

Hadron Production in 120 GeV/ c Proton-Carbon Interactions

by

Brant T Rumberger

B.A., Northeastern University, 2013

M.S., University of Colorado Boulder, 2016

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Committee Members:

Eric D. Zimmerman, Chair

Jeremy Darling

Alysia D. Marino

Dennis Perepelitsa

Kevin Stenson



Rumberger, Brant T (Ph.D., High-Energy Physics)

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Thesis directed by Prof. Eric D. Zimmerman

This thesis presents multiplicity measurements of $\pi^\pm$, $p/\bar{p}$, and $K^\pm$ in 120 GeV/ c proton-carbon interactions. The measurements were made using data collected at the NA61/SHINE experiment in 2016 and 2017. The experimental data sets were recorded using two complementary detector configurations, with increased phase space coverage in 2017 due to the addition of new subdetectors. This thesis details the design, construction, and performance of these detectors. Comparisons of the measured hadron multiplicities show significant discrepancies with popular Monte Carlo models, indicating the need for improvement in hadron production modeling. These multiplicity measurements can be used by Monte Carlo modeling groups to improve the models, and will also help current and future long-baseline neutrino oscillation experiments estimate neutrino beam fluxes with higher accuracy.

Dedication

To my parents and my brothers, for all of their loving support.

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Chapter 1

Introduction

1.1 Introduction to Neutrino Physics

Neutrinos have been a subject of study in particle physics for over ninety years. In 1930, Wolfgang Pauli postulated the neutrino as a solution to an issue with nuclear β -decay [1]. Nuclear β -decay was thought to be a two-body decay process, with the parent particle being the initial nucleus and the child particles being an electron and the nucleus with incremented atomic number. In two-body decays of type $A \rightarrow B + C$, the energy of the child particles in the center-of-mass frame are uniquely determined by the masses of the participating particles m_A , m_B & m_C . Thus, in any given nuclear β -decay, the emitted electron should be mono-energetic (assuming the initial nucleus is at rest). However, experiment shows that the electron is *not* mono-energetic; the predicted energy gives only the endpoint of the emitted electron energy spectrum [2]. Pauli concluded that a yet-to-be-detected third particle must be emitted along with the electron. This particle, dubbed the *neutrino* by Enrico Fermi, must be neutral and extremely light to produce the characteristic electron energy spectrum. With this postulation, the concept of the neutrino was born.

We now know the neutrino to be a fundamental particle with very small mass (< 1 eV) [3] that participates only in the weak and gravitational interactions. Many dedicated experiments have been conducted to measure neutrino properties, the results of which will be discussed in subsequent sections. To give a detailed description of neutrino properties, we must first discuss the Standard Model of Particle Physics.

1.1.1 The Standard Model of Particle Physics

The Standard Model of Particle Physics is an immensely successful description of the fundamental constituents of our universe. The power of the Standard Model has been demonstrated time and again over the past several decades with its prediction of the $W^\pm$ and Z bosons (discovered at CERN in 1983) [4], the top quark (discovered at the Tevatron in 1995) [5], and the Higgs boson (discovered at CERN in 2012) [6].

The Standard Model is a quantum field theory obeying the gauge symmetry $SU(3) \otimes SU(2)_L \otimes U(1)$. Each of these gauge symmetries specifies the structure of one of the interactions described by the Standard Model. The discussion of the constituents of the Standard Model will be separated into two groups: the **matter content** (fermions) and the **force carriers** (bosons).

- Matter Content (Fermions)

The fermionic content of the Standard Model constitutes all known matter and antimatter in the Universe¹. These particles, organized in the left three columns of Fig. 1.1, participate in the various fundamental interactions. The **quarks** in the top two rows and their antiparticle partners are the constituents of nuclear matter. The **leptons** in the bottom two rows and their antiparticle partners differ from the quarks in that they do not experience the strong force. Each charged lepton (electron, muon, tau) has a corresponding neutrino partner (electron neutrino, muon neutrino, tau neutrino). These pairs are coupled via charged-current interactions of the weak force.

Left-handed fermions of the same generation are paired together in *weak isospin* doublets, which are coupled together via the $W^\pm$ boson. Right-handed fermions (and left-handed anti-fermions) exist in weak isospin singlets, meaning that they do not interact via charged-current weak interactions.

Fermions, as constructed in the Standard Model, are massless. An explicit mass term in the fermionic Lagrangian would couple the left-handed and right-handed chiral components of the fermion wave functions, which would violate gauge symmetry. Interactions of the fermions with

¹ Apart from dark matter.

Standard Model of Elementary Particles

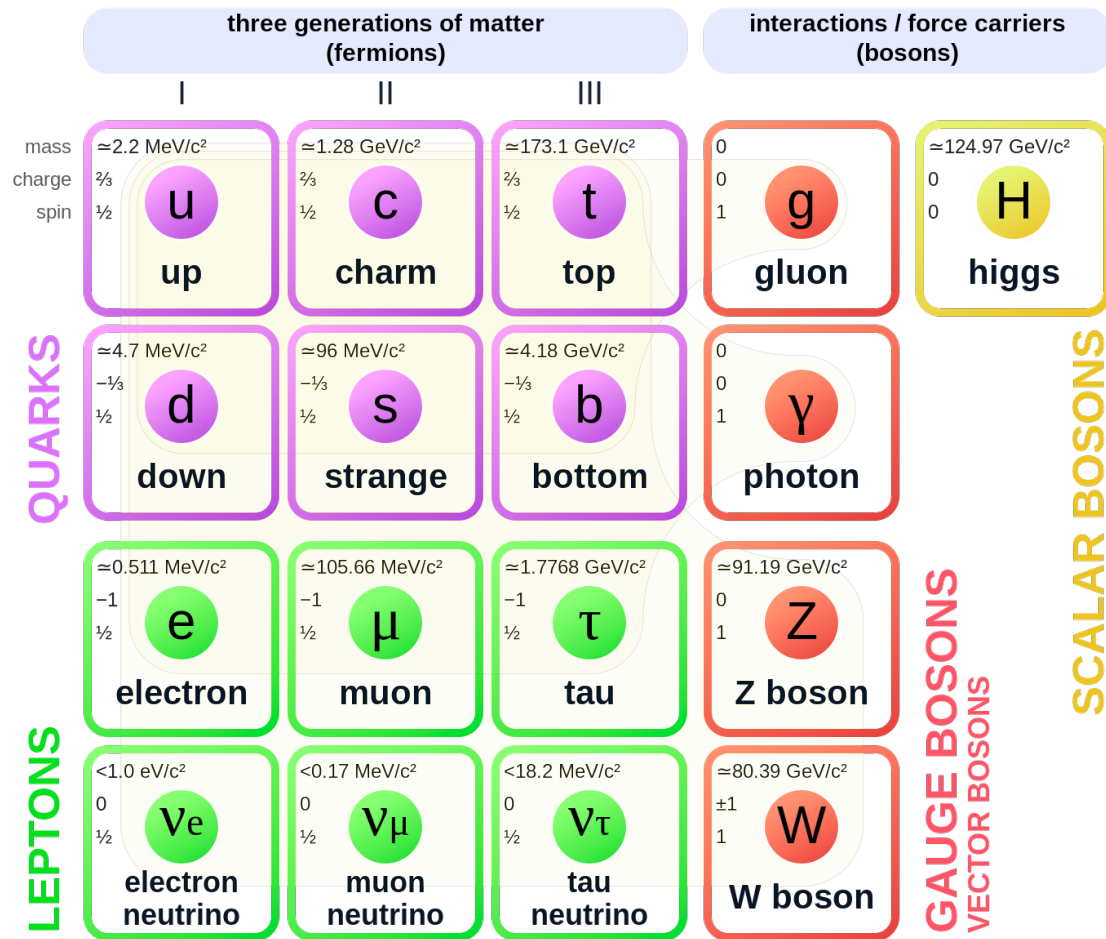


Figure 1.1: The Standard Model of particle physics. Reproduced from [7].

the Higgs field (after spontaneous symmetry breaking) have the form

$$\begin{aligned}\mathcal{L}_{Higgs-Lepton} &= -\lambda_\ell \left[\begin{pmatrix} \bar{\nu}_L & \bar{\ell}_L \end{pmatrix} \begin{pmatrix} 0 \\ v+h \end{pmatrix} \ell_R + \bar{\ell}_R \begin{pmatrix} 0 & v+h \end{pmatrix} \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \right] \\ &= -\lambda_\ell [\bar{\ell}_L \ell_R (v+h) + \bar{\ell}_R \ell_L (v+h)]\end{aligned}\tag{1.1}$$

where ℓ may be e , μ , or τ , and λ_ℓ is the Higgs-lepton coupling factor. Here v is the vacuum expectation value of the Higgs field after spontaneous symmetry breaking, and h is the remaining dynamic degree of freedom in the Higgs field. These terms have the effect of correcting the free fermion propagators, and act as mass terms for the fermions. The stronger the coupling of a given fermion to the Higgs field, the larger the mass of the fermion. Note that due to the left-handed $SU(2)$ doublet and right-handed $SU(2)$ singlet structure of the fermions, the Standard Model neutrino does not acquire a mass via Higgs interactions. As we will discuss in Sec. 1.2.2, neutrino masses have been observed experimentally, meaning that the Standard Model description of neutrinos is incomplete. The nature of the neutrino mass term is an area of ongoing research [8].

It is important to note that for the quarks and the neutrinos, the weak interaction eigenstates, or “flavor eigenstates,” are distinct from the mass eigenstates, which govern propagation through space-time. To move between these bases, a 3x3 rotation matrix must be invoked. In the quark sector, this matrix is known as the CKM matrix, and is nearly diagonal—that is, the mass eigenstates are very similar to the flavor eigenstates. For neutrinos, the corresponding matrix is the PMNS matrix, and is far from diagonal (see Fig. 1.2). The off-diagonal structure of the PMNS matrix may be a hint at some undiscovered physics.

- Force Carriers (Gauge Bosons)

The bosonic particles with spin 1 are the force mediators of the Standard Model interactions. Gauge bosons arise naturally when a gauge symmetry is imposed, and thus the interaction structure is dictated by the gauge symmetry associated with the boson.

The photon, mediator of the electromagnetic interaction, arises from a local $U(1)$ gauge symmetry. The photon is massless and couples to electrically-charged objects, the charged leptons

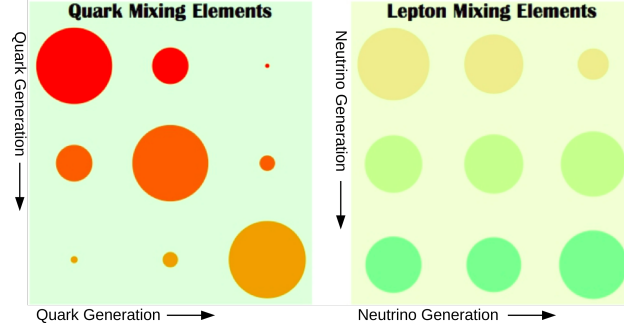


Figure 1.2: Visual comparison of the magnitude of flavor mixing in the quark sector (CKM matrix elements, left) and the neutrino sector (PMNS matrix elements, right). Flavor mixing is much more significant in the neutrino sector. Reproduced from [9].

and the $W^\pm$. The massless nature of the photon gives the electromagnetic force infinite range and the photon an infinite lifetime.

The W -bosons and the Z -boson, mediators of the weak force, arise from a local $SU(2)_L$ gauge symmetry². The fact that there are three independent mediators of the weak force is again a reflection of the gauge symmetry group structure, as there are three generators of $SU(2)$. These particles gain mass through interactions with the Higgs field (after spontaneous symmetry breaking). The massive nature of these force mediators restricts the range of the weak force and dictates that they have a finite lifetime. The non-Abelian group structure of $SU(2)$ leads to self-interaction terms in the Standard Model Lagrangian that couple the W^+ , W^- and Z at one vertex.

The gluons, mediators of the strong force, arise from a local $SU(3)$ gauge symmetry. The group structure dictates that there are 8 distinct gluons, each with two color indices. Gluons also have self-interaction terms due to the non-Abelian structure of $SU(3)$, since the commutator term $[A_\mu, A_\nu]$ in the field tensor does not vanish. Three-gluon and four-gluon interactions are possible, and it is this self-sourcing of the gluon fields that leads to quark confinement.

An important interaction that will be discussed in subsequent sections is the charged-current interaction of the weak force. The charged-current interaction couples lepton weak isospin doublets to the charged $W^\pm$ bosons through the interaction $\ell^\pm + \nu_\ell \rightarrow W^\pm$. Thus, the possible neutrino

² The L in $SU(2)_L$ indicates the curious nature of the weak interaction: it only couples to left-handed chiral states. This behavior means that the weak force maximally violates parity, which was somewhat unexpected at the time of its experimental discovery.

charged-current interaction Feynman diagrams are

$$(1.2)$$

As previously mentioned, these interactions occur in the interaction (or flavor) basis, as opposed to the mass eigenstate basis.

- Higgs Particle (Scalar Boson)

The Higgs boson and its coupling to other Standard Model particles is responsible for the generation of mass. The Higgs-fermion couplings, as demonstrated in Eq. 1.1, contribute Lagrangian terms quadratic in the massless fermion fields, which act as effective mass terms.

The Higgs boson is charged under the electroweak interaction, and exists in an $SU(2)_L$ doublet, as seen in the second line of Eq. 1.3. The covariant derivative in the kinetic term for the Higgs field couples the Higgs to the electroweak gauge fields:

$$D_\mu \phi = (\partial_\mu + igT^i W_\mu^i + i\frac{1}{2}g' B_\mu)\phi$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \quad (1.3)$$

Here, W_μ and B_μ are the massless gauge fields of the unbroken $SU(2)_L \otimes U(1)_Y$ symmetry, T^i are the generators of $SU(2)_L$, and g and g' are the coupling constants between the Higgs field and the massless fields.

By assigning the Higgs field a polynomial potential of the form

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \quad (1.4)$$

and tuning the coupling constants μ and λ , a double-welled rotationally-symmetric potential in the complex field ϕ may be constructed. The minimum of this potential is displaced from zero, and the

Higgs field develops a non-zero vacuum expectation value as a result. This phenomenon is known as *spontaneous symmetry breaking*. Once the symmetry is broken, a particular gauge can be chosen and the two-component complex Higgs field can be reduced to just one degree of freedom. The couplings to this remaining degree of freedom generate the mass terms for the Standard Model $W^\pm$ and Z bosons, while leaving the γ massless.

1.1.2 Neutrino Mass Models and Mixing

Despite the Standard Model's immense success in the quark and electroweak sectors, it fails to provide a complete picture of the neutrinos. As mentioned in the previous section, the Standard Model does not generate neutrino masses through the Higgs mechanism. Neutrino masses were not considered necessary to include, since right-handed neutrino singlets would not participate in any Standard Model interactions. Experimental discovery of neutrino flavor oscillation imply that the mass basis associated with propagation through spacetime is distinct from the flavor basis. Since there must be at least three distinct neutrino mass eigenstates, at least two of the neutrinos cannot be massless. This raises the question: How do we introduce neutrino mass into the Standard Model?

- Dirac Neutrino Mass

To extend the Standard Model in a minimal way, we may add Dirac neutrino masses. This is accomplished in an analogous way to how the up-type quark masses are generated. We consider the addition of the following Lagrangian term, which obeys the requisite $SU(2)$ and $U(1)$ symmetries of the electroweak interaction:

$$\begin{aligned}\mathcal{L}_{\text{Higgs-Neutrino}} &= -\lambda_\nu \left[\begin{pmatrix} \bar{\nu}_L & \bar{\ell}_L \end{pmatrix} \begin{pmatrix} v+h \\ 0 \end{pmatrix} \nu_R + \bar{\nu}_R \begin{pmatrix} v+h & 0 \end{pmatrix} \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} \right]. \\ &= -\lambda_\nu [\bar{\nu}_L \nu_R (v+h) + \bar{\nu}_R \nu_L (v+h)]\end{aligned}\tag{1.5}$$

Here we have used the following transformation property of the Higgs $SU(2)$ doublet Φ (still given

here post- spontaneous symmetry breaking):

$$\bar{\Phi} = i\sigma^2 \begin{pmatrix} 0 \\ v+h \end{pmatrix}^T = \begin{pmatrix} v+h \\ 0 \end{pmatrix}^T \quad (1.6)$$

where σ^2 is the second Pauli matrix. Much like the quark sector, adding in subsequent generation of neutrinos leads to the following term:

$$\mathcal{L}_{\text{Higgs-Neutrino}} = -\lambda_\nu \left[y_{ij} \begin{pmatrix} \bar{\nu}_{iL} & \bar{\ell}_{iL} \end{pmatrix} \begin{pmatrix} v+h \\ 0 \end{pmatrix} \nu_{jR} + y_{ij}^* \bar{\nu}_{iR} \begin{pmatrix} v+h & 0 \end{pmatrix} \begin{pmatrix} \nu_{jL} \\ \ell_{jL} \end{pmatrix} \right] \quad (1.7)$$

where y_{ij} is a dimensionless matrix of coupling constants. This matrix is in general non-diagonal. Diagonalizing it using appropriate unitary matrices yields the well-known PMNS matrix, which gives explicit relations between the neutrino flavor eigenstates and mass eigenstates:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}. \quad (1.8)$$

Here, $\nu_i, i = 1, 2, 3$ represent the neutrino mass eigenstates. The nature of the mass eigenstates being a superposition of flavor eigenstates allows for the phenomenon of *neutrino flavor oscillation* — the creation of a particular flavor neutrino that is later measured to be of a different flavor. This, as mentioned before, has been observed in a number of experiments [10] [11] [12]. A vast amount of experimental effort has been put into measuring the parameters of the PMNS mixing matrix. This will be discussed in Sec. 1.2.2.

The simple addition of Dirac neutrino mass is unsatisfying due to the small nature of the required coupling constants — on the order of 10^{-5} when compared to the next-heaviest particle, the electron. For this reason, another mass generation mechanism is suspected to be at work.

- Majorana Neutrino Mass

The electromagnetic neutrality of the neutrino allows us to consider another possible mass generation mechanism. The associated Lagrangian term, known as the Majorana mass term, consists of only two degrees of freedom. This is accomplished by pairing one chiral neutrino state with

its charge-conjugated state. Consider the following term:

$$\mathcal{L}_{\text{Majorana}} = -\frac{1}{2}m [\bar{\nu}_L^C \nu_L + \bar{\nu}_L \nu_L^C], \nu^C = \hat{C}\nu = \nu. \quad (1.9)$$

Here, $\hat{C} = i\gamma^2\gamma^0$ is the charge conjugation operator. This operator has the effect of exchanging a particle with its antiparticle. The condition in Eq. 1.9 is known as the Majorana condition, and a particle that satisfies this condition is its own antiparticle. Note that this is only possible for particles with zero electromagnetic charge, as the sign of the charge is reversed under $\hat{C}$.

- The Seesaw Mechanism

We now assemble and examine the permitted neutrino mass terms, including a Majorana term for the right-handed neutrino states in the Dirac mass term:

$$\begin{aligned} \mathcal{L}_{\text{mass}} &= -\frac{1}{2} [m_D \bar{\nu}_R \nu_L + m_D \bar{\nu}_L \nu_R + m_L \bar{\nu}_L^C \nu_L + m_R \bar{\nu}_R^C \nu_R] + h.c. \\ &= -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L^C & \bar{\nu}_R \end{pmatrix} \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} + h.c. \end{aligned} \quad (1.10)$$

Here $h.c.$ stands for Hermitian conjugate. The fields ν_L and ν_R are purely chiral fields, and do not represent the mass eigenstates due to the off-diagonal terms in the mass matrix in Eq. 1.10. We now note that the left-handed Majorana neutrino cannot be part of the Standard Model: Its charge under $SU(2)_L$ would result in coupling to the Higgs field via a non-renormalizeable dimension-5 operator or via a Higgs triplet, for which there is no experimental evidence. We thus set $m_L = 0$.

By diagonalizing the mass matrix, we obtain the physical neutrino masses m_1 and m_2 (note that this is for a single neutrino generation only):

$$U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} = \begin{pmatrix} m_R + \sqrt{m_R^2 + 4m_D^2} & 0 \\ 0 & m_R + \sqrt{m_R^2 + 4m_D^2} \end{pmatrix}. \quad (1.11)$$

Examining the case where a right-handed non-interacting neutrino is much heavier than the Dirac neutrino of the weak interaction, we obtain the following approximation for the mass eigenstates:

$$\begin{aligned} m_1 &\approx \frac{m_D^2}{m_R} \\ m_2 &\approx m_R. \end{aligned} \quad (1.12)$$

This is known as the Seesaw Mechanism, and is an attractive model explaining the lightness of the neutrinos we observe through the weak interaction. The Seesaw Mechanism postulates the existence of very heavy right-handed neutrinos that do not participate in the weak interaction and thus cannot be detected through Standard Model processes.

This particular neutrino mass model is known as the Type-I Seesaw Mechanism. Many other mechanisms for neutrino mass generation are possible, via seesaw mechanisms or different models [8].

1.2 Neutrino Experiments

The origin and nature of neutrino mass is one of the most exciting areas of research in modern high-energy physics, since the neutrino mass and mixing parameters are directly accessible with current technologies. This section will detail our present understanding of neutrino masses and mixing parameters.

1.2.1 Neutrino Flavor Oscillation

As mentioned in Sec. 1.1.2, neutrino flavor oscillation is an unexpected experimental discovery indicating that the current formulation of neutrinos in the Standard Model is incomplete. Neutrino oscillation also indicates that conservation of lepton flavor (e -ness, μ -ness, and τ -ness) is not a symmetry of the Standard Model. Here we will describe the phenomenon of neutrino flavor oscillation and discuss experiments that measure the oscillation parameters.

- Neutrino Oscillation Phenomenology

Consider a system consisting of two neutrino flavors. The flavor interaction eigenstates m_α, m_β are distinct from the mass eigenstates m_1, m_2 , and are related by a unitary transformation:

$$\begin{pmatrix} m_\alpha \\ m_\beta \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}. \quad (1.13)$$

The magnitude of the mixing between the states is described by the mixing angle θ ; no mixing occurs if $\theta = 0$ or $\pi/2$, while maximal mixing occurs if $\theta = \pi/4$. We express the neutrino flavor state time evolution using the free Hamiltonian:

$$|\nu_\alpha(t)\rangle = U_{\alpha i} e^{-i\hat{H}_i t} |\nu_i(0)\rangle$$

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos(\theta)e^{-iE_1 t}\nu_1 + \sin(\theta)e^{-iE_2 t}\nu_2 \\ -\sin(\theta)e^{-iE_1 t}\nu_1 + \cos(\theta)e^{-iE_2 t}\nu_2 \end{pmatrix} = e^{-iE_1 t} \begin{pmatrix} \cos(\theta)\nu_1 + \sin(\theta)e^{-i\Delta E t}\nu_2 \\ -\sin(\theta)\nu_1 + \cos(\theta)e^{-i\Delta E t}\nu_2 \end{pmatrix}. \quad (1.14)$$

We now consider the states as possessing approximately the same momentum. Since the neutrinos observed experimentally are all produced with a large energy compared to their rest mass ($\geq$ MeV vs. sub-eV), we make the approximation

$$\begin{aligned} \Delta E &= p \left(1 + \frac{m_1^2}{p^2} \right)^{1/2} - p \left(1 + \frac{m_2^2}{p^2} \right)^{1/2} \\ &\approx p \left(1 + \frac{m_1^2}{2p^2} \right) - p \left(1 + \frac{m_2^2}{2p^2} \right) = \frac{1}{2p} (m_1^2 - m_2^2) \approx \frac{\Delta m^2}{2E} \end{aligned} \quad (1.15)$$

where Δm^2 is the difference of the squared neutrino masses $m_1^2 - m_2^2$. Examining the squared overlap of one flavor state with another gives the probability of flavor oscillation $|\langle \nu_\alpha(0) | \nu_\beta(t) \rangle|^2$:

$$\begin{aligned} P(\nu_\alpha \rightarrow \nu_\beta) &= (\sin(\theta) \cos(\theta))^2 |1 - e^{i\frac{\Delta m^2 t}{2E}}|^2 \\ &= 2(\sin(\theta) \cos(\theta))^2 (1 - \cos(\Delta m^2 \frac{t}{2E})). \\ &= \sin(2\theta) \sin^2(\Delta m^2 \frac{L}{4E}) \end{aligned} \quad (1.16)$$

In the last line we have made the replacement $t \rightarrow L$, the distance traveled by the neutrino, due to the relativistic nature of the neutrinos. Equation 1.16 gives us intuition about the physics of neutrino oscillation: The amplitude of the oscillation probability is given by the mixing angle θ , while the frequency of the oscillation is given by the parameter $\Delta m^2 \frac{L}{4E}$. Two examples of two-neutrino flavor oscillation can be seen in Fig. 1.3.

As will be discussed in Sec. 1.2.2, it is useful to study the neutrino oscillation and survival probabilities as a function of neutrino energy, since experimental neutrino sources are not mono-energetic. In Fig. 1.4, we see the oscillation frequency scaling inversely with neutrino energy.

It is important to note that the oscillation frequency parameter $\Delta m^2 \frac{L}{4E}$ does not depend on the absolute neutrino mass, but the difference in squared masses. Thus, neutrino oscillation

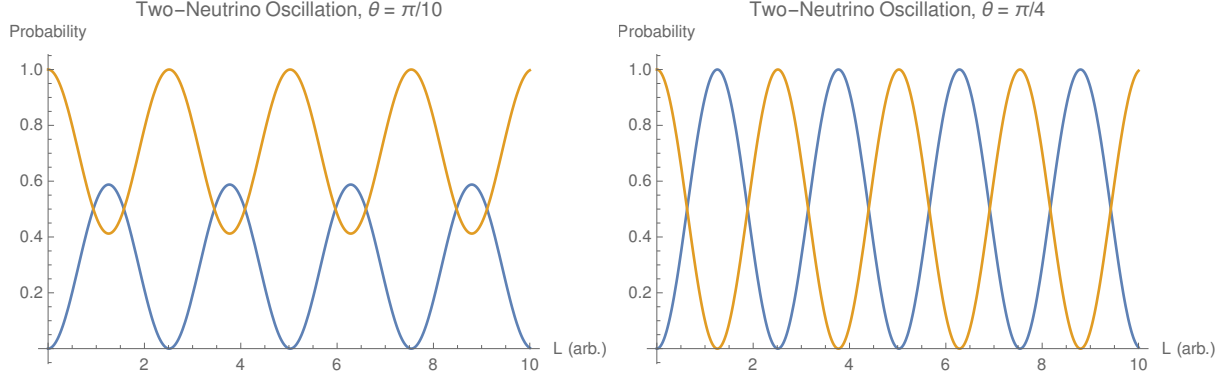


Figure 1.3: Example oscillation probabilities as a function of distance for $\Delta m^2/E = 5$, with a smaller mixing angle (left) and larger mixing angle (right). Orange lines represent survival probability $P(\alpha \rightarrow \alpha)$, blue lines represent oscillation probability $P(\alpha \rightarrow \beta)$.

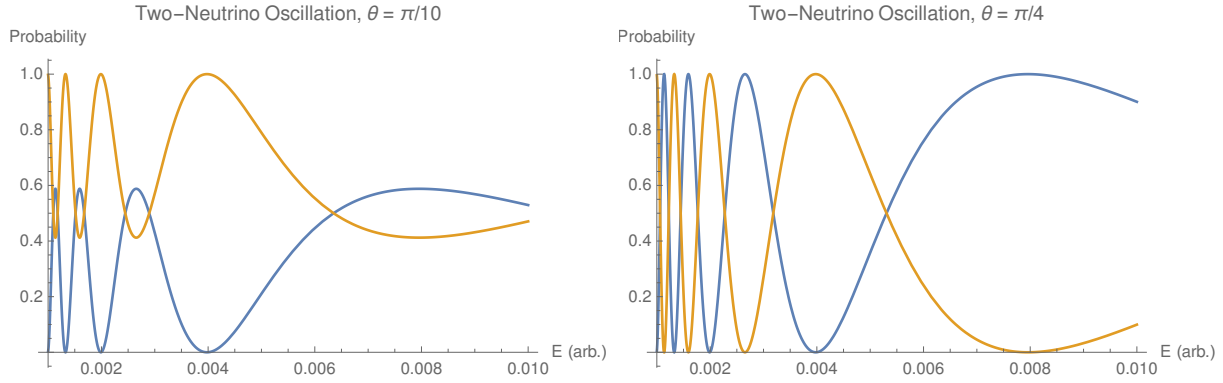


Figure 1.4: Example oscillation probabilities as a function of energy for $\Delta m^2 L = 0.05$, with mixing angles used in Fig. 1.3. Orange lines represent survival probabilities $P(\alpha \rightarrow \alpha)$, blue lines represent oscillation probabilities $P(\alpha \rightarrow \beta)$.

experiments are unable to determine the absolute neutrino mass scale. Additionally, the oscillation probability depends on the square of $\sin(\Delta m^2 \frac{L}{4E})$, meaning the sign of the mass difference cannot be determined. These two open questions — absolute neutrino mass scale and neutrino mass state ordering — are two major unresolved questions in neutrino physics.

- Three-Neutrino Mixing and the PMNS Matrix

Extending the simple two-neutrino mixing model to the three flavors of Standard Model neutrinos is relatively straightforward. The matrix relating the mass and flavor eigenstates, known as the PMNS matrix (see Sec. 1.1.2), is a 3×3 unitary matrix characterized by 4 free parameters: the

mixing angles θ_{12} , θ_{13} , and θ_{23} , and a complex CP-violating phase δ_{CP} ³. The organization of these parameters can be viewed in Eq. 1.17, where they have been split into separate mixing sectors. The mixing angles give the magnitude of mixing between flavor states, while the CP-violating phase describes the amount of CP violation in the neutrino sector, or how differently neutrinos and antineutrinos behave. Some mixing angles are named corresponding to the experimental method in which they were first measured: θ_{12} is known as the solar mixing angle, and θ_{23} is known as the atmospheric mixing angle. θ_{13} is measured by accelerator-based and reactor-based experiments.

$$\begin{aligned}
 U_{\text{PMNS}} &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{\text{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{23}c_{13} \end{pmatrix} \\
 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{\text{CP}}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{\text{CP}}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1.17)
 \end{aligned}$$

- Neutrino Mass Hierarchy

In order for three non-degenerate neutrino mass states to exist, at least two states must have non-zero mass. The difference between the state masses, referred to as the *mass splittings* Δm_{31}^2 , Δm_{32}^2 and Δm_{21}^2 (defined as $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$), contain one of the significant unanswered questions in neutrino physics. While the magnitudes of all three splittings are somewhat precisely known (see Table 1.1), only the sign of Δm_{21}^2 is known. The ν_3 mass may either be much larger or much smaller than the ν_1 and ν_2 masses, which are closely spaced. If ν_3 is determined to be much heavier than ν_1 and ν_2 , neutrino mass states would follow the *normal hierarchy*, so-called because the ν_1 state, with the largest ν_e component, would be the lightest one. If ν_3 is determined to be much lighter than ν_1 and ν_2 , neutrino mass states would follow the so-called *inverted hierarchy*.

³ Being a 3x3 complex unitary matrix, U_{PMNS} would in general be described by 9 free parameters ($\frac{2n^2}{2}$), partitioned into three mixing angles and six phases. We may, however, re-phase the neutrino fields and their corresponding lepton partners without affecting the weak interaction Lagrangian or the neutrino mass terms. This re-phasing may be done with respect to one reference phase, and this is what we refer to as δ_{CP} . If neutrinos are Majorana particles, however, this re-phasing scheme does not work for all neutrino fields, and the mixing matrix requires two more Majorana phases [13].

The sign of Δm_{21}^2 has an effect on neutrino oscillation in the solar neutrino sector. Oscillation probabilities are modified by the presence and density of matter in the Sun, and the neutrino flavor composition exiting the Sun depends on a combination of Δm_{21}^2 and $\cos(\theta_{12})^4$. Measurement of the octant of the angle θ_{12} combined with solar neutrino oscillation data can thus yield a measurement of the sign of Δm_{21}^2 .

1.2.2 Current Measurements of Neutrino Oscillation Parameters

All of the parameters in the PMNS mixing matrix and the two independent neutrino mass splittings Δm_{12}^2 and $|\Delta m_{32}^2|$ have been measured (apart from δ_{CP}), some with very high degrees of precision. The current best-fit values for each oscillation parameter can be seen in Table 1.1.

Parameter	Normal Ordering: Best-Fit $\pm 1\sigma$	Normal Ordering: 3- σ Range	Inverted Ordering: Best-Fit $\pm 1\sigma$	Inverted Ordering: 3- σ Range
$\sin^2 \theta_{12}$	$0.304^{+0.012}_{-0.012}$	0.269 – 0.343	$0.304^{+0.013}_{-0.012}$	0.269 – 0.343
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	31.27 – 35.86	$33.45^{+0.78}_{-0.75}$	31.27 – 35.87
$\sin^2 \theta_{23}$	$0.573^{+0.016}_{-0.020}$	0.415 – 0.616	$0.575^{+0.016}_{-0.019}$	0.419 – 0.617
$\theta_{23}/^\circ$	$49.2^{+0.9}_{-1.2}$	40.1 – 51.7	$49.3^{+0.9}_{-1.1}$	40.3 – 51.8
$\sin^2 \theta_{13}$	$0.02219^{+0.00062}_{-0.00063}$	0.02032 – 0.02410	$0.02238^{+0.00063}_{-0.00062}$	0.02052 – 0.02428
$\theta_{13}/^\circ$	$8.57^{+0.12}_{-0.12}$	8.20 – 8.93	$8.60^{+0.12}_{-0.12}$	8.24 – 8.96
$\delta_{\text{CP}}/^\circ$	197^{+27}_{-24}	120 – 369	282^{+26}_{-30}	193 – 352
$\frac{\Delta m_{21}^2}{10^{-5}\text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	06.82 – 8.04	$7.42^{+0.21}_{-0.20}$	6.82 – 8.04
$\frac{\Delta m_{3l}^2}{10^{-3}\text{eV}^2}$	$2.517^{+0.026}_{-0.028}$	2.435 – 2.598	$-2.498^{+0.028}_{-0.028}$	-2.581 – -2.414

Table 1.1: Current best-fit neutrino oscillation parameters as of July 2020, assuming normal hierarchy (left) and inverted hierarchy (right). Reproduced from [14].

⁴ More on matter effects in neutrino oscillation in the following chapter

- Solar Neutrino Sector: θ_{12} and Δm_{12}^2

The solar neutrino sector parameters $\sin^2(\theta_{12})$ and Δm_{12}^2 are measured by predicting the neutrino flavor content of fusion-produced neutrinos in the Sun and comparing the prediction to the solar neutrino flux measured on Earth⁵. The first measurements by the Homestake experiment in the 1960s discovered a discrepancy between the estimated solar neutrino flux and the measured electron neutrino flux [15]. Various subsequent experiments sensitive to charged-current and neutral-current neutrino interactions were performed, transforming solar neutrino measurements from the indication of a “problem” to a powerful means of measurement. Solar neutrino observatory results, combined with the results from reactor-based neutrino experiments, produced the measurements $\Delta m_{12}^2 \approx 7.4 \times 10^{-5} \text{eV}^2$ and $\sin^2(\theta_{12}) \approx 0.3$ [3].

- Atmospheric Neutrino Sector: θ_{23} and Δm_{23}^2

The atmospheric neutrino sector parameters θ_{23} and Δm_{23}^2 are measured by comparing the expected neutrino flux produced in cosmic ray interactions in the atmosphere with the neutrino flux measured on the earth. Experiments also compare the neutrino flux measured as a function of zenith angle, enabling a baseline-dependent measurement of neutrino flux. The atmospheric neutrino sector is distinct from the solar neutrino sector in that the energy range of detected neutrinos is significantly different — while solar neutrino energies span the range $\mathcal{O}(0.1) - \mathcal{O}(10) \text{MeV}$, atmospheric neutrino energies span a much larger range of $\mathcal{O}(0.1) \text{GeV} - \text{TeV}$. Definitive evidence supporting neutrino flavor oscillation came from measurements by the Super Kamiokande experiment [10], the results of which showed a large discrepancy between the measured and expected upward-traveling ν_μ flux directed through the earth (see Fig. 1.5). The discrepancy was removed once models accounted for $\nu_\mu \rightarrow \nu_\tau$ oscillations. Current best-fit results for the atmospheric neutrino sector are $\sin^2(\theta_{23}) \approx 0.57$ and $\Delta m_{32}^2 \approx 2.5 \times 10^{-3} \text{eV}^2$ [14]⁶.

- Accelerator Neutrino Sector: θ_{13} and δ_{CP}

⁵ The earth is, in fact, bathed in solar neutrinos, at a rate of about 6×10^{10} neutrinos per cm^2 per second [3].

⁶ Best-fit results reported for normal mass-ordering.

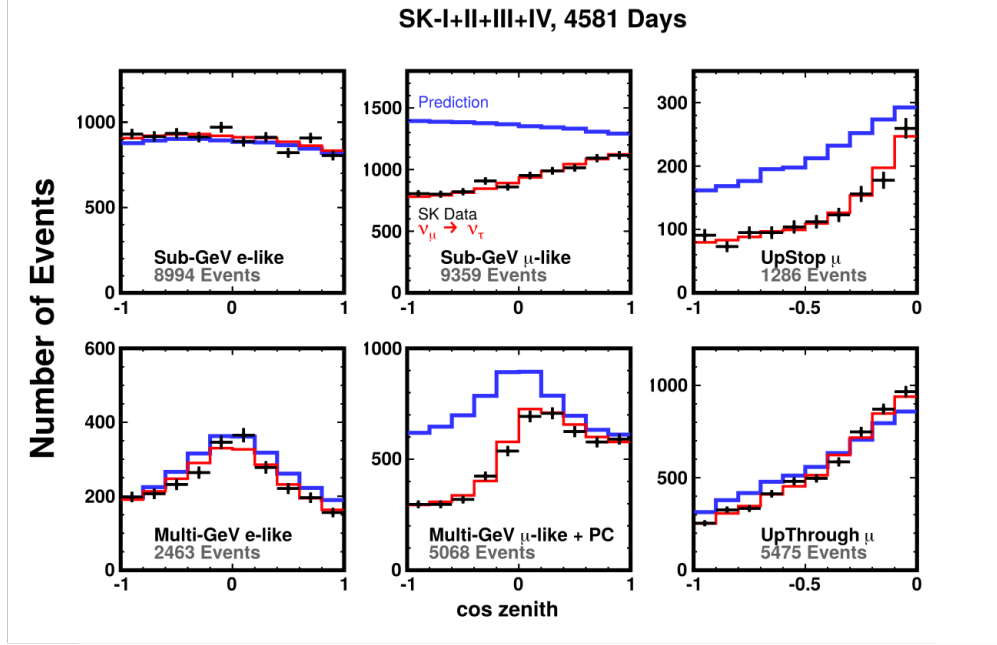


Figure 1.5: Atmospheric oscillation data from the Super Kamiokande experiment. Atmospheric neutrino event measurements pictured in black. Zenith angle is the polar angle as measured from the Super Kamiokande detector. Monte Carlo expectation assuming no flavor oscillation is given by the blue line, while Monte Carlo expectation assuming oscillations using the best-fit parameters is given by the red line. Reproduced from [10].

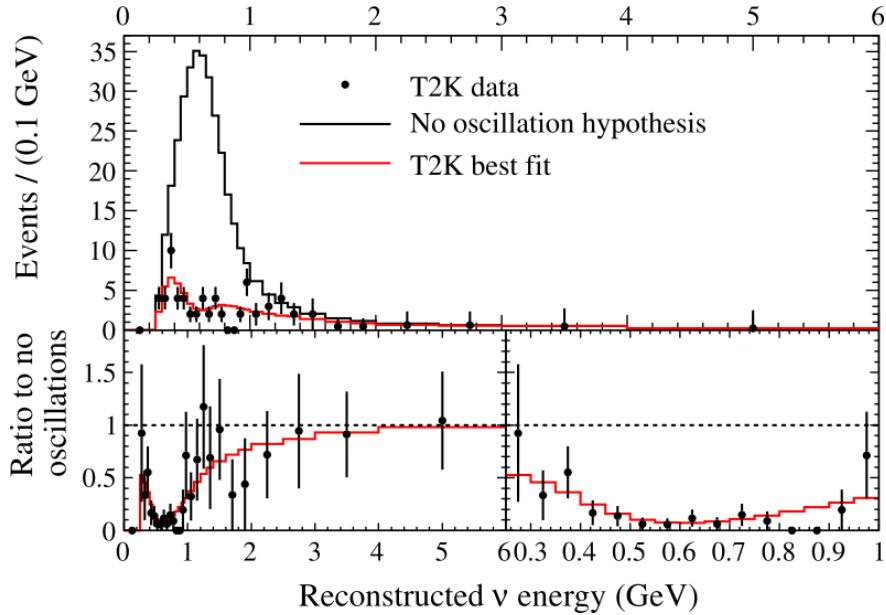


Figure 1.6: $\nu_\mu \rightarrow \nu_x$ Oscillation results reported by T2K. Black line represents estimated number of muon neutrino events with no neutrino oscillation. Data points are in black. Note the energy dependence of the oscillation spectrum, compared to Fig. 1.4. Reproduced from [11].

The final neutrino oscillation parameters, θ_{13} and δ_{CP} , are measured using human-made beams of neutrinos produced at particle accelerators. This type of experiment will be the focus of Chapter 2. In contrast to the previous two neutrino sectors, accelerator-based neutrino experiments have the freedom to choose the value of L/E in the oscillation frequency parameter. This provides enhanced neutrino samples at or near the oscillation maxima and minima, improving experimental signatures of neutrino oscillation or survival. Key measurements in long-baseline neutrino experiments are ν_μ disappearance and ν_e appearance from ν_μ , the latter of which is generated by a non-zero value of θ_{13} . Measurement of ν_e appearance at the T2K experiment can be seen in Fig. 1.6. Long-baseline neutrino beams have the ability to alter the beam generation process and produce a $\bar{\nu}_\mu$ -rich beam, enabling the measurement of δ_{CP} . The current best-fit values for the accelerator neutrino sector parameters are $\sin^2(\theta_{13}) \approx 0.3$ and $\delta_{\text{CP}} \approx 195^\circ$ ⁷.

It should be noted that flavor mixing in the lepton sector is drastically different than in the quark sector: the CKM matrix is nearly diagonal, while the PMNS matrix is distinctly non-diagonal. Two of the mixing angles in the lepton sector, θ_{12} and θ_{23} , are nearly maximal. The CP-violating phase δ_{CP} may be large enough to significantly impact the behavior of antineutrinos compared to neutrinos. These characteristics make the neutrino sector a very exciting potential source of new physics. In particular, CP-violation is a necessary condition in order to generate baryon asymmetry, which accounts for the prevalence of matter over antimatter in our universe [16].

⁷ Best-fit results reported for normal mass-ordering. It is important to note that the $3\text{-}\sigma$ range for δ_{CP} is $120^\circ\text{--}369^\circ$, meaning that δ_{CP} is not yet measured with a high degree of precision.

Chapter 2

Long-Baseline Neutrino Oscillation Experiments

This chapter describes in more detail the physics of long-baseline neutrino oscillation experiments, including results from current long-baseline experiments and prospects from future ones. In addition, this chapter introduces hadron production measurements for long-baseline experiments, and explains how these results are used to improve the quality of oscillation physics results.

2.1 Oscillation Parameter Sensitivity

To explore the experimental sensitivity of long-baseline neutrino oscillation experiments, we turn to the equation governing two-neutrino flavor oscillations. Here we convert back from natural units, obtaining a useful parameterization of the two-neutrino oscillation equation [8]:

$$\begin{aligned} P_{\nu_\alpha \rightarrow \nu_\beta} &= \sin(2\theta) \sin^2 \left(\Delta m^2 \frac{L}{4E} \right) \\ &\approx \sin(2\theta) \sin^2 \left(1.27 \frac{(\Delta m^2/\text{eV}^2)(L/\text{km})}{(E/\text{GeV})} \right). \end{aligned} \tag{2.1}$$

In the three-flavor oscillation picture, the survival and oscillation probabilities involve multiple oscillation parameters. In general, the time evolution of neutrino flavor eigenstates is given in terms of U_{PMNS} as

$$|\nu_\alpha(t)\rangle = e^{-iE_i t} U_{\alpha i}^* |\nu_i\rangle \tag{2.2}$$

where ν_α are the flavor eigenstates and ν_i are the mass eigenstates. The oscillation probability $P_{\nu_\alpha \rightarrow \nu_\beta}$ is given in terms of the PMNS matrix elements by [3]

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \delta_{\alpha\beta} - 4 \sum_{i < j}^n \Re [U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \sin^2 \left(1.27 \frac{(\Delta m^2 / \text{eV}^2)(L/\text{km})}{(E/\text{GeV})} \right) - 2 \sum_{i < j}^n \Im [U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j}] \sin \left(2.54 \frac{(\Delta m^2 / \text{eV}^2)(L/\text{km})}{(E/\text{GeV})} \right). \quad (2.3)$$

Here $\delta_{\alpha\beta}$ is the Kronecker delta. Let us examine this oscillation equation in the context of long-baseline oscillation experiments. As will be discussed in subsequent sections, the vast majority of neutrinos produced at accelerator complexes are of the flavor ν_μ . Focusing on the ν_μ term and examining survival probability, Eq. 2.3 becomes

$$P_{\nu_\mu \rightarrow \nu_\mu} \approx 1 - 4|U_{\mu 3}|^2(1 - |U_{\mu 3}|^2) \sin^2 \left(1.27 \frac{\Delta m_{32}^2 L}{E} \right) = 1 - (\sin^2(2\theta_{23}) \cos^4(\theta_{13}) + \sin^2(2\theta_{13}) \sin^2(\theta_{23})) \sin^2 \left(1.27 \frac{\Delta m_{32}^2 L}{E} \right). \quad (2.4)$$

Here we have taken the limit $|\Delta m_{21}^2| \ll |\Delta m_{32}^2|$ and made the approximation $|\Delta m_{31}^2| \approx |\Delta m_{32}^2|$. We have also made explicit use of the unitarity of U_{PMNS} . Thus, long-baseline survival measurements are sensitive to three oscillation parameters simultaneously (to leading order): θ_{23} , θ_{13} , and Δm_{23}^2 . Since θ_{13} is relatively small, the first term dominates the measured oscillations.

The transition probability $P_{\nu_\mu \rightarrow \nu_e}$ is the compliment of the survival probability, and thus measures the same oscillation parameters. If we focus on oscillation to one specific species of neutrino, for example ν_e , we can enhance the measurement of specific oscillation parameters by removing terms:

$$P_{\nu_\mu \rightarrow \nu_e} \approx \sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin^2 \left(1.27 \frac{\Delta m_{32}^2 L}{E} \right). \quad (2.5)$$

Finally, accelerator-based neutrino beams have the capability to create beams primarily composed of $\bar{\nu}_\mu$, as well. By comparing the probabilities $P_{\nu_\mu \rightarrow \nu_\mu}$ and $P_{\nu_\mu \rightarrow \nu_e}$ to $P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu}$ and $P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e}$, experimenters can measure the CP-violating phase δ_{CP} and quantify the difference between neutrino and antineutrino oscillations.

2.2 Neutrino Beam Creation

Neutrino beam creation and focusing is a multi-step process. Beamlines are typically constructed at existing accelerator facilities, where protons may be extracted from the accelerator and directed toward a neutrino beam facility. A schematic for the planned Long-Baseline Neutrino Facility at Fermilab can be seen in Fig. 2.1.

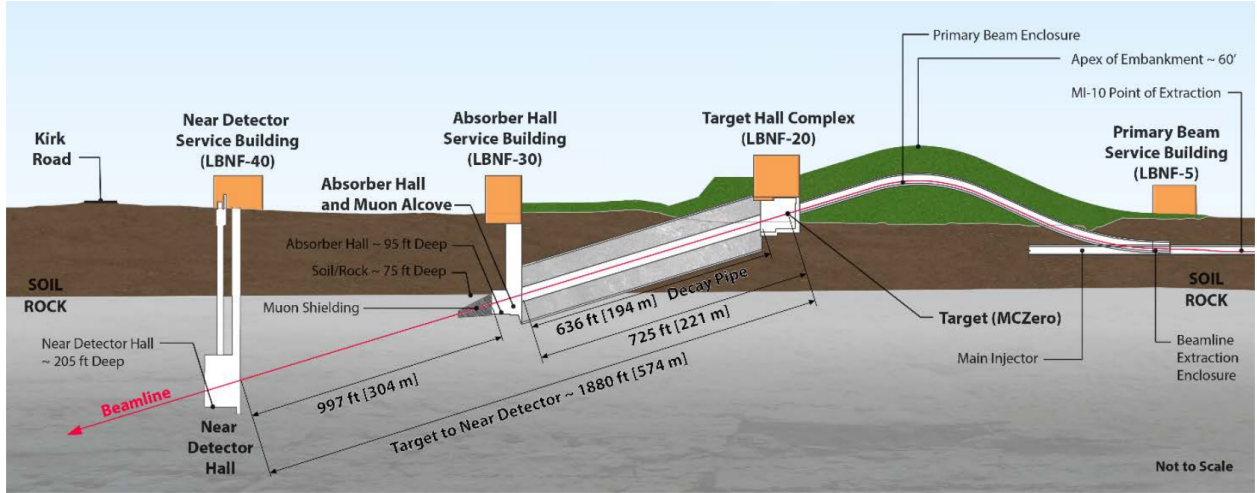


Figure 2.1: Schematic overview of the Long Baseline Neutrino Facility beamline at Fermilab, which will provide the neutrino beam for DUNE and the short-baseline neutrino program. Reproduced from [17].

First, a target material, usually carbon, is struck with an intense beam of medium-energy protons ($\mathcal{O}(1-100)$ GeV) extracted from an accelerator. This results in an intense beam of hadrons — mostly charged pions — traveling in the general direction of the neutrino detectors. These hadrons are allowed to decay in a vacant tunnel as they propagate. Many of these decays produce neutrinos, which then travel on uninhibited toward the neutrino detectors. Residual hadrons and muons are stopped by hadron absorbers. The primary reactions resulting in neutrino beam production can be seen in Table 2.1.

The hadrons produced in the initial target interactions will have some inherent divergence. The charged hadrons can be focused by a series of toroidal focusing elements known as magnetic “horns.” These horns tend to focus hadrons of the desired charge and de-focus hadrons of the

Neutrino-Producing Decay Channel	Branching Fraction (%)
$\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$	99.98
$\pi^\pm \rightarrow e^\pm + \nu_e(\bar{\nu}_e)$	0.01
$K^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$	64
$K^\pm \rightarrow \pi^0 + e^\pm + \nu_e(\bar{\nu}_e)$	5
$K^\pm \rightarrow \pi^0 + \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$	3
$K_L^0 \rightarrow \pi^\pm + e^\pm + \nu_e(\bar{\nu}_e)$	41
$K_L^0 \rightarrow \pi^\pm + \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$	27
$\mu^\pm \rightarrow e^\pm + \nu_e(\bar{\nu}_e) + \bar{\nu}_\mu(\nu_\mu)$	~ 100

Table 2.1: Relevant neutrino-producing decay channels. Branching ratios from [3].

opposite charge, helping to increase overall neutrino beam intensity and purity.

The final neutrino beam composition depends on the initial beam energy, target characteristics, and focusing scheme. Each experiment possesses its own specific physics goals, but most share similar beam requirements: a high-purity ν_μ or $\bar{\nu}_\mu$ beam with low flavor contamination from non- ν_μ neutrinos and a relatively low spread in energy. The expected neutrino beam composition for DUNE can be seen in Fig. 2.2.

The final neutrino beam composition directly depends on the initial hadronic interactions in the target and other beamline elements. In order to estimate the amount of neutrino flavor oscillation, the initial neutrino flux must be estimated precisely. Due to this, neutrino-producing hadronic interactions become a leading systematic uncertainty in neutrino oscillation analyses. This will be the subject of Sec. 2.4.

2.3 The Deep Underground Neutrino Experiment (DUNE)

The Deep Underground Neutrino Experiment (DUNE) is an international, multi-decade neutrino oscillation experiment planned to begin data taking around 2029 [19]. It is a long-baseline oscillation experiment with a baseline of 1300 km spanning from near Chicago, Illinois to the Black Hills, South Dakota. The proton beam energy and target material are under discussion, but a

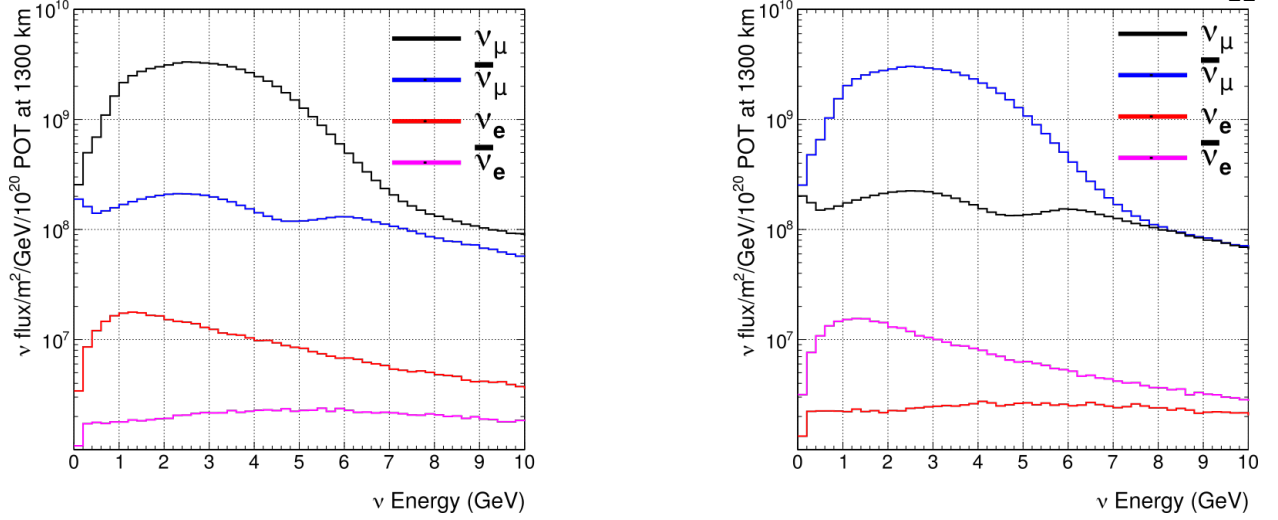


Figure 2.2: Expected un-oscillated neutrino flux for the LBNF neutrino facility. Neutrino focusing mode is shown on the left, while antineutrino focusing mode is shown on the right. The LBNF neutrino energy distribution is broader than past long-baseline neutrino experiments in order to measure both the first and second oscillation maxima. 10^{20} protons on target (POT) at 120 GeV corresponds to 0.1 MW·year. Nominal beam power is 1.2 MW. Reproduced from [18].

likely option is 120 GeV/c protons on a carbon target. If these characteristics are selected, the data analyzed in this thesis will be directly applicable to estimating the neutrino beam composition for DUNE.

2.3.1 DUNE Physics Goals

The primary DUNE scientific goals encompass many remaining issues in neutrino oscillation physics, along with some additional goals:

- Determine the neutrino mass ordering (the sign of Δm_{31}^2)
- Definitively measure the CP-violating phase δ_{CP}
- Precisely test the three-flavor neutrino oscillation paradigm
- Search for proton decay
- Detection and measurement of ν_e flux from a core-collapse supernova

One of the major strengths of DUNE is its oscillation baseline. The neutrino oscillation equations above consider neutrino oscillation in vacuum only. These equations are modified when considering oscillations in the presence of matter. In matter, an additional interaction term must be added to the Hamiltonian. The presence of electrons in matter (as opposed to other lepton flavor) results in a different interaction term for ν_e as opposed to $\nu_{\mu,\tau}$, as well as for $\bar{\nu}_e$. This interaction term describes forward elastic scattering of electrons and the electron neutrinos: While any neutrino flavor may exchange a Z^0 with any electron during propagation, only a ν_e may exchange a W^- with any electron and transform into an electron itself. This is known as the Mikhaev-Smirnov-Wolfenstein (MSW) effect [20].

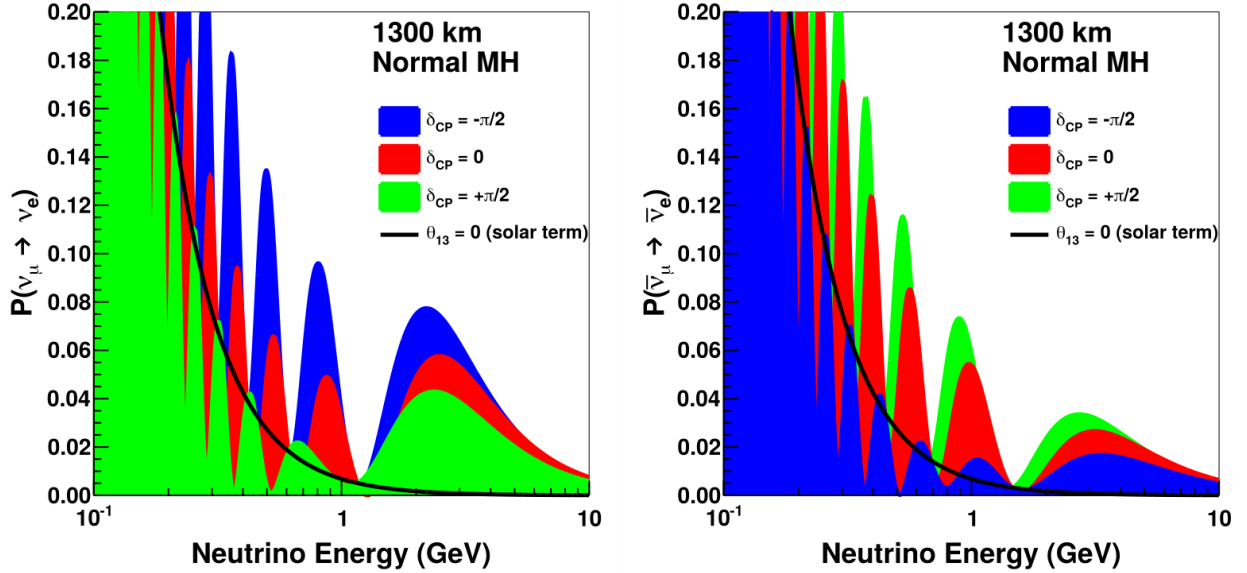


Figure 2.3: Expected DUNE oscillation probabilities for neutrino mode (left) and antineutrino mode (right). The calculation is shown for three possible values of δ_{CP} . Discrepancies between left and right are due to a combination of δ_{CP} and the MSW effect. Normal Mass Hierarchy is assumed. Reproduced from [18].

The additional term in the Hamiltonian is

$$V_e = \pm 2\sqrt{2}G_F N_e E \quad (2.6)$$

where G_F is the Fermi constant and N_e is the electron density. Neutrinos receive a positive-signed term, while antineutrinos receive a negative-signed term. The resulting transition probability

$P_{\nu_\mu \rightarrow \nu_e}$ is given by [18]

$$\begin{aligned}
P_{\nu_\mu \rightarrow \nu_e} \approx & \sin^2(2\theta_{13}) \sin^2(\theta_{23}) \frac{\sin^2(\Delta_{31} - aL)}{(\Delta_{31} - aL)^2} \Delta_{31}^2 \\
& + \sin^2(2\theta_{23}) \sin^2(2\theta_{13}) \sin^2(2\theta_{12}) \frac{\sin^2(\Delta_{31} - aL)}{(\Delta_{31} - aL)} \Delta_{31} \frac{\sin(aL)}{aL} \Delta_{21} \cos(\Delta_{31} + \delta_{\text{CP}}) \\
& + \cos^2(\theta_{23}) \sin^2(2\theta_{12}) \frac{\sin^2(aL)}{(aL)^2} \Delta_{21}^2
\end{aligned} \tag{2.7}$$

where $\Delta_{ij} = \Delta m_{ij}^2 L / 4E$ and $a = V_e / 4$. The quantities a and δ_{CP} switch signs for antineutrino transition probability. Thus, the MSW effect induces a neutrino-antineutrino oscillation asymmetry even with zero-valued δ_{CP} .

Since the second term in Eq. 2.7 depends directly on the squared mass differences Δm_{31}^2 , the neutrino mass hierarchy can be determined by measuring the $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ event rates. Figure 2.3 shows the expected probabilities for various values of δ_{CP} .

CP violation may be measured by examining the matter-antimatter asymmetry $\mathcal{A}_{\text{CP}}$:

$$\begin{aligned}
\mathcal{A}_{\text{CP}} &= \frac{P_{\nu_\mu \rightarrow \nu_e} - P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e}}{P_{\nu_\mu \rightarrow \nu_e} + P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e}} \\
&\approx \frac{\cos(\theta_{23}) \sin(2\theta_{12}) \sin(\delta_{\text{CP}})}{\sin(\theta_{23}) \sin(\theta_{13})} \left(\frac{\Delta m_{21}^2 L}{4E} \right) + \text{matter effects}
\end{aligned} \tag{2.8}$$

where terms proportional to higher powers of Δm_{21}^2 have been omitted [21]. If a measurement of δ_{CP} is made that is inconsistent with the measured value of $\mathcal{A}_{\text{CP}}$, the three-flavor neutrino oscillation model may be insufficient. DUNE will provide both a measurement of $\mathcal{A}_{\text{CP}}$ and δ_{CP} , paving the way for this comparison.

After an exposure of 300 kt · MW · year, corresponding to 3.5 years of neutrino-mode data taking and 3.5 years of antineutrino-mode data taking, DUNE will be able to exclude or measure significant portions of the δ_{CP} parameter space. Figure 2.4 shows the 3- σ and 5- σ sensitivity levels for all possible values of δ_{CP} . The measurable CP-violation signal varies as a function of this parameter and vanishes for $\delta_{\text{CP}} = -\pi, 0$, and π , corresponding to the regions where the sensitivity (red band) drops to 0.

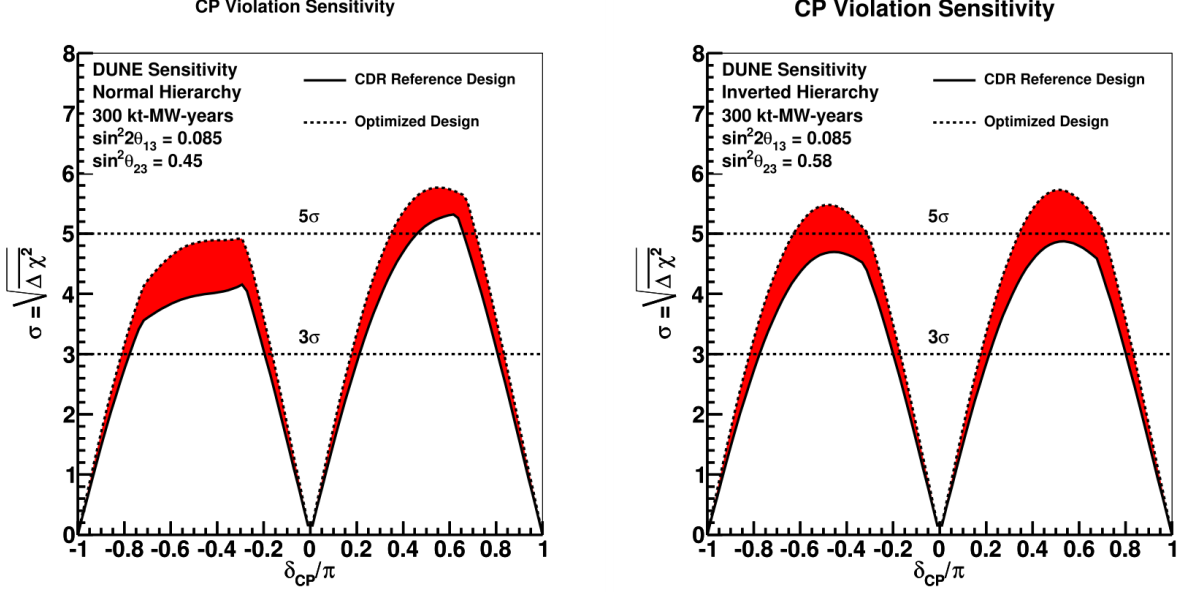


Figure 2.4: Expected DUNE experimental sensitivity to the CP-violating phase δ_{CP} after a total exposure time of seven years. δ_{CP} sensitivity for the Normal Hierarchy is shown on the left and for the inverted hierarchy is shown on the right. Reproduced from [18].

2.3.2 DUNE Neutrino Detectors

The DUNE experiment will use a near-detector and far-detector strategy common to long-baseline neutrino oscillation experiments. The near detector measures the neutrino beam composition before oscillation, and the far detector measures the beam composition after oscillation. This concept is used in order to cancel systematic uncertainties by taking the ratio of measured neutrino events as a function of energy¹. Each detector possesses independent physics objectives in addition to the primary DUNE physics goals.

2.3.2.1 DUNE Near Detector

The main physics goal of the DUNE near detector is to constrain the various systematic uncertainties in the oscillation analysis. In practice, this means making precise flavor content measurements in the portion of the neutrino beam sampled by the near detector, and making

¹ As will be discussed in Section 2.4, the systematic uncertainties do not entirely cancel in this scheme, mainly due to the differences in solid angle covered by the near and far detectors.

cross-section measurements of neutrinos on the target nuclei used in the far detector.

The DUNE near detector will be composed of several detectors located in a complex approximately 575 meters from the neutrino source and 60 meters below the surface of the Earth [22].

The planned components are:

- Liquid argon detector (ND-LAr)
- Gaseous argon detector (ND-GAr)
- Movable support system for ArgonCube and MPD (PRISM)
- System for on-Axis Neutrino Detection (SAND)

The upstream liquid argon detector serves to measure neutrino interactions on identical target nuclei to those in the Far Detector. Interactions in the upstream detector will be measured in the gaseous argon detector, a so-called Multi-Purpose Detector (MPD) which will be surrounded by a superconducting magnet in order to provide momentum determination. The core of the MPD is foreseen to be a high-pressure gaseous argon (HP-GAr) TPC. The TPC will be surrounded by a segmented electromagnetic calorimeter in order to reconstruct neutral pion decays. This calorimeter will be made of scintillator tiles and strips, which will be read out with silicon photomultipliers (SiPMs). The MPD will also be instrumented with a muon spectrometer in order to separate charged pions from muons, as this is difficult to achieve based solely on dE/dx . The muon detector technology has not yet been selected, but will be installed outside of the magnet. There will be at least two layers of absorber before the muon detectors to block pions not captured by the electromagnetic calorimeter.

A key component of the near detector design is the so-called DUNE-PRISM concept [22], involving the movement of the liquid argon and gaseous argon detectors transverse to the neutrino beam. By making measurements at different off-axis beam locations, the DUNE near detectors will build a neutrino flux profile as a function of off-axis angle. This will enable the isolation of beam flux and neutrino cross-section on the measured event rate. Since the neutrino energy spectrum is

strongly-dependent on the off-axis angle [23], measurements for a wide range of neutrino energies will be possible.

The final component of the DUNE near detector system is the System for on-axis Neutrino Detection (SAND). This detector component will serve as an online beam monitor for beam stability as a function of time. The SAND detector will include a scintillator-based tracker within a magnet system in order to perform momentum measurement and a calorimeter for total energy measurement. These detector components will allow SAND to measure the on-axis neutrino energy spectrum.

2.3.2.2 DUNE Far Detector

The DUNE far detector system will be composed of four 10 kiloton argon-based time projection chambers (TPCs) located 1300 km from the beam source and 4850 feet below the surface of the earth [24]. The first of these TPCs are planned to be liquid argon (LAr)-based. Measuring $12\text{ m} \times 14\text{ m} \times 58.2\text{ m}$, these will be the largest cryogenic TPCs ever constructed. Figure 2.5 demonstrates the sheer scale of these TPCs, which possess a drift length of 3.5 m. The large dense volume of liquid argon will act as both a target material for neutrino interactions and an active detector material, as produced charged particles will ionize argon atoms as they travel through the detectors. The detectors' TPC design will allow for three-dimensional track reconstruction with sub-centimeter resolution, as well as particle identification via specific ionization energy loss (dE/dx) measurement [24].

The high spatial resolution of the DUNE single-phase² far detectors is a result of the readout wireplane scheme and wire spacing. Four layers of readout wireplanes with various orientations are placed near the anode positions in the LAr volume. The wireplanes are biased so that all but the final plane are transparent to drifting electrons ionized by charged particles. The drifting electrons induce a signal on each plane and are collected by the final plane. A schematic of this process can

² “Single-phase” differentiates these detectors from so-called “dual-phase” detectors, which use liquid argon for a drift medium and gaseous argon in the amplification region.

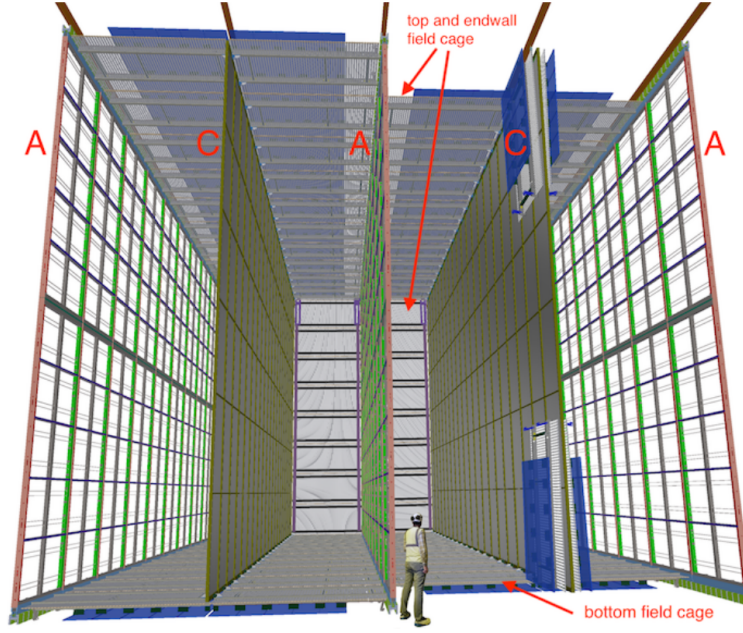


Figure 2.5: Graphic rendering of one of the DUNE single-phase LAr TPC modules. Anodes and cathodes providing the 500 V/cm drift field are labeled “A” and “C.” Reproduced from [24].

be seen in Fig. 2.6.

Drift times through the liquid argon will be on the order of milliseconds, with drift provided by a 500 V/cm electric field produced at a cathode held at -180 kV. The task of triggering the data acquisition system and determining individual event t_0 is accomplished by using the scintillating properties of argon. When argon atoms are ionized, they emit photons with a wavelength of 127 nm. This prompt scintillation light is detected by photodetectors on the outer regions of the TPC and can be used to begin the triggering and data acquisition process.

The DUNE LArTPC program is very ambitious in both detector design and engineering. A proof-of-concept detector known as ProtoDUNE-SP [25] has been constructed and installed in the CERN experimental hall EHN1. This detector is a scaled-down version of the planned DUNE single-phase LArTPC detectors, with an active volume of $7.2 \times 6.0 \times 6.9$ m³. ProtoDUNE-SP began operation in 2018 at CERN. One of its events can be seen in Fig. 2.6. The success of the ProtoDUNE-SP project and the quality of the recorded TPC data has demonstrated the effectiveness of DUNE’s planned single-phase LArTPC system.

