

Effective model of black hole electromagnetic interactions

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We develop simple effective model of black hole event horizon electromagnetic interactions. The event horizon is represented as a membrane located at constant value of lapse possessing arbitrary complex conductivity. We derive important relations governing electromagnetic interactions of membrane and consider implications of changed boundary conditions on horizon on formation of jets and imaging of black holes. Modified version of Znajek condition is derived. Power output of jet is calculated; it appears to change between those of black hole jet and pulsar one. Images of membrane surrounded by several types of accretion disks are produced; nonzero reflection coefficient of membrane leads to appearance of new features observable inside the black hole shadow.

I. INTRODUCTION

Nowadays there are no doubts that general relativity is a right theory of gravity, at least at low energies. One of its most striking predictions is the existence of black holes, which are regions of spacetime bounded by the so called event horizon, spacetime surface that can be crossed by matter and radiation only inwards.

There are a lot of indirect evidences for existence of black holes, e.g. supermassive black hole influence on motion of nearby stars, observations of relativistic jets from active galactic nuclei and accretion disks around quasars, or recent observation of gravitational waves from black hole merger, but event horizon of black hole was never directly observed by the moment. This lack of direct observations leaves room for possible violations of general relativity in regions close to event horizon[1, 2].

However, such an experimental vacuum soon will be hopefully broken by the Event Horizon Telescope (EHT) project, which is a global network of millimeter and submillimeter telescopes aiming at imaging of Sagittarius A*, the supermassive black hole in the center of our galaxy, on horizon scales using very-long baseline interferometric techniques[3]. If EHT will detect any violations of general relativity in near horizon region of Sgr A*, one will need to somehow determine, to which model, modifying general relativity, such a violation points out. Generally one will need to simulate black hole imaging for each such model to separate those that are allowed by the newly obtained experimental data.

There are numerous ways, in which general relativity can be modified and therefore it is desirable to have some effective model of black hole event horizon, that can account for any such modification and fit observational data with a finite number of horizon parameters,

which can be than related to parameters of underlying gravitation theories. Such effective model is a near-horizon strong-field counterpart of parametrized post-Newtonian formalism, developed in order to describe possible violations of general relativity on the basis of weak gravitational field observations[4].

So our task is to develop such an effective theory of black hole event horizons and it was accomplished for electromagnetic interactions of black holes as reported in this notes. The note is organized as follows: in the second section we describe our parametrization of event horizon and derive some of its implications on horizon electrodynamics; then we consider two examples of application of formalism developed, for formation of black hole jets in the third sections and for black hole imaging in the fourth. In fifth section we summarize our results and discuss directions for future work. There are also two appendices: in first we reviewed some facts from theory of geodesic motion in Kerr spacetime and describe our setup for ray tracing and in the second we derive some results mentioned in the next section.

II. MODEL SETUP

Let us now describe the model of black hole event horizon we propose. Since we are not interested in modifications of general relativity on large distances from event horizon, we consider the external geometry to be those of general relativistic black hole. Here we consider only stationary axisymmetric case, which is uniquely described by the Kerr metric, that can be written as follows:

$$ds^2 = -\frac{\rho^2 \Delta}{\Sigma^2} dt^2 + \Sigma^2 \rho^2 \sin^2 \theta (d\varphi - \omega dt)^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2, \quad (1)$$

where $\rho^2 = r^2 + a^2 \cos^2 \theta$, $\Delta = r^2 - 2Mr + a^2$, $\Sigma^2 = (r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta$ and $\omega = \frac{2aMr}{\Sigma^2}$. Our effective model is the extension of the so called stretched horizon formalism[26], that is we assume that horizon is replaced by corotating membrane located at some finite (though very small) distance from Kerr horizon. It is natural to assume that membrane inherits symmetries of spacetime that is it is stationarity and axisymmetric.

Following [26] the membrane is chosen as the surface of constant lapse $\alpha = \frac{\rho\sqrt{\Delta}}{\Sigma}$, since such a surface has some desirable features. First, it is obviously a constant redshift surface in the sense that observers at infinity perceive signals emitted from different points of this surface with the same redshift $z = \frac{1}{\alpha} - 1$. Second, membrane of constant lapse is free of tangential stresses, just as it would be natural for a stationary surface. The derivation of this and the following properties of membrane are given in appendix B.

The value of lapse on membrane should be very small for it to be close to event horizon in order to avoid modifications of general relativity on large distances from event horizon. It leads to some consequences on interactions of membrane with electromagnetic field. First, since observers on membrane have very large redshift, they perceive any incident electromagnetic field as those of ingoing plane wave in the following sense[26]:

$$\mathbf{E}_{||} = \mathbf{n} \times \mathbf{B}_{||} + O(\alpha), \quad \mathbf{B}_{||} = -\mathbf{n} \times \mathbf{E}_{||} + O(\alpha), \quad (2)$$

where $\mathbf{E}_{||} = (0, E_{(\theta)}, E_{(\varphi)})$ and $\mathbf{B}_{||} = (0, B_{(\theta)}, B_{(\varphi)})$ are parallel to membrane components of electric and magnetic field vectors respectively and $\mathbf{n}$ is the normal to surface. Note that it is more convenient here to work with space vectors. Moreover, it appears that this plane wave falls on membrane almost normally.

It was shown in [24, 27] that black hole event horizon interactions with electromagnetic field can be thought of as if it is carrying some surface charge density σ_H and current density $\mathbf{j}_H$ with specific resistivity $\rho_H = \frac{1}{4\pi}$ as it follows from requirement of regularity of electromagnetic field, measured by geodesic observer, on horizon, reflecting the fact that event horizon is the perfect absorber. In our model we extend this picture allowing for arbitrary complex specific resistivity of our membrane ρ_M . It is equivalent to allowance of reflection of electromagnetic waves from membrane with reflection coefficient κ and phase shift ψ given by

$$\kappa = \left| \frac{4\pi - \rho_M}{4\pi + \rho_M} \right|^2, \quad \psi = \arctan \frac{8\pi \operatorname{Im} \rho_M}{|\rho_M|^2 - 16\pi^2}. \quad (3)$$

In the case $\rho_M = 0$ one obtains $\kappa = 0$, $\psi = 0$ as it is for usual event horizon. Note that nonzero phase shift makes no sense for constant electromagnetic field. Therefore we will further consider only membrane with real resistivity. Moreover, we allow only physical values of reflection coefficient $0 < \kappa < 1$ and thus allowed values of resistivity lie in range $\rho_M \in [4\pi, \infty]$.

Ultimately the boundary condition on membrane has the form

$$\mathbf{B}_{||} = -\frac{4\pi}{\rho_M} \mathbf{n} \times \mathbf{E}_{||} + O(\alpha), \quad (4)$$

generalizing those (2) for nonreflecting stretched horizon to correspond to superposition of ingoing and outgoing plane waves. This boundary condition can be written invariantly as

$$\begin{aligned} F_{\mu\nu} \left(\frac{4\pi}{\rho_M} u^\nu e^{(\theta)\mu} + \varepsilon^{\rho\sigma\mu\nu} u_\rho e_\sigma^{(\varphi)} \right) &= O(\alpha), \\ F_{\mu\nu} \left(\frac{4\pi}{\rho_M} u^\nu e^{(\varphi)\mu} + \varepsilon^{\rho\sigma\mu\nu} u_\rho e_\sigma^{(\theta)} \right) &= O(\alpha). \end{aligned} \quad (5)$$

We consider two examples of applications of event horizon model developed: formation of jets and optical imaging of black holes.

III. JET FORMATION

As a first example of application of membrane model to investigation of electromagnetic properties of black holes we consider formation of relativistic jets sourced by black holes.

Black hole jets were first successfully explained by Blandford and Znajek [21]. The explanation proposed is that black hole surrounded by accretion disk possess strong magnetosphere sourced by currents in disk; the charges in magnetosphere are produced by the Ruderman-Sutherland[25] 'spark-gap' process, a cascade production of electron-positron pairs in force-free region of black hole magnetosphere induced by component of electric field (which is induced due to rotation of black hole) along magnetic field; these charges are further accelerated by magnetosphere forming the jet.

First we describe the model of black hole magnetosphere proposed. It is assumed that black hole, accretion disk and magnetosphere are stationary and axisymmetric. Moreover, since electrical conductivity of plasma in the disk is so high, that the electric field vanishes in plasma rest frame and the magnetic field is frozen into accretion disk, it is assumed that region of magnetosphere surrounding disk is degenerate, that is inside it the condition $F_{[\mu\nu}F_{\rho\sigma]}$ (that is $\mathbf{E} \cdot \mathbf{B} = 0$ in flat spacetime) is fulfilled. Charged particles are produced in part of the region of degeneracy outside the accretion disk by means of Ruderman-Sutherland (or perhaps any other) mechanism. Particles there are rare enough to exert no significant force on magnetic field lines, but are dense enough for region becoming force-free, that is satisfying the condition $F_{\mu\nu}j^\nu$ (or equivalently $\rho\mathbf{E} + \mathbf{j} \times \mathbf{B} = 0$ and $\mathbf{E} \cdot \mathbf{j} = 0$). Outside the region of degeneracy region of acceleration of charged particles lies. We do not concern ourselves with details of acceleration processes assuming that all the power beign transported to acceleration region from region of degeneracy is used to accelerate particles. Therefore one should calculate power flux from region of degeneracy in order to calculate power output of jet.

Blandford and Znajek developed perturbative formalism for construction of force-free magnetospheres in Kerr spacetime. We however interested in investigation of influence of generalization of boundary conditions on event horizon rather on general relations governing black hole magnetospheres than on explicit solutions. General theory of stationary axisymmetric black hole magnetospheres was developed by Mcdonald and Thorne[22] using 3+1 formalism for description of electrodynamics. It was further reconsidered by Gralla and Jacobson[30] in covariant fashion. We follow the latter approach to investigate magnetospheres around our reflecting membrane.

