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Mathematical Descriptions of Axially Varying Penning Traps for the Antimatter Experiment: Gravity, Inferometry, and Spectroscopy

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1. Abstract

Antimatter, though proposed in 1933, is still not well understood. AEgIS aims to study how antihydrogen interacts with the earths gravitational feild. This experiment will enhance our understanding of the matter-antimatter asymmetry in our universe. This paper discusses a proposed Penning-Malmberg trap with a magnetic mirror, which will hold C_2^- for sympathetic cooling of antiprotons. The trap, which is critical to the cooling process of the antihydrogen, can be characterized by the separatrix between trapped and un-trapped particles. This paper applies analytical processes previously used to define the separatrix of pure electron plasmas to molecular plasmas. Our trap design is based on the desired conditions (density, particle number, field strength, trap size) of the high field region. The initial application of a semi-analytical method applied to our trap defines the trap potential difference at $\tilde{0}$.6V. The separatrix is defined in both the high and low field region by an ellipse and a hyperbola respectively. This shape varies from the separatrix of a Penning trap with a constant axial magnetic field. Now that we have successfully applied this method to our C_2^- trap and defined the high and low field separatrices, we may move forward with a better understanding of our proposed cooling system.

2. Introduction

The Antimatter Experiment: Gravity, Interferometry, and Spectroscopy (AEgIS) project at CERN aims to study the interaction of antimatter with the earth's gravitational field. In order to do this calculation, AEgIS needs very cold antimatter: on the order of a micro Kelvin. Currently, AEgIS is considering cooling the antiprotons sympathetically with C_2^-. Cooling of the molecules depends on confining the plasma in a Penning trap with a magnetic mirror, the details of which are discussed in this paper.

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3. Background

C_2^-. The electronic transitions of the carbon molecules allow the plasmas to be cooled effectively with laser cooling methods. For the cooling process, the C_2^- is held in a Penning trap with a varying axial magnetic field. This is a modification of a traditional Penning trap. AEgIS is currently considering Sisyphus, Doppler, and evaporative cooling.

In Sisyphus cooling, the particles move to a potential maximum, using up some of their kinetic energy, and a laser drops them to a lower energy state so that the potential energy is dissipated. This results in a lower kinetic energy and a lower temperature. Doppler cooling uses the Doppler effect to tune lasers to Carbon molecules moving towards the laser. The photons of the laser then impart their momenta on the carbon, which results in a retarding force. In evaporative cooling, the highest energy molecules are excited, ionized and hence removed from the system. This lowers the net energy of the Carbon molecules in the trap.

To take part the Sisyphus cooling process, the carbon molecules must bounce from one end of the trap to another. In order to avoid being confined in one of the trap regions, the molecules must obey certain velocity constraints.

A function called the separatrix defines the difference between trapped and un-trapped particles. There are distinct separatrices for the high- and low-field regions. Particles with high perpendicular velocities cannot penetrate into the high field region. Conversely, particles with small perpendicular velocities cannot escape the high field region.

4. Methodology

The aim of my work was to define the separatrices for AEgIS's proposed C_2^- trap. The axially varying Penning trap can be described mathematically through a series of steps.

To characterize the plasma in the trap, we first recognize two 'constants' of motion, the total energy and the canonical azimuthal momentum. In real plasmas there are asymmetries in the field and collisions with background gas that allow constant loss of energy and momentum[3]. However, these constants still define the global thermal distribution $f(H - \omega P_\theta)$ of the plasma. This distribution defines the density, which follows a Boltzmann distribution based on $f(H - \omega P_\theta)$ where

$$f(H - \omega P_\theta) = f\Big(\frac{p_z^2}{2m} + \frac{p_r^2}{2m} + \frac{(p_\theta - \omega r)^2}{2m} - e\phi(r,z) + \frac{m}{2}\omega(\omega_c - \omega)r^2\Big) \longrightarrow f(-e\phi(r,z) + \frac{m}{2}\omega(\omega_c - \omega)r^2)$$

The momenta terms of the Hamiltonian are neglected because they are negligible in size compared to the other terms.

