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Neutral kaon detection asymmetry with charm hadron decays at LHCb

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Introduction

The violation of the CP symmetry, which is the combination of charge conjugation C and spatial parity P , is predicted in the Standard Model of Particle Physics. The only CP violation source is a single complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix, which describes the mixing between quarks of different generations. The CKM mechanism has proven to be very successful in describing and predicting CP violation in the neutral flavoured meson systems, but it is not enough to explain the cosmological observations, which indicate a universe dominated by matter with only a negligible amount of antimatter. Consequently, the particle physics community actively seeks new sources of CP violation that might have played a crucial role in the evolution of our universe.

The violation of CP symmetry has been first observed in the K meson system in 1964 [1] and then discovered in B^0 mesons in 2001 [2, 3] and in B_s^0 mesons in 2013 [4]. Since CP violation effects are extremely small in D mesons, the first evidence of CP violation in the charm sector, in the golden channels $D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow K^+K^-$, has been achieved by the LHCb experiment only recently, in 2019 [5]. The study of charm physics is particularly interesting as it provides the only probe of CP violation in the up-type quark sector. Furthermore, theoretical uncertainties on non-perturbative strong interaction effects prevent a rigorous assessment of its compatibility with the Standard Model. Therefore, increasing the precision of observed CP -violating effects in charm decays is essential for advancing our understanding.

Searching for CP -violating effects in the charm sector is experimentally challenging. It requires precise knowledge of all contributions to particle-antiparticle asymmetries in the channels of interest, down to a few parts in 10^{-4} . Common approaches rely on data and necessitate calibration channels where CP violation is much smaller than the statistical and systematic uncertainties involved. Some of the most abundant calibration channels include a neutral kaon in their final states, such as $D^+ \rightarrow K_S^0\pi^+$, $D_s^+ \rightarrow K_S^0K^+$ and $D^0 \rightarrow K_S^0\pi^+\pi^-$. Therefore, modelling and precisely determining the asymmetry due to the presence of neutral kaons in the final state of two-body or many-body charm decays is of paramount importance and forms the main subject of this thesis work.

This thesis presents the high-precision measurement of the asymmetry in detecting neutral kaons, K^0 and $\bar{K}^0$, produced in charm hadron decays. This asymmetry depends on the neutral kaon's decay time and is primarily due to three physical phenomena: the different interaction of a K^0 and a $\bar{K}^0$ with matter, CP violation in the $K^0 - \bar{K}^0$ mixing, and the interference of the Cabibbo-Favoured (CF) and Doubly Cabibbo-Suppressed (DCS) charm hadron decay amplitudes with the $K^0 - \bar{K}^0$ mixing. An original contribution of this thesis is the modelling of the CP violation arising from the third effect, which occurs because a D meson can produce both a $\bar{K}^0$, via the CF process, and a K^0 , via the DCS process. Since both the K^0 and $\bar{K}^0$ have access, through mixing, to the the same $\pi^+\pi^-$ CP -even final state, the two amplitudes interfere, providing an additional source of CP violation affecting the neutral kaon detection asymmetry. The analysis uses data samples produced in pp collisions at a center-of-mass energy of 13 TeV, collected with the LHCb

detector during the LHC Run 2 (2016, 2017, 2018) and corresponding to an integrated luminosity of 5.4 fb^{-1} . The neutral kaon detection asymmetry is measured using about 50 million $D^+ \rightarrow K_S^0 \pi^+$ and $D^- \rightarrow K_S^0 \pi^-$ decays. The measured particle-antiparticle asymmetry in $D^+ \rightarrow K_S^0 \pi^+$ decays, where CP violation is negligible, arises from three contributions: the different production cross section of a D^+ and a D^- , the different detection efficiency of the π^+ and π^- coming directly from the D meson decay, and the neutral kaon detection asymmetry. The first two contributions are removed by using the $D^+ \rightarrow \phi \pi^+$ decay sample, where the particle-antiparticle asymmetry is similar except for the neutral kaon detection asymmetry term.

The measured values of the neutral kaon detection asymmetry at different K_S^0 decay times are compared with an analytical model of the system's time evolution. This model, developed for past charm CP violation measurements [6, 7, 8], predicts the detection asymmetry based on the amount of detector material crossed by neutral kaon candidates. The effects included so far were the *regeneration* of K_S^0 , due to the interaction of the neutral kaons with matter, and the CP violation in the $K^0 - \bar{K}^0$ mixing. As original contribution, this thesis extends the model to account for an additional CP violation source, due to the interference between the CF and the DCS amplitudes in the $D^+ \rightarrow K_S^0 \pi^+$ decay [9]. These effects are parameterised by the ratio of the DCS to CF amplitudes, in particular by the modulus r_π and the relative strong phase δ_π , defined as

$$\frac{\mathcal{A}(D^+ \rightarrow K^0 \pi^+)}{\mathcal{A}(D^+ \rightarrow \bar{K}^0 \pi^+)} \equiv r_\pi e^{i\delta_\pi}.$$

This additional contribution has a significant impact, especially at high decay times, and can no longer be neglected in current and future high-precision measurements. The relevant parameters describing this interference are measured for the first time directly from data, representing a novel and original piece of work.

This thesis also presents the extension of the measurement of $A_{CP}(D^+ \rightarrow \phi \pi^+)$ to the 2018 Run 2 data sample, corresponding to about 2 fb^{-1} of data, not used in the published analysis of Ref. [8]. Data samples collected in 2016 and 2017 are also reanalysed to align selection and analysis strategies across the three data samples. Thanks to the determination of r_π and δ_π , $D^+ \rightarrow K_S^0 \pi^+$ decays with neutral kaons reconstructed outside the acceptance of the vertex detector are included in the analysis, allowing a more robust estimate of systematic uncertainties related to the knowledge of the neutral kaon detection asymmetries.

In summary, this thesis provides a high-precision measurement of neutral kaon detection asymmetry, extending the currently used model to include CF and DCS interference effects. These advancements are applied to measure the interference parameters in the $D^+ \rightarrow K_S^0 \pi^+$ decay channel and to measure with the full Run 2 dataset the $A_{CP}(D^+ \rightarrow \phi \pi^+)$ observable. The implications of these results are substantial for advancing current and future high-precision CP violation measurements in particle physics.

Chapter 1

Mixing and CP violation in the flavoured neutral mesons

This chapter briefly describes the phenomenology of mixing of neutral flavoured mesons, namely K^0 , D^0 , B^0 , and B_s^0 , and of CP violation in the Standard Model of particle physics. The first sections present a general overview of the model used to describe flavour oscillations and the phenomenology of CP violation in the four neutral mesons systems. The last section focuses on neutral kaons, which are the system of interest of this thesis, and the features of their time evolution and decay in matter are discussed in detail.

1.1 Introduction

In the Standard Model, flavoured neutral mesons present a unique phenomenon, due to the features of quarks weak interactions: the *oscillation* or *mixing* of a particle into its own antiparticle. Since mixing would violate the electromagnetic charge and the baryonic number conservations, particle-antiparticle oscillations are forbidden for all charged and neutral baryons, charged leptons, and charged mesons. On the other hand, the Dirac or Majorana nature of neutrinos remains yet to be determined. In this situation, particle-antiparticle mixing can only happen for neutral mesons. In addition, it is necessary that at least one flavour quantum number, such as strangeness, charmness or bottomness, is nonzero, so that particles and antiparticles are distinguishable. Neutral mesons containing top quarks are excluded since the top quark does not hadronise. These conditions leave only four mesons which can mix between particle and antiparticle states: $K^0(d\bar{s})$, $D^0(c\bar{u})$, $B^0(d\bar{b})$, and $B_s^0(s\bar{b})$.

Mixing of neutral mesons was first hypothesised in the kaon system. In 1955, Gell-Mann and Pais postulated that neutral kaons are produced in strong interactions as two distinct flavour eigenstates, K^0 and $\bar{K}^0$, which are different from the mass eigenstates K_S^0 and K_L^0 [10]. Only the eigenstates of the Hamiltonian, which determines the time evolution of the system, have distinct lifetimes and masses. The fact that K^0 and $\bar{K}^0$ are not eigenstates of the Hamiltonian leads to flavor oscillations. For instance, this means that a beam initially consisting purely of K^0 will, after some time, contain $\bar{K}^0$ as well. The long-lived neutral kaon K_L^0 was experimentally observed in 1957 by Lande, Lederman, and Chinowsky at the Brookhaven Cosmotron experiment [11]. Mixing in neutral B mesons has been observed for the first time in the $B^0 - \bar{B}^0$ system by the ARGUS experiment at DESY [12] and, with a big experimental effort to increase the time resolution, in 2006 the CDF collaboration has established evidence of mixing also in the $B_s^0 - \bar{B}_s^0$ system [13]. Finally, the first single-experiment observation of charm mixing was made by LHCb in 2013 [14].

Flavoured neutral mesons are the only¹ system in which there is evidence, at more than 5σ , of the violation of the CP symmetry in weak interactions. The discrete CP transformation is the combination of the charge conjugation C , which reverses the internal quantum numbers of all particles, and the spatial parity P , which reverses the sign of spatial coordinates. Hence, if CP is a symmetry of the universe, matter and antimatter are indistinguishable.

While strong and electromagnetic interactions preserve both P and C , and thus CP , weak interactions maximally violate both P and C . Violation of the CP symmetry in quark weak interactions was first observed in 1964 in neutral kaons and it was soon introduced in the Standard Model of Particle Physics. The only source of CP violation in the quark sector is a single complex phase in the quark-mixing matrix, also known as Cabibbo-Kobayashi-Maskawa (CKM) matrix [16, 17]. The CKM mechanism has proven to be very successful in describing and predicting CP violation in the neutral flavoured meson systems, but it is not enough to explain the cosmological observations, which show us a universe dominated by matter, with a really low percentage of antimatter. For this reason, the particle physics community is searching for new sources of CP violation, in the quark and in the neutrino sector, which could have played an important role in the evolution of our universe [18].

1.2 Quark weak interactions

Weak interactions are responsible for both the flavour oscillations and CP violation phenomena, which have been explained with the CKM mechanism. In the Standard Model (SM) the charged-current interaction of the three quark generations with the W boson is described by the following Lagrangian term:

$$\mathcal{L}_{W,q} = -\frac{g_W}{\sqrt{2}} \cdot (\bar{u}_L \ \bar{c}_L \ \bar{t}_L) \gamma^\mu W_\mu^+ V_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c. , \quad (1.1)$$

where $(u_L, d_L), (c_L, s_L), (t_L, b_L)$ are the three generations of the left chirality spinors of the quark mass eigenstates. From this lagrangian term we can see that the weak interaction eigenstates are a rotation, identified by the unitary matrix V_{CKM} , of the quark mass eigenstates:

$$\begin{pmatrix} d'_L \\ s'_L \\ b'_L \end{pmatrix} = V_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix}. \quad (1.2)$$

The V_{CKM} matrix elements describe the coupling between $W^\pm$ gauge bosons and quark mass eigenstates, also belonging to different generations, thereby governing flavour-changing interactions. Furthermore, the presence of complex conjugate pairs $V_{ij}^* \neq V_{ij}$, due to the existence of at least one complex phase in the matrix elements, allows for CP violation to occur in the charged weak interactions of quarks. The number of physical² independent real parameters of a complex unitary

¹It is worth mentioning that the T2K collaboration has found in 2020, for the first time ever, evidence at 3σ of CP violation in neutrino oscillations [15]. Thus, neutrino mixing will become in the future years, along to the quark sector, a powerful probe to understand more deeply the CP violation sources in the Standard Model.

²A unitary complex matrix has, in total, N^2 free real parameters. However, among them $2N - 1$ parameters are unphysical phases and do not need to be taken into account. In fact, one phase can be absorbed into each up or down type quark field, but changing all the quark fields by one common phase does not affect V_{CKM} . This leaves $(N - 1)^2$ physical free parameters.

$N \times N$ matrix is $(N - 1)^2$, among which $N(N - 1)/2$ are independent rotation angles between the N basis vectors and the remaining $(N - 1)(N - 2)/2$ free parameters are then complex phases. In the Standard Model three generation of quarks exist, thus the CKM matrix is a 3×3 complex unitary matrix with 4 free parameters: 3 rotation angles and 1 complex phase, responsible for CP violation.

A common parametrisation of the CKM matrix has been introduced by Wolfenstein [19] and the four real parameters used to describe V_{CKM} are: $\lambda \sim 0.22$, $A \sim 0.82$, $\rho \sim 0.22$, $\eta \sim 0.34$. The CKM matrix can be approximated up to order λ^4 as

$$V_{\text{CKM}} \simeq \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \quad (1.3)$$

We can see that, in this parametrisation, to have CP violating effects in the theory it is necessary that $\eta \neq 0$. In addition, it is clear that the magnitude of the V_{CKM} elements depends on different powers of λ according to a hierarchical structure:

- the elements on the diagonal are close to 1, thus the transitions between quarks of the same generations are favoured;
- the elements V_{us} , V_{cd} , V_{cb} and V_{ts} are of the order of $\lambda/\lambda^2 \sim 10^{-1} \div 10^{-2}$, thus the transitions between quarks of two close generations are suppressed;
- the elements V_{ub} and V_{td} are of the order of $\lambda^3 \sim 10^{-3}$, thus the transitions between quarks of the first and third generation are extremely suppressed.

Based on this hierarchical structure of the V_{CKM} matrix, the hadron decay modes can be divided into three categories, depending on which V_{CKM} elements appear in the dominant weak amplitudes:

- **Cabibbo-Favoured** processes, in which only the diagonal V_{CKM} elements appear in the Feynman diagram. An example is the $D^+ \rightarrow \bar{K}^0 \pi^+$ decay, whose tree level amplitude is proportional to $V_{cs} V_{ud}^*$;
- **Singly Cabibbo-Suppressed** processes, in which one non-diagonal V_{CKM} element appears in the Feynman diagram. An example is the $D^+ \rightarrow \phi \pi^+$ decay, whose tree level amplitude is proportional to $V_{cs} V_{us}^*$;
- **Doubly Cabibbo-Suppressed** processes, in which two non-diagonal V_{CKM} elements appear in the Feynman diagram. An example is the $D^+ \rightarrow K^0 \pi^+$ decay, whose tree level amplitude is proportional to $V_{cd} V_{us}^*$;

1.3 Mixing in flavoured neutral mesons systems

The theoretical formalism describing the time evolution and the mixing of a generic flavoured neutral meson is well established and it is described in depth in the literature, for example in Ref. [20]. Here, the most important results, relevant for this thesis work, are briefly overviewed.

In the following, a generic flavoured neutral meson is referred to as M^0 , which stands for K^0 , D^0 , B^0 and B_s^0 . A $M^0 - \bar{M}^0$ system can be described by a phenomenological model that involves non-relativistic quantum mechanics, which consists in a two level system. This simplified formalism can be derived using the Wigner-Weisskopf approximation [21], based on the fact that the times t under consideration in the weak evolution are much larger than the typical time scale of the strong

interaction and weak interactions are considered as a perturbation to the strong ones. The time evolution of the system is then determined by an effective Hamiltonian, $\mathbf{H}$, which describes the effects of the weak Hamiltonian at second order in perturbation theory. $\mathbf{H}$ can be split into an unperturbed part, whose eigenstates are the flavour eigenstates $|M^0\rangle$ and $|\bar{M}^0\rangle$ and the eigenvalues are both equal to the mass M , and a perturbation, the weak Hamiltonian $\mathbf{H}_w$. The generic state of the system can be written in the $\{|M^0\rangle, |\bar{M}^0\rangle\}$ basis as:

$$|\psi(t)\rangle = a(t)|M^0\rangle + b(t)|\bar{M}^0\rangle, \quad (1.4)$$

where $a(t)$ and $b(t)$ are time dependent coefficients. The initial state is given by a general superposition of the flavour eigenstates. The time evolution of a general initial state $|\psi(0)\rangle$ can be found solving the Schrödinger's equation:

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = \mathbf{H}|\psi(t)\rangle. \quad (1.5)$$

In the basis of the flavour eigenstates, the Hamiltonian of the system can be written as a 2×2 effective matrix, which is not hermitian since the neutral mesons can decay in a set of final states:

$$\mathbf{H} \equiv \mathbf{M} - \frac{i}{2}\mathbf{\Gamma}, \quad (1.6)$$

where $\mathbf{M}$ and $\mathbf{\Gamma}$ are hermitian matrices that describe the time evolution of the $M^0 - \bar{M}^0$ system, including the $(M^0, \bar{M}^0) \leftrightarrow (\bar{M}^0, M^0)$ transitions. In particular, $\mathbf{M}$ describes dispersive flavour transitions via off-shell intermediate states, while $\mathbf{\Gamma}$ describes the absorptive transitions, which occur through on-shell intermediate states. In general, $\mathbf{H}$ depends on 8 free real parameters, M_{ij} and Γ_{ij} , which can be computed from the unperturbed and weak Hamiltonian. If the time evolution described by $\mathbf{H}$ is invariant under some combinations of the discrete transformations P, C, T , the 8 parameters must respect some conditions, summarised in table 1.1. In the case of

Symmetry	Conditions
CPT	$M_{11} = M_{22} \equiv M \quad , \quad \Gamma_{11} = \Gamma_{22} \equiv \Gamma$
CP	$M_{11} = M_{22} \equiv M \quad , \quad \Gamma_{11} = \Gamma_{22} \equiv \Gamma \quad , \quad \Im[\Gamma_{12}/M_{12}] = 0$
T	$\Im[\Gamma_{12}/M_{12}] = 0$

Table 1.1: Summary of the conditions that the $\mathbf{H}$ free parameters must fulfil to have the reported symmetries in the theory.

weak interactions, the theory is invariant under CPT and the matrix elements can be written in the following form [20]:

$$M_{ij} = M \cdot \delta_{ij} + \langle M_i | \mathbf{H}_w | M_j \rangle + \sum_f P \frac{\langle M_i | \mathbf{H}_w | f \rangle \langle f | \mathbf{H}_w | M_j \rangle}{M - E_f}, \quad (1.7)$$

$$\Gamma_{ij} = 2\pi \sum_f \delta(M - E_f) \langle M_i | \mathbf{H}_w | f \rangle \langle f | \mathbf{H}_w | M_j \rangle, \quad (1.8)$$

where $|M_1\rangle \equiv |M^0\rangle$, $|M_2\rangle \equiv |\bar{M}^0\rangle$, and P indicates the principal part of the sum over the possible final states $|f\rangle$, each with energy E_f . To study the time evolution of neutral mesons it is useful to define the two eigenstates of the total Hamiltonian, $|M_{H,L}\rangle$, related to $|M^0\rangle$ and $|\bar{M}^0\rangle$ in the

following way:

$$|M_{H,L}\rangle \equiv p|M^0\rangle \mp q|\bar{M}^0\rangle \quad \text{with} \quad \frac{q}{p} = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}}. \quad (1.9)$$

The corresponding eigenvalues $\lambda_{H,L}$ are:

$$\lambda_{H,L} = M - \frac{i}{2}\Gamma \pm \frac{q}{p} \left(M_{12} - \frac{i}{2}\Gamma_{12} \right) \equiv m_{H,L} - \frac{i}{2}\Gamma_{H,L}, \quad (1.10)$$

where the real and imaginary parts $m_{H,L}$ and $\Gamma_{H,L}$ are related to M and Γ :

$$M = \frac{m_H + m_L}{2} \quad \text{and} \quad \Gamma = \frac{\Gamma_H + \Gamma_L}{2}. \quad (1.11)$$

The time evolution of initial flavour eigenstates $|M^0\rangle$ and $|\bar{M}^0\rangle$ can be found from Schrödinger's equation and using the fact that the eigenstates $|M_{H,L}\rangle$ evolve with a phase: $\exp(-i\lambda_{H,L}t)$. From this, we get the time evolution for a state that was initially a pure $|M^0\rangle$ or $|\bar{M}^0\rangle$:

$$\begin{cases} |M^0(t)\rangle &= g_+(t)|M^0\rangle + \frac{q}{p}g_-(t)|\bar{M}^0\rangle \\ |\bar{M}^0(t)\rangle &= \frac{p}{q}g_-(t)|M^0\rangle + g_+(t)|\bar{M}^0\rangle \end{cases} \quad \text{with} \quad g_{\pm}(t) = \frac{e^{-i\lambda_H t} \pm e^{-i\lambda_L t}}{2}. \quad (1.12)$$

Therefore, it is possible to derive the time-dependent expressions for the mixing and non-mixing probabilities, which are:

$$\mathcal{P}(M^0(t) \rightarrow M^0) = \mathcal{P}(\bar{M}^0(t) \rightarrow \bar{M}^0) = |g_+(t)|^2 \quad (1.13)$$

$$\mathcal{P}(M^0(t) \rightarrow \bar{M}^0) = \left| \frac{q}{p} \right| \cdot |g_-(t)|^2 \quad (1.14)$$

$$\mathcal{P}(\bar{M}^0(t) \rightarrow M^0) = \left| \frac{p}{q} \right| \cdot |g_-(t)|^2. \quad (1.15)$$

The probability of preserving the initial flavour is the same for both the particles and the anti-particles, whereas the mixing probability is different if the initial state is a M^0 or an $\bar{M}^0$ only if $|q/p| \neq 1$. Therefore, a measurement of this quantity different than 1 is a direct evidence of CP violation in the mixing in the $M^0 - \bar{M}^0$ system. The term $|g_{\pm}(t)|^2$ can be explicitly written as:

$$|g_{\pm}(t)|^2 = \frac{1}{2} e^{-\Gamma t} [\cosh(y\Gamma t) \pm \cos(x\Gamma t)], \quad (1.16)$$

where the mixing parameters x and y are defined as:

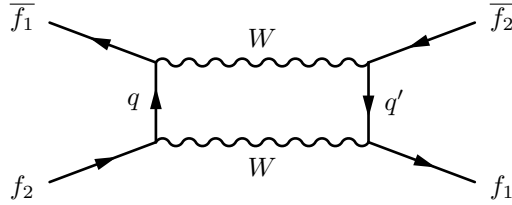
$$x \equiv \frac{m_H - m_L}{\Gamma} \quad \text{and} \quad y \equiv \frac{\Gamma_H - \Gamma_L}{2\Gamma}. \quad (1.17)$$

These two parameters determine the behaviour of the oscillations of neutral flavoured mesons. In particular, the x parameter, related to the mass difference between the two physical eigenstates, is proportional to the rate of the oscillations. On the contrary, the y parameter, proportional to the decay width difference, appears in the hyperbolic cosine term and determines the difference in the exponential decay trend.

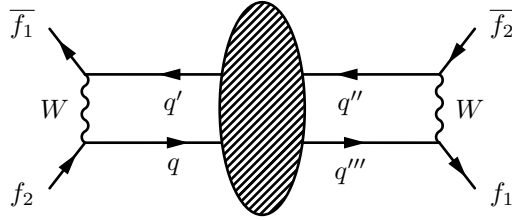
1.4 Mixing phenomenology in flavoured neutral mesons

Despite the mixing model being the same for all the neutral mesons systems, the values of x and y , which determine the behaviour of the oscillations, vary significantly between them, as shown in table 1.2 and figure 1.1. These big discrepancies depend on the different interactions contributing to the matrix elements responsible for the transitions, M_{12} and Γ_{12} . The amplitudes contributing to the mixing diagrams can be divided in two categories:

- *short-distance* contributions, which involve the exchange of virtual particles off the mass shell only, mainly W bosons and heavy t , c or b quarks. The box diagrams that describe the oscillations of $f_1 \bar{f}_2$ mesons with off shell q and q' quarks in the loop are roughly proportional to $(V_{qf_1} V_{qf_2}^*)(V_{q'f_2} V_{q'f_1}^*) \cdot (m_q/m_W)^2$, where m_q is the mass of the lightest quark. In the case of D^0 mesons, the dominant contribution is given by $q = q' = b$, while for down-type mesons we have $q = q' = c, t$.



- *long-distance* contributions, which are characterised by the exchange of hadrons on the mass shell. In this case the oscillation can be interpreted as the decay of the meson to a common final state, which then recombines to form the opposite flavour meson. For this reason the size of this contribution is proportional to the phase space shared by the meson and the anti-meson. In the following diagram, the blob stands for low-energy QCD interactions, possibly involving the exchange of hadrons on the mass shell.



The differences in the phenomenology of the four neutral mesons systems can be explained as follows.

- In **K^0 mesons**, M_{12} is dominated by short-distance contributions with the exchange of two charm quarks and, as a minor correction, top quarks. On the contrary, Γ_{12} is dominated by long-distance contributions, which can be easily computed, since there is a small number of possible K^0 final states and they are almost all shared by the $\bar{K}^0$. An interesting property of neutral kaons that has significant experimental implications is $y \approx 1$, which means that the two physical eigenstates have lifetimes differing by nearly three orders of magnitude. For this reason, historically the two eigenstates have been identified by their lifetimes and called, as discussed in detail in section 1.6, K-short (K_S^0 , with $\tau_S \sim 0.89 \times 10^{-10}$ s) and K-long (K_L^0 , with $\tau_L \sim 5.2 \times 10^{-8}$ s).

- The mixing in B^0 and B_s^0 systems is dominated by short-distance contributions, in particular by the exchange of two top quarks off the mass shell, thus the magnitude of M_{12} , i.e. of x , is much larger than that of Γ_{12} , i.e. of y , which is very small $\sim 10^{-2} - 10^{-3}$. More specifically, the long-distance contributions are suppressed because there is a large number of possible final states for the B^0 , but just a small fraction of them are accessible to the $\bar{B}^0$. In both the B^0 and B_s^0 systems, x has a large value and the flavour oscillation frequency is significantly higher in B_s^0 than in B^0 , due to the fact that $|V_{ts}| \gg |V_{td}|$.
- In D^0 mesons the short-distance contributions are extremely suppressed with respect to the other down-type neutral mesons, mainly because the masses of the internal down-type quarks in the box diagrams, which break the GIM cancellation at loop level, are much smaller than that of the top quark. The dominant long-range contributions are extremely difficult to compute in the case of D^0 mesons, because the number of accessible final states is larger than for the K^0 , but the c quark is not heavy enough to exploit power series expansions used in heavy quark effective field theories as in the case of B^0 and B_s^0 mesons. The long-range contributions are then computed under several assumptions, as shown in [22], and, the Standard Model theoretical predictions for mixing (and CP violation) parameters span several order of magnitudes, as discussed in [23]. The direct experimental effect is that both x and y are very small ($\sim 0.5\%$), thus the mixing is extremely difficult to observe, as shown in figure 1.1.

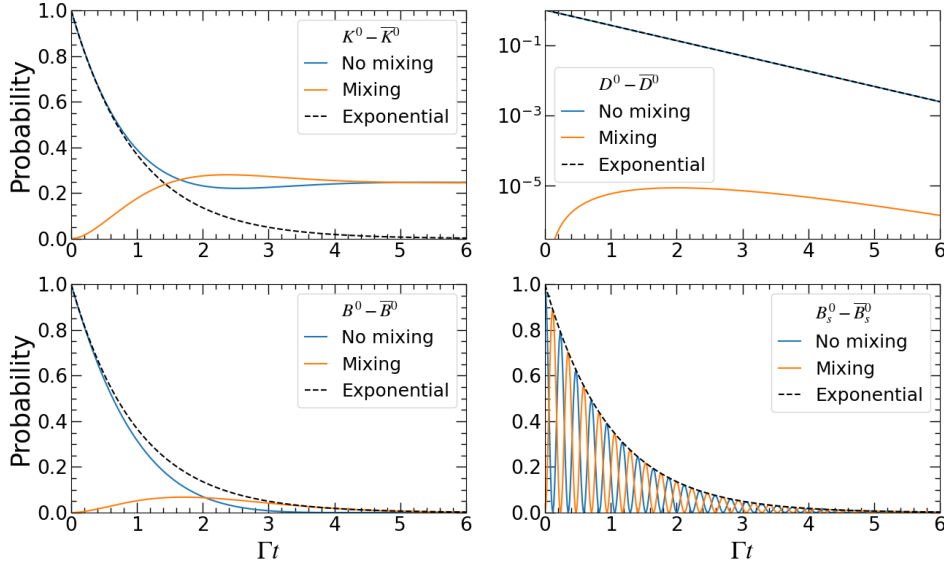


Figure 1.1: Mixing and no-mixing probability for the four neutral flavoured mesons as a function of time, assuming $|q/p| = 1$. In each plot, also an exponential function, which corresponds to the no-mixing probability in the case $x = y = 0$, is shown. To be noted that in the $D^0 - \bar{D}^0$ plot the probability is in logarithmic scale, since mixing effects are extremely suppressed, and the no-mixing probability is indistinguishable from the exponential.

System	x	y	Refs
$K^0 - \bar{K}^0$	-0.946 ± 0.004	0.99650 ± 0.00001	[24]
$D^0 - \bar{D}^0$	$(0.407 \pm 0.044)\%$	$(0.647 \pm 0.024)\%$	[25]
$B^0 - \bar{B}^0$	0.769 ± 0.004	0.001 ± 0.010	[25]
$B_s^0 - \bar{B}_s^0$	27.03 ± 0.09	0.126 ± 0.007	[25]

Table 1.2: Current experimental status of the mixing parameters in the flavoured neutral mesons systems in which flavour oscillations have been measured.

1.5 CP violation in flavoured neutral mesons

The presented model for time evolution of neutral flavoured mesons includes also the effects due to CP violation in the mixing, which depends on the condition $|q/p| \neq 1$. However, CP violation in the M^0 systems can also occur in the decay to a generic final state $M^0 \rightarrow f$. In this section, the different types of CP violating effects are presented and classified, with particular focus on the rich phenomenology of neutral mesons systems.

To observe CP violation in a generic process $i \rightarrow f$ it is not sufficient that the amplitudes of the process and of its CP conjugate are different. In fact, the observables we have access to are usually related to the squared modulus of the total amplitudes of the process and of its CP conjugate. The necessary condition to observe CP violation in the decay is then:

$$\text{observable } CPV \text{ in } i \rightarrow f \iff |\mathcal{A}(i \rightarrow f)|^2 \neq |\mathcal{A}(\bar{i} \rightarrow \bar{f})|^2. \quad (1.18)$$

It is easy to see that this condition holds only if these total amplitudes are given by the sum of at least two amplitudes with different strong and weak phases. In particular, the total amplitude must be written in the form

$$|\mathcal{A}(i \rightarrow f)|^2 = \left| \sum_k \mathcal{A}_k(i \rightarrow f) \right|^2, \quad (1.19)$$

where the amplitudes have different strong phases $\alpha_{s,k}$, which originate from final state interactions in the hadronic systems and don't change under CP conjugation, and weak phases $\alpha_{w,k}$, which are related to the complex phase in V_{CKM} and do change under CP conjugation. If we apply CP to these amplitudes, they transform in the following way:

$$\mathcal{A}_k(i \rightarrow f) \equiv |a_k| e^{i\alpha_{s,k}} e^{i\alpha_{w,k}} \xrightarrow{CP} \mathcal{A}_k(\bar{i} \rightarrow \bar{f}) \equiv |a_k| e^{i\alpha_{s,k}} e^{-i\alpha_{w,k}}. \quad (1.20)$$

Thus, the CP asymmetry in the $i \rightarrow f$ process can be written as:

$$\begin{aligned} A_{CP}(i \rightarrow f) &\equiv \frac{|\mathcal{A}(i \rightarrow f)|^2 - |\mathcal{A}(\bar{i} \rightarrow \bar{f})|^2}{|\mathcal{A}(i \rightarrow f)|^2 + |\mathcal{A}(\bar{i} \rightarrow \bar{f})|^2} \\ &= -\frac{2 \sum_{ij} |a_i| |a_j| \sin(\alpha_{s,i} - \alpha_{s,j}) \sin(\alpha_{w,i} - \alpha_{w,j})}{\sum_{ij} |a_i|^2 + |a_j|^2 + 2 |a_i| |a_j| \cos(\alpha_{s,i} - \alpha_{s,j}) \cos(\alpha_{w,i} - \alpha_{w,j})}, \end{aligned} \quad (1.21)$$

showing that $A_{CP}(i \rightarrow f) \neq 0$ only if there are at least two amplitudes with $\alpha_{s,i} \neq \alpha_{s,j}$ and $\alpha_{w,i} \neq \alpha_{w,j}$.

Classification of CP violating effects

In processes that involve neutral flavoured mesons (K^0, D^0, B^0, B_s^0) there can be three kinds of CP violation, which are schematically illustrated in figure 1.2:

1. CP violation in the decay, when the width of the processes involving particles and antiparticles are different. This type of CP violation can occur for any initial neutral or charged particle $M = M^0, M^+$, under the condition:

$$|\mathcal{A}(M \rightarrow f)|^2 \neq |\mathcal{A}(\bar{M} \rightarrow \bar{f})|^2. \quad (1.22)$$

The main observable to quantify CP violating effects is the direct CP asymmetry, defined as:

$$A_{CP}(M \rightarrow f) = \frac{|\mathcal{A}(M \rightarrow f)|^2 - |\mathcal{A}(\bar{M} \rightarrow \bar{f})|^2}{|\mathcal{A}(M \rightarrow f)|^2 + |\mathcal{A}(\bar{M} \rightarrow \bar{f})|^2}, \quad (1.23)$$

2. CP violation in the mixing, when the mixing probabilities are different for mesons and antimesons:

$$\mathcal{P}(M^0 \rightarrow \bar{M}^0) \neq \mathcal{P}(\bar{M}^0 \rightarrow M^0), \text{ that means } |q/p| \neq 1. \quad (1.24)$$

3. CP violation given by the interference between decay and mixing, possible only if the final state is accessible both by the particle and the antiparticle. The condition to have this third kind of CP violation with a CP invariant final state³ ($f = \bar{f}$) is:

$$\Im[\lambda_f] \neq 0 \quad \text{with} \quad \lambda_f \equiv \frac{q \cdot \mathcal{A}(\bar{M}^0 \rightarrow f)}{p \cdot \mathcal{A}(M^0 \rightarrow f)}. \quad (1.25)$$

The main observable to quantify CP violating effects in neutral mesons systems is the time-dependent CP asymmetry, which can be expressed as:

$$A_{CP}(M^0(t) \rightarrow f) = \frac{S_f \sin(x\Gamma t) - C_f \cos(x\Gamma t)}{\cosh(y\Gamma t) + (1 - C_f^2 - S_f^2) \sinh(y\Gamma t)}, \quad (1.26)$$

$$\text{with } C_f = \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \quad \text{and} \quad S_f = \frac{2\Im[\lambda_f]}{1 + |\lambda_f|^2}.$$

In particular, S_f describes the interference between mixing and decay and vanishes if $\Im[\lambda_f] = 0$. On the other hand, C_f is non-zero in the case of CP violation in the mixing or in the decay, since $|\lambda_f| = |q/p| \cdot |\mathcal{A}(M^0 \rightarrow f)/\mathcal{A}(\bar{M}^0 \rightarrow \bar{f})|^2 \neq 1$ if at least one of the two terms is different than one. Furthermore, if there is only CP violation in the decay, at $t = 0$ we find the expression for direct CP asymmetry in (1.23):

$$A_{CP}(M^0 \rightarrow f) \equiv A_{CP}(M^0(0) \rightarrow f) = -C_f = \frac{|\mathcal{A}(M^0 \rightarrow f)|^2 - |\mathcal{A}(\bar{M}^0 \rightarrow \bar{f})|^2}{|\mathcal{A}(M^0 \rightarrow f)|^2 + |\mathcal{A}(\bar{M}^0 \rightarrow \bar{f})|^2}, \quad (1.27)$$

³This type of CP violation can also occur for a final state accessible to both M^0 and $\bar{M}^0$ but not CP invariant. In this case, the condition becomes $\Im[\lambda_f] + \Im[\lambda_{\bar{f}}] = 0$

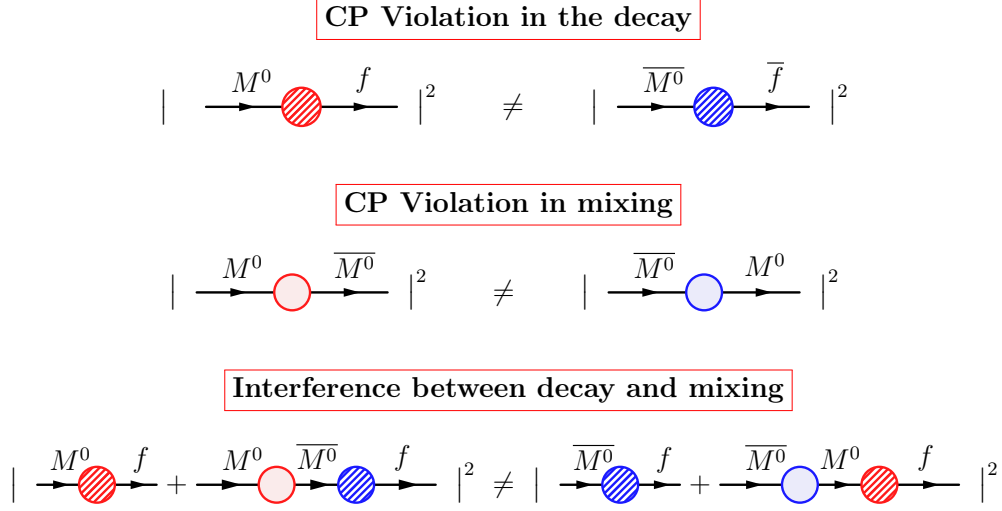


Figure 1.2: Schematic representation of the three possible types of CP violation in flavoured neutral mesons in a general $M^0 \rightarrow f$ decay process.

Phenomenology of CP violation in neutral mesons

CP violation has been first observed in the K meson system in 1964 [1] and then discovered in B^0 mesons in 2001 [2, 3] and in B_s^0 mesons in 2013 [4]. Since these effects are extremely small in D mesons, the first evidence of CP violation in the charm sector has been achieved only recently, in 2019 [5], by the LHCb experiment. In table 1.3 the current experimental status of CP violation in neutral mesons system is summarised.

In K mesons, all three types of CP violation have been observed. The first evidence of CP violation in the mixing occurred in 1964 [1], and it is characterised by the parameter ε^4 , currently measured to be $|\varepsilon| = (2.228 \pm 0.011) \times 10^{-3}$. The first evidence of direct CP violation in the decay of a neutral kaon into the CP even final states $\pi^+\pi^-$ and $\pi^0\pi^0$ was achieved in 1988 [26], when the double ratio R of the relative decay rates of the K_S^0 and K_L^0 into two charged and two neutral pions was measured to be different from one. This discovery was then confirmed in 1999 by the NA48 experiment at CERN and the KTeV experiment at Fermilab [27, 28], which measured a non-zero value of the parameter $\Re(\varepsilon'/\varepsilon)^5$.

In B mesons, evidence of CP violation in the interference between mixing and decay has been established at more than 5σ . However, no clear evidence of CP violation in the mixing itself has been observed. The first measurement of the time-dependent CP asymmetry in $B^0 \rightarrow J/\psi K_S^0$ decays has been achieved by the BaBar and Belle collaborations [2, 3], which measured, respectively⁶, $\sin(2\beta) = 0.59 \pm 0.14$ (stat) ± 0.05 (syst) and $\sin(2\phi_1) = 0.99 \pm 0.14$ (stat) ± 0.06 (syst). Direct CP violation in the B^0 system was first observed by the LHCb collaboration in 2013, through measurements of CP asymmetries in the decay of $B^0 \rightarrow K^+\pi^-$ [29]. In the B_s^0 system, direct CP violation was first established by the LHCb collaboration in 2013, in the $B_s^0 \rightarrow K^-\pi^+$ decay [4].

In the case of D mesons, only CP violation in the decay has been observed. In particular, in 2019

⁴ ε is related to the q/p ratio in the kaons system, as defined in section 1.6, and $|\varepsilon| \neq 1$ implies $|q/p| \neq 1$, i.e. CP violation in the $K^0\bar{K}^0$ mixing.

⁵ $\Re(\varepsilon)$ quantifies the CP violation in the mixing, while $\Re(\varepsilon')$ quantifies direct CP violation in $\pi\pi$ decay modes, due to the interference between the amplitudes into final states with different isospin.

⁶ ϕ_1 and β are both related to $\arg(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*)$ and are thus a measurement of CP asymmetry arising from the interference between $B^0 - \bar{B}^0$ mixing and decay in the tree-dominated $b \rightarrow c\bar{c}s$ transitions, such as into the abundant $J/\psi K_S^0$ CP odd final state.

the LHCb collaboration reported evidence for CP violation in the difference of time-integrated CP asymmetries in $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays, with the measured asymmetry difference $\Delta A_{CP} = [-15.4 \pm 2.9] \times 10^{-4}$ [5]. This marked the first observation of CP violation in charm decays.

System	CPV I type	CPV II type	CPV III type
$K^0 - \bar{K}^0$	$K \rightarrow \pi\pi$	$K \rightarrow \pi\pi$	$K \rightarrow \pi\pi$
$D^0 - \bar{D}^0$	$D^0 \rightarrow \pi^+\pi^-$ and $D^0 \rightarrow K^+K^-$	—	—
$B_{(s)}^0 - \bar{B}_{(s)}^0$	$B^0 \rightarrow K^+\pi^-, B_s^0 \rightarrow K^-\pi^+$	—	$B^0 \rightarrow J/\psi K_S^0$ $b \rightarrow c\bar{u}d, b \rightarrow c\bar{c}d\ldots$

Table 1.3: Review of the main channels used to measure CP violating effects at more than 5σ in the different neutral meson systems. More details can be found in Ref. [30].

1.6 The system of neutral kaons

So far, the presented model describes the time evolution and flavour oscillations of neutral mesons in vacuum before their decay. In the case of D^0 and $B_{(s)}^0$ mesons, the lifetimes of the two mass eigenstates almost coincide ($y \ll 1$) and are of the order of 0.4 ps and 1.5 ps respectively. For D^0 and $B_{(s)}^0$ mesons produced at hadron colliders, which are significantly boosted in the laboratory frame, this translates in an average distance before the decay of $\mathcal{O}(\text{mm} - \text{cm})$. For this reason, in the case of D^0 and $B_{(s)}^0$ mesons it is possible to neglect the presence of matter and to always assume their evolution to occur in vacuum before the decay process.

For neutral kaons, the situation is quite opposite: since $y \simeq 1$, the two mass eigenstates have very different lifetimes, both significantly larger than those of the other neutral flavoured mesons ($\tau_S \simeq 0.89 \times 10^{-10}$ s and $\tau_L \simeq 5.2 \times 10^{-8}$ s.) The average travelled distance, considering the neutral kaons boost, can be estimated to be $\beta\gamma c\tau_{S,L}$, with $c\tau_S \simeq 27$ mm and $c\tau_L \simeq 16$ m. In an experimental setup, $\beta\gamma$ can reach values of the order of 10 – 100, thus even a K_S^0 can travel an average distance before the decay of $\mathcal{O}(\text{m})$ ⁷. Since, experimentally, neutral kaons are reconstructed after having decayed into other particles, they can only be revealed after having travelled a significant distance inside the detector material. For this reason, it is necessary to build a model for the time evolution of neutral kaons in matter, taking into account also the contributions to the Hamiltonian given by the interaction between a K^0 or a $\bar{K}^0$ with nucleons.

The phenomenology of neutral kaon mesons is different from other neutral flavoured mesons, and therefore it is worth first introducing the notation used in literature to describe their time evolution in vacuum. Subsequently, the effect of the different interaction of a K^0 or a $\bar{K}^0$ with nucleons on the *mixing* phenomenon will be discussed and, finally, the expression for the neutral kaon detection asymmetry in matter will be derived. This is an essential ingredient of the work described in this thesis.

⁷The Lorentz factor $\beta\gamma$ is given by the ratio between the neutral kaon momentum and its mass. For instance, if we have a K_S^0 with $p \simeq 30$ GeV/c, its boost is $\beta\gamma \simeq 60$, thus the average travelled distance can reach a few meters.

1.6.1 Time evolution of neutral kaons in vacuum

A $K^0 - \bar{K}^0$ system can be described with the model introduced in section 1.3, where the generic state of the system can be written in the $\{|K^0\rangle, |\bar{K}^0\rangle\}$ basis as:

$$|\psi_K(t)\rangle = \alpha(t) |K^0\rangle + \bar{\alpha}(t) |\bar{K}^0\rangle. \quad (1.28)$$

In this basis, the Hamiltonian of the system in vacuum can be written as a 2×2 matrix, which fulfils the conditions required by the CPT symmetry,

$$\mathbf{H}_{\text{vacuum}} \equiv \mathbf{M} - \frac{i}{2} \mathbf{\Gamma} = \begin{pmatrix} M_K - \frac{i}{2} \Gamma_K & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{12}^* - \frac{i}{2} \Gamma_{12}^* & M_K - \frac{i}{2} \Gamma_K \end{pmatrix}. \quad (1.29)$$

The two eigenstates of the Hamiltonian in vacuum, $|K_{S,L}^0\rangle$, are usually written in terms of a complex parameter ε , related to the q and p parameters in (1.9):

$$\begin{cases} |K_S^0\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}} ((1+\varepsilon) |K^0\rangle - (1-\varepsilon) |\bar{K}^0\rangle), \\ |K_L^0\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}} ((1+\varepsilon) |K^0\rangle + (1-\varepsilon) |\bar{K}^0\rangle). \end{cases} \quad (1.30)$$

The parameter ε describes the CP violation in the mixing, since, if $\varepsilon \neq 0$, we have

$$\left| \frac{p}{q} \right| = \left| \frac{1+\varepsilon}{1-\varepsilon} \right| \neq 1. \quad (1.31)$$

In terms of the eigenstates, the $|K^0\rangle, |\bar{K}^0\rangle$ states can be written as:

$$\begin{cases} |K^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}} \cdot \frac{1}{1+\varepsilon} (|K_L^0\rangle + |K_S^0\rangle), \\ |\bar{K}^0\rangle = \sqrt{\frac{1+|\varepsilon|^2}{2}} \cdot \frac{1}{1-\varepsilon} (|K_L^0\rangle - |K_S^0\rangle). \end{cases} \quad (1.32)$$

The eigenvalues $\lambda_{S,L}$ are expressed as:

$$\lambda_{S,L} \equiv m_{S,L} - \frac{i}{2} \Gamma_{S,L}, \quad (1.33)$$

where their real and imaginary parts $m_{S,L}$ and $\Gamma_{S,L}$ are related to the matrix diagonal elements M_K and Γ_K :

$$M_K = \frac{m_S + m_L}{2} \quad \text{and} \quad \Gamma_K = \frac{\Gamma_S + \Gamma_L}{2}. \quad (1.34)$$

It is also useful to define another basis for the 2D states, composed by the CP eigenstates $|K_1^0\rangle$ and $|K_2^0\rangle$, defined as: $CP |K_1^0\rangle = + |K_1^0\rangle$ and $CP |K_2^0\rangle = - |K_2^0\rangle$. Using the convention

$CP |K^0\rangle = -|\bar{K}^0\rangle$, these eigenstates can be written as:

$$\begin{cases} |K_1^0\rangle \equiv \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}, \\ |K_2^0\rangle \equiv \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}}, \end{cases} \quad (1.35)$$

or, in terms of $|K_L^0\rangle$ and $|K_S^0\rangle$, as:

$$\begin{cases} |K_S^0\rangle = \frac{|K_1^0\rangle + \varepsilon |K_2^0\rangle}{\sqrt{1 + |\varepsilon|^2}}, \\ |K_L^0\rangle = \frac{|K_2^0\rangle + \varepsilon |K_1^0\rangle}{\sqrt{1 + |\varepsilon|^2}}. \end{cases} \quad (1.36)$$

This last relation shows that if $\varepsilon = 0$ the Hamiltonian eigenstates correspond to the CP ones ($K_S^0 = K_1^0$ and $K_L^0 = K_2^0$), i.e. CP is a symmetry of the theory.

1.6.2 Interaction of K^0 and $\bar{K}^0$ with matter

The evolution of neutral kaons in matter has been studied and modelled for the first time in 1955 by Pais and Piccioni [31] and Good [32], with the aim to explain the so-called *regeneration* of neutral kaons. A wide description of the phenomenon can be found in Ref. [33], but here the main features of interest to this thesis work are presented.

The K_S^0 regeneration phenomenon is based on the fact that strong interactions with matter are different for the two strangeness components (K^0 and $\bar{K}^0$) of a neutral kaon: if a pure beam of K_L^0 (composed at $\sim 50\% - 50\%$ by the flavour eigenstates) is sent into a layer of material, called *regenerator*, the K^0 and $\bar{K}^0$ components interact differently with matter, thus the relative strangeness composition of the state is altered. As a consequence, beyond the material it is possible to reveal a K_S^0 component, not present at the beginning. The K_S^0 component can be identified by its decay into two pions, a process that is extremely rare for the K_L^0 . Therefore, an increase in the number of observed two-pion decays beyond the regenerator provides clear evidence of K_S^0 regeneration (figure 1.3).

This phenomenon is due to the fact that the total interaction cross section of a $\bar{K}^0$ is higher than for a K^0 and that the phase shift inside matter is different for the K^0 and $\bar{K}^0$ wavefunctions. In particular, neutral kaons can interact in two main ways with nucleons: absorption, which makes the $\bar{K}^0$ component disappear faster than the K^0 one, and elastic scattering, which alters the relative phase of the K^0 and $\bar{K}^0$ states.

In the case of absorption, both K^0 and $\bar{K}^0$ are allowed to create charged kaons, respectively $K^0 + p \rightarrow n + K^+$ and $\bar{K}^0 + n \rightarrow p + K^-$, with similar cross sections. However, a $\bar{K}^0$ can also create hyperons, mainly through the processes $\bar{K}^0 + n \rightarrow \Lambda^0 \pi^0$ and $\bar{K}^0 + p \rightarrow \Lambda^0 \pi^+$. In contrast, the equivalent process is forbidden to a K^0 , because a hyperon containing a $\bar{s}$ quark and with baryon number $\mathcal{B} = +1$ does not exist. Therefore, a $\bar{K}^0$ can be absorbed in matter with higher probability than a K^0 .

The regeneration due to elastic scattering can be divided in two categories: *coherent* regeneration, in which the contributions of all the scattering centres of the material add up coherently in the forward direction and the state of the regenerator material remains unchanged; *incoherent*

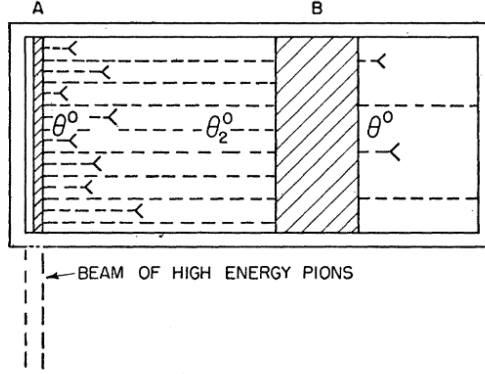


FIG. 1. Schematic diagram showing the regeneration of θ_1^0 events in a multiplate cloud chamber. The symbol $---<$ indicates the decay: $\theta_1^0 \rightarrow \pi^+ + \pi^-$.

Figure 1.3: Schematic illustration of the K_S^0 regeneration, taken from the original paper [31]. Until the 1960s, the neutral kaons were referred to as θ^0 . In this representation, a high-energy π^- beam interacts with nucleons, in the thin target A, and produces K^0 mesons, for example in the $\pi^- p \rightarrow \Lambda^0 K^0$. The K_S^0 component of the K^0 beam decays after a short distance into two pions, leaving a pure K_L^0 beam. The K_L^0 beam enters then in the thick absorber B, in which the $\bar{K}^0$ component is strongly reduced. For this reason, the neutral kaon beam emerging from B contains now also a K_S^0 component, revealed by its $K_S^0 \rightarrow \pi^+ \pi^-$ decay.

or *diffraction* regeneration, arising from the contribution of individual scattering centres, due to the difference in the finite-angle scattering probability for a K^0 and a $\bar{K}^0$. This second type of regeneration has been neglected in this thesis, following the work described in Ref. [34], thus from now on the considered interactions of neutral kaons with matter are absorption and forward elastic scattering.

To model the time evolution of a $K^0 - \bar{K}^0$ system in matter, it is necessary to include in the total Hamiltonian another term, which describes the interaction with the nuclei of the medium. This new term takes into account the fact that a K^0 and a $\bar{K}^0$ have a different interaction cross section with nucleons and, since it can't induce flavour transitions, it can be written in the $(|K^0\rangle, |\bar{K}^0\rangle)$ basis in a diagonal form:

$$\mathbf{H}_{\text{matter}} \equiv \chi = \begin{pmatrix} \chi_{K^0} & 0 \\ 0 & \chi_{\bar{K}^0} \end{pmatrix}, \quad (1.37)$$

where χ_{K^0} and $\chi_{\bar{K}^0}$ are complex coefficients related to the interaction between neutral kaons and the nucleons in the material. In particular, their imaginary part describes absorption in the material and the real part phase shifts (dispersion) due to elastic scattering.

The time evolution of neutral kaons in matter can be determined by solving Schrödinger's equation, using both the $\mathbf{H}_{\text{vacuum}}$ and $\mathbf{H}_{\text{matter}}$ contributions, which can be expressed in the $|K^0\rangle, |\bar{K}^0\rangle$ basis (see (1.28)) as:

$$i \frac{\partial}{\partial t} \begin{pmatrix} \alpha(t) \\ \bar{\alpha}(t) \end{pmatrix} = \begin{pmatrix} M - \frac{i}{2}\Gamma + \chi_{K^0} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma + \chi_{\bar{K}^0} \end{pmatrix} \begin{pmatrix} \alpha(t) \\ \bar{\alpha}(t) \end{pmatrix}. \quad (1.38)$$

The solution to this equation most common in literature is expressed in the $|K_S^0\rangle, |K_L^0\rangle$ basis,

defining the coefficients $\alpha_S(t)$ and $\alpha_L(t)$ to describe a generic state of the system:

$$|\psi(t)\rangle = \alpha_S(t) |K_S^0\rangle + \alpha_L(t) |K_L^0\rangle. \quad (1.39)$$

From Schrödinger's equation one can then derive Good's equations:

$$i \frac{\partial \alpha_L(t)}{\partial t} = \left(\lambda_L + \frac{\chi_{K^0} + \chi_{\bar{K}^0}}{2} \right) \alpha_L(t) + \left(\frac{\chi_{K^0} - \chi_{\bar{K}^0}}{2} \right) \alpha_S(t), \quad (1.40)$$

$$i \frac{\partial \alpha_S(t)}{\partial t} = \left(\lambda_S + \frac{\chi_{K^0} + \chi_{\bar{K}^0}}{2} \right) \alpha_S(t) + \left(\frac{\chi_{K^0} - \chi_{\bar{K}^0}}{2} \right) \alpha_L(t), \quad (1.41)$$

which have the following solutions (Ref. [34]):

$$\alpha_S(t) = e^{-i\Sigma \cdot t} \left(\alpha_S(0) \cos \Omega t + i \frac{\alpha_S(0)\Delta\lambda - \alpha_L(0)\Delta\chi}{2\Omega} \sin \Omega t \right), \quad (1.42)$$

$$\alpha_L(t) = e^{-i\Sigma \cdot t} \left(\alpha_L(0) \cos \Omega t - i \frac{\alpha_L(0)\Delta\lambda + \alpha_S(0)\Delta\chi}{2\Omega} \sin \Omega t \right), \quad (1.43)$$

where the following parameters are defined:

- $\Delta\lambda \equiv \lambda_L - \lambda_S = \Delta m - \frac{i}{2}\Delta\Gamma$, which depends on the eigenvalues of the vacuum Hamiltonian term;
- $\Delta\chi \equiv \chi_{K^0} - \chi_{\bar{K}^0}$, which depends on the material properties and will be estimated in the following paragraphs;
- $\Omega = \frac{1}{2} \sqrt{\Delta\lambda^2 + \Delta\chi^2}$, which determines the time dependence of α_S and α_L ;
- $\Sigma = \frac{1}{2} (\lambda_S + \lambda_L + \chi_{K^0} + \chi_{\bar{K}^0})$, which is a common complex exponential term to $\alpha_S(t)$ and $\alpha_L(t)$, describing the absorption of the strangeness components as well as the decay of the mass eigenstates. In particular, the decay width of the neutral kaon state in any final state is proportional to:

$$|e^{-i\Sigma t}| = \exp \left(-\frac{t}{2} \left(\frac{\Gamma_S + \Gamma_L}{2} + \Im(\chi_{K^0} + \chi_{\bar{K}^0}) \right) \right),$$

However, the quantity of interest in this thesis, as explained in section 1.6.3, is the neutral kaon asymmetry, defined in (1.51) as the asymmetry between the decay widths of a K^0 and a $\bar{K}^0$ in a common final state. Since both the K^0 and $\bar{K}^0$ decay widths are proportional to the same $|\exp(-i\Sigma t)|^2$ term, any dependence on Σ cancels out. For this reason, Σ will be omitted in the rest of this thesis.

The difference $\Delta\chi = \chi_{K^0} - \chi_{\bar{K}^0}$ determines the time evolution of neutral kaons in matter, as shown in equations (1.42) and (1.43). In particular, $\Delta\chi$ is responsible for the K_S^0 regeneration phenomenon, observed for the first time in the Pais and Piccioni experiment: even if the initial beam has only the K_L^0 component, i.e. $\alpha_S(0) = 0$, $\alpha_S(t)$ can be non-zero thanks to the presence of a $\Delta\chi \neq 0$ term. The imaginary part of $\Delta\chi$ describes the regeneration due to absorption, while the real part describes the coherent regeneration due to dispersion (phase shift) of the K^0 and $\bar{K}^0$ states.

