

Corrections of high-order nonlinearities in the LHC and High-Luminosity LHC beam optics

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Dedicated to all the valiant protons, fearlessly colliding in the pursuit of scientific enlightenment. May their sacrifice forever be honored, as they create the sparks which unveil the mysteries of the universe.

Abstract

The impact of high-order nonlinear magnetic field errors on the performance of the Large Hadron Collider (LHC) and its planned High-Luminosity upgrade, the HL-LHC, has been extensively studied. Particularly, the presence of such errors in the Insertion Regions (IR) has shown significant repercussions due to the high beta-functions and feed-down to lower orders caused by crossing schemes. This thesis aims to explore different methods for effectively addressing these high-order errors, with the ultimate goal of identifying and correcting them to optimize beam optics and enhance machine performance.

Simulation studies are employed, using a novel and flexible correction algorithm developed during the course of this PhD research. Various strategies are investigated to improve corrections by targeting Resonance Driving Terms (RDTs) associated with diverse error sources. Notably, the algorithm accounts for the feed-down effects and avoids symmetry assumptions between the counter-circulating particle beams. Special attention is devoted to decapole and dodecapole errors, which have demonstrated detrimental effects on amplitude detuning due to feed-down based on previous measurements in the LHC. The anticipated increase in optics sensitivity to errors in the IRs of the HL-LHC further underscores the importance of addressing these errors.

The thesis also investigates the influence of misalignments within the IRs housing the detector experiments of both, LHC and HL-LHC, machine configurations, in particular in the final focusing triplets and the nonlinear corrector packages.

Correction options are evaluated, focusing on the utilization of the nonlinear corrector packages to address errors in the new separation and recombination dipoles in the HL-LHC, where increased decapole errors had been expected.

Experimental studies are conducted to validate the findings. These studies involve replicating the nonlinear errors anticipated in the HL-LHC by powering the correctors in the LHC. Additionally, significant efforts are dedicated to mitigating the feed-down effects arising from decapole and dodecapole field errors in order to minimize amplitude detuning, which plays a crucial role in maintaining optimal beam stability and performance. To address this challenge, novel corrections involving the operational implementation of dodecapole correctors in the IRs have been introduced for the first time. These corrections have demonstrated their efficacy in successfully reducing the undesired feed-down effects.

The results of these experiments provide valuable insights into the mitigation of high-order errors and contribute to the overall understanding of beam dynamics in advanced particle accelerators.

Keywords: Collider, Accelerator Physics, High Order, Decapole, Dodecapole, Nonlinear, Corrections, LHC, HL-LHC, Beam Optics, Magnets, Magnetic Fields

Zusammenfassung

Der Einfluss von Nichtlinearitäten höherer Ordnung der Magnetfelder auf die Leistung des Large Hadron Collider (LHC) und dessen geplante High-Luminosity-Aufrüstung, dem HL-LHC, wurde umfangreich untersucht. Insbesondere hat sich gezeigt, dass das Vorhandensein solcher Fehler in den Insertion Regions (IR) erhebliche Auswirkungen hat, bedingt durch hohe Beta-Funktionen und Feed-Down auf niedrigere Ordnungen aufgrund der Kreuzungsschemata. Augenmerk dieser Arbeit ist auf die Erforschung diverser Methoden zur effektiven Behandlung dieser Nichtlinearitäten höherer Ordnung gerichtet, mit dem Ziel, sie zu identifizieren und korrigieren, um die Strahloptik zu optimieren und die Maschinenleistung zu verbessern.

Simulationsstudien werden eingesetzt, in denen auch ein, im Rahmen dieser Promotionsarbeit entwickelter, neuartiger und flexibler Korrekturalgorithmus verwendet wird. Es werden verschiedene Strategien untersucht, um Korrekturen zu verbessern, indem mit verschiedenen Fehlerquellen assoziierte Resonanzantreibende Terme (RDTs) gezielt angegangen werden. Auch berücksichtigt der Algorithmus die Feed-Down Effekte und vermeidet Symmetrieannahmen zwischen den beiden gegenläufigen Teilchenstrahlen. Besondere Aufmerksamkeit gilt Dekapol- und Dodekapolfehlern, die in früheren Messungen im LHC schädliche Auswirkungen durch Feed-Down auf Amplituden-Detuning gezeigt haben. Die erwartete Erhöhung der Sensitivität der Optik gegenüber Fehlern in den IRs des HL-LHC unterstreicht weiter die Bedeutung der Behandlung dieser Fehler.

Auch wird in dieser Arbeit der Einfluss von Ausrichtungsfehlern der nichtlinearen Korrektorpakete und der final-focusing Triplets innerhalb der mit Detektor-Experimenten ausgestatteten IRs untersucht. Des Weiteren werden Korrekturoptionen mit Hilfe der nichtlinearen Korrektorpaketen entwickelt und bewertet, um Fehler in den neuen Separations- und Rekombinationsdipolen im HL-LHC zu kompensieren, bei denen vermehrt Dekapolfehler erwartet wurden.

Experimentelle Studien werden durchgeführt, um die Ergebnisse zu validieren. Diese Studien umfassen die Nachbildung der im HL-LHC erwarteten nichtlinearen Fehler, durch emulieren dieser Fehler mit Hilfe der Korrektoren im LHC. Darüber hinaus werden erhebliche Anstrengungen unternommen, um die Feed-Down Effekte von Dekapol- und Dodekapol-Feldfehlern zu mindern, mit dem Ziel Amplituden-Detuning zu minimieren, welches eine entscheidende Rolle im Erreichen der optimalen Strahlstabilität und -leistung spielt. Um diese Herausforderung anzugehen, wurden neuartige Korrekturalgorithmen eingeführt, die erstmals die Dodekapol-Korrektoren in den IRs im operationellen Betrieb ansteuern. Diese Korrekturen haben ihre Effektivität in der erfolgreichen Reduzierung der Feed-Down Effekte bewiesen.

Die Ergebnisse dieser Experimente liefern wertvolle Erkenntnisse zur Minderung von Fehlern höherer Ordnung und tragen zum besseren Verständnis der Strahldynamik in modernen und zukünftigen Teilchenbeschleunigern bei.

Schlagwörter: Teilchenbeschleuniger, Beschleunigerphysik, Hohe Ordnung, Dekapol, Dodekapol, Nichtlinear, Korrekturen, LHC, HL-LHC, Strahloptik, Magnete, Magnetische Felder

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Glossary

Nomenclature

AC-Dipole Dipole magnet attached to an AC power source with variable frequency and strength, thus allowing to impose forced oscillations on the beam. See [1].

ALICE (A Large Ion Collider Experiment) An experiment optimized to study heavy-ion collisions. See: alice.cern.

Amplitude detuning Tune change with the transversal amplitude of a particle. See [Section 2.2.6](#).

ATLAS (A Toroidal LHC ApparatuS) is the larger of two general-purpose detectors at the Large Hadron Collider (LHC). It investigates a wide range of physics, from the search for the Higgs boson to extra dimensions and particles that could make up dark matter. Although it has the same scientific goals as the CMS experiment, it uses different technical solutions and a different magnet-system design. See: atlas.cern.

ATS (Achromatic Telescopic Squeeze) A flexible mechanism to strongly reduce β^* in a symmetric or asymmetric way (i.e. without necessarily imposing the same β^* in both planes), while perfectly controlling the chromatic aberrations induced, namely the linear and nonlinear chromaticities, the off-momentum β -beating, and the spurious dispersion from the large crossing angle which is required at small β^* in the particular case of the Large Hadron Collider (LHC) and High-Luminosity Large Hadron Collider (HL-LHC) [2–5] .

Beam Short for “Particle Beam”. The name for the collection of all particles in an accelerator traveling in the same direction. In the Large Hadron Collider (LHC) there are two beams (Beam 1 and Beam 2) traversing in opposite directions and colliding at the Interaction Points (IPs).

Beta-function Value of the twiss parameter β as a function of longitudinal location. This value is closely related to the amplitude \mathcal{A} of the betatron-oscillations and hence the size of the beam at that longitudinal location s via the action J by $\mathcal{A}(s) = \sqrt{2J\beta(s)}$.

Beta-beating Relative difference of the β -function between measurement and model: $(\beta_{\text{Measured}} - \beta_{\text{Model}})/\beta_{\text{Model}}$.

Bunch Short for “Particle Bunch”. Due to the RF acceleration scheme, a continuous beam is not possible in a synchrotron and particles become bunched during acceleration.

Burn-off In a particle collider, burn-off is the slow loss of beam intensity in colliding beams due to inelastic particle-particle interactions.

CMS (Compact Muon Solenoid) a general-purpose detector at the Large Hadron Collider (LHC). It has a broad physics programme ranging from studying the Standard Model (including the Higgs boson) to searching for extra dimensions and particles that could make up dark matter. Although it has the same scientific goals as the ATLAS experiment, it uses different technical solutions and a different magnet-system design. See: cms.cern.

Courant-Snyder parameters Set of quantities describing the distribution of positions and momenta of the particles in a beam. Also known as twiss parameters. See also β -function.

D1&D2 D1 and D2 are the shorthands for the separation/recombination dipoles in the Insertion Regions (IRs) of the Large Hadron Collider (LHC) and High-Luminosity Large Hadron Collider (HL-LHC), diverting the beams from their respective beam-lines into the common aperture region around the Interaction Points (IPs) and vice-versa.

Experiment In the context of the Large Hadron Collider (LHC), an “experiment” denotes one of the particle physics detector experiments and its accompanying data-collecting research structure. The four largest detectors, ATLAS, CMS, ALICE, LHCb are located at the four Interaction Points (IPs) of the LHC.

Feed-down Particles passing off-center through a multipole field experience effects akin to the influence of lower-order multipoles due to the orbit offset. See [Section 2.2.2](#).

Flat-orbit Machine setup without any crossing bumps in the Insertion Regions (IRs).

Flat-optics Optics in which β^* is different between the two transversal planes. Usually it is much smaller in one than in the other. See also: round-optics.

Hadron Composite subatomic particles consisting of two or more quarks, held together by strong interactions.

IP (Interaction Point) Point at which the two counter-rotating beams interact, i.e. their particles collide. In the Large Hadron Collider (LHC) these are located in the center of Insertion Regions (IRs) 1, 2, 5 and 8 inside the detectors (“Experiments”).

IR (Insertion Region) Section between the arcs of a synchrotron. They contain the straight sections, housing larger facilities, such as detectors (“Experiments”), acceleration (“RF”), collimation and beam-dump.

LEP (Large Electron-Positron Collider) The LEP is largest electron-positron accelerator ever built and was located in the same tunnel that now houses the Large Hadron Collider (LHC). It provided a detailed study of the electroweak interaction, producing and observing millions of W- and Z- bosons.

Leveling Luminosity leveling in the Large Hadron Collider (LHC) describes the process of compensating the loss of beam intensity, created by burn-off to provide constant luminosity. Multiple methods are applied in the Large Hadron Collider (LHC) and High-Luminosity Large Hadron Collider (HL-LHC):

- *β^* leveling*: reduction of the β -function at the Interaction Point (IP).
- *crossing-angle leveling*: reduction of the crossing angles.
- *offset leveling*: reducing the beam offset.
- *crabbing*: re-orientation of the bunch within the crossing region, counteracting the “tilt” due to the crossing angle.

These are used in combination for an optimal control of luminosity.

LHCb (LHC-beauty) A specialized b-physics experiment, designed primarily to measure the parameters of CP violation in the interactions of b-hadrons (heavy particles containing a bottom quark). See: lhcbrun.org.

LHCf (LHC forward) An experiment made up of two detectors 140 m left and right of ATLAS, which uses particles thrown forward by collisions as a source to simulate and study cosmic rays in laboratory conditions. See: home.cern/science/experiments/lhcf.

Long shutdown Planned shutdown periods of the Large Hadron Collider (LHC) between runs spanning multiple years, as opposed to the Year-End Technical Stop (YETS), which only lasts a few months. This time is used for repairs and upgrades of the machine, as well as general maintenance.

Luminosity Is the ratio of the number of events, i.e. collisions, detected per cross-section, either per time interval (“instantaneous luminosity”) or in total (“integrated luminosity”, e.g. since the beginning of a year, a run or the total since the beginning of operation of the machine in question). Colloquially it is shortened to “lumi”.

MAD-X Current version of the Methodical Accelerator Design (MAD) framework developed in BE-ABP at CERN.

MD (Machine Development) Studies dedicated to improving the Large Hadron Collider (LHC), e.g. by trying to reveal error-sources or attempting their correction, or testing possible future procedures and machine configurations. As they usually involve “beam-time”, i.e. a dedicated measurement setup and shift in the CERN Control Center (CCC), a proposal needs to be written and they need to be approved by the LHC Machine Committee (LMC).

Normal magnets Magnet with a “normal” oriented multipole field with pole-tips off the horizontal axis. Opposed to a “skew” magnet of the same order n , for which the field is rotated by $\pi/(2n)$ rad.

Offline analysis Any analysis that is being done after the measurements are completed. In contrast, see: online analysis.

omc3 Python package maintained and applied by the Optics Measurements and Corrections (OMC)-Team to analyse measurements from different accelerators and calculate corrections. See [6].

Online analysis Any analysis that is being done in the CERN Control Center (CCC), i.e. at the same time as the measurements are performed during machine commissioning or Machine Development (MD). In contrast, see: offline analysis.

Optics Here “optics” refers to “accelerator beam optics”, describing the particle motion through an accelerator as defined by the elements (i.e. magnets) of the machine. As the behaviour of a particle beam in magnetic fields shows similarities to a light beam propagating through lenses and can be described with similar equations, a lot of theory and nomenclature has been borrowed from ray optics.

RF (Radio Frequency) Short for the acceleration part of a synchrotron, as the frequency of the accelerating electric field is usually in the radio-frequency range (at the LHC ≈ 400 MHz).

Round-optics Optics in which β^* is identical for both planes. See also: flat-optics.

Run Consecutive years of LHC-operation, separated by long shutdowns. Run 1 in 2008-2013, run 2 in 2015-2018 and run 3 started in 2022.

Skew magnets Magnet with a “skew” oriented multipole field, with pole-tips on the horizontal axis. Opposed to a “normal” magnet of the same order n , for which the field is rotated by $\pi/(2n)$ rad.

SVD cleaning Noise cleaning method used in the omc3 python package. Based on an Singular Value Decomposition (SVD), using the strongest modes to reconstruct the signal.

Triplet Name for the final focusing quadrupoles in the Insertion Region (IR) left and right of an Interaction Point (IP). In principle these consist of three magnets Q1–3 (starting with Q1 closest to the IP), where Q1 and Q3 have the same orientation (e.g. focusing) and are opposite to Q2 (e.g. defocusing), depending on beam, plane and side. In the Large Hadron Collider (LHC) Q2 is split into two magnets, while in the High-Luminosity Large Hadron Collider (HL-LHC) also Q1 and Q3 will each be split in two.

Twiss parameters See Courant-Snyder.

Acronyms

ABP	Accelerator Beam Physics Group
ALICE	A Large Ion Collider Experiment
ATLAS	A Toroidal LHC Apparatus
ATS	Achromatic Telescopic Squeeze
BBQ	Base Band Tune
BCTFR	(Fast) Beam Current TransFormer
BE	Beams Department
BLM	Beam Loss Monitor
BPM	Beam Position Monitor
CCC	CERN Control Center
CERN	Conseil Européen pour la Recherche Nucléaire
CMS	Compact Muon Solenoid
DA	Dynamic Aperture (<i>See Section 2.2.7</i>)
FASER	ForwArd Search ExpeRiment
FCC	Future Circular Collider
FiDeL	Field Model of the LHC
h.o.t	High(er) Order Terms
HL-LHC	High-Luminosity Large Hadron Collider
IP	Interaction Point
IPAC	International Particle Accelerator Conference
IR	Insertion Region
LEP	Large Electron-Positron Collider
LHC	Large Hadron Collider
LHCb	LHC-beauty
LHCf	LHC forward

LMC	LHC Machine Committee
LSA	LHC Software Architecture
MAD	Methodical Accelerator Design
MD	Machine Development
MoEDAL	Monopole and Exotics Detector at the LHC
ODR	Orthogonal Distance Regression
OMC	Optics Measurements and Corrections
PRAB	Physical Review Accelerators and Beams
PS	Proton Synchrotron
PTC	Polymorphic Tracking Code
RDT	Resonance Driving Term (<i>See Section 2.2.5</i>)
RF	Radio Frequency
RHIC	Relativistic Heavy Ion Collider
RMS	Root Mean Square
SND	Scattering and Neutrino Detector
SPS	Super Proton Synchrotron
SVD	Singular Value Decomposition (<i>See SVD cleaning</i>)
TOTEM	TOTAL Elastic and diffractive cross section Measurement
WISE	Windows Interface to Simulation Errors
YETS	Year-End Technical Stop

Symbols

J Action. One of the phase-space coordinates in the Courant-Snyder normalization and an invariant of (linear) motion. It is customary to relate most dependencies of optics-parameters to the also commonly used invariant $\epsilon = 2J$ instead. In this thesis the shorthand ϵ for this quantity is rarely used and $2J$ is usually written out. The action J is subscripted with x or y , giving the transversal plane, and can thus be distinguished from the skew magnetic field strength, which is subscripted with an integer n . *Unit:* m.

- B ρ** Beam rigidity. Used as a normalization factor for normal magnetic field strength and skew magnetic field strength. See [Section 2.2.1](#). *Unit:* T m.
- β^* β -function at the IP. Usually subscripted with x or y giving the transversal plane. *Unit:* m.
- fb** Femtobarn. Metric unit of area used in high-energy physics as a measure for the cross-section of particle interactions. “barn” was introduced as a convenient measure of a large target area for a nucleus during collision. In particle accelerators inverse barn (in the Large Hadron Collider (LHC) inverse femtobarn) are used to quantify integrated luminosity. $1 \text{ fb} = 10^{-43} \text{ m}^2 = 10^{-39} \text{ cm}^2$.
- J_n Skew magnetic field strength. Skew field component normalized to the beam rigidity. Usually it is subscripted with an integer n giving the field order and can thus be distinguished from the action, which is subscripted with its respective transversal plane. *Unit:* m^{-n} .
See [Section 2.2.1](#).
- K_n Normal magnetic field strength. Normal field component normalized to the beam rigidity. Usually subscripted with an integer n giving the field order. *Unit:* m^{-n} .
See [Section 2.2.1](#).
- Q Tune, the number of betatron oscillations per turn in a circular accelerator. Usually subscripted with x or y giving the transversal plane. See also: β -function.

1 Introduction

1.1 Motivation

Nonlinear corrections have been shown to be of great importance for successful operation of the Large Hadron Collider (LHC) [7–17] and other circular colliders [18–20] as they suppress resonances and can improve Dynamic Aperture (DA) as well as beam lifetime [21].

A strong focusing is needed to achieve a high collision rate at the Interaction Points (IPs) in Insertion Regions (IRs) 1, 2, 5 and 8 - housing the main experiments at the LHC, namely ATLAS, ALICE, CMS and LHCb respectively. This leads to an extremely large β -function in the surrounding IR and makes the beam optics in these regions especially susceptible to magnetic field errors. The final focusing triplets left and right of the IP are of particular interest in this regard and nonlinear corrector packages, for the local correction of magnetic fields up to dodecapole multipole order, are installed [22] in their immediate vicinity. Yet the local corrections come with a caveat: due to the common-aperture nature of this region, corrections need to be found valid for both beams at the same time. Especially with increasing crossing-angles at the IPs and decreasing β^* (the β -function at the IP), the nonlinear influences on the machine become more and more pronounced, leading to sub-optimal performance if not addressed [17, 23, 24].

Crossing-angle and β^* will be pushed beyond their current limits in new schemes with high-intensity beams designed to further increase collision rates, and therefore luminosity, in the upcoming High-Luminosity Large Hadron Collider (HL-LHC) [22, 25]: If higher order errors are not properly mitigated beam-quality and lifetime are expected to deteriorate: Current estimations show, that while the upgrade allows to push for even smaller β^* , e.g. due to the installation of new larger aperture magnets, these changes come at the price of even higher optics sensitivity [26], as the β -function in the triplet quadrupoles of IPs 1 and 5 will increase above 20 km [22, 26, 27], more than four times the current LHC value [24, 28]. At the same time, constraints on the orbit control will be tighter than for the LHC [29].

Apart from decreasing β^* in both planes, another method to further increase luminosity, and at the same time distribute radiation load on the magnets, will be applied in the HL-LHC: so called “flat-optics” [3, 30], in which the β -function in the IPs, will have different values in the vertical and horizontal plane respectively. Currently, only the symmetrical “round-optics” are used during collision in the machine (with the exception of special runs at high β^* [31]) and are assumed in simulation tests of the correction schemes.

While local corrections of sextupole and octupole errors have been successfully applied in the IRs in the past, corrections of higher order were not necessary for stable operation at the present settings [32]. To mitigate the harmful influence on the tune via third order chromaticity, per-beam corrections up to dodecapole errors have been successfully performed in the arcs of the LHC [15]. Yet, apart from simulations [33] no attempt has been made to correct errors of order higher than octupoles locally in the IRs, despite dedicated “normal-oriented” dodecapole correctors being installed [34]. This circumstance is due to the fact that these dodecapole fields are the first allowed harmonics of quadrupole magnets [35] such as the triplets, and therefore expected to be stronger than e.g. decapole fields. Their influence due to feed-down, i.e. affecting lower order multipole errors due to particles orbit offsets, have already been observed [36, 37]. Creating an environment in the LHC imitating the expected settings of the HL-LHC, has hinted at the growing significance of multipoles beyond octupoles, for example on second order amplitude detuning [38] and DA [32, 39]. Additionally, well corrected low-order effects, such as coupling between the transversal planes and amplitude detuning, are expected to suffer from feed-down in the future machine [21, 24]. Feed-down is not very well handled as, in the deployed correction strategy, corrections are applied from low orders upwards. Hence, interplay between the errors of different orders will lead to non-optimal corrections: if the contribution from feed-down is large, the correction at a lower order will be overcompensated and the assumed-to-be-corrected effects at that order reappear after high-order correction.

Consequently, the thesis aims at the understanding of the sources and effects of high-order nonlinearities, i.e. nonlinearities caused by decapole and dodecapole magnetic field errors, in the IRs of LHC and HL-LHC. The influence of these errors on beam stability and machine performance is investigated with the ultimate goal to test the feasibility of corrections and develop applicable correction strategies.

1.2 Content

In the following chapters I will analyse different aspects of the correction of high-order nonlinearities in the LHC and HL-LHC, with a main focus on the error-sensitive IRs. To allow the reader to follow the studies presented, they will be guided by [Chapter 2](#) into the captivating world of particle accelerators and accelerator physics. The required mathematical framework for understanding the measured data acquired and the correction approaches chosen is also layed out in the second half of [Chapter 2](#).

As the thesis has started during the long shutdown of the LHC (Dec. 2018 until end of 2021), [Chapters 3](#) to [6](#) are mainly concerned with the analysis of data collected throughout previous LHC runs. Extensive simulations have been developed and executed to fill the gaps in the collected data and explore setups which were not applicable to the current machine.

A correction algorithm and its implementation [40], which uses the local minimization of Resonance Driving Terms (RDTs) as target measures and is thus finding corrector settings which suppress resonances in the machine locally, is outlined in [Chapter 3](#). This

algorithm is simulation-based, meaning it requires full knowledge of the optics layout and the errors in the region under test, which need to be provided by an accelerator model and either magnetic measurements or educated guesses for the errors. The correction procedure, derived from an earlier implementation [7, 41], allows for the correction of optics from two different beams at the same time, without assumptions of symmetry between them, which allows to evaluate the corrections for the aforementioned “flat-optics”. Further, feed-down is now also taken into account and the targeted RDT can be freely chosen. These new features are put under test in the simulations presented in the remainder of the chapter: Of special interest are hereby the influence of “feed-down” (see Section 3.3) and the “flat-optics” configuration (see Section 3.5) on the corrections, the quality of which is determined by evaluating amplitude detuning and DA for different scenarios in both LHC and HL-LHC.

In Chapter 4, the influence of misalignments of the nonlinear-correctors and triplets in the IRs of the LHC and HL-LHC on amplitude detuning are investigated, quantifying their impact on the successful operation of the machines under these disturbances. The results were calculated utilizing MAD-X [42] simulations of the two accelerator configurations respectively and applying errors and misalignments to their simulated elements.

Magnetic measurements of prototypes for the new D1/D2 magnets for the HL-LHC triggered the simulation-study presented in Chapter 5. These magnets are located close to the triplets, on the far side of the IPs, and the β -function is hence still quite large. In this study the influence of the newly measured field errors on machine performance via evaluation of DA are put to the test and their correctability is explored. Goal of this study is to conclude whether the errors are acceptable for operation, or if improvements on the magnet design are necessary. The new correction algorithm from Chapter 3 is used for this study, as its new features, such as the selection of different target RDTs allowed to find the best possible correction under the given circumstances.

Chapter 6 contains a short summary of an Machine Development (MD) which aimed to emulate some of the nonlinearities expected to be found in the HL-LHC IRs in the LHC by using the nonlinear corrector package to artificially introduce these errors and give insights into machine behavior.

The restart of the LHC in 2022 has seen a very successful commissioning and fruitful MDs. During that time new measurements could be performed in the running machine to attack the feed-down from decapole and dodecapole fields to octupole fields, which was observed by a change in amplitude detuning upon turning on the crossing scheme in 2018 [36, 37, 43]. A correction scheme was developed, calculating dodecapole corrections directly from the feed-down to amplitude detuning. Before 2022, this scheme had been studied in simulations using the data from 2018, and later refined with new data and successfully tested and applied in the machine. The results of both, simulations and experiments, can be found in Chapter 7.

In Appendix A some of my additional work is mentioned, that has been related and been important to the overall investigations conducted in this thesis.

In the final Chapter 8, the main results and conclusions of this thesis are repeated

and critically reflected upon. A discussion is held, about what has been found and what is missing and it is pondered, how future machine operation has been changed by the advent of this thesis.

2 Background

The research into particle accelerators has started at the beginning of the 20th century [44], and has in the beginning been mainly driven to study high-energy physics, uncovering the building blocks of matter and producing environments recreating the earliest stages of the universe. In its core, any particle accelerator uses electromagnetic fields to speed up charged particles and keep them on predefined trajectories. Nowadays this technology is not only used to collide particles with each other or fixed targets, but also to use the generated synchrotron radiation, e.g. for spectroscopy and (medical) imaging, from designated light-sources [45, 46], as well as other medical applications, such as cancer treatment with proton beams [46, 47].

To facilitate the international research in high-energy physics and by that means also develop new accelerator technologies in the 1950s the Conseil Européen pour la Recherche Nucléaire (CERN) was founded with a peaceful mission:

“ ”

The Organization shall have no concern with work for military requirements and the results of its experimental and theoretical work shall be published or otherwise made generally available. [48]

Today, CERN is the largest particle physics laboratory in the world and houses a multitude of particle accelerators and experiments. A schematic of the whole complex is shown in Fig. 2.1: Visible is the complete injector chain for protons and ions leading up to the biggest of all accelerators, the Large Hadron Collider (LHC), which is discussed in the next section Section 2.1. Apart from this widely known machine, CERN is also home to a multitude of fixed-target experiments [49–54], utilizing particle beams from the various injectors. Notably, there is also the anti-matter factory, where in the Antiproton Decelerator (AD) and the Extra Low ENergy Antiproton ring (ELENA) produced anti-matter particles are slowed down to be used in further experiments [55, 56], as well as the Isotope Separator OnLine DEvice (ISOLDE) [57], where properties of exotic nuclei are produced and studied and forwarded to MEDICIS, “designed to to produce unconventional radioisotopes with the right properties to enhance the precision of both patient imaging and treatment” [47]. In addition, CERN provides an experimental platform for neutrino experiments [58], general accelerators for R&D, such as the CERN Linear Electron Accelerator for Research (CLEAR) [59], and is studying the possibility of charged particle acceleration by means of plasma wakefields driven by a proton bunch in the Advanced WAKefield Experiment (AWAKE) [60].

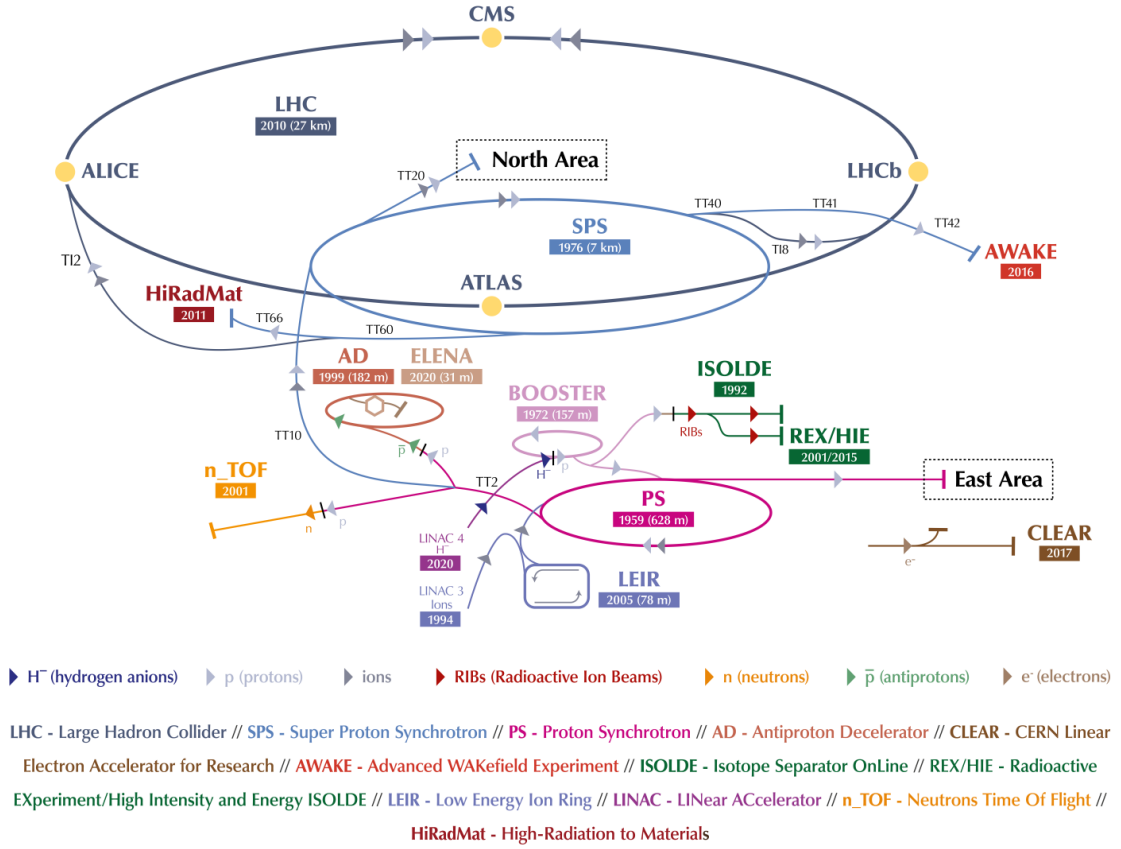


Figure 2.1. CERN Accelerator Complex [61].

2.1 The Large Hadron Collider

The best known and largest particle accelerator at CERN, reaching the currently highest collision energies in the world, is the LHC. The almost 27 km long accelerator was built between 1998 and 2008 into the tunnel of the former Large Electron-Positron Collider (LEP). Its main purpose is to study different theories of particle physics, to answer some of the fundamental open questions in physics, which in 2012 lead to the discovery of the Higgs-Boson [62, 63].

Overview. The LHC is a synchrotron, i.e. a circular accelerator, in which particles are guided in closed loops around the ring by magnetic fields, allowing to re-use acceleration and collision facilities at each turn. As the protons (or during special fills: heavy ions) have passed through the injector chain (Fig. 2.1) when injected into the LHC at 450 GeV, they are already nearly at the speed of light and have a revolution frequency of about 11 kHz.

In Fig. 2.2 a schematic layout of the LHC is shown: The LHC is divided into octants,

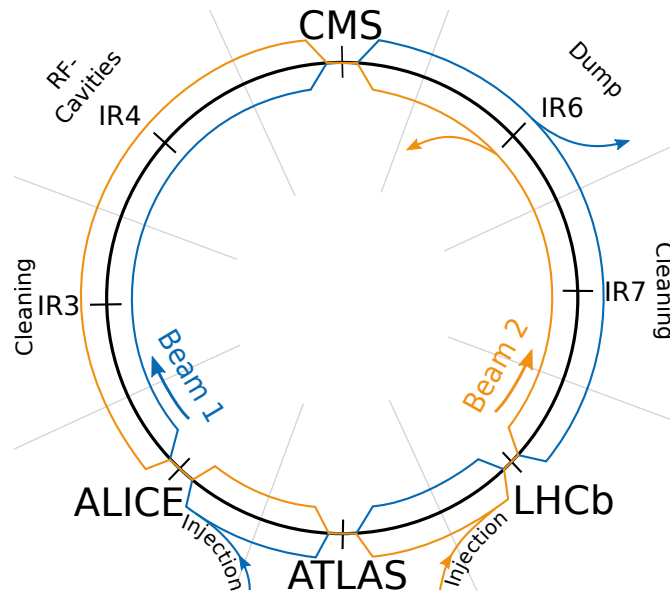


Figure 2.2. Schematic Layout of the LHC.

which reflect the rotational symmetry of the machine but are otherwise purely nominal and serve to aid in orientation and element nomenclature. It is further divided into arcs, in which the bending dipoles are located, and the Insertion Regions (IRs), housing straight sections which in turn contain the experiments and other special equipment.

The two particle beams, “Beam 1” and “Beam 2”, are injected in opposite directions, in IR2 and IR8 respectively, into separate beam-lines. IR4 contains the Radio Frequency (RF)-cavities, which accelerate the particles, IR3 and 7 on the other hand contain the cleaning facilities, for momentum- and betatron-cleaning respectively [64, 65]. In the IRs which contain the experiments, Interaction Points (IPs) are present: Here the two beam-lines converge, allowing the particles to collide. IR6 hosts the beam-dump, at which the beam is disposed after each fill.

Experiments. There are nine different experiments directly attached to the LHC. The four large detectors, marked in the schematic LHC layout of Fig. 2.2, are:

- ATLAS (A Toroidal LHC ApparatuS) [66] and CMS (Compact Muon Solenoid) [67] are the two biggest detectors and are built as general purpose detectors.
- LHCb (LHC-beauty) [68] is designed to measure the interactions of b-hadrons (containing bottom-quarks), to uncover the parameters of CP-violation.
- ALICE (A Large Ion Collider Experiment) [69] is optimized to focus on the properties of the quark-gluon plasma using heavy-ion collisions, to achieve insight into the physics of strongly interacting matter at extreme energies.

There are also five smaller experiments, namely:

- TOTEM (TOTAl Elastic and diffractive cross section Measurement) [70], close to CMS, is aiming at measurement of total cross section, elastic scattering, and diffraction processes.
- LHCf (LHC forward) [71], the purpose of which is to simulate cosmic rays by measuring particles generated in the “forward” direction of collisions close to the beams. It is located close to ATLAS.
- MoEDAL (Monopole and Exotics Detector at the LHC) [72], which will share an IP with LHCb and its goal is to detect magnetic monopoles or other highly ionizing stable and pseudo-stable particles.
- FASER (ForwArD Search ExpeRiment) [73], an experiment to search for new light and weakly-interacting particles, 480 m downstream of ATLAS.
- SND (Scattering and Neutrino Detector) [74], searching for collider neutrinos is also located close to ATLAS.

Energy and Luminosity. The most important parameters for the particle physicists running the experiments in the LHC are the energy and luminosity.

Higher energy of the particle beams allows to study interactions during which more particles are created as well as interactions leading to heavier particles.

The luminosity on the other hand is the coefficient for the collision rate in a particle-particle collider, i.e. the number of events per time per area:

$$L = \frac{R}{\sigma} \quad (2.1)$$

where R is the event rate and σ is the interaction cross-section. This “instantaneous luminosity” is therefore proportional to the number of particles per bunch ($N_{B1,B2}$) and inverse proportional to the beam-size at the collision point, i.e. the β -function at the IP (β^*).

$$L \propto \frac{N_{B1} \cdot N_{B2}}{\beta^*} \quad (2.2)$$

The “integrated luminosity” on the other hand is L integrated over time and is a useful value to characterize the performance of a particle accelerator: the higher this value, the more statistics are available for data-analysis and the more precise statements can be made about the measured quantities. In the LHC it is given in units of femtobarn.

Accelerators have therefore strived to maximize both of these parameters. In Fig. 2.3 the energy reach and design peak luminosities of past, modern and future particle accelerators is shown. The LHC can be found at the highest energy of the currently existing (filled markers) accelerators, but also close to the top in luminosity, presently only surpassed by the Super KEK-B [75], an electron-positron collider, which reached a peak luminosity of $4.65 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ in 2022, while the LHC is limited by heat deposition to the triplets at $2 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ [76].

The design parameters for the LHC can be found in Table 2.1.

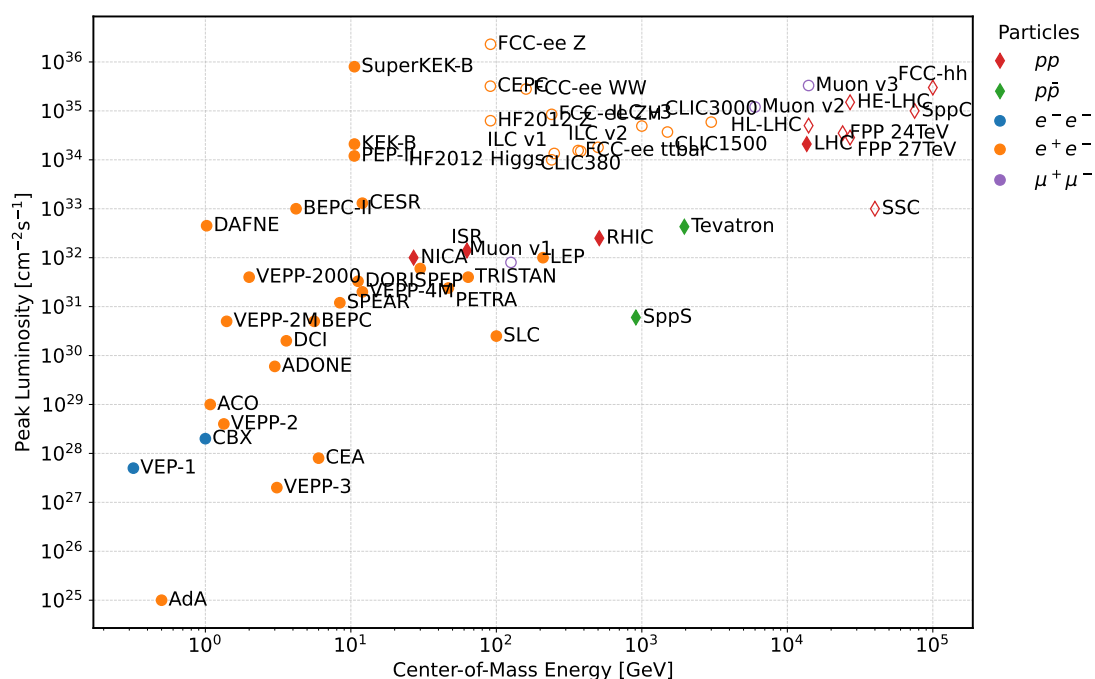


Figure 2.3. Chart of (design) peak luminosities of particle accelerators versus their energy reach. Already built machines are shown with filled markers, planned or suggested machines are shown with open markers. Data sources and more detailed charts can be found in [77].

Table 2.1. Selected LHC and HL-LHC design parameters [22]

		LHC (nominal)	HL-LHC (standard)
Beam energy in collision	[TeV]	7	7
Particles per bunch	[10^{11}]	1.15	2.2
Bunches per beam		2808	2760
Collisions in IP1 and IP5		2808	2748
Half-crossing angle in IP1 and IP5	[μ rad]	142.5	250
Minimum β^*	[m]	0.55	0.15
Normalized emittance ϵ_n	[μ m]	3.75	2.5
Peak ^a luminosity w/o crab cavities	[$10^{34} \text{ cm}^{-2} \text{ s}^{-1}$]	1.00	8.11
Peak ^a luminosity w/ crab cavities	[$10^{34} \text{ cm}^{-2} \text{ s}^{-1}$]	–	17.0
Events / Crossing		27	131

^a here: peak “virtual” luminosity, i.e. at the lowest β^* without other leveling schemes

Magnets. In a particle accelerator, the particles are guided on their circular orbit by magnetic fields, created by classic and super-conducting electro-magnets. Magnetic fields are in detail discussed in [Section 2.2.1](#), yet in general they are classified by their (even) number of poles and their orientation: either normal or skew. In magnets with a skew oriented multipole field two pole-tips are aligned with the horizontal axis of the magnet, whereas normal magnets of the same multipole order n are rotated by $\pi/(2n)$ rad. Schematic representations of magnets up to dodecapole order are illustrated in [Fig. 2.4](#). Depending on these parameters, they fulfill different roles in the accelerator, the most important ones are:

Bending Dipoles These are normal dipoles used as the bending magnets in a circular accelerator, forcing the particles on a circular orbit.

Dipoles Apart from the bending dipoles, normal and skew dipoles are installed, e.g. to control the orbit in the machine, either as orbit correctors, to steer the crossing scheme or to induce orbit bumps. Another important use-case are the separation/recombination dipoles, which transition the beam between the single- and dual-aperture sections of the collider.

Normal Quadrupoles These magnets are focusing in one transversal plane and defocusing in the other. A rotation of the field by 90° (usually achieved by inverting the direction of the powering current) results in switching the roles between the two planes. They are used control the transversal beam size along the beamline.

Skew Quadrupoles Skew Quadrupoles can be used to correct the coupling between the two transversal planes.

Normal Sextupoles Normal aligned magnets with six poles can be used to control the chromaticity of the accelerator and to correct sextupolar magnetic field errors, but have also been used in the LHC in combination with orbit-bumps to correct

the optics locally via feed-down [78].

Normal Octupoles Normal Octupoles are used to combat octupolar field errors, but also to stabilize the beam by inducing Landau Damping [79].

Skew sextupoles and skew octupoles as well as other higher order magnets are usually used as corrector magnets. The corrector packages in the IRs will be discussed below in this chapter.

Magnet names are usually abbreviated by a single letter, representing their multipole order, and an additional **S** in the case of a skew magnet. A full list of these abbreviated names can be found in Table 2.2.

Table 2.2. Magnet Order Abbreviations.

Name	Abbreviation	
	Normal	Skew
Dipole	B	BS
Quadrupole	Q	QS
Sextupole	S	SS
Octupole	O	OS
Decapole	D	DS
Dodecapole	T	TS

Layout and LHC Naming Conventions. Each of the octants of the LHC reaches from mid-arc to mid-arc and contains an IR as shown in Fig. 2.5. Based on this layout, elements of the LHC can be identified according to the following convention [80]:

$$(\text{TYPE}) (\text{SPECIAL}) . (\text{EXTRA}) (\text{HALF_CELL}) (\text{SIDE}) (\text{OCTANT}) . (\text{BEAM})$$

The entries in the definition above are defined as follows:

TYPE Type of element. In the context of this thesis the important ones are **M** for Magnets and **BPM** for Beam Position Monitors (BPMs) .

SPECIAL Optional subtype of an element, e.g. for magnets the abbreviation of their order (Table 2.2) is placed here, preceded by a **C** in case of corrector magnets and followed by **X**, **XF** in case of common aperture magnets (Table 2.3).

EXTRA Optional entry used to separate between otherwise identically named elements. This is used for example to distinguish between the two parts of the triplet magnets (MQXB.A and MQXB.B).

HALF_CELL Sequence number of the half-cell (see paragraph “Arcs” below) the element is part of. These start at the IP and are incremented by 1 outward until mid-arc.

SIDE L (left) or R (right), specifying which side of the closest IP the element is on.

OCTANT Single digit number from 1 to 8, specifying the octant the element is a part of.

BEAM Optional entry specifying which beam the element is part of. This can be either B1 or B2 unless the element is located inside of the common aperture region of the

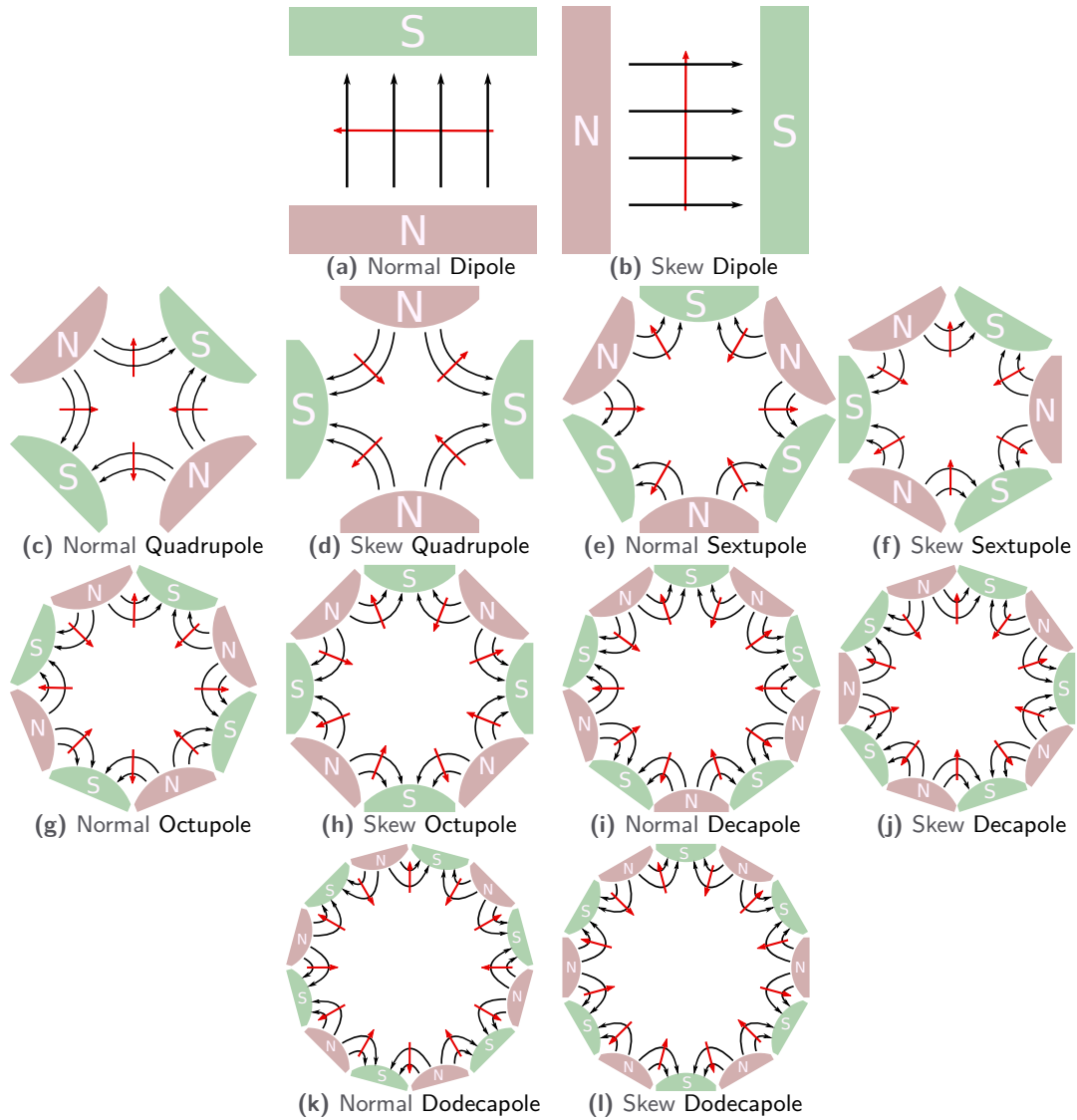


Figure 2.4. Schematics of magnetic multipoles. In black the magnetic fieldlines, which extend also to the inner part of the magnets, but drawing them there has been omitted for clarity of the figure. Red arrows indicate the direction of force on a positive charge moving out of the page towards the reader.

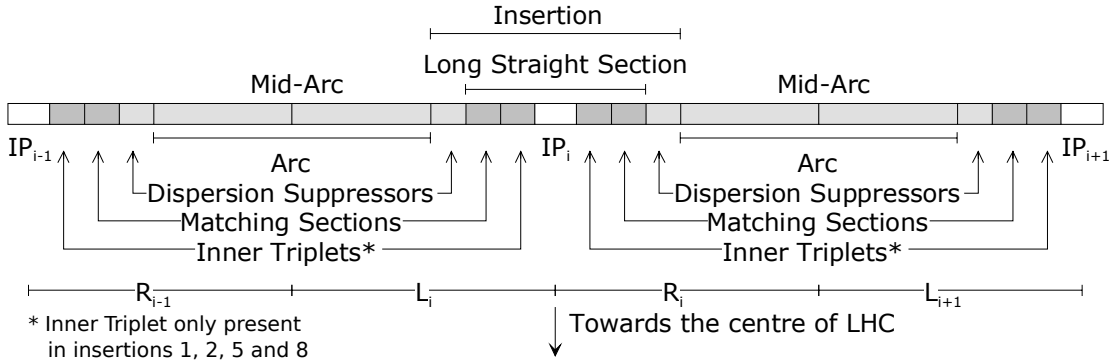


Figure 2.5. A more detailed LHC layout spanning two octants [80].

Table 2.3. Other Magnet Abbreviations.

Name	Abbreviation
(Tune) Trim Quadrupole	QT
Common aperture magnet	X, XF (suffix)
Triplet Magnet	A/B (suffix), e.g. MQXFA
Corrector Magnet	C (prefix), e.g. MCOX
<i>Shorthands in text:</i>	
Focusing/Defocusing Quadrupole	QF / QD
Triplet Quadrupole	Q1, Q2, Q3

IR in which case this part is omitted.

Arcs. The arcs themselves are each made up of 23 repeating cells (or 46 half-cells) and are ≈ 2.45 km long. Subsequent to each arc, dispersion suppressors are installed at the beginning of the IR to facilitate the transition between arc and straight section. Together with two additional quadrupole magnets (located in the straight sections), they bring the dispersion at the IP to zero [34].

The cells are based on FODO-cells (see Fig. 2.6) in which the drift spaces (0) are replaced with the bending dipoles. In reality, the cells contain more magnets, such as trim quadrupoles (MQT) allowing fine control of the tune, the lattice octupoles (MO), also called Landau octupoles, as they control Landau damping, and various corrector-magnets. A schematic is shown in Fig. 2.7.

Final-Focusing of the Beam. In the IRs in which the main experiments are located - ATLAS in IR1, ALICE in IR2, CMS in IR5 and LHCb in IR8 - the final-focusing quadrupoles are installed left and right of the IP. They are also called triplets (MQX) and bring the beam-size, and therefore β^* , down at the collision point and hence increase luminosity (see Eq. (2.2)).

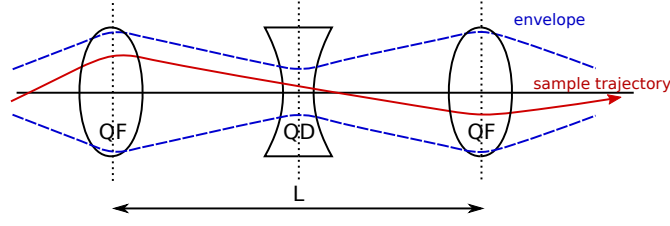


Figure 2.6. A schematic of a FODO-cell of length L , combining focusing (QF) and defocusing (QD) quadrupoles to achieve an overall focusing effect in both transversal planes. The blue line is representing envelope of all particles, which is proportional to the square-root of the β -function, while the red line depicts an exemplary trajectory of a single particle.

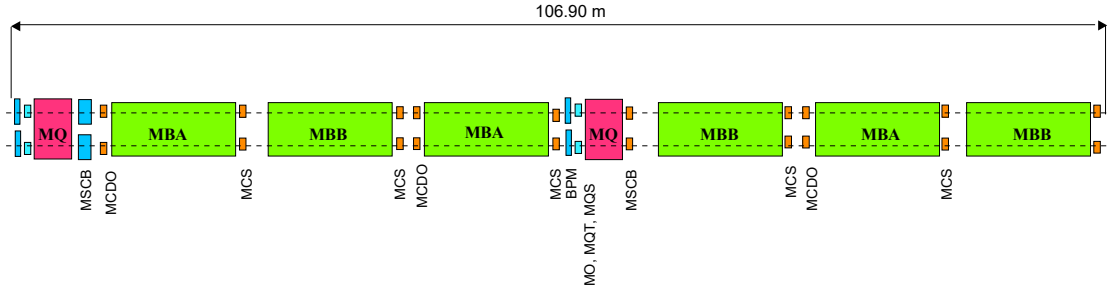


Figure 2.7. Arc-cell schematic of the LHC [34]: Based on a FBDB-cell, the MQ are the main quadrupoles and the MBA/MBB the bending dipoles. MQT (MQS) are the (skew) trim quadrupoles, MO the lattice octupoles. MSCB are combined sextupole and orbit corrector magnets, while the MCS are sextupole corrector spool-pieces. MCDO are combined octupole and decapole spool-piece corrector magnets. Also shown are the Beam Position Monitors (BPMs).

In Fig. 2.8 the elements and β -functions are shown using the example of IP5. In principle the triplets consist of three magnets Q1–3 (starting with Q1 closest to the IP), where Q1 and Q3 (MQXA) have the same orientation (e.g. focusing) and are opposite to Q2 (MQXB; e.g. defocusing), with the orientation depending on beam, plane and side. In the LHC Q2 is split into two magnets (MQXB.A and MQXB.B, see Fig. 2.9). To achieve maximum focusing strength, the β -function in the triplets is very high, in the LHC up to ≈ 10 km, while in the (non-ATS, see below) arcs the highest peak is at ≈ 180 m at the focusing quadrupoles.

In addition to this standard focusing scheme for accelerators, the LHC implements so-called Achromatic Telescopic Squeeze (ATS) optics. They provide a flexible mechanism to strongly reduce β^* in a symmetric or asymmetric way i.e. without necessarily imposing the same β^* in both planes. They have the advantage to be able to control the chromatic aberrations induced, namely the linear and nonlinear chromaticities, the off-momentum β -beating, and the spurious dispersion from the large crossing angles [2–5]. In this optics, the beam is first “pre-squeezed” using the triplets to the desired β^* value at the beginning of collisions. β^* -leveling, to compensate the burn-off (the loss of beam intensity due to collisions), in IP1 and IP5 is then executed using the quadrupoles in the

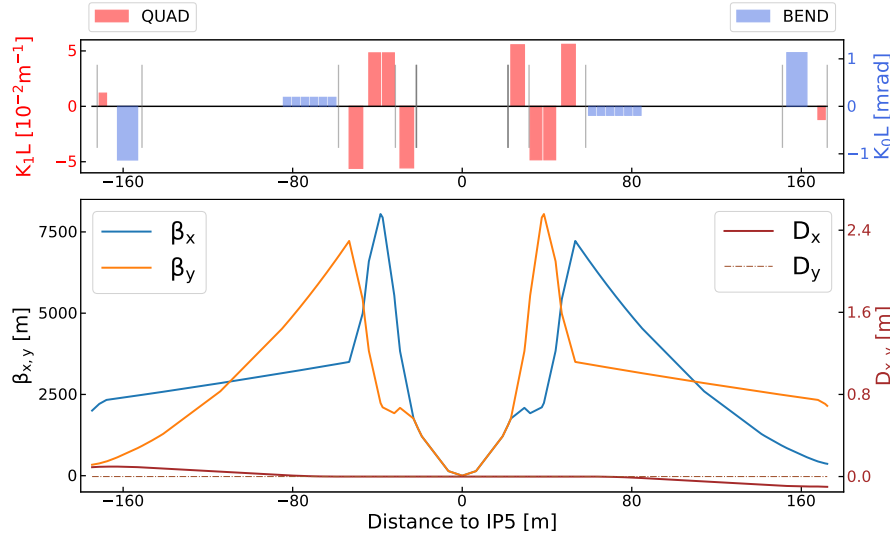


Figure 2.8. Simplified LHC layout with β -function and dispersion for Beam 1 in IR5 with $\beta^* = 30$ cm optics (generated with [81]).

adjacent IRs, which increases the β -function in the “ATS-arcs” in between the involved IR, namely arcs 81, 12, 45, and 56, performing an additional “tele-squeeze”.

Nonlinear Correctors. Due to the focusing scheme, leading to up to 10 km high β -functions as mentioned above and shown in Fig. 2.8, the optics are very sensitive to magnetic field errors in the area surrounding the IPs. To compensate for errors locally, both sides of the LHC IRs hosting experiments are equipped with linear and non-linear corrector packages. As shown in the schematics of Figs. 2.9 and 2.10, these packages are located within the common aperture region of the machines, between Q3 and the separation dipoles D1, and hence contain common magnets for the two beams. Any correction should therefore take the optics of both beams into account.

In the experimental IRs of the LHC (and in HL-LHC IR2 and IR8, see Section 2.1.1), nonlinear correctors for skew and normal sextupoles (a_3, b_3), skew and normal octupoles (a_4, b_4) and normal dodecapoles (b_6) are available.

Some correctors were defective in LHC run 2 and run 3: MCSSX.3L2, MCOX.3L2, MCOX.3L2, the skew sextupole, octupole and skew octupole correctors left of IP2, possibly due to a hit from a pilot beam; as well as MCOX.3L1, the skew octupole corrector left of IP1, probably due to powering issues. This is already reflected in the lattice used for simulations in this thesis [82].

Beam Position Monitors (BPMs). In the LHC there are around 500 Beam Position Monitors (BPMs) around the ring. These are usually dual-plane BPMs, meaning

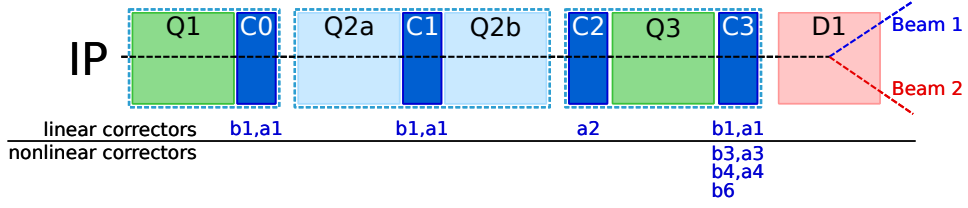


Figure 2.9. Schematic of the right hand side of a LHC IR and HL-LHC IR2 and IR8. Q1, Q2a/b and Q3 are the triplet quadrupoles. C0-C3 show the corrector packages with the field order to be corrected indicated. Light blue lines mark common cryostats. D1 is the separation dipole, diverging Beam 1 and Beam 2 to their respective beamlines.

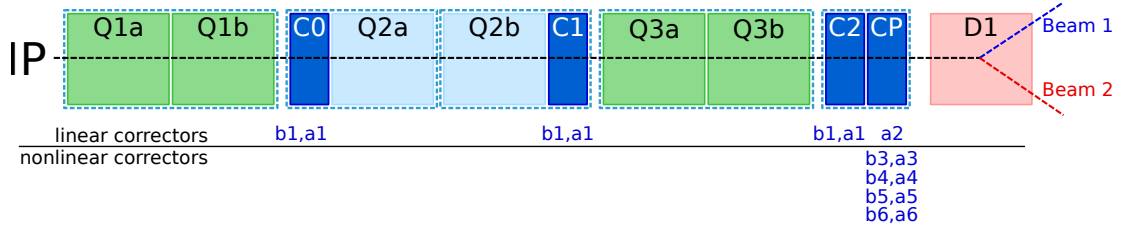


Figure 2.10. Schematic of the right hand side of HL-LHC IR1 and IR5. Q1a/b, Q2a/b and Q3a/b are the triplet quadrupoles. C0-C2 and CP show the corrector packages with the field order to be corrected indicated. D1 is the separation dipole, diverging Beam 1 and Beam 2 to their respective beamlines. Light blue lines mark common cryostats.

their electrodes can pick up the position of the beam centroid at each turn at their location [83], which is calculated from the induced voltage due to the passing charged particles. They deliver so called turn-by-turn data, i.e. transversal beam positions at all BPM locations for each turn during beam excitation, which is the main input for optics analysis (Section 2.3).

2.1.1 High luminosity LHC

In 2010, the “high-luminosity LHC project” was established at CERN, with the goal to enable full exploitation of the LHC’s physics potential [84]: After run 3, the LHC is going to be updated with the goal to achieve at least $5 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ peak luminosity, producing 250 fb^{-1} per year. This will result in a total of 3000 fb^{-1} after the planned 12 years of luminosity production, ten times the original total design luminosity of the LHC (currently expected to be 350 fb^{-1} after run 3).

To this end, major upgrades of the machine are envisioned during the long shutdown 3. Of main concern for this thesis are the upgrades in the experimental IRs, around ATLAS and CMS:

Triplets as well as the corrector magnets in the nonlinear corrector packages in these low- β^* regions receive high dosages of radiation, which can damage the material. The triplets are expected to be failing after 400 fb^{-1} to 700 fb^{-1} , while the end of the lifetime of the corrector magnets is estimated to be already at 300 fb^{-1} [22]. Due to the larger

β^* at ALICE and LHCb, and the limited duration of ion-ion collisions at ALICE per year, the integrated luminosity is much lower at these experiments and radiation damage is not expected to be a problem in their IRs. For the HL-LHC triplets and correctors in IR1 and IR5 will be replaced, and also upgraded: The new triplet magnets will have increased coil aperture of 150 mm, from 70 mm in the LHC [34], allowing to increase the β -function in the triplets by a factor of ≈ 2 compared to the LHC configuration, reducing the attainable β^* down to 15 cm [85]. They will also have an advanced cooling scheme to improve intervention flexibility of the cryogenic system and heat absorption, which will make them better suitable for higher peak luminosity. As shown in Fig. 2.10, the Q1 and Q3 (the MQXA) will be each replaced by two new magnets Q1a/b and Q3a/b (MQXFA.A and MQXFA.B).

While in IR2 and IR8, the available correctors will be the same as in the LHC-configuration (Fig. 2.9), in the HL-LHC's IR1 and IR5 the corrector package will be upgraded to also include skew and normal decapoles (a_5, b_5) as well as skew dodecapoles (a_6). They offer therefore a wider range in the type of field errors to correct, to account for the increase in the β -function, which makes the optics more sensitive to errors [22, 26, 27, 41].

To reduce radiation damage in the HL-LHC, a new asymmetric type of optics is envisioned, so called flat-optics [3, 30], in which β^* in the two transversal planes can have different values, usually differing by a factor of 2 or 4. Switching the plane of the smaller β^* (e.g. between runs or fills) results in a more distributed radiation deposition in the LHC magnets, while the optics also give rise to an additional increase in luminosity [30].

Among the other updates planned for the HL-LHC, illustrated in Fig. 2.11, are the new “crab-cavities”, which provide an additional way to increase luminosity: With this technology longitudinal rotation of bunches at the IPs is possible, enhancing the overlap of the colliding bunches [86]. The also upgraded collimation system will guarantee machine safety even at the increased beam intensities, and will incorporate novel crystal collimators for operations with ion beams [87].

2.2 Accelerator Physics Theory

To not repeat what others have already published in detail, this chapter is limited to a brief introduction of the theory required to appreciate the studies conducted, the measured data acquired and the correction approaches expounded in later chapters of this thesis. The interested reader may find more details in e.g. [89–92], and more specific introductions covering similar topics as presented here in [93, 94]. A general familiarity with classical Hamiltonian mechanics [95] is presumed.