The most general stationary axisymmetric degenerate electromagnetic field is described as

$$F = d\psi \wedge (d\psi_2 + d\varphi - \Omega(\psi)dt) \quad (6)$$

in terms of Euler potentials $\psi = \psi(r, \theta)$ and $\psi_2 = \psi_2(r, \theta)$. ψ has the sense of magnetic flux through loop $\mathcal{C}$ formed by flowing along ξ , ψ_2 determines bending of field line in azimuthal direction and Ω can be viewed as angular velocity of the field lines. It is useful to define polar current, that is current through disk $\mathcal{D}$, formed by flowing loop $\mathcal{C}$ along η , as

$$I = \int_{\mathcal{D}} j^\mu dS_\mu = \int_{\mathcal{C}} F^{\mu\nu} dS_{\mu\nu}, \quad (7)$$

where the Stokes theorem is used. For field described by (6) it takes value

$$I = 2\pi \frac{\Delta}{\rho^2} (\psi_{,\theta} \psi_{2,r} - \psi_{2,\theta} \psi_{,r}) \sin \theta. \quad (8)$$

ψ, ψ_2 and Ω are not independent in black hole magnetosphere due to presence of event horizon (or reflecting membrane) and boundary condition associated to it. One can uncover relation between them substituting (6) into (5). After a bit of algebra one is left with the requirement

$$d\psi_2 = \frac{f_1(r)}{\Delta} dr + f_2(\theta) d\theta, \quad (9)$$

where $f_2(\theta)$ is an arbitrary function and $f_1(r)$ is further limited by the condition on membrane

$$I = 2\pi \frac{\rho_M}{4\pi} (\Omega - \omega) \psi_{,\theta} \sqrt{\frac{g_{\varphi\varphi}}{g_{\theta\theta}}} + O(\alpha^2), \quad (10)$$

generalizing the so called *Znajek condition*[24] for black hole magnetospheres in case of usual event horizon. Appearance there of the factor $\frac{\rho_M}{4\pi}$ is the general result of our consideration of influence of changed boundary conditions on black hole magnetospheres.

Let us also calculate angular momentum and energy outflow of relativistic jet powered by Blandford-Znajek mechanism. Those can be found as

$$\mathcal{E} = \int_S T_{\mu\nu} \eta^\mu dS^\nu, \quad \mathcal{L} = \int_S T_{\mu\nu} \xi^\mu dS^\nu, \quad (11)$$

where S is closed hypersurface at infinity and $T_{\mu\nu}$ is the stress-energy tensor of electromagnetic field:

$$T_{\mu\nu} = F_{\mu\rho} F_\nu^\rho - \frac{1}{4} F_{\rho\sigma} F^{\rho\sigma} g_{\mu\nu}. \quad (12)$$

Using (6), (7) and supposing conservation of energy and angular momentum (due to force-freeness or vanishing of dissipation in symmetry directions, $F_{\mu\nu} J^\mu \xi^\nu = F_{\mu\nu} J^\mu \eta^\nu = 0$) one can express (11) as

$$\mathcal{E} = - \int_{\mathcal{P}} \Omega I d\psi, \quad \mathcal{L} = - \int_{\mathcal{P}} I d\psi, \quad (13)$$

where $\mathcal{P}$ is any poloidal curve, e.g. those along membrane.

We further provide an explicit example of calculation of energy and angular momentum outflow for monopole magnetosphere solution first found by Blandford and Znajek. To the leading term in a it coincides with the Michel monopole solution[31]:

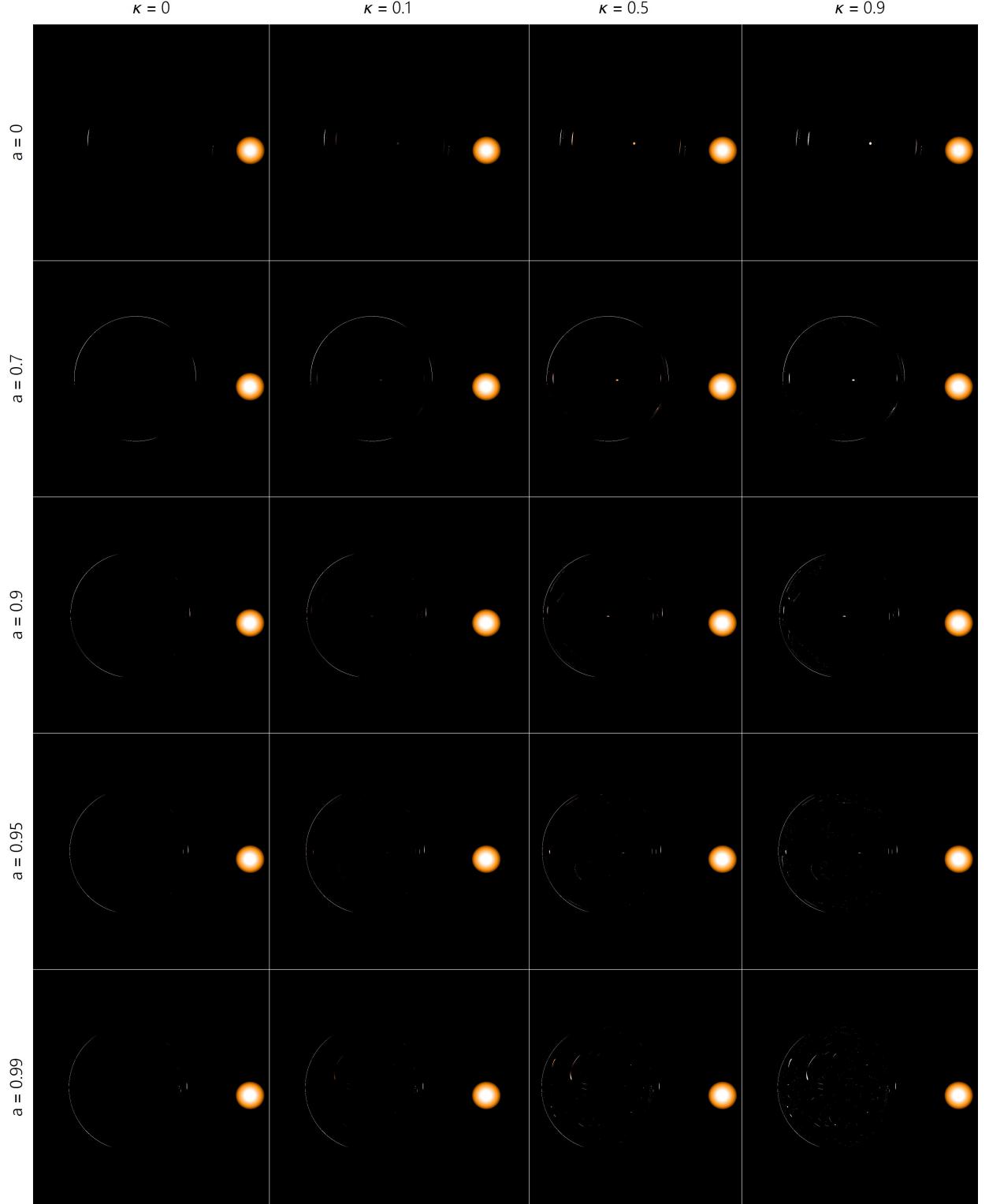


FIG. 1: Images of blob of luminous matter with mass $1M$ following Keplerian orbit with radius $10M$ over a black hole with mass $M = 1$ and lapse $\alpha = 0.01$ for a range of values of spin parameter a and reflection coefficient κ .

$$F = q \sin \theta \, d\theta \wedge \left(d\varphi - \Omega \left(dt - \frac{r^2 + a^2}{\Delta} dr \right) \right), \quad (14)$$

where q is monopole charge. Condition (10) leads for such configuration to limitation of field line angular velocity to be

$$\Omega = \omega \frac{\rho_M}{4\pi + \rho_M}, \quad (15)$$

that is it takes values between $\frac{\omega}{2}$ corresponding to black hole event horizon and ω corresponding to ideally conducting surface, e.g. those of neutron star[28]. Energy and angular momentum outflows thus can be calculated as

$$\mathcal{E} = \frac{8}{3} \pi q^2 \omega^2 \left(\frac{\rho_M}{4\pi + \rho_M} \right)^2, \quad \mathcal{L} = \frac{8}{3} \pi q^2 \omega^2 \frac{\rho_M}{4\pi + \rho_M}, \quad (16)$$

that is depending on the value of resistivity of membrane power output of jet takes values in range $\mathcal{E}_H < \mathcal{E}_M < 4\mathcal{E}_H$, that is between those for monopole magnetosphere around usual black hole and pulsar.

IV. IMAGING OF BLACK HOLES

We next consider implications of nonzero membrane reflection coefficient and lapse on direct observations of black holes. We simulate these observations by the means of ray tracing procedure, details of which are described in appendix A. We everywhere assume that observer is located at point with coordinates $r_0 = 50M$ and $\theta_0 = 0.95\frac{\pi}{2}$.

First we consider a couple of toy examples to study properties of reflection from membrane. As a first example we consider blob of luminous matter orbiting a black hole in order to study affection of nonzero membrane reflection coefficient on image of single point. We model blob as a spherical optically thin body made of matter with flat spectrum and following Keplerian orbit around black hole.