It is possible to express the χ_{K^0} and $\chi_{\bar{K}^0}$ coefficients as a function of the neutral kaons momentum p and of the material properties, namely the nucleon density n and the total interaction cross section of a K^0 or a $\bar{K}^0$ with nucleons, respectively σ_{K^0} and $\sigma_{\bar{K}^0}$. In quantum mechanics, working under Born's approximation, the matrix elements χ_{K^0} and $\chi_{\bar{K}^0}$ can be expressed in terms of the scattering amplitudes at a general angle θ in the interaction of a K^0 or a $\bar{K}^0$ with nucleons, respectively $f_{K^0}(\theta)$ and $f_{\bar{K}^0}(\theta)$. In this thesis, as already discussed, the scattering of neutral kaons at finite angles is neglected, thus χ_{K^0} and $\chi_{\bar{K}^0}$ depend only on the forward scattering amplitudes $f_{K^0}(0)$ and $f_{\bar{K}^0}(0)$:

$$\chi_{K^0} \equiv -\frac{2\pi n}{M_K} f_{K^0}(0) \quad \text{and} \quad \chi_{\bar{K}^0} \equiv -\frac{2\pi n}{M_K} f_{\bar{K}^0}(0), \quad (1.44)$$

where M_K is the neutral kaons mass and n is the nucleons density in the material, which can be expressed in terms of the density ρ and atomic weight A as:

$$n = \frac{\rho N_A}{A}, \text{ with } N_A \text{ the Avogadro number.} \quad (1.45)$$

A crucial parameter is the difference between the forward scattering amplitudes, $\Delta f \equiv f_{\bar{K}^0}(0) - f_{K^0}(0)$, which needs to be determined in order to estimate the parameter $\Delta\chi$ and the time evolution of neutral kaons in matter:

- the phase of Δf has been determined using the phase-power relation (Ref [35]):
 $\arg \Delta f = (-124.7 \pm 0.8)^\circ$.
- the imaginary part of Δf is related, via the optical theorem, to the difference between the total interaction cross section of a K^0 and a $\bar{K}^0$ with matter, including both the absorption and forward scattering contributions:

$$\Delta\sigma \equiv \sigma_{\bar{K}^0} - \sigma_{K^0} = \frac{4\pi}{p} \text{Im}(\Delta f), \quad (1.46)$$

where $\Delta\sigma$, describing the interaction between neutral kaons with momentum p and nucleons with atomic weight A , can be written (Ref. [35]) as:

$$\Delta\sigma(p, A) = \frac{23.2 \cdot A^{(0.758 \pm 0.003)}}{(p[\text{GeV}/c]^{0.614})} \text{ mb}. \quad (1.47)$$

Therefore, Δf can then be written as a function of the atomic weight of the material and of the neutral kaon momentum:

$$\Delta f(p, A) = \frac{p \Delta\sigma(p, A)}{4\pi \sin(\arg \Delta f)} e^{i \arg \Delta f} \propto p^{0.386} \cdot A^{0.758}. \quad (1.48)$$

Finally, the quantity of interest for the time evolution of neutral kaons in matter, $\Delta\chi$, can be expressed as:

$$\Delta\chi = \frac{2\pi\rho N_A}{M_K A} \Delta f \propto p^{0.386} \cdot \rho \cdot A^{-0.242}. \quad (1.49)$$

Parameter	Measured Value	Ref.
M_K	$(497.611 \pm 0.013) \text{ MeV}/c^2$	[36]
Δm	$(0.5293 \pm 0.0009) \hbar/s$	[36]
τ_S	$(0.8954 \pm 0.0004) \cdot 10^{-10} \text{ s}$	[36]
τ_L	$(5.116 \pm 0.021) \cdot 10^{-8} \text{ s}$	[36]
$ \varepsilon $	$(2.228 \pm 0.011) \times 10^{-3}$	[37]
$\arg \varepsilon$	$(43.5 \pm 0.5)^\circ$	[37]
$\arg \Delta f$	$(-124.7 \pm 0.8)^\circ$	[35]

Table 1.4: Experimental measurements of the parameters used to estimate the neutral kaon asymmetry in matter.

1.6.3 Decay rate and time-dependent asymmetry

The decay channel used in most of the experiments to identify a neutral kaon is in two charged pions, $|f\rangle = |\pi^+\pi^-\rangle$, which is a CP -even state. The $K^0/\bar{K}^0 \rightarrow \pi^+\pi^-$ decay is Cabibbo-Favoured, with a dominant tree-level amplitude which depends on the CKM matrix elements $V_{cs}V_{ud}^*$. For this reason, even though direct CP violation in this decay channel has been observed with a significance of more than 5σ , it is extremely suppressed, as shown by the measurement⁸ of:

$$\Delta_{\pi^+\pi^-} \equiv \frac{\Gamma(\bar{K}^0 \rightarrow \pi^+\pi^-) - \Gamma(K^0 \rightarrow \pi^+\pi^-)}{\Gamma(\bar{K}^0 \rightarrow \pi^+\pi^-) + \Gamma(K^0 \rightarrow \pi^+\pi^-)} = (-5.2 \pm 0.05) \times 10^{-6}.$$

The direct CP violation in the $K \rightarrow \pi^+\pi^-$ decay is significantly smaller than the typical uncertainties on the phenomena of interest in this thesis, namely CP asymmetries in charm decays, which have uncertainties of the order of $\mathcal{O}(10^{-4} - 10^{-3})$. Therefore, direct CP violation has been neglected in this thesis. This translates in assuming the following equality between the amplitudes, $\mathcal{A}(K^0 \rightarrow \pi^+\pi^-) = -\mathcal{A}(\bar{K}^0 \rightarrow \pi^+\pi^-)$, which can also be expressed in the form $\langle \pi^+\pi^- | K_2^0 \rangle = 0$. This means that the final state $\pi^+\pi^-$ is only accessible to the CP -even eigenstate $|K_1^0\rangle$. The decay rate into two pions of a general neutral kaon state $\psi_K(t)$ can thus be computed as:

$$\Gamma(\psi_K(t) \rightarrow \pi^+\pi^-) \propto |\langle \pi^+\pi^- | \psi_K(t) \rangle|^2 \propto \left| \alpha_S^{(\psi_K)}(t) + \varepsilon \alpha_L^{(\psi_K)}(t) \right|^2, \quad (1.50)$$

where $|\psi_K(t)\rangle = \alpha_S^{(\psi_K)}(t) |K_S^0\rangle + \alpha_L^{(\psi_K)}(t) |K_L^0\rangle$.

The expression of the decay rate as a function of the kaon decay time, derived above in equation (1.50), allows to compute the time-dependent asymmetry for any initial state $|\psi(0)\rangle$, given its coefficients in the $\{|K_S^0\rangle, |K_L^0\rangle\}$ basis, $\alpha_{S,L}^{(\psi_K)}(0)$. Considering as initial states the flavour eigenstates

⁸This measurement of the CP asymmetry in the $K^0 \rightarrow \pi^+\pi^-$ decay channel is based on the measurement of $\text{Re}(\varepsilon'/\varepsilon) = (16.3 \pm 1.6) \times 10^{-4}$, as discussed in section 8.3 of [20].

K^0 and $\bar{K}^0$, the time-dependent asymmetry can be written as:

$$\begin{aligned}
 A_{K_S^0}(t) &\equiv \frac{\Gamma(\bar{K}^0(t) \rightarrow \pi^+\pi^-) - \Gamma(K^0(t) \rightarrow \pi^+\pi^-)}{\Gamma(\bar{K}^0(t) \rightarrow \pi^+\pi^-) + \Gamma(K^0(t) \rightarrow \pi^+\pi^-)} \\
 &= \frac{\left| \alpha_S^{(\bar{K}^0)}(t) + \varepsilon \alpha_L^{(\bar{K}^0)}(t) \right|^2 - \left| \alpha_S^{(K^0)}(t) + \varepsilon \alpha_L^{(K^0)}(t) \right|^2}{\left| \alpha_S^{(\bar{K}^0)}(t) + \varepsilon \alpha_L^{(\bar{K}^0)}(t) \right|^2 + \left| \alpha_S^{(K^0)}(t) + \varepsilon \alpha_L^{(K^0)}(t) \right|^2},
 \end{aligned} \tag{1.51}$$

where the coefficients $\alpha_{S,L}^{(K^0)}(t)$ and $\alpha_{S,L}^{(\bar{K}^0)}(t)$ are determined using equations (1.42) and (1.43), and the initial state coefficients $\alpha_{S,L}^{(K^0)}(0)$ and $\alpha_{S,L}^{(\bar{K}^0)}(0)$ for the flavour eigenstates are (see Eq. (1.32)):

$$\begin{aligned}
 K^0 : \quad \alpha_S^{(K^0)}(0) &= \sqrt{\frac{1+|\varepsilon|^2}{2}} \cdot \frac{1}{1+\varepsilon} \quad \text{and} \quad \alpha_L^{(K^0)}(0) = \sqrt{\frac{1+|\varepsilon|^2}{2}} \cdot \frac{1}{1+\varepsilon}; \\
 \bar{K}^0 : \quad \alpha_S^{(\bar{K}^0)}(0) &= \sqrt{\frac{1+|\varepsilon|^2}{2}} \cdot \frac{-1}{1-\varepsilon} \quad \text{and} \quad \alpha_L^{(\bar{K}^0)}(0) = \sqrt{\frac{1+|\varepsilon|^2}{2}} \cdot \frac{1}{1-\varepsilon}.
 \end{aligned} \tag{1.52}$$

The time-dependent kaon asymmetry, due to mixing-induced effects, such as regeneration and CP -violation in the kaon system, is crucial for high-precision measurements of CP violation in charm hadrons, as discussed in chapter 2. The approximate model derived here needs some external inputs, including the precise knowledge of the material properties (density ρ and atomic weight A), and of the kinematics of neutral kaons passing through the detector material (momentum p and decay time t). These aspects are discussed in chapters 6 and 7.

This final result is based on several assumptions worth reiterating. First, as mentioned at the beginning of section 1.6.2, only forward elastic scattering is included in the matter Hamiltonian term of eq. (1.37). This approximation is particularly valid for ultra-relativistic kaons, for which deviations from a linear trajectory due to material interactions are negligible. Secondly, nucleons in the material are assumed to be homogeneously distributed and kaon energy variations due to interactions are neglected. While these assumptions are difficult to test purely from a theoretical point of view, they are widely accepted in the literature with great precision [34]. The compatibility between the model and the data, as shown in this thesis, can provide useful insights once all systematic uncertainties are properly accounted for, as discussed in chapter 7.

As an example, the time-dependent neutral kaon asymmetry is shown in figure 1.4, with the CP violation and material contributions. Since this formalism will be applied to study neutral kaons produced in charm hadron decays and reconstructed by the LHCb detector, typical values for the kaon decay-time interval ($t/\tau_{K_S^0} \in [0, 3]$), momentum ($p = 30 \text{ GeV}/c$), and material parameters are chosen.

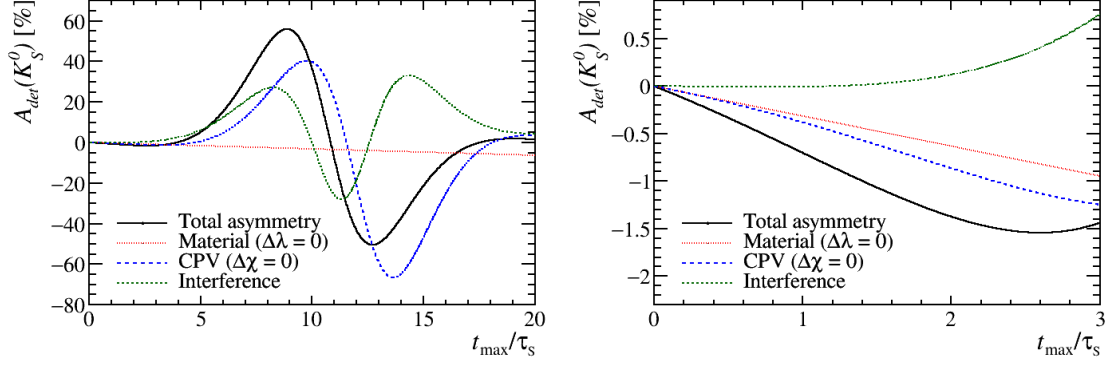


Figure 1.4: Prediction of $A_{K_S^0}(t)$, according to equation (1.51), on a wide time range (left) and on a time scale typical of an experimental environment (right). The asymmetry has been computed assuming a monochromatic distribution in momentum of the kaons ($p = 30$ GeV/c) and a homogeneous material with $A = 27$ and $\rho = 0.05$ g/cm³. The material contribution has been computed by setting $\Delta\lambda = 0$ and the CP violation contribution by setting $\Delta\chi = 0$, while the interference between the two has been estimated as the difference $A_{TOT} - (A_{ABS} + A_{CPV})$.

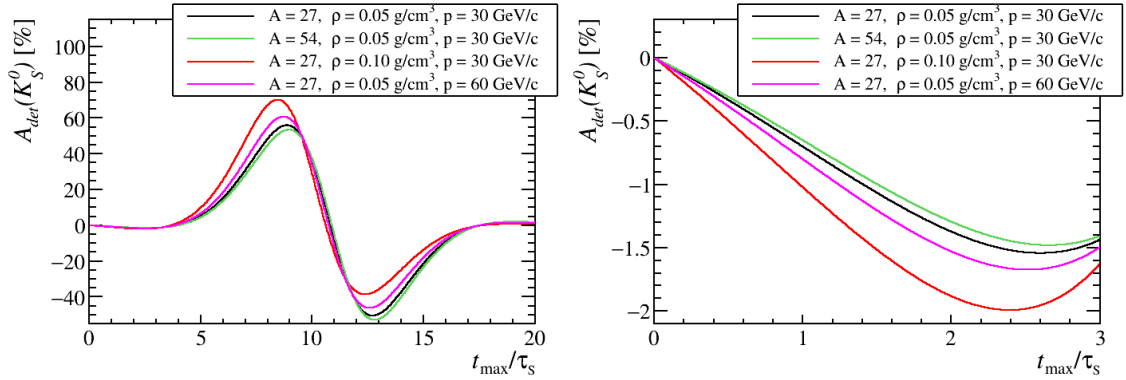


Figure 1.5: Neutral kaon asymmetry assuming different materials and neutral kaon momenta, on a wide time range (left) and on a time scale typical of an experimental environment (right). In particular, we can see the effect of doubling A , ρ and p with respect to the $A = 27, \rho = 0.05$ g/cm³ and $p = 30$ GeV/c hypothesis (black plot). The parameter to which the asymmetry is most sensitive is the material density, in agreement with the dependence $\Delta\chi \propto p^{0.386} \cdot \rho \cdot A^{-0.242}$.

Chapter 2

CP violation in charm decays and neutral kaon detection asymmetry

This chapter presents a short overview of the current state-of-the-art of measurements of CP violation in charm decays, with a particular focus on the CP violation in the decay, also known as direct CP violation. The experimental methodologies for performing these measurements in hadronic collisions are highlighted, along with the techniques for precisely subtracting production and detector-induced charge asymmetries using calibration channels, including those with neutral kaons in the final state. Particular attention is given to the Singly Cabibbo-Suppressed $D^+ \rightarrow \phi\pi^+$ decay, as a case study for measuring CP violation, and to the Cabibbo-favoured $D^+ \rightarrow K_S^0\pi^+$ decay, used as a powerful calibration channel in many measurements. Both decay processes are the main subjects of this thesis.

2.1 Introduction

High-precision measurements of CP violation in charm decays are of paramount importance because the physical asymmetries are expected to be small ($10^{-3} - 10^{-4}$) and any new observation of CP violation would provide valuable theoretical insights. Nowadays, experiments like LHCb at the Large Hadron Collider and Belle II at the SuperKEKB Collider can collect abundant and pure samples of charm hadron decay processes. When performing high-precision measurements with these abundant samples, extreme care is needed as any small experimental bias must be accounted for and eliminated. This experimental challenge requires a precise knowledge of all contributions to the particle-antiparticle asymmetries measured in the channels of interest. For instance, the measured 'raw' asymmetries, obtained by counting the number of reconstructed decays, do not depend solely on CP violation but can be influenced by various different charge-asymmetric effects. For example, different production cross sections of D mesons and the corresponding $\bar{D}$ antimesons can perfectly mimic a CP -violating asymmetry, as can any different response of the detector to the decay products of a D or of a $\bar{D}$, as discussed in section 2.2.

Both production and detection effects, commonly referred to as *nuisance asymmetries*, alter the measured particle-antiparticle asymmetry of the decays of interest. Therefore, several techniques are used to cancel such contributions and isolate the CP asymmetries of physical interest. These techniques cannot rely on simulation, which has limited accuracy, particularly in reproducing such subtle and small effects. Consequently, the most common approaches are based on data and require the use of one or more calibration channels, in which the CP violation is much smaller than the statistical and systematic uncertainties at play. These calibration decays are Cabibbo-Favoured

processes and have one or more particles in the initial or final state shared with the decay of interest. The former requirement ensures that no CP violation can be detected, as Cabibbo-Favoured transitions are largely dominated by a single amplitude, while the latter allows a robust cancellation of the nuisance asymmetries which are shared by the decay channel of interest.

Some of most abundant and used calibration channels include a neutral kaon in their final states, such as $D^+ \rightarrow K_S^0 \pi^+$, $D_s^+ \rightarrow K_S^0 K^+$ and $D^0 \rightarrow K_S^0 \pi^+ \pi^-$. The modelling and precise determination of the asymmetry due to the presence of neutral kaons in the final state of two-body or many-body charm decays are therefore of paramount importance and are the main subject of this thesis.

Current experimental status of CP violation in the charm sector

This section presents the current experimental status of direct CP violation in D^+ and D^0 decay modes, defined and characterised in section 1.3. The golden experimental observable to perform such measurements is the time-integrated CP asymmetry. In the case of D^+ mesons, in which no mixing can occur, this quantity is exactly equal to the direct CP asymmetry. In contrast, in D^0 decays, the time-integrated CP asymmetry differs from the direct CP asymmetry for small corrective terms, due to the $D^0 - \bar{D}^0$ mixing, which however have a negligible impact and can be cancelled with precision.

Currently the main experiments searching for CP violation in the decays of charmed hadrons can be classified in two categories on the basis of the hadron production mechanism: experiments installed at hadron colliders, such as LHCb (pp) and CDF ($p\bar{p}$), and experiments installed at e^+e^- colliders, such as Belle, Belle II, BaBar, CLEO-c and BESIII. Thanks to the high $c\bar{c}$ production cross section in pp collisions and excellent trigger capabilities, the LHCb experiment is currently the major player in this field, above all in the measurements involving final states with the presence of charged particles, while B -factories provide valuable contributions in decay modes with elusive particles in the final states, such as neutrinos, π^0 , and photons.

The first observation of CP violation in charmed hadrons decays dates back to 2019 using the singly Cabibbo-suppressed $D^0 \rightarrow \pi^+ \pi^-$ and $D^0 \rightarrow K^+ K^-$ decays. The LHCb Collaboration measured the difference between the time-integrated CP asymmetries of these two decay modes to be nonzero at more than 5σ [5]:

$$\Delta A_{CP} \equiv A_{CP}(D^0 \rightarrow K^+ K^-) - A_{CP}(D^0 \rightarrow \pi^+ \pi^-) = (-15.4 \pm 2.9) \times 10^{-4}.$$

This is a manifestation of CP violation in the decay. The first evidence of CP violation in a single decay channel of a D meson has been announced in 2022, thanks to the measurement of the time-integrated CP asymmetry in the $D^0 \rightarrow K^+ K^-$ channel alone [7]:

$$A_{CP}(D^0 \rightarrow K^+ K^-) = (6.8 \pm 5.4(\text{stat}) \pm 1.6(\text{syst})) \times 10^{-4}.$$

This observable allows the estimate of the direct CP asymmetry, $A_{CP}^d(D^0 \rightarrow K^+ K^-)$, being equal to it except for corrective terms¹, due to $D^0 - \bar{D}^0$ mixing, which can be estimated and subtracted. The direct asymmetry $A_{CP}^d(D^0 \rightarrow K^+ K^-)$ is found to be compatible with zero, but it allows, combined with the previous result for ΔA_{CP} , for the first evidence of a nonzero direct

¹The complete expression for the time-integrated asymmetry is $A_{CP}(D^0 \rightarrow K^+ K^-) = A_{CP}^d(D^0 \rightarrow K^+ K^-) + \Delta Y \cdot \langle t_{KK} \rangle / \tau_{D^0}$. In particular, the correction is proportional to the ΔY parameter, due to CP violation in the decay, in the mixing and in the interference, measured to be compatible with zero: $\Delta Y = (-0.089 \pm 0.113) \times 10^{-3}$ [36].

CP asymmetry in the $D^0 \rightarrow \pi^+\pi^-$ decay exceeding a significance of 3.8σ :

$$A_{CP}^d(D^0 \rightarrow \pi^+\pi^-) = (23.2 \pm 6.1) \times 10^{-4},$$

$$A_{CP}^d(D^0 \rightarrow K^+K^-) = (7.7 \pm 5.7) \times 10^{-4}.$$

So far, this is the only place where a non-zero CP violating asymmetry in the charm sector is observed. Experiments are extensively searching for CP -violating effects also in other D^0 , D^+ , D_s^+ , and charm baryon decay channels. A brief summary of the most relevant measurements of direct CP violation in charm decays is reported in table 2.1.

Decay channel	A_{CP} best measurement [$\times 10^{-3}$]	Experiment and Ref.
$D^0 \rightarrow K^+K^-$	0.77 ± 0.57	LHCb [7]
$D^0 \rightarrow \pi^+\pi^-$	2.32 ± 0.61	LHCb [7]
$D^+ \rightarrow \phi\pi^+$	0.05 ± 0.51	LHCb [8]
$D^+ \rightarrow K_S^0 K^+$	-0.09 ± 0.80	LHCb [8]
$D^0 \rightarrow \pi^0\pi^0$	-0.3 ± 6.5	Belle II [38]
$D^+ \rightarrow \pi^0\pi^+$	-13 ± 11	LHCb [39]
$D^0 \rightarrow K_S^0 K_S^0$	-31 ± 13	LHCb [40]
$D_s^+ \rightarrow \eta'\pi^+$	1 ± 14	LHCb [41]
$D^0 \rightarrow \phi\gamma$	-94 ± 66	Belle II [42]

Table 2.1: Summary of some of the most relevant measurements of direct CP violation in charm decays. The uncertainty on the reported direct A_{CP} measurements takes into account both the statistical and systematic contributions.

Among the various possibilities currently under investigation, particularly promising are the two-body Singly Cabibbo-Suppressed D^+ and D_s^+ final states, such as $D^+ \rightarrow \phi\pi^+$, where direct CP violation might arise from the interference between tree and loop-level diagrams in the Cabibbo-Suppressed $c \rightarrow d\bar{d}u$ and $c \rightarrow s\bar{s}u$ transition amplitudes. No evidence for CP violation has been found within a precision of a few units in 10^{-4} . The most precise measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$ was published in 2019, based on a data sample collected by the LHCb experiment in 2016 and 2017, during the Run 2 data-taking period, corresponding to about 3.8 fb^{-1} of integrated luminosity [8]:

$$A_{CP}(D^+ \rightarrow \phi\pi^+) = (0.05 \pm 0.42(\text{stat}) \pm 0.29(\text{syst})) \times 10^{-3}.$$

2.2 Measurement of CP violation in charm

The methodologies and definitions presented in this section are applicable to a general experimental environment, but the focus here is specifically on D^+ , D_s^+ , or D^0 mesons produced in hadron collisions, particularly in pp collisions. Unless explicitly specified, all equations are considered integrated over the decay time of the D meson.

The golden observable used to probe direct CP violating effects in a generic $D \rightarrow f$ decay is the CP asymmetry, defined as:

$$A_{CP}(D \rightarrow f) = \frac{\Gamma(D \rightarrow f) - \Gamma(\bar{D} \rightarrow \bar{f})}{\Gamma(D \rightarrow f) + \Gamma(\bar{D} \rightarrow \bar{f})}, \quad (2.1)$$

where Γ denotes the decay width of the respective process. From an experimental point of view, directly accessing this quantity is impossible. Instead, the raw asymmetry of the decay process of interest is estimated using the numbers of reconstructed events $N(D \rightarrow f)$ and $N(\bar{D} \rightarrow \bar{f})$:

$$A_{raw}(D \rightarrow f) = \frac{N(D \rightarrow f) - N(\bar{D} \rightarrow \bar{f})}{N(D \rightarrow f) + N(\bar{D} \rightarrow \bar{f})}. \quad (2.2)$$

This raw asymmetry incorporates various contributions, not solely the CP asymmetry of physical interest. If the initial state (e.g. pp) is not CP -invariant, the production cross sections of D and $\bar{D}$ mesons are different. This gives rise to a production asymmetry of the initial D meson, $A_P(D)$, defined in general as:

$$A_P(D) = \frac{\sigma(pp \rightarrow D) - \sigma(pp \rightarrow \bar{D})}{\sigma(pp \rightarrow D) + \sigma(pp \rightarrow \bar{D})}. \quad (2.3)$$

This asymmetry arises because the $\bar{c}$ quarks produced in pp interactions can combine preferentially with the valence quark of the colliding protons, whereas c quarks need an anti-quark to form D mesons. Consequently, a slight excess in the production of D^- and $\bar{D}^0$ mesons compared to their antiparticles can be expected.

In addition, since the experimental apparatus is made of matter, there is a detection asymmetry between the final state f and its CP -conjugate $\bar{f}$, denoted as $A_D(f)$. This asymmetry includes contributions from detector acceptance, track reconstruction, particle identification, and trigger requirements. Generally, the detection asymmetry is defined as a function of the total detection efficiency of a final state f :

$$A_D(f) = \frac{\varepsilon(f) - \varepsilon(\bar{f})}{\varepsilon(f) + \varepsilon(\bar{f})}. \quad (2.4)$$

The *raw* asymmetry can be written in terms of these *nuisance* asymmetries, exploiting the relations:

$$\begin{aligned} N(D \rightarrow f) &\propto \sigma(pp \rightarrow D) \cdot \Gamma(D \rightarrow f) \cdot \varepsilon(f) \\ &\propto (1 + A_P(D)) \cdot (1 + A_{CP}(f)) \cdot (1 + A_D(f)) \end{aligned} \quad (2.5)$$

$$\begin{aligned} N(\bar{D} \rightarrow \bar{f}) &\propto \sigma(pp \rightarrow \bar{D}) \cdot \Gamma(\bar{D} \rightarrow \bar{f}) \cdot \varepsilon(\bar{f}) \\ &\propto (1 - A_P(D)) \cdot (1 - A_{CP}(f)) \cdot (1 - A_D(f)). \end{aligned} \quad (2.6)$$

Inserting these relations in (2.2) and expanding all the terms we get:

$$A_{raw}(D \rightarrow f) = \frac{A_P(D) + A_{CP}(f) + A_D(f) + A_P(D) \cdot A_{CP}(f) \cdot A_D(f)}{1 + A_P(D) \cdot A_{CP}(f) + A_{CP}(f) \cdot A_D(f) + A_P(D) \cdot A_D(f)}. \quad (2.7)$$

If the magnitude of all these contributions is small (of the order of 1% or less) the total raw asymmetry can be approximated up to order $\mathcal{O}(10^{-6})$ as:

$$A_{raw}(D \rightarrow f) \approx A_P(D) + A_{CP}(D \rightarrow f) + A_D(f). \quad (2.8)$$

This equation shows that to accurately determine the CP asymmetry of a channel of interest, it is crucial to precisely identify and subtract nuisance asymmetries. Given the typical precision achievable in charm decay measurements, ranging from 10^{-3} to a few units in 10^{-4} , accurately

subtracting systematic effects cannot be achieved solely through realistic Geant-based simulations of the detector. Therefore, it is crucial to utilise one or more calibration decay modes with the same or very similar nuisance asymmetries as the channel of interest. The raw asymmetry of these calibration channels includes terms such as $A_P(D)$ and $A_D(f)$ (or their sum), which can be subtracted from $A_{raw}(D \rightarrow f)$ to isolate the observable of physical interest, $A_{CP}(D \rightarrow f)$. It is important to note that both production and detection asymmetries depend on the kinematics of the reconstructed particles. Therefore, to accurately correct for these effects, it is essential to equalise the distributions of the kinematic variables of the calibration channels to those of the decays of interest.

This approach relies on the linear expansion used to derive the raw asymmetry in equation (2.8), which is valid only for small nuisance asymmetries. Higher order corrections are very small and can be safely neglected, provided that the nuisance asymmetries remain on the order of a few percent across all allowed kinematic phase space.

2.3 State-of-the-art of $A_{CP}(D^+ \rightarrow \phi\pi^+)$

The measurement of the CP asymmetry $A_{CP}(D^+ \rightarrow \phi\pi^+)$, introduced in section 2.1 and described in Ref.[8], utilises promptly-produced D^+ decays reconstructed from proton-proton collision data collected by LHCb between 2015 and 2017, during Run 2, corresponding to about 3.8fb^{-1} of integrated luminosity. The experimental observable used to measure this direct CP asymmetry is the *raw* asymmetry, defined as:

$$A_{raw}(D^+ \rightarrow \phi\pi^+) = \frac{N(D^+ \rightarrow \phi\pi^+) - N(D^- \rightarrow \phi\pi^-)}{N(D^+ \rightarrow \phi\pi^+) + N(D^- \rightarrow \phi\pi^-)}. \quad (2.9)$$

$A_{raw}(D^+ \rightarrow \phi\pi^+)$ can be expressed using the linear approximation as defined in (2.8), in terms of the production and detection charge asymmetries:

$$A_{raw}(D^+ \rightarrow \phi\pi^+) \approx A_P(D^+) + A_{CP}(D^+ \rightarrow \phi\pi^+) + A_D(\pi^+), \quad (2.10)$$

where the detection asymmetry of the CP symmetric $\phi \rightarrow K^+K^-$ mode is neglected. In order to cancel these nuisance asymmetries and accurately measure $A_{CP}(D^+ \rightarrow \phi\pi^+)$, the Cabibbo-Favoured $D^+ \rightarrow K_S^0\pi^+$ decay is used as calibration mode². The raw asymmetry in this calibration channel can be written as:

$$A_{raw}(D^+ \rightarrow K_S^0\pi^+) \approx A_P(D^+) + A_D(\pi^+) + A_D(K_S^0) \quad (2.11)$$

where $A_D(K_S^0)$ is the neutral kaon detection asymmetry, integrated over the K_S^0 decay time. This term is determined separately and then used to correct the raw asymmetry of the calibration mode. The physics observable, $A_{CP}(D^+ \rightarrow \phi\pi^+)$, is then determined as:

$$A_{CP}(D^+ \rightarrow \phi\pi^+) = A_{raw}(D^+ \rightarrow \phi\pi^+) - [A_{raw}(D^+ \rightarrow K_S^0\pi^+) - A_D(K_S^0)] \quad (2.12)$$

²In this experimental context, the K_S^0 notation refers to a neutral kaon, initially produced as a flavour eigenstate, which evolves in the detector and decays in a pion pair $\pi^+\pi^-$. This notation is slightly imprecise since, as described in chapter 2, the mass eigenstate K_S^0 and the CP eigenstate K_1^0 do not coincide. Despite this imprecision, the notation is very common in literature, being used also by the Particle Data Group [37], and it will be employed in this thesis as well.

Neutral kaon detection asymmetry

The methodology employed to determine the neutral kaon asymmetry, as utilised in Ref.[8], was developed in 2014 (Refs.[6, 43]) and has remained largely unchanged over the years. It was also recently applied in the measurement of the time-integrated CP asymmetry in $D^0 \rightarrow K^+K^-$ decays [7]. The approach is based on the model presented in chapter 2, where the time evolution of neutral kaons in matter includes two primary sources of $K^0 - \bar{K}^0$ asymmetry:

- CP violation in $K^0 - \bar{K}^0$ mixing, quantified by the complex parameter ε ;
- $K^0 - \bar{K}^0$ asymmetry arising from the different interaction cross sections of the two flavour eigenstates with matter, quantified by the complex parameter $\Delta\chi$.

In this method, the initial state of the neutral kaon produced by $D^\pm$ mesons is assumed to be a pure flavour eigenstate, following the Cabibbo-Favoured decays $D^+ \rightarrow \bar{K}^0\pi^+$ and $D^- \rightarrow K^0\pi^-$. The asymmetry of neutral kaons in a selected data sample is estimated by reconstructing their trajectory in the detector material and computing their time-dependent asymmetry in the decay into two charged pions, using equation (1.51).

In figure 2.1, taken from Ref. [6], the prediction for the time-dependent neutral kaon detection asymmetry is overlaid with the observed asymmetry in $D^+ \rightarrow K_S^0\pi^+$ decays, collected by LHCb during Run 1. The measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$, published in Ref. [8], uses only short-lived neutral kaons ($t/\tau_{K_S^0} < 0.4$), which have decayed inside the LHCb vertex detector. The time-integrated neutral kaon detection asymmetry, $A_D(K_S^0)$, is estimated by integrating the prediction shown in the left plot of figure 2.1, yielding a value around -7×10^{-4} . In contrast, neutral kaons that are reconstructed beyond the vertex detector, which are not included in the analysis, typically have longer decay times and traverse a longer path inside the detector material. Consequently, the predicted asymmetry for these kaons can increase to a few percent, as illustrated in the right plot of figure 2.1.

The magnitude of $A_D(K_S^0)$ is particularly significant because measurements of CP violation in many charm decays are aiming for uncertainties well below the level of 10^{-3} . Therefore, understanding and accounting for the impact of longer-lived neutral kaons on asymmetry measurements is crucial for achieving high precision in these experiments.

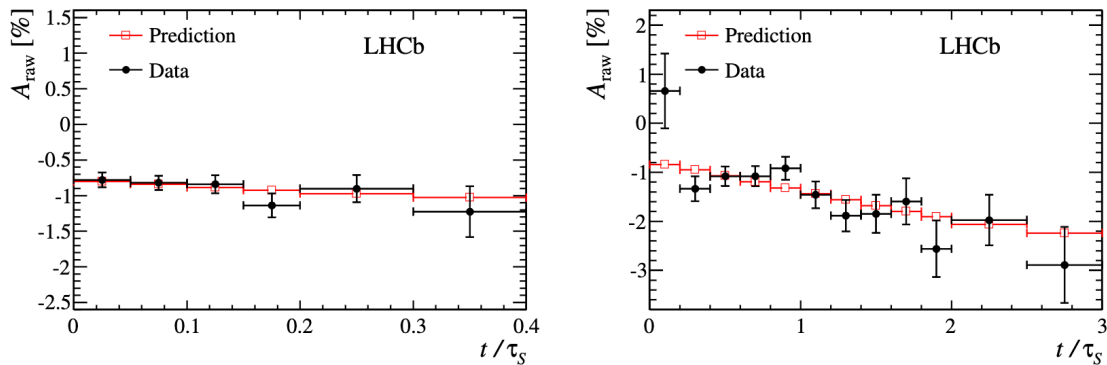


Figure 2.1: Time-dependent neutral kaon detection asymmetry predicted from the model and measured in $D^+ \rightarrow K_S^0\pi^+$ Run 1 data. The left side shows the predicted and measured asymmetry for neutral kaons decayed inside the LHCb vertex detector, with a decay-time range $t/\tau_{K_S^0} < 0.4$. On the right side are reported neutral kaons decayed beyond the vertex detector, thus the decay time range is wider, $t/\tau_{K_S^0} < 3$. The dependence of the measured asymmetry on the K_S^0 decay-time is well predicted by the model, also at high decay times.

2.4 Interference between favoured and suppressed decay

In the published measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$ [8], the method used to estimate the neutral kaon detection asymmetry in $D^+ \rightarrow K_S^0\pi^+$ decays considers only the Cabibbo-Favoured (CF) decay mode. In this channel, a D^+ meson decays to a $\bar{K}^0$ and a D^- meson decays to a K^0 , resulting in the final state $K_S^0\pi^\pm$. However, the final state $K_S^0\pi^\pm$ can also be reached by another tree-level amplitude, known as Doubly Cabibbo-Suppressed (DCS), where D^+ decays to $K^0\pi^+$ and D^- decays to $\bar{K}^0\pi^-$, as illustrated in figure 2.2.

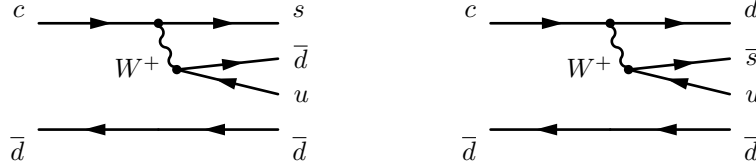


Figure 2.2: Left: Cabibbo-Favoured decay, with the tree-level amplitude determined by the V_{CKM} elements $V_{cs}^*V_{ud}$. Right: Doubly Cabibbo-Suppressed decay, with the tree-level amplitude determined by the V_{CKM} elements $V_{cd}^*V_{us}$.

The Doubly Cabibbo-Suppressed (DCS) amplitude is indeed suppressed relative to the Cabibbo-Favoured (CF) amplitude by a factor of about $|V_{cd}^*V_{us}/V_{cs}^*V_{ud}| \simeq 0.053$, and the ratio of their branching fractions is approximately 2.5×10^{-3} . Despite this suppression, the presence of both CF and DCS tree-level amplitudes contributing to the same $D^+ \rightarrow K_S^0\pi^+$ decay introduces the possibility of additional CP -violating effects. Specifically, in the $D^+ \rightarrow K_S^0\pi^+$ decay, the D^+ meson can produce a $\bar{K}^0$ through the CF process and a K^0 through the DCS process. Both K^0 and $\bar{K}^0$ can decay to the $\pi^+\pi^-$ final state through mixing, which are CP -even eigenstates. The interference between DCS and CF charm decays, coupled with K^0 - $\bar{K}^0$ mixing, has the potential to significantly impact the accurate estimation of $A_D(K_S^0)$, as highlighted in Ref. [9]. In this thesis, for the first time, the model for the neutral kaon detection asymmetry in matter, initially described in chapter 1, is extended to incorporate the interference effects between Cabibbo-Favoured and Doubly Cabibbo-Suppressed amplitudes in charm decays featuring a neutral kaon in the final state.

The strategy developed to incorporate the interference effect between CF and DCS decays applies broadly to any two-body³ decay process of the form $D \rightarrow K_S^0 f$, where both a $\bar{K}^0$ and a K^0 can appear in the final state due to different amplitudes. However, the specific focus of this thesis is on the $D^+ \rightarrow K_S^0\pi^+$ decay process. This choice is motivated by its significance in the measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$, where precise determination of the neutral kaon detection asymmetry, including the interference between CF and DCS contributions, is crucial for extracting accurate CP -violating parameters.

The interference between Cabibbo-Favoured and Doubly Cabibbo-Suppressed charm decays is characterised by two real parameters: the magnitude of the ratio between the CF and DCS amplitudes, r_π , and their relative strong phase, δ_π . These parameters are defined as:

$$\frac{\mathcal{A}(D^+ \rightarrow K^0\pi^+)}{\mathcal{A}(D^+ \rightarrow \bar{K}^0\pi^+)} \equiv r_\pi e^{i(\delta_\pi + \varphi)} \simeq r_\pi e^{i\delta_\pi}, \quad (2.13)$$

where φ is the relative weak phase given by $\varphi = \text{Arg}\{-V_{cd}^*V_{us}/V_{cs}^*V_{ud}\} = (-6.2 \pm 0.4) \times 10^{-4}$. This

³The developed framework could be applied also to many-body charm decays. However, it is necessary to keep in mind that such processes can occur through a large number of intermediate amplitudes, some yielding a K^0 and some other a $\bar{K}^0$. The extension of the presented formalism to many-body decays is not straightforward and requires additional effort to describe all the amplitudes involved.

phase parameterises direct CP violation in the $D^+ \rightarrow K_S^0 \pi^+$ decay, arising from the interference between CF and DCS amplitudes. However, the magnitude of this direct CP asymmetry is predicted to be approximately -9×10^{-5} [9], which is much smaller than the achievable experimental precision. Therefore, in the analysis described in this thesis φ has been neglected.

To account for the interfering contributions from Doubly Cabibbo-Suppressed (DCS) and Cabibbo-Favoured (CF) amplitudes in the $D^+ \rightarrow K_S^0 \pi^+$ decay mode, we need to parameterise the initial state of the neutral kaon in terms of its flavour eigenstates, $|K^0\rangle$ and $|\bar{K}^0\rangle$. This parameterisation allows us to compute the coefficients $\alpha_{S,L}^{(+)}(0)$ and $\alpha_{S,L}^{(-)}(0)$, which are essential for predicting the neutral kaon asymmetry (see equations (1.51), (1.42) and (1.43)). The initial state of a neutral kaon produced in a $D^+ \rightarrow K_S^0 \pi^+$ decay can be expressed as a superposition of $|K^0\rangle$ and $|\bar{K}^0\rangle$, weighted by the amplitudes of the CF and DCS processes:

$$|\psi_K^{(+)}(0)\rangle \propto \mathcal{A}(D^+ \rightarrow \bar{K}^0 \pi^+) |\bar{K}^0\rangle + \mathcal{A}(D^+ \rightarrow K^0 \pi^+) |K^0\rangle. \quad (2.14)$$

Introducing the parameters r_π and δ_π , the initial state can be rewritten as:

$$|\psi_K^{(+)}(0)\rangle \propto |\bar{K}^0\rangle + r_\pi e^{i\delta_\pi} |K^0\rangle. \quad (2.15)$$

On the contrary, for the CP -conjugate decay $D^- \rightarrow K_S^0 \pi^-$, assuming that CP violation in the charm decay is negligible⁴, the initial state can be expressed as:

$$\begin{aligned} |\psi_K^{(-)}(0)\rangle &\propto \mathcal{A}(D^- \rightarrow K^0 \pi^-) |K^0\rangle + \mathcal{A}(D^- \rightarrow \bar{K}^0 \pi^-) |\bar{K}^0\rangle \\ &\propto |K^0\rangle + r_\pi e^{i\delta_\pi} |\bar{K}^0\rangle. \end{aligned} \quad (2.16)$$

The expression for the neutral kaon detection asymmetry depends on the initial coefficients of the $|\psi_K^{(\pm)}(0)\rangle$ states in the $|K_S^0\rangle - |K_L^0\rangle$ basis, denoted as $\alpha_S^{(\pm)}(0)$ and $\alpha_L^{(\pm)}(0)$. These coefficients can be computed by performing a change of basis from equations (2.15) and (2.16). For the decay of the D^+ meson, the initial coefficients for the neutral kaon state are given by:

$$\alpha_S^{(+)}(0) = \sqrt{\frac{1 + |\varepsilon|^2}{2(1 + r_\pi^2)}} \cdot \left(\frac{-1}{1 - \varepsilon} + \frac{r_\pi e^{i\delta_\pi}}{1 + \varepsilon} \right), \quad (2.17)$$

$$\alpha_L^{(+)}(0) = \sqrt{\frac{1 + |\varepsilon|^2}{2(1 + r_\pi^2)}} \cdot \left(\frac{1}{1 - \varepsilon} + \frac{r_\pi e^{i\delta_\pi}}{1 + \varepsilon} \right). \quad (2.18)$$

while for the decay of the D^- meson:

$$\alpha_S^{(-)}(0) = \sqrt{\frac{1 + |\varepsilon|^2}{2(1 + r_\pi^2)}} \cdot \left(\frac{1}{1 + \varepsilon} - \frac{r_\pi e^{i\delta_\pi}}{1 - \varepsilon} \right), \quad (2.19)$$

$$\alpha_L^{(-)}(0) = \sqrt{\frac{1 + |\varepsilon|^2}{2(1 + r_\pi^2)}} \cdot \left(\frac{1}{1 + \varepsilon} + \frac{r_\pi e^{i\delta_\pi}}{1 - \varepsilon} \right). \quad (2.20)$$

⁴i.e. that $\mathcal{A}(D^+ \rightarrow \bar{K}^0 \pi^+) = \mathcal{A}(D^- \rightarrow K^0 \pi^-)$ and $\mathcal{A}(D^+ \rightarrow K^0 \pi^+) = \mathcal{A}(D^- \rightarrow \bar{K}^0 \pi^-)$ except for a common phase.

These initial state coefficients can be used to estimate the neutral kaon asymmetry, according to the following equation:

$$\begin{aligned}
 A_{K_S^0}(t) &\equiv \frac{\Gamma[\psi_K^{(+)}(t) \rightarrow \pi^+\pi^-] - \Gamma[\psi_K^{(-)}(t) \rightarrow \pi^+\pi^-]}{\Gamma[\psi_K^{(+)}(t) \rightarrow \pi^+\pi^-] + \Gamma[\psi_K^{(-)}(t) \rightarrow \pi^+\pi^-]} \\
 &= \frac{|\alpha_S^{(+)}(t) + \varepsilon\alpha_L^{(+)}(t)|^2 - |\alpha_S^{(-)}(t) + \varepsilon\alpha_L^{(-)}(t)|^2}{|\alpha_S^{(+)}(t) + \varepsilon\alpha_L^{(+)}(t)|^2 + |\alpha_S^{(-)}(t) + \varepsilon\alpha_L^{(-)}(t)|^2}.
 \end{aligned} \tag{2.21}$$

This equation allows for the analytical prediction of the neutral kaon detection asymmetry once a limited number of parameters are provided through the coefficients $\alpha_S^{(\pm)}(t)$ and $\alpha_L^{(\pm)}(t)$. These coefficients, derived in equations (1.42) and (1.43), incorporate the effects of material interactions, CP violation in the kaon system, and the interfering contributions from both Cabibbo-Favoured (CF) and Doubly Cabibbo-Suppressed (DCS) charm decay amplitudes.

2.5 Current knowledge of the r_π and δ_π parameters

The parameter δ_π is challenging to measure because amplitude phases are observable only through interference effects. However, while δ_π can only be predicted from a model describing strong contributions to CF and DCS decay amplitudes, a rough estimate of r_π can be inferred by considering only the weak tree-level diagram. In particular, r_π is approximately given by the ratio between the CKM matrix elements in the $c \rightarrow d\bar{s}u$ (DCS) and $c \rightarrow s\bar{d}u$ (CF) transitions:

$$|r_\pi|_{\text{weak}} = \left| \frac{V_{cd}^* V_{us}}{V_{cs}^* V_{ud}} \right| \simeq 0.053. \tag{2.22}$$

To obtain a more accurate estimate of r_π , and of δ_π , it is essential to consider the contributions from strong interactions involved in the formation of final meson bound states. This approach is detailed in Ref. [44], which employs the Factorisation-Assisted Topological Amplitudes (FAT) framework specifically for the $D^+ \rightarrow K_S^0 \pi^+$ decay mode. In Ref. [44], a parametrisation of strong amplitudes, incorporating parameters from the strong effective Hamiltonian, is presented for both $D^+ \rightarrow K_{S,L}^0 f$ and $D^0 \rightarrow K_{S,L}^0 f$ decays. These parameters are determined using a set of measured branching fractions from various decay modes. Ref. [9] utilises this model for the strong amplitudes to estimate r_π and δ_π in $D^+ \rightarrow K_S^0 \pi^+$ decays:

- $r_\pi = -0.073 \pm 0.004$;
- $\delta_\pi = -1.39 \pm 0.05$.

Throughout the thesis, these numerical results for r_π and δ_π are referred to as the *FAT prediction*. The convention of assigning a negative value to r_π aligns with the sign convention used in Ref. [44], which relates to the CP eigenstate convention $CP|K^0\rangle = -|\bar{K}^0\rangle$.

The FAT prediction achieves a relative accuracy of approximately 5% for r_π and about 3.6% for δ_π . Understanding the theoretical and experimental sources of these uncertainties and their reliability is challenging based on the information provided in Refs. [9, 44], as they are inherently model-dependent. The uncertainties stem from the factorisation approach employed to model the hadronic contributions to the various amplitudes. From an external perspective, one can

contrast the FAT prediction for r_π with an estimate based solely on weak diagrams, where hadronic contributions are neglected. This comparison reveals a discrepancy $\Delta r \approx 0.02$.

The magnitude of hadronic corrections to the CKM estimate $|r_\pi|_{\text{weak}}$ can be probed through decay processes of D^0 and D^+ mesons where the dominant tree-level amplitudes involve the same CF $c \rightarrow s\bar{d}u$ and DCS $c \rightarrow d\bar{s}u$ transitions as in the $D^+ \rightarrow K_S^0\pi^+$ decay. The ratio between the DCS and CF amplitudes in such decays is proportional to the same V_{CKM} elements ratio as r_π , with differences arising solely from the contribution of strong amplitudes involved in the specific process. Table 2.2 presents the ratio between the DCS and CF amplitudes for several decay modes that share the same weak content in both CF and DCS amplitudes.

Decay	Ratio	Ref.	Note
$D^+ \rightarrow K_S^0\pi^+$	0.073 ± 0.004	[9]	FAT prediction
$D^+ \rightarrow K^-\pi^+\pi^+$	0.072 ± 0.001	[36]	PDG Branching fractions
$D^+ \rightarrow K^-\pi^+\pi^+\pi^0$	0.139 ± 0.006	[36]	PDG Branching fractions
$D^0 \rightarrow K^-\pi^+$	0.059 ± 0.001	[36]	PDG Branching fractions
$D^0 \rightarrow K^-\pi^+\pi^0$	0.046 ± 0.002	[36]	PDG Branching fractions
$D^0 \rightarrow K^-\pi^+\pi^-\pi^+$	0.055 ± 0.001	[36]	PDG Branching fractions
$D^0 \rightarrow K^{*-}(892)\pi^+$	0.072 ± 0.004	[45]	Amplitude analysis $D^0 \rightarrow K_S^0\pi^+\pi^-$

Table 2.2: Absolute value of the ratio between the DCS and CF amplitudes for a selected set of $D \rightarrow Kf$ decays, where all decays result in a final state containing a charged kaon. The branching fractions of CF and DCS decays are measurable, allowing the estimation of the amplitude ratio as the square root of the branching fractions ratio. The only exception is the $D^0 \rightarrow K^{*-}(892)\pi^+$ decay, for which the ratio between the DCS and CF amplitudes was determined from an amplitude analysis of the $D^0 \rightarrow K_S^0\pi^+\pi^-$ channel.

Except for the measurement of the DCS/CF ratio in $D^0 \rightarrow K^-(892)\pi^+$ decays, all other entries in the table are based on direct measurements of branching fractions with no theoretical assumptions. The ratio for $D^0 \rightarrow K^-(892)\pi^+$ decays comes from an amplitude analysis of $D^0 \rightarrow K_S^0\pi^+\pi^-$ decays, which may introduce model-dependent uncertainties. These ratios differ only due to the hadronic contributions to the amplitudes and fall within the range of $[0.046, 0.139]$, containing the FAT prediction. When considering only two-body decays with a similar topology to the $D^+ \rightarrow K_S^0\pi^+$ decay mode, the range narrows to $[0.057, 0.080]$, allowing for a $\pm 2\sigma$ interval for associated uncertainties. This suggests that the contribution from hadronic corrections is minimal and indicates the reliability of the FAT prediction for r_π , with an accuracy level of $|r_\pi| \in [0.053, 0.093]$. This interval is centred on the FAT prediction and is slightly enlarged to contain the r_π estimate based solely on CKM factors of the tree amplitudes, $|r_\pi|_{\text{weak}}$.

For illustrative purposes, figure 2.3 shows the impact of the interference between CF and DCS charm decays on the time-dependent neutral kaon asymmetry, varying r_π and δ_π . Since this effect depends on time, it becomes more pronounced as the decay time of the neutral kaon increases.

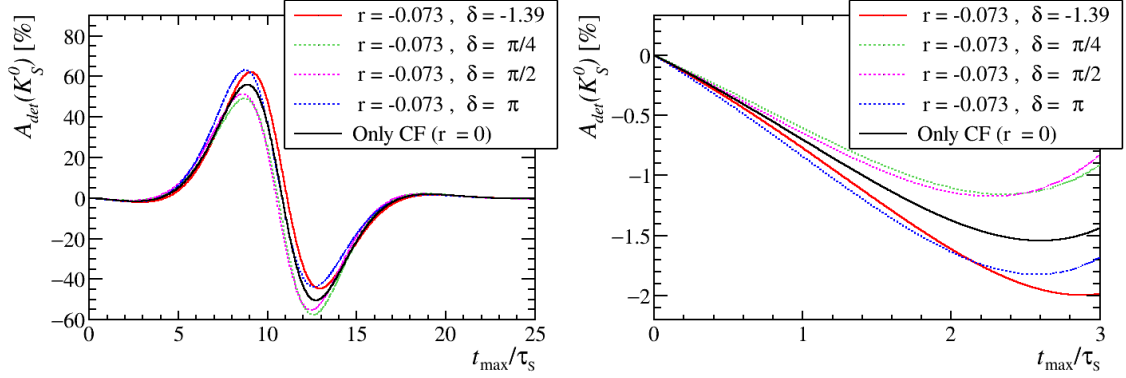


Figure 2.3: Prediction of $A_{K_S^0}(t)$ including also strong phase effects, for different r_π and δ_π values, according to equation (2.21), on a wide time range (left) and on a time scale typical of an experimental environment (right). The asymmetry is computed assuming a monochromatic distribution in momentum of the kaons ($p = 30 \text{ GeV}/c$) and a homogeneous material with $A = 27$, as in figure 1.4.

2.6 This thesis

This thesis work, for the first time, extends the model for neutral kaon detection asymmetry in matter described in chapter 1, including the contribution from the interference of Cabibbo-Favoured and Doubly Cabibbo-Suppressed decays in charm decays with a neutral kaon in the final state. Additionally, the relevant parameters describing this interference in the $D^+ \rightarrow K_S^0 \pi^+$ decay mode, r_π and δ_π , are directly measured from data. This is possible thanks to the high statistics charm decay samples collected by the LHCb experiment during the LHC Run 2 data-taking period. This work represents a novel and highly original contribution to the field.

Since determining r_π and δ_π necessitates the cancellation of other nuisance asymmetries and thus requires the simultaneous use of the $D^+ \rightarrow \phi \pi^+$ decays, this thesis also extends the measurement of $A_{CP}(D^+ \rightarrow \phi \pi^+)$ to the 2018 Run 2 data sample, corresponding to about 2 fb^{-1} of data that were not included in the published analysis of Ref. [8]. The 2016 and 2017 data samples are also reanalysed to ensure consistent selection and analysis strategies across all three data samples. Thanks to the work on determining r_π and δ_π , $D^+ \rightarrow K_S^0 \pi^+$ decays reconstructed outside the acceptance of the vertex detector are included in the analysis. This inclusion enables a more robust estimate of systematic uncertainties related to neutral kaon detection asymmetries. Several other aspects of the analysis are also improved and are described in the following chapters.

Chapter 3

The LHCb experiment at the LHC

This chapter provides an overview of the Large Hadron Collider (LHC) and the LHCb detector and trigger system during Run 2. The geometry and performances of the LHCb sub-detectors are discussed, with particular focus on the tracking, particle identification and calorimeter systems. The last section presents the LHCb trigger system, which is crucial in the selection of high statistics samples of charmed hadrons.

3.1 The Large Hadron Collider

The Large Hadron Collider (LHC) is a two-ring hadron collider, designed to accelerate protons and heavy ions, located at CERN (Conseil Européen pour la Recherche Nucléaire) and crossing the French-Swiss border near Geneva. The accelerator is hosted in a 27 km long and 100 m underground circular tunnel, previously occupied by the Large Electron Positron collider (LEP).

The LHC's primary objectives are to explore the properties of elementary particles and the fundamental forces that govern their interactions. Its key scientific goals include the Higgs Boson discovery, high precision measurements to test the Standard Model, the search for New Physics and the study of Quark-Gluon Plasma.

The LHC is composed of several critical components:

- **Accelerators:** The acceleration process in the LHC involves multiple stages, each progressively increasing the energy of the protons, as shown in figure 3.2:
 - **Proton Source and Linear Accelerator 2 (Linac 2):** protons are extracted from hydrogen gas using an electric field and initially accelerated in the Linear Accelerator 2 (Linac 2) to energies of 50 MeV;
 - **Proton Synchrotron Booster (PSB):** further accelerates protons from the Linac 2 up to 1.5 GeV;
 - **Proton Synchrotron (PS):** the protons are then transferred to the Proton Synchrotron, where their energy is increased to 25 GeV;
 - **Super Proton Synchrotron (SPS):** finally the Super Proton Synchrotron accelerates the protons to 450 GeV before injecting them into LHC.
- **Magnets:** The LHC uses a system of superconducting magnets to steer and focus the two opposite circulating proton beams. These magnets include superconducting NbTi dipole magnets for bending the beams along the circular path and quadrupole magnets for focusing the beams. In addition, protons are accelerated by radio-frequency cavities.

- **Beam Pipes:** Two separate beam pipes contain the proton beams travelling in opposite directions. The beams are kept in ultra-high vacuum conditions to minimise collisions with residual gas molecules.
- **Detectors:** The two beams collide in four interaction points where the four main LHC experiments detect and analyse their collision products. These experiments include ATLAS and CMS, general purpose detectors, ALICE, specifically designed to study quark-gluon plasma in heavy-ion collisions, and LHCb, focused to study CP violation and heavy flavour physics, mainly B and D mesons.

The two circulating beams are divided into *bunches* of 10^{11} protons each, which collide once in 25 ns, corresponding to a nominal bunch-crossing rate of 40 MHz during Run 2, from 2015 to 2018. A fundamental parameter to quantify the performance of a collider is the instantaneous luminosity $\mathcal{L}$, defined by the relation

$$\frac{dN}{dt} = \mathcal{L} \cdot \sigma(\sqrt{s}), \quad (3.1)$$

where $\frac{dN}{dt}$ is the rate of events of a specific process produced in pp collisions and $\sigma(\sqrt{s})$ is the cross section for the given process at the center-of-mass energy $\sqrt{s}$, which has been kept constant to 13 TeV during Run 2. The nominal peak instantaneous luminosity for pp collisions during Run 2 is $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$, which is fully exploited by ATLAS and CMS, in search for rare decays. On the contrary, the LHCb experiment studies processes with an average higher cross section and, in order to limit the input to the data storage system, the instantaneous luminosity is reduced to the level of $2 \div 4 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$. Since $\mathcal{L}$ is determined from the bunch crossing frequency and the overlap area between colliding bunches, this reduction is done by increasing the distance between the beam positions in the transverse plane.

$\mathcal{L}$ is also an indicator of the amount of statistics produced and analysed by an experiment: the integrated luminosity over a period of time t , referred to as $\int \mathcal{L} dt$, is an estimate of the ratio between the number of events produced for a specific process and the relative cross section. For instance, the integrated luminosity, divided by year, recorded by the LHCb experiment and the corresponding beam energy is shown in figure 3.1 and it amounts to 3.2 fb^{-1} during Run 1 and 5.9 fb^{-1} during Run 2, for a total of 9.1 fb^{-1} .

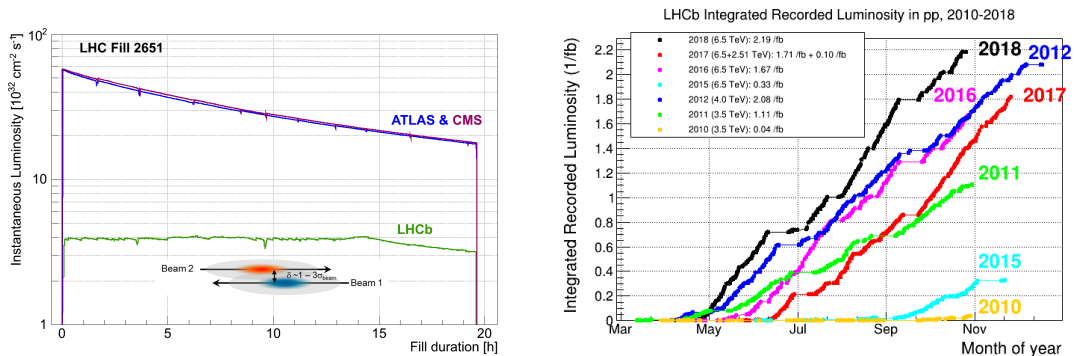


Figure 3.1: Left: Development of the instantaneous luminosity for ATLAS, CMS and LHCb during LHC fill 2651 (Run 1), taken from [46]. Right: integrated luminosity collected during Run 1 and Run 2 by the LHCb experiment.