Note. Most of content of this chapter has been published in similar form in either the background chapter of [96] or in [97], both of which have been exclusively written by myself and while the co-authors of these notes have provided contributions solely in the capacity of reviewers. Some of the more complicated derivations conducted in

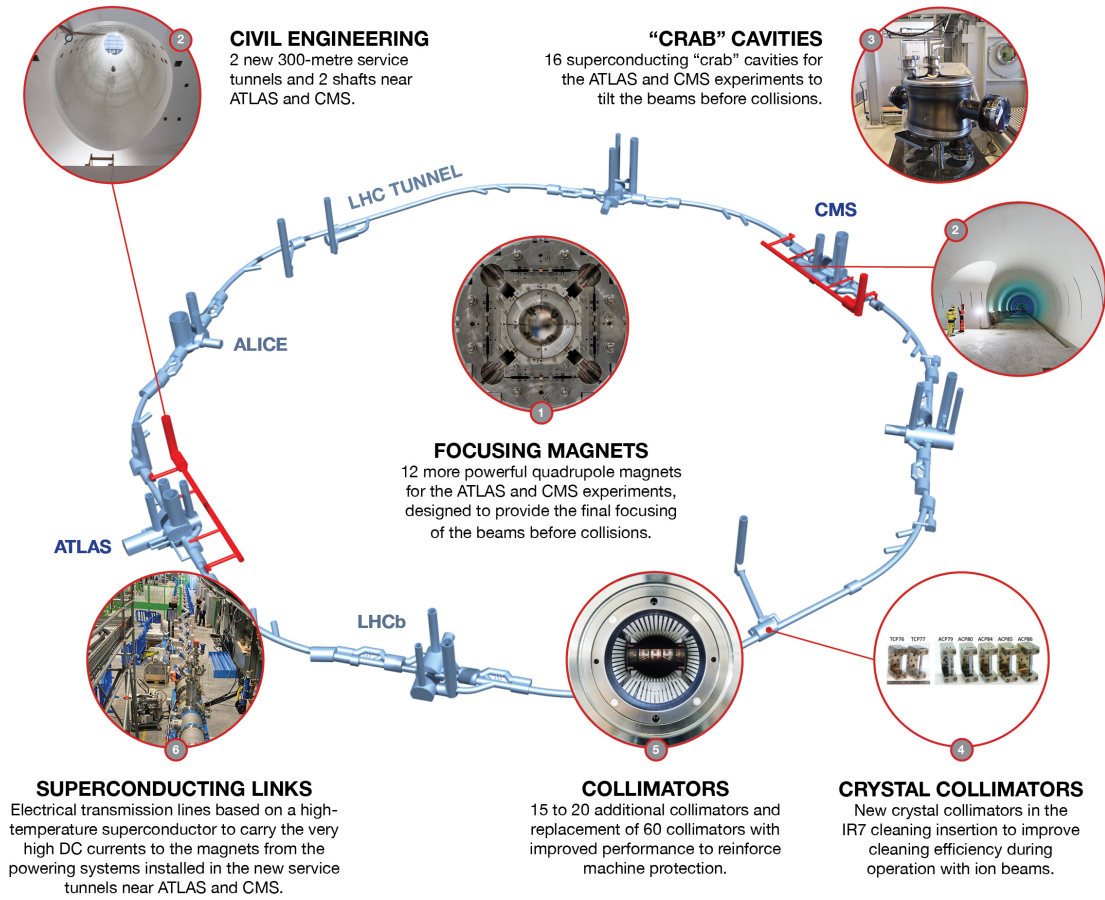


Figure 2.11. HL-LHC New Technologies [88].

the following sections are presented in more detail in the latter reference, to allow the reader can verify their validity with ease.

2.2.1 Magnetic Fields



In this thesis the *European Convention* for field indices is used, so the **indices for the magnetic fields components as well as for the field strengths begin at 1, indicating a dipole field**. This is in contrast to the *American Convention*, e.g. used in MAD-X, where the dipole field strength is assigned the index 0.



There are two different usages of J : One is a stand-in for the skew magnetic field strength (in MAD-X called KS) and has an integer subscript n showing the field order. The other one represents the action, the invariant of linear motion. The action will have a plane - x, y or z - as subscript.

Multipole Expansion

The multipole expansion of a general magnetic field \mathbf{B} in the planes x and y reads

$$B_y + iB_x = \sum_{n=1}^{\infty} (B_n + iA_n) (x + iy)^{n-1} . \quad (2.3)$$

The normal and skew field gradients B_n and A_n are therefore defined as derivatives of the complex field:

$$B_n + iA_n = \frac{1}{(n-1)!} \left. \frac{\partial^{n-1} (B_y + iB_x)}{\partial (x + iy)^{n-1}} \right|_{x=0, y=0} \quad (2.4)$$

The main field component of a 2N-pole magnet (i.e. the **only** component of a **perfect** 2N-pole magnet) is B_N for a normal and A_N for a skew magnet. The multipole expansion of the magnetic field for a non-perfect magnet are based around this main component. The coefficients b_n and a_n represent the normal and skew relative field errors, normalized to the main field component and at a reference radius r_{ref} :

$$B_y + iB_x = \begin{cases} B_N \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{r_{ref}} \right)^{n-1} , & \text{for normal magnets} \\ A_N \sum_{n=1}^{\infty} (b_n + ia_n) \left(\frac{x+iy}{r_{ref}} \right)^{n-1} , & \text{for skew magnets} \end{cases} . \quad (2.5)$$

b_n and a_n are dimensionless but usually given in 'units' of 10^{-4} . Therefore the normal and skew field component B_n and A_n of an normal 2N-pole magnet are then

$$B_n [\text{T m}^{1-n}] = B_N \cdot \frac{b_n}{r_{ref}^{n-1}} \quad \text{and} \quad A_n [\text{T m}^{1-n}] = B_N \cdot \frac{a_n}{r_{ref}^{n-1}} , \quad (2.6)$$

and equivalent for the normal and skew field components of a skew 2N-pole magnet with the main field A_N .

Field Normalization

The field gradients can furthermore be normalized to $B\rho$ (beam rigidity), the main dipole field B and its bending radius ρ (see Fig. 2.12):

$$K_n + iJ_n = \frac{1}{B\rho} \left. \frac{\partial^{n-1} (B_y + iB_x)}{\partial (x + iy)^{n-1}} \right|_{x=0, y=0} \quad (2.7)$$

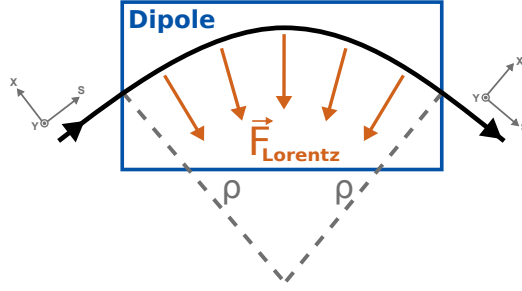


Figure 2.12. Schematic of the Lorentz force \vec{F}_{Lorentz} and bending radius ρ of a dipole with magnetic field along \vec{y} and a positive charged particle traversing along \vec{s} .

In the following q is the particle charge and p is the beam momentum. Also, with c the speed of light in vacuum and the velocity of a particle v , the relativistic Lorentz factor γ_L and the velocity ratio β_L are defined as

$$\gamma_L = \frac{1}{\sqrt{1 - \beta_L^2}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} . \quad (2.8)$$

The beam rigidity can then be calculated from the equality of the Lorentz force and the centripetal force of a particle with velocity v and its rest mass m_0 :

$$F_{\text{Lorentz}} = F_{\text{Centripetal}} \quad (2.9a)$$

$$\Rightarrow qvB = \frac{\gamma_L m_0 v^2}{\rho} = \frac{pv}{\rho} \quad (2.9b)$$

$$\Rightarrow B\rho = \frac{p}{q} . \quad (2.9c)$$

With Eqs. (2.7) and (2.9c), the field strengths K_n , J_n relate to B_n and A_n :

$$\begin{aligned} K_n [\text{m}^{-n}] &= \frac{q}{p} (n-1)! B_n, \quad \text{and} \\ J_n [\text{m}^{-n}] &= \frac{q}{p} (n-1)! A_n . \end{aligned} \quad (2.10)$$

2.2.2 Hamiltonian Description

The time dependent Hamiltonian of a particle of mass m , charge q and momentum \mathbf{p} in a static electro-magnetic field, with vector potential \mathbf{A} and scalar potential ϕ , is ([90] Eq. (2.22))

$$H = T - V = c\sqrt{|\mathbf{p} - q\mathbf{A}|^2 + m^2c^2} + q\phi. \quad (2.11)$$

After changing dependent variable to s in a curved trajectory reference system (x, y, s) ([90] Eq. (2.30)) the Hamiltonian becomes:

$$H = \frac{\delta}{\beta_L} - \left(1 + \frac{x}{\rho}\right) \sqrt{\left(\delta + \frac{1}{\beta_L}\right)^2 - (p_x - a_x)^2 - (p_y - a_y)^2 - \frac{1}{\beta_L^2 \gamma_L^2}} - \left(1 + \frac{x}{\rho}\right) a_s, \quad (2.12)$$

with the relativistic energy deviation from the total energy E of a reference particle

$$\delta = \frac{E}{cp} - \frac{1}{\beta_L} \quad (2.13)$$

and the scaled vector potential

$$\mathbf{a} = \frac{q}{p} \mathbf{A}. \quad (2.14)$$

Vector Potential

In the coordinates (x, y, s) of the curved trajectory reference system, the magnetic field \mathbf{B} becomes ([90] Eq. (3.36) - Eq. (3.38)):

$$\mathbf{B}(x, y, s) = \nabla \times \mathbf{A}(x, y, s) = \begin{bmatrix} \frac{\partial A_s}{\partial y} - \frac{1}{1+\frac{x}{\rho}} \frac{\partial A_y}{\partial s} \\ \frac{1}{1+\frac{x}{\rho}} \frac{\partial A_x}{\partial s} - \frac{\partial A_s}{\partial x} - \frac{A_s}{\rho+x} \\ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \end{bmatrix} \stackrel{\text{gauge}}{=}_{A_x=A_y=0} \begin{bmatrix} \frac{\partial A_s}{\partial y} \\ -\frac{\partial A_s}{\partial x} - \frac{A_s}{\rho+x} \\ 0 \end{bmatrix}. \quad (2.15)$$

For $n > 1$ (not a dipole) the path through the magnet center is straight and hence $\rho \rightarrow \infty$:

$$B_y + iB_x = -\frac{\partial}{\partial x} A_s + i \frac{\partial}{\partial y} A_s. \quad (2.16)$$

With $\Re[z]$ the real and $\Im[z]$ the imaginary part of any complex number z , it therefore follows that

$$A_s = -\Re \left[\sum_{n \geq 2} \frac{1}{n} (B_n + iA_n) (x + iy)^n \right]. \quad (2.17)$$

As can be seen by calculating the derivative:

$$\begin{aligned}
B_y + iB_x &= -\frac{\partial}{\partial x}A_s + i\frac{\partial}{\partial y}A_s \\
&\stackrel{\text{Eq. (2.17)}}{=} \Re \left[\sum_{n \geq 2} (B_n + iA_n)(x + iy)^{n-1} \right] - i\Re \left[\sum_{n \geq 2} i(B_n + iA_n)(x + iy)^{n-1} \right] \\
&= \Re \left[\sum_{n \geq 2} (B_n + iA_n)(x + iy)^{n-1} \right] + i\Im \left[\sum_{n \geq 2} (B_n + iA_n)(x + iy)^{n-1} \right] \\
&= \sum_{n \geq 2} (B_n + iA_n)(x + iy)^{n-1}, \tag{2.18}
\end{aligned}$$

which is the multipole expansion of Eq. (2.3) without the contributions from dipoles. Using Eqs. (2.14) and (2.17), the contribution h of the magnetic field to the Hamiltonian in Eq. (2.12) is then

$$\begin{aligned}
h &= \frac{q}{p} \Re \left[\sum_{n \geq 2} \frac{1}{n} (B_n + iA_n)(x + iy)^n \right] \\
&\stackrel{\text{Eq. (2.10)}}{=} \Re \left[\sum_{n \geq 2} (K_n + iJ_n) \frac{(x + iy)^n}{n!} \right] \tag{2.19}
\end{aligned}$$

For different orientations (normal, skew) and orders of magnetic field components, we can get the corresponding Hamiltonian, i.e. their contribution to the total Hamiltonian as

$$\text{normal Hamiltonian term:} \quad N_n = \frac{1}{n!} K_n \Re [(x + iy)^n] \tag{2.20a}$$

$$\text{skew Hamiltonian term:} \quad S_n = -\frac{1}{n!} J_n \Im [(x + iy)^n] \quad . \tag{2.20b}$$

Feed-Down

The effect of feed-down occurs when a particle beam's orbit passes off-center through a magnet, due to either a transverse misalignment of the magnet or an off-center closed orbit of the beam itself. In these cases, the magnetic field can be described as a composition of the current order main field plus lower order components, and can be understood by applying a Taylor expansion in x and y to Eq. (2.3) or, equivalently, to the Hamiltonian of the system in curvilinear (comoving) coordinates. The resulting additional components therefore cause the same effects on the beam as lower order sources would [91]. Using Eq. (2.3) and Eq. (2.10), and with $(\Delta x, \Delta y)$ being the orbit of the off-center particle, feed-down to field order $n \geq 1$ from fields up to order $n + P$

can be expressed as:

$$(K_n + iJ_n)^{\text{w/ feed-down}} = \sum_{p=0}^P (K_{n+p} + iJ_{n+p}) \frac{(\Delta x + i\Delta y)^p}{p!}. \quad (2.21)$$

Feed-down is in more detail described in [Appendix B](#), in which also [Eq. \(2.21\)](#) is derived.

2.2.3 Nonlinear Systems

2.2.4 Courant-Snyder Coordinates and Nonlinear Maps

The transformation of the phase-space coordinates of a particle propagating through an accelerator from s' to s can be described by means of maps $\mathcal{M}(s', s)$, each describing the transformation of the coordinates generated by the elements of the machine between the locations. These maps are symplectic, i.e. they preserve phase-space volume, and can be combined, describing the propagation through multiple elements of the accelerator [\[98\]](#). In a circular accelerator one can construct a one-turn-map \mathcal{M}_\circ , describing the coordinate transformation of one complete turn. For an ideal accelerator consisting of purely *linear* elements, one can use the Courant-Snyder formalism [\[99\]](#) to express the coordinates of the phase-space as a vector $\mathbf{c} = (c_{x,+}, c_{x,-}, c_{y,+}, c_{y,-})$ of complex coordinates in action J_u (the invariant of linear motion) and phase ϕ_u

$$c_{u,\pm} = \hat{u} \pm i\hat{p}_u = \sqrt{2J_u} e^{\mp i\phi_u}, \quad (2.22)$$

for $u \in \{x, y\}$, where \hat{u} and \hat{p}_u are the canonical position and momentum, related to the cartesian position u and momentum p_u via the α - and β -functions at any location s by

$$\begin{pmatrix} \hat{u}(s) \\ \hat{p}_u(s) \end{pmatrix} = \begin{pmatrix} 1/\sqrt{\beta_u(s)} & 0 \\ \alpha_u(s)/\sqrt{\beta_u(s)} & \sqrt{\beta_u(s)} \end{pmatrix} \begin{pmatrix} u(s) \\ p_u(s) \end{pmatrix}. \quad (2.23)$$

With [Eq. \(2.22\)](#) propagation through the accelerator is described by rotations, e.g. to a longitudinal location s by advancing the phase from s_0 to s by

$$\Delta\phi_u(s_0, s) = \phi_u(s) - \phi_u(s_0) = \int_{s_0}^s \frac{1}{\beta(s')} ds' \quad (2.24)$$

meaning the coordinates at s of a particle can be given as the initial coordinates at s_0 propagated to s

$$c_{u,\pm}(s) = \hat{u}(s) \pm i\hat{p}_u(s) = \sqrt{2J_u} e^{\mp i\phi_u(s)} = \sqrt{2J_u} e^{\mp i(\phi_{u,0} + \Delta\phi(s_0, s))} = \mathcal{M}_{u,\pm}(s_0, s) \cdot c_{u,\pm}(s_0), \quad (2.25)$$

where $\phi_{u,0} = \phi_u(s_0)$ is the initial phase of the particle, $J_u = J_u(s) \equiv J_u(s_0)$ its action and $\mathcal{M}_{u,\pm}$ are the components of $\mathcal{M} = (\mathcal{M}_{x,+}, \mathcal{M}_{x,-}, \mathcal{M}_{y,+}, \mathcal{M}_{y,-})$ propagating \mathbf{c} . In this linear system, the linear one-turn-map of an accelerator of circumference C is then

also a rotation $\mathcal{M}_o = R$ by $\Delta\phi_u(s, s+C) = 2\pi Q_u$, where Q_u are the *tunes* of the accelerator:

$$c_{u,\pm}(s+C) = \mathcal{M}_{u,\pm,o} \cdot c_{u,\pm}(s) = e^{\mp i 2\pi Q_u} c_{u,\pm}(s) = \sqrt{2J_u} e^{\mp i(\phi(s)+2\pi Q_u)} . \quad (2.26)$$

In [94, 100–103] this formalism is extended to derive a transformation, such that the propagation also through a *nonlinear* circular machine, i.e. a circular accelerator containing additional nonlinear elements, can be described by an amplitude dependent rotation and is shortly summarized here.

Using Lie-Algebra notation [98, 104] we can define an operator $: \cdot :$ as

$$: g := \sum_{u=x,y} \frac{\partial g}{\partial q_u} \frac{\partial}{\partial p_u} - \frac{\partial g}{\partial p_u} \frac{\partial}{\partial q_u} , \quad (2.27)$$

for a function $g(q_x, p_x, q_y, p_y)$ of canonical coordinates, e.g. positions/momenta (x, p_x, y, p_y) or actions/angles $(J_x, \phi_x, J_y, \phi_y)$. It is shown that the, now location dependent, nonlinear one-turn map can be expressed as

$$\mathcal{M}_o(s) = e^{:h(s):} R , \quad (2.28)$$

where R is still a rotation containing the linear contributions of the system and $h(s)$ is an integral of the Hamiltonians H_w of the nonlinear elements w of the accelerator. H_w are expressed as functions of powers of the normalized coordinates $c_{u,\pm}(s_w)$, but these coordinates propagated (linearly) backward from the origin s_w of the nonlinear sources to s . Or, maybe more intuitively, the particle at s is being propagated linearly through the ring, while experiencing the influence of the nonlinearities at the location of their origins.

If the nonlinearities H_w in the machine are small $h(s)$ can be approximated, truncating the Baker-Campbell-Hausdorff formula at first order of H_w

$$\begin{aligned} h(s) &\approx \oint_{\text{Ring}} H_w(s_w, s) ds_w = \\ &\sum_{jklm} \oint_{\text{Ring}} h_{jklm}(s_w) e^{i[(j-k)\Delta\phi_x(s, s_w) + (l-m)\Delta\phi_y(s, s_w)]} c_{x,+}^j(s_w) c_{x,-}^k(s_w) c_{y,+}^l(s_w) c_{y,-}^m(s_w) ds_w . \end{aligned} \quad (2.29)$$

The summation $jklm$ is conducted over all possible values of j, k, l and m ($\in \mathbb{Z}_{\geq 0}$). h_{jklm} are parts of the, in \mathbf{c} binomial expanded, Hamiltonians of all sources of multipole order $n = j + k + l + m$ (see Appendix A of [103]). The contribution h_{jklm} from magnetic fields of order $n \geq 2$ to the total Hamiltonian of the system is given e.g. in [103] Eq. (3.11) as

$$h_{jklm}(s) = -\Re \left[\frac{i^{l+m}}{j! k! l! m! 2^{j+k+l+m}} \beta_x(s)^{\frac{j+k}{2}} \beta_y(s)^{\frac{l+m}{2}} (K_n(s) + iJ_n(s)) \right] , \quad (2.30)$$

which is related to Eq. (2.19) via the binomial expansion.

2.2.5 Normal Form and Resonance Driving Terms

It has been neglected up until now that, with nonlinearities present in the machine, the phase-space is distorted and J_u is no longer the invariant of motion. Similar to Eq. (2.22), a new coordinate vector ζ can be found, depending on new action and angle coordinates I_u and ψ_u

$$\zeta_{u,\pm} = \sqrt{2I_u} e^{\mp i\psi_u}, \quad (2.31)$$

called normal form, in which the one-turn map \mathcal{M}_I can again be split into a nonlinear part and a rotation R , the same as in Eq. (2.28)

$$\mathcal{M}_I = e^{i h_I} R, \quad (2.32)$$

now with the Hamiltonian series h_I only dependent on I_u , making M_I independent of location. A variable transformation can be constructed, translating between \mathbf{c} and ζ , to make use of the simplicity of Eq. (2.32):

$$\mathcal{M}_o(s) = e^{i F(s)} \mathcal{M}_I e^{-i F(s)}. \quad (2.33)$$

$F(s)$ is calculated as

$$F(s) = \sum_{jklm} f_{jklm}(s) \zeta_{x,+}^j(s) \zeta_{x,-}^k(s) \zeta_{y,+}^l(s) \zeta_{y,-}^m(s). \quad (2.34)$$

The generating terms f_{jklm} of F are the so called RDTs and it is shown in [101] (also given in [103] Eq. (3.15)) that their relation to the $h_{jklm}(s)$ in Eq. (2.29) is

$$f_{jklm}(s) = \frac{h_{jklm}^{\text{Ring}}(s)}{1 - e^{i2\pi[(j-k)Q_x + (l-m)Q_y]}} = \frac{\oint_{\text{Ring}} h_{jklm}(s_w) e^{i[(j-k)\Delta\phi_x(s,s_w) + (l-m)\Delta\phi_y(s,s_w)]} ds_w}{1 - e^{i2\pi[(j-k)Q_x + (l-m)Q_y]}}. \quad (2.35)$$

Sometimes the here defined numerator $h_{jklm}^{\text{Ring}}(s)$ is already called RDT. In case of the condition

$$(j-k) \cdot Q_x + (l-m) \cdot Q_y = p, \quad p \in \mathbb{Z} \quad (2.36)$$

being fulfilled, f_{jklm} diverges, if there are sources present for that order (combinations of j, k, l, m with $j-k = l-m = 0$ do not have driving terms). In this case the system is in a resonant state, and hence unstable. The behaviour of the instability as the system approaches the resonance condition therefore depends on the strength of the multipole sources present in the machine. Resonances labeled (n_x, n_y) are driven by all h_{jklm} terms such that $n_x = j-k$ and $n_y = l-m$.

For each resonance (n_x, n_y) there is a spectral component in the turn-by-turn particle position data, which can be found at $-(n_x-1) \cdot Q_x - n_y \cdot Q_y$ (label: $(-n_x+1, -n_y)$) in the

spectrum of the horizontal plane and $-n_x \cdot Q_x - (n_y - 1) \cdot Q_y$ (label: $(-n_x, -n_y + 1)$) in the spectrum of the vertical plane [103]. The amplitude of the spectral lines is proportional to $|f_{jklm}(s)|$ [103], the terms are therefore easily accessible from measurements [13, 102, 105–107].

As long as there are no sources of order n , $|f_{jklm}(s)|$ are constant along s . At the location of a multipole source, the value of $|f_{jklm}(s)|$ changes, making them very well suited to build local observables [106]. This locality is used in the correction algorithm presented in Chapter 3, which is based on locally minimizing the RDTs in the IR.

2.2.6 Amplitude Detuning

Using the invariant of motion $\epsilon_u = 2J_u$, with the action J_u , the amplitude $\mathcal{A}_u(s)$ of a particle in the transversal plane $u \in \{x, y\}$ at longitudinal position s is

$$\mathcal{A}_u(s) = \sqrt{2J_u \beta_u(s)} = \sqrt{\epsilon_u \beta_u(s)}. \quad (2.37)$$

The dependency of the tune on amplitude is usually given with respect to this invariant and is therefore location independent. The detuning coefficients can be expressed via Taylor expansion of the tune Q_u around the unperturbed tune at zero-action Q_{u0} ; explicitly stated up to second order in amplitude:

$$\begin{aligned} Q_u(\epsilon_x, \epsilon_y) &= Q_{u0} + \frac{\partial Q_u}{\partial \epsilon_x} \epsilon_x + \frac{\partial Q_u}{\partial \epsilon_y} \epsilon_y \\ &+ \frac{1}{2!} \left(\frac{\partial^2 Q_u}{\partial \epsilon_x^2} \epsilon_x^2 + 2 \frac{\partial^2 Q_u}{\partial \epsilon_x \partial \epsilon_y} \epsilon_x \epsilon_y + \frac{\partial^2 Q_u}{\partial \epsilon_y^2} \epsilon_y^2 \right) \\ &+ \dots \end{aligned} \quad (2.38)$$

Further, ϕ_u and J_u are the canonical coordinates in the Courant-Snyder coordinate system of Section 2.2.4 and hence the Hamiltonian equations read:

$$\frac{\partial \phi_u}{\partial s} = \frac{\partial H}{\partial J_u}, \quad \frac{\partial J_u}{\partial s} = -\frac{\partial H}{\partial \phi_u} \quad (2.39)$$

With the definition of the tune as the total phase-advance we get

$$Q_u = \frac{1}{2\pi} \oint_{\text{Ring}} \frac{\partial \phi_u}{\partial s} ds \stackrel{\text{Eq. (2.39)}}{=} \frac{1}{2\pi} \oint_{\text{Ring}} \frac{\partial \langle H \rangle}{\partial J_u} ds, \quad (2.40)$$

where we need to calculate the expectation value $\langle \cdot \rangle$, as we are interested in the effect over many turns. The “contribution” to the tune of magnetic fields of order n with Hamiltonians $h_n \in \{N_n, S_n\}$ (see Eq. (2.20)) is

$$Q_u^{h_n} = \frac{1}{2\pi} \oint_{\text{Ring}} \frac{\partial \langle h_n \rangle}{\partial J_u} ds. \quad (2.41)$$

From Eq. (2.41) contribution to the detuning terms of Eq. (2.38) from any field orders

can be calculated, e.g. from normal octupoles to first order detuning

$$\frac{\partial Q_u^{N_4}}{\partial(2J_x)} = \frac{\partial}{\partial(2J_x)} \left(\frac{1}{2\pi} \oint_{\text{Ring}} \frac{\partial \langle N_4 \rangle}{\partial J_u} ds \right). \quad (2.42)$$

This is a lengthy process and has been done in detail in [97], which is why here only the final results are presented.

In first order, the linear detuning terms are governed by the octupole fields in the ring. Without forced oscillations (see Section 2.3), these terms read:

$$\frac{\partial Q_x^{N_4}}{\partial(2J_x)} = \oint_{\text{Ring}} \frac{K_4}{32\pi} \beta_x^2 ds \quad (2.43a)$$

$$\frac{\partial Q_x^{N_4}}{\partial(2J_y)} = \frac{\partial Q_y^{N_4}}{\partial(2J_x)} = \oint_{\text{Ring}} -\frac{K_4}{16\pi} \beta_x \beta_y ds \quad (2.43b)$$

$$\frac{\partial Q_y^{N_4}}{\partial(2J_y)} = \oint_{\text{Ring}} \frac{K_4}{32\pi} \beta_y^2 ds. \quad (2.43c)$$

In [10, 97] the influence of using forced oscillations for measurements of amplitude detuning is discussed and derived: The measured detuning will be dominated by the Hamiltonian terms dependent on the driven amplitude which leads to a factor of 2 in the measured direct terms (i.e. $\partial Q_x/\partial(2J_x)$ and $\partial Q_y/\partial(2J_y)$).

Similarly, the first-order contributions to the quadratic detuning terms are derived, which come from dodecapole fields:

$$\frac{\partial^2 Q_x}{\partial(2J_x)^2} = \oint_{\text{Ring}} \frac{K_6}{384\pi} \beta_x^3 ds \quad (2.44a)$$

$$\frac{\partial^2 Q_y}{\partial(2J_x)^2} = \frac{\partial^2 Q_x}{\partial(2J_x)\partial(2J_y)} = \oint_{\text{Ring}} -\frac{K_6}{128\pi} \beta_x^2 \beta_y ds \quad (2.44b)$$

$$\frac{\partial^2 Q_x}{\partial(2J_y)^2} = \frac{\partial^2 Q_y}{\partial(2J_x)\partial(2J_y)} = \oint_{\text{Ring}} \frac{K_6}{128\pi} \beta_x \beta_y^2 ds \quad (2.44c)$$

$$\frac{\partial^2 Q_y}{\partial(2J_y)^2} = \oint_{\text{Ring}} -\frac{K_6}{384\pi} \beta_y^3 ds, \quad (2.44d)$$

and with forced oscillations factors of 3 are seen for the direct terms $\partial^2 Q_u/\partial(2J_u)^2$ and factors of 2 for the mixed terms $\partial^2 Q_u/\partial(2J_a)\partial(2J_b)$ ($a, b \in \{x, y\}, a \neq b$). This decouples the terms in Eqs. (2.44b) and (2.44c), as the terms of form $\partial^2 Q_a/\partial(2J_b)^2$ ($a, b \in \{x, y\}, a \neq b$) will not change (see again [97] for the specific terms and [10, 108] for the derivation of generic expressions to arbitrary detuning order).

For brevity, in the following I will use the symbols

$$Q_{a,b} = \frac{\partial Q_a}{\partial(2J_b)} \quad \text{and} \quad Q_{u,ab} = \frac{\partial^2 Q_u}{\partial(2J_a)\partial(2J_b)}, \quad (2.45)$$

as well as short-hands for the coefficients

$$\tilde{\beta}_{a,b} = \begin{cases} \tilde{\beta}_{x,x} = \frac{\beta_x^2}{32\pi} \\ \tilde{\beta}_{x,y} = -\frac{\beta_x\beta_y}{16\pi} \\ \tilde{\beta}_{y,y} = \frac{\beta_y^2}{32\pi} \end{cases} \quad \text{and} \quad \tilde{\beta}_{u,ab} = \begin{cases} \tilde{\beta}_{x,xx} = \frac{\beta_x^3}{384\pi} \\ \tilde{\beta}_{x,xy} = -\frac{\beta_x^2\beta_y}{128\pi} \\ \tilde{\beta}_{x,yy} = \frac{\beta_x\beta_y^2}{128\pi} \\ \tilde{\beta}_{y,yy} = -\frac{\beta_y^3}{384\pi} \end{cases}, \quad (2.46)$$

with $a, b, u \in \{x, y\}$.

2.2.7 Dynamic Aperture

The area of the set of initial conditions in phase-space enclosed by the last connected stable invariant curve, defines the stability domain of particle motion [109]. In theory, Dynamic Aperture (DA) can, and often is, defined as the radius of a circle of equal area.

In practice, as the nonlinear character of the machine is usually small, the DA is considered to be the amplitude beyond which particle motion becomes unbounded. Particles beyond this amplitude, either due to their initial conditions or when being pushed (e.g. by beam excitation), will rapidly grow in amplitude and are eventually lost from the accelerator. The DA is hence of key concern for the successful machine operation and provides an important benchmark for the effectiveness of nonlinear corrections.

We differentiate between the “long term” DA, sometimes denoted as D_∞ , which indicates the DA after an infinite number of turns and defines the actual border of the stability domain, and the more practically obtainable D_N , the amplitude within which particles “survive” (i.e. circulate without being transported out of the physical aperture of the machine) for N turns in the accelerator. D_N is generally larger than D_∞ but will approach D_∞ as the number of turns increases and can for example be evaluated via tracking simulations. In the studies in this thesis $N = 100\,000$ was chosen as a compromise between a reasonably large number of turns and the duration of the tracking computations. Instead of the circle-of-equal-area definition above and in accordance with other tracking studies at the LHC, DA in this thesis is given as the minimum and average (with one standard deviation indicated) amplitude of surviving particles over different statistical machine realizations (so-called “seeds”) and, depending on the presentation, also over the probed angles.

2.3 Optics Measurements and Corrections

This thesis has been conducted within the Optics Measurements and Corrections (OMC)-team at CERN, whose mission is to optimize the performance of LHC and injector

optics, especially during the commissioning phases after the Year-End Technical Stop (YETS) and long shutdowns, and to conduct studies to ensure well corrected optics for future machine configurations.

One of the main quantifiers of (linearly) corrected optics is the β -beating, the relative difference of the measured β -function and the model:

$$\frac{\Delta\beta_u}{\beta_u}(s) = \frac{\beta_{u,\text{measured}}(s) - \beta_{u,\text{model}}(s)}{\beta_{u,\text{model}}(s)} \quad \text{with } u \in \{x, y\} \quad (2.47)$$

(examples for β -beating before and after a correction can be found in this thesis in Fig. 6.9 in Chapter 6; multiple successful corrections during commissioning 2022 and 2023 have been presented in [110, 111]).

Without any correction, the β -beating in the LHC would be above 100%. To ensure machine protection peak beating needs to be below 18%, an estimate based on aperture measurements [26, 112–114], as otherwise damage to the machine components can occur [115–117]. To enable an effective control of the optics, i.e. guarantee delivered luminosity [118], the β -beating should be in general even lower: In 2023, with squeezed optics at $\beta^* = 30$ cm, the peak beating was 10% and below (depending on plane and beam), with a Root Mean Square (RMS) of about 2% [111]. This tight control is also beneficial for wide range of other machine parameters and operational aspects, as improving control on the optics also improves the errorbars of other systems, e.g. the orbit control [119], relying on the machine to be close to the design parameters.

Increased accuracy will also feedback into the optics measurements, further increasing its reliability, as has been observed for example at the Base Band Tune (BBQ)-system, used to measure tune and coupling, which has shown to be of low quality in the presence of octupole errors [21].

In the following examples for the impact of field errors of various multipole order are presented and their correction approaches introduced.

2.3.1 Linear Errors

The LHC can be well described as a linear machine for many purposes. It is therefore not surprising, that the first step to allow for stable beams, is the application of linear optics corrections, which are concerned with the corrections of dipole and quadrupole errors. The influence of errors on this order are therefore changes in orbit, tune, coupling and changes in the β -beating. Our correction approach is based on an interplay between local and global correction approaches, as described in the following.

Local Corrections. These corrections are usually (but not exclusively) applied in the IRs for two reasons. Firstly, as mentioned before, the large β -functions make the optics very susceptible to local errors in this areas, but in the same manner also allow the IR-correctors to compensate errors with equally large impact. Secondly, due to the symmetry of the IRs, local errors might be hidden from global observables, yet still have an impact on the experiment close by. An example for the latter is given below.

Techniques for local corrections include the segment-by-segment method, in which measured optics parameters are fed into a MAD-X simulation and then propagated through the IRs to show the difference between the measured and model optics [120–122], and the method of finding local action-phase jumps to estimate the strengths of the quadrupole errors causing them [123].

Global Corrections. To compensate errors globally, a response matrix approach is used, where in a first step the linear response of the targeted parameter (e.g. β -beating) is evaluated for each corrector magnet via a MAD-X simulation. The resulting response matrix then used to solve the linear system of equations, fitting the model as close as possible to the measured data [124]. This leads to an “effective” global correction, which minimizes the difference between the measured and model optics, but - due to the degeneracy of the system - provides very little information about the location of the errors. This correction is usually performed to calculate the corrector powering of the (linear) correctors per beam, i.e. with magnets that can be steered per beam individually, such as the MQY quadrupoles (Q4, Q5 and in the past Q6) in the dual-aperture regions of the IRs.

Examples. In 2018 uncorrected coupling has lead to a 50% luminosity loss in ALICE. It could be recovered after thorough investigation and application of local corrections [125]. In the same year a luminosity imbalance between ATLAS and CMS could be seen, which had its origin in local coupling within the IR, that was not visible in the global coupling observable and could only be made visible by artificially breaking symmetry within the IR [17, 125–127].

2.3.2 Nonlinear Errors

Nonlinear errors are all magnetic field errors of multipole orders higher than that of quadrupoles. In the past mostly sextupole and octupole errors have been considered, but with the increase in luminosity the importance of “high-order nonlinear errors” (below) has become more prevalent.

Very often, the DA is affected by the presence of nonlinear errors, and their correction therefore leads to an improvement of beam lifetime. A thankful example are here the local normal octupole corrections in the IRs of ATLAS and CMS, which reduced the beam intensity loss drastically [17]. Longer beam lifetime leads to longer fills and therefore less time wasted on refills, which in turn leads to more integrated luminosity.

The corrections of nonlinear errors is often based on multiple optics measurements, e.g. at different settings of the machine, such as varied crossing angles or increased action. As changing machine parameters requires manual interaction, automation of measurements of the nonlinear errors is not trivial [11, 128]. In addition, the evaluation of the measured data needs in many cases still a lot of manual work.

Removing nonlinear influences from the optics leads to improvements in the linear control as well and a linear reoptimization of the optics can follow [17]. Through this

feedback loop, the linear and nonlinear optics can be iteratively improved.

2.3.3 High-Order Nonlinear Errors

The primary focus of this thesis revolves around addressing the corrections associated with “high-order nonlinear errors”. In this work “high-order” refers to all multipole orders higher than octupole, yet I am concerned with decapole and dodecapole corrections. While it has been so far assumed that the impact of even higher orders in the LHC and HL-LHC is negligible, the detrimental influence on the LHC of the two aforementioned orders has already been shown in measurements [43]. As mentioned in Section 2.1.1, due to the increased β -functions in the IRs, the effects of high-order field errors on the optics is expected to only become more prominent in the HL-LHC. This has been confirmed through simulations [33], where b_6 errors are shown to significantly increase amplitude detuning, particularly of second order.

Due to the smaller and sometimes more indirect influence of these errors even more and more precise measurements are required to determine their effect and in particular to calculate corrections [33]. Decapole corrections have been applied in the arcs of the LHC to reduce (third order) chromaticity [11, 129]. Motivated by the observed impact of high-order errors in the IRs through feed-down on first-order amplitude detuning [43], in this thesis (Chapter 7) corrections using dodecapole correctors in the IRs are presented, which have now been successfully integrated into the LHC operational cycle.

2.3.4 Used Software

MAD-X. The software package MAD-X [130], which is being developed at CERN, is a widely used general-purpose tool for optics design of accelerators. Elements are represented by their transfer maps (see Section 2.2.3) from which the full set of optics of the designed machine is calculated. It is not only utilized at CERN but has become one of the standard tools in the field. MAD-X provides access to the Polymorphic Tracking Code (PTC) [131], a symplectic integrator, and is employed for all simulations related to the LHC-optics. A well maintained optics repository [82] makes it possible to easily and quickly set up simulations for a multitude of machine configurations. Particularly useful for this thesis has been `cpymad` [132], a `python` wrapper to MAD-X which allows circumventing the restrictions of the MAD-X-programming language.

SixTrack. SixTrack [133, 134] is a 6D single-particle symplectic tracking code specifically designed to efficiently compute the trajectories of individual relativistic charged particles in circular accelerators for the purpose of studying DA and evaluating the performance of beam-intercepting devices, such as collimators. Additionally, the script collection SixDesk [135] is available, which serves as a wrapper for setting up and evaluating a grid of particles and assess DA.

OMC-Software. The heart of the analysis software developed within the OMC-Team forms the `python` package `omc3` [6]. It contains a significant portion of our analysis code

and is hosted on GitHub with continuous integration (CI) to ensure regular testing, as the ability to run it online without unforeseen errors in the CERN Control Center (CCC) is crucial. The package is constantly improved by members of the OMC-Team, developing new features and debugging issues.

The software performs various steps of analyses: At first a frequency analysis based on the harmonic analysis package `harpy` [136] is conducted, in which noisy measurement data is also rigorously cleaned. It uses the evaluated spectrum to reconstruct optics parameters by using analysis methods such as the N-BPM-method [137–139], β -from-amplitude [140, 141], and coupling analysis [106, 142, 143]. Finally, nonlinear optics parameters, such as amplitude detuning, are determined from these parameters [11, 144]. For more details on its functioning shown at the specific case of amplitude detuning-analysis refer to [Section 2.4.2](#).

In addition to the aforementioned software packages, the OMC-Team is also maintaining several other software packages [145–150]. I have contributed in some capacity to all of the referenced packages; further details about my major contributions are provided in [Appendix A.2](#). Any other software used within this thesis has been `python` scripts and packages developed by myself exclusively for the analysis of the given data. Some of these scripts have found their way into `omc3`, others into their own package, while again others were either too specialized to be useful to share or still need to be brought into a shareable format.

2.4 Optics Measurements Procedure

In this section, the detailed procedure of optics measurements is presented at the example of amplitude detuning measurements. This kind of measurements requires analysis of the measured turn-by-turn data at different kick amplitudes and fitting the tune-results. Other parameter of interest, such as β -beating or global coupling, often only require analysis at a single measurement point (yet usually two or more are used for improved statistics).

While in the beginning of LHC operation, measurements were often performed with free oscillations following a single dipole “kick” [151], in later years they were achieved using forced oscillations with an AC-Dipole [10, 144, 152, 153]. Optics measurements with AC-Dipoles had already been tested at the Relativistic Heavy Ion Collider (RHIC) [106, 154], and in the Super Proton Synchrotron (SPS) in preparation for LHC optics measurements [155–157].

The excitation, close to the tune frequency, is adiabatically ramped up and down to and from the desired amplitude [10, 158, 159]. Exciting single pilot bunches ($\approx 10^{10}$ protons) per beam, this method allows for measurements at top-energy without endangering machine safety and without beam decoherence or emittance growth. This means a single bunch can be excited multiple times, depending on machine setup and excitation amplitude, without having to go through the time-consuming process of dumping the blown-up beam, ramping down the magnets to injection settings, refilling the proton bunches, ramping to top-energy again and performing the optics-squeeze (i.e. the

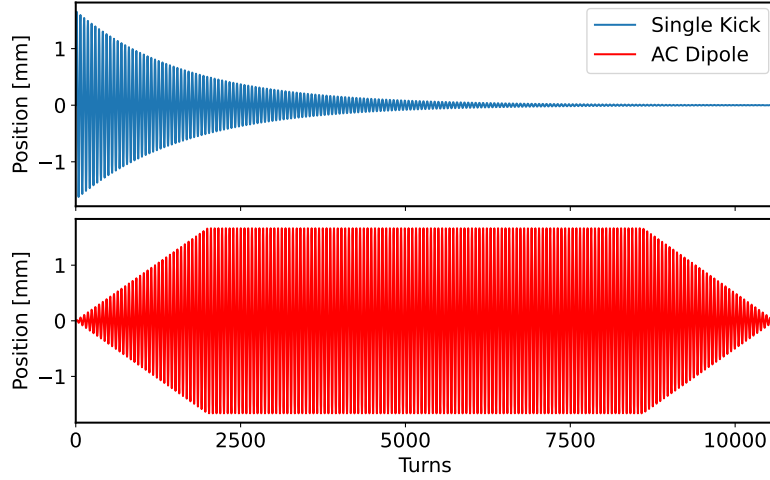


Figure 2.13. Illustration of orbit oscillations after a free kick (blue) and during AC-Dipole excitation (red).

reduction of β^* to the desired value). The transverse positions of the beam centroid are captured during the AC-Dipole’s flat-top (approximately 6600 turns, limited by the properties of the AC-Dipole) at each BPM [83] and then analysed. A visualization the orbit oscillations for both types of kicks, free (or “Single Kick”) and AC-Dipole, is presented in Fig. 2.13.

2.4.1 Amplitude Detuning Measurements

Due to the use of the AC-Dipole the first-order amplitude detuning direct terms $Q_{x,x}$ (Eq. (2.43a)) and $Q_{y,y}$ (Eq. (2.43c)) will be enhanced by a factor of 2, while the second-order direct terms $Q_{x,xx}$ (Eq. (2.44a)) and $Q_{y,yy}$ (Eq. (2.44d)) are enhanced by a factor of 3 [10]. All detuning data presented in this thesis are compensated for these factors and detuning terms quoted are the “free” detuning of the machine. In principle, also the second-order terms $Q_{x,xy}$ and $Q_{y,xy}$, measured with forced oscillations, would appear to be twice as large as their free counterparts, yet only the tune shift with the action of one plane has been assessed during the here presented studies and these terms have therefore not been investigated.

AC-Dipole excitations are generally performed in the two transverse planes at the same time to be able to measure both tunes. For detuning measurements the amplitude of one plane is kept low and constant, while slowly increasing the strength, i.e. action, of the other plane. The $\Delta Q = |Q_{\text{natural}} - Q_{\text{driven}}|$ of the plane with constant amplitude kicks is usually smaller than the other, to increase signal-strength of the tune line even at these low amplitudes by means of the decreased adiabaticity of the ramp. In the other plane the driven tune is kept slightly further apart to avoid exciting diagonal ($\Delta Q_x = \Delta Q_y$) resonances [160, 161]. The amplitude is increased within the amplitude range of the AC-Dipole only in small steps and only until first losses at the collimators

can be seen, to not risk a beam dump and to not lose beam intensity, which is needed to ensure good signals in the subsequent measurements. When the available kick amplitude has been exploited, the procedure is repeated with the roles of the planes swapped.

2.4.2 Data Analysis

The resulting turn-by-turn data is then processed by our python software tools [6]: Cleaning is done automatically via a specialized Singular Value Decomposition (SVD) and Fourier transform on the signal [136,162,163], followed by a combination of automated and manual cleaning steps to remove faulty BPMs and ascertain the correct tune-line in the spectrum [164]. In preparation for extensive amplitude detuning studies like the one presented, these latter methods have recently been extended with new features, such as automated BPM cleaning (via the *Outlier Filter* described below) and an option for manual spectral-line selection. This has allowed for fast online (i.e. during measurements) analysis of the data during the MD, which to this level had not been possible in the past.

Outlier Filter. The outlier filtering function [163] is utilized at multiple stages of the data analysis and operates in the following way: The function gets an array x of data of length n_x , which can be any scalar data but in the context of this chapter has been data of the tunes, either measured per BPM or a time series from the BBQ. It removes data points in the tails of the measured distribution, which are too populated due to the finite sample size n_x , assuming x_n to be samples from a normal distribution, specified by measured mean and standard deviation of the given data. A data point, outside of a user-specified limit, is removed if there is less than a 50 % chance that it stems from the specified normal distribution. This filtering method is first applied to remove BPMs for which the driven or natural tunes were not correctly identified and later to clean the tune data from the BBQ, used to establish the baseline for the machine tune at zero-action (see Section 2.4.2).

Action. The action J (we are actually interested in $2J$, as will be unambiguously stated in the given formulas) is calculated at each BPM at location s from the amplitude \mathcal{A}_{BPM} of the main line of the BPM-spectrum, i.e. the driven tune, via

$$2J_{\text{BPM}} = \frac{\mathcal{A}_{\text{BPM}}^2}{\beta_{\text{model}}(s)} . \quad (2.48)$$

Instead of the β -function from the model (as in [165]), the measured β values could also be used as shown in [141], yet this feature has not yet been implemented in the analysis code. Due to the low β -beating in the machine [118,119] using the model β -function is still justified.

The action is further rescaled to perform beam-based adjustments for residual BPM calibration errors [141]: the rescaling factor is the mean ratio of the β -function estimated from phases [166] and the β -function from amplitudes [165] at the arc BPMs per kick.

The error on the action $\delta 2J_{\text{BPM}}$ can be acquired from the error on the amplitude $\delta \mathcal{A}_{\text{BPM}}$, which in turn is estimated from the noise on the signal:

$$\delta \mathcal{A}_{\text{BPM}} = \sqrt{\frac{2}{N_{\text{Turns}}}} R_{\text{BPM}}^{(\text{cleaned})} . \quad (2.49)$$

N_{Turns} is the number of turns over which the measurement is recorded and $R_{\text{BPM}}^{(\text{cleaned})}$ is the estimated BPM-Resolution of the cleaned signal. Here, BPM-Resolution is defined as the standard deviation of the noise, which is first estimated by the difference between the cleaned signal and the original signal at the BPM (as $R_{\text{BPM}}^{(\text{raw})}$). On this estimate a coefficient has been matched via an empirical study, targeting e.g. the ratio of the noise floor levels in the frequency spectra of raw data and SVD-cleaned data:

$$R_{\text{BPM}}^{(\text{cleaned})} = \frac{1}{\sqrt{2} \times 10} R_{\text{BPM}}^{(\text{raw})} . \quad (2.50)$$

With Eqs. (2.48) and (2.49) we can calculate the error on the action:

$$\delta 2J_{\text{BPM}} = \frac{2 \mathcal{A}_{\text{BPM}} \delta \mathcal{A}_{\text{BPM}}}{\beta_{\text{model}}(s)} . \quad (2.51)$$

From these estimates at each BPM for actions $2J_{\text{BPMs}} = \{2J_{\text{BPM}_1}, \dots, 2J_{\text{BPM}_N}\}$ and corresponding errors $\delta 2J_{\text{BPMs}} = \{\delta 2J_{\text{BPM}_1}, \dots, \delta 2J_{\text{BPM}_N}\}$, we want to get an estimate for the actual action and the error we might have on this estimation.

We can use inverse-variance weighting to define an error-weighted average of a set of measurements $x = \{x_1, \dots, x_N\}$ and their associated errors $\delta x = \{\delta x_1, \dots, \delta x_N\}$

$$\langle x \rangle_{\delta} = \frac{1}{\sum_{n=1}^N \frac{1}{\delta x_n^2}} \sum_{n=1}^N \frac{x_n}{\delta x_n^2} = \frac{1}{S_{\delta}(x)} \sum_{n=1}^N \frac{x_n}{\delta x_n^2} , \quad (2.52)$$

defining also a sum-of-weights $S_{\delta}(x)$. Equation (2.52) is used to get an estimate of the action in the machine:

$$2J = \langle 2J_{\text{BPMs}} \rangle_{\delta} . \quad (2.53)$$

For the estimation of the error on this quantity, we can calculate the error-weighted variance on the data

$$\sigma_{\delta}^2(x) = \left\langle |x - \langle x \rangle_{\delta}|^2 \right\rangle_{\delta} \quad (2.54)$$

as well as the variance on $\langle x \rangle_{\delta}$ itself:

$$\sigma^2(\langle x \rangle_{\delta}) = \frac{1}{\sum_{n=1}^N \frac{1}{\delta x_n^2}} = \frac{1}{S_{\delta}(x)} , \quad (2.55)$$

and the effective sample size

$$N_\delta(x) = \frac{\left(\sum_{n=1}^N 1/\delta x_n^2\right)^2}{\sum_{n=1}^N (1/\delta x_n^2)^2}. \quad (2.56)$$

Also using the abbreviations $\sigma_\delta^2 = \sigma_\delta^2(2J_{\text{BPMs}})$, $N_\delta = N_\delta(2J_{\text{BPMs}})$ and $S_\delta = S_\delta(2J_{\text{BPMs}})$ the error on the action can then be estimated via the unbiased sum of the two above described variances:

$$\delta 2J_{\text{estimated}} = \sqrt{\frac{N_\delta}{N_\delta - 1} \left(\sigma_\delta^2 + \frac{1}{S_\delta} \right)}. \quad (2.57)$$

To account for the measurement's finite sample size, whilst still estimating the standard deviation of a normally distributed quantity (Student's t-distributed with an infinite number of degrees of freedom), the error value is corrected by a multiplicative coefficient. We calculate the coefficient $t_{\text{correction}}$ utilising the cumulative distribution function $F(s, N)$ of Student's t-distribution (with mean equal to 0 and spread equal to 1) as a function of the actual value s and the number of degrees of freedom N .

$$t_{\text{correction}} = F^{-1}(F(1, \infty), N_\delta), \quad (2.58)$$

which is a factor towards the limiting case $N \rightarrow \infty$ for a given confidence level corresponding to 1σ in the normal distribution.

$$\delta 2J = \delta 2J_{\text{estimated}} \times t_{\text{correction}}. \quad (2.59)$$

These statistical functions are part of the `omc3-package` [6]. The calculations are done individually for each transversal plane.

Tune. For each BPM the natural tune is identified within a given frequency window as the line with the largest amplitude in that window [136]. Again, the two transversal planes are handled identically and separately. The selected lines are manually confirmed in a spectral plot and can be modified if needed.

The measured natural tune Q_{natural} and its error $\delta Q_{\text{natural}}$ at the time of the excitation is calculated from the selected spectral lines by the mean and standard deviation over the BPMs. The natural tune at zero-action (Q_{u0} in Eq. (2.38)) is found around each excitation time from the BBQ data logged in the LHC, which removes the influence of possible tune drifts during the measurement duration. The BBQ data is first cleaned using a sliding window of 100 data points, which translates to roughly 15 seconds of data, and applying the *Outlier Filter* described above within each window. This removes measurement outliers of the BBQ as well as the excited tunes during the AC-Dipole operation, which lasts about 1 s. If a value is removed in any window, it will be completely excluded from analysis. From this filtered data the mean is calculated for each window, resulting in a moving average. This average is in turn subtracted from the data points to calculate a moving standard deviation using the same window lengths. The tunes

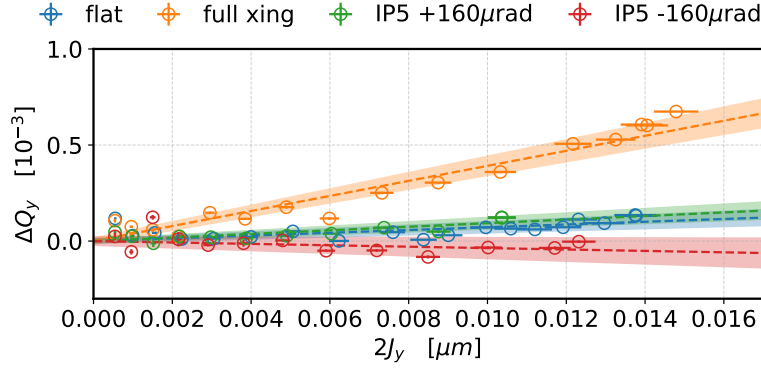


Figure 2.14. Example of the results of a detuning measurement. The data was taken from the study presented later in [Chapter 7](#). Shown are the scenarios of flat-orbit (blue), with full crossing scheme applied (orange), and with crossing only in IP5 at $+160\mu\text{rad}$ (green) and $-160\mu\text{rad}$ (red). Dashed lines show the linear fit to the data points, while the colored area covers one standard deviation of the fit. The constant part of each fit has been subtracted for plotting from the fit and its respective data points. These and more detuning results are summarized in [Table 7.2](#).

Q_{BBQ} are then the value of the moving average closest to the kick-times, while the error on those measurements δQ_{BBQ} are the values of the moving standard deviation at the same times. The tune values for the detuning-fit are then

$$\Delta Q_{\text{BBQ-corrected}} = Q_{\text{natural}} - Q_{\text{BBQ}} \quad (2.60)$$

$$\delta \Delta Q_{\text{BBQ-corrected}} = \sqrt{\delta Q^2 + \delta Q_{\text{BBQ}}^2} . \quad (2.61)$$

Fitting. The slope of the detuning $\Delta Q_{\text{BBQ-corrected}}$ over the actions $2J$ (from different kick-amplitudes) is fitted to a linear polynomial, to get the first-order detuning coefficients. The Orthogonal Distance Regression (ODR) algorithm [167] provided by the `scipy` python package [168] allows to weigh the fitting by their errors $\delta \Delta Q_{\text{BBQ-corrected}}$ and $\delta 2J$. ODR also produces an estimated error (one standard deviation) on the fit. In principle also a fit to second-order polynomials is possible, yet, as will be discussed in [Section 7.4.3](#), due to the low kick amplitudes in most of the measurements, no usable fits could be obtained.

Examples of the results of the measurement at different crossing angles are given in [Fig. 2.14](#): depicted are the vertical-plane tune shifts over the vertical-plane actions including their linear ODR fits for four different scenarios, all without dodecapole correction, as performed during the dodecapole-correction MD in 2022.

3 Nonlinear Corrections in the Insertion Regions

Note. The here presented nonlinear correction algorithm has been made accessible on [github](#) [40] together with a companion note [96] in which most of the here presented content can also be found.

The results of the studies conducted with the algorithm in [Sections 3.3](#) and [3.5](#) have been presented by me at the International Particle Accelerator Conference (IPAC) in 2021 and have been published in their proceedings [169, 170].

3.1 Motivation

The sensitivity of accelerator beam optics to magnetic errors depends directly on the β -function, which is highest in the around the IP with the lowest β^* .

Hence, studies of the possibility of correcting the non-linear magnetic errors in these regions in the have already been of significant importance during its design phase: It was envisaged to make use of the magnetic measurement data of the LHC magnets [171–173] to simulate the machine in MAD-X [130] and calculate the corrections to be used in the machine [7, 8, 174]. While these simulation-based corrections produced great results in the arcs [142] in the IRs discrepancies with corrections from beam-based measurements were observed [14]. The sources for these discrepancies are still not fully known. Apart from the successful arc-corrections, simulations have nevertheless been a useful tool for the estimation of linear and non-linear effects in the IRs [14, 17, 120, 175]. Magnetic-measurement based simulations have since supported the continuing endeavour to optimize the LHC machine performance with beam-based corrections in the IRs [16, 17, 33, 128, 176, 177], and continue to be an invaluable tool in studying future machine layouts, e.g. the installation of stronger magnets in the IR and the decrease of β^* in operation in the HL-LHC [22, 178], which is foreseen to result in even tighter constraints on residual errors.

At the same time the crossing-angle scheme of the collision optics creates large orbit bumps in the IRs, leading to feed-down effects, the influence of which have been observed and investigated in the LHC. For both, LHC and HL-LHC the need for corrections of this feed-down has been established [14, 15, 17, 26, 33, 128, 177, 179].

In [Section 3.2](#) an approach is described to perform local error corrections in the IRs while also addressing feed-down, based on minimizing local RDTs. This approach has been implemented into a `python`-package [40], and has been used for the simulation-studies presented in [Section 3.3](#), in which the possible correction of feed-down is in-

vestigated, [Section 3.4](#), in which corrections with feed-down is in detail studied for dodecapole errors, and [Section 3.5](#), in which the results of simulations with asymmetric optics are layed out.

3.2 Flexible Correction Script

To estimate the powering of the corrector magnets, a local correction scheme based on the RDTs in the IRs has been utilized [7, 41]. Up to now, the implementation of this scheme calculated the correction based on a single input optics, for either Beam 1 or Beam 2, and made use of symmetries between the beams to optimize the correction for both beams. Cases will occur in which this symmetry does not hold, e.g. through the introduction of feed-down, or the use of inherently asymmetric optics. An example for the latter is the flat-optics [3, 30], in which β^* in the two transversal planes no longer has identical values. These optics allow for a more distributed radiation deposition in the LHC magnets as well as an increase in luminosity [30]. Their feasibility has been studied during machine developments in the LHC [180] and preliminary analysis regarding their influence on corrections and amplitude detuning has been conducted [181]. A new and flexible version of the correction principle has been implemented [40], taking up to two optics into account and hence not relying on symmetry assumptions, allowing to target RDTs freely, as well as including feed-down into the calculations. The implementation allows for the feed-down from higher orders to the RDT to be corrected, as well as using the feed-down from higher order corrector magnets to correct for lower order errors.

3.2.1 Correction Principle

In this section the correction principle as implemented in the flexible correction algorithm v1.0.0 in [40] is described. The algorithm follows the simplifications of [Eqs. \(2.30\)](#) and [\(2.35\)](#) as outlined in [7, 41]:

- Only the contribution from elements in one IR to the RDTs are minimized. The integral in [Eq. \(2.35\)](#) includes therefore only IR elements.
- Constant coefficients of [Eq. \(2.35\)](#) are ignored, i.e. the numerator and actions as well as the coefficients depending only on j, k, l, m and any signs, as they are not needed for minimization, .
- The phase of one side of the IR is approximately constant, as $\beta(s)$ being very large in the triplets and therefore the integral [Eq. \(2.24\)](#) very small.
- The phase-advance between the left and right side of the IP is π .
- The RDT is evaluated locally at the entrance of the IR.

With these approximations [Eq. \(2.35\)](#) becomes a local, *effective* RDT to minimize within the IR:

$$f_{jklm}^{\text{IR}} = \int_{\text{IR}} \Re \left[i^{l+m} (K_n(s) + iJ_n(s)) \right] \beta_x(s)^{\frac{j+k}{2}} \beta_y(s)^{\frac{l+m}{2}} e^{i\pi n\theta(s-s_{\text{IP}})} ds, \quad (3.1)$$

with $\theta(x)$ being the Heaviside step function and s_{IP} the location of the IP within the IR. From Eq. (2.35), exponential function would actually contain $j - k + l - m = n - 2k - 2l$, which is even when n is even and odd when n is odd. As in this approximation, due to $e^{i\pi} = -1$, only the parity of the exponent is important (independent of the particular choices for j, k, l, m), n is used for simplicity.

The main concept of the correction is then to find the $K_n(s)$ and $J_n(s)$ of the corrector magnets, that minimize a set of given f_{jklm}^{IR} based on given optics.

As there are usually two correctors per multipole field available (one on each side of the IP), two combinations of $l + m$ and $j + k$ (the exponents of the β -function) can be corrected. As, due to the single-aperture nature of the magnets close to the IP, the correctors are responsible for the correction of both beams. In Section 3.2.1 it will be explained, how the β -exponents can be chosen such that the correction is valid for both beams, correcting only a single RDT per beam, unless the symmetry of the optics can be used to correct two RDTs.

Equation System

In our simulations, the input to the correction algorithm will be the output of TWISS (see “twiss parameter”) and **ESAVE** functions from MAD-X [42]. These are tables in which $K_n(s)$ and $J_n(s)$ are not continuous functions, but given as already integrated values $K_n L_w$, $J_n L_w$ ($K_{n-1}\text{L}$, $K_{n-1}\text{SL}$ in the terminology of MAD-X) for each element w . Values for the longitudinal position s_w , $\beta_{x,w}$, $\beta_{y,w}$ and the transversal orbit x_w, y_w , which will be important when calculating feed-down (see below), are also provided.

One way to get an estimate for the integral in Eq. (3.1), is *slicing* the lattice in MAD-X, i.e. approximating the magnets as single kicks surrounded by drift-spaces. Long magnets should be cut into multiple of these slices to increase accuracy. Corrector magnets on the other hand, which are in any case short compared to e.g. dipoles, can be represented by a single slice.

In this thin-lens approximation, Eq. (3.1) transforms into a sum over all elements (slices) w in the IR, which needs to be set (using “ $\stackrel{!}{=}$ ” to stress the intention) to zero to suppress the RDT:

$$f_{jklm}^{\text{IR}} = \sum_{w \in \text{IR}} \Re \left[i^{l+m} (K_n L_w + i J_n L_w) \right] \beta_{x,w}^{\frac{j+k}{2}} \beta_{y,w}^{\frac{l+m}{2}} (-1)^{n\theta(s_w - s_{\text{IP}})} \stackrel{!}{=} 0. \quad (3.2)$$

Splitting the elements into corrector elements \mathcal{C} and non-corrector elements $\text{IR} \setminus \mathcal{C}$, Eq. (3.2) transforms into:

$$\begin{aligned} & \sum_{w \in \mathcal{C}} \Re \left[i^{l+m} (K_n L_w + i J_n L_w) \right] \beta_{x,w}^{\frac{j+k}{2}} \beta_{y,w}^{\frac{l+m}{2}} (-1)^{n\theta(s_w - s_{\text{IP}})} \\ &= - \sum_{w \in \text{IR} \setminus \mathcal{C}} \Re \left[i^{l+m} (K_n L_w + i J_n L_w) \right] \beta_{x,w}^{\frac{j+k}{2}} \beta_{y,w}^{\frac{l+m}{2}} (-1)^{n\theta(s_w - s_{\text{IP}})} = -I_{jklm}, \end{aligned} \quad (3.3)$$

where I_{jklm} a shorthand for the sum over $\text{IR} \setminus \mathcal{C}$.

