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ADVANCEMENTS IN OPTICS CONTROL AT THE
CERN PROTON SYNCHROTRON

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EXTENDED SUMMARY

The CERN Proton Synchrotron (PS), inaugurated in 1959, represents a cornerstone of CERN's accelerator complex. As the oldest operational accelerator at CERN, the PS has played a pivotal role in advancing high-energy physics and remains critical to the injector chain supporting the Large Hadron Collider (LHC). Designed with a 100-meter-radius ring featuring alternating-gradient focusing magnets, the PS accelerates protons and ions for downstream experiments, contributing to groundbreaking discoveries in particle physics. Its versatility allows it to handle a wide range of beam types and energies, serving experiments like the Antiproton Decelerator, nTOF, and fixed-target research while also acting as a key injector for larger machines such as the Super Proton Synchrotron (SPS) and the LHC.

The PS faces unique challenges due to its age and the evolving demands of modern physics experiments. Many of its components must operate with high precision under increasingly complex conditions. Key challenges include the control of tune and chromaticity, accurate horizontal emittance measurements, and the identification and correction of resonances. These issues are further complicated by non-linearities in the magnetic lattice, space charge effects, and dispersive contributions that distort beam diagnostics.

The thesis addresses these challenges by introducing innovative methods that leverage emerging technologies and advanced modeling. For tune and chromaticity control, machine learning (ML) techniques were employed to improve the operation of Pole Face Windings (PFWs), allowing for dynamic adjustments across the magnetic cycle. Horizontal emittance measurements were enhanced using zero dispersion optics, which eliminate dispersive effects and enable more accurate beam diagnostics. Finally, Resonance Driving Terms (RDTs) were studied using forced oscillations from the transverse damper (ADT), enabling precise resonance correction and revealing inconsistencies in machine modeling.

The PFWs are integral to the PS's ability to manage tune and chromaticity throughout its magnetic cycle. These correction elements, embedded within the PS's combined function magnets, allow for precise adjustments to the machine's magnetic field, enabling control over the particle beam's oscillatory behavior.

However, the current PFW control system is plagued by significant limitations that hinder its operational flexibility and efficiency.

The PFW control system relies on response matrices and basic function settings to dictate the currents applied at various stages of the magnetic cycle. These response matrices, derived from empirical measurements, aim to correlate PFW currents with tune and chromaticity changes. However, they fail to account for the non-linear behavior of the PFWs, especially at higher energies. This leads to inaccuracies in tune corrections, particularly during transition crossing, a critical phase where beam stability is most vulnerable.

Additionally, the basic functions that provide initial current settings are rigid and not tailored for specific beam destinations. This rigidity necessitates extensive manual adjustments for modern beam types, making the process cumbersome and error-prone. Consequently, operators often avoid setting up new magnetic cycles, impeding the PS's ability to adapt to the demands of evolving experiments.

The thesis proposes a machine learning-based approach to overcome these challenges, introducing a prediction and control system for the PFWs. The solution leverages neural networks to predict tune and chromaticity based on PFW current settings, magnetic field values, and beam parameters. To address noise and inconsistencies in measurement data, data acquisition methods were also enhanced, including refined tune measurement techniques that mitigate coupling errors and noise. A physics-based loss function was also incorporated into the neural network's training process. This aligns the network's predictions with the underlying physics of the PS, reducing the likelihood of overfitting and improving accuracy.

Building on the prediction network, a control network was developed to dynamically adjust PFW currents in real time. This system would replace the static basic functions with a flexible, adaptive approach, enabling seamless operation across the magnetic cycle. Validation tests demonstrated that the control network outperformed the traditional response matrix approach, particularly in high-energy regions where previous systems struggled.

Zero dispersion optics address the fundamental challenge of accurately measuring horizontal emittance in the presence of dispersive effects, which convolute transverse and longitudinal beam distributions. In accelerators like the PS, this convolution complicates diagnostics, as traditional methods fail to separate the contributions of dispersion from the intrinsic beam properties. This issue is especially critical at low energies, where larger momentum spreads amplify the dispersive contribution, undermining emittance accuracy and obscuring beam characteristics like tail population.

To overcome this, the thesis designed and implemented zero dispersion optics using the Low Energy Quadrupoles (LeQs) in the PS ring. By carefully adjusting the quadrupoles, dispersion at key beam profile measurement locations was reduced to near zero. This enabled accurate emittance measurements without the presence of dispersive distortions. Experimental results confirmed that the horizontal emittance discrepancies between the PS Booster and PS, previously attributed to blow-up during transfer, were largely the result of differing dispersive contributions rather than genuine emittance growth. Moreover, the introduction of zero dispersion optics revealed overpopulated beam tails at low energies, which had been masked under normal optics.

These findings demonstrated that zero dispersion optics improve both the accuracy of horizontal emittance measurements and the ability to diagnose beam dynamics. At higher energies, where the momentum spread diminishes, comparisons with zero dispersion measurements at low energies validated that dispersive effects were effectively nullified. This comparative analysis confirmed the precision of the zero dispersion method and provided insights into beam tail evolution across energy levels.

For accelerators lacking the flexibility to implement zero dispersion optics, a numerical deconvolution method was developed. This technique uses measured beam profiles and longitudinal tomography to separate transverse and longitudinal distributions. Tests of the numerical method showed strong agreement with results obtained using zero dispersion optics, extending its applicability to machines like the PSB. This approach also offered significant improvements in identifying beam tails.

The interplay between dispersion and space charge effects emerged as an additional area of investigation. Space charge influences beam behavior in ways that standard dispersion measurements cannot capture. By combining experimental data with advanced modeling, the thesis validated space charge effects on dispersion, enhancing the overall understanding of their impact on beam dynamics.

Resonance Driving Terms (RDTs) represent critical tools for understanding and correcting non-linearities in particle accelerators. These terms quantify the driving strengths of resonances, which can cause beam instabilities, losses, and emittance growth. In the PS, the challenges of resonance correction are magnified by its low energy and high brightness, leading to significant space charge tune spreads. Historically, resonance studies relied on loss maps, wherein beam losses were measured as the machine's tune was scanned. Although effective, this method is time-consuming and lacks the precision required to quantify subtle sources of non-linear excitation.

To address this, the thesis utilized forced oscillations generated by the transverse damper (ADT) to measure RDTs with unprecedented accuracy. This innovative approach enabled the extraction of detailed resonance information from turn-by-turn data collected from the PS's Beam Position Monitors (BPMs). By analyzing the frequency spectra of these data, specific RDTs were identified and their amplitudes quantified.

A key case study involved the $3Q_y$ resonance, driven by the f_{0030} RDT. Historical issues with this resonance were linked to non-linearities introduced by shielding plates near the Beam Gas Ionization (BGI) monitor. The thesis demonstrated that RDT measurements could detect residual effects of these non-linearities even after corrective shielding was installed. The precision of the RDT-based method revealed subtleties that traditional loss maps could not, underscoring its diagnostic power.

Further investigations applied these techniques to correct the $2Q_x+1Q_y$ resonance, driven by the f_{2010} RDT. Using response matrices derived from model simulations, corrections were calculated and applied, reducing the resonance's impact on beam stability. This approach is complementary to the trial-and-error process inherent in loss map corrections, enabling faster and more efficient resonance mitigation. Post-correction measurements showed a marked reduction in beam losses, confirming the efficiency of the RDT-based method.

The sensitivity of RDT measurements was further enhanced through noise reduction techniques like Singular Value Decomposition (SVD) cleaning. These methods improved the signal-to-noise ratio in frequency spectra, allowing for the identification of weaker resonances. Additionally, discrepancies in BPM calibration were identified and corrected, ensuring the accuracy of subsequent measurements.

The work presented in this thesis addresses critical challenges in the operation of the CERN Proton Synchrotron. The machine learning-based control system for Pole Face Windings significantly improves the precision and flexibility of tune and chromaticity adjustments. By replacing static basic functions with dynamic predictions, it allows operators to address non-linearities and adapt more effectively to evolving experimental needs. Zero dispersion optics have improved horizontal emittance measurements by eliminating distortions from dispersive contributions. This advancement not only ensures accurate diagnostics but also reveals critical insights into beam tail evolution and space charge effects. This result is complemented by the development of a numerical deconvolution method, which extends these diagnostic capabilities to less flexible accelerators. The use of Resonance Driving Terms, measured through ADT-induced oscillations, has proven a new method for resonance correction

in machines that do not have a dedicated AC-Dipole. These measurements enable precise identification and mitigation of resonances, reducing beam losses and improving machine performance. By enhancing signal processing and addressing calibration discrepancies, the RDT-based approach streamlines the correction process and provides a powerful diagnostic tool for non-linear optics.

ZUSAMMENFASSUNG

Das CERN Protonen-Synchrotron (PS), das 1959 eingeweiht wurde, ist ein Eckpfeiler des Beschleunigerkomplexes des CERN. Als ältester in Betrieb befindlicher Beschleuniger am CERN hat das PS eine entscheidende Rolle bei der Weiterentwicklung der Hochenergiephysik gespielt und ist nach wie vor ein wichtiger Bestandteil der Injektorkette, die den Large Hadron Collider (LHC) unterstützt. Der PS ist mit einem Ring mit einem Radius von 100 Metern und Magneten mit Wechselgradientenfokussierung ausgestattet. Er beschleunigt Protonen und Ionen für nachgeschaltete Experimente und trägt so zu bahnbrechenden Entdeckungen in der Teilchenphysik bei. Dank seiner Vielseitigkeit kann er eine breite Palette von Strahltypen und -energien verarbeiten und dient Experimenten wie dem Antiprotonenbeschleuniger, nTOF und der Forschung mit festen Zielen, während er gleichzeitig als wichtiger Injektor für größere Anlagen wie das Super Proton Synchrotron (SPS) und den LHC dient.

Das PS steht aufgrund seines Alters und der sich entwickelnden Anforderungen moderner Physikexperimente vor besonderen Herausforderungen. Viele seiner Komponenten müssen mit hoher Präzision unter immer komplexeren Bedingungen arbeiten. Zu den wichtigsten Herausforderungen gehören die Kontrolle von Tune und Chromatizität, genaue Messungen der horizontalen Emittanz sowie die Identifizierung und Korrektur von Resonanzen. Diese Probleme werden durch Nichtlinearitäten im magnetischen Gitter, Raumladungseffekte und dispersive Beiträge, die die Strahldiagnose verzerren, zusätzlich erschwert.

In dieser Arbeit werden diese Herausforderungen durch die Einführung innovativer Methoden angegangen, bei denen neue Technologien und fortschrittliche Modellierung zum Einsatz kommen. Für die Tune- und Chromatizitätskontrolle wurden Techniken des maschinellen Lernens (ML) eingesetzt, um den Betrieb von Pole Face Windings (PFWs) zu verbessern und dynamische Anpassungen über den gesamten magnetischen Zyklus hinweg zu ermöglichen. Horizontale Emittanzmessungen wurden durch den Einsatz von Optiken mit Null-Dispersion verbessert, die dispersive Effekte eliminieren und eine genauere Strahldiagnose ermöglichen. Schließlich wurden die Resonanzantriebsbedingungen (RDTs) unter Verwendung erzwungener Schwingungen des Querdämpfers (ADT) untersucht, was eine präzise Resonanzkorrektur ermöglichte und Unstimmigkeiten

in der Maschinenmodellierung aufdeckte.

Die PFWs sind ein wesentlicher Bestandteil der Fähigkeit des PS, die Stimmung und die Chromatizität während des gesamten magnetischen Zyklus zu steuern. Diese Korrekturlemente, die in die kombinierten Funktionsmagnete des PS eingebettet sind, ermöglichen eine präzise Anpassung des Magnetfelds der Maschine und damit die Kontrolle über das Schwingungsverhalten des Teilchenstrahls. Das derzeitige PFW-Kontrollsystem hat jedoch mit erheblichen Einschränkungen zu kämpfen, die seine betriebliche Flexibilität und Effizienz beeinträchtigen.

Das PFW-Kontrollsystem stützt sich auf Reaktionsmatrizen und grundlegende Funktionseinstellungen, um die in den verschiedenen Phasen des magnetischen Zyklus angelegten Ströme zu diktieren. Diese aus empirischen Messungen abgeleiteten Reaktionsmatrizen zielen darauf ab, die PFW-Ströme mit der Stimmung und den Farbveränderungen zu korrelieren. Sie berücksichtigen jedoch nicht das nichtlineare Verhalten der PFWs, insbesondere bei höheren Energien. Dies führt zu Ungenauigkeiten bei den Stimmungskorrekturen, insbesondere während des Übergangs, einer kritischen Phase, in der die Strahlstabilität am meisten gefährdet ist.

Darüber hinaus sind die grundlegenden Funktionen, die die anfänglichen Stromeinstellungen liefern, starr und nicht auf spezifische Strahlziele zugeschnitten. Diese Starrheit macht umfangreiche manuelle Anpassungen für moderne Strahltypen erforderlich, was den Prozess umständlich und fehleranfällig macht. Infolgedessen vermeiden es die Bediener oft, neue magnetische Zyklen einzustellen, was die Fähigkeit des PS beeinträchtigt, sich an die Anforderungen der sich entwickelnden Experimente anzupassen.

In dieser Arbeit wird ein auf maschinellem Lernen basierender Ansatz zur Bewältigung dieser Herausforderungen vorgeschlagen, indem ein Vorhersage- und Steuerungssystem für die PFWs eingeführt wird. Die Lösung nutzt neuronale Netze zur Vorhersage von Tune und Chromatizität auf der Grundlage von PFW-Stromeinstellungen, Magnetfeldwerten und Strahlparametern. Um das Rauschen und die Unstimmigkeiten in den Messdaten zu beseitigen, wurden auch die Datenerfassungsmethoden verbessert, einschließlich verfeinerter Tune-Messverfahren, die Kopplungsfehler und Rauschen abmildern. Außerdem wurde eine physikalisch basierte Verlustfunktion in den Trainingsprozess des neuronalen Netzes integriert. Dadurch werden die Vorhersagen des Netzwerks an die zugrunde liegende Physik des PS angepasst, was die Wahrscheinlichkeit einer Überanpassung verringert und die Genauigkeit verbessert.

Aufbauend auf dem Vorhersagenetz wurde ein Steuernetz entwickelt, um die

PFW-Ströme in Echtzeit dynamisch anzupassen. Dieses System würde die statischen Grundfunktionen durch einen flexiblen, adaptiven Ansatz ersetzen und einen nahtlosen Betrieb über den gesamten magnetischen Zyklus hinweg ermöglichen. Validierungstests haben gezeigt, dass das Kontrollnetzwerk den traditionellen Antwortmatrix-Ansatz übertrifft, insbesondere in Hochenergiebereichen, in denen frühere Systeme Schwierigkeiten hatten.

Die Null-Dispersions-Optik befasst sich mit der grundlegenden Herausforderung der genauen Messung der horizontalen Emittanz bei Vorhandensein von Dispersionseffekten, die die transversalen und longitudinalen Strahlverteilungen falten. In Beschleunigern wie dem PS erschwert diese Faltung die Diagnose, da herkömmliche Methoden die Beiträge der Dispersion nicht von den intrinsischen Strahleigenschaften trennen können. Dieses Problem ist vor allem bei niedrigen Energien kritisch, wo größere Impulsspannungen den Dispersionsbeitrag verstärken, was die Genauigkeit der Emittanz untergräbt und Strahlcharakteristika wie die Schwanzpopulation verschleiert.

Um dieses Problem zu lösen, wurde im Rahmen dieser Arbeit eine Null-Dispersions-Optik unter Verwendung der Niedrigenergie-Quadrupole (LeQs) im PS-Ring entwickelt und implementiert. Durch die sorgfältige Einstellung der Quadrupole konnte die Dispersion an den Schlüsselstellen der Strahlprofilmessung auf nahezu Null reduziert werden. Dies ermöglichte genaue Emittanzmessungen ohne dispersive Verzerrungen. Die experimentellen Ergebnisse bestätigten, dass die horizontalen Emittanzdiskrepanzen zwischen dem PS Booster und dem PS, die zuvor auf ein Aufblähen während des Transfers zurückgeführt wurden, größtenteils auf unterschiedliche dispersive Beiträge und nicht auf einen echten Emittanzanstieg zurückzuführen waren. Darüber hinaus wurden durch die Einführung einer Optik mit Null-Dispersion überfüllte Strahlenschwänze bei niedrigen Energien aufgedeckt, die bei normaler Optik verborgen geblieben waren.

Diese Ergebnisse zeigen, dass Optiken mit Null-Dispersion sowohl die Genauigkeit der horizontalen Emittanzmessungen als auch die Fähigkeit zur Diagnose der Strahldynamik verbessern. Bei höheren Energien, wo die Impulsspanne abnimmt, bestätigten Vergleiche mit Null-Dispersions-Messungen bei niedrigen Energien, dass dispersive Effekte effektiv aufgehoben wurden. Diese vergleichende Analyse bestätigte die Präzision der Nulldispersionsmethode und lieferte Einblicke in die Entwicklung des Strahlenschwanzes über verschiedene Energiestufen hinweg.

Für Beschleuniger, die nicht über die Flexibilität verfügen, eine Null-Dispersions-Optik zu implementieren, wurde eine numerische Entfaltungsmethode entwick-

elt. Diese Technik nutzt gemessene Strahlprofile und longitudinale Tomographie, um transversale und longitudinale Verteilungen zu trennen. Tests der numerischen Methode ergaben eine hohe Übereinstimmung mit den Ergebnissen, die mit der Nulldispersionsoptik erzielt wurden, was ihre Anwendbarkeit auf Maschinen wie die PSB erweitert. Dieser Ansatz bietet auch erhebliche Verbesserungen bei der Identifizierung von Strahlschwänzen.

Das Zusammenspiel zwischen Dispersions- und Raumladungseffekten erwies sich als zusätzlicher Untersuchungsgegenstand. Die Raumladung beeinflusst das Strahlverhalten auf eine Art und Weise, die mit Standard-Dispersionsmessungen nicht erfasst werden kann. Durch die Kombination von experimentellen Daten mit fortschrittlicher Modellierung wurden in dieser Arbeit die Auswirkungen von Raumladungen auf die Dispersion validiert und das Gesamtverständnis ihrer Auswirkungen auf die Strahldynamik verbessert.

Resonanztreibende Terme (RDTs) sind wichtige Werkzeuge zum Verständnis und zur Korrektur von Nichtlinearitäten in Teilchenbeschleunigern. Diese Terme quantifizieren die Antriebsstärken von Resonanzen, die Strahlinstabilitäten, Verluste und Emittanzwachstum verursachen können. Im PS werden die Herausforderungen der Resonanzkorrektur durch die niedrige Energie und die hohe Helligkeit noch vergrößert, was zu beträchtlichen Raumladungsabstimmungsspreizungen führt. In der Vergangenheit stützten sich Resonanzstudien auf Verlustkarten, bei denen die Strahlverluste gemessen wurden, während die Stimmung der Maschine abgetastet wurde. Diese Methode ist zwar effektiv, aber zeitaufwändig und nicht präzise genug, um subtile Quellen nichtlinearer Anregung zu quantifizieren.

Um dieses Problem zu beheben, wurden in dieser Arbeit erzwungene Schwingungen, die durch den Querdämpfer (ADT) erzeugt wurden, zur Messung von RDTs mit noch nie dagewesener Genauigkeit verwendet. Dieser innovative Ansatz ermöglichte die Extraktion detaillierter Resonanzinformationen aus den Turn-by-Turn-Daten, die von den Beam Position Monitors (BPMs) des PS gesammelt wurden. Durch die Analyse der Frequenzspektren dieser Daten konnten spezifische RDTs identifiziert und ihre Amplituden quantifiziert werden.

Eine wichtige Fallstudie betraf die 3Qy-Resonanz, die durch den RDT f_{0030} angetrieben wird. Historische Probleme mit dieser Resonanz waren mit Nichtlinearitäten verbunden, die durch Abschirmplatten in der Nähe des BGI-Monitors (Beam Gas Ionization) verursacht wurden. In dieser Arbeit wurde gezeigt, dass RDT-Messungen Restauswirkungen dieser Nichtlinearitäten auch nach der Installation einer korrigierenden Abschirmung erkennen können. Die Präzision der FTE-basierten Methode zeigte Feinheiten auf, die herkömmliche Verlustkarten nicht aufzeigen konnten, was ihre diagnostische Leistung unterstreicht.

In weiteren Untersuchungen wurden diese Techniken zur Korrektur der 2Qx+1Qy-

Resonanz angewandt, die von der f_{2010} -RDT angetrieben wird. Mithilfe von Reaktionsmatrizen, die aus Modellsimulationen abgeleitet wurden, wurden Korrekturen berechnet und angewendet, die die Auswirkungen der Resonanz auf die Strahlstabilität verringerten. Dieser Ansatz ergänzt den Versuch-und-Irrtum-Prozess, der mit der Korrektur von Verlustkarten einhergeht, und ermöglicht eine schnellere und effizientere Resonanzminderung. Messungen nach der Korrektur ergaben eine deutliche Verringerung der Strahlverluste, was die Effizienz der RDT-basierten Methode bestätigt.

Die Empfindlichkeit der RDT-Messungen wurde durch Rauschunterdrückungstechniken wie die Singular Value Decomposition (SVD) weiter verbessert. Diese Methoden verbesserten das Signal-Rausch-Verhältnis in den Frequenzspektren und ermöglichten die Identifizierung schwächerer Resonanzen. Darüber hinaus wurden Unstimmigkeiten bei der BPM-Kalibrierung festgestellt und korrigiert, wodurch die Genauigkeit der nachfolgenden Messungen gewährleistet wurde.

Die in dieser Dissertation vorgestellte Arbeit befasst sich mit kritischen Herausforderungen beim Betrieb des CERN Protonen-Synchrotrons. Das auf maschinellem Lernen basierende Steuerungssystem für Polflächenwicklungen verbessert die Präzision und Flexibilität von Tune- und Chromatizitätseinstellungen erheblich. Indem es statische Grundfunktionen durch dynamische Vorhersagen ersetzt, ermöglicht es den Betreibern, Nichtlinearitäten zu berücksichtigen und sich effektiver an die sich entwickelnden experimentellen Anforderungen anzupassen. Optiken mit Null-Dispersion haben die horizontalen Emittanzmessungen verbessert, indem sie Verzerrungen durch dispersive Beiträge eliminiert haben. Dieser Fortschritt gewährleistet nicht nur eine genaue Diagnose, sondern liefert auch wichtige Erkenntnisse über die Entwicklung des Strahlenschweifs und Raumladungseffekte. Ergänzt werden diese Ergebnisse durch die Entwicklung einer numerischen Entfaltungsmethode, die diese Diagnosemöglichkeiten auf weniger flexible Beschleuniger erweitert. Die Verwendung von Resonanzantriebskräften, die durch ADT-induzierte Oszillationen gemessen werden, hat sich als neue Methode zur Resonanzkorrektur in Maschinen erwiesen, die nicht über einen speziellen AC-Dipol verfügen. Diese Messungen ermöglichen eine präzise Identifizierung und Entschärfung von Resonanzen, wodurch Strahlverluste verringert und die Leistung der Maschine verbessert werden. Durch die Verbesserung der Signalverarbeitung und die Beseitigung von Kalibrierungsdiskrepanzen rationalisiert der RDT-basierte Ansatz den Korrekturprozess und bietet ein leistungsstarkes Diagnosewerkzeug für nichtlineare Optik.

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INTRODUCTION

In the early days of high-energy particle physics, the quest to understand the fundamental building blocks of matter seemed like a daunting task. Imagine holding a solid object, theorizing that deep inside, there are even smaller components that define its essence, but having no direct way to observe them. The great scientific minds of the time pondered how to unveil these hidden components. Their solution, though seemingly crude, proved effective: they decided to smash the object against a wall and carefully study the fragments that remained. As the field advanced, this method evolved, sparking new disciplines in accelerator and detector physics. Instead of hurling particles against a wall, we now collide them with each other at immense energies. These collisions, far more controlled and powerful than those early experiments, have led to some of the most groundbreaking discoveries in modern science. The development of particle accelerators has since expanded beyond the realms of particle and nuclear physics, branching into fields like material science, industry, and medical applications.

1.1 The CERN accelerator complex

After World War II, the global scientific community reached a consensus to separate scientific advancements from commercial and political influences, championing the idea of "science for the sake of science." With this vision, the *Conseil Européen pour la Recherche Nucléaire* (CERN)[1] was established on the Franco-Swiss border near Geneva in 1954. CERN quickly became a pioneer of the Open Science philosophy[2], setting a new standard for scientific laboratories worldwide.

CERN operates eight particle accelerators[3], starting with three linear machines: Linac4 (L4), which accelerates hydrogen ions; Linac3 (L3), designed for lead ions; and the CERN Linear Electron Accelerator for Research (CLEAR). The remaining accelerators are circular, allowing particles to remain in the machines for extended periods. These include the Proton Synchrotron Booster (PSB), the Low Energy Ion Ring (LEIR), the Proton Synchrotron (PS), the Super Proton Synchrotron (SPS), and the Large Hadron Collider (LHC), currently the world's largest particle accelerator.

The LHC features a 27-kilometer circumference ring where two beams intersect at four points which are used to collide particles inside highly sophisticated independent

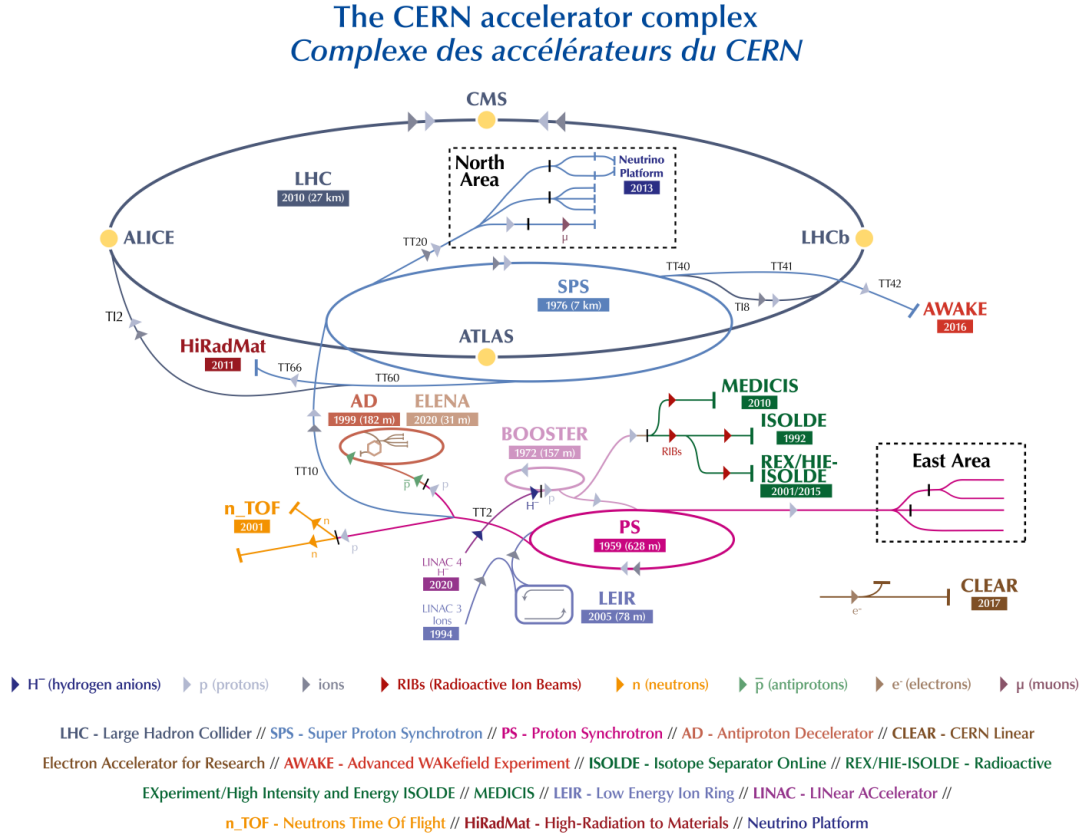


Figure 1.1: A schematic overview of the CERN accelerator complex from 2022. [5]

detectors: ATLAS, CMS, ALICE, and LHCb. In addition to the LHC experiments, CERN operates two particle decelerators, the Antiproton Decelerator (AD) and the Extra Low Energy Antiproton Decelerator (ELENA), alongside a comprehensive fixed-target physics program. This program includes the Common Muon and Proton Apparatus for Structure and Spectroscopy (COMPASS), Cosmics Leaving Outdoor Droplets (Cloud), the CERN Linear Electron Accelerator for Research (CLEAR), the Isotope mass Separator On-Line facility (ISOLDE), High-Radiation to Materials (HiRadMat), Neutron Time Of Flight (n_TOF), and the Advanced Proton Driven Plasma Wakefield Acceleration Experiment (AWAKE), among many other experiments in the NORTH and EAST Areas. Except for CLEAR and ISOLDE, all of these experiments rely on beam delivery from the PS, highlighting its important role within CERN.

This intricate network of accelerators and experiments, known collectively as the CERN accelerator complex, makes CERN the largest particle physics laboratory in the world, illustrated in Fig. 1.1. CERN is also the site for a proposed next step in high-energy physics in the Future Circular Collider (FCC)[4]. The FCC is envisioned to first operate as an electron-positron collider at the precise energy needed for Higgs boson production (FCC-ee), and later transition into a hadron collider (FCC-hh), capable of exploring an even broader range of higher energies.

1.2 Aim and outline of the thesis

The PS is the oldest operational particle accelerator at CERN, and its legacy presents unique challenges when integrating new advancements in accelerator physics[6, 7]. This integration is crucial, as the PS plays a pivotal role in the CERN accelerator complex, with almost all experiments relying on particles that have, at some point, been accelerated by the PS. The PS must exhibit immense flexibility in its beam parameters, and its ease of operation is directly linked to the overall performance of the entire CERN accelerator complex. This thesis addresses several critical aspects of PS operation, ranging from the generation of tune and chromaticity settings using Pole Face Windings (PFWs) to improve machine performance, to the measurement, correction, and understanding of resonances within the PS. Additionally, the thesis explores emittance reconstruction across all operational energy levels, which is essential for studying emittance limitations in the LHC injector chain and for more effectively meeting the luminosity targets set by the High-Luminosity LHC (HL-LHC) upgrade[8, 9]. The studies presented offer a balanced combination of beam-based experiments, simulation for benchmarking and improving the PS magnetic model, and the application of emerging techniques in accelerator physics. Together, these contributions aim to enhance the PS's capabilities and ensure its continued relevance within CERN's complex network of accelerators.

Chapter 2 of this thesis provides an introduction of the PS, detailing the layout of the machine. This chapter also covers the beam instrumentation and operational aspects of the PS, alongside relevant topics in beam dynamics that are required for understanding the studies presented in the thesis. It is assumed that the reader has a basic knowledge of accelerator physics. For those seeking to deepen their understanding, classical textbooks by K. Wille[10], M.G. Minty and F. Zimmermann[11], and S.Y. Lee[12] are recommended. Additionally, the lectures from the CERN Accelerator School[13] and the Joint Universities Accelerator School[14] offer an excellent introduction to the fundamentals of accelerator physics.

Chapter 3 focuses on the operational significance of the PFWs for controlling tune and chromaticity in PS. The current settings generator and control system present significant challenges, to the extent that setting up new magnetic cycles is often avoided when possible. However, recent advancements in machine learning have demonstrated its potential in addressing complex problems in accelerator physics. This chapter explores the application of machine learning techniques to overcome the difficulties associated with operating the PFWs, and showing how such approaches can be used for facilities constrained by similar tuning and chromaticity issues.

Chapter 4 addresses the challenges associated with measuring horizontal emittance in the PS, particularly at lower energies. These challenges arise from the convolution of the horizontal beam distribution with the longitudinal distribution due to the dispersive component. This chapter introduces a novel method that utilizes low-energy quadrupoles to induce dispersive excitations, effectively nullifying the dispersion precisely when the transverse beam distribution is measured. Using this method,

the beam profile measurements are more accurate, deepening our understanding of emittance and beam tail progression along the injector chain and allowing to study advancements in space charge models. Additionally, the zero dispersion optics developed here facilitate the creation of numerical deconvolution tools which is then expanded to other accelerators.

Chapter 5 explores the application of forced excitation techniques using an AC-dipole-like transverse damper and extensive turn-by-turn data to investigate resonance driving terms in the PS. Analysis of forced resonance driving terms, particularly with a dedicated AC-dipole, has shown promising results in the LHC. However, most accelerators lack such dedicated instrumentation, despite the broad interest in resonance driving terms. This chapter specifically examines skew sextupole resonances in the PS, attempting to demonstrate that the transverse damper, when used as an AC-dipole, allows for precise resonance driving term analysis. This approach not only provides an additional method for resonance correction in the PS but also is of interest for other CERN accelerators and beyond.

Finally, Chapter 6 provides a summary and review of the advancements made in the context of the PS and the contributions presented in this thesis.

THE CERN PROTON SYNCHROTRON

Thus the situation in December 1959 was that the synchrotron had worked successfully up to its design energy, and already beyond its design current, but with its builders and operators in a state of almost complete ignorance on all the details of what was happening at all stages of the acceleration process.

*First Quarterly Report of the Proton Synchrotron [15]
Anonymous author*

The Proton Synchrotron (PS) [16, 17] at CERN has embarked on a remarkable journey of discovery that spans over half a century. The PS's inaugural moment on November 24, 1959, marked a large milestone in the history of CERN and accelerator physics research. After 7 years of planning and construction in addition to eight weeks of rigorous trials and modifications, the PS achieved a monumental feat by accelerating protons to 25 GeV/c for the first time. Over the ensuing decades, the PS evolved from a groundbreaking accelerator to a cornerstone of particle physics research.

The PS has played multiple roles throughout its history, serving as a source of protons and ions for new fixed-target experimental areas and as an injector for the Intersecting Storage Rings (ISR), the Super Proton Synchrotron (SPS), and the Large Hadron Collider (LHC). Its versatility is further demonstrated by its ability to accelerate electrons and positrons for the Large Electron-Positron Collider (LEP), provide accelerated antiprotons for the ISR and the Super Proton-Antiproton Synchrotron (Sp $\bar{p}$ S), and decelerate antiprotons for the Low Energy Antiproton Ring (LEAR).

Today, the PS remains integral to almost all experiments at CERN. It provides beams to the Antiproton Decelerator (AD), the Neutron Time Of Flight (n_TOF) experiment, the fixed-target EAST area, it is a source of protons for the SPS NORTH Area experiments and continues to be a crucial part of the injector chain for the LHC. This makes the performance, flexibility, and control of the PS critical for CERN's operations and, with its long history and unique design, serves as an intriguing testbed for advancing studies in accelerator physics.

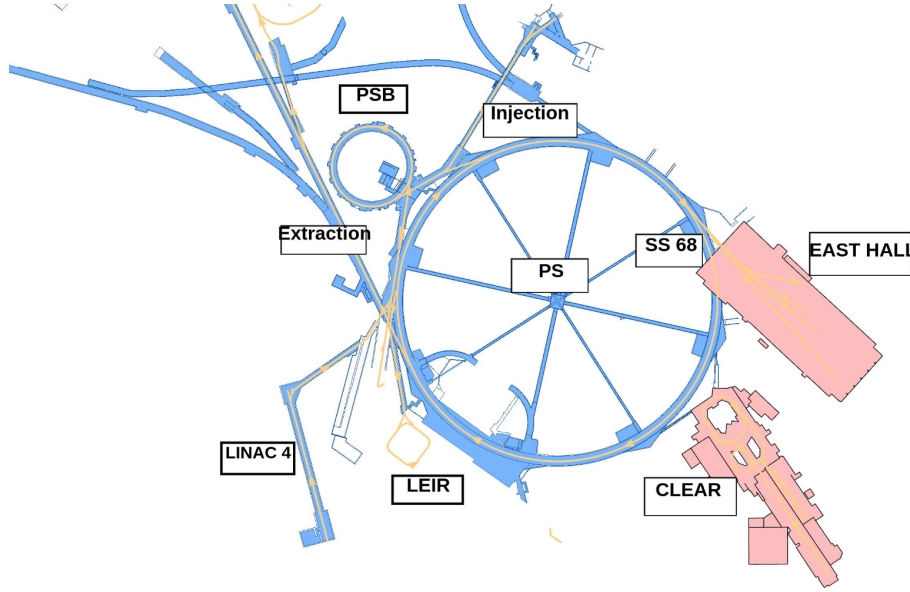


Figure 2.1: Layout of the Proton Synchrotron where the protons start the acceleration process in LINAC 4, going to the PSB before being injected to the PS. The beam can then be extracted to the experiments in the EAST area or n_TOF, or transferred to other machines like the AD and SPS.

2.1 Machine Layout

The 100-meter-radius ring of the PS (Fig. 2.1) consists of 100 combined function magnets each measuring 4.4 meters – also called the main units [18]. They possess a C-shaped structure which generates the bending dipole field but also introduces higher order focusing or defocusing fields, as shown in Fig. 2.2. Each main unit includes a closed (focusing) half-unit and an open (defocusing) half-unit, separated by a central junction. Open straight sections intersperse between the main units, serving as an area for beam instrumentation and other equipment and resulting into a ‘FOFDOD’-magnetic field pattern. Due to this design, the PS functions as an alternating-gradient machine, which was one of its main design requirements. As an alternating-gradient machine, it benefits from a significantly smaller vacuum chamber compared to other designs of the time, as the oscillations created by the focusing and defocusing pairs greatly reduce the maximum size of the particle beam. During construction, considerable emphasis was placed on the main units, resulting in remarkably reliable magnet operation. Over its 65 years of operation, there has been no significant breakdown of the combined function magnets. Alongside the PS building, these main units are one of the only original components still in use today. However, about half of the main units were refurbished in the 2000s due to the degradation of the insulation for the pole face windings on top of the main units.

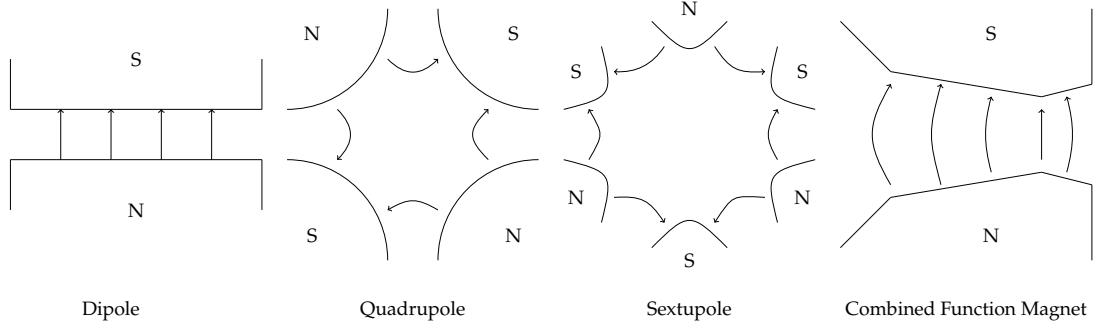


Figure 2.2: Illustration of the magnetic field of the combined function magnets.

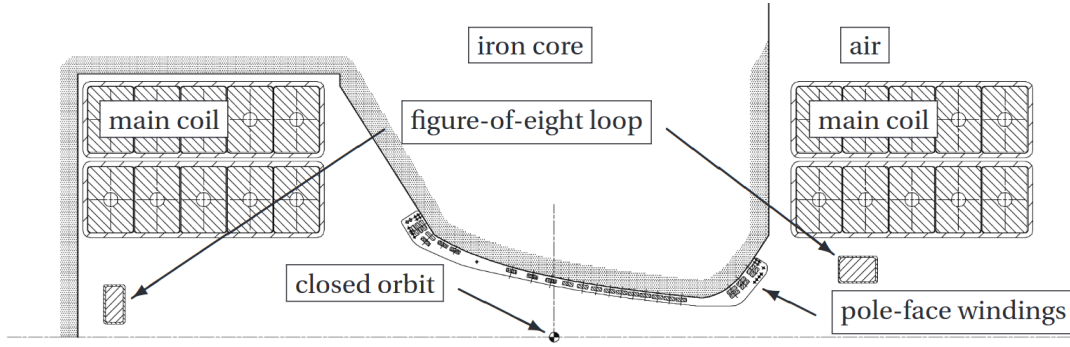


Figure 2.3: Location of the pole face windings in a cross-section of the PS main units. [20]

Due to the introduction of quadrupolar, sextupolar and even octupolar effects of the combined function magnets, other magnetic installations were foreseen to correct undesired multi-pole errors. Among these additional magnets are the low-energy quadrupoles (LeQ). The LeQs were installed in the 1950s to enhance tune control at injection energy and are therefore limited in magnet strength. There are currently 40 available in the PS but only 39 are used operationally to reduce the deviation of the design beam parameters. They are typically powered in series of each other but have the capability to be powered individually.

2.1.1 Pole face windings

As the LeQs' name suggests, they lack the power to be consistently used at higher energy. For higher energies, correction windings on top of the magnet poles of the main units are used, as shown on Fig. 2.3. These are called the pole face windings (PFW) and will act along the whole magnet unit whereas the LeQs can act as correctors at given locations in the straight sections [19]. The PFWs can create an additional multipole field of low amplitude to compensate for undesired field components originating from the shape of the main units. Since 1976 the PFWs were operationally used with a three-current mode where three circuits could be powered individually: the figure-of-eight loop, a focusing and a defocusing PFW.

The figure-of-eight loop (8L) comprises a single turn of a rigid copper conductor wound

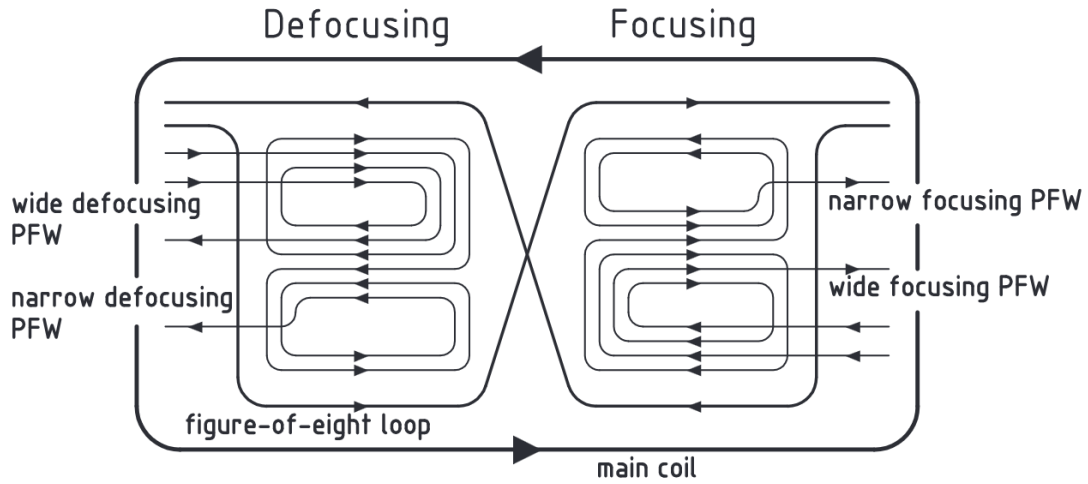


Figure 2.4: Sketch of the pole face windings. [21]

around the focusing half-part of the main unit in one direction and around the defocusing half-part in the opposite direction, intersecting at the central junction between the two halves to form the figure-of-eight loop. This leaves the total integrated dipole field unaffected but compensating a decrease in field gradient for the focusing half-unit with an increase in the defocusing half-unit. Consequently, the integrated field gradient in each half-unit can be adjusted to control both horizontal and vertical tunes of the machine.

The other circuit takes the form of plates positioned on the surfaces of the top and bottom pole pieces of the main unit, separately for the focusing (F) and defocusing (D) half-units. These pole-face winding plates consist of conductors, shaped to precisely fit the pole surface of five adjacent blocks. Their double-curved design follows the curvature of the main unit, with conductors arranged parallel to the hyperbolic pole surface. The winding incorporates four distinct circuits: two active circuits, named narrow (N) and wide (W), for correcting quadrupolar and sextupolar components, and two passive circuits for addressing eddy current effects. Since 2007, the PFWs have been upgraded to allow separate control of the narrow and wide circuits, effectively expanding the control currents to five: focusing narrow (FN), focusing wide (FW), defocusing narrow (DN), defocusing wide (DW), and the figure-of-eight loop (8L), as shown in Fig. 2.4.

Although PFWs were an integral part of the PS's original design, their use still presents challenges today [20–22]. A major issue arises from the fact that the PFWs are managed by two independent systems: the basic function settings generator and the PFW response matrices, which provide additional control. The basic functions dictate the currents applied at specific magnetic fields to ensure the survival of the beam during acceleration and extraction. However, these functions are identical for all beam destinations, limiting the PS's flexibility and necessitating extensive adjustments for most modern beam types. Another difficulty for using the PFWs is shown in Fig. 2.5, where the basic functions,

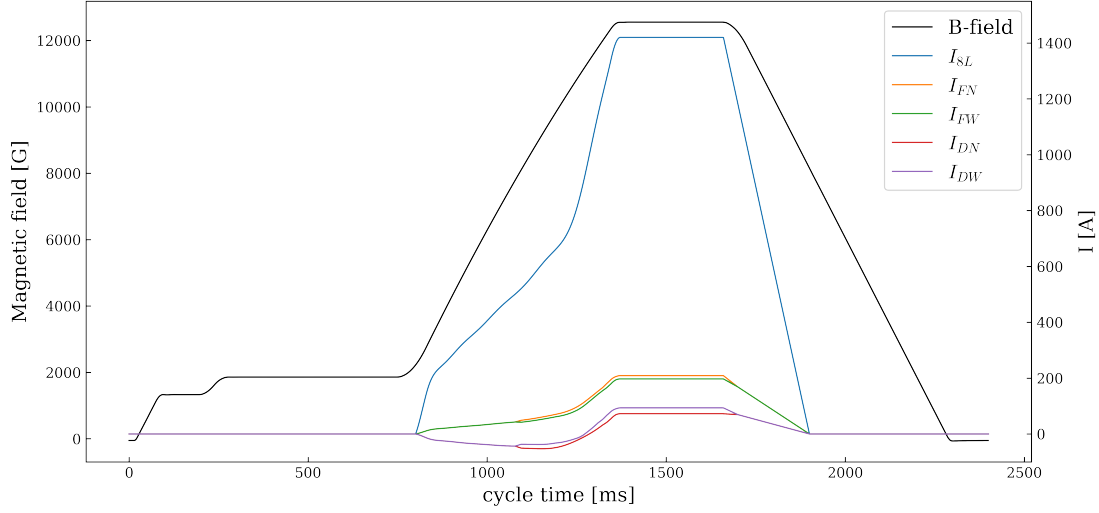


Figure 2.5: An operational magnetic cycle with the PFW basic functions in the PS. The black line shows the magnetic field of the main units in Gauss and the other lines are the currents of the PFW circuits.

alongside the PS's magnetic cycle, illustrate that the PFWs are not utilized at lower energies, where tune control is instead managed by the LeQs.