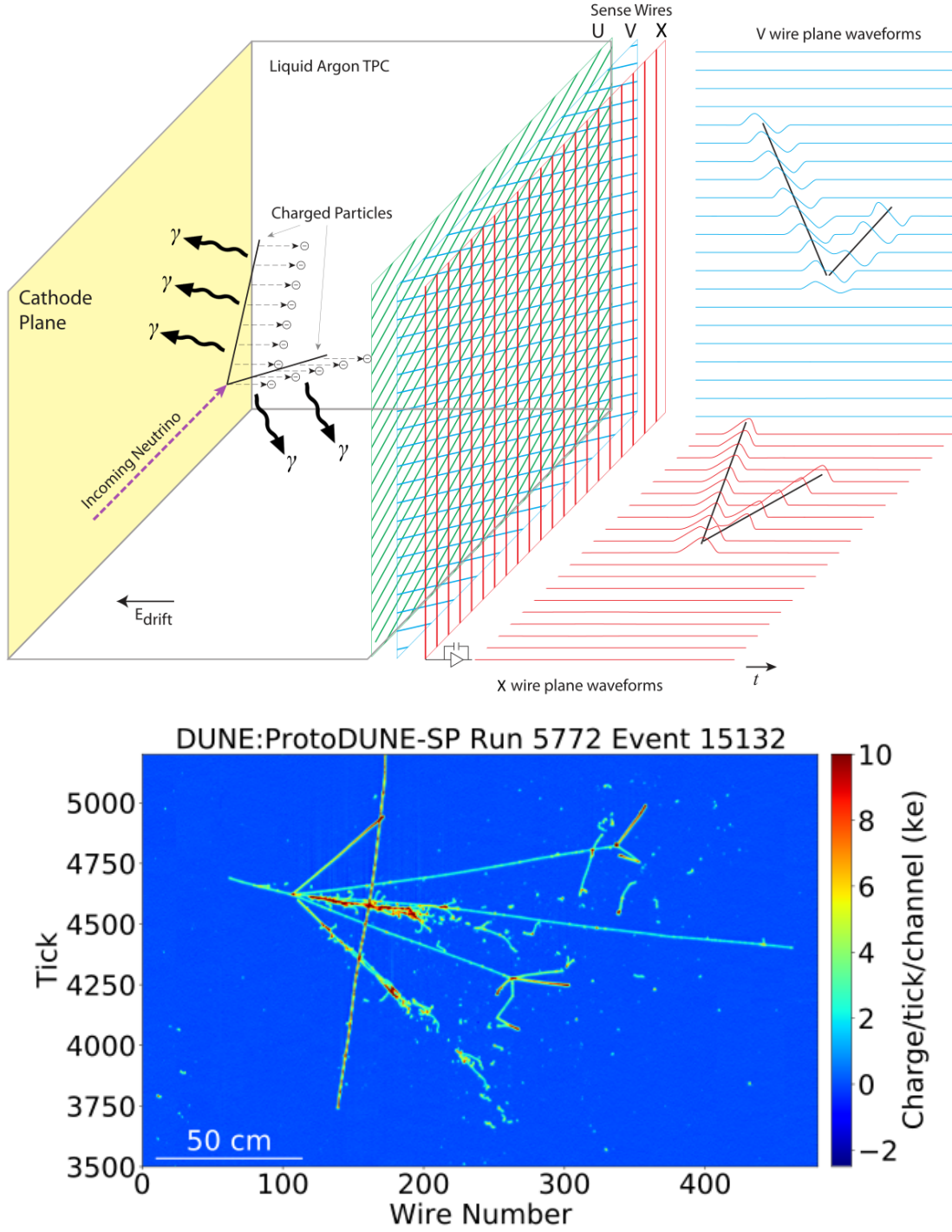


Figure 2.6: Top: A liquid argon TPC readout scheme from MicroBooNE, showing photon production, ionized electron drift, and signals recorded on TPC sense wires. Bottom: 6 GeV/ c pion candidate event recorded by the ProtoDUNE-SP detector, which acts as a proof-of-concept detector for the DUNE far detectors. Reproduced from [24] and [25].

2.4 Neutrino Beam Flux and Hadron Production Uncertainties

To achieve the stated physics goals, DUNE must have strict control over its systematic uncertainties. Uncontrolled systematic uncertainties of just a few percent have a dramatic impact on parameter measurement sensitivity. As can be seen in Fig. 2.7, reducing systematic uncertainties from 3% to 1% is equivalent to nearly doubling the experimental exposure time on the CP violation sensitivity. The current goal for $\nu_e(\bar{\nu}_e)$ normalization uncertainty is 2% [18].

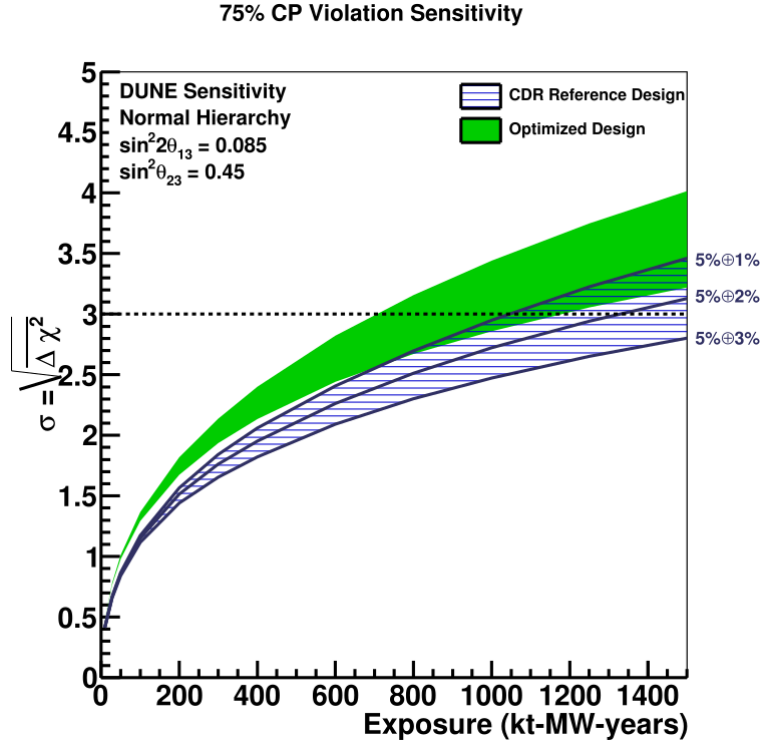


Figure 2.7: Impact of systematic uncertainties on CP violation sensitivity for 75% of δ_{CP} values. The 5% common to each curve refers to the absolute $\nu_\mu(\bar{\nu}_\mu)$ normalization uncertainty, while the other curves vary the absolute $\nu_e(\bar{\nu}_e)$ normalization. Reproduced from [18].

The prediction of neutrino beam flux at the near and far detectors is carried out via Monte Carlo simulation. These complex simulations are tasked with accurately predicting the neutrino flavor composition and energy spectra as seen by the near and far detectors, and uncertainties in this simulation are propagated to overall beam flux uncertainties. Two main sources of uncertainty in the beam Monte Carlo are variation in the beam optics ($\mathcal{O}(1\%)$) and uncertainties associated

with hadron production modeling ($\mathcal{O}(10\%)$). The hadron production uncertainty may be reduced by improving models or by using a Monte Carlo re-weighting scheme (see Section 2.5) using external constraint measurements. These are precisely the measurements presented in this thesis.

2.5 Beam Monte Carlo and Hadron Production Measurements

Neutrino beam Monte Carlo modeling is a critical part of oscillation experiment infrastructure, as inaccuracies in the prediction of neutrino flux at the near and far detectors can lead to biases in oscillation parameter measurements. These complex simulations include modeling the neutrino beam focusing elements and infrastructure, using accurate beamline element geometry details obtained from engineering specifications, and include the full chain of events beginning with incident particles and ending with produced neutrinos.

There are three main steps in a neutrino beam Monte Carlo:

- Primary interaction generation
- Primary particle propagation & reinteraction
- Re-weighting based on neutrino ancestors

In the first step, the initial primary hadronic interaction in the target is simulated. This is typically performed by using a dedicated hadronic cascade simulation model, such as FLUKA [26] or one from GEANT4 [27]. These models often do not accurately represent hadron production in the pertinent energy range, as will be demonstrated in Chapter 7.

The second step uses GEANT4 to propagate the initially produced particles through the beam facility, complete with realistic magnetic fields in the horns. This results in secondary and tertiary interactions in the target, the magnetic horns, support and cooling structures, and other beamline elements. The resulting hadrons and leptons subsequently decay, resulting in the simulated neutrino beam. A cartoon of this process can be seen in Fig. 2.8.

The final step in the simulation process is a re-weighting of the produced neutrino flux based on hadron production measurements. The neutrino beam content is re-weighted according to the

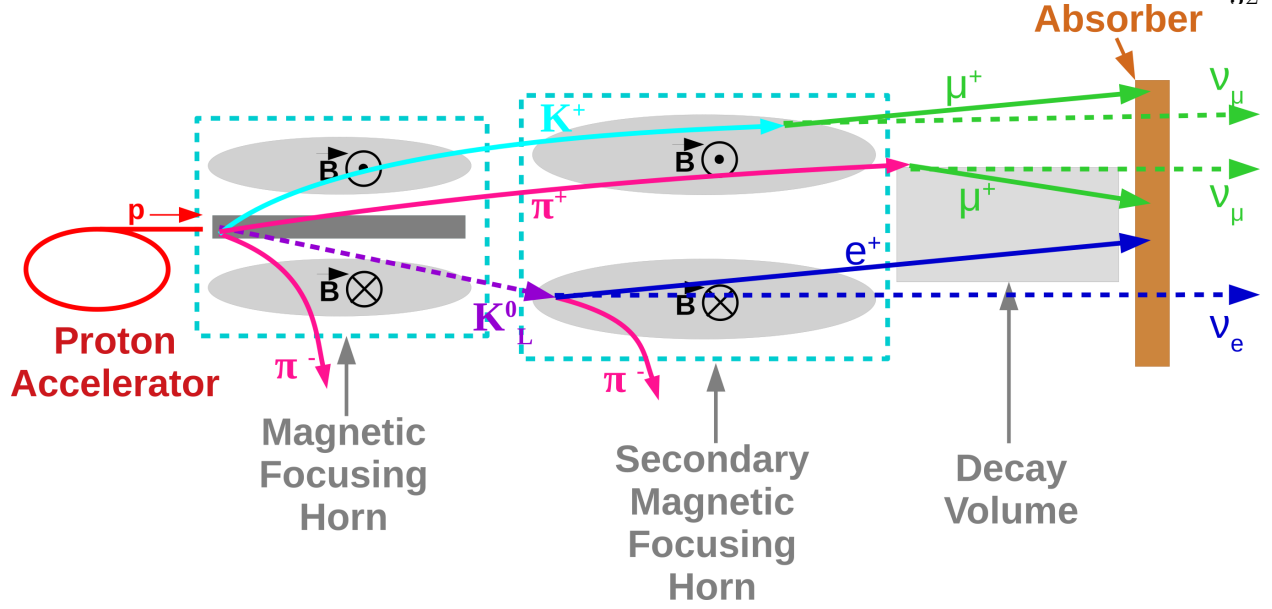


Figure 2.8: A cartoon of neutrino beam simulation with example interactions leading to neutrino creation. Neutrinos are produced both in primary interactions and subsequent reinteractions with beamline components.

hadron parents of the neutrinos. A given species of hadron s is re-weighted by the factor

$$w_s = \frac{m_s(p, \theta)^{\text{data}}}{m_s(p, \theta)^{\text{Monte Carlo}}} \quad (2.9)$$

where m_s is the multiplicity of the produced hadron, and p and θ are the kinematic properties of the hadron. If the Monte Carlo production multiplicity agrees with externally-measured multiplicities, this weighting factor is nearly 1. If there is a large discrepancy between external hadron production constraint data and the Monte Carlo predictions, the produced hadrons in the beam simulation will be scaled accordingly. This type of re-weighting is known as *thin-target re-weighting*.

Due to the wide variety of beamline materials and broad energy distributions of secondary hadrons, external constraint data is required involving many target materials, incident hadron species, and beam energies. NA61/SHINE is a facility capable of providing data for hadronic interactions of protons, $\pi^\pm$, and $K^\pm$ in the energy range of 13–350 GeV/ c on a wide variety of target materials. As the DUNE target interaction will likely be 120 GeV/ c protons on a carbon target, this is a critical reaction to record and analyze.

Varying the incident hadron species is critical due to the importance of reinteractions in

the beamline. As can be seen in Fig. 2.9, neutrinos produced at NuMI and traveling to the NO ν A detector are often not produced by secondary particles. A large fraction of ν_μ and ν_e in the neutrino beamline are produced in tertiary or subsequent reinteractions. Thus, hadron production measurements involving incident protons, pions, and kaons can improve neutrino beam flux predictions.

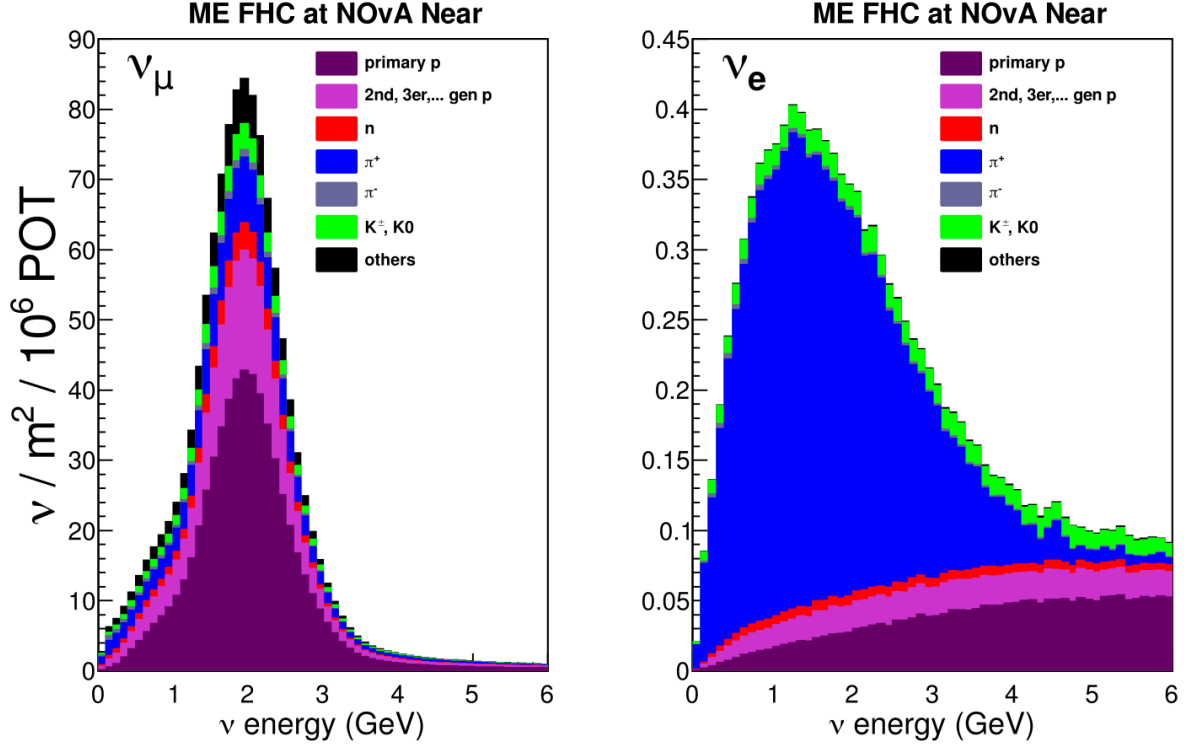


Figure 2.9: Muon and electron neutrino grandparents in the NuMI neutrino beam. Reproduced from [28].

An additional re-weighting technique, known as *replica target re-weighting*, is also very useful in improving beam flux predictions, but specific to individual experiments. This technique involves recording data using a replica target, identical to the one used in the neutrino beamline. The hadrons produced in the target are then directly measured, eliminating the need for simulating multiple reinteractions of hadrons inside the target. A slightly different Monte Carlo correction

factor is then used to re-weight hadron production:

$$w_s = \frac{m_s(p, \theta, z)^{\text{data}}}{m_s(p, \theta, z)^{\text{Monte Carlo}}} \quad (2.10)$$

where z is the binned z -position of the particle exiting the target. Experiments such as T2K have used both thin-target and replica-target re-weighting to improve their flux predictions [29]. Some example improvements made to the T2K flux prediction using hadronic interaction constraint data can be seen in Fig. 2.10.

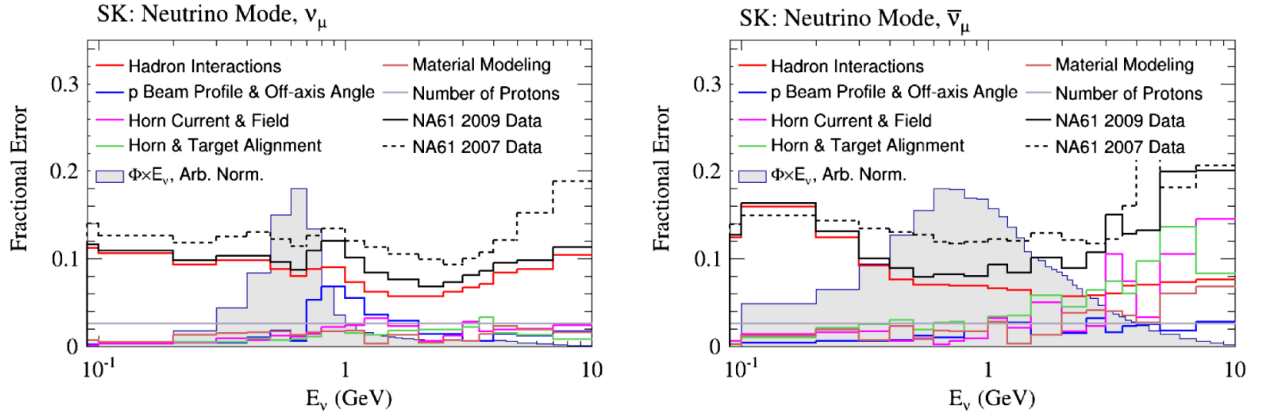


Figure 2.10: Influence of external hadron production constraint data on the T2K flux prediction. Sources of uncertainty are shown as a function of energy for ν_μ and $\bar{\nu}_\mu$ flux in neutrino mode. Reduction of several percent can be seen in the hadronic interaction uncertainties near the beam energy peak (shaded gray). Reproduced from [29].

2.6 Hadron Production Measurements: History

Hadron production measurements for neutrino experiments have been conducted for several decades at various facilities, including CERN and Fermilab [30]. Historically, these measurements were made at instrumented beam lines and yielded low phase space coverage limited to < 0.15 rad [31]. Later, dedicated experiments such as MIPP (Fermilab E907) and HARP performed hadron production measurements in larger regions of phase space [32] [33]. The MIPP experiment performed measurements using both thin targets and neutrino experiment replica targets. The NA49 experiment provided crucial higher-energy pion production measurements [34] and proton,

anti-proton, neutron, deuteron and triton production measurements [35] in 158 GeV/ c proton-carbon interactions.

Various parameterizations exist for extrapolating these limited measurements to various energies and target materials [36] [37], and are currently used to make hadron production predictions. As hadron production measurements are made in a variety of reactions across a range of energies, uncertainties associated with these extrapolating techniques can be reduced.

Chapter 3

The NA61/SHINE Experiment

NA61, also known as the SPS Heavy Ion and Neutrino Experiment (SHINE), is a multi-purpose fixed-target hadron spectrometer located at CERN's Super Proton Synchrotron (SPS)¹. NA61/SHINE possesses a rich physics program encompassing three areas of research: heavy ion physics, cosmic ray physics, and neutrino physics. The mostly TPC-based detector was in part inherited from the previous North Area experiment NA49, a heavy ion experiment active in the 1990s. Detector upgrades and improvements have been made since the NA49 era to increase phase space coverage and measurement capability.

Located on CERN's Prévessin campus, NA61/SHINE records reactions of 13-350 GeV/ c protons, charged pions, charged kaons, and heavy ions on various target materials. The neutrino physics program is dedicated to making hadron production measurements relevant to neutrino beam creation by measuring the produced particles that will eventually decay and become the neutrino beam. NA61 has a modifiable detector configuration, allowing for the measurement of both thin-target reactions and neutrino experiment replica target reactions.

This chapter will give an overview of the NA61/SHINE detector and facility. Detailed descriptions of each subdetector system will be left to the NA61/SHINE detector and DAQ papers [38] [39]. The Forward Time Projection Chambers (FTPCs) will be covered in greater detail here, as they play an important role in this thesis.

¹ The experiment is the 61st North Area experiment, hence the name NA61.

3.1 The NA61/SHINE Detector

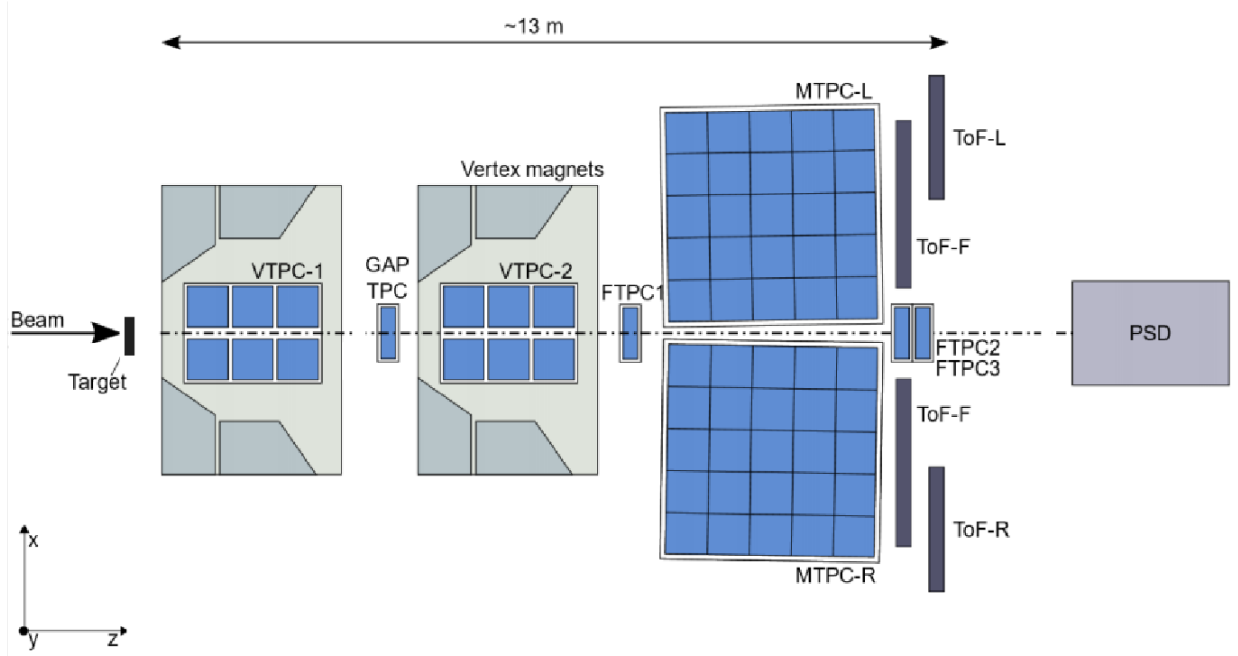


Figure 3.1: Overview of the NA61/SHINE detector. Detectors pictured: Vertex TPC 1 & 2 (VTPC-1 & VTPC-2), Main TPC Left & Right (MTPC-L & MTPC-R), Time-of-Flight wall Left & Right & Forward (TOF-L & TOF-R & TOF-F), Projectile Spectator Detector (PSD). Reproduced from [40].

NA61/SHINE is a large-acceptance hadron spectrometer. Its TPC-based tracking detectors are capable of recording charged particle trajectories and identifying particle species via specific ionization energy loss (dE/dx). It is a fixed-target experiment, accepting beam from CERN's SPS accelerator. The large array of SPS beam capabilities combined with target flexibility makes NA61/SHINE a very versatile detector, capable of studying many reactions across a wide range of energies.

3.1.1 Beam Facilities and Capabilities

NA61/SHINE is located on the H2 beamline in CERN's North Area. A schematic overview of the detector is shown in Fig. 3.1. The H2 beamline is fed by the Super Proton Synchrotron (SPS), which contains protons accelerated to 400 GeV/ c or various heavy ions. Protons and ions begin at rest in LINAC2 and LINAC3 respectively, and journey through the accelerator chain to

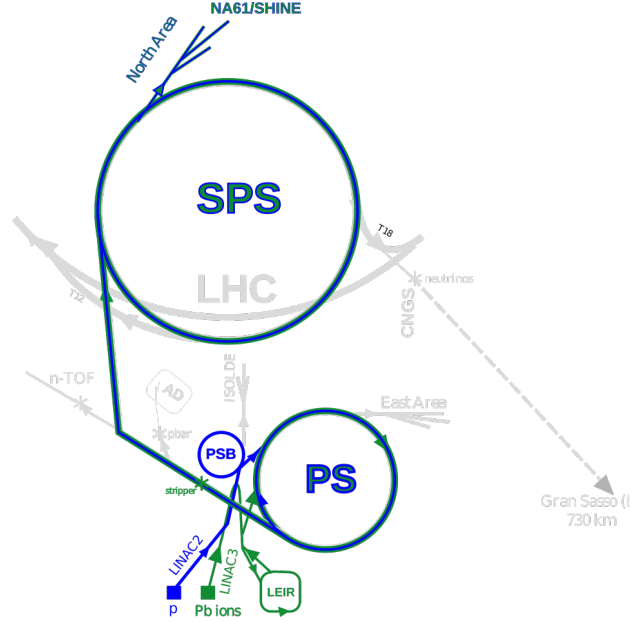


Figure 3.2: Overview of the SPS accelerator chain. Protons begin acceleration at LINAC2, while ions begin acceleration at LINAC3 and LEIR. NA61/SHINE's location off of the SPS ring is shown at the top of the figure. Reproduced from [38].

the SPS. The SPS is fed by the Proton Synchrotron (PS), which is fed by the Proton Synchrotron Booster (PSB) in the case of protons or the Low-Energy Ion ring (LEIR) in the case of heavy ions. A schematic of these accelerators can be seen in Fig. 3.2.

Beam in the SPS is used to feed the Large Hadron Collider (LHC) and for North Area experiments. To provide beam to the North Area, the SPS beam is de-bunched and slowly extracted over the course of several seconds. The beam travels through a series of bending and focusing magnets for approximately 1 km before being directed into a primary target facility. If primary protons are used, the 400 GeV/c protons are directed into the H2 beamline and transported to the experimental hall. If a secondary beam is desired, the primary protons are directed into a 1 m target, where a variety of particles are produced. Out of the array of produced particles, particles of the desired charge and momentum are selected using large dipole magnets. These particles are transported 535 m downstream to NA61/SHINE. An overview of this process can be seen in Fig. 3.3.

Secondary beams will contain a variety of charged particle species. In order to identify the

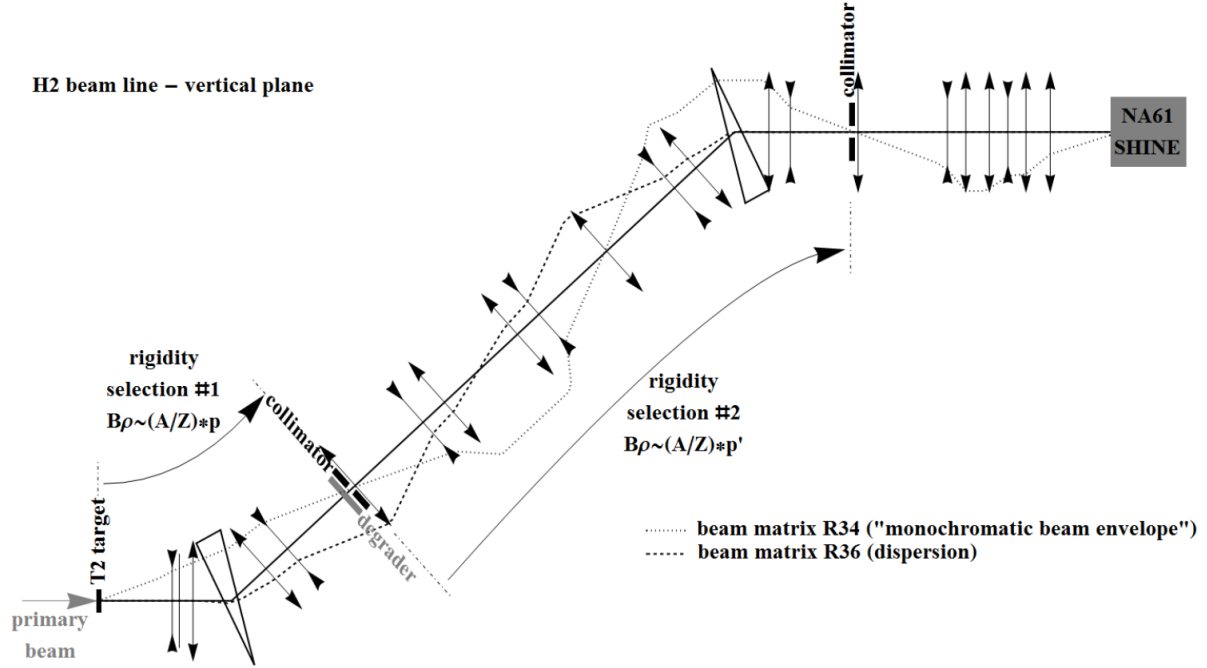


Figure 3.3: The H2 beamline and beam optics. The T2 production target is on the left. Lines with arrows on either end represent quadrupole magnets, used for focusing and de-focusing. Wedges represent dipole magnets, used for momentum selection. Approximate beam envelope and dispersion is shown in dashed lines. Reproduced from [38].

desired beam particle and trigger when it is detected, a particle identification detector is placed upstream of NA61/SHINE. This detector, known as the Cherenkov Differential Counter with Achromatic Ring Focus (CEDAR), is equipped with 8 photomultiplier tubes (PMTs) and a photon selection focusing system such that Cherenkov radiation from only one species of particle will be directed into the PMTs at once. By tuning the gas pressure inside the detector and changing the index of refraction, individual particle species can be selected with high accuracy [38].

3.1.2 Beam Detectors and Trigger

Several detectors and scintillators placed in the H2 beamline provide measurement of individual beam particle trajectory and allow for trigger construction. An example of a typical thin-target configuration of these detectors can be seen in Fig. 3.4.

Three gaseous strip detectors known as Beam Position Detectors (BPDs) provide transverse (x,y) position measurements of beam particles at $z = -3620$ cm, -1490 cm, and -670 cm in the global detector coordinate system. The nominal target z-location is -581 cm. Requirements on the quality of the BPD measurements help remove beam particles with trajectories that miss the target or have upstream interactions and scatter out of the beamline.

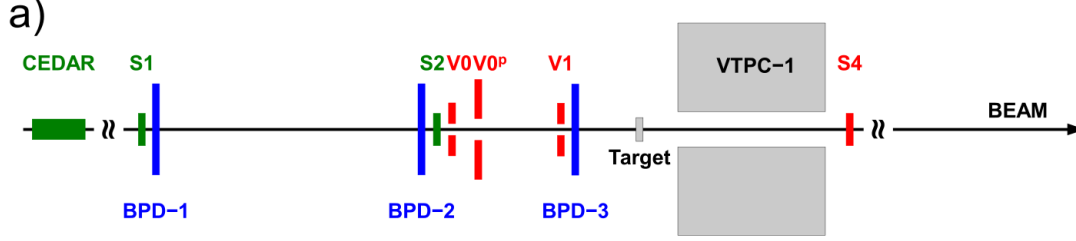


Figure 3.4: Beam trigger configuration “a” used for thin-target data taking. Beam position detectors pictured in blue, positive trigger components pictured in green, vetoing trigger components pictured in red. Reproduced from [38].

The trigger is composed of signals from several scintillators and the CEDAR positive identification signal. The scintillators provide a fast triggering signal requiring no reconstruction that instructs the DAQ to begin recording data. S1, a 6×6 cm square scintillator equipped with 4 PMTs, and S2, a 2.8 cm diameter scintillator equipped with 1 PMT, ensure that the trajectory of the beam particle is approximately acceptable. A finer trajectory selection is later performed with reconstructed BPD data. V0, V0P and V1, veto scintillators with holes of diameter 1 cm, 2 cm, and 0.8 cm, respectively, ensure that the beam particle does not scatter out of the beamline before hitting the target and that no coincident beam halo is present. The S4 scintillator, located at $z = -211$ cm and possessing a diameter of 2 cm, is an interaction selection scintillator: if a significant interaction occurs, the beam particle will scatter out of the nominal beamline and miss this counter.

The four main triggers (MT) are defined as:

- T1 (identified beam particle): $\text{CEDAR} \cdot \text{S1} \cdot \text{S2} \cdot \overline{\text{V0}} \cdot \overline{\text{V0P}} \cdot \overline{\text{V1}}$
- T2 (identified beam particle interaction): $\text{CEDAR} \cdot \text{S1} \cdot \text{S2} \cdot \overline{\text{V0}} \cdot \overline{\text{V0P}} \cdot \overline{\text{V1}} \cdot \text{S4}$

- T3 (unidentified beam particle): $S1 \cdot S2 \cdot \overline{V0} \cdot \overline{V0^P} \cdot \overline{V1}$
- T4 (unidentified beam particle interaction): $S1 \cdot S2 \cdot \overline{V0} \cdot \overline{V0^P} \cdot \overline{V1} \cdot \overline{S4}$

A prescaling module accepts triggers according to a set reduction factor, or prescale factor. The prescale factors are set such that the majority of recorded data is of type T2 (identified beam particle interaction), and a small fraction is an unbiased sample of T1, T3 or T4 triggers.

3.1.3 Time Projection Chambers (TPCs)

Time Projection Chambers (TPCs) form the basis of NA61/SHINE's tracking system. These large (1–4 m long) Ar/CO₂ (90%/10%) ionization detectors provide three-dimensional space point measurement of a charged particle's trajectory. The detectors also provide ionization measurements, allowing for particle identification via dE/dx . A cartoon of the TPC operational principle can be seen in Fig. 3.5.

The NA61/SHINE detector contains 8 TPCs as can be seen in Fig. 3.1, with each addressing different experimental requirements. Vertex TPC 1 & 2 (VTPC1 & VTPC2) provide 72 track position and dE/dx measurements for particles traversing the length of the chambers. These chambers sit inside superconducting dipole magnets VTX-I & VTX-II, enabling momentum estimation from the curvature of tracks in these detectors. The magnetic fields point in the $-y$ direction, and the magnets have a combined bending power of 9 T·m. Main TPCs Left & Right (MTPCL & MTPCR) provide 90 additional track measurements, increasing particle separation capability via dE/dx . The Gap TPC and Forward TPCs (GTPC, FTPC1, FTPC2 & FTPC3) sit directly in the beamline and measure the momentum and dE/dx of forward-going particles that pass through the gap between MTPCL & MTPCR. Particle species separation via TPC dE/dx can be seen in 3.6.

The drift velocity in each TPC is recorded as a function of time using dedicated drift velocity monitoring detectors. These detectors are filled with diverted chamber gas and measure the drift velocity using α sources. Drift velocity measurements are recorded and stored at 10 minute intervals [38].

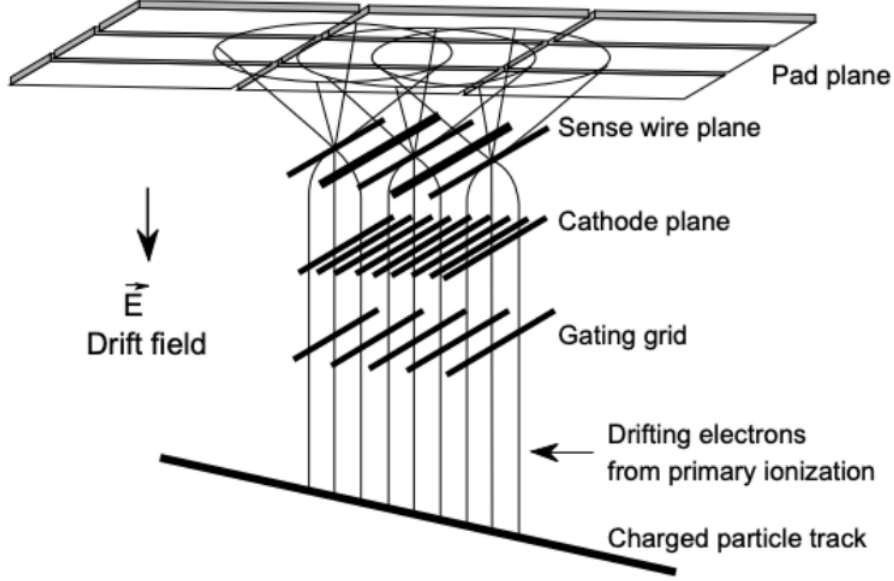


Figure 3.5: Cartoon of the TPC operational principle. A charged particle ionizes the Ar/CO₂ chamber gas as it passes through. The uniform electric drift field pushes ionized electrons to the readout wireplanes, where the signal is amplified at the high-voltage sense wire plane via Townsend avalanche. The induced signal on the printed circuit board pad plane is measured by front-end electronics. The cathode plane provides a termination to the chamber drift volume and prevents ion back-flow from the sense wires. The gating grid screens electrons when the trigger is inactive, preventing over-amplification and damage to the fragile sense wires in high-rate environments.

Significant data reconstruction and calibration efforts must be made to convert raw TPC information into meaningful analysis-level data. This will be further discussed in Chapter 4.

3.1.4 Time of Flight (ToF) Walls

Time of Flight (ToF) walls are employed to separate charged particle tracks in the region of overlapping Bethe-Bloch curves ($\sim 1\text{-}5\text{ GeV}/c$). Three ToF walls exist in NA61/SHINE: ToF-Left and ToF-right (ToF-L & ToF-R), which cover part of the MTPCL & MTPCR acceptance, are segmented in both x and y . The Forward ToF (ToF-F) consists of long scintillator bars instrumented on both ends with a PMT. In the ToF-F, the hit position along the scintillator bar is inferred by taking the difference in the signal arrival times in the upper and lower PMTs. Particle species separation via ToF can be seen in 3.7.

Since the ToF-L & ToF-R are segmented in y , the scintillators can be used as an absolute

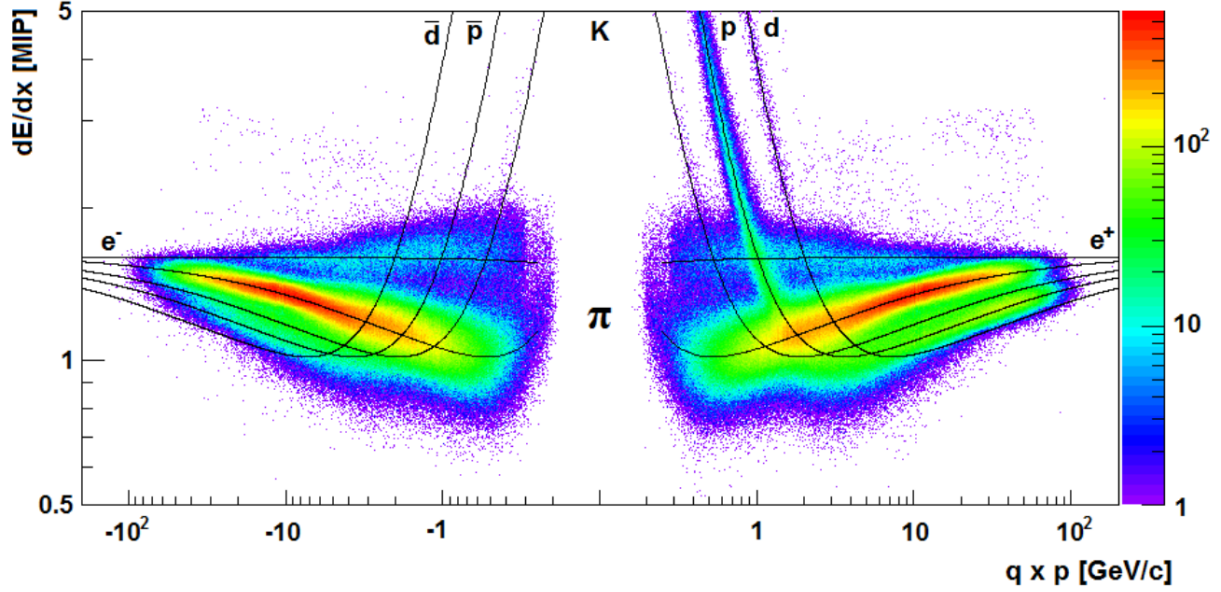


Figure 3.6: Capability of particle identification via specific energy loss (dE/dx) in NA61/SHINE TPCs. Positively-charged particles are shown on the right, while negatively-charged particles are shown on the left. Data shown is 80 GeV/c protons colliding with protons in a liquid hydrogen target vessel. Z -axis corresponds to track counts. Note: GTPC and FTPCs are not included on this plot. Reproduced from [38].

reference for MTPC track y -positions. This is a critical concept for TPC drift velocity calibration, which will be discussed in Chapter 5. Apart from drift velocity calibration, the ToF walls were not used in this analysis².

3.1.5 Projectile Spectator Detector (PSD)

The Projectile Spectator Detector (PSD) is a forward hadronic sampling calorimeter used for measuring the energy of forward-traveling spectator particles in nucleus-nucleus interactions. Measurement of projectile spectator energy can yield an estimation of the number of participating nucleons in a heavy ion interaction, and allows for event centrality estimation. The PSD is included at the trigger level in heavy-ion interactions, allowing for the selection of interesting central events and the rejection of peripheral collisions.

² The forward Time of Flight covers the most pertinent region of phase space for this analysis, but the raw data format was incompatible with the current SHINE software framework. Once the ToF data is included in SHINE, the analysis may be extended to cover Bethe-Bloch crossing regions.

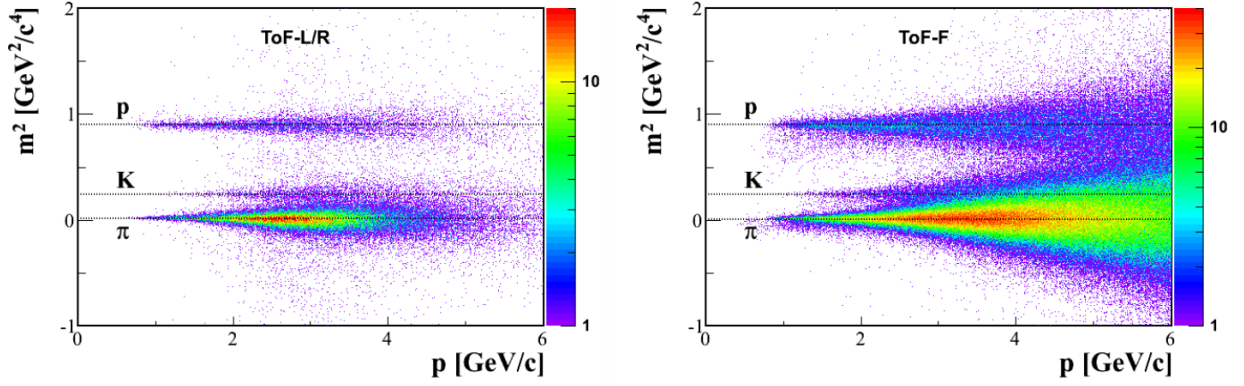


Figure 3.7: Capability of particle identification via Time of Flight (ToF). ToF L & R data is shown in left plot, and ToF-F data is shown in right plot. Z-axis corresponds to track counts. As track momentum increases, finite time resolution of the ToF walls becomes significant and particle separation grows more difficult. Reproduced from [38].

The PSD is composed of 44 modules, each individually composed of layers of lead and scintillator. Conversion in the lead plates leads to light production in the scintillator, which is then transmitted via wavelength-shifting fibers to photodetectors. As this analysis does not include events with heavy-ion projectile spectators, the PSD was not used.

3.2 The Forward Time Projection Chambers (FTPCs)

The Forward Time Projection Chambers (FTPCs) are a critical part of the analysis contained in this thesis. A significant portion of the author's thesis work was dedicated to the design, construction, installation and analysis of these TPCs. The details of the chambers and their operation will be described in this section. Further details can be found in [40].

The FTPC project was a joint collaboration between the US collaborators on NA61 and the Wigner Institute in Budapest, Hungary. The purpose of these chambers was to provide measurement capability in a significant region of uninstrumented phase space — up to and including the beam momentum. The beam gap in the VTPCs & MTPCs was designed to shield the TPC amplification wireplanes from the heavily-ionizing beam region, which can overwhelm and burn the sense wires in ion beam conditions. Forward-going particles are interesting to neutrino exper-

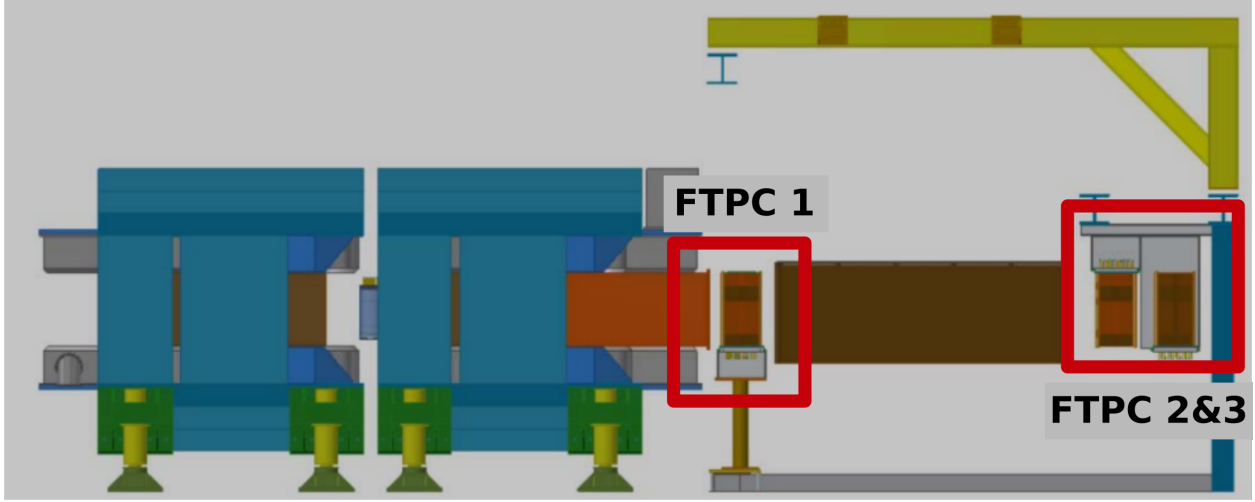


Figure 3.8: Locations of FTPCs in NA61/SHINE. Side view of detector is shown.

iments, such as DUNE, since a significant fraction of neutrinos are produced by these particles or products of their re-interactions. Thus, the decision was made to close the forward acceptance gap by constructing three new chambers to be placed in the beamline. The FTPC locations can be seen in Fig. 3.8. The impact of these chambers on NA61/SHINE's forward acceptance can be seen in Fig. 3.9. TPC technology was selected due to the previous experience of the collaboration with TPC wireplane construction, the desire for both momentum and dE/dx measurement, and the availability of spare front end electronics.

3.2.1 Tandem Field Cage Concept

In addition to closing the phase space gap for neutrino ancestor measurements, the FTPCs provided a detector R&D opportunity. Due to the potentially high-rate region the detectors are situated in, a method of rejecting pileup beam tracks arriving before or after the triggered beam track was devised. This so-called *tandem field cage* concept is achieved by flipping the orientation of successive drift field cages with respect to one another, resulting in opposite electron drift directions. When tracks are projected to their corresponding positions in each chamber using the trigger time, particles associated with the trigger will appear continuous, while particles not associated with the trigger will appear broken. Figure 3.10 shows a cartoon of this concept.

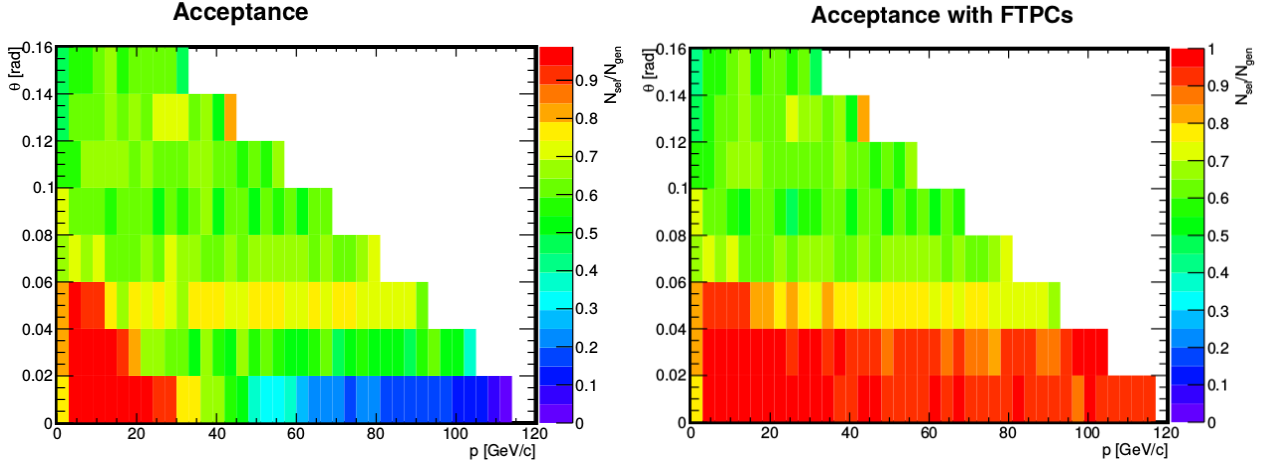


Figure 3.9: Demonstration of expected NA61 detector acceptance in 120 GeV/c proton-carbon interactions before and after the addition of the FTPCs, as obtained from Monte Carlo simulation. N_{gen} denotes the number of generated tracks in a given $[p, \theta]$ bin, and N_{sel} means the number of tracks that generated sufficient measurement points to pass preliminary track selection in that bin. Left panel: NA61 acceptance before the integration of the FTPCs. Right panel: NA61 acceptance with FTPCs installed.

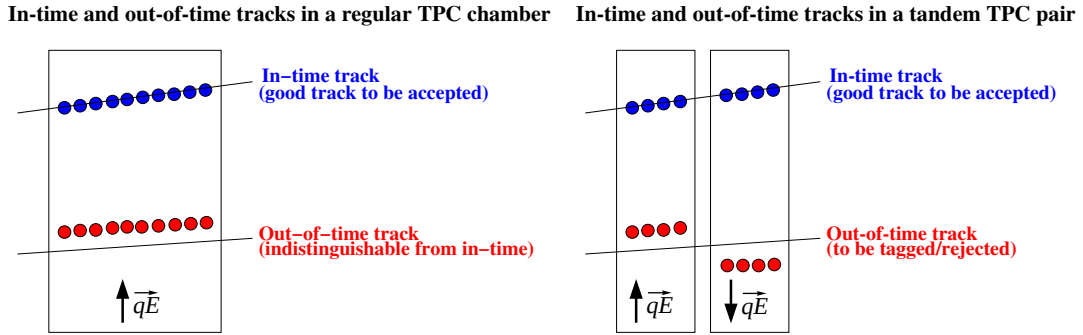


Figure 3.10: Cartoon showing the tandem-TPC concept. Colored dots represent reconstructed TPC measurements. Left panel: in a regular TPC, in-time tracks and the out-of-time particle tracks are indistinguishable. Right panel: in a tandem-TPC pair the drift directions ($q\vec{E}$) are not the same in the two drift volumes, and therefore the segments of an out-of-time particle track already start to drift apart by the time of the arrival of the interaction trigger. Thus, by their discontinuity, the out-of-time particle tracks become distinguishable from true in-time tracks.

3.2.2 Detector Layout

Significant consideration was given to the configuration and layout of the forward tracking system. Due to limited space in the existing detector, only two regions were available for the inclusion of forward detectors: between VTPC2 and the MTPCs, and downstream of the MTPCs.

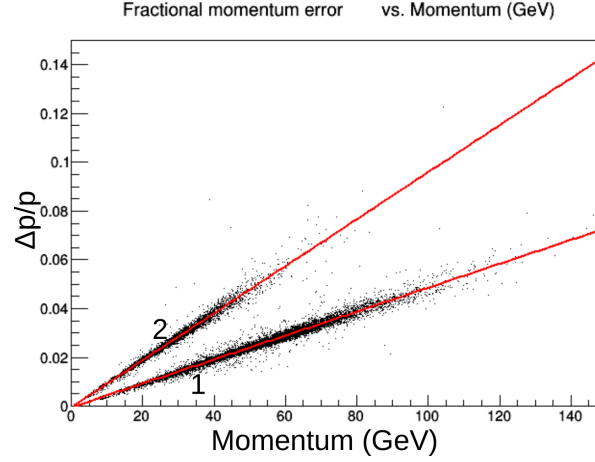


Figure 3.11: Expected FTFC momentum resolution in two scenarios: 1) With FTFC1 included. 2) Without FTFC1 included. The main contributing factor to momentum resolution is angular resolution, with which FTFC1 greatly assists.

The space between VTPC2 and the MTPCs is significantly more limited, and justification was needed in order to include FTFC1 in the forward tracking design. In the end, FTFC1 was included in order to achieve the target momentum resolution of less than 10% at 100 GeV/ c and to perform adequate track matching between the GTPC and FTFC2/3. Figure 3.11 compares the momentum resolution with and without the inclusion of FTFC1. The multiplicity in FTFC2 & FTFC3 is often larger than in FTFC1 due to particle production on the support structures for the MTPCs. As a result, FTFC1 is critical for track matching with the GTPC. More details on tracking with the FTFCs system and GTPC will come in chapter 4.

3.2.3 Field Cage and Gas Envelope

Another critical design element was the gas buffer volume. Diffusion of oxygen and other contaminants through gas volume walls can significantly detract from dE/dx resolution, as primary ionization will combine with these contaminants and will not be detected. In order to maintain gas purity, a double-walled gas envelope was designed. The inner portion of the gas envelope contains fresh chamber gas, pumped through holes in the perforated copper-G10 cathode. The gas is then forced through perforations in the field cage support near the wireplane and up into the protective

buffer volume, where it is then directed toward the chamber exhaust (see Fig. 3.12). This design expels the majority of contaminants that diffuse through the outer Mylar windows and into the buffer volume gas by directing this contaminated gas to the exhaust.

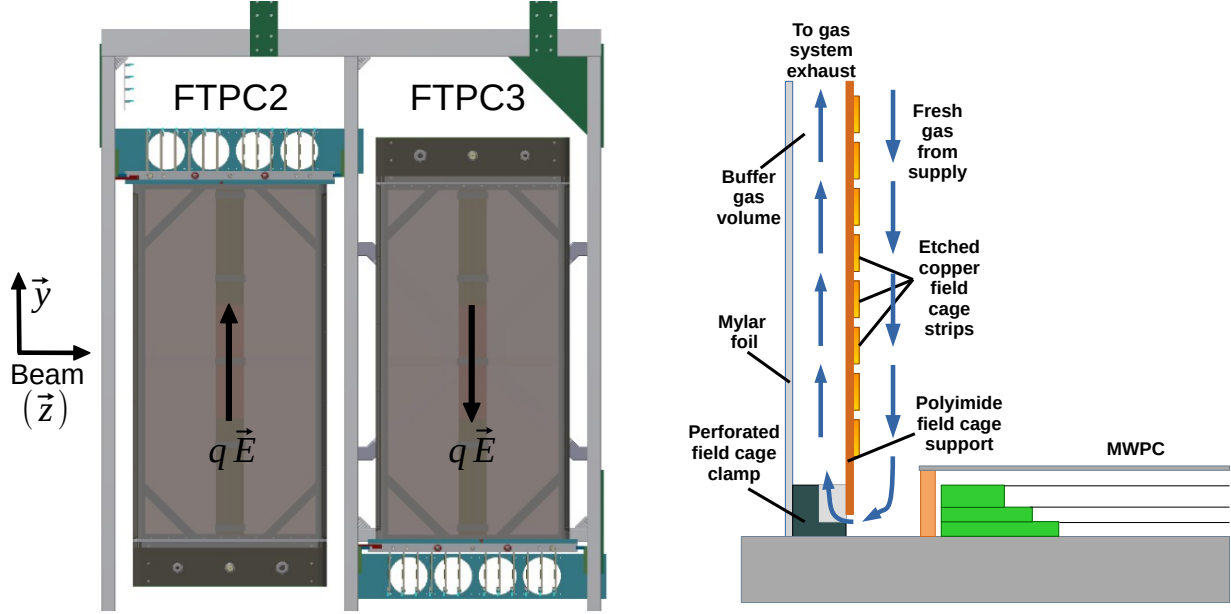


Figure 3.12: FTPC gas envelope and buffer volume design. Left: FTPCs 2 and 3 rendering in joint support. The gas volumes are the gray regions. Right: Cross-sectional view through buffer volume and field cage showing gas flow.

The FTPC field cage was designed as part of the gas envelope system. The cage itself is composed of 200 4.5 mm wide copper strips, separated by etched 0.5 mm slots, which are printed on a flexible Kapton sheet³. The Kapton, copper strips, and outer Mylar windows contribute just 0.34% radiation lengths per FTPC. The field cage strips are joined by 1 M Ω resistors. The cathode is kept at 17.3 kV, giving a potential difference of 86.5 V between strips and a nominal resistor chain current of 86.5 μ A. An image of the FTPC2 field cage, resistors and perforated cathode can be seen in Fig. 3.13. Stability of the Kapton field cages was difficult to achieve at this scale, as the sides were quite large (up to 1 m x 0.71 m). Future designs could benefit from a field cage tensioning system. The current suspension system relies on the mechanical rigidity of the Noryl

posts, to which the Kapton support is adhered.



Figure 3.13: Image of FTPC2 field cage and resistors and gas connections during assembly.

3.2.4 Wireplane and Readout Pads

Additional critical design parameters include wireplane spacing, wire gauge, and pad geometry. The wireplanes were designed to have the capabilities of the existing NA61/SHINE TPC readout planes. These multiwire proportional chambers (MWPCs) possess three wire planes spaced above the readout pad printed circuit board (PCB). Nearest to the PCB is the amplification plane, consisting of sense wires and field wires, where ionization is collected and amplified via Townsend avalanche. An image charge of the avalanche is induced on the readout pads, and this signal is amplified by the front-end electronics. In order to keep the amplification voltage reasonable, the sense wire radius is kept very small, so that the $\frac{1}{r}$ behavior of the electric field results in sufficient electron acceleration near the wire. The sense wires are thus the thinnest wire, at 22 μm in diameter. They are constructed from gold-plated tungsten, due to tungsten's high tensile strength and melting point. The thicker wires are constructed from a Cu-Be alloy. A full description of the wire properties can be found in Table 3.1, and a schematic of the wire spacings can be seen in Fig. 3.14.

The FTPC pad planes were designed with conservation of electronics channels in mind. Each FTPC contains 12 padrows, translating to a maximum of 12 trajectory and dE/dx measurements

³ The etched Kapton field cages began as continuous 2.8 m x 0.5 m sheets of copper-coated 75 μm -thick Kapton. Two such sheets were joined together to create the 1 m field cage. The 0.5 mm slots between the field cage strips were etched by Advanced Circuits.

wire type	material	diameter [μm]	tension [g]	pitch [mm]	distance to Pad Plane [mm]
Sense Wire	Au plated W	22	19	4	3.62
Field Wire	Cu-Be alloy	114	38	4	3.62
Cathode Grid	Cu-Be alloy	66	29	1	6.82
Gating Grid	Cu-Be alloy	66	29	2	13.24

Table 3.1: Main geometric and manufacturing parameters of the FTPC wire planes.

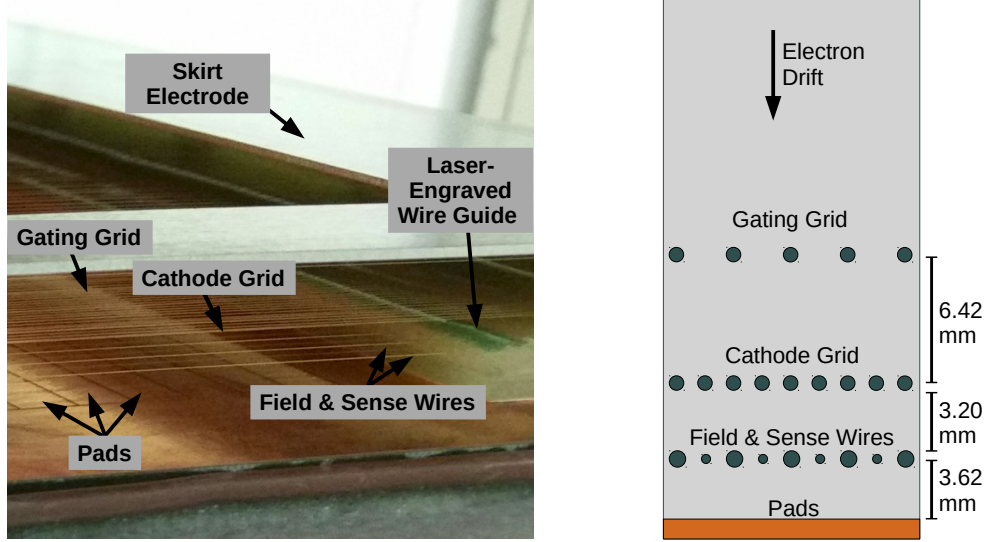


Figure 3.14: The structure of an FTPC amplification and readout wire plane. Left: image of constructed MWPC and individual wireplanes. The Gating Grid, Cathode (Frisch) Grid, and the Field Wire / Sense Wire Grid wire planes are stretched above the Pad Plane. The precision of the Field Wire / Sense Wire pitch is set by a laser engraved plastic wire guide. All sharp metal structures, potentially sources of corona discharges, are covered by a protective epoxy resin layer, as seen in the left figure. Right: Wireplane design cross-sectional view.

per chamber. FTPC1, being the critical piece of the forward lever arm directly after the GTPC, was instrumented with uniform pad sizes throughout the chamber. FTPC2 and FTPC3 were designed with two pad sizes: a high-resolution pad size of 4 mm on the outer part of the chamber (the upstream end of FTPC2 and the downstream end of FTPC3), and a low-resolution pad size of 6.6 mm on the inner part of the chamber (the downstream end of FTPC2 and the upstream end of FTPC3). By increasing the pad width, we reduced the demand for front-end electronics⁴. The pad

layouts can be seen in Fig. 3.15, and the parameters are found in Table 3.2.

The expected binary $d/\sqrt{12}$ resolutions given by the high-resolution and low-resolution pad sizes are 1.1 mm and 1.9 mm, respectively, where d is the pad width. However, the reconstruction algorithm for position measurements uses the charge amplitude recorded on each pad, enabling a better resolution than given by the naïve $d/\sqrt{12}$ estimation by a factor $f(N_{pads}, S/N)$ which is a function of the number of pads sampling the signal N_{pads} and the signal-to-noise ratio S/N . MTPC pads that are 3 mm wide result in approximately 200 μm point resolution, giving a signal-to-noise ratio factor of ~ 4.3 . Using this factor, we expect FTPC point resolution of $\sim 260 \mu\text{m}$ and $\sim 440 \mu\text{m}$ for the high-resolution and low-resolution sectors, respectively.

chamber name	sensitive area width [mm]	sensitive area length [mm]	pad width [mm]	pad length [mm]
FTPC1	512	480	4	40
FTPC2 upstream sector	640	240	4	40
FTPC2 downstream sector	640	240	6.6	40
FTPC3 upstream sector	640	240	6.6	40
FTPC3 downstream sector	640	240	4	40

Table 3.2: Parameters of the FTPC readout pad planes.

3.2.5 FTPC System Performance

The FTPCs were constructed over the period of 2016-2017 and installed in July 2017. After installing the FTPCs, the primary goal of increasing NA61/SHINE’s forward acceptance was achieved, as can be seen in Fig. 3.16. This increase in acceptance can be compared to the expected acceptance increase shown in Fig. 3.9. The final three neutrino data reactions of 2017 were taken with full forward phase space coverage, including the 120 GeV/ c proton-carbon data analyzed in this thesis⁵.

⁴ In retrospect, physics analyses would have benefited from a uniform pad size throughout each chamber. A varying pad size causes significant changes to the dE/dx corrections for clusters on each padrow, and effectively chops each dE/dx measurement in FTPC2 and FTPC3 into two parts, each containing only six dE/dx measurements. Such a small number of measurements significantly decreases the dE/dx resolution, making calibration of each half of these chambers very difficult. Lesson learned!

⁵ See Section 3.3

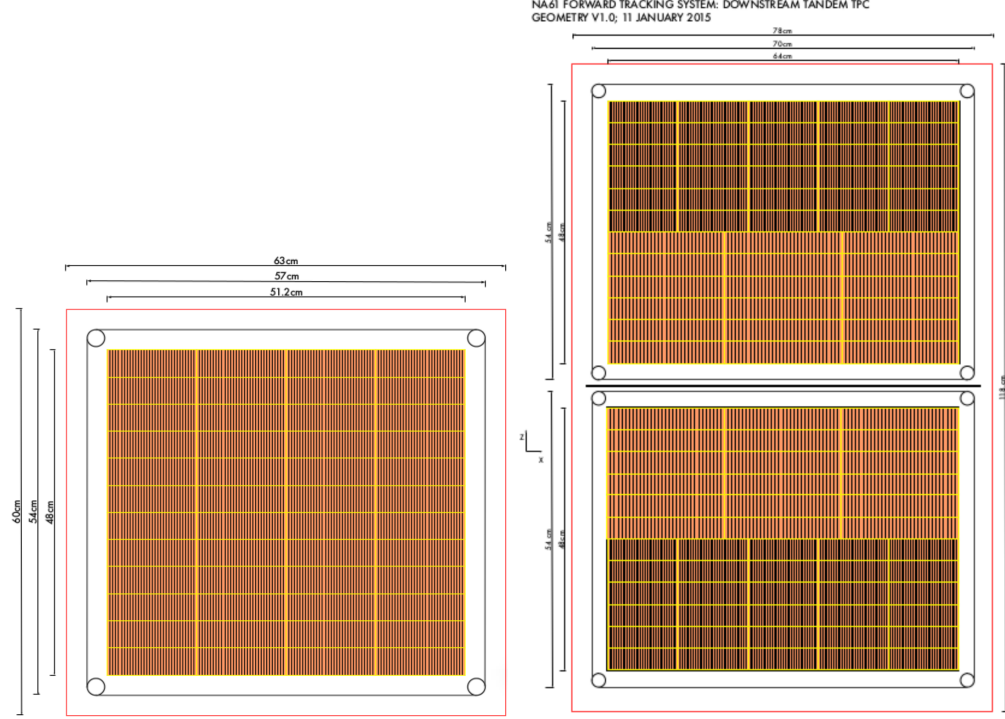


Figure 3.15: Schematic overviews of the FTTPC pad planes. FTTPC1's uniform high-resolution 3 mm pad plane is pictured on the left, while the non-uniform FTTPC2 & FTTPC3 pad planes can be seen on the right. The high-resolution sectors of FTTPCs 2 & 3 are oriented to be on the outside of the chamber, giving the outer reconstructed points in the tandem TPC system smaller uncertainties than the inner reconstructed points.

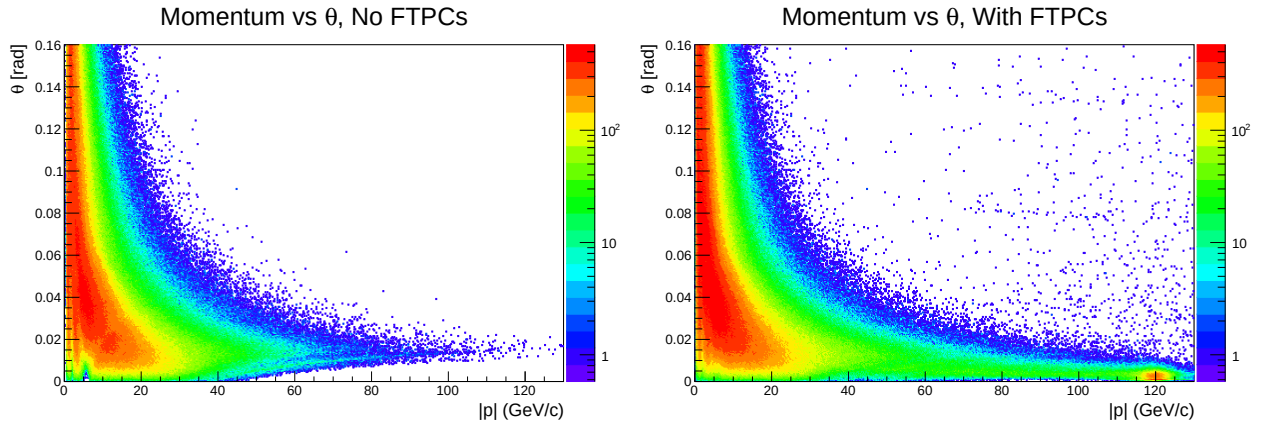


Figure 3.16: Reconstructed track phase space coverage in recorded 120 GeV/c proton-carbon interactions with and without inclusion of FTTPC tracks. Left panel: tracks passing minimal cuts without inclusion of FTTPC measurements. Right panel: tracks passing minimal cuts including FTTPC tracks. Note the beam spot at 120 GeV/c.

In addition, the tandem-TPC concept significantly reduces backgrounds related to out-of-time tracks. Figure 3.17 demonstrates the background reduction measured during the 2017 120 GeV/ c proton-carbon data taking period, in which the typical beam intensity ranged from 1 to 25 beam particles per recorded event. Figure 3.17 shows events from this period separated into three intensity regimes: low beam intensity (1-5 beam particles per event), medium beam intensity (6-10 beam particles per event), and high beam intensity (more than 10 beam particles per event). Each intensity regime shows the distribution of track position mismatch Δy between the tandem pairs, where the non-drift direction coordinate of the tracks are required to match ($|\Delta x| \leq 1$ cm). The peaks at $\Delta y = 0$ cm correspond to in-time tracks, while the continuous background corresponds to random matches of out-of-time track pieces. As the beam intensity increases, so does the background of out-of-time tracks.

This continuous background extends throughout the chamber in y , i.e. along the drift coordinate, for the full 100 cm of detector fiducial volume. If the tandem concept were not used, these tracks would match both in x and in y and thus would provide a significant background for data analysis. The method presented in Figure 3.17 can also be used for purity estimation of the out-of-time rejection of the tandem concept by extrapolating the out-of-time background passing to the region under the Δy cut of the tandem pair.

For more technical information and details, the reader is referred to [40].

3.3 Recorded Neutrino Physics Reactions, 2016-2018

This section details the reactions taken for neutrino hadron production analyses from 2016-2018. In 2016 and 2017, thin-target data was taken. The final three reactions were recorded with the FTPCs installed and operational. In 2018, replica target data was taken using the NO ν A replica target, which was shipped to CERN. The author served as run coordinator at points during each of these data taking periods. Reactions are recorded in Tables 3.3, 3.4 and 3.5.

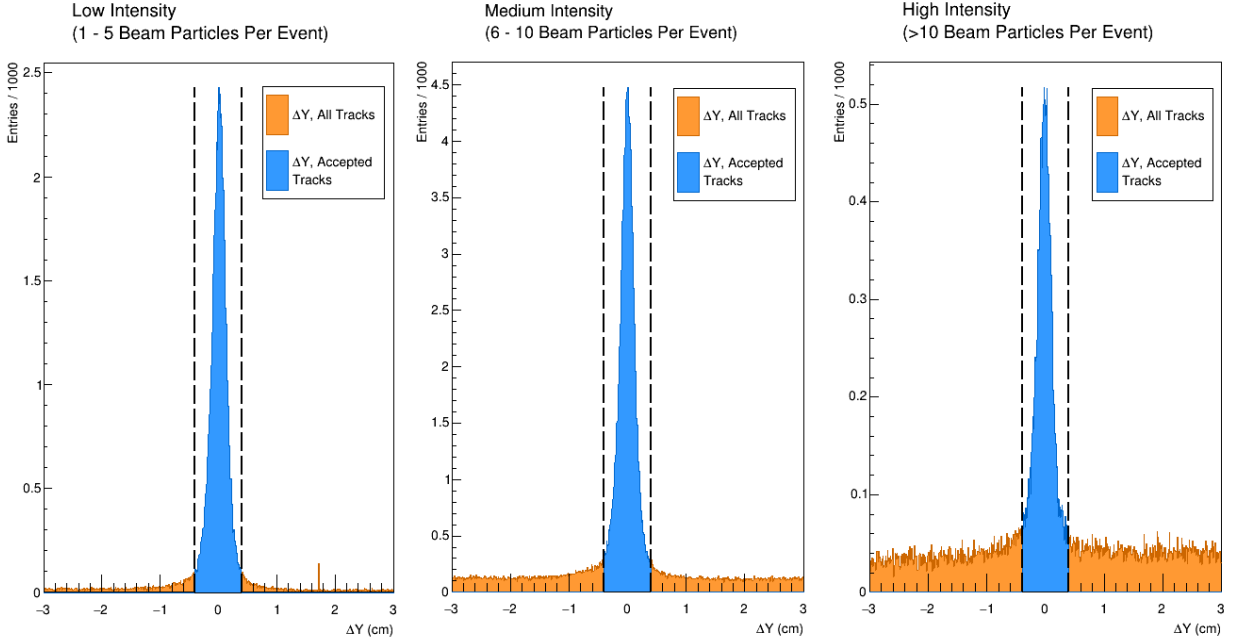


Figure 3.17: Background rejection with the tandem concept. The peak in each plot at 0 cm corresponds to in-time particles, while the continuous background (orange) corresponds to out-of-time particles. The out-of-time track background increases with beam intensity. The measured out-of-time fraction of tracks for each intensity (ratio of tracks in orange histogram to all reconstructed tracks) is 56% (low-intensity), 76% (medium-intensity) and 83% (high-intensity). For a standard TPC configuration, these tracks would be indistinguishable from the in-time tracks.