To make the solution analytically tractable we make several simplifications. The first step of my work was to ensure that the assumptions presented in the literature applied to our system. Both papers depend on the assumption that the axial magnetic field has almost no radial dependence.

This works for non-steep magnetic gradients. Our magnetic field rises from 0.45T to 0.8T across 9 cm of the trap. Though this is a fairly steep gradient, the error produced by this generalization is small.

Then, we model the magnetic field as a piecewise continuous function with two regions. Poisson's equation could, ideally, be solved as a function of z, but again this is quite complicated. This simplifies the calculations by treating the system as two semi-infinite axially constant traps, which are more straightforward to solve. Knowing that the entire plasma rotates with the same frequency, known as the rigid rotor frequency, and that the density follows the Boltzmann distribution, we can relate the two regions.

In order to solve the system, I began with the ideal conditions for the high field region, which we defined.

1. Density: $n_0 = 10^{13}$ particles/m^2
2. Trap Length: 10cm
3. Number of particles: 10^7

Since the solution to the Poisson equation- in the case without radial dependence, axial dependence, or axial variance of the magnetic field- is known to be a Bessel function with an imaginary argument, I could solve for the frequency given the density, the number of particles, and an estimate for the physical shape of the trap. Modeling the trap first as a single cylinder and then as two cylinders with radii r_h and $\sqrt{\eta} r_h$ generated a reasonable guess for the plasma radii.

A rough correlation between the radius and the Debye length in the high and low field is known ($\sqrt{\eta}$). This allows us to estimate the low field values. This, combined with the rotor frequency can be used to estimate the low field density. The solutions are checked for self-consistency and the process is reiterated until stable values are found.

5. Discussion

The initial study of the assumptions made in the literature suggested that they might not be fully valid for our trap. Our trap has a steep magnetic gradient that calls into question the assumption that the trap can be treated as a piecewise continuous plasma without radial dependence of the axial magnetic field.

However, difficulties rapidly arise when one tries to determine the off axis field of the trap. The vector potential A_θ, and hence the radial dependence, is based on the geometry and magnetization of the magnets used to create the field. The Halbach arrangement that AEgIS plans to use has a complicated scheme of ring magnets with multiple magnetization directions. Off axis magnetic fields are difficult to calculate from permanent magnets and would make further calculations unfeasible. Moving forward, the plasma was treated as a piecewise continuous function without radial dependence in the magnetic field.

In order to apply the method used in Gopalan's thesis[1], a few modifications had to be made to the method. The literature begins with low field assumptions and solves for high field region. Simple algebra allowed us to determine the plasma characteristics based on desired aspects of the high field region.

The initial conditions defined earlier in this paper were used to calculate the rigid rotor frequency of our C_2^- plasma. Preliminary results show that the plasma has a velocity of roughly 293 kHz.

A preliminary solution– without iteration and stabilization of the solution– found a trap potential of 0.58V. This could then be applied to the equation for a separatrix. We know that increasing magnetic fields repel particles with increasing perpendicular velocities. Similarly, particles with low energy are blocked from returning to the low field region. Conservation of momentum and energy can be applied to define the separatrices[1] between trapped and free particles.

1. Low Field Region

$$v_\perp^2(v_\|) = \frac{1}{\beta}(v_\|^2 + \frac{2e\Delta\phi}{kT})$$

2. High Field Region

$$v_\perp^2(v_\|) = \frac{1+\beta}{\beta}(\frac{2e\Delta\phi}{kT} - v_\|^2)$$

The preliminary separatrix (Figure 1) is based off of a potential difference of 0.58 V. A second run returned a value of 0.73 V (Figure 2). The separatrices of these two runs differ based on this factor. The of the semi-major axis of the trap, the parallel velocity, increases from 5.1 m/s to 5.7 m/s and the major axis rises from 6.7 m/s to 7.5 m/s. The higher potential in the second run directly relates to the increase in the size of the high field separatrix.

In order to obtain a stable value for the potential and the separatrix, the process will have to be repeated many times to take into account various physical affects that change the relationship between the high and low field features. However, the data we have currently is a start for characterizing and optimizing trap conditions.

