



UNIVERSITÀ DEGLI STUDI DI GENOVA
Dipartimento di Fisica
Corso di Dottorato di Ricerca, XXIX ciclo



Towards measuring gravity on neutral antimatter

Ph.D. thesis

Candidate: Ruggero Caravita

Tutor: Dr. Gemma Testera, INFN
Dr. Michael Doser, CERN

Abstract

One of the antimatter community current biggest efforts is to measure gravity on a neutral antimatter system, towards a first test of Equivalence Principle - a cornerstone of General Relativity. Any deviation from the expected identical behaviour with respect to normal matter would be an indication of new physics.

This thesis explores the possibility to measure gravity with cold antihydrogen ($\bar{\text{H}}$) produced with a novel pulsed scheme based on a charge-exchange reaction between antiprotons and Rydberg positronium (Ps). $\bar{\text{H}}$ and Ps are currently the only neutral antimatter systems available in quantities at low temperatures, and both are being considered for measuring gravity.

Several results towards producing pulsed charge-exchange $\bar{\text{H}}$ were obtained in the context of the AEGIS experiment. Positronium was produced with 35% efficiency in a dedicated setup, where spectroscopy of the $1^3S \rightarrow 3^3P$ transition and subsequent excitation to Rydberg levels were performed with $\sim 15\%$ efficiency (bandwidth limited). Ps was produced also in the cryogenic $\bar{\text{H}}$ production region of AEGIS (with $\sim 25\%$ efficiency), where $\sim 10^5$ antiprotons were also stored for minutes after capture, cooling, radial compression and transfer. An estimate showed that 3 – 40 $\bar{\text{H}}$ atoms per measurement run could be formed in the present setup.

The experimental work motivated two detailed theoretical studies. A numerical code for calculating Rydberg Ps energy levels and optical spectra in arbitrary (strong) external fields, based on a non-perturbative approach, was developed. Results well reproduced high-resolution spectroscopic data from experiments, and also demonstrated the feasibility of measuring both Ps axial and transverse velocity components using only transverse laser probes. A novel numerical method for calculating single-species non-neutral plasma equilibria in cylindrical trap at low T was also developed, showing higher robustness and faster convergence to solution ($\propto T^{-1}$) than currently available methods ($\propto T^{-2}$).

Finally, the combined availability of the intense Ps source and the $1^3S \rightarrow 3^3P$ laser made possible to observe the 2^3S metastable state of positronium by optical decay from 3^3P , with a measured efficiency of $\sim 1\%$. This result represents a first step towards a gravity measurement on metastable 2^3S positronium.

Sommario

La misura della gravità terrestre su un sistema di antimateria neutro è una fra le mete attualmente più ambite dalla comunità della fisica dell'antimateria. Essa permetterebbe la prima verifica diretta del Principio di Equivalenza in forma debole, alla base della Relatività Generale, su un sistema di antimateria: una qualunque deviazione dalla perfetta simmetria con la materia costituirebbe una chiara indicazione di nuova fisica.

Questa tesi indaga la possibilità di misurare la gravità con atomi di anti-idrogeno freddo ($\bar{\text{H}}$), ottenuto tramite un innovativo schema di produzione impulsato basato sullo scambio di carica fra antiprotoni e positronio (Ps) eccitato a livelli di Rydberg. $\bar{\text{H}}$ e Ps sono attualmente gli unici sistemi di antimateria neutra che possano essere sintetizzati a bassa temperatura in elevate quantità, e sono entrambi oggetto di ricerca nella direzione di misurare la gravità.

Diversi risultati sono stati ottenuti, nel contesto dell'esperimento AEGIS, verso la produzione impulsata di anti-idrogeno per scambio di carica. Atomi di positronio sono stati prodotti con efficienza del 35% in un apparato dedicato per lo studio spettroscopico della transizione $1^3S \rightarrow 3^3P$ e la sua successiva eccitazione a livelli di Rydberg (entrambi ottenuti con efficienza del $\sim 15\%$, limitata dalla banda del laser). Il positronio è stato inoltre prodotto a temperature criogeniche in prossimità della zona di produzione dell'anti-idrogeno di AEGIS, con efficienza $\sim 25\%$. $\sim 10^5$ antiprotoni sono stati separatamente accumulati per diversi minuti nella trappola di produzione di $\bar{\text{H}}$ in seguito alla loro cattura, raffreddamento, compressione radiale e trasferimento.

Due studi teorici dettagliati sono stati intrapresi a seguito di questo lavoro sperimentale. E' stato sviluppato un codice di calcolo numerico dei livelli energetici e degli spettri ottici del positronio a livelli Rydberg in campi esterni arbitrari (e intensi), basato su un approccio non-perturbativo. I risultati hanno riprodotto fedelmente i dati ottenuti in esperimenti di spettroscopia ad alta risoluzione, e hanno dimostrato la fattibilità di misurare entrambe le componenti della velocità del positronio (parallela ed ortogonale al campo magnetico) utilizzando solo laser trasversali. E' stato inoltre sviluppato un nuovo metodo numerico per il calcolo della distribuzione di equilibrio di un plasma non-neutro a bassa temperatura T , che ha dimostrato maggior robustezza e più rapida convergenza alla soluzione ($\propto T^{-1}$) rispetto a metodi precedentemente pubblicati ($\propto T^{-2}$).

Infine, la disponibilità di una sorgente di Positronio intensa e di un laser per la transizione $1^3S \rightarrow 3^3P$ ha consentito di rivelare atomi di positronio nello stato eccitato metastabile 2^3S a seguito del decadimento ottico da 3^3P , con un'efficienza di produzione di circa $\sim 1\%$. Questo risultato costituisce un primo passo verso una futura misura di gravità su positronio metastabile.

It is a very remarkable thing that gravitation, a subject that began with Newton (...) is still relevant to some of the most profound questions in fundamental physics.

E. G. Adelberger et al., *Torsion balance experiments: a low-energy frontier of particle physics*, Progr. Part. Nucl. Phys 62 (2009) 102-134

Contents

I	Introduction	1
1	Why measuring gravity on neutral antimatter?	3
1.1	Universality of the free-fall and the Equivalence Principle	3
1.2	Experimental tests of the Equivalence Principle	3
1.3	Availability of neutral antimatter	9
2	How to measure gravity with neutral antimatter?	13
2.1	The moiré deflectometer/Talbot-Lau interferometer	14
2.2	Pulsed antihydrogen production	20
2.3	Introducing AEGIS	23
II	AEGIS	27
3	Layout of the experiment	29
3.1	Overview	29
3.2	Magnet system	33
3.3	Vacuum and cryogenics	34
3.4	Traps system	36
3.5	Detectors	41
3.6	Data acquisition	42
3.7	Positron system	44
3.8	Laser system	45
4	Positronium excitation to Rydberg levels	47
4.1	AEGIS positron system	47
4.2	Positron transport, bunching and focusing	52
4.3	Positronium formation and detection	55

4.4	Laser system for Ps $1 \rightarrow 3 \rightarrow \text{Ry}$ excitation	59
4.5	Positronium excitation to 3^3P	69
4.6	Positronium excitation to Rydberg levels	76
5	Positronium formation in AEgIS	81
5.1	Positron transfer to the main traps	82
5.2	Positronium formation via plasma manipulations	93
5.3	Positronium formation via direct injection	102
6	Antiprotons preparation in the $\bar{\text{H}}$ production trap	113
6.1	Electron plasma preparation	113
6.2	Antiproton capture	122
6.3	Electron cooling	127
6.4	Electron-antiproton plasma compression	135
6.5	Ballistic transfer of antiproton/electron plasmas	146
7	Towards pulsed $\bar{\text{H}}$ production	161
7.1	Experimental results in a summary	161
7.2	Expected $\bar{\text{H}}$ production rate	162
III	Special topics	171
8	Non-perturbative calculation of Positronium energy levels and optical spectra in uniform strong external fields	173
8.1	The problem of Ps energy levels in strong external fields	174
8.2	Calculation of optical transition probabilities	194
8.3	Towards measuring Ps axial velocity with transverse laser probes	205
9	Non-neutral plasma thermal equilibrium in cylindrically symmetric arbitrary confining potentials	211
9.1	The problem of the autoconsistent plasma potential	211
9.2	A spectral method for Poisson's equation in cylindrical coordinates	216
9.3	Numerical implementation of the spectral Poisson solver	222
9.4	Numerical solution of the autoconsistent plasma potential problem	226
9.5	Numerical results	230
10	Positronium excitation to the 2^3S metastable state	245

10.1 Theory	247
10.2 Experimental	250
10.3 Analysis	257
10.4 Results and discussion	267
10.5 Towards efficient production of 2^3S metastable positronium	272
Conclusions	277
Acknowledgments	281
Bibliography	285
List of Acronyms	iii
List of Figures	vii
List of Tables	xiii

Part I

Introduction

Chapter 1

Why measuring gravity on neutral antimatter?

1.1 Universality of the free-fall and the Equivalence Principle

The main scientific goal beyond directly measuring an antimatter body free-fall trajectory in a known (Earth's) gravitational field is testing the principle of universality of the free-fall, one of the cornerstones of our understanding of gravitation.

After Galileo's notorious observation that the free-fall trajectory of a body is independent of its mass (presented in *Discorsi intorno a due nuove scienze*) the principle of universality of the free-fall, which in Newtonian physics terms requires the exact equivalence between inertial and gravitational masses $m_i = m_g$, has never been disproved as of today.

Later, it was promoted by Einstein from principle to fundamental axiom, who founded around it (together with the constancy of c in vacuum) the theoretical framework of special and general relativity. In Einstein's formulation, universality of free-fall comes as a consequence of the more general Equivalence Principle (EP), which states that *a homogeneous gravitational field is locally equivalent to a uniformly accelerated reference frame* (cit. [1]). Universality of free-fall is, in this view, the most direct prediction of the Equivalence Principle.

1.2 Experimental tests of the Equivalence Principle

When put in the experimental context, the Equivalence Principle is customary distinguished in a Strong form and a Weak form (see [1]). If, in a free-fall experiment, $m_i = m_g$ is tested including the gravitational binding energy contribution to the mass (usually true only for masses

from the scale of stellar objects), the experiment is said to be testing the Equivalence Principle in its Strong form (or Strong Equivalence Principle (SEP)). Vice-versa, if the gravitational contribution to the binding energy is neglected, the experiment tests the Equivalence Principle in its Weak form (or Weak Equivalence Principle (WEP)).

In this section, a summarized review of the present state-of-the-art in testing experimentally the Equivalence Principle is given.

1.2.1 Tests of the Equivalence Principle with matter

The equivalence between gravitational and inertial masses has been tested very accurately with matter systems. A complete review of its experimental verifications is found in C. Will's periodically updated review article (see [2], from which Figure 1.1 is taken), which discusses the most significant Strong and Weak Equivalence Principle tests performed during the last century.

In order to compare experiments using very different systems (spanning from atomic to stellar scale), the EP-violation parameter η is commonly adopted (also called the Eötvös parameter), defined as the differential acceleration between the two bodies under examination divided by the average. η , in terms of Newtonian physics, is defined as ([3])

$$\eta = 2 \frac{|a_1 - a_2|}{|a_1 + a_2|} = 2 \frac{|(m_g/m_i)_1 - (m_g/m_i)_2|}{|(m_g/m_i)_1 + (m_g/m_i)_2|} \quad (1.2.1)$$

Torsion balance experiments

Starting from Eötvös's original experiment of 1909, measuring $\eta < 10^{-8}$, torsion balances are historically the tool par excellence for testing the Weak Equivalence Principle, as they provide a very good control over the systematics.

The most stringent limit of WEP violation by torsion balances was performed as of writing by the Eöt-Wash group in 2012 (see [3]). The outcome of the experiment on a Be-Ti torsion balance was (to 95 % confidence level)

$$\eta_{weak} < (0.3 \pm 1.8) \cdot 10^{-13} \quad (1.2.2)$$

Lunar laser-ranging

A series of accurate measurements of Earth-Moon's distance were conducted by means of laser ranging since the first placement on the lunar ground of retroreflector arrays by the Apollo 11,

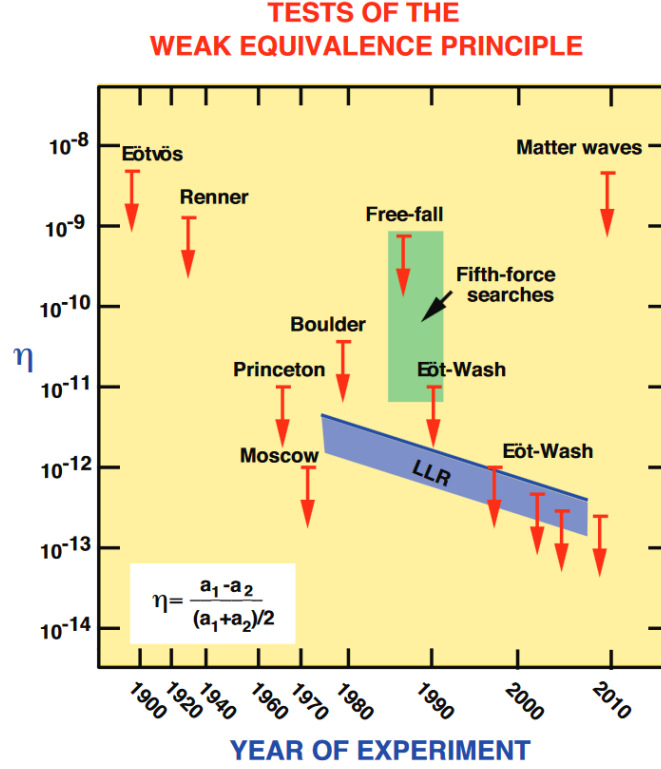


Figure 1.1: Overview of the Equivalence Principle tests with matter as of 2014 (from [2]). The Eötvös parameter η on the systems under examination (see text) is plotted versus the year of the test.

14 and 15 missions. The wide set of recorded lunar laser-ranging data allowed to test very accurately Moon's free-fall trajectory around the Sun and the Earth, leading to a test of the Universality of Free Fall and Equivalence Principle in its Strong form using planetary bodies [4].

The result of the UFF test yielded, to 95 % confidence level

$$\left(\frac{m_g}{m_i}\right)_{Earth} - \left(\frac{m_g}{m_i}\right)_{Moon} = (-1.0 \pm 1.4) \cdot 10^{-13} \quad (1.2.3)$$

which can be translated into a SEP violation test, yielding (to 95 % confidence level)

$$\eta_{strong} < (4.4 \pm 4.5) \cdot 10^{-4} \quad (1.2.4)$$

Atom interferometry experiments

A very active research community is currently increasing the limits on the validity of the Weak Equivalence Principle at atomic scale on different elements. The most stringent test of WEP at the time of writing was performed by the Wuhan group using a two-species atom interferometer loaded with the rubidium isotopes $^{85}\text{Rb} - ^{87}\text{Rb}$ [5].

$$\eta_{weak} < (2.8 \pm 3.0) \cdot 10^{-8} \quad (1.2.5)$$

Another significative test has been performed by the Firenze group using two strontium isotopes with different total spin $^{88}\text{Sr} - ^{87}\text{Sr}$ (thus also limiting spin-gravity couplings) measuring the Bloch oscillation frequency in a vertical optical lattice [6].

$$\eta_{weak} < (0.2 \pm 1.6) \cdot 10^{-7} \quad (1.2.6)$$

A further series of accurate measurements of η are planned for the next years in the International Space Station microgravity [7] and using a 10 m-long atomic fountain [8].

1.2.2 Tests of Equivalence Principle with antimatter

Contrarily to the case of matter systems, experimental bounds on the validity of the Weak Equivalence Principle ¹ set using antimatter are either extremely loose or model-dependent, as a consequence of the technical difficulties in producing and handling it.

In this section, available direct and indirect limits on the validity of the WEP, obtained using antimatter systems. are briefly reviewed.

ALPHA's rough direct limit

In 2013, the ALPHA collaboration at CERN Antiproton Decelerator (AD) reported a first rough measurement of the gravitational to inertial mass ratio $F = m_g/m_i$ measured on 434 trapped antihydrogen atoms released from their octupole magnetic trap [9] over two years of data taking. ALPHA excluded, to 95 % confidence level, that $F > 110$ if normal gravity is assume and $F < -65$ if gravity with a reversed sign of g is assumed. As of the time of writing, this is the sole direct test of WEP on a cold antimatter system.

In order to extrapolate an Eötvös parameter for comparing these results to matter systems ones, the measured acceleration can be compared to Earth's gravitational acceleration on normal

¹unfortunately only in the weak form, as antimatter stellar bodies have never been observed.

matter g . ALPHA's measured acceleration on antihydrogen is obtained upon inverting the relation $F = m_g/m_i = g/a$. Substituting a and g in the definition of η , with $F = 110$ (normal gravity), Equation 1.2.1, yields

$$\eta_{weak} = 2 \frac{|g/F - g|}{|g/F + g|} = 2 \frac{|1/F - 1|}{|1/F + 1|} = 1.96 \sim 2 \quad (1.2.7)$$

This result shows that ALPHA's measurement practically did not set any limit on the Weak Equivalence Principle on cold antimatter. Moreover, as the described technique relies on the number of available antiatoms, any further improvement is subjected to the $1/\sqrt{N}$ law, which is hard to be achieved by increasing the experimental time.

Attempts with charged antimatter systems

In 1965, Witteborn and Fairbank investigated experimentally the possibility of measuring the gravitational free-fall of electrons and positrons in a vertical drift tube. In their two published seminal papers [10, 11], they discussed the feasibility of a WEP test on charged particles, and the null result of their experiment as a consequence of the dominant systematic effects²

In 1986, the PS200 collaboration at CERN Low-Energy Antiproton Ring (LEAR) proposed to measure Earth's gravitational force on antiprotons using a free-fall experiment similar to the one pioneered by Witterborn and Fairbank [15]. PS200 developed several techniques to take control over the very tough systematic effects of such kind of experiments [16], and was still in its design phase - working on reducing electric patch-effect [17] - when LEAR was shutdown from CERN in favour of the AD program, focused on antihydrogen production.

TRAP's redshift measurement of $p/\bar{p}$ cyclotron frequencies

In 1999, the former TRAP collaboration at CERN LEAR published a result on Physical Review Letters [18] with a very stringent comparison of proton-antiproton cyclotron frequencies in trap, measuring

$$\frac{(q/m)_{antiproton}}{(q/m)_{proton}} = 1 + (0.9 \pm 0.9) \cdot 10^{-11} \quad (1.2.8)$$

This measurement, which was meant as an accurate direct test of CPT symmetry between matter and antimatter [18], was also interpreted (controversially) as a WEP test using the

²examples of systematic effects encountered by Witteborn and Fairbank were electric field patch effect [12], Schiff-Barnhill effect [13] and residual charge on materials, as one electron charge at 5 m distance equals gravity [14].

following argument. If CPT symmetry is assumed valid, and protons are assumed not to violate the Equivalence Principle, the result of the experiment constraints the maximum gravitational redshift of cyclotron frequencies in the absolute gravitational potential according to [19]

$$\frac{\Delta\omega_c}{\omega_c} = 3(F - 1)\frac{\Delta U}{c^2} \quad (1.2.9)$$

where ΔU is the gravitational potential energy difference between empty flat space-time and the local position of the observation (i.e. inside a supercluster of galaxies). TRAP collaboration's measurement in [18] would yield

$$F - 1 < 10^{-6} \quad (1.2.10)$$

This test was criticized using two arguments. First, the assumption of CPT validity between proton and antiprotons had not been verified independently³. Secondly, it relies on the accurate estimation of the absolute gravitational potential at the observer's position - which comes with all the modellistic assumptions of gravitation at larger than galactic scales, Dark Matter above everything.

TRAP's measurement is usually not considered an appropriate WEP test, but rather a stringent CPT test.

SN1987A

The observation of the neutrino burst coming from the supernova explosion SN1987A, measured by the Kamiokande-II detector [21], lead to hypothesize an indirect WEP test on the $\nu - \bar{\nu}$ system. As no different time-of-arrival for ν and $\bar{\nu}$ was observed, it could be concluded then that the Shapiro delay in the Galaxy gravitational field had to be the same for both neutrinos and antineutrinos. Thus, both ν and $\bar{\nu}$ satisfy the Equivalence Principle to a precision proportional to the 13s of the measured supernova light burst over the 4.8 months of light flight time. If accepted, this test would limit WEP violations on matter and antimatter to the $\sim 10^{-6}$ accuracy [22].

This WEP test was openly contradicted by Unnikrishnan and Gillies in [23]. The argument used was the following: since neutrinos are ultrarelativistic particles, the Shapiro delay of the two species does not depend on their rest masses, but (almost) entirely on their momenta; thus, no real sensitivity to violations of $m_i = m_g$ is, in fact, available.

³now, after [20], this argument is superseded by events.

1.3 Availability of neutral antimatter

The experimental difficulties experienced by Witteborn and Fairbank [10, 11] and by the PS200 collaborators [16] in controlling systematic effects to directly measure gravity on charged particles, lead to conclude that neutral antimatter systems are, generally, a more preferable choice. Before discussing the technical aspects of a free-fall experiment using neutral antimatter to test the Weak Equivalence Principle (see 2), a review of the present availability of suitable low-energy neutral antimatter systems is found useful.

As of today, only four neutral antimatter systems have been produced at temperatures $\lesssim$ eV in good amounts: antihydrogen, the stable bound state of $\bar{p}$ and e^+ ; antiprotonic helium, the metastable (lifetime $\sim$ ms) bound state of ${}^4\text{He}^+$ and $\bar{p}$; Positronium (Ps), the unstable bound state of e^+ and e^- ; Protonium, the unstable bound state of p and $\bar{p}$.

The most interesting of these sources for the purpose of measuring gravity are antihydrogen in first instance, for being stable, massive and purely made of antiparticles, and positronium in second instance, for being purely leptonic, easier to synthesize and also very useful in producing $\bar{H}$.

Two active research lines are currently pursuing the task of measuring directly gravity on those two systems and set a first stringent WEP limit with neutral antimatter: on $\bar{H}$, the AEGIS, GBAR and ALPHA-g collaborations at CERN AD; on Ps, the UCL-London group and the ETH-Zurich.

1.3.1 Antihydrogen

Antihydrogen is, as of today, the only available neutral antimatter system made entirely of antimatter constituents that can be synthesized at $\sim$ K temperatures [24]. Moreover, it is stable (its lifetime in the ground state should be virtually infinite, as for hydrogen) and $\times 2000$ less sensitive to external field gradients than Ps, thanks to its higher mass.

Producing antihydrogen depends almost entirely on the possibility to get low-energy antiprotons, which in the last 20 years have been available only at CERN Antiproton Decelerator, delivered to the experiments in spills of $3 \cdot 10^7$ pbar/spill every 110 s.

The antihydrogen saga began with its milestone first formation and detection in 2002 by the ATHENA collaboration ([25]) followed right away by the ATRAP collaboration independent confirmation ([26]). Since then, the AD community achieved many milestones towards studying and manipulating low temperature $\bar{H}$ at increasing degrees of precision: from ATRAP's first laser-controlled $\bar{H}$ formation [27], to ALPHA's first confinement of $\bar{H}$ in an octupole magnetic trap ([24]), to ASACUSA's first beam formation [28], to ALPHA's spectroscopy of $\bar{H}$'s 1S-2S

line [20].

Besides the improvements in manipulation and detection techniques, $\bar{\text{H}}$ is still produced by the AD experiments in almost the same amounts of 2002, in the order of $\sim 10^4$ atoms per 10 min at the most [24]. Obtaining very cold $\bar{\text{H}}$ atoms with temperatures below K has proved to be a very difficult task, as the temperature of the initial antiprotons has never been measured colder than 3.5 K [29].

Presently, a new decelerator named ELENA is being commissioned in the AD hall to furtherly reduce $\bar{\text{p}}$ energy from 5.3 MeV to 100 keV, thus allowing the experiments to get up to a factor $\times 100$ more antiprotons for upcoming applications.

1.3.2 Positronium

Positronium, contrarily to $\bar{\text{H}}$, is a very light and very short-lived atom⁴: only 143 ns in its ground 1^3S state. Many efforts are necessary to increase its lifetime, such as laser-excitation to long-lived Rydberg states [30, 31, 32], where it survives de-excitation and annihilation $\sim$ ms. Its only two advantages are being a purely leptonic system (hadronic contributions to energy levels are orders of magnitude smaller than antihydrogen [33]) and being much easier to synthesize in quantities than $\bar{\text{H}}$. Efficient Ps conversion and emission into vacuum has been subject of intense research over all last decade, mostly made possible by technological advances in trap-based techniques for positron plasma synthesis from intense β^+ sources [34, 35] and material physics studies for building efficient positron-positronium converters (see, among the many, [36, 37, 38, 39, 40, 41]). These developments in trap-based techniques for positron applications were, for example, at the basis of the first antihydrogen formations of ATHENA and ATRAP [25, 26, 27].

Production of dense Ps clouds in vacuum carved out the way for many new dedicated experiments. As Ps is a purely leptonic system, many precise spectroscopical surveys were carried out for QED tests ([42, 43, 44, 45, 46, 30, 47, 31], among the many). Another example are the accurate annihilation lifetime measurements [48, 49, 50], which gave rise to the famous ortho-Positronium (o-Ps) puzzle due to the (now solved) disagreement between experiments and QED predictions of $5.2 - 9.1\sigma$ [51, 52]. Studies of molecular Ps-Ps interactions were also undertaken (see [53]), as well as proposals towards Bose-Einstein condensation of Ps [54] as way to build a γ -ray laser [55, 56]. Finally, several papers studied the possibility to perform gravity tests on Ps as a symmetric matter-antimatter probe [57, 58, 59].

Ps has been used also to achieve efficient Ps-mediated antihydrogen synthesis, as already mentioned above [27]. The $\bar{\text{H}}$ production method via charge-exchange with Rydberg Ps offers higher production cross-section (see 2.2.2) than the usual three-body recombination technique

⁴actually, it is the lightest electromagnetic bound-state in the Standard Model.

developed by ATHENA [25], but requires an intense source of Rydberg Positronium.

Current sources based on accumulating e^+ from an intense β^+ source are able to store and deliver spills of $10^7 - 10^8 e^+$ within a few minutes of accumulation ⁵. Mesoporous silica targets convert $\sim 30\%$ of those positrons into thermal positronium re-emitting it into vacuum [40] to be used by the experiments (see 4.3 for further details).

⁵a more intense source of e^+ , based on a low-energy LINAC for pair-production, is currently under development for the GBAR experiment [60]

Chapter 2

How to measure gravity with neutral antimatter?

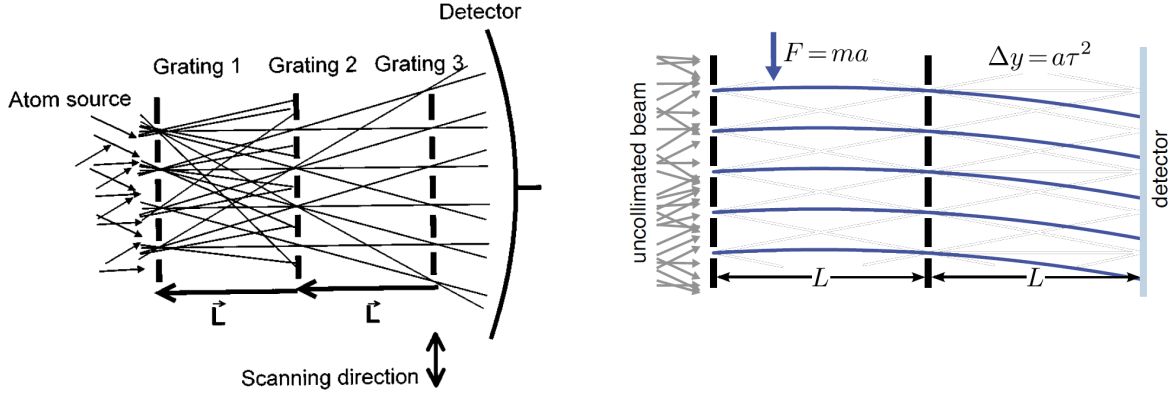
The aim of this chapter is to bring theory to practice and describe - in a summary - a method to observe neutral antimatter atoms free-fall in Earth's gravitational field. The method described here has been chosen by the AEGIS collaboration to attempt a first gravity measurement on antihydrogen ($\bar{\text{H}}$).

The chosen experimental methodology (possibly the most direct and least model dependent for a test of universality of the free-fall) consists of letting the (anti-)atoms annihilate on a position-sensitive detector after free-falling for a time Δt with uniform acceleration $-g$. From the vertical displacement of their parabolic trajectories $\Delta y = -1/2 g \Delta t^2$, g is obtained given the time-of-flight Δt .

As vertical displacement detector, a tool from atom optics is borrowed: the moiré deflectometer - Talbot-Lau interferometer, used within its field as inertial sensor on non-monochromatic beams [61]. In the first section of this chapter, the physical model beyond a moiré deflectometer - Talbot-Lau interferometer inertial sensitivity is outlined, in order to clarify the main requirement imposed on the antihydrogen source, i.e. being pulsed.

The $\bar{\text{H}}$ pulsed formation scheme adopted by AEGIS, based on a charge-exchange between trapped antiprotons and Rydberg positronium, is then shortly reviewed in the subsequent section.

Finally, a suitable experimental scheme for implementing the pulsed $\bar{\text{H}}$ production via charge-exchange, at the basis of the current setup of AEGIS, is given.



(a) moiré deflectometer as trajectory selector

(b) principle of a free-fall measurement

Figure 2.1: A classical moiré deflectometer selects particles trajectories from a non-monochromatic source to produce a regular periodic distribution of counts on the detector plane (panel (a), from [61]). Particles, as they free-fall through the gratings, get deflected by the action of gravity, inducing a phase shift of the observed periodic distribution (panel (b), from [62]).

2.1 The moiré deflectometer/Talbot-Lau interferometer

A moiré deflectometer (first described by M. Oberthaler in [61]) consists of a set of two regular gratings with the same periodicity d set in front of a position-sensitive detector, the three being equally spaced by a distance L (see Fig. 2.1).

As in the case of its wave-mechanical counterpart, the Talbot-Lau interferometer, a distribution of atoms from a non-monochromatic source, after flying through the gratings and hitting the detector surface, produces a periodic pattern with the same period d of the gratings and an arbitrary phase ϕ . The sensitivity to external accelerations originates from the fact that any deviation from the linear trajectory, due to the presence of an external force, induces a position shift of the detected distribution on the detector plane, which is observed as a phase shift of the periodic pattern. By taking the difference of the measured phase shift ϕ against a reference value ϕ_0 , obtained with a reference sample, the value of the differential acceleration felt by the atoms can be obtained:

$$\Delta\phi = \phi - \phi_0 = -\frac{2\pi}{d}a\tau^2 \quad (2.1.1)$$

In order to work out a from Equation 2.1.1, two informations are needed: the vertical position of the atom annihilation and its time-of-flight in the deflectometer τ .

The vertical position of an $\bar{\text{H}}$ annihilation can be measured in two ways (as depicted in Figure

2.1): either adopting a position-sensitive detector able to resolve annihilation vertices far better than the grating periodicity d , or using a third movable grating to perform a full-period vertical scan with a simple counting detector. Examples of high-resolution detectors considered for this application were studied during the last years by AEGIS collaborators (see [63, 64, 65, 66])

Measuring the time-of-flight in the deflectometer τ translates mainly into a constraint imposed on the chosen $\bar{H}$ production mechanism. The arrival time on the detector t_1 can be easily accessed experimentally by tagging in time the byproducts of the $\bar{H}$ annihilation (for example using a fast scintillation tracker, see [67]). The knowledge of the free-fall initial time t_0 , on the contrary, has to be somewhat already known a-priori, for example adopting a pulsed $\bar{H}$ beam formation scheme (as discussed in the following section of this chapter).

2.1.1 Phase reference from near-field diffraction of light

A convenient choice for measuring the reference phase shift ϕ_0 without external forces is to shine a beam of light through the gratings. If a monochromatic laser source is adopted, even a set of two regular gratings well within the classical regime for atoms at $\sim$ K temperature will behave as an near-field interferometer for light. The regular shadow pattern on the detector surface is substituted by a near-field interference pattern known as Talbot pattern, after its first observation in 1836 by H. F. Talbot [68]. Interferometers based on the near-field Talbot effect are similarly named Talbot-Lau interferometers (after [69]).

Talbot patterns are observed in the Fresnel/Rayleigh diffraction regime from coherent and semi-coherent sources (see Figure 2.2, obtained using the technique described in [70]). Unlike the case of far-field interference patterns, the visibility¹ of Talbot fringes depends upon the distance between last grating and detector. If the detector distance is chosen at integer multiples of the characteristic Talbot length $L_T = d^2/\lambda$, Talbot fringes are observed with maximum visibility and almost reproducing the grating square-wave profile (upon bandwidth limitations), as a consequence of a coherent rephasing of the interfering waves. An example of fringe visibility as a function of the z distance within the interval $z \in [0, 2L_T]$ is shown in Figure 2.2, inset.

Talbot effect solves automatically two practical problems: it provides the necessary phase reference ϕ_0 on the detector plane and offers a neat solution to adjust and monitor gratings and detector alignments. If all distances L are set as an integer multiple of the Talbot length L_T

¹the definition of visibility adopted here is

$$\mathcal{V} = \frac{\max[I_n] - \min[I_n]}{\max[I_n] + \min[I_n]} \quad (2.1.2)$$

where max and min span on the uniformly sampled intensity signal I_n obtained by binning the continuous intensity signal $I(x)$ with either a pixel detector (as in Figure 2.1.b) or by vertically displacing by regular intervals a third grating in front of a summing detector (as in Figure 2.1.a).

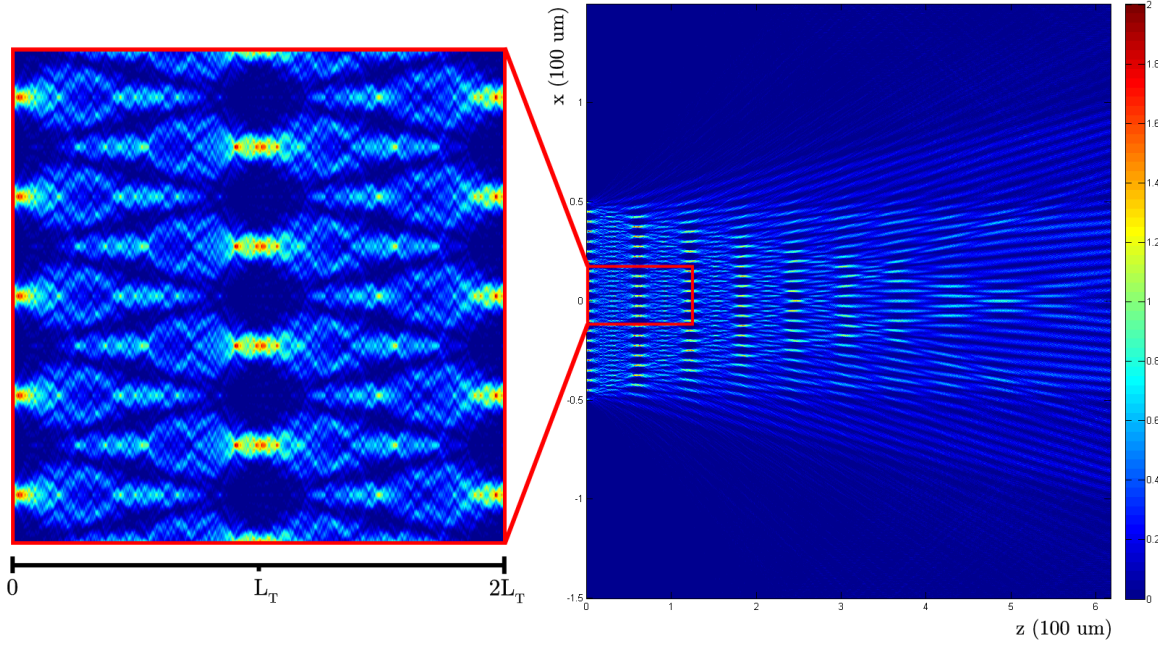


Figure 2.2: Near-field diffraction of a finite-size regular grating: light intensity distribution as a function of the x - z position, showing the consequence of light interference as a function of the z distance from the grating. The so-called *Talbot carpet* is a consequence of periodic rephasing of light, which happens regularly after the grating at integer multiples of the Talbot length L_T (see text).

², the mutual alignment of the different components can be optimized by varying the distances with microactuators, seeking the maximum visibility of the Talbot fringes. This alignment technique guarantees an axial position sensitivity of about $L_T/10$, which for a $d = 20 \mu\text{m}$ grating illuminated by a red laser diode ($\lambda = 650 \text{ nm}$) is about $62 \mu\text{m}$.

2.1.2 Sensitivity to an external acceleration

The possibility to adopt a classical moiré deflectometer/Talbot-Lau interferometer as inertial sensor for measuring gravity from the free-fall of antihydrogen atoms has been considered first in the AEGIS experiment proposal [71, 72], then in P. Bräunig doctoral thesis [62], where a theoretical model to calculate deflectometer inertial sensitivity in the classical and matter-wave regime has been developed.

The technical feasibility of such a measurement within the context of an antihydrogen experiment was also demonstrated by using a miniaturized version of a moiré deflectometer installed

²varying the λ of the reference laser could be the best option

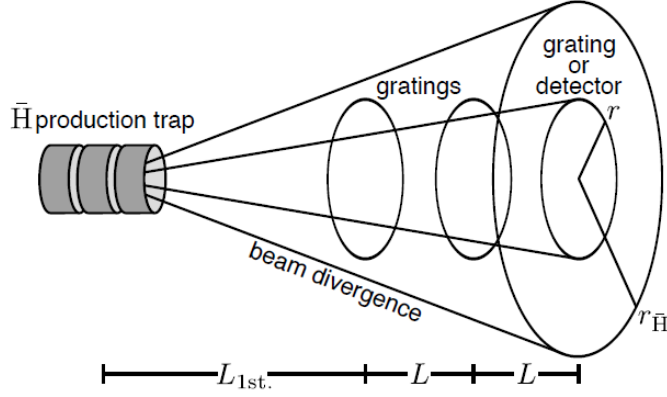


Figure 2.3: Sketch of the moiré deflectometer/interferometer geometry layout in front of the antihydrogen source. From [62]

in front of a nuclear emulsion detector, all exposed to a beam of antiprotons of the AEgIS experiment. A mean force of $F = 530 \pm 50_{\text{stat}} \pm 350_{\text{sys}}$ aN acting on the 4.5 m s^{-1} $\bar{\text{p}}$ emerging from the degrading system of AEgIS was observed with this mini-moiré deflectometer (see [73]). This measured force, interpreted as a stray Lorentz force of electric or magnetic origin, induced an average shift of the periodic pattern on the detector of $\Delta y_{\text{mean}} = (9.8 \pm 0.9_{\text{stat}} \pm 6.4_{\text{sys}}) \mu\text{m}$, akin to the shift due to gravity that would be observed with low-velocity antihydrogen. An increase in sensitivity of 11 orders of magnitude is expected when using $\sim 500 \text{ m s}^{-1}$ $\bar{\text{H}}$: 8 orders from the longer interrogation time due to the slower velocity of the particles, and three orders of magnitude from the increased size of the detector.

Following the theoretical derivation in P. Bräunig thesis [62], the minimal inertial sensitivity of a moiré deflectometer/Talbot-Lau interferometer positioned downstream of a $\bar{\text{H}}$ source (as outlined in Figure 2.3) can be expressed as a function of the following constructive parameters:

d - the grating periodicity;

$\mathcal{V}$ - the visibility of the periodic fringes, see Equation 2.1.2;

η - the gratings open fraction;

r - the radius of the gratings/detector, assumed equal;

τ - the time-of-flight $\tau = L/v$

$L_{1\text{st}}$ - the distance between the $\bar{\text{H}}$ source and the first grating;

L - the grating separation;

β - the forward boost parameter ($\beta = v_{\parallel}/v_{\perp} \geq 1$; $\beta = 1$ stands for isotropical emission);

k_b - the Boltzmann constant;

T - the $\bar{\text{H}}$ source temperature, assumed as a Maxwellian distribution;

m - the mass of the $\bar{\text{H}}$ atom;

N_{det} - the number of detected atoms;

N_{prod} - the number of produced atoms.

The minimal detectable acceleration, written as a function of the number of detected atoms and their time-of-flight in the deflectometer, results

$$a_{\min} = \frac{d}{2\pi \mathcal{V} \tau^2 \sqrt{N_{\text{det}}}} \quad (2.1.3)$$

Alternatively, it can be expressed as a function of the number of produced atoms by an isotropical thermal source at temperature T

$$a_{\min} = \underbrace{\frac{d}{2\pi \mathcal{V} \eta r}}_{\text{gratings}} \cdot \underbrace{\frac{2(L_{1\text{st}} + 2L)}{L^2}}_{\text{geometry}} \cdot \underbrace{\beta}_{\text{boost}} \cdot \underbrace{\frac{2k_b T}{m} \frac{1}{\sqrt{N_{\text{prod}}}}}_{\bar{\text{H}} \text{ source}} \quad (2.1.4)$$

This formula is valid both for the classical regime ($L \ll L_T$) and the interferometric regime ($L \gtrsim L_T$). All parameters in the formula are free, with the exception of the fringe visibility $\mathcal{V}$, which depends by the gratings open fraction η , the grating periodicity d and the spatial coherency of the incoming beam.

The dependency of the fringe visibility on the gratings open fraction η , for the case of a completely incoherent source, is discussed in [61]. For $\eta \leq 0.25$, the fringe pattern visibility is 100%; then progressively drops to $\mathcal{V} = 0$ as η reaches 0.5 (see Figure 2.4, panel a, top). As the source gets more coherent, the limiting value of η for getting $\mathcal{V} = 1$ gets closer to 0.5, up to the limit of a perfectly coherent beam for which $\mathcal{V} = 1$ also for $\eta = 0.5$ (i.e. the case of a laser beam discussed above) ³. A good compromise is usually found for $\eta = 0.3$, for which the fringe visibility of the completely incoherent case is $\sim 80\%$ and the minimal detectable acceleration - resulting also from the transmittivity of the two gratings - is close to the minimum value (see Figure 2.4, panel a, bottom).

The dependency of the fringe visibility on the gratings periodicity d was the object of study of a subsequent internal report circulated among the AEgIS collaboration by P. Bräunig and A. Demetrio [75], where $\mathcal{V}$ was studied as a function of d in presence or absence of gravity in both

³this happens also for thermal sources as the $L_{1\text{st}}$ distance grows, as a consequence of Van Cittert-Zernike theorem [74].

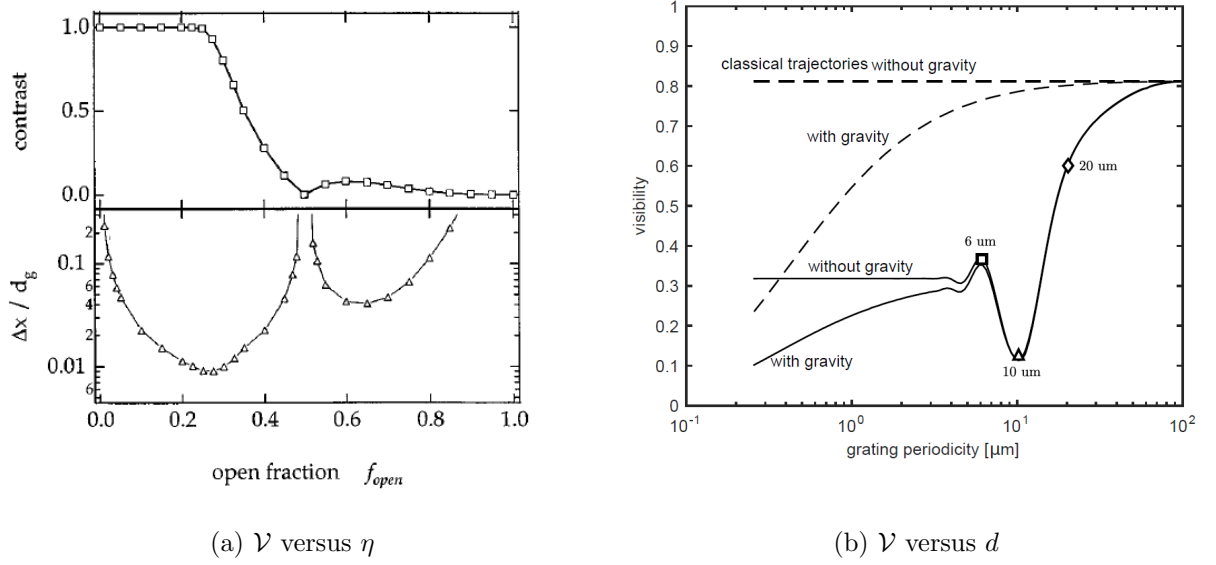


Figure 2.4: Visibility of moiré deflectometer fringes from a completely incoherent source. Panel (a), from [61], shows the fringe visibility as a function of the gratings open fraction (above) and the minimal fraction of a period that can be resolved with N given particles (below). Panel (b), courtesy of P. Bräunig and A. Demetrio [75], shows the fringe visibility as a function of the gratings periodicity both in the classical moiré regime (dashed curves, with and without gravity) and the Talbot-Lau interferometer regime (solid curves, with and without gravity).

classical and wave regimes, using $\eta = 0.3$ and $T = 10$ K. The visibility was found a complex function of the periodicity (shown in Figure 2.4, panel b) leaving the purely classical trajectory regime for $d < 50 \mu\text{m}$.

The number of detected antihydrogen atoms for reaching a relative precision of 1 % on gravity using Equation 2.1.3 were estimated in [62, p. 41] to be about ~ 2000 , assuming a periodicity $d = 40 \mu\text{m}$, 50 mm gratings separated by 0.6 m and 1 m distant from the source, an axial velocity of the atoms of 500 m s^{-1} and no systematic effects from magnetic gradient forces.

This reference value could be converted into a flux of produced atoms by the source using Equation 7.2.14 at various temperatures [62, p. 42]. A gravity measurement would start getting feasible when the total integration time is of the order of tens of measurement days, at most, which (according to [62, p. 42]) is achieved with a flux of about $100 \bar{\text{H}}$ atoms every 110 s at 4 K temperature.

This reference numbers have to be considered as a starting point for optimizing the $\bar{\text{H}}$ source. The most critical ones, according to Equation 7.2.14, are $\bar{\text{H}}$ temperature, as the measurement time scales quadratically $\propto T^{-2}$, and number of produced atoms per shot N_{prod} , which is linear

in the integration time.

2.2 Pulsed antihydrogen production

The gravity measurement scheme outlined in the last section, based upon measuring the time of flight of $\bar{\text{H}}$ atoms between its annihilation time on the detector surface and the starting instant of its free-fall inside the deflectometer, requires knowledge of the atoms launch instant with a precision $\ll$ than the characteristic flight time within the deflectometer $\tau = L/v \approx 100 \mu\text{s}$.

This could be achieved experimentally only in two ways: decoupling the $\bar{\text{H}}$ formation time from the free-fall measurement time by trapping the atoms, if a time-indeterministic production mechanism is adopted, or producing and accelerating the atoms in-flight in a pulsed production scheme.

2.2.1 Antihydrogen production schemes

At the time of writing, antihydrogen has only been produced by experiments in the AD community adopting experimental techniques based on stochastic collisional processes: three-body and radiative recombinations during $\bar{\text{p}}\text{-e}^+$ plasmas mixing [25, 26, 76, 77]



and the double charge-exchange with Rydberg cesium [27]



where the $*$ symbol indicates highly-excited (Rydberg) atomic states.

Antihydrogen trapping was shown possible using an octupolar magnetic trap [24], which could be subsequently opened by controlled magnetic field quenching to release the trapped atoms and perform a gravity measurement (the method used for the first attempt of measuring gravity discussed before [9]).

Another possibility would be to adopt the same charge-exchange reaction between antiprotons and Rydberg-excited positronium demonstrated in [27] as a basis for a pulsed $\bar{\text{H}}$ formation scheme. Antihydrogen would be produced by sending a pulse of Rydberg positronium atoms, exploiting the larger production cross-section of the Ps-mediated $\bar{\text{H}}$ production channel (which

is about $\propto a_0^2 n^4$, where a_0 is the Bohr radius and n is the principal quantum number of Ps, as discussed in [71, p. 30] and in the next paragraph).

Another advantage of this method, shared with ATRAP's double charge-exchange, is the negligible heating applied to the $\bar{p}$ plasma. No movement of the trap potential is required, differently to the traditional mixing scheme, which instead requires significant efforts to keep plasma temperatures low during recombination (see [78, 20]).

2.2.2 Charge-exchange cross-section calculation

A detailed set of numerical calculations of the $\bar{H}$ charge-exchange production cross-section, both in field-free and for a 1 T magnetic field, were performed using the Classical Trajectory Montecarlo (CMTC) approach (see [79], also used in [71]).

The calculation method consisted in numerically integrating the classical equations of motion of an electron and a positron in external fields⁴, initially positioned on a classical elliptical orbit⁵ associated to a choice of the n, l quantum numbers with random angular orientation, far away from an antiproton position (set in the axis origin). The center of mass velocity v_{cm} was chosen so that the atom trajectory was hitting the antiproton with a specific impact parameter b (expressed, in the following formulas, in atomic units). The production cross-section was thus calculated by measuring, over many realizations of the initial conditions, the probability to end up with the e^+ bounded to the $\bar{p}$, multiplied by the impact surface.

$$\sigma = \pi (a_0 b_{\text{max}})^2 \frac{N_{\bar{H}}}{N_{\text{tot}}} \quad (2.2.3)$$

The cross-section was evaluated versus many different parameters, among which a central role was played by the relative velocity of the Ps atom with respect to the positron in the orbit k_v , defined as

$$k_v = \frac{2}{\alpha c} n_{\text{Ps}} v_{\text{cm}} \quad (2.2.4)$$

where $v_{e^+} = 1/2n_{\text{Ps}}$ is the velocity of the e^+ in the orbit in atomic units⁶.

The effect of an uniform magnetic field along the $\hat{z}$ axis was taken into account by adiabatically switching on the field in the simulation after the initial atomic state was prepared, to address the problem of semi-chaotic classical orbits of Ps in a strong external fields (see [79, p. 9] and [81] for more detailed discussions).

⁴by means of an adaptive-stepsizes 4th-order explicit Runge-Kutta method [80]

⁵the classical orbit approximation is valid only for Rydberg atoms, with principal quantum number $n \gg 1$

⁶eq. (5) in the published paper [79, p. 3] is a typo.

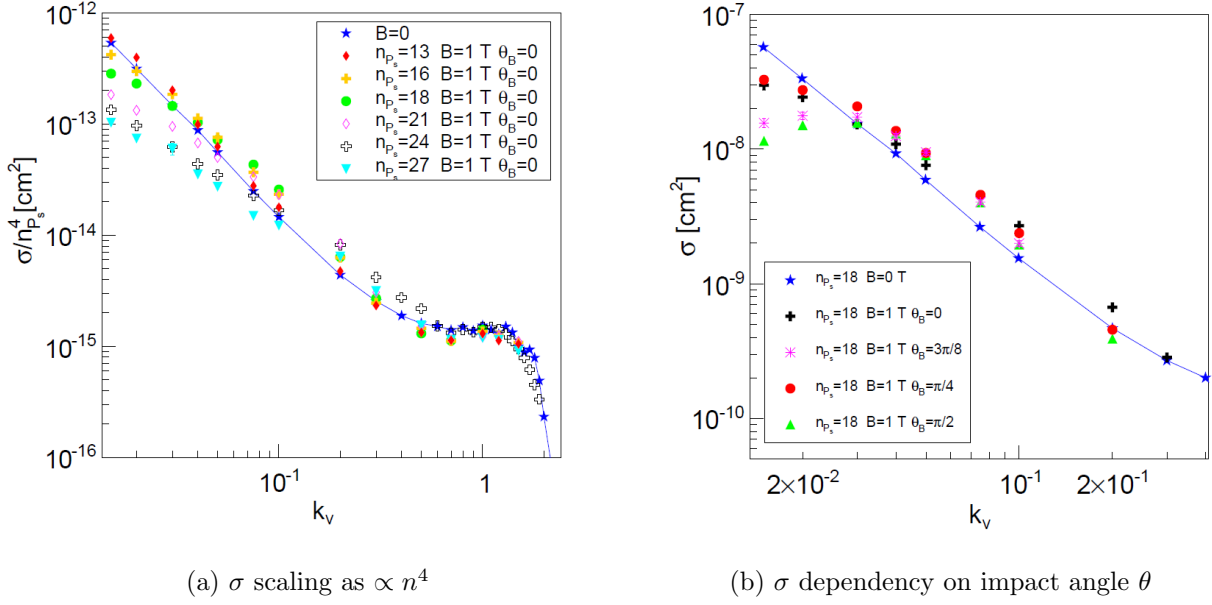


Figure 2.5: Summary of classical-trajectory Monte Carlo calculations of $\bar{\text{H}}$ production cross-section via charge-exchange with Rydberg positronium. Panel (a) shows the dependency of the σ/n^4 ratio as a function of the reduced Ps velocity (see text) for various initial principal quantum numbers. Panel (b) shows the dependency of the cross-section by the flight angle of Ps with respect to the magnetic field $\vec{B}$. From [79]

Among the many simulations performed in [79], two are of particular importance for the present discussion. Figure 2.5.a shows the $\sigma/n_{p_s}^4$ ratio as a function of the relative velocity k_v , for Ps states spanning in the interval $n \in [13, 27]$ in a 1 T magnetic field. The cross-section scales as $\propto n^4$ for almost the entire considered region of k_v , with a moderate reduction only for k_v approaching 10^{-2} . Figure 2.5.b shows that the cross-section does not significantly depend on the incidence angle θ of the Ps atom with respect to the $\vec{B}$ field (at the most a factor of 2 at very low relative velocities). This consideration opens up the possibility to send Ps atoms on the $\bar{p}$ cloud with an angle, without affecting significantly $\bar{\text{H}}$ production cross-section.

Finally, simulations showed that the $\bar{\text{H}}$ atoms are produced on average on a Rydberg level with principal quantum number $n_{\bar{\text{H}}} \sim \sqrt{2} n_{p_s}$, corresponding to the same binding energy of the e^+ in the initial Ps [79, p. 5]. A coarse control over the produced antihydrogen quantum states distribution could be envisaged by tuning the Ps Rydberg state, under the limitations set by Ps level mixing in the high external magnetic field.

2.2.3 Antihydrogen initial velocity

Antihydrogen atoms produced with this pulsed scheme would possess an initial center of mass velocity resulting from the collision kinematics described in [71, p. 31], which is mostly dominated by the $\bar{p}$ initial velocity inside the cloud.

Typical $\bar{p}$ cloud densities obtained in AD experiments of $10^5 - 10^6 \text{ cm}^{-3}$ are low enough for the $\bar{p}$ velocity to be dominated by the thermal velocity (for reference, $\sim 500 \text{ m s}^{-1}$ for a 10 K plasma). Thus, if no further acceleration is applied to the produced atoms, this pulsed charge-exchange scheme would be well approximated by an isotropical thermal source with $\beta = 1$ inside Equation 7.2.14.

Accelerating the atoms to increase the measurement solid angle would be possible via Stark acceleration of the Rydberg $\bar{\text{H}}$ atoms, as described in [71], thus effectively increasing the β of the source. However, despite the increase in the number of detected atoms, this would actually worsen the free-fall acceleration measurement, as already commented in P. Bräunig thesis [62, p. 42], as the sensitivity gain by the higher number of detected atoms scales as $\propto \beta^{-1}$, while the reduced flight time in the deflectometer reduces it by a factor $\propto \beta^2$.

2.3 Introducing AEGIS

Formed in 2006 and accepted in 2008, AEGIS is a CERN-based scientific collaboration aiming to perform a direct test of the universality of the free-fall on a beam of cold antihydrogen, following the logic presented here. It is located into the AD experimental facility, for its sole availability of cold antiprotons, where it gets the beam of antiprotons since 2012 - its first commissioning run.

The AEGIS collaboration chose to adopt a moiré deflectometer/Talbot-Lau interferometer as free-fall detector and the pulsed $\bar{\text{H}}$ formation scheme described in the last section as its primary source of antihydrogen.

The outline of the experimental scheme is shown in Figure 2.6: $\bar{\text{H}}$ is produced by sending a pulse of Rydberg positronium atoms over a plasma of very cold antiprotons (0.1 K, nominally) confined in a Malmberg-Penning trap. Rydberg positronium is obtained by two-step laser excitation of a cloud of thermalized Ps emerging from a suitable matter converter [40], where a spill of positrons were implanted at $\sim \text{keV}$ energy a few ns earlier. Thanks to the increased optical lifetimes of Ps in Rydberg levels ($\sim 26 \mu\text{s}$ for $n = 19$, see [82]), the matter converter can be displaced at $\sim \text{cm}$ distance with respect to the $\bar{p}$ without inducing significant losses.

A subsequent Stark acceleration of the produced $\bar{\text{H}}$ atoms could be optionally introduced, if

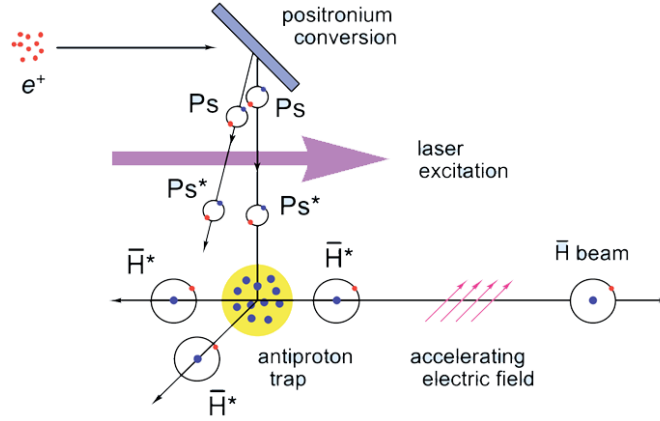


Figure 2.6: Outline of AEGIS experimental scheme.

found necessary⁷.

In symbols



A more detailed discussion on the charge-exchange reaction can be found in a following chapter 7. A full analysis of this experimental method is presented in the experiment proposal [71] and subsequent publications [72, 83].

2.3.1 Status and short-term goals

In its first phase, the objective of the experiment is to succeed forming $\bar{\text{H}}$ by pulsed charge-exchange and characterize the source intensity with the limitations of its present geometry. Subsequently, the pulsed source has to be optimized in view of performing a gravity measurement with a moiré deflectometer, which (as discussed above) imposes strict boundaries on the minimum number of produced atoms versus their temperature.

In order to first attempt a pulsed antihydrogen production, three intermediate substeps have to be achieved:

1. demonstrate the feasibility of in-flight laser excitation to Rydberg levels of a cloud of

⁷the 3^3P level of positronium was selected as intermediate level for the laser excitation process, instead of the 2^3P level already demonstrated in literature [30], as some novel scientific output from the spectroscopy of the third excited state could be obtained

positronium emerging from the matter converter chosen for the cryogenic vacuum of AEGIS;

2. re-do the same operations inside the main setup of the experiment, designed also to catch and store antiprotons from the AD using a set of Malmber-Penning traps in a high magnetic field and ultra-high cryogenic vacuum;
3. in parallel, capture, cool, compress, transfer and store as many antiprotons as possible (10^5 nominally) in the $\bar{\text{H}}$ production region, right below the Ps converter.

The aim of the following section of this thesis is to first outline the current hardware setup of the experiment, then present the experimental work conducted in these three years on the way a first pulsed antihydrogen production via charge-exchange.

Part II

AEGIS

Chapter 3

Layout of the experiment

In this chapter, a sketched description of AEgIS from hardware and software point of views (as well as its most important subsystems) is given, in order to establish a common terminology used throughout the following chapters to describe experiments with positrons, positronium and antiprotons.

3.1 Overview

AEgIS, as one of the five currently active experiments within the AD hall at CERN¹, is physically located in its experimental zone inside the ring of the AD, where it gets the antiprotons from (see Figure 3.1). Antiprotons are produced by collisions of a 26 GeV proton beam from CERN Proton Synchrotron (PS) against a solid Ir target, collected inside the storage ring and cooled both axially and transversally by a combination of stochastic cooling and electron cooling (see Figure 3.1, inset). When active, AD delivers spills of about $3 \cdot 10^7$ antiprotons every 110 s to one experiment at a time, in shifts of 8 hours per day in a period from May to November. Antiprotons are delivered to the experiments using magnetic transfer lines with an energy of 5.3 MeV in bunches of about ~ 100 ns FWHM. The technique used by the majority of the experiments in AD to capture antiprotons consist of reducing their axial energy by letting them hit a thin foil of Al, called *degrader*, installed at the entrance of the traps. About 50 % of the initial antiprotons survive annihilation and a few % of these leave the degrader with energies below 10 keV, which can be reached by ordinary high-voltage electrodes.

¹along with ALPHA, ASACUSA, ATRAP and BASE. GBAR was approved but not yet installed.

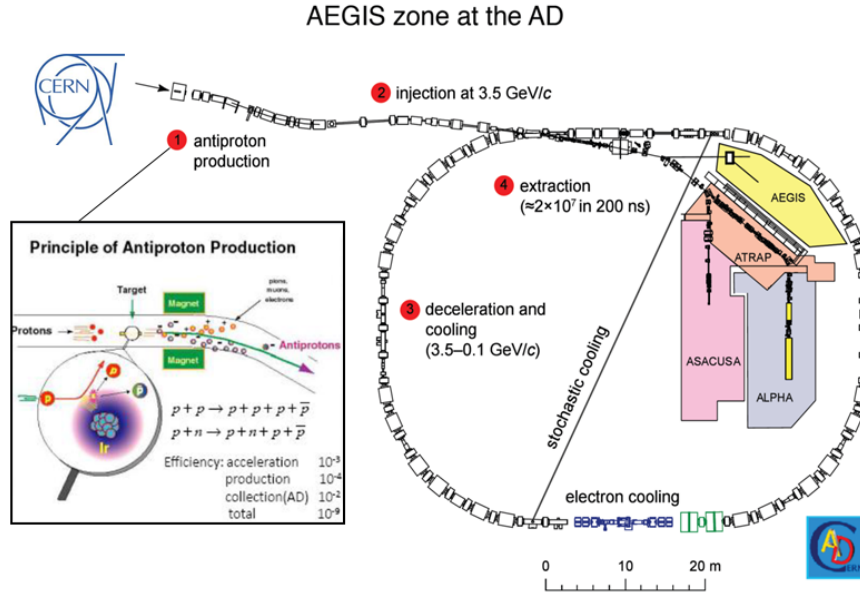


Figure 3.1: Overview of AEGIS placement within the Antiproton Decelerator (AD) ring. Inset show the antiproton production mechanism and efficiencies.

3.1.1 Hardware overview

From the hardware point of view, AEGIS is a complex system built around its principal vacuum chamber hosting the Malmberg-Penning traps for manipulating non-neutral plasmas of antiprotons, positrons and electrons in a high magnetic field, cryogenic and ultra-high vacuum environment. Most of the complexity of the experiment lies inside the main vacuum chamber, which hosts the cryostats, insulation and ultra-high vacuum regions, as well as the magnets, traps and detectors. All other systems (requiring neither proximity to the traps nor vacuum nor cryogenic temperature) are placed outside the main cryostat: the laser system on the side; the positron system above the AD injection line (connected to the main cryostat with a 45° tilted magnetic transfer line); data acquisition, traps electronics and detector electronics in external rack.

The main vacuum chamber, whose cut-open is shown in Figure 3.2, is divided into two separate regions with different magnetic fields: a 4.46 T region is used to catch antiprotons; a 1.0 T region for antihydrogen formation. Each magnetic field is generated by a superconducting Niobium Titanium alloy (Nb:Ti) solenoid cooled down by liquid Nitrogen (lN) and liquid Helium (lHe) to 4.2 K ².

Since the 4.46 T magnet was originally designed to operate at 5.0 T, the shorter nomenclature 5 Tesla (5T) and 1 Tesla (1T) is adopted for brevity from now on for addressing individual

components belonging to one or the other section of the experiment.

3.1.2 Antihydrogen production region

The antihydrogen production region, sketched in Figure 3.3, is the most down-stream part of the trap structure with respect to the AD injection line. It is composed of a custom-designed Penning trap for storing cold antiprotons, whose electrodes present an grid in the upper part to let Rydberg Ps atoms enter in the trap. Positronium is produced by implanting positrons into an off-axis mesoporous silica converter installed vertically above the antiproton trap on a 60° inclined support. Laser pulses for Rydberg excitation are sent on a orthogonal transfer line as Ps atoms propagates into vacuum towards the antiprotons. The entire region is surrounded by Fast Annihilation Cryogenic Tracker (FACT), the main antihydrogen formation detector, composed of 800 scintillating fibers in 4 cylindrical layers.

3.1.3 Software overview

From the software point of view, the experiment is managed by several control systems interacting together via network (see Table 3.1) ³. When running experiments, AEgIS is managed by the trap control system, which acts as master controller and dispatches commands to the other systems; all time-critical operations are synchronized on the same master clock distributed by the realtime computer in the traps electronics.

During data taking periods, AEgIS performs series of *measurement runs* with particles (electrons, positrons, antiprotons or combinations of the three). A *run* is a well defined set of operations (usually including particles loading, manipulation and measurement) that is completely specified by a sequence description file (in the form of an Excel spreadsheet) processed by the trap control system.

3.1.4 AEgIS coordinate system

It is convenient to introduce now the AEgIS coordinate system, defined as a right-hand cartesian system with the $\hat{z}$ axis directed and oriented as the antiproton beam (*downstream*), the $\hat{y}$ axis

²the intricacy of the apparatus visible in Figure 3.2 derives in big part from the three requirements cited above ($B \sim T$, vacuum $< 10^{-13}$ mbar, $T < 10$ K), which were source of incredible complications in terms of geometry design, materials selection, thermal insulation and electrical/magnetical shielding, and very often required dedicated technological developments as off-shelf components available on scientific market did not satisfy all three requirements.

³most of the control systems are written using NI LabView[®] and communicate using network-published LabView shared variables, TCP/IP commands and Windows network file sharing.

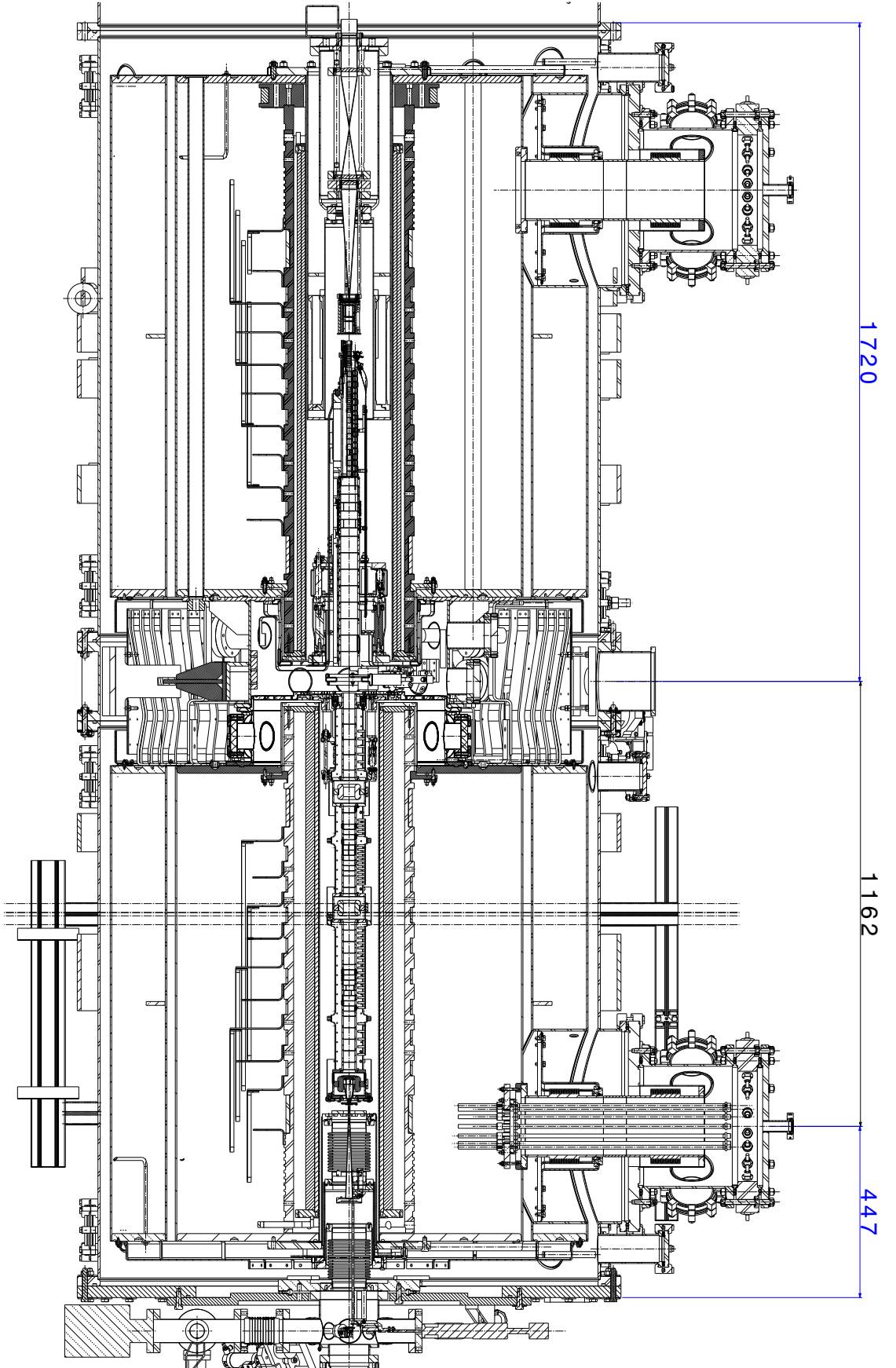


Figure 3.2: Cut-open view of AEGIS main cryostat. The two regions of the vacuum chamber, associated to the two different magnetic field regions, are distinguishable. All units are mm.

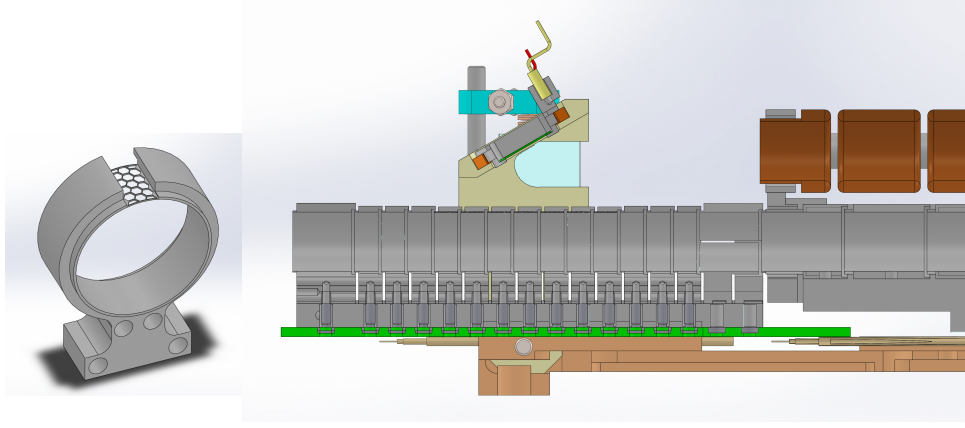


Figure 3.3: Sketch of AEGIS antihydrogen production region. The off-axis positronium converter is placed above the antiproton storage trap, whose electrodes are manufactured with tiny apertures to let Rydberg Ps atoms fly in. The Rydberg-exciting laser impinges on the Ps cloud orthogonally w.r.t. the view; the last reflecting prism has been removed in the drawing for clarity. The laser beams alignment is monitored by observing the spots on MACOR screen in the back.

Subsystem	Software architecture
Trap system	LabView/Windows + LabView/RTOS
Magnet system	LabView/Windows + PLC
Data acquisition	Custom/Linux
Vacuum and cryogenics	LabView/Windows
Positron system	LabView/Windows
Laser system	Python/Windows + FPGA

Table 3.1: List of AEGIS software control systems

pointing vertically upwards and the $\hat{x}$ consequently determined so that $\hat{z} = \hat{x} \times \hat{y}$. The origin of the system is defined as the geometrical center of the central region flange, which connects 5T and 1T sections of the vacuum chamber (see Figure 3.2 for a visual reference).

This reference system is adopted in the next sections for discussing placement of individual components inside the experiment.

3.2 Magnet system

AEGIS twofold magnet system has been designed to fulfil two apparently contradicting requirements: a magnetic field as high as possible in the capture region to maximize $\bar{p}$ radial confinement

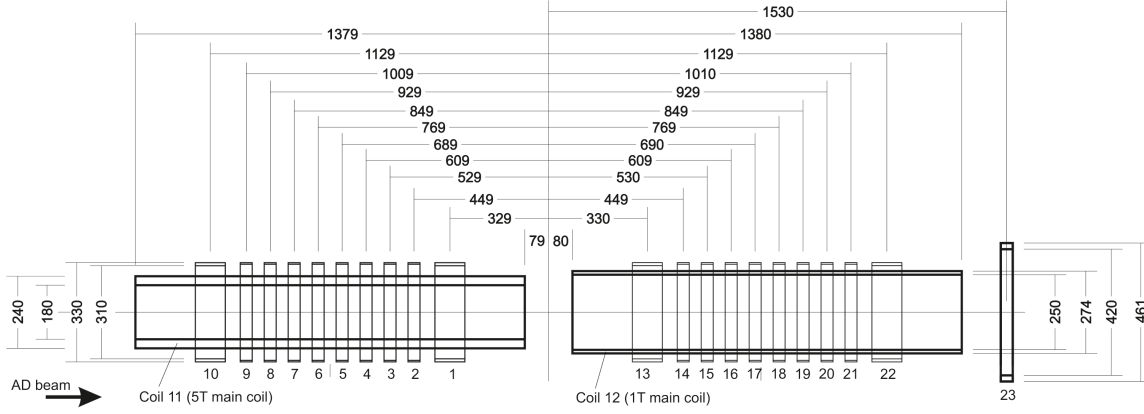


Figure 3.4: AEGIS magnets coil configuration. All measurement are mm in the AEGIS system of coordinates.

and electron cooling efficiency (whose driving process, cyclotron radiation, scales as $\propto B^2$) and as small as possible in the antihydrogen production trap to limit atomic level mixing of Rydberg positronium and antihydrogen in high external fields [71].

The solution was obtained spatially separating capture region from antihydrogen production region: AEGIS main magnets were designed to have two distinct 1T and 5T field regions, each with a homogeneity target level of one part in 10^{-4} . The final system is constituted of two main superconducting Nb:Ti solenoids and a two groups of axial correction coils for proper field termination (see Figure 3.4).

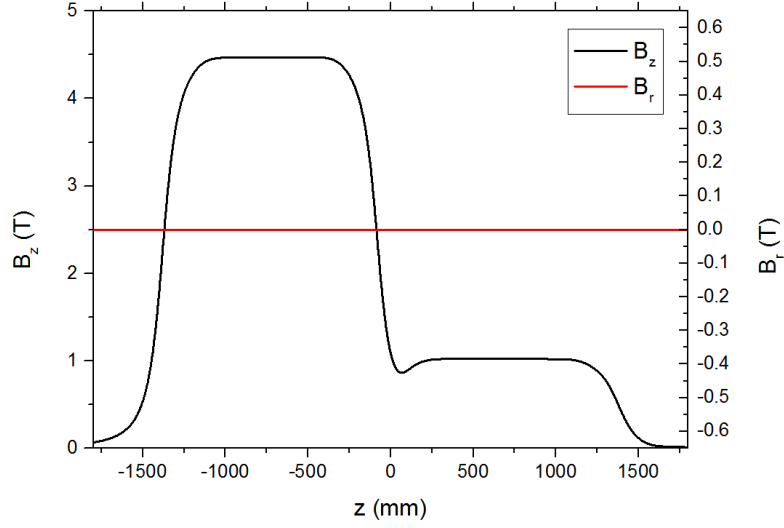
Another important requirement in AEGIS magnet design was mutual alignment and the possibility to align each magnet with respect to the internal traps, which is crucial to obtain long plasma lifetimes. Both magnets have been aligned during production with respect to the central region mounting flange with an angular accuracy of $300 \mu\text{rad}$.

The result of an OPERA simulation of the average magnetic field in the trap region is shown in Figure 3.5. The 5T magnet is routinely operated at a conservative field value, 4.46 T, to reduce the risk of magnetic quenching.

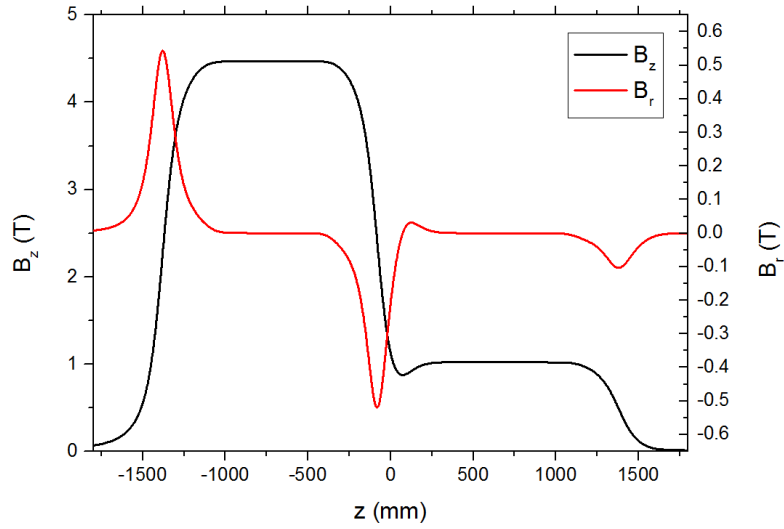
3.3 Vacuum and cryogenics

Storage of $\bar{p}$ requires state-of-the-art ultra-high vacuum in particle traps, as the lifetime of antiprotons is ultimately limited by annihilations on residual gases ². Vacua of $< 10^{-13}$ mbar

²mostly hydrogen, with the highest annihilation cross-section.



(a) Magnetic field on-axis



(b) Magnetic field off-axis

Figure 3.5: AEgIS simulated magnetic field as a function of the $\hat{z}$ position on the trap axis (a) and 50 mm off-axis (b). Both field components parallel to the axis (in black) and transverse to the axis (in red) are shown.

are common in order to achieve macroscopic lifetimes ³.

AEgIS vacuum system has been designed to use the cryogens needed to bring the magnets to their superconducting state (9 K or less for Nb:Ti) to cool also the entire ultra-high vacuum region. Both 5T and 1T cryostats use at the same liquid nitrogen and liquid helium to keep the magnets cold. The external surface of the cryostat is at nitrogen temperature, 77 K, whereas the internal bore tube is at 4 K. This freely makes all internal surfaces cryo-pump the residual gases in the traps, with the sole exceptions of helium and hydrogen.

An Outer Vacuum Chamber (OVC) was realized around the magnet cryostats to minimize heat conduction. Superinsulation material was placed around the entire external surface of the cryostat to minimize heat transfer through radiation. All electrical, optical and thermal connections are conveyed through two series of vacuum feed-throughs (external-to-OVC and OVC-to-Ultra-High Vacuum (UHV)).

AEgIS reaches 10^{-13} mbar or better in the trap region thanks to the combined pumping action of cryo-surfaces, four big getter pumps for hydrogen (three are inside the central region, one outside) and a ionic pump installed outside the main vacuum chamber for helium. The main pumping station of the UHV vessel is installed in the entrance upstream region of the experiment, which takes the name of *sun* after its shape. The full closing, leak-testing, pumping, baking and cool-down procedure is complex and requires about one month ⁴.

3.4 Traps system

The AEgIS trap system hardware consists of two parts: the traps electrode structure, installed inside the high vacuum region, and the control electronics on the outside of the vacuum chamber.

3.4.1 Trap regions

The 102 electrodes composing AEgIS Malmberg-Penning traps are divided into different trap regions located either in the 5T magnetic region (for antiproton and positron catching) or in the 1T magnetic field region (for antihydrogen production). Electrode configuration and logical trap regions are sketched in Figure 3.7 and listed in Table 3.2.

³the record at the time of writing is held by the BASE collaboration with 1.5 y [84]

⁴this long closing time (resulting eventually from the monolithic design of the main vacuum chamber) is a serious unforeseen drawback of the experiment design. AEgIS, as well as all other experiments in AD, experiences recurrent vacuum incidents due to the difficulty of keeping such high levels of vacuum. Due to its very long vacuum recovery time, however, AEgIS is often forced to loose beam time to recover vacuum. Even worse, if an hardware fault happens during run time, the decision whether to fix it or not becomes extremely difficult and painstaking.

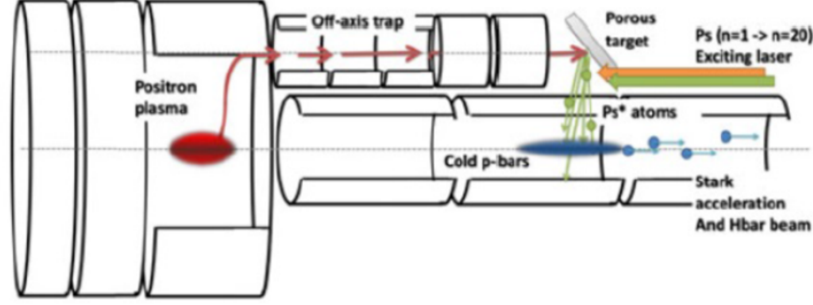


Figure 3.6: Sketch of the original envisaged use of AEGIS 1T traps.

5T traps are divided into two smaller trap regions, named Catch (C) and Positron (P) ⁵, with shorter electrodes to shape harmonic potentials and sectorized rotating-wall electrodes in their center. C and P traps are terminated by high-voltage electrodes reaching up to 9 kV (HV1 to HV3, in upstream to downstream order), used to catch antiprotons and positrons.

A series of Transfer (T) electrodes connect 5T traps to 1T traps and convey the particles as they traverse the non-homogeneous region of the magnetic field. A fourfold sectorized 50 mm-long electrode is installed in the center of the transfer section (at $z = 0$) on a movable actuator for particle steering (B0), which allows also a Faraday cup (CRFC) to be put in its place.

1T traps are more sophisticated than 5T traps. The first in the stack, named Big (B), was originally envisaged to perform resonant diocotron excitation of the positron plasma to load ballistically an Off-axis (OffAx) high-voltage trap, which could be raised to a few kV for implanting positrons in the positronium converter (see the original sketched scheme in Figure 3.6 and [85]). This trap was removed during 2016 experiment upgrade, meant to simplify positron manipulations procedures ⁶ by adopting a direct acceleration-and-injection scheme with magnetic off-axis steering (see 5.3.1).

The following trap in the upstream to downstream direction is conversely named On-axis (OnAx). It is made of smaller diameter electrodes with respect to the Big trap. A capping electrode with two holes (one on the axis, one in front of the off-axis trap) provides proper field termination at the junction. The OnAx trap region, which apparently has no specific use but transferring $\bar{p}$ to the $\bar{H}$ production region, has been devoted to dedicated experiments

⁵besides their original names, P and C regions are general purpose traps. In fact, they are normally used conversely from what the name suggests: C trap for positrons, P trap for antiprotons.

⁶which were found too complex for reliable operation of the experiment, during the campaign of positron capture and manipulation studies described in chapter 5).

on antiprotons: a cryogenic amplifier tuned on $\bar{p}$ cyclotron motion is installed on a twofold sectorized electrode. OnAx is followed by the final Production or Ultracold (UC) trap of the antihydrogen production region, already shown in 3.1 (see Figure 3.3), whose electrodes have a grid opening in the upper part to let Ps atoms fly in. A cryogenic amplifier, tuned in the range of electron plasma axial bounce frequency, is installed as well in the UC trap to resistively cool e^- which subsequently lower the $\bar{p}$ temperature. This trap is the only one in the entire stack without a sectorized rotating-wall electrode in the center, but as upstream endcap (UC1).

Further downstream, few cm away from the last UC electrode, three field-ionization electrodes and a fixed Micro-channel plate (MCP) detector are positioned, terminating the field lines ⁷. This detection structure is meant to prove unambiguously $\bar{H}$ formation via field-ionization. Its MCP, labelled 1T micro-channel plate (1TMCP), is now being used mostly for plasma diagnostics and positron steering.

3.4.2 Control electronics

Electrode voltage polarization is managed by a custom-made trap controller connected to the electrodes via filtered vacuum connections. The control electronics is made of several sub-components dedicated to the different trap manipulations that needs to be performed. Bias voltages are applied using accurate Digital-to-Analog Converter (DAC) boards, allowing independent voltages to be set on all electrodes in a range between -200 V to 200 V with 16-bit resolution (corresponding to 6 mV) with a delay of $150\text{ }\mu\text{s}$ and a time-resolution of $1\text{ }\mu\text{s}$. An accurate and versatile pulser system allows train of pulses of variable length (between 1 ns to $64\text{ }\mu\text{s}$) and of the same voltage amplitude and resolution of the DACs to be summed to the bias voltage of some selected electrodes. A programmable, multi-channel sinusoidal waveform synthesizer adds on the bias voltage of sectorized electrodes pure phase-shifted sinusoids up to 5 V in amplitude in a frequency range from 0 Hz to 30 MHz , allowing rotating-wall radial manipulations to be performed. It also features frequency, amplitude and mixed sweeps. Finally, the trap controller electronics also produces the high resolution programmable triggers to synchronize all other equipments (laser, positron system, detectors) to trap operations.

The trap electronics is externally programmed by a NI PXI Embedded Real-Time controller (RT) computer via asynchronous communication. All operations to be performed by the traps (electrode voltages, triggers to raise, pulses to send) are stored in a buffer and periodically downloaded by the master Field-Programmable Gate Array (FPGA) in the trap controller, which takes care of executing the specified operation at the absolute time of the master clock

⁷in place of the Stark accelerator and the gratings system for gravity measurements, which will come in the future.

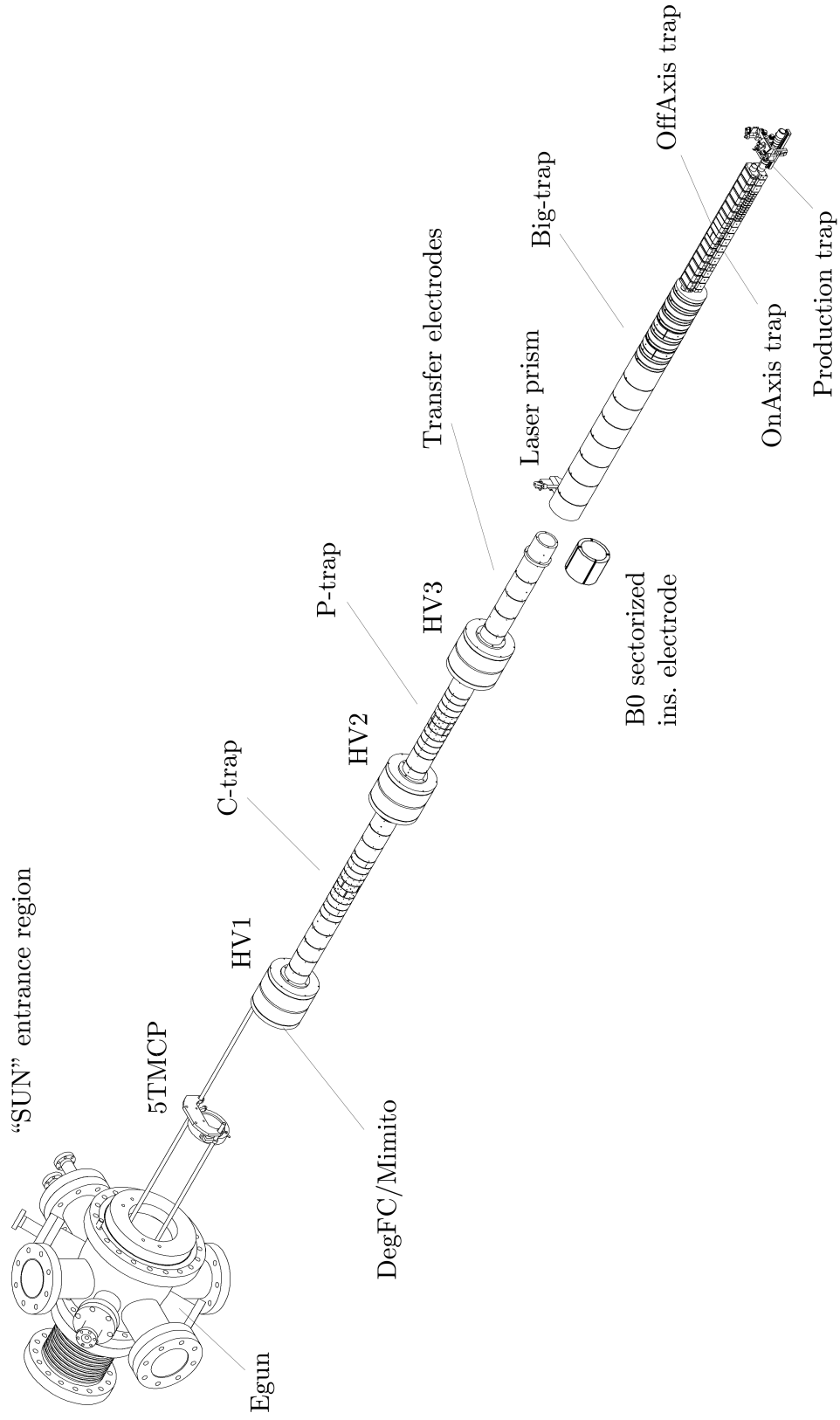
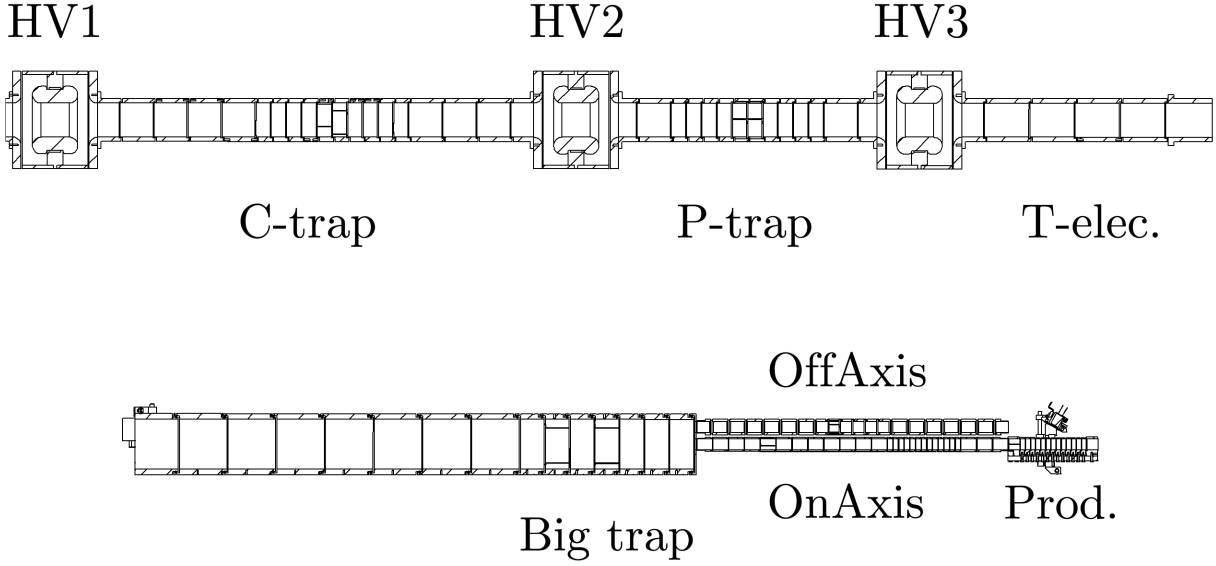


Figure 3.7: Overview of the AEGIS Malmber-Penning trap system.



(a) Cut-open view of the AEGIS Malmber-Penning trap system.

Trap region	Electrodes	D [mm]	B [T]
C	C1 to C19	30	4.5
P	P1 to P13	30	4.5
T	T1 to T6	30	4.5 – 0.8
Big	B1 to B15	44	0.8 – 1.0
OnAxis	OnAx1 to OnAx28	10	1.0
*OffAxis	OffAx1 to OffAx28	10	1.0
Prod.	UC1 to UC15	10	1.0

(b) Details of the main trap regions

Table 3.2: Summary of the main AEGIS Malmberg-Penning trap regions. The sectorized B0 electrode, installed on a movable actuator between the end of the transfer region and the beginning of the Big trap, was omitted.

specified by the RT in a specific programming word. In this manner, all trap operations are time-deterministic and procedures are reproducible. The 1ppm GPS-disciplined master clock generator is installed withing the same crate of the RT, as well as several general purpose I/O boards, counters and acquisition cards for providing precise triggers and acquiring trap diagnostic detectors.

3.5 Detectors

AEgIS features many different particle detectors expleting different functions, some of which were already mentioned in the former sections of this chapter.

Moving in a downstream to upstream order, the first detector found is the field ionization detector MCP (1TMCP). The 1TMCP consists of a 1' MCP assembly in chevron configuration with a P67 fast phosphor screen on the back. The light information from the phosphor screen is conveyed by a magnifying telescopic system to an Hamamatsu Flash4 v2 CMOS camera with very high quantum efficiency ($\sim 87\%$). This detector proved to be a crucial piece of diagnostics for all charged particles in the experiment, as it images plasmas of either electrons, positrons and antiprotons with a very good definition and mechanical stability. Moreover, it allows in principle for unambiguous proof of antihydrogen formation through its three field-ionization electrodes and single particle sensitivity.

A few cm upstream the 1TMCP the main antihydrogen formation detector FACT, is found. It is the only detector with some vertex reconstruction capabilities, composed of 800 $\hat{z}$ -stacked rings of scintillating fiber disposed in four concentric layers around the UC trap with an active region of about 300 mm in length. Each scintillating fiber is coupled to a plain fiber that carries the scintillation light to the external flange of the experiment, where it is read by an Multi-Pixel Photon Counter (MPPC) detector at room temperature, discriminated and send to a readout FPGA (for more details on FACT, see [67]).

Next in the list, 12 external scintillator slabs are installed around the external surface of the vacuum chamber to monitor $\bar{p}$ and e^+ annihilations. Each scintillating slab is half-cylinder shaped, about 100 mm in width, 20 mm in depth and made of plastic polystyrene doped with POPOP wavelength shifter. It covers a solid angle of about 3 % with respect to an annihilation happening in its center (excepting the two most upstream slabs which are twice the length). Each slab is optically coupled to two heavily shielded photo-multipliers tubes counted in coincidence within a window of 50 ns to exclude the background. Among the 24 PMTs coupled to the external scintillators, two in particular have a privilegiate role: PMT20 and PMT2. Both PMTs are acquired on a parallel acquisition chain using a fast 1 Gsample/s oscilloscope in the trap control electronics, to measure the positron annihilation signal during transfer to the main

traps.

In the central region of the experiment, a two-sided, sectorized Faraday cup detector is installed on the movable actuator shared with the B0 electrode, allowing an accurate estimation of the total charge contained in a plasma.

Going further upstream, the degrader/beam monitor structure is found at the entrance of the 5T traps, installed on movable actuator. It is composed by a wafer of two thin aluminum foils (respectively 50 μm and 100 μm , used both as degraders and as non-sectorized Faraday cups) and a thinned ($\sim 53 \mu\text{m}$) version of the MIMOTERA silicon detector (see [86]) used as beam monitor.

Finally, the last detector is the 5T micro-channel plate (5TMCP), a 2" MCP in chevron configuration with a P67 phosphor screen and metallized surface, able to image the entire full radius of the traps. This MCP is installed on a tiltable flip mount, controlled from pressure actuators. Light is conveyed on a Hamamtsu Orca R2 camera by an insertable mirror just out of the main vacuum chamber.

AEgIS features many other secondary detectors with accessory diagnostic roles, all listed in Table 3.3.