It is important to note, that each corrector is defined by either $K_n L$ or $J_n L$, so that per order n and orientation (normal, skew) only a limited set of correctors is left. Normally there are two of these correctors in the LHC/HL-LHC, i.e. one per IP side, and $\mathcal{C} = \{cl, cr\}$, a left (cl) and a right (cr) corrector element. With the definitions

$$\begin{aligned} b_{jklm}^{(cl)} &= i^{l+m} \beta_{x,cl}^{\frac{j+k}{2}} \beta_{y,cl}^{\frac{l+m}{2}} \\ b_{jklm}^{(cr)} &= (-1)^n i^{l+m} \beta_{x,cr}^{\frac{j+k}{2}} \beta_{y,cl}^{\frac{l+m}{2}}, \end{aligned} \quad (3.4)$$

Eq. (3.3) can be split into a two equation system

$$\begin{aligned} \begin{pmatrix} b_{jklm}^{(cl)} & b_{jklm}^{(cr)} \end{pmatrix} \begin{pmatrix} K_n L_{cl} \\ K_n L_{cr} \end{pmatrix} &= -I_{jklm} \quad \text{if } l+m \text{ even,} \\ \begin{pmatrix} i b_{jklm}^{(cl)} & i b_{jklm}^{(cr)} \end{pmatrix} \begin{pmatrix} J_n L_{cl} \\ J_n L_{cr} \end{pmatrix} &= -I_{jklm} \quad \text{if } l+m \text{ odd,} \end{aligned} \quad (3.5)$$

each of which can be easily extended to include multiple RDTs, e.g. with $j+k+l+m = j'+k'+l'+m' = n$ and $l+m \equiv l'+m' \pmod{2}$:

$$\begin{aligned} \begin{pmatrix} b_{jklm}^{(cl)} & b_{jklm}^{(cr)} \\ b_{j'k'l'm'}^{(cl)} & b_{j'k'l'm'}^{(cr)} \end{pmatrix} \begin{pmatrix} K_n L_{cl} \\ K_n L_{cr} \end{pmatrix} &= - \begin{pmatrix} I_{jklm} \\ I_{j'k'l'm'} \end{pmatrix} \quad \text{if } l+m \text{ (and } l'+m') \text{ even,} \\ \begin{pmatrix} i b_{jklm}^{(cl)} & i b_{jklm}^{(cr)} \\ i b_{j'k'l'm'}^{(cl)} & i b_{j'k'l'm'}^{(cr)} \end{pmatrix} \begin{pmatrix} J_n L_{cl} \\ J_n L_{cr} \end{pmatrix} &= - \begin{pmatrix} I_{jklm} \\ I_{j'k'l'm'} \end{pmatrix} \quad \text{if } l+m \text{ (and } l'+m') \text{ odd.} \end{aligned} \quad (3.6)$$

These linear equation systems can be solved or optimized for $K_n L_{cl,cr}$ or $J_n L_{cl,cr}$ by standard algorithms.

Dual Optics

In the original implementation of the algorithm in [7], the optics of only one beam could be given and the algorithm was making use of the symmetries of the β -function in the IR

$$\begin{aligned} \beta_x^{(B1)}(s) &= \beta_y^{(B2)}(s) \\ \beta_y^{(B1)}(s) &= \beta_x^{(B2)}(s), \end{aligned} \quad (3.7)$$

which is true for round-optics as shown in Fig. 3.1a, to calculate a correction valid for both beams. With this symmetry the effective RDT Eq. (3.1) simply switches the β

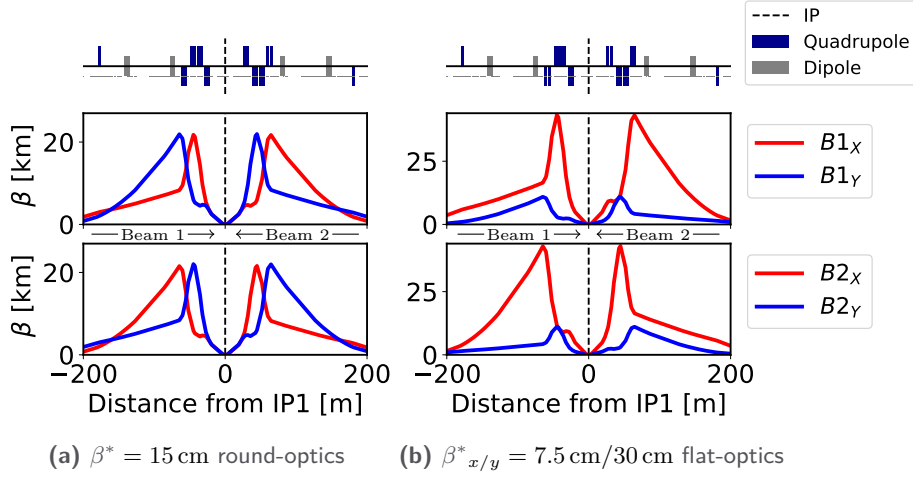


Figure 3.1. HL-LHC β -functions in the IR around IP1.

exponents between the beams, for which we will use the subscript $jklm^*$:

$$f_{jklm^*}^{\text{IR (B1)}} \stackrel{\text{Eq. (3.1)}}{=} \int_{\text{IR}} \Re \left[i^{l+m} (K_n(s) + iJ_n(s)) \right] \beta_x^{(\text{B1})}(s)^{\frac{l+m}{2}} \beta_y^{(\text{B1})}(s)^{\frac{j+k}{2}} e^{i\pi n \theta(s-s_{\text{IP}})} ds \stackrel{\text{Eq. (3.7)}}{=} f_{jklm}^{\text{IR (B2)}} . \quad (3.8)$$

As mentioned in the previous section, with the two correctors per field type (order and orientation), we can perfectly correct two RDTs locally in the IR. When

$$j + k \equiv l + m \pmod{2} , \quad (3.9)$$

i.e. both f_{jklm} and f_{lmjk} have the same orientation (skew or normal) RDTs, two RDTs per beam can be corrected, as $|i^{j+k}| = |i^{l+m}|$. Therefore $|f_{jklm^*}^{\text{IR}}| = |f_{lmjk}^{\text{IR}}|$ within the optics of the same beam and we can choose $f_{j'k'l'm'}^{\text{IR}} = f_{lmjk}^{\text{IR}} (= f_{jklm^*}^{\text{IR}} \text{ or } -f_{jklm^*}^{\text{IR}})$ in Eq. (3.6). If, on the other hand, $j + k$ and $l + m$ are of different parity, only one RDT f_{jklm}^{IR} per beam can be corrected, by targeting f_{jklm}^{IR} and $f_{j'k'l'm'}^{\text{IR}} = f_{jklm^*}^{\text{IR}}$ in the given optics.

Example To correct b_4 , the two RDTs f_{4000} and f_{0040} can be targeted, as both are normal octupole RDTs. $j + k$ and $l + m$ are both even ($i^{j+k} = i^{l+m}$), so correcting either one in one optics, will correct the respective other, in the other optics, e.g.

$$\begin{aligned} f_{4000}^{\text{IR (B1)}} &= f_{0040}^{\text{IR (B1)}} = f_{0040}^{\text{IR (B2)}} , \quad \text{and} \\ f_{0040}^{\text{IR (B1)}} &= f_{4000}^{\text{IR (B1)}} = f_{4000}^{\text{IR (B2)}} . \end{aligned} \quad (3.10)$$

When correcting normal sextupole errors (b_3) on the other hand, we cannot target f_{3000} and f_{0030} as the latter targets a skew sextupole RDT. We can still target one RDT in each Beam, also by using just one optics

$$\begin{aligned} f_{3000}^{\text{IR (B1)}} &= f_{3000*}^{\text{IR (B2)}} , \quad \text{and} \\ f_{3000*}^{\text{IR (B1)}} &= f_{3000}^{\text{IR (B2)}} . \end{aligned} \quad (3.11)$$

There are optics in which Eq. (3.7) does not hold true anymore. For example, in contrast to round-optics, in which the β -function at the IP ($\beta^* = \beta(s_{\text{IP}})$) is equal for both transversal planes ($\beta_x^* = \beta_y^*$), there exists also the flat-optics, for which $\beta_x^* \neq \beta_y^*$ (i.e. the beam shape is not round at the IP) [3, 30]. A realization of flat-optics, foreseen to be used in the HL-LHC, is shown in Fig. 3.1b.

The straightforward way to not to rely on Eq. (3.7), is to use the optics for both beams in the correction and construct Eq. (3.6) from both

$$\begin{pmatrix} b_{jklm}^{(cl, B1)} & b_{jklm}^{(cr, B1)} \\ b_{jklm}^{(cl, B2)} & b_{jklm}^{(cr, B2)} \end{pmatrix} \begin{pmatrix} K_n L_{cl} \\ K_n L_{cr} \end{pmatrix} = - \begin{pmatrix} I_{jklm}^{(B1)} \\ I_{jklm}^{(B2)} \end{pmatrix} . \quad (3.12)$$

Feed-Down

Feed-down is discussed in Section 2.2.2 and Appendix B and Eq. (2.21) can be included easily into the definition of the effective RDT Eq. (3.1)

$$\begin{aligned} f_{jklm}^{\text{IR}} &= \\ \int_{\text{IR}} \Re \left[i^{l+m} \sum_{q=0}^{\infty} (K_{n+q}(s) + iJ_{n+q}(s)) \frac{(\Delta x(s) + i\Delta y(s))^q}{q!} \right] &\beta_x(s)^{\frac{j+k}{2}} \beta_y(s)^{\frac{l+m}{2}} (-1)^{n\theta(s-s_{\text{IP}})} ds , \end{aligned} \quad (3.13)$$

and can therefore easily include it when building the equation systems Eq. (3.5), Eq. (3.6) or Eq. (3.12).

Not only can feed-down be used to calculate the influence of field errors of orders larger than n on the RDTs of order n , i.e. by contributing to the integral I_{jklm} on the right-hand side of the equation systems, but it can also be used to calculate the strengths of correctors of orders $n_{\text{Corrector}} > n$ to counteract the RDTs at order n via feed-down, by including it on the left-hand side: The matrix elements of the corrector coefficients in Eq. (3.4) will then also contain the feed-down coefficient

$$z_p = \frac{(\Delta x + i\Delta y)^p}{p!} \quad (3.14)$$

with p being the order of feed-down from the corrector to the RDT, i.e.

$$p = n_{\text{Corrector}} - n . \quad (3.15)$$

As $z_p \in \mathbb{C}$, this makes the evaluation of the real part in Eq. (3.3), needed to split the equation system into two, separating the correctors (Eq. (3.5)), less straightforward and more cases need to be considered. Inserting

$$\begin{aligned}
 & \Re \left[i^{l+m} (K_{n+p} L_w + i J_{n+p} L_w) \frac{(\Delta x L_w + i \Delta y L_w)^q}{q!} \right] \\
 \text{Eq. (3.14)} \quad &= \Re \left[i^{l+m} (K_{n+p} L_w + i J_{n+p} L_w) \cdot z_p \right] \\
 &= \Re \left[i^{l+m} (K_{n+p} L_w + i J_{n+p} L_w) (\Re[z_p] + i \Im[z_p]) \right] \\
 &= \Re \left[i^{l+m} [(K_{n+p} L_w \cdot \Re[z_p] - J_{n+p} L_w \cdot \Im[z_p]) + i(K_{n+p} L_w \cdot \Im[z_p] + J_{n+p} \cdot \Re[z_p])] \right]
 \end{aligned} \tag{3.16}$$

into Eq. (3.3) yields the equation system

$$\begin{aligned}
 & \begin{pmatrix} \Re[z_p] \cdot b_{jklm}^{(cl)} & \Re[z_p] \cdot b_{jklm}^{(cr)} & -\Im[z_p] \cdot b_{jklm}^{(cl)} & -\Im[z_p] \cdot b_{jklm}^{(cr)} \end{pmatrix} \begin{pmatrix} K_{n+p} L_{cl} \\ K_{n+p} L_{cr} \\ J_{n+p} L_{cl} \\ J_{n+p} L_{cr} \end{pmatrix} = -I_{jklm} \\
 & \hspace{15em} \text{for even } l+m, \\
 & \begin{pmatrix} i \Im[z_p] \cdot b_{jklm}^{(cl)} & i \Im[z_p] \cdot b_{jklm}^{(cr)} & i \Re[z_p] \cdot b_{jklm}^{(cl)} & i \Re[z_p] \cdot b_{jklm}^{(cr)} \end{pmatrix} \begin{pmatrix} K_{n+p} L_{cl} \\ K_{n+p} L_{cr} \\ J_{n+p} L_{cl} \\ J_{n+p} L_{cr} \end{pmatrix} = -I_{jklm} \\
 & \hspace{15em} \text{for odd } l+m.
 \end{aligned} \tag{3.17}$$

In the case of $p = 0$, Eq. (3.17) transforms back to Eq. (3.5), as $\Re[z_0] = 1$ and $\Im[z_0] = 0$. For simplicity, we can redefine b_{jklm} to include the additional factors directly into the coefficients

$$b_{jklm,p} = \begin{cases} \Re[z_p] \cdot b_{jklm} & \text{if normal corrector and } l+m \text{ even,} \\ \Im[z_p] \cdot b_{jklm} & \text{if normal corrector and } l+m \text{ odd,} \\ -\Im[z_p] \cdot b_{jklm} & \text{if skew corrector and } l+m \text{ even,} \\ \Re[z_p] \cdot b_{jklm} & \text{if skew corrector and } l+m \text{ odd} \end{cases} \tag{3.18}$$

and take z_p into account whenever $p > 0$.

Including feed-down into the equation system also allows therefore to correct multiple orders of RDTs with the same correctors, as well as correcting RDTs with correctors of multiple orders. Equation (3.5) can hence not only be extended “vertically” by correcting for multiple beam optics (Eq. (3.12)) and different RDTs (Eq. (3.6)) at the same time,

but also “horizontally”, by adding more correctors, e.g.

$$\begin{pmatrix} b_{jklm}^{(cl)} & b_{jklm}^{(cr)} & b_{jklm,p}^{(cl)} & b_{jklm,p}^{(cr)} \\ b_{j'k'l'm'}^{(cl)} & b_{j'k'l'm'}^{(cr)} & b_{j'k'l'm',p}^{(cl)} & b_{j'k'l'm',p}^{(cr)} \end{pmatrix} \begin{pmatrix} K_n L_{cl} \\ K_n L_{cr} \\ K_{n+p} L_{cl} \\ K_{n+p} L_{cr} \end{pmatrix} = - \begin{pmatrix} I_{jklm} \\ I_{j'k'l'm'} \end{pmatrix} \quad (3.19)$$

3.2.2 Summary

An improved algorithm to correct nonlinear errors by locally compensating effective RDTs in the IRs has been derived and implemented, overcoming the rigidity of previous implementations and giving the user more control over the correction. Its main features include the option to target arbitrary RDTs, include more than one beam optics, and either include feed-down into the RDTs to be corrected, or using the feed-down from the corrector magnets themselves for compensation. The code and implementation details of this algorithm can be found in [40] and [96] respectively.

3.3 Feeddown Correction

3.3.1 Introduction

Extensive tracking studies have been performed, investigating the influence of feed-down on correction and therefore on machine performance by examining their effect on DA, for the (at the point in time when the studies were performed) nominal collision-optics scenarios for the LHC and HL-LHC as described in Section 3.3.2. The results are presented in Section 3.3.3. To correct the feed-down from higher orders accurately, the calculation is done from highest to lowest RDT order and including the evaluated corrector strengths into the subsequent feed-down.

3.3.2 Simulation Setup

The effects of including feed-down in the optimization of machine performance were investigated in tracking simulations and evaluated by their influence on the DA. Optics, errors and corrections were first set up from cpyrad [132], a python interface to MAD-X [130], and the actual tracking was performed via SixTrack [134] within the SixDesk [135] wrapper from the resulting configurations.

The machines were set-up in MAD-X as summarized in Tables 3.1 and 3.2. The sequence for the respective machine was loaded and optics with $\beta_{x,y}^* = 30$ cm in the LHC and $\beta_{x,y}^* = 15$ cm in HL-LHC were initialized. The orbit was set to either the full crossing scheme (see Table 3.2) specific to the respective machine or flat-orbit, i.e. crossing and separation set to zero in all IRs. In the latter case, no feed-down effects are present and hence this case can be used as reference for optimal feed-down compensation. Then, one of the 60 realizations of the magnetic model from 2015 Windows Interface to Simulation Errors (WISE) [182, 183] tables, was applied to skew and normal fields from sextupole to

Table 3.1. Simulation Setup

Machines	LHC	HL-LHC
Beams	1 and 2	
Energy	6.5 TeV	
Orbit	see Table 3.5	
β^*	30 cm	15 cm
RDTs	$F_{0003} F_{1002} F_{3001} F_{4000} F_{0006}$ $F_{0003*} F_{1002*} F_{1003} F_{0004} F_{6000}$	as LHC + $F_{0005} F_{5000} F_{5001}$ $F_{0005*} F_{5000*} F_{1005}$
Magnetic Field Errors	$a_3, b_3, a_4, b_4, a_5, b_5, a_6, b_6, a_7, b_7, a_8, b_8$ from 60 WISE-Seeds	
DA-Tracking	100'000 turns, $2\sigma - 30\sigma$ in 2σ steps, 11 angles, $\Delta p/p = 2.7 \cdot 10^{-4}$	

hexadecapole order. In the RDTs used for correction, "*" refers to RDTs with switched β -exponents (as introduced in Eq. (3.8)) to correct for the other beam. In the HL-LHC, three additional RDTs can be corrected, as there are three extra orders of correctors (a_5, b_5 and a_6 [39]) planned to be installed. Feed-down was calculated up to the second order, when included into the correction. Finally, coupling was minimized and the tunes were matched. The SixDesk environment generated the initial conditions for the particles to be tracked by SixTrack from the resulting machine setup. For the tracking, simulated particles were evenly distributed over 11 angles in one quadrant of the $x - y$ plane and from 2σ to 30σ in buckets of 2σ in amplitude. Within each bucket, 60 particles were initialized with a relative momentum deviation of $2.7 \cdot 10^{-4}$ and were tracked for 100'000 turns.

Survival or loss of these particles determines whether the point was counted as stable or unstable and the minimum DA could then be determined per angle and seed. In the DA plots shown (Figs. 3.2a and 3.2d), we can find therefore the results of the simulations with the statistics over the seeds: the mean DA is presented as a thick line, the standard deviation as the area surrounding it and the extrema by dashed lines.

Amplitude detuning to first and second order in action is evaluated via the PTC [131] module in MAD-X and the violin plots of Figs. 3.2b, 3.2c, 3.2e and 3.2f show again the statistics of the simulation results: the mean and extreme values appear as vertical bars in the plots, while the distribution (kernel density estimation) is given by the colored area, of which one standard deviation is highlighted. As we want to correct the absolute value of the detuning, the results are presented as the change in detuning magnitude before and after correction.

3.3.3 Results

Representative simulation results are shown in Fig. 3.2. For brevity only Beam 1 is discussed, but very similar results were also obtained for Beam 2 (Fig. 3.3). As expected, the DA results summarised in the first row (Figs. 3.2a and 3.2d), show that the flat-orbit scenario (blue) delivers the best performance, compared to the scenarios introducing

Table 3.2. Orbit Setup. “Crossing” refers to the Half-Crossing Angle. The Values are shown for Beam 1, in Beam 2 the signs depend on the orbit-symmetry

			IP1		IP2		IP5		IP8	
			H	V	H	V	H	V	H	V
LHC	Crossing	[μ rad]	–	160	–	200	160	–	-250	–
	Separation	[mm]	-0.55	–	1.4	–	–	0.55	–	-1.0
HL-LHC	Crossing	[μ rad]	–	250	–	170	250	–	-200	–
	Separation	[mm]	0.75	–	-1.0	–	–	0.75	–	-1.0

the crossing scheme and therefore feed-down effects into the machine. Not taking these effects into account, lowers the DA in both machines by about 3σ (orange). Calculating the corrections according to Eq. (3.17) with feed-down (green), recovers some of the performance in the HL-LHC simulations. In the LHC setup, the DA is much less affected. It deteriorates even slightly for Beam 2.

Looking at improvements of the first and second order amplitude detuning of the two machines (Figs. 3.2b, 3.2c, 3.2e and 3.2f), we see that in the LHC the correction performs much better when including feed-down. One could see from residual detuning (not shown, due to space limit), that including feed-down corrects closer to the values of the error-free machine and also, that in general the direct terms are better corrected than the cross terms. In the HL-LHC the bulk of the corrections improve the detuning similarly.

The simulations have been shown to be very sensitive to coupling in the setup. As the coupling is only compensated after correction but the detuning is taken as the difference between before and after applying the correction, the outliers might be explained by the uncorrected coupling in the machine. Further investigations need to be performed. The differences between LHC and HL-LHC behaviour are also not yet understood.

3.3.4 Summary

The incorporation of feed-down into simulation based RDT optics corrections in the IRs of the LHC and HL-LHC has been investigated through extensive tracking simulations. While no significant impact on DA in the LHC is observed, there are good indications that first and second order amplitude detuning corrections will profit from considering feed-down effects in the calculation of corrections. On the contrary, in the HL-LHC a clear improvement is seen on the DA when considering the feed-down, but no improvement is seen in the amplitude detuning. These results should be taken with caution, as the corrections are done on magnets common to both beams, and the influence on the other beam is neglected within the scope of this chapter. It has been shown, that the DA of the other beam deteriorates when feed-down is factored in, as its influence can be opposite between beams as discussed in Section 3.5.

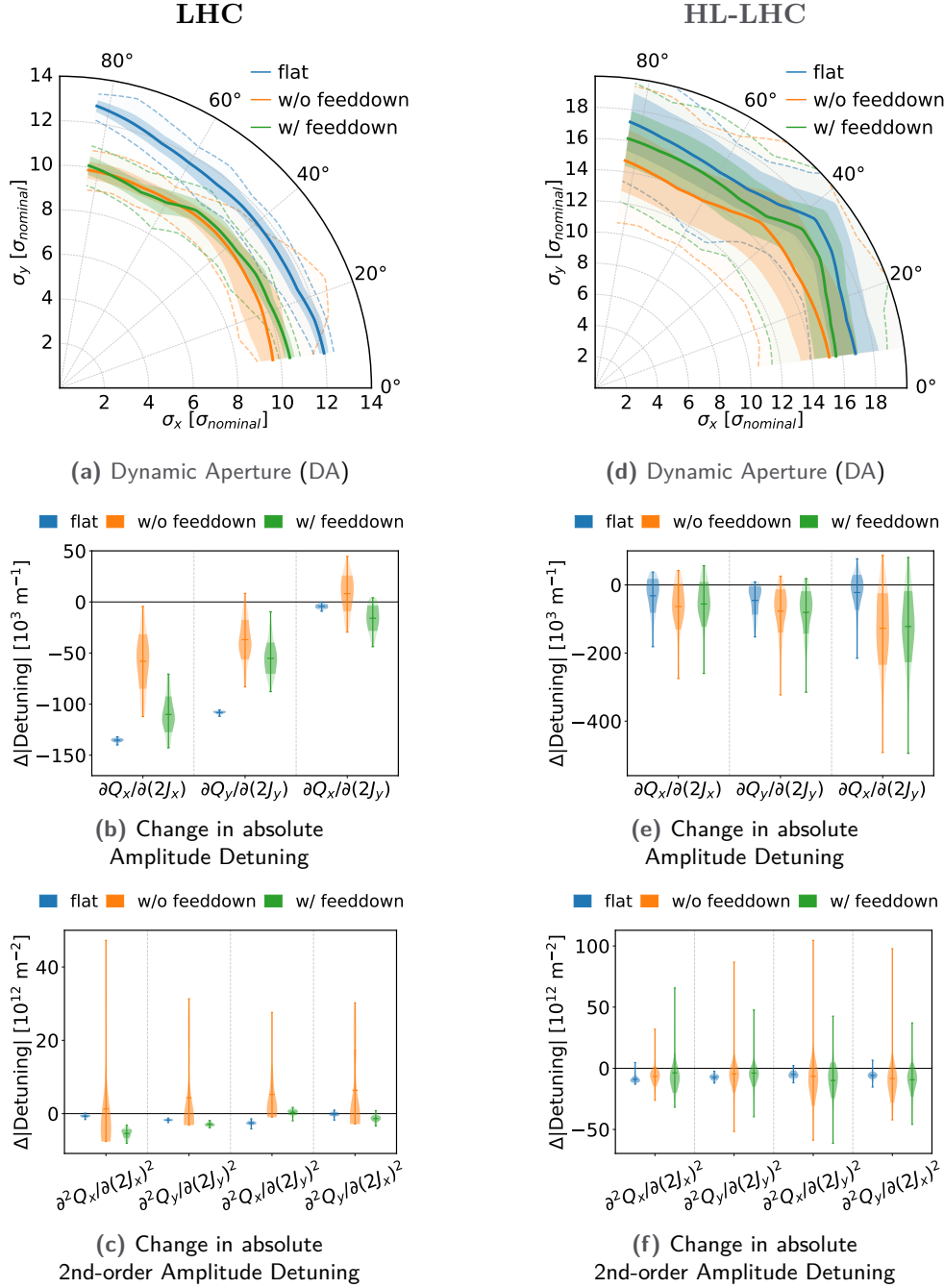


Figure 3.2. Simulation results for Beam 1 LHC (left) and HL-LHC (right) after correction. The scenarios are flat-orbit (blue), or the full crossing scheme applied and ignoring feed-down in the corrections (orange) or including feed-down into the calculations (green). The statistics over the error realizations are shown by their mean (thick line/central horizontal bar) one standard deviation ([strong] colored area) and extrema (dashed lines/horizontal end-bars). The shape of the violins shows the distribution.

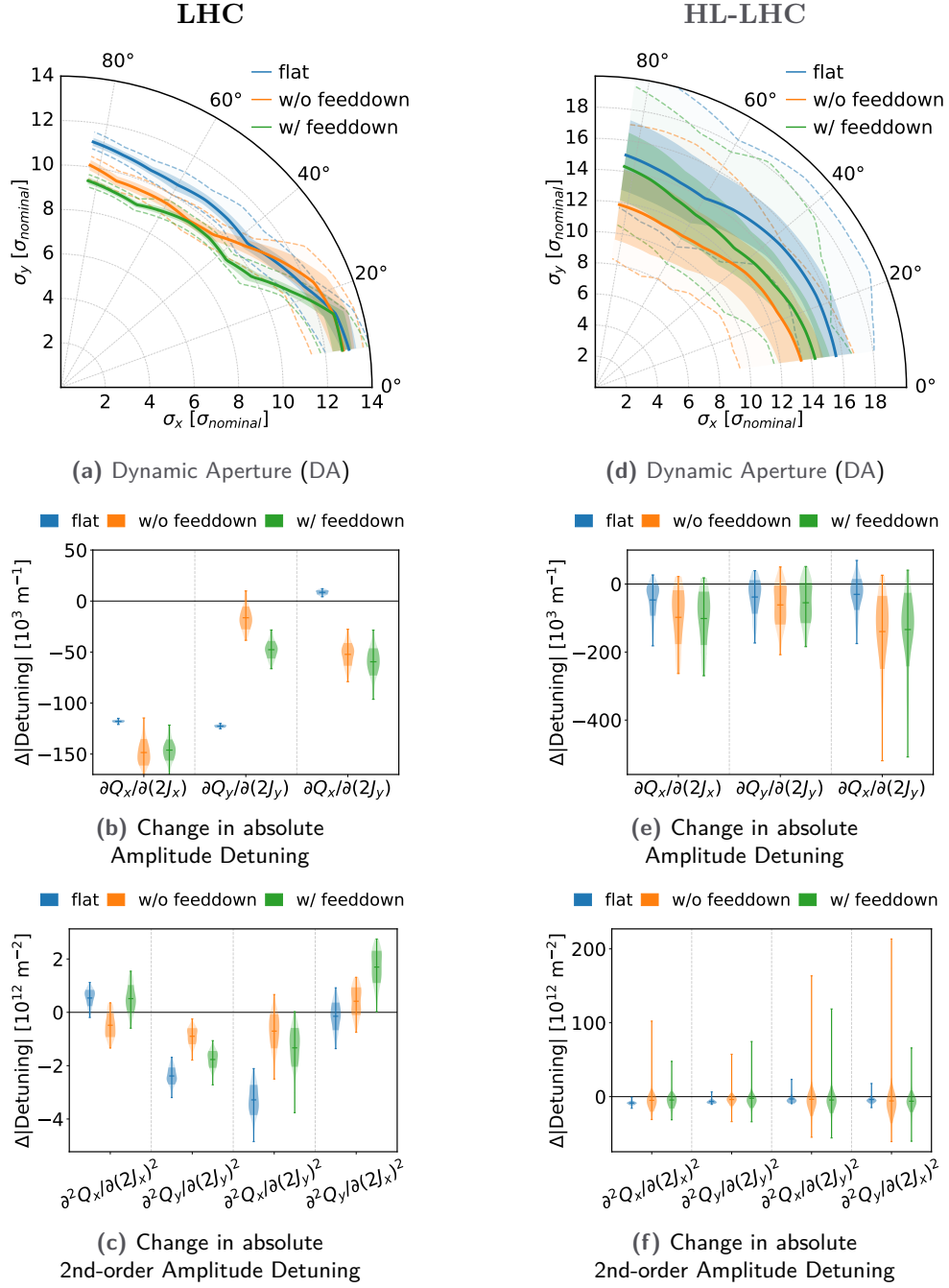


Figure 3.3. Simulation results for Beam 2 LHC (left) and HL-LHC (right) after correction. The scenarios are flat-orbit (blue), or the full crossing scheme applied and ignoring feed-down in the corrections (orange) or including feed-down into the calculations (green). The statistics over the error realizations are shown by their mean (thick line/central horizontal bar) one standard deviation ([strong] colored area) and extrema (dashed lines/horizontal end-bars). The shape of the violins shows the distribution.

3.4 Dodecapole Error Correction

3.4.1 Introduction

In the context of correcting feed-down, of special interest are the correction of dodecapole errors and their feed-down to octupole fields. This interest was put into focus with the measurements performed during LHC commissioning in 2018, where upon changing the crossing scheme from flat-orbit (i.e. no crossing angles active) to a collision scheme, an increase in amplitude detuning was measured [36,37]. Further investigation [43] during MD sessions confirmed this finding and revealed the main contribution to be feed-down from high-order errors to the octupole fields, due to the crossing schemes in IP1 and IP5.

In [Chapter 7](#) corrections are developed based on these measurements and finally also tested in the actual machine. As a precursor to the experimental study, tracking simulations have been conducted, evaluating the possibility of performing these corrections in the first place and to assess the influence on amplitude detuning and DA. The results of this study are presented here.



For brevity, the normal dodecapole nonlinear IR correctors are assigned the shorthand MCT and the normal octupole nonlinear IR correctors are called MCO. This follows the LHC naming scheme as described in [Section 2.1](#), in which magnet elements starting with M, followed by C for corrector magnet and then T for dodecapole order and O for octupole order. To indicate that we are talking about the common aperture magnets in the crossing-regions, usually also a X is appended (i.e. MCTX and MCOX). As in the here presented studies only correctors in the IRs are used, the X is omitted.

3.4.2 Simulation Setup

The simulations for the LHC have been set up and run as described in [Section 3.3](#) and summarized in [Table 3.3](#). The major difference to the previous section is, that instead of the full range of magnetic errors, only normal dodecapole errors (b_6 , see [Section 2.2.1](#)) are applied from the WISE-tables and then corrected via the IR nonlinear correction script of [Section 3.2](#).

As correction targets, the normal dodecapole RDTs F_{6000} , F_{0006} and normal octupole RDTs F_{4000} , F_{0004} were used. These are the default RDT-targets for normal dodecapole and normal octupole fields, as they also correct the direct terms of second and first order amplitude detuning. This can be seen from the β -exponents of e.g. [Eq. \(3.1\)](#):

- F_{6000} , F_{0006} scale with β_x^3 and β_y^3 respectively and therefore correct the direct terms of second-order amplitude detuning, which scales with the same powers ([Eqs. \(2.44a\)](#) and [\(2.44d\)](#)).
- F_{4000} , F_{0004} scale with β_x^2 and β_y^2 respectively and therefore correct the direct terms of first-order amplitude detuning, also scaling with these powers ([Eqs. \(2.43a\)](#)

Table 3.3. Simulation Setup

Machine	LHC
Beams	1, 2
Energy	6.5 TeV
β^*	30 cm
Orbit	same as in Table 3.5 (for LHC)
WISE-Seeds	60
Magnetic Field Errors	b_6
Corrected RDTs	F_{4000} or F_{6000}
DA-Tracking	
Turns	100'000 turns
Amplitudes	4σ - 30σ in 2σ steps,
Angles	11 angles
$\Delta p/p$	$2.7 \cdot 10^{-4}$

and (2.43c)).

After the application of b_6 errors as well as after correction the first and second order amplitude detuning (see Section 2.2.6) were calculated via PTC. The following correction scenarios are evaluated:

Uncorrected No corrections applied.

b_6 by MCT The b_6 RDTs are targeted directly by normal dodecapole correctors (MCT).

b_4 by MCO The b_4 RDTs are targeted, generated by the feed-down from the applied b_6 errors, using normal octupole correctors (MCO).

b_4 by MCT The b_4 RDTs are targeted, generated by the feed-down from the applied b_6 errors, using now the feed-down from normal dodecapole correctors (MCT).

With the investigated scenarios we are covering current and possible future correction schemes: It is expected, that correcting b_6 errors directly with the normal dodecapole correctors is the most efficient, as this will compensate all of their influences. The difficulty in pursuing this approach in the real machine is that direct observables, such as dodecapole RDTs and second-order amplitude detuning, are hard to measure, due to the high-order of the dodecapoles, and are often indistinguishable from noise. The second scenario, using the normal octupole correctors to correct the feed-down from b_6 , is the approach currently established in the LHC. This approach is achieved by measuring and correcting first-order amplitude detuning, yet it may not be able to correct all harmful influences of the b_6 errors, as e.g. second-order amplitude detuning is not affected. The third approach is a combination of the former two: using the normal dodecapole

correctors, which are available but so far unused in the LHC, to target the feed-down to an order at which observables have proven to be measurable, here b_4 magnetic fields.

3.4.3 Results

Shown in Figs. 3.4a and 3.4d are the simulated DAs over the different simulation instances (WISE-seeds), gathered into mean (thick line), standard deviation (colored area) and extrema (dashed lines).

Targeting F_{6000} with the MCTs (orange) increases the DA from the uncorrected (blue) case by $5\sigma - 10\sigma$, neglecting a few outlier-seeds pushing the minimum DA all the way down to around 10σ , close to the “Uncorrected” case. The correction of F_{4000} by means of the MCOs (green) leaves the DA almost unchanged in Beam 1, whereas in Beam 2 it increases slightly by $1\sigma - 2\sigma$. Targeting F_{4000} with the MCTs (red) increases DA almost as well as targeting F_{6000} directly, but only in the vertical plane. In the horizontal plane on the other hand, the DA in this case is close to the DA after correction with MCOs.

In Figs. 3.4b, 3.4c, 3.4e and 3.4f first- and second-order amplitude detuning is shown as change (Δ) of the absolute value of the detuning compared to the “Uncorrected” case. This means that negative values show a decrease in the absolute detuning value, i.e. detuning gets corrected, while positive values indicate an increase, meaning worsening of the detuning. As targeting F_{4000} with MCOs corrects first-order amplitude detuning almost perfectly, the difference in detuning from the correction corresponds approximately to the respective absolute values of the detuning before correction. The plots therefore also show that first-order amplitude detuning was already higher in Beam 2 compared to Beam 1 before correction, e.g. the mean $Q_{x,x}$ was $\approx 85 \times 10^3 \text{ m}^{-1}$ in Beam 2 but only $\approx 30 \times 10^3 \text{ m}^{-1}$ in Beam 1. It makes therefore more sense to compare the two beams qualitatively than quantitatively.

There are many things to unpack: First of all, we see that first-order detuning is not well corrected, if at all, in the “ b_6 by MCT” scenario: in the direct horizontal term of Beam 1 there is barely a correction on the average and the direct vertical and cross-terms are deteriorated after the “correction”. This behaviour is not surprising, as the first-order detuning is not directly targeted in this scenario and rather corrected “by accident”. The other two scenarios are targeting F_{4000} and therefore the direct terms of the first-order detuning. No matter the corrector choice (MCO or MCT), both correct the detuning very well and, apart from $Q_{x,x}$ in Beam 1, in which the correction with the MCTs seems to be slightly better, identically. The difference between these two corrections, and therefore probably also the difference in DA, comes from their effect on second-order amplitude detuning: While the correction with the MCOs barely affects this order of detuning (K_4 only contributes to it in second order [97]), the MCTs have a visible impact. As expected, because feed-down from b_6 to b_4 is corrected with feed-down from b_6 to b_4 , this impact is correcting in nature. One of the assumptions of this local correction is that we can correct the errors coming from the whole IR with counter-measures at the two points of the correctors. While this seems to be true for the here presented scenario, sadly, this assumption might not always hold, e.g. for optics where the phase-advances between IR-elements are too large or if the orbit at the correctors are not representative for the

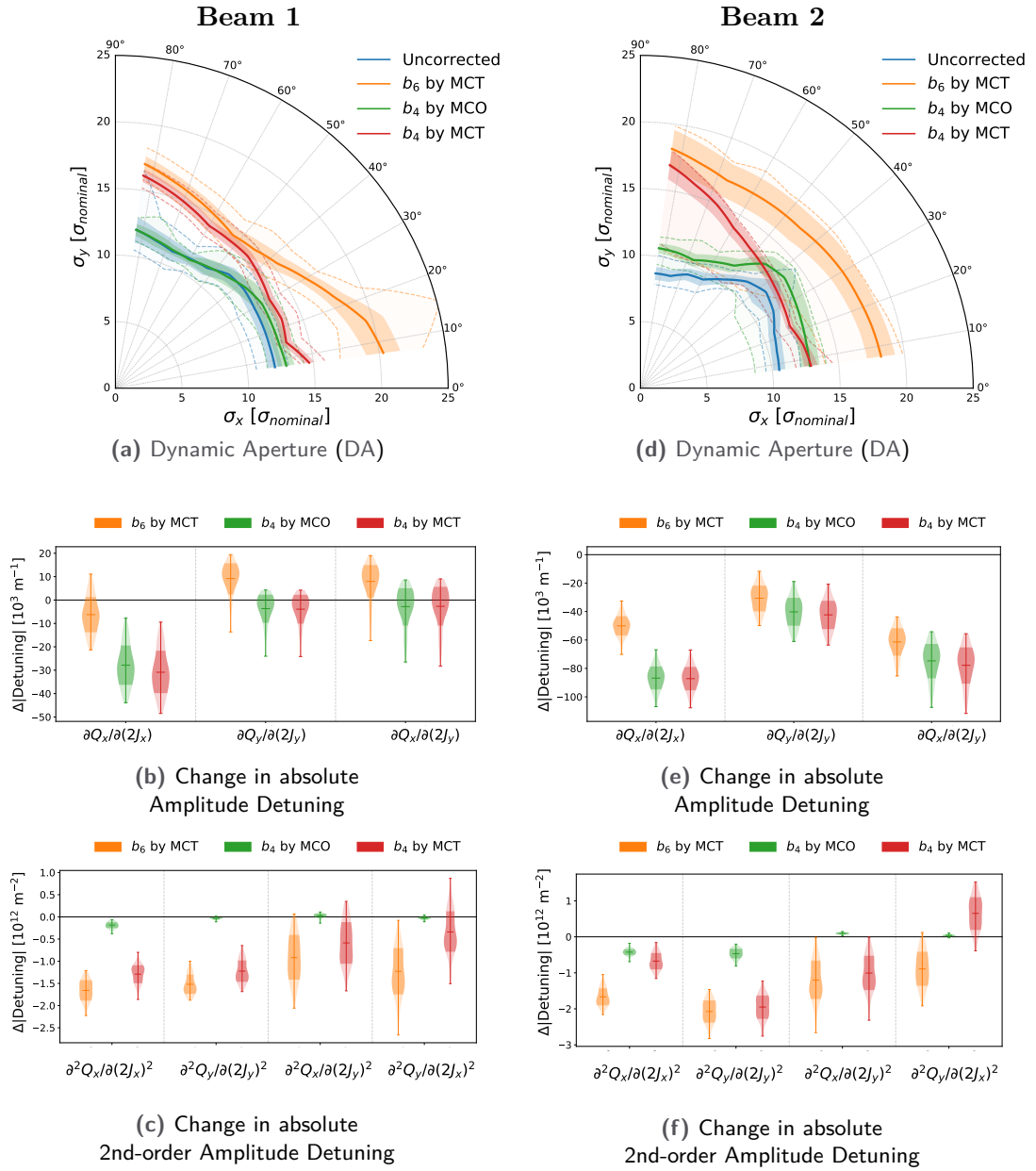


Figure 3.4. Simulation results for simulation studies with b_6 errors in Beam 1 (left) and Beam 2 (right) with different correction scenarios: uncorrected (blue), targeting b_6 with normal dodecapole correctors (orange), targeting feed-down to b_4 with normal octupole correctors (green), targeting feed-down to b_4 with normal dodecapole correctors. The statistics over the error realizations are shown by their mean (thick line/central horizontal bar) one standard deviation ([strong] colored area) and extrema (dashed lines/horizontal end-bars). The shape of the violins shows the distribution.

orbit at sources of the errors. The latter might be the reason that we see that targeting b_4 -RDTs does not as effectively correct second-order amplitude detuning as targeting b_6 -RDT would. Terms for vertical action seem to be better corrected their horizontal counterparts. Especially $Q_{y,xx}$ (and in particular Beam 2) suffers from detrimental effects, hinting at an incompatible horizontal orbit between error-sources and correctors. This is likely the explanation for the lower DA observed for that scenario in Figs. 3.4a and 3.4d in that plane.

Nonetheless, this type of correction is very promising, as provides an optimal first-order amplitude detuning correction and additionally shows improvements in most of the terms of second-order amplitude detuning. Also, DA improvement in the horizontal plane is at least as good as for the correction scenario with MCOs and almost perfect in the vertical plane.

3.4.4 Summary and Outlook

The results confirm, that correcting b_6 errors with the normal dodecapole correctors would be an optimal solution with respect to DA and second-order amplitude detuning, yet it is detrimental to first-order detuning. Using the normal octupole correctors on the other hand, we can correct the feed-down to first-order amplitude detuning perfectly, yet will not improve DA.

The third scenario of correcting the feed-down to b_4 with the feed-down from the normal dodecapole correctors appears to be the most promising of the investigated scenarios. It has also the advantage of already well established observables in the LHC, such as first-order amplitude detuning, which can be well corrected with this approach. On the other hand it comes with the caveat, that the DA in the horizontal plane and the second-order detuning can be spoiled.

Additional studies should be performed, in which combinations of the different targets and correctors are tested. For example, the option of using both MCO and MCT correctors at the same time to correct b_6 errors has not been investigated here. Also the interaction with additional b_4 errors is a topic of great interest. Looking at individual orders, together with the insights of asymmetric optics (see Section 3.5), might also help to explain the results seen in Section 3.3, where including feed-down for all orders did not necessarily improve DA.

These simulation studies could not be performed within this PhD due to time constraints, and instead the correction of high-order errors via feed-down from the dodecapole-correctors has been studied experimentally and is presented in Chapter 7.

3.5 Correction of Asymmetric Optics

3.5.1 Introduction

Before this study was launched and the new correction script (Section 3.2) developed, the implementation of this scheme calculated the correction based on a single input optics, for either Beam 1 or Beam 2, and made use of symmetries between the beams,

to optimize the correction for both. Cases will occur in which this symmetry does not hold, e.g. through the introduction of feed-down (Section 3.3), or the use of inherently asymmetric optics. An example for the latter are flat-optics [3, 30], in which β^* in the two transversal planes no longer has identical values (see Fig. 3.1). These optics allow for a more distributed radiation deposition in the LHC magnets as well as an increase in luminosity [30]. Their feasibility has been studied during machine developments in the LHC [180] and preliminary analysis regarding their influence on corrections and amplitude detuning has been conducted [181]. Extensive tracking studies have been performed, using the new correction script and comparing the single- and combined-optics corrections, the latter are now not relying on symmetry assumptions, as discussed in Section 3.2.1. The results are presented here.

3.5.2 Simulation Setup

To compare the effects of different correction methods on the DA, the optics, errors and corrections were first set up from within the cymad [132] wrapper of MAD-X [132] and then tracking studies were performed via SixTrack [134] within the SixDesk [135] environment from the resulting configurations.

The machines are initialized in MAD-X by loading their sequence and application of the optics of interest: Round-optics were utilized with $\beta_{x,y}^* = 30$ cm and $\beta_{x,y}^* = 15$ cm for the LHC and HL-LHC respectively. The investigated flat-optics were $\beta_x^* = 60$ cm, $\beta_y^* = 15$ cm (in IP1, switched planes in IP5) in the LHC and $\beta_x^* = 7.5$ cm, $\beta_y^* = 30$ cm (in IP1, switched planes in IP5) in the HL-LHC. Afterwards one of 60 realizations of the magnetic field errors from the 2015 WISE [182, 183] tables, based on the magnetic measurements mentioned in the introduction, was applied to skew and normal fields, from sextupole to hexadecapole order. A conservative systematic $b_6 = -4$ value was assumed for the HL-LHC. Either of the *single* beam optics or both beam optics *combined* were then chosen to compute the corrections. The RDTs utilized for the corrections can be found in Table 3.4. "+" indicates that these RDTs were corrected in addition to the ones specified before. In the HL-LHC, three additional RDTs can be corrected, as there are three extra orders of correctors (a_5, b_5 and a_6 [39]) planned to be installed (Fig. 2.10) Feed-down was included into the corrections up to second order on the transverse displacements. After a final coupling correction and matching of the tunes, the resulting configuration was passed to SixDesk, which generates the initial conditions for the particles to be tracked by SixTrack: The particles were evenly distributed over 11 angles in one quadrant of the $x - y$ plane and from 2σ to 30σ in buckets of 2σ in amplitude (all values given in σ of the nominal beam). Within each bucket, 60 particles were initialized, with a relative momentum deviation of 2.7×10^{-4} , and being tracked for 100'000 turns. Survival or loss of these particles determines whether the point was counted as stable or unstable. The minimum DA could then be derived per angle and seed. An overview of the simulation parameters are found in Tables 3.4 and 3.5.

Table 3.4. Simulation Setup

Machines	LHC	HL-LHC
Beams	1 and 2	1 and 2
Energy	6.5 TeV	6.5 TeV
β^*	round: flat: 60/15 cm 30 cm,	round: flat: 15 cm, 7.5/30 cm
Combined Optics RDTs	$F_{0003} F_{1002} F_{3001} F_{4000} F_{0006}$	$+ F_{0005} F_{5000} F_{5001}$
Single Optics RDTs	$+ F_{0003}^* F_{1002}^* F_{1003} F_{0004} F_{6000}$	$+ F_{0005}^* F_{5000}^* F_{1005}$
Magnetic Field Errors	$a_3, b_3, a_4, b_4, a_5, b_5, a_6, b_6, a_7, b_7, a_8, b_8$ from 60 WISE-Seeds	
DA-Tracking	100'000 turns, $2\sigma - 30\sigma$ in 2σ steps, 11 angles, $\Delta p/p = 2.7 \cdot 10^{-4}$	

Table 3.5. Orbit Setup. The values are given for Beam 1 round-optics. The magnitude of the values are the same for Beam 2, but signs depend on the orbit-symmetries. In LHC flat-optics the crossing planes are switched in IP1 and IP5. “Crossing” is given as the crossing half-angles.

			IP1		IP2		IP5		IP8	
			H	V	H	V	H	V	H	V
LHC	Crossing	[μ rad]	–	160	–	200	160	–	–250	–
	Separation	[mm]	–0.55	–	1.4	–	–	0.55	–	–1.0
HL-LHC	Crossing	[μ rad]	–	250	–	170	250	–	–200	–
	Separation	[mm]	0.75	–	–1.0	–	–	0.75	–	–1.0

3.5.3 Results

Results of simulations are shown in the DA plots in Figs. 3.5 and 3.6, with the statistics over the error realizations: the thick lines show the mean DA, the standard deviation is indicated by the area surrounding it and the dashed lines mark the extrema. LHC simulation results are presented in the left columns, while the right columns show HL-LHC results.

As seen from the first rows (Beam 1: Figs. 3.5a and 3.5d, Beam 2: Figs. 3.6a and 3.6d), in case of flat-optics without including feed-down, the corrections as calculated from the other beam optics (orange) are performing as well as the ones calculated from the same beam optics (blue) and from both optics combined (green). This confirms the findings of the preliminary study in [181]. Omitted here are the results from round-optics without feed-down, which also show identical DA for all three corrections; this is not surprising, as the symmetry considerations were made for exactly this case.

Including feed-down, the symmetry assumptions break. While for the LHC round-optics, the split does not favour either correction in Beam 1 (Fig. 3.5c), it follows the tendencies of the other three cases in Beam 2 (Fig. 3.6c), which is more pronounced: Correcting via the other beam’s optics results in a much worse DA (orange) than the corrections from the beams own optics (blue). Including both beams into the correction

can recover some of the lost performance (green), especially in the LHC. The difference between LHC and HL-LHC round-optics might stem from the additional correctors in the HL-LHC, which improve the feed-down correction for the same beam, yet add more invalid corrections for the other beam. Not shown here are the results of Beam 2, as they display the same behaviour if not otherwise specified.

When including feed-down into the correction, the best correction for both beams is therefore calculated from both beam optics. While incorporation of feed-down could be beneficial for the correction of a single beam, as seen in Figs. 3.5d to 3.5e, the overall correction of both beams was always negatively impacted.

3.5.4 Summary and Outlook

Extensive tracking simulations have been performed to investigate how much influence the breakdown of the symmetry considerations utilized in the non-linear corrections of beam optics has on the common correction of both beams in the LHC and HL-LHC. While it has been assumed that the use of flat-optics will deteriorate the correction on the other beam, this is not the case. Only when feed-down is considered in the correction scheme we can see a split between the effectiveness of the corrections, as calculated from the optics of the other beam, which can be partly compensated by using both beam optics to compute the correction. In the LHC, this behaviour can be observed only with flat-optics, while in the HL-LHC both flat- and round-optics are affected.

Future studies including feed-down corrections, e.g. as described in Sections 3.3 and 3.4, can profit from combined optics corrections. Trying to optimize corrections for both beams with additional RDTs might also lead to further improvements.

3.6 Overall Conclusion and Outlook

In Section 3.2 corrections of nonlinear errors by locally minimizing effective RDTs in the IRs have been derived and implemented in a new python package [40, 96], overcoming the rigidity of previous implementations and giving the user more control over the correction. Its main features, to include feed-down and more than one beam optics have been explored to the LHC and HL-LHC and have been presented in Sections 3.3 and 3.4 (feed-down) and Section 3.5 (dual optics).

Both can improve specific facets of the correction, e.g. feed-down has a significant influence on amplitude detuning in the LHC while not effecting much DA, and vice versa for the HL-LHC. It has also been shown, that including feed-down in the corrections has an effect on the symmetry assumptions, when using asymmetric optics, such as flat-optics, and in these cases both optics should be given to the algorithm. On a more practical note, the results from Section 3.4 show promise in correcting b_6 errors with the dodecapole correctors by targeting the feed-down to b_4 and lay the simulation foundation to Chapter 7.

Due to the broad parameter space and the computational effort of tracking studies, the simulations performed within the scope of this chapter, while giving a first insight

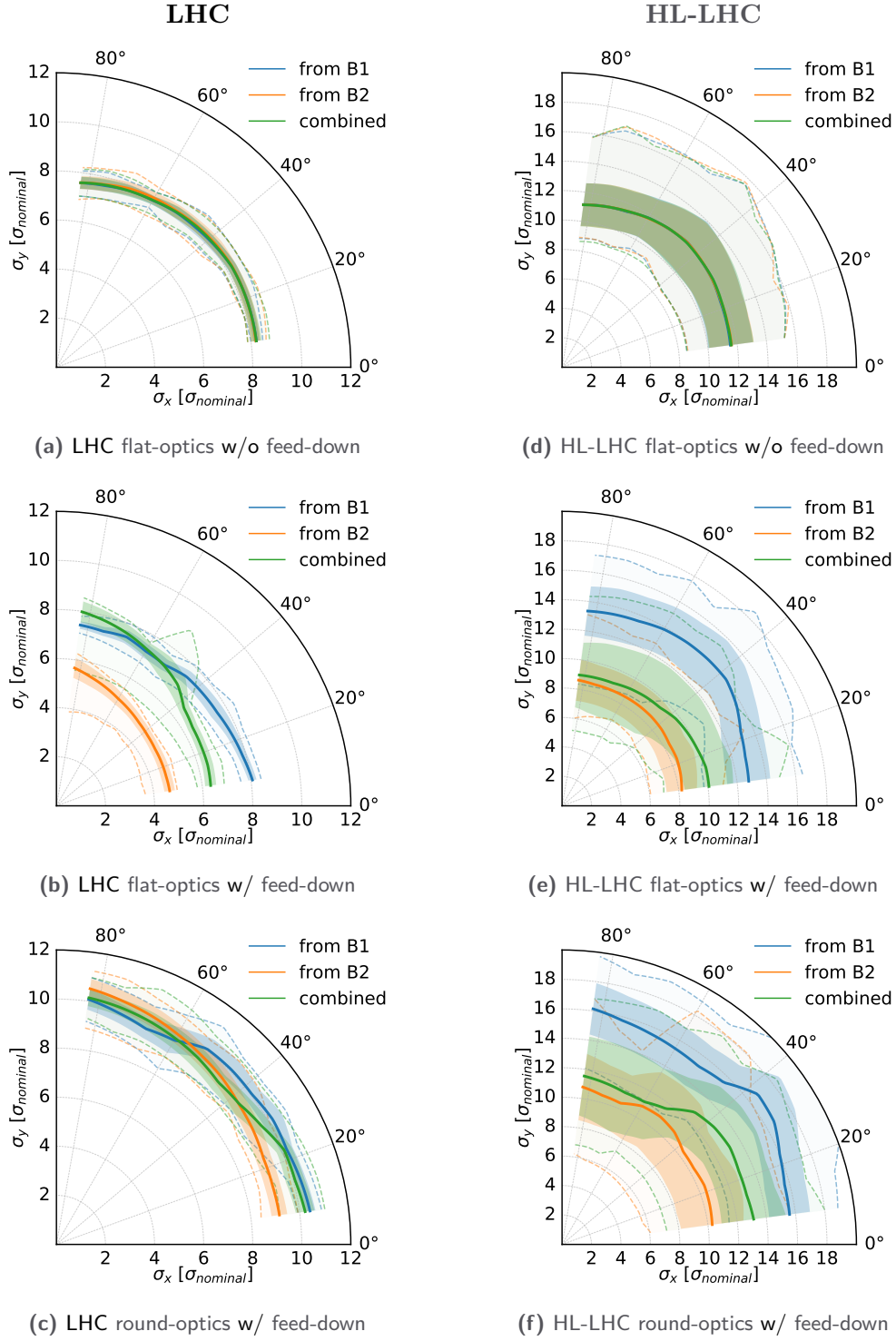


Figure 3.5. DA results for the LHC (left) and HL-LHC (right) Beam 1 after applying the corrections from the same beam optics ('from B1', blue), the other beam optics ('from B2', orange), and from both optics combined (green). The thick line indicates the mean over the realizations, while the dashed lines show extrema and the area covers one standard deviation.

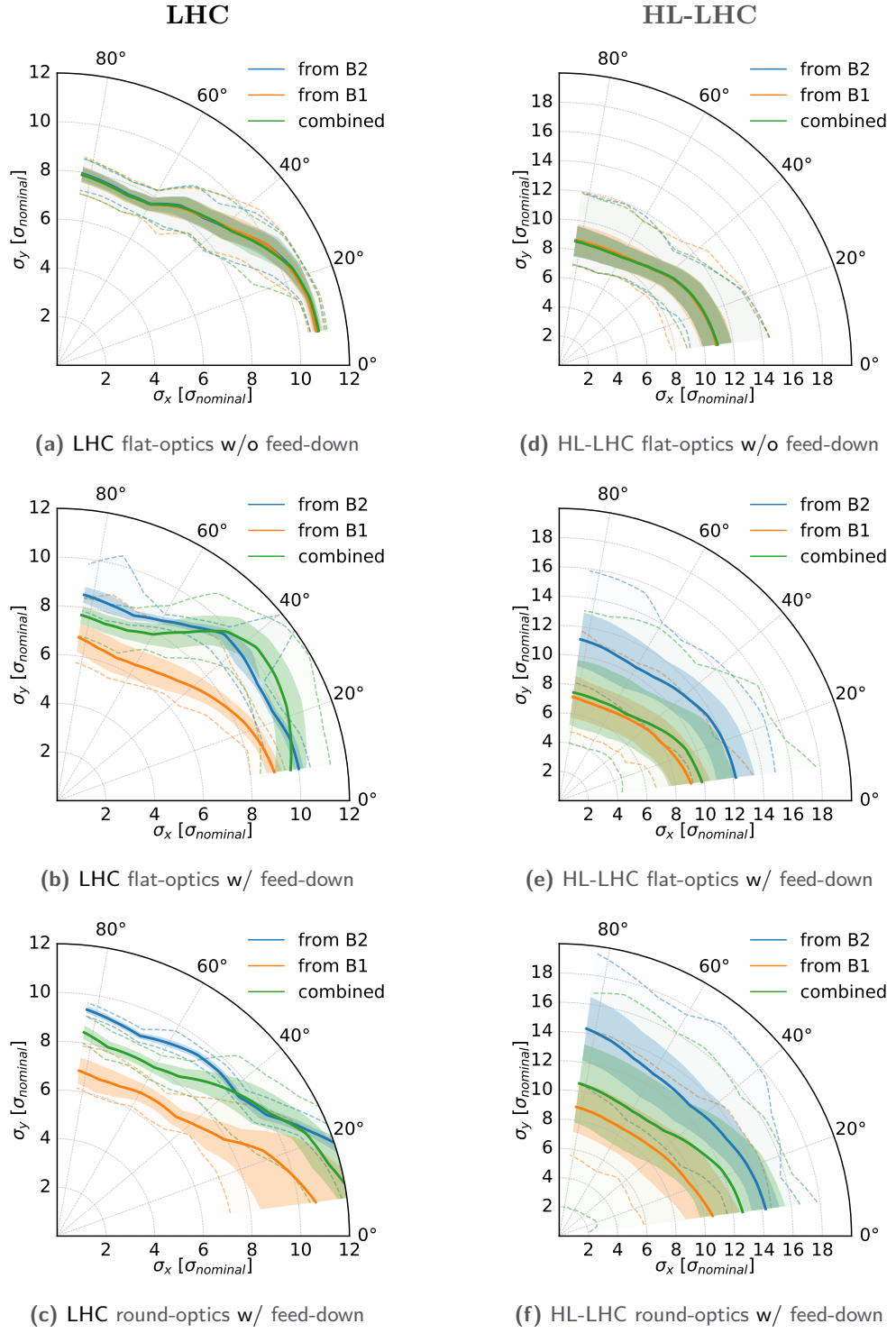


Figure 3.6. DA results for the LHC (left) and HL-LHC (right) Beam 2 after applying the corrections from the same beam optics ('from B2', blue), the other beam optics ('from B1', orange), and from both optics combined (green). The thick line indicates the mean over the realizations, while the dashed lines show extrema and the area covers one standard deviation.

into the possible corrections, are far from exhaustive. Additional studies should be performed, in which combinations of feed-down and dual optics corrections are applied and in which different combinations of RDT targets and correctors are tested.

4 Amplitude Detuning from Misalignments of Nonlinear-Corrector and Triplet Magnets in the LHC and HL-LHC

Note. The results of the here presented study have partly been presented in a meeting of the “HiLumi Workpackage 2” [175] as well as fully documented in a technical note [184].

4.1 Introduction

The goal of the study presented in this chapter was to investigate amplitude detuning under the influence of misalignments in the nonlinear-correctors and triplets in the IRs of the LHC and the HL-LHC, and the impact on the successful operation of the machine under these perturbations. The results were calculated utilizing simulations of the two accelerator configurations respectively and applying errors and misalignments to their simulated elements.

These studies were designed to deepen the understanding of previous findings in the LHC: On multiple different occasions, feed-down effects at different locations around the ring and in different planes were observed. The influence of feed-down in general has been observed, investigated and used to verify corrections in the LHC and will be crucial for commissioning of the HL-LHC [14, 15, 17, 33, 38, 128, 177].

Application of b_4 corrections have a detrimental impact on the a_3 and b_3 correction in Beam 1, as seen during a vertical crossing angle scan in the ATLAS-IR (IP1) already back during LHC run 1 [14], hinting to an orbit offset of Beam 1 in the normal octupole correctors. During LHC run 2, similar deterioration of the a_3 and b_3 corrections upon simultaneous application of a_4 corrections was measured in both beams during a vertical crossing angle scan in IP1 [17]. In this case - as both beams were involved equally, a misalignment of the skew octupole corrector was assumed to be the culprit. On both of these occasions, the observed behavior could be explained through simulations incorporating beam offsets of around 1 mm [185] in the respective magnets. Furthermore, tune shifts, during a sextupole-corrector polarity check with crossing angles applied, hinted towards an unexpected orbit or magnet misalignment with respect to the outermost triplet quadrupole (Q3) in the CMS-IR (IP5) [186].

The accuracy of the position alignment of the nonlinear correctors in the HL-LHC is given to be smaller than 1 mm. Hence, this limit is also used as an upper bound in other simulations, e.g. for DA studies as in [187]. Therefore, also in the study at hand, misalignments of the corrector magnets are assumed to be uniformly distributed within a ± 1 mm interval.

The triplets themselves, on the other hand, may also experience misalignments. In [188, 189] the residual beam offsets from magnetic center of the triplets are given as 0.5 mm RMS. These values were published after the simulations for the note at hand were already completed, here utilizing misalignments within a Gaussian distribution of $\sigma = 0.4$ mm. This is nevertheless deemed to be sufficiently precise.

In Section 4.2, the methods used to achieve the detuning results are outlined: Section 4.2.1 describes the simulation setup and parameters, while Section 4.2.2 introduces the formulas used to determine amplitude detuning from the simulation results.

The results are then presented in Section 4.3, which is split in two parts. Section 4.3.1 shows the influence of misaligning the nonlinear-correctors on amplitude detuning, while Section 4.3.2 presents similarly the outcome upon misaligning the triplets.