6. Figures and Tables

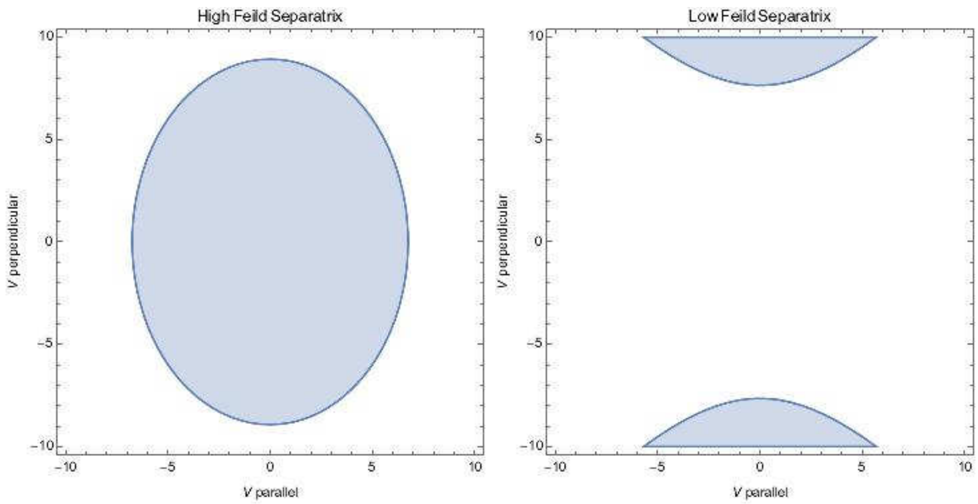

Fig. 1.— High and low field separatrices for the first run. $\Delta\phi = 0.58$V

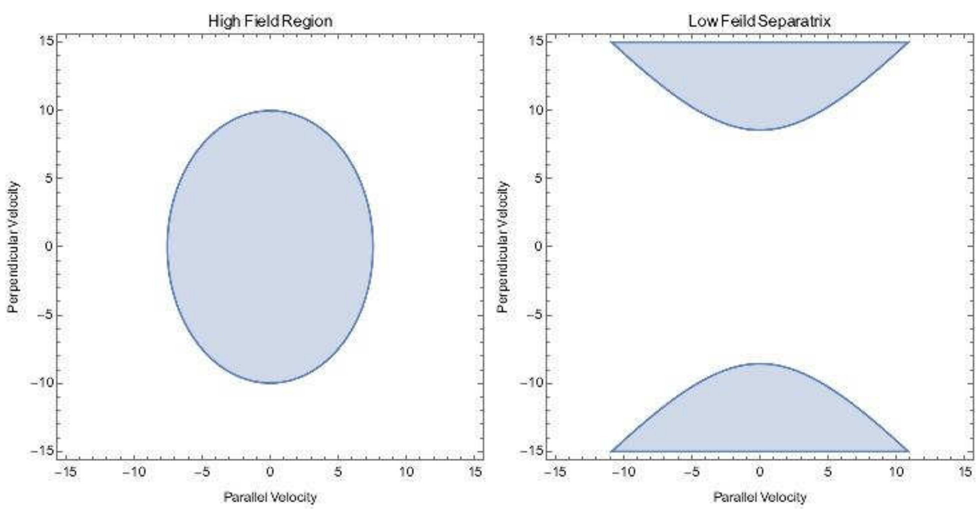

Fig. 2.— High and low field separatrices for the second run. $\Delta\phi = 0.73$V

7. References

1. R. Gopalan. PhD Thesis. University of California Berkley, 1998.

2. J. Fajans. *Physics of Plasmas.* Vol 10. No. 5. May 2003. 1209.

3. K. Gomberoff, J. Fajans, J. Wurtele, A. Friedman, D.P. Grote, R.H. Cohen, and J.-L. Vay, Simulation studies of non-neutral equilibria in an electrostatic trap with a magnetic mirror, Phys. Plasmas, 14 052107, 2007.