Images of black hole for different values of its spin parameter a and reflection coefficient κ are presented at Fig. 1. In the first column one can find images of blob around membrane with zero κ , that is conventional black hole. There usual feature of black hole lensing can be observed: multiple images, corresponding to direct ray and to rays that traveled several times around the black hole, exist around black hole shadow, which is circular for Schwarzschild black hole and gets more and more squashed as spin parameter increases (exact expression for form of shadow can be found in appendix A). As reflection coefficient increases new images appear inside the shadow. In case of low values of spin parameter or low reflection coefficients they can be readily identified with images corresponding to rays, which reflected after being radiated and to those that reflected and traveled several times around the hole. For high values of a when κ increases new features of images appear, structure of which

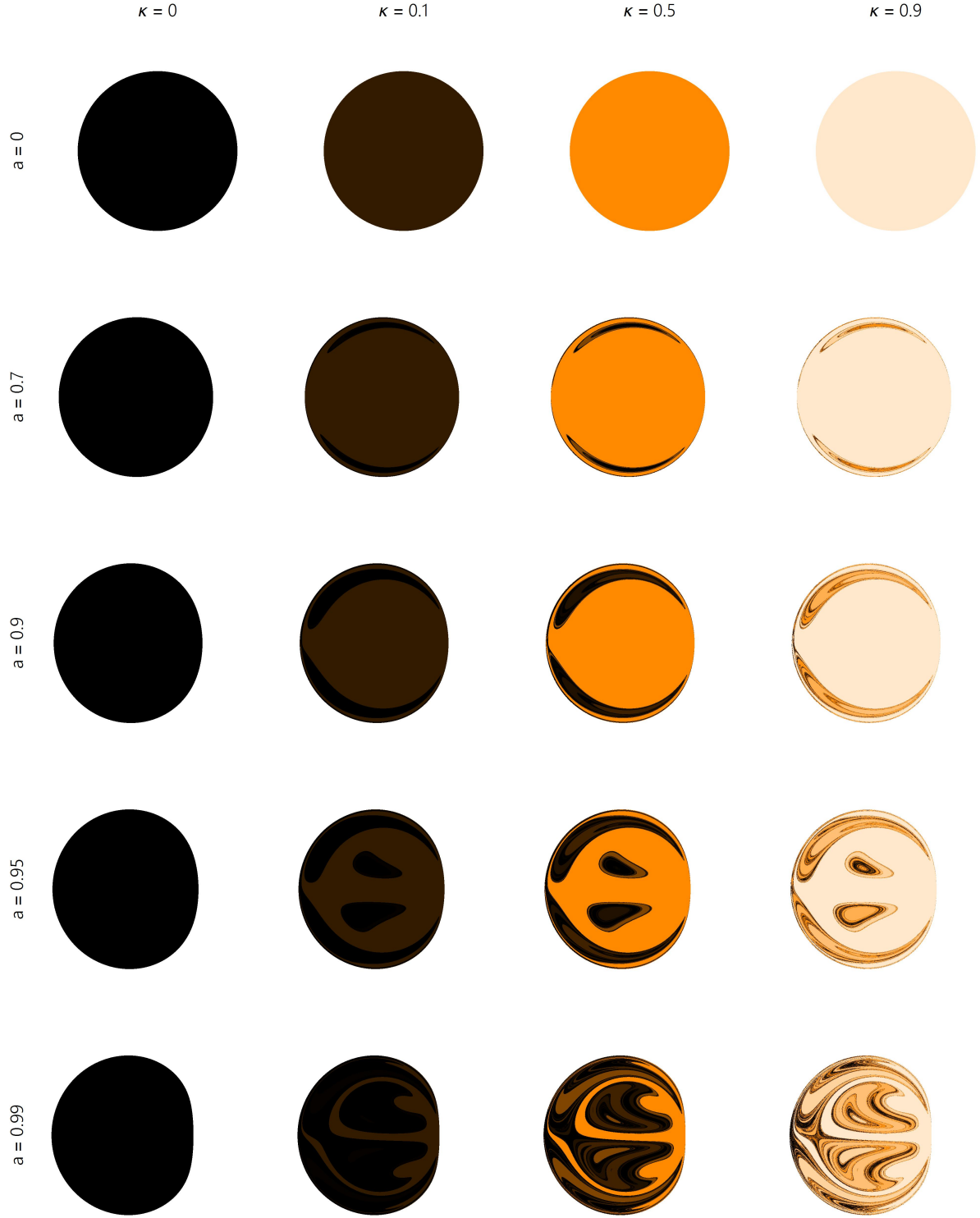


FIG. 2: Images of uniformly illuminated membrane with mass $M = 1$ and lapse $\alpha = 0.01$ for a range of values of spin parameter a and reflection coefficient κ .

cannot be understood so easily. In order to elucidate this behavior we consider the next example.

As a second example we consider black hole surrounded by light sphere with constant brightness. Images of black hole for different values of its spin parameter a and reflection coefficient κ are presented at Fig. 2. For low spin parameters one can see just what expected: specific intensity in shadowed region is κ times those of emitter. But for high enough values of spin parameter regions of shadow with lower intensity than should be for single reflection from membrane appear (for $a \simeq 0.7$, however already for $a = 0.5$ such regions can be distinguished at the edge of shadow). Those regions correspond to multiple reflections from membrane. In the case of Schwarzschild black hole multiple reflections are not possible since escape cones are circular and therefore rays that hit membrane always escape. But as spin parameter grows escape cones get more and more deformed, and, therefore part of rays hitting membrane at some point though belong to escape cone at that point miss it after reflection, leading to second reflection (see (A13), (A14)). If after second reflection ray again misses escape cone it will be reflected again and so on. This is illustrated on Fig. 8, where escape cones for a photon on a membrane are presented.

For high values of spin parameter a regions of multiple reflection exhibit very complex structure, see Fig. 3. One can observe that that distribution of specific intensity behaves self-similarly, though reasons of such a behavior are unclear and definitely deserve further investigation. It is not clear if possible number of reflections is bounded or not. Maximal number of reflections found in our numerical experiments is 187, though numerics are not quite reliable for such a long trajectory.

We also investigate time delays of various rays coming to observer. Their distribution is presented at Fig. 4. One can see that this distribution has the same pattern as those for intensity. This is not surprising, since close to horizon the redshift is very large and time delay grows as $\ln(r - r_H) \sim \ln \alpha$ when approaching horizon. Thus each reflection greatly increases time delay. It hints that multiply reflected rays and self-similar pattern formed by them can be observed only in long-term observations.

Having considered two toy examples of light sources around reflecting membrane and understood basic features of their images, we now turn to more realistic surroundings of black holes.

A. Accretion disks

Most of black holes formed as a result of gravitational collapse should possess accretion disk around it, and, what is more important, accretion disks and processes originated from them, such as jets, are the only directly observable evidences of black hole existence. Therefore we further consider several examples of black hole surrounded by accretion disks.

Since the pioneering work by Shakura and Sunaev [7] there we proposed a lot of models

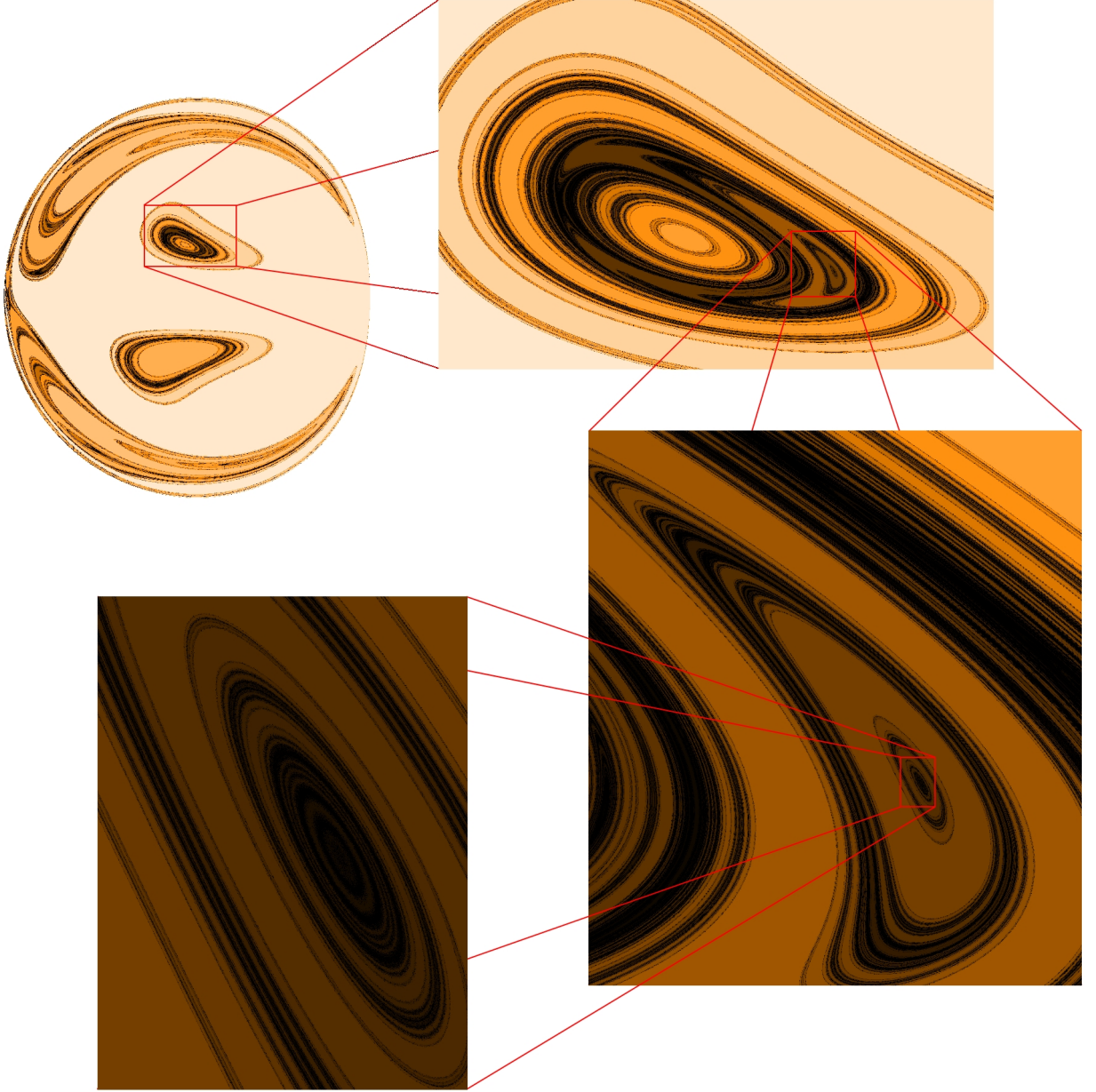


FIG. 3: Magnification of image of uniformly illuminated membrane with mass $M = 1$, spin parameter $a = 0.95$, lapse $\alpha = 0.01$ and reflection coefficient $\kappa = 0.9$.