These limitations can be partially addressed by the PFW control scheme, which was developed using response matrices that relate changes in tune and chromaticity to PFW currents. These matrices are derived from measurements taken at different magnetic fields, theoretically allowing their application throughout the acceleration cycle. However, in practice, the control system frequently fails at higher energies, revealing its shortcomings. The measurements and fits of the response matrices are shown in Fig. 2.6. The rigid basic functions, coupled with the unreliable response matrices, make PS operation cumbersome, particularly for non-standard beams. One of the studies presented in this thesis focuses on creating a unified control system to integrate and build on these existing tools.

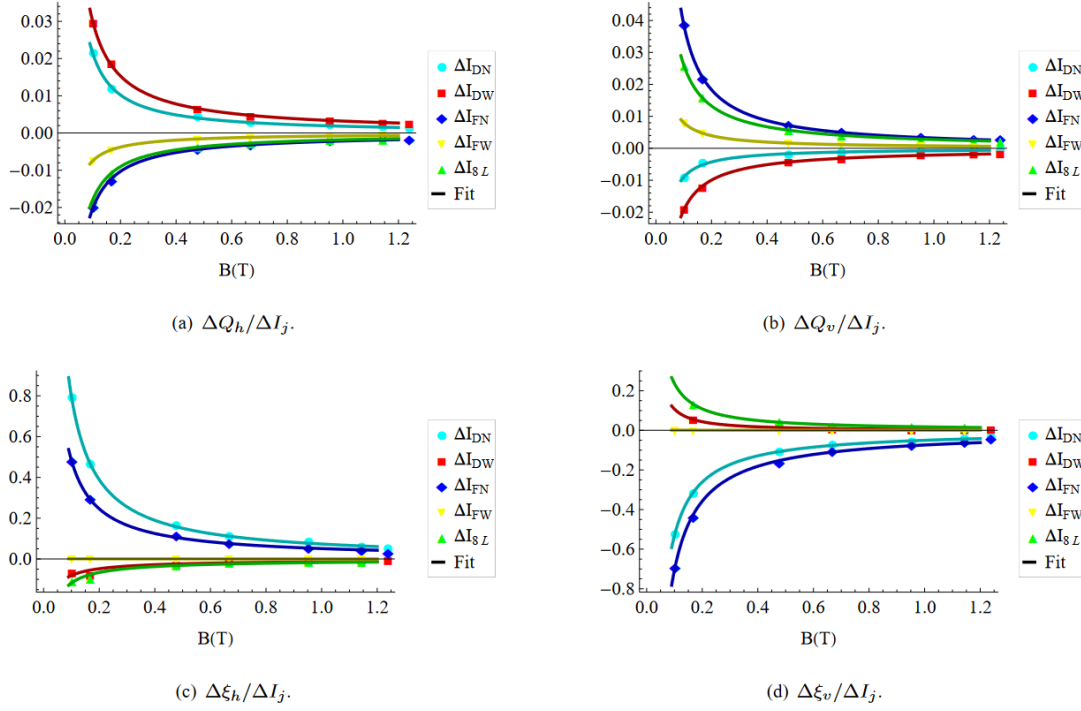


Figure 2.6: The shift of the tune and normalised chromaticity $\xi = \frac{1}{Q} \frac{\partial Q}{\partial \delta p}$ of the PS resulting from an increase of 1A of current in each of the PFW circuits at different magnetic fields. These measurements are used to create the PFW response matrices currently used in operations. [23]

2.2 Physical concepts and beam instrumentation

Every 1.2 to 3.6 seconds, the PS injects, accelerates, and extracts a new set of particles. Each destination for these particle bunches requires specific beam parameters, such as beam size, number of particles per bunch, and momentum spread. To achieve this, it is essential to understand the characteristics of a particle beam circulating in a synchrotron and how to measure them using beam instrumentation. The following sections will discuss these characteristics, focusing on the various tools and methods used in this thesis.

2.2.1 Beam position and dispersion

As the first alternating-gradient accelerator, designing the PS also meant advancing the physics of accelerators. Notable amongst these advancements is the theory of the elliptical distribution of particles in phase space, depicted in Fig. 2.7, due to the alternating quadrupolar gradient of the main units, developed by Courant and Snyder while being part of the CERN Proton Synchrotron Group [24]. They concluded that positions x and velocities x' of particles along one dimension behave according to

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon, \quad (2.1)$$

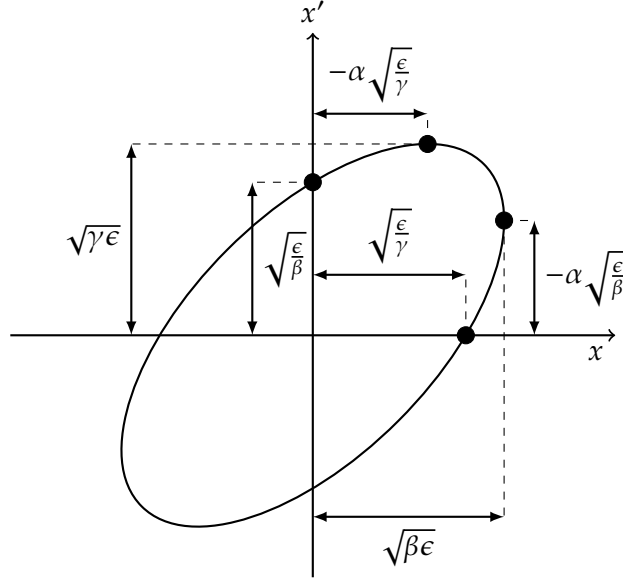


Figure 2.7: Illustration of the horizontal phase space ellipse.

where $\pi\epsilon$ is the area of the ellipse, termed the emittance, and α , β and γ are the Courant-Snyder, or Twiss parameters. For a deeper understanding of these parameters we'll have to look at the solutions of the linear equations of motion for circular accelerators, with $u = x$ or y . These equations are:

$$u(s) = \sqrt{\epsilon} \sqrt{\beta_u(s)} \cos\left(\int_0^s \frac{ds}{\beta_u(s)} + \phi_u\right) \quad (2.2)$$

$$u'(s) = -\sqrt{\frac{\epsilon}{\beta(s)}} \left[\sin\left(\int_0^s \frac{ds}{\beta_u(s)} + \phi_u\right) - \frac{1}{2} \frac{d\beta(s)}{ds} \cos\left(\int_0^s \frac{ds}{\beta_u(s)} + \phi_u\right) \right]. \quad (2.3)$$

Here, ϕ_u is the initial phase of the particles, and the β -function is a measure of the transverse amplitude of the particles at longitudinal position s . Based on the properties of the phase space ellipse, the following relations between the Courant-Snyder parameter can be found:

$$\begin{aligned} \alpha(s) &= -\frac{1}{2} \frac{d\beta(s)}{ds} \\ \gamma(s) &= \frac{1 + \alpha^2(s)}{\beta(s)}. \end{aligned} \quad (2.4)$$

It is important to note that the β -function is determined by the location and strength of the quadrupoles. Furthermore, it is clear from the equation above that the particles are going to move along the phase-space ellipse a number of times during one rotation in the accelerator. The number of transverse oscillations a single particle makes during one turn is called the tune of that particle Q_u . The tune and the first derivative of the tune in function of the momentum offset $\frac{\Delta p}{p}$ to the reference momentum is given by:

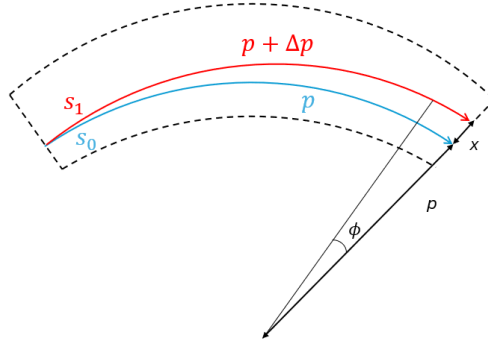


Figure 2.8: Bending effects of a dipole on particles with a momentum deviation Δp resulting in a positional shift x .

$$Q_u = \frac{1}{2\pi} \int_s^{s+L} \frac{ds}{\beta_u(s)} \quad (2.5)$$

$$Q'_u = \frac{\partial Q_{meas}}{\partial \frac{\Delta p}{p}}.$$

This first derivative is called the chromaticity of the beam. Only the first-order Q' and second-order $Q'' = \frac{\partial^2 Q}{\partial (\frac{\Delta p}{p})^2}$ chromaticities are non-negligible in the PS, for the scenarios presented in this thesis. The momentum difference will also cause a positional shift due to the bending effects of a dipole being dependent on the momentum of the particles, as illustrated in Fig. 2.8. Due to the bending effects of a dipole, an additional longitudinal term needs to be added to Eq. 2.2:

$$D_u(s) \frac{\Delta p}{p}, \quad (2.6)$$

where D_u is called the dispersion function. Both chromaticity and dispersion are now defined, and both effects are created by the presence of momentum deviations. Collectively, these effects are referred to as chromatic aberrations, which play a significant role in the behavior of particle beams in accelerators.

Beam position monitors

It will be proven possible to measure the β -function, tune Q_u and dispersion of the beam all from measurements of the transverse position u . These measurements need to be done in a non-destructive way, as they are repeated many times and in many locations in the ring. The beam instrumentation used for this are the beam position monitors (BPMs) [25, 26]. They operate by estimating the difference in induced voltage between two metal plates that are placed equidistant to the nominal orbit as seen in Fig. 2.9, where the sum of the voltages is used as a normalization, so the measurement

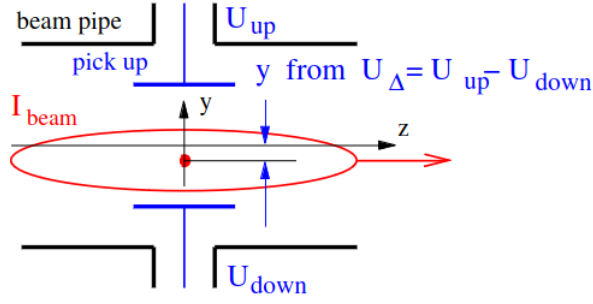


Figure 2.9: Illustration of the working of a BPM. [25]

is intensity independent. An additional scaling factor A is required to have a good calibration of the BPM, resulting in the following relation between position u and the voltages of opposite plates.

$$u = A_u \frac{\Delta U_u}{\sum U_u} + E, \quad (2.7)$$

where E is an additional error that accounts for misalignments. Typically, four pick-up plates are positioned crosswise along the beam pipe wall to get measurements of both planes.

The BPMs in the PS can perform measurements every turn. From this turn-by-turn data, the tune is inferred by performing a Fast Fourier Transformation (FFT). Since the tune represents the number of oscillations the particles make each turn, the fractional tune will manifest as the largest peak on the frequency spectrum, unless there is a large closed orbit which will present itself in the spectrum as a peak at $Q = 0$ [27]. For operational purposes, designated high-precision Base-Band-Tune (BBQ) BPMs or pick-ups are used specifically for tune measurements [28]. These specialized BPMs are not utilized for other analyses in the PS.

Given the high importance of accurate tune measurements for machine operation, a stripline kicker called the transverse damper (ADT) is employed [29, 30]. The ADT kicks or "chirps" the beam, resulting in higher amplitudes in the BPM data, less noisy frequency spectra, and more precise tune measurements. The precision of these measurements increases with the number of turns used in the spectrum analysis, typically 2048 turns are used in the PS, allowing for measurements every 6 ms.

The PS has 54 BPMs spread quasi-symmetrically around its ring, excluding the two BBQ BPMs that are used to do tune measurements. These BPMs provide turn-by-turn data that, when analyzed, can infer the β -function and dispersion, provided the signal-to-noise ratio is sufficiently high. To enhance the signal-to-noise ratio, a forced excitation similar to the excitation used for tune measurements can be employed. While a single kick is used for tune measurement, periodic kicks used for this type of analysis

can adiabatically increase and decrease the excitation amplitude, preventing emittance blow-up. The resulting forced oscillations are then transformed using an FFT to estimate the phase ϕ of the BPM.

The β -function can be reconstructed based on either the amplitude of all BPMs or the phase of neighboring BPMs. The β reconstructed from the phase measured at subsequent BPMs i , j , and k , is given by:

$$\beta_i^\phi = \frac{\varepsilon_{ijk} \cot \phi_{i,j} + \varepsilon_{ikj} \cot \phi_{i,k}}{\varepsilon_{ijk} \frac{M11_{i,j}}{M12_{i,j}} + \varepsilon_{ikj} \frac{M11_{i,k}}{M12_{i,k}}}, \quad (2.8)$$

where $\phi_{i,j}$ is the phase advance between BPM i and BPM j , ε_{ijk} is the Levi-Civita symbol, and $M(i, j)$ is the matrix representation of the magnetic elements between the two BPMs. The relation between the coordinates of both BPMs is given by:

$$\begin{bmatrix} u_j \\ u'_j \end{bmatrix} = \begin{bmatrix} M11 & M12 \\ M21 & M22 \end{bmatrix} \begin{bmatrix} u_i \\ u'_i \end{bmatrix}. \quad (2.9)$$

This expression is independent of calibration errors or excitation amplitude, relying solely on the accuracy of the phase between BPMs and the magnetic model of the accelerator. β_i^ϕ is often the best method for measuring the β -function using BPMs. However, if the phase values approach $n\pi$, Eq. 2.8 becomes infinite. In such cases, the β -function based on BPM amplitudes is used:

$$\beta_i^A = \frac{A_i^2}{2J}, \quad (2.10)$$

where A_i is the amplitude of the signal at BPM i , and J is the action of the particle beam. This action is usually increased by a dipole kick to the beam for better measurements and can be calculated by averaging the signal amplitude to β -function ratio of all BPMs:

$$2J = \left\langle \frac{A^2}{\beta} \right\rangle. \quad (2.11)$$

Thus, the expression for β^A at one BPM depends on the systematic calibration of all BPMs.

Finally, dispersion can be measured by controlling the momentum deviation $\frac{\Delta p}{p}$ in the machine. In the PS, the revolution frequency F_{rev} can be altered from its reference value F_{rev}^0 through Radio-Frequency (RF) manipulations, which are coupled with momentum deviation via:

$$\eta = \frac{1}{\gamma_r} - \frac{1}{L} \oint \frac{D}{\rho} ds \quad (2.12)$$

$$\frac{\Delta p}{p} = \frac{1}{\eta} \frac{F_{rev}^0 - F_{rev}}{F_{rev}^0}, \quad (2.13)$$

where γ_r is the relativistic gamma, ρ is the orbit radius, η is the slip factor, and L is the circumference of the machine. By measuring the beam position for different revolution

frequencies and using the appropriate equations, the dispersion can be determined. Note that a dispersion measurement via BPM is also dependent on the signal amplitude and therefore on the calibration factor of the BPM.

2.2.2 Longitudinal dynamics

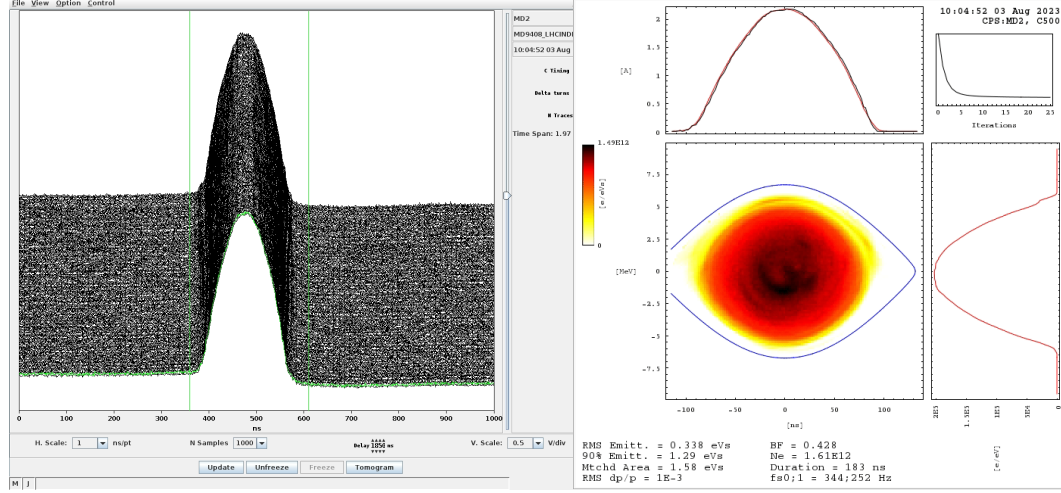
The effects of the longitudinal behavior on the transverse beam position have been defined above. The longitudinal behavior of a beam, governed by the accelerating cavities or RF-system of an accelerator, constitutes another branch of accelerator physics that will be briefly touched upon in this thesis. The longitudinal distribution is coupled with the transverse behavior through the momentum differences of the particles in the beam and the dispersion of the accelerator. While the center-of-mass momentum deviation $\frac{\Delta p}{p}$ is used for dispersion calculations, it does not account for the individual momenta of the particles. Therefore, the root-mean-square (RMS) value of all the momenta of the particles compared to the average momentum is defined as the momentum spread of the beam $\frac{\Delta p_{RMS}}{p}$. This parameter is crucial in the context of this thesis and can be reconstructed through tomography [31].

In the context of accelerator physics, tomography is the process of reconstructing the longitudinal phase space from one-dimensional bunch profile measurements. The longitudinal bunch profile is measured for consecutive turns by wall current monitors, as shown in Fig. 2.10a. With knowledge of the RF parameters used in the machine and the resulting synchrotron motion, these profiles represent different angle projections of the same phase space. By appropriately combining these one-dimensional profiles, the two-dimensional phase space can be reconstructed. Refinements of this initial approximation are achieved iteratively through back projections with the original profiles. The automation of this measurement process has been facilitated by the development of the tomoscope application by the PS operators, where the reconstructed phase space of the longitudinal distributions shown in Fig. 2.10a are presented in Fig. 2.10b.

In terms of RF manipulations, the PS is a unique machine within the LHC injector scheme. For beams destined for the LHC the bunches undergo a process called triple splitting before the acceleration process, where each bunch is split into three [32, 33]. An example of the longitudinal distributions during this splitting process is shown in Fig. 2.11. After acceleration, these tripled bunches are further split into four, resulting in a total of 12 bunches being extracted for every single bunch injected into the PS.

2.2.3 Beam width and emittance

In many accelerators designed for collision experiments, the most important parameter is luminosity, which quantifies the machine's ability to produce useful particle interactions. For two identical Gaussian colliding beams, the luminosity $\mathcal{L}$ is expressed as



(a) Screenshot of the tomoscope application in the PS. The consecutive black lines are the longitudinal distributions. (b) Screenshot of the resulting phase space reconstruction by tomography.

Figure 2.10: Tomography of a oscillation-free longitudinal distribution in the PS.

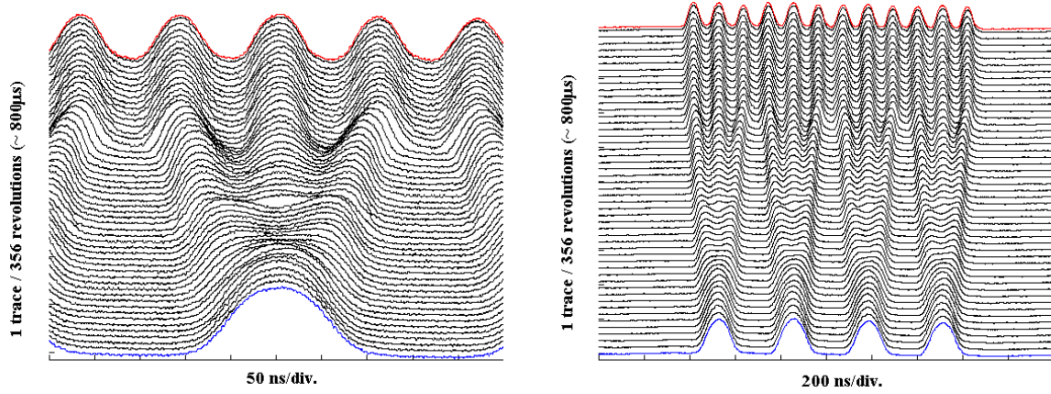


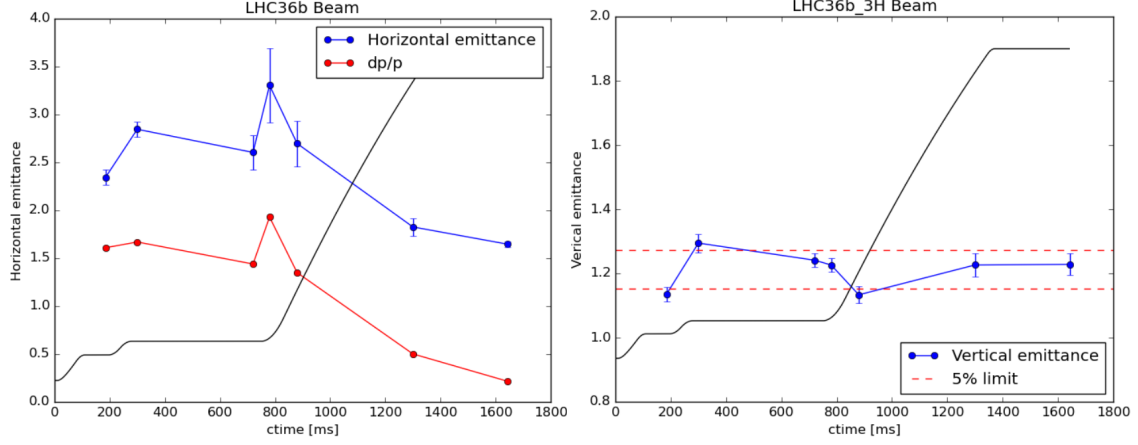
Figure 2.11: The longitudinal distributions of 4 PS bunches undergoing triple splitting. [34]

$$\mathcal{L} = \frac{N_1 N_2 F_{rev} N_b}{4\pi\sigma_x\sigma_y} \quad (2.14)$$

where N_1 and N_2 are the number of particles per bunch, N_b is the number of bunches, and σ_x and σ_y are the transverse beam widths. Achieving high luminosity is essential for maximizing collision rates, and optimizing the beam widths is a key factor in this. Assuming the beam is purely betatronic, the beam width can be inferred from the Courant-Snyder parameters and their elliptical properties to find

$$\sigma_{u;\beta}(s) = \sqrt{\epsilon_u \beta_u(s)}. \quad (2.15)$$

However, the beam size will be affected by chromatic aberrations similarly to the transverse position and tune. These aberrations will be governed by the dispersion and the momentum spread $\frac{\Delta p_{RMS}}{p}$ of the beam. These effects contribute an additional term



(a) The horizontal emittance measured along the PS cycle shown together with the $\frac{\Delta p_{RMS}}{p}$ measurements and the magnetic field in black. (b) The vertical emittance measured along the PS cycle with the magnetic field.

Figure 2.12: Emittance measurements in the PS along the magnetic cycle. [35]

to the beam width:

$$\sigma_{u;D}(s) = D_u(s) \frac{\Delta p_{RMS}}{p}. \quad (2.16)$$

By assuming that the beam distributions are Gaussian in both the transverse and longitudinal planes, and that the betatron and dispersive terms are uncorrelated, the total beam size $\sigma_{u,tot}^2$ is given by

$$\sigma_{u,tot}^2 = \epsilon_u \beta_u(s) + D_u^2(s) \left(\frac{\Delta p_{RMS}}{p} \right)^2. \quad (2.17)$$

To optimize luminosity during collisions, precise emittance measurements and a thorough understanding of the transverse beam profile along the injector chain are essential for the LHC. Throughout this thesis, emittance values are calculated using Eq. 2.17. In the PS, the vertical dispersion is nearly zero, meaning vertical emittance measurements are unaffected by dispersive contributions. However, horizontal dispersion is present throughout the PS lattice, impacting horizontal emittance measurements.

Figure 2.12 illustrates typical horizontal and vertical emittance measurements in the PS. The data show that horizontal emittance closely follows the relative momentum spread $\frac{\Delta p_{RMS}}{p}$, whereas vertical emittance exhibits no such correlation. This indicates that dispersive effects significantly influence horizontal emittance measurements, particularly at lower energies where the momentum spread is larger. Although Eq. 2.17 is designed to account for dispersive contributions, the observed discrepancies suggest that its underlying assumptions, namely, that both transverse and longitudinal distributions are Gaussian and uncorrelated, do not fully hold in the PS. Consequently, an alternative method for measuring horizontal emittance is required.

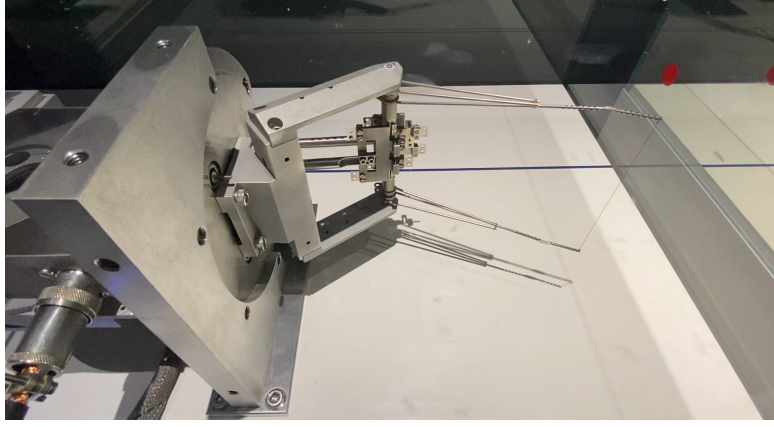


Figure 2.13: *Photo of the wire scanner on display in the CERN control center visit point.*

The challenge of horizontal emittance measurements in the PS and PSB has been the subject of numerous studies [36–41]. In this thesis, a novel method for accurately measuring horizontal emittance in the PS is introduced. The advantages of this approach for the LHC injector chain are demonstrated, and the potential for extending this method to other accelerators with dispersive lattices is explored.

Wire scanners

For emittance measurements in the PS, the beam width σ is derived from the transverse distribution and the $\frac{\Delta p_{RMS}}{p}$ is a result from a tomograph. However, getting the β -function and dispersion at the transverse beam distribution measurement location is a time-intensive measurement, so these optics are typically taken from a MAD-X model with a 5% uncertainty assumed on the values.

The transverse distribution or profile can be measured in multiple ways. In the PS, the main tool to measure the beam profile is the wire scanner (WS) [42, 43]. Another possible way is using beam gas ionization (BGI) chambers [44, 45], these are relatively new and still being commissioned as of the time of writing of this thesis.

A WS is the combination of detection photomultipliers and a rotating mechanism that rapidly crosses a thin carbon wire through the beam (Fig. 2.13). The interaction of the beam and the wire will produce a shower of secondary particles through a scintillator which are then detected using the photomultipliers. There are currently five wire scanners installed in the PS, three for the horizontal and two for the vertical plane. A WS can be used to do two measurements in one cycle, once passing the wire into the vacuum chamber and the other returning to its original position. Given the interaction of the wire with the beam, some of the particles will become lost. These losses are often negligible compared to the size of the beam but a wire scanner measurement is therefore considered a destructive measurement.

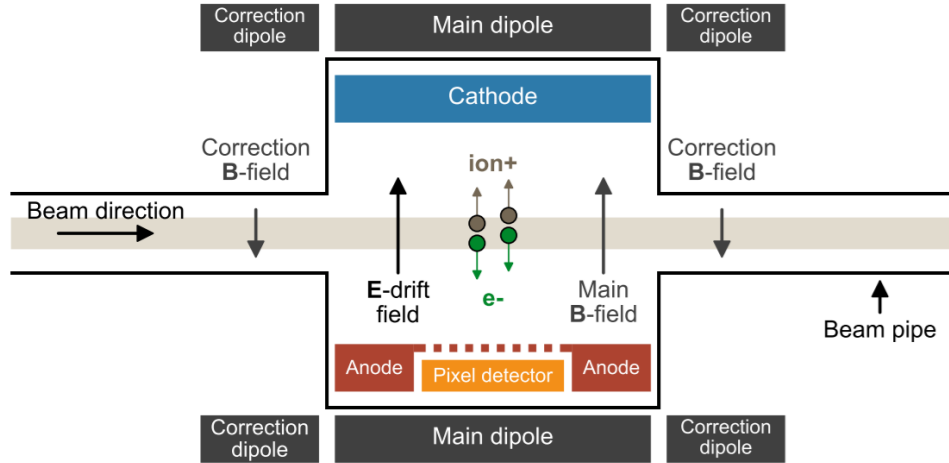


Figure 2.14: Schematic overview of a beam gas ionization chamber. [45]

Beam gas ionization

The BGI measurements will not affect the beam, and is the non-destructive alternative to the WS. Once the BGI becomes fully commissioned, this tool is planned to become the new operational standard. These monitors operate by detecting electrons liberated through the ionization of residual or injected gas by the beam, as depicted in Fig. 2.14. Positioned in SS 82 and 84 for the horizontal and vertical planes respectively, each monitor focuses on measuring the beam profile of its plane. The electrons generated by the ionization process in the vacuum chamber are accelerated towards a pixel detector by an applied electric drift field. To ensure accurate measurements, a dipole magnet situated outside the vacuum chamber creates a magnetic field parallel to the electric drift field, guiding the electrons towards the detector and minimizing transverse spatial drift. Additionally, a pair of correction dipoles located outside the instrument works to nullify the magnetic field produced by the transverse field dipole, effectively closing any potential bumps in the beam path. Due to the non-destructive nature of the BGI, a continuous measurement is performed during one cycle. This could serve as one high resolution measurement of the beam profile or these measurements can be split into multiple profiles to measure the effect of changes to beam optics on the profiles.

2.2.4 Intensity and beam losses

The intensity of the beam in the PS is measured using beam current transformers (BCT). BCTs are toroidal transformers through which the beam passes. The induced current in the windings of the transformer is proportional to the charge of the beam, and therefore proportional to the number of particles in the beam. The BCT are used in unison with the beam loss monitors (BLM). BLMs are used to measure both the location and amount of beam losses.

2.2.5 Resonances and resonance driving terms

The vertical and horizontal tunes discussed earlier in the chapter play an important role in determining beam stability. A simple illustration of this can be found by considering a machine with an integer tune. In such a system, the beam would experience kicks from the dipoles at the same point in phase space on every turn. Even a minor longitudinal misalignment, strength variation, manufacturing error or calibration error in one of the dipoles would introduce an unintended kick, which, with each turn, would incrementally destabilize the beam's trajectory until the aperture of the beam pipe is hit. This phenomenon is known as a resonance, and when primarily caused by dipoles, it is referred to as a dipole resonance.

Since perfect alignment of all magnets in an accelerator is impossible and different types of magnets will influence different tune values, understanding and correcting for resonances is required. Resonances occur when the tunes of the machine satisfy the condition:

$$aQ_x + bQ_y = p, \quad \text{where} \quad a, b, p \in \mathbb{N}. \quad (2.18)$$

To achieve a better understanding for resonances, it is helpful to examine how particle coordinates evolve as they pass through the magnetic elements of an accelerator [27, 46–48].

Linear lattice

The magnetic elements of a linear lattice can be approximated using transfer maps. For any two positions, 1 and 2, the matrix $M_{1 \rightarrow 2}$ represents the combined effect of all magnetic elements between these positions:

$$\begin{pmatrix} u \\ p_u \end{pmatrix}_2 = M_{1 \rightarrow 2} \begin{pmatrix} u \\ p_u \end{pmatrix}_1. \quad (2.19)$$

The cumulative effects of all the linear transfer maps around a complete turn are described as the one turn map $M_{s \rightarrow s+C}$. This one turn map can be derived from the equations of motion (Eqs. 2.2 and 2.3) and leads to

$$M_{s \rightarrow s+C} = \begin{pmatrix} \cos 2\pi Q_u + \alpha_u \sin 2\pi Q_u & \beta_u \sin 2\pi Q_u \\ -\gamma_u \sin 2\pi Q_u & \cos 2\pi Q_u - \alpha_u \sin 2\pi Q_u \end{pmatrix}. \quad (2.20)$$

By transforming this one turn map to a linearly normalized coordinate system, the dependencies on the optics function at location s can be eliminated. This new coordinate system is known as the Courant-Snyder coordinates:

$$\begin{aligned} \hat{u} &= \sqrt{2J_u} \cos(\phi_u) \\ \hat{p}_u &= -\sqrt{2J_u} \sin(\phi_u). \end{aligned} \quad (2.21)$$

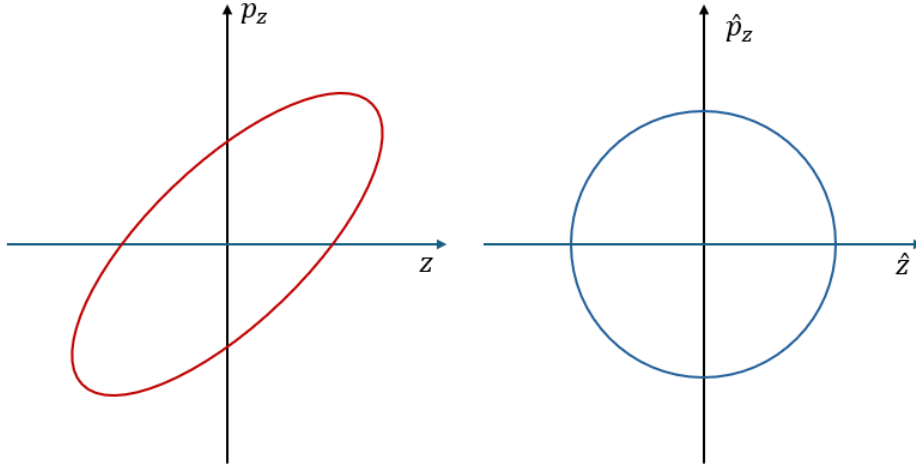


Figure 2.15: Phase space described in both regular and normalized coordinates.

Here J_u is the action of the particles. The result of the transformation

$$\begin{pmatrix} \hat{u} \\ \hat{p}_u \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{\beta_u}} & 0 \\ \frac{\alpha_u}{\sqrt{\beta_u}} & \sqrt{\beta_u} \end{pmatrix} \begin{pmatrix} u \\ p_u \end{pmatrix} \quad (2.22)$$

is a circular phase space where the motion is described as a simple rotation, as shown in Fig. 2.15. This can be best perceived through the simplification of the normalized one turn map $\hat{M}_{s \rightarrow s+C}$:

$$\hat{M}_{s \rightarrow s+C} = \begin{pmatrix} \cos 2\pi Q_u & \sin 2\pi Q_u \\ -\sin 2\pi Q_u & \cos 2\pi Q_u \end{pmatrix}. \quad (2.23)$$

This description holds for a linear lattice. However, to fully characterize resonances, non-linearities need to be included in our description. Unfortunately, non-linear elements can not be represented by simple matrices as their magnetic field does not depend linearly with the transverse position in the magnetic field.

Non-linear lattice

Non-linear elements in accelerators can be described using Lie algebra in the thin-lens approximation. The Lie transformation representing these elements is the unevaluated Poisson bracket:

$$:H := \frac{\partial H}{\partial x} \frac{\partial}{\partial p_x} - \frac{\partial H}{\partial p_x} \frac{\partial}{\partial x}. \quad (2.24)$$

where H is the Hamiltonian of the system. Using this approach, the transfer map for a particle's coordinates between positions 1 and 2, with only non-linear elements between

them, is expressed as:

$$\begin{pmatrix} x \\ p_x \\ y \\ p_y \end{pmatrix}_2 = e^{L:H} \begin{pmatrix} x \\ p_x \\ y \\ p_y \end{pmatrix}_1, \quad (2.25)$$

where L is the length of the non-linear element. In this derivation, the length is ignored for simplicity and can be reintroduced in the final expression if needed. It is important to note that this expression uses physical, not normalized, coordinates. Expanding the exponential term yields

$$e^{H:}x = x + [H, x] + \frac{1}{2!}[H, [H, x]] + \dots \quad (2.26)$$

Typically, this expansion is used up to first order but the second order term is shown for completeness. For a general multipole of order n , the Hamiltonian is given by

$$H = \Re \left[\sum_{n>1} (K_n + iJ_n) \frac{(x + iy)^n}{n!} \right]. \quad (2.27)$$

where K_n and J_n are the strengths of the multipole components. To describe the complete dynamics of a system with both linear and non-linear elements, the combined one turn map $\mathcal{M}$ is constructed as

$$\mathcal{M} = e^{H_{w+2}:} \cdot M_{W+2} \cdot e^{H_{w+1}:} \cdot M_{W+1} \cdot e^{H_w:} \dots e^{H_1:} \cdot M_1, \quad (2.28)$$

where M_W represents the linear transfer maps, and H_w represents the Hamiltonians of the non-linear elements at various locations w , as shown in Fig. 2.16. This expression can be simplified by switching to normalized coordinates, where the linear maps reduce to simple phase-space rotations. Consequently, the one-turn map in normalized coordinates becomes

$$\mathcal{M} = \prod_{w=1}^W e^{\hat{H}_w:} R, \quad (2.29)$$

where R is a single matrix that describes the linear motion over one turn.

Further simplification is possible using the properties of Lie operators and the Campbell-Baker-Hausdorff theorem, which allows for approximations of the exponential terms. Specifically, to first order, the product of exponentials simplifies as:

$$e^{H_W:} \approx e^{H_w:} e^{H_{w+1}:}, \quad (2.30)$$

leading to the combined Hamiltonian:

$$H_W = H_w + H_{w+1}. \quad (2.31)$$

Thus, the full one turn map reduces to a single Lie transformation:

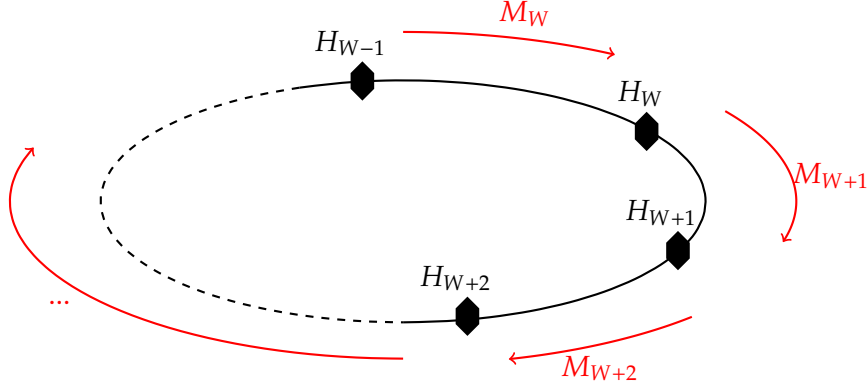


Figure 2.16: Illustration of an accelerator lattice with linear M_W and non-linear H_W elements.

$$\mathcal{M} = e^{iH_W} R. \quad (2.32)$$

At this point, filling in the Hamiltonian that describes a multipole in normalized coordinates $\hat{H}$ is the next step toward understanding resonances. This Hamiltonian can be derived by expanding the multipole Hamiltonian (2.27) using the binomial theorem:

$$H = \Re \left[\sum_{p=0}^n \frac{(K_{n-1} + iJ_{n-1})}{(n-p)!n!} x^p (iy)^{(n-p)} \right]. \quad (2.33)$$

Followed by moving to complex normalized coordinates $u = \frac{\sqrt{\beta_u}}{2}(h_{u,-} + h_{u,+})$ where $h_{u,\pm} = \hat{u} \pm i\hat{p}_u$. Here, indices $p = j + k$ and $n - p = l + m$ can be substituted, leading to

$$\hat{H} = \Re \left[\sum_{\substack{p=j+k \\ n-p=l+m}}^{n=j+k+l+m} \frac{(K_{n-1} + iJ_{n-1})}{(l+m)!(j+k)!} \beta_x^{\frac{j+k}{2}} \beta_y^{\frac{l+m}{2}} \frac{i^{l+m}}{2^{j+k+l+m}} (h_{x,-} + h_{x,+})^{j+k} (h_{y,-} + h_{y,+})^{l+m} \right]. \quad (2.34)$$

The binomial theorem can be used here again to expand around $(h_{u,-} + h_{u,+})$

$$\hat{H} = \sum_{j,k,l,m=0}^{n=j+k+l+m} - \frac{K_{n-1}\Omega(l+m) + iJ_{n-1}\Omega(l+m+1)}{j!k!l!m!2^{j+k+l+m}} \beta_x^{\frac{j+k}{2}} \beta_y^{\frac{l+m}{2}} i^{l+m} h_{x,-}^j h_{x,+}^k h_{y,-}^l h_{y,+}^m \quad (2.35)$$

where the observation was made that the normal terms K_{n-1} would be real if $l + m$ is even and the skew terms J_{n-1} are selected if $l + m$ is odd. Therefore, the selection function Ω was introduced which is 1 for even values and 0 for odd values. To convert this general multipole Hamiltonian to $\hat{H}_W$ which is present in the non-linear transfer maps, the phase advance between a reference point and the propagating magnet $\Delta\phi_{w,u}$ needs to be added

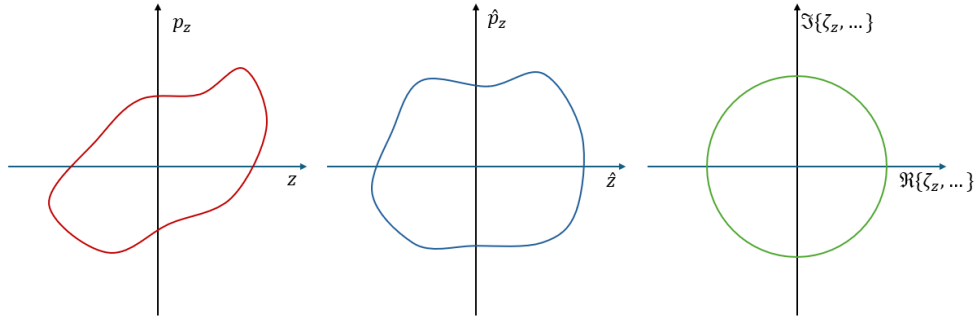


Figure 2.17: Exaggerated phase space distorted by non-linearities described in regular, normalized and normal form coordinates.

$$\hat{H}_W = \sum_{w=1}^W \sum_{j,k,l,m=0}^{n=j+k+l+m} h_{w,jklm} e^{i[(j-k)\Delta\phi_{w,x} + (l-m)\Delta\phi_{w,y}]} h_{w,x,-}^j h_{w,x,+}^k h_{w,y,-}^l h_{w,y,+}^m, \quad (2.36)$$

with

$$h_{w,jklm} = -\frac{K_{w,n-1}\Omega(l+m) + iJ_{w,n-1}\Omega(l+m+1)}{j!k!l!m!2^{j+k+l+m}} i^{l+m} \beta_{w,x}^{\frac{j+k}{2}} \beta_{w,y}^{\frac{l+m}{2}}. \quad (2.37)$$

The non-linear one turn map for normalized space is found, but phase space is perturbed due to the non-linear motion and this makes calculating turn-by-turn motion impractical. Similarly to the transformation to normalized coordinates, another transformation can be made to have an unperturbed circular phase space again called normal form coordinates $\zeta_{u,\pm} = \sqrt{2I_u} e^{\mp i\Delta\psi_u}$, with I_u and ψ_u the newly defined action and phase in the normal form coordinate system. The transformation from regular to normalized to normal form coordinates is illustrated in Fig. 2.17. Analogously to Eq. 2.35 this new transformation will be performed using generating function F

$$F = \sum_{j,k,l,m=0}^{n=j+k+l+m} f_{jklm} \zeta_{x,-}^j \zeta_{x,+}^k \zeta_{y,-}^l \zeta_{y,+}^m. \quad (2.38)$$

This transformation will reduce to

$$e^{-:F} e^{:\hat{H}:} e^{:RF:} R = e^{:H(I_x, I_y):} R. \quad (2.39)$$

Using the properties of Lie operators and using the Campbell-Baker-Hausdorf theorem up to first order again the following relation is found

$$f_{jklm} = \frac{h_{jklm}}{1 - e^{2\pi i[(j-k)Q_x + (l-m)Q_y]}} = \frac{\sum_w h_{w,jklm} e^{i[(j-k)\Delta\phi_{w,x} + (l-m)\Delta\phi_{w,y}]}}{1 - e^{2\pi i[(j-k)Q_x + (l-m)Q_y]}}. \quad (2.40)$$

These normal form coordinates can now describe the one turn map as an amplitude dependent rotation. The turn-by-turn motion can thus be constructed in normal form coordinates using the generating function F as a function of the turn number.

$$\zeta_{u,\pm} = e^{iF} \zeta_{u,\pm}(N) \quad (2.41)$$

$$\begin{aligned} \zeta_{x,-} = & \sqrt{(2I_x)} e^{i(2\pi Q_x N + \psi_{x,0})} \\ & - 2i \sum_{jklm} j f_{jklm} (2I_x)^{\frac{j+k-1}{2}} (2I_y)^{\frac{l+m}{2}} \times \\ & e^{i[(1-j+k)(2\pi Q_x N + \psi_{x,0}) + (m-l)(2\pi Q_y N + \psi_{y,0})]} \end{aligned} \quad (2.42)$$

$$\begin{aligned} \zeta_{y,-} = & \sqrt{(2I_y)} e^{i(2\pi Q_y N + \psi_{y,0})} \\ & - 2i \sum_{jklm} l f_{jklm} (2I_x)^{\frac{j+k}{2}} (2I_y)^{\frac{l+m-1}{2}} \times \\ & e^{i[(k-j)(2\pi Q_x N + \psi_{x,0}) + (1-l+m)(2\pi Q_y N + \psi_{y,0})]} \end{aligned} \quad (2.43)$$

Some interesting conclusions can be made from the normal form coordinates. First of all, Eq. 2.40 re-establishes the resonance condition (Eq. 2.18) in the following form:

$$(j - k)Q_x + (l - m)Q_y = p, \quad p \in \mathbb{N} \quad (2.44)$$

as a function of the tunes Q_x , Q_y and the type of multipole magnet in which case f_{jklm} , also known as the resonance driving terms (RDTs), goes to infinity. Some resonances will also be more prominent than others depending on the magnetic structure of the accelerator and the order of the resonance. This can be well visualized with the use of a loss map, where the working point is varied in a specific area using quadrupoles, usually around the operational working point, and it is recorded how much intensity is lost at each tune value. In Fig. 2.18, a loss map recorded in 2020 of the PS is shown together with the resonances fulfilling Eq. 2.44 up to fourth order.

