scales with momenta $p \sim M$ are naturally included in matching coefficients and in Section 2.2.2 it will be shown that the soft field also decouples at the lagrangian level at LO, leading to a full factorization of the partonic cross section. The soft logarithms and Coulomb singularities are then naturally resummed by solving the RG equations. Before discussing this in further detail we briefly examine the reasons for considering heavy SUSY particle production in the rest of this chapter.

1.3 SUSY and the MSSM

Despite the good agreement of the Standard Model (SM) with a wealth of experimental data, both empirical reasons (e.g. the observation of dark matter) and theoretical arguments (such as the naturalness problem and the desire for gauge coupling unification) point to physics beyond the SM. One of the most thoroughly studied extensions of the SM is Supersymmetry (SUSY), which will be briefly reviewed in this section (see [22] for a more thorough review). SUSY is an extension of the Poincare space-time symmetry and can be shown to be the only extension possible [23]. The Poincare algebra is enlarged by Weyl spinorial supercharges $Q, \bar{Q}_i$, satisfying anti-commutation relations. These supercharges relate particles of different spins and one speaks of $\mathcal{N} = 1, 2, \dots$ SUSY where $\mathcal{N}$ denotes the amount of spinorial charges that are added to the space-time Poincare algebra. Since extended $\mathcal{N} = 2$ or higher SUSY can be shown to be not chiral [24], the only possible realistic SUSY extension of the SM is $\mathcal{N} = 1$ SUSY. At its heart, $\mathcal{N} = 1$ SUSY posits that every fundamental particle has a so-called *superpartner* with spin $1/2$ difference, or in other words SM bosons have bosinos as superpartners, while SM fermions have sfermions as superpartners.

There are many reasons for considering a SUSY extension of the SM and among them three stand out, which are briefly explained in what follows. The main reason to consider a SUSY extension is that it leads to the stabilization of the Higgs boson mass against large higher order radiative corrections. In the SM, the Higgs boson mass is not protected from large radiative corrections by any symmetry,² contrary to fermion masses which are protected by chiral symmetry. The Higgs mass gets radiative corrections from fermion loops such as quark loops or possibly other, more massive yet undiscovered fermions as shown in the left graph of Figure 1.2. Such a diagram contributes to the Higgs mass a term of the form:

$$\delta_{\text{fermion};f} m_H^2 \sim -y_{ffH} \int^{\Lambda_{\text{UV}}} \text{Tr} \left[\frac{d^4 k}{(\not{k} - m_f)^2} \right] \sim -y_{ffH} \Lambda_{\text{UV}}^2 + \mathcal{O}(m_f^2). \quad (1.11)$$

²In fact this statement is true for any scalar field in any QFT.

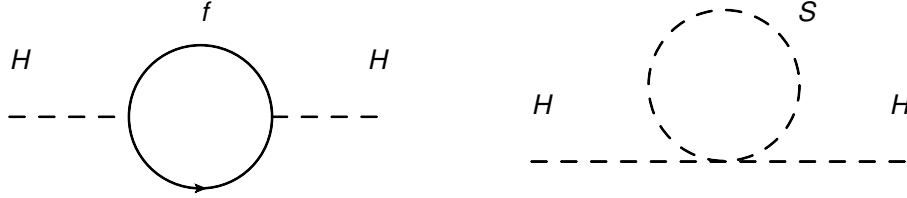


Figure 1.2: Divergent radiative corrections contributing to the Higgs mass. The left diagrams contains a fermion loop, while the right diagram its scalar superpartner loops.

The extra factor of (-1) appears because of the fermion loop. The cut-off Λ_{UV} is the maximum energy up to where we naturally expect the SM to be valid. Beyond $\Lambda_{UV} \sim 10^{16}$ GeV we expect new physics such as possibly gauge coupling unification and a *Grand Unifying Theory* (GUT) description of particle physics. This leads to a naturally large Higgs mass of the order of Λ_{UV} , which is much larger than the weak scale ~ 100 GeV and would require fine-tuning. This is called the *Hierarchy problem* and is remedied by SUSY. In the case of SUSY the loop diagrams from the superpartners coupled to the Higgs also need to be taken into account as shown on the right in Figure 1.2.³ Since the fermion loops carry an extra factor of (-1) this just might result in a cancellation of the contribution in Eq. (1.11). Such a scalar loop diagram contributes to the Higgs mass a term of the form:

$$\delta_{\text{scalar}:S} m_H^2 \sim \lambda_{HHSS} \int^{\Lambda_{UV}} \frac{d^4 k}{k^2 - m_S^2} \sim \lambda_{HHSS} \Lambda_{UV}^2 + \mathcal{O}(m_S^2). \quad (1.12)$$

It can be shown that under the assumption of SUSY, $\lambda_{HHSS} = y_{fH} := \lambda$ and accordingly the leading power terms cancel. The subleading logarithmic term equals:

$$\sim \lambda(m_S^2 - m_f^2) \log \Lambda_{UV}, \quad (1.13)$$

which is of order of m_H^2 and would not require any fine-tuning as long as the difference in masses of the superpartners are of the order of $\mathcal{O}(1 \text{ TeV})$. This reasoning holds true for any other pairs (f, S) , including the Higgs boson itself and its superpartner.

The *Minimal Supersymmetric Standard Model* (MSSM) is the simplest $\mathcal{N} = 1$ SUSY extension of the SM. An extra Higgs doublet is added to the SM and the resulting spectrum is doubled by the superpartners while at the same time R -parity is imposed.⁴ Besides solving the Hierarchy problem, the MSSM has two extra advantages. First and foremost, because of R -parity the *lightest superpartner* (LSP) is stable and if it is also a weakly coupled particle it is a perfect candidate for *dark matter* (DM). Recent experimental results indicate that either neutralino or gravitino LSP's

³In $\mathcal{N} = 1$ SUSY, a massive Dirac fermion has technically two complex scalar superpartners.

⁴ R -parity is a global product symmetry which essentially gives the SM superpartners a charge (-1) and the usual SM particles a charge $(+1)$.

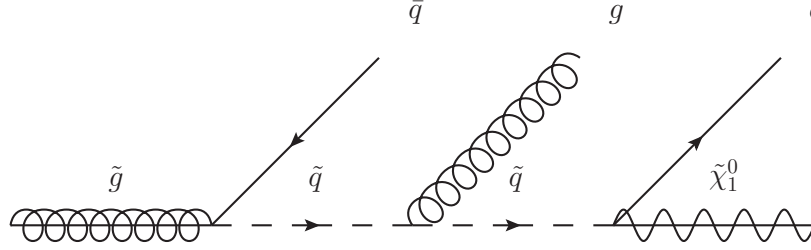


Figure 1.3: A gluino decays to an antiquark and squark, which after emitting a gluon decays to the neutralino LSP.

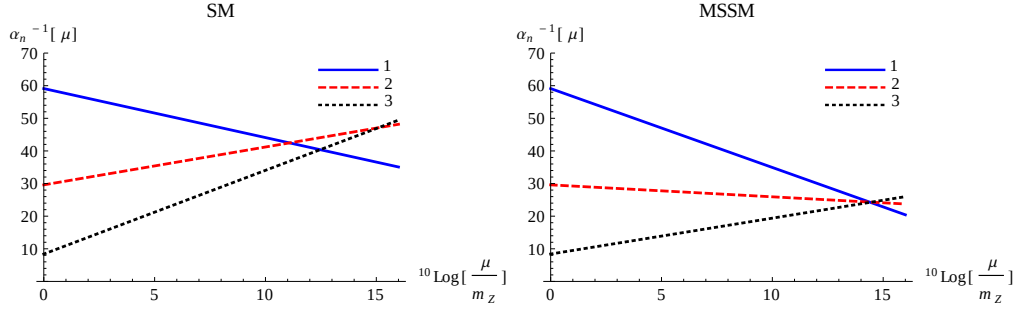


Figure 1.4: The one-loop RG flow of the inverse squared coupling constants in the SM (left) and MSSM (right). The thick blue and dashed red lines denote the (inverse of the) EW squared coupling constants $\alpha_1 = (5/3)g_{U(1)_Y}^2/4\pi$ and $\alpha_2 = g_{SU(2)_L}^2/4\pi$ respectively. The dotted black line denotes the (inverse of the) squared strong coupling constant $\alpha_3 = g_s^2/4\pi$.

are very good DM candidates [25]. Every superpartner would in that case decay to the neutralino or gravitino LSP as shown for example for a gluino decaying in Figure 1.3. Secondly, in the MSSM there is (almost) gauge coupling unification which points to a GUT description. This takes place at a unification energy of approximately 10^{16} GeV as can be seen in Figure 1.4 where the one-loop renormalization group flow of the inverse squared coupling constants for SM (left) and MSSM (right) are shown.

Arguably, the main disadvantage of MSSM is that it introduces a large amount of unobserved superpartners. If SUSY is an exact symmetry, the superpartners will have the same mass as their SM counterparts. However, since these particles have not been observed yet, it means that SUSY has to be *softly broken*. In practice, the effective soft SUSY-broken lagrangian contains about 100 extra parameters besides the usual SM parameters, which makes calculations much more complicated. To deal with this the phenomenologists and model builders consider simplified and

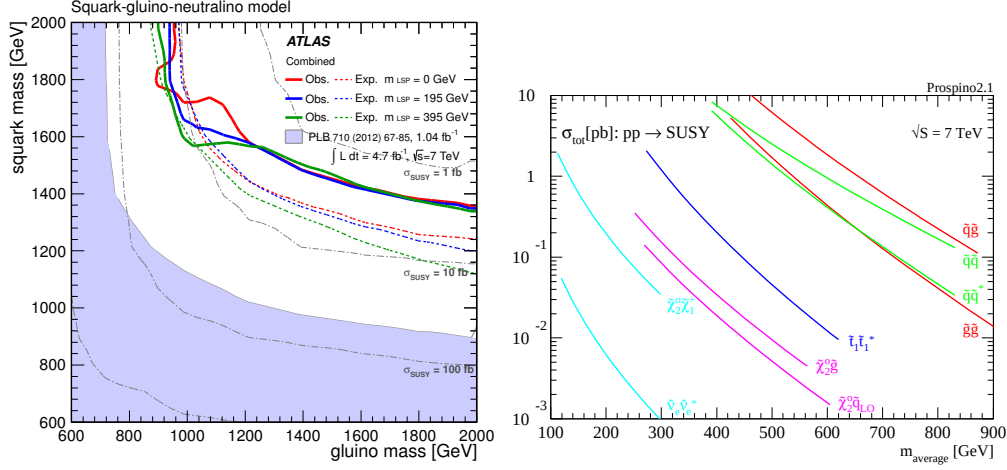


Figure 1.5: Left: Exclusion bounds of the squark and gluino masses at $\sqrt{s} = 7$ TeV from the Atlas group [32] for three neutralino LSP models with different LSP masses. Right: A plot that can be found at [33] of the NLO SUSY pair production cross section at $\sqrt{s} = 7$ TeV and produced with the PROSPINO tool [34, 35].

constrained SUSY models [26]. The search for SUSY at the TeV scale is a central part of the physics program of the Large Hadron Collider (LHC) at CERN and experimental searches for SUSY have also been performed at LEP and Tevatron in various final-state signatures, see [27] for a review. Recent LHC results put a lot of constraints on the parameter space of the MSSM or at least its simplified models [28]. In particular these searches have put strong model dependent exclusion bounds on the squark and gluino masses of simplified models, see for example the left graph in Figure 1.5 for exclusion bounds from Atlas for neutralino LSP models with different masses. In order to efficiently restrict the vast parameter space of the MSSM, model independent predictions and methods are required, such as the (mostly) model independent resummation formalisms of threshold enhanced corrections. For squark- and gluino-pair production the tightest mass bounds generically arise from jets+missing energy signatures, where the two LHC experiments have set lower bounds on the mass of squarks and gluinos of about 800 GeV-1 TeV [29, 30], depending on the precise underlying theoretical model assumed. Once upgraded to its nominal energy of 14 TeV the LHC should be sensitive to squark and gluino masses of up to 3 TeV [31]. These bounds reintroduce a fine-tuning, albeit much smaller, *little hierarchy problem* as seen from Eq. (1.13). In the context of the MSSM, and of any other R -parity conserving model, the supersymmetric partners of the SM particles are produced in pairs. Squarks and gluinos, coupling strongly to quarks and gluons, have typically the highest production rates (see the right graph of Figure 1.5) and therefore require a closer study in the following section.

1.4 Coloured SUSY production at the LHC

SUSY searches and, if squarks and gluinos are discovered, the measurement of their properties rely on a precise theoretical understanding of the production mechanism and on accurate predictions of the observables used in the analysis. From the theoretical point of view, the simplest of such observables is the total production cross section, on which we will focus in this thesis. For QCD mediated processes, such as squark- and gluino-pair production, the Born cross section is notoriously affected by large theoretical uncertainties, such that the inclusion of at least the next term in the expansion in the strong coupling constant α_s is mandatory for a reliable prediction. *Next-to-leading order* (NLO) SUSY QCD corrections for production of squarks and gluinos were computed in [36] and implemented in the program PROSPINO [34,35]. The corrections are large, up to 100% of the tree-level result, and lead to a significant reduction of the scale dependence of the cross section. The size of the $\mathcal{O}(\alpha_s)$ corrections to the cross section raises the question of the magnitude of unknown higher-order QCD corrections, and makes it desirable to include at least the dominant contributions beyond NLO. In Chapter 2 we will see that in the threshold region the partonic cross section is dominated by the soft-gluon emission off the initial- and final-state coloured particles and by Coulomb interactions of the two non-relativistic heavy particles, which give rise to singular terms of the form $\alpha_s \ln^{2,1} \beta$ and α_s/β respectively and in this work these corrections will be resummed to all orders in α_s .

Electroweak contributions were also investigated [37–42], but found to be much smaller than the QCD contributions, less than 5 – 10% of the Born result and therefore in this thesis we only consider QCD interactions. The QCD sector of the SM is:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}G_a^{\mu\nu}G_{\mu\nu}^a + \sum_i \bar{q}_i(\not{D} - im)q_i, \quad (1.14)$$

where G and q denote the gluon and the quark fields respectively while the sum runs over the flavours i of the quark particles. The complete MSSM lagrangian is large, but its QCD sector is much smaller and makes up the SUSY-QCD (SQCD) lagrangian:

$$\begin{aligned} \mathcal{L}_{\text{SQCD}} = & \mathcal{L}_{\text{QCD}} + \sum_{i,\sigma} ((D_\mu \tilde{q}_{i\sigma})^\dagger (D^\mu \tilde{q}_{i\sigma}) - m_{\tilde{q}_{i\sigma}}^2 \tilde{q}_{i\sigma}^\dagger \tilde{q}_{i\sigma}) + \frac{1}{2} \bar{\tilde{g}}_a (\not{D}_{ab} - im_{\tilde{g}_a} \delta_{ab}) \tilde{g}_b \\ & + g_s \sqrt{2} \sum_i \bar{\tilde{g}}_a (-\tilde{q}_{iL}^\dagger P_L + \tilde{q}_{iR}^\dagger P_R) \mathbf{T}^a q_i + \text{c.c.} + \mathcal{O}(\tilde{q}^4). \end{aligned} \quad (1.15)$$

In the above SQCD lagrangian the squark and gluino fields are denoted as $\tilde{q}$ and $\tilde{g}$ respectively. The sums run over the flavour i and the helicity type $\sigma = L, R$ of the squarks. Here we have left out the four-vertices of the squarks for simplicity, which are not needed at tree level for LHC processes. Throughout this thesis we only consider mass-degenerate light-flavour squarks, i.e. $m_{\tilde{q}_{i\sigma}} \equiv m_{\tilde{q}}$ for $i = u, d, s, c, b$ while the stops are treated separately.

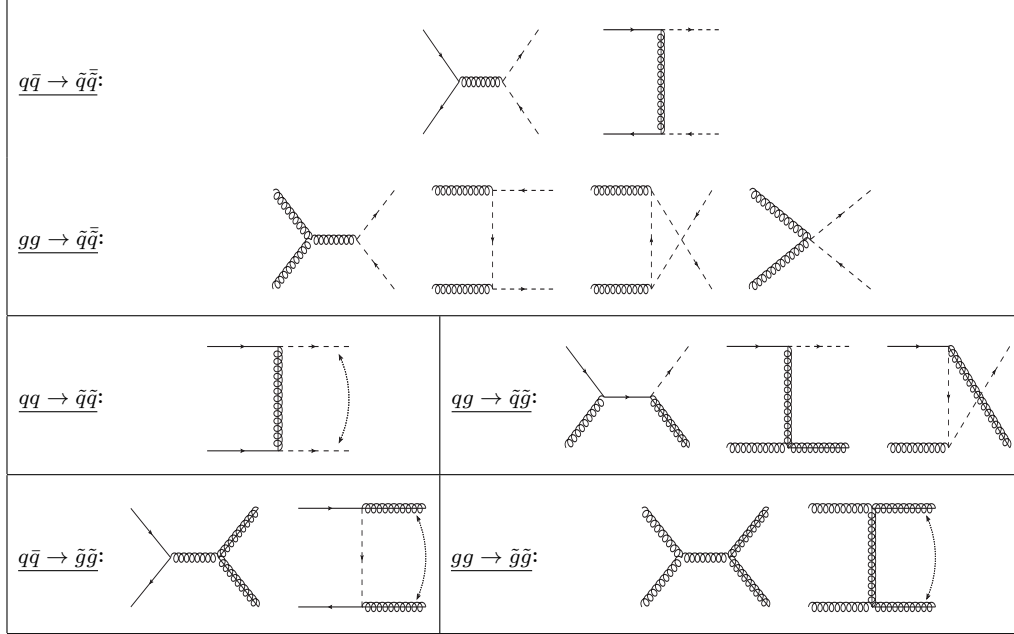


Table 1.1: All tree-level Feynman diagrams contributing to the processes of squark and gluino production at the LHC as shown in (1.16). The particles are represented by the common Feynman propagator lines found in the literature. Solid line: (anti-)quark. Curly line: gluon. Dashed line: (anti-)squark. Solid plus curly line: gluino.

The above SQCD model allows at leading order [43–45] for the following partonic channels which contribute to the production of light-flavour squarks and gluinos:

$$\begin{aligned}
 gg, q_i \bar{q}_j &\rightarrow \tilde{q} \bar{\tilde{q}}, \\
 q_i q_j &\rightarrow \tilde{q} \bar{\tilde{q}}, & \bar{q}_i \bar{q}_j &\rightarrow \bar{\tilde{q}} \tilde{\tilde{q}}, \\
 g q_i &\rightarrow \tilde{g} \tilde{q}, & g \bar{q}_i &\rightarrow \tilde{g} \bar{\tilde{q}}, \\
 gg, q_i \bar{q}_i &\rightarrow \tilde{g} \tilde{g},
 \end{aligned} \tag{1.16}$$

where $i, j = u, d, s, c, b$. The corresponding tree level Feynman diagrams can be found in [36] and are reproduced in Table 1.1. At NLO further partonic processes contribute to the cross section. To keep the notation as simple as possible, in (1.16) we have suppressed the helicity and flavour indices of the squarks.

To illustrate the relative magnitude of the various processes depending on the squark and gluino masses, the ratio of the total hadronic cross section for the processes (1.16) to the total inclusive cross section for squark and gluino production $\sigma_{\text{SUSY}} = \sigma_{PP \rightarrow \tilde{q} \bar{\tilde{q}} + \tilde{q} \tilde{q} + \tilde{g} \tilde{g} + \tilde{g} \bar{\tilde{g}}}$ is shown

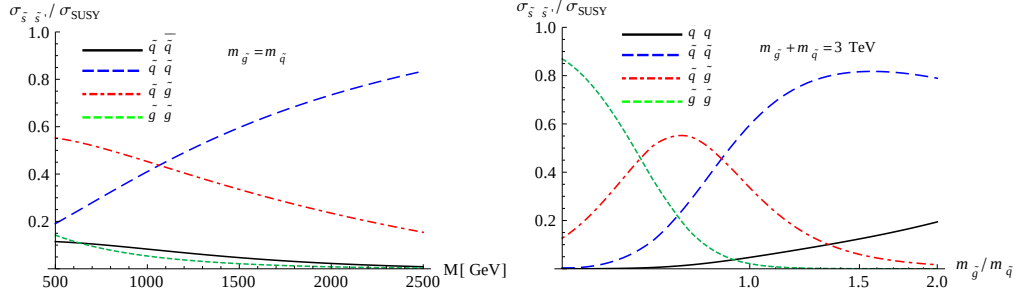


Figure 1.6: Ratio of the LO production cross sections for the processes (1.16) to the total Born production rate of coloured particles, σ_{SUSY} , for the LHC with $\sqrt{s} = 8$ TeV. Left: Mass dependence for a fixed mass ratio $m_{\tilde{q}} = m_{\tilde{g}} = M$. Right: Dependence on the ratio $m_{\tilde{g}}/m_{\tilde{q}}$ for a fixed average mass $(m_{\tilde{q}} + m_{\tilde{g}})/2 = 1.5$ TeV.

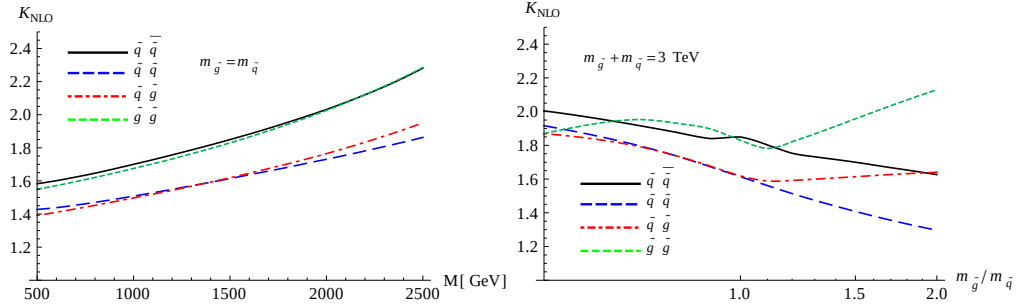


Figure 1.7: NLO K -factor for the processes (1.16) at the LHC with $\sqrt{s} = 8$ TeV. Left: Mass dependence for a fixed mass ratio $m_{\tilde{q}} = m_{\tilde{g}} = M$. Right: Dependence on the ratio $m_{\tilde{g}}/m_{\tilde{q}}$ for a fixed average mass $(m_{\tilde{q}} + m_{\tilde{g}})/2 = 1.5$ TeV.

in Figure 1.6 for the LHC with $\sqrt{s} = 8$ TeV centre-of-mass energy. The charge-conjugate final states are also included in the total SUSY production cross section σ_{SUSY} . Here and throughout this chapter we take the factorization scale equal to the average mass of the heavy pair $\mu_f = M$. From the left-hand side plot, showing the relative contributions of the various processes as a function of a common squark and gluino mass, it can be seen that squark-squark and squark-gluino production are by far the dominant channels over the full mass range considered. In the right-hand side plot, the relative contributions are shown as a function of the squark-gluino mass ratio for average mass $\frac{m_{\tilde{g}} + m_{\tilde{q}}}{2} = 1.5$ TeV and it is seen that only for gluinos that are significantly lighter than squarks, gluino-pair production becomes the dominant production channel. In Figure 1.7 we show the K -factor $K_{\text{NLO}} = \sigma_{\text{NLO}}/\sigma_{\text{LO}}$ for the SUSY-QCD corrections for the various production

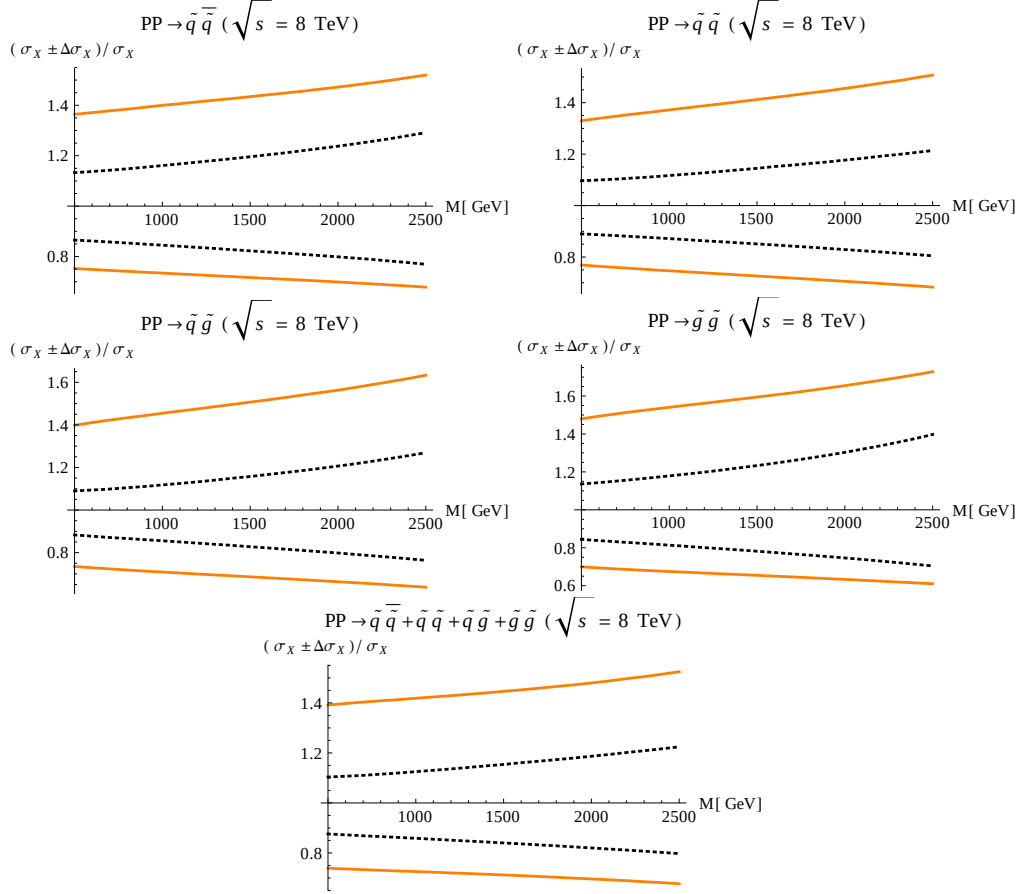


Figure 1.8: Factorization uncertainty of the LO (solid orange) and NLO (dotted black) at the LHC with $\sqrt{s} = 8$ TeV. All cross sections are normalized to one at the central value of the factorization scale.

processes as obtained from PROSPINO [34].⁵ The corrections are positive and enhance the cross section from 40% for squark-gluino production with light sparticle masses up to 100% or larger for squark-antisquark and gluino-pair production at large sparticle masses.

The hadronic cross section is factorization scale independent as it is defined in Eq. (1.3). A factorization scale dependence is only introduced after truncating the series expansion in α_s of the partonic cross section and parton luminosity. For this reason, by including NLO and higher

⁵To see the genuine size of the NLO corrections, the K -factors have been computed using the NLO PDFs for the Born cross sections.

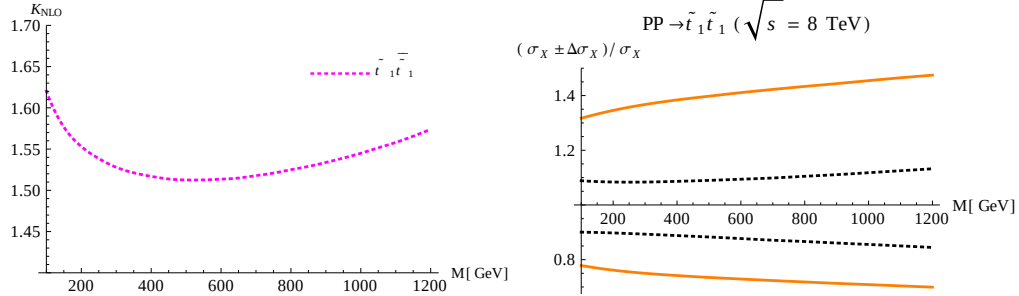


Figure 1.9: Left: NLO K -factor for stop-pair production at $\sqrt{s} = 8$ TeV as a function of the stop mass. Right: Factorization uncertainty of the LO (solid orange) and NLO (dotted black) at the LHC with $\sqrt{s} = 8$ TeV, normalized to one at the central value of the factorization scale.

orders the factorization scale dependence of the hadronic cross section is generally reduced. The large NLO corrections alluded to by Figure 1.7 are thus expected to drastically reduce the scale dependence. This is shown in Figure 1.8 for all four processes and the total coloured SUSY process with $\sqrt{s} = 8$ TeV at the LHC, where the factorization scale dependence is plotted for the LO (solid orange) as well as the NLO (dotted black) cross section. In the plots the factorization scale was varied between $M/2 \leq \mu_f \leq 2M$ and the corresponding positive and negative error resulting from this scale variation plotted and normalized to the corresponding default cross section $\sigma(\mu_f = M)$. The uncertainty at LO is reduced from approximately 50% to about 20% at NLO at large masses. Similarly, an improvement in the theoretical prediction of the cross section and the reduction in the scale dependence is also expected when including even higher order corrections such as the previously mentioned threshold enhanced terms. This error reduction for the threshold enhanced corrections is indeed found (cf. Figure 3.9).

Beside the channels listed in (1.16), we will also consider stop-pair production in this thesis:

$$gg, q_i \bar{q}_i \rightarrow \tilde{t}_j \tilde{t}_j^*, \quad (1.17)$$

where only the initial states appearing at leading order have been shown. The NLO SUSY-QCD corrections have been computed in [35] and are implemented in PROSPINO [34]. Contrary to the light-flavour squark case, in most scenarios the mixing of the two weak eigenstates $\tilde{t}_L, \tilde{t}_R$, and the mass difference of the resulting mass eigenstates, $\tilde{t}_1 = \tilde{t}_L \cos \theta_{\tilde{t}} + \tilde{t}_R \sin \theta_{\tilde{t}}$, $\tilde{t}_2 = -\tilde{t}_L \sin \theta_{\tilde{t}} + \tilde{t}_R \cos \theta_{\tilde{t}}$, is non-negligible. Off-diagonal production of the mass eigenstates, e.g. $\tilde{t}_1 \tilde{t}_2^*$, appears at NLO in SUSY-QCD, and through electroweak contributions. It is therefore suppressed compared to diagonal production [35, 46] and will not be considered here. It must also be mentioned that because of the absence of a significant top-quark component inside the nucleon the processes $\tilde{g}\tilde{t}$ and $\tilde{t}\tilde{t}^*$ first contribute to the cross section at NLO and NNLO respectively, and are thus numerically suppressed. The NLO K -factor for the process $PP \rightarrow \tilde{t}_1 \tilde{t}_1$ is shown on the left

in Figure 1.9 for a centre-of-mass energy of 8 TeV, and the mass range $M = 100 - 1200$ GeV. As can be seen, NLO corrections are in the 50 – 60% range. The right hand side of Figure 1.9 shows the factorization scale dependence for the light stops. The inclusion of NLO corrections again reduces the scale dependence from about 50% to 20% at large masses and even larger reductions are expected from the resummation of threshold enhanced terms (cf. Figure 3.12). The predictions for the second mass eigenstate differ only in the fixed-order NLO results, and for a given mass the numerical difference between the cross sections for $\tilde{t}_1\tilde{t}_1$ and $\tilde{t}_2\tilde{t}_2$ production is below 2% for the mass range considered in this work. We therefore omit results for the process $PP \rightarrow \tilde{t}_2\tilde{t}_2$ in this thesis.

1.5 Outline of the thesis

In this thesis we present full *Next-to-leading log* (NLL) and partial *Next-to-next-to-leading log* (NNLL) results for mass degenerate light-flavour squarks, gluino and stop pair production at the LHC. The content of the rest of the thesis is organized as follows. In Chapter 2 we present the theoretical foundation needed for our numerical results given in Chapters 3 to 5. The resummed partonic cross section is shown to factorize and the relevant RG equations are presented.

Chapter 3 is based on [47] and contains our complete numerical results for stable squarks, gluinos and stops at NLL order with $\sqrt{s} = 8$ TeV and $\sqrt{s} = 14$ TeV at the LHC. We briefly review the resummation procedure for the case of squarks and gluinos at NLL. The threshold enhanced corrections are found to be large, 5% – 170% depending on the process with errors reduced to $\pm 10\%$. The chapter also contains some benchmark points for $\sqrt{s} = 7$ TeV at the LHC.

Chapter 4 is based on [48] and studies the effects of unstable squarks and gluinos on our NLL results from Chapter 3 at $\sqrt{s} = 8$ TeV at the LHC. We find that for squark and gluino widths $\Gamma/M \lesssim 5\%$ the effects are small and contained within the total uncertainty of the zero-width results. For larger widths a higher order treatment becomes necessary. Finite-width effects on the total SUSY production rate are found to be negligible on the whole range of squark and gluino widths considered.

Chapter 5 contains some unpublished partial numerical results at NNLL order with $\sqrt{s} = 8$ TeV and $\sqrt{s} = 14$ TeV at the LHC. Only the soft logarithms are resummed at NNLL with LO hard functions and are found to drastically reduce the errors involved with the resummation procedure. We conclude that for a comparison with the Mellin-space resummation approach [49, 50] it is necessary to take the NNLL soft corrections into account.

In Chapter 6 we present our conclusions and an outlook of the thesis. The appendices contain some NLL and NNLL resummation and expansion formulas used throughout the thesis and a discussion of the soft scale involved with the resummation procedure.

CHAPTER 2

Theory of soft and Coulomb resummation

There have been many papers on improving theoretical predictions for coloured heavy pair particle production at hadron colliders by calculating higher-order threshold corrections in QCD. These can be generally classified into two groups: the works of [49–57] perform a resummation of only soft logarithms in so-called *Mellin-space*, while taking the first Coulomb correction into account. In such a formalism a Mellin transformation of the cross section is performed, after which the partonic cross section is factorized and resummed. Contrary to the more traditional Mellin-space formalism, the approach developed in [5, 58], which is based on soft-collinear effective theory (SCET) and potential non-relativistic QCD (pNRQCD), resummation is performed directly in *momentum space* via renormalization-group evolution equations [11, 59, 60]. As was mentioned in the previous chapter, the corrections that are enhanced near threshold are given by two contributions, Coulomb corrections, that occur in powers of α_s/β , and threshold logarithms of the form $\alpha_s^n \ln^m \beta$. As a result, the expansion in the strong coupling constant must be reorganized near threshold, since both types of corrections can become of order one,

$$\alpha_s \ln \beta \sim 1, \quad \frac{\alpha_s}{\beta} \sim 1. \quad (2.1)$$

These threshold enhanced corrections are studied in this chapter and we provide the technical details necessary for their *combined* resummation directly in momentum space. The combined soft-Coulomb resummation formalism in momentum space of [5, 58] is based on a factorization of the hard-scattering total cross section for partonic subprocesses of the type (1.2). It can be shown that near the partonic threshold $\hat{s} \sim (M_{\bar{s}} + M_{\bar{s}'})^2$, the partonic cross section factorizes into three contributions, a hard function H , a soft function W containing soft gluons to all orders, and a potential function J summing Coulomb-gluon exchange. The corresponding factorization formula has been derived in [5] for S-wave dominated production processes up to NNLL accuracy. The strength of the resummation formalism lies in the fact that its derivation depends purely on the structure of QCD interactions in the soft, potential and collinear regions of phase space. Therefore the formalism is applicable to any S-wave processes which arise in the SM and its

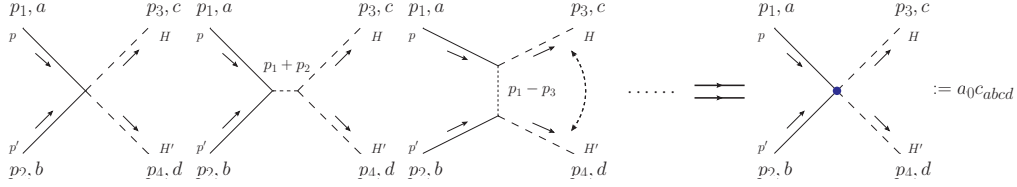


Figure 2.1: The hard contributions to pair production. The dots represent higher loop corrections where the momenta inside the loops are in the hard region $k \sim M$.

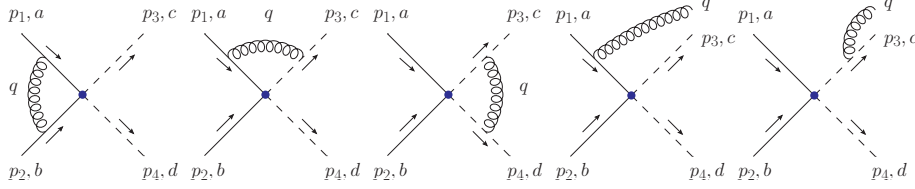


Figure 2.2: The one-loop virtual and real gluon corrections. The self-energy corrections are not shown.

extensions. This includes all production processes of squarks and gluinos, apart from quark-antiquark initiated stop-antistop production, that proceeds through a P-wave. In this chapter we first review the derivation of the S-wave case given in [5] and extend the applicability of the formalism to stop-antistop production [47].

The chapter is organized as follows. In Section 2.1 we study one-loop diagrams that contribute to the production cross section near threshold and introduce the large soft logarithmic and Coulomb corrections. Section 2.2 discusses the effective field theories required to perform the combined resummation of the threshold enhanced terms in momentum space. In Section 2.3 we start by reviewing the factorization and resummation formula derived in [5] for S-wave processes and then show that it also holds true for stop pair production. Finally, we discuss the scales and errors involved with the resummation formalism that is eventually used in later chapters.

2.1 Threshold enhanced corrections

Throughout this chapter we consider the production of two *coloured* heavy particles H, H' according to (1.2). Near threshold, H and H' have a small non-relativistic velocity β as observed in the centre of mass frame. In this frame the modes of the interacting particles with momenta of the order $k \sim M \gg \Lambda_{QCD}$ decouple from the dynamical degrees of freedom. These modes may thus be integrated out and the resulting effective vertices are to be matched to the full theory in a perturbative fashion. The effective vertex of two collinear modes and two heavy potential modes will be represented by blue dots throughout this chapter as shown in Figure 2.1. In what

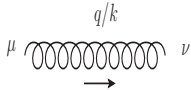
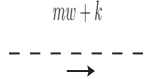
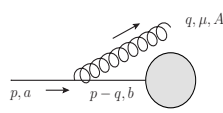
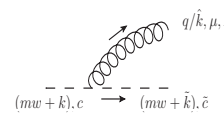
	$\frac{-ig_{\mu\nu}}{q^2+i.0} / \frac{-ig_{\mu\nu}}{-\vec{k}^2+i.0}$		$\frac{i}{2mk_0-\vec{k}^2+i.0}$
	$\frac{-g_s p_\mu}{p \cdot q - i.0} T_{ba}^{(r)A} \times \frac{p-q, b}{\rightarrow}$		$-2ig_s m w_\mu T_{cc}^{(R)A}$

Table 2.1: The Feynman diagrams for the soft, potential and collinear fields.

follows we have integrated out all the hard modes and the contribution of the effective vertex to the Feynman rules is denoted as $a_0 c_{abcd}$. Here c_{abcd} is the colour tensor of the hard production process and the matching coefficient a_0 is a series in α_s which depends on the masses of the produced particles. The matching coefficient $a_0 = C\mathcal{O}_{\{\alpha\}}$ contains the Lorentz structure $\{\alpha\}$ of the hard scattering amplitude (cf. Eq. (2.15)), but in the following we suppress all Lorentz indices. The incoming particles p, p' are assumed to be massless and moving along opposite collinear directions $n_\pm := (1, \pm \vec{n})$, $\vec{n}^2 = 1$ with momenta $p_{1,2} = (\sqrt{s}/2)n_{+,-}$. The centre of mass energy is $\sqrt{s}$. Near threshold, the two outgoing heavy particles are non-relativistic with momenta $p_{3,4} = M_{H,H'} w + k_{1,2}$, where $w^\mu = (1, 0, 0, 0)$ is the time-like direction of the produced particles at rest and $(k_{1,2})_0 \sim M\beta^2$, $\vec{k}_{1,2} \sim M\beta$ are *potential* momenta. In this chapter, $M := (M_H + M_{H'})/2$ will denote the average mass of the produced heavy particle pair.

A study of one-loop gluon corrections as shown in Figure 2.2 reveals poles in the gluon propagator in the soft, potential, collinear and hard region. Since the hard contributions are already included in the effective vertex we conclude that near threshold only the following regions in phase space contribute in the EFT [5]:

- Soft (final state and gluons): $k \sim M\beta^2$
- Potential⁶ (final states H, H' and gluons): $k_0 \sim M\beta^2, \vec{k} \sim M\beta$
- Collinear (initial states p, p') along $n_\pm$: $k \cdot n_\pm \sim M$, $k \cdot n_\mp \sim M\beta^2$, $k \cdot n_\perp \sim M\beta$

The relevant Feynman rules for one-loop calculations corresponding to these fields are shown in Table 2.1. In this table and throughout this chapter we denote momenta in the collinear, potential and soft regions by the letters p, k and q respectively. The incoming collinear fields are represented by solid lines, while the produced heavy particles are dashed. Table 2.1 is derived by expanding the full Feynman rules in β and keeping the LO term. One observes that the emission or absorption of a soft gluon by collinear fields proceeds according to the usual *eikonal*

⁶The total momenta of the heavy final states themselves are not in the potential region, but their variation on top of their rest momenta $M_{H,H'} w$ is.

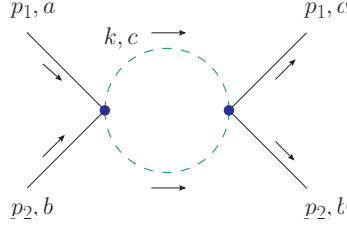


Figure 2.3: The tree level forward-scattering diagram.

Feynman rules. Furthermore, note that there are no 3-vertices containing a potential and collinear leg in the EFT. The reason for this is that the momentum of the third leg would be the sum of a potential and collinear momentum, which would neither be soft, potential nor collinear and has thus already been integrated out. The Feynman rules are valid for collinear quarks and gluon fields. The quarks and gluons belong to the representations $r = \mathbf{3}$ and $r = \mathbf{8}$ respectively of the QCD colour group $SU(3)$, while the produced particles H, H' are assumed to be heavy and to belong to the colour representation R, R' of $SU(3)$. If the colour tensor of any (incoming or outgoing) particle is $T_{ba}^{(r)A}$, then that of its antiparticle equals $T_{ba}^{(\bar{r})A} = -T_{ab}^{(r)A}$.

Let us now start by calculating the tree level production cross section near threshold. As we will do here and throughout this chapter, we calculate the cross section from the imaginary part of the forward-scattering amplitude $pp' \rightarrow HH'X \rightarrow pp'$ as shown for example in Figure 2.3 at tree level. In that figure and the remaining ones in this chapter we drop the standard $M_{H,H'}w$ term when denoting the momentum flow through the produced heavy pair. The green colours denote the fact that the momentum (difference) flow k is in the potential region. Using the Feynman rules in Table 2.1 one finds for the tree level forward-scattering amplitude:

$$\mathcal{A}^{(0)}(\epsilon) = a_0^2 c_{abcd} c_{abcd} \tilde{\mu}^{2\epsilon} (-i) \int \frac{d^d k}{(2\pi)^d} \frac{i}{(2M_H k_0 - \vec{k}^2 + i.0)} \frac{i}{(2M_{H'}(E - k_0) - \vec{k}^2 + i.0)}.$$

We remind the reader that a_0 contains Lorentz indices while in a_0^2 these are contracted. Here $E = \sqrt{\hat{s}} - 2M$ denotes the kinetic energy of the produced pair in the centre of mass frame and $\tilde{\mu} = e^{\gamma_E/2} \mu / \sqrt{4\pi}$ is the usual dimensional regularization factor in the $\overline{\text{MS}}$ prescription. For simplicity we define $v_0 := a_0^2 c_{abcd} c_{abcd}$. The spherical coordinate transformation in $d = 4 - 2\epsilon$ dimensions satisfies:

$$\begin{aligned} \int d^d k f(k) &= \int_0^{2\pi} d\phi_{1-2\epsilon} \int_{-1}^1 dy (1-y^2)^{-\epsilon} \int_0^\infty d|\vec{k}| |\vec{k}|^{2-2\epsilon} \int_{-\infty}^\infty dk_0 f(k_0, |\vec{k}|, y, \phi), \\ \int_0^{2\pi} d\phi_{1-2\epsilon} &= \frac{2\pi^{1-\epsilon}}{\Gamma(1-\epsilon)}, \quad \int_{-1}^1 dy (1-y^2)^{-\epsilon} = \frac{\sqrt{\pi} \Gamma(1-\epsilon)}{\Gamma(3/2-\epsilon)}, \end{aligned} \quad (2.2)$$

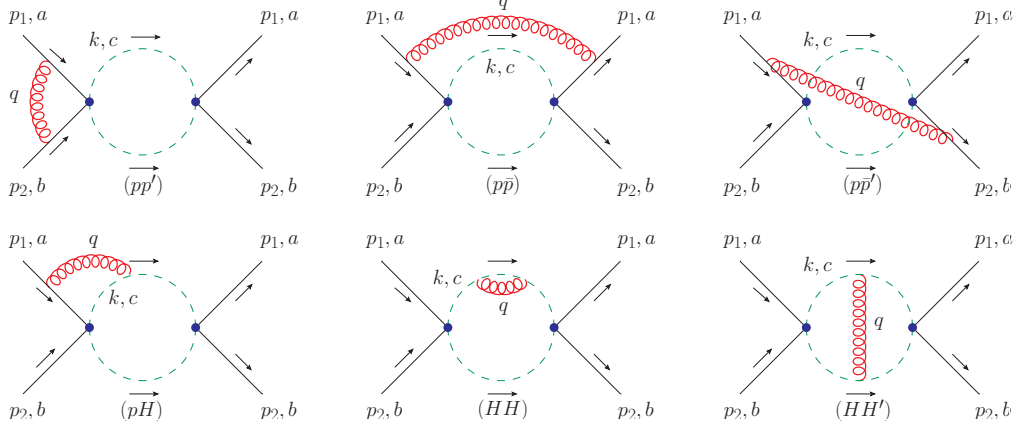


Figure 2.4: The one-loop soft gluon corrections to the forward-scattering amplitude.

where $y = \cos(\theta)$ denotes the cosine of the polar angle θ and ϕ the azimuthal angle. Performing first a contour integral over k_0 and then going to spherical coordinates with Eq. (2.2) gives:

$$\mathcal{A}^{(0)}(\epsilon) = v_0 \frac{(-2e^{-\gamma_E} \mu^{-2} m_r E)^{-\epsilon}}{16\sqrt{\pi}\Gamma(3/2 - \epsilon) \cos(\pi\epsilon)} \sqrt{\frac{-m_r E}{2M^2}} = v_0 \frac{\sqrt{-m_r E}}{8\sqrt{2}\pi M} (1 + \mathcal{O}(\epsilon)), \quad (2.3)$$

where $m_r := M_H M_{H'}/(M_H + M_{H'})$ is the reduced mass. The cross section equals the imaginary part of the forward-scattering amplitude:

$$\hat{\sigma}^{(0)}(\epsilon) = \text{Im}[\mathcal{A}^{(0)}(\epsilon)] = -\tilde{v}_0 \frac{(2e^{-\gamma_E} \mu^{-2} m_r E)^{-\epsilon}}{16\sqrt{\pi}\Gamma(3/2 - \epsilon)} \sqrt{\frac{m_r E}{2M^2}} = -\tilde{v}_0 \frac{\sqrt{m_r E}}{8\sqrt{2}\pi M} (1 + \mathcal{O}(\epsilon)), \quad (2.4)$$

where we defined $\tilde{v}_0 = a_0^2 c_{abcd} c_{abcd}^*$. Note that since a_0 is imaginary, its square is negative and thus the cross section in (2.4) is positive as one would expect.

Next we will consider higher loop corrections to the desired process (1.2). Collinear and hard gluon corrections are needed for renormalization purposes but their finite terms themselves contribute logarithms of μ/M that are not large, since typically $\mu \sim M$. These finite terms of the collinear and hard corrections are already assumed to be included in the PDF's and in the coefficients of the effective operators respectively. Therefore let us calculate the gluon corrections in the soft and potential region.

