

Schottky Signal Analysis: Tune and Chromaticity

Ondine CHANON, CERN, BE-BI-QP

Supervisors: M. Wendt, CERN and M. Betz, CERN

Summer internship report, July 4th-September 2nd, 2016.

Abstract—Schottky monitors are used to determine important beam parameters in a non-destructive way. The Schottky signal is due to the internal statistical fluctuations of the particles inside the beam. In this report, after explaining the different components of a Schottky signal, an algorithm to compute the betatron tune is presented, followed by some ideas to compute machine chromaticity. Everything explained here has been tested on LHC data.

I. SCHOTTKY SIGNAL

A. Longitudinal Schottky signal

A single particle rotating in a circular accelerator will produce a short current pulse in the Schottky monitor each time the particle passes it. In the frequency domain, in an ideal system, this corresponds to Dirac pulses at every multiple of f_0 , the revolution frequency of the particle. Figure 1 shows the longitudinal signal induced by 2 particles with revolution frequencies f_0 and $f_1 = f_0 + \Delta f$, while Figure 2 generalizes it to N particles with revolution frequency spread $f_0 \pm \Delta f$. With this signal, it is possible to measure the mean revolution frequency f_0 and the distribution of the particles as a function of the frequency, and thus the energy.

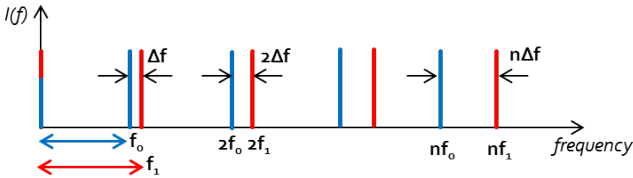


Fig. 1. 2 particles longitudinal Schottky signal, unbunched beam.

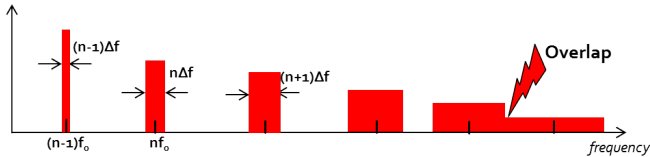


Fig. 2. N particles longitudinal Schottky signal, unbunched beam.

B. Transverse Schottky signal

For the particles to follow a specified trajectory, quadrupole magnets are used. In circular accelerators, the ideal trajectory is a circle; but in practice, because of the focusing-defocusing effect of the magnets, particles oscillate in the transverse plane around this perfect trajectory. The number of oscillations for a complete revolution is called *betatron tune* Q with frequency $f_\beta = qf_0$. For the beam to be stable, resonance effects need

to be avoided: some values of the tune are forbidden, such as the integers, and here comes the importance of carefully computing q , the non-integer part of the tune. Figure 3 shows first the longitudinal signal for one particle having the ideal trajectory, and then the superposition of longitudinal and transverse signals for a slightly shifted particle.

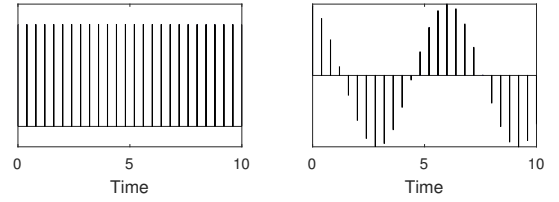


Fig. 3. One particle, time domain, unbunched beam Schottky noise.

The corresponding dipole moment (amplitude modulation due to both terms of the betatron oscillation) writes:

$$D(t) = a_0 \cos(2\pi f_\beta t + \phi) \left[A + B \sum_{n=1}^{\infty} \cos(2\pi n f_0 t) \right] \\ = C \left[\cos(2\pi q f_0 t + \phi) + \sum_{n=1}^{\infty} \cos[(n+q)2\pi f_0 t + \phi] \sum_{n=1}^{\infty} \cos[(n-q)2\pi f_0 t + \phi] \right].$$

with a_0, A, B, C some constants, and ϕ some random initial phase. So in frequency domain, for each particle, one Dirac pulse is replaced by two, as showed in Figure 4.