A second property that can be observed from Eqs. 2.42 and 2.43 concerns the location, amplitude and phase of lines in the frequency spectrum of the normal form coordinates. The first and largest line will come from the first term, this is equal to the tune and is also called the fundamental line. The secondary lines, which come from the summation term, depend on the type of non-linear magnet and the tunes. Their amplitude and location in the spectrum is given by

$$H(1 - k + l, m - l) = 2j |f_{jklm}| (2I_x)^{\frac{j+k-1}{2}} (2I_y)^{\frac{l+m}{2}} \quad (2.45)$$

$$V(k - j, 1 + m - l) = 2l |f_{jklm}| (2I_x)^{\frac{j+k}{2}} (2I_y)^{\frac{l+m-1}{2}}, \quad (2.46)$$

where the amplitude of the RDT line is set by the action of the turn-by-turn motion and how much a given resonance is perturbing this motion. It is important to note that the horizontal and vertical spectral lines only appear if respectively $j \neq 0$ and $l \neq 0$. As example, the f_{2010} RDT would appear in the horizontal spectrum at $H = -Q_x - Q_y$ and in the vertical spectrum at $V = -2Q_x$ and this example RDT would grow in amplitude if the working point came closer to $2Q_x + Q_y = p$, with p an integer or if the actions I_x and I_y would increase by introducing a continuous excitation to the beam.

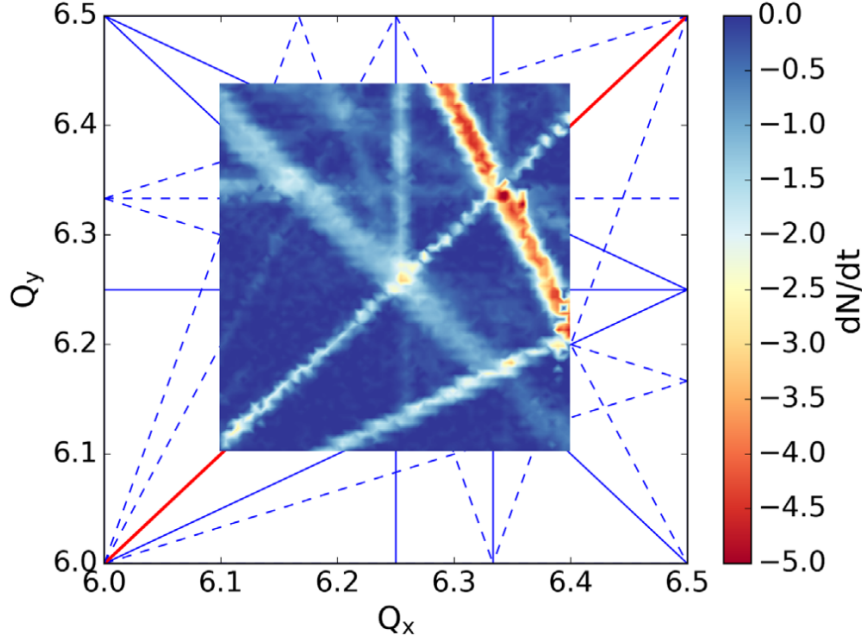


Figure 2.18: Beam loss map in the transverse tune space color coded to the loss rate. Theoretical resonance lines up to fourth order are plotted. The skew resonances are shown with dashed lines and the normal resonances with solid lines. [49]

2.2.6 Space charge

In circular accelerators or storage rings, additional electromagnetic fields arise beyond those created by the magnets or the accelerating RF system. These typically undesirable forces can be categorized as space charge [50–52] and beam-beam effects. Beam-beam effects are not applicable to the PS, but space charge effects are applicable as they result from the electromagnetic interactions of neighboring particles in a beam or from the interaction of the particle beam with the surrounding vacuum pipe wall. The self-field's order of magnitude is larger than the wall-effects in the PS, so "space charge" will refer to these self-fields in the context of this thesis.

The electric field for a particle at position $\vec{r}$ and charge q is given by Coulomb's law:

$$\vec{E}(\vec{r}) = \frac{q}{4\pi\epsilon_0} \sum_i \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|^3}, \quad (2.47)$$

with ϵ_0 the permittivity of vacuum. Transforming this to the electric field in the transverse plane of a coasting beam at longitudinal position s with beam size $a_0(s)$ in both planes, results in:

$$E_r(s) = \begin{cases} \frac{\rho(s)}{2\epsilon_0} r & \text{if } r \leq a_0 \\ \frac{\rho(s)a_0^2(s)}{2\epsilon_0} \frac{1}{r} & \text{if } r \geq a_0, \end{cases} \quad (2.48)$$

where $\rho(s)$ is the charge density. Further generalization to an axi-symmetric non-uniform beam where the beam size in both planes is the same, yields:

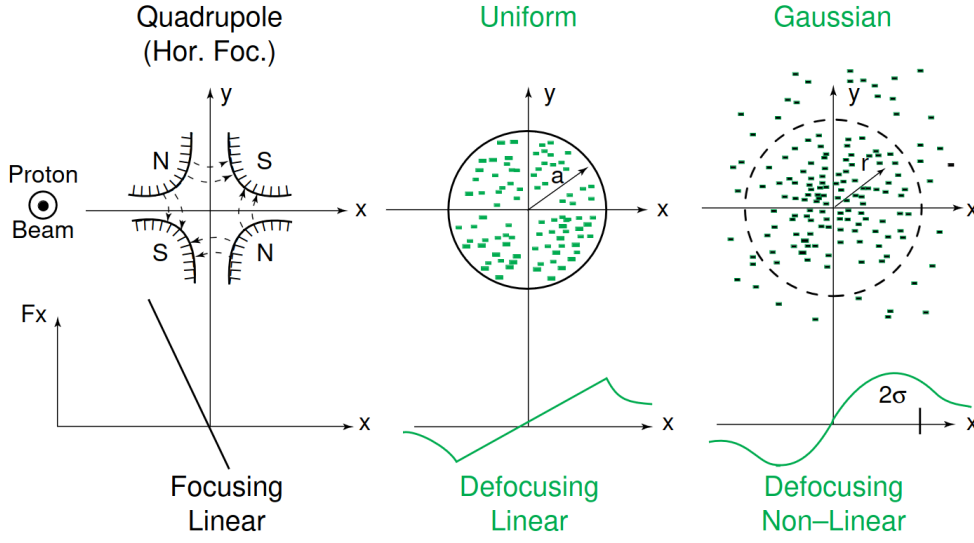


Figure 2.19: Focusing/defocusing force F_x vs. x of a quadrupole (left), and of space charge dominated beams: uniform (center) and Gaussian (right). [51]

$$E_r(r, s) = \frac{1}{\epsilon_0} \frac{1}{r} \int_0^r \rho(r', s) r' dr'. \quad (2.49)$$

The particle distributions for a uniform and Gaussian beam are shown in Fig. 2.19. The figure also shows the electric force $F_x = qE_x$ of a focusing quadrupole, showing the similarities between the two. The beams that are of interest for this study have a Gaussian distribution where the beam size in both planes is significantly different. For the case where the beam sizes in the transverse planes are a_0 and b_0 with $a_0 \neq b_0$ the equation for the electric field needs to account for different axial symmetries. This is achieved by defining the iso-density parameter T :

$$T = \frac{x^2}{a_0^2} + \frac{y^2}{b_0^2}. \quad (2.50)$$

Using the relation between the charge line density λ , the beam distribution $n(x, y)$ and the charge density $\rho(x, y)$:

$$\rho(x, y) = \lambda n(x, y) = \lambda \frac{\hat{n}(T)}{\pi a_0 b_0}. \quad (2.51)$$

The electric field for the horizontal plane is described by:

$$E_x = \frac{\lambda}{2\pi\epsilon_0} x \int_0^\infty \frac{\hat{n}(\hat{T})}{(a_0^2 + t)^{3/2} (b_0^2 + t)^{1/2}} dt. \quad (2.52)$$

Here, $\hat{T} = \frac{x^2}{a_0^2 + t} + \frac{y^2}{b_0^2 + t}$. Focusing on the particles close to the beam center, where $|x|, |y| \ll a_0, b_0$, $\hat{T}$ can be approximated to zero. Solving the integral in Eq. 2.52 results in electric field:

$$E_x = \frac{\lambda \hat{n}(0)}{\pi \epsilon_0} \frac{1}{a_0(a_0 + b_0)} x. \quad (2.53)$$

Including this electric field into the equations of motion of a single particle leads to the equivalent of an extra defocusing quadrupole in the machine.

$$\frac{d^2 x}{ds^2} + \left[k_{1,x}(s) - \hat{n}(0) \frac{qI}{2\pi \epsilon_0 m \gamma_r^3 \beta_r^3 c^3} \frac{2}{a_0(a_0 + b_0)} \right] x = 0, \quad (2.54)$$

$$\frac{d^2 y}{ds^2} + \left[k_{1,y}(s) - \hat{n}(0) \frac{qI}{2\pi \epsilon_0 m \gamma_r^3 \beta_r^3 c^3} \frac{2}{b_0(b_0 + a_0)} \right] y = 0. \quad (2.55)$$

Here, I represents the beam current. Knowing that a change in quadrupole strength $\Delta k_{1,x}(s)$ coincides with a change in tune

$$\Delta Q_x = \frac{1}{4\pi} \beta_x(s) \Delta k_{1,x}(s), \quad (2.56)$$

a particle in the longitudinal center of the beam will experience a space charge tune-shift given by

$$\Delta Q_x = \frac{1}{4\pi} \int_0^L \beta_x(s) \hat{n}(0) \frac{2K}{a(s)[a(s) + b(s)]} ds. \quad (2.57)$$

Here the perveance $K = \frac{qI}{2\pi \epsilon_0 m \gamma_r^3 \beta_r^3 c^3}$ is defined, and changes to the beam sizes along the circumference are taken into account. Using the approximation $Q_x = \frac{R}{\langle \beta_x \rangle}$ with R the circumference of the machine and by substituting the beam size as $\langle a \rangle = \sqrt{\epsilon_x \langle \beta_x \rangle}$ in the absence of dispersive contribution, the integral can be solved to find the incoherent space charge tune-shift

$$\Delta Q_x = -\frac{R^2}{Q_x} \hat{n}(0) K \frac{1}{\sqrt{\epsilon_x \langle \beta_x \rangle} (\sqrt{\epsilon_x \langle \beta_x \rangle} + \sqrt{\epsilon_y \langle \beta_y \rangle})}. \quad (2.58)$$

This space charge tune-shift diminishes with relativistic factors $\beta_r^2 \gamma_r^3$, making it negligible in high-energy machines like the LHC. However, in the PS, tune spreads across multiple resonances are common, as shown in Fig. 2.20, where the characteristic "neck-tie" pattern illustrates how particles at different energies experience varying tune-shifts. This leads to substantial beam losses due to synchrotron motion, which causes particle energies to oscillate around their reference value. As a result, the space charge tune-shift fluctuates for individual particles, causing them to repeatedly cross resonances. This is another component to why it is important to understand and control resonances in the PS.

2.2.7 Transition crossing

For low-energy accelerators, when the momentum of a particle increases, the revolution frequency of that particle also increases. This relationship is illustrated in Eq. 2.13 and is repeated here for clarity:

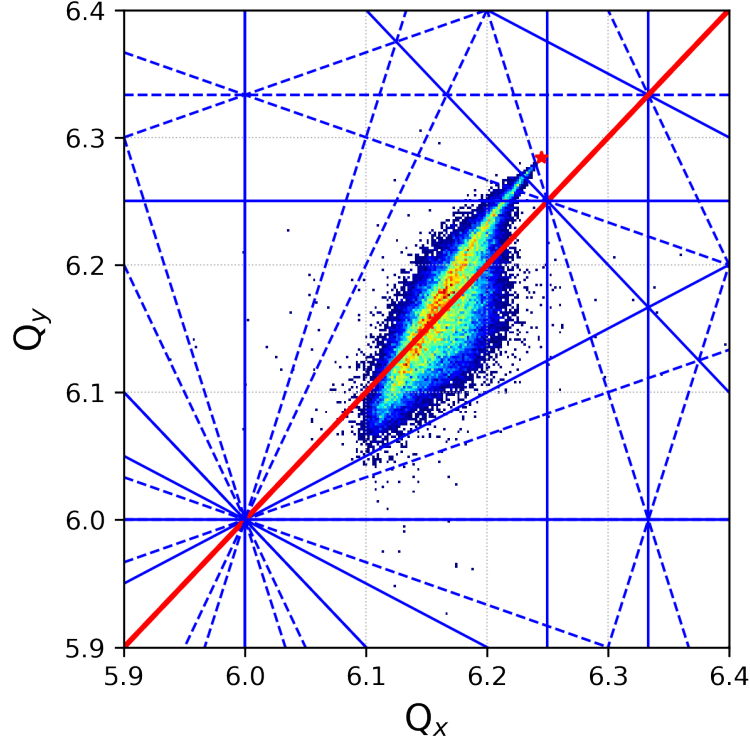


Figure 2.20: Space charge tune spread of a simulated beam at injection energies in the PS.

$$\frac{\Delta p}{p} = \frac{1}{\eta} \frac{F_{rev}^0 - F_{rev}}{F_{rev}^0},$$

with

$$\eta = \frac{1}{\gamma_r} - \frac{1}{L} \oint \frac{D}{\rho} ds.$$

The slip factor η is positive for low-energy machines, as the relativistic gamma γ_r remains low. However, as γ_r increases during the acceleration process, the sign of η can change, leading to a decrease in revolution frequency with increasing momentum. The energy value where η changes sign is known as the transition energy, and crossing this energy generally has negative effects on beam quality for several reasons. Just as space charge effects create a tune spread in the transverse plane, the bunch length in the longitudinal plane is also impacted. When η changes sign during transition, the bunch length can shift abruptly, potentially causing emittance blow-up as particles struggle to adapt to the rapid change in the RF bucket shape. Moreover, due to the momentum spread within the beam, not all particles experience the transition at the same time, further destabilizing the beam during this phase. As a result, transition crossing is inherently unstable for any accelerator that encounters it, including the PS [6]. Even with methods to reduce the negative effects of transition crossing that will not be discussed here, it has long been suspected as a possible source of PS beam degradation [53].

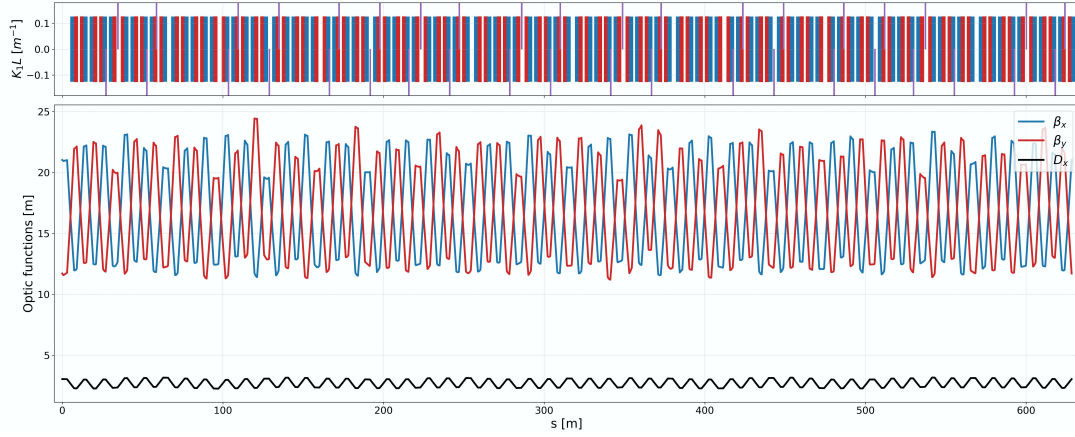


Figure 2.21: Top: positional overview of focusing (blue) and defocusing (red) main units and quadrupoles (purple) in the PS ring. Bottom: β -functions and horizontal dispersion at the working point of $(Q_x, Q_y) = (6.214, 6.238)$ at flat bottom energy. Since the model vertical dispersion along the PS ring is zero, this is excluded from the plot.

2.3 Operational beams

A general overview of the parameter values in the PS introduced in this chapter will be provided. The magnetic structure of an accelerator determines the β and dispersion functions of the machine, and once the β -function is established, the tune is also set. The model optics of the PS lattice for beams destined for the LHC are presented in Fig. 2.21, where the tunes are $(Q_x, Q_y) = (6.214, 6.238)$. This combination of horizontal and vertical tunes is called the working point. The PS operates between working points of $(6.0, 6.0)$ and $(6.5, 6.5)$, avoiding resonances where possible. Beams with different destinations have individual optical requirements and consequently different working points, but generally, the β and dispersion functions remain similar across various operational settings.

The PS operates using a magnetic supercycle, an example of which is shown in Fig. 2.22. The individual magnetic cycles are visible as white horizontal lines in the top half of the figure. The beam is injected when the magnets are at their lowest value, denoted as bottom energy. The beam is then accelerated or ramped up to top energy before being extracted. The magnetic supercycle shown in the plot consists of 43 magnetic cycles, each taking 1.2 seconds, synchronized with its injector, the Proton Synchrotron Booster (PSB). Some destinations or tests require only one cycle before extraction, such as the Multi-Turn Extraction (MTE) and neutrino time-of-flight (TOF) beams. Others, like the beams destined for the East Area (EAST), require two basic periods. Beams going to the LHC, although not shown in the figure, use three cycles. This supercycle is constantly looped, and during machine development (MD), it is often important for the subsequent cycle to remain consistent since the magnetic hysteresis loop of the previous cycle could affect the next one.

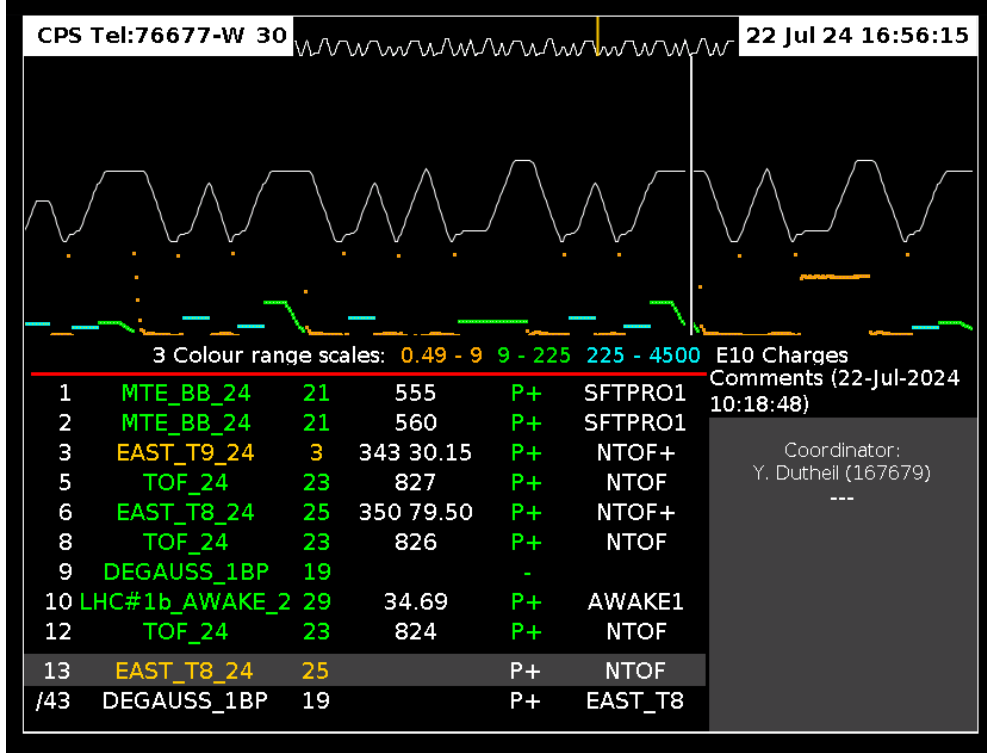


Figure 2.22: A screenshot from the CPS Vistar, showing the magnetic supercycle in the PS.

For the experimental measurements conducted in this thesis, the LHC and LHCINDIV cycles are used. Their significant characteristics at bottom energy, relevant to the work done, are displayed in Table 2.1. The most notable difference between the two cycles is that the LHCINDIV cycle has lower intensity, $\frac{\Delta p_{RMS}}{p}$, and the PFWs are unused at bottom energy. The beam characteristics of the LHCINDIV cycle make it a perfect test bed for new studies. After the initial tests were successful, the higher intensity operational LHC cycle was used.

Parameter	LHCINDIV	LHC
N_b [10^{10} ppb]	2-15	1080
$\frac{\Delta p_{RMS}}{p}$ [e^{-3}]	0.5	1.5
(Q_x, Q_y)	(6.21,6.245)	(6.214,6.238)
(ξ_x, ξ_y)	(-5.28,-7.14)	(0.71,-2.87)
$\beta_{x;WSH54}$	12.436	12.697
$\beta_{x;WSH68}$	13.149	12.817
$D_{x;WSH54}$	2.343	2.323
$D_{x;WSH68}$	2.427	2.344

Table 2.1: The differences of interest in beam parameters and optics between an LHC and LHCINDIV cycle at flat bottom energy.

2.4 Models and codes

The PS is modeled using MAD-X (Methodical Accelerator Design) [54], a widely used software for designing, simulating, and optimizing particle accelerators. MAD-X is employed across all CERN accelerators to benchmark and analyze optical configurations at various energy levels throughout the acceleration process. For the remainder of this thesis, any reference to the PS Model pertains to the MAD-X model. MAD-X uses its own scripting language, but for ease of use and enhanced automation, the `cpymad` Python wrapper is utilized [55]. While MAD-X supports multi-particle tracking, it does not inherently include space charge effects. For simulations where space charge forces are relevant, `pyORBIT` is used instead. `pyORBIT` is a Python interface for ORBIT (Objective Ring Beam Injection and Tracking) [56], which allows the integration of space charge nodes into the PS model. In these simulations, tracked particles receive kicks based on their relative positions to others, approximating the continuous effect of space charge through discrete nodes. With a sufficient number of space charge nodes, an accurate simulation of space charge effects can be achieved.

Additionally, for beam optics measurements and corrections, CERN's Optics Measurements and Corrections (OMC) team has developed a Java-based analysis tool for multi-turn measurement data [57]. Amongst the things that this tool can compute are beam optics and resonance driving terms. In this thesis, whenever the beta-function or resonance driving terms of the PS are measured, the OMC Graphic User Interface (GUI) is used for data analysis.

TUNE AND CHROMATICITY CONTROL OF THE POLE FACE WINDINGS

Machine learning (ML) has recently emerged as a promising tool in accelerator physics, offering new possibilities where traditional methods have often struggled [58–62]. Key parameters in complex systems, such as tune and chromaticity for particle accelerators, can have non-linear dependencies that complicate their control. As accelerators continue to increase in both energy and complexity, these challenges become more pronounced, requiring more sophisticated approaches to machine control. ML, especially neural networks, offers a promising solution by dynamically adapting to a wide range of operating conditions.

In the case of the PS, tune and chromaticity control is especially difficult due to the limitations of traditional correction methods in the PFW basic functions and response matrices introduced in 2.1.1. The empirical response matrices do not account for the non-linear behavior of the PFWs at higher energies, making this control model inefficient for corrections above transition energy, while the basic functions provide no settings below this energy threshold. Rather than refining these outdated models, a novel ML-based approach is proposed, that unifies the PFW control system and settings generator. By using neural networks to control the PFW currents based on the desired tune and chromaticity inputs, this method offers a more flexible control system, while holding on to the same level of accuracy. Once validated, this model can serve as a universal tool for settings generation and control, streamlining operational tasks and improving performance, especially at higher energies.

The application of ML in the PS not only addresses these specific challenges but also demonstrates the broader potential of such techniques to enhance control in particle accelerators.

Neural network concepts

Neural networks consist of multiple layers of processing units, or neurons, as depicted in Fig. 3.1. Each neuron in a network takes a linear combination of inputs from the previous layer, applies an activation function, and propagates its output to the next layer.

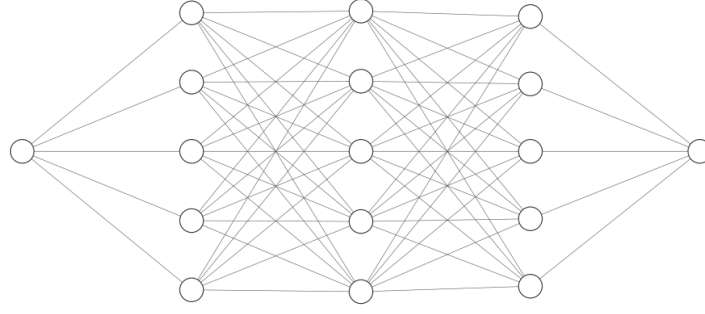


Figure 3.1: Simplified schematic of a neural network with one input layer, three hidden layers with 5 neurons, and one output layer.

The most commonly used activation function is the Rectified Linear Unit (ReLU) [63], defined by the equation:

$$\sigma_j(a_j) = \max(0, a_j), \quad (3.1)$$

where a_j represents the linear combination of the inputs from the nodes of the previous layer and the corresponding weights assigned by the network.

The first layer, known as the input layer, and the final layer, known as the output layer, function differently from the hidden layers, which are the layers in between. The input layer serves as the entry point for data into the network, while the output layer generates the network's predictions, which are then compared to the true values (labels). Training a neural network requires a substantial amount of labeled data for which both the input and output (label) are known. The network's predictions are compared to these labels using a loss function, typically mean square error (MSE) for numerical data. The network's weights are updated based on this loss using gradient descent optimization. The update rule for the weights x in the network is given by:

$$x_{i+1} = x_i - \alpha \nabla_x f(x), \quad (3.2)$$

where α is the learning rate, which determines the step size for each update. This process is repeated iteratively until the network converges to a local minimum in the loss function.

For this study, the python version of TensorFlow is used to build, train, and test the neural networks [64]. TensorFlow is a widely used deep learning framework that is well-suited for large-scale machine learning tasks in accelerator physics. The neural networks in this study are implemented using Keras [65], TensorFlow's high-level API. A fully connected feedforward neural network architecture is chosen, where each neuron in a layer is connected to every neuron in the subsequent layer and the ReLU activation function is used for all hidden layers. Training is performed using the Adam optimizer [66], a gradient descend-based optimization method that adjusts learning rates individually for each parameter. This initial loss function is based on the Mean

Network parameters	Values
Number of hidden layers	9
Neurons per layer	70
Activation function	ReLU
Training/validation split	80/20
Optimizer	Adam
Learning rate	0.003
Loss function	MSE

Table 3.1: *The neural network parameters used for the tune and chromaticity predictions of the PFWs*

Squared Error (MSE) between outputs and labels of the network. The final trained models are saved and deployed using TensorFlow's SavedModel format, which would allow for easy integration into the PS control system. The specific parameters and architecture of the neural network used in this study are detailed in Table 3.1.

3.1 Initial considerations for a Q and Q' prediction network

At the outset of the studies presented in this thesis, a first attempt was made to control the tune and chromaticity in the PS using historical data that had been collected in 2018 [67], where the PFW response was measured during acceleration instead of at different energy plateaus. These measurements were then split into high-energy and low-energy sections by transition crossing. During this measurement campaign, four currents were held at 0A while the fifth was set to five distinct levels. At each setting, the $\frac{dp}{p}$ of the beam was slightly varied between each new injection, or shot, to obtain a precise chromaticity estimate according to Eq. 2.5. In each shot, the tune was measured every 7ms equating to 183 measurements in both planes with 100 $\frac{dp}{p}$ variations for each PFW current setting, as seen on Fig. 3.2, leading to over 385,000 total data points.

This extensive dataset enables the application of machine learning to develop a high-precision prediction model, which will serve as a foundation before progressing to a control model. The focus on prediction is due to the fact that such a model takes the machine settings in the PFW currents as inputs and the measured tunes and chromaticities as outputs, making it easier to train and validate compared to a control model, where these inputs and outputs are reversed. The underlying assumption is that if the prediction network performs well, the inverse control network should also deliver good results.

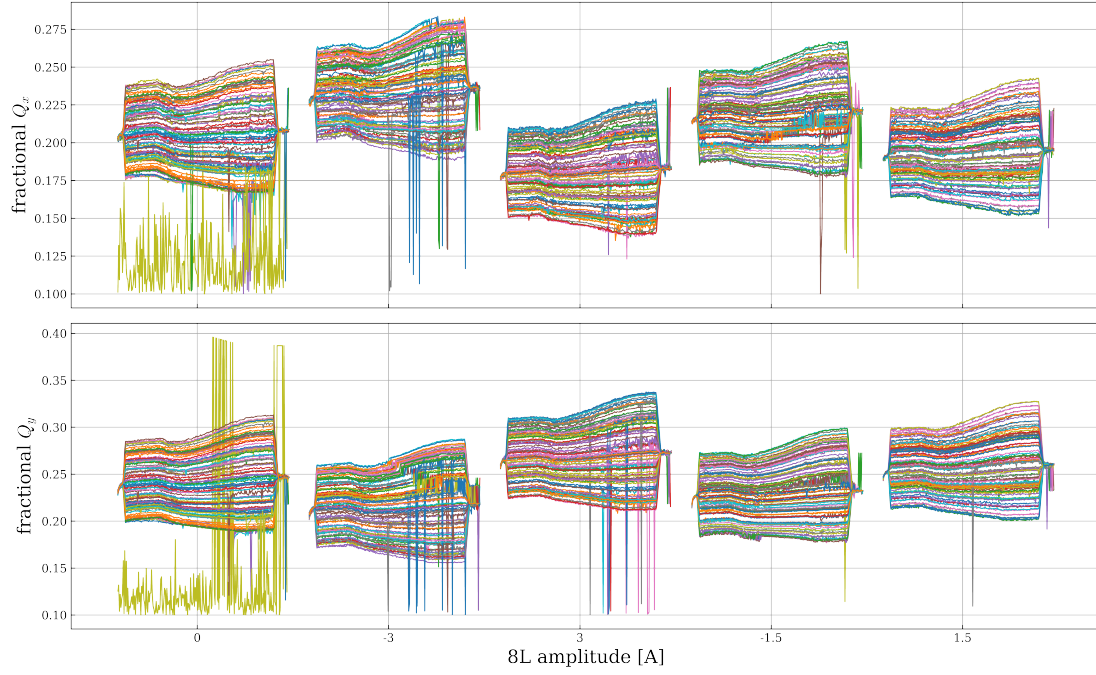


Figure 3.2: The tune measurements obtained from the automatic Base Band Tune (BBQ) analysis 7ms apart for the 8L-PFW settings where every line represent a new injection into the PS. Many faulty measurements can be seen in sudden peaks or dips in tune over a few milliseconds. There are also cases where the planes are coupled, leading to the same tune value for both planes. Additionally, there is visible noise between subsequent measurements in the same shot.

A standard neural network, denoted $\mathbf{F}$, is used for this task, with its parameters outlined in Table 3.1. The network is trained to model the following relationship:

$$\mathbf{F} \begin{pmatrix} B \\ \frac{dp}{p} \\ I_{DN} \\ I_{FN} \\ I_{DW} \\ I_{FW} \\ I_{8L} \end{pmatrix} = \begin{pmatrix} Q_x \\ Q_y \\ Q'_x \\ Q'_y \end{pmatrix}, \quad (3.3)$$

where the inputs are the magnetic field B , momentum deviation $\frac{\Delta p}{p}$, and PFW currents, while the outputs are the horizontal and vertical tunes (Q_x, Q_y) and their chromaticities (Q'_x, Q'_y).

Initial attempts to train the neural network were unsuccessful, with the issue traced to inconsistencies in the tune data provided by the PS. Although generally reliable, the standard data acquisition system produced faulty data under the specific demands of this study. To ensure the network was trained on accurate data, the measured tunes and momentum deviations underwent custom post-processing to remove errors and achieve the required precision. In contrast, the magnetic field and PFW current values, being machine settings, already had the necessary accuracy for this analysis.

The custom post-processing techniques developed for this study primarily focus on refining the tune data by removing outliers and decoupling any mixed signals. Additionally, a detailed analysis of the momentum deviation $\frac{\Delta p}{p}$ was performed for each tune measurement to ensure data accuracy.

3.1.1 Tune coupling cleaning

The tunes in the PS are measured using the Base-Band tune system (BBQ) [28], where the transverse damper (ADT) generates a small excitation, called a chirp, in both planes for a number of turns every few milliseconds. This excitation is recorded by dedicated pick-ups and analyzed using a Fast Fourier Transform (FFT). Ideally, the tune should produce the largest peak in its respective frequency spectrum. However, in practice, the tune of one plane can be present in the other plane's frequency spectrum and occasionally the tune of the wrong plane is selected in the tune measurement.

This is illustrated in Fig. 3.3, where the measurement at 966ms after injection selects the correct tunes in both planes but the next measurement at 973ms, the horizontal tune is selected in the vertical spectrum as "correct" tune. This effect, known as coupling, is typically mitigated by placing peak detection windows around the set tunes. However, given the wide range of PFW and momentum deviation settings used during the data acquisition, limiting the tunes to specific detection windows is not feasible which is why an extra analysis is required.

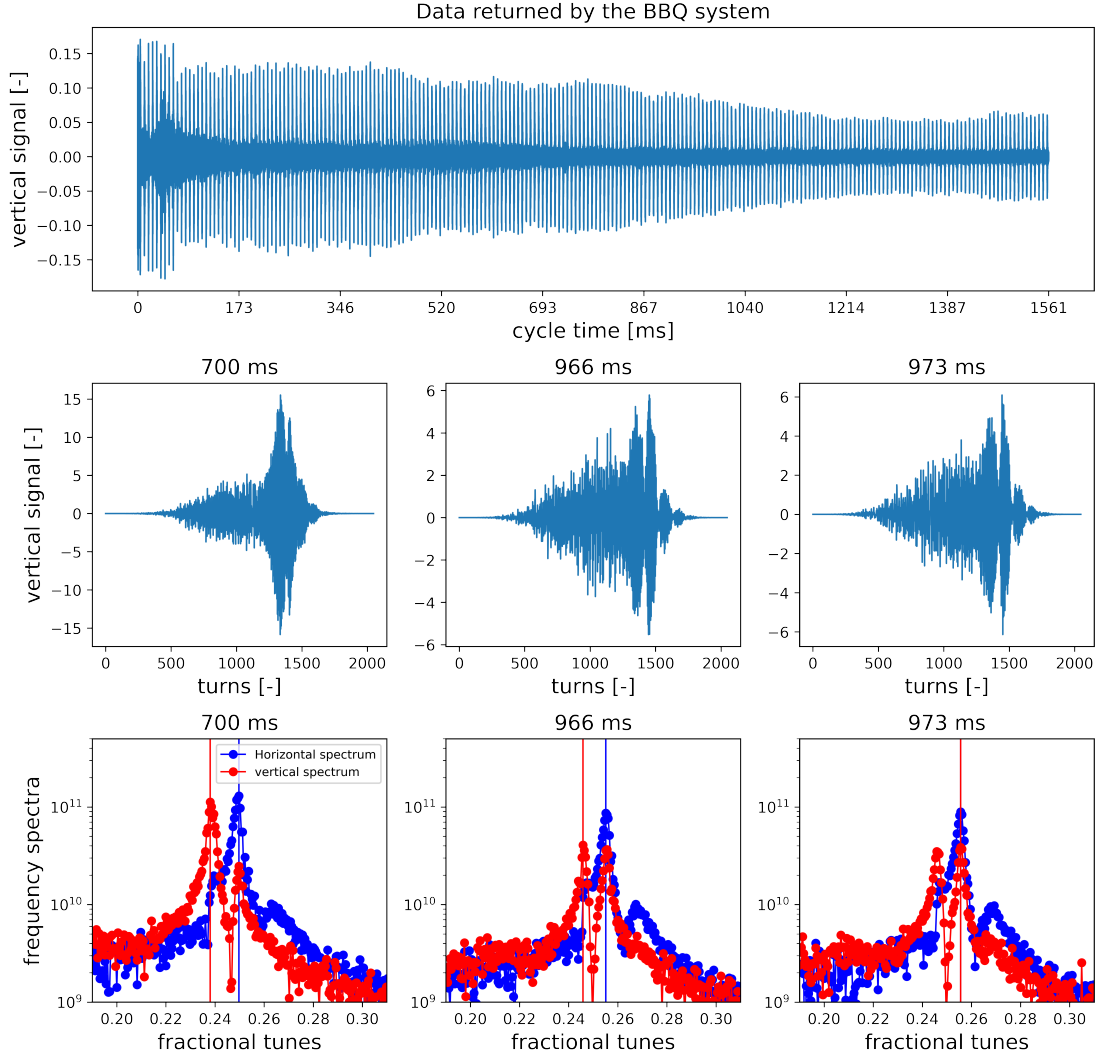


Figure 3.3: The top plot displays the raw oscillation signal from the vertical pick-up, showing the injection oscillations and the excitations due to the chirp occurring every 7 ms. In the plots below, three tune measurements are windowed and analyzed. The largest peak in each frequency spectrum is highlighted with colored vertical lines. At 966 ms after injection, the horizontal excitation (red) is prominently visible in the vertical spectrum (blue). In the following measurement, the horizontal peak becomes larger than the vertical peak, resulting in a coupled measurement.

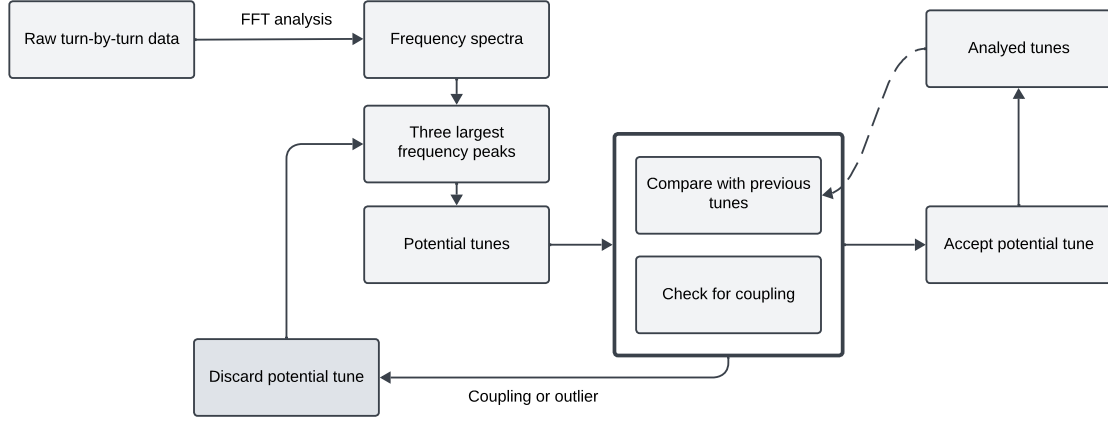


Figure 3.4: A flowchart showcasing the process of cleaning one tune measurement for the training of the prediction network.

If coupling or general outliers in the data are present, the correct tune can still be extracted from the frequency spectrum, as the correct tune is likely the second or third largest peak in the spectrum. Therefore, the raw turn-by-turn signal from the pick-ups is re-analyzed using an FFT, and the three largest peaks in the spectrum are recorded. The largest peak in the spectrum is selected as the potential tune. If the potential tune in both planes are identical, or if the different values between previous tune measurements and the potential tune is unrealistically large, the next largest peak in the spectrum is selected as the new potential tune and the process is repeated. If the potential tune fits the criteria, the tune values are updated accordingly; if none of the recorded peaks fit the criteria, this measurement is discarded. This process is illustrated in Fig. 3.4 for clarity and the final cleaned tune values used for training the network are shown in Fig. 3.5.

3.1.2 Momentum analysis

The described network requires the $\frac{dp}{p}$ for every measurement. This parameter is extracted from the machine using the revolution frequency f_{rev} as given by Eq. 2.13. The nominal revolution frequency f_{rev}^0 is taken from the first shot of every PFW-setting, before the radial steering of the machine is varied. This method of selecting the nominal revolution frequency f_{rev}^0 is the same as the one used for chromaticity measurements during operation.

With the $\frac{dp}{p}$ for every measurement, the betatronic tunes and chromaticity can be separated from the total tune measurements by using Eq. 2.5 through linear regression. The data is grouped according to cycle time and PFW setting before doing the chromaticity analysis, as shown in Fig. 3.6. Using this method, every data point now also has a chromaticity value, making the data ready to be used in the training of a neural network.

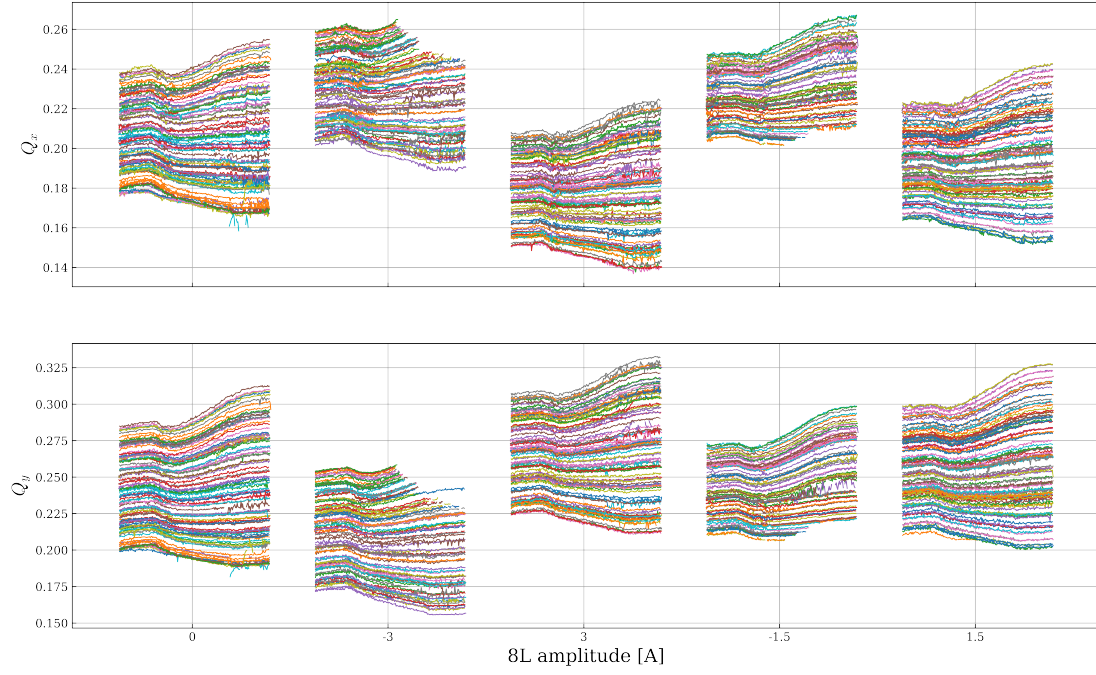


Figure 3.5: The tune measurements for the 8L-PFW settings after post-processing and cleaning.

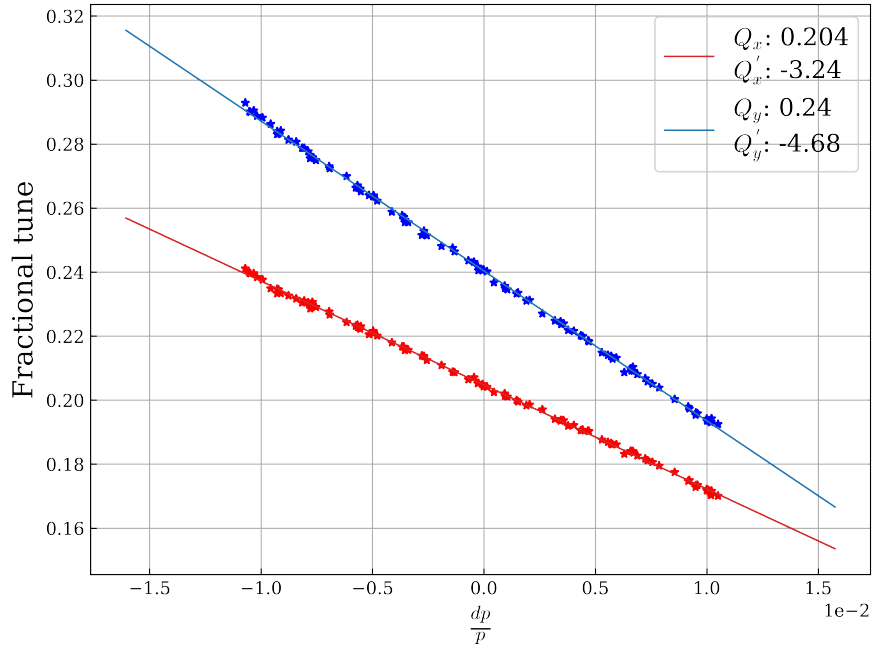


Figure 3.6: A chromaticity Q' measurement in both planes where the change in measured tune with respect to $\frac{dp}{p}$ equals the chromaticity according to Eq. 2.5. The measured chromaticity is assigned to every tune in this plot. This analysis is repeated for all 183 measurements in one PFW setting.