3.6 Data acquisition

The role of AEgIS Data Acquisition (DAQ) system is to manage run opening/closing and data gathering centrally, so that all data from a run can be collected by the various detectors in the same place. As many other experiments at CERN, Root files are used as data containers. The chain of events during a run is roughly the following:

1. the trap controller sends a *run start* request;
2. DAQ opens a new incremental run number, starts listening for connections on a TCP/IP port linked to the run number and communicates to the trap controller the new run number;
3. as it receives the new run number, the trap controllers starts to execute trap operations in the current sequence;
4. in the meantime, the DAQ starts receiving TCP connection on the open port by the many other systems in AEgIS asking to save data;
5. connection requests are handled in parallel (by different machines) and the output Root file is regenerated every 10s for online analysis purposes;

Name	z [mm]	Details	Primary use
Custom detectors			
FACT	—	800 SciFi + MPPCs	$\bar{H}$ formation
Mimito	-1100	thin MIMOTERA	Imaging beam mon.
Micro-channel plate assemblies			
5TMCP	-1350	insertable, 2-inch	Full-radius plasma img.
1TMCP	+977	fixed, 1-inch	Precision plasma img.
Coincidence scintillators			
SCI12 to SCI2324	-960 to 960	12 external scint. slabs	$\bar{p}$ annih. counting
Digitized scintillators			
PMT2	-960	integral charge	e^+ annih. signal
PMT20	+795	integral charge	e^+ annih. signal
FACTDigi	+795	24 analog FACT fibers	e^+ annih. signal
CsI	—	2 CsI crystals	e^+ annih. signal
Movable CsI	—	2 movable CsI crystals	e^+ transfers
Faraday cups			
DegFC	-1100	two-sided, 1+1 sect.	Integrated charge meas.
CRFC	0	two-sided, 5+13 sect.	Sectorized charge meas.
5TMCPIn	-1345	one-sided, 1 sector	Charge meas. on MCP
1TMCPIn	+977	one-sided, 1 sector	Charge meas. on MCP
Non-destructive plasma detectors			
Radial cryoamp	—	OnAx10	$\bar{p}$ cyclotron motion
Axial cryoamp	—	UC8	e^- axial motion
Plasma modes board	—	any electrode	Plasma axial modes in traps
Laser detectors			
Laser/pos synchro	—	2 SciFi + APD	laser/ e^+ synchronization
MACOR screen	—	10 x 10 mm	laser alignment

Table 3.3: Full list of AEGIS particle detectors

6. trap controller issues a *run stop* request;
7. DAQ sets ready for opening a new run, but keeps listening for new data belonging to the old run for some seconds, then stops listening for connections of the old TCP/IP port.

The main DAQ server structures the received data using one different tree per detector, with the detector name as key. Data is copied automatically on a CERN tape library for long term archiving.

Raw data acquisition is demanded to individual detectors ⁸, with the sole exception of coincidence scintillators, digitized PMTs and FACT fibers, which are acquired directly within the main DAQ environment.

The main DAQ server also manages continuous acquisition of environmental monitors and subsystem settings. A dedicated service with its own TCP/IP port and output Root file is used for this purpose.

3.7 Positron system

The AEgIS positron system (sketched in 3.8) consists of a trap-based apparatus whose goal is to make use of positrons produced by a strong β^+ source and accumulate them in quantities in a magnetic storage trap. This is achieved by reducing their initial β -decay energy in two subsequent stages: a solid Neon moderator (deposited in front of the source) and a cooling trap ⁹ filled with a certain pressure of cooling gas, which provides a means of friction. The cooling trap also collects positrons in small spills named *pulses*, which are periodically transferred to the magnetic accumulator trap, which keeps accumulating them until an extraction triggers is received.

The AEgIS positron accumulator delivers on average $2 \cdot 10^7$ e^+ in ~ 6.5 min of accumulation time with a 11 mCi source.

A more detailed description of the system is found in 4.1 within the context of the experiments conducted in transferring positrons to the main traps.

⁸as acquisition chains are very different per type of detector

⁹commonly known as Surko trap after the pioneering work of prof. Cliff Surko and his group at UCSD (see [34, 35])

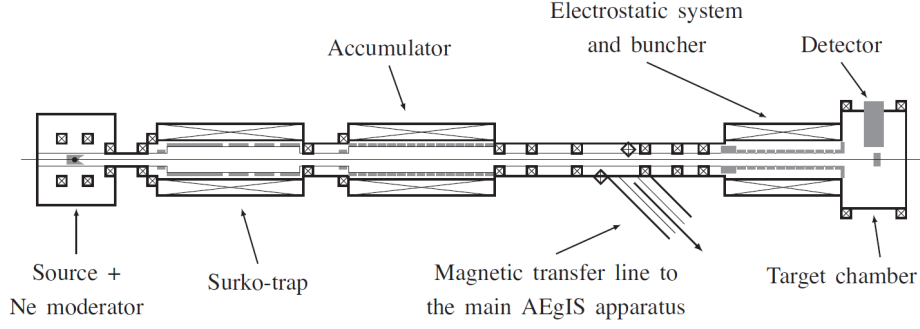


Figure 3.8: Sketch the AEGIS positron system: ^{22}Na moderated source, buffer-gas cooling trap, magnetic accumulator and transfer lines to either the main antihydrogen experiment or the dedicated Ps spectroscopy chamber, itself composed by an electrostatic transport/bunching system and the experimental chamber. A more detailed description can be found in 4.1

3.8 Laser system

Production of antihydrogen through charge-exchange benefits significantly from having Positronium atoms in highly excited (Rydberg) states, both for increasing the reaction cross-section and for extending Ps atoms lifetime in vacuum (as discussed in [71, 45]).

The goal of the AEGIS laser system is to convey as many Ps atoms leaving the converter as possible to Rydberg levels, without depositing a significant amount of energy in the cryogenic region of the experiment. The AEGIS laser system is, in fact, composed of two pulsed lasers systems both for exciting Positronium to Rydberg levels. The first laser system (sketched in Figure 3.9) is meant for efficient Rydberg Ps excitation for antihydrogen formation. It produces broadband pulses to cover the Doppler-profile of 100 K Ps and uses 3^3P as intermediate level for excitation to Rydberg levels as it is expected to deposit about 30% less energy than 2^3P in the vacuum chamber ($\lambda_3 = 205\text{ nm}$ and $\lambda_{\text{Ry}} = 1710\text{ nm}$). The second laser system, conversely, is narrow-band, uses 2^3P as intermediate level and is tunable over a wide range of wavelengths ($\lambda_2 = 243\text{ nm}$ and $\lambda_{\text{Ry}} = 750\text{ nm}$). It is meant for trying a first attempt of laser-cooling on Positronium.

The two laser pulses are let into the vacuum chamber by two viewports and transported to the antihydrogen production region by a prism transfer line suitable for both wavelengths. Laser alignments are monitored by observing their image on a small MACOR screen on the back of the target. Their temporal synchronization is verified at the target position using two auxiliary FACT scintillating fibers able to both scintillate on Ps annihilation and photo-convert UV radiation.

A more detailed description on the laser system is found in 4.4, contextually to the experiments

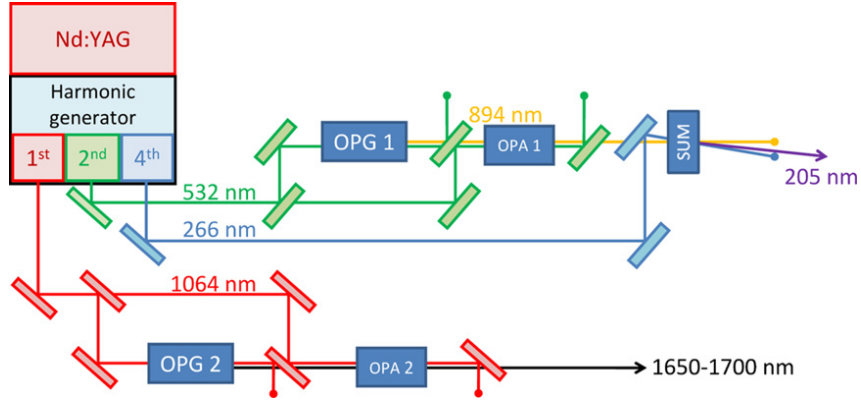


Figure 3.9: Sketch of AEgIS laser system for $1 \rightarrow 3 \rightarrow \text{Ry}$ excitation of Ps. From [87].

of Rydberg excitation for $\bar{\text{H}}$ production conducted in the dedicated chamber for positronium studies.

Chapter 4

Positronium excitation to Rydberg levels

In this chapter the series of experiments conducted in a dedicate chamber for showing broadband laser excitation of positronium (Ps) to Rydberg levels (suitable for charge-exchange $\bar{\text{H}}$ synthesis) are presented.

First, spectroscopy of the third excited state 3^3P of Ps was conducted, assessing the effectiveness of the broadband laser excitation scheme chosen for AE $\bar{\text{g}}$ IS. A measurement of the Ps cloud transverse velocity/temperature was also obtained, which could be used for future studies on the production/emission into vacuum properties of positronium converters.

Subsequently, 3^3P Ps atoms were furtherly excited to Rydberg states ($n = 15 - 17$) by means of a second broadband laser pulse.

A detailed presentation of Ps physics in external fields and the theoretical/numerical approach to the calculations needed to compute theoretical spectra of positronium is presented in chapter 8, rather than within the present discussion, which is focused on the experimental details.

The results presented here, which constitutes a significant part of the experimental work performed during this thesis, were the subject of two publications from the AE $\bar{\text{g}}$ IS collaboration on *Nuclear Instruments and Methods in Physics Research, section B* [88] and *Physical Review A* [32].

4.1 AE $\bar{\text{g}}$ IS positron system

AE $\bar{\text{g}}$ IS positron system is outlined in Figure 4.1. It consists mainly of three elements: a neon-moderated source, a buffer-gas cooling trap and (optionally) a magnetic accumulator for positron

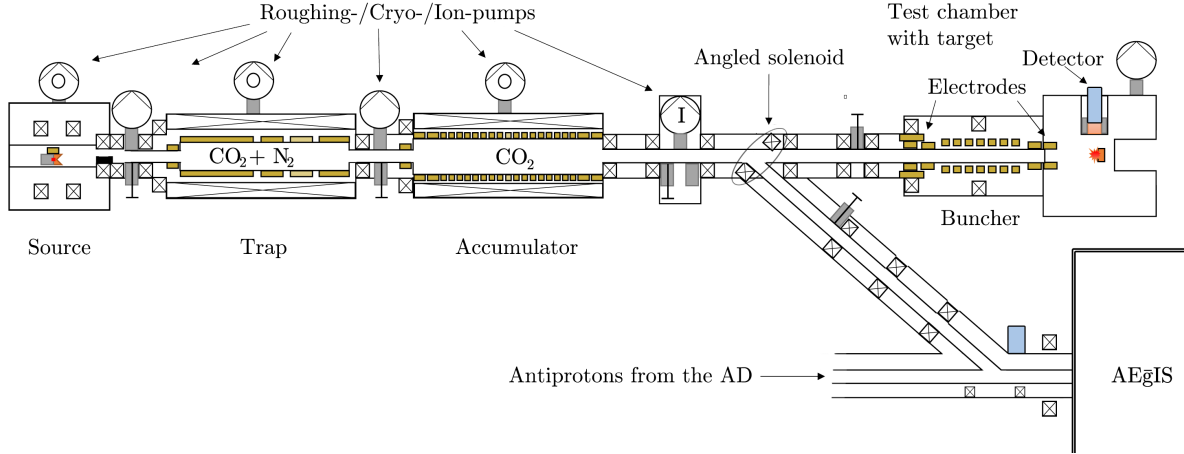


Figure 4.1: AEgIS positron system. Positrons leave the ^{22}Na source region, after solid-Ne moderation, to be cooled by collisions in a Surko-type, buffer-gas trap and be subsequently stored in a magnetic accumulation. Sectorized electrodes are present both in the trap and accumulator regions to provide rotating-wall radial compression during the cooling and accumulation phases. Two magnetic transfer lines then carry the particles to the main AEgIS apparatus or to the positronium test chamber.

storage.

4.1.1 Positron source and trap

The ^{22}Na positron source [89] consists in a 1.0 cm diameter, 11 mCi tablet¹ installed on a metallic support (the *coldhead*) cooled down to 7.0 K via an two-staged He cryocooler. In front of the source, a thin layer of frozen Ne is deposited on a parabolic cone. The solid Ne layer acts as a moderator for the outcoming fast positrons, which are slowed from an energy of ~ 0.2 MeV down to an energy of a few eV [89].

The slow positrons are then transferred by a magnetic field into a three-staged Surko-trap, where they get cooled by collisions with N_2 buffer gas. Optimal buffer gas pressure determination is crucial, as the optimal number of positrons is achieved with a tradeoff of annihilations on the buffer gas and efficient cooling. The combined efficiency of moderation and transport in optimal conditions for the AEgIS system has been measured to be $\eta_{mod} = 2.5 \cdot 10^{-3}$ [90].

Positrons are extracted from the cooling trap every 0.15 s with an energy of about 17 eV. The number of positrons in each bunch has been measured to be $2.7 \pm 0.3 \cdot 10^4$. The trapping-extracting efficiency is measured to be $\eta_{trap} = 1.4 \cdot 10^{-1}$, as the ratio between positrons injected

¹At the time of writing, the ^{22}Na source was replaced by a stronger, 50 mCi tablet, to increase the number of accumulated positrons to 10^8 per shot, i.e. the nominal value required by AEgIS in the proposal of the experiment.

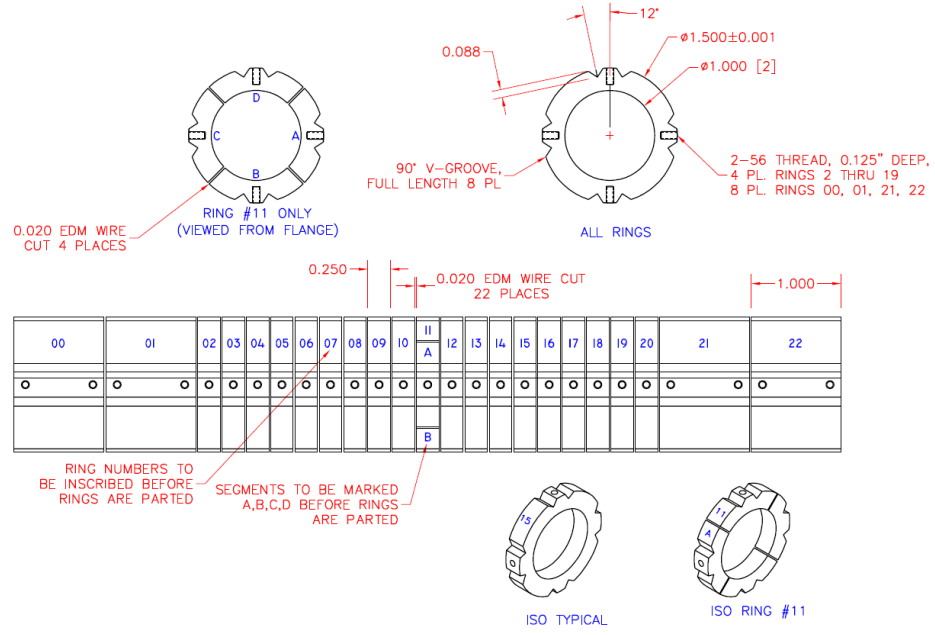


Figure 4.2: Technical drawing of positron accumulator trap, showing electrode geometry and sectorized (rotating-wall) electrode placement.

and released from the trap [90].

4.1.2 Positron accumulation

Extracted bunches are magnetically transported to a 0.1 T magnetic field accumulation trap, where many pulses are stored for several hundred of seconds, thanks to a low base pressure (low 10^{-10} mbar) filled with a small amount buffer-cooling gas ($5 \cdot 10^{-8}$ mbar).

The accumulator trap is a multiring cylindrical trap with 23 electrodes of 1.27 cm in radius (see Figure 4.2). The central 19 electrodes have 0.64 cm length, including the gap, for shaping an harmonic region with accuracy; 2 longer electrodes of 2.53 cm terminate the trap on both ends.

Longitudinal confinement is ensured by a harmonic potential about 14 V deep. Virtually infinite radial confinement of the plasma is ensured by the combined action of the 0.1 T magnetic field, generated by a high homogeneity air-cooled electro-magnet, and a rotating electric field applied through the central electrode of the trap, which is segmented into four azimuthal sections, using the rotating-wall technique [91, 92]. A cooling mechanism other than cyclotron cooling, which is extremely inefficient in the low 0.1 T field, is provided by letting some cooling buffer gas (N_2) diffuse from the Surko trap into the accumulator.

Number of positrons accumulated in the trap were measured by opening the trap on the

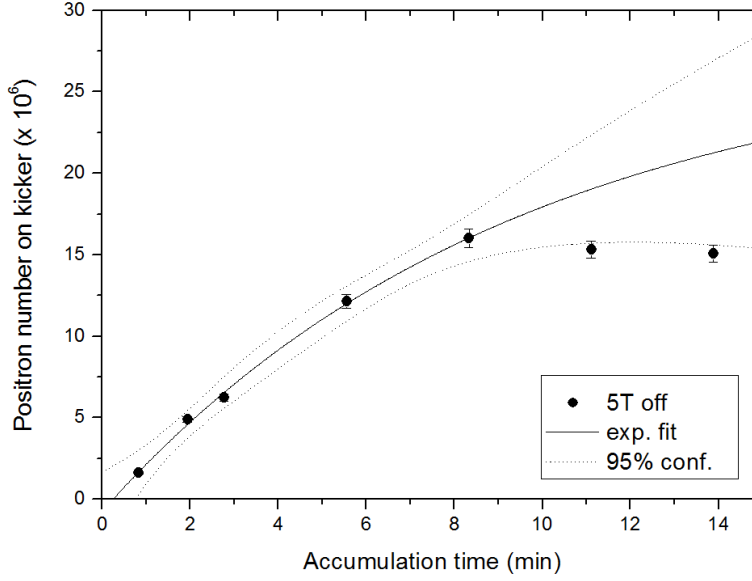


Figure 4.3: Absolute number of positrons (according to the calibration discussed in 5.1.3) extracted from the magnetic accumulator versus the number of pulses received from the Surko-trap. The first five points are fitted with an exponential growth, showing the confidence bands at 95 %. The last two points do not belong to the same behaviour (for unknown reasons). At the time of the experiment, the installed ^{22}Na source intensity was 11 mCi.

transfer line side and letting positrons annihilate on an obstacle. Annihilation gamma rays due to the entire bunch of positrons annihilating a few ns were detected using a calibrated (CsI) scintillating crystal coupled to a photodiode, in order to reconstruct the intensity of the gamma source (see 4.1.4).

In Figure 4.3 the number of positrons stored in the accumulator as a function of number of pulses from the trap and time are shown. Up to $8.0 \cdot 10^7$ positrons have been stored in around 450 s (roughly corresponding to 3000 pulses from the trap) with both the main magnets of AEgIS turned off and magnetic transfer lines to nearby experiments (mostly ATRAP's and ALPHA's) also turned off. Due to their proximity to the positron accumulator, this stray magnetic fields affects the storage performances reducing the maximum number of positrons by about a factor of 2.

4.1.3 Fast ejection from the accumulator

The stored positron cloud stored had to be ejected in a very short time (a few ns) from the trap for allowing both ballistic recapture of the bunch in the main AEgIS traps and Ps formation

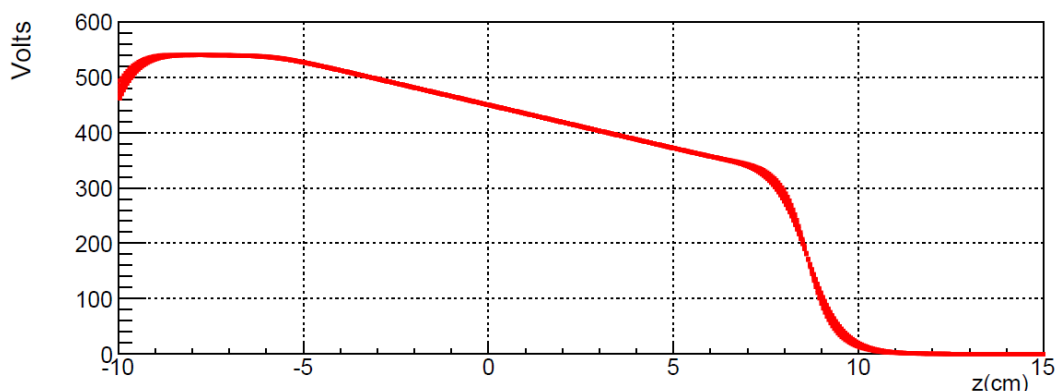


Figure 4.4: Drawing of the accumulator trap potential at the instant of ejecting positrons from the trap. From [93].

studies with the Single-Shot Positron Annihilation Lifetime Spectroscopy (SSPALS) technique. Extraction from the trap is performed by a dedicated electronics which uses a series of 450 V-rated fast MOSFETs to add to the harmonic trap voltage a linear ramp in a few ns time. This potential linear ramp causes the particle to feel almost instantly an uniform electric field in the axial direction, thus uniformly accelerating to an axial energy set by the voltage value at the particles' position. A drawing of the trap potentials at e^+ ejection is given in Figure 4.4.

The ejection electronics box allowed quick modification of the linear ramp slope by acting on a single variable resistor, choosing the maximum voltage of the ramp. Two settings are normally used: 120 V ejection energy for experiments in the dedicated Ps chamber and 300 V ejection energy (as in in Figure 4.4) for transferring positrons towards AEgIS.

4.1.4 Positron number at the exit of the accumulator

The absolute number of positrons per spill was measured at the exit of the accumulator in two ways.

1. Detecting the annihilation peak on a factory-calibrated CsI detector.
2. Fitting a Poisson distribution on the counts detected by a collimated scintillator placed far from the source.

The first measurement method was obtained placing a calibrated CsI(Tl) detector above the flange facing an aluminum dumper placed in front of the accumulator. As CsI(Tl) is not fast enough to resolve the annihilation peak time structure (its relaxation time of about 1 μ s is much bigger than the annihilation peak width ~ 20 ns), the measured peak amplitude is proportional

to the total amount of annihilations occurring at the opening of the trap. A calibration table, provided by the manufacturer of the detector, was used to work out the number of γ -rays at 511 keV equivalent to the amplitude of the recorded peaks. Solid angle and material absorption corrections were applied, considering a point source of monochromatic γ -rays and a 2 cm thick steel flange of the vacuum chamber.

The measurement was repeated varying the number of pulses from the Surko-trap stored in the accumulator, before the spill occurred. The resulting function was fitted with a logistic curve $N(n) = p_1(1 - \exp(-n/p_2))$ and its linear approximation for low number of pulses considered to work out the value of accumulated positron per pulse $\alpha = p_1/p_2$.

$$\alpha = (3.23 \pm 0.34_{stat}) \cdot 10^4 \text{ e}^+/\text{s} \quad (4.1.1)$$

The second measurement scheme used a 2" x 4" Sodium Iodide (NaI) crystal placed about 150 cm far from the positrons annihilation point with a 5 cm thick collimator in front of it, letting only γ -rays from an aperture of 2 cm diameter hit the surface of the scintillator. The detector was viewing the annihilation peak under an angle of $1.18 \cdot 10^{-5}/4\pi$ sr. With such low solid angle, the number of γ detected by the detector was in the order of unity per shot, thus allowing a direct comparison of the statistical distribution of detected photoelectrons with a Poissonian distribution, expected for a radioactive decay process with very low counting rates.

Two measurements were performed, with 100 spills of 100 pulses and 100 spills of 1000 pulses, resulting

$$\alpha = (2.1 \pm 1.1_{stat}) \cdot 10^4 \text{ e}^+/\text{s} \quad (4.1.2)$$

Results of this method were found in agreement with the CsI(Tl) measurement, and provide an independent estimate of the number of positrons leaving the accumulator.

4.2 Positron transport, bunching and focusing

The dedicated test chamber used to perform Ps excitation studies, outlined in Figure 4.5.a, consists in a switchable magnetic/electrostatic transfer line including a buncher and multi-viewport vacuum chamber where the target performing Ps conversion is installed on a movable sample holder. The buncher, key element in the electrostatic transfer line shown in Figure 4.5.b, is composed by a series of 28 electrodes designed to:

- Extract positrons from the magnetic field of the accumulator and drive them towards the target chamber, which benefits of a magnetic field-free environment, particularly conven-

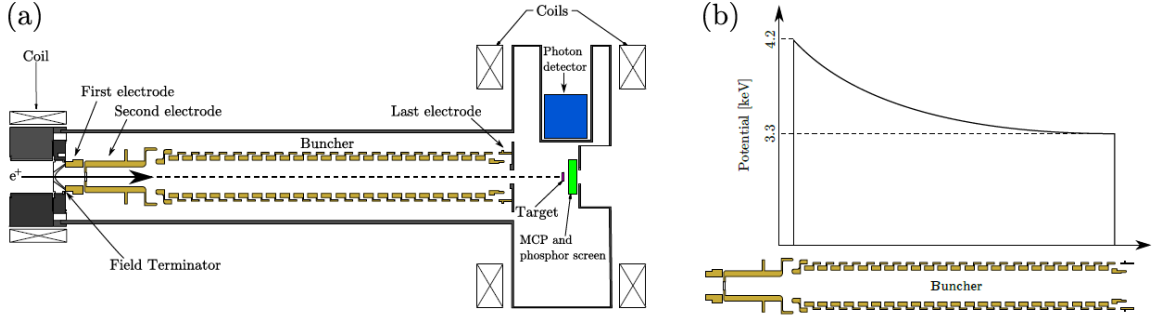


Figure 4.5: Outline of the dedicated Ps formation and spectroscopy chamber (panel a) and the electrostatic buncher used for magnetic field-free positron transport (panel b).

tient for performing spectroscopic studies of Rydberg states since positronium is highly sensitive to magnetic fields inducing motional Stark splitting of the energy levels;

- Accelerate positrons to the required energy to implant at a depth in the converting material that allows efficient thermalization (usually the energy is in the range of a few keV - see [40] or next section for further details);
- Spatially focus the positron cloud on the target, with a spot smaller than 5.0 mm and a time spread of less than 10 ns. Time and spatial compression are both of particular importance for laser-excitation studies to relax the requirements in terms of laser spot size (and thus, energy) to cover the full extent of the Ps cloud.

During the first transport experiments, positrons were let annihilate on the Al target holder used as a dumper. The γ -rays generated by the annihilation were observed using a fast scintillation detector composed of a PbF_2 crystal (diameter 20 mm, length 60 mm) coupled to a Hamamatsu R11265-100 PMT placed 4.0 cm from the target. A microchannel plate (MCP) assembly (Hamamatsu F2222-21P25 + phosphor screen P46) was placed ~ 1 cm on the back of the target holder to monitor the positron bunch shape at the target position.

Bunching experiments were conducted applying a fast, high voltage pulse of 4.2 kV in amplitude and 35 ns in time to the buncher as the positron cloud ejected from the accumulator was flying through. The control electronics of the buncher was constituted by a chain of gold resistors [94] in parallel to the electrodes shaping the potentials on the electrodes so to get common bias voltage of 3300 V superimposed to a semiparabola of 900 V amplitude. These setting values were chosen and optimized according to a SIMION[®] 8 simulation of the particle flight in the buncher, under the approximation of negligible interaction among the particles [95].

The best results obtained transporting and time-bunching positrons are shown in Figure 4.6.

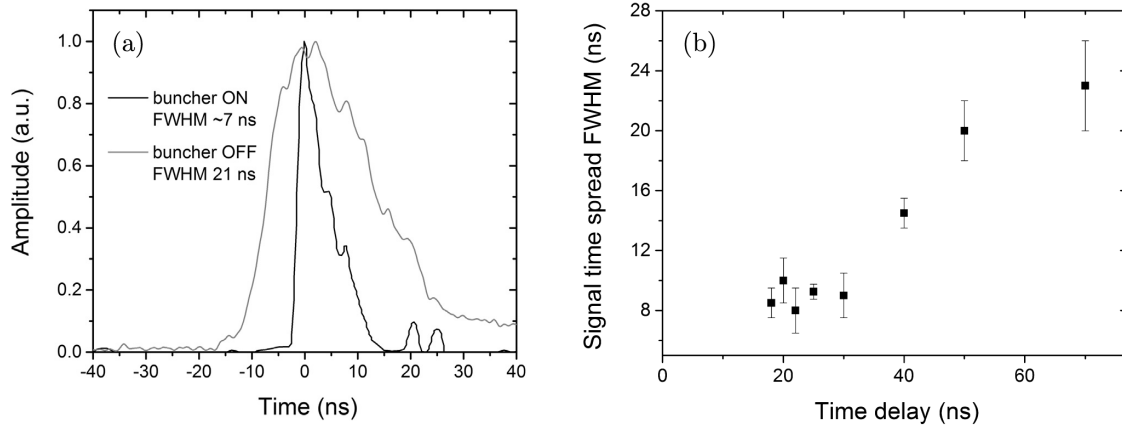


Figure 4.6: (a) Comparison of the two annihilation curves obtained with and without the electrostatic buncher, showing bunching and time-focusing of the positron cloud on the target. (b) Full-width at half-maximum of the annihilation peak on the target as a function of the time delay of the HV pulse applied on the buncher.

A time compression of the annihilation distribution from 21.0 ± 4.0 ns FWHM to 7.0 ± 1.0 ns FWHM was observed by optimizing the HV pulse delay time with respect to the launch from the accumulator.

Subsequently, the Al dumper was removed and positrons were made annihilate directly on the MCP surface to observe the beam spatial distribution on its phosphor screen with a CCD camera (Hamamatsu Orca R2). Focusing was optimized by changing voltage on the last electrodes of the buncher, and a minimum size of 6 mm diameter at 95% of the integrated signal was obtained.

It is worth mentioning now that turning on a magnetic field for spectroscopic purposes in the target chamber (see 4.5) has an effect on the positron beam shape. Two beam profiles, obtained without magnetic field and with a magnetic field of 250 G (the reason of this particular value is clarified in 4.5), are shown in Figure 4.7. The diameter at 95% of the integrated signal reduces to 3.5 mm with the magnetic field on, against the 6.0 mm found in the case without the field. This is an expected consequence of the field presence, which provides better radial confinement to the particles and effectively impedes Coulomb expansion in the transversal degrees of freedom.

A throughout description of design and simulations of the electrostatic buncher can be found in L. Di Noto PhD thesis [95].

In conclusion, the electrostatic transfer line/buncher was able to convey spills of $1.2 \pm 0.1 \cdot 10^7$ $e^+{}^1$ accelerated to 3.5 keV with a temporal spread of 8 ns and a spatial full-diameter of 3.5 mm every 3 min on the target region for positronium formation studies. The overall transfer efficiency

¹measured at the exit of the accumulator

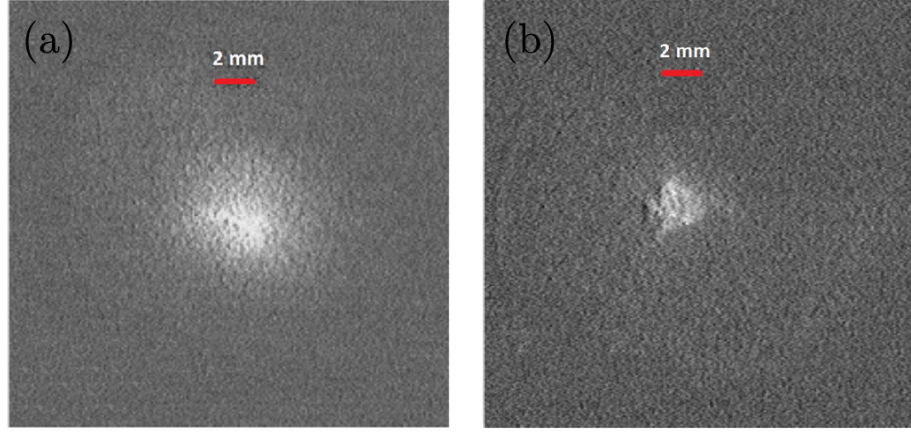


Figure 4.7: Comparison of the positron beam radial distributions with and without magnetic field in the target chamber: (a) magnetic field off (panel b) 250 G magnetic field on. The beam shape decreases as the magnetic field increases as it provides radial confinement, absent in the no-field case.

of the electrostatic buncher was estimated using a CsI detector and found to be $\sim 30\%$.

4.3 Positronium formation and detection

The Ps formation mechanism chosen by AEgIS uses a semiconductor converter where positrons are implanted at about keV energy in the surface. High-energy positrons scatter in the bulk of the material progressively losing their energy, then diffuse until they either annihilate in the bulk or find the surface and are re-emitted into vacuum in form of positron or positronium, according to the probability to capture a surface electron (a sketch of the possible interaction processes positrons can incur when implanted in a semiconductor is shown in Figure 4.8, panel (a)).

Among the various materials considered as positron-Positronium converters for antihydrogen production [96], the choice was limited by the constraint that the target has to be operated at very low temperatures (nominally 100 mK). The choice of silicon as semiconductor is motivated for its highest efficiency in producing Ps in the material bulk at low temperatures [97, 39, 98]. Cooling of produced Ps is achieved by collisions inside specifically engineered nanometric pores electrochemically etched in the surface of the target and subsequently oxidized to set a positive work-function barrier to the produced atoms in the vacuum volume of the pore [38]. Mesoporous silica targets showed efficient production of Ps atoms thermalized at the same temperature of the target or at minimum temperature associated to the energy of the quantum potential well

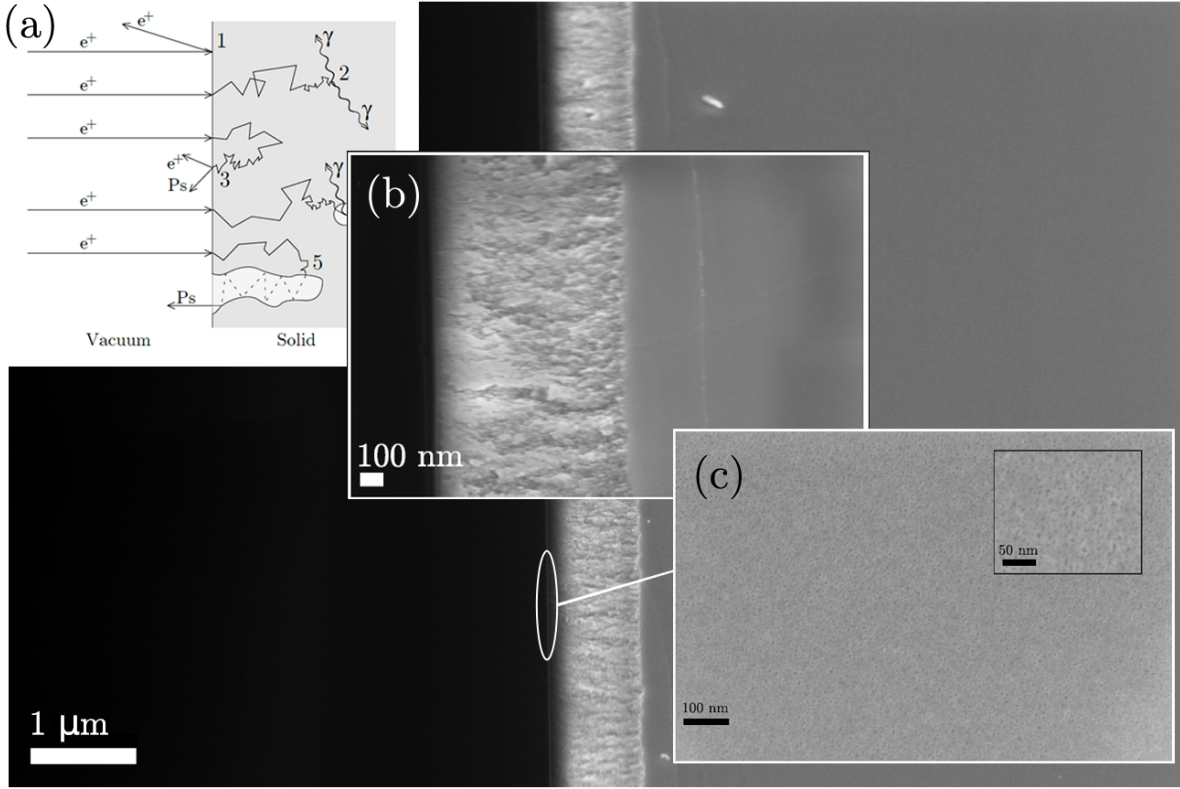


Figure 4.8: SEM images of the p-type (111) silica converter used for thermalized positronium production in AEgIS. Plots from [99].

of a single pore [39, 40].

Positronium converters developed and tested for AEgIS used three types of silicon: p-type with (100) and (111) crystal orientations and n-type with (100) crystal orientation. Regular silica nanochannels were obtained on the surface with electrochemical etching and oxidation and measured (via Scanning Electron Microscope (SEM) imaging) to have a diameter of 5 – 8 nm and length of 1 – 2 μm (see Figure 4.8, panel (b) and Figure 4.8, panel (c)). More details on the manufacturing process of the used samples are found in I. Larsen and B. Rienaecker master theses [99, 100]. One sample per category was positioned on the movable target holder to assess Ps yield.

The detection technique used to prove Ps formation and vacuum emission is known in positron literature as Single-Shot Positron Annihilation Spectroscopy (SSPALS). It is based on detecting o-Ps annihilation in vacuum, whose longer lifetime of 142 ns than the 125 ps of para-Positronium (p-Ps) produces a clean exponential distribution of γ -rays at later times with respect to the positron implantation instant. This γ -ray distribution can be observed by means of a fast scintillation detector.

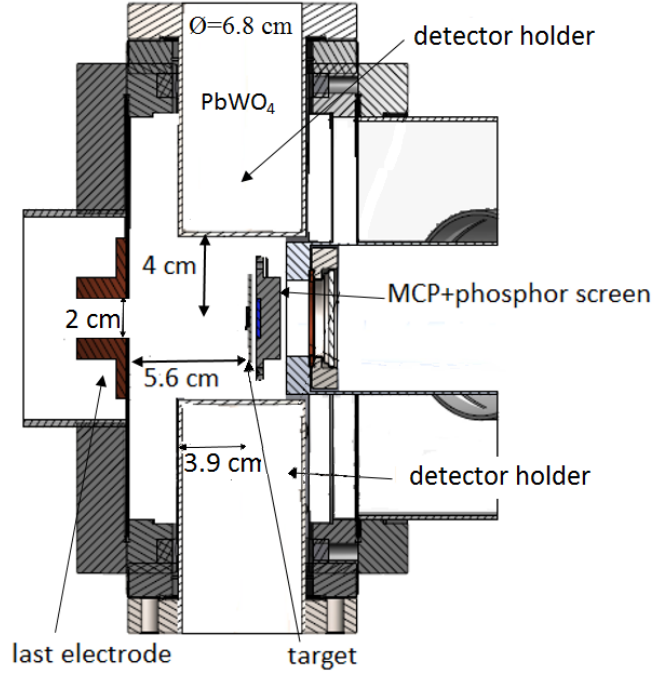


Figure 4.9: Detail of the positronium spectroscopy chamber target region: placement of the target holder with respect to the last electrode of the electrostatic buncher (5.6 cm apart) and the detector holder hollow flange. From [32].

For the set of experiments described in this and the next paragraphs, a PbWO_4 crystal ($25 \times 25 \times 20 \text{ mm}^3$) coupled to a Hamamatsu R11265-100 PMT was used. A special hollow viewport (see Figure 4.9 and Figure 4.5, panel (a)) with a 10 cm socket hosted the detector right above the sample holder, at a distance of 4.0 cm from the center of the crystal. This setup combined scintillation light detection efficiency, significantly higher than a Cherenkov crystal such as PbF_2 , and fast time response (FWHM of single photon signal of 4 ns). The signal from the PMT was divided in a Mini-Circuits ZFRSC-2050B 50% – 50% splitter, terminated to 50Ω and acquired with two channels of a Tektronix TDS5054B, 500 MHz 8-bit oscilloscope at high-Z. One channel, set to 1 V div^{-1} , acquired the *prompt peak* of p-Ps annihilations in the bulk of the target. The other one, with a scale of 100 mV div^{-1} , recorded the low-amplitude part of the signal at later times due to o-Ps annihilations in vacuum. A low-pass filter was added to attenuate the noise below 100 MHz.

Waveforms were transferred automatically on a computer where a reconstruction algorithm was applied to the data. The two channels were combined together to a single, higher-dynamics waveform by first adjusting the voltage scales, then refining using the data of the superposition region by imposing that the value and its first derivatives were to be equal at the junction point, and finally selecting the windows of high dynamics for each single channel.

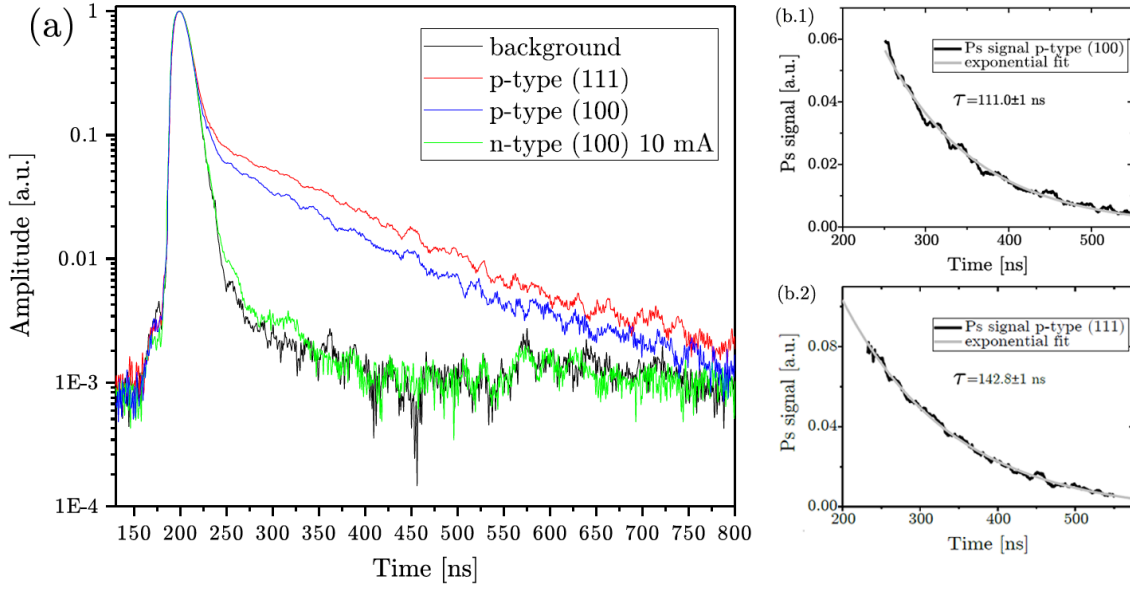


Figure 4.10: Positronium formation and lifetime by silica targets. Normalized SSPALS spectra of different samples showing the comparison of different yields of Ps (panel a). Measured lifetime of o-Ps for silicon p-type (100) (panel b.1) and silicon p-type (111) (panel b.2). Plots from [99]

50 shots from the accumulator (with the buncher always operated at 3.3 keV implantation energy) were acquired and the SSPALS signals averaged to produce the lifetime curves. Normalization to the amplitude of the prompt peak was applied before averaging, to remove the 7% amplitude fluctuation due to varying number of positrons from the accumulator.

Background subtracting and fitting the curves with an exponential model

$$y(t) = A + B \exp(-(t - t_0)/\tau) \quad (4.3.1)$$

gives the Ps lifetime τ and amplitude $y_0 = y(t_0)$. Amplitudes, rescaled to measurements performed at the University in Trento on p-type (100) samples [39, 40] gave a relative estimation of the yield of Ps produced normalized on the number of ingoing positrons [99]. Results obtained are reported in Table 4.1.

Substrate	Ps yield	Ps lifetime τ
p-type (100)	35 %	111.0 ± 1.0 ns
p-type (111)	40 %	142.8 ± 1.0 ns
n-type (100)	0 %	

Table 4.1: Summary of Ps yields and lifetimes for the different SiO₂ targets tested.

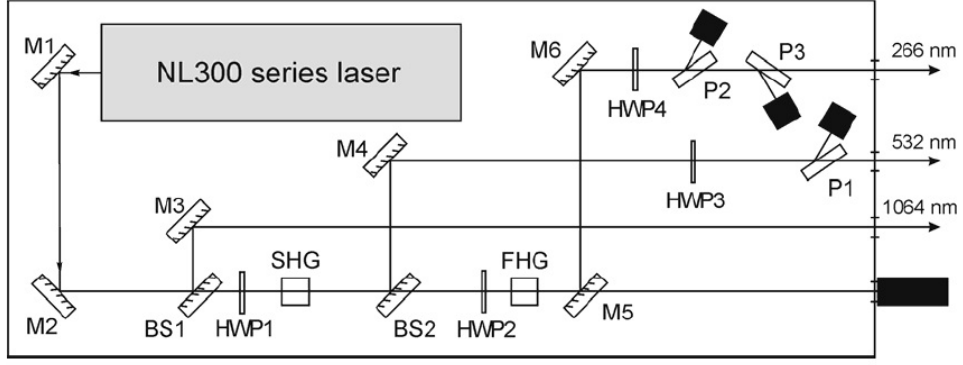


Figure 4.11: Sketch of the EKSPLA pump laser. From [105].

In conclusion, the p-type (111) target was selected for installation into AEgIS and further Positronium experiments for its higher conversion efficiency and lifetime. The lifetime value, close and compatible to the theoretical $\tau_{3\gamma} = 142.04(1)$ [101], is a good indication that a significant fraction of the produced Ps is emitted into vacuum.

4.4 Laser system for $Ps\ 1 \rightarrow 3 \rightarrow Ry$ excitation

AEgIS laser system for Ps Rydberg excitation, described in detail in [87, 102, 103] and the author's master thesis [104], consists of two separate optical setups, fed by the same pump, for generating the two wavelengths required to perform $1^3S \rightarrow 3^3P$ and $3^3P \rightarrow Ry$ transitions, respectively in the Ultra-Violet (UV) at 205.045 nm and Infra-Red (IR) between 1650 nm and 1720 nm.

4.4.1 Pump laser setup

The pump is constituted by a custom laser setup (sketched in Figure 4.11) from the Lithuanian manufacturer EKSPLA, including a pump cavity, a Second Harmonic Generation (SHG) and a Fourth Harmonic Generation (FHG). The Q-switched Yttrium-Aluminum Garnet (YAG) cavity was realized in an unstable confocal configuration, providing single-mode TEM^{00} pulses with hat-top profile of 0.8 J and 4.0 ± 0.4 ns at 1063.8 nm with 30 GHz linewidth.

4.4.2 UV laser for $1^3S \rightarrow 3^3P$ experiments

The Deep Ultra-Violet (DUV) setup for exciting Ps to 3^3P (shown in Figure 4.12) was designed around the use of a Barium Borate (BBO) crystal to produce the required 205 nm light via Sum-Frequency Generation (SFG) with a few nm tunability. An effective choice is to mix the

fourth YAG harmonic at 266 nm with 894 nm photons.

$$\text{SFG } 894 \text{ nm}_{(o)} + 266 \text{ nm}_{(o)} \longrightarrow 205 \text{ nm}_{(e)} \text{ type II} \quad (4.4.1)$$

The 894 nm beam is produced by a Periodically-Poled Potassium Titanyl Phosphate (PPKTP) crystal acting as an Optical Parametric Generation (OPG) in the quasi-phase matching configuration, fed by a small amount of light from the second harmonic of the YAG at 532 nm.

$$\text{OPG } 532 \text{ nm}_{(o)} \longrightarrow 894 \text{ nm}_{(o)} + 1313 \text{ nm}_{(o)} \text{ type II} \quad (4.4.2)$$

Subsequently, two BBO crystals perform Optical Parametric Amplification (OPA) of the produced, low energy seed 894 nm. Again, 532 nm light from the second YAG harmonics is used for pumping the OPA crystals.

$$\text{OPA } 532 \text{ nm}_{(e)} \longrightarrow 894 \text{ nm}_{(o)} + 1313 \text{ nm}_{(o)} \text{ type I} \quad (4.4.3)$$

The SFG crystal was producing in normal working conditions about 120 μJ of 205 nm light at the beginning of the transfer line to the experimental chamber, with the possibility to tune the a wavelength in range between 204 nm to 206 nm according to the temperature set for the OPG crystal and optical alignments on the table.

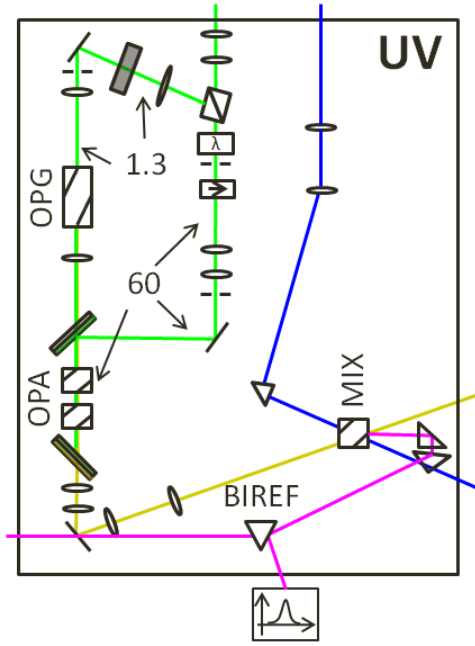
4.4.3 IR laser for $3^3P \rightarrow \text{Ry}$ experiments

The IR setup for the subsequent $3^3P \rightarrow \text{Ry}$ excitation (shown in Figure 4.13) is similar in conception to the UV setup, but does not require a SFG step. It uses directly a PPKTP crystal as OPG to generate a seed of IR in the wavelength range 1650 – 1700 nm, corresponding to $n = 14 - 23$ levels of Ps. The OPG in this case is pumped from the YAG first harmonic at 1064 nm.

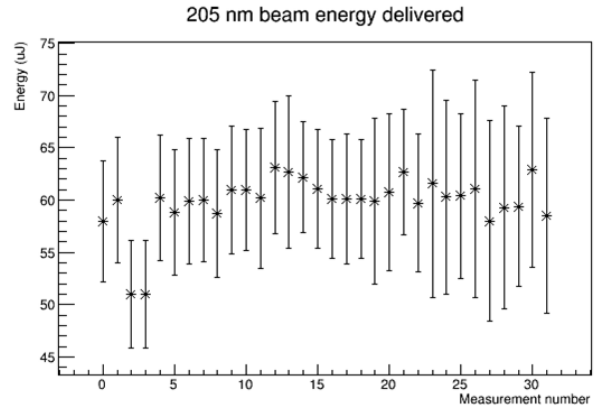
$$\text{OPG } 1064 \text{ nm}_{(o)} \longrightarrow 1700 \text{ nm}_{(o)} + 2844 \text{ nm}_{(o)} \text{ type II} \quad (4.4.4)$$

Then, two Potassium Titanyl-Phosphate (KTP) crystals are used as OPA for increasing the seed energy. Also in this case, a fraction of the YAG first harmonic is used for pumping the crystals.

$$\text{OPA } 1064 \text{ nm}_{(o)} \longrightarrow 1700 \text{ nm}_{(o)} + 2844 \text{ nm}_{(e)} \text{ type I} \quad (4.4.5)$$

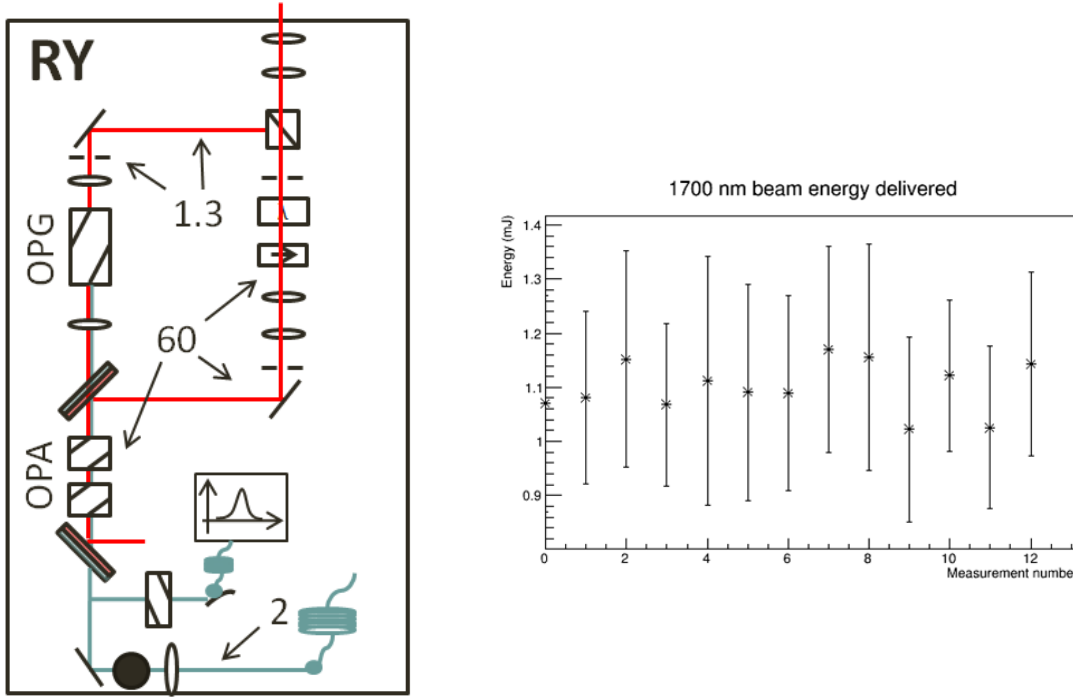


(a) UV laser scheme (energies in mJ)



(b) Energy delivered

Figure 4.12: (a) sketch of the UV laser setup used to generate 205 nm light for $1^3S \rightarrow 3^3P$ spectroscopy (b) energy delivered as a function of the measurement day during $Ps\ 3^3P$ spectroscopy measurements.



(a) Rydberg laser scheme (energies in mJ)

(b) Energy delivered

Figure 4.13: (a) sketch of the IR laser setup used to generate 1700 nm light for $3 \rightarrow 15 - 17$ spectroscopy (b) energy delivered as a function of the measurement day during Ps Rydberg spectroscopy measurements.

Wavelength tunability was obtained by controlling the OPG crystal in temperature and consequently adjusting the OPA phase matching alignment. Varying the crystal temperature in the range $0 - 200^\circ\text{C}$, wavelength tuning in the range $1675 - 1720\text{ nm}$ was achieved, corresponding to the Rydberg levels interval $n = 14 - 21$. The pulse energy was constant up to 1705 nm , with a 20 % reduction at shorter wavelengths.

4.4.4 Photoionization laser

A third laser line with an intense pulse at 1064 nm was prepared to selectively photoionize the atoms in the 3^3P state (shown in Figure 4.14). The photionization pulse was prepared from the spurious outlet of the YAG pump containing both 1064 nm and 532 nm by removing with two dichroic mirrors the 532 nm componen. The final IR pulse was 4 ns in full-width and contained 55 mJ of 1064 nm . It was superimposed to the 205 nm pulse both in time, with a precision $\sim \text{ns}$ by means of an optical delay line, and in space by increasing the its size so to completely cover

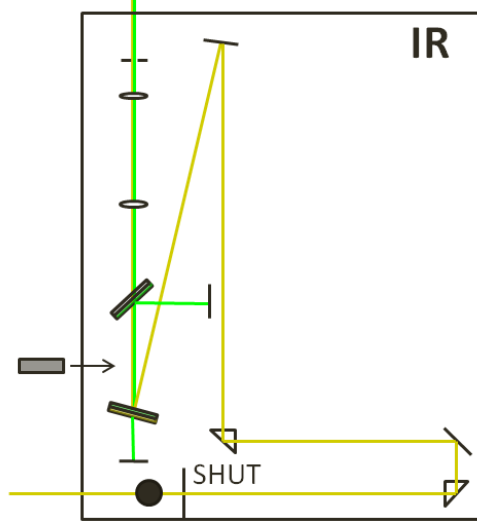


Figure 4.14: Schematics of the IR photoionization laser obtained selectively removing the 532 nm component from the spurious outlet of the YAG pump (energies are given in mJ).

the excitation pulse area (hat-top profile, circular shape, 8 mm in diameter).

4.4.5 Transfer lines to the experiments

All laser systems described in the last paragraphs are physically located in AEgIS laser hut, which is placed aside of the main cryostat. It consists of two laser tables installed vertically one above the other (see Figure 4.15). The bottom table hosts the laser YAG pump, the 205 nm UV laser and the 1700 nm IR laser. The upper table, instead, is reserved for the $1 \rightarrow 2 \rightarrow Ry$ laser system, and holy hosts the IR photoionization line and the launch optics towards the spectroscopy chamber.

Two optical transfer lines transport the laser beams from the lower table to spectroscopy chamber (see [104]) and to the main AEgIS apparatus (see [106]). The transfer line to the spectroscopy chamber is constituted of four broadband, DUV mirrors from CVI Melles-Griot with a measured reflectivity of 95% at 205 nm and 90% reflectivity at 1700 nm. The high-power photoionization pulse was transported on a prism transfer line to reduce the risk of damaging the broadband mirrors.

At the experimental chamber viewport, the UV laser pulse had the following specifications: horizontal polarization (i.e. perpendicular to the sample); an asymmetric, nearly Gaussian temporal profile with a FWHM of $t_{UV} \approx 1.5$ ns, a Gaussian-like spectral profile with $\sigma_{laser} \approx 2\pi \times 48$ GHz and a slightly elliptical spatial shape, with $FWHM_{vertical} = 6mm$ and $FWHM_{horiz} =$

4mm. The UV energy was kept at 54 ± 5 μJ energy in all different wavelength setups by adjusting the amount of power fed into the OPA by means of an half-wave plate and Brewster window.

The transfer line to AEGIS is conceptually identical to the one to the spectroscopy chamber, but for the reflective elements, which are all uncoated Fused Silica (FS) prisms in total reflection configuration. In this way, all wavelengths can be transported by the same transfer line, at the price of a slight absorption of the UV (FS prisms absorb around 10 % of the energy). For superimposing the two lasers together outside the main vacuum cryostat, a dispersive 60° CaF_2 prism was used. The optical path superposition is realized using normal dispersion of CaF_2 , i.e. two different indices of refraction at 205 nm and 1600 nm. This difference, in terms incidence angles, is about 4.3° for FS and 2.3° for CaF_2 , large enough to easily place two 1-inch mirrors, one for each wavelength, at a distance of about 30 cm from the prism. This technique, envisaged for the prism transfer line that should carry both lasers inside AEGIS, has the advantage of sharing the same optics for transporting both wavelengths, at the price of about 20 % of the UV power absorbed/reflected by the prism and making the polarization elliptical. It has been tested and used during the Rydberg excitation campaign of o-Ps described further through in this chapter.

4.4.6 Alignment and synchronization

Laser beams were aligned on the target region by monitoring their position on a 1-inch MACOR[®] screen placed inside the vacuum region a few cm away from the target (see Figure 4.16). The beam position was acquired and monitored each sequence of 30 shots with a camera looking at the screen from a 45° viewport. The camera allowed a positioning of the beam with an estimated resolution of mm; once good signal conditions were found, alignment of the beams was not altered anymore to avoid systematical effects.

The 205 nm beam alignment was found critical both in time and space to obtain efficient laser excitation, as expected from Montecarlo calculations. A shift in positioning of the order of mm resulted in a major loss of signal of the order of 30%: for this reason, significant efforts were put in keeping the alignment conditions stable during the whole scan in wavelength. On the contrary, the alignment of the photoionization beam was not crucial, as long as it overlapped the excitation laser.

Positrons and laser were synchronized by a custom FPGA-based device, which was developed to compensate the Q-switched cavity pulse build-up time, and the possibility to adjust the mutual synchronization providing a time resolution of 2 ns with a jitter < 600 ps. The time delay between the 1^1S annihilation peak and the laser pulses was set according to the Montecarlo

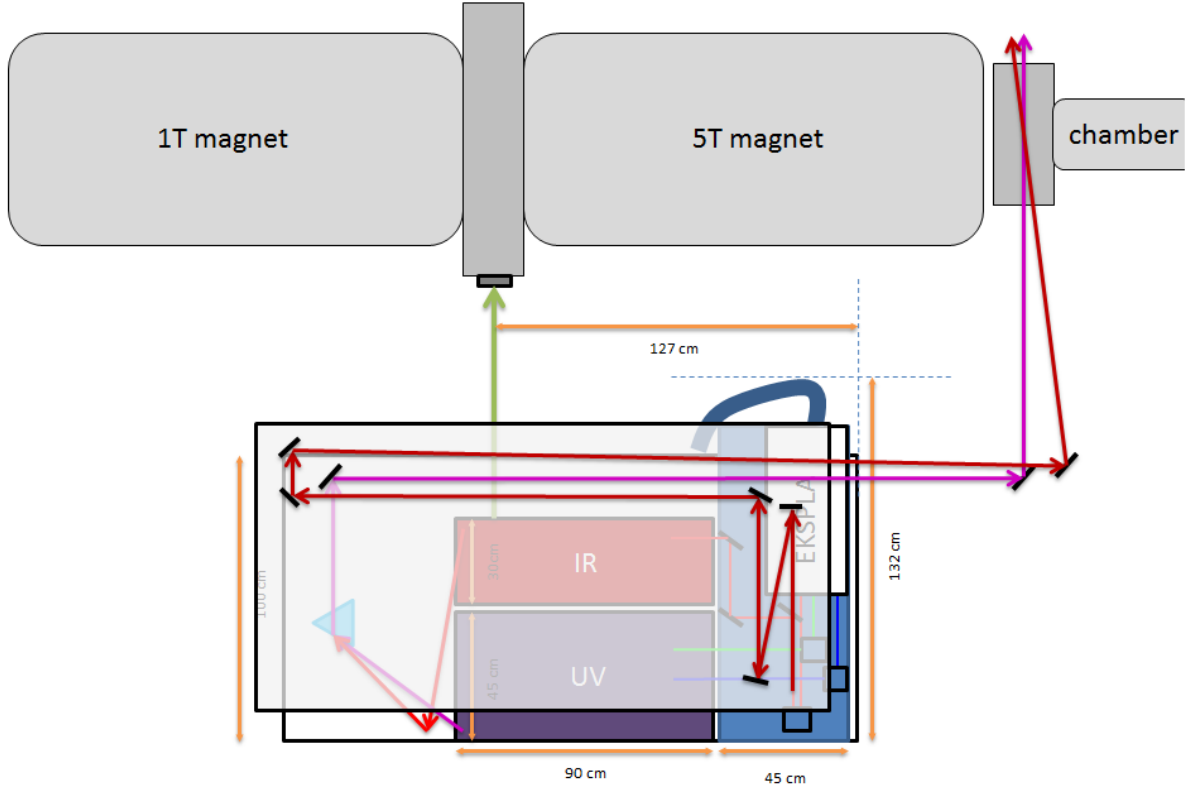


Figure 4.15: Scheme of laser transfer lines to the main experiment and to the spectroscopy chamber. AEgIS laser hut is composed of two tables vertically placed one above the other: the bottom one contains the laser system for $1 \rightarrow 3 \rightarrow Ry$ spectroscopy, the upper the laser system for $1 \rightarrow 2 \rightarrow Ry$ (not depicted here), the photoionization line and the optics for launching in the spectroscopy chamber.

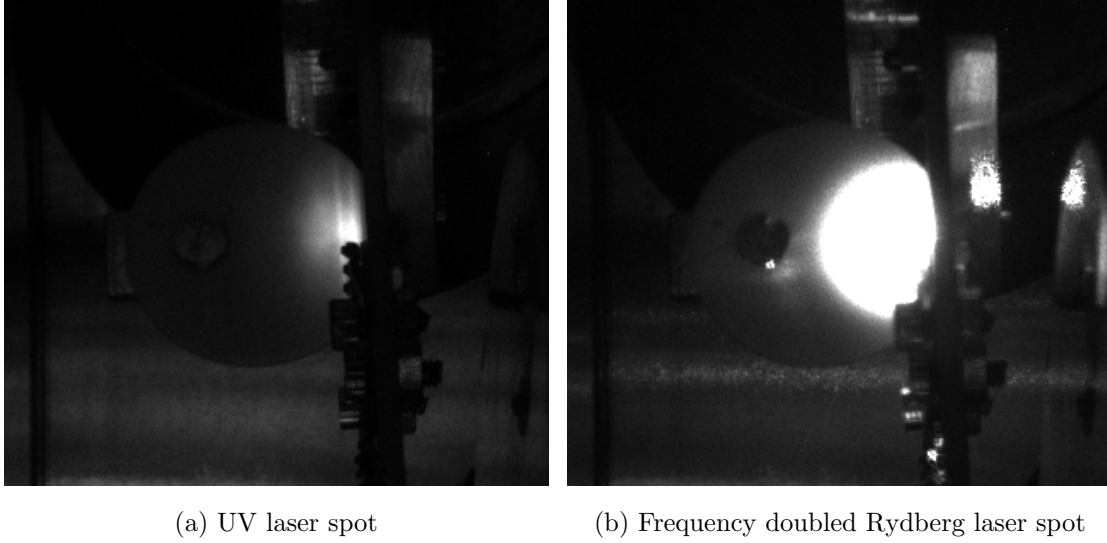


Figure 4.16: Alignment of the UV laser (panel a) and frequency-doubled IR laser for Rydberg excitation (panel b) on the MACOR screen on the back of the sample holder, as observed by the monitoring camera installed on a 45° tilted viewport.

estimations to 16 ns.

4.4.7 UV spectrum diagnostics and calibration

The wavelength of the UV pulse was monitored using a spurious reflection from the CaF_2 prism of the transfer line. A fiber Avantes AvaSpec 3648-USB2 wavemeter, with instrumental resolution declared by the manufacturer of ± 0.005 nm, was used to acquire the spectrum.

In the SFG crystal near-field regime, the UV pulse shows a peculiar spatial wavelength distribution which has a linear dependence over the horizontal position in the beam (see Figure 4.18). This displacement is caused by the non-null divergence of the beams entering in the SFG crystal, which determines a slightly different phase-matching as a function of the horizontal position. This displacement is completely smeared-out in the far-field from the crystal at the end of the transfer line. From a practical point of view, induces an additional source of broadening of the laser line.

It's worth noticing that this effect potentially gives control over the produced UV laser bandwidth. For further studies of Ps excitation efficiency as a function of the laser bandwidth, an effective experimental procedure would be to vary the divergence of the YAG 4-th harmonic using a movable telescope, thus inducing a different distribution of the wavelengths. This distribution in the far-field is convoluted to the temporal linewidth of the 266 nm (which is the low-end limit of the technique), determining in part the final bandwidth of the pulse.

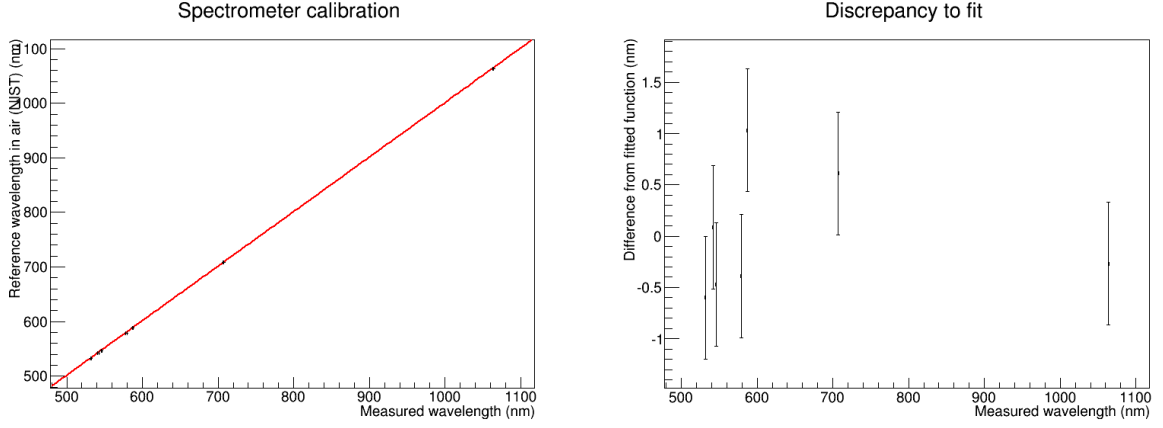


Figure 4.17: IR spectrometer calibrations. Linear fit of five Hg peaks and two YAG lines (panel a) and residual errors from the resulting fit (panel b).

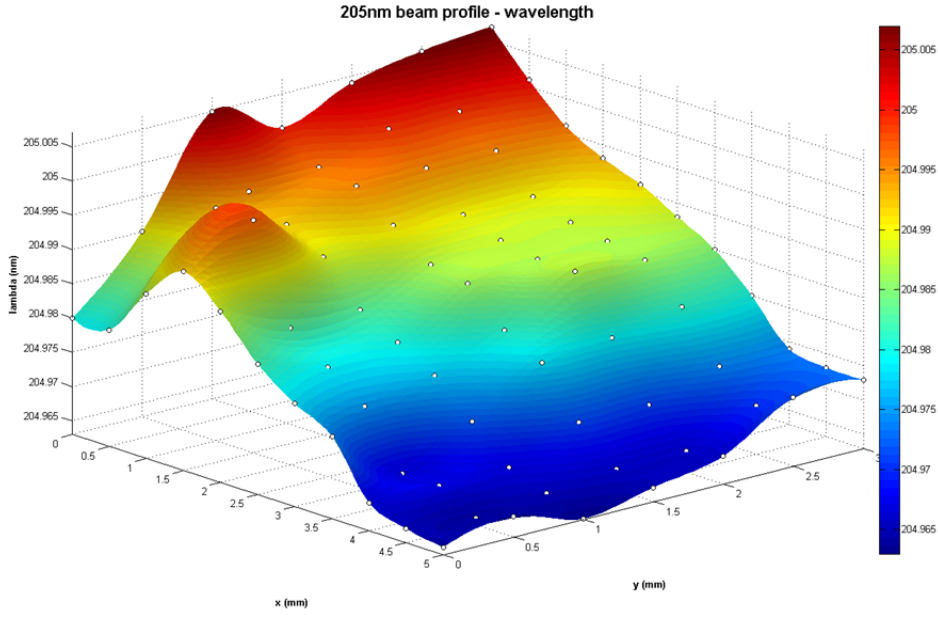
The factory calibration of the spectrometer was verified right before the start of the measurement campaigns on a precise hydrogen 1S-2S laser source at 243.13466 ± 0.00003 nm of the ALPHA collaboration². The Avantes spectrometer showed an offset miscalibration of -0.069 ± 0.005 nm, corresponding to 350 GHz, that was compensated correcting the calibration coefficients programmed in the firmware.

4.4.8 IR spectrum diagnostics and calibration

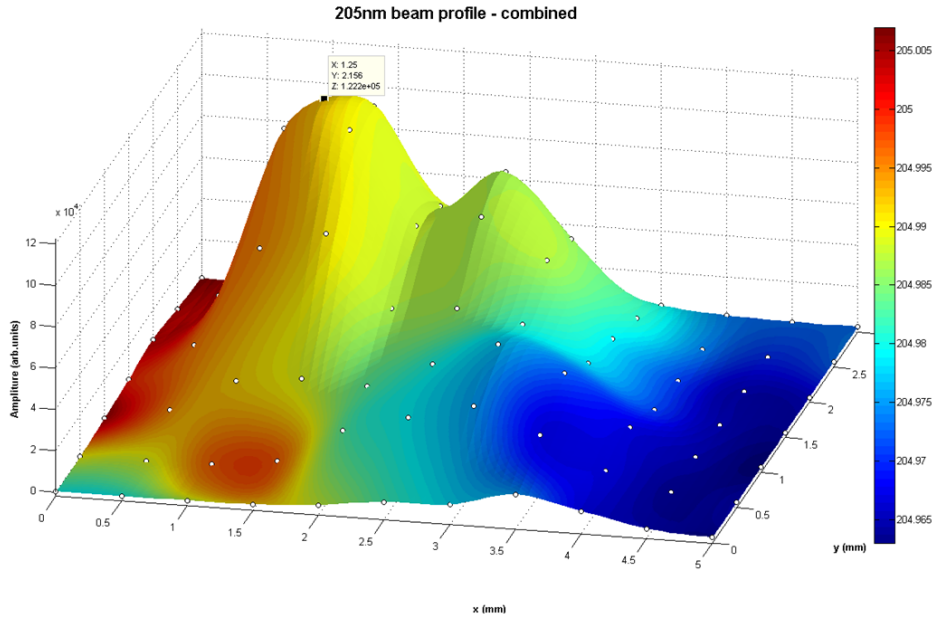
The wavelength of the IR pulse was assessed by inserting in the already aligned laser line a Lithium Niobate (LiNb) doubling crystal, whose acceptance bandwidth, 0.98 THz, is much higher than the measured bandwidth of the IR pulse (100 – 200GHz depending on the OPA alignment). The role of this crystal was to map the 1650 – 1720nm wavelength range in the detection range of a commercial Thorlabs CCS175 spectrometer, without altering significantly its spectral shape. The phase-matching angle of such crystal was optimized for each wavelength used by maximizing the outgoing beam intensity, in order not to incur in systematics effects related to limiting crystal acceptance angle.

The CCS175 spectrometer was used during the data taking with its original factory calibrations. At the end of the whole measurement campaign, a spectrum of a mercury lamp was acquired with high statistics, using the same set of calibration coefficients as for acquiring the experimental data. Five mercury peak positions in the range 541 – 707nm were identified. Together with the theoretical position of the first two YAG lines (531.91 nm and 1064.82 nm),

²the author acknowledges and is grateful to the ALPHA experiment colleagues for their kindness in letting us cross-calibrate the wavemeter on their laser



(a) UV spot spatial wavelength distribution



(b) UV spot spatial intensity distribution

Figure 4.18: UV laser pulse wavelength (a) and intensity (b) distributions in the near-field region of the SFG crystal. The wavelength distribution (a) shows a linear dependence of the wavelength on the horizontal position. This effect is mainly caused by the divergence of the incoming 266 nm beam, the wide spectrum of the 894 nm pulse of about 1 THz and the wide acceptance bandwidth of the BBO crystal used for SFG. The intensity distribution (b) shows an irregular intensity profile, which was mainly caused by some damages on the crystal surface due to the intense UV power.

these Hg peaks were used to fit a linear model ³ to the National Institute of Standards and Technology (NIST) reference values for mercury in air (see Figure 4.17.a), so to account a-posteriori for a non-ideal spectrometer calibration. A shift in the measured wavelength in the range 532–1064 nm versus the reference NIST lines of 1.14 ± 0.86 nm was observed, while on the contrary the linearity of the spectrometer was found compatible with unity within the statistical sensitivity. The statistical uncertainty for a single measurement in this wavelength range was found to be very close to what declared by the spectrometer specifications (0.57 nm measured versus 0.6 nm declared by manufacturer), and mostly dominated by the instrument resolution (see Figure 4.17.a). Spectral data values from the spectrometer were corrected accordingly to the linear model obtained from this calibration.

4.5 Positronium excitation to 3^3P

The present experiments aimed to demonstrate the first laser excitation of positronium to the 3^3P excited state. Two independent measurement campaigns were conducted. The first one aimed to show $n = 3$ excitation by detecting the effect of magnetic quenching, i.e. the reduction in the overall amount of atoms annihilating on the o-Ps decay channel as a result of the excitation to 3^3P and subsequent decay to 1^1S in a magnetic field. The second measurement campaign used a second laser pulse for selectively photoionize 3^3P and detect the decrease of the number of o-Ps atoms.

4.5.1 Magnetic quenching experiment

In absence of magnetic field, only 3^3P Ps atoms are populated by the laser from 1^3S , as only transitions with $\Delta L = 1$, $\Delta S = 0$ are allowed. These atoms then decay radiatively back to 1^3S in about 10.5 ns.

In presence of a magnetic field, differently, due to mixing of 3^3P sub-states with 3^1P sub-states (with the exception of 3^3P_{10}) the laser populates also sublevels with $S = 0$, which can decay back to 1^1S and incur into fast (~ 125 ps) annihilation. This effect, known as *magnetic quenching* of energy levels, can be observed using SSPALS by measuring the relative amounts of 1^3S atoms at later times (with respect to the laser pulse instant) with and without the laser excitation pulse. Dedicated simulations showed that the effect is maximum for a magnetic field intensity of 250 G, for which 15.5% of the excited atoms do not decay back to the o-Ps state (see Figure 4.19).

³higher-order polynomials and different fitting functions were also considered for fitting the measured peaks and discarded, since they all scored worse in the χ^2 test.

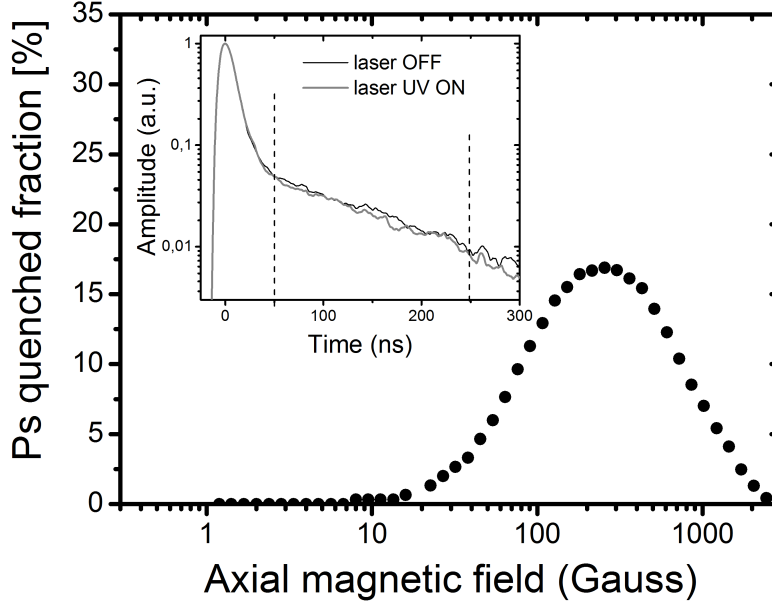


Figure 4.19: Simulation of the expected reduction in the o-Ps fraction due to magnetic quenching assuming 100 % Doppler coverage of the excitation laser (main figure) and results of the magnetic quenching experiment (inset). The two SSPALS spectra shown were obtained averaging 50 positrons shots each. From [32]

The comparison of the relative amounts of o-Ps atoms is obtained from SSPALS spectra by evaluating the areas below the o-Ps 142 ns tail f_{OFF} and f_{ON} , in a window between 50 ns and 250 ns after the prompt peak, and evaluating the S parameter

$$S = \frac{f_{\text{OFF}} - f_{\text{ON}}}{f_{\text{OFF}}} \quad (4.5.1)$$

Two long tails of o-Ps were acquired with 50 shots statistics in presence and absence of the laser excitation pulse. Then, the S parameter was calculated on the average area of each curve. In order to remove potential systematic errors due to temporal drifts in the number of positrons from the accumulator (usually due to moderator aging, which happens over a time scale of a few hours), a leapfrog data taking scheme was adopted (one measurement acquired with laser and one without laser - sometimes alternated).

The SSPALS spectra obtained in this experiments are reported in the inset of Figure 4.19, showing a small reduction of the o-Ps annihilations in the considered window (50 – 250 ns after the prompt peak). The observed reduction of o-Ps signal due to magnetic quenching was $S = 3.6 \pm 1.2 \%$, about 3σ away from a null result.

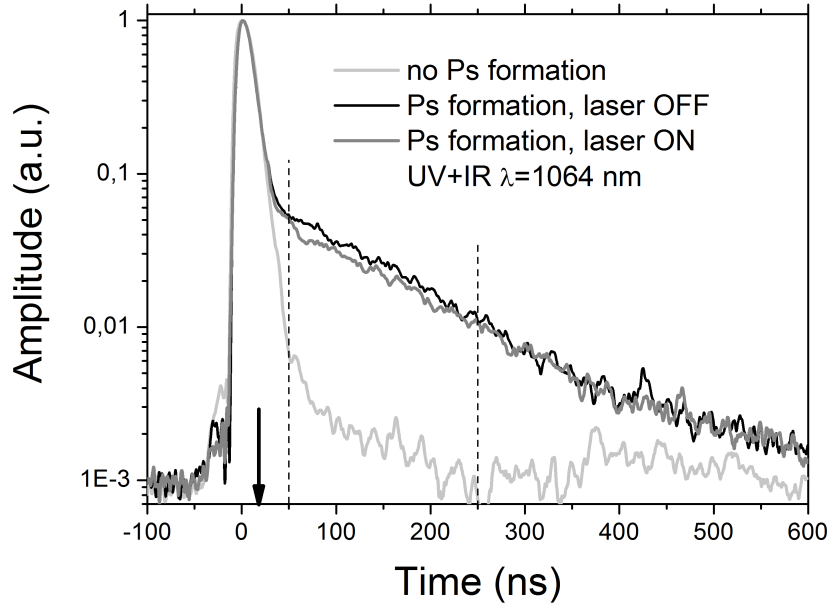


Figure 4.20: Average SSPALS spectrum of Ps in vacuum with and without the UV+IR excitation+photoionization lasers (205.05 nm + 1064 nm). A reduction of the o-Ps annihilation signal due to the effect of the lasers is clearly visible. Each spectrum is the average of 15 single shots. The arrow marks the time when the laser is shot on the Ps cloud (16 ns after the peak). The window between 50 ns and 250 ns was used to evaluate the S signal. From [32]

4.5.2 Laser photoionization experiment

A more striking evidence of the excitation of the 3^3P level of Ps was achieved by adding a photoionization 1064 nm pulse to the magnetic quenching experiment setup. Since the laser power on the photoionization line was almost one order of magnitude higher than the energy required to saturate the transition (about 55 mJ), near 100% efficiency in selectively photoionizing 3^3P atoms was achieved. This resulted in a significantly higher reduction of the observed signal, which became clearly visible in a SSPALS spectrum (see Figure 4.20, with an average signal of $S = 15.5 \pm 1.1$ %).

This higher levels of signal allowed a more systematic scan of the 3^3P level to be performed by varying the UV laser wavelength. The procedure used was the following:

1. The SFG crystal angle was set to the desired wavelength. The temperature was kept constant at 40 °C.
2. The phase-matching angle of the OPA crystals amplifying the 894 nm pulse was adjusted in order to match the frequency needed for the new setting of the SFG. This step was

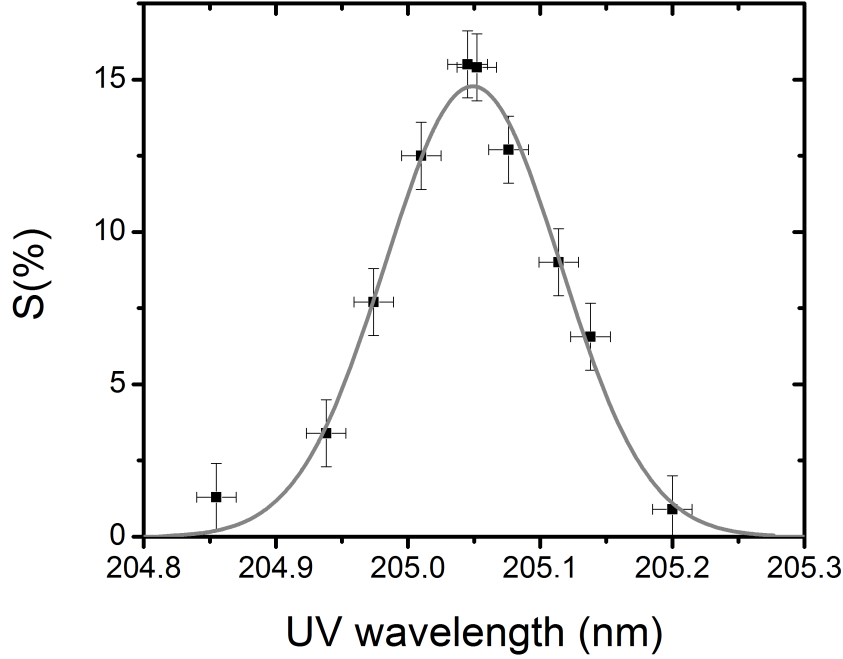


Figure 4.21: Wavelength scan of the $1^3S \rightarrow 3^3P$ transition of Ps obtained by varying the UV laser wavelength at constant energy and IR laser. Each point has been obtained by computing the S signal on a sample 15+15 SSPALS signal/background spectra. Statistical errors (on y axis) and total spectrometer uncertainty (statistical + systematic on the x axis) are reported. The continuous line is obtained from a Gaussian fit (see text). From [32]

performed alternating on the two crystals, starting always from the second, until the maximum energy throughput was obtained.

3. The energy of the UV pulse was adjusted to the average used level of 55 μJ at the entrance of the chamber by acting on a Brewster window varying the pump power in the OPA crystals.
4. An alignment check on the MACOR screen was performed, to verify the presence and alignment of both laser beams.

Results of the wavelength scan are shown in Figure 4.21. A very distinct, Gaussian-shaped peak was observed, centred around the expected wavelength for the $n = 3$ peak of 205.045 nm in vacuum.

Finally, two saturation curves showing the behaviour of the excitation signal as a function (separately) of the UV and IR laser energies were acquired to assess whether the energies used were in the saturation region of the transition (and thus determine the laser excitation efficiency).

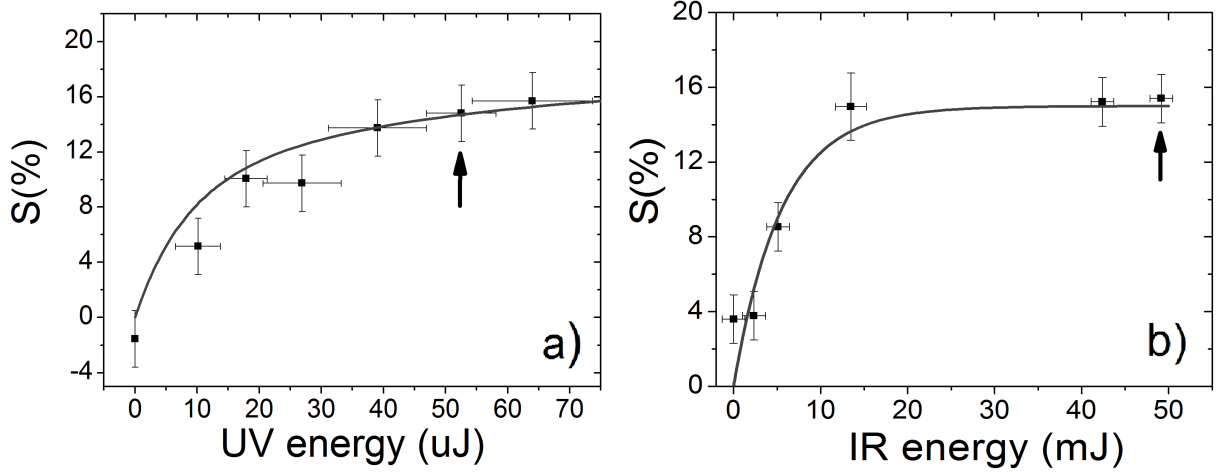


Figure 4.22: Signal obtained from the photoionization experiment as a function of the UV laser pulse energy (IR was kept at 50 mJ) and as a function of the IR laser pulse energy (UV was kept at 55 μ J). Arrows mark the laser energies used in all other data shown here. The continuous lines result from fitting a three-level rate equation model on the data (see text). From [32]

The two saturation curves were obtained varying the UV and IR pulse energies separately from the normal working condition used to acquire the spectra shown in Figure 4.20 and Figure 4.21. While varying the UV energy was obtained easily changing the amount of power fed in the OPA crystals, reducing the IR power on the photoionization line was made complex by the absence of optical components offering tunability on the line. A viable solution was obtained inserting three neutral filters in the optical line, each with a different optical density, in all possible combinations. Results are shown in Figure 4.22. Apart from the IR pulse energy, expected and found well above saturation, also the UV pulse energy was found to be already in the flat part of the saturation curve - even if, as it was already observed by Cassidy et al. in [30], the exponential model is unproper to model the UV saturation curve as it does not properly account for the beam tails at a lower intensity. Also back-of-the-shelf calculations of the required laser energy (see [104]) in the approximation of a circular beam indicated that $\sim 32 \mu$ J of UV energy were enough to saturate the transition with a laser spot of 8 mm FWHM.

4.5.3 Discussion and modelling

The ratio of the experimental to the theoretical efficiencies are $3.6/17 \approx 0.21$ and $15.5/93 \approx 0.17$ for quenching and photoionization, respectively. The reasonable agreement between the two ratios indicates that the experimental efficiency is around 17 – 21% of the maximum theoretical one. In the present setup, the main limitations are the laser spectral width, which does not cover the entire Doppler profile of the Ps cloud emitted from a room-temperature target, and

the limited geometrical overlap of the laser spot and the cloud.

Fitting a Gaussian to the wavelength scan points yields a central value for the 3^3P line of

$$\lambda_3^{\text{exp}} = 205.050 \pm 0.02_{\text{sys}} \pm 0.005_{\text{stat}} \text{ nm} \quad (4.5.2)$$

which is consistent to the theoretical α^4 -corrected value of

$$\lambda_3^{\text{the}} = 205.0474 \text{ nm} \quad (4.5.3)$$

calculated using Positronium Transitions Calculator (PsTC) calculation tool in a magnetic field of 250 G (see chapter 8).

The excitation efficiency can be promptly estimated. Since the $3^3P \rightarrow$ continuum step is strongly saturated, almost all of the atoms excited to 3^3P are immediately photoionized. Thus, the S value found when photoionizing, 15%, can be directly seen as the total excitation efficiency. To separate the different contributions to the total efficiency from the geometrical and spectral overlap efficiencies, a simple three-level rate equations model can be used, which neglects spontaneous emission from $n = 3$ and assumes square laser pulses.

Under the assumption that Ps atoms are very quickly ionized once they reach $n = 3$, the probability for an atom to be photoionized as a function of time is

$$p(t) = 1 - \exp(-\gamma_3 t) \quad (4.5.4)$$

where $\gamma_3 = \gamma_3(\vec{r}, \vec{v}, \delta)$ is the UV photons absorption rate, which depends on:

- the intensity of the laser pulse at the atom position (and thus, indirectly, over its position $\vec{r}$);
- the velocity of the atom $\vec{v}$, which cause Doppler shift of the transition frequency;
- the laser frequency detuning with the atom at rest, δ .

As the UV laser linewidth $\sigma_3 \approx 2\pi \cdot 48 \text{ GHz}$ is much bigger than the natural linewidth of the transition ($\sim 0.1 \text{ GHz}$), the annihilation rate ($\sim 0.01 \text{ GHz}$) and the hyperfine splitting structure ($\sim 3 \text{ GHz}$), the absorption rate γ_3 is modelled by a simple incoherent excitation process:

$$\gamma_3(\vec{r}, \vec{v}, \delta) = \pi \Omega^2(\vec{r}) \frac{1}{\sqrt{2\pi\sigma_3^2}} \exp\left(-\frac{(\delta - \vec{k}_3 \cdot \vec{v})^2}{2\sigma_3^2}\right) \quad (4.5.5)$$

where $\vec{k}_3 = 2\pi/\lambda_3 \hat{k}$ is the UV laser wave number, $\hat{k}$ is the laser propagation axis and $\Omega(\vec{r})$ is the Rabi frequency, that depends on the local laser intensity and thus the position of the Ps atom $\vec{r}$.

Averaging over the (Gaussian) Ps velocity distribution in the laser propagation axis $\rho(v)$ and spatial distribution of the atoms at a certain time $\phi(t, \vec{r})$ is needed to work out the final excitation probability.

$$p(t, \vec{r}, \delta) = \int p(t, \vec{r}, v\hat{e}_3, \delta) \rho(v) \phi(t, \vec{r}) dv d^3r \quad (4.5.6)$$

The problem of giving an explicit form to $\phi(t, \vec{r})$, in order to carry out the integration, is generally complex. A way to attack the problem would be to hypothesize a spatial distribution for the atoms $\phi(t, \vec{r})$ under a certain model, carry out the integration over d^3r , and fit the free parameters of the model over the experimental data to test the hypothesis. This approach would provide very interesting insights over the cloud distribution of positions and velocities as it emerges from the target, and is surely of big interest for experiments, such as AEgIS, involved in Ps physics. It is, however, out of the scope of the present analysis, which aims to extract informations only from the three independent observations performed in the present experiments and reported in Figure 4.21 and Figure 4.22.

A preferable easier approach is to absorb all the spatial-dependent terms in 4.5.6 into two free parameters: η , modelling the geometrical overlapping efficiency, and Ω , as the peak value for the Rabi frequency $\Omega(\vec{r})$. The probability is then rewritten as

$$\begin{aligned} p(t, \delta) &= \eta \int p(t, v\hat{e}_3, \delta) \rho(v) dv \\ &= \eta \int_{-\infty}^{+\infty} p(t, v\hat{e}_3, \delta) \sqrt{\frac{1}{2\pi\sigma_v^2}} \exp\left(-\frac{v^2}{2\sigma_v^2}\right) dv \end{aligned} \quad (4.5.7)$$

where $\sigma_v = \sqrt{k_b T/m}$ is the standard deviation of the velocity along the laser propagation axis $\hat{k}$. Equation 4.5.7 was used to fit the data shown in Figure 4.21 and Figure 4.22. The resulting free parameters obtained from the fitting are given in Table 4.2.

It's worth noticing that the line shape of Figure 4.21 is almost entirely dominated by the thermal distribution of Ps, as $\sigma_v \approx 10^5 \text{ m s}^{-1}$. The transverse velocity component is in the same range of the mean velocity component observed in the experiments of [40] and [107]. This finding can be explained by two effects: the type of silicon bulk used for the present sample, (111) oriented, that is expected to exhibit nanochannels with a 45° orientation with respect to the surface normal and the randomization of Ps trajectories as they hit the irregular walls of the nanochannels (as observed by SEM imaging). This two effects together could indeed produce a

Parameter	Value	Unit	Description
Ω	$5.4 \cdot 10^{10}$	Hz	Rabi frequency
d	1.70	—	Dipole strength
σ_v	10^5	m s^{-1}	Transverse o-Ps velocity
T	1300	K	Transverse o-Ps temperature
η	0.80	—	Geometrical overlap
γ_{ion}	$5.0 \cdot 10^9$	s^{-1}	Max. ionization rate
σ_{ion}	$3.1 \cdot 10^{-17}$	cm^2	Photoionization cross-section

Table 4.2: Parameter values obtained from fitting the model discussed in the text with the results of $n = 3$ spectroscopy experiments.

more isotropic cloud where the transverse velocity distribution is of the same order of magnitude of the axial velocity distribution.

Comparing the laser bandwidth of $\sigma_3 = 2\pi \cdot 48 \text{ GHz}$ and the total Doppler broadened Ps linewidth of $k\sigma_v \approx 2\pi \cdot 470 \text{ GHz}$, the excitation efficiency is expected to reach up to 10% because of the convolution of the two spectral distributions. As a consequence, the main limitation of the present setup for efficient Ps excitation comes from either the limited laser bandwidth compared or the broad Doppler profile.

As a final result, the photoionization cross-section by IR photons found from the fit is $\approx 3.1 \cdot 10^{-17} \text{ cm}^2$, in good agreement with the theoretical value of $3.5 \cdot 10^{-17} \text{ cm}^2$.

4.6 Positronium excitation to Rydberg levels

A follow-up measurement campaign, conducted right after the completion of 3^3P excitation experiment, was conducted to demonstrate Rydberg-states excitation of o-Ps using 3^3P as intermediate level, as proposed by AEgIS to be used for antihydrogen production.

4.6.1 Experimental

The experimental setup is the same as for the 3^3P photoionization experiment, with the only difference of the photoionization IR laser, which was replaced by the mid-IR 1650 – 1700 nm for $3 \rightarrow \text{Ry}$ excitation. The UV wavelength was set on resonance of the $1^3S \rightarrow 3^3P$ transition ($\lambda = 205.045 \text{ nm}$, $E = 54 \pm 5 \mu\text{J}$) and monitored hourly to compensate eventual temperature drifts. The second laser wavelength was scanned in the interval associated to the levels $n = 15 - 18$, i.e. 1686 – 1712 nm, keeping its energy constant at $E_r = 1.1 \pm 0.1 \text{ mJ}$.

The Rydberg excitation signal was obtained averaging 15+15 SSPALS spectra with/without excitation laser in a leapfrog scheme to compensate eventual secular drifts in the number of positrons by the accumulator. The S parameter was calculated on the averaged spectrum in the time window 50 – 250 ns and 300 – 600 ns, since Ps atoms flying in the chamber with an approximate velocity of 10^5 m s^{-1} should reach the first obstacle (4 – 6 cm from the target) about 400 – 600 ns after the prompt peak.

For conducting this experiment, the UV transfer line with DUV mirrors was modified as discussed in 3.8, sending both laser pulses on a dispersive prism to superimpose the optical paths before the transfer line to the experimental chamber. It's important to underline that doing so, the polarization of the lasers was slightly altered, from a pure π -pulse to a mixed $(1/3\sigma + 2/3\pi)$ -pulse as the polarization changed from linear to elliptical. The biggest complication of adding the mid-IR laser pulse was introducing the spectrum diagnostics on the IR line, since the LiNb doubling crystal had to be inserted in the line every time a spectrum or alignment measurement had to be performed. This problem was avoided by using a very stable mirror flip mounts for both the doubling crystal and the wavemeter mirror.

The (rather complex) experimental procedure used to acquire a set of 15+15 shots was the following:

1. Assuming the IR laser to be stable and optimized in the previous state, the initial wavelength was measured by inserting the doubling LiNb crystal and the wave meter flip mirror. The phase-matching of the LiNb was adjusted maximizing the second harmonics energy yield (to avoid introducing a systematic due to the non-infinite acceptance bandwidth of the doubling crystal).
2. Varying the temperature on the OPG of the IR line, wavelength was set half-distance away from the destination value.
3. Then, the phase-matching of the OPA crystals was optimized maximizing the energy yield. Doing so, the peak of maximum gain was moved under the hood by the second half-distance needed to get to the destination value.
4. Again, the spectrum was measured by inserting flip mirror and LiNb crystal and adjusting its phase-matching.
5. After a good energy/wavelength was found on the IR line, UV energy and spectrum was verified and, in case, corrected (usually once/twice per day).
6. Alignments of the two lines were checked on the MACOR screen. To verify the IR alignment, again the doubling crystal was introduced in the optical line to bring the mid-infrared in the wavelength range detectable by the alignment camera.