Reaction	Recorded Number of Events
60 GeV/ c π^+ + C	3.0×10^6
60 GeV/ c π^+ + Be	2.7×10^6
120 GeV/ c p + C	4.6×10^6
120 GeV/ c p + Be	2.5×10^6
60 GeV/ c p + C	3.1×10^6
60 GeV/ c p + Al	3.5×10^6
60 GeV/ c p + Be	2.2×10^6

Table 3.3: Neutrino reaction data recorded in 2016.

Reaction	Recorded Number of Events	FTPCs Installed
60 GeV/ c π^+ + Al	2.52×10^6	No
30 GeV/ c π^+ + C	2.19×10^6	No
60 GeV/ c π^- + C	3.57×10^6	No
120 GeV/ c p + C	2.57×10^6	Yes
120 GeV/ c p + Be	3.97×10^6	Yes
60 GeV/ c p + C	3.12×10^6	Yes

Table 3.4: Neutrino reaction data recorded in 2017.

Reaction	Recorded Number of Events	FTPCs Installed
120 GeV/ c p + NO ν A (Summer)	18.1×10^6	Yes
120 GeV/ c p + NO ν A (Autumn)	3.22×10^6	Yes

Table 3.5: Neutrino reaction data recorded in 2018.

Chapter 4

Experimental Data and Reconstruction

This chapter will detail the nature of NA61/SHINE experimental data and the reconstruction software used. The reconstruction software will be explained in detail, as the majority of it was developed by the author for the purpose of completing this analysis.

4.1 Data Acquisition, Trigger, and Raw Detector Data

The NA61/SHINE The Data Acquisition system (DAQ) is a critical piece of detector infrastructure. The DAQ orchestrates the data taking process, collecting and organizing the raw data from each subdetector system and storing it on tape. Since physically interesting events only occur for a few percent of beam particles that pass through the target, the trigger and the DAQ work together to store data for these interesting events only. The incident beam particle rate ranges from 1 - 100 kHz, and the DAQ records data at a rate of 100 Hz. A comprehensive overview of the NA61/SHINE DAQ can be found in [39]. A schematic overview of the DAQ can be seen in Fig. 4.1.

The trigger signal is formed by constructing a boolean combination of fast scintillator signals as described in Section 3.1.2. One of the four photomultipliers attached to the S1 beam counter, known as S1_1, possesses a time resolution of 30 picoseconds and serves as a pre-trigger signal for the CAMAC [41] and ToF readout electronics¹. The ToF electronics have a 300 nanosecond timeout, and if the pre-trigger signal is not confirmed by a Main Trigger signal (T1, T2, T3 or T4), the stored data is cleared. The S1_1 PMT also serves as a timing reference for the CAEN 1190

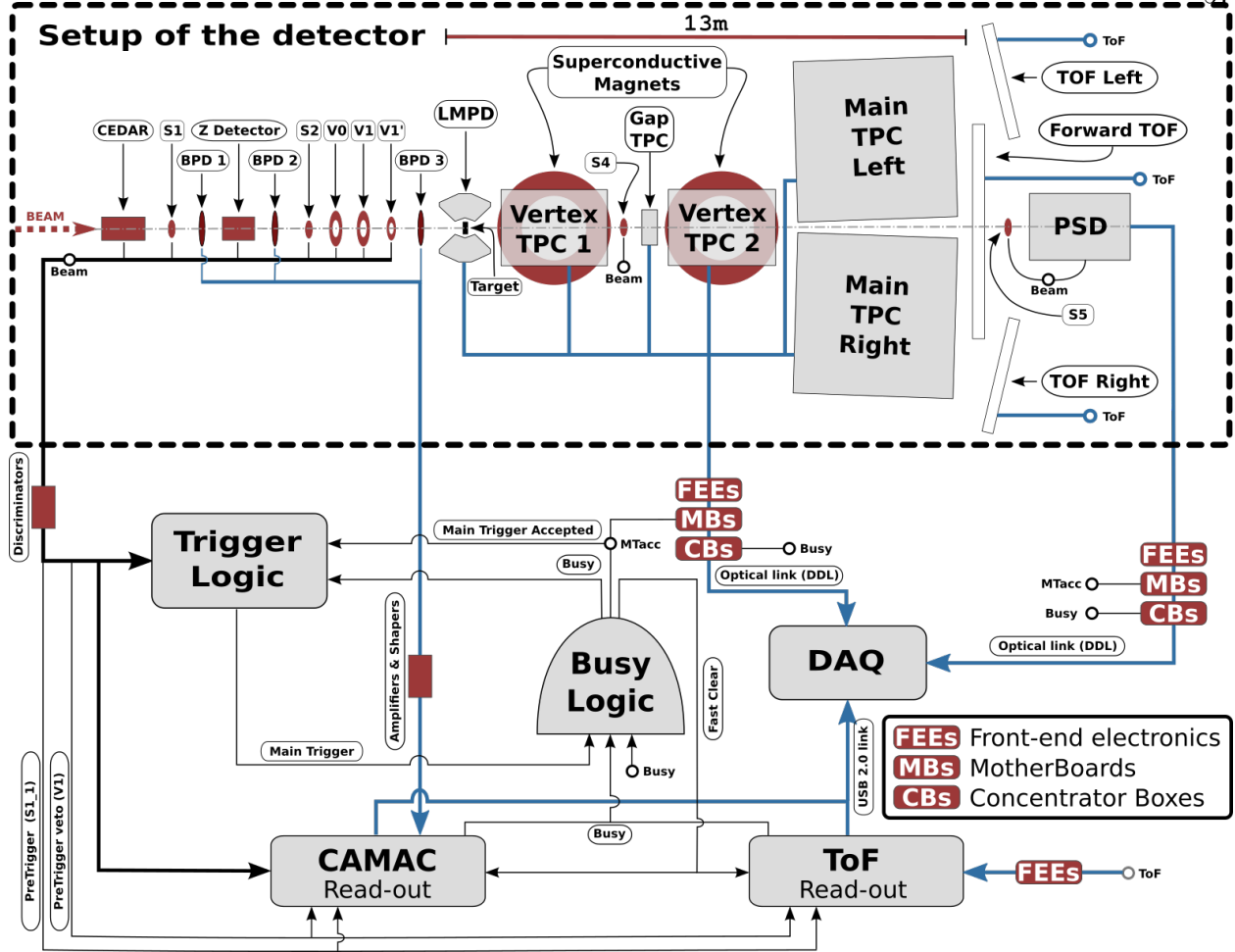


Figure 4.1: Schematic overview of the NA61/SHINE DAQ. The trigger logic, DAQ, and busy logic work with the trigger signals and current readout status to decide whether or not to issue a Main Trigger Accepted signal and begin detector readout. Reproduced from [39].

Multi-Hit TDC.

The TPC readout electronics can be broken down into four logical components:

- Front-End Electronics (FEEs)
- Motherboards (MBs)
- Concentrator Boxes (CBs)
- Detector Data Link (DDL)

¹ The CAMAC readout electronics acquire data from the beam counters, BPDs, and scaler modules, around 300 channels total.

The FEE cards couple directly to the TPC readout pad planes, amplifying and shaping the image charges of sense wire avalanches and storing them in a Switched Capacitor Array (SCA). Each card serves 32 TPC pads. The channels are served by two *chips*, which amplify 16 channels. The chip designation will re-surface in the calibration chapter, where the chip gains must be calculated and corrected for.

Each motherboard serves 24 FEE cards. The boards are responsible for accepting FEE data, building a sub-event structure corresponding to the motherboard ID, performing pedestal subtraction, suppressing noise, and applying zero suppression. Pedestal subtraction is performed by acquiring randomly-triggered events between accelerator spills, corresponding to measuring the electronic baselines of the pre-amplifiers, calculating an average baseline, and subtracting the baseline from each FEE channel. For further random noise suppression, a threshold of 5 ADC (Analog-to-Digital converter) counts is applied, corresponding to $\approx 2\sigma$ of pedestal fluctuation noise. This threshold has a severe impact on dE/dx resolution and is later corrected for, as will be discussed in the Calibration chapter.

The CBs concentrate data from up to 32 motherboards. The boxes serialize incoming motherboard data and prepare the data for transfer to the central DAQ computer. The CBs also accept commands from the central DAQ computer and distribute them to the motherboards, such as pedestal table values or operational mode changes.

The data from the CBs is sent to the central DAQ via the Detector Data Links (DDLs), which are optical links capable of transferring large amounts of serialized data. The links operate in parallel via PCI-express Readout Receiver Cards (RORCs²) in the central DAQ machine. Each link is capable of an upload and download bandwidth of 200 MB/s.

² RORCs and the high-speed optical serial links were developed for the ALICE experiment, a TPC-based LHC experiment.

4.1.1 TPC Raw Data

TPC raw data arrives at the Central DAQ machine pre-packaged by the TPC motherboards. These packaged units of raw data are known as *TPC BOS (Bank Organization System) [42] banks*,³ and contain individual identifiers signifying their origin in the TPC system. The TPC channels are organized according to TPC identifier (TPC Id), sub-sector identifier (sector Id), and padrow identifier (padrow Id). Within each padrow, the individual pad channels contain 256 time slices of raw data, with each of these pixels containing the measured ADC value. Each padrow can be visualized as a two-dimensional cross-section of a TPC's fiducial volume, spanning the encompassed pads in x and the sampled drift volume in y . Figure 4.2 shows an example of the raw padrow data.

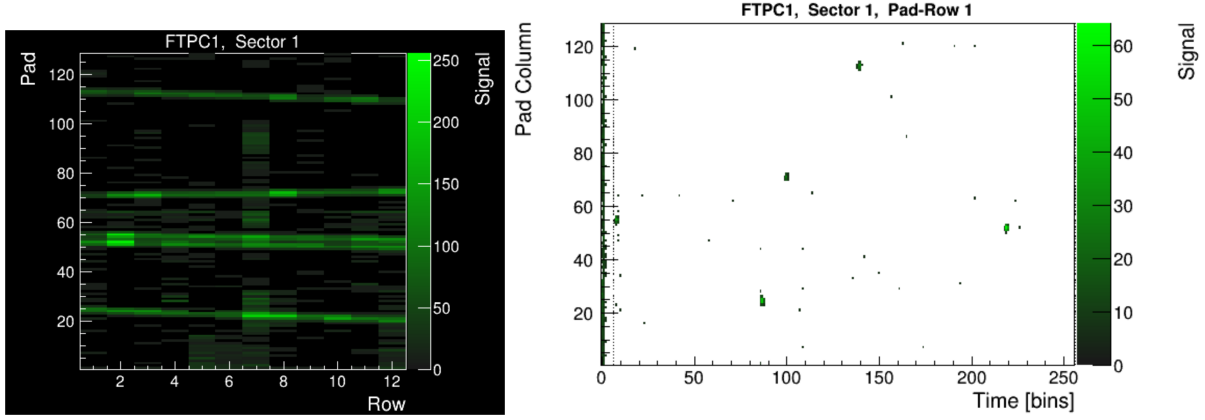


Figure 4.2: Visualization of raw TPC data. Left: Sector-wide view of raw data. A TPC sector contains several padrows, each serving as a cross-section through the TPC fiducial volume. Track-like structures are visible in this view, which is effectively an $x - z$ projection of the data. Right: Padrow-wide view of first padrow in this sector. Small groups of collected charge are visible. This is effectively an $x - y$ view of collected charge integrated over the pad length in z . A critical task of the reconstruction and calibration software will be accurately determining the $x - y$ origin of these groups of deposited charge.

Data compression via zero-suppression is critical for TPC data, as many of the TPC pad-timebin pixels are unoccupied. By using a simple algorithm replacing zeroes with one zero and the number of zeroes to follow, event size is reduced from 100 MB to 1-5 MB. This enables reasonable data transfer and storage time. 1 GB *chunks* of raw data, corresponding to ~ 300 events, are

³ TPC raw data BOS banks are identified through the MBRD and PADR identifiers. The MBLV identifier signifies TPC electronics low-voltage and other housekeeping data.

transferred to the CERN Advanced Storage (CASTOR) tape system by the DAQ machine.

4.1.2 Other Raw Data

Raw data from the other subdetectors is collected and stored by the DAQ as well. These detectors have their own corresponding BOS banks: TOFD banks for ToF data, PSDD banks for PSD data, MTDC banks for multi-hit TDC data, and CAMC banks for BPD and trigger data. BPD, ToF, and PSD data is passed to the corresponding reconstruction modules during data processing.

4.2 SHINE Reconstruction Software

This section will detail the process of converting raw detector data, such as ADC counts and TDC counts, to meaningful physics analysis objects, such as clusters, tracks, and vertices. These objects possess concrete physical attributes recorded in terms of observables, such as position and momentum, and the corresponding uncertainties.

The modules described in this section run within the SHINE Software Framework, known as **SHINE**. **SHINE** is a C++-based reconstruction, calibration, and analysis software framework. It includes an abstract detector description and interface, allowing users to easily obtain time-dependent detector parameters and calibration constants such as detector position, TPC drift velocity, and atmospheric pressure. The framework also contains many useful functionalities, such as coordinate system transformation and unit translation. The SHINE Offline Event (SHOE) data structure is the main method of storing higher-level raw and reconstructed data. An example of some SHOE structures can be seen in Fig. 4.3. For more details on the **SHINE** software framework, see [43].

The SHINE Reconstruction Chain will be presented using a bottom-up approach, similar to the manner in which it was designed: The description will begin with raw detector-level data reconstructed into three-dimensional point measurements, which are in turn reconstructed into local TPC tracks. These tracks are combined between TPCs into global tracks, and well-measured tracks

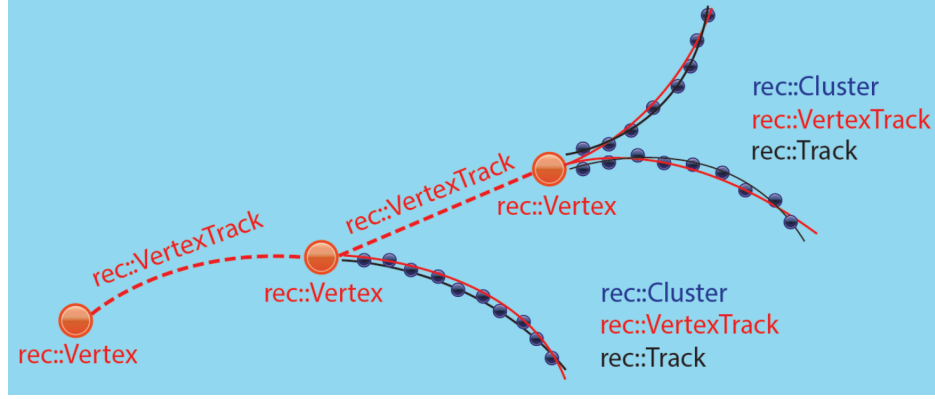


Figure 4.3: Visualization of abstract reconstructed objects stored in a SHOE file. Point measurements are known as *Clusters*; combinations of clusters representing charged particle trajectories are known as *Tracks*; Tracks with a measured point of origin are known as *Vertex Tracks*; The measured point of origin is known as a *Vertex*. Vertex Tracks can also represent neutral particles whose charged decay products were measured. Reproduced from [43].

are extrapolated toward the target and used for vertex estimation. An overview of the process can be seen in Fig. 4.4.

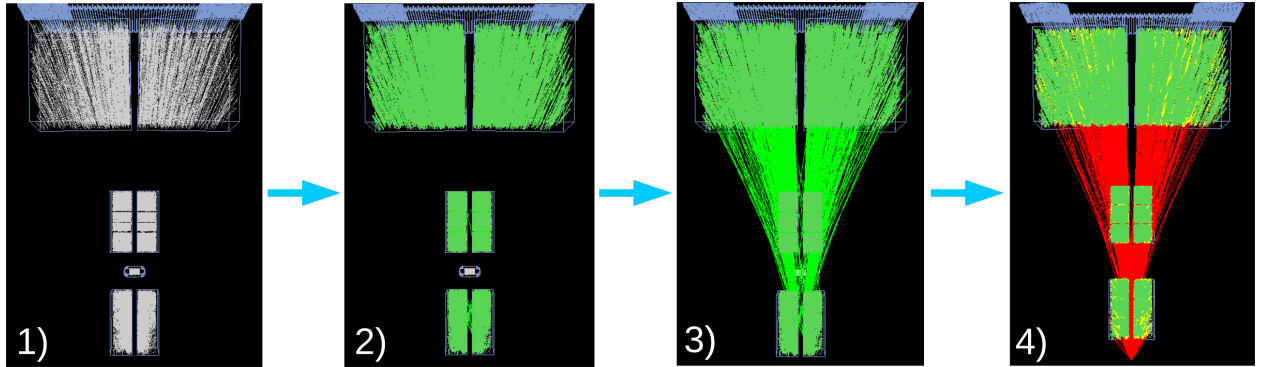


Figure 4.4: An overview of the major SHINE Reconstruction Chain steps. From left to right: 1) TPC clusterization, 2) Local Track Finding, 3) Global Track Merging, 4) Vertex Finding. Grey points represent TPC clusters, green points represent TPC clusters associated with tracks, yellow points represent TPC clusters associated with vertex tracks, green lines represent track measurements, and red lines represent vertex track measurements.

4.2.1 TPC Clusterization

The first step in TPC raw data reconstruction is known as *clusterization*, or making three-dimensional point measurements in TPC fiducial volumes. This process takes as input raw pad-

timebin pixel signals⁴, as can be seen in Fig. 4.2, and produces as output a set of reconstructed three-dimensional points called *clusters*. The clusterization algorithm is broken into two parts: Cluster Finding, in which contiguous groups of signal are found and marked for analysis, and Cluster Summarizing, where the groups of signal are converted into position and charge measurements.

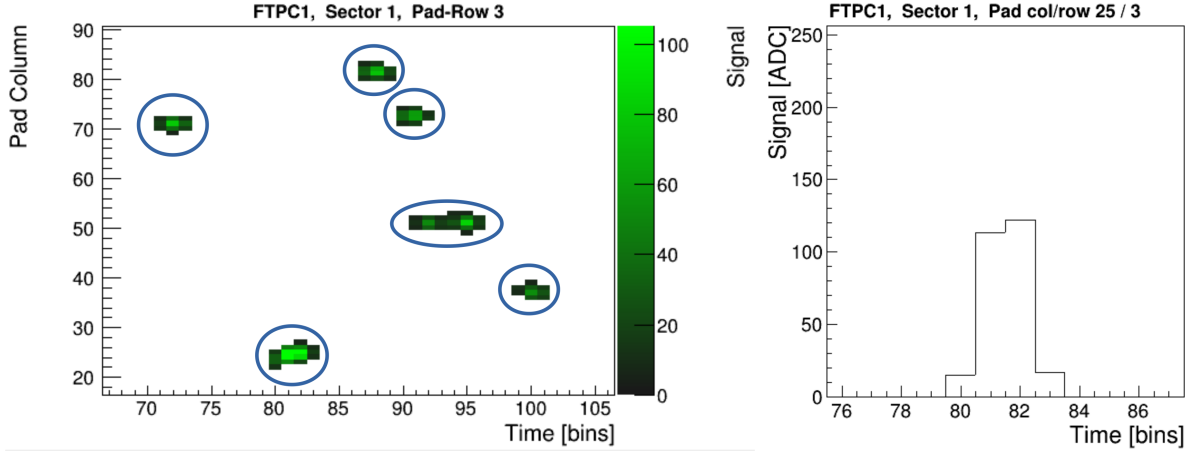


Figure 4.5: Visualization of the TPC clusterization process. Left: The cluster finding algorithm finds and stores contiguous pixels with ADC value greater than a threshold (usually 5 ADC counts). Note that this figure shows a potential shortcoming of this algorithm — the wide cluster contains two distinct peaks, possibly originating from two charged particle tracks. A cluster separation algorithm could be applied here to resolve the individual ionization points. Right: A one-dimensional view of ADC values recorded on a single TPC pad.

- Cluster Finding

Each two-dimensional pad/pixel plane corresponding to a TPC padrow serves as the input for cluster finding (see Fig. 4.5). The algorithm loops through the pixels on the plane and begins a list of contiguous pixels when a pixel is found whose ADC value exceeds the threshold for beginning clusterization. Each neighboring pixel is checked, and if the ADC value exceeds a separate continuation threshold, it is added to the contiguous list. The list is ended when no more pixels satisfy this criteria. This process is repeated until each pixel has been checked.

- Cluster Summarizing

⁴ TPC padrows represent a two-dimensional pixelated plane, with pad boundaries segmenting the plane in X and timebin boundaries segmenting the plane in Y.

The next part of the clusterization algorithm converts the contiguous groups of pad/timebin pixels into point measurements. A simple weighted mean is calculated in both dimensions to reflect the cluster center of gravity:

$$x = \frac{\sum_i \text{ADC}_i x_i}{\sum_i \text{ADC}_i}, \quad y = \frac{\sum_i \text{ADC}_i y_i}{\sum_i \text{ADC}_i}. \quad (4.1)$$

Here, the subscript i runs over all pixels contained in the cluster, x is the pad x-coordinate in the local TPC coordinate system, and y is the timebin y-coordinate in the local TPC coordinate system⁵. Given that these measurements use a centroid calculation over a uniformly-segmented detection region, the (x, y) uncertainties $\sigma_{(x,y)} \approx \frac{d}{f(S/N)\sqrt{12}}$, where $f(S/N)$ is a factor related to the signal-to-noise ratio of the measured ADC values. The experimentally-measured cluster position uncertainties are $\sim 200 \mu\text{m}$.

Several other values are calculated and recorded. The total collected charge $q = \sum_i \text{ADC}_i$ is stored for dE/dx analysis. The second moment of weighted position is also calculated:

$$\begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{pmatrix} = \begin{pmatrix} E[x^2 - \langle x \rangle^2] & E[xy - \langle xy \rangle] \\ E[xy - \langle xy \rangle] & E[y^2 - \langle y \rangle^2] \end{pmatrix} \quad (4.2)$$

where the expectation values denoted $E[\dots]$ are weighted by the pixel ADC values. The resulting matrix, denoted the shape covariance matrix, describes the orientation of the cluster imaged on the padrow. If the track passes at a large angle in x or y , the corresponding eigenvector will be large. Instantaneous single-pad electronic noise (appearing as a large cluster in y with little extent in x) may be detected and rejected using this method.

The uncertainties associated with cluster measurement are difficult to calculate from the raw pad/timebin pixel plane alone, as they depend on the track angles, which are not known at the time of clusterization. There are options to overcome this, such as calculating the cluster uncertainties after performing local tracking and re-calculating the track parameters, or parameterizing the

⁵ It is important to note that the y-coordinate is a combination of several values: the pad's total $t0$ (itself a combination of global detector $t0$, chamber-wise $t0$ and pad-by-pad $t0$), time bin width (200 nanoseconds when running in standard 256 timebin mode, or 512 nanoseconds when running in 512 timebin mode), and chamber drift velocity.

cluster uncertainties as a function of cluster size and shape. For this thesis, a flat overestimated uncertainty of $(1 \text{ mm}, 0.1 \mu\text{s} \times v_{\text{drift}})$ was applied in (x, y) . This can be improved.

4.2.2 TPC Local Tracking

After the clusterization step, TPC data exists in the form of (x, y) position measurements located on the planes associated with TPC padrows⁶. The next task, known as *local tracking*, aims to identify which clusters are associated with a given charged particle, group clusters accordingly, and perform a track fit to each distinct group. This is a notoriously difficult problem in high-multiplicity events. The SHINE Local Track Finder was designed to operate quickly and efficiently in both low-multiplicity events, comprising all neutrino data events, and high-multiplicity events, corresponding to heavy ion reactions. The technical implementation of the SHINE Local Track Finder is a cellular-automaton-based track seeding algorithm [44] combined with a Kalman-Filter-based track fitting and extension algorithm. The process is broken up into three distinct parts: *Tracklet seeding*, *Tracklet fitting*, and *Tracklet extension*⁷.

- Cellular-Automaton-Based Tracklet Seeding

Local Tracking begins with a track seeding algorithm. This is a pattern recognition algorithm that serves to identify point measurements that were likely created by the same charged particle. We will use the term *track-like* to express that a collection of measurements likely share a common parent track. In this section we will refer to measurements as *clusters*, padrow geometrical planes as *detection planes* (see Fig. 4.6), and abstract connections joining pairs of clusters as *links*. The seeding process used is referred to as a cellular automaton algorithm due to its operation on nearest-neighbor measurements only. The algorithm creates a network of nearest-neighbor connections and then evolves the network according to a set of rules, much like the cellular automaton Game of Life [45].

⁶ At this point, the cluster positions have been transformed into the global detector coordinate system, accounting for relative rotation of the TPC padrows or TPC chambers. The padrow geometrical plane definitions exist in their rotated forms in the global coordinate system.

⁷ *Tracklets* refer to portions of local tracks that may or may not represent the full local track. Once extended, they become *local tracks*.

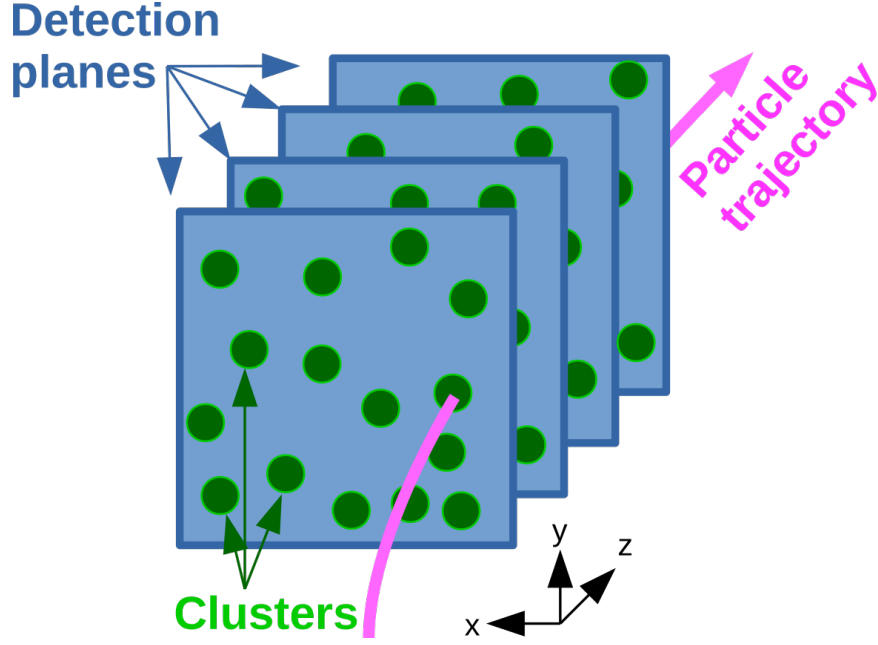


Figure 4.6: Visualization of relevant objects in this section: A charged particle trajectory (pink) passes through several detection planes (blue), leaving energy deposits in the form of clusters (green). Detection planes are transverse to the z -axis in the local TPC coordinate system.

The tracklet seeding algorithm is relatively simple by design, in order to reduce the required computation time. The procedure is as follows: First, links are formed between clusters on adjacent detection planes. Links are created in pairs, as shown in Fig. 4.7. Each cluster possesses a forward-pointing link joining it to a cluster on the nearest downstream detection plane, and backward-pointing link joining it to a cluster on the nearest upstream detection plane. The algorithm relies on the assumption that clusters created by track-like objects will have nearby clusters on adjacent detection planes.

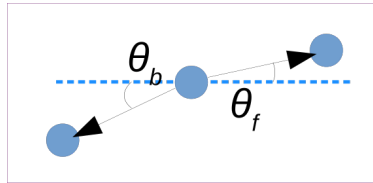


Figure 4.7: Relevant angles in the tracklet seeding step. θ_f is the forward-pointing angle, while θ_b is the backward-pointing angle. Angles in both the $x-z$ and $y-z$ planes are considered.

The seeding algorithm must select which forward and backward link to form, in the case

that there are many clusters upstream and downstream of the one being considered. In order to perform this selection, a list of possible combinations is constructed. These possible combinations are sorted according to a sorting metric m_{link} :

$$m_{\text{link}} = \sqrt{\Delta\theta_{xz}^2 + \Delta\theta_{yz}^2} \quad (4.3)$$

where $\Delta\theta_{xz}$ and $\Delta\theta_{yz}$ are $\theta_f - \theta_b$ in the $x - z$ and $y - z$ planes, respectively. The quantity m_{link} increases as the change in angle increases in either the $x - z$ or $y - z$ planes. The links that minimize this quantity, representing the straightest of the possible combinations, are stored⁸.

At the end of the link creation phase, the best forward-pointing and backward-pointing link for each cluster is stored. The next phase of track seeding is the *filtering phase*, where the list of links is updated according to nearby links. For a given cluster, the forward-pointing link is examined and the pointed cluster is studied. If this cluster's backward-pointing link does not point to the original cluster, the link is purged. The process is repeated for the backward-pointing link. These two-way links, or so-called *strong links*, are stored as final connections between clusters. Figure 4.8 shows an example of potential links to be formed (left) and strong links stored after the filtering phase (right).

Sets of clusters joined by strong links are stored in a C++ Standard Library (STL) Vector of **TrackCandidate** objects, a custom class containing the clusters and information concerning the links, such as the average and standard deviation of link angles in the $x - z$ and $y - z$ planes. These quantities could be used to reject outliers in the set of clusters, but this functionality is not presently used, as the subsequent steps sufficiently reject errant clusters.

- Tracklet Fitting

After the seeding step is complete, the collections of clusters are tested using a track model. This model performs a fit to the clusters, returning a set of track parameters and an average cluster-track residuals in the $x - z$ and $y - z$ planes. The model used depends on whether or not the TPC

⁸ This procedure works equally well for TPCs in magnetic fields, where track projections in the bending plane are not straight. Weights can be applied when calculating m to favor minimizing the non-bending plane $\Delta\theta$ over the bending plane $\Delta\theta$. This was studied and found not to improve overall track seeding.

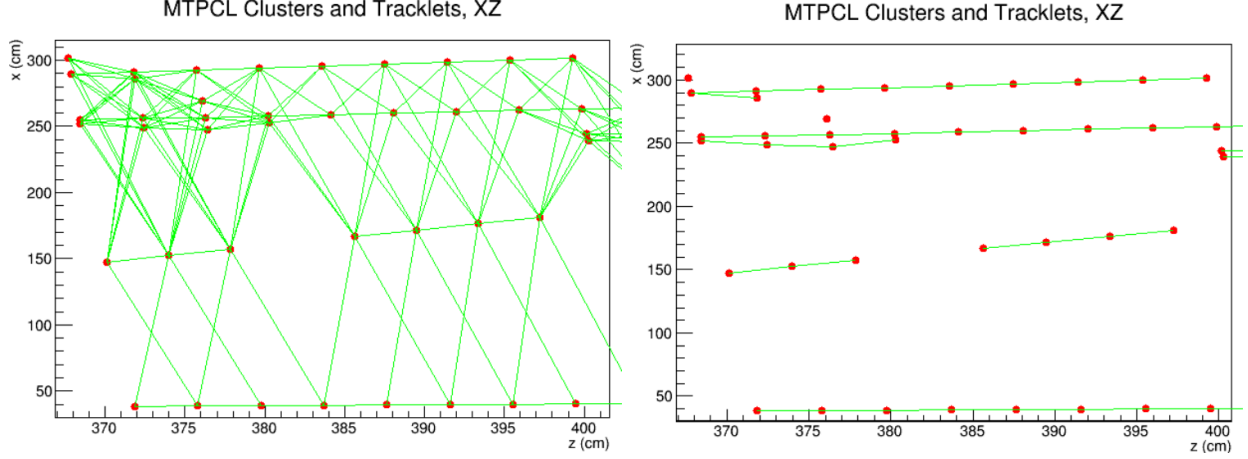


Figure 4.8: Visualization of the tracklet seeding process. Left: All links that could potentially be formed joining clusters on adjacent detection planes. Right: Stored links after application of a filtering algorithm, likely connecting clusters from the same track.

considered is inside one of the superconducting magnets. If the TPC is far from the magnetic fields, a simple least squares straight line fit is used. The least squares fit is a simple regression obtained by minimizing the χ^2 , and can be performed in a single-pass, reducing computation time:

$$\begin{aligned}
 m_{xz} &= \frac{N \sum_i z_i x_i - \sum_i z_i \sum_i x_i}{N \sum_i z_i^2 - \left(\sum_i z_i \right)^2} \\
 b_{xz} &= \frac{\sum_i x_i \sum_i z_i^2 - \sum_i z_i x_i \sum_i z_i}{N \sum_i z_i^2 - \left(\sum_i z_i \right)^2}, \\
 \end{aligned} \tag{4.4}$$

$$\begin{aligned}
 m_{yz} &= \frac{N \sum_i z_i y_i - \sum_i z_i \sum_i y_i}{N \sum_i z_i^2 - \left(\sum_i z_i \right)^2} \\
 b_{yz} &= \frac{\sum_i y_i \sum_i z_i^2 - \sum_i z_i y_i \sum_i z_i}{N \sum_i z_i^2 - \left(\sum_i z_i \right)^2}.
 \end{aligned}$$

Here m_{xz} and b_{xz} are the slope and intercept in the $x - z$ plane, and m_{yz} and b_{yz} are the slope and intercept in the $y - z$ plane.

These fits cannot, of course, determine the track momentum, as they assume zero curvature.

Momentum estimates for these tracks will follow in the Global Track Matching phase, when they are joined with tracks passing through the magnetic fields.

If the TPC is in the magnetic field, a different track model is used. The user may select either a parabolic least-squares fit in the bending plane or a full Kalman filter fit. The Kalman filter fit is the default fit for physics analysis, while the parabolic track fit can be used for fast online tracking and noise cluster rejection. The NA61/SHINE Kalman filter is a sophisticated fitting algorithm returning accurate track parameter estimates in one or two passes.

Once the track fit has been performed, the track parameters are stored at the upstream end of the track. This position does not necessarily correspond to the most upstream cluster position, as the track fit will be close to, but not exactly located at, this cluster's position. The quality of the fit is then tested by examining the average residual between the clusters and the track fit at each detection plane. If the average residual exceeds a user-defined threshold value,⁹ the track candidate is rejected and the clusters are released.

- Tracklet Extension

Once the track candidates are fit and poorly-fit candidates are rejected, each chamber contains a collection of track segments likely representing physical tracks. However, many of the full tracks spanning each TPC can be broken up into small segments. The loss of any cluster along a track will break the track seeding, due to its nearest-neighbor philosophy. Several primary phenomena are responsible for cluster loss.

First and foremost, electronic noise often affects front-end card performance. Due to the age and nature of the front-end electronics, a large fraction of the TPC channels will record noise during each event. This noise often prevents a reasonably-shaped cluster from being reconstructed by embedding the would-be cluster pixels in a sea of transient ADCs. Electronic noise can also be reconstructed as a reasonably-shaped cluster, which may be attached to a collection of physical

⁹ This parameter is tuned via Monte Carlo studies. It is unsatisfying to use absolute parameters like this instead of the track fit χ^2 value, but the χ^2 cannot be used until the cluster uncertainties are treated properly. This is a planned major improvement.

clusters during track seeding and throw off the track fit.

Another major source of cluster loss is finite detector resolution. If two tracks pass over a padrow in close proximity, the deposited signals will begin to overlap. Up to a point, these overlapping clusters can be examined and split, a process in the NA49 code referred to as *cleaving*. This overlapping phenomenon can be seen in Fig. 4.5, where the widest cluster in the left image contains two distinct ADC peaks. If the peaks are unresolvable, at least one track will be broken.

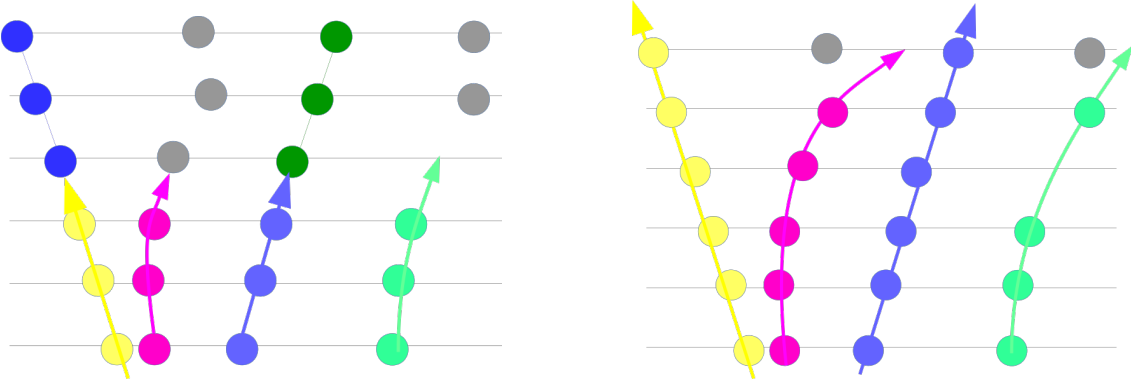


Figure 4.9: Visualization of the tracklet extension process before (left) and after (right) tracklet extension. Left: Yellow and blue track segments are compared to nearby track segments, while pink and seafoam tracks are extended in search of compatible lone clusters. Grey clusters are not associated with any tracks. Right: Compatible track segments are merged and re-fit, and compatible clusters are added to tracks and enter the track fit.

Track extension is performed in two steps: *track segment merging*, in which compatible sets of track segments are merged and re-fit into a single track, and *cluster pickup*, in which stray clusters not associated with any track are attached to compatible tracks. Images of these two processes can be seen in Fig. 4.9.

Track segment merging is performed in several steps. First, track segments are organized according to the number of measurements, from longest to shortest. The longest segments are extended first, as their track parameters are measured more accurately than shorter segments. The segments are extrapolated to each detection plane in the TPC not currently spanned by the track segment, and other segments close to the extrapolated points are collected into a list. This list is

organized according to a sorting metric m_{track} :

$$m_{\text{track}} = \sqrt{w_x \Delta x^2 + w_y \Delta y^2 + w_A \Delta A^2 + w_B \Delta B^2 + w_{q/p} \Delta q/p^2} \quad (4.5)$$

where Δx is the difference between the extrapolated segment x-position and the found segment x-position, or the *track mismatch* in x , Δy is the track mismatch in y , ΔA is the track mismatch in A , or $\frac{p_x}{p_z}$, ΔB is the track mismatch in B , or $\frac{p_y}{p_z}$, and $\Delta(q/p)$ is the track mismatch in $q/|p|$. These are the five track parameters, completely describing the kinematics of a track. Each of these parameters receives a weight factor, w_i , tempering the contribution of each parameter's mismatch to the overall metric m_{track} ¹⁰.

Once the list of found segments (referred to as *merge candidates*) is constructed and organized by compatibility, the list is looped through and tracks below a threshold compatibility are merged with the original track segment. This process is repeated until all track segments have been tested for merge possibility.

- Detection Plane Grids

As the number of clusters increases, the combinatorics from considering each possible formable link during the seeding phase quickly renders local tracking prohibitively time-consuming. The combinatorics can be greatly reduced if links are considered only between nearby clusters¹¹. This is accomplished via the Detection Plane Grid structures, which serve as data compartmentalization objects. These structures are integral to the entire TPC reconstruction process. An illustration of their operation can be seen in Fig. 4.10.

The grid spacing in x and y are determined depending on the cluster density:

$$w_{x,y} = \frac{l_{x,y}}{d\sqrt{N_{\text{cl}}}} \quad (4.6)$$

¹⁰ These weights are again obtained from Monte Carlo, which I also find unsatisfying. In the future when the cluster and track uncertainties are fixed, these weight factors should be replaced by the extrapolation uncertainty and measurement uncertainty of the extrapolated and found tracks, respectively.

¹¹ Concern over biasing accepted tracks is warranted, but if the grid cells are kept reasonably large, this data segmentation amounts to ignoring links between clusters $\mathcal{O}(10\text{cm})$ apart, which is quite reasonable.

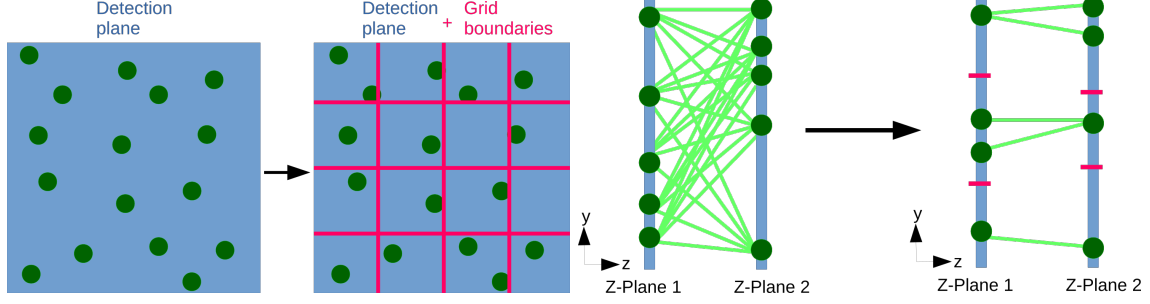


Figure 4.10: Illustration of the Detection Plane Grid data compartmentalization scheme. After the bin widths in x and y are determined, an individual STL container is created for each bin and assigned a coordinate corresponding to the center of the bin.

where $w_{x,y}$ is the bin width in x and y , $l_{x,y}$ is the length of the detection plane in x and y , N_{cl} is the number of clusters on the detection plane, and d is a tuneable parameter of order 1. Each subsequently created bin is assigned an (x, y) coordinate corresponding to the center of the bin and contains an STL container holding references to the clusters within the bin boundaries. Other STL containers are created for the bin, holding track segments and tracks. Objects in the vicinity of point (x_o, y_o) are obtained by querying the Detection Plane Grid object, providing the central point (x_o, y_o) and a range (dx, dy) . The grid will return objects contained in the range $[(x_o - dx, y_o - dy), (x_o + dx, y_o + dy)]$. Grid bins are lower-bound inclusive.

The querying function is used during track seeding, track segment extension, and global track merging, as each of these processes involve extrapolation to neighboring detection planes and suffer in computation time when combinatorics become large. Thus, when any TPC object is created, be it a cluster, track segment, track, or simulated hit, the object is placed into its corresponding container on the detection plane grid.¹²

4.2.3 TPC Global Tracking

After the Local Tracking step is completed, the Global Tracking module begins attempting to merge compatible tracks between TPCs. This module extrapolates local TPC track parameters

¹² Note here that the z-position of the cluster need not correspond to the z-position of the Detection Plane Grid. This is the case in the VTPCs, where clusters measured on padrows can be generated at differing z-positions. This will be explained in a subsequent chapter.

to adjacent chambers, searches for compatible tracks, selects the best match if multiple candidates exist, and merges the two tracks (see Fig. 4.11). This merging procedure is performed in a particular order, satisfying the following algorithm requirements:

- Prioritization of extrapolation to adjacent TPCs
- Extrapolation from higher track density regions to lower track density regions
- Extrapolation only between tandem TPC pairs with opposing drift directions

The procedure works from downstream to upstream, extrapolating in the downstream direction. First, FTTPC2 tracks are extrapolated to FTTPC3, then FTTPC1 tracks are extrapolated to FTTPC2, followed by MTPC tracks being extrapolated to FTTPC1 and FTTPC2. VTPC1 tracks are then extrapolated to the MTPCs and FTTPC1, and VTPC1 tracks are extrapolated to the GTPC, VTPC2, FTTPC1, and the MTPCs. VTPC2 receives special designation in that it may extrapolate upstream, in order to merge well-measured VTPC2 tracks with poorly-measured GTPC or short VTPC1 tracks.

This merging procedure involves track parameter extrapolation over distances of $\mathcal{O}(m)$, during which errors in track parameter measurement will become magnified. For this reason, adjacent TPCs are extrapolated to first, and compatible tracks are merged. These re-measured tracks will possess more accurate track parameters, which will subsequently be easier to extrapolate to other chambers.

To describe the merging procedure, we consider two TPCs containing several local tracks each. TPC 1, the upstream TPC, contains track segments A_1 , B_1 and C_1 . TPC 2, the downstream TPC, contains track segments A_2 , B_2 , and C_2 . Track labeled with the same letter, such as A_1 and A_2 , were produced by the same charged particle and should be merged. Track A_1 is extrapolated to the first detection plane in TPC 2, and the Detection Plane Grid is queried for track objects near the extrapolated point. If no tracks are found, the track parameters are extrapolated to the next detection plane, and the process is repeated. Any nearby tracks found during the extrapolation procedure are stored in a list of possible merge candidates.

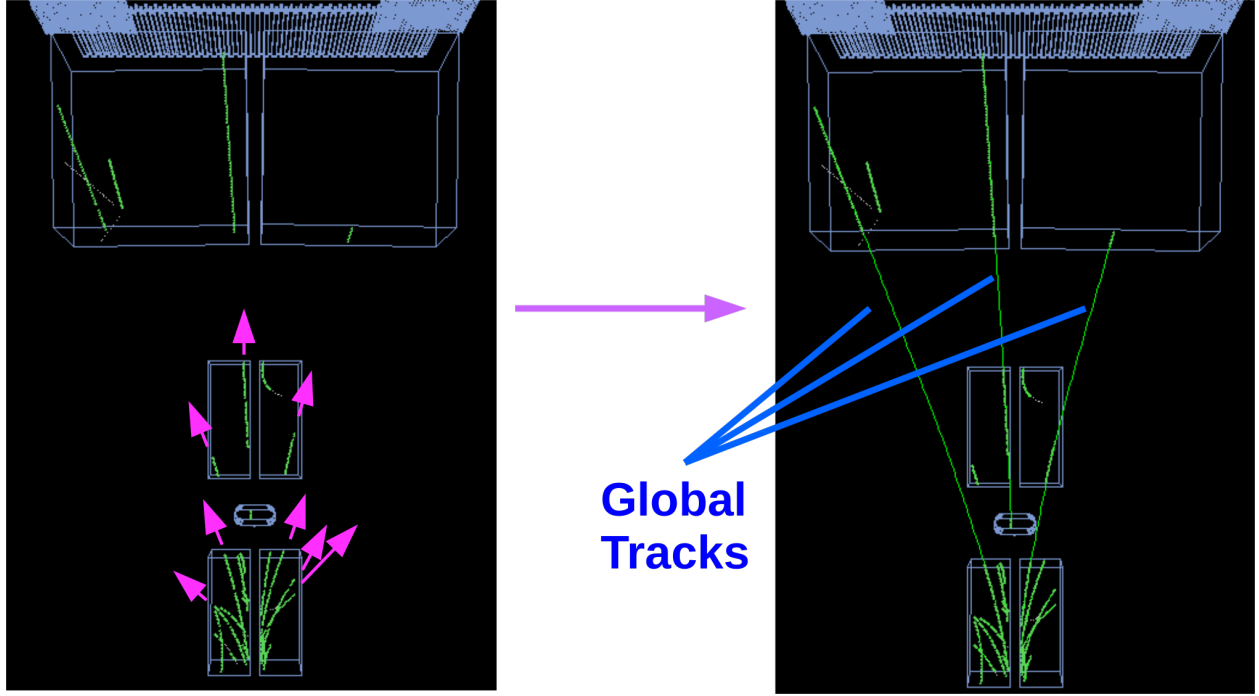


Figure 4.11: Illustration of a low-multiplicity event before and after global tracking. Left: Local tracks (green) have been found and fit in each TPC. Pink arrows indicate direction of extrapolation. Right: Compatible tracks between TPCs have been merged and re-fit.

Once the list of merge candidates is assembled, the tracks are organized by a compatibility metric, in direct analogy with local track segment merging. In our example, if track A_1 has extrapolated close enough to tracks A_2 , B_2 and C_2 , their three sets of track parameters will be compared to those of track A_1 . The most-compatible track is then selected from the list and merged.¹³ If no track candidates have a compatibility metric above a threshold value, no tracks are merged. Global tracking performance is shown in Fig. 4.12.

4.2.4 TPC Vertex Finding

Once the TPC tracking algorithms are complete, the origins of the tracks must be determined in order to complete a meaningful analysis. The vertex finding and track refitting algorithms used in this thesis analysis are the *Main Vertex Finder and Fitter*, which seeks to identify tracks

¹³ Of course, in high track density environments, detector resolution impacts the track merging procedure and tracks created by different charged particles will be merged. This is the main source of efficiency loss in Global Tracking.

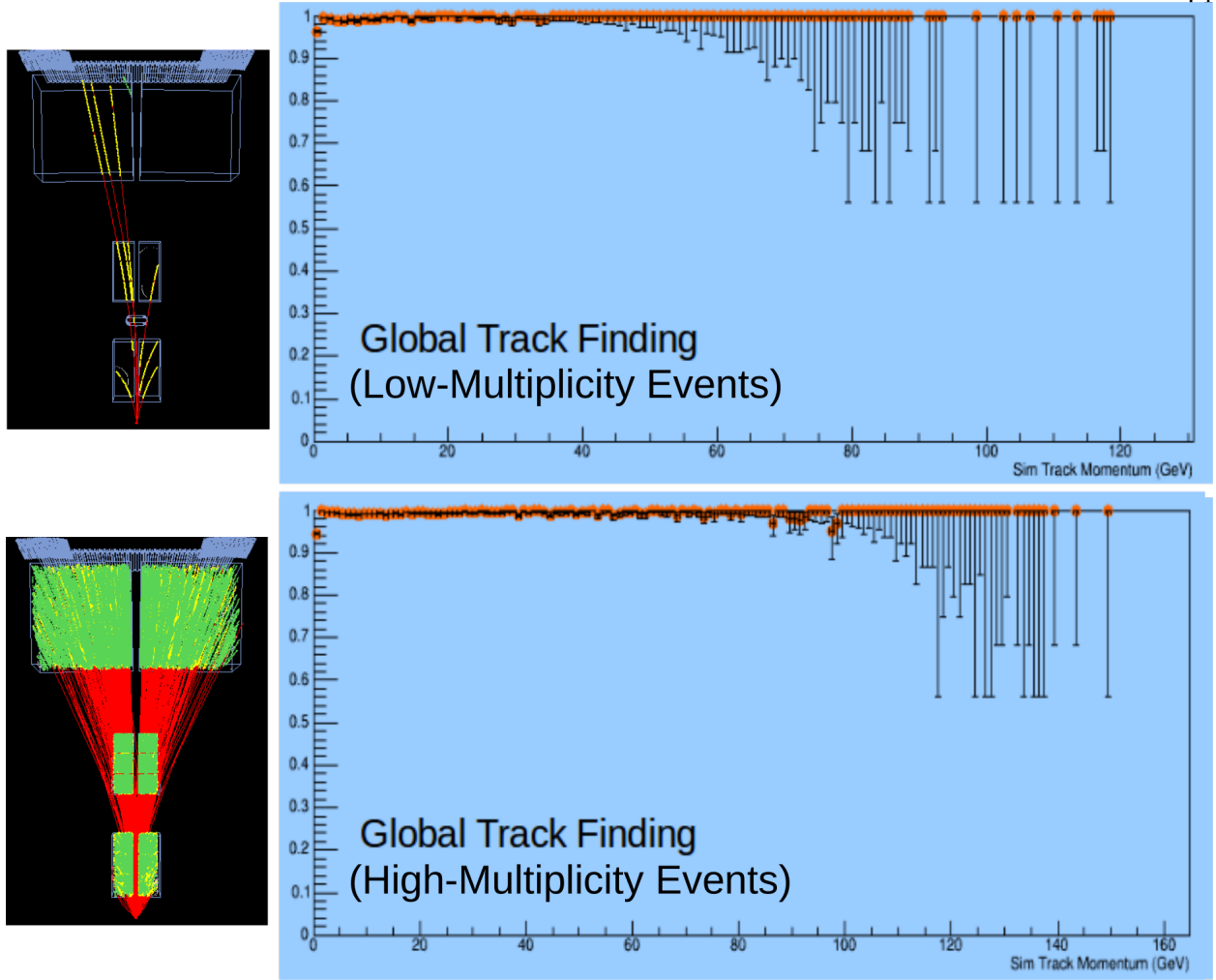


Figure 4.12: Tracking efficiency after the Global Tracking procedure. Obtained from comparison to Monte Carlo. Top plot: uniform tracking efficiency of near 100% is observed above several hundred MeV/ c tracks in low-multiplicity events (see image left of upper plot) Integrated efficiency is over 99% above 1 GeV/ c . Bottom plot: Slightly lower yet similar performance is observed in high-multiplicity events (see image left of lower plot), where the track multiplicity is $\mathcal{O}(100)$ times larger. Integrated efficiency is 97% above 1 GeV/ c .

originating in the primary beam-target interaction, and the V^0 *Finder and Fitter*, which seeks to identify displaced vertices corresponding to neutral particle decays.

4.2.4.1 Main Vertex Finding and Fitting

Proper identification and measurement of the main interaction vertex is critical to performing an accurate physics analysis. In the SHINE software framework, **VertexTracks** are fundamental

analysis objects. The measured vertex position is used as a constraint when estimating **VertexTrack** momentum, and can impact the measurement significantly.¹⁴ The algorithm for identifying the main vertex works in the following manner. Tracks are extrapolated backward to the target z -center, and their extrapolated track parameters are collected. A cut is imposed on extrapolated (x, y) position:

$$\left(\frac{x_i - x_v}{\sigma_{x,i}}\right)^2 + \left(\frac{y_i - y_v}{\sigma_{y,i}}\right)^2 < \sigma_{\max}^2. \quad (4.7)$$

Here, (x_i, y_i) is the track position at the target center, $\sigma_{x,i}$ and $\sigma_{y,i}$ are the uncertainties in extrapolated track position, and (x_v, y_v) is the main vertex seed position given by the intersection of the BPD beam track with the target. $\sigma_{\max}$ is a parameter denoting the track deviation from the vertex seed position.

Once compatible tracks are selected, a vertex position is estimated. Two options are currently available for position estimation: a one-dimensional fit in z , and direct assignment of the beam-target intersection¹⁵.

The direct vertex assignment is suitable for low-multiplicity events, as the vertex position resolution increases with the number of tracks in a given event. This makes target-out subtraction important, as it is not possible to precisely identify the origin of tracks, and they may be produced in out-of-target interactions close to the target position.

The one-dimensional vertex z -fit is performed by adjusting the vertex z -position, while using the BPD track as a constraint on x and y . An objective function f_{obj} is constructed and minimized by varying z_v :

$$f_{\text{obj}}(z_v) = \sum_{i \in T} \left\{ \ln \left(1 + \frac{(x_i(z_v) - x_v)^2}{2\sigma_{x,i}^2(z_v)} \right) + \ln \left(1 + \frac{(y_i(z_v) - y_v)^2}{2\sigma_{y,i}^2(z_v)} \right) \right\}. \quad (4.8)$$

Here, all tracks in the fit are summed over. This equation is an arbitrary construction, with the natural logarithm of the typical χ^2 contribution term taken to temper the impact of outliers, and 1 added inside the logarithm to prevent misbehavior when the χ^2 term becomes small. A visual

¹⁴ This is especially true for forward-going tracks, like the ones measured by the GTPC/FTPCs.

¹⁵ There is a fully three-dimensional least-squares-based fitter available, but this has been used only for data quality assessment. The resolution is not as high as the other two fitters.

representation of this minimization process is reproduced in Fig. 4.13.

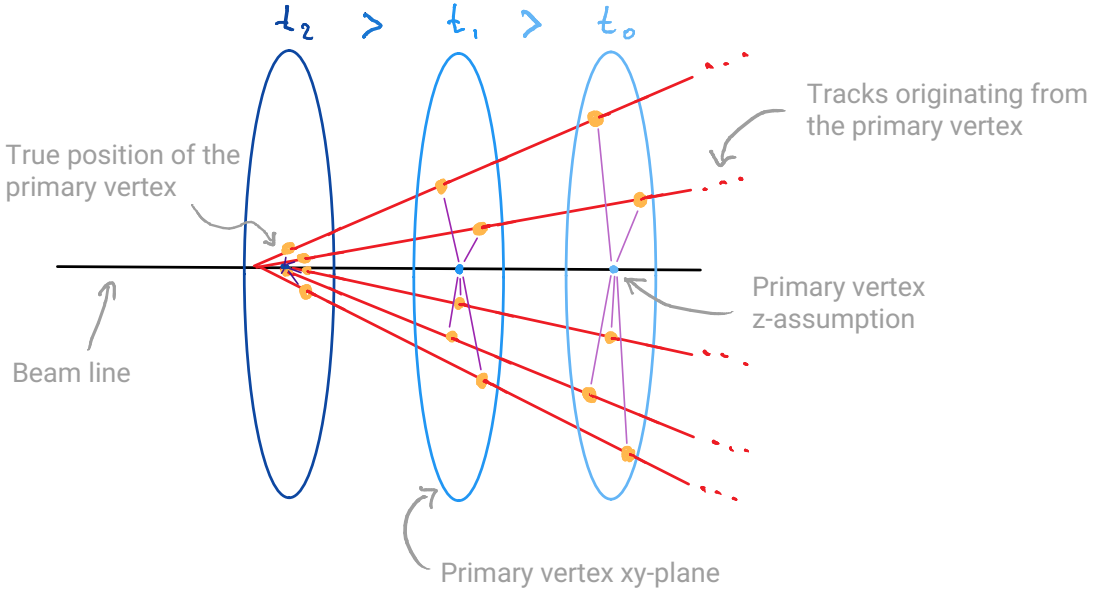


Figure 4.13: Visualization of the one-dimensional Main Vertex Fitter algorithm. Three minimization steps t_0 , t_1 , and t_2 are pictured. This algorithm gives vertex z -resolution of approximately 2 mm in the presence of 30 tracks, and 1 mm in the presence of 60 tracks. Image courtesy of Ivan Pidhurskyi.

4.2.4.2 V^0 Finding and Fitting

Once the Main Vertex has been identified, vertices corresponding to short-lived neutral particle decays are searched for. The simplest such search for such V^0 decays (named for their signature V-like structure in the detector) involves studying all positively and negatively charged tracks, extrapolating them towards each other and deciding whether or not they extrapolate sufficiently close. This brute-force method involves extrapolating each positively-charged track to each negatively-charged track and calculating the Distance of Closest Approach (DCA), and thus becomes an unreasonably expensive search method when the number of tracks grows large. In low-multiplicity events, such as the ones studied in this thesis analysis, the method works reasonably quickly.

If two tracks possess a DCA less than some threshold value (typically 5 cm), a common

vertex position is fit for and each track's momentum is re-fit using this vertex as a constraint. The combined momenta are vector-summed, and the resulting momentum is assigned to the neutral particle trajectory, which is created between the main vertex and the decay vertex. The momentum vector is pointed back to the main vertex in order to calculate an impact parameter, or the mismatch in (x, y) at the Main Vertex z -position.

The V^0 Finder & Fitter considers all tracks within an event, even if the tracks are considered attached to the Main Vertex. This is due to the finite z -resolution of the Main Vertex position. Tracks with small $\mathcal{O}(\text{mm-cm})$ decay lengths are often assigned to the Main Vertex. Additional cuts to improve V^0 sample purity, such as invariant mass cuts, are applied at the analysis level.

4.2.5 ToF Hit Matching

The Time of Flight Left & Right (ToF-L & ToF-R) walls sit directly downstream of MTPCL and MTPCR. As will be discussed in a subsequent chapter, the segmentation of the ToF-L and ToF-R in the vertical dimension enables their use as a physical reference for TPC drift velocity calibration. The y -positions of TPC tracks depend on the TPC's drift velocity, which can vary significantly with time. A reference is needed in order to calibrate the drift velocity. A crucial part of the reconstruction chain, then, is the TPC-ToF hit matching module, ToFHitMatcher.

This module works by examining well-fit vertex tracks with segments in the MTPCs. To get rid of any pre-calibration bias originating from global track merging, the MTPC track portion is isolated and re-fit to a straight line. This fit line is then extrapolated to the beginning of the ToF wall, in a coarse extrapolation. Since the wall is not planar [38], a second finer step is required for accurate extrapolation. The nearest scintillator to the coarse extrapolation is found, and a finer extrapolation is made to its central position. The scintillator nearest to this fine extrapolation point is recorded.

The recorded scintillator is then checked for the presence of a hit. Its time-to-digital (TDC) and charge-to-digital (QDC) DAQ values are obtained, and a pedestal cut is applied to the QDC value in order to ensure sufficient signal in the scintillator. A `ToFMass SHINE` object is then created,

storing the scintillator ID, position, and total path length from the main vertex to the scintillator. This information is later used during ToF calibration to estimate particle mass.

Chapter 5

Data Calibration

This chapter will explain the various calibration steps of NA61/SHINE experimental data, using the calibration of this analysis data set as an example. The major calibration components are TPC tracking calibration (BPD-TPC alignment, TPC point residuals calibration, TPC drift velocity calibration), and TPC dE/dx calibration. The author is responsible for developing the SHINE-Native dE/dx calibration software framework, and contributed significantly to the TPC drift velocity calibration and TPC point residuals calibration.

5.1 Tracking-Based Corrections

The following calibration steps involve data-derived corrections to detector point measurements, and thus most significantly impact charged particle tracking.

5.1.1 TPC Drift Velocity Calibration

Precise knowledge of the TPC drift velocity is critical for determining the TPC cluster positions in the drift dimension. Drift velocity is continuously monitored using dedicated detectors in the TPC gas cage [38], and often undergoes variations of up to 5%. These changes are induced by changes in atmospheric conditions, such as sudden pressure variations or nightly drops in temperature. A summer thunderstorm's impact will be visible in the recorded drift velocities.

The goal of the drift velocity calibration is to measure the drift velocity to the 0.1% level. In order to achieve this level of precision, we perform a tracking-based calibration procedure. The

procedure simultaneously measures each TPC's chamber-to-DAQ t_0 offset and y-position, as these values also influence track position measurement in the drift dimension.

A TPC point measurement's local y-position (in the drift dimension) is projected back from the sense wire plane, as given by

$$y_c = y_{\text{wp}} - v_d(t_0 + w_t(t_{\text{center}} - 0.5)) \quad (5.1)$$

where y_c is the cluster y-position (cm), y_{wp} is the sense wire plane y-position (cm), v_d is the chamber drift velocity, t_0 is the combined cable and electronics timing offset from the DAQ (μs), w_t is the width of a timebin (μs), and t_{center} is the amplitude-weighted central timebin from cluster reconstruction. A factor of 0.5 is subtracted from the central timebin number to set the measurement in the center of the timebin. All positions are in the local chamber coordinate system, so that the direction of the projection is purely in the y -dimension.

The factor t_0 is further broken up:

$$t_0 = t_{0 \text{ global}} + t_{0 \text{ chamber}} + t_{0 \text{ pad-by-pad}} \quad (5.2)$$

The global t_0 is fixed such that VTPC1's chamber t_0 is equal to -94.6 ns by historical convention. Each chamber possesses a unique cable- and electronics-dependent chamber t_0 , which can be thought of as the delay between the DAQ's sending of the $\text{MT}_{\text{accept}}$ signal and the execution of the command by the chamber electronics. These values range from $\mathcal{O}(10 - 100) \text{ ns}$. The pad-by-pad t_0 is a fine correction due to motherboard-FEC cable lengths and further electronics channel delays ($\mathcal{O}(1 - 10) \text{ ns}$). Each t_0 value must be precisely measured in order to properly perform tracking.

The drift velocity and t_0 calibration steps, known as Stages 1-4, will now be described.

5.1.1.1 Stage 1

The first calibration stage involves calibrating the multi-hit TDC (MHTDC) unit. The $\text{MT}_{\text{accept}}$ and S1_1 signal offset must be calculated in order to align the trigger signals, and the MHTDC time conversion factor must be measured. These values are simultaneously obtained by

plotting the S_{11} arrival time at the MHTDC, with the knowledge that this spectrum should be uniform in the interval of $[0, 40]$ ns (see Fig. 5.1).

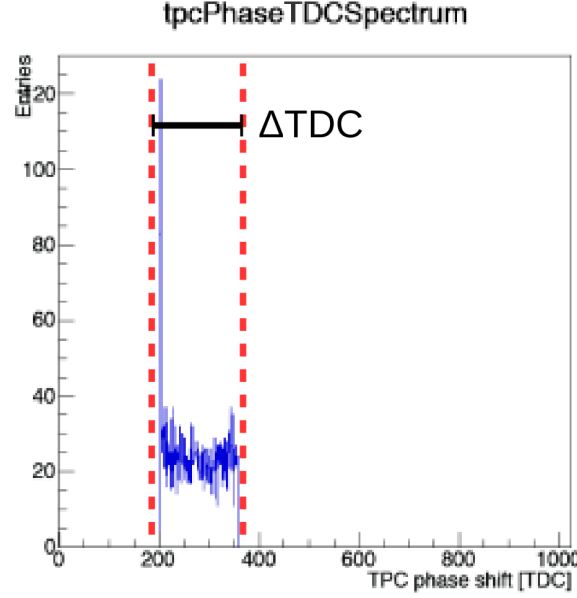


Figure 5.1: Distribution of S_{11} signal arrival time at the MHTDC detector, also referred to as the "TPC Phase". The absolute position is used to measure the MHTDC- S_{11} offset, and the span is used to measure the TDC time conversion factor.

5.1.1.2 Stage 2

The second calibration step is significant in both concept and computation. This step employs local TPC tracking and extrapolation to neighboring chambers in order to measure each chamber's drift velocity. Track matching and the subsequent calculations are performed in a time-dependent manner, resulting in a time-dependent drift velocity measurement. The following illustrations will show the procedure at discrete time steps.

A poorly-calibrated drift velocity will result in distortion of the TPC fiducial volume in the drift dimension. This is illustrated in Fig. 5.2. The volume may be stretched in the case of an excessive large drift velocity or compressed in the case of an excessively small drift velocity. This distinction between too-large and too-small drift velocities results in an expected distribution of track match deviations along the chamber's drift dimension, and this can be used to calibrate the

drift velocity. If the distortion is too large and the drift velocity is unreasonably far from its true value, track matching is not possible, and some rough pre-correction is required.

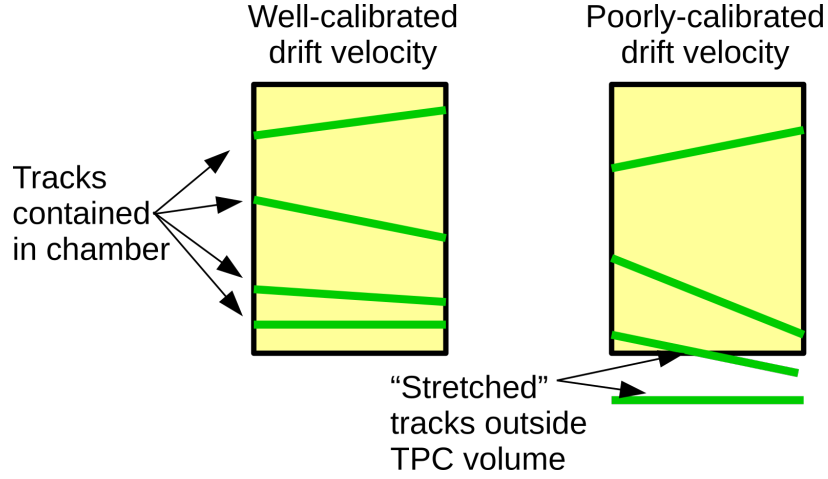


Figure 5.2: Illustration of mis-calibrated TPC drift velocity. Here the poorly-calibrated drift velocity is too large, resulting in a stretched fiducial volume. The drift velocity can be corrected by matching tracks to a well-calibrated reference and calculating the mismatches as a function of Y .

At the start of the procedure, no drift velocities are calibrated, and an external reference is required. This is available via the ToF-L and ToF-R, which are segmented in the TPC drift dimension.¹ By collecting distributions of matched ToF hit positions with MTPC track y -positions, we obtain the “stretch” or “compression” factor for the MTPCs relative to the absolute position of the ToF scintillators (see Fig. 5.3). This factor can be expressed as $\frac{\Delta y}{y}$, where Δy is the track-to-ToF mismatch in the drift dimension and y is the y -position of the track.

In practice, MTPCL is first calibrated using ToF-L, and chambers are subsequently calibrated to each other using the ordering shown in Fig. 5.4. To compute the time-dependent calibration constant, a coarsely-binned histogram of track Δy vs y values is assembled. The slope of this histogram will correspond to the drift velocity calibration factor. When a sufficient number of histogram entries is collected to make an estimate of the slope, the collection is halted and a slope measurement is made. In order to reduce statistical variation in the measurements, a minimum threshold is applied to each histogram at 20% of its maximum value. The slope is then recorded

¹ although with a large segmentation of 2 cm, as shown in the left image in Fig. 5.3

and used to correct the drift velocity measurement for the pertinent time period. An example of track mismatches along the drift dimension can be seen in Fig. 5.5.

² Significant contamination of the fiducial volume gas would lead to undesirable effects such as charge quenching as a function of drift length. Effects such as these are not seen in data, as will be discussed in Section 5.2 – to the contrary, charge resolution is measured to be better than some of the older TPCs. Thus we conclude that the fiducial volume gas is largely unaffected.

and calculate an additive correction factor in a similar manner.

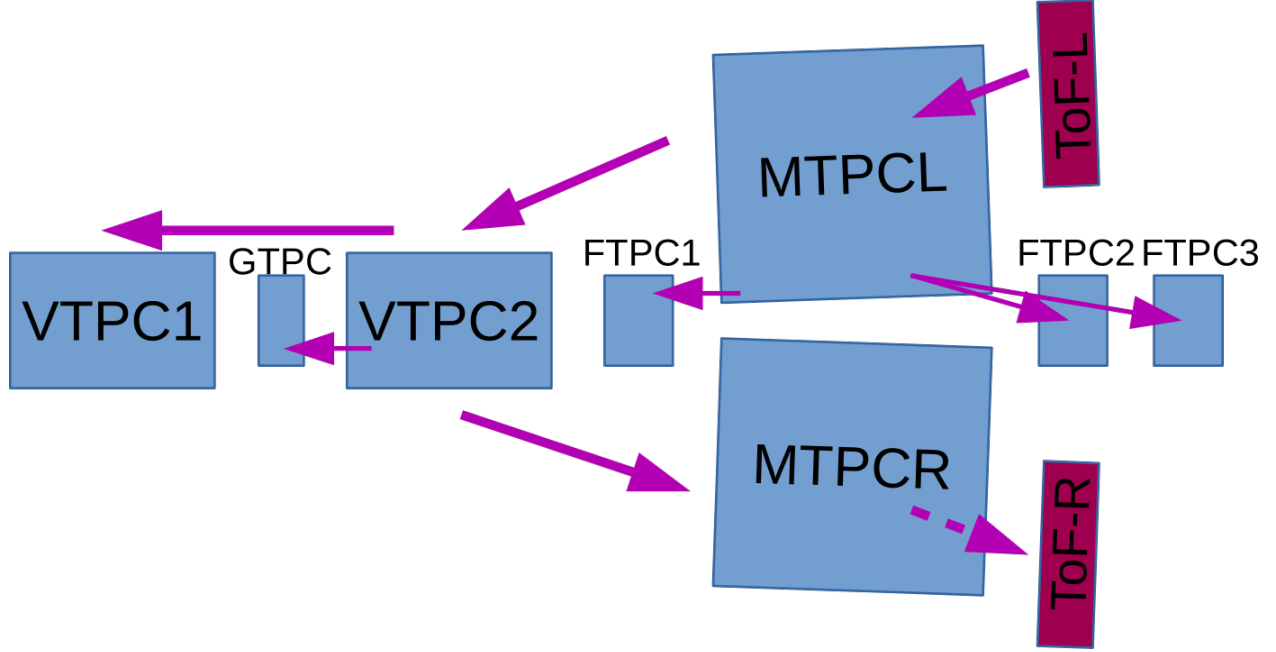


Figure 5.4: Calibration order of the TPCs. VTPC2 is used as a reference for most chambers. MTPCR is the final chamber calibrated, and the MTPCR-to-ToF-R track matching can be used as a cross-check to evaluate the consistency of the calibration.