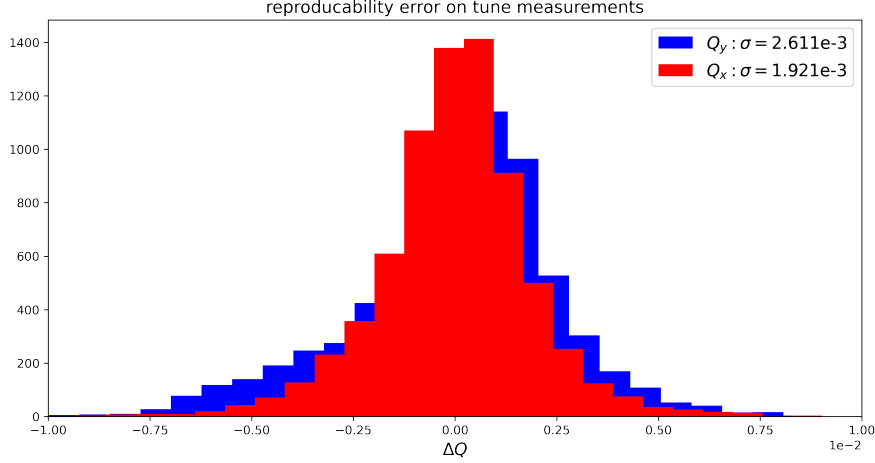


Figure 3.7: *The variation of the measured tunes for identical PFW settings at constant magnetic field.*

3.1.3 Neural network results

Even after cleaning the tune measurements, there remains noise present in the data, due to the limited precision of the beam instrumentation of the PS. This noise can be illustrated by comparing the measured tunes for identical PFW settings at a constant magnetic field. The resulting distribution of the tune differences is shown in Fig. 3.7. This distribution serves as a reference for the "goodness" of the resulting prediction network. If the network's predictions exhibit the same variation relative to the measured values as the reproducibility error, this indicates that the network is correctly trained and can be tested.

The difference between predicted tunes $Q_{pred} = Q_{\beta;pred} + \frac{dp}{p} Q'_{pred}$ and measured tunes is shown in Fig. 3.8. The predictions are similar to the measured tunes but significant improvements can be made. It is clear that the variation between prediction and measurements is larger than the reproducibility of the tune measurements, indicating that the network introduces additional errors. To investigate the source of these discrepancies, whether they arise from tune prediction, chromaticity prediction, or both, new beam-based measurements were conducted. These measurements, not used in the training of the network, serve exclusively for validation purposes.

The results are presented in Fig. 3.9a and 3.9b. The first figure compares the measured and predicted tunes and chromaticities. The chromaticity predictions perform well, especially considering that chromaticity is generally a smaller effect compared to the tune. However, the betatronic tune predictions consistently deviate by a constant value, indicating that the assumed nominal momentum used by the network is ill-defined. Currently, the nominal momentum is set to the momentum of the first shot for each PFW configuration, resulting in small shifts in $\frac{dp}{p}$ across all data points. This discrepancy explains why the chromaticity predictions remain accurate while the predicted tunes exhibit an offset.

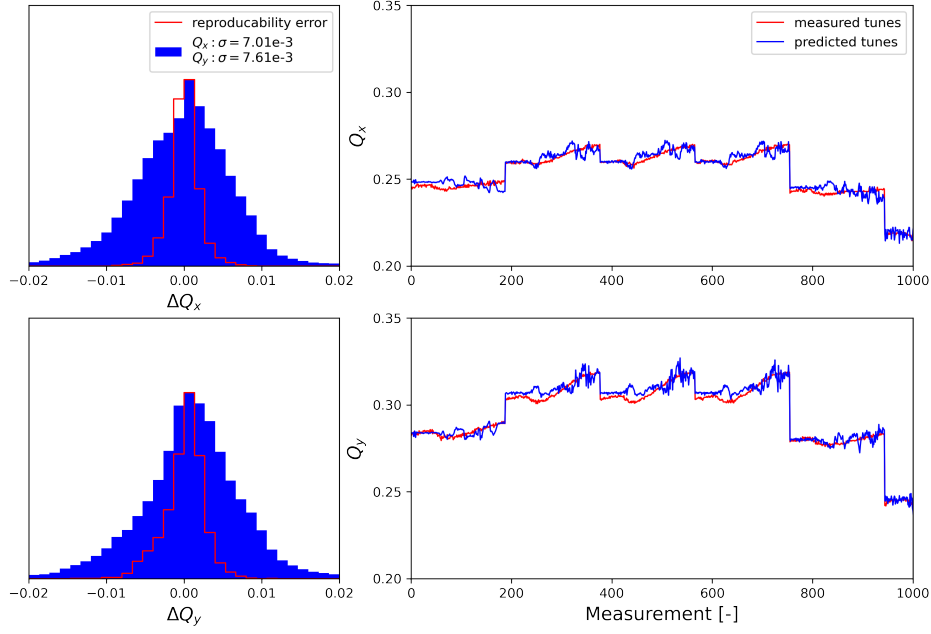
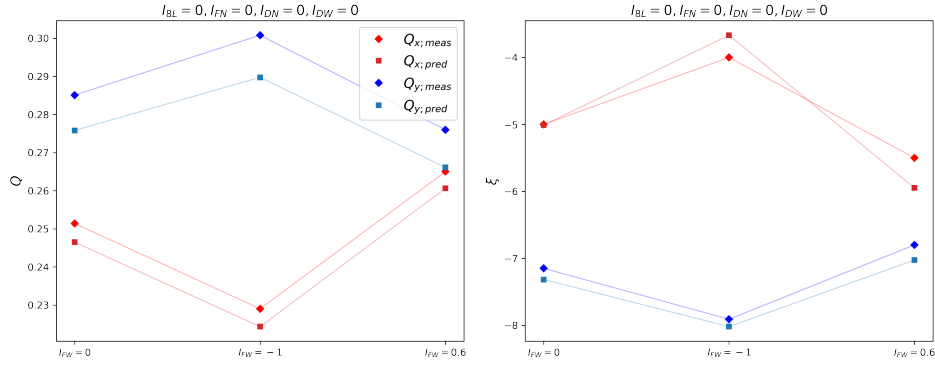
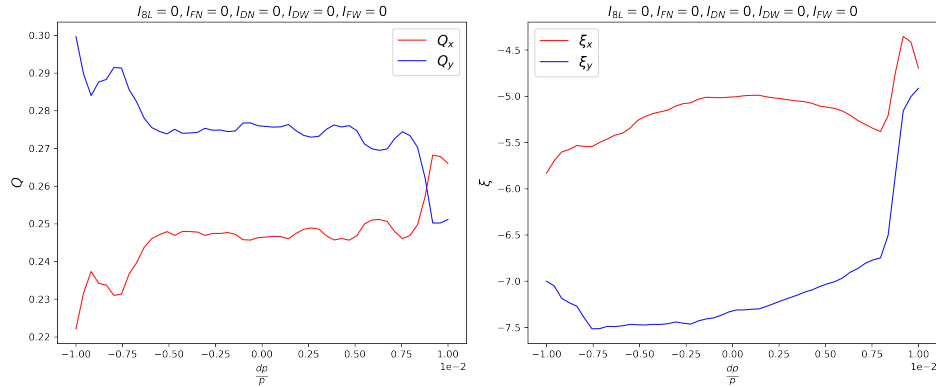


Figure 3.8: Comparison of the network predictions and the 2018 training data. On the left the variation of predictions are depicted while on the right, 1000 data points and their predictions are shown.



(a) Comparison of the network predictions and new beam-based measurements for 3 PFW settings that are not used in the training of the network.



(b) Network results for a constant PFW setting and magnetic field but varying $\frac{dp}{p}$ indicating the presence of second-order chromaticity in the training data.

Figure 3.9: Results from the neural network based on the 2018 measurements.

In the next iteration, the network needs a well-defined zero point for $\frac{dp}{p}$. Fig. 3.9b shows a dependence between the predicted betatronic tunes and chromaticities on the input $\frac{dp}{p}$. This relationship suggests that second-order chromaticity effects are present in the training data. Ideally, the betatron tune and chromaticity should remain constant across different $\frac{dp}{p}$ values, indicating that $\frac{dp}{p}$ may not be a suitable input parameter for the network.

Furthermore, it is important to acknowledge that neural networks are notoriously poor at extrapolating beyond the range of the training data. To ensure accurate predictions, the training dataset must encompass a wide range of betatron tunes and chromaticities. The current dataset, which utilizes only 21 PFW settings, is insufficient in this regard, so this requires an expansion of the training data to improve the network's performance.

3.2 Improvements to the neural network structure and momentum analysis

To address the issues concerning $\frac{dp}{p}$ and the operational range of the neural network, both the network and the training data need updates. In the previous approach, $\frac{dp}{p}$ was calculated based on the change in revolution frequency. However, due to the undefined nominal revolution frequency, a new figure-of-merit is chosen for momentum calculations. The Mean Radial Position $\langle x \rangle$ (MRP) is used to calculate the momentum deviation, as it has a well-defined nominal value. The MRP is based on the mean value of the BPM measurements over the entire orbit and can be converted to $\frac{dp}{p}$ using Eq. 2.16 and the average dispersion along the ring $\langle D_x \rangle$:

$$\frac{dp}{p} = \frac{\langle x \rangle}{\langle D_x \rangle}. \quad (3.4)$$

The average dispersion is calculated based on the MAD-X optics model. Another change to the network concerns its architecture. Ideally, the momentum deviation $\frac{dp}{p}$ would be a label instead of an input variable since the output tunes and chromaticities should not be affected by this value. However, It is crucial that the $\frac{dp}{p}$ is used during the training of the network. This can be achieved by using a physics-based loss function. A physics-based loss function incorporates the known behavior of problems into the neural network's training process [68]. By aligning the loss function with the physics of the problem, which in this case is the chromatic expansion of the tune, the network achieves more realistic results and reduces the likelihood of over-fitting on the training data. The used loss function is:

$$L = \sqrt{\left[Q_{x;meas} - \left(Q_{x;\beta} + \frac{dp}{p} Q'_x + \frac{dp^2}{p} Q''_x \right) \right]^2 + \left[Q_{y;meas} - \left(Q_{y;\beta} + \frac{dp}{p} Q'_y + \frac{dp^2}{p} Q''_y \right) \right]^2}. \quad (3.5)$$

Here the measured tunes and momentum deviation $\frac{dp}{p}$ are used as labels instead of pre-analyzing the deconvoluted tunes and chromaticities. Consequently, the mapping $\mathbf{F}$ is transformed into:

$$\mathbf{F} \begin{pmatrix} B \\ I_{DN} \\ I_{FN} \\ I_{DW} \\ I_{FW} \\ I_{8L} \end{pmatrix} = \begin{pmatrix} Q_x \\ Q_y \\ Q'_x \\ Q'_y \\ Q''_x \\ Q''_y \end{pmatrix}. \quad (3.6)$$

The second-order chromaticities are included in the network for more degrees of freedom as it is sometimes present in the PS but too small to consistently measure.

PFW circuit	Low energy [A]		high energy [A]		ramp [ms]	
	min	max	min	max	min	max
8L	-40	25	1170	1590	-8	12
DN	-5	5	32	153	-35	20
DW	-10	7	53	154	-25	25
FN	-5	10	180	250	-16	18
FW	-11	7	140	245	-40	40

Table 3.2: The extreme values of the PFW circuits used during data acquisition. Low and high energy sections are offset in current, while the ramp is varied in timing.

The final issue to address is the operational range of the network. To resolve this, new training data must be acquired. The focus of data acquisition will shift from using a small number of PFW settings with many $\frac{dp}{p}$ values per setting to using many different PFW settings with a small number of $\frac{dp}{p}$ values. This shift in focus is largely facilitated by the physics-based loss function, which eliminates the need to deconvolute the tunes and chromaticities before training the network. The magnetic cycle used for acquiring new training resembles operational cycles more closely and follows the basic functions shown in Fig. 2.5. This cycle can be split into three sections: low energy, ramp and high energy.

Both the low and high energy sections will be sampled similarly, as the B-field remains constant in these sections. The PFW currents will be set to random values at the beginning and end of each section, and will vary linearly between these two values along the constant B-field. These currents remain unchanged for 7 shots, during which the radial steering is varied, resulting in multiple measurements of the same PFW settings for different $\frac{dp}{p}$.

The ramp section poses a challenge. Here, the beam crosses the transition energy, which can cause significant losses if the tunes and chromaticities deviate too much from their nominal values. To mitigate this, the ramp section will be sampled by randomly offsetting the start timings of the PFW current ramps relative to the B-field ramp. This approach increases the likelihood of beam survival while still allowing for some variation in the tunes along the ramp. The PFW currents and $\frac{dp}{p}$ from one night of measurements are shown in Fig. 3.10, where the different sampling methods used for each section is clearly visible. The PFW currents are randomly varied between their minimum and maximum values during each section without causing huge losses. These extreme values of the PFW currents are measured before data acquisition began and are shown in table 3.2.

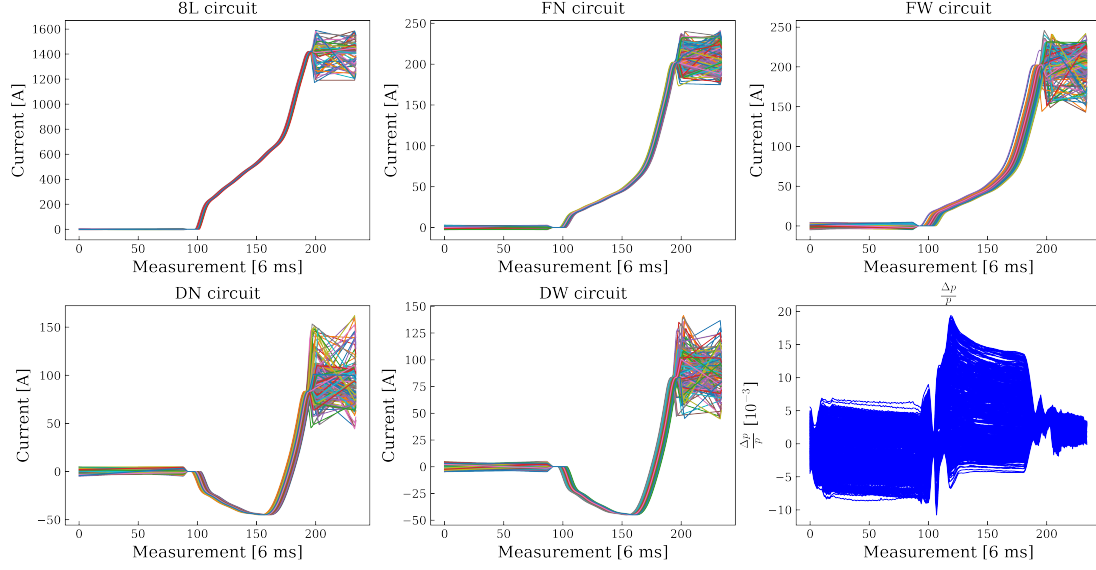


Figure 3.10: The PFW currents and $\frac{dp}{p}$ calculated with Eq. 3.4 sampled during one night of data acquisition. The PFW values are being varied at random between the different shots between the values shown in table 3.2.

3.2.1 Neural network results using the mean radial position

The PFW currents were scanned each night over the course of one workweek, resulting in approximately 500,000 data points collected during the second measurement campaign. The inputs and outputs of the network have underwent a change, as explained above, but the other parameters remain the same as in table 3.1. The results of the network are shown in Fig. 3.11. The variation of the predictions to the measurements are much more similar to the reproducibility error on the measurements themselves. This indicates that the changes that were made to the network vastly improved the training process.

The same low-energy comparison performed for the previous network is repeated and shown in Fig. 3.12a. Switching to the MRP as the figure-of-merit in the training of the neural network effectively resolves the tune prediction issues observed in Fig. 3.9a, demonstrating the importance of adopting a well-defined nominal momentum. However, a discrepancy emerges in the chromaticities. In Fig. 3.12a, the chromaticities of the new beam-based measurements are calculated using the revolution frequency as the figure-of-merit, which is the standard method in the PS. The predictions, however, do not align with these measurements. Conversely, when the MRP method is applied to both the neural network training data and the beam-based measurements (as shown in Fig. 3.12b), the chromaticities align significantly better. This shows that the relation $\frac{dp}{p} = \frac{\langle x \rangle}{\langle D_x \rangle}$ introduces a scaling error in the momentum deviation. Since the neural network predictions ultimately need to match the results obtained from the revolution frequency method, further refinement is necessary to bridge this gap and ensure accurate chromaticity predictions based on the standard approach.

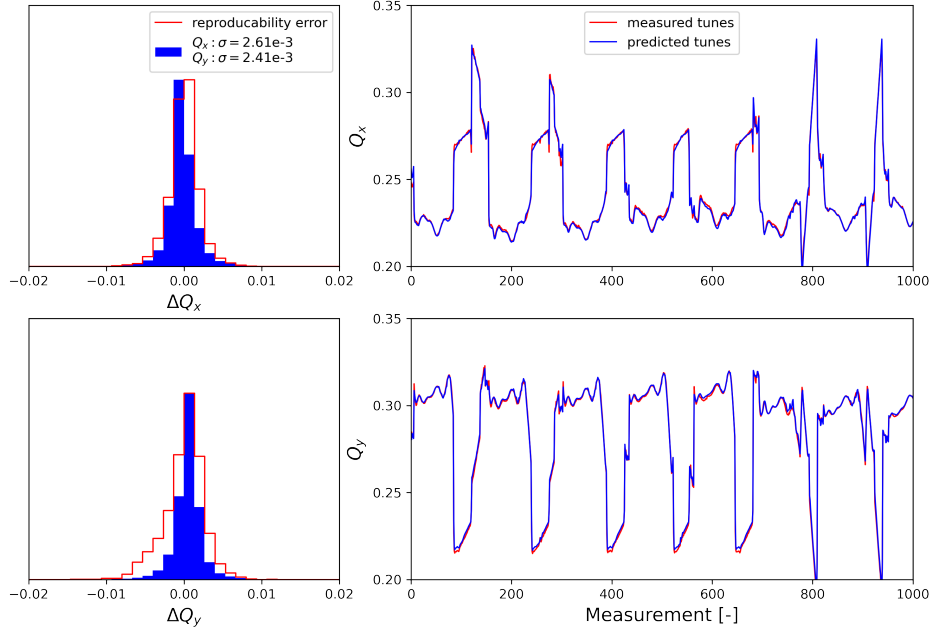
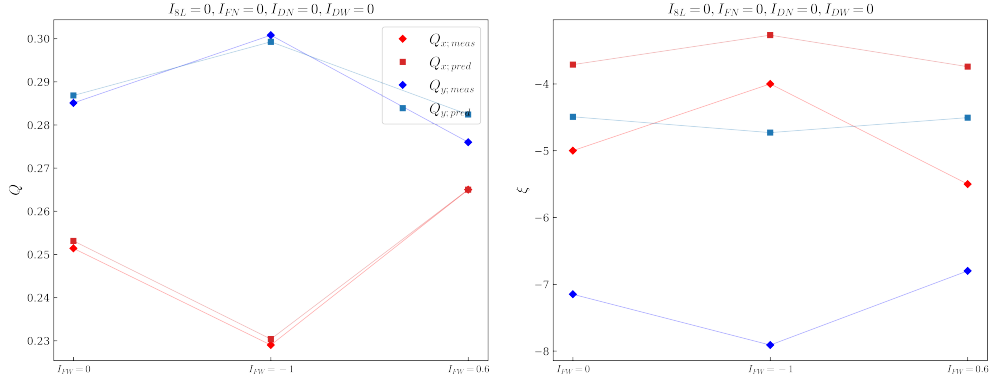
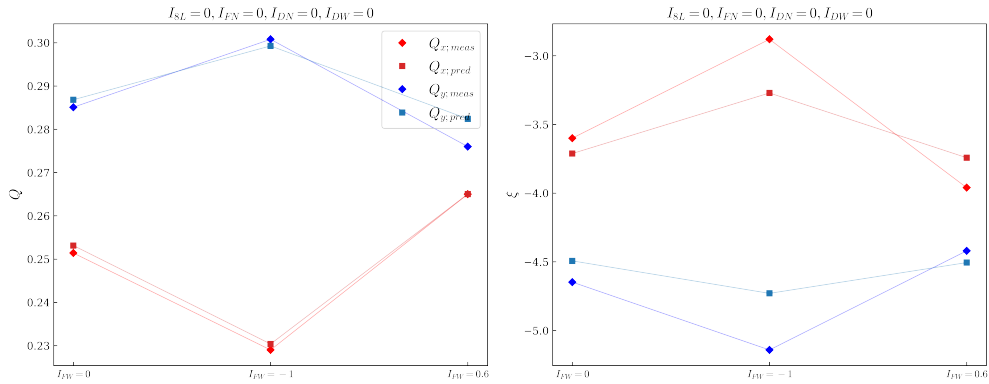


Figure 3.11: Comparison of the network predictions and the new training data.



(a) Comparison of the network predictions and new beam-based measurements for 3 PFW settings that are not used in the training of the network. The chromaticity analysis of the beam-based measurements were done using the revolution frequency.



(b) The same comparison as the figure above but the chromaticity of the new measurements are performed using the mean radial position as the figure-of-merit.

Figure 3.12: First results from the neural network based on the second measurement campaign.

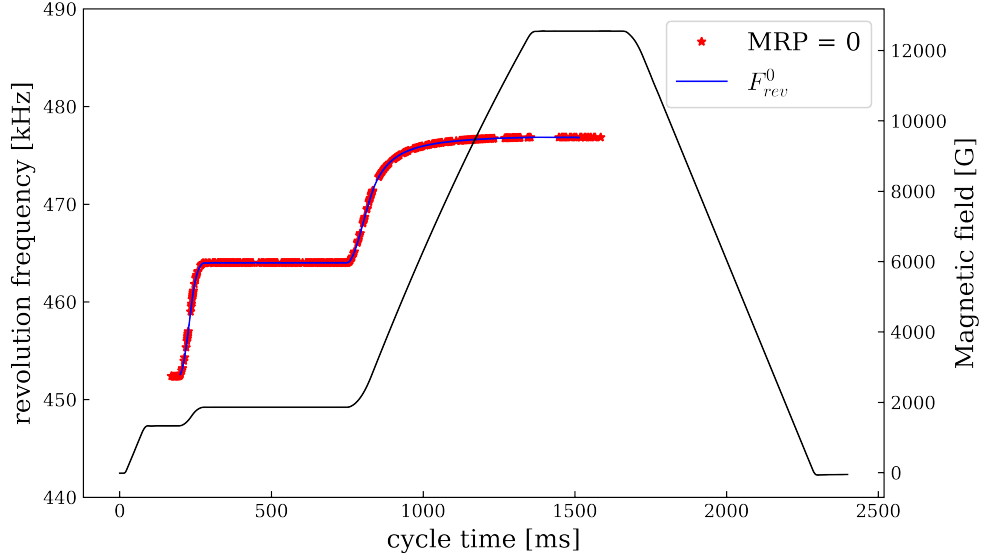


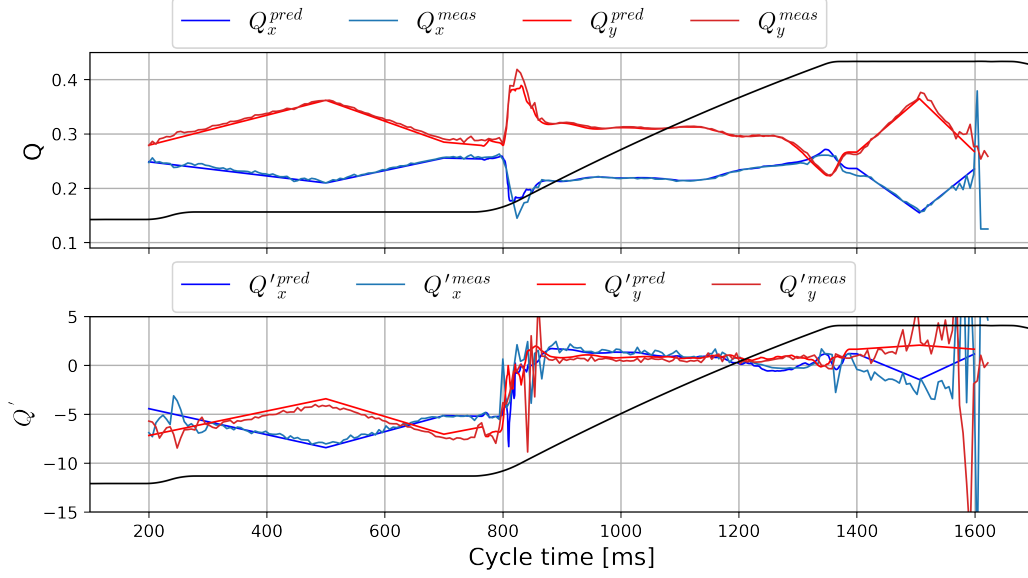
Figure 3.13: The nominal revolution frequency dependant on the B-field based on the datapoints where the MRP is equal to zero.

3.2.2 Neural network results using the revolution frequency

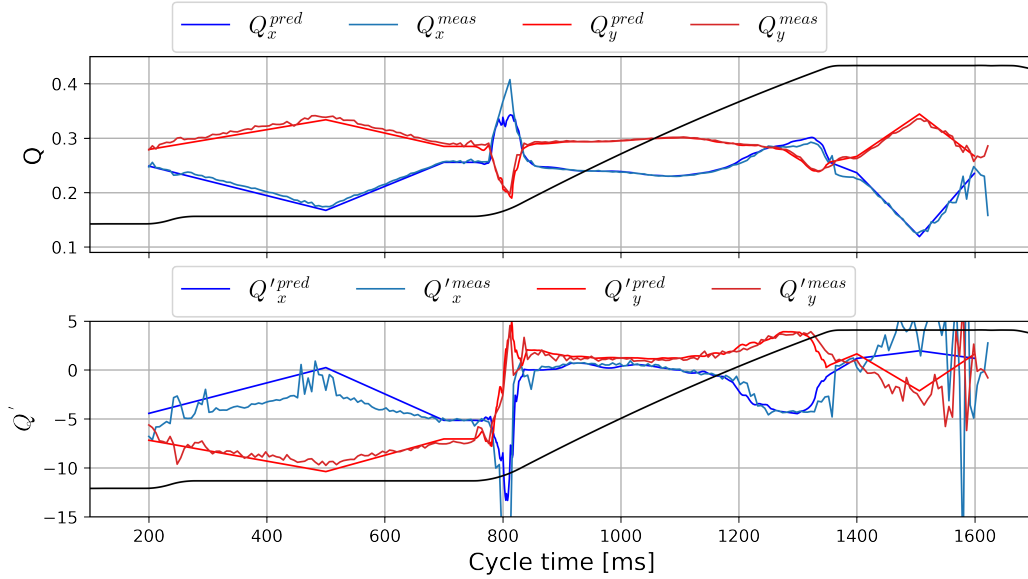
Since using the revolution frequency to calculate $\frac{dp}{p}$ has no scaling issue, the two methods will be combined to define a nominal revolution frequency where the MRP is equal to zero. During the measurement campaign the revolution frequency was also recorded and the training data of the network includes cases where the MRP is approximately zero at different B-fields. These cases are used to create a polynomial fit, as shown in Fig. 3.13. This approach leverages the accuracy of the revolution frequency method while ensuring consistency with the MRP-based method, effectively combining the strengths of both techniques to improve the overall accuracy of $\frac{dp}{p}$ calculations.

After retraining the network with the updated $\frac{dp}{p}$ values, the spread between the predicted and measured tunes remains nearly identical to that of the previous network. However, a significant improvement is seen in the chromaticity discrepancy present in the earlier model. To highlight the enhanced performance of the new network, instead of showcasing only a few PFW configurations at low energy, entire cycles are depicted in Fig. 3.14, where one of the PFW circuits is varied within one shot. The predicted tunes and chromaticities show good alignment with the measured values.

The figure also reveals substantial noise in the chromaticity measurements, even when more than 15 points are used for analyzing a single PFW setting. This highlights the inherent challenges of performing chromaticity measurements throughout the entire cycle. By using the physics-based loss function this challenge is mitigated since this analysis is no longer required, and the outputs of the prediction network are taken to be the tune and chromaticity of the machine for the next steps of this study.



(a) Neural network predictions versus measured tunes and chromaticities for varying I_{DW} .



(b) Neural network predictions versus measured tunes and chromaticities for varying I_{FW} .

Figure 3.14: Final neural network predictions along the whole cycle for varying PFW settings during the 3 stages of a shot: low energy, ramp and top energy. Only two circuits (DW & FW) are shown but the other tested settings performed comparatively.

3.3 PFW controller

With the successful completion and validation of the PFW prediction network, the next step is to develop a PFW controller network. This controller network is tasked with determining the optimal PFW settings to move to target tunes and chromaticities. To accomplish this, the output tunes and chromaticities of the prediction network are used as the training data for the controller network, effectively inverting the prediction network as such:

$$F^{control} \begin{pmatrix} B \\ Q_x \\ Q_y \\ Q'_x \\ Q'_y \end{pmatrix} = \begin{pmatrix} I_{DN} \\ I_{FN} \\ I_{DW} \\ I_{FW} \\ I_{8L} \end{pmatrix} \quad (3.7)$$

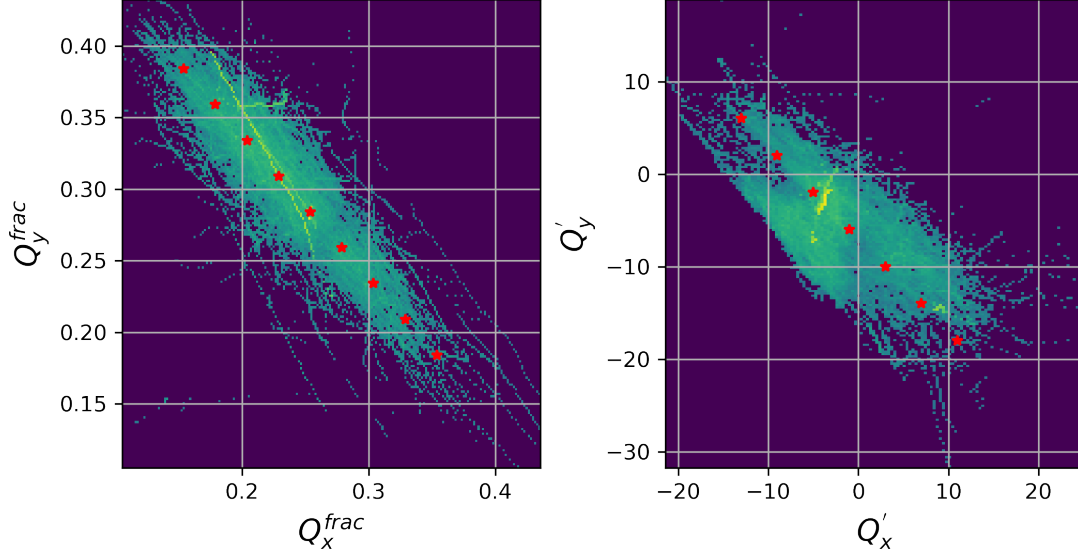
The initial iterations of the controller network are trained using data at a constant magnetic field, focusing on the areas of the PS magnetic cycle where the most comprehensive training data was available: the low energy plateau and the high energy plateau. By concentrating on these regions of the magnetic cycle, the performance of the controller network can be assessed and compared to the performance of the PFW response matrices that are currently being used to control the PFWs, allowing to confirm the network's applicability in these specialized use-cases before extending the training to the entire magnetic cycle.

3.3.1 Controller network at constant magnetic field

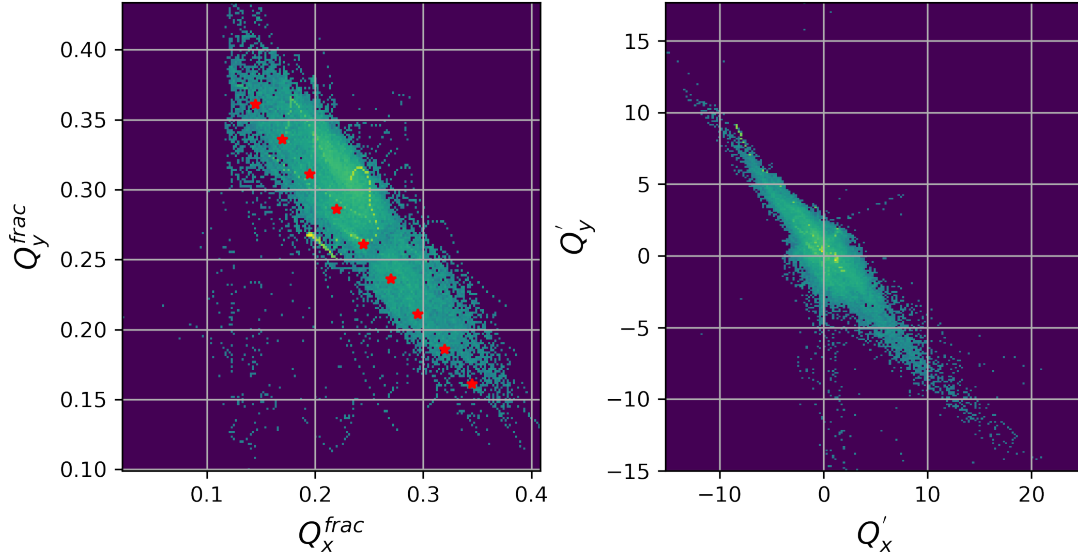
For the control network, the architecture mirrors that of the prediction network, with one key modification: the loss function reverts to using Mean Squared Error (MSE) instead of the physics-based loss function employed previously. With the tunes and chromaticities fully separated by the prediction network, the control network also has no need to have $\frac{dp}{p}$ as an input or label.

With this approach, the first control neural networks are constructed. An important aspect of developing the control network is to ensure its accuracy within the tune and chromaticity ranges for which training data is available. Neural networks are known to perform poorly when required to extrapolate beyond the data they have been trained on, so the control network performance can only be validated within the bounds of the existing data. These bounds are depicted in Fig. 3.15, where all the tunes and chromaticities of the training data is displayed as green dots and the selected targets to test the control network against the currently employed matrix response scheme are also highlighted in red.

The test results for the low energy network are presented in Fig. 3.16. In both tune and chromaticity control test cases, the neural network demonstrates performance that



(a) Depiction of the low energy tune and chromaticity training data used for training the network in green and the selected test targets in red.



(b) Depiction of the high energy tune and chromaticity training data used for training the network in green and the selected test targets in red.

Figure 3.15: The tune and chromaticity training data of the PFW control network at constant magnetic fields. The control network should work within the ranges of this training data. In red, the targets for the test of the control networks are highlighted.

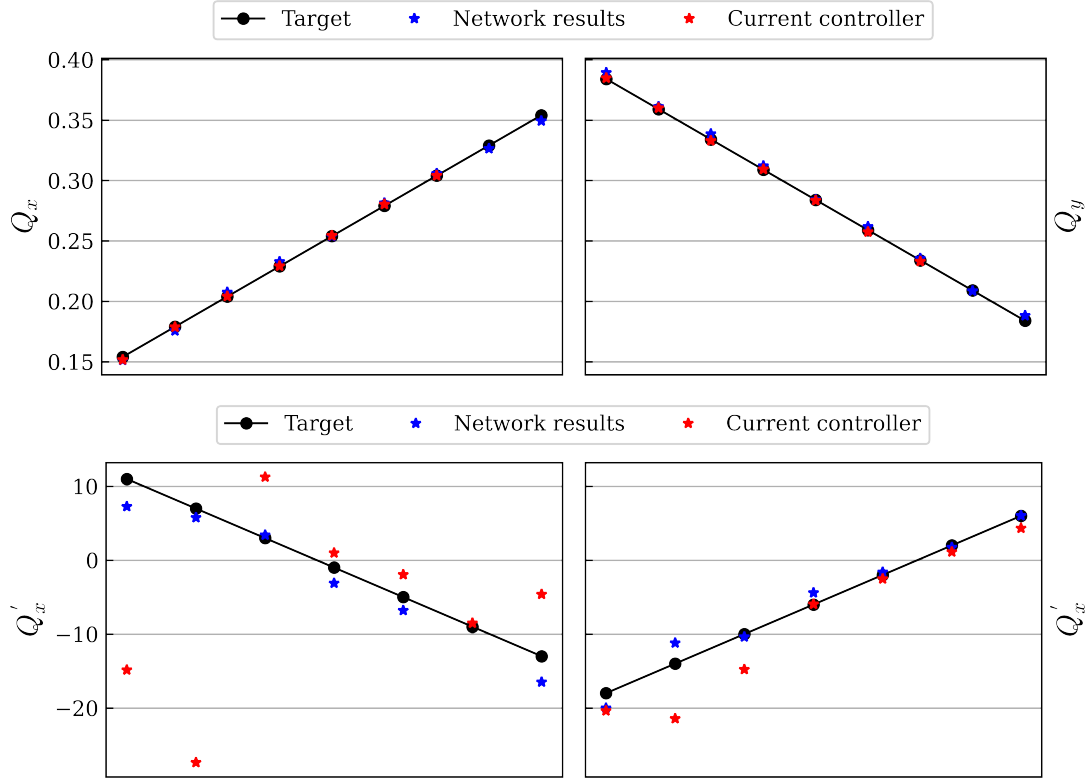


Figure 3.16: *Tunes and chromaticity controlled by the PFW control neural network at low energy in Blue. Additionally, the results from the PFW response matrices currently used in operations (current controller) are shown in red for comparison.*

is equal to, if not better than, the current PFW control system's response matrices at low energy. When examining the high energy network results in Fig. 3.17, it is evident that the network even outperforms the existing PFW controller for the target tunes. These measurements prove the feasibility of developing a PFW settings generator and controller using neural networks.

The next step involves training a network to operate effectively across the entire magnetic cycle. However, it was discovered that the PFW currents at high energy are much larger than those at low energy, causing the network to focus predominantly on the high energy portion during training. To address this, normalization of the PFW currents across the full magnetic field is necessary. The initial normalization attempts using beam rigidity were unsuccessful as the PFW currents at high energy remain disproportionately large. This discrepancy is likely due to saturation of the PFWs in this region, indicating that a more refined normalization approach is required to achieve balanced training across the entire cycle.

3.3.2 B-field dependant controller network

The data used to train both the prediction and control networks was obtained by varying the PFW currents as much as possible around the baseline settings without losing the

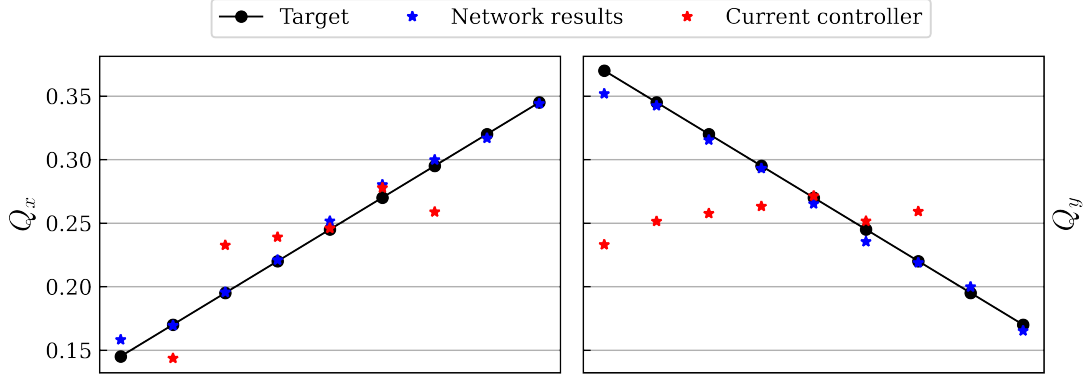


Figure 3.17: Tunes and chromaticity controlled by the PFW control neural network at high energy in Blue. Additionally, the results from the PFW response matrices currently used in operations (current controller) are shown in red for comparison.

beam. To improve the network's performance across the full magnetic field range, a new normalization method was developed based on these baseline functions, $I_{PFW}^{basic}(B)$:

$$I_{PFW}^*(B) = A_{PFW} \frac{I_{PFW}(B) - I_{PFW}^{basic}(B)}{I_{PFW}^{basic}(B) + \varepsilon_{PFW}}. \quad (3.8)$$

In this equation, A_{PFW} and ε_{PFW} are additional factors unique to each circuit, selected to ensure that the variance $\sigma(I_{PFW}^*(B))$ at each magnetic field value is comparable. This approach leads to equal weighting during network training across the entire magnetic field spectrum. The term ε_{PFW} is particularly important, as it prevents $I_{PFW}^*(B)$ from approaching infinity when the baseline function approaches zero, thereby stabilizing the normalization process.

With this normalization in place, a neural network was developed capable of predicting the PFW currents needed to achieve a given tune and chromaticity across the entire magnetic cycle. A performance test of this network is shown in Fig. 3.18. The results indicate that while the network successfully generates accurate settings at both low and high energy, noticeable deviations occur during the beam's ramping phase. These oscillations around the set tunes closely resemble those observed when using the PFW basic functions, suggesting that they are artifacts from the training data collection process.

To optimize this network and make it fully operational, additional training data must be gathered across various operational cycles. Furthermore, an improved data acquisition technique for the ramp region is essential to address these tune oscillations and enhance the network's overall performance.

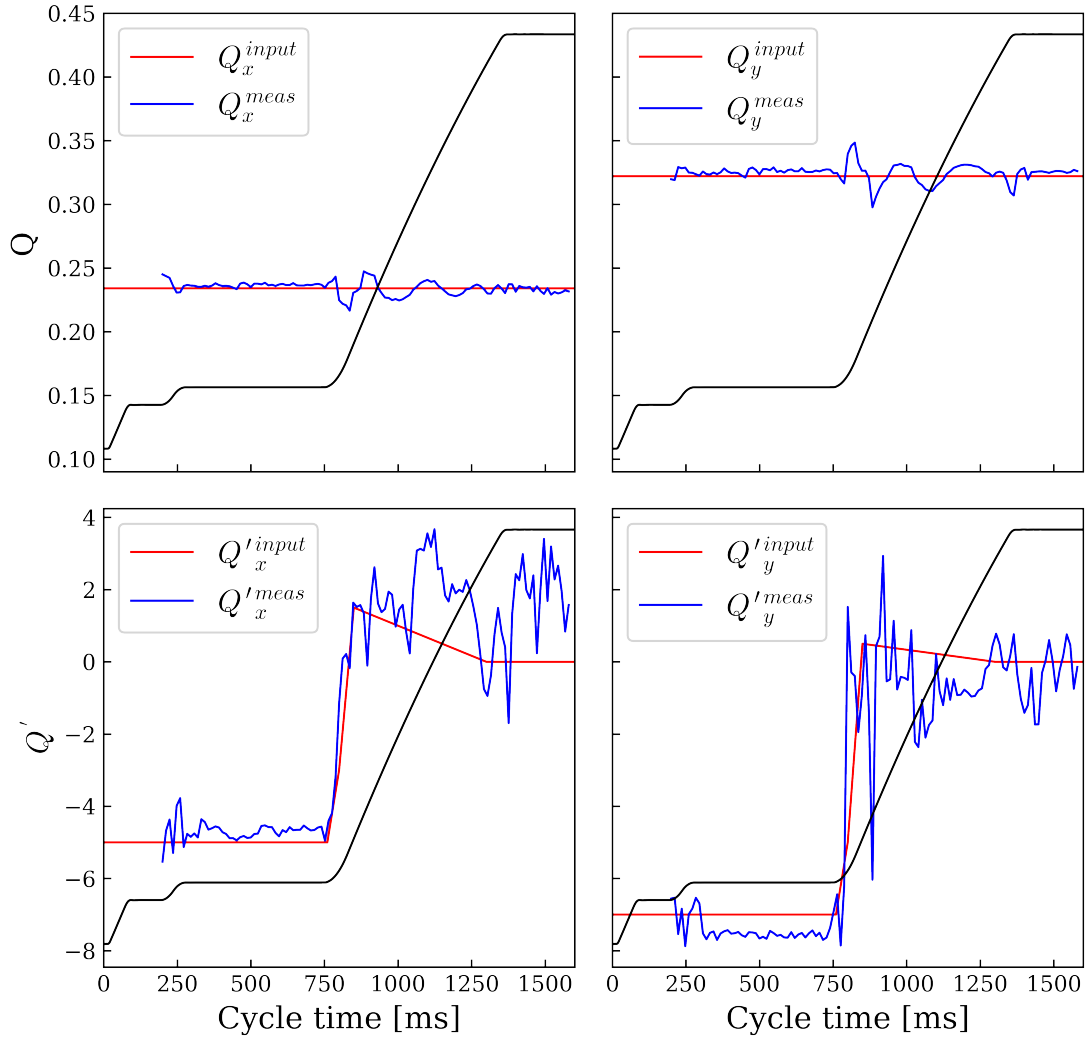


Figure 3.18: The tunes and chromaticities along the cycle where the targets are shown in red (Q^{input} & Q'^{input}) and the blue data shows the measured data (Q^{meas} & Q'^{meas}) where the PFW settings were generated with the control network.