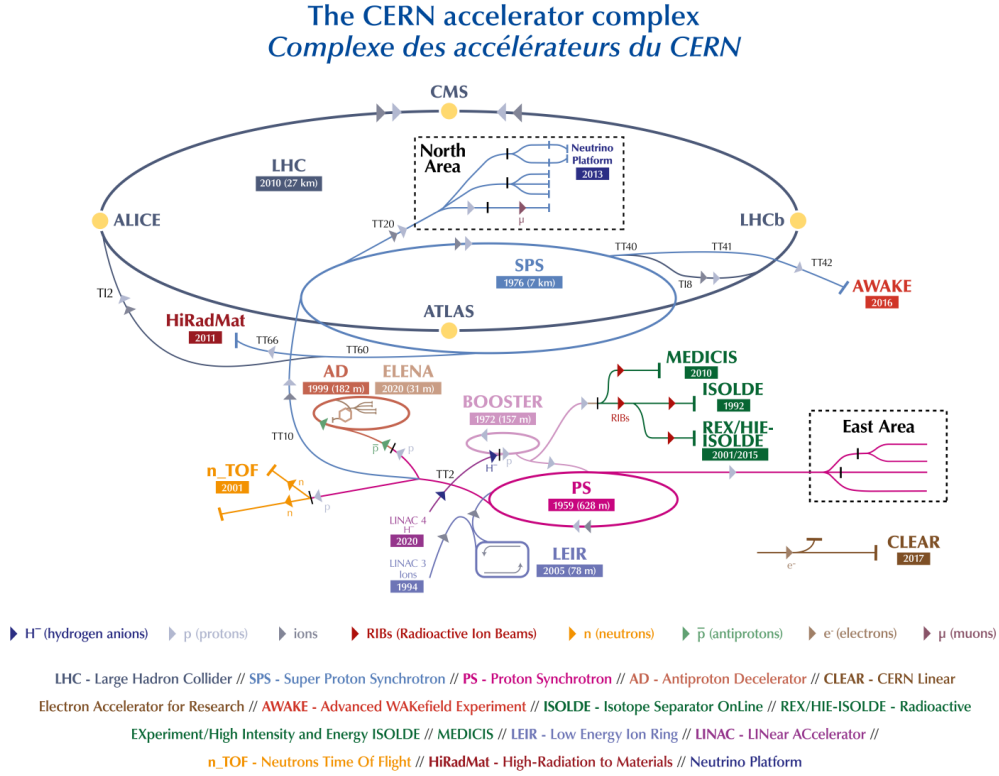


Figure 3.2: Scheme representing the CERN accelerator complex, taken from [47]. Most of the smaller accelerators in the chain have their own experimental halls where beams are used for experiments at lower energies.

3.2 The LHCb detector

LHCb is one of the four main LHC experiments designed primarily to study the properties and interactions of heavy-flavoured hadrons containing beauty and charm quarks with high precision. The primary goals of the LHCb detector include studying CP violation, measuring crucial parameters of heavy flavour physics such as masses, lifetimes, and mixing parameters, and searching for rare decays.

The LHCb detector is configured as a single-arm forward spectrometer (Figure 3.4), with a forward angular acceptance between 10 mrad and 300 mrad (250 mrad) in the horizontal (vertical) plane. This corresponds to a pseudorapidity acceptance of $1.8 < \eta < 4.9$, with η related to the angle with respect to the beam axis, θ , as $\eta \equiv -\log(\tan(\theta/2))$. The LHCb acceptance is complementary to the one of a general purpose detector, such as CMS and ATLAS, which is $|\eta| < 2.4$. In particular, LHCb is optimised to detect particles produced at small angles relative to the beamline. In fact, the production cross sections of $c\bar{c}$ and $b\bar{b}$ in parton strong interactions peak in the forward and backward directions, as shown in figure 3.3.

The main components of the LHCb detector in Run 2¹, shown in figure 3.4, were:

- **Vertex Locator (VELO)**, designed for precise measurement of primary vertices from pp

¹The analysis presented in this thesis utilises data collected during the LHC Run 2 (2015-2018). Therefore, this chapter describes the features of the LHCb detector as they were during this period. It is important to note that after 2018, all the subdetectors underwent several upgrades and now have different features.

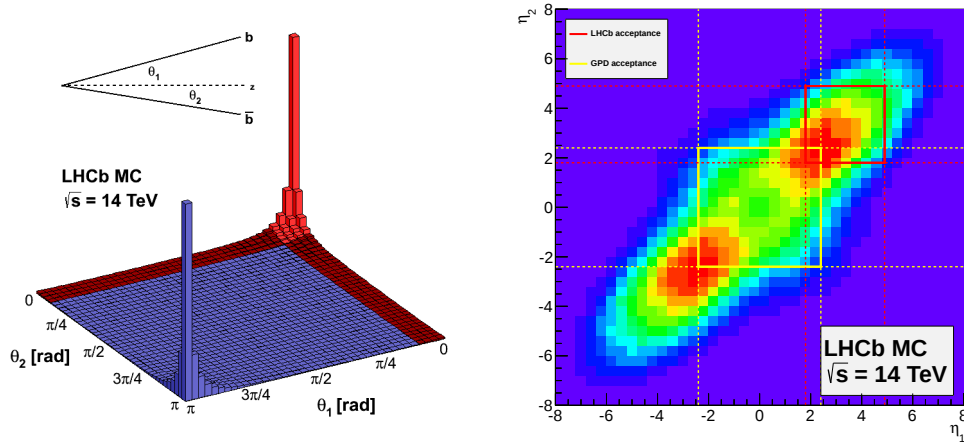


Figure 3.3: Left: distribution of $b\bar{b}$ production cross section in pp collisions at $\sqrt{s} = 14$ TeV as a function of their polar angles θ_1 and θ_2 with respect to the beam axis. The red region represents the LHCb angular acceptance, defined as $1.8 < \eta < 4.9$, and it contains 27 % of b or $\bar{b}$ quarks and 24 % of $b\bar{b}$ pairs. Right: color map of the $b\bar{b}$ production cross section dependence on η_1 and η_2 , showing the acceptance of LHCb and of a General Purpose Detector (GPD). This simulation has been performed with the PYTHIA software and the figures have been taken from Ref. [48].

collisions and displaced decay vertices of beauty and charmed hadrons. It consists of silicon strip detectors arranged very close to the proton-proton interaction point, providing very precise spatial resolution.

- **Tracking System**, meant for measuring the momentum and trajectory of charged particles. The components are:
 - **Tracker Turicensis (TT)**: large area strip detector, located upstream of the magnet to provide a measurement before the particle is deflected.
 - **Tracking Stations (T1,T2,T3)**: each composed by an Inner Tracker (IT), made of silicon strips, covering the region close to the beamline, and an Outer Tracker (OT), made of straw drift tubes, covering the outer region.
- **RICH Detectors (Ring Imaging Cherenkov)**, aimed at providing particle identification by measuring the Cherenkov radiation emitted by charged particles as they travel through a medium. The components are:
 - **RICH1**: located upstream the magnet, used for low-momentum particle identification.
 - **RICH2**: located downstream the magnet, used for high-momentum particle identification.
- **Magnet**, which bends the trajectories of charged particles to measure their momenta. The magnet creates a strong magnetic field perpendicular to the beamline, with a bending power of 4 Tm.
- **Calorimeters**, aimed at measuring the energy of particles and providing additional particle identification. The components are:
 - **Scintillating Pad and Preshower detectors (SPD/PS)**: useful for distinguishing between electrons, hadrons, and photons.

- **Electromagnetic Calorimeter (ECAL):** measures the energy of electrons and photons.
- **Hadronic Calorimeter (HCAL):** measures the energy of hadrons.
- **Muon Stations (M1-M5),** with the goal of identifying muons, which are typically the only particles to penetrate the entire detector. Composed of several layers of multiwire proportional chambers interspersed with iron absorbers.

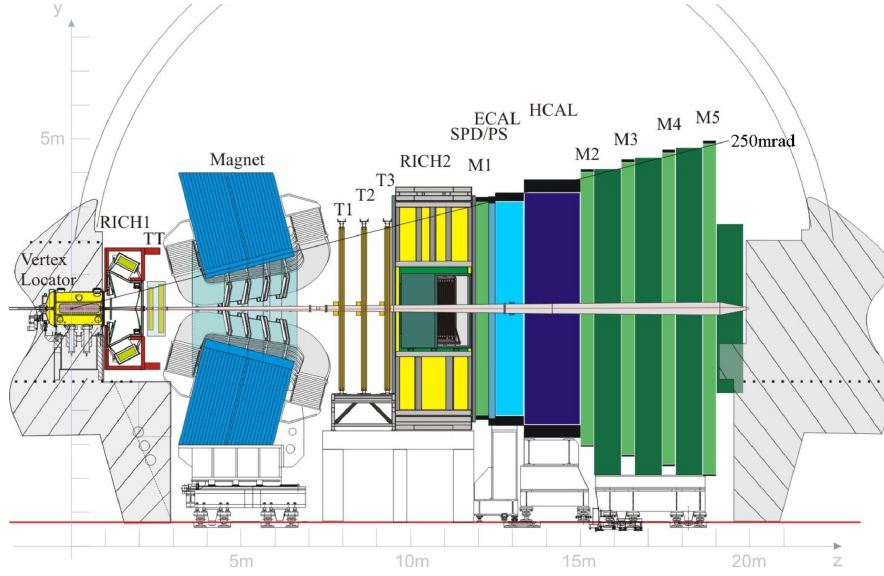


Figure 3.4: Schematic representation of the LHCb detector during Run 2 - side view in the zy plane. The right-handed coordinate system used has z pointing to the beams direction, y pointing upwards and x pointing to the center of the LHC. The pp collisions occurs nominally in the origin of the axis system.

3.3 The LHCb tracking system

During Run 2, the LHCb tracking system was composed by three main subdetectors: the Vertex Locator (VELO), the Tracker Turicensis (TT) and the Tracking Stations ($T_{1,2,3}$, each comprising Inner Tracker (IT) and an Outer Tracker (OT)). Each subdetector was composed of several modules, mainly silicon strips or gaseous drift tubes, with a geometrical disposition optimised to maximise acceptance and performance, while minimising the occupancy, i.e. the particle flux in each module. When a charged particle crosses a module, it interacts with the material and leaves a *hit*, which can be used by track finding algorithms to reconstruct its trajectory. The spatial hit resolution and hit identification efficiency are summarised in table 3.1.

Vertex Locator (VELO)

The Vertex Locator (VELO) [49] is a critical component of the LHCb detector, designed to provide precise measurements of particle trajectories close to the pp interaction point. Its primary role is to reconstruct the vertices of particle decays, which is crucial for identifying the displaced decay vertices of heavy flavoured hadrons. In particular, due to their finite lifetime and their boost, b and c hadrons travel for a mean distance of about $\mathcal{O}(\text{mm-cm})$ before decaying. The geometry and design of the VELO are optimised to achieve very precise spatial resolution and high efficiency.

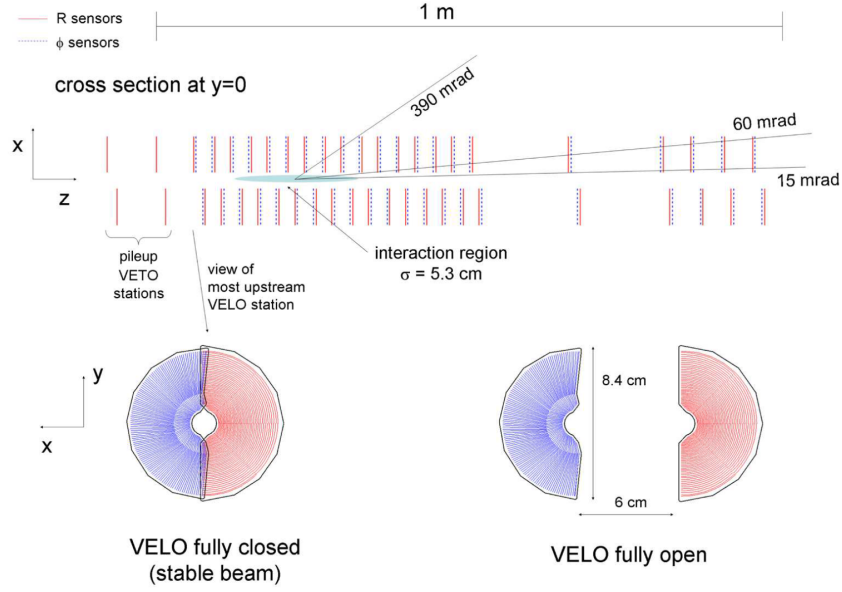


Figure 3.6: Cross section in the (x, z) plane of the VELO silicon sensors, at $y = 0$, with the detector in the fully closed position. The front face of the first modules is also illustrated in both the closed and open positions. Figure taken from Ref. [50]

Tracker Turicensis (TT)

The Tracker Turicensis (TT) [51] is a silicon microstrip detector positioned immediately before the dipole magnet and after RICH1 in the LHCb experiment. This strategic placement allows it to detect low-momentum tracks that would otherwise be deflected out of acceptance by the magnetic field and to reconstruct long-lived particles like K_S^0 and Λ , which typically decay outside the VELO region. The TT detector significantly contributes to the momentum resolution of tracks reconstructed in the VELO and T stations, providing reference segments for combining tracks from different detector stations and improving overall track and momentum resolution.

The TT detector spans a rectangular area approximately $150 \text{ cm} \times 130 \text{ cm}$, covering the full LHCb acceptance. It consists of four planar layers grouped into two pairs, TTa and TTb, with each pair separated by about 30 cm along the beam axis, as shown in 3.7. These layers are configured in an "x-u-v-x" arrangement: the first and fourth layers have vertical strips, while the second and third layers have strips tilted with respect to the y axis by $+5^\circ$ and -5° , respectively. This geometry enhances the resolution in the $x - z$ plane, crucial for tracking charged particles, while also using information on the y direction to reduce the background from fake tracks. Each TT layer is made up of several $500 \mu\text{m}$ thick sensor modules, each subdivided into 512 strips at a $183 \mu\text{m}$ pitch.

Tracking Stations (T1, T2, T3)

The T-stations, T1, T2 and T3, are three tracking stations placed downstream the magnet, with the primary goal of measuring of the deflection angle in the magnet, and, from this, determining the momentum of charged particles.

Each of the three T-Stations consists of four layers arranged in an "x-u-v-x" configuration, as for the TT, covering an active area of $6 \text{ m} \times 5 \text{ m}$. Due to the high occupancy near the beamline, each station is divided in two parts, the Inner Tracker (IT) and the Outer Tracker (OT):

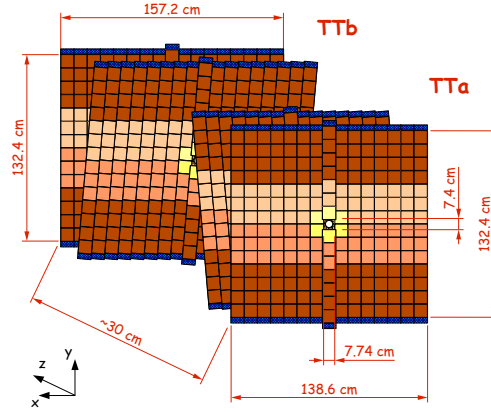


Figure 3.7: Schematic representation of the TT detector geometry: the TTa is composed of the first (vertical) and second (tilted by $+5^\circ$), while the TTb of the third (tilted by -5°) and fourth (vertical). Figure taken from Ref. [52].

- **Inner Tracker (IT)** [53]: positioned close to the beamline, it uses silicon strip detectors arranged in a cross-shaped geometry around the beam pipe. Each IT station features four boxes, as shown in 3.8, each containing four detection layers made of silicon strips with a pitch of $196\ \mu\text{m}$.
- **Outer Tracker (OT)** [54]: covering the larger part of the T Stations, it employs a gas detector technology consisting of drift tubes with a diameter of 4.9 mm and a length of 2.4 m. Their geometry is shown in 3.8. These tubes are filled with a gas mixture of $\text{Ar}/\text{CO}_2/\text{O}_2$ (70/28.5/1.5%) and have a drift time below 50 ns, crucial for the LHC's 25 ns bunch spacing.

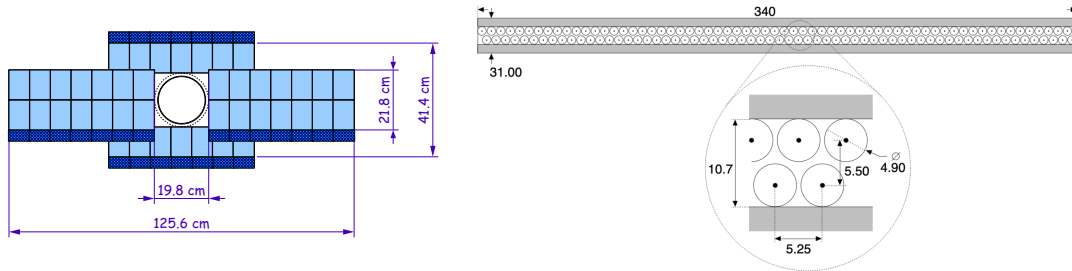


Figure 3.8: Left: geometrical layout of a layer of the Inner Tracker [52]. The circle in the middle is the beam pipe. Right: front view of an OT module, with a zoom on the drift tubes [46].

Dipole magnet and track reconstruction

The LHCb detector is provided with a warm non superconducting dipole magnet, placed between the TT and the first tracking station T1, necessary to measure the momentum of charged particles. The magnet is formed by two coils, slightly inclined with respect to the beam axis, in order to better match the acceptance of the LHCb detector. The main component of the magnetic field is along the y axis, with a maximum intensity of about 1.1 T, as shown in figure 3.9, and a total bending power of $\int B dl = 4\ \text{Tm}$.

The bending plane of the magnet can be assumed to be the xz plane and opposite charged particles are bent into opposite x regions of the detector, especially if they have low momentum. This effect gives rise to relevant detection asymmetries, thus, to partially correct for this problem, the polarity of the magnet is inverted about every two weeks between two configurations: *MagUp*, in which the magnetic field points upwards, and *MagDown*.

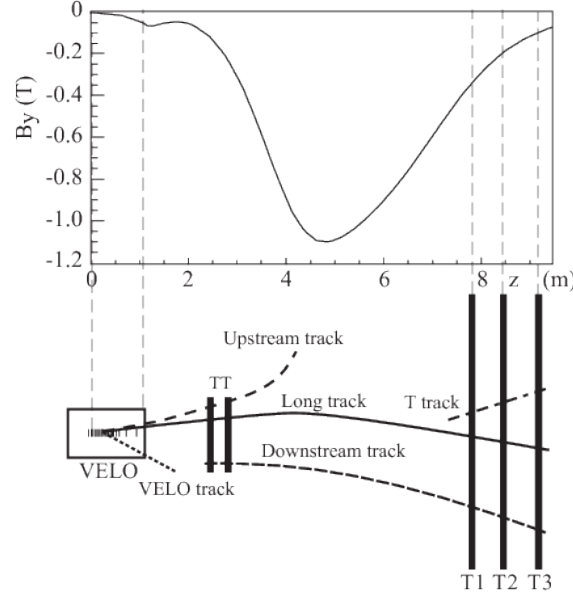


Figure 3.9: Top: y component of the magnetic field, in the *MagDown* configuration, as a function of z . Bottom: sketch of the various track types (VELO, Upstream, T tracks, Downstream, Long). Figure taken from Ref. [46].

A track is the reconstructed trajectory of a charged particles, built with specific pattern recognition and fitting algorithms from the hits left in the different tracking detectors. The tracks are divided in five categories, based on the subdetectors they left hits in, as shown in figure 3.9 :

- **VELO tracks:** reconstructed with only VELO hits and can be used to find the primary vertex. These tracks can be used as seeds, being extended to the other subdetectors to form upstream or long tracks.
- **Upstream tracks:** reconstructed with VELO and TT hits, usually generated by particles with low momentum bent out of acceptance by the magnetic field.
- **T tracks:** reconstructed with hits only in the T-stations, used as seeds for downstream and long tracks.
- **Downstream tracks:** reconstructed with hits in the TT and the T-stations but not in the VELO. They typically correspond to the decay products of long-lived particles which decay between the VELO and the TT such as the K_S^0 .
- **Long tracks:** reconstructed with hits in all three of the tracking detectors: VELO, TT, and T-stations. Long tracks have the best momentum resolution, as shown in figure 3.10, thus these are the main track type used in physics analyses.

The tracking performances are summarised in table 3.2. In particular, Primary Vertex (PV) refers to the pp collision vertex, while the Impact Parameter (IP) of a track is defined as its closest distance to the PV.

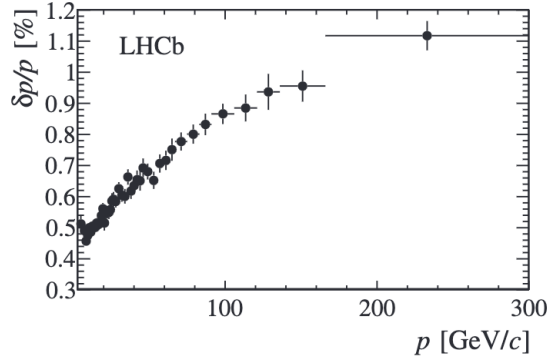


Figure 3.10: Relative momentum resolution versus momentum for long tracks in Run 1 data obtained using $J/\Psi \rightarrow \mu^+ \mu^-$ decays [46].

Quantity	Interval
Track finding efficiency	$\gtrsim 96\%$
Primary Vertex resolution	$\sigma_{x,y} \sim 7 \div 35 \mu\text{m}$
Impact Parameter resolution	$\sigma_x \sim 10 \div 80 \mu\text{m}$
Mass resolution	$\sigma_m/m \sim 0.5\%$
Momentum	$\sigma_p/p \sim 0.5 \div 1\%$
Decay Time resolution	$\sigma_t \sim 50 \text{ fs}$

Table 3.2: Tracking systems performances in the measurement of the physical quantities of interest [46].

3.4 The LHCb particle identification system

The Particle Identification (PID) system is crucial for distinguishing between the different types of long-lived particles and it is composed of several subdetectors: RICH1, RICH2, the calorimeter system (SPD/PS, ECAL, HCAL), and the muon stations. The PID information differs for charged particles, mainly $\pi^\pm, K^\pm, e^\pm, \mu^\pm$, and neutral particles, such as photons and π^0 . For charged PID, information from all the subdetectors is grouped into a single powerful observable: $DLL_{x\pi} \equiv \log \mathcal{L}(x) - \log \mathcal{L}(\pi)$, defined as the difference between the logarithm of the global likelihood, computed using tracking and PID sub-detectors information, under a specific identity hypothesis x ($\mathcal{L}(x)$, where $x = K, p, \mu, e$) and under the hypothesis that the track is a pion ($\mathcal{L}(\pi)$). In contrast, for neutral PID dedicated neural networks can be trained on simulation data to distinguish photons from electrons, hadrons and neutral pions, primarily using information from the calorimeters.

Ring Imaging Cherenkov (RICH) detectors

To identify charged particles, mainly to discriminate between pions, kaons and protons, LHCb employs two Ring Imaging Cherenkov detectors (RICH) [55]. The first detector is installed immediately after the VELO, while the second is positioned after the Tracking Stations. The Cherenkov effect occurs when a charged particle travels through a medium faster than the speed of light in that medium, c/n , where n is the refractive index. This causes the emission of Cherenkov radiation

photons at a specific angle, θ_C , relative to the particle's direction, given by:

$$\cos \theta_C(p, m) = \frac{1}{n\beta} = \frac{1}{n} \sqrt{1 + \left(\frac{mc}{p}\right)^2}, \quad (3.2)$$

where m and p are the particle mass and momentum. Since the momentum is measured by the tracking system, the Cherenkov angle can help identify particles with different masses, as shown in the left part of figure 3.11. In the LHCb RICH system, θ_C is measured directly from the radius of the Cherenkov ring, which is the projection of the light cone around the particle's trajectory onto the photon detector plane.

Equation (3.2) shows that in the limit of $p \gg mc$ the Cherenkov angle becomes independent of the mass $\cos \theta_C(p) \sim 1/n$, materials with a refraction index closer to 1 have a better separation power for high momenta particle. On the other hand, n determines the lower momentum threshold for which we can see Cherenkov radiation, given by $\beta > 1/n$: the higher n , the wider the low momentum acceptance of the RICH detector. The two LHCb RICH detectors have different refractive indices, in order to maximally extend the momentum acceptance for PID estimation:

- **RICH1**: placed upstream the magnet and aimed at identifying lower momentum particles before they are bent by the magnet. The radiation media are aerogel ($n = 1.03$) and C_4F_{10} ($n = 1.0014$), resulting in a momenta range of $1 \div 60$ GeV/ c ;
- **RICH2**: placed downstream the magnet and aimed at identifying higher momentum particles. The radiation medium is CF_4 ($n = 1.0005$), resulting in a momenta range of $15 \div 100$ GeV/ c .

The geometry of the two RICH detectors is shown in figure 3.12: the radiators are placed around the beampipe, in the LHCb acceptance, and the Cherenkov photons are brought by a complex mirror system in the photon detector area, shielded by the magnetic field. The photons positions are reconstructed by a lattice of Hybrid Photon Detectors (HPDs).

The RICH detectors play a critical role in the PID of charged hadrons such as pions, kaons, and protons. The efficiency and misidentification probabilities between $K\pi$ and $p\pi$ as a function of momentum are shown in figure 3.13, demonstrating that PID performance is poorer at low momenta, outside the RICH2 acceptance, and at very high momenta, beyond the RICH1 acceptance and in the limit $p \gg M_{K/p}$.

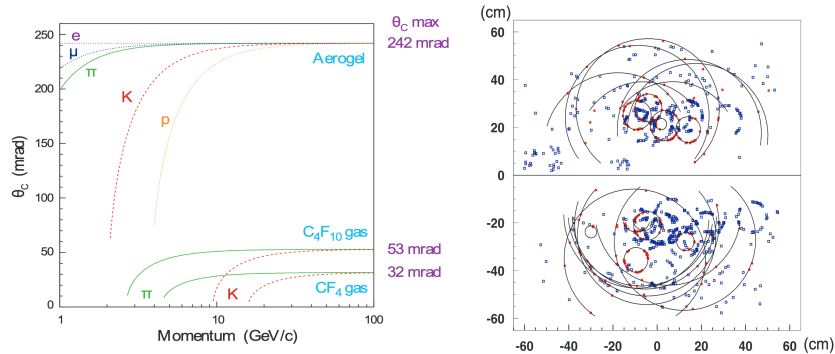


Figure 3.11: Left: Cherenkov angle versus particle momentum for different charged particles in the three radiators used in the LHCb RICH system (aerogel, C_4F_{10} and CF_4). Right: Display of a typical LHCb event in RICH1, with the reconstructed Cherenkov rings superimposed to the photon detectors response [46].

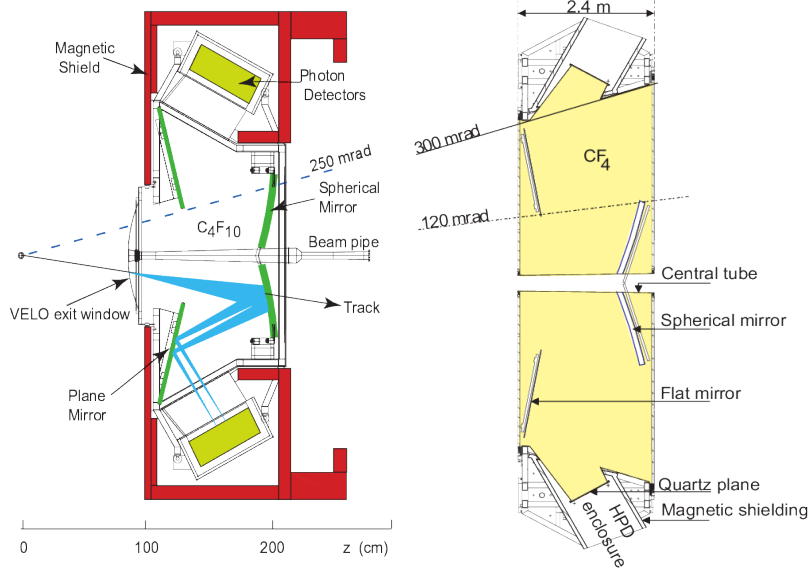
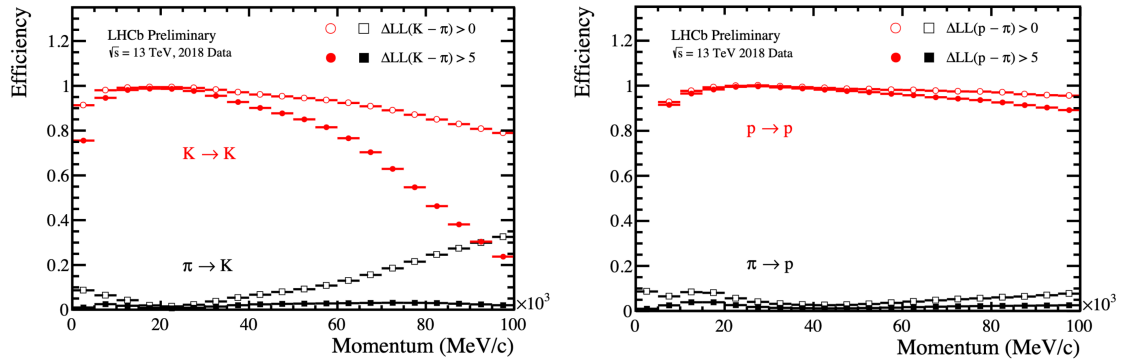


Figure 3.12: The optical system of RICH1 (left) and RICH2 (right) [56].

Figure 3.13: Performance of the loose and tight cuts on the DLL variable, as a function of track momentum, for kaon-pion separation and proton-pion separation in 2018 LHCb data [57].

Calorimeters

The calorimetric system [58], composed of four subdetectors, is designed to measure the energy of charged and neutral particles and to identify electrons, photons, and hadrons, as shown in figure 3.14. In addition, due to its fast response in energy measurement, the calorimetric information is used by the low-level hardware trigger, as explained in 3.5. The four subdetectors that make up the calorimetric system are:

- The **Scintillating Pad/PreShower Detector (SPD/PS)** consists of a 15 mm lead converter, 2.5 radiation length-thick, sandwiched between two almost identical planes of rectangular scintillator pads with high granularity. The SPD detects charged particles through the scintillation light they produce, with its primary goal being to distinguish electrons from photons. The PS, placed beyond the lead plate, is used to separate electromagnetic showers from hadrons;
- The **Electromagnetic Calorimeter (ECAL)** is composed of alternating layers of lead and scintillator tiles, corresponding to a total of 25 radiation lengths and 1.2 nuclear interaction

length, ensuring that electromagnetic showers are completely contained within it, while the majority of hadrons interact minimally. The energy resolution is given by $\sigma(E)/E = 0.8\% \oplus 9\%/\sqrt{E[\text{GeV}]}$;

- The **Hadronic Calorimeter (HCAL)** is composed of alternating slabs of iron and scintillator tiles, corresponding to only 5.6 nuclear interaction lengths due to space limitations in the LHCb cavern. The energy resolution is very poor $\sigma(E)/E = 9\% \oplus 69\%/\sqrt{E[\text{GeV}]}$, because the HCAL does not fully contain the hadronic shower.

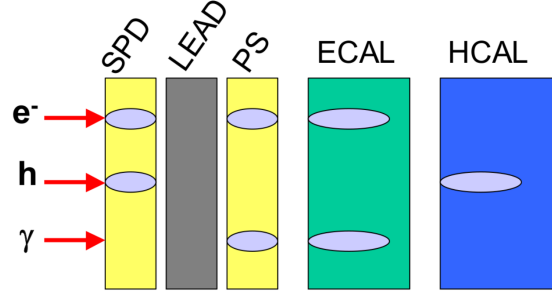


Figure 3.14: Main energy deposits for different particles in the different subdetectors of the calorimetric system.

Muon detectors

The five Muon Stations (M1-5) [59] have the primary goal of identifying muons and, thanks to their fast response, they are used along with the calorimetric system in the low-level hardware trigger. Each station is equipped with 276 multi-wire proportional chambers (MWPCs), with the exception of the inner part of M1, which is subject to the highest radiation, and is equipped with 12 GEM detectors. Stations M2 to M5 are placed downstream of the calorimeters and are interleaved with 80 cm thick iron absorbers, which can only be traversed by high momentum muons, as shown in figure 3.15.

Each station is divided into four regions with increasing distance from the beam axis, in which the detector and readout features have been optimised independently for the five stations. For example, stations M1, M2 and M3, used by the trigger to determine the track direction and the p_T of the candidate muon, have a higher x granularity than stations M4 and M5, whose main purpose is the identification of penetrating particles.

The muon stations are also used as an input to the charged PID algorithms: if a track has at least two hits in M2-M5, the muon likelihood is computed and used as a particle identification discriminator.

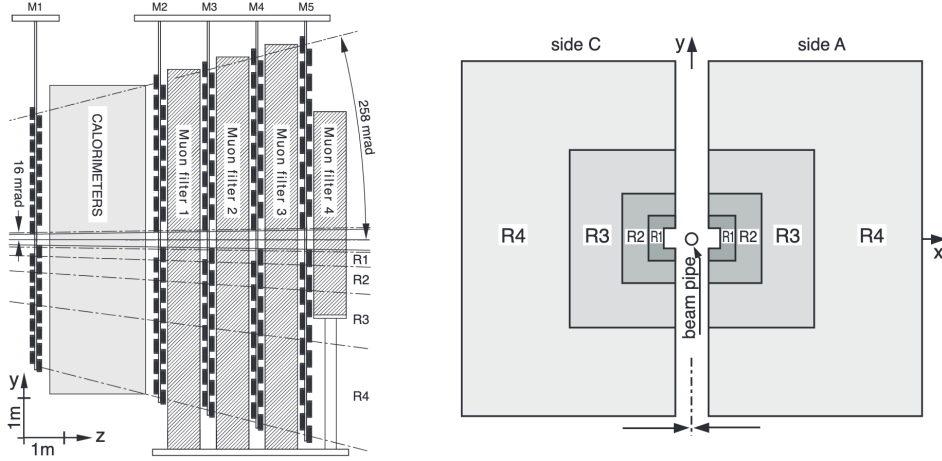


Figure 3.15: Left: Side view of the five LHCb Muon Stations, where *Muon filter* refers to the 80 cm thick iron absorbers. Right: station layout in the xy plane, with the four regions R1–R4 [50].

3.5 The LHCb trigger

The LHCb experiment has the primary goal of identifying hadrons containing b and c quarks produced in pp collisions. The production cross-sections for $b\bar{b}$ and $c\bar{c}$ pairs are quite substantial ($\sigma(pp \rightarrow c\bar{c}X) \sim 1.5$ mb); however, the total inelastic cross section can be higher by 2-3 orders of magnitude, resulting in a low signal-to-background ratio. Furthermore, the typical collision rates are usually very high, $\mathcal{O}(10)$ MHz, and the events are characterised by a large number of tracks. However, only a limited amount of data, corresponding to 12.5 kHz in Run 2, could be written to storage. This requires the employment of an efficient trigger system, able to identify in real-time the interesting events and discard the rest.

The LHCb trigger system [60] is designed to operate at the LHC's bunch crossing frequency of 40 MHz and is structured into multiple levels, each building upon the previous one. In Run 2, the LHCb trigger was divided into three levels, summarised in figure 3.16:

- **Level-0 Trigger (L0):** this initial trigger level is based on custom electronics designed to perform preliminary event filtering, reducing the input rate from about 40 MHz to an output rate of 1 MHz. The L0 trigger utilises information exclusively from the calorimeters and the muon system, making decisions based on the transverse energy or momentum of the particles in the event.
- **High Level Trigger 1 (HLT1):** this software-based trigger level reduces the event rate from 1 MHz to about 110 kHz. HLT1 performs a partial reconstruction of events, based on tracking information only, leveraging characteristics specific to beauty and charm hadrons such as large p_T and significant impact parameters (IP).
- **High Level Trigger 2 (HLT2):** the final trigger level, entirely software-based. It performs a full reconstruction of events, taking into account the information from all the subdetectors and ultimately reducing the event rate to approximately 12.5 kHz.

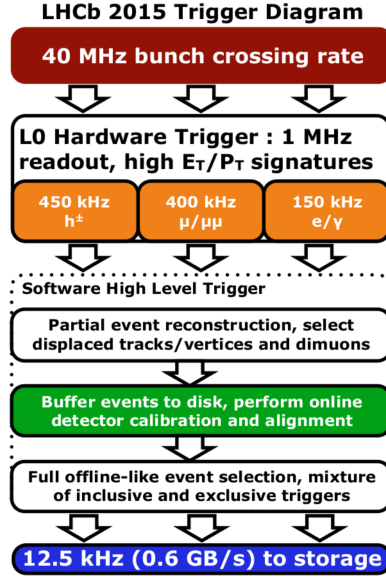


Figure 3.16: LHCb trigger diagram during Run 2. Figure taken from [61].

Level-0 Trigger (L0)

The first level hardware trigger (L0) has the goal of reducing the rate of events from 40 MHz to 1 MHz with a maximum latency of 4 μs. For this reason, due to their fast response, only the calorimeter and muon systems are used as an input to three independent triggers, running in parallel on FPGAs:

- **L0Hadron**: uses information from the HCAL detector and from the SPD and PS systems to identify hadrons. It accepts events with at least one cluster, composed of 2×2 cells of the HCAL, having sufficient transverse energy, defined as:

$$E_T = \sum_{i=1}^4 E_i \sin \theta_i, \quad (3.3)$$

with E_i the energy deposited in the i^{th} cell and θ_i the angle between the z axis and a line connecting the average pp interaction point to the centre of the cell. The E_T threshold has varied in the different data taking years between 3.4 GeV and 3.7 GeV.

- **L0Photon/L0Electron**: uses information from the SPD, PS, and ECAL detectors about the deposited energy and photon-electron discrimination. Events are accepted if at least one cluster, composed of 2×2 cells of the ECAL, has E_T exceeding a specified threshold, which has been varied in the data taking years around 2.4 GeV for electrons and 2.8 GeV for photons.
- **L0Muon/L0Dimuon**: uses information from the five muon stations, to reconstruct tracks and measure the p_T . The trigger decision is based on the two muon candidates with the largest p_T : either the largest p_T must be above the L0Muon threshold, varied between 1.35 GeV/ c and 2.8 GeV/ c , or the product of the largest and second largest p_T values must be above the L0DiMuon threshold, varied between $(1.3 \div 1.5 \text{ GeV}/c)^2$.

High Level Trigger (HLT)

Events selected by L0 are transferred for further selection to the Event Filter Farm (EFF), where the HLT filters and reconstructs, initially partially and then completely, the events. The HLT is written in the same framework as the software used in the offline reconstruction of events for physics analyses and it runs in parallel on thousands of physical cores. The HLT is divided into two successive steps, HLT1 and HLT2, each with several lines designed to reconstruct and select specific decay topologies:

- **HLT1** processes the trajectories of charged particles, starting from VELO tracks, to identify primary and secondary vertices. Next, the VELO tracks are combined with TT hits, to form upstream tracks, and finally with IT and OT hits, for which the search window is defined by the maximum possible deflection of charged particles with p_T larger than 500 MeV/ c , in order to only select *long* tracks with high momentum. Subsequently, all tracks are fitted with a Kalman filter to obtain the optimal parameter estimate, mainly the p_T and impact parameter, used in the HLT1 selection, composed of ~ 20 lines. The output rate of the HLT1 step is reduced to 110 kHz.
- The **HLT2** full event reconstruction consists of three major steps: the track reconstruction of charged particles, of neutral particles, and, finally, particle identification (PID). The information of the tracking detectors, exploited in the first step, is then combined with calorimeters, to distinguish photons electrons, and π^0 mesons, and with the RICH information, to identify charged hadrons. There are approximately 500 HLT2 trigger lines, each applying specific cuts to identify a particular decay mode.

Due to the vast amount of data generated, the primary constraint on the LHCb event rate stored to disk is the availability of storage space. Between Run 1 and Run 2, upgrades to the Event Filter Farm (EFF) enhanced its computing power and memory capacity. This upgrade significantly increased the number of events saved for offline analysis². This upgrade, combined with the strategy to buffer the data completely between the first and second software trigger levels, extended the allocated time for the HLT2 reconstruction process. Furthermore, automated alignment and calibrations performed in real-time now enable offline-quality information to be computed within the trigger system. As a result of these improvements, a new data stream was introduced in HLT2, known as the **Turbo stream** [62, 63, 64]. The Turbo stream selectively stores only the triggered HLT2 candidates while discarding the information of the rest of the event. This approach drastically reduces the event size, as an entire raw event weighs around 70 kb, whereas the relevant particles of interest usually amount to only 5 kb. The Turbo stream comprises nearly all of the charm exclusive lines, including the ones employed in this thesis, and it accounts for about 5 kHz of the total 12.5 kHz HLT2 output rate.

²The total buffer space of the EFF, around 10 PB, is sufficient to store the information of all the events selected by the L0 trigger during about ten days of continuous data-taking. Considering the operational duty cycle of the LHC, this allocation allows approximately 50 ms and 800 ms for the execution of the HLT1 and HLT2 triggers, respectively, with an HLT1 output rate of 140 kHz. For comparison, during the 2011–2012 period, the computing time available to the HLT2 was approximately 30 ms, even with a reduced HLT1 output rate of 100 kHz

Chapter 4

Data samples and selection

This chapter describes the trigger and offline requirements used to select pure samples of $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ decays. Particular attention is given to the equalisation of the requirements between samples, which is crucial for ensuring a precise cancellation of nuisance asymmetries. Finally, the obtained yields for each decay channel are determined.

4.1 Decay channels of interest for the analysis

This master's thesis work has two main goals: first, the high precision estimation of the neutral kaon detection asymmetry in charm decays, using the $D^+ \rightarrow K_S^0 \pi^+$ decay channel as case study, and, second, the measurement of the direct CP asymmetry in the $D^+ \rightarrow \phi \pi^+$ decay channel with the full LHCb Run 2 data samples. Therefore, the first step of the analysis is the selection of pure $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ decay samples, with the aim of reducing background candidates.

The data samples available for the analysis were produced in pp collisions at a centre-of-mass energy of $\sqrt{s} = 13$ TeV during the LHC Run 2 (2015-2018). The datasets used in this analysis were collected in the years 2016, 2017, and 2018, in both the LHCb magnet configurations, *MagUp* and *MagDown* (see section 3.3) and correspond to a total integrated luminosity of 5.4 fb^{-1} . In the following chapters, the datasets, divided by year and polarity, are referred to as $yypp$, where yy refers to the year (16, 17, 18) and pp refers to the magnet polarity (Up and Dw). The stored events used in the analysis have passed the L0, HLT1, and HLT2 trigger selections described in section 4.2. After these trigger requirements, additional *offline* selections were applied for each decay channel to further suppress backgrounds and to reduce nuisance asymmetries, as described in section 4.3.

The topology of both $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ decays, where the parent particle is a charged $D^\pm$ meson, produced promptly in pp collisions, is shown in figure 4.1. Since the proper decay time of the D^+ meson is $\tau_{D^+} \simeq 1.04 \text{ ps}$ and the average D^+ momentum is $70 \text{ GeV}/c$, the average travelled distance before the decay is $d_{D^+} \simeq 1.25 \text{ cm}$. Thanks to the good vertex resolution of the VELO, it is then possible to distinguish the decay vertex of the D^+ from the primary vertex (PV) of the pp collision, as all the charged particles produced by the D^+ have a non-negligible impact parameter with respect to the PV.

In $D^+ \rightarrow \phi \pi^+$ decays, ϕ mesons are reconstructed from their strong decay into a charged kaon pair, $\phi \rightarrow K^+ K^-$ ¹. As a consequence, $D^+ \rightarrow \phi \pi^+$ decays can be identified as three long tracks, two opposite charged kaons and one pion, all coming from the same displaced vertex. To suppress the combinatorial background due to random associations of three tracks, the invariant mass of

¹The width of ϕ mesons is about $4.25 \text{ MeV}/c^2$ and the branching ratio into the $K^+ K^-$ final state is 49.1% [36].

the K^+K^- pair is required to be close to M_ϕ and the invariant mass of the $K^+K^-\pi^+$ triplet is required to be close to M_{D^+} .

In $D^+ \rightarrow K_S^0\pi^+$ decays, neutral kaons are reconstructed from their weak decay into a charged pion pair, $K_S^0 \rightarrow \pi^+\pi^-$. However, since the K_S^0 lifetime is $\tau_{K_S^0} = 0.89 \times 10^{-10}$ s and the K_S^0 momentum has a wide distribution between 3 GeV/c and 200 GeV/c, the distance travelled by a neutral kaon before decaying can vary between a few centimeters and a few meters. The LHCb detector reconstructs two types of neutral kaons candidates:

- the Long $K_S^0(LL)$ candidates, in which the neutral kaon decays inside the VELO detector and both pions are long tracks. The $K_S^0(LL)$ proper decay time belongs to the $[0, 0.4\tau_{K_S^0}]$ interval;
- the Downstream $K_S^0(DD)$ candidates, in which the neutral kaon decays between the VELO and the TT detector and both pions are downstream tracks. The $K_S^0(DD)$ proper decay time belongs to the $[0, 3\tau_{K_S^0}]$ interval.

For this reason, this D^+ decay channel can be reconstructed by two different HLT2 lines, one for $K_S^0(LL)$ candidates and the other for $K_S^0(DD)$ candidate.

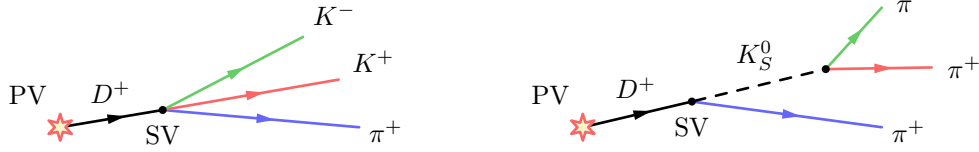


Figure 4.1: Scheme of the topology of the $D^+ \rightarrow \phi\pi^+$ (left) and $D^+ \rightarrow K_S^0\pi^+$ (right) decays.

Overview of used observables in the offline analysis

Before presenting the trigger and offline requirements applied to select pure data samples of $D^+ \rightarrow \phi\pi^+$ and $D^+ \rightarrow K_S^0\pi^+$ decay candidates, it is convenient to introduce and define a series of variables that will be utilised in the offline analysis.

- Candidate variables, related to the spatial position of production and decay vertices:
 - **Primary Vertex (PV)**: the reconstructed point of interaction for the pp collision;
 - **Decay Vertex (DV)**: the reconstructed point for the decay of a particle;
 - **Displacement vector ($\vec{d}$)**: vector connecting the PV to the DV:

$$\vec{d} = \overrightarrow{DV} - \overrightarrow{PV}$$

- **Flight Distance (FD)**: modulus of the displacement vector $\vec{d}$;
- **Decay Time (t)**: defined as the momentum $\vec{p}$ and mass m in the relation:

$$\vec{d} = \vec{\beta}\gamma ct = \frac{\vec{p}}{m}ct;$$

- **Impact Parameter (IP)**: the distance of closest approach to the PV of the particle, defined from the displacement vector $\vec{d}$ and the momentum direction $\hat{p}$ as:

$$\text{IP} = \left| \vec{d} - (\vec{d} \cdot \hat{p}) \hat{p} \right|;$$

- **Direction angle (θ_{DIRA})**: the angle between the momentum of a particle $\vec{p}$ and its displacement vector $\vec{d}$, defined as $\cos \theta_{\text{DIRA}} = \hat{p} \cdot \hat{d}$;
- **Distance Of Closest Approach (DOCA)**: the minimum distance between two tracks.
- Kinematic variables, which characterise the energy and momentum of charged and neutral reconstructed particles:
 - **Transverse momentum (p_{T})**: modulus of the projection of the momentum of a particle on the transverse plane xy ;
 - **Pseudorapidity (η)**: defined as $\eta = -\log \tan(\theta/2)$, with θ the polar angle between the trajectory of a particle and the z axis;
 - **Azimuthal angle (φ)**: angle of the momentum of a particle on the xy plane;
 - **Curvature (k)**: quantity inversely proportional to the curvature radius in the dipole magnetic field, defined as:

$$k = \frac{1}{\sqrt{p_z^2 + p_x^2}};$$

- **Emission angles ($\theta_{x,y}$)**, defined as:

$$\theta_{x,y} = \arctan \left(\frac{p_{x,y}}{p_z} \right).$$

- Track variables, describing the quality of the reconstructed track-particles (also referred as tracks):
 - **Track ghost probability**: the output of a multivariate classifier evaluating the probability that the track does not correspond to the trajectory of a real particle;
 - **Flight Distance χ^2 (χ_{FD}^2)**: the χ^2 -distance between the PV and DV, defined from the reconstructed vectors $\overrightarrow{\text{PV}}$ and $\overrightarrow{\text{DV}}$ and their covariance matrices $\mathbf{cov}_{\text{PV}}$ and $\mathbf{cov}_{\text{DV}}$ as:

$$\chi_{\text{FD}}^2 = \left(\overrightarrow{\text{DV}} - \overrightarrow{\text{PV}} \right)^T (\mathbf{cov}_{\text{DV}} + \mathbf{cov}_{\text{PV}})^{-1} \left(\overrightarrow{\text{DV}} - \overrightarrow{\text{PV}} \right)$$
 - **Impact Parameter χ^2 (χ_{IP}^2)**: difference between the χ^2 of the PV fit with and without the track under consideration. This quantity is correlated to the IP, but the main difference is that it takes into account also the track uncertainties.
- PID variables, used to distinguish charged pions, kaons and protons, already introduced in section 3.4:
 - **$DLL_{K\pi}$** : given a particle, is the difference between the logarithm of the likelihood for the K hypothesis and for the π hypothesis;

- $DLL_{p\pi}$: given a particle, is the difference between the logarithm of the likelihood for the p hypothesis and for the π hypothesis.
- Trigger variables, used to select the particles that fire the L0 and HLT1 trigger lines:
 - **Trigger On Signal (TOS)**: candidates in which the trigger fired on at least one of the particles in the decay chain of the candidate under consideration;
 - **Trigger Independent of Signal (TIS)**: candidates in which the trigger fired on a particle that does not belong to the decay chain of the candidate under consideration.

4.2 Trigger selections

The LHCb trigger chain consists of multiple stages, where events or candidates of interest can be stored permanently in different paths and lines both at L0 and HLT levels. In particular, a decay process could have been accepted and subsequently written to disk, such as a TOS or TIS candidate, depending on various trigger lines throughout the chain. This is especially relevant for fully reconstructed hadronic decays, such as those examined in this thesis, where the efficiency of the L0 requirements is notably less than 100%, when the decay of interest occurs within the detector acceptance. Therefore, at the offline level, the selection of which triggers are fired or not can be optimised depending on the specific type of measurement being conducted. For instance, a measurement of CP violation, aiming for very high precision, would tend to use candidates triggered by lines with minimal detector- and selection-induced charge asymmetries. In contrast, a search for a rare decay mode would favour inclusive requirements to maximise the total reconstruction efficiency.

4.2.1 L0 selection

The main goal of the L0 trigger selection at the offline level is to minimise detection asymmetries in the $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ samples. Candidates are selected independently of whether the bachelor pion has fired the L0 trigger to keep the π^+ detection asymmetries small, enabling the linear approximation of the *raw* asymmetry in (2.8). Candidates with the `L0_Hadron_TOS` flag on the π^+ exhibit large detection asymmetries because the LHCb calorimeter, located downstream of the dipole magnet, requires that the transverse energy of the firing track exceeds a certain threshold. For example, if a π^+ and a π^- are produced by a D^+ and a D^- in the same vertex with the same momentum, their trajectories will bend in opposite directions due to the magnetic field. If the π^+ crosses the center of the calorimeter with low transverse energy while the π^- hits the left or right end with high transverse energy, only the π^- will fire the L0 hadron trigger, resulting in significant detection asymmetry. This effect can be observed in figure 4.2 for the $D^+ \rightarrow K_S^0(LL)\pi^+$ sample. When requiring `L0_Hadron_TOS` on the π^+ , large regions in the $p_x(\pi^+) - p_z(\pi^+)$ plane show *raw* asymmetries of the order of 10%-100%.

To mitigate this, two classes of candidates, where the trigger is fired independently of the bachelor π^+ , are selected for the analysis:

- candidates in which the L0 trigger has been fired independently on any particle produced in the decay of the D^+ , i.e. the requirement is `Dplus_LOGlobal_TIS`, where `LOGlobal` is the logic OR of all the L0 triggers;
- candidates in which the `L0Hadron` trigger has been fired by at least one of the two $\pi^\pm$ or $K^\pm$ produced by the K_S^0 and ϕ respectively. Since the K_S^0 and the ϕ are neutral particles and are indistinguishable in the decay of a D^+ or a D^- , triggering on their decay products does not introduce any detection asymmetries.

The final L0 requirements are then (Dplus_LOGlobal_TIS OR KS0_LOHadron_TOS) for the $D^+ \rightarrow K_S^0 \pi^+$ sample and (Dplus_LOGlobal_TIS OR Phi_LOHadron_TOS) for the $D^+ \rightarrow \phi \pi^+$ one. Such requests discard a percentage of signal candidates of about 23% in the $D^+ \rightarrow K_S^0(LL)\pi^+$ sample, 36% in the $D^+ \rightarrow K_S^0(DD)\pi^+$, and 22% in the $D^+ \rightarrow \phi \pi^+$, but they reduce significantly the π^+ detection asymmetries.

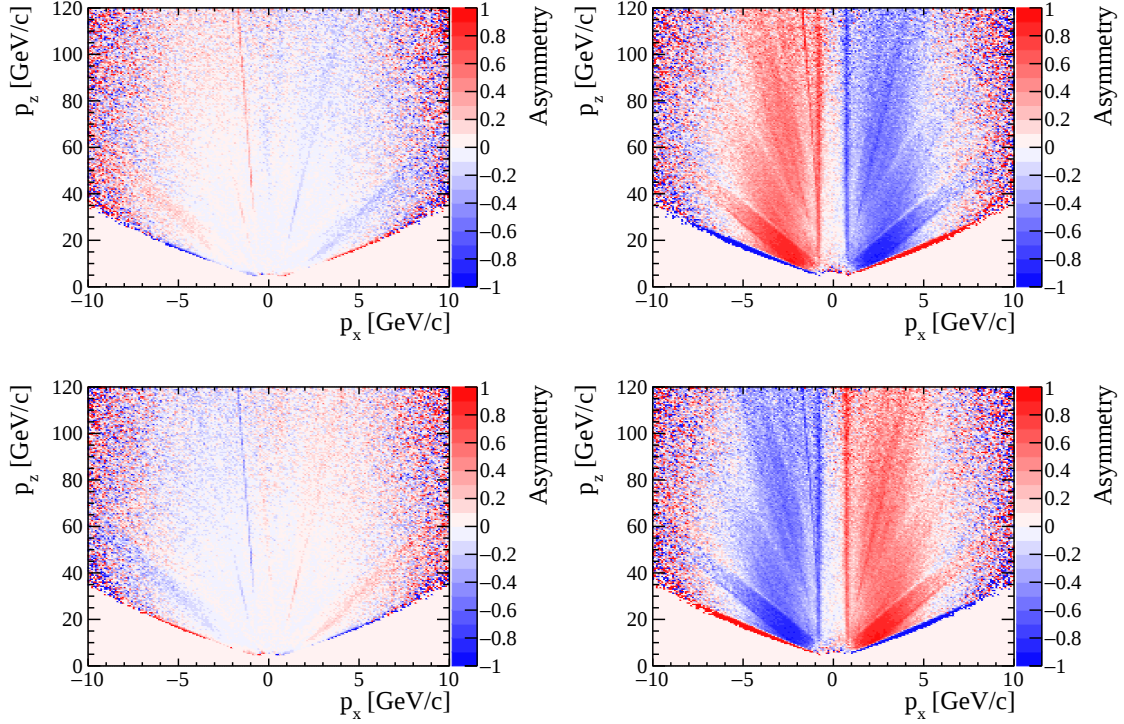


Figure 4.2: Distribution of the *raw* asymmetry in the $D^+ \rightarrow K_S^0(LL)\pi^+$ sample in the $p_x(\pi) - p_z(\pi)$ plane for the *MagUp* (first row) and *MagDown* data samples (last row) requiring different L0 conditions: on the left plots, the selection is independent on whether the bachelor pion has fired the L0 trigger (`Dplus_LOGlobal_TIS OR KSO_LOHadron_TOS`), while on the right plots the complementary selection is shown (`piplus_LOGlobal_TOS AND NOT ( Dplus_LOGloba_TIS OR KSO_LOHadron_TOS)`). Since the asymmetry scale is the same in all the plots, we can see that the request `piplus_LOGlobal_TOS` introduces significant detection asymmetries in certain regions on the kinematic space, which depend on the magnet polarity.

4.2.2 HLT1 selection

At the HLT1 level, the decay channels of interest in this analysis can be selected by two possible algorithms: `Hlt1TrackMVA`, which looks for single long tracks, and `Hlt1TwoTrackMVA`, which selects couples of long tracks coming from the same vertex. In this analysis, however, only candidates in which the bachelor pion has fired the `Hlt1TrackMVA` line, i.e. that respect the `piplus_Hlt1TrackMVA_TOS` requirement, have been included. This choice has been made due to the different topology of our decays of interest:

- in the $D^+ \rightarrow \phi\pi^+$ channel, the candidate is identified by three long tracks (K^+ , K^- , bachelor pion) which come from the same vertex, thus the `Hlt1TrackMVA` and `Hlt1TwoTrackMVA` could be triggered by any of them;
- in the $D^+ \rightarrow K_S^0\pi^+$ channel there is a bachelor pion long track and two displaced pion tracks, which could be long or downstream. For the K_S^0 which have decayed close to the D^+ vertex, the situation is similar to the $D^+ \rightarrow \phi\pi^+$ channel, but in the case of long lived K_S^0 the topology is much different and HLT1 can be fired only by the `Hlt1TrackMVA` line on the bachelor pion.

Since in a large component of the $D^+ \rightarrow K_S^0 \pi^+$ sample HLT1 can only be fired with the `piplus_Hlt1TrackMVA_TOS` condition, this request has been extended also to the $D^+ \rightarrow \phi \pi^+$ sample. This is necessary because the HLT1 trigger selection can introduce detection asymmetries and, if the requirements are the same in both samples, such effects cancel out properly. This request is conservative and it results in a significant statistics loss, discarding about 23% of the $D^+ \rightarrow \phi \pi^+$ candidates and 43% of the $D^+ \rightarrow K_S^0(LL)\pi^+$ candidates, while leaving the $D^+ \rightarrow K_S^0(DD)\pi^+$ sample unaffected. This strict condition could be relaxed in order to include also candidates in which the bachelor π^+ has not fired the HLT1 trigger, but this would require additional studies on the introduced detection asymmetries and on their cancellation.

The `Hlt1TrackMVA` algorithm looks for single long tracks with good reconstruction quality, high momentum and large IP from all the PVs. The reconstruction quality and the high momentum of the tracks is required with the following conditions:

- the long tracks must have at least 9 hits in the VELO;
- the track fit has $\chi^2/\text{ndof} < 2.5$;
- the ghost probability is small ($< 0.2 - 0.4$);
- the momentum fulfils $p > 3 - 5 \text{ GeV}/c$.