2.1.1 Soft region

The one-loop soft corrections to the forward-scattering amplitudes are shown in Figure 2.4. The red colouring of the gluon lines denotes the fact that the momentum q is taken in the soft region.

The effective vertices and the colour matrices are neglected at first and are taken into account later. The diagrams (pp') and $(p\bar{p})$ are zero because (pp') leads to a scaleless integral and $(p\bar{p}) \sim p_1^2 = 0$ as follows from the Feynman rules in Table 2.1. The kinematic part of the Feynman diagrams (pH) and $(p\bar{p}')$ equal:

$$\begin{aligned}
(\Delta\mathcal{A})_{s,kin}^{(pH)} &= \tilde{\mu}^{2\epsilon}(-i) \int \frac{d^4k}{(2\pi)^4} \int \frac{d^dq}{(2\pi)^d} \frac{-g_s p_{1\mu}}{(p_1 \cdot q - i.0)} \frac{-ig^{\mu\nu}}{(q^2 + i.0)} \frac{i(-2ig_s M_H w_\nu)}{(2M_H k_0 - \vec{k}^2 + i.0)} \times \\
&\quad \frac{i}{(2M_H(k_0 + q_0) - \vec{k}^2 + i.0)} \frac{i}{(2M_{H'}(E - k_0 - q_0) - \vec{k}^2 + i.0)}. \\
(\Delta\mathcal{A})_{s,kin}^{(p\bar{p}')} &= \tilde{\mu}^{2\epsilon}(-i) \int \frac{d^4k}{(2\pi)^4} \int \frac{d^dq}{(2\pi)^d} \frac{-g_s p_{1\mu}}{(p_1 \cdot q - i.0)} \frac{-ig^{\mu\nu}}{(q^2 + i.0)} \frac{-g_s p_{2\nu}}{(p_2 \cdot q - i.0)} \times \\
&\quad \frac{i}{(2M_H k_0 - \vec{k}^2 + i.0)} \frac{i}{(2M_{H'}(E - k_0 - q_0) - \vec{k}^2 + i.0)}.
\end{aligned}$$

Performing contour integrals for k_0, q_0 in the upper and lower planes respectively and using (2.2):

$$\begin{aligned}
(\Delta\mathcal{A})_{s,kin}^{(pH)} &= \frac{\pi^3 g_s^2 \tilde{\mu}^{2\epsilon}}{m_r M (2\pi)^{d+4}} \frac{2\pi^{1-\epsilon}}{\Gamma(1-\epsilon)} \int_{-1}^1 dy \frac{(1-y^2)^{-\epsilon}}{1-y} \int_0^\infty \frac{q^{-2\epsilon} k^2 dk dq}{(E - \frac{k^2}{2m_r})(E - q - \frac{k^2}{2m_r})} \\
&= \mathcal{A}^{(0)}(0) \frac{\alpha_s}{4\pi} \left\{ \frac{1}{\epsilon^2} + \frac{4 - 2L_{-E}}{\epsilon} + 2L_{-E}(L_{-E} - 4) + 16 + \frac{13\pi^2}{12} + \mathcal{O}(\epsilon) \right\}. \\
(\Delta\mathcal{A})_{s,kin}^{(p\bar{p}')} &= -2(\Delta\mathcal{A})_{s,kin}^{(pH)}, \tag{2.5}
\end{aligned}$$

We have defined $L_X := \log(\frac{8X}{\mu})$ and in the second line we expanded in ϵ and factored out the tree level amplitude (2.3). The remaining Feynman diagrams (HH) and (HH') equal:

$$\begin{aligned}
(\Delta\mathcal{A})_{s,kin}^{(HH)} &= \tilde{\mu}^{2\epsilon}(-i) \int \frac{d^4k}{(2\pi)^4} \int \frac{d^dq}{(2\pi)^d} \frac{i}{(2M_H k_0 - \vec{k}^2 + i.0)} \frac{i(-2ig_s M_H w_\mu)}{(2M_H(k_0 + q_0) - \vec{k}^2 + i.0)} \\
&\quad \times \frac{-ig^{\mu\nu}}{(q^2 + i.0)} \frac{i(-2ig_s M_H w_\nu)}{(2M_H k_0 - \vec{k}^2 + i.0)} \frac{i}{(2M_{H'}(E - k_0) - \vec{k}^2 + i.0)}. \\
(\Delta\mathcal{A})_{s,kin}^{(HH')} &= \tilde{\mu}^{2\epsilon}(-i) \int \frac{d^4k}{(2\pi)^4} \int \frac{d^dq}{(2\pi)^d} \frac{i}{(2M_H k_0 - \vec{k}^2 + i.0)} \frac{i(-2ig_s M_H w_\mu)}{(2M_H(k_0 + q_0) - \vec{k}^2 + i.0)} \\
&\quad \times \frac{-ig^{\mu\nu}}{(q^2 + i.0)} \frac{i(-2ig_s M_{H'} w_\nu)}{(2M_{H'}(E - k_0) - \vec{k}^2 + i.0)} \frac{i}{(2M_{H'}(E - k_0 - q_0) - \vec{k}^2 + i.0)}.
\end{aligned}$$

Performing contour integrals for k_0, q_0 and again using (2.2) results in:

$$\begin{aligned}
(\Delta\mathcal{A})_{s,kin}^{(HH)} &= \frac{-\pi^3 g_s^2 \tilde{\mu}^{2\epsilon}}{m_r M (2\pi)^{d+4}} \frac{2\pi^{1-\epsilon}}{\Gamma(1-\epsilon)} \int_{-1}^1 dy (1-y^2)^{-\epsilon} \int_0^\infty \frac{q^{1-2\epsilon} k^2 dk dq}{(E - \frac{k^2}{2m_r})^2 (E - q - \frac{k^2}{2m_r})} \\
&= \frac{2\epsilon}{1-2\epsilon} (\Delta\mathcal{A})_{s,kin}^{(pH)} = \mathcal{A}^{(0)}(0) \frac{\alpha_s}{4\pi} \left\{ \frac{2}{\epsilon} + 12 - 4L_{-E} + \mathcal{O}(\epsilon) \right\}. \\
(\Delta\mathcal{A})_{s,kin}^{(HH')} &= 2(\Delta\mathcal{A})_{s,kin}^{(HH)}. \tag{2.6}
\end{aligned}$$

In order to discuss the colour structure it is very useful to consider a basis of colour tensors $\{c^{R_\alpha}\}$ as given in [58], where R_α is an irreducible representation appearing in the colour decomposition of the heavy pair product representation $R \times R'$. The colour structures for the production of a final state in the representation R_α then equal:

$$\begin{aligned}
(\Delta\mathcal{A})_{s,col}^{(pH)} &= 2(T_{\bar{a}a}^{(r)A}\delta_{\bar{b}b} + T_{\bar{b}b}^{(r')A}\delta_{a\bar{a}})c_{\bar{a}\bar{b}cd}^{R_\alpha}(T_{\bar{c}c}^{(R)A}\delta_{d\bar{d}} + T_{\bar{d}d}^{(R')A}\delta_{c\bar{c}})c_{abcd}^{R_\alpha*} \\
(\Delta\mathcal{A})_{s,col}^{(p\bar{p}')} &= 2T_{\bar{a}a}^{(r)A}T_{\bar{b}b}^{(r')A}c_{\bar{a}\bar{b}cd}^{R_\alpha}c_{abcd}^{R_\alpha*} \\
(\Delta\mathcal{A})_{s,col}^{(HH)} &= c_{abcd}^{R_\alpha}T_{\bar{c}c}^{(R)A}T_{\bar{d}d}^{(R)A}c_{\bar{a}\bar{b}cd}^{R_\alpha*} + c_{abcd}^{R_\alpha}T_{\bar{d}d}^{(R')A}T_{\bar{c}c}^{(R')A}c_{\bar{a}\bar{b}cd}^{R_\alpha*} = C_R + C_{R'} \\
(\Delta\mathcal{A})_{s,col}^{(HH')} &= c_{abcd}^{R_\alpha}T_{\bar{c}c}^{(R)A}T_{\bar{d}d}^{(R')A}c_{\bar{a}\bar{b}cd}^{R_\alpha*}
\end{aligned} \tag{2.7}$$

The expressions for (pH) and $(p\bar{p}')$ do not strictly depend on the colour basis chosen and can be expressed in terms of the Casimir invariants by the use of colour conservation:

$$(T_{\bar{c}c}^{(R)A}\delta_{d\bar{d}} + T_{\bar{d}d}^{(R')A}\delta_{c\bar{c}})c_{\bar{a}\bar{b}cd}^{R_\alpha} = (T_{\bar{a}a}^{(r)A}\delta_{\bar{b}b} + T_{\bar{b}b}^{(r')A}\delta_{a\bar{a}})c_{\bar{a}\bar{b}cd}^{R_\alpha} \tag{2.8}$$

By using (2.8), the sum of (pH) and $(p\bar{p}')$ when including the colour structure equals:

$$(\Delta\mathcal{A})_s^{(pH)} + (\Delta\mathcal{A})_s^{(p\bar{p}')} = (\Delta\mathcal{A})_{s,kin}^{(pH)} \left((\Delta\mathcal{A})_{s,col}^{(pH)} - 2(\Delta\mathcal{A})_{s,col}^{(p\bar{p}')} \right) = 2(C_r + C_{r'}) (\Delta\mathcal{A})_{s,kin}^{(pH)}.$$

In order to simplify the colour structure of the remaining diagrams it is useful to choose the above so-called s -channel colour basis [61], in which case one can show:

$$(\Delta\mathcal{A})_s^{(HH)} + (\Delta\mathcal{A})_s^{(HH')} = (\Delta\mathcal{A})_{s,kin}^{(HH)} \left((\Delta\mathcal{A})_{s,col}^{(HH)} + 2(\Delta\mathcal{A})_{s,col}^{(HH')} \right) = C_{R_\alpha} (\Delta\mathcal{A})_{s,kin}^{(HH')}.$$

The total one-loop soft corrections in Eq. (2.5) and (2.6) contain double as well as single poles. After adding the hard contributions the double poles cancel while the single poles in ϵ survive. As alluded to before, the single poles cancel when taking the corrections coming from collinear splittings of the initial states into account [9, 10], which are included in the PDF's. In other words, in practice one may simply subtract the ϵ poles and use the renormalized $\overline{\text{MS}}$ PDF's. Keeping only the finite terms, we conclude that the total one-loop soft gluon corrections to the tree level cross section equals:

$$\begin{aligned}
\Delta\hat{\sigma}_s^{R_\alpha,(1)} &= 2(C_r + C_{r'})\text{Im}\left[(\Delta\mathcal{A})_{s,kin}^{(pH)}\right] + C_{R_\alpha}\text{Im}\left[(\Delta\mathcal{A})_{s,kin}^{(HH')}\right] \\
&= \frac{\alpha_s}{4\pi}\hat{\sigma}^{(0)}(0)\left\{4(C_r + C_{r'})\left(L_E^2 - 4L_E + 8 - \frac{11\pi^2}{24}\right) + 4C_{R_\alpha}(3 - L_E)\right\}.
\end{aligned} \tag{2.9}$$

Eq. (2.9) agrees with the results given for example in [62] and as anticipated contains soft logarithms of the form $L_E^{(2,1)} = \log^{(2,1)}(8E/\mu) \sim \log^{(2,1)}(\beta)$.

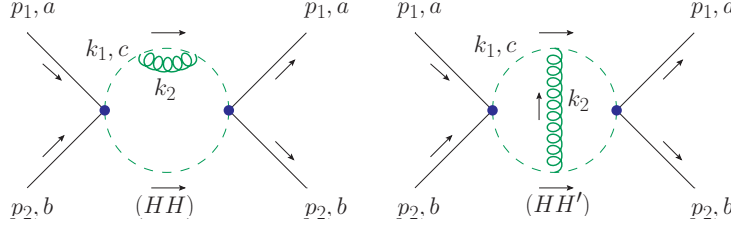


Figure 2.5: The relevant one-loop potential (Coulomb) gluon corrections to the forward-scattering amplitude.

2.1.2 Potential region

The one-loop corrections in the potential region are shown in Figure 2.5. The diagram (HH) does not contribute because the $k_{2,0}$ integral is scaleless with no poles in the upper plane. The green gluon lines denotes the fact that the gluon momentum k_2 is taken in the potential region. Neglecting the colour structure and effective vertex again, one finds:

$$\begin{aligned}
 (\Delta\mathcal{A})_{p,kin}^{(HH')} &= \tilde{\mu}^{2\epsilon}(-i) \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^d k_2}{(2\pi)^d} \frac{i}{(2M_H k_{1,0} - \vec{k}_1^2 + i.0)} \times \\
 &\quad \frac{i(-2ig_s M_H w_\mu)}{(2M_H(k_{1,0} + k_{2,0}) - (\vec{k}_1 + \vec{k}_2)^2 + i.0)} \frac{-ig^{\mu\nu}}{(-\vec{k}_2^2 + i.0)} \times \\
 &\quad \frac{i(-2ig_s M_{H'} w_\nu)}{(2M_{H'}(E - k_{1,0}) - \vec{k}_1^2 + i.0)} \frac{i}{(2M_{H'}(E - k_{1,0} - k_{2,0}) - (\vec{k}_1 + \vec{k}_2)^2 + i.0)}.
 \end{aligned}$$

Performing contour integrals for $k_{1,0}, k_{2,0}$ in the upper planes results in:

$$(\Delta\mathcal{A})_{p,kin}^{(HH')} = \frac{\pi^2 g_s^2 \tilde{\mu}^{2\epsilon}}{2m_r M (2\pi)^{d+4}} \int d^3 \vec{k}_1 d^{d-1} \vec{k}_2 \frac{1}{(E - \frac{\vec{k}_1^2}{2m_r})^2 (E - \frac{(\vec{k}_1 + \vec{k}_2)^2}{2m_r}) \vec{k}_2^2}. \quad (2.10)$$

The colour tensor is of course the same as for the soft diagram (HH') in the previous section:

$$(\Delta\mathcal{A})_{p,col}^{(HH')} = (\Delta\mathcal{A})_{s,col}^{(HH')} = \frac{1}{2}(C_{R_\alpha} - (\Delta\mathcal{A})_{s,col}^{(HH)}) = \frac{1}{2}(C_{R_\alpha} - (C_r + C_{r'})) \equiv D_{R_\alpha}. \quad (2.11)$$

The remaining integrals in (2.10) can be performed with Feynman parameters. The resulting one-loop correction to heavy pair production equals:

$$\begin{aligned}
 (\Delta\mathcal{A})_p^{R_\alpha,(1)} &= D_{R_\alpha} \mathcal{A}^{(0)}(0) \frac{\alpha_s \sqrt{m_r}}{\sqrt{-2E}} \left\{ \frac{1}{\epsilon} - \left(\log \left(\frac{-8m_r E}{\mu^2} \right) - 2 + \log(4) \right) + \mathcal{O}(\epsilon) \right\}. \\
 (\Delta\hat{\sigma})_p^{R_\alpha,(1)} &= \text{Im}[(\Delta\mathcal{A})_p^{R_\alpha,(1)}] = \frac{\alpha_s}{4\pi} \hat{\sigma}^{(0)}(0) \left(-\frac{2\sqrt{2m_r} \pi^2 D_{R_\alpha}}{\sqrt{E}} \right). \quad (2.12)
 \end{aligned}$$

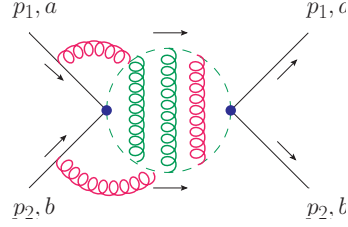


Figure 2.6: Higher-loop soft and potential (Coulomb) gluon corrections to the forward-scattering amplitude.

Note that the ϵ pole drops out in this case after taking the imaginary part of $(\Delta\mathcal{A})_p^{R_\alpha, (1)}$. As expected, the one-loop gluon corrections coming from the potential region contribute an inverse Coulomb term $1/\sqrt{E} \sim 1/\beta$.

2.1.3 Summation and interferences

The total sum of the one-loop soft and Coulomb gluon contributions from Eq. (2.9) and (2.12) is:

$$\Delta\hat{\sigma}_{s+p}^{R_\alpha, (1)} = \frac{\alpha_s}{4\pi} \hat{\sigma}^{(0)} \left\{ -\frac{2\sqrt{2m_r}\pi^2 D_{R_\alpha}}{\sqrt{E}} + 4(C_r + C_{r'}) \left(L_E^2 - 4L_E + 8 - \frac{11\pi^2}{24} \right) + 4C_{R_\alpha} (3 - L_E) \right\}, \quad (2.13)$$

which agrees with the known one-loop corrections given for example in [5]. These logarithmic and Coulombic inverse power corrections are large near threshold $\beta \ll 1$ and *need* to be taken into account at NLO. Feynman diagrams with more loops as shown for example in Figure 2.6 lead to even higher powers in the logarithms and inverse powers of β which need to be resummed to all orders when close to threshold. In order to do this it is useful to introduce a parametrization as in (2.1) and to reorganize the series expansion of the partonic cross section in α_s as:

$$\hat{\sigma}_{pp'}(\hat{s}) \sim \hat{\sigma}_{pp'}^{(0)} \sum_{k=0} f_k(\beta) \alpha_s^k \xrightarrow{\text{reorganize}} \hat{\sigma}_{pp'}^{(0)} \sum_{k,l=0} \tilde{f}_{klm} \left(\frac{\alpha_s}{\beta} \right)^k \alpha_s^l \sum_{m=0}^{2l} \ln^m \beta.$$

Resumming all enhanced contributions to all orders using eikonal methods [51, 53, 63, 64] and non-relativistic field theory [4], the cross section assumes the form:

$$\begin{aligned} \hat{\sigma}_{pp'} &\propto \hat{\sigma}^{(0)} \sum_{k=0} \left(\frac{\alpha_s}{\beta} \right)^k \exp \left[\underbrace{\ln \beta g_0(\alpha_s \ln \beta)}_{(\text{LL})} + \underbrace{g_1(\alpha_s \ln \beta)}_{(\text{NLL})} + \underbrace{\alpha_s g_2(\alpha_s \ln \beta)}_{(\text{NNLL})} + \dots \right] \\ &\times \{ 1 \text{ (LL,NLL)}; \alpha_s, \beta \text{ (NNLL)}; \alpha_s^2, \alpha_s \beta, \beta^2 \text{ (NNNLL)}; \dots \}. \end{aligned} \quad (2.14)$$

Here the label (LL) stands for leading logarithm, the (NLL) for next-to-leading logarithm etcetera and they denote the order to which one performs the resummation. A resummation to order $N^k\text{LL}$ contains per definition all logarithmic and Coulomb corrections up to fixed $N^k\text{LO}$. For squark and gluino production, the corrections beyond next-to-leading order (NLO) can be large [47], becoming as large as the NLO cross sections for the case of heavy gluinos as we will later see in Chapter 3. The inherent hierarchy of the scales $M \gg M\beta \gg M\beta^2 \gg \Lambda_{QCD}$ involved near threshold makes the combined efficient resummation of these large logarithms and Coulomb singularities possible with the EFT machinery, as shown in the next section.

2.2 Effective field theory description

Contrary to the traditional Mellin-space formalism, in the approach developed in [5, 58], which is based on soft-collinear effective theory (SCET) and potential non-relativistic QCD (pNRQCD), resummation is performed directly in momentum space via renormalization-group evolution equations [11, 59, 60]. In this section we will review the methods described in [5, 11, 58–60]. In particular, the relevant effective lagrangians will be motivated and at the end we discuss the decoupling of the soft gluons [5].

We consider again the production of a heavy pair H, H' in the collision of two collinear particles p, p' . Close to threshold, the final state may still contain soft modes X . The initial and final state fields will be denoted as ϕ_c and ψ respectively. After integrating out the hard scales we are left with an EFT, whose relevant dynamical fields are fields with momenta in the collinear, soft and potential region. In this EFT the inclusive amplitude for the production of H and H' can be expressed as:

$$\mathcal{A}(pp' \rightarrow HH'X) = \sum_l C^{(l)}(\mu) \langle HH'X | \mathcal{O}^{(l)}(\mu) | pp' \rangle. \quad (2.15)$$

The $C^{(l)}(\mu)$ are the matching coefficients found by matching to the full theory. In Eq. (2.15) and in the rest of this chapter we will suppress the spin and colour indices of the fields and matching coefficients. The sum runs over different effective operators $\mathcal{O}^{(l)}$ and is in practice a power series in the non-relativistic velocity β . The two lowest power effective operators are the S-wave and P-wave production operators:

$$\begin{aligned} \mathcal{O}^{(0)}(\mu) &= \mathcal{O}_S(\mu) = [\phi_c \phi_{\bar{c}} \psi^\dagger \psi'^\dagger](\mu) \\ \mathcal{O}^{(1)}(\mu) &= \mathcal{O}_P(\mu) = [\phi_c \phi_{\bar{c}} \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}_\top \right) \psi'^\dagger](\mu). \end{aligned} \quad (2.16)$$

The $\top$ sign denotes the orthogonal component to w^μ . The P-wave operator is β suppressed compared to the S-wave operator because of the extra derivative. We note that for stop-antistop production via quark-antiquark fusion, the first non-vanishing matrix element in the sum is that

of the P-wave operator [65]. In other words one speaks of P-wave stop production in the quark-antiquark channel which results in a β^2 suppressed cross section compared to that of the gluon-gluon channel (see Sections 2.3.2 and 3.1.3).

2.2.1 SCET and pNRQCD

The kinematic parts of the Feynman rules in Table 2.1 only depend on the regions of the momenta involved and not on the identity of the produced coloured particles. In other words, there should exist an effective field theory which describes the dynamics of the collinear, soft and potential fields, whose form only depends on the QCD interactions and not on the particular hard process, which are included via the above matching coefficients. The Feynman rules derived from this EFT should agree with those given in Table 2.1. Since the soft and collinear regions only couple indirectly via soft-gluon exchanges, the desired EFT can be interpreted as two different EFT's, one describing the soft-collinear interactions and one describing the soft-potential interactions.

The EFT describing the soft-collinear interactions is called SCET [66] and its lagrangian at LO in β in the position-space formalism [18, 67] is given by:

$$\begin{aligned} \mathcal{L}_{SCET} = & \bar{q}_c \left(i n \cdot D_s + i \not{D}_{\perp c} \frac{1}{i \bar{n} D_c} i \not{D}_{\perp c} \right) \frac{\not{n}}{2} q_c - \frac{1}{2} \text{tr} \left(F_c^{\mu\nu} F_{\mu\nu}^c \right) + (n, c \longleftrightarrow \bar{n}, \bar{c}) \\ & + \bar{q}_s i \not{D}_s q_s - \frac{1}{2} \text{tr} \left(F_s^{\mu\nu} F_{\mu\nu}^s \right) + \mathcal{O}(\beta\text{-suppressed}). \end{aligned} \quad (2.17)$$

Here, q are the quark fields which couple to gluons via the covariant derivatives. The indices c and $\bar{c}$ denote a collinear field⁷ to $n := n_+$ and $\bar{n} := n_-$ respectively. The soft fields are labeled by the index s .

In order to find the EFT describing the soft-potential interactions one proceeds in two steps. First, after integrating the hard scales out, the EFT for the heavy non-relativistic particles is called NRQCD [68–70]. The NRQCD lagrangian for the heavy particles at LO in β is [71]:

$$\mathcal{L}_{NRQCD} = \Psi^\dagger \left(i D^0 + \frac{\vec{D}^2}{2M_s} \right) \Psi + \Psi'^\dagger \left(i D^0 + \frac{\vec{D}^2}{2M_{s'}} \right) \Psi' + (\Psi, \Psi' \rightarrow \bar{\Psi} \bar{\Psi}') + \mathcal{O}(1/M). \quad (2.18)$$

The non-relativistic field $\Psi^{(\prime)}$ annihilates the heavy particle $H^{(\prime)}$ while the field $\bar{\Psi}^{(\prime)}$ annihilates its antiparticle. Here, $\Psi^{(\prime)}$ is a combination of semi-soft modes with momenta $u \sim m\beta$ as well as potential modes and it couples to semi-soft gluons as well as potential gluons. The semi-soft modes may not appear in the final states because of energy conservation. Furthermore, external massless quarks and gluons are on-shell and may not have momenta in the potential region. Therefore, one may integrate all semi-soft modes and massless potential modes out, which results

⁷The initial state operators ϕ_c in (2.16) are related to the collinear fields q_c, A_c . Both the quark and gluon initial state operators are defined in terms of the corresponding SCET fields and dressed with collinear Wilson lines (cf. Equations (2.22) and (2.39)) such that they are invariant under the collinear gauge transformations [5].

in the EFT called pNRQCD [19–21, 72]. Its lagrangian describes the soft-potential interactions and at LO in β equals:

$$\begin{aligned} \mathcal{L}_{pNRQCD} = & \psi^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2M_H} + \frac{i\Gamma_H}{2} \right) \psi + \psi'^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2M_{H'}} + \frac{i\Gamma_{H'}}{2} \right) \psi' \\ & + \int d^3\vec{r} [\psi^\dagger T^{(R)a} \psi](\vec{r}) \left(\frac{\alpha_s}{r} \right) [\psi'^\dagger T^{(R')a} \psi'](0) + \mathcal{O}(\beta\text{-suppressed}). \end{aligned} \quad (2.19)$$

Here we have included the finite widths Γ, Γ' of the heavy particles H, H' . The effect of the potential gluons is included via the non-local effective operator in the second line which takes the form of a Coulomb potential between the two outgoing heavy particles.

The total lagrangian for the massless soft, collinear and heavy potential modes is the sum of the SCET lagrangian (2.17) and the pNRQCD lagrangian (2.19):

$$\mathcal{L}_{tot} = \mathcal{L}_{SCET}(q_c, q_{\bar{c}}, q_s, A_s, A_c, A_{\bar{c}}) + \mathcal{L}_{pNRQCD}(\psi, \psi', A_s). \quad (2.20)$$

To summarize, the initial states are described by SCET, the final heavy states by pNRQCD and they interact indirectly via soft gluons which appear in the covariant derivatives.

2.2.2 Soft gluon decoupling

The factorization formula derived in the next section is based on the fact that the soft gluons can be decoupled by the following field transformations [5]:

$$q_c \rightarrow S_n^{(3)} q_c, \quad A_c \rightarrow S_n^{(8)} A_c, \quad (n, c \longleftrightarrow \bar{n}, \bar{c}), \quad \psi \rightarrow S_w^{(R)} \psi, \quad (\psi, R \longleftrightarrow \psi', R'). \quad (2.21)$$

The soft Wilson lines S are defined as path exponentiated soft gluon fields along the directions of propagation:

$$\begin{aligned} S_n^{(r)} &= \text{P exp} \left[ig_s \int_{-\infty}^0 dt n \cdot A_s^a(x + nt) T^{(r)a} \right], \\ S_w^{(R)} &= \text{P exp} \left[-ig_s \int_0^\infty dt w \cdot A_s^a(x + wt) T^{(R)a} \right]. \end{aligned} \quad (2.22)$$

As a result of the field redefinitions in Eq. (2.21) the covariant soft derivatives in the SCET and pNRQCD lagrangians (2.19, 2.17) are replaced by ordinary derivatives:

$$D_s^0 \rightarrow \partial^0.$$

Therefore at LO in β , the soft, collinear and potential fields in the total EFT lagrangian are fully decoupled:

$$\mathcal{L}_{tot} = \mathcal{L}_{SCET}(q_c, A_c) + \mathcal{L}_{SCET}(q_{\bar{c}}, A_{\bar{c}}) + \mathcal{L}_{pNRQCD}(\psi, \psi') + \mathcal{L}_s + \mathcal{O}(\beta\text{-suppressed}).$$

In order to do a NLL order calculation the above LO lagrangian can be safely used. Naïvely one would expect that at NNLL order the β -suppressed terms are needed. However, in [5, 73] it was shown that for a NNLL calculation, adding the sub-leading Coulomb and non-Coulomb potential to the pNRQCD lagrangian suffices, while the sub-leading soft-potential and soft-collinear interactions do not contribute. In other words, up to NNLL the soft, collinear and potential fields are effectively fully decoupled at the lagrangian level. This decoupling forms the basis for the factorization of the hard, soft and potential contributions in the next section.

2.3 Factorization and resummation

In the previous section we saw that near threshold the relevant degrees of freedom, i.e. the potential, soft and collinear modes do not directly couple to LO in β . In the same way that the decoupling of the hard contributions at non-relativistic energies results in a factorization of the amplitude as in Eq. (2.15), one expects that this decoupling of the soft gluons leads to some form of factorization of the amplitude and cross section as well. In [5] it was shown how the partonic cross section for S-wave production processes factorizes into three contributions, a hard function H , a soft function W and a potential function J . We start by reviewing the factorization for S-wave processes given in [5] and afterwards we discuss the applicability of the formalism to stop production in Section 2.3.2.

2.3.1 S-wave processes

After performing the field redefinitions (2.21), the amplitude (2.15) for the S-wave operator factorizes as [5]:

$$\mathcal{A}(pp' \rightarrow HH'X) = \langle 0 | \phi_c | p \rangle \langle 0 | \phi_{\bar{c}} | p' \rangle \sum_{R_\alpha} C_{pp' \rightarrow HH'X} \langle HH' | \psi^\dagger \psi'^\dagger | 0 \rangle \langle X | \mathcal{S}^{R_\alpha} | 0 \rangle, \quad (2.23)$$

where the sum now runs over the basis of colour tensors $\{c^{R_\alpha}\}$ referred to in Section 2.1.1 and the soft Wilson lines are contained in:

$$\mathcal{S}^{R_\alpha} := c^{R_\alpha} S_n^{(r)} S_n^{(r')} S_w^{(R)\dagger} S_w^{(R')\dagger}. \quad (2.24)$$

The corresponding cross section can be shown to similarly factorize [5]:

$$\begin{aligned} \sigma_{pp' \rightarrow HH'X}(s) &= \frac{1}{2s} \sum_X \int dP S_X dP S_{HH'} \sum_{\text{pol, colour}} |\mathcal{A}(pp' \rightarrow HH'X)|^2 \\ &= \sum_{\text{pol, colour}} \int d^4z e^{-2iMw \cdot z} \langle p | \phi_c^\dagger(z) \phi_c(0) | p \rangle \langle p' | \phi_{\bar{c}}^\dagger(z) \phi_{\bar{c}}(0) | p' \rangle \\ &\quad \frac{1}{2s} \sum_{i, i' \in \{R_\alpha\}} (C_i C_{i'}^*) \langle 0 | [\psi' \psi](z) [\psi^\dagger \psi'^\dagger](0) | 0 \rangle \langle 0 | \mathcal{S}^{i'*}(z) \mathcal{S}^i(0) | 0 \rangle. \end{aligned} \quad (2.25)$$

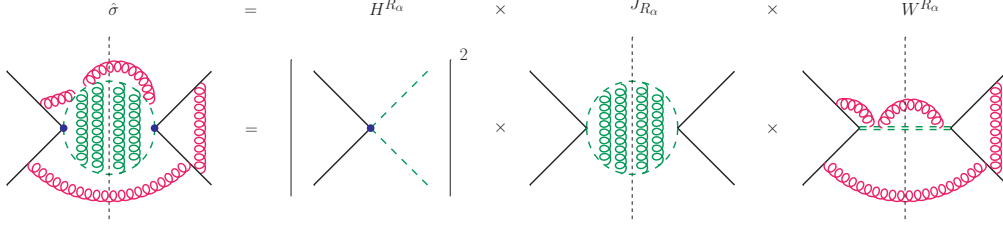


Figure 2.7: A diagrammatic interpretation of the factorization formula (2.26).

The matrix elements of the collinear fields can be rewritten in terms of PDF's. Therefore one finds that the partonic cross section also factorizes, which is subsequently found by removing the PDF's. By choosing the s -channel colour basis as in [58] the matrix element $\langle 0 | \mathcal{S}^{i'*}(z) \mathcal{S}^i(0) | 0 \rangle$ diagonalizes. Transforming to the momentum representation and subsequently defining the hard, soft and potential matrix elements as separate functions H , W and J respectively, one can finally express the factorized partonic cross section as [5]:

$$\hat{\sigma}_{pp'}(\hat{s}, \mu) = \sum_{R_\alpha} H_{pp'}^{R_\alpha}(m_{\bar{q}}, m_{\bar{g}}, \mu) \int d\omega J_{R_\alpha}(E - \frac{\omega}{2}) W^{R_\alpha}(\omega, \mu). \quad (2.26)$$

Here $E = \sqrt{\hat{s}} - 2M$ is the energy relative to the production threshold and the sum is over the colour representations of the heavy particle pair. The integral runs over the energy $\omega/2$ of the soft gluons in the final state. The kinetic energy of the heavy pair in the centre of mass frame is therefore $E - \frac{\omega}{2}$, which enters the argument of the potential function J . The soft function then depends only on the colour representations of the initial-state partons and the irreducible representation R_α of the produced pair appearing in the decomposition of the product of the two heavy particle colour representations, but not on the individual heavy particle representations R, R' . This is in agreement with the picture that soft-gluon radiation is only sensitive to the total colour charge of the slowly moving pair [64]. Since the decoupling of the soft gluons was shown up to NNLL order, the factorization formula (2.26), which is roughly expressed in terms of diagrams in Figure 2.7, is of course only valid up to NNLL order. In the rest of this section we explain the functions separately appearing in (2.26).

The soft function W contains the large logarithms from the contributions of the soft gluons. In the s -channel colour bases, the soft function $W(z, \mu)$ can be expressed in position-space as:

$$W^{R_\alpha}(z, \mu) = \langle 0 | \bar{T}[S_w^{R_\alpha} S_n^{(r')\dagger} S_n^{(r)\dagger}](z) T[S_n^{(r)} S_n^{(r')} S_w^{R_\alpha\dagger}](0) | 0 \rangle. \quad (2.27)$$

This means that one can calculate the soft correction by simply considering the heavy pair as a single final state, which simplifies the calculations considerably [61]. Note that the s -channel basis was also used in Section 2.1.1. For NLL resummation only the tree level soft function is

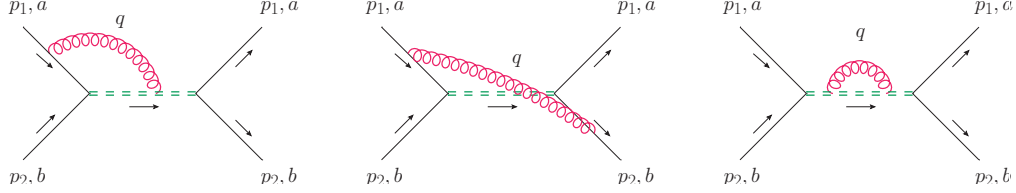


Figure 2.8: One loop soft gluon corrections in the EFT picture.

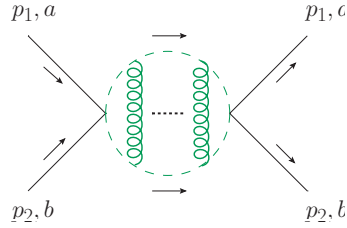


Figure 2.9: Higher loop potential gluon corrections.

needed: $W^{(0)} = \delta(\omega)$, while the one-loop soft function is required at NNLL and can be calculated from the forward-scattering diagrams shown in Figure 2.8. Here one can use the Feynman rules in Table 2.1, where the mass of the double dashed potential line should be taken as $2M$. One finds [58] in momentum space:

$$W^{R_\alpha}(\omega, \mu) = \delta(\omega) + \frac{\alpha_s(\mu)}{4\pi} \left\{ \frac{-2\Gamma(-\epsilon)}{\Gamma(-2\epsilon)} \left(\frac{C_r + C_{r'}}{\epsilon} + \frac{C_{R_\alpha}}{1-2\epsilon} \right) \frac{1}{\omega} \left(\frac{\mu}{\omega} \right)^{2\epsilon} \right\} + \mathcal{O}(\alpha_s^2). \quad (2.28)$$

By convoluting the one-loop soft function (2.28) with the trivial potential function (see below) according to the factorization formula (2.26) one can reproduce our previous results (2.9).

The potential function J contains the contribution of the potential gluon exchanges between the heavy pair and is expressed as:

$$J_{R_\alpha} = \langle 0 | [\psi' \psi](z) [\psi^\dagger \psi'^\dagger](0) | 0 \rangle = 2\text{Im}[G_C^{R_\alpha}(0, 0; E)]. \quad (2.29)$$

The S-wave Green's function $G_C^{R_\alpha}$ of the Schroedinger equation with the leading Coulomb potential resums the Coulomb singularities $(\alpha_s/\beta)^n$ that are sufficient for a NLL calculation [74]:

$$G_C^{R_\alpha}(0, 0; E) = -\frac{(2m_r)^2}{4\pi} \left\{ \sqrt{-\frac{E}{2m_r}} + (-D_{R_\alpha} \alpha_s) \left[\frac{1}{2} \log \left(-\frac{8m_r E}{\mu^2} \right) - \frac{1}{2} + \gamma_E \right. \right. \\ \left. \left. + \psi \left(1 - \frac{-D_{R_\alpha} \alpha_s}{2\sqrt{-E/(2m_r)}} \right) \right] \right\}. \quad (2.30)$$

The NLO term in the $\overline{\text{MS}}$ -subtracted Green's function (2.30) agrees with our previous results in Eq. (2.12) up to real terms. The difference comes from the fact that for the above Green's function the final state loop-momentum has been calculated in $d = 4 - 2\epsilon$, while in our case we performed the loop calculation in $d = 4$. This difference does not contribute to the potential function. The function ψ is the polygamma function and contains the higher order potential loop corrections as shown in Figure 2.9. Above threshold the potential function is continuous while below threshold it is made up of a sum of bound states [5]:

$$J_{R_\alpha}(E) = \frac{(2m_r)^2 \pi D_{R_\alpha} \alpha_s}{2\pi(e^{\pi D_{R_\alpha} \alpha_s \sqrt{2m_r/E}} - 1)} \theta(E) + 2 \sum_{n=1}^{\infty} \delta(E - E_n) \left(\frac{-2m_r D_{R_\alpha} \alpha_s}{2n} \right)^3 \theta(-E), \quad (2.31)$$

with bound-state energies equal to:

$$E_n = -\frac{2m_r \alpha_s^2 D_{R_\alpha}^2}{4n^2}. \quad (2.32)$$

At NLL (2.31) suffices, while at NNLL also the NLO and non-Coulomb Green's functions are required [75]. One can set the Coulomb corrections to zero by taking $D_{R_\alpha} = 0$ and this results in the trivial potential function $J^{(0)}(E) = \frac{(2m_r)^2}{2\pi} \sqrt{\frac{E}{2m_r}}$.

The hard function H is the only process-dependent ingredient and is related to the hard matching coefficients $C_{pp' \rightarrow HH'X}$ by:

$$H^{R_\alpha}(\mu) = \frac{1}{2\hat{s}} N^p N^{p'} |C_{pp' \rightarrow HH'X}|^2. \quad (2.33)$$

Here, N^p equals the helicity average of the polarization wave function of the incoming collinear parton p in SCET. In practice the hard function is calculated at a specific order k in α_s , which is required for a $N^k\text{LL}$ resummation, by expanding the factorized expression (2.26) to $N^k\text{LO}$ and matching it to the exact $N^k\text{LO}$ cross section evaluated near threshold [75]. For example, in Chapter 3 we will see that the tree level hard function is simply related to the LO cross section near threshold as expressed in Eq. (3.6), while the NLO hard function can be found by matching Eq. (3.4) to the exact NLO cross section expanded near threshold.

The hard function satisfies the evolution equation [58, 76]:

$$\frac{dH^{R_\alpha}(\mu)}{d \ln(\mu)} = \left(\gamma_{\text{cusp}}(C_r + C_{r'}) \ln \left(\frac{4M^2}{\mu^2} \right) + 2\gamma_{R_\alpha}^V \right) H^{R_\alpha}(\mu), \quad (2.34)$$

where γ_{cusp} and $\gamma_{R_\alpha}^V$ are (cusp-)anomalous dimensions. The soft function satisfies an evolution equation as well, which can be found by using the hard function RG equation (2.34), the DGLAP

equation (1.6) and the fact that the hadronic cross section is scale independent:

$$\begin{aligned} \frac{d}{d \ln \mu} W_i^{R_\alpha}(\omega, \mu) = & -2 \left[\gamma_{\text{cusp}}(C_r + C_{r'}) \ln \left(\frac{\omega}{\mu} \right) + 2\gamma_{W_i}^{R_\alpha} \right] W_i^{R_\alpha}(\omega, \mu) \\ & - 2\gamma_{\text{cusp}}(C_r + C_{r'}) \int_0^\omega d\omega' \frac{W_i^{R_\alpha}(\omega', \mu) - W_i^{R_\alpha}(\omega, \mu)}{\omega - \omega'}, \end{aligned} \quad (2.35)$$

where $\gamma_{W_i}^{R_\alpha}$ is an anomalous dimension. The fact that the RG equations are all connected is intuitively clear since the ϵ -poles from the soft contributions cancel with (some of) those of the hard and collinear contributions, such that their scale dependences should be related via RG equations. By solving the hard and soft RG equations at initial scales μ_h and μ_s respectively, the factorization formula can be expressed up to NNLL order as [5]:

$$\begin{aligned} \sigma^{\text{res}}(\hat{s}, \mu_f) = & \sum_{S=|s-s'|}^{s+s'} \sum_{R_\alpha} H^{R_\alpha}(\mu_h) U_{R_\alpha}(M, \mu_h, \mu_s, \mu_f) \left(\frac{2M}{\mu_s} \right)^{-2\eta} \\ & \times \tilde{s}^{R_\alpha}(\partial_\eta, \mu_s) \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \int_0^\infty d\omega \frac{J_{R_\alpha}(E - \omega/2)}{\omega} \left(\frac{\omega}{\mu_s} \right)^{2\eta} \Big|_{\eta=2a_\Gamma(\mu_s, \mu_f)}. \end{aligned} \quad (2.36)$$

Note that by solving the evolution equations one naturally introduces the hard and soft scales μ_h and μ_s . The value of these are taken such as to make a perturbative calculation of the hard and soft function at μ_h and μ_s possible. This will be further discussed in Section 2.3.3. The function $\tilde{s}^R(\rho, \mu)$ is the Laplace transform of the soft function W . Using (2.28) one finds up to one loop:

$$\begin{aligned} \tilde{s}^{R_\alpha}(\rho, \mu) &= \int_0^\infty d\omega e^{-t\omega} W^{R_\alpha}(\omega, \mu) \\ &= 1 + \frac{\alpha_s(\mu)}{4\pi} \left((C_r + C_{r'}) \left(\rho^2 + \frac{\pi^2}{6} \right) - 2C_{R_\alpha}(\rho - 2) \right) + \mathcal{O}(\alpha_s^2), \end{aligned} \quad (2.37)$$

where $t = 1/(e^{\gamma_E} \mu e^{\rho/2})$. The evolution functions U and a_Γ both depend on the (cusp-)anomalous dimensions which control the RG equations. These and other ingredients needed for Eq. (2.36) are all given in Appendix A.

2.3.2 P-wave processes

In this section we derive the factorization formula (2.26) for the specific case of stop-antistop production from quark-antiquark annihilation that proceeds in a P-wave state. While the aim is to establish resummation at NLL accuracy, the arguments suggest that the combined soft-Coulomb resummation can be performed at NNLL accuracy as well. Our reasoning applies to partonic subprocesses with a leading P-wave contribution, as relevant for stop-antistop production, but not to P-wave contributions to S-wave dominated subprocesses.

Following Section 2.2 [5] the scattering process $q\bar{q} \rightarrow t\bar{t} + X$ is described in terms of an effective field theory using soft-collinear effective theory (SCET) [18, 66, 67, 77] for the initial state quarks and potential non-relativistic QCD (pNRQCD) [19–21, 72] for the stop and antistop. In this framework, the scattering amplitude in the full MSSM is expressed in terms of expectation values of EFT operators multiplied by short-distance coefficients C , that contain the dependence on hard modes not present in the EFT (e.g. gluinos, hard off-shell gluons):

$$\mathcal{A}(q\bar{q} \rightarrow t\bar{t}X) = \sum_{\ell, R_\alpha} C^{(\ell), R_\alpha}(\mu) \langle t\bar{t}X_s | \mathcal{O}^{(\ell), R_\alpha}(\mu) | q\bar{q} \rangle_{\text{EFT}}. \quad (2.38)$$

The series in ℓ accounts for the threshold expansion in powers of β , while $R_\alpha = 1, 8$ are the irreducible colour representations of the stop-antistop pair. Due to the threshold kinematics, $(k_1 + k_2)^2 \sim 4M^2$, no collinear modes can appear in the final state X_s , that only contains soft modes. Both the short-distance coefficients $C^{(\ell), R_\alpha}$ and the operators $\mathcal{O}^{(\ell), R_\alpha}$ can carry open Lorentz and spin indices, that have been suppressed. For NLL resummation, only the colour-octet state is relevant.⁸ For the case of P-wave production, the leading $\ell = 0$ term in the expansion of the scattering amplitude (2.38) vanishes and the first non-vanishing term occurs for $\ell = 1$, i.e. it is suppressed by $\mathcal{O}(\beta)$ compared to the S-wave case. The P-wave production operator for a colour-octet state reads (cf. (2.16))

$$\mathcal{O}_\nu^{(1), 8}(\mu) = \frac{1}{\sqrt{2}} [(\bar{\xi}_c W_c) T^a (W_c^\dagger \xi_c)] \left[\psi^\dagger T^a \left(-\frac{i}{2} \overleftrightarrow{D}_{\nu, \top} \right) \chi \right](\mu). \quad (2.39)$$

Here the field $\psi^\dagger(\chi)$ is a non-relativistic field that creates the stop (antistop) while $\xi_c(\bar{\xi}_c)$ is a collinear (anti-collinear) field that destroys the initial-state quark with momentum $\sim \sqrt{\hat{s}}/2 n$ (antiquark with momentum $\sim \sqrt{\hat{s}}/2 \bar{n}$), with light-cone vectors $n^2 = \bar{n}^2 = 0$, $n \cdot \bar{n} = 2$. The $W_c(W_{\bar{c}})$ are collinear (anti-collinear) Wilson lines summing up collinear gluon emission to all orders [66, 77]. $D_\top$ denotes the projection of the covariant derivative $D_\mu = (\partial_\mu - ig_s T^a A_\mu^a(x))$ in the direction orthogonal to the timelike velocity w^μ of the stop-antistop pair.

At this stage, the operator (2.39) is written in NRQCD and contains the full gluon field A . This does not yet incorporate a systematic expansion in β . For this purpose we adopt potential non-relativistic QCD (pNRQCD) [19–21, 72] as we did in Section 2.2.1, where only soft gluons with momenta scaling as

$$q \sim M\beta^2 \quad (2.40)$$

and the non-relativistic fields with so-called potential momenta scaling as

$$k_0 \sim M\beta^2, \quad \vec{k} \sim M\beta \quad (2.41)$$

⁸ At NLO a finite, non-logarithmic colour-singlet contribution is generated by real-gluon emission [78] that is suppressed by $\alpha_s\beta^2$ compared to the leading P-wave contribution, and therefore not even relevant at NNLL. Since there is no leading S-wave contribution to $q\bar{q} \rightarrow t\bar{t}$, the situation is simpler than for P-wave quarkonium production and decay, which gets contributions both from colour octet S-wave and singlet P-wave channels [69, 79].

are retained in the EFT, while potential gluons with momenta scaling like (2.41) and modes with momenta of order $M\beta$ are integrated out. After this step, the covariant derivative in (2.39) contains only the soft gluon field A_s with the same scaling as the soft momentum,

$$A_s^\mu \sim M\beta^2. \quad (2.42)$$

Therefore the term $g_s A_{s,\mu\top}$ in the covariant derivative is suppressed by a factor $g_s\beta$ compared to the derivative term $\partial_{\mu\top} \sim \beta$ and the production operator (2.39) at leading power in pNRQCD is simply given by

$$\mathcal{O}_\nu^{(1),8}(\mu) = \frac{1}{\sqrt{2}} [(\bar{\xi}_c W_c) T^a (W_c^\dagger \xi_c)] \left[\psi^\dagger T^a \left(-\frac{i}{2} \overleftrightarrow{\partial}_{\nu,\top} \right) \chi \right](\mu). \quad (2.43)$$

The potential, collinear and anti-collinear fields interact only through the exchange of soft gluons. These interactions can be removed from the leading effective-theory Lagrangians by the same decoupling transformation of the collinear [77, 80] and potential [5] fields as in Eq. (2.21). Since the new, transformed fields do not interact with each other in the leading effective Lagrangians, the scattering amplitude assumes a factorized form:

$$\begin{aligned} \mathcal{A}(q\bar{q} \rightarrow \tilde{t}_i \bar{\tilde{t}}_i X) &= \left(-\frac{i}{2\sqrt{2}} \right) C_\nu^{(1),8}(\mu) \langle 0 | (W_c^\dagger \xi_c)_{j_1} | q \rangle \langle 0 | (\bar{\xi}_c W_c)_{j_2} | \bar{q} \rangle \\ &\times \langle X_s | (S_n^\dagger T^a S_n)_{j_2 j_1} (S_w^\dagger T^a S_w)_{j_3 j_4} | 0 \rangle \langle \tilde{t}_i \bar{\tilde{t}}_i | \psi_{j_3}^\dagger \overleftrightarrow{\partial}_{\top,\mu} \chi_{j_4} | 0 \rangle. \end{aligned} \quad (2.44)$$

Note that the factorization (2.44) holds only at leading power in the β -expansion, since the field redefinition does not remove the interaction with the spatial components of the soft gluons in the P-wave production operators, or in the subleading effective Lagrangians. We will argue below that these corrections are not relevant at NLL accuracy for stop-antistop production from quark-antiquark annihilation.