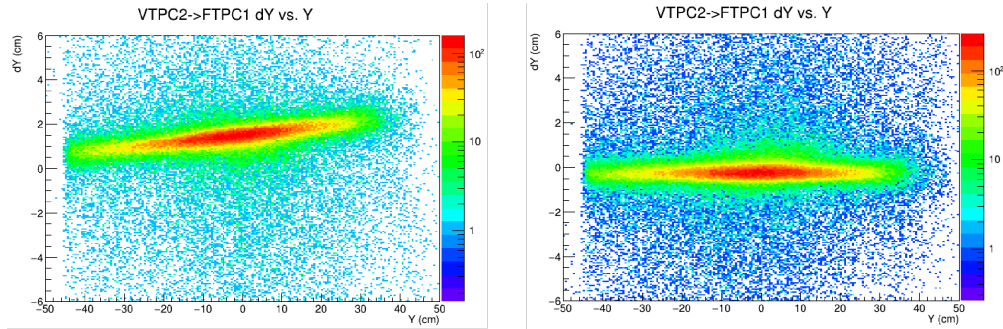


Figure 5.5: FTPC1 drift velocity calibration plots. Left: Track mismatch plotted as a function of track y -position (Δy vs. y) before calibration. A large slope is visible, indicating an incorrectly-calibrated drift velocity. Right: After three iterations of Stages 2 - 4, the Δy vs. y slope is calibrated away. A chamber is considered calibrated when the absolute value of this slope is less than 0.001 for the entire data period.

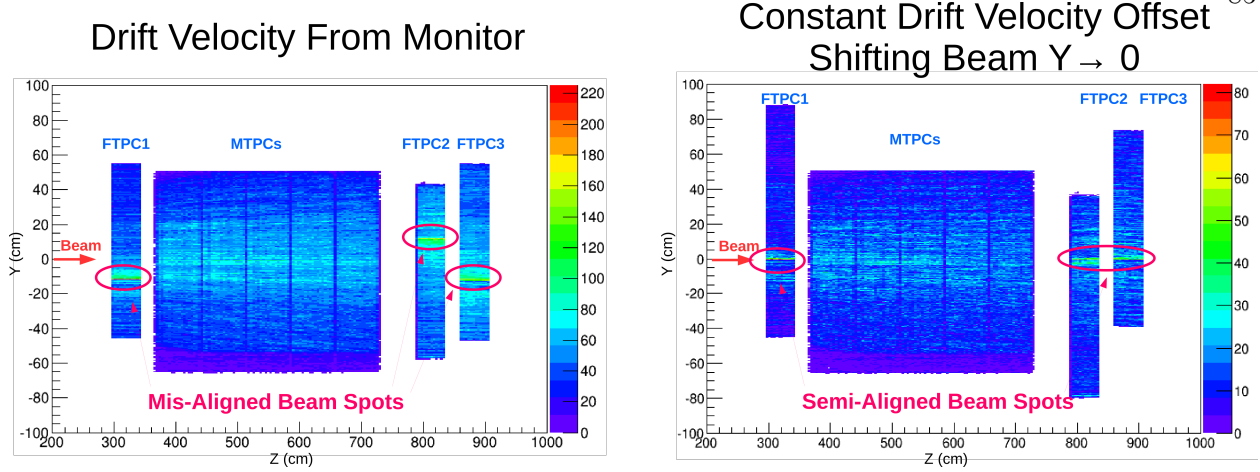


Figure 5.6: FTPC & MTPC cluster occupancy prior to drift velocity calibration. Left: Clusters reconstructed using drift velocity given by the drift velocity monitoring system. Right: Drift velocity manually adjusted to push the beam spot near the vicinity of the true beam position. This rough initial calibration allowed fine calibration to begin.

5.1.1.3 Stage 3

Once the drift velocity is calibrated, the chamber and global t_0 value can be measured.³ This is done by examining tracks that pass through the cathode, which provide information about the total chamber drift length. Tracks that exit the chamber through the cathode share a common plane of final point measurements. To select these points, we plot the last point position on each track. To avoid selecting tracks that pass through the downstream or outer TPC faces, we select only tracks whose final points are not on the most downstream TPC padrows.

Plotting the raw drift times of these clusters will yield a histogram with a peak at the total chamber drift time, which is the cathode-wireplane distance divided by the calibrated drift velocity. By subtracting off the nominal cathode-wireplane distance and fitting the t_0 peak with a Gaussian function, we obtain the chamber t_0 . By fixing VTPC1's chamber t_0 to its nominal value, we also obtain the global t_0 .

The t_0 measurement can be obscured by the presence of various chamber phenomena, which will also contribute peak-like structures to the drift time histogram. Gating grid noise, originating

³ The pad-by-pad t_0 is measured by using a calibration pulser that injects charge into the TPC field wires. This is a separate procedure.

from electronic noise present at the beginning of each data acquisition window, shows a peak at an unphysical negative t_0 measurement. This is caused by the gating grid “ringing” after being opened to allow primary ionization through to the sense wires. Off-time particles present a continuous background and a peak at the beam y-position drift time, which corresponds to an unreasonably large t_0 measurement. These phenomena are highlighted in Fig. 5.7

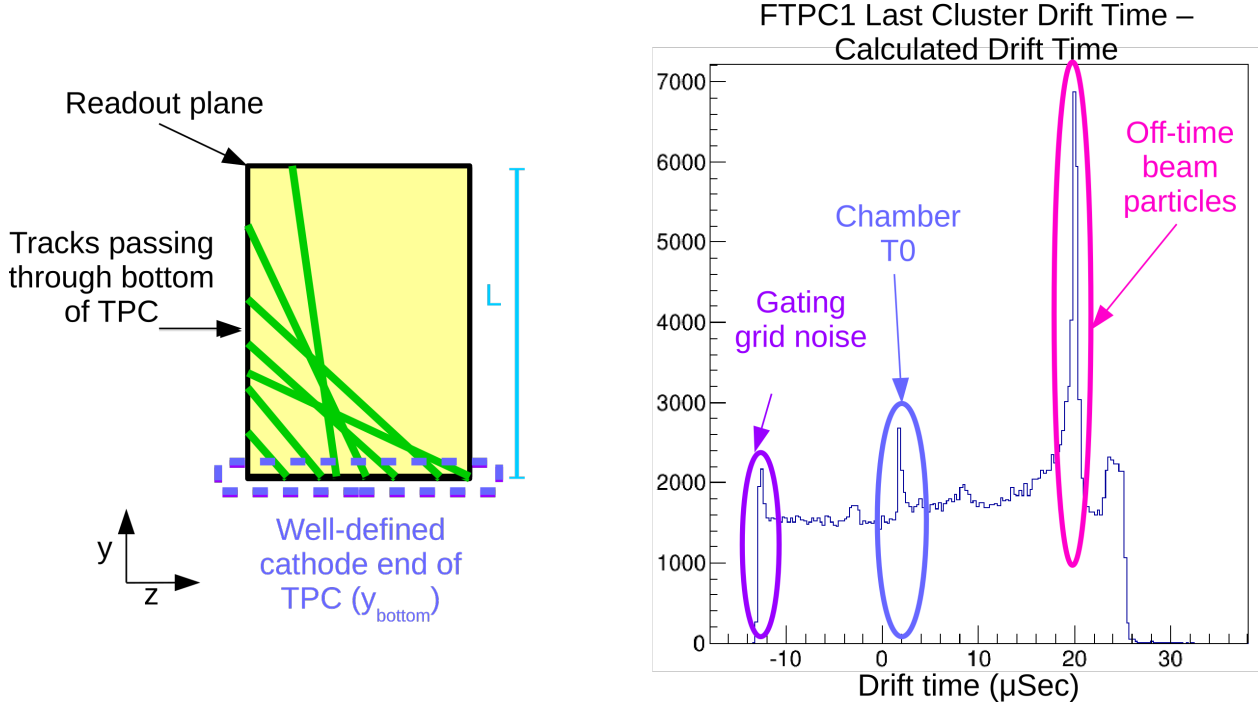


Figure 5.7: Stage 3: t_0 measurement procedure. Left: Illustration of tracks whose final points correspond to the cathode y-position. These points’ y-positions, when histogrammed, will produce a peak at the cathode y-position. Right: Drift time of clusters like the ones highlighted on the left, with nominal chamber drift time subtracted. The true t_0 peak, which should be $\mathcal{O}(\mu\text{s})$, is visible in the center.

5.1.1.4 Stage 4

Once the chamber drift velocity and t_0 values are measured, any residual offset between TPCs is attributed to physical chamber displacement. This is measured by fitting the flat Stage 2 Δy vs. y plots and obtaining a constant offset. This offset is typically on the order of millimeters.

Drift velocity values typically do not converge after the first round of calibration, and the

stages are performed in an iterative manner. Stage 1 is not repeated, since it is decoupled from the track y -position measurements. Stages 2 - 4 are iterated until the Δy vs. y slope obtained is within calibration tolerance. The data set analyzed in this thesis required three iterations.

5.1.2 BPD-TPC Calibration

As was discussed in the Reconstruction Software section, the BPDs play a critical role in TPC vertex reconstruction. Mis-measured BPD positions significantly impact track momentum measurement. The BPDs are often moved between data acquisition periods for various detector configuration changes, and thus must be calibrated using data. The BPD-TPC calibration procedure is performed by extrapolating well-measured VTPC1 tracks to the target z -position and subtracting the fitted vertex position. This value is known as the impact parameter. By histogramming the impact parameter components, we obtain distributions that can be fit for offsets in x and y . The resulting value is used to correct each BPD's position. BPD-TPC alignment plots before and after calibration can be seen in Fig. 5.8

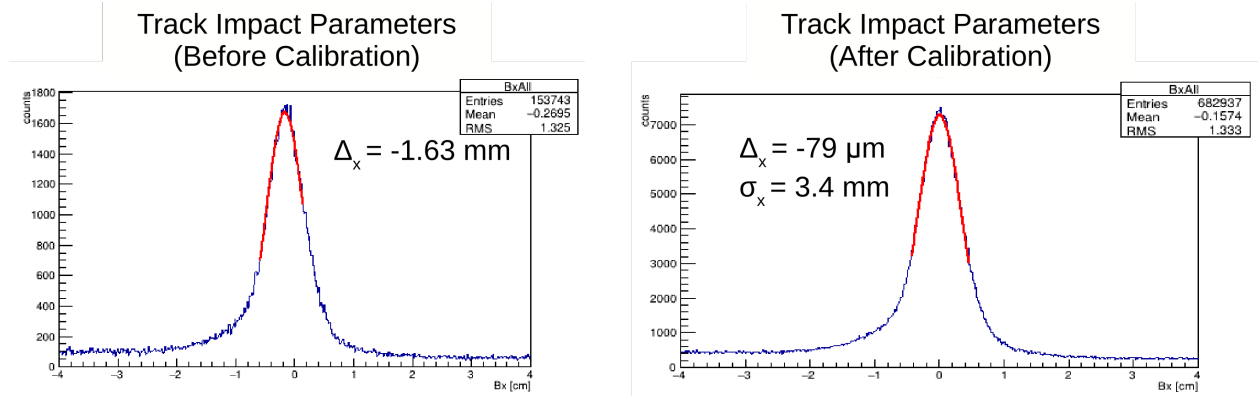


Figure 5.8: Effect of BPD-TPC calibration. Left: Track impact parameter x -component before BPD-TPC alignment. Right: Impact parameter distribution after alignment. The measured offset from the left distribution was applied as a position correction to each BPD. BPD-TPC alignment is considered sufficient when the measured offset is less than 200 μ m, the point resolution of a TPC cluster.

5.1.3 TPC Residuals Calibration

Design and construction conditions inevitably lead to local position distortions within TPCs. These distortions can be the result of electric field inhomogeneity due to field cage properties, mechanical component sag, or insufficient corrections for the $E \times B$ effect. Our strategy for measuring and removing these local distortion relies on well-measured vertex tracks saturating the chamber fiducial volumes. The data-derived TPC Residuals correction is calculated by binning the three-dimensional chamber volumes and calculating an (x,y) correction for each bin. The bin size is determined by the track statistics in parts of the detector being corrected, and range from 2 - 8 cm in x and y .

To calculate the residuals corrections, vertex tracks are extrapolated to each constituent cluster z -position.⁴ The difference between the extrapolated track position and the cluster measurement position, referred to as the track-cluster residual, are calculated. This track-cluster residual is and stored in a three-dimensional histogram representing the chamber fiducial volumes. Once all data has been analyzed, the average residual in each bin is computed for both the x - and y - dimensions. These residuals are stored in the database and applied after clusterization in a subsequent round of reconstruction.

An improvement to the standard TPC residuals procedure used in this analysis is the capability to run the correction iteratively. Since the data-derived residuals are directly coupled to one-another, the initial correction application often does not reduce the overall residuals to an acceptable level (less than 200 μm). Running the correction with one additional iteration reduced the overall residuals to less than 100 μm . The results of the first and second iterations can be seen in Figs. 5.9 and 5.10.

⁴ Extrapolation to the z -position, and not the padrow plane, is necessary due to the $E \times B$ correction in the VTPCs: post-correction, clusters on a given padrow will define a curved surface.

Residuals (Pre-Correction)

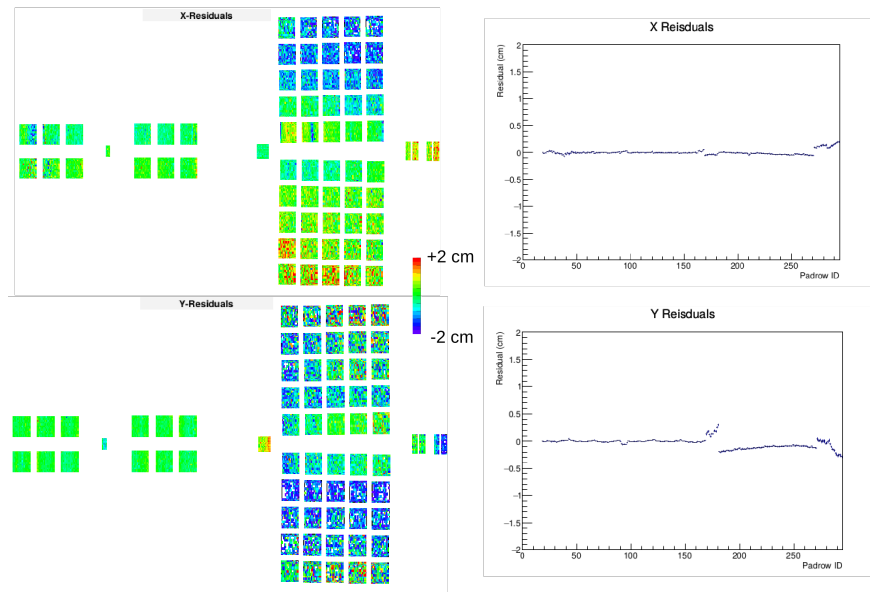


Figure 5.9: TPC cluster - vertex track residuals before corrections are calculated.

Residuals (Post-Correction)

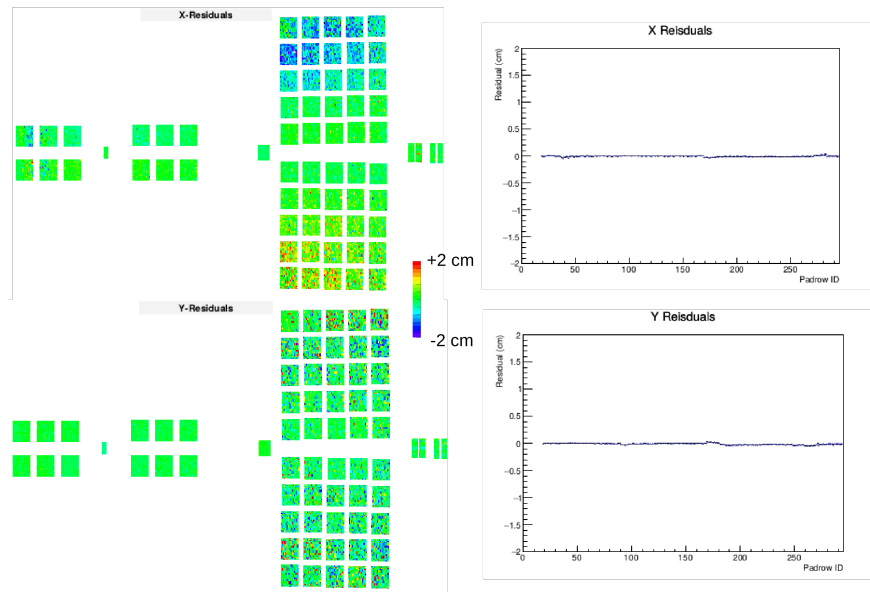


Figure 5.10: TPC cluster - vertex track residuals after second iteration of residuals correction.

5.2 TPC dE/dx Calibration

The following calibration steps are data-derived corrections to TPC charge measurements, and most significantly impact TPC dE/dx measurements. Since the clusterization process involves charge-weighting of pads and timebins, these steps also impact tracking.

5.2.1 Krypton⁸³ Gain Calibration

Before fine charge calibration can begin, large variations in electronics channel gain must be corrected for. These variations, mostly due to fluctuations in front-end card pre-amp response, can be as large as 80%. In order to measure these variations, a physical charge reference is used. The widely-used krypton gain calibration technique [46] was employed at NA61/SHINE in 2018 (see Fig. 5.11). This technique involves introducing a small amount of Kr⁸³ isotope via a Rb⁸³ source. The Kr⁸³ possesses a well-known decay spectrum of six peaks, with the most prominent peak at 41.6 keV.

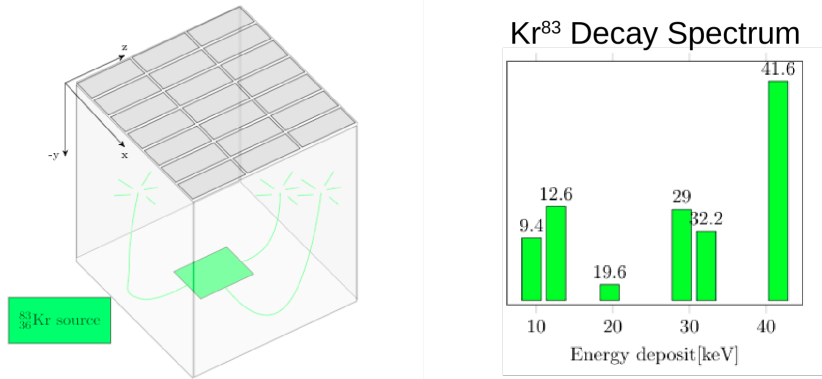


Figure 5.11: Schematic overview of the krypton calibration technique. Left: Chamber fiducial volume with radioactive Kr⁸³ introduced into the chamber gas. Right: Expected prominent decay peaks. The main calibration peak is the most prominent peak at 41.6 keV. Reproduced from [47].

By introducing a sufficient amount of Kr⁸³ and collecting decay ionization events in the TPC gas, we are able to reconstruct the decay spectrum for each electronics channel and identify the prominent peak. This peak is then used to determine a relative gain factor for each electronics

channel⁵. An overview of the effects of the krypton calibration technique for FTPC1 can be seen in Fig. 5.13. The charge resolution of the 41.6 keV peak is improved by a factor of ~ 2.5 . The reconstructed Kr^{83} spectra pre- and post-calibration can be seen in the lower plots of Fig. 5.13, where each of the six prominent decay peaks can be identified after calibration.

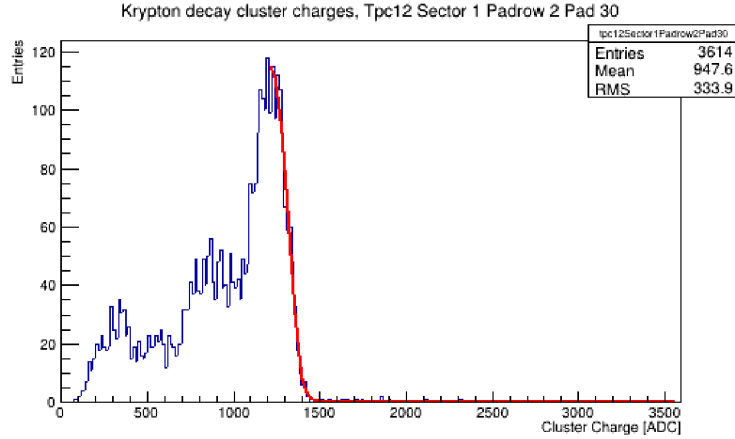


Figure 5.12: Example spectrum fit showing detected upper edge. The steepness of this falling edge within one electronics channel corresponds to the resolution capability for identifying the spectrum edge.

Considerations must be made when taking krypton decay data. The TPC amplification voltage must be tuned such that the reconstructed cluster charges are within the dynamic range of the front-end electronics. This factor differs significantly from the voltages used during standard data taking. The activity level and amount of source should be selected such that each channel samples a sufficient number of decays (~ 1000). Lastly, TPC amplification sectors with varying pad sizes (such as FTPC2 and FTPC3) should vary the amplification voltage during data taking, as the different pad sizes will have different optimal amplification voltages.

One software improvement made for this analysis is the creation of a krypton spectrum processing and gain calculation software package. The new software package, called **KryptonAnalyzer**, extracts the spectra for each channel and detects the position of the upper edge of the krypton spectrum. This edge detection method performs much better than a peak fit algorithm when

⁵ Ideally, the krypton calibration technique would be used to determine the sector gain constants for the dE/dx calibration. However, the krypton calibration requires a different amplification factor in order to not saturate the readout electronics, making this impossible (unfortunately).

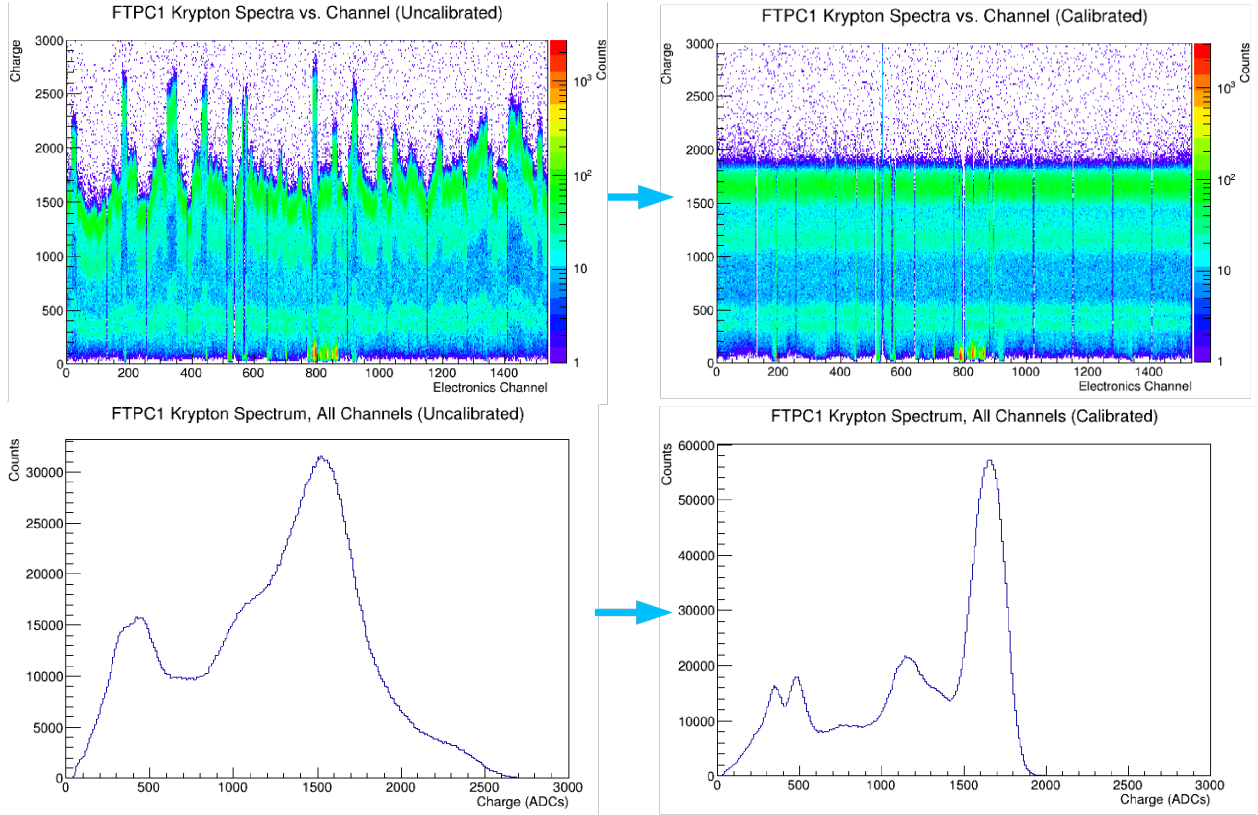


Figure 5.13: Krypton calibration results for FTPC1. Upper row: Raw electronics channel gain (ADC) before (left) and after (right) calibration. Some problematic clusters of electronics channels are visible near channel 800, likely due to aging front-end electronics. Lower row: Total Kr^{83} decay spectrum for all electronics before (left) and after (right) calibration. All six expected decay peaks are identifiable.

low-statistics electronics channels are calibrated. The function used is of the form

$$f(x) = \frac{a}{1 + e^{b(x-c)}} \quad (5.3)$$

where a , b and c are fit parameters. The fit parameter c , which has units of charge, corresponds to the spectrum edge position and is used as the gain factor. The parameter b corresponds to the steepness of the edge, and a is a normalization factor. An example fit using the edge detection method can be seen in 5.12.

5.2.2 SHINE dE/dx Calibration Chain

The following calibration steps remove various systematic distortions to TPC charge measurement. The four calibration steps are collectively known as the SHINE dE/dx Calibration Chain. The software was developed and tested by the author and has recently been fully automated by Antoni Marcinek of Jagiellonian University. The Time-dependence, y-Dependence, and Chip Gain corrections are conceptually similar to the previous NA61 dE/dx calibration steps, while the Sector Constants correction is entirely new. The analysis presented in this thesis is the first to use the results of the SHINE-native dE/dx calibration chain. The calibration target for each correction is to reduce the relevant distortions to below 1%.

5.2.2.1 Time-Dependence Correction

The first and simplest effect to correct for is the time-dependent variation of TPC charge amplification gains. Each *TPC sector*, defined as independent groups of padrows served by one high-voltage supply channel, will possess a unique time variation that must be corrected for. Sources of these time variations include high-voltage supply channel drift, atmospheric condition changes, and beam intensity fluctuations.

A large portion of time-dependence is removed in the reconstruction module `TPCDEDXCalculatorSG`, which corrects all cluster charges uniformly using a time-dependent atmospheric pressure measurement. The time-dependence correction may be thought of as a residual correction on top of this

correction.

To calculate the time-dependent gains, all charges associated with TPC tracks are recorded with corresponding timestamp and sector number. Once the charge data is stored, the program `TimeDependenceCalculator` constructs a correction table binned in time according to the user's desire. The default correction bin width is 5 minutes. Each bin contains a structure storing the charge information and a lightweight C++ object that performs an incremental weighted average.

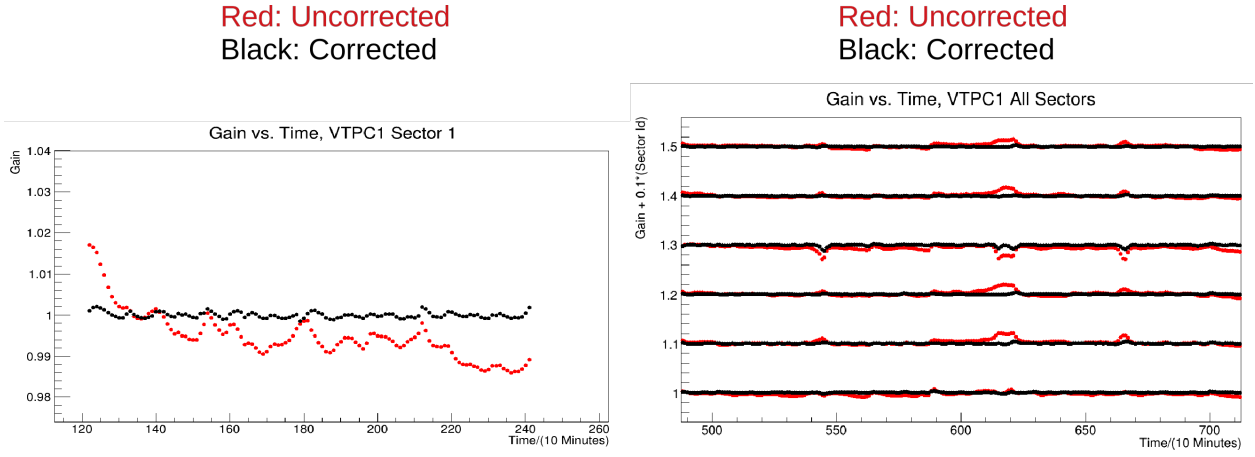


Figure 5.14: Uncorrected and corrected TPC sector gain. Left. VTPC1 sector 1 gain, un-corrected (red) and corrected (black). Post-correction gain variations are less than 1%. Right: Gain factors for all 6 VTPC1 amplification sectors, showing independent time variations.

Large variations often occur in each bin due to statistical fluctuations associated with charge deposition. In order to resolve trends at the 1% level, some sort of smoothing operation must be performed. The smoothing operation implemented is a Gaussian smoother, which uses charges from nearby time bins to temper fluctuations. The Gaussian Smoother algorithm calculates each gain measurement using unique weights for each charge. The calculated gain is given by

$$g(t) = \frac{\sum_i w_i q_i}{\sum_i w_i}, \quad w_i = e^{-\frac{(t_i - t)^2}{2\sigma_s^2}} \quad (5.4)$$

where $g(t)$ is the time-dependent gain measurement, t is the time bin center corresponding to the gain measurement, t_i is the timestamp for an individual charge, σ_s is the user-defined smoothing width (with a default value of 15 minutes). These gains are then normalized using the average of all gain measurements, converting them to a dimensionless correction factor. Reconstruction is

re-run with these gain factors included in the charge calculation, and the correction is checked by re-calculating the gain factors. The effect of including this calibration step can be see in Fig. 5.14.

5.2.2.2 Y-Dependence Correction

The second correction step is the y-dependence or drift dependence correction. This calibration step removes systematic distortions associated with charge loss during drift. These distortions, pictured in Fig. 5.15, scale with drift distance, or distance to the sense wire plane. The main sources of charge loss during drift are charge cloud diffusion and electron loss via O_2 or H_2O attachment.

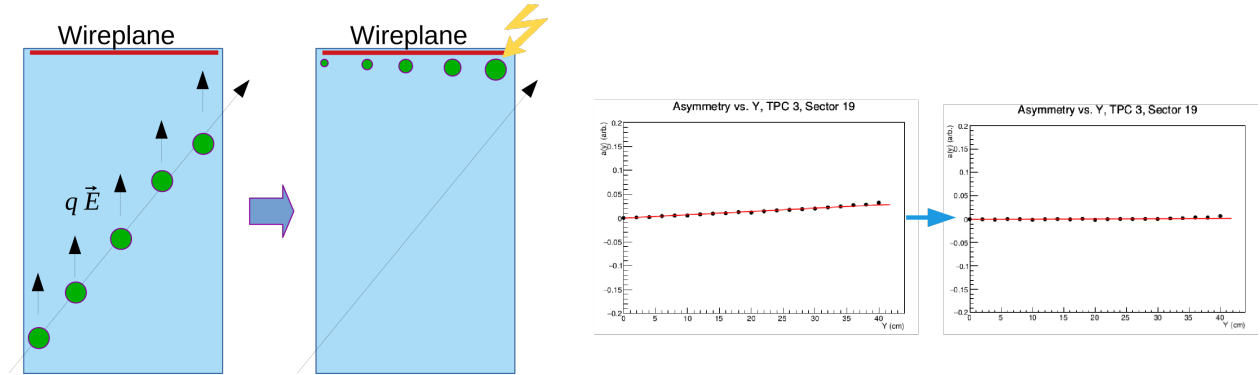


Figure 5.15: Y-dependence correction. Left: Illustration of charge loss effect due to drift. Marker size indicates charge magnitude. Right: Uncorrected (left) and corrected (right) y-dependence for an MTPC sector. Red line indicates fit for drift loss coefficient.

Charge cloud diffusion describes the process of groups of electrons from the same charged track spreading and diffusing as they drift toward the wireplane. If the charge diffuses significantly, some pad-timebin pixels will contain a charge fraction that may drop below the zero-suppression electronics threshold. This charge will not be recorded by the TPC electronics, and is lost. This effect, known as the *threshold effect*, is largely corrected for in the reconstruction module `TPCDEDXCalculatorSG` using a parameterization based on track angles⁶. Charge loss due to electron attachment is caused by electron clouds drifting near electro-negative chamber gas impurities (most commonly H_2O and O_2).

⁶ This correction is not available for the FTTPCs or GTTPC, since the lookup tables were created for the MTPCs and VTTPCs only, but similar y-dependence correction factors are observed for these chambers.

Charge y -dependence is calculated using the assumption that track production physics is symmetric in ϕ , the angle in the (x, y) plane. Thus, track characteristics and deposited charge should be symmetric in y about the beam position $y = 0$. We then model charge loss during drift as an exponential process and expand to first-order in y :

$$q(y) = f(y)e^{-cy} = f(y) \left(1 - cy + \frac{(cy)^2}{2} + \dots \right) \approx f(y)(1 - cy) \quad (5.5)$$

where $q(y)$ is the measured y -dependent charge, $f(y)$ describes the distribution of charge from production physics, and c is the constant describing the exponential suppression of charge due to drift loss. We now consider a quantity called the *asymmetry factor* a , defined as

$$a(y) = \frac{q(y) - q(-y)}{q(y) + q(-y)} = \frac{(1 + cy) - (1 - cy)}{(1 + cy) + (1 - cy)} = -cy \quad (5.6)$$

where $q(-y)$ is the measured charge at y -value $-y$. To first-order in y , this quantity is the drift distance multiplied by the exponential suppression factor. We thus calculate this asymmetry factor using data and correct for first-order charge loss effects by re-scaling the charge by the factor $(1 - cy_c)$, where y_c is the cluster y -position. c is obtained by fitting $a(y)$ with a linear model, as can be seen in Fig. 5.15. The effect of the correction can also be seen in the figure.

5.2.2.3 Sector Constants Correction

The most difficult and computationally-intensive dE/dx calibration task is the determination of absolute sector amplification, known as the Sector Constant. This gain is defined as the factor converting raw cluster ADC counts to pion dE/dx value given by the Bethe-Bloch, normalized to the pion minimum ionizing particle (MIP) value⁷. Without this gain determination, clusters along a given track would have amplification variation of more than 200%, and particle identification via dE/dx would not be possible.

In the process of including dE/dx calibration routines in the SHINE software framework, the Sector Constants correction was completely re-designed. The correction is run with the following

⁷ With this normalization, the Bethe-Bloch curves for each particle species will have a minimum value of 1, as will be seen on all subsequent dE/dx plots. Since this normalized dE/dx is unit-less, the Sector Constant possesses units of ADC counts.

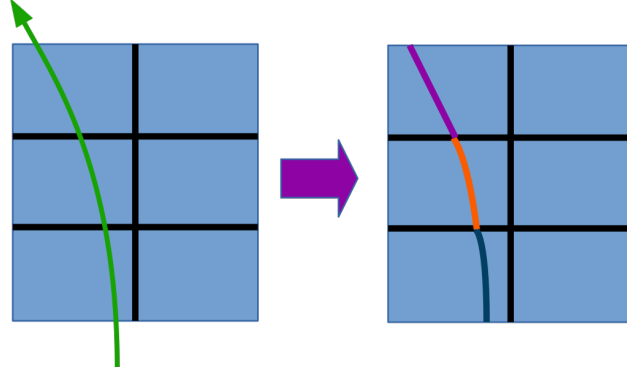


Figure 5.16: Illustration of the Sector Constants calculation concept. Well-measured tracks are broken up into tracklets spanning one TPC amplification sector.

steps:

- (1) Split tracks into track segments spanning one sector
- (2) Calculate $[0,50]\%$ truncated mean dE/dx of sufficiently long tracks⁸
- (3) Store dE/dx and momentum magnitude distributions for each TPC sector
- (4) Separate distributions into fine momentum bins
- (5) Perform multi-species dE/dx fit in each momentum bin
- (6) Record pion mean dE/dx as Sector Constant estimate
- (7) Combine well-measured estimates for each TPC sector

Steps 1 - 3 are performed in a SHINE module using reconstructed data as input. Well-measured vertex track objects are split by sector into *tracklets* and the truncated mean charge of each tracklet is calculated (see Fig. 5.16). This procedure allows for the study of individual sector gains. Track dE/dx samples are measured by converting cluster charge to dE/dx using track length over padrows, which depends on padrow geometry and track parameters. Histogramming a given track's dE/dx samples will show the characteristic *straggling function*⁹ associated with energy

⁸ See next section for detail of the truncated mean.

⁹ See Chapter 7 for more details on dE/dx and straggling functions.

deposition, often approximated using a Landau or Moyal distribution [3]. The [0,50]% truncated mean statistic is used to temper the long tail of the straggling function, and distributions of track truncated means for a given particle species tend to follow an asymmetric Gaussian distribution whose mean is given by the Bethe-Bloch formula. The asymmetric Gaussian provides a much easier functional form for analysis fitting.

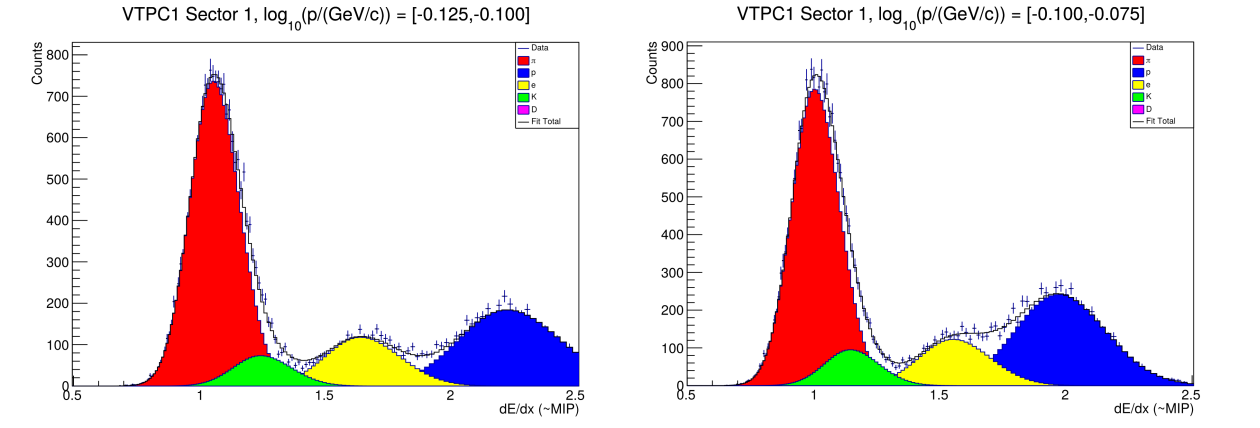


Figure 5.17: Example particle species fits in adjacent momentum bins during Sector Constant calculation. Particle species are identifiable by eye given Bethe-Bloch expectations in this momentum span: Protons, in the $1/\beta\gamma$ Bethe-Bloch region, move rapidly as a function of momenta; electrons are near the high- $\beta\gamma$ saturation value; pions and kaons are in the relativistic rise region. Deuterons are not significantly observed in this region of phase space. The pion mean position is used as the Sector Constant estimate for these momentum bins.

Once the tracklet dE/dx is recorded and stored in a file, the `SectorConstantsCalculator` program begins its analysis. Tracklets in each sector are binned into fine momentum slices ($\Delta \log(p/(\text{GeV}/c)) = 0.025$), and in each bin a five-species dE/dx fit is performed. This fit returns the fractional yields for electrons, pions, kaons, protons and deuterons, as well as the deviations from the expected Bethe-Bloch dE/dx values for each species. The details of the fitter will be provided in Chapter 7. An example fit to a collection of tracklets can be seen in Fig. 5.17.

For calibration, the relevant quantity obtained from these fits is the pion Bethe-Bloch deviation, $\Delta\pi - BB$. This quantity, along with the Bethe-Bloch value for pions, is precisely what is required to normalize the tracklet dE/dx to the pion MIP value. By performing fits in each of the fine momentum slices, multiple estimates of the Sector Constant are obtained. These estimates and

their fit uncertainties are subsequently combined to produce a final Sector Constant estimate.

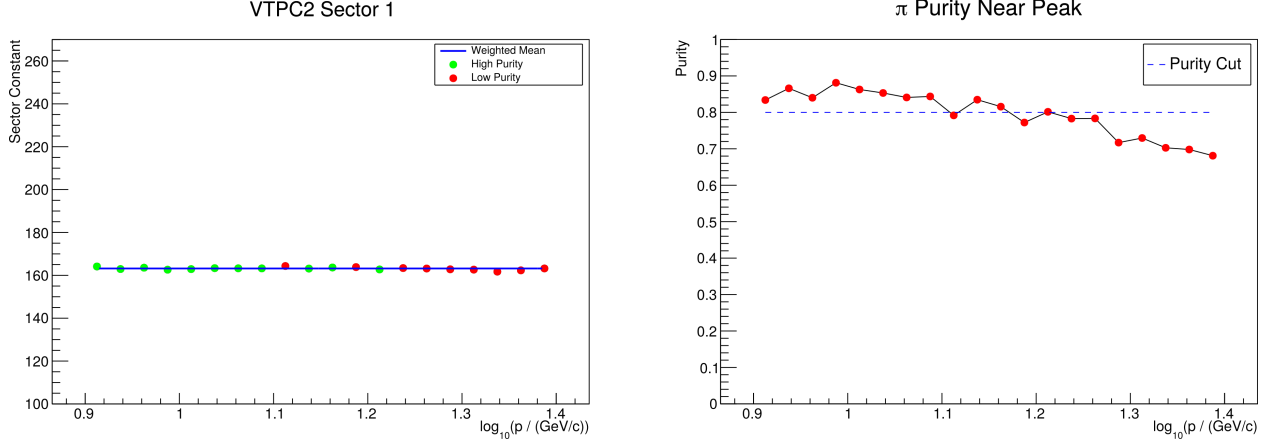


Figure 5.18: Left: Measured Sector Constant estimates for one TPC sector. High-purity points are shown in green and enter into the final value calculation. Right: Pion purity in the vicinity of the pion peak for each fit. Purity cut of 80% shown.

In order to reduce the bias in pion Bethe-Bloch deviation, we consider only regions of momentum in which the pion distribution is well-separated from other particle species. This is implemented by calculating the *pion purity* near the pion peak, or the fraction of fit-yielded pions to total particles. This quantity is calculated within the full-width half-maximum region of the fit pion distribution. Momentum bins with a pion purity less than 80% are not considered in the Sector Constant calculation, as can be seen in Fig. 5.18.

5.2.2.4 Chip Gain Correction

The final correction calculated is known as the Chip Gain correction. It is termed this because each four adjacent electronics channels share a preamplifier chip, and these chips often vary in gain. In practice, the Chip Gain Correction corrects for systematic charge loss as a function of x and z , which is caused by fiducial volume effects, sagging or broken amplification wires, or electronics variations. Visual representations of VTPC1 chip gains before and after correction can be seen in 5.19.

The Chip Gain factors are calculated by binning charge by chip ID and normalizing each track

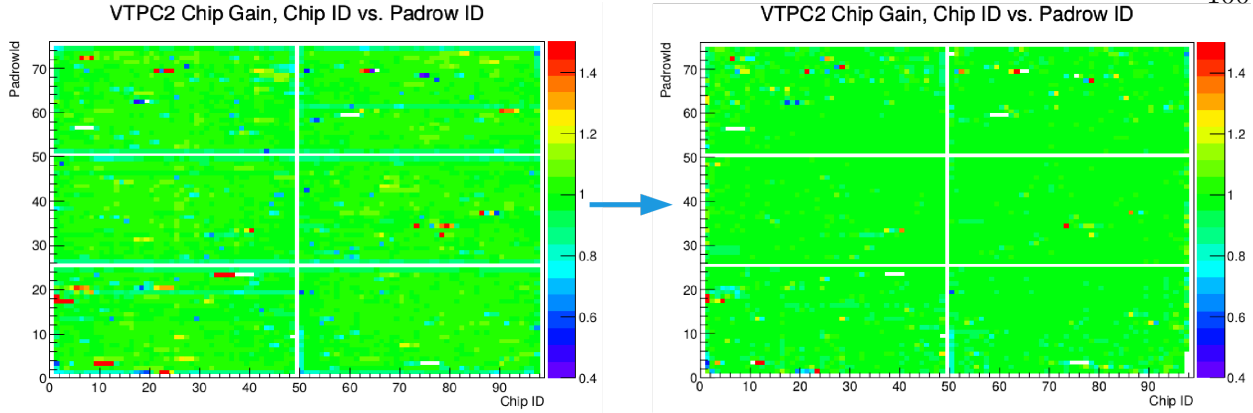


Figure 5.19: Visualization of Chip Gain corrections for VTPC2. Chips can be thought of as binned TPC pads, and this view represents an $x - z$ gain map, with x in the horizontal dimension, z in the vertical dimension. Left: Chip gain corrections calculated from data. Two low-gain or broken amplification wires are visible in the bottom-left and top-right sectors as horizontal blue stripes. Minor fiducial volume effects are visible around each sector boundary. Right: Chip-gains calculated after applying corrections. Some chips (dark red and dark blue) cannot be recovered, likely due to malfunctioning electronics.

charge to the track's mean dE/dx ¹⁰. For each chip, a lightweight incremental average calculator accepts charges and calculates a gain factor. After all charges are included, the gain factors for each sector are normalized to the mean gain factor. This operation is performed in order to de-couple the overall sector gain factor from the chip gains, which are directly coupled to the Sector Constants.

Chip gain also serves as a goodness test for TPC front-end electronics: if the calculated chip gain is significantly above or below the nominal value of 1, clusters associated with the chip are ignored during dE/dx calculations. This helps to increase the quality of dE/dx measurements.

Due to the direct coupling between the Sector Constants and the Chip Gains, the standard calibration procedure first calculates rough Sector Constants in order to obtain reasonable Chip Gain estimates. Once the Chip Gains are calculated and applied, the Sector Constants are recalculated for more precise estimates, as some systematic spread in charge due to lack of Chip Gain corrections is removed.

¹⁰ It is important to use the track mean dE/dx instead of the truncated mean, as we are comparing members of the mean calculation to the statistical value. We don't want to normalize charge values that would have been truncated away to the mean of the non-truncated charges.

5.2.2.5 dE/dx Calibration Results

The performance of the dE/dx calibration is evaluated by measuring the pion Bethe-Bloch deviation across phase space. For perfectly-calibrated data, this value should be zero – the fit pion distribution should be located exactly at the pion Bethe-Bloch expectation. In practice this is not true, as calibration imperfections and biases exist. A deviation at the few-percent level is expected.

A good performance test of the new SHINE-native dE/dx calibration is to compare the obtained Bethe-Bloch deviations to the values given by the previous dE/dx calibration software, the so-called “legacy” dE/dx calibration technique. As can be seen in Figs. 5.20 and 5.21, the SHINE-native calibration outperforms the legacy calibration in almost all regions of phase space. One-dimensional plots showing profiles of these two-dimensional histograms give bin-by-bin comparisons. The new dE/dx calibration is thus deemed fit for analysis use.

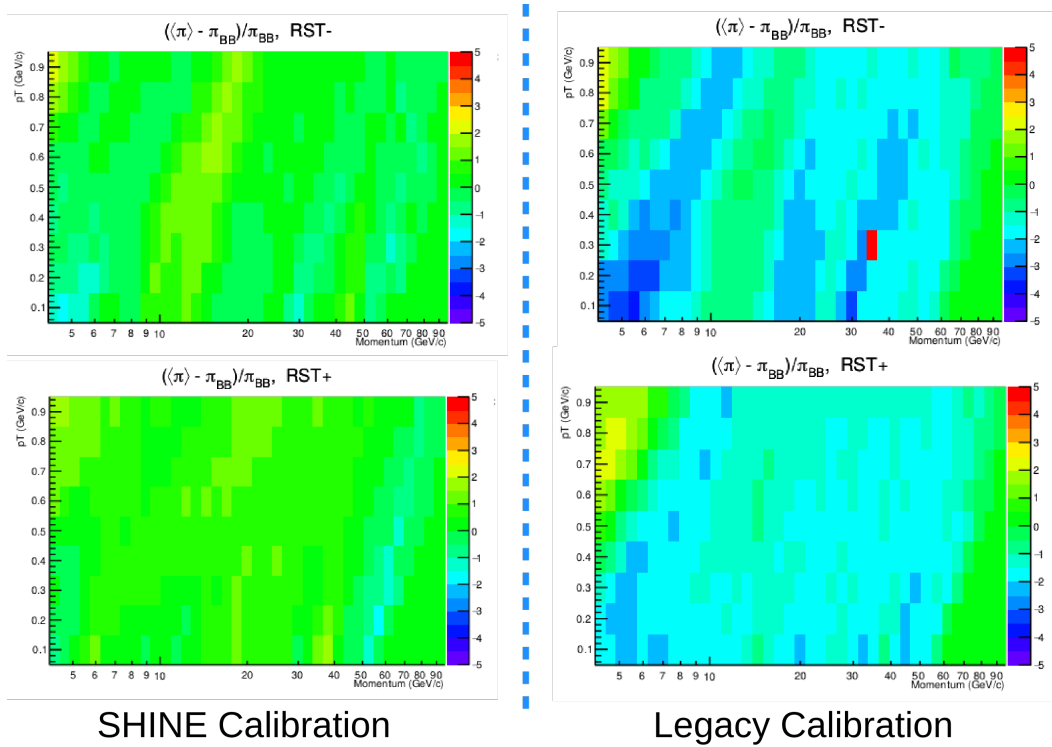


Figure 5.20: Pion Bethe-Bloch deviation as a function of $\log(p)$ and p_T gives performance evaluation of the dE/dx calibration. Left: New SHINE-native dE/dx calibration results. Right: “legacy” dE/dx calibration procedure results. The new calibration outperforms the old calibration at the percent level across phase space.

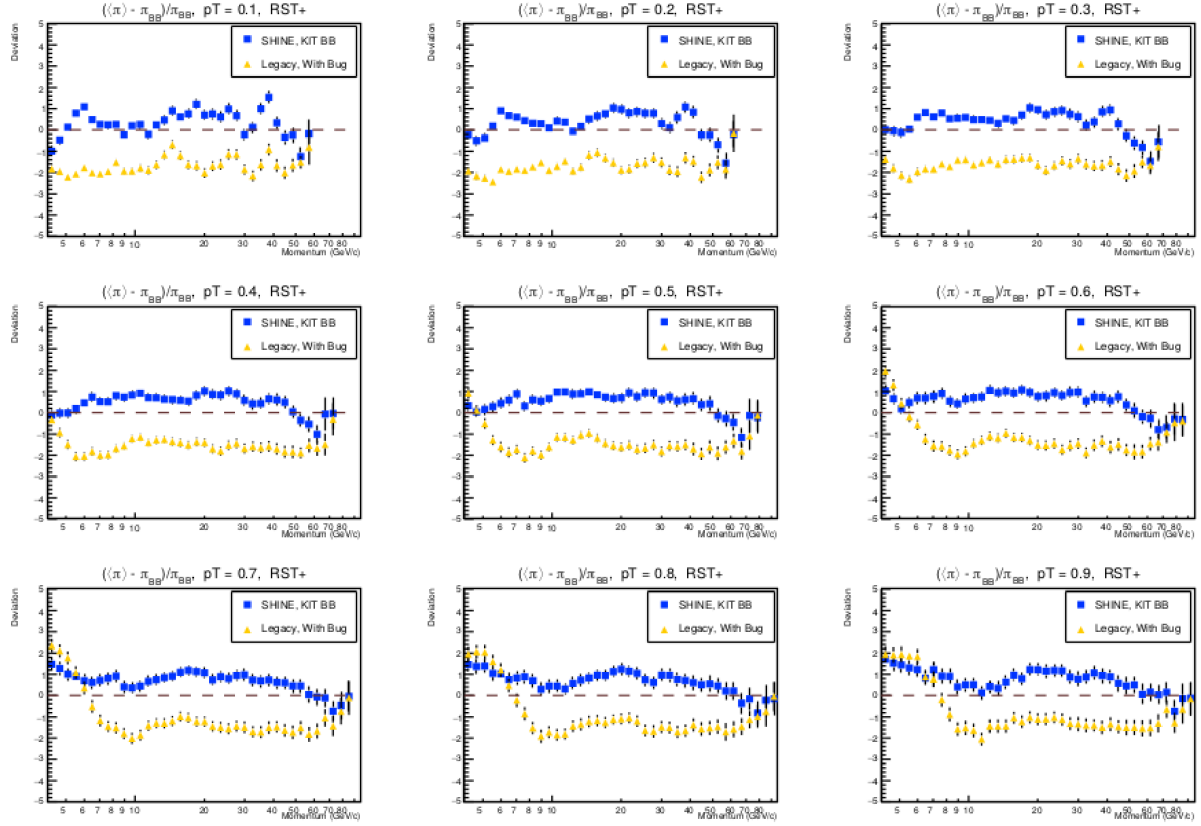


Figure 5.21: One-dimensional slices of the above plots for one track topology for direct bin-by-bin value comparison. The legacy calibration had a bug concerning initial cluster threshold correction – hence the plot legend.

Chapter 6

Total Cross-Section Analysis

This chapter will define the various process cross-sections discussed in this analysis. Various sources define the inelastic cross-section components differently; for the purposes of this thesis, the cross-section definitions given in this chapter will be used. The chapter will present the total production cross-section measurements for 120 GeV/ c proton-carbon interactions and discuss the measurements from complementary 2016 and 2017 data sets.

6.1 Cross-Section Definitions

The total cross-section for a hadron-nucleus interaction is given by the sum of the elastic and inelastic cross-sections. The inelastic cross-section can be further broken up into two classes of interactions: those that produce new hadrons, defined as *production interactions*, and those that do not produce new hadrons, defined as *quasi-elastic interactions*. Quasi-elastic interactions often involve fragmentation of the nucleus (see Fig. 6.1).

$$\sigma_{\text{total}} = \sigma_{\text{elastic}} + \sigma_{\text{inelastic}} \tag{6.1}$$

$$\sigma_{\text{inelastic}} = \sigma_{\text{production}} + \sigma_{\text{quasi-elastic}}. \tag{6.2}$$

6.2 Cross-Section Calculations

The cross-section measurements are derived from the trigger and interaction measurements, which are calculated using the T1 (identified beam particle) and T2 (identified beam particle with

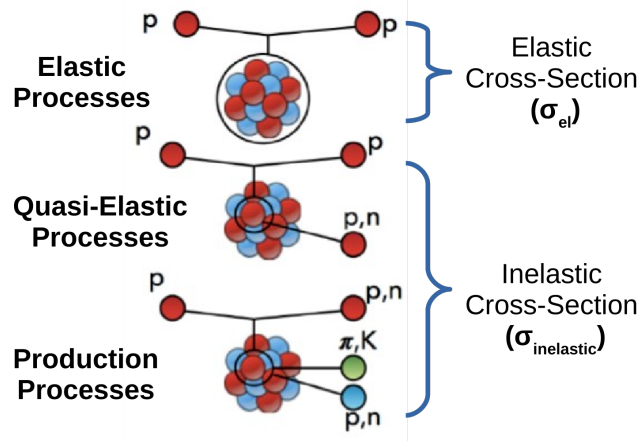


Figure 6.1: Cross-section processes as defined for this thesis.

no signal in the S4 scintillator) event samples. A trigger-tagged T2 event as defined in Chapter 3 does not necessarily indicate an interaction in the target. The T2 event sample is contaminated with out-of-target interactions and non-interactions in which the beam missed the S4 scintillator, while the T1 event sample is contaminated with interactions in which particles associated with the interaction passed through the S4 scintillator.

In order to calculate the trigger cross-section associated with interactions in the target, we perform *target-out subtraction*. This technique involves recording a small amount of interaction data with identical detector characteristics but the target removed. The trigger probabilities for each target configuration are the ratio of the total number of prescaled T1 triggers to the total number of T2 triggers that are also prescaled T1 triggers:

$$P_{\text{In/Out}} = \frac{N(T1_{\text{pre}})_{\text{In/Out}}}{N(T2 \ \&\& \ T1_{\text{pre}})_{\text{In/Out}}}. \quad (6.3)$$

We may then calculate the target interaction trigger probability P_{trig} , which removes contributions from out-of-target interactions:

$$P_{\text{trig}} = \frac{P_{\text{In}} - P_{\text{Out}}}{1 - P_{\text{Out}}}. \quad (6.4)$$

This probability can be used to calculate the trigger cross-section. For thin targets, the interaction probability can be obtained by weighting the interaction cross-section σ_{int} by the number of

potentially interacting nuclei:

$$P_{\text{int}} = n L \sigma_{\text{int}} \quad (6.5)$$

where n is the number density of target nuclei ($1/\text{cm}^3$) and L is the target length (cm). The number density can be expressed as a combination of the material density ρ , atomic number A , and Avogadro's Number:

$$n = \frac{\rho N_A}{A}. \quad (6.6)$$

The interaction cross-section can then be re-written as

$$\sigma_{\text{int}} = \frac{A}{\rho N_A L_{\text{eff}}} P_{\text{int}} \quad (6.7)$$

where L_{eff} is the *effective target length*, which corrects the target length for exponential beam attenuation in the target using the absorption length λ_{abs} :

$$L_{\text{eff}} = \lambda_{\text{abs}}(1 - e^{-L/\lambda_{\text{abs}}}). \quad (6.8)$$

When the target length is small compared to the absorption length, this reduces to the target length L . Using the effective target length, the trigger cross-section is given by

$$\sigma_{\text{trig}} = -\frac{A}{\rho N_A L} \ln(1 - P_{\text{int}}). \quad (6.9)$$

6.3 Production Cross-Section Measurements

NA61/SHINE recorded 120 GeV/ c proton-carbon interactions in 2016 and again in 2017 after the addition of the FTPCs. The two data sets are complementary, as two different magnetic field settings were used and the phase space coverage differs. Due to a trigger misconfiguration in 2017, the prescaled T1 data set was not correctly recorded. Thus, the 2016 cross-section measurement was used for this analysis, with the 2017 data analyzed in a different way as a cross-check for data consistency.

6.3.1 2016 Measurement

This section reports the results of Yoshiakzu Nagai's 2016 120 GeV/ c proton-carbon cross-section analysis. These results were published in 2019 [48]. Since the author of this thesis did not perform the analysis, the details are left to the paper. The final result enters the multiplicity calculations in Chapter 7.

6.3.1.1 Event selection

Events selected for the total cross-section analysis are prescaled (unbiased) identified proton beam tracks with well-measured beam trajectories. The beam trajectory cut is implemented by requiring that the three BPDs measure a well-fit track and the most downstream BPD measures a reasonable cluster. The beam track must also be aimed at the S4 scintillator. A schematic showing extrapolated beam tracks and the S4 cut can be viewed in Fig. 6.2. Table 6.1 shows the resulting event counts and interaction probabilities for the target-in and target-out data sets.

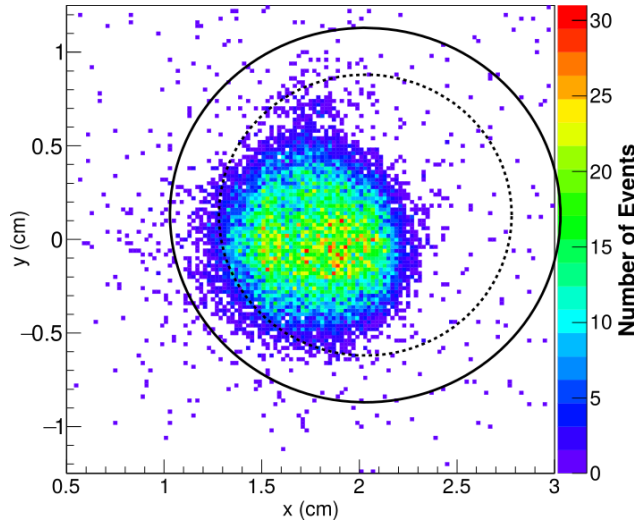


Figure 6.2: 120 GeV/ c beam tracks extrapolated to the S4 plane. Beam tracks shown here are correspond to the 120 GeV/ c proton-beryllium reaction, taken just after the 120 GeV/ c proton-carbon reaction and possessing very similar beam characteristics.

The trigger cross-section σ_{trig} contains contributions from elastic, quasi-elastic and production

Target Status	Analysis Events (After Cuts)
Target-In	214k
Target-Out	117k

Table 6.1: Event counts for 2016 120 GeV/ c proton-carbon cross-section analyses. Reproduced from [48].

Monte Carlo Correction Factors				
f_{prod}	f_{el}	σ_{el} (mb)	f_{qe}	σ_{qe} (mb)
0.926	0.085	65.1	0.425	23.3

Table 6.2: FTFP_BERT Monte Carlo corrections for 2016 120 GeV/ c proton-carbon cross-section analyses. Elastic processes correspond to QBBC Monte Carlo. Reproduced from [48].

events that scatter out of the S4 scintillator:

$$\sigma_{\text{trig}} = f_{\text{prod}} \cdot \sigma_{\text{prod}} + f_{\text{qe}} \cdot \sigma_{\text{qe}} + f_{\text{el}} \cdot \sigma_{\text{el}} \quad (6.10)$$

Here f_{prod} is the fraction of production events that scatter out of S4, f_{qe} is the fraction of quasi-elastic events that scatter out of S4, f_{el} is the fraction of quasi-elastic events that scatter out of S4. and In order to measure the production cross-section, Monte-Carlo corrections are calculated to remove the contributions to σ_{trig} from elastic and quasi-elastic processes:

$$\sigma_{\text{prod}} = \frac{1}{f_{\text{prod}}} (\sigma_{\text{trig}} - f_{\text{qe}} \cdot \sigma_{\text{qe}} - f_{\text{el}} \cdot \sigma_{\text{el}}) \quad (6.11)$$

The correction factors can be seen in table 6.2. The resulting cross-section and statistical uncertainty were measured to be

$$\sigma_{\text{prod}} = 227.1 \pm 3.4 \text{ millibarn.} \quad (6.12)$$

Systematic uncertainties considered were the target density, the S4 scintillator size and position, and Monte-Carlo interaction modeling. The calculated systematic uncertainties can be seen in table 6.3. The resulting uncertainty was calculated to be

$$\Delta\sigma_{\text{prod}} = \pm_{13.1}^{4.2} \text{ millibarn.} \quad (6.13)$$

This value is used to calculate identified charged particle multiplicities, as will be explained in the following chapter.

Systematic Uncertainty Breakdown (mb)			
Target Density	S4 Size & Position	Elastic Model	Other model
± 1.8	$\pm_{3.2}^{2.0}$	$\pm_{0.4}^{0.0}$	$\pm_{14.1}^{0.0}$

Table 6.3: Systematic uncertainties estimated for 2016 120 GeV/ c proton-carbon cross-section analysis. Reproduced from [48].

6.3.2 2017 Measurement

The cross-section analysis was repeated using the 2017 120 GeV/ c proton-carbon data set. Unfortunately, a detector mis-configuration was discovered, and the total cross-section measurement was not possible to make¹. Specifically, the unbiased beam particle event sample (T1_{pre}) was rendered unusable. This is the event sample normally used to calculate total cross-sections. The beam particle interaction event sample (T2) was recovered by re-creating the trigger criteria using beam counter signals, which were properly recorded, making the multiplicity analysis possible.

In order to extract the necessary trigger information from the 2017 data set, a different source was used. The Detector Control System (DCS) records the *trigger scaler* values, or the raw counts corresponding to each trigger type integrated over each spill. These counts are made before prescaling of the data, meaning that they are a nearly unbiased sample. Unfortunately, using the trigger scaler counts eliminates the ability to perform event-by-event cuts, such as the BPD or S4 cut. To ensure the quality of these measurement values, an interaction probability was calculated for this reaction and compared against the probability obtained from the 2016 cross-section analysis.

The raw scaler-based interaction probabilities are susceptible to changes in beam conditions, mostly due to the fraction of beam particles which do not interact but miss the S4 scintillator. Beam condition changes are usually discrete, and lead to periods with shifted interaction baselines. To reduce the beam condition impact on the interaction probabilities, separate beam condition periods were defined. If the mean target-in interaction probability ($\frac{T_2}{T_1}$) changes by more than 5%, a new period is defined (see Fig. 6.3). For each interaction period, a target trigger probability is

¹ The configuration issue was the trigger signal output connection to the DAQ trigger pattern unit input board. The input GATE signal cable, which triggers an update of the T1, T2, T3, T4, T1_{pre}, T2_{pre}, T3_{pre}, and T4_{pre} registers, was switched with the T1_{pre} signal cable.

calculated as given by equation 6.4.

Beam Condition Period	$P_{\text{target-in}} (\%)$	$P_{\text{target-out}} (\%)$	$P_{\text{trig}} (\%)$
Period 1	3.39 ± 0.08	0.43 ± 0.03	2.97 ± 0.08
Period 2	3.6 ± 0.1	0.64 ± 0.04	2.99 ± 0.1
Period 3	3.50 ± 0.08	0.57 ± 0.03	2.94 ± 0.09
Combined	3.47 ± 0.05	0.51 ± 0.02	2.97 ± 0.05

Table 6.4: Target-in, target-out, and trigger probabilities calculated for three beam condition periods, and their combined value. The data entering these calculations can be seen in Figs. 6.3 - 6.6.

The calculated interaction probabilities for the three periods differ from their combined value by less than 1%, and agree with the 2016 in-target trigger probability of 2.967 ± 0.041 [48]. The consistency of these measurements lends credibility to the scalers analysis technique and to the quality of the data set. The target-in and target-out probabilities calculated here are used in the multiplicity calculation.

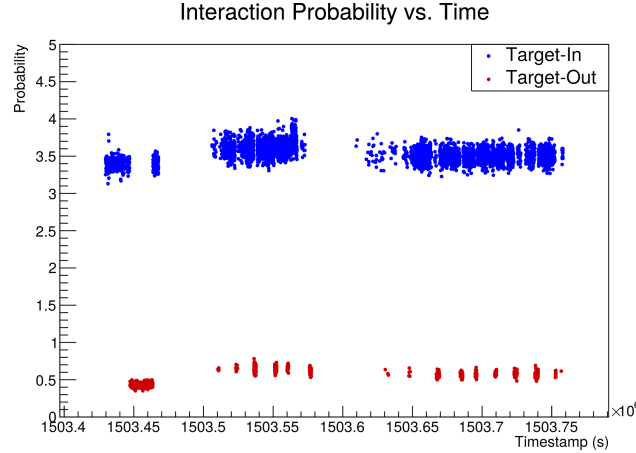


Figure 6.3: Interaction probability ($\frac{T_2}{T_1}$) for the 2017 data-taking period. Three distinct regions of target-in probability are visible. Interaction probabilities for these three periods are calculated separately using the respective target-in and target-out measurements.

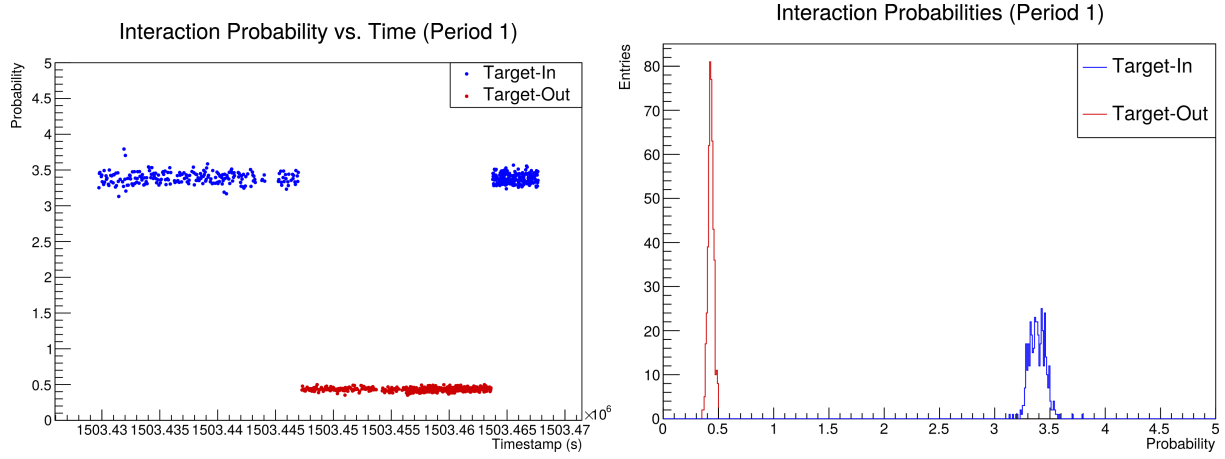


Figure 6.4: Left: Trigger scaler interaction probability ($\frac{T^2}{T_1}$) as a function of time for beam condition period 1. Right: One-dimensional projection integrated over time. In-target trigger probability calculated for this data sample is 2.97 ± 0.08 %.

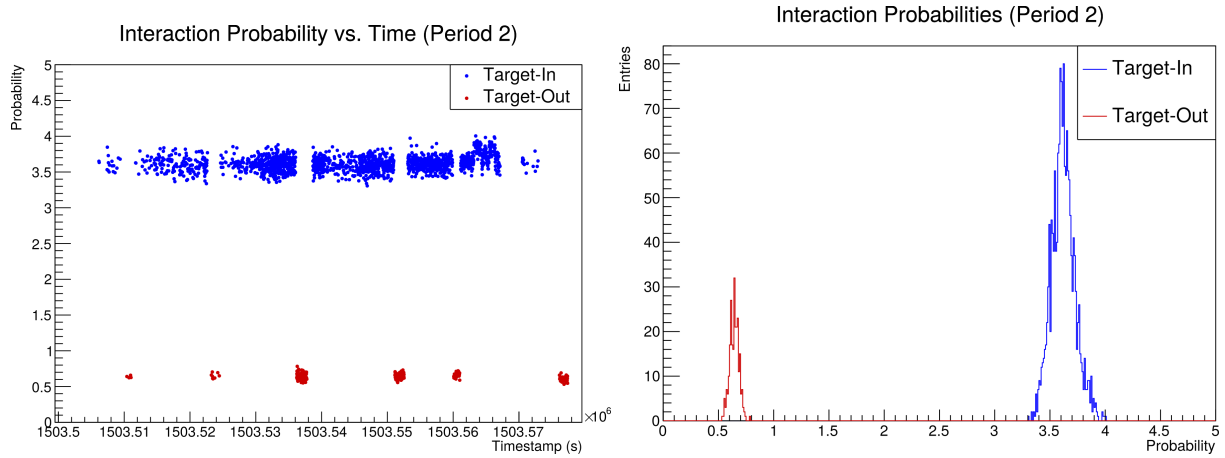


Figure 6.5: Left: Trigger scaler interaction probability ($\frac{T^2}{T_1}$) as a function of time for beam condition period 2. Right: One-dimensional projection integrated over time. In-target trigger probability calculated for this data sample is 2.99 ± 0.1 %.