4.2 Simulation Setup

4.2.1 Simulation Description

MAD-X [42] simulations for the LHC and HL-LHC are performed in order to analyse amplitude detuning due to feed-down, where the triplets in the IRs and the multipole correctors in the IRs are misaligned, one at a time. The respective lattices for LHC and HL-LHC are created from the standard LHC (run 2 2018) and HL-LHC (V1.4) sequences and optics files. The relevant optics parameters can be found in Table 4.1. The elements of the thick lattice are converted to a thin lattice using the available slicing macros [190], as this is necessary for the error correction in the IRs. Magnetic field errors are then applied in all straight sections using the 60 different seeds of the magnetic error model [183, 191, 192]. A schematic view of an IR had been shown in Figs. 2.9 and 2.10. To obtain the strengths of individual errors, simulations were run applying sextupole, octupole, decapole and dodecapole errors (SODT), decapole and dodecapole errors only (DT), sextupole and octupole errors only (SO), and all four errors individually (S/O/D/T). The abbreviations in parentheses are used in the summary tables Tables 4.2 and 4.3 in the results chapter Section 4.3. The assigned errors in the triplets and D1 are corrected by locally minimizing RDTs via the nonlinear correctors (Section 2.1) using the “old” corrections scripts [7, 193, 194], as the flexible IR nonlinear correction, as described in Section 3.2, had not been implemented at the time when the study was performed.

In the first study, the correctors are misaligned up to ± 1 mm with 50 misalignment value realizations in both transverse planes, using a uniform random distribution.

Amplitude detuning is evaluated, via the PTC-twiss module directly in MAD-X, before and after the misalignments, from which the difference is taken to remove the detuning that does not stem from the feed-down. Change in amplitude detuning from misalignments is also calculated in python scripts implementing the equations described in Section 4.2.2 reading values from during the simulation saved misalignment errors and corrector strengths.

In the second study, the triplets were misaligned within a truncated Gaussian distri-

bution with $\sigma = 0.4$ mm and a cutoff at 2.5σ . Because of the sensitive optics at the triplets, these misalignments would require orbit corrections before running PTC-twiss, corrections which in turn would interfere with the aim of the study of having off-center beams in the first place. In this second study, the amplitude detuning was therefore calculated by the analytical formulas only, inserting the saved misalignments into the equations in [Section 4.2.2](#).

Consequently, all misalignments are assigned independently for each quadrupole and corrector magnet, hence the Q1a/b - Q3a/b ([Figs. 2.9](#) and [2.10](#)) of the triplets were given different misalignments, as well as each nonlinear corrector magnet. Particular care was taken to misalign associated slices of an element with the same amount.

Table 4.1. Machine setup.

	LHC	HL-LHC
β^* [m]	0.30	0.20 ^a and 0.30 ^b
Energy [TeV]	6.5	7.0
Orbit	flat-orbit	
Q_x, Q_y	0.31, 0.32	

^a Initial target for run 4. ^b chosen to compare with LHC

4.2.2 Analytical amplitude detuning calculation

In addition to the amplitude detuning from PTC, or when the triplet misalignments did prevent stable optics, detuning was also calculated from the misalignment errors via feed-down to octupole components. In the following K_N/J_N are again the normalized magnetic field strength of the normal/skew field components of order N as in [Eq. \(2.10\)](#). $K_N L/J_N L$ are the strengths integrated over the length L (usually of a single element), which can be output per element from MAD-X. Feed-down to $K_4 L$ is calculated from the beam displacements dx , dy at decapoles and dodecapoles. Note that here the misalignments are of opposite sign as compared to the ones stated in the MAD-X error-file, as in MAD-X they describe the displacement of the element [\[130\]](#), while the formulas [Eqs. \(4.1a\) to \(4.1d\)](#) describe the displacement of the beam from the center of the element. In case of studies with non-zero orbit, the orbit-displacement would need to be taken into account as well.

$$K_4 L^{(K_5 L)} = dx \cdot K_5 L \quad (4.1a)$$

$$K_4 L^{(J_5 L)} = -dy \cdot J_5 L \quad (4.1b)$$

$$K_4 L^{(K_6 L)} = \frac{1}{2} (dx^2 - dy^2) \cdot K_6 L \quad (4.1c)$$

$$K_4 L^{(J_6 L)} = -dx \cdot dy \cdot J_6 L, \quad (4.1d)$$

which can be derived by first order perturbation of the Hamiltonian terms in dx and dy . As in the simulations dx and dy are taken from independent distributions with zero-mean and equal variance for both planes, the resulting expectation value for all four feed-down terms is also zero (see [Appendix C.2](#)). Contribution to amplitude detuning coefficients from the misaligned elements w are then retrieved, calculating first the feed-down to K_4 via [Eq. \(4.1\)](#) and then [Eq. \(2.43\)](#):

$$\frac{\partial Q_x}{\partial 2J_x} = \sum_w \frac{K_4 L_w}{32\pi} \beta_x^2(s_w) \quad (4.2a)$$

$$\frac{\partial Q_x}{\partial 2J_y} = - \sum_w \frac{K_4 L_w}{16\pi} \beta_x(s_w) \beta_y(s_w) \quad (4.2b)$$

$$\frac{\partial Q_y}{\partial 2J_y} = \sum_w \frac{K_4 L_w}{32\pi} \beta_y^2(s_w) , \quad (4.2c)$$

where s_w marks the location of the (end of the) element, i.e. here the values of the β -function as given per element by the TWISS command of MAD-X are used.

4.3 Results

4.3.1 Corrector Misalignments

PTC vs Feed-down calculations

The difference between amplitude detuning of aligned and misaligned correctors from PTC-twiss (**PTC**) as well as the feed-down-calculated (**FD**) detuning, are discussed in this section. They are summarized in [Table 4.2](#) (LHC) and [Table 4.3](#) (HL-LHC at $\beta^* = 20$ cm). The columns indicate the different magnetic field error orders applied to the triplets. Simulations were performed having sextupole, octupole, decapole and dodecapole errors applied together (SODT), and sextupole and octupole (SO), and decapole and dodecapole (DT) errors applied separately, as well as sextupole (S) and octupole (O) errors individually. As the nonlinear-correction script treats the field orders independently, only the nonlinear-corrector magnets of the applied field orders are powered, meaning that even though no distinction was made as to which correctors magnets to misalign, only the ones correcting for the applied field errors contribute to the optics. In the case of the LHC simulations an additional study, in which coupling was corrected (CC) before the evaluation of detuning, was performed. The results are given as mean and standard deviation of the 3000 samples, from error instances and misalignment realizations, in units of 10^3 m^{-1} .

In these two tables, we see that when there are only decapole and dodecapole errors involved (DT), the values from PTC agree very well with the values as calculated from the analytical formulas. We also see that the values from the analytical formulas are consistent for all investigated setups, which means that including the other errors does not change the β -function noticeably.

As mentioned above, we expect the simulations to be distributed around the correction

Table 4.2. Summary of amplitude detuning from simulations of the LHC with misaligned nonlinear correctors. Given are the mean and standard deviation of the 3000 realizations per simulation setting. The settings differ in the orders of field errors, as combinations of sextupole (S), octupole (O), decapole (D), and dodecapole (T) errors were applied in the triplets. In addition, coupling correction (CC) was performed after misalignments in one setup. Detuning was evaluated either from PTC simulations (**red**, top) or analytical feed-down calculations (**blue**, bottom). The detuning coefficients have been abbreviated by $Q_{a,b} = \partial Q_a / \partial(2J_b)$.

		SODT [10^3 m^{-1}]	DT [10^3 m^{-1}]	SO [10^3 m^{-1}]	S [10^3 m^{-1}]	O [10^3 m^{-1}]	SODT w/ CC [10^3 m^{-1}]
Beam 1	$Q_{x,x}$	-0.02 ± 2.72 0.00 ± 0.53	0.00 ± 0.53 0.00 ± 0.53	-0.03 ± 2.68 —	-0.04 ± 0.05 —	0.08 ± 0.81 —	0.08 ± 0.99 0.00 ± 0.53
	$Q_{y,y}$	-0.02 ± 5.62 0.00 ± 0.52	0.00 ± 0.52 0.00 ± 0.52	-0.02 ± 5.60 —	-0.02 ± 0.05 —	0.15 ± 0.81 —	0.14 ± 1.01 0.00 ± 0.52
	$Q_{x,y}$	0.52 ± 8.13 0.00 ± 0.68	0.00 ± 0.68 0.00 ± 0.68	0.56 ± 8.10 —	0.06 ± 0.07 —	0.27 ± 1.16 —	0.29 ± 1.46 0.00 ± 0.68
Beam 2	$Q_{x,x}$	-0.13 ± 6.02 0.00 ± 0.52	0.00 ± 0.52 0.00 ± 0.52	-0.12 ± 5.99 —	-0.05 ± 0.06 —	0.09 ± 0.50 —	0.08 ± 0.63 0.00 ± 0.52
	$Q_{y,y}$	-0.02 ± 5.62 0.00 ± 0.53	0.00 ± 0.53 0.00 ± 0.53	0.02 ± 2.86 —	-0.06 ± 0.06 —	0.16 ± 0.30 —	0.14 ± 0.57 0.00 ± 0.53
	$Q_{x,y}$	0.62 ± 8.96 0.00 ± 0.68	0.00 ± 0.68 0.00 ± 0.68	0.62 ± 8.92 —	0.11 ± 0.12 —	0.26 ± 1.03 —	0.29 ± 1.07 0.00 ± 0.68

Table 4.3. Summary of amplitude detuning from simulations of the HL-LHC with misaligned nonlinear correctors. Given are the mean and standard deviation of the 3000 realizations per simulation setting. The settings differ in the orders of field errors, as combinations of sextupole (S), octupole (O), decapole (D), and dodecapole (T) errors were applied in the triplets. Detuning was evaluated either from PTC simulations (**red**, top) or analytical feed-down calculations (**blue**, bottom). The detuning coefficients have been abbreviated by $Q_{a,b} = \partial Q_a / \partial(2J_b)$.

		SODT [10^3 m^{-1}]	DT [10^3 m^{-1}]	SO [10^3 m^{-1}]	S [10^3 m^{-1}]	O [10^3 m^{-1}]
Beam 1	$Q_{x,x}$	-0.03 ± 1.57 0.02 ± 1.57	0.02 ± 1.56 0.02 ± 1.56	-0.06 ± 0.13 —	-0.07 ± 0.11 —	0.01 ± 0.06 —
	$Q_{y,y}$	-0.05 ± 1.58 0.01 ± 1.58	0.01 ± 1.58 0.01 ± 1.58	-0.06 ± 0.12 —	-0.07 ± 0.11 —	0.01 ± 0.04 —
	$Q_{x,y}$	0.02 ± 1.74 -0.02 ± 1.74	-0.02 ± 1.73 -0.02 ± 1.73	0.15 ± 0.26 —	0.13 ± 0.21 —	0.01 ± 0.14 —
Beam 2	$Q_{x,x}$	-0.07 ± 1.59 -0.01 ± 1.58	-0.01 ± 1.58 -0.01 ± 1.58	-0.07 ± 0.16 —	-0.07 ± 0.12 —	0.01 ± 0.11 —
	$Q_{y,y}$	-0.08 ± 1.56 -0.02 ± 1.56	-0.02 ± 1.56 -0.02 ± 1.56	-0.07 ± 0.14 —	-0.08 ± 0.13 —	0.01 ± 0.04 —
	$Q_{x,y}$	0.18 ± 1.76 0.02 ± 1.73	0.02 ± 1.73 0.02 ± 1.73	0.17 ± 0.25 —	0.15 ± 0.24 —	0.02 ± 0.06 —

without misalignments when using the analytical formula, due to the zero-mean nature of the feed-down. We can see that this is true for the calculated values for the LHC

(Table 4.2), while there is a very small (10 m^{-1} to 20 m^{-1}) mean detuning for the HL-LHC. This might stem from the finite sample size, and agrees with the values from PTC-twiss when only decapole and dodecapole (DT) values are involved. Therefore, any larger shift in the mean, as observed on misalignments when including sextupole and octupole errors, comes from feed-down of these errors to the linear optics. In fact, any influence of the misalignments of sextupole and octupole correctors must come from the change in the linear optics, as the orbit does not factor into their contribution to amplitude detuning. We see that this is rather strong in the LHC, while for the HL-LHC PTC and analytical equations are still very much in line, apart from deviations in the mean of up to 60 m^{-1} and a stronger influence in Beam 2 $\partial Q_x / \partial (2J_y)$, yet still less than in the LHC. Some of the detuning in the LHC can be mitigated by applying coupling correction before each PTC evaluation, yet the remaining standard deviation is still twice the expected detuning from feed-down, and the means are also non-zero. Confusingly, simulations with sextupole errors (S) or octupole errors (O) alone do not show these large spreads.

Feeddown at $\beta^* = 30 \text{ cm}$

After this initial simulation showed an approximately threefold increase in amplitude detuning spread going from LHC at $\beta^* = 30 \text{ cm}$ to HL-LHC $\beta^* = 20 \text{ cm}$, the initial target for run 4 (at the time of publishing: 15 cm), it was decided to repeat the HL-LHC simulations at $\beta^* = 30 \text{ cm}$, to have a direct comparison between the two machines with similar optics. As the detuning from PTC and from analytical calculations agreed very well, only the latter were repeated, with decapole and dodecapole errors applied. The summarized results can be found in Table 4.4. Here only the standard deviation of the distribution is given, as the mean is within $\pm 0.01 \times 10^3 \text{ m}^{-1}$ in all cases. The flat-orbit measurement from MD3311 [43] is also given for comparison, which is also illustrated in Fig. 4.1. As the LHC only has b_6 correctors, K_6L is the only contributor. In the HL-LHC, on the other hand, the main contributors are K_5L and J_5L , while the dodecapole contributions are much smaller, as is expected from the power of the misalignments as shown in Eqs. (4.1a) to (4.1d).

There is a $\approx 40\%$ increase of detuning spread in the direct terms and a $\approx 30\%$ increase in the cross terms when going from LHC to HL-LHC. Interestingly, the K_6 contribution, which is responsible for all the detuning seen in the LHC, is a factor of 10 smaller in the HL-LHC when using directly the errors from the 2015 WISE table. In a slightly modified simulation, the systematic part of the MQXF b_6 errors was manually increased from the values in the error table, to -4 , as this is expected to be the next reference value. In this case, the detuning from K_6 quadruples, but still remains less than half the value in the LHC. In total this change of b_6 increases the spread of the detuning only by around 4% , and is hence not of larger importance at these small values.

Compared to the detuning currently present in the machine, both of these spreads are still not only within the tolerable range but even barely at the minimum accuracy of the measurement. It is therefore questionable if the influence of the misalignments can be observed in amplitude detuning measurements at all. This point is further illustrated in

Table 4.4. Simulation summary for corrector misalignments from feed-down calculations for decapole and dodecapole fields to K_4L for LHC and HL-LHC at $\beta^* = 30$ cm. Values are given as the standard deviation of the distribution from misaligning realizations. Sums are calculated over individual simulations, and are hence the root sum square of the value of the field contributions. For reference also the latest detuning measurement at flat-orbit from MD3311 is given. The detuning coefficients have been abbreviated by $Q_{a,b} = \partial Q_a / \partial(2J_b)$.

		Beam 1					Beam 2	
		Sum	K_5L	J_5L	K_6L	J_6L	Sum	
		[10^3 m $^{-1}$]	[10^3 m $^{-1}$]	[10^3 m $^{-1}$]	[10^3 m $^{-1}$]	[10^3 m $^{-1}$]	[10^3 m $^{-1}$]	
$Q_{x,x}$	LHC	± 0.53	–	–	± 0.53	–	± 0.52	
	HL-LHC	± 0.69	± 0.43	± 0.55	± 0.05	± 0.02	± 0.70	
	HL-LHC ($b_6 = -4$)	± 0.72	± 0.43	± 0.55	± 0.22	± 0.02	± 0.73	
	LHC (MD3311)	0.8 ± 0.5					-7.5 ± 0.5	
$Q_{y,y}$	LHC	± 0.52	–	–	± 0.52	–	± 0.53	
	HL-LHC	± 0.70	± 0.48	± 0.52	± 0.05	± 0.02	± 0.69	
	HL-LHC ($b_6 = -4$)	± 0.73	± 0.48	± 0.52	± 0.22	± 0.02	± 0.72	
	LHC (MD3311)	-3 ± 1					6 ± 1	
$Q_{x,y}$	LHC	± 0.69	–	–	± 0.69	–	± 0.69	
	HL-LHC	± 0.77	± 0.51	± 0.59	± 0.05	± 0.02	± 0.77	
	HL-LHC ($b_6 = -4$)	± 0.79	± 0.51	± 0.59	± 0.24	± 0.02	± 0.79	
	LHC (MD3311)	8 ± 28					-2 ± 1	

Fig. 4.1, where the distribution is shifted to the mean of the amplitude detuning without the misalignments, and in Fig. 4.2, in which instead of showing the full distribution, only the standard deviation is shown. In these two figures (Figs. 4.1 and 4.2), we see how small the spread from misalignments is for all detuning terms in both beams and machines, compared to the measurement data from MD3311 [43].

Comparison with MO powering

In the LHC, one of the main sources, but also regulators, for amplitude detuning are the Landau Octupoles (MO) [195]. It is therefore common to compare expected or measured amplitude detuning to the detuning coming from the MOs, giving the values in equivalent MO-powering.

Figure 4.3 shows that the amplitude detuning from MO-powering at 570 A is more than 30 times larger in the direct terms and more than 10 times larger in the cross terms for both machine configurations. This further underlines, that corrector misalignment contributions to amplitude detuning are very small. In fact, their equivalent MO powering is 2 A in the direct terms and 4 A in the cross term (see Table C.1).

Comparison between IPs

A quick check of how the amplitude detuning is distributed between the IPs has been performed on the simulation results. As is to be expected from the β -functions in the

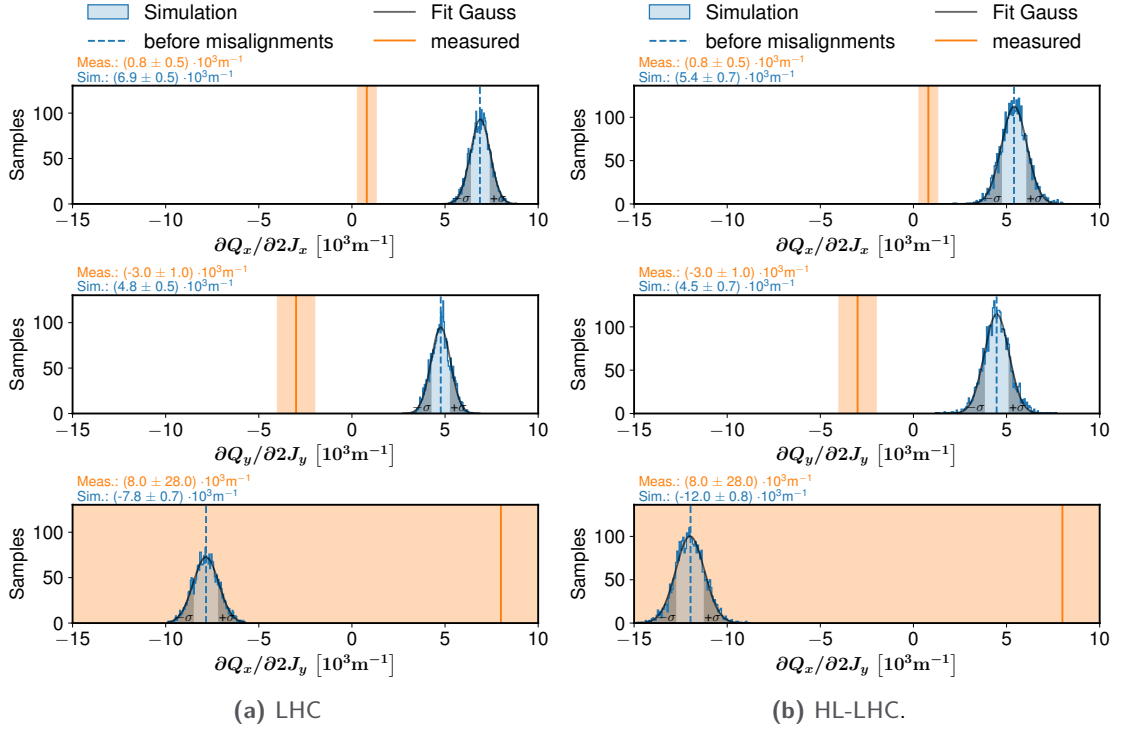


Figure 4.1. Amplitude detuning calculated from corrector misalignments from feed-down to K_4 for Beam 1 in the LHC and in the HL-LHC at $\beta^* = 30$ cm, the latter with MQXF $b_6 = -4$. The distributions are shifted to the detuning value before misalignments and compared to measurement from MD3311 [43].

machines, almost all seen amplitude detuning stems from IP1 and IP5, while IP2 and IP8 contribute very little. The contributions from IP1 and IP5 are skewed towards IP5 for the direct y term and the cross term in the LHC, as well as the direct x term in the HL-LHC.

A detailed summary of the results is shown in Table 4.5

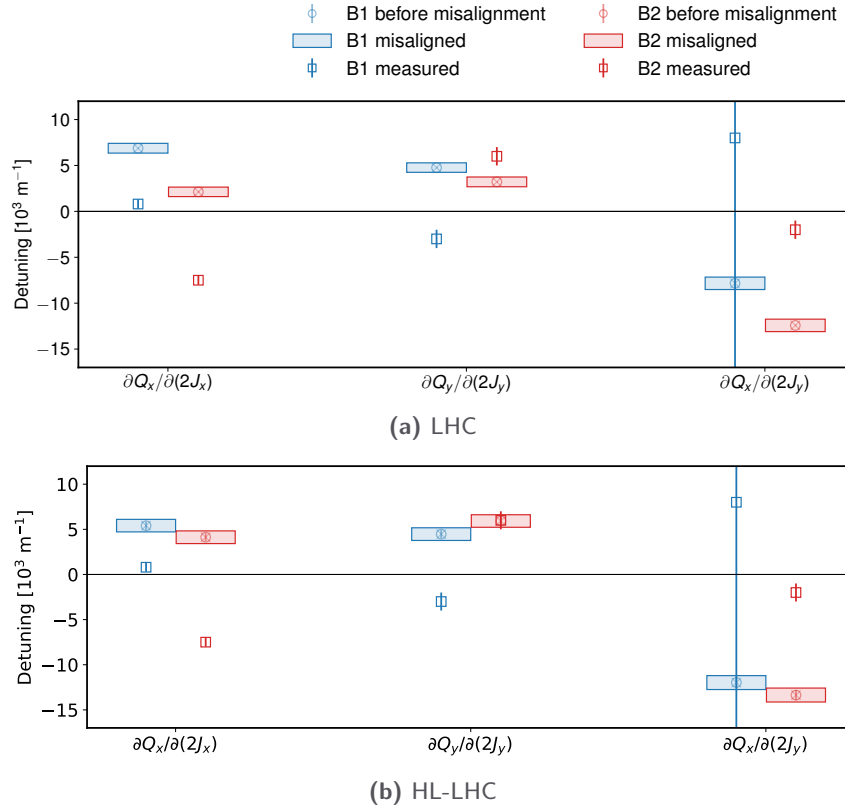


Figure 4.2. Results from the detuning simulations with corrector misalignments at $\beta^* = 30$ cm. The simulated detuning before misalignments is shown as circles with error-bars indicating the spread over the simulation results. The mean and standard deviation of the detuning from feeddown after corrector misalignments is shown as a rectangle around an x-marker. Also shown, with a square marker and errorbar, is the latest amplitude detuning measurement at flat-orbit [43]. All three detuning terms are presented for both beams (blue and red) and for both machine configurations (LHC and HL-LHC).

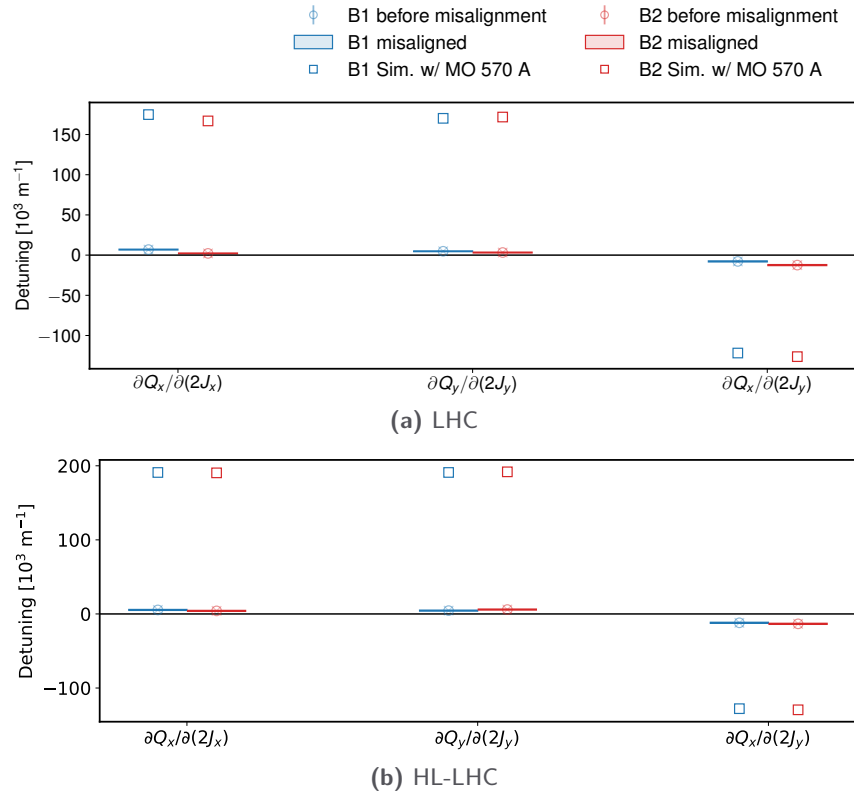


Figure 4.3. Misalignment simulation results presented in the same manner as in Fig. 4.2, but compared to the results from simulations of the same optics, with the Landau octupoles powered to 570 A (square).

Table 4.5. Simulation results split into contributions from different IPs for corrector misalignments from feed-down calculations for decapole and dodecapole fields to K_4L for LHC and HL-LHC at $\beta^* = 30$ cm. Values are given as the standard deviation of the distribution from misaligning realizations. Sums are calculated over individual simulations, and are hence the root sum square of the value of the field contributions. The detuning coefficients have been abbreviated by $Q_{a,b} = \partial Q_a / \partial(2J_b)$.

		Beam 1				
		Sum	IP1+5	IP1	IP5	IP2+8
		[10^3 m^{-1}]	[10^3 m^{-1}]	[10^3 m^{-1}]	[10^3 m^{-1}]	[10^3 m^{-1}]
$Q_{x,x}$	LHC	± 0.53	± 0.53	± 0.40	± 0.35	± 0.001
	HL-LHC	± 0.69	± 0.69	± 0.43	± 0.55	± 0.001
	HL-LHC ($b_6 = -4$)	± 0.72	± 0.72	± 0.46	± 0.56	± 0.001
$Q_{y,y}$	LHC	± 0.52	± 0.52	± 0.17	± 0.49	± 0.005
	HL-LHC	± 0.70	± 0.70	± 0.50	± 0.50	± 0.005
	HL-LHC ($b_6 = -4$)	± 0.73	± 0.73	± 0.53	± 0.52	± 0.005
$Q_{x,y}$	LHC	± 0.69	± 0.68	± 0.39	± 0.55	± 0.000
	HL-LHC	± 0.77	± 0.77	± 0.52	± 0.57	± 0.005
	HL-LHC ($b_6 = -4$)	± 0.79	± 0.79	± 0.54	± 0.59	± 0.005

4.3.2 Triplet Misalignments

Results

Table 4.6. Simulation summary for triplet misalignments from feed-down calculations for decapole and dodecapole fields to K_4L for LHC and HL-LHC at $\beta^* = 30$ cm. Values are given as the standard deviation of the distribution from misaligning realizations. Sums are calculated over individual simulations, and are hence the root sum square of the value of the individual contributions. For reference also the latest detuning measurement at flat-orbit from MD3311 is given. The detuning coefficients have been abbreviated by $Q_{a,b} = \partial Q_a / \partial (2J_b)$.

		Beam 1					Beam 2
		Sum	K_5L	J_5L	K_6L	J_6L	Sum
		$[10^3 \text{ m}^{-1}]$	$[10^3 \text{ m}^{-1}]$	$[10^3 \text{ m}^{-1}]$	$[10^3 \text{ m}^{-1}]$	$[10^3 \text{ m}^{-1}]$	$[10^3 \text{ m}^{-1}]$
$Q_{x,x}$	LHC	± 2.73	± 2.52	± 0.90	± 0.26	± 0.47	± 2.88
	HL-LHC	± 0.46	± 0.32	± 0.32	± 0.03	± 0.01	± 0.45
	HL-LHC ($b_6 = -4$)	± 0.46	± 0.32	± 0.32	± 0.08	± 0.01	± 0.46
	LHC (MD3311)	0.8 ± 0.5					-7.5 ± 0.5
$Q_{y,y}$	LHC	± 2.88	± 1.23	± 2.59	± 0.32	± 0.10	± 2.73
	HL-LHC	± 0.45	± 0.31	± 0.33	± 0.03	± 0.01	± 0.46
	HL-LHC ($b_6 = -4$)	± 0.46	± 0.31	± 0.33	± 0.08	± 0.01	± 0.46
	LHC (MD3311)	-3 ± 1					6 ± 1
$Q_{x,y}$	LHC	± 2.64	± 2.00	± 1.69	± 0.33	± 0.35	± 2.64
	HL-LHC	± 0.48	± 0.33	± 0.34	± 0.03	± 0.01	± 0.48
	HL-LHC ($b_6 = -4$)	± 0.48	± 0.33	± 0.34	± 0.09	± 0.01	± 0.48
	LHC (MD3311)	8 ± 28					-2 ± 1

As discussed in [Section 4.2.1](#), when misaligning the triplets the simulation becomes unstable, due to the high β -functions at their locations and their strong magnetic fields, meaning that PTC-twiss does not converge anymore. For this reason, all results in this section are obtained from the manual feed-down calculations only.

For the same reasons, i.e. high β -functions and strong fields, it is expected that triplet misalignments will have a much larger impact on amplitude detuning as compared to misalignments from the correctors. While the results in [Table 4.6](#) show that this is true for the LHC, the HL-LHC shows only about 20% of the detuning of the LHC. So while the detuning spread of the LHC from triplet misalignments is a factor of 4-5 that of corrector misalignments, the HL-LHC shows only 60% – 70% of that detuning. While differences in the assigned errors, as explored in [Section 4.3.2](#), are one explanation for this result, another point that factors into this is the split of Q1 and Q3 into two magnets in the HL-LHC ([Fig. 2.10](#)): these are misaligned independently. So while the A and B contributions from the two magnets of the HL-LHC Q1 and Q3 can be opposing, the full length of the single Q1 and Q3 magnets show constructive detuning contributions in the LHC. This illustrated in [Fig. 4.4](#).

[Table 4.7](#) shows the different contributions from the IPs, which, following the same argument as in [Section 4.3.1](#), are almost exclusively coming from IP1 and IP5. Here, the

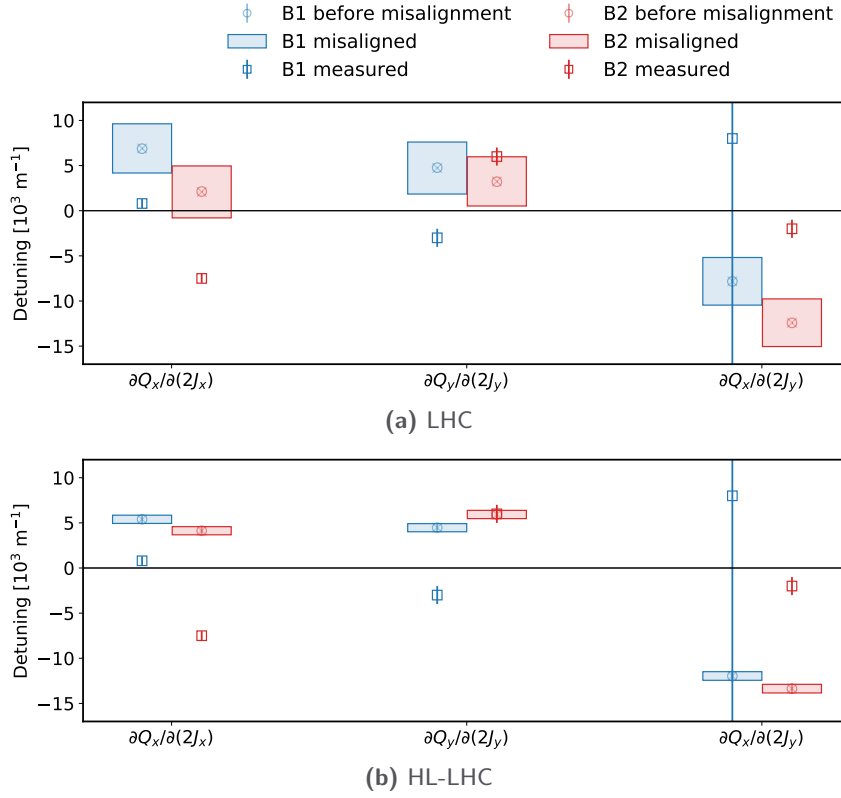


Figure 4.4. Results from the detuning simulations with triplet misalignments at $\beta^* = 30$ cm. The simulated detuning before misalignments is shown as circles with error-bars indicating the spread over the simulation results. The mean and standard deviation of the detuning from feeddown after triplet misalignments is shown as a rectangle around an x-marker. Also shown, with a square marker and errorbar, is the latest amplitude detuning measurement at flat-orbit [43]. All three detuning terms are presented for both beams (blue and red) and for both machine configurations (LHC and HL-LHC).

direct x term in the LHC is dominant in IP5, while the direct y term is more prominent in IP1. In the HL-LHC, IP1 and IP5 contributions are split evenly.

Similar arguments about the insignificance of these values as compared to the equivalent current of the MOs, (see also [Section 4.3.1](#)) can be made, even for the larger detuning in the LHC - which is now at around 12 A (10 A) equivalent MO current at flattop (at $\beta^* = 30$ cm) in the direct terms and 15 A (13 A) in the cross term (see [Table C.1](#)).

Table 4.7. Simulation results for triplet misalignments from feed-down calculations for decapole and dodecapole fields to K_4L for LHC and HL-LHC at $\beta^* = 30$ cm, split into contributions from different IPs. Given are the standard deviations of the distribution from misaligning and error realizations. Sums are calculated over individual simulations, and are hence the root sum square of the value of the individual contributions. The detuning coefficients have been abbreviated by $Q_{a,b} = \partial Q_a / \partial (2J_b)$.

		Beam 1				
		Sum	IP1+5	IP1	IP5	IP2+8
		$[10^3 \text{ m}^{-1}]$	$[10^3 \text{ m}^{-1}]$	$[10^3 \text{ m}^{-1}]$	$[10^3 \text{ m}^{-1}]$	$[10^3 \text{ m}^{-1}]$
$Q_{x,x}$	LHC	± 2.73	± 2.73	± 1.03	± 2.49	± 0.02
	HL-LHC	± 0.46	± 0.46	± 0.32	± 0.33	± 0.02
	HL-LHC ($b_6 = -4$)	± 0.46	± 0.46	± 0.32	± 0.33	± 0.02
$Q_{y,y}$	LHC	± 2.88	± 2.88	± 2.52	± 1.49	± 0.02
	HL-LHC	± 0.45	± 0.45	± 0.32	± 0.32	± 0.02
	HL-LHC ($b_6 = -4$)	± 0.46	± 0.46	± 0.33	± 0.33	± 0.02
$Q_{x,y}$	LHC	± 2.64	± 2.64	± 1.77	± 1.99	± 0.02
	HL-LHC	± 0.48	± 0.48	± 0.34	± 0.34	± 0.02
	HL-LHC ($b_6 = -4$)	± 0.48	± 0.48	± 0.34	± 0.34	± 0.02

Error Distribution

As the difference in spread between LHC and HL-LHC is quite significant, its validity was checked by comparing the integrated field errors between the two machines directly (instead of their amplitude detuning). As expected, in almost all cases, the LHC values were spread over higher, sometimes much higher, values. An example of these distributions can be seen in Fig. 4.5, where the Q1 (MQXA), the triplet quadrupole directly right of IP5, is shown. For the HL-LHC the two parts of the Q1 (MQXFA.A1R5 and MQXFA.B1R5) have been combined to allow comparison with the LHC Q1.

A full overview over the distributions in the triplets can be found in Appendix C.3.

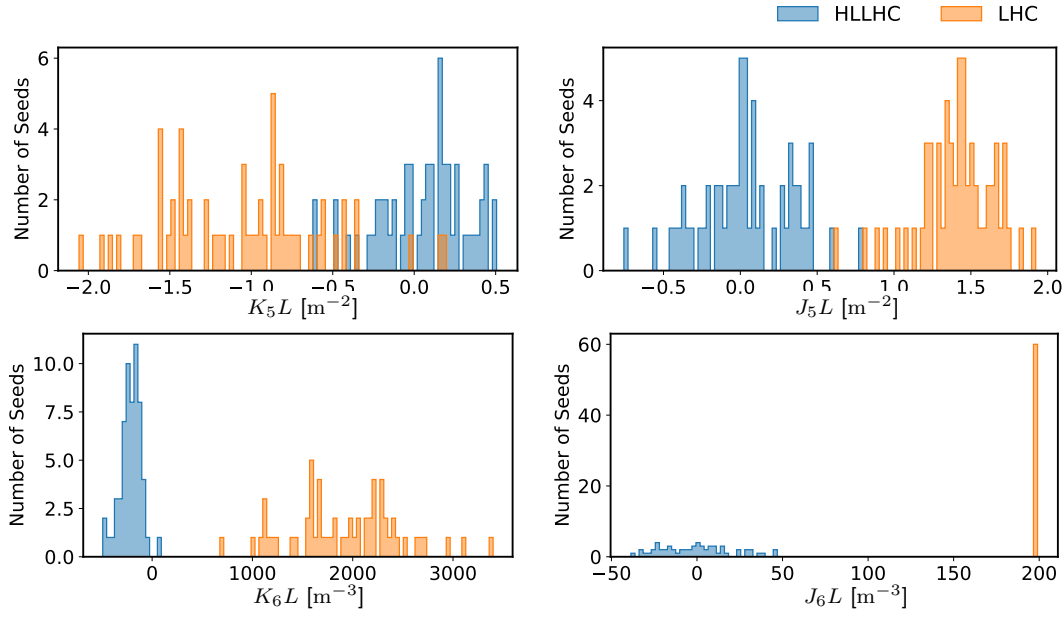


Figure 4.5. Comparison in the integrated field error distributions between LHC MQXA.1R5 (orange) and HL-LHC (systematic $b_6 = -4$) MQXFA.(A+B)1R5 (blue). For (A+B) the contributions from the two magnets have been summed.

4.4 Conclusion

Simulations as described in [Section 4.2](#) for LHC and HL-LHC have been performed to investigate feed-down to amplitude detuning from misalignments of the nonlinear correctors in the IRs, as well as from misalignments of the triplet magnets themselves. The results in [Section 4.3](#) show a detuning spread around the non-misaligned value. The two machine configurations show very similar detuning in case of corrector misalignments, yet misaligning the triplets significantly increases the amplitude detuning in the LHC, while the values in the HL-LHC decrease slightly. For all investigated cases, detuning stems almost completely from IP1 and IP5, whereas IP2 and IP8 contribute only marginally. The detuning spread is very small when compared to other sources of amplitude detuning, e.g. from Landau Octupole powering, in all investigated cases and hence, is of no great concern for the operation of the machines.

Updated Findings. After the studies in this chapter had been finalized, it was found that due to the quadratic nature of [Eq. \(4.1c\)](#) the results here presented are only valid at flat-orbit. It has been found that, for the LHC with full crossing scheme, the detuning from feed-down is dominated by the influence of dodecapoles, with the same misalignments then causing a detuning spread 10–15 times larger than the spread seen in this chapter (see [Chapter 7](#)). The current study should therefore be revisited for full-crossing setups in both LHC and HL-LHC as well as for the nominal β^* of 15 cm.

5 Corrections of Systematic Normal Decapole Field Errors in the HL-LHC Separation/Recombination Dipoles

Note. The main results of this chapter have been in a condensed form presented by me at IPAC in 2022 and published in their proceedings [196].

Acknowledgement. F.F. Van der Veken[©] conducted precursor studies [197] and made his MAD-X simulation setup available to me, without which this study would not have been possible. His help has further been indispensable in ironing out the mistakes I made in transforming his template into my `cpymad` evaluation setup.

5.1 Introduction

5.1.1 Motivation

During magnetic measurements it was found out that the b_5 values of the new D2 recombination dipoles (aka MBRD) in the HL-LHC [22, 25] might have a systematic b_5 component of up to 11 units [198]. Based on earlier studies it was projected that the current correction scheme [7, 41] will not be able to compensate these and that this will lead to a degradation of 0.5σ to 1σ in the DA [22, 199]. In fact, contributions from D2 have not been incorporated into the correction scheme, as D2 is a dual aperture magnet, while the b_5 corrector is single aperture. Additionally, it is placed far away from the corrector package (CP, see Figs. 2.10 and 5.1 and [22]), leading to a large phase advance, lower β -function and different orbit. Hence it was uncertain, if the D2 inclusion would be beneficial to DA (see [199]). It was also investigated, whether the hitherto used RDT f_{5000} was the optimal choice (see Section 5.1.2). Of further concern was the interplay between D2 and the other separation dipole, D1 (aka MBX). The new D1 is expected to have a systematic b_5 component of 6 units to 7 units [198]. Due to the opposite beam bending directions (see Fig. 5.1) in the two magnets, their contribution to the resonance driving terms will partly compensate. This is explored in Section 5.2. Tracking simulations have been run to estimate the influence of the measured errors and to explore for their correction. The setup is described in Section 5.3 and their findings are presented in Section 5.4.

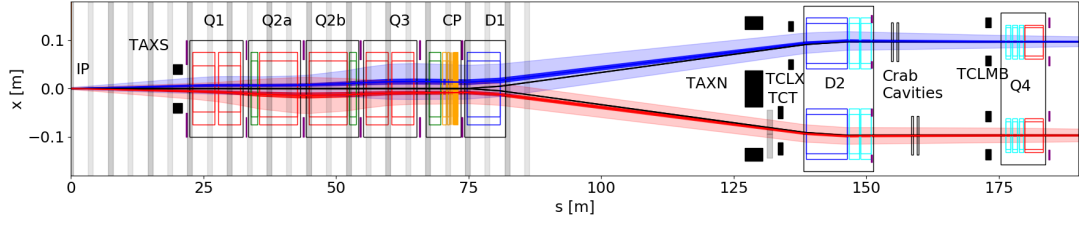


Figure 5.1. HL-LHC IR layout between the IP and Q4. [22]

5.1.2 Closest Resonances

At collision energy, the HL-LHC has a design working point with fractional tunes of $Q_x = 0.31, Q_y = 0.32$. As seen in the tune diagram of Fig. 5.2, the closest normal decapole resonances are $(1, -4)$, and $(5, 0)$, which correspond to the RDTs f_{1004} and f_{5000} , respectively. $(3, -2)$ driven by f_{3002} has also been partly investigated. With $(1, -4)$ being the closest to the working point, one might assume that its influence might be the strongest and its correction the most beneficial. As the current implementation [194] on the other hand, chooses f_{5000} as correction target, its suitability was confirmed and the correction compared to targeting f_{1004} . The details are presented in the Section 5.4.

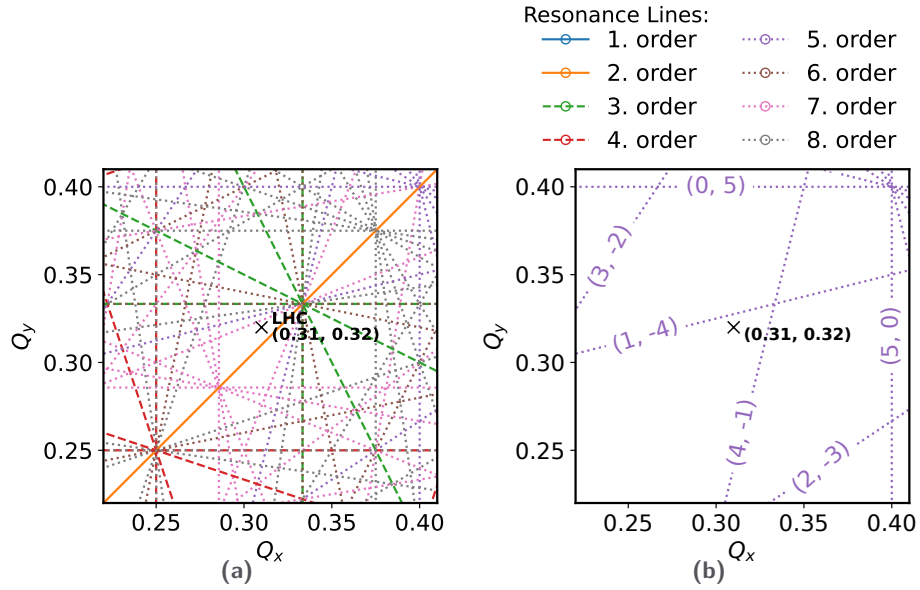


Figure 5.2. Tune diagram with resonances a) up to 8th order and b) only of 5th order.

5.1.3 Phase-Advance to D2

There are two main reasons, why it was not clear if D2 can be included into the nonlinear corrections in the IR: For one, it is a dual-aperture magnet and therefore only the

average error between the two apertures can be corrected. The other reason is the phase advance from the IP. One of the assumptions of the correction algorithm (see [Section 3.2](#)) used in this study is a constant phase advance on each side of the IP and a π -phase advance between the sides. Due to the very high β -functions around the triplets ([Fig. 5.3](#)), this condition should be well met until D1. As seen in [Fig. 5.4](#), D2 ([mbrd.4r1.b1](#)) has in fact the largest phase advance from the corrector ([mcdfx.3r1](#)) than any other shown magnet.

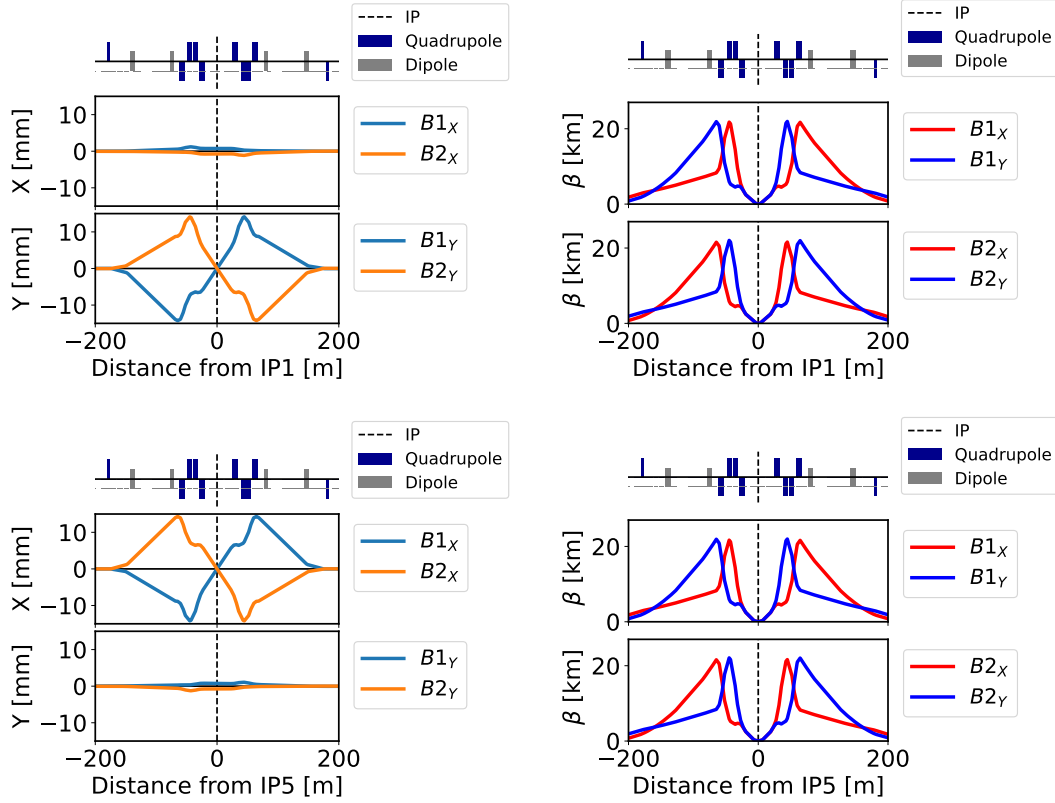


Figure 5.3. Orbit and β -function of the nominal HL-LHC around IP1 and IP5.

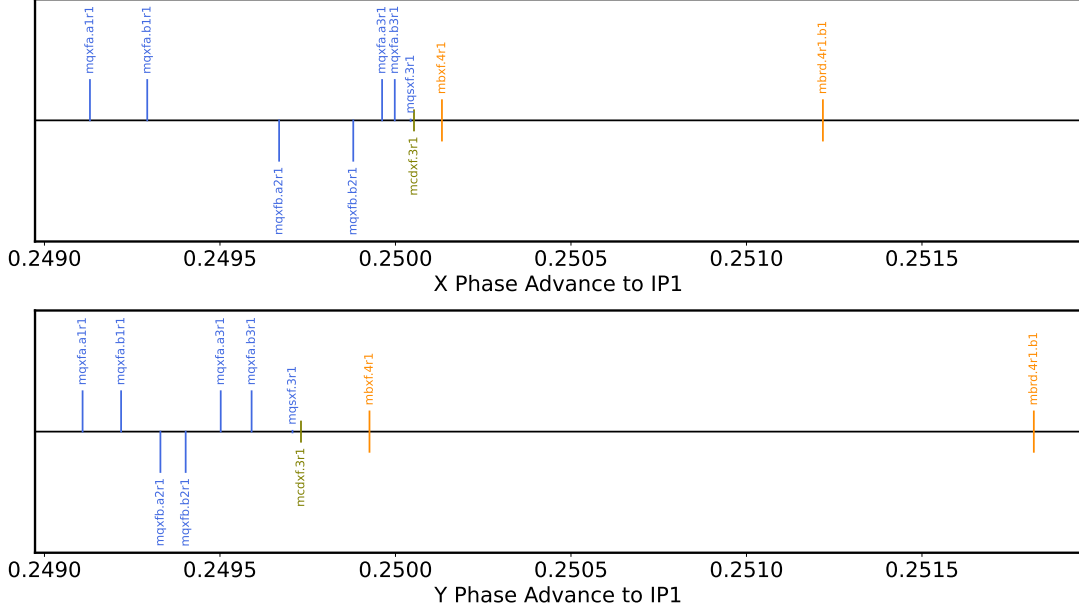


Figure 5.4. Phase advance (in units of 2π) of triplets, corrector, D1 and D2 on the right hand side of IP1 for HL-LHC 15 cm optics.

5.2 Compensation

One part of the investigation was to check, whether the D1 and D2 b_5 errors might compensate each other. The reasoning behind this is, that D1 and D2 have opposite bending angles (see Fig. 5.1). In fact, they have an integrated dipole strength value $K_1 L$ of equal magnitude, but with opposite signs

$$K_1^{D1} L = -K_1^{D2} L, \quad (5.1)$$

even though the magnets have different lengths L and reference radii r_{ref} (Table 5.1). In the dipoles, conversion from the field components to the magnetic strengths K_n is, according to Eqs. (2.6) and (2.10), with K_1 as the main field component, i.e. $b_1 = 1$:

$$K_n = K_1 \cdot \frac{b_n}{r_{ref}^{n-1}} \cdot (n-1)!, \quad (5.2)$$

Table 5.1. Magnet length and reference radius for D1 and D2.

		D1	D2
L	[m]	6.27	7.78
r_{ref}	[mm]	50	35

Table 5.2. Approximate ratio of the terms in Eq. (5.3).

	f_{5000}	f_{1004}	f_{3002}
r_{ref} -fraction		4.2^{-1}	
β -fraction	12.8	9.8	8.4
Total	3.1	2.4	2.0

and equivalently for $K_n L$ using $K_1 L$.

From Eq. (3.1) we can check the required ratio between D1 and D2 b_5 for perfect compensation. As the sign of $K_1 L$ changes between D1 and D2 (see Eq. (5.1)), we require the contributions from the magnets to the RDT ($f_{jklm}^{D1/D2}$) to be equal in magnitude to reach compensation of the D1 and D2 b_5 errors:

$$-\frac{f_{jklm}^{D1}}{f_{jklm}^{D2}} = \left(\frac{r_{ref}^{D1}}{r_{ref}^{D2}} \right)^{-4} \cdot \frac{\sum_{w \in D1} \beta_x^{\frac{j+k}{2}}(s_w) \beta_y^{\frac{l+m}{2}}(s_w)}{\sum_{w \in D2} \beta_x^{\frac{j+k}{2}}(s_w) \beta_y^{\frac{l+m}{2}}(s_w)} \cdot \frac{b_5^{D1}}{b_5^{D2}} \stackrel{!}{=} 1 \quad (5.3)$$

The terms were evaluated for the D1 and D2 in IP1 and IP5 separately, yet came to the same results. That a sum of the β -functions left and right is sufficient occurs, as the sign of K_1 in Eq. (5.2) changes between the sides as does the sign of $e^{i\pi n \Theta(s_w - s_{IP})}$ in Eq. (3.1). As the β -function may vary within one magnet, we usually use a thin lattice in MAD-X and sum over the individual slices.

The approximate ratios of the individual terms in Eq. (5.3) for squeezed 15 cm optics are given in Table 5.2. To achieve cancellation of the RDTs within one IP within the approximation described, b_5 of D2 needs to be ≈ 3.1 times larger than b_5 of D1 for f_{5000} and 2.4 times for f_{1004} .

The ratio of the estimated values from [198], i.e. 11 units for D2 b_5 and 6–7 units for D1 b_5 are with factors of ≈ 1.6 – 1.8 too low for total compensation, yet at least some noticeable effect is to be expected.

5.3 Simulation Setup

The DA studies were performed with AutoSix [150], a SixDesk [134] wrapper, on top of a HL-LHC V1.4 MAD-X [42] setup.

5.3.1 MAD-X Setup

The HL-LHC lattice was created from the default sequence and magnetic field errors with crossing angles enabled at their nominal values, and at 7 TeV squeezed to $\beta^* = 15$ cm. Instead of the default error-tables for D2, a special set of errors were loaded with zeroed systematic errors and doubled random errors. The systematic b_5 values of the D1 ($b_{5,sys}^{D1}$) and D2 ($b_{5,sys}^{D2}$) were set to 0 units, 5 units, 10 units or 15 units to cover the

estimated values as given in [198] with some additional margin. Other magnetic errors were realized from 60 different seeds, corresponding to either measurement errors from the WISE-tables [182] or random Gaussian distributed errors, leading to 60 different instances of the ring lattice for each scan point.

The magnetic errors of the main dipoles were corrected with the standard correction tools [200], while the magnetic errors of the insertion region were corrected by the correction script presented in Section 3.2, which allows for the flexibility of targeting different RDTs as needed in this study. The correction is based on the approximate RDT calculation described in [7, 41], and the symmetry between the beams is used as described in Section 3.2.1, meaning the RDTs are always corrected in pairs f_{jklm} and f_{jklm*} (see Eq. (3.8)).

As D1 is already integrated into the standard corrections, only the change of its systematic b_5 field component is considered. D2 has not been integrated into the nonlinear correction, due to its phase-advance and lower β -function (see Section 5.1.3), and it is either included “as a whole” in the corrections or omitted. Apart from the systematic b_5 error of the scan, the modelled D2 contains only random errors distributed over ± 4 units so that any internal compensation would be visible in the seeds, as discussed in [197]. Three different correction scenarios are investigated to distinguish between the origin and effects of the different error sources:

- **D1 b_5 & D2 uncorrected:** The standard corrections of the machine are performed, without the errors under test as described in the next two points.
- **correct D1 b_5 :** As above, the standard corrections are performed, now including the systematic b_5 of D1. D2 is still not corrected.
- **correct D1 b_5 & D2:** D2 is here also fully included into the correction scheme. To calculate the correction the average error between the two apertures is considered.

This workflow is depicted in Figs. 5.5 and 5.6.

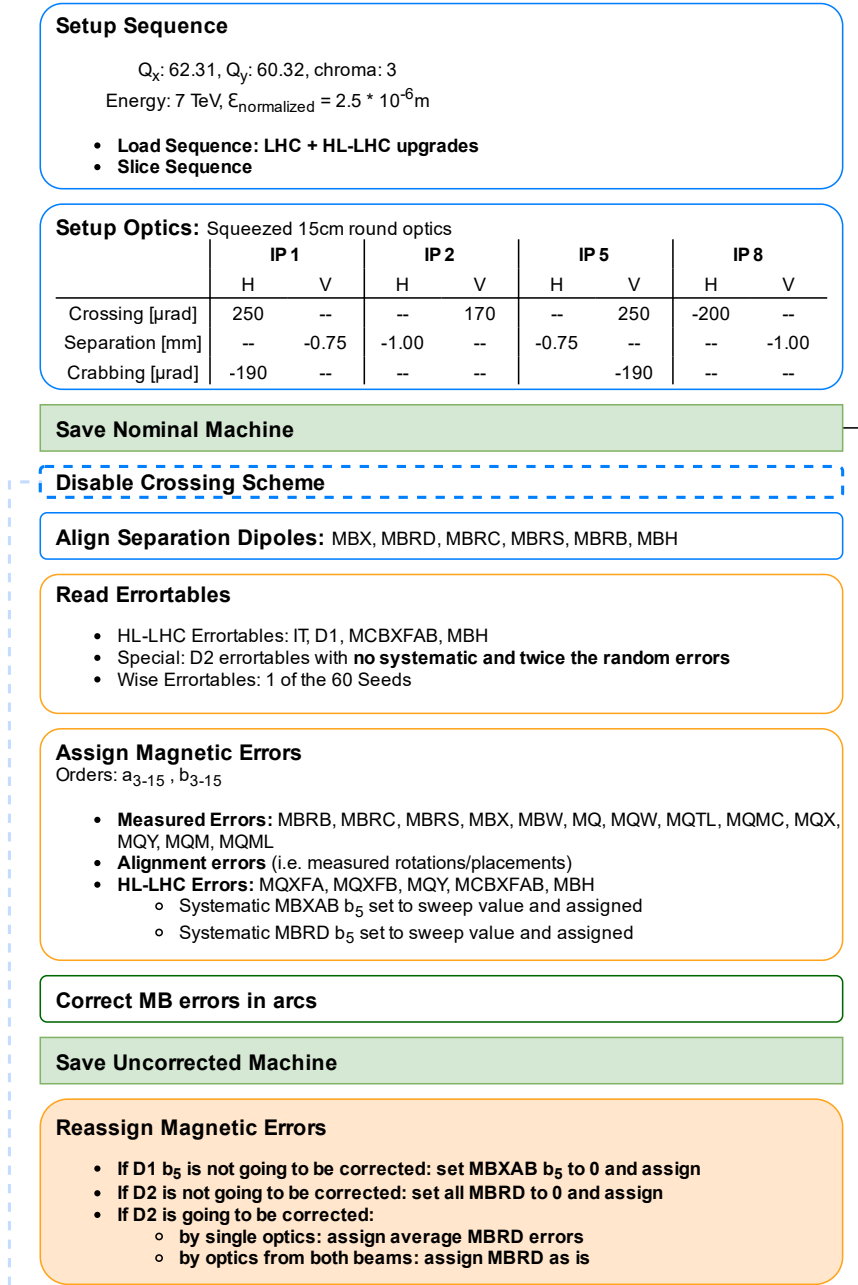


Figure 5.5. MAD-X workflow (part 1).

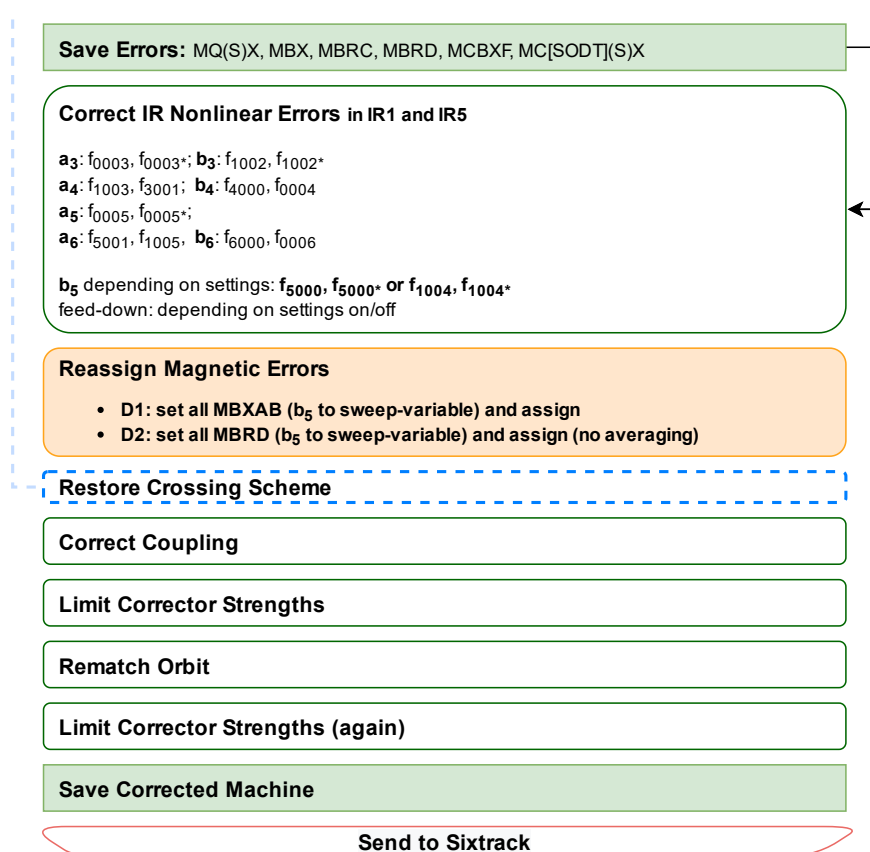


Figure 5.6. MAD-X workflow (part 2).

5.3.2 Tracking Setup

Particles are initialized over 11 angles in the transverse plane for amplitudes between 2σ and 20σ (30σ for flat-orbit) in intervals of 2σ containing 30 particle-pairs each. These are tracked for 100'000 turns with SixTrack [134] on the CERN HTCondor Cluster, after which the surviving particles are registered and their amplitude calculated (via the “ALOST2”-method). The DA per angle can then be found by the largest amplitude of the surviving particles.

Statistics parameters, such as extrema, mean and standard deviation of the DA are gathered over the resulting 60 error instances and angles per scan point and correction scenario.

A mistake had been made in the setup of the energy and normalized emittance as the values for LHC (6.5 TeV and $3.75\mu\text{m}$) have been used instead of the HL-LHC values (7 TeV and $2.5\mu\text{m}$). As these values had only been applied in the tracking part of the simulation, this could be corrected by rescaling the resulting amplitudes by

$$\sqrt{\frac{\varepsilon_{\text{LHC}}}{\varepsilon_{\text{HL-LHC}}}} = \sqrt{\frac{\varepsilon_{n,\text{LHC}} \cdot \beta_{\text{HL-LHC}} \cdot \gamma_{\text{HL-LHC}}}{\varepsilon_{n,\text{HL-LHC}} \cdot \beta_{\text{LHC}} \cdot \gamma_{\text{LHC}}}}. \quad (5.4)$$

ε and ε_n are the emittance and normalized emittance, while β and γ are the relativistic speed ratio and Lorentz factor from the respective energies of the machines.

5.4 Results

5.4.1 D2 Sweep

For both beams, a sweep at 0 units, 5 units, 10 units and 15 units over the systematic b_5 in D2 was performed, while the systematic b_5 in D1 was fixed at 0 units and 7 units. These values were chosen to approximately reflect the magnetic measurements introduced in Section 5.1.1.