Using the representation of the scattering amplitude (2.44) and following the steps discussed in detail in Section 2.3.1 [5] one derives the factorization formula

$$\hat{\sigma}_{q\bar{q}}(\hat{s}, \mu) = H_{q\bar{q}}^8(\mu) \int d\omega J_8^P(E - \frac{\omega}{2}) W^8(\omega, \mu). \quad (2.45)$$

Here the potential function for P-wave production in a colour octet state is given by

$$J_8^P(q) = \frac{1}{2} \int d^4 z e^{iq \cdot z} \langle 0 | [\chi^\dagger \overleftrightarrow{\partial}_{\top,\mu} T^a \psi](z) [\psi^\dagger T^a \overleftrightarrow{\partial}_{\top,\mu} \chi](0) | 0 \rangle, \quad (2.46)$$

while the colour-octet soft function (defined in terms of the squared matrix element of the soft Wilson lines in (2.44)) is identical to that appearing for S-wave production [58]. The factorization formula resembles that for S-wave production, up to the replacement of the potential function by the appropriate P-wave expression (see Eq. (3.13) for the NLL case).

We now show that no corrections to (2.45) appear at NNLL. As far as *soft-gluon effects* are concerned, we have argued that the corrections to the production operator (2.43) from the subleading $A_{s,\top}$ -terms in (2.39) are of order $g_s\beta$. Since the potential function for S-P-wave interference vanishes, there can be no contributions to the total cross section where one $A_\top$ -operator interferes with a leading operator (2.43). The squared $A_\top$ -term gives a correction to the cross section of the order⁹ $\Delta\sigma \sim \sigma^0 \alpha_s \beta^2 \{1, \ln \beta\}$. These corrections are at most of order N³LL in the combined counting $\alpha_s/\beta \sim 1$ and $\alpha_s \ln \beta \sim 1$.

Following [5] one sees that insertions of higher-order *soft-collinear* interactions into the matrix element (2.38) are at least suppressed by $\mathcal{O}(\beta^2)$ and therefore are beyond NNLL. For P-wave production it has to be shown in addition that no contributions to the $\ell = 1$ operators in (2.38) are generated from partonic channels with leading S-wave ($\ell = 0$) contributions. In the EFT language they would correspond to operators where one or both of the quark/antiquark fields are replaced by (anti)collinear gluon fields. A non-vanishing matrix element of such an operator in the $|\bar{q}q\rangle$ initial state requires splittings of a collinear quark into a collinear gluon and a soft quark that are mediated by $\mathcal{O}(\beta)$ -suppressed interactions in the SCET Lagrangian [67] (recall that no collinear final state particles appear at partonic threshold). Since there is no qg -initiated production of stop-antistop pairs at leading order, two subleading splittings are required so these contributions to the amplitude are suppressed by $g_s^2\beta$ compared to the $\ell = 1$ term. The resulting real corrections to the total cross section are therefore even further suppressed compared to the subleading soft effects.

Finally, subleading *soft-potential effects* due to chromoelectric $\vec{x} \cdot \vec{E}_s$ interactions [21] have been shown to be beyond NNLL in the S-wave case [5] since the potential function with a single insertion of an operator $\sim \vec{x}$ vanishes by rotational invariance. The same argument holds for the P-wave potential function.¹⁰ Furthermore, as mentioned before, due to the absence of a leading S-wave contribution to the quark-antiquark channel, no mixing of S-wave and P-wave states by subleading interactions appears. Therefore, while we didn't perform an exhaustive study of effects beyond NLL, we do not encounter an obstruction for NNLL resummation for the quark-antiquark initial-state contribution to stop-antistop production. Note, however, that Coulomb resummation at this accuracy requires the computation of the NLO P-wave Coulomb Green's function for scalar particles.

2.3.3 Scale choices and corresponding uncertainties

The above resummed partonic cross section (2.26) depends on a number of scales related to the factorization of hard, soft and Coulomb effects. The dependence on these scales would cancel

⁹We have allowed for logarithmic corrections in the generalized potential function from contractions of the two gluon insertions. A simple estimate indicates, however, that these are in fact absent.

¹⁰A possible interference of a single $A_\top$ -term in the production operator with a chromoelectric vertex might lead to a non-vanishing potential function but would be of the same order $\alpha_s \beta^2 \{1, \ln \beta\}$ as the soft contributions considered above.

in the exact result, but a residual dependence remains at a given logarithmic order. Our default choice for the factorization and hard scales are $\mu_f \equiv M$ and $\mu_h \equiv 2M$, respectively. On the other hand, the resummation of all effects related to Coulomb exchange requires that the scale in the potential function J_{R_α} is chosen of the order of $\sqrt{2m_r M} \beta$, which is the typical virtuality of Coulomb gluons. A more detailed analysis shows in fact that for an attractive Coulomb potential the Coulomb scale freezes when $\beta \sim |D_{R_\alpha}| \alpha_s$, due to bound-state formation. We thus choose the scale in J_{R_α} to be

$$\mu_C = \text{Max} \left\{ 2\alpha_s(\mu_C) m_r |D_{R_\alpha}|, 2\sqrt{2m_r M} \beta \right\}. \quad (2.47)$$

Note that, for a repulsive potential, $D_{R_\alpha} > 0$, no bound states arise, so that (2.47) is not completely justified. However in this case resummation of Coulomb corrections leads to small effects, and J_{R_α} vanishes for small β , so that the precise choice of μ_C in this limit has a negligible numerical impact on predictions of the cross section.

The choice of the soft scale μ_s presents some subtleties. The exponentiation of all $\ln \beta$ -terms in the partonic cross section would require a choice $\mu_s \sim M\beta^2$. However a running scale leads to strong oscillations of the cross section for small β , due to the prefactor $e^{-2\gamma_E \eta} / \Gamma(2\eta)$ in the resummation formula (2.36), amplified by the factor $\omega^{2\eta}$ and terms in the function U_{R_α} , and eventually hits the Landau pole of the strong coupling constant α_s when $\beta \rightarrow 0$. To overcome these problems two different approaches have been used in the literature:

Fixed μ_s : In [11, 59, 60] the choice of a fixed soft scale was advocated. Such a scale is determined from the minimization of the one-loop *soft* corrections to the hadronic cross section,

$$0 = \mu_s \frac{d}{d\mu_s} \sum_{p,p'} \int_{\tau_0}^1 d\tau L_{pp'}(\tau, \mu_s) \frac{\hat{\sigma}_{pp',\text{soft}}^{(1)}(\tau s, \mu_s)}{\sigma_{N_1 N_2}^{(0)}(s, \mu_s)}. \quad (2.48)$$

In this approach threshold logarithms are resummed in an average sense and not locally at the level of the partonic cross section. However one can argue that, for threshold dominated processes, the choice (2.48) preserves the hierarchy between the soft and short-distance scales and that logarithmic corrections $\log(1 - \tau_0)$ to the *hadronic* cross section are correctly resummed. This was the method adopted in [5] for resummation of the squark-antisquark production cross section. The explicit value of the scales determined with the minimization procedure (2.48) are given in Eq. (B.1) in Appendix B.

Running μ_s : For the NNLL resummation of soft effects in $t\bar{t}$ production presented in [75] a different approach was adopted. There a running soft scale,

$$\mu_s^> = k_s M \beta^2, \quad (2.49)$$

was used in the interval $\beta > \beta_{\text{cut}}$, and replaced by a fixed soft scale

$$\mu_s^< = k_s M \beta_{\text{cut}}^2 \quad (2.50)$$

below the cutoff. With this scale choice, logarithms of β are exponentiated locally in the *partonic* cross section in the large- β region, where the use of a fixed soft scale cannot be a priori justified. On the other hand if β_{cut} is not too big, in the lower interval the *hadronic* cross section is in fact dominated by logarithms of β_{cut} , as can be explicitly checked by convoluting the partonic cross section with toy parton luminosities [60], so that the use of a fixed scale once again correctly resums the dominant logarithms. The precise value of β_{cut} is chosen through the prescription described in [75], which is reviewed in Appendix B. The default choice for the prefactor k_s adopted in this thesis is $k_s = 1$. We have observed that, for the coloured SUSY processes, the NLL expression and its NNLO expansion are generally stable against variations of k_s for this choice.¹¹

The two possible choices of the soft scale μ_s just discussed are one of the ambiguities associated with threshold resummation. Others are related to the choice of the hard and Coulomb scales and to power-suppressed terms which are not controlled by resummation. Additionally, one has to consider the ambiguity arising from the choice of the factorization scale μ_f . The latter clearly also applies to the fixed-order NLO result. Thus, to reliably ascertain the residual uncertainty of the fixed-order and resummed results we present in this thesis, we adopt the following procedure:

- **Scale uncertainty:** for both the NLO and NLL result the factorization scale μ_f is varied between half and twice the default value, i.e. $M/2 < \mu_f < 2M$. For the NLL result, this is done keeping the other scales μ_h , μ_C , μ_s and the parameters β_{cut} and k_s fixed.
- **Resummation uncertainty:** both hard and Coulomb scales are varied between half and twice the default values, i.e. $M < \mu_h < 4M$ and $\mu_C^{(0)}/2 < \mu_C < 2\mu_C^{(0)}$, where $\mu_C^{(0)}$ is the solution of the implicit equation (2.47). In addition, for the NLL implementation with a fixed soft scale, μ_s is varied between half and twice its default value, while for the running-scale implementation uncertainties related to the choice of β_{cut} and k_s are estimated according to the procedure given in [75] (and reviewed in Appendix B). Finally, as anticipated below, we take the difference in parametrizing the resummed cross section in terms of β or $\hat{E}$ as a measure of the effect of power-suppressed terms. All the scales and the parameters β_{cut} and k_s are varied one at the time keeping the other fixed to their central values, and the resulting errors are summed in quadrature.
- **PDF uncertainty:** we estimate the error due to uncertainties in the PDFs using the 68% confidence level eigenvector set of the MSTW08NLO PDFs [8].

¹¹Our choice of k_s deviates from the one adopted in [75], where $k_s = 2$. This corresponds to resumming some of the $\ln 2$ terms in the fixed-order cross section alongside the threshold-enhanced $\ln \beta$ contributions.

An additional source of error arises from the uncertainty on the α_s -determination. This effect has been found to be of the order of 3% for the NLO cross sections of squark-squark, squark-antisquark and squark gluino production and up to 8% for gluino-pair production [50]. We expect a similar uncertainty of the NLL results in Chapter 3.

In the following chapters we will often refer to the sum in quadrature of scale and resummation uncertainty as “total theoretical uncertainty”. Note that the terminology adopted here differs slightly from the one used for $t\bar{t}$ production in [75] where the errors from variation of the hard and Coulomb scales, and of the soft scale for the fixed-scale implementation, had been incorporated into the scale uncertainty, while we consider them as resummation ambiguities. Additionally, in [75] independent and simultaneous variations of the factorization and renormalization scale have been considered, whereas in this thesis we identify the factorization and renormalization scales and vary them as one scale, i.e. $M/2 < \mu_f \equiv \mu_r < 2M$. This is the default procedure implemented in the numerical code `PROSPINO` used for the computation of the fixed-order NLO result [34].

Finally, in practice one is interested in the effects beyond a fixed order $Y = N^k\text{LO}$. Usually one takes $k = 1$ for NLL and $k = 2$ for NNLL. For this reason, in the following chapters we will consider the *matched* cross section, which by definition coincides with the exact result up to a fixed $N^k\text{LO}$:

$$\begin{aligned}\sigma_{N_1 N_2 \rightarrow H H' X}^{\text{matched}}(s) &= \sum_{p, p'=q, \bar{q}, g} \int_0^1 d\tau L_{pp'}(\tau, \mu_f) \hat{\sigma}_{pp'}^{\text{matched}}(s\tau, \mu_f), \\ \hat{\sigma}_{pp'}^{\text{matched}} &= \hat{\sigma}^{\text{res}} - \hat{\sigma}_{(Y)}^{\text{res}} + \hat{\sigma}_Y.\end{aligned}\tag{2.51}$$

where $\hat{\sigma}_{(Y)}^{\text{res}}$ is the expansion of the resummed partonic cross section up to $Y = N^k\text{LO}$ and $\hat{\sigma}_Y$ denotes the exact fixed order partonic cross section. In this way, the matched cross section will also contain contributions outside the threshold region, which can in fact be non-negligible (cf. Figure 1.1).

NLL resummation for SUSY-QCD

Resummation of soft logarithms for squark and gluino production at the next-to-leading (NLL) logarithmic accuracy have been presented in [50, 54–57] using the Mellin-space resummation formalism developed by [51, 53, 63, 64]. Recently the same formalism has been extended to NNLL order [81] and applied to squark-antisquark production [49]. These works do not resum Coulomb corrections to all orders, though the numerically dominant terms are accounted for at fixed order. All-order resummation of Coulomb contributions and bound-state effects for squark-gluino and gluino-gluino production were on the other hand investigated in [82–84], without the inclusion of soft resummation. In [85, 86] partial NNLL resummation of soft logarithms has been used to construct approximated NNLO results for the squark-antisquark production cross section. The formalism for the combined resummation of soft and Coulomb corrections that was reviewed in Section 2.3 has been applied to NLL resummation of squark-antisquark production [5], and NNLL resummation of $t\bar{t}$ hadroproduction [75]. The combined soft-Coulomb effects have been found to be sizeable for the case of squark-antisquark production [5] and lead to a reduction of the scale uncertainty, as has been observed as well in [49]. Note that to obtain the total *hadronic* cross section, the partonic cross section is convoluted with parton luminosity functions. The convolution scans over regions where β is not necessarily small, unless M is close to the hadronic centre-of-mass energy s . Hence, in these regions the threshold-enhanced terms cannot be expected *a priori* to give the dominant contribution to the cross section. However, one often finds after convoluting with the parton luminosity that the threshold contributions give a reasonable approximation to the total hadronic cross section (this will be shown in Figures 3.1 and 3.3 in Section 3.1.1 and 3.1.3 for the case of squark and gluino production and stop pair production respectively), so resummation is also relevant for improving predictions of the hadronic cross section.

In this chapter we extend the results given in [5] to the remaining production processes for squarks and gluinos at NLL accuracy, i.e. squark-squark, squark-gluino and gluino-gluino production. We also consider separately the production of pairs of stops, which required the extension shown in Section 2.3.2 of the formalism presented in [5] to particles pair-produced

in a P-wave state. The chapter is organized as follows: in Section 3.1 we give an overview of squark and gluino production processes, set up the calculation and briefly review the resummation formalism we employ, listing the ingredients needed for NLL resummation. Numerical results for the cross sections are presented in Section 3.2, including predictions for a representative set of the benchmark points proposed in [26]. We follow in Section 3.3 with a comparison of our results to the results using the Mellin-space formalism [56, 57]. Finally in Section 3.4 we present our conclusions and outlook.

3.1 Theoretical framework

We perform a NLL resummation of soft logarithms and Coulomb corrections to the partonic cross section, counting both α_s/β and $\alpha_s \ln \beta$ as quantities of order one, where $\beta = (1 - 4M^2/\hat{s})^{1/2}$ is the heavy-particle velocity. Our predictions include all corrections to the Born cross section of the schematic form (cf. Eq. (2.14))

$$\hat{\sigma}_{pp'}^{\text{NLL}} \propto \hat{\sigma}^{(0)} \sum_{k=0} \left(\frac{\alpha_s}{\beta} \right)^k \exp \left[\ln \beta g_0(\alpha_s \ln \beta) + g_1(\alpha_s \ln \beta) \right]. \quad (3.1)$$

A resummation at NNLL accuracy in the counting $\alpha_s \ln \beta \sim 1$, $\alpha_s/\beta \sim 1$, which is beyond the scope of this chapter but has recently been performed for top-pair production [75], would include in addition terms of the form $\exp[\alpha_s g_2(\alpha_s \ln \beta)]$ and corrections of order $\mathcal{O}(\alpha_s)$ relative to the NLL cross section, including NLO corrections to the Coulomb potential and other higher-order potentials, as well as the non-logarithmic one-loop hard corrections. The recent NNLL calculation of squark-antisquark production [49] included the corrections of the g_2 -type related to soft corrections and the hard $\mathcal{O}(\alpha_s)$ corrections, but kept only the $(\alpha_s/\beta)^1$ -term in the sum over k . In Section 3.1.1 we collect some facts about the production processes of gluinos and the superpartners of the light quarks at NLO while the formalism employed for the NLL resummation is reviewed in Section 3.1.2. The production of stop pairs is included in 3.1.3, while details about the choice of the soft scale in the momentum-space resummation formalism and our procedure to estimate the remaining theoretical uncertainty were discussed in 2.3.3.

It is understood that in the predictions for the cross sections presented below the contributions of the ten light-flavour squarks ($\tilde{u}_{L/R}$, $\tilde{d}_{L/R}$, $\tilde{c}_{L/R}$, $\tilde{s}_{L/R}$, $\tilde{b}_{L/R}$) are always summed over. Furthermore, the ten scalars are assumed to be degenerate in mass, with the common light-flavour squark mass given by $m_{\tilde{q}}$. In the following, the charge-conjugate subprocesses for squark-squark and gluino-squark productions will be included in our results. As input for the convolution (1.3) we will use the MSTW08 set of PDFs [8] at the appropriate perturbative order (the LO PDFs for Born-level predictions and the NLO PDFs for the NLO and NLL results) and set the factorization scale to the average mass of the produced sparticles, $\mu_f = M$. We use a set of PDFs with an improved accuracy at large x provided to us by the MSTW collaboration that has also been

employed for the NLL results in [50]. Explicit expressions for resummation functions appearing in the NLL cross sections are provided in Appendix A, while Appendix B contains some details on the scales used in the momentum-space resummation and on our method to estimate ambiguities in the resummation procedure.

3.1.1 NLO cross sections

Since the focus of this work is on higher-order corrections that are enhanced in the threshold limit $\beta \rightarrow 0$, we consider here the corresponding terms appearing at NLO. To this end, we decompose the partonic cross section $\hat{\sigma}_{pp'}$ into a complete colour basis, and parametrize the higher-order corrections as

$$\hat{\sigma}_{pp'} = \sum_{R_\alpha} \hat{\sigma}_{pp'}^{(0), R_\alpha} \left\{ 1 + \frac{\alpha_s}{4\pi} f_{pp'}^{(1), R_\alpha} + \dots \right\}. \quad (3.2)$$

The sum is over the irreducible colour representations appearing in the decomposition of the product representation $R \otimes R' = \sum R_\alpha$, where R, R' are the $SU(3)$ representations of the two final-state particles. The relevant decompositions for squark and gluino production are given by

$$\begin{aligned} \tilde{q}\tilde{q} : \quad & 3 \otimes \bar{3} = 1 \oplus 8, \\ \tilde{q}\tilde{q} : \quad & 3 \otimes 3 = \bar{3} \oplus 6, \\ \tilde{q}\tilde{g} : \quad & 3 \otimes 8 = 3 \oplus \bar{6} \oplus 15, \\ \tilde{g}\tilde{g} : \quad & 8 \otimes 8 = 1 \oplus 8_s \oplus 8_a \oplus 10 \oplus \bar{10} \oplus 27. \end{aligned} \quad (3.3)$$

The explicit basis tensors for the various representations have been constructed in [58] (see also [55, 56]), where it has been shown that an s -channel colour basis based on the decompositions (3.3) is advantageous for the all-order summation of soft-gluon corrections. In Eq. (3.2) $\hat{\sigma}_{pp'}^{(0), R_\alpha}$ represents the tree-level cross section for a given process in colour channel R_α , while the $f_{pp'}^{(1), R_\alpha}$ are colour-specific NLO scaling functions. The colour-separated Born cross sections for squark and gluino production are available in [55–57]. The NLO scaling functions, on the contrary, are only known numerically in their colour-averaged form [36]. However, a simple formula is available for the threshold limit of the NLO scaling functions, containing all the threshold-enhanced contributions, for arbitrary colour representation R_α [73]:

$$\begin{aligned} f_{pp'}^{(1), R_\alpha} = & -\frac{2\pi^2 D_{R_\alpha}}{\beta} \sqrt{\frac{2m_r}{M}} + 4(C_r + C_{r'}) \left[\ln^2 \left(\frac{8M\beta^2}{\mu_f} \right) + 8 - \frac{11\pi^2}{24} \right] \\ & - 4(C_{R_\alpha} + 4(C_r + C_{r'})) \ln \left(\frac{8M\beta^2}{\mu_f} \right) + 12C_{R_\alpha} + h_{pp'}^{(1), R_\alpha} + \mathcal{O}(\beta), \end{aligned} \quad (3.4)$$

with M the average mass of the two particles produced (1.7), while m_r denotes the reduced mass, $m_r = M_{\tilde{s}} M_{\tilde{s}'} / (M_{\tilde{s}} + M_{\tilde{s}'})$. C_r , $C_{r'}$ and C_{R_α} are the Casimir invariants for the colour

$\tilde{q}\tilde{q}$	$D_1 = -\frac{4}{3}$	$D_8 = \frac{1}{6}$		
$\tilde{q}\tilde{q}$	$D_{\bar{3}} = -\frac{2}{3}$	$D_6 = \frac{1}{3}$		
$\tilde{g}\tilde{q}$	$D_3 = -\frac{3}{2}$	$D_{\bar{6}} = -\frac{1}{2}$	$D_{15} = +\frac{1}{2}$	
$\tilde{g}\tilde{g}$	$D_1 = -3$	$D_8 = -\frac{3}{2}$	$D_{10} = 0$	$D_{27} = 1$

Table 3.1: Numerical values of the coefficients of the Coulomb potential (3.5) for squark and gluino production processes. Negative values correspond to an attractive potential.

representations of the initial-state particles, p and p' , and for the irreducible representation R_α of the SUSY pair. The coefficients D_{R_α} of the Coulomb potential for the production of heavy particles in $SU(3)$ representations R and R' in the colour channel R_α are given in terms of the quadratic Casimir operators for the various representations:

$$D_{R_\alpha} = \frac{1}{2}(C_{R_\alpha} - C_R - C_{R'}), \quad (3.5)$$

where negative values correspond to an attractive Coulomb potential, positive values to a repulsive one. The numerical values for the representations relevant for squark and gluino production can be found in [5, 87] and are collected in Table 3.1. The coefficient $h_{pp'}^{(1),R_\alpha}$ is the one-loop contribution to the hard matching coefficient appearing in Eq. (2.26), and represents the only process-specific quantity in Eq. (3.4). It has been obtained recently for squark-antisquark production and gluino-gluino production [49, 83] and for the other two processes in [88]. The knowledge of $h_{pp'}^{(1),R_\alpha}$ is required for NNLL resummation [49], but not at NLL accuracy as considered here, so the $h_{pp'}^{(1),R_\alpha}$ will be always set to zero in the following. The NLO scaling functions agree with our previous results for the one-loop soft and potential corrections in Eq. (2.13) after setting the one-loop hard contributions $h_{pp'}^{(1),R_\alpha}$ to zero. Furthermore, using the Born cross sections for the different colour channels [55, 56] and (3.4) one can reproduce the threshold expansions of the NLO corrections in [36].¹²

In Figure 3.1 we study to which extent the full NLO corrections as obtained from PROSPINO are approximated by the singular NLO corrections, obtained by dropping all constant terms from (3.4), including $\ln 2$ terms, and convoluting the resulting partonic cross section (3.2) with the parton luminosities. For the Born cross sections $\hat{\sigma}_{pp'}^{(0),R_\alpha}$ in (3.2) the exact expressions, without use of the threshold approximation, have been kept, but colour channels with a vanishing threshold limit of the Born cross section at leading order in β have been dropped. For the case of degenerate

¹²In [36] there is a typo in the sign of the Coulomb correction for like-flavour $\tilde{q}\tilde{q}$ production, i.e. the function f_{qq}^{V+S} in eq. (54). Also note that [36] choose to expand the cross section of $\tilde{q}\tilde{q}$ production in the variable $\tilde{\beta} = \sqrt{1 - 4m_{\tilde{q}}m_{\tilde{g}}/(s - (m_{\tilde{q}} - m_{\tilde{g}})^2)} \approx \beta\sqrt{M/2m_r}$ which leads to the appearance of additional $\ln(m_r/M)$ terms. For this process they also observe an apparent non-factorization of the colour-averaged NLO threshold corrections from the Born cross section. This is nevertheless consistent with (3.4) since the Born $\tilde{q}\tilde{g}$ cross section in the colour-triplet channel is not proportional to that for the other channels [56].

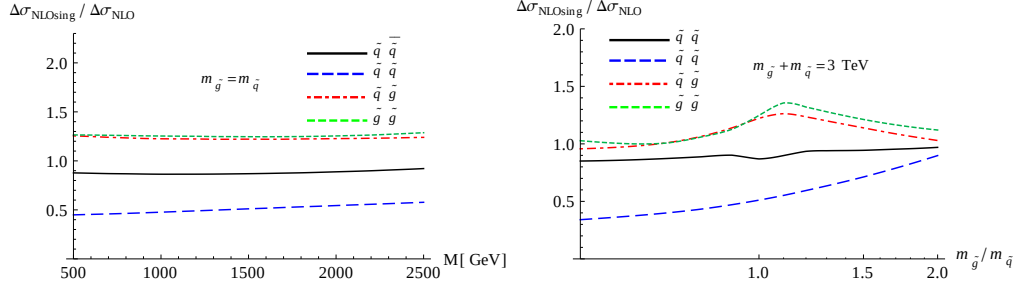


Figure 3.1: Ratio of the singular NLO contributions obtained from (3.4) to the exact NLO corrections for the LHC with $\sqrt{s} = 8$ TeV. Left: Mass dependence for a fixed mass-ratio $m_{\tilde{q}} = m_{\tilde{g}} = M$. Right: Dependence on the ratio $m_{\tilde{g}}/m_{\tilde{q}}$ for a fixed average mass $(m_{\tilde{q}} + m_{\tilde{g}})/2 = 1.5$ TeV.

squark and gluino masses it is seen that the difference of the threshold-enhanced contributions to the full NLO corrections is at the 10 – 30% level over the whole mass range considered, with the exception of the squark-squark production channel where the threshold contributions account for only 40 – 50% of the full NLO corrections. For $m_{\tilde{g}} > m_{\tilde{q}}$ the singular terms overestimate the corrections for the processes involving gluinos, while the agreement for squark-antisquark and squark-squark production improves. For $m_{\tilde{g}} < m_{\tilde{q}}$ the singular terms approximate the full corrections very well for all processes apart from squark-squark production.¹³ Comparing to Figure 1.6, it is seen that the singular terms capture the NLO corrections to the dominant processes for larger mass ratios (i.e. squark-squark production for $m_{\tilde{g}} \sim 2m_{\tilde{q}}$ and gluino-pair production for $m_{\tilde{g}} \sim 0.5m_{\tilde{q}}$) rather well. For degenerate squark and gluino masses the quality of the threshold approximation for the dominant squark-squark and squark-gluino processes is somewhat worse. In all cases, the inclusion of the threshold enhanced NLO corrections in addition to the Born terms improves the agreement with the full NLO results. This motivates the computation of the higher-order threshold-enhanced terms through resummation, as performed in the remainder of this work.

3.1.2 NLL soft-gluon and Coulomb resummation

Next, we briefly review the formalism for the combined resummation of soft- and Coulomb-gluon corrections [5, 58] given in Chapter 2 and provide the relevant ingredients for squark and gluino production specifically at NLL accuracy. We also discuss some features of the implementation that differ from that used previously for squark-antisquark production in [5].

¹³Note that one could improve the threshold approximation by including the constant terms in (3.4) once the coefficients $h_{pp'}^{(1), R\alpha}$ are known. This could be particularly relevant for the squark-squark process where the threshold contributions to the NLO cross section are relatively small due to the smaller colour charges involved and an accidental cancellation of Coulomb corrections between the same-flavour and different-flavour production channels.

We have seen in Section 2.3.3 that the natural scale for the evaluation of the hard function in (2.26), leading to well-behaved higher-order corrections, is of the order of $\mu_h \sim 2M$, while the natural scale for soft-gluon radiation is of the order of $\mu_s \sim M\beta^2$. We use the momentum-space resummation formalism of [11, 59, 60] and described in Section 2.3 to evolve the soft and hard functions from their natural scales to the factorization scale μ_f used for the evaluation of the parton distribution functions, commonly taken to be of the order of $\mu_f \sim M$. In this way, logarithms of $\mu_s/\mu_f \sim \beta^2$ are summed to all orders. The precise prescription for the choice of the soft scale adopted in our calculation has been discussed in 2.3.3. The exchange of multiple Coulomb gluons can be summed up using the method of Coulomb Green's functions in non-relativistic QCD [4, 89, 90].

For resummation at NLL accuracy, the leading-order hard and soft functions are required as fixed-order input to the evolution equations. The LO soft function is trivial, $W^{(0)R_\alpha}(\omega) = \delta(\omega)$. The leading-order hard functions are obtained from the threshold limit of the Born cross section for the colour channel R_α [5],

$$\hat{\sigma}_{pp'}^{(0),R_\alpha}(\hat{s}) \underset{\hat{s} \rightarrow 4M^2}{\approx} \frac{(2m_r)^2}{2\pi} \sqrt{\frac{E}{2m_r}} H_{pp'}^{(0),R_\alpha}. \quad (3.6)$$

Although the Born-cross sections in the threshold limit appear on the left-hand side in (3.6), we keep the exact expressions in our numerical implementation, so the hard functions in practice depend on $\hat{s}$. This incorporates some higher-order terms in β , albeit not systematically.¹⁴ Here our current treatment differs from that used for squark-antisquark production in [5] where only the threshold limit of the Born cross section was used to compute the hard function.

For the resummation of Coulomb corrections, we use results for the non-relativistic Coulomb Green's function obtained for top-quark production at electron-positron colliders and stop production [4, 65] that were reviewed in Section 2.3. For positive values of E and vanishing decay widths of the sparticles, the S-wave potential function is given by the Sommerfeld factor

$$J_{R_\alpha}(E) = \frac{(2m_r)^2 \pi D_{R_\alpha} \alpha_s}{2\pi} \left(e^{\pi D_{R_\alpha} \alpha_s \sqrt{\frac{2m_r}{E}}} - 1 \right)^{-1}, \quad E > 0, \quad (3.7)$$

with the coefficients D_{R_α} of the Coulomb potential given in (3.5). For an attractive potential, a series of bound states develops below threshold with energies

$$E_n = -\frac{2m_r \alpha_s^2 D_{R_\alpha}^2}{4n^2}. \quad (3.8)$$

¹⁴This corresponds to the treatment of [55–57], up to the fact that we set the hard function for a given production and colour channel to zero if the Born cross section vanishes at threshold at leading order in β , even if the full Born cross section for this channel is non-vanishing. This affects the subprocesses $q\bar{q} \rightarrow \tilde{g}\tilde{g}$ in the singlet channel and $q_i q_i \rightarrow \tilde{q}_i \tilde{q}_i$ in the triplet channel. The numerical effect is however negligible.

Their contribution to the S-wave potential function is given by

$$J_{R_\alpha}^{\text{bound}}(E) = 2 \sum_{n=1}^{\infty} \delta(E - E_n) \left(\frac{2m_t(-D_{R_\alpha})\alpha_s}{2n} \right)^3 \quad E < 0. \quad (3.9)$$

For sufficiently broad squarks and gluinos with decay widths exceeding the binding energy of the would-be bound states, $\Gamma_{\tilde{s}} + \Gamma_{\tilde{s}'} > |E_1 - E_2|$, the bound state poles are smeared out by the finite lifetimes.¹⁵ We consider here a situation where the widths of the squarks and gluinos are large enough to prevent the formation of bound states, but small enough that the use of a narrow-width approximation is justified, which is the case for SUSY scenarios with moderate mass ratios of squarks and gluinos where $\Gamma_{\tilde{s}}/M_{\tilde{s}} \sim 1\%$. The contributions to the total cross section below the nominal production threshold, $\hat{s} = 2M$, can then be included by setting the sparticle widths to zero and including the bound-state poles (3.9). For other observables, the finite width can be taken into account to a first approximation by the replacement $E \rightarrow E + i(\Gamma_{\tilde{s}} + \Gamma_{\tilde{s}'})/2$ in the potential function, see e.g. [82–84] for recent studies of the invariant-mass spectrum of gluino-pair and squark-gluino production. The study of finite-width corrections for larger decay widths (e.g. for gluino masses $m_{\tilde{g}} \gtrsim 1.4m_{\tilde{q}}$) is considered in Chapter 4. In the numerical results presented in this work, the contributions of the bound state poles for $E < 0$ will always be included in our default implementation, and are convoluted with the resummed soft function as described in [75]. Note that in the previous results for squark-antisquark production [5] the bound-state corrections have been added without soft-gluon resummation.

The resummed cross section at NLL accuracy is obtained by inserting the potential function (3.7) and the solutions to the evolution equations of the hard and soft functions [58, 76] into the factorization formula (2.26). Using the solutions in momentum-space obtained in [60], the NLL cross section is written as (cf. Eq. (2.36))

$$\hat{\sigma}_{pp'}^{\text{NLL}}(\hat{s}, \mu_f) = \sum_{R_\alpha} H_{pp'}^{(0), R_\alpha}(\mu_h) U_i(M, \{\mu\}) \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)} \int_0^\infty d\omega \frac{J_{R_\alpha}(M\beta^2 - \frac{\omega}{2})}{\omega} \left(\frac{\omega}{2M} \right)^{2\eta}. \quad (3.10)$$

Here the label i jointly refers to the colour of the initial-state partons and the representation R_α of the sparticle pair. The function $\eta = \frac{2\alpha_s(C_r + C_{r'})}{\pi} \ln(\mu_s/\mu_f) + \dots$ contains single logarithms, while the resummation function U_i sums the Sudakov double logarithms $\alpha_s \log^2 \frac{\mu_h}{\mu_f}$ and $\alpha_s \log^2 \frac{\mu_s}{\mu_f}$. Explicit expressions up to NNLL accuracy are given in Appendix A. For $\mu_s < \mu_f$ the function η is negative and the factor $\omega^{2\eta-1}$ in the resummed cross section (3.10) has to be

¹⁵The actual formation of bound states is possible if the decay widths of the sparticles are smaller than those of the bound state. For long-lived gluinos, or light stops without allowed two-body decays, higher-order corrections to bound-state production have been recently obtained in [78, 91]. Since the bound states decay into gluon or photon pairs, this scenario leads to very different collider signatures compared to the missing-energy signatures of continuum production and is not considered further here.

understood in the distributional sense, as discussed in detail in [75]. We have used the non-relativistic expression $E = M\beta^2$ in the argument of the potential function, that is valid near the partonic threshold (1.8). This follows the default treatment of top-pair production in [75] and leads to the customary expansion of the cross section in β (see Eq. 3.4). We also perform this replacement in the definition of the hard functions (3.6). The difference between this default implementation and the results obtained by consistently keeping the expression $E = \sqrt{s} - 2M$ (as in the previous results for squark-antisquark production [5]) will be used to estimate the effect of subleading terms in the cross section, as discussed in Section 2.3.3.

In order to assess the importance of the Coulomb corrections and to compare to the results of the Mellin approach [56] we will also present results without Coulomb resummation, obtained by inserting the trivial potential function $J^{(0)}(E) = \frac{(2m_r)^2}{2\pi} \sqrt{E/2m_r}$ into (3.10). In this approximation, that we will denote by NLL_{s+h} , a fully analytical expression for the resummed cross section can be obtained:

$$\hat{\sigma}_{pp'}^{\text{NLL}_{s+h}} = \sum_{R_\alpha} \hat{\sigma}_{pp'}^{(0)}(\hat{s}, \mu_h) U_i(M, \mu_h, \mu_s, \mu_f) \frac{\sqrt{\pi} e^{-2\eta\gamma_E}}{2\Gamma(2\eta + \frac{3}{2})} \beta^{4\eta}. \quad (3.11)$$

Since contributions to the cross section from outside the threshold region can be numerically non-negligible, we match the NLL resummed cross section to the fixed-order NLO calculation by subtracting the NLO expansion of the NLL expression and adding back the full NLO corrections:

$$\hat{\sigma}_{pp'}^{\text{matched}}(\hat{s}) = \left[\hat{\sigma}_{pp'}^{\text{NLL}}(\hat{s}) - \hat{\sigma}_{pp'}^{\text{NLL}(1)}(\hat{s}) \right] + \hat{\sigma}_{pp'}^{\text{NLO}}(\hat{s}), \quad (3.12)$$

where $\hat{\sigma}_{pp'}^{\text{NLO}}(\hat{s})$ is the fixed-order NLO cross section obtained in standard perturbation theory, as implemented in PROSPINO [34], and $\hat{\sigma}_{pp'}^{\text{NLL}(1)}$ is the resummed cross section expanded to NLO, as given in [5]. The total hadronic cross section at NLL is then obtained by convoluting (3.12) with the parton luminosity, as in (1.3).

3.1.3 Stop-antistop production

Contrary to the production of a light squark-antisquark pair, stop-pair production in the $q\bar{q}$ channel cannot be mediated by a t -channel diagram like in Figure 3.2(a), due to the extreme suppression of top-quark PDFs inside the proton. As a result, at LO in QCD a stop-antistop pair is produced in a P-wave state in quark-antiquark collisions. As was shown in Section 2.3.2, the resummation formalism can be extended in a straightforward way to $q\bar{q}$ -initiated stop-antistop production at NLL, and the only modification of (2.26) is the replacement of the potential function by that appropriate for P-wave processes. For stable particles, above threshold the result is given by [65] (see also [92])

$$J_{R_\alpha}^P(E) = 2m_r E \left(1 + \frac{(\alpha_s D_{R_\alpha})^2 2m_r}{4E} \right) J_{R_\alpha}(E), \quad E > 0. \quad (3.13)$$

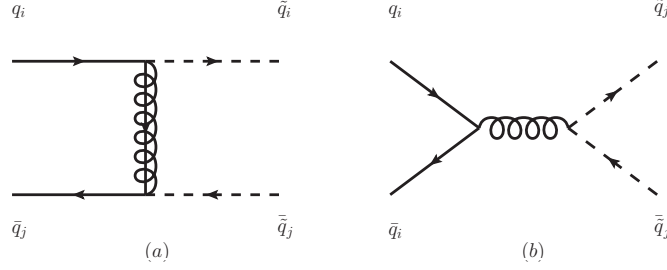


Figure 3.2: Tree-level diagram topologies contributing to $q_k \bar{q}_l \rightarrow \bar{q}_i q_j$.

The bound-state contributions of the P-wave Green's function can be found in [65] but are not needed here, since only the repulsive colour-octet channel appears in our application to stop-pair production. By expanding (3.13) in the strong coupling constant one obtains the coefficients of the fixed-order Coulomb corrections. While the one-loop Coulomb correction agrees with that for S-wave production (3.4), the second Coulomb correction $\frac{(D_{R_\alpha} \alpha_s)^2}{12} (\pi^2 + 3) / \beta^2$ differs from the S-wave case. Due to the different normalization of the P-wave Green's function, the definition of the leading-order hard functions for P-wave production reads

$$\hat{\sigma}_{pp',P}^{(0),R_\alpha}(\hat{s}) \underset{\hat{s} \rightarrow 4M^2}{\approx} \frac{(2m_r)^4}{2\pi} \sqrt{\left(\frac{E}{2m_r}\right)^3} H_{pp'}^{(0),R_\alpha}, \quad (3.14)$$

where the P-wave hard function now has mass dimension -6 , contrary to the S-wave hard function in Eq. (3.6) which carries a mass dimension -4 .

In addition to the combined soft/Coulomb resummation, we will again consider the NLL_{s+h} approximation where the trivial P-wave potential function $J^{(0)}(E) = \frac{(2m_r)^4}{2\pi} (E/2m_r)^{3/2}$ is used in the resummation formula, leading to the analytical result

$$\hat{\sigma}_{pp',P}^{\text{NLL}_{s+h}} = \sum_{R_\alpha} \hat{\sigma}_{pp'}^{(0)}(\hat{s}, \mu_h) U_i(M, \mu_h, \mu_s, \mu_f) \frac{3\sqrt{\pi} e^{-2\eta\gamma_E}}{4\Gamma(2\eta + \frac{5}{2})} \beta^{4\eta}. \quad (3.15)$$

In analogy to the S-wave result (3.4), one can use the resummation formalism to obtain the threshold-enhanced one-loop scaling functions for P-wave production in the colour channel R_α from initial-state partons in the representations r and r' :

$$\begin{aligned} f_{pp',P}^{(1)R_\alpha} = & -\frac{2\pi^2 D_{R_\alpha}}{\beta} \sqrt{\frac{2m_r}{M}} + 4(C_r + C_{r'}) \left[\ln^2 \left(\frac{8E}{\mu_f} \right) + \frac{104}{9} - \frac{11\pi^2}{24} \right] \\ & - 4 \left(C_{R_\alpha} + \frac{16}{3} (C_r + C_{r'}) \right) \ln \left(\frac{8E}{\mu_f} \right) + \frac{44}{3} C_{R_\alpha} + h_i^{(1)}(\mu_f). \end{aligned} \quad (3.16)$$

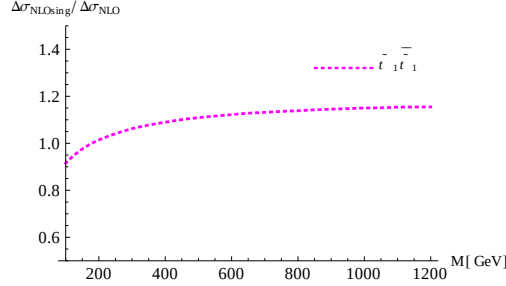


Figure 3.3: Ratio of the singular NLO contributions obtained from Eqs. (3.4) and (3.16) to the full NLO cross section for $PP \rightarrow \tilde{t}_1 \tilde{t}_1$ at the LHC with $\sqrt{s} = 8$ TeV.

In agreement with [57] one finds that the coefficient of the single logarithms related to initial-state radiation is multiplied by a factor of $\frac{4}{3}$ compared to the S-wave case while the double logarithm and the logarithms related to final state radiation proportional to C_{R_α} are unchanged. In addition, the constant terms are different which is irrelevant at NLL accuracy but has to be taken into account if one aims to extract the one-loop hard function from a computation of the NLO cross section. In Figure 3.3 we study the accuracy of the threshold approximation defined by inserting the NLO singular terms, obtained from Eqs. (3.4) and (3.16) by dropping constant terms, into (3.2). The ratio of the singular NLO corrections to the full NLO corrections to the hadronic cross section obtained using PROSPINO is shown in Figure 3.3. Analogously to squark-antisquark production, the threshold terms provide an excellent approximation of the full NLO result.

3.2 Numerical results

In this Section we present numerical results for the cross sections of the five SUSY processes introduced in Section 3.1.1 and 3.1.3. In Section 3.2.1 we discuss the impact of the NLL soft and Coulomb corrections on the central value of the total cross sections and the uncertainties for the production of light-flavour squarks and gluinos. In Section 3.2.2 we provide predictions for a selection of the benchmark points defined in [26]. The results for stop-antistop production are presented in 3.2.3. In order to facilitate the use of our results, the arXiv submission of the corresponding paper [47] (see also [93]) includes grids with predictions for the LHC with $\sqrt{s} = 7$ and 8 TeV, for light-flavour squark and gluino masses from 200 – 2000 GeV and stop masses from 100 – 1000 GeV (200 – 2500 GeV and 100 – 1200 GeV, respectively, for $\sqrt{s} = 8$ TeV). We also provide a *Mathematica* file containing interpolations of the cross sections with an accuracy that is typically better than $\sim 1\%$, and at worst 1 – 2% for almost degenerate masses or 2 – 3% for the lower boundaries of the intervals where $m_{\tilde{q}}, m_{\tilde{g}} < 400$ GeV.

3.2.1 Squark and gluino production at NLL

To illustrate how different classes of corrections contribute to the total cross section, we introduce three different NLL implementations:

- **NLL**: our default implementation. Contains the full combined soft and Coulomb resummation, Eq. (3.10), including bound-state contributions below threshold, Eq. (3.9). For the soft scale we adopt the running scale given in Eqs. (2.49), (2.50).
- **NLL_{no BS}**: as above, but without the inclusion of bound-state effects.
- **NLL_{s+h}**: this implementation includes resummation of soft and hard logarithms only, without Coulomb resummation. This is obtained using Eqs. (3.11) and (3.15).

The three NLL approximations defined above are always matched to the exact NLO results computed with `PROSPINO`, according to (3.12). As input for the convolution with the parton luminosity functions, Eq. (1.3), we adopt the MSTW08NLO PDF set [8] and the associated strong coupling constant $\alpha_s(M_Z) = 0.1202$. Unless otherwise specified, the parameter r , defined as

$$r = \frac{m_{\tilde{g}}}{m_{\tilde{q}}}, \quad (3.17)$$

is set to one.

We start presenting results for the NLL K -factor, defined as

$$K_{\text{NLL}} = \frac{\sigma^{\text{matched}}}{\sigma^{\text{NLO}}}, \quad (3.18)$$

where σ^{matched} is our matched result for one of the NLL implementations defined in the beginning of this Section and σ^{NLO} the fixed-order NLO result obtained using `PROSPINO`. The NLL- K -factor for LHC with 8 TeV centre-of-mass energy is plotted in Figure 3.4, for the four light-squark/gluino production processes and the mass range $m_{\tilde{q}} = m_{\tilde{g}} = 500\text{--}2500$ GeV. The results for $\sqrt{s} = 14$ TeV and the mass range $m_{\tilde{q}} = m_{\tilde{g}} = 500\text{--}3000$ GeV are given in Figure 3.5. The NLL corrections for our default implementation (solid blue lines) can be large, with corrections to the fixed-order NLO results of up to 170% in the upper mass range for gluino-gluino production at 8 TeV. The higher-order effects are smaller, but still sizeable, for the other three processes, due to the smaller colour charges involved in squark-antisquark, squark-squark and squark-gluino production. Furthermore, for a fixed SUSY mass the K_{NLL} -factor decreases from 8 to 14 TeV, consistently with the expectation that at lower centre-of-mass energies the threshold region plays a more prominent role.