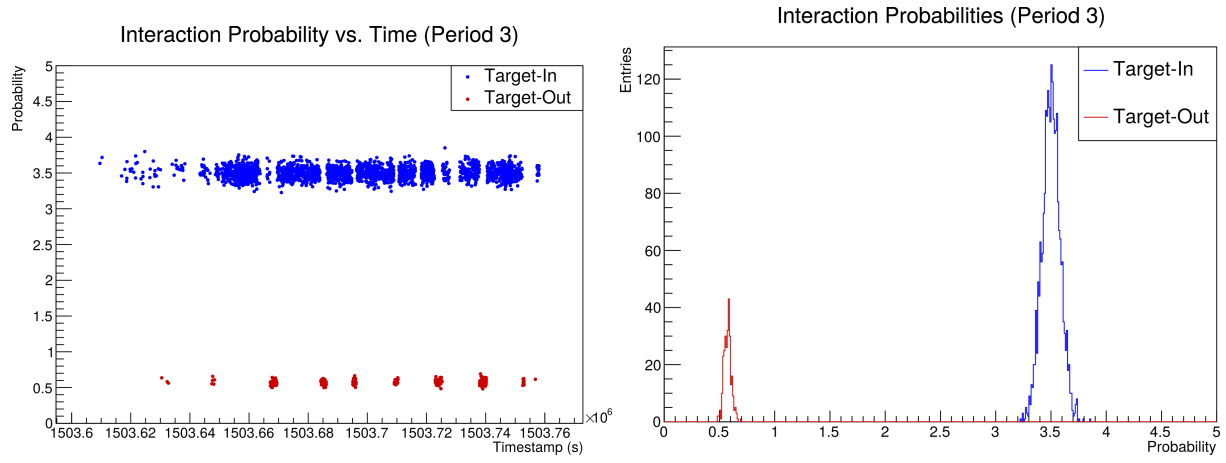


Figure 6.6: Left: Trigger scaler interaction probability ($\frac{T_2}{T_1}$) as a function of time for beam condition period 3. Right: One-dimensional projection integrated over time. In-target trigger probability calculated for this data sample is 2.94 ± 0.09 %.

Chapter 7

Charged Hadron Analysis

This chapter details the charged hadron analysis of both the 2016 and 2017 120 GeV/ c proton-carbon data sets. The methods of dE/dx analysis and fitting will be discussed first, followed by the details of the analyses, including binning choices, track selection, Monte-Carlo corrections, and the resulting differential multiplicities.

7.1 Particle Identification via dE/dx

Charged particles passing through matter will deposit energy stochastically along their trajectory. This deposition occurs via various electromagnetic interactions, mainly ionization and atomic excitation. The mean energy loss per unit length is a quantity known as $\langle dE/dx \rangle$. This quantity depends on the charge of the particle, the particle's speed relative to the speed of light (v/c or β), and properties of the material being traversed. The dominant interactions a particle experiences can be separated into several regimes based on the particle's energy (see Fig. 7.1). We will focus on the *Bethe* region near the minimum ionization location, which covers the pertinent NA61/SHINE-accepted phase space. For other regions, the reader is referred to the PDG [3]. We refer to the curve describing $\langle dE/dx \rangle$ as the Bethe-Bloch curve, given by a Bethe-Bloch model such as Eq. 7.2. Near the minimum ionization point, this function has two distinct regions: Below the minimum ionization point, where $\langle dE/dx \rangle$ increases approximately as $1/\beta^2$, and above the minimum ionization point, where $\langle dE/dx \rangle$ increases approximately as $\ln(\beta\gamma)$ where γ is the Lorentz factor.

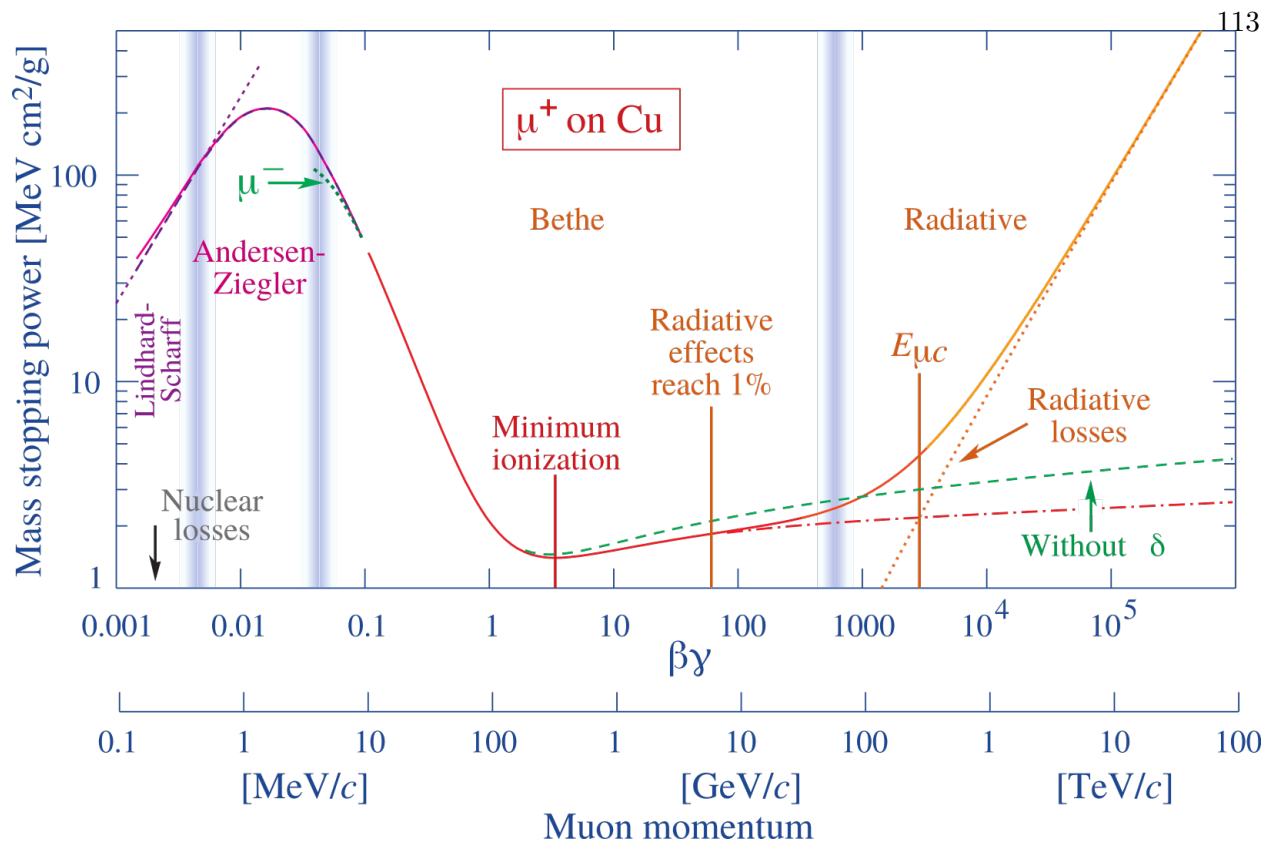


Figure 7.1: Specific energy loss ($-dE/dx$) for muons in copper, showing four distinct energy loss regimes. The region used for particle identification spans from $\beta\gamma \sim 0.1$ – 1000 . Reproduced from [3].

When plotted along a common momentum axis, the Bethe-Bloch curves for particles of different mass are displaced. This is the key concept involved in performing particle identification via dE/dx . In regions of momentum with well-separated Bethe-Bloch curves for different particle species, statistical separation of the species is possible. In regions where Bethe-Bloch curves cross, separation is not possible and other distinguishing characteristics, such as time-of-flight, are typically used to separate particle species. For this analysis, time-of-flight data is unfortunately not yet available, and regions of Bethe-Bloch overlap within 5% are not considered for analysis.

The value given by Bethe-Bloch models describes the *mean* dE/dx for a given particle species. The stochastic energy deposition process distributes particle dE/dx about this mean value according to a probability distribution known as a *straggling function*. Various parameterizations of straggling functions exist, including the Landau and Moyal distributions [49]. These probability

distributions all possess a characteristic long tail toward large energy deposition (see Fig. 7.2). dE/dx samples along a track measured at NA61/SHINE are distributed according to such a function, sampled $\mathcal{O}(10 - 100)$ times using cluster charges. In order to temper the long tail behavior of these distributions, track $\langle dE/dx \rangle$ is calculated using the [0,50]% *truncated mean* of its constituent cluster charges, where only the smallest 50% of charge values enter the calculation. By using this method, dE/dx samples from particles of the same type and momentum approximately follow an asymmetric Gaussian distribution (see Fig. 7.3). This distribution can be fit using minimization software in a much less computationally-intensive manner.

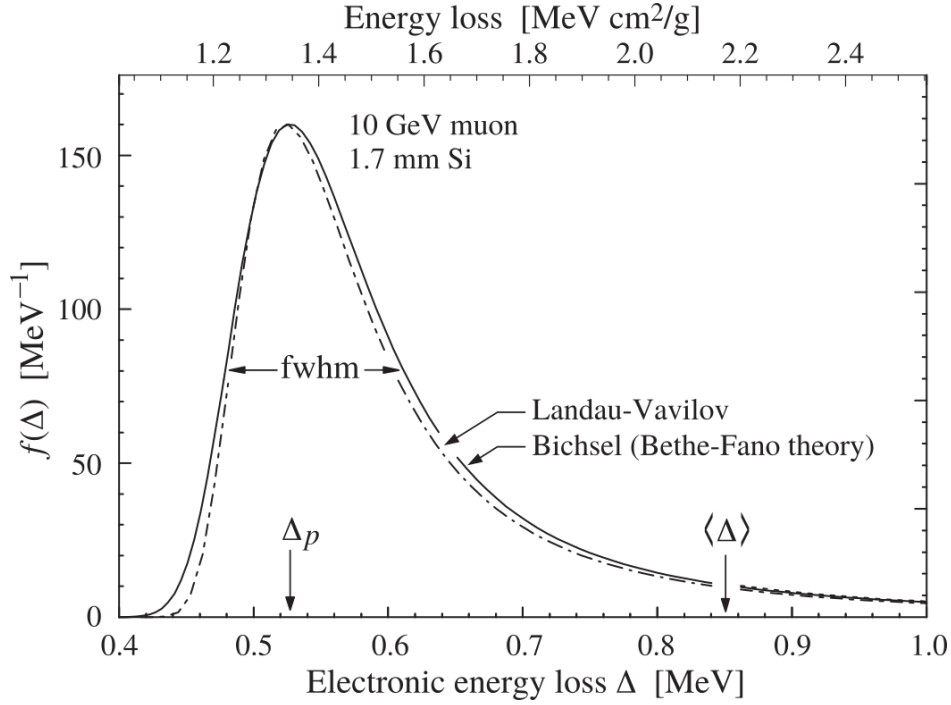


Figure 7.2: Straggling functions for 10 GeV muons on silicon. Most-probable energy loss is denoted by Δp , while mean dE/dx is denoted by $\langle \Delta \rangle$. Two different straggling function models, Landau and Bichsel, are compared. Reproduced from [3].

Particle $\langle dE/dx \rangle$ may be parameterized as a function of $\beta\gamma$, or equivalently p/mc , where p is the particle momentum, m is the particle mass, and c is the speed of light. The Bethe-Bloch equation [3] describes the region from approximately 40 MeV/ c to 6 GeV/ c for pions:

$$-dE/dx = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta}{2} \right]. \quad (7.1)$$

Here, $K = 0.307 \text{ MeV cm}^2 / \text{mol}$, z is the incident charge number of the particle, Z and A are respectively the atomic number and mass of the absorber, m_e is the electron mass, T_{max} is the maximum kinetic energy that may be imparted to a free electron, I is the ionization potential of the absorber, and δ is an energy-dependent density correction.

Experimentally, it is useful for this quantity to be normalized such that the $\langle dE/dx \rangle$ corresponding to the minimum ionization point is unity. It is also convenient to approximate the Bethe-Bloch formula with a parameterized version whose parameters may be fit to experimental data. This is useful in the case where experimental absorbers are not entirely pure and may not be accurately described by equation 7.2. One possible version, known as the *ALEPH parameterization* [50], is defined as

$$-\frac{\langle dE/dx \rangle}{\langle dE/dx \rangle_{\text{MIP}}} = \frac{p_0}{\beta^{p_3}} \left[p_1 - \beta^{p_3} - \ln \left(p_2 + \frac{1}{(\beta\gamma)^{p_4}} \right) \right] \quad (7.2)$$

where parameters p_0, p_1, p_2, p_3 and p_4 are fit to experimental dE/dx data. Other parameterizations also exist, and tabulated functions may be used. The “Legacy” NA61/SHINE dE/dx analyses used a tabulated Bethe-Bloch function, while this analysis uses a parameterized version. The version used must be consistent between the calibration step, in which TPC gains are normalized to the selected Bethe-Bloch equation, and the analysis step, where fits to calibrated data are performed.

7.1.1 dE/dx Fitting and Fit Model

To distinguish between charged particle species in a region of phase space, a maximum-likelihood five-species dE/dx fit is performed. This fit involves minimizing a set of 21 parameters to best describe the dE/dx data using five asymmetric Gaussian functions, with one function representing a fit species. The species fit for are $e^\pm$, $\pi^\pm$, $K^\pm$, $p/\bar{p}$, and $D/\bar{D}$. We now detail the aspects of the fit model used to construct the likelihood function.

The asymmetric Gaussian distribution is given by

$$f(x, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2} \left(\frac{x-\mu}{\delta\sigma} \right)^2} \quad \delta = \begin{cases} 1-d, & x \leq \mu \\ 1+d, & x > \mu \end{cases}. \quad (7.3)$$

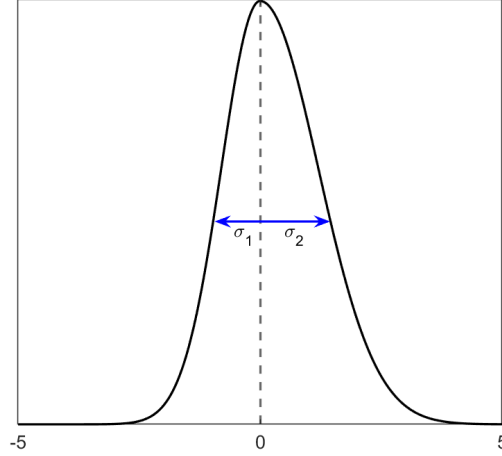


Figure 7.3: Asymmetric Gaussian function demonstrating distinct widths $\sigma_1 = (1 - d) \cdot \sigma$ and $\sigma_2 = (1 + d) \cdot \sigma$ above and below function most probable value μ . Example function properties are $\mu = 0$, $\sigma = 1$, $d = 0.2$.

We use this model to describe the distribution of dE/dx samples ϵ_i along a given track. The peak of this distribution, μ , can be expressed using the mean dE/dx value $\langle \epsilon \rangle$:

$$\mu = \langle \epsilon \rangle - \frac{4d\sigma}{\sqrt{2\pi}}. \quad (7.4)$$

The distribution width σ of a single track depends on the number of clusters along the track (N_{cl}), the mean track dE/dx $\langle \epsilon \rangle$, a base width σ_0 , and a scaling parameter α describing the scaling dependence on $\langle \epsilon \rangle$:

$$\sigma = \frac{\sigma_0 \langle \epsilon \rangle^\alpha}{\sqrt{N_{cl}}}. \quad (7.5)$$

Various parts of TPCs are described by different base widths, due to TPC pad geometries and differences in calibration. The fit model contains four base σ_0 parameters:

- $\sigma_{0 \text{ }_{UP}}$: Upstream VTPC1 sectors 1 & 4
- $\sigma_{0 \text{ }_{V}}$: All other VTPC sectors
- $\sigma_{0 \text{ }_{M}}$: MTPC sectors
- $\sigma_{0 \text{ }_{F}}$: FTPC sectors

The dE/dx width of a single track depends on the base widths and the number of measurements in each of the corresponding TPC sectors:

$$\sigma = \frac{1}{\sqrt{\frac{N_{\text{cl U P}}}{\sigma_0 \text{ U P}} + \frac{N_{\text{cl V}}}{\sigma_0 \text{ V}} + \frac{N_{\text{cl M}}}{\sigma_0 \text{ M}} + \frac{N_{\text{cl F}}}{\sigma_0 \text{ F}}}} \langle \epsilon \rangle^\alpha. \quad (7.6)$$

At this point, the model consists of the sum of five such asymmetric Gaussians whose widths vary according to the number of measurements on constituent tracks and the regions of the detector the tracks pass through. Let us consider the case in which only the five species yields are fit for. The likelihood function is given by

$$LL(\text{data}; Y_e^\pm, Y_\pi, Y_K^\pm, Y_P^\pm, Y_D^\pm) = \prod_i^{i \in \text{data}} \left(\sum_j \frac{Y_j}{\sqrt{2\pi}\sigma_i} e^{-\frac{1}{2} \left(\frac{\epsilon_i - \mu_j}{\delta\sigma_i} \right)^2} \right), \quad j \in e^\pm, \pi^\pm, K^\pm, p^\pm, D^\pm \quad (7.7)$$

where Y_j are the species fit yields, σ_i is the width corresponding to a single track dE/dx measurement, and ϵ_i is the track dE/dx . The sum i runs over all tracks, but must be separated for positive and negative tracks, as each track charge contributes only to the respective species yield. The likelihood function in this form contains 10 free parameters, corresponding to the five positive and five negative particle species. Taking the logarithm of this likelihood function (denoted using LL) and separating the track charges, the log-likelihood becomes

$$LL(\text{data}; Y_e^\pm, Y_\pi, Y_K^\pm, Y_P^\pm, Y_D^\pm) = \sum_i^{i \in + \text{ tracks}} \left(\sum_j \frac{Y_j}{\sqrt{2\pi}\sigma_i} e^{-\frac{1}{2} \left(\frac{\epsilon_i - \mu_j}{\delta\sigma_i} \right)^2} \right) + \sum_k^{k \in - \text{ tracks}} \left(\sum_l \frac{Y_l}{\sqrt{2\pi}\sigma_k} e^{-\frac{1}{2} \left(\frac{\epsilon_k - \mu_l}{\delta\sigma_k} \right)^2} \right), \quad \begin{cases} j \in e^+, \pi^+, K^+, p^+, D^+ \\ l \in e^-, \pi^-, K^-, p^-, D^- \end{cases}$$

TPC dE/dx calibration, as described in Chapter 5, is an imperfect process. To allow for variations in the calibration, 10 additional fit parameters are introduced to the model. Calibration constants accounting for misalignment of the various species dE/dx to the Bethe-Bloch function

are given by X_j^i such that the mean $dE/dx \langle \epsilon \rangle$ for each species can be expressed as

$$\langle \epsilon \rangle_j^i = \begin{cases} \langle \epsilon \rangle_{\text{BB}} + X_j^\pi & \text{if } i = \pi_j \\ \langle \epsilon \rangle_{\text{BB}} + X_j^\pi + X_j^{e, K, p, D} & \text{if } i \neq \pi_j \\ j \in +, - \end{cases} \quad (7.8)$$

The widths of the positive and negative tracks are allowed to vary independently, and receive an overall multiplicative fit factor. In addition, due to the greater amount of scattering undergone by electrons in comparison to the heavier fit species, a multiplicative width fit factor is assigned to electrons only. Both of these modify the width calculated using the number of clusters in various parts of the detector as express in 7.6:

$$\sigma_j^i = \begin{cases} f_j f_e \sigma & \text{if } i = e_j \\ f_j \sigma & \text{if } i \neq e_j \\ j \in +, - \end{cases} \quad (7.9)$$

In addition, the width d and the base dE/dx widths σ_0^i , $i \in \text{Up, V, M, F}$ are allowed to vary. A constraint may be placed on the yields, demanding that the fractional yield values sum to 1:

$$\sum_{i \in \text{species}} Y_j^i = 1, \quad j \in +, - \quad (7.10)$$

The log-likelihood function may now be minimized by varying the free model parameters using minimization software such as Minuit [51]. The objective function passed to Minuit (referred to according to convention as FCN) is

$$FCN = 2(-LL + \text{constraints}). \quad (7.11)$$

Here, the constraints represent the soft constraints implemented in the fit. The fit is a 21-parameter fit¹, requiring external constraints to prevent the calibration and shape parameters from freely

¹ The full parameter count: 4 positive yield parameters Y_+^i (5 species minus 1 normalization constraint), 3 negative yield parameters Y_-^i (negative deuterons are not considered), 4 shape parameters f_+ , f_- , f_e , d , 6 calibration parameters X_j^i , and 4 base widths σ_0^i .

running and, for example, swapping the order of Bethe-Bloch locations for particle species. The specific soft constraints used in the fits were enforced using the penalty terms

constraints =

$$\begin{aligned} & \left(\frac{f_+}{0.01} \right)^2 + \left(\frac{f_-}{0.01} \right)^2 + \left(\frac{f_+ - f_-}{0.01} \right)^2 + \left(\frac{f_e}{0.05} \right)^2 + \left(\frac{f_d}{0.005} \right)^2 + \\ & \left(\frac{X_+^\pi}{0.01} \right)^2 + \left(\frac{X_-^\pi}{0.01} \right)^2 + \left(\frac{X_+^\pi - X_-^\pi}{0.01} \right)^2 + \left(\frac{X^e}{0.001} \right)^2 + \left(\frac{X^K}{0.001} \right)^2 + \left(\frac{X^P}{0.001} \right)^2 + \left(\frac{X^D}{0.001} \right)^2 + \\ & \left(\frac{\sigma_0^{\text{Up}} - (\sigma_0^{\text{Up}})_0}{0.01} \right)^2 + \left(\frac{\sigma_0^{\text{V}} - (\sigma_0^{\text{V}})_0}{0.01} \right)^2 + \left(\frac{\sigma_0^{\text{M}} - (\sigma_0^{\text{M}})_0}{0.01} \right)^2 + \left(\frac{\sigma_0^{\text{F}} - (\sigma_0^{\text{F}})_0}{0.01} \right)^2. \end{aligned}$$

Many of these soft constraint values were informed by a previous analysis [52]. Others were obtained by iteratively fitting in all kinematic bins while fixing and releasing individual parameters. The base dE/dx widths have different values for the 2016 and 2017 data sets, as can be seen in Table 7.1.

The resulting fit yields provide an estimate of particle yield by species in each kinematic analysis bin. These yields will be the basis for the identified multiplicity calculations discussed in the following section.

Parameter	2016 Base Value	2017 Base Value	Uncertainty
σ_0^{Up}	0.56	0.68	0.06
σ_0^{V}	0.43	0.46	0.02
σ_0^{M}	0.6	0.46	0.01
σ_0^{F}	<i>N/A</i>	0.44	0.01

Table 7.1: Base dE/dx widths for 2016 and 2017 dE/dx fits.

7.2 2016 & 2017 Analyses

The procedure detailed in Section 7.1 was used to calculate proton, pion and kaon multiplicities resulting from 120 GeV/ c protons interacting with a carbon target. Two complementary data sets were recorded by NA61/SHINE, each with a different detector configuration. The 2016 data set was taken using the full-strength magnetic field, while the 2017 data set was taken with

the half-strength magnetic field and after the addition of the FTPCs. This section describes the multiplicity analyses and compares the 2016 and 2017 multiplicity results.

7.2.1 dE/dx Analysis Binning

Traditionally at NA61/SHINE, multiplicity analysis results are presented in a two-dimensional phase space of reconstructed track momentum p and reconstructed track production angle θ . This is due to the dependence of particle dE/dx on momentum: If binning is sufficiently fine in the momentum dimension, the dE/dx will remain approximately constant in the analysis bin, enabling the likelihood fitter to function well. Particle dE/dx does not strongly depend on production angle θ^2 .

Due to differences in particle production by species, separate binning schemes in momentum are used for the $\pi^\pm$, $p/\bar{p}$ and $K^\pm$ analyses. Pions, being the most abundantly-produced particles, receive the finest binning, while protons and kaons have increasingly coarser binning. Momentum bin boundaries were determined using Monte Carlo and requiring more than 1000 tracks of the species selected per kinematic bin. Regions of Bethe-Bloch overlap were excluded within $\pm 5\%$ of the crossings ($\pm 6\%$ for Kaons), since particle species cannot be determined when the curves are closer than can be resolved. These exclusion regions are $[1.64, 2.02]$ GeV/ c ($\pi^\pm$), $[1.64, 4.32]$ GeV/ c ($p/\bar{p}$), and $[0.95, 4.32]$ GeV/ c ($K^\pm$). Narrow bins between Bethe-Bloch crossings in proton and kaon analysis bins were studied, but in the end were omitted from the final analysis due to low track occupancy.

The binning in θ was kept uniform for each particle species, and boundaries in θ were selected to match boundaries used in previous analyses to ease comparison. The angular binning is finer than previous analyses in order to assist with neutrino flux estimation in θ .

Uniform binning schemes were used for each particle species in both the 2016 and 2017 analyses. Resulting bin occupancies for the 2017 analysis can be seen in Figs. 7.4–7.6.

² Due to idiosyncrasies in dE/dx calibration, track dE/dx may indeed depend on kinematic properties other than just p . If the calibration is performed to a sufficient level, these dependencies will not impact the dE/dx fits.

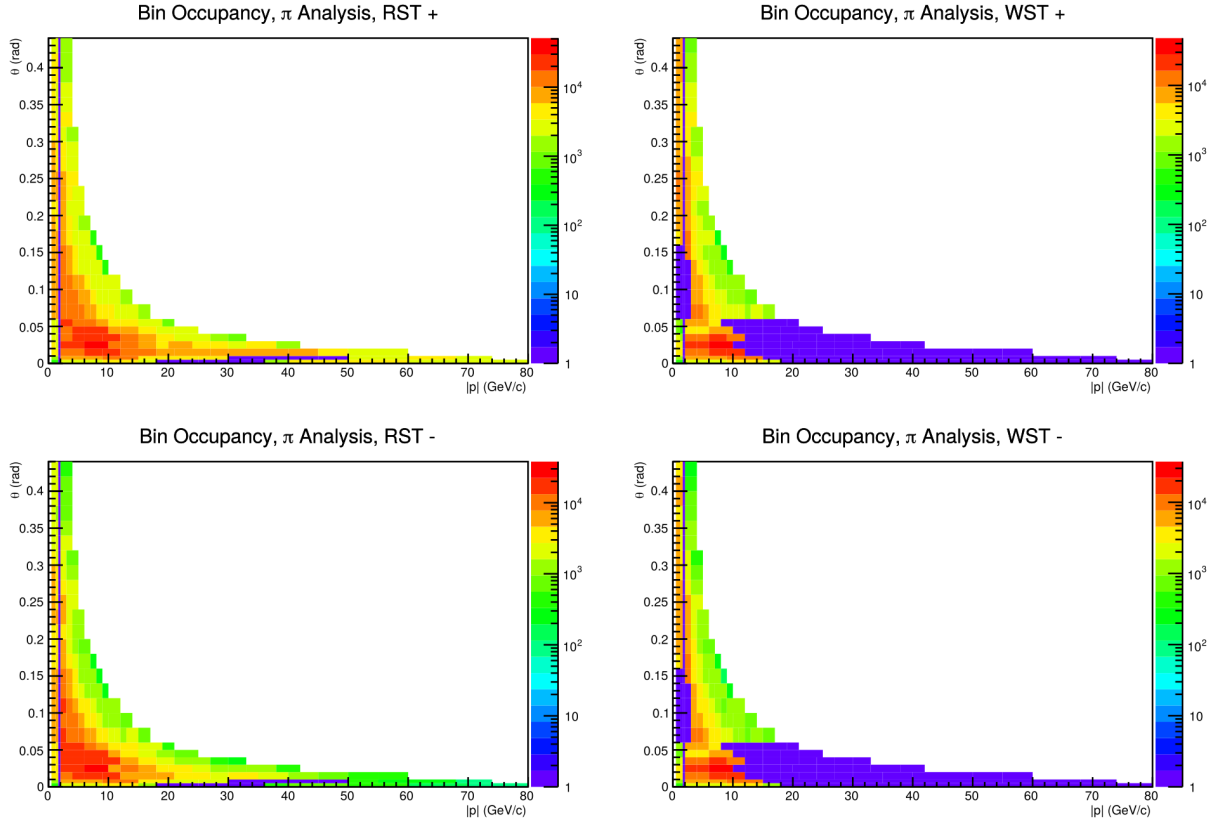


Figure 7.4: Bin occupancy for $\pi^\pm$ analysis (2017 dataset). Top row: positive track occupancy for right-side tracks (left) and wrong-side tracks (right). Bottom row: negatively-charged track occupancy for right-side tracks (left) and wrong-side tracks (right). Vertical purple lines represent the region rejected due to Bethe-Bloch proximity.

7.2.2 Track Topologies

In NA61/SHINE, tracks are separated into two distinct topologies, known as *right-side tracks* (RSTs) and *wrong-side tracks* (WSTs). Due to differences in how clusters of the two track topologies are measured, differences in kinematic variables and dE/dx may arise between the track types. Right-side tracks are emitted from the target such that they are bent by the magnetic field in the same direction they were produced, while wrong-side tracks are bent in the opposite direction that they were produced (see Fig. 7.7). Another way of expressing this is by combining the x-component of the momentum p_x and the track charge q :

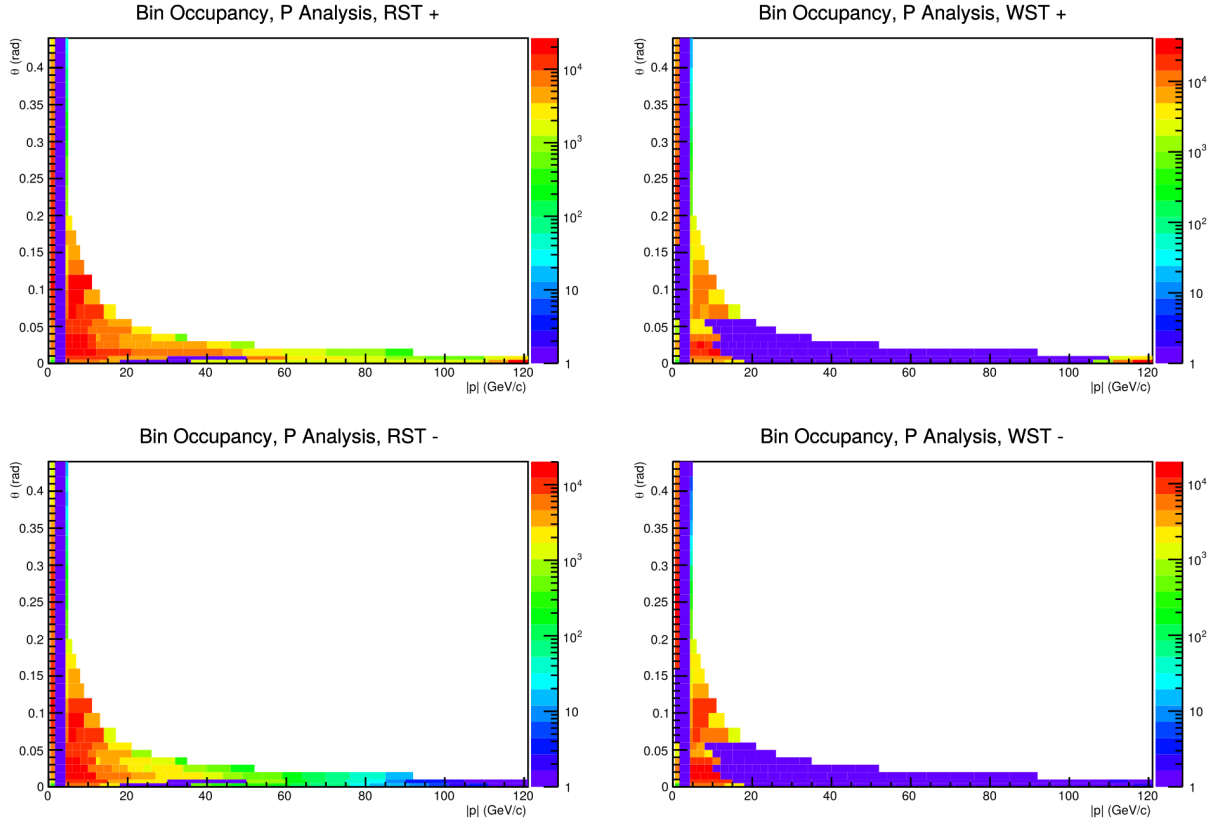


Figure 7.5: Bin occupancy for $p/\bar{p}$ analysis (2017 dataset). Top row: positive track occupancy for right-side tracks (left) and wrong-side tracks (right). Bottom row: negatively-charged track occupancy for right-side tracks (left) and wrong-side tracks (right). Momentum binning is more coarse than $\pi^\pm$ binning and extends to higher momentum in the forward region. Vertical purple lines represent the region rejected due to Bethe-Bloch proximity.

$$\begin{cases} q \cdot p_x > 0 & (\text{RST}) \\ q \cdot p_x < 0 & (\text{WST}) \end{cases} \quad (7.12)$$

The difference between these track topologies arises from the TPC pad geometry. VTPC pads and some MTPC pads are angled with respect to the beamline in order for tracks to traverse padrows parallel to the pads. Tracks parallel to TPC pads have a smaller associated charge distortion with each cluster measurement, while tracks crossing padrows with a large angle tend to produce distorted clusters. By tilting the TPC pads, one of the two track topologies is aligned with the pad directions. The other topology, however, will have a large angle with respect to the pads. The aligned tracks are the right-side tracks, while the large-angle tracks are the wrong-side tracks.

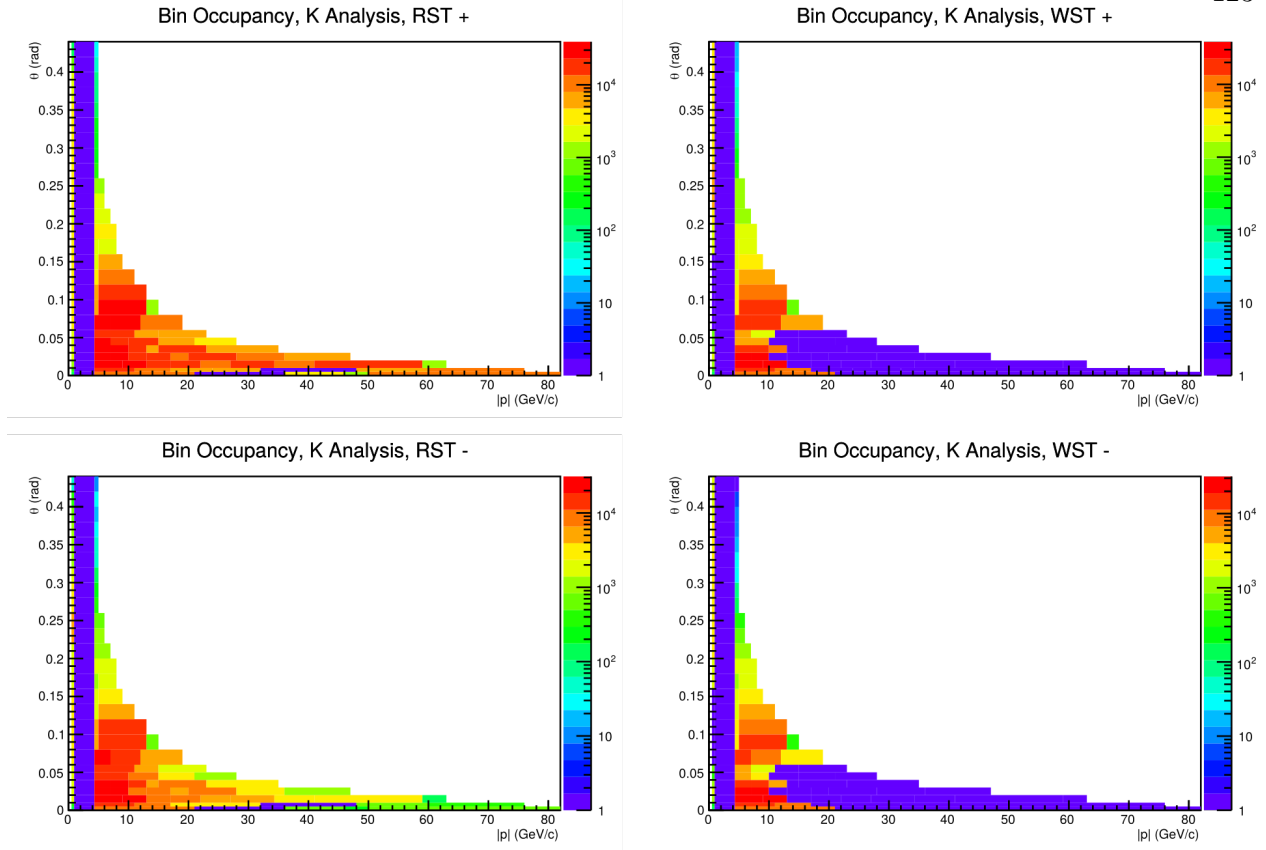


Figure 7.6: Bin occupancy for $K^\pm$ analysis (2017 dataset). Top row: positive track occupancy for right-side tracks (left) and wrong-side tracks (right). Bottom row: negatively-charged track occupancy for right-side tracks (left) and wrong-side tracks (right). Momentum binning for $K^\pm$ analysis is the most coarse. Vertical purple lines represent the region rejected due to Bethe-Bloch proximity.

Typically in NA61/SHINE, RST tracks are used for analysis and WST tracks are used as cross-checks on the obtained values. This is the case for the analyses presented here.

7.2.3 Event & Track Selection

Two levels of selection are applied to events entering analysis. At the top level, events must pass selection criteria. These are:

- Well-measured BPD track
- Beam track pointed at S4

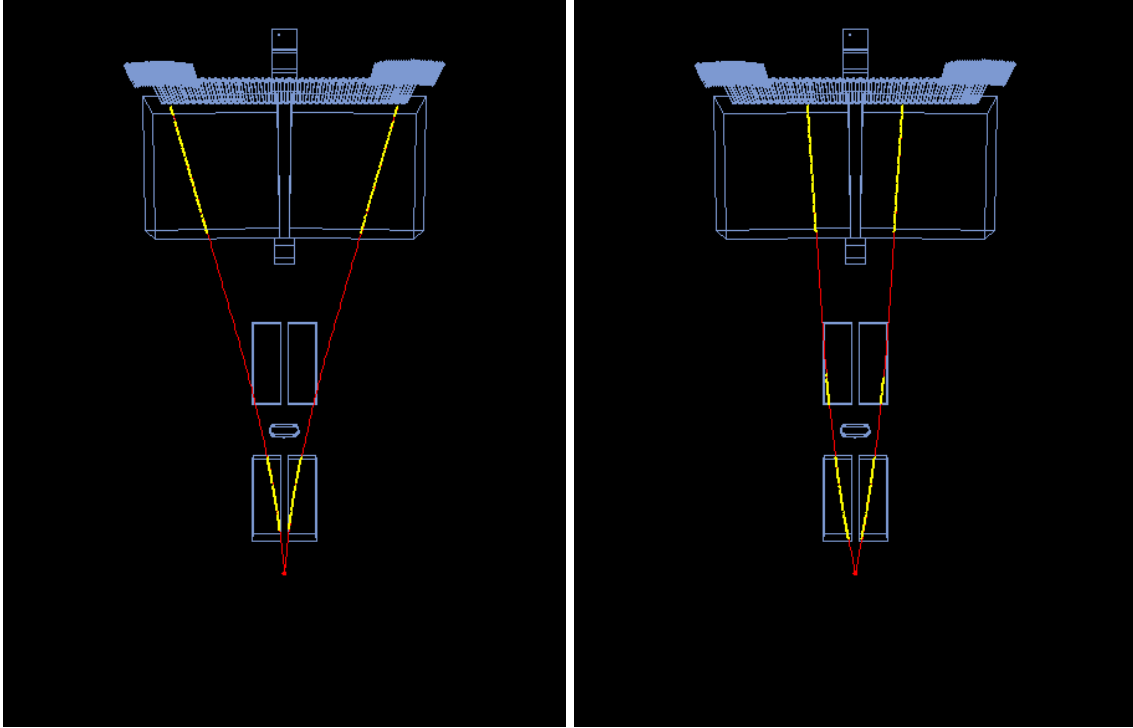


Figure 7.7: Visual comparison of right-side tracks (left) and wrong-side tracks (right). Left: The tracks curve through the magnetic field in the same direction they were emitted from the target. The left track is positively-charged, while the right track is negatively-charged. Right: The tracks curve through the magnetic field in the opposite direction they were emitted from the target. The left track is negatively-charged, while the right track is positively-charged.

- Beam particle separated from others by at least 800 nanoseconds

The BPD cut requires a properly-fit line in the BPDs, which may be constructed from two BPDs only in the case of a malfunctioning BPD. The cut also requires a well-measured BPD cluster in BPD3 just upstream of the target, which helps to remove events involving beam tracks scattering out of the beamline.

The S4 cut removes events with beam tracks pointed outside of the S4 counter, which appear similar to T2 interaction events during analysis. The selected requirement is that the beam track be within 0.95 cm of the center of the S4 counter, which has a radius of 1 cm.

The beam particle separation cut, or WFA cut, mitigates errors of track mis-association with the main vertex. If multiple beam particles arrive closely-spaced in time, tracks from each interaction may be associated with the main vertex, leading to multiplicity counting errors. Tracks

spaced in time by 800 ns will be displaced in the drift dimension by $1 - 2$ cm, allowing for rejection with an impact parameter cut. The effects of each cut on the number of analysis events can be seen in Table 7.2.

Within each event, **VertexTrack** objects are processed with additional selection criteria:

- Impact Parameter cut: $\sqrt{b_x^2 + b_y^2} < 2$ cm
- Phi cut (detector acceptance cut)
- Total cluster cut: $N_c > 15$
- Momentum measurement cluster cut (topology cut): $N_{\text{VTPC}} \geq 10$ or $N_{\text{GTPC}} \geq 4$
- Bethe-Bloch crossing cut
- dE/dx cut: $dE/dx < 2$ if $p \geq 2.2$; $dE/dx < \langle dE/dx \rangle_D + 1$ if $p < 2.2$

The impact parameter cut ensures that stray tracks and secondary tracks are not mis-associated with the main vertex. Impact parameter (b_x, b_y) is defined as the extrapolation mismatch between a track and its vertex position. The detector acceptance cut, or phi cut, will be explained in the following section. The total cluster cut ensures that track kinematic properties can be sufficiently estimated, and that enough clusters are present to calculate track dE/dx . The momentum measurement cut ensures that tracks contain a sufficient number of clusters in detectors surrounded by the magnetic field in order to estimate track momentum. The Bethe-Bloch crossing cut ensures that particle species dE/dx are sufficiently separated to perform analysis fits. Lastly, the dE/dx cut removes particles with unreasonable reconstructed energy deposition³.

The effect of each cut on the number of analysis tracks can be seen in Table 7.3. Additionally, dE/dx distributions may be seen in Figs. 7.8.

³ Tracks may be assigned an unreasonable amount of dE/dx for a variety of reasons, including large errors in kinematic property estimation, unreasonable track-by-track calibration corrections, or being doubly-charged.

Event Cut	Accepted Events (2016 Analysis)	Effect	Accepted Events (2017 Analysis)	Effect
(No Cuts)	3227564	–	1782561	–
Beam Aimed at S4	2869603	89%	1732342	97%
Beam Particle Separation (WFA)	2869603	85%	1732342	90%
BPD Fit Accepted	2081112	64%	1496062	84%

Table 7.2: Impact of event cuts on T2 analysis events for 2016 and 2017 analyses.

Track Cut	Accepted Tracks (2016 Analysis)	Effect	Accepted Tracks (2017 Analysis)	Effect
(No Cuts)	18003744	–	9882087	–
Beam Aimed at S4	16300911	91%	9746244	99%
Beam Particle Separation (WFA)	15581029	87%	9064039	92%
BPD Fit Accepted	11915226	66%	8519357	86%
Impact Parameter Cut	11008272	61%	7732587	78%
Detector Acceptance Cut	6984156	39%	3898672	39%
Total Cluster Cut	6858125	38.1%	3752966	38%
Topology Cut	6847901	38.0%	3711075	37.5%
Bethe-Bloch Separation Cut	6479734	36%	3486717	35.3%
dE/dx Cut	6466573	35.9%	3474245	35.2%

Table 7.3: Impact of track cuts on analysis tracks for 2016 and 2017 $\pi^\pm$ analysis.

7.2.4 Track Acceptance

The NA61/SHINE detector acceptance is highly-dependent on track kinematic quantities p , θ and ϕ . In order to perform track analysis only in regions of high detector acceptance, a three-dimensional acceptance map is used to reject tracks with poor acceptance due to lack of 4π coverage. Different acceptance maps are used depending on detector configuration (magnetic field setting and TPC configuration).

To create these acceptance maps, a dedicated Monte Carlo sample was produced for each of the 2016 and 2017 detector configurations. These Monte Carlo samples included the standard detector descriptions, but involved protons and anti-protons with flat distributions in (p, θ, ϕ) , with $p \in [0.2, 120]$ GeV/ c , $\theta \in [0, 0.5]$ radians, and $\phi \in [-180, 180]$ degrees. By comparing the simulated and reconstructed track quantities, a three-dimensional map of acceptance as a function of p and

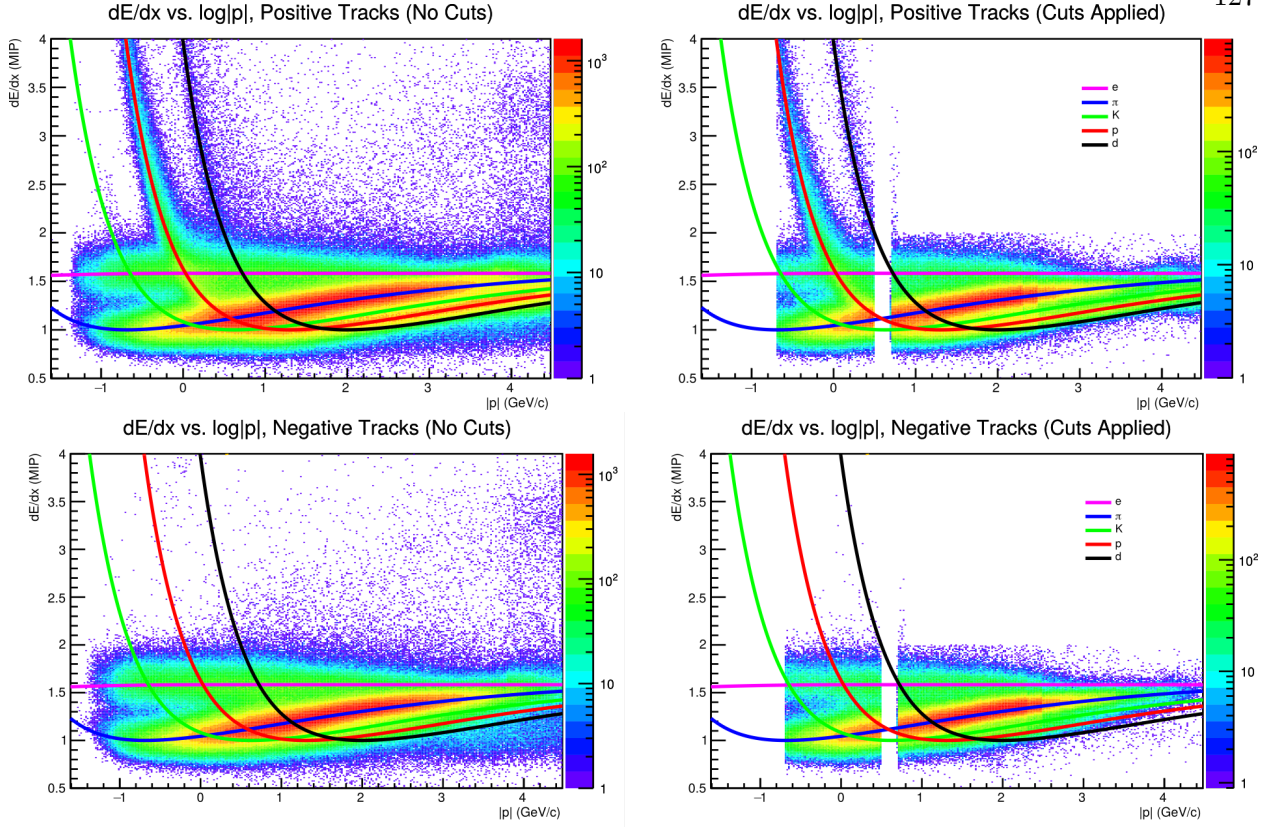


Figure 7.8: Track dE/dx vs. $\log|p|$ — before application of track quality cuts (left) and after application of track quality cuts (right). The top row shows positively-charged tracks, while the bottom row shows negatively-charged tracks. Nominal Bethe-Bloch dE/dx values are overlaid for electrons, pions, kaons, protons & deuterons.

ϕ is created for each angular θ bin. These maps are used to select tracks for entering the analysis. The maps and the overlaid allowed regions are shown in Fig. 7.9. The allowed regions were chosen conservatively, and additional regions showing data and Monte-Carlo acceptance discrepancies were rejected. Monte-Carlo acceptance corrections are used to estimate the impact of these cuts, as will be discussed in the following section.

7.2.5 Correction Factors

Tracks in regions of phase space removed by various cuts are accounted for using Monte-Carlo corrections. This involves simulating the pertinent interaction and comparing simulated tracks to reconstructed tracks as a function of phase space. The impact of each cut is studied on the simulated

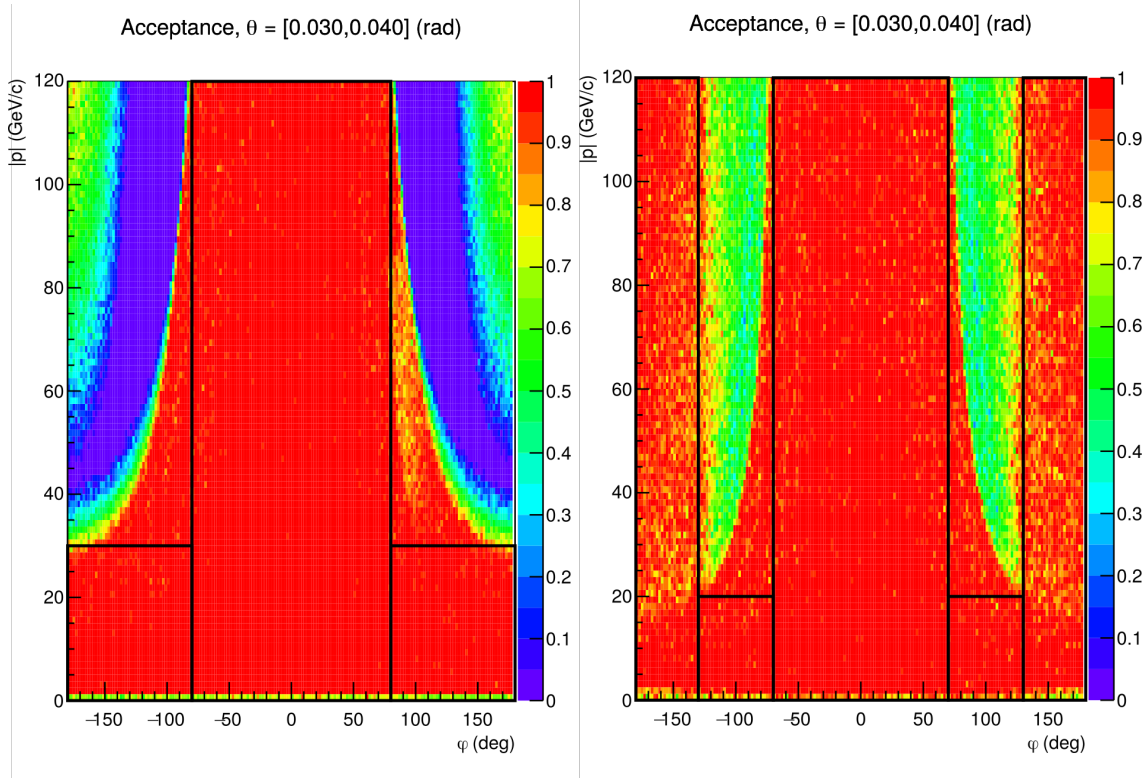


Figure 7.9: Visualization of acceptance cuts for one angular bin. Color axis represents track reconstruction efficiency. Left: 2016 acceptance cuts. Allowed regions are indicated by black outlines surrounding high-efficiency regions. Right: 2017 acceptance cuts. Allowed regions are again indicated by black outlines surrounding high-efficiency regions.

track sample, and a bin-by-bin correction factor is obtained. In this analysis, charged tracks are treated uniformly whenever possible, with the aim of decreasing model dependence.

The total charged track Monte-Carlo correction factor for a given analysis bin k may be broken down into its constituent parts:

$$c_k = \frac{N(\text{Simulated charged tracks from production events})_k}{N(\text{Selected reconstructed charged tracks})_k} = c_{\text{acc}} \times c_{\text{rec. eff.}} \times c_{\text{sel. eff.}} \quad (7.13)$$

These corrections are calculated by counting the number of simulated charged tracks in each analysis bin and dividing by the number of accepted reconstructed sim tracks in each bin. This factor may be applied to reconstructed data to estimate the total number of charged tracks using the number of selected reconstructed tracks. Correction factors for right-side π^+ , p and K^+ tracks are shown for the 2016 and 2017 analyses in Figs. 7.10–7.12.

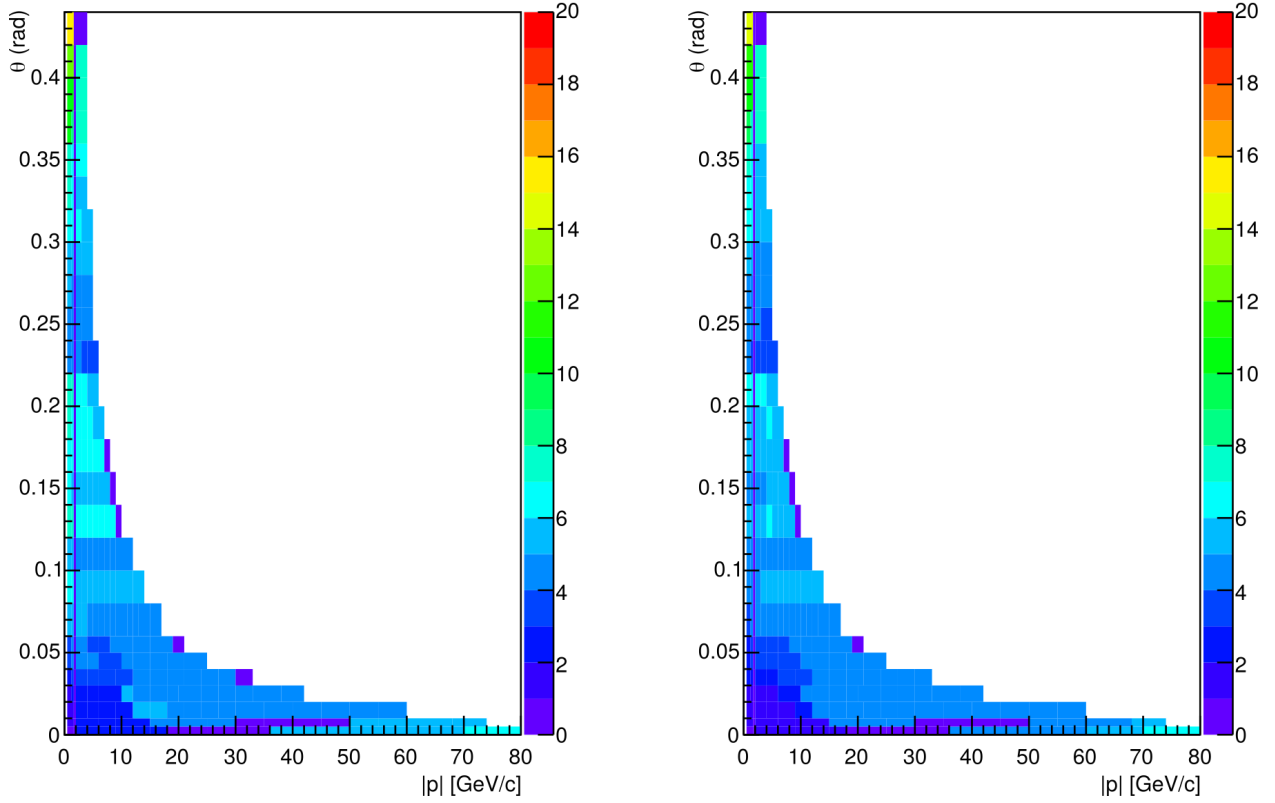


Figure 7.10: Monte-Carlo correction factors for the charged pion analysis. Feed-down corrections are factored out of the total correction.

7.2.5.1 Feed-Down Correction

Feed-down tracks are secondary tracks produced in weak decays after the primary interaction, as shown in Fig. 7.13. Feed-down tracks are difficult to distinguish from production tracks if the secondary vertex is located near the main vertex. The feed-down contribution may be estimated using Monte-Carlo or constrained using a V^0 analysis⁴. In order to treat this correction separately, it is factored out from the total charged track correction factor. Particle species must now be considered – the $\pi^\pm$ feed-down correction will be significantly different from the $p/\bar{p}$ feed-down correction. The correction is defined as

$$(c_k^i)_{\text{Feed-Down}} = \frac{(n_{\text{rec}}^i)_k - (n_{\text{Feed-Down}}^i)_k}{(n_{\text{rec}}^i)_k}, \quad i = \pi^\pm, p/\bar{p}, K^\pm. \quad (7.14)$$

⁴ A V^0 analysis is planned for this dataset in the near future.

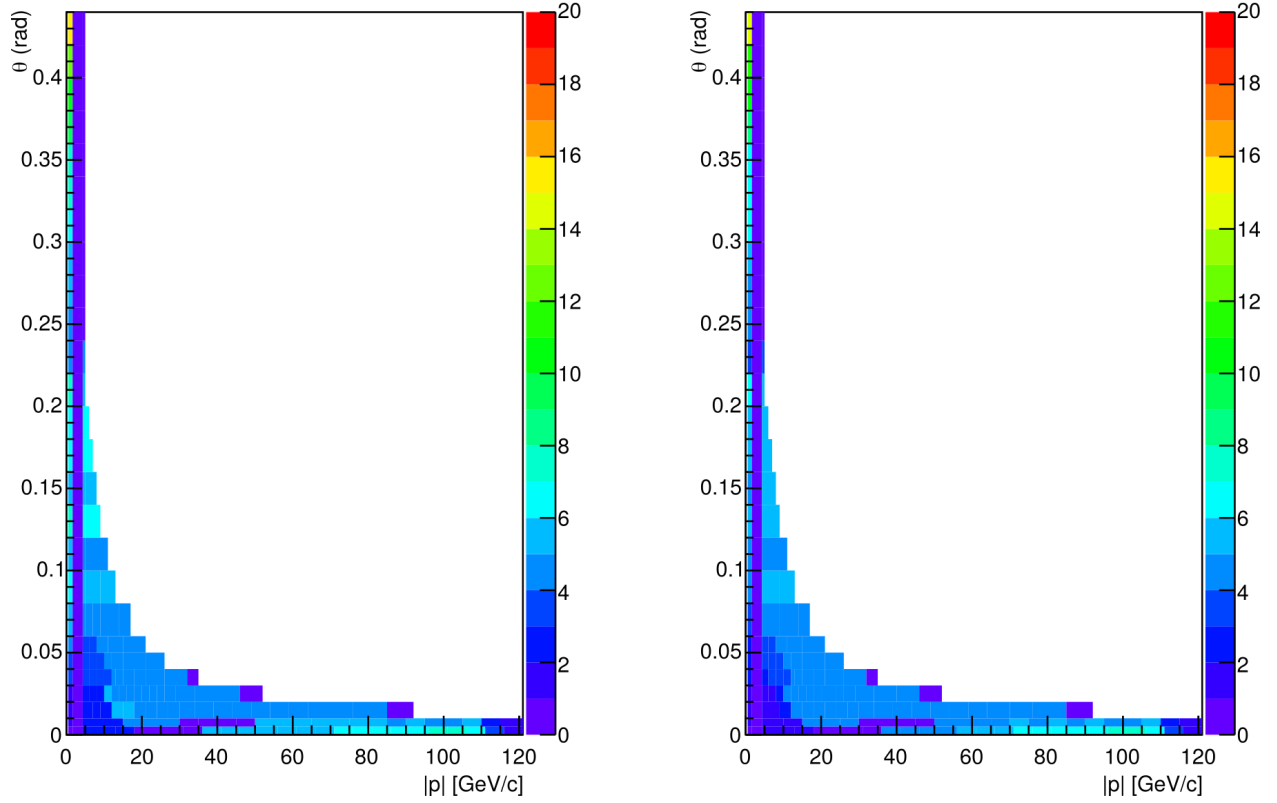


Figure 7.11: Monte-Carlo correction factors for the proton analysis. Feed-down corrections are factored out of the total correction.

Feed-down corrections for protons and pions can be seen in Fig. 7.14. The spectra presented in this thesis will contain Monte-Carlo-based feed-down corrections only.

7.2.5.2 dE/dx Fit Bias Correction

An additional correction was calculated to remove biases introduced during the dE/dx fitting procedure. This is again a species-dependent correction, adjusting the number of fit tracks for a given species. To estimate these biases, a dedicated dE/dx Monte Carlo was made. Using the full fit results from each analysis bin, the fit parameters were read in and varied, and the individual track dE/dx in each bin was re-simulated with the varied parameters. The resulting dE/dx distributions were then re-fit and studied. The difference in simulated and fit particle yields was recorded and

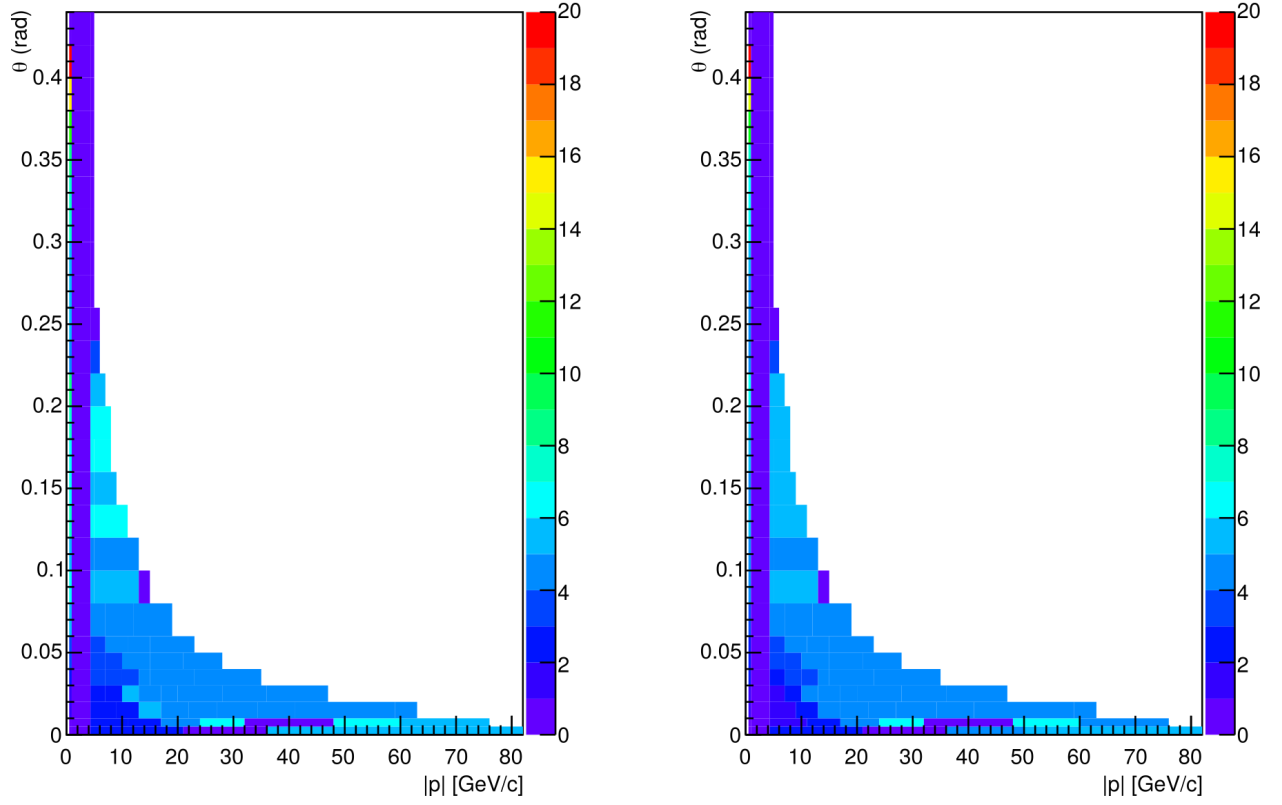


Figure 7.12: Monte-Carlo correction factors for the charged kaon analysis. Feed-down corrections are factored out of the total correction.

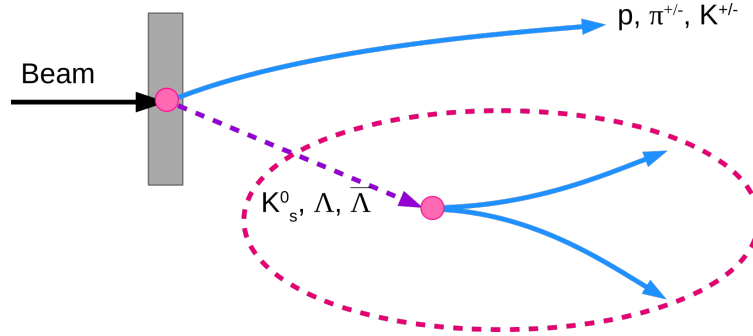


Figure 7.13: Example feed-down interaction (circled).

the mean of the values was taken as the fit bias. The explicit correction is given by

$$c_i^{\text{Fit}} = \frac{1}{N_{\text{trials}}} \sum_{i=1}^{N_{\text{trials}}} \left(\frac{y_i^{\text{fit}} - y_i^{\text{true}}}{y_i^{\text{true}}} \right). \quad (7.15)$$

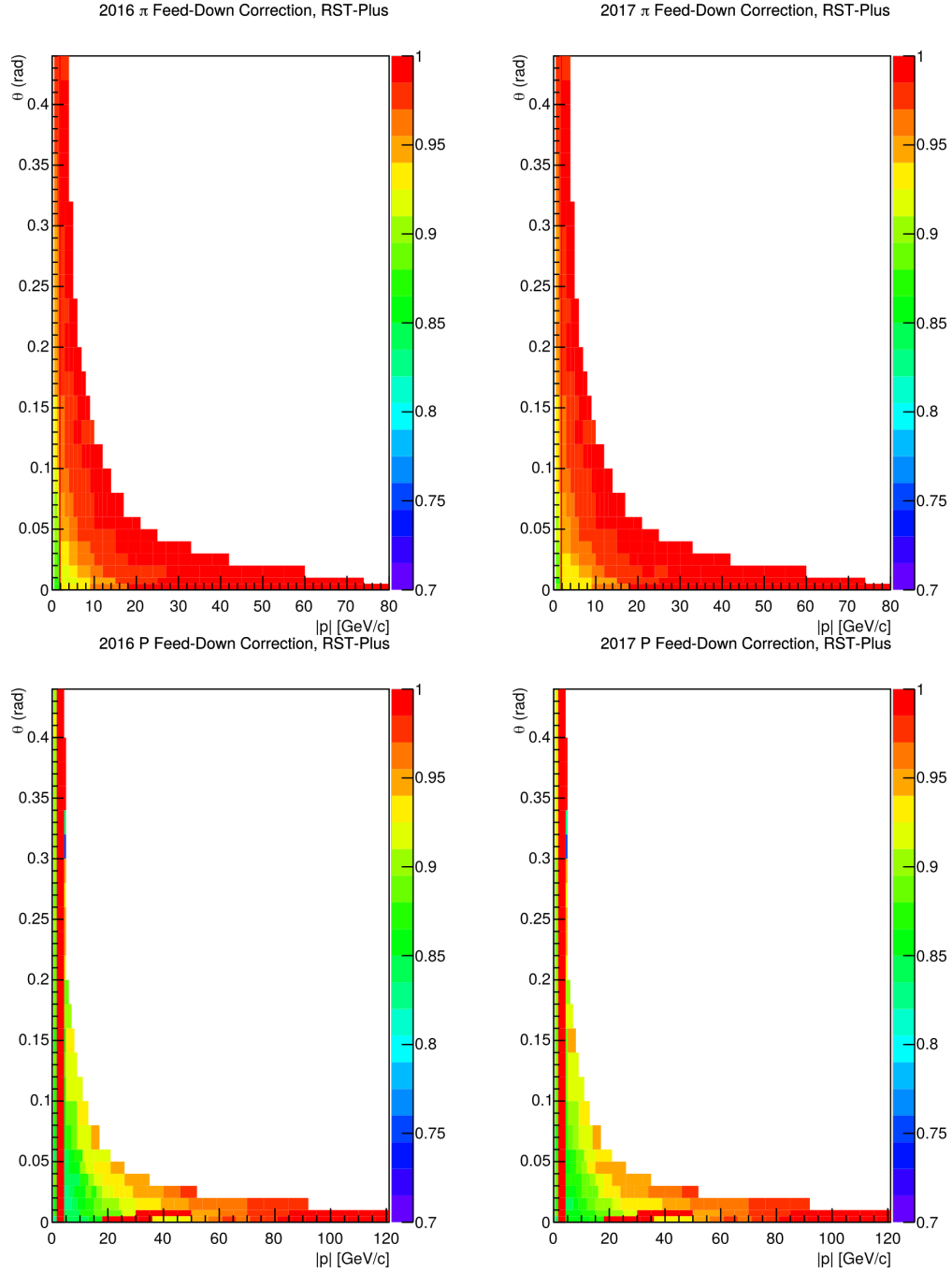


Figure 7.14: Feed-down correction factors for charged pion and proton analyses. Feed-down contributions to charged kaons are near 1% and were not considered.

The standard deviation of the differences was also recorded and taken as the fit uncertainty. An individual example fit can be seen in Fig. 7.16, while the resulting fit biases can be seen in Fig. 7.15.

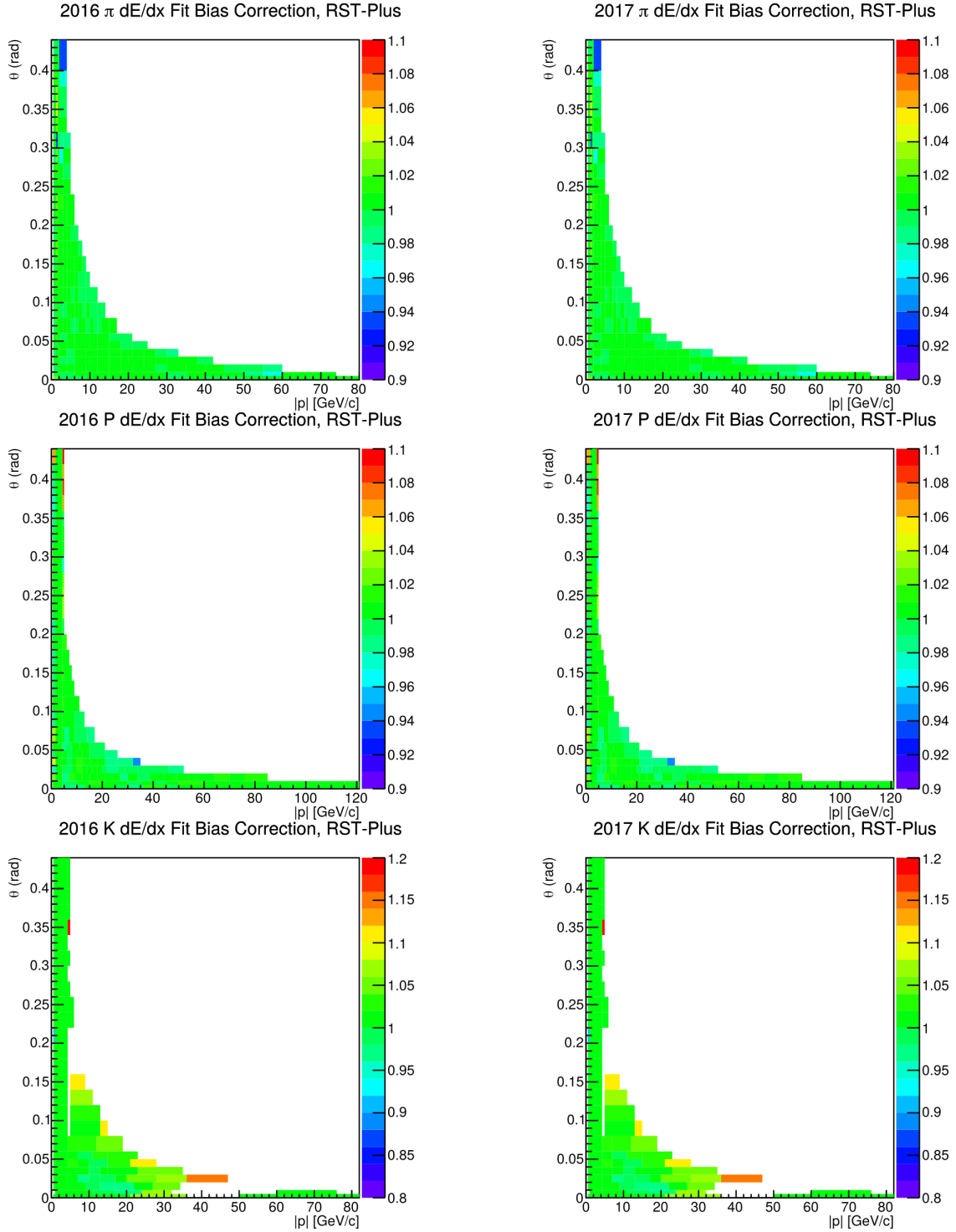


Figure 7.15: Fit bias corrections for π^+ (top row), p (middle row), and K^+ (bottom row). Biases are most severe for the charged kaon analysis.