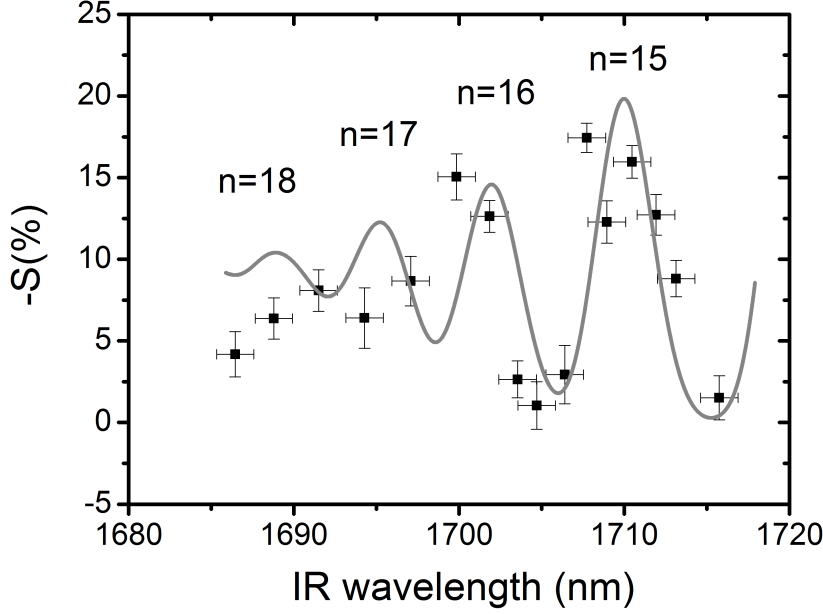


Figure 4.23: Wavelength scan of Ps $n = 15 - 17$ Rydberg levels obtained by varying the IR laser wavelength at constant energy and constant UV, 3^3P laser. Each point has been obtained by computing the S signal on a sample 15+15 SSPALS signal/background spectra. Statistical errors (on y axis) and total spectrometer uncertainty (statistical + systematic on the x axis) are reported. The continuous line is the theoretical prediction of the PsTC code rescaled to the height of the $n = 15$ peak.

7. Energies of the UV and IR line were verified at the entrance of the experimental chamber and, in case, corrected by acting on the half-wave plates on the two pump line of the OPA systems.

4.6.2 Discussion

The resulting S signal as a function of the IR wavelength is shown in Figure 4.23.

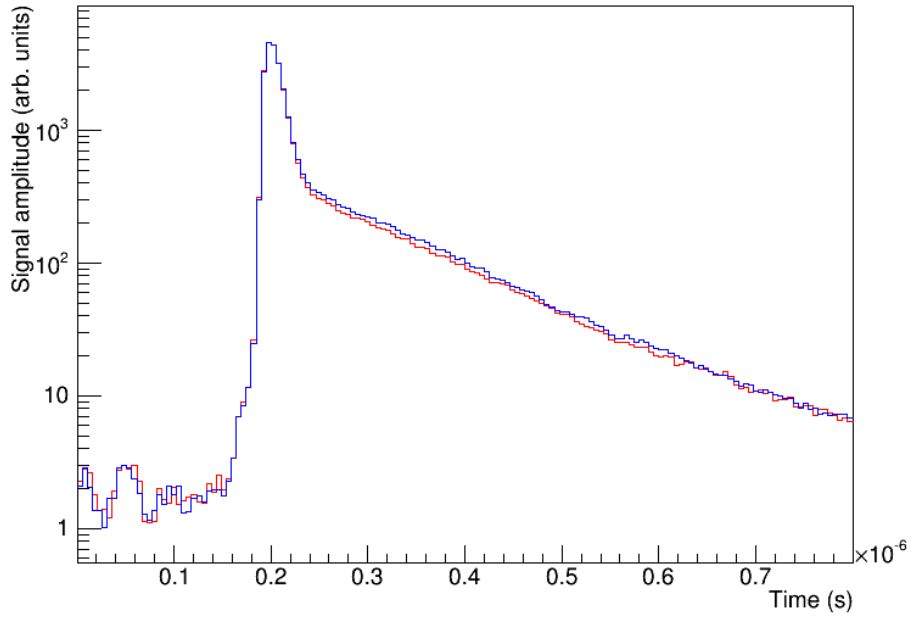
Despite the significant broadening of Rydberg lines, the $n = 15$ transition was observed clearly isolated. For higher states, different n manifolds start to overlap in a continuum of energy levels, as expected [45] and observed also in [30]. Several contributions cause such a broadened spectrum: the already-observed $k\sigma_v \approx 2\pi \cdot 470$ GHz Doppler-broadening; the contribution from the Stark effect due to the 600 V m^{-1} electric field of the last lens; the Ps-specific contribution of motional-Stark effect in the external field of 250 G. The resulting $n = 15$ central wavelength

has been determined to be

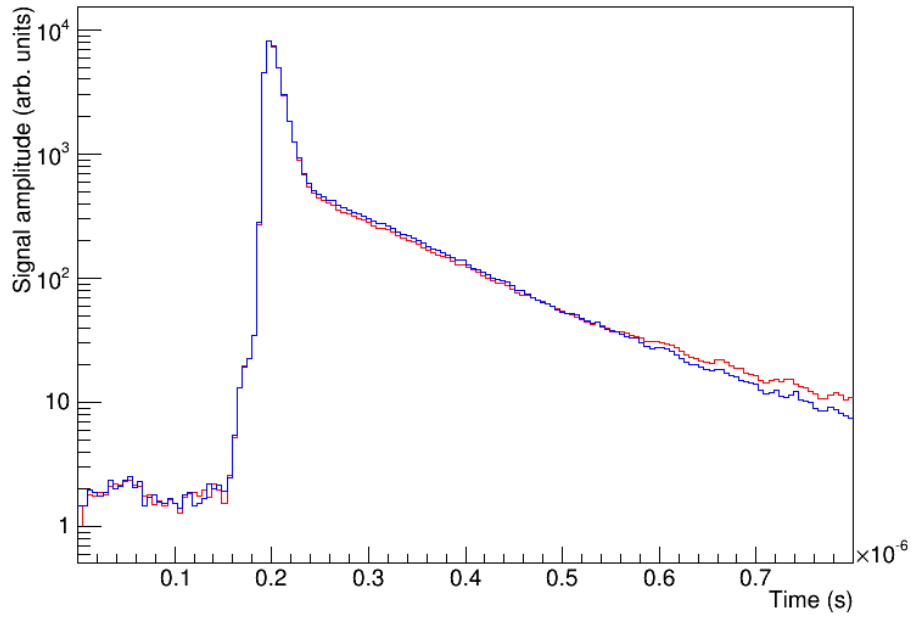
$$\lambda_{15}^{\text{exp}} = 1710.0 \pm 1.14_{\text{sys}} \pm 0.37_{\text{stat}} \text{ nm} \quad (4.6.1)$$

This result is out from the zero-field excitation line central value for $n = 15$, expected to be at 1708.63 nm, but still within estimated experimental errors (for an accurate comparison of this experimental data to a theoretical Ps spectrum, see 8.2.4).

A final comparison of all averaged photoionization and Rydberg SSPALS spectra is shown in Figure 4.24 for later use). An estimate of the Ps atom velocity in the direction of the magnetic field was obtained from the time-of-flight observed in average Rydberg spectra Figure 4.24, panel (b), knowing the position of the first obstacle on the flight path (the last lens of the buncher, placed 5.6 cm away from the target). The retarded annihilation signal due to Rydberg atoms starts being observed 400 ± 50 ns after the prompt peak, corresponding to an axial velocity of $v_{\text{Ps}} = (1.38 \pm 0.12) \cdot 10^5 \text{ m s}^{-1}$.



(a) $1 \rightarrow 3 \rightarrow \text{Ionized}$



(b) $1 \rightarrow 3 \rightarrow \text{Rydberg}$

Figure 4.24: Comparison of averaged SSPALS spectra obtained in all photoionization runs (panel a) and al Rydberg spectroscopy runs (panel b). Each panel shows the comparison of the averaged measurements without laser (blue curve) and with laser (red curve).

Chapter 5

Positronium formation in AEgIS

After the successful milestone achievement of producing and exciting Ps to Rydberg states in the spectroscopy chamber (described in the last chapter), the next objective on the way to pulsed $\bar{H}$ formation was forming and detecting positronium in the antihydrogen production region inside the main experiment.

The history beyond this achievement was not as linear as for other results presented in this thesis, as a change of the experimental strategy was introduced. The original method envisaged by AEgIS to form positronium [71] consisted in ballistically transfer positrons to the main traps, move them off-axis through autoresonant excitation of the plasma diocotron motion, recapture in an off-axis trap that could be biased to high-voltage and extraction on the target.

A much simpler strategy than the original scheme was found along the way. By introducing a long acceleration electrode in the last section of the positron magnetic transfer line, e^+ were sent directly on the target at keV energies without the need of retrapping in the main traps (thus the name *direct injection*). This method made use of the possibility of storing high number of positrons in the magnetic accumulator and the experience gained in in-flight acceleration of e^+ bunches (see 4.3).

The chapter was thus organized as follows. The shared methodology between the two strategies, i.e. positron magnetic transferring, is described in the first section of the chapter. Then, the results obtained following the original way of re-trapping and manipulating e^+ plasmas in AEgIS main traps are summarized, as some achievements (such as e^+ accumulation in high numbers) could be of use for future experiments. Then, the new experimental strategy is presented together with its results.

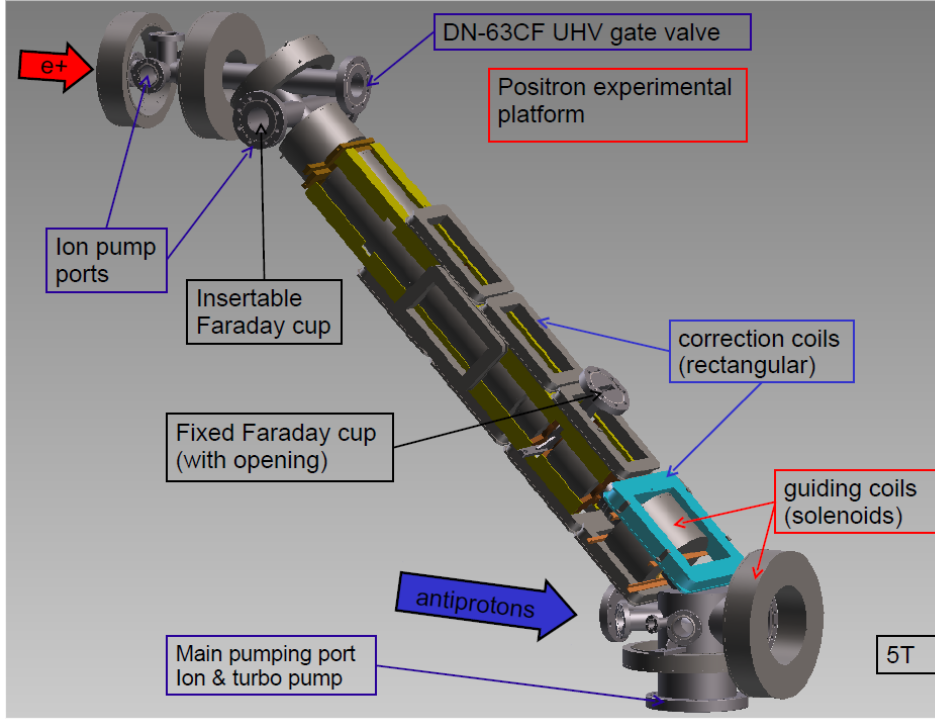


Figure 5.1: Technical drawing of $AE\bar{g}IS$ positron transfer line. From [108].

5.1 Positron transfer to the main traps

The aim of this section is to describe the part of the positron transfer procedure from the magnetic accumulator (described in 4.1) to the main experiment which was not significantly altered by the adoption of the direct injection scheme. Results presented here were obtained during $AE\bar{g}IS$ first commissioning run with e^+ in 2014, and were perfected during the following two years.

5.1.1 Transfer line

The main $AE\bar{g}IS$ positron transfer line (sketched in Figure 5.1), thoroughly described in [108], is 45° tilted, about 2.5 m long and consists of a sequence of coils producing the guiding magnetic field between 0.05 T and 0.25 T. A series of correction coils in Helmholtz-racetrack configuration allow the beam to be steered through the entire length and, most importantly, at the extraction point. A set of detailed simulations performed with OPERA[©] were used to design the transfer line and optimize the currents for optimal transfer [109]. An high-vacuum level of 10^{-10} mbar was maintained during positron transfer operations, and an automatic opening gate valve is installed at the end of the tilted region separating the vacua, in order not to continuously pollute the ultra-high vacuum of $AE\bar{g}IS$.

5.1.2 Positron transfer diagnostics

Positrons were measured during the setting up of the transfer procedure by letting them annihilate at different z -positions along the transfer line and using scintillators to evaluate their relative numbers.

The principal detectors used for this purpose were two movable CsI:Ti scintillators coupled to photodiodes (which could be positioned in different places along the transfer line to determine if losses were present) and some of the 12 AEGIS plastic scintillator slabs around the main cryostat. For those selected slabs, a second 1 ns acquisition chain digitizing the analog waveform output of the PMTs was setup in parallel to the coincidence acquisition chain.

Two PMTs were selected: one at the entrance of the experiment (PMT3) for monitoring annihilations on the degrader structure, and one above the 1TMCP (PMT20) for monitoring annihilations at the end of the experiment.

The acquisition chain was setup as follows: the two free $50\ \Omega$ outlets of PMT3 and PMT20 were connected to a 10 dB calibrated attenuator, routed through a Phillips linear fanin/fanout module to limit the maximum amplitude of the signal ($\pm 3\text{ V}$), connected to a 4x2 high-bandwidth multiplexer where the analog signals were distributed to a National Instruments PXI5152, 300 GHz, 1 Gsample digitizer which featured two channels with 8bit resolution and 256 Mbyte of onboard memory for long timed acquisitions.

The digitizer was set to acquire in DC with $50\ \Omega$ termination to ground for $20\ \mu\text{s}$ with 1 ns resolution in an amplitude window of -3.8 V to 0.2 V . The trigger instant was set at 55 % of the acquisition window to acquire the baseline on the same time interval of the signal. The second half of the resulting digitized waveform was processed subtracting the average of the first half (assumed to be background), and the error of the average was propagated as an estimate of the noise floor. The total charge measured by the scope (usually expressed in pC) was thus calculated by integrating the background subtracted signal in time, according to

$$Q_{10\text{ dB}} = \frac{10^{12}}{50\ \Omega} \int V(t) dt \quad (5.1.1)$$

5.1.3 Absolute calibration of the number of positrons

The reconstructed charge $Q_{10\text{ dB}}$ is, in good approximation, proportional to the number of annihilating positrons N_{e^+} , until the scintillation material is far from saturation and its photomultiplier operates in the linear regime (a reference measurement of the saturation behaviour of PMT20 is shown in Figure 5.12).

Different strategies can be adopted to reconstruct N_{e^+} from the measured value of $Q_{10\text{dB}}$. One is to work out (either by means of Montecarlo simulations or by experimental determination with radioactive sources) the average charge observed from a single 511 keV γ originated at the annihilation position. Another is to cross-calibrate the detector with an independent estimate of the absolute number.

The calibration described here has been performed a-posteriori using the electrical pickup signal acquired on a long cylindrical electrode installed during the technical modifications (see 5.3), which provided an independent absolute reference of the number of particles.

Pickup electrodes are a common tool used in accelerator physics as beam non-destructive diagnostics [110]. Its principle of measurement is based on Shockley-Ramo's theorem, which established a relation between the instantaneous current I induced on an electrode of arbitrary geometry by the motion of a charge q at velocity $\vec{v}(t)$ and position $\vec{r}(t)$

$$I(t) = -q \vec{v}(t) \cdot \vec{\nabla} W(\vec{r}(t)) \quad (5.1.2)$$

where $W(\vec{x})$ is the so-called weighting potential, i.e. the electrostatic potential induced by the electrode when removing the flying charge, setting the potential to 1 V on the electrode surface S and all other conductors to ground.

$$\nabla^2 W(\vec{x}) = 0 \quad W|_S = \delta \quad (5.1.3)$$

This theorem is more powerful than it looks at first glance, and allows the total charge Q of a particle cloud to be measured independently of the pickup electrode geometry, the particles' velocity and their time distribution. First, it can be simplified to a monodimensional problem by neglecting the radial components of velocity $v_z \gg v_r$

$$I(t) = -Q v_z(t) \frac{dW}{dz}(z(t)) \quad (5.1.4)$$

Performing an integration on time between $-\infty$ and t_0 , assumed to be the instant at which the particles are sitting in the center of the electrode, gives

$$\int_{-\infty}^{t_0} I(t) dt = -Q \int_{-\infty}^{t_0} v_z(t) \frac{dW}{dz}(z(t)) dt \quad (5.1.5)$$

Substituting the definition of velocity $v_z(t) = dz/dt$ and changing the integration variable

form t to $z(t)$ yields

$$\begin{aligned}\int_{-\infty}^{t_0} I(t)dt &= -Q \int_{-\infty}^{t_0} \frac{dW}{dz}(z(t)) \frac{dz}{dt} dt \\ &= -Q \int_{-\infty}^{z_0} \frac{dW}{dz} dz \\ &= -Q [W(z_0) - W(-\infty)]\end{aligned}\tag{5.1.6}$$

where $z_0 = z(t_0)$ is the center of the electrode, and particles are assumed to start their flight at $z = -\infty$. The final relation Equation 5.1.6, derived here in the monodimensional case, is in fact more general and valid also in the full 3D case (for a throughout derivation in the full 3D case, see [111] and [112, chap. 5]). $W(-\infty)$ is customary set to 0 by defining the ground potential at the borders of the integration domain. All the geometrical dependence of the formula is absorbed within the value of the weighting potential at the center of the electrode, $W(z_0)$, which for the case of a long cylinder is ≈ 1 with very good approximation. Thus

$$\int_{-\infty}^{t_0} I(t)dt = -Q\tag{5.1.7}$$

The integral of $I(t)$ can be measured by digitizing the voltage drop on a known resistor with a fast oscilloscope and performing the integration offline. Neither the weight potential W nor the particles velocity v_z nor the exact electrode geometry enter in the calculation of to the final value of Q . Contributions from the weight potential W has to be considered only when extracting time informations from $I(t)$.

An acquisition chain of the pickup electrode was set up as follows (see Figure 5.3, panel (a)). The electric signal from the electrode was connected through an impedance-matching voltage divider to the input of a 50 Ω ZPUL-30P wideband amplifier from Minicircuits with minimum 29 dB linear gain from 2.5 kHz to 700 MHz ¹. The amplified signal was subsequently acquired using the same PXI5152 300 MHz 1 Gsample digitizer used form PMT20.

The absolute number of positrons was thus obtained using Equation 5.1.7

$$Q = e_0 N_{e^+} = -\frac{1}{G R_0} \int_{t_0-35 \text{ ns}}^{t_0+35 \text{ ns}} V(t)dt\tag{5.1.8}$$

where G is the V-V gain of the amplifier (~ 19), $R_0 = R_1 R_3 / (R_1 + R_2 + R_3)$ is the reading resistance ($\sim 20 \Omega$), t_0 is the pickup signal peak and 35 ns is half the flight time of 270 eV e^+ in a 70 cm tube.

¹the actual measured value of the amplifier gain is 34.5 dB, corresponding to a V-V gain of $\times 54$. This measurement was unavailable at the time of writing of this chapter; all plots were thus obtained with the previous gain of 29 dB as declared by the manufacturer, and should be rescaled by a factor of 2

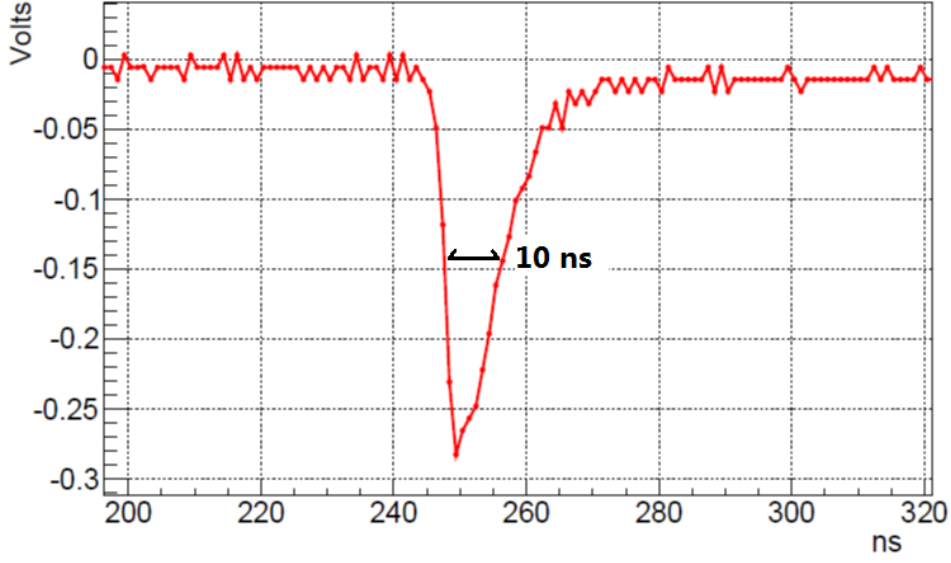


Figure 5.2: Time distribution of the e^+ bunch at the drift tube entrance, as measured from a fast NE111 plastic scintillator. The observed FWHM is ~ 10 ns

A set of runs contemporary acquiring the absolute charge on the kicker tube and the annihilations signal on the PMT20 (an example of such signals is shown in Figure 5.3) were acquired in order to work out the cross-calibration factor.

$$Q_0^{-1} = (497 \pm 43_{\text{stat}}) \text{ pC}^{-1} \quad (5.1.9)$$

All the numbers of e^+ quoted in this chapter were obtained with

$$N_{e^+} = Q_0^{-1} \left(\sqrt{10} \cdot Q_{10 \text{ dB}} \right) \quad (5.1.10)$$

A further consistency check was performed using Shockley-Ramo's theorem in its monodimensional time-depending form (Equation 5.1.4) to calculate the theoretical time impulse response of the drift tube, in order to compare it to the observed time structure of the e^+ spill on a fast scintillator. The measured signal length on the drift tube had to be consistent with a bunch of e^+ flying at an average velocity v_z relative to their measured axial energy of 270 eV (see Figure 5.7, panel (b)).

A plastic NE111 scintillator coupled to an XP2020 photomultiplier was used to acquire the reference time length of the e^+ spill at the drift tube entrance (see Figure 5.2), found to be ~ 10 ns FWHM.

The weighting potential $W(z)$ was calculated using the Sine-Hankel transform solver developed

to calculate accurate trap potentials, described in 9.3. The accurate geometry of the drift tube electrode and its outer vacuum chamber at ground potential were given as input to the code, setting all potentials on the conductors to ground and the drift tube potential to 1 V ². The resulting calculated $W(z)$ profile is shown in Figure 5.4, panel (b). The time impulse response was then promptly obtained from Equation 5.1.4 calculating the discrete derivative of $W(z)$ on the axis of the electrode and substituting the value of v_z associated to 270 eV.

The comparison between an example measured signal with the calculated time impulse response is shown in Figure 5.4, panel (a). The theoretical curve has been shifted and normalized to the signal peak value for better clarity. The signal shape is found in well consistent to the predicted time shape from Shockley-Ramo's theorem. The measured signal length is - as expected - slightly longer than the theoretical impulse response, as a consequence of the time distribution of the e^+ spill of ~ 10 ns that broadens the pick-up signal. The two rising edges are almost identical, given that some pickup noise from an unknown source is superimposed to the signal in the bottom part.

5.1.4 Magnetic reflection

A transition between low to high magnetic fields, such as the one between the end of the transfer line and the main AEgIS 4.5 T field, introduces the risk of partial/total magnetic reflection. A fraction of positron could loose all axial kinetic energy and be reflected back towards the accumulator, thus remaining trapped between a long, spurious trap between the magnetic mirror and the last endcap electrode inside the accumulator.

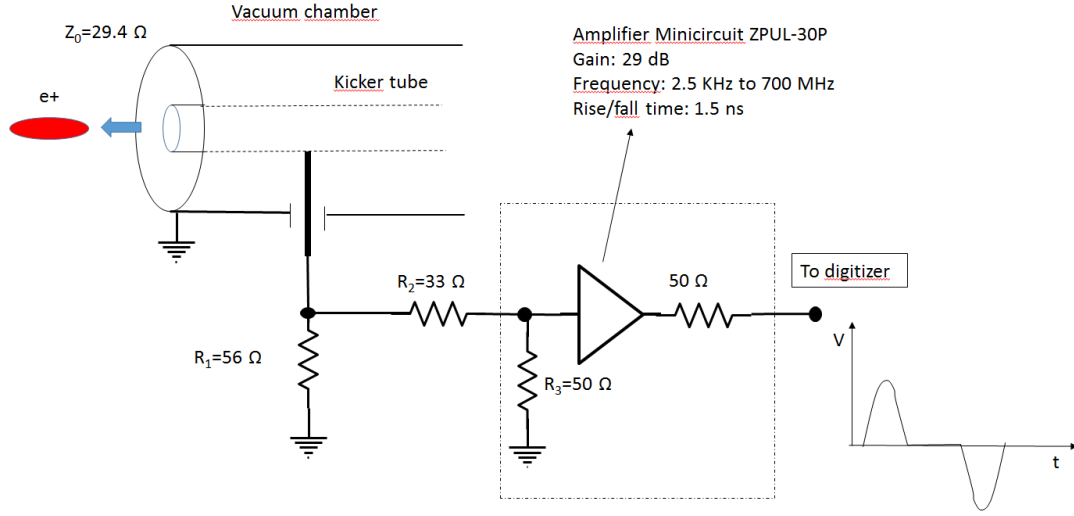
The occurring of magnetic reflection was both modeled theoretically and measured experimentally to properly choose the axial transfer energy of the e^+ spill.

Two quantities are conserved during particle motion in a non-homogeneous magnetic field: the total energy of the particle E and its magnetic momentum μ (see [113, chap. 12, sec. 5])

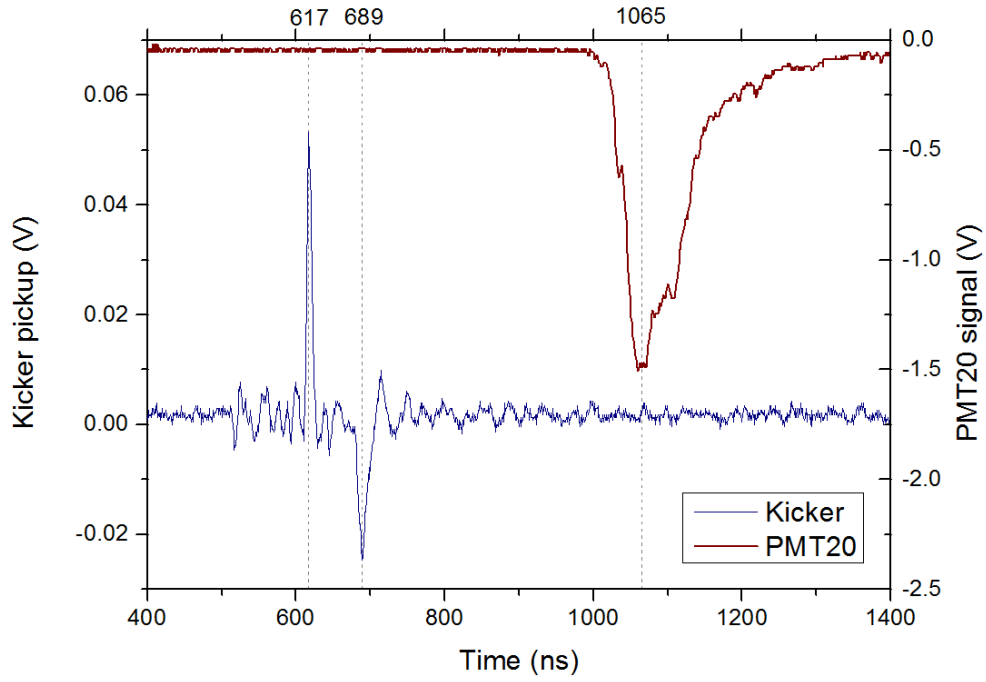
$$\begin{aligned}\mu &= \frac{mv_{\perp}^2}{2B} \\ E &= qV(z) + \frac{1}{2}mv_{\parallel}^2 + \frac{1}{2}mv_{\perp}^2\end{aligned}\tag{5.1.11}$$

where $V(z)$ is the potential along the axis, $v_{\perp}$ and $v_{\parallel}$ are the velocity components perpendicular and parallel to the magnetic field, B' is the entering field 4.5 T and B is the accumulator magnetic field 0.1 T. Denoting V_0 the potential in the center of the accumulator trap and with

²see 5.3.1 for details about the dimensions

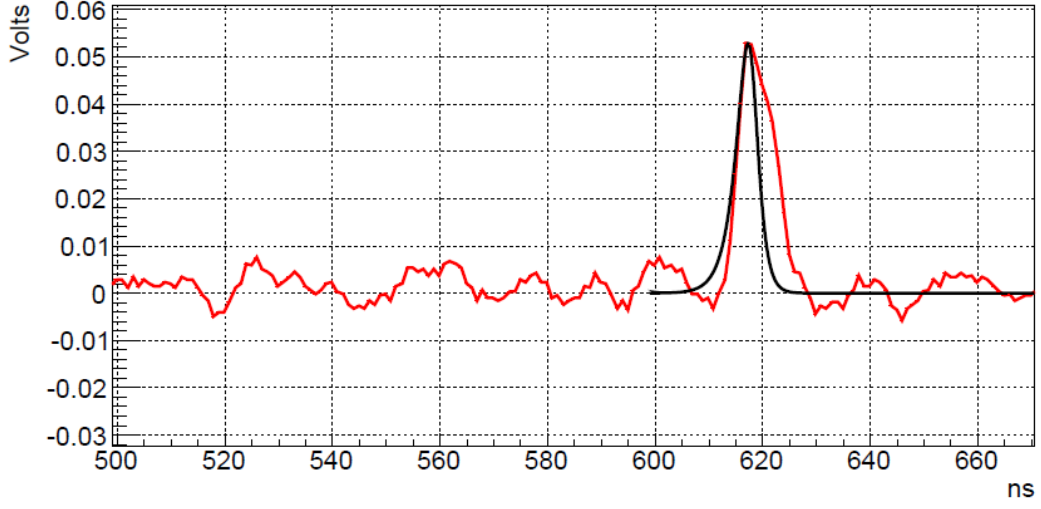


(a) Acquisition chain scheme for the drift tube pick-up electrode

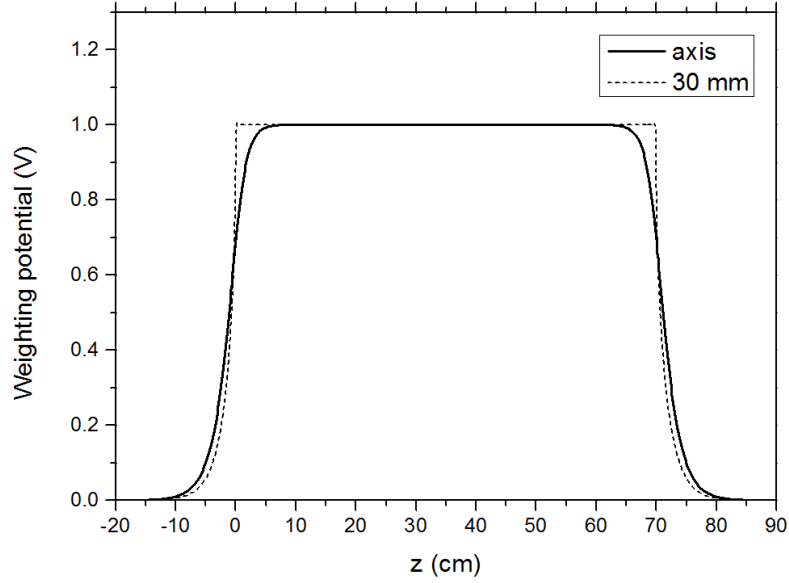


(b) Comparison of contemporary e^+ signals on drift tube (blue) and PMT20 (red).

Figure 5.3: Cross-normalization of PMT20 signal on the electrical pick-up signal of a long drift tube for e^+ acceleration. Panel (a) shows the acquisition chain used to detect the signals, panel (b) compares the signal on the drift tube tube and the PMT20.



(a) Comparison of the measured pick-up signal (red) with the theoretical time impulse response of a single charge flying through the drift tube at 300 eV (black). The theoretical model has shifted and rescaled to the maximum value of the signal.



(b) Weighting potential $W(z)$ of the drift tube electrode (see Equation 5.1.2)

Figure 5.4: Comparison of the measured time structure of the e^+ bunch, measured by the drift tube, with the theoretical signal obtained from Shockley-Ramo's theorem 5.1.2 (panel a). The weighting potential $W(z)$ of the drift tube (panel b) was calculated using the Sine-Hankel transform method described in 9.3.

at the quantities related to the final position, the following relations hold:

$$\begin{aligned} \left(\frac{v'_{\perp}}{v_{\perp}}\right)^2 &= \frac{B'}{B} \\ v_{\parallel}^2 + v_{\perp}^2 &= \frac{2q}{m}V_0 + v_{\parallel}^2 + v_{\perp}^2 \end{aligned} \quad (5.1.12)$$

Combining the two and rearranging the terms to expose the axial velocity at the trap entrance

$$v_{\parallel}^2 = \frac{2q}{m}V_0 + v_{\parallel}^2 + \left(1 - \frac{B'}{B}\right)v_{\perp}^2 \quad (5.1.13)$$

Particles start getting reflected for $v_{\parallel}^2 = 0$, while are still transmitted for $v_{\parallel}^2 > 0$. Defining $E_{\parallel} = qV_0 + 1/2mv_{\parallel}^2$ as the initial axial energy and $E_{\perp} = 1/2mv_{\perp}^2$ as the initial radial energy at the exit of the accumulator and imposing the total reflection condition $v_{\parallel}^2 = 0$ to the formula above, an expression for the energy ratio is obtained

$$\frac{E_{\perp}}{E_{\parallel}} = \left(\frac{B'}{B} - 1\right)^{-1} \quad (5.1.14)$$

Experiments conducted with $2.0 \cdot 10^6$ e^+ , ramping the extraction energy of positrons V_0 in the range 150 - 300 eV showed that when positrons were ejected from the accumulator with an energy lower than 225 eV, transfer efficiency started to reduce, coming to a complete reflection of the positron beam for $V_0 \approx 175$ eV. Direct observation of spurious annihilation signals around the transfer line at later times confirmed the hypothesis that e^+ stay trapped within a long spurious trap. Results of a measurement in the number of positrons as a function of the axial transfer energy are shown in Figure 5.5.

For these values of V_0 the kinetic energy term in the definition of the axial energy can be neglected $E_{\parallel} = qV_0 + 1/2mv_{\parallel}^2 \approx qV_0$, as the axial energy inside the plasma is well below V_0 (the maximum trap depth is 14 V). Thus, substituting into 5.1.14, the radial energy can be evaluated

$$E_{\perp} = \left(\frac{B'}{B} - 1\right)^{-1} qV_0 \quad (5.1.15)$$

The interval in which magnetic mirroring is partially to totally observed, 175 – 225 eV, maps to an initial radial energy of

$$E_{\perp}^{0.1T} = 3.9 - 5.1 \text{ eV} \quad (5.1.16)$$

This measured $E_{\perp}$ interval is of the same order of magnitude of the accumulator trap potential depth ~ 14 eV. The eventuality that $E_{\perp}$ could be related to the number of accumulated

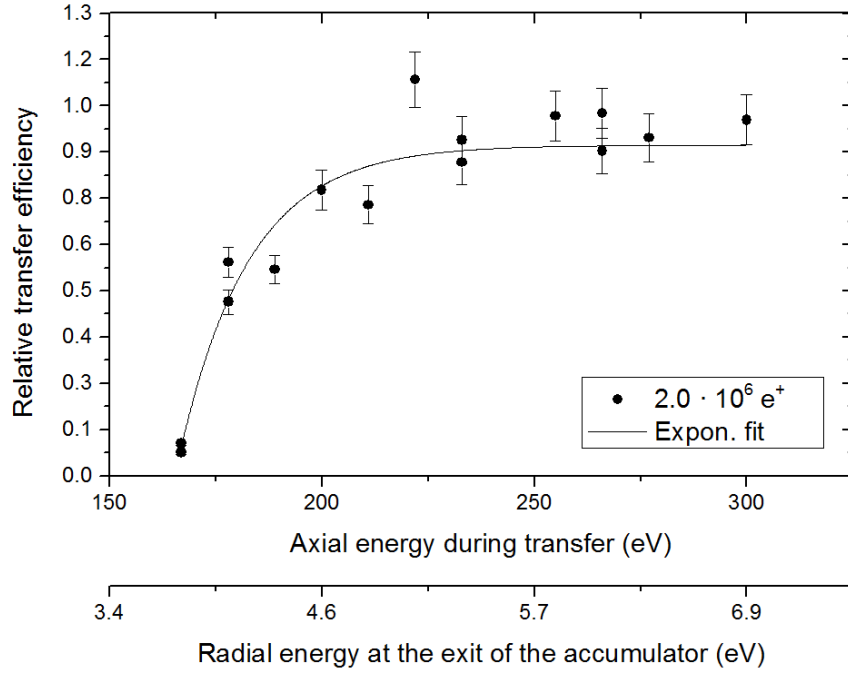


Figure 5.5: Observation of the magnetic mirroring effect at the junction between transfer line and main 4.5 T AEgIS field. Relative number of transmitted positrons, as observed by PMT20, as a function of the axial energy V_0 (as described in the text). Positrons are all reflected with $V_0 < 175$ eV. Exponential fit is a guide for the eye.

positrons, and that the observed saturation behaviour of the number of stored positrons in the accumulator (see Figure 4.3) could come from a too shallow potential depth of the harmonic potential, in view of this measured values of $E_{\perp}$, should be considered for future experiments with high numbers of accumulated positrons ³.

This radial energy could come from potentially three sources:

1. a radial component of the accelerating electric field used to launch e^+ , which could be estimated by an accurate calculation of the trap potential at the moment of the launch;
2. an initial plasma temperature of the order of $\sim eV$, which is possible as the RW drive is constantly applied in the accumulator and cooling goes only through collisions with the buffer gas;
3. a fraction of the cloud space-charge potential transferred to the radial degrees of freedom by the Coulomb expansion during the flight.

In conclusion of this section, the positron ejection potential V_0 was set to the maximum allowed setting of 300 eV to prevent mirroring effects.

5.1.5 Steering and characterizing the spill

Positrons were let fly through the traps to annihilate on the surface of the 1TMCP to image their spatial distribution and properly align the spill. e^+ were positioned on the observed electron plasma center of the C-trap, designated as the optimal capture region as it is slightly longer than the P-trap. An example of positron beam steering is shown in Figure 5.6.

Following proper alignment of the beam, the total charge measured by the PMT20 could be adopted as a measurement of the number of positrons, as the beam was not chopped anymore and annihilations were all occurring all at the same z position.

The positron spill was subsequently characterized (see Figure 5.7, panel (a)). The precise timing of the bunch arrival at the traps entrance and its time length were measured by applying a positive 1 kV pulse applied on the HV1 electrode, which could be varied in time by steps of 12.5 ns. The number of transmitted positrons were measured by means of PMT20. The entire (5-95 %) positron bunch was contained within an interval of 50 ns, whereas the arrival time on electrode HV1 with respect to the accumulator trigger was found 512 ns.

The axial velocity and energetic spread of the spill were measured by setting a bias voltage on the HV2 electrode used as barrier. Similarly, the number of transmitted positrons were

³the number of positrons used to perform this measurement was about half the maximum number that could be stored in the accumulator during this measurement day.



Figure 5.6: Example of positron beam steering on the 1TMCP, acquired with different current settings on the last vertical and horizontal correction coils of the transfer line. The cut on the lower right edge is the border of OnAx1, used as visual reference for beam movements.

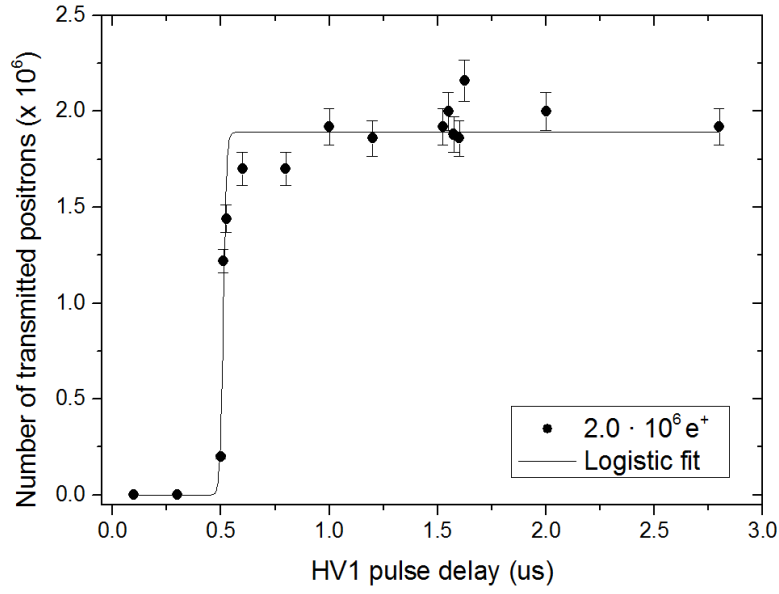
measured by means of PMT20. Results, shown in Figure 5.7, panel (b), are consistent with a launch energy of about 300 eV and an overall energy spread of about 50 V. This energy spread is related to the plasma length inside the accumulator and the steepness of the applied parabolic potential profile to extract the spill from the accumulator.

5.2 Positronium formation via plasma manipulations

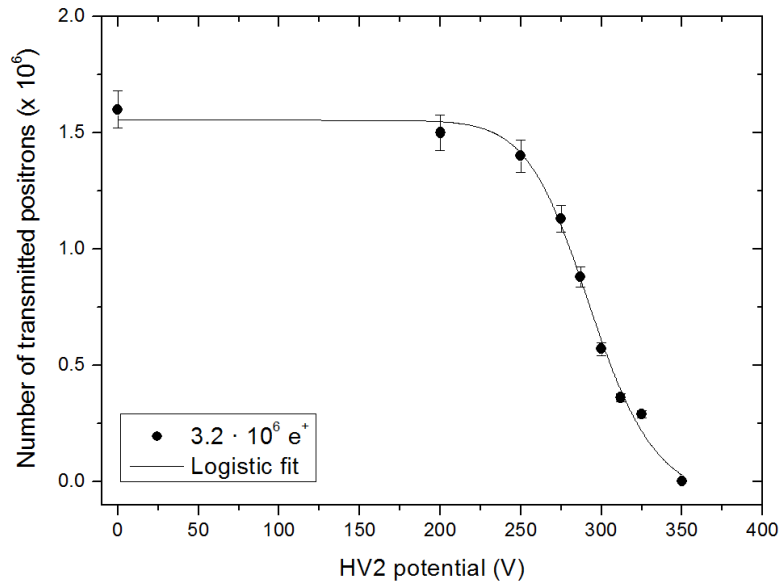
The original experimental strategy envisaged for forming Ps inside AEgIS [71] consisted in a long sequential list of trap operations:

1. positrons accumulation and transfer through the magnetic transfer line to the 5T traps;
2. e^+ re-trapping, cooling and recompression using rotating-wall;
3. adiabatic transfer to the 1T Big trap;
4. off-axis radial displacement by means of autoresonant excitation of the plasma diocotron mode;
5. synchronous launch during diocotron and ballistic recatch in the off-axis trap;
6. lift of the bottom trap potential to kV, and ejection of plasma on the target.

This section goes sequentially through this list up to autoresonant diocotron excitation, which was demonstrated on electrons but not attempted with positrons during the early 2015 positron run, when these measurements were collected. The necessity of performing positronium spectroscopy and excitation to Rydberg levels before the beginning of 2015 antiproton run set the limit to the available time for performing e^+ experiments in the main traps.



(a) Time distribution at HV1



(b) Energy distribution at HV2

Figure 5.7: Positron spill characterization: (a) time distribution at the traps entrance, as measured by applying a 1 kV positive pulse on HV1 electrode and observing the transmitted number of e^+ on PMT20, (b) positron energy distribution. Logistic fits are guides for the eye.

Working with e^+ inside the main traps became also very time consuming around the end of the positron run two concurrent reasons: an hardware misalignment between 5T and 1T traps, which made transfer procedures more difficult when the number of positrons was low, and a reduced availability of positrons experienced when the magnetic fields of AD and the other experiments were turned on.

Both problems were later solved, the former by two trap alignment campaigns using both particles and an in-house built interferometric optical alignment tool, the latter by encasing the entire structure of the positron source, Surko trap and accumulator (described in 4.1) in a mu-metal box for shielding external magnetic field disturbances.

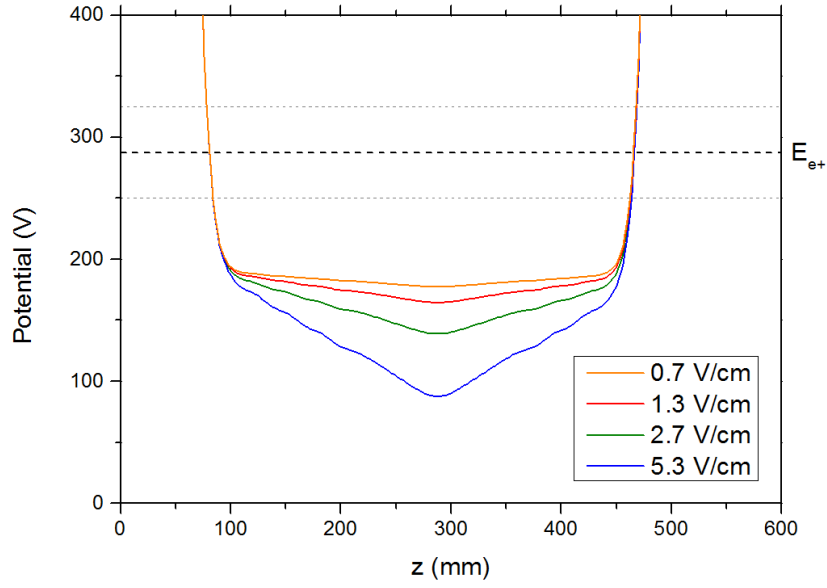
5.2.1 Positron capture

After the arrival time of e^+ was determined (see Figure 5.7), the positron spill was captured in the C-trap region of the 5T traps, i.e. the trap region at the entrance of the 5T magnet composed of 7 electrodes two radii long and a central region of 10 electrodes one radius long (see 3.4 and Figure 3.7 in particular). The trapping procedure consisted of biasing the HV2 high-voltage electrode between the C-trap region and the P-trap region (see Figure 3.7) to positive 1 kV, and fast-switching the HV1 electrode at the entrance of the C-trap to the same potential using a fast 5 kV switch from Behlke.

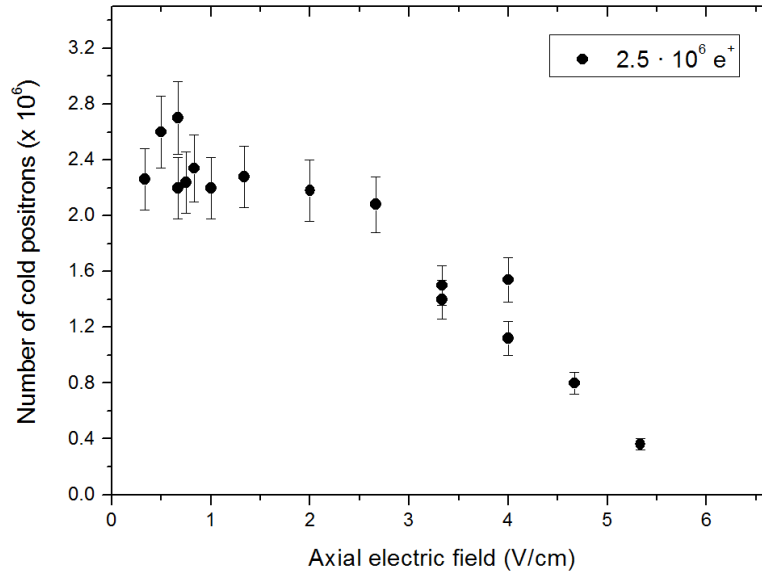
Before triggering the positron accumulator to send the e^+ bunch, a symmetric double-linear potential profile was shaped over the entire length of the trap (electrodes from C1 to C19) to collect all positrons under the two rotating-wall electrodes at the center of the short electrodes region (electrodes C10 and C11) (see Figure 5.8, panel (a)). The entire trap was biased to maximum setting of the external electronics, 190 V, to reduce as much as possible the axial energy of the incoming 270 eV e^+ .

A study of the cooling trap potential profile was conducted in order to establish the optimal steepness of the cooling trap to collect all e^+ under the central rotating-wall electrodes. The steepness was varied by adjusting the bias voltages according to their relative electrodes length, in order the potential on the axis to best approximate a linear trend, while keeping the two extremal electrodes (C1 and C19) at the maximum setting of 190 V to help the e^+ bunch slow down (see Figure 5.8, panel (a)).

The timing of the trap closing pulse applied on HV1 was set at about 2/3 of the total e^+ bunch flight time between HV1 and HV2 and back. This precaution was taken in order to avoid introducing e^+ losses due to a bad combination of a too tight timing of the closing pulse combined with the different average axial energies of the e^+ bunch for the different cooling traps tested.



(a) On-axis potential of the e^+ cooling trap for different choices of the trap steepness.



(b) Cooling efficiency versus axial electric field

Figure 5.8: Positron trapping efficiency as a function of the axial electric field in the initial part of the double-linear cooling trap used to collect positrons.

Results of this study are summarized in Figure 5.8, panel (b). The overall cooling efficiency was constant for low values of the cooling trap steepness, then started decreasing as the steepness went above a threshold proportional to the initial positron density.

This is a consequence of both the higher energy e^+ had to dissipate to end in the bottom of the trap, as its depth was increased, and the reduced axial to radial energy transfer from collisions at higher plasma temperatures (see the discussion in 6.3 and the scaling of the collision rate in Equation 6.3.4 as $T^{3/2}$). The more the cooling trap depth was increased, the more this effect was pronounced, up to the point where radial expansion of the cloud start happening on a faster time scale than energy dissipation through collisions. As a consequence, a fraction of e^+ was left uncooled (mostly at high radii) and a smaller fraction was observed at the ejection of the cold fraction from the trap.

A steepness value of 1 V cm^{-1} was typically found as a good compromise for positron numbers in the range $10^6 - 10^7$.

5.2.2 Positron cooling efficiency

The trapping and cooling efficiency was measured by opening the low-voltage trap with a fast pulse towards the 1TMCP to measure the e^+ number on PMT20 and image the distribution at the same time. The efficiency showed saturation with a $\tau \approx 10 \text{ s}$ (see Figure 5.9, black). More interesting, also the plasma radius exhibited saturation on the same time scale (see Figure 5.9, white).

This behaviour was interpreted as a consequence of the low density of the transported e^+ plasma. The exchange of heat between axial and radial degrees of freedom in a plasma is governed by the Coulomb collisions among the particles (see 6.3.1 for details). If the plasma density is low, the heat exchange could happen on a slower time scale than radial energy dissipation through cyclotron radiation, which for a positron plasma in a 4.46 T field is 0.13 s (see Equation 6.3.4). For the same reason, the coldest point of a rarefied plasma in non-thermal equilibrium is expected to be in the center, where the density is at maximum.

The ability to effectively trap and cool positrons was strongly dependent by the number of particles in the transferred plasma. Typically, if the accumulated number of positrons reached $\sim 1.0 \cdot 10^6$ or below, the cooling time was observed in the region of the many tens of second, forcing the operator to work in the non-saturation regime of the cooling efficiency.

This made transfer and cooling procedures dangerously fluctuating, as any tiny change in the accumulator condition, such as the effectiveness of its rotating-wall or the age of the source moderator, varied the cooling efficiency in a very non-linear way. Among the various causes for

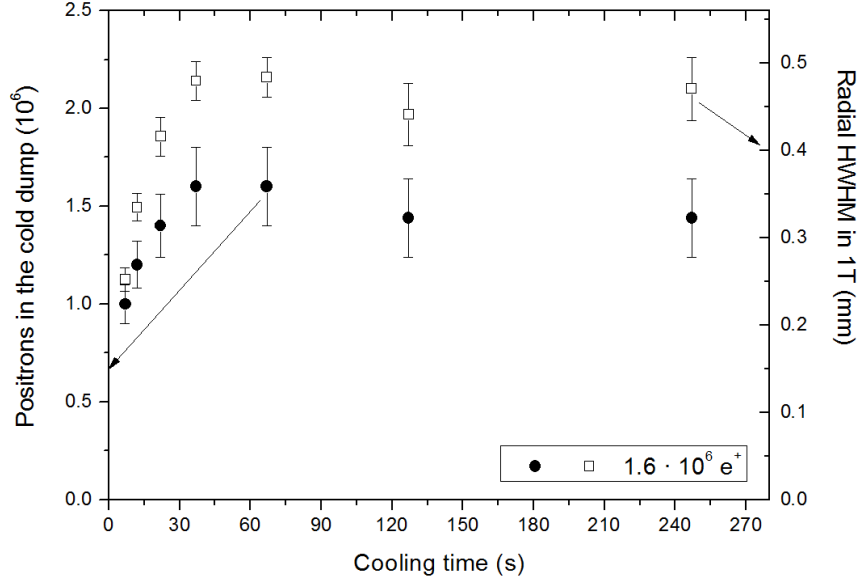


Figure 5.9: Positron cooling efficiency vs time (black dots) and positron cloud radius HWHM vs time (white dots).

accumulator instability, one that was clearly identified was the introduction of external magnetic field disturbances in the few G range.

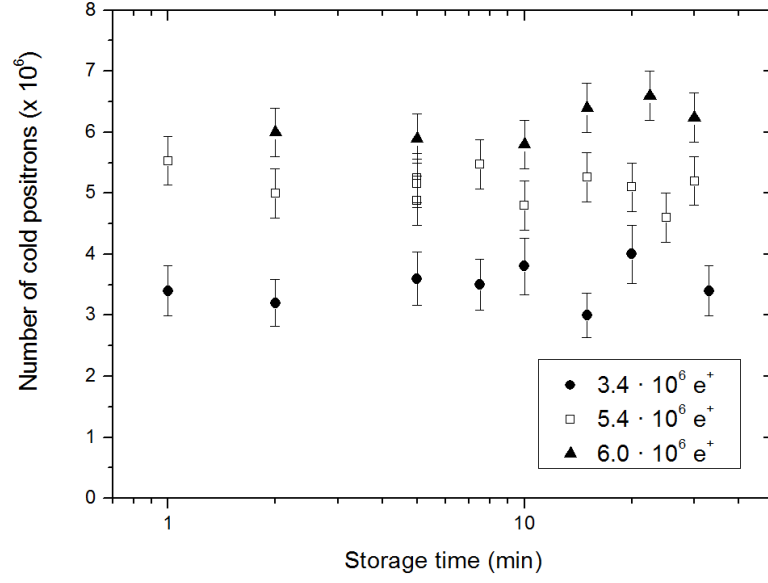
To avoid working in this unstable regime, at least $2.0 \cdot 10^6 e^+$ in the plasma were used, as well as 60 s of cooling time. Under these conditions (or better), trapping and cooling efficiencies close to 100% were normally observed⁴. The measurement was performed comparing the annihilation signals of cold e^+ plasmas ejected from the trap with direct transfers without trapping.

5.2.3 Positron storage, rotating-wall and stacking

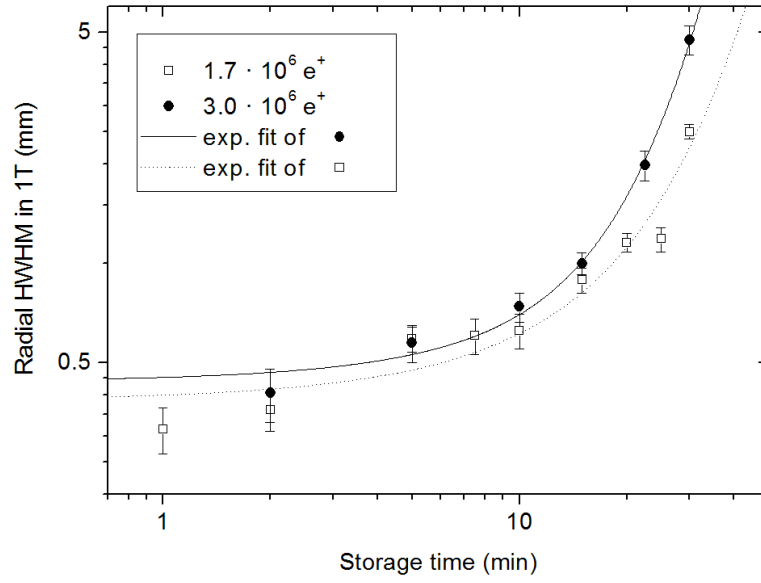
Positron plasmas could be stored in the UHV C-trap for ~ 30 min without observing any decrease in the number of particles, as shown in Figure 5.10, panel (a). Spontaneous expansion of the e^+ plasma was however observed following a roughly exponential growth on a time scale of $\tau \approx 10$ min, regardlessly of the number of particles in the cloud (Figure 5.10, panel (b)).

Rotating-wall (RW) was introduced to counteract this expansion and enable longer accumulation times. A rough scan in frequency within the interval 0.5 MHz - 4.0 MHz with a drive amplitude of 0.5 V applied for 60 s is showed in Figure 5.11. For frequencies ≥ 4.0 MHz, the plasma did not react anymore to the drive. The typical RW settings used for storing plasmas

⁴meaning that all positrons getting through the magnetic transition were effectively trapped



(a) Number of e^+ versus storage time in 5T C-trap



(b) Radius of the e^+ cloud versus storage time in 5T C-trap

Figure 5.10: Number of positrons (a) and plasma radius on 1TMCP (b) from the 5T C-trap as a function of the storage time.

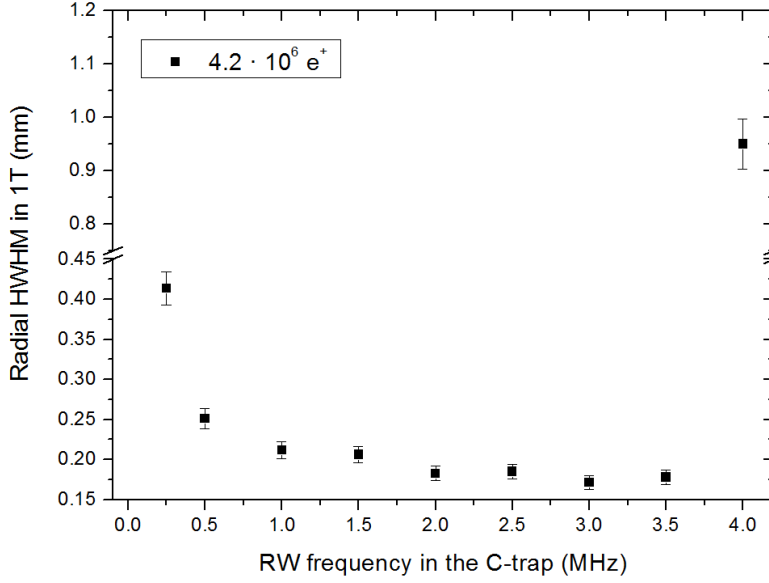


Figure 5.11: Observed positron plasma radius vs rotating-wall applied frequency.

for long times was 0.5 MHz at 0.25 V amplitude.

A first trial in stacking many e^+ shots from the accumulator, to test if the goal of $\sim 10^8$ accumulated positrons of AEGIS proposal [71] could be practically reached, was attempted with the low ^{22}Na source intensity at the time of the measurements: 9 mCi. This rather low source would have required an accumulation times of about two hours and half with a fresh moderator to reach $\sim 10^8$ positrons. The RW drive was kept at constant frequency 0.5 MHz and amplitude 0.25 V to compensate the radial expansion.

In order to measure the annihilation of an high number of e^+ at the trap opening, both PMT20 and PMT3 information were analyzed. As PMT3 was connected to a scintillator slab at the entrance of the 5T cryostat, it detected the annihilation peak under a much smaller solid angle than PMT20, which started to exhibit significant saturation with $1.5 \cdot 10^7$ e^+ annihilations. The informations of the two detectors were combined by cross-calibrating them in the region between $0.5 - 1.0 \cdot 10^7$ e^+ , and the rescaled charge of PMT3 was used for higher number of positrons.

The feasibility of long positron accumulations was verified stacking up to 15 shots, for a total of one hour of accumulation (see Figure 5.12). The accumulation was found well within linearity still after one hour of accumulation, and from the slope of a linear fit the trapping and cooling efficiency per shot value of $(87 \pm 7.5_{\text{stat}}) \%$ was estimated.

This measurement set the AEGIS e^+ record accumulation of $(4.5 \pm 0.3_{\text{stat}}) \cdot 10^7$ positrons, about half of the target goal. With the more intense 50 mCi source subsequently installed, this

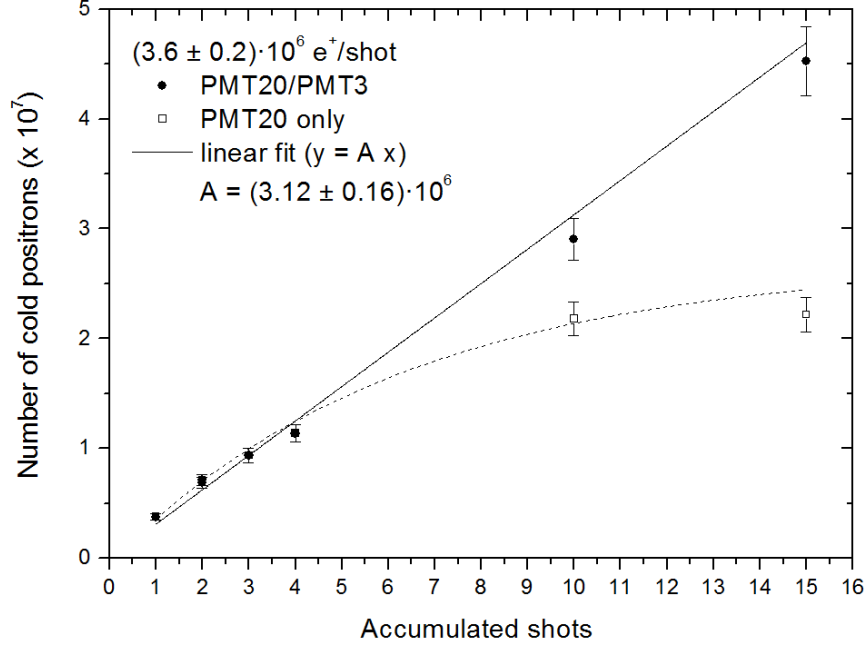


Figure 5.12: Number of positrons measured with PMT20 and PMT3 (black) and PMT20 only (white) as a function of the number of e^+ accumulator shots stacked in the 5T C-trap. Each shot took 5.5 min of accumulation. Saturation of PMT20 starts being observed when the measured charge $Q_{10\text{dB}} > 8000 \text{ pC}$.

result could be obtained with $< 30 \text{ min}$ of accumulation.

5.2.4 Adiabatic transfer to the Big-trap

Subsequently, the procedure for adiabatically transfer the e^+ plasma to the 1T Big-trap was developed. The procedure consisted in shifting along $\hat{z}$ the harmonic section of positron cooling trap (initially consitued by the four central electrodes of the C-trap C9-C12) through the entire 5T stack and over. This procedure was quickly developed and showed no particular technical difficulties:⁵ the positron transfer efficiency to the 1T Big-trap was found to be $\sim 100\%$ within experimental uncertainty already during the first trials.

Once in the Big-trap, e^+ were collected in a harmonic region under the B9-B13 electrodes. Two storage time studies of the particle number and the particle size as a function of the storage time, akin to the ones performed in the 5T C-trap, were repeated to assess the arrival conditions. No e^+ losses were observed within 30 min of storage, as for the C-trap (see Figure 5.13, panel

⁵but for the accurate synchronization of the two master FPGAs of the 5T and 1T trap controllers

(a)). The observed radius of the positron cloud was found significantly expanded after the transport. The average radius of the plasma on the 1TMCP was 1.6 mm, almost regardlessly of the initial plasma conditions in the 5T traps (in both size and numbers). On the contrary, no further spontaneous radial expansion was occurring on the time scale of 30 min (see Figure 5.13, panel (b)).

The radial expansion of the e^+ cloud vanished abruptly if stopping the adiabatic transfer before the end of the T section (still in the 5T field). A radial misalignment between the 5T and 1T traps stack of about ~ 1.5 mm was hypothesized as the most reasonable explanation, and later it was directly observed by means of an optical alignment tool.

Rotating-wall recompression in the Big-trap was subsequently attempted to reduce again the plasma size. Results, in this case, were less encouraging: RW was only rarely grasping the e^+ plasma, mostly at higher frequencies within the range 5 – 15 MHz and low amplitude settings 0.05 – 0.2 V. For higher settings of the amplitude, RW-induced plasma expansion was observed.

A more reproducible condition was found collecting two stacks from the accumulator before transferring to the Big-trap, then performing recompression with 18 MHz and 0.2 V. The plasma was compressed to 0.35 mm, close to the original diameter observed from the 5T, in about 50 s (see Figure 5.14) if at least $4 \cdot 10^6$ e^+ were present.

The clear cause of these difficulties in recompressing e^+ in the Big-trap was not clearly identified. It was hypothesized, as a primary responsible, the roughly $\times 100$ lower plasma density, consequence of the radial expansion of the cloud, which - combined to the $\times 20$ slower cooling rate due to the different magnetic field - made it very difficult for the plasma to dissipate the heat introduced with a strong rotating-wall drive.

The entire procedure to get a dense plasma of e^+ in the 1T Big-trap, suitable for a first attempt of autoresonant diocotron excitation was, at this stage, already quite complex, time consuming and hardly reproducible. Two accumulator shots had to be stacked in the C-trap with its RW on to compensate the expansion (2×5.5 min of accumulation plus 1 min for RW) in order the subsequent 1T RW to be working (other 1.5 min). Every measurement took about 15 min, and was subjected to frequent irreproducibilities caused by moderator ageing happening at a rate about 7% per hour.

5.3 Positronium formation via direct injection

Three arguments mainly motivated the change of strategy in view of the e^+ direct injection scheme, adopting magnetic steering for off-axis displacement and electrostatic acceleration to reach the requested keV target implantation energies:

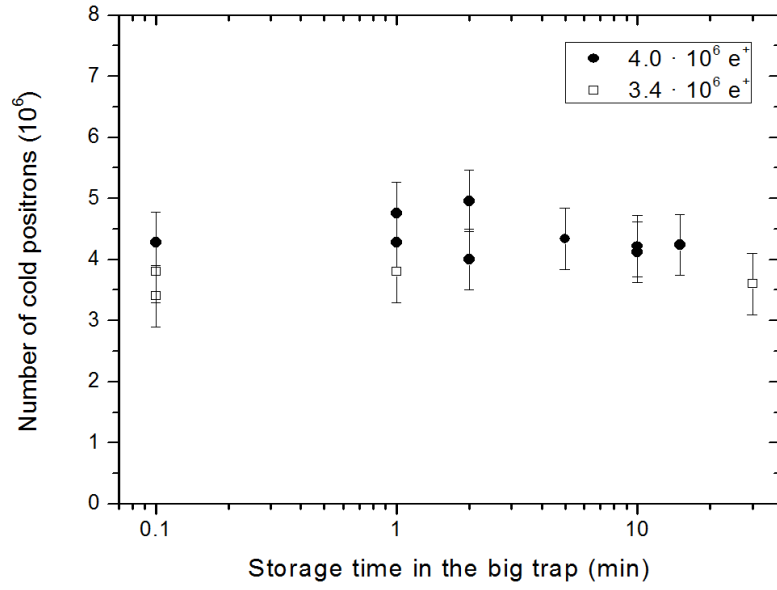
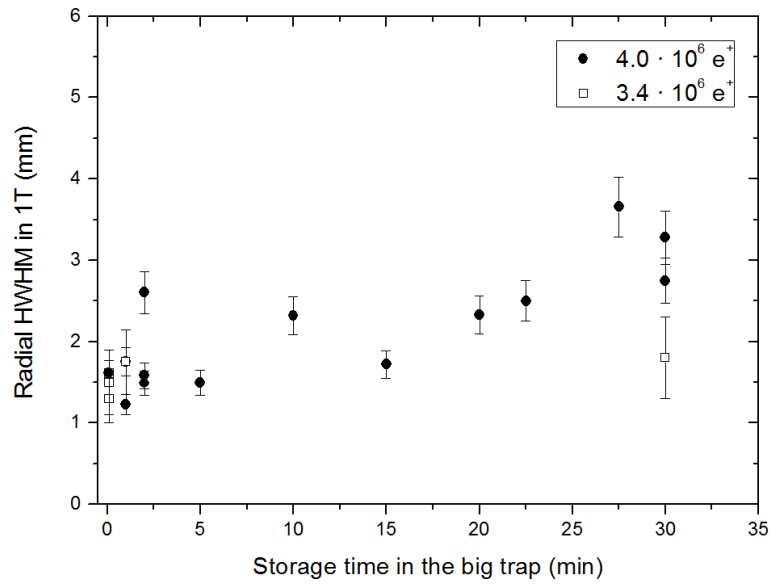

 (a) Number of e^+ versus storage time in 1T Big-trap

 (b) Radius of the e^+ cloud versus storage time in 1T Big-trap

Figure 5.13: Number of positrons (a) and plasma radius on 1TMCP (b) from the 1T Big-trap as a function of the storage time. A radial expansion to ~ 1.5 mm with high fluctuation observed (see text).

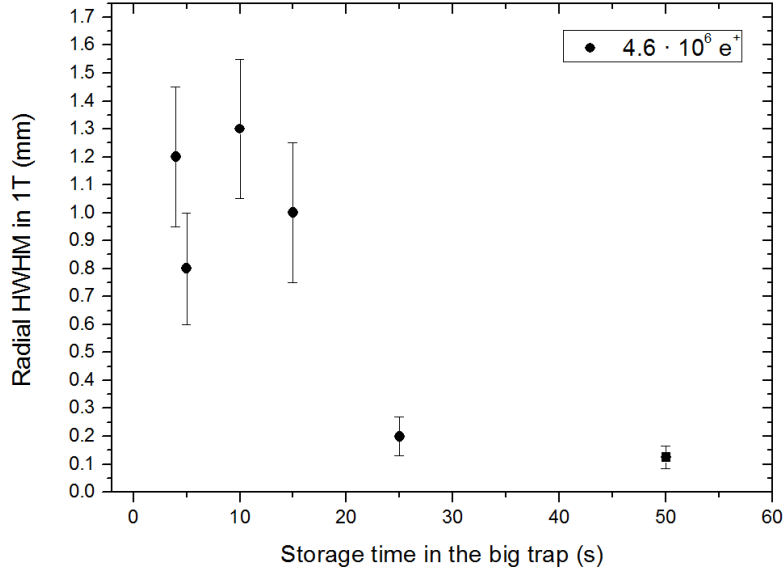


Figure 5.14: Observed e^+ plasma radius on the 1TMCP vs time of RW drive application. Two stacks of $\sim 2.5 \cdot 10^6$ positrons were used per measurement.

1. a missing working procedure for transferring efficiently positrons to the 1T traps, as discussed above;
2. the strong argument of significantly reduce the procedural complexity of the original scheme (thus increasing the overall reliability);
3. the parallel demonstration in the dedicated setup of Ps formation through electrostatic in-flight acceleration and bunching (see 4.2 and [88]).

5.3.1 Technical modifications

The technical solution adopted for letting the e^+ enter in the main apparatus with enough vertical displacement to end up 2.0 cm above the axis in the 1 T field⁶ was to modify the vacuum chamber at the end of the 45° tilted section of the transfer line to let the positrons fly through the entrance region of the experiment with a more off-axis trajectory (see Figure 5.15). This required a major modification of the set of magnetic field correction coils in the bending region (as well as their power supplies).

The OffAxis-trap structure was also removed, and in its place a single, long electrode of ~ 30 cm length and 10 mm diameter was placed, which was meant as a pick-up electrode for measuring the time distribution of the e^+ cloud.

⁶also retaining the possibility to steer the beam on the trap axis

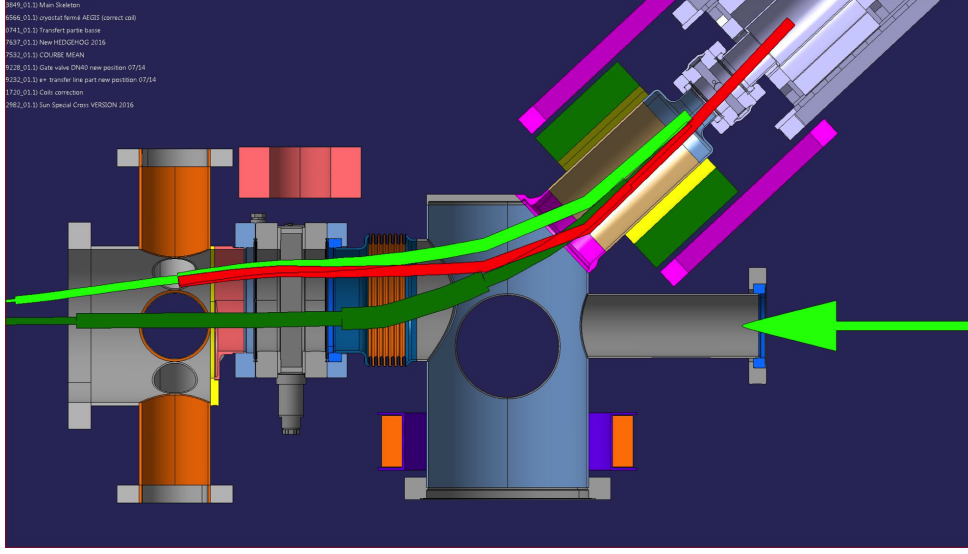


Figure 5.15: Scheme of the modified final part of the transfer line and entrance chamber of the experiment to allow the off-axis injection of positrons. From [114].

The positron spill in-flight acceleration has been studied and simulated in [93], where the two possibilities of a simple kicker tube and a multi-ring parabolic buncher were considered. As no further time focusing was found necessary, the simpler kicker solution was adopted for its simplicity. The final layout consisted in a 70 cm tube of 30 mm inner diameter and 52 mm outer shielding diameter inserted in the vacuum chamber of the last section of the magnetic transfer line. The tube was designed as a coaxial waveguide with $29.4\ \Omega$ impedance ($50\ \Omega$ were impossible due to mechanical constraints). The inner tube was connected through a coaxial vacuum feed-through to a 8 kV, 200 A switch from Behlke with 4 – 5 ns measured risetime, fast enough to raise the high-voltage within the 71 ns of e^+ flight time in the tube at $\sim 270\text{ eV}$ (see Figure 5.7, panel (b)).

The initial working set for the new correction coils was determined by a SIMION[®] simulation, then tuned following the upstream to downstream order using the movable CsI detector in different positions along the transfer line to minimize the e^+ losses. As positrons were made pass through the kicker tube, their electrical pickup signal was observed on the other (multiplexed) channel of the PXI5152 1 Gsample digitizer used for PMT20, connected to the tube by an impedance-matched, $\times 19\text{ V/V}$ gain, 700 MHz wideband amplifier (see 5.1.3).

Positrons were then injected into the main cryostat, and the correct steering was set seeking for an image of the distribution on the 1TMCP. An independent confirmation of the correct steering was given by the correct time delay observed on PMT20. The spill was found elliptic in the vertical direction, $\sim 0.7\text{ mm} \times 1.5\text{ mm}$. The horizontal position was set in the center of the trap by acting on the last horizontal correction coil current.

5.3.2 Positron centering on the target

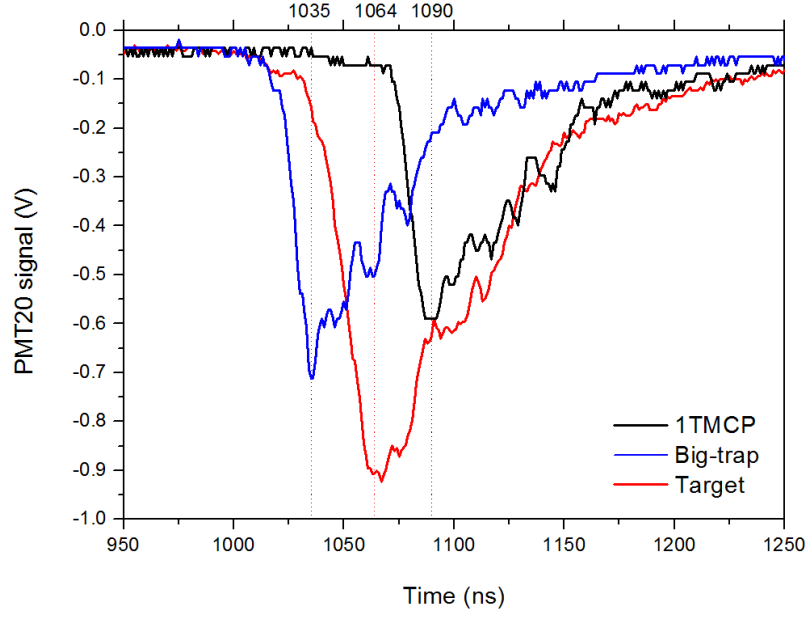
The positron spill was coarsely centered on the target as follows. The beam position was moved upwards progressively while observing the image on the 1TMCP, until it started getting chopped by the upper part of the first 10 mm diameter electrode in the smaller-diameter trap stack (the capping electrode at the junction between the Big-trap and the OnAxis-trap, as shown in Figure 5.6). Observing the annihilation peak on the PMT20 as the spill was hitting the electrode, it was found anticipated by 60 ns (see Figure 5.16, panel (a)), in good agreement with an annihilation occurring ~ 65 cm upwards with respect to the 1TMCP surface. The current on the vertical coil was further reduced, until the PMT20 peak was again found delayed, now by 30 ns, in agreement with an annihilation on the target structure, that lies 33 cm away from the end of the Big-trap.

The fine tuning of the vertical position was done using a small PbF_2 Cherenkov crystal, read by a prototype Hamamatsu MPPC suitable for operation in a cryogenic region at about ~ 15 K, which was installed on the top of the former OffAxis-trap structure, roughly ~ 4 cm away from the target (on a prolongation of the 30° tilted axis of the target, see Figure 5.17). This Cherenkov detector, as a consequence of its compact size and vicinity to the target, was much more sensitive than PMT20 to the annihilation position. The amplitude of the PbF_2 signal as a function of the vertical correction coil current is shown in Figure 5.16, panel (b), compared to amplitude of PMT20 delayed peak obtained with the same settings.

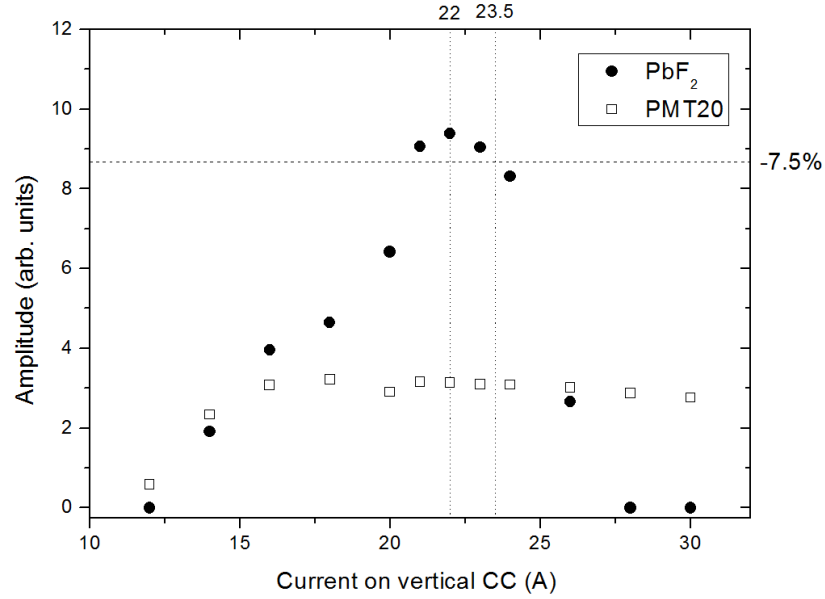
The target position was identified interpreting the signal amplitude detected on the PbF_2 in view of the complex geometry of the antihydrogen production region (see Figure 5.17, panel (a) for a visual reference). By going up with the alignment (i.e. reducing the current on the vertical correction coil), the detected signal quickly bumped close to the maximum value at 22 A, interpreted as the beam hitting the target structure, then linearly dropped with a first slope up to 20 A and continuing to decrease with a second slope as the beam went higher. This second linear trend has been imputed to a small angular misalignment of the long pickup electrode which causes the e^+ spill to annihilate at different z positions (see Figure 5.17, inset).

The installed target⁷ is 12 mm in length and its center is 44 mm away from the PbF_2 front face. The expected signal amplitude drop due to solid angle difference for a movement of 12 mm is 7.5% if the front face distance 44 mm is considered in the solid angle calculation. If the detector baricenter is considered instead, which is 65 mm distant from the target center, the expected signal drop drops to 3.4%. In Figure 5.16, panel (b), a 7.5% signal decrease is marked by a vertical dashed line, which correspond to the roughly linear region of decrease in the peak region. The most likely placement of the target, in terms of current setting of the coil, was found between 22.0 A (closest point to PbF_2) and 23.5 A (farthest point from PbF_2).

⁷which is the same used for Ps excitation experiments, as it was thoroughly tested



(a) PMT20 amplitude versus time for different vertical steerings.



(b) Comparison of PMT20 and PbF_2 amplitudes versus vertical coil current. The vertical line corresponds to the used setting; the horizontal line marks the expected decrease due to detector solid angle of a 1.0 cm movement.

Figure 5.16: Positron centering on the target (a) coarse steering performed by measuring the annihilation delay time on PMT20 (b) fine steering using the signal of the small PbF_2 detector placed on the top of the OffAxis-trap structure. From [114].

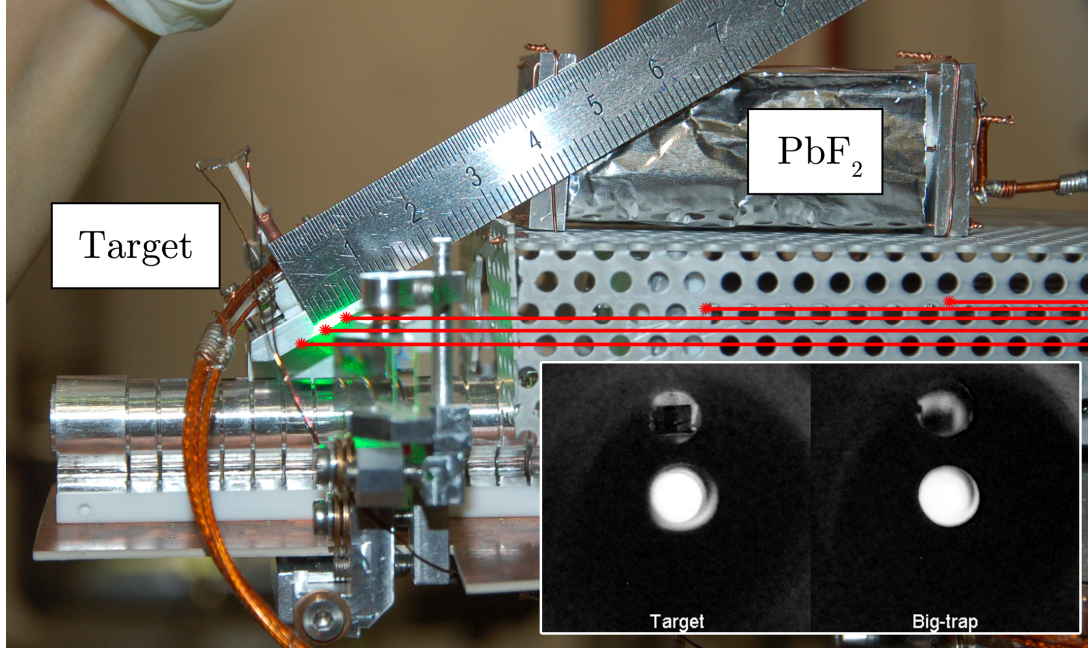


Figure 5.17: Side view of the $\bar{H}$ production region with the PbF_2 installed and hypothesized e^+ trajectories. Inset shows the front view from the e^+ perspective.

5.3.3 Positron acceleration

The subsequent step to form positronium (following the same logic as 4.3) consisted in accelerating the e^+ cloud to $\sim$ keV energy by raising the high-voltage potential on the kicker tube as all the e^+ spill was contained within its ends.

The correct timing was determined performing a small scan in steps of 12.5 ns of the time delay for opening the high-voltage switch in the time window indicated by the pickup signal and the PMT20 (Figure 5.3). When the potential was switched on too early, positrons were reflected back towards the accumulator. Viceversa, when the kicker tube was switched on too late, a fraction of positrons transited without being accelerated, observed as a peak later in time on PMT20. With proper synchronization, almost 100% of the spill of positrons was accelerated on the target position. The observed distribution got sharper in time as the acceleration voltage increased, as a consequence of the shorter flight time Figure 5.18.

5.3.4 Positronium formation and detection in $AE\bar{g}IS$

The technique used to observe positronium formation in the antihydrogen formation region was the same adopted in the dedicated setup experiments (see 4.3). A sequence of SSPALS shots acquired with PMT20 was 50%-50% splitted and digitized by two channels of a 2.5 GHz, 10 bit oscilloscope with high (100 mV/division) and low (1 V/division) resolution. The resulting

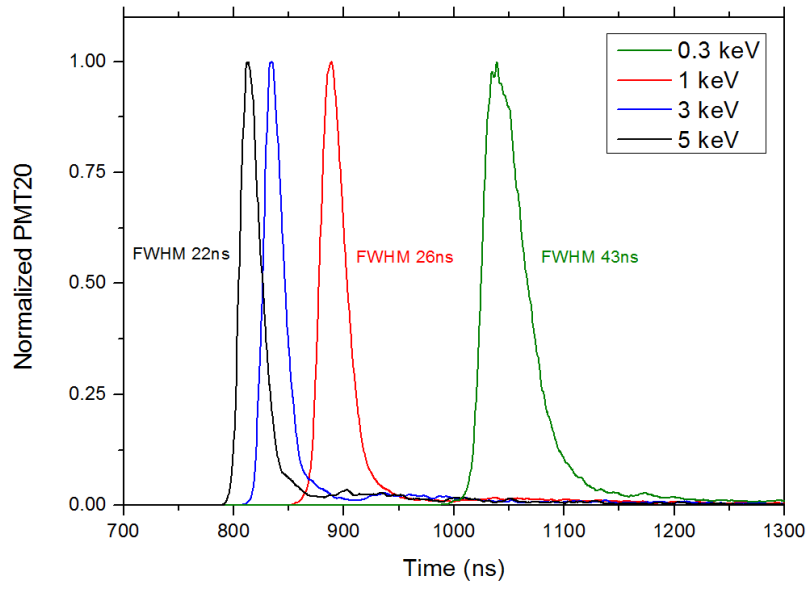


Figure 5.18: Acceleration of the e^+ spill by applying a fast HV pulse on the kicker electrode. Comparison of normalized PMT20 signals obtained with different acceleration voltages, shown in different colours. The FWHM reduces as the voltage increases due to the shorter flight time. From [114].

spectra were joined together, normalized on the peak amplitude and averaged.

The main difference between the two experiments were the types of detector adopted: in the former case, a fast inorganic scintillator $PbWO_4$, in the present case, the organic plastic scintillator slab coupled to PMT20. The reason for this choice was mostly its high solid angle ($\sim 3\%$) and relatively fast decay time (~ 4 ns), despite organic plastics exhibit slow-decaying scintillation light components with $\tau \sim 100$ ns of the order of 1 % (see [99, p. 58], literature examples are found in [115, p. 136] and [116]), which makes them not the ideal choice for SSPALS.

Due to the presence of many obstacles on the flight path of Ps (the closest electrode of $\bar{H}$ formation trap is just 1.0 cm distant from the center of the target), the signal due to positronium formation is expected to be distorted from the neat exponential long tail with $\tau = 142$ ns observed in free space (as in Figure 4.10), as a consequence of earlier annihilations on the obstacles. The complex geometry of the $\bar{H}$ formation region makes it difficult to predict the expected shape of the distribution without a numerical calculation.

Magnetic quenching of the 1^3S sublevel with $m_j = 0$ is also expected, due to the presence of the 1 T field, causing a 33% reduction in the amount of formed o-Ps.

Two measurement runs were performed with 5 kV implantation energy, the first with the target heated to $\sim$ room temperature ($T = 220$ K), the second with the target at the same cryogenic temperature of the environment ($T = 20$ K). In both cases, 45 acquisitions were averaged and compared to the reference impulse response of the plastic scintillator coupled to PMT20.

Results are shown in Figure 5.19. A long tail starting at about 10 % of the peak amplitude, inconsistent with the PMT20 impulse response (about 1 % in that region), has been observed independently of the temperature choice. It exhibits rough exponential behaviour in its first 150 ns with a decay time of $\tau \sim 100$ ns, compatible with Ps formation and subsequent annihilation on the $\bar{H}$ formation trap electrodes (1.0 cm is equivalent to 100 ns, assuming an axial velocity of $1.0 \cdot 10^5$ m s $^{-1}$ according to the measurements performed in 4.5).

The initial amplitude of this long tail is almost the same as the one observed in free space, around 10% of the peak in both cases. This suggests that the expected 33% reduction from magnetic quenching could be hidden by early annihilations on the electrodes.

The need of deeper understanding of Ps annihilation positions in $\bar{H}$ formation region motivates further research on Ps emission distributions in space and velocity from nanochanneled silica targets of the type installed in the experiment (p-type 111). This experimental data could be obtained in the dedicated spectroscopy chamber by scanning in space and time with the photoionization spectroscopy method described in 4.5.

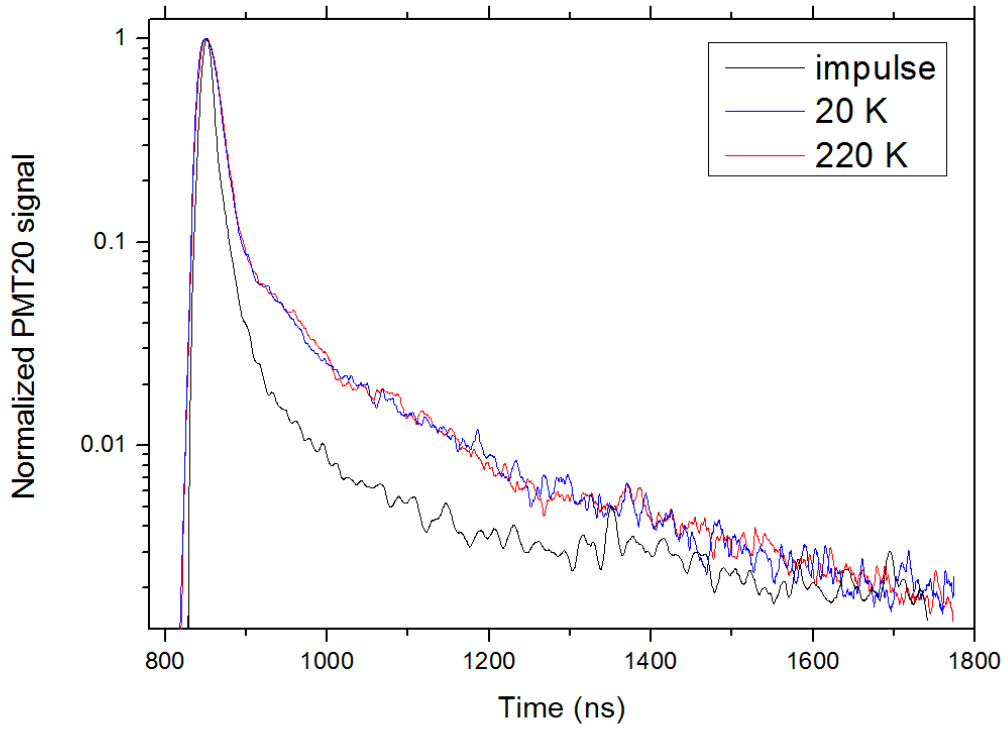


Figure 5.19: Observation of positronium formation from the target installed inside the AEGIS antihydrogen formation region. The resulting averaged SSPALS spectra for two different settings of the target temperature (see text) are pictorially compared to the impulse response of the plastic scintillator used for detection to emphasize its different lineshape. From [114].

Chapter 6

Antiprotons preparation in the $\bar{H}$ production trap

In this chapter the experimental procedure developed to load $\propto 10^5$ antiprotons in the final $\bar{H}$ production trap is described.

This procedure, which took years to be set up, is composed of several steps which were sequentially developed, commissioned and optimized for efficiency: capture of antiprotons, electron cooling, compression of mixed antiproton/electron plasmas to sub-cm diameters and transfer to the $\bar{H}$ production trap.

6.1 Electron plasma preparation

Electrons, at first glance, are a second-tier player in antimatter experiments, where all the emphasis is put on $\bar{H}$, $\bar{p}$ and e^+ ¹. Everyday practice, however, suggest something different, that manipulating $\bar{p}$ necessitate precise manipulations and control of e^- plasmas. There are three reasons for this to be true:

1. the electrons provide the only efficient cooling mechanism for antiprotons during any type of particle manipulation;
2. due to the $e^-/\bar{p}$ mass ratio, electron plasma dynamics is much faster than $\bar{p}$ plasma dynamics;
3. the number of e^- that can be practically achieved in traps is orders of magnitude higher

¹usually they only get mentioned explicitly as the cooling mechanism adopted for cooling the very energetic $\bar{p}$ cloud right after capture

than the number of $\bar{p}$ ($\gtrsim 10^8$ against $\lesssim 10^5$).

Setting up a reproducible procedure to load plasma of electrons, with a distribution as close as possible to ideality, is a key step for all subsequent antiproton manipulations.

6.1.1 Electron source

The primary electron source of AEGIS, i.e. the so-called *electron gun*, lies on a structure installed on an insertable high-vacuum linear actuator of the SUN entrance region of the 5T region, at $z = -1710$ mm (see Figure 3.7 for a visual reference). It consists of a barium oxide thermoionic filament mounted in an insulated hollow metal cage used as extractor electrode. A custom designed current-regulated power supply in the trap electronics rack delivers the 1130 mA current accurate to 1 mA to power up the filament and applies bias voltages to both filament and extraction electrode in the range provided by a standard DAC board (± 200 V).

During standard operation, the electron gun outputs an e^- current between $10 - 100 \mu\text{A}$, depending on the filament temperature and, consequently, the uptime. Typical settings are 1130 mA drive current and an extraction voltage of $\Delta V_{fil-ext} = 80$ V. The intensity of the produced current is monitored either by inserting the Central Region Faraday Cup (CRFC) (which terminates the current to ground through the input impedance of the reading amplifier).

6.1.2 Electron loading to the trap space-charge limit

The standard electron loading procedure consists of shaping the destination trap with a positive flat bottom (usually at 30 V) and the downstream endcap to -180 V to reflect the primary current back towards the electron gun. Reflected electrons loose part of their energy by Coulomb collisions with incoming electrons and can end up trapped in the positive part of the trap, which ultimately ends up filled with an e^- plasma up to its space-charge limit. The number of trapped electron is further increased ramping slowly up the upstream endcap at the same potential of the reflecting endcap while the primary current is still present (see Figure 6.1).

6.1.3 Measuring the electron number

The standard procedure for measuring the number of electrons consisted of letting the e^- escape by lowering one of the two sides of the trap (either partially or completely) and observing the electrical signal deposited on a Faraday cup detector. A Faraday cup detector (also FC, in a shorter nomenclature) at such low energies of the impinging particles and in presence of a strong magnetic field that provides radial confinement, is nothing but a simple metallic foil used as a

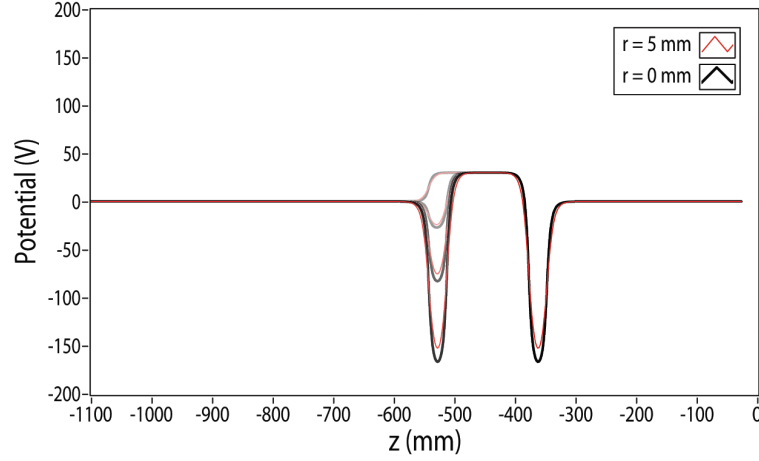


Figure 6.1: Potentials of the 5T trap electrodes during the standard electron loading procedure. The total potential ramp is performed in 4 s.

charge collector. Installed Faraday cups are all made of gold plated copper, to reduce the risk of surface oxidizing, with the exception of the degrader Faraday cup, which is made of aluminum. The central region Faraday cup is also split in different rings and sectors (different when used from the 5T or the 1T side), to provide some spatial sensitivity. A complete list of the available Faraday cups is given in Table 3.3.