The effect of including Coulomb resummation and its interference with soft resummation is on average as large as (or even larger than) the effect of pure soft and hard corrections, as can be seen comparing our default implementation NLL with NLL_{s+h} (dashed red lines). Pure soft

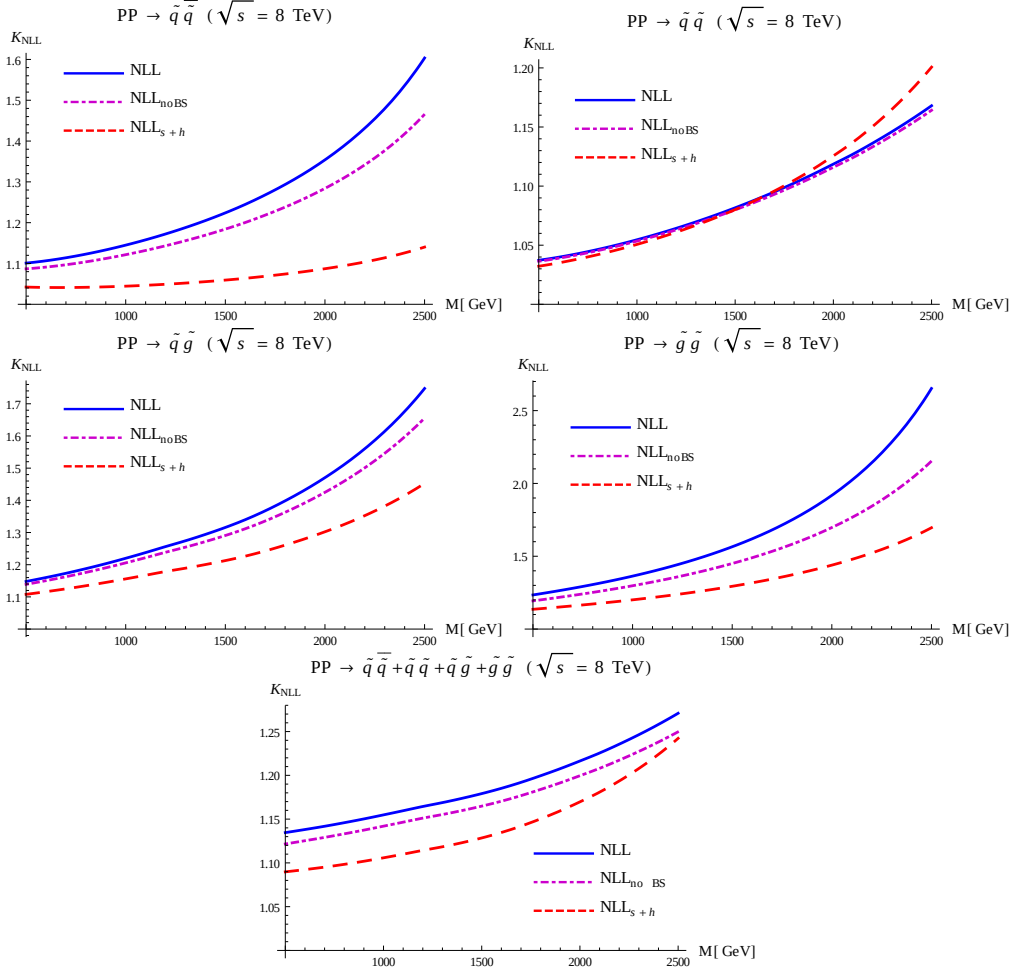


Figure 3.4: NLL K -factor for squark-antisquark (top-left), squark-squark (top-right), squark-gluino (centre-left) and gluino-gluino (centre-right) production at LHC with $\sqrt{s} = 8$ TeV, and for the sum of the four processes (bottom). The plots show K_{NLL} as a function of M for different NLL approximations: NLL (solid blue), NLL_{noBS} (dot-dashed purple) and NLL_{s+h} (dashed red). See the text for explanation.

contributions beyond $\mathcal{O}(\alpha_s)$ amount to 5 – 70% of the fixed-order NLO result, depending on the mass and process considered, whereas pure Coulomb effects and interference of soft and Coulomb corrections can amount to up to 100%. An exception to this is the squark-squark production process, where the effect of Coulomb corrections is small. This particular behaviour originates

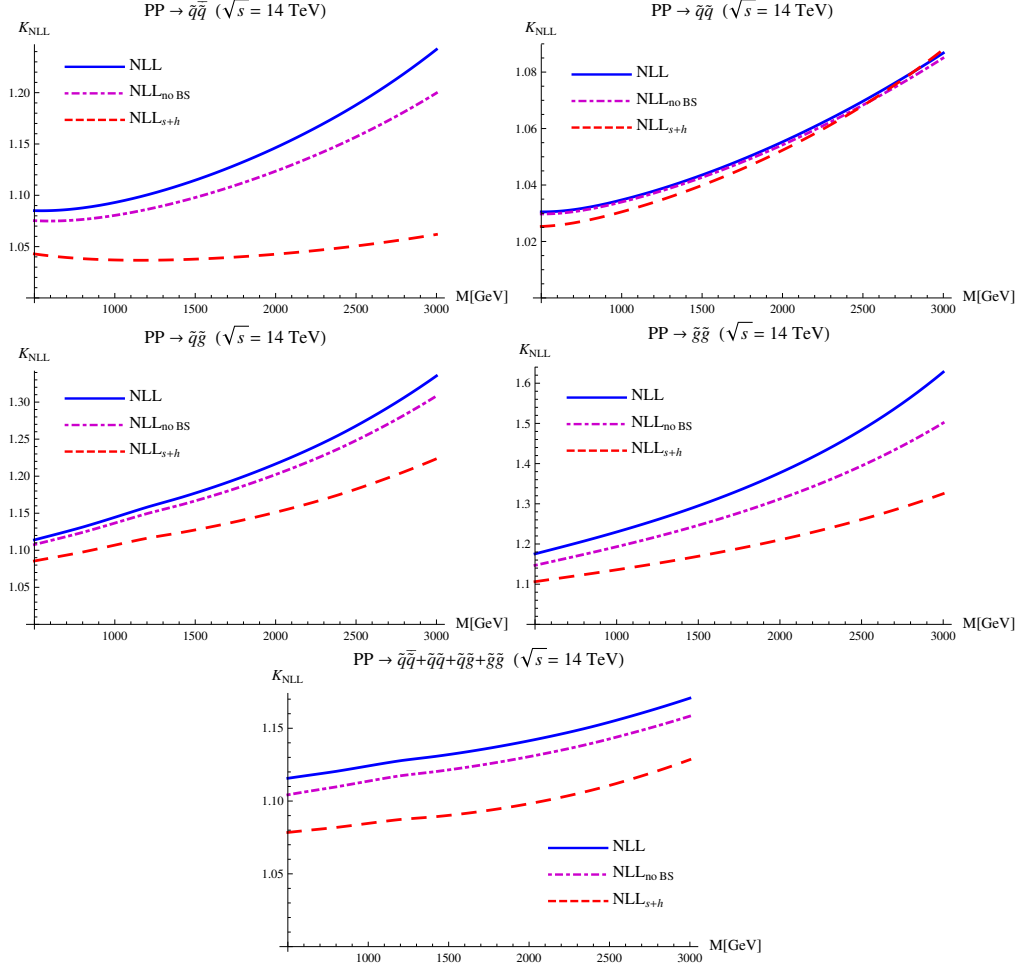


Figure 3.5: NLL K -factor for squark-antisquark (top-left), squark-squark (top-right), squark-gluino (centre-left) and gluino-gluino (centre-right) production at LHC with $\sqrt{s} = 14$ TeV, and for the sum of the four processes (bottom). The plots show K_{NLL} as a function of M for different NLL approximations: NLL (solid blue), NLL_{no BS} (dot-dashed purple) and NLL_{s+h} (dashed red). See the text for explanation.

from cancellations between the cross sections for same-flavour squark production, where the repulsive colour-sextet channel is numerically dominant and gives rise to negative $\mathcal{O}(\alpha_s^2 \ln^2 \beta/\beta)$ corrections, and different-flavour squark production, where the corresponding term is positive, due to the dominance of the attractive colour-triplet channel.

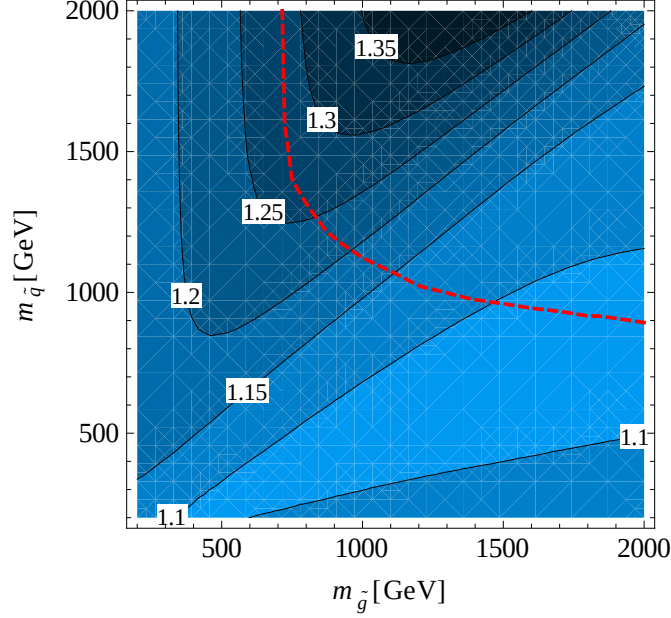


Figure 3.6: NLL K -factor for the total SUSY production rate at LHC with $\sqrt{s} = 8$ TeV as a function of the gluino mass $m_{\tilde{g}}$ and average squark mass $m_{\tilde{q}}$. The dashed line corresponds to the most recent exclusion limit at $\sqrt{s} = 7$ TeV presented in [29].

For squark-antisquark, squark-gluino and gluino-gluino production, a significant portion of the total Coulomb and soft-Coulomb corrections originates from bound-state effects below threshold. These correspond to the difference between the NLL and NLL_{no BS} (dot-dashed purple) curves in the plots. For squark-antisquark and squark-gluino production bound-state corrections amount to 2 – 15% of the fixed-order NLO cross section, whereas for gluino-gluino production they can be as large as 50%.

Figure 3.6 shows the NLL K -factor for the total SUSY production rate at the 8 TeV LHC as a contour plot in the $(m_{\tilde{g}}, m_{\tilde{q}})$ -plane. The r -dependence of the total resummed cross section arises from an interplay of the r -dependence of the single-process cross sections and of the relative dominance of the four subprocesses for a given r . The largest K -factor is obtained for $m_{\tilde{q}} \sim 2$ TeV and $m_{\tilde{g}} \sim 1.4$ TeV, with corrections of 40% to the NLO cross section. The plot shows also the recent exclusion limit at $\sqrt{s} = 7$ TeV published by the ATLAS collaboration in [29] assuming a simplified model of a massless neutralino, a gluino octet and degenerate squarks of the first two generations, while all the other supersymmetric particles, including stops and sbottoms, are decoupled by giving them a mass of 5 TeV. The limits are therefore not directly comparable to our

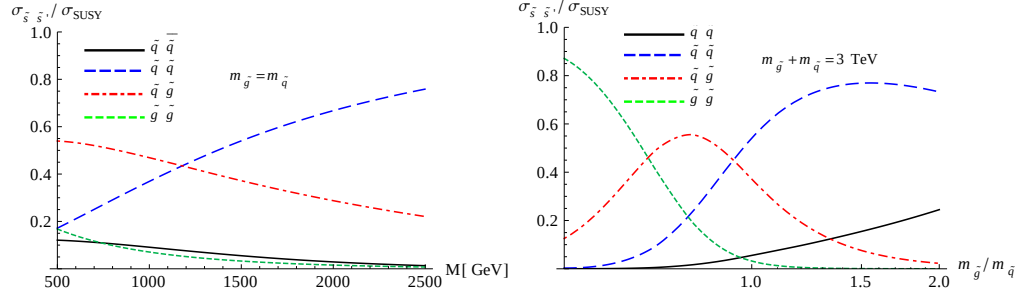


Figure 3.7: Ratio of the NLL production-cross sections for the processes (1.16) to the total NLL rate of coloured sparticle production σ_{SUSY} for the LHC with $\sqrt{s} = 8$ TeV. Left: Mass dependence for a fixed mass-ratio $m_{\tilde{g}} = m_{\tilde{q}} = M$, Right: Dependence on the ratio $m_{\tilde{g}}/m_{\tilde{q}}$ for a fixed average mass $(m_{\tilde{q}} + m_{\tilde{g}})/2 = 1.5$ TeV.

results which treat the sbottom as degenerate with the light-flavour squarks, but are shown here as an indication of the current LHC reach. We do not attempt to estimate how resummation would affect the determination of this limit. However, one can observe that in the large squark-mass region the exclusion limit crosses regions with a K -factor of the order 1.3, where resummation effects on the limit extraction might be relevant.

Given the large effect of resummation, especially for squark-gluino and gluino-gluino production, it is interesting to study how the relative contribution of the four production processes to the total SUSY production rate is modified by the inclusion of NLL corrections. This is shown in Figure 3.7. The qualitative behaviour of the relative contribution of the four different processes is very similar to the LO result (Figure 1.6). However at large masses one can notice an enhancement of the squark-gluino production rate compared to the squark-squark channel (left plot), as one would expect from the larger NLL K -factor for the first processes. For a fixed average squark and gluino mass of 1.5 TeV (right plot) the relative ratios are basically unchanged for moderate values of $r = m_{\tilde{g}}/m_{\tilde{q}}$, though one observes a significant enhancement of the squark-antisquark cross section for large gluino masses ($r = 2$).

It is interesting to study how the choice of a fixed or running soft scale affects the NLL resummed cross section, especially in view of the uncertainties just discussed. In Figure 3.8 we plot the NLL K -factor, defined in Eq. (3.18), as a function of a common SUSY mass $M = m_{\tilde{g}} = m_{\tilde{q}}$ for the four processes listed in (1.16), for a centre-of-mass energy of 8 TeV (the situation for $\sqrt{s} = 14$ TeV is qualitatively similar). Results for the stop-pair production process and the total SUSY production rate are also shown. The thick lines represent the central values for the two implementations, whereas the bands (delimited by thinner lines) correspond to the resummation uncertainty as defined above. The central values are in good agreement for squark-antisquark

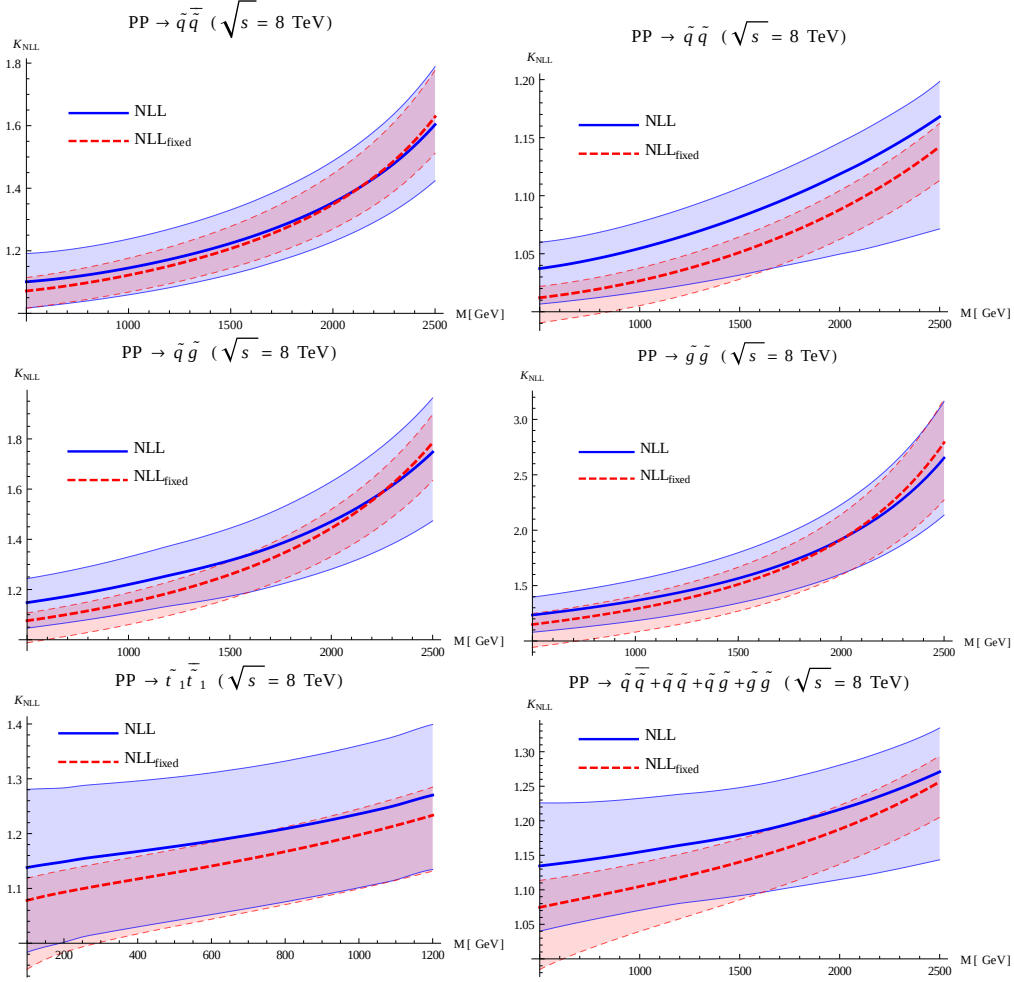


Figure 3.8: Resummation uncertainty for the NLL resummed result with a running soft scale (NLL, solid blue) and a fixed soft scale (NLL_{fixed}, dashed red) for squark-antisquark (top-left), squark-squark (top-right), squark-gluino (centre-left), gluino-gluino (centre-right), stop-antistop (bottom-left) production and the inclusive gluino and light-flavour squark cross section (bottom-right) at LHC with $\sqrt{s} = 8$ TeV. The central line represents the K -factor for the default scale choice, while the band gives the resummation uncertainty associated with the result. See text for explanation.

and gluino-pair production, and for squark-gluino production at larger masses. For squark-squark production the agreement is less satisfactory, especially for smaller masses. This is consistent with

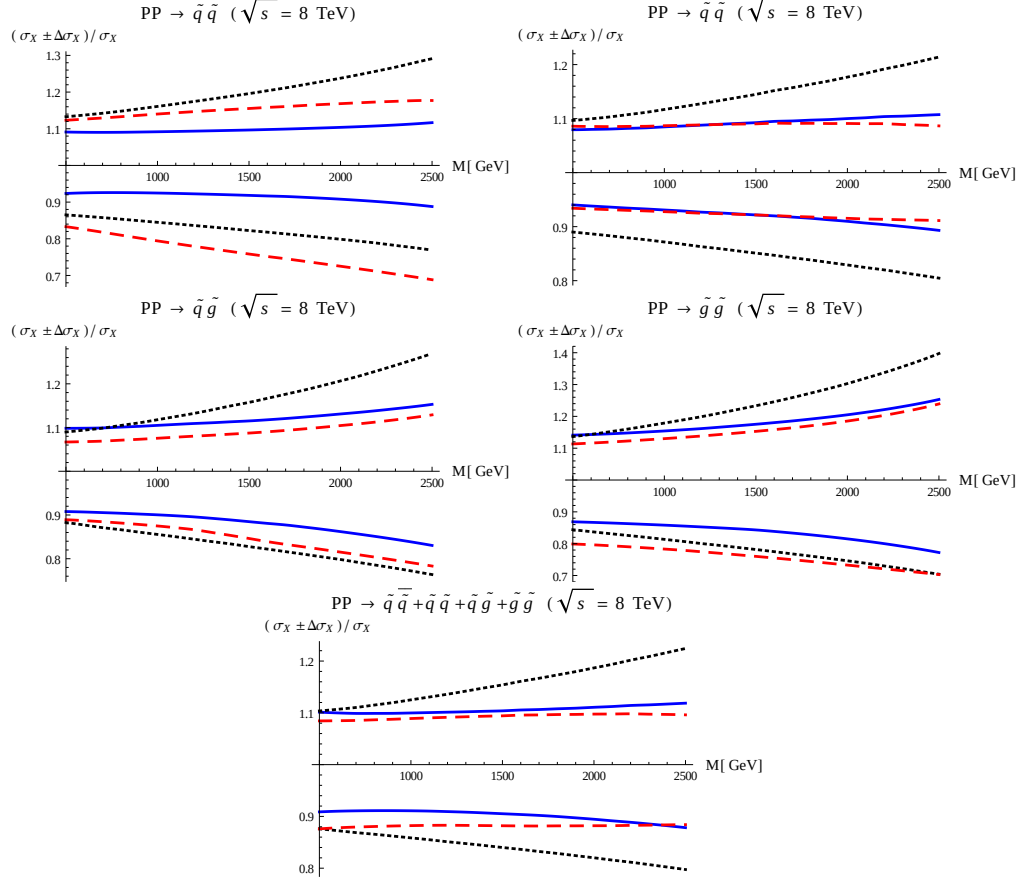


Figure 3.9: Total theoretical uncertainty of the NLO approximation (dotted black), full NLL resummed result (solid blue) and NLL_{s+h} (dashed red) at the LHC with $\sqrt{s} = 8$ TeV. All cross sections are normalized to one at the central value of the scales.

the observation from Figure 3.1 that the NLO corrections for squark-squark production are not as dominated by the threshold contributions as those for the other processes. In all cases, however, the two different NLL predictions are consistent with each other once the uncertainty associated with the resummation procedure is taken into account. It can also be seen that the uncertainty band for the fixed-scale implementation $\text{NLL}_{\text{fixed}}$ is mostly contained inside the uncertainty band of the running-scale result, with the possible exception of the small-mass region.

As a result of including the threshold-enhanced higher-order corrections, one expects that the uncertainty due to missing perturbative corrections is reduced compared to the NLO results.

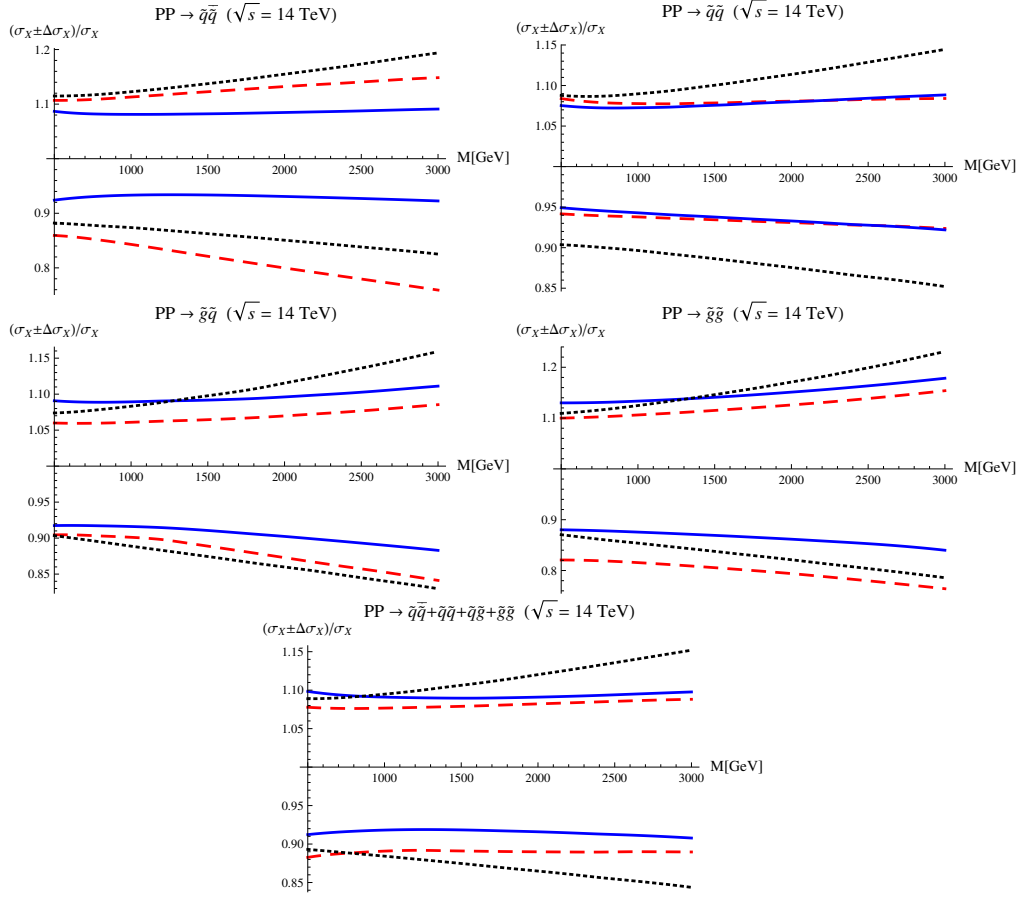


Figure 3.10: Total theoretical uncertainty of the NLO approximation (dotted black), full NLL resummed result (solid blue) and NLL_{s+h} (dashed red) at the LHC with $\sqrt{s} = 14$ TeV. All cross sections are normalized to one at the central value of the scales.

While for NLO the theoretical uncertainty arises from scale-variation only, the total theoretical error of the NLL results is obtained by adding scale and resummation uncertainties in quadrature, as was defined in Section 2.3.3. The uncertainty bands for NLO, NLL_{s+h} and NLL approximations are shown in Figure 3.9 for the LHC with $\sqrt{s} = 8$ TeV and in Figure 3.10 for the LHC with $\sqrt{s} = 14$ TeV. In all plots the cross sections are normalized to unity at the central values of the scales and other input parameters. It is evident that the combined resummation of soft and Coulomb effects (NLL, solid blue) generally leads to a significant reduction of theoretical uncertainties compared

to the NLO result (dotted black), especially in squark-antisquark and squark-squark production, where the error is reduced by a factor 2 or more in the large-mass region. The behaviour of NLL_{s+h} (dashed red) is more process-dependent, with basically no uncertainty reduction compared to the fixed-order NLO result for squark-antisquark production, and moderate effects for squark-gluino and gluino-gluino production. For squark-squark production (and, as a consequence of the dominance of squark-squark production, for the total SUSY production rate) the uncertainties of NLL_{s+h} and NLL are very similar, due to the smallness of Coulomb effects in this particular production channel. The large reduction of the scale dependence for squark-antisquark production by soft-Coulomb interference effects is consistent with recent NNLL studies in this channel [49] that include the first Coulomb correction.

3.2.2 Benchmark points for SUSY searches at LHC

In addition to the grid files found online with the arXiv submission [47] (see also [93]), we here present numerical predictions for some benchmark points at the LHC with 7 TeV centre-of-mass energy, in order to illustrate the effect of our NLL results on the production cross sections. We employ the sets of benchmark points defined in [26], that are compatible with recent LHC bounds and other data such as $b \rightarrow s\gamma$, but not necessarily with constraints from the anomalous magnetic moment of the muon, or from the dark matter relic abundance. We consider the seven lines in the constrained MSSM (CMSSM) parameter space defined in [26] and one line for the minimal gauge mediated SUSY breaking (mGMSB) scenario. For each line we selected one benchmark point expected to be relevant for 5 fb^{-1} of data and a second point relevant for $10 - 30 \text{ fb}^{-1}$, where we naively extrapolate the reach of the 1 fb^{-1} LHC data [29, 30] that exclude CMSSM benchmark points and simplified models with total SUSY production cross sections of the order of $\sigma_{\text{SUSY}} \gtrsim 0.04 \text{ pb}$ for $m_{\tilde{q}} \lesssim m_{\tilde{g}}$ and $\sigma_{\text{SUSY}} \gtrsim 0.1 \text{ pb}$ for $m_{\tilde{q}} > m_{\tilde{g}}$. The mGMSB scenario we have selected has a quasi-stable neutralino as next-to-lightest SUSY particle (NLSP), so a similar reach as for CMSSM-type scenarios are expected. Since only the squark and gluino masses are relevant for the production cross sections, we have chosen points with a reasonable spread of masses and mass ratios, covering the range of average sparticle masses, $1.3 \text{ TeV} \lesssim (m_{\tilde{g}} + m_{\tilde{q}})/2 \lesssim 1.5 \text{ TeV}$, and the mass ratios $0.75 \lesssim \frac{m_{\tilde{g}}}{m_{\tilde{q}}} \lesssim 1.12$. This mass range is also compatible with the estimated discovery reach [94] of $m_{\tilde{g}} \sim 1.3 \text{ TeV}$ (1.6 TeV) for $m_{\tilde{q}} \sim m_{\tilde{g}}$ at 5 fb^{-1} (30 fb^{-1}) and $m_{\tilde{g}} \sim 0.8 \text{ TeV}$ (1 TeV) for $m_{\tilde{q}} \gg m_{\tilde{g}}$ in a CMSSM scenario with $\tan\beta = 0.45$, $A_0 = 0$. The remaining families of benchmark scenarios introduced in [26] tend to have very similar mass ratios as our selected points. Therefore the relative contributions of the different production channels and the effect of the higher-order QCD corrections will be similar, although the decay chains and the resulting collider signatures can be very different. For some scenarios lighter squarks and gluinos than the ones considered here might still be allowed, for instance in GMSB with a stau NLSP. Predictions for such scenarios can be obtained by an interpolation of the grid files provided with the arXiv submission of the paper [47] (see also [93]).

Point	m_0	$m_{1/2}$	$m_{\tilde{g}}$	$m_{\tilde{q}}$	$\sigma_{\text{SUSY}}(\text{pb}), \sqrt{s} = 7 \text{ TeV}$			
					NLO	NLL	Δ_{PDF}	K_{NLL}
10.1.3	150	600	1357	1209	$0.91^{+0.13}_{-0.14} \times 10^{-2}$	$1.04^{+0.10}_{-0.09} \times 10^{-2}$	+4.2% -2.9%	1.15
10.1.4	162.5	650	1461	1300	$4.17^{+0.60}_{-0.64} \times 10^{-3}$	$4.79^{+0.45}_{-0.42} \times 10^{-3}$	+4.2% -3.0%	1.15
10.2.2	225	550	1255	1130	$1.83^{+0.25}_{-0.27} \times 10^{-2}$	$2.10^{+0.20}_{-0.18} \times 10^{-2}$	+4.2% -3.0%	1.15
10.2.5	300	700	1569	1412	$1.66^{+0.26}_{-0.27} \times 10^{-3}$	$1.92^{+0.18}_{-0.17} \times 10^{-3}$	+4.4% -3.1%	1.16
10.3.2	350	525	1209	1115	$2.21^{+0.30}_{-0.33} \times 10^{-2}$	$2.54^{+0.24}_{-0.23} \times 10^{-2}$	+4.4% -3.2%	1.15
10.3.3	400	600	1367	1261	$6.32^{+0.92}_{-0.98} \times 10^{-3}$	$7.31^{+0.71}_{-0.66} \times 10^{-3}$	+4.4% -3.3%	1.16
10.4.2	850	400	983	1160	$3.48^{+0.57}_{-0.59} \times 10^{-2}$	$4.24^{+0.48}_{-0.43} \times 10^{-2}$	+7.0% -6.2%	1.22
10.4.4	1050	500	1207	1427	$3.65^{+0.68}_{-0.66} \times 10^{-3}$	$4.59^{+0.54}_{-0.50} \times 10^{-3}$	+8.5% -7.4%	1.26

Table 3.2: CMSSM benchmark points and total inclusive SUSY production cross sections for $\tan \beta = 10$, $A_0 = 0$. All masses are given in GeV.

The SUSY breaking parameters and the resulting mass spectrum of the coloured SUSY particles for the selected points is shown in Tables 3.2, 3.3 and 3.4 together with our best NLL predictions for the total cross section for light-flavour squark and gluino production (including simultaneous soft-gluon and Coulomb resummation as well as bound-state effects). Here $m_{\tilde{q}}$ denotes the average mass of all squarks except the stops, following the setup of [49, 95]. The low-scale mass parameters have been generated using SUSY-HIT [96] employing SuSpect 2.41 [97] with the standard model input $m_t = 172.5 \text{ GeV}$, $\alpha_s(m_Z) = 0.1172$, $\overline{m}_b(m_b) = 4.25 \text{ GeV}$.¹⁶ The cross sections for the separate squark and gluino production processes are shown in Tables 3.5 and 3.6. The stops are always heavier than $m_{\tilde{t}_1} > 750 \text{ GeV}$ for the considered benchmark points so direct stop-antistop production will be out of reach at the LHC with $\sqrt{s} = 7 \text{ TeV}$ (for some of the benchmark points, it might be possible to discover them in the gluino-decay products). Therefore we will give results for stop-antistop production separately in Section 3.2.3.

For comparison, NLO results obtained using PROSPINO are also shown in the tables. Our setup agrees with the one in [50, 95] and the NLO results agree at the expected one-percent level or better with the results obtained from an interpolated grid using NLL-fast [95]. For the NLO results the scale uncertainty is estimated by varying the factorisation scale in the interval $M/2 < \mu_f < M$, with the renormalization scale set equal to the factorization scale. The tables also include the relative PDF uncertainties of the NLO cross sections. Based on experience with

¹⁶ Note that in the mass-spectra quoted in [26] the masses are rounded to 5 GeV accuracy and only the light-flavour squark masses are averaged. Furthermore version 2.3 of SuSpect has been used for the CMSSM benchmark points and SOFTSUSY for the mGMSB benchmark points.

Point	m_0	$m_{1/2}$	$m_{\tilde{g}}$	$m_{\tilde{q}}$	$\sigma_{\text{SUSY}}(\text{pb}), \sqrt{s} = 7 \text{ TeV}$			
					NLO	NLL	Δ_{PDF}	K_{NLL}
40.1.2	345	550	1259	1144	$1.66^{+0.23}_{-0.25} \times 10^{-2}$	$1.91^{+0.18}_{-0.17} \times 10^{-2}$	$+4.3\%$ -3.1%	1.15
40.1.4	375	650	1468	1325	$3.48^{+0.52}_{-0.55} \times 10^{-3}$	$4.02^{+0.38}_{-0.36} \times 10^{-3}$	$+4.4\%$ -3.1%	1.15
40.2.2	600	500	1172	1153	$1.90^{+0.28}_{-0.29} \times 10^{-2}$	$2.22^{+0.23}_{-0.21} \times 10^{-2}$	$+5.0\%$ -3.8%	1.17
40.2.5	750	650	1492	1460	$1.34^{+0.22}_{-0.23} \times 10^{-3}$	$1.59^{+0.17}_{-0.16} \times 10^{-3}$	$+5.3\%$ -4.0%	1.19
40.3.1	1000	350	886	1182	$5.30^{+0.96}_{-0.94} \times 10^{-2}$	$6.63^{+0.77}_{-0.71} \times 10^{-2}$	$+8.4\%$ -7.8%	1.25
40.3.5	1200	450	1111	1446	$5.29^{+1.06}_{-1.00} \times 10^{-3}$	$6.86^{+0.83}_{-0.77} \times 10^{-3}$	$+10\%$ -9.3%	1.30

Table 3.3: CMSSM benchmark points and total inclusive SUSY production cross sections for $\tan \beta = 40$, $A_0 = -500 \text{ GeV}$. All masses are given in GeV.

Point	Λ_{SUSY}	$m_{\tilde{g}}$	$m_{\tilde{q}}$	$\sigma_{\text{SUSY}}(\text{pb}), \sqrt{s} = 7 \text{ TeV}$			
				NLO	NLL	Δ_{PDF}	K_{NLL}
mGMSB2.2.1	1.2×10^5	943	1142	$4.55^{+0.76}_{-0.77} \times 10^{-2}$	$5.57^{+0.63}_{-0.57} \times 10^{-2}$	$+7.3\%$ -6.4%	1.22
mGMSB2.2.4	1.5×10^5	1154	1408	$5.04^{+0.95}_{-0.92} \times 10^{-3}$	$6.38^{+0.76}_{-0.69} \times 10^{-3}$	$+8.9\%$ -7.9%	1.27

Table 3.4: mGMSB benchmark points and total inclusive SUSY production cross sections for $\tan \beta = 15$, $N_{\text{mess}} = 1$, $M_{\text{mess}} = 10^9 \text{ GeV}$. All masses and scales in GeV.

top-pair production [75], we expect that these agree with the relative PDF uncertainties of the NLL results at the relevant accuracy. Note that, due to correlations, the PDF uncertainty of the total SUSY production cross section is not equal to the sum of the uncertainties of the individual channels [50]. For the NLL results we quote the total uncertainty including scale and resummation uncertainties, as discussed in Section 2.3.3. In this case, the uncertainty of the total SUSY production cross section was obtained by neglecting correlations and adding the uncertainties of the individual production channels linearly, which gives a conservative upper bound.

As can be seen from Tables 3.2, 3.3 and 3.4, for the benchmark points with $m_{\tilde{q}} \lesssim m_{\tilde{g}}$ the NLL correction to the inclusive squark and gluino production cross section is in the 15 – 19%-range, which is consistent with Figure 3.6. For these points the theoretical uncertainty is reduced from $\pm 14\text{--}15\%$ at NLO to $\pm 9\text{--}10\%$ at NLL, in agreement with the behaviour seen in Figure 3.9. The PDF uncertainty is also small for the benchmark points with $m_{\tilde{q}} < m_{\tilde{g}}$, since only the relatively precisely known quark PDFs are relevant at leading order for the dominant squark-squark production channel. For scenarios where the gluinos are lighter than the squarks, the NLL corrections grow to 20-30% and the theoretical uncertainty at NLL is at the 11-12% level. These

Point	$\tilde{s}\tilde{s}'$	$\sigma(pp \rightarrow \tilde{s}\tilde{s}')(\text{pb}), \sqrt{s} = 7 \text{ TeV}$			
		NLO	NLL	Δ_{PDF}	K_{NLL}
10.1.3	$\tilde{q}\tilde{q}$	$7.26^{+1.34}_{-1.25} \times 10^{-4}$	$8.80^{+0.84}_{-0.73} \times 10^{-4}$	$+11\%$ -11%	1.21
	$\tilde{q}\tilde{q}$	$6.22^{+0.81}_{-0.88} \times 10^{-3}$	$6.72^{+0.55}_{-0.48} \times 10^{-3}$	$+3.8\%$ -2.8%	1.08
	$\tilde{q}\tilde{g}$	$2.06^{+0.30}_{-0.35} \times 10^{-3}$	$2.69^{+0.32}_{-0.32} \times 10^{-3}$	$+11\%$ -10%	1.31
	$\tilde{g}\tilde{g}$	$0.77^{+0.17}_{-0.17} \times 10^{-4}$	$1.26^{+0.24}_{-0.22} \times 10^{-4}$	$+25\%$ -23%	1.64
10.1.4	$\tilde{q}\tilde{q}$	$2.91^{+0.56}_{-0.52} \times 10^{-4}$	$3.59^{+0.35}_{-0.31} \times 10^{-4}$	$+13\%$ -12%	1.23
	$\tilde{q}\tilde{q}$	$3.02^{+0.41}_{-0.44} \times 10^{-3}$	$3.28^{+0.27}_{-0.24} \times 10^{-3}$	$+3.9\%$ -2.8%	1.09
	$\tilde{q}\tilde{g}$	$0.83^{+0.13}_{-0.15} \times 10^{-3}$	$1.11^{+0.13}_{-0.14} \times 10^{-3}$	$+12\%$ -11%	1.33
	$\tilde{g}\tilde{g}$	$2.54^{+0.59}_{-0.57} \times 10^{-5}$	$4.36^{+0.87}_{-0.81} \times 10^{-5}$	$+28\%$ -25%	1.72
10.2.2	$\tilde{q}\tilde{q}$	$1.60^{+0.29}_{-0.27} \times 10^{-3}$	$1.92^{+0.18}_{-0.16} \times 10^{-3}$	$+9.9\%$ -9.6%	1.20
	$\tilde{q}\tilde{q}$	$1.17^{+0.15}_{-0.16} \times 10^{-2}$	$1.25^{+0.10}_{-0.09} \times 10^{-2}$	$+3.7\%$ -2.7%	1.07
	$\tilde{q}\tilde{g}$	$4.81^{+0.67}_{-0.79} \times 10^{-3}$	$6.18^{+0.71}_{-0.72} \times 10^{-3}$	$+9.7\%$ -9.2%	1.28
	$\tilde{g}\tilde{g}$	$2.27^{+0.48}_{-0.48} \times 10^{-4}$	$3.56^{+0.65}_{-0.61} \times 10^{-4}$	$+23\%$ -22%	1.57
10.2.5	$\tilde{q}\tilde{q}$	$0.96^{+0.19}_{-0.18} \times 10^{-4}$	$1.21^{+0.12}_{-0.10} \times 10^{-4}$	$+14\%$ -14%	1.26
	$\tilde{q}\tilde{q}$	$1.26^{+0.18}_{-0.19} \times 10^{-3}$	$1.38^{+0.12}_{-0.11} \times 10^{-3}$	$+4.0\%$ -3.0%	1.10
	$\tilde{q}\tilde{g}$	$3.00^{+0.51}_{-0.55} \times 10^{-4}$	$4.11^{+0.51}_{-0.53} \times 10^{-4}$	$+13\%$ -12%	1.37
	$\tilde{g}\tilde{g}$	$0.80^{+0.20}_{-0.19} \times 10^{-5}$	$1.44^{+0.30}_{-0.28} \times 10^{-5}$	$+31\%$ -27%	1.81
10.3.2	$\tilde{q}\tilde{q}$	$1.88^{+0.34}_{-0.32} \times 10^{-3}$	$2.24^{+0.21}_{-0.18} \times 10^{-3}$	$+9.8\%$ -9.2%	1.19
	$\tilde{q}\tilde{q}$	$1.33^{+0.17}_{-0.19} \times 10^{-2}$	$1.43^{+0.12}_{-0.10} \times 10^{-2}$	$+3.7\%$ -2.7%	1.07
	$\tilde{q}\tilde{g}$	$6.49^{+0.89}_{-1.05} \times 10^{-3}$	$8.30^{+0.94}_{-0.95} \times 10^{-3}$	$+9.4\%$ -8.9%	1.28
	$\tilde{g}\tilde{g}$	$3.70^{+0.77}_{-0.77} \times 10^{-4}$	$5.69^{+1.03}_{-0.96} \times 10^{-4}$	$+22\%$ -21%	1.54
10.3.3	$\tilde{q}\tilde{q}$	$4.40^{+0.85}_{-0.78} \times 10^{-4}$	$5.37^{+0.52}_{-0.45} \times 10^{-4}$	$+12\%$ -11%	1.22
	$\tilde{q}\tilde{q}$	$4.23^{+0.58}_{-0.62} \times 10^{-3}$	$4.58^{+0.39}_{-0.34} \times 10^{-3}$	$+3.8\%$ -2.9%	1.08
	$\tilde{q}\tilde{g}$	$1.58^{+0.24}_{-0.27} \times 10^{-3}$	$2.08^{+0.24}_{-0.25} \times 10^{-3}$	$+11\%$ -10%	1.32
	$\tilde{g}\tilde{g}$	$0.69^{+0.16}_{-0.15} \times 10^{-4}$	$1.13^{+0.22}_{-0.21} \times 10^{-4}$	$+26\%$ -23%	1.64
10.4.2	$\tilde{q}\tilde{q}$	$1.25^{+0.22}_{-0.21} \times 10^{-3}$	$1.48^{+0.14}_{-0.12} \times 10^{-3}$	$+10\%$ -9.9%	1.19
	$\tilde{q}\tilde{q}$	$1.09^{+0.16}_{-0.16} \times 10^{-2}$	$1.16^{+0.12}_{-0.10} \times 10^{-2}$	$+3.7\%$ -2.7%	1.07
	$\tilde{q}\tilde{g}$	$1.82^{+0.29}_{-0.31} \times 10^{-2}$	$2.30^{+0.24}_{-0.23} \times 10^{-2}$	$+8.3\%$ -8.0%	1.27
	$\tilde{g}\tilde{g}$	$4.55^{+0.99}_{-0.94} \times 10^{-3}$	$6.37^{+0.95}_{-0.88} \times 10^{-3}$	$+17\%$ -17%	1.40
10.4.4	$\tilde{q}\tilde{q}$	$0.85^{+0.17}_{-0.16} \times 10^{-4}$	$1.07^{+0.10}_{-0.09} \times 10^{-4}$	$+15\%$ -15%	1.25
	$\tilde{q}\tilde{q}$	$1.29^{+0.22}_{-0.21} \times 10^{-3}$	$1.40^{+0.16}_{-0.13} \times 10^{-3}$	$+4.0\%$ -3.0%	1.08
	$\tilde{q}\tilde{g}$	$1.86^{+0.35}_{-0.34} \times 10^{-3}$	$2.47^{+0.28}_{-0.27} \times 10^{-3}$	$+11\%$ -10%	1.33
	$\tilde{g}\tilde{g}$	$4.12^{+1.02}_{-0.91} \times 10^{-4}$	$6.19^{+0.99}_{-0.90} \times 10^{-4}$	$+22\%$ -20%	1.50

Table 3.5: NLL results for CMSSM benchmark points for $\tan\beta = 10$, $A_0 = 0$.