In addition, a multivariate selection is applied in the $p_T - \chi_{\text{IP}}^2$ plane, in order to select particles produced by heavy hadrons. The requirement is given by (4.1), with the momenta in GeV/c and α a parameter which has been changed during the data acquisition between 1.1, 1.6 and 2.3. The region on the $p_T - \chi_{\text{IP}}^2$ plane selected by this condition is shown in figure 4.3.

$$\{p_T > 25 \wedge \chi_{\text{IP}}^2 > 7.4\} \vee \left\{ [1 < p_T < 25] \wedge \left[\ln \chi_{\text{IP}}^2 > \ln 7.4 + \frac{1}{(p_T - 1.0)^2} + \alpha \left(1 - \frac{p_T}{25} \right) \right] \right\} \quad (4.1)$$

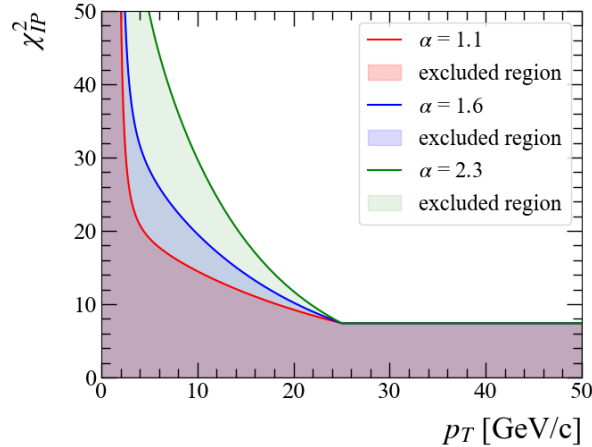


Figure 4.3: Region selected and excluded by the `Hlt1TrackMVA` multivariate request on the $p_T - \chi_{\text{IP}}^2$ plane for the different values of α used in the Run 2 data taking.

4.2.3 HLT2 selection

The lines used at the HLT2 level to select the decays of interest are:

- `Hlt2CharmHadDpToKmKpPipTurbo` for the $D^+ \rightarrow \phi\pi^+$;
- `Hlt2CharmHadDp2KS0Pip_KS0YYTurbo` for the $D^+ \rightarrow K_S^0\pi^+$, where YY stands for LL or DD in the case of long or downstream K_S^0 .

The selection requirements of these lines have been summarised in tables 4.1 and 4.2. Both lines select prompt D^+ candidates, with a mass close to the *PDG* value, a small θ_{DIRA} and a finite Decay Time from the PV. All the tracks are required to be displaced, i.e. with an high χ_{IP}^2 , and with high transverse momentum.

The `Hlt2CharmHadDpToKmKpPipTurbo` line is specifically designed to find three² good quality long tracks (Track Fit $\chi^2/\text{ndof} < 3$) which come from the same vertex, identified by a vertex fit with $\chi^2/\text{ndof} < 6$. Requirements on Particle Identification variables, to distinguish kaons and pions, are used, in order to reject residual combinatorial and physics backgrounds.

Particle	Quantity	Conditions
D^+	Mass	[1789, 1949] MeV/ c^2
	θ_{DIRA}	< 10 mrad
	Vertex Fit χ^2/ndof	< 6
	Decay Time wrt PV	> 0.4 ps
	χ_{FD}^2	—
all D^+ decay products	$p_{\text{T}}(K^+) + p_{\text{T}}(K^-) + p_{\text{T}}(\pi^+)$	> 3 GeV/ c
	p_{T}	> 250 MeV/ c
	χ_{IP}^2	> 4
2 out of 3	p_{T}	> 400 MeV/ c
1 out of 3	χ_{IP}^2	> 10
	p_{T}	> 1 GeV/ c
	χ_{IP}^2	> 50
π_{tr}^+	Track Fit χ^2/ndof	< 3
	Track ghost probability	< 0.4
	p_{T}	> 200 MeV/ c
	p	> 1 GeV/ c
	$DLL_{K\pi}$	< 5
K^+ and K^-	Track Fit χ^2/ndof	< 3
	Track ghost probability	< 0.4
	p_{T}	> 200 MeV/ c
	p	> 1 GeV/ c
	$DLL_{K\pi}$	> 5

Table 4.1: HLT2 cuts for $D^+ \rightarrow K^+K^-\pi_{tr}^+$

²To note that this line does not require the charged kaons to come from a ϕ decay, but accepts any $K^+K^-\pi^+$ final state. Charged kaons not coming from a ϕ have been discarded offline, as described in 4.3.

The `Hlt2CharmHadDp2KS0Pip_KS0YYTurbo` line looks for a good quality π^+ long track and a neutral kaon candidate, reconstructed from the intersection between two long or two downstream tracks, with a vertex fit $\chi^2 < 30/\text{ndof}$. Also in this case, the D^+ vertex fit must have a good chi-square ($\chi^2/\text{ndof} < 10$). The bachelor pion must fulfil the same PID requirement as in the `Hlt2CharmHadDpToKmKpPipTurbo` line. Regarding the neutral kaons, the requested conditions differ slightly between the $K_S^0(LL)$ and $K_S^0(DD)$ candidates, due to the different range and resolution on the interesting quantities. For both neutral kaon types, the $\pi^+\pi^-$ invariant mass is requested to be close to the *PDG* value (within $35 \text{ MeV}/c^2$ for the $K_S^0(LL)$ and $64 \text{ MeV}/c^2$ for the $K_S^0(DD)$), and the decay time must be larger, respectively, than 2 ps and 0.5 ps. The $K_S^0(LL)$ need to have the decay vertex z coordinate inside the VELO ($DV_z \in [-100, 500] \text{ mm}$), while $K_S^0(DD)$ decayed in the VELO and in the RICH1 ($DV_z \in [300, 2275] \text{ mm}$) are accepted.

Particle	Quantity	Conditions
D^+	Mass	$[1789, 2049] \text{ MeV}/c^2$
	θ_{DIRA}	$< 17.3 \text{ mrad}$
	Vertex Fit χ^2/ndof	< 10
	Decay Time wrt PV	$> 0.25 \text{ ps}$
	χ_{FD}^2	> 30
all D^+ decay products	$p_T(K_S^0) + p_T(\pi^+)$	$> 2 \text{ GeV}/c$
	χ_{IP}^2	> 36
π_{tr}^+	Track Fit χ^2/ndof	< 3
	Track ghost probability	< 0.4
	p_T	$> 200 \text{ MeV}/c$
	p	$> 1 \text{ GeV}/c$
	$DLL_{K\pi}$	< 5
$K_S^0(LL)$	$ m(\pi\pi) - m_{K_S^0}^{pdg} $	$< 35 \text{ MeV}/c^2$
	Decay Time wrt PV	$> 2 \text{ ps}$
	DV_z	$[-100, 500] \text{ mm}$
	Vertex Fit χ^2/ndof	< 30
$K_S^0(LL)$ decay products	Track Fit χ^2/ndof	< 3
	χ_{IP}^2	> 36
$K_S^0(DD)$	$ m(\pi\pi) - m_{K_S^0}^{pdg} $	$< 64 \text{ MeV}/c^2$
	Decay Time wrt PV	$> 0.5 \text{ ps}$
	DV distance from the PV along z	$> 400 \text{ mm}$
	DV_z	$[300, 2275] \text{ mm}$
	Vertex Fit χ^2/ndof	< 30
$K_S^0(DD)$ decay products	Track Fit χ^2/ndof	< 4
	p_T	$> 175 \text{ MeV}/c$
	p	$> 3 \text{ GeV}/c$

Table 4.2: HLT2 cuts for $D^+ \rightarrow K_S^0 \pi_{tr}^+$

4.3 Offline selections of $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ decays

In addition to the trigger selection, additional conditions are requested in each decay channel, in order to equalise the selections among all data samples, to remove large asymmetry regions and to further suppress combinatorial backgrounds, misidentified particles and secondary decays. All the offline conditions applied to the $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ samples have been summarised in tables 4.3 and 4.4. The optimisation of some of these cuts has been explained in the following paragraphs, while others have been taken from published LHCb analyses (mainly from Ref. [8] and Ref. [7]).

Conditions on $M(\pi^+ \pi^-)$ and $M(K^+ K^-)$

The HLT2 line used to select $D^+ \rightarrow \phi \pi^+$ candidates looks for a non-resonant charged kaons pair in the final state. Therefore, only candidates with kaons produced in the $\phi \rightarrow K^+ K^-$ decay have been selected, by requiring their invariant mass to be within $10 \text{ MeV}/c^2$ from the *PDG* ϕ mass. Furthermore, to additionally reduce combinatorial background in the $D^+ \rightarrow K_S^0 \pi^+$ samples, the HLT2 conditions on the invariant mass of the pions coming from a K_S^0 have been tightened, as shown in figure 4.4.

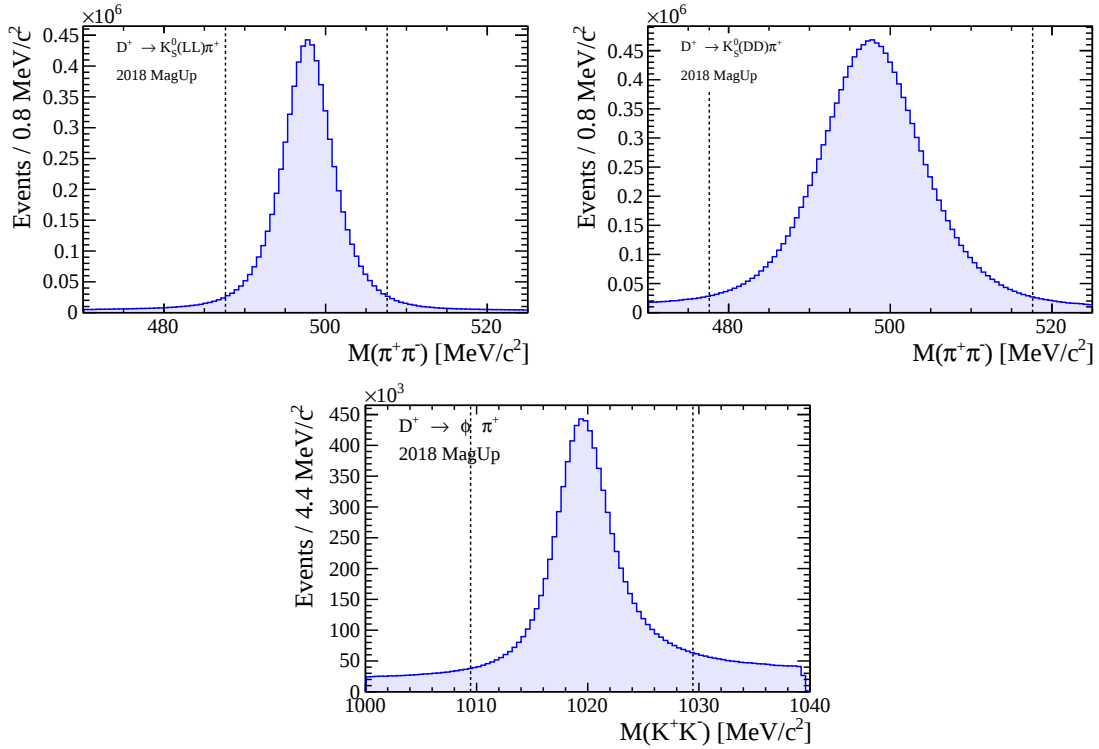


Figure 4.4: Distribution of $M(\pi^+ \pi^-)$ around the K_S^0 resonance (top) and of $M(K^+ K^-)$ around the ϕ resonance (bottom). The vertical lines show the cut around the $M_{K_S^0}$ and M_ϕ mass.

Equalisation of the kinematic variables edges

In order to equalise the selections among all data samples, some additional cuts on the kinematic variables of the D^+ and of the bachelor π^+ have been applied. In particular, the one that determines the bigger loss in statistics in the $K_S^0(LL)$ sample is the request $p_T(D^+) > 2.8 \text{ GeV}/c^2$, necessary

to equalise the $p_T(D^+)$ threshold of the $D^+ \rightarrow K_S^0 \pi^+$ sample to the one of the $D^+ \rightarrow \phi \pi^+$ sample, as shown in the upper right subplot of figure 4.5.

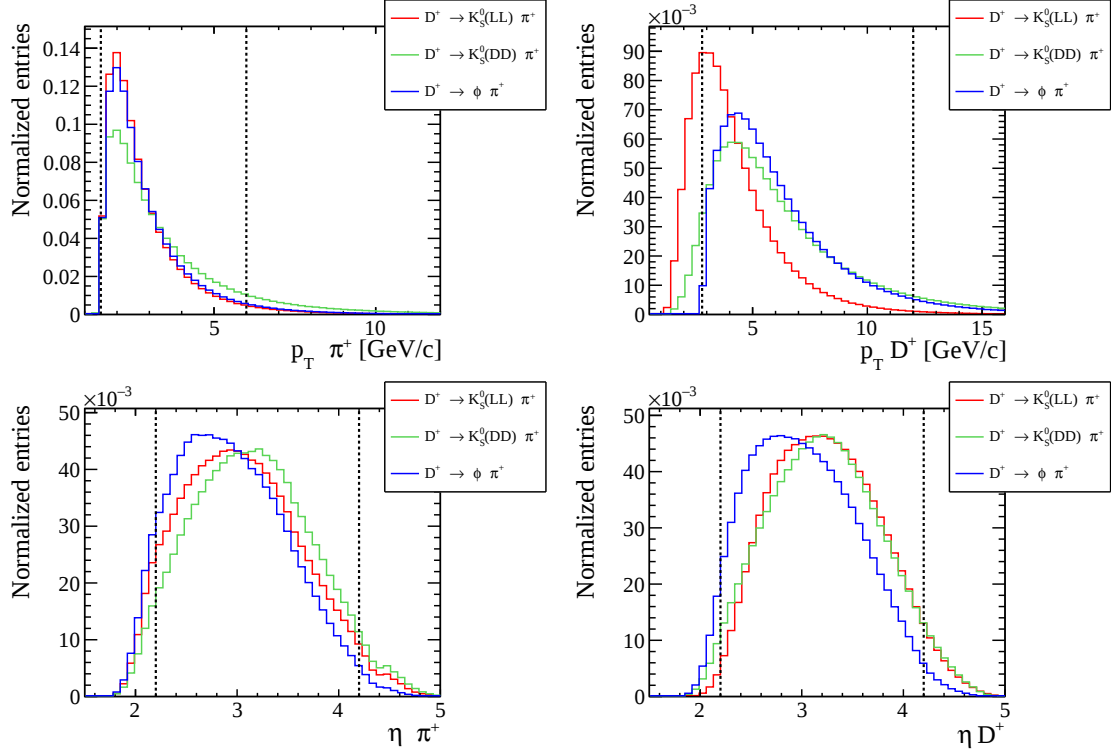


Figure 4.5: Kinematic variables distributions in the $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ samples. The dashed vertical lines represent the edges chosen for the analysis.

Fiducial cuts on the bachelor $\pi^\pm$ kinematic variables

Additional cuts on the bachelor $\pi^\pm$ kinematics are needed to exclude high detection asymmetry regions. In fact, the magnetic field bends pions of different charge in opposite directions. If we consider low momentum pions produced at large angles in the xz plane, one charge is much more likely to remain within the LHCb sub-detectors acceptance than the other. This effect gives rise to a large π^+ detection asymmetry in certain kinematic regions, in which the assumption that the total *raw* asymmetry can be written as the sum of nuisance and CP effects, as in (2.8), is not valid. For this reason, the problematic kinematic regions, in which the detection asymmetries are higher than a few tens percent, have been removed, so that the linear approximation in (2.8) is valid up to $\mathcal{O}(10^{-4})$.

In figure 4.6 it is possible to observe the distribution of the *raw* asymmetry in the $D^+ \rightarrow K_S^0(LL)\pi^+$ channel in the $\theta_x(\pi^+) - \theta_y(\pi^+)$ and $\theta_x(\pi^+) - k(\pi^+)$ planes for the *MagUp* and *MagDown* data samples. It is clear that these effects are mainly due to the bending of charged tracks in the magnetic field, because the change in polarity translates into a change of sign of the *raw* asymmetry in all the kinematic regions. From these plots it is possible to identify the regions with large $A_D(\pi^+)$:

- large asymmetry region for small momentum (large k) and large θ_x particles in the θ_x, k space, in which the $\pi^\pm$ are bent in a dead region of the detector:

$$\text{DeadRegion} = k[\text{GeV}^{-1}] > 0.3 - |\theta_x|,$$

- large asymmetry region close to the origin in the θ_x, θ_y plane, in which the $\pi^\pm$ are bent into the beampipe close to the interaction region:

$$\text{BeamPipeOrigin} = \left(\frac{\theta_x}{0.027} \right)^2 + \left(\frac{\theta_y}{0.017} \right)^2 < 1,$$

- large asymmetry regions for small θ_y values, in which the $\pi^\pm$ are bent into the conical beampipe inside the RICH detector:

$$\text{BeamPipeVertical}_1 = |\theta_y| < 0.001$$

$$\text{BeamPipeVertical}_2 = |\theta_y| < 0.005 \text{ and } 0.06 < |\theta_x| < 0.1.$$

The fiducial cuts on the $\pi^\pm$ kinematics can then be summarised as

$$\begin{aligned} \text{FidCuts} = & \text{!DeadRegion} \wedge \\ & \text{! (BeamPipeOrigin} \vee \text{BeamPipeVertical}_1 \vee \text{BeamPipeVertical}_2), \end{aligned} \quad (4.2)$$

and their effect on the removal of the large asymmetry regions can be seen in figure 4.7.

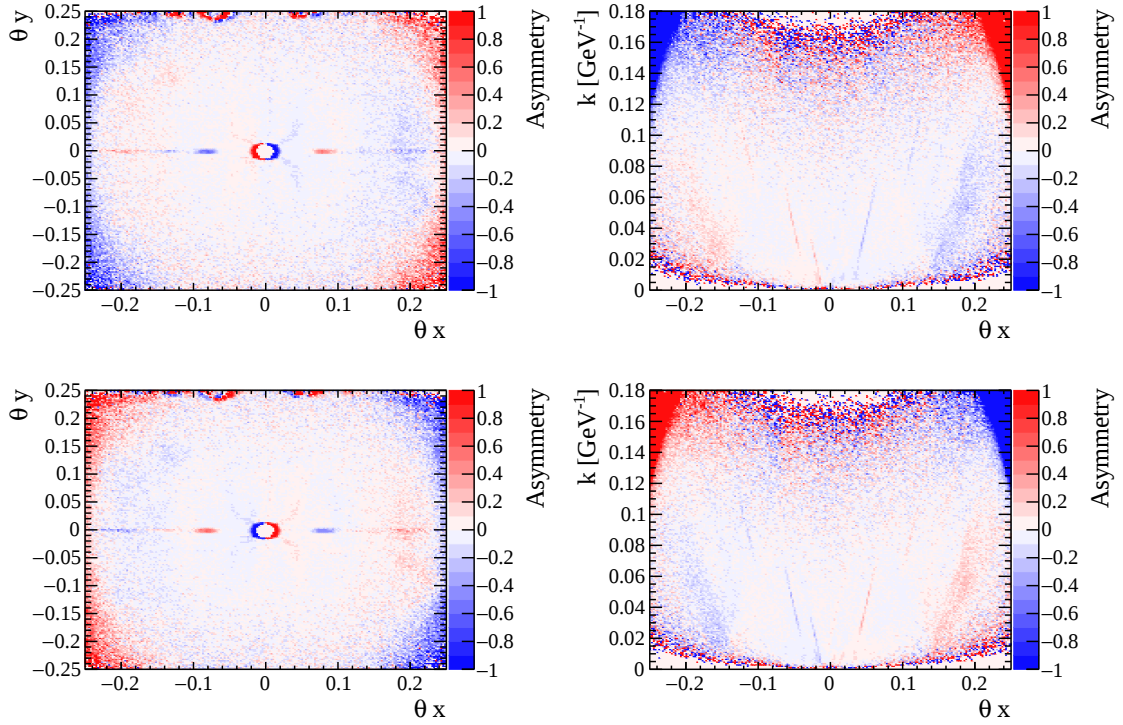


Figure 4.6: Distribution of $A_{raw}(D^+ \rightarrow K_S^0(LL)\pi^+)$ in the $\theta_x(\pi^+) - \theta_y(\pi^+)$ (top) and $\theta_x(\pi^+) - k(\pi^+)$ (bottom) plane for the *MagUp* (left) and *MagDown* (right) data samples before the application of fiducial cuts.

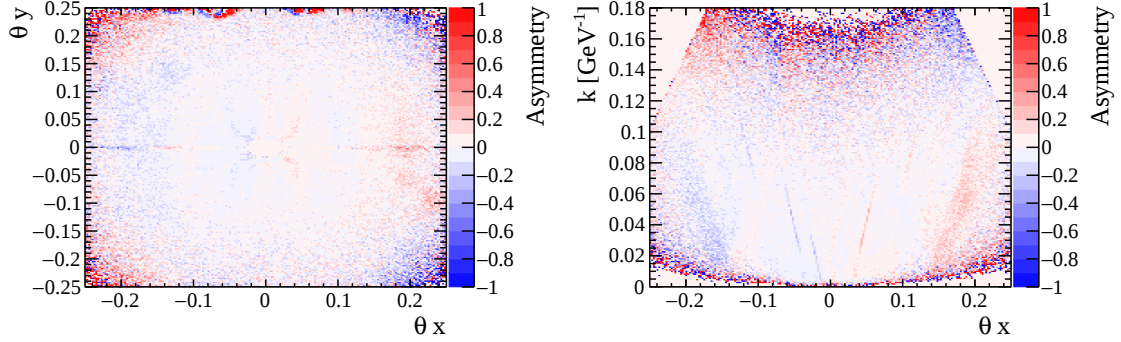


Figure 4.7: Distribution of the *raw* asymmetry in the $D^+ \rightarrow K_S^0(LL)\pi^+$ sample in the $\theta_x(\pi^+) - \theta_y(\pi^+)$ (left) and in the $\theta_x(\pi^+) - k(\pi^+)$ (right) space after the fiducial cuts in (4.2), for the *MagDown* data samples.

Cut on the D^+ impact parameter

When considering D^+ *prompt* decays produced in the primary vertex of pp interactions, candidates where the D^+ was produced in the decay of a B meson constitute a relevant background and are named *secondary* decays. Such background candidates are given by a true D^+ meson, thus their invariant mass falls in the M_{D^+} peak, but they have a different production asymmetry³ than the prompt decays. Therefore, the contamination from secondary decays introduces a bias in the *raw* asymmetries, as discussed and quantified in 5.4.

A useful quantity to reject secondary backgrounds is the impact parameter (IP) of the D^+ , which is null, within the experimental resolution, for a prompt particle and on average non-null for secondary particles. A cut in the form $\text{IP}(D^+) < \text{IP}_{\text{thr}}$ can both reduce the fraction of secondary decays and the combinatorial background, discarding candidates given by the association of random tracks not coming from the D^+ decay vertex.

However, the distribution of $\text{IP}(D^+)$ varies significantly in the ϕ , $K_S^0(LL)$, and $K_S^0(DD)$ samples, as shown in figure 4.8. In particular, while in the $D^+ \rightarrow \phi\pi^+$ and $D^+ \rightarrow K_S^0(LL)\pi^+$ samples it is possible to identify the prompt peak and secondary tail at large $\text{IP}(D^+)$ values, in the $D^+ \rightarrow K_S^0(DD)\pi^+$ the resolution on $\text{IP}(D^+)$ is worse and the two populations are indistinguishable.

Regarding the $D^+ \rightarrow K_S^0(LL)\pi^+$ sample, the $\text{IP}(D^+)$ cut has been optimised to minimise the secondary fraction and to select the prompt region. The $\text{IP}(D^+)$ threshold has been set to $100\ \mu\text{m}$, where the prompt peak and the secondary tail become distinct (4.8). On the contrary, for the $D^+ \rightarrow K_S^0(DD)\pi^+$ sample, it is impossible to distinguish the secondary component, thus the $\text{IP}(D^+)$ cut has been optimised to maximise the significance $\mathfrak{z}$, defined as

$$\mathfrak{z} = \frac{S}{\sqrt{S+B}}, \quad (4.3)$$

where S and B are the signal and background yields in the $D^+ \rightarrow K_S^0(DD)\pi^+$ sample, estimated from the distribution of the invariant mass $m(K_S^0\pi^+)$ around the D^+ peak (figure 4.8).

Regarding the sample of $D^+ \rightarrow \phi\pi^+$ candidates, it is necessary to set a requirement on the

³The production asymmetry of secondary D^+ mesons is due mainly to the different production cross sections of $\bar{B}$ and B mesons in pp collisions. $A_P(\bar{B}^0)$ and $A_P(B^-)$ have been measured by the LHCb collaboration [65, 66] and they are in general different than the production asymmetries of prompt D^+ mesons because of the different strong processes at play.

IP(D^+) observable, capable to reproduce an effect, on secondaries, as similar as possible of the requirement utilised for the $D^+ \rightarrow K_S^0 \pi^+$, both for LL and DD K_S^0 candidates. Since the ϕ meson in the final state is intrinsically different from the long-lived K_S^0 meson, it is necessary to artificially emulate the K_S^0 decay length (and therefore its decay time) in the $D^+ \rightarrow \phi \pi^+$ decays. This procedure is discussed in detail in chapter 5 and it is one of the original contribution to this type of analyses developed in this thesis.

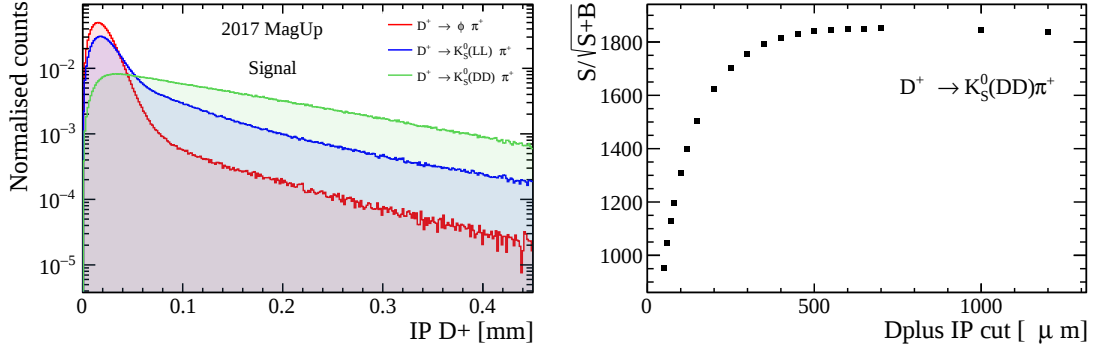


Figure 4.8: Left: distribution of the impact parameter of the D^+ , IP(D^+), in the three decays of interest. Right: dependence of the significance s on the IP(D^+) cut IP_{thr} for $D^+ \rightarrow K_S^0(DD) \pi^+$ decays. The chosen IP(D^+) threshold for the $K_S^0(DD)$ sample is 350 μm , value for which s reaches a *plateau*.

Particle	Quantity	Conditions
Trigger	L0	LOGlobal TIS on D^+ or L0Hadron TOS on ϕ
	HLT1	Hlt1TrackMVA TOS on π^+
D^+	η	[2.2, 4.2]
	p_T	[2.8, 12] GeV/ c
	k	[0.005, 0.06] (GeV/ c) ⁻¹
	IP	see section 5.4
π^+	$DLL_{K\pi}$	< 5
	$DLL_{p\pi}$	< 5
	η	[2.2, 4.2]
	p_T	[2.8, 12] GeV/ c
	k	[0.005, 0.06] (GeV/ c) ⁻¹
	θ_x, θ_y, k	fiducial cuts (4.2)
ϕ	$ m(KK) - m_\phi^{pdg} $	< 10 MeV/ c^2

Table 4.3: Offline cuts for $D^+ \rightarrow \phi \pi^+$

Particle	Quantity	Conditions
Trigger	L0 HLT1	LOGlobal TIS on D^+ or L0Hadron TOS on K_S^0 Hlt1TrackMVA TOS on π^+
D^+	η p_T k IP	[2.2, 4.2] [1.5, 6] GeV/ c [0.01, 0.12] (GeV/ c) $^{-1}$ $\leq 100 \mu\text{m}$ (LL) $\leq 350 \mu\text{m}$ (DD)
π^+	$DLL_{K\pi}$ $DLL_{p\pi}$ η p_T k θ_x, θ_y, k	< 5 < 5 [2.2, 4.2] [2.8, 12] GeV/ c [0.005, 0.06] (GeV/ c) $^{-1}$ fiducial cuts (4.2)
$K_S^0(LL)$	$ m(\pi\pi) - m_{K_S^0}^{pdg} $ η $t/\tau_{K_S^0}$ FD wrt D^+ SV	$< 10 \text{ MeV}/c^2$ [2.2, 4.2] [0, 0.4] $> 20 \text{ mm}$
$K_S^0(DD)$	$ m(\pi\pi) - m_{K_S^0}^{pdg} $ η $t/\tau_{K_S^0}$	$< 20 \text{ MeV}/c^2$ [2.2, 4.2] [0, 3]

Table 4.4: Offline cuts for $D^+ \rightarrow K_S^0 \pi^+$

4.4 Total signal yields

After the application of all online and offline requirements, combinatorial background is still present in all data samples, as shown in the distribution of the invariant mass of the $K_S^0 \pi^\pm$ or $\phi \pi^\pm$ pairs around the $D^\pm$ mass, M_{inv} (figure 4.9). In order to estimate the signal yields in each channel, the residual background under the peak can be removed with a sideband subtraction procedure. The signal region is defined as a $\sim 3\sigma$ interval around the peak, while the sideband regions as symmetric intervals with respect to the peak, as shown in figure 4.9. For this preliminary estimate, background candidates in the signal and sideband regions are assumed to behave in the same way. They are therefore removed inside the signal region, by assigning to each candidate a proper weight, used to fill all the histograms inspected in the analysis procedure. This weight depends on the value of the measured invariant mass M_{inv} and is defined in equation (4.4): it is 1 for all the candidates in the signal region, while for candidates in the sideband region it is given by the ratio of the number of background candidates in the sideband region ($N_{bkg,side}$) and the number of background candidates inside the signal region ($N_{bkg,sig}$).

$$w_s^i = \begin{cases} 1 & \text{if } M_{inv} \in \text{signal region,} \\ -\frac{N_{bkg,side}}{N_{bkg,sig}} & \text{if } M_{inv} \in \text{sideband region,} \\ 0 & \text{otherwise.} \end{cases} \quad (4.4)$$

The signal and sideband intervals are reported in table 4.5 and are determined from the width of the mass peaks in the used channels, which have different resolution on the invariant mass. The

Sample	Signal region	Left sideband	Right sideband
$D^+ \rightarrow K_S^0 \pi^+$	$m_D \pm 20 \text{ MeV}/c^2$	$[1814.66, 1834.66] \text{ MeV}/c^2$	$[1904.66, 1924.66] \text{ MeV}/c^2$
$D^+ \rightarrow \phi \pi^+$	$m_D \pm 15 \text{ MeV}/c^2$	$[1819.66, 1834.66] \text{ MeV}/c^2$	$[1904.66, 1919.66] \text{ MeV}/c^2$

Table 4.5: Signal and sideband regions for the $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ samples, all symmetric around the D^+ mass, $m_D = 1869.66 \text{ MeV}/c^2$.

combinatorial background has a linear shape to a very good approximation. The sideband regions are symmetric around the signal peak and are chosen to have the same width as the signal region. This allows to approximately have $N_{bkg,side} \simeq N_{bkg,sig}$, thus the sideband weights are equal to -1 for all the samples.

Figure 4.9 shows the distribution of the invariant mass of the three used data samples, while table 4.6 reports the total yields of signal candidates used in the analysis, divided by year and magnet polarity. We can see that the mass peaks in the $D^+ \rightarrow \phi \pi^+$ and $D^+ \rightarrow K_S^0(LL)\pi^+$ samples are extremely clean, with a background fraction in the signal region of about 5%. On the contrary, the $D^+ \rightarrow K_S^0(DD)\pi^+$ has a significantly higher combinatorial background contamination, of around 25% in the signal region.

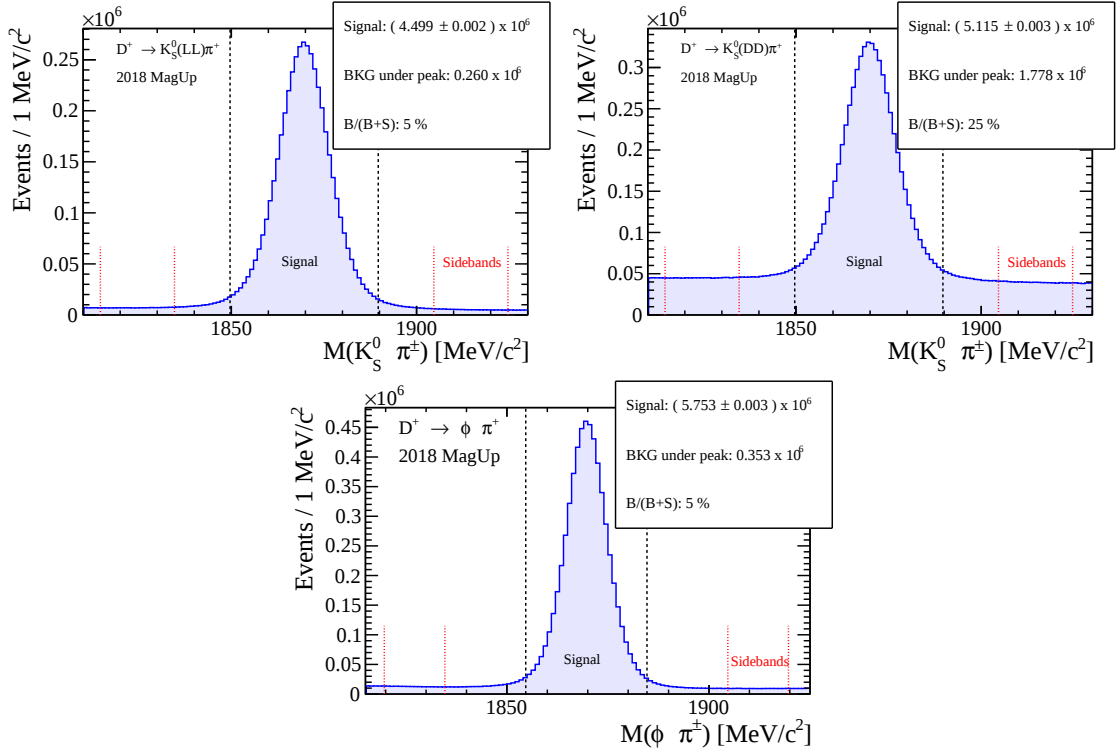


Figure 4.9: Distribution of $M(K_S^0 \pi^\pm)$ (top) and of $M(\phi \pi^\pm)$ (bottom) around the $D^\pm$ mass resonance for the 2018 MagUp dataset. In the legend, *BKG under peak* refers to the number of background candidates in the signal region $N_{bkg,sig}$, thus $B/(B+S)$ is the background fraction in the signal region.

Dataset	$D^+ \rightarrow K_S^0(LL)\pi^+$	$D^+ \rightarrow K_S^0(DD)\pi^+$	$D^+ \rightarrow \phi\pi^+$
18Up	4.5 M	5.1 M	5.8 M
18Dw	4.2 M	4.7 M	5.3 M
17Up	3.9 M	4.4 M	4.9 M
17Dw	4.0 M	4.7 M	5.1 M
16Up	3.2 M	3.6 M	3.9 M
16Dw	3.8 M	4.1 M	4.7 M
total	23.6 M	26.6 M	30.1 M

Table 4.6: Yields of the samples in the different datasets divided by year and magnet polarity. The invariant mass distributions for all datasets divided by year and magnet polarity are reported in appendix [A](#).

Chapter 5

Cancellation of nuisance asymmetries

This chapter illustrates the strategy developed to subtract the nuisance asymmetries in the $D^+ \rightarrow K_S^0 \pi^+$ sample by using the $D^+ \rightarrow \phi \pi^+$ sample as calibration. At first, the kinematic weighting procedure to equalise the momentum distributions of the D^+ and of the bachelor pion is presented. It follows a discussion on the contamination of D^+ mesons produced in bottom hadron decays, which introduce a bias related to their different production asymmetries that cannot be removed in the kinematic weighting procedure. Finally, the developed method to measure the neutral kaon detection asymmetry based on the determination of raw asymmetries in the signal and calibration sample is presented.

5.1 Nuisance asymmetries and analysis overview

The main goal of this thesis is to study the time dependence of neutral kaon detection asymmetry in charmed meson decays, specifically focusing on the $D^+ \rightarrow K_S^0 \pi^+$ channel. The strategy developed to extract the quantity of interest, $A_D(K_S^0)$, involves using the $D^+ \rightarrow \phi \pi^+$ decay as a calibration sample to cancel the nuisance asymmetries in the channel of interest. The *raw* asymmetry in the $D^+ \rightarrow K_S^0 \pi^+$ channel can be expressed, using the formalism and the approximations developed in section 2.2, as:

$$A_{raw}(D^+ \rightarrow K_S^0 \pi^+) = A_P(D^+) + A_D(\pi^+) + A_D(K_S^0), \quad (5.1)$$

while for the $D^+ \rightarrow \phi \pi^+$ channel we have:

$$A_{raw}(D^+ \rightarrow \phi \pi^+) = A_P(D^+) + A_D(\pi^+) + A_{CP}(D^+ \rightarrow \phi \pi^+). \quad (5.2)$$

The production asymmetry of the D^+ meson, measured in Ref. [67], is found to be $A_P(D^+) = (-0.96 \pm 0.26 \pm 0.18)\%$, not showing any significant dependence on the p_T and η of D^+ mesons. This slight excess in the production of the antimesons is due to the fact that $\bar{c}$ quarks might combine with the valence quark of the colliding protons, whereas the same is not true for c quarks. It is also worth mentioning that the measurement of Ref. [67] is performed using data from pp collisions at 7 TeV, while the analysis described in this thesis is based on data produced with pp collisions at 13 TeV, therefore a slightly lower production asymmetry is expected. A detailed study of all charged particle detection asymmetries, with a focus on their momentum dependence, is provided in Ref. [68]. Here, only the most significant effects are briefly overviewed. The main source of the bachelor pion detection asymmetry is the vertical dipole magnetic field, which bends oppositely charged particles in opposite directions. Since the used data samples are collected,

with similar yields, both in the *MagUp* and *MagDown* dipole magnet configurations, the detection asymmetries due to such effects are drastically reduced, to the percent level, when averaging across all years and magnet polarities. Furthermore, the phase-space regions with acceptance asymmetries larger than few tens percent are removed from the data sample applying the fiducial requirements described in 4.3. Despite all these measures, there remain residual asymmetries, on the order of a few percent, which do not depend on the dipole magnet polarity but are mainly due to tracking efficiencies and detector alignment.

Both the $A_P(D^+)$ and $A_D(\pi^+)$ terms in the raw asymmetries of the two samples depend on the momentum distributions of the D^+ and π^+ , respectively. Therefore, if the kinematic distributions of the D^+ and π^+ are equalised between the two samples, the $A_P(D^+)$ and $A_D(\pi^+)$ terms will cancel out to a very high degree of precision when taking the difference between their raw asymmetries. This cancellation is much more precise than any of the uncertainties involved. Consequently, the neutral kaon detection asymmetry can be precisely determined by calculating the difference between the raw asymmetry of the $D^+ \rightarrow K_S^0 \pi^+$ channel and that of the control sample. This difference can be expressed as:

$$\begin{aligned} \Delta A_{K_S^0} &\equiv A_{raw}(D^+ \rightarrow K_S^0 \pi^+) - A_{raw}(D^+ \rightarrow \phi \pi^+) \\ &= A_D(K_S^0) - A_{CP}(D^+ \rightarrow \phi \pi^+), \end{aligned} \quad (5.3)$$

where the last term does not depend on the K_S^0 decay time and thus has no impact on the time-dependence of $A_D(K_S^0)$.

To study the dependence of this quantity on the K_S^0 decay time, the $D^+ \rightarrow K_S^0 \pi^+$ sample is divided into 6 decay-time bins for the $K_S^0(LL)$ and 7 decay-time bins for the $K_S^0(DD)$. To cancel the nuisance asymmetries in all 13 K_S^0 decay-time bins, the $D^+ \rightarrow \phi \pi^+$ candidates are also split into 13 sub-samples, each corresponding to one K_S^0 decay-time bin. This **splitting** procedure is described in detail in section 5.2. As discussed above, it is also necessary to equalise the kinematics of the D and π mesons between the $D^+ \rightarrow \phi \pi^+$ decay channel and the $D^+ \rightarrow K_S^0 \pi^+$ decay channel. Therefore, after the splitting, an iterative **kinematic weighting** procedure is performed independently for each of the 13 sub-samples. This procedure is described in detail in section 5.3. Finally, after the splitting and kinematic weighting procedures, the **raw asymmetries** are determined independently in all 13 K_S^0 decay-time bins for the $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ samples. This is achieved through a simultaneous fit to the invariant mass distributions of the positively and negatively charged D meson candidates, as detailed in section 5.5.

The output of this procedure is the observable $\Delta A_{K_S^0}(t)$ for each of the 13 K_S^0 decay-time bins, computed as the difference between the raw asymmetries of the corresponding K_S^0 and ϕ sub-samples, as defined in equation (5.3). This quantity is used to constrain the models of the time dependence of the neutral kaon asymmetry in LHCb, to measure $A_{CP}(D^+ \rightarrow \phi \pi^+)$ and to determine the CF-DCS interference parameters in the $D^+ \rightarrow K_S^0 \pi^+$ decay, r_π and δ_π , as presented in chapter 6.

5.2 Splitting into K_S^0 decay-time bins

In order to perform a time-dependent analysis of the neutral kaon detection asymmetry, the $D^+ \rightarrow K_S^0 \pi^+$ sample is split into independent sub-samples based on different K_S^0 decay-time bins. Specifically, the events are divided into 13 sub-samples, 6 for the $D^+ \rightarrow K_S^0(LL)\pi^+$ sample and 7 for the $D^+ \rightarrow K_S^0(DD)\pi^+$ sample, accordingly to the measured value of $t/\tau_{K_S^0}$. The not uniform binning scheme is determined by the decay-time distributions of the $K_S^0(LL)$ and $K_S^0(DD)$ candidates, shown in figure 5.1: $t/\tau_{K_S^0} \in [0.00, 0.05, 0.10, 0.15, 0.20, 0.30, 0.40]$ for long decays and

$t/\tau_{K_S^0} \in [0.0, 0.4, 0.6, 1.0, 1.4, 1.8, 2.2, 3.0]$ for downstream decays. The kinematic variables of interest, i.e. the 3D momenta of the D^+ and of the π^+ , are correlated with the K_S^0 decay time, by means of the K_S^0 momentum. As an example, figure 5.2 shows how the distributions of p_T and η of the D^+ and π^+ , in the $D^+ \rightarrow K_S^0 \pi^+$ sample, significantly vary as a function of the different K_S^0 decay-time bins. In order to ensure a precise cancellation of the nuisance asymmetries, it is

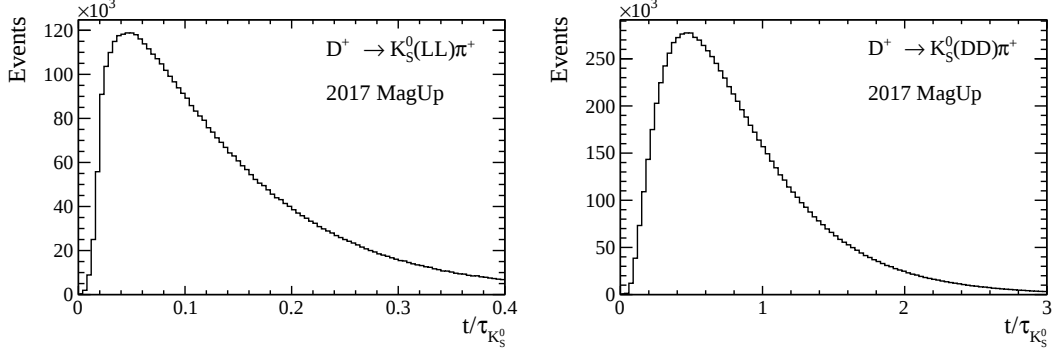


Figure 5.1: Decay time distributions, in units of the K_S^0 lifetime, $\tau_{K_S^0}$, for the long (on the left) and downstream (on the right) samples.

crucial that the $D^+ \rightarrow \phi \pi^+$ calibration sample is also split into 13 sub-samples, by assigning to each event a random K_S^0 decay-time bin, between 1 and 13. In order to exploit the full statistical power of the $D^+ \rightarrow \phi \pi^+$ sample, the assignment of the random decay-time bin is performed trying to reproduce some of the correlations between the decay-time bin and the kinematic distributions observed in the $D^+ \rightarrow K_S^0 \pi^+$ sample. The assignment of the random decay-time bin proceeds with the following steps:

1. a multidimensional histogram, $H^b(\vec{k})$, is filled with some of the kinematic variables, $\vec{k}$, for each decay-time bin, b , of the $D^+ \rightarrow K_S^0 \pi^+$ sample (combinatorial background is subtracted). The three chosen kinematic variables are $\vec{k} = (\eta(D^+), \eta(\pi^+), p_T(D^+))$, as these variables have the most different distributions between the $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ samples, as shown in figure 4.5;
2. given a candidate i in the $D^+ \rightarrow \phi \pi^+$ sample with kinematic variables equal to $\vec{k}^i$, the random decay-time bin b is randomly extracted from a discrete distribution, taking values $\in [1, 13]$. The probability for the candidate i to get assigned to the decay-time b is given by

$$p(b|\vec{k}^i) = \frac{H^b(\vec{k}^i)}{\sum_{b'} H^{b'}(\vec{k}^i)}.$$

These two steps can be heuristically understood, for example, by considering the extreme scenario in which a specific corner of the kinematic variables phase-space in the $D^+ \rightarrow K_S^0 \pi^+$ is exclusively populated in the decay-time bin $b = 1$. In this case, any $D^+ \rightarrow \phi \pi^+$ candidate that falls into the same phase-space corner is automatically assigned to decay-time bin $b = 1$. A different splitting procedure might assign this $D^+ \rightarrow \phi \pi^+$ candidate to a different decay-time bin $b \neq 1$, resulting in a loss of statistical power. This is because the weighting procedure described in subsection 5.3 would weight this candidate to 0, as the kinematic region it falls into would be empty in the $D^+ \rightarrow K_S^0 \pi^+$ bin $b \neq 1$.

Even in less extreme cases, this splitting procedures ensures that the initial kinematic distributions $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ samples already agree quite well in each decay-time bin. This

alignment simplifies the weighting procedure described in subsection 5.3.

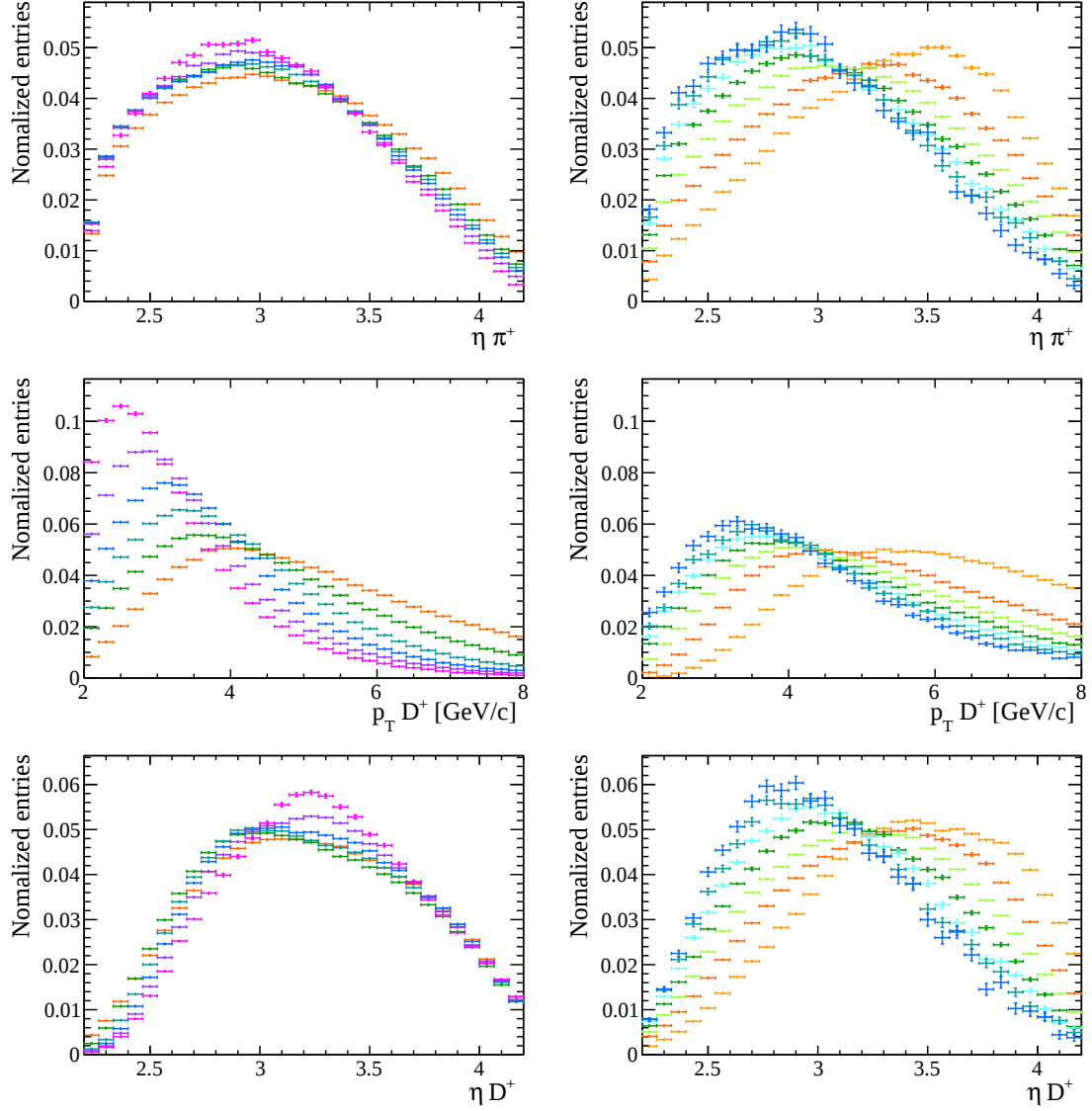


Figure 5.2: Distributions of $\eta(D^+)$, $\eta(\pi^+)$, and $p_T(D^+)$ variables for the $D^+ \rightarrow K_S^0 \pi^+$ sample in the different long (left) and downstream (right) K_S^0 decay-time bins. These distributions are reported for mere illustration purposes and they are chosen among all the kinematic variables because they have the greatest variations in the different K_S^0 decay-time bins.

Legend: t/τ bins for $K_S^0(LL)$

- t/τ in $[0.00, 0.05]$
- t/τ in $[0.05, 0.10]$
- t/τ in $[0.10, 0.15]$
- t/τ in $[0.15, 0.20]$
- t/τ in $[0.20, 0.30]$
- t/τ in $[0.30, 0.40]$

Legend: t/τ bins for $K_S^0(DD)$

- t/τ in $[0.0, 0.4]$
- t/τ in $[0.4, 0.6]$
- t/τ in $[0.6, 1.0]$
- t/τ in $[1.0, 1.4]$
- t/τ in $[1.4, 1.8]$
- t/τ in $[1.8, 2.2]$
- t/τ in $[2.2, 3.0]$

5.3 3D kinematic weighting

After the division into 13 independent sub-samples, a kinematic weighting procedure is applied in each sub-sample (or bin), with the aim of equalising the 3D momentum distributions of the D^+ and of the bachelor π^+ of the $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ candidates. The optimal option would be to perform a simultaneous 6D weighting in the space identified by the three-dimensional momenta of the D^+ meson and of the bachelor pion. However, this is not very practical even if the size of the available data sample is large. Therefore, an iterative procedure of multiple and sequential 3D weighting is chosen. A simple binned approach is used, instead of more sophisticated ones based on Multivariate or Machine Learning tools, in order to have full control of the procedure. It is worth mentioning that this is not a complex task, so there was no need to use much more powerful tools which, while increasing the quality of the result, would have made the quantification of possible systematic effects more difficult. Two sets of variables describing the 3D momentum, related to the components p_x, p_y, p_z , have been explored and used in the weighting procedure: (p_T, η, ϕ) and (k, θ_x, θ_y) , defined in 4.1. Therefore, the total number of kinematic variables equalised is 12, 6 for the π^+ momentum and 6 for the D^+ momentum. They are strongly correlated as three kinematic variables of a given particle uniquely determine the other three variables of the same particle, however this is not a problem for the purpose of a weighting. The first set is well suited to model effects due to the production mechanism of the D mesons, while the second probe with higher granularity all the corners of the phase space where detector-induced and acceptance effects are present.

The iterative kinematic weighting procedure is composed of several subsequent stages, in which the 3D distribution of a chosen triplet of kinematic variables is equalised between the two samples, as described below. In each step, a different triplet of variables, extracted from the 12 momenta variables, 6 for the π^+ and 6 for the D^+ , is equalised. The total procedure requires six subsequent steps and the triplets equalised in each step are:

- | | |
|--------------------------------------|--|
| 1. $k(\pi^+), p_T(D^+), \eta(D^+)$ | 4. $p_T(\pi^+), p_T(D^+), \eta(D^+)$ |
| 2. $p_T(\pi^+), \eta(\pi^+), k(D^+)$ | 5. $k(\pi^+), k(D^+), \phi(D^+)$ |
| 3. $\phi(\pi^+), k(\pi^+), k(D^+)$ | 6. $p_T(\pi^+), p_T(D^+), \eta(D^+)$. |

Each step of the kinematic weighting procedure consists in the equalisation of the 3D background-subtracted distributions in three chosen kinematic variables of a $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ sub-sample. This equalisation is done by assigning to each event of the $D^+ \rightarrow \phi \pi^+$ sample a weight w^i , which is used to fill all the histograms of the analysis. Assuming that the n -th step is aimed at equalising three kinematic variables, $\vec{v} = (v_1, v_2, v_3)$, the weight $w_{(n)}^i$ for a $D^+ \rightarrow \phi \pi^+$ event i , with measured kinematic variables $\vec{v}^i = (v_1^i, v_2^i, v_3^i)$, is defined as:

$$w_{(n)}^i = w_{(n-1)}^i \cdot \frac{h_{K_S^0}^{(n-1)}(\vec{v}^i)}{h_{\phi}^{(n-1)}(\vec{v}^i)}, \quad (5.4)$$

where:

- $w_{(n-1)}^i$ is the weight assigned to the candidate i of the $D^+ \rightarrow \phi \pi^+$ data sample in the $(n-1)$ -th step of the kinematic weighting (the initial weights are those from the sideband-subtraction procedure, defined in (4.4));

- $h_{K_S^0}^{(n-1)}(\vec{v}^i)$ is the content of the normalised 3D $\vec{v}$ histogram of the $D^+ \rightarrow K_S^0 \pi^+$ sample, filled with weights $w_{(n-1)}^i$, in the bin that contains the $\vec{v}^i$ values;
- $h_{\phi}^{(n-1)}(\vec{v}^i)$ is the content of the normalised 3D $\vec{v}$ histogram of the $D^+ \rightarrow \phi \pi^+$ sample, filled with weights $w_{(n-1)}^i$, in the bin that contains the $\vec{v}^i$ values;
- any bin in which at least one of the decay channels, i.e. $D^+ \rightarrow \phi \pi^+$ or $D^+ \rightarrow K_S^0 \pi^+$, has less than 10 candidates is excluded by setting its weight to zero, in order to avoid large statistical fluctuations in the kinematic weights.

In each step, to ensure the success of the kinematic weighting procedure, *acceptance* weights are assigned to $D^+ \rightarrow K_S^0 \pi^+$ candidates: these weights are 0 for all the candidates falling into a 3D space bin with weight zero and 1 otherwise. The kinematic weighting procedure is stopped when all the kinematic variables distributions are in agreement, i.e. when the ratio between the normalised histograms in each kinematic variable of the $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ sample is compatible with 1, within the statistical uncertainties, in all the bins.

In each kinematic weighting step, the assignation of weights different than 1 to the candidates determines a loss in statistical power, which can be quantified for a data sample as:

$$S_w = \frac{\sum w_i^2}{(\sum w_i)^2}. \quad (5.5)$$

This quantity is an estimate of the variance of the *raw* asymmetry in that sample after the kinematic weighting step. In the ideal case in which all the weights are equal to 1 and there is no background $S_w = 1/N$, with N the number of signal candidates in the sample. For this reason, $1/S_w$ can be interpreted as an effective number of signal candidates, which takes into account the presence of combinatorial background, after the kinematic weighting. The percentage loss of statistical power between the initial weights and the final weights can then be defined as

$$R_w = \frac{1/S_w^{(rew)}}{1/S_w^{(init)}}. \quad (5.6)$$

For illustration purposes, figures 5.4 and 5.5 show the distributions of the $\eta(D^+, \pi^+)$ and $p_T(D^+, \pi^+)$ in different K_S^0 decay-time bins before and after the kinematic weighting. The reported figures are produced only using the 2017 MagUp data sample, but the result is similar in all the other Run 2 datasets. The percentage loss of statistical power due to the iterative bin-by-bin kinematic weighting procedure for the 2017 MagUp sample is reported in table 5.1. More plots and tables evaluating the performance of the weighting procedure can be found in appendix B.

Sample	$\sum w_i^{(init)}$	$1/S_w^{(init)}$	$\sum w_i^{(rew)}$	$1/S_w^{(rew)}$	R_w
$D^+ \rightarrow \phi \pi^+$	4.50	3.52	3.96	2.20	0.63
$D^+ \rightarrow K_S^0(LL)\pi^+$	3.51	3.17	1.90	1.75	0.55
$D^+ \rightarrow K_S^0(DD)\pi^+$	4.03	2.53	2.90	2.04	0.81

Table 5.1: Statistics loss in the iterative kinematic weighting procedure for 2017 MagUp sample. In the first two columns, the quantities are computed using the sideband subtraction weights. All the reported quantities are in units of 10^6 .