Faraday cups directly measure the amount of charge deposited on their front face. Each sector is directly connected through a few meters cable to the input of a high-impedance operational amplifier. The cable acts as a capacitor of about 100 pC per meter, integrating the total charge deposited on the front face. Typical Faraday cups used for measuring the total number of particles in the plasma were typically the Degraded Faraday Cup (DegFC) and the 1T micro-channel plate Faraday cup (1TMCPIn), as they covered the full view of 5T and 1T traps respectively, the CRFC was used when radial sensitivity was required (see 3.5 for an overview of the placement of such detectors).

A typical signal of e^- of a Faraday cup detector (in this case, DegFC) is shown in Figure 6.2. A set of linear amplifiers for each FC channel were installed close to the vacuum feedthroughs. Their gain ($G \approx 53$ in amplitude) was absolutely calibrated against a common source, constituted by a precise μ s pulser of known amplitude.

The number of electrons was reconstructed in two ways. The first method consisted in fitting the exponential tail due to the amplifier RC circuit discharge to reconstruct the discharge τ of the RC circuit in the amplifier. As a precise R of 100 k Ω was used, the capacitance C can be

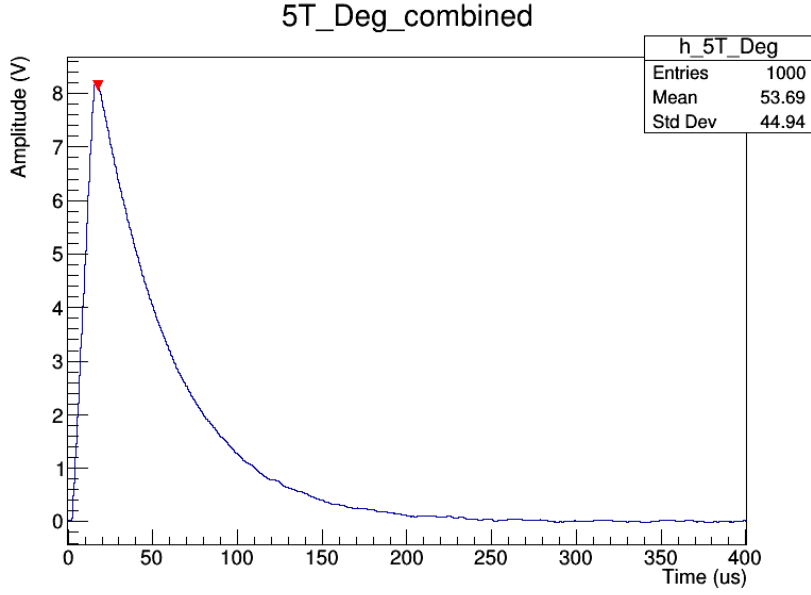


Figure 6.2: Example of electron signal acquired on the DegFC. The number of electrons was estimated by fitting the exponential tail and integrating the area (see text).

reconstructed by

$$C = \frac{\tau}{R} \quad (6.1.1)$$

The total charge collected by the amplifier is obtained by impling the definition of capacitance $C = Q/V_0$ where V_0 is the amplitude at the initial time of the discharge. The value of V_0 was obtained extrapolating the exponential fit function to t_0 , the raising edge of the signal. Combining the equations and dividing by the elementary charge e_0 the number of electrons is obtained.

$$N_1 = \frac{1}{e_0} \frac{V_0 \tau}{RG} \quad (6.1.2)$$

This method comes with two drawbacks: for low number of electrons the exponential fit fails to converge, and the method is not applicable at all when the trap opening time t_0 cannot be properly defined, as in the case of a slow opening of the trap or a train of pulses (used for some antiproton manipulations).

These drawbacks are solved by introducing a second method, consists in integrating the area below the curve to obtain the total charge observed by the amplifier on its input resistor:

$$Q(t) = \int_{t_0}^{t_1} I(t) dt = \frac{1}{R} \int_{t_0}^{t_1} V(t) dt \quad (6.1.3)$$

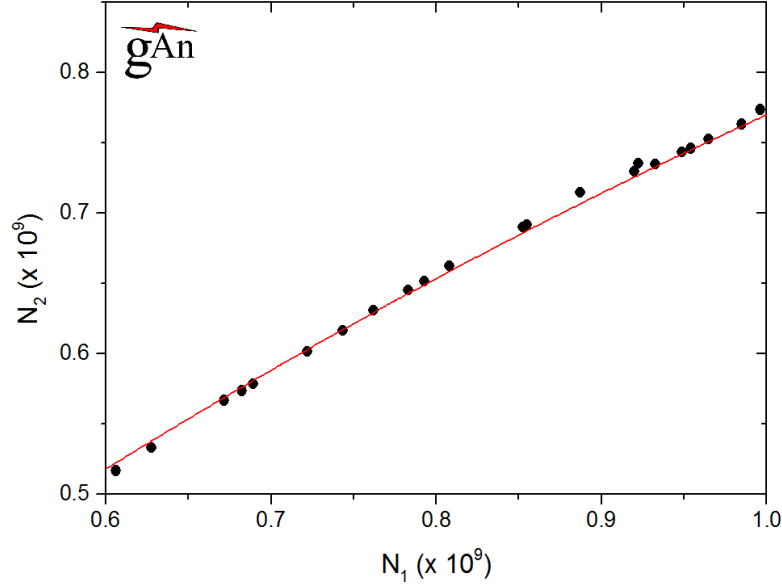


Figure 6.3: Comparison of the number of electrons measured with the fit method N_1 and the integral method N_2 . The resulting curve was fitted with a second order polynomial to obtain a cross-calibration model (see text).

Assuming linearity of both amplifier and digitizer (which does not hold for $N \gtrsim 10^9$) and correcting by the gain of the amplifier G , the number of electrons is obtained.

$$N_2 = \frac{1}{e_0} \frac{1}{RG} \int_{t_0}^{t_1} V(t) dt \quad (6.1.4)$$

A comparison of the two different estimate methods on a common dataset obtained varying the number of electrons in the region of expected saturation is shown in Figure 6.3.

The results of the two estimation methods were fitted with a second order polynomial function. The fit yields

$$\begin{aligned} N_2 = & (-3.710 \cdot 10^6 \pm 7.12 \cdot 10^6) \\ & + (1.012 \pm 0.014) \cdot N_1 \\ & + (-2.387 \cdot 10^{-10} \pm 0.065 \cdot 10^{-10}) \cdot N_1^2 \end{aligned}$$

For $N < 2.0 \cdot 10^8$, the quadratic term becomes negligible and two methods give equivalent results. The intercept is compatible with zero within the error bar, and the slope of the linear

term is compatible with unity within the error bar and the two methods give the same result within a 5 % error.

As the signal grows and reaches $N \approx 10^9$, the two estimate method fail to reconstruct the number of electrons consistently as a non-linear distortion effect start to play a role. This distortion effect was identified in peak clamping due to the digitizer reaching the maximum value.

This systematic error mostly affects the integral method, where all the data points are treated equally. On the contrary, it does not affect the fit method, as it uses only the data points in the tail region.

As a conclusion remark, the integral method is preferred for measuring the number of electrons for $N < 1.0 \cdot 10^8$ for its higher flexibility, whereas the fit method is used for $N > 1.0 \cdot 10^8$. The discrepancy among the two reconstructed methods is considered as the statistical error in reconstructing the electron number.

6.1.4 Reducing the electron number

The loading procedure was usually combined with a partial opening of the loading trap on one side (called *reduction step*) to chop the space-charge of the plasma. This procedure was meant both to reduce the number of electrons linearly (in the approximation of a flat Malmberg trap) and increase the reproducibility of the loading procedure in terms of number of e^- .

It was implemented as follows: right after e^- loading, an adiabatic reshaping of the trap bottom from 30 V to -10 V was applied, then one of the two endcaps was partially lowered to the desired value for 1.0 s to limit the maximum space charge of the plasma. Finally the trap was opened completely with a fast pulse and the number of e^- observed on the DegFC. Results are shown in Figure 6.4.

First of all, any reduction from the nominal endcap voltage of -180 V to about -70 V does not vary significantly the number of particles observed in the final extraction. Then, particles start getting extracted from the trap axis according (initially) to their thermal equilibrium distribution. Finally, for reduction voltages between -58 V and -35 V the reduction is almost linear as expected.

Fitting the linear reduction region and extrapolating over the entire interval gives two interesting consistency checks of the electron loading scheme. First, the trap bottom value of -10 V is found consistent within 95 % confidence level against the linear extrapolation. Secondly, the voltage level at which the linear extrapolation encounters the flat region of the number of electrons, ≈ -70 V, could also be expected. The filament is kept at -30 V with respect to the ground

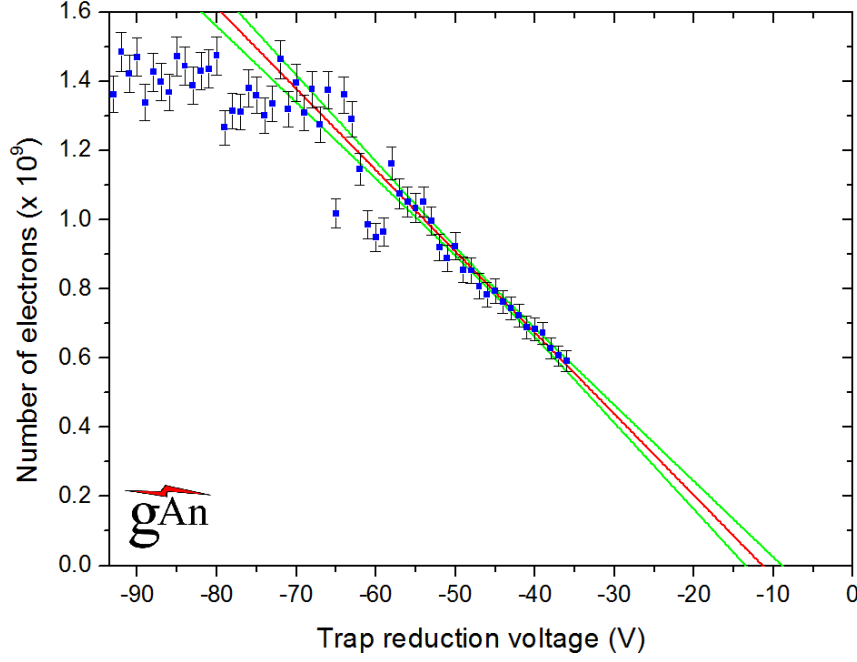


Figure 6.4: Number of electrons loaded in the P-trap and observed on the DegFC as a function of the applied voltage on the endcap electrode during the reduction phase, used for lowering the particles number (see text). Blue points are obtained with the fit method, red points with the integral method. Error bars show the statistical error only.

of the traps during e^- loading and the trap bottom is 30 V. If the endcap raising manoeuvre is performed slowly enough one could expect to load particles inside the trap up to the point they start being reflected back from the space charge potential, which happens when e^- fill the trap up to -30 V. Consistently, when reducing their number, particles start leaving the trap 60 V away from the trap bottom.

Electron number reduction with the described was used routinely for selecting a reproducible number of electrons. It has a drawback, however, that is worth at least mentioning. As it removes a significant fraction of particles from the axis (transporting no angular momentum), the plasma is forced to reequilibrate varying its overall radius trying to conserve $L \propto \sum_i r_i^2$ during the process. This is thought as the responsible of annoying e^- tails at high radii, which limits $\bar{p}$ compression efficiency.

6.1.5 Radial control with the rotating-wall technique

The RW technique is an essential tool for non-neutral plasma manipulations, first developed to counteract outward plasma transport (see [117, 118, 91, 92]). It consists of perturbing the plasma at equilibrium by superimposing on the trap potential a rotating electric field through an N -fold sectorized electrode, whose sectors are driven by sinusoidal phase-locked RF signals shifted in phase by $2\pi/N$. The resulting electric field co-rotates with the plasma at the drive frequency ω_d , generally different from the plasma rotation frequency at equilibrium in the unperturbed trap field ω_p . Within certain limits this frequency difference causes the plasma to readjust in the trap varying its angular momentum to match the frequency of the external drive. The radial equilibrium distribution changes accordingly to the different total angular momentum depending on ω_d ; in this way, very different radii and central densities can be obtained.

Fourfold sectorized electrodes for RW applications are installed in all main AEGIS trap sections, as already mentioned before (see Table 6.1 for the reference list). The multichannel controller based on AD9959 DDS DACs installed within the trap electronics synthesizes the pure sine waves/frequency sweeps with a maximum 2.5 V amplitude, 10bit resolution in a frequency range from DC to 50 MHz.

An example of RW compression is shown in Figure 6.5. An electron plasma of about 10^8 e^- is first loaded in the P-trap region with the procedure described above. Subsequently, the RW electric field is turned on at constant frequency and amplitude for a fixed amount of time. The trap is opened right after switching off the RW field and the resulting plasma distribution is observed on the 1TMCP.

Trap region	RW electrodes
C	C10, C11
P	P7, P8
Big	B10, B12
OnAxis	OA12
Prod.	UC1

Table 6.1: List of RW sectorized electrodes

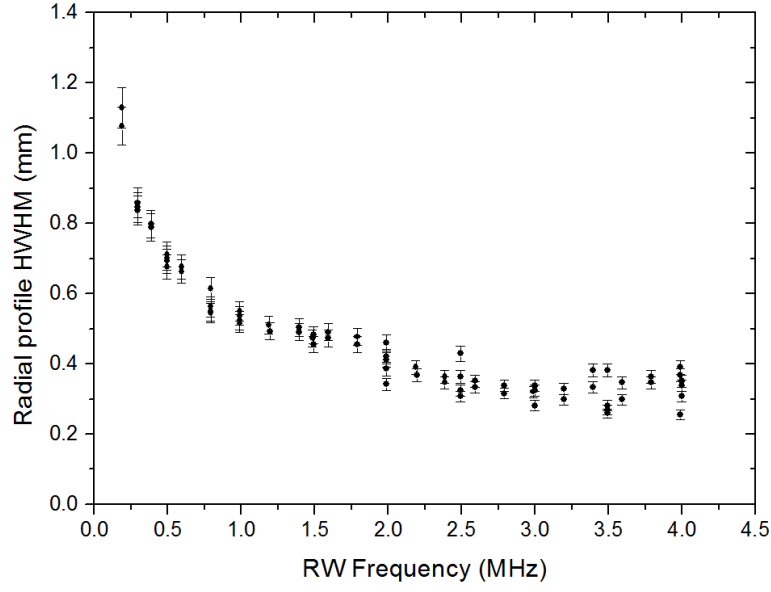
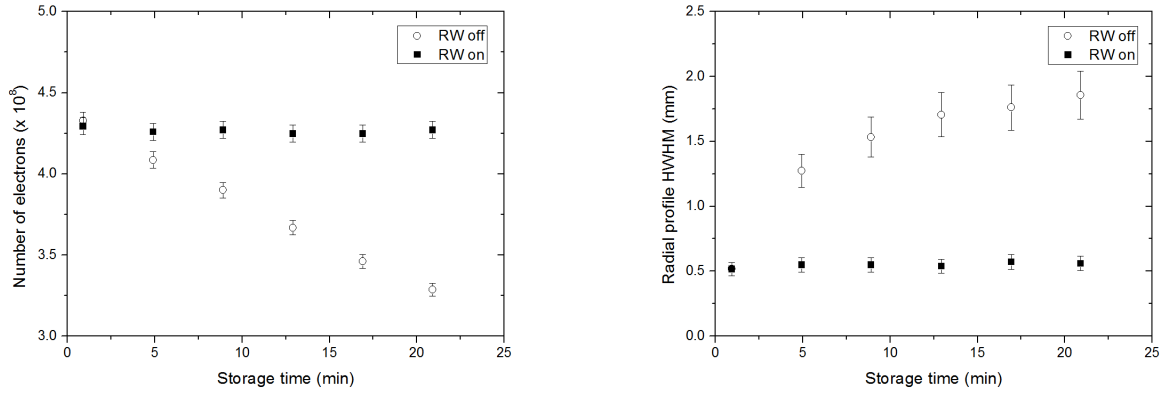

 (a) e^- radius versus frequency

 (b) e^- number and radius versus time

Figure 6.5: Example of rotating-wall compression of electron plasmas in the 5T P-trap region. Plasma radii were measured by imaging the plasma on the 1TMCP, while particle numbers were measured with the DegFC.

6.2 Antiproton capture

The first part of the experimental procedure to prepare plasmas of antiprotons in the final trap consists of catching a fraction of the high-energy $\bar{p}$ from AD using high-voltage electrodes, keep them in traps for macroscopic times and estimate the overall catching efficiency.

6.2.1 Antiproton catching procedure

AEgIS experimental procedure to collect antiprotons from AD was developed during the first 2012 run and never significantly altered since then (see [119, 120]). It consists of catching ballistically the low-energy ($\sim$ keV) fraction of $\bar{p}$ emerging from a series of energy-degrading foils using two high-voltage electrodes acting as endcaps while the magnetic field provides radial confinement. A constant negative bias voltage is applied to the downstream endcap (usually HV3) and a fast-rising negative voltage is applied with the right timing to the upstream endcap (HV1).

As antiprotons dissipate almost no energy through brehmsstrahlung on practical time scales², without an external cooling mechanism captured $\bar{p}$ conserve their keV initial energy until the moment they get extracted from the trap. This is performed opening the trap towards the degrader structure where the antiprotons annihilate by slowly ramping down the high-voltage on HV1. Pions resulting from the annihilations are detected by the external scintillator slab SCI12 and counted, providing the reference signal used for the estimation of the amount of collected $\bar{p}$.

The $\bar{p}$ catching efficiency depends mainly on four parameters: degrader thickness; trap closing time; potential applied on the high-voltage electrodes and beam steering.

Beam steering was verified and adjusted each day of measurement with $\bar{p}$ (or anytime the conditions in AD were changed) by tuning the current on the last dipole and quadrupole magnets on AEgIS extraction line. Steering was first set observing the annihilation signal on the degrader structure by means of a fast HPD detector coupled to a plastic scintillator (see Figure 6.6). Then, the fine tuning of the position was performed maximizing the number of antiprotons captured.

AEgIS incorporates since 2015 a beam monitor based on 50 μm thinned version of the MI-MOTERA detector [86], named MIMITO. This detector was able to image the beam at low AD intensities; however, strong saturation occurred at nominal AD intensities due to a too-high energy deposition in the material. The detector was found unreliable as a beam intensity normalization tool, and practically not much more usable than a veto for the presence of the spill.

²energy loss scales as $1/m^2$, so the process for $\bar{p}$ is $\approx 2.5 \cdot 10^{-7}$ less efficient than e^-

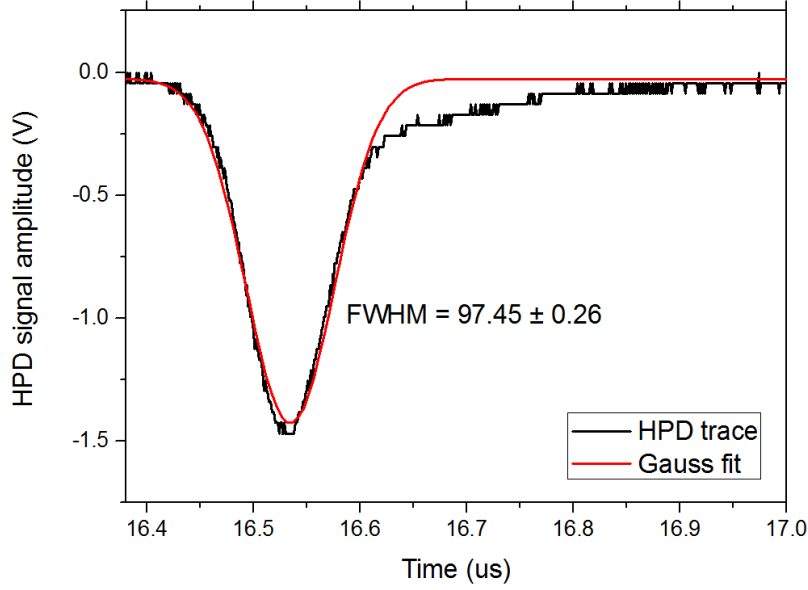


Figure 6.6: Antiproton spill time distribution on the degrader recorded by a fast HPD detector. The zero of the time axis corresponds to the AD trigger instant. Peak fitted with a Gaussian function to extract a reference FWHM.

The optimal degrader thickness was determined first by Monte-Carlo simulations of the $\bar{p}$ energy deposition process in the degrader structure, which is composed of MIMITO wafered between the two aluminum foils that act both as degraders and as two-sided Faraday cup (the DegFC) ³.

Trap closing time was set according to the annihilation signal on the HPD detector (see Figure 6.6) and optimized by varying the opening time of the HV1 negative high-voltage switch and observing the amount of trapped $\bar{p}$ on SC12 with the maximum usable trapping voltage (9 kV) (see Figure 6.8.b).

6.2.2 Antiproton number determination

The determination of the absolute number of antiprotons annihilating inside the experiment after an extraction from the traps was performed calibrating the external scintillators slabs in coincidence (counting) mode.

³an optimal degrader thickness scan was performed during 2015 run by inserting two extra aluminum degrading foils. Results showed that the best setting would be found with additional 4 μm . The collaboration decided however not to take the risk of permanently gluing this extra foil.

In order to convert a number of counts on external scintillator slabs to the actual number of antiprotons annihilating inside the experiment, three numbers need to be known or modeled: the number of minimum ionizing particles (or MIP)⁴ produced per antiproton annihilation event (mostly pions); the detection efficiency of the slab to a MIP and the probability for the particle to reach the detector and be above threshold after absorption from the materials.

The first item, the number of MIPs per event, has been object of dedicated research for many years, and now is modeled to some extent by most Montecarlo simulation packages; on average, each $\bar{p}$ produces 2.6 MIPs per annihilation event on aluminum (see [121, p. 66] for the experimental data).

The detection efficiency of the scintillators to a minimum ionizing particles was obtained experimentally. All PMTs in the scintillator slabs were cross-calibrated in gain by acquiring spectra of cosmic rays for long time on a QDC counter. All recorded spectra were equalized adjusting the voltage of the different PMTs. Subsequently, the intrinsic detection efficiencies for minimum ionizing particles (and the background counting rate) were measured for each slab in coincidence mode by means of the same calibration source with and without the main magnetic field. All efficiencies were found within 90 % and 97 %, with an average background rate of 100 ± 10 Hz.

Finally, the energy deposit of a MIP in the external scintillators was modeled using a Geant4-VMC Montecarlo simulation using FRITIOF of the material budget between the annihilation point considered and each scintillator slab (see Figure 6.7 for a visual comparison of the real and simulated geometries and [122] for a detailed comparison on available Montecarlo models for $\bar{p}$ annihilations at low energy). Corrections due to annihilation position within the experiment and presence of cryogenic liquids were considered.

Results from 2056 antiproton catch-and-count runs performed during 2013 were compared to the MC estimates for 5T scintillator slabs with the calibration procedure described above (see [123]). An high fluctuation in the reconstructed number of $\bar{p}$ was observed among the different scintillators. Especially, the counts-to-pbar number reconstructed by SC12 and SC34 (bigger and closest the annihilation point on the degrader structure) was found consistent among the two, but smaller than the number reconstructed by the two (smaller and farther away) slabs SC56 and SC78 by about 50 % (see Table 6.2).

For the rest of this chapter, all the reported number of antiprotons will be referred to the most pessimistic estimate of SC12 using the conversion factor $\times 12.7$, as this detector has also the biggest solid angle for annihilations on the degrader structure. All measurements of antipro-

⁴to distinguish them from the heavily ionizing particles, usually massive particles resulting from nuclear fragmentation which are stopped by a very small budget of material and do not reach external detectors

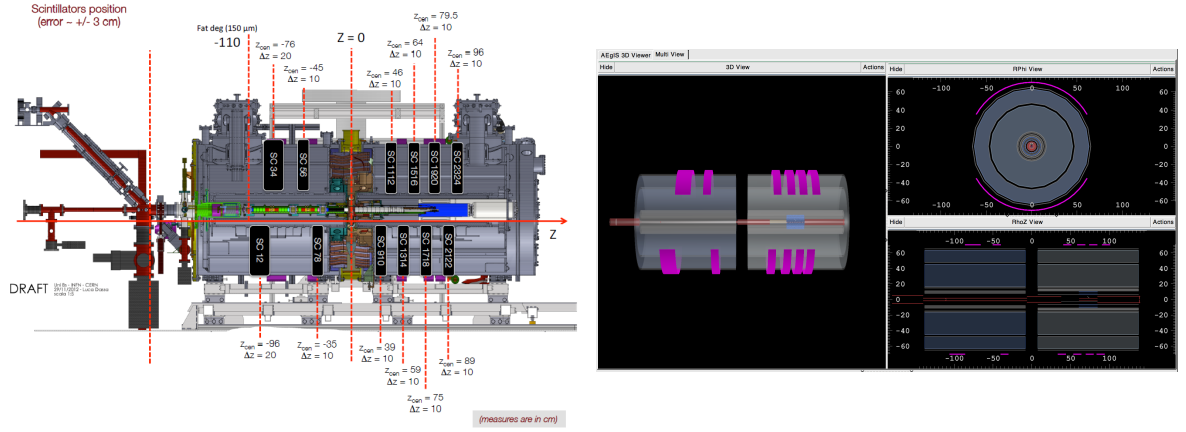


Figure 6.7: Comparison of the real geometry vs the one used in the Montecarlo simulation adopted to calibrate the external scintillators (from [123]).

tons number comes with an estimated systematic error of at least 50 % so to account for the disagreement among the different slabs.

Scintillator	Counts-to- $\bar{p}$	$\bar{p}$ number
SC12	12.7	$1.27 \cdot 10^5$
SC34	14.7	$1.39 \cdot 10^5$
SC56	52.5	$2.04 \cdot 10^5$
SC78	56.8	$1.93 \cdot 10^5$

Table 6.2: Counts to antiprotons conversion factors per scintillator slab and reconstructed number of $\bar{p}$ in 2056 catch and count runs (from [123]).

6.2.3 Antiproton catching results

Results obtained in catching antiprotons are summarized in Figure 6.8.

In Figure 6.8.a the number of antiprotons captured normalized on the declared AD intensity is shown for the 2015 and 2016 runs. The reported error bar is only the statistical fluctuation among the different runs as a result of the AD intensity fluctuation. Since the reconstructed number of antiprotons, as discussed above, was obtained by multiplying the counts on the scintillators by the factor $\times 12.7$, a systematic overall uncertainty of 50 % should be considered. The number of captured $\bar{p}$ as a function of the trapping voltage is almost perfectly fitting with

an exponential curve saturating at

$$\begin{aligned}\eta_{\text{catch}}^{2015} &= 2.22\% \pm 0.10_{\text{stat}} \pm 1.1_{\text{sys}} \\ \eta_{\text{catch}}^{2016} &= 1.98\% \pm 0.06_{\text{stat}} \pm 1.0_{\text{sys}}\end{aligned}\tag{6.2.1}$$

for the 2015 and 2016 runs respectively. These numbers are consistent with the capture efficiency declared by the ALPHA experiment of about 1% with a 5 kV capture potential (see [124]). The slight reduction in the catching efficiency among 2015 and 2016 is not explained, as the AEGIS degrading system was left untouched. A slight variation in the AD spill energy, time or spatial distribution is the most probable cause.

Figure 6.8.b the relative number of antiprotons captured as a function of high-voltage closing time is shown, providing some interesting insights on the $\bar{p}$ capture mechanism. After some initial null values, a sudden bump and a plateau in the capture signal are observed between 16.3 μs and 17.5 μs . The signal then slowly degrades in a long tail for longer closing times of the trap.

This observed signal behaviour can be understood if the dynamics of the flying particles at different axial energies is considered. An emerging $\bar{p}$ from the last degrader foil at the instant $t_0 = 16.534 \pm 0.04 \mu\text{s}$ ⁵ with energy $E = e_0 V$ has to fly twice the distance between HV1 and HV3 $L = 0.763 \text{ m}$ in order to reach HV1 again to escape capture before the high-voltage is risen. The HV1 closing time t sets the minimum axial energy a $\bar{p}$ needs to possess in order to escape capture through the relation

$$V = \frac{2m_p}{e_0} \frac{L^2}{(t - t_0)^2}\tag{6.2.2}$$

This formula (plotted in Figure 6.8.c1) can be used to work out the time corresponding to the 9 kV voltage applied on the electrodes $t_1 = 17.696 \mu\text{s}$. This value, shown in Figure 6.8.b in the vertical dashed line, corresponds to the end of the flat plateau region, whose length can now be interpreted as the time spent by 9 keV $\bar{p}$ to fly from HV1 to HV3 and back.

Figure 6.8.c2 shows the same data points of Figure 6.8.b, plotted as a function of the corresponding energy calculated through the relation above. Here the evidence that only antiprotons with energies of 9 keV or less can be trapped is even more striking, as the right part of the plot (with respect to the vertical line at 9 keV) is consistent with a flat plateau. The distribution of $\bar{p}$ leaving the degrader is found almost exponential in the region of energy below 2 keV.

To conclude this section, the actual limitation of AEGIS in catching antiprotons is set by the

⁵see Figure 6.6; the length of the spill was considered as fluctuation, as its time distribution is roughly Gaussian in shape.

maximum voltage reachable with negative high-voltage power supplies and fast switch. Both high-voltage electrodes and trap length would sustain higher voltage level up to ~ 25 kV, possibly more than doubling the number of captured $\bar{p}$.

6.3 Electron cooling

The second part of the experimental procedure to prepare plasmas of antiprotons in the final trap consists of reducing the captured antiprotons energy by means of sympathetic electron cooling down to $\sim$ eV, where only low-voltage potential manipulations are required.

6.3.1 Principle of sympathetic antiproton cooling on electrons

The concept of sympathetic antiproton cooling through electron radiative cooling, widely adopted within AD experiments for its effectiveness and simplicity (see [125]), consists of exploiting electron plasmas high emitted power through cyclotron radiation as a means of friction to dissipate the keV energy of the antiprotons right after capture, using Coulomb collision between the two plasmas to transfer the energy from $\bar{p}$ to e^- .

The text-book principle beyond electron radiative cooling is that a single charge left alone in a trap dissipates energy spontaneously via cyclotron radiation as it feels a net acceleration from the confining potentials. The consequent reduction of energy per unit time is calculated for both the axial and radial energy components from the power emitted by an accelerated charge (see [113] and [108]), and yields

$$\begin{aligned}\frac{dE_z}{dt} &= -\gamma_z E_z(t) \\ \frac{dE_r}{dt} &= -\gamma_r E_r(t)\end{aligned}\tag{6.3.1}$$

where γ_z and γ_r are given by

$$\begin{aligned}\gamma_z &= \frac{e_0^2 \omega_z^2}{6\pi\epsilon_0 m c^3} = 6.25 \cdot 10^{-10} \text{ Hz} \cdot \left(\frac{m}{m_e}\right)^{-1} \cdot \left(\frac{\omega_z}{10 \text{ MHz}}\right)^2 \\ \gamma_r &= \frac{e_0^4 B^2}{3\pi\epsilon_0 m^3 c^3} = 0.39 \text{ Hz} \cdot \left(\frac{m}{m_e}\right)^{-3} \cdot \left(\frac{B}{1 \text{ T}}\right)^2\end{aligned}\tag{6.3.2}$$

In nominal AEGIS conditions (4.46 T magnetic field, axial frequency $\omega_z = 2\pi \times 30$ MHz, [108]) the characteristic damping times for the radial and axial motions for e^- and $\bar{p}$ are summarized in Table 6.3.

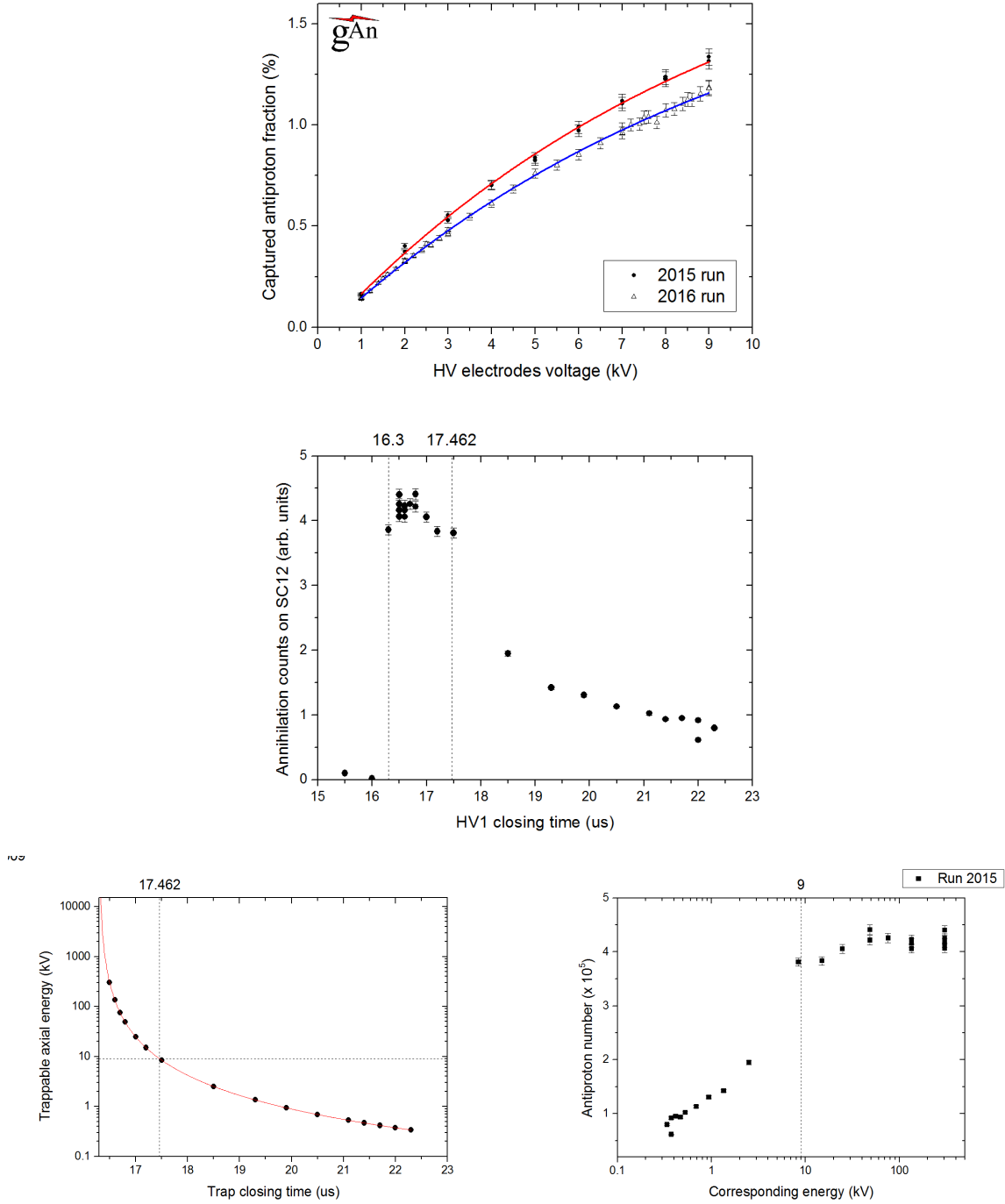


Figure 6.8: Antiproton catching results. (a) Number of antiprotons captured as a function of the high-voltage set on the endcaps. (b) Number of antiprotons captured and observed on SCI12 after 5 s storage as a function of the trap endcap closing time. (c.1) Maximum axial energy of trappable antiprotons as a function of the chosen trap closing time. (c.2) Same data of 2. plotted in energy instead of time, showing consistency with the applied 9 kV on the electrodes. Only statistical error bars are shown (see text).

	4.5 T	1.0 T
$\gamma_r^{-1}(e^-)$	0.13 s	2.6 s
$\gamma_z^{-1}(e^-)$	54 days	54 days
$\gamma_r^{-1}(\bar{p})$	25 years	500 years
$\gamma_z^{-1}(\bar{p})$	262 years	262 years

Table 6.3: Characteristic radial and axial cyclotron (synchrotron) cooling times for electrons and antiprotons in typical AEGIS fields

In the case of single particles, only the radial motion of e^- is cooled effectively through radiation. In a highly-collisional plasma, however, since the frequent Coulomb collisions among the particles act as an efficient heat transfer mechanism from the axial to the radial degrees of freedom, also the axial energy gets efficiently dissipated (see [126]).

In the case of a mixed e^- and $\bar{p}$ plasma, as discussed in [125, 127], under the approximation of two radially overlapping clouds with different axial lengths $\alpha = L_e/L_{\bar{p}} < 1$, the cooling process can be modeled by two rate equations linking together the instantaneous plasma temperatures T_e and $T_{\bar{p}}$

$$\begin{aligned}\frac{dT_{\bar{p}}}{dt} &= -\alpha\gamma_c(T_{\bar{p}} - T_e) \\ \frac{dT_e}{dt} &= \alpha\gamma_c\frac{N_{\bar{p}}}{N_e}(T_{\bar{p}} - T_e) - \gamma_r(e^-)(T_e - T_0)\end{aligned}\tag{6.3.3}$$

The first equation includes a single term modelling the outgoing transfer of heat through collisions, happening at the rate γ_c . The second equation consist of two terms: the first one modelling the ingoing transfer of heat (here normalized by the densities n_e and n_p); the second the dissipation through cyclotron radiation at rate $\gamma_r(e^-)$ and with limit temperature T_0 of sole electrons in the trap.

An analytic expression for the collision rate γ_c is found in [127]

$$\gamma_c^{-1} = 4\pi\epsilon_0^2 \cdot \frac{3m_em_pc^3}{8\sqrt{2\pi}e_0^4} \cdot \frac{1}{n_e \log(\Lambda)} \cdot \left(\frac{k_BT_{\bar{p}}}{m_{\bar{p}}c^2} + \frac{k_BT_e}{m_ec^2} \right)^{\frac{3}{2}}\tag{6.3.4}$$

where the factor $\log(\Lambda)$, the so-called Coulomb logarithm, models the dependence of the impact parameter of the collisions as a function of the mutual temperatures. An analytical expression for a two-component plasma was derived by Spitzer [128].

$$\Lambda = 4\pi \left(\frac{\epsilon_0 k_B}{e_0^2} \right)^{\frac{3}{2}} \cdot \sqrt{\frac{T_e}{n_e}} \cdot \left(T_e + \frac{m_e}{m_{\bar{p}}} T_{\bar{p}} + 2\sqrt{\frac{m_e}{m_{\bar{p}}}} \sqrt{T_e T_{\bar{p}}} \right)\tag{6.3.5}$$

Numerical simulations obtained upon integration of this set of rate equations with realistic initial plasma densities were performed during the early stages of the ATHENA experiment by D. Manuzio in her master thesis (see [127]). The expected cooling time for a mixed $\bar{p}$ - e^- plasma was found in the range of the tens of seconds with an electron density of $5 \cdot 10^7 \text{ cm}^{-3}$ or higher and a density ratio $n_e/n_p \leq 10^3$.

This model gives adequate order of magnitude estimates for the expected sympathetic cooling time in a realistic case. However, fitting this simplified model on real data at energies of $\sim \text{eV}$ or below requires attention, as it lies upon the approximation of clouds overlapping being independent of temperatures, which is not usually true neither in the axial direction (antiprotons fall in the shorter cooling trap of the electrons), nor in the radial direction (at plasma temperatures $\lesssim \text{eV}$ spatial decoupling of the two particles distributions due to centrifugal separation starts being observed, as shown further in this section, envisaging a more complex modelling strategy, including plasma radial transport mechanisms).

6.3.2 Outline of the experimental procedure

The antiproton electron cooling experimental procedure is the combination of the e^- loading and $\bar{p}$ catching procedure described above. A plasma of electrons is first prepared in the P-trap region between the electrodes P3 and P12; after reducing the electrons number (usually with a -40 V reduction step) the trap is adiabatically reshaped to a flat Malmberg trap with a bottom at 140 V .

Antiprotons, as they are captured and bounce between HV1 and HV3, tranverse the e^- plasma loosing a fraction of their energy by collisions, ultimately ending up trapped together with electrons in the same low-voltage P3-P12 trap.

It's worth introducing now a terminology that will be used frequently from now on: the remaining fraction of $\bar{p}$ with keV energies will be referred to as the *hot* fraction, to distinguish it from the *cold* $\bar{p}$ which are the ones ending up in the low-voltage trap.

The hot $\bar{p}$ fraction is removed from the trap after some time cooling with electrons by lowering the HV1 electrode. Hot antiprotons annihilate on the degrader surface and are counted by external scintillators to keep catching and cooling monitored. Then, also the small trap is reshaped and opened in different ways depending on the type of measure under interest:

- for measuring the cold $\bar{p}$ number, the trap is opened slowly towards the degrader to let $\bar{p}$ escape one by one from the trap and avoid pile-up effects on external scintillators;
- for measuring the cold $\bar{p}$ plasma spatial distribution, electrons are first removed by quickly

opening the trap with a train of (usually 10) 41.6 ns pulses applied to one endcap⁶. The trap is then quickly opened again by another train of (usually 10) 2.66 μ s pulses either towards the 5TMCP or the 1TMCP;

6.3.3 Electron cooling efficiency

The electron cooling efficiency was obtained by calculating the ratio of cold annihilations observed by SC12 versus the total number of cold and hot annihilations.

$$\eta_c = \frac{N_C}{N_C + N_D} \quad (6.3.6)$$

Two measurements of the cooling efficiency as a function of the cooling time⁷ were collected at the beginning of 2015 and 2016 AEGIS runs with two different initial electron conditions (shown in Figure 6.9). Each measurement was fitted with an exponential function (found well modelling the data) to extract the cooling time τ_c and the maximum cooling efficiency $\eta_c|_{\max} = \eta_c(t \rightarrow \infty)$.

The maximum cooling efficiency was found

$$\begin{aligned} \eta_c^{2015}|_{\max} &= 0.72 \pm 0.03_{\text{stat}} \\ \eta_c^{2016}|_{\max} &= 0.986 \pm 0.006_{\text{stat}} \end{aligned} \quad (6.3.7)$$

and cooling time

$$\begin{aligned} \tau_c^{2015} &= (21.3 \pm 2.7_{\text{stat}}) \text{ s} \\ \tau_c^{2016} &= (9.0 \pm 0.7_{\text{stat}}) \text{ s} \end{aligned} \quad (6.3.8)$$

The result of AEGIS 2016 run was compatible with 100 % cooling efficiency. It was achieved by increasing the cooling electron number (from $\sim 5.5 \cdot 10^8$ to $\sim 8.5 \cdot 10^8$) and the electron trap depth (from 30 V to 140 V). However, as discussed in the next section, when rotating-wall compression of antiprotons had to be applied as successive step to electron cooling, this almost 100 % electron cooling efficiency was lowered to $\sim 55\%$. This came as a consequence of the necessity to apply a low-frequency rotating-wall drive on the electron plasma during electron cooling as an active external source of heat, to keep the plasma at high temperature and avoid centrifugal separation.

⁶electrons, having a much faster dynamics than antiprotons due to the mass ratio, leave the trap almost instantly from the perspective of $\bar{p}$ s

⁷i.e. the time spent by the $\bar{p}$ together with the cooling e^- , before the high-voltage endcap HV1 was opened to let the hot antiprotons escape

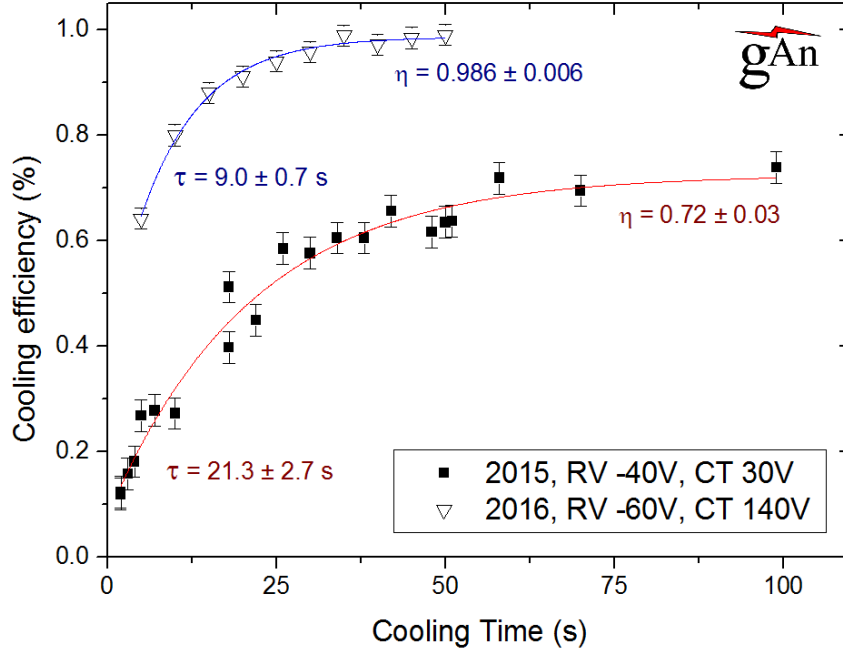


Figure 6.9: Antiproton cooling efficiency observed as a function of the time spent by the $\bar{p}$ together with electrons before opening the high-voltage trap endcaps. Two measurements are shown, obtained in the 2015 and 2016 AEGIS runs with different cooling electrons conditions: $5.5 \cdot 10^8$ (black) and $8.5 \cdot 10^8$ (white), and different reduction voltage (RV) and coolign trap potential (CT). No external heating was applied.

Typical standard settings of 2015 and 2016 runs for the electron cooling time were $t_{\text{cool}} = 40 - 50$ s.

6.3.4 Cold antiproton lifetime and vacuum

Cold antiprotons lifetime is well-known to be bound to the vacuum quality inside the traps. Measuring vacua of $\sim 10^{-13}$ mbar or less in a high-magnetic and cryogenic region is an experimental problem that currently has no technological solution. One possibility is to make indirect measurements of the vacuum quality from the outside using a commercial ion gauge placed in the SUN entrance region reaching 10^{-11} mbar.

Alternatively, cold $\bar{p}$ lifetime can be used as ultra-high vacuum gauge (a widely adopted solution within the AD community). Antiproton plasmas lifetimes can be interpreted as an absolute vacuum measurement, given that the relative content of residual gases is known⁸ to

⁸residual gas is almost entirely made of hydrogen and helium at typical operating pressures and temperatures

account for the different $\bar{p}$ annihilation cross-sections on light elements [121, p. 9, 32].

With nominal vacuum conditions, cold antiproton lifetimes of several tens of minutes are usually achieved. An example is shown in Figure 6.10, panel (a), where the storage time of cold antiprotons for early 2015 and 2016 runs is shown⁹. If vacuum worsens (spontaneously, due surface degassing, or as consequence of vacuum leak), cryogenic surfaces saturation could occur causing an immediate reduction of cryogenic pumping, usually observed in a reduced cold storage time of antiprotons independent of the position of the storage trap (see Figure 6.10, panel (b)).

6.3.5 Centrifugal separation of electron-antiproton plasmas

Centrifugal separation is a well-known multi-species non-neutral plasma effect commonly observed in laser-cooled trapped ions experiments (see, for instance, [129]). It occurs when the plasma mixture approaches low temperatures: species with different masses distribute in the trap radially so that larger masses sit at larger radii, as a consequence of the centrifugal force $F_r \propto m\omega_r^2 r$ proportional to the mass (to many extents equivalent to a mechanical centrifuge [130]).

It has been observed also in mixed plasmas of electrons and antiprotons by the ATRAP collaboration [130] and ALPHA collaboration [131]. According to the simplified model reported in [130] and [131], the separation occurs as the temperature of the mixed plasma approaches the centrifugal energy difference

$$T_{sep} = \frac{1}{2k_B}(m_p - m_e)\omega_p^2 r^2 \approx \frac{m_p e_0^2}{8\epsilon_0^2 k_B} \left(\frac{n_e r_p}{B} \right)^2 \quad (6.3.9)$$

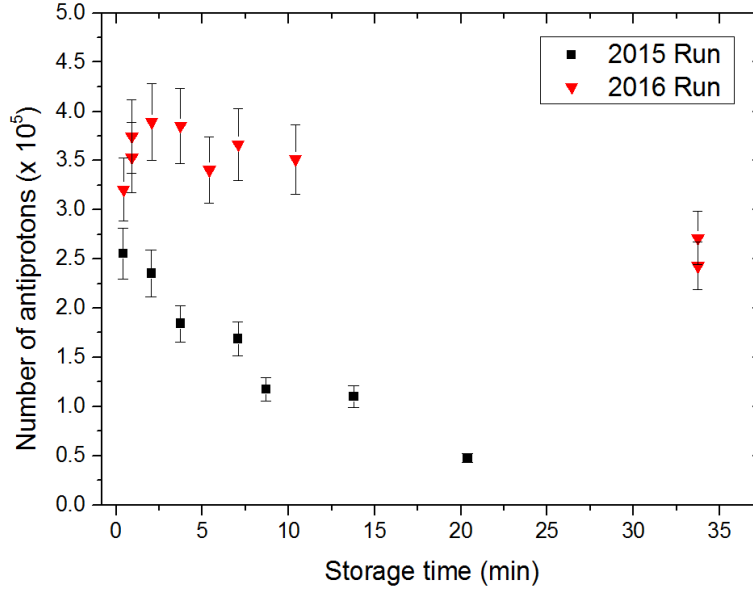
under the two approximations of $m_p \gg m_e$ and $\omega_p \approx e_0 n_e / (2\epsilon_0 B) \ll \omega_c$ valid in the long plasma limit.

Establishment of centrifugal separation has been observed in AEGIS after standard electron cooling procedures. A comparison of the radial profiles of an antiproton plasma with its electron cooling plasma is shown in Figure 6.11, panel (a). The two shown radial profiles were obtained with the same plasmas in two subsequent runs (once imaging e^- , once $\bar{p}$), radially averaging the 2D distributions acquired on the 1TMCP.

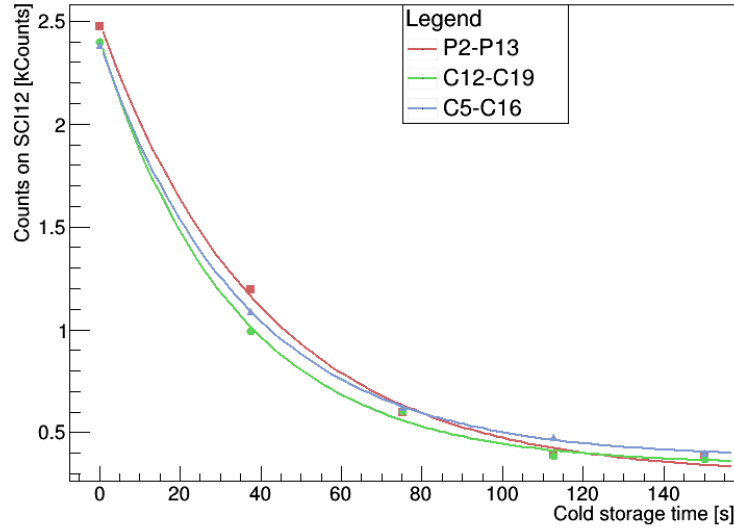
In Figure 6.11, panels (b-c), the centrifugal separation formation dynamics is shown as a function of the electron cooling time (2D distributions and their radial profiles). The experimental

of AEGIS

⁹the vacuum quality in early 2016 run was higher than during 2015 run, where an open vacuum leak was later found in the experiment



(a) Cold $\bar{p}$ lifetime under good vacuum conditions.



(b) Cold $\bar{p}$ lifetime under bad vacuum conditions.

Figure 6.10: (a) cold antiproton lifetime in 5T P-trap in 2015 (black) and 2016 (white) runs. (b) cold antiproton lifetime in different regions of 5T traps under bad vacuum conditions. Plotted the relative height of the $\bar{p}$ signal as a function of the storage time.

procedure was the same described in the last paragraph to measure the cooling efficiency. The $\bar{p}$ distribution was sent over the 5TMCP to image the entire plasma. An estimate using Equation 6.3.9 of the separation temperature indicated $T_{\text{sep}} = 15.3 - 41.7 \text{ K}$ after $\sim 70 \text{ s}$ spent together with e^- in the cooling trap.

If, on one side, observing centrifugal separation was a positive indication that antiprotons were effectively cooled by electrons, it however caused $\bar{p}$ to spatially migrate to a radius where the underlying electron distribution density was orders of magnitude lower. Since the collision rate γ_c is proportional to the electron density n_e (see Equation 6.3.4), sympathetic cooling is expected to become less efficient by the same ratio.

6.4 Electron-antiproton plasma compression

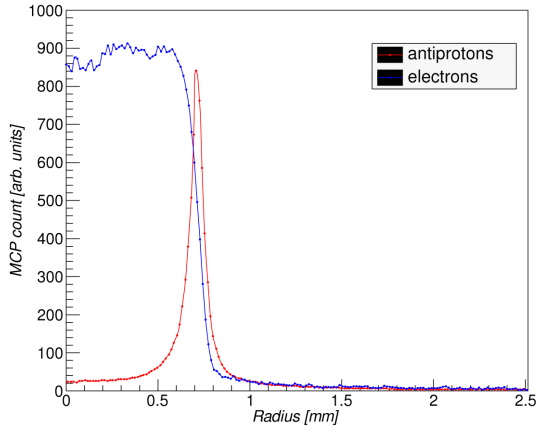
The third part of the procedure to prepare a plasma of antiprotons in the final production trap consisted of reducing the $\bar{p}$ cloud radial dimension to $\approx \text{mm}$ radius to allow the plasma to be transferred to the 10mm-diameter 1T traps.

The method adopted here is similar to the one developed by the ALPHA collaboration (see also [132], also outlined in [124]). It consists of applying rotating-wall to the mixed $\bar{p}$ - e^- plasma as if electrons were alone in the trap, i.e. assuming that the $\bar{p}$ contribution to the total plasma density is negligible due to their low number.

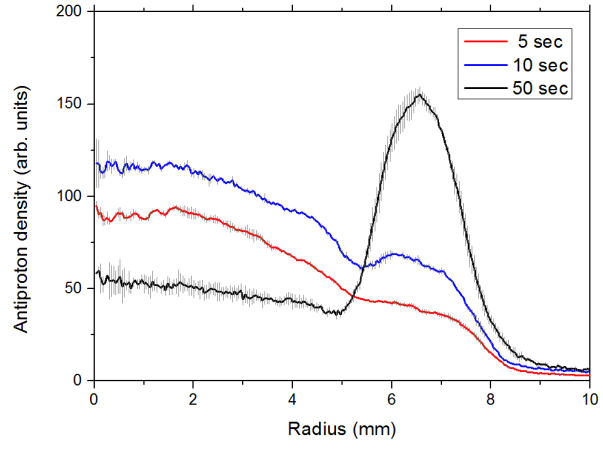
Antiprotons, whose radial equilibrium with electrons is maintained by the frequent Coulomb collisions between the two species, will tend to follow the electron distribution as it changes density during the application of the RW drive, if the density change happens slow enough to let the plasma re-equilibrate.

The conditions for applying rotating-wall with success on $\bar{p}$ - e^- plasma were found empirically using the experience gained in compressing electron to sub-mm radii (see 6.1.5). The first experiments, conducted on centrifugally separated $\bar{p}$ - e^- plasmas (already mentioned in 6.3.5 and shown in Figure 6.11), gave poor results. Even if electrons in the center were nicely compressed to small plasmas of $\sim \text{mm}$, antiprotons lying on external rings were observed to follow the electrons only partially. Instead of ending up in a significant fraction in the same compressed core as electrons, antiprotons assumed a peculiar distribution with a small compressed core of the same radius of the e^- plasma and a large pedestal with the same external radius as the original ring.

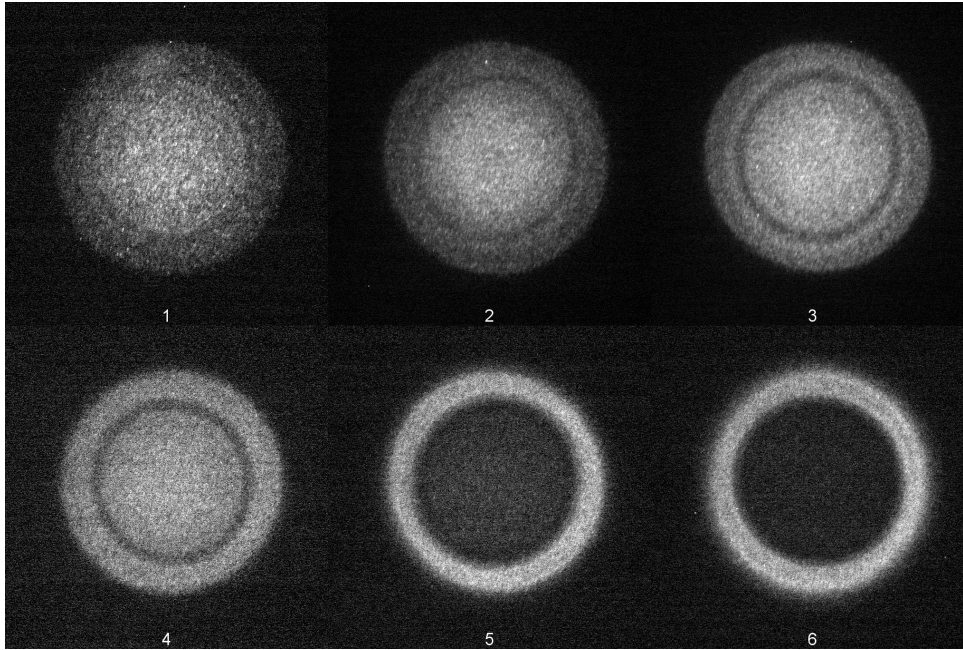
Some time was spent exploring rotating-wall parameters (frequency, amplitude and time) using centrifugally separated $\bar{p}$ - e^- plasmas without success in removing the pedestal and increasing the fraction of $\bar{p}$ in the core. The drawn conclusion was that centrifugal separation had to



(a) $e^-/\bar{p}$ radial profiles



(b) $\bar{p}$ only radial profiles



(c) 2D distributions at different cooling times

Figure 6.11: Establishing of centrifugal separation in a mixed electron/antiproton plasma. a) radial profile of antiproton and electron distributions imaged in two subsequent runs with the same trapping conditions (courtesy of D. Krasnický) b-c) distribution of antiprotons observed on the 5TMCP after electron cooling and removal of electrons for different cooling times (2 s, 5 s, 10 s, 20 s, 50 s, 70 s) and comparison of the radial profiles obtained at 5 s, 10 s, 50 s.

prevented from the beginning (i.e. during electron cooling): it was causing both the antiprotons to drift in the opposite direction as wished, and to equilibrate at a radius where the electron density was orders of magnitude lower than the center. A lower electron density caused the $\bar{p}$ - e^- collision rate Equation 6.3.4, i.e. the characteristic time for the plasmas to exchange energy by collisions, to reduce by the same amount (as it depends linearly on the electron density n_e). This reduced the antiprotons' capability to follow electrons during their compression, and a fraction was left behind forming the pedestal.

The establishing of centrifugal separation between the two species was prevented by introducing an external source of heat that kept the electron plasma at an higher temperature than the separation temperature T_{sep} (see Equation 6.3.9). Antiprotons, which spontaneously thermalize to the same temperature of electrons according to the model described in 6.3.1, were thus impeded to cool down and separate during electron cooling.

Various technical solutions could be adopted for heating up the electron plasma. Due to limitations of the available hardware, the same RF generator used for rotating-wall was used. Applying a low-frequency rotating-wall drive to the electron plasma for 30s before capture and keeping it on during electron cooling was effective in keeping the plasmas hot and remove centrifugal separation (a typical used setting was 300 kHz frequency and 2 V amplitude).

This solution was promptly obtained with the available hardware and had the extra advantage of precisely setting the initial density and radius of the cooling electrons. It came, however, with the price of reducing the radial overlap between electrons and antiprotons during electron cooling. The almost 100 % cooling efficiency reported in 6.3.3 (and shown in Figure 6.9) was reduced, on average, to $\sim 55\%$. In principle, 100 % cooling efficiency could be recovered studying a different method to keep the plasmas at higher temperature than T_{sep} , without altering the electron plasma initial radius.

As soon as centrifugal separation of the electron-antiproton plasma was prevented introducing this low-frequency rotating-wall drive on electrons before and during electron cooling, the antiproton distribution at the end of electron cooling was found overlapping the one of electrons. Subsequent antiproton compression procedures started showing promising results, as the pedestal was removed and $\bar{p}$ were effectively compressed to the same core of electrons. Antiproton compression procedures, described in the following sections, were studied and developed from this starting point.

6.4.1 Direct rotating-wall compression of mixed electron-antiproton plasmas

The first procedure studied to compress antiprotons to sub-mm radii used a single step of rotating-wall compression at constant frequency and amplitude on the initial non-separated $\bar{p}$ -

e^- plasma after electron cooling with heating (see the discussion above). The rotating-wall frequency and amplitude were simply changed in real-time to the values used for compression without turning off the drive (this possibility is a nice feature offered by rotating-wall controllers based on a Direct Digital Synthesis (DDS) DAC, which allows the frequency to be changed arbitrarily while phase is kept coherent by a local oscillator).

After applying rotating-wall directly in the electron cooling P2-P13 trap (made 300 V deep to avoid the risk of axial losses), a fast axial reshaping to the shorter P3-P9 trap (roughly half the length than P2-P13, still with two pulsable electrodes as endcaps) was introduced right before particle extraction. This step was found necessary for a physics reason: the P2-P13 trap, constituted of 10 short electrodes of the P-trap region (see 3.4 and Figure 3.7), had to be opened with such a long train of pulses in order to remove all electrons and image the antiproton distribution alone, that a fraction of antiprotons was also ejected from the trap. On the contrary, no losses were observed when removing electrons from the P3-P9 trap, so this trap was adopted as standard ejection trap during antiproton compression studies (a detailed discussion of P3-P9 trap potential is given in 9.5.1 and shown in Figure 9.4).

The full sequential list of operations in the first version of the antiprotons compression procedure was the following:

- standard electron loading with reduction of the number of electrons (reduction voltage -40 V, see 6.1.4);
- rotating-wall on electrons before $\bar{p}$ capture to keep the e^- plasma at high temperature and a controlled radius/density (RW drive $t_{RW} = 30$ s, 300 kHz, 2 V), see 6.1.5;
- $\bar{p}$ capture with e^- cooling in a 140 V deep trap located the P2-P13 region for 40 s, see 6.2;
- removal and measurement of the hot $\bar{p}$ fraction from the trap on SC12 for monitoring the capture conditions;
- increase of the trap depth to 300 V to eliminate the risk of axial losses during compression;
- change of rotating-wall frequency and amplitude in a range from 300 kHz to 2.7 MHz to compress the mixed plasma;
- reshaping of the RW trap to the shorter P3-P9 trap used for particles ejection in ~ 2 s with -10 V trap bottom;
- electrons ejection with 30 short pulses of 41.6 ns to let all e^- leave the trap, either towards the 1TMCP (to image the e^- plasma) or towards the DegFC (to count the e^-);
- opening of the trap to let also $\bar{p}$ escape, either with 10 long pulses of 2.66 μ s towards the 1TMCP (to image the $\bar{p}$ plasma) or with a slow potential ramp towards the DegFC to count the annihilations on SC12.

Using this procedure, $\sim 50\%$ of the initial cooled antiprotons were compressed together with e^- up to a limiting HWHM radius of 0.8 mm, corresponding to a drive frequency $f_{RW} = 2.7$ MHz

(see Figure 6.12, panel (a)). Antiprotons were observed to follow the electron cloud during compression, as no difference between the two radii was observed. A compression limit of about ~ 0.8 mm was observed, after which no further change was obtained increasing the drive frequency. At even higher frequencies than 2.7 MHz, compression was suddenly not obtained at all, similarly to what observed with positrons in the C-trap (see Figure 5.11) and by other experiments in the field (see, for example, [133]).

Centrifugal separation of the compressed $\bar{p}$ - e^- plasma quickly established in the P3-P9 trap after compression. Plasma were observed to be completely separated after only 1 s between switching off of the RW drive and electron plasma extraction (see, as an example, Figure 6.11, panel (a)).

6.4.2 Rotating-wall compression reducing the electron number

The first procedure to compress antiprotons, described in the last section, showed a compression limit of 0.8 mm, which was overcome by developing a second procedure which removes part of the initial cooling electrons from the trap after electron cooling was complete. The argument beyond this partial electron removal lies in a simplified model of the action of a rotating-wall drive on the cloud of particles, assuming the $\bar{p}$ contribution to the total plasma density and potential to be negligible.

Rotating-wall has been observed in literature to be working in various regimes according to the different couplings between plasma and RF drive (see [117, 118, 134] for experimental and theoretical works on rotating-wall coupling to plasma oscillation modes, and [133] for the so-called strong drive regime). Abstracting from the details of each regime in which rotating-wall operates, the overall effect is always to establish a proportionality relation between the external RF drive frequency f_{RW} and the plasma rotation frequency $f_{\text{E} \times \text{B}}$ of the type

$$f_{\text{E} \times \text{B}} = \alpha + \beta f_{\text{RW}} \quad (6.4.1)$$

where α and β are connected to the specific type of coupling between plasma and drive. For example, the simple equality $f_{\text{E} \times \text{B}} = f_{\text{RW}}$ holds if RW operates in the strong drive regime ($\alpha = 0$ and $\beta = 1$), as discussed in [133]¹⁰.

$f_{\text{E} \times \text{B}}$, for typical particle densities that can be achieved in Malmberg-Penning traps (up to

¹⁰expressions for α and β when the coupling goes through plasma modes are more intricate, and are found also in [133], which itself refers to [134].

$\approx 10^{16} \text{ cm}^{-3}$), depends linearly upon the electron plasma density n_e ¹¹

$$f_{\text{E} \times \text{B}} = \frac{e_0}{4\pi\epsilon_0 B} n_e \quad (6.4.5)$$

Equating 6.4.1 and 6.4.1 yields a general linear relation between the rotating-wall drive frequency f_{RW} and the electron density n_e

$$n_e = \frac{4\pi\epsilon_0 B}{e_0} (\alpha + \beta f_{\text{RW}}) \quad (6.4.6)$$

In the approximation of a infinite long electron plasma column at $T \rightarrow 0$, the density is simply the ratio between the number of particles N_e and the cylindrical volume occupied $V_p = \pi r_p^2 L_p$, where r_p is the cylinder radius and L_p the cylinder length. Substituting and rearranging the terms gives

$$r_p^2 = \frac{N_e e_0}{4\pi^2 \epsilon_0 B} \cdot \frac{1}{\alpha + \beta f_{\text{RW}}} \quad (6.4.7)$$

This simplified model shows that the application of a rotating-wall drive at frequency f_{RW} , whatever its working regime, fixes the plasma density according to Equation 6.4.6. If the total number of electrons in the trap N_e is reduced keeping the frequency constant, the final plasma radius is also reduces $\propto \sqrt{N_e}$ as a result of 6.4.7. This lead to develop a second procedure based on the partial removal of the initial electrons after electron cooling, when they are no more necessary in high amounts to cool effectively antiprotons through collisions (Equation 6.3.4).

¹¹following the derivation of D. Dubin in [135, p. 99], an explicit expression for the plasma rotation gyro-frequency $\omega_{\text{E} \times \text{B}}$ is obtained solving the equation of force balance in the trap. Calling $\Omega_c = e_0 B / m_e$ the cyclotron gyro-frequency and $\omega_p = \sqrt{(e_0^2 / \epsilon_0 m) n_e}$ the plasma frequency, the plasma rotation frequency is

$$\omega_{\text{E} \times \text{B}} = \frac{\Omega_c \pm \sqrt{\Omega_c^2 - 2\omega_p^2}}{2} \quad (6.4.2)$$

The square root is now expanded in series using the approximation $\Omega_c \gg \omega_p$. This approximation is valid for all practical values of the density n_e , since for $B = 4.46 \text{ T}$, the cyclotron gyro-frequency for electrons is $\Omega_c = e_0 B / m_e = 2\pi \times 124 \text{ GHz}$, and even with a very high electron density of $n_e \propto 10^{16} \text{ m}^{-3}$, the plasma gyro-frequency $\omega_p = \sqrt{(e_0^2 / \epsilon_0 m) n_e} \approx 0.9 \text{ GHz}$. Thus

$$\omega_{\text{E} \times \text{B}} = \Omega_c \pm \Omega_c \left(1 - \frac{1}{2} \frac{\omega_p^2}{\Omega_c^2} \right) \quad (6.4.3)$$

Choosing the solution with lower rotation gyro-frequency (i.e. with minus sign), and substituting the definition of plasma gyro-frequency $\omega_p^2 = e_0^2 n_e / (\epsilon_0 m)$, yields

$$\omega_{\text{E} \times \text{B}} = \frac{e_0}{2\epsilon_0 B} n_e \quad (6.4.4)$$

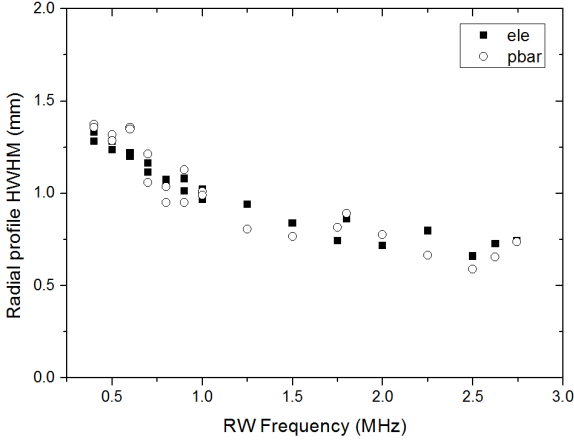
that is the linear relation between $f_{\text{E} \times \text{B}}$ and n_e seeked

The full sequential list of operations in this second version of the antiprotons compression procedure was the following:

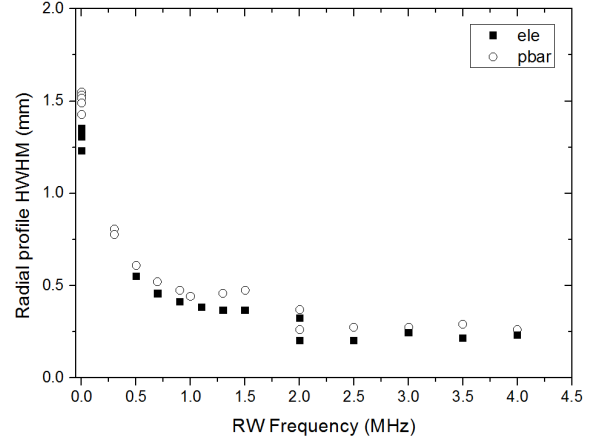
- standard electron loading with reduction of the number of electrons (reduction voltage -40 V, see 6.1.4);
- rotating-wall on electrons before $\bar{p}$ capture to keep the e^- plasma at high temperature and a controlled radius/density (RW drive $t_{RW} = 30$ s, 300 kHz, 2 V), see 6.1.5;
- $\bar{p}$ capture with e^- cooling in a 140 V deep trap located the P2-P13 region for 40 s, see 6.2;
- removal and measurement of the hot $\bar{p}$ fraction from the trap on SC12 for monitoring the capture conditions;
- removal of $\sim 3/4$ of the cooling electrons by temporarily splitting the trap in two asymmetric sections of 2 and 7 of the original 10 electrodes and removal of all e^- from the longer section;
- increase of the trap depth to 300 V to avoid axial losses during compression;
- application of two RW steps using two different constant frequencies and amplitudes;
- reshaping of RW trap to the shorter P3-P9 trap, with two pulsable electrodes as endcaps, in ~ 2 s with -10 V trap bottom;
- electron ejection with 30 short pulses of 41.6 ns to let all e^- leave the trap, either towards the 1TMCP (to image the e^- plasma) or towards the DegFC (to count the e^-);
- opening of the trap to let also $\bar{p}$ escape, either with 10 long pulses of 2.66 μ s towards the 1TMCP (to image the $\bar{p}$ plasma) or with a slow potential ramp towards the DegFC to count the annihilations on SC12.

This second procedure was able to compress $\sim 45\%$ of the initial cooled antiprotons up to a limiting HWHM radius of 0.3 mm, corresponding to a drive frequency of the second rotating-wall step of $f_{RW2} = 4$ MHz (see Figure 6.12, panel (b)). Antiprotons were observed as well to follow the electron cloud during compression, as no difference between the two radii was observed.

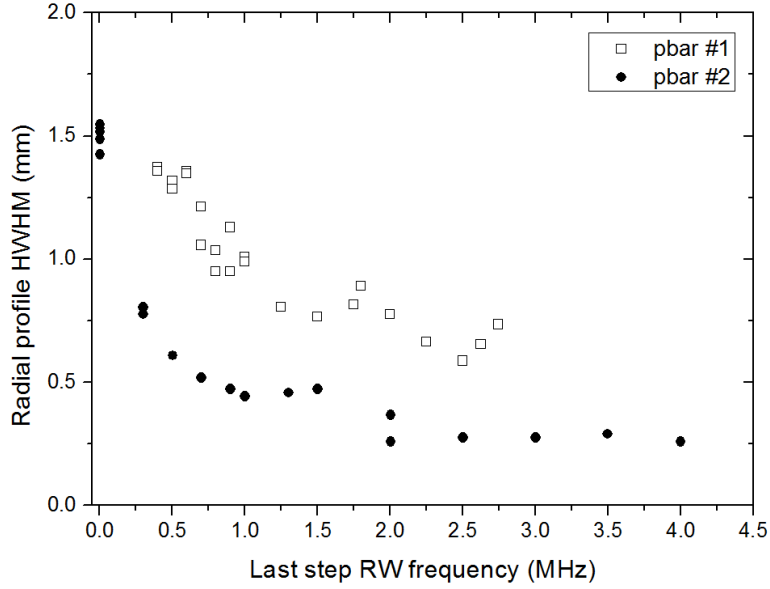
This second procedure was selected for developing the $\bar{p}$ ballistic transfer procedures to the 1T production trap. At the beginning, 0.8 MHz were used as standard setting for the last RW frequency step, equivalent to a plasma radius HWHM of ~ 0.5 mm. Then, after the procedure was developed, the best compression results were used to achieve the best storage time results.



(a) Comparison of $\bar{p}$ and e^- radii for procedure #1



(b) Comparison of $\bar{p}$ and e^- radii for procedure #2



(c) Comparison of the $\bar{p}$ radii for the two compression procedures

Figure 6.12: Results of antiproton compression procedures with/without initial cooling electrons removal. Statistical error bars are smaller than the markers.

6.4.3 Centrifugal separation after rotating-wall

A neat, direct measurement of centrifugal separation forming up dynamics after the end of antiproton RW in the compression procedure described above was made impossible by the necessity to change trap from P2-P13, used for electron cooling and compression, to P3-P9, used for particle ejection. This reshaping operation, which was necessary to remove the e^- without incurring in $\bar{p}$ losses and image either one or the other distribution, was significantly altering the plasma densities. Thus, it would have been very difficult to disentangle the forming up dynamics in P2-P13 from spurious heating effects introduced by the reshaping operation.

This measurement was indeed obtained during the 2016 run, after the installation of a fourfold sectorized electrode in place of the B0 electrode of the Big-trap. This electrode, meant for correcting the 5T and 1T trap misalignment during ballistic transfers by inducing a controlled $\vec{E} \times \vec{B}$ drift of the $\bar{p}$ cloud (see 6.5.2), allowed also to spatially separate antiprotons from their cooling e^- s as they were flying towards the 1TMCP, after a single extraction from the untouched cooling trap. This is because $\bar{p}$ drift for more time than their cooling e^- because of their different mass: in this way, the species can be separated by several mm (see Figure 6.20).

The mutual extraction was performed using a new pulser controller developed for $\bar{p}$ ballistic transfers, which was able to synthesize long pulses up to 64 μ s suitable for both e^- s and antiprotons.

The compression procedure was modified so that the time delay between RW switching off and plasma extraction could be varied. Results, shown in Figure 6.13, confirm that centrifugal separation is established with characteristic times of $\propto s$. This results are in line with the one observed in [131, p. 2], which reports plasma distributions very similar to the ones observed in AEGIS. Furthermore, the characteristic separation time has been seen varying from < 1 s to ~ 5 s throughout the 2016 run. An unknown source of heat was supposed responsible for this variation, but not clearly identified.

Performing further analyses of these distributions, calculating their hollowness and comparing them to a two components plasma thermal equilibrium numerical model, as done in [131], could lead to an accurate measurement of the cooling dynamics of the two plasmas. This motivated developing a numerical code for plasma thermal equilibria (see 9).

6.4.4 Observation of $l > 1$ diocotron instabilities

Plasma ring decays through multiple vortices induced by the diocotron instability (the plasma equivalent of the Kelvin-Helmoltz instability for fluids, see [136, chap. 6]) were observed on single-component plasma of electrons in many literature works [137, 138].

Vortices decays were never reported for antiprotons by experiments in the AD so far. The only evidence of an antiproton ring decay through the diocotron instability found in literature was reported by the ALPHA collaboration [131, p. 5], which observed a $\bar{p}$ ring collapse through a $l = 1$ diocotron instability without vortices, when all the cooling electrons were removed from the trap.

In AEGIS, centrifugally separated $\bar{p}$ rings were routinely observed decaying through diocotron instabilities with $l \geq 2$. The many vortices progressively smoothed out until the center of the original ring was completely filled. An example of a ring decay dynamics was performed on the 1TMCP with a compressed plasma of antiprotons using the partial removal of cooling electrons procedure. Antiprotons were ejected from the trap with a variable delay time with respect to their cooling electrons, to image the evolution of the ring distribution (as shown in Figure 6.14), $\bar{p}$ ring decay through the $l = 2$ and $l = 3$ instabilities were observed for this particular initial distributions. Higher order instabilities (up to $l = 15$) were observed varying the size of the initial ring distribution.

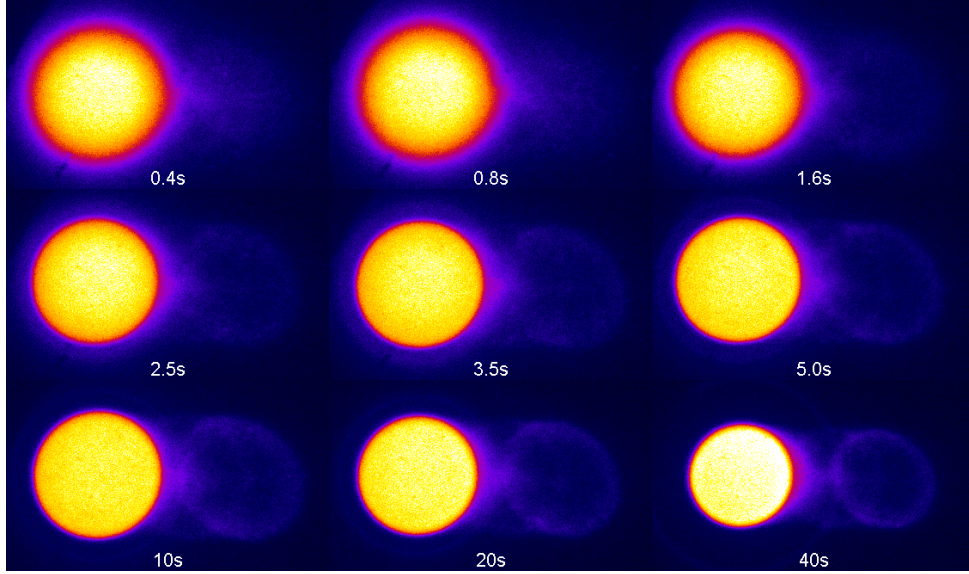


Figure 6.13: Observation of e^- and $\bar{p}$ distribution dynamics on the 1TMCP using the $\vec{E} \times \vec{B}$ electrode (see text) as a function of the cooling time spent together after RW drive was switched off. Centrifugal separation establishes after ~ 5 s.

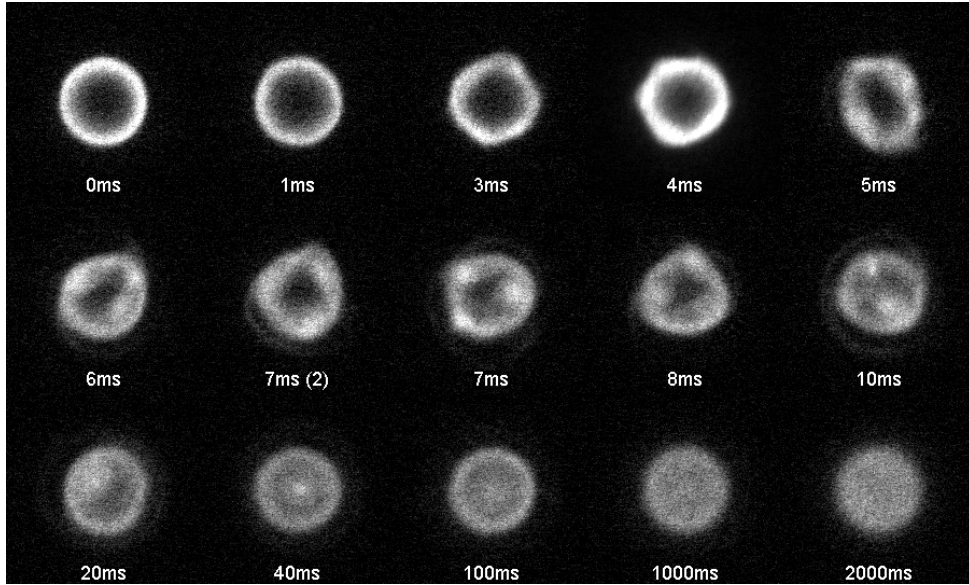


Figure 6.14: Observation of $\bar{p}$ ring distribution evolution on the 1TMCP after removal of the central cooling electrons. The ring decays forming vortices due to diocotron instability, ultimately ending up in a uniform disk in ~ 100 ms.

6.5 Ballistic transfer of antiproton/electron plasmas

The final part of the experimental procedure to prepare a plasma of antiprotons in the final trap consisted of transferring in-flight the compressed $\bar{p}$ plasma in 5T traps to the 1T final traps by means of accurate synchronized voltage pulses applied on the electrodes. In this so-called *ballistic transfer* procedure, antiprotons are treated in subsequent operations performed quickly one after the other (see Figure 6.15 for a sketch of the three different phases):

1. $\bar{p}$ are first prepared in a short launch trap in the 5T P-trap region (P9-P13, see 3.4) using three short electrodes as trap bottom and a pulsable electrode as downstream endcap. At the same time, a second trap used for recatching is prepared in the 1T production trap (OnAx28-UC15) with a pulsable electrode as upstream endcap (see Figure 6.15, upper panel);
2. electrons are disbanded from the launch trap with 5 short pulses applied on the upstream endcap, just before ($\sim 150\mu\text{s}$) $\bar{p}$ are launched by the trap with a single, long pulse. At the same time, a ns synchronized pulse is applied also to the upstream gate of the recatching trap to open it, whose length is carefully selected so that its falling edge arrived at the right time to re-trap the $\bar{p}$. Antiprotons, as they fly through the entire trap stack, are furtherly accelerated by biasing all electrodes to 100 V (or in some cases even higher) to make them travel as quick as possible. The $\bar{p}$ bunch is also aligned in the $\hat{x} - \hat{y}$ plane to the exact axis of the 1T traps by inducing a controlled $\vec{E} \times \vec{B}$ radial drift during the flight inside the sectorized B0 electrode, as described in 6.5.2 (see Figure 6.15, middle panel);
3. antiprotons, as they arrive in the region of the 1T production trap, are slowed down by the bias voltage applied on the electrodes of the bottom region of the recatching trap. When they are all contained within the recatching trap, the upstream endcap is raised again to the previous voltage as the end of its opening pulse is reached, trapping the bunch in the recatching region (see Figure 6.15, bottom panel);.

6.5.1 Launch trap

Transferring in-flight and recatching antiprotons strongly benefits from having the bunch short in length $\Delta z'$ and time $\Delta t'$ at the moment of arrival in the recatching trap and, possiblity, with the narrowest axial energy distribution ΔE as possible. The first two requirements, clearly, are due to the fact that the spatial extent of the recatching trap is limited (roughly 8 cm). The shorter the bunch in both space and time, the easier would be to find the right timing to close the electrode without reflecting back a part the of the bunch.

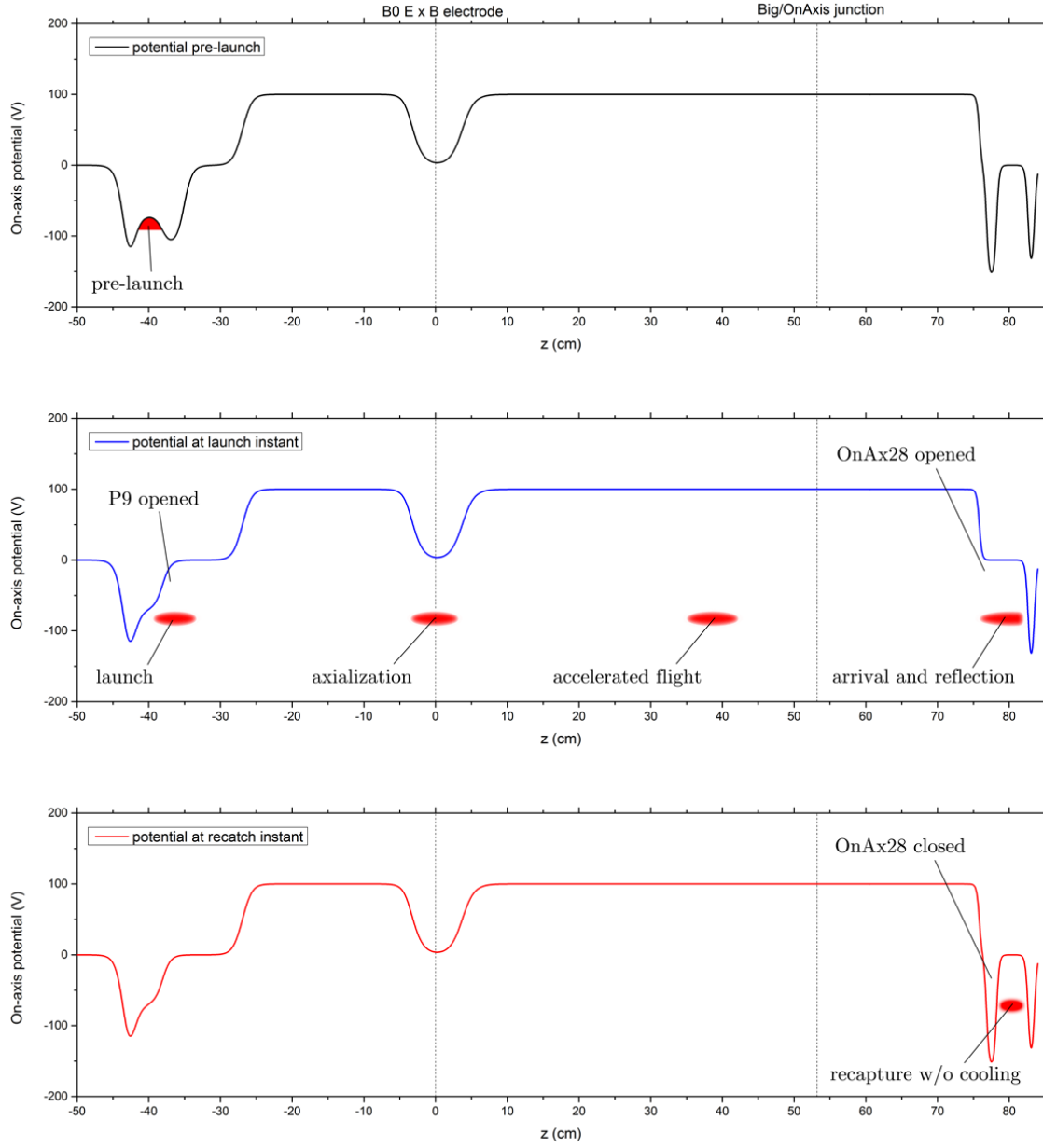


Figure 6.15: Sketch of the antiproton in-flight ballistic transfer procedure.

One could argue, as already quickly mentioned above, that $\Delta t'$ for closing the trap can be always made longer slowing down the bunch by a bias voltage on the flat part of the recatching trap. This argument is applicable only up to the point $\bar{p}$ start being reflecting back, which happens when the slowing-down potential touches from below the axial energy distribution of the flying cloud. In order not to preclude the possibility to recatch 100 % of the arriving particles, the slowing potential must be set some ΔE away from the average axial energy.

The value of the axial energy spread ΔE is - at first sight - set by the plasma equilibrium distribution inside the launch trap and the potential ramp used to extract the plasma. Particles at different z positions accelerate up to an axial energy equal to the potential they sit into at the moment of the launch: for example, an ideal plasma cylinder of 3 cm, extracted with a linear potential ramp of 10 V cm^{-1} , would show an energy spread $\Delta E = 30 \text{ V}$. Yet, one gains nothing in varying the launch potential ramp to reduce ΔE (for example, making it less steep), as the time distribution Δt would be then increased. This is a direct consequence of Liouville's theorem ($m\Delta v_z \Delta z = \Delta E \Delta t = \text{const}$) which governs the evolution of the distributions, as the total energy is conserved through the flight. Thus, the only real levers to change the initial energy in the trap E and its spread ΔE are given by varying the initial plasma conditions in the launch trap.

Antiprotons, after being radially compressed to $\propto \text{mm}$ with the procedure described above, were axially squeezed from the long Malmberg trap used for rotating-wall compression (P2-P13) to a short trap (P9-P13) composed of three short electrodes as bottom part of the trap (P10-P12) and two pulsable electrodes as endcaps, where the upstream endcap (P9) is again a short electrode and the downstream endcap (P12) is a long one.

The choice of this launch trap was preferable for two reasons. First, its short length reduced as much as possible the $\bar{p}$ bunch initial spatial length Δz . Furthermore, the choice of a long electrode as downstream endcap was done on purpose, as its influence on the trap potential is roughly twice as much stronger than short electrodes. The application of a single launch pulse on P12 is effective in applying an axial accelerating electric field over the whole length of the P10-P12 trap where the $\bar{p}$ plasma sits (compare Figure 6.15, upper and middle panels for a visual reference), and so the required time Δt for extracting all antiprotons is also minimized. An analogous technical solution to minimize Δt would be to apply many contemporary pulses to all electrodes of the launch trap (similarly to what is done in the positron accumulator at e^+ extraction, see 4.1.3 and Figure 4.4), but this is technically more complex to achieve.

The launch energy E of the $\bar{p}$ was chosen according to the potential applied on the three bottom electrodes P10-P12. In principle, the higher E , the shorter the transit time in the free-flight region, the lower the spatial expansion of the cloud. Practically, ballistic transfer experiments were conducted using 70 eV launch energy, which was close to the highest setting

that could be achieved with the endcaps set at -115 V (the limitation was given by P9, which was close to maximum operating value at -180 V ; as it is a short electrode, the potential in its center is roughly $\sim 70\%$ of the potential on the borders, see Figure 6.15). The bunch was furtherly accelerated during its flight through the $\sim 1\text{ m}$ traps stack by biasing all the electrodes in the stack by 100 V , with the exception of B0 (marked with a vertical dashed line in Figure 6.15, which requires a longer transit time for effective axialization (see 6.5.2)).

A train of 5 pulses of 41.6 ns length was applied on the P9 electrode $400\text{ }\mu\text{s}$ before transferring antiprotons to remove the electrons present in the trap at the moment of the launch, in order to avoid ending up with an electron plasma trapped withing a spurious trap along the trap stack. The minimal number of pulses to be used was determined imaging the centrifugally separated plasma on the 1TMCP and using B0 to displace $\bar{p}$ from e^- (as discussed in 6.4.3 and shown in Figure 6.13). For $N_p \geq 5$ pulses, only the $\bar{p}$ distribution was observed, while a fraction of e^- was still present for $0 < N_p < 5$.

The number of antiprotons on SC12 after electron removal was compared to the number obtained after $\bar{p}$ compression, before reshaping to P9-P13, and it was found unchanged. In this way, the setting up of the launch trap was verified not to be source of losses of antiprotons, despite the temporary opening for electron removal.

Two measurements of the $\bar{p}$ bunch energy spread ΔE and time spread Δt were performed using OnAx28 electrode as a timed gate and the peak of the detected by the 1TMCP, relative to the transmitted fraction of antiprotons. In the former case, the electrode potential was varied within the interval -90 V and -50 V observing the transmitted fraction (see Figure 6.16, panel (a)). In the latter case, the electrode was completely blocking, and the time was varied between 6000 ns and 7500 ns after the launch (see Figure 6.16, panel (b)).

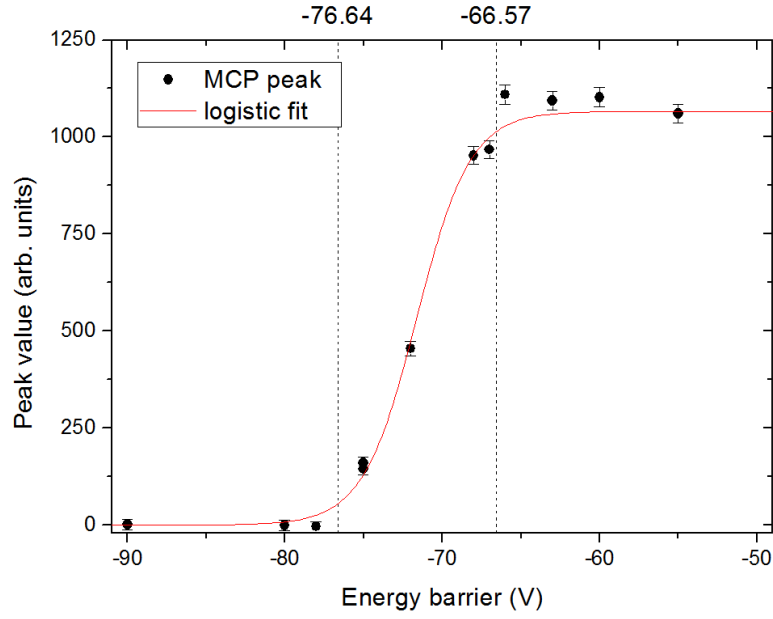
The average transfer energy and the total energy spread 5-95 % were measured to be

$$\begin{aligned} E_0 &= (71.63 \pm 0.28_{\text{stat}}) \text{ eV} \\ \Delta E &= (10.07 \pm 0.40_{\text{stat}}) \text{ eV} \end{aligned} \tag{6.5.1}$$

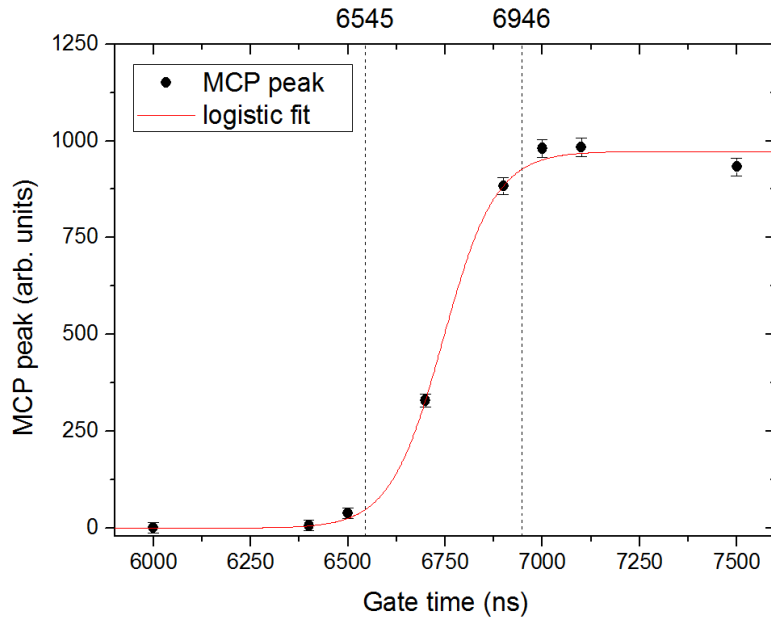
The time-of-arrival at OnAx28 and the total time spread 5-95 % were instead found

$$\begin{aligned} t_0 &= (6744 \pm 8_{\text{stat}}) \text{ ns} \\ \Delta t &= (401 \pm 11_{\text{stat}}) \text{ ns} \end{aligned} \tag{6.5.2}$$

These values were used to set the proper slowing potential and the proper timing to close OnAx28.



(a) $\bar{p}$ energy spread at OnAx28



(b) $\bar{p}$ time spread at OnAx28

Figure 6.16: Measurement of the $\bar{p}$ bunch energy spread (panel a) and time spread (panel b) using the OnAx28 electrode, just before entering the recatching trap region. In both cases, the signal is obtained using the peak value measured by the 1TMCP.