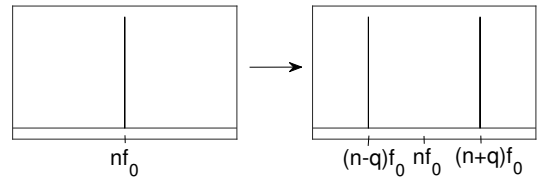


Fig. 4. One particle, frequency domain, unbunched beam longitudinal (left) and transverse (right) Schottky noise.

C. Bunched beams Schottky signal

For bunched beams, the particles pass through radio-frequency cavities, and thus also oscillate at the synchrotron frequency f_s . The dipole moment is then a combination of the betatron oscillation (amplitude modulation) and the synchrotron oscillation (time modulation):

$$D(t) = C \cos(2\pi q f_0 t + \phi) \cos[2\pi f_0 t + D \sin(2\pi f_s t + \psi)].$$

As before, the second cosine can be replaced by two other ones, splitting further the betatron side-bands into synchrotron satellites separated by f_s . This is represented in Figure 5 for a single particle.

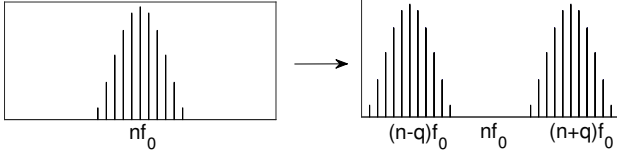


Fig. 5. One particle, frequency domain, bunched beam Schottky noise.

Schottky signals are usually measured at a high measurement frequency $f_c = hf_0$. Figure 6 shows a typical low resolution Schottky signal from the LHC, where each bunch is composed of $1.15e^{11}$ particles.

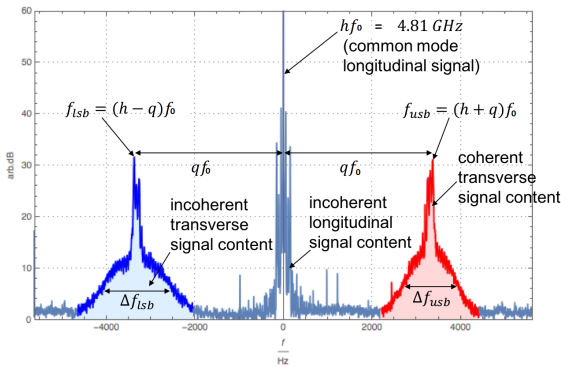


Fig. 6. Example of LHC Schottky spectrum at injection energy.

If the revolution frequency decreases, the signal is shifted to the left. If q increases, the side-bands move away from the common mode f_c , and inversely. When many particles with different revolution frequencies and different tunes are present, this gives rise to a lower and an upper side-bands with different widths, Δf_{lsb} and Δf_{usb} , as explained in Figure 7 with 2 particles. *Chromaticity* Q' is defined as:

$$Q' = \eta \cdot \left(\frac{\Delta f_{lsb} - \Delta f_{usb}}{\Delta f_{lsb} + \Delta f_{usb}} \cdot \frac{f_c}{f_0} + q \right).$$

II. BETATRON TUNE: PEAK FINDER ALGORITHM

Steps of the algorithm:

- Remove baselines.
- Cubic spline interpolation, if more precision is needed.
- Find the maximum within a tiny window around the middle of the spectrum in order to get the common mode longitudinal signal frequency. Adapt the tune axis in consequence to have exactly 0 on it.
- Use symmetry and periodicity of the signal to extract the zones of interest where the tune can be found. These zones are highlighted in grey in Figure 8 and correspond to a tune in $[0.28, 0.33]$ since we know that the beam is out if the tune is outside of this interval.

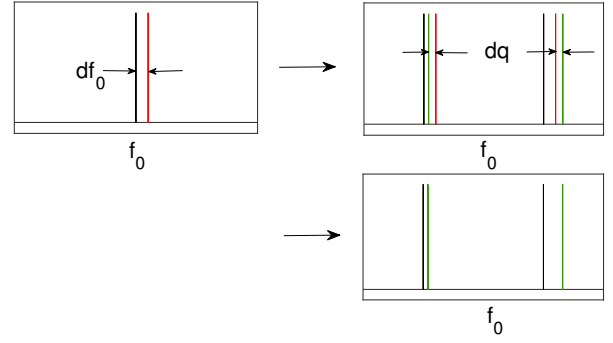


Fig. 7. Up-left: 2 particles with different revolution frequencies, longitudinal signal. Up-right: in black and red, 2 particles with different revolution frequencies, transverse signal; in black and green, 2 particles with different revolution frequencies and different tunes, transverse signal. Down: apparition of incoherent transverse signal content.