Compensation. Sweeping over the systematic b_5 values of D2 shows the expected compensation between $b_{5,sys}^{D1}$ and $b_{5,sys}^{D2}$ as discussed in Section 5.2. Figure 5.7 shows a clear increase in DA with raising the systematic D2 b_5 value when setting the systematic b_5 of D1 to 7 units and no correction (blue). This effect is barely visible after applying corrections (green), where at first only a slight increase in DA is seen with increasing $b_{5,sys}^{D2}$ (until around 5 units), after which the DA decreases again falling by 2σ in minimum DA in Beam 1. In Beam 2, there is a dip in the minimum DA at $b_{5,sys}^{D2} = 5$ units (Fig. 5.7b), which is discussed in Section 5.4.1 and can be attributed to an outlier seed/angle combination. As the mean is similar to Beam 1, I argue that this beam shows a more realistic behavior.

Compensation between D1 and D2 is also perceptible in the values of the applied corrector strengths in Fig. 5.8a: The corrector powering is strongest at $b_{5,sys}^{D2} = 0$ units and decreases with larger $b_{5,sys}^{D2}$. Extrapolating from this plot, one can also estimate the

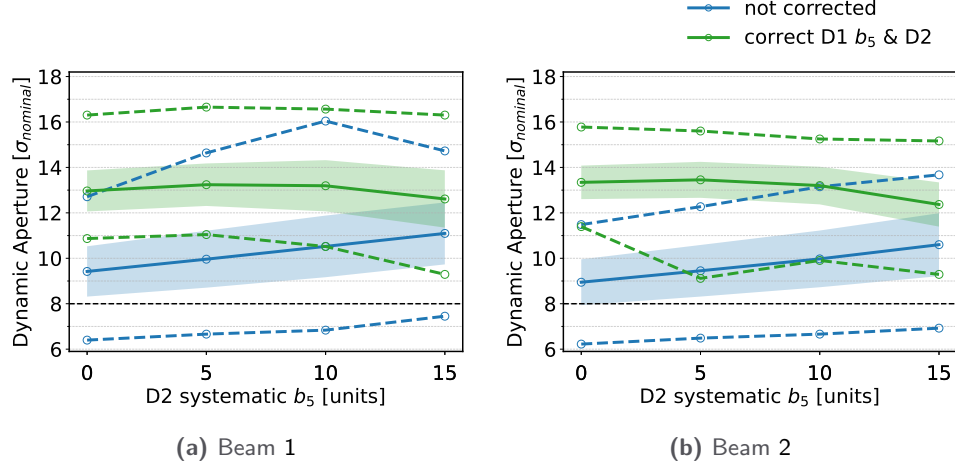


Figure 5.7. DA simulation results for two correction scenarios after corrections (b_5 corrected via f_{5000}) with $b_{5,sys}^{D1} = 7$ units, sweeping over $b_{5,sys}^{D2}$. Thick lines show the mean over all seeds and angles, dashed line the extrema in the same set. The colored area marks the standard deviation.

best compensation for a value of around 20 units, which agrees with the values calculated in Table 5.2. Despite the compensation effect, the minimum DA without correction (dashed blue line at the bottom) is always below the 8σ needed to avoid degradation of beam intensity [22].

In contrast, at zero systematic b_5 in D1, as in the results presented in Figs. 5.9a and 5.9b, the DA is much higher at $b_{5,sys}^{D2} = 0$ units but falls off from there, in the minimum as well as in the mean value. Unsurprisingly, the corrector strengths in Fig. 5.8b show the lowest values at $b_{5,sys}^{D2} = 0$ units and linearly follow its increase.

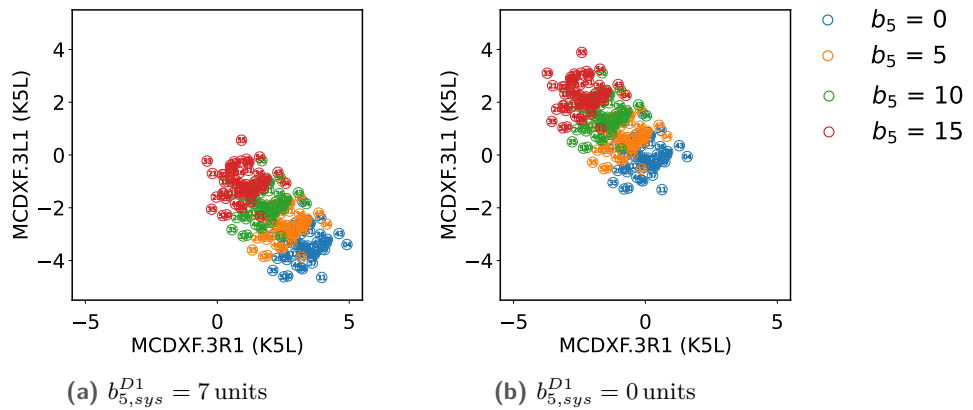


Figure 5.8. Beam 1 b_5 corrector strength in IP1 for different values of $b_{5,sys}^{D2}$, correcting for f_{5000} .

Correction. As in Fig. 5.7, Fig. 5.9 shows the DA over the D2 systematic b_5 .

First checking the influence of the D1 b_5 correction, we look at the cases with these errors present ($b_{5,sys}^{D1} = 7$ units) and compare the DA results without correction in Fig. 5.7 with the corrected cases in Figs. 5.9c and 5.9d. We see that correcting for D1 b_5 (orange data in Figs. 5.9c and 5.9d) increases the DA by up to 4σ without $b_{5,sys}^{D2}$, but we see a steep decrease in DA, losing 1σ (1.5σ) in minimum (mean) DA at the expected error value of $b_{5,sys}^{D2} = 11$ units. In the cases where no D1 b_5 errors are present (Figs. 5.9c and 5.9d), there is obviously no difference if we correct for them or not.

Including now D2 into the corrections (green data in all of Fig. 5.9), we can recover about 1σ in minimum DA (excluding the outliers for Beam 2 at $b_{5,sys}^{D1} = 7$ units and $b_{5,sys}^{D2} = 0$ units and 5 units). As the DA loss is even present at $b_{5,sys}^{D2} = 0$ units in the minimum DA, but not in the mean DA, the loss must therefore be originating from the random errors. The curve of the mean DA is flatter in the “D1 b_5 & D2” corrected cases, compared to the minimum DA, in the investigated range, yet also shows DA loss of around 0.5σ (Figs. 5.9c and 5.9d) to 1σ (Figs. 5.9a and 5.9b) from 10 units to 15 units. On the other hand, there is no change in the mean DA between $b_{5,sys}^{D2} = 0$ units and the expected value at around 11 units. In conclusion, a partial correction of the b_5 in D2 is in fact possible.

Outliers. The only case, in which including D2 correction seem to spoil the DA, is in Beam 2 at $b_{5,sys}^{D1} = 7$ units and $b_{5,sys}^{D2} = 5$ units. Figure 5.10b shows that the mean of each seed at this point is above 12σ , there can therefore be only a few values, that is DA at specific angles, that are responsible for this outlier. And indeed, the angular distribution in Fig. 5.11d shows a single seed with low DA in the horizontal plane (below 30°). The DA for the other cases, which do not show any such extreme outliers, are given for comparison in Figs. 5.11 and 5.12.

A Note about Compensation. Interestingly, Fig. 5.9 also seems to show, that the DA is slightly better in the case **with** some $b_{5,sys}^{D1}$ present in the machine, which can then be slightly compensated by present $b_{5,sys}^{D2}$, in addition to the correction by the correctors, as long as it is about the same value. As the improvement is only marginal, while on the other hand it has been shown that some combinations of errors can lead to a devastating drop in DA, one should not rely on this effect in the real machine and try to operate at the lowest systematic b_5 in D1 and D2 possible.

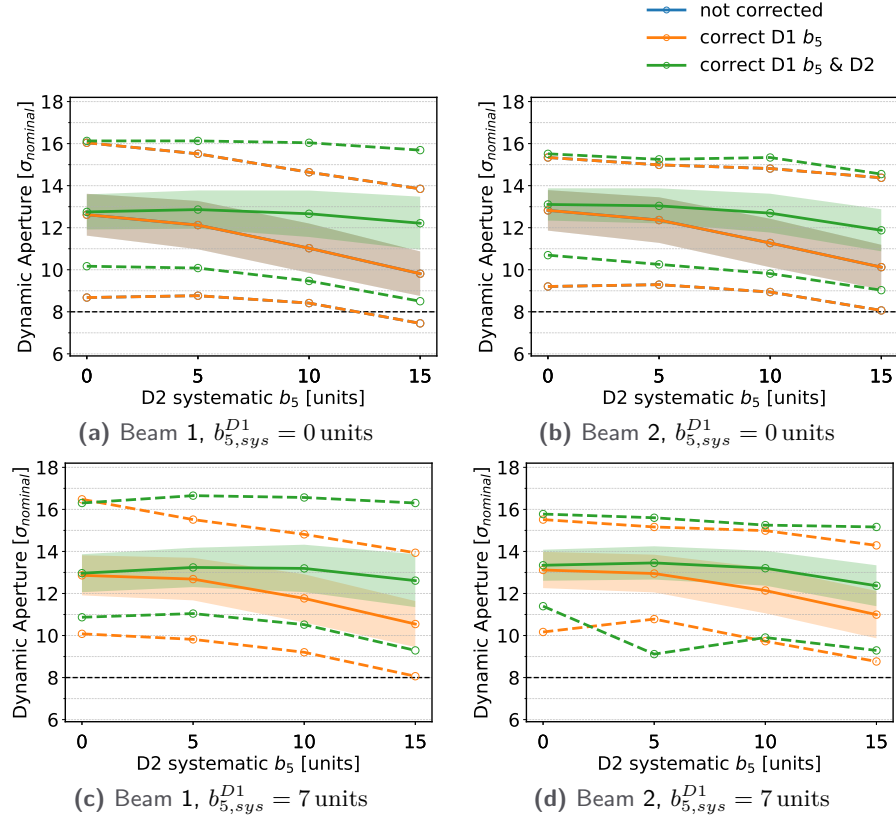


Figure 5.9. DA simulation results for the correction scenarios after corrections (b_5 corrected via f_{5000}) with $b_{5,sys}^{D1} = 0$ units and 7 units, sweeping over $b_{5,sys}^{D2}$. The “not corrected” case is here only shown for $b_{5,sys}^{D1} = 0$ units (for $b_{5,sys}^{D1} = 7$ units see Fig. 5.8) and it is not really visible as, due to $b_{5,sys}^{D1} = 0$ units, it coincides with “correct D1 b_5 ”. Thick lines show the mean over all seeds and angles, dashed line the extrema in the same set. The colored area marks the standard deviation.

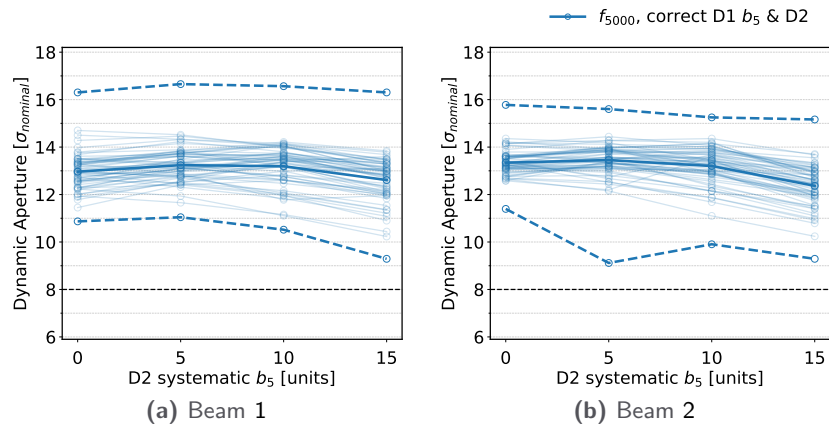


Figure 5.10. DA simulation results per seed, as well as mean (thick) and extrema (dashed), with D1 and D2 b_5 correction (b_5 corrected via f_{5000}) and $b_{5,sys}^{D1} = 7$ units, sweeping over $b_{5,sys}^{D2}$.

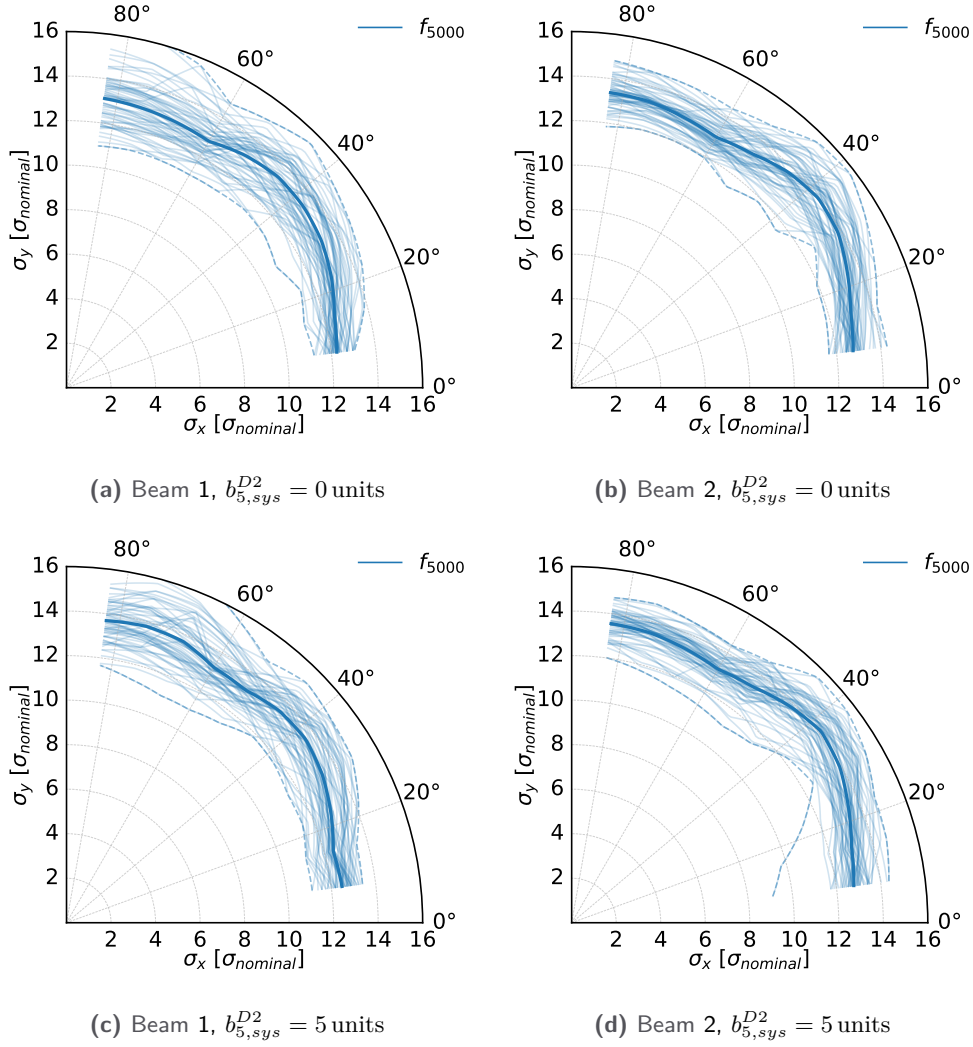


Figure 5.11. DA simulation results in both planes, with D1 b_5 and D2 correction (b_5 corrected via f_{5000}) and $b_{5,sys}^{D1} = 7$ units, $b_{5,sys}^{D2} = 0$ units and 5 units. The thick line shows the mean, the dashed line the extrema, over all seeds per angle. Individual seeds are shown as semi-transparent lines.

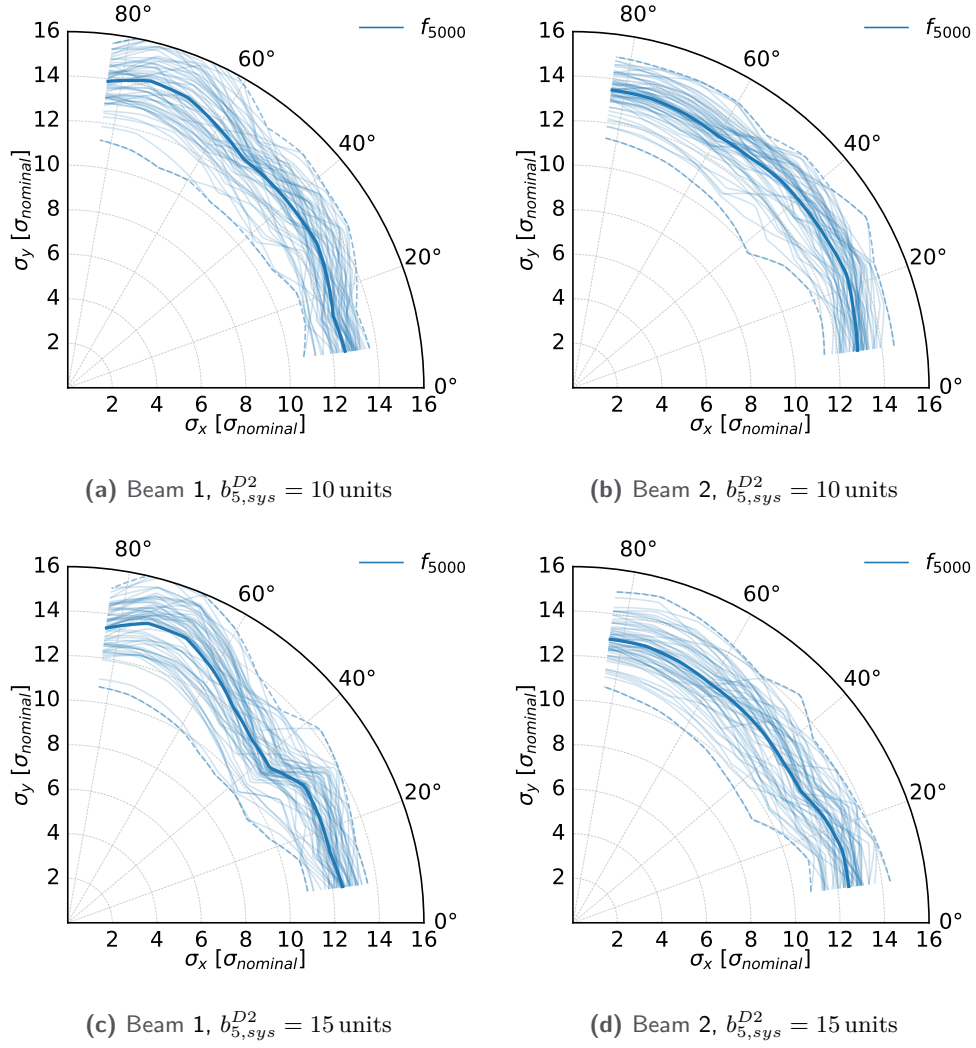


Figure 5.12. DA simulation results in both planes, with D1 b_5 and D2 correction (b_5 corrected via f_{5000}) and $b_{5,sys}^{D1} = 7$ units, $b_{5,sys}^{D2} = 10$ units and 15 units. The thick line shows the mean, the dashed line the extrema, over all seeds per angle. Individual seeds are shown as semi-transparent lines.

5.4.2 D1 Sweep

To examine the influence of the systematic b_5 in D1 in combination with the D2 correction in more detail, a sweep of 0 units, 5 units, 10 units and 15 units has also been performed for this parameter. Due to the extensive duration of the tracking studies, only the correction cases “not corrected” and “correct D1 & D2” (see Section 5.3) were considered.

The effect of b_5 on DA is stronger than for D2, as can be seen in the “not corrected” (blue) case in Figs. 5.13a and 5.13b, which falls off rapidly with $b_{5,sys}^{D1}$ to 5σ to 6σ at 10 units to 15 units.

The corrections on the other hand are the more effective, about 5σ can be recovered at $b_{5,sys}^{D1} = 10$ units, and we see similar curves to the ones observed in the D2 sweep in Section 5.4.1: While the mean DA stays constant up until $b_{5,sys}^{D1} = 10$ units and decreases

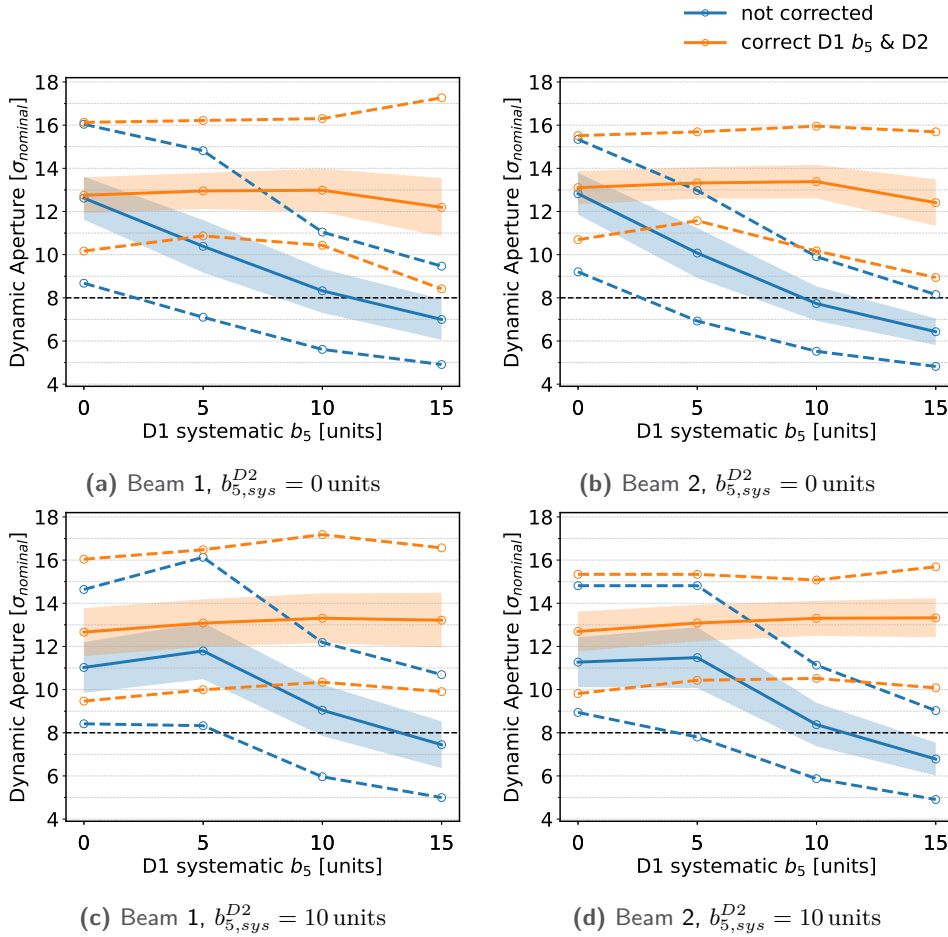


Figure 5.13. Dynamic Aperture (DA) simulation results for two correction scenarios after corrections (b_5 corrected via f_{5000}) with $b_{5,sys}^{D2} = 0$ units (top) and $b_{5,sys}^{D2} = 10$ units (bottom), sweeping over $b_{5,sys}^{D1}$. Thick lines show the mean over all seeds and angles, dashed line the extrema in the same set. The colored area marks the standard deviation.

by 1σ at $b_{5,sys}^{D1} = 10$ units, the minimal DA increases by 1σ within the first 5 units but decreases at towards the end of the sweep range until about 2σ below the starting value.

Introducing a $b_{5,sys}^{D2}$ of 10 units (Figs. 5.13c and 5.13d) leads to a compensation effect as well, which shifts the “peak” of the curve to the right, seen in the mean of the “not corrected” case, which increases between $b_{5,sys}^{D1} = 0$ units and 5 units, but especially in the corrected case, in which the DA - mean and minimal - expands with increasing $b_{5,sys}^{D1}$, reverting this trend only after $b_{5,sys}^{D1} = 10$ units, ending the sweep slightly above its starting value. As the introduction of systematic b_5 in D2 reduced the starting value at 0 units (as also seen in Section 5.4.1), the largest minimum DA for this case is still 0.5σ to 1σ below the minimum DA in case of $b_{5,sys}^{D2} = 0$ units.

5.4.3 Comparing RDTs

As discussed in Section 5.1.2, of further interest was the influence of the choice of the targeted RDT for correction on DA. In this section, the results of simulations targeting either f_{5000} (and f_{5000*}), the default value, or f_{1004} (and f_{1004*}), the closest resonance line according to Section 5.1.2, are presented. In a preliminary study f_{3002} (and f_{3002*}) had been targeted as well, yet had performed the worst and their results are not presented here.

All other RDTs are targeted as before (see Fig. 5.5). The DA results sweeping over the D2 values at a systematic b_5 of D1 of 7 units are compared.

With Crossing Angle

In this section, the simulation was set up with the crossing angle scheme for 15 cm squeezed round-optics, as described in Fig. 5.5. Figure 5.14 shows the DA results of the tracking simulations after the nonlinear IR corrections:

- In the case of the systematic D1 b_5 and D2 not being corrected of Figs. 5.14a and 5.14b, the DA results over the whole D2 sweep are very similar for both RDT targets. Hence, any differences we will see in other setups will come from the systematic b_5 in D1 and D2.
- When $b_{5,sys}^{D1}$ is corrected as in Figs. 5.14c and 5.14d, targeting f_{1004} performs slightly worse in the mean DA, but has a big outlier in minimum DA of Beam 1 at $b_{5,sys}^{D2} = 5$ units being more than 1σ lower. Looking at this into detail in Fig. 5.15 we see that this comes from a single seed (Fig. 5.15b), which is more than 2σ below the average DA of the seeds over all angles. In other configurations, e.g. at $b_{5,sys}^{D2} = 10$ units (Fig. 5.15c), we do not see such an outlier.
- In the case of the full correction, Beam 1 in Fig. 5.14e shows a clear preference in mean and minimum DA of correcting f_{5000} over f_{1004} . The mean DA between the two drifts apart with increasing $b_{5,sys}^{D2}$ to almost 1σ difference at the end of the sweep. Beam 2 in Fig. 5.14f shows a similar behaviour in the mean, yet the minimum DA follows no clear pattern. In Fig. 5.16 the details of this are shown:

The behaviour of the f_{5000} at 5 units (Figs. 5.16a and 5.16c) is due to a dramatic decrease of DA in a single seed in the horizontal plane, as discussed in Section 5.4.1. In contrast, the dip in correcting f_{1004} at 0 units in Fig. 5.16b looks more like a numerical issue, as a sharp drop in DA for two seeds at a single angle in Fig. 5.16d, while otherwise being at similar values as the f_{5000} correction (see Fig. 5.11b). Ignoring these data points, Beam 2 would show the same behaviour as Beam 1.

So with crossing angles applied, targeting f_{5000} , as has been done in the past, is of advantage for DA over targeting f_{1004} , despite the latter being the closer resonance (as discussed in Section 5.1.2). To investigate the source of the difference, we can look at the effective RDTs before and after correction (their nominal value is zero). Figure 5.17 show that no matter which RDT is targeted, when correcting all errors, both f_{5000}^{IP} and f_{1004}^{IP} are well corrected. Shown is only IP1, but IP5 yields identical results. The targeted RDT is corrected a better, but not for all seeds - hence the big spread in this cases, while the values over the seeds for the respective other RDT are closer together. Comparing the different correction scenarios, we see that correcting everything but D1 b_5 and D2 does not decrease the mean RDT value, but reduces the spread between seeds. This is due to the fact, that the correction algorithm in this case does not know about the errors it is not supposed to correct and creates corrections that do not take any compensation from these errors into account. This is especially visible when including only d1 b_5 but not D2 into the correction, which does not change the $|f_{5000}^{\text{IP}}|$ results (Fig. 5.17), yet both RDT targets make the total $|f_{1004}^{\text{IP}}|$ (Fig. 5.18) worse. That this is not an error of the RDT-correction has been checked by verifying that the effective RDT values, based on the elements given to the correction algorithm, is suppressed. It can also be seen in the case at $b_{5,sys}^{D2} = 0$ units and $b_{5,sys}^{D1} = 7$ units and $b_{5,sys}^{D1} = 0$ units, shown in Fig. 5.19 for $|f_{1004}^{\text{IP}}|$ with targeting the same RDT: The correction is very effective only if there is no other errors, unknown to the correction algorithm, present in the IR! Otherwise, the “correction” might even increase the value of the RDT in the IR, as seen in the case of $|f_{1004}^{\text{IP}}|$. The fact that after each full correction in Figs. 5.18 and 5.19 the distribution of $|f_{1004}^{\text{IP}}|$ resembles closely the distribution of the case, where the respective error is not present (i.e. “correct D1 b_5 D2” in Fig. 5.18 bottom compared to Fig. 5.19 top; and “correct D1 b_5 ” in Fig. 5.19 top compared to Fig. 5.19 bottom) instills trust in the quality of the RDT correction.

As both RDT targets correct both RDTs very well in the “correct D1 b_5 & D2” scenario, the difference seen between the two targets in DA must have another origin. As decapole fields are a first order source for third order chromaticity [97], looking at this global machine parameter is useful to qualify the b_5 correction. The chromaticity is calculated independent from the correction algorithm by the PTC-module [131] of MAD-X. Results for all correction scenarios, as well as for the fully uncorrected (“before corr.”) and “nominal” case, at $b_{5,sys}^{D1} = 7$ units and $b_{5,sys}^{D2} = 10$ units are shown in Fig. 5.20.



One data point in Beam 1 “before corr.” in both of the RDT targets was ignored, as well as one point in Beam 2 “D1 b_5 & D2” of f_{1004} , as their values were more than 1000× bigger than all other values.

An important factor influencing DA is amplitude detuning, which on one hand can be used to stabilize the beam via the effect of Landau Damping [79], can push particles with increasing amplitude towards resonances and has been shown to be detrimental to beam lifetime and DA [15, 17, 195]. It has been one of the targets of non-linear corrections in the LHC throughout operation [10, 17, 36, 37, 128, 144, 151, 153, 176, 177, 201–203] (summary in [43]). First order amplitude detuning has also been calculated by the PTC-module in MAD-X and the results for all correction scenarios, as well as for the fully uncorrected (“before corr.”) and “nominal” case, are shown in Fig. 5.21. All detuning terms seem to be well corrected, independent of targeted RDT. In fact, only in the “D1 b_5 & D2” correction scenario we can find obvious differences between targeting either RDT, exposed in Fig. 5.22 (bottom). Targeting f_{1004} performs slightly but visibly worse in all three detuning terms for both beams, and this increased residual amplitude detuning is very likely the culprit for the differences we see in DA. Amplitude detuning from decapole fields is created via feed-down [14, 97] from horizontal off-center trajectories (off magnet-center that is) which are present in the IR due to the crossing angle scheme. Therefore a possible explanation hypothesis is, that targeting f_{5000} targets the feed-down to amplitude detuning in a similar fashion as targeting f_{4000} targets amplitude detuning directly. The contributions of the elements w at s_w to the effective RDT to be corrected are (from Eq. (3.1)):

$$f_{5000}^{IP} = \sum_w K_5 L(s_w) \beta_x^{5/2}(s_w) e^{i\pi\Theta(s_w - s_{IP})} \quad (5.5)$$

$$f_{1004}^{IP} = \sum_w K_5 L(s_w) \beta_x^{1/2}(s_w) \beta_y^2(s_w) (-1)^{\Theta(s_w - s_{IP})} \quad (5.6)$$

The contribution to amplitude detuning from $K_5 L$ feed-down on the other hand read:

$$\begin{aligned} \frac{\partial Q_x}{\partial(2J_x)} &= \sum_w \Delta x(s_w) \frac{K_5 L(s_w)}{32\pi} \beta_x^2(s_w) \\ &\approx \frac{\sqrt{2J_x}}{32\pi} \sum_w K_5 L(s_w) \beta_x^{5/2}(s_w) (-1)^{\Theta(s_w - s_{IP})} \end{aligned} \quad (5.7)$$

$$\begin{aligned} \frac{\partial Q_x}{\partial(2J_y)} &= - \sum_w \Delta x(s_w) \frac{K_5 L(s_w)}{16\pi} \beta_x(s_w) \beta_y(s_w) \\ &\approx - \frac{\sqrt{2J_x}}{16\pi} \sum_w K_5 L(s_w) \beta_x^{3/2}(s_w) \beta_y(s_w) (-1)^{i\pi\Theta(s_w - s_{IP})} \end{aligned} \quad (5.8)$$

$$\begin{aligned} \frac{\partial Q_y}{\partial(2J_y)} &= \sum_w \Delta x(s_w) \frac{K_5 L(s_w)}{32\pi} \beta_y^2(s_w) \\ &\approx \frac{\sqrt{2J_x}}{32\pi} \sum_w K_5 L(s_w) \beta_x^{1/2}(s_w) \beta_y^2(s_w) (-1)^{\Theta(s_w - s_{IP})} \end{aligned} \quad (5.9)$$

Where, due to the orbit also in this configuration being closed, the normalized coordinates were used for $\Delta x(s_w) \approx \sqrt{2J_x \beta_x(s_w)} (-1)^{\Theta(s_w - s_{IP})}$, with the phase approximation as

used for the effective RDTs, which here also reflect the sign-change of Δx on the two sides of the IP. We see that targeting f_{5000} (Eq. (5.5)) corrects also for the direct horizontal term (Eq. (5.7)), while targeting f_{1004} (Eq. (5.6)) corrects for the direct vertical term (Eq. (5.9)). As we were hoping to see a difference between the RDTs that will be visible in all three amplitude detuning terms, feed-down to b_4 alone cannot be the sole explanation.

This finding is supported by looking again at the DA at fixed $b_{5,sys}^{D2}$ and sweeping over $b_{5,sys}^{D1}$ for both RDT targets, as done in Fig. 5.23: Ignoring the behaviour of the minimum DA in Beam 2 at $b_{5,sys}^{D2} = 0$ units of Fig. 5.23b, which is again caused by single seeds, as seen by the mean DA and in Fig. 5.23e, we see that the deviation between targeting f_{5000} and f_{1004} is only increasing with increasing $b_{5,sys}^{D2}$ but stays relatively stable with increasing $b_{5,sys}^{D1}$.

Nonetheless, to sort out the influence of feed-down, the simulations have been repeated without the crossing scheme at flat-orbit and are presented in “[With Flat Orbit](#)” below.

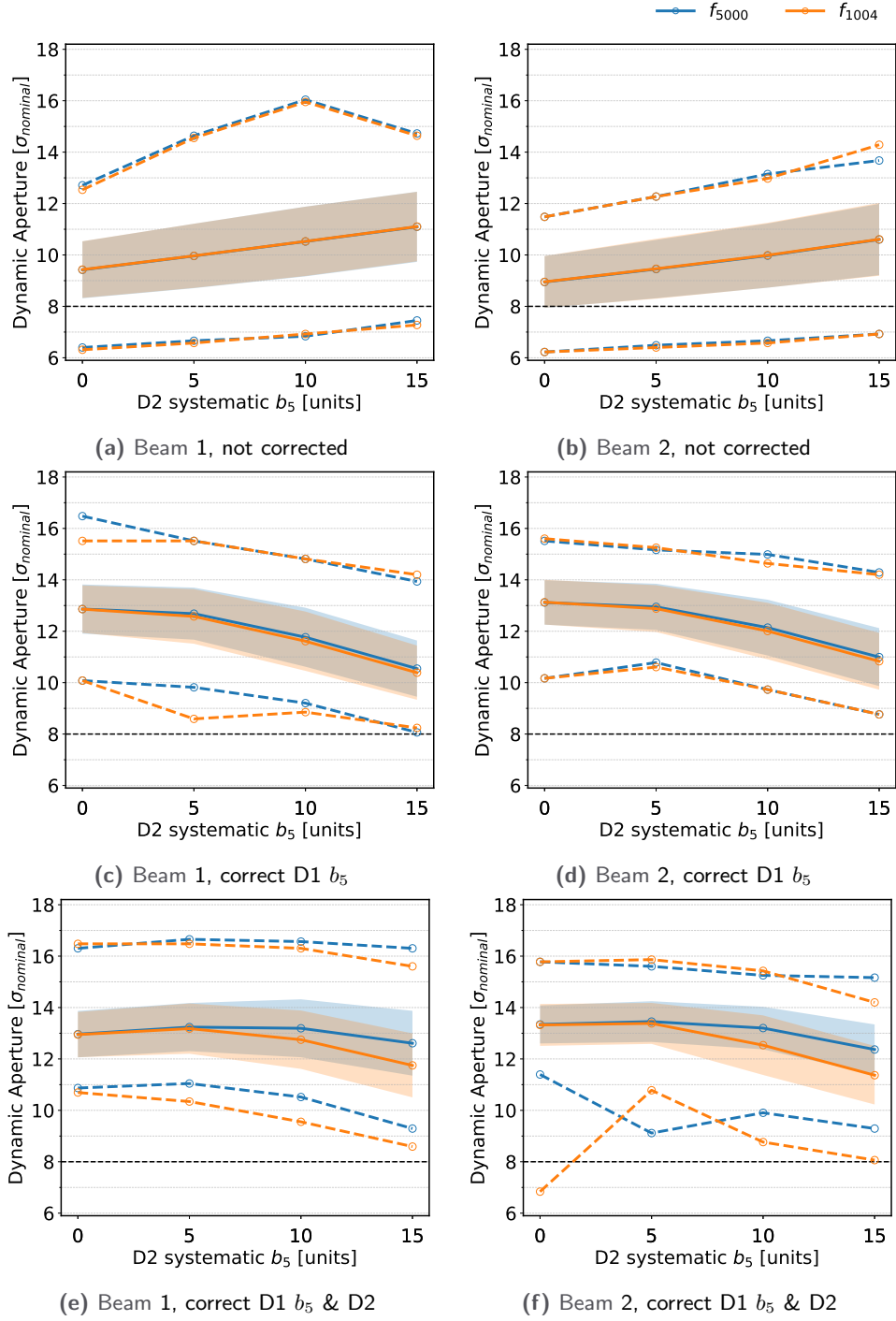


Figure 5.14. Dynamic Aperture (DA) simulation results for corrections with different targeted RDTs for b_5 , with $b_{5,sys}^{D1} = 7$ units, sweeping over $b_{5,sys}^{D2}$. Thick lines show the mean over all seeds and angles, dashed line the extrema in the same set. The colored area marks the standard deviation.

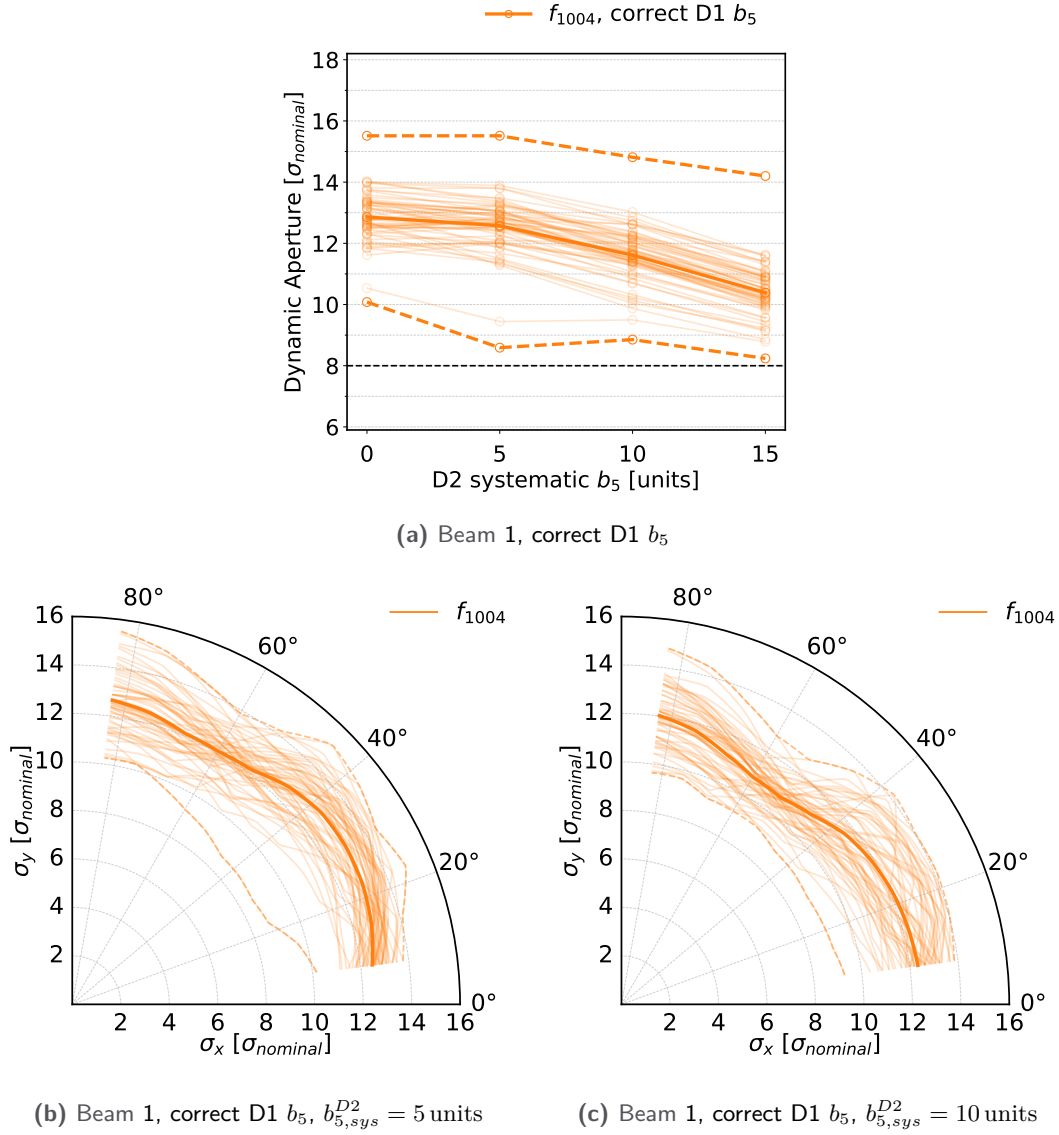


Figure 5.15. Dynamic Aperture (DA) simulation results for correction scenario “D1 b_5 ” targeted RDT f_{1004} for b_5 correction, with $b_{5,sys}^{D1} = 7$ units of Beam 1. The bold line indicates the mean, the dashed lines the extrema over all seeds (and angles in a)), while the semi-transparent lines show the results of the individual seeds. The full sweep over $b_{5,sys}^{D2}$ is shown in a). In b) and c) the DA at $b_{5,sys}^{D2} = 5$ units and 10 units is shown per angle.

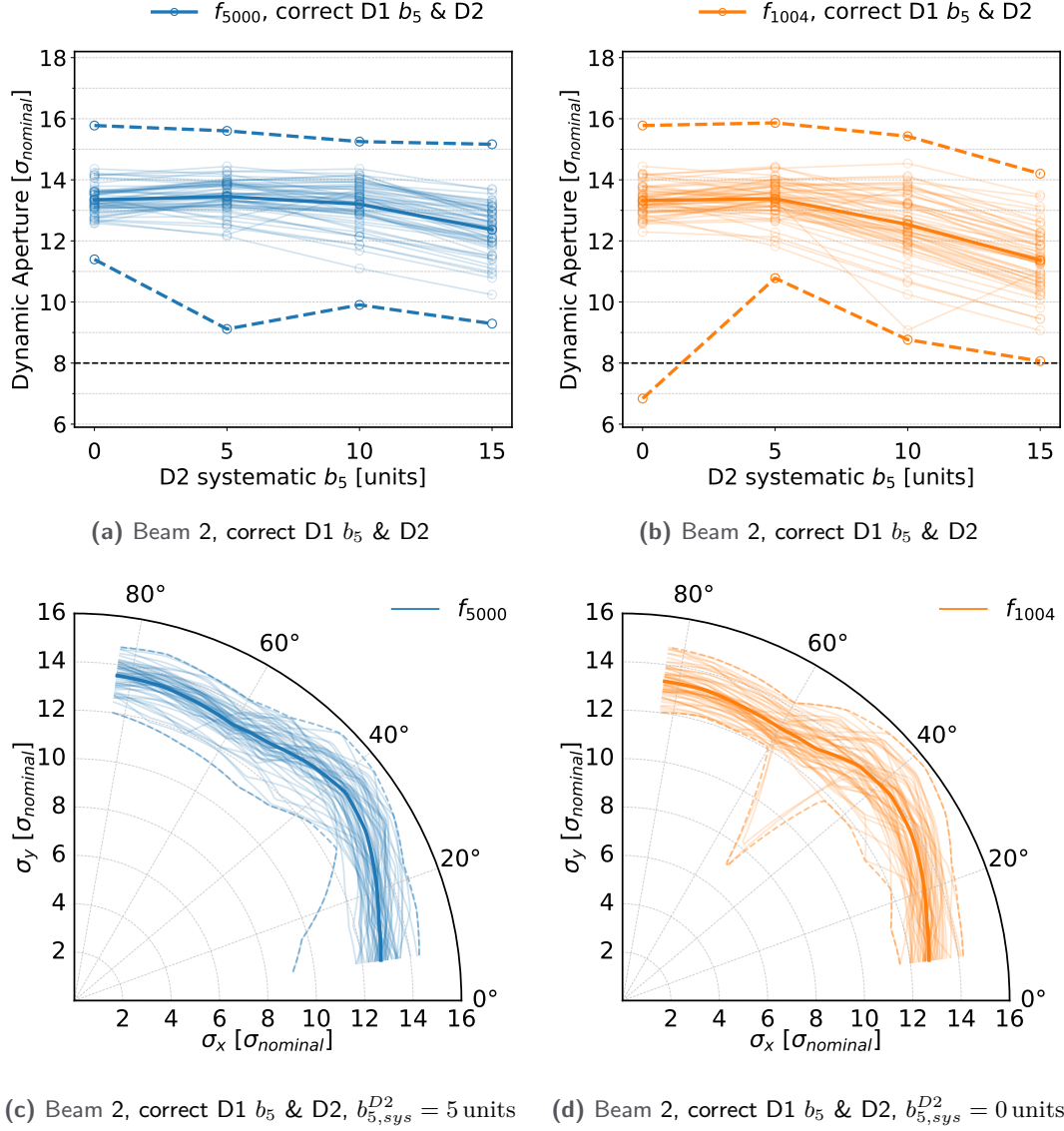


Figure 5.16. Dynamic Aperture (DA) simulation results for correction scenario "D1 b_5 & D2" for the targeted RDTs f_{5000} (left) and f_{1004} (right), with $b_{5,sys}^{D1} = 7$ units of Beam 2. The bold line indicates the mean, the dashed lines the extrema over all seeds (and angles in a) and b)), while the semi-transparent lines show the results of the individual seeds. The full sweep over $b_{5,sys}^{D2}$ is shown in a) and b). In c) and d) the DA at $b_{5,sys}^{D2} = 5$ units and 0 units is shown per angle.

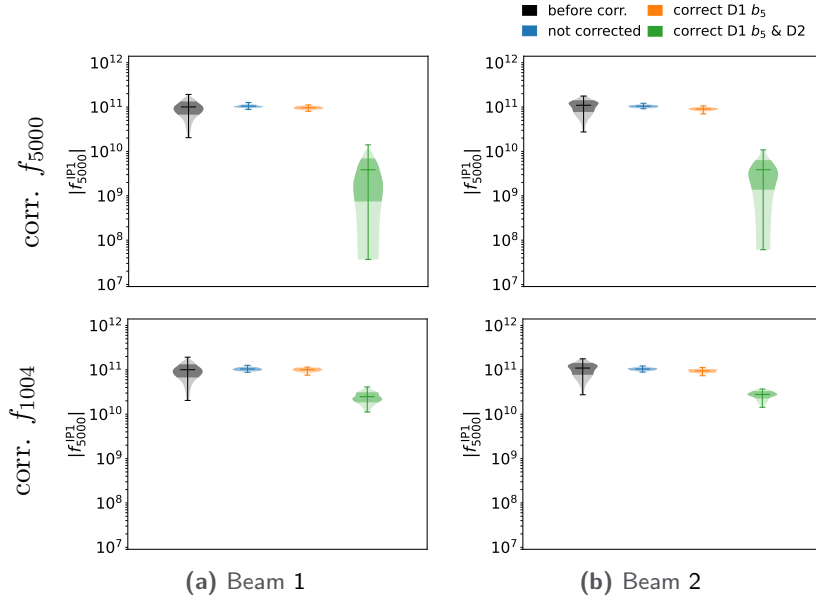


Figure 5.17. Absolute value of f_{5000}^{IP1} for the uncorrected machine (*before corr.*) as well as the three correction scenarios, targeting either f_{5000} (top) or f_{1004} (bottom) for b_5 correction, at $b_{5,sys}^{D1} = 7$ units and $b_{5,sys}^{D2} = 10$ units. The violins show a gaussian kernel density estimation of the realizations. Mean and extrema are indicated and one standard deviation emphasised.

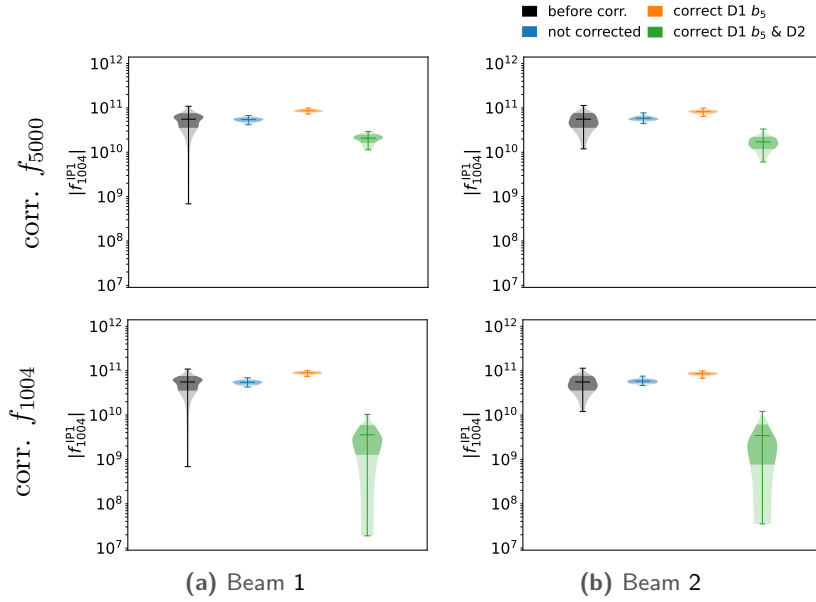


Figure 5.18. Absolute value of f_{1004}^{IP1} for the uncorrected machine (*before corr.*) as well as the three correction scenarios, targeting either f_{5000} (top) or f_{1004} (bottom) for b_5 correction, at $b_{5,sys}^{D1} = 7$ units and $b_{5,sys}^{D2} = 10$ units. The violins show a gaussian kernel density estimation of the realizations. Mean and extrema are indicated and one standard deviation emphasised.

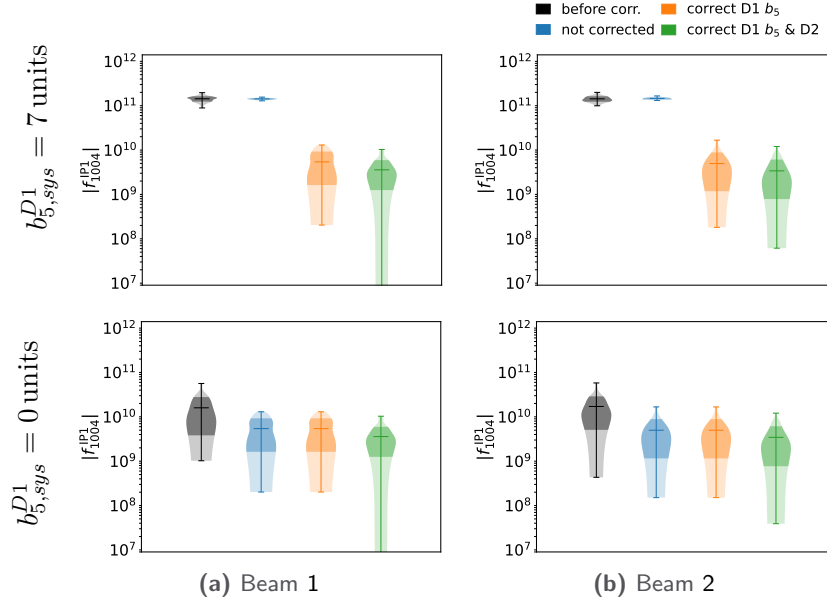


Figure 5.19. Absolute value of f_{1004}^{IP1} for the uncorrected machine (*before corr.*) as well as the three correction scenarios, targeting f_{1004} for b_5 correction, at $b_{5,sys}^{D1} = 7$ units (top) or $b_{5,sys}^{D1} = 0$ units (bottom) and $b_{5,sys}^{D2} = 0$ units. The violins show a gaussian kernel density estimation of the realizations. Mean and extrema are indicated and one standard deviation emphasised.

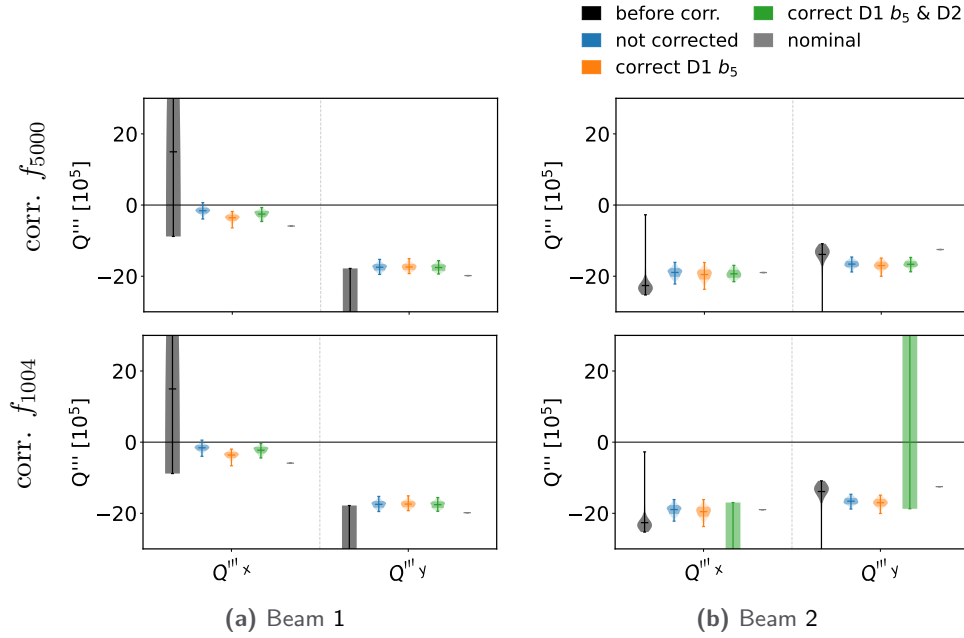


Figure 5.20. Third order Chromaticity for the uncorrected machine (*before corr.*) as well as the three correction scenarios, targeting either f_{5000} (top) or f_{1004} (bottom) for b_5 correction, at $b_{5,sys}^{D1} = 7$ units and $b_{5,sys}^{D2} = 10$ units. The violins show a gaussian kernel density estimation of the realizations. Mean and extrema are indicated and one standard deviation emphasised.

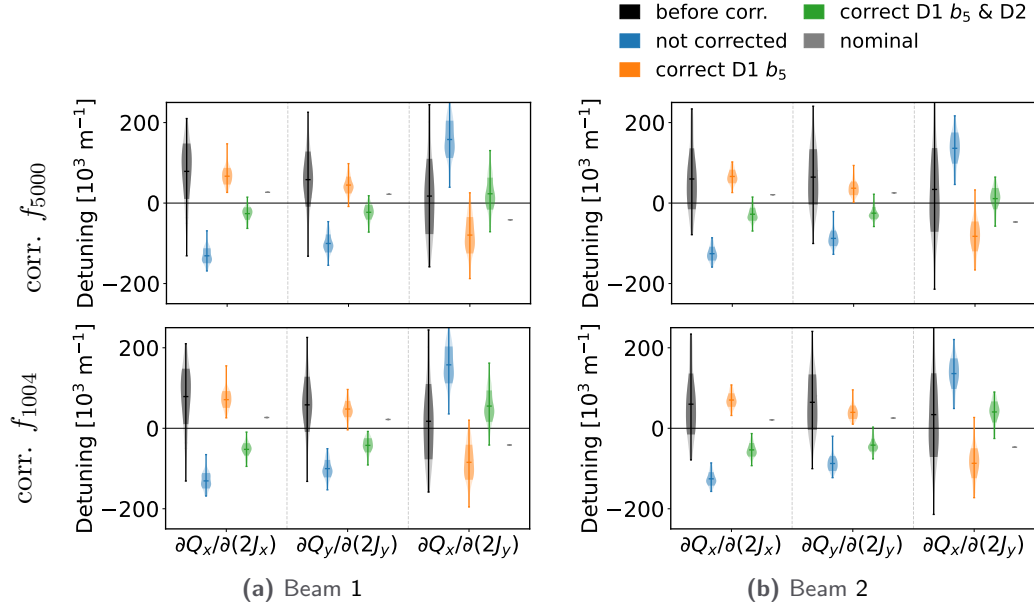


Figure 5.21. First order amplitude detuning for the uncorrected machine (*before corr.*) as well as the three correction scenarios, targeting either f_{5000} (top) or f_{1004} (bottom) for b_5 correction, at $b_{5,sys}^{D1} = 7$ units and $b_{5,sys}^{D2} = 10$ units. The violins show a gaussian kernel density estimation of the realizations. Mean and extrema are indicated and one standard deviation emphasised.

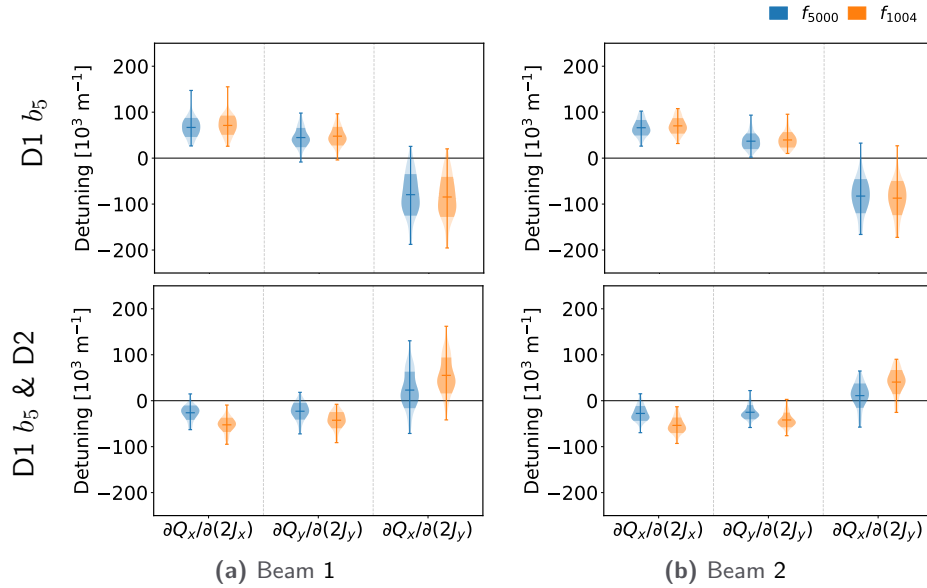


Figure 5.22. As Fig. 5.21, scenarios “D1 b_5 ” and “D1 b_5 & D2” only.

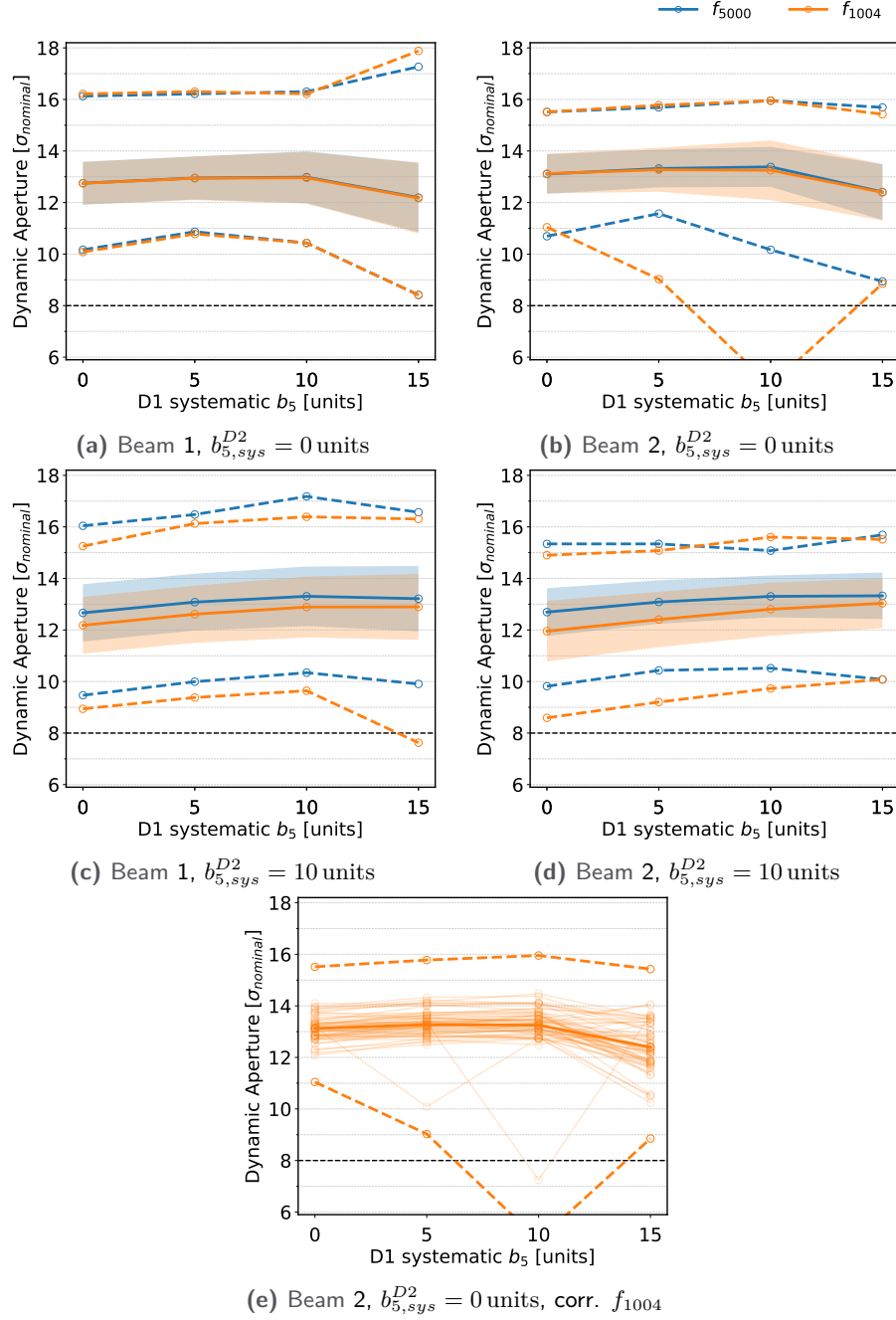


Figure 5.23. Dynamic Aperture (DA) simulation results for "correct D1 b_5 & D2" with different targeted RDTs for b_5 , at $b_{5,sys}^{D2} = 0$ units (top) and $b_{5,sys}^{D2} = 10$ units (middle), sweeping over $b_{5,sys}^{D1}$. Thick lines show the mean over all seeds and angles, dashed line the extrema in the same set. The colored area marks the standard deviation. Plot e) shows the f_{1004} result for $b_{5,sys}^{D2} = 0$ units of Beam 2 from b) in detail, the semi-transparent lines being the mean of individual seeds.