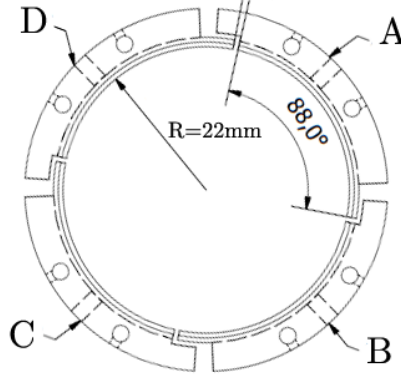


Figure 6.17: Sketch of sectorized B0 electrode showing the 45° tilted sectors with respect to AEgIS system of coordinates.

6.5.2 Antiprotons axialization through controlled $\vec{E} \times \vec{B}$ drift

Antiprotons position in the transverse plane had to be corrected in order to match the center of the 1T production trap, as a result of a mechanical misalignment between 5T and 1T trap axes, which could not be mechanically corrected. The solution was found replacing the first electrode of the Big-trap, B0, to a fourfold sectorized electrode, which was used to induce a constant dipole electric field in the transverse plane.

In this way a controlled $\vec{E} \times \vec{B}$ drift of antiprotons could be introduced. On the contrary, a $\times 43$ smaller displacement of the electrons position was expected due to the different transit time, consequence of the mass ratio.

The four electrode sectors were divided into two couples, labelled AC and BD. As the B0 electrode is 45° degrees azimuthally rotated with respect to the AEgIS reference $\hat{x}$ and $\hat{y}$ directions, the couple AC induces a drift in the $(\hat{y} - \hat{x})/2$ direction, the couple BD in the $(\hat{x} + \hat{y})/2$ direction (the drift direction is orthogonal to the sectors orientation, see Figure 6.17).

A first set of calibration measurements were performed using the $\bar{p}$ to measure the displacement obtained as a function of the potential V_0 applied on the sectors. Two separate measurements for the AC and BD couples were performed: if $+V_0$ was applied on sector A, $-V_0$ was applied on sector C in order to have the closest to ideal dipolar field. Antiprotons were launched with 70 eV energy without any electrons. The resulting ring-shaped image was imaged on the 1TMCP and the center of the $\bar{p}$ distribution was measured using a weighted average.

The center of the $\bar{p}$ distribution was found to be linearly proportional to the applied voltage on both electrode couples as expected (see Figure 6.18, panel (a)). The measured mm to V

conversion factor for an axial transfer energy of 70 eV was calculated from the measured linear interpolations, and found to be 1.38 mm/100V.

This calibration factors allowed the antiproton spill to be aligned with respect to the measured center of the 1T production trap using electrons. Proper alignment of the spill was easily obtained measuring the $\bar{p}$ displacement when all sectors were at 0 V on the 45° rotated frame of reference of the electrode, then calculating the voltage to be applied by means of the V $\rightarrow$ mm conversion.

This technique proved to a powerful tool for the recatching procedures, as it allowed to significantly reduce diocotron motion of the recaptured antiprotons and their subsequent radial expansion (Figure 6.20).

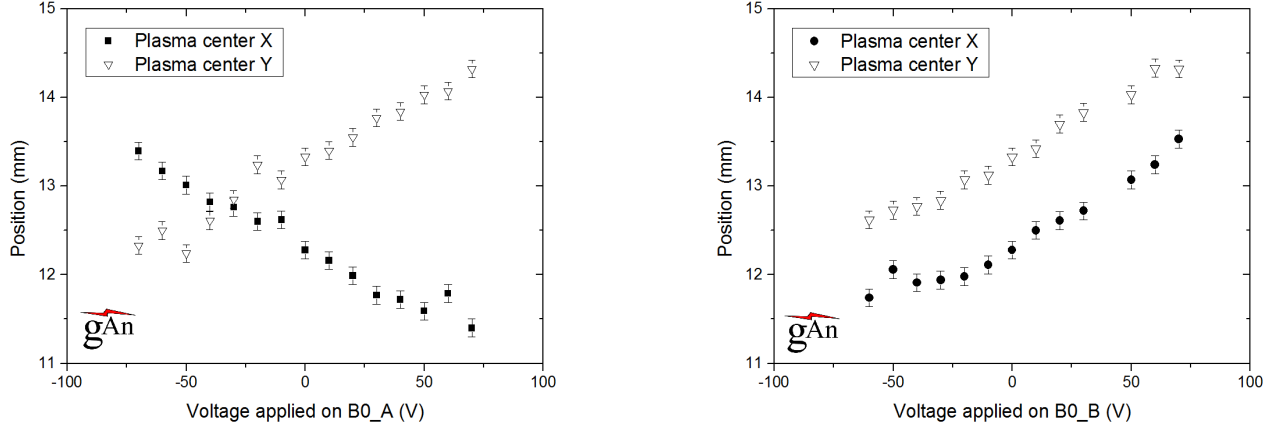
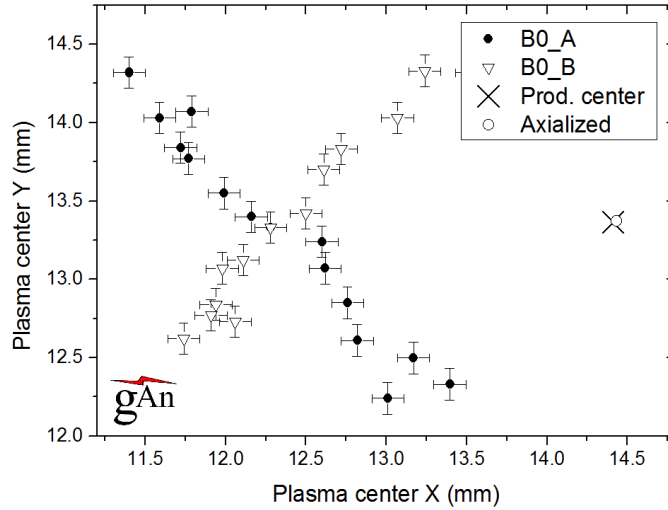
6.5.3 Recatching

Antiproton recatching in the 1T production trap was easily realized thanks to a deeply upgraded, in-house built FPGA-based pulse controller¹² which featured: 4x4 programmable multiplexed output (for a total 4 of parallel pulses on 16 possible electrodes); -200 V to 200 V voltage range with a vertical precision of 6 mV; single pulses from 1 ns to 64 μ s with a time precision of 1 ns; trains of pulses of arbitrary number with selectable duty cycle; accurate synchronization between the different channels with a precision of 1 ns to 3 ns (in-board or between boards); 2x4 programmable multiplexing of the input triggers for the allocation of the channels to the 5T and 1T trap controllers. The pulse controller was configured to act as an independent state machine configured and armed via RS485.

This state-of-art piece of electronics made the development of recatching procedures straightforward. The procedure was developed initially without axialization observing the annihilations of a fraction of the launched $\bar{p}$ at different z positions along the trap stack. The main detector sused were the 1TMCP and the digitized PMT2, as PMT20 was found saturating due to its much higher solid angle for annihilations happening on the 1TMCP. The digitized plastic scintillators provided a 4 ns time resolution diagnostic over the $\bar{p}$ annihilation peaks, and the digitizer was triggered synchronously with respect to the launch from the 5T traps.

Three annihilation peaks were clearly identified on PMT2 traces, according to $\bar{p}$ flight velocity of 71 eV and accounting for a 5.5 μ s delay of the acquisition (see Figure 6.19, upper plot), due to spurious antiprotons annihilations along the traps stack. Respectively, on the B0 electrode structure, at the junction between Big-trap and OnAxis-trap, where the electric field is not homogeneous (as shown also in Figure 6.15), and the last due to the antiprotons annihilating on the 1TMCP surface. The losses due to the B0 electrode were minimized optimizing the

¹²designed from scratch, implemented and tested by V. Lagomarsino


 (a) Plasma measured positions versus voltage applied on the two 45° tilted directions


(b) Result of the axialization procedure

Figure 6.18: Results of antiproton axialization procedure using controlled $\vec{E} \times \vec{B}$ drifts. Panel (a) shows the calibration measurements used to determine the mm V^{-1} conversion factor in the two 45° degrees rotated directions of the electrode sectors. Panel (b) shows the positions of the plasma centers in the $\hat{x} - \hat{y}$ plane for the calibration measurements, the expected center of the production trap and the final position of the $\bar{p}$ after axialization.

alignment of the actuator holding the B0 electrode, while the losses at the traps junction were significantly reduced when axialization was introduced. When antiprotons are reflected back from OnAx28, the image disappears on the 1TMCP and its annihilation peak on the spectrum as well (see Figure 6.19, bottom plot). Two other peaks at later times are observed on the

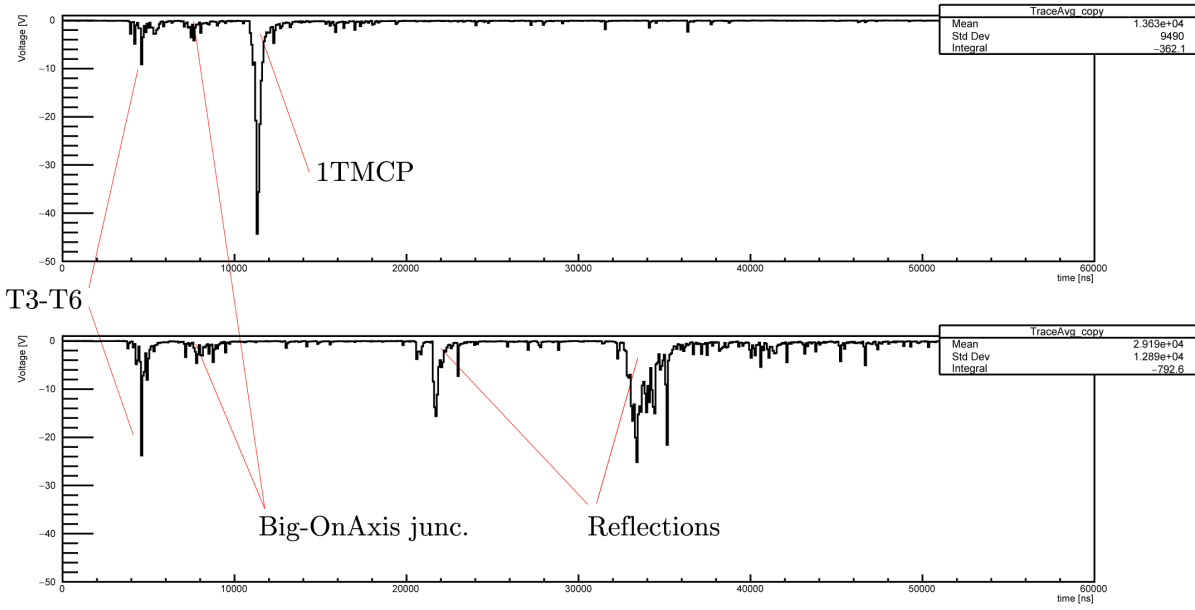
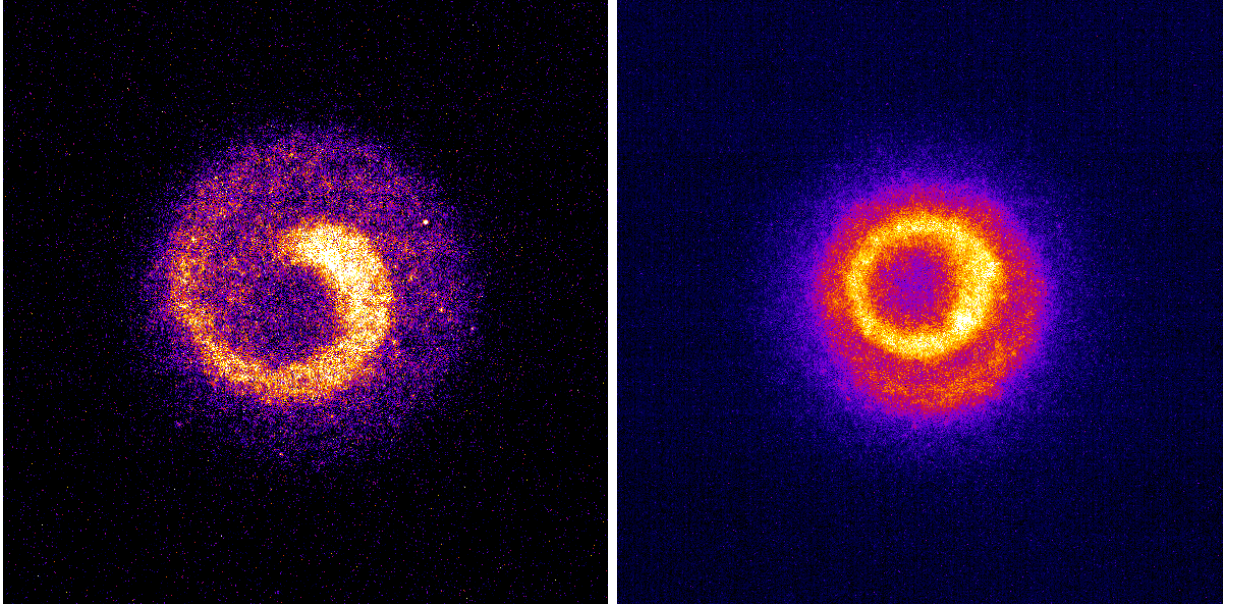


Figure 6.19: Antiproton annihilations along the flight path, as observed by PMT1/PMT2 averaged, while imaging the bunch on the 1TMCP (upper spectrum) and while reflecting the bunch back from OnAx28 (bottom). In the first case, the three identifiable peaks are due to spurious annihilations in the transfer section between 5T and 1T, the Big-OnAxis trap junction and the MCP. In the latter case, the MCP peak is substituted by two later peaks: $\bar{p}$ did respectively one and two others round trips in the long spurious trap incrementing their $\vec{E} \times \vec{B}$ drift each pass, ending up annihilating on the Big-OnAxis junction.

spectrum, consistent with two fractions of antiprotons doing respectively one and two others round trips in the long spurious trap between P9 and OnAx28, incrementing their $\vec{E} \times \vec{B}$ drift each pass B0, and finally ending up annihilating on the Big-OnAxis junction.

Recatching was first realized setting a bias voltage of 160 V on OnAx28 and superimposing a 8000 μ s, -160 V pulse, while letting the production trap bottom to ground. The length of this pulse was chosen according to the flight time measured in Figure 6.16. A second pulse applied on UC15 32 μ s later was used to open the production trap towards the 1TMCP. The $\bar{p}$ annihilation peak associated to the 1TMCP surface highly decreased, in favour of a new, delayed peak observed ~ 35 μ s later. Consistently, an image from the 1TMCP taken 32 μ s later showed a distribution of antiprotons decaying on the $l = 1$ diocotron instability ejected from the production trap (see Figure 6.20, panel (a)). The observation of this image constituted the first evidence of $\bar{p}$ recapture from the 1T production trap.

As the axialization was carefully tuned, the $l = 1$ diocotron instability almost disappeared in favour of a symmetric ring-shaped distribution (Figure 6.20, panel (b)), was much closer to the



(a) antiprotons 32 μs after recatching without axialization tuning (b) antiprotons 32 μs after recatching with axialization tuning

Figure 6.20: Reduction of $\bar{p}$ diocotron motion as an effect of the axialization procedure.

original ring-shaped distribution of $\bar{p}$ due to centrifugal separation in the 5T traps.

6.5.4 Recatching efficiency

The experimental difficulty in measuring the efficiency of the recatching operation, during its early stages of development, mainly consisted in dealing with the consequences of ballistically transferring a ring of antiprotons in the production trap (caused by the original centrifugal separation in 5T traps, as depicted in Figure 6.20), removed in a later development. Annular density distributions are unstable and tend to decay spontaneously to density distributions peaked on the axis through the diocotron instability, with measured characteristic times $\propto 10$ ms (as discussed in 6.4.4 and shown in Figure 6.14). This made the plasma unstable in the production trap, which quickly enlarged on a time scale of $\propto 50$ ms. The physical reason is simply angular momentum conservation: a cloud of charged particles, not subjected to an external torque, conserves $L \propto \sum_i^N r_i^2$ despite of spontaneous density variations in time (see the discussion in 9.1.1 and Equation 9.1.3). Since, in the case of ring decays, a significant fraction of the initial charge in the annulus ends up filling the hole, a proportional amount of charge has to fly outwards, causing the cloud to expand, hit the electrodes and annihilate.

The efficiency of the recatching procedure was thus estimated with the scintillator slab SC2122, which detects annihilations due to radial losses in the production trap under the same solid angle

as annihilations on the surface of the 1TMCP. Antiprotons were kept into the production trap for 2 s after recapture, then ejected slowly towards the 1TMCP to count the remaining fraction of particles.

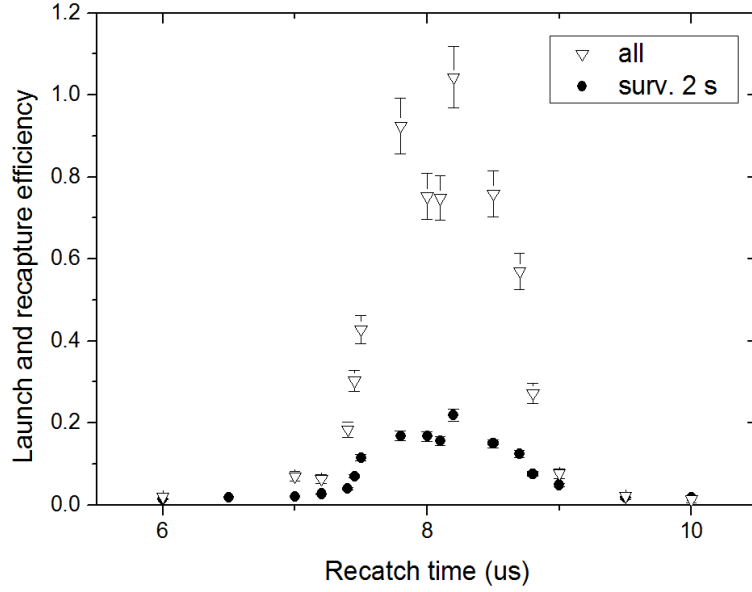
Coincidence counts observed by SC2122 after recatching were compared to number of coincidence counts on SC12 from the 5T before the launch. A cross-calibration of the two detectors' measured numbers $N_{SC12} = \eta \cdot N_{SC2122}$ was performed using 27 measurement runs with very stable AD conditions from the 5T P-trap. These measurements were collected ejecting slowly antiproton once towards 1TMCP, once towards the DegFC, were they were counted in coincidence by SC2122 and SC12. Three different numbers of antiprotons were used (varying the capture voltage between 3, 6, 9 kV) to extrapolate a linear fit and exclude systematic effects due to different backgrounds of the two detectors. The measured cross-calibration factor was found to be $\eta = 1.13 \pm 0.01$.

Results of this measurement, normalized to the hot extraction after $\bar{p}$ cooling, are shown in Figure 6.21. The overall recatching efficiency was found to be $\sim 80\%$. The higher part of the distribution is almost flat, as for the case of $\bar{p}$ capture (see Figure 6.8), which is consistent with a $\bar{p}$ bunch shorter in time/space than the production trap. This result is consistent with roughly 20% losses of $\bar{p}$ on B0 and Big-trap/OnAxis-trap junction, as indicated by scintillators spectra in Figure 6.19, and a 100 % efficient capture of the $\bar{p}$ that make through the 10 mm diameter restriction.

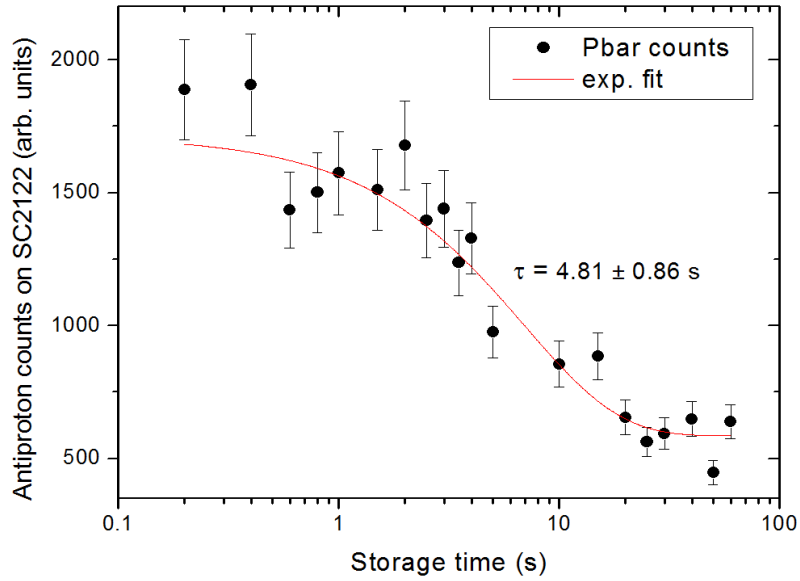
Around 25 % of the recaptured $\bar{p}$ were seen surviving after 2 s in the production trap, as a consequence of the fast radial losses due to centrifugal separation. An independent confirmation that 75 % of $\bar{p}$ were dying on the electrodes was obtained with a long 1 s acquisition of the 1TMCP held at high-gain, which observed the products of the annihilations at the electrode radius transported by the magnetic field lines, able to overcome the barrier of -180 V of UC15 (see Figure 6.22).

Ballistic transfer procedure was progressively refined throughout the end of 2016 AEgIS run. Initial centrifugal separation in the 5T trap was removed by working on the 5T $\bar{p}$ rotating-wall settings, obtaining a much better compressed $\bar{p}$ plasma without centrifugal separation before launch. This avoided the fast losses due to radial expansion induced by the ring collapse, as well as allowing much longer storage times to be achieved with antiprotons. Antiprotons survived many minutes in the production trap alone without electrons, without incurring in detectable losses (see Figure 6.23).

Stacking of different shots from AD was also demonstrated to increase the available number of $\bar{p}$ in the production trap in view of the long accumulation times necessary to store enough positrons to attempt $\bar{H}$ formation. 3 stacks of $\bar{p}$ were transferred ballistically to the production



(a) Recatching efficiency



(b) Storage time of the surviving fraction

Figure 6.21: Antiproton ballistic recatching efficiency with axialization (panel a) and storage time measurement of the recaptured antiprotons (panel b). Total annihilations observed from the production trap, comprehensive of radial losses an extraction on 1TMCP, are shown as white triangles. The sole annihilations on the 1TMCP are shown as black dots.

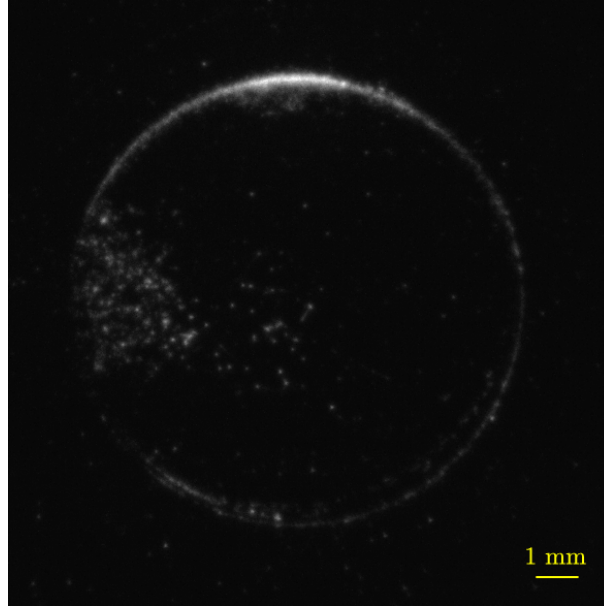


Figure 6.22: Long 1 s acquisition of the 1TMCP kept at high gain, while $\bar{p}$ are held in the production trap right after recapture. Products of the $\bar{p}$ annihilations are observed to form a ring of the same size as the production trap electrode, with an excess in the upper part due to the grid for positronium entrance.

trap, corresponding to $1.8 \cdot 10^5$ antiprotons with sub-mm radius.

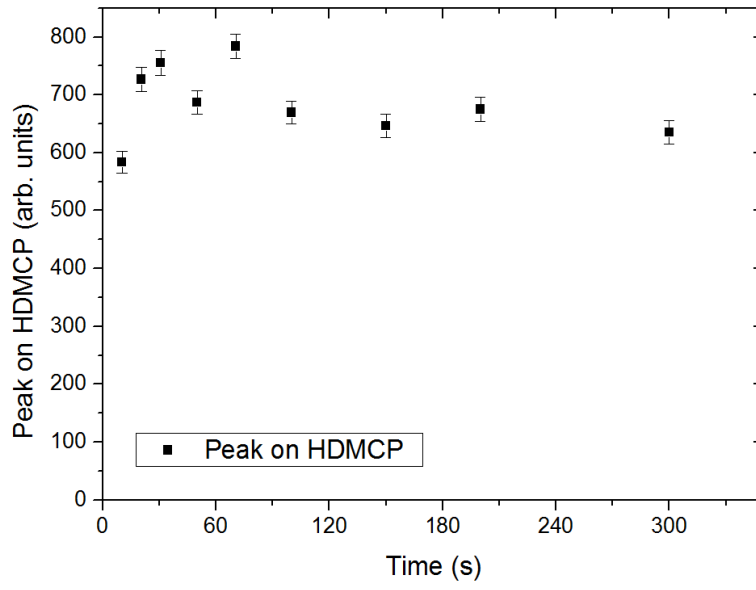


Figure 6.23: Number of antiprotons observed from the production trap on the 1TMCP after a variable storage time. The procedure used here for recatching uses a non-centrifugally separated $\bar{p}$ plasma.

Chapter 7

Towards pulsed $\bar{\text{H}}$ production

Antihydrogen atoms formed with a pulsed scheme are very promising for future tests of the Equivalence Principle on a purely antimatter system. The knowledge of their formation instant opens up the possibility to directly measure the local gravitational acceleration from the vertical displacement of their free-fall parabolic trajectory detected by a position sensitive detector $\Delta y = g\Delta t^2$. Adopting a trajectory selector such as the moiré deflectometer/Talbot-Lau interferometer, a first $\Delta g/g \approx 1\%$ measurement is feasible also using a non-monochromatic beam produced by a thermal source, with just a few thousands detected atoms.

As discussed in the introduction 2.2, finding a working experimental procedure to produce antihydrogen with knowledge of its formation instant is an open technical/research problem which currently has no demonstrated solution. Among the experiment in the AD community, AEgIS is currently the only one investing in this direction; other experiments are either relying on the traditional three-body recombination scheme, or developing alternative methods still without knowledge of the $\bar{\text{H}}$ formation instant (e.g. the ATRAP collaboration, which first demonstrated charge-exchange with positronium [27]).

By demonstrating both antiproton storage in high numbers in the production trap and positronium formation by the target installed in the antihydrogen production region, AEgIS is almost ready to attempt a first pulsed formation of antihydrogen through charge-exchange. What number of $\bar{\text{H}}$ atoms could be expected with the current antiproton and positronium conditions? A simplified model for answering quantitatively what numbers could be expected is presented in the final section, after drawing the conclusions of the thesis experimental work.

7.1 Experimental results in a summary

As a brief summary of the results presented in last chapter:

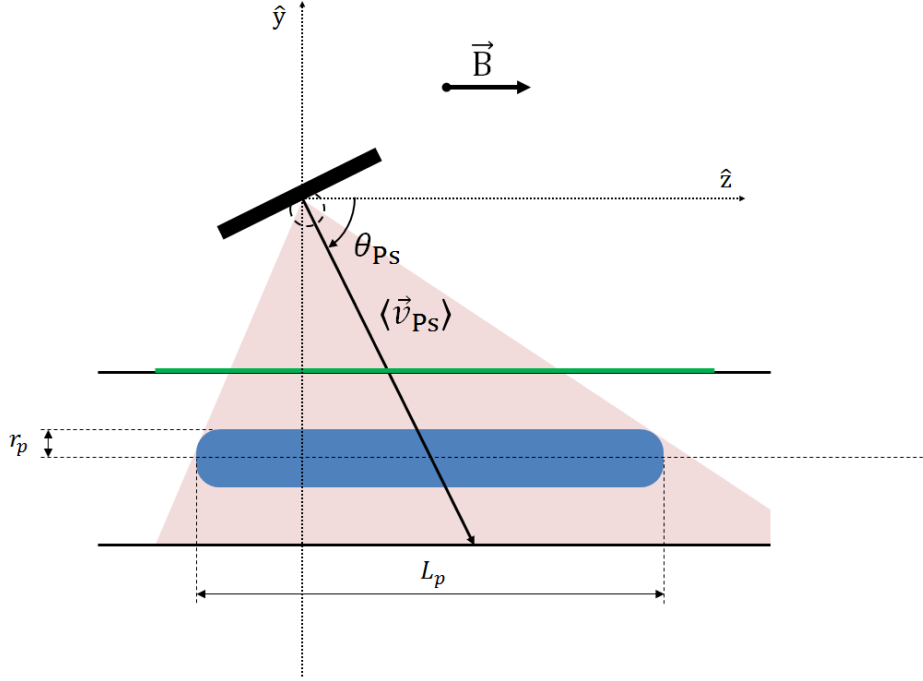
- A reliable and reproducible electron loading procedure, showing complete control over the plasma density (number of particles, length and radius), has been setup (see 6.1);
- An antiproton catching procedure was setup and completely understood, allowing approximately $3.6 \cdot 10^5$ $\bar{p}$ to be trapped per AD shot (see 6.2);
- Electron cooling of antiprotons was demonstrated with an efficiency of 100 % without active heating of the plasmas, and an efficiency of 55% performing rotating wall on electrons to keep the plasmas at high temperature and avoid centrifugal separation (see 6.3);
- A working antiproton rotating-wall compression procedure was demonstrated, with a compression efficiency to 1 mm radius of about 45%, achieved by removing partly the initial cooling electrons from the trap (see 6.4);
- An in-flight antiproton transfer procedure was developed, demonstrating a launch and re-catching efficiency in the production trap of 80% (see 6.5);
- Antiprotons were stacked, transferred and kept in the production trap for several minutes, without observing significant losses.

All the efficiencies combined give an overall 20% efficiency (less, if the initial $\sim 2\%$ capture efficiency is not taken into account) which is consistent with the final observed number of about $6 \cdot 10^4$ available $\bar{p}$ per AD shot in the production trap. There is even some room for improvement, since the initial cooling efficiency of electrons could be in principle increased to 100 %, if the radial overlap between electrons and antiprotons is optimized by studying a more efficient mechanism to heat up the plasmas. Moreover, also the antiproton compression efficiency could be increased, since it has been seen to be strongly depending on the quality of the electron plasma.

With up to 3 stacks of $\bar{p}$ successfully transferred to the production trap, corresponding to $1.8 \cdot 10^5$ antiprotons with sub-mm radius able to survive for minutes in the production trap, AEGIS demonstrated a factor of 2 more antiprotons than expected in the original proposal [71]. After this long setting-up work, the experiment is now ready, from the antiproton point of view, for antihydrogen formation through charge-exchange.

7.2 Expected $\bar{H}$ production rate

$N_{\bar{p}}$ antiprotons are assumed to be stored in a flat Malmberg region in the center of the production trap, where the $w = 2$ mm wide grid with $\eta_{\text{grid}} = 0.8$ transmittivity to let Ps atoms enter in the trap is installed (see Figure 7.1 for a visual reference and 3.1 for further details on the


 Figure 7.1: Side view sketch of the $\bar{H}$ production experiment ($\hat{z} - \hat{y}$ plane)

$\bar{H}$ production trap). The $\bar{p}$ plasma equilibrium distribution of a flat Malmberg trap is well approximated by a cylinder of radius r_p , length L_p and average particle density $n_{\bar{p}} = N_{\bar{p}}/(\pi r_p^2 L_p)$.

N_{e^+} positrons are then assumed to be implanted in the center of the target, which lies above the production trap at a vertical distance $l_0 = 10$ mm from the grid and $l_1 = 15$ mm from the trap axis (see Figure 7.1). The target is inclined with an angle $\theta_{Ps} = \pi/3$ with respect to the magnetic field axis. A cloud of ortho-positronium is assumed to be emitted (with a e^+ -oPs conversion efficiency of $\eta_{\text{conv}} = 0.35$) from a point-like source located in the center of the e^+ implantation position, with a Maxwellian distribution of velocities (in modulo) as from an thermal source with temperature $T = 1300$ K (see 4.5 and [40]).

The two Rydberg excitation lasers are set parallel to the $\hat{x}$ axis tangentially to the target, as in the experiments conducted in 4.5 and 4.6. The $1 \rightarrow 3$ laser is assumed to be set on the transition resonance $\lambda_3 = 205.047$ nm with null detuning $\delta = 0$, in order to address the Ps atoms with null velocity component on the $\hat{x}$ -axis (see Figure 7.3 for a pictorial view).

The number of formed antihydrogen atoms can be estimated starting from the definition of cross-section

$$\frac{dN_{\bar{H}}}{dt} = \sigma n_{\bar{p}} v_{\text{rel}} N_{Ps^*} \quad (7.2.1)$$

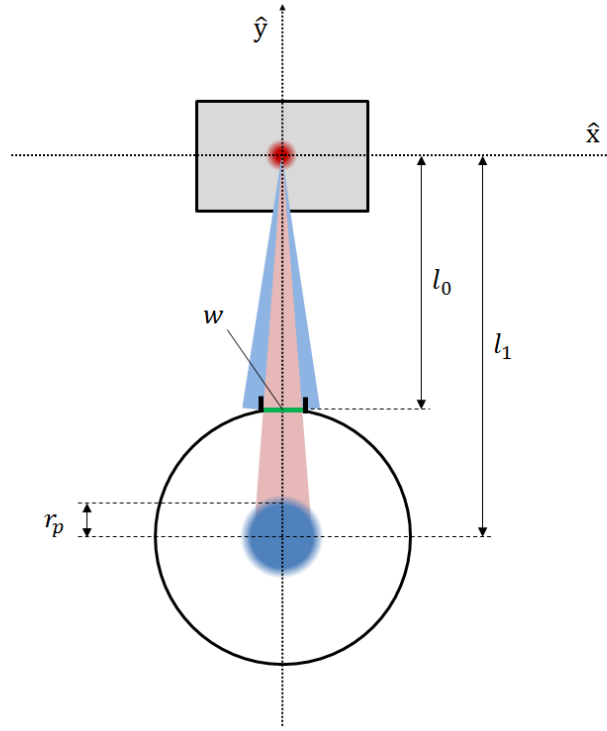


Figure 7.2: Front view sketch of the $\bar{H}$ production experiment ($\hat{z} - \hat{y}$ plane)

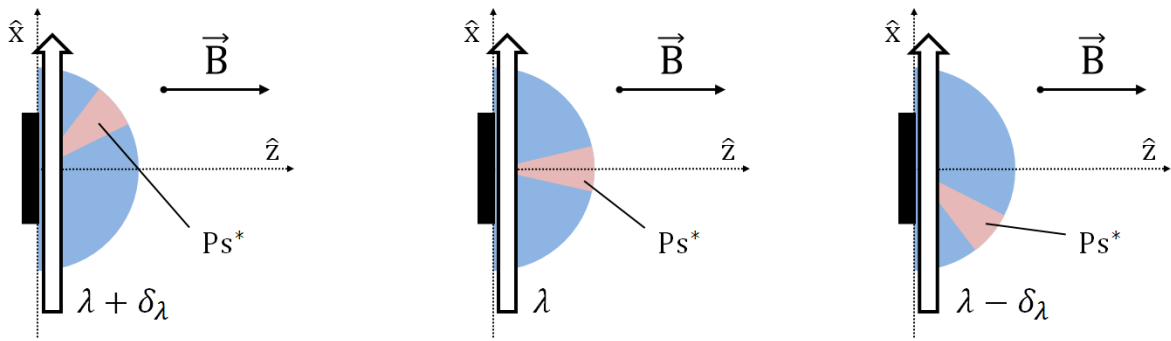


Figure 7.3: Top view sketch of the effect of sending the $n = 3$ laser with a detuning $\delta_\lambda \neq 0$ on the Ps sample, as emitted from an ideal isotropical source

where N_{Ps^*} is the number of Rydberg Ps atoms that make it through the grid and interact with the $\bar{p}$ plasma and v_{rel} is the relative velocity between Ps and $\bar{p}$.

In order to find $N_{\bar{H}}$, one should first integrate Equation 7.2.1 on the interaction time, i.e. the average time-of-flight of Ps atoms within the $\bar{p}$ cloud. The only quantity depending on time is v_{rel} , the relative velocity between antiprotons and Ps, which can be approximated by the average center-of-mass velocity of Ps atoms $v_{\text{rel}} \approx \langle v_{\text{Ps}} \rangle$ if antiprotons in the trap have a temperature $T \ll \text{eV}$.

The only integral to be calculated is thus $\int_V \langle v_{\text{Ps}} \rangle dt$. The average Ps velocity $\langle v_{\text{Ps}} \rangle$ is tilted, with respect to the vertical direction, by $\theta_{\text{Ps}} = \pi/3$ (see Figure 7.1). The integral ends up being equal, with good approximation, to the average interaction length between a vertical ray and a planar ellipse distribution in the θ_{Ps} tilted plane, with major semiaxis $r_p / \sin(\theta_{\text{Ps}})$ and minor semiaxis r_p . The integral results

$$\int_V \langle v_{\text{Ps}} \rangle dt = \frac{\pi r_p}{2 \sin(\theta_{\text{Ps}})} \simeq 1.81 r_p \quad (7.2.2)$$

The number of antihydrogen atoms is thus

$$N_{\bar{H}} = \frac{\pi \sigma r_p}{2 \sin(\theta_{\text{Ps}})} n_{\bar{p}} N_{\text{Ps}^*} \quad (7.2.3)$$

The antiproton density $n_{\bar{p}}$ can be expressed as a function of the plasma parameters using the expression of the plasma density in a flat Malmberg trap $n_{\bar{p}} = N_{\bar{p}} / (\pi r_p^2 L_p)$

$$N_{\bar{H}} = \frac{\sigma}{2 \sin(\theta_{\text{Ps}})} \frac{N_{\bar{p}}}{r_p L_p} N_{\text{Ps}^*} \quad (7.2.4)$$

The $\bar{H}$ production cross-section σ is obtained from the Classical Trajectory Monte-Carlo (CTMC) calculations presented in the introduction (see 2.2.2). In the region of k_v (recall its definition, Equation 2.2.4) between 0.5 and 1.6 the $\bar{H}$ formation cross-section is almost flat $\sigma \simeq 1.5 \cdot 10^{-15} \text{ cm}^2 \cdot n_{\text{Ps}}^4$ (see Figure 2.5, panel (a)), whereas it drops to $\sigma \sim 10^{-16} \text{ cm}^2 \cdot n_{\text{Ps}}^4$ for $k_v = 2$.

The value that should be considered for the average Ps axial velocity, according to the experiments conducted in 4.5 and 4.6, should be $\langle v_{\text{Ps}} \rangle = 1.4 \cdot 10^5 \text{ m s}^{-1}$. This value was obtained from the analytical evaluation of the most probable speed from a Maxwell distribution with $T = 1300 \text{ K}$

$$\langle v_{\text{Ps}} \rangle = \sqrt{\frac{2 k_B T}{m_{\text{Ps}}}} \simeq 1.4 \cdot 10^5 \text{ m s}^{-1} \quad (7.2.5)$$

and is in good agreement with the axial velocity estimate from the time-of-flight velocity of Rydberg Ps atoms annihilating on the first obstacle in their path, obtained during the Rydberg excitation campaign (see 4.6 and Figure 4.24, panel (b)).

This velocity corresponds, for $n = 18$ positronium (close to the maximum that can be currently achieved), to a value of $k_v = 2.3$, that is already beyond the drop in the cross-section happening at $k_v = 2.0$, and the cross-section results

$$\sigma^{1300\text{ K}} = 0.5 \cdot 10^{-15} \text{ m}^2 \quad (7.2.6)$$

With $\langle v_{\text{Ps}} \rangle = 1.0 \cdot 10^5 \text{ m s}^{-1}$, corresponding to $k_v = 1.6$ and a Ps temperature of 650 K, the cross-section becomes

$$\sigma^{650\text{ K}} = 1.0 \cdot 10^{-14} \text{ m}^2 \quad (7.2.7)$$

Thus, a reduction of Ps temperature from the present 1300 K to 650 K would increase the cross-section of almost a factor of 20. A lower temperature should be achieved in a straightforward with a cryogenic target, as discussed in [40, 39].

The final ingredient to evaluate $N_{\bar{H}}$ is the number of Rydberg positronium atoms able to interact with the $\bar{p}$ cloud, which can be written as a product of several efficiencies

$$N_{\text{Ps}^*} = \eta_{\text{grid}} \cdot \eta_{\text{geom}} \cdot \eta_{\text{laser}} \cdot \eta_{\text{conv}} \cdot N_{e^+} \quad (7.2.8)$$

An initial cloud of N_{e^+} positrons is assumed to impinge on the target and produce a $\eta_{\text{conv}} = 2/3 \times 0.35$ fraction of ortho-positronium atoms (magnetic quenching kills 1/3 of the o-Ps population), emitted from the converter *non-isotropically*, but with the same velocity distribution that yielded the measurements obtained during the $1 \rightarrow 3$ campaign 4.5 (the target is the same).

The $1 \rightarrow 3 \rightarrow 18$ lasers are assumed to excite $\eta_{\text{laser}} = 0.15$ of the total amount of produced o-Ps atoms to the $n = 18$ Rydberg level with the laser set on resonance¹. η_{grid} is simply the 0.8 transmittivity of the grid. η_{geom} , the geometrical probability for a Rydberg Ps atom to reach the aperture of the $\bar{H}$ production trap, still has to be evaluated.

Since the $1 \rightarrow 3$ laser has a bandwidth $\sigma_L = 48 \text{ GHz}$ much smaller than the Doppler profile, only a thin slice of the transverse velocity distribution of Ps can be addressed by the laser, according to the chosen detuning δ from the center of the $1 \rightarrow 3$ line at $\lambda_3 = 205.047 \text{ nm}$ (see Figure 7.3). The most effective choice for producing $\bar{H}$ efficiently is to set $\delta = 0$, for which

¹ $\eta_{\text{laser}} = 0.15$ is itself the product of the geometrical laser coverage $\eta_{\text{cov}} \simeq 0.8$ and the spectral bandwidth coverage $\eta_{\text{BW}} \simeq 0.2$

$\langle v_{\text{Ps}} \rangle_x = 0$. In this condition, the maximum amount of atoms addressable by the laser travels in the $\hat{y}-\hat{z}$ plane towards the grid. The grid has a maximum angular acceptance set by the angle $\phi^{\text{max}} = \text{asin}(w \sin(\theta_{\text{Ps}})/2l_0) = 86.7 \text{ mrad}$, for atoms flying in the $\hat{y}-\hat{z}$ plane on an average trajectory tilted by $\theta_{\text{Ps}} = \pi/3$ with respect to $\hat{z}$ (see Figure 7.1 for a reference). This angle corresponds to the maximum radial velocity a Ps atom (with average total velocity $\langle v_{\text{Ps}} \rangle$) could possess in order to find the aperture

$$v_x^{\text{max}} = \langle v_{\text{Ps}} \rangle \sin(\phi^{\text{max}}) = \langle v_{\text{Ps}} \rangle \frac{w \sin(\theta_{\text{Ps}})}{2l_0} \quad (7.2.9)$$

which, for $\langle v_{\text{Ps}} \rangle = 1.4 \cdot 10^5 \text{ m s}^{-1}$ results $v_x^{\text{max}} = 1.21 \cdot 10^4 \text{ m s}^{-1}$. Only Ps atoms having $|v_x| < 1.21 \cdot 10^4 \text{ m s}^{-1}$ can thus enter in the $\bar{H}$ production trap.

The distribution of transverse velocities in the Rydberg Ps cloud is chosen by the bandwidth σ_L of the excitation laser, which cuts a slice $\propto \sigma_L$ of the broad Doppler distribution of velocities (centered in $\delta = 0$). The η_{geom} efficiency is given by the fraction of this distribution that possesses a radial velocity component smaller (in modulo) than v_x^{max} , suitable for entering through the aperture.

In order to evaluate η_{geom} , the laser frequency distribution $S(\nu) = \text{Gauss}(c/\lambda_3, \sigma_L)$ has to be first expressed as a function of v_x . Then, η_{geom} can be simply calculated as the fraction of the area under the distribution $S'(v_x)$ satisfying $|v_x| < v_x^{\text{max}}$.

The link between transverse velocity $v_{\parallel}$ and laser frequency is set by the non-relativistic Doppler formula $v_{\parallel} = c(\nu_{\text{obs}}/\nu_3 - 1)$, which can be rewritten in the more compact expression $v_{\parallel} = \lambda_3 \delta\nu$ substituting $\nu_{\text{obs}} = \nu_3 + \delta\nu$. The laser Gaussian spectral profile is thus transformed into a velocity distribution by

$$\frac{1}{\sigma_L \sqrt{2\pi}} \exp\left(-\frac{1}{2} \frac{\delta\nu^2}{\sigma_L^2}\right) \longrightarrow \frac{1}{\lambda_3 \sigma_L \sqrt{2\pi}} \exp\left(-\frac{1}{2} \frac{v_{\parallel}^2}{\lambda_3^2 \sigma_L^2}\right) \quad (7.2.10)$$

This expression can now be integrated for $|v_x| < v_x^{\text{max}}$ to calculate η_{geom}

$$\eta_{\text{geom}} = \frac{1}{\sigma_L \sqrt{2\pi}} \int_{-v_x^{\text{max}}}^{v_x^{\text{max}}} \exp\left(-\frac{1}{2} \frac{v^2}{\lambda_3^2 \sigma_L^2}\right) dv = \text{Erf}\left(\frac{v_x^{\text{max}}}{\sqrt{2} \lambda_3 \sigma_3}\right) \quad (7.2.11)$$

The result of this calculation can be numerically evaluated for $v_x^{\text{max}} = 1.21 \cdot 10^4 \text{ m s}^{-1}$, $\lambda_3 = 205.045 \text{ nm}$ and $\sigma_L = 48 \text{ GHz}$, and results

$$\eta_{\text{geom}}|_{\text{AEgIS}}^{1300 \text{ K}} = 0.78 \quad (7.2.12)$$

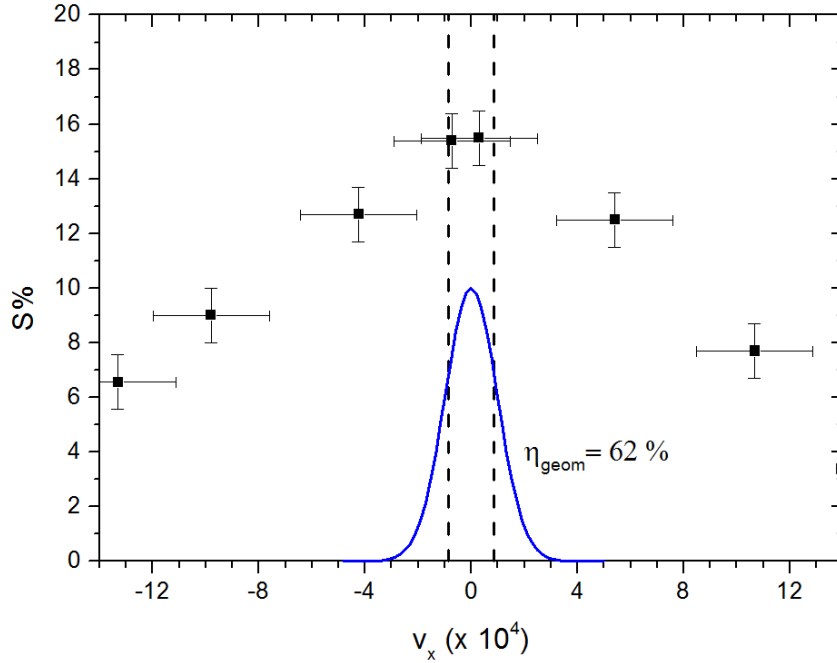


Figure 7.4: Measured distribution of transverse Ps velocities during the $1 \rightarrow 3$ measurement campaign (black dots), superimposed with the drawing of the $1 \rightarrow 3$ laser spectrum with $\sigma_L = 48 \text{ GHz}$ and null detuning (solid blue line) and the acceptance of the grid for $\langle v_{\text{Ps}} \rangle = 1.0 \cdot 10^5 \text{ m s}^{-1}$. The geometrical acceptance is found $\eta_{\text{geom}} = 0.62$.

If $\langle v_{\text{Ps}} \rangle = 1.0 \cdot 10^5 \text{ m s}^{-1}$ (corresponding to Ps at $T = 600 \text{ K}$) is considered instead, the maximum transverse velocity becomes $v_x^{\text{max}} = 0.87 \cdot 10^4 \text{ m s}^{-1}$) and the value of η_{geom} becomes

$$\eta_{\text{geom}}|_{\text{AEgIS}}^{650 \text{ K}} = 0.62 \quad (7.2.13)$$

Figure 7.4 gives a visual reference of this calculation. The experimental data of the $1 \rightarrow 3$ campaign (which gave 15% excitation efficiency with laser set on resonance) are plotted as a function of v_x , superimposed to the calculated $S'(v_x)$ with null detuning and two vertical lines marking the maximum grid acceptance.

Assembling all together, the number of Rydberg Ps making it through the grid results

$$N_{\text{Ps}^*} = \underbrace{0.8}_{\text{grid}} \times \underbrace{0.78}_{\text{geometry}} \times \underbrace{0.15}_{\text{exc. } \delta=0} \times \underbrace{0.25}_{\text{conversion}} \times N_{e^+} \quad (7.2.14)$$

This formula can be made compact by multiplying all the efficiencies together and defining

an overall e^+ utilization efficiency $N_{Ps^*} = \eta_{e^+} \cdot N_{e^+}$, which results

$$\eta_{e^+}^{1300\text{K}} = 0.0234 \quad (7.2.15)$$

Substituting into Equation 7.2.4 (and introducing the superscript T to keep track of the Ps source temperature considered) the number of formed $\bar{H}$ atom results

$$N_{\bar{H}}^T = \frac{\sigma^T}{2 \sin(\theta_{Ps})} \frac{N_{\bar{p}}}{r_p L_p} \eta_{e^+}^T N_{e^+} \quad (7.2.16)$$

for $N_p = 1.8 \cdot 10^5$ antiprotons (obtained stacking three AD shots, see 7.1), $N_{e^+} = 1.0 \cdot 10^8$ (which can be achieved using the current setup of the accumulator with the new upgraded 50 mCi source, within the 500 s required for a full cycle of $\bar{p}$ handling), a plasma radius of $r_p = 1.5$ mm and a plasma length of $L_p = 30$ mm, the number of formed $\bar{H}$ atoms per shot results

$$N_{\bar{H}}^{1300\text{K}} = 3.0 / \text{shot} \quad (7.2.17)$$

whereas for a 650 K Ps one would get

$$N_{\bar{H}}^{650\text{K}} = 40 / \text{shot} \quad (7.2.18)$$

where a shot takes ~ 500 s. The main limitation of the present scheme comes from the inefficient use it makes of positrons, as a consequence of the poor solid angle overlap between the positronium cloud emitted from the target and the antiproton plasma in the trap, which is of the order of 2%. A future upgrade of the experiment should envisage re-designing the antihydrogen production region in view of increasing the geometrical overlap.

A factor could be gained increasing the target Rydberg level from the conservative choice of $n = 18$. Ionization in strong 1 T magnetic field occurs at $n = 27$ for 100 K Ps, according to [45]. Setting for example $n = 23$ could be suitable choice, that would increase the cross-section and the number of produced $\bar{H}$ by a factor $\times 2.7$, increasing the number of $\bar{H}$ to ~ 100 /shot. A possibility to overcome the ionization limit that could be considered would be to reduce the magnetic field in the 1T magnetic region.

There isn't much room for improvement on the laser geometrical coverage and bandwidth: increasing the bandwidth of the UV laser from the actual $2\pi \times 48$ GHz would only increase the excitation efficiency in Doppler bands outside the angular acceptance of the $\bar{H}$ production trap grid.

Finally, an alternative solution to increase the solid angle overlap between positronium and antiprotons could be Rydberg positronium focusing using a Stark lens. This solution, which

would require a dedicated theoretical and experimental study, could potentially provide a way to increase the spatial separation between the mesoporous matter converter and the $\bar{p}$ plasma without incurring in further solid angle losses.

Part III

Special topics

Chapter 8

Non-perturbative calculation of Positronium energy levels and optical spectra in uniform strong external fields

This chapter describes a novel non-perturbative numerical method for calculating the energy levels structure of positronium in presence of strong uniform electric and magnetic fields with arbitrary orientations, based on the efficient numerical evaluation and diagonalization of the exact Hamiltonian matrix in presence of fields. This method led to the development of a numerical code for calculating the exact energy levels, transition probabilities, optical spectra and laser excitation dynamics of Positronium at Rydberg levels (named Positronium Transitions Calculator, or PsTC). This code is, to the knowledge of the author, the first to calculate the exact positronium energy structure on Rydberg levels in arbitrary external electric and magnetic fields. The use of state-of-the-art numerical techniques made the code fast and efficient: a single execution of an energy levels calculation of Rydberg Ps equires up to one second of computation time on a modern laptop computer.

First, the problem of calculating positronium energy levels in external fields is presented. Two main ingredients are necessary in order to evaluate the Ps Hamiltonian matrix in presence of external fields: a calculation of positronium energy levels in absence of fields and an analytical calculation of Zeeman, Stark and diamagnetic interactions matrix elements on the unperturbed Hamiltonian eigenstates. Results obtained are compared with similar results found in literature and applied to the calculation of the Ps energy structure at Rydberg levels, of interest for AEGIS and other experiments in the field.

Subsequently, the resulting structure of energy levels was used for the calculation of Ps optical transition probabilities in strong fields and the optical absorption spectrum of Ps. Theoretical absorption spectra of Ps in Rydberg levels, obtained with PsTC, were compared to the experimental survey discussed in 4.6 and the more refined spectroscopic survey in [31], with good agreement in both cases.

Finally, the possibility to accurately simulate Ps Rydberg spectra in realistic experimental conditions shined some new light on the possibilities offered by the motional Stark effect - prerogative of positronium and almost absent for massive atoms. A novel method for measuring simultaneously both Ps axial and radial velocities, using only transverse laser beams, is discussed.

The calculation technique described in this chapter can be extended in a straightforward way to hydrogen, antihydrogen and other hydrogenoid atoms. Furthermore, it was found promising, thanks to its fast computing speed, for a future implementation of a positronium tracking code in strong fields accounting for pseudo-momentum conservation, to be adopted for future experiments with Ps in presence of a magnetic environment.

8.1 The problem of Ps energy levels in strong external fields

Solutions to the problem of calculating the energy levels of positronium - necessary for a proper theoretical comparison of the spectroscopic data collected by the many experiments in the field (see [139] for an historical review) - have been proposed by many authors in literature with increasing accuracy [140, 141, 142, 143, 144, 145]. Perturbation theory proved to be an effective tool for such calculations at low magnetic fields (of the order of $\lesssim 0.01$ T [141, p. 7], roughly equivalent - in terms of splitting of the energy levels - to electric fields of $\lesssim 1000$ V m⁻¹). In presence of stronger external fields, however, perturbation theory fails even at low values of the n quantum number (such as the case of $n = 2$, discussed in [141]): the number of terms in the perturbative expansions that should be considered becomes so high that the adoption of a numerical method is preferable.

An adopted numerical method for calculating the non-perturbative energy levels of atoms and molecules in strong external fields consists of solving the three-dimensional, time-independent Schrödinger Partial differential equation (PDE) with a brute-force numerical solver. A frequent choice is the Finite-Elements Method (FEM): an example of such methodology, applied to the problem of positronium energy levels in external fields, is discussed to some extent in [146]¹. The choice of solving the energy levels problem using a PDE solver, however, shows some significant drawbacks. The three-dimensional space has to be discretized on a suitable spatial grid to

¹a similar calculation for helium energy levels is found in the same proceedings collection, [147]

perform the calculation, which should resolve the zeroes of the eigenfunctions - thus requiring roughly n^3 nodes. The calculation performed in [146], for instance, adopted a Cartesian grid of $10^5 a_0 < x < 10^5 a_0$ and $0 < y, z < 10^5 a_0$ and an iterative algorithm to converge to the solution.

The numerical method developed here is based on the idea of diagonalizing numerically the full-interaction Hamiltonian matrix of Ps in fields, originally discussed in Curry's work [141]. This method, re-considered in more recent years by Castelli and Villa [45, 102], has the advantage of a very straightforward numerical implementation, consisting in the diagonalization of a big sparse matrix for which optimized algorithms and scalable computing libraries are available.

8.1.1 The method in a summary

The problem of calculating the energy levels of positronium consists of determining the eigenspace of its Hamiltonian matrix, i.e. solving the time-independent Schrödinger eigenvalue equation

$$(H_0 + H_I) |\psi\rangle = E |\psi\rangle \quad (8.1.1)$$

where H_0 is the Hamiltonian matrix of positronium in absence of the external fields and H_I is the interaction Hamiltonian due to the fields presence². One is interested in finding the bound state solutions to Equation 8.1.1, associated to its discrete optical spectrum, so H_0 and H_I are discrete matrices whose elements can be written as

$$\begin{aligned} H_0^{(i,j)} &\leftrightarrow \langle \psi_i | H_0 | \psi_j \rangle \\ H_I^{(i,j)} &\leftrightarrow \langle \psi_i | H_I | \psi_j \rangle \end{aligned} \quad (8.1.2)$$

where $|\psi_i\rangle$ are the elements of any numerable orthonormal basis of the space, arbitrarily chosen for performing the calculation.

The solutions of Equation 8.1.1 are the eigenvalues $E^{(i)}$ and eigenvectors $|E^{(i)}\rangle$ of the Hamiltonian matrix (on the basis $|\psi_i\rangle$), satisfying

$$(H_0 + H_I) |E^{(i)}\rangle = E^{(i)} |E^{(i)}\rangle \quad (8.1.3)$$

If one chooses the eigenvectors of the unperturbed Hamiltonian H_0 as the basis vectors of the

²here only the internal degrees of freedom of Ps are considered: $|\psi\rangle$ indicates the wave function of the relative orbit of e^- and e^+ around their center-of-mass.

space, $|\psi_i\rangle = |E_0^{(i)}\rangle$, the matrix elements become

$$\begin{aligned} H_0^{(i,j)} &\leftrightarrow \langle E_0^{(i)} | H_0 | E_0^{(j)} \rangle = \delta_{i,j} E_0^{(i)} \\ H_I^{(i,j)} &\leftrightarrow \langle E_0^{(i)} | H_I | E_0^{(j)} \rangle \end{aligned} \quad (8.1.4)$$

where H_0 has been simplified to a diagonal matrix having its eigenvalues on the principal diagonal. Equation 8.1.4 offers the possibility to use the a-priori knowledge of the eigenvalues/eigenvectors of H_0 (known, for example, from a previous calculation, either numerical or analytical) to reduce the complexity of the diagonalization of the full Hamiltonian $H_0 + H_I$, as only the matrix elements of H_I have to be explicitly calculated and diagonalized.

Equation 8.1.4 implicitly defines a recursive algorithm for solving Equation 8.1.1 in a non-perturbative way. The eigenspace of the final matrix is obtained by first choosing an arbitrary system of coordinates, then progressively diagonalizing piece by piece the Hamiltonian matrix from the biggest to the smallest contribution to the eigenvalues³. No further approximation is introduced in this process, but for the necessary limitation of the matrices size (which is equivalent to limiting the otherwise infinite-dimensional Hilbert space). This doesn't affect at all the calculations for n manifolds far away from ionization limit.

In other words, this implies solving first the Schrödinger eigenvalue problem in the case

$$H_0 |\psi_0\rangle = E_0 |\psi_0\rangle \quad (8.1.5)$$

finding its eigenvalues $E_0^{(i)}$ and eigenvectors $|E_0^{(i)}\rangle$ and using them as basis vectors for calculating the matrix elements of the interaction Hamiltonian H_I .

8.1.2 Positronium energy levels in absence of fields

This section deals with the calculation of the energy levels of positronium in absence of external fields, first in the non-relativistic approximation (to introduce the spherical basis linked to the radial and orbital quantum numbers), then in the relativistic case (introducing spin operators).

Energy levels of positronium, being primarily determined by the Coulombian potential between electron and positron, are similar to those of a hydrogen atom, with a reduced mass $\mu = m_e/2$. The simplest description (without spin) of Ps energy levels is obtained solving the time-

³considering it a-posteriori, this process is curiously similar to what has been done historically by the people who were calculating atomic energy levels by hand.

independent Schrödinger's equation with the non-relativistic, spherically symmetric Hamiltonian

$$H_0 = \frac{\vec{p}^2}{2\mu} - \frac{1}{4\pi\epsilon_0} \frac{e_0^2}{r} \quad (8.1.6)$$

The eigenspace of this Hamiltonian is the well-known n -manifold of energy levels of hydrogen, whose eigenvalues are given by the Rydberg formula

$$E(n) = -\frac{E_{ha}}{2n^2} \quad (8.1.7)$$

where $E_{ha} = \mu c^2 \alpha^2$ is the Hartree energy for Ps, which is equal to $E_{ha} = 13.6$ eV, one half of the corresponding value for hydrogen. The corresponding main transitions of Ps are reported in Table 8.1. The orthonormal basis used to solve Schrödinger PDE in spherical coordinates associated to H_0 (using separation of variables) is constituted by orthogonal Laguerre polynomials $L_n^a(x)$ (for the radial part) and spherical harmonics $Y_{lm_l}(\theta, \varphi)$ (for the angular part). The orthonormal basis of kets $|n, l, m_l\rangle$ is defined by

$$\begin{aligned} \psi_{n,l,m_l}(r, \theta, \varphi) &= R_{n,l}(r) \cdot Y_{lm_l}(\theta, \varphi) \\ R_{n,l}(r) &= a_0^{-3/2} \frac{2}{n^2} \frac{((n-1-l)!)^{1/2}}{((n+l)!)^{3/2}} F_{n,l} \left(\frac{2r}{na_0} \right) \\ F_{n,l}(x) &= x^l e^{-x/2} (n+l)! L_{n-l-1}^{2l+1}(x) \end{aligned} \quad (8.1.8)$$

Transition	Wavelength λ	Transition	Wavelength λ
1 $\rightarrow$ 2	243.00 nm	3 $\rightarrow$ 4	3749.2 nm
1 $\rightarrow$ 3	205.04 nm	3 $\rightarrow$ 8	1908.7 nm
1 $\rightarrow$ 4	195.40 nm	3 $\rightarrow$ 12	1749.6 nm
1 $\rightarrow$ 5	189.85 nm	3 $\rightarrow$ 15	1708.6 nm
2 $\rightarrow$ 3	1312.2 nm	3 $\rightarrow$ 16	1700.0 nm
2 $\rightarrow$ 6	820.14 nm	3 $\rightarrow$ 17	1693.0 nm
2 $\rightarrow$ 9	766.88 nm	3 $\rightarrow$ 18	1687.1 nm
2 $\rightarrow$ 12	749.84 nm	3 $\rightarrow$ 19	1682.2 nm
2 $\rightarrow$ 15	742.21 nm	3 $\rightarrow$ 20	1678.0 nm
2 $\rightarrow$ 18	738.12 nm	3 $\rightarrow$ 23	1668.7 nm

Table 8.1: Wavelength of Ps transition lines in absence of fields

The energy levels of positronium are refined by introducing relativistic corrections from Dirac

theory. Following Bethe and Salpeter [140, p. 115]), the Hamiltonian of Ps in relativistic theory can be written as the sum

$$H'_0 = H_0 + H_1 + H_2 + H_3 + H_4 + H_5 \quad (8.1.9)$$

composed of the sum of H_0 and the five extra terms [140, p. 191], which can be treated as perturbations (as they are smaller by two power of α than H_0)

$$\begin{aligned} H_1 &= -\frac{1}{4c^2 m_e^3} p^4 \\ H_2 &= \frac{e_0^2}{2m_e^2 c^2} \frac{1}{r} (p^2 + p_r^2) \\ H_3 &= 3 \frac{\mu_0 e_0}{m_e c} \frac{\hbar}{r^3} \vec{L} \cdot \vec{S} \\ H_4 &= \frac{i e_0^2 \hbar}{4c^2 m_e^2} \vec{p} \cdot \vec{\nabla} \left(\frac{1}{r} \right) \\ H_5 &= 4\mu_0^2 \left\{ \frac{8\pi}{3} \delta^{(3)}(\vec{r}) \vec{S}_1 \cdot \vec{S}_2 + \frac{1}{r^3 (2l+3)(2l-1)} \cdot \left[\left(2 \vec{S}_1 \cdot \vec{S}_2 + \frac{3}{2} \right) L^2 - \frac{3}{2} \vec{S} \cdot \vec{L} - \frac{3}{2} (\vec{S} \cdot \vec{L})^2 \right] \right\} \end{aligned} \quad (8.1.10)$$

where $\vec{S} = \vec{S}_1 + \vec{S}_2$, $\vec{L} = \vec{r} \times \vec{p}$, $p_r = \vec{p} \cdot \hat{r}$, μ_0 is the Bohr magneton. As H_1 , H_2 and H_4 do not depend on any spin operator, they can be evaluated in terms of n , l and m using the non-relativistic eigenfunctions of H_0 . H_3 and H_5 , the spin-orbit and hyperfine terms (which in the case of Ps is of the same order as the fine structure), conversely depend by on spin operators and require a proper basis for the spin to be defined.

The relativistic corrected, α^4 accurate energy levels of Ps reads (see [140, p. 117])

$$\begin{aligned} E(n, l, s, j) &= -\frac{E_{ha}}{2n^2} + \frac{E_{ha}}{4n^3} \alpha^2 \left[\frac{11}{8n} - \frac{4}{2l+1} + \right. \\ &\quad \left. \delta_{s,1} \left[\delta_{l,0} \frac{14}{3} + \frac{2(1-\delta_{l,0})}{l(2l+1)(l+1)} \begin{pmatrix} -\frac{(l+1)(3l-1)}{2l-1}, & \text{if } j = l-1 \\ -1, & \text{if } j = l \\ +\frac{l(3l+4)}{2l+3}, & \text{if } j = l+1 \end{pmatrix} \right] \right] \Big\} \end{aligned} \quad (8.1.11)$$

If further accuracy is required (for example, if comparing theory to results obtained from an accurate spectroscopic survey), a more modern calculation of the energy levels (including QED

corrections), accurate to $m\alpha^5$, was carried in [142] and [144] for the $n = 1$ and $n = 2$ manifolds only.

Positronium eigenstates of relativistic theory, corresponding to the energy levels of Equation 8.1.11, are usually indicated with the quantum numbers n, l, s, j, m . The corresponding eigenstates can be written using the bra-ket notation as $|n, l, s, j, m\rangle$. This ordering of quantum numbers is common choice in literature, and corresponds to the so-called Russell-Saunders LS angular momentum coupling scheme (see [148]).

The quantum numbers j, m derive from the sum of orbital and spin angular momenta $\vec{J} = \vec{L} + \vec{S}$. The angular part of Ps eigenstates is usually expanded into products of spherical harmonics $Y_{lm_l}(\theta, \varphi)$ (as in the non-relativistic case) and spin functions χ_{sm_s} (introduced in relativistic theory). This is equivalent to expanding the ket $|n, l, s, j, m\rangle$ in terms of the factorized kets $|n, l, m_l\rangle |s, m_s\rangle$, which are the natural description of the eigenstates in absence of hyperfine interactions. The expansion coefficients are the well-known Clebsch-Gordan coefficients defined as

$$C_{m_l m_s}^j = \langle n, l, m_l | \langle s, m_s | \cdot | n, l, s, j, m \rangle \quad (8.1.12)$$

and the series expansion of $|n, l, s, j, m\rangle$ (which will be of use later on in the chapter) is:

$$|n, l, s, j, m\rangle = \sum_{m_l=-l}^l \sum_{m_s=-s}^s \delta_{m, m_l+m_s} C_{m_l m_s}^j |n, l, m_l\rangle |s, m_s\rangle \quad (8.1.13)$$

In Ps literature it is not unfrequent, however, to find calculations adopting the different SL angular momentum coupling scheme (see [148]), where the quantum numbers are indicated in the different order n, s, l, j, m and the eigenstates as the kets $|n, s, l, j, m\rangle$, which offers some advantages in writing matrix element formulas. The two methods indeed differs only by a permutation of the matrix row/columns and an overall sign factor. A suitable conversion formula for matrix elements between the two coupling schemes, is found for example in [149]

$$\langle n, l, s, j, m | T | n', l', s', j', m' \rangle = (-1)^{j-j'-l+l'-s+s'} \langle n, s, l, j, m | T | n', s', l', j', m' \rangle \quad (8.1.14)$$

Whether not specified, in the following sections the LS scheme is adopted.

8.1.3 Positronium energy levels in strong electric and magnetic fields

This section describes the non-perturbative method used to calculate positronium energy levels in uniform arbitrary electric and magnetic fields, that lead to the development of the PsTC

calculation tool.

The spherical basis plus spin that diagonalizes the unperturbed Hamiltonian H'_0 in relativistic theory (derived in the last section) was first adopted. The unperturbed Hamiltonian matrix H'_0 was then built populating the diagonal elements with the energies of Ps sublevels in absence of external fields calculated using Equation 8.1.11.

The choice of a specific spherical basis allowed analytical expressions of the perturbing matrix elements due to external fields, to be derived. The perturbed Hamiltonian matrix was subsequently built summing the diagonal matrix calculated in the first step with the analytical non-diagonal terms due to the presence of external fields. The final step consists of diagonalizing numerically the perturbed matrix to obtain both the exact eigenvalues (i.e. sublevel energies) and the exact eigenvectors (expressed as linear combinations of the initial basis vectors).

Positronium Hamiltonian in uniform electric and magnetic fields

The non-relativistic Hamiltonian of positronium in uniform external electric and magnetic fields (without considering spin contributions for the moment) is ([150])

$$H = \frac{\vec{p}^2}{2\mu} - \frac{e^2}{4\pi\epsilon_0 r} + e\vec{v} \cdot (\vec{r} \times \vec{B}) - \frac{e^2}{2M} (\vec{r} \times \vec{B})^2 - e\vec{r} \cdot \vec{E}_{\text{ext}} \quad (8.1.15)$$

where $M = m_e + m_p = 2m_e$, $\vec{r}$ is the relative position of the particles and $\vec{v}$ is the center of mass velocity. Due to the equality of masses, the linear Zeeman factor proportional to the orbital angular momentum of the system $\vec{L} = \vec{r} \times \vec{p}$ is absent for Ps. Note also that $\vec{v}$ is not rigorously a constant of motion (see [81] for further details).

The linear term in $\vec{B}$ can be rewritten as

$$e\vec{v} \cdot (\vec{r} \times \vec{B}) = -e\vec{r} \cdot (\vec{v} \times \vec{B}) = -e\vec{r} \cdot \vec{E}_{\text{MS}}, \quad (8.1.16)$$

where the non-relativistic transformations of external electric and magnetic fields in the Ps atom rest frame were used.

$$\vec{B}' = \vec{B} \quad \vec{E}' = \vec{E} + \vec{v} \times \vec{B}$$

The $\vec{E}_{\text{MS}} = \vec{v} \times \vec{B}$ term is the so-called *motional Stark* term, and can be interpreted as an electric field self-induced by the Ps motion in the magnetic environment, acting on Ps exactly as an external electric field orthogonal both to the magnetic field and the positronium trajectory (thus destroying axial symmetry). It results being the dominant contribution to the Ps energy

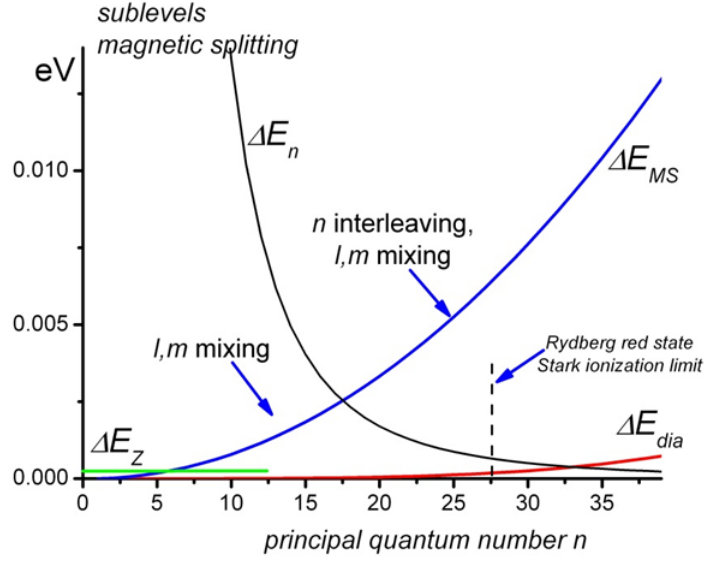


Figure 8.1: Contributions from the different Hamiltonian perturbation terms to the final energy splitting of Ps in external fields. From [45].

levels for high n , in the typical experimental conditions of $B = 1$ T and $v \sim 10^4 - 10^5$ m s $^{-1}$. Constancy of the center of mass velocity $\vec{v}$ is assumed from now on, a statement which is well realized for low values of n , but could fail for Rydberg levels approaching the ionization limit in strong magnetic field (an example of trajectories in such a case is found, for example, in [79]).

The term quadratic in $\vec{B}$ is the so-called *diamagnetic or quadratic Zeeman effect* and its contribution turns out to be negligible in the case of Ps compared to the motional Stark term [45] (see Figure 8.1).

The interaction between magnetic field and the intrinsic magnetic dipole moment of electron and positron still remains to be added. Denoting $\vec{\sigma}_e$ and $\vec{\sigma}_p$ are the 2×2 Pauli matrices describing electron and positron spin

$$\sigma_j = \begin{pmatrix} \delta_{j,z} & \delta_{j,x} - i\delta_{j,y} \\ \delta_{j,x} + i\delta_{j,y} & -\delta_{j,z} \end{pmatrix} \quad (8.1.17)$$

and μ_B the Bohr magneton, the final Hamiltonian of Ps in external fields reads

$$H = H_0 - e\vec{r} \cdot (\vec{E}_{\text{ext}} + \vec{E}_{\text{MS}}) + \mu_B \vec{B} \cdot (\vec{\sigma}_e - \vec{\sigma}_p) - \frac{e^2}{4m_e} (\vec{r} \times \vec{B})^2 \quad (8.1.18)$$

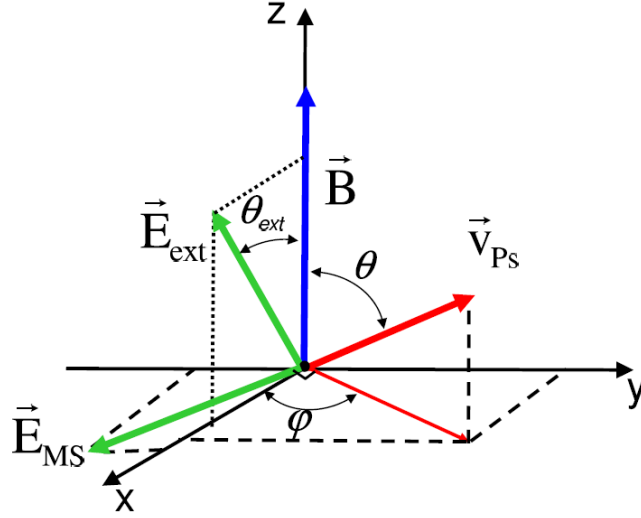


Figure 8.2: Reference cartesian coordinate system used within PsTC for a moving atom in external magnetic field and electric fields. The Ps atom is set at the origin of the axes.

Reference coordinate system

The reference coordinate system (in the laboratory frame) chosen for the calculation is shown in Figure 8.2. The magnetic field is assumed to define the $\hat{z}$ direction, while the external electric field is set on the $\hat{x} - \hat{z}$ plane, separated with the angle θ_{ext} from the vertical $\hat{z}$ axis. Positronium velocity vector $\vec{v}_{Ps}$ is let free, with arbitrary norm and orientation (defined by the two angles φ_{Ps} and θ_{Ps}).

It is convenient to identify the two components of Ps velocity under interest: the axial velocity $v_{\parallel} = v_{Ps} \cdot \cos(\theta_{Ps})$ and radial velocity $v_{\perp} = v_{Ps} \cdot \sin(\theta_{Ps})$. The Ps velocity vector is thus expressed as:

$$\vec{v}_{Ps} = \begin{pmatrix} v_{\perp} \cos(\varphi_{Ps}) \\ v_{\perp} \sin(\varphi_{Ps}) \\ v_{\parallel} \end{pmatrix} \quad (8.1.19)$$

Calling $E_{MS} = Bv_{\perp}$ and using the definition of the motional Stark term (see Equation 8.1.16), the total electric field is expressed in this coordinate system as:

$$\vec{E} = \vec{E}_{ext} + \vec{E}_{MS} = \begin{pmatrix} E_{ext} \sin(\theta_E) + E_{MS} \sin(\varphi_{Ps}) \\ -E_{MS} \cos(\varphi_{Ps}) \\ E_{ext} \cos(\theta_E) \end{pmatrix} \quad (8.1.20)$$

Zeeman interaction matrix element

The first interaction term of Equation 8.1.18 considered is the Zeeman term

$$H_Z = \mu_B \vec{B} \cdot (\vec{\sigma}_e - \vec{\sigma}_p) \quad (8.1.21)$$

An analytical expression for an arbitrary matrix element $\langle n, l, s, j, m | H_Z | n', l', s', j', m' \rangle$ is sought on the basis vectors defined above, in order to populate the numerical Hamiltonian matrix with the off-diagonal terms. The detailed calculation of this matrix element can be found in [143] using the SL coupling in place of the Russell-Saunders used here. Converted to LS coupling by using Equation 8.1.14, it results (expressed in terms of Wigner's 3j and 6j symbols).

$$\begin{aligned} \langle n, l, s, j, m | H_Z | n', l', s', j', m' \rangle = & 2\mu_B B \delta_{nn'} \delta_{ll'} \delta_{mm'} \delta_{s,1-s'} (-1)^{2j+l'-s+s'+m} \\ & \cdot \sqrt{3(2j+1)(2j'+1)} \begin{pmatrix} j & 1 & j' \\ -m & 0 & m' \end{pmatrix} \begin{Bmatrix} s & l & j \\ j' & 1 & s' \end{Bmatrix} \end{aligned} \quad (8.1.22)$$

Stark interaction matrix element

The second interaction term of Equation 8.1.18 considered is the Stark term due to the total electric field

$$H_S = -e \vec{r} \cdot \vec{E} \quad (8.1.23)$$

Again, an analytical expression for an arbitrary matrix element $\langle n, l, s, j, m | H_S | n', l', s', j', m' \rangle$ is sought on the basis vectors defined above, in order to populate the numerical Hamiltonian matrix with the off-diagonal terms. The detailed calculation of this matrix element is more complex than the Zeeman term, as it involves the decomposition and summation on the spherical components of $\vec{E}$ and $\vec{r}$, and is off the scope of the present section (for further details, see the more detailed discussions in [143, 102, 150] .

The Stark matrix element, converted to LS coupling by using Equation 8.1.14, results

$$\begin{aligned} \langle n, l, s, j, m | H_S | n', l', s', j', m' \rangle &= e \delta_{ss'} \delta_{l'l \pm 1} (-1)^{1+m+l_m+j-j'-l+l'+s'} \sqrt{l_m(2j+1)(2j'+1)} \\ &\cdot R_{nl, n'l'} \left\{ \begin{matrix} s & l & j \\ 1 & j' & l' \end{matrix} \right\} \sum_{q=-1}^1 (-1)^q E_{-q} \left(\begin{matrix} j & 1 & j' \\ -m & q & m' \end{matrix} \right) \end{aligned} \quad (8.1.24)$$

where q is a spherical index spanning between $(-1, 0, 1)$, $l_m = \max(l, l')$, $R_{nl, n'l'}$ is the expectation value of the r operator on the radial eigenfunctions (see the following section on radial matrix elements, 8.1.3), and E_{-q} is the spherical expansion of the electric field, given by

$$E_{-q} = \sqrt{\frac{4\pi}{3}} |\vec{E}| Y_{1,-q}(\theta_E, \varphi_E), \quad (8.1.25)$$

It's worth noticing that, differently from the Zeeman term, the Stark matrix element is able to mix sublevels on different n shells, due to the presence of the radial matrix element.

Diamagnetic interaction matrix element

The last interaction term of Equation 8.1.18 considered is the diamagnetic (or quadratic Zeeman) term

$$H_D = -\frac{e^2}{4m_e} (\vec{r} \times \vec{B})^2 \quad (8.1.26)$$

An analytical expression for an arbitrary matrix element $\langle n, l, s, j, m | H_D | n', l', s', j', m' \rangle$ is again sought on the basis vectors defined above. A detailed calculation of this matrix element was not found in published results. An unpublished calculation due to Castelli [151] using the SL coupling was used for the present case. Converted to LS coupling by using Equation 8.1.14, it results:

$$\begin{aligned} \langle n, l, s, j, m | H_D | n', l', s', j', m' \rangle &= \frac{e^2 B^2}{6m_e} Q_{nl, n'l'} \delta_{ss'} \delta_{mm'} (-1)^{s'+2l'+2j+m} \\ &\cdot \sqrt{(2j+1)(2j'+1)(2l+1)(2l'+1)} \\ &\cdot \left[\left(\begin{matrix} j' & 0 & j \\ m & 0 & -m \end{matrix} \right) \left(\begin{matrix} l & 0 & l' \\ 0 & 0 & 0 \end{matrix} \right) \left\{ \begin{matrix} l & s & j \\ j' & 0 & l' \end{matrix} \right\} \right. \\ &\quad \left. - \left(\begin{matrix} j' & 2 & j \\ m & 0 & -m \end{matrix} \right) \left(\begin{matrix} l & 2 & l' \\ 0 & 0 & 0 \end{matrix} \right) \left\{ \begin{matrix} l & s & j \\ j' & 2 & l' \end{matrix} \right\} \right] \end{aligned} \quad (8.1.27)$$

where $Q_{nl,n'l'}$ is the expectation value of the r^2 operator on the radial eigenvalues (see the following section on radial matrix elements, 8.1.3). As in the case of the Stark matrix element, also the diamagnetic matrix element is able to mix sublevels on different n shells, due to the presence of a radial matrix element.

Radial matrix elements

The calculation of radial matrix elements (also known as Gordon radial integrals), involves the calculation of the integrals

$$R_{nl,n'l'} = \langle n, l | r | n', l' \rangle = \int_0^\infty dr r^3 R_{n,l}(r) R_{n',l'}(r) \quad (8.1.28)$$

$$Q_{nl,n'l'} = \langle n, l | r^2 | n', l' \rangle = \int_0^\infty dr r^4 R_{n,l}(r) R_{n',l'}(r) \quad (8.1.29)$$

where $R_{n,l}(r)$, defined in Equation 8.1.8, are complicated functions of Legendre polynomials. Some analytical results can be found in literature for the calculation of such integrals (see for example [152]), obtained using some assumptions on the values of n, n', l, l' (formulas for $n' = n$ and $n' = n + 1$ are available).

For the present case, a numerical approach was preferred, in order to evaluate $R_{nl,n'l'}$ and $Q_{nl,n'l'}$ for arbitrary values of the indices (see next section).

8.1.4 Numerical calculation of Positronium energy levels in strong fields

The first numerical problem to be addressed is the proper evaluation of radial matrix elements, which involves solving the complicated integrals Equation 8.1.28 and Equation 8.1.29 on the basis of Legendre functions. A different approach has been found necessary in the present case, if one wants to properly address the problem of calculating the energy levels of Rydberg Ps in presence of high magnetic field. High B fields of the order of 1 T cause severe Stark broadening of the energy levels and mixing among sublevels of many different n manifolds for $n > 15$. In these cases, the Hamiltonian matrix of many n levels needs to be diagonalized simultaneously in order to properly account for the mixing of nearby n manifolds (an example of such a case is discussed in [45] or shown in Figure 8.3).

A numerical approach for tabulating the radial matrix elements was adopted. A quadrature method for definite integrals [153, 80] based on a 2nd order Gauss-Legendre quadrature was developed for integrating Equation 8.1.28 and Equation 8.1.29 (the choice of Legendre quadrature

was found very appropriate for integrating functions based on Legendre polynomials). Results were verified against the ones in [140] also re-evaluated in [145], and found in perfect agreement.

The second numerical problem consists in evaluating Wigner 3j and 6j symbols. Explicit expression of 3j and 6j symbols are very cumbersome and can be found in [148] and [154]. For the present use, the values of these complicated combinatorial functions were tabulated using Mathematica[©] to a high values of l and j and cached, to avoid the high computational cost of evaluating them at runtime.

Instead of giving an explicit expression for calculating Clebsch-Gordan coefficients Equation 8.1.12, it was found more convenient to relate them as well to the Wigner 3j symbol. The advantage of using always 3j symbols is that they are more general with respect to Clebsch-Gordan coefficients, as they appear in most problems where the addition of two angular momenta is considered. The relation linking the two can be written as

$$\langle n, l, m_l | \langle s, m_s | \cdot | n, l, s, j, m \rangle = (-1)^{l-s+m} \sqrt{2j+1} \begin{pmatrix} l & s & j \\ m_l & m_s & -m \end{pmatrix} \quad (8.1.30)$$

The perturbed Hamiltonian matrix was finally assembled according to Equation 8.1.18

The diagonal terms of the matrix were populated with the energies of the unperturbed levels (Equation 8.1.11); the off-diagonal terms were obtained by summing the Zeeman matrix element (Equation 8.1.22), the Stark matrix element (Equation 8.1.24) and the diamagnetic matrix element (Equation 8.1.27). The size of the final H matrix for a single n manifold is equal to the number of sublevels: $4n^2 \times 4n^2$.

The final Hamiltonian matrix was fed into a state-of-the-art dense matrix eigensolver algorithm for determining the eigenvalues $E^{(i)}$ and eigenvectors $|E^{(i)}\rangle$ of Equation 8.1.4. The chosen eigensolver was LAPACK, since it is supported by the convenient high-level C++ linear algebra library Armadillo [155]⁴.

A pictorial drawing of the non-null elements disposition in the Hamiltonian matrix is shown in Figure 8.3, panel (a), for the selected case of $n = 15$ in a strong 1 T magnetic field (i.e. the matrix is 900×900). The matrix is evidently sparse, as the vast majority of the elements is null.

The code was developed to allow side levels with different n to be considered in the diago-

⁴There is currently neither the need for higher performance nor the necessity of a sparse matrix library, which could handle bigger matrix sizes. Higher performances would be promptly obtained replacing the current 32bit single-core version of LAPACK with an optimized CPU-based library (such as Intel TBB) or with a scalable linear algebra package that could be run on clusters (such as ScaLAPACK) or with a GPU-accelerated library (such as cuBLAS). For future use on higher n states (for example the case of antihydrogen in Rydberg levels produced by charge-exchange with Ps, for which $n_H = \sqrt{2}n_{Ps}$) an optimized sparse matrix solution on CPU (such as FILTLAN or CUSP) or on GPU (such as cuSPARSE) could be also considered.

nalization, to account for possible overlaps between different n manifolds at high external fields (see the case of Figure 8.6, panel (a)). This was obtained over-sizing the Hamiltonian matrix of the level under consideration to a bigger matrix including the Hamiltonians of an interval of energy levels spanning from $n - n_s$ to $n + n_s$, with n_s being the number of side levels considered. This enlarged matrix was thus constituted by the Hamiltonians of each n manifold, appearing as blocks on the principal diagonal, and all off-diagonal interaction terms between different manifolds. An example of such matrix is shown in Figure 8.3, panel (b), where the levels interval $n = 14 - 16$ in a high 1 T magnetic field is considered.

This expedient was found necessary for correctly calculating the levels structure of Rydberg levels in presence of strong fields that cause mixing of nearby energy levels, such as the case depicted in Figure 8.6, where the effect of nearby manifolds is not negligible. The effect becomes stronger and stronger the closer the selected state to the ionization limit, which for a 1 T field occurs at $n \sim 28$ [102]. For n manifolds very close to the ionization limit, this way of performing the levels calculation becomes extremely time consuming, as the energy levels are all mixed in a quasi-continuum of energy levels (as discussed in [45]).

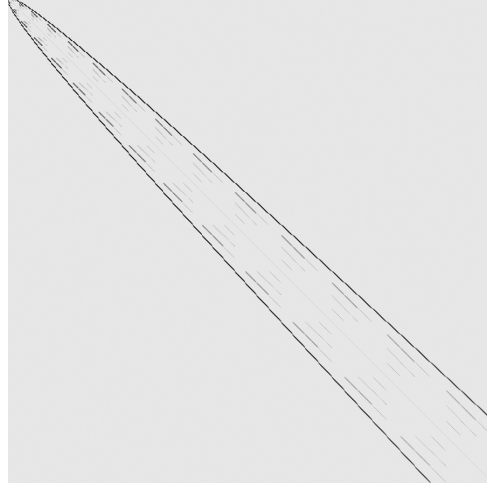
8.1.5 Numerical results

This section first presents the numerical experiments conducted to validate the present method against results found in literature. Subsequently an example of Ps energy levels calculation on $n = 15, 16, 17$ Rydberg levels, using the same experimental conditions of the spectroscopy chamber and in AEGIS $\bar{\text{H}}$ production region, is shown. Finally, some performance considerations of the Hamiltonian generation and diagonalization are drawn for future use.

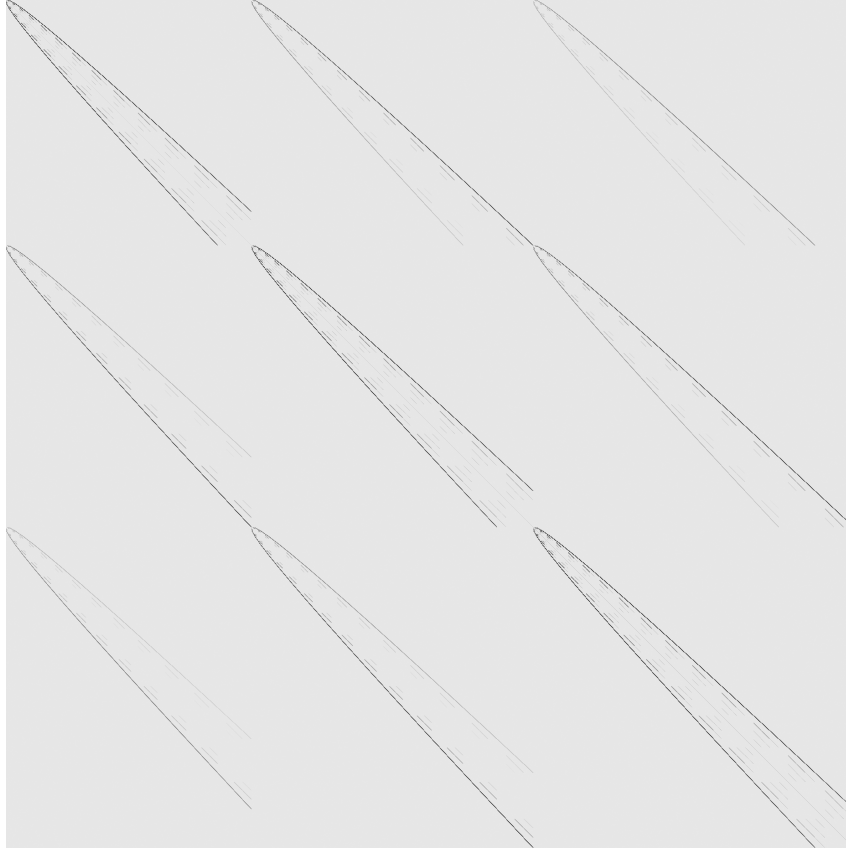
Comparison with published literature

A direct comparison of Ps energy levels structure calculated from PsTC against published results was possible for the $n = 2$ level, which was studied non-perturbatively by Curry [141] including Zeeman, Stark and motional Stark contributions for both Ps atoms at rest and in motion. This comparison allowed to verify independently the calculations of both Zeeman and Stark matrix elements.

First, the energy levels of $n = 2$ positronium at rest $v_{\text{Ps}} = 0$ were calculated as a function of the external magnetic field in the range $0 - 0.75$ T, without the presence of an external electric field. The comparison of the results found in [141] and obtained from PsTC is shown in Figure 8.4. The perfect matching of the two results verifies the correctness of the calculation of the Zeeman matrix element, as no electric field is present in this case.



(a) Hamiltonian of $n = 15$ only



(b) Hamiltonian of the levels $n = 14 - 16$ including cross-manifold terms

Figure 8.3: Visualization of the Hamiltonian matrix of the $n = 15$ manifold (panel (a)) and of the manifolds $n = 14 - 16$ including cross-manifolds interaction terms (panel (b)). Null matrix elements are shown in light gray, whereas non-null elements are in stronger shades. The two matrices are respectively 900×900 and 2708×2708 in size. As one could notice, they are both sparse. Under the conditions considered ($B = 1 \text{ T}$, $E = 500 \text{ V cm}^{-1}$, $v_{\text{Ps}} = 1.0 \cdot 10^5 \text{ m s}^{-1}$), the effect of side levels cannot be neglected during an accurate levels evaluation.

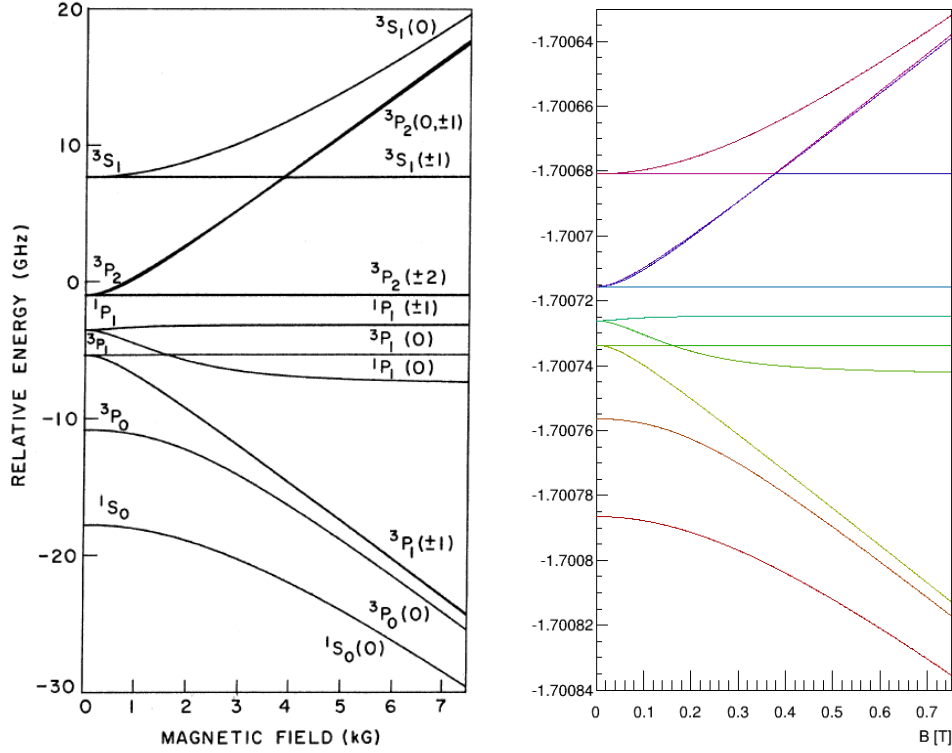
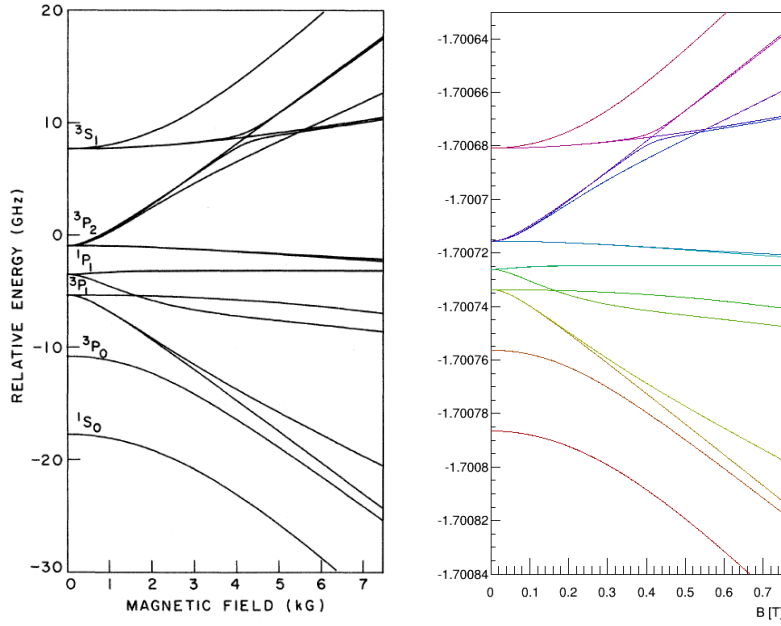


Figure 8.4: Comparison of literature calculations of Ps levels on the $n = 2$ manifold with the atom at rest (left, from [141]) and the present code (right). The plot shows the energy structure of Ps as a function of the axial magnetic field; the right plot is given in eV.