- Superpose the 4 zones, taking care of flipping the ones that need to be flipped because of symmetry. In that way, noise is reduced and the peak is more clearly identifiable.
- Smooth the signal without smearing out the peaks. Two different smoothers are proposed:
 - Savitzky-Golay smoother: fitting successive sub-sets of adjacent data points with a low-degree polynomial by the method of linear least squares. For polynomials of degree 0, this corresponds to a moving average filter. Polynomials of degree 2 take the curvature into account, while polynomials of degree 3 take inflexion points into account. The window size of the adjacent points considered should be set.
 - Smoother based on Markov chains: it is a statistical method to compute the probability to find a peak in each position. A relative height of the candidate peaks and their approximative width should be set, as well as a window for the peak search (see [6]).

The methods are represented and compared in Figure 9.

- Find the maximum. The corresponding value of the tune is the non-integer part of the tune we are looking for.

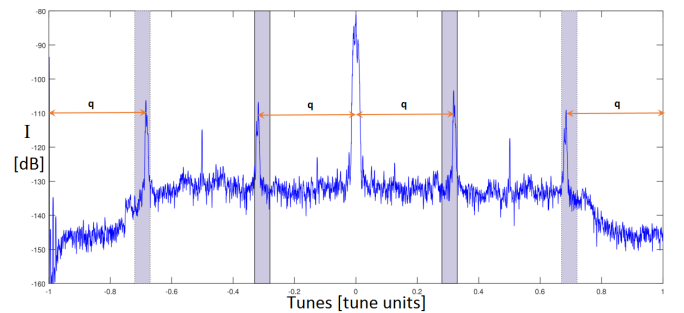


Fig. 8. Selection of the zones of a 2-period Schottky signal where the peak is present. Real data LHC, 22-Jul-2016 18:04:56, B1H.

Some parameters need to be tuned: window size, degree of the Savitzky-Golay smoother, approximative height and width of the searched peak for the Markov-based smoother. They heavily depend on the signal and the acquisition resolution.

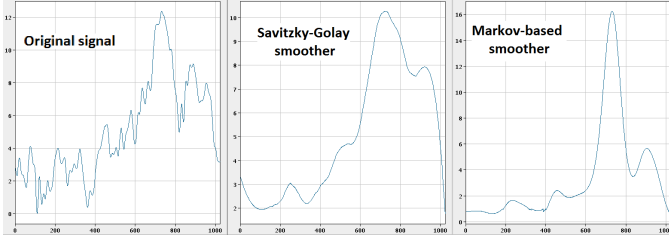


Fig. 9. Comparison of the 2 smoothers: Markov-based smoother has been designed to find peaks and smear out everything else, while Savitzky-Golay smoother keeps the global shape of the signal.

Note that the smoothing window should not be too large, otherwise two synchrotron satellites may mix and a peak in the middle of two synchrotron satellites may be found instead of the correct one. However, it should not be too small neither in order to reduce noise as much as possible without alternating too much the signal.

III. CHROMATICITY COMPUTATION BY CURVE FITTING

A. Algorithm

- Remove baselines.
- Cubic spline interpolation, if more precision is needed.
- Extract the zones of interest, i.e. consider only the zones around the humps: $[-0.4, -0.2]$ and $[0.2, 0.4]$, separately.
- Apply a light smoothing filter that does not smear out the humps: Savitzky-Golay smoother of degree 3 is fine.
- Compute the analytical envelope, i.e. get the local maximum within small windows moving along the spectrum.
- Fit both curves (the lower and the upper side-bands) with a suitable function, find the maximum and get the full width at half maximum (FWHM).
- With both FWHM, compute the chromaticity.

B. Fitting choices

A least square optimization is used for the fitting. To obtain the best fit as possible, a robust algorithm has been used, i.e. some weights are given to every point depending on their distance to the actual tried fitting curve. The coherent bands need to be excluded from the fit, not being part of the incoherent transverse signal content. To do so in a generic way on any input spectrum, consider the previously found betatron tune and exclude all points around it within a defined window. This is showed in Figure 10. In a further work, this window should be dynamically determined, but as for now, it is fixed to 0.016 (tune units).