for black hole accretion disks depending on parameters of accretion disk and physical processes involved. There exist models of different complexity ranging from highly approximate analytic ones to full 3D general relativistic magnetohydrodynamic simulations[8]. Since we are interested not in modeling real black hole accretion disk images, but rather in understanding properties of images of reflecting membrane and highlighting crucial new features of such images, we content ourselves with simple analytic models of accretion. We consider models falling into three classes of accretion disk theories: thick disk, thin disk and

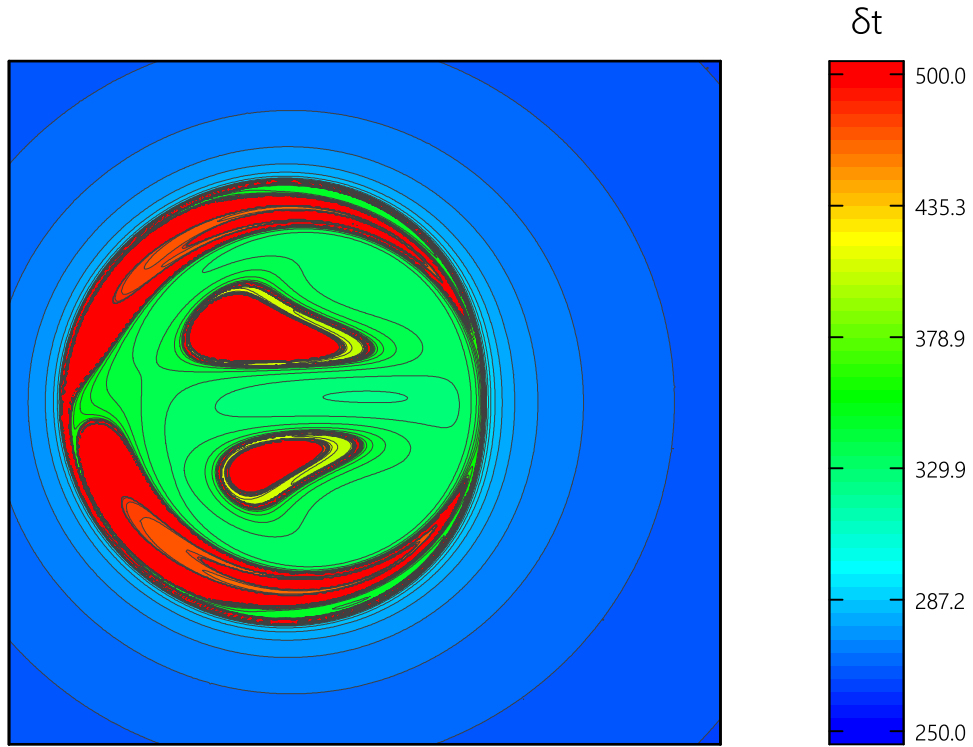


FIG. 4: Distribution of time δt for ray to achieve observer from luminous surface at $r = 200M$ around membrane with mass $M = 1$ and spin parameter $a = 0.95$ at lapse $\alpha = 0.01$.

advection-dominated accretion flow.

1. Novikov-Thorne accretion disk

Model proposed by Shakura and Sunaev [7] belongs to class of thin (though optically thick) disk models. Its general relativistic version was later worked out by Novikov and Thorne [9] and further improved in [11, 32]. The model is believed to be a good approximation for intermediate accretion rate ($\dot{M} < \dot{M}_{\text{Edd}}$) largely opaque disks. It can be used for description e.g. accretion disk around black hole binaries and luminous active galactic nebulae.

The model is based on the following assumptions:

- disk is lying in equatorial plane of black hole;
- disk is thin in the sense that for its thickness H at radial distance r condition $\frac{H}{r} \ll 1$

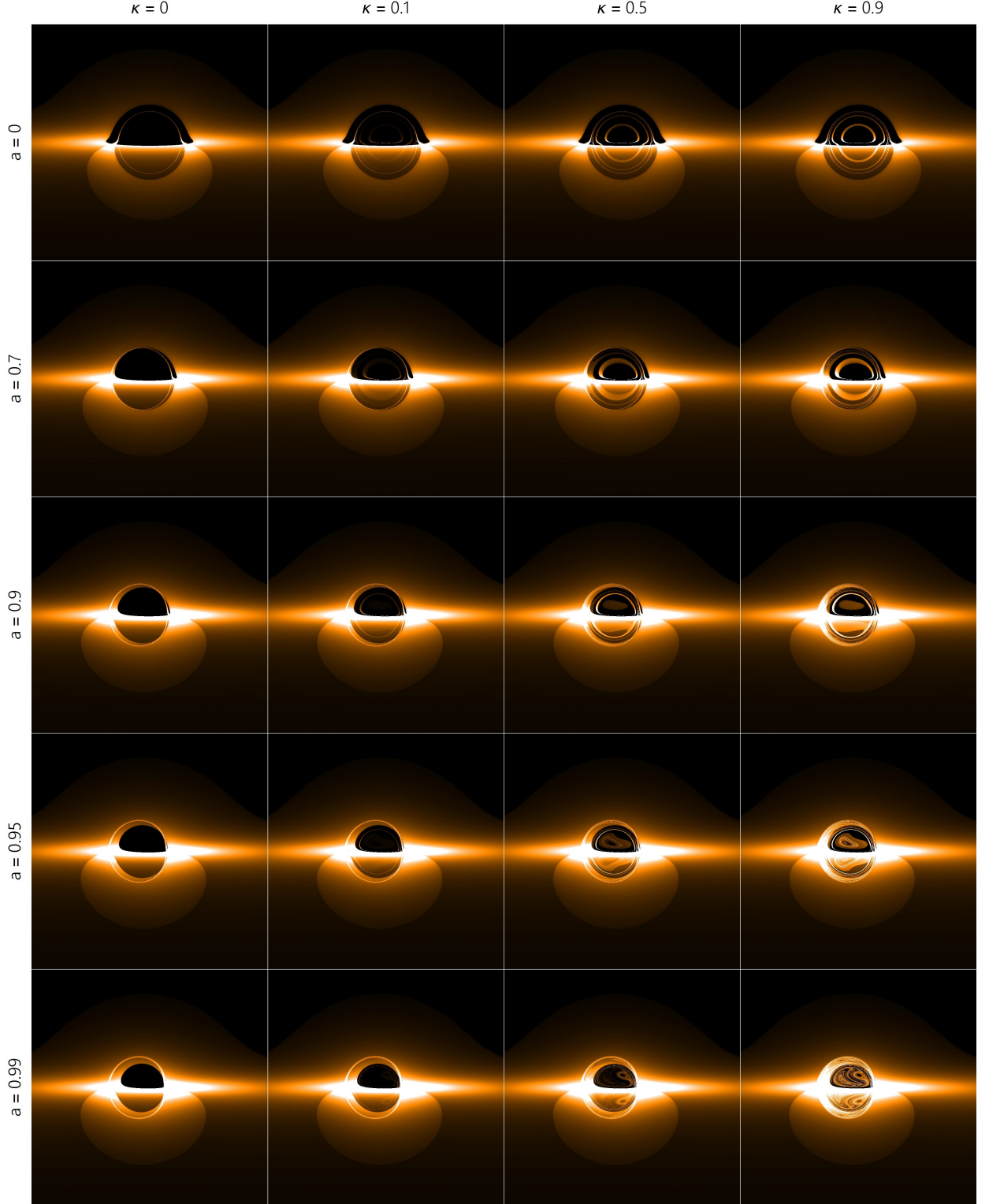


FIG. 5: Images of membrane with mass $M = 1$ and lapse $\alpha = 0.01$ surrounded by thin accretion disk for a range of values of spin parameter a and reflection coefficient κ . The values of viscosity α parameter and accretion rate $\dot{M}$ are $\alpha = 0.2$ and $\dot{M} = 0.00024\dot{M}_{\text{Edd}}$.

is satisfied;

- matter of the disk is a single component relativistic viscous fluid with stress-energy tensor of form

$$T^{\mu\nu} = \rho(1 + \Pi)u^\mu u^\nu + ph^{\mu\nu} + S^{\mu\nu} + u^\mu q^\nu + q^\mu u^\nu, \quad (17)$$

where ρ is fluid's rest-mass density, Π is its specific internal energy, p is its pressure, $h^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$ is the projector on fluid frame, $q^\mu = -\lambda h^{\mu\nu} (T_{,\nu} + T a_\nu)$ (here T is temperature, λ is heat conduction coefficient and $a_\nu = u_\nu \nabla^\nu u_\mu$ is fluid's acceleration) is transverse energy flux and $S^{\mu\nu} = -\eta \sigma^{\mu\nu}$ is symmetric, transverse and traceless anisotropic tensor of viscous stresses, where η is viscosity and $\sigma^{\mu\nu}$ is the shear tensor defined as

$$\sigma_{\mu\nu} = h_\mu^\alpha h_\nu^\beta \nabla_{(\beta} u_{\alpha)} - \frac{1}{3} h_{\mu\nu} \nabla_\alpha u^\alpha. \quad (18)$$

For viscosity the Shakura-Sunyaev α -prescription is assumed, that is in fluid frame viscous stress tensor has only one independent component $S_{(r)(\varphi)} = \alpha p$. Total pressure include radiation and gas (which is assumed to be ideal) pressure:

$$p = p_{\text{rad}} + p_{\text{gas}}, \quad \text{where } p_{\text{gas}} = \frac{k_B \rho}{m_p} T \quad \text{and} \quad p_{\text{rad}} = \frac{4}{3} \sigma_{\text{SB}} T^4, \quad (19)$$

where k_B is Boltzmann's constant, m_p is mass of proton and σ_{SB} is Stephan-Boltzmann's constant.