3.4 Summary

In this chapter, the application of machine learning to address a longstanding challenge in accelerator operations is explored: the precise and effective manipulation of the PFWs to generate settings and control tune and chromaticity in the PS. Initial attempts to use BBQ tune measurements directly in a conventional neural network were unsuccessful, prompting several key improvements to the process.

First, data acquisition and cleaning methods were adapted to better align with the requirements of ML tools, ensuring higher-quality inputs for training and testing. A physics-based loss function was introduced, which simplified the training and testing process by eliminating the need to pre-analyze chromaticity and expanding the operational range of the training data. Additionally, systematic errors in the prediction model were mitigated by defining a zero point for the revolution frequency using the mean radial position. Building on this foundation, a magnetic field-dependent control network was implemented by normalizing the PFW currents with their basic functions. This allowed for control across a range of operating conditions, including otherwise problematic areas such as above and below transition energy.

While the PS served as a test case for these developments, the lessons learned and methods introduced have broader implications for accelerator physics. The success of this ML-based approach highlights its advantages over traditional correction methods, particularly in addressing the highly complex nature of circuits that induce a variety of multipole responses and therefore do not have a simple linear response in tune and chromaticity.

ZERO DISPERSION OPTICS

Accurately measuring emittance becomes challenging in accelerators where dispersion is present [36–41], as the standard equation for emittance calculation (Eq. 2.17) may not fully account for dispersive effects as shown in chapter 2. This is particularly problematic in accelerators like the PSB and PS within the LHC injector chain, where the horizontal dispersion is larger than zero everywhere in the ring, complicating studies of horizontal emittance blow-up. Given that preventing emittance blow-up and having detailed measurement of the beam shapes allows for the maximization of LHC luminosity, addressing this issue is important [9, 69–71].

The PS offers a very flexible lattice for developing methods to counteract the effects of dispersion in emittance calculations, thanks to the presence of numerous Low Energy Quadrupoles (LeQs) arranged quasi-symmetrically around the ring, which provide better control over the linear optics. Additionally, recent improvements in turn-by-turn data acquisition have equipped the PS with the tools needed for high-precision optics measurements [29].

In this study, an approach is explored to reduce dispersion at the locations of beam profile measurements, while maintaining close control over the other optics. An initial investigation, carried out by the author [72, 73], demonstrated that zero dispersion can be achieved at these measurement points using MAD-X models of the PS. Building on this, a more generalized approach to minimizing dispersion is formulated and tested directly in the machine.

Through these zero dispersion measurements, the evolution of emittance and beam tail populations over the PS magnetic cycle can be assessed more accurately. Additionally, the potential applications for space charge and a numerical deconvolution technique will be explored with the possibility of expanding these studies to other accelerators.

4.1 Optics design

The development of zero dispersion optics relies on the ability to shift dispersion through adjustments in quadrupole strength. With 40 individually powered quadrupoles installed in the PS ring, it is theoretically possible to fine-tune these strengths to force the

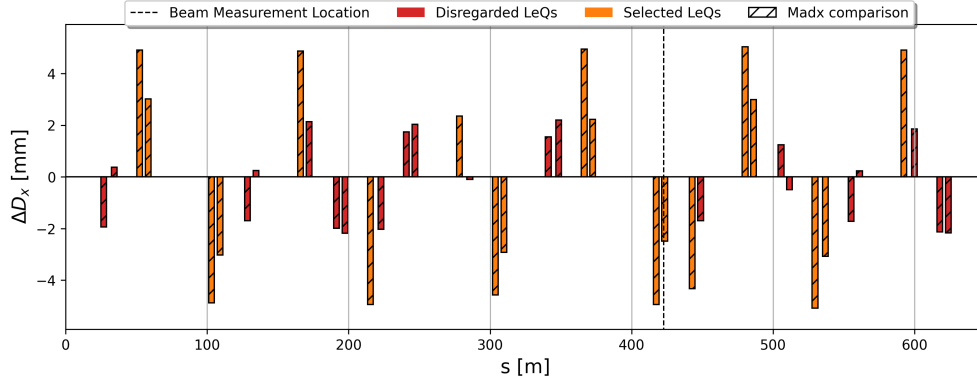


Figure 4.1: The resulting dispersion shifts ΔD_x at a beam measurement location for a consistent quadrupolar strength change Δk_1 across all 40 LeQs in the PS. The 15 LeQs that produce the largest shifts are selected for creating zero dispersion optics at this beam measurement location.

dispersion to zero at low energies. However, such adjustments would cause significant deviations of other optical parameters, such as the β -function, from their nominal values. From studies of linear imperfections, it is well understood that changes in quadrupole strength have a directly proportional effect on the β -function. However, the impact of quadrupole adjustments on dispersion has not yet been fully quantified. This relationship is derived in Appendix A, where it is shown that dispersion shifts are also directly proportional to changes in quadrupole strength. This simplifies the problem by allowing us to focus on the LeQs that have the greatest effect on dispersion, thereby minimizing the overall quadrupolar adjustments needed to transition between normal and zero dispersion optics.

Figure 4.1 illustrates the changes in dispersion at WSH68 resulting from a 0.01 increase in quadrupole strength across different LeQ locations. Both the dispersion-beating equation (Eq. A.26) and MAD-X simulations are represented, highlighting the 15 LeQs that exert the most influence on the dispersion at WSH68.

By using these 15 LeQs and their corresponding dispersion shifts at the horizontal beam profile measurement location, the precise quadrupole strengths needed to achieve zero dispersion at this point can be determined. The resulting optics, with all other magnets at their nominal settings, are termed "zero dispersion optics" for this location. This method can be applied to each horizontal beam measurement point around the PS ring. The optical functions for both the nominal and zero dispersion settings at WSH68 at low energies are simulated in MAD-X and shown in Fig. 4.2.

It is important to note that the LeQs used for zero dispersion optics are only effective at injection energies, as their quadrupolar strength is insufficient for use at extraction energies and special considerations need to be made to achieve zero dispersion optics for energies in between, these considerations will be specifically explained later in this chapter. Fortunately, as illustrated in Fig. 2.12a, the $\frac{\Delta p_{RMS}}{p}$ decreases significantly at

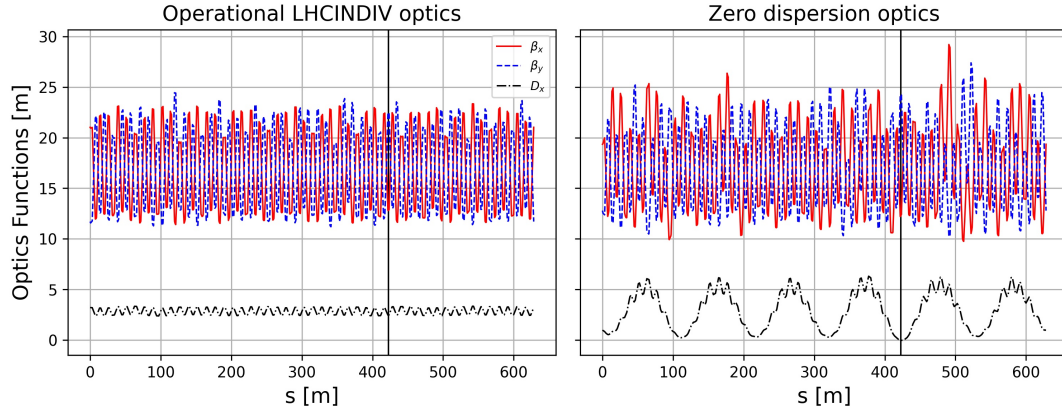


Figure 4.2: The β -functions and dispersion function for an operational PS beam and at zero dispersion optics from single-particle MAD-X simulations. 15 LeQs are used to go to zero dispersion optics creating a dispersive wave over the whole ring that reaches exactly zero at the beam measurement location WSH68, indicated by the vertical black line.

higher energies in the PS, diminishing the dispersive effect on emittance calculations. Given the reduced influence of dispersion at these higher energies, emittance measurements at this stage are considered accurate. This also presents an excellent opportunity to compare emittances measured with zero dispersion optics at low energies to those measured at high energies.

4.2 Optics implementation

The zero dispersion optics are now defined for each of the horizontal WS locations and for the BGI, and to achieve better emittance measurements it is important to properly monitor the value of these optics in the machine. Additionally, since the beam must initially be injected with nominal optics, a method for smoothly transitioning between nominal and zero dispersion optics is necessary. To facilitate this, a "zero dispersion knob" is defined in machine operations. This knob represents the gradual adjustment of quadrupole strengths from the nominal optics to the zero dispersion optics. A knob value of zero corresponds to nominal optics, while a value of one represents the fully implemented zero dispersion optics. In the following section, detailed measurements of the optical functions are taken at various settings of the zero dispersion knob. For each measurement, the beam starts at nominal optics (knob = 0), and the knob is shifted to the desired value to achieve the corresponding optic configuration.

4.2.1 Dispersion measurements

Beam-based measurements of dispersion are typically conducted by varying the beam's momentum deviation, $\frac{\Delta p}{p}$, using Eq. 2.13, and measuring the resulting shift in beam position. In the PS, the theoretical vertical dispersion is zero, so only methods to measure the horizontal position are discussed. The horizontal position can be measured using the BPMs or directly at the wire scanners. The mean value of the transverse beam profile μ is obtained from the wire scanners, note that μ does not represent the phase advance in these results. This mean value of the beam profile will also shift according to the momentum deviation of the beam, as illustrated in Fig. 4.3.

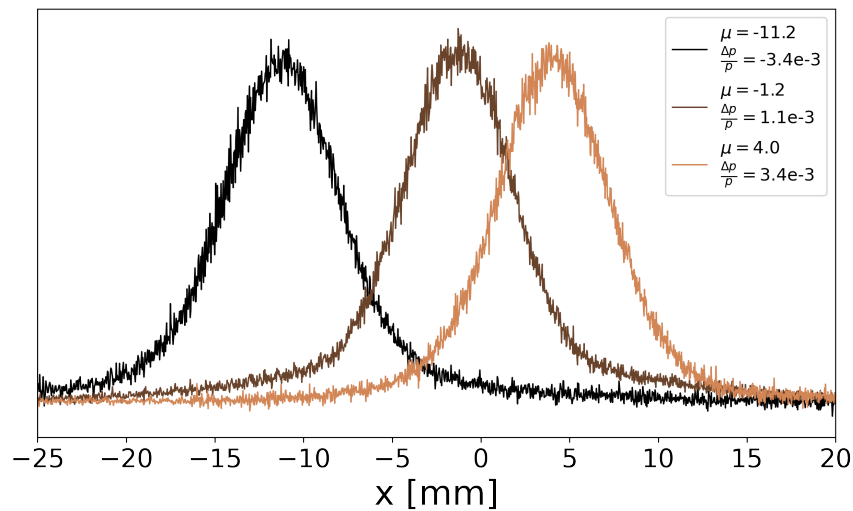


Figure 4.3: Three horizontal beam profile measurements using WSH54 at different $\frac{\Delta p}{p}$ shifting the mean value of the beam profile.

Given the relation $x = \frac{\Delta p}{p} D_x$, where x is the horizontal position and D_x is the horizontal dispersion, the dispersion can be measured using the wire scanners and these results are compared with measurements from the neighboring BPMs. This comparison helps validate the MAD-X model and determine which measurement tool provides the most accurate dispersion measurements.

The measured dispersions at all wire scanners are displayed in Fig. 4.4 for nominal optics. It is important that these measurement properly align with the MAD-X model since if the MAD-X model does not represent the optics of the machine, the zero dispersion optics developed through this model will be compromised. As shown in the figure, the wire scanner measurements consistently fall within the uncertainty range of the MAD-X model's predictions, confirming the reliability of the wire scanner data for this study. However, the BPM measurements deviate from the model. This discrepancy of the BPM measurements are consistent with the known calibration uncertainties inferred from other optics studies and will be further addressed in Chapter 5.

The vertical dispersion was also examined using the wire scanners, as any non-zero vertical dispersion could introduce uncertainty into the vertical emittance measurements. These results, shown in Fig. 4.5, reveal a small vertical dispersion at WS85, though it remains negligible compared to the horizontal dispersion.

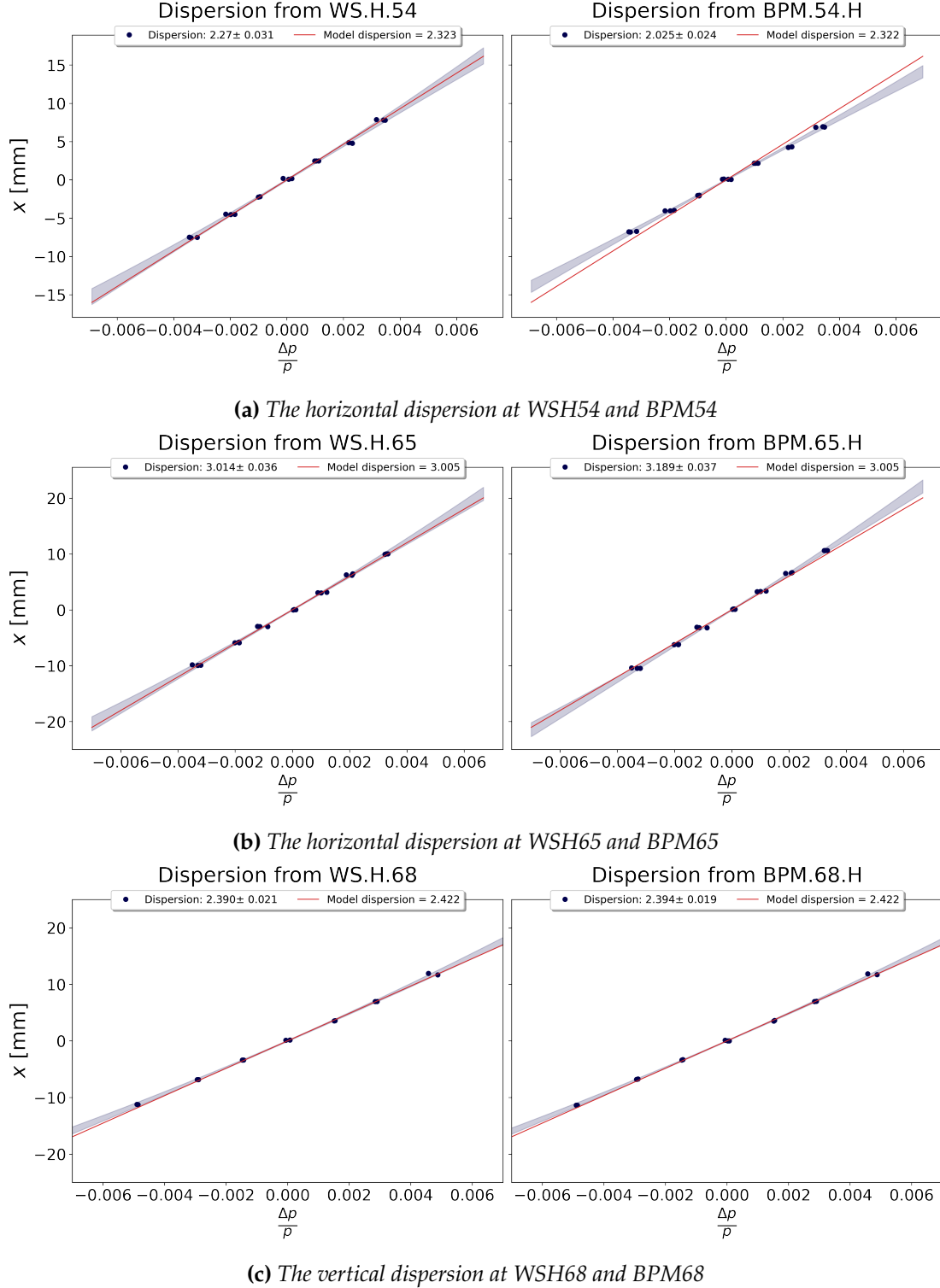


Figure 4.4: The measured dispersion of the horizontal beam profile measurement location and at the neighboring BPMs, compared to the MAD-X model dispersion at the same location.

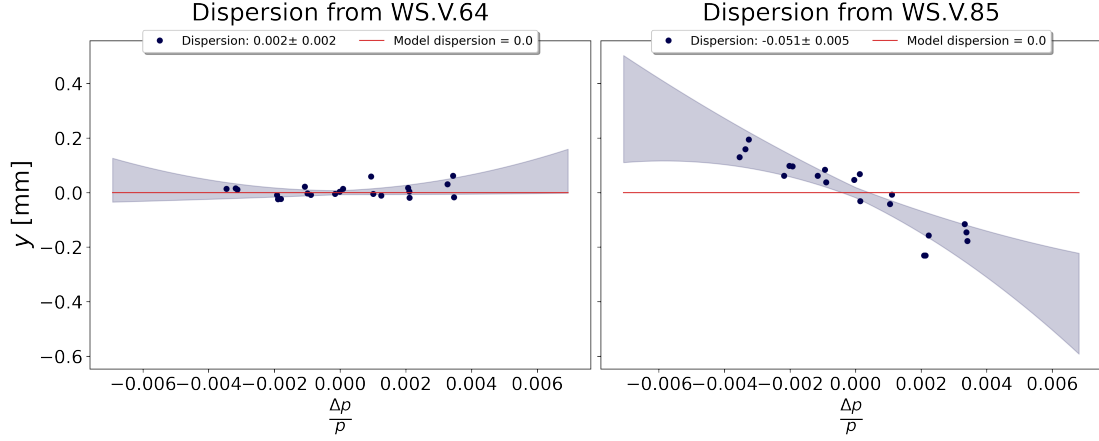


Figure 4.5: The measured dispersion of the vertical beam profile measurement locations WSV64 and WSV85.

4.2.2 Shifting between nominal optics and zero dispersion optics

As mentioned earlier, the linear optics of the machine will be carefully monitored during the transition from nominal to zero dispersion optics to understand their impact on a PS beam. Since the zero dispersion optics are defined using a single-particle model, these measurements are repeated with beams subject to varying space charge effects. This is achieved by injecting beams with different transverse emittances into the PS. Beams with smaller emittances experience stronger space charge effects, as the same number of particles is confined to a smaller area, while beams with larger emittances experience weaker space charge effects. The tested emittances were $\epsilon_x = \epsilon_y = 1 \times 10^{-6}$ [mm rad], $\epsilon_x = \epsilon_y = 1.5 \times 10^{-6}$ [mm rad] and $\epsilon_x = \epsilon_y = 2 \times 10^{-6}$ [mm rad]. For each of these test beams, the tomoscope was used to measure the longitudinal distribution and the RMS momentum deviation $\frac{\Delta p_{RMS}}{p}$. In all cases, the momentum deviation was approximately 1.01×10^{-3} , which is large enough to have a significant dispersive effect on the beam width.

Figure 4.6 shows the dispersion for different values of the zero dispersion knob. As the knob value increases, the dispersion gradually decreases, reaching zero around a knob setting of 1.3 for all test beams. Zero dispersion is not achieved at a knob value of 1 because the knob was originally defined for LHC optics, where the PFWs are active at low energies, whereas these measurements were conducted on LHCINDIV beams, where the PFWs are inactive.

This marks the first successful measurement of a transverse beam profile with zero horizontal dispersion in the PS. The effect of this dispersion decrease on the widths of the beam profiles is illustrated in Fig. 4.7, along with a summary of the dispersion progress. As the dispersion decreases, the beam widths also decrease until the dispersion drops below zero, at which point the beam width begins to increase again. To understand this behavior Eq. 2.17 can be used, adapted here for convenience:

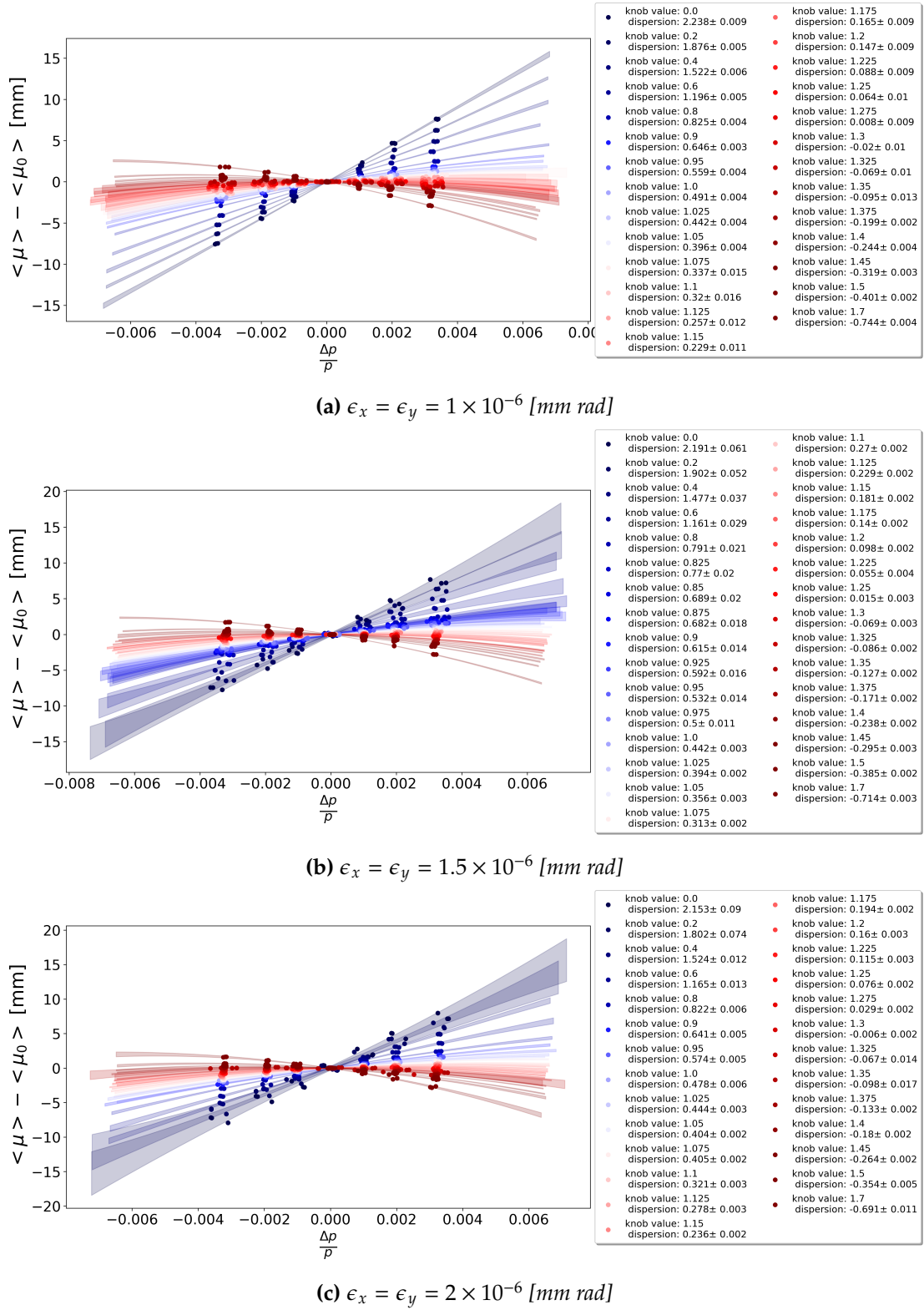


Figure 4.6: The measured dispersion for the three test beams at WSH68 while going to zero dispersion optics and beyond.

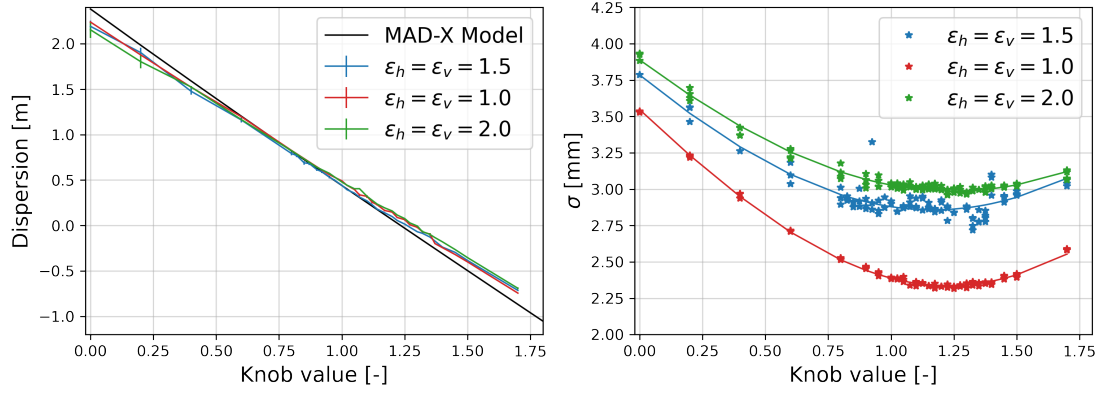


Figure 4.7: The summarized dispersion measurements from Fig. 4.6 compared to the MAD-X model dispersion on the left plot. The right plot shows the beam width σ measured in function of the zero dispersion knob.

$$\sigma_x = \sqrt{\epsilon_x \beta_x + \left(\frac{\Delta p_{RMS}}{p} \right)^2 D_x^2}. \quad (4.1)$$

To use this equation, and to identify any significant changes to the other linear optics aside from the dispersion, the β -function must also be measured while the knob is being shifted. This is done using turn-by-turn data measured at the adjacent BPM located 0.5 m from the wire scanner. The results are shown in Fig. 4.8. The measured β -function at the BPM closely matches the model β , within a 1% margin. Since the wire scanner and the beam position monitor are only separated by a small drift space, it is unlikely that a β -error is introduced in this small region. Therefore, it is assumed that the β -function at the WS is also within 1% of its model value.

The behavior of the beam widths over the different knob values in Fig. 4.7 can now be explained, considering that the horizontal emittance and RMS momentum deviation remain constant, the β -function changes with only 7-8% from its nominal value, and the dispersion value crosses zero where the beam widths reach a minimum value. The measured and MAD-X model dispersions are similarly close to each other, indicating that the linear optics change is within the expected ranges when switching between normal optics and zero dispersion optics. The dispersion measurements show minimal distinction between the different emittance settings, confirming that the linear optics across various configurations are stable.

Although not shown here, the other wire scanners were also able to perform beam profile measurements at their respective zero dispersion optics. Also, the zero dispersion optics at BGI82 led to a significant reduction in beam width (Fig. 4.9) but due to the ongoing commissioning of the BGI, no further measurements were done on this instrument. The measurements at the BGI closely mirror the behavior of the beam profiles measured at

the WS between the two settings, as shown in Fig. 4.10 where the beam profile with zero dispersion optics appears narrower compared to the beam profile with normal optics. Overall, this section demonstrates the successful achievement of dispersion-free beam width measurements in the PS. In the next section, the resulting changes in emittance measurements will be presented and compared to emittance measurements taken at other points in the injector chain.

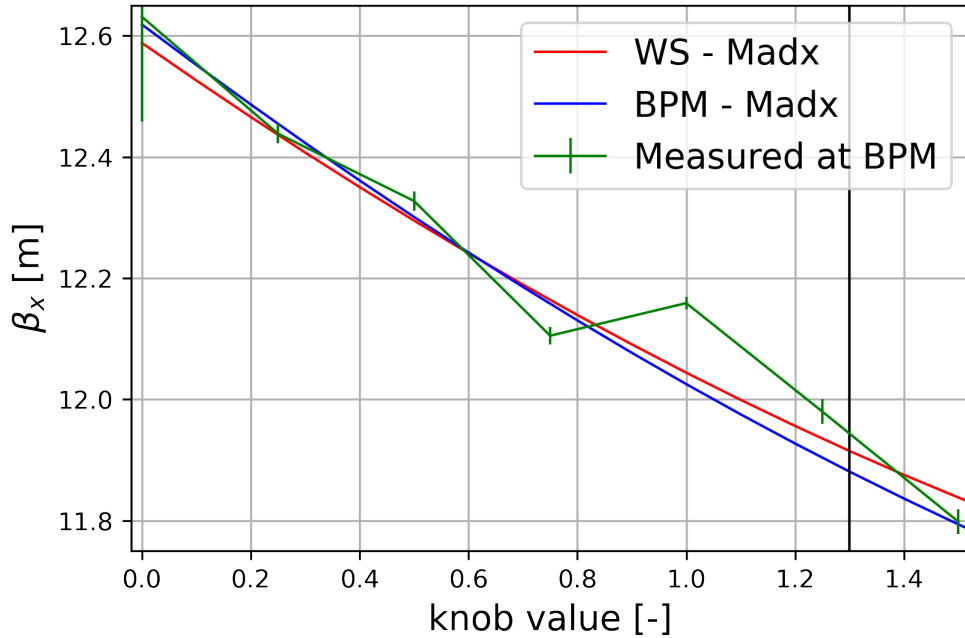


Figure 4.8: The horizontal β -function measured at the neighboring BPM to WSH68 in green at different knob values. The MAD-X model β -functions at the BPM and WS are shown in blue and red respectively.

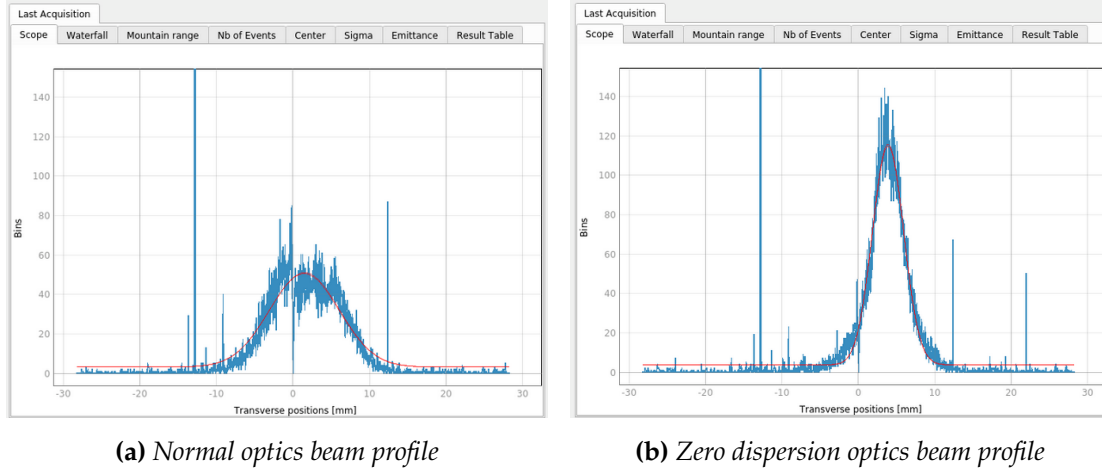


Figure 4.9: The beam profiles measured at BGI82 using normal optics and zero dispersion optics, showing a beam width reduction using zero dispersion optics.

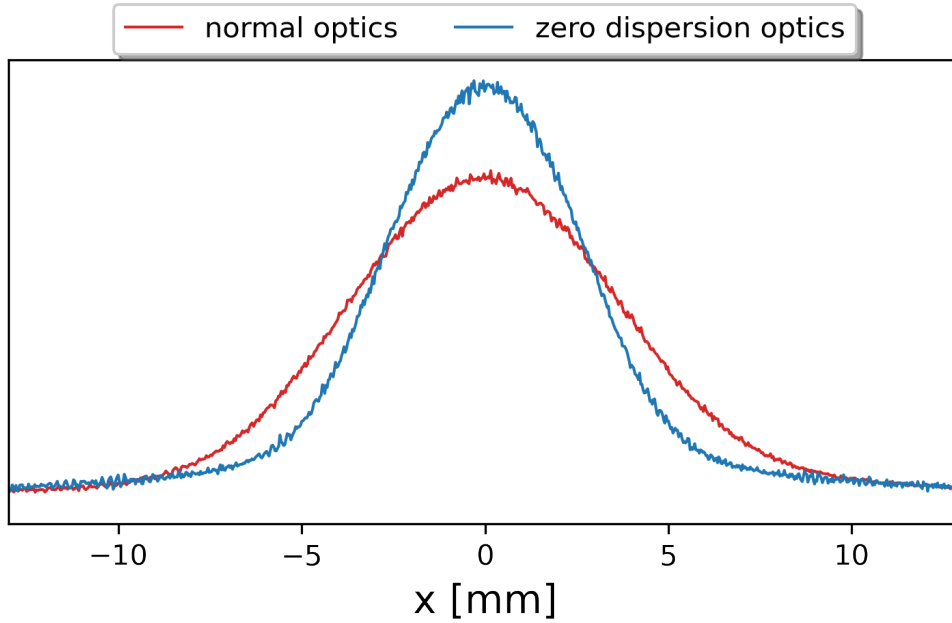


Figure 4.10: The beam profiles measured in one shot for normal optics and zero dispersion optics using a wire scanner.

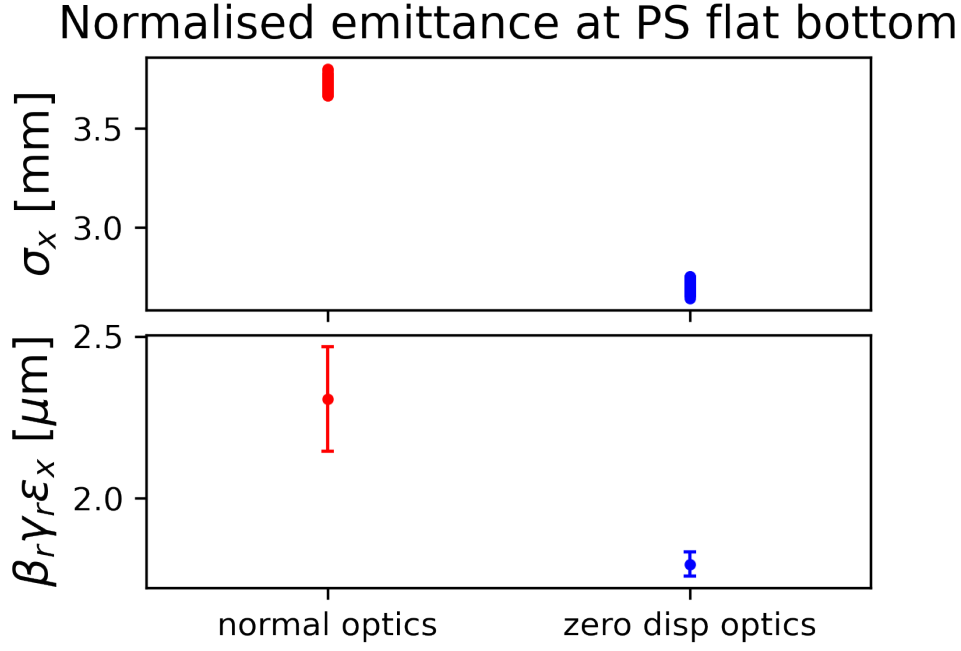


Figure 4.11: The measured beam widths and resulting emittance estimate at PS bottom energy with 200e10 particles per bunch (ppb) using the normal optics and zero dispersion optics over a hundred shots of an LHCINDIV cycle.

4.3 Effects on the emittance measurement

The implementation of zero dispersion optics has led to a noticeable reduction in beam width compared to nominal optics. By measuring at zero dispersion, the horizontal beam distribution is naturally deconvoluted from the longitudinal beam distribution. Consequently, the difference of the calculated emittance is studied between the WS measurements at nominal optics, where the horizontal beam distribution is convoluted but the emittance is analytically deconvoluted according to Eq. 4.1, and the WS measurements at zero dispersion optics. If the two emittance measurements do not align, this would confirm that Eq. 4.1 is not valid, as expected from Chapter 2 (Fig. 2.12).

To measure the difference in emittance of these two settings, beam profile measurements are repeated across 100 injection-extraction cycles to account for the shot-to-shot emittance variations in the PS. First, the emittance was measured using the normal optics; then, the optics were ramped to zero dispersion for a subsequent emittance measurement on the same cycle. This ensures that the same 100 shots are used for both emittance estimates. The results of this process are shown in Fig. 4.11. As expected, the beam widths at zero dispersion optics are reduced compared to those at nominal optics, but more importantly, the calculated emittances are significantly different between the two. This further proves that deconvolution of horizontal and longitudinal distributions is not fully captured by Eq. 4.1 but the use of zero dispersion optics does indeed achieve this de-

convolution. Additionally, by eliminating the longitudinal contribution to the emittance measurement, the uncertainty on the emittance evolution has been significantly reduced.

One potential concern when switching between nominal and zero dispersion optics is the possibility of emittance blow-up. To investigate this, the emittance of an LHC-type beam is measured with nominal optics before and after applying zero dispersion optics. Although horizontal emittance measurements at low energy are generally less reliable, any significant emittance increase should still be detectable. The measured horizontal emittances were $5.4 \pm 0.4 \mu\text{m}$ before using zero dispersion optics and $5.4 \pm 0.4 \mu\text{m}$ after, showing no emittance blow-up. The vertical emittances showed similar values, providing no evidence of an emittance increase due to the implementation of zero dispersion optics.

While the reduced emittance at zero dispersion optics is a promising result, the ultimate confirmation of whether these optics truly allow for accurate low-energy emittance measurements in the PS would be to compare the horizontal emittance at zero dispersion with the emittance measured at high energies, where the momentum deviation $\frac{\Delta p}{p}$ is minimal.

4.3.1 Emittance progression between PSB and PS top energies

To compare the emittances between low and high energy in the PS, operational LHC-type beams are used because accurate knowledge of the emittance progression along the injector chain has long been a concern for the LHC luminosity. For additional comparison, the emittance at the PSB's top energy was also measured, given that the PSB-PS transfer has been suspected as a potential source of emittance blow-up.

The results of these measurements are presented in Fig. 4.12. Emittances between zero dispersion measurements at low energy and top energy show a small increase but are otherwise very consistent across the three different measured intensities. This result gives strong confidence that the zero dispersion optics allow for WS measurements of the deconvoluted horizontal beam distribution, which leads to accurate horizontal emittance measurements at bottom energy of the PS. The slight emittance increase between bottom and top energies could be due to the small dispersive contribution still present at top energies ($\frac{\Delta p_{RMS}}{p} = 0.245 \times 10^{-3}$) or due to emittance blow-up during transition crossing.

As the benefit of the zero dispersion optics has been proven at low energy in the PS, it will be interesting to explore what can be learned with this new tool, starting with the emittances of the PSB. To compare the emittances of the PSB to those in the PS, it is important to note that the standard dispersion in the PSB is $1.469 \pm 0.008 \text{ m}$ with $\frac{\Delta p_{rms}}{p} = 1.53 \times 10^{-3}$, while the dispersion in the PS at bottom energy is $2.249 \pm 0.028 \text{ m}$ with the same $\frac{\Delta p_{rms}}{p}$. This causes the dispersive contribution in the PSB to be lower

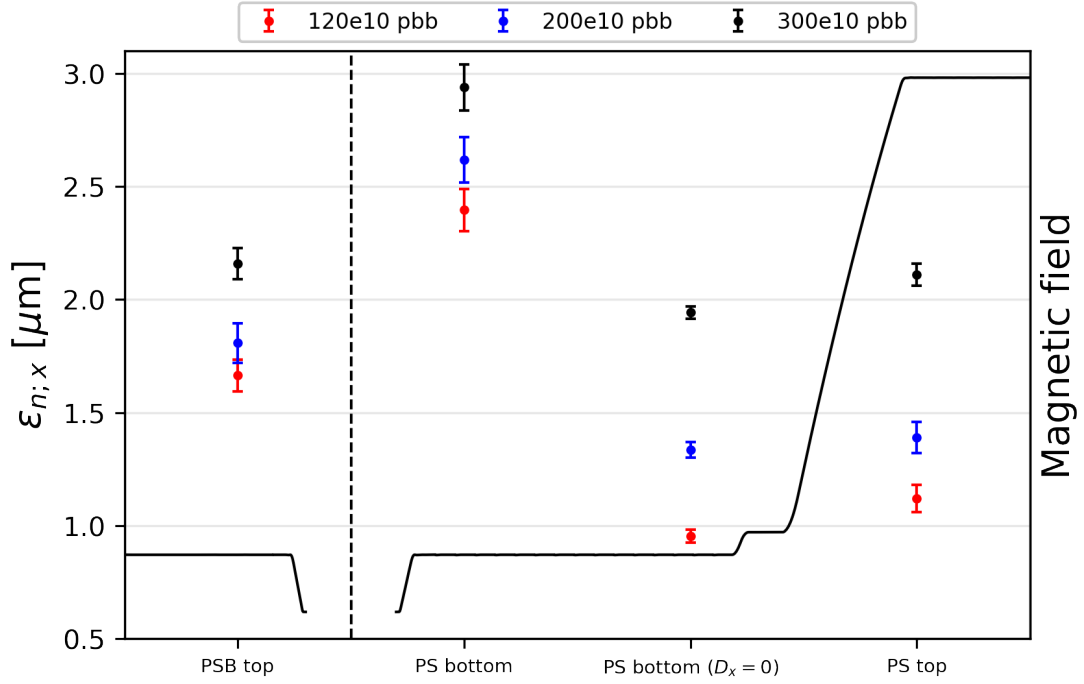


Figure 4.12: The measured emittances of an operation LHC-cycle between PSB and PS top energies with measurements of normal optics at PS bottom energy and with zero dispersion optics for three different beam intensities. The extraction of the PSB, injection in the PS and magnetic field of the two accelerators are highlighted in black for clarity.

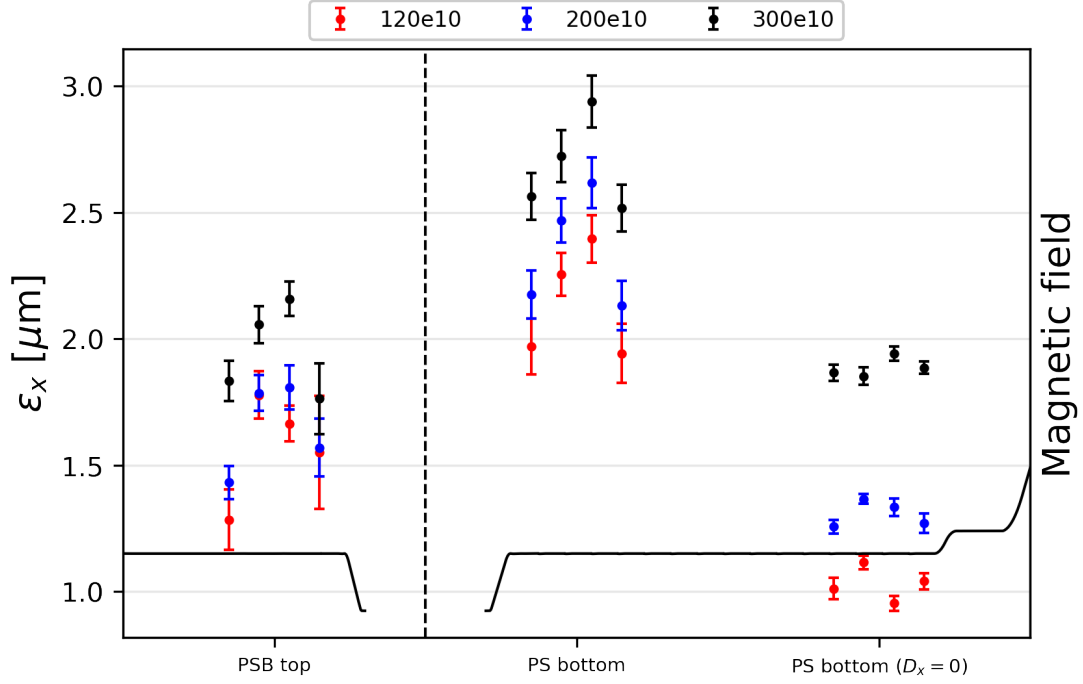


Figure 4.13: The measured emittances between the four PSB rings at PSB top energy and at PS bottom energy. The rings are plotted in quick succession from one to four. The extraction of the PSB, injection in the PS and magnetic field of the two accelerators are highlighted in black for clarity.

than in the PS. The measured emittance increase from the PSB to the PS, which was previously thought to be a sign of emittance blow-up, appears to be due to the difference in dispersive contribution, since the results from Fig. 4.12 suggest that the measured emittance increases depending on the dispersive contribution.

4.3.2 Emittance differences between the PSB rings

The measurements conducted so far have always been performed on a single bunch from PSB ring 3. This is standard practice for machine development measurements in the PS. However, the operational beams that go to the LHC typically have six bunches at PS flat bottom, injected from all four rings of the PSB. Although the four PSB rings are designed to produce beams with identical parameters, inconsistencies have been observed in practice, which in turn could lead to inconsistencies of the bunch-by-bunch luminosity of the LHC. Therefore, the improved zero dispersion measurements in the PS will be used to compare the transverse distribution of the PSB rings.

The resulting emittances are shown in Fig. 4.13. At PSB top energy and with normal optics at PS bottom energy, there are noticeable differences in emittance between the four PSB rings. The emittances of the outer rings, ring 1 and ring 4, are consistently lower than those of the inner rings, ring 2 and ring 3, for every intensity setting. This relationship between the inner and outer rings disappears when using zero dispersion optics at PS bottom energy.

Since the zero dispersion optics separate the horizontal from the longitudinal distribution, this suggests that the horizontal distributions from the four PSB rings are sufficiently similar, and the discrepancies observed under normal optics arise when the measured beam profile is convoluted with the longitudinal distribution. From this, it can be concluded that the observed differences between the PSB rings are not due to variations in the transverse distributions, but rather the result of the longitudinal distribution being convoluted with the transverse distribution due to the effects of dispersion.