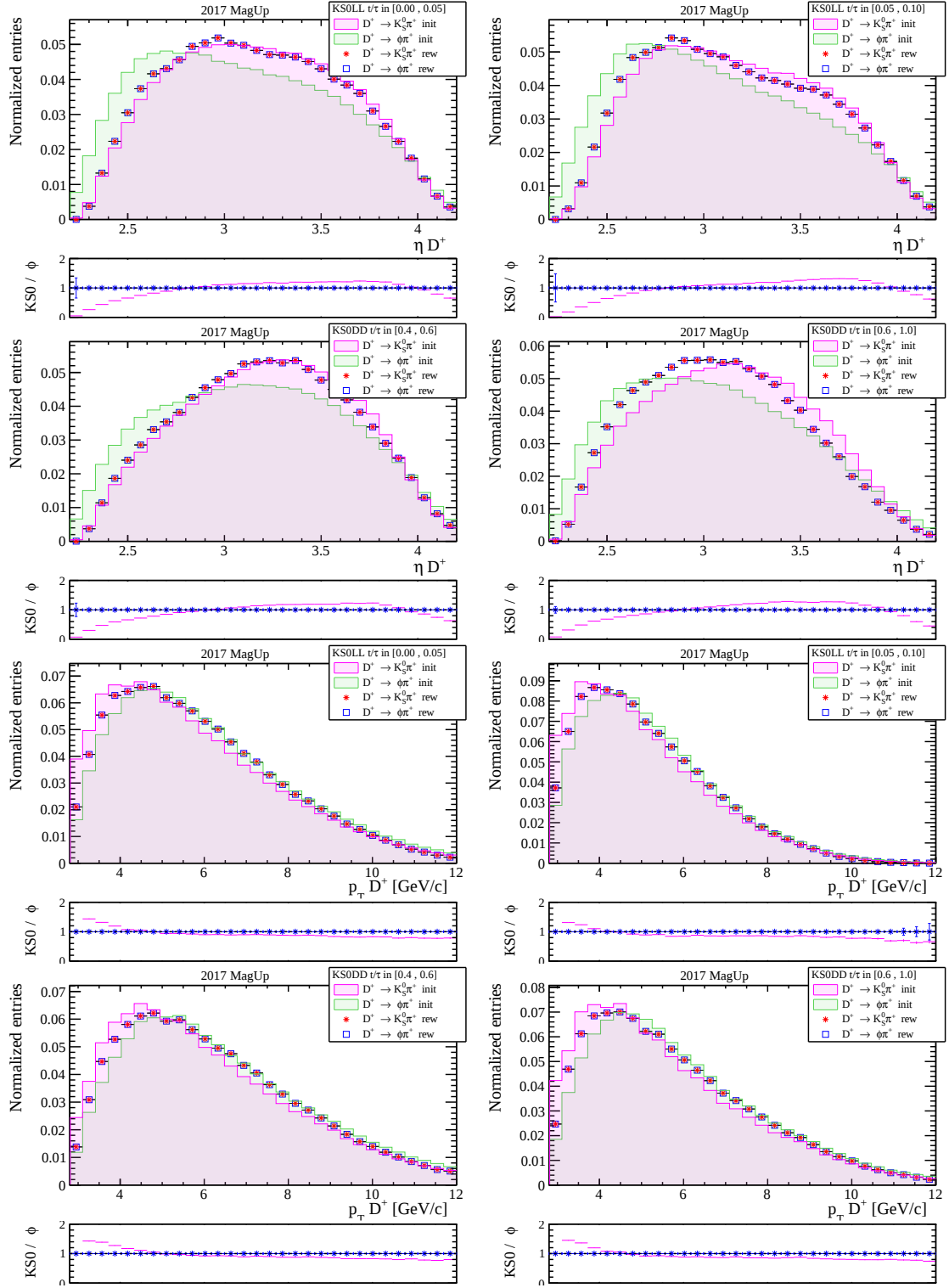


Figure 5.4: Distributions of the kinematic variable $\eta(D^+)$ and $p_T(D^+)$ in some of the K_S^0 decay-time bins before and after the kinematic weighting. The initial histograms have been filled with sideband subtraction weights. On the bottom part of each plot, the ratio between the ϕ and K_S^0 histograms is shown before (pink line) and after (blue line) the weighting procedure.

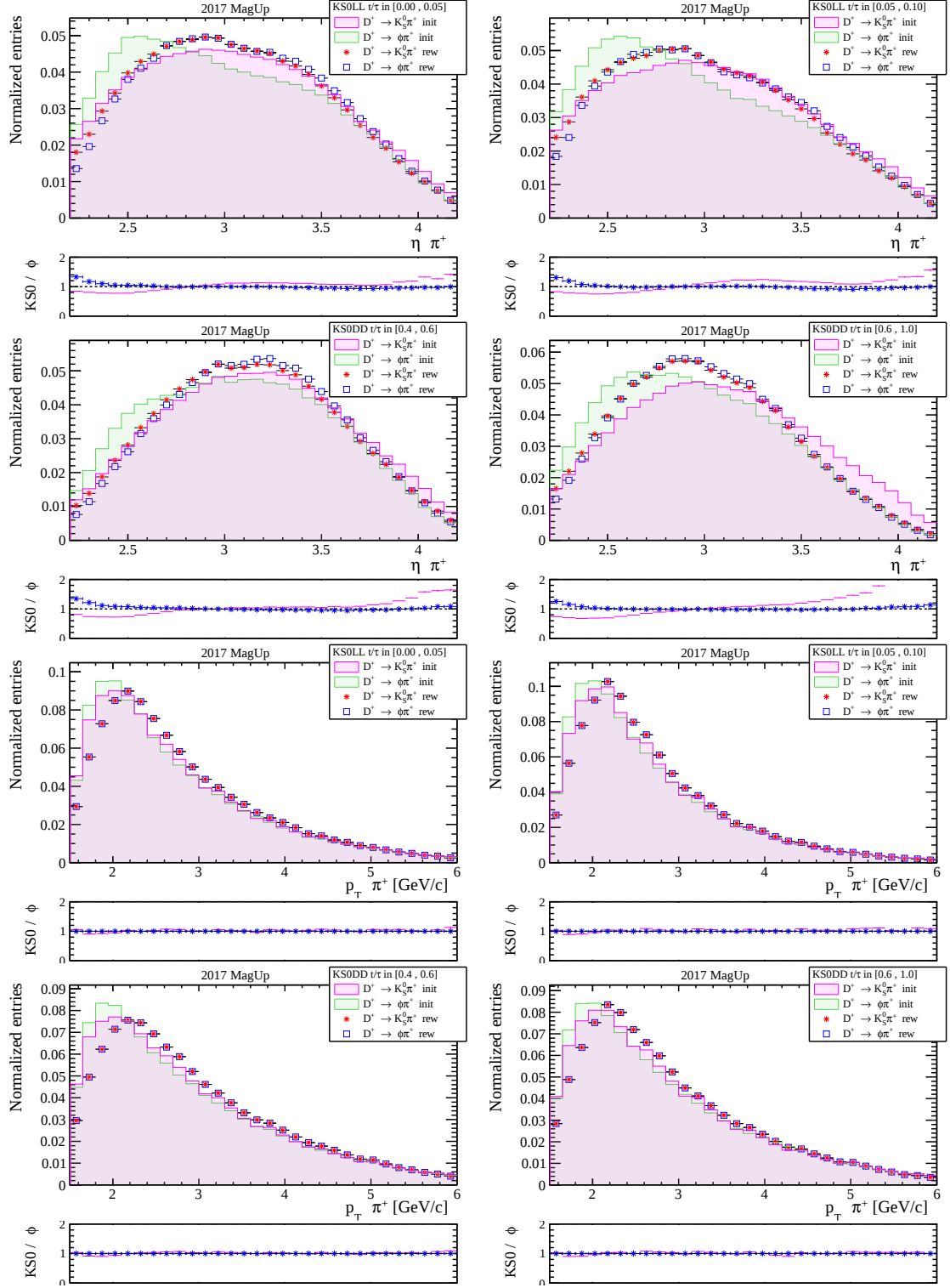


Figure 5.5: Distributions of the kinematic variable $\eta(\pi^+)$ and $p_T(\pi^+)$ in some of the K_S^0 decay-time bins before and after the kinematic weighting. The initial histograms have been filled with sideband subtraction weights. On the bottom part of each plot, the ratio between the ϕ and K_S^0 histograms is shown before (pink line) and after (blue line) the weighting procedure.

5.4 Contamination from D mesons produced in B decays

The developed strategy to measure $A_D(K_S^0)$ and $A_{CP}(D^+ \rightarrow \phi\pi^+)$ relies on the assumption that, after the equalisation of the D^+ and π^+ kinematics, all the nuisance asymmetries cancel out when computing the difference $A_{raw}(D^+ \rightarrow K_S^0\pi^+) - A_{raw}(D^+ \rightarrow \phi\pi^+)$ in each K_S^0 decay-time bin. Thanks to the sideband subtraction procedure at the beginning of the weighting, the contribution of the combinatorial background to the kinematic variables distributions is negligible, meaning that the equalisation concerns only the signal events. However, as stated in section 4.3, the data samples contain another type of background: secondary D^+ mesons originating from a B meson decay. These background events comprise true D^+ mesons, thus their invariant mass falls in the M_{D^+} peak, but they exhibit different production asymmetries compared to prompt decays.

In particular, the production asymmetry of secondary D^+ mesons is primarily due to the different production cross sections of $\bar{B}$ and B mesons in pp collisions. $A_P(\bar{B}^0)$ and $A_P(B^-)$, defined in analogy with $A_P(D^+)$ in (2.3), have been measured by the LHCb collaboration [65, 66] and they are in general different than the production asymmetries of prompt D^+ mesons. As a result, the contamination from secondary decays introduces a bias in the $\Delta A_{K_S^0}$ observable, in a given K_S^0 decay-time bin. This bias can be estimated as follows:

$$\begin{aligned}\Delta_{sec} &= A_P(D^+ \rightarrow K_S^0\pi^+) - A_P(D^+ \rightarrow \phi\pi^+) \\ &\approx \left[\left(1 - f_{sec}^{K_S^0}\right) A_P(D^+) + f_{sec}^{K_S^0} A_P(B) \right] - \left[\left(1 - f_{sec}^\phi\right) A_P(D^+) + f_{sec}^\phi A_P(B) \right] \\ &\approx (A_P(B) - A_P(D^+)) \cdot (f_{sec}^{K_S^0} - f_{sec}^\phi),\end{aligned}\quad (5.7)$$

where $f_{sec}^{K_S^0}$ and f_{sec}^ϕ are the secondary fractions in the K_S^0 and ϕ samples in the considered K_S^0 decay-time bin and $A_P(B)$ is the production asymmetry of B mesons in pp collisions. Since $A_P(B) \neq A_P(D^+)$, this bias can be reduced by minimising the difference $(f_{sec}^{K_S^0} - f_{sec}^\phi)$ in each K_S^0 decay-time bin.

The background from secondary D^+ mesons can be identified using the distribution of the impact parameter of the D^+ , $IP(D^+)$, which is on average nonzero for secondary D^+ mesons. However, the distribution of the $IP(D^+)$ varies significantly in the $K_S^0(LL)$ and $K_S^0(DD)$ samples, as shown in figure 4.8. In particular, while the prompt peak and secondary tail at high $IP(D^+)$ values can be distinguished in the $D^+ \rightarrow \phi\pi^+$ and $D^+ \rightarrow K_S^0(LL)\pi^+$ channels, the resolution on the $IP(D^+)$ is worse in the $D^+ \rightarrow K_S^0(DD)\pi^+$ channel, making the two populations indistinguishable.

The requirement on the $IP(D^+)$ in the $D^+ \rightarrow K_S^0\pi^+$ data sample is described in 4.3, with the optimal threshold set to $100\,\mu\text{m}$ for the long sample and $350\,\mu\text{m}$ for the downstream sample. Minimising $(f_{sec}^{K_S^0} - f_{sec}^\phi)$ in each K_S^0 decay-time bin can be achieved by applying an $IP(D^+)$ cut to the $D^+ \rightarrow \phi\pi^+$ after splitting it into different K_S^0 decay-time sub-samples. This approach accounts for the differences between the long and downstream neutral kaons:

- For the $D^+ \rightarrow \phi\pi^+$ events assigned to $K_S^0(LL)$ bins, a cut of $IP(D^+) < 100\,\mu\text{m}$ is applied. This reduces the secondary fractions in both the $K_S^0(LL)$ and ϕ samples, making their difference negligible and minimising corrections to the production asymmetries due to secondary decays. The impact of this approach and the associated systematic uncertainty are studied in detail in section 7.1.
- For the $D^+ \rightarrow \phi\pi^+$ events assigned to $K_S^0(DD)$ bins, a different strategy is required due to the high uncertainty on the $IP(D^+)$, which prevents clear separation of secondary and prompt components. The objective here is to align f_{sec}^ϕ closely with $f_{sec}^{K_S^0(DD)}$. To achieve this, the $IP(D^+)$ distribution in the $D^+ \rightarrow \phi\pi^+$ sample is artificially smeared to mimic the

$\text{IP}(D^+)$ distribution in the $D^+ \rightarrow K_S^0(DD)\pi^+$ sample. Following this adjustment, a cut on $\text{IP}(D^+)_{\text{smear}} < 350 \mu\text{m}$ is applied to the $D^+ \rightarrow \phi\pi^+$ sample to match the effect of the $\text{IP}(D^+)$ cut on the $K_S^0(DD)$ sample, ensuring $f_{\text{sec}}^\phi \simeq f_{\text{sec}}^{K_S^0(DD)}$. The systematic uncertainty related to this method is detailed in section 7.1.

5.4.1 Smearing of $\text{IP}(D^+)$ in $D^+ \rightarrow \phi\pi^+$ decays

To understand why the uncertainty on the $\text{IP}(D^+)$ variable is greater for the $D^+ \rightarrow K_S^0(DD)\pi^+$ candidates, we can examine the decay topology, represented in figure 5.6. The $\text{IP}(D^+)$ variable is determined using information from the Primary Vertex (PV), the direction of the reconstructed momentum of the D^+ ($\mathbf{p}(D^+)$), and the position of the D^+ decay vertex, referred to as the Decay Vertex (DV). The uncertainties in estimating the PV location and $\mathbf{p}(D^+)$ are comparable between $D^+ \rightarrow K_S^0\pi^+$ and $D^+ \rightarrow \phi\pi^+$ decays. However, the resolution on the DV increases with the flight distance of the K_S^0 in the final state, particularly for the downstream sample.

The uncertainty in locating the DV of the D^+ is significantly influenced by the K_S^0 flight distance, particularly for downstream decays, due to several contributing factors:

1. The distance from the DV to the decay vertex of the neutral kaon can range from 350 mm to 2400 mm for $K_S^0(DD)$ decays, as shown in figure 5.7. The DV of the D^+ is determined by intersecting the momenta of the K_S^0 and bachelor π^+ , where errors in the direction of the K_S^0 are amplified for longer flight distances, as shown in 5.6.
2. The uncertainty on the K_S^0 decay vertex depends on its flight distance. $K_S^0(DD)$ candidates are reconstructed using tracks from the $\pi^+\pi^-$ pair, originating from the TT without VELO hits. Longer flight distance indicates that the K_S^0 decayed further from the PV and closer to the TT hits, thus the uncertainty in identifying the intersection of the π^+ and π^- tracks is smaller. Conversely, shorter K_S^0 flight distances indicate that the $\pi^+\pi^-$ pair was detected farther from its origin, amplifying the error in the K_S^0 decay vertex.

Thus, the decay vertex of $K_S^0(DD)$ candidates has a bigger uncertainty for shorter flight distances, because the $\pi^+\pi^-$ can be revealed up to two meters far from their production point. When determining the decay vertex of the D^+ , the uncertainty on the K_S^0 decay vertex and direction gets amplified and this results in a significantly enlarged D^+ DV uncertainty, which determines the wide $\text{IP}(D^+)$ distribution shown in figure 4.8.

Therefore, to impose a similar criterion on the D^+ vertex position as applied in the $K_S^0(DD)$ candidates, the spatial position of the Decay Vertex (DV) for $D^+ \rightarrow \phi\pi^+$ candidates is artificially adjusted within each decay-time bin. This adjustment aims to align the distribution of the $\text{IP}(D^+)$ variable with that observed in $K_S^0(DD)$ decays and it involves smearing the reconstructed spatial position of the DV. It takes into consideration that the momentum of the bachelor π^+ is accurately known, particularly in the $K_S^0(DD)$ sample. Consequently, the smeared DV is constrained to lie close to the direction of the reconstructed π^+ momentum, accounting for tracking uncertainties.

The decay vertex position and the direction of the ϕ meson are precisely determined thanks to the VELO sub-detector and the proximity to the Primary Vertex (PV). To emulate less precise measurements in 3D space and displaced DV positions from their true locations, the variables can be artificially smeared using 3D covariance matrices derived from $D^+ \rightarrow K_S^0\pi$ candidates on an event-by-event basis.

However, this approach is computationally intensive and requires detailed modelling of all tracking characteristics, which may not always be necessary. Therefore, a simplified yet effective method is preferred: directly smearing the reconstructed DV position by adding a displacement along the direction of the bachelor $\vec{p}(\pi^+)$. The magnitude of this displacement is randomly sampled

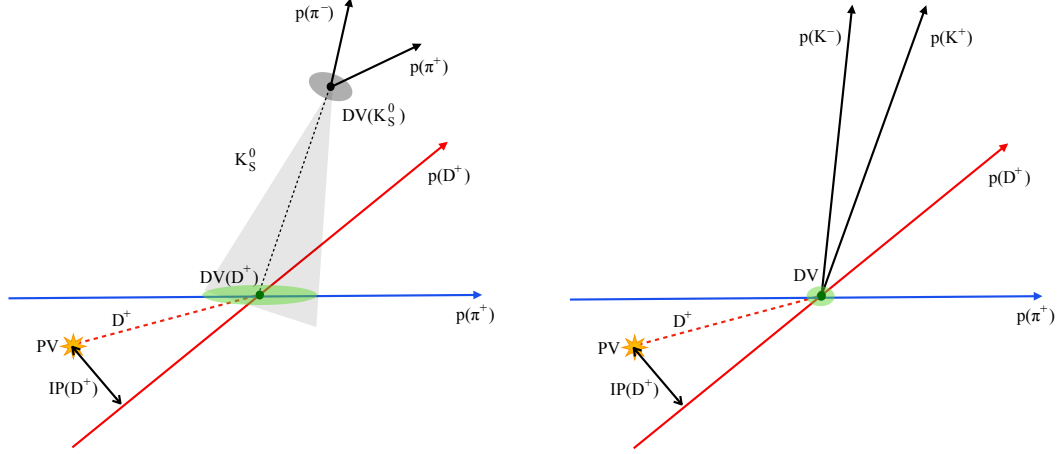


Figure 5.6: Schematic representation of the impact parameter of D^+ mesons in $D^+ \rightarrow K_S^0 \pi^+$ (left) and $D^+ \rightarrow \phi \pi^+$ (right) decays. In both cases, the momentum of the D^+ is reconstructed from the momenta of three stable particles in the final state: the π^+ and the products of the $K_S^0 (\rightarrow \pi^+ \pi^-)$ or of the $\phi (\rightarrow K^+ K^-)$ decays. In the left scheme, the uncertainty on the D^+ decay vertex is dominated by the errors on the direction and decay vertex of the K_S^0 .

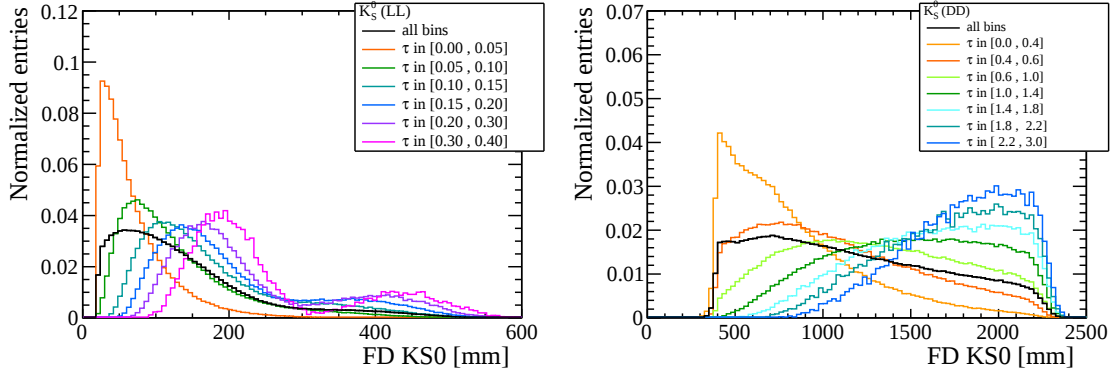


Figure 5.7: Distribution of the flight distance of the K_S^0 in the different $K_S^0(LL)$ and $K_S^0(DD)$ decay-time bins. A strong positive correlation between the FD and the decay time is clearly visible.

from a Gaussian distribution with a mean of zero and a standard deviation σ_{smear} . Subsequently, the smeared $IP(D^+)$ is calculated for each event using the reconstructed PV position, the measured momentum direction of the D^+ , $\hat{p}_D$, and the newly smeared DV position:

$$\overrightarrow{DV}_{smear} = \overrightarrow{DV} + \alpha \hat{p}_\pi \quad \text{with } \alpha \sim G(0, \sigma_{smear}) \quad (5.8)$$

$$\vec{d}_D = \overrightarrow{DV}_{smear} - \overrightarrow{PV} \quad (5.9)$$

$$IP(D^+)_{smear} = \left| \vec{d}_D - (\vec{d}_D \cdot \hat{p}_D) \hat{p}_D \right| \quad (5.10)$$

The best value for the width σ_{smear} is determined separately for each $K_S^0(DD)$ decay-time bin, by maximising the agreement between the distribution of $IP(D^+)_{smear}$ in the $D^+ \rightarrow \phi \pi^+$ candidates and that of $IP(D^+)$ in the $D^+ \rightarrow K_S^0(DD) \pi^+$ candidates.

Figure 5.8 shows some examples of $IP(D^+)_{smear}$ distributions in different $K_S^0(DD)$ decay-time

bins. Despite the simplicity of the smearing strategy, they all show a good agreement with the $\text{IP}(D^+)$ distribution in the $K_S^0(DD)$ sample. This is fully satisfactory for the purpose of equalising a selection requirement between two data samples. The values found for σ_{smear} are also reported in figure 5.8. They decrease for higher $K_S^0(DD)$ decay times. Therefore, the uncertainty on reconstructing the position of the decay vertex of the K_S^0 , which decreases with the flight distance in the downstream sample, is the dominant contribution to the resolution of the IP.

After having built in the presented way the $\text{IP}(D^+)_{\text{smear}}$ distribution for $D^+ \rightarrow \phi\pi^+$ decays, the cut $\text{IP}(D^+)_{\text{smear}} < 350 \mu\text{m}$ is applied. The effect of this requirement on the original $\text{IP}(D^+)$ histogram can be seen in figure 5.9: the efficiency, defined as the ratio between the $\text{IP}(D^+)$ histogram after and before the application of the $\text{IP}(D^+)_{\text{smear}}$ cut, is very high, close to 90%, in the prompt region and it starts decreasing when approaching $350 \mu\text{m}$. This shows that the number of secondary events with high impact parameter is reduced by this cut, but also about 10% of prompt candidates is discarded.

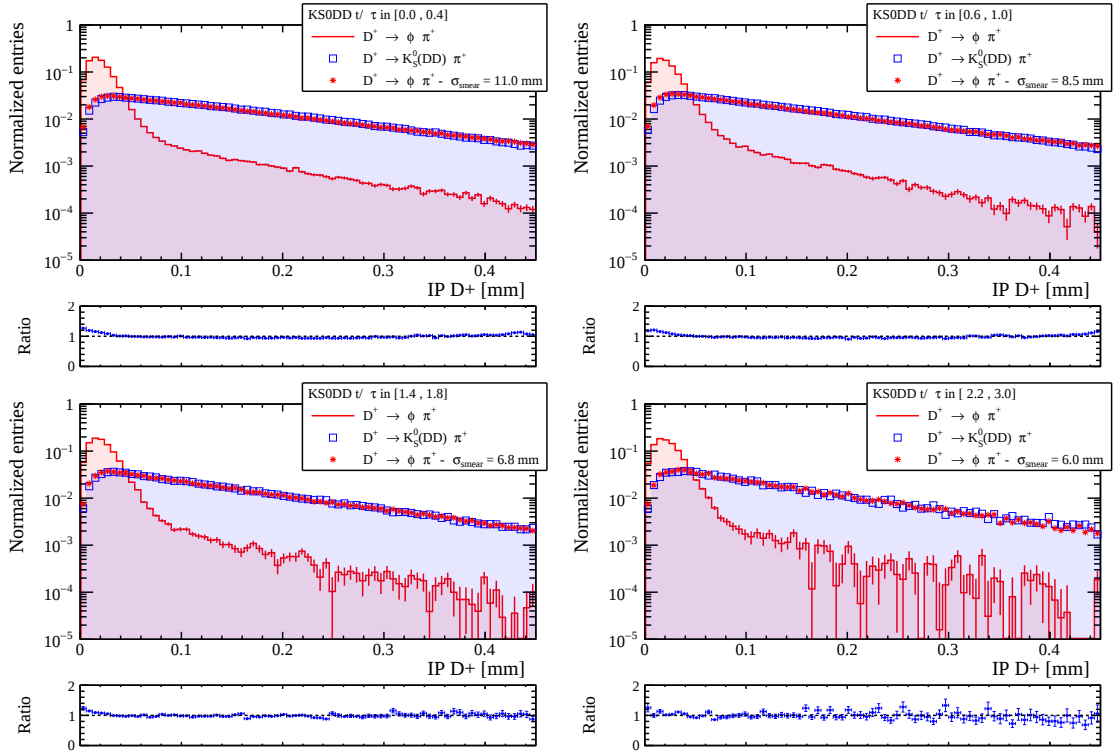


Figure 5.8: Distribution of $\text{IP}(D^+)$ for the $D^+ \rightarrow \phi\pi^+$ (red) and $D^+ \rightarrow K_S^0(DD)\pi^+$ (blue) samples in four $K_S^0(DD)$ decay-time bins. The red star histograms show the distribution of the smeared $\text{IP}(D^+)$ with the σ_{DV} that allows to reach the best agreement with the IP distribution in the $K_S^0(DD)$ sample. On the bottom of each plot the ratio between the $\text{IP}(D^+)_{\text{smear}}$ distribution in the ϕ sample and the $\text{IP}(D^+)$ in the $K_S^0(DD)$ sample is shown.

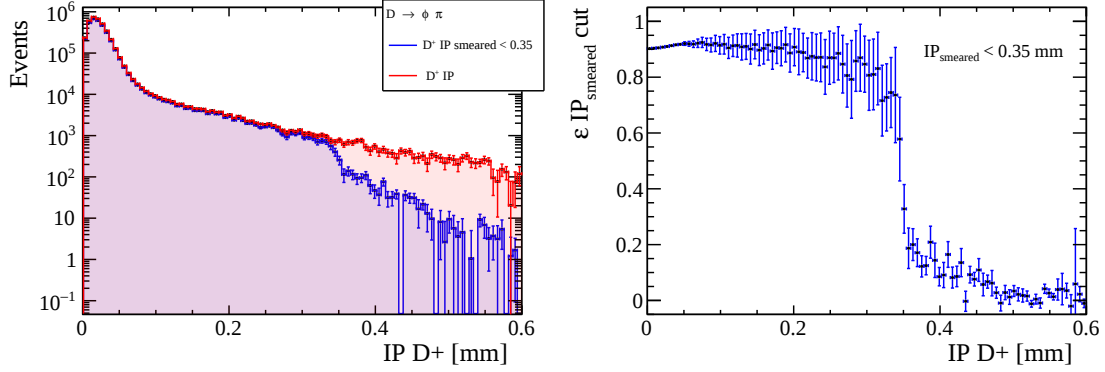


Figure 5.9: Effect of the cut $\text{IP}(D^+)_{\text{smeared}} < 350 \mu\text{m}$ to the distribution of $\text{IP}(D^+)$ in $D^+ \rightarrow \phi\pi^+$ decays. Left: distribution of $\text{IP}(D^+)$ before and after the cut on $\text{IP}(D^+)_{\text{smeared}}$. Right: ratio between the $\text{IP}(D^+)$ distribution after and before the cut on $\text{IP}(D^+)_{\text{smeared}}$, to quantify the efficiency of the cut. The fraction of prompt events discarded by the cut is of the order of 10%.

5.5 Determination of the raw asymmetries

All ingredients are now ready to measure $\Delta A_{K_S^0}(t)$ in each K_S^0 decay-time bin, which is given by the difference between the *raw* asymmetries of the $D^+ \rightarrow K_S^0\pi^+$ and $D^+ \rightarrow \phi\pi^+$ channels. These are determined from a fit to the invariant mass distributions for the positively and negatively D meson candidates, $M(K_S^0\pi^\pm)$ and $M(\phi\pi^\pm)$, respectively.

5.5.1 Simultaneous mass fits to determine *raw* asymmetries

A robust approach to determine the raw asymmetries $A_{\text{raw}}(D^+ \rightarrow K_S^0\pi^+)$ and $A_{\text{raw}}(D^+ \rightarrow \phi\pi^+)$ in each K_S^0 decay-time bin is a simultaneous fit to the invariant mass distributions of the positive and negative D candidates. The probability density functions (pdfs), for the D^+ and D^- samples, are:

$$F^\pm(m; \alpha^\pm, \beta^\pm) = N_{\text{sig}} \cdot \frac{1 \pm A_{\text{sig}}}{2} \cdot F_{\text{sig}}(m; \alpha^\pm) + \frac{1 \pm A_{\text{bkg}}}{2} \cdot F_{\text{bkg}}(m; \beta^\pm), \quad (5.11)$$

where A_{sig} , A_{bkg} , and N_{sig} are shared parameters between the D^+ and D^- pdfs. In particular, the first two terms represent, respectively, the asymmetry between the positively and negatively charged particles in the signal (A_{sig}) and background (A_{bkg}) components. Furthermore, since F_{sig} is normalised to unity, N_{sig} represents the total number of signal events in the two samples. On the contrary, for simplicity F_{bkg} has not been normalised to unity and the number of background events, which has never been used in the analysis, is given by its integral in the fit interval.

The fit is binned and minimises the total χ^2 , given by the sum of the D^+ and D^- χ^2 terms. Thus, the raw asymmetries in the different decay-time bins are separately measured for the $D^\pm \rightarrow K_S^0\pi^\pm$ and the $D^\pm \rightarrow \phi\pi^\pm$ samples with two simultaneous fits and then subtracted to obtain $\Delta A_{K_S^0}$. The used pdfs, both for signals and backgrounds, are empirical, with shape parameters determined from the fit itself. This is not a big issue in this analysis, since prominent peaks are distributed over an almost flat background and no significant differences between the two flavours are found. However, several fit models are tested, and the systematic uncertainties associated to the mass modelling are described in section 7.1. The terms $\alpha^\pm$ and $\beta^\pm$ are vectors of free shape parameters for the signal and background pdfs: most of them are shared between the D^+ and D^- samples, except for a few, which are free to float between the positively and negatively charged particles and will be referred to as $x^\pm$. The chosen central fit model has the following features:

- F_{sig} is given by a Johnson S_U distribution, defined by its mean (μ), its standard deviation (σ), its skewness (δ) and its kurtosis (λ) as:

$$J(x; \mu, \sigma, \delta, \lambda) = \frac{e^{-r^2/2}}{2\pi w \sigma \lambda \sqrt{z^2 + 1}},$$

$$\text{with } w = \frac{e^{\lambda^2} - 1}{2\sqrt{e^{\lambda^2} \cosh 2\delta\lambda + 1}}, \quad (5.12)$$

$$z = \frac{x - \mu}{w\sigma} + \sqrt{e^{\lambda^2}} \sinh \delta\lambda,$$

$$r = -\delta + \frac{\text{asinh} z}{\lambda}.$$

In the central model, the mean is free to float between the D^+ and D^- samples ($\mu^\pm \equiv \mu \pm \delta_\mu$), in order to account for the misalignment of the tracking layers of the detector, which can cause an offset between the momenta measured for particles with opposite charge. The other shape parameters are shared between the two samples. The total number of parameters used to describe the two signal functions is then 5: $\alpha^\pm = \{(\mu \pm \delta_\mu), \sigma, \delta, \lambda\}$.

- F_{bkg} is a 1st grade polynomial, defined by 2 parameters, which are shared between the two samples: $\beta^\pm = \{p_0, p_1\}$.

Considering also the 3 parameters introduced above, $N_{sig}, A_{sig}, A_{bkg}$, the total number of parameters used to perform the fit is then 10.

Fits are performed separately for each Run 2 dataset, divided by year and dipole magnet polarity, in order to have a better control of the nuisance detection asymmetries. In fact, as a consequence of the detector left-right asymmetry, different detection asymmetries are expected when using the magnet with opposite polarities. Moreover, minor upgrades of the LHCb detector and of its trigger conditions among the years can also have an impact on detection asymmetries. Figure 5.10 shows the fit results, for the 2017 MagUp dataset, in different K_S^0 decay-time bins for the $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ samples.

5.5.2 Time-dependence of $\Delta A_{K_S^0}(t)$

After the measurement of the *raw* asymmetries of all the 13 K_S^0 and ϕ sub-samples, it is possible to measure the quantity of interest of this thesis, $\Delta A_{K_S^0}(t)$, defined in equation (5.3). Except for a constant term, this observable is equal to the time-dependent neutral kaon detection asymmetry, $A_D(K_S^0, t)$. As discussed above, the measurement of the *raw* asymmetries in the $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ data samples, and therefore that of their difference $\Delta A_{K_S^0}(t)$, are performed separately for each year and magnet polarity dataset, in order to precisely cancel the nuisance asymmetries. Furthermore, in order to avoid any experimenter's bias to the measurement of the constant term $A_{CP}(D^+ \rightarrow \phi \pi^+)$, all the $\Delta A_{K_S^0}(t)$ values are *blinded*, i.e. shifted by a unknown offset, shared by all K_S^0 decay-time bins and for all the Run 2 datasets, extracted randomly between -10% and 10% . The blinded measurements of $\Delta A_{K_S^0}(t)$, divided by year and magnet polarity, are reported in figure 5.11, where a satisfactory agreement among the different datasets, in each K_S^0 decay-time bin, is found. This compatibility can be quantified by the global χ^2 , reported in the plots, which results in a *p-value* of 72%. This result is expected since all the differences $\Delta A_{K_S^0}(t)$ no longer depend on either the detector effects or the D meson production mechanisms. All the Run 2 datasets are therefore combined by performing the average over the years and magnet polarities of the asymmetry differences, in each K_S^0 decay-time bin, and the obtained result is reported in figure 5.12.

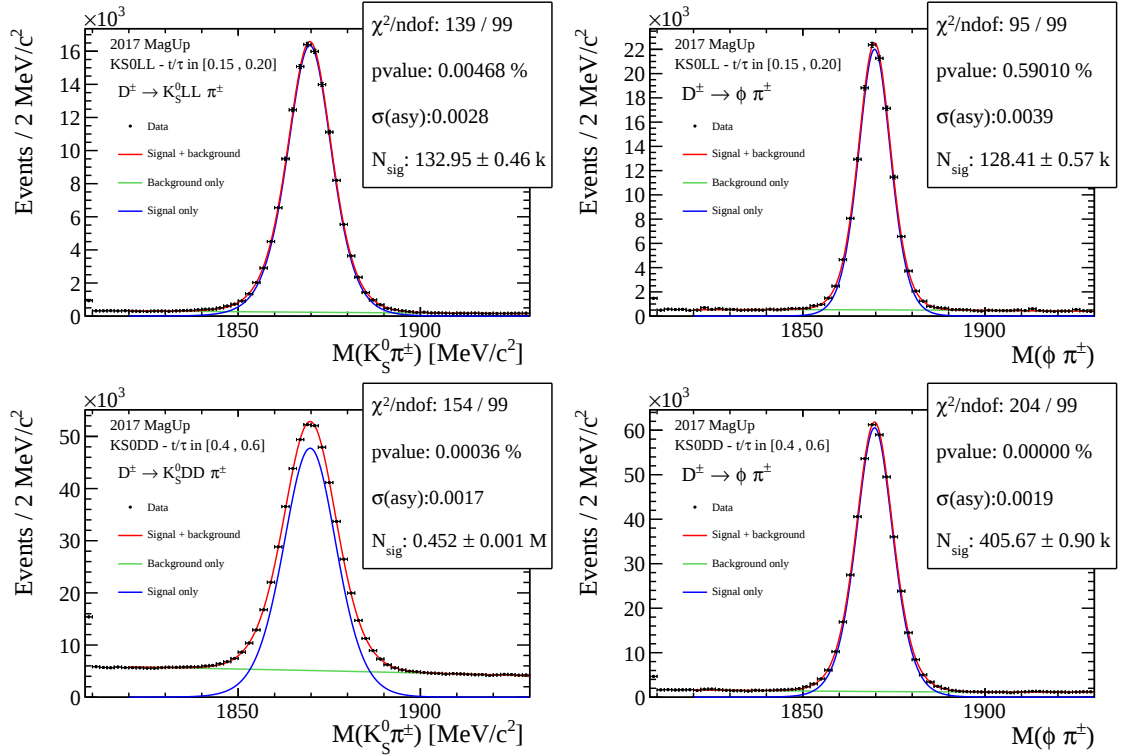


Figure 5.10: Simultaneous mass fits in two K_S^0 decay-time bins for the $D^+ \rightarrow K_S^0 \pi^+$ (left) and $D^+ \rightarrow \phi \pi^+$ (right) samples separately, for the 2017 MagUp dataset. In the legends, the total number of signal events, N_{sig} , and the uncertainty on the signal asymmetry $\sigma(A_{sig})$ are reported. The histogram and fit functions shown in each plot are given by the sum of the D^+ and D^- components.

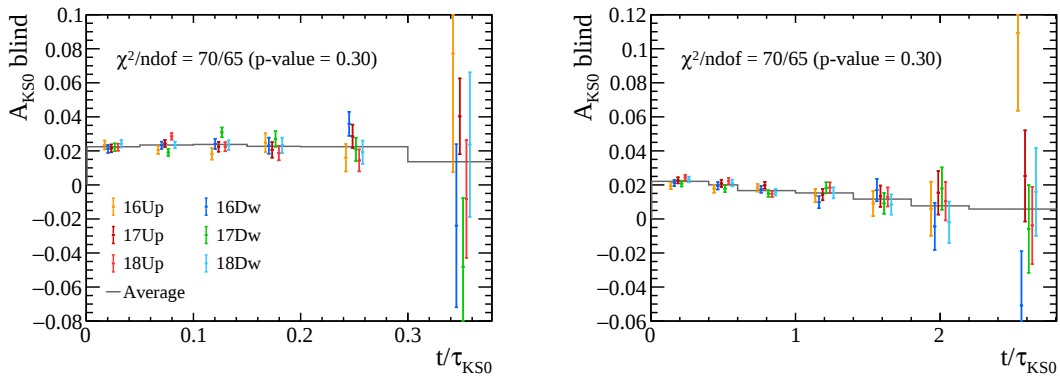


Figure 5.11: Compatibility between the measurements of $\Delta A_{K_S^0}(t)$ in the different long (left) and downstream (right) bins with all the six Run 2 datasets.

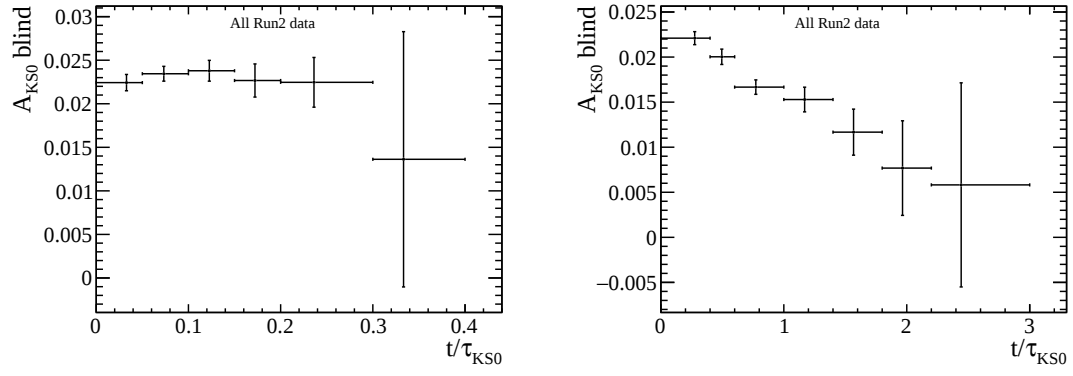


Figure 5.12: $\Delta A_{K_S^0}(t)$ measurements in the different long (left) and downstream (right) bins, obtained averaging on all the Run 2 datasets.

Chapter 6

Time-dependent neutral kaon asymmetry

This chapter presents the core focus of this master's thesis, which is centred on modelling the time-dependent neutral kaon detection asymmetry in $D^+ \rightarrow K_S^0 \pi^+$ decays, $A_D(K_S^0, t)$. At first, detailed discussions are presented on the methodology used to predict these effects, using the models presented in chapters 1 and 2 and exploiting the knowledge of the LHCb material map. A novel aspect introduced in this thesis is the inclusion of a new interference contribution, which is discussed in depth. Furthermore, the thesis reports the measurement of the relative magnitude, r_π , and strong phase difference, δ_π , between the Cabibbo-Favoured and Doubly Cabibbo-Suppressed amplitudes, governing this interference process. The methodology employed to obtain this result represents an original contribution, developed entirely within this thesis. Finally, the model for $A_D(K_S^0, t)$ is applied to measure $A_{CP}(D^+ \rightarrow \phi \pi^+)$ using the full LHCb Run 2 dataset.

6.1 Time-dependent neutral kaon asymmetry templates

The method employed to predict the time-dependent neutral kaon detection asymmetry is based on the phenomenological model introduced in chapters 1 and 2. This methodology has been developed for the measurement of $A_{CP}(D^0 \rightarrow K^+ K^-)$ in 2014 (Refs [6, 43]) and has since been used in more recent analyses as well [8, 7]. This method estimates the detection asymmetry for each reconstructed K_S^0 candidate, accounting for its momentum and trajectory within the LHCb detector. The asymmetry of a K_S^0 candidate is determined iteratively by reconstructing its time evolution using the phenomenological framework introduced in section 1.6. Initially, this method considered only CP violation in the $K^0 - \bar{K}^0$ mixing and the K_S^0 regeneration, due to the different interaction cross sections of K^0 and $\bar{K}^0$ mesons with the detector material. This thesis extends the methodology by introducing, for the first time, a new source of CP violation: the interference between Cabibbo-Favoured and Doubly Cabibbo-Suppressed charm decays and the $K^0 - \bar{K}^0$ mixing, presented in section 2.4.

Given a reconstructed K_S^0 , its path in the LHCb detector can be determined based on its production and decay vertices, as well as its momentum. The distribution of material within the LHCb detector is not homogeneous, and detailed descriptions of the detector geometry and material distribution have been implemented using the GEANT4 framework [69, 70]. In this analysis, the software-based LHCb detector description is assumed to be accurate at a level of precision much higher than the statistical uncertainties at play. Therefore, any systematic errors arising from material characterisation, as discussed further in section 7.4, are considered negligible. Once

the distribution of the material in the detector is known, the asymmetry $A_D(K_S^0)$ can be estimated using an iterative procedure, as detailed in Ref. [6].

Steps of the iterative procedure to estimate the K^0 asymmetry

The asymmetry for the k^{th} K_S^0 candidate in the dataset, given its momentum and its path, can be estimated using equation (2.21). The fundamental ingredients in this equation are the coefficients describing the composition of the neutral kaon state, $\alpha_{S,L}^{(\pm)}(t_k)$, at the time of its decay into the $\pi^+\pi^-$ final state. The starting point to compute these coefficients are the equations (1.42) and (1.43), which assume that the material through which the time evolution occurs is homogeneously distributed, with atomic weight A and density ρ . The approach to calculate these coefficients while considering the trajectory of a candidate in the detector is described in the following steps, which are illustrated in figure 6.1.

- The LHCb detector is described by a structure of geometrical volumes, each made of a specific material. Each volume is assigned a **MEDIUM** number defining the properties of the contained material, assumed to be homogeneously distributed within the volume. The reconstructed path of each K_S^0 candidate is divided into n small steps, whose boundaries are defined by intersections between the K_S^0 path and the detector volumes. The properties of the material, namely the atomic weight A and density ρ , in each step can be accessed using the **MEDIUM** number associated to the crossed volume. Each path step corresponds to a proper time range $[t_{i-1}, t_i]$, with $t_0 = 0$ and t_n equal to the k^{th} K_S^0 candidate decay time t_k .
- In the first step, the initial amplitudes of the neutral kaon, $\alpha_{S,L}^{(\pm)}(0)$, are estimated according to equations (2.17) and (2.20) and they depend on the r_π and δ_π parameters.
- In each subsequent step, the amplitudes $\alpha_{S,L}^{(\pm)}(t_i)$ are estimated according to equations (1.42) and (1.43), using as initial values the amplitudes in the previous step $\alpha_{S,L}^{(\pm)}(t_{i-1})$. This computation considers the time interval $\Delta t = t_i - t_{i-1}$ and estimates $\Delta\chi$ using the average A and ρ of the detector material in that path step.

This method enables the computation of the coefficients $\alpha_{S,L}^{(\pm)}(t_k)$, which describe the $K_S^0 - K_L^0$ composition of the neutral kaon state at the time of its decay. With these coefficients, the predicted neutral kaon asymmetry for the k^{th} K_S^0 candidate, $A_{K_S^0}^{(k)}(t_k)$, can be computed as in equation (2.21).

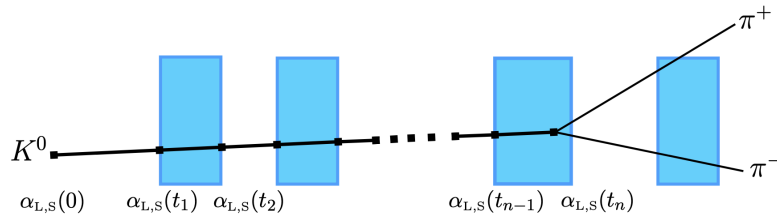


Figure 6.1: Simplified representation of the iterative procedure to compute the asymmetry for a neutral kaon candidate. The path of the neutral kaon is divided into several steps, in which the material distribution is assumed to be homogeneous. In this picture, the blue boxes are made of a different material than the space in between. In each step, the coefficients $\alpha_{S,L}$ are computed iteratively. Figure taken from Ref. [6].

Once the asymmetry is determined for each K_S^0 candidate in the data sample, the predicted asymmetry in a K_S^0 decay-time bin is computed as the weighted average of the asymmetries of all the

K_S^0 candidates in the considered bin, using the kinematic weights. This averaging uses kinematic weights coming from the weighting and splitting procedure described in section 5.3:

$$A_{pred}(K_S^0, t) = \frac{\sum_k w_k A_{K_S^0}^{(k)}(t_k)}{\sum_k w_k}. \quad (6.1)$$

The output of this procedure is a template for the time-dependent neutral kaon detection asymmetry in the $K_S^0(LL)$ and $K_S^0(DD)$ decay-time bins, built with a chosen pair of (r_π, δ_π) . As an example, the templates built with a set of r_π and δ_π values are shown in figure 6.2. We can see that the templates built with $r_\pi = 0$ (only CF) and with the FAT prediction ($r_\pi = -0.073$ and $\delta_\pi = -1.39$) are really close to each other, in all the K_S^0 decay-time bins. In contrast, for other strong phases, such as $\delta_\pi = \pi/4$ and $\delta_\pi = \pi/2$, the discrepancy is larger, especially at high decay times.

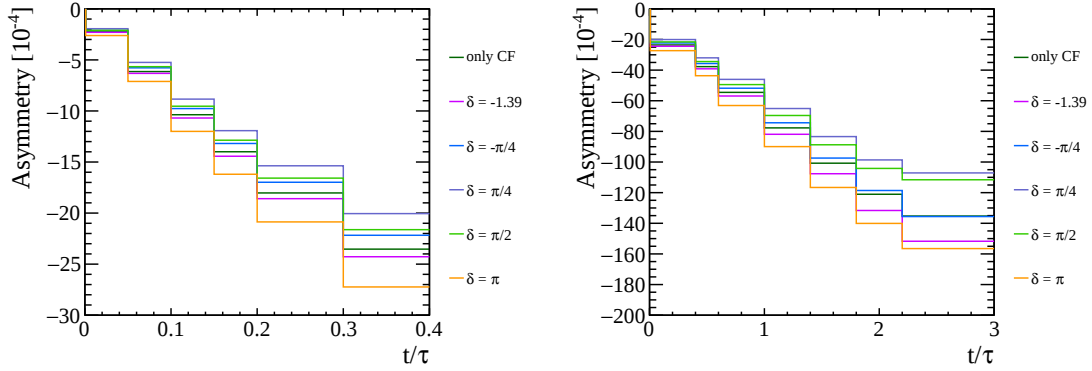


Figure 6.2: Templates built with the 2017 MagUp dataset in $K_S^0(LL)$ (left) and $K_S^0(DD)$ (right) bins, using $r_\pi = 0$ (only CF) and $r_\pi = -0.073$ and several strong phases reported in the legend.

Test of the agreement between the prediction and the data

For each dataset, the predicted neutral kaon detection asymmetry is estimated using the procedure described above, making a scan on several (r_π, δ_π) pairs. Since the predicted asymmetry is always null at $t/\tau = 0$, in order to compare it to the $\Delta A_{K_S^0}(t)$ measured in data, it is necessary to shift the templates by an offset, which is related to $A_{CP}(D^+ \rightarrow \phi\pi^+)$ and, therefore, to the blinding constant according to equation (5.3). The optimal offset for each (r_π, δ_π) template is found by minimising the χ^2 with respect to data: given a tested offset, the χ^2 is computed from the comparison between the 13 $\Delta A_{K_S^0}(t)$ measurements and the time-dependent prediction, given by the (r_π, δ_π) template shifted by the tested offset; this procedure allows to study the dependence of the χ^2 on the offset, given a (r_π, δ_π) pair, as shown in figure 6.4. The optimal offset is then simply estimated by minimising the χ^2 . As an example, figure 6.3 shows the agreement of several templates, each shifted by its optimal offset, and data.

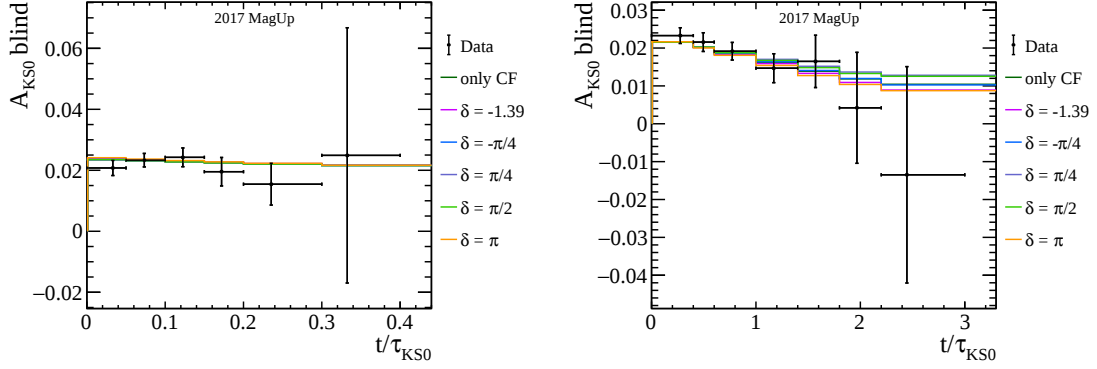


Figure 6.3: Comparison between the templates with $r_\pi = -0.073$ and the $K_S^0(LL)$ and $K_S^0(DD)$ data.

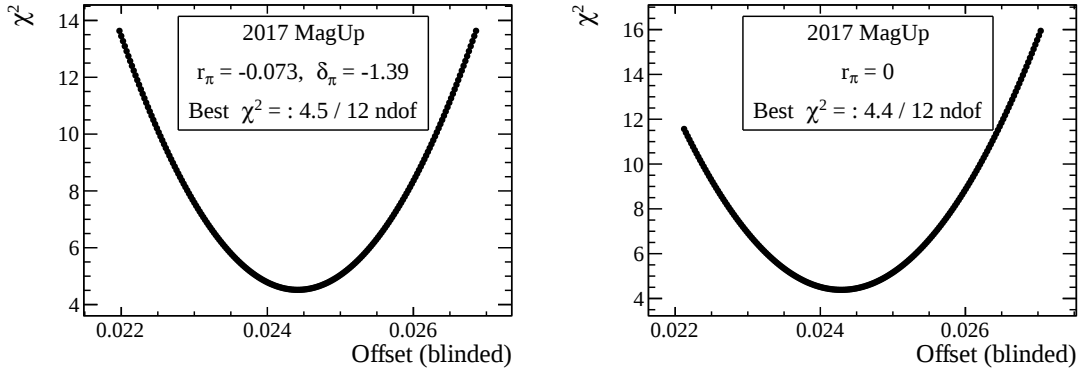


Figure 6.4: Plot of the χ^2 in the comparison between 2017 MagUp data and templates as a function of the offset at fixed values of (r_π, δ_π) . The optimal offset for each (r_π, δ_π) pair has been found minimising the χ^2 .

6.2 Measurement of r_π and δ_π

As discussed in chapter 2, the effects due to the interference between the Cabibbo-Favoured and Doubly Cabibbo-Suppressed amplitudes in the $D^+ \rightarrow K_S^0 \pi^+$ decay channel are included for the first time in the time-dependent model of neutral kaon detection asymmetry. Such interference effects are exploited to measure the relevant parameters, the modulus ratio r_π and the relative strong phase δ_π . This section presents the strategy developed to build confidence intervals in the $r_\pi - \delta_\pi$ space, which is based on the agreement of the templates, built for a set of tested (r_π, δ_π) pairs, with the $\Delta A_{K_S^0}(t)$ data. Because of the peculiarity of the statistical problem, it is decided to use the *Plugin* method (also referred to as the $\hat{\mu}$ method) [71, 72]. This is used to determine the confidence level of a chosen set of parameters, in this case r_π and δ_π , in the presence of nuisance parameters, in this case the offset, using a frequentist approach.

First of all, a set of templates for the time-dependent asymmetry, choosing the values of (r_π, δ_π) on a grid in the intervals $r_\pi \in (-1, 0)$ and $\delta_\pi \in (-\pi, \pi)$, are generated, using the procedure described in section 6.1. The differences among the templates with different (r_π, δ_π) pairs are shown in figure 6.5. In particular, as expected, the dependence on the relative strong phase δ_π increases in magnitude for larger r_π values. This effect results in a loss of sensitivity to the δ_π parameter in the region at $r_\pi \sim 0$, as shown in the following sections.

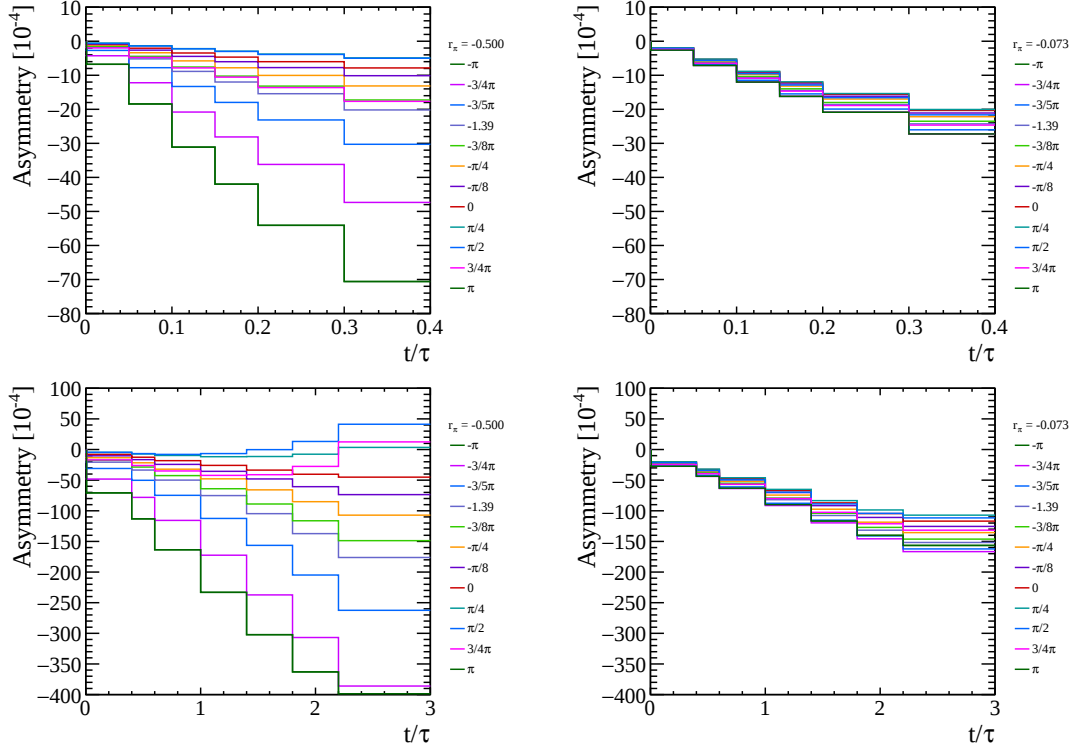


Figure 6.5: Set of templates for δ_π free to vary and r_π fixed at a large value -0.500 (left) and at the FAT prediction value -0.073 (right) in long (top) and downstream (bottom) bins. For large values of r_π , different strong phases result in very different templates, especially at large K_S^0 decay time values.

6.2.1 Plugin method

The model which describes the time-dependence of the 13 $\Delta A_{K_S^0}(t)$ measurements depends on three parameters: r_π , δ_π , and the offset q , which is treated as a nuisance parameter (at least in this first part of the analysis). The global Likelihood of the 3 parameters can be written as:

$$\mathcal{L}(\mathbf{X}|\boldsymbol{\alpha}) = \prod_{t=1}^{13} p(\Delta A_{K_S^0}(t); \boldsymbol{\alpha}), \quad (6.2)$$

where we define:

- $\mathbf{X} = \{\Delta A_{K_S^0}(t)\}_{t=1}^{13}$ the set of the 13 time-dependent asymmetry measurements;
- $\boldsymbol{\alpha} = \{r_\pi, \delta_\pi, q\}$ the set of the three parameters;
- $p(\Delta A_{K_S^0}(t); \boldsymbol{\alpha})$ the p.d.f. of the asymmetry measurement in the K_S^0 decay-time bin t .