With Flat Orbit

To investigate the effect of feed-down on the targeted RDT, some of the simulations from the last section “[With Crossing Angle](#)” have been repeated with a flat-orbit setup. The sweep over the systematic D2 b_5 has been repeated only at $b_{5,sys}^{D1} = 7$ units and only the two scenarios “not corrected”, in which all errors but the systematic D1 b_5 and D2 are corrected, and the scenario “correct D1 b_5 & D2”, have been tracked. Additionally the RDT f_{3002} (f_{3002*}) has been targeted for correction, corresponding to the (3, -2) resonance seen in [Section 5.1.2](#).

The DA results of the tracking studies are shown in [Fig. 5.24](#): Unsurprisingly, for the “not corrected” scenario ([Figs. 5.24a](#) and [5.24b](#)), the targeted RDT does not make a difference, as it did not with orbit either. The DA is also at very similar values as they had been with crossing scheme. In the “correct D1 b_5 & D2” scenario ([Figs. 5.24c](#) and [5.24d](#)), the results change: First of all, the DA is now about 4σ larger than it had been [With Crossing Angle](#). In the mean DA, targeting f_{1004} yields the best results at high $b_{5,sys}^{D2}$, followed by targeting f_{3002} and then f_{5000} , which is in line with our expectations from looking at the tune diagram [Fig. 5.2](#). In the minimum DA we see the results from targeting f_{1004} and f_{5000} very close, one outperforming the other by a margin, depending on which beam we look at. f_{5000} shows in both beams the worst minimum DA of $0.5\sigma - 1\sigma$ below the best performing target at $b_{5,sys}^{D2} = 11$ units. For all three targeted RDTs, the effect on amplitude detuning is the same, as seen in [Fig. 5.25](#).

Conclusion and Following Studies

We have seen there is only little difference between targeting different RDTs when only D1 is corrected, yet f_{5000} performs better than f_{1004} when including D2, in contrast to the expectation from [Section 5.1.2](#). Investigation of the residual amplitude detuning after correction, which is often an important factor for DA in the LHC [[151](#), [195](#)], indicates that it is the feed-down to b_4 from the b_5 in D2 due to the crossing orbit, which is better corrected targeting f_{5000} . It is not clear why this is the case, as the two RDTs simply target different terms of the amplitude detuning, yet we see all terms improving when choosing f_{5000} . It nonetheless hints at b_5 feed-down still being present after correction, due to the orbit difference between the source (D2) and the corrector. D1, on the other hand, is very close to the corrector with negligible orbit difference. Turning the crossing angles off, as done in [Figs. 5.24](#) and [5.25](#), shows the expected behaviour of targeting f_{1004} resulting in larger mean DA and equal residual amplitude detuning.

Attempting to fix the residual feed-down and improving the correction at D2, the advanced features of the correction algorithm ([Section 3.2](#)) to use both optics as input (accounting for the dual-aperture in D2) and to target feed-down were utilized. As this study had been going on for already longer than planned, only preliminary tests could be performed, which were inconclusive. Their results are presented in the next two sections.

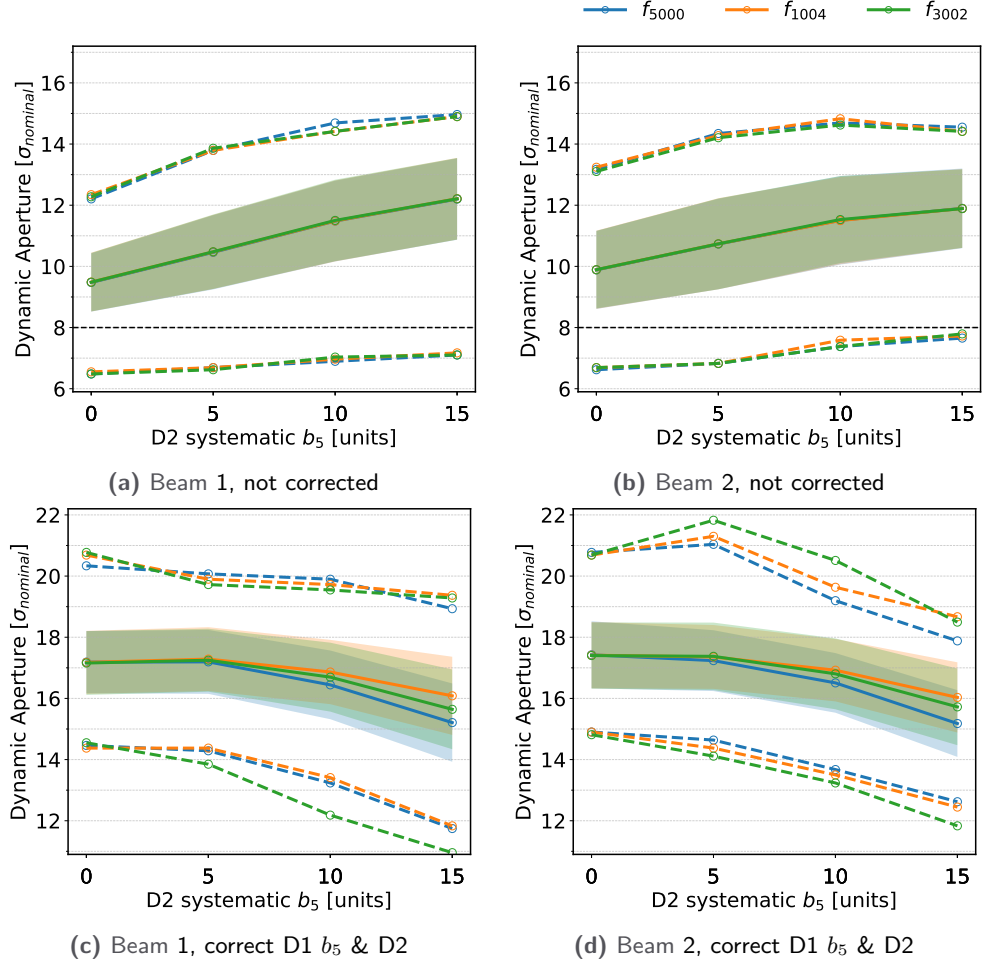


Figure 5.24. Dynamic Aperture (DA) simulation results with flat-orbit for corrections with different targeted RDTs for b_5 , with $b_{5,sys}^{D1} = 7$ units, sweeping over $b_{5,sys}^{D2}$. Thick lines show the mean over all seeds and angles, dashed line the extrema in the same set. The colored area marks the standard deviation.

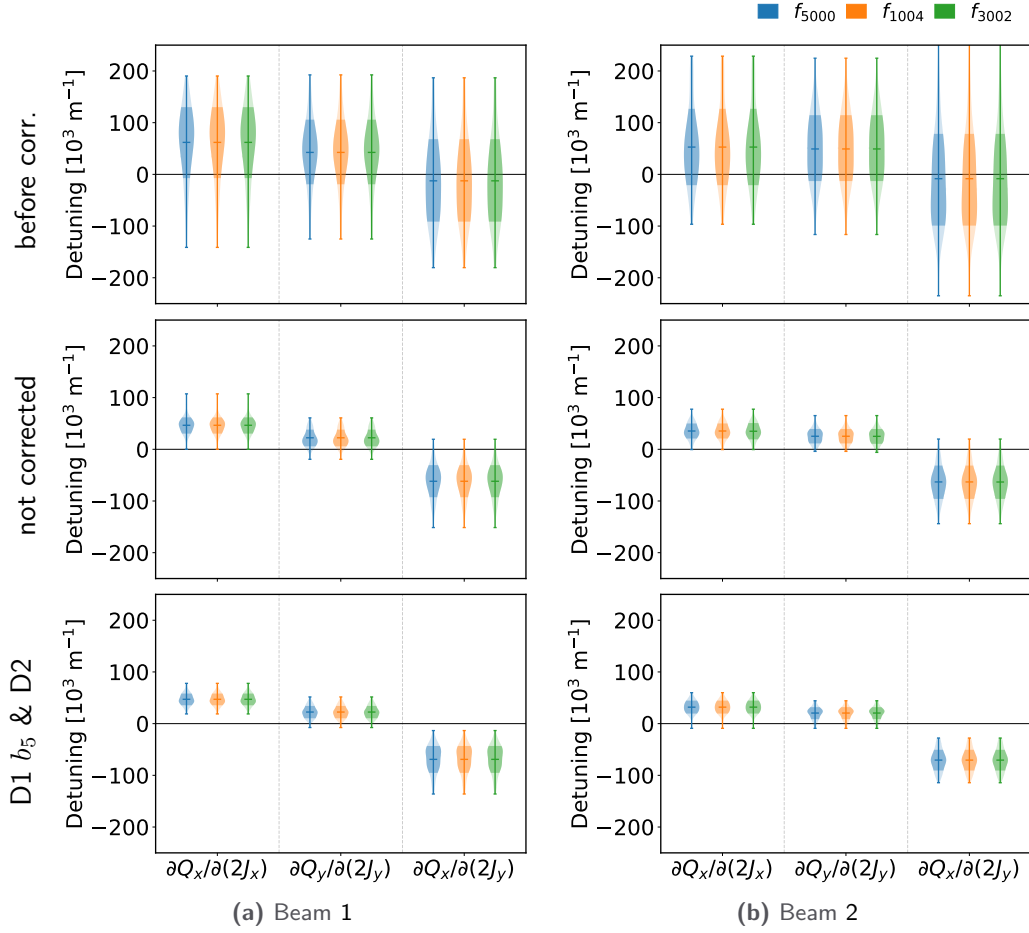


Figure 5.25. First order amplitude detuning for the uncorrected machine (*before corr.*, top) as well as the correction scenarios "not corrected" (middle) and "D1 b_5 & D2" (bottom), targeting either f_{5000} , f_{1004} or f_{3002} for b_5 correction, at $b_{5,sys}^{D1} = 7$ units and $b_{5,sys}^{D2} = 10$ units. The violins show a gaussian kernel density estimation of the realizations. Mean and extrema are indicated and one standard deviation emphasised.

5.4.4 Correcting Both Beams

Using the feature of the correction algorithm to provide the two optics of the beams independently (see Section 3.2.1), the need to supply an average error in the single-aperture D2 can be eliminated and the actual errors can be targeted. Figure 5.26 shows that, equivalently to the results in Section 3.5, the DA becomes worse when comparing to the beam from which the single-optics correction (based on the assumed symmetries of f_{jklm} and f_{jklm*}) is calculated.

These results by itself are not so useful, as in fact (as seen in Section 3.5), the improvement is only visible when comparing to the quality of the correction on the “other” beam. To come to a conclusion the following test are required first:

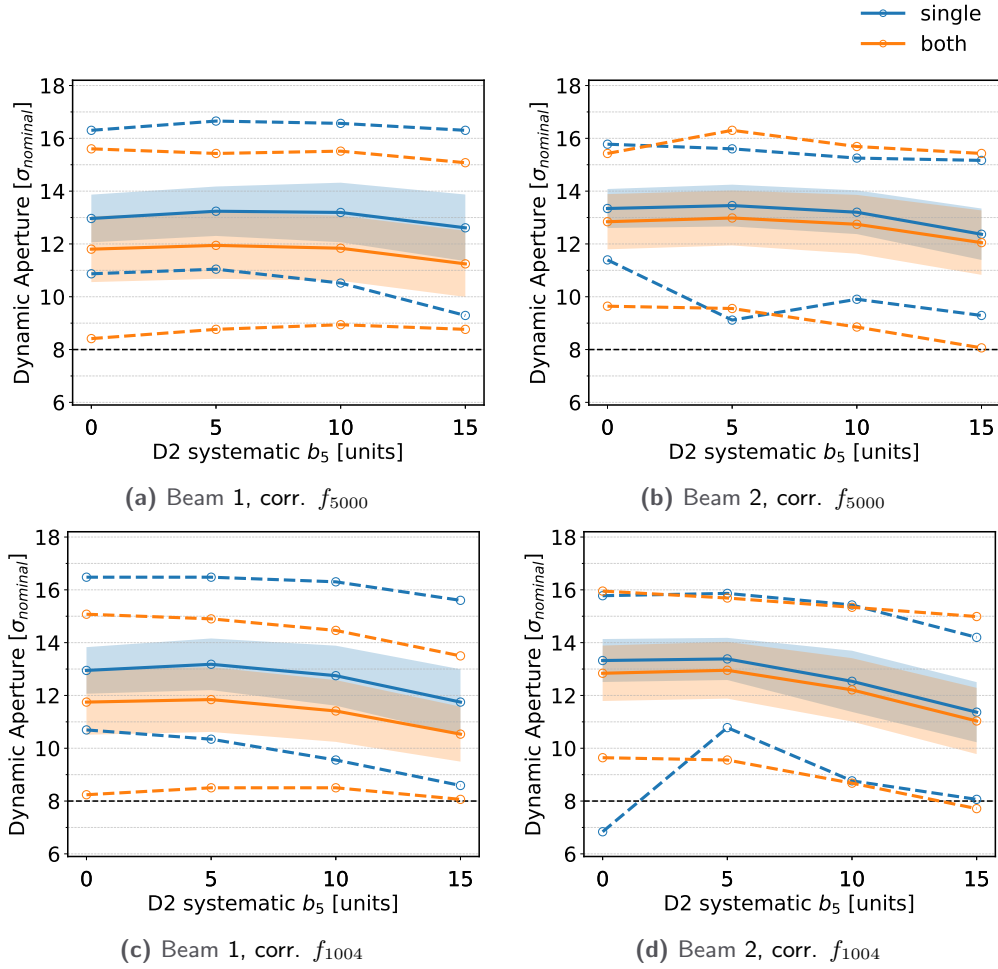


Figure 5.26. Dynamic Aperture (DA) simulation results with corrections based on single optics or both beam optics correcting D1 b_5 and D2, with $b_{5,sys}^{D1} = 7$ units, sweeping over $b_{5,sys}^{D2}$. Thick lines show the mean over all seeds and angles, dashed line the extrema in the same set. The colored area marks the standard deviation.

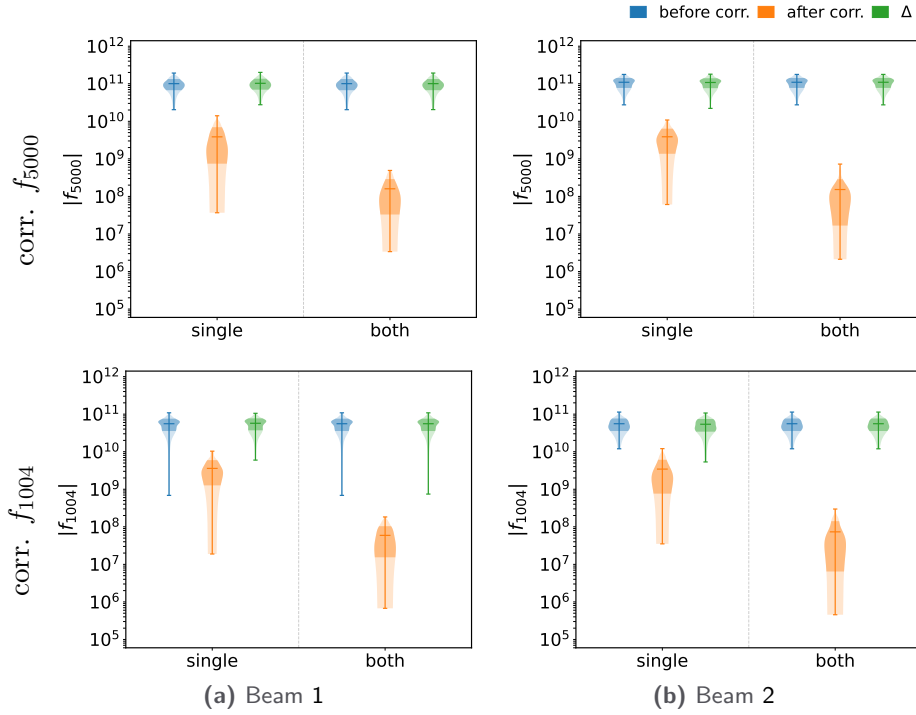


Figure 5.27. Absolute value of f_{5000}^{IP1} (top) and f_{1004}^{IP1} (bottom), targeting the respective RDT for b_5 correction at $b_{5,\text{sys}}^{D1} = 7$ units and $b_{5,\text{sys}}^{D2} = 10$ units. Displayed are corrections based on *single* optics or *both* beam optics of the uncorrected machine (*before corr.*), the correction scenario “D1 b_5 & D2” (*after corr.*) and the per-seed difference between the two (Δ). The violins show a gaussian kernel density estimation of the realizations. Mean and extrema are indicated and one standard deviation emphasised.

- Run and compare to “other beam” corrections.
- Check differences in correction values between Beam 1 and Beam 2 for “single” correction.

Due to a lack of time (as these studies require some time to setup, run and analyse), these could not yet been performed, and are therefore still open questions to be answered.

There are several hints, that the quality of the corrections are very good, already in these results: Figure 5.27 shows that the correction on the RDT itself actually better when using both optics, which is not expected, especially in view of the DA results of Fig. 5.26. Also the amplitude detuning, shown in Fig. 5.28 is well corrected, and correction is even improved for Beam 2 with “both” optics used.

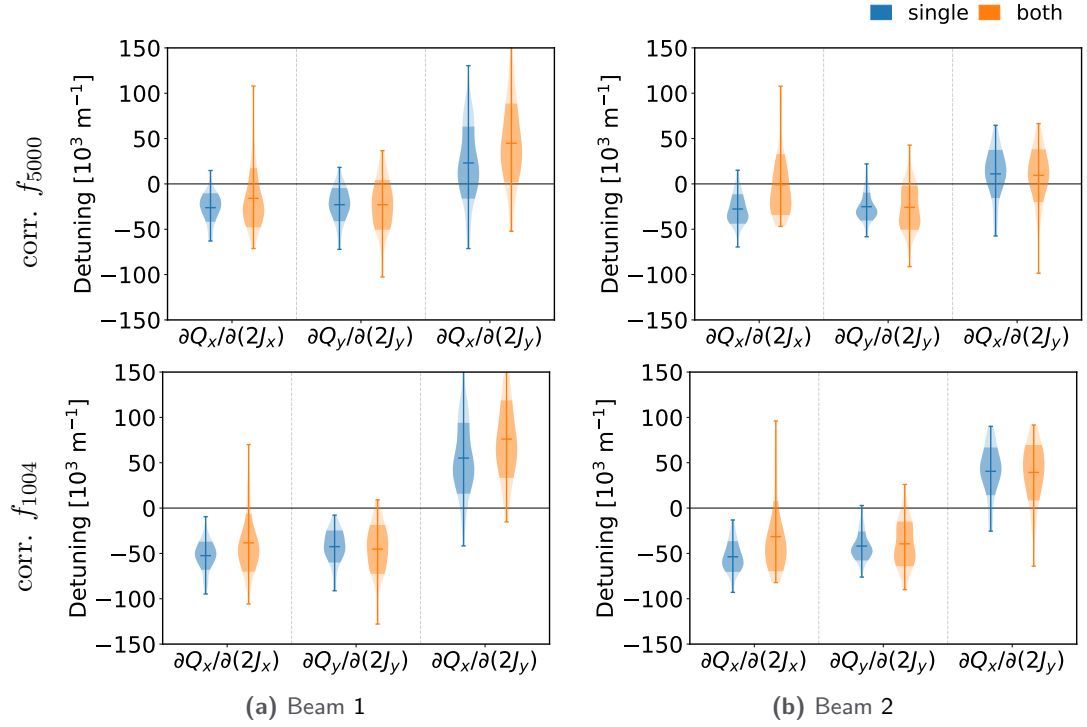


Figure 5.28. First order amplitude detuning with corrections based on *single* optics or *both* beam optics for the uncorrected machine (*before corr.*) and the correction scenario “D1 b_5 & D2”, targeting either f_{5000} (top) or f_{1004} (bottom) for b_5 correction. At $b_{5,sys}^{D1} = 7$ units and $b_{5,sys}^{D2} = 10$ units. The violins show a gaussian kernel density estimation of the realizations. Mean and extrema are indicated and one standard deviation emphasised.

5.4.5 Correcting Feed-Down

Mirroring Section 3.3, it was tried to also include feed-down (up to second order) into the corrections. The results shown in Fig. 5.29 are not very motivating: The minimum DA shrinks by 2 or more σ , independent of the RDT targeted. At a quick glance, no root cause for this behaviour could be found and a deeper dive into the details of the RDT correction and amplitude detuning would be necessary. Due to the large amount of time needed for further investigation, this has been put on hold and should be reevaluated at a later date.

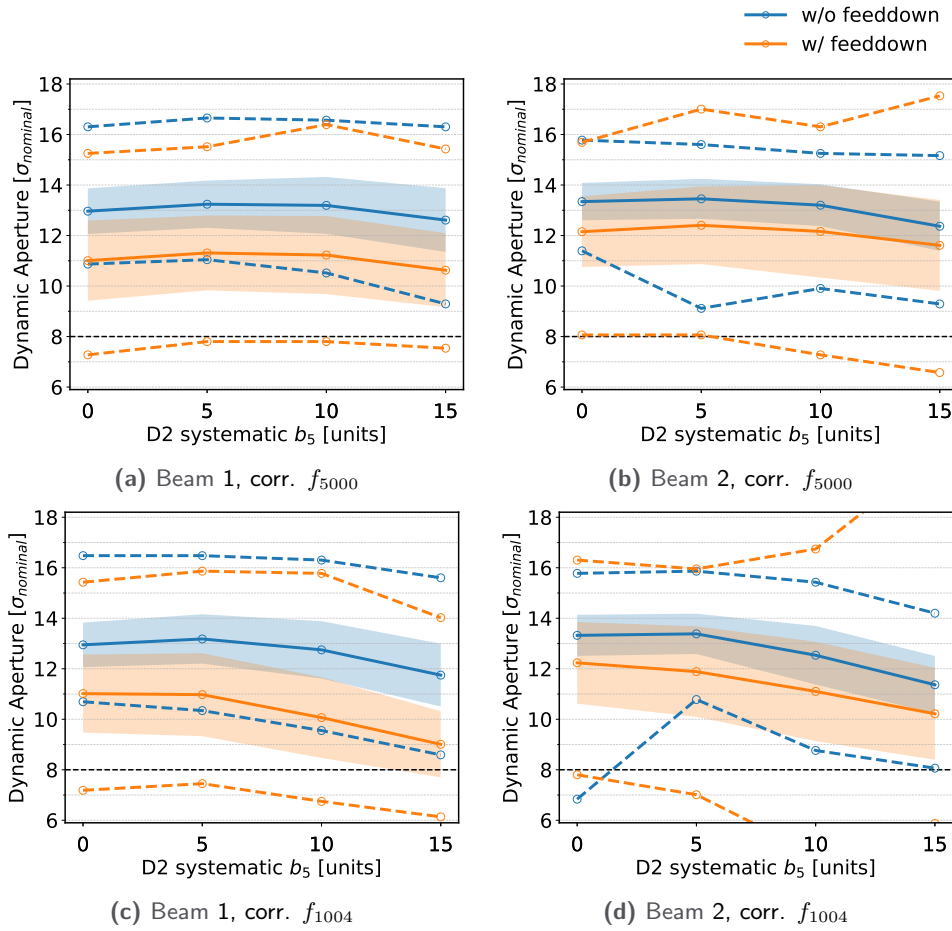


Figure 5.29. Dynamic Aperture (DA) simulation results without and with two orders of feed-down correction correcting D1 b_5 and D2, with $b_{5,sys}^{D1} = 7$ units, sweeping over $b_{5,sys}^{D2}$. Thick lines show the mean over all seeds and angles, dashed line the extrema in the same set. The colored area marks the standard deviation.

5.5 Conclusion

It has been shown that inclusion of D2 into the nonlinear correction scheme for the high-luminosity insertion region of the HL-LHC is beneficial as its magnetic errors are partly corrected by the nonlinear corrector package. Full correction of D2 b_5 errors cannot be achieved due to its dual-aperture nature, its phase advance and orbit difference to the corrector package. Compensation between the b_5 components of D1 and D2 is present, but alone is insufficient to achieve the required DA. With crossing orbit bumps in the IRs, the current RDT target f_{5000} is superior to f_{1004} , as feed-down to amplitude detuning is better corrected, the reasons for which could not be determined. For flat-orbit, f_{1004} is superior as expected from proximity of tunes to the resonance. Further improvements of the correction might be possible, including both optics and feed-down in the corrections, yet the preliminary studies in this direction were, for unknown reasons, not very promising.

Further studies on this topic have been dropped due to time constraints of this PhD, but also because improvements to the magnet design were proposed, reducing the expected b_5 errors in D1 and D2 to 1 unit and 2 units respectively [204]. This aligns well with the results presented, indicating the errors should be kept below 5 units to allow correction and not spoil DA.

6 Replication of HL-LHC Dynamic Aperture in the LHC

Note. In this chapter parts of an MD-note are presented, which has been published as a public note at CERN in [205].

Acknowledgements. While the MD proposal [206] has been prepared by E.H. Maclean[✉] and me, the different working points at which the machine was probed for resonances under forced oscillations had been prepared by F.S. Carlier[✉] and were in part already discussed in his PhD-Thesis [94]. The actual measurements were performed by the team of scientists listed as authors in [205], yet the analysis presented of the measured data has been performed by myself.

6.1 Motivation and procedure overview

In July 2017, MD2158 [38] was performed to explore the prospects for the HL-LHC linear and non-linear optics commissioning by enhancing sextupole and dodecapole sources in the ATLAS and CMS IR. While overall the MD was successful, measurements during that MD suffered from the loss of AC-Dipole adiabaticity [159,207] under the influence of the strong skew sextupoles (a_3) in the IR. In consequence, this led to dramatic blow-up of the beam after - even small - AC-Dipole kicks. Furthermore, following studies of the influence of strong dodecapole errors (b_6) were complicated by the blown up beam. Questions were raised on how this effect will limit the ability to perform linear optics commissioning [144,208] in the HL-LHC, taking into account the strong nonlinear errors anticipated at low- β^* [17,24,128,209].

Low β^* is provided by using flat-optics with $\beta^* = 60 \text{ cm}/15 \text{ cm}$, which may be used in the HL-LHC as well [3,30,209]. Using this configuration also allows to follow up on the results from the flat-optics MD program (MD2148) [125,210]. After checking for residual coupling in the machine, the initial part of MD3312 was thus dedicated to re-visit linear optics corrections in the flat-ATS scheme. In particular, the waist of the β -function in IP5, was shifted to correct the error on β^* , which has been tested by luminosity scans and K-modulation procedures [211].

Procedure. High-order errors were artificially introduced with the MCOX, MCOSX, MCSX, MCSSX and MCTX in IR1 and IR5 [7]. Beam 1 has been used to study amplitude detuning throughout the MD, while DA measurements were performed on Beam 2. For the amplitude detuning study vertical and horizontal kicks were executed independently,

kicking only lightly in the respective opposite plane to observe coupling effects. The DA measurements on the other hand were performed by diagonal kicks at different working points of the tunes. Goal of the DA measurements was to observe the impact of strong nonlinear errors in the low- β IRs on the DA of forced oscillations [161], not only to quantify the expected limits during HL-LHC linear optics commissioning mentioned above, but also to allow benchmarking of DA simulations. Both studies were executed at different tune settings, i.e. working points in the tune diagram (see Figs. 6.14 and 6.19), to probe the influence of various resonances.

To avoid blow-up and distortion of the beam by the influence of the lower orders, especially at high kick amplitudes, as seen in the past, the individual influence of b_6 has been studied first. After successful measurements, b_4, a_3, b_3 and a_4 were trimmed in as well, and further amplitude detuning and DA kicks were conducted.

Unfortunately, instead of probing close to the diagonal resonances as before (working point 3 in Fig. 6.19a), we kicked accidentally almost directly on the $-4Q_x + Q_y$ resonance (working point 3 in Fig. 6.19b), a dodecapole resonance. With all correctors powered, the beams were dumped due to too high losses during the first excitation, and could not be refilled due to time constraints. For this reason, studies with crossing angles to measure feed-down, as described in the MD request [206], could not be conducted.

Nonetheless, measurements during MD3312 generated a plethora of data, revealing crucial insights into the physics of HL-LHC beam optics as shown in the following chapters.

6.2 Measurement Summary

6.2.1 Full Procedure

A detailed timeline of the MD procedure is provided by Tables 6.1 and 6.2. Key parameters of the MD are summarized in Table 6.3.

6.2.2 Results

Influence of Tune-Feedback on ATS-Optics

The machine was set up with the same optics as during MD2148 [210], apart from the separation bumps, which were zero during MD2148. The β -beating was then measured while at the same time checking for coupling. Comparison to the β -beating from MD2148 showed a difference of about 5%.

Simulations were run to check whether this deviation could arise from orbit change (Figs. 6.1 and 6.2). Yet, these could not explain the observed difference (Figs. 6.3 and 6.4).

Intensive investigations exposed powering differences of the MQTs [212], regulated by the tune-feedback [213, 214]. Further simulations concluded that the ATS optics [2–5] utilized in the LHC to achieve low β^* , and especially the flat-optics, are susceptible to these changes (Figs. 6.5 and 6.6), as in these optics larger β -functions and are used

Table 6.1. MD Time-line (Part 1). Key measurements are shown in **bold**.

12:00→13:50	MD Setup. Get machine to end-of-squeeze, flat-optics, 6.5TeV.
13:50→14:05	Coupling Measurements: <div>Beam 1: $(-2 + 12i) \times 10^{-4}$</div> <div>Beam 2: $(-14 + 6i) \times 10^{-4}$</div>
14:05→14:15	K-Modulation: IP5, 5A right: 14:07:10→14:09:50 left: 14:09:50→14:12:29
14:15→14:44	Trim waist-shift for β^* correction. Trim: <i>LHCBEAM/2018_global_atl_flat_b1_for_ip5_waist</i> Stepwise 0% to 80%.
14:44→14:50	K-Modulation: IP5, 5A right: 14:44:00→14:46:30 left: 14:46:40→14:49:20
14:30→14:50	β-beating measurements. <div>Beam 1 kickgroup: <i>b1_after_betastar_corr</i></div> <div>Beam 2 kickgroup: <i>b2_afterbetastarcorr</i></div>
14:55→15:02	Trimming RCTX , 60A Trim: <i>LHCBEAM/2018_MD4_replicatingHL_b6</i> to 1.0
Beam 1	Amplitude Detuning
15:07	Working point at: $Q_{x,y}$: 0.283, 0.31 $Q_{x,y}^{ACD}$: 0.271, 0.325
15:07→15:23	Vertical kicks for amplitude detuning. (10% horizontal) Kickgroup: <i>b1_for_amplitude_detuning_vertical</i>
15:23→15:30	Horizontal kicks for amplitude detuning. (10% vertical) Kickgroup: <i>b1_for_amplitude_detuning_horizontal</i>
15:45	Moving working point to: $Q_{x,y}$: 0.283, 0.31 $Q_{x,y}^{ACD}$: 0.275, 0.30
15:45→15:45	Vertical kick for amplitude detuning. (10% horizontal) <i>One kick only!</i> Kickgroup: <i>b1_for_amplitude_detuning_vertical_newwp</i>
Beam 2	Dynamic Aperture
15:10→15:20	DA measurements. Kickgroup: <i>Beam2_after_b6</i> $Q_{x,y}$: 0.28, 0.31 $Q_{x,y}^{ACD}$: 0.272, 0.32
15:25→15:40	DA measurements. Kickgroup: <i>Beam2_after_b6_inverted_QyDelta</i> $Q_{x,y}$: 0.28, 0.31 $Q_{x,y}^{ACD}$: 0.272, 0.30

Table 6.2. MD Time-line (Part2). Key measurements are shown in **bold**.

Beam 2	Dynamic Aperture (<i>continued</i>)
15:40→15:55	DA measurements. Kickgroup: <i>Beam2_after_b6_CouplingWorkingPoint</i> $Q_{x,y}$: 0.28, 0.31 $Q_{x,y}^{ACD}$: 0.292, 0.30
15:55→16:05	DA measurements. Kickgroup: <i>Beam2_after_b6_OriginalWorkingPoint</i> $Q_{x,y}$: 0.28, 0.31 $Q_{x,y}^{ACD}$: 0.272, 0.32
16:15→16:20	Trimming RCOX Trim: <i>LHCBEAM/2018_MD4_replicatingHL_b4</i> to -0.74
16:20→17:00	Trimming RCSSX, RCSX, RCSOX Trim: <i>LHCBEAM/2018_MD4_replicatingHL_a3</i> to 0.85 Trim: <i>LHCBEAM/2018_MD4_replicatingHL_b3</i> to 1.0 Trim: <i>LHCBEAM/2018_MD4_replicatingHL_a4</i> to 1.0
17:00→17:03	Wirescans, average emittance: Beam 1 x : 4.04 μm , y : 3.89 μm Beam 2 x : 2.55 μm , y : 4.39 μm
Beam 1	Amplitude Detuning
17:14	Working point: $Q_{x,y}$: 0.283, 0.31 $Q_{x,y}^{ACD}$: 0.271, 0.325
17:14→17:30	Vertical kick for amplitude detuning. (1% horizontal) Kickgroup: <i>b1_for_amplitude_detuning_allcor_vertical</i>
Beam 2	Dynamic Aperture
17:14→17:22	DA measurements. Kickgroup: <i>Beam2_after_ALL_Correctors</i> $Q_{x,y}$: 0.274, 0.31 $Q_{x,y}^{ACD}$: 0.266, 0.30
17:14→17:22	DA measurements. One kick only! Kickgroup: <i>Beam2_after_ALL_Correctors_OriginalWorkingPoint</i> $Q_{x,y}$: 0.274, 0.31 $Q_{x,y}^{ACD}$: 0.266, 0.32
17:29→17:30	DA measurements. One kick → dump! Kickgroup: <i>Beam2_after_ALL_Correctors_CouplingWorkingPoint</i> $Q_{x,y}$: 0.274, 0.31 $Q_{x,y}^{ACD}$: 0.286, 0.32

Table 6.3. Key MD parameters.

Objective:	Replication of HL-LHC Dynamic Aperture and Amplitude Detuning.
MD #:	3312
Operators:	Markus Albert, Matteo Solfaroli Camillocci
Fill #:	7391
Beam Process:	MD → SQUEEZE-6.5TeV-ATS-65cm-60_15cm-2017_V1_ATSFlat@526_[END]
Date:	30.10.2018
Start Time:	12:00
End Time:	17:30
Optics:	R2017aT65_A60_15C15_60A10mL300
Crossing:	No crossing
Separation:	[0.3 / 1.4 / 0.3 / -1] mm in [IP1 / IP2 / IP5 / IP8] , <i>Plane:</i> [V / V / H / H]
Offset:	No offset

in every second arc (the “ATS arcs”), as well as a the shift in phase advance between MQTs from the 90° phase advance in the non-ATS configuration, which minimizes the generated β -beating. The seen β -beating difference in Beam 1 can be fully explained by the powering change as seen in Fig. 6.7. In Beam 2, as shown in Fig. 6.8, the agreement is not as good as for Beam 1, yet this can be attributed to the change in beating being small here and a noisy measurement.

These findings have been presented at the LHC Machine Committee (LMC) meeting [215]. It was concluded that a strategy was needed for the tune-feedback to take these effects for ATS optics into account in the future, for example by excluding the MQTs in the ATS arcs from the tune feedback.

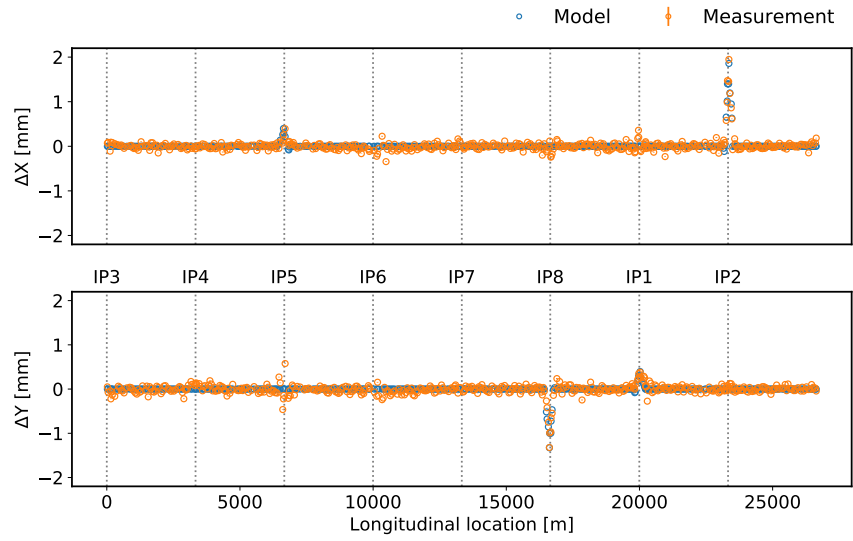


Figure 6.1. Orbit difference between the two MDs in Beam 1, by measurement (orange) and model via MAD-X simulation (blue).

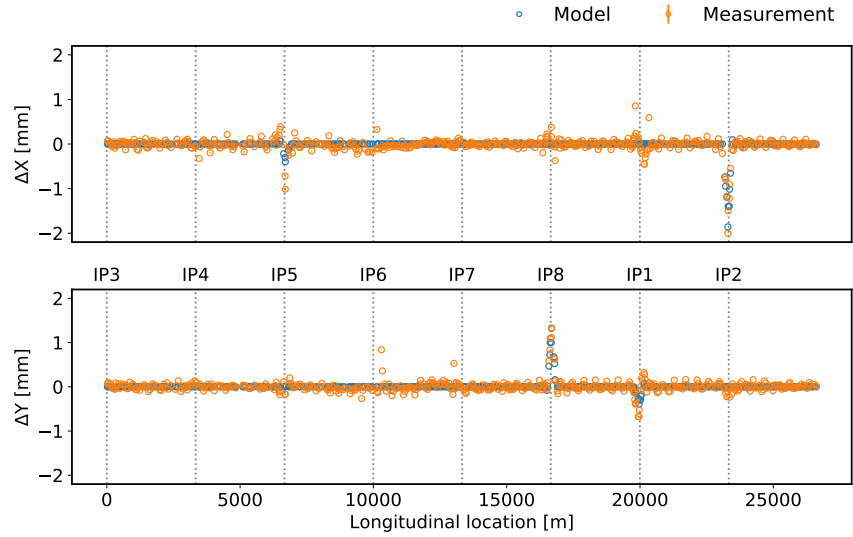


Figure 6.2. Orbit difference between the two MDs in Beam 2, by measurement (orange) and model via MAD-X simulation (blue).

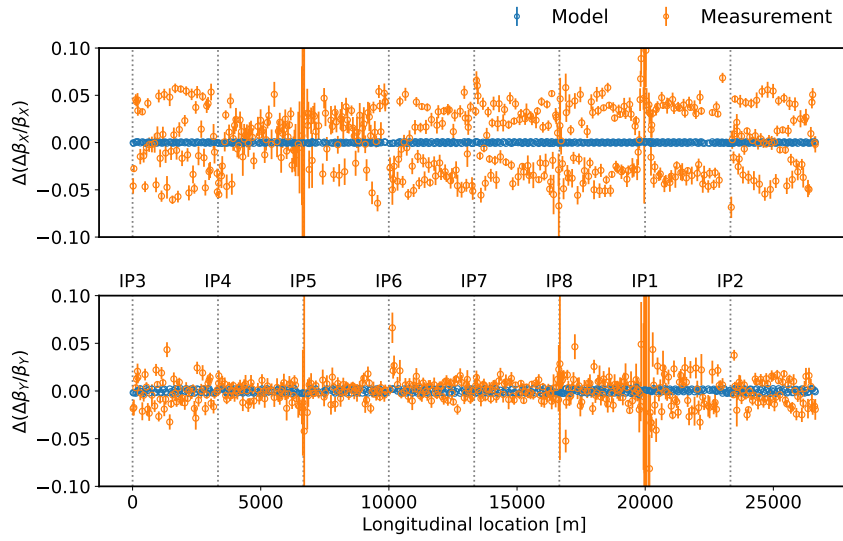


Figure 6.3. β -beating difference between the two MDs in Beam 1, by measurement (orange) and model via MAD-X simulation (blue).

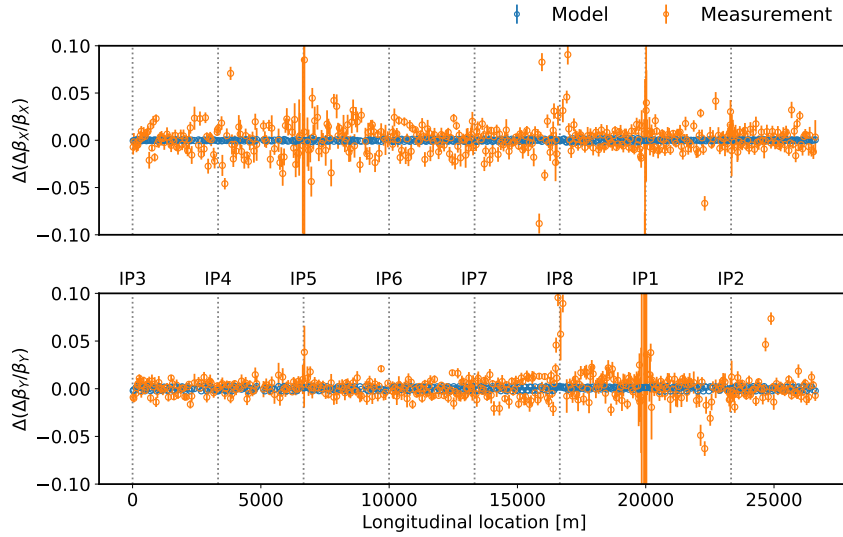


Figure 6.4. β -beating difference between the two MDs in Beam 2, by measurement (orange) and model via MAD-X simulation (blue).

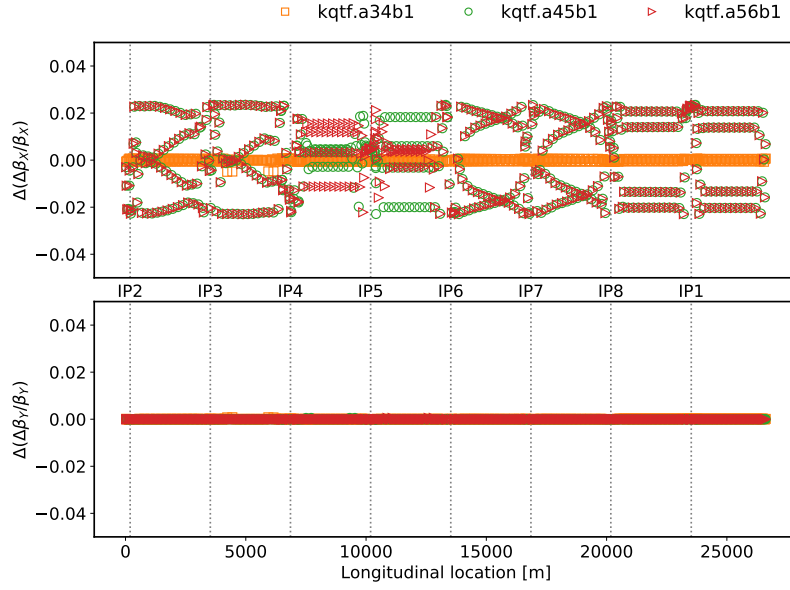


Figure 6.5. Simulation results of β -beating differences from ΔK values in Beam 1. Compared are the results for **kqtf.a56b1** and **kqtf.a45b1**, which are in the ATS arcs and have both a $\Delta K \approx 5.4 \times 10^{-5}$ and **kqtf.a34b1** with $\Delta K \approx 9.0 \times 10^{-5}$. Despite the larger ΔK of the latter, its influence on β is much smaller as arc34 is not an ATS arc.

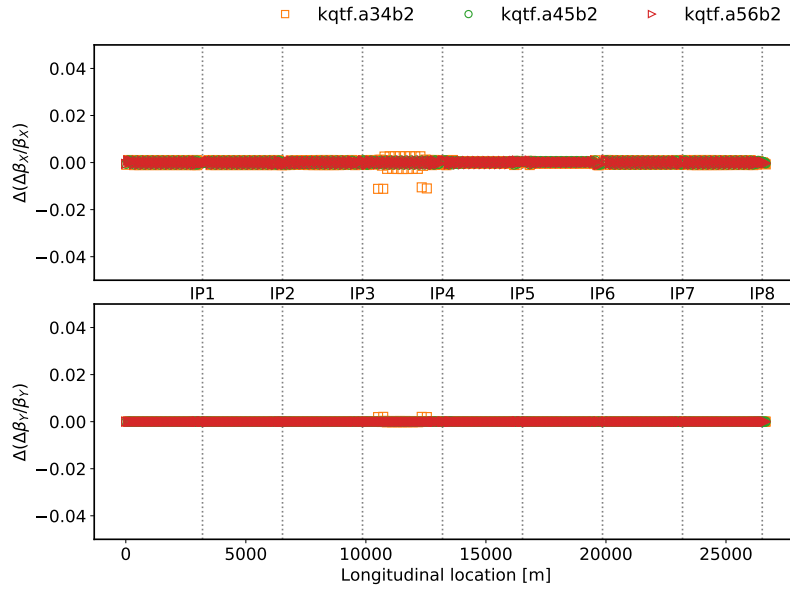


Figure 6.6. Simulation results of β -beating differences from ΔK values in Beam 2. Compared are the results for **kqtf.a56b2** and **kqtf.a45b2**, which are in the ATS arcs and have both a $\Delta K = 0.2 \times 10^{-5}$ and **kqtf.a34b2** with $\Delta K \approx 18.2 \times 10^{-5}$. The influence on β of all magnets is small compared to Beam 1 (Fig. 6.5), as their change in K is either small, or - in the latter case - they do not lie in the ATS arcs.

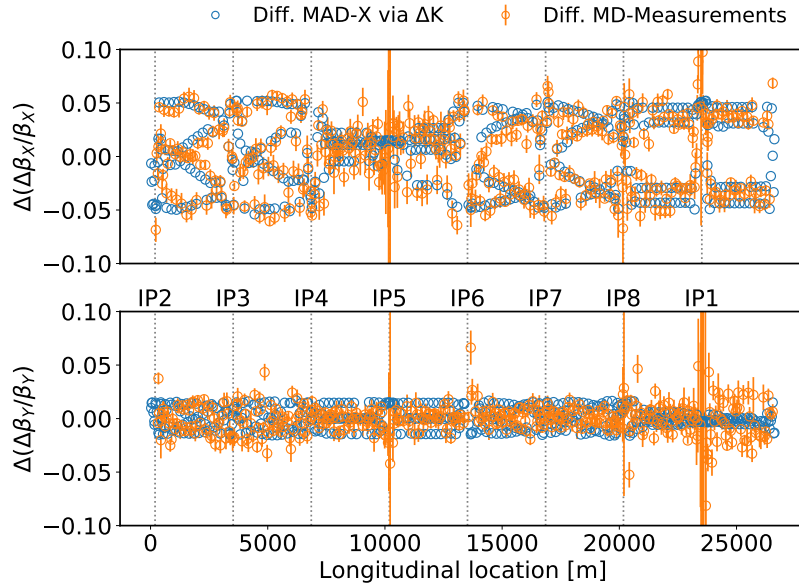


Figure 6.7. Comparison between the difference in Beam 1 in β -beating from MAD-X simulations from the different MQT powering and orbit between the two MDs.

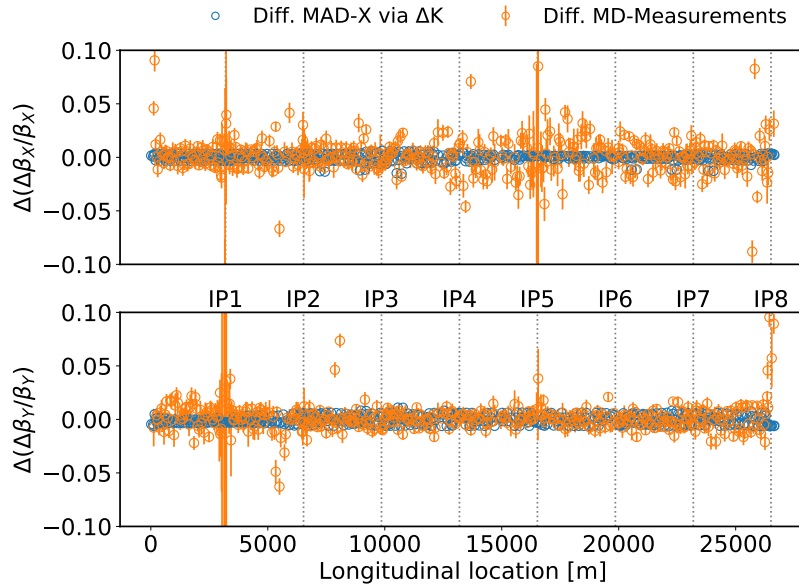


Figure 6.8. Comparison between the difference in Beam 2 in β -beating from MAD-X simulations from the different MQT powering and orbit between the two MDs.

Waist correction

During the flat-optics MD2148 in June a waist shift of about ± 7 cm, in the X/Y plane respectively, was observed in Beam 1 in IP5 (see Table 6.4). As in June, K-modulation was performed in the present MD to check on the current status of this waist shift, which turned out to be much larger this time: Shifts between 12 cm and 26 cm were found, changing the β^* in the IP drastically (Table 6.4).

A correction, using the global correctors in the machine, was calculated on the fly by means of the iterative correction functionality [124] of the Beta-Beat-Gui [216]. Despite many tries, the waist shift correction was always predicted to induce some additional β -beating in the machine. As the β -beating was already 5% higher than expected (see Section 6.2.2), the correction (named *LHCBEAM/2018_global_atl_flat_b1_for_ip5_waist*) was trimmed in stepwise and only to 80%. This corrected the waist shift successfully, as can be seen in Table 6.4. The increase in β -beating, dominant in the vertical plane of Beam 1, is shown in Figs. 6.9 and 6.10.

Table 6.4. Results from K-modulation in IP5 with 4.5 A from MD2148 and this MD, before and after trimming the global correction for the observed waist shift.

	Beam 1			
	β^* [cm]		Waist [cm]	
	X	Y	X	Y
MD2148	18.0 ± 1.2	59.6 ± 0.2	-6.7 ± 1.2	6.7 ± 1.0
MD3312 before trim	24.1 ± 1.5	69.9 ± 3.5	-11.6 ± 0.9	22.1 ± 4.5
MD3312 after trim	16.9 ± 0.5	64.0 ± 0.4	-5.2 ± 0.7	4.0 ± 2.6
	Beam 2			
	β^* [cm]		Waist [cm]	
	X	Y	X	Y
MD2148	15.3 ± 0.1	61.2 ± 0.2	1.8 ± 0.4	3.2 ± 1.1
MD3312 before trim	51.4 ± 7.2	70.7 ± 9.1	23.2 ± 2.1	-25.8 ± 9.0
MD3312 after trim	15.3 ± 0.3	61.9 ± 1.1	1.7 ± 1.0	6.6 ± 4.0

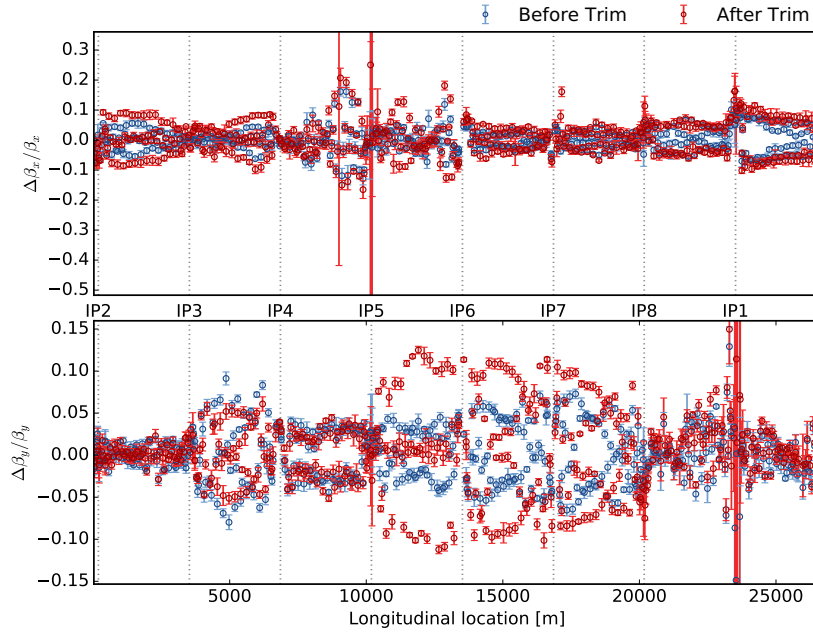


Figure 6.9. β -beating in Beam 1 before and after waist-correction trim for IP5.

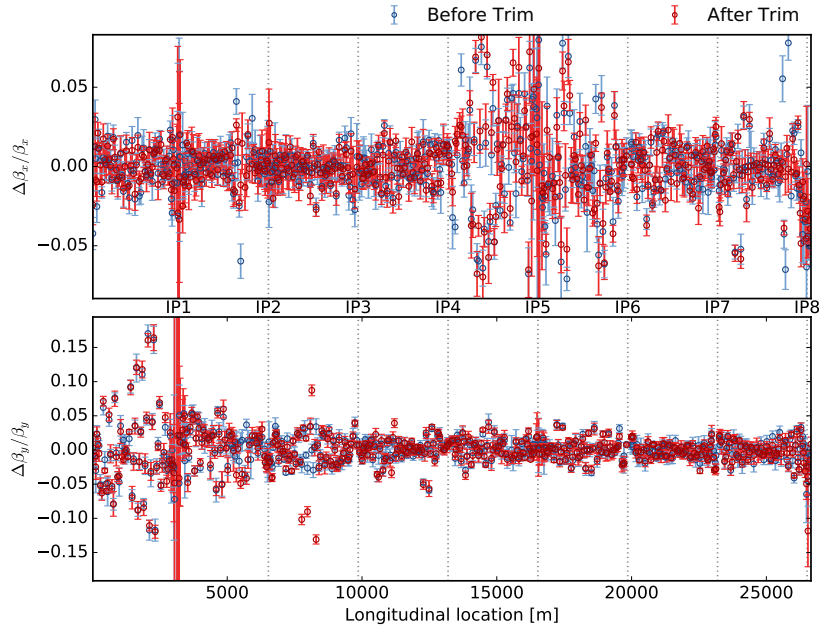


Figure 6.10. β -beating in Beam 2 before and after waist-correction trim for IP5.

High-Order Field Error Trimming

The MD was split into two parts: First only the expected normal dodecapolar errors (b_6) were trimmed to simulate errors in the HL-LHC. Knob name, trim value and circuit

can be found in Table 6.1. After finishing measurements at this setting, sextupolar and octupolar errors (a_3 , b_3 , a_4 , b_4) were put into the machine on top. Knob names, trims and circuit can be found in Table 6.2.

The change in beam lifetime, extracted from the Beam Loss Monitor (BLM), and in beam intensity, extracted from the (Fast) Beam Current TransFormeR (BCTFR), are shown in Figs. 6.11 to 6.13.

Despite a slight recovery of lifetime after trimming in the b_6 knob, overall there is a lifetime depletion of 25-30% (Fig. 6.11). While the trim of the b_4 knob does not influence the lifetime at all (Fig. 6.12), the trimming of the remaining knobs (Fig. 6.13) leads to a pronounced deterioration of beam lifetime again, which the beams do not fully recover from, leading to a loss in lifetime of 20-25%.

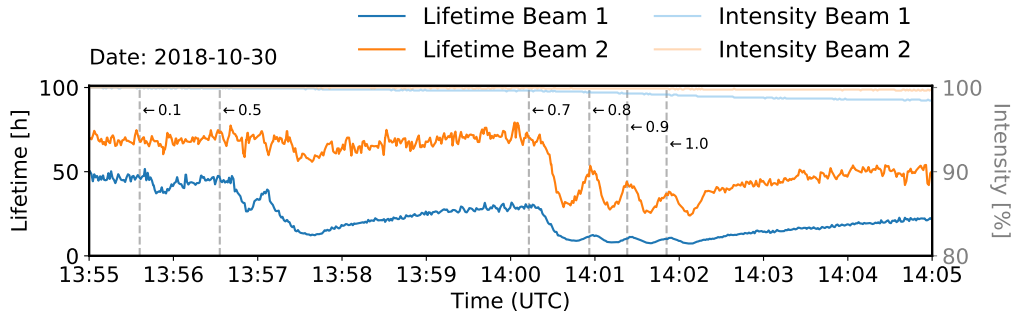


Figure 6.11. Beam lifetime (from BLMs) and intensity (from BCTFR) during trimming of b_6 knob. The markers show the current trim values.

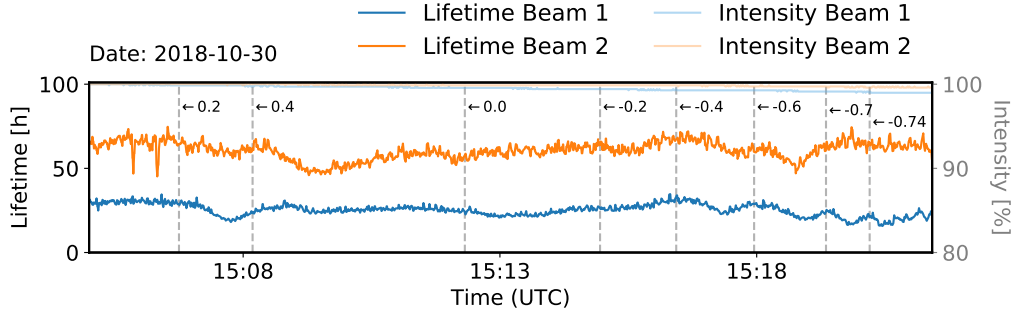


Figure 6.12. Beam lifetime (from BLMs) and intensity (from BCTFR) during trimming of b_4 knob. The markers show the current trim values.

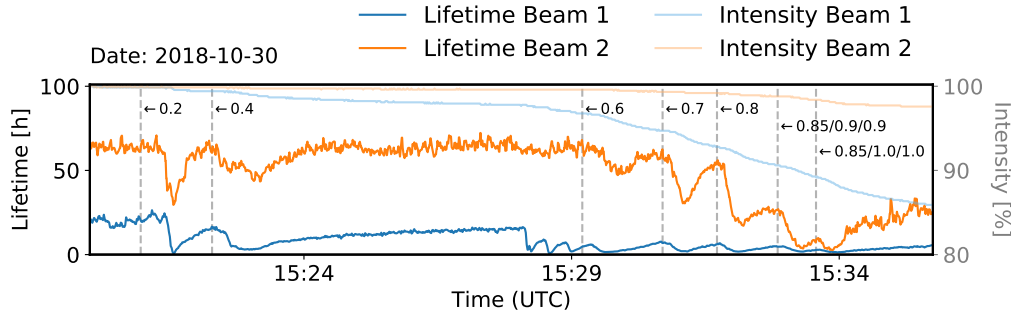


Figure 6.13. Beam lifetime (from BLMs) and intensity (from BCTFR) during trimming of a_3 , b_3 and a_4 knobs. The markers show the current trim values. Multiple values correspond to the $a_3/b_3/a_4$ knobs respectively.

Amplitude Detuning

All amplitude detuning measurements were done using the AC-Dipole (“ACD”). Its influence on the amplitude detuning direct terms, as described in [Section 2.4.1](#), will need to be corrected for. See [\[43\]](#) for a detailed description of previous amplitude detuning measurements.

The measurements were conducted at two different working points of the AC-Dipole, while keeping the natural tunes constant at the injection tunes, as shown in the tune diagram in [Fig. 6.14](#).

Beam Lifetime. The beam lifetime measurements were very successful and conclusions could already be drawn from the beamlifetime: At the original AC-Dipole working point at $Q_{x,y}^{ACD} = 0.271, 0.325$, (working point 1 in [Fig. 6.14](#)) it was possible to kick to peak-to-peak amplitudes useful for analysis of 2.6 mm vertical and 1.6 mm horizontal (at the $\beta = 180$ m BPMs in the arcs). From the recovery of the beam lifetime in [Fig. 6.15](#), we can see that the losses during these kicks are recovering very slowly, whereas after changing the working point to $Q_{x,y}^{ACD} = 0.275, 0.3$ (working point 2 in [Fig. 6.14](#)) the losses recover fast after the kick. After applying all magnetic field errors expected in the HL-LHC, only vertical kicks could be performed, as the beams were dumped due to triggering the safety measures with an unfortunate kick in Beam 2. Up to then a kick amplitude of only 1.9 mm vertically could be reached, with again only slowly recovering losses. It is not clear what causes the difference between the slow and fast losses in this case, during excited kicks fast losses, only during the excitation period, are the expected behaviour for both hitting a dynamic and a physical aperture, as the beam is brought back to the closed orbit after each kick.

Detuning. Analysis of the amplitude detuning, similar to [\[43\]](#), has been attempted, yet despite the good kick-amplitudes during the first part of the measurements, the acquired data is very noisy and the natural tune could not be accurately identified in

the spectrum. A more detailed analysis with the new software tools [6] or getting the tune of the machine directly from the spectrum of the BBQ-BPM [217, 218] could turn out to be fruitful.

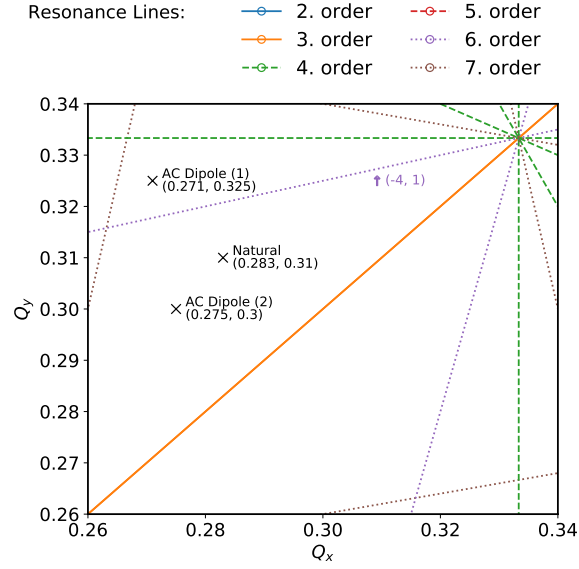


Figure 6.14. Tune diagram for the working points set during amplitude detuning measurements. The natural tunes were fixed at the injection tunes, while the AC-Dipole tunes were set first to (1), then (2) and back to (1) for the last measurement.

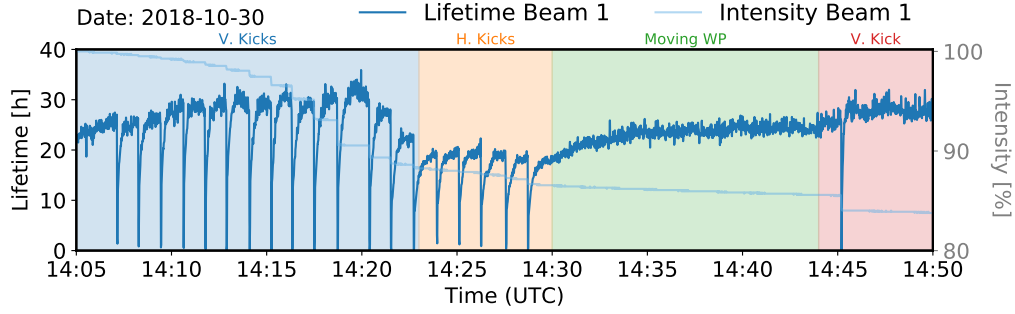


Figure 6.15. Beam lifetime (from BLMs) and intensity (from BCTFR) during the first part of the kicks for amplitude detuning in Beam 1.