Point	$\tilde{s}\tilde{s}'$	$\sigma(pp \rightarrow \tilde{s}\tilde{s}')(\text{pb}), \sqrt{s} = 7 \text{ TeV}$			
		NLO	NLL	Δ_{PDF}	K_{NLL}
40.1.2	$\tilde{q}\tilde{q}$	$1.41^{+0.26}_{-0.24} \times 10^{-3}$	$1.68^{+0.16}_{-0.14} \times 10^{-3}$	$+10\%$ -9.7%	1.20
	$\tilde{q}\tilde{q}$	$1.05^{+0.13}_{-0.15} \times 10^{-2}$	$1.13^{+0.09}_{-0.08} \times 10^{-2}$	$+3.7\%$ -2.7%	1.07
	$\tilde{q}\tilde{g}$	$4.46^{+0.63}_{-0.73} \times 10^{-3}$	$5.74^{+0.66}_{-0.67} \times 10^{-3}$	$+9.8\%$ -9.2%	1.29
	$\tilde{g}\tilde{g}$	$2.18^{+0.46}_{-0.46} \times 10^{-4}$	$3.42^{+0.63}_{-0.58} \times 10^{-4}$	$+23\%$ -22%	1.57
40.1.4	$\tilde{q}\tilde{q}$	$2.28^{+0.45}_{-0.41} \times 10^{-4}$	$2.83^{+0.27}_{-0.24} \times 10^{-4}$	$+13\%$ -13%	1.24
	$\tilde{q}\tilde{q}$	$2.51^{+0.35}_{-0.37} \times 10^{-3}$	$2.73^{+0.23}_{-0.21} \times 10^{-3}$	$+3.9\%$ -2.9%	1.09
	$\tilde{q}\tilde{g}$	$7.25^{+1.16}_{-1.28} \times 10^{-4}$	$9.71^{+1.17}_{-1.21} \times 10^{-4}$	$+12\%$ -11%	1.34
	$\tilde{g}\tilde{g}$	$2.37^{+0.57}_{-0.54} \times 10^{-5}$	$4.08^{+0.81}_{-0.75} \times 10^{-5}$	$+28\%$ -25%	1.72
40.2.2	$\tilde{q}\tilde{q}$	$1.34^{+0.26}_{-0.23} \times 10^{-3}$	$1.60^{+0.16}_{-0.13} \times 10^{-3}$	$+10\%$ -9.7%	1.19
	$\tilde{q}\tilde{q}$	$1.04^{+0.14}_{-0.15} \times 10^{-2}$	$1.11^{+0.10}_{-0.08} \times 10^{-2}$	$+3.7\%$ -2.7%	1.07
	$\tilde{q}\tilde{g}$	$6.74^{+0.97}_{-1.11} \times 10^{-3}$	$8.65^{+0.98}_{-0.98} \times 10^{-3}$	$+9.4\%$ -8.9%	1.28
	$\tilde{g}\tilde{g}$	$5.48^{+1.15}_{-1.14} \times 10^{-4}$	$8.28^{+1.46}_{-1.35} \times 10^{-4}$	$+22\%$ -20%	1.51
40.2.5	$\tilde{q}\tilde{q}$	$6.14^{+1.34}_{-1.17} \times 10^{-5}$	$7.75^{+0.79}_{-0.67} \times 10^{-5}$	$+15\%$ -15%	1.26
	$\tilde{q}\tilde{q}$	$9.01^{+1.41}_{-1.43} \times 10^{-4}$	$9.87^{+0.94}_{-0.82} \times 10^{-4}$	$+4.0\%$ -3.1%	1.10
	$\tilde{q}\tilde{g}$	$3.64^{+0.64}_{-0.67} \times 10^{-4}$	$4.98^{+0.61}_{-0.62} \times 10^{-4}$	$+13\%$ -12%	1.37
	$\tilde{g}\tilde{g}$	$1.82^{+0.46}_{-0.42} \times 10^{-5}$	$3.14^{+0.62}_{-0.57} \times 10^{-5}$	$+29\%$ -26%	1.72
40.3.1	$\tilde{q}\tilde{q}$	$1.00^{+0.18}_{-0.17} \times 10^{-3}$	$1.19^{+0.11}_{-0.10} \times 10^{-3}$	$+11\%$ -10%	1.19
	$\tilde{q}\tilde{q}$	$0.95^{+0.15}_{-0.15} \times 10^{-2}$	$1.01^{+0.12}_{-0.09} \times 10^{-2}$	$+3.7\%$ -2.7%	1.06
	$\tilde{q}\tilde{g}$	$2.83^{+0.48}_{-0.49} \times 10^{-2}$	$3.57^{+0.37}_{-0.35} \times 10^{-2}$	$+7.9\%$ -7.5%	1.26
	$\tilde{g}\tilde{g}$	$1.42^{+0.31}_{-0.29} \times 10^{-2}$	$1.93^{+0.27}_{-0.25} \times 10^{-2}$	$+15\%$ -15%	1.36
40.3.5	$\tilde{q}\tilde{q}$	$7.05^{+1.45}_{-1.30} \times 10^{-5}$	$8.81^{+0.87}_{-0.75} \times 10^{-5}$	$+15\%$ -15%	1.25
	$\tilde{q}\tilde{q}$	$1.14^{+0.20}_{-0.19} \times 10^{-3}$	$1.23^{+0.15}_{-0.12} \times 10^{-3}$	$+4.2\%$ -3.0%	1.08
	$\tilde{q}\tilde{g}$	$2.87^{+0.55}_{-0.53} \times 10^{-3}$	$3.78^{+0.41}_{-0.40} \times 10^{-3}$	$+10\%$ -9.8%	1.32
	$\tilde{g}\tilde{g}$	$1.21^{+0.29}_{-0.26} \times 10^{-3}$	$1.75^{+0.26}_{-0.24} \times 10^{-3}$	$+19\%$ -18%	1.45
mGMSB2.2.1	$\tilde{q}\tilde{q}$	$1.48^{+0.26}_{-0.25} \times 10^{-3}$	$1.76^{+0.17}_{-0.14} \times 10^{-3}$	$+10\%$ -9.6%	1.19
	$\tilde{q}\tilde{q}$	$1.26^{+0.19}_{-0.19} \times 10^{-2}$	$1.34^{+0.15}_{-0.12} \times 10^{-2}$	$+3.8\%$ -2.6%	1.06
	$\tilde{q}\tilde{g}$	$2.43^{+0.39}_{-0.41} \times 10^{-2}$	$3.07^{+0.32}_{-0.31} \times 10^{-2}$	$+8.1\%$ -7.6%	1.26
	$\tilde{g}\tilde{g}$	$7.17^{+1.54}_{-1.46} \times 10^{-3}$	$9.92^{+1.45}_{-1.35} \times 10^{-3}$	$+17\%$ -16%	1.38
mGMSB2.2.4	$\tilde{q}\tilde{q}$	$1.04^{+0.21}_{-0.19} \times 10^{-4}$	$1.29^{+0.13}_{-0.11} \times 10^{-4}$	$+14\%$ -14%	1.24
	$\tilde{q}\tilde{q}$	$1.52^{+0.26}_{-0.25} \times 10^{-3}$	$1.65^{+0.19}_{-0.15} \times 10^{-3}$	$+4.1\%$ -2.9%	1.08
	$\tilde{q}\tilde{g}$	$2.67^{+0.49}_{-0.49} \times 10^{-3}$	$3.52^{+0.39}_{-0.38} \times 10^{-3}$	$+10\%$ -9.9%	1.32
	$\tilde{g}\tilde{g}$	$0.74^{+0.18}_{-0.16} \times 10^{-3}$	$1.09^{+0.17}_{-0.15} \times 10^{-3}$	$+20\%$ -19%	1.47

Table 3.6: NLL results for CMSSM benchmark points for $\tan \beta = 40$, $A_0 = -500$ and mGMSB benchmark points.

	$\sigma(pp \rightarrow \tilde{t}_1 \bar{\tilde{t}}_1)(\text{pb}), \sqrt{s} = 7 \text{ TeV}$			
$m_{\tilde{t}_1}$	NLO	NLL	Δ_{PDF}	K_{NLL}
100	$4.18^{+0.63}_{-0.59} \times 10^2$	$4.77^{+0.87}_{-0.66} \times 10^2$	$+2.0\%$ -2.4%	1.14
200	$1.28^{+0.17}_{-0.18} \times 10^1$	$1.47^{+0.25}_{-0.19} \times 10^1$	$+3.1\%$ -4.1%	1.15
300	$1.28^{+0.17}_{-0.18}$	$1.49^{+0.24}_{-0.19}$	$+4.4\%$ -5.3%	1.17
400	$2.12^{+0.28}_{-0.31} \times 10^{-1}$	$2.50^{+0.38}_{-0.30} \times 10^{-1}$	$+5.7\%$ -6.4%	1.18

Table 3.7: Stop-antistop production cross sections at $\sqrt{s} = 7 \text{ TeV}$. All masses are given in GeV.

features can be understood by considering the relative contributions of the different production processes in Figure 3.7 and Tables 3.5 and 3.6. For $m_{\tilde{q}} \lesssim m_{\tilde{g}}$, squark-pair production with moderate NLL corrections in the 6-10% range is the dominant production channel, with a non-negligible contribution from squark-gluino production with larger NLL corrections of the order of 30%. For $m_{\tilde{g}} < m_{\tilde{q}}$ the roles are reversed, resulting in larger NLL corrections to the total rate and a slightly larger theoretical uncertainty, as can be seen from the uncertainties of the different production channels in Figure 3.9. Due to the large uncertainties in the current gluon PDF sets for large x , the PDF error for these points rises to the 10%-level. In gluino-pair production, especially for large gluino masses, the PDF error can even become $\sim 30\%$. At a mass ratio $\frac{m_{\tilde{g}}}{m_{\tilde{q}}} \lesssim 0.75$, as realized for the 40.3.1 and 40.3.5 points, gluino-pair production overtakes squark-pair production as the second-most important channel. For these points, the gluinos are relatively light, and the NLL corrections to gluino-pair production are at most 45%. For benchmark points with heavier gluinos, the NLL corrections can grow to 70-80%, but the gluino-pair production rate is negligible for these points. For the moderate mass ratios considered here, squark-antisquark production is always much suppressed, but would have the second-largest cross section for $\frac{m_{\tilde{g}}}{m_{\tilde{q}}} \rightarrow 2$.

3.2.3 Results for stop-antistop production

In this Section we present results for resummation for stop-antistop production $PP \rightarrow \tilde{t}_i \bar{\tilde{t}}_i$. At LO the cross section depends only on the stop mass, while at NLO it also presents a (much smaller) dependence on the mixing angle $\theta_{\tilde{t}}$ and the squark and gluino masses. For definiteness, we follow [50] and fix all the parameters, except for the stop mass, to the values corresponding to the CMSSM benchmark point 40.2.4 with SUSY breaking parameters $m_0 = 700 \text{ GeV}$, $m_{1/2} = 600 \text{ GeV}$, $A_0 = -500 \text{ GeV}$ and $\tan \beta = 40$ [26]. Using SUSY-HIT [96] we obtain the relevant input parameters $m_{\tilde{g}} = 1386 \text{ GeV}$, $m_{\tilde{q}} = 1358$ and $\cos \theta_{\tilde{t}} = 0.39$. The numerical dependence on these parameters has been studied in [57] and found to be negligible. As discussed in Section 3.1.3 we will limit ourselves to results for the lighter mass-eigenstate $\tilde{t}_1$. Numerical

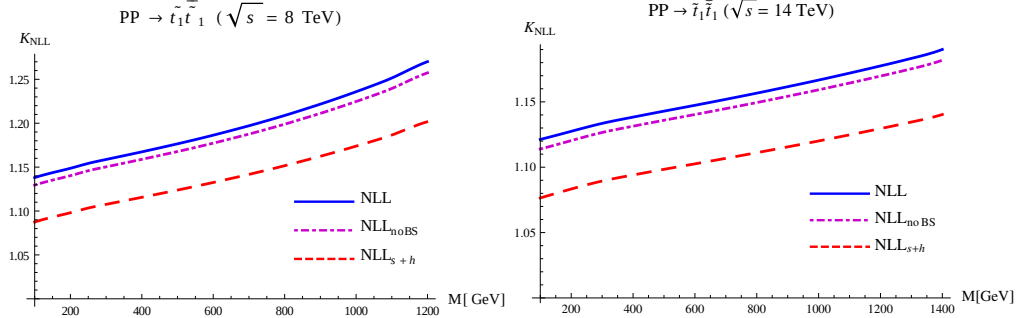


Figure 3.11: K_{NLL} for stop-antistop production at the LHC at $\sqrt{s} = 8$ TeV (left) and $\sqrt{s} = 14$ TeV (right) for different NLL approximations: NLL (solid blue), $\text{NLL}_{\text{no BS}}$ (dot-dashed purple) and NLL_{s+h} (dashed red).

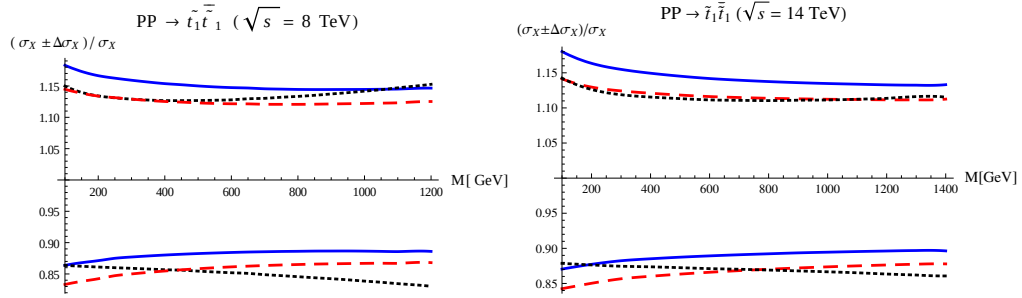


Figure 3.12: Total theoretical uncertainty of the NLO approximation (dotted black), full NLL resummed result (solid blue) and NLL_{s+h} (dashed red) for $\tilde{t}_1 \tilde{t}_1^*$ at the LHC with $\sqrt{s} = 8$ TeV (left) and $\sqrt{s} = 14$ TeV (right). All cross sections are normalized to one at the central value of the scales.

results for the lightest stop mass eigenstates for masses in the 100-400 GeV range are presented in table 3.7 at a centre-of-mass energy $\sqrt{s} = 7$ TeV.

The NLL K -factor is plotted as a function of the mass in Figure 3.11. For the stop-mass range considered here the resummation effects are in the 15-27% range at the LHC with $\sqrt{s} = 8$ TeV and at the 12-20%-level at a centre-of-mass energy of 14 TeV. The NLL-corrections are therefore moderate for the mass-ranges accessible at the LHC, but larger than for light-flavour squark-antisquark production for the same masses (c.f. figure 3.4). Contrary to the latter, for stops the most substantial contribution is given by pure soft resummation, with Coulomb effects, including bound-state corrections, in the 5% range of the NLO result. As the partonic cross sections for the gluon-fusion channel are identical for the light-flavour and top squarks, the different behaviour can be attributed to the quark-antiquark induced channel. For the light-flavour squarks it is

dominated by S-wave colour-singlet production with a large, attractive Coulomb potential. For stop production the colour-singlet channel is absent, and only the P-wave colour-octet channel with a smaller repulsive Coulomb potential contributes. The increased relative size of the soft corrections compared to the light-flavour squarks follows from the P-wave suppression of the quark-antiquark channel and the resulting dominance of the gluon-fusion channel with larger soft corrections due to the colour factors $C_A = N_C$.

The total theoretical uncertainty of the fixed-order NLO result and the resummed cross section, with and without Coulomb resummation, is compared in Figure 3.12. The width of the uncertainty band for NLL and NLL_{s+h} is similar, consistent with the observed small effect of Coulomb resummation. One can also notice that the resummed results shows almost no uncertainty reduction compared to the NLO result, except for the high-mass range.

3.3 Comparison with Mellin-space results

Results for NLL resummation of soft logarithms for squark and gluino production have been presented earlier in [50, 54–57]. These works adopt the so-called Mellin-space formalism, in which the threshold logarithms are exponentiated in Mellin-moment space, where singular terms appear as logarithms of the Mellin-moment N , and the resummed cross section is then numerically inverted back to momentum-space. In this Section we compare these earlier predictions to the momentum-space formalism adopted here at a centre-of-mass energy $\sqrt{s} = 7$ TeV, using the numerical code `NLL-fast` [95] to compute the Mellin-space resummed cross sections. Since `NLL-fast` provides results for soft-resummation only (i.e. no Coulomb effects beyond $\mathcal{O}(\alpha_s)$ are included), for the comparison we introduce two additional NLL implementations:

- **NLL_s** : this implementation includes NLL resummation of soft logarithms, but no Coulomb or hard effects beyond $\mathcal{O}(\alpha_s)$. This is achieved using Eqs. (3.11) and (3.15) and setting $\mu_h = \mu_f$. For the soft scale we adopt the running scale given in Eqs. (2.49), (2.50).
- **$\text{NLL}_{s,\text{fixed}}$** : as above, but with the running scale replaced by the fixed soft scale μ_s determined from Eq. (2.48).

The K -factor for NLL_s (solid blue), $\text{NLL}_{s,\text{fixed}}$ (dashed red) and the Mellin-space result (green dots) from `NLL-fast` are shown in Figure 3.13. For clarity, we do not show the resummation uncertainties of the NLL_s and $\text{NLL}_{s,\text{fixed}}$ results that are significant, especially for smaller masses, as can be anticipated from the full NLL-results including Coulomb resummation in Figure 3.8. Even though the difference between NLL_s and $\text{NLL}_{\text{Mellin}}$ can be sizable, the envelope of the three curves in the plots in Figure 3.13 is well within the theoretical uncertainties of the predictions, and is thus correctly accounted for by our estimate of the intrinsic ambiguities of NLL resummation.

The comparison of our default running-scale implementation NLL_s with $\text{NLL}_{\text{Mellin}}$ shows different features for the various production channels. For squark-antisquark and gluino-pair

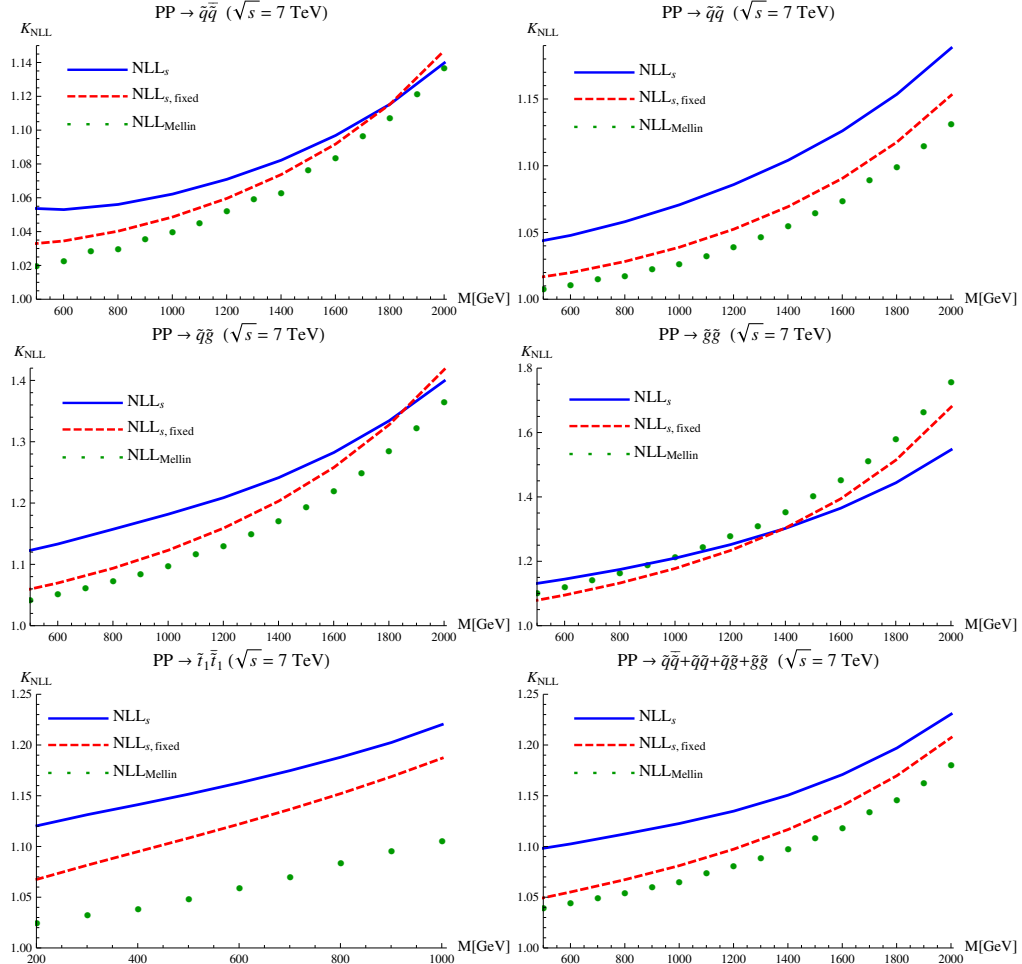


Figure 3.13: Comparison of soft resummation in the momentum-space formalism adopted in this work and in the conventional Mellin-space approach for squark-antisquark (top-left), squark-squark (top-right), squark-gluino (centre-left), gluino-gluino (centre-right), stop-antistop (bottom-left) production and the inclusive gluino and light-flavour squark cross section (bottom-right) at LHC with $\sqrt{s} = 7$ TeV. The plots show the K -factor for our default running-scale implementation (NLL_s , solid blue), for the fixed-scale implementation ($\text{NLL}_{s, \text{fixed}}$, dashed red) and for the Mellin-space result ($\text{NLL}_{\text{Mellin}}$, green dots) obtained with NLL-fast . See the text for more details.

production the agreement is overall reasonable, although the behaviour as a function of the mass is different in both cases, leading to a better agreement at large masses in the former case and at smaller masses in the latter. For squark-gluino production, the agreement also improves at larger masses. In contrast, for the squark-squark production channel, there is a constant shift, and the differences are sizable over the whole mass range. Since the NLL corrections are small for this production channel, the discrepancies in the cross-section predictions are however less important. For all processes, the fixed-scale momentum-space results $\text{NLL}_{s,\text{fixed}}$ are closer to the Mellin-space results, with similar magnitude and mass-dependence of the corrections. Since NLL_s and $\text{NLL}_{\text{Mellin}}$ both resum threshold logarithms ($\ln \beta$ and $\ln N$ respectively) appearing in the partonic cross sections, while $\text{NLL}_{s,\text{fixed}}$ resums logarithms in the hadronic cross section, this behaviour is somewhat counter-intuitive and deserves further studies in the future.¹⁷ The spread of the three predictions for stop production is comparable to the one observed for the other SUSY processes in the same mass range. In this case, however, the two momentum-space predictions with a fixed and running soft scale show a better agreement with each other.

It is interesting to note that the NNLL results for squark-antisquark production obtained in the N -space approach in [49] are very similar to our best prediction for this channel in Figures 3.4 and 3.5. Part of this agreement can be attributed to the fact that Ref. [49] includes the interference of the first Coulomb correction with higher-order soft corrections, which give a dominant contribution to our full NLL predictions (i.e. to the difference between the red and blue curves in Figures 3.4 and 3.5), and to the difference between the NLL and NNLL results in [49]. However, our prediction includes higher-order Coulomb corrections and bound-state effects not included in the results of [49], while their results include one-loop hard corrections and NNLL soft corrections not included in ours. Therefore the good agreement is to some extent fortunate, and cannot necessarily be expected for the other processes where the Coulomb corrections are of less relative importance (in particular for squark-squark production), and where an NNLL analysis remains to be performed. Furthermore, a combined NNLL soft/Coulomb resummation as performed for top-quark pair production in [75] would include higher-order Coulomb effects included neither in our present predictions nor in [49].

3.4 Summary

We have performed a combined resummation of soft-gluon and Coulomb effects for all squark- and gluino-pair production channels at the LHC, including bound-state effects, based on the formalism reviewed in Section 2.3 and derived in [5, 58]. We have also applied the factorization and resummation formalism to the case of stop-antistop production with a quark-antiquark initial

¹⁷An analytic comparison of momentum-space resummation with fixed soft scale to N -space resummation has appeared recently for Drell-Yan production [98], but no such investigation for the running scale with a lower cutoff has been performed yet.

state, that proceeds through a P-wave. The corrections from higher-order Coulomb and bound-state effects and their interference with soft corrections are not included in previous predictions and can be sizeable, in particular for gluino-pair production, where they are as large as the soft-gluon corrections alone, and for squark-antisquark production where the effect is even bigger. For benchmark scenarios with moderate squark-gluino mass splitting the effect on the total inclusive squark and gluino production cross section is less pronounced but still relevant, with the total NLL corrections of the order of 15 – 30% of the NLO cross section. Therefore these predictions should be taken into account in the analysis of the LHC results. To facilitate the application of our results we provided numerical predictions for some of the benchmark points defined in [26], and include grid files with our results for squark and gluino masses in the 200-2000 GeV (200-2500 GeV) range at the 7 TeV (8 TeV) LHC that can be found online with the arXiv submission of the paper [47] or at [93].

Our results for soft-gluon resummation alone, obtained in the momentum-space resummation approach [11, 59, 60] and with the scale-setting procedure introduced in [75], agree within our estimate of resummation ambiguities with results obtained in the Mellin-space formalism [56, 57], although a more detailed study of the relation between the approaches and the different scale choices within the momentum space framework would be desirable.

In our analysis, the squarks and gluinos have been treated as stable, but contributions to the cross section from below the nominal production threshold have been included through bound-state corrections. This is expected to take the effect of small, but finite, widths to some extent into account. A more refined analysis is possible in our framework using a complex energy in the argument of the potential function in the factorization formula (2.26), as done in recent studies of the invariant mass spectrum of gluino-pair and squark-gluino production [82–84]. This finite-width effect on the combined NLL soft and Coulomb results will be studied in Chapter 4. A start to the extension to NNLL accuracy following Ref. [75] is given in Chapter 5.

Unstable SUSY-QCD: NLL resummation

In the calculations of threshold corrections the produced squarks and gluinos are usually treated as stable, while in most realistic SUSY models they are highly unstable and decay promptly.¹⁸ In a more realistic treatment including finite decay widths Γ , the threshold singularities are smoothed [4] since the decay width sets the smallest perturbative scale in the process. This raises the question of whether the resummation in Chapter 3, based on the counting (2.1) is appropriate in the presence of the new scale Γ and to which extent the large higher-order threshold corrections found for stable squarks and gluinos provide a reasonable approximation to the more realistic unstable case. One effect of the finite decay width is the emergence of a non-vanishing partonic cross section below the nominal production threshold $\hat{s} = 4M^2$, as discussed for stop-pair production in [65], and more recently for gluino pair [82, 83] and squark-gluino production [84]. In our previous combined resummation of threshold logarithms and Coulomb corrections for squark and gluino pair-production in Chapter 3 [5, 47] the contribution from the region below the nominal threshold to the total production cross section has been taken into account in the zero-width limit, where the smooth invariant-mass distribution turns into a series of would-be bound-state poles.

As we will show in this chapter, for moderate decay widths $\Gamma/\bar{m} \lesssim 5\%$, the finite-width effects on higher-order QCD corrections are small and within the error estimate of the NLL cross sections with soft and Coulomb resummation and bound-state corrections. This covers the relevant production channels in the MSSM, since larger decay widths only arise for heavier gluinos with a large gluino-squark mass splitting, in which case gluino production is suppressed compared to squark production. Therefore the treatment of NLL soft and Coulomb corrections in Chapter 3 [47] captures the dominant higher-order QCD effects also when the instability of squarks and gluinos is taken into account. Furthermore, the dominant effect of the finite decay widths arises at NLO, so the impact on higher-order corrections is small.

¹⁸We do not consider the case of stops and gluinos that are stable on collider timescales. For recent work on higher-order corrections in this case see [78, 91].

It should be emphasized that the aim of our study is the assessment of finite-width effects on QCD corrections beyond LO rather than a complete treatment at leading and next-to-leading order. Studies of other aspects of finite squark and gluino decay widths include that of non-resonant tree diagrams [99]; the accuracy of the narrow-width approximation for small mass-splittings of the decaying particle and its decay products [100, 101], the consistent treatment of off-shell effects in Monte-Carlo programs [102] and NLO corrections to production and decay for the squark-squark production process [103].

This chapter is organized as follows. In Section 4.1 we specify the considered production and decay processes and review the effective-theory treatment of unstable particles [104–107] and soft and Coulomb resummation near threshold [5]. In Section 4.2 we estimate the impact of the screening of threshold corrections by the finite decay width as well as the magnitude of non-resonant corrections. In Section 4.3 we study the invariant-mass spectrum for squark and gluino production processes at the LHC and the total production cross sections. We find that for moderate decay widths the finite-width corrections to the total cross section are within the residual uncertainty of the NLL calculation of Chapter 3 [47]. We summarize our findings in Section 4.4.

4.1 Theoretical framework

4.1.1 Production and decay of squarks and gluinos in SQCD

In this Section we consider the pair-production processes of squarks $\tilde{q}$ and gluinos $\tilde{g}$ at hadron colliders in supersymmetric QCD (SQCD) that proceed through the partonic production channels

$$pp' \rightarrow \tilde{s}_1 \tilde{s}_2 X, \quad (4.1)$$

where $\tilde{s}_i \in \{\tilde{q}, \bar{\tilde{q}}, \tilde{g}\}$ denote the two produced sparticles and $p, p' \in \{q, \bar{q}, g\}$ the initial-state partons. The average mass of the sparticle pair is given by¹⁹

$$\bar{m} = \frac{m_{\tilde{s}_1} + m_{\tilde{s}_2}}{2}. \quad (4.2)$$

In general, the produced squarks and gluinos can decay via long cascade decay chains to a final state of standard model particles and the lightest supersymmetric particle (assuming the latter is stable on detector time scales). For this initial study of finite-width effects we limit ourselves to squark and gluino decay in pure SQCD, i.e. we neglect the decay width of the lightest coloured

¹⁹Contrary to the previous and following chapters, in this chapter we will reserve M for the more massive of the two produced sparticles, i.e. $M := \text{Max}[m_{\tilde{s}_1}, m_{\tilde{s}_2}]$, which is assumed to decay to the other SQCD sparticle with mass m (cf. Section 4.2.1).

sparticle and consider the decay modes

$$\begin{aligned}\tilde{q} &\rightarrow q\tilde{g}, \quad \bar{\tilde{q}} \rightarrow \bar{q}\tilde{g}, \quad m_{\tilde{q}} > m_{\tilde{g}}, \\ \tilde{g} &\rightarrow q\bar{\tilde{q}}, \quad \tilde{g} \rightarrow \bar{q}\tilde{q}, \quad m_{\tilde{q}} < m_{\tilde{g}}.\end{aligned}\tag{4.3}$$

We treat the masses of the different squark flavours as degenerate and equal to $m_{\tilde{q}}$, while the gluino mass equals $m_{\tilde{g}} \neq m_{\tilde{q}}$. The full one-loop SUSY-QCD partial widths of the above decay processes have been calculated in [108]. The NLO corrections to the LO partial widths were found to be of the order of 30 – 50% for squarks and $\pm 10\%$ for gluinos and will not be given here. In order to estimate the finite-width effects we will employ the LO tree level SUSY-QCD widths,

$$\begin{aligned}\Gamma(\tilde{q} \rightarrow q\tilde{g}) &= \frac{\alpha_s C_F m_{\tilde{q}}}{2} \left(1 - \left(\frac{m_{\tilde{g}}}{m_{\tilde{q}}}\right)^2\right)^2, \quad m_{\tilde{q}} > m_{\tilde{g}}, \\ \Gamma(\tilde{g} \rightarrow q\bar{\tilde{q}}, \bar{q}\tilde{q}) &= \frac{\alpha_s n_f m_{\tilde{g}}}{2} \left(1 - \left(\frac{m_{\tilde{q}}}{m_{\tilde{g}}}\right)^2\right)^2, \quad m_{\tilde{q}} < m_{\tilde{g}}.\end{aligned}\tag{4.4}$$

Since the numerical values are only used for illustration of the finite-width effects, we only consider gluino decay into light-flavour and bottom squarks that are treated as mass-degenerate, i.e. we set $n_f = 5$. Therefore we neglect the decay into stops and the related additional dependence on the parameters of the stop sector unless stated otherwise. The effect of gluino decay to stops on our results is discussed briefly in Section 4.3.2. In the full MSSM, the lighter coloured sparticles are, of course, unstable with respect to electroweak two- or three-body decays $\tilde{q} \rightarrow \chi q$ and $\tilde{g} \rightarrow q\bar{q}\chi$ for charginos or neutralinos χ , which have been calculated in leading order in [109, 110] and for which state-of-the-art predictions are implemented in [96, 111–113]. These effects would be straightforward to include in our framework but would introduce a dependence on the electroweak parameters of the MSSM. Therefore we will use the tree-level SQCD decay widths (4.4) for the purpose of estimating the size of finite-width effects in higher-order QCD corrections. Since the electroweak decay widths are typically of the order $\Gamma/\bar{m} < 1\%$, this should not affect our qualitative conclusions. We therefore consider production and decay processes of squarks and gluinos at hadron colliders of the type

$$pp' \rightarrow \tilde{s}_1 \tilde{s}_2 \rightarrow (\tilde{s}'_1 p_1) (\tilde{s}'_2 p_2),\tag{4.5}$$

with massless partons $p_{1/2}$. Such four-body final states arise in SQCD for gluino pair production for the case $m_{\tilde{g}} > m_{\tilde{q}}$ or squark-(anti-)squark production for $m_{\tilde{q}} > m_{\tilde{g}}$. The treatment of a three-body final state that arises for squark-gluino production in our approximation should be obvious. In detail, the partonic production and decay processes of squarks and gluinos for the

two mass hierarchies are as follows:

$$\begin{aligned}
 m_{\tilde{g}} < m_{\tilde{q}} : \quad & gg, q_i \bar{q}_i \rightarrow \tilde{g}\tilde{g}, \\
 & q_i g \rightarrow \tilde{q}\tilde{q} \rightarrow q\tilde{g}\tilde{g}, \\
 & gg, q_i \bar{q}_j \rightarrow \tilde{q}\tilde{q} \rightarrow q\tilde{q}\tilde{g}\tilde{g}, \\
 & q_i q_j \rightarrow \tilde{q}\tilde{q} \rightarrow q\tilde{q}\tilde{g}\tilde{g}, \\
 m_{\tilde{g}} > m_{\tilde{q}} : \quad & gg, q_i \bar{q}_j \rightarrow \tilde{q}\tilde{q}, \\
 & q_i q_j \rightarrow \tilde{q}\tilde{q}, \\
 & q_i g \rightarrow \tilde{q}\tilde{q} \rightarrow \bar{q}\tilde{q}\tilde{q}, q\tilde{q}\tilde{q}, \\
 & gg, q_i \bar{q}_i \rightarrow \tilde{g}\tilde{g} \rightarrow \bar{q}\tilde{q}\tilde{q}\tilde{q}, q\tilde{q}\tilde{q}\tilde{q}
 \end{aligned} \tag{4.6}$$

where $i, j = u, d, s, c, b$ and all charge conjugated processes are understood.

The complete gauge-invariant set of Feynman diagrams for the production of the three- or four-body final states of “stable” particles (4.6) contain doubly-resonant diagrams, i.e. diagrams where the particles indicated in the first step of (4.6) appear as internal lines, and singly- or non-resonant diagrams, where only one or none of the unstable squarks or gluinos is present. Examples of these topologies are given in Figure 4.1. Beyond leading order, the production processes cannot be strictly separated since, e.g. the process $q_i g \rightarrow \bar{q}\tilde{q}\tilde{q}$ enters the real NLO corrections to $\tilde{q}\tilde{q}$ -production. Furthermore, the non-resonant diagrams can contain collinear singularities (e.g. the last topology given in Figure 4.1 that would cancel against higher-order virtual corrections and require taking kinematic cuts on the final state particles into account).

A full NLO QCD calculation of a process comparable to (4.5) including finite-width effects has been performed in the standard model for the case of $b\bar{b}W^+W^-$ production [114, 115], but not yet for an MSSM process. Very recently, a tool for automatic calculation of MSSM processes at NLO has been presented in [116], but so far applied to the on-shell processes (4.1). Short of a full NLO calculation, a well-defined approximation method for calculating radiative corrections to production and decay processes is based on the pole expansion [117, 118]. In this framework the total partonic production cross sections for the processes (4.6) is written in the form

$$\hat{\sigma}_{pp'}(\hat{s}) = \hat{\sigma}_{pp'}^{\text{res}}(\hat{s}) + \hat{\sigma}_{pp'}^{\text{non-res}}(\hat{s}). \tag{4.7}$$

The doubly-resonant cross-section $\hat{\sigma}^{\text{res}}$ is defined in a consistent way by an expansion of the S-matrix around the complex poles of the Dyson-resummed propagators of the unstable particles [117, 118]. At NLO it contains both factorizable corrections to production and decay, as well as non-factorizable corrections connecting production, propagation and decay stages. Non-factorizable corrections related to the final state cancel for the total cross section [119, 120], but not for differential distributions. The fully differential factorizable corrections to production and decay have recently been computed for squark-squark production [103]. For the remaining processes, at present only the corrections to the total production cross sections [36] and decay

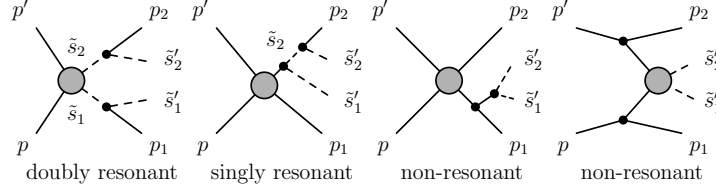


Figure 4.1: Examples of Feynman-diagram topologies contributing to the generic production and decay process (4.5). The second non-resonant diagram contains collinear singularities.

rates [108] are available. The “non-resonant” cross-section $\hat{\sigma}^{\text{non-res}}$ includes singly- and non-resonant Feynman-diagrams as well as contributions from doubly-resonant diagrams where one or both of the $\tilde{s}_1$ or $\tilde{s}_2$ lines are far off-shell.

4.1.2 Effective theory framework for unstable particles

For the investigation of the interplay of higher-order threshold corrections in QCD and finite squark and gluino lifetimes, we are interested in the partonic cross sections for the processes (4.6) near the partonic production threshold (1.8). For the precise definition of the resonant and non-resonant contributions to the cross section we adopt the effective-theory approach to unstable particles [104, 105], generalizing the treatment of W -boson pair production at an electron positron collider [106, 107]. The aim of the effective-theory approach is to provide a precise prediction of the partonic cross section for partonic centre-of-mass energies in the vicinity of the threshold, $\hat{s} - 4\bar{m}^2 \sim \bar{m}\Gamma$. This is achieved by a simultaneous expansion in the quantity

$$\delta = \frac{\hat{s} - 4\bar{m}^2}{\bar{m}^2} \approx \beta^2 \quad (4.8)$$

and the coupling constants. For power-counting purposes, we therefore set

$$\beta \sim (\Gamma/\bar{m})^{1/2} \sim \delta^{1/2}. \quad (4.9)$$

The leading-order resonant production cross-section for S-wave production processes is of the form

$$\hat{\sigma}_{pp'}^{\text{res}}(\hat{s}, \mu) \sim \alpha_s^2 \beta. \quad (4.10)$$

As we reviewed in Chapter 2, near the production threshold, the processes (4.5) can be treated in an effective theory where the light-partons are described by collinear fields ϕ, ϕ' in soft-collinear effective theory (SCET) [18, 66, 77] and the pair-produced squarks and gluinos are described by non-relativistic fields ψ, ψ' in potential non-relativistic QCD (pNRQCD) [20]. In the leading effective Lagrangian, the collinear and non-relativistic fields interact only through the exchange of soft gluons, which give rise to the non-factorizable corrections mentioned above.

$$i\mathcal{A}_{pp'} = \mathcal{O}_p^{(0)} + \mathcal{O}_p^{\dagger(0)} + \mathcal{O}_{4p}^{(0)}$$

Figure 4.2: Diagrammatic representation of the leading resonant and non-resonant contribution to the forward-scattering amplitude in unstable-particle effective theory.

Figure 4.3: Computation of the matching coefficient of the production operator for squark-antisquark production from the quark-antiquark initial state.

Figure 4.4: Computation of the matching coefficient of the four-parton operator for squark-antisquark production and decay. The first diagram is an example of a double-resonant diagram, the second of the interference of a double-resonant and a single-resonant diagram.

This effective theory has also been used for soft-gluon and Coulomb resummation in [5], where more details can be found.

In the EFT framework, the total partonic cross section for the process (4.5) is computed from the imaginary part of the partonic forward-scattering amplitudes $\mathcal{A}_{pp'}$ of the processes $pp' \rightarrow pp'$ which reads²⁰

$$i\mathcal{A}_{pp'}(\hat{s})|_{\hat{s} \sim 4\bar{m}^2} = \int d^4x \langle pp'|T [i\mathcal{O}_{\text{prod}}^\dagger(0)i\mathcal{O}_{\text{prod}}(x)]|pp'\rangle + \langle pp'|i\mathcal{O}_{4p}(0)|pp'\rangle. \quad (4.11)$$

The “production operator” $\mathcal{O}_{\text{prod}} = C_{\text{prod}} \phi \phi' \psi^\dagger \psi'^\dagger$ describes the resonant production of the $\tilde{s}_1 \tilde{s}_2$ -pair while the expectation value of the four-parton operators $\mathcal{O}_{4p} = C_{4p} \phi^\dagger \phi'^\dagger \phi \phi'$ describes the non-resonant contributions from pp' collisions. The two terms in (4.11) therefore correspond to the resonant and non-resonant cross sections defined earlier in (4.7), see Figure 4.2. The cross

²⁰As written, the forward-scattering amplitude is not infrared safe but requires mass factorization where matrix elements of the collinear fields are identified with parton distribution functions (PDFs).

section for a specific final state $\tilde{s}'_1 \tilde{s}'_2 p_1 p_2$ can be obtained by computing the imaginary part of the forward-scattering amplitude using unitarity cuts and selecting only cuts corresponding to the desired final state [106]. At leading order, this simply amounts to multiplying by the branching ratios $\text{BR}(\tilde{s}_1 \rightarrow \tilde{s}'_1 p_1)$ and $\text{BR}(\tilde{s}_2 \rightarrow \tilde{s}'_2 p_2)$. The matching coefficients C_{prod} of the production operators are computed from the on-shell amplitude for the processes $pp' \rightarrow \tilde{s}_1 \tilde{s}_2$ at the desired accuracy, as sketched for LO in Figure 4.3. The coefficients C_{4p} are calculated by expanding the forward-scattering amplitude in the hard momentum region, where all loop momenta are far off-shell.²¹ The leading non-resonant contribution to the processes (4.5) arises from the squared tree amplitudes of the processes $pp' \rightarrow \tilde{s}_1 \tilde{s}'_2 p_2$ and $pp' \rightarrow \tilde{s}'_1 p_1 \tilde{s}_2$ as sketched in Figure 4.4 for the example of squark-antisquark production and decay. The EFT calculation of the forward-scattering amplitude to order α_s and δ includes soft one-loop corrections, the one-loop corrections to C_{prod} as well as kinematic corrections due to subleading kinetic Lagrangian terms.

4.1.3 Soft and Coulomb resummation for the resonant contributions

In this Section we review the resummation formula in the case when the heavy produced particles undergo further decay. The resummation of soft logarithms and Coulomb corrections has been derived in [5] and reviewed in Chapter 2 by establishing a factorization of the doubly resonant contribution to the partonic cross section into hard, soft and non-relativistic (potential) matrix elements,

$$\hat{\sigma}_{pp'}^{\text{res}}(\hat{s}, \mu) = \sum_{R_\alpha} H_{pp'}^{R_\alpha}(m_{\tilde{q}}, m_{\tilde{g}}, \mu) \int d\omega J_{R_\alpha}(E + i\bar{\Gamma} - \frac{\omega}{2}) W^{R_\alpha}(\omega, \mu). \quad (4.12)$$

Here we have defined the average width

$$\bar{\Gamma} = \frac{1}{2}(\Gamma_{\tilde{s}_1} + \Gamma_{\tilde{s}_2}) \quad (4.13)$$

and R_α are the irreducible colour representations in the decomposition of the product of the colour representations of the sparticles $\tilde{s}_1$ and $\tilde{s}_2$. In analogy to the zero-width treatment in Chapter 3 [47], as a default we use the expression

$$E = \bar{m} \left(1 - \frac{4\bar{m}^2}{\hat{s}} \right) \quad (4.14)$$

for the energy of the squark or gluino pair which coincides with the non-relativistic expression $E = \bar{m}\beta^2$ for $\hat{s} > 4\bar{m}^2$. Therefore, throughout this chapter we refer to this treatment as the ‘ β -implementation’. In order to estimate ambiguities of the threshold approximation we will also use an ‘ E -implementation’ defined by $E = \sqrt{\hat{s}} - 2\bar{m}$ that agrees with (4.14) near threshold.

²¹Explicit calculations have been performed for W -pair production [106] and top-pair production [121, 122] at electron-positron colliders.

The hard functions $H_{pp'}^{R_\alpha}$ are related to the square of the matching coefficients C_{prod} . At LO they are proportional to the Born cross section at threshold.²² The soft functions W^{R_α} are matrix elements of soft-gluon Wilson lines and implement the eikonal approximation. Precise definitions can be found in Chapter 2 [5]. Threshold resummation of soft logarithms can be performed using renormalization group equations for the hard and soft function [58, 76, 81] that can be solved in Mellin-moment or momentum space [11]. The relevant anomalous dimensions for all squark and gluino production processes up to NNLL accuracy have been collected in Appendix A [5, 58].

The potential function J_{R_α} is an expectation value of the non-relativistic fields $\psi^{(0)}$, that have been decoupled from the soft gluons by a field redefinition and interact only through the exchange of Coulomb gluons. It is related to the zero-distance Green function of the non-relativistic Schrödinger equation with a Coulomb potential, $G_C^{R_\alpha(0)}(0, 0; \mathcal{E})$:

$$J^{R_\alpha}(\mathcal{E}) = 2 \text{Im} G_C^{R_\alpha(0)}(0, 0; \mathcal{E}), \quad (4.15)$$

with the Coulomb Green function including all-order gluon exchange in the $\overline{\text{MS}}$ -scheme [21, 123]:

$$\begin{aligned} G_C^{R_\alpha(0)}(0, 0; \mathcal{E}) = & -\frac{(2m_r)^2}{4\pi} \left\{ \sqrt{-\frac{\mathcal{E}}{2m_r}} + (-D_{R_\alpha})\alpha_s \left[\frac{1}{2} \ln \left(-\frac{8m_r\mathcal{E}}{\mu^2} \right) \right. \right. \\ & \left. \left. - \frac{1}{2} + \gamma_E + \psi \left(1 - \frac{(-D_{R_\alpha})\alpha_s}{2\sqrt{-\mathcal{E}/(2m_r)}} \right) \right] \right\}. \end{aligned} \quad (4.16)$$

Here γ_E is the Euler-Mascheroni constant and $m_r = m_{\tilde{s}_1} m_{\tilde{s}_2} / (m_{\tilde{s}_1} + m_{\tilde{s}_2})$. The coefficients $D_{R_\alpha} = \frac{1}{2}(C_{R_\alpha} - C_{R_1} - C_{R_2})$ of the Coulomb potential depend on the quadratic Casimir operators of the colour representations R_i of the sparticles $\tilde{s}_i$ and of the irreducible representation R_α in the decomposition of $R_1 \otimes R_2$. The trivial ($\alpha_s \rightarrow 0$) potential function for finite width obtained by neglecting the Coulomb corrections is given by

$$J^{R_\alpha(0)}(E + i\bar{\Gamma}) = \frac{m_r^{3/2}}{\pi} \sqrt{E + \sqrt{E^2 + \bar{\Gamma}^2}}. \quad (4.17)$$

The use of a complex energy $E + i\bar{\Gamma}$ takes the finite-width effects in the doubly-resonant cross section into account in leading order in the non-relativistic expansion [4].

For energies below the production threshold $E < 0$ and for $\bar{\Gamma} = 0$, the Coulomb Green function develops a series of bound-state poles at energies

$$E_n = -\frac{2m_r\alpha_s^2 D_{R_\alpha}^2}{4n^2}. \quad (4.18)$$

These contributions have been taken into account in the NLL predictions of Chapter 3 [47]. For unstable particles, the complex energy shift in the potential function in (4.12) leads to a smoothing

²²As in Chapter 3 [47], in our numerical results we compute the hard function from the full Born cross section instead of its threshold limit.

of the discrete set of bound-state poles (4.18) into a continuous non-vanishing contribution below the nominal production threshold.

4.2 Estimate of contributions to the cross section

In Section 4.1 we have discussed the treatment of finite-width effects (Section 4.1.2) and higher-order soft and Coulomb corrections for the case of unstable particles (Section 4.1.3). The resummation of threshold logarithms and Coulomb corrections was performed under the assumption (2.1) that a sizable contribution to the total hadronic cross section arises from the region where $\alpha_s \ln \beta \sim \frac{\alpha_s}{\beta} \sim 1$, leading to a reorganization of the perturbative expansion according to (2.14). For finite decay widths, the replacement $E \rightarrow E + i\bar{\Gamma}$ in the potential function leads to the screening of the threshold singularities, raising the question if the resummation based on the counting (2.1) is still justified. Furthermore, for an unstable particle the size of the non-resonant contribution to the cross section (4.7) compared to the higher-order corrections to the resonant contributions has to be addressed. In this Section we use power-counting arguments to estimate these effects and compare these expectations to explicit results for squark and gluino production.

4.2.1 Size of the decay width

To discuss the size of the screening effect and the non-resonant contributions, we consider the decay of the sparticle $\tilde{s}$ of mass M into a sparticle $\tilde{s}'$ of mass m and a massless parton p , mediated by the strong interaction. On kinematic grounds, the decay width is of the form

$$\Gamma(\tilde{s} \rightarrow \tilde{s}' p) = \alpha_s M (1 - x^2) f(x) \quad (4.19)$$

for some function f of the mass ratio

$$x \equiv \frac{m}{M}. \quad (4.20)$$

In the following, we consider the decay width (4.19) for two limiting cases:

a) Light decay product, $x \rightarrow 0$:

$$\Gamma/M \sim \alpha_s. \quad (4.21)$$

In this case, the expansion parameter δ of the effective theory (4.9) is estimated as

$$\delta \sim \beta^2 \sim \alpha_s. \quad (4.22)$$

b) Heavy decay product, $x \rightarrow 1$:

$$\Gamma/M \sim \alpha_s \times (1 - x^2)^\gamma, \quad (4.23)$$

where $\gamma = 1$ if the function $f(x)$ in (4.19) is of the order one, but $\gamma = 2$ for the case of squark and gluino decay (4.4). In this case, another small scale $\kappa \equiv (1 - x^2)^\gamma$ is present, and the hierarchy of scales is given by

$$\delta \sim \beta^2 \sim \alpha_s \kappa. \quad (4.24)$$

In particular, if for squarks and gluinos the numerical size of the mass ratio is such that $\kappa = (1 - (m/M)^2)^2 \sim 0.1$, i.e. $m/M \gtrsim 0.8$, we count $\delta \sim \alpha_s^2$.

For the phenomenology of unstable strongly-interacting particles, different scenarios have been identified (e.g. [82, 87]). For very narrow particles, bound states can form that subsequently decay into di-photon or di-jet final states and therefore lead to a very different phenomenology than the missing energy signatures usually employed in squark and gluino searches. In the MSSM this case is only relevant for gluinonium and stoponium production and will not be considered further here. For a sparticle decay width that is larger than the bound-state decay widths into di-photons or di-jets, but much smaller than the energy E_1 of the first bound-state (4.18), a few bound state-peaks can form, but the decay is dominated by the underlying sparticle decay, while for $\bar{\Gamma} \lesssim E_1$, bound-state effects enhance the cross section near threshold, but the different peaks might not be separated. Since $E_1 \sim \alpha_s^2$, bound-state peaks might be visible in the invariant-mass distribution below threshold for narrow sparticles in the category b), as we will study in Section 4.3.1. For $\bar{\Gamma} > E_1$ the bound-state effects are washed out, which we expect to be the case for broader sparticles in category a).