- fluid nearly follows circular geodesics with a small radial (non-geodesic) component of velocity produced by viscous stresses and responsible for accretion;
- fluid's specific internal energy Π is negligible;
- energy is released entirely in form of radiation;
- radial heat transfer inside the disc is negligible (including self-irradiation);
- optical opacity of the disk originates from free-free absorption and electron scattering:

$$\kappa = \kappa_{\text{ff}} + \kappa_{\text{es}}, \quad \text{where } \kappa_{\text{ff}} \sim \rho T^{-\frac{7}{2}} \quad \text{and} \quad \kappa_{\text{es}} = \text{const}; \quad (20)$$

- radial fluid velocity at innermost stable circular orbit equals sound speed (so called *sonic-ISCO* boundary condition as proposed in [11] contrary to the so called *no-torque* condition[10] originally used by Novikov and Thorne).

The dynamics of accretion disk is governed by the following equations:

$$\nabla_\mu (\rho u^\mu) = 0, \quad (21a)$$

$$h_{\mu\sigma} \nabla_\nu T^{\sigma\nu} = 0, \quad (21b)$$

$$u_\mu \nabla_\nu T^{\sigma\nu} = 0, \quad (21c)$$

which are fluid continuity equation, relativistic Navier-Stokes equation and energy conservation law respectively. Supplying this with energy transport law

$$4\sigma T^4 = \kappa \Sigma F, \quad (22)$$

where $\Sigma = 2\rho H$ is surface density of the disk and F is radiative energy flux from its surface, and using the listed assumptions one turns the system (21) into set of algebraic equations for structural and thermodynamical properties of disk. There can be obtained four analytical solutions for regions dominated by gas pressure and electron-scattering opacity ('edge region', closest to ISCO), radiation pressure and electron scattering opacity ('inner region'), gas pressure and electron scattering opacity ('middle region') and gas pressure and free-free absorption opacity ('outer region'). Their explicit form is rather complicated and can be found e.g. in [12].

There are only two free parameters for the model: mass accretion rate $\dot{M}$ and viscosity coefficient α .

Radiated energy flux is calculated in Novikov-Thorne model from assumption of it being equal to energy produced from viscous heating and therefore spectral distribution of radiation is not described by the model. We therefore assume spectrum of radiation to be thermal, as predicted by Shakura-Sunyaev prescription. It is interesting to note that since for high enough temperatures Planck distribution is

$$B_\nu = 2\nu^2 T, \quad (23)$$

it can be seen from (A10) that Doppler effect is compensated.

Images of black hole with thin accretion disk for different values of its spin parameter a and reflection coefficient κ are presented at Fig. 5. One can readily notice the novel observational feature of membrane model for low spin parameter or reflection coefficient values: appearance of several additional light rings inside the black hole's shadow specific intensity of which is determined by reflection coefficient. Central ring is an image of whole disk in rays reflected from membrane, while outer rings correspond to rays that also traveled several times around black hole. For high values of a and κ inside the shadow appears the complicated self-similar structure described in previous section, cf. Fig. 2. Ultimately in case of high spins and reflection coefficient membrane model predicts appearance of almost uniform spot inside the shadow instead of rings. However, as it was pointed out time delays

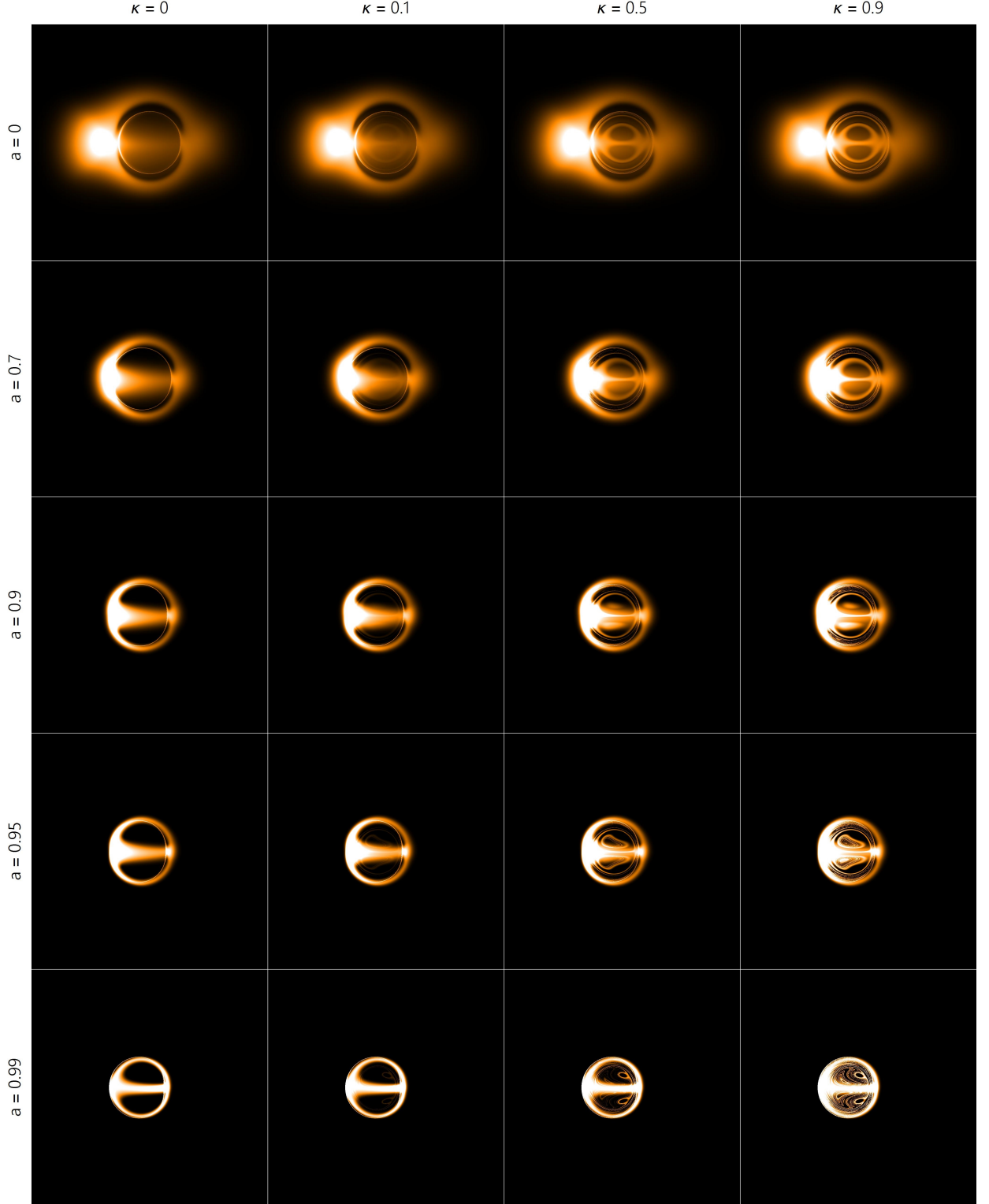


FIG. 6: Images of membrane with mass $M = 1$ and lapse $\alpha = 0.01$ surrounded by magnetized tori for a range of values of spin parameter a and reflection coefficient κ at 100 GHz. The values of angular momentum parameter λ , potential on the surface of disk W_s , ratio of gas and radiation pressure β , temperature at the center of torus T_c and electron density there n_c are $\lambda = 0.7$, $W_s = W_{\text{cusp}}$, $\beta = 1$, $T_c = 10^{11}$ K, $n_c = 10^7 \text{ cm}^{-3}$.

for ray forming this complex structure is much larger than those for surrounding rays and therefore for short-time observations only light ring pattern can be observed.

2. Komissarov magnetized tori

The second model we consider is a thick (though optically thin) disk one, the so called *magnetized tori* [13, 14]. This model is often used as initial profile numerical GRMHD simulations. It was also particularly used to successfully model Sgr A* millimeter spectrum [17], that is interesting in the context of its observation on Event Horizon Telescope [2].

The model is based on the following assumptions:

- accretion flow is stationary and axisymmetric;
- disk is made of barotropic perfect fluid with equation of state $p = Kw^k$ (here $w = p + \rho$ is enthalpy, K is polytropic constant and k is polytropic index);
- the flow is a pure rotation around black hole, that is its velocity is given by (A3);
- magnetic field is purely azimuthal, that is in fluid frame it has the form $B = b\partial_\varphi$;
- magnetic field and fluid are in approximate equipartition, that is for magnetic pressure $p_m = K_m \tilde{w}^k = \frac{b^2}{8\pi} = \beta p$;
- the disk is fully ionized.

The structure of disk is then governed by the following generalization of Boyer's condition [15] as it directly follows from stress-energy conservation:

$$W - W_s + \int_0^p \frac{dp}{w} + \int_0^{\tilde{p}_m} \frac{d\tilde{p}_m}{\tilde{w}} = 0, \quad (24)$$

where $\tilde{p}_m = \mathcal{L}p_m$ and $\tilde{w} = \mathcal{L}w$ with $\mathcal{L} = g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}$; W is the potential given by

$$W = \ln|u_t| + \int_l^{l_\infty} \frac{\Omega dl}{1 - l\Omega} \quad (25)$$

with Ω being an angular velocity and l being an angular momentum inside the disk, and W_s is the value of this potential on surface of disk (being zero pressure surface according to (24)). Thus points with $W > W_s$ are inside the torus. We further assume that disk is marginally stable, that is angular momentum is constant along the disk $l = l_0$.

There are points inside the torus, where fluid moves freely due to absence of pressure-gradient forces. Then it follows Keplerian orbit, angular velocity on which is given by (A4). Using relation between angular momentum and angular velocity (A2) one finds that there are two such points, one is located at $r_c > r_{ms}$ ('center' of disk) and the second is at $r_{mb} < r_{cusp} < r_{ms}$ ('cusp').