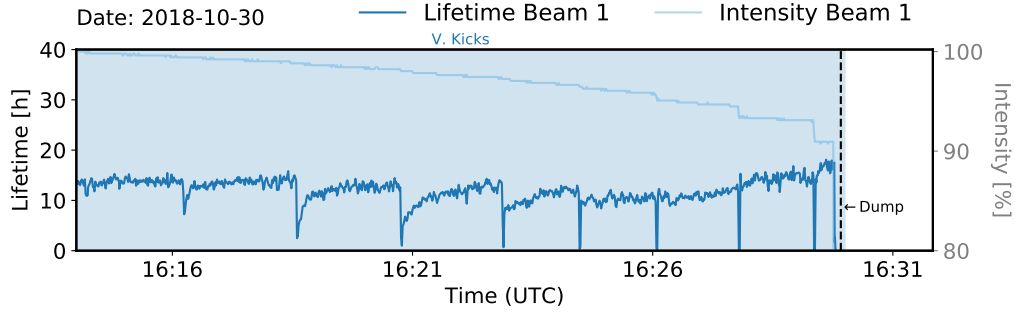


Figure 6.16. Beam lifetime (from BLMs) and intensity (from BCTFR) during the second part of the kicks for amplitude detuning in Beam 1.

Dynamic Aperture

In Fig. 6.17 diagonal kicks for measuring DA with b_6 errors applied at the first two working points $Q_{x,y}^{ACD} = 0.272, 0.32$ and $Q_{x,y}^{ACD} = 0.272, 0.30$ show rapid losses and a quick recovery of beam lifetime. While for single kicks this is hinting at hitting a physical aperture, for excited kicks this is the expected behaviour for both, as the beam is brought back to the closed orbit after each kick. On the other hand, the reached maximum peak-to-peak kick amplitudes (at the $\beta = 180$ m BPMs in the arcs) of 1.0 mm and 1.4 mm respectively are below optimum for analysis. At the third working point $Q_{x,y}^{ACD} = 0.292, 0.30$ higher amplitudes of 1.5 mm horizontal and 1.3 mm vertical were reached, and the lifetime recovery was much slower after each kick. With these amplitudes, the data looked more promising for DA analysis similar to [161], yet evaluation of the data still proved to be difficult and has evaded any attempt of analysis so far. Going back to the first working point, the lifetime behaviour from before returned and the maximum kick amplitude went back down to 1.0 mm.

After applying the remaining errors and switching to $Q_{x,y}^{ACD} = 0.292, 0.30$ larger kicks than before could be performed, up to 1.3 mm horizontal and 1.1 mm vertical, which is

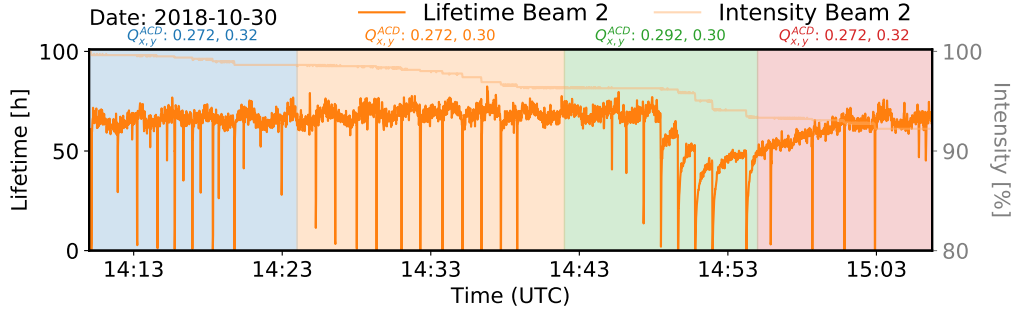


Figure 6.17. Beam lifetime (from BLMs) and intensity (from BCTFR) during the first part of the kicks for dynamic aperture in Beam 2.

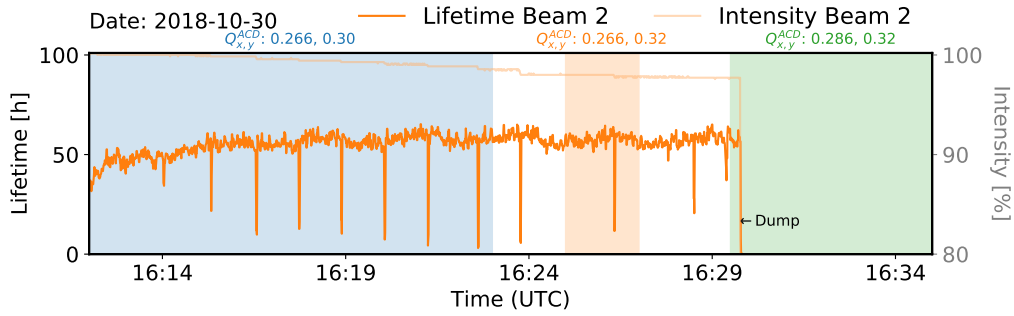


Figure 6.18. Beam lifetime (from BLMs) and intensity (from BCTFR) during the second part of the kicks for dynamic aperture in Beam 2.

still less than optimal. Beam lifetime in Fig. 6.18 shows again rapid recovery after each kick. Back near the original working point, at $Q_{x,y}^{ACD} = 0.266, 0.32$, only a single kick was performed, as 37% of losses were already seen at 2% AC-Dipole strength, which reached a peak-to-peak amplitude of less than 0.9 mm.

The last working point $Q_{x,y}^{ACD} = 0.286, 0.32$ was too close to the dodecapole resonance line $(-4, 1)$ (purple in Fig. 6.19b) and the beam was dumped due to high losses on the first kick.

Analysis of the data has been attempted, using the forced Dynamic Aperture (DA) algorithm described in [94, 161] and implemented in [149], but due to the quality of the data, no conclusive result could be found thus far.

Resonance Driving Terms

In addition to amplitude detuning and DA, the study of RDTs [102, 105, 152] can give valuable insight into the beam stability of the LHC and have been studied in the past [12, 13, 161, 219]. Recent research shows, that measurement and correction of specific RDTs is feasible and leads to improved machine performance [94, 219]. Analysis of the

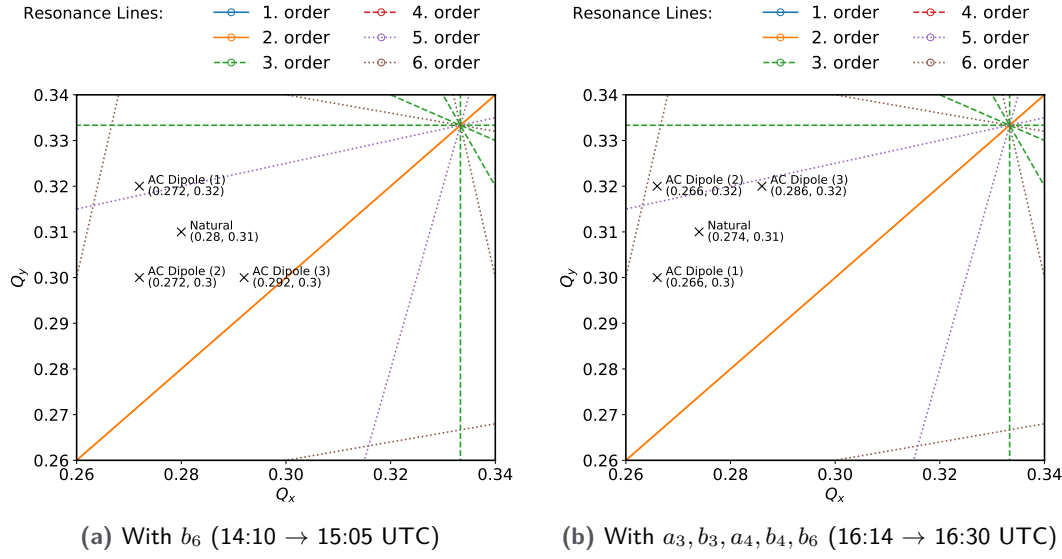


Figure 6.19. Tune diagrams for DA measurements.

data gathered during this MD is still to be performed and could provide first insights into the RDTs to be expected in the HL-LHC.

6.3 Conclusion

MD3312 has been a very demanding MD, yet lots of data has been collected to lead the way towards a successful HL-LHC commissioning.

Initial confusion about the increase in β -beating has revealed the strong influence of the tune-feedback when using ATS optics and shows that one has to be cautious when using these sensitive optics.

Another challenge of these advanced optics encountered during the MD was the observed waist-shift in IP5 which was successfully corrected on the fly. A correction crucial for keeping the luminosity high in the HL-LHC.

Further, this MD hints at an expected reduced beam lifetime originating from uncorrected high-order field errors. Analysis of these field errors will be an important part of the HL-LHC commissioning.

The applicability of two measures for non-linear commissioning, DA and amplitude detuning, was tested. Under the influence of high-order errors the choice of working-point is crucial, as the resonances can be so enhanced, that losses were too high at too low amplitudes for usable measurement data. At first glance amplitude detuning measurements were successful at two working points under test, yet the data proved to be too noisy for analysis. Detuning measurements after this MD followed the low ΔQ -procedure described in [Section 2.4.1](#) and produced much better data. DA studies proved to be even more difficult yet a seemingly working configuration was found, but is

still evading analysis.

The noisiness of the measurement data taken during this MD provides also a valuable insight into the challenges of future commissioning of the HL-LHC: Measurements with high-order nonlinear errors present in the machine will be even more demanding than in the LHC and will require additional preparation and new strategies, to establish consistent methods unperturbed by the presence of these errors.

In the LHC the current strategy for correction is an alternation of linear and nonlinear optics commissioning: linear optics are corrected first at flat-orbit to allow for the nonlinear commissioning to be successful. After correcting sextupole and octupole errors, the linear corrections can be re-optimized to account for the changed optics and improved measurement quality [17]. The experience in this MD show, that an adapted scheme is likely needed for HL-LHC, in which also the current non-linear correction is iterated upon, after correcting high-order errors.

7 Dodecapole correction in the LHC

Note. The results of the first half of this chapter has been presented by me at IPAC in 2022 and has been published in [220], while the second half contains a modified version of the peer-reviewed paper published with Physical Review Accelerators and Beams (PRAB) [221].

Acknowledgements. The data presented in this chapter has been measured by the people listed as authors in [221] in 2022 during LHC machine commissioning and during MD, with the exception of L. Malina^{ORCID} and T. Pugnati^{ORCID}.

L. Malina^{ORCID}'s contribution to the study has been manifold: He is the main author of the `omc3-python` package [6], in particular the harmonic analysis module `Harpy` [136], and has as such written a lot of the underlying functionality used to analyse the amplitude detuning data on which the presented corrections are based. He has further developed most of the statistical analysis functions and the outlier filter (described in Section 2.4.2), which have then be re-utilized by me for the action and tune extraction.

The error values used in the magnetic model of the simulation of Fig. 7.12 have been provided by T. Pugnati^{ORCID} and B. Dalena^{ORCID}.

7.1 Introduction and Motivation

Amplitude detuning, also known as “tune shift with amplitude”, describes the action-dependent tune shift in a storage ring and has significant impact on the tune footprint of the particle beams in the LHC, affecting DA and beam lifetime [17, 222]. It also influences the performance of beam instrumentation [17] and the control of Landau damping. Landau damping of head-tail instabilities is induced by dedicated octupole magnets (Landau Octupoles or MOs) in the arcs around the LHC [79].

To first order in field strength and first order in action, detuning is generated by normal-oriented octupole fields, weighted by powers of the β -functions at the octupole sources [195, 223] (see Section 7.2.1). Reduction of undesired detuning from octupole errors therefore directly relates to improved control of Landau damping, possibly reducing the necessary powering of the MOs.

It has been established, that in the LHC during proton-proton collisions, the main source of unwanted detuning are normal octupole errors from the quadrupole triplets [14, 17, 34, 177] left and right of the insertions housing the experiments of ATLAS at IP1 and CMS at IP5. The beam dynamics in the IRs around the IP are very sensitive to errors, due to their high β -functions, required to reach very small β^* , the value of the β -function at the IP. Error correction in the IRs has therefore been subject to

intensive studies during the design phase and in the actual machine, not only in the LHC [8, 14, 17, 174, 177, 224], but in most circular colliders, such as the Tevatron [225], RHIC [226], SuperKEKB [227] and in the FCC [228–230]. One of the upgrades envisioned for the HL-LHC [22], are triplets with larger aperture, increasing the coil aperture from 70 mm in the LHC [34] to 150 mm. This allows for a doubling of the β -function in the triplets, supporting β^* down to 15 cm [85] to raise collision rates and hence the name-giving high luminosity. Yet this change elevates the already high sensitivity to errors in this region [24, 26]. The search and testing for suitable correction targets for increasingly higher field orders has been a long ongoing journey at the LHC [11, 33, 94, 128, 177].

The regions of the LHC housing the triplets are occupied by common-aperture magnets, meaning both circulating beams, named Beam 1 and Beam 2, share a single aperture between the so-called separation dipoles, which divide the beams again into their individual beamlines. The nonlinear correction packages [34] are located close to the triplets [17] within this region. A schematic of an IR of the LHC is given in Fig. 2.9. Any correction needs hence to be valid for both beams.

One of the observables targeted to correct IR-errors has been amplitude detuning. Amplitude detuning is a global parameter and therefore cannot display the individual contribution of each IP to the total detuning. To determine the origin of the errors, local orbit bumps have been introduced during measurements, generating feed-down (see Section 2.2.2) to tune and coupling which could then be mitigated using the octupole correctors within the nonlinear correction packages in each IR. Performing this correction has also led to immediate improvements in the performance of beam instrumentation, e.g. in the accuracy of the coupling measurement and the signal-to-noise ratio of the BBQ [218, 231, 232] system [17].

During LHC commissioning in 2018, upon changing the crossing scheme from flat-orbit (i.e. no crossing angles active) to an collision scheme (see Table 7.1), an increase in amplitude detuning was measured (see Table 7.2 and [36, 37]). Further investigation [43] during MD sessions confirmed this finding and revealed the main contribution to be feed-down from high-order errors, i.e. decapole and dodecapole errors and above, to the octupole fields, due to the crossing schemes in IP1 and IP5. While feed-down to tune from dodecapole fields induced by orbit bumps had been observed in other colliders [18], there have been no measurements of feed-down to first-order amplitude detuning outside of the LHC.

As the influence of the feed-down of magnetic fields depends on the (small) orbit offset raised to the power of the field's order (see Section 2.2.2), field errors higher than dodecapole errors are ignored in the here presented study. Also sextupole errors and feed-down to them are omitted in the following discussion: not only are they less affected by feed-down from the high-order errors compared to octupole fields, they also contribute only in second order in field strengths to amplitude detuning [223] and their correction has been discussed in e.g. [14–17].

Dodecapole fields on the other hand are the first allowed harmonic of quadrupole magnets [35] and also contribute directly to second-order amplitude detuning (see Section 7.2.1). The harmful influence of decapole and dodecapole errors on DA and

beam lifetime in the upcoming HL-LHC has been shown in simulations and dedicated measurements, in which the normal dodecapole errors were artificially increased to replicate the HL-LHC conditions [33, 38, 169, 196, 205, 233].

In this chapter detuning measurements are presented, which have been performed in 2022 during LHC commissioning and in a dedicated MD session (with the ID MD6863), confirming the observed errors still to be present in LHC run 3. Important machine settings for these measurements are shown in Table 7.1 and a summary of the measured detuning terms can be found in Table 7.2. A correction option, previously only researched in simulations [196], is explored: Normal dodecapole errors in the IRs are corrected using the observed change in amplitude detuning from feed-down, by in turn utilizing the feed-down to normal octupoles from the dodecapole correctors in the nonlinear corrector packages of the IRs, which had been hitherto inactive during operation.

While measurements and corrections of magnetic fields up to dodecapole order, based on tune shifts using the feed-down artificially introduced by orbit bumps, had also been attempted at RHIC [18], these corrections were never (or only partially) used operationally, as they were detrimental to beam lifetime. On the other hand, a 4% increase of integrated luminosity was achieved using decapole and dodecapole correctors by targeting beam lifetime directly [234]. So while this is not the first time a particle collider uses dodecapole correctors operationally, it is the first time targeting the feed-down to amplitude detuning and hence also controlling Landau damping. For the LHC this is in fact also the first time these correctors are used in operation.

This chapter is structured as follows: In Section 7.2 additional theoretical background, notations and conventions are introduced, as well as the procedure for simulations and corrections outlined. This is followed by the actual presentation and discussion first of the simulation results in Section 7.3 and then of the experimental results in Section 7.4. Conclusions are drawn in Section 7.5, in which also outlook for further studies is suggested.

7.2 Background

The measurement procedure in this study is performed as explained in Section 2.4.1, resulting fit values and their standard deviation for all relevant measurement scenarios can be found in Table 7.2: the measurement results from 2018 which triggered the study, and from commissioning and the MD in 2022. A discussion of the data is presented in Sections 7.3 and 7.4.

7.2.1 Amplitude Detuning from Dodecaipoles

To numerically compute detuning integrals Eqs. (2.43) and (2.44) discussed in Section 2.2.6, a thin lens approximation is used in which multipole elements are split into kicks at s_w surrounded by drift spaces; the kick strength being $K_n L_w$ - the integrated strength $K_n(s)$ (see Eq. (2.10)) over the length L_w of (possibly partial) element w . β -function and orbit are then also approximated using the value at s_w . In this approxi-

Table 7.1. Machine Settings used during measurements. Crossing is given in half-angles. The horizontal plane is X, the vertical plane is Y. MD3311 is the Machine Development (MD) session in 2018 [43] first investigating the feed-down from high-order errors, MD6863 the session in 2022 dedicated to dodecapole corrections.

TUNE:		$Q_x = 62.28,$		$Q_y = 60.31$	
OPTICS:		$\beta^* = 30$ cm round-optics ($\beta^*_x = \beta^*_y$)			
Commissioning		28.04.2018		full crossing	
ENERGY:		6.5 TeV			
CROSSING:	IP-Plane	IP1-Y	IP2-Y	IP5-X	IP8-X
	μ rad	160	200	160	-250
SEPARATION:	IP-Plane	IP1-X	IP2-X	IP5-Y	IP8-Y
	mm	-0.55	1.4	0.55	1.0
MD3311		16.06.2018		IP5 crossing	
ENERGY:		6.5 TeV			
CROSSING:	IP-Plane			IP5-X	
	μ rad			160	
SEPARATION	IP-Plane			IP5-Y	
& OFFSET:	mm			0.55 -1.8	
Commissioning		09.05.2022		full crossing	
ENERGY:		6.8 TeV			
CROSSING:	IP-Plane	IP1-Y	IP2-Y	IP5-X	IP8-X
	μ rad	-150	0	150	0
SEPARATION:	IP-Plane	IP1-X	IP2-X	IP5-Y	IP8-Y
	mm	0	0	0	0
Commissioning		04.06.2022		full crossing	
& MD6863		24.06.2022		full crossing	
ENERGY:		6.8 TeV			
CROSSING:	IP-Plane	IP1-Y	IP2-Y	IP5-X	IP8-X
	μ rad	-160	200	160	-200
SEPARATION:	IP-Plane	IP1-X	IP2-X	IP5-Y	IP8-Y
	mm	0	1.0	0	1.0
MD6863		24.06.2022		IP5 crossing	
ENERGY:		6.8 TeV			
CROSSING:	IP-Plane	IP1-Y	IP2-Y	IP5-X	IP8-X
	μ rad	0	200	± 160	-200
SEPARATION:	IP-Plane	IP1-X	IP2-X	IP5-Y	IP8-Y
	mm	0	1.0	0	1.0

Table 7.2. Summary of amplitude detuning measurements from 2018 [36, 37, 43] at 6.5 TeV and $\beta^* = 0.3$ m and 2022 at 6.8 TeV and $\beta^* = 0.3$ m. Detailed orbit setups are described in Table 7.1 while here the measurements are identified by either “flat-orbit” (i.e. crossing bumps deactivated) or which of IP1 and IP5 are activated and at (“@”) which half-angle. If both are mentioned with different signs, the top sign refers to IP1 and the bottom sign to IP5. The four detuning terms are abbreviated as in Eq. (2.45): $Q_{a,b} = \partial Q_a / \partial (2J_b)$. Measurements for Beam 1 are shown in blue (top) and for Beam 2 in red (bottom). Where AC-Dipole kicks were used, the results have been corrected for the effect of forced oscillations [10].

	$Q_{x,x}$ [10^3 m^{-1}]	$Q_{y,x}$ [10^3 m^{-1}]	$Q_{x,y}$ [10^3 m^{-1}]	$Q_{y,y}$ [10^3 m^{-1}]
2018 MD3311	0.8 \pm 0.5	10 \pm 1	6 \pm 2	20 \pm 5
flat-orbit	−7.5 \pm 0.5	8 \pm 2	−2 \pm 1	6 \pm 1
2018 Commissioning	34 \pm 1	8 \pm 2	18 \pm 1	−38 \pm 1
IP1&5 xing @ +160 μ rad	−3 \pm 1	−10 \pm 3	−14 \pm 2	13 \pm 3
2018 MD3311	56 \pm 6	−9 \pm 15	−78 \pm 7	2.5 \pm 0.8
IP5 xing @ +160 μ rad	1.5 \pm 0.5	4 \pm 1	−4 \pm 3	12 \pm 1
2022 Commissioning w/o b_6	−15.4 \pm 0.9	32.2 \pm 0.2	33.7 \pm 1.0	−8.4 \pm 0.5
flat-orbit	−8.7 \pm 0.7	13 \pm 2	−3 \pm 2	10.0 \pm 0.9
2022 Commissioning w/o b_6	20 \pm 4	43 \pm 4	33 \pm 10	−10 \pm 3
IP1&5 xing @ \mp 150 μ rad	26.0 \pm 0.8	−31 \pm 3	−27 \pm 4	18 \pm 7
2022 Commissioning w/ b_6	−34 \pm 7	38 \pm 3	24 \pm 4	−6 \pm 1
flat-orbit	−15 \pm 2	10 \pm 3	6.3 \pm 0.4	−3.6 \pm 0.6
2022 Commissioning w/ b_6	−21 \pm 4	47 \pm 2	56 \pm 6	15 \pm 1
IP1&5 xing @ \mp 160 μ rad	−28 \pm 2	22 \pm 4	13 \pm 2	−7.8 \pm 0.5
2022 MD6863 w/o b_6	−18 \pm 2	32 \pm 2	22 \pm 4	−0.0 \pm 0.9
flat-orbit	−19.2 \pm 1.7	13.1 \pm 1.7	12 \pm 2	3.4 \pm 0.8
2022 MD6863 w/o b_6	9 \pm 2	36.8 \pm 2.0	27 \pm 2	2.1 \pm 1.0
IP1&5 xing @ \mp 160 μ rad	20.9 \pm 1.1	−39 \pm 2	−42.7 \pm 1.6	19.7 \pm 1.3
2022 MD6863 w/o b_6	23 \pm 2	1 \pm 2	−3.7 \pm 1.2	3.0 \pm 1.4
IP5 xing @ +160 μ rad	10.6 \pm 1.2	−8 \pm 3	−15.8 \pm 1.6	5.3 \pm 1.2
2022 MD6863 w/o b_6	8.9 \pm 1.4	4 \pm 3	−0.9 \pm 0.5	0.3 \pm 0.5
IP5 xing @ −160 μ rad	20.3 \pm 1.7	−15 \pm 4	−23.3 \pm 1.7	−1.5 \pm 1.6
2022 MD6863 w/ b_6	−12.7 \pm 1.0	33 \pm 2	30.1 \pm 1.0	17.5 \pm 1.4
IP1&5 xing @ \mp 160 μ rad	−46 \pm 4	31 \pm 2	34.5 \pm 1.4	−17.9 \pm 1.0

mation, the contribution to the first-order terms of Eq. (2.38) from octupole fields of all elements w with integrated strength K_4L_w can be calculated [100] as

$$\frac{\partial Q_x}{\partial(2J_x)} = \sum_w \frac{K_4L_w}{32\pi} \beta_x^2(s_w) \quad (7.1a)$$

$$\frac{\partial Q_x}{\partial(2J_y)} = \frac{\partial Q_y}{\partial(2J_x)} = \sum_w -\frac{K_4L_w}{16\pi} \beta_x(s_w)\beta_y(s_w) \quad (7.1b)$$

$$\frac{\partial Q_y}{\partial(2J_y)} = \sum_w \frac{K_4L_w}{32\pi} \beta_y^2(s_w) . \quad (7.1c)$$

Including feed-down, as in Eq. (2.21), from normal and skew decapoles (K_5L , J_5L) and normal and skew dodecapoles (K_6L , J_6L) we get due to the orbit $\Delta x(s_w)$, $\Delta y(s_w)$ at an element w

$$\begin{aligned} (K_4L_w)^{w/\text{ feed-down}} = & \\ & K_4L_w + \Delta x(s_w)K_5L_w - \Delta y(s_w)J_5L_w + \\ & \frac{1}{2} \left(\Delta x(s_w)^2 - \Delta y(s_w)^2 \right) K_6L_w - \Delta x(s_w)\Delta y(s_w)J_6L_w . \end{aligned} \quad (7.2)$$

With Eq. (2.44) in thin lens approximation, normal dodecapole magnetic fields contribute directly and linearly to the second-order detuning terms of Eq. (2.38) in the following manner:

$$\frac{\partial^2 Q_x}{\partial(2J_x)^2} = \sum_w \frac{K_6L_w}{384\pi} \beta_x^3(s_w) \quad (7.3a)$$

$$\frac{\partial^2 Q_y}{\partial(2J_x)^2} = \frac{\partial^2 Q_x}{\partial(2J_x)\partial(2J_y)} = \sum_w -\frac{K_6L_w}{128\pi} \beta_x^2(s_w)\beta_y(s_w) \quad (7.3b)$$

$$\frac{\partial^2 Q_x}{\partial(2J_y)^2} = \frac{\partial^2 Q_y}{\partial(2J_x)\partial(2J_y)} = \sum_w \frac{K_6L_w}{128\pi} \beta_x(s_w)\beta_y^2(s_w) \quad (7.3c)$$

$$\frac{\partial^2 Q_y}{\partial(2J_y)^2} = \sum_w -\frac{K_6L_w}{384\pi} \beta_y^3(s_w) . \quad (7.3d)$$

For brevity, from here on, the shorthands defined in Section 2.2.6 are used.

7.2.2 Correction Approach

The normal dodecapole corrector elements (**MCTX**) left and right of IP1 and IP5 are used to compensate for the measured detuning shift with crossing angle $\Delta Q_{a,b}$. As the contributions to detuning add up linearly, an equation system can be built with these

correctors as unknowns, targeting $-\Delta Q_{a,b}$:

$$-\begin{pmatrix} \Delta Q_{a,b}^{(B1)} \\ \Delta Q_{a,b}^{(B2)} \\ \vdots \end{pmatrix} = \begin{pmatrix} M_{a,b;L1}^{(B1)} & M_{a,b;R1}^{(B1)} & M_{a,b;L5}^{(B1)} & M_{a,b;R5}^{(B1)} \\ M_{a,b;L1}^{(B2)} & M_{a,b;R1}^{(B2)} & M_{a,b;L5}^{(B2)} & M_{a,b;R5}^{(B2)} \\ \vdots & \vdots & \vdots & \vdots \end{pmatrix} \cdot \begin{pmatrix} K_6 L_{L1} \\ K_6 L_{R1} \\ K_6 L_{L5} \\ K_6 L_{R5} \end{pmatrix}, \quad (7.4)$$

where the matrix elements are the detuning coefficients from Eq. (2.46) with feed-down from K_6 to K_4 (see Eq. (7.2))

$$M_{a,b;w}^{(BN)} = \frac{1}{2} \left(\Delta x_w^{(BN)^2} - \Delta y_w^{(BN)^2} \right) \tilde{\beta}_{a,b;w}^{(BN)}, \quad (7.5)$$

using the subscript short-hands LN_{IP} , RN_{IP} for the corrector elements left and right of $N_{IP} \in \{1, 5\}$. $BN \in \{B1, B2\}$ indicates the beam. Equation (7.4) can be extended, as hinted at by the vertical dots, to include multiple targeted detuning terms $\Delta Q_{a,b}$, as well as different setups, e.g. the same detuning term but with different crossing angles. The resulting equation system can then be solved, or the error on the solution minimized, by standard linear optimizers.

The measurement of the detuning cross-terms is, due to the low kick amplitude in the un-kicked plane, very challenging. Cross-term values in Table 7.2 for 2018 data had therefore been ignored for the simulated corrections in Section 7.3. Instead, they are either set to zero, or are transformed into inequalities, demanding $\Delta Q_{x,y}$ and $\Delta Q_{y,x}$ to be of negative value after correction, which, with the positive polarity of the Landau octupole currents in run 3, ensures beam stability [235]. This adds rows of inequalities to the equation system Eq. (7.4), which are used as boundary conditions while optimizing. This convex problem could be solved, using the python package CVXPY [236, 237].

In 2022 steps were taken to improve the cross-term measurement quality. In preparation of commissioning, the analysis was extended with 3D-fits, i.e. Q_a vs. J_x and J_y , to be able to evaluate the detuning of dual-plane kicks, which could have been performed to gain additional information. As this would have required additional kicks, it was first tried to improve the measurement by reducing ΔQ , the difference between forced- and natural tune (see also Section 2.4.1), which already led to clear cross-term tune signals in the spectrum. With the improvement in data quality, these cross-terms could be targeted by value instead of inequalities and all corrections are based on solving Eq. (7.4), targeting equality to the measured detuning shifts $\Delta Q_{a,b}$. During 2022 commissioning and the MD the detuning shifts were exclusively based on the difference between flat-orbit and with crossing in both IR1 and IR5 (see Table 7.1), due to the data from single IRs not yet being available at the time of the “online” correction calculations. These corrections could be tested and verified in the machine. After the data from

the measurements with individual crossing schemes in IR5 (again [Table 7.1](#)) had been analyzed, corrections were also calculated taking these into account and are discussed in [Section 7.4.2](#).

7.2.3 Correction Calculation

In simulations the nominal LHC is recreated in cypmad [132], a python wrapper for MAD-X [42], using the run 3 sequence and $\beta^* = 30$ cm round-optics. There are no errors included into the model. The orbit is set up according to the desired realization, as described in [Table 7.1](#).

From the optics functions, acquired using the TWISS command in MAD-X, and the measurement data the desired equation systems of [Eq. \(7.4\)](#) are built and a sum-of-squares minimization performed, either using CVXPY (in case of the simulation-study in [Section 7.3](#)) or using the (Moore-Penrose) pseudo-inverse of the matrix build from the $M_{a,b;w}$ derived from the model (in case of the study of 2022 data in [Section 7.4](#)). In the latter case uncertainties on the measurement-fits $\Delta Q_{a,b}$ are propagated to the resulting corrections. To check the validity of the calculations, the obtained corrector strengths are applied to the simulated lattice and the actual detuning change is determined from the PTC-module [131, 238] as well as from [Eqs. \(7.1\) and \(7.2\)](#).

The corrections can then be trimmed into the accelerator via the LHC Software Architecture (LSA) [239] and their actual effectiveness evaluated.

As the correction makes use of the normal dodecapole corrector magnets and is mostly aimed at correcting the b_6 (see [Section 2.2.1](#)) error components (see [Section 7.4.2](#)), the shorthand “ b_6 correction” is employed in the figures and tables of this chapter.

7.3 Simulation Results

A multitude of different correction scenarios have been investigated via simulations and a collection of the most meaningful correction attempts can be found in [Fig. 7.1](#). Shown are detuning values for correction, that is the opposite of the measured change in detuning between flat-optics and crossing scheme scenarios in [Table 7.2](#).

7.3.1 Correcting IP5

In a first attempt only the dodecapole correctors of IP5 were utilized, as the biggest contribution to detuning stems from this IP. The results are shown in scenarios A–C in [Fig. 7.1](#): When targeting a single term, e.g. the horizontal direct term (A), this specific term can be corrected very well, yet at the cost of the other terms. Especially, the large positive cross term in Beam 1, which is almost equal in magnitude to the corrected direct term, is of major concern for beam stability as mentioned above. Trying to match for the other terms, and in particular forcing the cross term to lower values (B: 0 m^{-1} , C: $-50 \times 10^3 \text{ m}^{-1}$), spoils the correction of the horizontal term drastically.

Target Scenarios						$[10^3 \text{ m}^{-1}]$
	$\Delta Q_{x,x}$		$\Delta Q_{x,y}$		$\Delta Q_{y,y}$	
	IP5	IP1	IP5	IP1	IP5	IP1
A	-55.2	-	-	-	-	-
	-9	-	-	-	-	-
B	-55.2	-	0	-	-6	-
	-9	-	0	-	-6	-
C	-55.2	-	-50	-	-6	-
	-9	-	-50	-	-6	-
D	-33.2	-	-	-	35	-
	-4.5	-	-	-	-7	-
E	-33.2	0	-	-	35	-
	-4.5	0	-	-	-7	-
F	-33.2	≤ 0	-	-	35	-
	-4.5	≤ 0	-	-	-7	-

Target Scenarios						$[10^3 \text{ m}^{-1}]$
	$\Delta Q_{x,x}$		$\Delta Q_{x,y}$		$\Delta Q_{y,y}$	
	IP5	IP1	IP5	IP1	IP5	IP1
G	-55.2	22	-	-	-6	41
	-9	4.5	-	-	-6	-1
	+ global as in D					
H	-55.2	22	0	0	-6	41
	-9	4.5	0	0	-6	-1
I	-55.2	22	≤ 0	≤ 0	-6	41
	-9	4.5	≤ 0	≤ 0	-6	-1
J	-55.2	22	≤ 20	≤ 20	-6	41
	-9	4.5	≤ 20	≤ 20	-6	-1
	+ global as in F					
K	-55.2	22	≤ 30	≤ 30	-6	41
	-9	4.5	≤ 30	≤ 30	-6	-1
	+ global as in F					

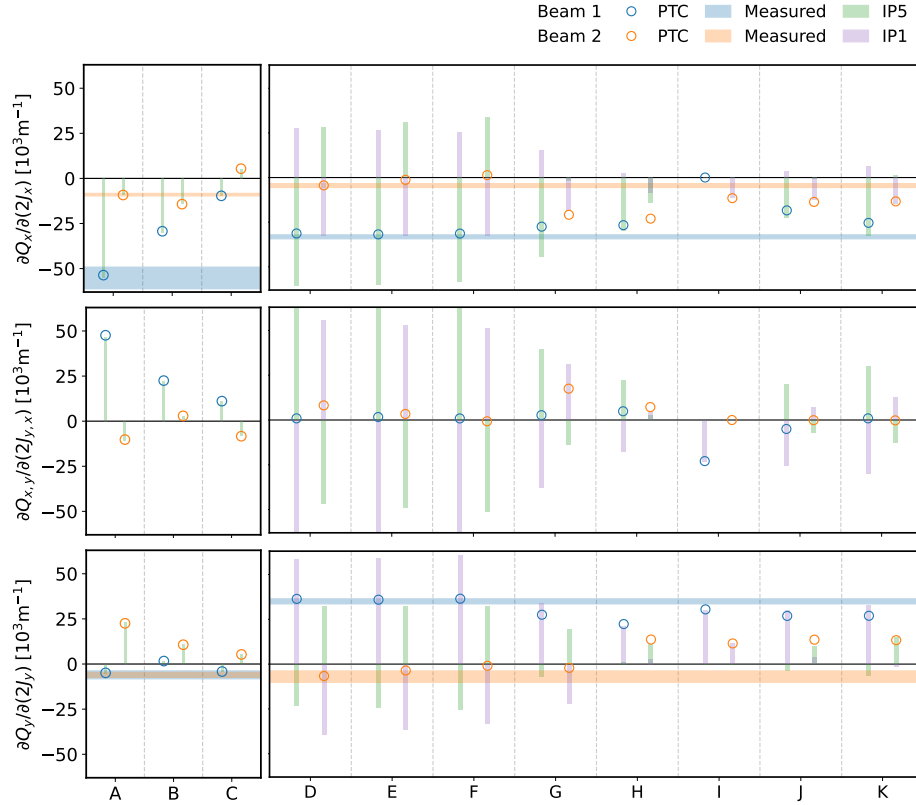


Figure 7.1. Change in amplitude detuning for the direct terms in the X- (top) and Y- (bottom) plane and for the cross term (middle) after applying the calculated corrections determined by the scenarios as given in the table. For A–C only the IP5 crossing scheme is applied, for D–K the full crossing is enabled (see Table 7.1). Circles show the results from PTC for Beam 1 (blue) and Beam 2 (orange), while vertical bars show the expected values as calculated by Eqs. (7.1a) to (7.1c), divided into contributions from IP5 (green) and IP1 (purple) and total. Where these bars overlap, they appear grey. Correction targets as measured are shown by the horizontal bands in the respective beam color.

7.3.2 Global Corrections of IP1 and IP5

The option to correct the detuning “globally” using all four correctors in IP1 and IP5 has been explored in the scenarios D–F in Fig. 7.1: The measured global detuning can be well corrected, with very little residual detuning (D), even in the (untargeted) cross-terms, which is still positive at a few 10^3 m^{-1} . Trying to reduce the cross-term residual by matching them to zero (E) or forcing them to be negative (F), spoils in turn the direct terms by a few 10^3 m^{-1} . The contribution of the Beam 1 horizontal direct term from IP5 also matches very well the measured value of around $-52 \times 10^3 \text{ m}^{-1}$, yet the other terms do not fit with the measured values in IP5.

Despite the good global agreement, we are hesitant to use them in the real machine, as they rely very heavily on the compensation between the two IPs. Other factors, such as DA, might be spoiled upon introducing large, locally uncorrected changes to the optics. It would therefore be favourable to control the local detuning using low corrector powering.

7.3.3 Localized Corrections of IP1 and IP5

Trying a local correction has proven to be very challenging, despite having 4 correctors available for matching, as seen in the attempts G–K in Fig. 7.1: Using all measured local and global values (G) gives large positive cross terms. Correcting for only the local terms while trying to keep the cross-terms low (H) does not match any of the terms very well, while forcing them to be negative (I) does not allow for a horizontal correction. Relaxing the cross term restriction locally but enforcing it globally (J and K) yields good compromises between all terms in the end, but spoils the cross term of Beam 1 in IP5.

The corrections from the scenario K look the most promising, as they replicate the measured local values best and allow for a compromise of the global correction of all measured terms, while keeping the cross terms locally limited. This correction uses 11% of the maximum corrector strength in the correctors L1 and R5 and only 1% in R1 and L5.

7.3.4 Simulation Conclusion

Several correction targets using the feed-down from the dodecapole correctors have been explored. While a perfect local correction of all three detuning terms is not feasible, controlling the detuning globally is possible and compromise solutions have been suggested, showing very promising results.

Better assessment of the cross term can help to improve upon the found corrections, but measuring these terms is very challenging.

7.4 Experimental Results

During the 2022 LHC commissioning and again for verification and improvement during MD, measurements were performed following the procedure described in [Section 2.4.1](#) at the top-energy of LHC run 3 of 6.8 TeV after squeezing the beam to $\beta^* = 30$ cm. All outcomes of these measurements are listed in [Table 7.2](#).

Despite similar machine settings some differences could be seen in the detuning measurements from different dates in 2022. Between measurements various corrections had been applied to the LHC, which are named in [Fig. 7.2](#). Yet, the expected detuning shifts from these do not seem to replicate the observed differences, as demonstrated for the flat-orbit measurements in [Fig. 7.2](#).

In contrast to the findings in 2018, large cross-term detuning is present in Beam 1 even at flat-orbit (see [Table 7.2](#)), the sources of which have not yet been identified. This is surprising, as between LHC run 1 and run 2 the octupole errors had not changed, and in both here presented cases, 2018 and 2022, corrections up to octupole order had been implemented beforehand. These findings have been presented to the LMC [240].

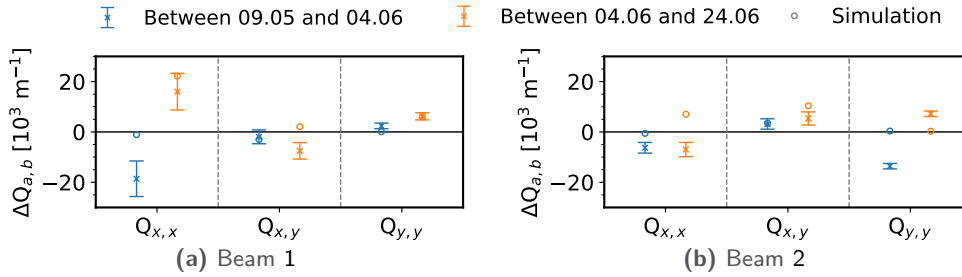


Figure 7.2. Differences of detuning at flat-orbit in measurement (crosses and errorbars) between the first two measurements (orange) and between the second and third measurement (blue) in 2022 compared to the simulated expected changes from differences in LHC machine settings (circles), using the calculated corrections of “MD6863 w/ IP5” in [Table 7.3](#) (with opposite signs) as error sources. The differences between the 09.05 and 04.06 were: crossing schemes in IP2 and IP8, skew and normal sextupole corrections in IP1 and IP5, as well as the “Commissioning 2022” correction, which was active at the time of the second measurement. Between 04.06 and 24.06 the dodecapole correction was deactivated again, but local corrections in the first single-aperture quadrupole (Q4) of Beam 2, corrections of arc 81 and arc 45 in Beam 1, and global β -beating and dispersion corrections for each beam, were applied. Additionally, a waist-shift correction had been applied in Beam 2 at IP5 [110].

7.4.1 Dodecapole Corrections

Detuning measurements were carried out first at flat-orbit and then with the full crossing scheme applied, to establish the shift in detuning between the two scenarios. Following [Sections 7.2.2](#) and [7.2.3](#) normal dodecapole corrections were calculated from the resulting detuning shifts, which were then in turn verified by repeating the measurements with powered dodecapole correctors. As an example, the measurement of the direct horizontal

term of Beam 2 during commissioning is shown in Fig. 7.3: In this instance it can be seen that the detuning shift $\Delta Q_{x,x}$ between flat-orbit (blue) and with crossing angles and dodecapole corrections applied (green) has been severely reduced over the detuning shift between flat-orbit (blue) and with crossing angles but without dodecapole corrections (orange). In Figs. 7.4, 7.5, 7.7, 7.9 and 7.10 discussed below, only these detuning shifts $\Delta Q_{a,b}$ are shown to illustrate the effectiveness of the corrections.

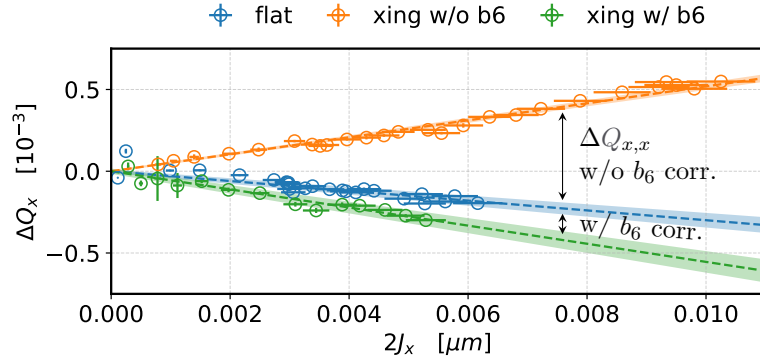


Figure 7.3. Data of the measured direct horizontal detuning term in Beam 2 during commissioning in 2022. The measured values can be found in Table 7.2. Shown are the scenarios of flat-orbit (blue), with full crossing scheme applied without dodecapole correction (orange), and with full crossing scheme applied with the “Commissioning 2022” dodecapole correction from Table 7.3 (green). Dashed lines show the linear fit to the data points, while the colored area covers one standard deviation of the fit. The constant part of each fit has been subtracted for plotting from the fit and its respective data points. The arrows show the detuning shifts $\Delta Q_{x,x}$ between each of the two “with crossing”-scenarios and flat-orbit.

During commissioning, the measurements were done on two separate occasions, allowing for “offline” analysis of the results, while during the MD the analysis and calculation of corrections were done “online”, that is during the measurement period. As the two first-order cross-terms (Eq. (7.1b)) are referring to the same physical quantity only one value was used in the corrections to give all three detuning terms (Eq. (7.1)) equal weight. For the corrections during commissioning these values were picked from one of the measurements, based on the quality of the measured data: At flat-orbit the measured value (Table 7.2) of $Q_{x,y}$ was used for Beam 1 and the value of $Q_{y,x}$ for Beam 2. With crossing enabled $Q_{y,x}$ for Beam 1 and $Q_{x,y}$ for Beam 2 were used. During the MD a different approach was elected and the average of the measured values $Q_{x,y}$ and $Q_{y,x}$ was designated as correction target. In Figs. 7.4, 7.5, 7.7, 7.9 and 7.10 the latter approach is used when the cross-term is presented.

The calculated corrective powering of the dodecapole magnets is presented in Table 7.3, including estimated errors on the correction from the errors on the fits of the measurement and also including the percentage of the maximum absolute powering of the magnets. We can see, that the corrections calculated in commissioning have almost equal powering between left and right in IR5 at 14 % of the maximum strength. There is a stronger

Table 7.3. Dodecapole-corrector strength values (K_6). In parenthesis the percentage (rounded to integers) of the maximum powering at 6.8 TeV is given. The “w/ IP5” and “w/o IP5” labels on the MD6863 corrections refer to whether the additional measurements of crossing-bumps only around IP5 were taken into account or not (see [Section 7.4.2](#)). Below also the β -functions at the correctors are given, rounded to meters. The β -symmetries are accurate to below 10 cm.

	IP1			
	MCTX.3L1		MCTX.3R1	
	$[10^3 \text{ m}^{-6}]$		$[10^3 \text{ m}^{-6}]$	
Commissioning 2022	-0.606 ± 0.715	(2%)	-2.696 ± 1.179	(7%)
MD6863 w/o IP5	1.269 ± 0.731	(3%)	-3.288 ± 0.577	(9%)
MD6863 w/ IP5	0.493 ± 0.192	(1%)	-3.982 ± 0.188	(11%)
	IP5			
	MCTX.3L5		MCTX.3R5	
	$[10^3 \text{ m}^{-6}]$		$[10^3 \text{ m}^{-6}]$	
Commissioning 2022	5.004 ± 0.752	(14%)	-5.053 ± 0.907	(14%)
MD6863 w/o IP5	6.367 ± 0.563	(18%)	-4.087 ± 0.782	(11%)
MD6863 w/ IP5	5.003 ± 0.132	(14%)	-5.032 ± 0.162	(14%)
$\beta_x^{(B1)}$ and $\beta_y^{(B2)}$	3494 m		7177 m	
$\beta_y^{(B1)}$ and $\beta_x^{(B2)}$	7177 m		3494 m	

powering in IR5 compared to IR1, where a maximum of only 7 % is reached in the right corrector. The corrections from the MD (“MD6863 w/o IP5”) are in general slightly increased in comparison: on average 1.5 % in IR1 and 0.5 % in IR5, reflecting increased detuning shifts (see below). Also, the left-right symmetry in IR5 is broken in these corrections, favouring the left corrector (at 18 %) over the right corrector (at 11 %). Including the IR5 measurements in the correction (“MD6863 w/ IP5”, see [Section 7.4.2](#)), the IR5 symmetry is restored, yet the strength of the corrector right or IP1 is further increased, while the strength of the left corrector almost vanishes.

As none of the powering values are above 18 % of the maximum powering of the magnets, we expect negligible impact on beam lifetime from the powering: they had to be powered to more than a quarter of their maximum strengths to show any measurable effect on lifetime (see [\[205\]](#)), which is discussed and confirmed in [Section 7.4.5](#).

[Figures 7.4](#) and [7.5](#) show the measured detuning change introduced by the crossing scheme during commissioning and MD respectively. In [blue](#) the detuning change without correction is presented and in both measurements the detuning from high-order errors can be observed. In contrast to what has been measured in 2018, where mostly Beam 1 had been affected, we now see in Beam 1 big changes only in the direct horizontal term ($Q_{x,x}$). In Beam 2 on the other hand, $Q_{x,x}$ and the cross-term $Q_{x,y}$ are majorly spoiled. The strongest feed-down was experienced in the cross-term of Beam 2 during MD, resulting in a change of detuning of more than $50 \times 10^3 \text{ m}^{-1}$ in absolute value.

Blue vertical bars show the expected detuning shift to be compensated by the correc-

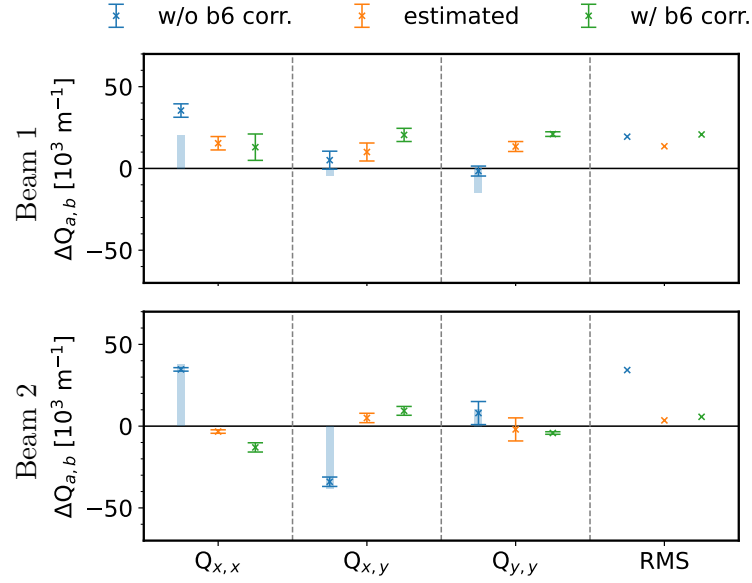


Figure 7.4. Detuning shifts as measured during commissioning 2022 between detuning with full crossing and at flat-orbit for all detuning terms and the error-weighted RMS over all terms. Measured values for the cross-terms $Q_{x,y}$ and $Q_{y,x}$ have been averaged. Shown in blue is the measurement without dodecapole correction applied as circle with error bars, and the detuning to be compensated by the correction as simulated via the bar. In orange the estimated value after correction is shown. The data in green is the actual measured detuning shift with crossing angle after dodecapole correction.

tion, based on the simulation results, and we see that for both corrections the measured data could not be matched perfectly. The expected residual detuning shifts after correction are presented in orange. The figures show that $Q_{x,y}$ and $Q_{y,y}$ in Beam 1 are expected to be slightly spoiled to achieve the partial correction of $Q_{x,x}$ in Beam 1 and very good correction of all terms in Beam 2. In the last column of the figures, the error-weighted RMS (a RMS where the mean is calculated in accordance with Eq. (2.52)) of all terms is shown, in which the net positive effect of the correction can be recognized easily: Before correction Beam 2 shows a RMS measured tune shift of $34 \times 10^3 \text{ m}^{-1}$ (commissioning) and $40 \times 10^3 \text{ m}^{-1}$ (MD6863) while the RMS of Beam 1 is much smaller at $19 \times 10^3 \text{ m}^{-1}$ (commissioning) and $11 \times 10^3 \text{ m}^{-1}$ (MD6863). In turn, Beam 2 is expected to be very well corrected, while Beam 1 will see a lesser improvement, if any.

In green the actual measured detuning shifts still present in the machine after correction are shown. In general, we see that for both corrections all terms have been “over-corrected”. This makes the RMS for most measurements worse than expected, yet still resulting in an overall significant improvement: Beam 1 has been slightly spoiled by $+1.3 \times 10^3 \text{ m}^{-1}$ (commissioning) and $+2 \times 10^3 \text{ m}^{-1}$ (MD6863), while Beam 2 has been drastically improved by $-29 \times 10^3 \text{ m}^{-1}$ (commissioning) and $-19 \times 10^3 \text{ m}^{-1}$ (MD6863).

In general, despite the observed discrepancies between the expectation from simulation and measurement, both tested corrections are well able to reduce the common detuning

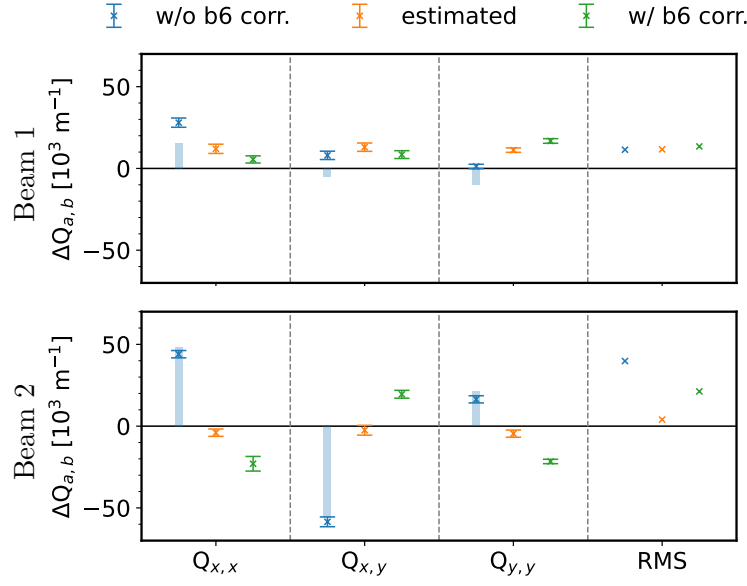


Figure 7.5. Detuning shifts during MD6863, presented in the same manner as the data in Fig. 7.4.

introduced by high-order errors in the two beams, and partially even out the contribution per beam.

7.4.2 Error Sources

In addition, studies were carried out during MD to identify the sources of the errors, i.e. whether their origin lies in decapole or dodecapole order and if they stem from IR1 or IR5. To attain this objective, the crossing bump at IP1 was kept deactivated, while measurements were performed with nominal crossing angle in IP5 and also with opposite angle sign.

Multipole Order. As there is only horizontal orbit in IP5 in our measurements (see Table 7.1), we know from Eqs. (7.1) and (7.2) using the abbreviations Eqs. (2.45)

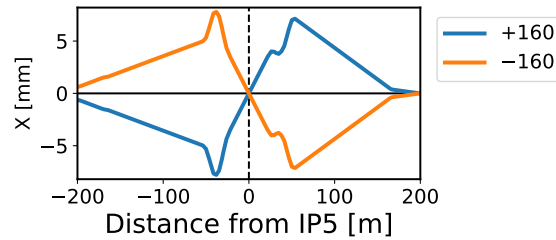


Figure 7.6. Horizontal orbit of Beam 1 in IR5 for crossing schemes with $\pm 160 \mu\text{rad}$ half-angles, demonstrating the symmetry with respect to the x -axis of the two scenarios.

and (2.46):

$$\begin{aligned}
Q_{a,b}^{\emptyset} &= \sum_w K_4 L_w \tilde{\beta}_{a,b;w} \\
Q_{a,b}^- &= \sum_w \left(K_4 L_w - \Delta x_w K_5 L_w + \frac{1}{2} \Delta x_w^2 K_6 L_w \right) \tilde{\beta}_{a,b;w} \\
Q_{a,b}^+ &= \sum_w \left(K_4 L_w + \Delta x_w K_5 L_w + \frac{1}{2} \Delta x_w^2 K_6 L_w \right) \tilde{\beta}_{a,b;w} ,
\end{aligned} \tag{7.6}$$

distinguishing between the cases “ \emptyset ” for flat-orbit, “+” for positive nominal crossing angle at IP5 and “−” for negative angle. Here it is also assumed that each element w sees an equal-in-magnitude but opposite in sign horizontal orbit offset x_w when switching crossing angle sign as shown in Fig. 7.6. The contributions to the detuning change in IR5

$$Q_{a,b}^+ - Q_{a,b}^{\emptyset} = \sum_w \left(\Delta x_w K_5 L_w + \frac{1}{2} \Delta x_w^2 K_6 L_w \right) \tilde{\beta}_{a,b;w} , \tag{7.7}$$

from each multipole order, can be calculated from the measured data using Eq. (7.6)

$$\begin{aligned}
\sum_w \Delta x_w K_5 L_w \tilde{\beta}_{a,b;w} &= \frac{1}{2} (Q_{a,b}^+ - Q_{a,b}^-) \\
\sum_w \frac{1}{2} \Delta x_w^2 K_6 L_w \tilde{\beta}_{a,b;w} &= \frac{1}{2} (Q_{a,b}^+ + Q_{a,b}^- - 2Q_{a,b}^{\emptyset}) .
\end{aligned} \tag{7.8}$$

The results are presented in Fig. 7.7, which clearly shows that the main contribution to the detuning change in IR5 with positive crossing (blue) stems from dodecapole sources, while the contribution from decapoles is only around 14 % in RMS. This finding is in line with the expectation from magnetic measurements and simulations [33] as dodecapole fields are the first allowed harmonic of quadrupole magnets [35].

Another insight we achieve from Fig. 7.7, is that in Beam 1 the contributions from b_5 and b_6 add up, whereas in Beam 2 they partly cancel each other, leading to the higher detuning shifts measured in Beam 1 with positive IP5 crossing. We also see, that the b_6 corrections applied in IR5, calculated by Eqs. (7.1) and (7.2) and shown as green vertical bars, compensate the b_6 contribution in Beam 2 very well, but in Beam 1 only manage to do so for the horizontal term. This could be because no distinction was made between contributions from IR1 and IR5 when calculating the correction, which could be overcome by targeting also the IR5 measurement in Eq. (7.4); another reason could be, that β -function and orbit at the corrector magnets are not representative for β and orbit at the actual sources, making it impossible to find a perfect match.

In addition to these uncertainties about the error sources, also uncertainties about the correctors themselves can play a role: Assuming the corrector magnets to be misaligned with Gaussian distributions of $\sigma = 1$ mm and a Gaussian distributed β -beating of $\sigma = 5$ %, the expected detuning can be calculated using the correction strengths “MD6863 w/ IP5” from Table 7.3 and Eq. (7.2). The resulting detuning distributions are shown in Fig. 7.8

and have a mean standard deviation, over terms and beams, of $14 \times 10^3 \text{ m}^{-1}$. Further simulations with the same parameters realizing only β -beating show a mean σ , over terms and beams, of $4 \times 10^3 \text{ m}^{-1}$ while simulations realizing only misalignments give a mean σ of $13 \times 10^3 \text{ m}^{-1}$, showing that including the measured β into calculations will not majorly improve the correction, as the uncertainty on alignments is the dominating factor. These values are compatible with the difference we see between expected and measured values after correction, meaning that there is a possibility that without better knowledge of the corrector (mis-)alignments we could be at the limit of the correction accuracy.

Interaction Region. Figure 7.9 is showing again the shift in detuning between full-crossing scheme and flat-orbit as measured during MD before dodecapole corrections (blue), but this time split into contributions from the IR around IP5 (orange) and IP1 (green), where the former comes from the measurement with positive crossing angle at IP5 and the latter is calculated from the difference between the full crossing scheme and the IR5 measurements. The plots show that contributions from IR1 change sign between beams, whereas in IR5 the sign of the detuning shift stays the same. This leads to the IR contributions adding up in Beam 2 and cancelling each other partly in Beam 1, the opposite effect as discovered between b_5 and b_6 contributions in IR5. The

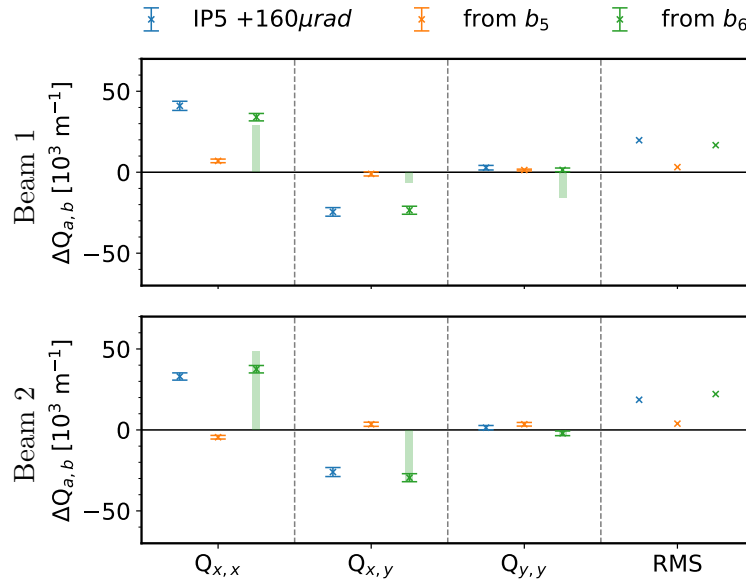


Figure 7.7. Detuning changes between the scenario of positive crossing angle at IP5 and flat-orbit as measured during MD. Shown are the contributions to the full detuning change (blue) from decapole sources (orange) and dodecapole sources (green) as calculated from Eq. (7.8), for all detuning terms and the error-weighted RMS over all terms. Vertical bars show the detuning compensated by the correction “MD6863 w/o IP5” in Table 7.3 according to Eqs. (7.1) and (7.2). Measured values for the cross-terms $Q_{x,y}$ and $Q_{y,x}$ have been averaged.

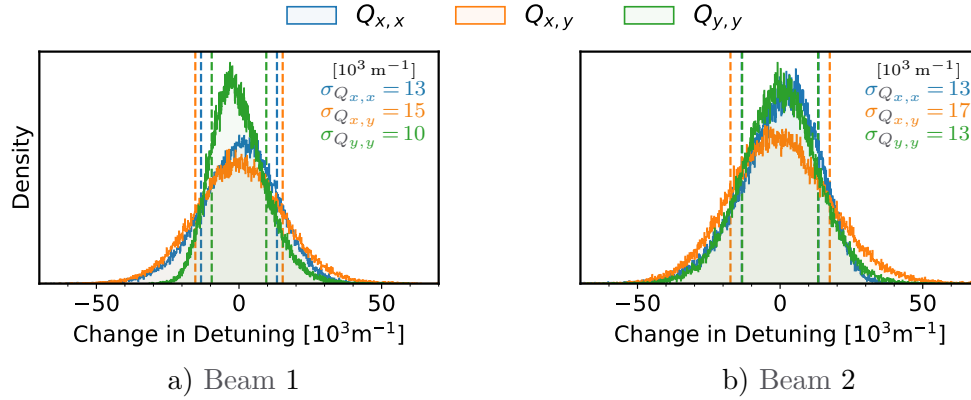


Figure 7.8. Histograms of detuning changes for 100'000 realizations of random misalignments and random β -beating at the corrector elements. Both parameters are taken from Gaussian distributions: The distribution parameters for the misalignments are $\mu = 0$ mm, $\sigma = 1$ mm and the for the β -ratio $\mu = 1$, $\sigma = 0.05$. The random values are applied individually to the corrector magnets left and right of IP1 and IP5. From each realization all three detuning terms were calculated from $Q_{a,b} = \sum_w \frac{1}{2} (\Delta x_w^2 - \Delta y_w^2) \tilde{\beta}_{a,b;w} K_6 L_w$ using the correction “MD6863 w/ IP5” in Table 7.3 for the strengths of the corrector elements w . The dashed vertical lines are set at the standard deviation of the resulting detuning distribution and their values are shown.

effect from different orders is shadowed by the effect from different IRs, leading to the higher measured detuning shift with the full crossing scheme in Beam 2.