4.3.3 Emittance blow-up due to transition crossing

Another issue that can be investigated using the zero dispersion optics is the potential for horizontal emittance blow-up during transition crossing, which had already been observed in the vertical plane [74] and represents a concern for LHC beam quality and luminosity. Transition crossing in the PS occurs around a magnetic field ranging between 2000 and 3000 Gauss, depending on the optics in the machine. This holds true when the zero dispersion optics are being used. To study transition crossing, a magnetic cycle is created that holds a constant magnetic field for a few hundred milliseconds at 1860 Gauss, just below transition, and at 4500 Gauss, just above transition. Due to the uncommon nature of this cycle and the necessity for a large $\frac{\Delta p_{rms}}{p}$ value, the RF gymnastics used for beams going to the LHC are also present in the following measurements. This means that the single bunch used in our measurements is split into three bunches on the 1860 Gauss plateau, as shown in Fig. 2.11. To ensure completeness, the emittance is measured before and after the triple splitting, as well as after transition on the 4500 Gauss plateau.

Another challenge with these measurements is the magnetic strength of the LeQs used to achieve zero dispersion. As the magnetic field increases, so does the beam rigidity, requiring the quadrupoles to exert more strength to achieve the same focusing effect. The zero dispersion knob defined for flat bottom energy ($\equiv 1332$ Gauss magnetic field) was tested just below transition with a higher knob value and zero dispersion was reached. However, above transition, new optics must be defined to achieve zero dispersion using the method described in this chapter, but with additional quadrupoles. The results of these measurements, performed with 200×10^{10} particles per bunch before triple splitting, can be found in Fig. 4.14.

Interestingly, the emittances at zero dispersion optics are consistent across the three measurement timings, showing only a slight decrease after triple splitting compared to before. This small decrease is likely due to a minor intensity loss (approximately 5%) when using zero dispersion optics during the splitting of the bunches and is not a concern. The fact that the horizontal emittance remains unchanged before and after transition crossing is intriguing, especially considering that similar studies on the vertical plane in the PS does exhibit emittance blow-up [74]. This finding could

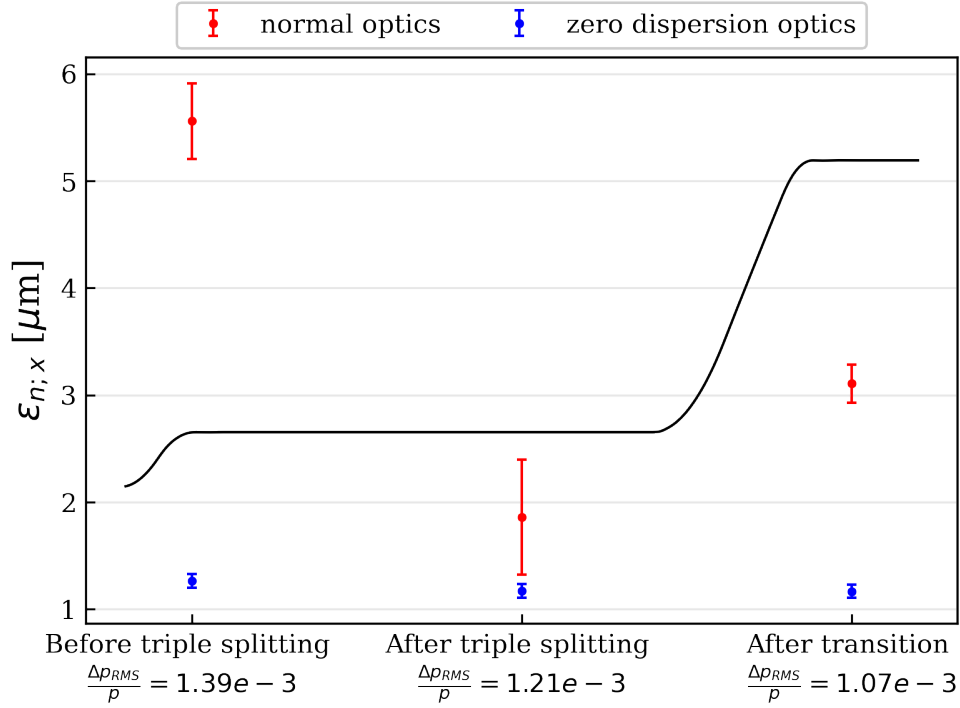


Figure 4.14: The measured emittances with an initial intensity of $200e10$ particles per bunch before and after transition crossing in the PS with the magnetic cycle highlighted in black for clarity.

help identify the cause of the vertical plane's emittance blow-up during transition, which would greatly benefit the LHC's luminosity. This finding also indicates that the emittance increase observed in Fig. 4.12 between zero dispersion flat bottom optics and flat top energy is due to the residual dispersive contribution at top energy, instead of an emittance blow-up.

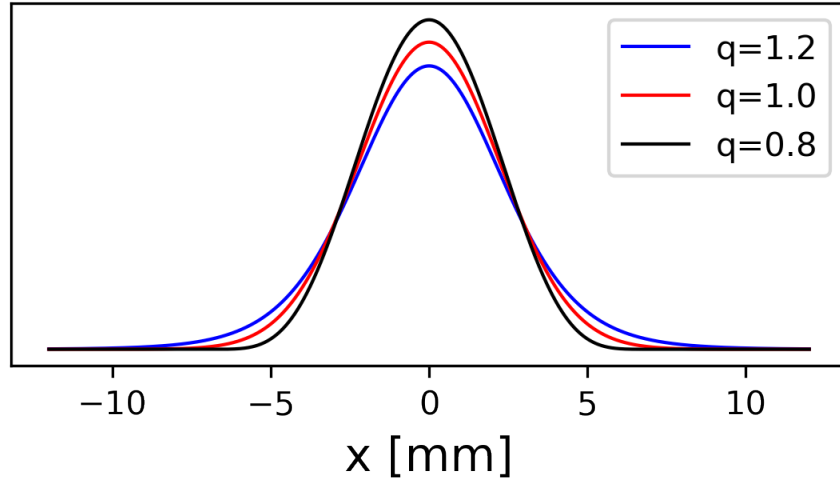


Figure 4.15: Illustration of the effect on the q -factor for q -Gaussian fits of WS beam profiles.

4.4 Effects on the beam tail population

The equation for luminosity (Eq. 2.14) assumes that the transverse beam distributions are Gaussian. However, beams provided to the LHC by the injector chain often deviate from a perfect Gaussian distribution, exhibiting increased populations in their tails [70]. The beam tail population has been a leading cause of lost luminosity in the LHC [75] and is therefore also studied in its injectors [76, 77]. Since the zero dispersion optics can deconvolute the horizontal beam distribution, they provide a valuable tool for studying how the horizontal beam tails behave under different optics settings and at various energy plateaus.

Beam profiles that follow a near-Gaussian shape can be characterized by the q -Gaussian function, where the q -parameter measures the weight of the distribution's tails. For $q < 1$, the tails are underpopulated; for $q = 1$, the distribution is Gaussian; and for $1 < q < 3$, the tails are overpopulated. The differences between these q -values are illustrated in Fig. 4.15. Using a q -Gaussian fit on the transverse beam profiles allows for easily distinguishable tail comparisons: the larger the q -factor, the larger the tail population.

4.4.1 Beam tail progression between PSB and PS top energies

Applying the q -Gaussian fit, the tails of the beam distribution can be studied along the PSB and PS. The same distributions used for Fig. 4.12 are analyzed to determine the corresponding q -factors, which are shown in Fig. 4.16.

At first glance, the tail evolution under normal optics suggests a significant increase in beam tails between bottom and top energy in the PS. However, when the zero dispersion optics measurements are included for comparison, it becomes clear that these large tails are already present at low energies. The tails at bottom energy are sometimes

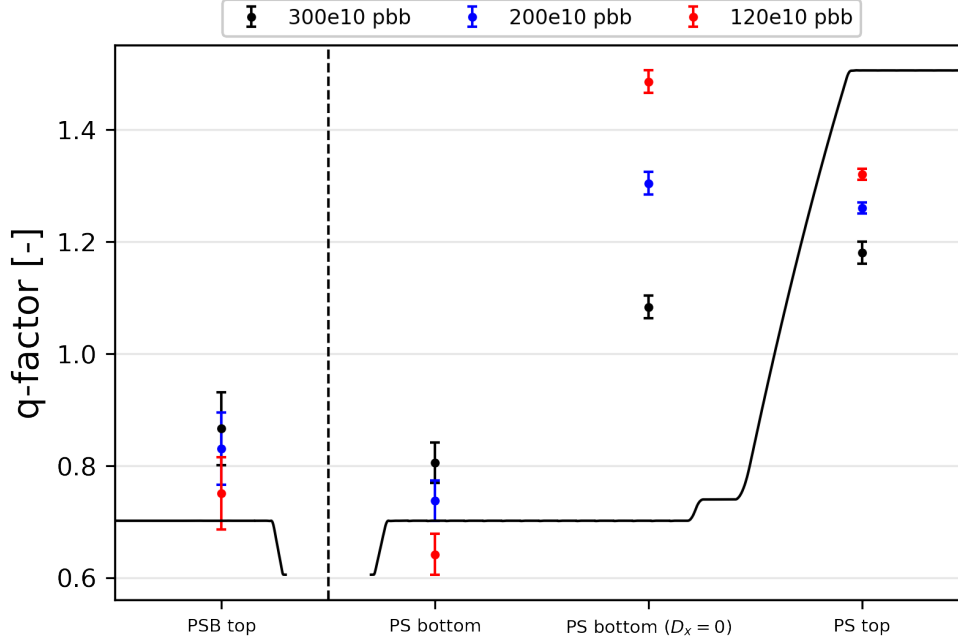


Figure 4.16: The measured q -factors of an operation LHC-cycle between PSB and PS top energies with measurements of normal optics at PS bottom energy and with zero dispersion optics for three different beam intensities. The extraction of the PSB, injection in the PS and magnetic field of the two accelerators are highlighted in black for clarity.

even larger than the tails at top energy, depending on the beam's intensity. This finding implies that the longitudinal convolution of the horizontal beam profile conceals the overpopulated tails. Another observation is that higher beam intensities result in distributions that appear more Gaussian, both with and without dispersive contributions.

To better understand the effects of dispersion on the beam tails, tracking simulations were performed. In these simulations, the transverse distribution of the beam was generated with a q -Gaussian distribution with $q = 1.25$ before transitioning to the zero dispersion optics used for the measurements. The results of these tracking studies, shown in Fig. 4.17, indicate that the q -factor measured on the beam profile with dispersion present shows underpopulated tails, even though the initial distribution was generated with overpopulated tails. Once the beam moves to zero dispersion optics, the initially generated tails reappear. This demonstrates that the zero dispersion optics reveal the true tails of the distribution. This further indicates that the tails, previously thought to result from transition crossing, are already present in the beam below transition.

To explore the effects of transition crossing on beam tails, a q -Gaussian fit is applied to the beam distributions and used for comparing emittance before and after transition from Fig. 4.14. The resulting q -factors, shown in Fig. 4.18, show that there is only a very small amount of tail generation over transition crossing. This suggests that the

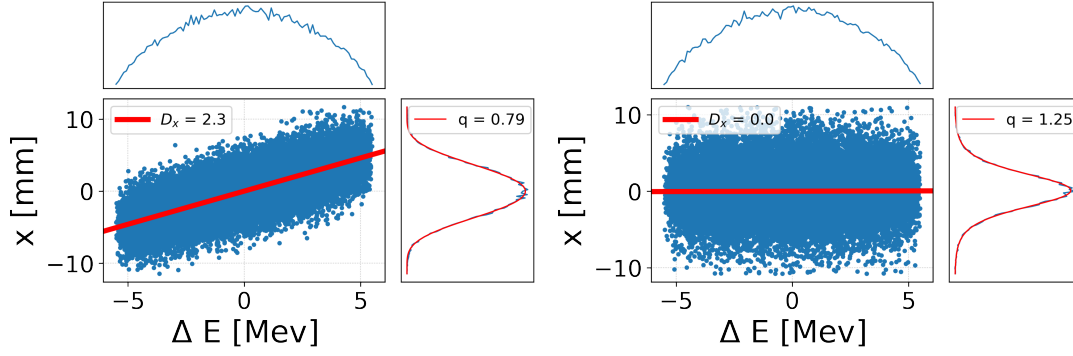


Figure 4.17: The beam distributions results of the tracking simulations at nominal dispersion and at zero dispersion, where the generated transverse distribution has a q -factor of 1.25. The longitudinal and transverse profiles based on the distributions are shown with a q -Gaussian fit on the transverse profile.

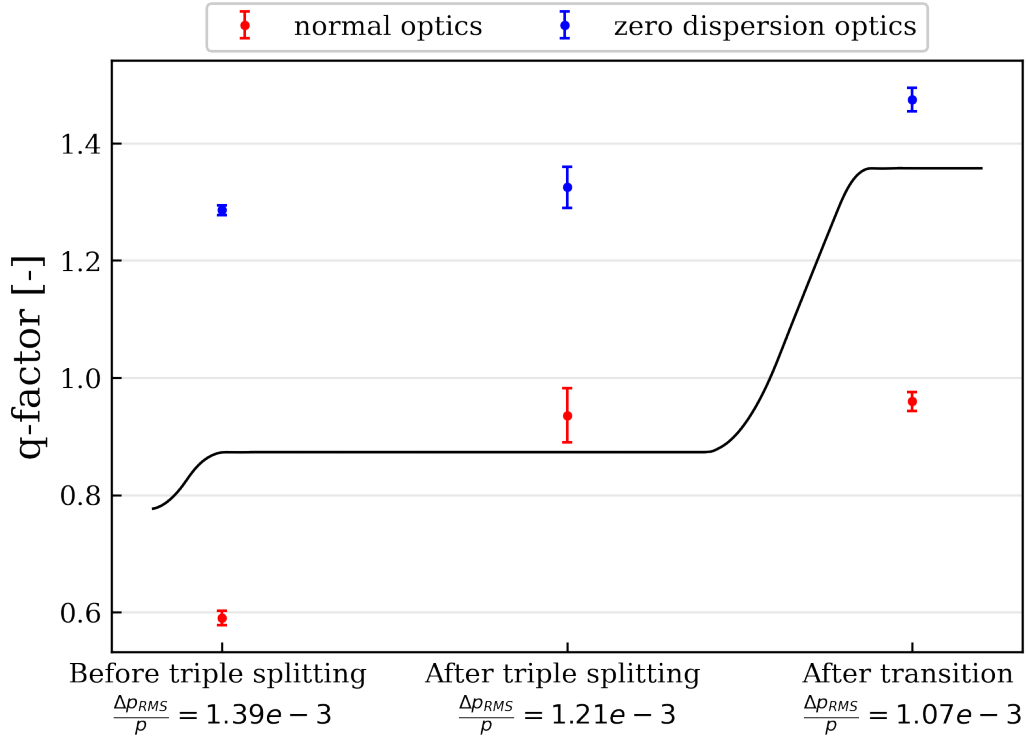


Figure 4.18: The beam tail progression with an initial intensity of 200×10^{10} particles per bunch before and after transition crossing in the PS with the magnetic cycle highlighted in black for clarity.

previously observed tail increases between bottom and top energies are primarily due to the removal of the dispersive contribution to the beam profile. The horizontal beam distribution in the PS thus has large beam tails but no emittance blow-up over transition crossing. This result is particularly relevant for ongoing studies of vertical beam tails in the PS, where the tails are suspected to become more populated during transition crossing.

4.5 Numerical approach to beam profile deconvolution

Accurate emittance measurements are of interest for many accelerators facing similar dispersive challenges, particularly at the PSB and SPS [69] at CERN, especially since zero dispersion optics showed clear advantages to emittance and beam tail studies. However, these machines lack the lattice flexibility required to design zero dispersion optics.

The PS offers a unique opportunity to address this challenge by enabling simultaneous measurements of both convoluted and deconvoluted horizontal beam distributions with nominal and zero dispersion optics respectively, while also reconstructing the longitudinal profile using tomographic techniques. This setup provides a valuable platform for studying alternative approaches to deconvolution of the transverse and longitudinal distributions in the presence of dispersion.

The current analytical method of deconvolution, based on Eq. 2.17, has proven insufficient. To address this, the following section explores a numerical approach to estimating the deconvoluted horizontal distribution. This method is then validated by comparing with measured deconvoluted profiles obtained through zero dispersion optics. If successful, this approach could be applied to other accelerators, particularly those that lack the flexibility to implement zero dispersion optics, providing a reliable alternative for precise beam profile measurements.

The proposed technique begins with an initial guess for the deconvoluted horizontal distribution, using either a Gaussian or a q-Gaussian distribution. This distribution is then convoluted with the longitudinal profile, and the resulting profile is compared to the measured convoluted distribution at normal optics. Based on the comparison, the guess for the deconvoluted distribution is iteratively updated, transforming this process into an optimization problem. The full method is illustrated in Fig. 4.19. A key component of this technique involves re-convoluting a horizontal distribution, measured under zero dispersion optics, with the longitudinal distribution.

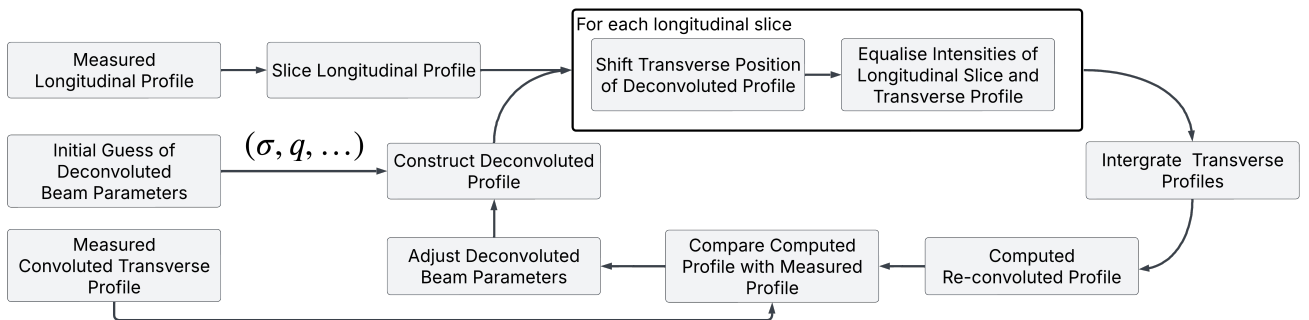


Figure 4.19: A flowchart showcasing the process of deconvoluting a normal optics beam profile from the longitudinal distribution.

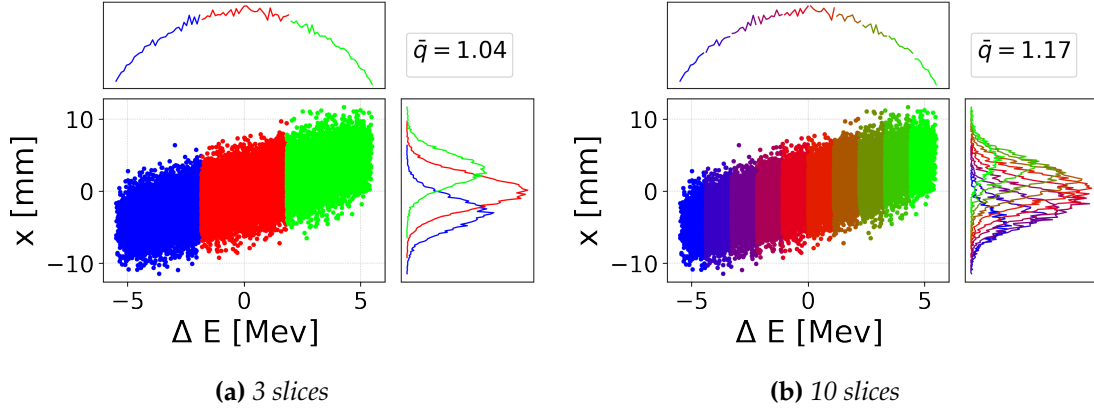


Figure 4.20: The simulated beam distribution with overpopulated tails ($q=1.25$) of a dispersive beam sliced based on their energy.

4.5.1 Beam profile re-convolution

An illustration of how the horizontal and longitudinal planes interact is provided through simulations in Fig. 4.17. In these simulations, while the transverse distribution is initially overpopulated, the resulting transverse beam profile exhibits underpopulated tails due to dispersive effects. To analyze this effect, the longitudinal distribution of the simulated beam is divided into slices, ensuring that the energy spread (ΔE) within each slice becomes smaller, as shown in Fig. 4.20. Starting with three slices, the transverse beam profiles corresponding to each longitudinal slice display Gaussian-like tails. However, when the longitudinal profile is divided into ten slices, the transverse beam profiles reveal overpopulated tails. This trend indicates that as the ΔE range within each slice decreases, the convolution of the slices weakens. Based on this observation, the re-convolution process is developed.

The re-convolution process of the horizontal distribution begins by dividing the longitudinal profile into a large number of slices (30 slices are used in the results presented here). For each slice, a copy of the deconvoluted horizontal profile is shifted according to the average ΔE of that slice and the corresponding dispersion. Additionally, the amplitude of the horizontal profile is weighted to match the intensity of the longitudinal slice. By summing and normalizing the resulting horizontal profiles from all slices, a re-convoluted horizontal profile is generated.

The result of this re-convolution is shown on the left side of Fig. 4.21. The right side of the figure displays the emittances and q -factors for the convoluted, deconvoluted, and numerically re-convoluted beam profiles for the PSB rings. The emittances of the re-convoluted profiles fall within the error bars of the convoluted emittances measured with nominal optics, demonstrating that the re-convolution process yields accurate results. Although the q -factors show a less precise match, the underpopulated tail, observable only when convolution is present, is effectively captured.

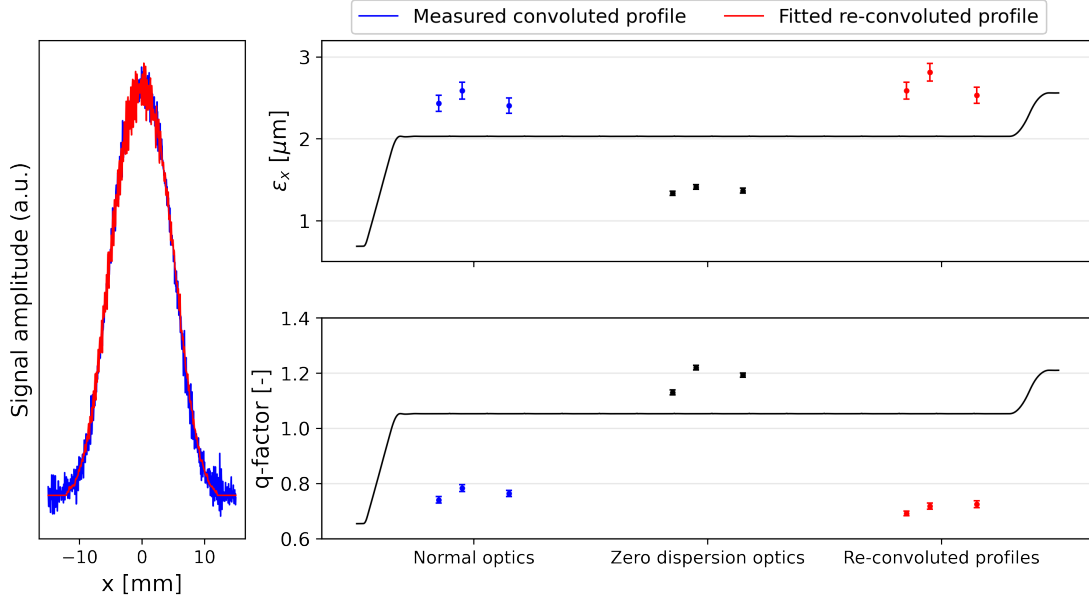


Figure 4.21: Left: The convoluted horizontal profile measured at normal optics compared to the fitted re-convoluted profile made through combining the deconvoluted profile measured during zero dispersion optics and the longitudinal distribution from tomography. Right: The emittances and q-values of the convoluted, deconvoluted and re-convoluted profiles for beams coming from PSB ring 1, 2 & 4.

4.5.2 Beam profile deconvolution

With the re-convolution process successfully tested and validated, the proposed deconvolution method can now be revisited, illustrated in Fig. 4.19. The deconvolution process should only need as input two measurements: a convoluted transverse beam profile measured at nominal optics and a longitudinal beam profile, given that the optics at the transverse beam profile measurement location are known. Therefore, an initial guess of the beam parameters, q-factor and beam width, is used for the deconvoluted transverse beam profile based on a q-Gaussian distribution. This initial guess will be convoluted with the measured longitudinal profile and compared with the measured convoluted transverse beam profile. Based on this comparison, the beam parameters of the initial guess will be updated and the process is repeated, leading to an optimization. The optimization process, employing the Powell method [78, 79], minimizes the difference between the measured and reconstructed profiles.

The resulting emittances and q-factors from the deconvolution process for three of the PSB rings are shown in Fig. 4.22. The emittances derived from the numerically deconvoluted profiles closely match those from the zero dispersion optics, indicating the effectiveness of this analytical approach. Significant improvements to beam tail estimations compared

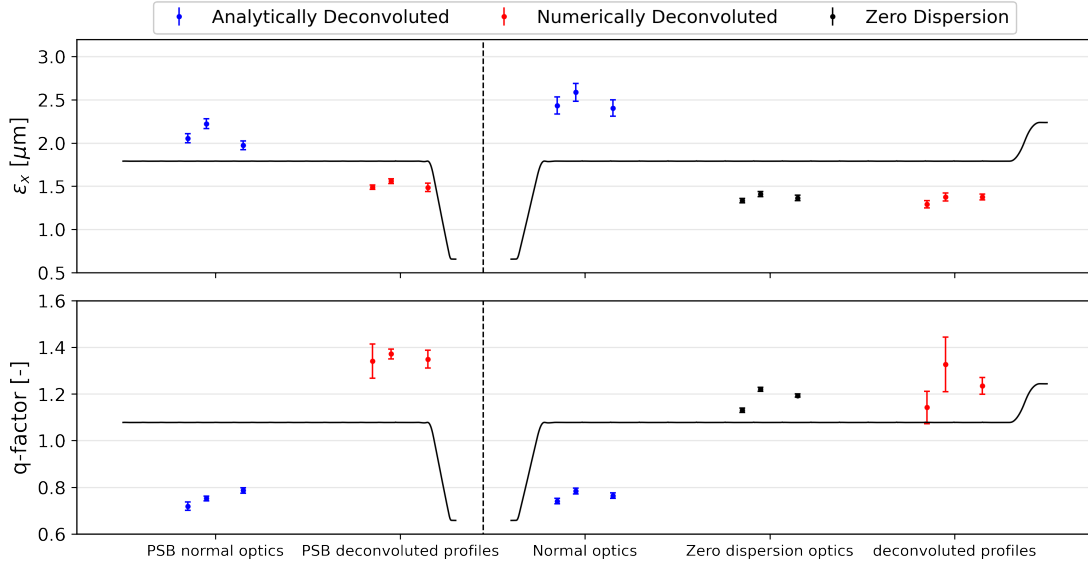


Figure 4.22: The analytically deconvoluted profiles (in red) in the PSB and the PS compared to the normal optics (in blue) and zero dispersion optics (in black) measurements for beams coming from PSB ring 1, 2 & 4.

to the usual methods are also observed, with the method successfully identifying the overpopulated tails in the beam even before transition crossing.

Given the success of this approach in the PS, the same technique was applied to the PSB emittances, also presented in Fig. 4.22. In the PSB, the emittances and beam tails are presented at normal optics in blue and using the numerical deconvolution in red, showing a decrease in emittance and an increase in beam tails. In the PS, similar behaviour is observed, with an additional measurement of the deconvoluted beam profile at zero dispersion optics in black. The results show slightly higher emittances from the numerical deconvolution in the PSB in red compared to the zero dispersion optics values in black from the PS. Furthermore, the q-factors in the PSB from the numerical deconvolution have similar error bars to those in the PS, and once again, the overpopulated tails are captured effectively.

This method demonstrates strong potential, especially considering the various opportunities for improvement. These include exploring different optimization algorithms, refining the re-convolution process, and optimizing based on the emittance and q-factor of the re-convoluted profiles rather than simply comparing signals. Further refinements to the convolution process are being studied and this technique has potential to be applied to other accelerators in the CERN injector chain and beyond.

4.6 Space charge effects on the dispersion

While the zero dispersion optics were implemented, a small but interesting result was seen in Fig. 4.7. On the figure, the beam widths for each emittance configuration are expected to reach a minimum value at the setting of the zero dispersion knob where the horizontal dispersion is equal to zero. However, the results of the measurements show that the beam width reaches a minimum value at a lower knob setting, observed to be between 88% - 97% of the knob setting where zero dispersion optics is reached. One could suspect that the change in β -function could lead to this deviation, however the β -function agrees well with model predictions and also continues to decrease in value as the zero dispersion knob is increased, so the effect on the beam width minimum would delay the minimum beam width minimum instead of advancing it.

Another suspect could be space charge effects, since the zero dispersion optics are designed with single-particle simulations and the result of space charge effects on these optics are unknown. The dispersion measurements via mean beam position are also blind to space charge, while the measurements of the beam width are sensitive to it. In this section, the effects of space charge is studied on the zero dispersion optics through multi-particle tracking simulations. In these simulations, the transverse and longitudinal positions of individual particles are known, as opposed to relying solely on beam-based measurements, which offer insights into the overall distributions. Afterwards, an attempt is made to include the space charge effects in the single-particle model to improve our understanding of the space charge effects on the zero dispersion optics.

4.6.1 Multi-particle space charge tracking simulations

To investigate the beam width minimum discrepancy that was seen when going to zero dispersion optics, the beam parameters of the three measured beams used in Fig. 4.7 are recreated for the simulations. These simulations will then yield nearly identical beams with distinct space charge tune spreads, allowing for a direct comparison with experimental data. The simulations will be conducted using pyORBIT [80, 81], a software that can use the standard MAD-X model of the PS as a series of matrix slices. During the tracking simulations, the particles experience individual space charge Coulomb kicks based on the current distribution of the bunch and the precise Hamiltonian between each matrix.

Leveraging the parallel computing capabilities of CERN's computer clusters, detailed space charge calculations can be achieved efficiently. To reduce computational time, 50,000 'macro'-particles are simulated, each representing the mass and charge of multiple real particles, thereby maintaining the full intensity and charge of the beam. These

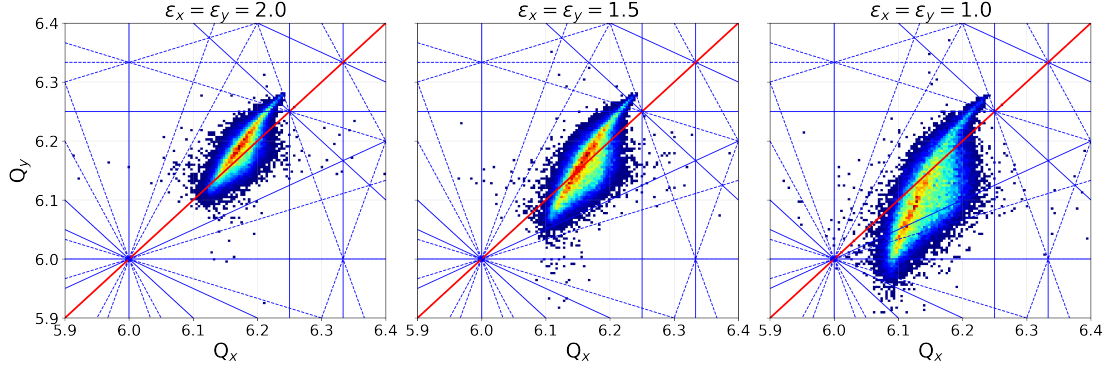


Figure 4.23: The tune spread of the particles in the multi-particle tracking space charge simulations after 100 turns for the three different emittance configurations.

particles are tracked for 3,000 turns: the first 500 turns at normal optics, followed by a ramping of the zero dispersion knob from 0 to 1.75 over 2,000 turns, and concluding with 500 turns at a dispersion value far below zero, without further optics changes.

The longitudinal distribution of the particles is generated using Beam Longitudinal Dynamics (BLonD) software based on the RF parameters in the PS. Meanwhile, the transverse distribution is set according to the required beam parameters, measured at zero dispersion optics in the PS. To validate the effects of space charge on the tune spread of each emittance configuration, the tune spread of the particles after 100 turns is shown in Fig. 4.23.

At this point a distinction is made between the model dispersion, where the dispersion is derived from the lattice optics and the effective dispersion that is experienced by the beam. Since the position and energy of each macroparticle in the simulations is known, the simulated dispersion will be calculated directly from the x to $\frac{\Delta E}{E}$ relation of all particles in the simulated beam, resulting in the effective dispersion. This method also avoids the need to vary the radial steering in the simulation, which would mimic the beam-based measurements. The large $\frac{\Delta p_{RMS}}{p}$ in the simulation makes this relation observable in the simulations, as demonstrated in Fig. 4.17.

Simulation results

The beam parameters are recorded at each turn during the simulation, and the evolution of the dispersion function is shown in Fig. 4.24. In these plots, the simulations with space charge kicks are shown in purple, while those without space charge kicks are in red. Lastly, the experimental measurements of the horizontal dispersion for each emittance setting are also on the figure. These experimental measurements are taken at specific values of the zero dispersion knob, which is why each measurement is able to correspond to a turn number of the simulation, since the same knob is used in the simulations.

When the beam circulates with normal optics, there is a small difference between the

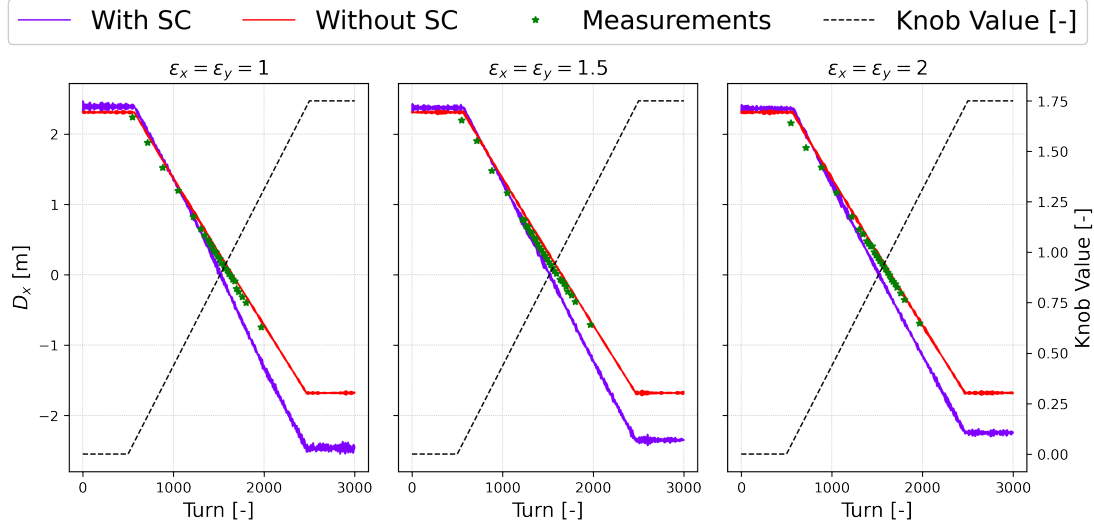


Figure 4.24: The dispersion progression of the pyORBIT multi-particle simulations with (purple) and without (red) space charge kicks. In green the experimental measurements of D_x are shown.

dispersion with and without space charge effects. Afterwards, when the quadrupoles start adjusting the beam to zero dispersion optics, this difference begins to increase to the point where the dispersion of the beam with space charge reaches zero almost a hundred turns before the beam simulation without space charge has zero dispersion. The experimental measurements which are based on the mean beam position, insensitive to space charge effects, move parallel to the simulations without space charge effects. Since the dispersion measured via the mean beam position has been proven to be equal to the model dispersion in previous sections (Fig. 4.4), it can be concluded that the model and effective dispersion in the absence of space charge forces are the same.

However, the effective dispersion of the machine can be expected to be dependent on space charge effects. To assess this, the beam widths, which are subjective to space charge effects, are also studied. These beam widths are shown in Fig. 4.25, using the same method as for the dispersion: simulations with space charge kicks are in purple, those without space charge kicks are in red, and the experimental measurements are in green. At normal optics, the beam widths vary depending on the initial horizontal emittance. The plot indicates a small difference between the initial beam widths in simulations with and without space charge kicks, which could be due to the observed difference in dispersion between the two space charge settings. In the final 500 turns, the beam widths in the space charge simulations return to values similar to those during the normal optics period. However, this is likely because the absolute value of the final dispersion closely resembles the initial dispersion, meaning the similarity between the initial and final beam widths holds no significant meaning.

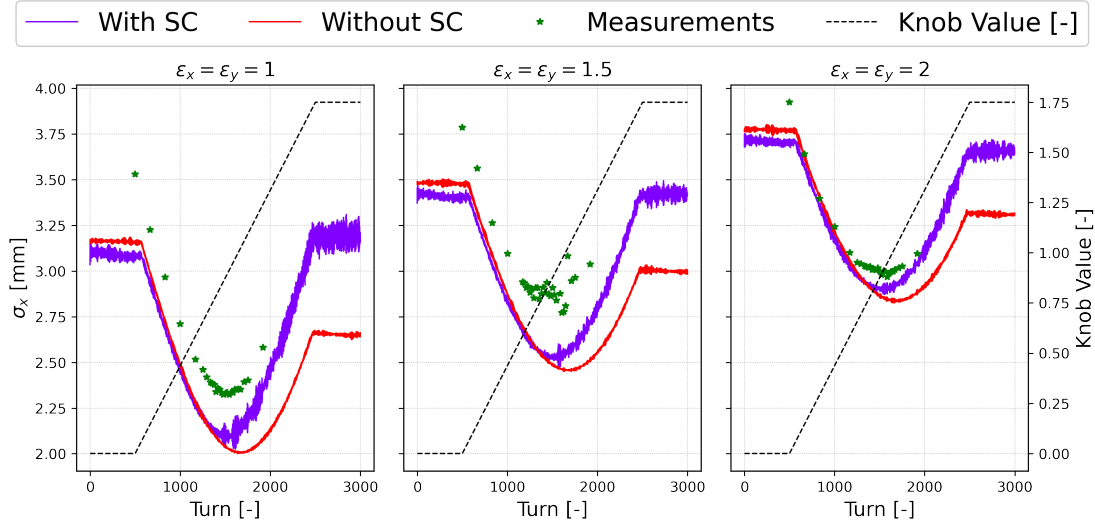


Figure 4.25: The beam width progression of the pyORBIT multi-particle simulations with (purple) and without (red) space charge kicks. In green the experimental measurements are shown.

As the quadrupoles ramp up and the zero dispersion point is crossed, the simulations with space charge effects reach a minimum beam width earlier than those without space charge effects. The experimental measurements of the beam width behave similarly to the simulations with space charge effects. This, combined with the observation that the measured dispersion behaves parallel to the simulations without space charge, helps explain the difference between the minimum beam widths and the zero dispersion point, by showing that the effective dispersion and the model dispersion will differ depending on the presence of space charge effects and this difference will increase due to the optics change of the zero dispersion knob. Note that this does not significantly change the findings from the zero dispersion optics study from earlier since the dispersion will still be negligibly small. However, this does open a new avenue of studying space charge models. This is because the magnified difference of the effective and model dispersion at zero dispersion optics and the effects of space charge on the optics can be distinguished through the space charge sensitive beam width measurements and space charge insensitive dispersion measurements.

4.6.2 Single-particle space charge model

Space charge effects influence the dispersion of the beam, causing the dispersion that the beam experiences to differ from the measured dispersion. This effect is practically immeasurable at normal optics, however it was clearly observed while using zero dispersion optics. This deviation can be understood by examining the influence of space charge on particle motion. In the introduction, it was made clear that space charge effects alters the effective defocusing properties of a magnetic lattice leading to a tune-shift. To replicate the expected tuneshift, extra defocusing multipoles are added

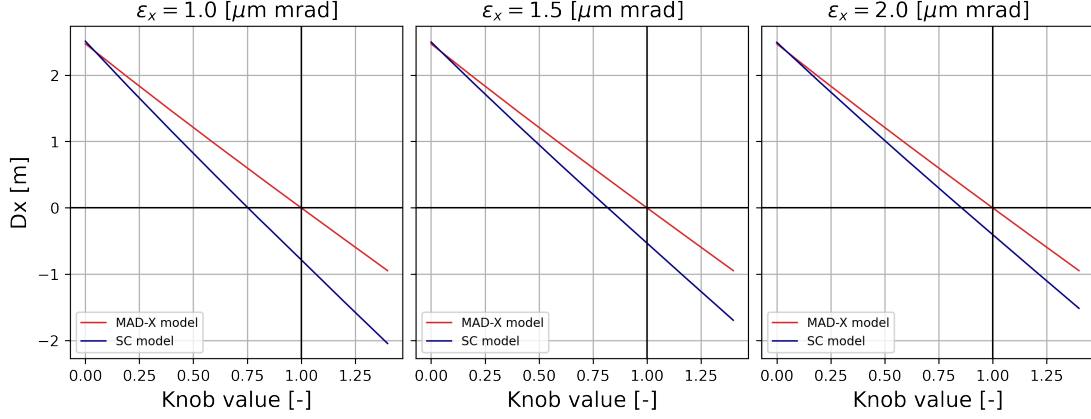


Figure 4.26: The dispersion value as WSH68 when going between normal optics, knob value = 0, and zero dispersion optics, knob value = 1, for both the standard PS MAD-X model and a model that includes the space charge tune-shift of the corresponding emittances.

to a linear MAD-X model in the horizontal plane. Since space charge is a continuous effect, the virtual multipoles need to be tightly packed and are therefore installed in each straight section, totaling 100 multipoles. The sum of their kicks produces a tune-shift as defined in 2.58 with one alteration: this tune-shift is defined for a coasting beam, while in the measurements and simulations, a bunched beam was used. To compare the difference between the two, the peak current of a bunched beam, I_{peak} is weighted to the current of a coasting beam with the same number of particles, $I_{average}$. This ratio is the bunching factor $B_f = \frac{I_{average}}{I_{peak}}$ which can be derived from the longitudinal distribution of the beam. In the case of the beams used in the PS, the bunching factor B_f is 0.43. Using this consideration, the results of the MAD-X model with the space charge tune-shift are shown in Fig. 4.26.

The figure qualitatively agrees with both measurements and simulations. At normal optics, the dispersion is slightly larger due to space charge effects. As the quadrupoles are ramped up towards zero dispersion optics, the dispersion approaches zero faster than in the standard model. In these cases, zero dispersion is reached at 75%-86% of the knob value where the standard model crosses zero. Beam-based measurements suggest that this relation is 88%-97%, while simulations show values between 88% and 90%.

This space charge model does not fully align with the previous findings, however the tune-shift that is considered here corresponds to the center of the beam, which experiences the largest effects. Another approach to define the space charge tune-shift would instead be to use the average space charge experienced by the beam. This tune-shift is derived from the Root-Mean-Square (RMS) values of the beam and the derivation is done in Appendix B. The space charge tune spread is then given by

$$\Delta\tilde{Q}_x = -\frac{R^2}{Q_x} \frac{K}{4} \frac{f}{2\tilde{x}(2\tilde{x} + 2\tilde{y})}, \quad (4.2)$$

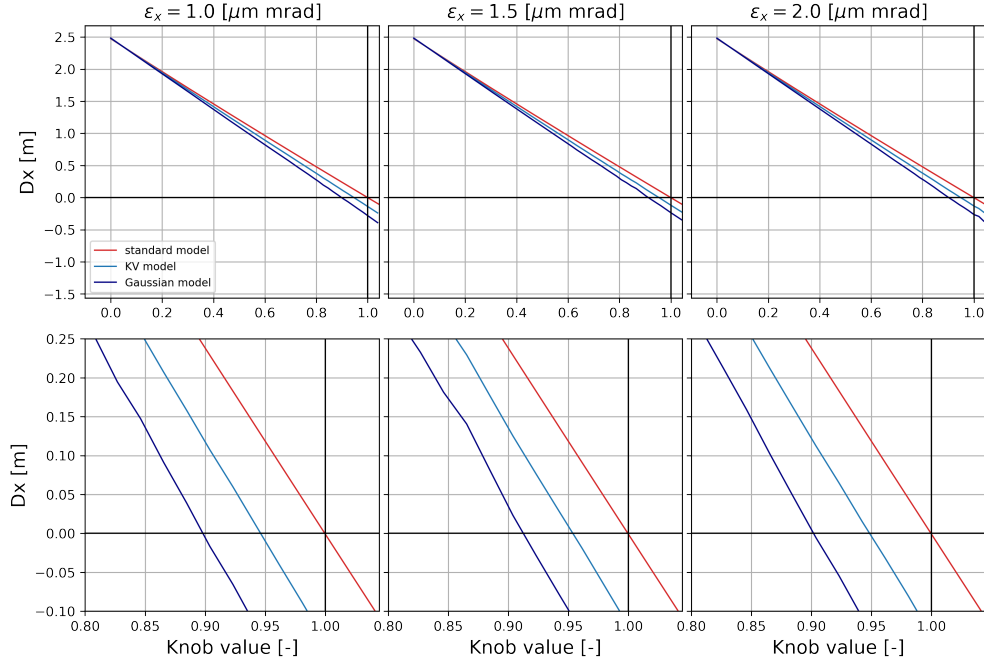


Figure 4.27: The dispersion for a MAD-X model that includes the space charge tune-shift for RMS equivalent beams where the RMS beam size $\tilde{x}$ is calculated from the measured data shown in Fig. 4.7 for a Gaussian (dark blue) and KV-type (light blue) beam. A standard MAD-X model without additional defocusing multipoles (red) is also shown.

where $\tilde{x}$ and $\tilde{y}$ are the RMS beam sizes, R is the radius of the accelerator, K is the perveance and the factor f depends on the type of distribution the beam has. Here, the case where the beam has a Kapchinsky-Vladimirsky (KV) distribution ($f=1$) and a Gaussian distribution ($f=2$) will be discussed.