We will now estimate the size of the Coulomb and soft corrections as well as the non-resonant contributions for these two limiting cases.

4.2.2 Coulomb corrections

In the two scenarios for the decay width, the n -th Coulomb correction is of the order

$$\left(\frac{\alpha_s}{\beta}\right)^n \sim \left(\frac{\alpha_s^2}{\delta}\right)^{n/2} \sim \begin{cases} \delta^{n/2} \sim \alpha_s^{n/2}, & \text{case a)} \\ \left(\frac{\alpha_s}{\kappa}\right)^{n/2}, & \text{case b)} \end{cases} \quad (4.25)$$

Therefore by power counting, Coulomb resummation is not necessary for case a), i.e. for particles with decay widths of the order of $\Gamma/M \sim \alpha_s \sim 10\%$. This exemplifies the screening of the threshold corrections by the finite lifetime. However, to reach NLO accuracy in $\delta \sim \alpha_s$ the second Coulomb correction has to be included. This is analogous to the case of W -pair production considered in [106] (up to the replacement of α by α_s). In the opposite limit of narrow particles where numerically $\frac{\Gamma}{M} \sim \alpha_s^2 \sim 1\%$ (case b), $\kappa \sim \alpha_s$ so the screening of the Coulomb corrections is not effective and Coulomb resummation should be performed as in the stable case.²³ Since in these

²³This is analogous to the well-known case of top-quark production at linear colliders, where the small decay width of the top is due to the electroweak nature of the decay, $\Gamma_t/m_t \sim \alpha_{EW} \sim \alpha_s^2$.

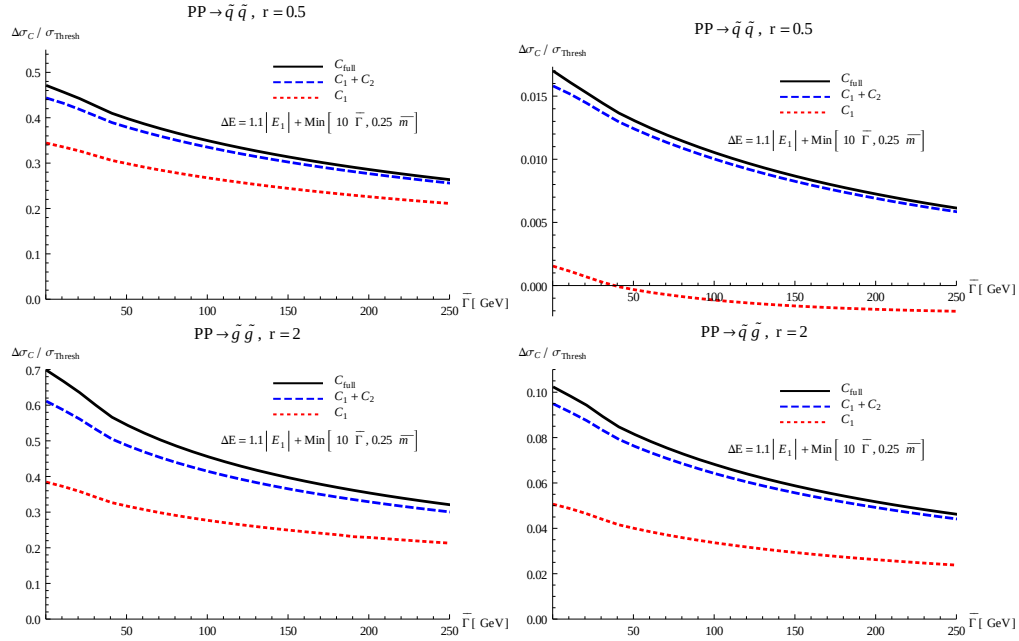


Figure 4.5: Size of the first Coulomb correction (red, dotted), first and second Coulomb corrections (blue, dashed) and resummed Coulomb corrections (black, solid) relative to the leading cross section in the effective theory as a function of $\tilde{\Gamma}$ and for $\tilde{m} = 1500$ GeV at the LHC with $\sqrt{s} = 8$ TeV. r is defined as $r = m_{\tilde{g}}/m_{\tilde{q}}$. For technical reasons the potential function is set to zero for $E < \Delta E$ as defined in (4.36).

formal counting arguments the numerical prefactors such as the strength of the Coulomb potential are not taken into account, the actual relevance of the Coulomb corrections has to be studied on a process-by-process basis. In Figure 4.5 we show the contribution of the first (C_1 , red dotted line) and second (C_2 , blue dashed line) Coulomb corrections, as well as the resummed Coulomb corrections (C_{full} , solid black line), to the total hadronic production cross sections as a function of the decay width for all four squark and gluino production processes. The Coulomb corrections are normalized by the approximation $\sigma_{\text{Thresh}}^{(0)}$ that is defined as the effective theory cross section (4.12) without soft-gluon resummation and using only the α_s^0 term of the potential function (4.17). One sees that for larger widths the Coulomb corrections are increasingly saturated by the first two Coulomb corrections, while the second Coulomb correction always gives a non-negligible contribution, in agreement with the general discussion above. However, the total contribution of the Coulomb corrections can be sizable, as for gluino-pair production, or numerically small, as for squark-squark production.

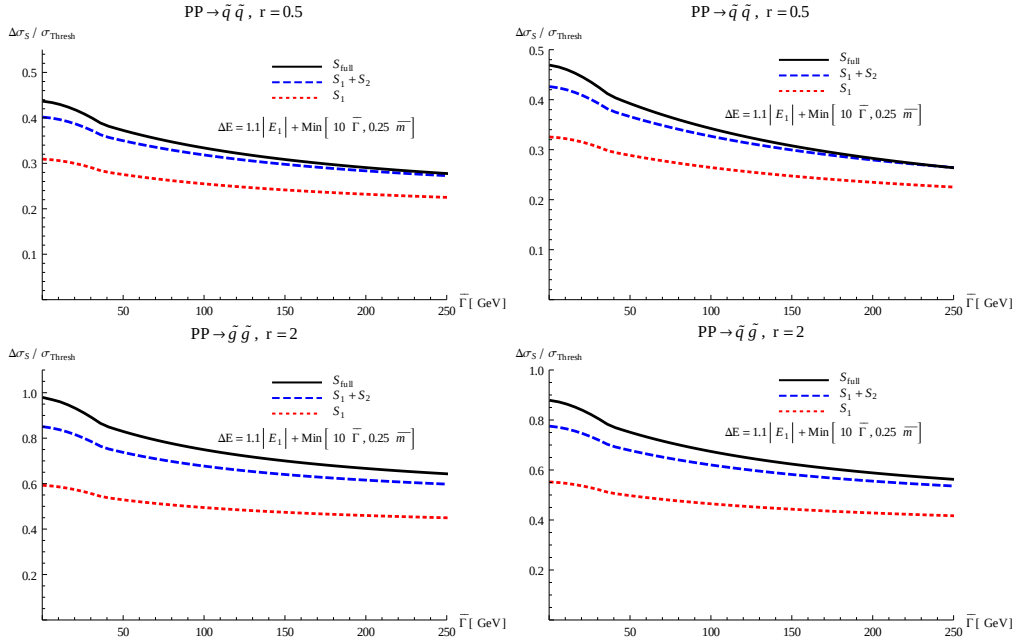


Figure 4.6: Size of the NLL soft corrections expanded to NLO (red, dotted), up to NNLO (blue, dashed) and the resummed NLL corrections (black, solid) relative to the leading cross section in the effective theory as a function of $\bar{\Gamma}$ and for $\bar{m} = 1500$ GeV at the LHC with $\sqrt{s} = 8$ TeV. The remaining definitions are as in Figure 4.5.

4.2.3 Soft corrections

The parametric scaling of soft corrections in the scenarios a) and b) considered in Section 4.2.1 is:

$$\ln \beta \sim \ln \sqrt{\delta} \sim \begin{cases} \frac{1}{2} \ln \alpha_s, & \text{case a)} \\ \frac{1}{2} (\ln \alpha_s + \ln \kappa) \sim \ln \alpha_s, & \text{case b)} \end{cases} \quad (4.26)$$

where in case b) we have again considered the case where numerically $\kappa \sim \alpha_s$. It is thus not obvious at a first glance in which scenarios resummation of soft-gluon corrections is necessary, if at all, since for $\alpha_s \sim 0.1$ one has $\alpha_s |\ln \alpha_s| \sim 0.23$. Clearly one would expect resummation effects to be more important for case b), where the screening effect due to a finite width is less effective. However the parametric scaling in the two scenarios is formally the same and equal, up to constant prefactors, to $\sim \ln \alpha_s$. As already stressed for the case of Coulomb corrections, the scaling argument which leads to (4.26) does not take into account the coefficients of the logs, which can be numerically large. This is particularly true for gluon-initiated squark-antisquark production and for squark-gluino and gluino-gluino production, where the Casimir invariants for

the adjoint and higher colour representations appear in the prefactors. Once again, a meaningful assessment of the importance of resummation can only be done on a process-by-process basis by a numerical analysis of different fixed-order contributions.

Figure 4.6 shows the effect of NLL soft resummation (S_{full} , solid black line) on the cross section of the four SUSY-production processes considered in this work and the contributions of the $\mathcal{O}(\alpha_s)$ (S_1 , red dotted line) and $\mathcal{O}(\alpha_s^2)$ (S_2 , blue dashed line) terms obtained from the expansion of the resummed cross section, as a function of the width $\bar{\Gamma}$. As in Figure 4.5 the cross sections are normalized to the leading threshold approximation. It can be seen that for small widths of order $\bar{\Gamma}/\bar{m} \sim \alpha_s^2$ resummation is numerically important for all processes (and more so for those where large colour charges are involved), with NNLO corrections of the order of 10 – 25% of the LO cross section and terms beyond NNLO as large as $\sim 10\%$ for squark-gluino and gluino-gluino production. One can therefore argue that in scenario b) it makes sense to keep the zero-width scaling $\alpha_s \ln \beta \sim 1$ for soft logarithms with the representation of the cross section given in (2.14).

At larger width ($\bar{\Gamma}/\bar{m} \gtrsim \alpha_s$) the bulk of NLL corrections is accounted for by the $\mathcal{O}(\alpha_s)$ terms, which correspond to a 30 – 50% contribution to the Born result. However NNLO terms still represent a correction of order 5 – 15% depending on the process considered. Consistent with the stronger screening effect expected for larger widths, higher-order soft corrections beyond NNLO are small, $\lesssim 5\%$ for gluino-gluino and squark-gluino production and below 1% for squark-antisquark and squark-squark production. We thus conclude that in case a) the relevance of soft resummation is not immediately clear, though the inclusion of higher-order corrections at NNLO might still be necessary to achieve an accuracy of order $\delta \sim \alpha_s$. It is interesting to note that for large widths the fixed-order one-loop soft correction is numerically about $\sqrt{\alpha_s} \sim 30\%$ of the tree-level threshold result, and the two-loop soft terms of order $(\sqrt{\alpha_s})^2 \sim 10\%$.

To summarize, in the two limiting scenarios one can give the following parametric representation of the cross section:

- a) Adopting for the soft logarithms the (approximate) scaling $\alpha_s \ln^2 \beta \sim \sqrt{\delta} \sim \sqrt{\alpha_s}$, which is motivated by the numerical results in Figure 4.6, and using (4.25), the cross section can be represented as

$$\hat{\sigma}_{pp'} \propto \hat{\sigma}^{(0)} \times \begin{cases} 1 & \text{LO} \\ \frac{\alpha_s}{\beta}, \alpha_s \ln^2 \beta & \text{N}^{1/2}\text{LO} \\ \alpha_s, \left(\frac{\alpha_s}{\beta}\right)^2, (\alpha_s \ln^2 \beta)^2, (\alpha_s \ln \beta)^2/\beta, \alpha_s \ln \beta, \beta^2 & \text{NLO} \\ \dots & \end{cases} \quad (4.27)$$

where the expansion in half-integer powers of δ is similar to the case of W -pair production at a linear collider [106]. There are no terms linear in β because they are known to average out to zero for the total cross section.

- b) For the case $\kappa \sim \alpha_s$, the counting is identical to the stable case.

While according to (4.27) resummation is parametrically not necessary for $\bar{\Gamma}/\bar{m} \gtrsim \alpha_s$, in practice it is not always clear for which numerical value of $\bar{\Gamma}$ one can switch from the prescription (2.14) to (4.27). This is particularly true for the transition region where $\alpha_s^2 \lesssim \bar{\Gamma}/\bar{m} \lesssim \alpha_s$. For this reason in Section 4.3 we will use the NLL implementation of (2.14) for arbitrary values of the width. This choice has the advantage of including, in both limits of a small and large width, all the terms relevant to achieving an accuracy of $\sim \delta$, with the exception of the β^2 contribution appearing at NLO in the counting (4.27). These are however correctly taken into account by matching the effective-theory result to the exact Born result including the leading finite-width and power suppressed effects, as explained in Section 4.2.4.

4.2.4 Non-resonant corrections

We now turn to an estimate of the size of non-resonant corrections neglected in present higher-order calculations that treat squarks and gluinos as stable. The leading non-resonant contributions as well as subleading kinetic corrections to the double-resonant cross section are both included in a calculation of the full Born cross section of a process of the form (4.5) using multi-leg Monte Carlo programs [99] so the strict EFT treatment is not necessary. Also the non-resonant diagrams are in general well-defined only with kinematic cuts, as mentioned above. A study of the full non-resonant effects for all squark and gluino production processes would therefore require a dedicated study taking the realistic selection cuts used by the LHC experiments into account, which is beyond the scope of this work. We will therefore limit ourselves to an estimate of the unknown radiative corrections to the non-resonant contributions that cannot easily be computed in a Monte Carlo program. These are given by higher-order corrections to the diagrams shown in Figure 4.4. The computation of these corrections is currently an ongoing effort for top-pair production at a linear collider [121, 122, 124, 125] and the computation for the case of squarks and gluinos is far beyond the scope of the present work. These corrections would also be included in a complete fixed-order NLO calculation of the process (4.5).

We consider again the two cases introduced in Section 4.2.1:

- a) Since in the calculation of the matching coefficients of the four-parton operator all the momenta are large compared to the scale δ , there is no enhancement of these corrections by resonant propagators and one expects them to be of the order [106]

$$\hat{\sigma}_{pp'}^{\text{non-res}} \sim \alpha_s^3 \sim \hat{\sigma}_{pp'}^{\text{res}} \times \frac{\alpha_s}{\beta}. \quad (4.28)$$

For the scaling (4.22) appropriate for $\Gamma/M \sim 10\%$, the leading non-resonant corrections are thus of the order $\delta^{1/2} \sim \sqrt{\Gamma/M}$, as the first Coulomb correction. The unknown radiative corrections to the non-resonant contributions (4.28) therefore are of the order $\alpha_s \sqrt{\Gamma/M} \sim \alpha_s^{3/2}$ relative to the leading resonant cross section, and therefore beyond NLO accuracy.

- b) For a small mass hierarchy leading to the scaling (4.24) the non-resonant contributions can be further expanded according to $\beta \ll \sqrt{\rho} \ll 1$, with $\rho = 1 - m/M$. For the case of top-quark production the leading non-resonant corrections were found to be of the order [122]

$$\hat{\sigma}_{pp'}^{\text{non-res}} \sim \hat{\sigma}_{pp'}^{\text{res}} \times \sqrt{\frac{\Gamma}{M\rho}}. \quad (4.29)$$

Using (4.23) we have

$$\frac{\Gamma}{M\rho} \sim \frac{\alpha_s(1-x^2)^\gamma}{(1-x)} \sim 2\alpha_s\kappa^{(\gamma-1)/\gamma}. \quad (4.30)$$

For squark and gluino decay $\gamma = 2$, so counting $\kappa \sim \alpha_s$ we find that the suppression of the non-resonant corrections is of the order of $(\frac{\Gamma}{M}\frac{1}{\alpha_s^{1/2}})^{1/2} \sim (\Gamma/M)^{3/8}$ compared to the leading resonant term. The unknown radiative non-resonant corrections are then of the order $(\frac{\Gamma}{M}\alpha_s^{3/2})^{1/2} \sim (\Gamma/M)^{7/8}$, i.e. of a similar magnitude as for the case $x \rightarrow 0$. These corrections are beyond NNLL accuracy.

To study the numerical impact of the non-resonant corrections, we consider the following approximations:

- $\hat{\sigma}_{\text{NWA}}^{(0)}$: The LO cross section calculated using the narrow-width approximation everywhere, i.e. the on-shell production cross section in the $\Gamma \rightarrow 0$ limit is multiplied by the decay branching ratios. This approximation is valid for $M \gg \Gamma$ and far above threshold [100]. In this work only the decay processes (4.3) are considered and thus $\hat{\sigma}_{\text{NWA}}^{(0)} = \hat{\sigma}_{\text{Tree}}^{(0)}(\Gamma = 0)$.
- $\hat{\sigma}_{\text{Thresh}}^{(0)}$: The threshold approximated LO cross section in the EFT, using only the α_s^0 term of the potential function (4.17). The unstable-particle momentum in the centre of mass frame is in the potential region (slightly off-shell).
- $\hat{\sigma}_{\text{Off}}^{(0)}$: The LO cross section obtained using the off-shell doubly-resonant diagrams and fixed-width Breit-Wigner propagators computed with WHIZARD [126] and validated with SHERPA [127] when possible. This implements tree-level off-shell effects and allows for unrestricted momenta of the unstable particles.²⁴

In Figure 4.7 the three different approximations of the partonic cross sections are shown for the example of squark-gluino production and subsequent squark decay as a function of the energy $E = \bar{m}\beta^2$ for two examples of the decay width. The plots for the other processes have the same behaviour as above and are not shown here. It can be seen that the EFT approximation $\hat{\sigma}_{\text{Thresh}}^{(0)}$ agrees with the off-shell results for small widths, but is systematically shifted for larger decay

²⁴The selection of doubly-resonant diagrams violates gauge invariance. However, we have verified for a selection of different covariant and axial gauges that gauge violation is numerically below 1% for the processes of interest in this work.

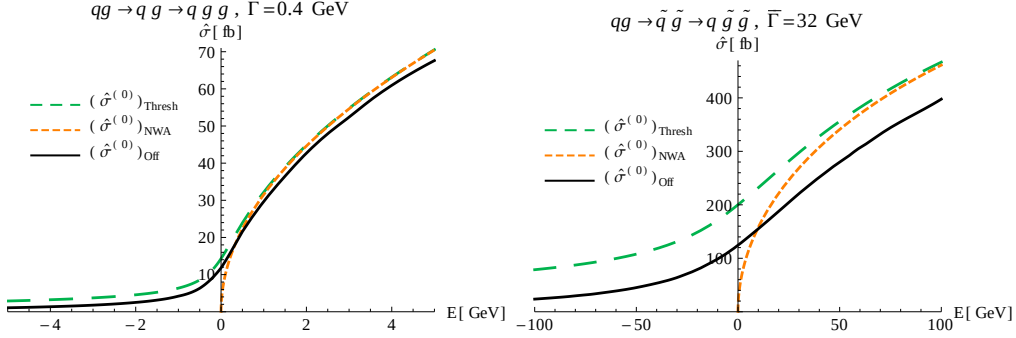


Figure 4.7: The partonic cross section for squark-gluino production and squark decay as a function of the energy $E = \bar{m}\beta^2$ at a fixed average mass $\bar{m} = \frac{1}{2}(m_{\tilde{q}} + m_{\tilde{g}}) = 1500$ GeV. Left: $r = \frac{m_{\tilde{g}}}{m_{\tilde{q}}} = 0.95$, $\bar{\Gamma} = 0.4$ GeV. Right: $r = 0.5$, $\bar{\Gamma} = 32$ GeV.

width. This is the expected effect of the non-resonant contribution to the Born cross section, that is of the form $\hat{\sigma}_{pp'}^{\text{non-res}} \sim \frac{\bar{m}^2}{s} \text{Im}C_{4p}$, and has also been observed in [106, 121, 124]. Note that while we do not include truly singly or non-resonant diagrams in the off-shell approximation, the off-shell momentum configurations of doubly-resonant diagrams contribute to the non-resonant part of the cross section and, in fact, provide the dominant numerical effect in the case of W or top production [106, 121, 122]. Therefore the shift observed in Figure 4.7 provides an estimate of the non-resonant corrections. For $\bar{\Gamma}/\bar{m} = 32/1500 = 0.02$ the nonresonant correction is of the order of 40%, which is of the same order of magnitude, but somewhat larger, than the estimate $(\Gamma/M)^{1/2} - (\Gamma/M)^{3/8} = 14\% - 24\%$. We then estimate that the uncalculated corrections due to higher-order non-resonant effects are of the order of $\lesssim 5\%$.

4.3 Application to squark and gluino production at LHC

In this Section we present results for finite-width effects on the soft and Coulomb NLL corrections to squark and gluino production at the LHC in the setup discussed in Section 4.1. In Section 4.3.1 we give results for the invariant-mass distributions, while in Section 4.3.2 we discuss our implementation and present numerical results for the total cross sections.

4.3.1 Invariant-mass distributions

In order to discuss the finite-width effects on the production cross sections, we first consider the invariant-mass distribution. Similarly to the total cross section (4.12), it satisfies a factorization

formula near the production threshold [5] (see also e.g. [82])

$$\frac{d\sigma_{NN'}^{\text{res}}(\hat{s}, \mu)}{dM_{\tilde{s}_1\tilde{s}_2}} = \sum_{R_\alpha} J_{R_\alpha}(M_{\tilde{s}_1\tilde{s}_2} - 2\bar{m} + i\bar{\Gamma}) \sum_{p,p'=q,\bar{q},g} 2H_{pp'}^{R_\alpha}(\mu) \times \int_{\tau_0}^1 d\tau L_{pp'}(\tau, \mu) W^{R_\alpha}(2(\sqrt{\hat{s}} - M_{\tilde{s}_1\tilde{s}_2}), \mu), \quad (4.31)$$

where $\tau_0 = M_{\tilde{s}_1\tilde{s}_2}^2/s$. We have defined the parton luminosity in terms of the PDFs $f_{p/N}$ for a parton p in a nucleon N :

$$L_{pp'}(\tau, \mu) = \int_0^1 dx_1 dx_2 \delta(x_1 x_2 - \tau) f_{p/N_1}(x_1, \mu) f_{p'/N_2}(x_2, \mu). \quad (4.32)$$

In our numerical results we use the NLL-resummed soft function that can be found in Chapter 3 [47]. Since we are interested in the invariant-mass spectrum near threshold we deviate somewhat from the default treatment of the total cross section and always express the hard function in terms of the threshold-approximation of the Born cross section and use a constant Coulomb scale, defined as solution to the equation $\mu_C = 2\alpha_s(\mu_C)m_r|D_{R_\alpha}|$, and constant soft scales μ_s that are process-specific and determined in Chapter 3 [47].

Consistent with the treatment of the total cross section discussed in Section 4.1.3, as a default we use the expression (4.14) for the energy of the squark or gluino pair, i.e. we write the argument of the soft function W^{R_α} in (4.31) as

$$\sqrt{\hat{s}} - M_{\tilde{s}_1\tilde{s}_2} \rightarrow \bar{m} \left(1 - \frac{4\bar{m}^2}{\hat{s}} \right) + 2\bar{m} - M_{\tilde{s}_1\tilde{s}_2}. \quad (4.33)$$

As for the total cross section, this will be referred to as the ‘ β -implementation’. Note that the soft function vanishes for negative argument which implies that the lower boundary of the τ integral is given by

$$\tau_0 = \frac{4\bar{m}^2}{s} \frac{\bar{m}}{3\bar{m} - M_{\tilde{s}_1\tilde{s}_2}}. \quad (4.34)$$

In order to estimate ambiguities of our treatment, we also introduce an ‘ E -implementation’ keeping the unexpanded expression $\sqrt{\hat{s}} - M_{\tilde{s}_1\tilde{s}_2}$ in the argument of the soft function. In order not to introduce artificial differences in the two implementations due to a different boundary of the τ integral, we perform the replacement $M_{\tilde{s}_1\tilde{s}_2} \rightarrow M'_{\tilde{s}_1\tilde{s}_2} \equiv \bar{m}(3 - 4\bar{m}^2/M_{\tilde{s}_1\tilde{s}_2}^2)$ in the E -implementation and consider the differential cross section with respect to $M'_{\tilde{s}_1\tilde{s}_2}$.²⁵

Based on experience with W -pair production [106] we do not expect the leading resonant approximation to be valid significantly below the nominal threshold, where higher-order terms

²⁵The differential cross section in Eq. (4.31) includes a factor $dM_{\tilde{s}_1\tilde{s}_2}/dM'_{\tilde{s}_1\tilde{s}_2} = (M_{\tilde{s}_1\tilde{s}_2}/2\bar{m})^3$ after replacing $M_{\tilde{s}_1\tilde{s}_2} \rightarrow M'_{\tilde{s}_1\tilde{s}_2}$ for the E -implementation.

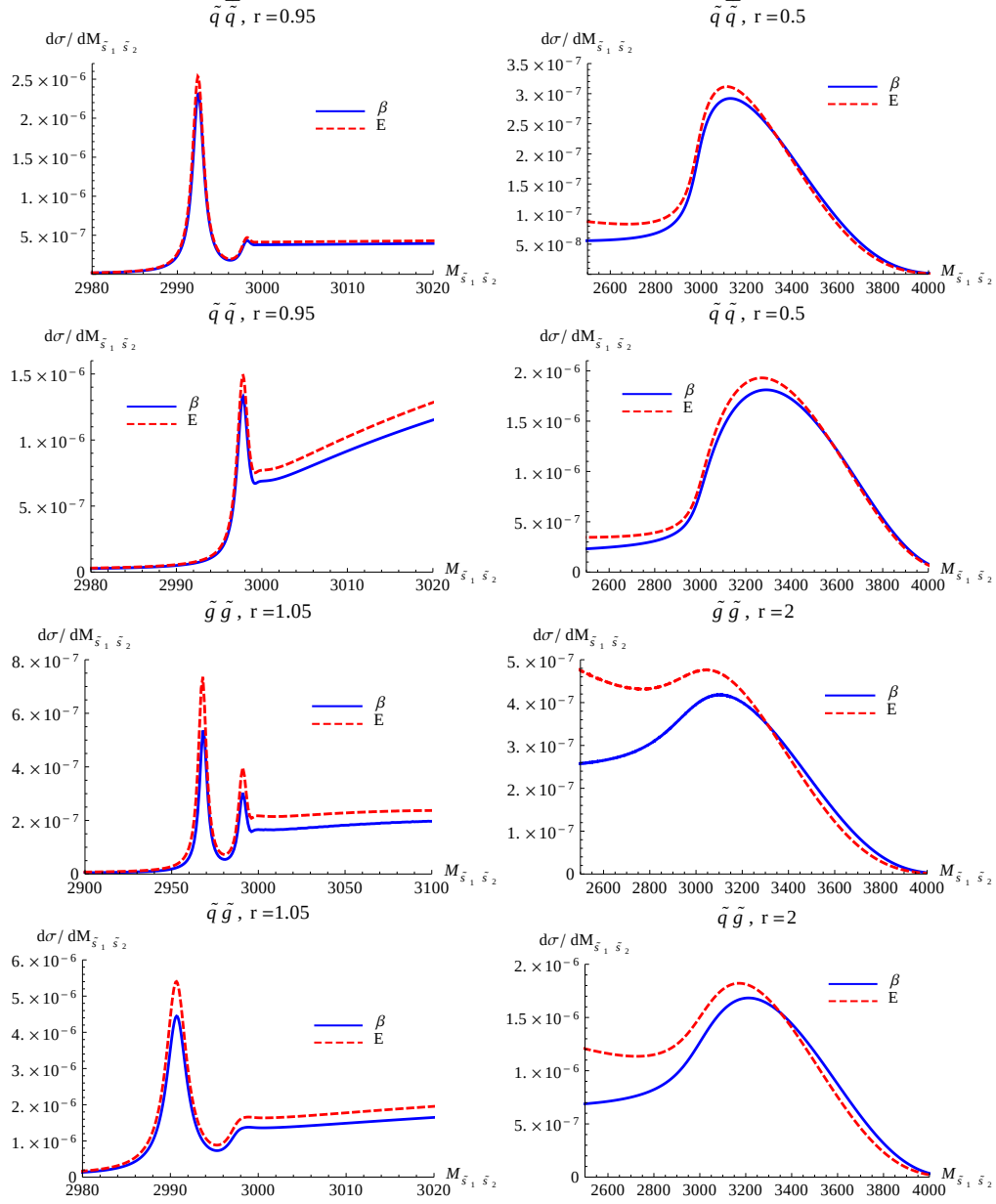


Figure 4.8: Invariant-mass distributions for an average mass $\bar{m} = 1500$ GeV of the produced sparticle pair and a centre-of-mass energy $\sqrt{s} = 8$ TeV for the β (blue, solid) and E (red, dashed) implementation as defined below (4.33). Left-hand plots are examples of case b) ($\bar{\Gamma}/\bar{m} \sim \alpha_s^2$), right-hand plots case a) ($\bar{\Gamma}/\bar{m} \sim \alpha_s$).

in the non-relativistic expansion and non-resonant contributions become important. The differences between the default β and the E approximation provide an estimate for the size of the higher-order relativistic effects. Note that because of soft radiation (as taken into account by the convolution of the soft function with the parton luminosity) also the invariant mass distribution directly at threshold is sensitive to larger partonic centre-of-mass energies and the associated ambiguities in the threshold expansion.

In Figure 4.8 the invariant-mass distributions for the four relevant squark and gluino production processes (4.6) are shown for $\bar{m} = 1500$ GeV at a centre-of-mass energy $\sqrt{s} = 8$ TeV for two different mass ratios corresponding to the cases a) and b) discussed in Section 4.2.1. For the case $m_{\tilde{g}} > m_{\tilde{q}}$ we consider $r = m_{\tilde{g}}/m_{\tilde{q}} = 2$ and $r = 1.05$, corresponding to $\Gamma_{\tilde{g}}/m_{\tilde{g}} = 12\%$ and $\Gamma_{\tilde{g}}/m_{\tilde{g}} = 0.2\%$, respectively. For the case $m_{\tilde{g}} < m_{\tilde{q}}$ we consider $r = 0.5$ and $r = 0.95$, corresponding to $\Gamma_{\tilde{q}}/m_{\tilde{q}} = 3\%$ and $\Gamma_{\tilde{q}}/m_{\tilde{q}} = 0.05\%$, respectively. As explained in Section 4.2, we will perform threshold resummation also for the case of larger decay widths, although it might not be strictly necessary in this case.

In the small-width case, i.e. the left-hand plots in Figure 4.8 where $r = 0.95$ or $r = 1.05$, the ambiguities in our prediction as estimated by the difference of the E - and β -implementations are moderate. Depending on the process, the different regimes mentioned in Section 4.2.1 are observed: for the given choice of parameters, a single would-be bound-state peak appears for squark-squark production, where $\bar{\Gamma} = 0.9$ GeV $\lesssim E_1 = 1.5$ GeV, while two peaks are visible in gluino-pair production where $\bar{\Gamma} = 3$ GeV $\ll E_1 = 30$ GeV. In the remaining two processes the second bound-state peak is less pronounced, but still visible.

In the case of larger widths, i.e. the right-hand plots in Figure 4.8 with $r = 0.5$ or $r = 2$, the decay widths are always much larger than the bound-state energy, so no peaks below threshold are observed. It can be seen that the invariant-mass distributions do not vanish for $M_{\tilde{s}_1\tilde{s}_2}$ far below the nominal threshold, $M_{\tilde{s}_1\tilde{s}_2} = 3000$ GeV. This behaviour arises as a combination of the rise of the PDFs for small x and the behaviour of the potential function in 4.31, that scales as $J_{DR} \propto \bar{\Gamma}/(2\bar{m} - M_{\tilde{s}_1\tilde{s}_2})^{1/2} \rightarrow \bar{\Gamma}/(2\bar{m})^{1/2}$ for $M_{\tilde{s}_1\tilde{s}_2} \rightarrow 0$, which follows from the LO (trivial) potential function (4.17). While for the default β -implementation the distributions approach a flat plateau far below threshold, in the E -implementation one observes a (clearly unphysical) rise for larger widths for processes involving gluinos.²⁶ In fact, as already pointed out, far below threshold the LO potential function is not expected to be a good approximation, and the inclusion of non-resonant contributions and relativistic corrections are needed for a realistic description of the invariant-mass distributions in this region, similar to what was seen for the partonic cross sections in Figure 4.7.

The difference between the E - and β -implementations is amplified due to the soft function in (4.31), $W^{R_\alpha}(\omega) \sim \omega^{-1+2\eta}$ with $\eta < 0$. In the E -implementation the argument of the soft

²⁶Note that this is not an artifact of our use of $M'_{\tilde{s}_1\tilde{s}_2}$ for the E -implementation. In fact, for the original formula (4.31) an even stronger rise is observed, since smaller x values are probed in the parton luminosity due to smaller values of τ_0 .

function (left-hand side of Eq. (4.33)) approaches zero faster than the corresponding quantity in the β -implementation (right-hand side of Eq. (4.33)) as the lower integration boundary τ_0 is approached, and the soft function W diverges faster.²⁷ If soft corrections are not taken into account, the soft function reduces to a delta function, $W^{R_\alpha}(\omega) \propto \delta(\omega)$. In this case we have observed a substantial reduction in the difference between the E - and β -implementation also for large widths.

From the results presented in this Section we conclude that our approximation is reasonably accurate for processes with decay widths of the order of $\bar{\Gamma}/\bar{m} \lesssim 5\%$. As we will argue in Section 4.3.2, this covers the phenomenologically-relevant MSSM parameter-space regions. Our effective-theory treatment becomes inadequate for larger decay widths, so our results in this regime should be considered as indicative only.

4.3.2 Total cross sections

In order to compute the total cross section for a finite decay width, in principle the ω -integral in (4.12) has to be performed up to the maximal energy $\omega = 2\sqrt{\hat{s}}$ since the potential function J_{R_α} is defined for arbitrary negative values of the partonic energy E . However, as mentioned above, far below threshold (i.e. $\omega \gg 2E$) the leading threshold approximation becomes insufficient and higher-order and non-resonant contributions become relevant. This was shown by the difference of the $\hat{\sigma}_{\text{thresh}}^{(0)}$ and $\hat{\sigma}_{\text{Off}}^{(0)}$ curves for large widths in Figure 4.7 and by the large difference of the β - and E -implementations for the invariant-mass spectrum and the unphysical behaviour of the latter below threshold, as seen in Figure 4.8. Since the non-resonant and higher-order non-relativistic corrections for squark and gluino production are not available beyond LO, we introduce a cut-off in the potential function:

$$J_R(E + i\bar{\Gamma}) = 2\text{Im} [G_R(E + i\bar{\Gamma})] \theta(E + \Delta E), \quad \Delta E > 0, \quad (4.35)$$

with E equal to the expression (4.14) as our default implementation. The parameter ΔE cuts off the convolution of the soft and potential function at $\omega = 2(E + \Delta E)$. In order to capture the bound-state peaks, this cut-off is chosen as

$$\Delta E = 1.1|E_1| + \overline{\Delta E}, \quad (4.36)$$

where E_1 is the energy of the lowest-lying bound-state peak, see Eq. (4.18). The prefactor 1.1 multiplying E_1 ensures that the bound-state contribution is always fully included. As a default for $\overline{\Delta E}$ we take

$$\overline{\Delta E_0} := \text{Min}[10\bar{\Gamma}, 0.25\bar{m}]. \quad (4.37)$$

A related cutoff $\Delta E = |E_1| + 3\bar{\Gamma}$ was used for gluino-pair production in [82] where the focus was on narrow gluinos with decay widths at most of the order of $\Gamma_{\tilde{g}} \sim |E_1| \sim m_{\tilde{g}}\alpha_s^2$. In the case

²⁷This is further enhanced by the rise of the PDFs at the lower integration boundary.

of larger decay widths, the value of $\Delta\bar{E} = 10\bar{\Gamma}$ becomes of the order of $m_{\tilde{g}}$, where we expect the threshold approximation to become invalid. Therefore we have chosen the cutoff $\Delta\bar{E} \sim 0.25\bar{m}$ in the case of larger decay widths. The uncertainty in choosing $\Delta\bar{E}$ is estimated by varying it around its default value:

$$\Delta\bar{E}_0/2 \leq \Delta\bar{E} \leq 2\Delta\bar{E}_0. \quad (4.38)$$

The hadronic cross sections are obtained by convoluting the partonic cross sections with the parton luminosity (4.32)

$$\sigma_{N_1 N_2 \rightarrow \tilde{s}\tilde{s}' X}(s) = \int_{\tau_0}^1 d\tau \sum_{p,p'=q,\bar{q},g} L_{pp'}(\tau, \mu_f) \hat{\sigma}_{pp'}(\tau s, \mu_f), \quad (4.39)$$

where the lower integration boundary depends on the cutoff,

$$\tau_0(\Delta E) = \frac{4\bar{m}^2}{s} \frac{1}{1 + \Delta E/\bar{m}}. \quad (4.40)$$

In the E -implementation, the lower integration boundary is instead $\tau_0(\Delta E) = (2\bar{m} - \Delta E)^2/s$. In order not to introduce numerical differences due to different integration boundaries, in the E -implementation we use the same numerical value for τ_0 as for the β -implementation, and adjust the value of ΔE in the potential function accordingly.²⁸

While the Born amplitudes for the production and decay processes of squarks and gluinos can be treated exactly with Monte Carlo programs [99], the NLO corrections for the production processes are known only in the narrow-width approximation, as implemented in `PROSPINO` [34]. In order to take these exactly-known contributions into account, we match our finite-width NLL results to the sum of the Born cross section including finite-width effects and the NLO corrections in the narrow-width approximation. In practice, we approximate the Born cross section by $\sigma_{\text{Off}}^{(0)}$ defined in Section 4.2.4 and computed with `Whizard`. The total hadronic cross sections at NLO for finite widths are thus approximated as:

$$\sigma_{\text{NLO}}(\bar{\Gamma}) = \sigma_{\text{Off}}^{(0)}(\bar{\Gamma}) + (\sigma_{\text{NLO,NWA}} - \sigma_{\text{NWA}}^{(0)}). \quad (4.41)$$

Here $\sigma_{\text{NLO,NWA}}$ is the NLO result obtained with `PROSPINO` and $\sigma_{\text{NWA}}^{(0)}$ is the leading-order production cross section for stable squarks or gluinos (see Section 4.2.4). Analogously to the narrow-width case in Chapter 3 [47], the NLL resummed cross section is then matched to $\sigma_{\text{NLO}}(\bar{\Gamma})$:

$$\begin{aligned} \sigma_{\text{NLL}}^{\text{matched}}(\bar{\Gamma}) &= \Delta\sigma_{\text{NLL}}(\bar{\Gamma}) + \sigma_{\text{NLO}}(\bar{\Gamma}), \\ \Delta\sigma_{\text{NLL}}(\bar{\Gamma}) &= \sigma_{\text{NLL}}(\bar{\Gamma}) - \sigma_{\text{NLL, LO}}(\bar{\Gamma}) - \sigma_{\text{NLL, NLO}}(\bar{\Gamma} = 0). \end{aligned} \quad (4.42)$$

²⁸This is related to the replacement of the invariant mass $M_{\tilde{s}_1 \tilde{s}_2} \rightarrow M'_{\tilde{s}_1 \tilde{s}_2}$ for the E -implementation in Section 4.3.1.

Here $\sigma_{\text{NLL}, (\text{N})\text{LO}}$ is the expansion of the NLL resummed cross section to (N)LO accuracy. For comparison we will also consider results where the decay width is set to zero in the higher-order corrections, i.e.

$$\sigma_{\text{NLL}}^{\text{matched}}(0) = \Delta\sigma_{\text{NLL}}(0) + \sigma_{\text{NLO}}(\bar{\Gamma}). \quad (4.43)$$

The result (4.42) is our best current prediction. It could be made more realistic by replacing the off-shell approximation for the tree cross section by a full LO simulation including acceptance cuts, which we leave for further investigation. Contrary to the NLO cross section $\sigma_{\text{NLO}}(\bar{\Gamma})$, the matched NLL cross section $\sigma_{\text{NLL}}(\bar{\Gamma})$ does contain finite-width effects at NLO from the resummed Coulomb and soft corrections. It is also interesting to consider only the finite-width corrections beyond NLO. This shows the size of the effects that are beyond a possible future exact NLO calculation of the processes (4.6). For this purpose we consider the alternative matching (cf. Eq. (C.1))

$$\begin{aligned} \sigma_{\text{NLL}}^{\text{matched},(2)}(\bar{\Gamma}) &= \Delta\sigma_{\text{NLL}}^{(2)}(\bar{\Gamma}) + \sigma_{\text{NLO}}(\bar{\Gamma}), \\ \Delta\sigma_{\text{NLL}}^{(2)}(\bar{\Gamma}) &= \sigma_{\text{NLL}}(\bar{\Gamma}) - \sigma_{\text{NLL}, \text{LO}}(\bar{\Gamma}) - \sigma_{\text{NLL}, \text{NLO}}(\bar{\Gamma}). \end{aligned} \quad (4.44)$$

For the results shown in this Section we take as a representative average mass $\bar{m} = 1500$ GeV and the setting is the LHC at a centre-of-mass energy of $\sqrt{s} = 8$ TeV. We use the β -implementation as default and the scales are taken to be the same as in the zero-width case Chapter 3 [47]: $\mu_f = \bar{m} = 1500$ GeV, $\mu_h = 2\bar{m} = 3000$ GeV and a running soft and Coulomb scale. We note that the soft and Coulomb scales are always fixed below threshold. The LO widths in Eq. (4.4) for the squarks and gluinos are used.

Figure 4.9 shows the ratio of the matched NLL cross section for finite widths (4.42) to that for the zero-width case (4.43) (red, dot-dashed) as a function of the gluino-to-squark mass ratio. The green band gives the total uncertainty for the zero-width NLL result, including ambiguities in soft, hard, factorization and Coulomb-scale choices and the difference of the E - and β -implementation, added up in quadrature as discussed in detail in Chapter 3 [47]. The difference of the E - and β -implementation alone is given by the blue band. The red band indicates the ambiguities related to our implementation of the finite-width effects, as estimated by the $\bar{\Delta}\bar{E}$ variation as described in Eq. (4.38) and the difference of the E - and β -implementations, summed in quadrature. The plots also show the ratio of the average decay width to the average mass of the produced particles.

It can be seen that for all processes the finite-width result including uncertainties lies within the uncertainty estimate of the NLL calculation for stable squarks and gluinos if $\bar{\Gamma}/\bar{m} \lesssim 5\%$. For larger widths, large corrections are observed for the gluino-gluino and squark-gluino processes, where the ratio $\bar{\Gamma}/\bar{m}$ grows up to 12%, and the finite-width corrections become of the order of 20%. In this case the uncertainties due to the treatment of finite-width effects are of a similar magnitude as those due to scale and resummation ambiguities. This indicates that corrections beyond the leading $E \rightarrow E + i\bar{\Gamma}$ replacement, such as higher-order non-relativistic corrections and non-resonant corrections become relevant. A more accurate treatment would require the

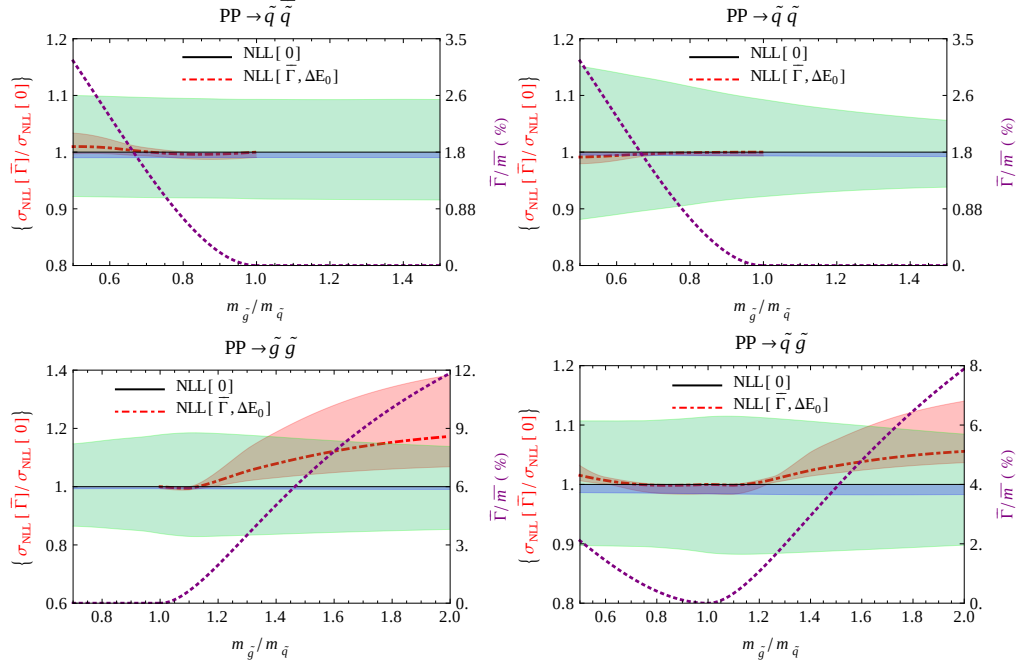


Figure 4.9: The plots show the ratio $\sigma_{\text{NLL}}^{\text{matched}}(\bar{\Gamma})/\sigma_{\text{NLL}}^{\text{matched}}(0)$ of the matched finite-width NLL cross section (4.42) to the zero-width approximation (4.43) (red dot-dashed, left-hand vertical axis) for $\bar{m} = 1500$ GeV and $\sqrt{s} = 8$ TeV and the ratio $\bar{\Gamma}/\bar{m}$ in per-cent (purple dotted, right-hand vertical axis). Both quantities are plotted as a function of the gluino-to-squark mass ratio. The NLL result is for default $\Delta E = \Delta E_0$. The red band represents the error from ΔE variation and E vs β uncertainty added in quadrature, the green band the total error for zero width, the blue band the E vs β uncertainty for the zero-width case.

matching of the NLL results to the exact NLO QCD cross sections for the full $\tilde{q}\tilde{q}q$ or $\tilde{q}\tilde{q}qq$ -processes and the inclusion of the $N^{3/2}\text{LO}$ corrections in the counting (4.27) that are expected to be of the order $\delta^{3/2} \sim (\bar{\Gamma}/\bar{m})^{3/2} \sim 5\%$, both of which are not available at the moment. Nevertheless it is interesting to anticipate such a future matching to a full NLO calculation by using the matching prescription (4.44) where the finite-width effects are included purely beyond NLO. The results are shown in Figure 4.10, where the colour coding used for the lines is the same as in Figure 4.9. It is seen that the finite-width effects on the central values, as well as ambiguities due to the ΔE variation and the E - and β -implementation difference, are much smaller than for the matching (4.42). In particular, the total uncertainty bands related to the finite-width implementation (red) lie now fully within the total zero-width NLL error bands (green). The difference of the plots in Figure 4.9 and Figure 4.10 indicates the potential impact of a full NLO

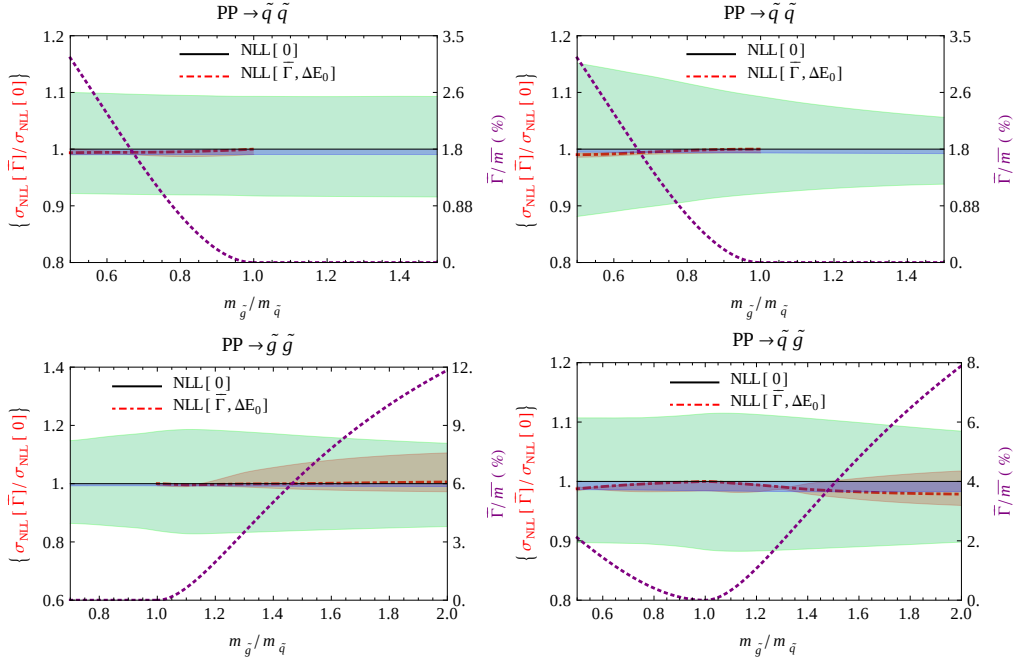


Figure 4.10: Same as in Figure 4.9, but for the ratio $\sigma_{\text{NLL}}^{\text{matched},(2)}(\bar{\Gamma})/\sigma_{\text{NLL}}^{\text{matched}}(0)$ of the matched cross section (4.44) including only finite-width effects beyond NLO to the zero-width approximation (4.43). See Figure 4.9 and the text for explanation.