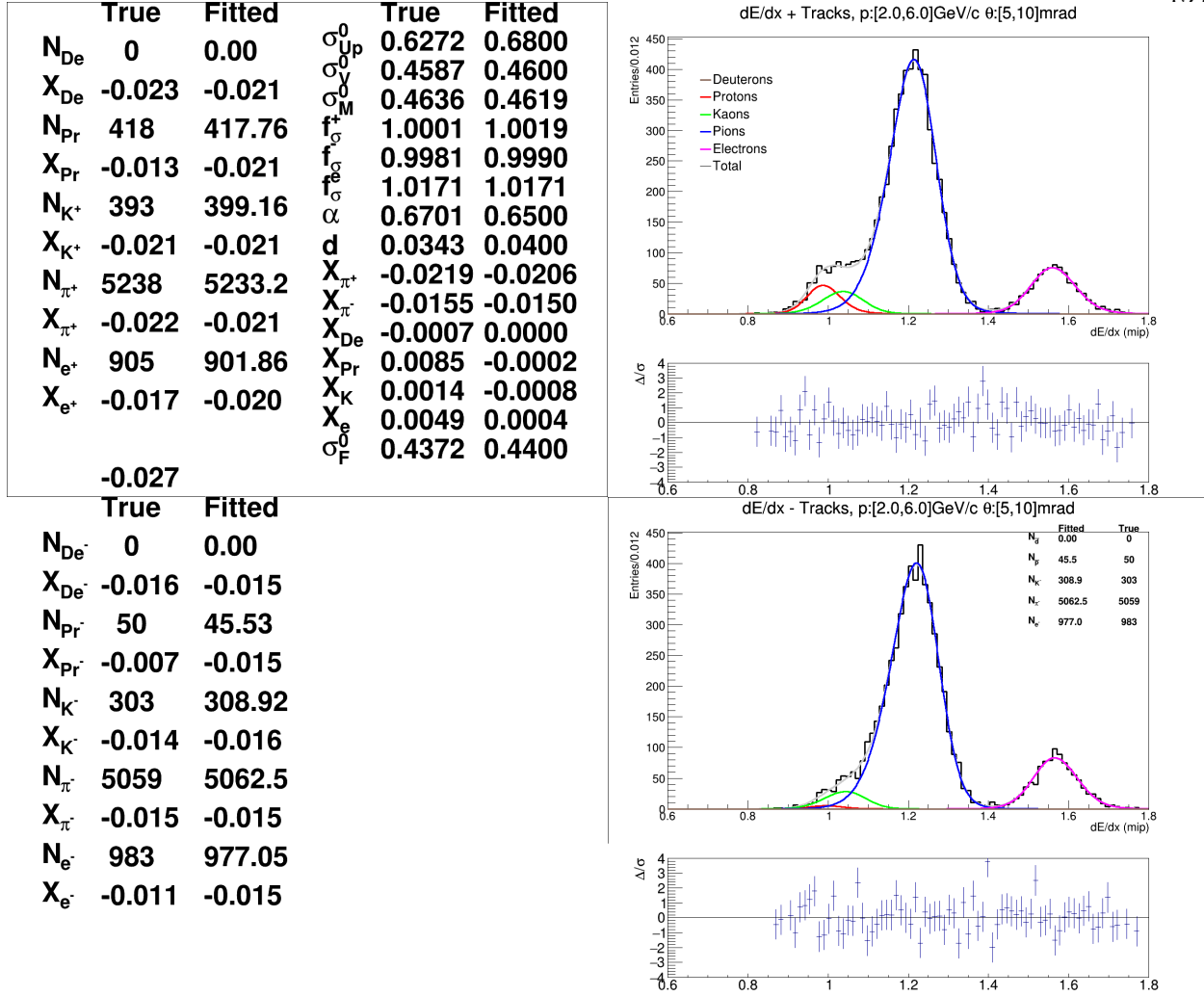


Figure 7.16: Example dE/dx fit to simulated data. True number of particles by species and calibration constants are compared to fitted values. The differences between the true and fitted values are used to calculate the bin-by-bin dE/dx fit biases.

7.2.6 Multiplicity Calculations

The differential particle multiplicity $\frac{d^2 n_i}{dp d\theta}$ is defined as the average number of particles produced in a given kinematic bin i per unit momentum per unit angle in a production interaction:

$$\frac{d^2 n_i}{dp d\theta} = \frac{y_i}{N_{\text{prod}} \Delta p \Delta \theta} \quad (7.16)$$

where y_i is the number of observed particles of a given species in bin i , N_{prod} is the number of production interactions, and $\Delta p \Delta \theta$ is the size of the kinematic bin. The species count y_i is

determined from the fraction of tracks in each bin attributed to a particular species of particle, as can be seen in Fig. 7.17.

The number of production events must be deduced from the total number of triggers, which contain production, elastic, and quasi-elastic events. We again break down the trigger cross-section using the equation from Chapter 6:

$$\sigma_{\text{trig}} = f_{\text{prod}} \cdot \sigma_{\text{prod}} + f_{\text{qe}} \cdot \sigma_{\text{qe}} + f_{\text{el}} \cdot \sigma_{\text{el}}. \quad (7.17)$$

From this, we can relate the fraction of triggered events which were production events to the total number of triggered events:

$$\frac{(N_{\text{prod}})_{\text{trig}}}{N_{\text{trig}}} = \frac{f_{\text{prod}} \cdot \sigma_{\text{prod}}}{\sigma_{\text{trig}}}. \quad (7.18)$$

$(N_{\text{prod}})_{\text{trig}}$ will serve as our multiplicity normalization. Re-writing the multiplicity calculation, we obtain

$$\frac{d^2 n_i}{dp d\theta} = \frac{\sigma_{\text{trig}}}{f_{\text{prod}} \sigma_{\text{prod}}} \frac{y_i}{N_{\text{trig}} \Delta p \Delta \theta}. \quad (7.19)$$

In order to remove out-of-target interactions that reconstruct to an event's main vertex, we perform a *target-out subtraction* using the data sample with the target removed from the detector. The target-in and target-out yields y_i^{In} and y_i^{Out} may then be combined with the ratio of the target-in and target-out interaction rates $P_{\text{In}}/P_{\text{Out}}$, defined as ϵ [52]:

$$\frac{y_i}{N_{\text{trig}}} = \frac{1}{1 - \epsilon} \left(\frac{y_i^{\text{In}}}{N_{\text{trig}}^{\text{In}}} - \frac{\epsilon y_i^{\text{Out}}}{N_{\text{trig}}^{\text{Out}}} \right). \quad (7.20)$$

The bin-by-bin correction $c_i^{\text{total}} = c_i^{\text{MC}} c_i^{\text{Fit}} c_i^{\text{Feed-Down}}$, composed of Monte-Carlo corrections, fit bias corrections, and feed-down corrections, may now be applied. The fraction of production events scattered out of S4, f_{prod} , is taken into account in the Monte-Carlo acceptance correction, since all production events are stored regardless if the S4 scintillator was hit. The multiplicity calculation becomes

$$\frac{d^2 n_i}{dp d\theta} = \frac{c_i^{\text{total}} \sigma_{\text{trig}}}{(1 - \epsilon) \sigma_{\text{prod}} \Delta p \Delta \theta} \left(\frac{y_i^{\text{In}}}{N_{\text{trig}}^{\text{In}}} - \frac{\epsilon y_i^{\text{Out}}}{N_{\text{trig}}^{\text{Out}}} \right). \quad (7.21)$$

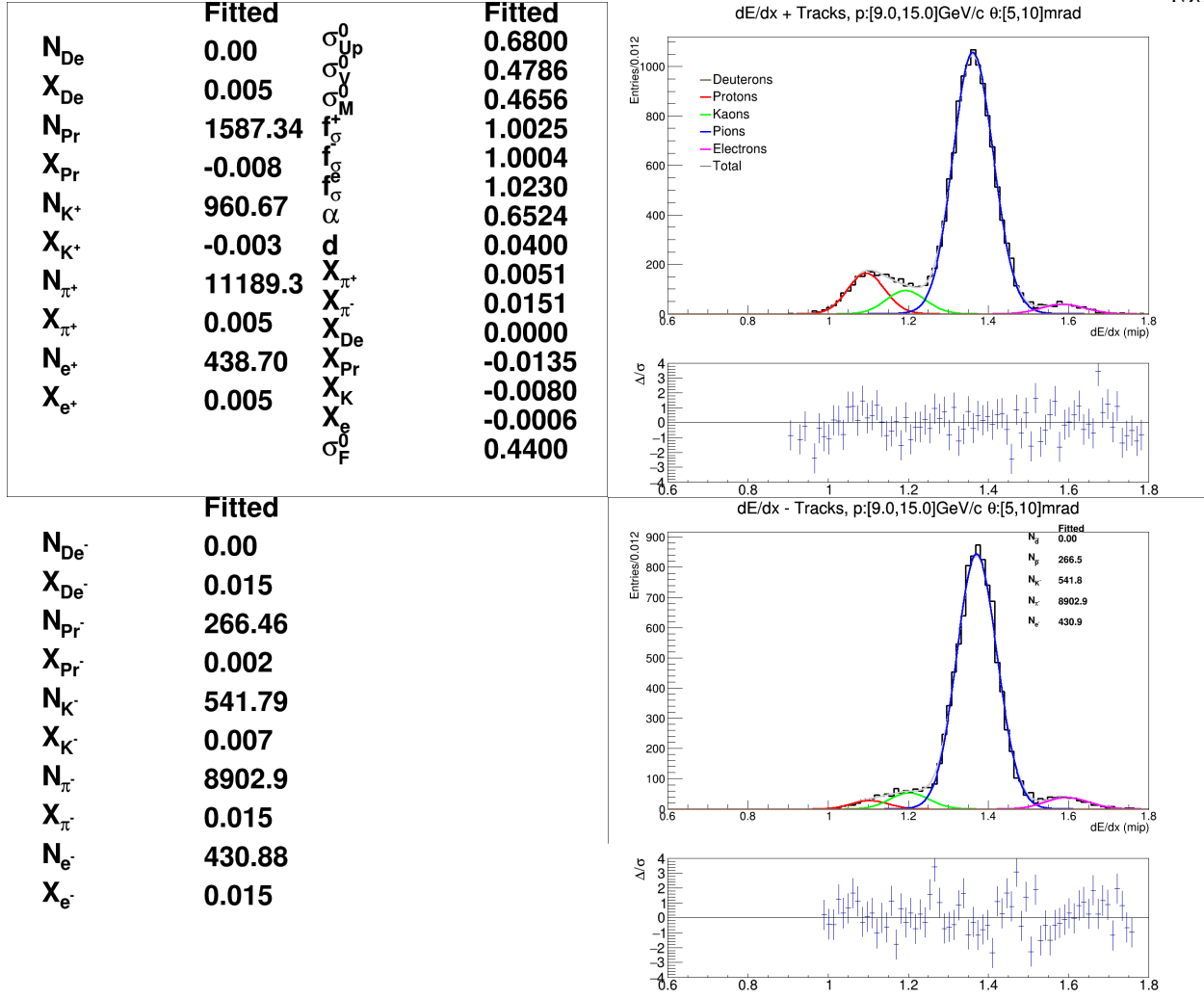


Figure 7.17: Example dE/dx fit to experimental data, showing number of fit particle per species and fit calibration constants. The number of particles enter the species multiplicity calculations.

7.2.7 Results

This section presents the results of the 2016 and 2017 identified multiplicity analyses. The two data sets are compared as a cross-check. Multiplicities for $\pi^\pm$, $p/\bar{p}$ and $K^\pm$ are presented and compared to select Monte-Carlo multiplicity predictions. The subsequent values and uncertainties are combined into a final multiplicity measurement in Chapter 8.

7.2.7.1 2016 - 2017 Comparison

Due to the two different detector configurations in 2016 and 2017, the data sets receive vastly different Monte-Carlo, feed-down, and dE/dx correction factors. Corroboration of the two analyses supports the final measurement. A comparison of the two multiplicity measurements, as well as select Monte-Carlo multiplicity predictions, for $\pi^\pm$, $p/\bar{p}$ and $K^\pm$ are shown in Fig. 7.18–7.30. The error bars represent statistical and systematic uncertainty.

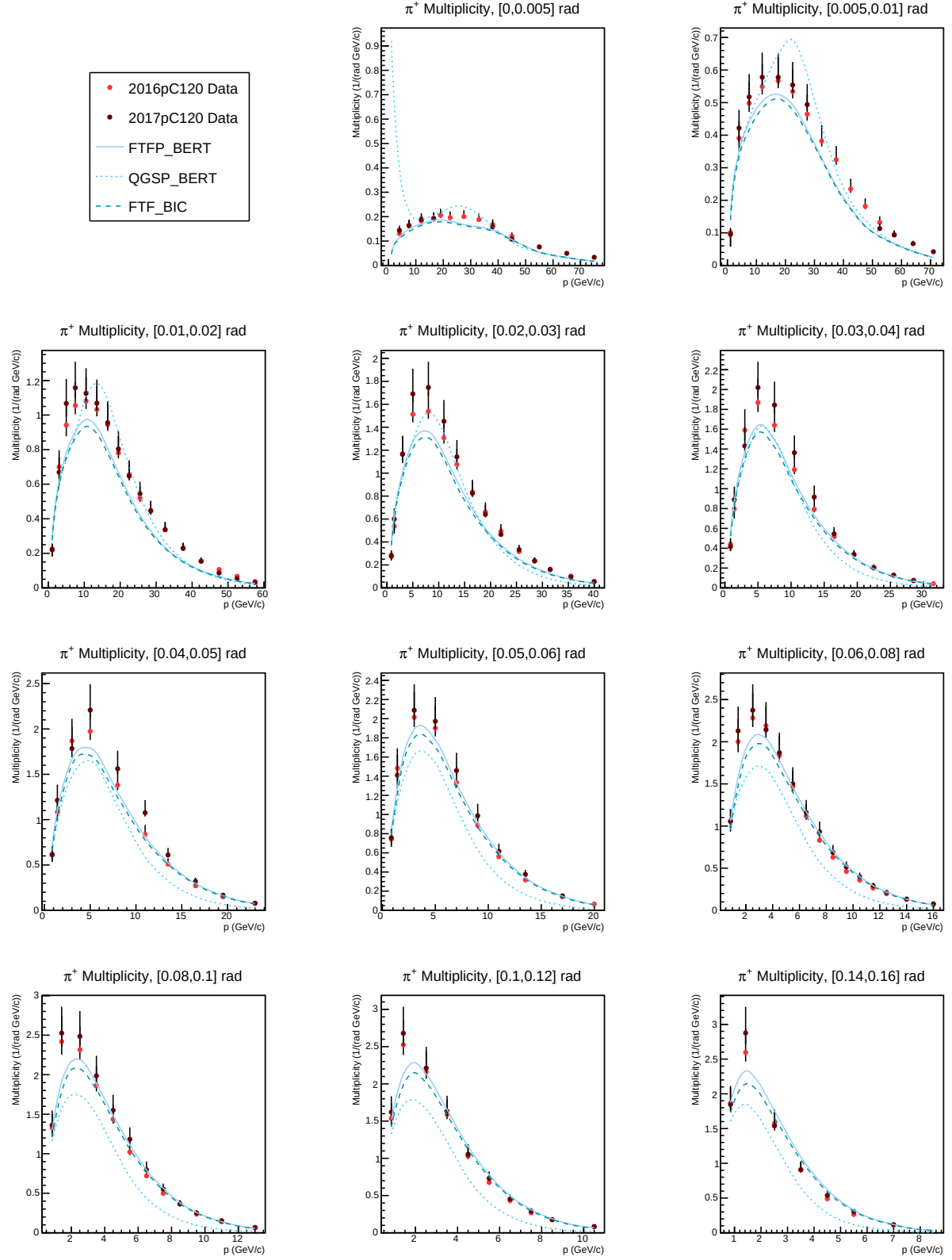


Figure 7.18: π^+ multiplicity measurements from 2016 & 2017 data sets from 0.0–0.16 rad.

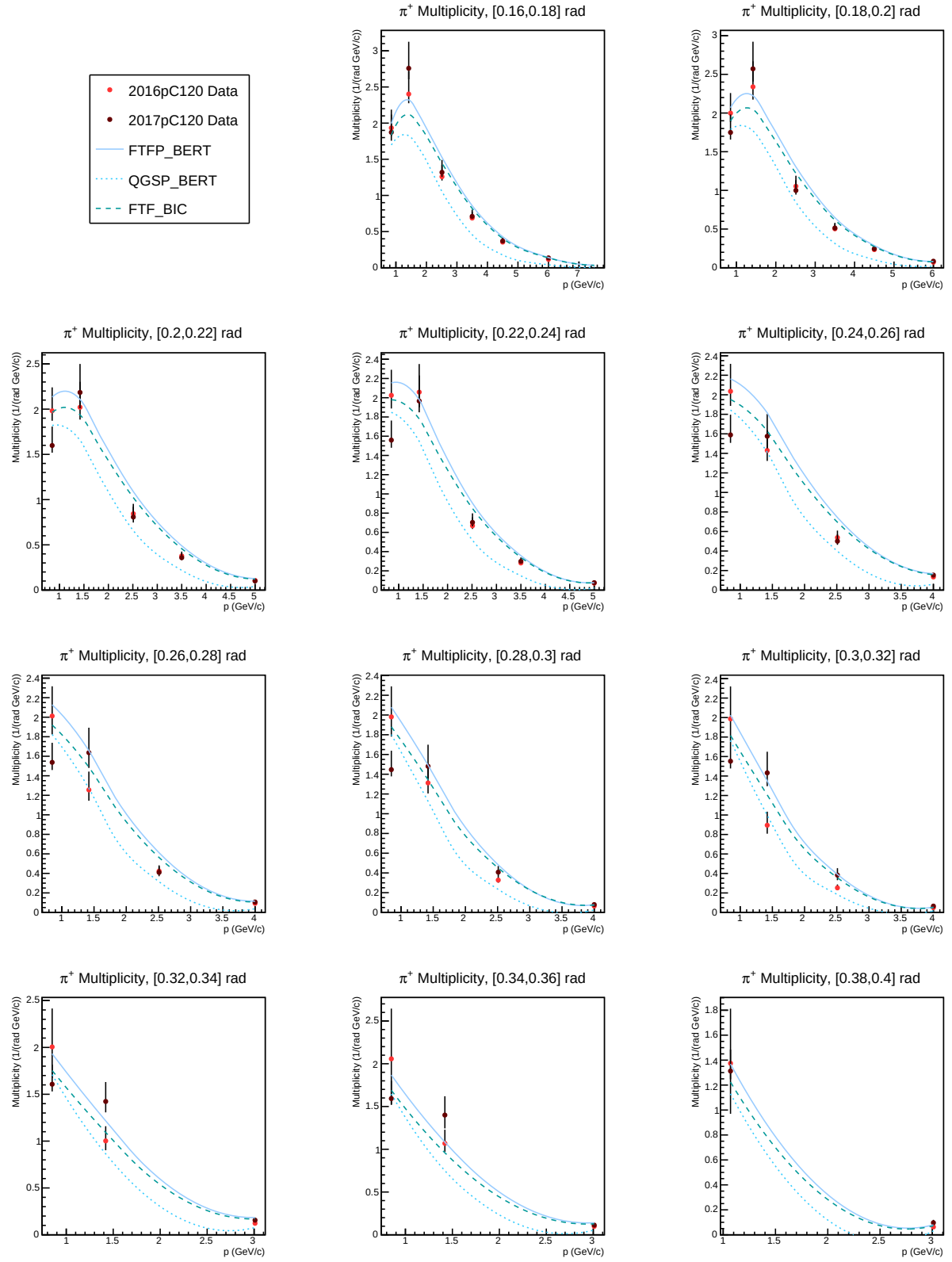


Figure 7.19: π^+ multiplicity measurements from 2016 & 2017 data sets from 0.16–0.4 rad.

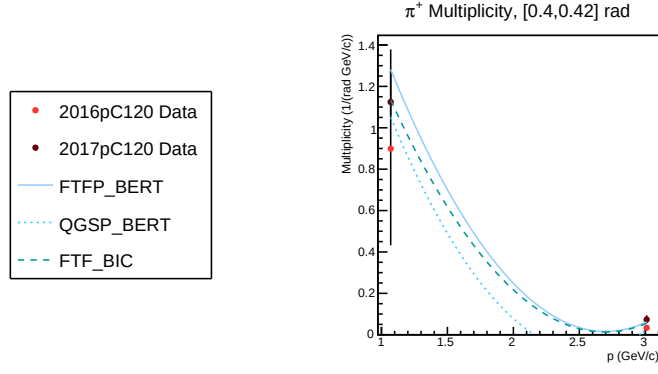


Figure 7.20: π^+ multiplicity measurements from 2016 & 2017 data sets from 0.40–0.42 rad.

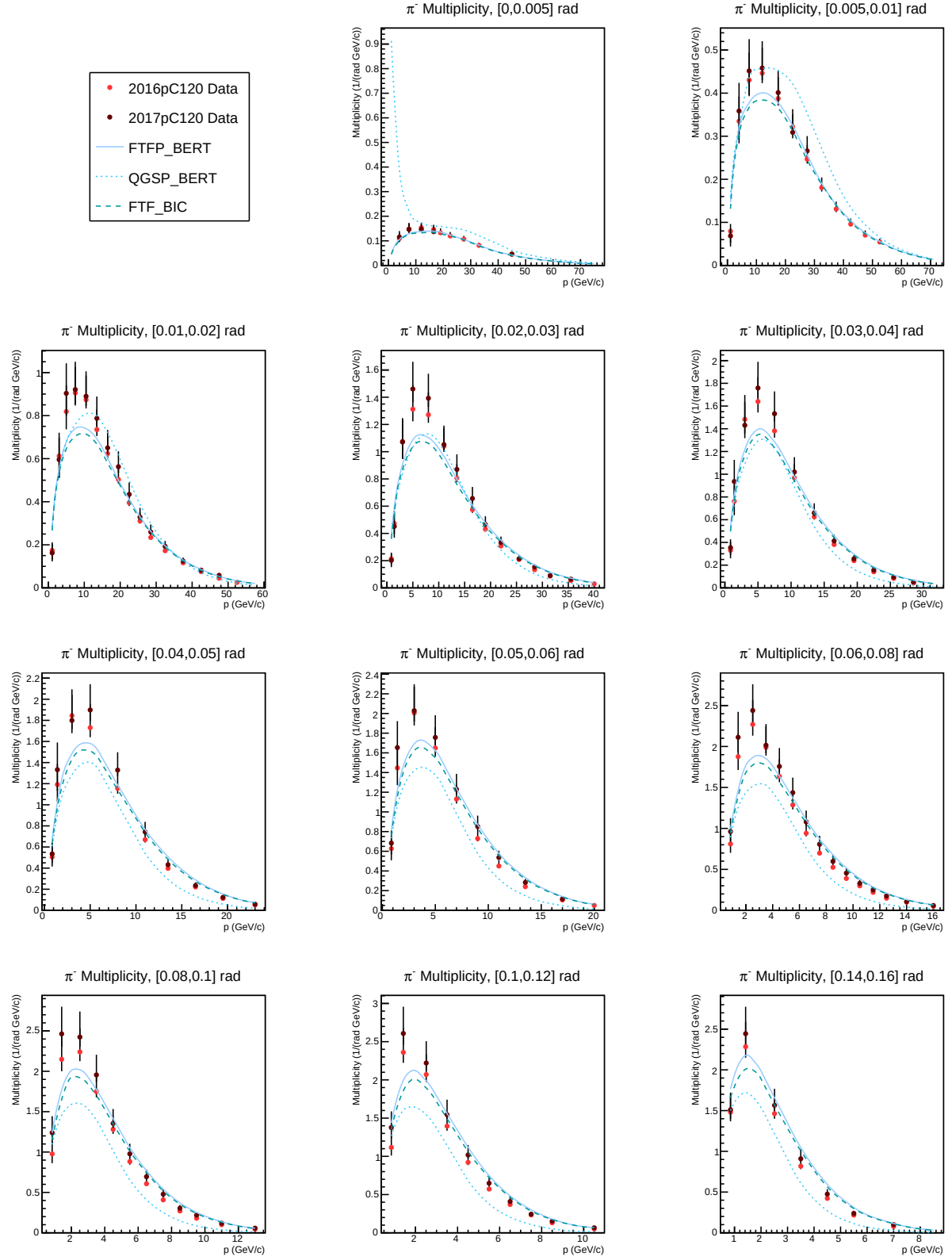


Figure 7.21: π^- multiplicity measurements from 2016 & 2017 data sets from 0.0–0.16 rad.

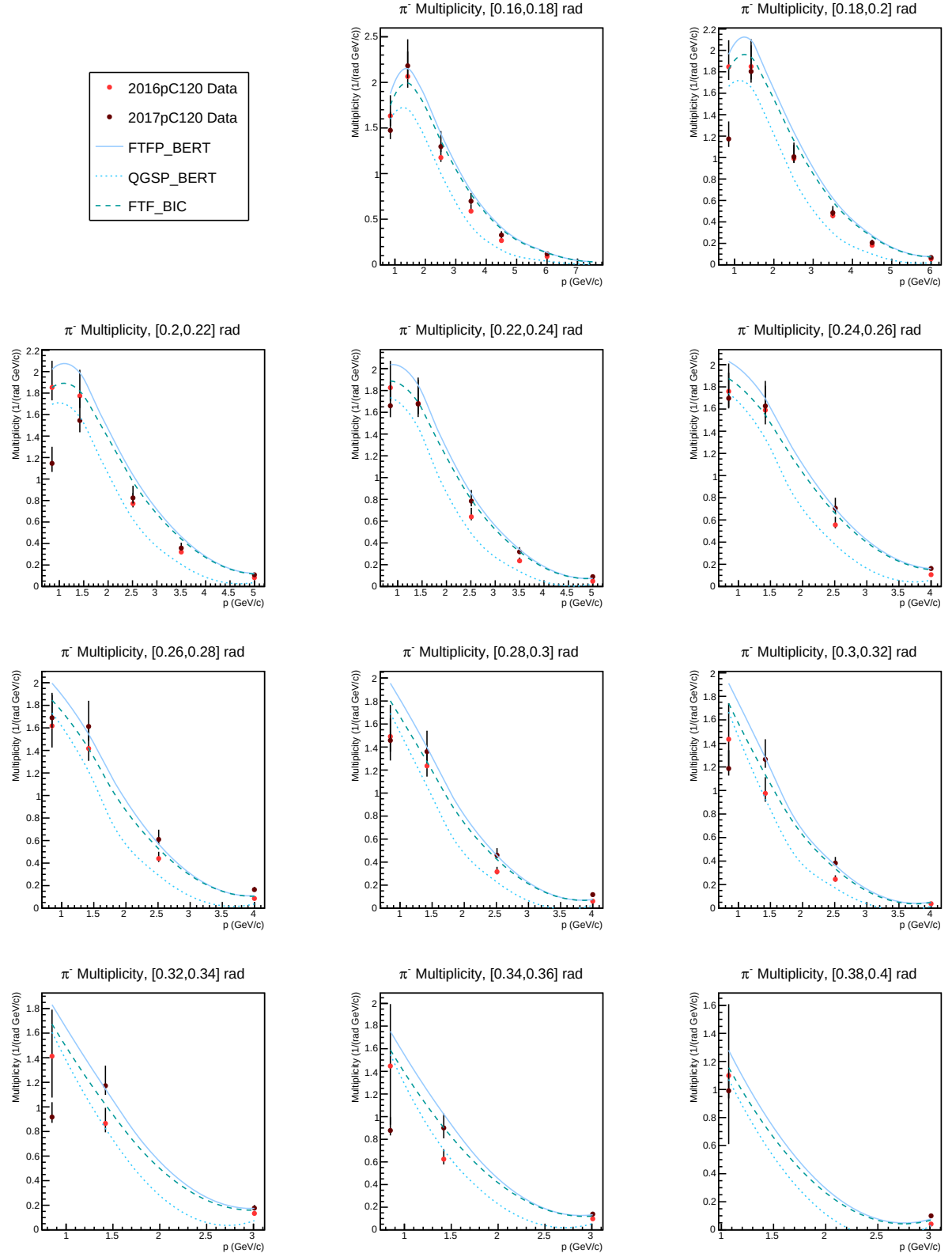


Figure 7.22: π^- multiplicity measurements from 2016 & 2017 data sets from 0.16–0.4 rad.

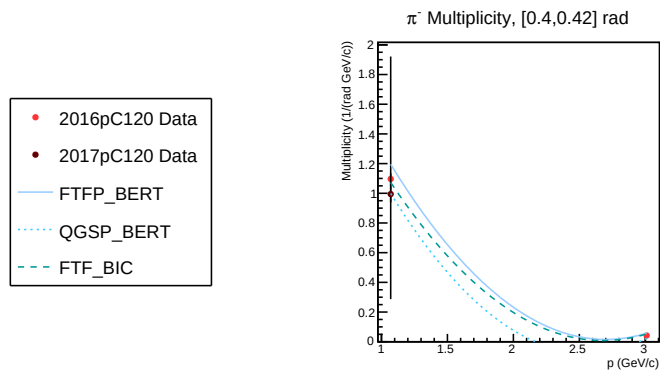


Figure 7.23: π^- multiplicity measurements from 2016 & 2017 data sets from 0.40–0.42 rad.

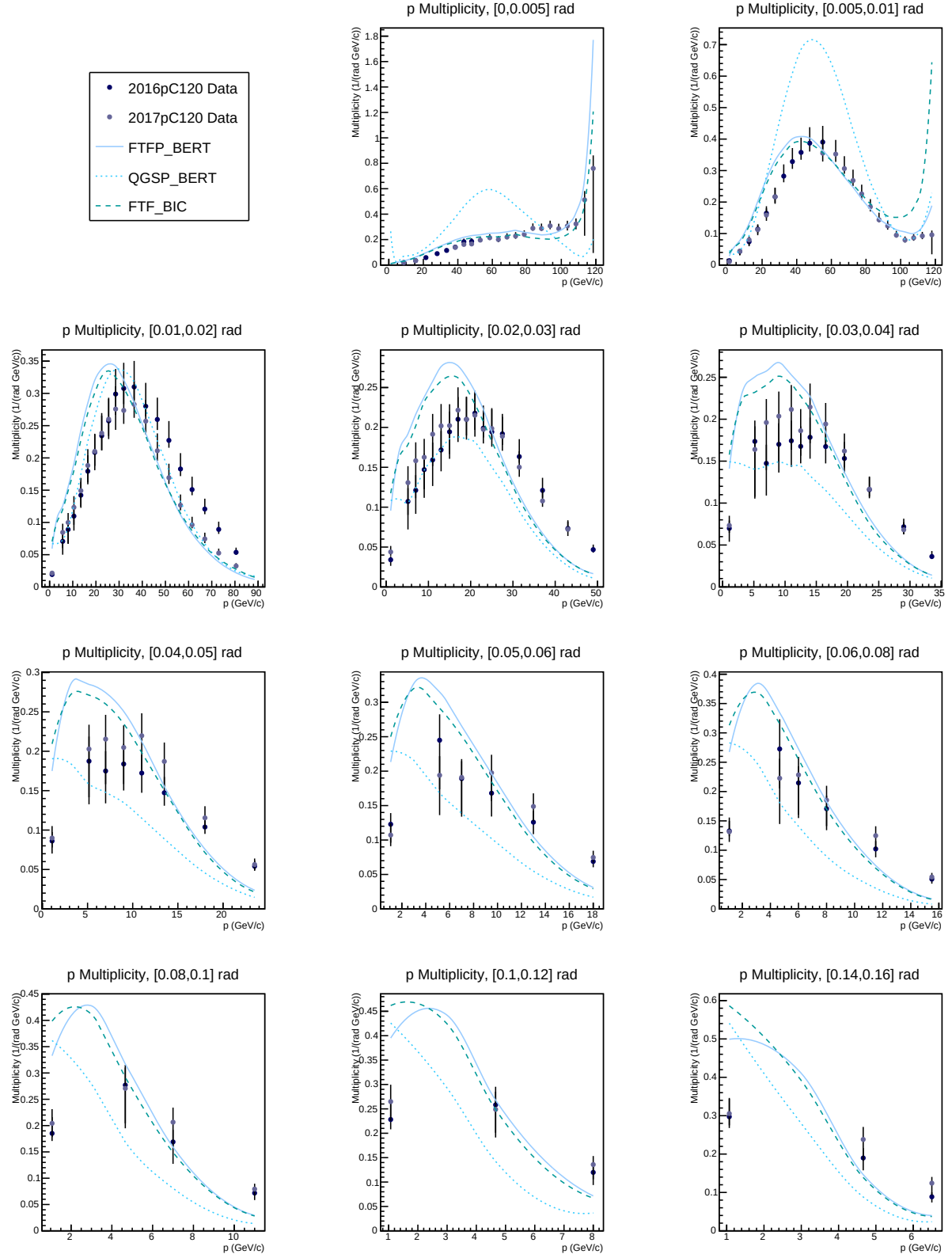


Figure 7.24: p multiplicity measurements from 2016 & 2017 data sets from 0.0–0.16 rad.

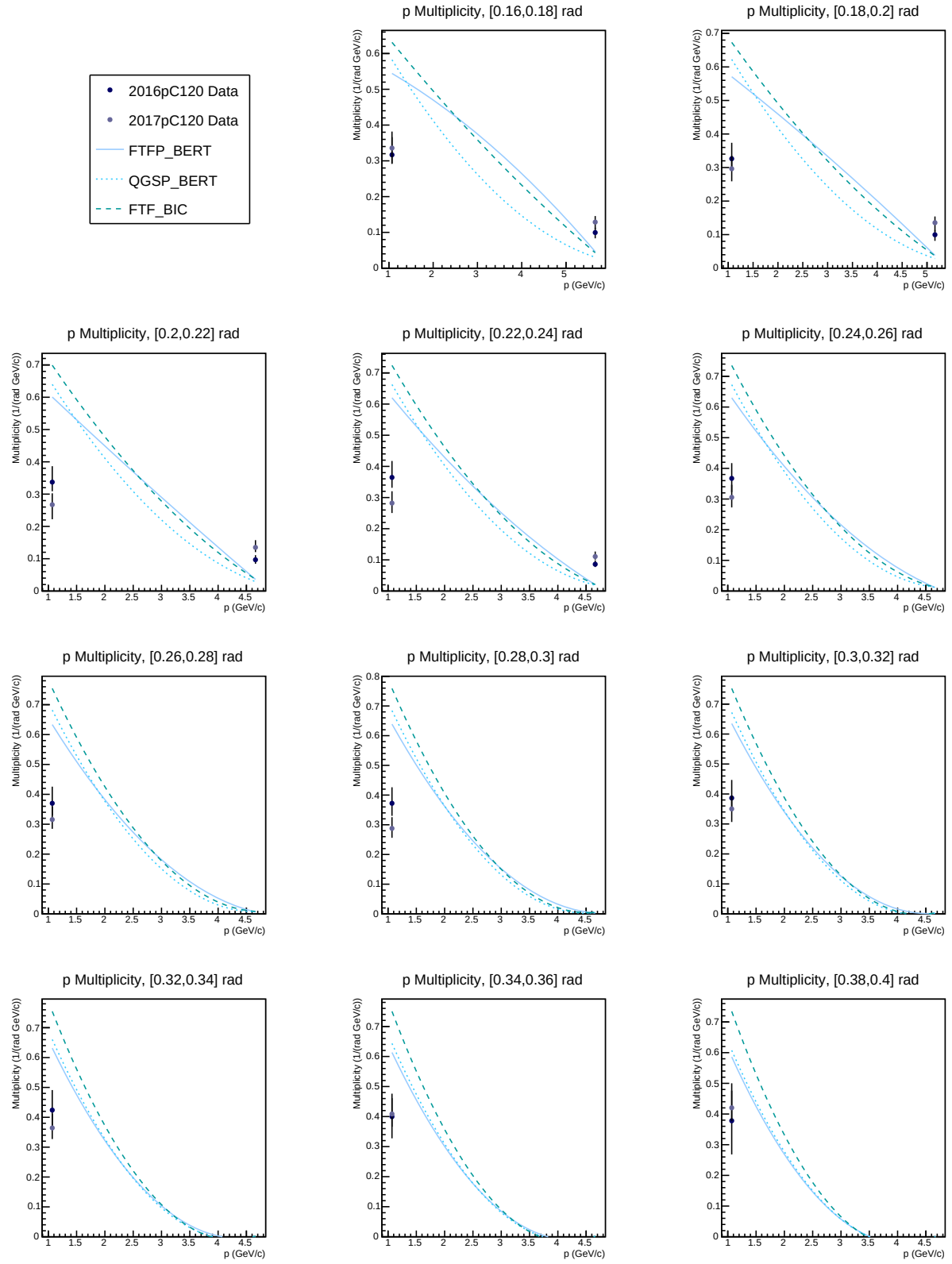


Figure 7.25: p multiplicity measurements from 2016 & 2017 data sets from 0.16–0.4 rad.

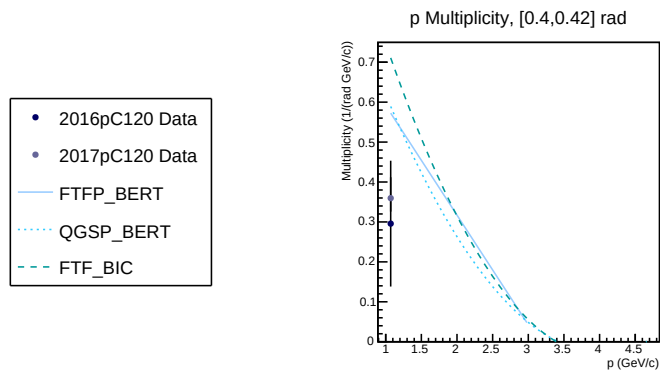


Figure 7.26: p multiplicity measurements from 2016 & 2017 data sets from 0.40–0.42 rad.

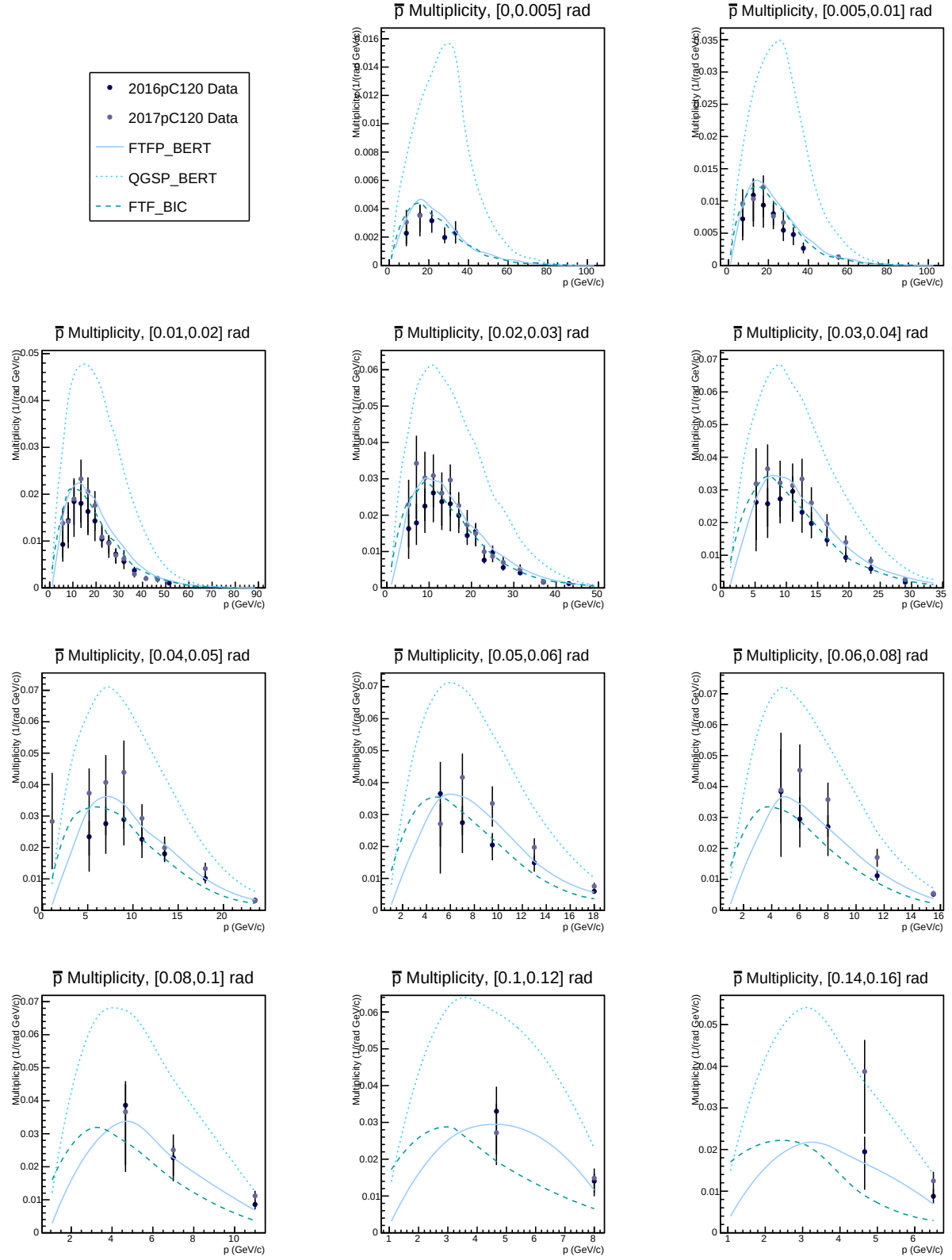


Figure 7.27: $\bar{p}$ multiplicity measurements from 2016 & 2017 data sets from 0.0–0.16 rad.

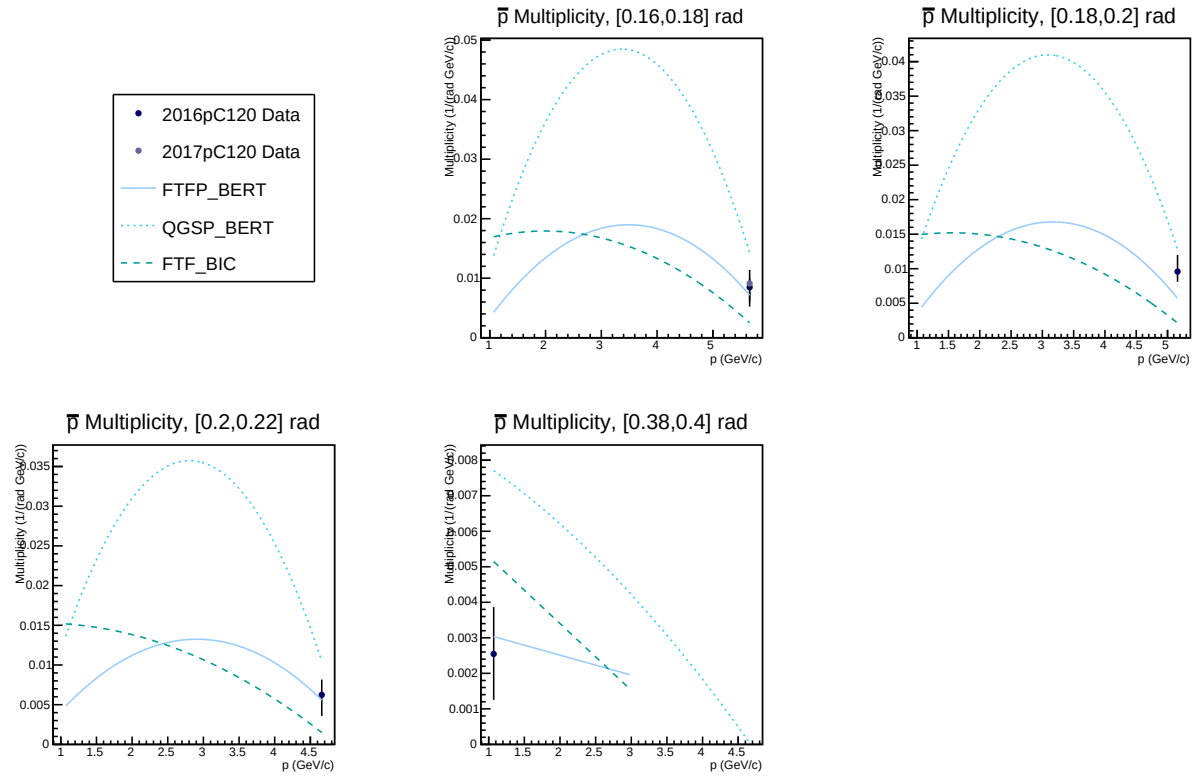


Figure 7.28: $\bar{p}$ multiplicity measurements from 2016 & 2017 data sets from 0.16–0.4 rad.

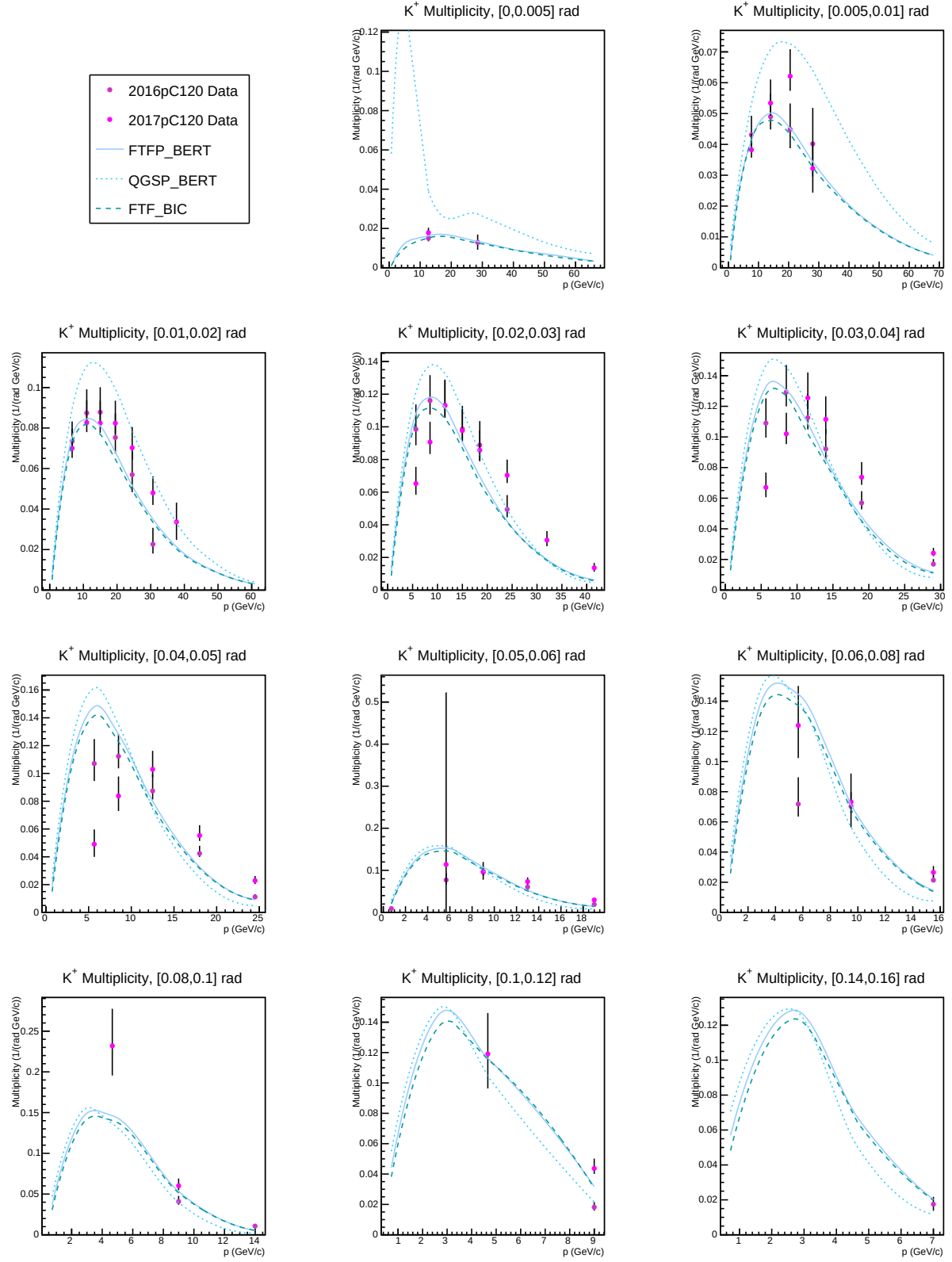


Figure 7.29: K^+ multiplicity measurements from 2016 & 2017 data sets from 0.0–0.16 rad.

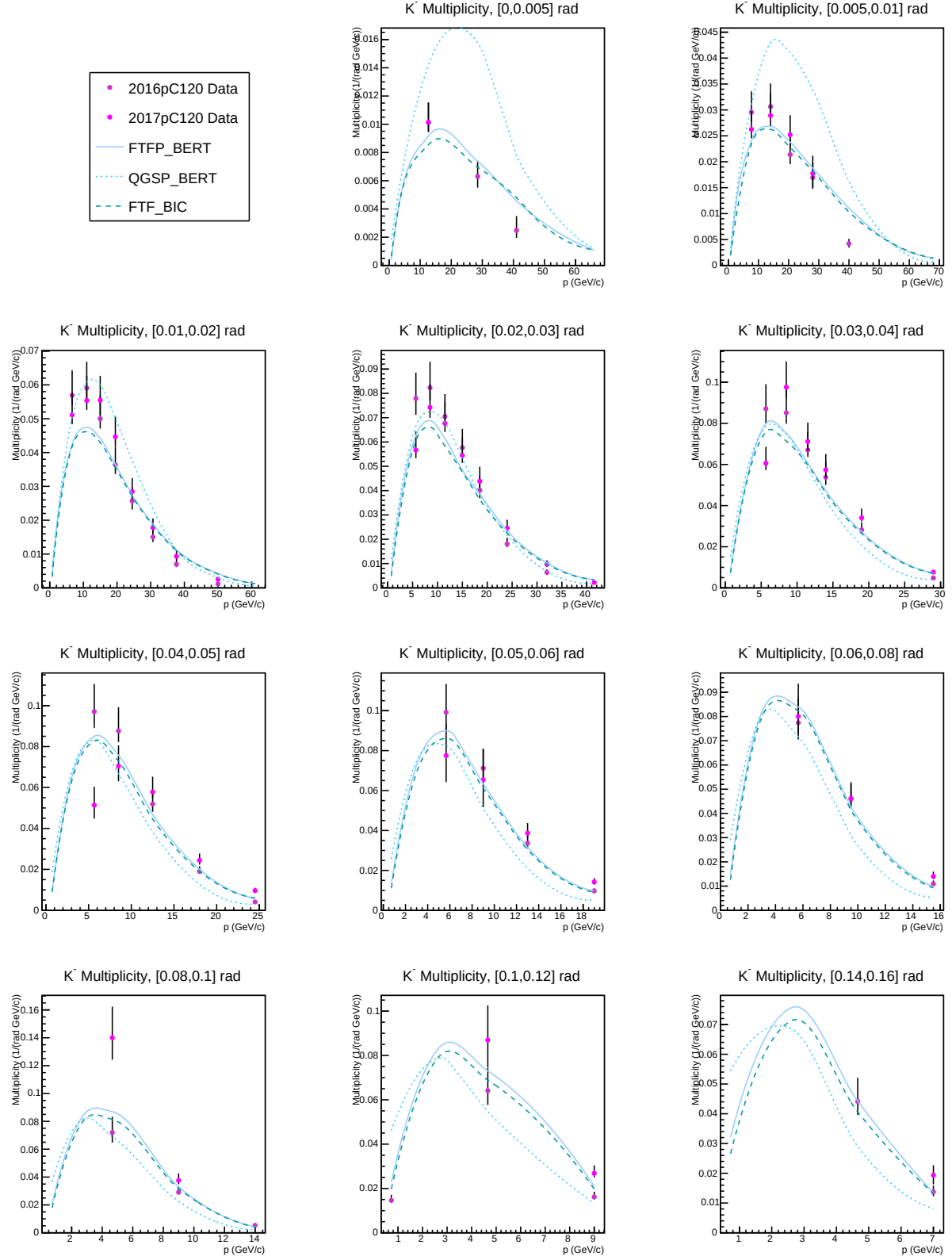


Figure 7.30: K^- multiplicity measurements from 2016 & 2017 data sets from 0.0–0.16 rad.

7.3 Systematic Uncertainties

Systematic uncertainties from several different effects on measured identified multiplicities were considered. This section will detail the sources of uncertainty considered and show the individual contributions to total systematic uncertainty in each analysis bin. The breakdown of the contributions of each uncertainty to the total uncertainty can be seen in Fig. 7.31–7.55. Bins with total uncertainty equal to 0 correspond to bins with no multiplicity measurement.

7.3.1 Production Cross-Section Uncertainty

The 120 GeV/c proton-carbon production cross-section measurement was reported with a highly asymmetric systematic uncertainty. The upper and lower uncertainty values were propagated through the multiplicity analysis in order to obtain the associated uncertainty on the multiplicity spectra. The result is a uniform fractional uncertainty on each measurement of (+12.6,-3.6)%.

7.3.2 dE/dx Fit Model Uncertainty

In Section 7.2.5.2, the procedure for determining dE/dx fit bias in each analysis bin was discussed. The systematic uncertainty associated with the fitting routine was evaluated using a similar procedure: Calibration parameters were independently smeared using a Gaussian distribution whose width corresponded to the RMS of each parameter from fits to the data. As the dE/dx simulation parameters were varied and the simulated data re-fit, the standard deviation of fit biases in each bin was calculated:

$$\sigma_i^{\text{Fit}} = \sqrt{\frac{1}{N_{\text{trials}}} \sum_{i=1}^{N_{\text{trials}}} \left(\frac{y_i^{\text{fit}} - y_i^{\text{true}}}{y_i^{\text{true}}} - \left\langle \frac{y^{\text{fit}} - y^{\text{true}}}{y^{\text{true}}} \right\rangle \right)^2} \quad (7.22)$$

This standard deviation of fractional multiplicity given by 50 Monte Carlo simulations was taken as the uncertainty associated with the fitting routine and was propagated to the measured multiplicities.

7.3.3 Physics Modeling Uncertainty

The Monte-Carlo correction factors are based off of a given physics model. Varying the underlying physics model will lead to different correction factors. The central values for the Monte-Carlo corrections were determined using the FTFP_BERT physics list, which has shown to produce results more consistent with measured data than others. Three other physics models, FTF_BIC, QGSP_BERT, and QBBC were substituted in independent Monte-Carlo samples, and the multiplicities were re-calculated with these correction factors. The difference from the nominal multiplicities was taken as a systematic uncertainty.

7.3.4 Feed-Down Uncertainty

Monte-Carlo corrections are relied on to correct for contributions from weakly-decaying particles to the identified multiplicity spectra. The most significant contributors for feed-down decays are K_S^0 for pions and $\Lambda^0/\bar{\Lambda}^0$ for pions and protons. The production of these parent particles varies significantly between physics models, incurring a large systematic uncertainty. An uncertainty of $\pm 50\%$ was assigned to the fraction of feed-down particles in each kinematic bin. The adjusted correction was propagated to the final multiplicity spectra, and the difference from the nominal multiplicities was taken as a systematic uncertainty. When the V^0 analysis is performed for this data set, the results will be used to constrain and significantly reduce these uncertainties.

7.3.5 Reconstruction Uncertainty

Though the NA61/SHINE detector calibration routine removes most alignment issues, the procedure is not perfect. Any remaining misalignment of detectors or mis-calibration can have an impact on the resulting multiplicity spectra. In order to evaluate the uncertainty associated with reconstruction, the central positions of crucial TPCs were adjusted in the detector Monte-Carlo. The magnitude of the adjustments was selected by examining the final TPC residuals as shown in Chapter 5. Adjustments used can be seen in Table 7.4. The resulting uncertainties for each scenario were added in quadrature to obtain the final reconstruction uncertainty.

Chamber	2016 central x -position shift	2017 central x -position shift
VTPC1	100 μm	100 μm
VTPC2	100 μm	100 μm
GTPC	100 μm	50 μm
FTPC1	-	50 μm

Table 7.4: x -position shifts for reconstruction uncertainty calculation.

7.3.6 Selection Uncertainty

Upon comparing track characteristics between reconstructed Monte Carlo and recorded data, a discrepancy was found between the average number of clusters per track. The simulated tracks contain 5 - 10 % more clusters than tracks from data. This is likely because we do not simulate faulty front-end electronics channels in the detector or periodic detector noise. These two effects often lead to cluster loss, as the cluster structures become difficult to distinguish from background noise. In order to compensate for this effect, the Monte Carlo corrections were re-calculated after artificially reducing the number of clusters on the simulated track by 15% for a conservative estimate. The resulting Monte Carlo corrections were used to re-calculate the multiplicity measurements, and the difference was taken as a systematic uncertainty.

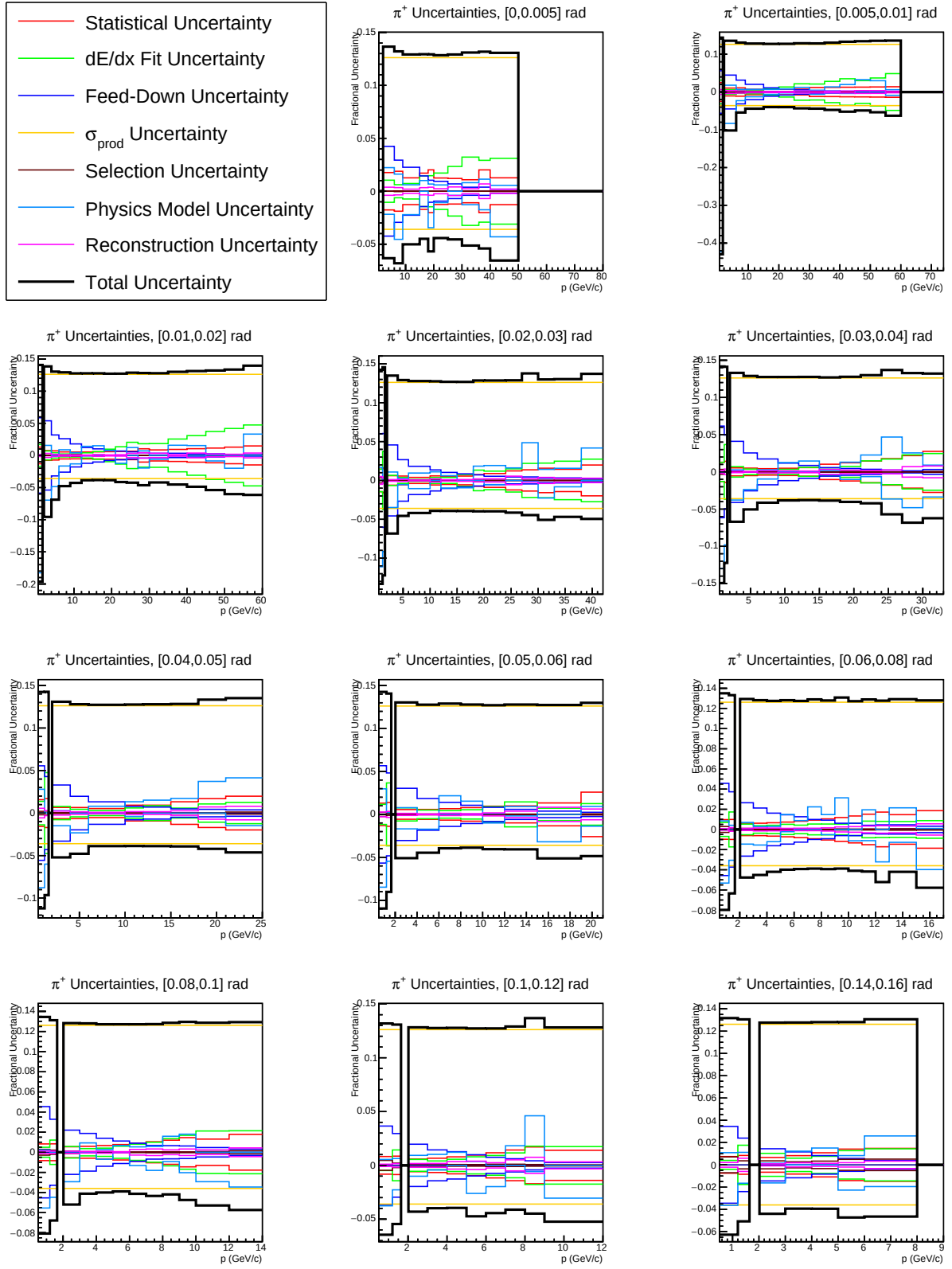


Figure 7.31: π^+ 2016 analysis uncertainty breakdown from 0.0–0.16 rad.

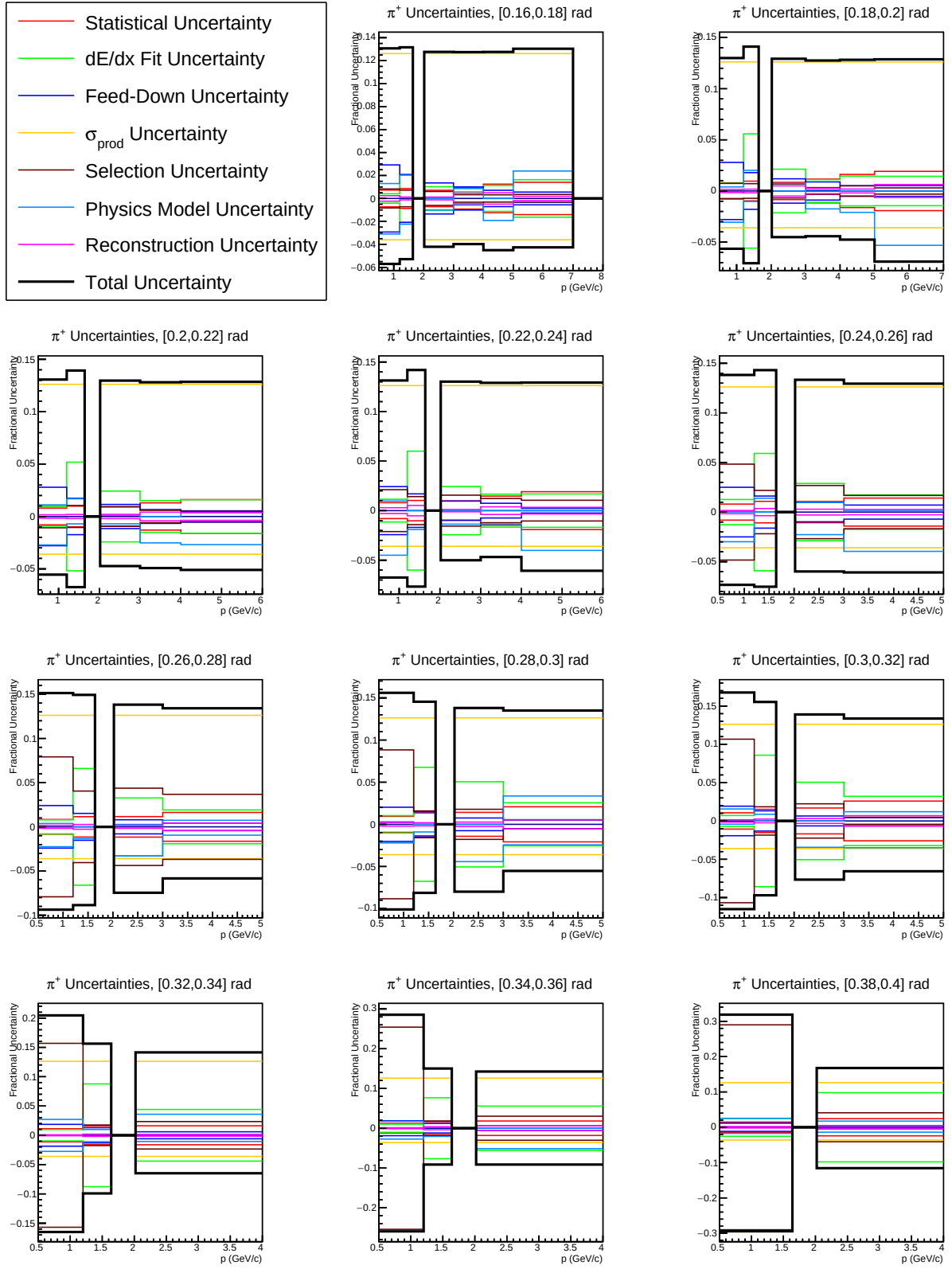


Figure 7.32: π^+ 2016 analysis uncertainty breakdown from 0.16–0.40 rad.

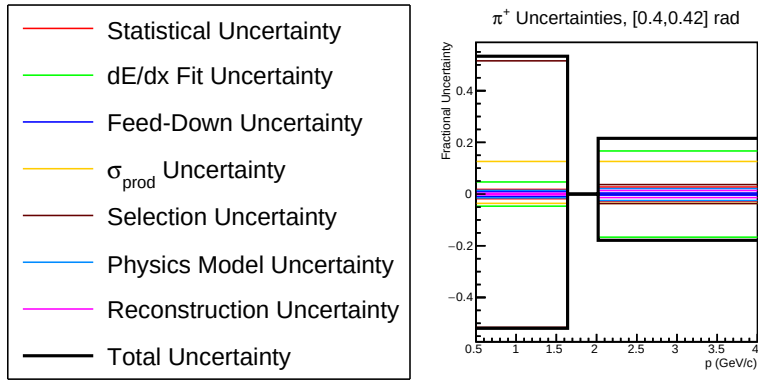


Figure 7.33: π^+ 2016 analysis uncertainty breakdown from 0.40–0.42 rad.

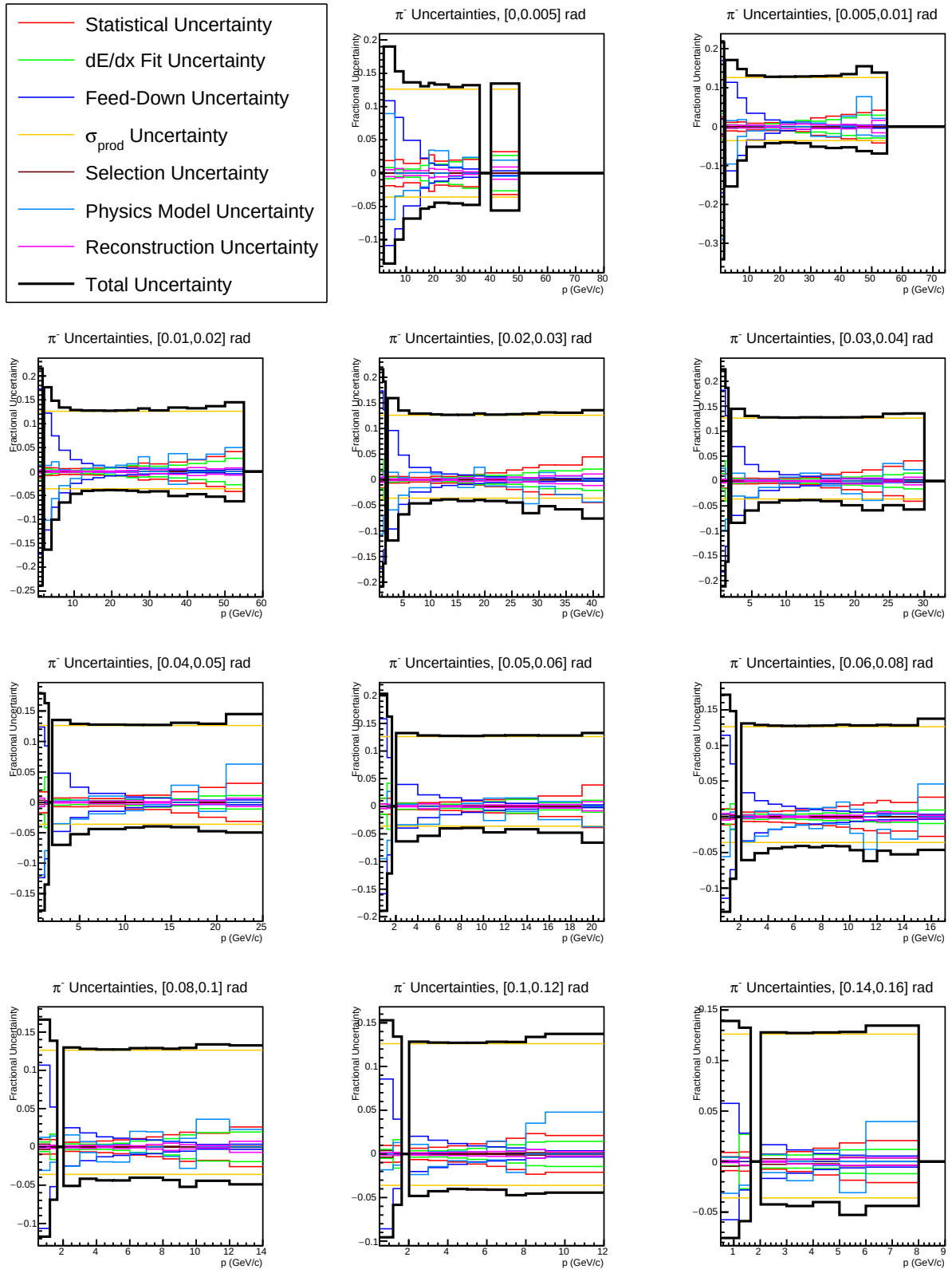


Figure 7.34: π^- 2016 analysis uncertainty breakdown from 0.0–0.16 rad.