Assuming that each $\Delta A_{K_S^0}(t)$ measurement has a Gaussian distribution, with the mean given by the prediction of the model in the bin t and the standard deviation given by its statistical uncertainty σ_t , the $p(\Delta A_{K_S^0}(t); \boldsymbol{\alpha})$ term can be written as:

$$p(\Delta A_{K_S^0}(t); \boldsymbol{\alpha}) = \frac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\frac{(\Delta A_{K_S^0}(t) - \text{model}(t, \boldsymbol{\alpha}))^2}{2\sigma_t^2}\right). \quad (6.3)$$

From the dependence of the global likelihood on the three parameters, it is possible to select the regions in the $r_\pi - \delta_\pi$ space in which the model describes better our measurements. A key ingredient to build confidence intervals in the $r_\pi - \delta_\pi$ space is to quantify the agreement between model and data as a function of the (r_π, δ_π) parameters, treating the offset as a nuisance parameter. A suitable statistics commonly used in Goodness of Fit problems with nuisance parameter is the *Logarithm Likelihood Ratio* (LLR), which will be referred to in this thesis as $\Delta\chi^2$, defined for each (r_π, δ_π) pair as:

$$\begin{aligned}\Delta\chi^2(r_\pi, \delta_\pi) &\equiv -2 \log \frac{\sup_q \mathcal{L}(\mathbf{X}|r_\pi, \delta_\pi, q)}{\sup_{r'_\pi, \delta'_\pi, q} \mathcal{L}(\mathbf{X}|r'_\pi, \delta'_\pi, q)} \\ &\equiv \min_q \chi^2(r_\pi, \delta_\pi, q|\mathbf{X}) - \min_{r'_\pi, \delta'_\pi, q} \chi^2(r'_\pi, \delta'_\pi, q|\mathbf{X}),\end{aligned}\tag{6.4}$$

where we define the χ^2 function of a parameters triplet α , given the set of measurements $\mathbf{X}$, as:

$$\chi^2(\alpha|\mathbf{X}) = \sum_{t=1}^{13} \left(\frac{\Delta A_{K_S^0}(t) - \text{model}(t, \alpha)}{\sigma_t} \right)^2 + \text{constant}.\tag{6.5}$$

Equation (6.4) can be computed from two ingredients: the χ^2 for the tested (r_π, δ_π) pair minimised with respect to the offset, for the value $\hat{q}_{(r_\pi, \delta_\pi)}$, and the global minimum of the χ^2 on the allowed (r_π, δ_π, q) space, found for the triplet $\hat{\alpha} = \{\hat{r}_\pi, \hat{\delta}_\pi, \hat{q}\}$.

The $\Delta\chi^2(r_\pi, \delta_\pi)$ map is a useful quantity to perform a Goodness of Fit test, i.e. to quantify the agreement between the model, with a specific (r_π, δ_π) pair, and data. In particular, given a specific (r_π, δ_π) pair, it is possible to define a *p-value*, $p(r_\pi, \delta_\pi)$, as the probability of obtaining, with another set of measurements $\mathbf{X}'$, a $\Delta\chi^2$ higher than what obtained with the original dataset $\mathbf{X}$. A critical region $\mathcal{C}(CL)$, given by (r_π, δ_π) values excluded with confidence level CL , can finally be built from the condition:

$$(r_\pi, \delta_\pi) \in \mathcal{C}(CL) \iff p(r_\pi, \delta_\pi) < CL.\tag{6.6}$$

The accepted region is given by the complementary condition:

$$(r_\pi, \delta_\pi) \in \mathcal{A}(CL) \iff p(r_\pi, \delta_\pi) \geq 1 - CL.\tag{6.7}$$

The only remaining ingredient is to compute the *p-value* for each (r_π, δ_π) pair, used to exclude the critical region at a given CL . To do so, there are two possible approaches, based on different assumptions on the distribution of $\Delta\chi^2$:

- the *Profile Likelihood* method, in which the assumption that the $\Delta\chi^2$ is distributed as a $\chi^2(\text{ndof} = 2)$, deriving directly from Wilks Theorem [73], is made. In this case, the *p-value* can be computed as:

$$p(r_\pi, \delta_\pi) = \int_{\Delta\chi^2(r_\pi, \delta_\pi)}^{+\infty} \chi^2(t; \text{ndof} = 2) dt;\tag{6.8}$$

- the *Plugin* method, which is a generalisation of the previous case with no assumptions on the distribution of $\Delta\chi^2$. Following the approach described in [72], the steps to compute the *p-value* of a specific (r_π, δ_π) pair are:

1. $\Delta\chi^2(r_\pi, \delta_\pi)$ is computed as described in (6.4). In particular, the offset which minimises

- the χ^2 for the fixed (r_π, δ_π) pair is referred to, from now on, as $\hat{q}_{(r_\pi, \delta_\pi)}$;
2. N_{toy} pseudo-experiments are then randomly generated, starting from the model with the tested (r_π, δ_π) pair and their best-fit offset $\hat{q}_{(r_\pi, \delta_\pi)}$. Each pseudo-experiment consists in a set of 13 measurements $\mathbf{X}^{(MC)} \equiv \{\Delta A_{K_S^0}(t)^{(MC)}\}_{t=1}^{13}$, where each value of $\Delta A_{K_S^0}(t)$ is randomly extracted, using the p.d.f. defined in (6.3), with the parameters set to $\alpha \equiv \{r_\pi, \delta_\pi, \hat{q}_{(r_\pi, \delta_\pi)}\}$;
 3. for each pseudo-experiment, the quantity $\Delta\chi^2^{(MC)}(r_\pi, \delta_\pi)$ is computed, optimising every time the offset separately and finding for each pseudo-experiment the global χ^2 minimum for $\hat{\alpha}^{(MC)}$. This allows to build the distribution of $\Delta\chi^2^{(MC)}(r_\pi, \delta_\pi)$, computed with different sets of generated measurements;
 4. the p -value of the (r_π, δ_π) pair can finally be computed as the tail of the $\Delta\chi^2^{(MC)}(r_\pi, \delta_\pi)$ distribution above the $\Delta\chi^2(r_\pi, \delta_\pi)$ measured in data, i.e. as the fraction of pseudo-experiments with $\Delta\chi^2^{(MC)} > \Delta\chi^2(r_\pi, \delta_\pi)$:

$$p(r_\pi, \delta_\pi) = \frac{N(\Delta\chi^2^{(MC)}(r_\pi, \delta_\pi) > \Delta\chi^2(r_\pi, \delta_\pi))}{N_{toy}}. \quad (6.9)$$

In our case, since we want to explore a region close to the $r_\pi = 0$ boundary, where we loose sensitivity to the δ_π parameter, we can't assume that the $\Delta\chi^2$ is distributed as a $\chi^2(\text{ndof} = 2)$, thus it is necessary to use the *Plugin* method.

Computation of $\Delta\chi^2(r_\pi, \delta_\pi)$, treating the offset as a nuisance parameter

The first step to build confidence intervals in the $r_\pi - \delta_\pi$ parameters space is to quantify the agreement of each (r_π, δ_π) template with data, i.e. by computing the $\Delta\chi^2(r_\pi, \delta_\pi)$ statistics defined in (6.4). The term $\Delta\chi^2(r_\pi, \delta_\pi)$ is computed as the difference between the χ^2 , computed as in (6.5) with fixed values for r_π and δ_π and minimised with respect to the offset, and the global minimum χ^2 . This procedure is done simultaneously over all Run 2 datasets, using the averaged $\Delta A_{K_S^0}(t)$ measurements shown in 5.12. In figure 6.6, the procedure to minimise the χ^2 with respect to the offset for two fixed (r_π, δ_π) pairs is shown. In figure 6.7 the distribution of the $\Delta\chi^2$, defined in (6.4), for each (r_π, δ_π) point in the scan is shown. It is evident from this plot, as expected, that for values of r_π closer to 0 the sensitivity on the strong phase is lost, while for higher r_π only few values of δ_π have good agreement with data.

Generation of the pseudo-experiments for each (r_π, δ_π) pair

A pseudo-experiment consist in a set of 13 $\Delta A_{K_S^0}(t)$ measurements, each extracted random from the p.d.f reported in 6.3, which consists in a Gaussian with σ given by the uncertainty on the measured asymmetry in that bin and μ given by the prediction for that bin of the template with the tested (r_π, δ_π) pair and the best-fit offset $\hat{q}_{(r_\pi, \delta_\pi)}$. As an example, figure 6.8 shows the time-dependence of a pseudo-experiment, built for $r_\pi = -0.073$ and $\delta_\pi = -1.39$, (blue points) and data (black points). For each (r_π, δ_π) pair in the scan, $N_{toys} = 10000$ pseudo-experiments have been generated. In figure 6.9 the distributions of the asymmetry in four chosen decay-time bins for the pseudo-experiments generated with $r_\pi = -0.073$ and $\delta_\pi = -1.39$ (FAT prediction) are shown.

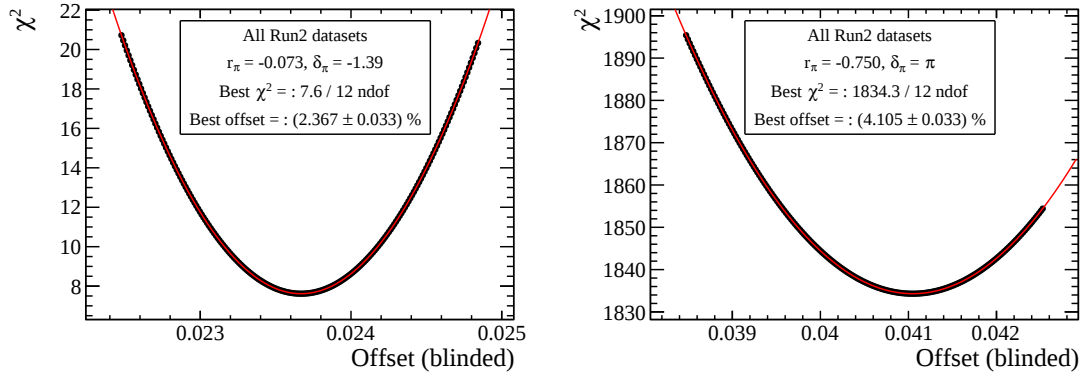


Figure 6.6: Optimisation of the offset for two different (r_π, δ_π) values for the full Run 2 dataset. The minimum of the χ^2 with respect to the offset and the optimal offset are reported in the plot.

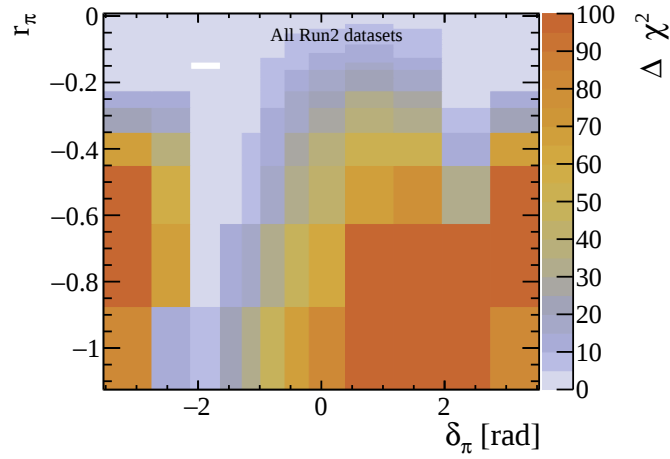


Figure 6.7: Map of the $\Delta\chi^2$ for each δ_π, r_π template, using all Run 2 datasets.

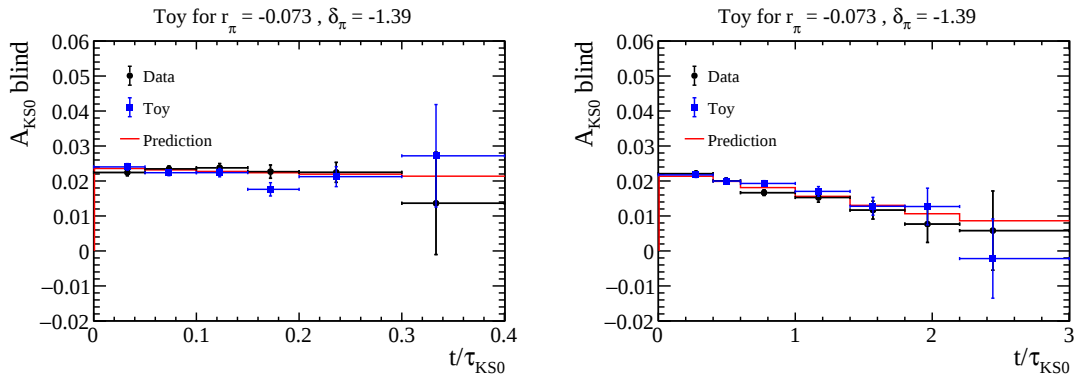


Figure 6.8: Comparison between the data and one pseudo-experiment for the $r_\pi = -0.073, \delta_\pi = -1.39$ scan point. The red histogram is the template for this (r_π, δ_π) point, with the optimal offset.

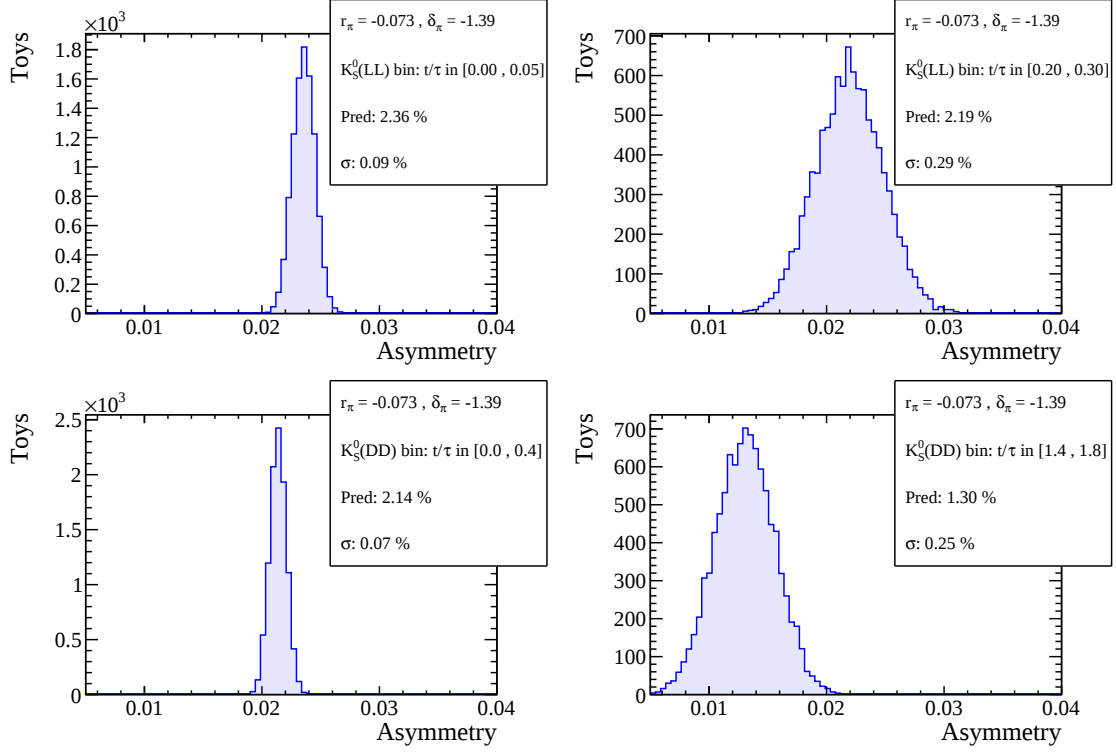


Figure 6.9: Distribution of the pseudo-experiments in the different decay-time bins for the $r_\pi = -0.073$, $\delta_\pi = -1.39$ point. In the boxes are reported the prediction of the template and the uncertainty on $\Delta A_{K_S^0}(b)$, used to generate the pseudo-experiments.

Distribution of $\Delta\chi^2^{(MC)}$ for different (r_π, δ_π) pairs

After having generated $N_{toys} = 10000$ pseudo-experiments for each (r_π, δ_π) point in the scan, the distribution of $\Delta\chi^2^{(MC)}(r_\pi, \delta_\pi)$ is produced. Given a (r_π, δ_π) pair, $\Delta\chi^2^{(MC)}$ for a single pseudo-experiment is computed as the difference between the χ^2 minimised with respect to the offset, with fixed (r_π, δ_π) , and the χ^2 minimised with respect to the three parameters free to vary in the space. In figure 6.10 the distribution of $\Delta\chi^2^{(MC)}$ for different (r_π, δ_π) pairs is shown. We can see that at high r_π values there is a peak at $\Delta\chi^2 = 0$, which is an artefact related to the fact that the scan is performed on a discrete set of templates. In particular, for the majority of toys with high r_π values the global minimum χ^2 is found for the same (r_π, δ_π) pair with which the toys were generated. This artefact, however, does not affect the computation of the p -value, given by the integral of the tail of this distribution.

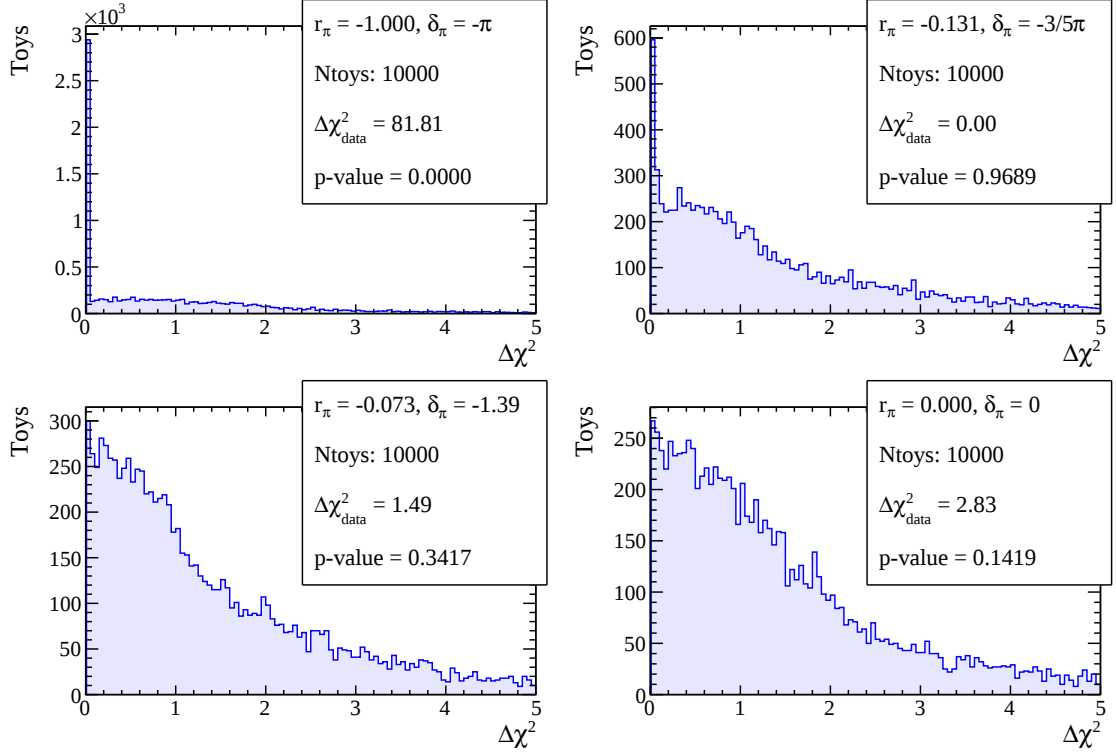


Figure 6.10: Distribution of $\Delta\chi^2$ (MC) for the pseudo-experiments built for different (r_π, δ_π) pairs. In the top right boxes the $\Delta\chi^2$ measured on all Run 2 datasets and the final $1 - CL$ value are reported.

6.2.2 Results: contour lines for r_π and δ_π

The output of the *Plugin* method is a map of the p -value in the $r_\pi - \delta_\pi$ space, shown in the left part of figure 6.11 using all Run 2 datasets. Based on the p -value, it is possible to define the excluded (6.6) and accepted (6.7) regions, by applying a threshold which depends on the chosen confidence level CL . The right part of 6.11 shows the confidence intervals built applying three different $1 - CL$ thresholds to the p -value map, which correspond to 1, 2 and 3 equivalent Gaussian standard deviations. It can be seen that the FAT prediction falls in the 1σ interval, but also the templates with $r_\pi = 0$, i.e. neglecting the presence of DCS charm decays, are in agreement with data. This result only takes into account statistical uncertainties on the $\Delta A_{K_S^0}(t)$ measurements. Systematic uncertainties are discussed and added to this p -value map in section 7.3, by repeating the whole procedure with enlarged errors given by the quadrature sum of the statistical and systematic components. This is a very important and interesting result. For the first time, a model-independent experimental measurement, free of any theoretical uncertainty, has access to this type of information. This is unique and opens the doors to a new research line in LHCb, completely unexpected until today.

The resolution on the $\Delta A_{K_S^0}(t)$ measurements, and consequently on r_π and δ_π , is constrained by the limited statistics of the calibration sample $D^+ \rightarrow \phi\pi^+$. To improve precision, as a secondary task, the thesis proposes to use the $D_s^+ \rightarrow \phi\pi^+$ decay as an additional calibration channel. It is assumed, and experimentally verified, that, after equalising the kinematics of D^+ and D_s^+ , the difference in production asymmetries between D^+ and D_s^+ mesons remains constant across K_S^0 decay-time bins. Thus, including the D_s^+ calibration sample does not bias the measured

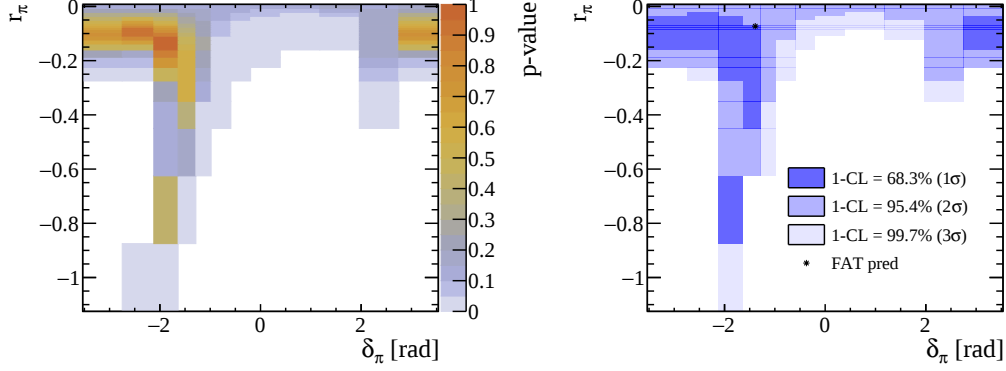


Figure 6.11: Left: map of the p -value computed with the plugin method for different r_π, δ_π values in the scan, using the full Run 2 data sample. Right: confidence intervals applying different $1 - CL$ thresholds to the p -value. We can see that the FAT prediction is in the 1σ interval.

time-dependence of $A_D(K_S^0)$. This study to improve the precision on r_π and δ_π is reported in Appendix C, where it is shown that the statistical uncertainties on the $\Delta A_{K_S^0}(t)$ measurements are all reduced by a factor ~ 0.7 . The confidence intervals in the $r_\pi - \delta_\pi$ space are recalculated using the *Plugin* method, now including the D_s^+ calibration sample to measure $\Delta A_{K_S^0}(t)$. The result for the p -value map and for the confidence intervals is reported in 6.12.

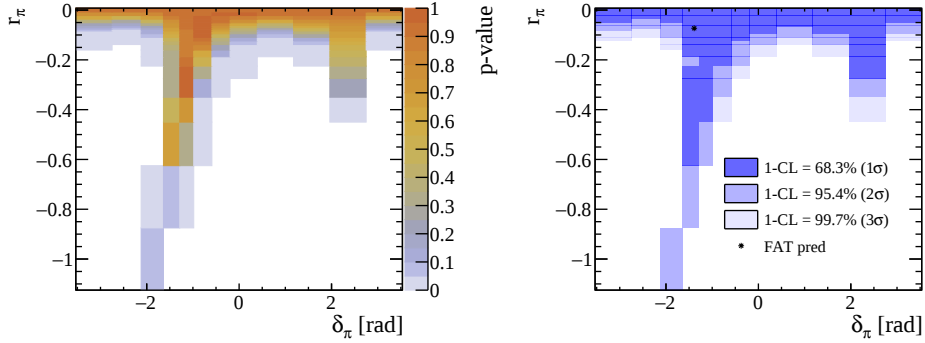


Figure 6.12: Left: map of $1 - CL$ computed with the plugin method for different r_π, δ_π values in the scan, including the D_s^+ calibration channel. Right: confidence intervals for different CL thresholds. We can see that the FAT prediction is still in the 1σ interval.

6.3 Measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$

As a by-product of the measurement of r_π and δ_π this thesis also presents the measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$ using the full Run 2 dataset. As described in section 2.3, the most recent measurement of this quantity was published in 2019 by the LHCb collaboration, using 2015-17 data (Ref.[8]), corresponding to an integrated luminosity of 3.8 fb^{-1} :

$$A_{CP}(D^+ \rightarrow \phi\pi^+) = [0.05 \pm 0.42(\text{stat}) \pm 0.29(\text{syst})] \times 10^{-3}.$$

This measurement uses the $D^+ \rightarrow K_S^0\pi^+$ decay as calibration channel, but only the $K_S^0(LL)$ dataset is included in the analysis. As in this thesis, the quantity of interest is estimated from the time-integrated *raw* asymmetries difference:

$$A_{CP}(D^+ \rightarrow \phi\pi^+) = A_{\text{raw}}(D^+ \rightarrow \phi\pi^+) - [A_{\text{raw}}(D^+ \rightarrow K_S^0\pi^+) - A_D(K_S^0)]. \quad (6.10)$$

The time-integrated neutral kaon detection asymmetry $A_D(K_S^0)$ is estimated from the predicted asymmetry for each $K_S^0(LL)$ candidate, computed as described in section 6.1 only considering CF charm decays, i.e. fixing $r_\pi = 0$.

This thesis presents a new measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$, reanalysing 2016 and 2017 data and including the 2018 data sample. The new measurement takes into account also the newly introduced effect due to the CF-DCS interference in the $D^+ \rightarrow K_S^0\pi^+$ charm decay, thus the model for the neutral kaon detection asymmetry is now more accurate even at large decay times. This enables the inclusion of the $K_S^0(DD)$ sample, in which such interference effects are more relevant, and to perform a more robust estimate of systematic uncertainties related to the knowledge of the neutral kaon detection asymmetries. The developed analysis strategy consists in measuring $A_{CP}(D^+ \rightarrow \phi\pi^+)$ directly from the offset of the best-fit model to the $\Delta A_{K_S^0}(t)$ plot. In fact, the time-dependent neutral kaon detection asymmetry vanishes at zero K_S^0 decay time, in the assumption of negligible direct CP violation in the $K^0/\bar{K}^0 \rightarrow \pi^+\pi^-$ decay, hence the offset can be expressed as:

$$\begin{aligned} \Delta A_{K_S^0}(t=0) &= A_D(K_S^0, t=0) - A_{CP}(D^+ \rightarrow \phi\pi^+) \\ &= -A_{CP}(D^+ \rightarrow \phi\pi^+). \end{aligned} \quad (6.11)$$

Several approaches to include the effect of the interference between CF and DCS charm decays in the estimate of the best fit offset are presented in this thesis. At first, working under the assumption that the model-dependent FAT prediction for r_π and δ_π is accurate, $\Delta A_{K_S^0}(t=0)$ is estimated as the offset that minimises the χ^2 with data for the template with $r_\pi = -0.073$ and $\delta_\pi = -1.39$, as described in section 6.3.1. However, in order to have a measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$ independent on the strong amplitudes models used in the FAT approach, $\Delta A_{K_S^0}(t=0)$ is also measured with a *Profile Likelihood* method, treating r_π and δ_π as nuisance parameters to be optimised directly on data. This strategy is described in detail in section 6.3.2 and it is independent of the model used to describe the strong amplitudes in the FAT approach.

6.3.1 Template fit fixing (r_π, δ_π) at the FAT prediction

This first estimate of $A_{CP}(D^+ \rightarrow \phi\pi^+)$ is performed with the strategy to optimise the offset for a fixed template described in section 6.1. In this case, the template with the FAT prediction ($r_\pi = -0.073, \delta_\pi = -1.39$) is used. The relative uncertainty on both parameters from FAT is small, 5% for r_π and about 3.6% for δ_π , thus the systematic uncertainty due to these errors is

negligible. The optimal offset is therefore determined by simply minimising the χ^2 , while its uncertainty is determined assuming that the likelihood of the offset is Gaussian, so that around the minimum the χ^2 can be written as:

$$\chi^2(q) \simeq \chi_{min}^2 + \left(\frac{q - q_{best}}{\sigma_q} \right)^2. \quad (6.12)$$

The fit is performed independently for each dataset divided by year and magnet polarity and the results are in good agreement with each other, as shown in figure 6.14. The final blinded result for $A_{CP}(D^+ \rightarrow \phi\pi^+)$, averaged among all Run 2 datasets, is:

$$A_{CP}(D^+ \rightarrow \phi\pi^+)_{\text{FAT}}^{\text{blind}} = (2.367 \pm 0.033)\%.$$

The statistical uncertainty of this result, 3.3×10^{-4} , is lower than what found in the published article ($\sigma_{stat} = 4.2 \times 10^{-4}$ in Ref. [8]), thanks to the inclusion of the 2018 dataset¹ and the $D^+ \rightarrow K_S^0(DD)\pi^+$ sample.

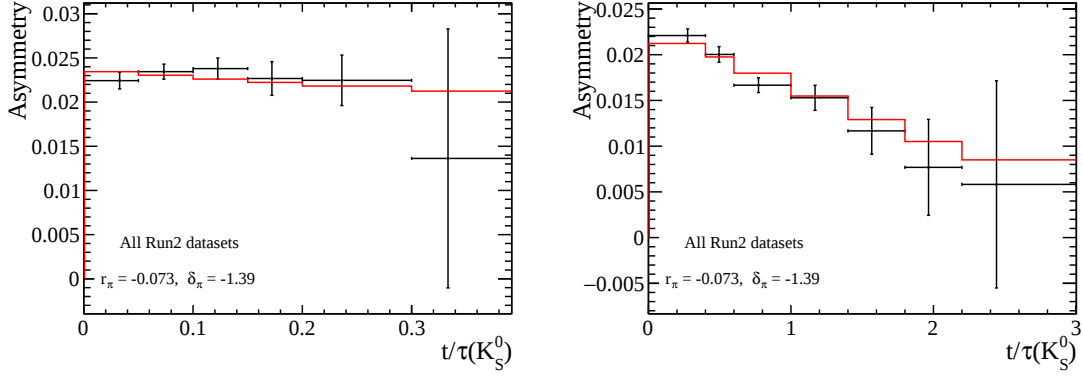


Figure 6.13: Template with $(r_\pi = -0.073, \delta_\pi = -1.39)$ with the offset that minimises the χ^2 , using all the Run 2 datasets.

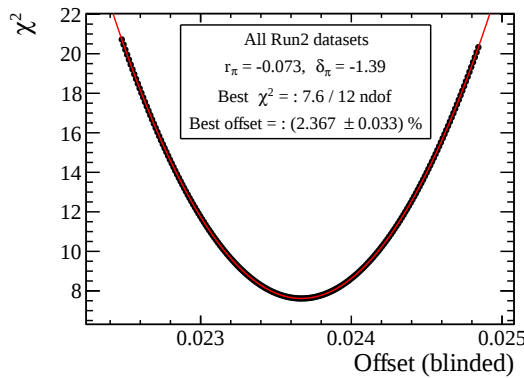


Figure 6.14: Results of the optimal offset for the template with $(r_\pi = -0.073, \delta_\pi = -1.39)$ simultaneously on all Run 2 datasets.

¹The analysed integrated luminosity is now 5.4 fb^{-1} , larger by a factor 1.4 than the 3.8 fb^{-1} used in [8].

6.3.2 Profile Likelihood fit, using r_π and δ_π as nuisance parameters

As discussed in section 6.2, our model for the time-dependent neutral kaon detection asymmetry depends on three parameters: the offset, q , and the shape parameters r_π and δ_π , which in the previous section have been fixed to the FAT prediction. This section proposes an alternative method to measure $A_{CP}(D^+ \rightarrow \phi\pi^+)$ that does not take any external input on r_π and δ_π , which would depend on the employed strong amplitudes model.

The chosen strategy to measure the best fit offset without any external inputs is to treat both r_π and δ_π as a nuisance parameters, to be optimised directly on data. The relative strong phase δ_π is left free to vary in the whole $[-\pi, \pi]$ interval, while r_π is constrained to be in $[-0.093, -0.053]$, estimated in section 2.4 as an interval, centered in the FAT prediction, much larger than any correction due to hadronic contributions. The statistical treatment is analogous to what described in section 6.2, with the only difference that now the parameter of interest is the offset q and the nuisance parameters are r_π and δ_π . With a similar strategy, it is possible to compute the Goodness of Fit statistics $\Delta\chi^2(q)$, defined as

$$\Delta\chi^2(q) \equiv \min_{r_\pi, \delta_\pi} \chi^2(r_\pi, \delta_\pi, q) - \min_{q', r_\pi, \delta_\pi} \chi^2(r_\pi, \delta_\pi, q'). \quad (6.13)$$

The best-fit value for the offset is simply given by the condition $\Delta\chi^2(q_{best}) = 0$, i.e. requiring that q_{best} minimises globally the χ^2 . On the other hand, for the estimate of the uncertainty on q_{best} it is necessary to work under an assumption for the distribution of $\Delta\chi^2(q)$. The most rigorous statistical treatment would require to compute this distribution using the *Plugin* method, generating a set of pseudo-experiments. However, since r_π is constrained to have a non-zero value and there is no loss of sensitivity on δ_π , the approach chosen is a *Profile Likelihood* method, working under the assumption that $\Delta\chi^2(q)$ is distributed as a χ^2 (ndof = 1). In this case, the confidence interval for the offset q equivalent to one Gaussian standard deviation, $\mathcal{J}_q(1\sigma)$, can be identified by the condition:

$$\Delta\chi^2(q) \leq 1 \quad \forall q \in \mathcal{J}_q(1\sigma). \quad (6.14)$$

Following the approach used by the MINOS algorithm [74], it is possible to define asymmetric errors on q (σ_q^d and σ_q^u) according to the following relations:

$$\Delta\chi^2(q_{best} - \sigma_q^d) = 1 \quad \text{and} \quad \Delta\chi^2(q_{best} + \sigma_q^u) = 1, \quad (6.15)$$

so that the 1σ confidence interval is given by $\mathcal{J}_q(1\sigma) = [q_{best} - \sigma_q^d, q_{best} + \sigma_q^u]$. The *Profile Likelihood* method is implemented using only the templates with $r_\pi \in [-0.093, -0.053]$ and the result for the minimisation of $\chi^2(q)$ is shown in figure 6.15.

$$A_{CP}(D^+ \rightarrow \phi\pi^+) \Big|_{\text{Profile Likelihood}}^{\text{blind}} = (2.413^{+0.034}_{-0.051}) \cdot \%$$

This result is compatible, within 1σ , with the result obtained using only the FAT prediction template. The statistical uncertainty is now worse than what found in the previous section, fixing r_π and δ_π at the FAT prediction. It is important to remember, however, that this measurement is completely independent on any strong amplitudes model used to predict r_π and δ_π , since the interval chosen for the parameters is much larger than the uncertainties estimated with the FAT approach.

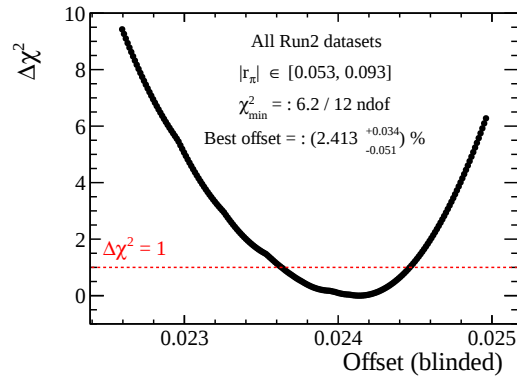


Figure 6.15: $\chi^2(q)$ plot using all the Run 2 datasets. We can see that the $\chi^2(q)$ has an asymmetric shape around the minimum, thus the asymmetric uncertainties $\sigma_q^{u,d}$ have been estimated as in (6.15).

Chapter 7

Systematic uncertainties and consistency checks

This chapter addresses the systematic uncertainties in measuring the DCS-CF interference parameters in the $D^+ \rightarrow K_S^0 \pi^+$ decay mode, r_π and δ_π , as well as the direct CP asymmetry in the $D^+ \rightarrow \phi \pi^+$ decay channel, $A_{CP}(D^+ \rightarrow \phi \pi^+)$. Additionally, the knowledge of the LHCb material map is discussed, with particular attention on the impact of uncertainties in the material budget within the LHCb detector description on the neutral kaon asymmetry models.

7.1 Systematic uncertainties on the measurements of $\Delta A_{K_S^0}(t)$

The main sources of systematic uncertainty on the determination of the 13 $\Delta A_{K_S^0}(t)$ measurements are due to the choice of the model used to perform the simultaneous mass fits, in order to measure the *raw* asymmetries in each decay-time bin, and to the contamination of D^+ mesons coming from B decays, which alter the production asymmetry. Table 7.1 reports the systematic uncertainties in each K_S^0 decay-time bin, divided by source. The dominant contribution is given by the choice of the fit model, which however depends on the statistical fluctuations of the various fit results. For this reason, this source of systematic uncertainty is expected to decrease as the size of the analysed samples increases.

Model for the simultaneous mass fits

The *raw* asymmetries of the $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ channels computed in each decay-time bin depend on the model used to describe signal and background in the simultaneous mass fits. A set of different mass models, capable to satisfactorily describe data, are used to perform the alternative mass fits, to measure the *raw* asymmetries, and to compute the $\Delta A_{K_S^0}$ values. The systematic uncertainty is assigned as the maximum dispersion of the obtained results, as shown as an example in figures 7.1 and 7.2. The size of this systematic uncertainty is affected by the statistical fluctuations of the asymmetry results in the various fits, hence it is expected to decrease as the number of signal events increases.

The used models are summarised in table 7.2 and they are chosen to test different background shapes, namely first and second grade polynomials and a double exponential, defined as:

$$F_{bkg}(m; \lambda_1, \lambda_2, f) = \lambda_1 e^{-\lambda_1 m} + f \lambda_2 e^{-\lambda_2 m}. \quad (7.1)$$

Furthermore, in the second and third models, all the shape parameters for the signal and for the background pdfs are allowed to float between the D^+ and the D^- sample, while in the first and fourth only the mean of the Johnson S_U is not shared.

$t/\tau_{K_S^0}$ bin	Secondaries	Fit model	σ_{sys}	σ_{stat}	σ_{tot}
$K_S^0(LL)$ bins					
[0.00, 0.05]	0.9	3.3	3.3	9.4	9.9
[0.05, 0.10]	0.6	6.8	6.8	8.5	10.9
[0.10, 0.15]	0.7	12.8	12.8	11.9	17.5
[0.15, 0.20]	1.11	12.7	12.7	19.0	22.9
[0.20, 0.30]	1.47	20.7	20.7	28.5	35.2
[0.30, 0.40]	3.13	47.7	47.7	146.5	154.1
$K_S^0(DD)$ bins					
[0.0, 0.4]	0.2	3.2	3.2	7.2	7.9
[0.4, 0.6]	0.2	2.7	2.7	8.5	8.9
[0.6, 1.0]	0.2	3.1	3.1	8.0	8.6
[1.0, 1.4]	0.2	7.1	7.1	13.7	15.5
[1.4, 1.8]	0.2	5.9	5.9	25.5	26.1
[1.8, 2.2]	0.2	23.8	23.8	52.5	57.7
[2.2, 3.0]	0.2	52.7	52.7	113	125.5

Table 7.1: Systematic uncertainties on $\Delta A(K_S^0)$ in all the K_S^0 decay-time bins, in units of 10^{-4} . In the last columns, the statistical and total (given by the quadrature sum of σ_{syst} and σ_{stat}) uncertainties on $\Delta A(K_S^0, t)$ using all the Run 2 datasets are reported.

Model label	Parameters	Signal Model	Background Model
$JSU1(10)$	10	Johnson S_U all shared except for μ	first grade polynomial all shared
$JSU2(17)$	17	Johnson S_U all free	second grade polynomial all free
$JSU1(15)$	15	Johnson S_U all free	first grade polynomial all free
$JSU2exp(12)$	12	Johnson S_U all shared except for μ	double exponential all shared

Table 7.2: Features of the models used to estimate the systematic uncertainty, with the labels reported in the figures. The notes below the signal and background models refer to the parameters, which can be shared between the D^+ and D^- samples or free to vary. The first model, $JSU1(10)$, is used to measure the central values of $\Delta A_{K_S^0}(t)$, as discussed in 5.5.

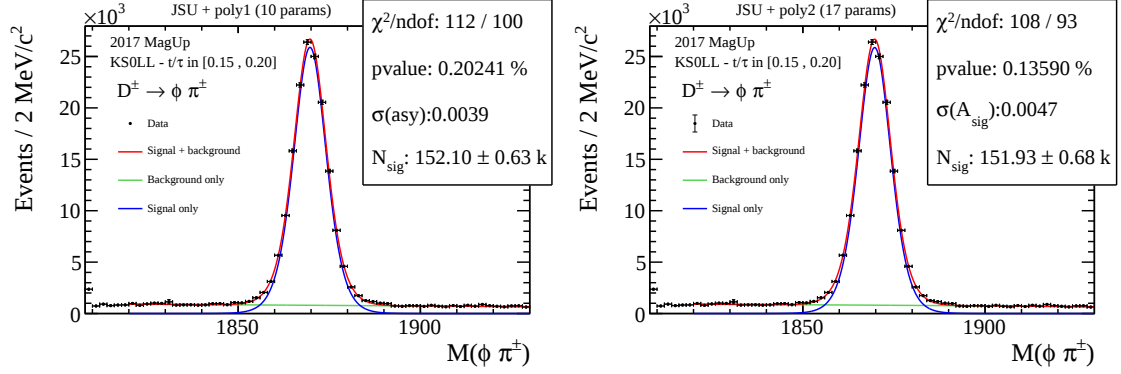


Figure 7.1: Fit results for $A_{raw}(D^+ \rightarrow \phi \pi^+)$ in the fourth $K_S^0(LL)$ bin with two different models for the 2017 MagUp dataset.

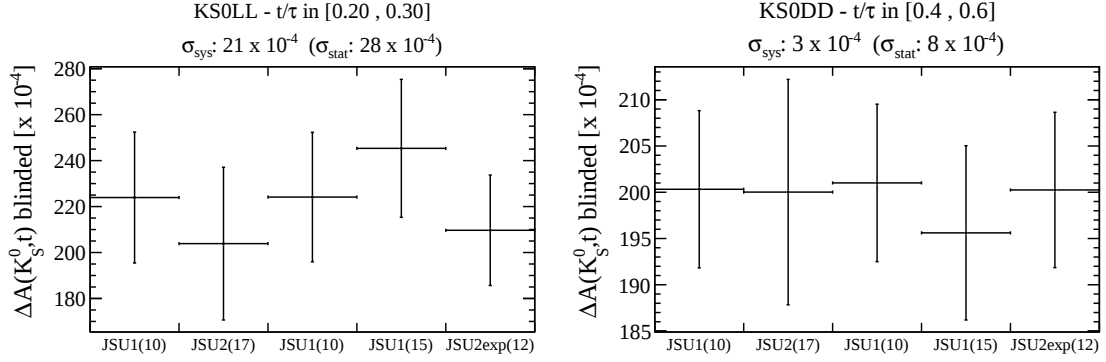


Figure 7.2: Dispersion between the $\Delta A_{K_S^0}(t)$ measurements with the different fit models in two K_S^0 decay-time bins. Statistical uncertainty for that bin is also reported.

Secondaries in the $K_S^0(LL)$ bins

Secondary D^+ mesons, produced in the decay of a B meson, exhibit different production asymmetries compared to prompt D^+ mesons. Consequently, they introduce a bias on $\Delta A_{K_S^0}$, as estimated in section 5.4:

$$\Delta_{sec} = (A_P(B) - A_P(D^+)) \cdot (f_{sec}^{K_S^0} - f_{sec}^\phi). \quad (7.2)$$

In the case of the $K_S^0(LL)$ bins, Δ_{sec} is estimated directly from the $IP(D^+)$ distributions in each decay-time bin following the approach adopted in Ref [8]:

- the difference of the production asymmetries $A_P(B) - A_P(D^+)$ is assumed to be approximately equal to the difference $A_{sec} - A_{prompt}$, where:
 - A_{sec} is the raw asymmetry in the region $IP(D^+) > 150 \mu\text{m}$;
 - A_{prompt} is the raw asymmetry in the region $IP(D^+) < 100 \mu\text{m}$.

While A_{sec} is equal to the raw asymmetry of a pure sample of secondary decays, A_{prompt} receives a small contribution, at the level of 3-4%, from the contamination of secondary decays, which is neglected in the determination of Δ_{sec} .

- The relative fraction of secondary decays below the prompt peak f_{sec} , i.e. for $\text{IP}(D^+) < 100 \mu\text{m}$, is estimated in both the $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ samples. This fraction is measured using a realistic cocktail of simulated D^+ mesons produced in $B^0, B^+, B_s^0 \rightarrow D^+ X$ decays, where the 3D distribution of p_T, η, ϕ of the D candidates is weighted to match that of the background-subtracted data candidates having $\text{IP}(D^+) > 150 \mu\text{m}$. This allows a fine-tuning of the simulated D^+ momentum, by exploiting a kinematic space in data, populated by only secondary decays. The $\text{IP}(D^+)$ distribution of simulated secondary decay candidates is overlapped to the same distribution of background-subtracted data, where the normalisation for the simulated distribution is chosen to match the data distribution in the region $\text{IP}(D^+) > 150 \mu\text{m}$, as shown in figure 7.3.

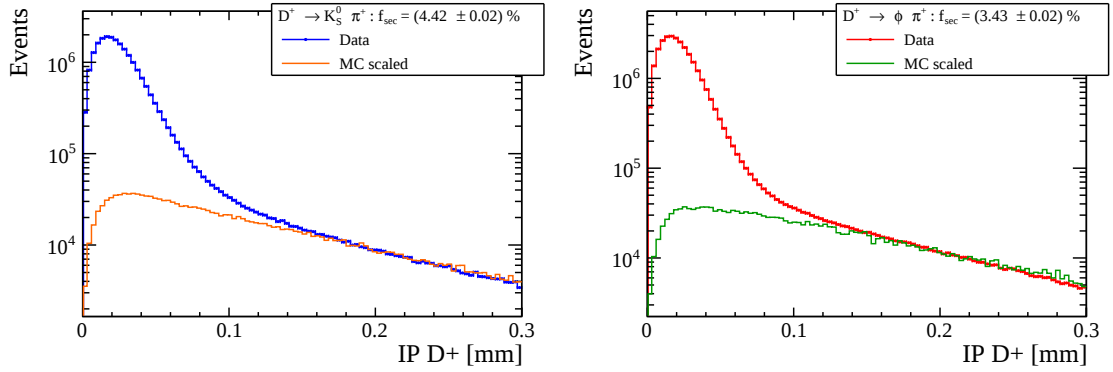


Figure 7.3: Comparison between the $\text{IP}(D^+)$ distribution of data and the scaled secondary simulation in the $D^+ \rightarrow K_S^0(LL)\pi^+$ (left) and $D^+ \rightarrow \phi\pi^+$ (right) samples, using all decay-time bins and Run 2 datasets. The estimated secondary fraction in the prompt region is also reported.

The bias introduced by secondary decays is estimated at first using the full statistics of the $D^+ \rightarrow K_S^0(LL)\pi^+$ sample and of the $D^+ \rightarrow \phi\pi^+$ sub-samples assigned to $K_S^0(LL)$ bins. This is done to verify that Δ_{sec} is compatible with zero using the full available statistics. The result is $\Delta_{sec} = (0.29 \pm 0.18) \times 10^{-4}$, as shown in the left panel of figure 7.4. The systematic uncertainty due to the contamination on secondary decays instead is estimated using the same strategy but repeated independently in each $K_S^0(LL)$ decay-time bin. The measured values of Δ_{sec} are reported in table 7.3 for all the $K_S^0(LL)$ decay-time bins, using the requirement $\text{IP}(D^+) < 100 \mu\text{m}$. The bias is found to be compatible with zero, with uncertainties in the range $[0.60 - 3.13] \times 10^{-4}$, well below the corresponding statistical uncertainties. The systematic uncertainty is thus estimated as the uncertainty on each Δ_{sec} measurement. As an example, the right panel of figure 7.4 shows the ingredients used to estimate the bias introduced by the contamination from secondary decays in the first $K_S^0(LL)$ decay-time bin: the secondary fractions in the prompt regions are estimated separately for the $D^+ \rightarrow K_S^0(LL)\pi^+$ and $D^+ \rightarrow \phi\pi^+$ samples, using the scaled simulated samples; the difference between the prompt and secondary D^+ production asymmetries is estimated as the average of the difference $A_{sec} - A_{prompt}$ in the two samples.

Secondaries in the $K_S^0(DD)$ bins

For the $D^+ \rightarrow K_S^0(DD)\pi^+$ sample, due to the poor resolution on $\text{IP}(D^+)$, it is not possible to distinguish the secondary and prompt components, and thus to measure $f_{sec}^{K_S^0}$ and A_{sec} directly from data, as done in the $D^+ \rightarrow K_S^0(LL)\pi^+$ sample. However, thanks to the smearing procedure performed on the $\text{IP}(D^+)$ variable of the calibration sample and the consequent equalisation of the requirement on both samples, the bias due to the contamination of secondary decays can be

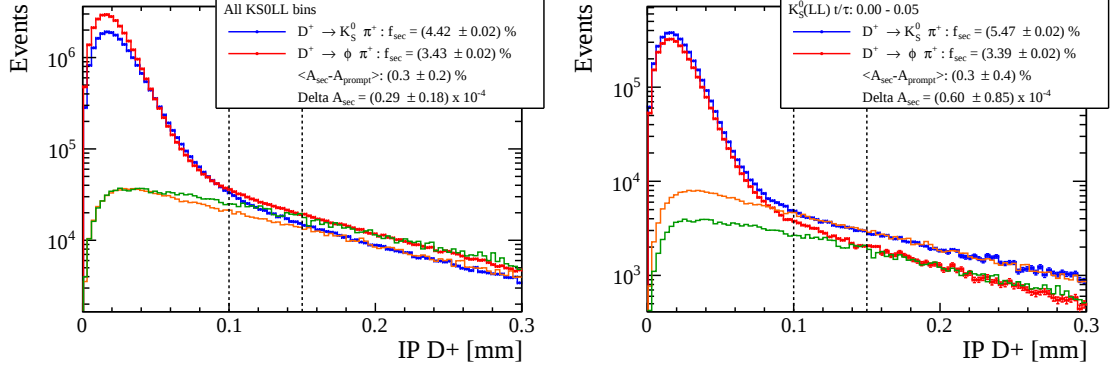


Figure 7.4: Estimate of the Δ_{sec} bias in two different K_S^0 decay-time bins. Both the data and the scaled MC $IP(D^+)$ distributions in the $K_S^0(LL)$ and $D^+ \rightarrow \phi\pi^+$ samples are shown. The dashed lines identify the prompt ($IP(D^+) < 100 \mu\text{m}$) and secondary ($IP(D^+) > 150 \mu\text{m}$) regions.

$K_S^0(LL)$ bin	$\Delta_{sec} [10^{-4}]$	$A_{sec} - A_{prompt}$	$f_{sec}^{K_S^0} - f_{sec}^{\phi}$	stat $[10^{-4}]$
[0.00, 0.05]	(0.60 ± 0.85)	$(0.3 \pm 0.4) \%$	$(2.08 \pm 0.03) \%$	9.4
[0.05, 0.10]	(0.48 ± 0.60)	$(0.3 \pm 0.4) \%$	$(1.46 \pm 0.03) \%$	8.5
[0.10, 0.15]	(0.12 ± 0.73)	$(0.1 \pm 0.6) \%$	$(1.21 \pm 0.02) \%$	11.9
[0.15, 0.20]	(0.49 ± 1.11)	$(0.4 \pm 0.9) \%$	$(1.22 \pm 0.02) \%$	19.0
[0.20, 0.30]	(-0.24 ± 1.47)	$(-0.2 \pm 1.2) \%$	$(1.19 \pm 0.02) \%$	28.5
[0.30, 0.40]	(-3.79 ± 3.13)	$(-4.0 \pm 3.3) \%$	$(0.94 \pm 0.03) \%$	146.5
All bins	(0.29 ± 0.18)	$(0.3 \pm 0.2) \%$	$(0.99 \pm 0.02) \%$	-

Table 7.3: Estimate of the systematic uncertainty due to the contamination of secondary decays in $K_S^0(LL)$ bins. As a comparison, the statistical uncertainty on $\Delta A_{K_S^0}(t)$ in the corresponding bin is reported in the last column.

assumed to be negligible, since $f_{sec}^{K_S^0} \approx f_{sec}^{\phi}$. The level of accuracy of this approximation can be verified using data itself by exploiting the $D^+ \rightarrow K_S^0(LL)\pi^+$ sample in the following way:

- the $IP(D^+)$ distribution of the full $D^+ \rightarrow K_S^0(LL)\pi^+$ sample is smeared with the procedure described in section 5.4.1, in order to reproduce as closely as possible the time-integrated $IP(D^+)$ distribution of the $D^+ \rightarrow K_S^0(DD)\pi^+$ sample. This stage emulates a sample of $D^+ \rightarrow K_S^0(DD)\pi^+$ candidates, starting from the $D^+ \rightarrow K_S^0(LL)\pi^+$ candidates, where the relative fraction of secondary decay component can be determined by inspecting the $IP(D^+)$ observable with no artificial smearing;
- the $IP(D^+)$ distribution of the full $D^+ \rightarrow \phi\pi^+$ sample is also smeared and, in this case, σ_{smear} is optimised to maximise the agreement with the $IP(D^+)_{smear}$ distribution of the $D^+ \rightarrow K_S^0(LL)\pi^+$ sample generated in the previous step;
- the $IP(D^+)_{smear} < 350 \mu\text{m}$ requirement is applied to both the $D^+ \rightarrow K_S^0(LL)\pi^+$ and $D^+ \rightarrow \phi\pi^+$ smeared samples;
- the bias Δ_{sec} is determined from the $IP(D^+)$ distributions of the $D^+ \rightarrow K_S^0(LL)\pi^+$ and $D^+ \rightarrow \phi\pi^+$, by looking at the observables with no smearing:
 - $A_{sec} - A_{prompt}$ is estimated as the difference between the *raw* asymmetries in the prompt and secondary regions, as done in the previous case;

- f_{sec} is now defined as the relative fraction of secondary decays in the whole interval $\text{IP}(D^+) \in [0., 0.6 \text{ mm}]$ and its determination is a bit more complex. For both the $D^+ \rightarrow K_S^0(LL)\pi^+$ and $D^+ \rightarrow \phi\pi^+$ samples, it is measured by exploiting the simulated cocktail of D^+ from b -hadron decays. The absolute normalisation of the $\text{IP}(D^+)$ distribution of secondary decays is done as before, looking at the tail of the impact parameter without any smearing and any requirement. Afterwards, the effect of the requirement $\text{IP}(D^+)_{\text{smear}} < 350 \mu\text{m}$ is reproduced by weighting the distribution of the $\text{IP}(D^+)$ observable (not smeared this time) with the associated efficiency. This latter is extracted directly from data by inspecting the impact parameter distribution with and without the application of the requirement on the smeared observable, as shown in figure 7.5. Finally, the relative fraction f_{sec} is measured from the number of prompt and secondary decays in the $\text{IP}(D^+) \in [0., 0.6 \text{ mm}]$ interval, estimated using the data and simulated candidates, as shown in figure 7.6.

The bias Δ_{sec} is then computed as $(A_{sec} - A_{prompt}) \cdot (f_{sec}^{K_S^0} - f_{sec}^{\phi})$ and the associated systematic uncertainty is reported in table 7.4. In particular, Δ_{sec} is found to be compatible with zero, with an uncertainty of the order of about 2×10^{-5} , and, consequently, completely negligible in our analysis. This is the first time that downstream $K_S^0(DD)$ are directly used to measure a CP -violating asymmetry with such a high precision. The method developed to bound any possible subtle systematic uncertainty originating from the contamination of secondary D^+ meson decays is a new and original piece of work of this thesis.

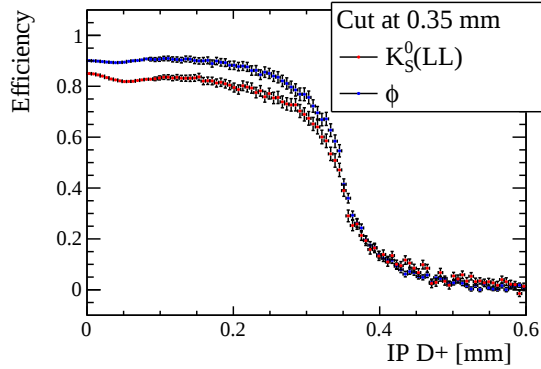


Figure 7.5: Efficiency of the $\text{IP}(D^+)_{\text{smear}} < 350 \mu\text{m}$ cut for both $D^+ \rightarrow K_S^0(LL)\pi^+$ and $D^+ \rightarrow \phi\pi^+$ samples. The efficiency is estimated as the ratio between the $\text{IP}(D^+)$ distribution after and before the cut on $\text{IP}(D^+)_{\text{smear}}$. The fraction of prompt events discarded by the cut is of the order of 10% for the $D^+ \rightarrow K_S^0(LL)\pi^+$ and 15% for the $D^+ \rightarrow \phi\pi^+$ sample.

$\text{IP}(D^+)$ cut	$\Delta_{sec} [\times 10^{-4}]$	$A_{sec} - A_{prompt}$	$f_{sec}^{K_S^0} - f_{sec}^{\phi}$
$\text{IP}(D^+)_{\text{smear}} < 350 \mu\text{m}$	0.20 ± 0.24	$(0.14 \pm 0.17) \%$	$(1.42 \pm 0.04) \%$

Table 7.4: Estimate of the systematic uncertainty due to the contamination of secondary decays in $K_S^0(DD)$ bins.

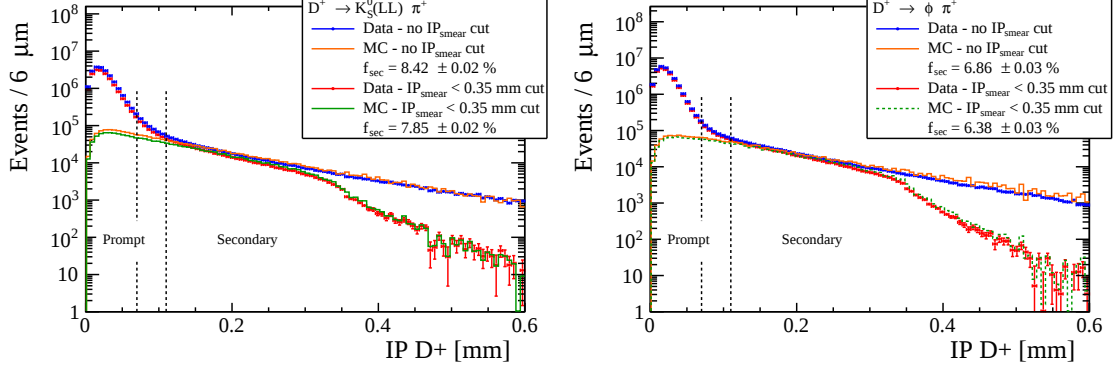


Figure 7.6: Distribution of $\text{IP}(D^+)$ in the $D^+ \rightarrow K_S^0(LL)\pi^+$ sample (left) and $D^+ \rightarrow \phi\pi^+$ sample (right) before and after the $\text{IP}(D^+)_{\text{smear}} < 350 \mu\text{m}$ cut. The prompt and secondary regions in which A_{prompt} and A_{sec} are estimated are displayed. In each figure, the scaled simulations, built with and without reproducing the effect of the $\text{IP}(D^+)_{\text{smear}}$ cut, are shown. These simulations are used to compute the secondary fractions before and after the cut, reported in the legends.