SQCD calculation for the processes (4.6) and shows that the finite-width effects beyond NLO are expected to be small.

To assess the size of the finite width corrections relative to the NLL soft and Coulomb corrections, in Figure 4.11 we compare the K -factors for our NLL result including finite decay widths to the result for stable squarks and gluinos (4.43). The K -factors relative to the NLO prediction are defined as $K_{\text{NLL}} = \sigma_{\text{NLL}}/\sigma_{\text{NLO}}$, where the finite-width approximation (4.41) is always used for the denominator. We also show the result NLL_{noBS} defined as the cross section in the stable case, excluding the bound-state contribution below threshold. In general, for relatively narrow particles with $\bar{\Gamma}/\bar{m} \lesssim 5\%$ the K -factors including finite-width effects are well approximated by those for the zero-width case including the bound-state contribution, with the exception of squark-gluino production and gluino-gluino production for the case $m_{\tilde{g}} > m_{\tilde{q}}$, where larger finite-width effects are observed.

The relatively large finite-width corrections for gluino-production processes for $m_{\tilde{g}} > m_{\tilde{q}}$ have to be compared to the contributions of these processes to the total SUSY production rate.

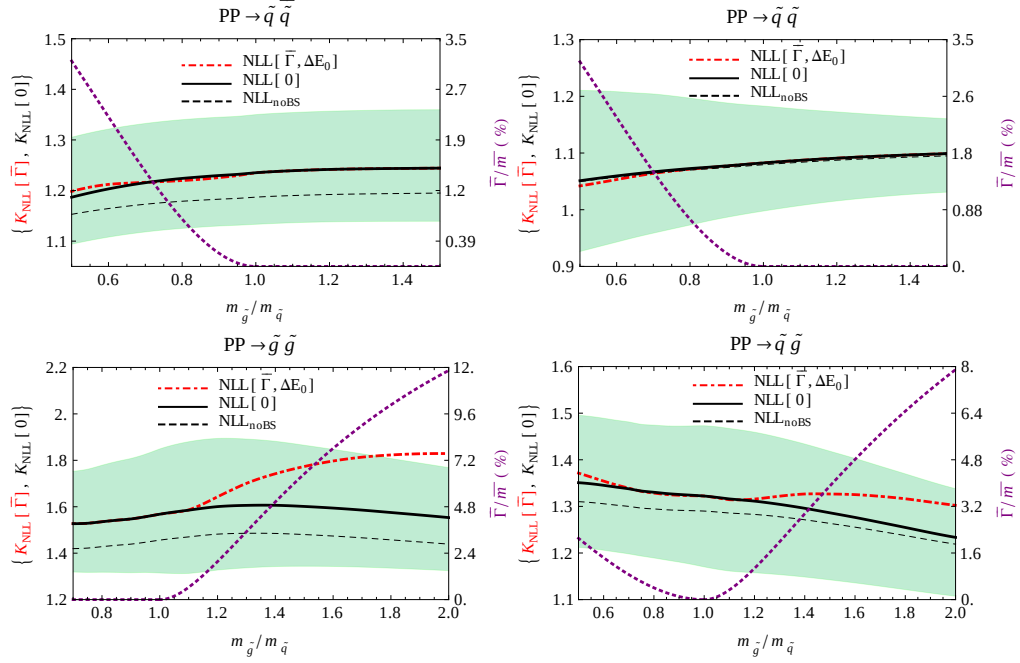


Figure 4.11: The K -factors of the NLL cross sections including (red dot-dashed) and excluding (black) the widths for default $\Delta E = \Delta E_0$ and $\bar{m} = 1500$ GeV at centre of mass energy $\sqrt{s} = 8$ TeV. Dashed black: NLL resummed cross sections excluding bound-state contributions. Green band: Total error for zero width.

This can be seen in Figure 4.12, where in the left plot the K_{NLL} -factor is shown for the finite-width and zero-width case. The effect of the QCD widths is seen to be negligible and the finite width can be safely set to zero when it comes to the total SUSY cross section. This is also expected since the numerically dominant processes are almost always those whose final-state particles are stable against SQCD decay, as can be seen from the right plot of Figure 4.12. The inclusion of electroweak widths is not expected to change our qualitative results. These can be dominating when $r \simeq 1$, but are only about the order of $\tilde{\Gamma}/\bar{m} \sim 1\%$ and thus the effects are expected to be negligible.

As mentioned in Section 4.1, in the study of finite-width effects considered in this work we have neglected the decay of gluinos to stops. We do not expect the inclusion of this additional decay channel to significantly change the results presented in this section, except in the case in which gluino production is the dominant SUSY production channel and the gluinos only decay to stops. This is for example the case for scenarios with the mass hierarchy $m_{\tilde{t}} < m_{\tilde{g}} \ll m_{\tilde{q}}$, with all squarks heavy except for stops. To have an estimate of the effects of a finite gluino width in this particular situation in Figure 4.13 we show the NLL cross section for gluino-gluino

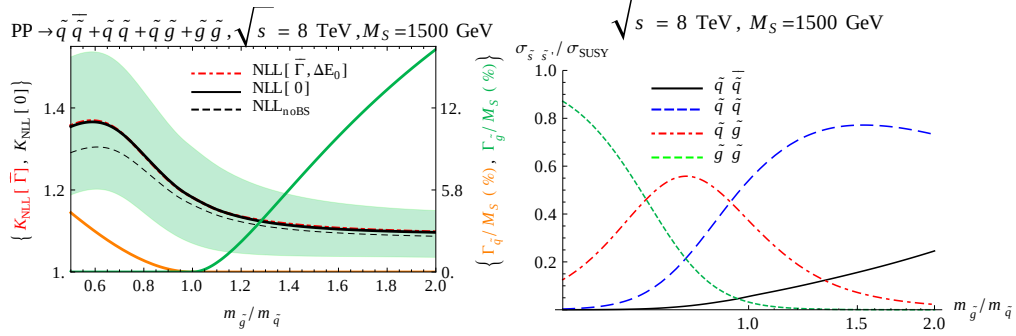


Figure 4.12: Left plot: The NLL K -factor including (dot-dashed red) and excluding (black) the widths for default $\Delta E = \Delta E_0$ and $M_S = (m_{\tilde{q}} + m_{\tilde{g}})/2 = 1500$ GeV at centre of mass energy $\sqrt{s} = 8$ TeV. Dashed black: NLL K -factor excluding bound-state contributions. Green band: Total error for zero width. The plot also shows the squark (thick orange) and gluino (thick green) widths (right vertical axis). Right plot: The ratio of the four relevant NLL cross sections to the total SUSY cross section.

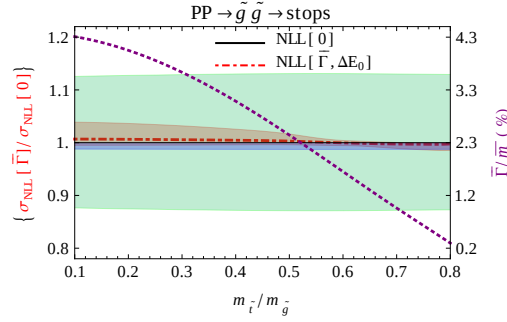


Figure 4.13: Ratio of the matched finite-width NLL cross section (4.42) to the zero-width approximation (4.43) for gluino-pair production and subsequent decay to stops, as a function of $m_{\tilde{t}}/m_{\tilde{g}}$ for a fixed gluino mass $m_{\tilde{g}} = 1000$ GeV, a common light-flavour squark mass of $m_{\tilde{q}} = 2000$ GeV. The error bands for finite- and zero-width curves are defined as in Figures 4.9 and 4.10.

production as a function of the ratio $m_{\tilde{t}}/m_{\tilde{g}}$ for a fixed gluino mass $m_{\tilde{g}} = 1000$ GeV, a common light-flavour squark mass of $m_{\tilde{q}} = 2000$ GeV and top mass $m_t = 174$ GeV. As in Figure 4.9 the cross section is normalized to the gluino-pair production cross section at zero width, and the green band represents the total uncertainty of the zero-width result, while the purple dashed curve gives the gluino width as a function of $m_{\tilde{t}}/m_{\tilde{g}}$. As can be seen from the plot the finite-width effects amount to a maximum of $\sim 1 - 2\%$ for small stop masses and the uncertainty associated with the finite-width result is contained in the zero-width case error band. We thus expect corrections to the plots in Figures 4.11, 4.12 to amount to at most few percents.

4.4 Summary

In this chapter we have considered corrections related to finite widths of squarks and gluinos on the NLL resummed results presented in Chapter 3 [47]. As anticipated, a finite decay width leads to a screening of higher-order soft and Coulomb corrections. However we have found that for decay widths of the order of $\bar{\Gamma}/\bar{m} \lesssim 5\%$, corresponding to gluino to squark mass ratios of $0.5 \lesssim m_{\tilde{g}}/m_{\tilde{q}} \lesssim 1.4$, finite-width effects on the NLL corrections are small and within the uncertainty of the soft and Coulomb resummation including bound-state effects performed in Chapter 3 [47].

For larger decay widths of the order of $\bar{\Gamma}/\bar{m} \sim \alpha_s$ non-resonant contributions and higher order non-relativistic corrections become important. In this case it becomes necessary to perform a calculation of order $\delta^{3/2} \sim \alpha_s \sqrt{\bar{\Gamma}/\bar{m}}$ in unstable particle-effective theory matched to an exact NLO calculation for three- or four-particle final states including finite-width effects, which should be feasible but challenging with current NLO technology. The accuracy of such a calculation is expected to be better than 5%. For the doubly-resonant contributions to the cross section, we have observed that finite-width corrections beyond NLO are small.

Because the pair production processes of the lighter coloured sparticles, which are stable with respect to SQCD, dominate the total SUSY cross section, finite-width effects on the total SUSY production rate are negligible on the whole range of squark and gluino widths considered.

NNLL soft resummation for SUSY-QCD

The previous two chapters contained the results of NLL soft and Coulomb resummation for coloured SUSY pair production at the LHC. This chapter serves as a follow-up in which we discuss and present partial results for NNLL resummation and its subsequent error reduction for the same processes. There are many new aspects which present themselves at NNLL. First of all, the evolution equations and Sudakov exponents at NNLL require knowledge of the next loop (cusp-) anomalous dimensions [60]. The form of these relevant up to NNLL have been gathered in Appendix A [58]. Secondly, at NNLL one requires the next order of the relevant functions in the resummation Eq. (2.26). In particular one requires the NLO Coulomb Green's function and the one-loop soft function, which are known for some time and can be found in [5, 21, 58, 90, 128]. The most difficult and time-consuming calculations are the process-dependent one-loop hard matching coefficients appearing in the one-loop hard functions. The one-loop hard matching coefficients in the Mellin-space formalism have been published for squark-antisquark in [49] and recently for the remaining three processes in [88]. By matching the NLO expansion of the resummation formula in the Mellin-space formalism to those in the momentum-space formalism (cf. Eq. (3.4)) one can in principle convert the one-loop hard matching coefficients from one formalism to the other. Finally, for a combined soft and Coulomb NNLL resummation one also needs the resummed non-Coulomb Green's function for the $\alpha_s(\frac{\alpha_s}{\beta})^m(\alpha_s \log(\beta))^n$ corrections (cf. Eq. (2.14)). At this point in time, these are only known for $n = 1$ and fermion-fermion pair production, relevant such as in top pair production [75].

It is not the purpose of this chapter to present the combined NNLL soft and Coulomb SUSY production cross sections. The main purpose of this chapter is to highlight the use and need of performing NNLL or higher order soft and Coulomb resummation in precision collider physics for the purpose of further error reduction. This has already been shown for top pair production in the momentum-space formalism [75] where a combined soft and Coulomb resummation was performed and for squark-antisquark production in the Mellin-space formalism [49], where besides the soft corrections the first Coulomb correction was included. In this chapter we present some partial NNLL soft resummed hadronic cross sections for all the coloured SUSY pair production

processes at the LHC in the momentum-space formalism, including stop-antistop production. More accurately, the Coulomb corrections are set to zero and moreover also the one-loop hard functions are neglected. Note that even though the one-loop matching coefficients for squark-antisquark have been published in [49], while for the other processes they can be found in the Ph.D. thesis [88], we will not use them here. Since we do not take the one-loop hard functions into account, these results are of course only a partial approximation to NNLL_s, which would also include the α_s -suppressed soft corrections. Nonetheless, it will be shown that the inclusion of these partial soft corrections are already enough to drastically lower the uncertainties related to the resummation procedure.²⁹ These results should serve as the first step towards a full NNLL resummation for coloured SUSY pair production.

The outline of this chapter is as follows. We start by reviewing in Section 5.1 the soft corrections and the partial subset that we will take into account in our numerically resummed results in this chapter, which we will afterwards show in Section 5.2 for squark, gluino and stop pair production. Finally, in Section 5.3 our conclusions are presented.

5.1 NNLL soft corrections

Let us consider the pure soft corrections without Coulomb corrections, that are set to zero by taking the LO potential function $J^{(0)}(E) = \frac{(2m_r)^2}{2\pi} \sqrt{E/(2m_r)}$ in the resummation formula (2.26), after which the resummed partonic cross section using the one-loop soft function given in Eq. (2.28) becomes:

$$\begin{aligned} \hat{\sigma}_{pp'}^{\text{NNLL}_{s+h}}(\hat{s}, \{\mu\}) &= \frac{(2m_r)^{3/2} e^{-2\gamma_E \eta}}{2\pi(2M)^{2\eta}} \sum_{R_\alpha} H_{pp'}^{R_\alpha}(\mu_h) U_i(\{\mu\}) \int d\omega \sqrt{E - \frac{\omega}{2}} W_{R_\alpha}^{(1)}(\omega, \mu_s, 2\eta), \\ W_{R_\alpha}^{(1)}(\omega, \mu_s, 2\eta) &= \frac{\omega^{2\eta-1}}{\Gamma(2\eta)} \left(1 + \frac{\alpha_s(\mu_s)}{\pi} \left((1 + H_{2\eta-1} - \log(\omega/\mu_s)) C_R \right. \right. \\ &\quad \left. \left. + \left((H_{2\eta-1} - \log(\omega/\mu_s))^2 + \pi^2/24 - H'_{2\eta-1} \right) (C_{r1} + C_{r2}) \right) \right), \end{aligned} \quad (5.1)$$

where H_n is the n th harmonic number and $\{\mu\}$ denotes the scales $\{\mu_f, \mu_h, \mu_s\}$. The above resummed partonic cross section includes all the exact NLL soft corrections of the form $\alpha_s^k \log^{2k}(\beta)$, $\alpha_s^k \log^{2k-1}(\beta)$ and their interactions. Besides these it also resums the higher order exact NNLL soft corrections $\alpha_s^k \log^{2k-2}(\beta)$, $\alpha_s^k \log^{2k-3}(\beta)$ and the resulting interference terms. The soft convolution with the trivial potential function can be performed analytically and the resummed

²⁹The inclusion of the one-loop hard matching coefficients are not expected to lower the uncertainties of the pure soft corrections by much, as long as the evolution equations are taken at NNLL order.

partonic cross section is then given as follows:

$$\hat{\sigma}_{pp'}^{\text{NNLL}_{s+h}}(\hat{s}, \{\mu\}) = \frac{m_r^{3/2} e^{-2\gamma_E \eta}}{2\pi^{1/2} (2M)^{2\eta}} \sum_{R_\alpha} H_{pp'}^{R_\alpha}(\mu_h) U_i(\{\mu\}) W_{R_\alpha}^{(1)}(2E, \mu_s, 2\eta + 3/2). \quad (5.2)$$

We recall Section 2.3.2 where it was shown that the resummation formula is also applicable to P-wave processes, relevant for the quark-antiquark channel for stop-antistop production. In this case the LO potential function equals $J_P^{(0)}(E) = \frac{(2m_r)^4}{2\pi} (E/(2m_r))^{3/2}$ and the above resummed partonic cross section can again be expressed analytically as follows:

$$\hat{\sigma}_{pp',P}^{\text{NNLL}_{s+h}}(\hat{s}, \{\mu\}) = \frac{3m_r^{5/2} e^{-2\gamma_E \eta}}{4\pi^{1/2} (2M)^{2\eta}} \sum_{R_\alpha} H_{pp'}^{R_\alpha}(\mu_h) U_i(\{\mu\}) W_{R_\alpha}^{(1)}(2E, \mu_s, 2\eta + 5/2). \quad (5.3)$$

The above two equations for the S- and P-wave processes agree with our previous results in Eq. (3.4) and (3.16) respectively if we neglect the NLO $\mathcal{O}(\alpha_s(\mu_s))$ contribution arising from the one-loop soft function. The resummed partonic cross sections in Eq. (5.2) and (5.3) can be expanded to NLO (see Appendix A) and one finds our previous results given in Eq. (3.11) and (3.15) respectively when setting the Coulomb correction to zero. These will be used in order to subtract and match to the exact NLO cross section calculated with PROSPINO [34]. The possible matching procedures that we consider are [75]:

$$\text{NNLL}_1 = \text{NNLL} - \text{NNLL}(\alpha_s) + \text{NLO}, \quad (5.4)$$

$$\text{NNLL}_2 = \text{NNLL} - \text{NNLL}(\alpha_s^2) + \text{NLO} + \text{NNLO}_{\text{app}}^{(2)}. \quad (5.5)$$

Here, $\text{NNLL}(\alpha_s)$ and $\text{NNLL}(\alpha_s^2)$ are the expansion terms of NNLL to NLO and NNLO respectively. The approximation NNLO_{app} , is found by expanding the above NNLL cross section to NNLO and keeping those terms that are accurately predicted at NNLL [73]. The upper index (2) refers then to the α_s^2 term in this expansion. The factorization and hard scales are taken as $\mu_f = k_f M$, $\mu_h = k_h M$ while the renormalization scale is fixed $\mu_r = \mu_f$. The only free parameter that is left is the soft scale μ_s , for which we again consider the fixed and running scale methods. The fixed soft scales are the same as those used for NLL resummation in the previous chapters. Here, the running soft scale is again taken as default and the corresponding choice of β_{cut} is similar to the NLL case and explained thoroughly in [75]. One considers the following cross sections:

$$\hat{\sigma}_{\bar{s}s'}(A_<, B_>, \beta_{\text{cut}}) = \hat{\sigma}_{\bar{s}s'}^{A_<} \theta(\beta_{\text{cut}} - \beta) + \hat{\sigma}_{\bar{s}s'}^{B_>} \theta(\beta - \beta_{\text{cut}}), \quad (5.6)$$

and minimizes the width of the envelope created by the eight different possible combinations of $A_< \in \{\text{NNLL}_1, \text{NNLL}_2\}$ and $B_> \in \{\text{NNLL}_{\text{app}}, \text{NNLL}_2, \text{N}^3\text{LO}_A, \text{N}^3\text{LO}_B\}$. The cross section N^3LO_A is defined as the N^3LO expansion of the NNLL cross section, while N^3LO_B is the N^3LO

expansion which only keeps the terms that are completely predicted at NNLL. As was previously the case when calculating NLL_{s+h} or NLL_s , we use the β_{cut} values found via the above procedure where the LO Green's function from Eq. (3.7) is included.³⁰ The related uncertainties are calculated in the same way as for the NLL case with the appropriate changes for NNLL and are not repeated here [75].

In the following we neglect the one-loop hard function and furthermore in the resummation formula (2.26) we take the hard scale equal to the factorization scale. We then use the evolution function $U(\{\mu\})$ including the higher order anomalous dimensions which correctly takes the factorization scale dependence at NNLL into account. In effect this means that we are considering a partial soft resummation and approximation of NNLL_s and we only resum the following terms (cf. Eq. (3.1)):

$$\hat{\sigma} \sim \exp\{\log(\beta)g_0(\alpha_s \log(\beta)) + g_1(\alpha_s \log(\beta)) + \alpha_s g_2(\alpha_s \log(\beta))\} \times \mathcal{O}(1). \quad (5.7)$$

The corrections which follow from α_s times the above would be captured by the one-loop hard function and are important and may even be large [49], though can in principle be included by simply multiplying our numerical partonic and hadronic cross sections below with the factor $(1 + (\alpha_s/(4\pi))h_{pp'}^{(1),R\alpha}(\mu_h))$.

5.2 Results for partial NNLL soft resummation

In this Section we will show our numerical results for resummation of the partial soft corrections, discussed at the end of the previous section, at the LHC with centre of mass energies $\sqrt{s} = 8$ TeV and $\sqrt{s} = 14$ TeV. Throughout this Section the one-loop hard functions have been set to zero and the squark-gluino mass ratio equals $r = \frac{m_{\tilde{g}}}{m_{\tilde{q}}} = 1$. We use the MSTW08NLO PDF set [8] for the parton distribution functions and the associated strong coupling constant $\alpha_s(M_Z) = 0.1202$ and furthermore use `PROSPINO` to calculate the exact NLO cross section. Analogous to the previous chapters, the results will be again presented in the form of the K -factor defined as:

$$K_X := \frac{X}{\text{NLO}}. \quad (5.8)$$

Plots of the K -factors for NLL, NLL_s and NNLL_s with a running soft scale at a centre of mass energy 8 TeV are shown in Figure 5.1. All four individual SUSY pair production processes are shown, together with stop-antistop production and the total SUSY pair production as a function of the average produced mass $M = (M_{\tilde{s}} + M_{\tilde{s}'})/2$. Equivalent plots were shown for the NLL case in Figure 3.4. For small average masses, the NNLL_s corrections are about the

³⁰For SUSY processes the NLO partonic cross section is not exactly known and instead approximated by NLO_{app} calculated with the NLO scaling functions from Eq. (3.4) and (3.16). In case we neglect the one-loop hard functions we set $h_{pp'}^{(1),R\alpha}(\mu_f)$ to zero both in NLO_{app} as well as NNLO_{app} .

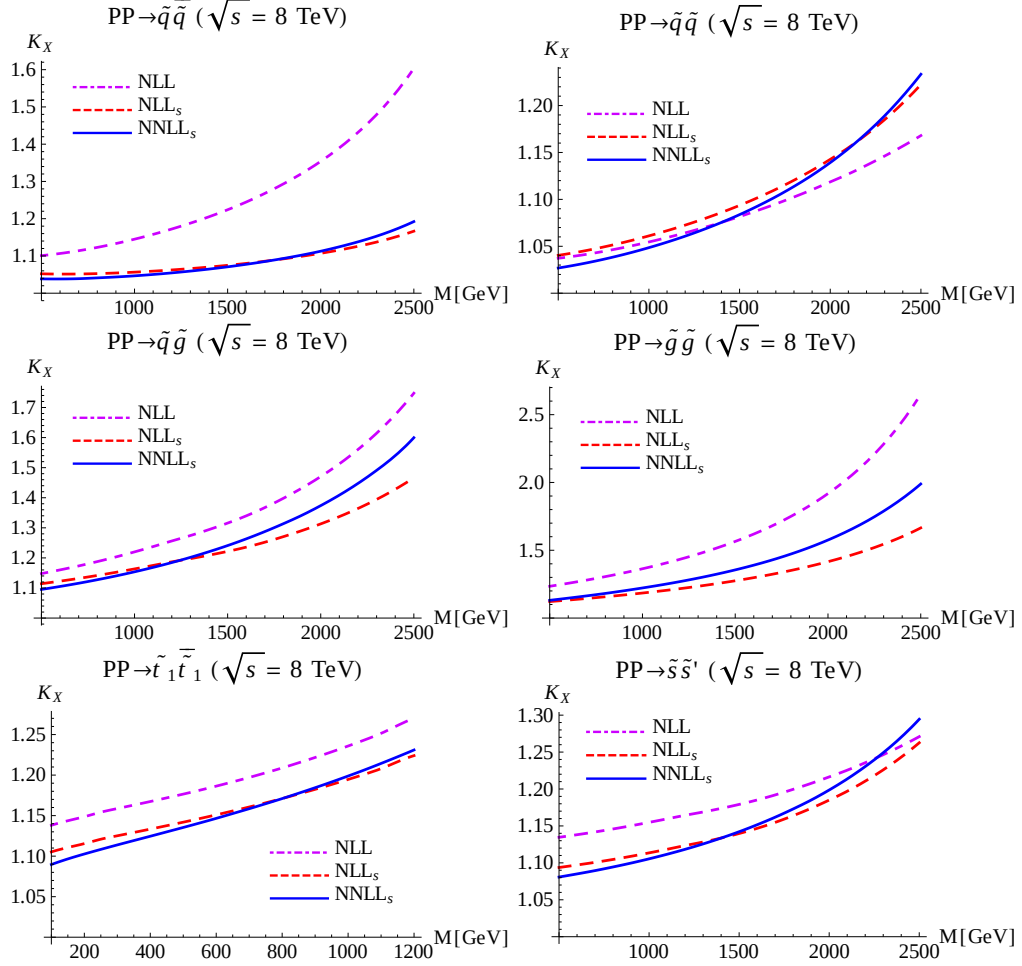


Figure 5.1: The K -factors as a function of M for squark-antisquark (top-left), squark-squark (top-right), squark-gluino (centre-left), gluino-gluino (centre-right), stop-antistop (bottom-left) and total SUSY (bottom-right) production with $\sqrt{s} = 8$ TeV at the LHC. The K -factors are shown for: NLL (dot-dashed purple), NLL_s (dashed red) and $NNLL_s$ (solid blue).

same size as the NLL_s corrections. As the average mass of the produced SUSY pair increases the threshold region plays a more dominant role and the pure soft corrections at NNLL order becomes larger than those at NLL order. The $NNLL_s$ corrections above NLL_s are between 1%

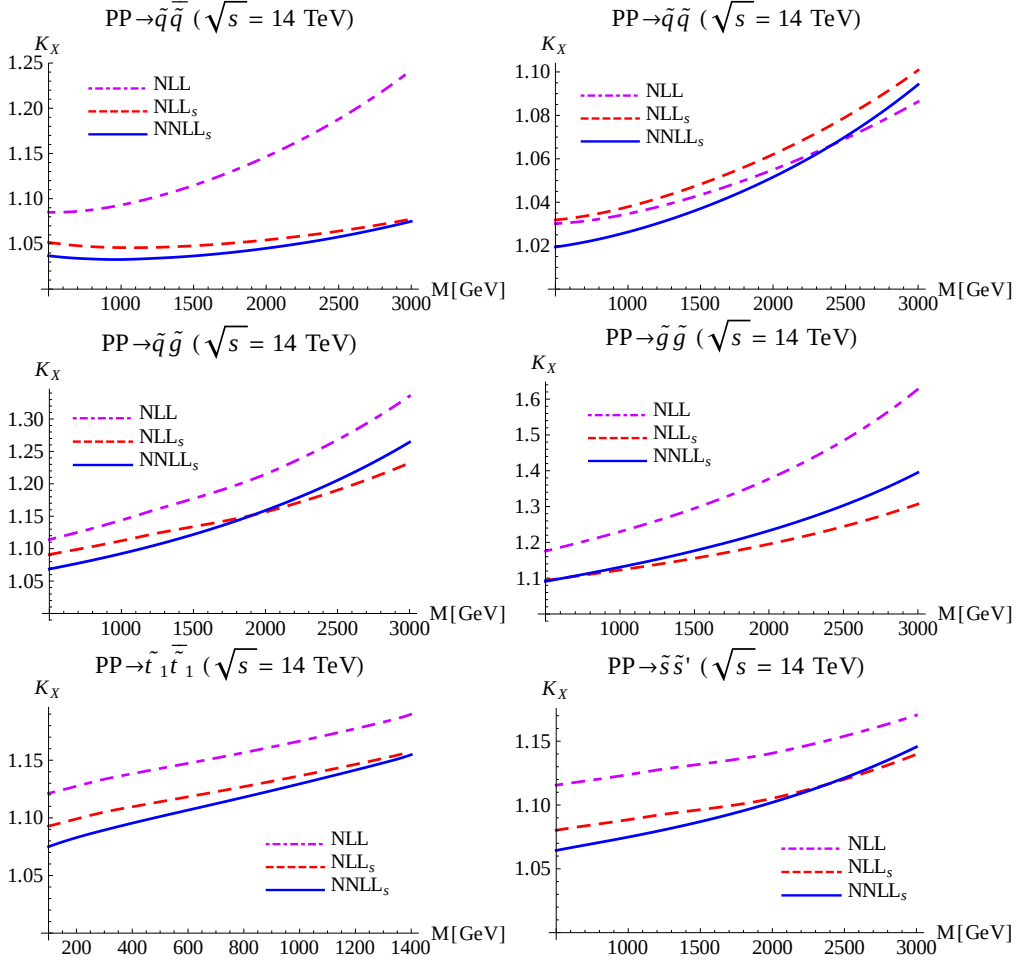


Figure 5.2: Similar to Figure 5.1, the K -factors are shown with $\sqrt{s} = 14$ TeV at the LHC for NLL (dashed purple), NLL_s (dashed red) and $NNLL_s$ (solid blue).

for the squark-squark process³¹ with its small colour charges and 30% of the NLO cross sections for the gluino-gluino process with its large colour charges at masses in the range of 2000 – 2500 GeV. The soft corrections for the total SUSY production process is enhanced by about 3% in this range. Similarly to the NLL soft corrections, the enhancement for stop-antistop is larger than for the squark-antisquark production process, which is a result of the larger relative contribution of

³¹The effect of the Coulomb corrections is negative in this case, as we have seen in our previous results for NLL in Chapter 3 [47].

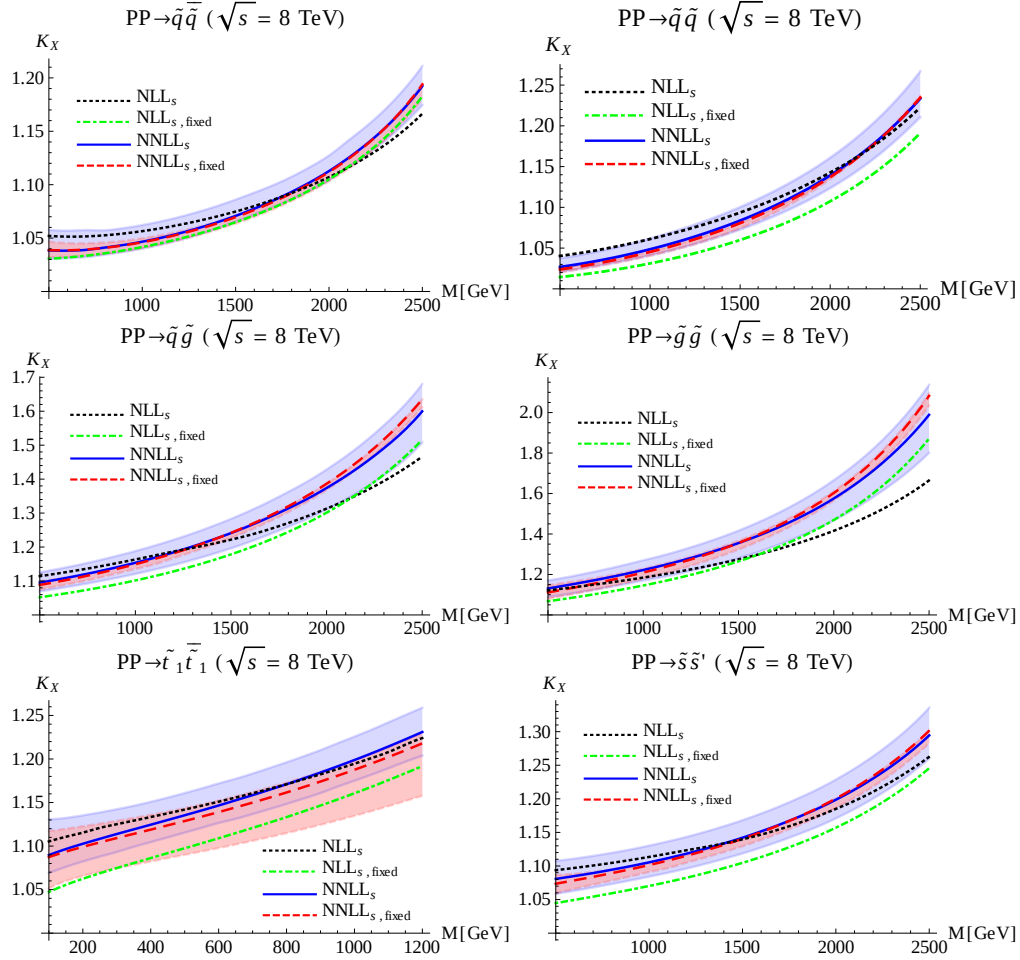


Figure 5.3: The K -factors when using a fixed or running soft scale, as a function of M for squark-antisquark (top-left), squark-squark (top-right), squark-gluino (centre-left), gluino-gluino (centre-right), stop-antistop (bottom-left) and total SUSY (bottom-right) production with $\sqrt{s} = 8$ TeV at the LHC. The K -factors are shown for: running soft scale NLL_s (dotted black), fixed soft scale NLL_s (dot-dashed green), running soft scale $NNLL_s$ (solid blue) and fixed soft scale $NNLL_s$ (dashed red). The resummation uncertainties are added for $NNLL_s$.

the gluon-gluon channel with its large colour charges.

The K -factors for NLL , NLL_s and $NNLL_s$ with a running soft scale at centre of mass energy 14 TeV are shown in Figure 5.2. There the pure soft corrections at $NNLL$ order are about 0 – 10%

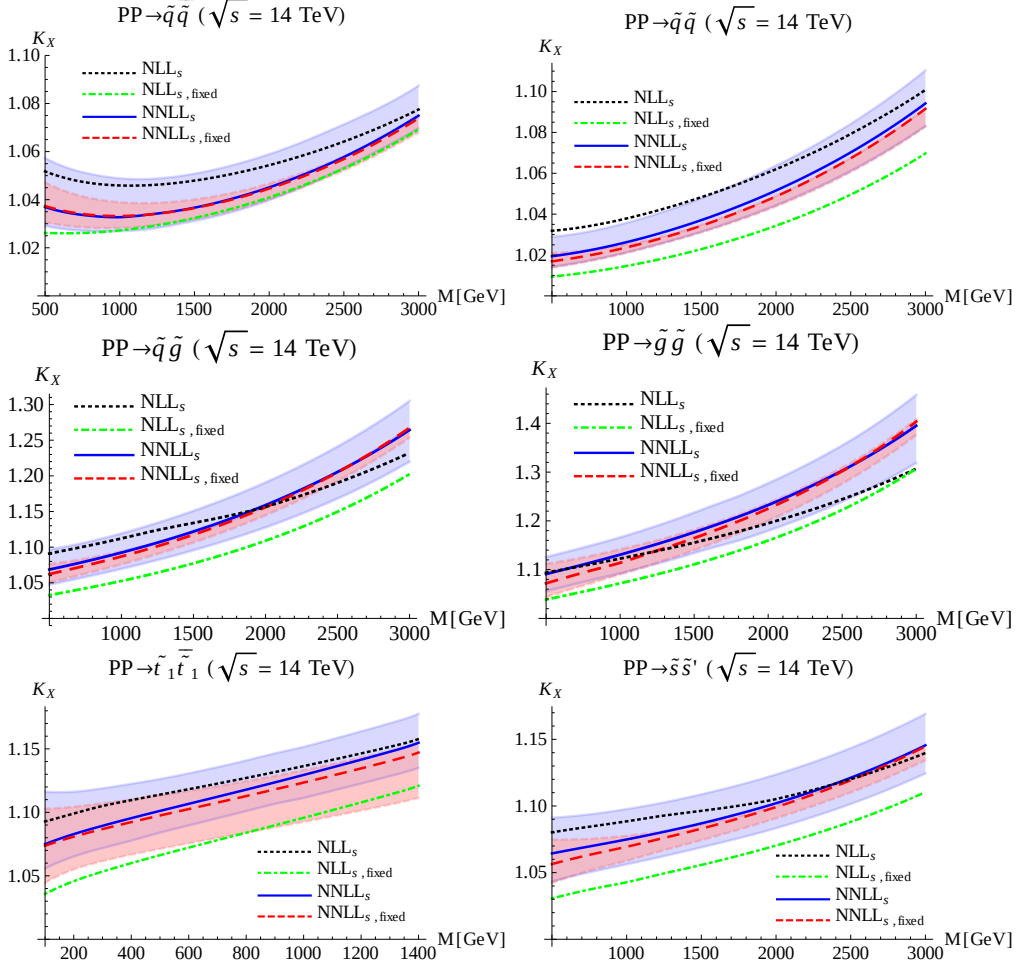


Figure 5.4: Similar to Figure 5.3, the K -factors and resummation uncertainties are shown for $\sqrt{s} = 14$ TeV at the LHC.

above those at NLL order in the large mass range, which is smaller than for the 8 TeV because the threshold region makes up a smaller fraction of the total phase space. For both sets of plots it is important to observe that the addition of Coulomb corrections and their interaction with the NLL soft corrections is more important because the corresponding contributions are larger than the NNLL pure soft corrections. This once more backs up and motivates the inclusion of the NLL and possibly higher order Coulomb corrections in combination with pure soft logs. It is interesting to consider the differences between the fixed and running soft scales. In Figures

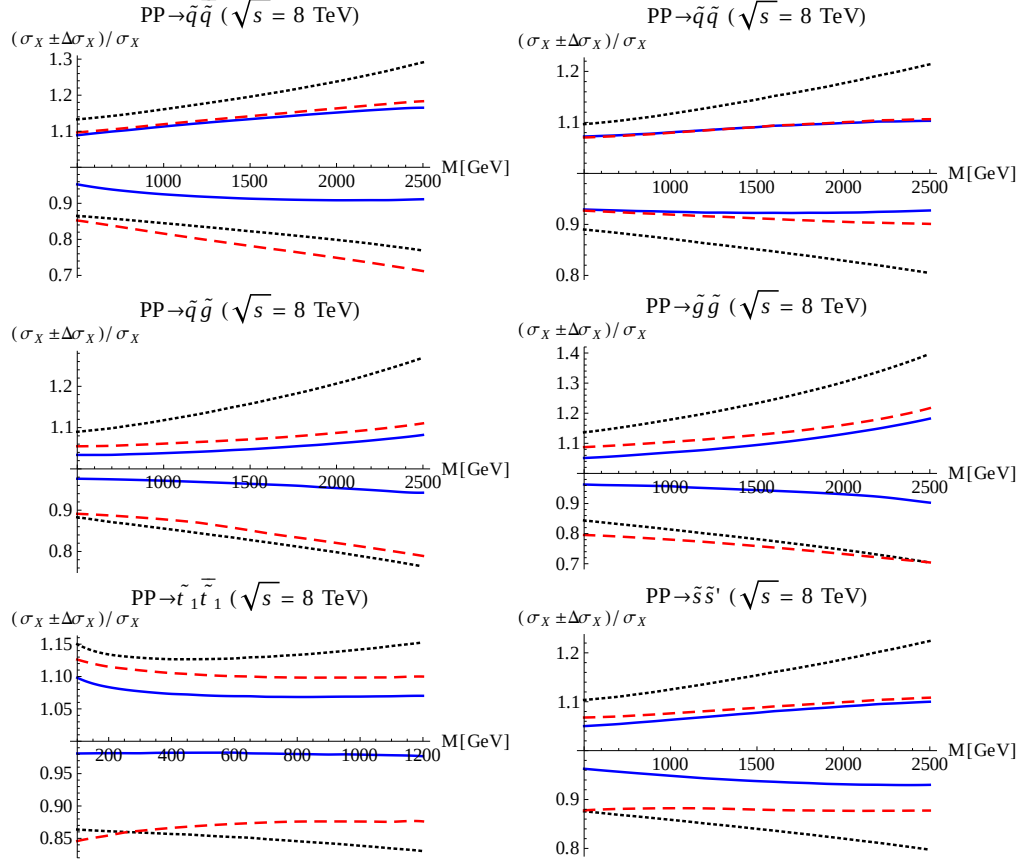


Figure 5.5: The total uncertainties normalized to the corresponding cross section with default scales as a function of M for squark-antisquark (top-left), squark-squark (top-right), squark-gluino (centre-left), gluino-gluino (centre-right), stop-antistop (bottom-left) and total SUSY (bottom-right) production with $\sqrt{s} = 8$ TeV at the LHC. The uncertainties are shown for: NNLL_s (solid blue), NLL_s (dashed red) and NLO (dotted black).

5.3 and 5.4 we plot the K -factors for NNLL_s using a fixed or running soft scale, together with their resummation uncertainty (cf. Figure B.1). The curves are plotted again as a function of the average produced mass M with the squark-gluino mass ratio taken as $r = 1$. At NNLL order the differences between the fixed and running scale plots are very small on the whole range of average masses. In particular, the resummation uncertainty envelopes of the fixed soft scale plots lie almost fully within those of the running soft scales. The envelopes of the running cases are larger than the fixed cases, mostly because of the β_{cut} variation error as was also seen in Chapter 3.

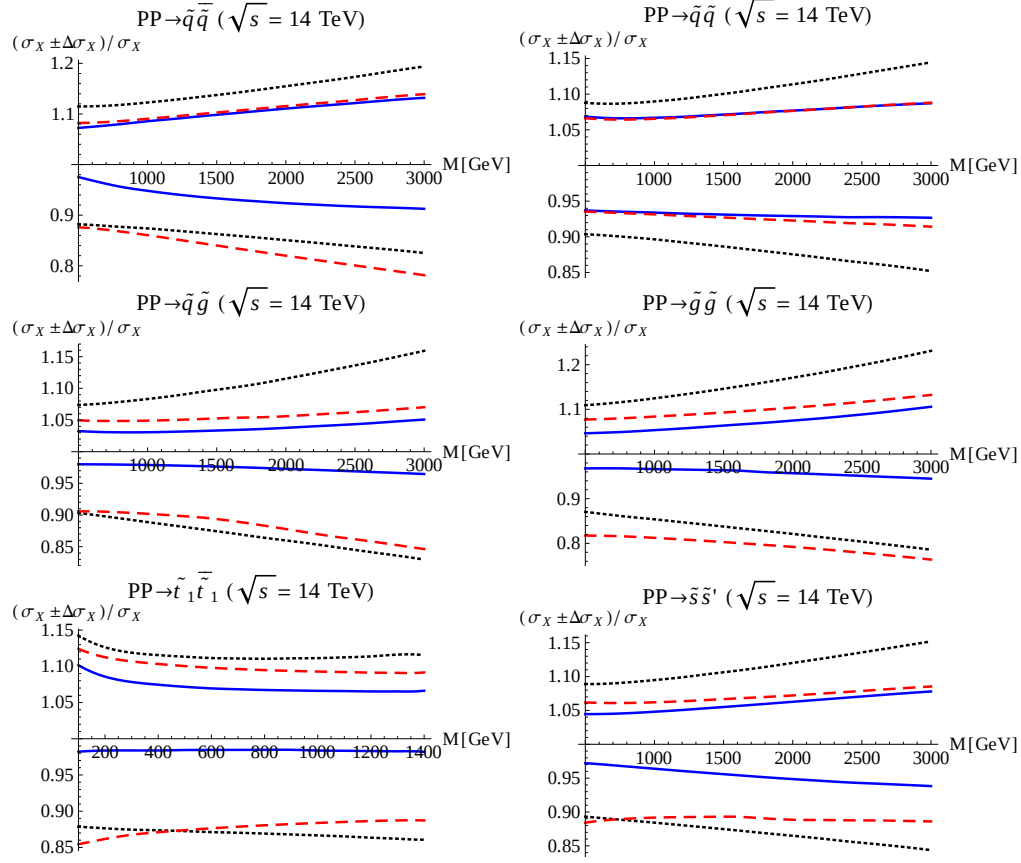


Figure 5.6: Similar to Figure 5.5, the total normalized uncertainties are shown for NNLL_s (solid blue), NLL_s (dashed red) and NLO (dotted black) with $\sqrt{s} = 14$ TeV at the LHC.

This running and fixed soft scale agreement is in stark contrast to the NLL_s plots, which still have a relatively large disagreement between the running and fixed soft scale plots even as the average produced mass increases. This implies that one has to include the soft corrections at NNLL in order to remove the ambiguity between the fixed and running soft scale choice. Again as we expect, because of the dominance of the threshold region, the differences at 8 TeV are smaller than those for the 14 TeV case.

The total uncertainties normalized to the corresponding cross section using default scales is given in Figure 5.5 at centre of mass energy $\sqrt{s} = 8$ TeV and Figure 5.6 at centre of mass energy $\sqrt{s} = 14$ TeV for NNLL_s (solid blue), NLL_s (dashed red) and fixed order NLO (dotted black). The inclusion of the soft corrections at NNLL_s order compared to NLL_s reduces the positive

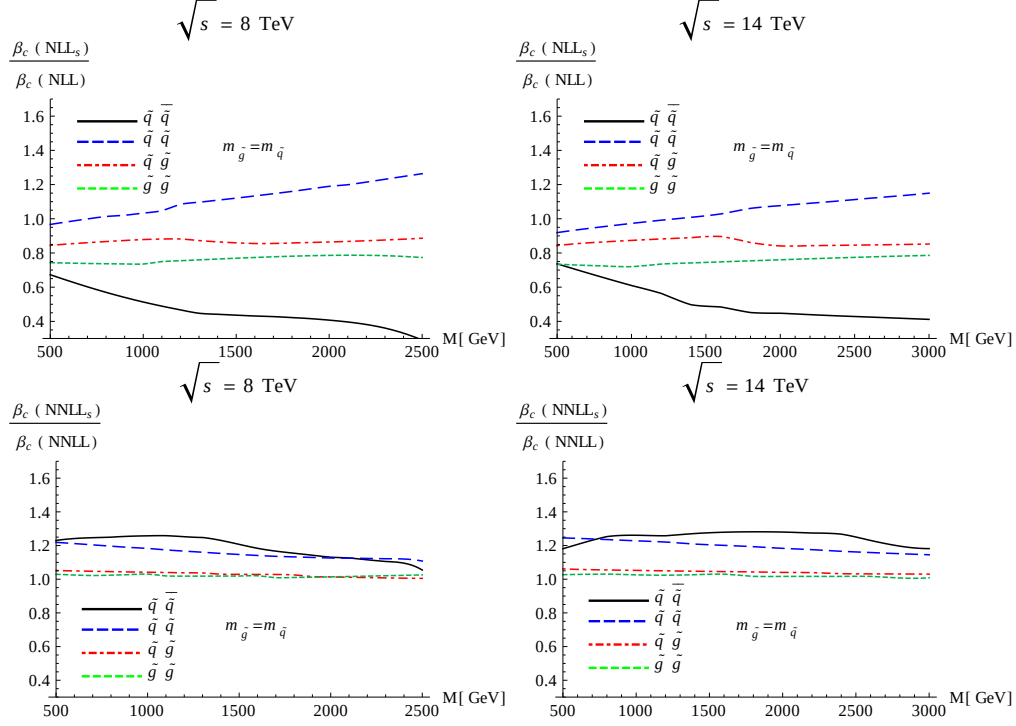


Figure 5.7: The ratio of $\beta_{\text{cut}}(\text{NLL}_s)$ to $\beta_{\text{cut}}(\text{NLL})$ at $\sqrt{s} = 8 \text{ TeV}$ (left) and $\sqrt{s} = 14 \text{ TeV}$ (right) and similarly the ratio of $\beta_{\text{cut}}(\text{NNLL}_s)$ to $\beta_{\text{cut}}(\text{NNLL})$ (down) for all four processes: squark-antisquark (solid black), squark-squark (dashed blue), squark-gluino (dot-dashed red) and gluino-gluino (dotted green).

errors slightly and the negative errors drastically. The reduction is enhanced at larger average produced masses and as one would expect the error reduction is not very large for squark-squark production because of the small colour charges involved. For the other processes, including stop-antistop, the total error gets reduced by approximately 50%. The total uncertainties for the total SUSY production cross section gets reduced by about 25%, to 8% of the NLO cross section and is approximately of the same order at 14 TeV.