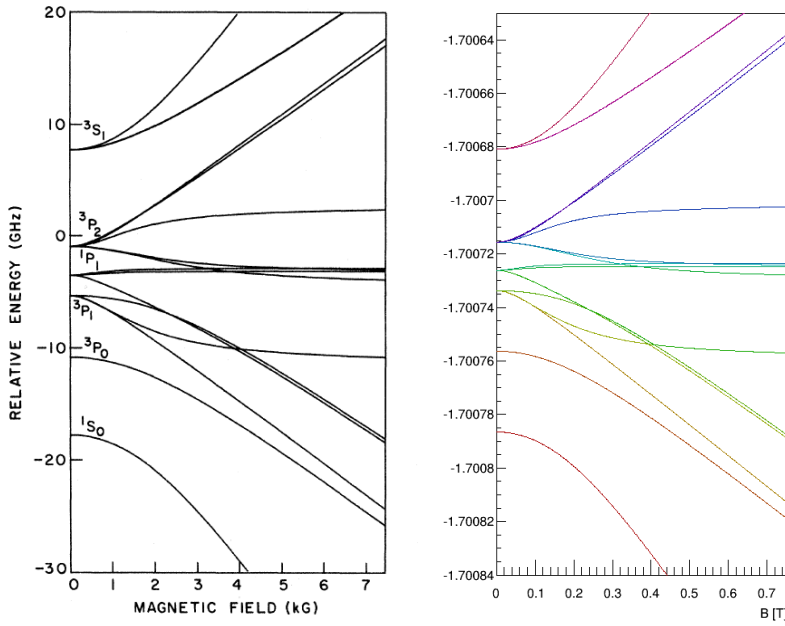
Subsequently, the energy levels of $n = 2$ positronium moving in the $\hat{y}$ direction (with the two velocities $1.07 \cdot 10^5 \text{ m s}^{-1}$ and $3.4 \cdot 10^5 \text{ m s}^{-1}$) were calculated as a function of the external magnetic field in the same range considered above. Again, the results of PsTC were compared to the same plots found in [141], as depicted in Figure 8.5. In this case, both Zeeman and Stark matrix elements play a role, as the velocity of Ps induces a motional Stark electric field. The perfect matching of the two results verifies independently also the correctness the Stark matrix element.

Calculation of accurate Ps energy structure at Rydberg levels

The method described in the last sections was applied to the calculation of the energy structure of Positronium at Rydberg levels in both experimental conditions of interest for AEgIS: the experimental conditions used during the Ps spectroscopy campaign ($B = 0.025 \text{ T}$, $E = 50\,000 \text{ V m}^{-1}$, $\vec{E} \parallel \vec{B}$, $v_{\text{Ps}} = 1.0 \cdot 10^5 \text{ m s}^{-1}$ and $\vec{v}_{\text{Ps}} \parallel \vec{B}$, see 4.5 and 4.6) and the experimental conditions to be



(a) $v_{Ps} = 1.07 \cdot 10^5 \cdot \hat{y} \text{ m s}^{-1}$



(b) $v_{Ps} = 3.4 \cdot 10^5 \cdot \hat{y} \text{ m s}^{-1}$

Figure 8.5: Comparison of literature calculations of Ps levels on the $n = 2$ manifold with the atom in motion (left, from [141]) and the present code (right). The comparison is shown for two different radial velocities (parallel to the y axis); the different magnitude of motional Stark splitting of energy levels in the two cases is evident. All plots show the energy structure of Ps as a function of the axial magnetic field; the right plots are given in eV.

used for antihydrogen production in the main experiment ($B = 1 \text{ T}$, $E = 1000 \text{ V m}^{-1}$, $\vec{E} \parallel \vec{B}$, $v_{\text{Ps}} = 1.0 \cdot 10^5 \text{ m s}^{-1}$ and $\theta_{\text{Ps}} = \pi/3$).

Results of these two calculations are shown in Figure 8.6 and Figure 8.7. The two $n = 15$ and $n = 16$ manifolds start to mix for $E = 40\,000 \text{ V m}^{-1}$ electric field and for $B = 0.5 \text{ T}$ magnetic field, respectively. The splitting of the energy levels obtained in the spectroscopy setup with $E = 50\,000 \text{ V m}^{-1}$ electric field was very similar to the splitting that should be expected inside AEgIS when performing excitation to Rydberg levels for antihydrogen production.

Performance considerations

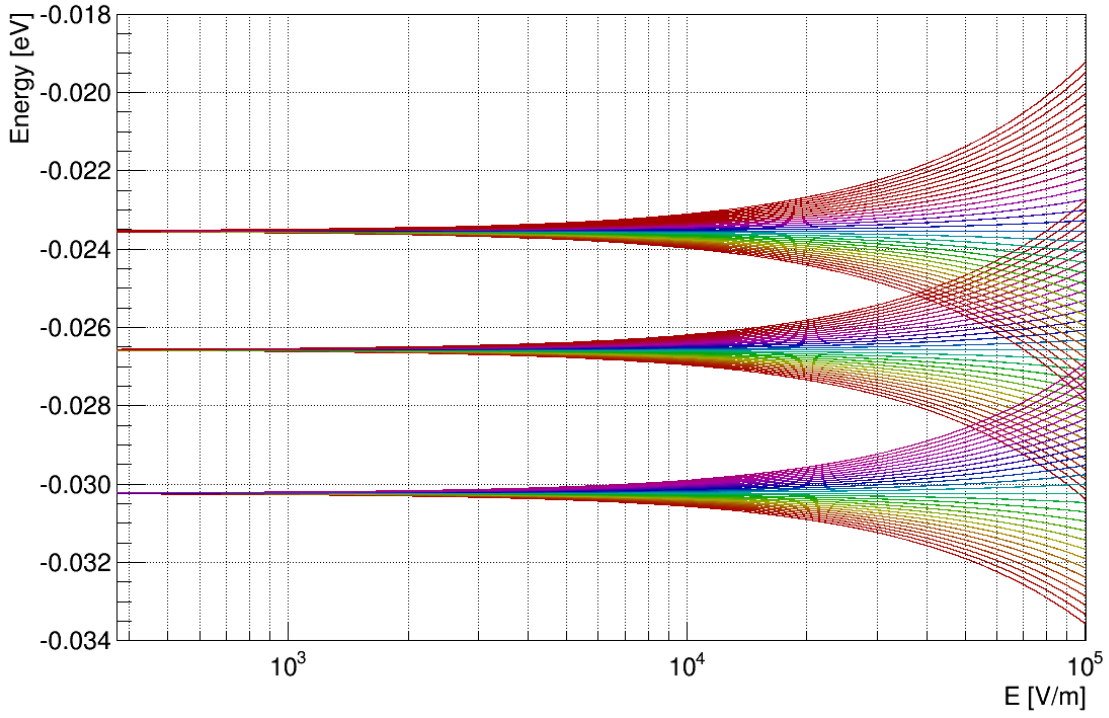
The numerical implementation of the non-perturbative energy levels calculation algorithm of PsTC, described in last paragraphs, was benchmarked to evaluate its computing speed in two test runs. The two time intervals elapsed for generating the Hamiltonian matrix and for its numerical diagonalization were measured during a single-core execution of the algorithm.

In the first run, the energy levels of a single n manifold were calculated for increasing values of the principal quantum number n . Results, summarized in Table 8.2, show that the diagonalization step takes roughly the same amount of time as the Hamiltonian matrix generation. The two times scales as $\propto n^{3.4}$, which is slightly less than the expected $o(n^4)$, since the tridiagonal eigensolver inside LAPACK is $o(N^2)$ [156] and the size of the matrix is $N = 4n^2$.

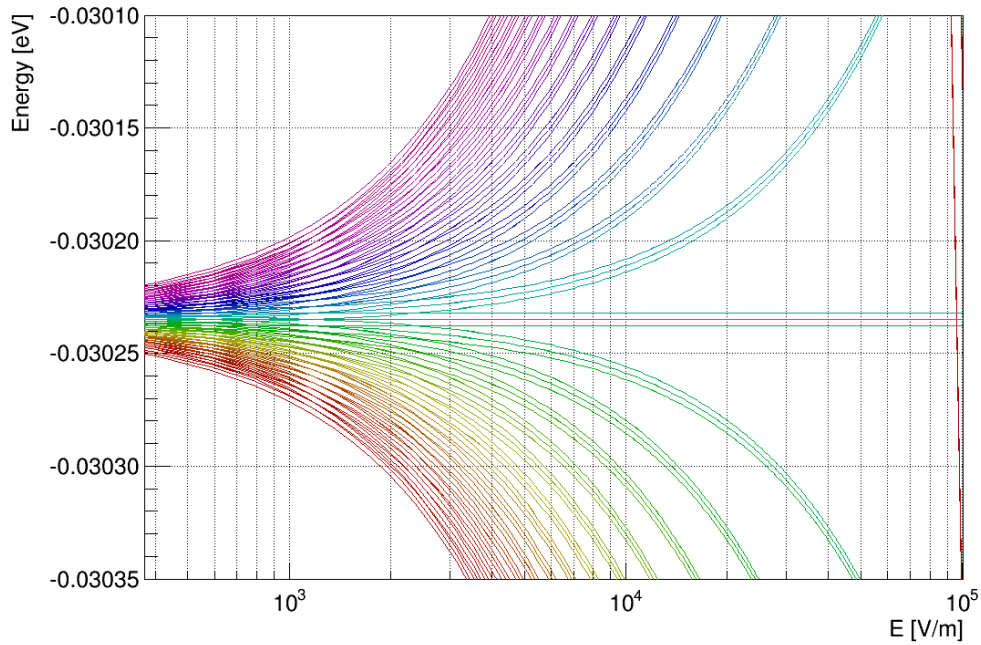
In the second run, the energy levels of a n manifold were calculated together with its two side manifolds, $n-1$ and $n+1$, including the cross-manifold interaction terms, of interest for accurate calculations on Rydberg levels when different manifolds are mixed. The resulting Hamiltonian matrix to be diagonalized has a size of $N = 4((n-1)^2 + n^2 + (n+1)^2)$. Results of this run are summarized in Table 8.3.

Energy level	H generation	H diagonalization
$n = 5$	6.6 ms	3.7 ms
$n = 10$	0.14 s	0.12 s
$n = 15$	1.0 s	1.2 s
$n = 20$	4.9 s	4.9 s

Table 8.2: Time elapsed for generating and diagonalizing the positronium Hamiltonian matrix of a single n manifold in strong external fields. Data obtained with a single-core execution of the algorithm on a Core i7-4720HQ CPU operating at 2.6 GHz.

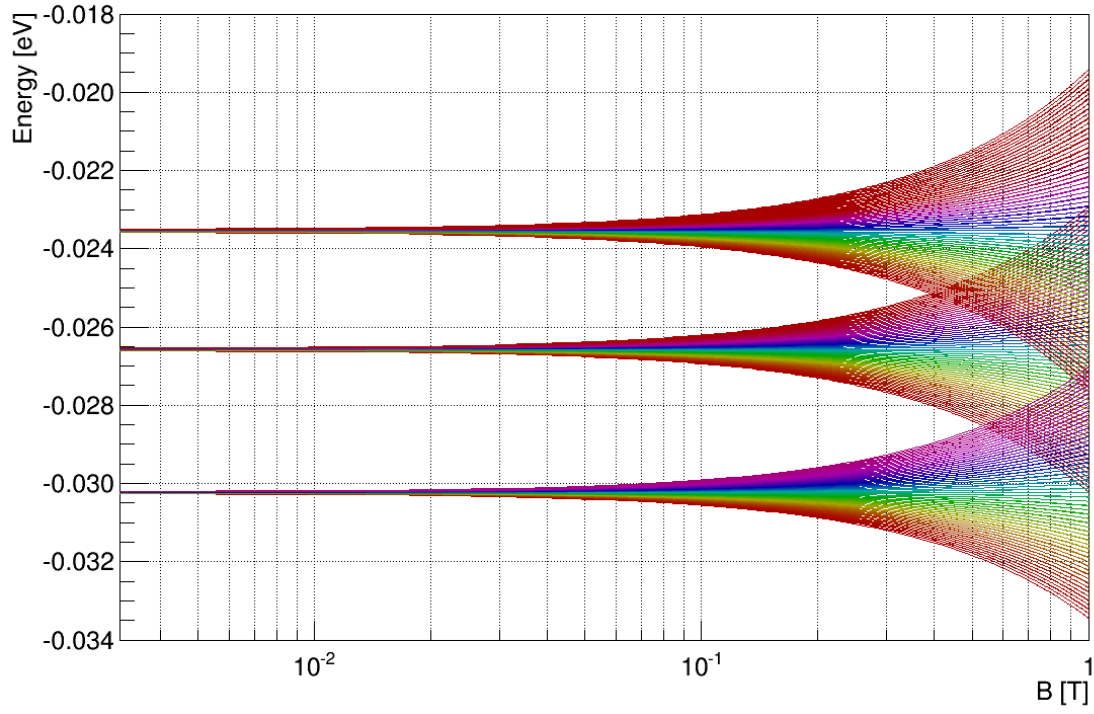


(a) Energy structure of Ps $n = 15 - 17$ manifolds vs applied electric field

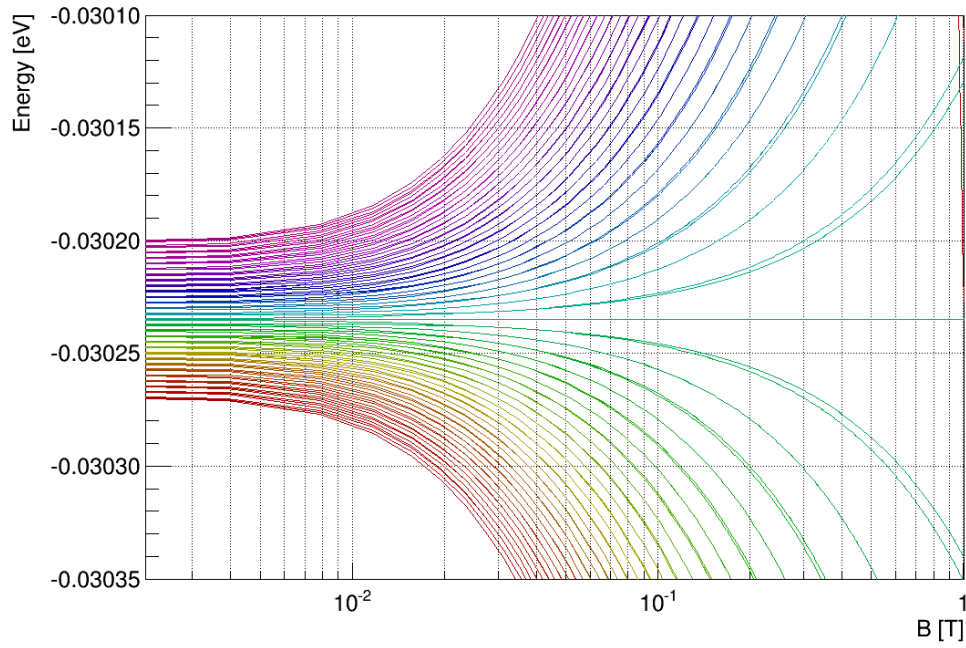


(b) Zoom on $n = 15$

Figure 8.6: Energy structure of Ps $n = 15, 16, 17$ Rydberg levels calculated in the experimental conditions of the dedicated spectroscopy chamber.



(a) Energy structure of Ps $n = 15 - 17$ manifolds vs applied magnetic field



(b) Zoom on $n = 15$

Figure 8.7: Energy structure of Ps $n = 15, 16, 17$ Rydberg levels calculated in the experimental conditions of the antihydrogen production region of AEGIS.

Energy level	H generation	H diagonalization
$n = 4 - 6$	85 ms	58 ms
$n = 9 - 11$	1.5 s	3.2 s
$n = 14 - 16$	10 s	36 s
$n = 19 - 21$	46 s	156 s

Table 8.3: Time elapsed for generating and diagonalizing the positronium Hamiltonian matrix of a n manifold with 2 side manifolds in strong external fields. Data obtained with a single-core execution of the algorithm on a Core i7-4720HQ CPU operating at 2.6 GHz.

8.2 Calculation of optical transition probabilities

This section treats the calculation of optical transition probabilities of positronium in strong fields. The generalized Einstein A and B coefficients, modelling absorption, spontaneous emission and stimulated emission, are first calculated. Then, the optical absorption spectrum of Ps on Rydberg levels is obtained using the calculated absorption rates and considering a Doppler broadening due to the transverse component of the atom velocity, as in spectroscopy experiments probe lasers are typically sent from the side. Finally, a comparison of the experimental results obtained in 4.6 and in the published spectroscopic survey [31] are presented.

8.2.1 Two-level broadband light-atom interaction model

A simple and effective way to model the interaction between a broadband light source and a two-level atomic system was first introduced by Einstein (see [157]). The model accounts for the following processes:

1. **Absorption**, modelling an atom transition $1 \rightarrow 2$ with the absorption of a photon, caused by the presence of the light source;
2. **Stimulated emission**, modelling an atom transition $2 \rightarrow 1$ with the emission of a photon, caused by the presence of the light source;
3. **Spontaneous emission**, modelling a spontaneous atom transition $2 \rightarrow 1$ with the emission of a photon, independent of the presence of the light source.

Two rate equations describe the dynamics of the atoms populations $N_1(t)$ and $N_2(t)$ as a function of time. Calling $\rho_{12}(t)$ the unit density of photons as a function of time, they are

defined as [158, p.12])

$$\begin{aligned}\frac{dN_1}{dt} &= -B_{12} N_1(t) \rho_{12}(t) + B_{21} N_2(t) \rho_{12}(t) + A_{21} N_2(t) \\ \frac{dN_2}{dt} &= B_{12} N_1(t) \rho_{12}(t) - B_{21} N_2(t) \rho_{12}(t) - A_{21} N_2(t)\end{aligned}\tag{8.2.1}$$

The coefficients A_{21} , B_{21} and B_{12} are the so-called *generalized Einstein coefficients* for the $1 \rightarrow 2$ transition. The coefficients are properties of the atomic system under consideration, and model respectively spontaneous emission (A_{21}), stimulated emission (B_{21}) and absorption (B_{12}). If the two levels under consideration are non-degenerate, the absorption and stimulated emission coefficients are equal in modulo $B_{21} = B_{12}$ [158, p. 13].

8.2.2 Calculation of generalized Einstein A and B coefficients

The non-perturbative calculation of Positronium energy levels and eigenstates allows the accurate calculation of generalized Einstein's spontaneous emission A and absorption/stimulated emission B coefficients for all sublevels inside the considered n manifolds. Explicit expression to calculate A and B coefficients are found in most atomic physics books (see, for example [158, p. 126])

$$A_{a \rightarrow b} = \frac{8\pi^2 e_0^2 \nu_{a \rightarrow b}^3}{\epsilon_0 \hbar c^3} \cdot \frac{|D_{a \rightarrow b}|^2}{3}\tag{8.2.2}$$

$$B_{a \rightarrow b} = \frac{\pi e_0^2}{\epsilon_0 \hbar^2} \cdot \left| \vec{\epsilon} \cdot \vec{D}_{a \rightarrow b} \right|^2\tag{8.2.3}$$

where $\nu_{a \rightarrow b}$ is the optical transition frequency between the initial a state and the final b state, $\vec{D}_{a \rightarrow b} = \langle a | \vec{r} | b \rangle$ is the dipole matrix element, $D_{a \rightarrow b}$ is its norm and $\vec{\epsilon}$ is the laser polarization vector.

One is thus interested in calculating the expectation values of the $\vec{\epsilon} \cdot \vec{r}$ operator using the non-perturbative eigenstates in strong fields. The components of the numerically calculated eigenvectors are used to express the final eigenstates as linear combinations of the basis vectors $|n, l, s, j, m\rangle$.

First, a suitable naming scheme is necessary to indicate the perturbed eigenstates, so far generically indicated as $|E^{(i)}\rangle$. The order in which the eigenstates are listed for strong external fields is, in general, different than the order they have with null external fields, as a consequence of the many crossings between sublevels (see, for instance, Figure 8.4 and Figure 8.5). The usual spectroscopic nomenclature is well-defined only for field-free calculations and low-order

perturbation theory calculations ⁵.

The naming scheme used here keeps only n as valid quantum number for labelling the perturbed eigenstates, and absorbs all other quantum numbers in a generic index i spanning over the $4n^2$ sublevels in that manifold ⁶.

$$|n, i\rangle \longleftrightarrow |E^{(i)}\rangle \quad (8.2.4)$$

Using this naming convention, the dipole matrix elements can be written as

$$\vec{\epsilon} \cdot \vec{D}_{a \rightarrow b} = \langle n, i | \vec{\epsilon} \cdot \vec{r} | n', i' \rangle \quad (8.2.5)$$

The scalar product can be rewritten in spherical coordinates, in a similar fashion to what was done calculating the Stark matrix element, 8.1.3):

$$\vec{\epsilon} \cdot \vec{r} = \sum_{q=-1}^1 (-1)^q r_q \epsilon_{-q} \quad (8.2.6)$$

where

$$r_q = \sqrt{\frac{4\pi}{3}} r Y_{1q} \quad (8.2.7)$$

and $\epsilon_q = [\epsilon_{\sigma+}, \epsilon_{\pi}, \epsilon_{\sigma-}]$ are the three polarization components expressed in spherical coordinates (see Figure 8.8 for a visual representation and 8.2.2 for further details). The spontaneous emission case is found setting all the three ϵ_q components to $1/\sqrt{3}$, i.e. averaging on all possible atom orientations. The matrix element is thus rewritten as

$$\vec{\epsilon} \cdot \vec{D}_{a \rightarrow b} = \sum_{q=-1}^1 (-1)^q \epsilon_{-q} \langle n, i | r_q | n', i' \rangle \quad (8.2.8)$$

For calculating the remaining matrix element $\langle n, i | r_q | n', i' \rangle$, one has to first expand the perturbed eigenvectors $|n, i\rangle$ in linear combinations of the unperturbed basis vectors $|n, l, s, j, m\rangle$, then change basis to $|n, l, m_l\rangle |s, m_s\rangle$ using the Clebsch-Gordan expansion Equation 8.1.13 and finally evaluate the expectation values between the $|n, l, m_l\rangle$ kets. The full calculation is quite cumbersome and omitted here for brevity.

⁵yet sometimes unpleasantly seen also in articles discussing calculations in strong fields, see for instance Figure 8.4, panel (a)

⁶this naming scheme clearly fails for severe mixing of the different n manifolds, where also n has to be dropped as a valid quantum number. See for instance Figure 8.7

The result is:

$$\langle n, i | r_q | n', i' \rangle = \sum_{m'_s, m'_l} \sum_{m_s, m_l} \delta_{m'_l+m'_s}^{m_l+m_s} C_{m_l, m_s}^j C_{m'_l, m'_s}^{j'} \cdot R_{nl, n' l+1} A_{\Delta m}^{\Delta l} \quad (8.2.9)$$

where $R_{nl, n'l'}$ are again the radial matrix elements discussed in 8.1.3 and $A_{\Delta m}^{\Delta l}$ are angular elements resulting from the product of spherical harmonics. $A_{\Delta m}^{\Delta l}$ are non-null only if the dipole selection rules are satisfied: $\Delta l = \pm 1$, $\Delta m = 0, \pm 1$. Their values can be calculated analytically for the two different values of Δl (see [140, pp. 253-254]).

For $\Delta l = +1$ transitions:

$$\begin{aligned} A_{\Delta m=+1}^{\Delta l=+1} &= \frac{1}{\sqrt{2}} \sqrt{\frac{(l+m+2)(l+m+1)}{(2l+3)(2l+1)}} \\ A_{\Delta m=0}^{\Delta l=+1} &= \sqrt{\frac{(l+1)^2 - m^2}{(2l+3)(2l+1)}} \\ A_{\Delta m=-1}^{\Delta l=+1} &= -\frac{1}{\sqrt{2}} \sqrt{\frac{(l-m+2)(l-m+1)}{(2l+3)(2l+1)}} \end{aligned} \quad (8.2.10)$$

For $\Delta l = -1$ transitions:

$$\begin{aligned} A_{\Delta m=+1}^{\Delta l=-1} &= -\frac{1}{\sqrt{2}} \sqrt{\frac{(l-m)(l-m-1)}{(2l+1)(2l-1)}} \\ A_{\Delta m=0}^{\Delta l=-1} &= \sqrt{\frac{l^2 - m^2}{(2l+1)(2l-1)}} \\ A_{\Delta m=-1}^{\Delta l=-1} &= \frac{1}{\sqrt{2}} \sqrt{\frac{(l+m)(l+m-1)}{(2l+1)(2l-1)}} \end{aligned} \quad (8.2.11)$$

Spherical polarization components and π -polarized and σ -polarized pulses

The spherical components of the polarization vector introduced above, $\vec{\epsilon}_q = [\epsilon_{\sigma+}, \epsilon_{\pi}, \epsilon_{\sigma-}]$, whose schematical drawing is shown in Figure 8.8, can be related to the Cartesian direction as

$$\vec{\epsilon}_{\sigma+} = \frac{\hat{x} + i\hat{y}}{\sqrt{2}} \quad \vec{\epsilon}_{\pi} = \hat{z} \quad \vec{\epsilon}_{\sigma-} = \frac{\hat{x} - i\hat{y}}{\sqrt{2}} \quad (8.2.12)$$

Most spectroscopy experiments on positronium used transverse probe laser pulses with linear polarization (examples are the experiments discussed in 4.5, 4.6, 10 and in literature [30, 31]. It's useful, for comparing experimental results to the theoretical predictions of PsTC, to introduce the following nomenclature: a π -polarized laser pulse exhibits linear polarization parallel to

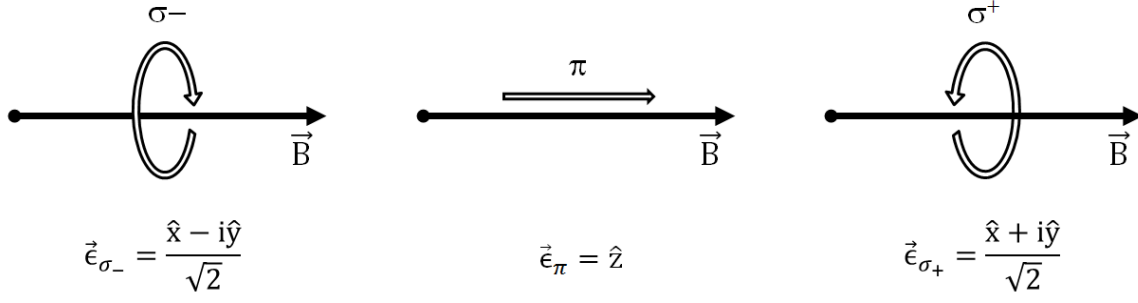


Figure 8.8: Visual representation of the spherical components of the polarization vector $\vec{\epsilon}$ and their decomposition on Cartesian space directions.

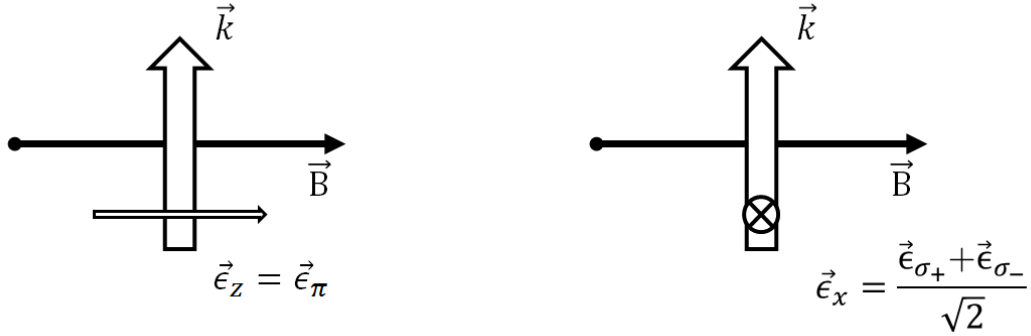


Figure 8.9: Schematic representation of linearly polarized laser pulses (coming from the transverse direction) with polarization vectors parallel (π -polarization, left) and orthogonal (σ -polarization, right) to the magnetic field direction. Below, their decomposition in spherical components of the polarization tensor.

the magnetic field axis; a σ -polarized laser pulse exhibits linear polarization orthogonal to the magnetic field axis (see Figure 8.9 for a schematic representation). The spherical polarization components, associated to π - and σ -pulses used in the calculations are

$$\vec{\epsilon}_\pi = [0, 0, 1] \quad \epsilon_\sigma = \left[\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0 \right] \quad (8.2.13)$$

8.2.3 Calculation of absorption spectra

Using the calculated values of the generalized Einstein A and B coefficients, it is now possible to calculate theoretical absorption spectra of positronium atoms as they interact with a probe spectroscopy laser, in order to compare them to published results of positronium spectroscopic

surveys on Rydberg levels. It is worth to distinguish the far non-saturated (linear) transition regime from the far saturated transition regime (already mentioned when discussing laser saturations in 4.5.2), as the two require different handling.

Positronium absorption spectrum in the linear regime

In the far non-saturated (or linear) transition regime, the probe laser - due to low laser fluence or atoms/laser interaction time - is unable to excite much of the initial population in the 1 state to the 2 state, thus $N_1(t) \gg N_2(t)$. Under this approximation, both spontaneous and stimulated emission terms in Equation 8.2.15 can be neglected, as they are both proportional to $N_2(t)$, and the probability to observe atoms in $N_2(t)$ is linearly proportional to the B_{12} coefficient.

$$\frac{dN_2}{dt} = B_{12} N_1(t) \rho_{12}(t) \quad (8.2.14)$$

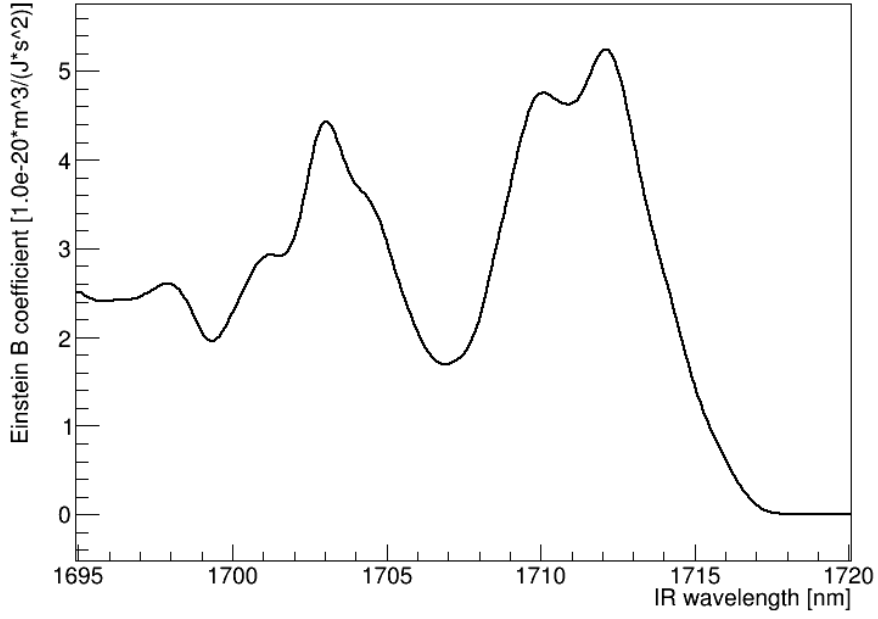
Positronium absorption spectra can be calculated, in the linear absorption regime, using the following procedure:

1. The full sublevels structure of initial and destination manifolds is first calculated using the non-perturbative energy levels calculation code (see 8.1);
2. Subsequently, absorption coefficients B_i , exact transition frequencies ν_i and wavelength λ_i of all allowed transitions (i.e. having $B \neq 0$) are calculated among sublevels of the initial and final manifolds ⁷;
3. Finally, the linear absorption spectrum is built on the frequency (or wavelength) domain by evaluating the convolution of the probe laser spectral profile with a series of Dirac-delta distributions ⁸ centered in ν_i (or λ_i) with amplitude B_i . The spectral profile is here assumed as a Gaussian distribution in frequency (wavelength), with $\sigma = \sqrt{\sigma_L^2 + \sigma_D^2}$ (or, equivalently, $\lambda_i^2/c\sigma$), where σ_L is the laser bandwidth and σ_D is the Doppler broadening of the transition due to the atom velocity distribution in the laser direction.

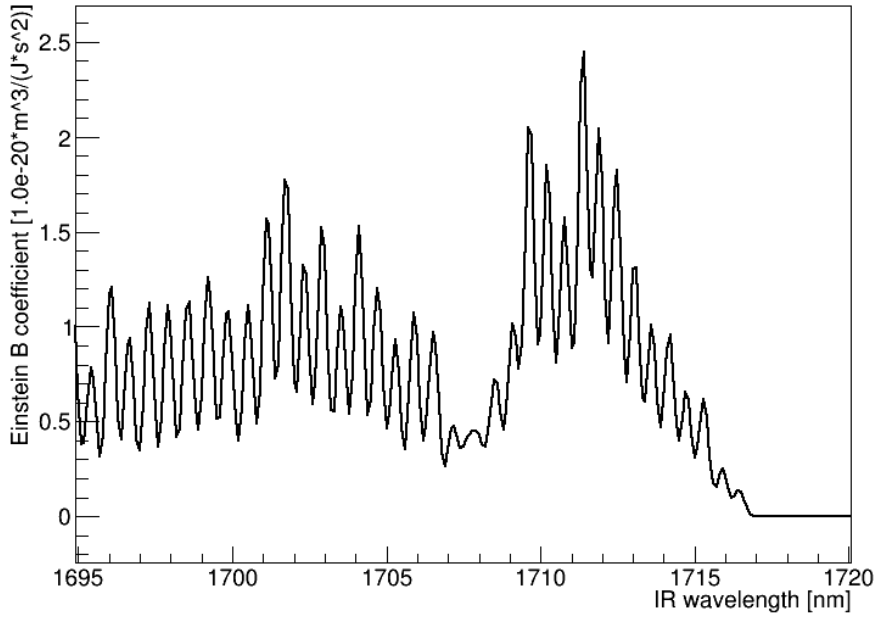
An example of calculated linear absorption spectra of Positronium is shown in Figure 8.10. Absorption spectra of Rydberg Ps atoms with two different velocity distributions, associated to two different temperatures (1200 K for the high-temperature case, 100 K for the low-temperature case), are reported for the $3 \rightarrow 15 - 17$ transition. When the Doppler broadening due to Ps temperature is reduced, the Stark structure of the energy levels becomes evident (see, for reference, the spectra measured in [31]).

⁷degeneracy is completely absent, as all sublevels are treated separately

⁸rigorously, a convolution with a Lorentzian distribution with FWHM equal to the natural linewidth should



(a) Maxwell-Boltzmann velocity distribution at $T = 1200$ K



(b) Maxwell-Boltzmann velocity distribution at $T = 100$ K

Figure 8.10: Comparison of Ps absorption spectra in the regime of far non-saturation, obtained for the $3 \rightarrow 15-17$ transition at two different sample temperatures of 1200 K (panel a) and 100 K (panel b), in ideal laser bandwidth conditions. Other settings were $B = 0$, $E = 1000 \text{ V cm}^{-1}$, σ -polarization and $\sigma_L = 0 \text{ GHz}$.

Positronium absorption spectrum in the far saturated regime

In the far saturated transition regime, the probe laser is imagined to be so intense that all terms not proportional to $\rho_{12}(t)$ in Equation 8.2.15 could be neglected. Thus

$$\begin{aligned}\frac{dN_1}{dt} &= -B_{12} \rho_{12}(t) (N_1(t) - N_2(t)) \\ \frac{dN_2}{dt} &= B_{12} \rho_{12}(t) (N_1(t) - N_2(t))\end{aligned}\tag{8.2.15}$$

It is straightforward to show that the populations on the N_1 and N_2 establish quickly a 50% – 50% equilibrium, regardless of the value of B_{12} . This is a consequence of the rapid balancement between absorption and stimulated emission, which go to equilibrium much faster than the dynamics in time of the laser pulse $\rho_{12}(t)$. In this regime, all spectral lines are expected to be observed with the same amplitude.

The calculation of saturated absorption spectra is more straightforward than linear absorption spectra. Here the accurate calculation of the transition frequencies ν_i and wavelengths λ_i (obtained from the exact eigenvalues of the Hamiltonian, see Equation 8.1.18) is sufficient: the spectrum is built on the frequency (or wavelength) domain by evaluating the convolution of the probe laser spectral profile with a series of Dirac-delta distributions centered in ν_i (or λ_i) with equal amplitudes.

The spectral profile is assumed as well as a Gaussian distribution in frequency (or wavelength), with $\sigma = \sqrt{\sigma_L^2 + \sigma_D^2}$ (or, equivalently, $\lambda_i^2/c\sigma$), where σ_L is the laser bandwidth and σ_D is the Doppler broadening of the transition due to the atom velocity distribution in the laser direction.

A visual comparison between a saturated absorption spectrum versus a linear absorption spectrum is shown in Figure 8.11, calculated for the example case of $2 \rightarrow 9 - 13$ Positronium transitions.

8.2.4 Comparison with experimental results

The accurate calculation of optical absorption spectra of Positronium, described in the last section (see 8.2.3), allowed the theoretical model to be tested on experimental data. Two spectroscopic surveys were considered: the accurate spectroscopic survey carried out on the $1 \rightarrow 2 \rightarrow 9 - 15$ excitation path by Wall et al. [31]⁹ and the data collected during the experimental campaign of laser excitation to Rydberg levels on the $1 \rightarrow 3 \rightarrow 14 - 16$ path in AEGIS (discussed in chapter 4.6).

be considered; this effect is negligible if the laser spectrum is $> \text{GHz}$

⁹the author acknowledges the kindness of the authors in sharing their experimental data

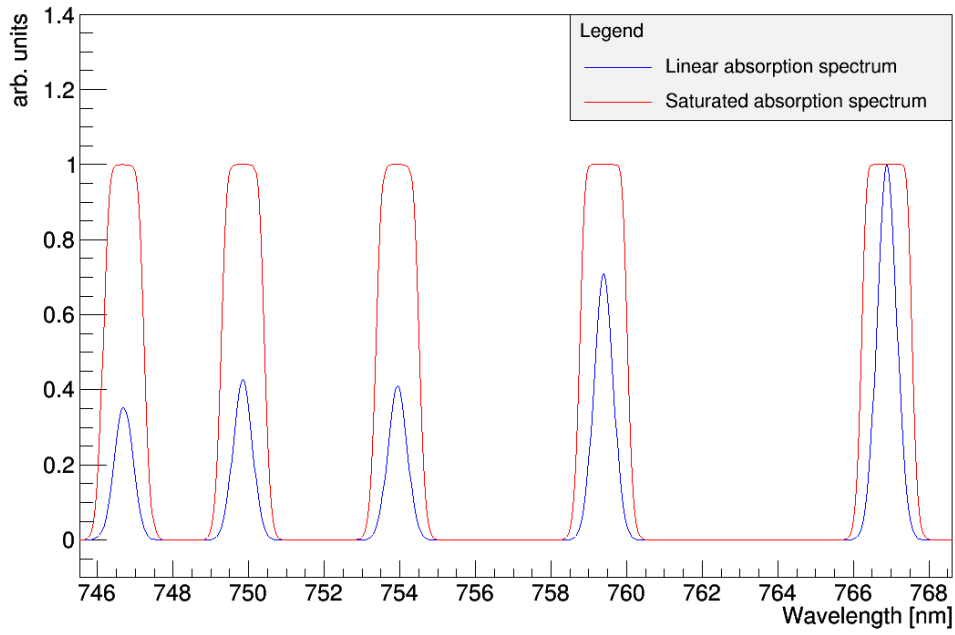


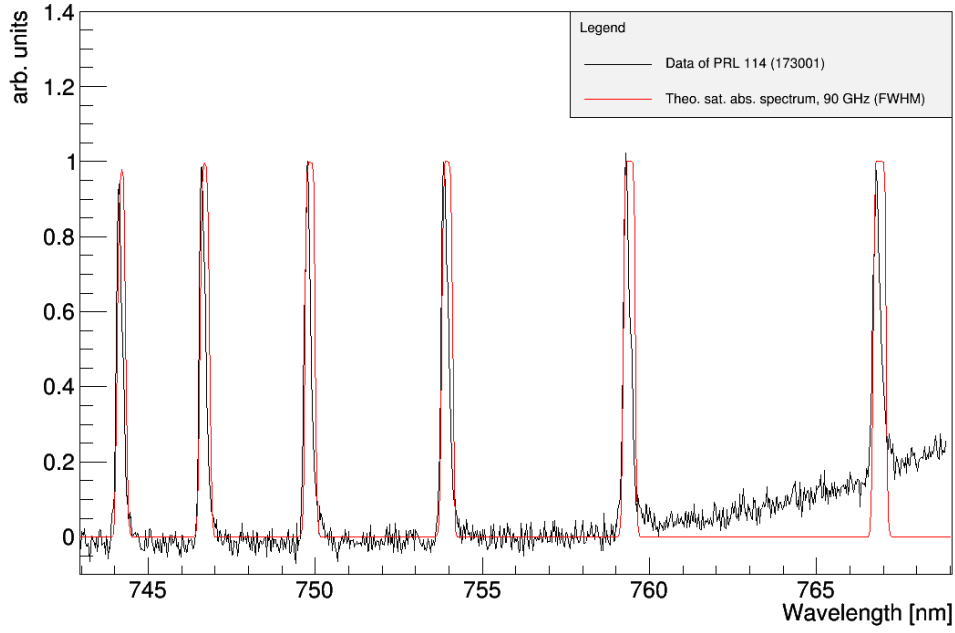
Figure 8.11: Comparison between linear absorption spectrum (blue) and a saturated absorption spectrum (red) calculated for the $2 \rightarrow 9 - 13$ Ps transitions, as described in the text. Power broadening of the saturated spectrum was also accounted in this example calculation.

In the first case, i.e. the spectroscopic survey of Wall et al. [31], the experimental conditions were the following. An axial magnetic field of $B = 0.01$ T was present for guiding the e^+ to the target, whereas the accelerating electric field was shielded with a grid (thus $E = 0$). The excitation and probe lasers were sent from the two opposite sides of the Ps sample. Their overall bandwidth was dominated by the ~ 90 GHz (FWHM) resolution of the $1 \rightarrow 2$ laser, which populated a set of sublevels on the $n = 2$ manifold. Both excitation lasers (respectively at 243 nm for the $1 \rightarrow 2$ transition and tunable in the range 729 – 769 nm) were operated in the saturation regime.

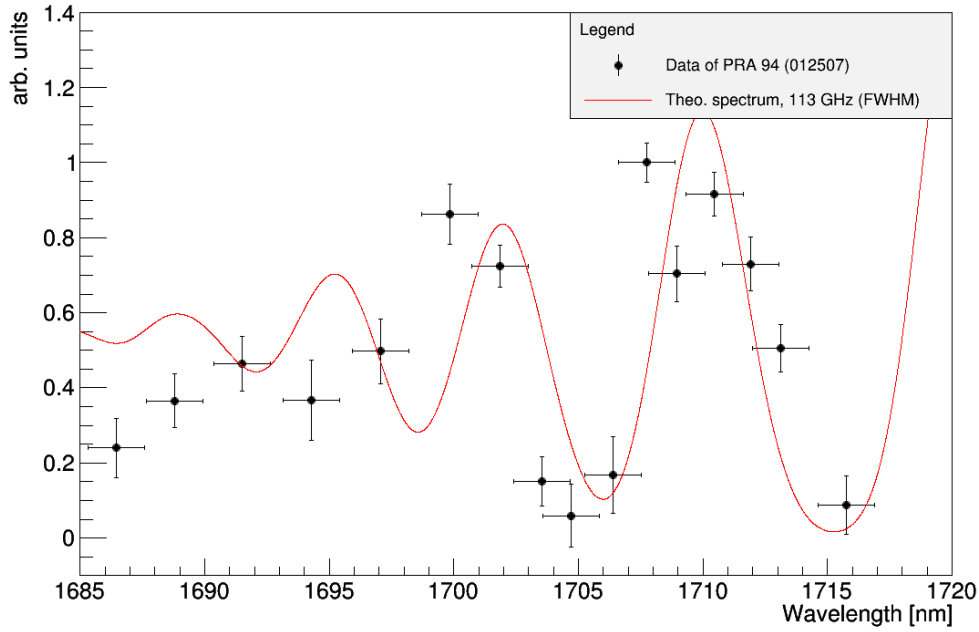
The comparison between PsTC saturated absorption spectrum and [31] experimental data is shown in Figure 8.12, panel (a). The consistency among the two results was found a striking evidence of the calculation correctness. The theoretical calculation correctly reproduced both the exact spectral peak positions (within a 90 GHz accuracy) and the bandwidth observed by the experiment. A small shift of the peaks centers, estimated in ~ 30 GHz, was noticed. It is unclear if this shift is somehow introduced by the model, or is due to an (indeed very small) offset in the calibration of the spectrometer used in [31].

In the second case, i.e. the spectroscopic survey conducted in AEgIS, the experimental conditions (already described in more details in 4.5 and 4.6) were different. A guiding magnetic field was present, $B = 0.025$ T (optimized for the magnetic quenching experiment, see 4.5), and an electric field parallel to the magnetic field as well, $E = 500$ V cm $^{-1}$, which induced a significant Stark broadening of the lines (see Figure 8.6). The excitation and probe lasers were sent from the same side of the Ps sample. Their bandwidth was dominated by the ~ 113 GHz (FWHM) resolution of the $1 \rightarrow 3$ laser. In this case, the $1 \rightarrow 3$ laser was operated in the near saturation regime, whereas the tunable IR laser $3 \rightarrow 14, 16$ was in the linear absorption regime (according to the calculations in [45, 87], a saturation measurement was not performed at the time).

A comparison between PsTC linear absorption spectrum and AEgIS experimental data is shown in Figure 8.12, panel (b). Here the consistency check is made more difficult by the scattering of the experimental points and the less-definite transition lines. The $n = 15$ line, which is clearly identified in the spectrum, is well reproduced by the linear spectrum, which also indicates $n = 15$ to be the last clearly identifiable peak. The $n = 16$ peak appears to be shifted towards lower wavelengths than predicted by the model, and no further peaks can be identified.



(a) PsTC saturated absorption spectrum vs. spectroscopic survey of [31]



(b) PsTC linear absorption spectrum vs. AEGIS spectroscopic survey 4.6

Figure 8.12: Comparison of absorption spectra calculated with PsTC versus the spectroscopic surveys performed in [31] and in the present work 4.6. In order to perform the comparison, the parameters in the calculation were set according to the two different experimental layouts (see text).

8.3 Towards measuring Ps axial velocity with transverse laser probes

The aim of this final section is to discuss a possible new method for measuring the velocity of a Ps sample in a strong magnetic field, sensible to both transverse and axial velocity components, and using only transverse laser pulses. This technique could be of particular interest for AEGIS to accurately characterize the positronium velocity distribution emitted from its cryogenic target, in view of its goal to excite positronium to Rydberg levels in the antihydrogen production region of the experiment.

In a typical spectroscopy experiment using a broadband laser excitation scheme, it is possible to measure the average velocity component of an atomic sample in the laser beam propagation direction by measuring the Doppler shift of a known transition. An estimate of the distribution of velocities in the laser beam propagation direction can be obtained as well, by considering the Doppler broadening of the line due to different velocities in the sample. An example of such case was discussed in 4.5, using a simple model of incoherent excitation process (recall Equation 4.5.5). In that experiment, the center of the line matched the theoretical center of the transition, meaning that no common velocity component was present. On the contrary, a severe broadening of the line was observed, caused by the broad distribution of velocities (with null average) of thermal Ps emitted from the target.

The possibility to measure the velocity component, orthogonal to the laser propagation direction, of a Ps sample on Rydberg levels is made possible by the motional Stark effect. Thus spurious electric field, felt by Ps atoms in their co-moving reference frame, is given by

$$\vec{E}_{\text{MS}} = \vec{v}_{\text{Ps}} \times \vec{B} \quad (8.3.1)$$

For a magnetic field oriented on the $\hat{z}$ axis, the motional Stark field experienced by a single particle is

$$\vec{E}_{\text{MS}} = \hat{x} v_y B_z - \hat{y} v_x B_z \quad (8.3.2)$$

A Ps atom moving parallel to $\vec{B}$, as in the case of a target set orthogonal to the $\hat{z}$ axis, has typically a velocity vector given by

$$\vec{v}_{\text{Ps}} = v_{\text{Ps}} \hat{z} \quad (8.3.3)$$

where velocity v_{Ps} indicates the velocity in modulo. Both $\hat{x}$ and $\hat{y}$ velocity components are

null in average, as well as the motional Stark field felt on average by the particles in the cloud

$$\langle \vec{E}_{\text{MS}} \rangle = \hat{x} \langle \vec{v}_{\text{Ps}} \rangle_y B_z - \hat{y} \langle \vec{v}_{\text{Ps}} \rangle_x B_z = 0 \quad (8.3.4)$$

The presence of the motional Stark field is source of broadening of the lines, according to the transverse velocity standard deviations $\sigma_{v_x} = \sigma_{v_y}$, typically equal for an isotropical source.

If the Ps converter target is now rotated downwards by an angle θ_{Ps} with respect to magnetic field direction around the $\hat{x}$ axis (as shown in Figure 8.13), the atoms parallel and transverse velocity distributions are mixed. All atoms in the cloud possess non-null velocity components both in $\hat{z}$ and in the $\hat{y}$ directions as

$$\vec{v}_{\text{Ps}}' = v_{\text{Ps}} [\cos(\theta_{\text{Ps}}) \hat{z} - \sin(\theta_{\text{Ps}}) \hat{y}] \quad (8.3.5)$$

Thus, all particles in the cloud feel a motional Stark electric field due to their extra transverse velocity component, as $\hat{y}$ lies in the plane orthogonal to the magnetic field. Averaging on the velocity distribution, one has

$$\langle \vec{E}_{\text{MS}}' \rangle = \hat{x} \langle v_{\text{Ps}} \rangle \sin(\theta_{\text{Ps}}) B_z \quad (8.3.6)$$

In this case, all Ps atoms in the cloud feel a bias electric field in the $\hat{x}$ direction as a consequence of their motion.

A theoretical evaluation of the effect of a rigid target rotation in a strong field on the $3 \rightarrow 15$ Rydberg line of Ps were carried out for single atoms (i.e. without averaging on velocity distributions) using PsTC. Being non-perturbative in the interaction Hamiltonian, the code automatically accounts for all terms in the perturbative series, including energy levels shifts due to atom polarizability. The effect of a rotation in a 1 T field to the energy levels of a Ps atom in motion at $1.0 \cdot 10^5 \text{ m s}^{-1}$ are shown in Figure 8.14, panel (a). In the $\theta_{\text{Ps}} = 0$ case, the shift from the unperturbed center of the line in Table 8.1 is consistent with the introduction of spin-orbit and hyperfine corrections. For $\theta_{\text{Ps}} > 0$, the transition line is both shifted towards higher wavelengths and progressively broadened, reaching a maximum for $\theta_{\text{Ps}} = \pi/2$. All simulations were performed considering a laser pulse with σ polarization. If θ_{Ps} is set to the constant value of $\pi/3$, as in AEgIS $\bar{\text{H}}$ production region, the broadening of the line can be varied by changing the value of the magnetic field around its nominal 1 T field, which could be of use for varying the magnitude of the effect or for increasing the visibility of the $n = 15$ line (see Figure 8.14, panel (b)).

A neat demonstration of the connection between total Ps velocity and broadening of Ps Rydberg lines, for a $\pi/3$ rotated target in a strong 1 T magnetic field, is shown in Figure 8.15,

where the two cases of parallel and rotated velocity vectors are compared for different magnitudes of a Ps atom velocity. Figure 8.15, panel (a) shows that in the $\theta_{\text{Ps}} = 0$ case, no broadening of the line due to the total velocity is expected, consistent with Equation 8.3.4. Viceversa, panel (b) shows the case of $\theta_{\text{Ps}} = \pi/3$, where sensitivity to the total velocity is shown both as a shift of the line center and its significant broadening, consistent with Equation 8.3.6.

In a rotated geometry, the motional Stark effect is a source of broadening of the lines different than the case where the target is set orthogonal to the magnetic field, as the velocity distributions are different. All calculations shown in Figure 8.14 and Figure 8.15 are performed per-atom, and do not include convolutions for the distribution of velocities.

The motional Stark field could be a valuable tool in a rotated geometry for measuring the total average Ps velocity $\langle v_{\text{Ps}} \rangle$, if a good modelling of the effect of $\vec{E}_{\text{MS}}$ is available. In particular, the highest sensitivity is expected in a strong magnetic field at Rydberg levels, where all other sources of levels splitting are negligible compared to the motional Stark shift (see Figure 8.1).

Comparing Rydberg spectra calculated from PsTC with accurate experimental data could provide a valid technique for measuring the average value of the Ps total velocity v_{Ps} using only transverse laser probes. This result, combined with a measurement of the velocity component in the laser beam propagation direction, could lead to the simultaneous determination of a Ps cloud velocity components parallel and orthogonal to the target orientation, by using

$$v_{\perp} = \sqrt{v_{\text{Ps}}^2 - v_{\parallel}^2} \quad (8.3.7)$$

The possibility to obtain an axial velocity measurement using a transverse laser probe is possibly an unique feature of Ps in strong fields, thanks to its lightness and the strength of the motional Stark effect - negligible for more massive systems. This method could be of interest for AEgIS, where the optical access is limited to the side due to geometrical constraints, for characterizing its cryogenic Ps source. An independent method would be to perform a time-of-flight measurement combined with a transverse velocity measurement using the $n = 3$ transition, but this approach would be more difficult due to the complex geometry in the $\bar{\text{H}}$ production region. Moreover, spectroscopy of Ps Rydberg levels in strong magnetic fields and, in particular, a dedicated study of the motional Stark effect of Ps energy levels has never been reported in literature (to the author's knowledge), and constitute an interesting item of research by themselves.

A first accurate spectroscopical survey of Rydberg levels in strong magnetic fields with a rotated target geometry can be performed in AEgIS without any deviation from its course towards charge-exchange $\bar{\text{H}}$ formation. Spectroscopic surveys of $n = 3$ and Rydberg levels around $n = 15$, combined with the theoretical results presented in this chapter, could provide

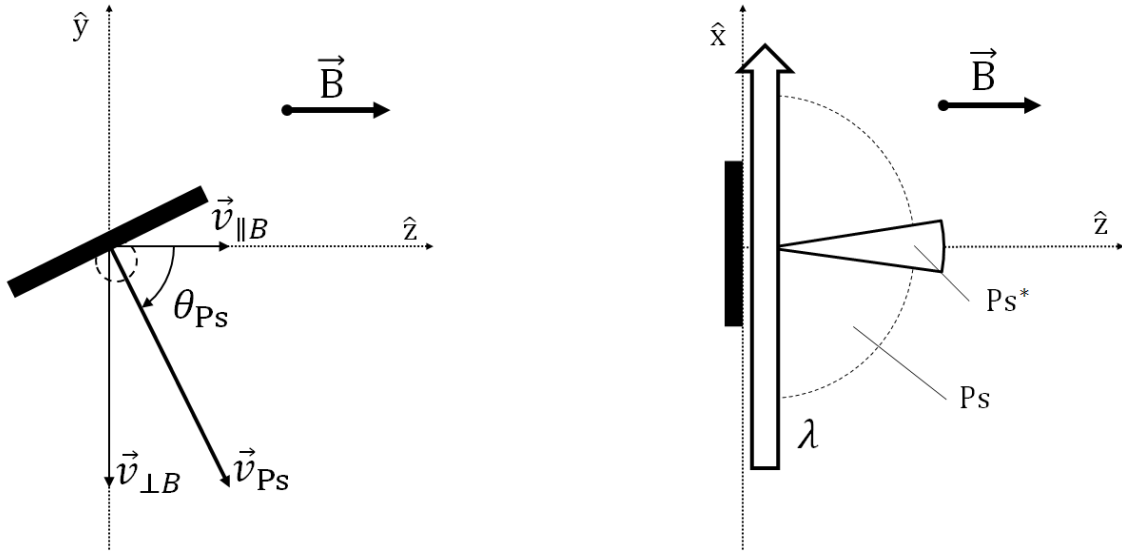


Figure 8.13: Sketched scheme of a tilted target Ps spectroscopy experiment in strong magnetic field. The target is rotated of an angle θ_{Ps} with respect to the $\hat{z}$ -axis, set parallel to the $\vec{B}$ field. Two laser pulses parallel to the $\hat{x}$ -axis hit the Ps sample from the side, exciting - due to a bandwidth narrower than the Doppler profile - only the distribution with $\langle v_x \rangle = 0$, if the laser is set with detuning $\delta = 0$.

insights on the distribution of velocities emitted from the cryogenic target and addressed by the lasers. In particular, several spectroscopic scans of the optical spectra around $n = 15$ with different values of the magnetic field should be considered, in order to discriminate the effect due to motional Stark broadening from other sources of broadening (see Figure 8.14, panel (B)).

A precise diagnostics of Ps velocity distributions addressed by the $1 \rightarrow 3 \rightarrow Ry$ lasers would be, for AEgIS, a precious tool for monitoring and optimizing Ps emission from the target. This is also important for correctly modelling charge-exchange production of $\bar{H}$ due to the strong dependance of the production cross-section from the Ps center-of-mass velocity¹⁰.

¹⁰see the discussion in 7.2

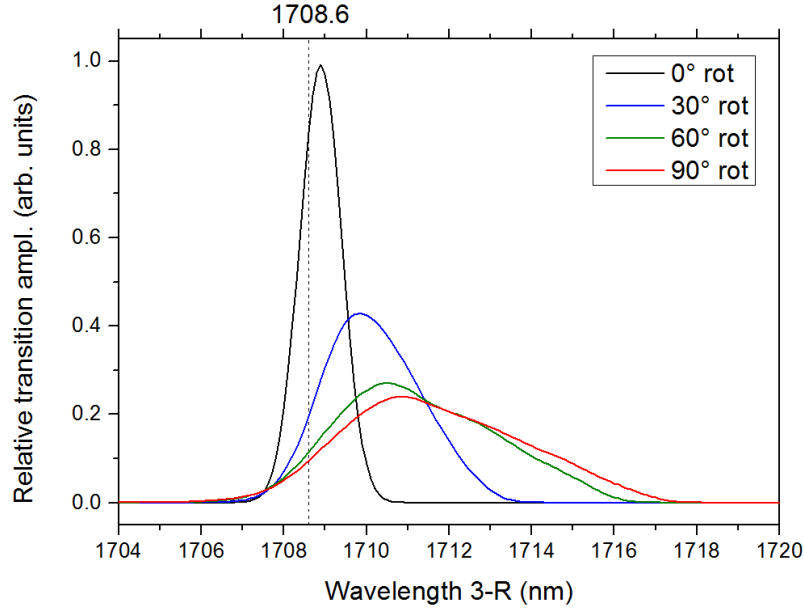
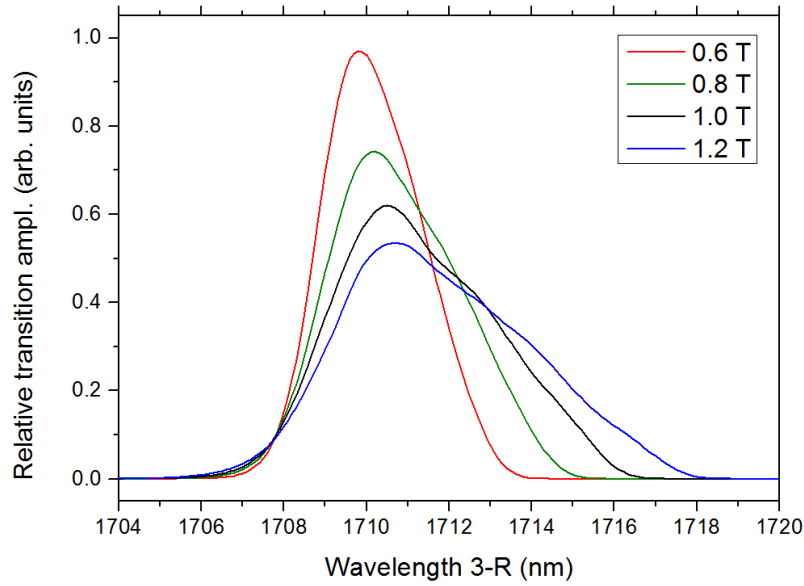
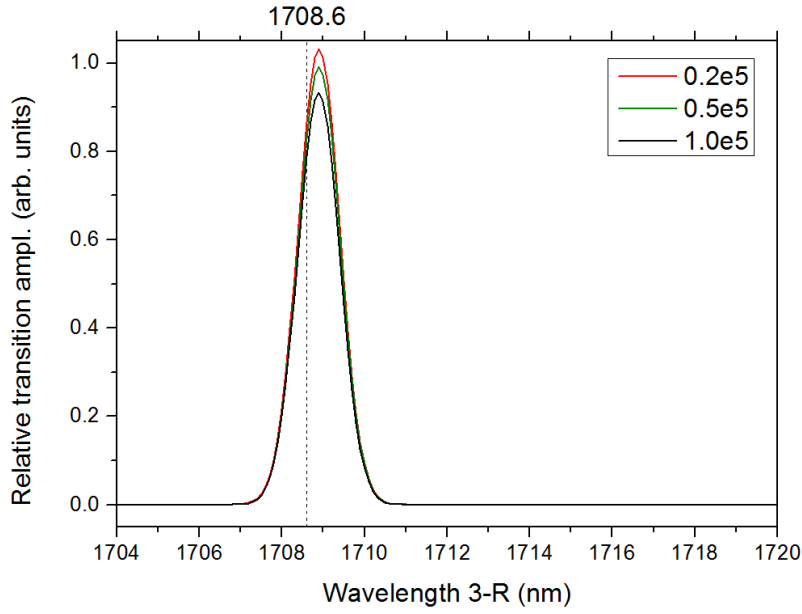
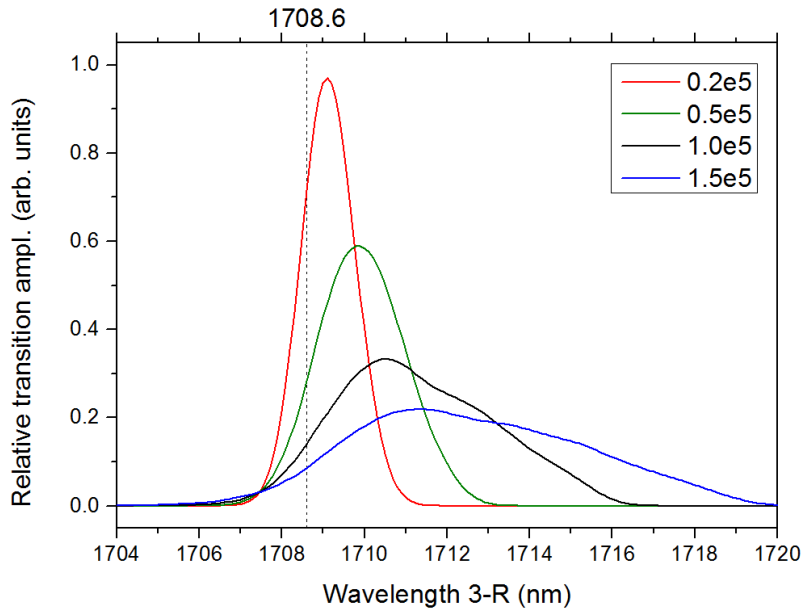

 (a) Shift and broadening of the $n = 15$ line vs. target rotation angle θ_{Ps}

 (b) Shift and broadening of the $n = 15$ line vs. magnetic field (in T)

Figure 8.14: Effect on the $n = 15$ *Ps* Rydberg line (of a single atom) of a change in the orientation angle of the velocity (panel (a)) and of a change in the magnetic field value for a fixed rotation angle of $\theta_{Ps} = \pi/3$ (panel (b)). The only source of broadening considered was the 48 GHz Doppler broadening due to the laser: no averaging on the velocity distribution was considered. The small shift from the 1708.6 nm zero-field value is meaningful, and comes from the non-perturbative treatment of the problem.



(a) Target orthogonal to magnetic field $\theta_{\text{Ps}} = 0$



(b) Target rotated by $\theta_{\text{Ps}} = \pi/3$, as in AEgIS $\bar{\text{H}}$ production region

Figure 8.15: Effect on the $n = 15$ Ps Rydberg line of a change in the norm of a single Ps atom velocity vector, for the two cases of a target orthogonal to the magnetic field (panel (a)) and rotated by $\pi/3$ on the $\hat{x}$ axis as sketched in Figure 8.13 (panel (b)). The only source of broadening considered was the 48 GHz Doppler broadening due to the laser: no averaging on the velocity distribution was considered. The small shift from the 1708.6 nm zero-field value is meaningful, and comes from the non-perturbative treatment of the problem.

Chapter 9

Non-neutral plasma thermal equilibrium in cylindrically symmetric arbitrary confining potentials

This chapter describes a novel numerical technique for solving the so-called *autoconsistent plasma potential* problem, which consists of calculating the thermal equilibrium distribution of a non-neutral plasma confined in arbitrary cylindrical confining potentials.

First, the problem of calculating a single-species plasma equilibrium distribution is described. A suitable calculation scheme to solve the resulting partial differential equations in cylindrical coordinates is then introduced, theoretically and numerically. The resulting technique is applied first to the calculation of the precise potential distribution for an arbitrary trap geometry with cylindrical symmetry, then to the calculation of plasma equilibrium distributions in arbitrary shape of the trap.

9.1 The problem of the autoconsistent plasma potential

The problem of calculating the equilibrium particle distribution for a completely charged, single-species plasma confined in a cylindrical trap by combined magnetic and electric fields has been discussed in many works in plasma literature, from [159], [160] to [135], and first solved numerically by [161]. The solution of this problem constitutes the basis for the analysis of the experimental data from experiments using cylindrical traps for axial confinement and an axial

static magnetic field for radial confinement.

9.1.1 Thermal equilibrium of a single-species non-neutral plasma

A collection of charged particles is named a non-neutral plasma when its equilibrium thermodynamics is dominated by the long-distance interactions among the constituents. This condition is usually reached in real charged particles systems as the temperature lowers and the density increases due to some mean of friction through a non-conservative force (among the many, emission of cyclotron radiation in a magnetic field, coupling to some colder system that carries away heat in form of kinetic motion, artificial dissipation of energy by active cooling - like sideband and laser cooling).

Thermodynamics of non-neutral plasmas is modeled, as in the case of gases, starting from its particle density distribution n . At macroscopic scale, two parameters are commonly used to describe equilibrium: the temperature of the cloud T and its Debye length λ_D , which is a specific observable of charged particle systems defined as the characteristic distance at which the cloud of moving electric charges shields the electric field produced by a charge in its center. The Debye length is connected to temperature T , the density of constituents in the cloud n and the charge of the particles q by

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{n q^2}} \quad (9.1.1)$$

The mathematical machinery of statistical mechanics developed to work out the thermodynamical behaviour of gases is used (see [136]) in direct analogy to work out the thermal equilibrium distribution of a cloud of charged particles, given the different microscopic model of interaction among the constituents (now including electric and magnetic interactions).

A collection of particles - even before being qualified or not as plasma - can be described by introducing the density distribution function $n(\vec{r}, \vec{v}, t)$ representing the number of particles found at position $\vec{r}$ and velocity $\vec{v}$ at the time t , which is expected not to depend on time anymore when equilibrium is reached. In the fluid case, n is worked out solving Boltzmann's equation with the constraint of maximum entropy. The analogue equation for plasmas is the Vlasov-Poisson equation, but also in the case of plasmas, the solution of this equation is a Boltzmann distribution of the type

$$n = \frac{1}{Z} \exp\left(-\frac{H}{k_B T}\right) \quad (9.1.2)$$

where H is the single particle Hamiltonian and Z is the partition function (density of states)

of the system.

It can be shown that the Hamiltonian of a charged particle system subjected to an uniform external magnetic field $\vec{B}$ is symmetric under rotations and thus, as a consequence of Noether's theorem, conserves the angular momentum p_θ in the $\vec{B}$ direction. Applying a Legendre transform to expose the conserved quantity is found equivalent to a change of reference system to one co-rotating with the cloud of particles at a gyro-frequency ω . Expanding the single-particle Hamiltonian, a throughout derivation (found for example in [136]) yields

$$f(\vec{r}, \vec{v}) = n_0 \left(\frac{m}{2\pi k_B T} \right)^{3/2} \exp \left[-\frac{1}{k_B T} \left(\frac{m}{2} (\vec{v} - \omega r \vec{\theta})^2 + q\phi(r, z) - \frac{m}{2} \omega (\omega_c + \omega) r^2 \right) \right] \quad (9.1.3)$$

where

$$\phi(r, z) = \phi^{tr}(r, z) + \phi^{pl}(r, z) \quad (9.1.4)$$

is the total potential due to both trap and plasma contributions, q is the signed charge of the particle ($q = e_0$ represents a proton/positron, $q = -e_0$ represents an electron/antiproton), $\omega_c = qB/m$ is the cyclotron frequency, n_0 is linked to the particle density at the center of the distribution and ω is the plasma rotation gyro-frequency on the trap symmetry axis.

It's worth noticing a consequence of the expected independence of equation 9.1.3 from time. The equilibrium distribution in the rotating frame shows no further evolution of its shape as it rotates around the trap axis, meaning that in the laboratory frame a plasma in thermal equilibrium behaves as a rigid body rotating with frequency ω . For this reason, the thermal equilibrium state of a non-neutral plasma is named *rigid rotor*.

The analytical spatial distribution of particles at thermal equilibrium is thus obtained integrating on velocities:

$$n(r, z) = \int f(\vec{r}, \vec{v}) d^3v = \bar{n} \exp \left[-\frac{1}{k_B T} \left(-q\phi(r, z) - \frac{m}{2} \omega_p (\omega_c + \omega_p) r^2 \right) \right] \quad (9.1.5)$$

This formula is a well established result in plasma literature [136] [161]. It links together the particle thermal equilibrium distribution $n(r, z)$ with the total electrical potential inside the trap $\phi(r, z)$, which still needs to be determined as it implicitly depends on $n(r, z)$ itself. The problem of determining $\phi(r, z)$ and $n(r, z)$ is well known in the plasma community as the *problem of the autoconsistent non-neutral plasma potential*. Its solution, either analytical or numerical, produces the desired modelling of the thermal equilibrium distribution of a trapped non-neutral plasma.

9.1.2 Plasma radius

The particle density 9.1.5 can be rewritten in a way so that it exposes experimental observables (see [160] and [161]).

To follow this purpose $\phi_0 = \phi(0, z_c)$, the value of the total electric potential at the center of the trap, is introduced. As third degree of freedom along the central particle density n_0 and T , a common choice is the plasma radius at half amplitude r_p , defined implicitly by solving $n(r_p, z_c) = n(0, z_c)/2$ in r_p .

The explicit r^2 dependence in 9.1.5 is conveniently rewritten in Gaussian form by also introducing the quantity σ defined as

$$\frac{1}{2\sigma^2} = \frac{1}{r_p^2} \left(\log(2) - \frac{q}{k_B T} (\phi(r_p, z_c) - \phi_0) \right) \quad (9.1.6)$$

With these definitions, the equilibrium distribution is rewritten as

$$n(r, z) = n_0 \exp \left(-\frac{q}{k_B T} (\phi(r, z) - \phi_0) \right) \exp \left(-\frac{r^2}{2\sigma^2} \right) \quad (9.1.7)$$

In the high temperature limit $k_B T \gg (\phi(r_p, z_c) - \phi_0)$, $\sigma(r_p)$ reduces to

$$\sigma(r_p) \approx \frac{1}{\sqrt{2 \log(2)}} r_p \approx 0.85 r_p \quad (9.1.8)$$

which is nothing but the definition of the Gaussian-sigma associated to the radius at half-maximum. An high-temperature non-neutral plasma is radially distributed as a Gaussian distribution with width $\sigma(r_p)$.

In the next paragraphs, a numerical way to calculate $\phi(r, z)$ inside 9.1.7 for a plasma in trap is described; the simplified 9.1.7 will be used in the calculations. It is worth mentioning now that as the mathematical distribution radius $\sigma(r_p)$ depends not only on the observable plasma radius r_p , but also on the space-charge potential (which is a-priori unknown) this poses a problem. A strategy to make the distribution end up with the wished observed radius r_p has to be found.

9.1.3 Calculation of axisymmetric non-neutral plasma equilibria

Equation 9.1.7 links together the number distribution of particles in the plasma cloud with the total electrical potential inside the trap $\phi = \phi^{tr} + \phi^{pl}$. The electrical potential map due to the electrodes ϕ^{tr} is assumed to be known, either from an analytical or a numerical calculation. As

already briefly introduced in the previous section, the contribution due to the particles ϕ^{pl} still needs to be determined and depends on the distribution of particles $n(r, z)$.

Another constraint is needed to close the system of equations and solve both for particle distribution n , total potential in the trap ϕ and particle potential ϕ^{pl} . This is given by Poisson equation for the plasma potential:

$$\nabla^2 \phi^{pl}(r, z) = -\frac{q}{\epsilon_0} n(r, z) \quad (9.1.9)$$

Equations 9.1.7, 9.1.9 and 9.1.4 form a closed system of non-linear partial differential equations, which can be solved numerically given the potential map of the trap ϕ^{tr} , the plasma temperature T , the plasma frequency ω_p and the central density parameter of the plasma n_0 .

In practice, it is more convenient to solve this problem in a slightly different form to simplify later on the comparison with experimental results. In a common non-neutral plasma apparatus, the accessible plasma observables are typically the total number of particles N , the plasma radial profile $h(r)$ and the temperature of the plasma T (accessible through few experimental techniques, among which the one introduced by Beck et al. [162] is the most frequently adopted). We will discuss in the next paragraphs how to adapt $n(r, z)$ in this view.

Analytical solutions of this rather complex, non-linear PDE system are found in literature only for an ideal harmonic potential (Penning) trap in the $T = 0$ limit (see [159] and [135]).

The general problem with arbitrary trap potentials requires numerical handling. This was done first by Spencer et al. in [161] by adopting an iterative approach based on Picard iterations with underrelaxation. In Spencer et al. prescription, the charge distribution is computed at each iteration step using 9.1.7 and the knowledge of the potential map $\phi^{pl}(r, z)$ obtained from the last iteration step. Then, the so-obtained charge distribution $n(r, z)$ is fed in a Poisson equation solver in cylindrical coordinates to update the potential map. Underrelaxation is applied to this iterative scheme to make the solution converge for any choiche of the input parameters. As method to solve Poisson's equation, two schemes based on the finite-differences method and the spline representation of the Laplace operator are discussed and compared.

The solution given by Spencer et al., however, shows some practical drawbacks. The approaches discussed to solve Poisson equation, i.e. the finite-differences method and a spline representation, suffer local numerical approximation problems when the spatial grid step approaches λ_D (especially for the finite-differences method). These issues seems mainly related by a non-optimal discretization scheme of the Laplacian, which causes numerical instabilities at the boundary of the plasma where the solution is almost discontinuous for grid spacings $d > \lambda_D$. In order the solver to converge to a stable solution, the grid step needs to be chosen according to λ_D (as given by 9.1.1) which scales in two dimensions as $\propto \lambda_D^{-2} \propto T^{-1}$.

The computation time, which already scales as the inverse temperature T^{-1} from the choice of the underrelaxation parameter ϵ (see [161]), takes another power of T^{-1} from the constraint of resolving the Debye length, ending up scaling $\propto T^{-2}$. If equilibrium is found at 1000 K in about 30 s on an ordinary, desktop PC, the same calculation at 100 K would require 50 min and about 2.5 days at 10 K.

Developing a method that drops the need of resolving the Debye length with the grid spatial size in order to achieve convergence would reduce the calculation complexity from $\propto T^{-2}$ to $\propto T^{-1}$, potentially achieving a tremendous speed-up of the calculation in the $T \rightarrow 0$ limit.

9.2 A spectral method for Poisson's equation in cylindrical coordinates

In this section an alternative method to solve Poisson equation in cylindrical coordinates than the one used in [161] is introduced, analogue to the Fourier transform method to solve Poisson equation in Cartesian coordinates. This method is based on a spectral decomposition of the Laplacian operator in cylindrical coordinates and the subsequent solution of an algebraic equation on the spectral domain.

9.2.1 Poisson equation in cylindrical coordinates

A proper diagonal representation for the Laplace operator in cylindrical coordinates has to be found in order to solve algebraically Poisson equation, with analogy to the $-1/k^2$ diagonal representation obtained in Cartesian coordinates on a Fourier-transformed domain.

The Laplacian operator in cylindrical coordinates is

$$\nabla^2 = \partial_r^2 + \frac{1}{r}\partial_r + \frac{1}{r^2}\partial_\theta^2 + \partial_z^2 \quad (9.2.1)$$

Assuming symmetry around the cylinder axis is equivalent to neglecting the azimuthal derivative term and dropping the relative coordinate θ .

$$\nabla^2 = \partial_r^2 + \frac{1}{r}\partial_r + \partial_z^2 \quad (9.2.2)$$

Thus, the Poisson equation in cylindrical coordinates is rewritten as

$$\left(\partial_r^2 + \frac{1}{r}\partial_r + \partial_z^2\right)\phi(r, z) = -\frac{q}{\epsilon_0}\rho(r, z) \quad (9.2.3)$$

This equation has to be solved in $\phi(r, z)$ given the charge distribution $\rho(r, z)$ and boundary conditions on the value of $\phi(r, z)$ at the borders of the domain. In particular, Dirichlet-type boundary conditions are chosen on the walls to define the ground potential $\phi = 0$ and Neumann-type boundary conditions are imposed on the axis of the trap, where the solution of the equation has to be continuous.

$$\begin{aligned}\phi(r, z_{min}) &= 0 \\ \phi(r, z_{max}) &= 0 \\ \phi(r_{max}, z) &= 0 \\ \partial_r \phi(0, z) &= 0\end{aligned}\tag{9.2.4}$$

In order to follow the purpose of representing the Laplacian operator as an algebraic quantity in cylindrical coordinates, a linearly independent set of solutions of the homogeneous version of the Poisson equation has to be identified, so that $\nabla^2 \phi = 0$, suitable to form a basis of the space (r, z) . In the language of calculus, this is equivalent to solving the homogeneous equation:

$$\left(\partial_r^2 + \frac{1}{r} \partial_r + \partial_z^2 \right) \phi(r, z) = 0\tag{9.2.5}$$

Using separation of variables $\phi(r, z) = R(r)Z(z)$ the problem is rewritten as

$$\left(\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} \right) Z(z) + \frac{d^2 Z}{dz^2} R(r) = 0\tag{9.2.6}$$

Dividing by $R(r)Z(z)$

$$\frac{1}{R(r)} \left(\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} \right) + \frac{1}{Z(z)} \frac{d^2 Z}{dz^2} = 0\tag{9.2.7}$$

The r and z terms were separated. Focusing the attention on the z -dependent term, since no other term in the differential equation depends on z , this term can be integrated first, and for what concerns the subsequent r integration it cannot but act as a (complex) number independent of r .

Thus, equation 9.2.32 can be split into two mono-dimensional equations

$$\begin{aligned}\frac{\ddot{Z}(z)}{Z(z)} &= a_z \\ \frac{\ddot{R}(r)}{R(r)} + \frac{1}{r} \frac{\dot{R}(r)}{R(r)} + a_z &= 0\end{aligned}\tag{9.2.8}$$

which can be solved independently.

9.2.2 Solution of the axial homogeneous equation

The axial equation

$$\ddot{Z}(z) = a_z Z(z) \quad (9.2.9)$$

is commonly solved on the Fourier domain upon substitution of the Fourier transforms of both $Z(z)$ and $\ddot{Z}(z)$

$$\begin{aligned} Z(z) &= \frac{1}{\sqrt{2\pi}} \int \tilde{Z}(k_z) e^{ik_z z} dz \\ \ddot{Z}(z) &= \frac{1}{\sqrt{2\pi}} \int -k_z^2 \tilde{Z}(k_z) e^{ik_z z} dz \end{aligned} \quad (9.2.10)$$

that yield, upon substitution, only an algebraic equation to be solved on the Fourier-transformed domain

$$-k_z^2 \tilde{Z}(k_z) = a_z \tilde{Z}(k_z) \quad (9.2.11)$$

which is satisfied if $a_z = -k_z^2$; this solves the axial homogeneous problem.

Here the complex exponential functions have been used as a set of linearly independent solutions for the axial equation. The Fourier transform is the map that transforms back and forth functions from the real domain to the spectral domain generated by the basis of linearly independent solutions. The z -component of the Laplace operator is represented diagonally on this domain by

$$\partial_z^2 \longrightarrow -k_z^2 \quad (9.2.12)$$

However, in the context of seeking a solution with Dirichlet boundary conditions of the type $Z(0) = 0, Z(L) = 0$, complex exponential functions are not the best choice as they naturally implement periodic boundary conditions $Z(0) = Z(L)$.

If, instead, only sine functions are used as spectral basis and the sine transform as the integral transform associated, then the problem of setting boundary conditions is automatically resolved because sines exhibit a zero for $z = 0$ and $z = L$ if the frequency k_z is chosen so that $k_z = n(2\pi/L)$, where n is an integer number.

Following this argument, the discretized version of the sine transform will be preferred for the

numerical implementation of the Poisson solver. It is defined as

$$\begin{aligned} Z(z) &= \frac{1}{\sqrt{2\pi}} \int \tilde{Z}(k_z) \sin(k_z z) dz \\ \ddot{Z}(z) &= \frac{1}{\sqrt{2\pi}} \int -k_z^2 \tilde{Z}(k_z) \sin(k_z z) dz \end{aligned} \quad (9.2.13)$$

Sine transform, cosine transform and Fourier transform are closely related to each other. Identity 9.2.12 holds in all the three cases; only the boundary conditions are implemented in different manners.

9.2.3 Solution of the radial homogeneous equation

The radial equation

$$\ddot{R}(r) + \frac{1}{r} \dot{R}(r) - k_z^2 R(r) = 0 \quad (9.2.14)$$

is a particular case of Bessel's differential equation

$$r^2 \ddot{R}(r) + r \dot{R}(r) + (r^2 - m^2) R(r) = 0 \quad (9.2.15)$$

where the azimuthal index m has been neglected (i.e. taken $m = 0$). The interest is to find non-divergent solutions to 9.2.14 as $r \rightarrow 0$, thus the family of Bessel functions of the first kind $J_a(r)$, finite and continuous in the origin, is a natural candidate ¹.

The aim is to represent the solution to 9.2.14 as a linear combination of basis functions - just like a Fourier series, but defined on the cylindrical domain.

$$R(r) \simeq \sum_{n=0}^{\infty} a_n |n\rangle \quad (9.2.17)$$

¹Bessel $J_a(r)$ will be used intensively in the next sections. These functions can be evaluated numerically with arbitrary precision by using their integral closed form valid for integer a

$$J_a(r) = \frac{1}{\pi} \int_0^\pi \cos(nt - r \sin(t)) dt \quad (9.2.16)$$

A numerical integration method based on Gauss-Legendre quadrature was implied, very close to the one described in [?]. Numerical experiments showed that the number of quadrature points that needs to be used to evaluate the integral effectively depends on the value of r . In particular, an effective choice was seen to be $N_{float} = 2r + 2$ for nominal single-precision numerical accuracy (7 digits) and $N_{double} = 8r + 8$ for double-precision (15 digits).

Bessel $J_a(r)$ can be used to construct such a basis, just as $\sin(x)$ and $\cos(x)$ are used on the Cartesian domain when constructing the Fourier transform.

Keeping into account the proper normalization over the $(0, +\infty)$ interval, denoting j_{an} as the n -th zero of the a -th Bessel function and introducing bra-ket notation for clarity (with a pedix a to keep track of the particular Bessel function chosen), the orthonormal basis vectors are defined as

$$|n\rangle_a = \frac{\sqrt{2}}{J_{a+1}(j_{an})} J_a(j_{an}r) \quad (9.2.18)$$

The scalar product between two generic functions defined on the radial domain is given by

$$\langle f|g\rangle = \int_0^\infty f(r)g(r)rdr \quad (9.2.19)$$

Note that the scalar product definition is independent of a . With such definition of scalar product, the orthonormality relation holds

$$\langle m|n\rangle_a = \delta_{mn} \quad (9.2.20)$$

With this new orthonormal set of solutions to 9.2.15, we are ready to rewrite the radial equation in an algebraic form. Let us define the operator ∇_r^2 as the radial part of the Laplacian operator

$$\nabla_r^2 = \partial_r^2 + \frac{1}{r}\partial_r \quad (9.2.21)$$

If we apply ∇_r^2 to a basis vector $|n\rangle_r$ and perform all derivatives, we get

$$\nabla_r^2 |n\rangle_a = -j_{an}^2 |n\rangle \quad (9.2.22)$$

The operator is linear, as it can be proven by applying ∇_r^2 to a linear superposition of basis vectors $A|n\rangle_r + B|m\rangle_r$

$$\nabla_r^2 (A|n\rangle_a + B|m\rangle_a) = -j_{an}^2 A|n\rangle - j_{am}^2 B|m\rangle \quad (9.2.23)$$

This result already proves that the radial homogenous equation 9.2.14 is verified for the special case of the basis vectors:

$$-j_{an}^2 |n\rangle_a - k_z^2 |n\rangle_a = 0 \quad (9.2.24)$$

A proper integral transform is now needed to decompose any function defined on the radial domain as a linear combination of the basis defined by 9.2.17, in order to solve the homogeneous 9.2.14 in the general case.

9.2.4 Continuous Hankel transform

The integral transform required to perform such spectral decomposition on the basis of Bessel's J_a functions is named Hankel transform (HT) (see [163, 164]). HT and its inverse HT^{-1} are defined as

$$\tilde{f}(\nu) = 2\pi \int_0^\infty f(r) J_a(2\pi\nu r) r dr \quad (9.2.25)$$

$$f(r) = 2\pi \int_0^\infty \tilde{f}(\nu) J_a(2\pi\nu r) \nu d\nu \quad (9.2.26)$$

Some properties of the Hankel transform are evident from its definition: it is a linear transform

$$\text{HT}[\alpha f + \beta g] = \alpha \text{HT}[f] + \beta \text{HT}[g] \quad (9.2.27)$$

it is the inverse of itself

$$\text{HT}[\text{HT}[f]] = \text{HT}^{-1}[\text{HT}^{-1}[f]] = f \quad (9.2.28)$$

and it diagonalizes the radial Laplacian operator

$$\nabla_r^2 \longrightarrow -k_r^2 = -(2\pi\nu)^2 \quad (9.2.29)$$

With the aid of the Hankel transform, any function on the radial domain can be represented as a (continuous) linear combination of Bessel J_a on the radial frequency domain.

Using the inverse definition 9.2.26, the radial Laplacian of an arbitrary function takes the form

$$\begin{aligned} \nabla_r^2 R(r) &= 2\pi \int_0^\infty \tilde{R}(\nu) \nabla_r^2 J_a(2\pi\nu r) \nu d\nu \\ &= -2\pi \int_0^\infty \tilde{R}(\nu) (2\pi\nu)^2 J_a(2\pi\nu r) \nu d\nu \end{aligned} \quad (9.2.30)$$

Substituting into 9.2.14 the radial homogeneous equation on the Hankel transformed domain becomes an algebraic equation as seeked

$$-4\pi^2\nu^2\tilde{R}(\nu) - k_z^2\tilde{R}(\nu) = 0 \quad (9.2.31)$$

9.2.5 Solution of the Poisson equation

The stage is set to solve to the non-homogeneous case. Poisson's equation in cylindrical coordinates can now be reduced to an algebraic equation by means of a combined sine transform on the z -axis and a Hankel transform on the r -axis (denoted SHT, as Sine-Hankel Transform), as the two coordinates are orthogonal.

$$\text{SHT} \left[\left(\partial_r^2 + \frac{1}{r} \partial_r + \partial_z^2 \right) \phi(r, z) \right] = -\frac{q}{\epsilon_0} \text{SHT}[\rho(r, z)] \quad (9.2.32)$$

Substituting the representations of the two Laplacians in the transformed domains 9.2.12 and 9.2.29 Poisson's equation can be rewritten as:

$$(-k_r^2 - k_z^2) \text{SHT}[\phi(r, z)] = -\frac{q}{\epsilon_0} \text{SHT}[\rho(r, z)] \quad (9.2.33)$$

Thus the solution on the real domain is obtained by applying on both sides the inverse transform.

$$\phi(r, z) = \text{SHT}^{-1} \left[\frac{q}{\epsilon_0 (k_r^2 + k_z^2)} \text{SHT}[\rho(r, z)] \right] \quad (9.2.34)$$

9.3 Numerical implementation of the spectral Poisson solver

Solving Poisson's equation using a combined sine-Hankel transform requires proper numerical algorithms for evaluating the transforms.

9.3.1 Discrete sine transform

Many accurate algorithms exists for computing the Fast Sine Transform (FST) in $o(n \log(n))$ steps, based on an adaptation of Cooley-Tukey's original algorithm to compute the Fast Fourier Transform (FFT) ([165]). The one adopted here for implementing the Poisson solver is the one described in Numerical Recipies, 3rd edition (see [?]).

The z grid was uniformly sampled between $z \in [z_{min}, z_{max}]$ with N_z points. The axial frequency ν_z grid associated to the real-space grid resulted

$$\nu_z \in \left[0, \frac{N_z}{2(z_{max} - z_{min})} \right] \quad (9.3.1)$$

9.3.2 Discrete Hankel transform

The Discrete Hankel Transform (DHT) have no $o(n \log(n))$ equivalent algorithms to FFT. The most efficient solution found in literature in terms of computational complexity is the algorithm described in [164], which has only a moderate performance gain with respect to the bare $o(n^2)$ in spite of a significantly higher complexity and implementation cost. The much neater and simpler algorithm described in [163] was preferred.

The trick to achieve good numerical stability in computing discrete Hankel transforms consists in choosing the r and ν grids not equispaced, as in the Cartesian case, but instead sampled over a non-uniform grid of N_r points spaced as the Bessel J_a zeroes j_{an} .

The real and transformed domains are thus assumed spanning on the limited intervals $r \in [0, r_{max}]$ and $\nu \in [0, \nu_{max}]$, where

$$\nu_{max} = \frac{j_{aN_r}}{2\pi r_{max}}$$

. All signals are assumed to be zero outside these intervals: $f(r \geq r_{max}) = 0$ and $\tilde{f}(\nu \geq \nu_{max}) = 0$.

From now on, signals are assumed to be sampled on the radial and the radial frequency grids according to

$$\begin{aligned} f_n &= f\left(\frac{j_{an}}{2\pi\nu_{max}}\right) \\ \tilde{f}_m &= \tilde{f}\left(\frac{j_{am}}{2\pi r_{max}}\right) \end{aligned} \quad (9.3.2)$$

The DHT_a can thus be defined by the discrete sum ([163])

$$\tilde{f}_m = \frac{1}{\pi\nu_{max}^2} \sum_{n=1}^{N_r} \frac{f_n}{J_{a+1}^2(j_{an})} J_a\left(\frac{j_{an}j_{am}}{j_{aN_r}}\right) \quad (9.3.3)$$

Similarly, the DHT_a^{-1}

$$f_n = \frac{1}{\pi r_{max}^2} \sum_{m=1}^{N_r} \frac{\tilde{f}_m}{J_{a+1}^2(j_{am})} J_a\left(\frac{j_{an}j_{am}}{j_{aN_r}}\right) \quad (9.3.4)$$

This method to compute DHTs has the advantage to offer an immediate way to cache transform coefficients and avoid the computational cost of evaluating Bessel J_a every time the transform has to be performed. Bessel J_a , as well as their zeroes, were evaluated only once with high precision and tabulated.

9.3.3 Algorithm for computing direct and inverse sine-Hankel transforms

The algorithm for computing direct sine-Hankel transforms of order a (SHT_a) of a signal f_{mn} sampled a bidimensional $r - z$ rectangular grid, using the prescriptions discussed above, is:

1. apply the FST to the rows of f_{mn} ;
2. transpose in-place the matrix;
3. apply the DHT_a to the former columns, now rows of f_{nm} ;
4. transpose back the matrix.

On the same line, the algorithm to compute the inverse sine-Hankel transform of order a (SHT_a^{-1}) of a signal $\tilde{f}_{mn}$ is:

1. transpose in-place the matrix;
2. apply the DHT_a^{-1} to the rows of $\tilde{f}_{mn}$;
3. transpose back the matrix;
4. apply the FST^{-1} to the former columns, now rows of $\tilde{f}_{nm}$.

9.3.4 Poisson's spectral solver

All the ingredients are now ready to setup the algorithm that solves numerically Poisson's equation 9.2.34 in cylindrical coordinates. Both potential $\phi(r, z)$ and charge distribution $\rho(r, z)$ are assumed to be sampled on the same grid.

The order a in all the DHTs still has to be chosen. Since in general a plasma charge distribution is non-null in the origin $\rho(r = 0, z) \neq 0$, the choice is forced: $a = 0$, since for $a > 0$ all Bessel $J_a(r)$ show a simple zero in $r = 0$.

SHT_0 is thus applied first to ρ_{mn} to compute $\tilde{\rho}_{mn}$. Then, each component of $\tilde{\phi}_{mn}$ is computed by

$$\tilde{\phi}_{mn} = \frac{q}{\epsilon_0} \frac{1}{(k_z)_m^2 + (k_r)_n^2} \tilde{\rho}_{mn} \quad (9.3.5)$$

where the values of k_z and k_r are constrained by the choice of the grid through

$$\begin{aligned}(k_z)_m &= \frac{2\pi m}{2(z_{max} - z_{min})} \\ (k_r)_n &= \frac{j_{0n}}{r_{max}}\end{aligned}\tag{9.3.6}$$

Finally SHT_0^{-1} is applied to $\tilde{\phi}_{mn}$ to work out the potential in the real space.

9.3.5 Numerical verifications

A first set of numerical experiments were performed to test and verify the numerical implementation of the DHT. DHT was verified comparing the numerical results to the (analytic) transform of the $\text{sinc}(r)$ function, as suggested in [163], and found perfectly consistent. Moreover, it was used to compute the impulse response of the 1D version of the cylindrical Poisson equation, physically modelling the potential produced by an infinite hollow cylinder of charge, to verify both stability of the solution and boundary conditions.

$$\nabla_r^2 \phi(r) = \alpha \delta(r - r_0)\tag{9.3.7}$$

An example of numerical result is given in Figure 9.1, showing the charge distribution and the resulting potential as a function of r . The algorithm computes the correct radial potential profile within an infinite hollow cylinder, which is flat and non-null in the center. Moreover, it also correctly reproduces the behaviour of the potential in the outer region of the cylinder, where it drops as $\log(r_0/r)$ in the empty space approximation, terminating it to ground at the boundaries.

It is worth noticing the ability of the spectral solver method based on the Hankel transform to solve Poisson's equation with radial non-linearities in the source distribution. Problems posed by boundary conditions on the electrodes, which were matter of discussion in [161], are here automatically solved.

A second set of numerical experiments were performed using the numerical implementation of the Sine-Hankel Transform (SHT) to verify also the impulse response of the 2D version of the cylindrical Poisson equation, which models a infinitesimal ring distribution of charge.

$$\nabla^2 \phi(r, z) = \alpha \delta(r - r_0) \delta(z - z_0)\tag{9.3.8}$$

An example of results obtained are shown in Figure 9.2. Here both r and z components of the calculated potential and charge distribution are shown. Also in this case, the boundary

conditions are correctly represented, as the potential is assumed at ground at the boundaries but for $r = 0$ where the solution is non-zero and continuous.

9.4 Numerical solution of the autoconsistent plasma potential problem

In order to solve numerically the autoconsistent plasma potential problem, the spectral solver of Poisson's equation developed in last sections had to be combined with the iterative underrelaxation method described in [161] to set up a convergent iterative fixed-point scheme.

The algorithm used for solving the autoconsistent plasma potential problem is (roughly) the following.