Additional systematic uncertainties

In this thesis, only two dominant sources of systematic uncertainty are considered and estimated. However, other potential effects that could introduce bias in the $\Delta A_{K_S^0}(t)$ measurements have not been studied due to time constraints. These additional potential bias sources are shown to be negligible in other published LHCb analyses involving the $D^+ \rightarrow K_S^0\pi^+$ and $D^+ \rightarrow \phi\pi^+$ channels [8, 7]. Additional sources of systematic uncertainty include the charged kaon detection asymmetry and inaccuracies in the kinematic weighting.

The detection asymmetry of the K^+K^- pair produced in $\phi \rightarrow K^+K^-$ decays is assumed to be negligible in this thesis. However, the two kaons have slightly different momentum distributions, as shown in figure 7.7, due to the presence of non-resonant background falling below the M_ϕ peak. Since the detection asymmetry of a charged kaon ($A_D(K^+)$) depends on its momentum, this difference leads to an asymmetry in the detection of the K^+K^- final state. This asymmetry is estimated in [8] using simulated $D^+ \rightarrow K^-\pi^+\pi^+$ decays to measure $A_D(K^+)$. The resulting bias on the raw asymmetry of the full $D^+ \rightarrow \phi\pi^+$ sample is found to be approximately 1.5×10^{-4} , which is smaller than the statistical and systematic uncertainties measured in this analysis.

The kinematic weighting procedure described in 5.3 is concluded when the distributions of the kinematic variables in all K_S^0 and ϕ sub-samples are in good agreement, as shown in appendix B. However, residual kinematic differences might induce a bias in the $\Delta A_{K_S^0}(t)$ measurements due to imperfectly cancelled nuisance asymmetries. This bias was estimated in [8] to be approximately 0.4×10^{-4} when using the full $D^+ \rightarrow K_S^0(LL)\pi^+$ and $D^+ \rightarrow \phi\pi^+$ samples. In this analysis, the kinematic distributions differ slightly from those in [8], but this estimate indicates that the bias is typically an order of magnitude smaller than the uncertainties at play.

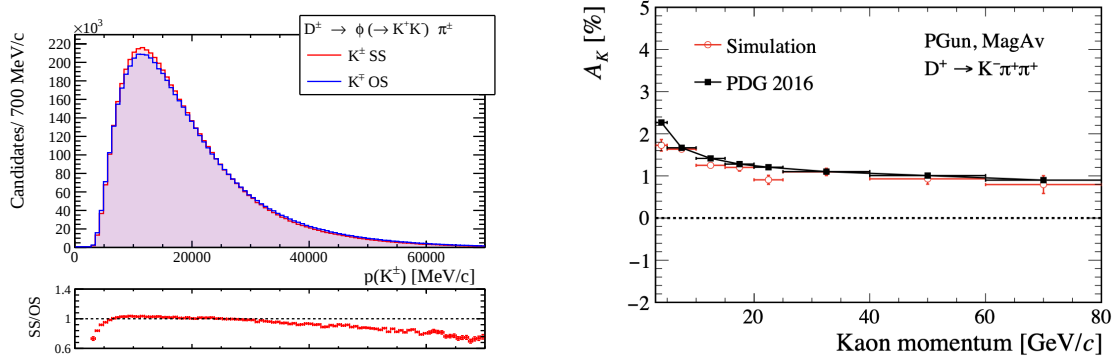


Figure 7.7: Left: distribution of the charged kaons momenta in the analysed $D^\pm \rightarrow \phi\pi^\pm$ sample. The red distribution refers to the charged kaons with the same sign (SS) of the π^+ , while the blue distribution is relative to the opposite sign (OS) charged kaons. The bottom plot shows the ratio between the SS and OS distributions. Right: simulated charged kaon detection asymmetry ($A_D(K^+)$) as a function of the kaon momentum, taken from Ref. [8].

7.2 Systematic uncertainty on $A_{CP}(D^+ \rightarrow \phi\pi^+)$

In section 6.3 the two developed methods to measure $A_{CP}(D^+ \rightarrow \phi\pi^+)$ are presented: the first uses the FAT predictions for both r_π and δ_π , while in the second r_π and δ_π are optimised on data using the *Profile Likelihood* method. This section presents the estimate of systematic uncertainties on both $A_{CP}(D^+ \rightarrow \phi\pi^+)$ measurements, which are summarised in table 7.5.

The main sources of systematic uncertainty accounted for in the measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$ are related to the measurement of the $\Delta A_{K_S^0}(t)$ asymmetry in the different decay-time bins. As discussed above, the systematic uncertainty on $\Delta A_{K_S^0}$ depends mainly on the choice of the fit model in the simultaneous mass fits, while the contamination from secondary decays plays a minor role, as it gives rise to systematic uncertainties smaller by one order of magnitude. For this reason, only the systematic uncertainties due to the choice of the fit models are taken into account for the measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$. To quantify such effects, the estimate of $A_{CP}(D^+ \rightarrow \phi\pi^+)$ is repeated on the $\Delta A_{K_S^0}(t)$ measurements built using the mass fits models described in section 7.1, with the two presented methods. The systematic uncertainty on $A_{CP}(D^+ \rightarrow \phi\pi^+)$ due to the fit model is then estimated as half the maximum deviation among such measurements. The systematic uncertainty on the FAT prediction result is $\sigma_{sys}^{(model)}(A_{CP}(D^+ \rightarrow \phi\pi^+)) = 0.6 \times 10^{-4}$, while systematic uncertainty on the *Profile Likelihood* method result is $\sigma_{sys}^{(model)}(A_{CP}(D^+ \rightarrow \phi\pi^+)) = 0.7 \times 10^{-4}$, as shown in figures 7.8 and 7.9.

Another source of uncertainty in the first method is due to the knowledge of δ_π and r_π parameters, since they are both fixed to the FAT prediction in the fit to the data. The associated systematic uncertainty is estimated as the maximum deviation of the offset by varying their values within $\pm 1\sigma$, where σ refers to the errors given by the FAT prediction, $\sigma_{r_\pi} = 0.004$ and $\sigma_{\delta_\pi} = 0.05$. It results in a maximum error of $\sigma_{sys}^{(model)}(A_{CP}(D^+ \rightarrow \phi\pi^+)) = 0.2 \times 10^{-4}$, which is much lower than the statistical uncertainty of the measurement, as expected.

In contrast, when using the *Profile Likelihood* method to determine the value of $A_{CP}(D^+ \rightarrow \phi\pi^+)$, the statistical uncertainty on the offset automatically accounts for the lack of knowledge of the free nuisance parameters. However, a rough estimate of the contribution to the total uncertainty due to r_π and δ_π parameters can be extracted by looking at the increase of the uncertainty with respect to what it is found by fixing both r_π and δ_π to the best knowledge, which is the FAT prediction. The statistical uncertainty found with the Profile Likelihood method is $(^{+3.4}_{-5.1}) \times 10^{-4}$, while that

obtained fixing the values of nuisance parameters is $\pm 3.3 \times 10^{-4}$. Thus, the contribution to the uncertainty due to r_π and δ_π can be roughly estimated from the difference of the squares of the two uncertainties:

$$\sigma_{sys}^{(r_\pi, \delta_\pi)} = \left(\begin{smallmatrix} +0.8 \\ -3.9 \end{smallmatrix} \right) \times 10^{-4}.$$

This contribution to the total uncertainty has a statistical nature, and it will decrease as the size of the available data samples will increase. Smaller error bars on the $\Delta A_{K_S^0}(t)$ measurements not only will better constrain the value of the offset, namely $A_{CP}(D^+ \rightarrow \phi\pi^+)$, but they will also drastically increase the capability of narrowing the allowed phase space of r_π and δ_π values. The associated systematic uncertainties are therefore expected to decrease as the statistical uncertainty of the observable of interest, always remaining at a lower level. It is worth mentioning that during Run 3 and Run 4, LHCb will collect much larger and more enriched data samples of decay modes with downstream $K_S^0(DD)$. This will largely impact on the determination of the neutral kaon detection asymmetry input parameters, such as those studied in this thesis.

Source	σ_{sys} FAT prediction [$\times 10^{-4}$]	σ_{sys} <i>Profile Likelihood</i> [$\times 10^{-4}$]
Fit Model	± 0.6	± 0.7
Secondaries	$\lesssim 0.1$	$\lesssim 0.1$
δ_π and r_π parameters	± 0.2	$\begin{smallmatrix} +0.8 \\ -3.9 \end{smallmatrix}$ ¹
Statistical uncertainty	± 3.3	$\begin{smallmatrix} +3.4 \\ -5.1 \end{smallmatrix}$
Total uncertainty	± 3.4	$\begin{smallmatrix} +3.5 \\ -5.2 \end{smallmatrix}$

Table 7.5: Systematic and total uncertainties for both measurements of $A_{CP}(D^+ \rightarrow \phi\pi^+)$ using the full Run 2 dataset. FAT refers to the model-dependent method, in which both r_π and δ_π are fixed to the FAT prediction, while *Profile Likelihood* refers to the measurement where δ_π and r_π are treated as nuisance parameters.

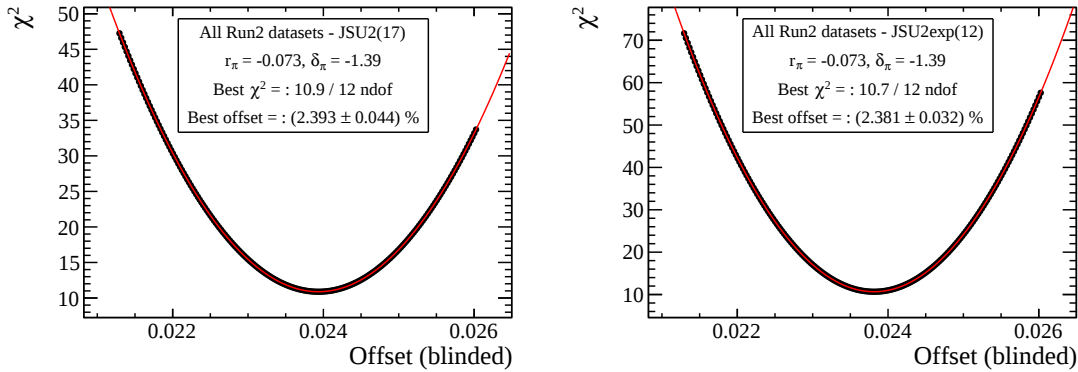


Figure 7.8: Dispersion between the blinded $A_{CP}(D^+ \rightarrow \phi\pi^+)$ results with the FAT prediction template fit method using the different fit models for the $\Delta A_{K_S^0}(t)$ measurements.

¹already included in the statistical uncertainty

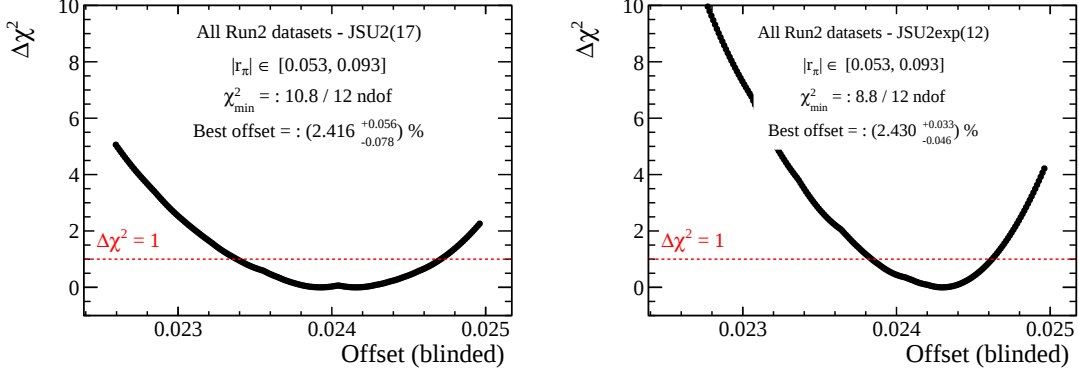


Figure 7.9: Dispersion between the blinded $A_{CP}(D^+ \rightarrow \phi\pi^+)$ results with the *Profile Likelihood* method using the different fit models for the $\Delta A_{K_S^0}(t)$ measurements.

7.3 Systematic uncertainty on the r_π, δ_π confidence intervals

The measurement of the parameters r_π and δ_π is achieved building a p -value map in the 2D parameter space, which serves as the basis for establishing 2D confidence intervals at any CL threshold. Section 6.2 presents the statistical methodology used to compute the p -value for any (r_π, δ_π) pair, based on comparing the 13 measurements of $\Delta A_{K_S^0}(t)$ with the time-dependent models. However, the initial p -value map, shown in 6.11, is computed using only the statistical uncertainties. For this reason, the p -value map is built again by means of the *Plugin* method incorporating now also the total uncertainty, which includes the systematic contributions, as detailed in table 7.1.

Figure 7.10 provides a comparison of the resulting p -value maps: the left panel shows p -values based solely on statistical uncertainties, while the right panel incorporates total uncertainties. As expected, a broader distribution of p -values across the $r_\pi - \delta_\pi$ parameter space is obtained. Despite this loss in sensitivity, the analysis still achieves high confidence in excluding large regions of the $r_\pi - \delta_\pi$ space.

Section 6.2 and appendix C present an additional measurement of r_π and δ_π using the abundant $D_s^+ \rightarrow \phi\pi^+$ calibration sample, which reduces the uncertainties on $\Delta A_{K_S^0}$ by a factor of 0.7. Due to time constraints, as this was a side project of the thesis, the systematic uncertainties for the new $\Delta A_{K_S^0}(t)$ measurements obtained with the D_s^+ calibration sample have not been estimated. The primary sources of systematic uncertainty are expected to be the same as those discussed in section 7.1, along with the bias in the time dependence of $A_D(K_S^0)$ caused by the different production asymmetry in the D_s^+ sample. This bias is studied in appendix C and is shown to be negligible. Due to the absence of an estimate for the systematic uncertainties, this measurement is not included in the final results reported in chapter 8. However, as a future development of this work, the measurement of r_π and δ_π could be repeated with the estimate and incorporation of systematic uncertainties.

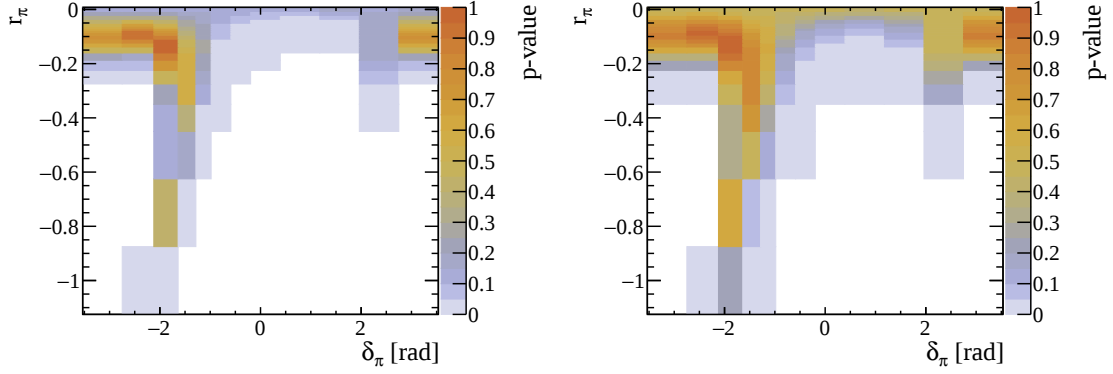


Figure 7.10: Map of the p -value computed with the plugin method for different r_π, δ_π values in the scan, using only the statistical uncertainties (left) and the total uncertainties (right).

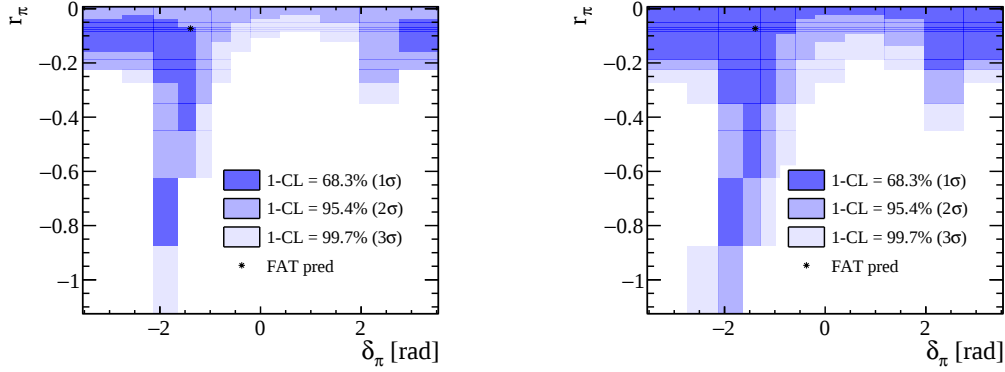


Figure 7.11: Confidence intervals, built applying different $1 - CL$ thresholds, as detailed in the legend, to the p -value map, using only the statistical uncertainties (left) and the total uncertainties (right).

7.4 Knowledge of the LHCb material map

In this analysis, it is assumed that the discrepancies between the LHCb detector software description and the real detector have a negligible impact on the measurements presented in this thesis, as on many other CP violation measurements in the charm sector [43, 7]. Over the years, the level of accuracy of the description of the LHCb detector material map has been extensively validated in various internal and published works [75, 76], even using charged kaons. In particular, Ref. [76] presents a detailed discussion on the uncertainties on the description of the material budget crossed by a particle passing through the LHCb detector. The associated overall uncertainty is conservatively estimated to be around 10%. Specifically, major contributions to this uncertainty arise from the description of the conical beam pipe inside the RICH1 and of the VELO, while the solid (aerogel) and gas (C_4F_{10}) radiators of the RICH1 are characterised with better precision ($\sim 1 - 2\%$). Figure 7.12 shows the estimated material budget present in VELO ($z \in [0, 770]$ mm) and RICH1 ($z \in [770, 2270]$ mm) detectors². The region allowed in the pseudo-rapidity of neutral kaons in the analysis of this thesis is $\eta_{K_S^0} \in [2.2, 4.2]$, as shown in figure 7.13, in order to specifically avoid these regions where the kaons pass through a lot of material in the vicinity of RICH1.

In the model used to predict the kaon asymmetry, the knowledge of the material is essential to

²As detailed in table 4.2, the HLT2 trigger selects $K_S^0(LL)$ candidates decayed in the interval $z \in [-100, 500]$ mm and $K_S^0(DD)$ candidates decayed in $z \in [300, 2275]$ mm.

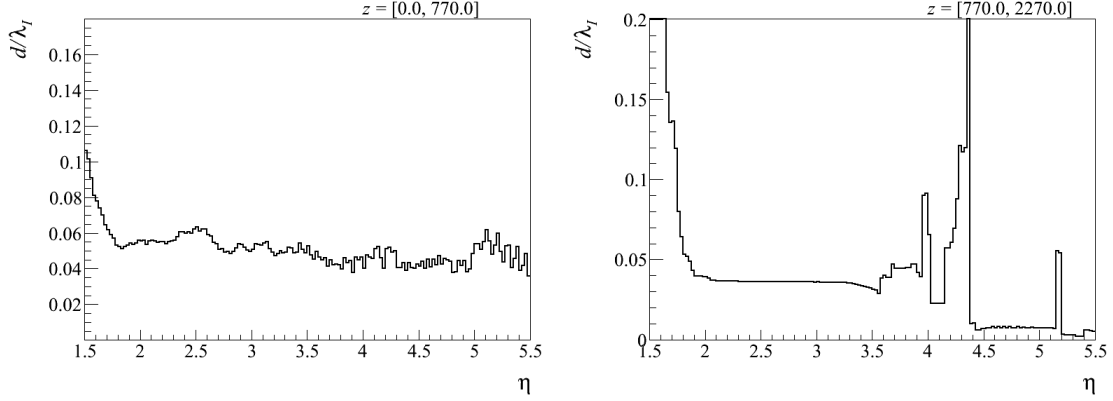


Figure 7.12: The amount of material or distance (d) that a particle has to traverse in the VELO (left) and in the RICH1 (right), in terms of hadronic interaction lengths (λ_I , inversely proportional to the material density ρ). Figure taken from [77], which is an internal page accessible only to the LHCb Collaboration members.

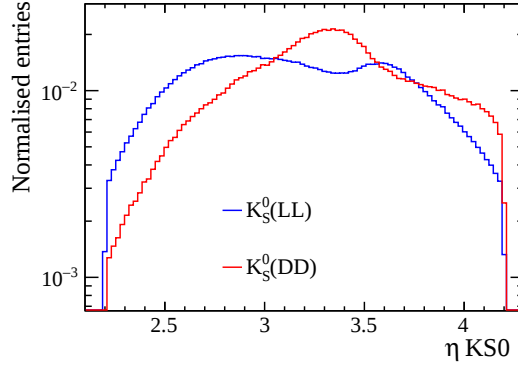


Figure 7.13: Distribution of $\eta(K_S^0)$ in the $D^+ \rightarrow K_S^0(LL)\pi^+$ and $D^+ \rightarrow K_S^0(DD)\pi^+$ samples used in this analysis. The cut $\eta_{K_S^0} \in [2.2, 4.2]$ is evident for both samples.

compute $\Delta\chi \propto \rho \cdot A^{-0.242}$ (see sections 1.6.2 and 1.6.3 for reference). In particular, the quantity to which the asymmetry is most sensitive is the material density ρ , therefore, the associated systematic uncertainty can be studied by looking at the variations of the templates when the density is varied. Following the approach used in Refs. [6, 43], the uncertainty due to the material map can be determined by shifting the material density, ρ , or the atomic weight, A , coherently in the different steps of the computation of the asymmetry by $\pm 10\%$. The asymmetry templates generated as a function of ρ and A are shown in figure 7.14 giving a quantitative size of the effect in each decay-time bin that can be directly compared with the statistical uncertainties obtained in our analysis. For each K_S^0 decay-time bin, the semi-dispersion between the asymmetry values computed with $\rho - 10\%$ and $\rho + 10\%$ or the $A - 10\%$ and $A + 10\%$ templates, is summarised in tables 7.6 and 7.7 and compared with the statistical uncertainty of the relative time bin. The impact of inaccuracies in reproducing the detector material is minimal, as highlighted by the results. This conclusion is further supported by the conservative nature of the approach for three main reasons: the $\pm 10\%$ variation from Ref. [76] is itself a conservative estimate; this variation is primarily driven by uncertainties in the material of the conical beam-pipe near the RICH1, placed in a pseudo-rapidity region removed from our data; more importantly, the effect is modelled coherently, which is very unlikely to occur in reality.

The impact of these 10% variations on the confidence intervals of r_π and δ_π parameters is expected to be negligible with the current data samples. The situation is slightly different for the measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$. In fact, the determination of the offset is dominated by the first bins in the long and downstream samples, which have lower uncertainties. While in the $K_S^0(LL)$ sample the effects due to the material are completely negligible, in the first $K_S^0(DD)$ bins they are of the order of a few units in 10^{-4} , comparable with the statistical uncertainties shown in table 7.7. However, they are always smaller than the statistical uncertainties by factors ranging from 3 to 25. Considering that this is a significant overestimate of the material budget uncertainty, the associated systematic error is therefore significantly smaller than that displayed in table 7.7, and can be safely neglected for the moment.

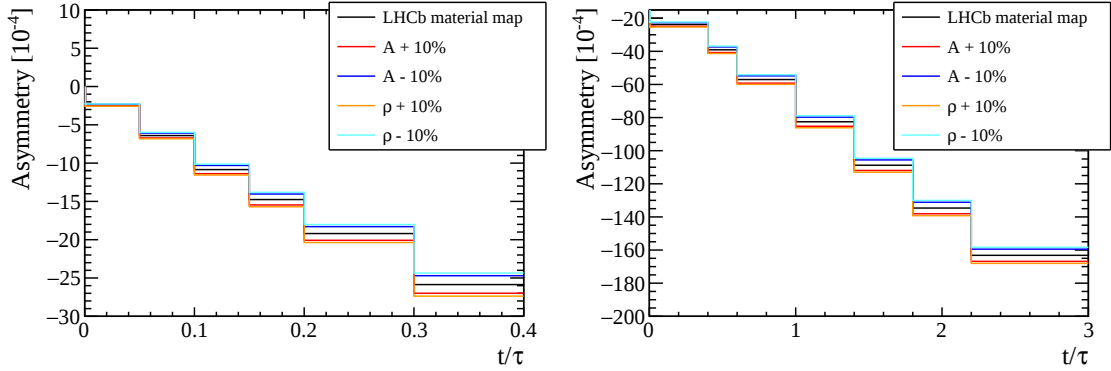


Figure 7.14: Comparison of the asymmetry templates in the long (left) and downstream (right) bins using the LHCb material map and shifting coherently all the densities of the materials in the map by +10% and -10%. In this simulation, r_π and δ_π have been fixed to the FAT prediction.

$K_S^0(LL)$ bin	$\sigma_{sys}(\rho \pm 10\%) [\times 10^{-4}]$	$\sigma_{sys}(A \pm 10\%) [\times 10^{-4}]$	$\sigma_{tot} [\times 10^{-4}]$
[0.00, 0.05]	0.14	0.11	9.9
[0.05, 0.10]	0.42	0.32	10.9
[0.10, 0.15]	0.71	0.54	17.5
[0.15, 0.20]	0.94	0.71	22.9
[0.20, 0.30]	1.17	0.89	35.2
[0.30, 0.40]	1.51	1.14	154.1

Table 7.6: Estimate of the systematic uncertainties on the K_S^0 time-dependent detection asymmetry due to the knowledge of the LHCb material for the different $K_S^0(LL)$ bins. As a comparison, the last column reports the total uncertainty in each bin, taken from table 7.1.

In principle it would be possible to treat ρ as an additional nuisance parameter in the fit to data, as done for r_π and δ_π . However, ρ is correlated with both r_π and δ_π , leading to a non negligible inflation of the statistical final uncertainty on $A_{CP}(D^+ \rightarrow \phi\pi^+)$. This can be certainly done, especially considering that this source of uncertainty is statistical in nature, similar to that of r_π and δ_π . Consequently, it will decrease as the size of the data samples increases. Nonetheless, this approach seems very conservative at present and is not adopted in this thesis.

Decay channels of D^+ and D^0 mesons with a neutral kaon in the final state, accessible solely through Cabibbo-Favoured amplitudes, provide an ideal system to assess the accuracy of the LHCb detector simulation and the impact of such uncertainties on the prediction of $A_D(K_S^0)$. For instance,

$K_S^0(DD)$ bin	$\sigma_{sys}(\rho \pm 10\%) [\times 10^{-4}]$	$\sigma_{sys}(A \pm 10\%) [\times 10^{-4}]$	$\sigma_{tot} [\times 10^{-4}]$
[0.0, 0.4]	1.5	1.1	7.9
[0.4, 0.6]	2.2	1.6	8.9
[0.6, 1.0]	2.9	2.2	8.6
[1.0, 1.4]	3.6	2.7	15.5
[1.4, 1.8]	4.2	3.2	26.1
[1.8, 2.2]	4.6	3.5	57.7
[2.2, 3.0]	4.9	3.7	125.5

Table 7.7: Estimate of the systematic uncertainties on the K_S^0 time-dependent detection asymmetry due to the knowledge of the LHCb material for the different $K_S^0(DD)$ bins. As a comparison, the last column reports the total uncertainty in each bin, taken from table 7.1.

in semileptonic D^+ and D^0 decays with a K_S^0 in the final state, such as the $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ and $D^0 \rightarrow K^{*-} (\rightarrow \bar{K}^0 \pi^-) \mu^+ \nu_\mu$, only the Cabibbo Favoured decay is allowed, as shown in figure 7.15. Hence, the study of the time-dependent asymmetry $A_D(K_S^0)$ in this decay mode would primarily probe two effects: the CP violation in the mixing, a well known phenomenon, and the interaction with the material, depending on the software-based LHCb material description. Any disagreement between the measured asymmetry and the models would highlight inaccuracies in the description of the LHCb detector. Detecting such discrepancies requires collecting a large and pure sample of these decays, which unfortunately have not been collected by the LHCb experiment in the past Runs. Thanks to the findings of this thesis work, it will be very important to immediately implement a new trigger line with the aim of specifically collecting these decays. Moreover, it is crucial for the future measurements to collect increasingly larger samples of decays with neutral kaons in the final state, reconstructed outside the acceptance of the vertex detector, to effectively constrain the model in a wide time range.

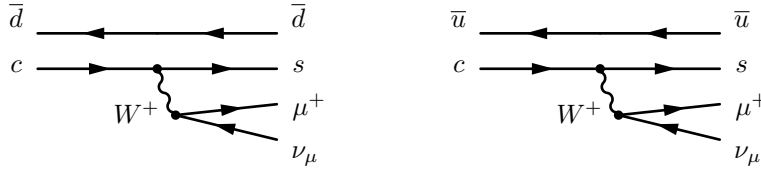


Figure 7.15: Feynman diagram for the $D^+ \rightarrow \bar{K}^0 \mu^+ \nu_\mu$ decay (left) and $D^0 \rightarrow K^{*-} \mu^+ \nu_\mu$ (right), in which the K^{*-} can be reconstructed from its strong decay into the $\bar{K}^0 \pi^-$ final state.

7.5 Consistency checks

Consistency checks are performed to identify and rule out possible systematic uncertainties not accounted for in the analysis. The measured time-dependence of $A_D(K_S^0)$ and the result for $A_{CP}(D^+ \rightarrow \phi \pi^+)$ must be independent of the running conditions, detector status, and dipole magnet polarity. To ensure this, the data set is divided into non-overlapping subsets based on the data-taking year and magnet polarity, and the analysis is repeated independently for each sub-sample.

The first type of check involves testing the compatibility of the results obtained from the six sub-samples, divided by data-taking year and magnet polarity. This check is sensitive to any

dependencies on running conditions, detector status, detector ageing, and trigger settings. The second type of check looks for any bias arising from residual detection asymmetries dependent on the magnet polarity, which may not cancel out precisely after performing the differences between the *raw* asymmetries in the two samples. To address this, the analysis is repeated using the *MagUp* and *MagDown* datasets separately.

7.5.1 Time-dependence of $A_D(K_S^0)$

In figure 5.11, the compatibility between the 13 $\Delta A_{K_S^0}(t)$ measurements obtained using the six Run 2 datasets is shown to be good, with a *p-value* of 30%. In order to look for any dependence on the magnet-polarity at higher precision, the $\Delta A_{K_S^0}(t)$ values are estimated using, separately, the *MagUp* or *MagDown* datasets. The results are shown in figure 7.16 for the $K_S^0(LL)$ and $K_S^0(DD)$ samples and the agreement is good in all the K_S^0 decay time bins, as highlighted by the *p-value* of 30%. The dependence of the 13 $\Delta A_{K_S^0}(t)$ measurements on the K_S^0 decay time is used as an input to compute the confidence intervals in the $r_\pi - \delta_\pi$. Therefore, this check ensures that also the measurement of r_π and δ_π is stable and independent on the detector conditions.

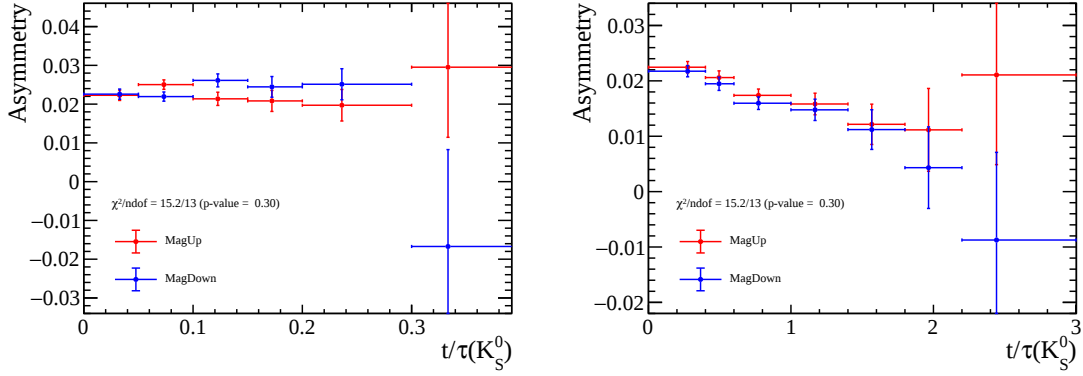


Figure 7.16: Comparison between the 13 $\Delta A_{K_S^0}(t)$ measurements using only the *MagUp* and *MagDown* datasets in the $K_S^0(LL)$ (left) and $K_S^0(DD)$ (right) bins.

7.5.2 Measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$

In order to verify the stability of the measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$ on the detector conditions, the procedure described in 6.3 is repeated separately on the different data samples, divided by year and dipole magnet polarity. This section only presents the results obtained with the *Profile Likelihood* method, treating r_π and δ_π as nuisance parameters. In figure 7.17 the results for the best fit blinded offset are reported separately for the six Run 2 datasets, divided by year and magnet polarity. The average offset, $(2.402 \pm 0.03)\%$, and the χ^2 is computed assuming symmetric uncertainties for the six measurements. In addition, figure 7.18 shows the $\Delta\chi^2$ dependence on the offset using only the *MagUp* or *MagDown* datasets.

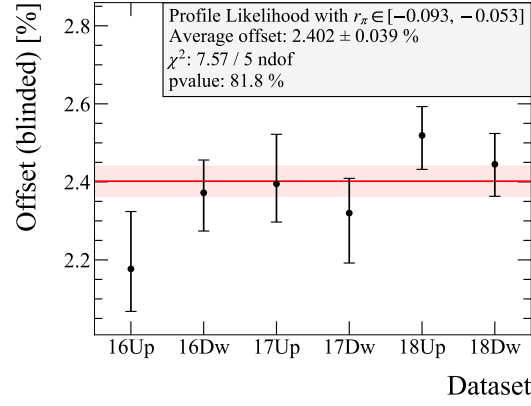


Figure 7.17: Right: Results of the optimal offset with the *Profile Likelihood* method for each dataset divided by year and magnet polarity. In the legend, the average blinded offset is shown and we can see that the statistical uncertainty is 3.9×10^{-4} . The statistical uncertainties on the different measurements are asymmetrical, but the χ^2 and the average are computed assuming them to be symmetric.

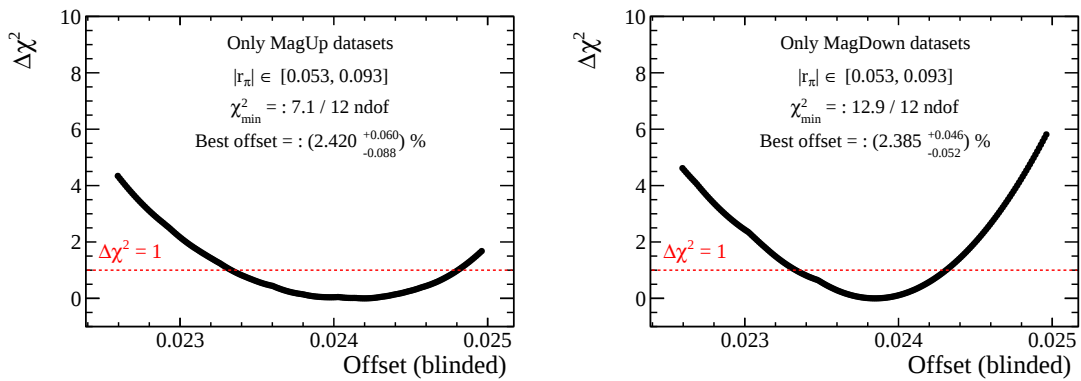


Figure 7.18: $\chi^2(q)$ plot using only the *MagUp* (left) and *MagDown* (right) datasets. The results are both compatible, within the statistical uncertainties, with each other.

Chapter 8

Results and conclusions

This chapter presents the main results of this thesis work: the confidence intervals in the two-dimensional space of the DCS-CF interference parameters in the $D^+ \rightarrow K_S^0 \pi^+$ decay mode, r_π and δ_π , and the measurement of the direct CP asymmetry in the $D^+ \rightarrow \phi \pi^+$ decay channel, $A_{CP}(D^+ \rightarrow \phi \pi^+)$. Finally, the future perspectives of the developed method and its impact on the current charm CP violation measurements are discussed.

8.1 Measurement of r_π and δ_π

This thesis presents, for the first time, a measurement of the parameters describing the interference of favoured and suppressed processes in the $D^+ \rightarrow K_S^0 \pi^+$ decay channel, namely r_π and δ_π , independent of any input from strong amplitude models. This novel measurement is made possible by extending the current models for neutral kaon detection asymmetry in matter to include contributions from the interference of Cabibbo-Favoured and Doubly Cabibbo-Suppressed charm decays with $K^0 - \bar{K}^0$ mixing. The parameters r_π and δ_π are measured thanks to these interference effects, which become significant at high values of the K_S^0 decay time.

The confidence intervals for the $r_\pi - \delta_\pi$ parameters are measured using a fully data-driven method, without any external inputs, except for the well-known physical observables describing the time evolution of neutral kaons in vacuum and in matter. These regions are identified by comparing the time-dependence of the neutral kaon detection asymmetry, measured with the full Run 2 dataset, with models generated using different values of r_π and δ_π . The final result, accounting for both statistical and systematic uncertainties, is shown in figure 8.1. Thanks to the large statistics of decays collected by the LHCb experiment during LHC Run 2 period (2015-2018), about 50 million $D^+ \rightarrow K_S^0 \pi^+$ and 30 million $D^+ \rightarrow \phi \pi^+$ candidates, a wide region in the $r_\pi - \delta_\pi$ space can be excluded with a high confidence level. The only available model-dependent estimate of these parameters comes from Ref. [9], which uses the Factorisation-Assisted Topological Amplitudes (FAT) approach. The FAT prediction is $r_\pi = -0.073 \pm 0.004$ and $\delta_\pi = -1.49 \pm 0.05$ and falls in the 1σ region of the allowed region found in this thesis work.

8.2 Measurement of $A_{CP}(D^+ \rightarrow \phi \pi^+)$

A by-product of this work is the measurement of the direct CP asymmetry in the $D^+ \rightarrow \phi \pi^+$ decay using the full LHCb Run 2 dataset, corresponding to an integrate luminosity of 5.4 fb^{-1} :

$$A_{CP}(D^+ \rightarrow \phi \pi^+) \Big|_{Profile \text{ Likelihood}}^{\text{blind}} = (2.413^{+0.034}_{-0.051}) \%,$$

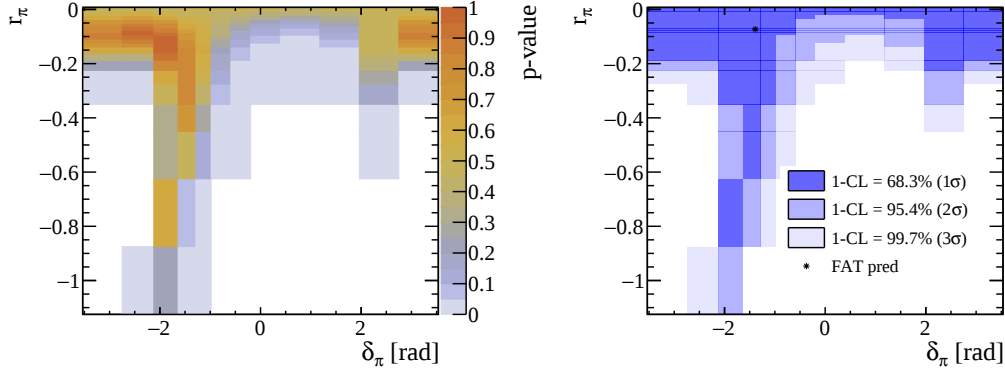


Figure 8.1: Left: map of the p -value computed with the *Plugin* method for different r_π and δ_π values. Result include both statistical and systematic uncertainties. Right: confidence intervals at 1, 2 and 3 σ , built applying different thresholds to the p -value map.

where the reported uncertainty accounts for both statistical and systematic contributions, while the central value is blinded to avoid any experimenter's bias. This slightly improves upon the published LHCb result [8], which reports a total uncertainty of 5.1×10^{-4} .

The analysis presented in this thesis includes new data from the 2018 data taking period, corresponding to about 2 fb^{-1} , and, for the first time, utilises data samples with $K_S^0(DD)$ candidates in the final state, reconstructed without using hits from the VELO vertex detector. The inclusion of $D^+ \rightarrow K_S^0(DD)\pi^+$ sample does not significantly enhance the measurement of the constant term, $-A_{CP}(D^+ \rightarrow \phi\pi^+)$, which is dominated by the $D^+ \rightarrow K_S^0(LL)\pi^+$ sample. However, it is crucial for accurately treating the kaon detection asymmetry and the associated uncertainty due to its modelling. The new method developed in this thesis includes, for the first time, DCS-CF interference effects, optimising the r_π and δ_π parameters directly from data without any external input. This requires the inclusion of the $D^+ \rightarrow K_S^0(DD)\pi^+$ sample in the analysis, to precisely constrain the newly introduced interference effects.

The contribution to the total uncertainty due to the determination of the neutral kaon detection asymmetry term is approximately 2.7×10^{-4} , primarily due to the limited knowledge of the parameters r_π and δ_π . This contribution is statistically dominated and will decrease as the size of future data samples increases, allowing better constraints on these parameters. In contrast, the published measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$ [8] reports a systematic uncertainty associated with the estimate of neutral kaon asymmetry of 0.2×10^{-4} . This contribution is lower than that estimated in this thesis, because the assumption $r_\pi = 0$ is made, and only the $K_S^0(LL)$ sample, where the interference effects are smaller, is used. Furthermore, the published measurement of $A_{CP}(K^+K^-)$ [7, 78] also employs the $D^+ \rightarrow K_S^0(LL)\pi^+$ decay as a calibration sample. In this published analysis, the systematic uncertainty is assessed by assuming a linear correction to the model, which is estimated using the higher decay times of the $K_S^0(DD)$ sample. The study reported in this thesis demonstrates that a linear term is insufficient to accurately describe effects such as those arising from CF-DCS interference.

8.3 Conclusions and future prospects

This thesis extends the current model used to predict neutral kaon detection asymmetry in matter by incorporating contributions from the interference between Cabibbo-Favoured (CF) and Doubly Cabibbo-Suppressed (DCS) charm decays, as well as $K^0 - \bar{K}^0$ mixing. This is a significant and original contribution to the charm sector, as many of the most relevant and precise measurements of

CP violation rely on calibration channels that include a neutral kaon in the final state. For instance, published measurements of $A_{CP}(D^0 \rightarrow K^+K^-)$ [7] and $A_{CP}(D^+ \rightarrow \phi\pi^+)$ [8] use $D^+ \rightarrow K_S^0\pi^+$ and $D_s^+ \rightarrow K_S^0K^+$ as calibration channels. This is particularly relevant today, as the LHCb-Upgrade I experiment has started to steadily collect data during Run 3 at an instantaneous luminosity of $2 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ with higher efficiency per fb^{-1} due to a new DAQ and trigger system. The goal is to collect an additional 14 fb^{-1} during LHC Run 3 (2024-2025) and 27 fb^{-1} during LHC Run 4 (2029-2032), in preparation for the LHCb-Upgrade II phase at even higher instantaneous luminosity ($\geq$ LHC Run 5) [79].

This thesis presents the first measurement of the r_π and δ_π parameters in the $D^+ \rightarrow K_S^0\pi^+$ decay mode, a novel achievement for an experiment like LHCb, which collects data from high-energy proton-proton collisions. To achieve this, several new and original methodologies were developed. In particular, this is the first time decays with $K_S^0(DD)$ in the final state have been used to make a high-precision measurement. The procedure developed to split and equalise the kinematics of the $D^+ \rightarrow \phi\pi^+$ candidates with those of the $D^+ \rightarrow K_S^0\pi^+$, in bins of K_S^0 decay time, is innovative and ensures reliable cancellation of both production and detection asymmetries of the D^+ and the bachelor pion. Closely linked to this point, it is also worth mentioning that the online and offline selection criteria for candidates were revisited and improved to ensure more robust cancellation of these nuisance asymmetries. The treatment of D^+ mesons originating from b -hadron decays and having a $K_S^0(DD)$ in the final state is novel in LHCb and has proven to be very reliable. The presence of this contamination had previously prevented the use of these decays in all prior high-precision measurements by LHCb. Lastly, due to the unique nature of the measurement, a modern statistical approach was employed to determine the confidence intervals of r_π and δ_π and to handle nuisance parameters in the determination of $A_{CP}(D^+ \rightarrow \phi\pi^+)$.

As a future development of this thesis work, it is worth mentioning that the LHCb collaboration is currently working on another measurement of the time-integrated CP violation, $A_{CP}(D^0 \rightarrow K^+K^-)$, using Run 2 data with a new method employing a single calibration channel, $D^0 \rightarrow K_S^0\pi^+\pi^-$ [80]. This new approach will provide a measurement almost independent of previous ones [7], serving as an important test ground of our current understanding of CP violation in charm decays. This is particularly promising for the future, when the precision will approach the level of 1×10^{-4} . Using a single calibration channel, which has a topology very similar to that of the signal from the point of view of the online selection, ensures a more reliable cancellation of all subtle nuisance asymmetries. The model developed in this thesis for neutral kaon detection asymmetry is a fundamental ingredient for precisely measuring the contribution from $A_D(K_S^0)$ using data itself, which is not removed in the difference between the two raw asymmetries.

Finally, neutral kaons reconstructed outside the acceptance of the vertex detector, $K_S^0(DD)$, play a crucial role in these analyses. In fact, the interference effects become significant at large values of the K_S^0 decay time, which are inaccessible to $K_S^0(LL)$, decayed within the VELO detector. Consequently, the LHCb collaboration is working extensively to increase the efficiency of downstream track reconstruction at early stages of the DAQ and trigger chain. In this regard, the RETINA project [81, 82, 83] aims to implement a specialised processor for real-time event reconstruction on the readout FPGA boards of the tracking detectors. The RETINA Downstream Tracker [84] will enable the online reconstruction at the full bunch crossing frequency of 30 MHz of tracks downstream the magnet, using raw hits from the new Scintillating Fibre sub-detector. This will significantly increase the efficiency for long-lived particles, such as the K_S^0 , benefiting analyses like the one presented in this thesis, which study channels with a neutral kaon in the final state.

Appendices

Appendix A

Mass distributions and yields

This appendix reports the distributions, divided by year and magnet polarity, of the $K_S^0\pi^\pm$ and $\phi\pi^\pm$ invariant masses in the four used data samples: $D^+ \rightarrow K_S^0(LL)\pi^+$, $D^+ \rightarrow K_S^0(DD)\pi^+$, $D^+ \rightarrow \phi\pi^+$, and $D_s^+ \rightarrow \phi\pi^+$. The total yields of signal candidates in each dataset, divided by year and magnet polarity, are reported in the plots.

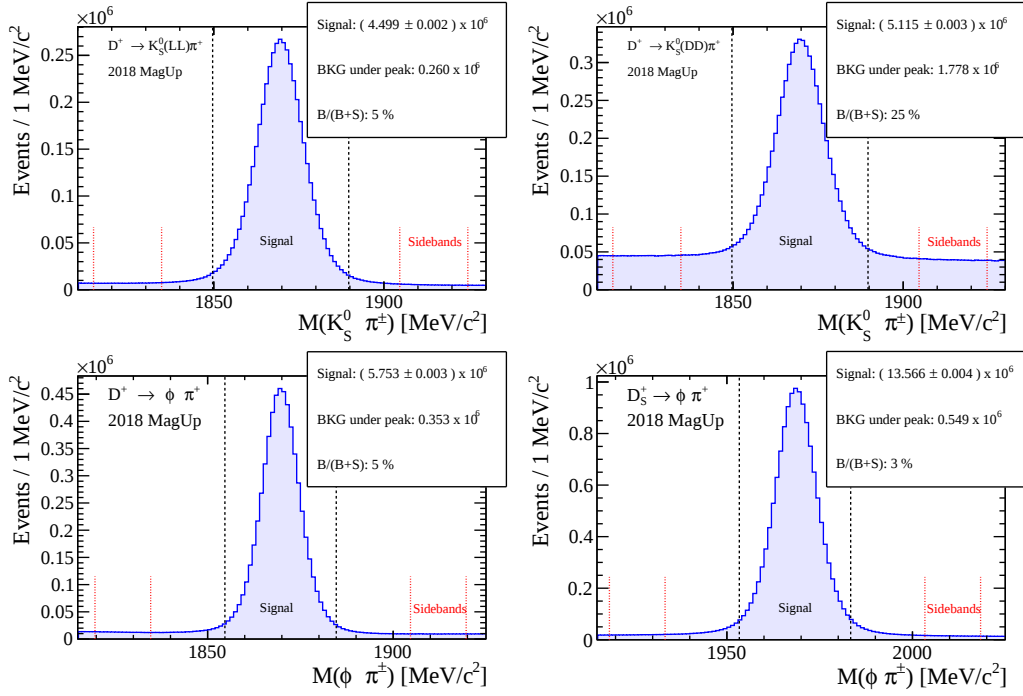


Figure A.1: Invariant mass distributions and yields for the 2018 MagUp data sample.

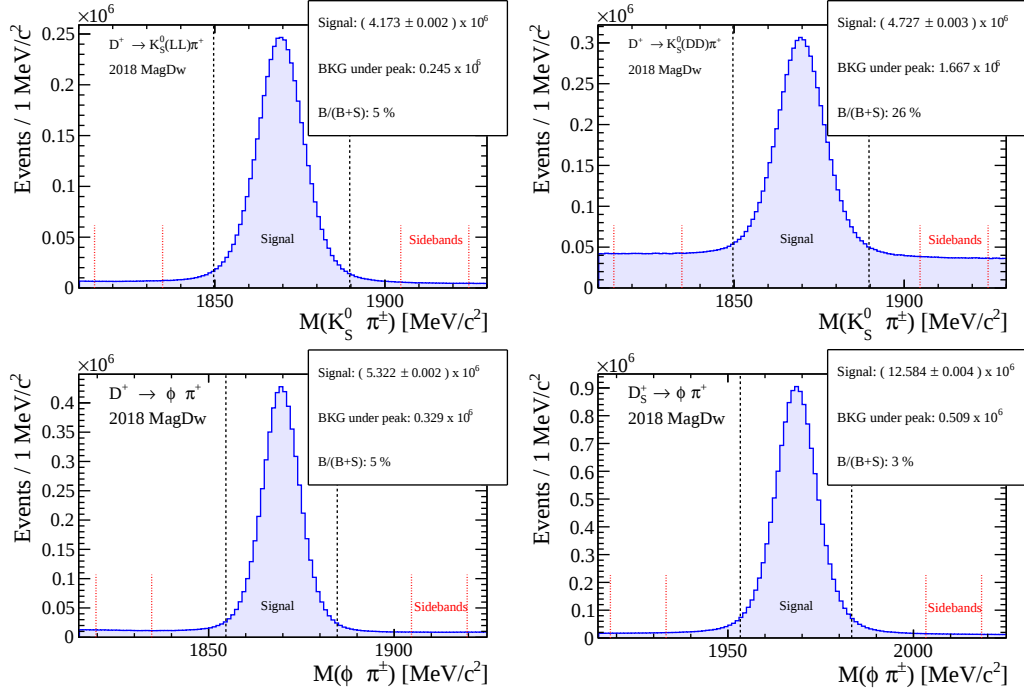


Figure A.2: Invariant mass distributions and yields for the 2018 MagDown data sample.

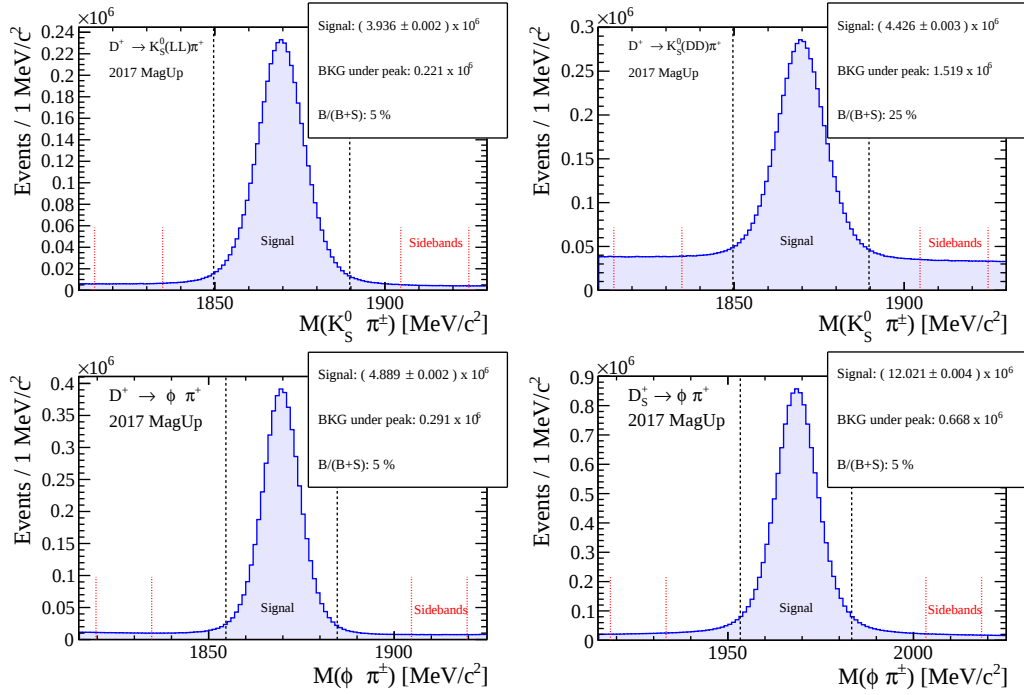


Figure A.3: Invariant mass distributions and yields for the 2017 MagUp data sample.

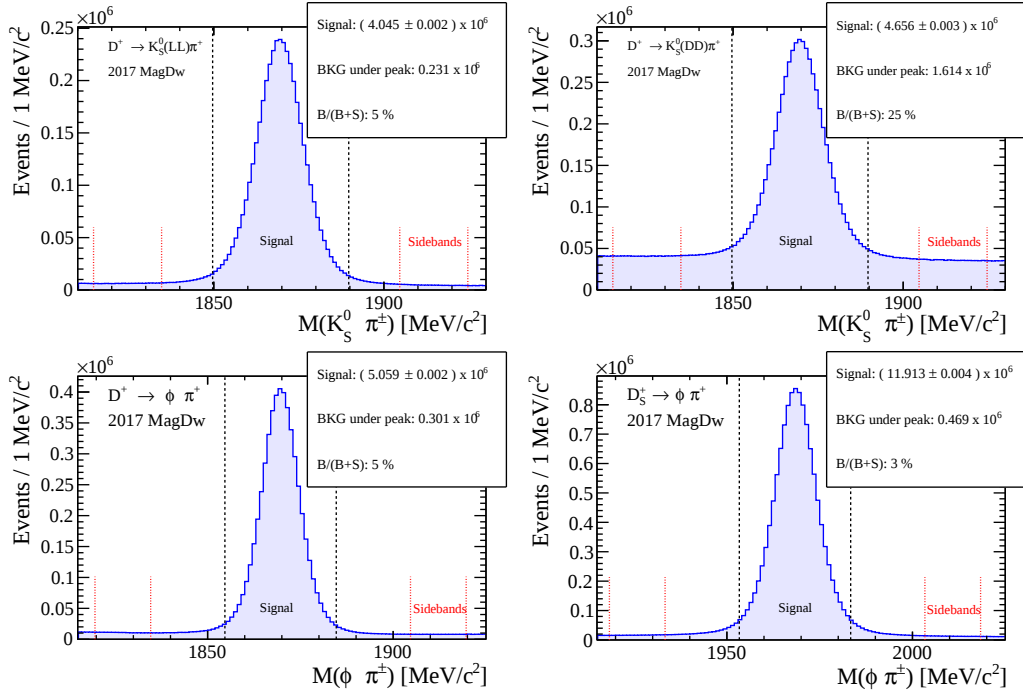


Figure A.4: Invariant mass distributions and yields for the 2017 MagDown data sample.

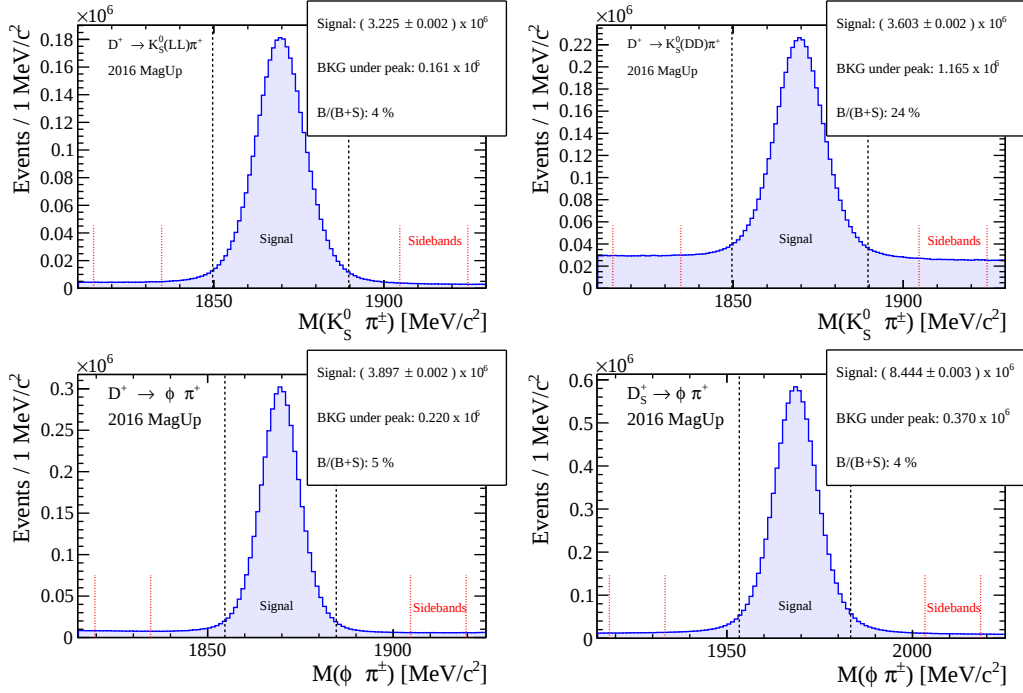


Figure A.5: Invariant mass distributions and yields for the 2016 MagUp data sample.