This drastic decrease when including higher soft corrections is partly because of the large error related to the β_{cut} variation for the NLL_s case. The β_{cut} value used for the soft cases NLL_s and NNLL_s are those derived by including the LO Green's function in the envelope (cf. Appendix B), called henceforth $\beta_{\text{cut}}(\text{NLL})$ and $\beta_{\text{cut}}(\text{NNLL})$ respectively. Hence, the β_{cut} value is not strictly speaking varied at exactly the minimum of the envelope corresponding to the pure soft cases NLL_s and NNLL_s , denoted here by $\beta_{\text{cut}}(\text{NLL}_s)$ and $\beta_{\text{cut}}(\text{NNLL}_s)$ respectively. We can thus expect a larger and more conservative estimate of the error involved with the β_{cut} variation.

The ratio of $\beta_{\text{cut}}(\text{NLL}_s)$ to $\beta_{\text{cut}}(\text{NLL})$ and $\beta_{\text{cut}}(\text{NNLL}_s)$ to $\beta_{\text{cut}}(\text{NNLL})$ is plotted in Figure 5.7 for centre of mass energies $\sqrt{s} = 8$ TeV (left) and $\sqrt{s} = 14$ TeV (right) for all four SUSY processes. The plots show there is a much better agreement between $\beta_{\text{cut}}(\text{NNLL})$ and $\beta_{\text{cut}}(\text{NNLL}_s)$ compared to $\beta_{\text{cut}}(\text{NLL})$ and $\beta_{\text{cut}}(\text{NLL}_s)$, because the β_{cut} dependence of the envelope width (cf. Figure B.1) is captured at NNLL relatively better than at NLL by pure soft logs than their interactions with the Coulomb singularities. This implies that one lies very close to the minimum width of the envelope when calculating NNLL_s using $\beta_{\text{cut}}(\text{NNLL})$ in comparison to the NLL case. Hence, the corresponding β_{cut} variation for NNLL_s is smaller than that of NLL_s and at NNLL it does not matter much if one uses $\beta_{\text{cut}}(\text{NNLL})$ or the more appropriate $\beta_{\text{cut}}(\text{NNLL}_s)$. This is expected, since not only do the uncertainties themselves decrease at higher orders of resummation but also the ambiguity in the determination of the resummation parameters and scales.

5.3 Summary

In this chapter we have outlined the first step towards a combined soft and Coulomb NNLL resummation for coloured SUSY pair production by considering only the pure soft corrections at NNLL in momentum space, in which case the soft-potential convolution can be performed analytically. We have performed a partial soft resummation for the four SUSY production processes, as well as stop-antistop production by neglecting the one-loop hard function and setting the hard scale equal to the factorization scale. Our results show that the pure soft corrections for the total SUSY production cross section at NNLL enhance those at NLL by 3% and 1% of the NLO cross section at $\sqrt{s} = 8$ TeV and $\sqrt{s} = 14$ TeV respectively for large masses. More importantly, these higher order corrections decrease the total uncertainty by approximately 25%, to about 8% of the NLO cross section and the difference between the fixed and running soft scale becomes negligible when including the soft corrections at NNLL order. These results imply that for a good comparison of soft corrections between the Mellin-space approach, it is very important to include the NNLL soft corrections.

Conclusions and outlook

In this thesis we have studied the inclusive production of squarks and gluinos close to threshold at the LHC. In particular we were interested in the inclusive *cross section* for the production of pairs of squarks and gluinos in the MSSM. Of all the sparticle production cross sections at the LHC, those of the squarks and gluinos are the largest because of their strong coupling to the initial quarks and gluons. Hence, if SUSY is ever to be found at the LHC, it might be very likely via missing transverse energy signals arising from the production of squark and gluino pairs and their subsequent decay to the LSP. For these reasons it makes sense to have an accurate prediction of their production cross sections in order to get better bounds on the MSSM. The contributions arising from EW interactions have been calculated to be of the order of 5 – 10% and were neglected in this thesis. Consequently, we have been effectively calculating with SQCD throughout much of the thesis.

In Chapter 2 we calculated the one-loop soft- and Coulomb-gluon corrections to the production of heavy coloured particles near threshold. It was shown that the cross section gets logarithmic contributions in the non-relativistic heavy pair velocity β from soft-gluon exchanges between the initial and final states. Similarly, the exchange of Coulomb-gluons between the final states leads to contributions that scale as inverse powers of β . Near threshold, when $\beta \ll 1$ these terms are large and the usual power series in α_s breaks down and needs to be reorganized. Hence, instead of expanding in powers of α_s , one needs to *resum* powers of $\alpha_s \ln(\beta) \sim 1$ and $\alpha_s/\beta \sim 1$ to all orders. Resumming particular subsets of the whole power series is usually classified as a N^k LL resummation.

Traditionally, particle physicists have performed resummation by first transforming the cross section to Mellin-space and then resumming the cross section by solving RG equations of the functions related to soft-gluon contributions. This is very similar to resumming self-energies or other diagrams by solving the renormalization group equations for the corresponding Green's functions. This method is an efficient way to sum the soft logarithms but there exists as of yet no consistent way to simultaneously sum the Coulomb singularities in this approach. Recently a new method of resummation performed directly in momentum space and based on EFT's has

been developed, which resums simultaneously the soft logarithms and Coulomb singularities. This method was reviewed and extended to the case of leading P-wave production processes in Chapter 2.

6.1 Coloured SUSY predictions for the LHC

The LHC finished its first phase of experiments at the beginning of 2013 with a centre-of-mass energy $\sqrt{s} = 8$ TeV and is expected to start again in 2014 – 2015 at much higher energies of $\sqrt{s} = 13, 14$ TeV. Therefore, in Chapter 3 we have presented NLL resummed results for squark and gluino pair production at both $\sqrt{s} = 8$ TeV and $\sqrt{s} = 14$ TeV. We found very large corrections beyond NLO ranging from 5 – 170% of the NLO cross section, depending on the process involved. The corrections for the total SUSY production process was found to be of the order of 15 – 30% while reducing the errors to $\pm 10\%$. Similar to previous calculations for top pair production [75], the Coulomb corrections were found to be large and thus we concluded that a *combined* resummation is necessary to get accurate predictions at the LHC. Comparing our EFT based results for soft-gluon resummation with those in Mellin-space [95] yielded some disagreements which were however still within the error bars. The results given in this chapter are expected to set stronger bounds on the squark and gluino masses in the MSSM.

The squarks and gluinos are known to be unstable and thus one may ask the question if our NLL resummation results shown in Chapter 4 are still valid after taking their decay widths into account. The corrections from the decay widths were found to be small and within the error bars of the NLL zero-width results as long as the widths were smaller than about 5% of the mass of the decaying parent particles. For larger widths, the finite-width results depart from the error bars of the zero width predictions. The reason for this is that the naïve replacement $E \rightarrow E + i\Gamma$ fails and one needs to take the exact NLO cross section including non-resonant diagrams into account. This can be seen from the invariant mass distributions which diverge below threshold for widths of the order $\Gamma/M \gtrsim 5\%$. The effects beyond NLO are however still small and within the zero-width error bars even at larger widths, which points towards the need of an exact NLO matching with non-resonant corrections. This is a challenge but seems feasible in the coming years as more general $2 \rightarrow 4$ NLO tools are developed. When it comes to the total SUSY production cross section, the widths of the squarks and gluinos can be safely set to zero, as most of the dominant processes are always stable.

6.2 Outlook

The next step would be to perform a complete NNLL calculation. One of the reasons for this is to get a better and more accurate prediction for the cross sections. As we saw in Chapter 3, the fixed and running scale results are still different at NLL, even if they are within the error bars. However,

in Chapter 5 we found that the differences between the fixed and running scale predictions for the pure soft corrections largely vanish when going to NNLL order. The NNLL corrections are expected to be large, as for example was found in the Mellin-space approach for the case of squark-antisquark production. The corrections on top of the NLL predictions were found to be of the order of 20 – 25% of the NLO cross section [49] and thus it would be interesting to see what the result would be for the other SUSY processes.

Furthermore, the difference between the pure NLL soft corrections calculated in the EFT approach and the Mellin-space approach are larger than one would have hoped for (but still within the error bars). Preliminary comparisons with the Mellin-space results for squark-antisquark production at NNLL order [49] points toward a much better agreement between the two approaches. Therefore, for a comparison with the Mellin-space approach it seems *necessary* to compare at NNLL order, which means that going to NNLL order is also necessary to get reliable predictions for the LHC. Eventually, in order to understand the differences between the Mellin-space and EFT approach it will become essential to perform a detailed analytic comparison of the two formalisms [129].

A natural extension of our predictions that is relevant at the LHC would be to include non-degenerate squark masses. For example, in most phenomenological models the mass of the sbottom is smaller than that of the other light flavour squarks. In addition, a proper implementation of the decay widths would require non-degenerate squark flavours and helicities. In particular, the width of the left helicity squark flavours may get large contributions from some EW decay channels. This might be interesting for the case of squark-gluino pair production for almost equal squark and gluino masses, which is a dominant SUSY channel whose corresponding decay widths are dominated by EW decay channels.

In the future it will be interesting to find ways to extend the resummation procedure to higher orders and to implement other corrections that are not currently included within the framework. One such improvement would be to include all the terms $\alpha_s(\alpha_s/\beta)^k(\alpha_s \ln(\beta))^l$ that are in fact necessary for a full NNLL resummation, for which the non-relativistic anomalous dimensions will be needed. It would also be useful to figure out a way to include the next-to-eikonal diagrammatic methods in the EFT resummation approach [130]. Finally, the strength of the resummation formalisms lies in the fact that they are very general and purely depend on the structure of QCD interactions [11, 58], so by studying these extensions we might also expand our knowledge of the colour structure of QCD itself.

APPENDIX A

Resummation functions and anomalous dimensions

In this Appendix we collect some of the resummation functions appearing in the factorization formulas (2.36) and (3.10). The evolution operator is given by (throughout this Appendix we replace the label R_α by i):

$$U_i(M, \mu_h, \mu_f, \mu_s) = \left(\frac{4M^2}{\mu_h^2} \right)^{-2a_\Gamma(\mu_h, \mu_s)} \left(\frac{\mu_h^2}{\mu_s^2} \right)^\eta \times \exp \left[4(S(\mu_h, \mu_f) - S(\mu_s, \mu_f)) - 2a_i^V(\mu_h, \mu_s) + 2a^{\phi, r}(\mu_s, \mu_f) + 2a^{\phi, r'}(\mu_s, \mu_f) \right] \quad (\text{A.1})$$

and $\eta = 2a_\Gamma(\mu_s, \mu_f)$. The labels r, r' denote the colour representation of the initial-state partons p, p' . Up to NNLL the functions S and a_Γ are given by [5, 60]

$$\begin{aligned} S(\mu_i, \mu_j) &= \frac{\Gamma^{(0)}}{8\beta_0^2} \left\{ \frac{4\pi}{\alpha_s(\mu_i)} \left(1 - \frac{1}{r} - \ln r \right) + \left(\frac{\Gamma^{(1)}}{\Gamma^{(0)}} - \frac{\beta_1}{\beta_0} \right) (1 - r + \ln r) + \frac{\beta_1}{2\beta_0} \ln^2 r \right. \\ &\quad + \frac{\alpha_s(\mu_i)}{4\pi} \left[\left(\frac{\beta_1 \Gamma^{(1)}}{\beta_0 \Gamma^{(0)}} - \frac{\beta_2}{\beta_0} \right) (1 - r + r \ln r) + \left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0} \right) (1 - r) \ln r \right. \\ &\quad \left. \left. - \left(\frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{\beta_0} - \frac{\beta_1 \Gamma^{(1)}}{\beta_0 \Gamma^{(0)}} + \frac{\Gamma^{(2)}}{\Gamma^{(0)}} \right) \frac{(1 - r)^2}{2} \right] \right\} \\ a_\Gamma(\mu_i, \mu_j) &= \frac{\Gamma^{(0)}}{4\beta_0} \left\{ \ln r + \frac{\alpha_s(\mu_i)}{4\pi} \left(\frac{\Gamma^{(1)}}{\Gamma^{(0)}} - \frac{\beta_1}{\beta_0} \right) (r - 1) \right\}, \end{aligned} \quad (\text{A.2})$$

where $r = \alpha_s(\mu_j)/\alpha_s(\mu_i)$ and where $\Gamma^{(i)} := (C_r + C_{r'})\gamma_{\text{cusp}}^{(i)}$ denotes the cusp-anomalous dimensions. The functions a_i^V and $a^{\phi, r}$ are given as in (A.2), where one needs to replace Γ with γ_i^V and $\gamma^{\phi, r}$. For NLL (NNLL) resummation one needs the two-loop (three-loop) cusp anomalous

dimensions and the one-loop (two-loop) anomalous dimensions given in the next section. The coefficients of the beta function which appear in Eq. (A.2) are:

$$\begin{aligned}\beta_0 &= \frac{11}{3}C_A - \frac{2}{3}n_l, \quad \beta_1 = \frac{34}{3}C_A^2 - \frac{10}{3}C_A n_l - 2C_F n_l. \\ \beta_2 &= \frac{2857}{54}C_A^3 + \left(2C_F^2 - \frac{205}{9}C_F C_A - \frac{1415}{27}C_A^2\right)T_F n_l + \left(\frac{44}{9}C_F + \frac{158}{27}C_A\right)T_F^2 n_l^2.\end{aligned}\quad (\text{A.3})$$

Anomalous dimensions

The cusp-anomalous dimensions appear in the RG equation (2.34) of the hard function and up to three loops equal [60]:

$$\begin{aligned}\gamma_{\text{cusp}}^{(0)} &= 4, \\ \gamma_{\text{cusp}}^{(1)} &= 4 \left[\left(\frac{67}{9} - \frac{\pi^2}{3} \right) C_A - \frac{20}{9} T_F n_l \right], \\ \gamma_{\text{cusp}}^{(2)} &= 4 \left[C_A^2 \left(\frac{245}{6} - \frac{134\pi^2}{27} + \frac{11\pi^4}{45} + \frac{22}{3} \zeta_3 \right) + C_A T_F n_l \left(-\frac{418}{27} + \frac{40\pi^2}{27} - \frac{56}{3} \zeta_3 \right) \right. \\ &\quad \left. + C_F T_F n_l \left(-\frac{55}{3} + 16\zeta_3 \right) - \frac{16}{27} T_F^2 n_l^2 \right],\end{aligned}\quad (\text{A.4})$$

where $T_F = 1/2$ is the Dynkin index of the fundamental representation of $SU(3)$ and $n_l = 5$ is the amount of light quark flavours. The explicit values of the Casimir invariants for the $SU(3)$ representations relevant for squark and gluino production are:

$$\begin{aligned}C_1 &= 0, \quad C_F = C_q = C_3 = \frac{4}{3}, \quad C_6 = \frac{10}{3}, \\ C_A &= C_g = C_8 = 3, \quad C_{10} = 6, \quad C_{15} = \frac{16}{3}, \quad C_{27} = 8.\end{aligned}\quad (\text{A.5})$$

The single-particle anomalous dimension is defined as $\gamma_i^V = \gamma^r + \gamma^{r'} + \gamma_{H,s}^{R_\alpha}$ and also appears in the RG equation (2.34) of the hard function. In [58] it was shown that the heavy particle soft anomalous dimension scales as $\gamma_{H,s}^{R_\alpha} = C_{R_\alpha} \gamma_{H,s}$ up to two-loop. The one- and two-loop soft anomalous-dimension coefficients appearing here are given by [58]:

$$\begin{aligned}\gamma_{H,s}^{(0)} &= -2, \\ \gamma_{H,s}^{(1)} &= -C_A \left(\frac{98}{9} - \frac{2\pi^2}{3} + 4\zeta_3 \right) + \frac{40}{9} T_F n_l.\end{aligned}\quad (\text{A.6})$$

On the other hand, the one- and two-loop anomalous dimensions γ^r of the initial state partons in γ_i^V are [131]:

$$\begin{aligned}
\gamma^{(0)q} &= -3C_F, \\
\gamma^{(1)q} &= C_F^2 \left(-\frac{3}{2} + 2\pi^2 - 24\zeta_3 \right) + C_A C_F \left(-\frac{961}{54} - \frac{11\pi^2}{6} + 26\zeta_3 \right) \\
&\quad + C_F T_F n_l \left(\frac{130}{27} + \frac{2\pi^2}{3} \right), \\
\gamma^{(0)g} &= -\beta_0 = -\frac{11}{3}C_A + \frac{4}{3}T_F n_l, \\
\gamma^{(1)g} &= C_A^2 \left(-\frac{692}{27} + \frac{11\pi^2}{18} + 2\zeta_3 \right) + C_A T_F n_l \left(\frac{256}{27} - \frac{2\pi^2}{9} \right) + 4C_F T_F n_l. \tag{A.7}
\end{aligned}$$

Finally, the anomalous dimensions $\gamma^{\phi,r}$ appear when solving the RG equations of the soft function. They are related to the splitting functions in the DGLAP equations (1.6) of the PDF's and up to two-loop are given by [60, 132]:

$$\begin{aligned}
\gamma^{(0)\phi,q} &= 3C_F, \\
\gamma^{(1)\phi,q} &= C_F^2 \left(\frac{3}{2} - 2\pi^2 + 24\zeta_3 \right) + C_A C_F \left(\frac{17}{6} + \frac{22\pi^2}{9} - 12\zeta_3 \right) \\
&\quad - C_F T_F n_l \left(\frac{2}{3} + \frac{8\pi^2}{9} \right), \\
\gamma^{(0)\phi,g} &= \beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_l, \\
\gamma^{(1)\phi,g} &= 4C_A^2 \left(\frac{8}{3} + 3\zeta_3 \right) - \frac{16}{3}C_A T_F n_l - 4C_F T_F n_l. \tag{A.8}
\end{aligned}$$

The above three anomalous dimensions are needed to compute $\gamma_{W,i}^{R_\alpha}$, which appeared in the soft evolution equation (2.35):

$$\gamma_{W,i}^{R_\alpha} = \gamma_{H,s}^{R_\alpha} + (\gamma^r + \gamma^{\phi,r}) + (\gamma^{r'} + \gamma^{\phi,r'}). \tag{A.9}$$

APPENDIX B

Determination of μ_s and β_{cut}

In this Appendix we give the values for the fixed soft scales that we used for NLL and NNLL resummation in Chapters 3 through 5. Furthermore, we discuss the determination of β_{cut} and their values for NLL resummation. The determination of the running soft scales for NNLL resummation is very similar and is briefly discussed in Section 5.1. For a more detailed discussion of the NNLL case we refer to [75]. The soft scale μ_s used in the fixed-scale implementation, (N)NLL_{fixed}, is defined by Eq. (2.48). For equal squark and gluino masses, $m_{\tilde{q}} = m_{\tilde{g}}$, the minimization procedure yields the following results:

$$\begin{aligned} \tilde{q}\tilde{q} : \quad & \mu_s = 132 - 175 \text{ GeV } (\sqrt{s} = 8 \text{ TeV}), \quad \mu_s = 146 - 376 \text{ GeV } (\sqrt{s} = 14 \text{ TeV}), \\ \tilde{q}\tilde{q} : \quad & \mu_s = 127 - 140 \text{ GeV } (\sqrt{s} = 8 \text{ TeV}), \quad \mu_s = 146 - 312 \text{ GeV } (\sqrt{s} = 14 \text{ TeV}), \\ \tilde{g}\tilde{q} : \quad & \mu_s = 114 - 148 \text{ GeV } (\sqrt{s} = 8 \text{ TeV}), \quad \mu_s = 131 - 310 \text{ GeV } (\sqrt{s} = 14 \text{ TeV}), \\ \tilde{g}\tilde{g} : \quad & \mu_s = 111 - 146 \text{ GeV } (\sqrt{s} = 8 \text{ TeV}), \quad \mu_s = 127 - 308 \text{ GeV } (\sqrt{s} = 14 \text{ TeV}). \end{aligned} \quad (\text{B.1})$$

The scales for a centre-of-mass energy of 8 TeV refers to a mass range $M = 500 - 2500$ GeV, while for a 14 TeV LHC the mass interval $M = 500 - 3000$ GeV was considered. For stop-antistop production we obtain the scales

$$\tilde{t}\tilde{t} : \quad \mu_s = 30 - 159 \text{ GeV } (\sqrt{s} = 8 \text{ TeV}), \quad \mu_s = 31 - 230 \text{ GeV } (\sqrt{s} = 14 \text{ TeV}) \quad (\text{B.2})$$

for the mass range $m_{\tilde{t}} = 100 - 1200$ GeV for a c.o.m. energy of 8 TeV and $m_{\tilde{t}} = 100 - 1400$ GeV at 14 TeV. In [60] it was suggested to fit the mass and energy dependence of the soft scales (B.1) by a function of the form

$$\mu_s = \frac{M(1 - \rho)}{\sqrt{a + b\rho}}, \quad (\text{B.3})$$

with $\rho = 4M^2/s$. The coefficients a and b for the different processes are given in Table B.1, and provide a fit to (B.1) with accuracy better than 5% for the mass range considered.

	LHC(8 TeV)	LHC(14 TeV)
$\tilde{q}\tilde{q}$	$a = 11.6909$ $b = 164.516$	$a = 11.5289$ $b = 171.856$
$\tilde{q}\tilde{q}$	$a = 10.5905$ $b = 271.353$	$a = 10.9981$ $b = 278.062$
$\tilde{g}\tilde{q}$	$a = 15.5624$ $b = 243.688$	$a = 15.3779$ $b = 267.763$
$\tilde{g}\tilde{g}$	$a = 16.7657$ $b = 243.352$	$a = 16.7483$ $b = 264.702$
$\tilde{t}\tilde{t}$	$a = 17.1015$ $b = 365.364$	$a = 15.7282$ $b = 481.045$

Table B.1: Coefficients of the fit (B.3) for the squark-gluino production processes at centre-of-mass energies of 8 and 14 TeV.

The procedure used to determine β_{cut} , which enters the definition of the soft scale used in our default NLL implementation, Eqs. (2.49) and (2.50), was explained in detail in [75]. Following [75] we introduce eight different cross sections

$$\hat{\sigma}_{pp'}(A_<, B_>, \beta_{\text{cut}}) = \hat{\sigma}_{pp'}^{A_<} \theta(\beta_{\text{cut}} - \beta) + \hat{\sigma}_{pp'}^{B_>} \theta(\beta - \beta_{\text{cut}}), \quad (\text{B.4})$$

defined using one of two possible matching prescriptions ($A_< \in \{\text{NLL}_1, \text{NLL}_2\}$) for the lower interval and one of four possible approximations ($B_> \in \{\text{NLL}_2, \text{NLO}_{\text{app}}, \text{NNLO}_A, \text{NNLO}_B\}$) for the upper interval. Here NLL_2 denotes our default approximation, Eq. (3.12), while NLL_1 is the resummed result matched to the Born instead of the NLO cross section,

$$\hat{\sigma}_{pp'}^{\text{NLL}_1}(\hat{s}) = \left[\hat{\sigma}_{pp'}^{\text{NLL}}(\hat{s}) - \hat{\sigma}_{pp'}^{\text{NLL}(0)}(\hat{s}) \right] + \hat{\sigma}_{pp'}^{\text{LO}}(\hat{s}). \quad (\text{B.5})$$

NLO_{app} represents the sum of the full Born cross section and the approximated NLO corrections given in (3.4), while the two NNLO approximations contain in addition the $O(\alpha_s^4)$ terms arising from the expansion of the NLL resummed result, including all of them (NNLO_A) or only the subset which is completely determined at NLL (NNLO_B). β_{cut} is then determined such that the width of the envelope of the eight different implementations $\hat{\sigma}_{pp'}(A_<, B_>, \beta_{\text{cut}})$ is minimal. The left plot of Figure (B.1) is an example of the NLL envelope created for the squark-antisquark process with $m_{\tilde{q}} = m_{\tilde{g}} = 1500$ GeV at $\sqrt{s} = 8$ TeV.³² For comparison, in the right plot we show the corresponding envelope created for the NNLL case (cf. Section 5.1). The procedure described

³²The values (B.6) are also used for the NLL_{s+h} approximation (3.11), instead of recalculating β_{cut} using that approx-

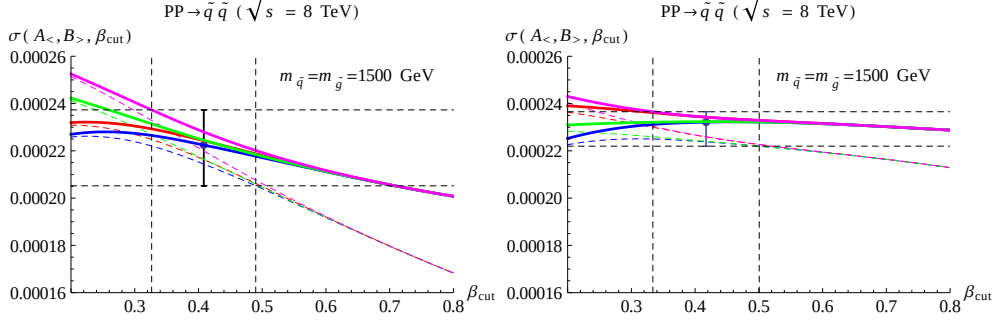


Figure B.1: Envelope created by the eight different implementations of Eq. (B.4). Left: NLL implementation. Right: NNLL implementation with the corresponding higher order cross sections.

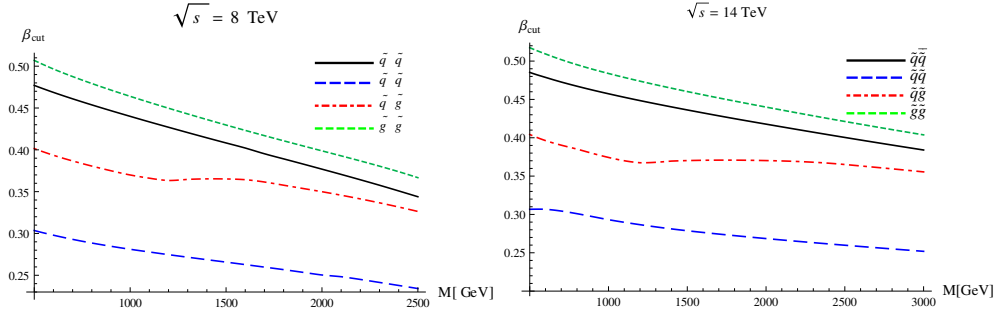


Figure B.2: Mass dependence of the parameter β_{cut} for the four SUSY production processes.

provides, for the default choice $k_s = 1$ and equal squark and gluino masses, the following values for β_{cut} :

$$\begin{aligned}
 \tilde{q}\tilde{q}^* : \quad & \beta_{\text{cut}}^{(7 \text{ TeV})} = 0.47 - 0.36, \quad \beta_{\text{cut}}^{(8 \text{ TeV})} = 0.48 - 0.34, \quad \beta_{\text{cut}}^{(14 \text{ TeV})} = 0.48 - 0.38, \\
 \tilde{q}\tilde{q} : \quad & \beta_{\text{cut}}^{(7 \text{ TeV})} = 0.30 - 0.24, \quad \beta_{\text{cut}}^{(8 \text{ TeV})} = 0.31 - 0.24, \quad \beta_{\text{cut}}^{(14 \text{ TeV})} = 0.31 - 0.25, \\
 \tilde{g}\tilde{q} : \quad & \beta_{\text{cut}}^{(7 \text{ TeV})} = 0.40 - 0.34, \quad \beta_{\text{cut}}^{(8 \text{ TeV})} = 0.40 - 0.33, \quad \beta_{\text{cut}}^{(14 \text{ TeV})} = 0.40 - 0.36, \\
 \tilde{g}\tilde{g} : \quad & \beta_{\text{cut}}^{(7 \text{ TeV})} = 0.50 - 0.39, \quad \beta_{\text{cut}}^{(8 \text{ TeV})} = 0.51 - 0.37, \quad \beta_{\text{cut}}^{(14 \text{ TeV})} = 0.52 - 0.40.
 \end{aligned} \tag{B.6}$$

imation in (B.4). Since the difference of NLL and NLL_{s+h} is used in order to assess the effect of Coulomb resummation, we consider it more meaningful to use the same β_{cut} for both in order not to obscure the genuine Coulomb effects by different scale choices.

The numbers refer to the usual mass range of $500 - 2500$ GeV at $\sqrt{s} = 8$ TeV and $500 - 3000$ GeV at $\sqrt{s} = 14$ TeV, and the exact mass dependence is plotted in Figure B.2. We have also included the results for the mass range of $500 - 2000$ GeV at $\sqrt{s} = 7$ TeV. For stop-antistop production the result is

$$t\bar{t} : \quad \beta_{\text{cut}}^{(7 \text{ TeV})} = 0.53 - 0.40, \quad \beta_{\text{cut}}^{(8 \text{ TeV})} = 0.53 - 0.39, \quad \beta_{\text{cut}}^{(14 \text{ TeV})} = 0.54 - 0.41, \quad (\text{B.7})$$

for the mass range $m_{\tilde{t}} = 100 - 1000$ GeV at 7 TeV, $m_{\tilde{t}} = 100 - 1200$ GeV at 8 TeV and $m_{\tilde{t}} = 100 - 1400$ GeV at 14 TeV. The theoretical ambiguities of the resummed result with the running scale are estimated as follows: i) the default implementation NLL_2 is parametrized in terms of $\hat{E}$ instead of β , ii) the NLL_2 result is computed for the values $k_s = 1/2, 2$, recomputing β_{cut} anew for each choice and iii), the envelope of the eight cross sections (B.4) is taken while β_{cut} is varied by $\pm 20\%$ (Figure B.1 shows the 20% variation and the corresponding error as horizontal and vertical dashed lines respectively). The uncertainties from the three sources are added in quadrature, and iii) gives the dominant contribution, in the $3 - 13\%$ range depending on the process, while i) and ii) are usually below 2%.

APPENDIX C

NLO scaling functions at finite width

In this Appendix we give the NLO scaling functions (cf. Eq. (3.4)) for the finite-width case. The scaling function at NLL order is found by expanding the NLL resummation formula to NLO when replacing $E \rightarrow E + i\Gamma$:

$$\begin{aligned} f_{pp'(i)}^{\text{NLL}(1)}(E, \Gamma) = & f_{pp'(i)}^{\text{NLL}(1)}(r_E, 0) + \frac{4\sqrt{2m_r}\pi D_R}{\sqrt{r_E}} \sqrt{1 + \Gamma_E^2} \tan^{-1}(\Gamma_E) - \left(8 \tanh^{-1}(\cos(\chi)) \right. \\ & \left. - 16\Gamma_E \left(\tan^{-1} \left(\frac{(r_E - E + r_{\Delta E} + \Delta E) \cos(\chi)}{2\Gamma} \right) - \tan^{-1}(\Gamma_E) \right) \right. \\ & \left. - 16\Gamma_{\Delta E} \sqrt{\frac{r_E(1 + \Gamma_E^2)}{r_{\Delta E}(1 + \Gamma_{\Delta E}^2)}} \right) (C_{r1} + C_{r2}) \log \left(\frac{\mu_s}{\mu_f} \right). \end{aligned} \quad (\text{C.1})$$

Here $f_{pp'(i)}^{\text{NLL}(1)}(E, 0)$ equals the zero-width result that can be found in the Appendix of [5] and we have also defined for simplicity:

$$r_y := \sqrt{y^2 + \Gamma^2}, \quad \Gamma_y := \frac{\Gamma}{y + r_y}, \quad \sin(\chi) := \frac{E + \Delta E}{r_E + r_{\Delta E}}.$$

Equation (C.1) is used to subtract the NLL cross section when considering the alternative matching (4.44). The above equation has singular logs in $E + \Delta E$, in the same way as the scaling function for zero width has them in E . This is seen by expanding (C.1) in $E + \Delta E$:

$$\begin{aligned} f_{pp'(i)}^{\text{NLL}(1)}(E, \Gamma) = & f_{pp'(i)}^{\text{NLL}(1)} \left(\frac{E + \Delta E}{4}, 0 \right) \Big|_{D_R=0} + 16(C_{r1} + C_{r2}) \log \left(\frac{\mu_s}{\mu_f} \right) \\ & + \frac{2\sqrt{2m_r}\pi D_R}{\sqrt{r_E}} \sqrt{1 + \Gamma_E^2} (2 \tan^{-1}(\Gamma_E) - \pi) + \mathcal{O}(E + \Delta E). \end{aligned} \quad (\text{C.2})$$

The NLO scaling function containing all the threshold-enhanced contributions is given on the next page in Eq.(C.3). It is found by using the exact one-loop soft function in the factorization formula and afterwards expanding it to NLO in α_s as explained in Chapter 3:

$$\begin{aligned}
f_{pp'(i)}^{(1)}(E, \Gamma) = & \frac{2\sqrt{2m_r}\pi D_R}{\sqrt{r_E}} \sqrt{1 + \Gamma_E^2} (2 \tan^{-1}(\Gamma_E) - \pi) + 4C_R \left(3 + \tanh^{-1}(\cos(\chi)) \right. \\
& + \log\left(\frac{\mu_f}{8r_E}\right) - 2\Gamma_E \left(\tan^{-1}\left(\frac{(r_E - E + r_{\Delta E} + \Delta E)\cos(\chi)}{2\Gamma}\right) - \tan^{-1}(\Gamma_E) \right) \Big) \\
& - (C_{r1} + C_{r2}) \left(32 \tan^{-1}(A_+)^2 + 16\Gamma_E \left(\tan^{-1}(\Gamma_E) \left(-2 + \log\left(\frac{8(E + \Delta E)}{\mu_f}\right) \right) \right. \right. \\
& \left. \left. - \tan^{-1}\left(\frac{(r_E - E + r_{\Delta E} + \Delta E)\cos(\chi)}{2\Gamma}\right) \left(-2 + \log\left(\frac{2(E + \Delta E)}{\mu_f}\right) \right) \right. \right. \\
& \left. \left. - \tan^{-1}(A_-) \log(4) + \tan^{-1}(A_+) \log\left(\frac{(r_E + r_{\Delta E})(1 + \cos(\chi))}{r_E}\right) \right) \right) \\
& + 2 \left(-8 \tanh^{-1}(\cos(\chi)) - \log(4) \log(8) + 8 \log\left(\frac{8r_E}{\mu_f}\right) + 4 \left(\tanh^{-1}(\cos(\chi)) \right. \right. \\
& \left. \left. + \log\left(\frac{E + \Delta E}{4r_E}\right) \right) \log\left(\frac{2(E + \Delta E)}{\mu_f}\right) + 2 \log\left(\frac{4(E + \Delta E)}{\mu_f}\right) \log\left(\frac{\mu_f}{E + \Delta E}\right) \right. \\
& \left. - 2 \log(4) \left(2 \log\left(\frac{r_E}{E + \Delta E}\right) + \log\left(\frac{(r_E + r_{\Delta E})(1 - \cos(\chi))}{r_E}\right) \right) \right. \\
& \left. - \log\left(\frac{(r_E + r_{\Delta E})(1 + \cos(\chi))}{r_E}\right)^2 \right) + 16 \operatorname{Re} \left[\operatorname{Li}_2 \left(2, \frac{1}{2} \left(1 - \sqrt{\frac{i\Gamma - \Delta E}{E + i\Gamma}} \right) \right) \right] \\
& - 16\Gamma_E \operatorname{Im} \left[\operatorname{Li}_2 \left(2, \frac{1}{2} \left(1 - \sqrt{\frac{i\Gamma - \Delta E}{E + i\Gamma}} \right) \right) \right] - 32 + \frac{\pi^2}{2} \\
& - 8 \left(C_R - 2(C_{r1} + C_{r2}) \left(-2 + \log\left(\frac{2(E + \Delta E)}{\mu_f}\right) \right) \right) \Gamma_{\Delta E} \sqrt{\frac{r_E(1 + \Gamma_E^2)}{r_{\Delta E}(1 + \Gamma_{\Delta E}^2)}}. \quad (C.3)
\end{aligned}$$

We have define the above in terms of:

$$A_{\pm} := \frac{\Gamma \tan(\chi)}{r_E \pm \frac{1}{2}(r_E + r_{\Delta E}) \cos(\chi) + \sqrt{r_E(r_E + r_{\Delta E})(1 \pm \cos(\chi))}}. \quad (C.4)$$

Equation (C.3) can be used to get an approximation of the NLO cross section to the production and subsequent decay of the heavy particles with average widths equal to Γ . Similarly to (C.1), Eq. (C.3) also contains singular logarithms in $E + \Delta E$ and it makes sense to thus expand it in

$E + \Delta E$ to find:

$$\begin{aligned}
 f_{pp'(i)}^{(1)}(E, \Gamma) = & f_{pp'(i)}^{(1)}\left(\frac{E + \Delta E}{4}, 0\right)\Big|_{D_R=0} + (C_{r1} + C_{r2})\left(16 \log\left(\frac{2(E + \Delta E)}{\mu_f}\right)\right. \\
 & \left. + \frac{4\pi^2}{3} - 32\right) + \frac{2\sqrt{2m_r}\pi D_R}{\sqrt{r_E}}\sqrt{1 + \Gamma_E^2}\left(2 \tan^{-1}(\Gamma_E) - \pi\right) - 8C_R \\
 & + \mathcal{O}(E + \Delta E).
 \end{aligned} \tag{C.5}$$

Here $f_{pp'(i)}^{(1)}(E, 0)$ equals the zero-width result given in Eq. (3.4).

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Zachte en Coulomb hersommatie bij de LHC - *samenvatting*

Als kleine kinderen hebben we waarschijnlijk allemaal al vaker met vergrootglazen en planten of bloemen gespeeld. Al snel komen we erachter dat als wij de bladeren van een bloem door het fijn geslepen, convexe glas bekijken, de groeven op de bladeren veel groter lijken. Op dezelfde manier, op de basisschool of thuis komen we vaak brillen tegen van leraren/leraressen, medeleerlingen of onze (groot-)ouders en soms gebeurt het dat wij zelf zulke convexe brilglazen nodig hebben om gemakkelijker naar het bord of bladeren en bloemen te kijken. Pas op de middelbare school leren leerlingen tijdens de natuurkundeles hoe deze brillen werken en kunnen zij tijdens de biologieles zelfs door nog sterkere vergrootglazen kijken, namelijk microscopen, en de bladeren van bloemen nog verder vergroten. Het blijkt dan dat de bladeren uit cellen zijn gemaakt, die weer uit kleinere onderdelen zoals de celkern gemaakt worden. In scheikunde leren wij vervolgens dat deze cellen uit moleculen en atomen bestaan die weer gezien kunnen worden door zogenaamde elektronenmicroscopen. Deze microscopen gebruiken als bron elektronen in plaats van licht. De bron, m.a.w. de elektronen worden nu niet met lenzen of spiegels gebogen, wat het geval is met normale microscopen, maar met elektromagnetische krachten. De krachtigste elektronenmicroscopen kunnen tegenwoordig atomen onderscheiden op de schaal van enkele Angström $\sim 10^{-10}$ meter. Op deze kleine schaal volgen de atomen de wetten van kwantummechanica. Aan het einde van de middelbare school leren de studenten tijdens de les over kwantummechanica in natuurkunde dat de atomen bestaan uit een kern met daaromheen rondvliegende elektronen. De kern is ongeveer 10.000 keer zo klein als een atoom en is veel te klein om met zelfs de krachtigste microscopen waargenomen te worden. De kern is zelf weer gemaakt uit protonen en neutronen met een breedte van ongeveer een femtometer $\sim 10^{-15}$ meter, die samen in de kern gehouden worden door de zogenaamde *sterke kernkracht (interacties)*.

Pas later op de universiteit leert men dat de protonen en neutronen uit drie nog kleinere deeltjes bestaan, die *quarks* heten en samen worden gehouden door het uitwisselen van zogenaamde *gluonen*. De interacties van quarks en gluonen voldoen aan de wetten van een kwantummechanische theorie die *Kwantumchromodynamica (QCD)* heet. Deze theorie wordt tegenwoordig getest bij de *Large Hadron Collider (LHC)* in CERN, de sterkste deeltjesversneller in de wereld, door op hoge snelheid protonen met elkaar te laten botsen met behulp van zeer sterke magneten. Net als

met een microscoop, fungeert de LHC dan als een sterk vergrootglas, maar in plaats van licht of electronen worden protonen als bron gebruikt en het specimen is in dit geval een ander proton. Vanwege de hoge snelheid van de protonen hebben zij een zeer kleine golflengte van ongeveer een femtometer, waardoor zij met een ander proton een interactie kunnen hebben. Een botsing tussen twee protonen kan dan opgevat worden als een botsing tussen de onderlinge quarks en gluonen waaruit de protonen gemaakt zijn. De sterkte van de quark-gluon interacties wordt uitgedrukt in een constante, namelijk de *sterke koppelingsconstante* $\alpha_s \sim 0.1$.

Protonbotsingen bij de LHC resulteren in een productie van een grote hoeveelheid deeltjes en experimentatoren meten daar de kans voor specifieke producties. Door gebruik te maken van QCD kan men de theoretische kans berekenen dat een specifieke verzameling deeltjes geproduceerd wordt.³³ De kans wordt uitgedrukt als een reeks in machten van de sterke koppelingsconstante α_s . Normaal gesproken is deze reeks een eindig getal (convergerend), maar sinds de jaren vijftig weet men dat bij de productie van *massieve* deeltjes de reeks niet zomaar convergeert. De reden hiervoor is dat de coëfficiënten van de reeks logaritmes en inverse machten bevat van de snelheid van de geproduceerde deeltjes. Voor massieve deeltjes is de snelheid heel klein en dus de coëfficiënten juist heel groot. Hiervoor moet de reeks opnieuw geordend worden en daarna weer gesommeerd. In de praktijk heet dit *hersommatie*. In hoofdstuk 2 leggen wij het formalisme van hersommatie uit dat wij in dit proefschrift hebben gebruikt. Door gebruik van zogenaamde *effectieve veldentheorieën* (EFT) kan men bewijzen dat de kans van productie uitgedrukt kan worden in drie factoren, die elk een deelreeks van de originele reeks bevatten. Voor meer details verwijzen wij naar hoofdstuk 2.

In de rest van het proefschrift hebben wij het EFT formalisme uit hoofdstuk 2 toegepast op een theorie die tegenwoordig getest wordt bij de LHC. Deze theorie heet het *Minimal Supersymmetric Standard Model* (MSSM) en is sinds de afgelopen dertig jaar bekender geworden vanwege de oplossingen die het biedt aan verscheidene fundamentele problemen. De MSSM behoort tot een verzameling kwantummechanische theorieën die een extra eigenschap hebben dat *supersymmetrie* (SUSY) heet. Deze theorieën worden ook bij de LHC getest en voorspellen dat elk deeltje wat we hebben geobserveerd in het universum, waaronder de quarks en gluonen, een paar vormt met een ander deeltje dat we tot nu toe niet hebben geobserveerd³⁴. Deze paarvorming is een symmetrie, net als bijvoorbeeld een paar van schoenen altijd samen gaan en men kan de ene schoen naar de andere transformeren en andersom door bijvoorbeeld een spiegel tussen de twee schoenen te zetten. In het bijzonder, het quark vormt een paar met een *squark* en het gluondeeltje een paar met een *gluino*. De twee deeltjes in een paar hebben dezelfde lading en sterke wisselwerking, maar verschillen in de kwantummechanische eigenschap die *spin* heet. De eigenschap spin is niet belangrijk bij ons verhaal, maar heeft te maken met hoe een deeltje zich gedraagt nadat men een rotatie erop uit voert. Aangezien wij de squarks en gluinos nog niet hebben kunnen

³³In technische termen berekent men de zogenaamde *werkzame doorsnede*, die gerelateerd is aan de kans.

³⁴Strikt genomen is dit $\mathcal{N} = 1$ SUSY.

observeren moeten zij heel massief zijn, anders zouden we hen allang hebben ontdekt.³⁵ Aan de andere kant, vanwege dezelfde sterke interacties, zullen de squarks en gluinos de sterke kernkracht voelen en dus samen in wisselwerking optreden via het uitwisselen van een gluon. Vanwege deze wisselwerking en het feit dat de squarks en gluinos massief zijn, zal de reeks van de kans voor hun productie grote coëfficiënten hebben, die dan als geheel gehersommeerd moet worden. In dit proefschrift hebben wij de productie van squarks en gluinos bij de LHC onderzocht met behulp van het EFT hersommatieformalisme van hoofdstuk 2.

In hoofdstuk 3 hebben wij een deel van de gehersommeerde reeks vergeleken met de som van de eerste en tweede term in de machtreeks (deze som heet **NLO**). De gehersommeerde reeks kan ongeveer twee of drie keer groter zijn dan de NLO som. Deze significante correctie betekent dat de gehersommeerde som gebruikt moet worden voor het verkrijgen van een juiste voorspelling voor de experimenten bij de LHC. In hoofdstuk 4 hebben wij het meer realistische geval bekeken waar de squarks en gluinos niet stabiel zijn, maar juist vervallen. We hebben gevonden dat voor het merendeel van de realistisch en interessante modellen onze resultaten van hoofdstuk 3, waar de squarks en gluinos stabiel waren genomen, nog steeds geldig zijn. Als laatste hebben wij in hoofdstuk 5 hogere orde correcties boven die van hoofdstuk 3 berekend. Deze correcties maken deel uit van de machtreeks en zorgen voor een veel kleinere foutmarge in onze resultaten. Dit betekent dat ze belangrijk zijn voor een juiste voorspelling van de kans voor de productie van squarks en gluinos bij de LHC.

³⁵Men spreekt van gebroken SUSY in het geval dat de massa's van de twee deeltjes in een paar niet hetzelfde zijn.

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Chris Wever,
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Curriculum Vitae

The author was born on April 19, 1985 in Oranjestad, Aruba. He attended high school at Colegio Arubano from 1997 to 2003 in Aruba. In 2003 he moved to the Netherlands to pursue a bachelor in Physics and Mathematics at Utrecht University, where he graduated in 2006. Afterwards, he followed the Theoretical Physics master program at Utrecht University and defended his master thesis in December, 2008. At the beginning of 2009 he started his PhD research at the Institute of Theoretical Physics at Utrecht University under the supervision of Prof. Gerard 't Hooft and Dr. Pietro Falgari. The PhD research was funded by a Mozaïek grant from the Dutch research organization NWO and the results are presented in this thesis.

Publications

The main chapters of this thesis are based on the following:

- Chapter 3:
[47] P. Falgari, C. Schwinn and C.S.P. Wever, *NLL soft and Coulomb resummation for squark and gluino production at the LHC*, *JHEP* **06** (2012) 052.
- Chapter 4:
[48] P. Falgari, C. Schwinn and C.S.P. Wever, *Finite-width effects on threshold corrections to squark and gluino production*, *JHEP* **01** (2013) 085.
- Chapter 5:
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