- the empty trap potential ϕ^{tr} is computed from geometry and electrodes polarizations V_i (as described in the next paragraph)
- the initial plasma potential ϕ^{pl} is initialized to zero
- Picard iterations with underrelaxation parameter $\epsilon \ll 1$ are performed (as described in [161]) until convergence is found or maximum iteration is reached

- ◊ the plasma density ρ_k is computed inserting the total potential at last iteration $\phi^{tot}_k = \phi^{tr} + \phi^{pl}_k$ into the analytical Boltzmann distribution (equation 9.1.5)

$$\rho_k = f(N, r_p, T, \phi^{tr} + \phi^{pl}_k)$$

using a numerical expedient to express f as a function of the plasma measurable radius r_p and number of particles N (described below)

- ◊ using the spectral Poisson solver in cylindrical coordinates, Poisson's equation

$$\nabla^2 \phi^*_k = -(q/\epsilon_0)\rho_k$$

is solved in ϕ^* , the potential associated to the current charge distribution ρ_k

- ◊ the new plasma potential is calculated as a perturbation of the current value ϕ^{pl}_k , i.e. by using the underrelaxation relation

$$\phi^{pl}_{k+1} = (1 - \epsilon)\phi^{pl}_k + \epsilon\phi^*$$

- ◊ the normalized Euclidean distance among the two subsequent plasma potential maps ϕ^{pl}_k and ϕ^{pl}_{k+1} is computed to measure the convergence distance

$$d = \frac{1}{d_0} \sqrt{\sum_{i=1}^{N_r N_z} ([\phi^{pl}_{k+1}]_i - [\phi^{pl}_k]_i)^2}$$

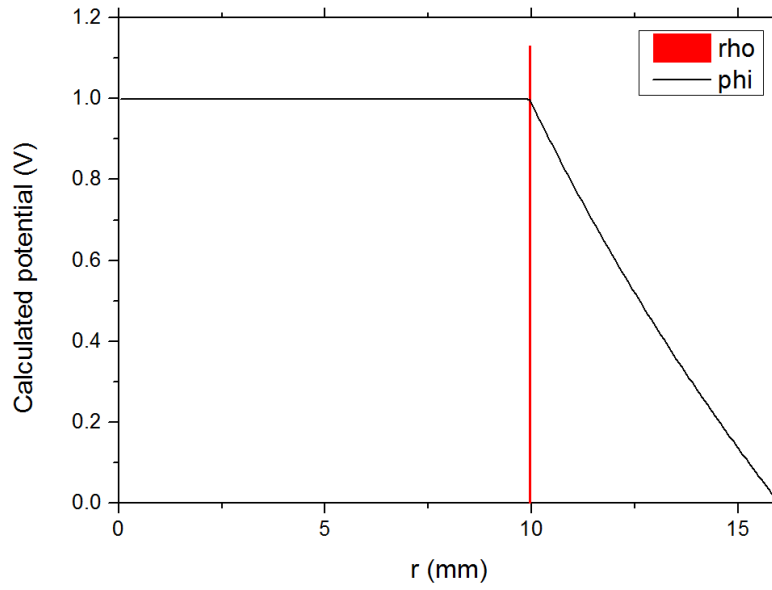


Figure 9.1: Calculated impulse response of the 1D Poisson equation 9.3.7 with the DHT method (see text). Comparison between the charge distribution (red bins) and the calculated potential profile (black line).

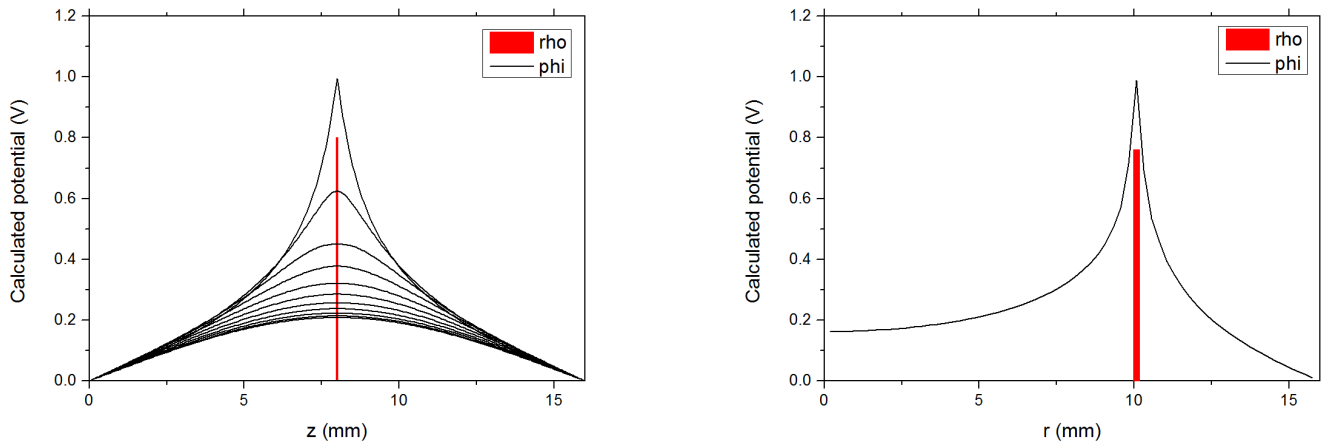


Figure 9.2: Calculated impulse response of the 2D Poisson equation 9.3.8 with the SHT method (see text). Projection of the 2D result on the z coordinate (left) and the r coordinate (right). Comparison between the charge distribution (red bins) and the calculated potential (black line).

and normalized versus the Euclidean distance of the first iteration against the initial potential

$$d_0 = \sqrt{\sum_{i=1}^{N_r N_z} ([\phi^{pl}_1]_i - [\phi^{pl}_0]_i)^2}$$

9.4.1 Calculating the empty trap potential

Calculating the empty trap potential map ϕ^{tr} in cylindrical coordinates is, by itself, a non-immediate task. For cylindrical coaxial electrodes, Bessel series expansions of ϕ^{tr} are available in literature (see for example [113]). Instead of using the series expansion, a method based on solving Poisson equation in cylindrical coordinates with Picard iterations similar to the one described above was adopted, as it offered a neat and verifiable test case for this numerical technique and also kept into account contributions of the external walls at ground potential.

The idea beyond the algorithm is to iteratively calculate the charge distribution on the internal surface of the electrodes that produces the potential inside the trap. This charge distribution has to induce a difference of potential between the electrode conductor volume and the external walls at ground equal to the electrode polarization voltage. Moreover, the potential within the conductor has to be uniform in the approximation of perfect conductor.

The algorithm used is the following:

- the charge distribution is initialized to 0 excepting at electrodes radii, where it is set as uniformly distributed on z over the electrode length (assumed to span between z^{min}_i and z^{max}_i) and infinitesimally broad in r (a Dirac delta at the electrode radius R_i).

$$\rho(z \in [z^{min}_i, z^{max}_i], r = R_i) = \alpha V_i$$

The parameter α is a potential-to-charge conversion unit (related to the electrode capacitance) to be determined, which is initially set to 1;

- the potential map is calculated with the SHT Poisson solver according to this initial charge distribution. The resulting potential map is renormalized at $r = R_i$ to the polarization voltage V_i ; all the renormalization constants are averaged to work out a suitable initial value for α ;
- Picard iterations with convergence parameter $\epsilon < 1$ are performed until convergence is found or maximum iteration is reached;
 - ◊ the new potential map ϕ^* is calculated using the current charge distribution ρ_{mn}^k solving the 2D Poisson problem using the SHT solver;

- ◇ the charge distribution on each electrode surface $z \in [z^{min}_i, z^{max}_i]$ and $r = R_i$ is updated according to the equation

$$\rho^{k+1}_{mn} = \rho^k_{mn} - \epsilon (\phi^*_{mn} - V_i)$$

where V_i is the requested polarization voltage for that electrode. This equation produces an exponential process that makes the potential within the electrode volume to converge uniformly to V_i if ϵ is chosen small enough;

- ◇ the normalized Euclidean distance is calculated (as described above) to check the convergence status.

The approach chosen here usually converges quickly and stably to the solution in few iterations. In perspective, it also offers a significative advantage over the static, analytical calculation: it would allow to solve at the same time for the plasma charge distribution and trap charge distribution if the Picard loops of the two separate algorithms would be merged together.

This could potentially overcome a limit of the autoconsistent plasma calculation technique described by [161], which does not take into account image charge effects. Instead, letting the two charge distributions adjust together in the trap, continuously imposing throughout the iterations both plasma constraints (number of particles, observed radius) and trap constraints (uniform potential over the electrodes), all mutual induction effects would be kept into account automatically. It would also be possible, in this way, to calculate the mirror charge induced by a real plasma on the electrode walls.

9.4.2 Experimental observables as input parameters

If the calculated distribution is required to exhibit half-maximum radius $r_p = R_p$ and total number of particles $N_p = N_0$ to verify the outcome of an experiment, two further constraint needs to be imposed.

The total number of particles is constrained by renormalizing the density with the integral over the entire grid at every step of the iteration:

$$\rho^k_{mn} \longrightarrow N_p \left(\sum_{mn}^{N_z, N_r} \rho^k_{mn} \right)^{-1} \rho^k_{mn} \quad (9.4.1)$$

Fixing the plasma radius to R_p has been found to be a much more difficult task. The prescription described in [161], which suggests to recalculate the plasma radius r_p at every iteration and use the new value to find the new plasma distribution, has been found (as already pointed out by the author) to be cause of instabilities for temperatures of ~ 300 K or lower.

A different approach based on slowly adjusting the plasma radius at each iteration step according to an exponential process has been found much more reliable.

- first, a scale parameter γ is initialized to 1 before starting to iterate;
- the current plasma radius r_p^k is evaluated at each iteration step by calculating the midplane radial profile of the charge distribution and extracting its FWHM;
- the new radius to be used for evaluating 9.1.7 is calculated according to $r_p^{k+1} = \gamma \cdot r_p^k$. If γ is left untouched from its initial value of 1, no action on the radius is undertaken;
- instead, the value of the radius conversion unit γ is left adapting throughout the iterations in order to minimize the distance between the actual radius r_p^k and the measured radius R_p , according to

$$\gamma^{k+1} = \gamma^k - \epsilon_r(r_p^k - R_p)$$

where ϵ_r is a convergency constant $\ll 1$, which was chosen, in most of the experiments conducted, as inversely proportional to the maximum number of Picard cycles $\epsilon_r = 1/(4N_{cy})$.

This technique proved to be effective in constraining the plasma to the desired value R_p even at low temperatures (~ 10 K), where the space-charge term contribution in 9.1.6 is much bigger than $k_B T$. An example of radial profile obtained with this technique is shown in Figure 9.10. According to the maximum number of iterations allowed N_{cy} and the target precision of the entire iteration, a more accurate convergence to the exact radius is to be expected. In the case of Figure 9.10, a target precision of 5% was requested.

9.5 Numerical results

This section briefly summarizes the numerical results obtained by the autoconsistent plasma potential algorithm based on the sine-Hankel transform (SHT) method, described in the sections above.

9.5.1 Validation of trap potential maps

Two numerical experiments were conducted to verify the correctness of the calculated trap potential maps with the iterative algorithm presented above: a simulation of a single electrode potential (P9) and a simulation of a realistic trap (P3-P9) frequently used in AEgIS for $\bar{p}$ manipulation procedures. In both cases, the results were compared to an already-available trap potential map calculated using Maxwell[©].

Results of the first numerical experiment are reported in Figure 9.3, showing both axial and radial profiles of the calculated potential using the SHT method at 4% and 1% requested relative accuracies (shown in black) and a 0.25 mm-accurate Maxwell[©] calculation (shown in red). The found discrepancy of the two highest potential points (worst case scenario) in the two cases is of 3.5% against 4% requested and 1.2% against 1% requested.

In the second experiment, the SHT method was used to calculate the realistic trap potential map of the P3-P9 trap used to compress $\bar{p}$ together with e^- , with the endcaps set at -160 V and the trap bottom at -10 V (see Figure 9.4, above). The resulting potential profiles in the z and r directions from the SHT method at 4% precision were again compared to the results obtained with a 2.0 mm-accurate Maxwell[©] calculation (see Figure 9.4, below). Results, shown in Figure 9.4, show a relative discrepancy of the highest potential point of 2% at $r = 0$ mm and of 5% at $r = 10$ mm, consistent with a requested 4% overall relative error.

Concluding, the SHT method shows almost perfect consistency to the Maxwell[©] reference calculation. In perspective of adopting the SHT method as a scientific-quality plasma solver, a validation with a more detailed reference Maxwell[©] simulation is required for reaching accuracies $< 1\%$.

9.5.2 Validation of plasma equilibrium distributions

The following step in verifying the results of the autoconsistent plasma potential solver consisted in cross-checking the produced plasma distributions against a literature result. The case of the ideal harmonic potential was adopted, as equilibrium distributions can be calculated analytically in the $T = 0$ case. Particles distribute in the center of the harmonic section of the trap in a spheroidal distribution, whose aspect ratio $\alpha = z_p/r_p$ is obtained upon inversion of the equation

$$\frac{1}{\alpha^2 - 1} Q_1^0 \left(\frac{\alpha}{\sqrt{\alpha^2 - 1}} \right) = \frac{\omega_p^2}{\omega_z^2} \quad (9.5.1)$$

where ω_p is the plasma frequency, ω_z is the axial oscillation frequency and Q_1^0 is an associated Legendre function of the second kind (see [135, pp. 103-104] for a detailed discussion) ¹

The procedure used to validate the iterative algorithm was the following:

1. the equilibrium distribution of a plasma of N_p particles in a given harmonic trap profile $V_t(z)$ was first calculated;

¹an already validated iterative code to solve 9.5.1 in α given the number of particles N_p , the plasma radius r_p and the trap potential 40 mm distant from the center of the parabola $V(40 \text{ mm})$ was already available at the time of writing.

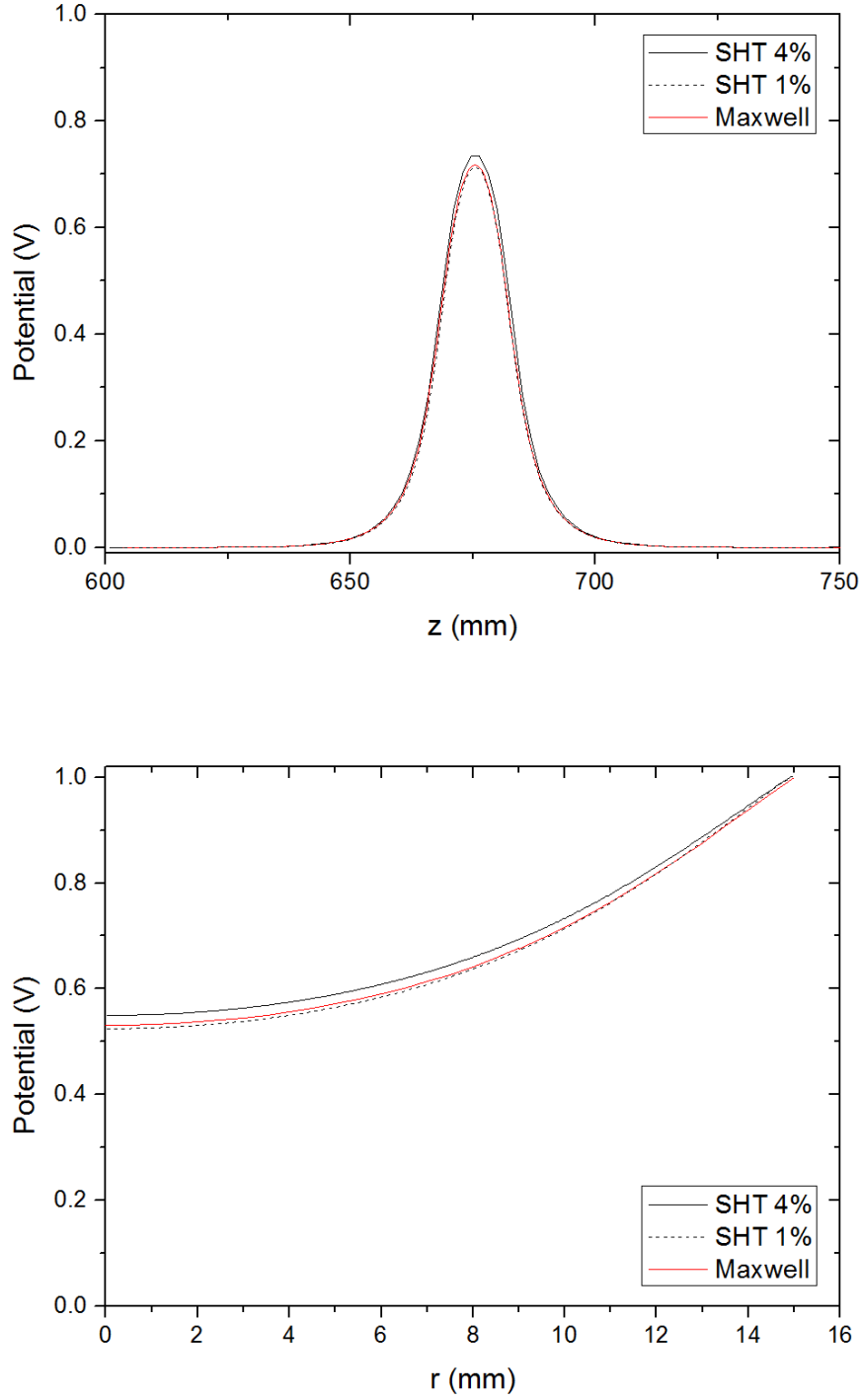


Figure 9.3: Comparison of the calculated potential profiles (axial above, radial below) of a single electrode (P9) with the SHT method in two different runs with 1 % (solid black curve) and 0.25 % (dashed black curve) target precisions against a 0.25 mm-accurate Maxwell[®] calculation (red curve).

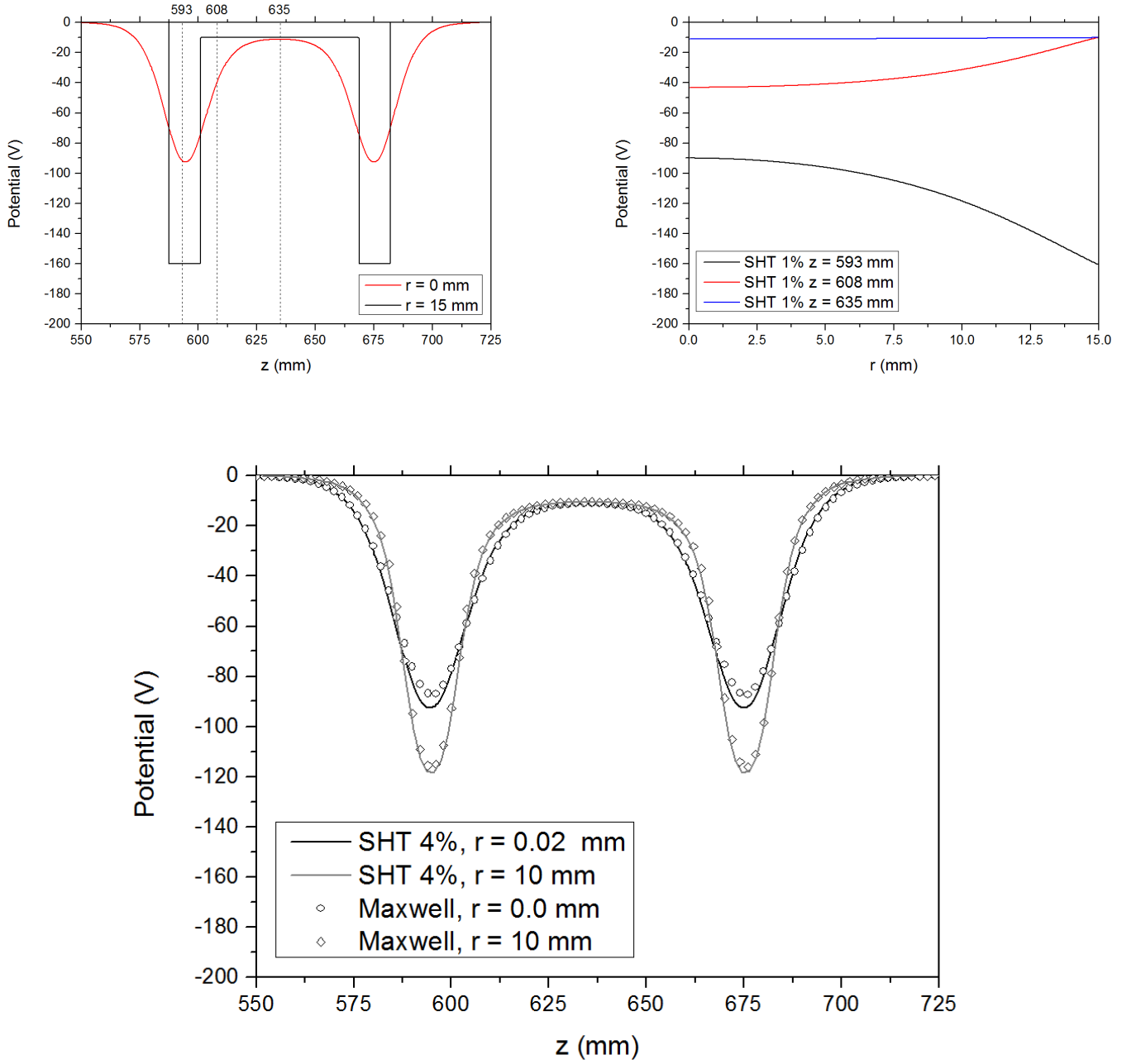


Figure 9.4: Above - trap potential profiles calculated with the SHT algorithm described in the text: plot of the potential profile versus z (left panel) and versus r (right panel) in the standard P3-P9 trap. Below - comparison of the calculated potential profiles (axial left, radial right) of the entire P3-P9 trap with the SHT method and a 0.25 mm-accurate Maxwell[®] calculation.

2. once the charge distribution was obtained from the iterative solver, the plasma radius r_p was estimated by measuring the FWHM of the plasma midplane radial profile;
3. the current values of N_p , r_p and trap parabolic profile $V_t(z)$ were used to solve 9.5.1 to obtain the theoretical estimate for the plasma half-length z_p ;
4. the obtained z_p value was finally verified on the axial profile of the calculated charge distribution.

An example of this procedure is depicted in Figure 9.5. Both axial and radial profiles of a plasma charge distribution ($N_p = 2 \cdot 10^7$ and $T = 2000$ K) are separately shown. Superimposed on the charge profiles, vertical lines show the measured plasma radius $r_p = 0.46$ mm and the calculated plasma half-length $z_p = 12.15$ mm. The resulting values for z_p were found consistent within the theoretical estimate to relative 1.2% precision, which is in very good agreement with the requested 1% accuracy of the iterative calculation.

9.5.3 Plasma distributions in arbitrary potentials with cylindrical symmetry

The biggest advantage of solving numerically the autoconsistent plasma equilibrium problem is that equilibrium distributions can be modeled for arbitrary shapes of the confining potentials. A neat example in the AEGIS context is given again by the P3-P9 trap used for antiproton manipulations, where the two endcaps (P3 and P9) are kept at -160 V and the bottom of the trap at -10 V. This trap is neither an ideal flat Malmberg trap nor an well-shaped harmonic trap, but is sitting somewhere in the middle (see Figure 9.4).

Two examples of equilibrium distributions found with the SHT method in P3-P9 are shown in Figure 9.6 for two different radii at constant number of particles. The ability to calculate plasma equilibrium in any potential opens up new interesting future possibilities, such as a theoretical accurate estimate of the Trivelpiece-Gould plasma modes frequencies [166] for calibrating RF plasma diagnostic tools [167, 108].

9.5.4 Plasma distributions in arbitrary trap geometries

Simulating equilibrium distributions in real-world cylindrical traps requires also the numerical code to handle electrodes with different radii (in the case of AEGIS, for instance, 5T and 1T traps are made of electrodes with very different radii: 22 mm of the Big trap against 15 mm of the 5T traps). This feature was unavailable in the method described in [161], where the electrode surface was set at the edge of the domain. The plasma equilibrium solver based on the SHT method has the non-negligible advantage of solving automatically this problem, since

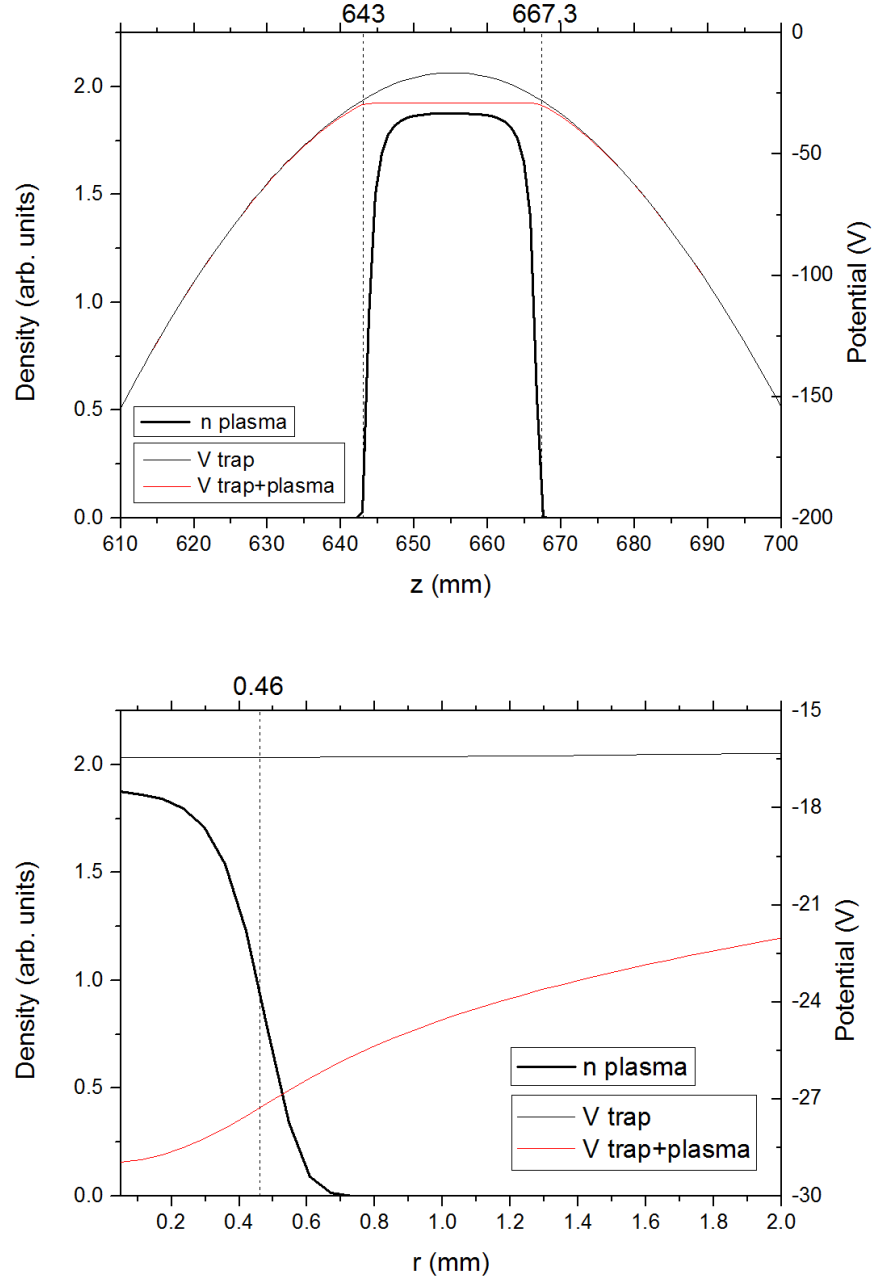


Figure 9.5: Comparison of plasma equilibrium profiles in a harmonic trap in the axial direction (above) and the radial direction (below) with theoretical estimates. First, the radius at half-maximum $r_p = 0.46$ mm is calculated from the radial density profile, shown as a vertical dashed line. r_p , together with the trap potential profile $V(z)$ (shown in the plot above as a thin black line) and the total number of particles $N_p = 2 \cdot 10^7$ yields a plasma length of 24.30 mm, represented in the plot above as two vertical dashed lines symmetric with respect to the parabola vertex.

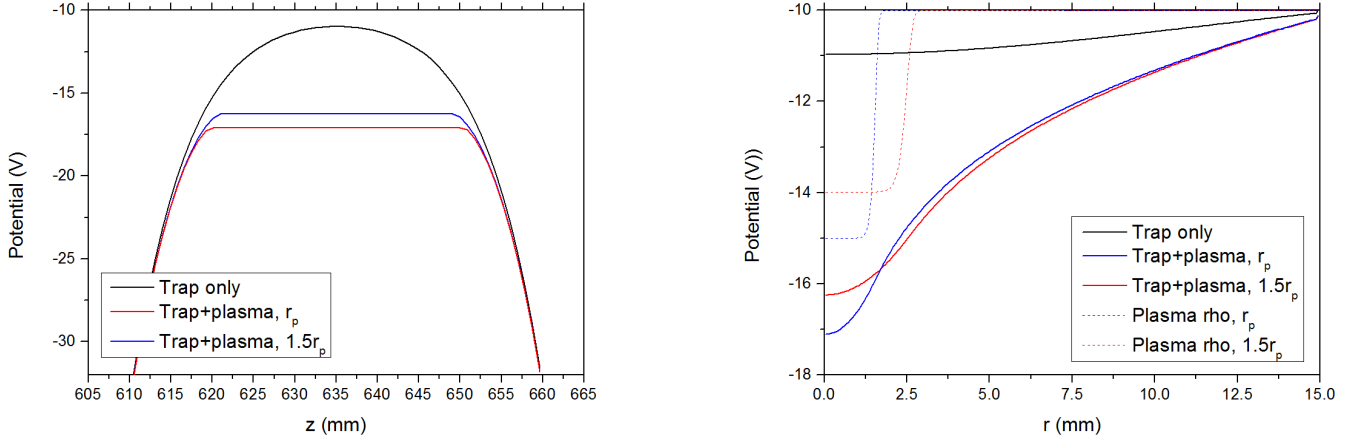


Figure 9.6: Above - example of 3D plasma charge distribution calculated in the P3-P9 trap. Below - comparison of the total potential in the trap with and without the presence of the plasma distribution, as calculated with the iterative algorithm described in the text. Plot of the potential profile versus z (left panel) and versus r (right panel) in the standard P3-P9 trap used for e^- with the endcaps set at -160 V and the trap bottom at -10 V. Reference plasma parameters are $N_p = 2 \cdot 10^7$, $r_p = 1.0$ mm and $T = 100$ K.

electrodes surfaces can be placed any radius as the Hankel transform handles without problems non-linearities in the radial charge distribution (as discussed before, see Figure 9.1 and Figure 9.2)). This feature gives the opportunity to calculate equilibrium distributions in arbitrary trap geometries, under the assumptions of cylindrical symmetry and co-axiality.

An example of such case is a typical trap potential profile used for electron cooling of antiprotons (see 6.3). Those traps are typically built with a set of low-voltage, smaller-radius electrodes terminated by two slightly separate bigger electrodes that can reach several kV (see [119] and [84]). Within the low-voltage section of the trap, a small positive potential well is first filled with e^- to provide the cooling mechanism to the antiprotons, which are captured using the high-voltage barriers. The interest is in modelling the equilibrium distribution of the electrons in the cooling trap at the instant of $\bar{p}$ injection, for example to estimate the geometrical overlapping of the e^- and $\bar{p}$ clouds.

Figure 9.7 and Figure 9.8 show the potential map of a simulated $\bar{p}$ cooling trap, where two high-voltage electrodes of $R_2 = 22$ mm radius terminate three equispaced low-voltage electrodes of $R_1 = 15$ mm radius, separated by a gap of 1 cm. The high-voltage electrodes are set to -0.5 kV, while the rightmost low-voltage electrode is set at 50 V. The parameters used for the plasma calculation were $N_p = 1.0 \cdot 10^7$ e^- , $T = 1000$ K and $r_p = 1.0$ mm.

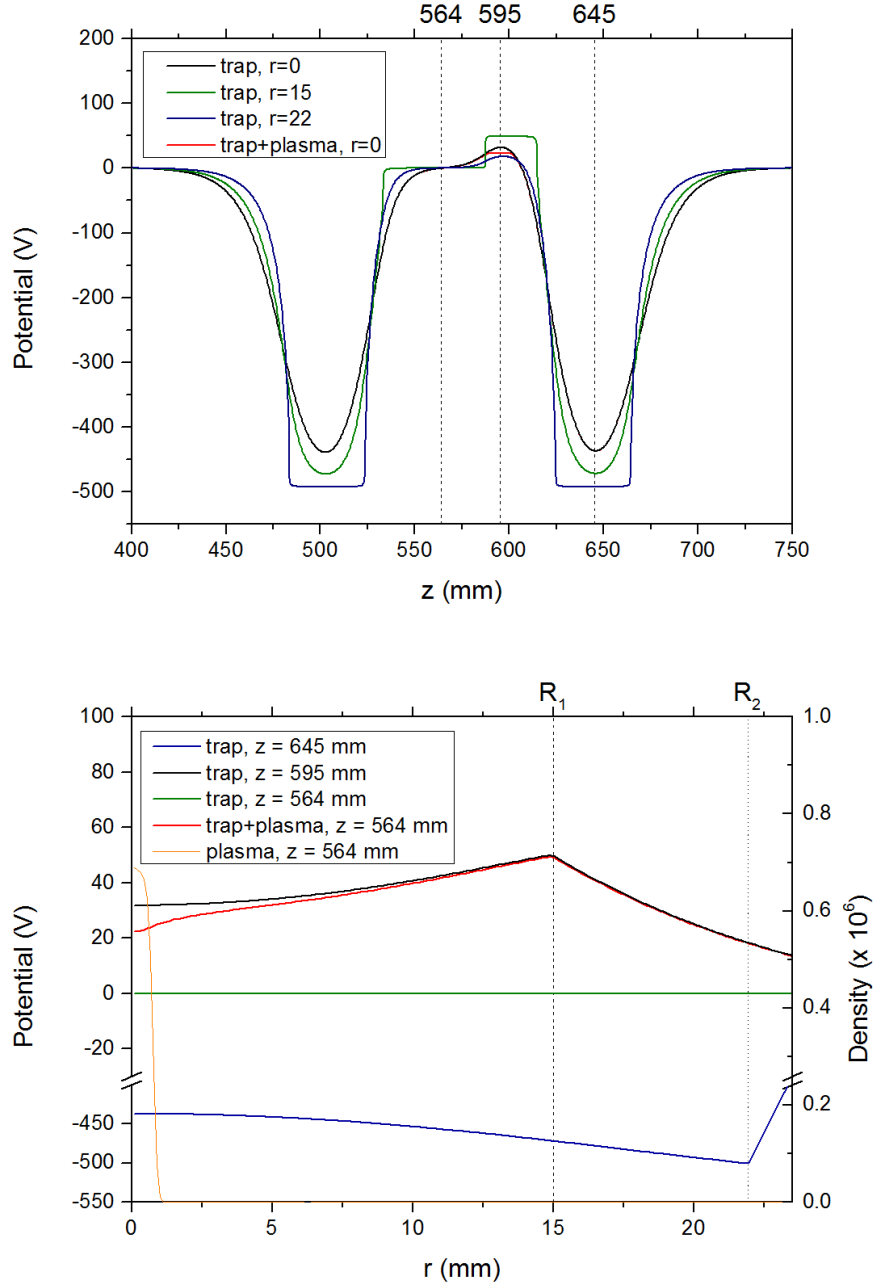


Figure 9.7: Comparison of trap potentials and plasma equilibrium profiles in a sample $\bar{p}$ cooling trap with complex electrode geometry. The above plot shows three axial trap potential profiles: on the trap axis (black), at the radius of the low-voltage electrodes R_1 (green) and at the radius of the high-voltage electrodes R_2 (blue). Each of the vertical dashed lines correspond to a radial profile of the potential shown in the plot below: at the high-voltage electrode (blue), at the center of the cooling trap (red and black respectively with and without particles) and in the flat part of the trap (green). Plasma distribution shown in orange.

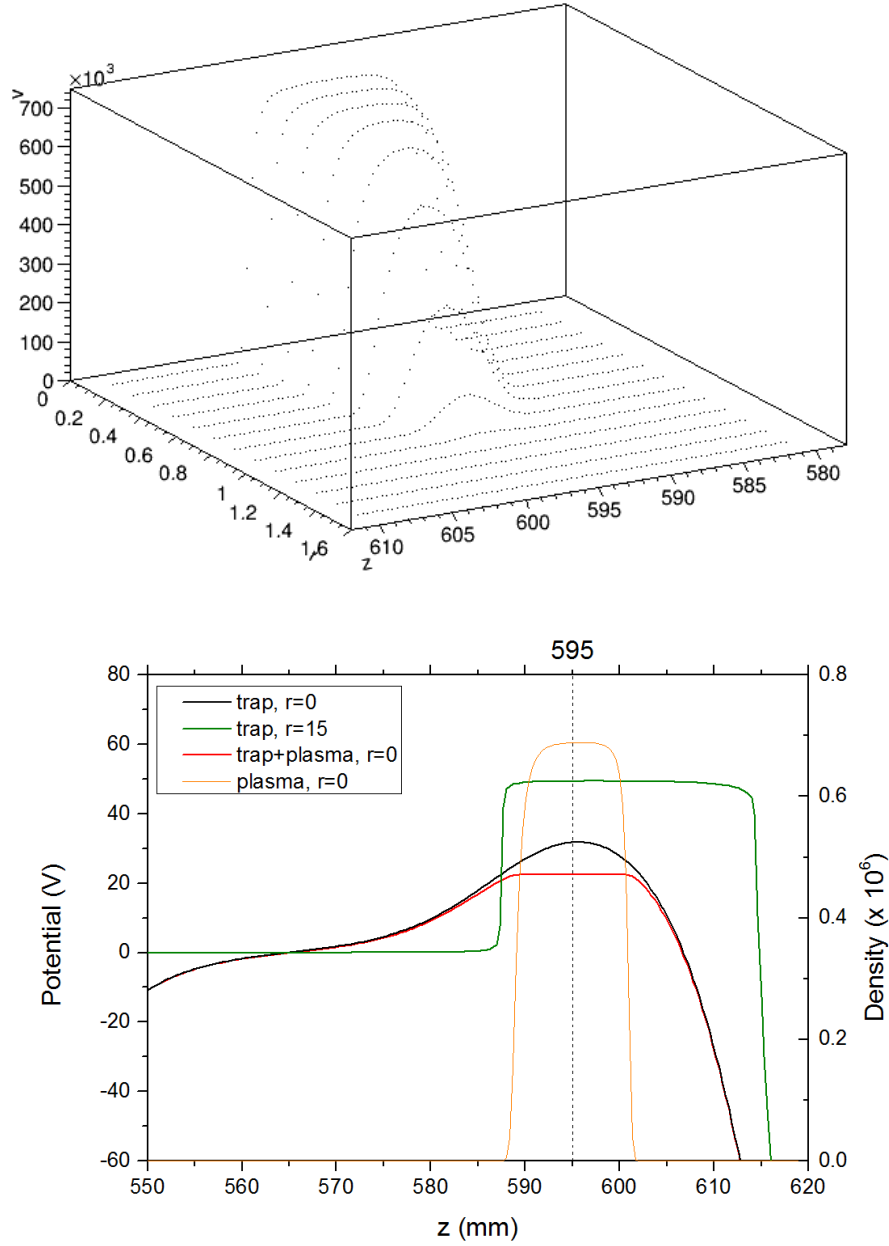


Figure 9.8: Above - 3D plot of the plasma equilibrium density calculated in a sample $\bar{p}$ cooling trap with complex electrode geometry as a function of r and z coordinates. Below - zoom in the trap region of Figure 9.7, above, showing the comparison of the plasma density profile (in orange), the empty trap potential on the axis (black), the empty trap potential at the radius of the low-voltage electrodes (green) and the total potential on the axis (red).

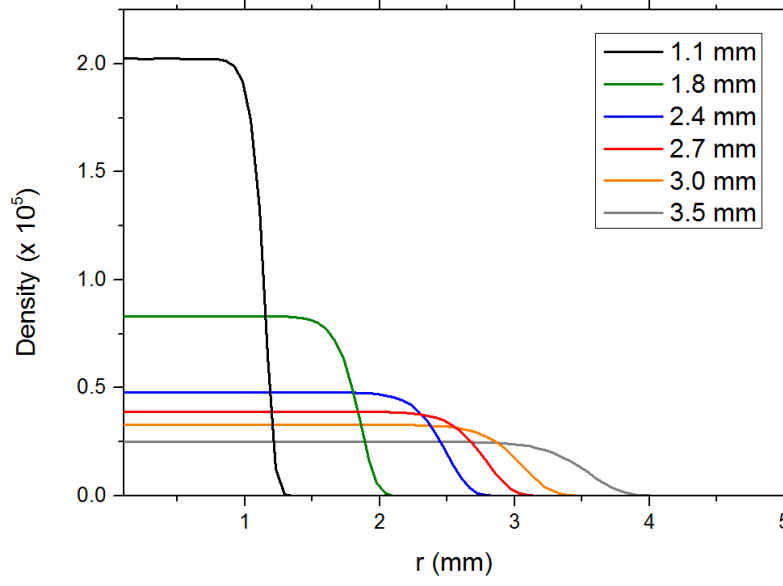


Figure 9.9: Comparison of different plasma midplane radial profiles for different requested plasma radii r_p at constant temperature ($T = 100$ K) and number of particles ($N_p = 2 \cdot 10^7$), showing that the plasma distribution converges to the requested radius. A target accuracy of 1 % was requested for these simulations.

9.5.5 Convergence to the requested plasma radius

In this paragraph, some reference results obtained in calculating plasma distributions at different values of the input parameters are summarized.

In Figure 9.9 the integrated radial profiles of plasmas with the same number of particles ($N_p = 2.0 \cdot 10^7$) and the same temperature ($T = 100$ K) are shown, proving the validity of the method chosen to force the iterative code to converge to the desired value of r_p . This method is expected to be put under stress at very low temperatures, as the space-charge term in 9.1.6 becomes more and more of weight.

Indeed, at very low temperatures (shown in Figure 9.10) the iterative method to fix r_p starts to fail, as an higher and higher discrepancy to the requested value is observed. However, this effect stays below 10% at 10 K temperature, which is already a significant achievement in low temperature for iterative autoconsistent plasma solvers.

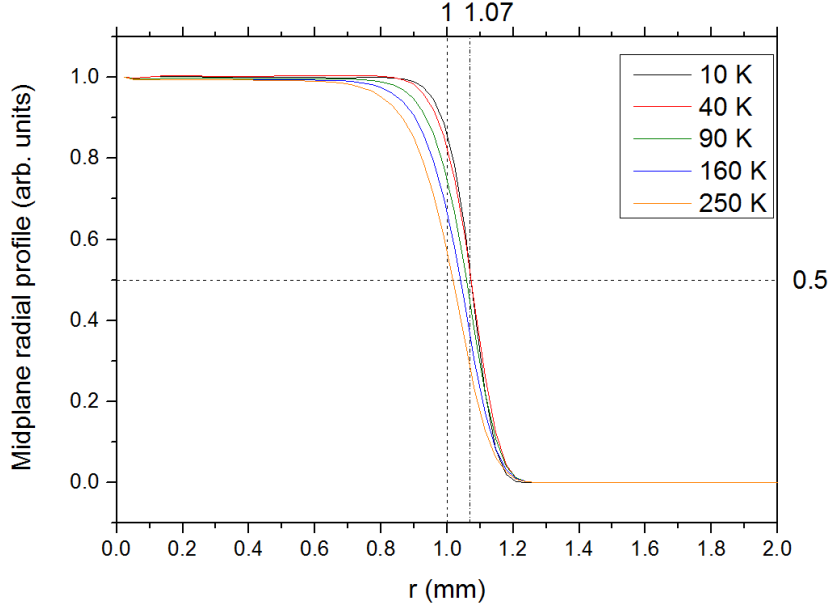


Figure 9.10: Comparison of different midplane radial profiles at low temperatures at constant plasma radius ($r_p = 1.0$ mm) and number of particles ($N_p = 2 \cdot 10^7$), showing the expected discrepancy of the solution radius r_p to the requested radius of $R_p = 1$ mm as a function of the temperature. A target accuracy of 1 % was requested for these simulations.

9.5.6 Midplane radial profile versus integrated radial profile

When comparing simulated radial profiles with experimental data collected with an MCP detector, midplane radial profiles used so far are a valid comparison tool only for plasmas stored in flat Malmberg traps. The proper observable is the integrated radial profile, which is obtained upon integration of the plasma charge distribution over z in the trap volume.

An example of the difference is shown in Figure 9.11, where the case of a plasma calculated in a non-Malmberg trap (P3-P9) is considered. In that case, an estimate of the plasma FWHM r_p using the midplane radial profile would yield a result that is 10 % higher than the integrated radial profile.

9.5.7 Convergence considerations

Finally, a series of numerical experiments were conducted to study the convergence behaviour of the autoconsistent plasma solver as a function of the temperature, in order to evaluate if convergency scales with temperature as T^{-1} (expected for the SHT method, as discussed before) and is almost independent on the choice of the other initial plasma parameters (N_p and r_p).

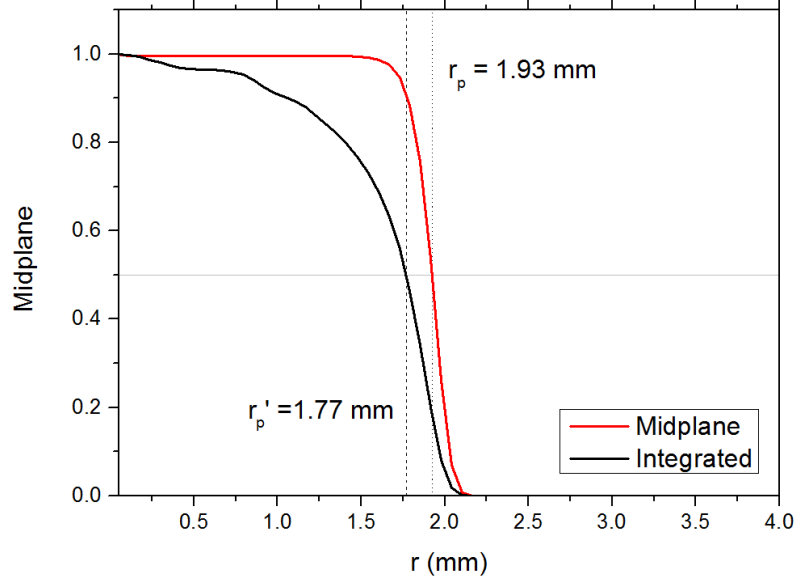


Figure 9.11: Comparison of the normalized midplane radial profile (white triangles) versus the normalized integrated radial profile (black squares) of a plasma at 50 K in P3-P9 almost harmonic trap. The difference in the extrapolated value of r_p with the FWHM becomes dependent to the aspect ratio α of the plasma.

Each measurement was performed calculating the plasma equilibrium distribution at 1 % precision with 0.1 mm radial resolution in the trap potential of the P3-P9 trap shown in Figure 9.6. The underrelaxation ϵ was chosen $\propto T^{-1}$ according to the prescription of [161] using the formula $\epsilon = \epsilon_0 T$. The number of iterations necessary to converge to 1 % accuracy was thus obtained for each found equilibrium and plotted versus the inverse temperature.

A total of four measurement runs were performed: with reference plasma conditions ($N_p = 2 \cdot 10^7$, $r_p = 1$ mm and $\epsilon_0 = 1 \cdot 10^{-5}$); with higher number of particles ($N_p = 5 \cdot 10^7$); with higher plasma radius ($r_p = 2$ mm); with higher underrelaxation parameter ($\epsilon_0 = 2 \cdot 10^{-5}$).

The summary plot of all convergence runs is shown in Figure 9.12 (above). In all evaluated conditions, the convergence behaviour was found linear in T^{-1} . Moreover, no significant dependence of the convergence for the other plasma parameters was found, but for a small increase in the low-temperature region which is imputed to slower convergence of the plasma radius to the requested value as the temperature lowers (see Figure 9.10).

As expected, doubling the underrelaxation parameter ϵ_0 makes the convergency go two times faster (as the slopes of the linear fits testify). Clearly, it is worth increasing ϵ_0 as much as possible to economize CPU time, without incurring in the risk of going very close to the unstable region

for the underrelaxation methods.

It is worth noticing, in conclusion, that the convergence behaviour as a function of the iteration for this particular algorithm, measured using the normalized Euclidean distance, has a very peculiar (non-exponential, non-powerlaw) shape, shown in Figure 9.12 (below). This probably comes from the strongly non-linear nature of the autoconsistent plasma equilibrium problem combined with the iterative scheme adopted for imposing the plasma radius constraint.

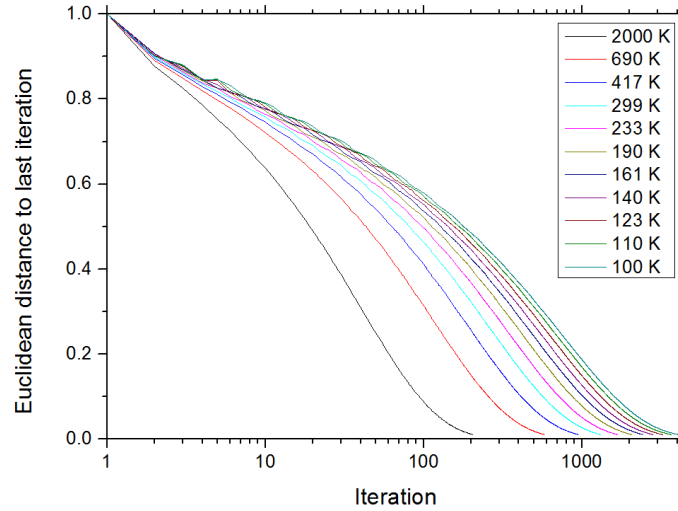
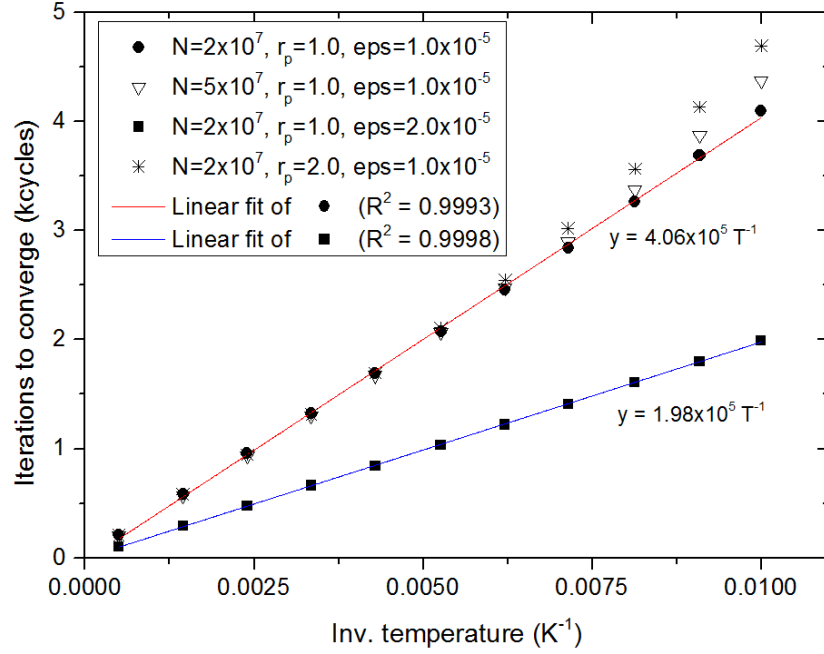


Figure 9.12: Above - number of iterations required to converge to 1 % precision as a function of the inverse plasma temperature T^{-1} , for different initial plasma conditions (circles, triangles and squares) and for a different choice of the underrelaxation parameter ϵ . The two curves with different ϵ are fitted with a linear model (red and blue) to show that the slope ratio is consistent with the ratio of the underrelaxation ϵ s. Below - normalized Euclidean distance (see text) as a function of the Picard cycle iteration for different plasma temperatures, showing the peculiar convergency curve for this algorithm.

Chapter 10

Positronium excitation to the 2^3S metastable state

Positronium and Helium show many similarities in their optical spectra, having both two particles with spin $1/2$ in their outer shell. Both atoms have para- and ortho- states, distinguished by the values of their total spin¹, forming respectively spin singlets and spin triplets (a comprehensive presentation of both He and Ps energy structures is found, for example, in Bethe and Salpeter book [140]). Moreover, both have optical metastable levels in their $n = 2$ manifold, 2^1S and 2^3S , that are characterized by an almost infinite optical lifetime in absence of external quenching fields. This is a consequence of electric dipole selection rules, which require $\Delta l = 1$ for the one-photon transition to be allowed: all spontaneous optical decays to sublevels of the $n = 1$ manifold are forbidden, as both starting and destination levels are in S -wave; the only permitted transitions go through very rare two-photon decays.

Real-world lifetimes of Ps metastable levels are, however, much smaller than their optical lifetimes (driven by two-photon decays processes) due to e^-e^+ annihilation into γ rays. Similarly to what happens to ground states, 2^1S atoms decay in 2γ whereas 2^3S atoms decay in 3γ , as a consequence of total angular momentum conservation. The resulting annihilation rates are three orders of magnitude different because of the smaller phase-space of a three body annihilation (see, for instance, [168, p. 171]). The lifetime of the 2^1S and 2^3S metastable states can be calculated by multiplying the lifetime of their relative ground state, respectively 125 ps and 142 ns, by n^3 , as the annihilation rate is proportional to the squared modulo of the electron

¹ $s = 0$ for para states, $s = 1$ for ortho states, being s the eigenvalue of the total spin operator $S^2 \rightarrow s(s+1)$, on a suitable basis where the total spin is diagonal

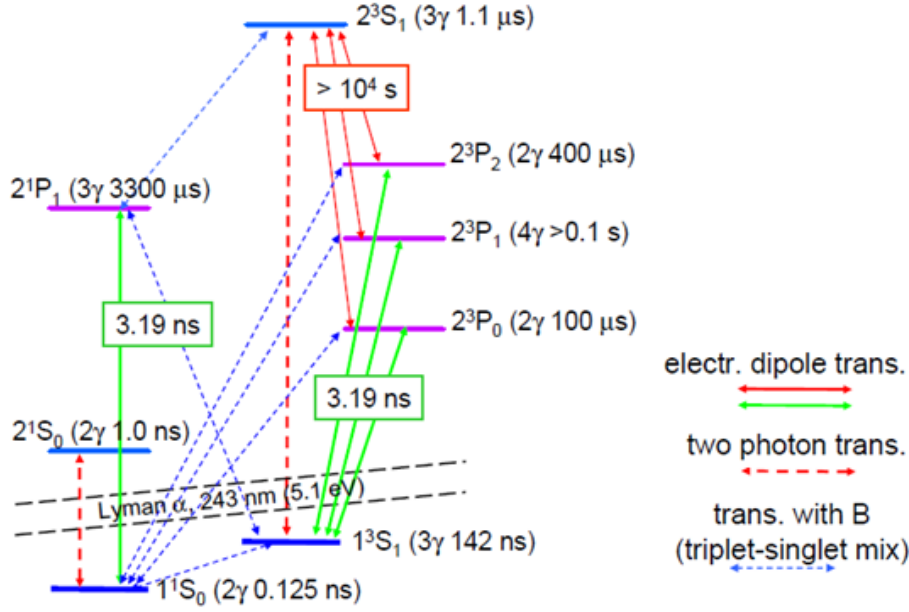


Figure 10.1: Level structure of positronium $n = 1$ and $n = 2$ levels in absence of external fields. Electrical dipole (optical) transitions are marked by solid lines (red within the same manifold, green among different manifolds) with their natural transition rate. Two-photon (forbidden) transitions are marked in dashed red. Transitions allowed only by singlet-triplet level mixing in magnetic field are marked in dashed blue lines. Leading annihilation process and its characteristic time is given within round brackets. Courtesy of F. Castelli

wavefunction in the origin $\tau_{\text{ann}}^{-1} \propto |\psi(0)|^2 \propto n^{-3}$. Thus:

$$\begin{aligned}\tau_{\text{ann}}(2^1S) &= 1.0 \text{ ns} \\ \tau_{\text{ann}}(2^3S) &= 1.1 \mu\text{s}\end{aligned}\tag{10.0.1}$$

A compact summary of positronium energy structure in absence of external fields is given in Figure 10.1, including the most important electric dipole transitions, two-photon transitions, annihilation rates and transitions in presence of an external magnetic field.

The metastable 2^3S triplet is of particular interest: it has $\times 8$ longer lifetime than ground-state ortho-Ps, and it does not feel the influence of external electric field gradients being in the S -wave. Its close companion 2^3S level of Helium is commonly used by experiments in the cold atoms community as a temporary storage level for subsequent manipulations of the atoms. It made Helium one of the first systems to be efficiently laser-cooled through Doppler cooling [169], and subsequently lead to the first demonstration of Bose-Einstein condensation of Helium [170, 171]. 2^3S of positronium could, in principle, provide an alternative to Doppler cooling

on $1^3S \rightarrow 2^3P$ (thoroughly described in P. Yzombard Ph.D thesis [172]), with the gain of the extended lifetime of 2^3S compared to 1^3S . This level has been also considered for observing Ps free-fall using an atom interferometer (see [58]) with analogy to a similar proposal on Rydberg levels (see [57]), which have much longer lifetime at the cost of a strong coupling to external field gradients.

This chapter reports the first observation of positronium atoms in their metastable 2^3S level by spontaneous radiative decay from 3^3P . Positronium atoms were excited first in 3^3P using the same experimental methodology reported in 4.5 (which lead to the first spectroscopical survey of Ps 3^3P level), then let annihilate on the spectroscopy chamber walls to observe a increase in the delayed annihilations, consequence of the $\times 8$ longer lifetime of 2^3S compared to 1^3S . Results obtained are inconsistent with a null-signal hypothesis with a 3σ confidence level.

10.1 Theory

Loading atoms in 2^3S was made possible by the availability of the $1 \rightarrow 3$ laser at 205 nm (see 4.4) developed for efficient Ps excitation to Rydberg levels for antihydrogen production in AEgIS. Positronium atoms laser-excited to 3^3P can only spontaneously decay back, in absence of external fields, to either 1^3S (ground state) or 2^3S (metastable). A theoretical calculation of the branching ratio of these two decay processes, in absence of fields, is given in [102].

$$\frac{3^3P \rightarrow 2^3S}{3^3P \rightarrow 1^3S} = 13.4\% \quad (10.1.1)$$

In the presence of an axial 250 G magnetic field and a parallel 600 V cm^{-1} electric field, such as the environment inside the spectroscopy chamber (due to necessity to transport e^+) the picture become more complex. Atoms on 3^3P can also decay back to 1^1S due to mixing of the triplet 3^3P substates with the singlet 3^1P substate. This extra decay process, caused by the presence of the magnetic field, is known as magnetic quenching (already discussed in 4.5). Thus, in the experimental conditions of the spectroscopy chamber, Ps atoms can follow three different possible paths:

$$\begin{array}{ll} 1^3S \rightarrow 3^3P \rightarrow 1^3S & \text{spontaneous decay} \\ 1^3S \rightarrow 3^3P \rightarrow 2^3S & \text{metastable formation} \\ 1^3S \rightarrow 3^3P \rightarrow 1^1S & \text{magnetic quenching} \end{array} \quad (10.1.2)$$

A calculation, in absence and presence of the fields, of the fraction of atoms following each of these three paths was performed using the PsTC code, which performs the full rate equations integration considering the processes of absorption, spontaneous emission, stimulated emission

and annihilation. PsTC accounts for the complete energy levels structure in external fields of Ps on the first three levels, assuming 100 % laser geometrical and spectral efficiencies. Three transition probabilities were thus calculated, each relative to one of the three possible optical decay paths mentioned above (Equation 10.1.2): η_{decay} , η_{meta} , η_{quench} . These three probabilities are bound by $\eta_{\text{decay}} + \eta_{\text{meta}} + \eta_{\text{quench}} = 1$ (no other decay channel for 3^3P is available).

The η_{decay} probability was calculated as the ratio between the number of atoms ending up back on 1^3S in the far future with respect to the laser excitation pulse versus the initial number of atoms on 1^3S before the laser pulse.

$$\eta_{\text{decay}} = \frac{1^3S \rightarrow 3^3P \rightarrow 1^3S}{1^3S} \quad (10.1.3)$$

For properly calculating this optical probability, the annihilation process from 1^3S into 3γ was disabled from the rate equations calculation. The 3γ annihilation process makes the final population on 1^3S decay exponentially with $\tau = 142$ ns. This process slowly removes atoms from the 1^3S population while the optical decay process $3^3P \rightarrow 1^3S$, with a characteristic time of ~ 30 ns, is still operating. This makes the final estimate of the final population on 1^3S both time-dependent and affected by a systematic error. Results of this calculation are given in Table 10.1.

Similarly, the η_{meta} efficiency was defined as the ratio between the number of atoms ending up on 2^3S in the far future with respect to the laser excitation pulse versus the initial number of atoms on 1^3S right before the laser pulse.

$$\eta_{\text{meta}} = \frac{1^3S \rightarrow 3^3P \rightarrow 2^3S}{1^3S} \quad (10.1.4)$$

Here, the annihilation process from 2^3S was disabled for properly estimating the optical probability (following the same argument used for η_{decay}). Results of this calculation are given in Table 10.2.

Finally, η_{quench} efficiency was calculated as the ratio between the number of atoms ending up annihilating in 2γ from 1^1S in the far future with respect to the laser excitation pulse, versus the initial number of atoms on 1^3S right before the laser pulse. Results are found in Table 10.3.

$$\eta_{\text{quench}} = \frac{1^3S \rightarrow 3^3P \rightarrow 1^1S}{1^3S} \quad (10.1.5)$$

	Vacuum	Exp. conditions
$\eta_{\text{decay}}^{\pi}$	90.6 %	73.1 %
$\eta_{\text{decay}}^{\sigma}$	87.8 %	76.7 %
$\eta_{\text{decay}}^{\text{unpol}}$	88.0 %	73.2 %

Table 10.1: Efficiency η_{decay} (see text) for different laser polarizations (π , σ and unpolarized light, see 8.2.2 for their definition) calculated using the PsTC code. The calculation is given both for vacuum and experimental conditions, set with $B = 0.025$ T, $E = 600$ V cm $^{-1}$, $v_{\text{Ps}} \propto 1200$ K all parallel in the $\hat{z}$ direction. Calculation error is estimated to be 5 %

	Vacuum	Exp. conditions
η_{meta}^{π}	9.42 %	7.15 %
$\eta_{\text{meta}}^{\sigma}$	12.2 %	10.4 %
$\eta_{\text{meta}}^{\text{unpol}}$	12.0 %	9.89 %

Table 10.2: Efficiency η_{meta} (see text) for different laser polarizations (π , σ and unpolarized light, see 8.2.2 for their definition) calculated using the PsTC code. The calculation is given both for vacuum and experimental conditions, set with $B = 0.025$ T, $E = 600$ V cm $^{-1}$, $v_{\text{Ps}} \propto 1200$ K all parallel in the $\hat{z}$ direction. Calculation error is estimated to be 5 %

	Vacuum	Exp. conditions
$\eta_{\text{quench}}^{\pi}$	0 %	19.8 %
$\eta_{\text{quench}}^{\sigma}$	0 %	12.9 %
$\eta_{\text{quench}}^{\text{unpol}}$	0 %	16.9 %

Table 10.3: Efficiency η_{quench} (see text) for different laser polarizations (π , σ and unpolarized light, see 8.2.2 for their definition) calculated using the PsTC code. The calculation is given both for vacuum and experimental conditions, set with $B = 0.025$ T, $E = 600$ V cm $^{-1}$, $v_{\text{Ps}} \propto 1200$ K all parallel in the $\hat{z}$ direction. Calculation error is estimated to be 5 %

The calculation of η_{meta} in free vacuum conditions,

$$\left. \frac{1^3S \rightarrow 3^3P \rightarrow 2^3S}{1^3S} \right|_{\text{vac}} = 12.0\% \quad (10.1.6)$$

can be cross-checked to an independent experimental measurement performed on hydrogen, which shares with Ps the same branching ratio for this particular transition. The measurement, carried over in [173], yielded 12%, in good agreement with the model estimate.

10.2 Experimental

The experimental methodology used for observing positronium atoms in their metastable 2^3S state was directly mutated from the one developed for observing long-lived Rydberg states of positronium, described in 4.6 (see also Figure 4.24 for a visual reference). It consisted of looking for an excess of annihilations at late times in SSPALS spectra when the 3^3P excitation laser was sent on the Ps sample. This expected excess of annihilations is a consequence of the longer lifetime of 2^3S atoms, which have an higher probability than 1^3S atoms to annihilate on the chamber walls.

The main difference between the two experiments, Rydberg and metastable, was the much lower expected signal in the latter case, roughly estimated to be $\sim 1\%$. This estimate was obtained by multiplying the measured 3^3P excitation efficiency of $\sim 15\%$ (when the excitation laser is set on-resonance, see Figure 4.20, and the photoionization probability is assumed 100 %) with the calculated η_{meta}^π decay probability for a laser with π polarization (7.2%, see Table 10.2).

Practically, the experiment was carried out acquiring a high-statistic sample of SSPALS spectra, with/without the presence of the UV 3^3P laser (the same type of measurements done during the magnetic quenching experiment described in 4.5), looking for an excess of annihilations in the same time window where the Rydberg signal was formerly observed (see Figure 10.7).

In order to reach the target precision of $\sim 1\%$, several technical upgrades were introduced with respect to the setup described in 4.5: the signal digitizer was replaced with one with higher accuracy; a second detector was added; the UV laser pulse energy was increased; a telescopic system was added for gaining control over the laser spot size. A careful measurement strategy for minimizing the systematic effects was also defined, and two measurement runs with and without metastable formation performed to carry out a comparative analysis.

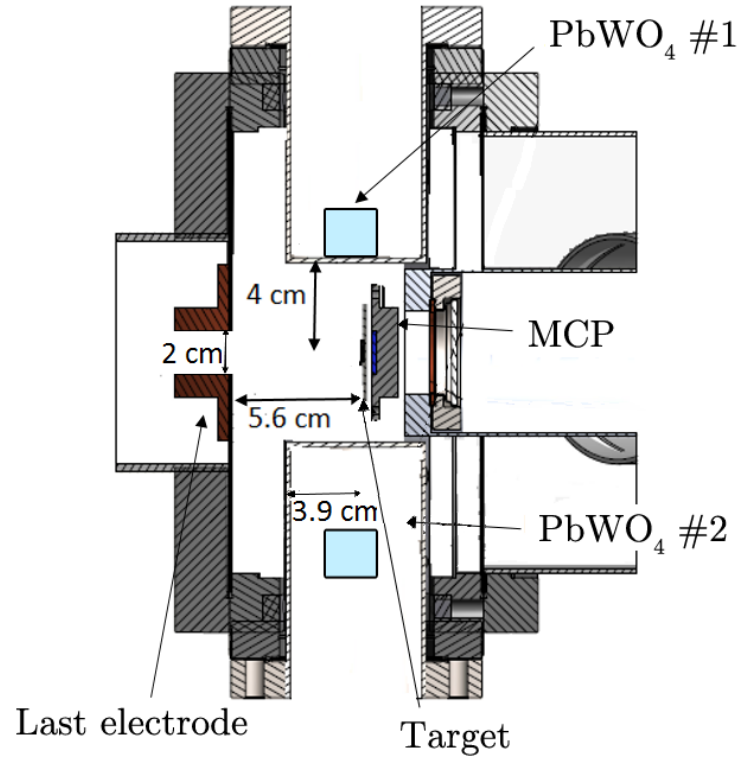


Figure 10.2: Experimental setup adopted for the metastable experiment: two PbWO_4 scintillating crystals coupled to PMTs were placed close to the target region, in the two hollow cylindrical target holders.

10.2.1 Positron setup and upgrades

A bunch of $\sim 10^6$ e^+ ejected from AEGIS positron accumulator (see 4.1) was implanted with 3.3 keV energy in a p-type (111) mesoporous silica converter target (see 4.3 and Table 4.1) at room temperature, installed in the center of the experimental chamber dedicated to Ps spectroscopy experiments (see 4.2 and Figure 4.5).

The same $20 \times 25 \times 25$ mm PbWO_4 scintillator crystal (coupled to an Hamamatsu R11265-100 photomultiplier tube) used for $1 \rightarrow 3$ SSPALS spectroscopy was placed 40 mm above the sample in a custom-made hollow cylindrical detector holder (see Figure 10.2 for a visual reference). A second identical $20 \times 25 \times 25$ mm PbWO_4 crystal (coupled to an Hamamatsu R11265-100 photomultiplier tube) was placed 60 mm below the sample in the symmetric detector holder in the bottom position.

Both photomultiplier signals were split by two Mini-Circuits ZFRSC-2050B 50% – 50% split-

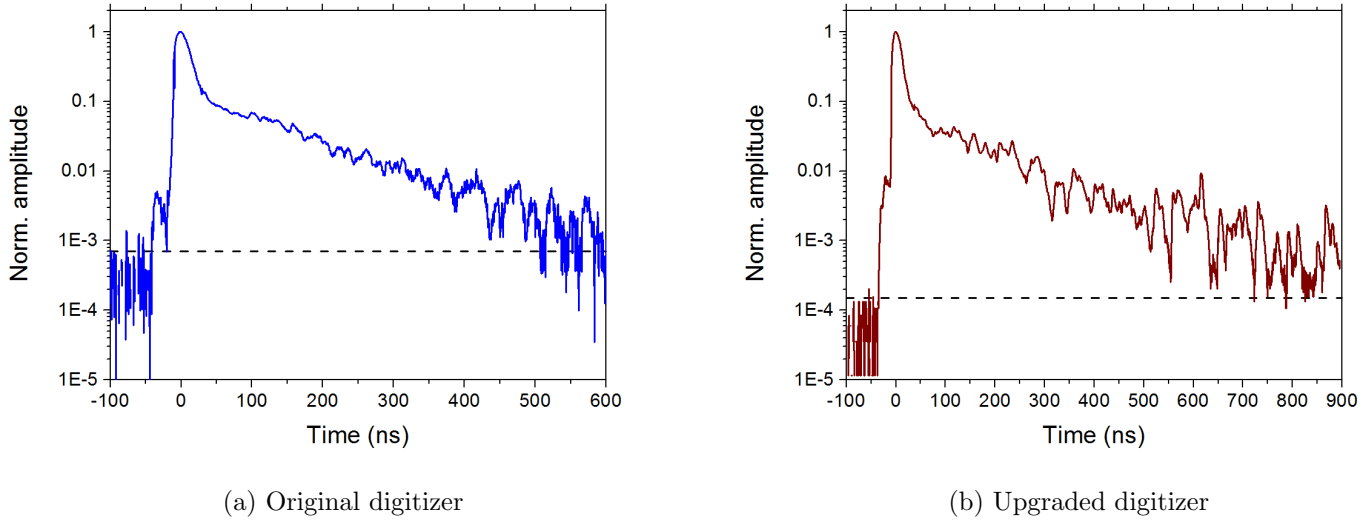


Figure 10.3: Example of SSPALS spectra acquired with the original 1.25 GHz, 8 bit digitizer (panel (a)) and the upgraded 2.5 GHz, 10 bit digitizer (panel (b)). The noise floor is marked with a dashed line.

ters and digitized (using two channels per photomultiplier) using an upgraded 10 bit, 2.5 GHz digitizer with high (100 mV/div) and low (1 V/div) resolutions. Thanks to its higher dynamics (10 bits instead of the 8 bits of the original digitizer used for the former spectroscopy measurements), this upgraded digitizer helped significantly reducing the shot noise floor of about a factor ~ 5 (see Figure 10.3 for a visual comparison of two example spectra). The final SSPALS spectra were obtained automatically by the data acquisition system combining the information of both high and low resolution channels, using the overlapping region as cross-normalization. The threshold for selecting either the low resolution or the high resolution channel was set to 10% of the maximum peak amplitude: above it, data from the low-resolution channel was used; below it, data from the high-resolution channel was used.

This particular is of importance later on when discussing the analysis procedure adopted for the measurements (see 10.3): special care was taken in using the data originated only from the high-resolution channel of the oscilloscope, having a smaller quantization error.

10.2.2 Laser setup and upgrades

The laser setup used for these measurements was essentially the same already adopted for the Ps excitation campaign to Rydberg levels, described in the details in 4.4, with some upgraded features.

The same OPG + OPA + SFG UV laser system described in 4.4.2 was used to generate a

pulse of laser light at 205.045 nm with 1.5 ns time length, a spectral width of $2\pi \times 48\text{GHz}$ and a polarization parallel to the magnetic field in the experimental region (π -polarization). Both crystals in the OPG stage (a PPKTP) and in the SFG stage (a BBO) were replaced with two newer and slightly bigger ones to increment the available generated pulse energy of about $\sim 50\%$.

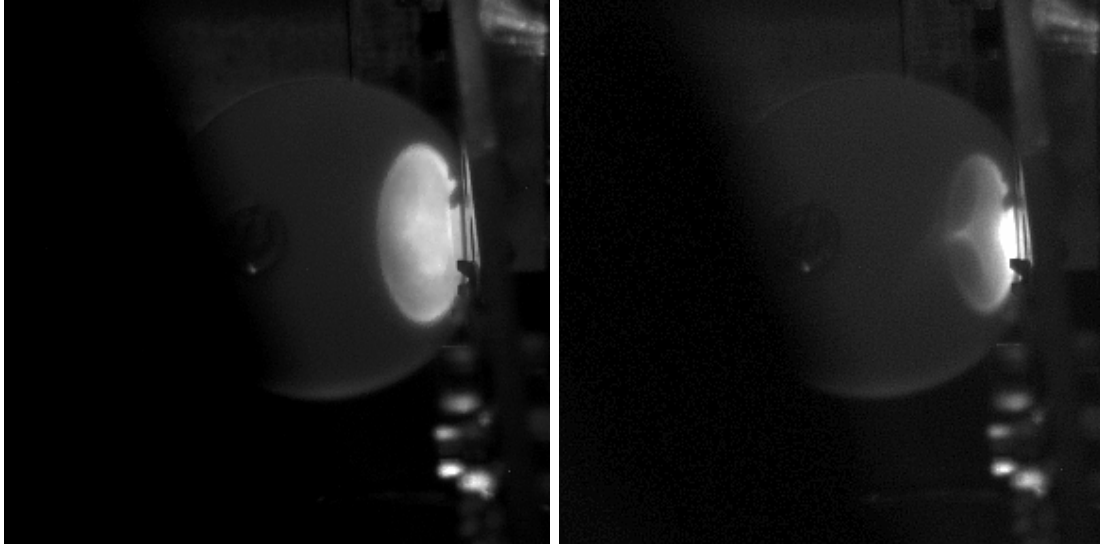
Moreover, the mirrors of the transport lines were replaced from the broadband aluminum mirrors from CVI to broadband aluminum mirrors from ACTON, whose reflectivity was slightly polarization-dependent and 5% higher per mirror than the ones from CVI. The entire transfer line was constituted of five of these mirrors, and the entire gain was about 15%.

This two improvements in the UV laser power made possible to introduce a telescopic system made of two uncoated fused silica lenses, which introduce a loss of $\sim 20\%$ of the power, close to the entrance viewport of the spectroscopy chamber. The telescopic system allowed to gain control over the spot shape and fine alignment of the beam. Despite the losses introduced by the telescope, 80 μJ of UV energy were normally measured at the viewport, about 25% more thanks to the upgrades, and well within the measured transition saturation regime (see Figure 4.22, panel (a)).

The introduction of the telescopic system allowed the laser power distribution to be optimized day-by-day for maximizing the Ps signal. The experimental procedure consisted in taking a few photoionization shots (using also the IR laser described in 4.4.4 to selectively photoionize the atoms excited to 3^3P , as described in 4.5.2) each day at the beginning of the measurements, and reach the nominal excitation efficiency of $> 15\%$ adjusting the spot with the telescope. Usually, the highest photoionization efficiency was obtained placing the last lens of the telescope out of the axis, to have the beam slightly elliptic in the vertical direction (see Figure 10.4, panel (a)), and rotated on its axis, so that most of the power ends up in front of the target (see Figure 10.4, panel (b)).

The laser power on the photoionization line was never a concern, on the contrary, as the maximum pulse energy available on that line is of 50 mJ - coming directly from the YAG cavity - well within the saturation region (see Figure 4.22, panel b). The photoionization laser spot was made on purpose very wide (20 mm $\times$ 20 mm) to cover completely the UV spot and relax the need of accuracy in the alignment. The alignment procedure of the photoionization laser is described in 4.4.6.

The power delivered on the UV line had to be very stable not to introduce a further systematic effect due to a signal drift in time. Frequent measurements of the UV power were carried out during the measurement days using a pulse energy meter from Coherent. The energy of 200 consequent UV shots was measured with a μJ accurate detecting head. The power of the UV laser fluctuates in power about 10% RMS shot-by-shot, so with 200 shots, the error of the



(a) Beam ellipticization

(b) Beam concentration

Figure 10.4: Example of ellipticization (panel a) and concentration (panel (b)) of the UV laser spot in front of the target. Ellipticization was achieved placing the final lens of the telescopic system off the beam axis; concentration was obtained by rotating it on its mounting. The ellipticization effect looks larger in the picture than reality, as the MACOR screen is 45° tilted downwards towards the camera.

average was about 0.7%. The average was then verified to be consistent within the average value measured at the beginning of the day. The UV laser was always found stable in the delivered power.

10.2.3 Measurement runs

Each measurement consisted of a shot of e^+ ejected from the accumulator, bunched and implanted in the mesoporous converter to form a Ps cloud intercepted by the synchronized laser pulses (see 4.4.6 for further details on Ps-laser synchronization). Both PbWO_4 detectors were acquired synchronously by the same digitizer triggered by the e^+ launch chain; the exact time of acquisition was saved as well, with a resolution of 1 s.

Two measurement runs with high statistics were conducted. During the first run, both excitation (UV) and photoionization (IR) lasers were sent synchronously on the Ps sample: the strong IR laser pulse inhibited the production of metastable 2^3S , thanks to the very high efficiency of photoionization, which has orders of magnitude higher branching ratio than optical decay to 2^3S . During the second run, the photoionization laser was blocked with an optical dumper, sending only the UV laser to excite 3^3P and produce 2^3S by spontaneous optical decay.

This experimental approach was found quite effective both in experimental and analysis terms. The advantage during analysis of having two datasets to compare, one with and one without the signal under interest, is described in more detail further in the chapter. From the experimental point of view, the possibility to introduce the IR photoionization laser at will allowed the efficiency of the 3^3P excitation to be measured and progressively optimized throughout the measurement days of the UV+IR run, seeking for the maximum photoionization signal.

During the metastable production run, moreover, photoionization served as a diagnostics for monitoring the excitation efficiency. This was an item of particular importance for having a reference value of the 3^3P excitation efficiency: every measurement day was started with a re-assessment of the Ps excitation conditions, shining both UV+IR in order to verify that at least 15% of photoionization signal was present. Then, the IR laser was dumped introducing the optical dumper without touching at all the UV line. The excitation efficiency was kept under constant monitoring throughout the measurements by temporary re-introducing the IR laser (every now and then) and verifying the signal conditions using photoionization.

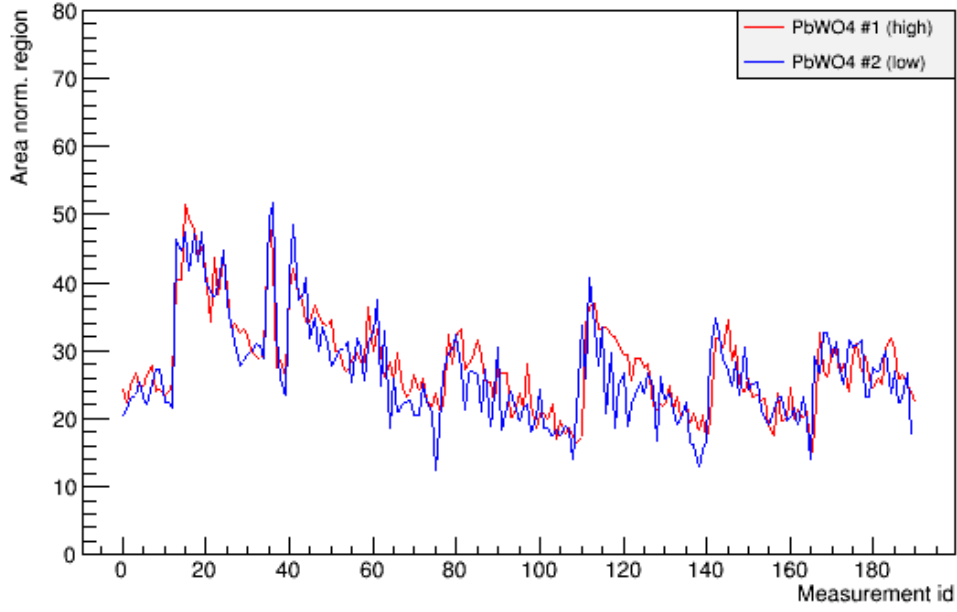
A final exploratory run with lower statistics was performed rotating the UV laser polarization by 90° , from π to σ , with an extra aluminum mirror. The metastable production signal is expected to be $\sim 45\%$ higher in this case (see Table 10.2).

10.2.4 Methodology for limiting systematics due to moderator ageing

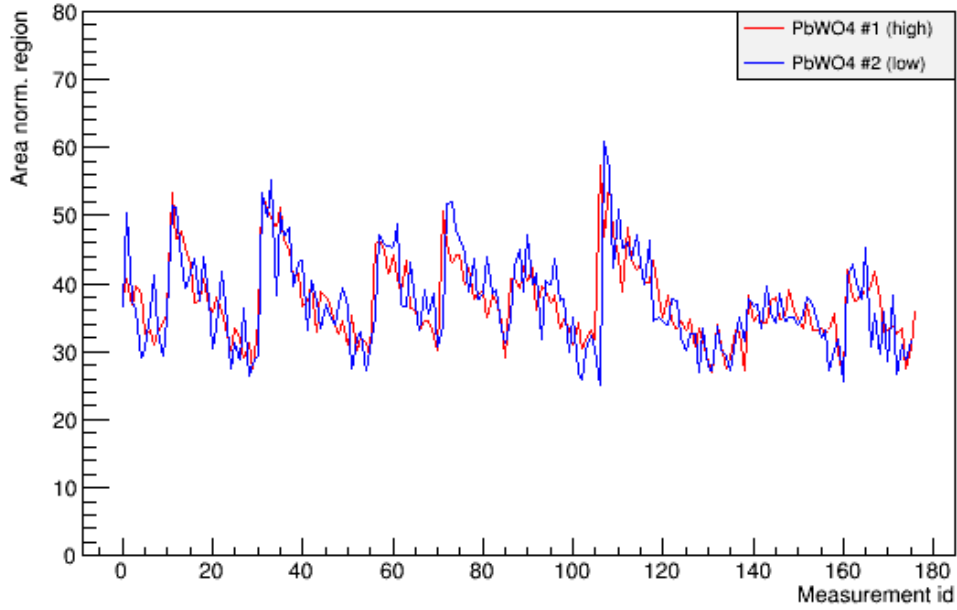
An alternating laser/no laser acquisition scheme was necessary to account for the drift of the number of e^+ delivered from the accumulator as a function of the time during the day, typical of neon-moderated positron sources (see 4.1). Neon moderators get polluted by the buffer gas used for e^+ cooling in the Surko trap, causing the number of positrons to reduce by $\sim 7\% \text{ h}^{-1}$. Moderators usually have to be re-grown every 6 hours of operation of the e^+ source to deliver the optimal number of positrons. This drift of the positron number can be up to 50 % after 6 hours of operation.

An example of such behaviour is shown in Figure 10.5, where signal amplitudes of time-sorted measurements are shown as a function of the measurement number, for the photoionization and metastable runs respectively. The abrupt peaks in the signal are caused by the moderator growths; then the e^+ number degrades progressively on a roughly exponential decay.

Moderator ageing required special care both in experimental and analysis procedures to reach 1 % precision. Each run consisted of about 350 measurements, which were collected alternating one shot with the laser and one shot blocking the laser with a hard obstacle, which provided the reference amount of Ps formed at that given moment in time. Two adjacent measurements in this alternating sequence of no-laser/laser measurements were separated by the e^+ accumulation



(a) Photoionization (UV + IR) run



(b) Metastable (UV only) run

Figure 10.5: Comparison of signal integrals measured by the upper detector (red) and lower detector (blue) in the normalization region (see text) as a function of the measurement number in the two photoionization (panel (a)) and metastable (panel (b)) measurement runs (data of the lower detector were cross-normalized as described in 10.3.6). All measurements were time ordered as described in the text. Moderator ageing is responsible of the multi-peaks structure observed in the data.

time, typically of about 200 s.

During the photoionization run, most measurements were collected keeping the same alternating order, starting from a measurement without the laser, followed by one with the laser. The average time interval between measurement with laser versus its reference measurement without laser, $t_{\text{laser}} - t_{\text{no laser}}$, was about 212 s, as shown in Figure 10.6, panel (a) (sometimes is higher due to a lower availability of e^+ , requiring higher accumulation time).

Differently, during the metastable formation run, the alternating order was sometimes inverted, collecting roughly as many measurements in one way as in the other (see Figure 10.6, panel (b)). This trick made the average time interval $t_{\text{laser}} - t_{\text{no laser}}$ drop to $23 \pm 13.7_{\text{stat}}$, thus cancelling at first-order systematic effects linked to time, such as moderator ageing or (unobserved) laser power drifts (see 10.3.2 for a throughout discussion).

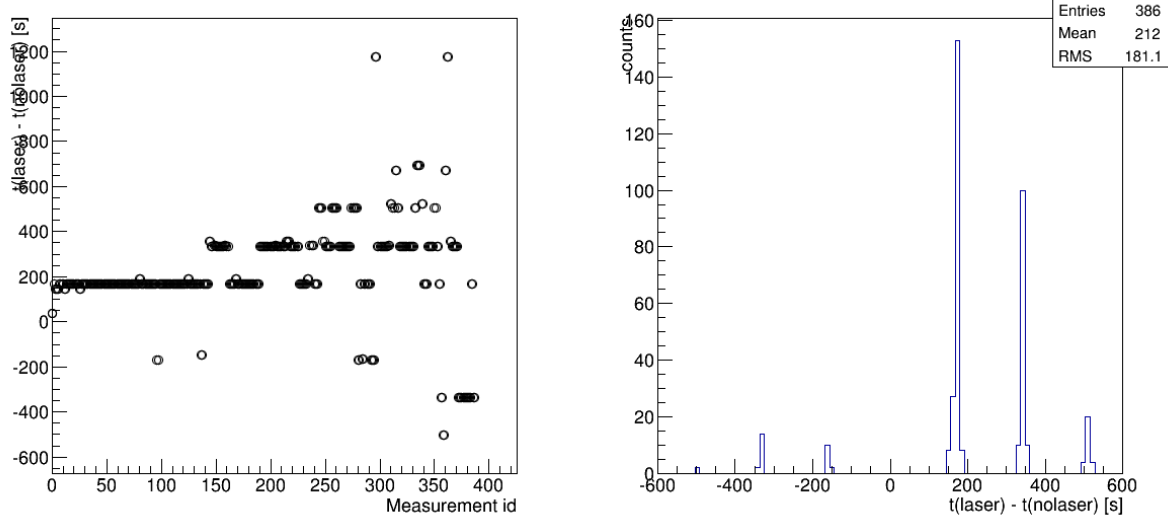
10.3 Analysis

The data analysis method used for assessing the presence of a fraction of metastable 2^3S positronium atoms was based on the formulation of a null hypothesis and its subsequent statistical invalidation (*modus tollens*).

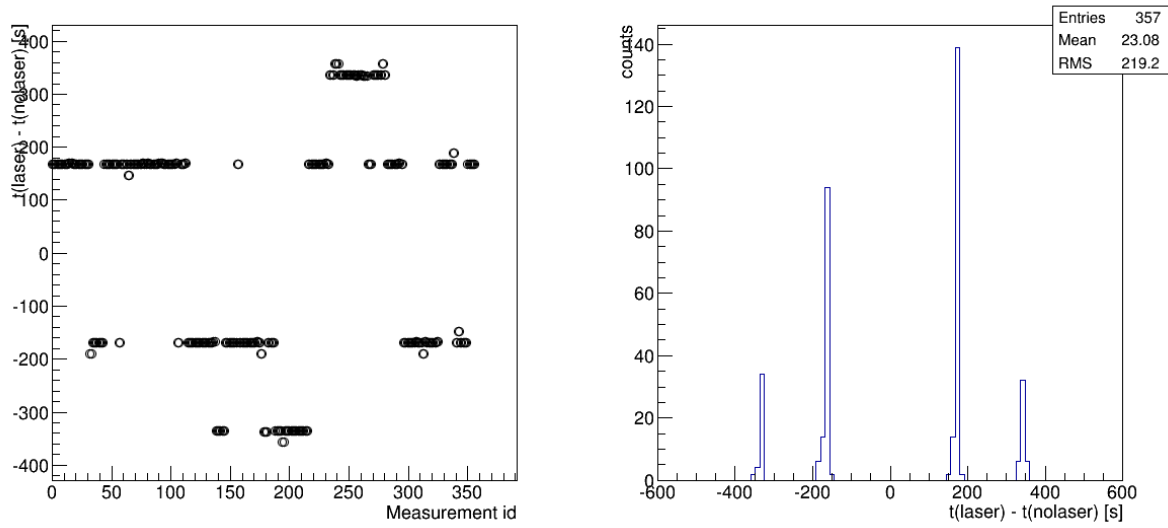
This null hypothesis consisted in postulating that, for some experimental reason not under control or some inaccuracy in the theoretical modelling, no long-lived fraction of Ps was actually formed by sending the 3^3P laser pulse, and that all the atoms excited to 3^3P either decayed spontaneously back to 1^3S or quenched on 1^1S incurring in rapid annihilation. In other words, the $1^3S \rightarrow 3^3P \rightarrow 2^3S$ process in Equation 10.1.2 was postulated to be either non-existing or followed by further optical decays or direct annihilations into γ -rays.

The consequence of this hypothesis is that the only expected action of the $1 \rightarrow 3$ laser would be to kill a fraction of the initial 1^3S atom population through $1^3S \rightarrow 3^3P \rightarrow 1^1S$ with probability ξ , in general different than η_{quench} calculated above, as the metastable formation process in Equation 10.1.2 would be ineffective. All non-quenched atoms excited to 3^3P would quickly decay back to 1^3S with a spontaneous optical decay time of $\tau_3 = 13$ ns.

In such a hypothetical scenario, the only observed effect in SSPALS spectra would be a reduction of the o-Ps signal with respect to the case when the laser is blocked. The signal when the laser is sent should be found proportionally reduced by the same amount in all time regions, with a relative difference to the spectra without laser equal to the initial population reduction ξ . Putting it in symbols, calling $A_{\text{ON}}(t)$ a SSPALS spectrum with the presence of the laser and $A_{\text{OFF}}(t)$ a SSPALS without the presence of the laser, this so-called *proportionality hypothesis*



(a) Photoionization (UV + IR) run



(b) Metastable (UV only) run

Figure 10.6: Distribution of time delays among measurements with and without lasers, for both photoionization (panel (a)) and metastable (panel (b)) runs.

can be written as (see 10.3.3 paragraph for its derivation)

$$A_{\text{ON}}(t) = (1 - \xi) A_{\text{OFF}}(t) \quad (10.3.1)$$

Under this null or proportionality hypothesis, the action of the $1 \rightarrow 3$ laser alone would be akin - but for a lower efficiency - to the combined action of UV plus photoionization lasers. When the photoionizing laser is added, indeed, metastable atoms are surely not formed even if all processes in Equation 10.1.2 are considered, as the ionization probability is so higher than any optical decay probability that all atoms are immediately ionized as soon as they reach 3^3P .

Instead of directly comparing the raw signals $A(t)$, the analysis was carried out using their integrals $\int A(t)dt$ on selected time windows in order to further reduce statistical fluctuations, on the same line to what already done in 4.5 (with some important differences, discussed in details in a following paragraph 10.3.5). This possibility is granted by the linearity of Equation 10.3.1, that can be reformulated as

$$\int_{t_1}^{t_2} A_{\text{ON}}(t)dt = (1 - \xi) \int_{t_1}^{t_2} A_{\text{OFF}}(t)dt \quad (10.3.2)$$

The proportionality parameter ξ could be measured from the inversion of Equation 10.3.2

$$\xi^{[t_1, t_2]} := 1 - \frac{\int_{t_1}^{t_2} A_{\text{ON}}(t)dt}{\int_{t_1}^{t_2} A_{\text{OFF}}(t)dt} \quad (10.3.3)$$

Equation 10.3.3 was used to measure the proportionality parameter ξ in two different time windows, named *control* and *signal* regions (see 10.3.4), for each couple of e^+ shots with/without lasers originated by the same detector in both photoionization and metastable runs. Under the proportionality hypothesis, the measured value of ξ should be found the same in both.

All the measured values of ξ in control ξ^c and signal ξ^s regions were averaged and statistically tested for compatibility. In principle, it was sufficient to find a single case of statistical incompatibility to prove the proportionality hypothesis wrong, as ξ - being independent of time under the no 2^3S formation hypothesis - has to be found equal for any choice of the windows.

As already mentioned in the experimental part (see 10.2.4), special care had to be taken to compare runs in close time proximity, to avoid introducing further (correlated) fluctuations due to the moderator ageing. The analysis procedure used here measured ξ every two adjacent runs in the time-sorted sequence with and without laser, that originated the plots shown in Figure 10.5 and Figure 10.6.

A throughout derivation of Equation 10.3.1, showing the approximations used to derive it, is found in a following paragraph (10.3.3). An initial value of ξ can be quantitatively estimated,

when the sole UV laser is used, to be $\xi_{\text{UV}} \simeq \eta_3 \cdot \eta_{\text{quench}} \approx 1\%$; when both UV and IR lasers are used, $\xi_{\text{UV+IR}} \simeq \eta_3 \cdot \eta_{\text{ion}} \approx 15\%$.

10.3.1 Details of the analysis procedure

The analysis procedure described here, based on the ξ parameter defined in Equation 10.3.3, was practically implemented as follows:

1. two samples of N spectra with/without laser were collected, A_{ON}^i and A_{OFF}^i ;
2. spectra were divided in two categories, one per detector, keeping shots with and without laser separated in two different lists;
3. each category was time ordered, so to compare couple of shots with/without laser collected with the same detector at instants as close as possible in time (as shown in Figure 10.5);
4. the ξ_i parameter was calculated per couple of shots using Equation 10.3.3 in both control and signal regions using

$$\xi_i^{[t_1, t_2]} = 1 - \frac{\int_{t_1}^{t_2} A_{\text{ON}}^i(t) dt}{\int_{t_1}^{t_2} A_{\text{OFF}}^i(t) dt}$$

5. the obtained values of ξ_i^c and ξ_i^s were put in two histograms to calculate their average values, $\langle \xi^c \rangle$ and $\langle \xi^s \rangle$, and errors of the average, σ_ξ^c and σ_ξ^s .

10.3.2 Insensitivity of ξ to moderator ageing

In presence of a time drift $\alpha(t)$ starting from $t = 0$, $\alpha(0) = 1$, on the scale of several minutes caused by moderator ageing (see Figure 10.5) that multiplies both sequences with/without laser, the integrals in the definition of ξ (Equation 10.3.3) are rewritten as

$$\begin{aligned} \int_{t_1}^{t_2} \alpha(t) A_{\text{OFF}}(t) dt &\simeq \alpha(t_i) \int_{t_1}^{t_2} A_{\text{OFF}}(t) dt \\ \int_{t_1}^{t_2} \alpha(t) A_{\text{ON}}(t) dt &\simeq \alpha(t_i + \delta t_i) \int_{t_1}^{t_2} A_{\text{ON}}(t) dt \end{aligned} \tag{10.3.4}$$

where t_i is the instant when the shot without laser is collected and δt_i is the accumulation time between it and its companion shot with laser. Substituting into the definition, ξ_i becomes

$$\xi_i^{[t_1, t_2]} = 1 - \frac{\alpha(t_i)}{\alpha(t_i + \delta t_i)} \cdot \frac{\int_{t_1}^{t_2} A_{\text{ON}}^i(t) dt}{\int_{t_1}^{t_2} A_{\text{OFF}}^i(t) dt} \tag{10.3.5}$$

The sensitivity to a signal drift is absorbed in the drift prefactor

$$D(t, \delta t) = \frac{\alpha(t_i)}{\alpha(t_i + \delta t_i)} \quad (10.3.6)$$

For the case of a drift caused by moderator ageing, the maximum measured drift is a $\sim 7\%$ loss per hour when the moderator is fresh. Since typical shot repetition times δt_i are of the order of a few minutes at maximum (see Figure 10.6), the approximation $\delta t_i \ll h$ is valid and the prefactor can be replaced by its series expansion in δt truncated to the first order:

$$\frac{\alpha(t_i)}{\alpha(t_i + \delta t_i)} \approx 1 - \frac{\alpha'(t_i)}{\alpha(t_i)} \delta t + o(\delta t^2) \quad (10.3.7)$$

Substituting the values for the worst case scenarios, when the moderator is new and the drift is at maximum, $\alpha'(0) = 0.07 \text{ h}^{-1}$, $\alpha(0) = 1$, and the two average values of δt obtained in the photoionization and metastable runs (shown in Figure 10.6), this prefactor is found

$$\begin{aligned} D_{\text{photo}} &= 100 - 0.41\% \\ D_{\text{meta}} &= 100 - 0.045\% \end{aligned} \quad (10.3.8)$$

In conclusion, the experimental methodology described in 10.2.4 and the analysis methodology based on Equation 10.3.3 is found effective in limiting the systematic effect caused by moderator ageing to better than 0.5% for the photoionization run and 0.05% for the metastable run.

10.3.3 Derivation of the proportionality relation

Calling $N_{\text{ON}}(t)$ the population of Ps atoms on 1^3S as a function of time when the laser is sent, $N_{\text{OFF}}(t)$ the population of atoms on 1^3S when the laser is blocked and ξ the probability for atoms on 1^3S to follow the excitation path $1^3S \rightarrow 3^3P \rightarrow 1^1S$, the comparison of the populations in the two cases can be written as

$$N_{\text{ON}}(t) = (1 - \xi) N_{\text{OFF}}(t) \quad (10.3.9)$$

where $t_0 = 0 \text{ ns}$ is set as the instant when Ps is formed, $t_L = 16 \text{ ns}$ is the instant at which the laser is optionally sent, and t is considered $\gg t_0$ to let the atoms spontaneously decay from 3^3P with a characteristic time $\tau_3 = 13 \text{ ns}$.