One can now calculate thermodynamical quantities for matter at some point of disk as

$$\begin{aligned} K &= w_c^{1-k} (W_c - W_s) \frac{k-1}{k} \frac{\beta_c}{1+\beta_c}, \quad K_m = \frac{\mathcal{L}_c^{1-k}}{\beta_c} K, \\ w &= w_c \left(\frac{W - W_s}{W_c - W_s} \frac{K + K_m \mathcal{L}_c^{k-1}}{K + K_m \mathcal{L}_c^{k-1}} \right)^{\frac{1}{k-1}}, \\ \rho &= w - K w^k, \quad T = T_c \left(\frac{\rho}{\rho_c} \right)^{k-1}. \end{aligned} \quad (26)$$

So far model include six free parameters: concentration of electrons, temperature and ratio of gas and radiation pressure n_c , β_c , T_c in the center of disk, angular momentum parameter $\lambda = \frac{l-l_{ms}}{l_{mb}-l_{ms}}$, potential on the surface of disk W_{in} and polytropic index k .

We assume that distribution of electrons in the disk is purely thermal and since electrons are thermal is given by Maxwell-Juttner distribution:

$$n(\gamma) = \frac{n}{\theta_e} \frac{\gamma \sqrt{\gamma^2 - 1}}{K_2\left(\frac{1}{\theta_e}\right)} \exp\left(-\frac{\gamma}{\theta_e}\right), \quad (27)$$

where $\gamma = \frac{1}{\sqrt{1-v^2}}$ is the Lorentz factor, $\theta_e = \frac{k_B T}{m_e}$ and K_2 is the modified Bessel function of the second kind. Radiation of torus is then assumed to be purely synchrotron (specific emissivity in that case can be found e.g. in [16]).

Images of black hole surrounded by magnetized tori for different values of its spin parameter a and reflection coefficient κ are presented at Fig. 6. One can readily observe the same features as in Novikov-Thorne case: for low values of a or κ additional light rings appear, while at high spin and reflection coefficient there is an almost uniform spot inside the shadow of black hole. Thus one can conclude that appearance of light rings inside the shadow of black hole is the general pattern for any distribution of matter localized close to equatorial plane in a wide range of parameters of reflecting membrane.

3. *Advection-dominated accretion flow*

The last accretion disk model under consideration is of *advection dominated accretion flow* or ADAF (also called *radiative inefficient accretion flow* or RIAF) type. For solutions of this type nearly all viscously dissipated energy is advected into the black hole rather than radiated. Thus such solutions are invoked when the luminosity (as well as mass accretion rates) are small. They also appear to be hot (with temperature close to virial), optically thin and quasi-spherical. The ADAFs are usual choice for consideration of Sgr A* due to its low luminosity.

The particular model we employ is the one used by Broderick et al.[18, 19] following[20], where the steady-state, height-averaged hydrodynamic equations are solved in the presence

of a self-similar mass outflow and an Shakura-Sunyaev α viscosity prescription. The accreting plasma is assumed to include only a thermal electron component, which is much cooler than the ions due to the weakness of the electron-ion Coulomb coupling, with non-thermal component neglected. The accretion flow has a Keplerian velocity distribution and population of thermal electrons with density and temperature

$$n_e = n_0 \left(\frac{r}{2M} \right)^{-1.1} e^{-\frac{1}{2} \cot^2 \theta}, \quad T_e = T_0 \left(\frac{r}{2M} \right)^{-0.84}, \quad (28)$$

where n_0 and T_0 are characteristic concentration and temperature of electrons, and an azimuthal magnetic field in approximate equipartition with matter of disk:

$$B^2 = \frac{\pi M m_p}{\beta r} n_e, \quad (29)$$

where β is the gas to magnetic field pressure ratio. We neglect the power-law distributed electrons and again assume that radiation spectrum is purely synchrotron. There are thus three free parameters in the model: n_0 , T_0 and β .

Images of black hole surrounded by advection-dominated accretion flow for different values of its spin parameter a and reflection coefficient κ are presented at Fig. 7. Due to rather different structure of accretion flow (it is not localized in equatorial plane, but rather is quasi-spherical) observed picture in ADAF case differs from those for thin and thick disks. At any values of spin parameter almost whole shadow appears to be luminous, with specific intensity of outer space to those of black hole shadow ratio governed by reflection coefficient. At high values of reflection coefficient, however, some structure can be resolved, which is presented by dark spots for lower a and already mentioned self-similar structure at higher spins.

V. CONCLUSION

In this work we proposed an effective model for description of electromagnetic interactions of black holes. The model is represented by surface of constant value of lapse endowed with an arbitrary complex resistivity and thus described by just two parameters. We then consider several examples of applications of our model for astrophysical phenomena in order to understand to which new predictions such generalization of concept of event horizon leads, and if it is compatible with results of astronomical observations.

We have considered implications of changed boundary conditions on black hole magnetospheres and power output of jets. The resulting power output lies between those for black hole and neutron star, that is it is compatible with current astrophysical observations and may be as well a testable prediction of our model.

We also considered imaging of membranes for various surroundings, including star orbiting black hole, uniformly illuminating surface and three common types of accretion flows. The general prediction there is the appearance of some light features inside the shadow of

black hole in the case of nonzero reflection coefficient. Those are light rings in the case of structures localized close to equatorial plane of black hole or uniform illumination for structures close to spherical. For high values of spin some self-similar structure were observed which are not well understood yet. Soon it hopefully will be possible to compare our results with observations of Event Horizon Telescope in order to determine allowed range of parameters of model.

This work can be considered as the introductory part of project of construction of effective model of black hole event horizon. Further it should be extended by inclusion of gravitational interactions of membrane (and as well its dynamics) and maybe with other fields as well (e.g. including treatment of black hole 'hair'). It is desirable to develop lagrangian formalism for event horizon extending [33].

APPENDIX A: RAY TRACING IN KERR SPACETIME

In this appendix we present equations of geodesic motion in Kerr spacetime and basics of our approach to imaging of objects in it.

Equations of motion in Kerr spacetime can be written as follows[29]:

$$\rho^2 \dot{r} = \pm \sqrt{R}, \quad \text{where } R = E(r^2 + a^2) - \Delta [Q + (aE - L)^2 + m^2 r^2], \quad (\text{A1a})$$

$$\rho^2 \dot{\theta} = \pm \sqrt{\Theta}, \quad \text{where } \Theta = Q - \cos^2 \theta \left(a^2(m^2 - E^2) + \frac{L^2}{\sin^2 \theta} \right), \quad (\text{A1b})$$

$$\rho^2 \dot{t} = \frac{E}{\Delta} [(r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta] - \frac{2Mar}{\Delta} L, \quad (\text{A1c})$$

$$\rho^2 \dot{\varphi} = \frac{2Mar}{\Delta} E + L \frac{\Delta - a^2 \sin^2 \theta}{\Delta \sin^2 \theta}. \quad (\text{A1d})$$

Here m is the rest mass of particle, $E = \eta^\mu p_\mu$ is a particle's energy and $L = \xi^\mu p_\mu$ is its angular momentum, where η^μ and ξ^μ are Killing vectors corresponding to time translations and rotations respectively. $Q = K^{\mu\nu} p_\mu p_\nu$ is the fourth integral of motion, the so called Carter's constant, connected with the hidden symmetry of Kerr spacetime corresponding to Killing tensor $K_{\mu\nu}$.

For any observer one can define its angular velocity $\Omega = \frac{u^\varphi}{u^t}$ and angular momentum $l = -\frac{u_\varphi}{u_t}$, which are interrelated as

$$\Omega = -\frac{g_{tt}l + g_{t\varphi}}{g_{\varphi\varphi}l + g_{\varphi\varphi}}, \quad l = -\frac{-g_{tt} + \Omega g_{t\varphi}}{-g_{\varphi\varphi} + \Omega g_{\varphi\varphi}}. \quad (\text{A2})$$

Thus for such an observer velocity is given by $u^\mu = (u^t, 0, 0, \Omega u^t)$ with

$$u^t = \sqrt{\frac{g_{tt}g_{\varphi\varphi} - g_{t\varphi}^2}{g_{\varphi\varphi} + 2l g_{t\varphi} + l^2 g_{tt}}}. \quad (\text{A3})$$