The RMS tune-shift based on the beam sizes of the beam-based experimental measurements, is used in the single-particle space charge model and the results are presented in Fig. 4.27. The results of the RMS tune-shifts based on the beam sizes of the multi-particle tracking simulations are shown in Fig. 4.28 where a direct comparison of the dispersion between the single particle space charge model and the space charge simulations can be made. The resulting dispersion evolution for both the simulation and the measurements shows a significant improvement in similarity compared to the previous attempt. In the case of the simulation, it is possible to directly compare the space charge dispersion from the tracking simulation and the MAD-X model. Up until zero dispersion is reached, the KV model demonstrates the best agreement with the simulations.

The difference between the space charge dispersion and the measured dispersion, deduced from the minimum beam width, is less precise but still shows a good agreement with the KV distributed MAD-X model. This indicates that the dispersion experienced by the beam can be modeled using RMS equivalent space charge effects. Despite the limitations in the accuracy of the measurements, the overall agreement supports the

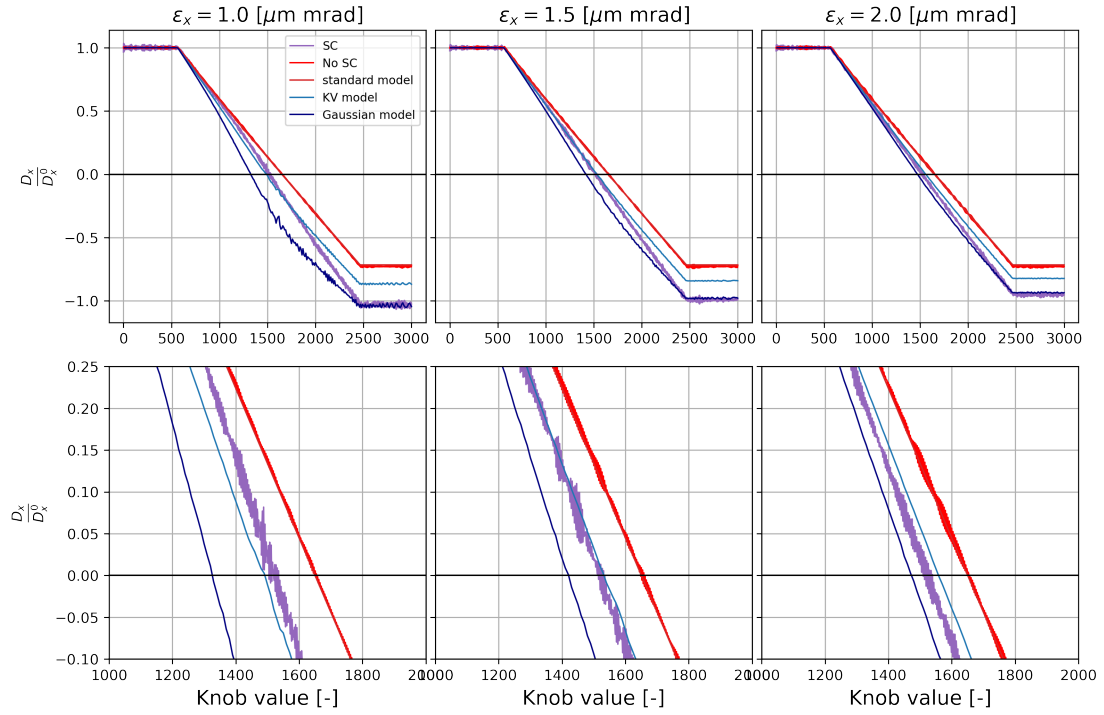


Figure 4.28: The dispersion for a MAD-X model that includes the space charge tune-shift for RMS equivalent beams where the RMS beam size $\tilde{x}$ is calculated from the simulated tracking data (purple) shown in Fig. 4.24 for a Gaussian (dark blue) and KV-type (light blue) beam. A standard MAD-X model without additional defocusing multipoles (light red) and tracking data without space charge effects (dark red) are also shown, they overlap on the plot.

conclusion that the KV model provides a reliable representation of the space charge effects on dispersion.

4.7 Summary

This chapter presents the design, implementation and success of the zero dispersion optics and their role in improving horizontal emittance measurements in the PS. Non-zero dispersion at beam measurement locations complicates emittance calculations in many accelerators, as it convolutes the transverse and longitudinal beam distributions. Analytical deconvolution methods often prove ineffective, yet accurate measurements of beam emittance and tails in the LHC injector chain are essential for maximizing the LHC's luminosity.

To mitigate these dispersive effects, zero dispersion optics were designed and implemented at injection energies using the PS's LeQs. These optics successfully eliminated dispersive contributions at beam profile measurement points. At higher energies, the natural reduction of dispersive effects due to the smaller $\frac{\Delta p_{RMS}}{p}$ enables meaningful comparisons, confirming the effectiveness of zero dispersion optics. These comparisons also revealed that the horizontal emittance discrepancies between the PSB and PS are primarily caused by the different dispersion of each machine, rather than actual emittance blow-up. The findings also confirm that the horizontal distributions from the PSB rings are consistent, and the observed variations under normal optics come from the difference of the longitudinal distributions.

The study also explored the impact of zero dispersion optics on beam tail measurements. Under normal optics, beam tails appeared to grow significantly at higher energies. However, zero dispersion measurements revealed that these large tails are already present at low energies but are concealed by longitudinal convolution. Additionally, transition crossing was shown to have minimal impact on horizontal beam tail generation, challenging prior assumptions that tail populations increase significantly during this process.

To extend the benefits of these findings to less flexible accelerators where zero dispersion optics may not be feasible, a numerical deconvolution method was developed to estimate the true transverse beam profile. This method produced emittance values and tail distributions closely matching those obtained using zero dispersion optics. When applied to the PSB, it successfully identified overpopulated tails and greatly improved emittance accuracy compared to measurements under normal optics. These results show the potential of numerical deconvolution and future refinements, such as improved optimization algorithms and re-convolution techniques, which could enhance its performance and broaden its applicability to other machines in the CERN injector chain and other synchrotrons.

The interplay between dispersion and space charge effects was also explored. While space charge significantly affects beam width measurements, it does not influence the measured dispersion, which is derived from the mean beam position. Through this, it is possible to distinguish the impact of space charge on the optics, which is greatly amplified at zero dispersion optics. This distinction enabled a validation of space charge models using experimental data where the RMS KV-distributed model showed strong agreement between measurements and simulations.

The study of zero dispersion optics in the PS significantly improves the accuracy of horizontal emittance measurements and provides valuable insights into beam tail evolution and space charge dynamics. Both the zero dispersion optics and numerical deconvolution techniques have broad implications for accelerators, offering a practical and effective alternative for precise transverse beam profile measurements.

RESONANCE DRIVING TERMS

The PS is a low-energy, high-brightness machine causing a generally large space charge tune spread and therefore multiple resonance crossings for the particles in the beam. The interplay of chromaticity, space charge and synchrotron motion can also lead to trapping of individual particles in nonlinear resonances [82]. All this can lead to beam losses and emittance growth, so the resonances in the PS have been a heavily studied subject with both beam-based measurements and simulations [49, 83–85]. These resonances have traditionally been measured by quickly crossing the resonance through a tune scan along parallel lines. During each crossing, intensity losses were observed and linked to the corresponding working point, effectively creating a loss map. This method allowed for the identification and correction of heavily excited resonances by individually varying the corresponding magnet types and measuring the effect on the loss map. By bringing the losses to a local minimum, the resonance could be corrected. The loss map method leads to good correction of the resonance but is very iterative and time-consuming, and it does not give information about the influence of potential sources of non-linearities.

Historically, resonance driving terms (RDTs) have been measured by kicking the beam with a fast kicker magnet and analyzing the excited turn-by-turn data. Measuring the RDTs in this way proved to be difficult due to rapid decoherence of the excitation altering the measured resonance lines in the spectrum [27], space charge affecting this decoherence and the possibility of beam dumps after large single kicks. All these issues made it difficult to benchmark non-linear optic models of the machine. However, past studies have shown that longer periods of forced oscillations, driven by an AC-Dipole, enable linear and non-linear optics measurements, including measurements of RDTs [86–92]. These forced oscillations facilitate a cleaner and more straightforward method for resonance measurements and correction. Although the PS lacks an AC-Dipole, it is equipped with a transverse damper (ADT) typically used for tune measurements [29]. This damper can also create forced oscillations, albeit at a much lower amplitude than an AC-Dipole.

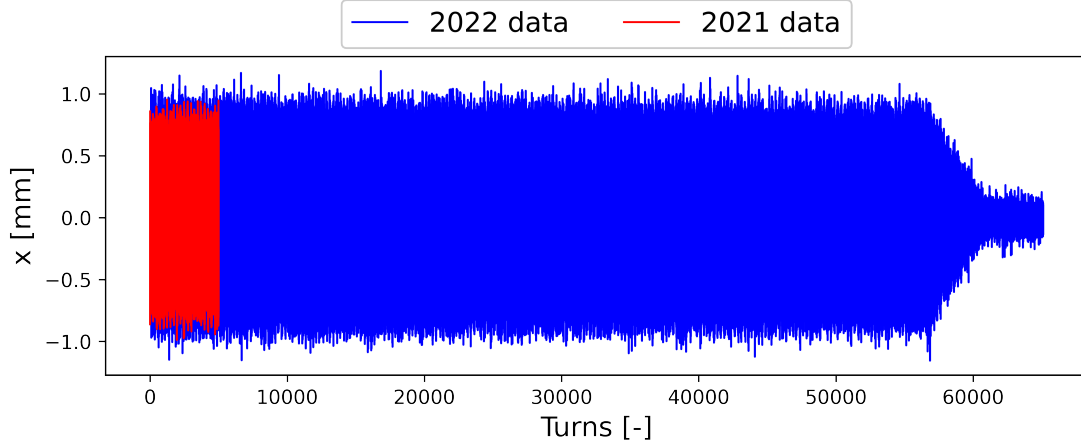


Figure 5.1: Turn-by-turn data of vertical BPM 03 using the 2021 recording method (5,000 turns) in red and showing the 2022 method (65,000 turns) in blue after a software update in the end of 2021. The ramping down of the forced oscillation caused by the ADT is visible after turn number 58,000.

In Figure 5.1, the forced oscillation turn-by-turn data of one BPM, excited by the ADT, is shown using the acquisition method from 2021, which can record up to 5,000 turns. The turn-by-turn data of each measurement was automatically analyzed and this greatly reduced the number of turns that could be recorded between shots. On the figure, the data acquisition from 2022 is also shown where due to a software update, it was made possible to only record the turn-by-turn data without doing the automatic analysis allowing for a much higher number of recorded turns. However, when recording more turns than shown here with this new data acquisition method, data from the next shot can appear in the turn-by-turn data, deteriorating the results. The figure also illustrates the difference between the signal of forced oscillations by the ADT from turns 0-58,000, where the peak-to-peak variation is around 2 mm, and the signal during free oscillations at turns greater than 60,000, where the peak-to-peak variation is smaller than 0.6 mm.

The goal of this chapter is to use the ADT to create forced oscillation turn-by-turn data, recorded with the upgraded turn-by-turn application, and evaluate the feasibility of accurately measuring the RDTs in the PS. This evaluation aims to enhance the PS model and resonance corrections, with an additional objective of refining this method for future studies. The chapter begins with an amplitude calibration of the Beam Position Monitors (BPMs), followed by the measurement of the f_{0030} skew sextupole RDT ($3Q_y$ resonance). Historically, this resonance has been strongly excited due to the installation of additional shielding to the BGI monitor and was previously measured and corrected using the loss map method [93]. Although the shielding has since been replaced to reduce excitation, applying the inverse correction is expected to produce a significant RDT, providing a suitable test case for the initial attempts of the new method.

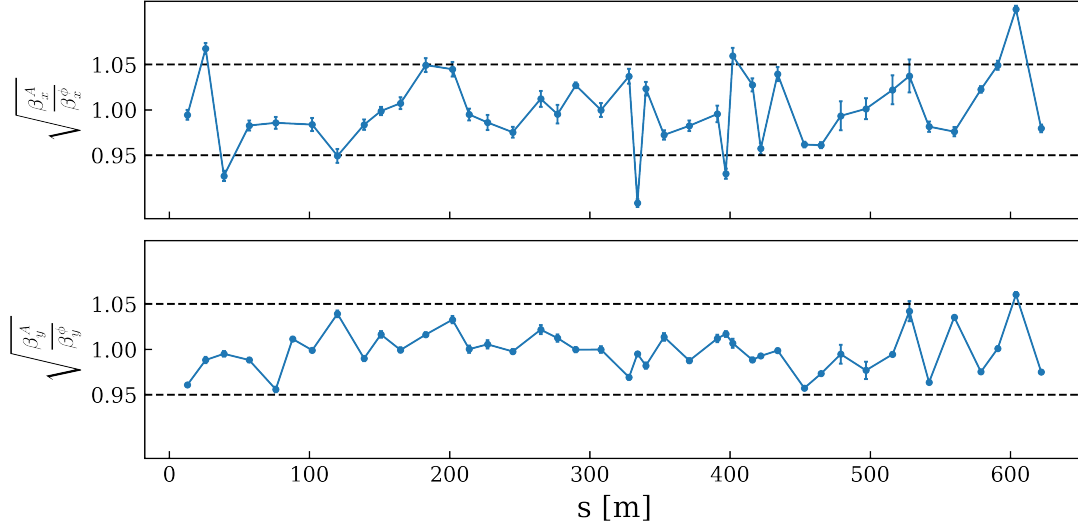


Figure 5.2: The relationship $\sqrt{\frac{\beta_y^A}{\beta_y^\phi}}$ for all the BPMs in the PS, which is as a measure of the amplitude calibration error of the BPMs.

Following the measurement, tracking simulations are performed to verify the accuracy of PS models, expanding this study to other RDTs as well. Finally, the chapter discusses potential strategies to reduce noise levels in the frequency spectra, enabling more accurate measurement of the optics and RDTs in future studies.

5.1 BPM calibration

In Chapter 4, Section 4.2.1, it was noticed in Fig. 4.4 that the dispersion measured with the BPM signals differs from the model dispersion, even though the model dispersion and the dispersion measured from the neighboring WS, separated by a 0.5 m drift space, agree with each other. This discrepancy suggests that the BPMs' calibrations are incorrect. To address this potential calibration error, Chapter 2, Section 2.2.1, is referred to, as it was introduced that the linear optics β -function can be determined via the phase advance between adjacent BPMs, β^ϕ (Eq. 2.8), or the amplitude of the excited betatron oscillations at a given BPM normalized to the action of the inferred from all BPMs, β^A (Eq. 2.10). Since β^ϕ is not dependent on the calibration factor while β^A is dependent, the relative calibration of each BPM can be improved by adjusting the calibration factor so that these two methods of calculating the β -function align [26, 94].

Using turn-by-turn data similar to Fig. 5.1, the relationship $\sqrt{\frac{\beta_y^A}{\beta_y^\phi}}$ is calculated, providing an estimate of the missing calibration factor for the BPMs. The results of this measurement are shown in Fig. 5.2. It is evident that a significant number of BPMs have calibration errors, with some exceeding 5%. By applying these new calibration factors, the relative BPM calibration can be corrected, resulting in the error estimates presented

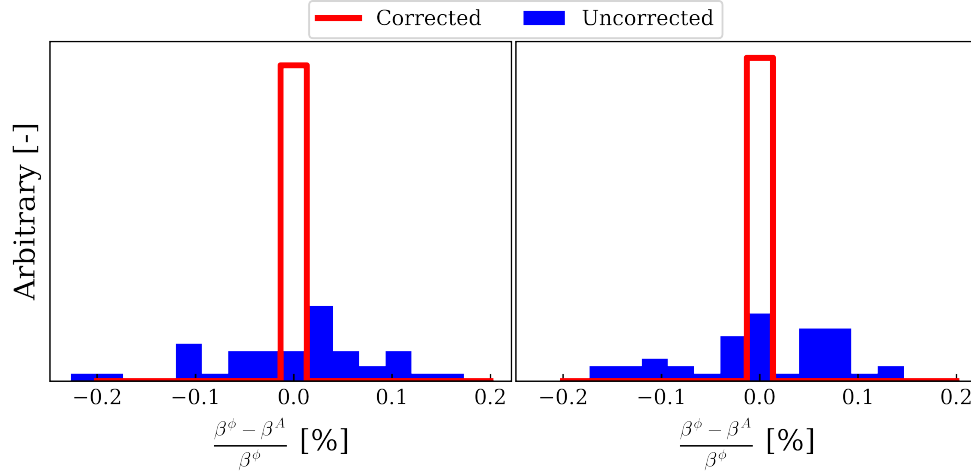


Figure 5.3: The β -beating comparison between the amplitude and phase methods using the uncorrected turn-by-turn data and the newly calibrated data.

in Fig. 5.3. This shows a marked improvement in calibration accuracy. Additionally, the dispersion measured from the BPMs in Fig. 4.4 now agree with the model and are consistent with the measured dispersion at the wire scanners. These findings demonstrate that turn-by-turn data from forced oscillations induced by the ADT can be an important tool for BPM calibrations.

	Natural tune	Driven tune	$V(0, -2)$
x	0.2575	0.252	\
y	0.325	0.33	0.34

Table 5.1: *The natural and driven tunes used for the f_{0030} measurements.*

5.2 ADT-based RDT measurements

With improved BPM calibration, it is now possible to investigate whether the non-linear effects of the PS can be accurately measured with the forced oscillation method using the smaller excitations of the ADT compared to an AC-Dipole. To test this, an attempt is made to measure the skew sextupole resonances of the PS, which have historically posed challenges to its operation [93].

5.2.1 The f_{0030} RDT

In 2021, large beam losses were observed at the $3Q_y$ resonance, associated with the BGI introduced in Chapter 2, Section 2.2.3 [95]. It was determined from the magnetic model of the PS that the newly installed shielding plates of the BGI introduced significant non-linearities. At the time, corrections were performed using a loss scan. The BGI shielding plates have since been redesigned and replaced [96]. This does provide an interesting test case with a large excitation of the $3Q_y$ resonance when the corrections from 2021 are inverted and activated in the machine. This is desirable since a larger excitation of the resonance is easier to measure. If successful, an attempt will be made to measure the naturally driven $3Q_y$ resonance with and without the BGI active to assess whether these smaller excitations of the $3Q_y$ resonance are detectable and to evaluate the impact of the new BGI shielding design on this RDT.

The natural and driven tunes used in these measurements are listed in Table 5.1. In general, the closer the natural and driven tunes are to each other and to the resonance, the more accurate the RDT measurement will be. However, this is countered by the losses induced by the resonance and the overlap of natural and driven tunes, making RDT measurements particularly challenging.

The resulting frequency spectra for the three configurations discussed above are shown in Fig. 5.4, it is worth noting that this result is from only one BPM, and the visibility of the RDT lines can vary from BPM to BPM. From Eq. 2.43, it is known that the amplitude of the RDT, in this case f_{0030} , is directly correlated with how much the RDT line perturbs the linear motion of the machine, and therefore how much the resonance is excited. Notably, the naturally driven RDT when the BGI is off is comparable to the noise level in the spectrum, while in both other cases, the RDT is evident, especially when using the inverse corrections of 2021. The noise in the turn-by-turn data must be reduced using Singular Value Decomposition (SVD) cleaning [97] to attempt to separate the naturally driven RDT from the noise floor. While this is a well-established technique for

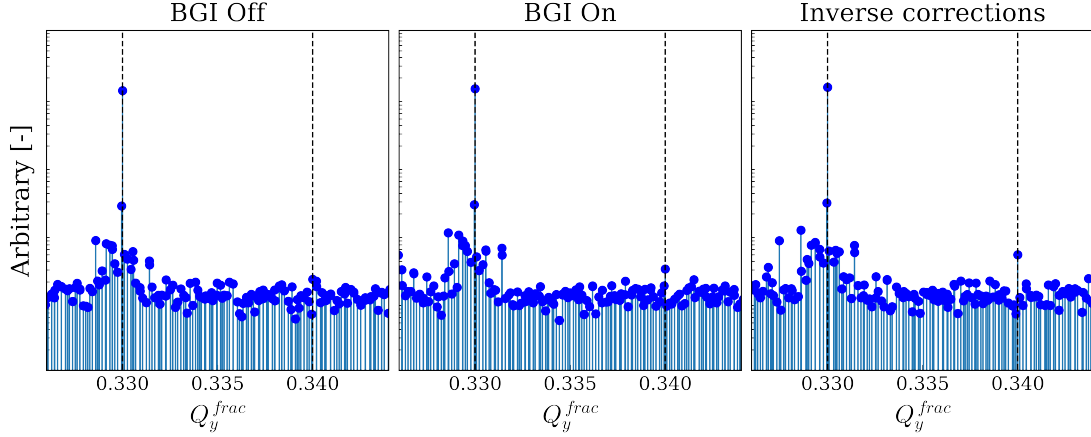


Figure 5.4: The frequency spectrum of BPM10 of the naturally driven f_{0030} RDT, when BGI84 is on and off next to the inverse correction of the previous BGI shielding. The spectra are focused around the driven fractional tune ($Q_y^{frac} = 0.33$) and the f_{0030} line ($Q_y^{frac} = 0.34$).

AC-Dipole data, additional care is used to adapt it for the ADT case.

SVD cleaning

SVD cleaning is commonly used to identify faulty BPMs [98] by analyzing the temporal and spatial variations of each singular mode. Modes exhibiting variations above a certain threshold can be excluded from the analysis. Additionally, global noise reduction can be achieved by retaining only the strongest singular modes in the turn-by-turn data, where small singular nodes tend to correspond to uncorrelated noise between BPMs. This predetermined number of retained modes, referred to as the SVD cut, is particularly important for this study.

However, overly aggressive cleaning risks removing meaningful signals along with the noise. To evaluate how varying levels of SVD cleaning impact the small f_{0030} RDT, Fig. 5.5 presents the frequency spectrum for BPM 10, along with the corresponding amplitude and phase of the RDT. If the amplitude decreases or the phase deviates significantly as the SVD cut becomes more restrictive, it would indicate that genuine signal is being removed. When the SVD cut decreases, the noise floor also seems to decrease. At the same time, a consistent phase evolution of the RDT line is observed where the phase of the line oscillates 19 times, which is equal to $3Q_y$ and thus expected from this resonance. Furthermore, the amplitude of the inferred RDT shows insignificant differences between the SVD cuts while the natural f_{0030} RDT becomes clearly separated from the noise floor. Since this aggressive cleaning reduces the noise floor without compromising the signal, it will be adopted as the standard for the remainder of this chapter.

Figure 5.6 presents the f_{0030} RDT measurements after SVD cleaning for all BPMs around the ring under three conditions: naturally driven (BGI off), BGI on, and with inverse 2021

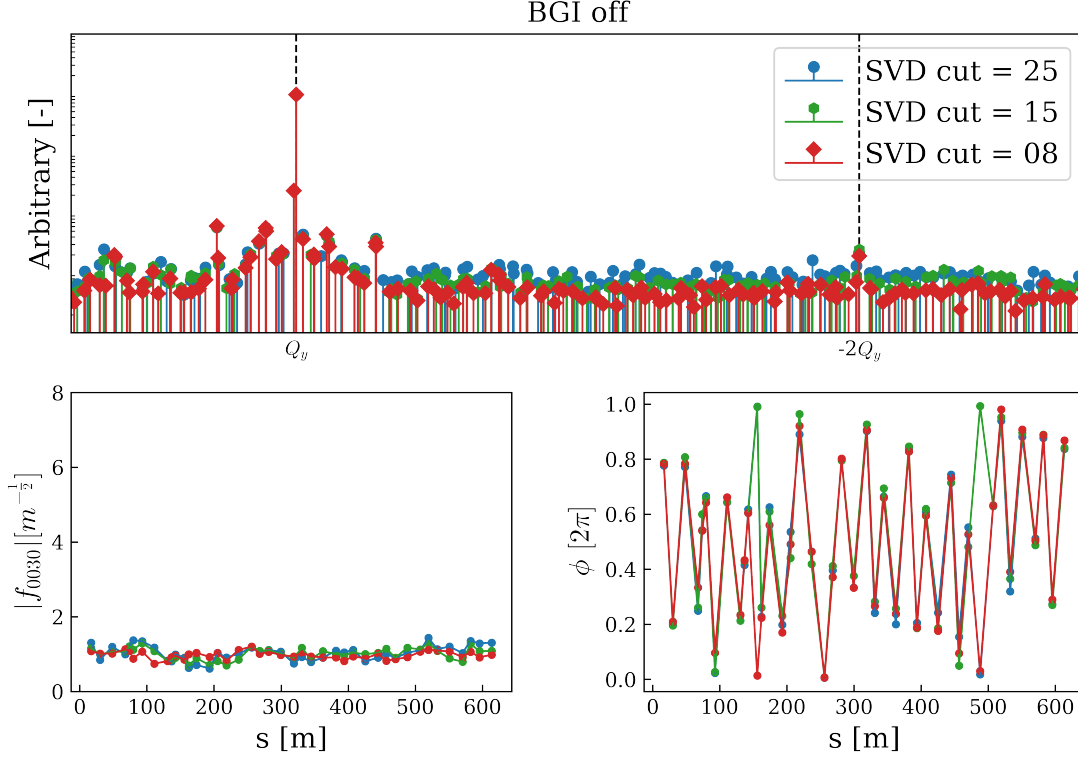
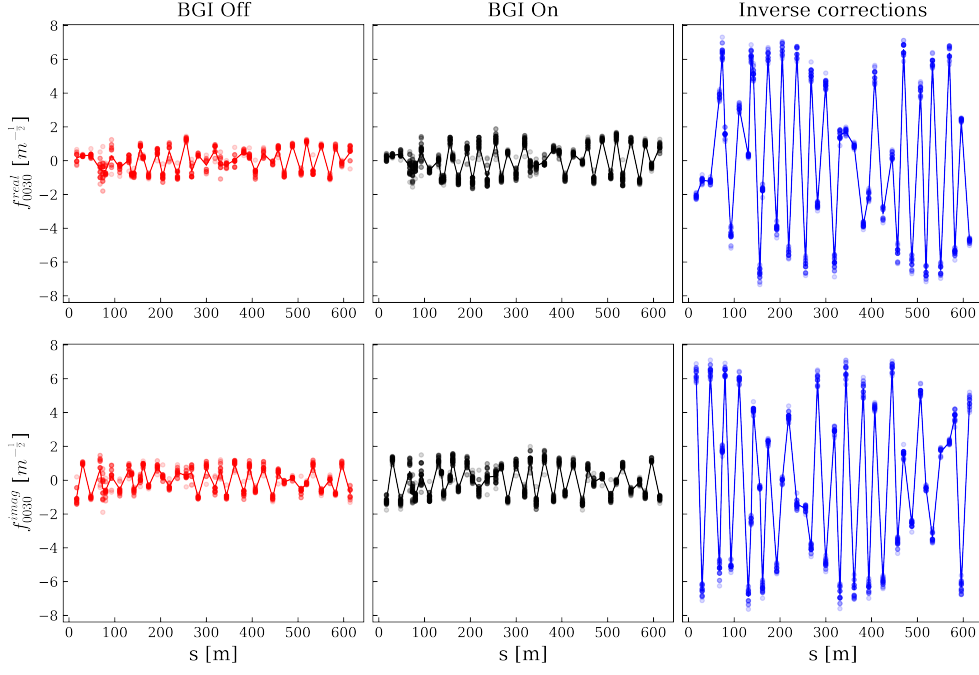


Figure 5.5: The frequency spectrum of BPM10 of the naturally driven f_{0030} RDT and the resulting amplitude and phase over all BPMs for different SVD cleanings.

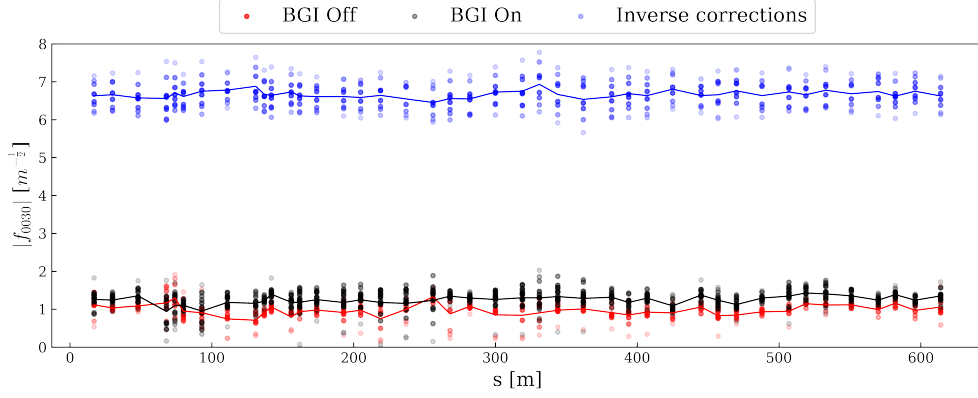
corrections applied. The small shot-to-shot variation in the RDT measurements across most BPMs confirms that meaningful signals were successfully extracted, where large variations could indicate that noise is being measured instead. Even for the naturally driven RDT, only a small section of the ring seems to contain noisy data, indicating that the SVD worked well in cleaning the measurements.

Notably, the results indicate that the redesigned BGI shielding has significantly reduced its impact on the f_{0030} RDT, although a small residual effect remains when the BGI is turned on. This subtle influence could not have been detected using the traditional loss map method, as crossing the $3Q_y$ resonance did not induce losses in either the "BGI on" or "BGI off" configurations.

These findings underline the advantage of using RDT measurements for studying suspected sources of non-linear excitation. Unlike loss maps, RDTs provide a more sensitive diagnostic tool, capable of detecting and quantifying residual errors that may otherwise go unnoticed. These measurements also demonstrate the feasibility of using forced oscillation turn-by-turn data from an ADT to measure RDTs in accelerators. This approach opens up new possibilities for non-linear optics studies in accelerators equipped with an ADT but lacking an AC-Dipole. The potential of the ADT based RDT measurements for resonance correction studies will be explored in the following sections.



(a) The real and imaginary part of the f_{0030} RDTs.



(b) The amplitude of the f_{0030} RDTs.

Figure 5.6: Multiple shots of the f_{0030} RDT for the naturally driven RDT (BGI Off), when the BGI is on and using the inverse corrections of the BGI shielding excitation.

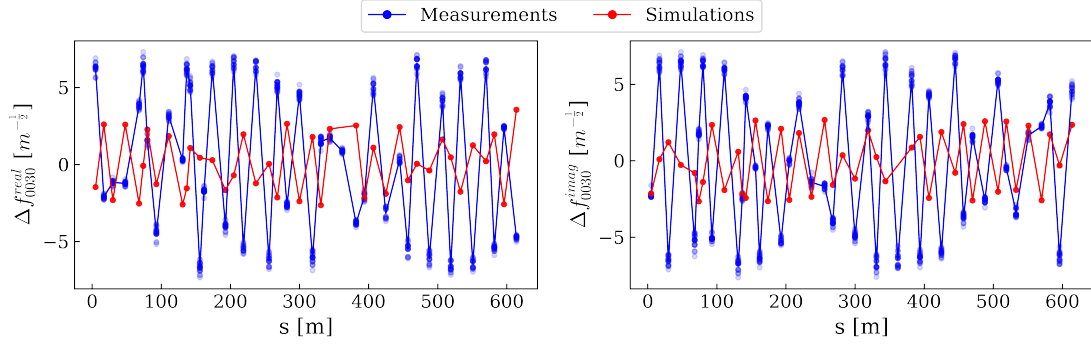


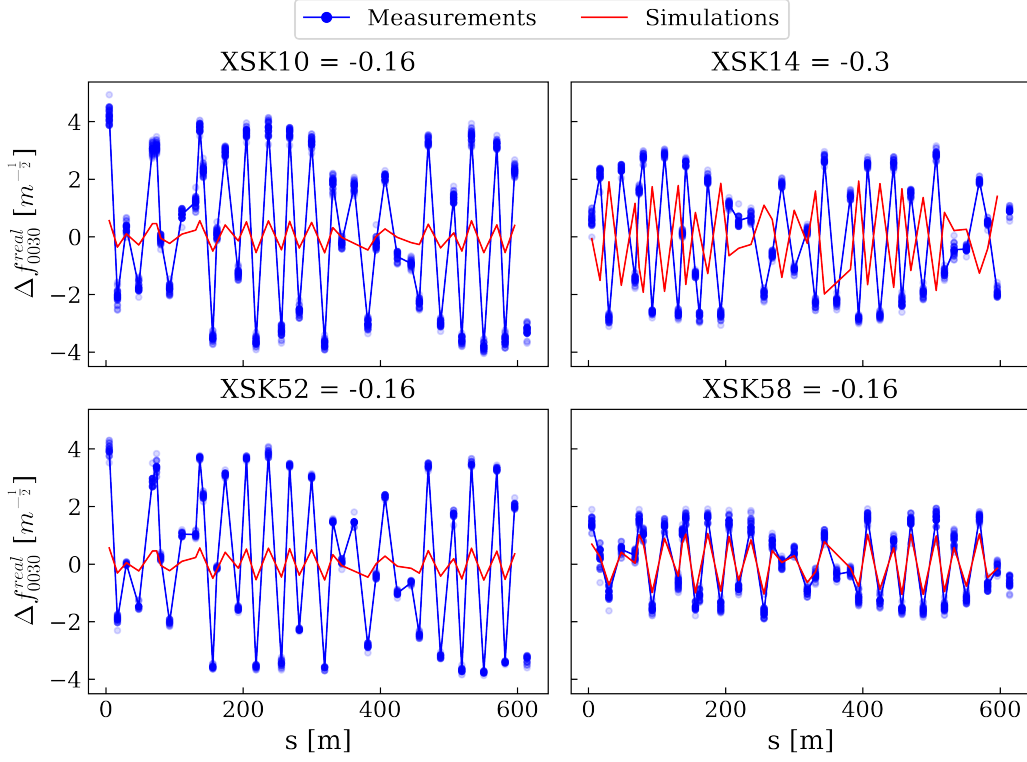
Figure 5.7: The difference between the naturally driven f_{0030} with the excited RDT using the inverted BGI shielding corrections for both the measurements and the tracking simulations.

5.3 Model improvement

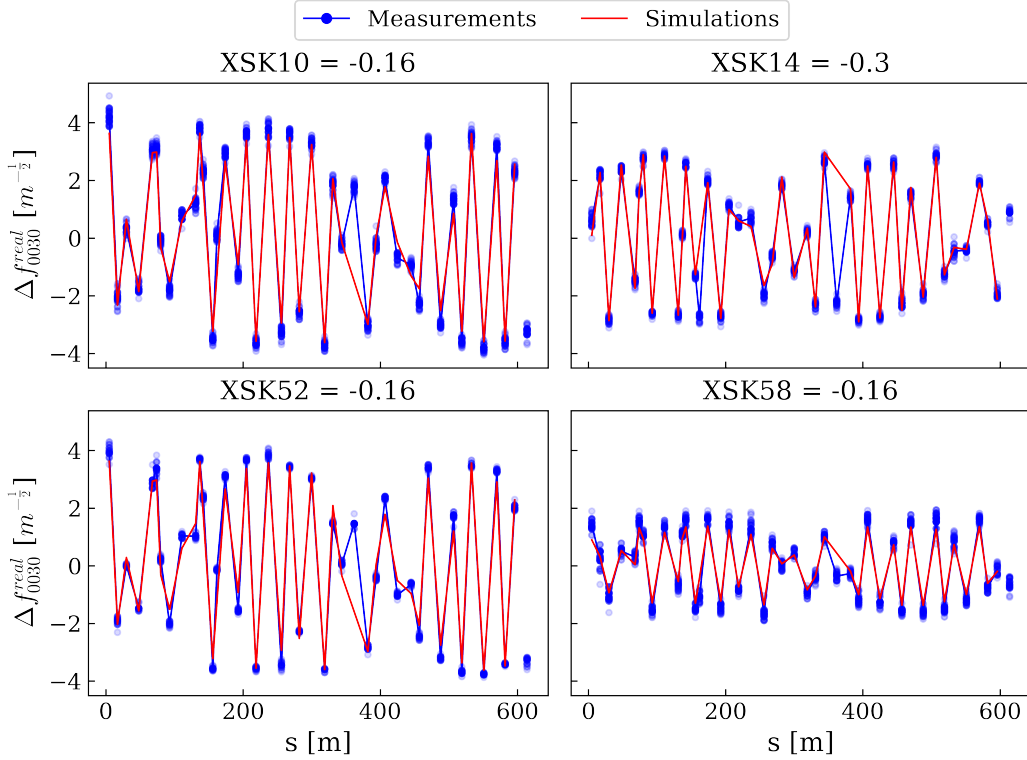
For future resonance studies in the PS, it is important to replicate these RDTs in the MAD-X model, specifically modeling the effect of the four skew sextupoles on the RDT. To achieve this, MAD-X is used to track a particle's behavior over 10,000 turns with the skew sextupoles off, mirroring the setup of the naturally driven RDT measurement. Another tracking simulation is then conducted using the inverted BGI shielding corrections. The difference between the inverted corrections RDT and the naturally driven RDT for both simulations and measurements should align if the skew sextupoles are accurately modeled. The results are shown in Fig. 5.7, and they do not agree at all, indicating an inconsistency between the MAD-X model and the machine.

To further study the effect of individual skew sextupoles on the f_{0030} RDT, additional measurements were conducted where the strengths of each skew sextupole (abbreviated as XSK in the PS) were altered individually, rather than all at once as was the case for the inverted BGI shielding corrections. This approach allows for a detailed comparison between the real and imaginary parts of the measured and simulated RDT at various strengths of each skew sextupole. The results of these measurements are shown in Fig. 5.8a. Only the real part of the RDT is displayed for one of the strengths measured, as the imaginary part and other skew sextupole strength settings yielded similar data.

Although the tracking simulations and measurements do not align again, the oscillations of the real part of the RDT are in phase between the measurements and the simulations, indicating that the sextupole strengths used in the simulation are off by a factor. These factors were calculated by taking the mean of the $\frac{f_{0030}^{\text{measured}}}{f_{0030}^{\text{simulated}}}$ values of the real and imaginary part of multiple measurements at each BPM. To do this, it is important to note that the same linear response was seen for all measurements performed at different strengths of that skew sextupole. The final factors are presented in Table 5.2. When the tracking simulations are repeated with these factors applied to the strengths of



(a) Comparison between the measurements and tracking simulations using the same skew sextupole strengths.



(b) Comparison between the measurements and tracking simulations where the tracking simulations include the extra factors presented in Table 5.2.

Figure 5.8: The differences between the real part of the naturally driven f_{0030} RDT and the excited RDT using the individual skew sextupoles (XSK) for both the measurements and the tracking simulations.

	XSK10	XSK14	XSK52	XSK58
factor	6.5	-1.5	6.5	1.5

Table 5.2: The missing factors between the model skew sextupole (XSK) strengths and the skew sextupole strengths in the machine.

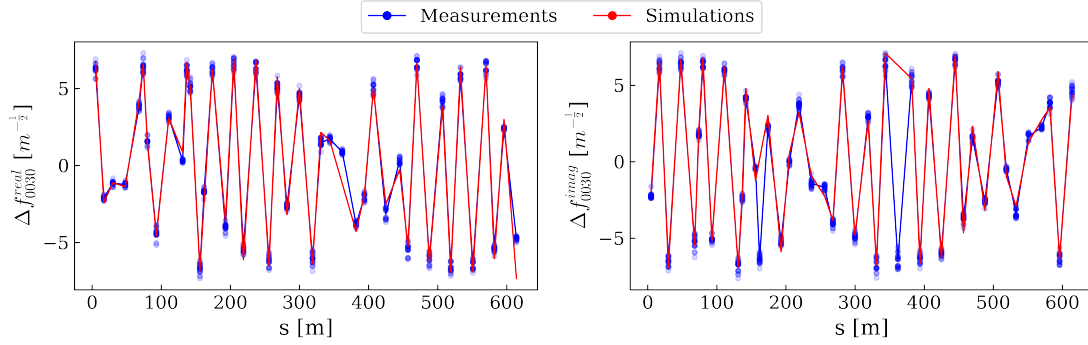


Figure 5.9: The difference between the naturally driven f_{0030} with the excited RDT using the inverted BGI shielding corrections for both the measurements and the tracking simulations with the updated factors from Tab. 5.2.

the skew sextupoles, the agreement between measurements and simulations improves significantly, as shown in Fig. 5.8b. As a final test of these factors, the effect of the inverted BGI shielding correction on the RDT was simulated using these adjusted factors. The result is shown in Fig. 5.9, demonstrating that the measurements and simulations now fully agree. These factors also proved consistent when other skew sextupole resonances, in particular the f_{2010} RDT, were measured and compared to simulations.

Discovering a discrepancy between the machine implementation and the MAD-X model is important for the future operation of the PS. After the RDT study pointed to a problem with these circuits further investigation revealed that the PS control system expects to set the skew sextupole strengths via integrated strengths, however the wrong length value had been set in the control system for skew sextupoles XSK10 and XSK52. The length of these sextupoles is 0.15 m, resulting in a factor 6.66 difference between the actual skew sextupole strength and the one set in the PS control system, resolving the factor mismatch between measurements and simulations for these sextupoles. The skew sextupoles XSK12 and XSK58 are of a different design and their factor mismatch remains unclear, although the opposite sign for XSK14 points to a cable inversion in the machine. With this finding, an improvement has been made to accurately calibrate and model the machine's components to ensure precise and effective operation.

	Natural tune	Driven tune	$H(-1, -1)$	$V(-2, 0)$
x	0.314	0.319	0.322	\
y	0.354	0.359	\	0.362

Table 5.3: The natural and driven tunes used for the f_{2010} measurements.

	XSK10	XSK14	XSK52	XSK58
strengths	0.0074	0.0433	0.0138	0.0014

Table 5.4: The skew sextupole strengths used by the MAD-X model to develop the f_{2010} RDT correction.

5.4 f_{2010} RDT Correction

The effects of skew sextupoles on the RDT line driven by these elements have now been successfully simulated, enabling the possibility of performing fast online corrections for this resonance type. Such a correction was first applied to the f_{0030} , in which the RDT was corrected down to the noise floor. However, the naturally driven $3Q_y$ resonance is already very small, and did not induce any beam losses, and no differences were observable in the loss maps between the corrected and uncorrected states.

As a result, using this resonance to showcase the efficiency and advantages of RDT-based corrections over loss map-based corrections would be ineffective. Instead, the correction will focus on the $2Q_x + 1Q_y$ resonance, driven by the f_{2010} RDT. This resonance caused the most significant beam losses in the PS during 2020 (Fig. 2.18 [49]) and is also a skew sextupole resonance. The natural and driven tunes, along with the locations of the RDT lines corresponding to these tunes, are listed in Table 5.3.

For this resonance, only the naturally driven RDT is measured. Two lines will be visible: one in the horizontal spectrum and one in the vertical spectrum. The subsequent measurement and analysis process mirrors the method used for the f_{0030} RDT. The results are shown in Fig. 5.10, where it is notable that both lines have approximately double the amplitude of the previously measured naturally driven f_{0030} RDT. These measurements were also used to reconfirmed the skew sextupole factors presented in Table 5.2.

Using the calibration factors for skew sextupole current to strength conversion identified in the previous section it is possible to make corrections to this RDT line using response matrices derived from model simulations. These matrices enable an analytical calculation of the skew sextupole correction strengths through calculating the pseudo-inverse of the response matrix and multiplying it with the measured data from the BPMs. This approach provides a fast online correction method, eliminating the need for the trial-and-error process required in the loss map correction method.

The skew sextupole strengths that lead to a correction are listed in Table 5.4, with the resulting RDT measurements shown in Fig. 5.11. The RDT line in the horizontal

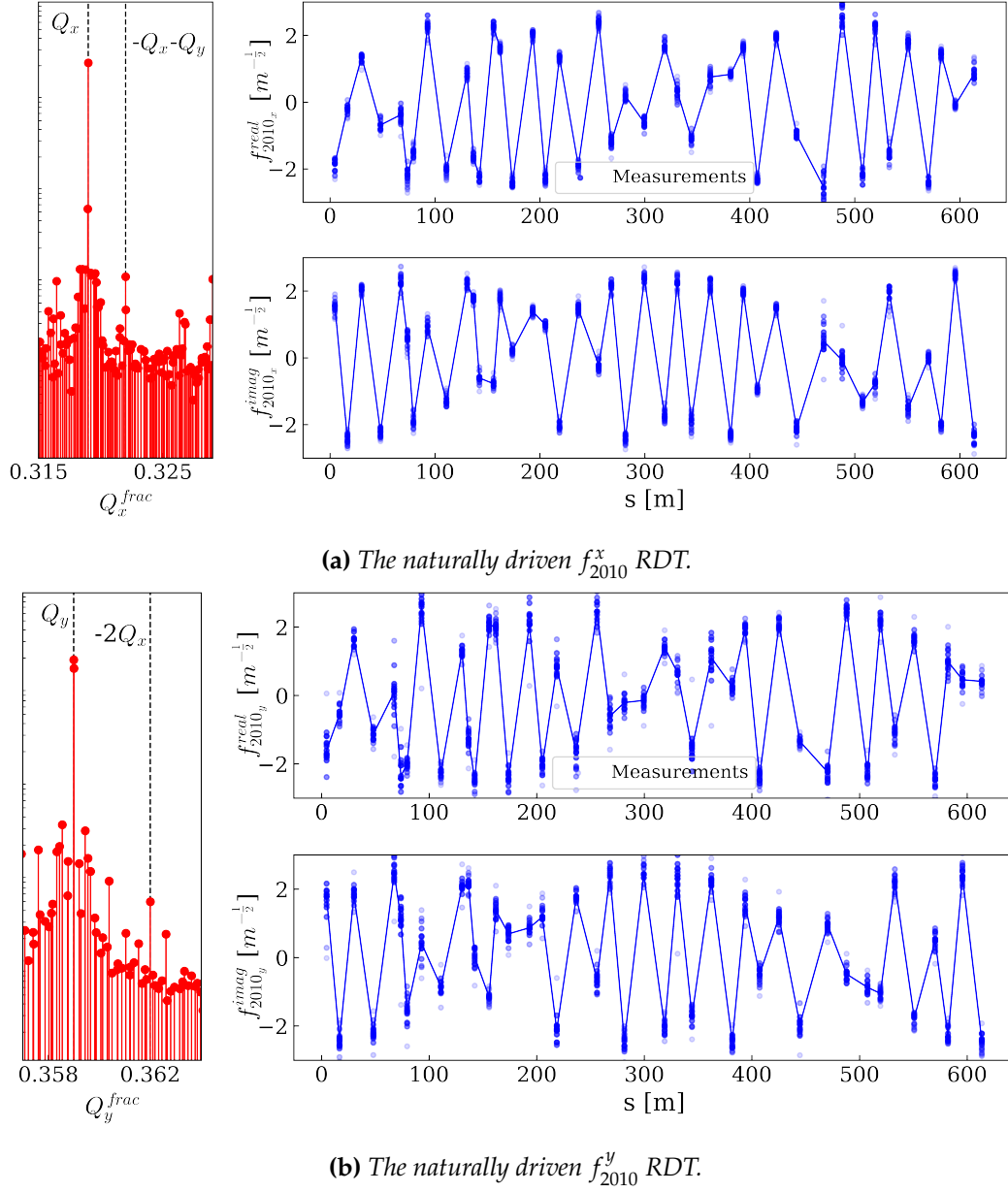


Figure 5.10: The naturally driven f_{2010} RDTs with the frequency spectrum of BPM10 on the right and the shot-to-shot variation of all BPMs to the left.