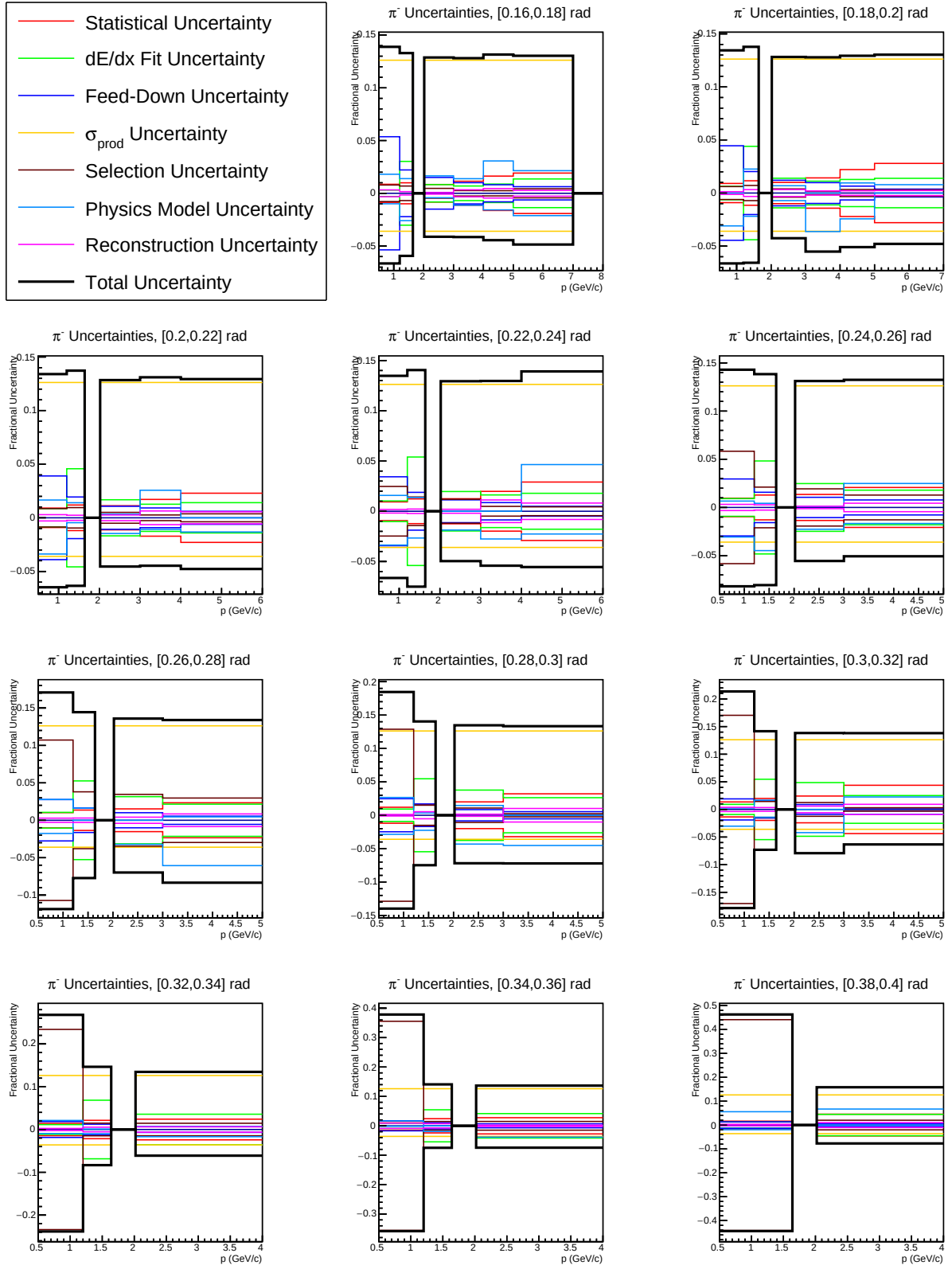


Figure 7.35: π^- 2016 analysis uncertainty breakdown from 0.16–0.40 rad.

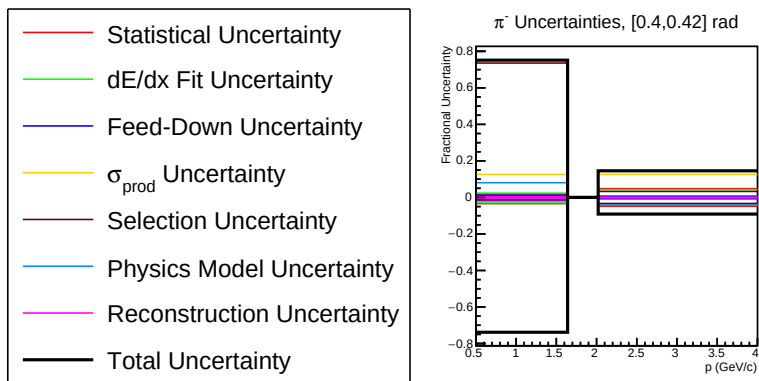


Figure 7.36: π^- 2016 analysis uncertainty breakdown from 0.40–0.42 rad.

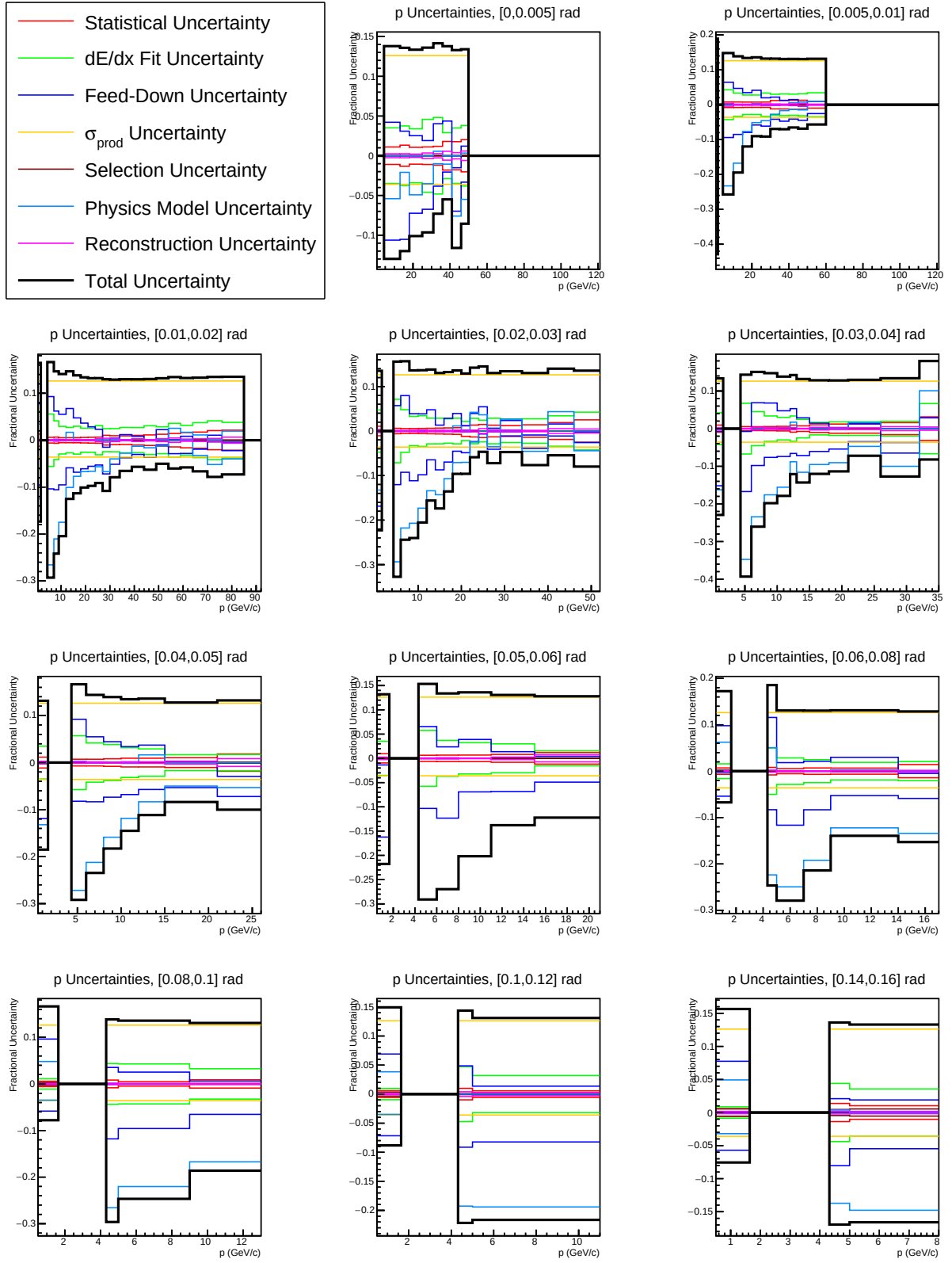
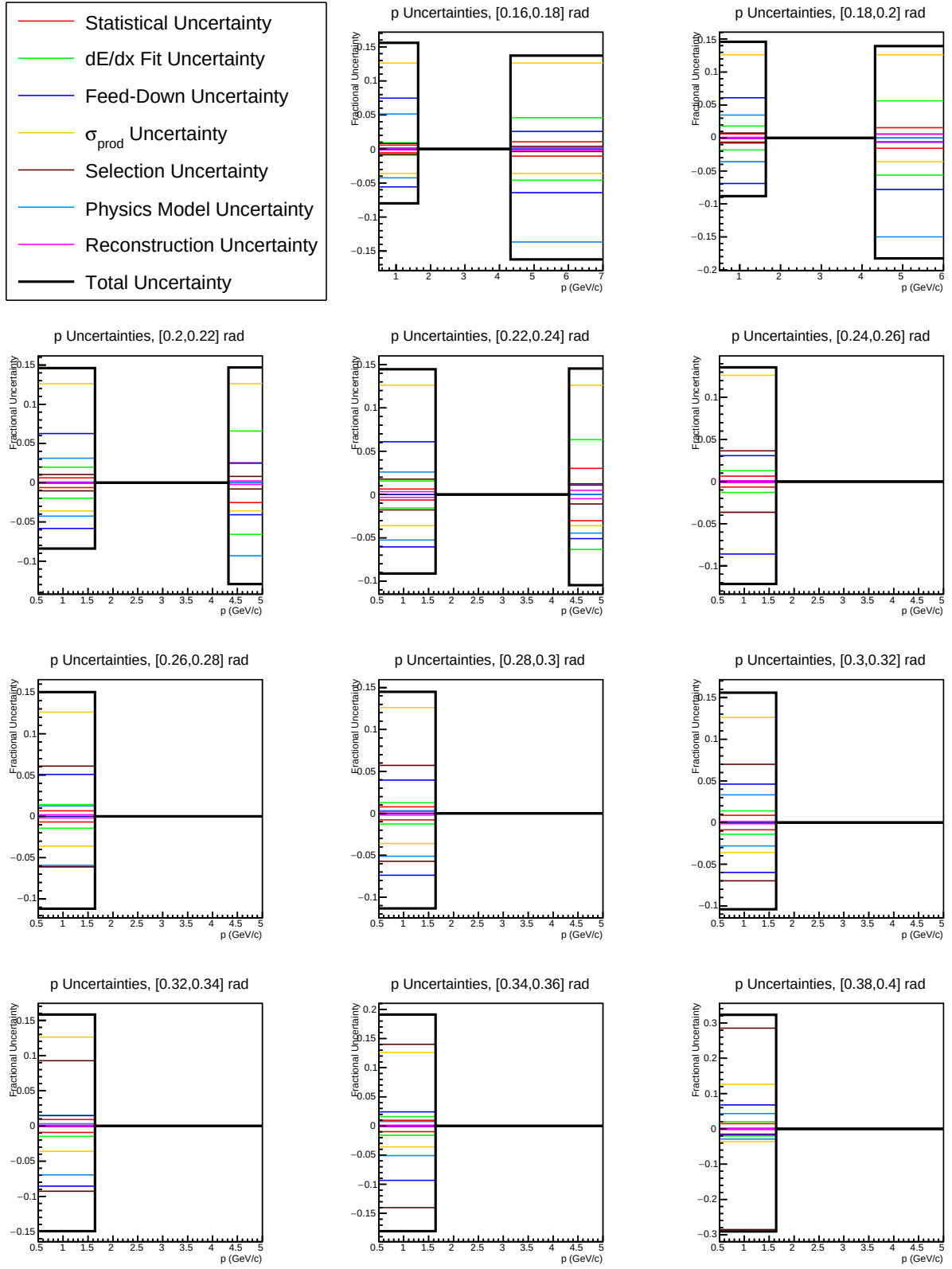


Figure 7.37: p 2016 analysis uncertainty breakdown from 0.0–0.16 rad.

Figure 7.38: p 2016 analysis uncertainty breakdown from 0.16–0.40 rad.

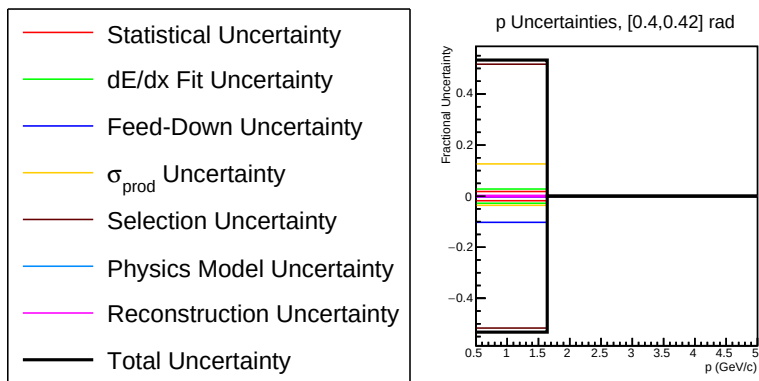


Figure 7.39: p 2016 analysis uncertainty breakdown from 0.40–0.42 rad.

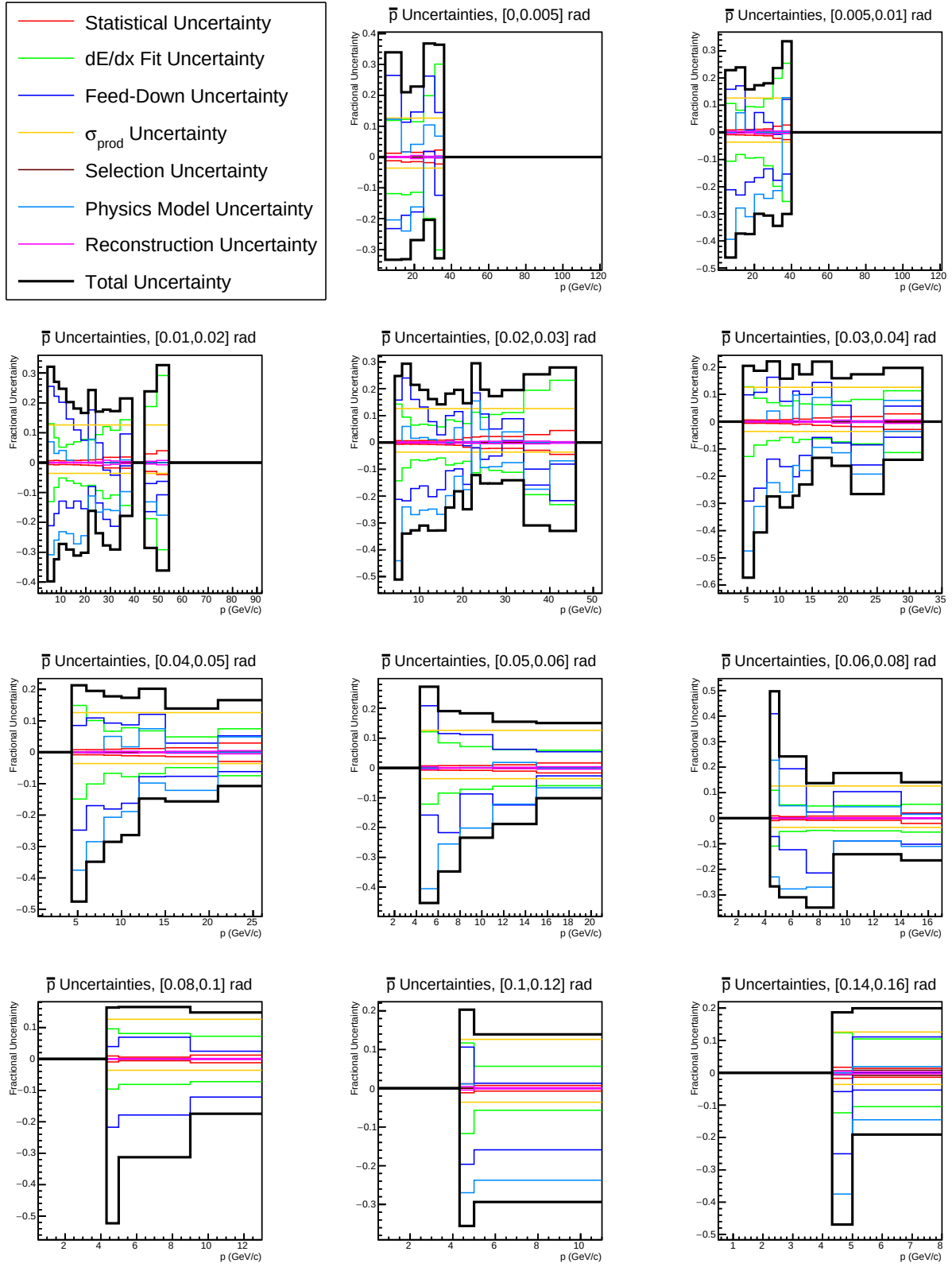


Figure 7.40: $\bar{p}$ 2016 analysis uncertainty breakdown from 0.0–0.16 rad.

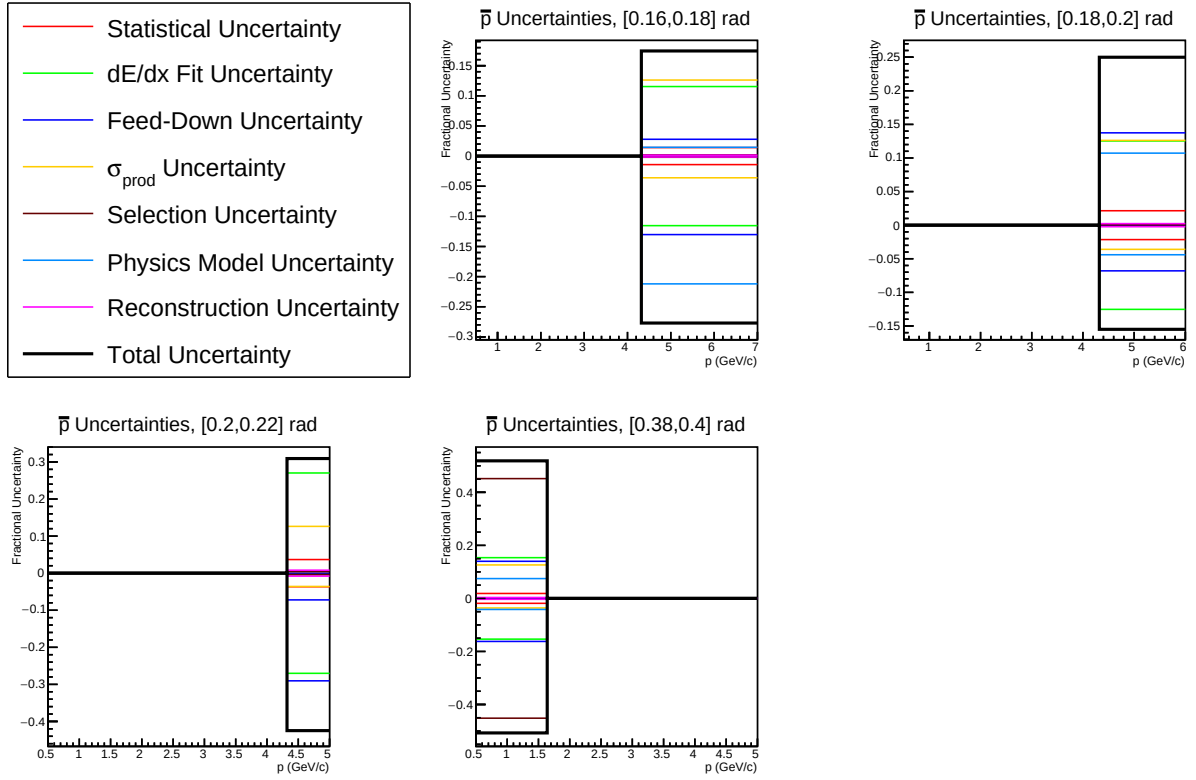


Figure 7.41: $\bar{p}$ 2016 analysis uncertainty breakdown from 0.16–0.40 rad.

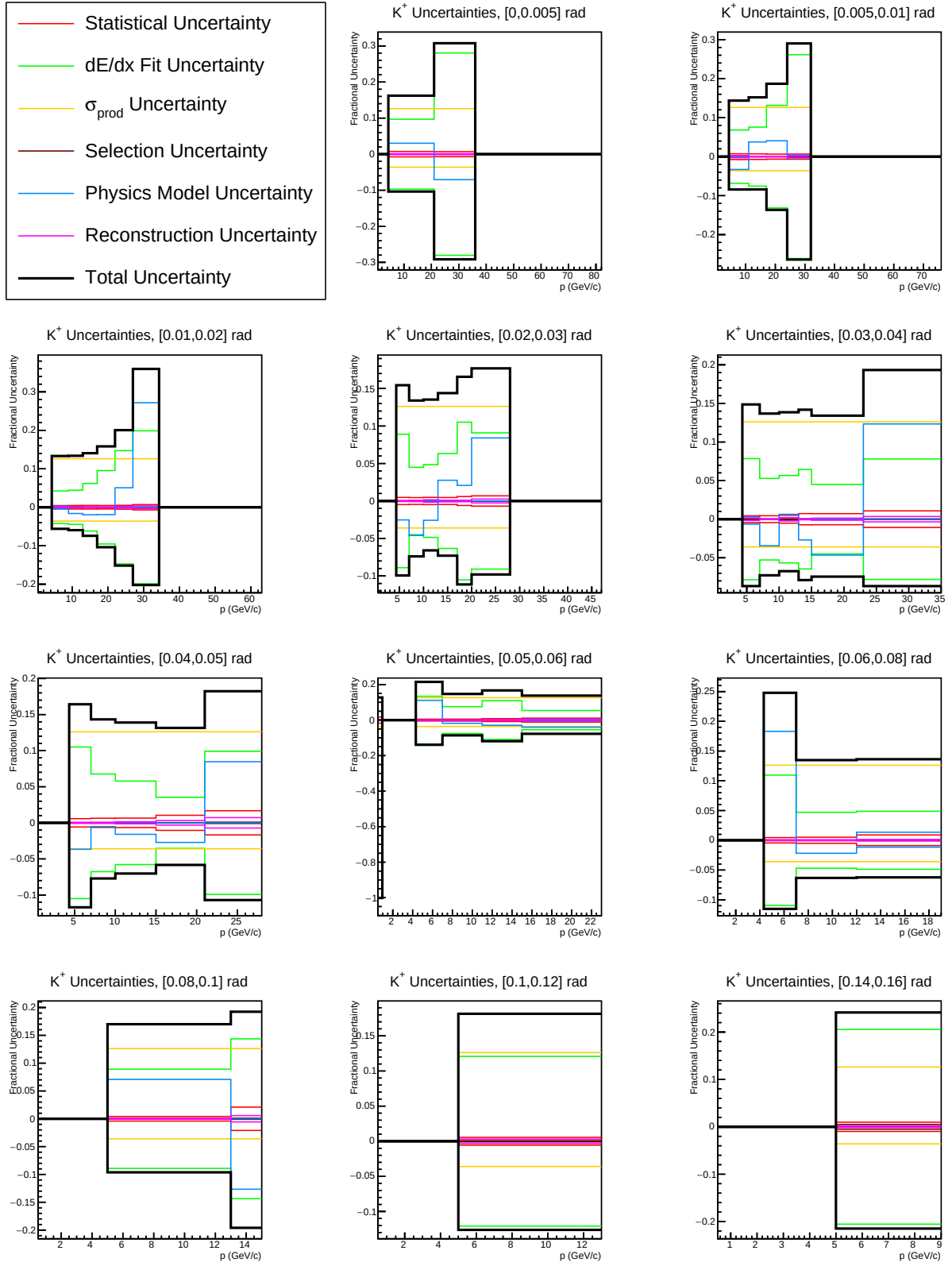
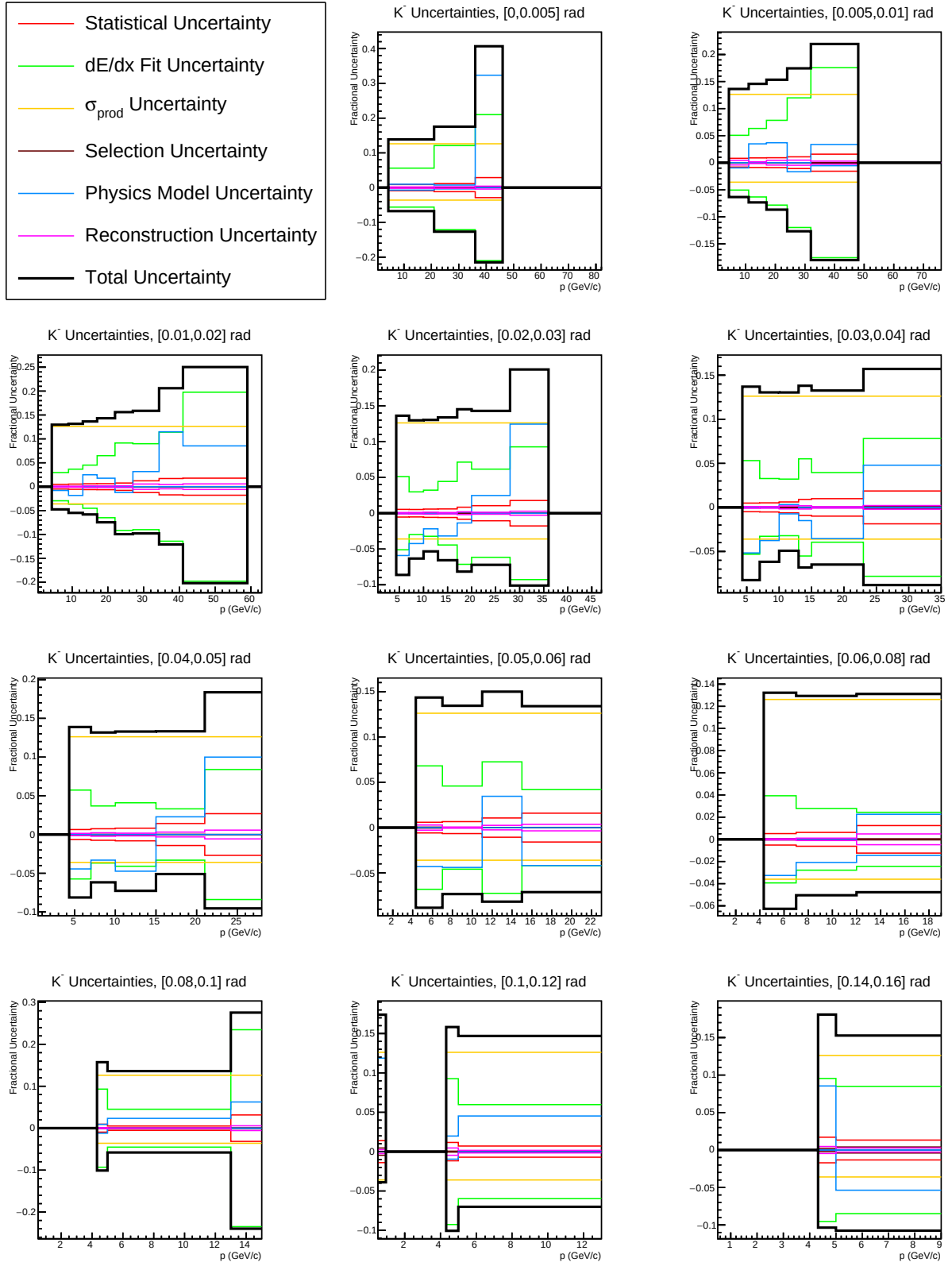


Figure 7.42: K^+ 2016 analysis uncertainty breakdown from 0.0–0.16 rad.

Figure 7.43: K^- 2016 analysis uncertainty breakdown from 0.0–0.16 rad.

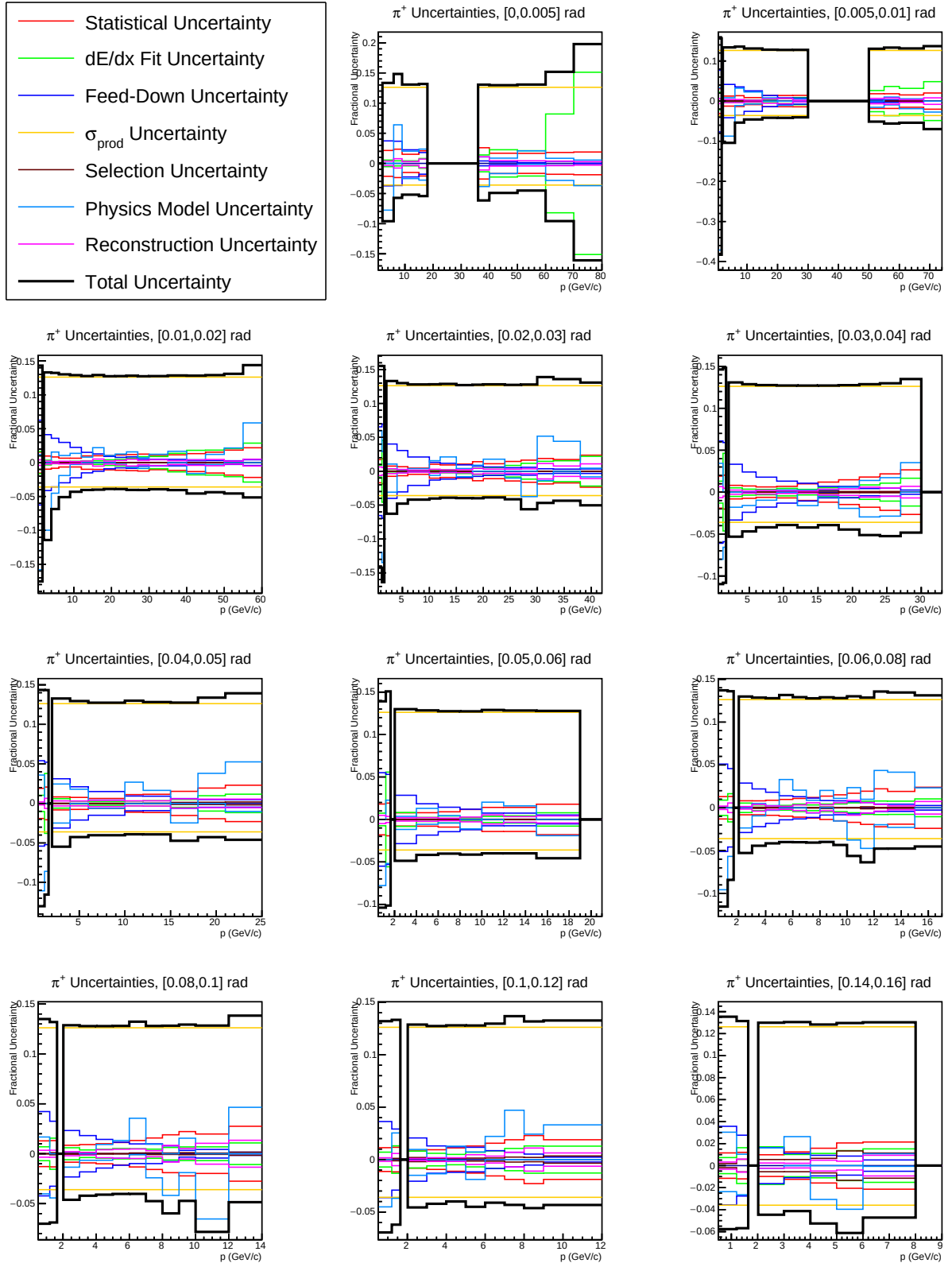


Figure 7.44: π^+ 2017 analysis uncertainty breakdown from 0.0–0.16 rad.

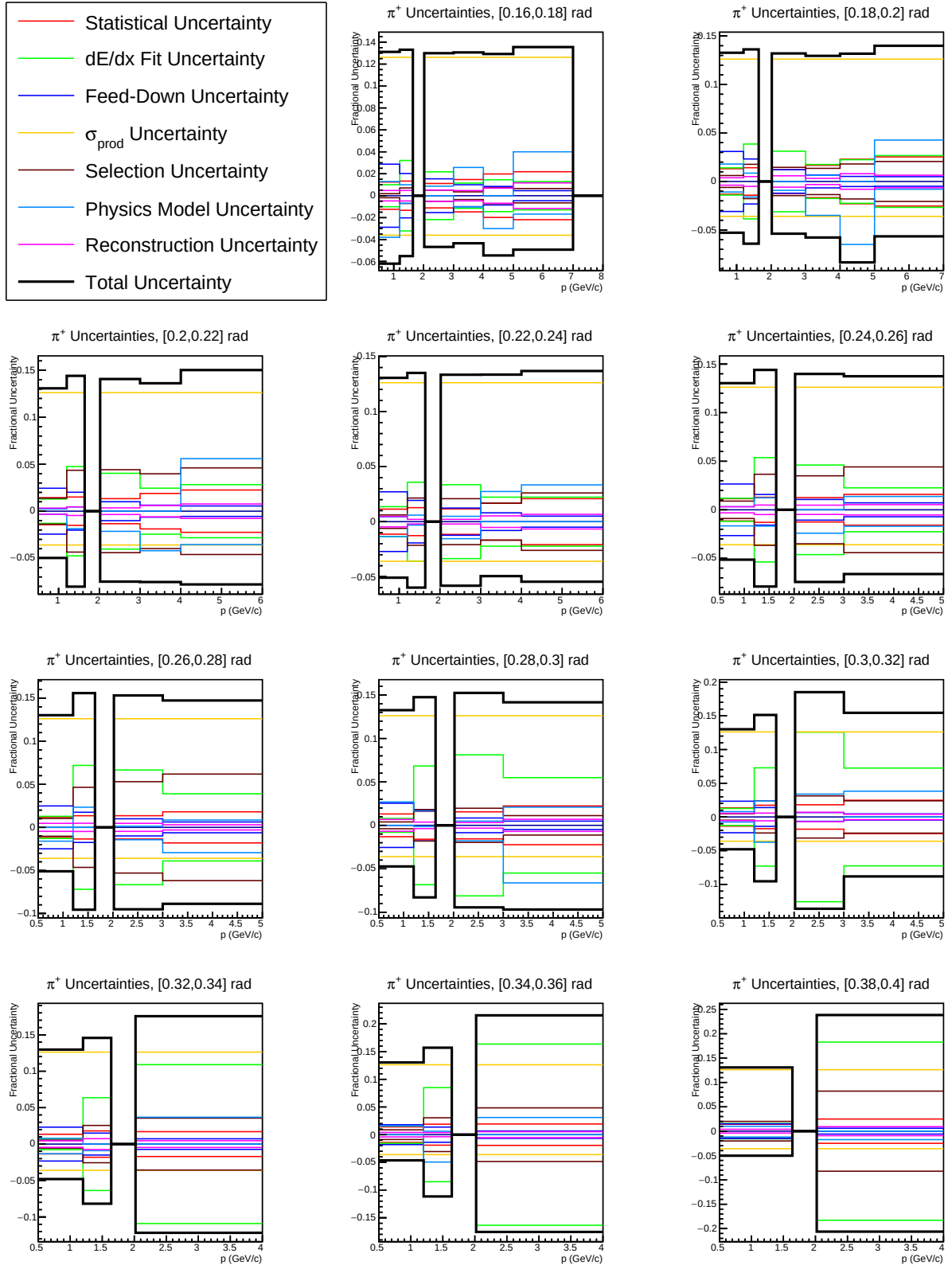


Figure 7.45: π^+ 2017 analysis uncertainty breakdown from 0.16–0.40 rad.

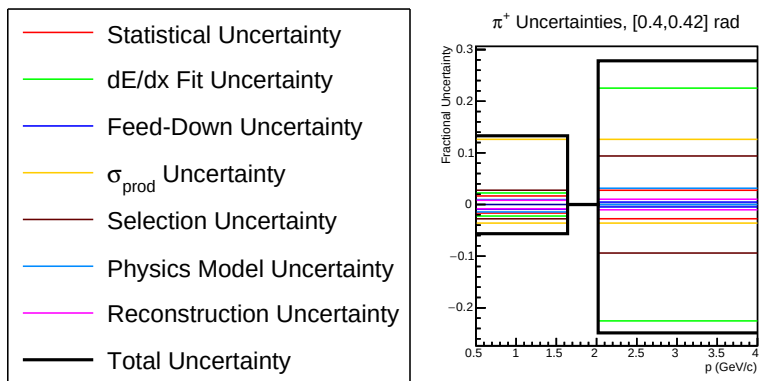


Figure 7.46: π^+ 2017 analysis uncertainty breakdown from 0.40–0.42 rad.

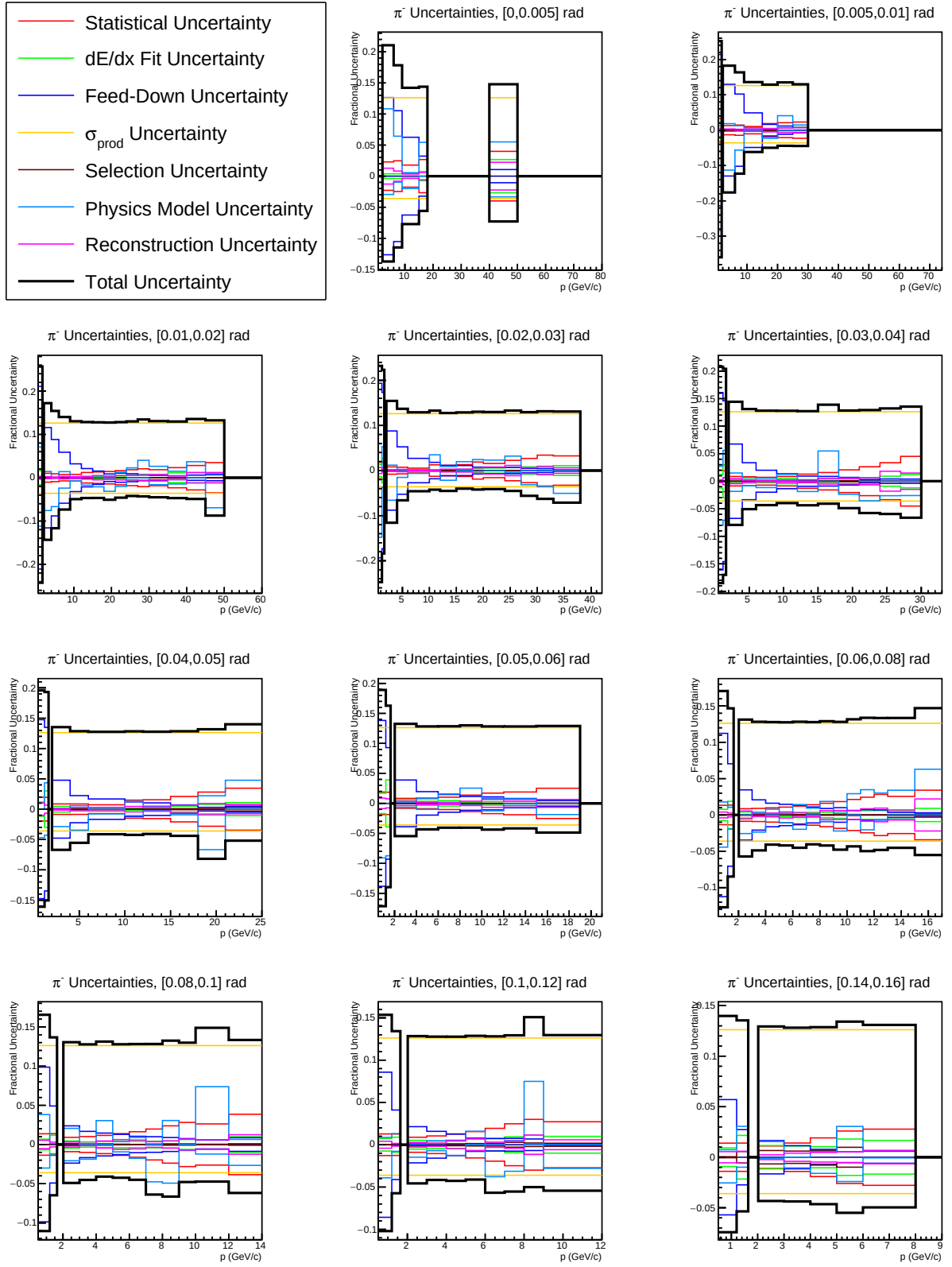


Figure 7.47: π^- 2017 analysis uncertainty breakdown from 0.0–0.16 rad.

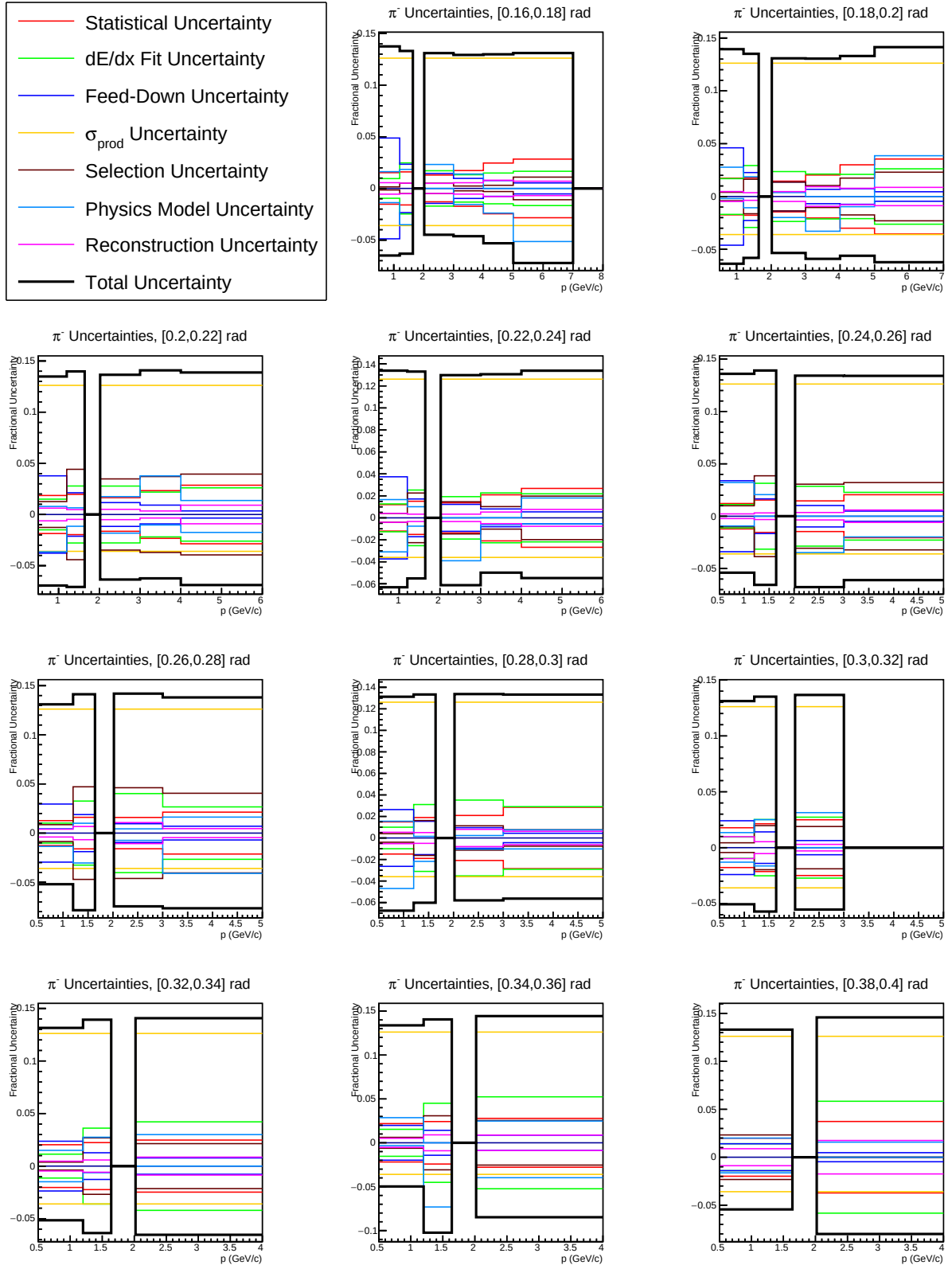


Figure 7.48: π^- 2017 analysis uncertainty breakdown from 0.16–0.40 rad.

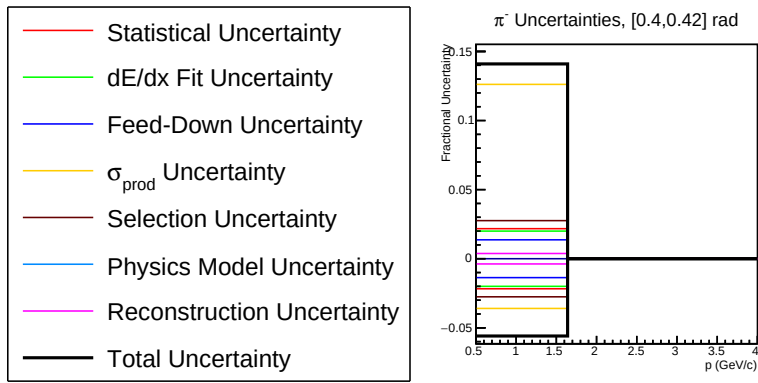


Figure 7.49: π^- 2017 analysis uncertainty breakdown from 0.40–0.42 rad.

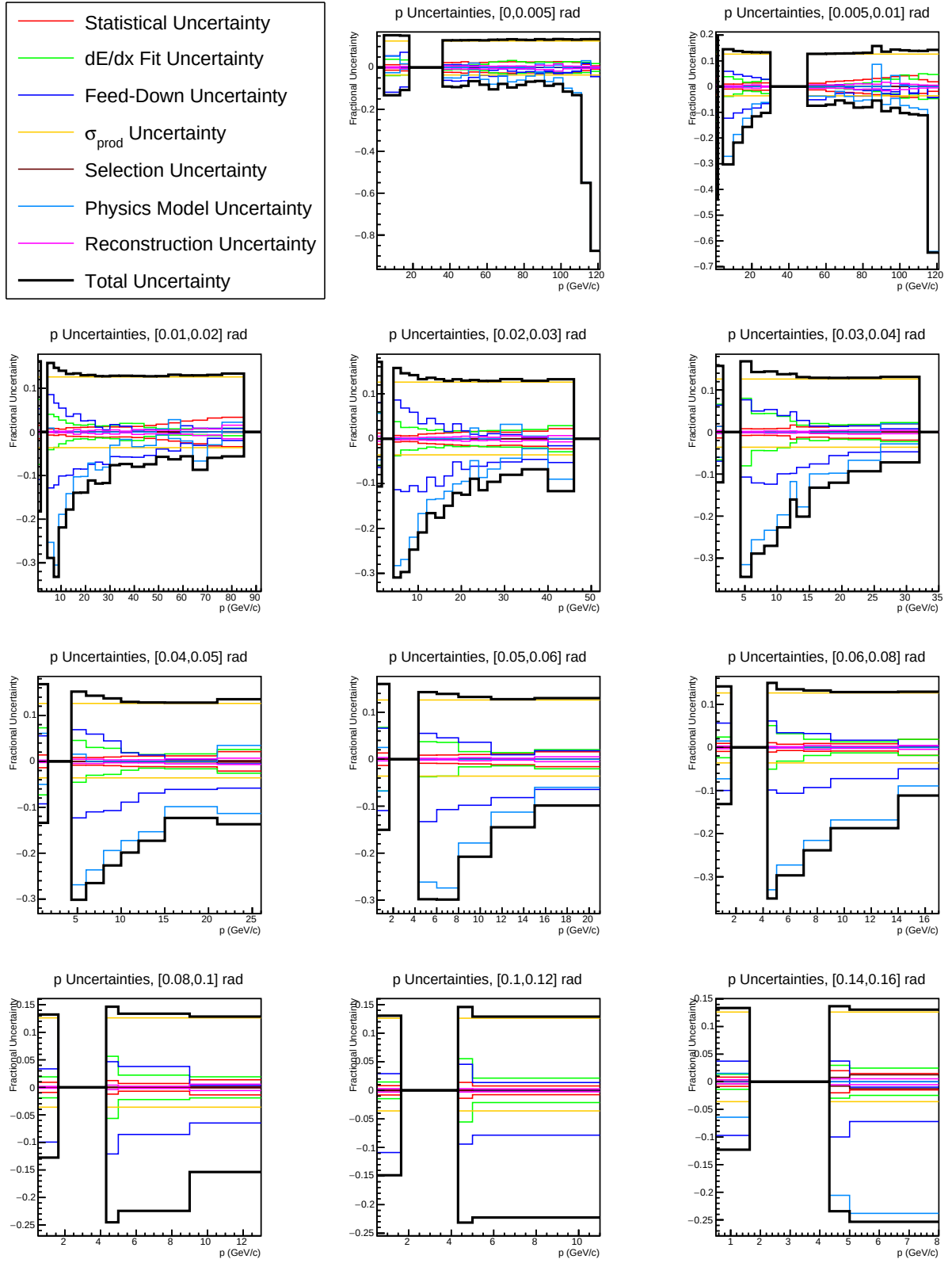
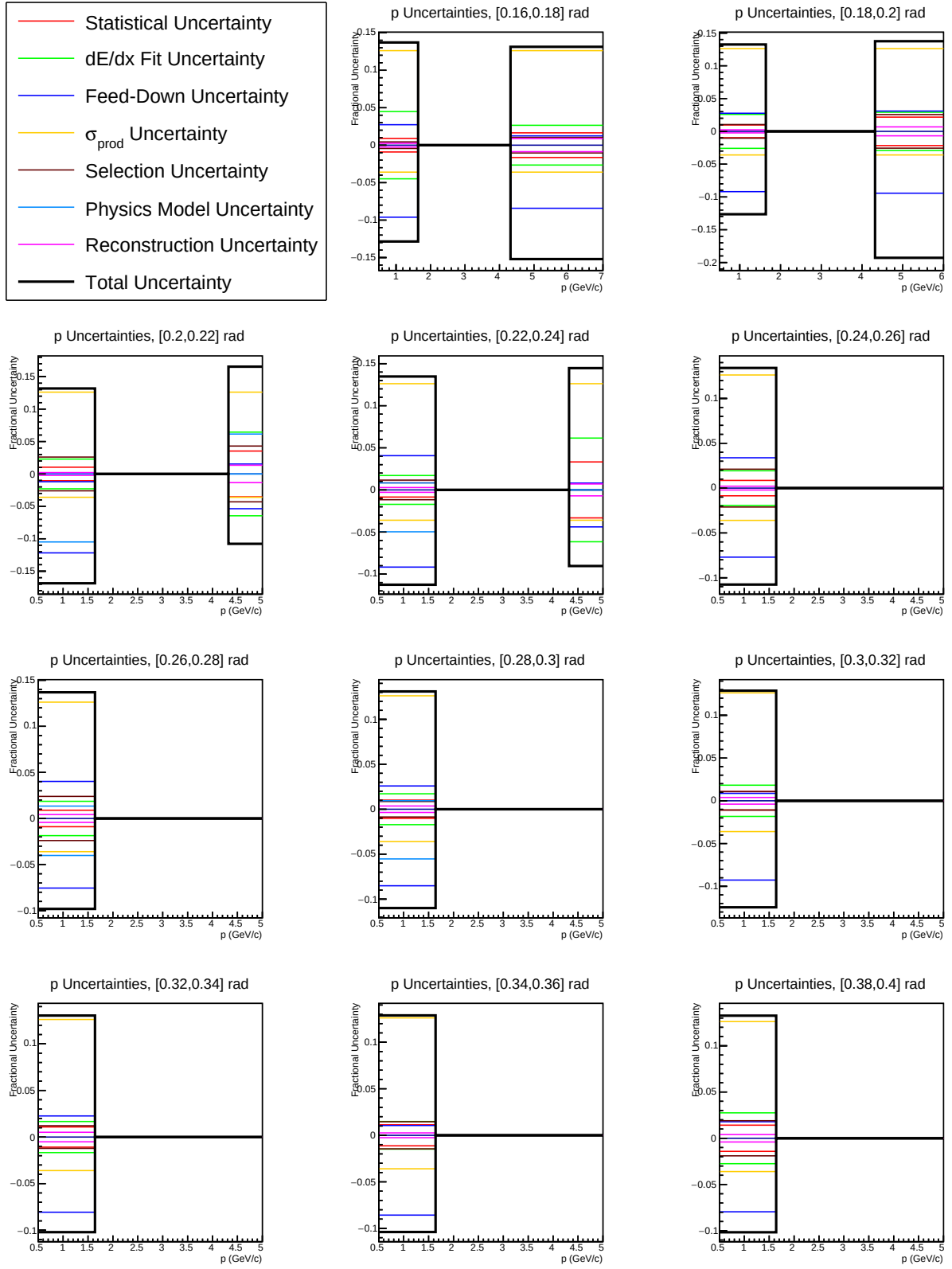


Figure 7.50: p 2017 analysis uncertainty breakdown from 0.0–0.16 rad.

Figure 7.51: p 2017 analysis uncertainty breakdown from 0.16–0.40 rad.

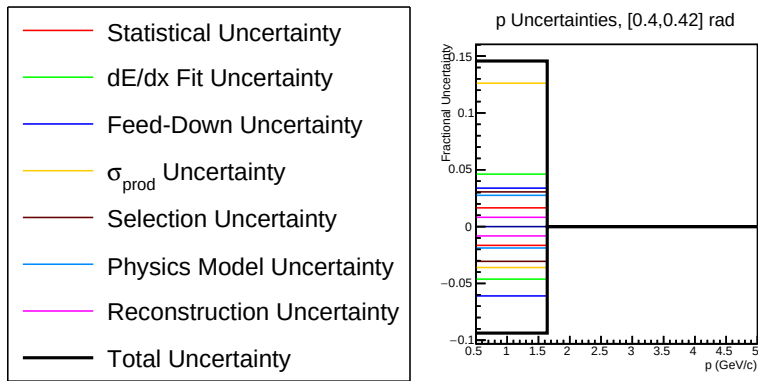


Figure 7.52: p 2017 analysis uncertainty breakdown from 0.40–0.42 rad.

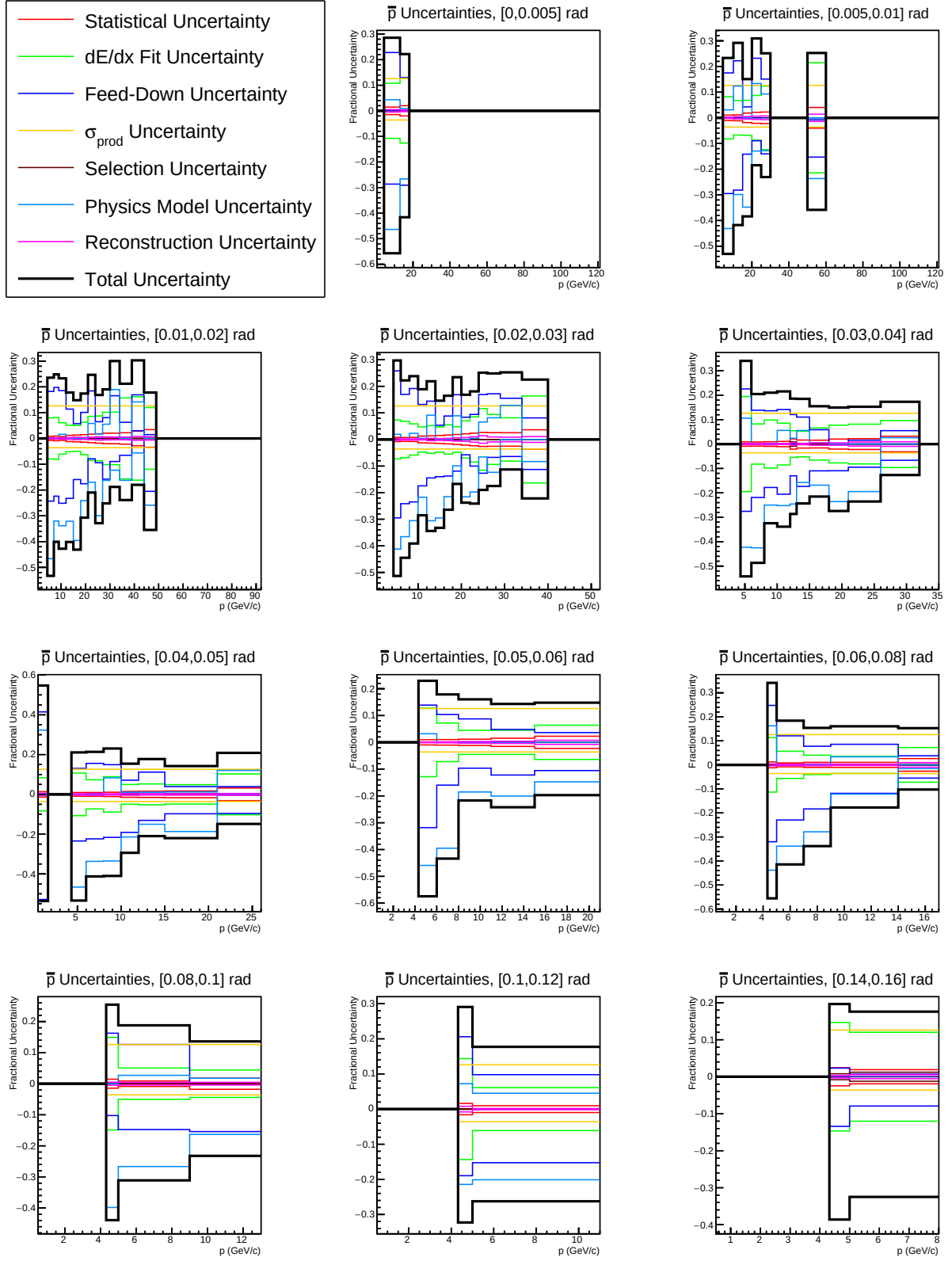


Figure 7.53: $\bar{p}$ 2017 analysis uncertainty breakdown from 0.0–0.16 rad.

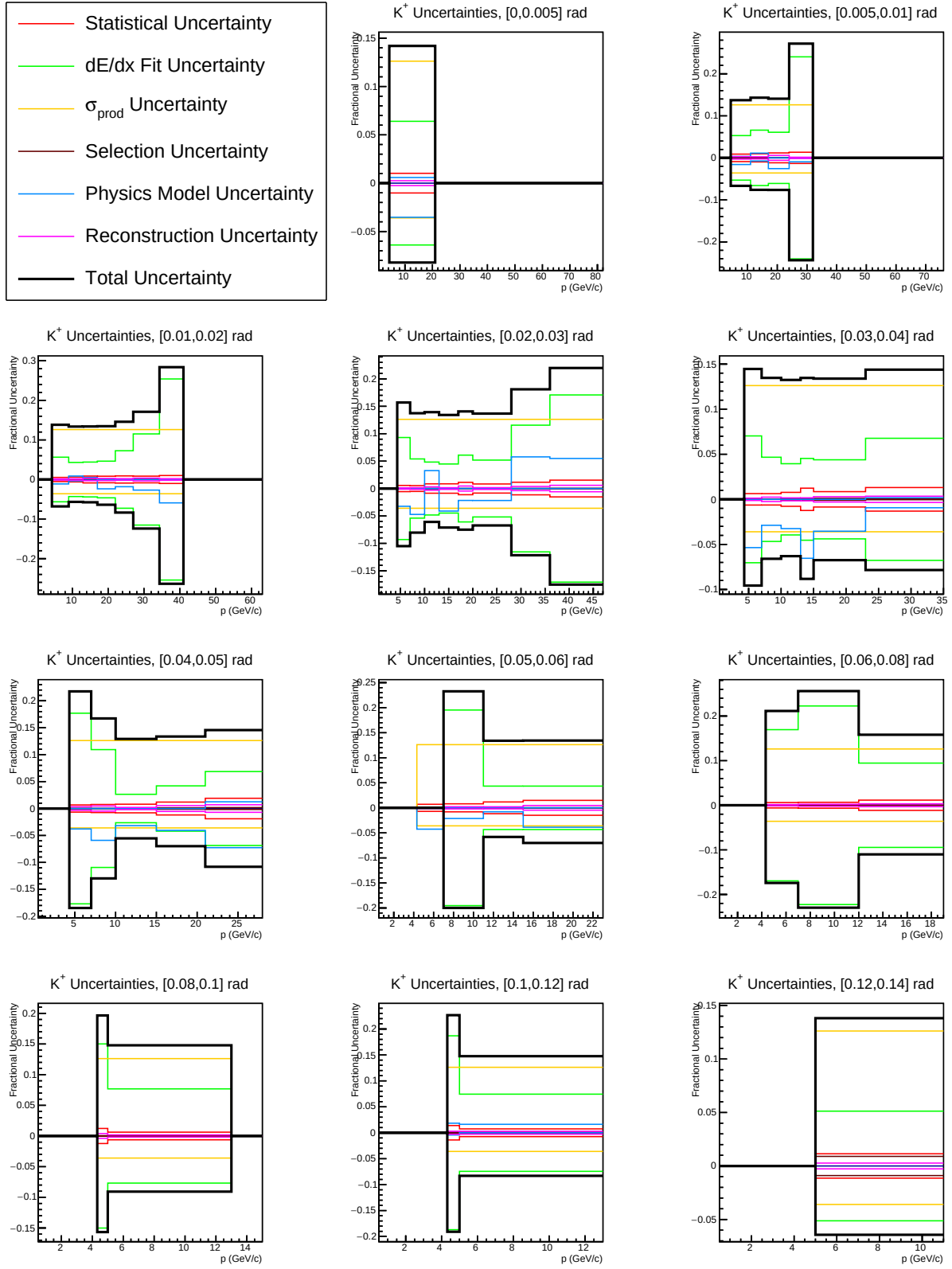


Figure 7.54: K^+ 2017 analysis uncertainty breakdown from 0.0–0.16 rad.

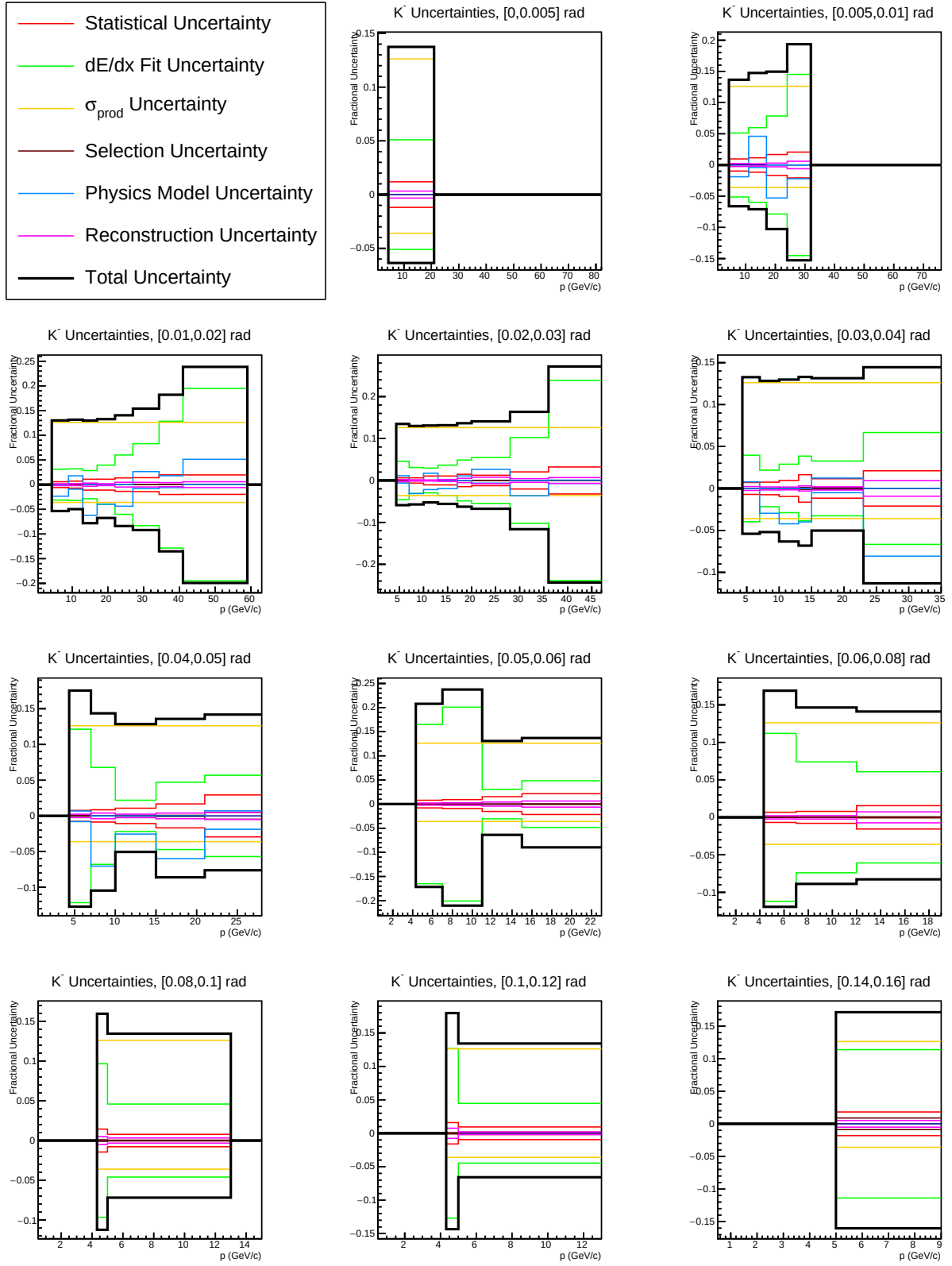


Figure 7.55: K^- 2017 analysis uncertainty breakdown from 0.0–0.16 rad.

Chapter 8

Combined Multiplicity Results

This chapter details the combination of independent multiplicity measurements from 2016 and 2017 into a final set of measurements. Uncorrelated and correlated uncertainties are treated, and special consideration is taken to combine measurements with asymmetric uncertainties. Final multiplicity measurements and systematic uncertainties are presented.

8.1 Combined Measurements

In order to combine the measurements from the 2016 and 2017 analyses reported in Chapter 7, the systematic uncertainties must be carefully considered. Correlated uncertainties between the two sets of measurements must be separated and treated specially, while uncorrelated uncertainties can be used to weight the individual measurements. The final measurements are presented in the form

$$\hat{x}_i \pm \hat{\sigma}_i \pm \hat{\Delta}_i \tag{8.1}$$

where $\hat{x}_i$ is the weighted measurement obtained from the 2016 & 2017 central values, $\hat{\sigma}_i$ is the combined uncorrelated uncertainty, and $\hat{\Delta}_i$ is the uncertainty associated with correlated systematic uncertainties between the 2016 & 2017 analyses.

8.1.1 Uncorrelated Uncertainties

The statistical uncertainties are uncorrelated and treated as such. Furthermore, due to differences in detector acceptance and calibration procedures, the dE/dx fit uncertainty, reconstruction uncertainty, and selection uncertainty are treated as uncorrelated.

The uncorrelated uncertainties are rendered asymmetric by the selection and reconstruction uncertainties, which are asymmetric by construction. The weighted average calculation must take into account these asymmetries [53]. Considering a measurement with asymmetric uncertainties, we define the following quantities:

$$\sigma = \frac{\sigma^+ + \sigma^-}{2}, \quad \alpha = \frac{\sigma^+ - \sigma^-}{2}, \quad A = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} \quad (8.2)$$

where σ^+ is the upper uncertainty and σ^- is the lower uncertainty. σ , α , and A are referred to as the mean uncertainty, difference, and asymmetry, respectively. If the measurement and its uncertainties represent an asymmetric Gaussian distribution whose most probable value is centered at 0 and widths are given by σ^+ and σ^- , the mean of the distribution will be α .¹ This bias in mean value is taken into account using the following procedure. The weighted mean of the points $\hat{x}$ is given by

$$\hat{x} = \frac{\sum_i w_i (x_i - \alpha_i)}{\sum_i w_i}, \quad w_i = \frac{1}{\sigma_i^2 + 2\alpha_i^2} \quad (8.3)$$

and the combined uncertainty $\hat{\sigma}$ is given by

$$\hat{\sigma} = \frac{\sum_i w_i^2 V_i}{(\sum_i w_i)^2}, \quad V_i = \sigma_i^2 + 2\alpha_i^2. \quad (8.4)$$

Note that as the asymmetry α approaches 0, the weighted mean and uncertainty approach their common definitions.

8.1.2 Correlated Uncertainties

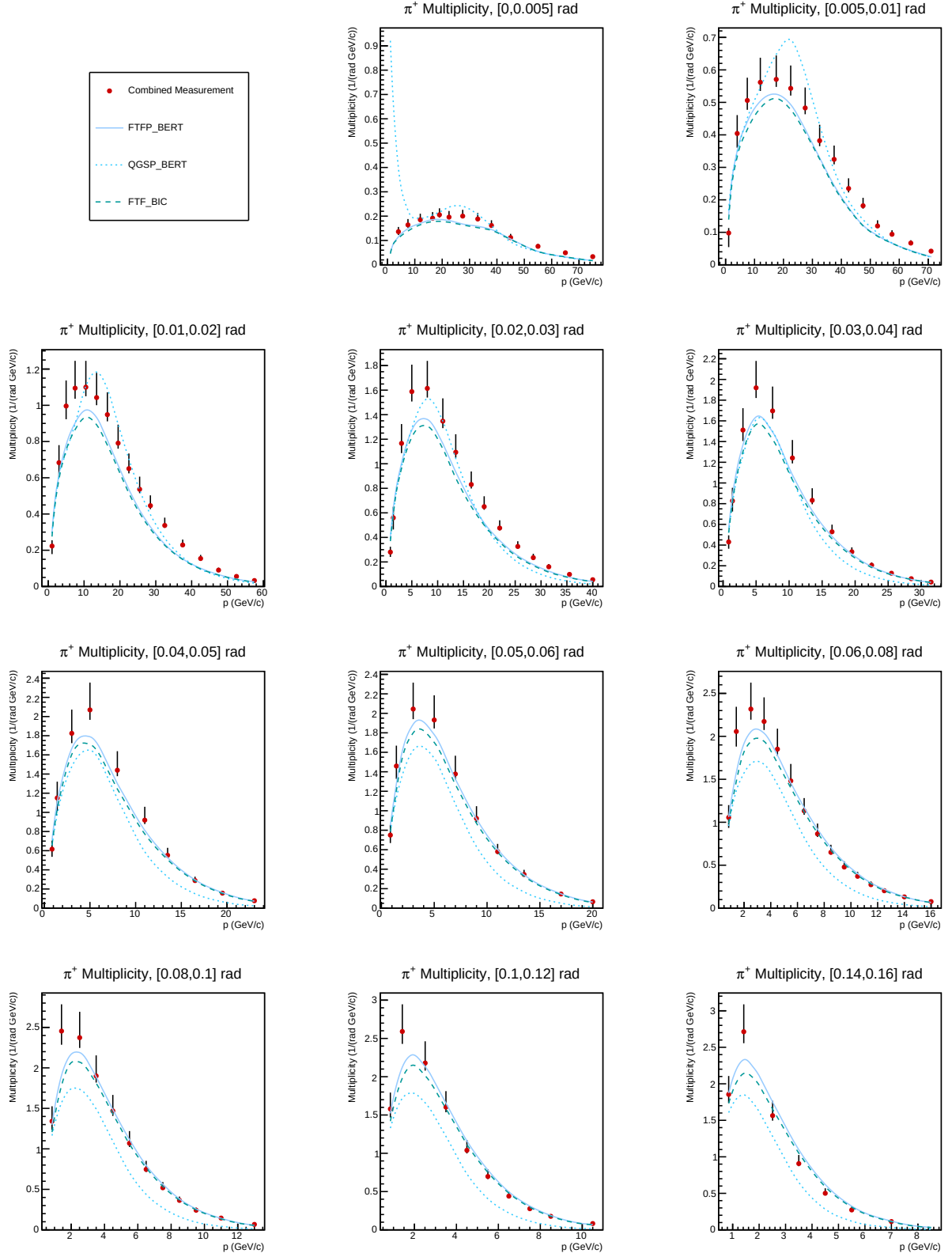
The model uncertainties and feed-down uncertainties originate from the same underlying distribution of simulated particles. When the physics simulation model is changed, this distribution is altered identically for the 2016 and 2017 data sets. The associated uncertainties are therefore considered to be correlated between the two measurements. In order to select the associated value for each uncertainty, the maximum fractional upper and lower uncertainty was taken from the two data sets, similarly to how the model uncertainties were calculated for the individual data sets.