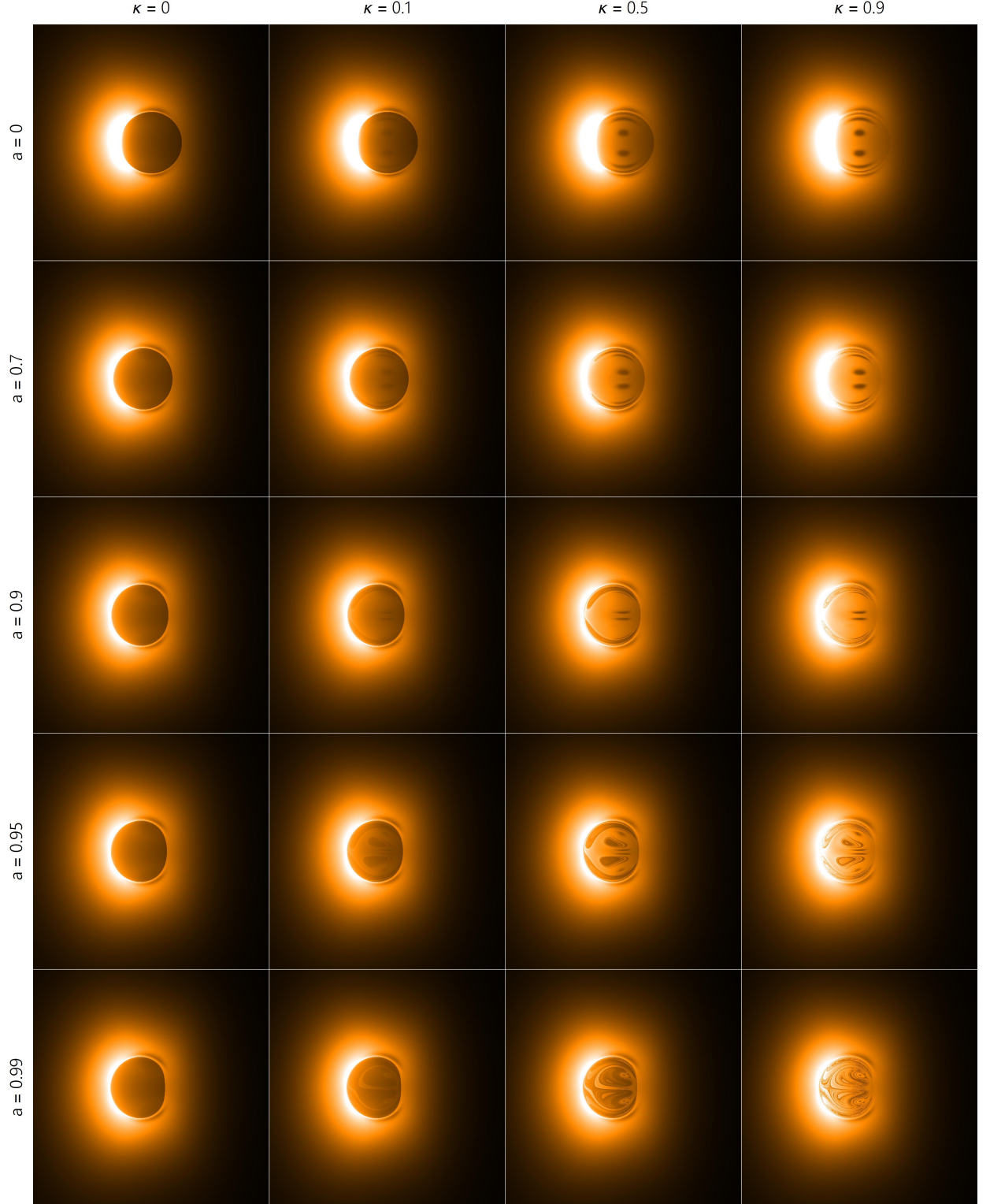


FIG. 7: Images of membrane with mass $M = 1$ and lapse $\alpha = 0.01$ surrounded by advection-dominated accretion flow for a range of values of spin parameter a and reflection coefficient κ at 100 GHz. The values of ratio of gas and radiation pressure β , characteristic temperature T_0 and electron density n_0 are $\beta = 1$, $T_0 = 10^{11}$ K, $n_0 = 10^7 \text{ cm}^{-3}$.

For observer following Keplerian orbit (that is circular orbit in equatorial plane) the angular velocity and angular momentum can be represented as

$$\Omega_K = \frac{\sqrt{M}}{r^{\frac{3}{2}} + a\sqrt{M}}, \quad l_K = \sqrt{M} \frac{r^2 - 2a\sqrt{rM} + a^2}{r^{\frac{3}{2}} - 2M\sqrt{r} + a\sqrt{M}}. \quad (\text{A4})$$

We also need to define several characteristic distances in Kerr spacetime. Those are horizon radius r_H , radius of *circular photon orbit* r_{ph} , radius of *marginally bound orbit*, that is orbit outside which circular Keplerian orbits exist, r_{mb} , and radius of *marginally stable orbit*, or *innermost stable circular orbit* (ISCO) that is orbit outside which stable circular Keplerian orbits exist, r_{ms} , given by

$$r_H = M \left(1 + \sqrt{1 - a_*^2} \right), \quad (\text{A5a})$$

$$r_{\text{ph}} = 2M \left(1 + \cos \left[\frac{2}{3} \arccos a_* \right] \right), \quad (\text{A5b})$$

$$r_{\text{mb}} = 2M \left(1 - \frac{a_*}{2} + \sqrt{1 - a_*} \right), \quad (\text{A5c})$$

$$r_{\text{ms}} = M \left(3 + Z_2 - \sqrt{(3 - Z_1)(3 + Z_1 + 2Z_2)} \right), \quad (\text{A5d})$$

where $a_* = \frac{a}{M}$, $Z_1 = 1 + (1 - a_*^2)^{\frac{1}{3}} \left[(1 + a_*)^{\frac{1}{3}} + (1 - a_*)^{\frac{1}{3}} \right]$ and $Z_2 = \sqrt{3a_*^2 + Z_1^2}$.

We use the following procedure in order to construct images of would-be event horizon: for each pixel of resulting image we evolve ray from observer point, located far enough from hole, backward. Observer is located at point (r_0, θ_0) and assumed to have zero angular momentum relative to infinity, that is it is *zero angular momentum observer*. The observer frame's basis can be expanded in the coordinate basis as follows (see e.g. [5]):

$$e_{(t)} = \zeta \partial_t + \gamma \partial_\varphi, \quad \text{where } \zeta = \sqrt{\frac{g_{\varphi\varphi}}{g_{t\varphi}^2 - g_{tt}g_{\varphi\varphi}}} \quad \text{and} \quad \gamma = -\frac{g_{t\varphi}}{g_{tt}}\zeta, \quad (\text{A6a})$$

$$e_{(r)} = A^r \partial_r, \quad \text{where } A^r = \frac{1}{\sqrt{g_{rr}}}, \quad (\text{A6b})$$

$$e_{(\theta)} = A^\theta \partial_\theta, \quad \text{where } A^\theta = \frac{1}{\sqrt{g_{\theta\theta}}}, \quad (\text{A6c})$$

$$e_{(\varphi)} = A^\varphi \partial_\varphi, \quad \text{where } A^\varphi = \frac{1}{\sqrt{g_{\varphi\varphi}}}. \quad (\text{A6d})$$

One can readily check that points which are in rest in this frame has zero angular momentum in coordinate frame (though nonzero angular velocity due to frame-dragging).

Momentum of photon measured by observer is connected to those in coordinate frame as $p^{(i)} = e_\mu^{(i)} p^\mu$. It is directly related to position of point corresponding to that photon on image (x, y) as

$$x = -r_0\beta \quad \text{and} \quad y = r_0\alpha, \quad (\text{A7})$$

where (α, β) is the pair of observational angles of a photon, which are connected to observed momentum as

$$p^{(r)} = p^{(t)} \cos \beta \cos \alpha, \quad (\text{A8a})$$

$$p^{(\theta)} = p^{(t)} \sin \alpha, \quad (\text{A8b})$$

$$p^{(\varphi)} = p^{(t)} \sin \beta \cos \alpha. \quad (\text{A8c})$$

Thus one finally arrives to initial conditions corresponding to point on image observed at angles (α, β) :

$$t(0) = 0, \quad p_t(0) = E \left(\frac{1 + \gamma \sqrt{g_{\varphi\varphi}} \sin \beta \cos \alpha}{\zeta} \right), \quad (\text{A9a})$$

$$r(0) = r_0, \quad p_r(0) = E \sqrt{g_{rr}} \cos \beta \cos \alpha, \quad (\text{A9b})$$

$$\theta(0) = \theta_0, \quad p_\theta(0) = E \sqrt{g_{\theta\theta}} \sin \alpha, \quad (\text{A9c})$$

$$\varphi(0) = 0, \quad p_\varphi(0) = E \sqrt{g_{\varphi\varphi}} \sin \beta \cos \alpha. \quad (\text{A9d})$$

Evolution of ray is determined by numerical solution of system of ordinary differential equations (A1) with initial condition (A9) using 8th order Dormand-Prince method with adaptive step size. Colour of pixel of image, corresponding to some ray, is determined by specific intensity, integrated along the ray. Due to Liouville's theorem phase space particle density of photons is conserved along ray [4] and therefore invariant specific intensity at given frequency ν_0 can be defined as[6]

$$\mathcal{I}_{\nu_0} = \int \frac{\nu_0^3}{\nu^3} j_\nu dl = E^2 \int \frac{j_\nu}{(u^\mu p_\mu)^2} d\lambda, \quad (\text{A10})$$

where j_ν is specific emissivity in emitter's frame, u^μ is emitter's velocity and dl is proper length along ray trajectory in the comoving frame. Instead of posterior integration of ray specific intensity along its trajectory we introduce equation

$$\frac{d\mathcal{I}_\nu}{d\lambda} = K \frac{j_\nu}{\nu^2}, \quad (\text{A11})$$

where K is accumulated reflection coefficient, which equals 1 initially, into set of equations considered.

When ray hits membrane it is reflected and K is multiplied by membrane reflection coefficient κ . Reflection is assumed to be specular in membrane frame, that is photon's momentum in membrane frame is transformed under reflection as