While the horizontal term shows a much larger contribution from IR5, the other terms are similar in absolute contribution from both IRs, leading to fairly equal RMS contributions from both IRs. This is in contrast to the behaviour observed in 2018, where the main contribution had come from IR5; see Table 7.2 and [43]. The expected detuning to be compensated by the corrections, as calculated by Eqs. (7.1) and (7.2), are presented again through the vertical bars in Fig. 7.9: in total (blue) and split into the contributions per IR (orange and green). The correction seems to be good locally in only half of the terms, that is in the horizontal direct term of Beam 1 and in the cross-terms of Beam 2 as well as Beam 2's vertical direct term. Despite stronger powering of the IR5 correctors, over-correction of IR5 and under-correction of IR1 is only visible in Beam 2's $Q_{x,x}$ and $Q_{x,y}$, signifying that the difference in powering is an effect of the different orbit and β -functions at the correctors of the IRs and not of different error distributions between the IRs.

Improved Corrections. Additional context about the locality and field order can be supplied to improve upon the corrections as discussed so far. This information, gained through the measurement of activating the crossing bump at the IPs individually, as well as with opposite sign, can be incorporated either through boundary restrictions on the linear equation system, as had been attempted in [220], or by including the measurements as additional targets into Eq. (7.4). The latter has been tested in simulations, and the

results, presented in Fig. 7.10, are very promising: Despite the additional restrictions on the correction the estimated RMS does not change compared to the “global” correction. On the other hand, in all terms the correction is able to target b_6 in IP5 and the contribution per IP more closely than the previous correction, increasing locality of the correction.

These corrections have since been incorporated operationally into the LHC, replacing the commissioning corrections which were used from 04.06.2022 to 02.10.2022. Yet, due to the LHC running now for luminosity production, they could not be in detail investigated by amplitude detuning measurements.

7.4.3 Second-Order Detuning

Up until now, corrections were calculated based on the assumption that the second-order terms in amplitude in Eq. (2.38) are negligible and the main contribution to detuning from dodecapoles comes from feed-down to first-order detuning. Assuming the dodecapole fields from the correctors to be representative for the IR, just with opposite sign, the “MD6863 w/ IP5” correction calculated in the previous chapter and Eq. (7.3) could be used to calculate their contributions to second-order detuning. It was found that for large kicks, that is at $0.02\text{ }\mu\text{m}$, the contributions could be up to 50 % of the total measured detuning.

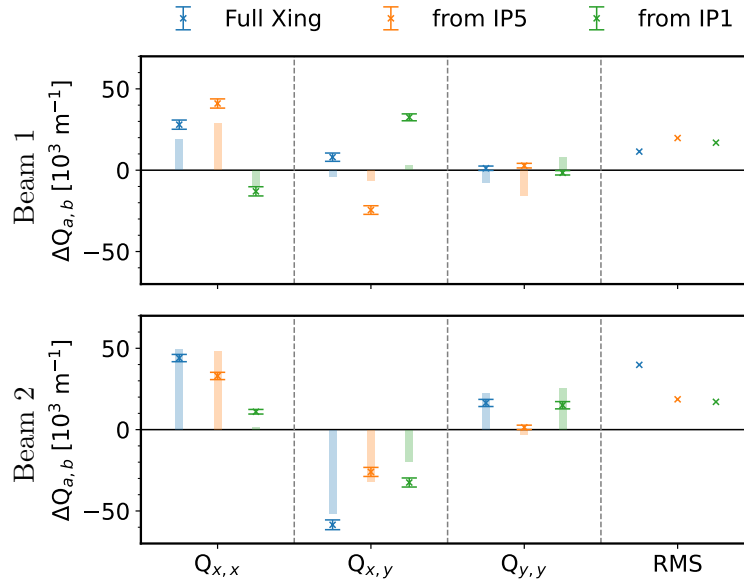


Figure 7.9. Detuning changes as measured during MD without dodecapole correction between the detuning with full crossing and flat-orbit (blue) as well as the scenario of positive crossing at IP5 (orange), for all detuning terms and the error-weighted RMS over all terms. For convenience, the detuning change on activating the crossing scheme at IP1 (green) has been calculated from the difference of the two measurements. Vertical bars show the detuning compensated by the correction “MD6863 w/o IP5” in Table 7.3 according to Eqs. (7.1) and (7.2).

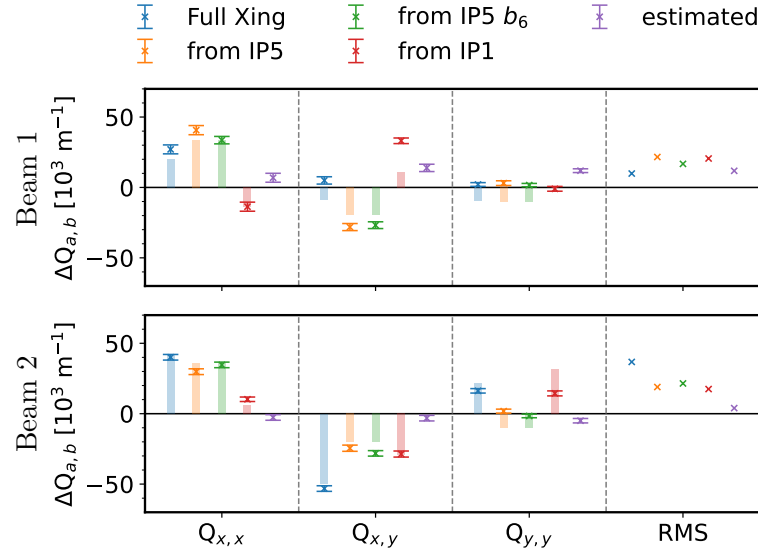


Figure 7.10. Detuning changes as measured during MD without dodecapole correction between the detuning with full crossing and flat-orbit in blue, as well as the measured individual contributions from the IR5 dodecapole fields in green, and the total contributions from IR5 (orange) and IR1 (red), as well as the estimated value after correction in purple. The data is presented for all detuning terms and the error-weighted RMS over all terms. Measured values for the cross-terms $Q_{x,y}$ and $Q_{y,x}$ have been averaged.

Table 7.4. Summary of amplitude detuning measurements after subtracting second-order contributions calculated from the “MD6863 w/ IP5” corrections (Table 7.3) following the conventions in Table 7.2. In parenthesis the difference (Δ) to the original detuning values are given.

	$Q_{x,x}$ [10^3 m^{-1}]			$Q_{y,x}$ [10^3 m^{-1}]			$Q_{x,y}$ [10^3 m^{-1}]			$Q_{y,y}$ [10^3 m^{-1}]		
		(Δ)			(Δ)			(Δ)			(Δ)	
2022 MD6863 w/o b_6	-22 ± 2	(-4)		37 ± 2	(+5)		27 ± 4	(+4)		-7.7 ± 0.7	(-7.7)	
flat-orbit	-27.1 ± 1.8	(-8)		19 ± 3	(+6)		23.9 ± 2.0	(+12)		-13.1 ± 0.8	(-16.5)	
2022 MD6863 w/o b_6	6 ± 2	(-4)		42.2 ± 2.0	(+5)		30 ± 2	(+3)		-4.1 ± 1.0	(-6.1)	
IP1&5 xing @ $\mp 160 \mu\text{rad}$	17.5 ± 1.3	(-3.4)		-43 ± 3	(-4)		-31 ± 3	(+12)		4.8 ± 0.8	(-14.9)	
2022 MD6863 w/o b_6	16 ± 2	(-7)		9 ± 3	(+7)		0.3 ± 1.4	(+4.0)		-6.2 ± 1.1	(-9.2)	
IP5 xing @ $+160 \mu\text{rad}$	5.8 ± 1.2	(-4.8)		-4 ± 3	(+5)		-6.5 ± 1.9	(+9)		-7.7 ± 0.6	(-13.0)	
2022 MD6863 w/o b_6	3.0 ± 1.0	(-5.9)		8 ± 4	(+5)		3.2 ± 0.5	(+4.1)		-9.0 ± 0.4	(-9.2)	
IP5 xing @ $-160 \mu\text{rad}$	15.2 ± 1.8	(-5)		-11 ± 3	(+4)		-12.4 ± 1.4	(+10.9)		-16.4 ± 1.8	(-15)	

To determine the influence of the second-order detuning on the first-order fits, two approaches have been tested: In the first approach, the measurement data has been fit with a second-order polynomial, still using ODR to include the measurement uncertainties. The second-order coefficients could in principle then also be used to extent the equation system Eq. (7.4), adding new rows for the $\Delta Q_{a,bc}^{(BN)}$ on the left hand side, and new matrix

Table 7.5. Dodecapole-corrector strength values (K_6). In parenthesis the percentage of the maximum powering at 6.8 TeV is given. Shown are again the values for the *Improved Corrections* of “MD6863 w/ IP5” as in Table 7.3 as well as the corrector strenths calculated in the same manner from the detuning values of Table 7.4, with the second-order contributions subtracted.

	IP1			
	MCTX.3L1 [10^3 m^{-6}]		MCTX.3R1 [10^3 m^{-6}]	
Original values	0.493 ± 0.192	(1%)	-3.982 ± 0.188	(11%)
Second-order subtracted	0.816 ± 0.197	(2%)	-4.232 ± 0.202	(12%)
	IP5			
	MCTX.3L5 [10^3 m^{-6}]		MCTX.3R5 [10^3 m^{-6}]	
Original values	5.003 ± 0.132	(14%)	-5.032 ± 0.162	(14%)
Second-order subtracted	5.363 ± 0.142	(15%)	-5.011 ± 0.167	(14%)

elements from Eq. (7.3)

$$M_{a,bc;w}^{(BN)} = \tilde{\beta}_{a,bc;w}^{(BN)} \quad (7.9)$$

on the right hand side. As for example shown in Fig. 7.11, second-order polynomial fitting lead only to reasonable results where high kick-amplitudes could be reached (Fig. 7.11 top), yet lead to an overfitting on measurements where there maximum amplitude was below $0.01 \mu\text{m}$ and in conclusion to unreasonable and unusable results for most measurements (Fig. 7.11 bottom). This approach has hence been discarded for the data at hand, but could be usable in future measurements if consitent large amplitude excitations can be achieved.

In the second approach, the values of the “MD6863 w/ IP5” corrections (Table 7.3) at the correctors were used to calculate the tune shift from second order in amplitude at each conducted kick. These contributions have then been subtracted from the measured data, neglecting the errorbars on the corrections, but accounting for any coefficients due to the forced oscillations [10]. Subsequently, first-order polynomial fits were applied to the resulting data (e.g. shown in Fig. 7.11). The found detuning coefficients are given in Table 7.4, which also provides the differences to the original data in Table 7.3. Despite the changes in detuning coefficients, the corrections evaluated from these and given in Table 7.5, are still within 10 % of the originally calculated corrections, apart from the corrector left of IP1 which almost doubled in strength, but is still lower than a quarter of the powering of the other correctors.

Interestingly, the correction strengths has overall been increased by this second approach, hinting at a compensation between first and second order terms. This approach could be extended to an iteration scheme, in the hopes that the corrector values will converge towards an optimal value. Due to time constraints and because the correction has not dramatically changed after this first iteration, this fine tuning

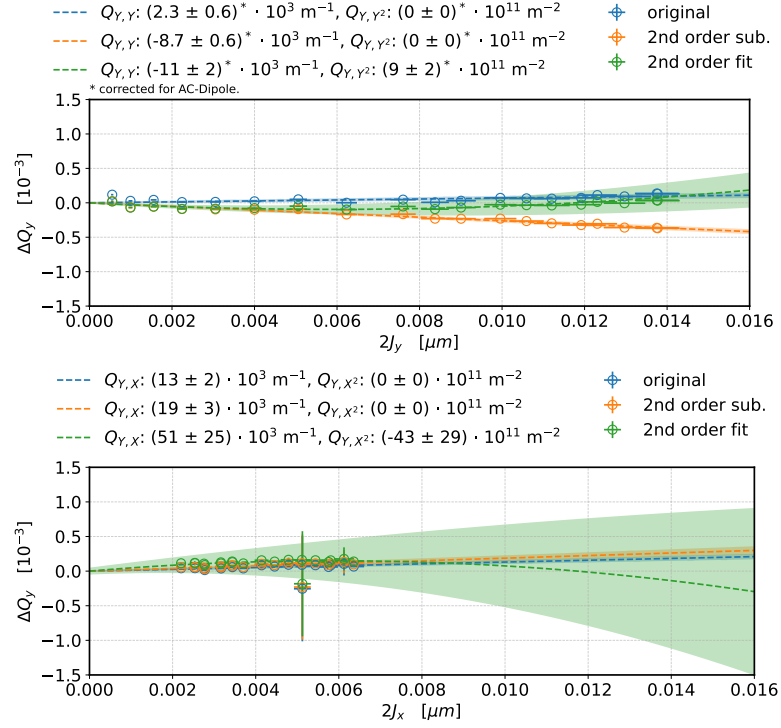


Figure 7.11. Data of the measured vertical detuning in Beam 2 without dodecapole correction at flat-orbit from MD. Shown are linear polynomial fits on original measured data (blue) and the measured data after subtraction of the second-order contribution as calculated from the “MD6863 w/ IP5” corrections in Table 7.3 (orange). In green, the original data is shown again, but here a second-order polynomial fit is performed. Dashed lines show the polynomial fit to the data points, while the colored area covers one standard deviation of the fit. The constant part of the fit has been subtracted from the fit and its respective data points.

has not been conducted. In fact, as most corrector values are within errorbars of each other (apart from the left corrector in IP5), it is doubtful that any difference would be perceptible given the accuracy of the measurements.

7.4.4 Comparison with the magnetic model

Magnets in the LHC have undergone magnetic measurements before installation, the results of which have been collected in the Field Model of the LHC (FiDeL) [241] which is the basis for the so called Windows Interface to Simulation Errors (WISE) [182]. The errors on the field measurements lead to 100 different realizations, or “seeds”, of the errors and are used to simulate realistic error scenarios in the LHC and HL-LHC.

In the past, differences had been found between corrections obtained from simulation and beam-based methods [242]. To test the agreement for the dodecapole corrections, a basic simulation has been set up, following the description in Section 7.2.3, but including dodecapole errors from the WISE tables. Previous studies have shown that, as the β -function for squeezed optics varies dramatically in the IR, the MQXA magnets of the triplets [34] should be split into hard-edge bodies and heads [26, 243, 244]. This procedure is applied to the simulation at hand. Errors are then locally corrected in IR1 and IR5 by minimizing dodecapole RDTs in the IRs using the correction algorithm [40] presented in Section 3.2. This means the correction target is inherently different to the beam-based measurements.

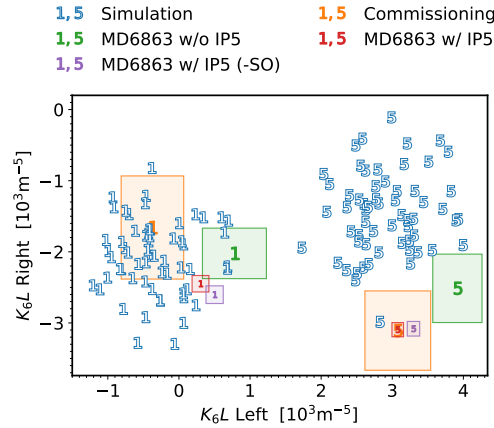


Figure 7.12. Comparison of dodecapole corrections from simulations based on the magnetic model (blue), with the corrections calculated from the beam-based measurements of feed-down to amplitude detuning. The markers express the IR of the correctors: “1” for IR1 and “5” for IR5. In the simulation b_6 errors are assigned from 100 different WISE-seeds and corrected by minimizing dodecapole RDTs locally in the IRs. The plot shows the integrated strength of the right dodecapole corrector over the integrated strength of the left dodecapole corrector. The values of the beam-based corrections are as given in Tables 7.3 and 7.5, where “(-SO)” refers to the values with second-order detuning subtracted, with their uncertainties represented by their surrounding area in the same colors.

Figure 7.12 shows the simulated corrections in blue, compared to the three differ-

ent beam-based corrections as in Table 7.3 from commissioning (orange), from MD without the IR5 measurements (green) and from the *Improved Corrections* including IR5 measurements (“MD6863 w/ IP5” in Table 7.3, red). For completeness also the in Section 7.4.3 explored correction values after subtracting the second-order contributions is shown (purple). While the beam-based corrections in IR1 fall right into the results from different seeds, they appear to be slightly stronger in the right corrector of IR5, yet still very close to the strength evaluated from some of the seeds. Hence, we find beam-based and magnetic measurements to agree very well within their uncertainties.

7.4.5 Beam Lifetime

During the trims of the dodecapole corrections into the machine, beamlifetime has been extracted from the BLMs and beam intensity from the BCTFR and are shown in Fig. 7.13.

There is no obvious change on these parameters detectable after powering the dodecapole corrections, neither during commissioning (top plot) nor during MD (middle plot), when only non-colliding pilot bunches were circulating in the LHC. During operation, i.e. with colliding beams and a full filling scheme, trimming out the commissioning corrections and trimming in the *Improved Corrections* including IR5 measurements, the lifetime drops slightly during the trims and recovers thereafter, as seen in the bottom plot of Fig. 7.13. The histograms in Fig. 7.14 show that the mean beam-lifetime in the 41 s before trimming out the old corrections, between the two corrections and after trimming in the new corrections is preserved. This behaviour is expected in the LHC, where any change in parameters can lead to a temporary change in expected beam lifetime. There is no additional loss of beam intensity, apart from the visible burn-off due to collisions.

These measurements are in line with the measurements performed during MD3312 [205] and the studies performed in [33], in which the dodecapole correctors were powered to higher values before an impact on beam lifetime became visible.

7.5 Conclusion and Outlook

In this chapter, the results of the amplitude detuning measurements conducted during the LHC commissioning 2018 and for run 3 in 2022 and in dedicated machine development sessions, have been presented, which were performed to identify and correct detuning originating in the feed-down from high-order nonlinear errors in the IRs. Dodecapole corrections were calculated, and for the first time integrated into the LHC operation, effectively reducing the RMS tune shift over all terms and both beams, therefore allowing e.g. for tighter control of Landau damping. Corrections calculated via this beam-based method agree very well with corrections calculated from local RDT minimization in simulations using the magnetic model. Incorporation of the corrections did not spoil beam-lifetime.

It was shown that with the measurement method at hand sources of the detuning

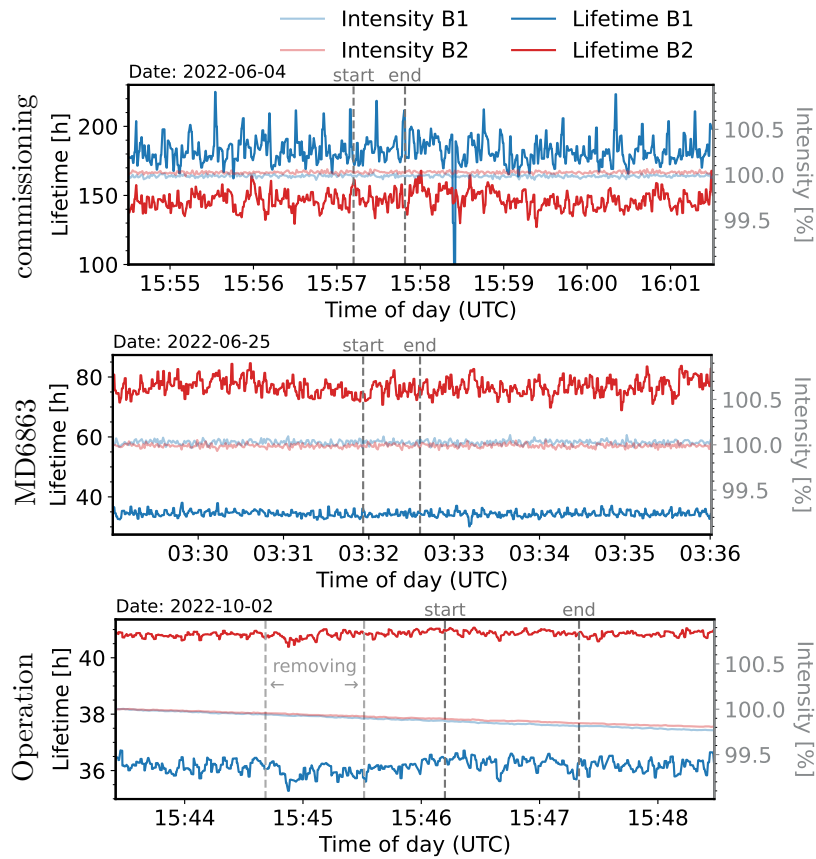


Figure 7.13. Beam lifetime (from BLMs) and intensity (from BCTFR, normalized to the first datapoint) before and after the trim of the dodecapole corrections during commissioning 2022 (top), MD (middle). The bottom plot shows the application of the “MD6863 w/ IP5” corrections (Table 7.3), during operation, after trimming out (“removing”) the previously incorporated dodecapole correction from commissioning. Start and end of the trimming processes are marked in the plot.

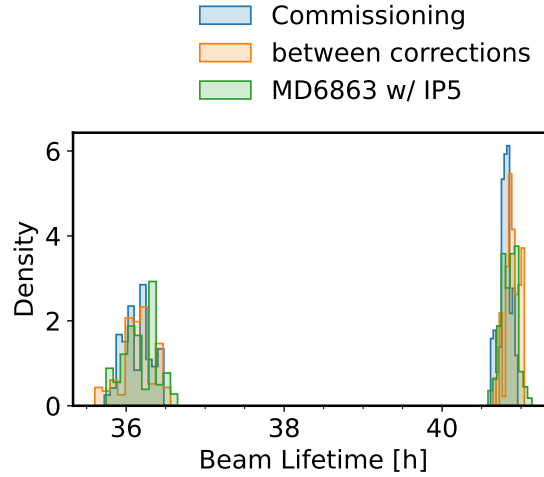


Figure 7.14. Beam lifetime as histograms of the BLM data of 41 s intervals before trimming out the commissioning correction (blue), between corrections (orange) and after trimming in the improved correction, including all measurements from MD. The left cluster shows data from Beam 1 while the right cluster shows Beam 2 data. The same data over time can be seen in the bottom plot of Fig. 7.13.

could be identified: In IR5 a clear origin from dodecapole errors has been seen and it could also be concluded that both IRs contribute equally to the observed detuning. To advance the corrections in the future, one could even further improve on the locality of the correction by targeting dodecapole contributions in both IRs specifically. To accomplish this, one more measurement with opposite-sign crossing angles is needed, either with crossing bumps around IP1 only, or opposite-sign full-crossing in both IRs.

As the presented results show, magnetic field errors up to dodecapole order can be identified using feed-down to amplitude detuning as observable. This can be achieved with our current measurement methods. For HL-LHC these results are vital, as the influence of high-order nonlinearities is foreseen to be increased. Not only confirms our study the expectations on the harmful influence of dodecapole errors presented [33], but also establishes that the therein described effort for high-order field error measurements can be reduced: If the quality of the measurements is good, i.e. there is a low error-bar on the datapoints, calculating the the dodecapole contribution from only three data points yields reliable results and can lessen the in [33] estimated workload. In case of larger error-bars measurements at additional working-points might be necessary to establish a statistically meaningful fit. Distinction between the different orders will become of more importance with the HL-LHC upgrade, as they can be targeted individually by the improved nonlinear corrector package, containing correctors for all normal and skew fields up to dodecapole order.

Of further interest is also the study of the change in free and forced DA upon applying the dodecapole corrections [161, 245].

8 Conclusion and Outlook

8.1 Summary

After introducing the reader into the topic of accelerator physics in general, and optics measurements and corrections in particular, the detrimental influence of high-order nonlinear errors on the machine performance of the LHC have been shown in this thesis. It has been established how errors, especially in the IRs, impact the beam optics, due to the high- β -functions in these regions. With the even more pronounced sensitivity of the optics after the high-luminosity upgrade HL-LHC, the impact of the IR errors will increase, as has been demonstrated in this thesis as well. The attempt to regain control of the optics lead to the need to correct high-order errors locally: With the nonlinear correction script introduced in [Chapter 3](#), simulations could be run evaluating the influence of errors and the possibility to correct them. Its capability to flexibly target RDT and its advanced features, such as including feed-down and dual-optics, lead to a wider range of possible correction approaches than viable before. While the algorithm has been successfully tested and applied to a multitude of scenarios, providing valuable insights into the correctability of nonlinear errors, the now opened correction space is too wide to be fully explored within the content of this thesis. The correction script will be an invaluable tool to investigate for optimal corrections in the future.

Triggered by recent findings in the Beams- and Technology-Departments at CERN, simulation studies were performed in [Chapters 4](#) and [5](#): Expected misalignments of triplets and corrector magnets in the IRs of LHC and HL-LHC and their influence on amplitude detuning was evaluated, and the influence of measured decapole errors and their correctability in the new separation and recombination dipoles (D1/D2) for the HL-LHC were explored. These studies have yielded valuable insights and feedback for the relevant working groups. It can be inferred that, at least at flat-orbit, misalignments did not pose significant issues. However, it is apparent that the decapole errors cannot be completely rectified by the corrector magnets and should ideally be addressed during the magnet design phase. Notably, the CERN magnet group has successfully accomplished this task.

The MD dedicated to replicate nonlinear errors in the HL-LHC, with the nonlinear magnets available in the LHC, was presented in [Chapter 6](#). While in the end only a part of the measured data could be evaluated, the MD provided nonetheless invaluable understanding of the constraints high-nonlinear errors pose on the measurement techniques due to their destabilizing effects on the optics. This showed the necessity to not only correct high-order errors for the sake of stable collisions, but also to be even able to perform optics measurements in the first place. E.g.: while there may not be

any immediate visible effect on beam quality measures such as beam-lifetime, different behaviour of the beam under the influence of excited kicks, performed to measure the optics, are visible and can lead to degradation of measurement quality.

The insights from this MD on how to prepare for sensitive measurements successfully was applied, choosing a small ΔQ between the natural and excited tunes and making use of the newly implemented detuning analysis tools, which allow for quick online analysis and therefore immediate feedback of the quality of the measurement data. This lead to the first ever dodecapole correction in the LHC presented in [Chapter 7](#). This correction, which uses the highest-order nonlinear correctors available in the LHC IRs, was triggered by earlier studies, seeing local feed-down to amplitude detuning in IR1 and IR5. This feed-down has first been studied in simulations and could then be mitigated with great success in the actual machine, leading to a reduction in the RMS of the measured detuning. Due to time limitations, measurements of its influence on DA could not yet be performed.

While the implementation of these corrections undoubtedly brings about a qualitative enhancement in beam optics, articulating the quantitative outcomes is not as straightforward. This complexity arises from the pronounced influence of decapole and dodecapole errors on the machine state, including parameters like β^* and crossing-angles. These evaluations can only be performed for specific scenarios, often only temporarily present (such as during β^* -leveling), requiring individual tracking simulations for each configuration. Additionally, there is no direct correlation between the values for amplitude detuning and beam lifetime or DA, making it impossible to make quantitative statements about the latter two parameters based on improvements in the former. Instead, the corrections are leading to an enhanced optics control, such as enlarged control of Landau damping and improved quality of lower-order corrections, which in turn are requisites to further enhancements in machine performance.

8.2 Conclusion

The correction of high-order nonlinearities in accelerator beam optics is crucial for the performance of present day and future high-energy and high-luminosity colliders and therefore of significant interest to the accelerator community in general and immediate interest to the LHC and its high-luminosity upgrade.

Under the umbrella of correcting high-order nonlinear errors in particular in the LHC and HL-LHC IRs, I have conducted a plethora of studies, via simulations as well as experimental, shedding light into the impacts of these errors on machine performance but also in turn using the impact as target to establish new corrections mitigating the influence of these errors on the optics.

In particular, the experimental studies have been faced with unprecedented challenges with respect to the quality of the data, due to the observables under test being highly sensitive to noise and the influence of machine parameters. These difficulties could be largely overcome by the novel measurement techniques and analysis tools developed in the context of this thesis. While corrections of sextupole and octupole errors have been

established in the LHC and other contemporary colliders, the correction of dodecapole errors had been largely unexplored.

As this was the first time a quantitative measurement and correction of dodecapole errors had been successfully performed in a particle accelerator, the algorithms used for the evaluation of the error-strengths from the measured data and for the calculation of the correction, had to be newly developed and implemented. A new technique, based on measuring the feed-down from decapole and dodecapole errors in the IRs 1 and 5 to amplitude detuning, has been successfully applied. This led to the first ever operational powering of the dodecapole correctors in the LHC and in turn to a significant measurable decrease in amplitude detuning from feed-down. While these corrections had not been required in the past to achieve stable machine operation of the LHC, they enhance control of the optics of this current machine. They will be necessary in the context of the HL-LHC upgrade and other future colliders aiming to surpass the LHC in luminosity and energy, due to their increased sensitivity of the optics to high-order errors. An understanding of the origins and influences of these errors, their principal correctability and the development of new methods of corrections is invaluable for the successful operation of these future colliders. It had been demonstrated and laid out in this thesis, how the uncorrected expected high-order errors can prohibit useful optics measurements and options for corrections. They are therefore immediately useful and crucial for successful HL-LHC commissioning.

During the course of this thesis I have improved the way we can locally correct IR errors in simulations by developing and implementing a new correction algorithm, broadening the explorable correction-parameter space and thus opening the way to future studies. This algorithm has been exhaustively used throughout this thesis to conduct a wide range of studies, providing a new basis for correcting decapole- and dodecapole errors in the LHC, as these high-order errors had been observed to cause perturbations on the optics.

My work conducted in this thesis has further been valuable for the planning of the mentioned commissioning of the HL-LHC. I have shown, on one hand, that without thorough preparation the feasibility of performing measurements as well as the quality of measurement data taken will be significantly affected by the presence of high-order errors. We now have a clearer understanding of the detrimental effects of these errors and this thesis provides tools to overcome them, e.g. by choosing different working points, tools for quick online analysis of the measured data and the new algorithm to correct high-order errors. In addition, I have shown that the effect of misalignments of the triplets and correctors in the IRs as well as the influence of decapole errors in the separation and recombination dipoles will not pose a major problem to the operation of the HL-LHC.

In summary, although the presence of high-order nonlinear errors presents significant yet intriguing challenges to the optics control and operation of LHC and HL-LHC, it is an important outcome of this thesis that none of these challenges are insurmountable. Theory, tools, results and experiences within this thesis in addition to the information found in the referenced published literature provide an encouraging outlook to the

successful operation of future high-energy and high-luminosity particle colliders.

8.3 Outlook

Numerous chapters in this thesis delve into the fundamentals of each topic, shedding light on various aspects and answering many of the posed questions, yet also opening new pathways for investigations. In particular, the newly developed nonlinear IR-error correction script presented in [Section 3.2](#) allows the easy exploration of a vast correction-parameter space. While also interesting, not all diverging routes could be fully explored due to the inherent time constraints of a PhD thesis and the focus on the problems at hand. The following still unanswered questions are presenting exciting avenues and fertile ground for future studies, continuing on the path layed out in this thesis:

Is amplitude detuning still the best target? Feed-down to amplitude detuning has been used in the LHC as target for high-order correctors, mainly because their influence was visible there and this measure was easily accessible in the machine. With the expected increase in optics sensitivity in the HL-LHC, possible new measures might emerge, like decapole and dodecapole RDTs in the IRs, allowing to use the new high-order nonlinear correctors to target the errors more directly and with less measurement effort.

DA studies with dodecapole correction. [Chapter 7](#) paves the way to an in-depth experimental study, exploring the change of DA after applying the dodecapole corrections. This could be done by blowing up the beam at the end-of-fill and testing the losses with dodecapole corrections applied, and studying the additional losses after trimming them out.

Misalignment studies with crossing scheme. It has been shown in [Section 7.4.2](#) that the misalignment studies performed in [Chapter 4](#) only capture the flat-orbit configuration of the machine and are, due to the quadratic dependency of the feed-down on the orbit offset for dodecapole errors to b_4 , not directly transferrable to a machine configuration with crossing schemes. To get the full picture, the study can be extended with crossing angles enabled.

Differences between LHC and HL-LHC in the misalignment study. From the results in [Section 4.3](#), the question arose, why the LHC seems to be more sensitive to the studied misalignments and could not be sufficiently clarified. It is of great interest, whether the sensitivity of the HL-LHC changes with newer optics versions or if there are important errors sources missing in the currently provided optics setups.

Optimal correction strategy given feed-down and dual optics for different machine settings. Including dual-optics and feed-down in the IR corrections has been studied individually ([Sections 3.3](#) and [3.5.3](#)), it is possible, that an optimal strategy using both could be found. This future study will require extensive tracking simulations,

to uncover the interdependencies between the different error sources and their influences as well as the optimal targets for each corrector.

Optimal correction strategy for high-order errors. The possibility of correcting feed-down from b_6 errors to b_4 with the dodecapole correctors has been explored in [Section 3.4](#), yet whether an optimal correction strategy within the correction-parameter space can be found, has not yet been established. Of particular interest here can be the use of both b_4 and b_6 correctors at the same time, as well as the influence of additional error sources (e.g. octupole and decapole errors).

Improvements to the flexible correction algorithm. Further additions to the correction algorithm ([Section 3.2](#)) itself are also an option for future improvement. One such addition could be including the full phase-advance information to the calculation of the RDT, enabling not only the “effective” RDT of [Eq. \(3.1\)](#), but also the actual RDT of [Eq. \(2.35\)](#) as correction target.

Synergy with Xtrack The nonlinear IR-error correction script presented in [Section 3.2](#) can be used in cooperation with XTrack [\[246\]](#), to improve the setup and run time of future tracking studies. Compatibility with this new tracking software is straight-forward, as XTrack also employs MAD-X or cpmad for the setup of the machine. In fact, the use of the new python package will not only extend the correction capabilities of the tracking studies, but also solve long-standing issues and compatibility problems with the current pre-compiled fortran scripts.

Automatization. Measurement of amplitude detuning is a tedious procedure and requires many beam excitation to arrive at useable results. It is envisioned, that this process could be speed-up by automating the kick-procedure. In a similar manner, the calculation of the dodecapole correction, performed “manually” in [Chapter 7](#), should be automated and extended to encompass the additional orders of corrector magnets in the HL-LHC, which can largely reduce the commissioning effort.

Alternative methods to use the data from MD3312. Due to the noisiness of the data gathered during MD3312 as shown in [Chapter 6](#), analysis of the data has not been possible with the currently implemented methods. New methods have been suggested, such as collecting data from additional sources, e.g. the BBQ-BPMs. Together with attempts on improved cleaning of the data, this could lead to a successful evaluation of the data already gathered, but also, more crucially, to new techniques usable during the commissioning of the HL-LHC.

All of these points pose interesting topics for future studies and I am looking forward contributing to or at least seeing the results of these explorations.

Appendix

A Miscellaneous

In this appendix smaller tasks and results are listed which I performed during the PhD-period and which are related to the thesis topic, yet do not merit their own chapter.

A.1 Error-Tables Check

The magnetic model as defined in WISE error-tables [182], regularly used in these simulations, had been under scrutiny around the same time, as their values seem to differ between different versions (2011 [247] and 2015 [183]), as well as from so called “ID-Cards” (data tables) provided by the CERN magnets group, responsible for acquiring this data. The discrepancies have been summarized in a note [248] and were discussed and partly identified as additional effects of the *beam screen* [249], incorporated in some of the tables but not in others. As this issue is closely linked to differences between the magnetic model and beam-based measurements [242, 250] this resulted in further, still ongoing, investigations by the magnet group into the matter.

A.2 Software Development

AutoSix. I developed AutoSix [150], a `python`-wrapper to ease usage of SixTrack [133, 134], a software to run particle tracking studies in accelerators, and its submission system SixDesk [135]. AutoSix allows straight forward dispatching of parametric studies and solves some limitations and bugs currently present in SixDesk itself. SixTrack and SixDesk are very powerful, but their handling is quite (and unnecessarily) complex; yet they were of great value - essential even - to study DA. Recently, they have largely been replaced by Xsuite [246], making AutoSix hopefully obsolete, as it should be easier to submit the same studies with this new package.

JobSubmitter. Together with M. Hofer[Ⓜ] and contributions from F. Soubelet[Ⓜ], I wrote JobSubmitter [150], another `python`-package for submitting parametrized studies to the queueing system HTCondor, which is installed on the computing cluster at CERN. This package has proven to be very popular amongst colleagues, due to its simplicity.

Optics-Functions. The `optics-functions` are a `python`-package to calculate various optics parameters from MAD-X TWISS-command outputs, such as RDTs and coupling. This package is based on the theory developed in [251].

omc3. In addition to the amplitude detuning analysis utilized in this thesis, several other contributions to the OMC-team's software [6, 149] have been performed. While many contributions were bugfixes and maintenance, some time was spend on upgrading from `python2` to `python3` and testing the two major correction schemes, e.g. used in future commissioning: there is the global corrections calculations via response matrix [124], and the yet still unfinished calculation of local corrections via the segment-by-segment approach [121, 122]. Also, many plotting scripts, e.g. to create publication-grade plots of amplitude detuning and the results of optics measurements, were added and the graphical user-interface, used to start the `python` scripts and visualize the results, was made compatible with `python3` (together with L. Malina^{id}) and got overhauled in the process.

Another major update in preparation for amplitude detuning measurements in 2022 has also been the implementation of 3D-fits to be able to analyse detuning data from measurements with forced-oscillations increasing in strengths in both planes. This has been done, to improve on the measurement quality of the amplitude detuning cross-terms, which had been very noisy in the past. Yet, these kicks were never performed during the measurements in 2022, as with optimization of the difference between forced and natural tune, good measurement results could already be achieved, without the cost of additional dual-plane kicks (see [Chapter 7](#)).

Documentation. Together with F. Soubelet^{id} I created a new web-page for the OMC-team [252], in which useful information about and for the team is collected and which serves as wiki, guide and resource collection, and as entrypoint for future members.

B Feed-Down

Feed-down, the effect that occurs when a particle beam's orbit passes off-center through a magnet, has been mentioned shortly in [Section 2.2.2](#). Here, it is discussed in more detail and the there presented [Eq. \(2.21\)](#) is derived.

Mathematically, feed-down can be understood by applying a first-order Taylor expansion to the Hamiltonian [Eq. \(2.19\)](#) in the curvilinear (comoving) coordinate system and cartesian transversal coordinates

$$h(x, y) = \Re \left[\sum_{n=2}^{\infty} (K_n + iJ_n) \frac{(x + iy)^n}{n!} \right] \quad (\text{B.1})$$

for a beam centroid traversing the magnet at $\Delta x(s), \Delta y(s)$:

$$\begin{aligned} & h(x + \Delta x, y + \Delta y) \\ &= \Re \left[\sum_{n=2}^{\infty} (K_n + iJ_n) \frac{((x + \Delta x) + i(y + \Delta y))^n}{n!} \right] \\ &\stackrel{\text{Taylor}}{=} \Re \left[\sum_{n=2}^{\infty} (K_n + iJ_n) \frac{\sum_{p=0}^n \frac{1}{p!} \frac{n!}{(n-p)!} (x + iy)^{n-p} (\Delta x + i\Delta y)^p}{n!} \right] \\ &= \Re \left[\sum_{n=2}^{\infty} (K_n + iJ_n) \sum_{p=0}^n \frac{(x + iy)^{n-p} (\Delta x + i\Delta y)^p}{p! (n-p)!} \right] \quad (\text{B.2}) \\ &\stackrel{\text{sort by } (x+iy)^n}{n \rightarrow n+p} \Re \left[\sum_{n=0}^{\infty} \sum_{p=\max(2-n, 0)}^{\infty} \frac{1}{p! n!} (K_{n+p} + iJ_{n+p}) (x + iy)^n (\Delta x + i\Delta y)^p \right] \\ &= \Re \left[\sum_{n=0}^{\infty} \left(\sum_{p=\max(2-n, 0)}^{\infty} (K_{n+p} + iJ_{n+p}) \frac{(\Delta x + i\Delta y)^p}{p!} \right) \frac{(x + iy)^n}{n!} \right] \end{aligned}$$

For brevity (s) is omitted, but h, K_n, J_n, x, y and $\Delta x, \Delta y$ are all dependent on the longitudinal location. From [Eqs. \(B.1\) and \(B.2\)](#) one can see that magnetic field strengths of order $n \geq 2$ without offset are replaced by a sum depending on the higher order field strengths scaled by powers of the offset. These higher order fields of $n + q$ therefore “feed down” to the field strengths of order n , showing the same effects on the beam as these lower orders would. As seen in [Eq. \(B.2\)](#), feed-down to field order $n \geq 2$

from fields up to $n + P$ can be calculated by:

$$(K_n + iJ_n)^{\text{w/ feed-down}} = \sum_{p=0}^P (K_{n+p} + iJ_{n+p}) \frac{(\Delta x + i\Delta y)^p}{p!}. \quad (\text{B.3})$$

Fields feed-down can also have an effect on the scalar field ($n = 0$) and dipole fields ($n = 1$). As these fields structure does not follow the structure in Eq. (B.1), Eq. (B.3) is not applicable. In the context of RDTs n (see Section 2.2.5) is always larger than 2 and we can use Eq. (B.3).

Individual Terms. So far we have looked at how a Hamiltonian of order n is build from sources of orders $n + q$, but sometimes it can make sense to look at it the other way around: How are sources of order n affecting Hamiltonian terms of order $n - q$ via feed-down? This is quickly explored looking at individual Hamiltonian terms in the following:

As a convention, terms that have the general coefficients as Hamiltonian terms of a certain order n (as in Eqs. (2.20a) and (2.20b)), but contain K_m and J_m values of a different order m are denoted by adding these in parenthesis in the superscript to the Hamiltonian terms name, e.g. $N_n^{(J_m)}$.

From Eqs. (2.20a) and (2.20b) we can calculate the derivatives with respect to x, y

$$\frac{\partial}{\partial x} N_n^{(L_m)} \stackrel{\text{Eq. (2.20a)}}{=} \frac{1}{n!} L_m \Re \left[n(x + iy)^{n-1} \right] = \frac{1}{(n-1)!} L_m \Re \left[(x + iy)^{n-1} \right] = N_{n-1}^{(L_m)} \quad (\text{B.4a})$$

$$\frac{\partial}{\partial y} N_n^{(L_m)} \stackrel{\text{Eq. (2.20a)}}{=} \frac{1}{n!} L_m \Re \left[in(x + iy)^{n-1} \right] = -\frac{1}{(n-1)!} L_m \Im \left[(x + iy)^{n-1} \right] = S_{n-1}^{(L_m)} \quad (\text{B.4b})$$

$$\frac{\partial}{\partial x} S_n^{(L_m)} \stackrel{\text{Eq. (2.20b)}}{=} -\frac{1}{n!} L_m \Im \left[n(x + iy)^{n-1} \right] = -\frac{1}{(n-1)!} L_m \Im \left[(x + iy)^{n-1} \right] = S_{n-1}^{(L_m)} \quad (\text{B.4c})$$

$$\frac{\partial}{\partial y} S_n^{(L_m)} \stackrel{\text{Eq. (2.20b)}}{=} -\frac{1}{n!} L_m \Im \left[in(x + iy)^{n-1} \right] = -\frac{1}{(n-1)!} L_m \Re \left[(x + iy)^{n-1} \right] = -N_{n-1}^{(L_m)} \quad (\text{B.4d})$$

with L_m as a stand-in for either J_m or K_m . The Taylor expansion of N_n and S_n therefore

reads

$$\begin{aligned}
& N_n(x + \Delta x, y + \Delta y) = \\
& N_n(x, y) \\
& + N_{n-1}^{(K_n)}(x, y) \Delta x + S_{n-1}^{(K_n)}(x, y) \Delta y \\
& + \frac{1}{2} N_{n-2}^{(K_n)}(x, y) (\Delta x^2 - \Delta y^2) + S_{n-2}^{(K_n)}(x, y) \Delta x \Delta y \\
& + \frac{1}{6} N_{n-3}^{(K_n)}(x, y) (\Delta x^3 - 3 \Delta x \Delta y^2) + \frac{1}{6} S_{n-3}^{(K_n)}(x, y) (3 \Delta x^2 \Delta y - \Delta y^3) \\
& + \frac{1}{24} N_{n-4}^{(K_n)}(x, y) (\Delta x^4 - 6 \Delta x^2 \Delta y^2 + \Delta y^4) + \frac{1}{6} S_{n-4}^{(K_n)}(x, y) (\Delta x^3 \Delta y - \Delta x \Delta y^3) \\
& + \frac{1}{120} N_{n-5}^{(K_n)}(x, y) (\Delta x^5 - 10 \Delta x^3 \Delta y^2 + 5 \Delta x \Delta y^4) \\
& + \frac{1}{120} S_{n-5}^{(K_n)}(x, y) (5 \Delta x^4 \Delta y - 10 \Delta x^2 \Delta y^3 + \Delta y^5) \\
& + h.o.t
\end{aligned} \tag{B.5}$$

$$\begin{aligned}
& S_n(x + \Delta x, y + \Delta y) = \\
& S_n(x, y) \\
& + S_{n-1}^{(J_n)}(x, y) \Delta x - N_{n-1}^{(J_n)}(x, y) \Delta y \\
& + \frac{1}{2} S_{n-2}^{(J_n)}(x, y) (\Delta x^2 - \Delta y^2) - N_{n-2}^{(J_n)}(x, y) \Delta x \Delta y \\
& + \frac{1}{6} S_{n-3}^{(J_n)}(x, y) (\Delta x^3 - 3 \Delta x \Delta y^2) - \frac{1}{6} N_{n-3}^{(J_n)}(x, y) (3 \Delta x^2 \Delta y - \Delta y^3) \\
& + \frac{1}{24} S_{n-4}^{(J_n)}(x, y) (\Delta x^4 - 6 \Delta x^2 \Delta y^2 + \Delta y^4) - \frac{1}{6} N_{n-4}^{(J_n)}(x, y) (\Delta x^3 \Delta y - \Delta x \Delta y^3) \\
& + \frac{1}{120} S_{n-5}^{(J_n)}(x, y) (\Delta x^5 - 10 \Delta x^3 \Delta y^2 + 5 \Delta x \Delta y^4) \\
& - \frac{1}{120} N_{n-5}^{(J_n)}(x, y) (5 \Delta x^4 \Delta y - 10 \Delta x^2 \Delta y^3 + \Delta y^5) \\
& + h.o.t
\end{aligned} \tag{B.6}$$

C Appendix to Amplitude Detuning from Misalignments of Nonlinear-Corrector and Triplet Magnets in the LHC and HL-LHC

C.1 Landau-Octupole Powering Reference

Table C.1. Amplitude detuning change between 0 A and 570 A MO powering as determined by simulations.

	$\partial Q_x / \partial(2J_x)$ [10^3 m^{-1}]	$\partial Q_y / \partial(2J_y)$ [10^3 m^{-1}]	$\partial Q_x / \partial(2J_y)$ [10^3 m^{-1}]
LHC flattop	+140	+136	-100
LHC $\beta^* = 30 \text{ cm}$	+168	+166	-114
HL-LHC $\beta^* = 30 \text{ cm}$	+186	+186	-116

C.2 Mean of Distributions

When simulating the misalignments, which are then used in Eqs. (4.1a) to (4.1d) to calculate feed-down, we have two independent variables X and Y to be used for dx and dy . They are taken from the same zero-mean distribution, either normal or uniform, i.e. their expectation values are zero

$$\mathbb{E}[X] = \mathbb{E}[Y] = 0 \quad (\text{C.1})$$

and their variances are equal

$$\mathbb{V}[X] = \mathbb{V}[Y] . \quad (\text{C.2})$$

The mean of Eqs. (4.1a) and (4.1b) is obviously zero as they only depend on one variable each. For Eq. (4.1c) we can use the definition of the variance:

$$\mathbb{E}[X^2] = \mathbb{V}(X) + \mathbb{E}[X]^2 . \quad (\text{C.3})$$

And then

$$\mathbb{E}[X^2 - Y^2] = \mathbb{E}[X^2] - \mathbb{E}[Y^2] = \mathbb{V}(X)^2 + \mathbb{E}[X]^2 - \mathbb{V}(Y)^2 - \mathbb{E}[Y]^2 = 0 . \quad (\text{C.4})$$

For Eq. (4.1d), as the two random variables are independent:

$$\mathbb{E}[XY] = \mathbb{E}[X] \cdot \mathbb{E}[Y] = 0 . \quad (\text{C.5})$$

Hence the expectation value for all Eqs. (4.1a) to (4.1d) is zero in the simulations.

C.3 Triplet Error Distributions

Full comparison plots between the LHC and HL-LHC, with systematic $b_6 = -4$, integrated field error distributions in the triplets of IP1 and IP5. Where the magnets are split into A and B, the contributions have been summed. The distributions are over the different WISE seeds for the LHC and the different error instances of the errortable [191] for the HL-LHC.

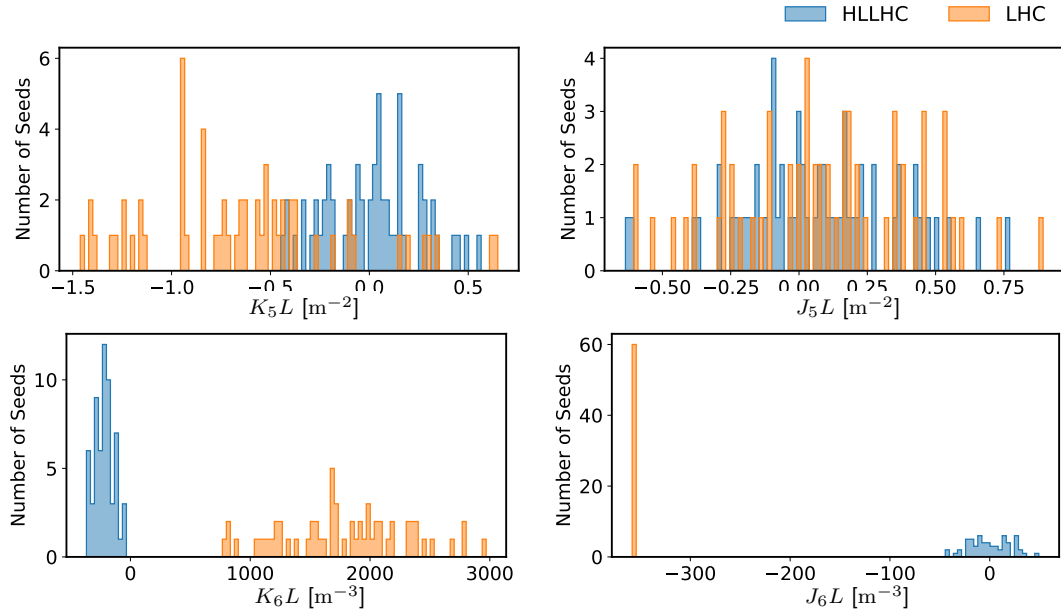


Figure C.1. Comparison in the integrated field error distributions between LHC MQXA.1R1 (orange) and HL-LHC (systematic $b_6 = -4$) MQXFA.(A+B)1R1 (blue). For (A+B) the contributions from the two magnets have been summed.

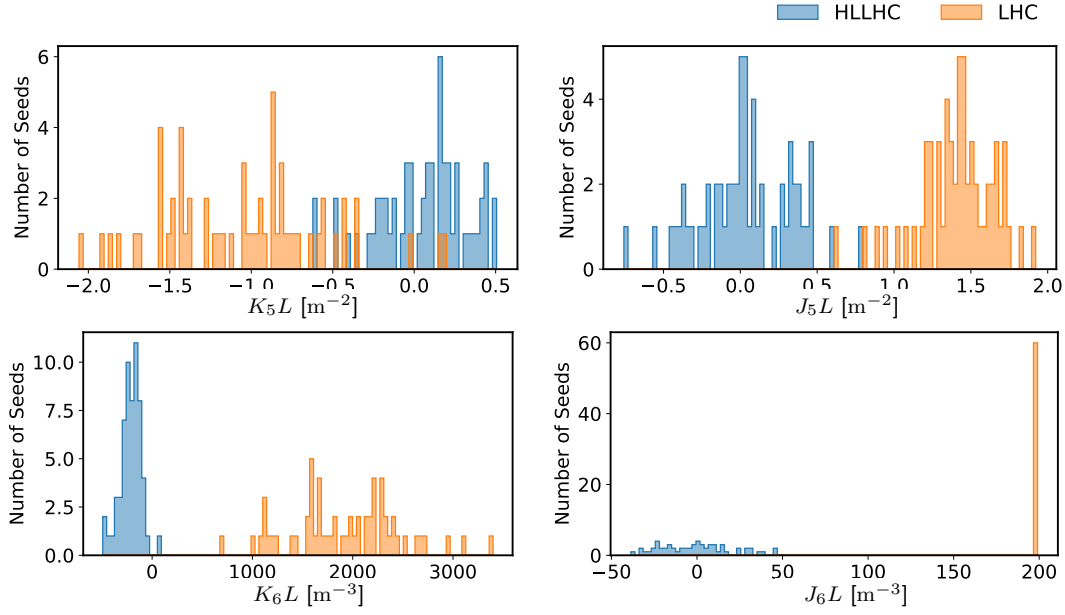


Figure C.2. Comparison in the integrated field error distributions between LHC MQXA.1R5 (orange) and HL-LHC (systematic $b_6 = -4$) MQXFA.(A+B)1R5 (blue). For (A+B) the contributions from the two magnets have been summed.

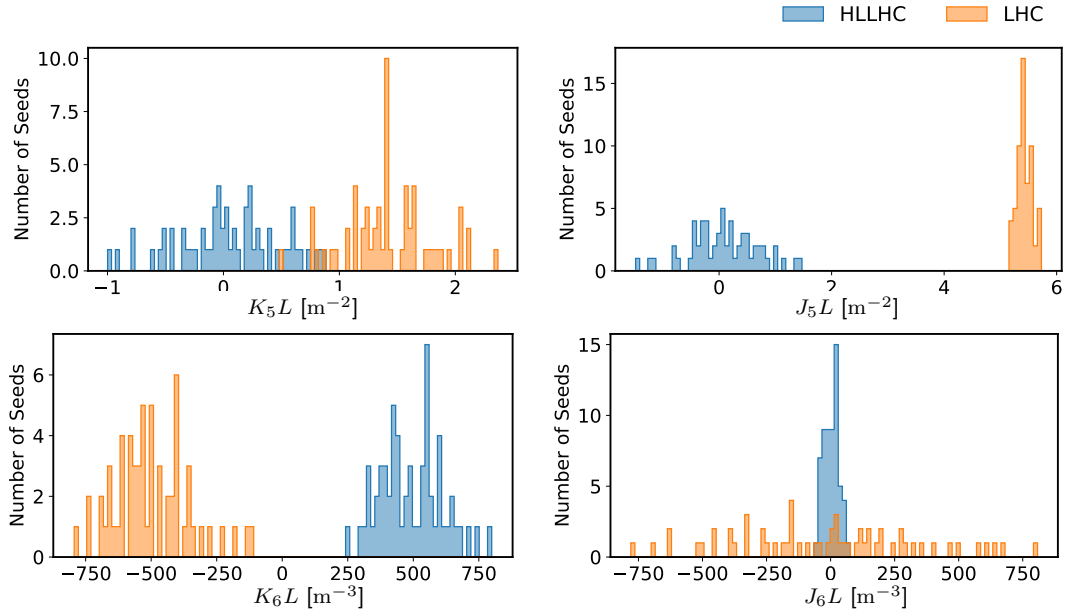


Figure C.3. Comparison in the integrated field error distributions between LHC MQXB.(A+B)2R1 (orange) and HL-LHC (systematic $b_6 = -4$) MQXFB.(A+B)2R1 (blue). For (A+B) the contributions from the two magnets have been summed.

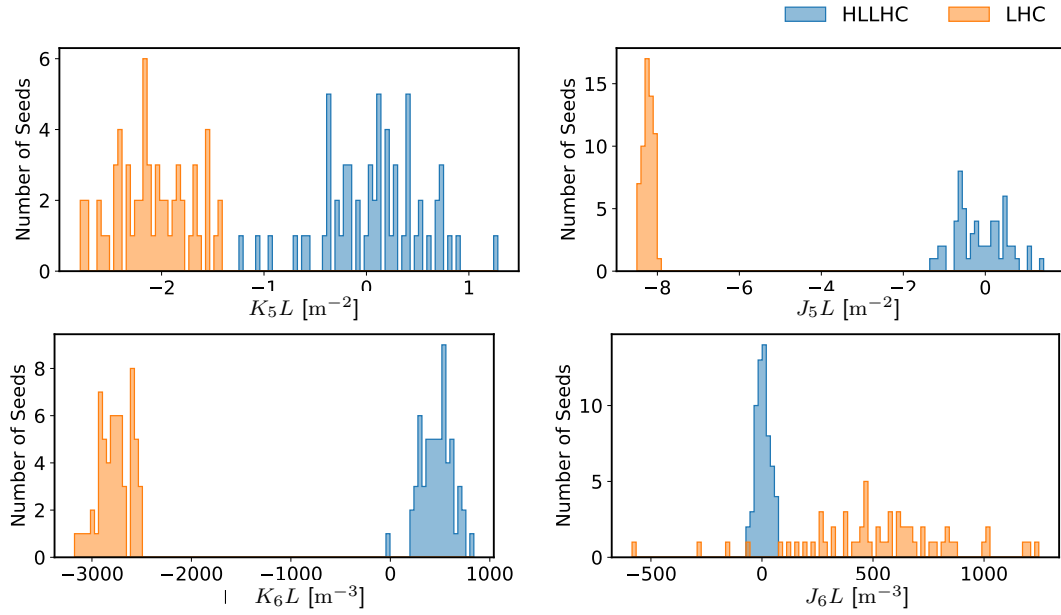


Figure C.4. Comparison in the integrated field error distributions between LHC MQXB.(A+B)2R5 (orange) and HL-LHC (systematic $b_6 = -4$) MQXFB.(A+B)2R5 (blue). For (A+B) the contributions from the two magnets have been summed.

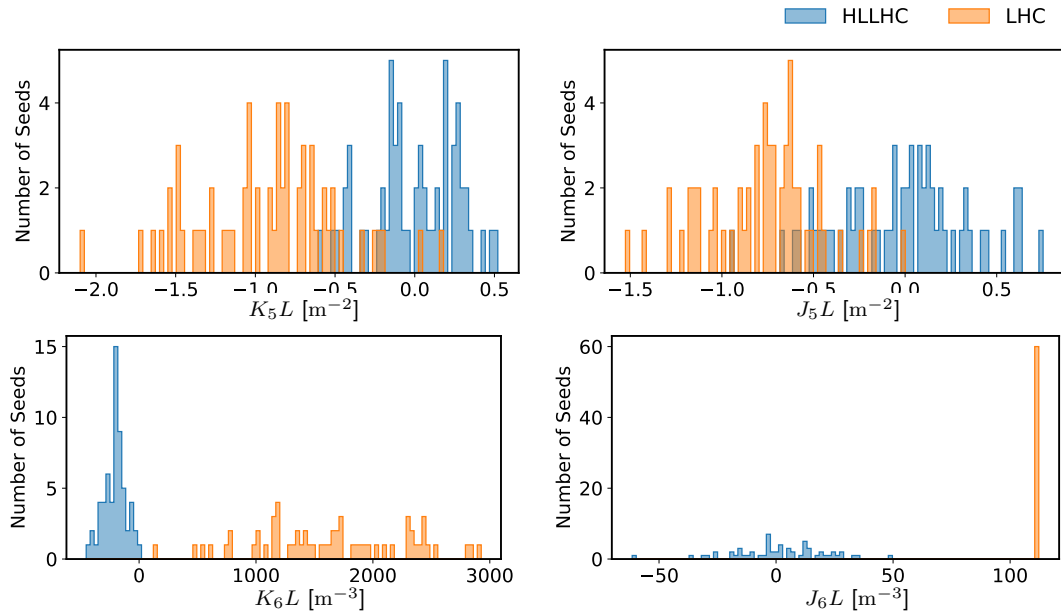


Figure C.5. Comparison in the integrated field error distributions between LHC MQXA.3R1 (orange) and HL-LHC (systematic $b_6 = -4$) MQXFA.(A+B)3R1 (blue). For (A+B) the contributions from the two magnets have been summed.

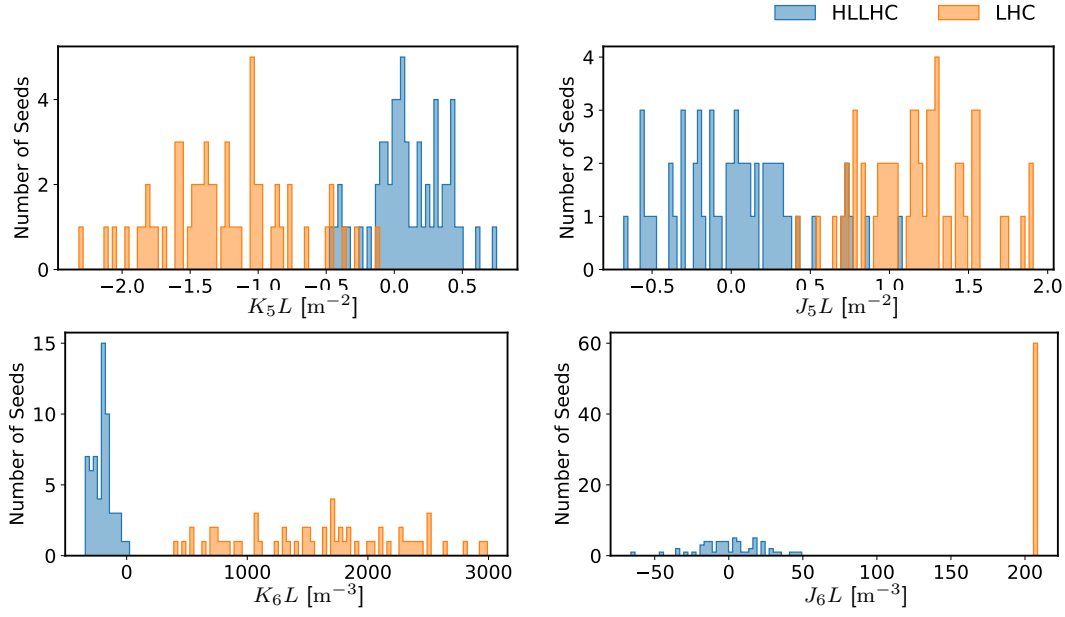


Figure C.6. Comparison in the integrated field error distributions between LHC MQXA.3R5 (orange) and HL-LHC (systematic $b_6 = -4$) MQXFA.(A+B)3R5 (blue). For (A+B) the contributions from the two magnets have been summed.

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As all scientific work, this thesis stands on the shoulders of giants, or at least on a huge pile of work that came before, lifted up by a lot of colleagues and friends, without whom the studies presented herein would not have been possible.

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Selbstständigkeitserklärung

Ich erkläre, dass ich die Dissertation selbständig und nur unter Verwendung der von mir gemäß §7 Abs. 3 der Promotionsordnung der Mathematisch-Naturwissenschaftlichen Fakultät, veröffentlicht im Amtlichen Mitteilungsblatt der Humboldt-Universität zu Berlin Nr. 42/2018 am 11.07.2018 angegebenen Hilfsmittel angefertigt habe.

Genf, den 07.09.2023

Joschua Dilly

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