Both $N_{\text{ON}}(t)$ and $N_{\text{OFF}}(t)$, being populations on 1^3S , decay exponentially with $\tau_{\text{ortho}} = 142 \text{ ns}$

following

$$\frac{dN(t)}{dt} = -\frac{1}{\tau_{\text{ortho}}} N(t) \quad (10.3.10)$$

Substituting into the precedent relation and dividing both terms by $-\tau_{\text{ortho}}$ gives

$$\frac{dN_{\text{ON}}(t)}{dt} = (1 - \xi) \frac{dN_{\text{OFF}}(t)}{dt} \quad (10.3.11)$$

The number of atoms decaying in the unit time can be bound to the electric signal acquired from the digitizer as follows. The instantaneous electric signal $A(t)$ observed from one of the two detectors is proportional to the total energy deposited by the γ -rays interacting with the scintillating material in the unit time. Calling E_γ the average energy deposited per γ -ray produced by an o-Ps annihilation and $N_{\text{det}}(t)$ the number of γ -rays depositing their energy in the crystal, and assuming the detector under consideration to be operating in its linear regime with a V MeV^{-1} proportionality factor α , the instantaneous signal $A(t)$ (assuming instantaneous time response, which in reality is of $\sim 8 \text{ ns}$), is written as

$$A(t) = \alpha E_\gamma N_{\text{det}}(t) \quad (10.3.12)$$

$N_{\text{det}}(t)$ can be bound to the number of atoms annihilating in the unit time, $dN(t)/dt$, by taking into account the number of γ -rays emitted per disintegration (3, for o-Ps annihilations), the geometrical solid angle of the detector and the spatial distribution of the annihilations as a function of time, during the flight of the atoms. For the purpose of this discussion, all the geometrical dependency can be put inside a single efficiency term $\beta(t)$:

$$N_{\text{det}}(t) = 3 \cdot \frac{dN(t)}{dt} \cdot \beta(t) \quad (10.3.13)$$

Substituting Equation 10.3.12 and Equation 10.3.13 into Equation 10.3.11 one gets

$$\frac{1}{\beta_{\text{ON}}(t)} A_{\text{ON}}(t) = (1 - \xi) \frac{1}{\beta_{\text{OFF}}(t)} A_{\text{OFF}}(t) \quad (10.3.14)$$

The geometrical efficiency, in first approximation, does not depend by the presence or absence of the laser pulse. The absorption of a 205 nm photon causes a negligible momentum transfer in the transverse direction of

$$p_{\text{UV}} = \hbar k_{\text{UV}} = 3.2 \cdot 10^{-27} \text{ kg m s}^{-1} \quad (10.3.15)$$

whereas a Ps atom moving at $v_{\text{Ps}} \approx 10^5 \text{ m s}^{-1}$ in the transverse direction (see Table 4.2) has a total momentum

$$p_{\text{Ps}} = 2m_e v_{\text{Ps}} = 1.8 \cdot 10^{-25} \text{ kg m s}^{-1} \quad (10.3.16)$$

Thus, the trajectory shift induced by the presence of the laser can be neglected $\beta_{\text{ON}}(t) \approx \beta_{\text{OFF}}(t)$, and the formula simplified to

$$A_{\text{ON}}(t) = (1 - \xi) A_{\text{OFF}}(t) \quad (10.3.17)$$

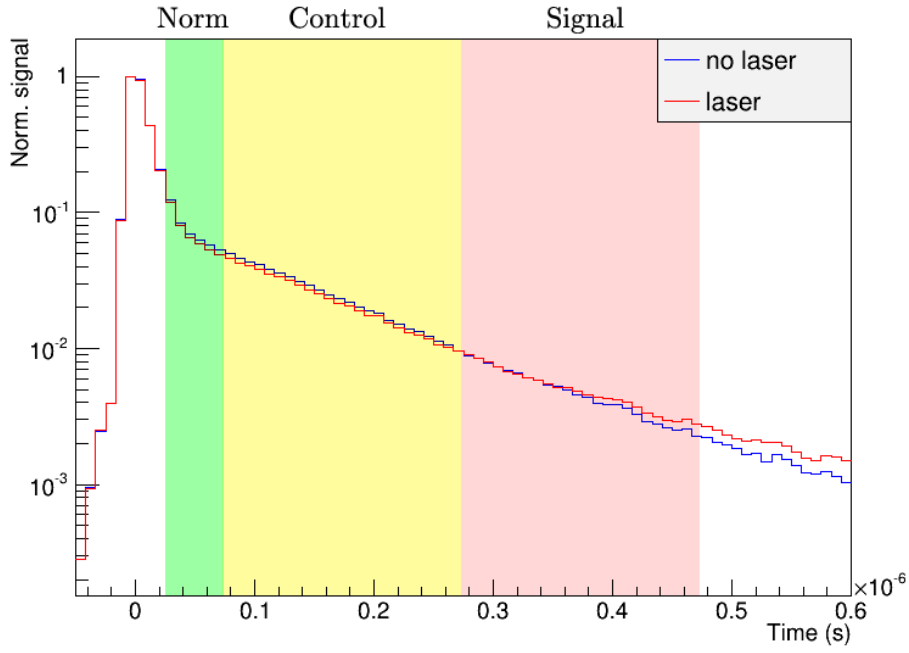
10.3.4 Definition of control and signal regions

The best candidates for control and signal regions were mutated from the $1 \rightarrow 3 \rightarrow \text{Ry}$ measurement campaign, whose average was already shown in Figure 4.24, panel (b). As control window, the region of the spectra in which Ps free-flight was observed was selected. As signal region, the subsequent part of the spectra where the delayed annihilations of long-lived Rydberg atoms were observed was used (see Figure 10.7, panel (a)).

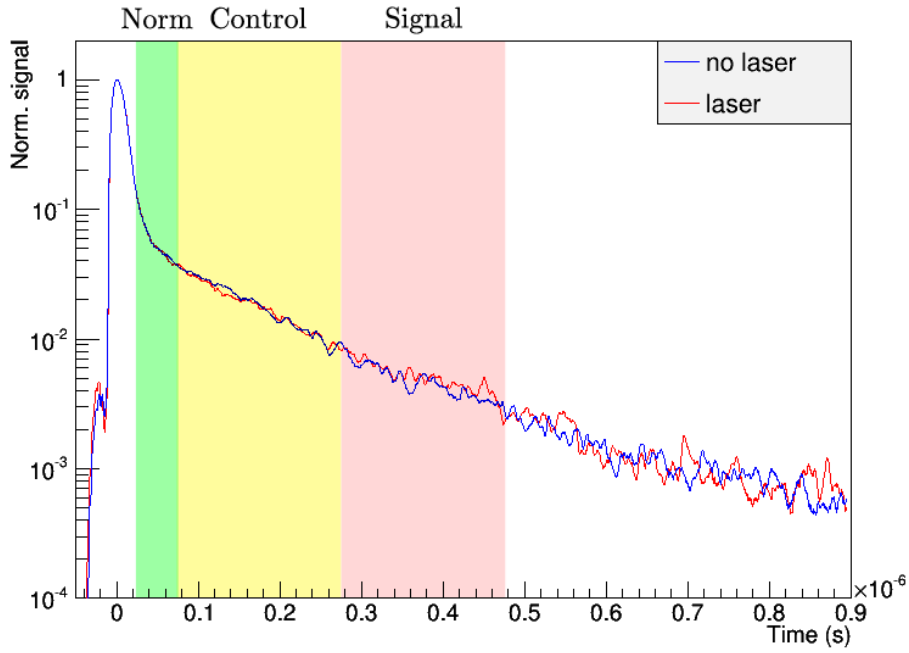
The windows start/end times, mutated from the old spectra, had to be adjusted according to the upgrades in the data acquisition chain which, for example, introduced a positive time shift of about 105 ns and varied the length of the acquisition. The spectra were thus first normalized and synchronized on the position of the prompt peak, and subsequently the windows properly defined (see Figure 10.7, panel (b) for the mapping of the windows defined on panel (a) on the new spectra).

An extra region, called *detector normalization* region, was defined in the portion of the spectra which was unused so far. The integral of this region was used to cross-calibrate the two PbWO_4 detectors, which observe Ps annihilations under two different solid angles. This cross-calibration procedure is - in principle - not necessary for the analysis using Equation 10.3.3, which only uses relative features extracted from spectra acquired by the same detector. It is useful, however, when absolute numbers, such as efficiencies, have to be estimated. Details of this cross-calibration procedure are given in the next paragraph (see 10.3.6).

The precise definitions of the normalization, control and signal regions are found in Table 10.4. All the values relative to the time at which the maximum of the prompt peak is observed, used as common $t = 0 \text{ ns}$ reference between old and new spectra.



(a) Definition of the norm/control/signal regions on the averaged SSPALS spectra of the $1 \rightarrow 3 \rightarrow \text{Ry}$ measurement run



(b) Mapping of the norm/control/signal regions on an example average of 30 SSPALS spectra acquired with the upgraded data acquisition system.

Figure 10.7: Definition of norm/control/signal regions on the averaged SSPALS spectra obtained during the $1 \rightarrow 3 \rightarrow \text{Ry}$ measurement campaign (panel (a), see 4.6) and mapping of the regions on the spectra obtained with the upgraded DAQ (panel (b)).

Window	Start	End	Length
Norm	25 ns	75 ns	50 ns
Control	75 ns	275 ns	200 ns
Signal	275 ns	475 ns	200 ns

Table 10.4: Definition of the detector normalization, control and signal time windows in SSPALS spectra.

10.3.5 Relation between ξ and S parameters

The analyses based on the ξ parameter (defined in Equation 10.3.3 and calculated using the procedure described in 10.3.1) and the S parameter defined in Equation 4.5.1 (used for the campaigns 4.5 and 4.6, and frequently found in Ps literature, for example [30]) are close relatives. The difference between the two, which has a profound impact on the results, lies in the sequence of operations performed, and specifically when the averaging step is performed.

The former analysis, based on S , was performed using the following procedure:

1. two samples of N spectra with/without laser were collected, A_{ON}^i and A_{OFF}^i ;
2. all spectra in the samples were normalized using a feature extracted from the spectra themselves, typically the maximum value of the prompt peak;
3. all spectra with and without laser were averaged together to calculate the two averaged spectra $\langle A_{\text{ON}} \rangle(t)$ and $\langle A_{\text{OFF}} \rangle(t)$;
4. the integrals over the time window of interest $t \in [t_1, t_2]$ were calculated on the averaged spectra;

$$f_{\text{ON}}^{[t_1, t_2]} = \int_{t_1}^{t_2} \langle A_{\text{ON}} \rangle(t) dt$$

$$f_{\text{OFF}}^{[t_1, t_2]} = \int_{t_1}^{t_2} \langle A_{\text{OFF}} \rangle(t) dt$$

5. the S parameter was calculated using Equation 4.5.1

$$S^{[t_1, t_2]} = \frac{f_{\text{OFF}}^{[t_1, t_2]} - f_{\text{ON}}^{[t_1, t_2]}}{f_{\text{OFF}}^{[t_1, t_2]}} = 1 - \frac{f_{\text{ON}}^{[t_1, t_2]}}{f_{\text{OFF}}^{[t_1, t_2]}}$$

The comparison between signals with/without laser is here performed *after* averaging. The argument of insensitivity to a time drift of the signals, discussed in 10.3.2, does not hold anymore. The drift $\alpha(t)$ is averaged together with the spectra, introducing further fluctuations in the final results from a correlated source of noise that could be avoided.

There are here two possible courses of action: either changing observable in favour of one insensitive to time drifts, such as ξ , or performing a de-trending operation of the data, estimating $\alpha(t)$ from the a-priori knowledge of the moderator growing instants and correcting the data accordingly. The first of these two options was preferred for this analysis in view of its simplicity.

10.3.6 Detectors cross-calibration

An a-posteriori cross-calibration of the two PbWO_4 was studied in order to be able to combine optimally spectra measured with both detectors for extracting absolute informations. This is a necessity imposed by the different solid angle under which the two observe o-Ps annihilations (see 10.2.1 and Figure 10.2).

The integrals in the normalization region of all the spectra acquired during both photoionization and metastable runs (for its definition, see 10.3.4 and Figure 10.7) were used to fit a linear $y = mx$ model and work out the geometrical correction factor m to compensate the solid angle difference. For this purpose, only the measurements without laser were used: results of this fitting procedure, shown in Figure 10.8 panel (a), gave:

$$m = 0.2931 \pm 0.0022_{\text{stat}} \quad (10.3.18)$$

This value was used to correct the data obtained from the second PbWO_4 . This correction was independently verified as follows. All signals obtained by the two detectors in the control region of the spectra without the laser, for both photoionization and metastable runs, were fitted as well with a linear $y = m'x$ model. An ideal calibration with infinite measurements would have yielded $m' = 1$; the found value was

$$m' = 1.0241 \pm 0.0073_{\text{stat}} \quad (10.3.19)$$

which is slight tension with the expected value of 1 with a 3σ difference. The small discrepancy underlines that the statistic used to verify the cross-calibration model was high enough to show a systematic effect not properly accounted by the simple $y = mx$ model chosen for cross-calibrating: this systematic can be an imperfect background subtraction of the second detector, or an imperfect joining of the low-resolution and high-resolution parts of the spectra.

It is important to remark that this calibration is not required for the ξ analysis presented in the following paragraphs, which rely only on the comparison of relative features extracted from couples of spectra from the same detector. A cross-calibration is on the contrary necessary for the performing S analysis, which requires normalizing the spectra before averaging. Cross-calibration is also necessary for working out absolute features from the spectra, such as Ps

formation probabilities.

10.4 Results and discussion

A total of 193 e^+ shots were collected during the photoionization run, and 179 during the metastable run (with π polarization). The averaged peak-normalized SSPALS spectra of these high statistics runs are shown in Figure 10.9 for a visual reference. The proportional decrease of $\sim 15\%$ of the photoionization run is clearly visible by eye (panel (a) of Figure 10.9), whilst in the metastable case, no relevant feature between the spectra with/without laser could be immediately identified by looking at the logarithmic signals.

The collected measurements were then analyzed using the procedure described above (see 10.3.1) to calculate the values of the proportionality parameter ξ in the control and signal regions. Figure 10.10 shows the histogram and the evolution as a function of the measurement number of the calculated value of ξ in the control and signal regions for both the photoionization and the metastable runs. No visible dependency on time correlated to the e^+ shot intensity fluctuation (shown in Figure 10.5) is observed as expected, consistent with the discussion in 10.3.2.

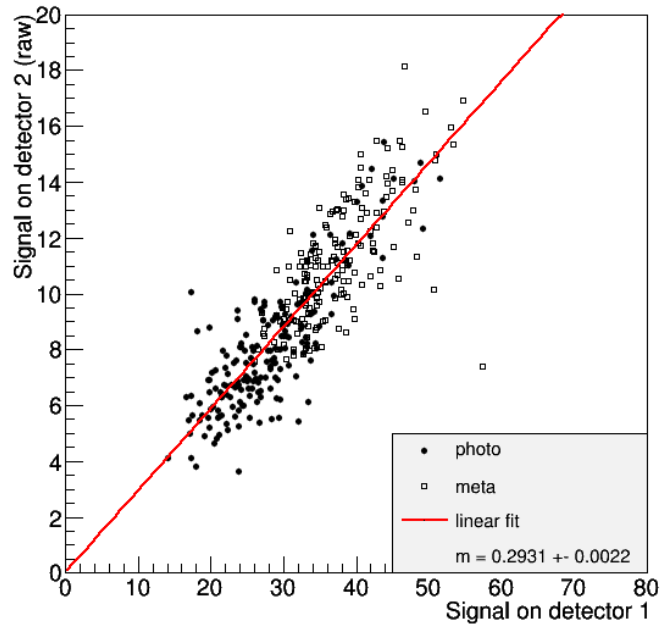
The final average values $\langle \xi_c \rangle$ and $\langle \xi_s \rangle$ for the two runs were obtained from the histograms shown in Figure 10.10. As statistical fluctuation, the error of the average was considered. The found values for the photoionization run are

$$\begin{aligned}\xi_{\text{photo}}^c &= 13.2 \pm 0.56_{\text{stat}}\% \\ \xi_{\text{photo}}^s &= 14.1 \pm 1.01_{\text{stat}}\%\end{aligned}\tag{10.4.1}$$

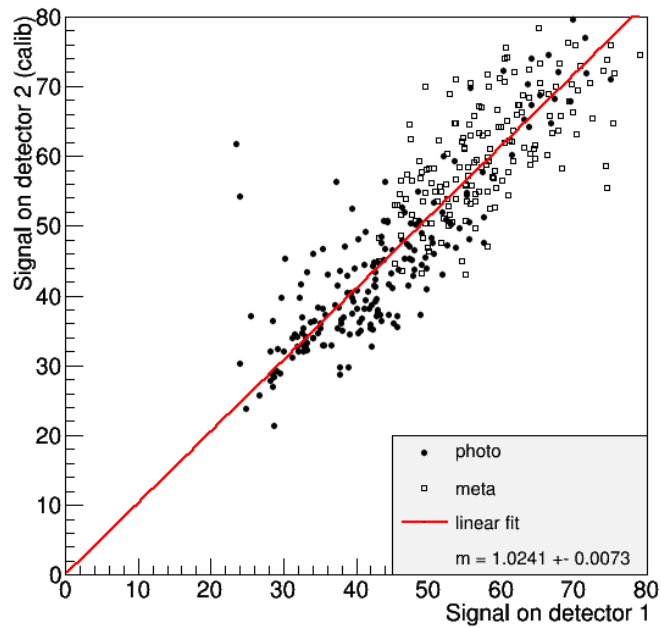
consistent among each other with a P-value of 44%, equivalent to 0.77σ . For the photoionization run, the proportional reduction hypothesis was found in agreement with the data as expected. Differently, the found values for the metastable run were

$$\begin{aligned}\xi_{\text{meta}}^c &= 1.81 \pm 0.54_{\text{stat}}\% \\ \xi_{\text{meta}}^s &= -2.22 \pm 1.14_{\text{stat}}\%\end{aligned}\tag{10.4.2}$$

which are inconsistent among each other with a P-value of 0.14%, equivalent to 3.2σ . Thus, the metastable run was found inconsistent with the proportionality (null) hypothesis formulated above to more than the 3σ level. The conclusion was that a small fraction of long-lived atoms was present, annihilating in the same time window of the SSPALS spectra where the annihilations from Rydberg atoms were already observed. The found results for the two high-statistics runs are visually compared in Figure 10.11.

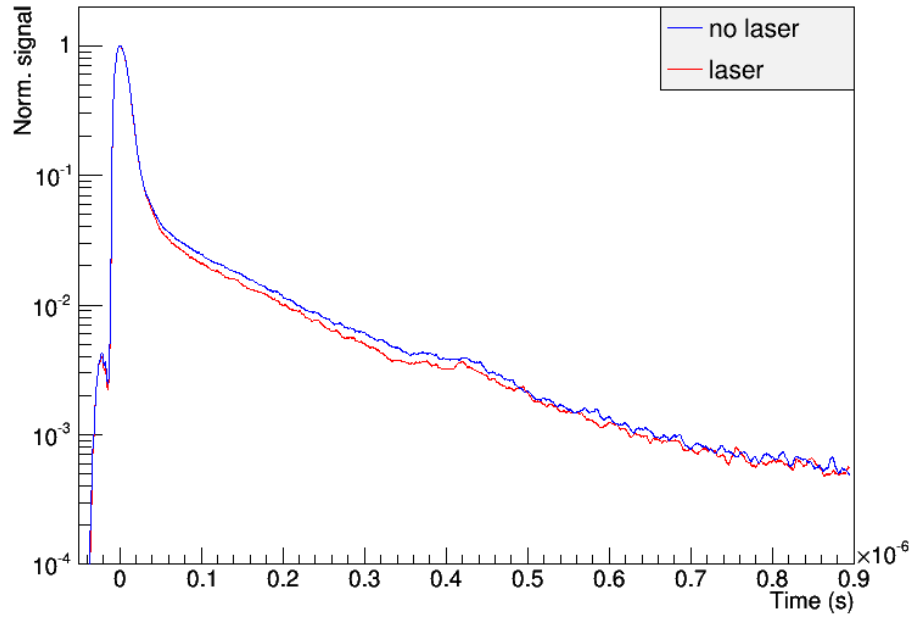


(a) Cross-calibration of the two detectors

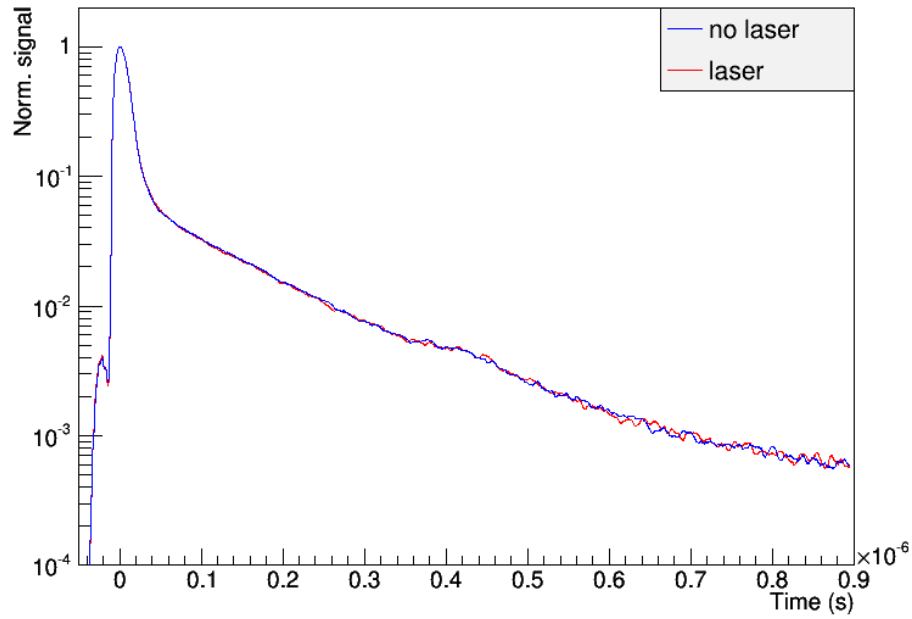


(b) Verification of detectors linearity after cross-calibration

Figure 10.8: Cross-calibration of the two PbWO_4 detectors. Panel (a) shows the cross-normalization data and the fitted linear model on used to work out the cross-calibration scale factor m . Panel (b) shows an independent comparison of signals obtained by the two detectors using the value of m from the calibration procedure.

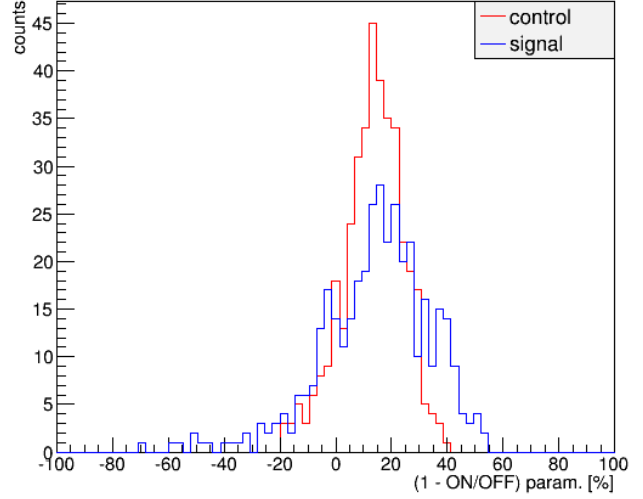
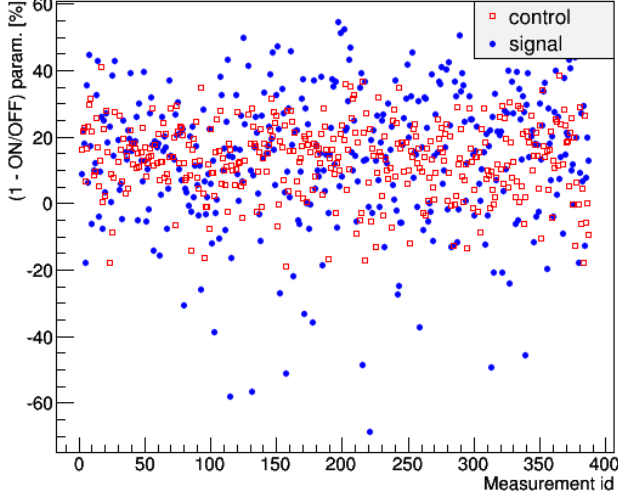


(a) Average of 386 photoionization (UV + IR) normalized SSPALS spectra

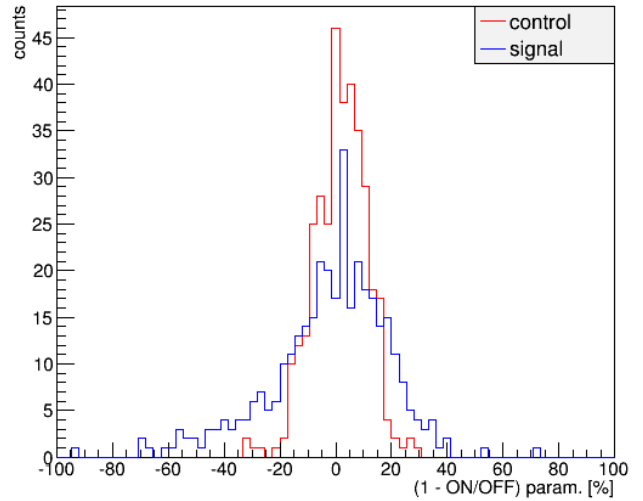
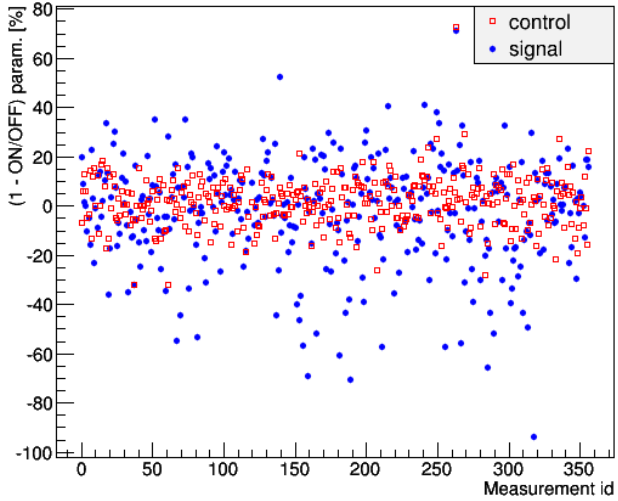


(b) Average of 358 metastable (UV only) normalized SSPALS spectra

Figure 10.9: Averaged normalized SSPALS spectra of photoionization runs (panel (a)) and metastable runs (panel (b)). Whilst the proportional signal decrease of 15% consequence of $1 \rightarrow 3$ excitation and photoionization is clearly identifiable, an excess of signal due to delayed 2^3S annihilations signal cannot be identified directly by eye.



(a) Evolution of the ξ parameter during the 386 photoionization (UV + IR) measurements



(b) Evolution of the ξ parameter during the 357 metastable (UV only) measurements

Figure 10.10: Calculation of the ξ parameter (see text) for the control (red) and signal (blue) regions, respectively for the photoionization run (panel (a)) and the metastable run (panel (b)). The evolution of ξ as a function of the measurement is shown on the left; its cumulative histograms on the right.

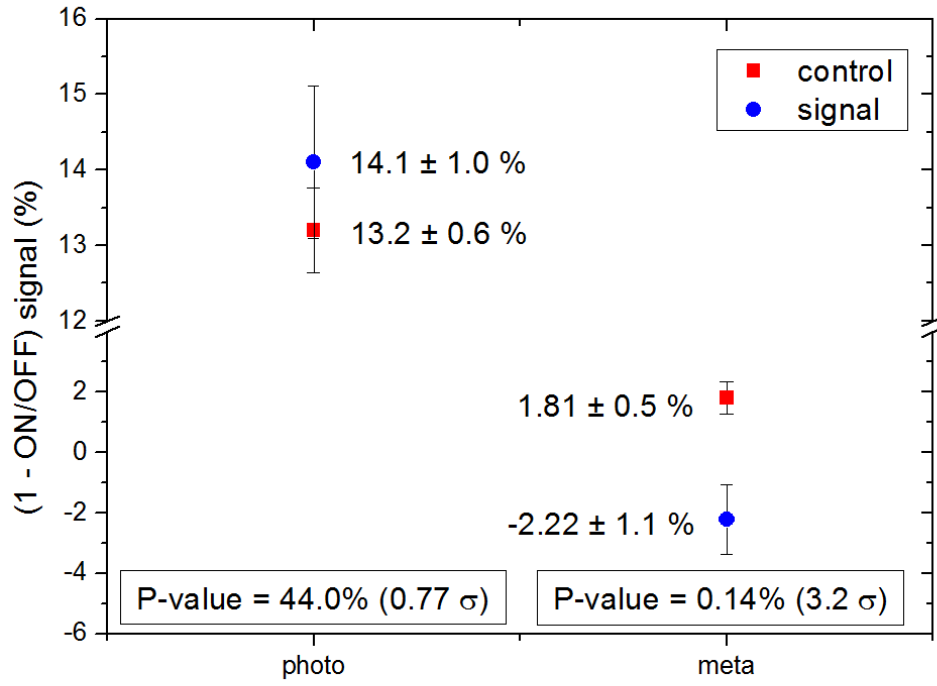


Figure 10.11: Statistical tests of the areas proportionality hypothesis for the control and signal regions, obtained using the data collected in the photoionization (left) and metastable (right) high-statistics runs. The areas within control and signal regions are found proportional in the photoionization case. On the contrary, the proportionality hypothesis is excluded in the metastable case to 3σ level.

The behaviour of the statistical significance was studied as a function of the position of signal region, to further verify that the analysis was not emphasizing a local maximum of statistical significance ². The signal region in Figure 10.7, panel (b), was rigidly shifted towards earlier times at expenses of the width of the control region, re-evaluating the analysis of the entire datasets at chosen position. The width of the control region was varied, whereas the width of the signal window was kept constant.

Results of this study for the photoionization and metastable runs are shown in Figure 10.12. The statistical significance to the proportionality hypothesis stays constantly close to zero for the photoionization measurements, for all choices of the signal window. Viceversa, the statistical significance of the metastable run is progressively but uniformly reduced, showing that the 3σ level found for the choice of signal region mutated from the Rydberg measurements is not a local excess, but grows progressively as the time is increased.

This found behaviour is again consistent with production of a long-lived fraction of Ps atoms. The more away from the prompt peak, the more relatively significant this long lived fraction becomes with respect to the remaining o-Ps fraction. Bringing this argument to the limit, after $1.0\ \mu\text{s}$, almost no remaining o-Ps fraction should be found, yet the metastable population would be only halved - accounting for almost 100 % of the observed signal.

Unfortunately, due to technical limitations of the apparatus used for these measurements, this striking evidence could not be obtained, due to the presence of obstacles close to the target production region. In a further upgrade of the experiment, it should be considered leaving more free space around the target region to let the o-Ps decay in flight and enhance the 2^3S signal.

10.5 Towards efficient production of 2^3S metastable positronium

As of today, the scientific interest in producing metastable 2^3S positronium in high amounts has been, curiously, very limited. A lot of experimental effort has been dedicated to efficiently produce and manipulate Ps on long-lived Rydberg levels, where annihilation is almost entirely suppressed by the size of the orbital and the atoms can be easily manipulated by electric field gradients.

The Rydberg Ps research line was motivated, besides pure scientific interest and $\bar{\text{H}}$ production via charge-exchange (see 2.2), by the proposed gravity measurement on Rydberg positronium by Mills and Leventhal in a seminal paper on the 2002 special issue of NIMB on positronium [57]. This paper was followed by many works towards gravity on Ps, first by the former Mills-Cassidy

²as the choice of the signal regions was not arbitrary, but decided on an independent set of data (as described in 10.3.4), possible reductions of the statistical significance from the *look-elsewhere effect* were not considered.

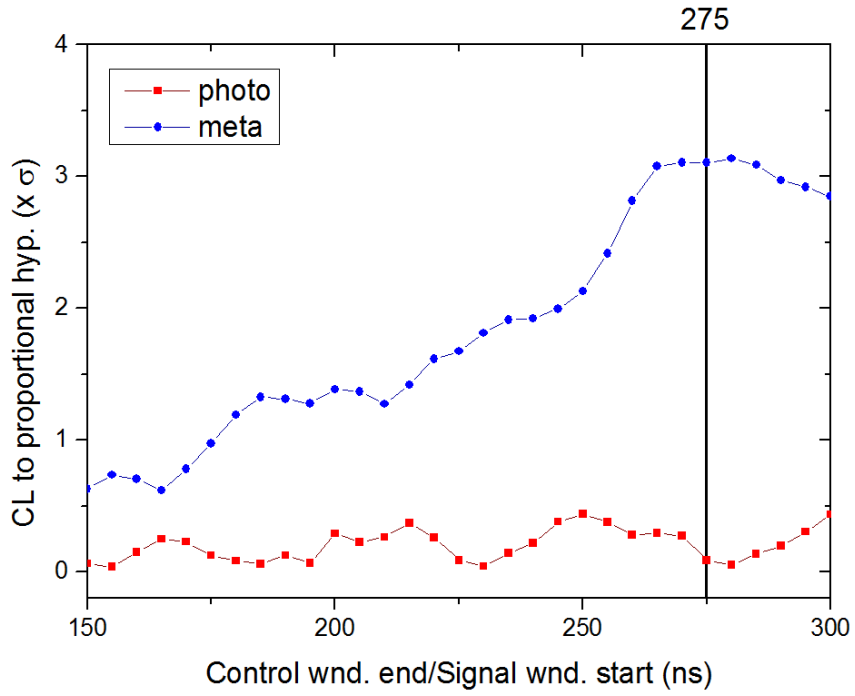


Figure 10.12: Behaviour of the statistical significance of the proportionality hypothesis for the photoionization (red) and metastable (blue) runs as a function of the choice of the signal window (varying the end of the control region/start of the signal region). The value marked with a solid vertical line corresponds to the 275 ns value mutuanted from the averaged SSPALS spectra from the $1 \rightarrow 3 \rightarrow \text{Ry}$ measurement (see Figure 10.7, panel (a)).

group [46, 30, 47], now by the current Cassidy group at UCL [59, 31].

Curiously, this very prolific research line obscured completely another proposal of gravity measurement on Ps, made by M. Oberthaler on the same special issue of NIMB, using an atom interferometer and metastable 2^3S positronium [58]. The outcome of Oberthaler's study was that an atom interferometer based on Bragg scattering, using the $2^3S \rightarrow 3^3P$ transition of Ps, could yield an inertial sensitivity of a fraction of Earth's g with an incident flux of 10^6 Ps/s for a measurement lasting one day. According to Fig. 3 in [58], the optimal interaction time would be of $2.2\mu\text{s}$ which, for a Ps moving at 10^5 m s^{-1} , translates to a distance of $\sim 20\text{ cm}$ - within reach in terms of mechanical feasibility.

Metastable 2^3S positronium is thus a recognized alternative tool for measuring gravity on positronium. The questions if metastable 2^3S could potentially be as powerful as Rydberg positronium in measuring gravity raises. There are a few pros and cons for the two choices:

1. **Pro-metastable - sensitivity to external fields:** 2^3S , being on the S-wave of a low- n manifold, is almost insensitive to external electric and magnetic fields. This does not happen to Rydberg states, which are both very sensitive to electric field gradients *and* very sensitive to magnetic fields due to the motional Stark effect, prerogative of positronium. If one wants to use positronium on Rydberg levels for a gravity measurement, external fields have to be cancelled or very well controlled.
2. **Pro-metastable - positron transport:** The sensitivity to external fields of Ps on Rydberg levels either impose tight constraints on the method used to transport and implant e^+ or requires dedicated efforts to transport Rydberg atoms far away from the target (as demonstrated by [174]). Magnetic transport should be clearly avoided for motional Stark effect considerations; transport electric fields should be switched off with fast switches as Ps is formed, to avoid introducing unwilling electrical deflections to Rydberg atoms. 2^3S , on the other hand, being very weakly affected by both magnetic and electric fields, could potentially allow the interferometer to be placed very close to the production region.
3. **Pro-Rydberg - optical lifetime:** positronium lifetime on Rydberg states can be extremely long, even milliseconds. Yet, the lifetime of Rydberg states that could be practically obtained via two-step laser excitation (used both in the present study and [30]) is not as long. The optical lifetime of $n = 20$ with $l = 0$ is $28.8\mu\text{s}$, the one of $n = 20$ with $l = 0$ $8.4\mu\text{s}$, which is about a factor of 20 more than 2^3S annihilation time.
4. **Pro-Rydberg - control with fields:** Reversing the argument used above, Ps on Rydberg states can be controlled using gradients of electric fields [174], possibility unavailable for 2^3S .

5. **Pro-Rydberg - higher excitation efficiency:** excitation to Rydberg states is 100% efficient with the use of a broadband laser - i.e. populating an high amount of sublevels (see [30]). A realistic metastable excitation efficiency of an upgraded version of the present experiment could be of to 10% (see below).
6. **Pro-metastable - cleanliness:** atoms on the metastable level either spontaneously annihilate or stay on three 2^3S sublevels, without any momentum transfer due to spontaneous optical decay. This doesn't happen to Rydberg atoms (even if a narrow excitation path is preferred to broadband excitation), as they can spontaneously decay following a huge amount of optical paths.

The question that rises is then, is it feasible to produce order $\propto 10^6$ Ps atoms on the 2^3S state per shot, with minor modifications to the present setup?

One's answer to this question would be affirmative. Shining both 205.045 nm and an extra 1312.5 nm laser on the 1^3P Ps sample would stimulate the optical decay from 3^3P to 2^3S , thus effectively creating a three levels system pumped with laser light. In first approximation, if the transitions are closed (i.e. no magnetic field present, possible thanks to the electrostatic transport 4.2), the population of the three sublevels would equally distribute $1/3$ $1/3$ $1/3$.

The 2^3S excitation efficiency in this scenario could be estimated from the current 15% 3^3P excitation efficiency in the two levels case, assumed to be in saturation (see Figure 4.22, panel (a)). The 15 % efficiency, divided by the 50 % population fraction for a saturated two levels system and multiplied by the 33 % population fraction for a saturated three levels system, becomes 10 %.

It would probably be enough to introduce a 1312.5 nm laser with enough power to be in saturation to populate 2^3S with $\sim 10\%$ efficiency. With an initial shot of $3 \cdot 10^7$ e^+ , an o-Ps production efficiency of 30 % (see Table 4.1) and a 10% excitation efficiency, 10^6 atoms in the 2^3S state would be obtained.

A simple and effective way to build a 1312.5 nm laser and test efficient 2^3S production of 10^6 atoms per shot, would be to replace the PPKTP crystal in the Rydberg laser line (described in 4.4.4) in favour of one able to synthesize 1312.5 nm laser light, and adjust the OPA consequently. One should go for it.

Conclusions

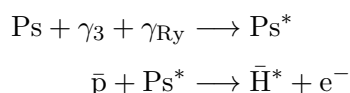
The direct observation of an antimatter body free-fall in a known gravitational field would lead to the first verification of one of the cornerstones of General Relativity with antimatter: the Equivalence Principle (in its weak form). Any deviation from the expected identical behaviour with respect to normal matter would be a indication of new physics.

The antimatter research community has been aiming to observe the gravitational free-fall of antimatter bodies in Earth's gravitational field for long time, since the early experiments on charged antiparticles (respectively, positrons and antiprotons) by Witteborn and Fairbanks and the PS200 collaboration at CERN's Low Energy Antiproton Ring. These were followed, at the present time, by a new generation of experiments aiming to gravity on cold neutral antimatter.

Producing cold neutral antimatter in a suitable way for attempting a first gravity measurement is now within reach. There are two promising candidate systems for such an attempt: antihydrogen (after its milestone production at CERN's Antiproton Decelerator in early 2000) and positronium (after the recent development of efficient matter converters able to synthesize cold Ps from e^+ with high efficiency).

Each of those two systems shows advantages and disadvantages, the former being more massive and less sensitive to external fields, the latter being purely leptonic and easily produced in quantities at low temperatures without the need of a sophisticated particle decelerator.

During the course of this thesis, the possibility to aim at the first of those two systems, antihydrogen, was explored. Most part of the experimental work was dedicated towards pulsed charge-exchange production of $\bar{H}$ according the strategy of the AEGIS experiment, in whose context this thesis was developed:



Several original results were achieved. Positronium was produced in a dedicated setup, where laser spectroscopy of its $n = 3$ level was conducted, as well as its excitation to Rydberg levels.

Positron plasmas were subsequently transferred to the main experiment, leading to the first formation of positronium from the cryogenic target installed in the $\bar{H}$ production region. In parallel, antiprotons were captured, sympathetically cooled with electrons, compressed to mm radii using rotating-wall, transferred and accumulated in the final $\bar{H}$ production trap. These results will lead, in a near future, to a first attempt of pulsed charge-exchange $\bar{H}$ formation. An estimate showed that 3 – 40 antihydrogen atoms per e^+ shot could be formed in the present setup.

This bulk of experimental work motivated the development of two original theoretical numerical models. A numerical code for calculating thermal equilibria of single-species non-neutral plasmas in arbitrary cylindrical traps was built around a novel technique for solving Poisson equation in cylindrical coordinates. Thanks to it, the computational complexity of the iterative calculation of plasma equilibria dropped from $\propto T^{-2}$ found in literature to $\propto T^{-1}$, leading to an accurate modelling of single-species plasma equilibria down to temperatures $\sim K$ in a reasonable computing time. As a future development, this code can be upgraded to the calculation of two-species plasma equilibria in a straightforward way.

The problem of calculating non-perturbatively positronium energy levels in arbitrary electric and magnetic fields was also addressed. A code, named PsTC, based on the numerical diagonalization of the full-interaction Hamiltonian of Ps in fields was developed, leading to the non-perturbative calculation of atomic levels and optical transition probabilities of positronium in arbitrary external fields. This model correctly reproduced both the measured optical spectra of Rydberg positronium presented in this thesis, and the results of more accurate spectroscopical surveys of Ps on Rydberg levels found in literature. It also lead to formulate the hypothesis that both radial and transverse positronium velocity components (with respect to the B field) could be measured at the same time using the combined measurement of $n = 3$ Doppler broadening and Rydberg levels motional Stark broadenind in a strong B field. As a future development, this code could be extended with minor modifications to calculate the optical spectra of hydrogen and antihydrogen in arbitrary external fields, and to other hydrogenoid atoms as well.

The combined availability of an intense positronium source (thanks to the choice of the $\bar{H}$ formation mechanism based on charge-exchange) and an accurate theoretical model of pulsed-laser/positronium interactions were, in a serendipitous way, the main causes beyond a result towards a first gravity measurement with positronium as well. The 2^3S metastable state of positronium was observed after spontaneous optical decay of atoms laser-excited to $n = 3$. This system has been proposed as an alternative to long-lived Rydberg states for observing gravity with Ps using an atom interferometer. The required source intensity for such an application could be obtained in the present AEGIS setup by adding a further laser pulse at 1312 nm, stimulating the decay transition from $n = 3$. This laser light could be simply synthesized using the present

laser apparatus of AEGIS, with a minor modification of the optical setup.

As future perspectives, there are many interesting items of research within reach in AEGIS during the next years. Performing positronium Rydberg levels spectroscopy in a strong 1T field could lead to a viable technique for measuring contemporaneously Ps radial and transverse velocities, using Doppler and motional Stark broadenings. Showing pulsed $\bar{H}$ production via charge-exchange with positronium would be both a significant milestone towards a first direct measurement of gravity with antihydrogen and a proof of concept for future experiments. Demonstrating the feasibility of an intense metastable positronium source would open up several possibilities in further optical manipulations motivated from the cold atoms community, and make a step towards a future gravity measurement with positronium using an atom interferometer.

Whichever the way, measuring gravity on neutral antimatter will happen in the next years, for the time is ripe.

Acknowledgments

There were many people that contributed - directly or indirectly - to this work. I feel I have to thank almost every present and past member of AĒIS for something; it's hard to keep track of everything that happened in almost five years of work, counting also the first two years of master thesis which laid the ground to the Ph.D. So, I wish to start my acknowledgments by thanking all the people in AĒIS, with whom I had the honor to work aside (or ski together): **S. Aghion, A. Belov, G. Bonomi, P. Bräunig, R. S. Brusa, M. Caccia, C. Canali, F. Castelli, G. Cerchiari, S. Cialdi, D. Comparat, G. Consolati, L. Dassa, A. Demetrio, L. Di Noto, M. Doser, C. Evans, R. Ferragut, J. Fesel, A. Fontana, O. K. Forslund, S. Gerber, M. Giammarchi, F. Guatieri, S. Haider, I. L. Jernelv, A. Kellerbauer, O. Khalidova, D. Krasnický, V. Lagormarsino, C. Malbrunot, S. Mariazzi, L. Marx, Z. Mazzotta, F. Moia, G. Nebbia, N. Pacifico, D. Pagano, L. Penasa, F. Prelz, M. Prevedelli, L. Resch, B. Rienäcker, A. Rotondi, L. Smestad, G. Sobrero, F. Sorrentino, M. Subieta Vasquez, I. Tietje, G. Testera, P. Yzombard and N. Zurlo.**

Now, it is time for some special thanks.

It's kind of obvious to thank your own tutor for all the help and support given throughout the years. I think one should not thank someone for what it's his (or her, in my case) duty to do, yet the first person I wish to thank is indeed **Gemma Testera**. Gemma has an incredible passion for this work which she trasmits to the others. Together with her always encouraging attitude - making anyone feel free to speak at her same level - she went far above the bare duties of tutor a countless number of times.

I have to thank **Daniel Krasnický** in many respects. First of all, for continuously stimulating me in getting the most out of my ideas. Secondly, for his incredible attention in giving credit to other people contributions, a quality that I always admired. Working together side by side (and phone by phone), throughout those years, was really a pleasure: we surely have made ourselves hated (and I believe we still do) by most inhabitants of L202 and bld. 93, for how much we use

to talk.

With **Sebastiano Mariazzi**, we shared an apartment together, a lot of transpiration and many great moments: I won't forget the first time we saw a Ps-laser interaction signal. I won't forget as well living for a week in the middle of the winter in the super-civilized Switzerland lighted only by candles and warmed with boiling water. I won't forget the breakfast table discussions in the morning too, the long chitchats on almost any topic and "a noni, mo viene zia no?!".

When we were all struggling after laser and positronium not very willing to cooperate, it was **Giancarlo Nebbia** that stood aside a young Ph.D student (me) and his half shot-in-the-dark idea of changing experimental strategy. He was a constant source of motivation for the following year, during the long and painstaking publishing of the article and beyond. If it wasn't for Giancarlo, who put some meals on the table and some serious fire under my back at the right moment, we probably wouldn't have managed to observe metastable positronium.

I tried my best to make **Michael Doser** kill me after my many bureaucratic disasters, yet he kept sustaining me with financial and personal support during all these years, starting with a six months contract just after the graduation and continuing with a CERN Doctoral position (and even more). I could have never managed to stay so much time working hard on AEgIS without his support, motivation and flexibility.

A special and transversal thank goes to **Stefan Haider**, for always being the judicious hand where the brain failed. And for saving me from jail in one or two occasions. If I ended the Ph.D. with some serious knowledge in analog and radiofrequency electronics, it's **Vittorio Lagormarsino** the sole responsible: thanks for always finding the time for sharing your immense passion for it.

I wish to give a special thank to **Stefano Mazzoleni** and his group, **Francesco Giannino** and **Fabrizio Cartenì** at the Dipartimento di Agraria, Università degli Studi di Napoli for teaching me agent-based modelling, making me discover Simile[©] and constantly flooding my brain with new ideas: if it weren't for all the discussions we had, PsTC would not exist.

I also wish to thank **Alberto Alonso** and **David Cassidy**, who kindly shared the data published in [31] for the comparison of the results of PsTC. With the very stimulating discussions we had in Lyon, **Markus Oberthaler** was the inspirer of our first observation of positronium metastable state. **Nicola Zurlo** kindly shared with me part of his computing time on the CINECA HPC clusters. **Germano Bonomi** co-authored the (now official) analysis code of AEgIS that is beyond many plots in this thesis, to which now also **Francesco Guatieri** is contributing, after being the mind behind the data acquisition of the experiments with positronium. It has been a pleasure to discuss with **Andrea Demetrio** in these years about the many

difficulties of a first gravity measurement. To **Pauline Yzombard**, **Zeudi Mazzotta**, and **Giovanni Cerchiari** I owe a lot for all their help - distributed over many years - with the laser system for positronium excitation and spectroscopy (other than being nice friends and tolerating me under stress). For all the thesis-related physics discussions we had in those years in L202, I wish to thank **Luca Pagani**, **Simone Copello** and **Laura Marini**.

Finally, I wish to thank all who put an effort reviewing this work. First of all, thanks to **Daniel Comparat** and **Eberhard Widmann** for kindly accepting being external reviewers. Thanks to **Daniel Krasnický**, **Gemma Testera** and **Sebastiano Mariazzi** for your precious inputs and suggestions, which helped significantly improving the work, and to **Fabrizio Castelli** for an accurate reviewing of the theoretical work on Ps in only one day.

C'è una lunga lista di amici vicini e lontani (in senso geografico) con cui ho trascorso del tempo fantastico in questi anni. A Genova, un grande abbraccio al fantastico di gruppo con cui abbiamo trascorso così tanti gran momenti insieme: Luchino, Lauretta, Coppe, Fede, Chiaretta, il Pres, SimoMarco, Lollo, Marzia, Ruben, Sivo, Cate, Fra, Volpe, Paolo, Gulli e Matte. Un grazie speciale va ad Ilaria, per un aiuto fondamentale in una situazione difficile. A Milano e Ginevra, un grazie alla vecchia guardia, Andre e Ari, Boes (che un grazie in più per aver messo su una casa insieme se lo merita tutto) e Ila, Cerotti e lo spirito santo, Luca e Gemma, Giacomo e Chiaretta, Giò e Miranda, Giacomino e Giuli. Un abbraccio a Ele, a cui ancora devo il 90 % del mio guardaroba. E poi, un grazie enorme per tutto il vostro infinito sostegno va a tutti i Mat's, in particolare Pier, Tina, Carlo e Silvia.

A tutti quelli che ci hanno sostenuto in un momento molto, molto, difficile, grazie di cuore. In particolare, Giusi e Cosimino; Giusi e Simona; Pippo, Graziella e Alice; doc Azzerboni; Laura e Luciana; Maurizio e Daniela; Dani e Sissa; Andrea (a cui va anche il merito di avermi fatto scoprire le radio software, che ho usato su AEgIS), Vittoria ed Enrico, Luchino ed Ale.

E per finire, un grazie alle tre persone che mi hanno dato costante supporto in tutti questi lunghi mesi di duro lavoro: Fio (che mi ha sopportato), Beppe (che ha tenuto duro) e Ale (che non è scappata a gambe levate).

Bibliography

- [1] E. G. Adelberger, J. H. Gundlach, B. R. Heckel, S. Hoedl, and S. Schlamminger. Torsion balance experiments: A low-energy frontier of particle physics. *Prog. Part. Nucl. Phys*, (62):102–134, 2009.
- [2] C. M. Will. The confrontation between general relativity and experiment. *Living Review of Relativity*, (17):4, 2014.
- [3] T. A. Wagner, S. Schlamminger, J. H. Gundlach, and E. G. Adelberger. Torsion-balance tests of the weak equivalence principle. *Classical and Quantum Gravity*, (29):184002, 2012.
- [4] J. G. Williams, S. G. Turyshev, and D. H. Boggs. Progress in lunar laser ranging tests of relativistic gravity. *Phys. Rev. Lett.*, (93):261101, 2004.
- [5] L. Zhou, S. Long, et al. Test of Equivalence Principle at 10^{-8} level by a dual-species double-diffraction Raman atom interferometer. *Phys. Rev. Lett.*, (115):013004, 2015.
- [6] M. G. Tarallo, T. Mazzoni, N. Poli, D. V. Sutyryn, X. Zhang, and G. M. Tino. Test of Einstein Equivalence Principle for 0-spin and half-integer-spin atoms: Search for spin-gravity coupling effects. *Phys. Rev. Lett.*, (113):023005, 2014.
- [7] J. Williams, S. Chiow, and H. Yu, M. adn Müller. Quantum test of the equivalence principle and space-time aboard the International Space Station. *New Journal of Physics*, (18):025018, 2016.
- [8] Alex Sugarbaker. *Atom interferometry in a 10m fountain*. PhD thesis, Stanford University, 8 2014.
- [9] ALPHA and A. E. Charman. Description and first application of a new technique to measure the gravitational mass of antihydrogen. *Nature Communications*, 4:1785, 2013.
- [10] F. C. Witteborn and W. M. Fairbank. Experimental comparison of the gravitational force on freely falling electrons and metallic electrons. *Phys. Rev. Lett.*, (19):1049, 1967.

- [11] F. C. Witteborn and W. M. Fairbank. Experiments to determine the force of gravity on single electrons and positrons. *Nature*, (220):436, 1968.
- [12] C. Herring and M. H. Nichols. Thermoionic emission. *Rev. Mod. Phys.*, (21):185, 1949.
- [13] L. I. Schiff and M. V. Barnhill. Gravitation-induced electric field near a metal. *Phys. Rev.*, (151):1067, 1966.
- [14] Michael Holzcheiter. Gravitational measurements on charged particles: can it be done? talk at the 3rd Workshop on Antimatter and Gravity, London (UK), 2015.
- [15] The PS200 collaboration. A measurement of the gravitational acceleration of the antiproton. *CERN/PSCC/86-2*, page PSCC/P94, 1986.
- [16] T. W. Darling, F. Rossi, G. I. Opat, and G. F. Moorhead. The fall of charged particles under gravity: A study of experimental problems. *Rev. Mod. Phys.*, 64(1):237–257, 1992.
- [17] Michael Holzcheiter. Test of the weak equivalence principle from particle-antiparticle cyclotron frequencies. talk at the 2nd Workshop on Antimatter and Gravity, Bern (CH), 2013.
- [18] G. Gabrielse et al. Precision mass spectroscopy of the antiproton and proton using simultaneously trapped particles. *Phys. Rev. Lett.*, 82(3198), 1999.
- [19] R. J. Hughes and M. H. Holzcheiter. Constraints on the gravitational properties of antiprotons and positrons from cyclotron-frequency measurements. *Phys. Rev. Lett.*, 66(854), 1991.
- [20] M. Ahmadi et al. Observation of the 1s-2s transition in trapped antihydrogen. *Nature*, 0(0), 2016.
- [21] K. Hirata et al. Observation of a neutrino burst from the supernova SN1987A. *Phys. Rev. Lett.*, (58):1490, 1987.
- [22] S. Pakvasa, W. A. Simmons, and T. J. Weiler. Test of equivalence principle for neutrinos and antineutrinos. *Phys. Rev. D*, (39):1761, 1989.
- [23] C. S. Unnikrishnan and G. T. Gillies. Falling right while moving slow: true tests of the weak equivalence principle for antiparticles. *Int. J. Mod. Phys. D*, (21):1242016, 2012.
- [24] G. B. Andersen et al. Confinement of antihydrogen for 1000 seconds. *Nature Physics*, 7:558–564, 2011.
- [25] M. Amoretti et al. Production and detection of cold antihydrogen atoms. *Nature*, 419(6906):456–459, 2002.

-
- [26] G. Gabrielse et al. Background-free observation of cold antihydrogen with field-ionization analysis of its states. *Phys. Rev. Lett.*, 89(213401), 2002.
- [27] C. H. Storry et al. First laser-controlled antihydrogen production. *Phys. Rev. Lett.*, 93(263401), 2004.
- [28] N. Kuroda et al. A source of antihydrogen for in-flight hyperfine spectroscopy. *Nature Communications*, (5):3089, 2014.
- [29] G. Gabrielse et al. Adiabatic cooling of antiprotons. *Phys. Rev. Lett.*, 106(073002), 2011.
- [30] D. B. Cassidy, T. H. Hisakado, H. W. K. Tom, and A. P. jr Mills. Efficient production of Rydberg positronium. *Phys. Rev. Lett.*, (108):043401, 2012.
- [31] T. E. Wall, A. M. Alonso, B. S. Cooper, A. Deller, S. D. Hogan, and D. B. Cassidy. Selective production of Rydberg-Stark states of positronium. *Phys. Rev. Lett.*, (114):173001, 2015.
- [32] S. Aghion et al. Laser excitation of the $n = 3$ level of positronium for antihydrogen production. *Phys. Rev. A*, 012507(94), 2016.
- [33] Paolo Crivelli. Testing antigravity with positronium 1s-2s spectroscopy. talk at the 2nd Workshop on Antimatter and Gravity, Bern (CH), 2013.
- [34] T. J. Murphy and C. M. Surko. Positron trapping in an electrostatic well by inelastic collisions with nitrogen molecules. *Phys. Rev. A*, 46:5696, 1992.
- [35] J. R. Danielson, D. H. E. Dubin, R. G. Greaves, and C. M. Surko. Plasma and trap-based techniques for science with positrons. *Rev. Mod. Phys.*, 87:247–306, 2015.
- [36] H. Saito, Y. Nagashima, T. Hyodo, and T. Chang. Detection of paramagnetic centers on a-SiO₂ grain surfaces using positronium. *Phys. Rev. B*, (52):R689, 1995.
- [37] D. B. Cassidy, K. T. Yokoyama, S. H. M. Deng, D. L. Griscom, H. Miyadera, H. W. K. Tom, C. M. Varma, and A. P. jr Mills. Positronium as a probe of transient paramagnetic centers in a-SiO₂. *Phys. Rev. B*, 75(8):085415, 2007.
- [38] S. Mariazzi, A. Salemi, and R. S. Brusa. Positronium cooling into nanopores and nanochannels by phonon scattering. *Phys. Rev. B*, 78(8), 2008.
- [39] S. Mariazzi, P. Bettoti, S. Larcheri, L. Toniutti, and R. S. Brusa. High positronium yield and emission into the vacuum from oxidized tunable nanochannels in silicon. *Phys. Rev. B*, 81:235418, 2010.
- [40] S. Mariazzi, P. Bettoti, and R. S. Brusa. Positronium cooling and emission in vacuum from nanochannels at cryogenic temperature. *Phys. Rev. Lett.*, 104:243401, 2010.

- [41] G. Consolati, R. Ferragut, A. Galarneau, F. Di Renzo, and F. Quasso. *Chem. Soc. Rev.*, 2011.
- [42] S. Chu, A. P. jr Mills, and J. L. Hall. Measurement of the positronium 1s-2s interval by Doppler-free two-photon spectroscopy. *Phys. Rev. Lett.*, (52):1689, 1984.
- [43] M. S. Fee, A. P. jr Mills, S. Chu, D. M., E. D. Shaw, K. Danzmann, R. J. Chichester, and D. M. Zuckerman. Measurement of the positronium 1s-2s interval by continuous-wave two-photon excitation. *Phys. Rev. Lett.*, 70(10), 1993.
- [44] M. S. Fee, S. Chu, A. P. jr Mills, R. J. Chichester, D. M. Zuckerman, E. D. Shaw, and K. Danzmann. Measurement of the positronium 1s-2s interval by continuous-wave two-photon excitation. *Phys. Rev. A*, 192(48), 1993.
- [45] F. Castelli, I. Boscolo, S. Cialdi, M. D. Giammarchi, and D. Comparat. Efficient positronium laser excitation for antihydrogen production in a magnetic field. *Phys. Rev. A*, 78:052512, 2008.
- [46] D. B. Cassidy, T. H. Hisakado, H. W. K. Tom, and A. P. jr Mills. Laser excitation of positronium in the Paschen-Back regime. *Phys. Rev. Lett.*, (106):173401, 2011.
- [47] D. B. Cassidy, T. H. Hisakado, H. W. K. Tom, and A. P. jr Mills. Positronium hyperfine interval measured via saturated absorption spectroscopy. *Phys. Rev. Lett.*, (109):073401, 2012.
- [48] C. I. Westbrook, D. W. Gidley, R. S. Conti, and A. Rich. New precision measurement of the orthopositronium decay rate: a discrepancy with theory. *Phys. Rev. Lett.*, 58:1328, 1987.
- [49] C. I. Westbrook, D. W. Gidley, R. S. Conti, and A. Rich. Precision measurement of the orthopositronium vacuum decay rate using the gas technique. *Phys. Rev. A*, (40):5489, 1989.
- [50] J. S. Nico, D. W. Gidley, A. Rich, and P. W. Zitzewitz. Precision measurement of the orthopositronium decay rate using the vacuum technique. *Phys. Rev. Lett.*, (65):1344, 1990.
- [51] G. S. Adkins, R. N. Fell, and J. Sapirstein. Order a^2 corrections to the decay rate of orthopositronium. *Phys. Rev. Lett.*, 84:5086, 2002.
- [52] R. S. Vallery, P. W. Zitzewitz, and D. W. Gidley. Resolution of the orthopositronium lifetime puzzle. *Phys. Rev. Lett.*, 90(20):203402, 2003.

-
- [53] D. B. Cassidy and A. P. jr Mills. The production of molecular positronium. *Nature*, 195(449), 2007.
- [54] A. P. jr Mills. Positronium molecule formation, Bose-Einstein condensation and stimulated annihilation. *Nucl. Instr. Meth. Phys. Res. B*, 192:107–116, 2002.
- [55] A. P. jr Mills, D. B. Cassidy, and R. G. Greaves. Prospects for making a Bose-Einstein condensed positronium annihilation gamma ray laser. *Materials Science Forum*, 445-446:424–429, 2004.
- [56] H. K. Avetissian, A. K. Avetissian, and G. F. Mkrtchian. Self-amplified gamma-ray laser on positronium atoms from a Bose-Einstein condensate. *Phys. Rev. Lett*, 113:023904, 2014.
- [57] A. P. jr Mills and M. Leventhal. Can we measure the gravitational free fall of cold Rydberg state positronium? *Nucl. Instr. Meth. Phys. Res. B*, 192:102–106, 2002.
- [58] M. K. Oberthaler. Anti-matter wave interferometry with positronium. *Nucl. Instr. Meth. Phys. Res. B*, (192):129, 2002.
- [59] D. B. Cassidy and S. D. Hogan. Atom control and gravity measurements using Rydberg positronium. *International Journal of Modern Physics: Conference Series*, 30:1450259, 2014.
- [60] L. Liskay, P. Comini, et al. Linac-based positron source and generation of a high density positronium cloud for the GBAR experiment. *Journal of Physics: Conference Series*, 443:012006, 2013.
- [61] M. K. Oberthaler, S. Bernet, E. M. Rasel, J. Schmiedmayer, and A. Zeilinger. Inertial sensing with classical atomic beams. *Phys. Rev. A.*, (54):3165, 1996.
- [62] Philippe H. M. Bräunig. *Atom Optical Tools for Antimatter Experiments*. PhD thesis, Ruperto-Carola University of Heidelberg, Heidelberg, 12 2014.
- [63] S. Aghion et al. Detection of low energy antiproton annihilations in a segmented silicon detector. *J. Instrum.*, (9):P06020, 2014.
- [64] S. Aghion et al. Prospects for measuring the gravitational free-fall of antihydrogen with emulsion detectors. *J. Instrum.*, (4):082, 2013.
- [65] M. Kimura et al. Development of nuclear emulsions with 1 μ m spatial resolution for the AEGIS experiment. *Nucl. Instrum. Methods Phys. Res., Sect. A*, (732):325–329, 2013.
- [66] P. Scamporrì et al. Development of nuclear emulsions operating in vacuum for the AEGIS experiment. *J. Instrum.*, (9):C01061, 2014.

- [67] J. Storey et al. Particle tracking at 4K: the Fast Annihilation Cryogenic Tracking (FACT) detector for the AEgIS antimatter gravity experiment. *Nucl. Instrum. Methods Phys. Res., Sect. A*, 732:437–441, 2013.
- [68] Henry Fox Talbot. Facts relating to optical science. *Philos. Mag.*, 4(9), 1836.
- [69] E. Lau. Beugungserscheinungen an doppellrastern. *Annalen der Physik*, (437):401–407, 1948.
- [70] Ruggero Caravita. Simulazione massivamente parallela di campi speckle generati da radiazione di sincrotrone. Bachelor’s thesis, Università degli Studi di Milano, 4 2010.
- [71] G Drobychev, P Nedelec, D Sillou, G Gribakin, H Walters, G Ferrari, M Prevedelli, G M Tino, M Doser, C Canali, C Carraro, V Lagomarsino, G Manuzio, G Testera, S Zavatarelli, M Amoretti, A G Kellerbauer, J Meier, U Warring, M K Oberthaler, I Boscolo, F Castelli, S Cialdi, L Formaro, A Gervasini, G Giammarchi, Antonio Vairo, G Consolati, A Dupasquier, F Quasso, Hinko Henry Stroke, A S Belov, S N Gninenko, V A Matveev, V M Byakov, S V Stepanov, D S Zvezhinskij, M De Combarieu, P Forget, P Pari, L Cabaret, D Comparat, G Bonomi, A Rotondi, N Djourellov, M Jacquey, M Büchner, G Trénec, J Vigué, R S Brusa, S Mariazzi, S Hogan, F Merkt, A Badertscher, P Crivelli, U Gendotti, and A Rubbia. Proposal for the AEgIS experiment at the CERN antiproton decelerator (Antimatter Experiment: Gravity, Interferometry, Spectroscopy). Technical Report SPSC-P-334. CERN-SPSC-2007-017, CERN, Geneva, Jun 2007.
- [72] A. Kellerbauer et al. Proposed antimatter gravity measurement with an antihydrogen beam. *Nucl. Instrum. Methods Phys. Res., Sect. B*, 266:351–356, 2008.
- [73] S. Aghion et al. A moiré deflectometer for antimatter. *Nature Comms.*, 5(4538), 2014.
- [74] Max Born and Emil Wolf. *Principles of Optics (4th ed.)*. Pergamon Press, 1970.
- [75] P. Bräunig, A. Demetrio, S. Müller, and M. K. Oberthaler. Limits on a gravity measurement with a non-collimated antihydrogen source. Technical report, Kirchoff-Institute for Physics, Heidelberg, 2015.
- [76] G. B. Andersen et al. Production of antihydrogen at reduced magnetic field for anti-atom trapping. *Jour. Phys. B*, 41:011001, 2008.
- [77] Y. Enomoto et al. Synthesis of cold antihydrogen in a cusp trap. *Phys. Rev. Lett.*, (105):243401, 2010.
- [78] G. B. Andersen et al. Autoresonant excitation of antiproton plasmas. *Phys. Rev. Lett.*, 106:025002, 2011.

-
- [79] D. Krasnický, R. Caravita, C. Canali, and G. Testera. Cross-section for Rydberg antihydrogen production via charge exchange between Rydberg positronium and antiprotons in magnetic field. *Phys. Rev. A*, (94):022714, 2016.
- [80] Arieh Iserles. *A First Course in the Numerical Analysis of Differential Equations, 2nd Edition*. Cambridge University Press, 2008.
- [81] P. Schmelcher. Regularity and irregularity in the centre of mass motion of the positronium atom in a magnetic field. *Journal of Physics B: Atomic, Molecular and Optical Physics*, 25(11):2697, 1992.
- [82] A. Deller, A. M. Alonso, B. S. Cooper, S. D. Hogan, and D. B. Cassidy. Measurement of Rydberg positronium fluorescence lifetimes. *Phys. Rev. A*, (93):062513, 2016.
- [83] M. Doser et al. Exploring the WEP with a pulsed cold beam of antihydrogen. *Classical and Quantum Gravity*, 29(18):183009, 2012.
- [84] C. Smorra et al. A reservoir trap for antiprotons. *International Journal of Mass Spectrometry*, 389:10–13, 2015.
- [85] C. Canali, C. Carraro, D. Krasnický, V. Lagomarsino, L. Di Noto, G. Testera, and S. Zavatarelli. Off-axial plasma displacement suitable for antihydrogen production in aegis experiment. *Eur. Phys. J., Sect. D*, 3(65):499–504, 2011.
- [86] L. Badano et al. *Proceedings of DIPAC 2007*, 2007. [Online; accessed 09-April-2016].
- [87] S. Cialdi, I. Boscolo, F. Castelli, F. Villa, G. Ferrari, and M.G. Giammarchi. Efficient two-step positronium laser excitation to Rydberg levels. *Nucl. Instrum. Methods Phys. Res. B*, 269(13):1527 – 1533, 2011.
- [88] S. Aghion et al. Positron bunching and electrostatic transport system for the production and emission of dense positronium clouds into vacuum. *Nucl. Instrum. Methods Phys. Res., Sect. B*, 362:86–92, 2015.
- [89] A. P. jr Mills and E. M. Gullikson. Solid neon moderator for producing slow positrons. *Appl. Phys. Lett.*, 17(49):1121, 1986.
- [90] S. Mariazzi et al. AEGIS experiment: towards antihydrogen beam production for antimatter gravity measurements. *Eur. Phys. J. D*, 41(68), 2014.
- [91] R. G. Greaves and C. M. Surko. Radial compression and inward transport of positron plasmas using a rotating electric field. *Phys. Plasmas*, 8(5):1879–1885, 2001.

- [92] C. M. Surko and R. G. Greaves. Emerging science and technology for antimatter plasmas and trap-based beams. *Phys. Plasmas*, 5(11):2333, 2004.
- [93] G. Testera, V. Lagomarsino, et al. Off-axis e^+ injection: extraction of e^+ from the accumulator and launch. Technical report, Istituto Nazionale di Fisica Nucleare, Genova, 16146 Italy, 2016.
- [94] L. Penasa, L. Di Noto, M. Bettonte, S. Mariazzi, G. Nebbia, and R. S. Brusa. Positron bunching system for producing positronium clouds into vacuum. *Journal of Physics: Conference series*, 505:012031, 2014.
- [95] Lea Di Noto. *Positronium in AEgIS experiment: study on its emission from nanochanneled samples and design of a new apparatus for Rydberg excitations*. PhD thesis, University of Trento, Dept. of Physics, 1 2014.
- [96] R. Ferragut, A. Calloni, A. Dupasquier, et al. Positronium formation in porous materials for antihydrogen production. *Journal of Physics: Conference Series*, 225:012007, 2010.
- [97] Y. Nagashima, Kakimoto, et al. Thermalization of free positronium atoms by collisions with silica-powder grains, aerogel grains, and gas molecules. *Phys. Rev. A*, (52):258–265, 1995.
- [98] D.B. Cassidy, P. Crivelli, et al. Positronium cooling in porous silica measured via doppler spectroscopy. *Phys. Rev. A*, 81(1):012715, 2010.
- [99] Ine Larsen Jernelv. Optimisation of positron-positronium conversion and positronium laser excitation. Master’s thesis, Norwegian University of Science and Technology, 7 2015.
- [100] Benjamin Rienäcker. Investigation of positron/positronium converter targets at AEgIS (CERN). Master’s thesis, University of Applied Sciences, Munich, 3 2015.
- [101] Y. Kataoka, S. Asai, and T. Kobayashi. First test of $o(\alpha^2)$ correction of the orthopositronium decay rate. *ICEPP Report*, (2), 2008.
- [102] Fabio Villa. *Laser system for positronium excitation to Rydberg levels for AEgIS experiment*. PhD thesis, Università degli Studi di Milano, 1 2012.
- [103] Francesco Ferri. Progettazione e realizzazione di un sistema laser per l’eccitazione efficiente del positronio a livelli Rydberg. Bachelor’s thesis, Università degli Studi di Milano, 1 2012.
- [104] Ruggero Caravita. Laser apparatus for exciting positronium in AEgIS positronium spectroscopy experiment. Master’s thesis, Università degli Studi di Milano, 4 2013.

-
- [105] EKSPLA, Savanoriu Av. 237, LT-02300 Vilnius, Lithuania. *NL300 series laser manual*, 2008.
- [106] Giovanni Cerchiari. Optical system for positronium excitation to measure gravity acceleration of antimatter in AEgIS experiment. Master's thesis, Università degli Studi di Milano, 4 2013.
- [107] S. Mariazzi, L. Di Noto, G. Nebbia, and R. S. Brusa. Collisional cooled ps emitted into vacuum from silica-based porous materials: experiment to measure the ps cooling time. *Journal of Physics: Conference Series*, 618:012039, 2015.
- [108] Daniel Krasnický. *Antiproton capture and cooling for production of cold antihydrogen*. PhD thesis, Università degli Studi di Genova, 3 2013.
- [109] Daniel Krasnický. *Development of positron transport and injection for the AEgIS experiment*. PhD thesis, Czech Technical University in Prague, 2009.
- [110] U. Raich. Beam diagnostics and instrumentation. CERN Accelerator school, <https://cas.web.cern.ch/cas/Holland/PDF-lectures/Raich/Raich.pdf>, 2005.
- [111] B. Bruyneel. Weighting potentials. EGAN School, Liverpool, <http://ns.ph.liv.ac.uk/EGAN/files/tuesday/Bruyneel-EGAN-School-P1>, 2011.
- [112] Walter Blum, Werner Riegler, and Luigi Rolandi. *Particle Detection with Drift Chambers*. Springer-Verlag Berlin Heidelberg, 2008.
- [113] John David Jackson. *Classical Electrodynamics, 3rd edition*. John Wiley and sons, 2015.
- [114] S. Aghion et al. Progress report on the AEgIS experiment (2016). CERN SPSC report, 2016.
- [115] G. Laustriat. The luminescence decay of organic scintillators. *Molecular Crystals*, 4:127–145, 1968.
- [116] R.S. Woolf, A. L. Hutcheson, et al. Comparing the response of PSD-capable plastic scintillator to standard liquid scintillator. *Nucl. Instr. Meth. Phys. Res. A*, 784:80–87, 2015.
- [117] X. P. Huang, F. Andereg, E. M. Hollmann, C. F. Driscoll, and T. M. O'Neil. Steady-state confinement of non-neutral plasma by rotating electric fields. *Phys. Rev. Lett.*, (78):875, 1997.
- [118] F. Andereg, E. M. Hollmann, and C. F. Driscoll. Rotating field confinement of pure electron plasmas using Tivelpiece-Gould modes. *Phys. Rev. Lett.*, (78):4875, 1998.

- [119] D. Krasnický et al. AEGIS experiment commissioning at CERN. *AIP Conf. Proc.*, (1521):144, 2013.
- [120] R. Caravita et al. Advances in antiproton and positron trapping in the AEGIS experiment. *in publication*, 2015.
- [121] G. Bendiscioli and D. Kharzeev. Antinucleon-nucleon and antinucleon-nucleus interaction. a review of experimental data. *Rivista del Nuovo Cimento*, 10(6), 1994.
- [122] Alice Magnani. Geant4 simulation of a silicon detector for the antihydrogen g measurement at the AEGIS experiment. Master’s thesis, Università di Pavia, via Bassi 12, 12 2012.
- [123] G. Bonomi, A. Fontana, D. Pagano, and F. Prelz. External scintillators: a comparison between data and MC. Technical report, Istituto Nazionale di Fisica Nucleare, sezione di Pavia-Brescia, 2015.
- [124] M. Knoop, N. Madsen, and R. C. Thompson. *Physics with Trapped Charged Particles: Lectures from the Les Houches Winter School*, chapter 8, pages 219–238. Imperial College Press, 57 Shelton Street, Covent Garden, London, 1 edition, 2014.
- [125] X. et al. Feng. Capture and cooling of antiprotons. *Hyperfine Interactions*, (109):145, 1997.
- [126] F. Anderegg, C. F. Driscoll, D. H. E. Dubin, and T. M. O’Neil. Measurement of correlated-enhanced collision rates using pure ion plasmas. *Phys. Plasmas*, (17):055702, 2010.
- [127] Daniela Manuzio. Ottimizzazione dell’efficienza di cattura e di cooling di antiprotoni nell’esperimento ATHENA. Master’s thesis, Facoltà di Scienze M. F. N, Università di Genova, via Dodecaneso, 33, Genova, 2000.
- [128] L. J. Spitzer. *Physics of Fully Ionized Gases*. Wiley and Sons, 1962.
- [129] J. J. Bollinger, D. J. Wineland, and Daniel H. E. Dubin. Non-neutral ion plasmas and crystals, laser cooling, and atomic clocks. *Phys. Plasmas*, 1(5):1403–1414, 1994.
- [130] G. Gabrielse et al. Centrifugal separation of antiprotons and electrons. *Phys. Rev. Lett.*, 105(213002), 2010.
- [131] G. B. Andersen et al. Centrifugal separation and equilibration dynamics in an electron-antiproton plasma. *Phys. Rev. Lett.*, 106:145001, 2011.
- [132] G. B. Andersen et al. Compression of antiproton clouds for antihydrogen trapping. *Phys. Rev. Lett.*, 100:203401, 2008.

-
- [133] J. R. Danielson and C. M. Surko. Torque-balanced high-density steady states of single-component plasmas. *Phys. Rev. Lett.*, 94:035001, 2005.
- [134] E. M. Hollmann, F. Anderegg, and C. F. Driscoll. Confinement and manipulation of non-neutral plasmas using rotating wall electric fields. *Phys. Plasmas*, (7):2776–2789, 2000.
- [135] D. Dubin and T. M. O’Neil. Trapped nonneutral plasmas, liquids and crystals (the thermal equilibrium states). *Rev. Mod. Phys.*, 71(1):87–172, 1999.
- [136] Ronald C. Davidson. *Physics of Nonneutral Plasmas*. Imperial College Press and World Scientific Publishing, 2001.
- [137] C. F. Driscoll, D. Z. Jin, D. A. Schecter, and D. H. E. Dubin. Vortex dynamics of 2d electron plasmas. *Physica C*, 21(369), 2002.
- [138] G. Bettega, B. Paroli, R. Pozzoli, and M. Romé. Excitation of the $l=3$ diocotron mode in a pure electron plasma by meand of a rotating electric field. *Journal of Applied Physics*, (105):053303, 2016.
- [139] A. Rich. Recent experimenta advances in positronium research. *Rev. Mod. Phys.*, 53:127, 1981.
- [140] D. A. Gurnett and A. Bhattacharjee. *Introduction to Plasma Physics: With Space and Laboratory Applications*. Cambridge University Press, 1999.
- [141] M. Curry. Combined Zeeman and motional Stark effects in the first excited state of positroninm. *Phys. Rev. A*, 7:447, 1973.
- [142] M. L. Lewis and V. W. Hughes. Higher-order relativistic contributions to the combined Zeeman and motional Stark effects in positronium. *Phys. Rev. A*, 8:625, 1973.
- [143] C. D. Dermer and J. C. Weisheit. Perturbative analysis of simultaneous Stark and Zeeman effects on $n=1$ $n=2$ radiative transitions in positronium. *Phys. Rev. A*, 40:5526, 1989.
- [144] Pineda A. and J. Soto. Potential NRQED: The positronium case. *Phys. Rev. D*, 59:016005, 1999.
- [145] S. D. Hogan. Calculated photoexcitation spectra of positronium Rydberg states. *Phys. Rev. A*, 87:063423, 2013.
- [146] J. Ackermann. Self-adaptive finite element techniques for stable bound matter-antimatter systems in crossed electric and magnetic fields. In P. Schmelcher and W. Schweizer, editors, *Atoms and Molecules in Strong External Fields*, pages 83–88. Kluwer Academic Publishers, 2002.

- [147] M. Braun, W. Schweizer, and H. Elster. Helium data for strong magnetic fields obtained by finite elements calculations. In P. Schmelcher and W. Schweizer, editors, *Atoms and Molecules in Strong External Fields*, pages 83–88. Kluwer Academic Publishers, 2002.
- [148] A. R. Edmonds. *Angular momentum in quantum mechanics, 3rd edition*. Princeton University Press, 1974.
- [149] R. D. Cowan. *The theory of atomic structure and spectra*. University of California Press, 1981.
- [150] F. Castelli. Notes on positronium physics. unpublished note, 2013.
- [151] F. Castelli. A quadratic angular function and its relation with diamagnetic effects. unpublished note, 2013.
- [152] L. D. Landau and E. M. Lifšic. *Kvantovaja mehanika: nereljativistskaja teorija, vol. 3 of Teoreticeskaja fizika*. Nauka Moskva, 1989.
- [153] William H Press, Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery. *Numerical Recipes: The Art of Scientific Computing, 3rd edition*. Cambridge University Press, 3 edition, 2007.
- [154] I. I. Sobelman. *Atomic spectra and radiative transitions, 2nd edition*. Springer-Verlag, 1992.
- [155] C. Sanderson and R. Curtin. Armadillo: a template-based C++ library for linear algebra. *Journal of Open Source Software*, 1:26, 2016.
- [156] J. W. et al. Demmel. Performance and accuracy of LAPACK’s symmetric tridiagonal eigensolvers. <http://www.netlib.org/lapack/lawnspdf/lawn183.pdf>, 2017.
- [157] Albert Einstein. Zur quantentheorie der strahlung. *Physikalische Zeitschrift*, 18:121–128, 1917.
- [158] Christopher J. Foot. *Atomic Physics*. Oxford University Press, 2005.
- [159] T. M. O’Neil and C. F. Driscoll. Transport to thermal equilibrium of a pure electron plasma. *Phys. Fluids*, 22(266), 1979.
- [160] S. A. Prasad and T. M. O’Neil. Finite length thermal equilibria of a pure electron plasma column. *Phys. Fluids*, 22(2):278–281, 1979.
- [161] Ross L. Spencer, S. N. Rasband, and Richard R. Vanfleet. Numerical calculation of axisymmetric non-neutral plasma equilibria. *Phys. Fluids B*, 5(12):4267–4272, 1993.

-
- [162] B. R. Beck, J. Fajans, and J. H. Malmberg. Temperature and anisotropic-temperature relaxation measurements in cold, pure-electron plasmas. *Phys. Plasmas*, 3(4), 1996.
- [163] M. Guizar-Sicairos and J. C. Gutiérrez-Vega. Computation of quasi-discrete Hankel transforms of integer order for propagating optical wave fields. *Opt. Soc. of America*, 21(1):53–58, 2004.
- [164] A. Townsend. A fast analysis-based discrete Hankel transform using asymptotic expansions. *arXiv*, page 1501.01652v2, 2015.
- [165] J. W. Cooley and J. W. Tukey. An algorithm for the machine calculation of complex Fourier series. *Math. Comput.*, (19):297–301, 1965.
- [166] J. J. Bollinger, Joseph N. Tan, Wayne M. Itano, D. J Wineland, and D. H. E. Dubin. Nonneutral ion plasmas and crystals in Penning traps. *Physica Scripta*, 1995(T59):352, 1995.
- [167] C. Carraro. *Charged particles manipulation and diagnostic for sub-eV antihydrogen production in ATHENA experiment*. PhD thesis, Università degli Studi di Genova, 3 2002.
- [168] Michael E. Peskin and Daniel V. Schroeder. *An Introduction to Quantum Field Theory*. Perseus books, 1995.
- [169] A. Aspect, N. Vansteenkiste, and R. Kaiser. Preparation of a pure intense beam of metastable helium by laser cooling. *Chemical Physics*, 145:307–315, 1990.
- [170] F. Pereira Dos Santos, J. Léonard, J. Wang, C. J. Barrelet, F. Perales, E. Rasel, C. S. Unnikrishnan, M. Leduc, and C. Cohen-Tannoudji. Bose-Einstein condensation of metastable helium. *Phys. Rev. Lett.*, 86:3459, 2001.
- [171] A. Robert, O. Sirjean, A. Browaeys, J. Poupard, S. Nowak, D. Boiron, C. Westbrook, and A. Aspect. A Bose-Einstein condensate of metastable atoms. *Science*, 292:5516, 2001.
- [172] Pauline Yzombard. *Laser cooling and manipulation of antimatter in the AEgIS experiment*. PhD thesis, Laboratoire Aimé Cotton, Université Paris-Sud, 11 2016.
- [173] K. C. Harvey. Slow metastable atomic hydrogen beam by optical pumping. *Journal of Applied Physics*, 53:3383, 1982.
- [174] A. Deller, A. M. Alonso, B. S. Cooper, S. D. Hogan, and D. B. Cassidy. Electrostatically guided Rydberg positronium. *Phys. Rev. Lett.*, (117):073202, 2016.

List of acronyms

5T 5 Tesla - label for specifying components in the part of the vacuum chamber hosting the (up to) 5 T magnet, currently operated at 4.5 T

1T 1 Tesla - label for specifying components in the part of the vacuum chamber hosting the 1 T magnet

5TMCP 5T micro-channel plate

1TMCP 1T micro-channel plate

5TMCPIn 5T micro-channel plate Faraday cup

1TMCPIn 1T micro-channel plate Faraday cup

AD Antiproton Decelerator - CERN dedicated decelerator for low-energy antiproton experiments

ADC Analog-to-Digital Converter

B Big trap - trap region in the 1T magnetic field used for off-axis positrons movement.

BBO Barium Borate

C Catch trap - trap region in the 5T magnetic field used for positron catching and cooling.

CD Cold dump

CRFC Central Region Faraday Cup

CsI Caesium Iodide

CTMC Classical Trajectory Monte-Carlo - simulation technique of particle behaviour in external fields based on the integration of the kinetical ODEs mutated from Lagrange's equations of motion

DAC Digital-to-Analog Converter

DAQ Data Acquisition

DegFC Degraded Faraday Cup

DDS Direct Digital Synthesis - a method used in digital electronics to generate an arbitrary periodic waveform using a single time reference

DFT Discrete Fourier Transform

DHT Discrete Hankel Transform

DUV Deep Ultra-Violet

EP Equivalence Principle

FACT Fast Annihilation Cryogenic Tracker - main antihydrogen formation detector in AEGIS, constituted by 800 scintillating fibers in a four layers setup read by MPPCs

FC Faraday Cup

FEM Finite-Elements Method

FFT Fast Fourier Transform - algorithm for computing discrete Fourier transforms with $o(n \log(n))$ computational complexity.

FHG Fourth Harmonic Generation

FST Fast Sine Transform

FPGA Field-Programmable Gate Array

FS Fused Silica

HD Hot dump

HPD Hybrid Photo Detector

IR Infra-Red

LEAR Low-Energy Antiproton Ring

LiNb Lithium Niobate

LN liquid Nitrogen

lHe liquid Helium

MCP Micro-channel plate

MOSFET Metal-Oxide-Semiconductor Field Effect Transistor

MPPC Multi-Pixel Photon Counter

NaI Sodium Iodide

Nb:Ti Niobium Titanium alloy

NIST National Institute of Standards and Technology

KTP Potassium Titanyl-Phosphate

ODE Ordinary Differential Equation

OffAx Off-axis trap - trap region in the 1T magnetic field for launching positrons off-axis on the positronium converter target.

OnAx On-axis trap - trap region in the 1T magnetic field for transferring antiprotons in the production region and for antiproton cooling experiments.

OPA Optical Parametric Amplification

OPG Optical Parametric Generation

OVC Outer Vacuum Chamber - AEGIS experiment insulation vacuum chamber.

P Positron trap - trap region in the 5T magnetic field used for antiproton and electron manipulations.

PDE Partial differential equation

PMT Photo-Multiplier Tube

PMT2 Digitized photo-multiplier tube used to acquire positron annihilation signal at the entrance of the 5T cryostat

PMT20 Digitized photo-multiplier tube used to acquire positron annihilation signal at the end of the 1T cryostat

PPKTP Periodically-Poled Potassium Titanyl Phosphate

Ps Positronium - the short-lived atomic bound state of electron and positron. Annihilates spontaneously in 2γ in 125 ps or 3γ in 142 ns according to its atomic total spin state.

p-Ps para-Positronium - singlet 1^1S state of Positronium, with a lifetime of 125 ps and annihilating into two monochromatic, back-to-back γ -rays at 511 keV energy.

- o-Ps** ortho-Positronium - triplet 1^3S state of Positronium, with a lifetime of 142 ns and annihilating into three non-monochromatic γ -rays with a roughly triangular spectrum from 0 to 511 keV peak.
- PsTC** Positronium Transitions Calculator - name given to the calculation tool developed to numerically evaluate energy levels, transition probabilities and spectra of Positronium in arbitrary external fields.
- PS** Proton Synchrotron - CERN circular synchrotron active since 1959, in use for LHC injection line.
- RT** NI PXI Embedded Real-Time controller - real-time computer running Windows, which runs the real-time part of the trap control software.
- RW** Rotating-wall
- SEM** Scanning Electron Microscope - type of electron beam microscope detecting backscattered, secondary and Auger electrons from the sample surface to produce its nanometric precision heightmap.
- SHG** Second Harmonic Generation
- SHT** Sine-Hankel Transform
- SEP** Strong Equivalence Principle
- SFG** Sum-Frequency Generation
- SSPALS** Single-Shot Positron Annihilation Lifetime Spectroscopy - detection technique used to prove positronium formation. It consists in detecting ortho-Ps annihilations in vacuum, whose longer lifetime of 142 ns compared to the 125 ps of para-Ps produces a clean exponential distribution of γ -rays that can be observed by fast scintillation detector.
- T** Transfer electrodes - trap region between 5T and 1T magnetic fields to adiabatically transport particles.
- UC** Production or Ultracold trap - trap region in the 1T magnetic field designed for antihydrogen formation through charge exchange.
- UHV** Ultra-High Vacuum region
- UV** Ultra-Violet
- WEP** Weak Equivalence Principle
- YAG** Yttrium-Aluminum Garnet (Neodimium-doped)

List of Figures

1.1	WEP tests on matter systems	5
2.1	The moiré deflectometer	14
2.2	Near-field diffraction and Talbot effect after a regular grating	16
2.3	Geometrical model of Hbar source - detector	17
2.4	Visibility of moiré fringes	19
2.5	Summary of CTMC charge-exchange cross-section calculations	22
2.6	Outline of AEgIS experimental scheme	24
3.1	AEgIS overview inside A.D.	30
3.2	AEgIS main cryostat	32
3.3	AEgIS antihydrogen production region	33
3.4	AEgIS magnets configuration	34
3.5	AEgIS magnetic field	35
3.6	AEgIS original 1T traps usage	37
3.7	Overview of AEgIS Malmber-Penning traps 1	39
3.8	AEgIS positron system (sketched)	45
3.9	Sketch of the laser setup	46
4.1	AEgIS positron system (full)	48
4.2	Technical drawing of positron accumulator trap	49
4.3	Positrons number vs accumulation time	50
4.4	Accumulator trap potential during ejection	51

4.5	Buncher and chamber for Ps experiments	53
4.6	Positron bunching: time compression	54
4.7	Positron bunching: spatial focusing	55
4.8	Mesoporous silica positronium converter	56
4.9	Zoom on spectroscopy chamber target region	57
4.10	Ps formation and lifetime in silica targets	58
4.11	Sketch of the laser pump setup	59
4.12	Sketch of the $1^3S \rightarrow 3^3P$ laser setup	61
4.13	Sketch of the Rydberg laser setup	62
4.14	Sketch of the Rydberg laser setup	63
4.15	Sketch of laser transfer lines	65
4.16	Example of UV and IR laser alignments	66
4.17	IR spectrometer calibration curves	67
4.18	UV laser wavelength/intensity distributions	68
4.19	Ps atoms excitation to 3^3P using magnetic quenching	70
4.20	Ps atoms excitation to 3^3P using selective photoionization	71
4.21	UV wavelength scan of 3^3P level using selective photoionization	72
4.22	UV and IR energy scan of 3^3P level	73
4.23	IR wavelength scan of $n = 15 - 17$ Rydberg levels	78
5.1	Positron transfer line	82
5.2	Positron transfer: time distribution at drift tube position	86
5.3	Positron transfer: cross-normalization of PMT20 on pick-up electrode	88
5.4	Positron transfer: time structure of positron spill on pick-up electrode	89
5.5	Positron transfer: magnetic mirroring effect	91
5.6	Positron transfer: steering on the 1TMCP	93
5.7	Positron transfer: energy distribution	94
5.8	Positron trapping: efficiency vs trap shape	96
5.9	Positron trapping: cooling efficiency and radius vs time	98

5.10	Positron storage in C-trap	99
5.11	Positron rotating-wall in C-trap	100
5.12	Positron stacking in C-trap	101
5.13	Positron storage in 1T Big-trap	103
5.14	Positron rotating-wall in 1T Big-trap	104
5.15	Direct positron injection: transfer line trajectory	105
5.16	Direct positron injection: target centering	107
5.17	Direct positron injection: target centering with PbF ₂	108
5.18	Direct positron injection: acceleration	109
5.19	Direct positron injection: positronium formation	111
6.1	Electron loading procedure	115
6.2	Example of electron signal on a Faraday cup	116
6.3	Electron number estimation	117
6.4	Linearity of the electron reduction procedure	119
6.5	Rotating-wall compression of electrons in P-trap	121
6.6	Antiproton spill time distribution	123
6.7	Geometry for MC antiproton number estimation	125
6.8	Antiproton catching results	128
6.9	Electron cooling efficiency	132
6.10	Cold antiproton lifetime in 5T traps	134
6.11	Centrifugal separation of antiproton/electron plasmas	136
6.12	Antiproton compression results	142
6.13	Centrifugal separation dynamics after RW	145
6.14	Centrifugal separation dynamics after RW	145
6.15	Scheme of antiproton in-flight ballistic transfer	147
6.16	Recatching antiprotons: energy and time spreads	150
6.17	Sketch of the sectorized B0 electrode	151
6.18	Summary of $E \times B$ steering results	153

6.19	Recatching antiprotons: spectra of digitized scintillators	154
6.20	Effects of the antiproton axialization	155
6.21	Recatching antiprotons: recatching efficiency and initial storage time	157
6.22	Recatching antiprotons: radial losses on the electrodes	158
6.23	Recatching antiprotons: storage time in the production trap	159
7.1	Side view sketch of the $\bar{H}$ production experiment ($\hat{z} - \hat{y}$ plane)	163
7.2	Front view sketch of the $\bar{H}$ production experiment ($\hat{z} - \hat{y}$ plane)	164
7.3	Sketch of the excited Ps distribution vs laser detuning	164
7.4	Acceptance of the grid on the transverse velocity plane	168
8.1	Contributions to splitting of Ps levels	181
8.2	Reference coordinate system used in PsTC	182
8.3	Visualization of the full Hamiltonian matrix for the $n = 15$ manifold	188
8.4	Ps levels comparison with literature: atom at rest	189
8.5	Ps levels comparison with literature: atom in motion	190
8.8	Schematic representation of spherical polarizations	198
8.9	Schematic representation of σ -polarized and π -polarized laser pulses	198
8.11	Comparison of linear and saturated absorption spectra	202
8.12	Comparison of absorption spectra with experimental data	204
8.13	Scheme of the tilted target spectroscopy experiment	208
8.14	Motional Stark induced Ps transitions shift vs target rotation and B	209
8.15	Motional Stark induced Ps transition shift vs Ps velocity	210
9.1	Solution of the 1D radial Poisson equation	227
9.2	Solution of the 2D cylindrical Poisson equation	227
9.3	Accuracy of SHT method: comparison with Maxwell (single electrode)	232
9.4	Accuracy of SHT method: comparison with Maxwell (whole trap)	233
9.5	Accuracy of the plasma solver: harmonic trap theory comparison	235
9.6	Trap-plasma potential maps for standard P3-P9 trap	236

9.7	Plasma in an arbitrary trap: calculated potentials	237
9.8	Plasma in an arbitrary trap: calculated equilibrium	238
9.9	Plasma midplane radial profile at different r_p	239
9.10	Plasma midplane radial profile at low T	240
9.11	Comparison of midplane versus integrated radial profiles	241
9.12	SHT method convergence versus temperature	243
10.1	Theory: Ps transitions from $n = 1$ to $n = 2$	246
10.2	Experimental: setup for the metastable experiment	251
10.3	Experimental: upgrades in SSPALS spectra acquisition	252
10.4	Experimental: concentration of the UV laser spot	254
10.5	Analysis: data time ordering	256
10.6	Analysis: distribution of time delays among laser and nolaser measurements . . .	258
10.7	Analysis: definition of the signal regions	264
10.8	Analysis: detectors cross-calibration	268
10.9	Results: averaged SSPALS spectra	269
10.10	Results: evolution and distribution of the proportionality parameter	270
10.11	Results: final test of the proportionality hypothesis	271
10.12	Discussion: statistical significance vs control/signal windows position	273

List of Tables

3.1	List of AEGIS software control systems	33
3.2	AEGIS Malmberg-Penning trap regions	40
3.3	Full list of AEGIS particle detectors	43
4.1	Ps yield and lifetime in SiO ₂ targets	58
4.2	Fit results of $n = 3$ spectroscopy experiments	76
6.1	List of RW sectorized electrodes	120
6.2	Counts to antiprotons conversion factors	125
6.3	Table of typical cyclotron cooling times	129
8.1	Wavelength of main Ps transition lines in absence of fields	177
8.2	Measured performance of the numerical method: single n manifold	191
8.3	Measured performance of the numerical method: multiple n manifolds	194
10.1	Calculated efficiency of Ps spontaneous decay process	249
10.2	Calculated efficiency of Ps metastable formation process	249
10.3	Calculated efficiency of Ps magnetic quenching process	249
10.4	Definition of control/signal regions	265