$$p_\mu \rightarrow p_\mu - 2(p^\nu n_\nu) n_\mu, \quad (\text{A12})$$

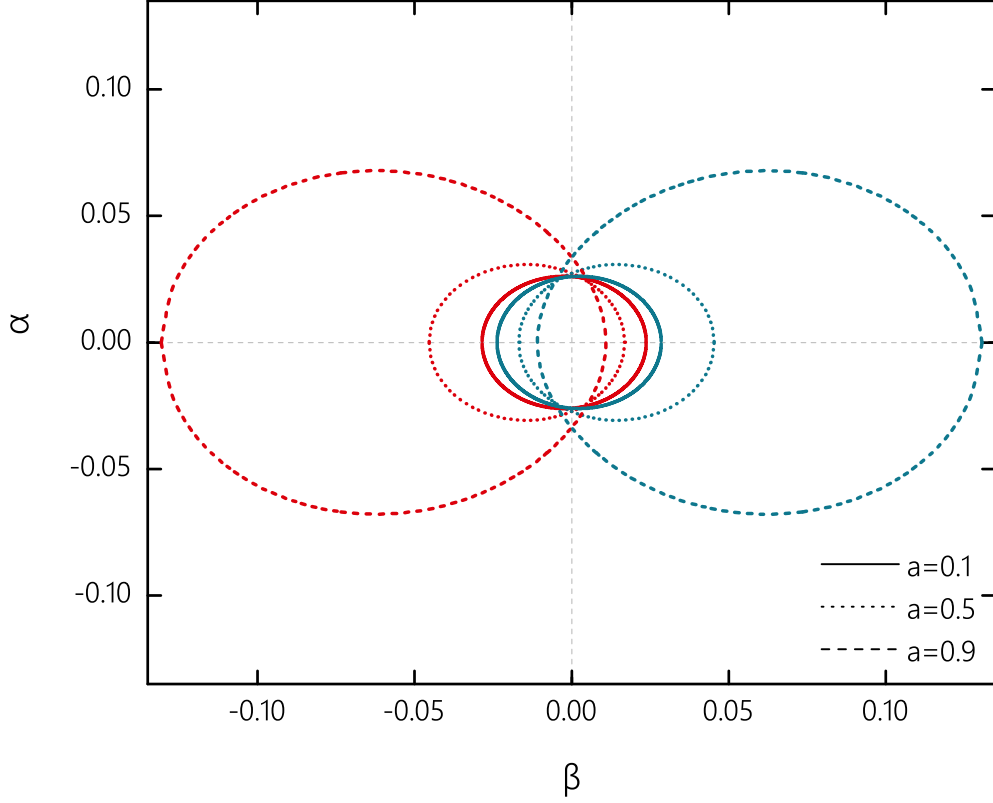


FIG. 8: Escape cones for a photon at equator of a membrane with $\alpha = 0.01$ as visible by observer on a membrane for a range of values of spin parameters a and $M = 1$. Blue curves is for cones of infalling rays and red curves are for reflected ones.

where n_ν is normal to membrane's surface. Since membrane is stationary and axially symmetric this transformation and transformation between corotating and coordinate frames are orthogonal in sense that they deal with different components of p_μ .

There is a range in a space of integrals of motion of photon around Kerr black hole such, that if photon has integrals of motion from this range then its trajectory necessary cross event horizon, that is photon will fall on black hole (if it is moving towards the black hole) or will escape on infinity (if it is moving outwards). The directions of motion of particle corresponding to this range form the so called *escape cone*. The boundary of this region is defined by condition of trajectory of photon being an orbit of constant radius, that is $R = 0$, $\frac{dR}{dr} = 0$. From this condition one can derive the following expressions for reduced integrals of motion $\lambda = \frac{L}{E}$ and $\eta = \frac{Q}{E^2}$:

$$\lambda = -\frac{R^3 - 3R^2 + a^2R + a^2}{a(R-1)}, \quad \eta = -\frac{R^3(R^3 - 6R^2 + 9R - 4a^2)}{a^2(R-1)^2}, \quad (\text{A13})$$

where R lies in a range of possible radii of spherical photon orbit, $R \in [r_1, r_2]$, which are defined as roots of η

$$r_{1,2} = 2 \left(1 + \cos \left(\frac{2}{3} \arccos \left[\mp \frac{|a|}{M} \right] \right) \right). \quad (\text{A14})$$

One can then calculate limiting values of momentum of photon in coordinate or observer's frame from (A1) and (A6) and directing angles of escape cone from (A8). The escape cone at position of observer also defines the edge of shadow of black hole as it is visible by him. Edge of shadow on image can be given parametrically as

$$y = r_0 \arcsin \left[\pm \frac{p^{(\theta)}}{p^{(t)}} \right], \quad x = r_0 \arctan \frac{p^{(\varphi)}}{p^{(r)}}, \quad (\text{A15})$$

where $p^{(i)}$ are given by (A1), (A6), (A13) with R as parameter.

APPENDIX B: PROPERTIES OF STRETCHED HORIZON

In this appendix we derive properties of stretched horizon discussed in section 2. The stretched horizon is assumed to be corotating, stationary and axisymmetric. Let us also require it to be free of tangential stresses, that is membrane acceleration to be orthogonal to its worldvolume. This condition can be written as follows:

$$e_\mu^i u^\nu \nabla_\nu u^\mu = 0, \quad (\text{B1})$$

where e_μ^i are basis of tangent vectors of membrane, and u^ν is the velocity of corotating or zero angular momentum observer, which has the following form:

$$u^\mu = \frac{1}{\alpha} (\eta^\mu + \omega \xi^\mu). \quad (\text{B2})$$

Using Killing equation and requirements of stationarity and axisymmetry of membrane one obtains that

$$\nabla_\nu u_\mu = -\frac{1}{\alpha} (\nabla_\mu \eta_\nu + \omega \nabla_\mu \xi_\nu) + e_\nu^i \left(\frac{\xi_\nu}{\alpha} \nabla_i \omega - u_\mu \nabla_i \ln \alpha \right). \quad (\text{B3})$$

Substituting this into (B1) and using conditions of zero angular momentum, stationarity and axisymmetry after a bit of algebra one is left with

$$\nabla_j \ln \alpha = 0, \quad (\text{B4})$$

that is there should be $\alpha = \text{const}$ along the surface.

We choose a reference frame of observer on membrane to have basis $(u^\mu, n^\mu e^{(\theta)\mu} e^{(\varphi)\mu})$, where u^μ is observer's velocity (B2), n^μ is normal to membrane defined as $n^\mu = \frac{\nabla^\mu \Phi}{\sqrt{\nabla_\mu \Phi \nabla^\mu \Phi}}$, where $\Phi = \frac{\rho\sqrt{\Delta}}{\Sigma} - \alpha$, and $e^{(\theta)\mu}, e^{(\varphi)\mu}$ are a couple of vectors orthogonal to them, which we choose in the form

$$\begin{aligned} e^{(\varphi)} &= \frac{1}{\sqrt{g_{\varphi\varphi}}} \partial_\varphi, \\ e^{(\theta)} &= \frac{1}{\rho\sqrt{1 + \frac{\Psi^2}{\Delta}}} (\Psi \partial_r + \partial_\theta), \quad \text{where } \Psi = -\frac{2\Delta\Sigma\partial_\theta\rho + \rho\Sigma\partial_\theta\Delta - 2\Delta\rho\partial_\theta\Sigma}{2\Delta\Sigma\partial_r\rho + \rho\Sigma\partial_r\Delta - 2\Delta\rho\partial_r\Sigma}. \end{aligned} \quad (\text{B5})$$

For our further consideration it is useful to take into account that corresponding value of lapse on membrane is $\alpha \ll 1$. Using definition of lapse one can derive the following relation between Boyer-Lindquist coordinates and lapse:

$$r = r_H + \frac{2M^2(2Mr_H - a^2)}{2M^2r_H - a^2(M + \sqrt{M^2 - a^2\sin^2\theta})} \alpha^2 + O(\alpha^4). \quad (\text{B6})$$

It is straightforward now to calculate momentum of photon reached membrane as measured by observer on membrane, $p^{(i)} = e_\mu^{(i)} p^\mu$ using (A1), (B5) and (B6). One then obtains that

$$\frac{p^{(\theta)}}{p^{(r)}} = O(\alpha), \quad \frac{p^{(\varphi)}}{p^{(r)}} = O(\alpha), \quad (\text{B7})$$

that is any light ray hits membrane almost normally.

Let us consider for a moment nonreflecting membrane, that is usual stretched horizon. Electromagnetic field measured by observer on membrane is diverging in limit $\alpha \rightarrow 0$, since observer is infinitely accelerated then. This is not the case, however, for geodesic observers, for which the field on membrane is regular. The basis of reference frame of such an observer with momentum u^μ can be written as $(u^\mu, e^{(r')\mu} e^{(\theta')\mu} e^{(\varphi')\mu})$, where u^μ is given in (A1) and other vectors can be determined from condition of orthonormality of tetrad. One can then calculate components of electromagnetic field tensor in membrane observer frame: $F_{(i)(j)} = F_{(i')(j')} e_{(i)}^{(i')} e_{(j)}^{(j')}$, in order to find that electric $E_{(i)} = F_{(i)(j)} u^{(j)} = F_{(i)(t)}$ and magnetic $B_{(i)} = \varepsilon_{(i)(j)(k)(l)} F^{(k)(l)} u^{(j)} = \varepsilon_{(i)(t)(k)(l)} F^{(k)(l)}$ fields are connected to those measured by free falling observer as

$$\begin{aligned} E_{(\theta)} &= \frac{1}{\alpha} (E_{(\theta')} - B_{(\varphi')}) + O(\alpha), & B_{(\varphi)} &= \frac{1}{\alpha} (B_{(\varphi')} - E_{(\theta')}) + O(\alpha), \\ B_{(\theta)} &= \frac{1}{\alpha} (B_{(\theta')} - E_{(\varphi')}) + O(\alpha), & E_{(\varphi)} &= \frac{1}{\alpha} (E_{(\varphi')} - B_{(\theta')}) + O(\alpha), \end{aligned} \quad (\text{B8})$$

that is leading terms of magnetic and electric field components parallel to membrane are not independent but rather connected as (2). Moreover, it appears that $E_{(r)} = O(1)$, $B_{(r)} =$

$O(1)$, which again confirms that the electromagnetic waves fall on membrane almost normally.

We then allows for resistivity of membrane not equaling 4π . Then the condition (2) on near-membrane field no longer holds, since field detected by free falling observer is no longer regular. Let us calculate reflection coefficient for a membrane with resistivity ρ_M . We assume that field on membrane can be separated into incident (i) and reflected (r) plane wave components as follows:

$$\mathbf{E}_{||} = \mathbf{E}_{||i} - \mathbf{E}_{||r}, \quad \mathbf{B}_{||} = \mathbf{B}_{||i} + \mathbf{B}_{||r}. \quad (\text{B9})$$

Substituting this into Ohm's and Ampere's laws

$$\begin{aligned} \mathbf{E}_{||} &= \rho_M \mathbf{j}_M, \\ \mathbf{B}_{||} &= 4\pi \mathbf{j}_M \times \mathbf{n} \end{aligned} \quad (\text{B10})$$

one after a bit of algebra obtains (3) and boundary condition (4).

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