Multiple fits have been tried:

- 1) Gaussian: $ae^{-((x-b)/c)^2} + d$ (see also [1]);
- 2) Sum of two Gaussian;
- 3) Fourier series: $a_0 + \sum_{i=1}^m (a_i \cos(cx) + b_i \sin(cx))$;
- 4) Sum of sine: $a_0 + \sum_{i=1}^m a_i \sin(b_i x + c_i)$;
- 5) Cauchy peak: $b / (\pi(b^2 + (x-a)^2))$;
- 6) Sum of two Cauchy peaks;
- 7) Smoothing spline: piecewise polynomials;
- 8) Exponential, 4th power: $ae^{-((x-b)/c)^4} + d$;

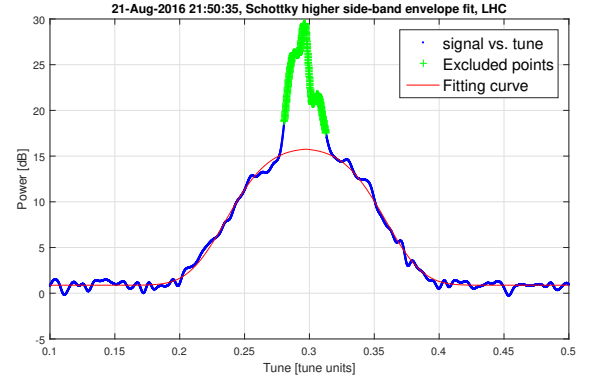


Fig. 10. Points exclusion and envelope fitting of an incoherent side-band.

- 9) Modified exponential, 4th power: the problem of the original exponential, 4th power, comes from the fact that it has a non-realistic flat top. Consequently, this modified version has been created by adding a parameter corresponding to a window around the mean that is cut out. The remaining two parts are then glued together. This is showed in Figure 11.

The difficulty of fitting 3) and 4) is to determine a good m . Those two, together with 7) might require the optimization of a high number of coefficients, that makes the algorithm computationally expensive.

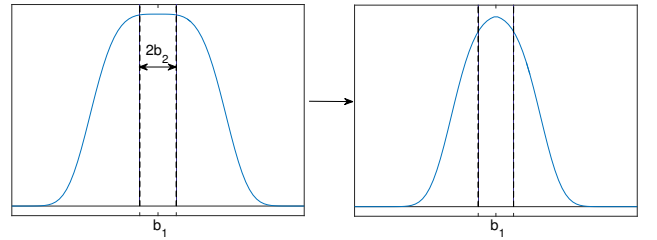


Fig. 11. Construction of the modified exponential, 4th power.

C. Some results

The analyse has been done at injection energy. Very small changes in the parameters influence very strongly the width of the fitting curves. The result of the previously explained procedure is represented in Figure 12, using the modified exponential, 4th power, fitting.

The results for this configuration are reported in Table I, together with the adjusted R^2 of each fit. The machine was here set to a (large) chroma equal to 20 on both vertical and horizontal planes, but this might not be the exact value. Instead, the results on Table II are reported on a spectra for which the machine was set to a (small) chroma equal to 2 but measured to 2.3 on both planes. For the fits 3) and 4), m has been chosen equal to 3 and 5 respectively, for both configurations.

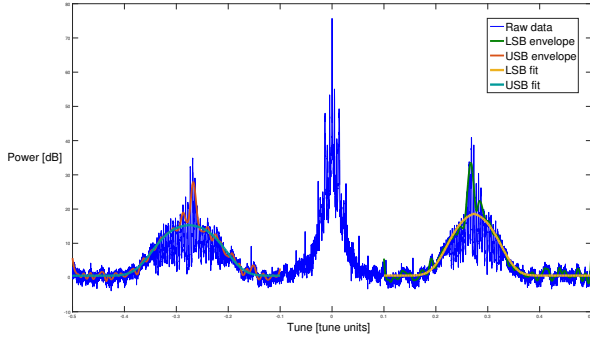


Fig. 12. Schottky side-bands fitting with the modified exponential, 4th power, LHC data 21-Aug-2016 21:00:26, B1H.