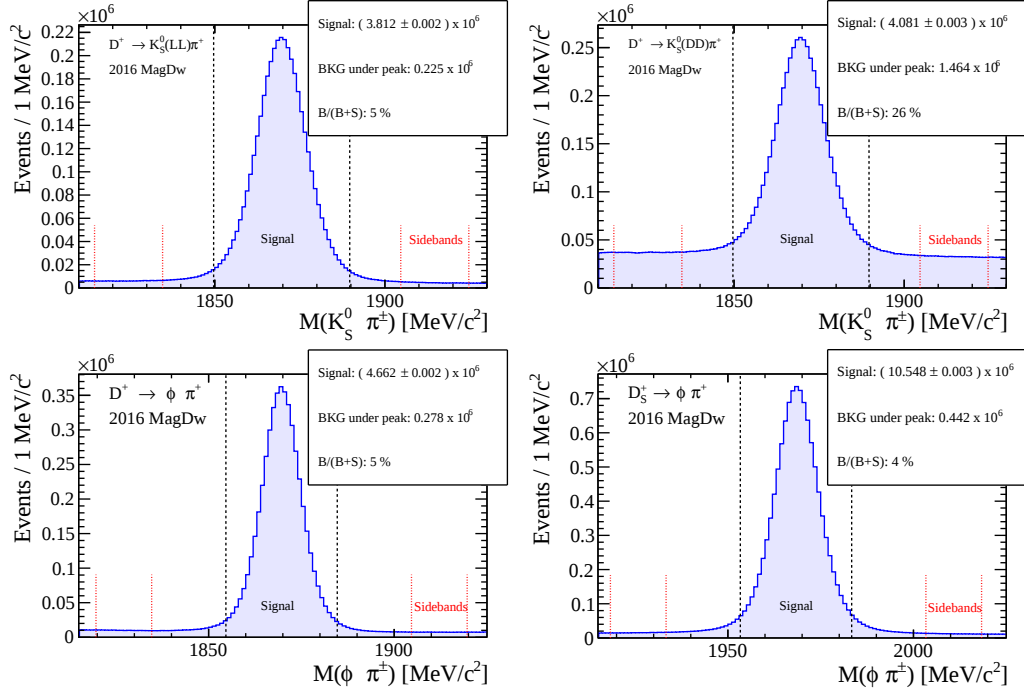


Figure A.6: Invariant mass distributions and yields for the 2016 MagDown data samples.

Appendix B

Kinematic weighting

This appendix reports some additional details on the kinematic weighting procedure, described in section 5.3. Figure B.1 reports the distribution of the π^+ curvature in four K_S^0 decay time bins. Figures B.2 and B.3 show time-integrated kinematic variables distributions before and after the kinematic weighting procedure.

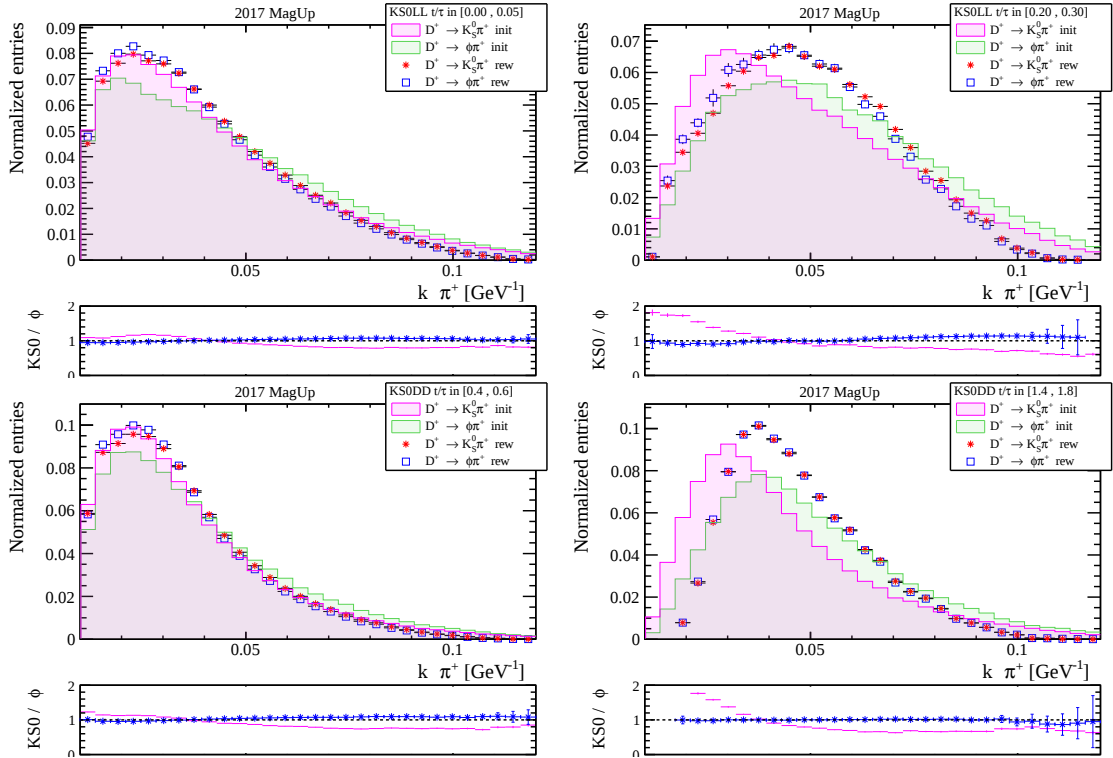


Figure B.1: Distributions of the curvature of the π^+ in several K_S^0 decay time bins.

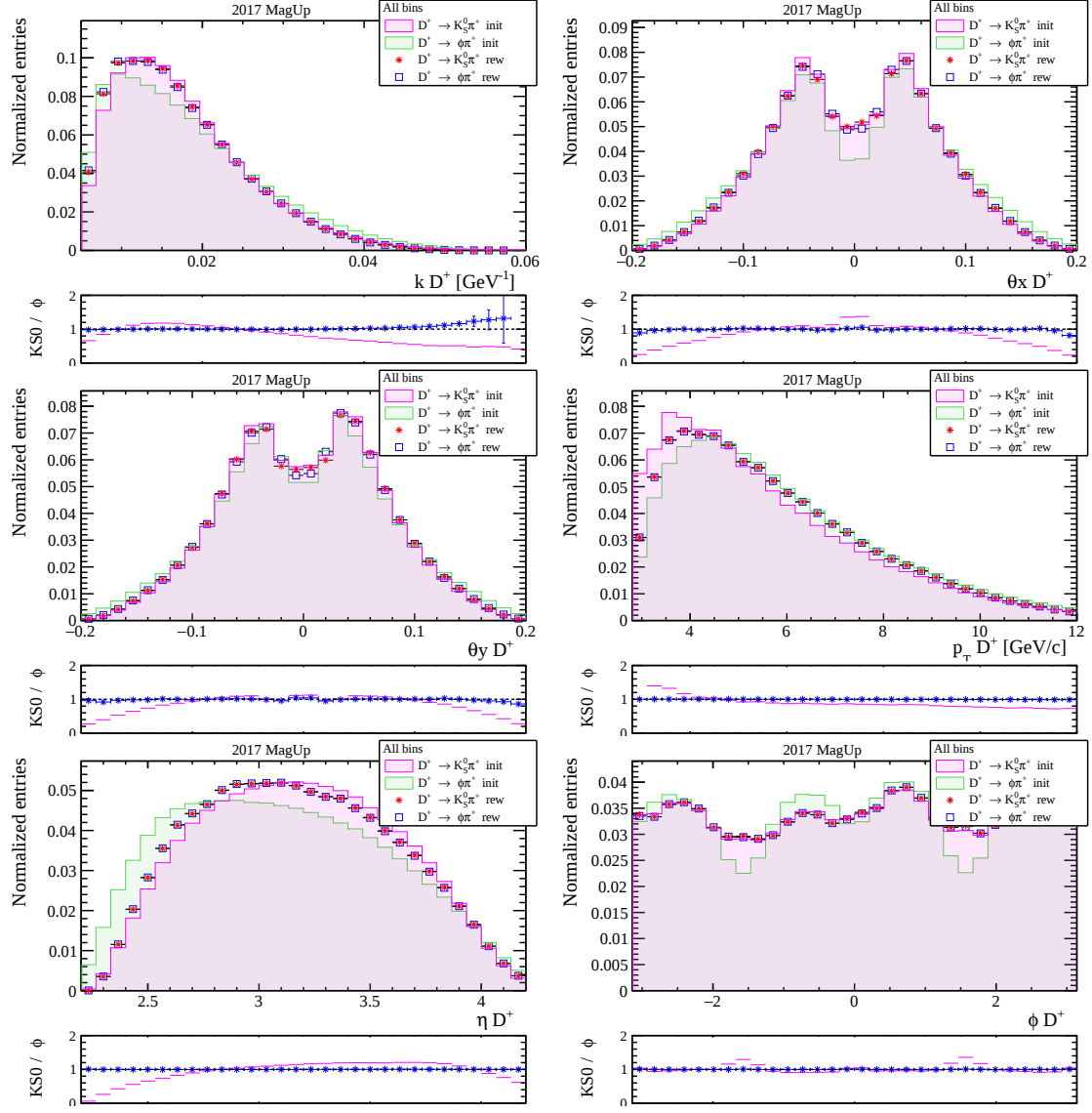


Figure B.2: Distributions of the kinematic variables of the D^+ , using the time-integrated $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ samples. The initial histograms have been filled with sidebands subtraction weights. On the bottom part of each plot, the ratio between the $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ histograms is shown before (pink line) and after (blue line) the weighting procedure.

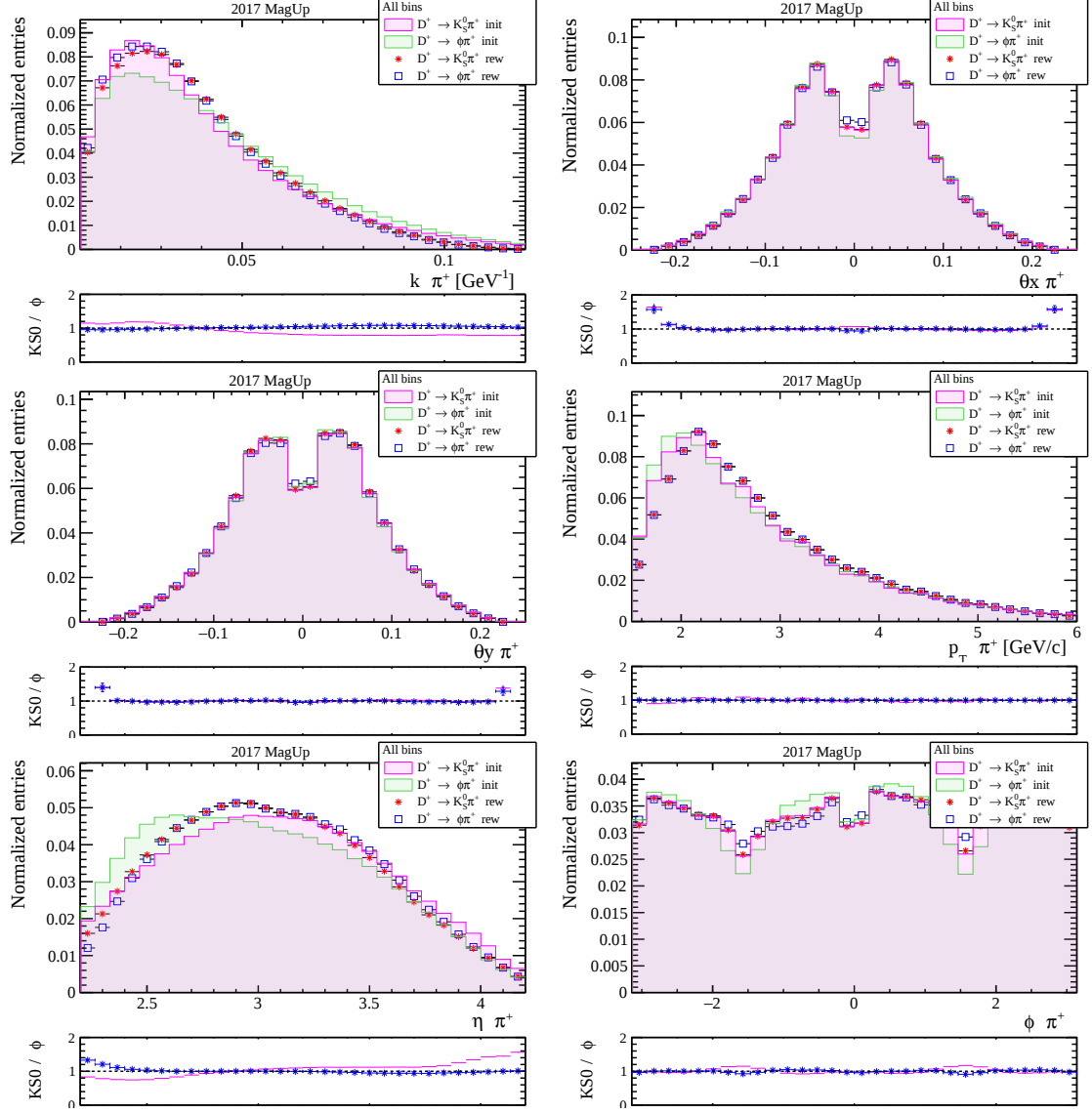


Figure B.3: Distributions of the kinematic variables of the π^+ , using the time-integrated $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ samples. The initial histograms have been filled with sidebands subtraction weights. On the bottom part of each plot, the ratio between the $D^+ \rightarrow K_S^0 \pi^+$ and $D^+ \rightarrow \phi \pi^+$ histograms is shown before (pink line) and after (blue line) the weighting procedure.

Appendix C

Increase of the resolution on r_π and δ_π using $D_s^+ \rightarrow \phi\pi^+$ decays

One of the aims of this thesis is to estimate with the highest precision the time-dependence of the K_S^0 asymmetry. However, the resolution on $A_D(K_S^0, t)$ is limited by the statistics of the calibration sample, $D^+ \rightarrow \phi\pi^+$. In fact, as we can see in table 4.6, the yield of the $D^+ \rightarrow K_S^0\pi^+$ sample, including both long and downstream samples, is about twice the one of the calibration sample, thus the statistical uncertainty on $\Delta A_{K_S^0}(t)$ is dominated by the $D^+ \rightarrow \phi\pi^+$ channel.

One solution to have higher precision on the time-dependence of the K_S^0 asymmetry would be to include the $D_s^+ \rightarrow \phi\pi^+$ sample as calibration channel. In fact, this decay is topologically identical to the $D^+ \rightarrow \phi\pi^+$ one, except for slightly different kinematic distributions, due to the difference between the D^+ and D_s^+ masses ($M_{D_s^+} - M_{D^+} \simeq 98.69 \text{ MeV}/c^2$), and a different production asymmetry of the initial state.

The strategy to include the $D_s^+ \rightarrow \phi\pi^+$ as a calibration channel without biasing the time dependence of $A_D(K_S^0)$ relies on the assumption that, after an equalisation of the D meson and bachelor pion kinematic distributions between the $D^+ \rightarrow \phi\pi^+$ and $D_s^+ \rightarrow \phi\pi^+$ samples, the difference between the production asymmetries of the D^+ and D_s^+ mesons does not depend on any kinematic variable. In fact, when computing the difference between the *raw* asymmetries of the $D^+ \rightarrow K_S^0\pi^+$ signal sample and $D_s^+ \rightarrow \phi\pi^+$ calibration samples after the splitting and weighting procedure, we have:

$$\Delta A_{K_S^0}(t) \Big|_{D_s^+} = A_D(K_S^0, t) + A_P(D^+) - (A_{CP}(D_s^+ \rightarrow \phi\pi^+) + A_P(D_s^+)) \quad (\text{C.1})$$

and, if the assumption holds, $A_P(D^+) - A_P(D_s^+) - A_{CP}(D_s^+ \rightarrow \phi\pi^+)$ is a constant term among all the K_S^0 decay time bins, which differ only for the kinematic variables distributions, and thus does not bias the time dependence of $A_D(K_S^0, t)$.

Therefore, when using as a calibration sample both the D^+ and D_s^+ channels, the offset is not a direct measurement of $A_{CP}(D^+ \rightarrow \phi\pi^+)$, since it includes also the D_s^+ production asymmetry, but the time-dependence remains unbiased, with increased resolution.

Selection and yields of $D_s^+ \rightarrow \phi\pi^+$ decays

To avoid any difference in the nuisance asymmetries between the $D^+ \rightarrow \phi\pi^+$ and $D_s^+ \rightarrow \phi\pi^+$ samples, all the offline cuts chosen for the former, summarised in table 4.3, have been applied to the latter. The sideband subtraction weights, used to fill the background-subtracted histograms

for the kinematic weighting procedure, have been defined analogously to the $D^+ \rightarrow \phi\pi^+$ sample, taking into account the $D^+ - D_s^+$ mass difference, as shown in table C.1 and in figure C.1. In table C.2, the total yields, including the D_s^+ calibration sample, are reported. We can see that the statistics in the $D_s^+ \rightarrow \phi\pi^+$ sample is about twice the $D^+ \rightarrow \phi\pi^+$ one, thus the total yield of the calibration channel increases by a factor of three.

Sample	Signal region	Left sideband	Right sideband
$D^+ \rightarrow \phi\pi^+$	$m_D \pm 15 \text{ MeV}/c^2$	[1819.66, 1834.66] MeV/c^2	[1904.66, 1919.66] MeV/c^2
$D_s^+ \rightarrow \phi\pi^+$	$m_{D_s} \pm 15 \text{ MeV}/c^2$	[1918.35, 1933.35] MeV/c^2	[2003.35, 2018.35] MeV/c^2

Table C.1: Signal and sideband regions for the $D^+ \rightarrow \phi\pi^+$ and $D_s^+ \rightarrow \phi\pi^+$ samples. For the latter, the signal and sideband regions are symmetric around the D_s^+ mass $m_{D_s} = 1968.35 \text{ MeV}/c^2$

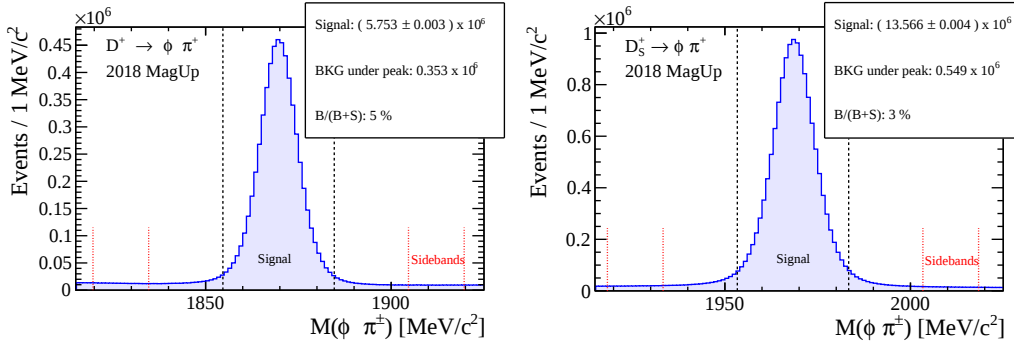


Figure C.1: Left: Distribution of $M(\phi\pi^+)$ around the D^+ resonance. Right: Distribution of $M(\phi\pi^+)$ around the D_s^+ resonance.

Dataset	$D^+ \rightarrow K_S^0(LL)\pi^+$	$D^+ \rightarrow K_S^0(DD)\pi^+$	$D^+ \rightarrow \phi\pi^+$	$D_s^+ \rightarrow \phi\pi^+$
18Up	4.7 M	5.8 M	6.4 M	13.6 M
18Dw	4.4 M	5.3 M	6.0 M	12.6 M
17Up	4.1 M	5.0 M	5.5 M	12.0 M
17Dw	4.2 M	5.3 M	5.7 M	11.9 M
16Up	3.4 M	4.1 M	4.4 M	8.4 M
16Dw	4.0 M	4.7 M	5.2 M	10.5 M
total	24.8 M	30.2 M	33.2 M	69.0 M

Table C.2: Yields of the samples in the different datasets divided by year and polarity.

Comparison between raw asymmetries in kinematic variables bins in the D^+ and D_s^+ samples

The condition to include the D_s^+ sample as a calibration channel without biasing the time dependence of $A_D(K_S^0)$ is that $A_P(D^+) - A_P(D_s^+)$ does not depend on any kinematic variable.

The first step to fulfil this condition is equalising the D meson and bachelor pion kinematics between the $D^+ \rightarrow \phi\pi^+$ and $D_s^+ \rightarrow \phi\pi^+$ samples. This has been done with an iterative kinematic weighting procedure, analogous to what described in section 5.3.

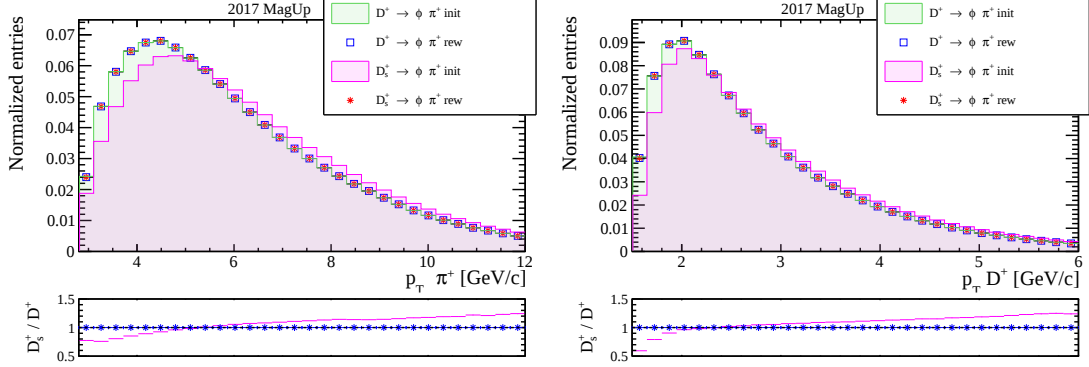


Figure C.2: Result of the iterative kinematic weighting procedure between the $D^+ \rightarrow \phi\pi^+$ and $D_s^+ \rightarrow \phi\pi^+$ samples for the 2017 MagUp sample. In the bottom, the ratio between the D_s^+ and D^+ normalised histograms before and after the weighting is shown.

After the kinematic weighting, it is necessary to verify the assumption that $A_P(D^+) - A_P(D_s^+)$ is a constant. Since the bachelor pions have the same kinematic distributions, the $A_D(\pi^+)$ terms are the same in the D^+ and D_s^+ sample, thus $A_P(D^+) - A_P(D_s^+)$ can be estimated as the difference of the D^+ and D_s^+ raw asymmetries. This difference has been computed in different kinematic variables bins and was found to be compatible with zero within 3σ in all bins, as shown in figure C.3. Therefore, the assumption that $A_P(D^+) - A_P(D_s^+)$ is a constant in the D meson and bachelor pion kinematic variables has been verified and the inclusion of the D_s^+ calibration channel should not affect the time dependence of $A_D(K_S^0)$.

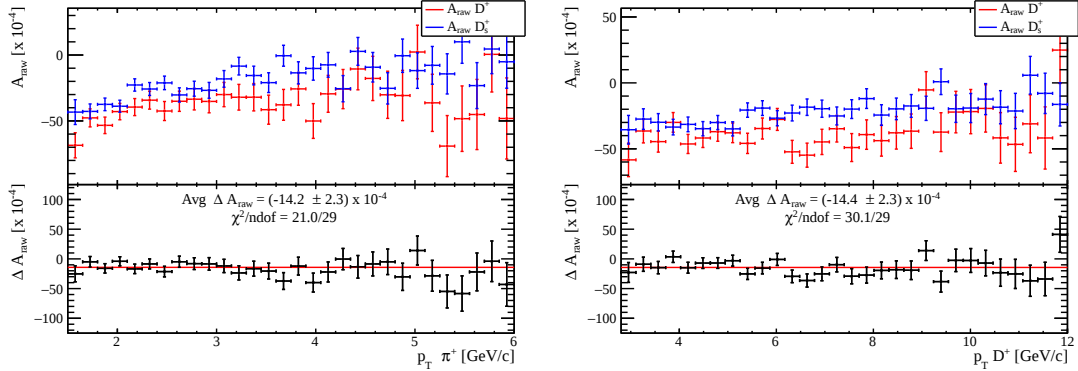


Figure C.3: Comparison between the $D^+ \rightarrow \phi\pi^+$ and $D_s^+ \rightarrow \phi\pi^+$ raw asymmetries in different kinematic variables bins, using all Run 2 datasets. In the plots, also the average difference between the raw asymmetries and the χ^2 of ΔA_{raw} with a constant model are shown.

Analysis results including the D_s^+ calibration sample

After having equalised the kinematics between the $D^+ \rightarrow \phi\pi^+$ and $D_s^+ \rightarrow \phi\pi^+$ samples and having verified that the difference between their production asymmetries is a constant, the full analysis described in chapter 5 has been repeated using as calibration channel both the joint D^+ and D_s^+ samples.

In particular, the kinematic weighting and splitting procedure has been performed as described in sections 5.2 and 5.3, with the only difference that the initial weights for the joint D^+ and D_s^+ calibration sample are not the sideband subtraction weights, but the weights computed to equalise

the kinematics between the $D^+ \rightarrow \phi\pi^+$ and $D_s^+ \rightarrow \phi\pi^+$ samples.

Determination of *raw* asymmetries and of $\Delta A_{K_S^0}(t)$

After having split the calibration sample into 13 bins, the next step is the measurement of the raw asymmetry in the two equalised samples, $A_{raw}(D_{(s)}^+ \rightarrow \phi\pi^+)$. This has been done with a simultaneous mass fit on the positively and negatively charged particles. However, to include the $D_s^+ \rightarrow \phi\pi^+$ sample it is necessary to slightly change the strategy to determine the raw asymmetry on the calibration sample, since its mass peak and shape parameters are different from those of the D^+ sample. The method used consists in a simultaneous fit with shared parameters on four distinct datasets: $D^+ \rightarrow \phi\pi^+$, $D^- \rightarrow \phi\pi^-$, $D_s^+ \rightarrow \phi\pi^+$ and $D_s^- \rightarrow \phi\pi^-$. The four fit functions, with indices $\pm$ and d, s to distinguish the four datasets, are still in the form:

$$F_{d/s}^\pm(m; \alpha_{d/s}^\pm, \beta_{d/s}^\pm) = N_{sig,d/s} \cdot \frac{1 \pm A_{sig}}{2} \cdot F_{sig}(m; \alpha_{d/s}^\pm) + N_{bkg,d/s} \cdot \frac{1 \pm A_{bkg,d/s}}{2} \cdot F_{bkg}(m; \beta_{d/s}^\pm) \quad (C.2)$$

where the raw asymmetry A_{sig} is a shared parameter by the four samples. The others parameters are free to vary between the $D^\pm$ and $D_s^\pm$ samples, even though some shape parameters are shared between all samples. The fit has been performed by minimising the total $\chi_{TOT}^2 = \chi^2(D^+) + \chi^2(D^-) + \chi^2(D_s^+) + \chi^2(D_s^-)$.

The chosen fit model is still a Johnson S_U for the signal and a 2nd grade polynomial for the background:

- $F_{sig,d}^\pm$ is a Johnson S_U with parameters $\alpha_d^\pm = \{(\mu_d \pm \delta_{\mu,d}), \sigma_d, \delta, \lambda\}$: the fit is made with 5 shared parameters between the positive and negative samples, with δ and λ shared with the $D_s^\pm$ sample;
- $F_{sig,s}^\pm$ is a Johnson S_U with parameters $\alpha_s^\pm = \{(\mu_s \pm \delta_{\mu,s}), \sigma_s, \delta, \lambda\}$: the fit is made with 5 shared parameters between the positive and negative samples, with δ and λ shared with the $D^\pm$ sample;
- $F_{bkg,d/s}^\pm$ is always a 2nd grade polynomial with 3 parameters shared between the positive and negative particles: $\beta_{d/s}^\pm = \{p_{0,d/s}, p_{1,d/s}, p_{2,d/s}\}$;
- the signal yield $N_{sig,d/s}$ and background asymmetry $A_{bkg,d/s}$ parameters are free to be different in the $D^\pm$ and $D_s^\pm$ samples.

The total number of free parameters is thus 19: 8 for the signal shapes, 6 for the background shapes, 2 for the signal yields, 2 for the background asymmetries and 1 for the signal asymmetry. An example of simultaneous mass fits in one K_S^0 decay time bin is shown in figure C.4.

After the measurement of *raw* asymmetries in the signal and calibration sample, the time-dependent neutral kaon detection asymmetry has been estimated in each K_S^0 decay time bin as in (C.1). The result achieved using all the Run 2 datasets can be seen in figure C.5.

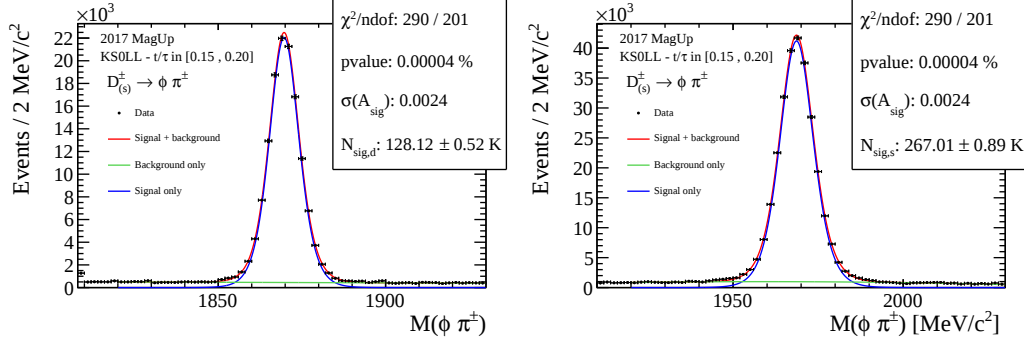


Figure C.4: Simultaneous mass fits in the fourth $t/\tau_{K_S^0}$ bin on the $D_s^\pm \rightarrow \phi\pi^\pm$ (left) and $D_s^\pm \rightarrow \phi\pi^\pm$ (right) samples, 2017 MagUp dataset. Here the sum of the $D_{(s)}^+$ and $D_{(s)}^-$ histograms and fit functions are shown.

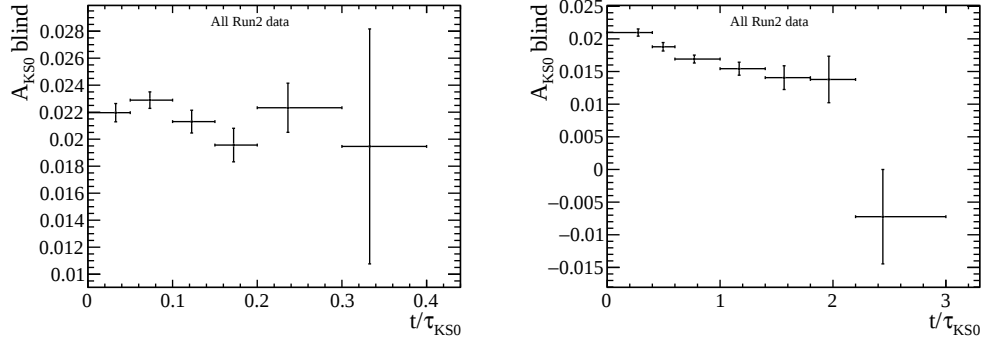


Figure C.5: $\Delta A_{K_S^0}(t)$ plot, using all Run 2 datasets, including the D_s^+ calibration sample, with the asymmetries estimated with the simultaneous mass fits method.

$K_S^0(LL)$ bin	$\Delta A_{K_S^0} _{D^+}$	$\Delta A_{K_S^0} _{D^++D_s^+}$
[0.00, 0.05]	224.3 ± 9.4	219.6 ± 6.8
[0.05, 0.10]	234.5 ± 8.5	228.9 ± 6.1
[0.10, 0.15]	237.8 ± 11.9	213.0 ± 8.4
[0.15, 0.20]	226.7 ± 19.0	195.6 ± 12.4
[0.20, 0.30]	224.6 ± 28.5	223.3 ± 18.2
[0.30, 0.40]	136.3 ± 146.5	194.6 ± 87.0

Table C.3: Results for the central blinded $\Delta A_{K_S^0}$ values and the statistical uncertainties in each $K_S^0(LL)$ decay time bin using only the D^+ calibration sample and including the D_s^+ sample. The statistical uncertainties are all reduced by a factor ~ 0.7 .

Results: confidence intervals for r_π, δ_π

Taking as input the 13 asymmetry measurements, the confidence intervals in the $r_\pi - \delta_\pi$ space have been computed with the *Plugin* method, in the same way as discussed in the previous section. In particular, figure C.6 shows the comparison between the $\Delta\chi^2(r_\pi, \delta_\pi)$ map computed taking as input the asymmetries measured with only the D^+ calibration sample and with the inclusion of the D_s^+ sample. We can see that the two distributions are in agreement with each other, thus

$K_S^0(DD)$ bin	$\Delta A_{K_S^0} _{D^+}$	$\Delta A_{K_S^0} _{D^++D_s^+}$
[0.0, 0.4]	221.0 ± 7.2	209.7 ± 5.5
[0.4, 0.6]	200.3 ± 8.5	187.9 ± 6.5
[0.6, 1.0]	166.7 ± 8.0	169.1 ± 6.1
[1.0, 1.4]	152.9 ± 13.7	154.3 ± 9.9
[1.4, 1.8]	116.7 ± 25.5	140.6 ± 18.2
[1.8, 2.2]	76.8 ± 52.5	137.8 ± 35.6
[2.2, 3.0]	58.2 ± 114	-72.3 ± 72.3

Table C.4: Results for the central blinded $\Delta A_{K_S^0}$ values and the statistical uncertainties in each $K_S^0(DD)$ decay time bin using only the D^+ calibration sample and including the D_s^+ sample. The statistical uncertainties are all reduced by a factor ~ 0.7 .

the assumption that the inclusion of the D_s^+ sample does not affect the time-dependence of the asymmetry holds.

Figure C.7 shows the results for the p -value distribution in the $r_\pi - \delta_\pi$ space and the confidence intervals built taking the same thresholds as discussed above.

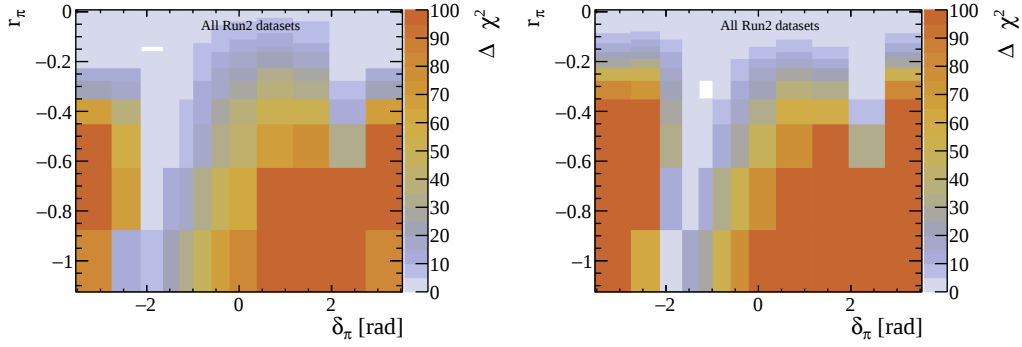


Figure C.6: Comparison between the $\Delta\chi^2$ distribution using only the D^+ calibration sample (left) and including also the D_s^+ sample (right). We can see that, as expected, the resolution on the r_π and δ_π parameters increases when using a larger calibration sample.

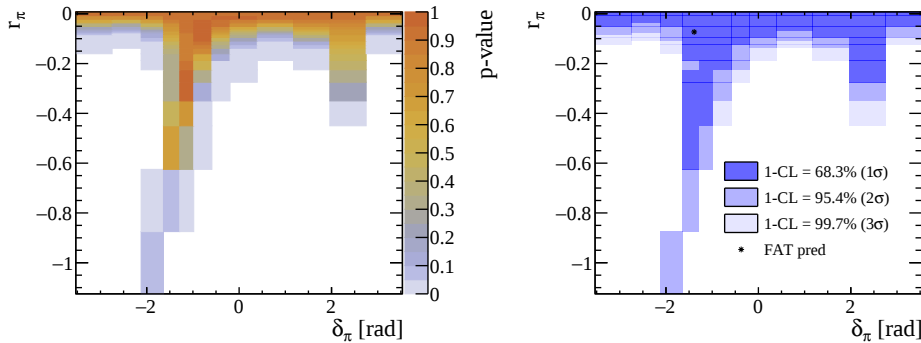


Figure C.7: Left: map of $1 - CL$ computed with the plugin method for different r_π, δ_π values in the scan, including the D_s^+ calibration channel. Right: confidence intervals for different CL thresholds. The FAT prediction still falls inside the 1σ region.

Bibliography

- [1] J. H. Christenson et al. “Evidence for the 2π Decay of the K_2^0 Meson”. In: *Phys. Rev. Lett.* 13 (July 1964), pp. 138–140. DOI: 10.1103/PhysRevLett.13.138.
- [2] B. Aubert et al. “Observation of CP violation in the B^0 meson system”. In: *Phys. Rev. Lett.* 87 (2001), p. 091801. DOI: 10.1103/PhysRevLett.87.091801.
- [3] K. Abe et al. “Observation of large CP violation in the neutral B meson system”. In: *Phys. Rev. Lett.* 87 (2001), p. 091802. DOI: 10.1103/PhysRevLett.87.091802.
- [4] R. Aaij et al. “First observation of CP violation in the decays of B_s^0 mesons”. In: *Phys. Rev. Lett.* 110.22 (2013), p. 221601. DOI: 10.1103/PhysRevLett.110.221601.
- [5] R. Aaij et al. “Observation of CP violation in charm decays”. In: *Phys. Rev. Lett.* (2019). URL: <https://arxiv.org/abs/1903.08726>.
- [6] S. Stahl. “Measurement of CP asymmetry in muon-tagged $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays at LHCb”. PhD thesis. Heidelberg University, 2014.
- [7] R. Aaij et al. “Measurement of time-integrated CP asymmetry in $D^0 \rightarrow K^-K^+$ decays”. In: *Phys. Rev. Lett.* (2022). URL: <https://arxiv.org/pdf/2209.03179.pdf>.
- [8] R. Aaij et al. “Search for CP Violation in $D_s^+ \rightarrow K_S^0\pi^+$, $D^+ \rightarrow K_S^0K^+$, and $D^+ \rightarrow \phi\pi^+$ Decays”. In: *Phys. Rev. Lett.* 122 (19 May 2019), p. 191803. DOI: 10.1103/PhysRevLett.122.191803. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.122.191803>.
- [9] Di Wang, Fu-Sheng Yu, and Hsiang-nan Li. “ CP Asymmetries in Charm Decays into Neutral Kaons”. In: *Phys. Rev. Lett.* 119 (18 Oct. 2017), p. 181802. DOI: 10.1103/PhysRevLett.119.181802. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.119.181802>.
- [10] M. Gell-Mann and A. Pais. “Behavior of Neutral Particles under Charge Conjugation”. In: *Phys. Rev.* 97 (5 Mar. 1955), pp. 1387–1389. DOI: 10.1103/PhysRev.97.1387. URL: <https://link.aps.org/doi/10.1103/PhysRev.97.1387>.
- [11] K. Lande, L. M. Lederman, and W. Chinowsky. “Report on Long-Lived K^0 Mesons”. In: *Phys. Rev.* 105 (6 Mar. 1957), pp. 1925–1927. DOI: 10.1103/PhysRev.105.1925.2. URL: <https://link.aps.org/doi/10.1103/PhysRev.105.1925.2>.
- [12] H. Albrecht et al. “Observation of $B^0 - \bar{B}^0$ mixing”. In: *Physics Letters B* 192.1 (1987), pp. 245–252. ISSN: 0370-2693. DOI: [https://doi.org/10.1016/0370-2693\(87\)91177-4](https://doi.org/10.1016/0370-2693(87)91177-4).
- [13] A. Abulencia et al. “Measurement of the $B_s^0 - \bar{B}_s^0$ Oscillation Frequency”. In: *Phys. Rev. Lett.* 97 (6 Aug. 2006), p. 062003. DOI: 10.1103/PhysRevLett.97.062003. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.97.062003>.
- [14] R. Aaij et al. “Observation of $D^0 - \bar{D}^0$ Oscillations”. In: *Phys. Rev. Lett.* 110 (10 Mar. 2013), p. 101802. DOI: 10.1103/PhysRevLett.110.101802. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.110.101802>.

-
- [15] K. Abe et al. “Constraint on the matter-antimatter symmetry-violating phase in neutrino oscillations”. In: *Nature* 580.7803 (2020), pp. 339–344. DOI: 10.1038/s41586-020-2177-0. URL: <https://doi.org/10.1038/s41586-020-2177-0>.
 - [16] Nicola Cabibbo. “Unitary Symmetry and Leptonic Decays”. In: *Phys. Rev. Lett.* 10 (12 June 1963), pp. 531–533. DOI: 10.1103/PhysRevLett.10.531. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.10.531>.
 - [17] Makoto Kobayashi and Toshihide Maskawa. “CP-Violation in the Renormalizable Theory of Weak Interaction”. In: *Progress of Theoretical Physics* 49.2 (Feb. 1973), pp. 652–657. ISSN: 0033-068X. DOI: 10.1143/PTP.49.652. eprint: <https://academic.oup.com/ptp/article-pdf/49/2/652/5257692/49-2-652.pdf>. URL: <https://doi.org/10.1143/PTP.49.652>.
 - [18] Michael Dine and Alexander Kusenko. “Origin of the matter-antimatter asymmetry”. In: *Rev. Mod. Phys.* 76 (1 Dec. 2003), pp. 1–30. DOI: 10.1103/RevModPhys.76.1. URL: <https://link.aps.org/doi/10.1103/RevModPhys.76.1>.
 - [19] Lincoln Wolfenstein. “Parametrization of the Kobayashi-Maskawa Matrix”. In: *Phys. Rev. Lett.* 51 (21 Nov. 1983), pp. 1945–1947. DOI: 10.1103/PhysRevLett.51.1945. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.51.1945>.
 - [20] Marco S. Sozzi. *Discrete symmetries and CP violation: from experiment to theory*. Oxford graduate texts. New York, NY: Oxford Univ. Press, 2008. DOI: 10.1093/acprof:oso/9780199296668.001.0001. URL: <https://cds.cern.ch/record/1087897>.
 - [21] V. Weisskopf and E. Wigner. “Over the natural line width in the radiation of the harmonius oscillator”. In: *Z. Phys.* 65 (1930), pp. 18–29. DOI: 10.1007/BF01397406.
 - [22] T. Gershon, J. Libby, and G. Wilkinson. “Contributions to the width difference in the neutral D system from hadronic decays”. In: *Physics Letters B* 750 (2015), pp. 338–343. ISSN: 0370-2693. DOI: <https://doi.org/10.1016/j.physletb.2015.08.063>. URL: <https://www.sciencedirect.com/science/article/pii/S0370269315006644>.
 - [23] Alexey A. Petrov. “Charm mixing in the Standard Model and Beyond”. In: *International Journal of Modern Physics A* 21.27 (2006), pp. 5686–5693. DOI: 10.1142/S0217751X06034902. eprint: <https://doi.org/10.1142/S0217751X06034902>. URL: <https://doi.org/10.1142/S0217751X06034902>.
 - [24] Particle Data Group et al. “Review of Particle Physics”. In: *Progress of Theoretical and Experimental Physics* 2020.8 (Aug. 2020), p. 083C01. DOI: 10.1093/ptep/ptaa104. URL: <https://doi.org/10.1093/ptep/ptaa104>.
 - [25] HFLAV. “Averages of b-hadron, c-hadron, and tau-lepton properties”. In: *Eur. Phys. J. C* 81 (2021). updated results and plots available at <https://hflav.web.cern.ch/>, p. 226. URL: <https://hflav.web.cern.ch/>.
 - [26] H. Burkhardt et al. “First evidence for direct CP violation”. In: *Physics Letters B* 206.1 (1988), pp. 169–176. ISSN: 0370-2693. DOI: [https://doi.org/10.1016/0370-2693\(88\)91282-8](https://doi.org/10.1016/0370-2693(88)91282-8). URL: <https://www.sciencedirect.com/science/article/pii/0370269388912828>.
 - [27] V. Fanti et al. “A new measurement of direct CP violation in two pion decays of the neutral kaon”. In: *Phys. Lett. B* 465 (1999), pp. 335–348. DOI: 10.1016/S0370-2693(99)01030-8.
 - [28] A. Alavi-Harati et al. “Observation of Direct CP Violation in $K_{S,L} \rightarrow \pi\pi$ Decays”. In: *Phys. Rev. Lett.* 83 (1 July 1999), pp. 22–27. DOI: 10.1103/PhysRevLett.83.22. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.83.22>.
 - [29] R. Aaij et al. “First observation of CP violation in the decays of B^0 mesons”. In: *Phys. Rev. Lett.* 110 (2013), p. 221601. DOI: 10.1103/PhysRevLett.110.221601.
-

- [30] Particle Data Group. “CP Violation in the Quark Sector”. In: *Particle Data Group Review* 2023 (2023). URL: <https://pdg.lbl.gov/2023/reviews/rpp2023-rev-cp-violation.pdf>.
- [31] A. Pais and O. Piccioni. “Note on the Decay and Absorption of the θ^0 ”. In: *Phys. Rev.* 100 (5 Dec. 1955), pp. 1487–1489. DOI: 10.1103/PhysRev.100.1487. URL: <https://link.aps.org/doi/10.1103/PhysRev.100.1487>.
- [32] Myron L. Good. “Relation between Scattering and Absorption in the Pais-Piccioni Phenomenon”. In: *Phys. Rev.* 106 (3 May 1957), pp. 591–595. DOI: 10.1103/PhysRev.106.591. URL: <https://link.aps.org/doi/10.1103/PhysRev.106.591>.
- [33] P. K. Kabir. *The CP Puzzle; strange decays of the neutral kaon*. London: Academic Press, 1968.
- [34] W. Fetscher et al. “Regeneration of arbitrary coherent neutral kaon states: A new method for measuring the K^0 anti- K^0 forward scattering amplitude”. In: *Z. Phys. C* 72 (1996), pp. 543–547. DOI: 10.1007/s002880050277.
- [35] A. Gsponer et al. “Precise Coherent K_S Regeneration Amplitudes for C, Al, Cu, Sn, and Pb Nuclei from 20 to 140 GeV/c and Their Interpretation”. In: *Phys. Rev. Lett.* 42 (1 Jan. 1979), pp. 13–16. DOI: 10.1103/PhysRevLett.42.13. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.42.13>.
- [36] *2023: Particle Listings*. URL: https://pdg.lbl.gov/2023/listings/contents_listings.html.
- [37] *CP Violation in K_L^0 Decays*. URL: <https://pdg.lbl.gov/2023/reviews/rpp2023-rev-cp-viol-kl-decays.pdf>.
- [38] N. K. Nisar et al. “Search for CP Violation in $D^0 \rightarrow \pi^0 \pi^0$ Decays”. In: *Phys. Rev. Lett.* 112 (21 May 2014), p. 211601. DOI: 10.1103/PhysRevLett.112.211601. URL: <https://link.aps.org/doi/10.1103/PhysRevLett.112.211601>.
- [39] R. Aaij et al. “Search for CP violation in $D_{(s)}^+ \rightarrow h^+ \pi^0$ and $D_{(s)}^+ \rightarrow h^+ \eta$ decays”. In: *JHEP* 06 (2021), p. 019. DOI: 10.1007/JHEP06(2021)019. arXiv: 2103.11058 [hep-ex].
- [40] R. Aaij et al. “Measurement of CP asymmetry in $D^0 \rightarrow K_S^0 K_S^0$ decays”. In: *Phys. Rev. D* 104.3 (2021), p. L031102. DOI: 10.1103/PhysRevD.104.L031102. arXiv: 2105.01565. URL: <https://cds.cern.ch/record/2765931>.
- [41] R. Aaij et al. “Measurement of CP asymmetries in $D_{(s)}^+ \rightarrow \eta \pi^+$ and $D_{(s)}^+ \rightarrow \eta' \pi^+$ decays”. In: *JHEP* 04 (2023), p. 081. DOI: 10.1007/JHEP04(2023)081. arXiv: 2204.12228. URL: <https://cds.cern.ch/record/2807820>.
- [42] Tara Nanut. “Search for CPV in D^0 decays to radiative and hadronic decays at Belle”. In: *PoS HQL 2016* (2017), p. 051. DOI: 10.22323/1.274.0051.
- [43] R. Aaij et al. “Measurement of CP asymmetry in $D^0 \rightarrow K^+ K^-$ and $D^0 \rightarrow \pi^+ \pi^-$ decays”. In: *Journal of High Energy Physics* 2014.7 (July 2014). ISSN: 1029-8479. DOI: 10.1007/jhep07(2014)041. URL: [http://dx.doi.org/10.1007/JHEP07\(2014\)041](http://dx.doi.org/10.1007/JHEP07(2014)041).
- [44] Di Wang et al. “ $K_S^0 - K_L^0$ asymmetries in D -meson decays”. In: *Physical Review D* 95.7 (Apr. 2017). ISSN: 2470-0029. DOI: 10.1103/physrevd.95.073007. URL: <http://dx.doi.org/10.1103/PhysRevD.95.073007>.
- [45] T. Aaltonen et al. “Measurement of CP -violation asymmetries in $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ ”. In: *Physical Review D* 86.3 (Aug. 2012). DOI: 10.1103/physrevd.86.032007. URL: <http://dx.doi.org/10.1103/PhysRevD.86.032007>.

-
- [46] R. Aaij et al. “LHCb Detector Performance”. In: *Int. J. Mod. Phys. A* 30 (2015), p. 1530022. DOI: 10.1142/S0217751X15300227. arXiv: 1412.6352. URL: <https://cds.cern.ch/record/1978280>.
- [47] Ewa Lopienska. “The CERN accelerator complex, layout in 2022. Complexe des accélérateurs du CERN en janvier 2022”. In: (2022). General Photo. URL: <https://cds.cern.ch/record/2800984>.
- [48] Christian Elsasser. *$\bar{b}b$ production angle plots*. URL: https://lhcb.web.cern.ch/lhcb/speakersbureau/html/bb%5C_ProductionAngles.html%7D%7Bhttps://lhcb.web.cern.ch/lhcb/speakersbureau/html/bb%5C_ProductionAngles.html%7D%7D.
- [49] P. R. Barbosa-Marinho et al. *LHCb VELO (Vertex Locator): Technical Design Report*. Technical design report. LHCb. Geneva: CERN, 2001. URL: <https://cds.cern.ch/record/504321>.
- [50] A. A. Alves Jr et al. “The LHCb Detector at the LHC”. In: *Journal of Instrumentation* 3.08 (Aug. 2008), S08005. DOI: 10.1088/1748-0221/3/08/S08005. URL: <https://dx.doi.org/10.1088/1748-0221/3/08/S08005>.
- [51] R. Antunes-Nobrega et al. *LHCb reoptimized detector design and performance: Technical Design Report*. Technical design report. LHCb. Geneva: CERN, 2003. URL: <https://cds.cern.ch/record/630827>.
- [52] Silicon Tracker UZH group. *Lhcb silicon tracker - material for publications*. URL: <http://lhcb.physik.uzh.ch/ST/public/material/index.php..>
- [53] P. R. Barbosa-Marinho et al. *LHCb inner tracker: Technical Design Report*. Technical design report. LHCb. Geneva: CERN, 2002. URL: <https://cds.cern.ch/record/582793>.
- [54] P. R. Barbosa-Marinho et al. *LHCb outer tracker: Technical Design Report*. Technical design report. LHCb. Geneva: CERN, 2001. URL: <https://cds.cern.ch/record/519146>.
- [55] S Amato et al. *LHCb RICH: Technical Design Report*. Technical design report. LHCb. Geneva: CERN, 2000. URL: <https://cds.cern.ch/record/494263>.
- [56] R. Antunes-Nobrega et al. *LHCb reoptimized detector design and performance: Technical Design Report*. Technical design report. LHCb. Geneva: CERN, 2003. URL: <https://cds.cern.ch/record/630827>.
- [57] R. Aaij et al. “Global PID performance for charged particles”. In: (2020). URL: <https://cds.cern.ch/record/2718739>.
- [58] S. Amato et al. *LHCb calorimeters: Technical Design Report*. Technical design report. LHCb. Geneva: CERN, 2000. URL: <https://cds.cern.ch/record/494264>.
- [59] A. A. Alves Jr et al. “Performance of the LHCb muon system”. In: *Journal of Instrumentation* 8.02 (Feb. 2013), P02022. DOI: 10.1088/1748-0221/8/02/P02022. URL: <https://dx.doi.org/10.1088/1748-0221/8/02/P02022>.
- [60] R. Aaij et al. “Design and performance of the LHCb trigger and full real-time reconstruction in Run 2 of the LHC. Performance of the LHCb trigger and full real-time reconstruction in Run 2 of the LHC”. In: *JINST* 14.04 (2019), P04013. DOI: 10.1088/1748-0221/14/04/P04013. arXiv: 1812.10790. URL: <https://cds.cern.ch/record/2652801>.
- [61] LHCb collaboration. *Trigger Schemes*. URL: <http://lhcb.web.cern.ch/lhcb/speakersbureau/html/TriggerScheme.html>.
- [62] R Aaij et al. “The LHCb trigger and its performance in 2011”. In: *Journal of Instrumentation* 8.04 (Apr. 2013), P04022. DOI: 10.1088/1748-0221/8/04/P04022. URL: <https://dx.doi.org/10.1088/1748-0221/8/04/P04022>.
-

- [63] R. Aaij et al. “Tesla: An application for real-time data analysis in High Energy Physics”. In: *Computer Physics Communications* 208 (2016), pp. 35–42. ISSN: 0010-4655. DOI: <https://doi.org/10.1016/j.cpc.2016.07.022>. URL: <https://www.sciencedirect.com/science/article/pii/S0010465516302107>.
- [64] S. Benson et al. “The LHCb Turbo Stream”. In: *Journal of Physics: Conference Series* 664.8 (Dec. 2015), p. 082004. DOI: 10.1088/1742-6596/664/8/082004. URL: <https://dx.doi.org/10.1088/1742-6596/664/8/082004>.
- [65] R. Aaij et al. “Measurement of the $B^\pm$ production asymmetry and the CP asymmetry in $B^\pm \rightarrow J/\psi K^\pm$ decays”. In: *Phys. Rev. D* 95.5 (2017), p. 052005. DOI: 10.1103/PhysRevD.95.052005. arXiv: 1701.05501 [hep-ex].
- [66] F. Ferrari on behalf of the LHCb Collaboration. “Production asymmetries of b and c hadrons at LHCb”. In: *Journal of Physics: Conference Series* 770.1 (Nov. 2016), p. 012005. DOI: 10.1088/1742-6596/770/1/012005. URL: <https://dx.doi.org/10.1088/1742-6596/770/1/012005>.
- [67] R. Aaij et al. “Measurement of the $D^{+/-}$ production asymmetry in 7 TeV pp collisions”. In: *Phys. Lett. B* 718 (2013), pp. 902–909. DOI: 10.1016/j.physletb.2012.11.038. arXiv: 1210.4112 [hep-ex].
- [68] Laurent Dufour and Jeroen Van Tilburg. *Decomposition of simulated detection asymmetries in LHCb*. Tech. rep. document accessible only by the members of the LHCb collaboration. Geneva: CERN, 2018. URL: <https://cds.cern.ch/record/2304546>.
- [69] Sebastien Ponce et al. *Detector Description Framework in LHCb*. 2003. arXiv: physics/0306089 [physics.ins-det].
- [70] *GEANT4 CERN project*. URL: <http://cern.ch/geant4>.
- [71] Bodhisattva Sen, Matthew Walker, and Michael Woodroffe. “On the unified method with nuisance parameters”. In: *Statistica Sinica* 19 (Jan. 2009), pp. 301–314.
- [72] R. Aaij et al. “Measurement of the CKM angle γ from a combination of LHCb results”. In: *Journal of High Energy Physics* 2016.12 (). ISSN: 1029-8479. DOI: 10.1007/jhep12(2016)087. URL: [http://dx.doi.org/10.1007/JHEP12\(2016\)087](http://dx.doi.org/10.1007/JHEP12(2016)087).
- [73] S. S. Wilks. “The Large-Sample Distribution of the Likelihood Ratio for Testing Composite Hypotheses”. In: *The Annals of Mathematical Statistics* 9.1 (1938), pp. 60–62. ISSN: 00034851. URL: <http://www.jstor.org/stable/2957648> (visited on 06/14/2024).
- [74] F. James and M. Roos. “Minuit - a system for function minimization and analysis of the parameter errors and correlations”. In: *Computer Physics Communications* 10.6 (1975), pp. 343–367. ISSN: 0010-4655. DOI: [https://doi.org/10.1016/0010-4655\(75\)90039-9](https://doi.org/10.1016/0010-4655(75)90039-9). URL: <https://www.sciencedirect.com/science/article/pii/0010465575900399>.
- [75] A. Davis et al. *Measurement of the $K^-\pi^+$ two-track detection asymmetry in Run 2 using the Turbo stream*. Tech. rep. Geneva: CERN, 2017. URL: <https://cds.cern.ch/record/2284097>.
- [76] R. Aaij et al. “Measurement of the track reconstruction efficiency at LHCb”. In: *Journal of Instrumentation* 10.02 (Feb. 2015), P02007. DOI: 10.1088/1748-0221/10/02/P02007. URL: <https://dx.doi.org/10.1088/1748-0221/10/02/P02007>.
- [77] LHCb collaboration. *Tracking Efficiency and Absorption Length*. URL: <https://twiki.cern.ch/twiki/bin/viewauth/LHCb/TrackingEffAbsLength>.
- [78] Serena Maccolini. “Search for direct CP violation in charm neutral meson decays at LHCb”. PhD thesis. Università di Bologna, 2022.

- [79] LHCb collaboration. *Framework TDR for the LHCb Upgrade II: Opportunities in flavour physics, and beyond, in the HL-LHC era*. Tech. rep. 2021.
- [80] Nico Kleijne. “A new method to measure the time-integrated CP asymmetry in singly-Cabibbo suppressed $D^0 \rightarrow K^+K^-$ and $D^0 \rightarrow \pi^+\pi^-$ decays at LHCb.” Pisa U., 2020. URL: <https://cds.cern.ch/record/2743432>.
- [81] LHCb collaboration. *Project RETINA web page*. URL: <https://web.infn.it/RETINA/index.php/en/>.
- [82] LHCb collaboration. *LHCb Data Acquisition Enhancement TDR*. Tech. rep. Geneva: CERN, 2024. URL: <https://cds.cern.ch/record/2886764>.
- [83] M. Morello et al. *Proposal for FPGA-based tracking in the LHCb downstream region*. Tech. rep. Geneva: CERN, 2024. URL: <https://cds.cern.ch/record/2888549>.
- [84] R. Cenci et al. “Real-time reconstruction of long-lived particles at LHCb using FPGAs”. In: *Journal of Physics: Conference Series* 1525.1 (Apr. 2020), p. 012101. ISSN: 1742-6596. DOI: 10.1088/1742-6596/1525/1/012101. URL: <http://dx.doi.org/10.1088/1742-6596/1525/1/012101>.