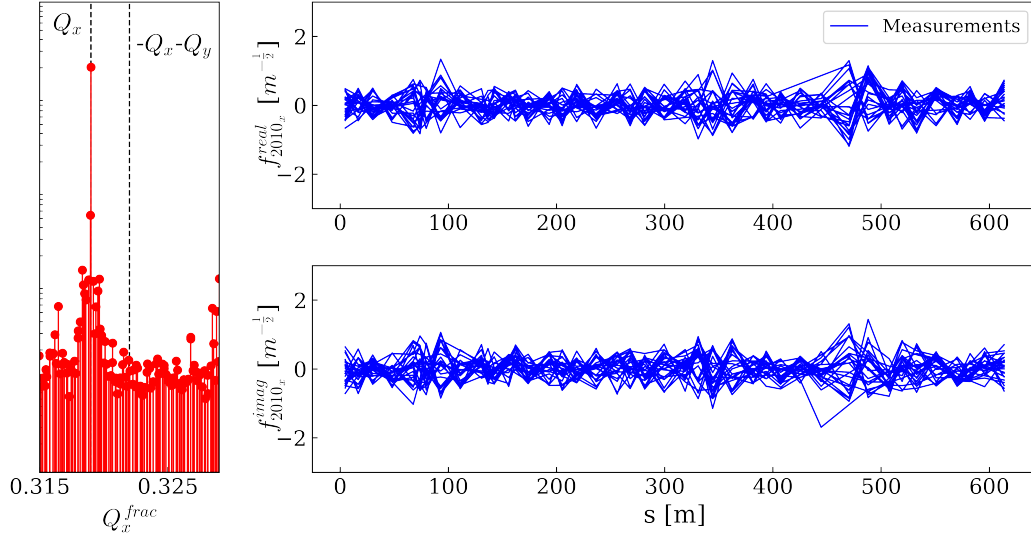
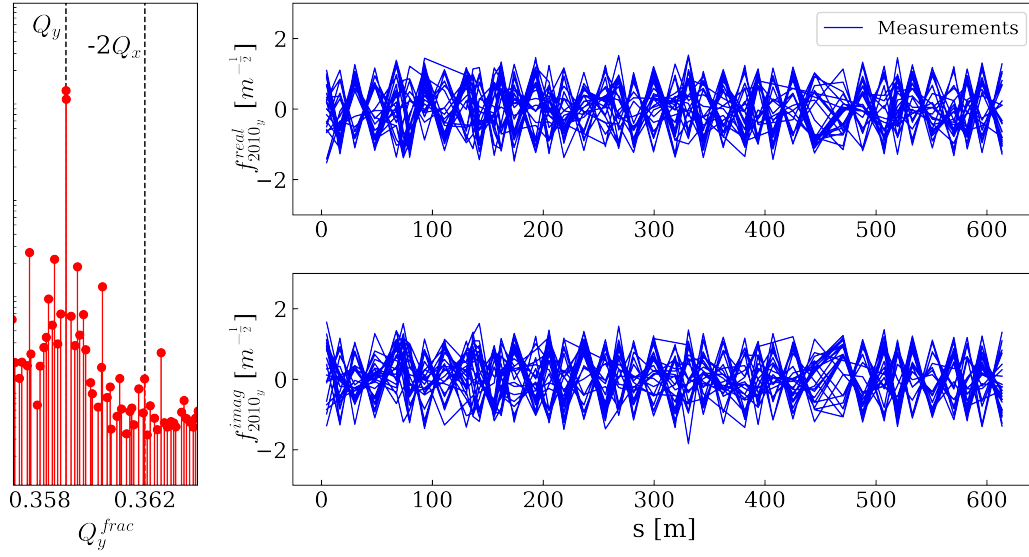
(a) The corrected f_{2010}^x RDT.(b) The corrected f_{2010}^y RDT.

Figure 5.11: The corrected f_{2010} RDTs with the frequency spectrum of BPM10 on the right and the shot-to-shot variation of all BPMs to the left.

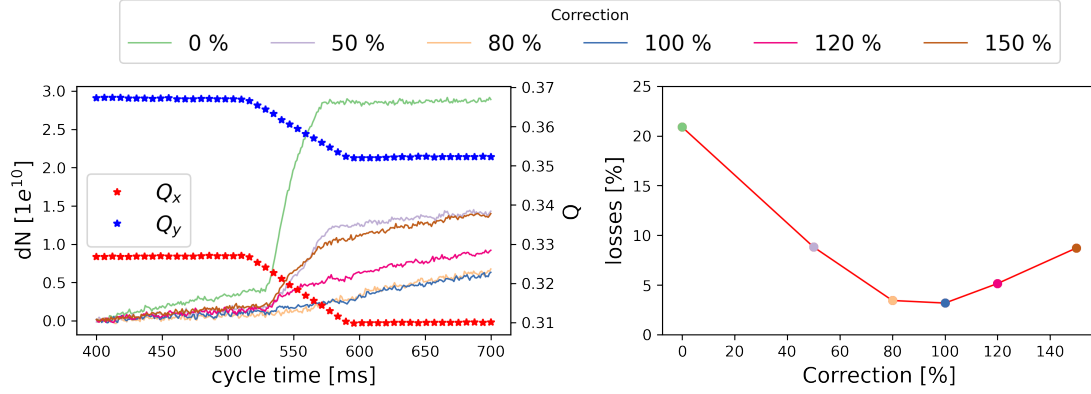


Figure 5.12: The intensity losses when crossing the $2Q_x+1Q_y$ resonance for different percentages of corrections presented in Tab. 5.4.

spectrum is fully corrected to the noise level, as indicated by the lack of agreement between different shots. The vertical RDT line still shows remnants, though its amplitude is greatly reduced. Some phase agreement between a few shots indicates that, depending on the noise of each measurement, the RDT can be either seen or just noise is picked up.

Given that the naturally driven f_{2010} RDT produces significant losses when the resonance is crossed, it is interesting to assess how the correction affects these losses. The result of this measurement is shown in Fig. 5.12. This figure on the left shows the time variation of the tune trims applied to cross the resonance in red and blue, with the measured intensity loss of different percentages of the correction in the other colours at the same times. The right plot on the figure shows the percentage of the intensity which was lost in the crossing of the resonance. No correction shows a significant beam loss of $\approx 20\%$, this decreases to a minimum of 5 % when the full correction is applied. There is a substantial reduction in losses, demonstrating that the correction effectively mitigates the impact of the $2Q_x+1Q_y$ resonance. This proves that resonances in the PS can be corrected quickly and effectively using the forced oscillations from the ADT. This method would also allow to correct multiple resonances of the same type using the same correction given that the correctors are sufficiently in phase with the errors, even though it was not tested in this study.

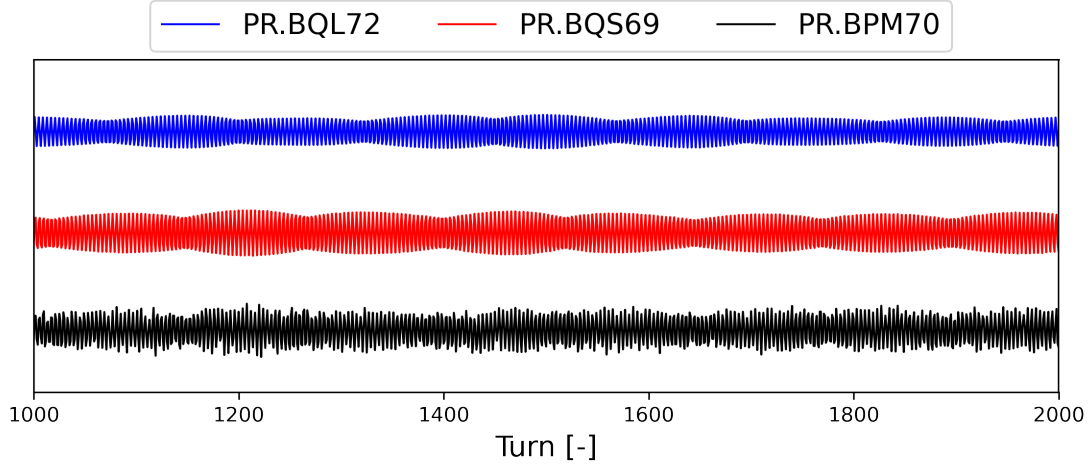


Figure 5.13: *The turn-by-turn data recorded by the two specialized BBQ pick-ups (PR.BQL72 & PR.BQS69) and from their neighboring BPM70.*

5.5 BBQ noise reduction

From the previous sections, it is clear that the noise level in the frequency spectrum is a crucial factor in determining how efficiently the forced oscillation turn-by-turn data can be utilized. This noise level is influenced by the action of the ADT kicks, the number of turns used in the analysis, and the precision of the BPMs. As mentioned before, the ADT has a limited kick strength, and the amount of turn-by-turn data that can be collected is also constrained by the machine operation. A possible method to further reduce the noise in the frequency spectrum, would be to use specialized BBQ pick-ups, which typically have less noise, as a high-precision BPM. The difference in noise is illustrated by the turn-by-turn data in Fig. 5.13.

In principle, if turn-by-turn data with very low noise from the BBQ pick-up could be incorporated alongside the regular BPMs, the SVD cleaning might be able to more effectively clean uncorrelated noise. The main challenge in this approach is synchronizing the timing of both turn-by-turn data sets, as even a single turn's error can deteriorate the analysis rather than improve it. Another issue is the difference in amplitude between the pick-ups and the BPMs, as they are not typically used together, which can lead to significant variations in their recorded amplitudes.

To address these issues, the extraction kicker is employed to kick the beam near the end of the recorded BPM turn-by-turn data after using the ADT to make forced oscillations. The extraction kicker is a fast kicker that can operate within a single turn. By using the large excitation visible in both the BPM data and the pick-up data, the timing can be synchronized accurately at the 1 turn level. The action of the ADT-excitations before the 1 turn kick is used for amplitude scaling between the BBQ pick-ups and the regular

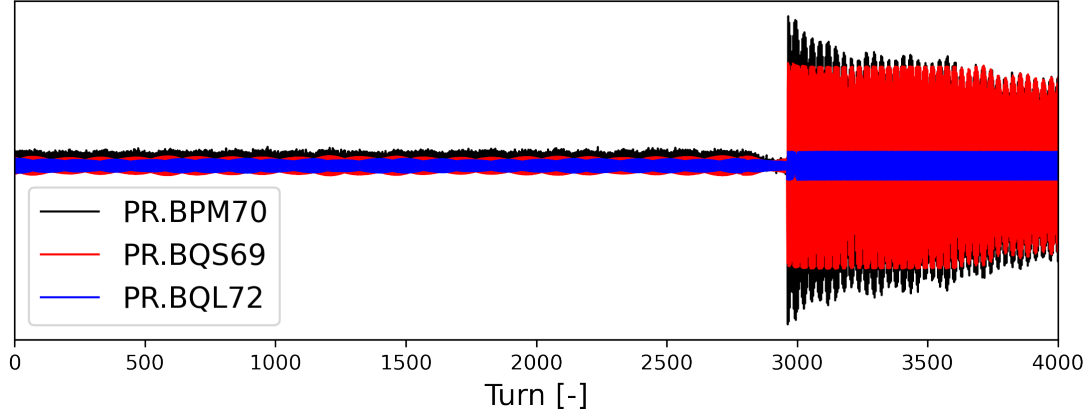


Figure 5.14: The synchronised and scaled turn-by-turn data from the BBQ pick-ups to BPM70.

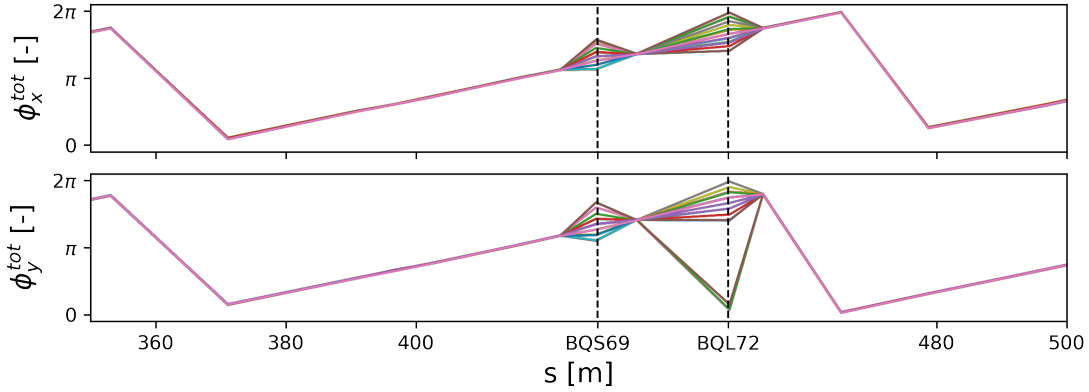


Figure 5.15: The total phase advance along the PS ring, calculated based on the turn-by-turn data using the BPMs and the BBQ pick-ups. The locations of the pick-ups are highlighted.

BPMs. The synchronised timing and amplitude data are shown in Fig. 5.14 where the forced ADT-oscillations end near turn 2800 and the single kick happens shortly after.

The turn-by-turn data from the pick-ups is now ready for use in spectrum analysis. However, they were initially being removed by the SVD cleaning process, which identified them as faulty BPMs. Upon further inspection, it was discovered that the phases of the newly introduced pick-ups were misaligned with the expected phase advance. This issue is demonstrated in Fig. 5.15, which shows that the phase at the locations of the pick-ups is not only incorrect but also varies by a fraction of the RF timing from shot-to-shot.

The issue of shot-to-shot phase variation was reported and addressed during the end-of-year shutdown. Despite these efforts, the following year the phase differences persisted, halting the progress of this study. Resolving this issue is valuable because once it is addressed, integrating the BBQ pick-ups into the forced oscillation turn-by-turn data

holds significant promise for increasing the precision of RDT measurements, for both the PS and other machines where BPM noise is a more prominent issue.

5.6 Summary

This chapter demonstrates the utility of using forced oscillation turn-by-turn data from an ADT to measure RDTs and highlights their advantages for accelerators lacking an AC-Dipole. The studies based on ADT forced oscillations not only enhance the understanding of non-linear optics but also offer broader implications for improving beam diagnostics and resonance correction strategies across various machines.

The use of ADT-induced oscillations enabled a recalibration of the PS BPMs by correcting relative calibration errors, some of which exceeded 5%. These corrections led to improved agreement of the measured dispersion with the theoretical model and the values measured by the wire scanners. Additionally, discrepancies between the PS control system and the MAD-X model were discovered. These investigations revealed errors in skew sextupole calibration, including a misconfigured length parameter in the control system and potential cabling issues. Resolving these discrepancies aligned simulations with measurements, enhancing the accuracy of the machine model and highlighting the value of RDT-based studies in diagnosing hardware and software inconsistencies.

ADT-based RDT measurements also proved useful in identifying and quantifying non-linear sources. For instance, the subtle residual effects of the redesigned BGI shielding on the f_{0030} RDT, which have been undetected using loss maps, were successfully measured by the RDTs. Furthermore, using the f_{2010} RDT, skew sextupole corrections could be analytically calculated via response matrices, resulting in a significant reduction of losses caused by the $2Q_x+1Q_y$ resonance. By enabling accurate resonance identification, machine model improvements, and efficient resonance corrections, this approach establishes ADT-based RDT measurements as a promising diagnostic tool for accelerators without a dedicated AC-Dipole.

CONCLUSION

In this thesis, several challenges and advancements in accelerator physics have been addressed, with a focus on the operational improvements and beam diagnostics in the PS, with broader implications for synchrotrons worldwide. Each chapter presented novel methods and insights that contribute to the field.

In Chapter 3, the difficulties of controlling combined function magnets in the PS for tune and chromaticity correction were addressed. Traditionally, these challenges have hindered operational flexibility, particularly in the development of new operational cycles, due to the complex nature of the Pole Face Windings (PFWs) control circuits. This chapter explored the application of machine learning techniques to overcome these challenges. By introducing a physics-based loss function and improvements in data acquisition and cleaning to a neural network, a tune and chromaticity control system was developed, enabling precise PFW adjustments across a wide range of operational conditions, including challenging regions like above and below transition energy. While these methods were tested on the PS, their success demonstrates the potential of machine learning to address non-linear control challenges in accelerator physics more broadly.

Chapter 4 tackled the complexities of horizontal emittance measurements at low energies, where dispersive contributions convolute the transverse and longitudinal beam distributions. This has historically been a key challenge to understand beam degradation through the injector chain and affects the luminosity of the LHC. To address this, zero dispersion optics were designed and implemented in the PS using low-energy quadrupoles. These optics successfully eliminated dispersive effects at measurement locations, allowing for more accurate emittance and beam tail measurements. The study revealed that horizontal emittance discrepancies previously observed between the PS Booster and PS were primarily due to differences in dispersion rather than actual emittance blow-up. Furthermore, zero dispersion optics showed that overpopulated beam tails were already present at low energies but obscured by longitudinal convolution, while they were previously thought to emerge during transition crossing.

To extend these improvements to less flexible accelerators, a numerical deconvolution method was developed, producing emittance and tail measurements closely matching those obtained using zero dispersion optics. This method was successfully applied

to the PSB, identifying overpopulated tails and improving emittance accuracy. Furthermore, the interplay between space charge effects and dispersion was investigated, validating space charge models using experimental data. This chapter demonstrated that zero dispersion optics and numerical deconvolution provide practical and effective alternatives for precise transverse beam profile measurements, with broad applicability to other accelerators in the CERN injector chain and beyond.

In Chapter 5, the use of forced oscillation turn-by-turn data from a transverse damper (ADT) was explored to measure Resonance Driving Terms (RDTs). These studies addressed the challenge of resonance analysis and correction in accelerators without a dedicated AC-dipole, which includes most machines. By utilizing ADT-induced oscillations, relative calibration errors in Beam Position Monitors (BPMs) were corrected, significantly improving the alignment of measured dispersion with the theoretical model. Additionally, discrepancies in skew sextupole calibration were identified and resolved, improving the accuracy of the machine model and demonstrating the diagnostic power of RDT-based studies.

The sensitivity of RDT measurements was further highlighted by the detection of residual effects from the BGI shielding, which were not observed using traditional loss maps. Using the f_{2010} RDT, corrections were analytically calculated and applied online, leading to a substantial reduction in beam losses caused by the $2Q_x+1Q_y$ resonance. These results demonstrated that ADT-based RDT measurements enable efficient resonance identification and correction, offering a faster and more precise alternative to loss map methods. Moreover, this approach has significant implications for accelerators without AC-dipoles, providing a versatile diagnostic tool for studying non-linear optics and improving machine performance.

The findings of this thesis demonstrate the potential of advanced diagnostic and correction techniques to address long-standing challenges in accelerator physics. Machine learning-based control offers a practical solution for tune and chromaticity corrections in highly complex scenarios, while zero dispersion optics and numerical deconvolution methods enable more accurate beam measurements and a deeper understanding of beam dynamics. ADT-based RDT measurements provide a sensitive and efficient tool for resonance analysis and correction, extending their applicability of forced RDT analysis to machines lacking dedicated AC-Dipoles. Collectively, these advancements represent significant contributions to the field, improving the precision, flexibility, and efficiency of accelerator operations, and paving the way for broader adoption of these methods in particle accelerators.

APPENDICES

APPENDIX A: GENERAL FORMULAE FOR DISPERSION-BEATING

The following derivation is the unpublished work of M. Giovannozzi, presented to the author through personal correspondence.

In the following the main formulae to evaluate the β -beating in a magnetic lattice are collected, according to the notation used in [99]. If a distribution of quadrupole errors $\delta k(s)$ is given, then this will generate the following

$$\Delta Q_z = \mp \frac{1}{4\pi} \int_0^C ds \beta_z(s) \delta k(s) \quad (\text{A.1})$$

$$\frac{\Delta \beta_z(s)}{\beta_z(s)} = \mp \frac{1}{2 \sin \pi Q_z} \int_s^{s+C} d\sigma \beta_z(\sigma) \delta k(\sigma) \cos 2 [\mu_z(\sigma) - \mu_z(s) - \pi Q_z] \quad (\text{A.2})$$

where C stands for the ring circumference, $z = x, y$ and the upper - sign should be used for the horizontal plane.

The value of the horizontal dispersion function is given by

$$D(s) = \frac{\sqrt{\beta_x(s)}}{2 \sin \pi Q_x} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \cos [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] \quad (\text{A.3})$$

where $\rho_0(s)$ stands for the local radius of curvature.

The expression for the dispersion-beating can be found by assuming a given distribution of errors $\delta k(s)$ and using Eqs. A.1 and A.2. The following relations can be found

$$\sqrt{\beta_x(s) + \Delta \beta_x(s)} \approx \sqrt{\beta_x(s)} + \frac{1}{2} \frac{\Delta \beta_x(s)}{\sqrt{\beta_x(s)}} \quad (\text{A.4})$$

$$\frac{1}{\sin \pi (Q_x + \Delta Q_x)} \approx \frac{1}{\sin \pi Q_x} (1 - \pi \Delta Q_x \cot \pi Q_x) \quad (\text{A.5})$$

$$\begin{aligned} \cos [(\mu_x(\sigma) + \Delta \mu_x(\sigma)) - (\mu_x(s) + \Delta \mu_x(s)) - \pi (Q_x + \Delta Q_x)] &\approx \cos [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \\ &+ (-\Delta \mu_x(\sigma) + \Delta \mu_x(s) + \pi \Delta Q_x) \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] \end{aligned} \quad (\text{A.6})$$

and the error on the phase advance can be found as

$$\Delta\mu_x(s) = - \int_0^s d\sigma \frac{1}{\beta_x(\sigma)} \frac{\Delta\beta_x(\sigma)}{\beta_x(\sigma)} \quad (\text{A.7})$$

Equation A.3 can be rewritten to evaluate the dispersion-beating. Considering that $\widehat{D}(s) = D(s) + \Delta D(s)$ one obtains

$$\begin{aligned} \widehat{D}(s) &\approx \frac{1}{2 \sin \pi Q_x} (1 - \pi \Delta Q_x \cot \pi Q_x) \left(\sqrt{\beta_x(s)} + \frac{1}{2} \frac{\Delta\beta_x(s)}{\sqrt{\beta_x(s)}} \right) \times \\ &\times \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \left(\sqrt{\beta_x(\sigma)} + \frac{1}{2} \frac{\Delta\beta_x(\sigma)}{\sqrt{\beta_x(\sigma)}} \right) \times \\ &\times \{ \cos [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + (-\Delta\mu_x(\sigma) + \Delta\mu_x(s) + \pi \Delta Q_x) \times \\ &\times \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] \} \end{aligned} \quad (\text{A.8})$$

Retaining only first-order contributions to the dispersion beating, where by this we mean first-order in the errors δk , one can recast Eq. A.8 in the following form

$$\begin{aligned} \widehat{D}(s) &\approx \frac{1}{2 \sin \pi Q_x} \left(\sqrt{\beta_x(s)} + \frac{1}{2} \frac{\Delta\beta_x(s)}{\sqrt{\beta_x(s)}} - \pi \Delta Q_x \cot \pi Q_x \sqrt{\beta_x(s)} \right) \times \\ &\times \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \left[\left(\sqrt{\beta_x(\sigma)} + \frac{1}{2} \frac{\Delta\beta_x(\sigma)}{\sqrt{\beta_x(\sigma)}} \right) \cos [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \right. \\ &\left. + \sqrt{\beta_x(\sigma)} (-\Delta\mu_x(\sigma) + \Delta\mu_x(s) + \pi \Delta Q_x) \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] \right] \\ &\approx D(s) + \frac{\pi \Delta Q_x}{2 \sin \pi Q_x} \sqrt{\beta_x(s)} \left[\int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \right. \\ &\left. - \cot \pi Q_x \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \cos [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] \right] + \\ &+ \frac{\sqrt{\beta_x(s)}}{2 \sin \pi Q_x} \left[\frac{1}{2} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \frac{\Delta\beta_x(\sigma)}{\sqrt{\beta_x(\sigma)}} \cos [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \right. \\ &+ \Delta\mu_x(s) \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \\ &\left. - \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \Delta\mu_x(\sigma) \sqrt{\beta_x(\sigma)} \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \right. \\ &\left. + \frac{1}{2} \frac{\Delta\beta_x(s)}{\beta_x(s)} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \cos [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] \right] \end{aligned} \quad (\text{A.9})$$

Considering now the normalized dispersion, one obtains

$$\begin{aligned}
\frac{\Delta D(s)}{\sqrt{\beta_x(s)}} &\approx \frac{1}{4 \sin \pi Q_x} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \frac{\Delta \beta_x(\sigma)}{\sqrt{\beta_x(\sigma)}} \cos [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \\
&+ \frac{\pi \Delta Q_x}{2 \sin \pi Q_x} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \\
&+ \frac{1}{2 \sin \pi Q_x} \left[\Delta \mu_x(s) \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \right. \\
&- \left. \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \Delta \mu_x(\sigma) \sqrt{\beta_x(\sigma)} \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] \right] + \\
&+ \left[\frac{1}{2} \frac{\Delta \beta_x(s)}{\beta_x(s)} - \pi \Delta Q_x \cot \pi Q_x \right] \frac{D(s)}{\sqrt{\beta_x(s)}}
\end{aligned} \tag{A.10}$$

Deriving the dispersion function with respect to s yields

$$\begin{aligned}
D'(s) &= \frac{\beta'_x(s)}{4 \sin \pi Q_x \sqrt{\beta_x(s)}} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \cos [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \\
&+ \frac{\mu'_x(s) \sqrt{\beta_x(s)}}{2 \sin \pi Q_x} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] \\
&= -D(s) \frac{\alpha_x(s)}{\beta_x(s)} + \frac{1}{2 \sin \pi Q_x \sqrt{\beta_x(s)}} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x]
\end{aligned} \tag{A.11}$$

where use has been made of the following fundamental relations

$$\mu'_z(s) = \frac{1}{\beta_z(s)} \quad \alpha_z(s) = -\frac{1}{2} \beta'_z(s) \tag{A.12}$$

Using Eq. A.11, we can recast some of the terms in Eq. A.10 in a different form, namely

$$\begin{aligned}
\frac{\pi \Delta Q_x}{2 \sin \pi Q_x} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] &= \\
&= \pi \Delta Q_x \sqrt{\beta_x(s)} \left(D'(s) + D(s) \frac{\alpha_x(s)}{\beta_x(s)} \right)
\end{aligned} \tag{A.13}$$

and

$$\begin{aligned}
\frac{\Delta \mu_x(s)}{2 \sin \pi Q_x} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] &= \\
&= \Delta \mu_x(s) \sqrt{\beta_x(s)} \left(D'(s) + D(s) \frac{\alpha_x(s)}{\beta_x(s)} \right)
\end{aligned} \tag{A.14}$$

and finally one obtains

$$\begin{aligned}
 \frac{\Delta D(s)}{\sqrt{\beta_x(s)}} &\approx \frac{1}{4 \sin \pi Q_x} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \frac{\Delta \beta_x(\sigma)}{\beta_x(\sigma)} \sqrt{\beta_x(\sigma)} \cos [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \\
 &- \frac{1}{2 \sin \pi Q_x} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \Delta \mu_x(\sigma) \sqrt{\beta_x(\sigma)} \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \\
 &+ \frac{1}{2} \frac{\Delta \beta_x(s)}{\beta_x(s)} \frac{D(s)}{\sqrt{\beta_x(s)}} + \sqrt{\beta_x(s)} \left(D'(s) + D(s) \frac{\alpha_x(s)}{\beta_x(s)} \right) (\pi \Delta Q_x + \Delta \mu_x(s)) + \\
 &- \pi \Delta Q_x \cot \pi Q_x \frac{D(s)}{\sqrt{\beta_x(s)}}
 \end{aligned} \tag{A.15}$$

The first two terms in Eq. A.15 provide a non-zero contribution only when ρ_0 is finite, i.e. in dipoles or whenever a non-zero dipolar feed-down effect arises. The other terms provides contributions proportional to complicated functions of the optical parameters. This indicates that even in the case of gradient errors that do not induce any tune change, the normalized dispersion could still be affected by the β -beating and phase-advance beating.

All the integrals can be replaced by sums assuming the discrete character of the errors' distribution, namely $\delta k(s) = \sum_{i=1}^N k_i \delta(s - s_i)$, where $\delta(s)$ is the Dirac's delta. In this case, Eqs. A.1 and A.2 can be recast in the following form

$$\Delta Q_z = \mp \frac{1}{4\pi} \sum_{i=1}^N k_i \beta_{z,i} \tag{A.16}$$

$$\frac{\Delta \beta_z(s)}{\beta_z(s)} = \mp \frac{1}{2 \sin \pi Q_z} \sum_i k_i \beta_{z,i} \cos 2 [\mu_{z,i} - \mu_z(s) - \pi Q_z] \tag{A.17}$$

where the notation $\beta_{z,i} = \beta_z(s_i)$ has been used (and similarly for μ). An expression for $\Delta \mu_x(s)$ can also be derived in the following form

$$\Delta \mu_x(s) = \frac{\sin \mu_x(s)}{2 \sin \pi Q_x} \sum_i k_i \beta_{x,i} \cos 2 \left[\mu_{x,i} - \frac{1}{2} \mu_x(s) - \pi Q_x \right] \tag{A.18}$$

where use has been made of some interesting integrals, i.e.

$$\int_0^s d\sigma \frac{\cos 2\mu_x(\sigma)}{\beta_x(\sigma)} \quad \text{or} \quad \int_0^s d\sigma \frac{\sin 2\mu_x(\sigma)}{\beta_x(\sigma)} \tag{A.19}$$

that can be computed thanks to the change of variable $\mu_x(\sigma) = t$ and the first relation in Eq. A.12. Note that the symbol $\sum_i$ indicates a sum starting from the index i satisfying $s_i \geq s$.

The dispersion-beating in its final form reads

$$\begin{aligned}
\frac{\Delta D(s)}{\sqrt{\beta_x(s)}} \approx & -\frac{1}{8 \sin^2 \pi Q_x} \sum_i k_i \beta_{x,i} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \cos 2 [\mu_{x,i} - \mu_x(\sigma) - \pi Q_x] \times \\
& \times \cos [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \\
& + \frac{1}{8 \sin^2 \pi Q_x} \sum_i k_i \beta_{x,i} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \sin 2 [\mu_{x,i} - \mu_x(\sigma) - \pi Q_x] \times \\
& \times \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \\
& - \frac{1}{4 \sin \pi Q_x} \frac{D(s)}{\sqrt{\beta(s)}} \sum_i k_i \beta_{x,i} \cos 2 [\mu_{x,i} - \mu_x(s) - \pi Q_x] + \\
& + \frac{\sin \mu_x(s)}{2 \sin \pi Q_x} \sqrt{\beta_x(s)} \left(D'(s) + D(s) \frac{\alpha_x(s)}{\beta(s)} \right) \sum_i k_i \beta_{x,i} \cos 2 \left[\mu_{x,i} - \frac{1}{2} \mu_x(s) - \pi Q_x \right] + \\
& - \frac{1}{4 \sin \pi Q_x} \sqrt{\beta_x(s)} \left(D'(s) + D(s) \frac{\alpha_x(s)}{\beta(s)} \right) \sum_i k_i \beta_{x,i} \sin 2 [\mu_{x,i} - \pi Q_x] + \\
& - \frac{1}{4} \left[\sqrt{\beta_x(s)} \left(D'(s) + D(s) \frac{\alpha_x(s)}{\beta_x(s)} \right) - \cot \pi Q_x \frac{D(s)}{\sqrt{\beta_x(s)}} \right] \sum_{i=1}^N k_i \beta_{x,i}
\end{aligned} \tag{A.20}$$

Equation A.20 can be recast in a different form by using the following relationship

$$\begin{aligned}
& \cos 2 [\mu_{x,i} - \mu_x(\sigma) - \pi Q_x] \cos [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] = \\
& = \frac{1}{2} \cos [2\mu_{x,i} - \mu_x(\sigma) - \mu_x(s) - 3\pi Q_x] + \frac{1}{2} \cos [2\mu_{x,i} - 3\mu_x(\sigma) + \mu_x(s) - \pi Q_x] = \\
& = \frac{1}{2} \cos [2(\mu_{x,i} - \mu_x(s) - 2\pi Q_x) - (\mu_x(\sigma) - \mu_x(s) - \pi Q_x)] + \\
& + \frac{1}{2} \cos [2(\mu_{x,i} - \mu_x(s) - 2\pi Q_x) - 3(\mu_x(\sigma) - \mu_x(s) - \pi Q_x)]
\end{aligned} \tag{A.21}$$

where, each of the two cosine terms can be further expressed as

$$\begin{aligned}
& \cos [2(\mu_{x,i} - \mu_x(s) - 2\pi Q_x) - (\mu_x(\sigma) - \mu_x(s) - \pi Q_x)] = \\
& = \cos 2 [\mu_{x,i} - \mu_x(s) - 2\pi Q_x] \cos [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \\
& + \sin 2 [\mu_{x,i} - \mu_x(s) - 2\pi Q_x] \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x]
\end{aligned} \tag{A.22}$$

and

$$\begin{aligned}
& \cos [2(\mu_{x,i} - \mu_x(s) - 2\pi Q_x) - 3(\mu_x(\sigma) - \mu_x(s) - \pi Q_x)] = \\
& = \cos 2 [\mu_{x,i} - \mu_x(s) - 2\pi Q_x] \cos 3 [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] + \\
& + \sin 2 [\mu_{x,i} - \mu_x(s) - 2\pi Q_x] \sin 3 [\mu_x(\sigma) - \mu_x(s) - \pi Q_x]
\end{aligned} \tag{A.23}$$

Similarly, one can write

$$\begin{aligned}
 & \sin 2 [\mu_{x,i} - \mu_x(\sigma) - \pi Q_x] \sin [\mu_x(\sigma) - \mu_x(s) - \pi Q_x] = \\
 & = -\frac{1}{2} \cos [2\mu_{x,i} - \mu_x(\sigma) - \mu_x(s) - 3\pi Q_x] + \frac{1}{2} \cos [2\mu_{x,i} - 3\mu_x(\sigma) + \mu_x(s) - \pi Q_x] = \\
 & = -\frac{1}{2} \cos [2(\mu_{x,i} - \mu_x(s) - 2\pi Q_x) - (\mu_x(\sigma) - \mu_x(s) - \pi Q_x)] + \\
 & + \frac{1}{2} \cos [2(\mu_{x,i} - \mu_x(s) - 2\pi Q_x) - 3(\mu_x(\sigma) - \mu_x(s) - \pi Q_x)].
 \end{aligned} \tag{A.24}$$

and also in this case Eqs. A.22 and A.23 can be used to further simplify the previous expressions. If all these identities used in Eq. A.20, one obtains

$$\begin{aligned}
 \frac{\Delta D(s)}{\sqrt{\beta_x(s)}} & \approx -\frac{1}{8 \sin^2 \pi Q_x} \sum_i k_i \beta_{x,i} \int_s^{s+C} \frac{d\sigma}{\rho_0(\sigma)} \sqrt{\beta_x(\sigma)} \times \\
 & \times \cos [2(\mu_{x,i} - \mu_x(s) - 2\pi Q_x) - (\mu_x(\sigma) - \mu_x(s) - \pi Q_x)] + \\
 & - \frac{1}{4 \sin \pi Q_x} \left[\frac{D(s)}{\sqrt{\beta_x(s)}} \cos 2\mu_x(s) + \sqrt{\beta_x(s)} \left(D'(s) + D(s) \frac{\alpha_x(s)}{\beta_x(s)} \right) \sin 2\mu_x(s) \right] \times \\
 & \times \sum_i k_i \beta_{x,i} \cos 2(\mu_{x,i} - \pi Q_x) + \\
 & - \frac{1}{4 \sin \pi Q_x} \left[\frac{D(s)}{\sqrt{\beta_x(s)}} \sin 2\mu_x(s) + \sqrt{\beta_x(s)} \left(D'(s) + D(s) \frac{\alpha_x(s)}{\beta_x(s)} \right) (1 - \cos 2\mu_x(s)) \right] \times \\
 & \times \sum_i k_i \beta_{x,i} \sin 2(\mu_{x,i} - \pi Q_x) + \\
 & - \frac{1}{4} \left[\sqrt{\beta_x(s)} \left(D'(s) + D(s) \frac{\alpha_x(s)}{\beta_x(s)} \right) - \cot \pi Q_x \frac{D(s)}{\sqrt{\beta_x(s)}} \right] \sum_{i=1}^N k_i \beta_{x,i}
 \end{aligned} \tag{A.25}$$

A final step can be performed by further transforming the integral in Eq. A.25 and one obtains

$$\begin{aligned}
\frac{\Delta D(s)}{\sqrt{\beta_x(s)}} \approx & -\frac{\cos(2\mu_x(s) + \pi Q_x)}{2 \sin \pi Q_x} \left[\frac{D(s)}{\sqrt{\beta_x(s)}} \cos \pi Q_x + \sqrt{\beta_x(s)} \left(D'(s) + D(s) \frac{\alpha_x(s)}{\beta_x(s)} \right) \sin \pi Q_x \right] \times \\
& \times \sum_i k_i \beta_{x,i} \cos 2(\mu_{x,i} - \pi Q_x) + \\
& -\frac{1}{2 \sin \pi Q_x} \left[\frac{D(s)}{\sqrt{\beta_x(s)}} \sin(2\mu_x(s) + \pi Q_x) \cos \pi Q_x + \sqrt{\beta_x(s)} \left(D'(s) + D(s) \frac{\alpha_x(s)}{\beta_x(s)} \right) \times \right. \\
& \times \left. \left(\frac{1}{2} - \sin(2\mu_x(s) + \pi Q_x) \sin \pi Q_x \right) \right] \times \\
& \times \sum_i k_i \beta_{x,i} \sin 2(\mu_{x,i} - \pi Q_x) + \\
& -\frac{1}{4} \left[\sqrt{\beta_x(s)} \left(D'(s) + D(s) \frac{\alpha_x(s)}{\beta_x(s)} \right) - \cot \pi Q_x \frac{D(s)}{\sqrt{\beta_x(s)}} \right] \sum_{i=1}^N k_i \beta_{x,i}
\end{aligned} \tag{A.26}$$

APPENDIX B: THE RMS EQUIVALENT SPACE CHARGE TUNE-SHIFT

The following derivation follows the CERN accelerator school proceedings on space charge in circular machines by G. Franchetti [52].

A particle beam can be characterized by its second-order moments, with that the center of mass of the beam as the origin these moments:

$$\begin{aligned}\langle x^2 \rangle &= \frac{1}{N} \sum_i^N x_i^2, \\ \langle x'^2 \rangle &= \frac{1}{N} \sum_i^N x_i'^2, \\ \langle xx' \rangle &= \frac{1}{N} \sum_i^N x_i x_i'.\end{aligned}\tag{B.1}$$

From these moments the following quantities can be defined: the RMS beam size $\tilde{x}$, RMS beam emittance $\tilde{\epsilon}_x$, $\tilde{x}'$ and $\tilde{x}''$, respectively defined as

$$\begin{aligned}\tilde{x} &= \sqrt{\langle x^2 \rangle} \\ \tilde{\epsilon}_x &= \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2} \\ \tilde{x}' &= \frac{\langle xx' \rangle}{\tilde{x}} \\ \tilde{x}'' &= \frac{\langle xx' \rangle \tilde{x}^2 + \langle x'^2 \rangle \tilde{x}^2 - \langle xx' \rangle^2}{\tilde{x}^3} \\ &= \frac{\langle xx' \rangle}{\tilde{x}} + \frac{\tilde{\epsilon}_x^2}{\tilde{x}^3}.\end{aligned}\tag{B.2}$$

The linear equations of motion (Eq. 2.54) can be written using these the following relation becoming

$$\tilde{x}'' + k_{1;x}(s)\tilde{x} - \frac{2K}{a(a+b)}\tilde{x} - \frac{\tilde{\epsilon}_x^2}{\tilde{x}^3} = 0. \quad (\text{B.3})$$

To predict the envelope equations, the relationship between a, b and $\tilde{x}, \tilde{y}$ has to be found. This changes however on which beam distribution is used. Here, two types of distribution are discussed, a Kapchinsky-Vladimirsky (KV) distribution and Gaussian distribution.

For a KV distribution:

$$\begin{aligned} n(x, x', y, y') &= \frac{1}{\pi^2 \epsilon_x \epsilon_y} \delta\left(\frac{\epsilon_x}{\epsilon_x} + \frac{\epsilon_y}{\epsilon_y} - 1\right) \\ a &= 2\tilde{x} \\ b &= 2\tilde{y}. \end{aligned} \quad (\text{B.4})$$

For a Gaussian distribution:

$$\begin{aligned} n(x, x', y, y') &= \frac{1}{4\pi^2 \epsilon_x \epsilon_y} e^{-\frac{1}{2}\left(\frac{\epsilon_x}{\epsilon_x} + \frac{\epsilon_y}{\epsilon_y}\right)} \\ a &= \tilde{x} \\ b &= \tilde{y}. \end{aligned} \quad (\text{B.5})$$

With ϵ_x, ϵ_y scaling factors defining the geometrical extension of the distribution. The KV distribution has the interesting property that it is stationary for linear lattices, therefore when this beam circles around the ring the beam type will remain a KV distribution. This is not the case for Gaussian distributed beams which are only stationary for short time-scales and yield non-linear space charge forces for linear lattices. The equation of motion with non-linear space charge forces using the electric field described Eq. 2.52 becoming

$$\frac{d^2x}{ds^2} + k_{1;x}(s)x - Kx \int_0^\infty \frac{\hat{n}(\hat{T})}{(a^2 + t)^{3/2}(b^2 + t)^{1/2}} dt = 0, \quad (\text{B.6})$$

leading to

$$\langle xx' \rangle = -k_{1;x} \langle x^2 \rangle + K \left\langle x^2 \int_0^\infty \frac{\hat{n}(\hat{T})}{(a^2 + t)^{3/2}(b^2 + t)^{1/2}} dt \right\rangle. \quad (\text{B.7})$$

It was proven by F.J. Sacherer [100] that this last term is equal to $\frac{K}{2} \frac{\tilde{x}}{\tilde{x} + \tilde{y}}$ for matched beams, like the KV and Gaussian distribution that are begin discussed. Therefore using Eq. B.2 results in

$$\tilde{x}'' = -k_{1;x}\tilde{x} + \frac{K}{2} \frac{1}{\tilde{x} + \tilde{y}} + \frac{\tilde{\epsilon}_x^2}{\tilde{x}^3}. \quad (\text{B.8})$$

This shows that for a matched beam in which the beam distribution does not change, the RMS envelope will evolve in the same way independently of beam distribution type. Furthermore, two beams that have the same RMS moments at a given point will have the

same RMS envelope evolution. This relation is called RMS equivalence. The tune-shift as a function of the RMS beam size $\tilde{x}$ and RMS emittance $\tilde{\epsilon}_x$ is given by

$$\Delta Q_x = -\frac{R^2}{Q_x} \hat{n}(0) K \frac{1}{2} \int_0^\infty t \hat{n}(t) dt \frac{1}{2\tilde{x}(2\tilde{x} + 2\tilde{y})}, \quad (\text{B.9})$$

using Eq. 2.58 and the following relations

$$\tilde{x}^2 = \epsilon_x \beta_x(s) \frac{1}{2} \int_0^\infty t \hat{n}(t) dt, \quad (\text{B.10})$$

$$\tilde{\epsilon}_x = \epsilon_x \frac{1}{2} \int_0^\infty t \hat{n}(t) dt. \quad (\text{B.11})$$

The final peak tune-shift is

$$\Delta Q_x = f \Delta \tilde{Q}_x, \quad (\text{B.12})$$

where the tune shift of a Kapchinsky-Vladimirsky beam RMS equivalent beam is given by

$$\Delta \tilde{Q}_x = -\frac{R^2}{Q_x} \frac{K}{4} \frac{1}{2\tilde{x}(2\tilde{x} + 2\tilde{y})}, \quad (\text{B.13})$$

and factor f is distribution dependent with $f = 1$ for a KV distribution and $f = 2$ for a Gaussian distribution.

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