Bold values are the closest ones to the expected chromaticities. We can see that the Gaussian works quite well, together with the two versions of the exponential, 4th power. At the opposite, any Cauchy fitting gives very bad results. Fit 7) fits well the curve since it has been designed for it, but it does not give specially good results. Indeed, in this case, the FWHM is not necessarily very well related to the width of the humps, since the fit is not a distribution. This is also true for the fits 3) and 4).

TABLE I
RESULTS FOR A LARGE CHROMATICITY SET TO 20

Fit	Q' B1H	Q' B1V	adj. R^2 B1H lsb	adj. R^2 B1V lsb	adj. R^2 B1H usb	adj. R^2 B1V usb
1)	19.2700	23.3377	0.9658	0.9845	0.9604	0.9893
2)	18.5511	88.1447	0.9579	0.9744	0.9382	0.9898
3)	23.5773	32.5569	0.9773	0.1153	0.9699	0.9938
4)	15.7298	55.4508	0.9763	0.9941	0.9771	0.9948
5)	25.7116	38.4209	0.9415	0.9523	0.9322	0.9616
6)	NaN	38.4269	0.1147	0.9523	0.9321	0.9616
7)	18.6704	28.0782	0.9809	0.9868	0.9756	0.9807
8)	17.0330	19.5982	0.9727	0.9915	0.9656	0.9919
9)	19.6552	23.4172	0.9756	0.9931	0.9675	0.9934

TABLE II
RESULTS FOR A SMALL CHROMATICITY SET TO 2, MEASURED 2.3

Fit	Q' B1H	Q' B1V	adj. R^2 B1H lsb	adj. R^2 B1V lsb	adj. R^2 B1H usb	adj. R^2 B1V usb
1)	1.6573	2.3742	0.9599	0.9855	0.9574	0.9827
2)	8.4375	2.4487	0.9586	0.9846	0.9494	0.9757
3)	2.5441	61.0792	0.9732	0.0794	0.9699	0.9949
4)	34.6706	0.2839	0.9799	0.9942	0.9801	0.9951
5)	2.7916	2.7579	0.9275	0.9578	0.9273	0.9513
6)	2.7880	2.7622	0.9275	0.9578	0.9274	0.9513
7)	2.6913	1.5315	0.9808	0.9960	0.9774	0.9928
8)	0.7795	2.0257	0.9682	0.9894	0.9606	0.9906
9)	2.3656	1.4061	0.9702	0.9930	0.9659	0.9941

IV. CONCLUSION

The peak detection algorithm to compute the betatron tune works with a precision from 10^{-3} to 10^{-4} at injection energy and at top energy, and it has been implemented as a FESA class in order to get the tune online. However, an ever better precision would be needed in order to be sensitive to collimator setting changes. An idea would be to consider the synchrotron lines, align them, and add to the present algorithm a relative tune computation. A solution has also to be found in order to be able to compute the tune during the ramps.

To compute chromaticity, there is still a lot of work to do. What is presented here are just first ideas on which can be based more research. More precision and a better reliability are needed, so more tests need to be done. A future work would be to optimize in a more precise way all the parameters, finding methods to determine them dynamically. The fact that the integral of both humps should be equal could also be used to ameliorate the results.

Some work has been also done to simulate Schottky signals from theoretical time domain formulas, given in [2], [3] and [5]. We strongly believe in the power of such simulations to help the computation of beam or machine parameters such as betatron tune or chromaticity.

REFERENCES

- [1] M. Betz, M. Wendt, T. Lefvre, *Summary of LHC MD 377: Schottky pick-up*, CERN, Geneva, Switzerland, 2015.
- [2] D. Boussard, *Schottky Noise And Beam Transfer Function Diagnostics*, CERN, Geneva, Switzerland, 1986.
- [3] F. Caspers, J. Tan, *Schottky Signal for Longitudinal and Transverse Bunched Beam Diagnostics*, CERN, Geneva, Switzerland, 2009.
- [4] M. Favier, *Schottky status - BI day*, CERN, Geneva, Switzerland, 2012.
- [5] T. Linnekar, *Schottky Beam Instrumentation*, CERN- PE-ED 001-92, CERN, Geneva, Switzerland, 1992.
- [6] Root documentation, *TSpectrum class reference*, <https://root.cern.ch/doc/master/classTSpectrum.html#a5ba181a45b117934b48c4ef5f78d0b2b>.