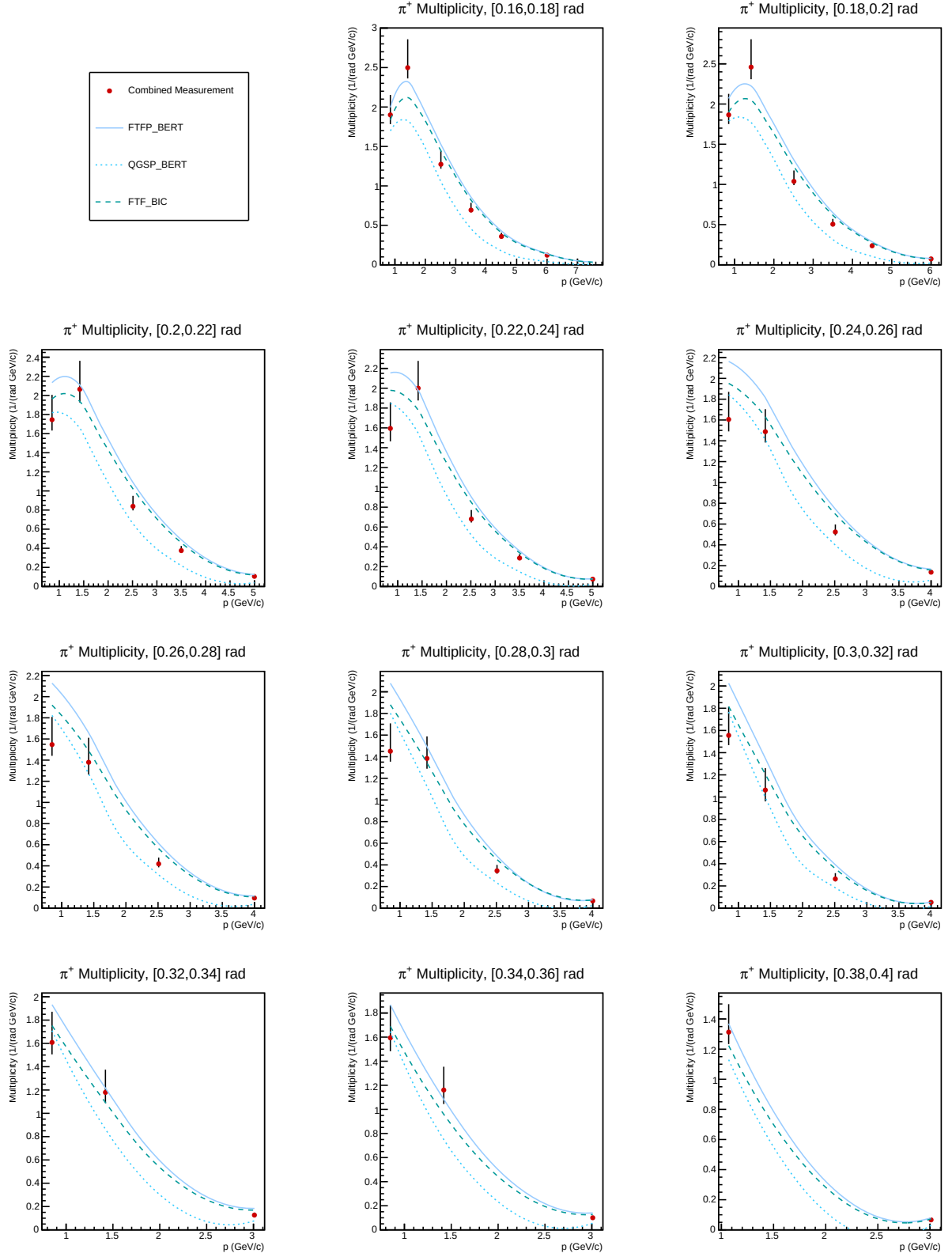
¹ It should be noted that the mean of the distribution will depend on the interpretation of the uncertainties and the underlying distribution assumed. The asymmetric Gaussian is a simple model. See [53] for more details.

The production cross-section uncertainty is uniformly applied to both the 2016 and 2017 measurements, and is therefore entirely correlated. This uncertainty was applied after the weighted measurement calculation. The final systematic uncertainty is the combination of the production cross-section uncertainty, model uncertainty, and feed-down uncertainty added in quadrature.

8.1.3 Results

This section presents combined multiplicity plots with statistical uncertainties and all systematic uncertainties, shown in Figs. 8.1– 8.12.

Figure 8.1: π^+ multiplicity measurements from 0.0–0.16 rad.

Figure 8.2: π^+ multiplicity measurements from 0.16–0.4 rad.

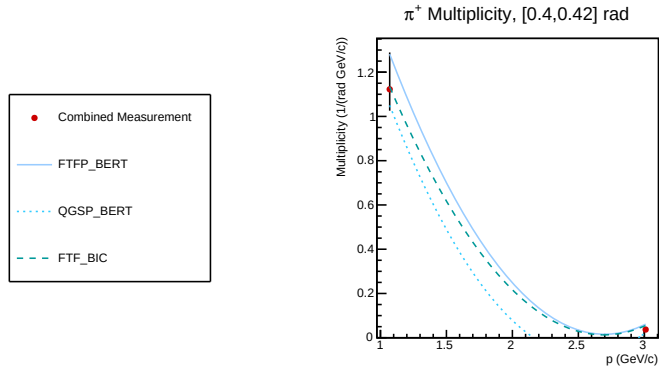
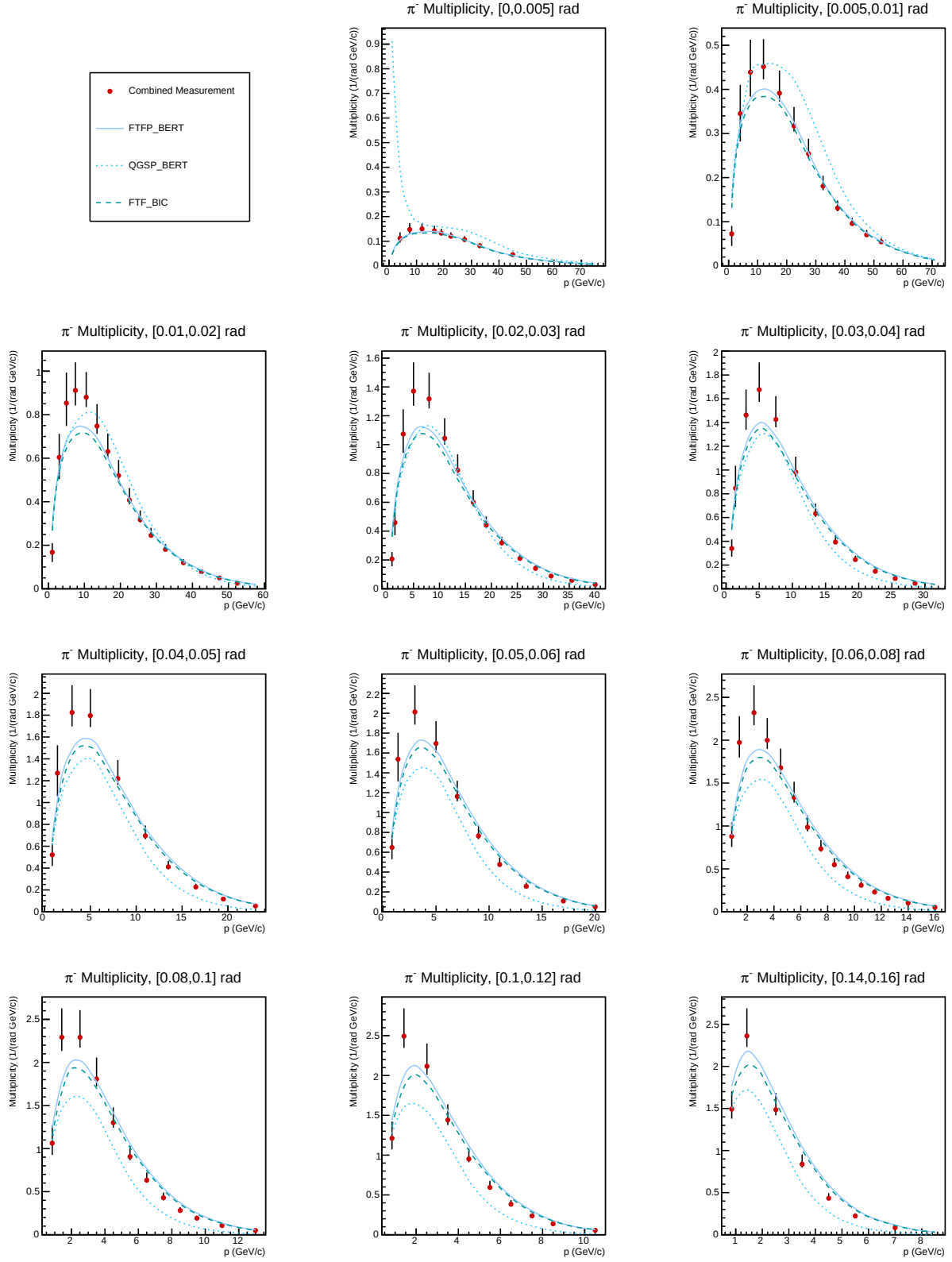
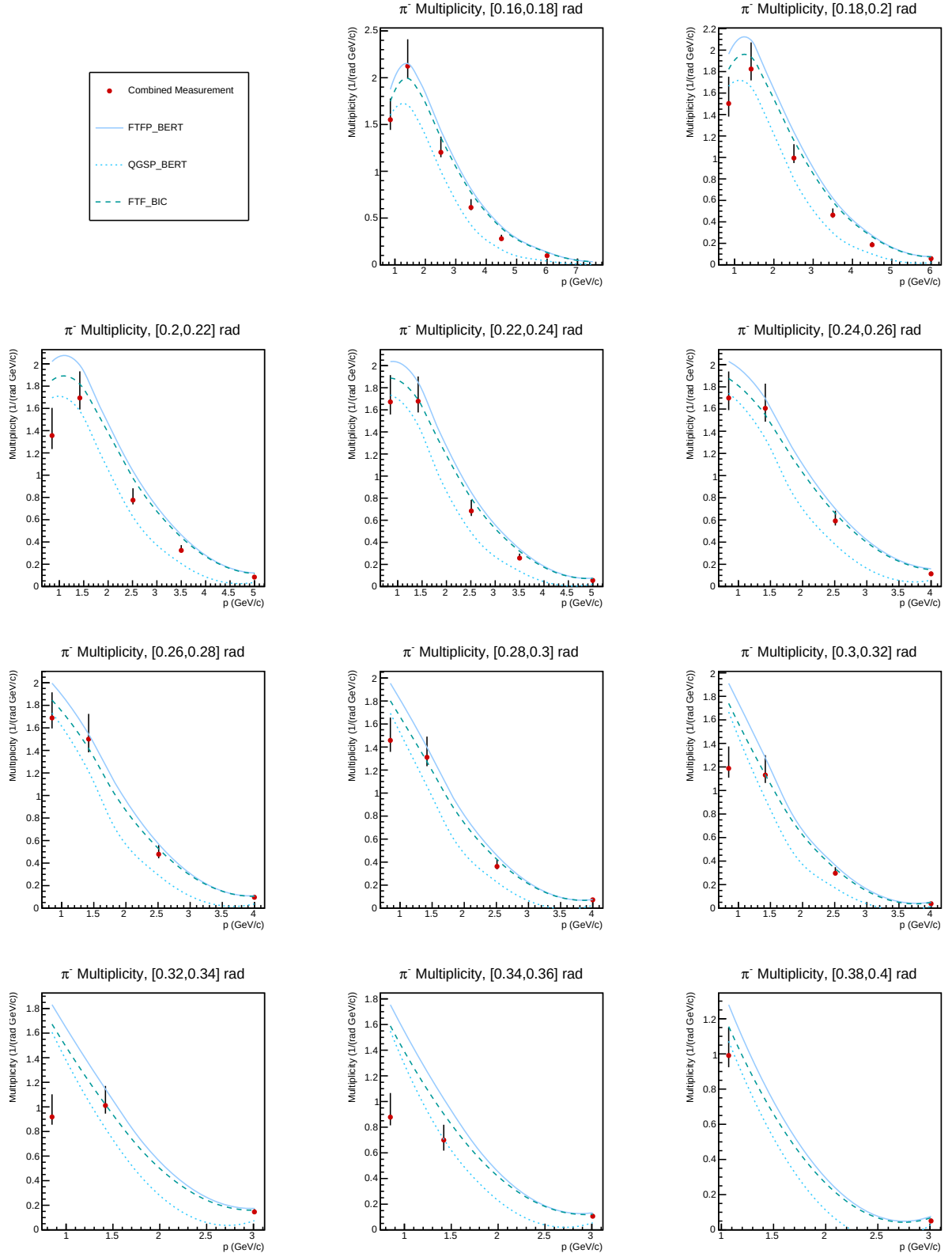


Figure 8.3: π^+ multiplicity measurements from 0.40–0.42 rad.

Figure 8.4: π^- multiplicity measurements from 0.0–0.16 rad.

Figure 8.5: π^- multiplicity measurements from 0.16–0.4 rad.

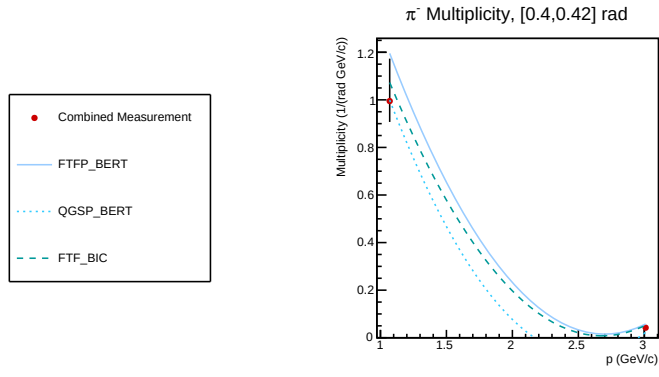
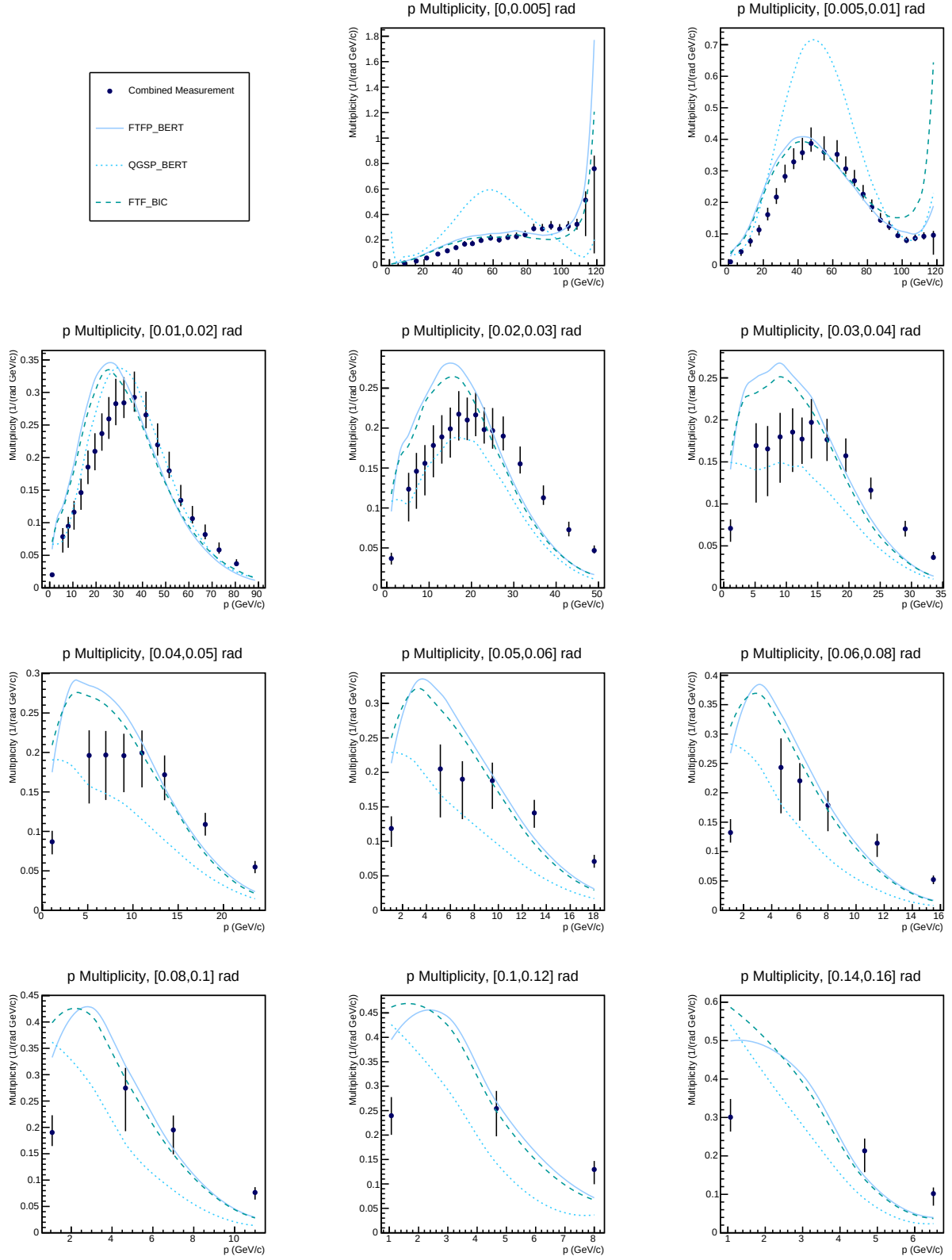
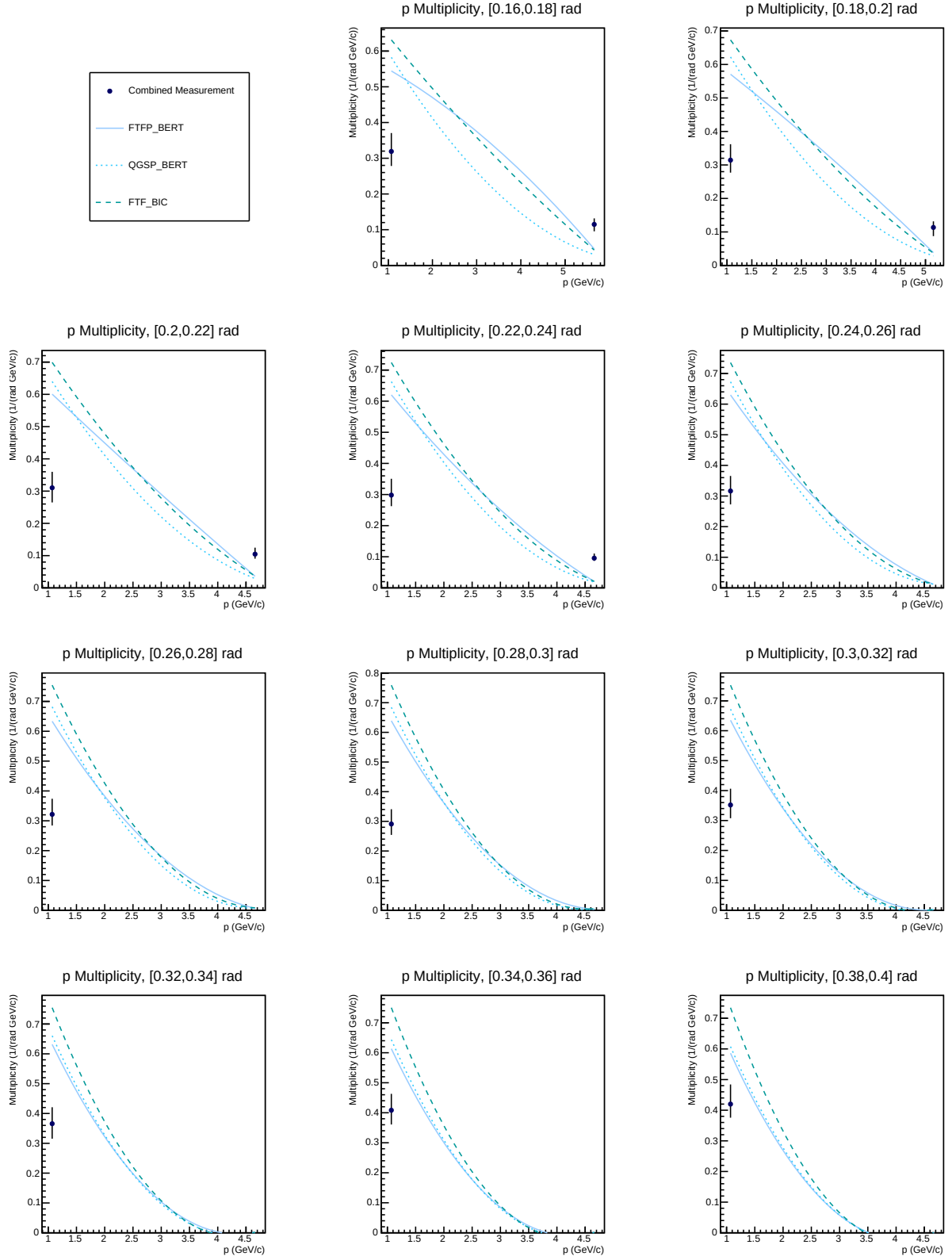


Figure 8.6: π^- multiplicity measurements from 0.40–0.42 rad.

Figure 8.7: p multiplicity measurements from 0.0–0.16 rad.

Figure 8.8: p multiplicity measurements from 0.16–0.4 rad.

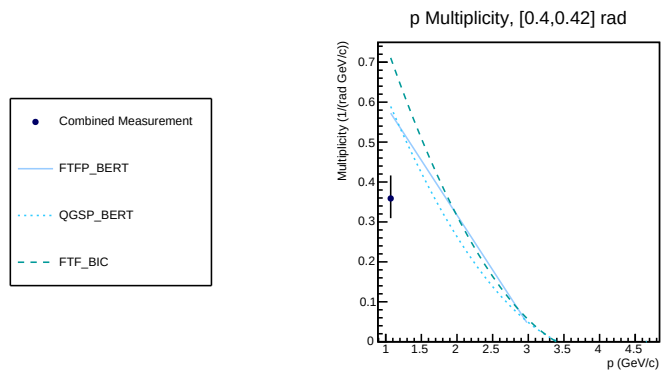
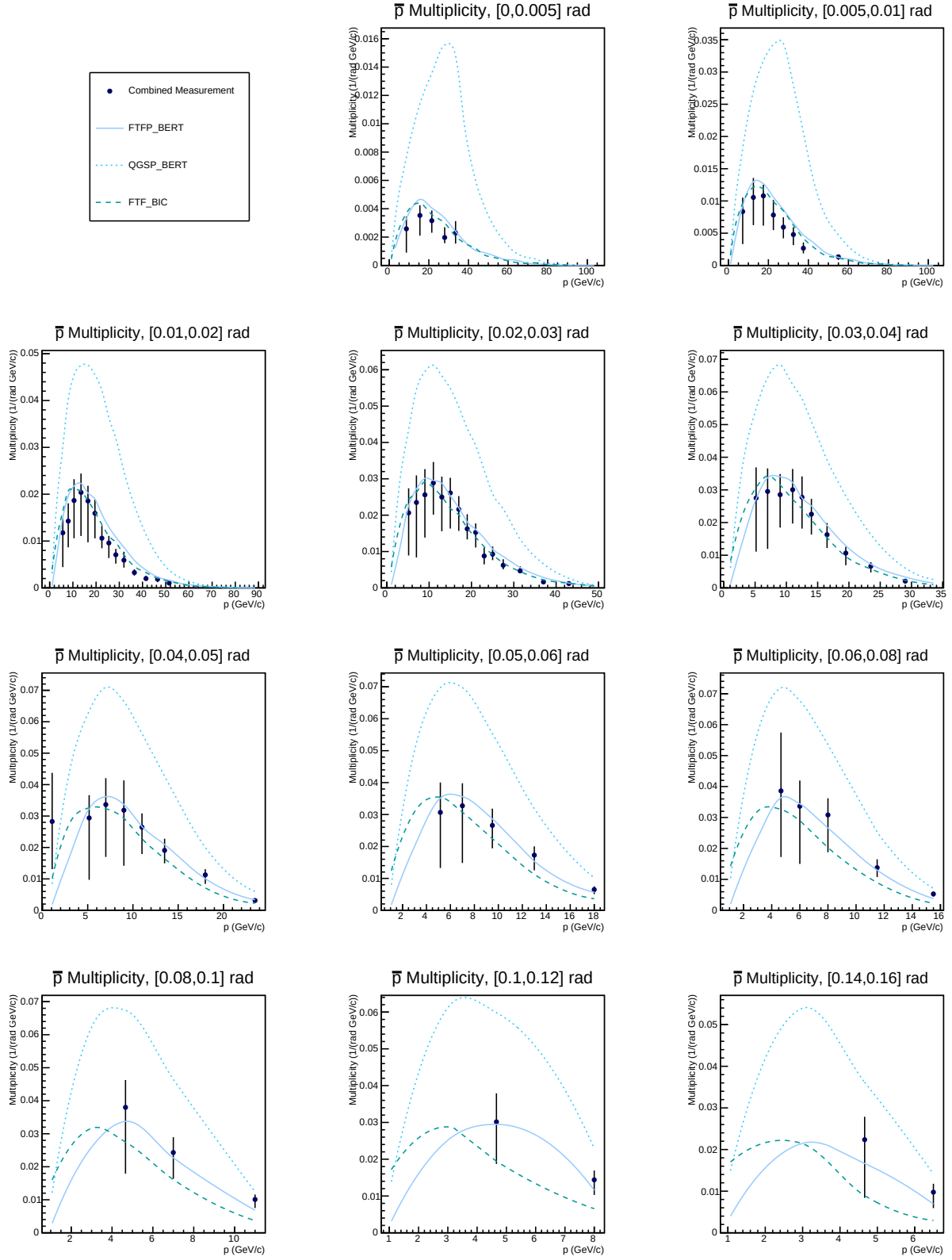
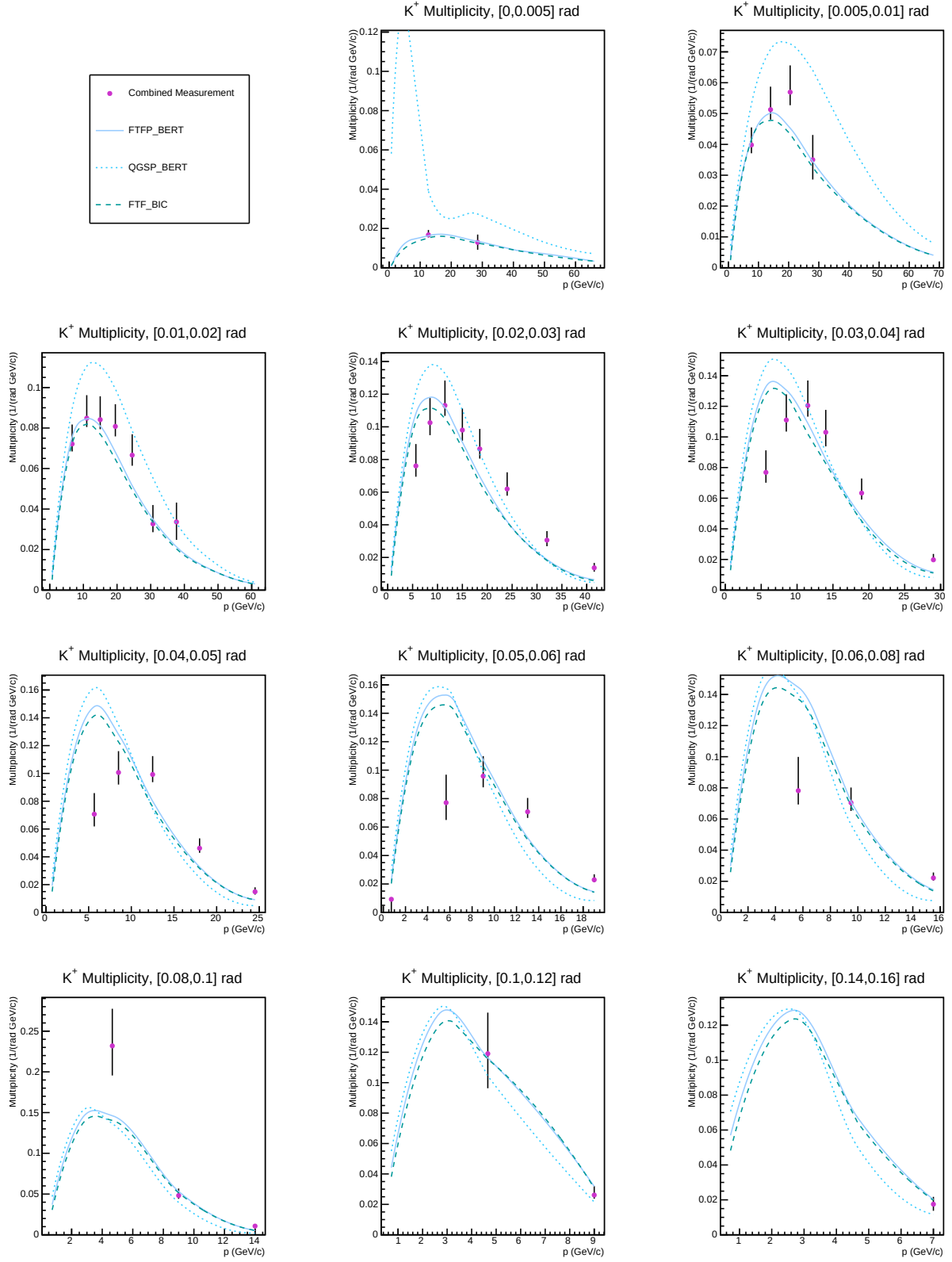
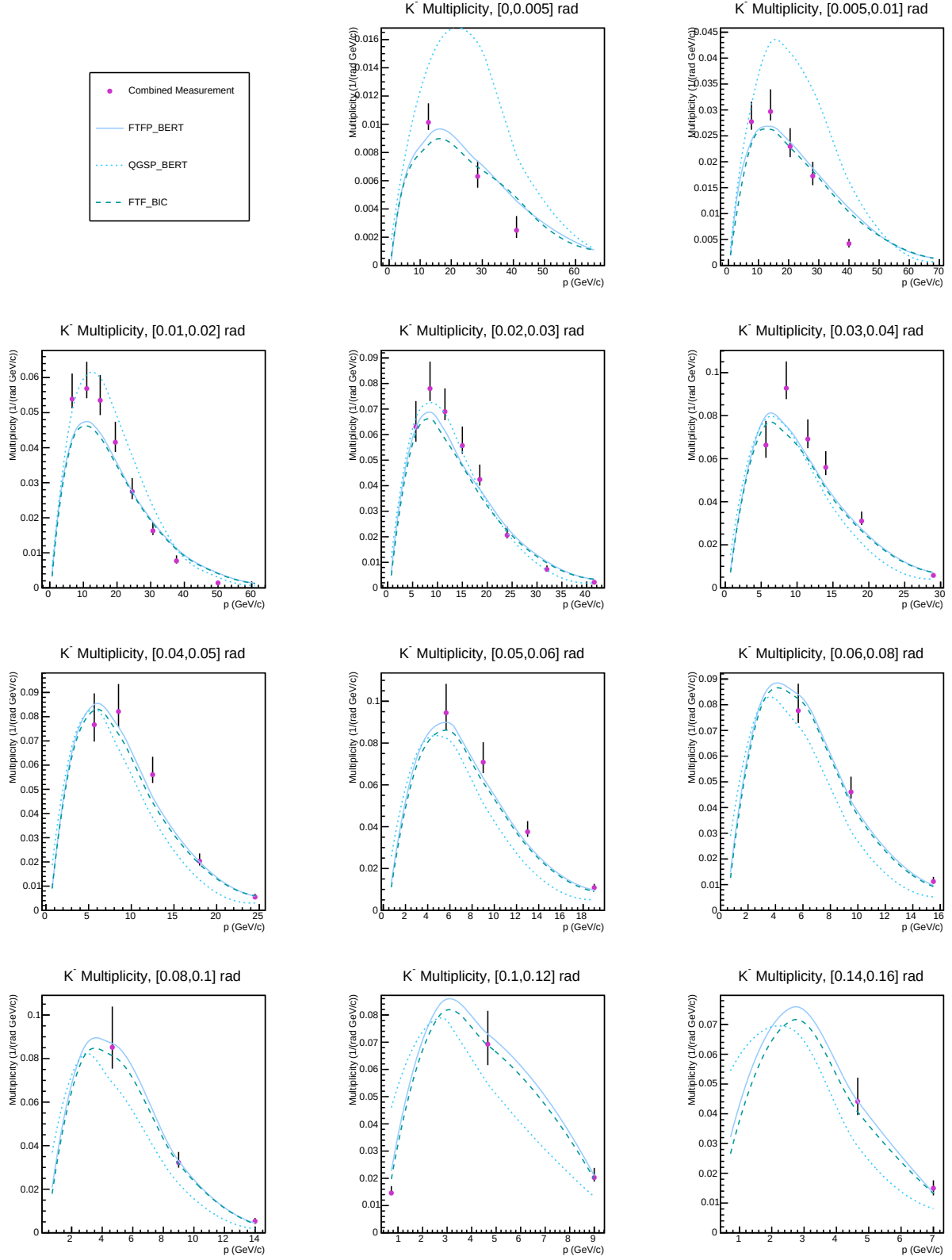


Figure 8.9: p multiplicity measurements from 0.40–0.42 rad.

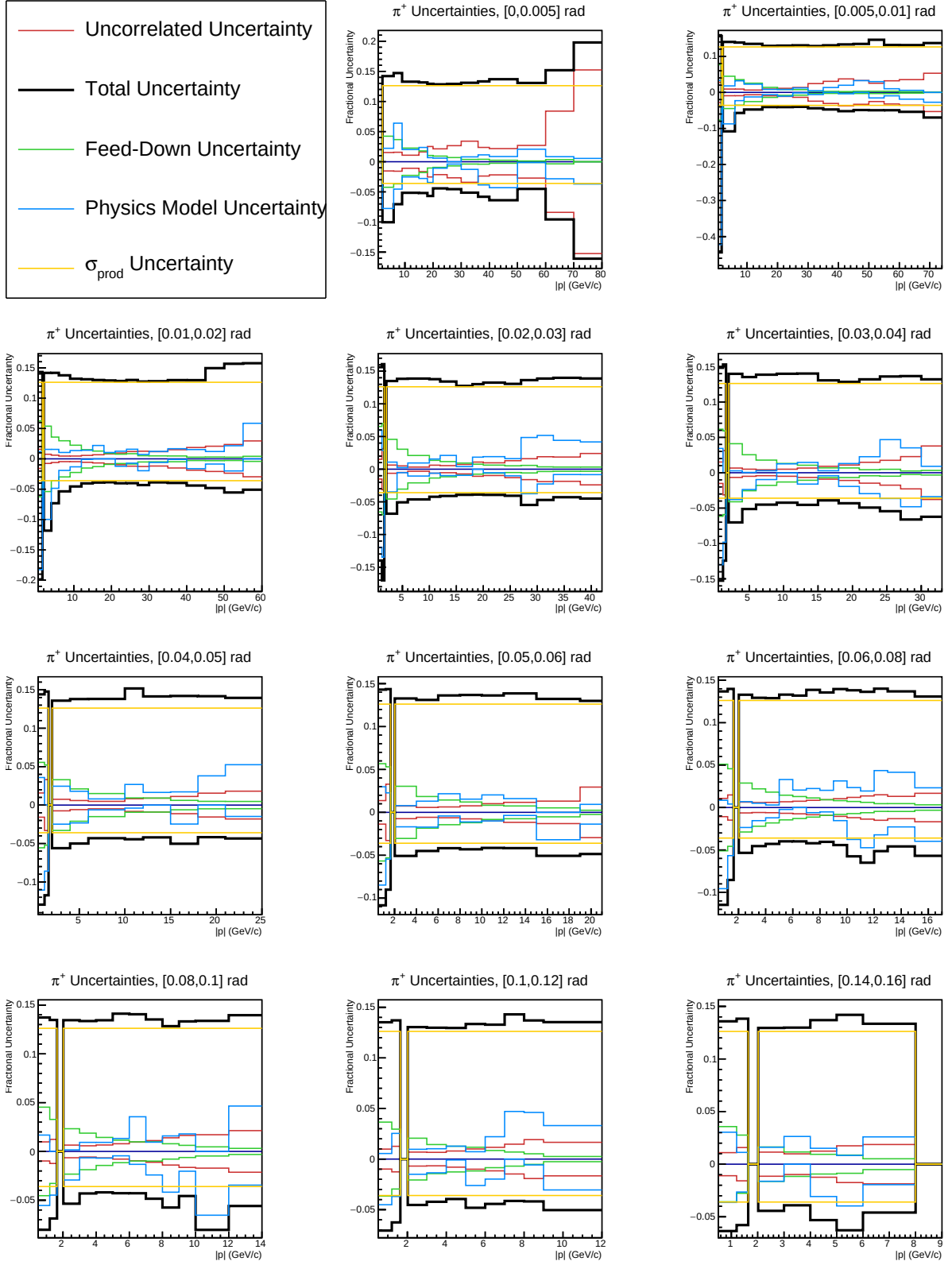
Figure 8.10: $\bar{p}$ multiplicity measurements from 0.0–0.16 rad.

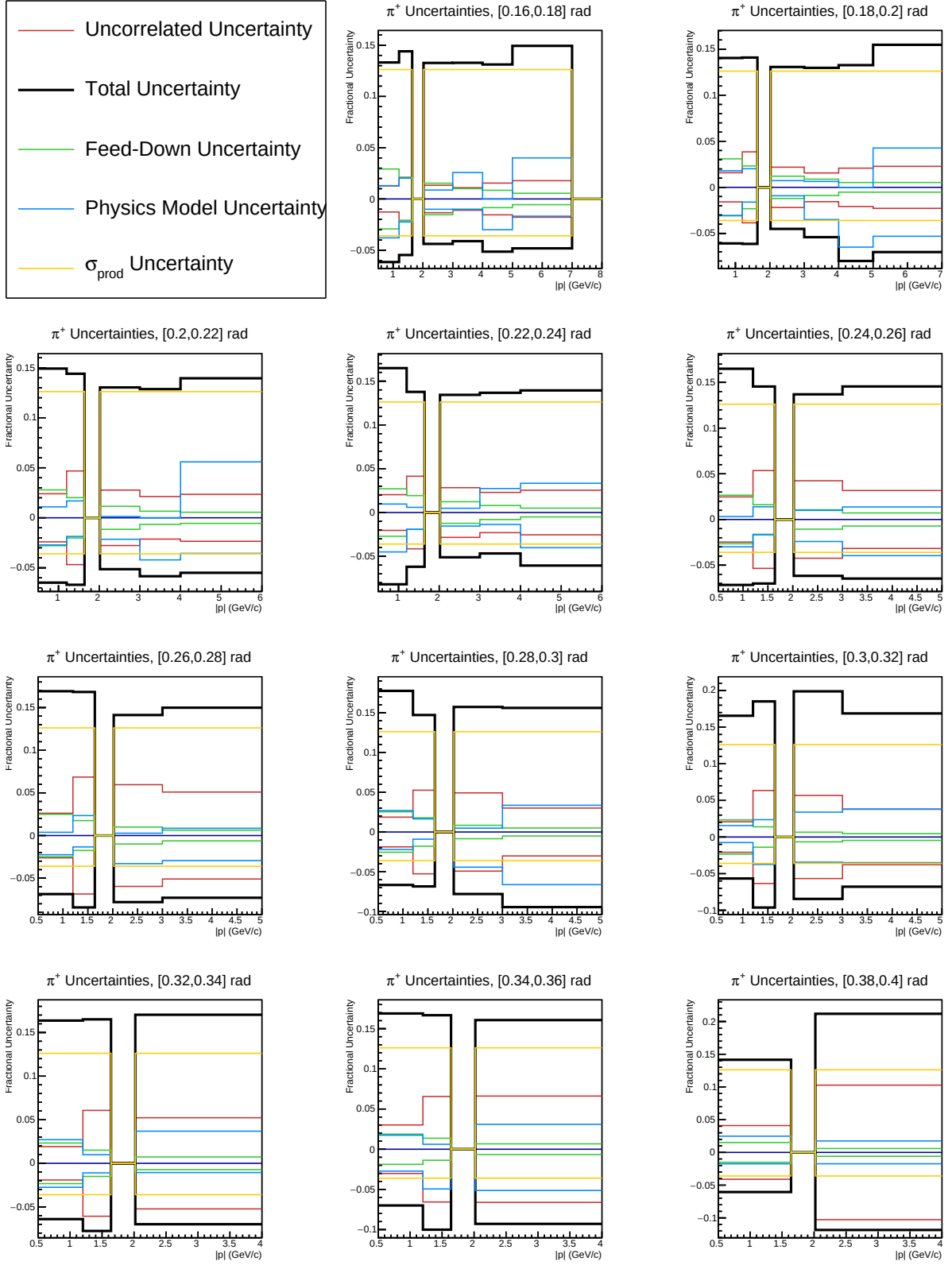
Figure 8.11: K^+ multiplicity measurements from 0.0–0.16 rad.

Figure 8.12: K^- multiplicity measurements from 0.0–0.16 rad.

8.1.4 Systematic Uncertainties

This section presents breakdowns of combined uncorrelated and correlated systematic uncertainties, shown together in Figs. 8.13 - 8.24.

Figure 8.13: π^+ uncertainties from 0.0–0.16 rad.

Figure 8.14: π^+ uncertainties from 0.16–0.4 rad.

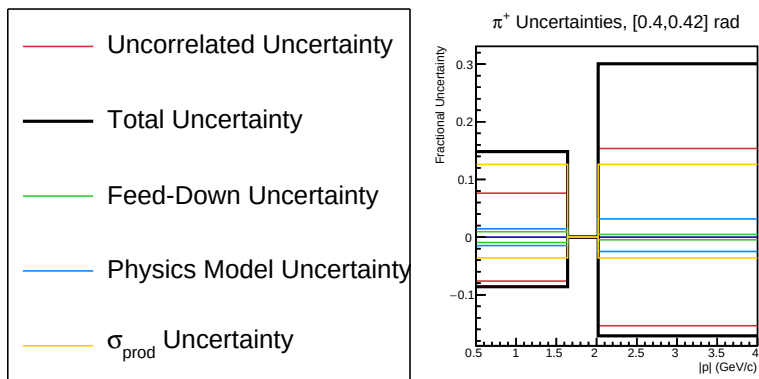
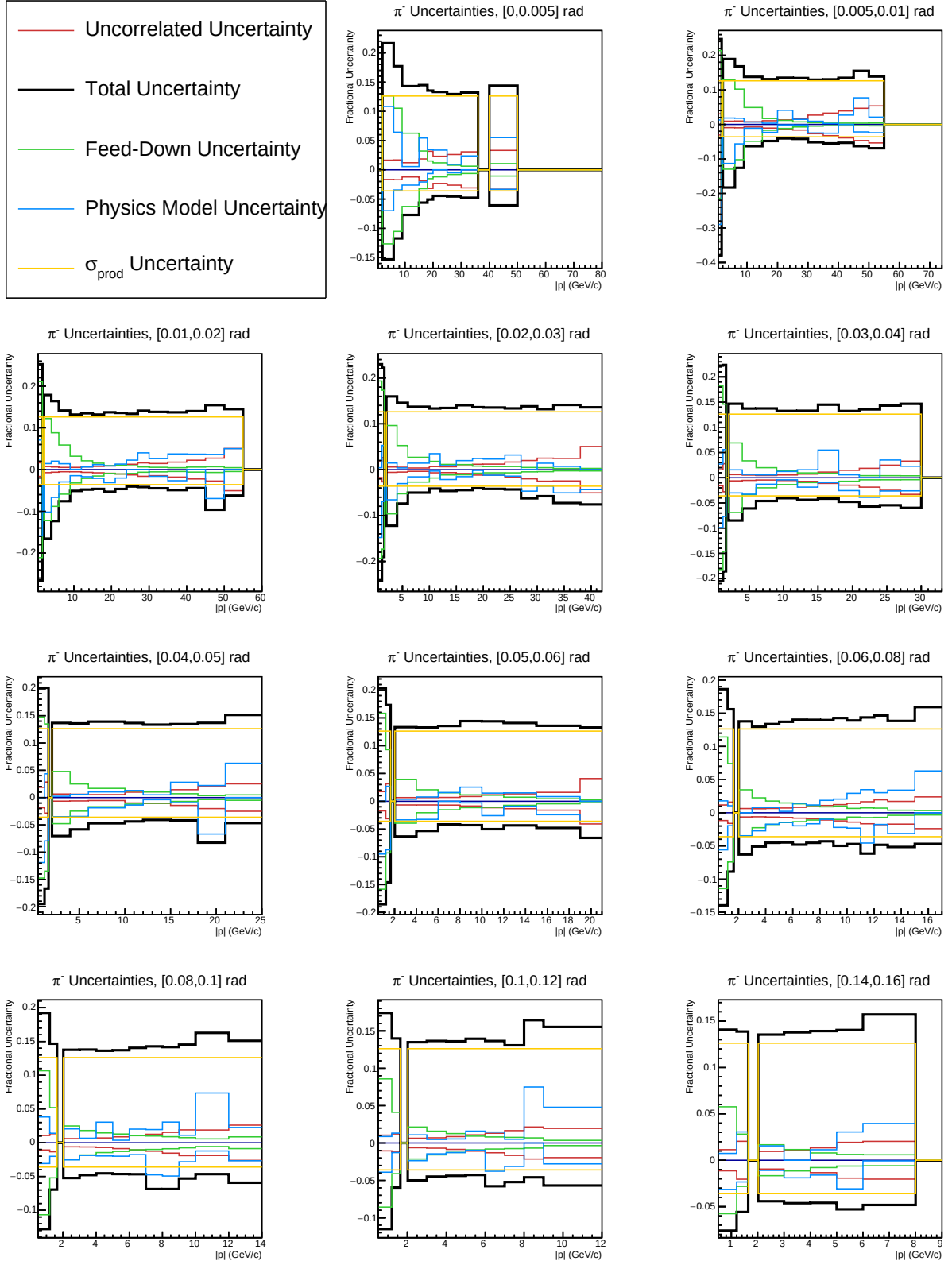
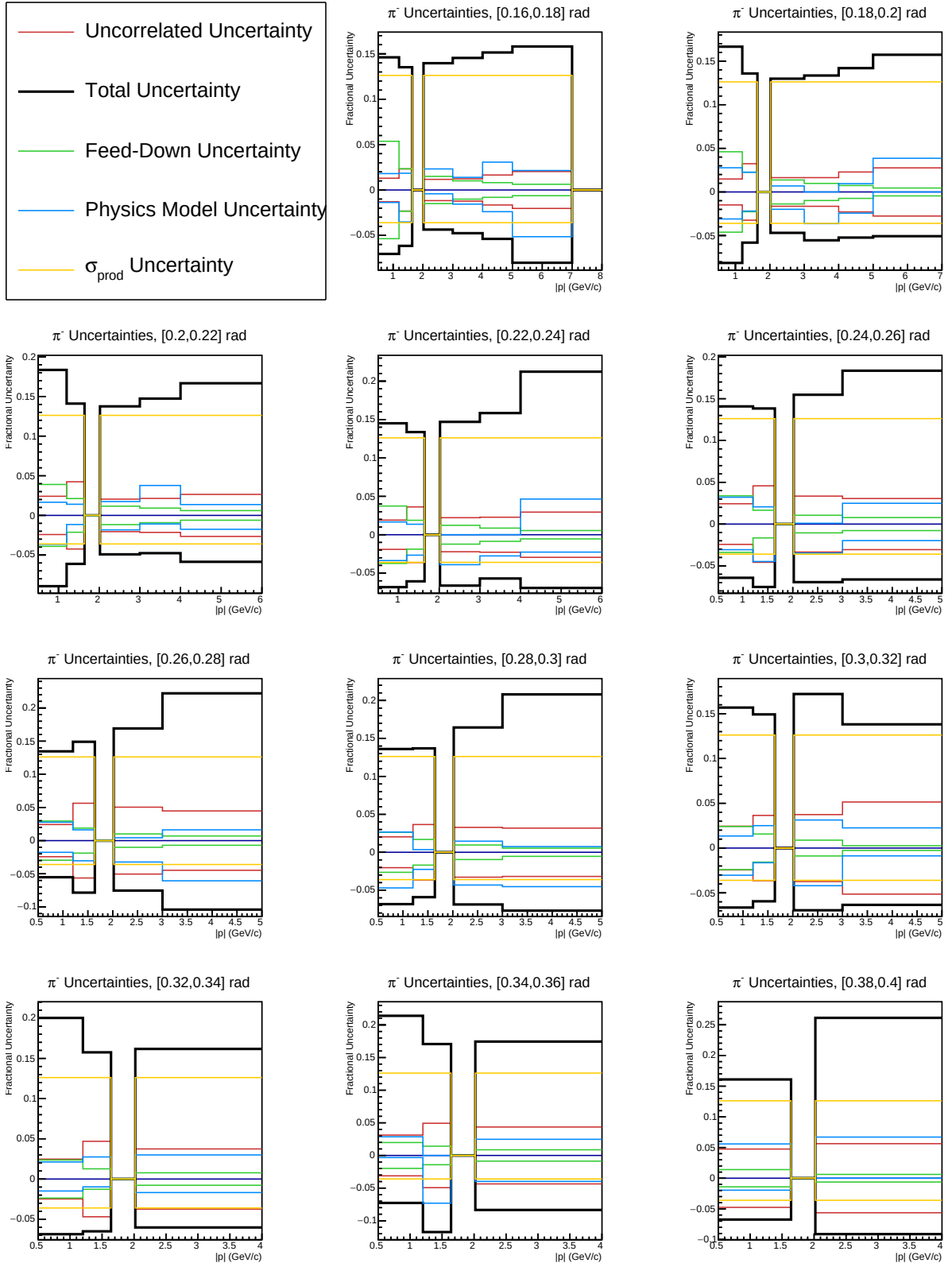


Figure 8.15: π^+ uncertainties from 0.40–0.42 rad.

Figure 8.16: π^- uncertainties from 0.0–0.16 rad.

Figure 8.17: π^- uncertainties from 0.16–0.4 rad.

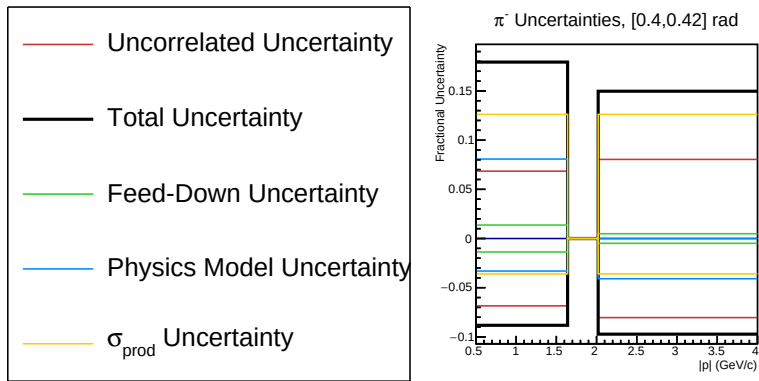
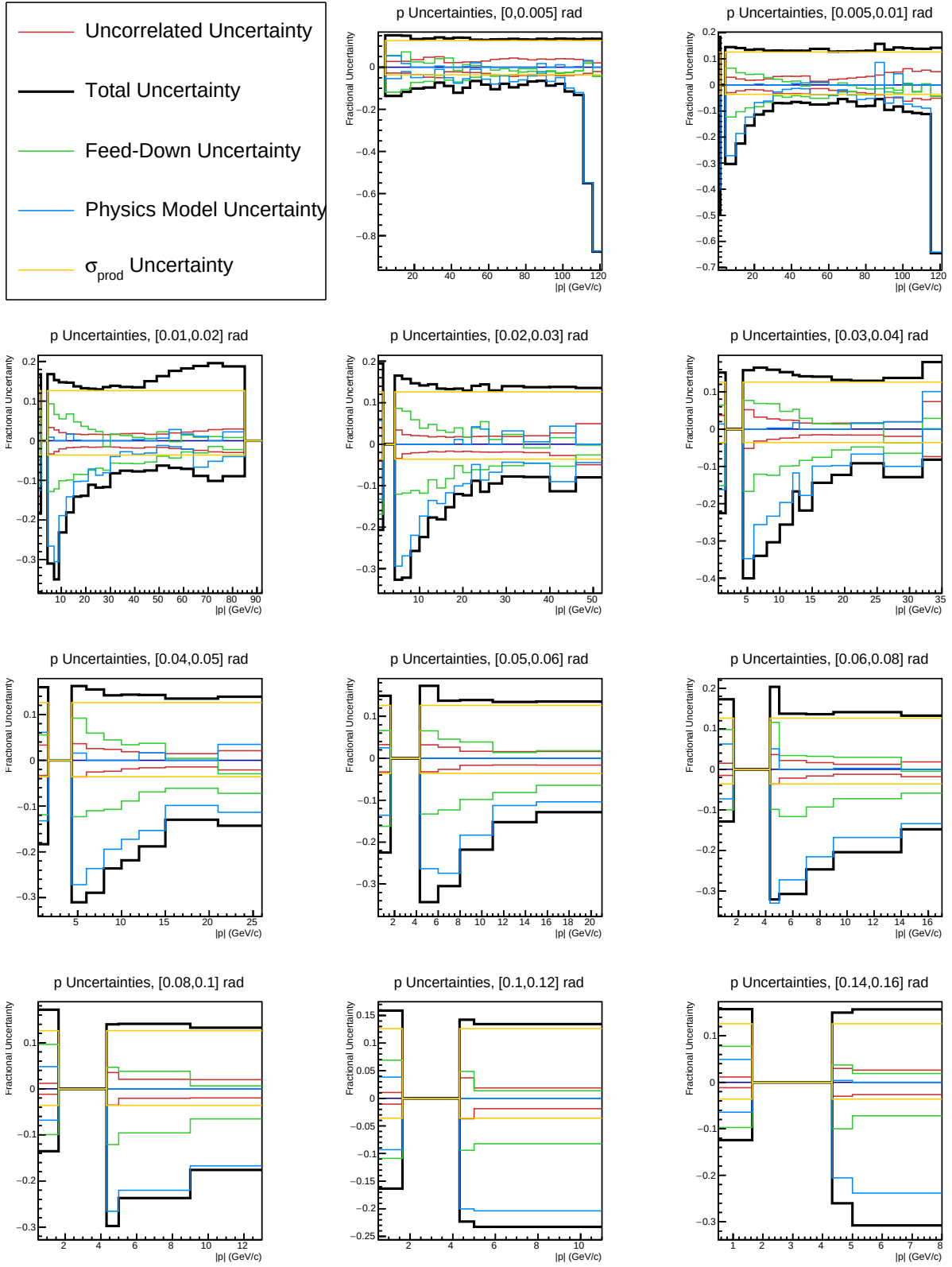
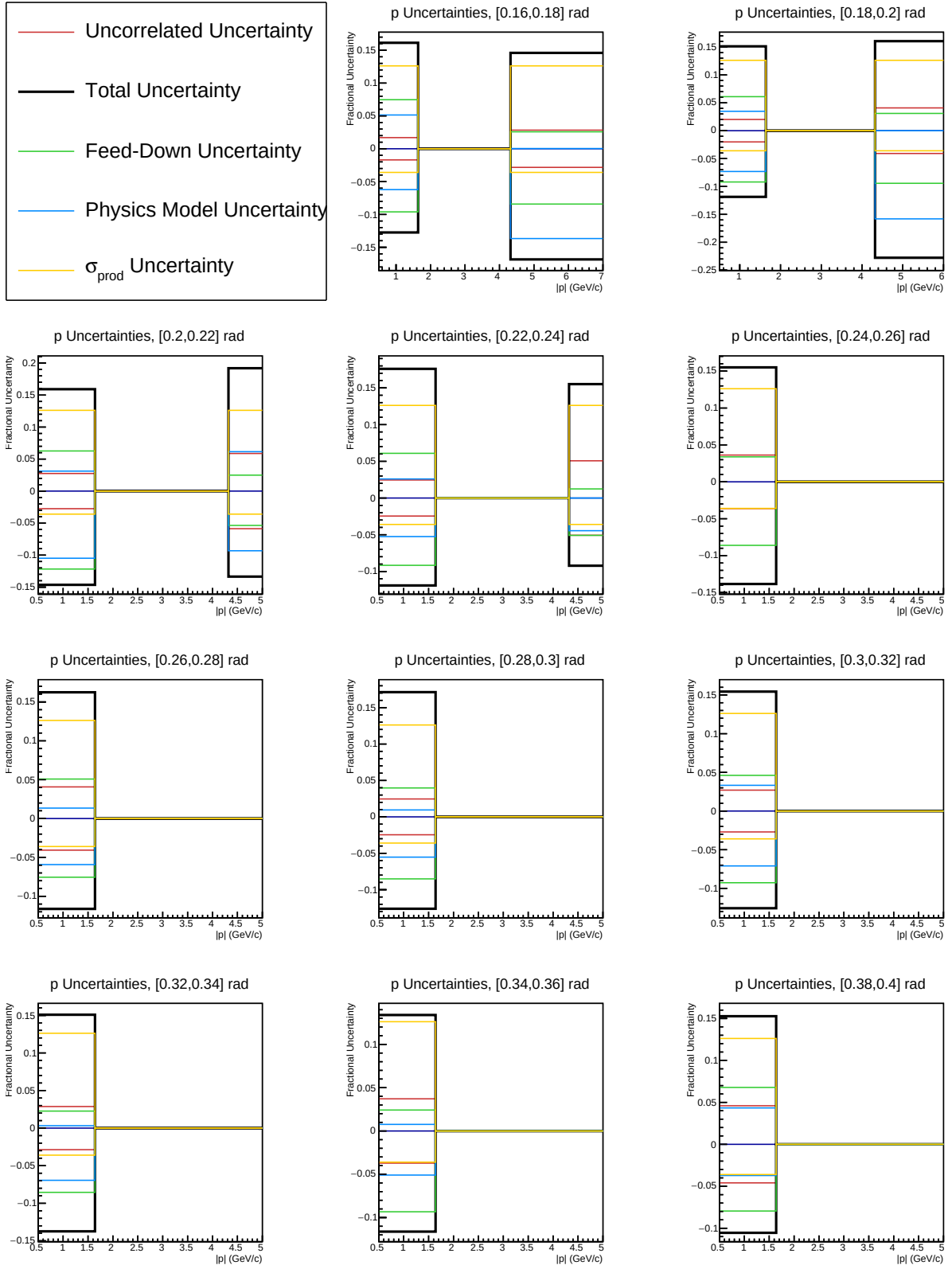


Figure 8.18: π^- uncertainties from 0.40–0.42 rad.

Figure 8.19: p uncertainties from 0.0–0.16 rad.

Figure 8.20: p uncertainties from 0.16–0.4 rad.

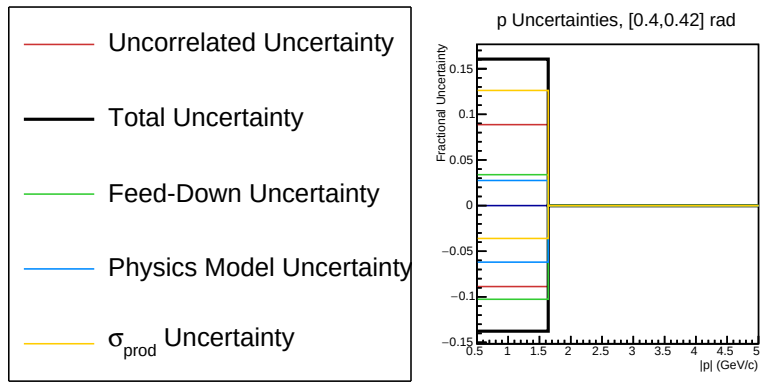
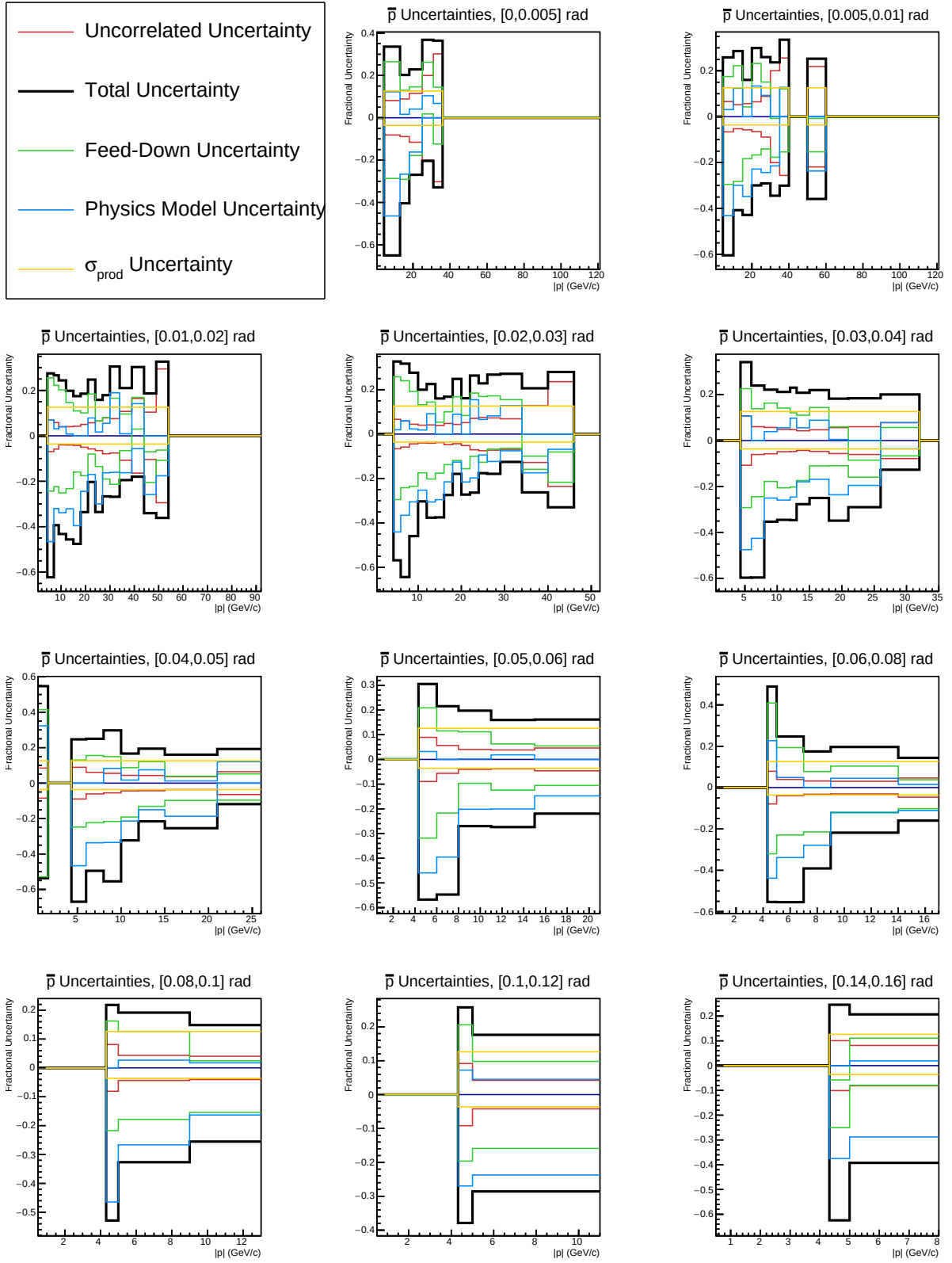
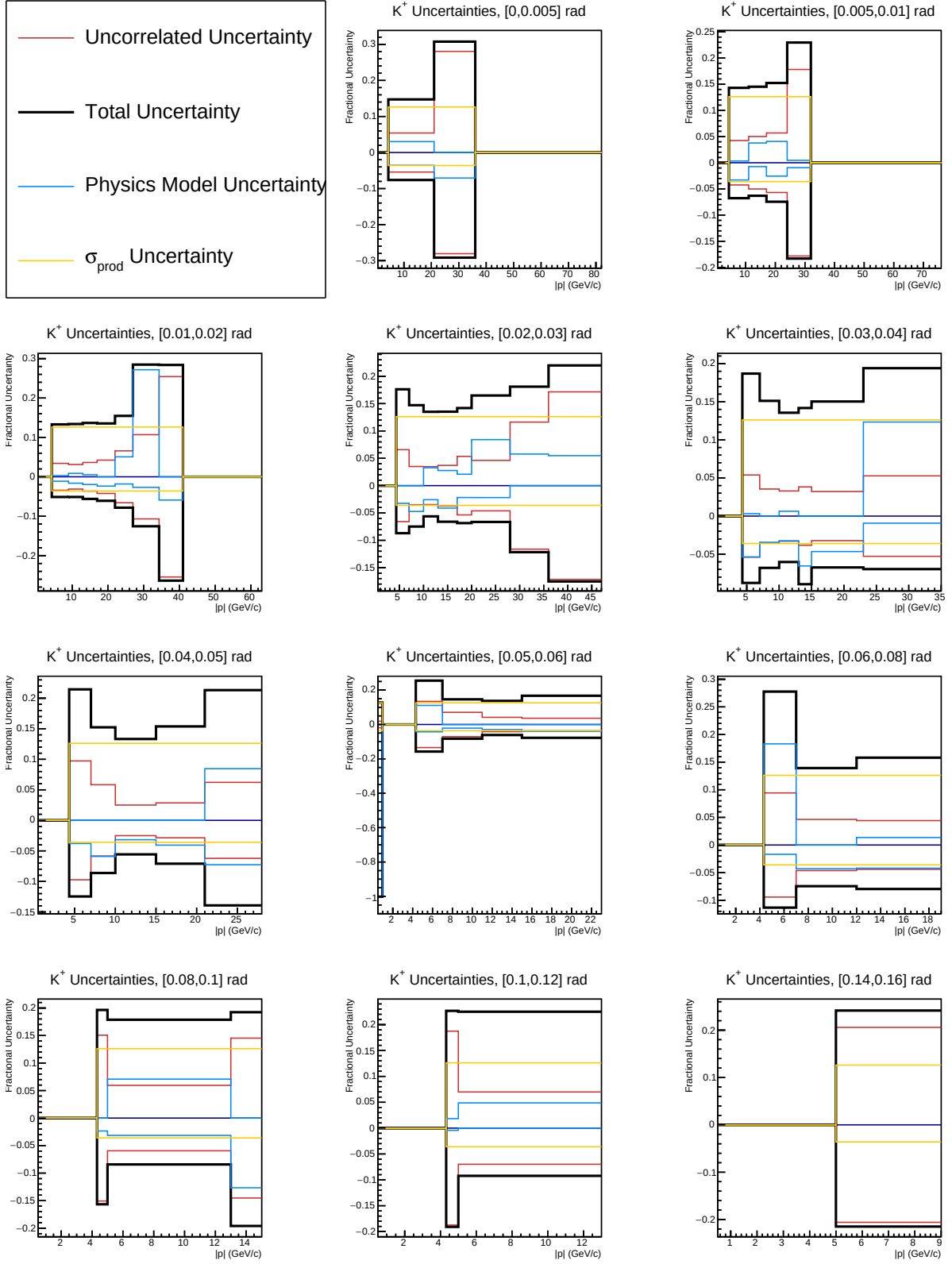
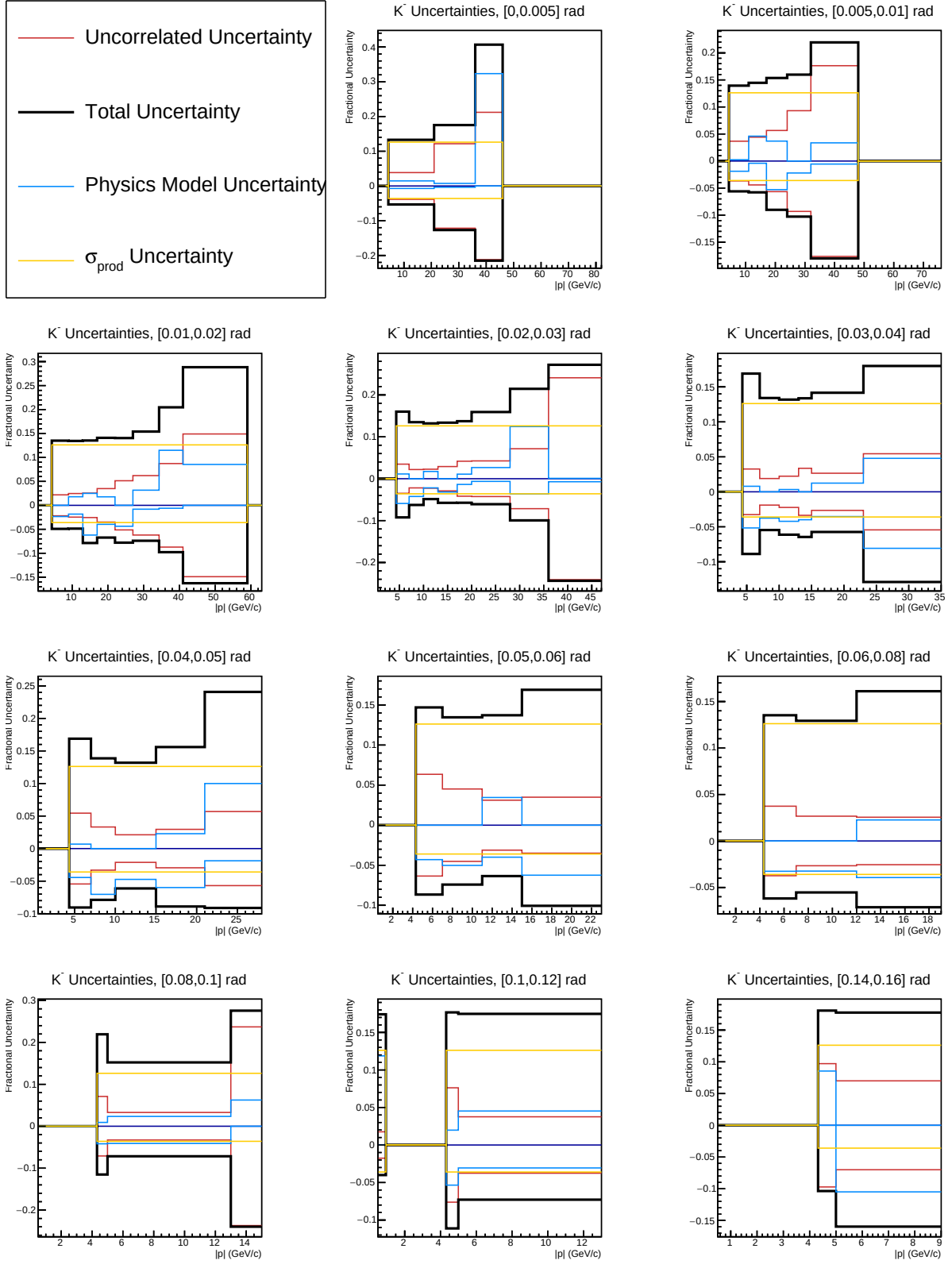


Figure 8.21: p uncertainties from 0.40–0.42 rad.

Figure 8.22: $\bar{p}$ uncertainties from 0.0–0.16 rad.

Figure 8.23: K^+ uncertainties from 0.0–0.16 rad.

Figure 8.24: K^- uncertainties from 0.0–0.16 rad.

Chapter 9

Conclusion

This thesis presents multiplicity spectra for $\pi^\pm$, $p/\bar{p}$, and $K^\pm$ produced in 120 GeV/c proton-carbon interactions. This reaction is likely to be selected as the primary interaction beginning the neutrino production process for the DUNE experiment, and is the primary interaction for the currently-operating NO ν A experiment. Previously, hadron production at this energy was estimated using extrapolation techniques from existing data at other energies. With the publication of these multiplicities, the expected hadron production at DUNE – and the resulting neutrino beam content – may be estimated more accurately.

This thesis also presents the first analysis performed with the new Forward Time Projection Chambers at NA61/SHINE, allowing for hadron production measurement up to the beam momentum in the low-angle region. The impact of these additional chambers can be seen in the first two angular bins of the proton multiplicity spectra.

The measured particle multiplicities are compared to three GEANT4 implementations of physics models: FTFP_BERT, QGSP_BERT, and FTF_BIC. Significant discrepancies are observed between the multiplicity predictions and measurements across phase space. The measurements presented here may be used to tune the Monte-Carlo models and lead to their improvement [54] [55].

Comparisons between three physics model predictions and the measured multiplicities are shown in Figs. 9.1– 9.3. In general, for pion production FTFP_BERT agrees best with measurements, but underestimates π^+ production by 20–40% above 30 GeV/c and below 0.05 rad. For protons, antiprotons, and $K^\pm$ discrepancies over 50% are present in all three models.

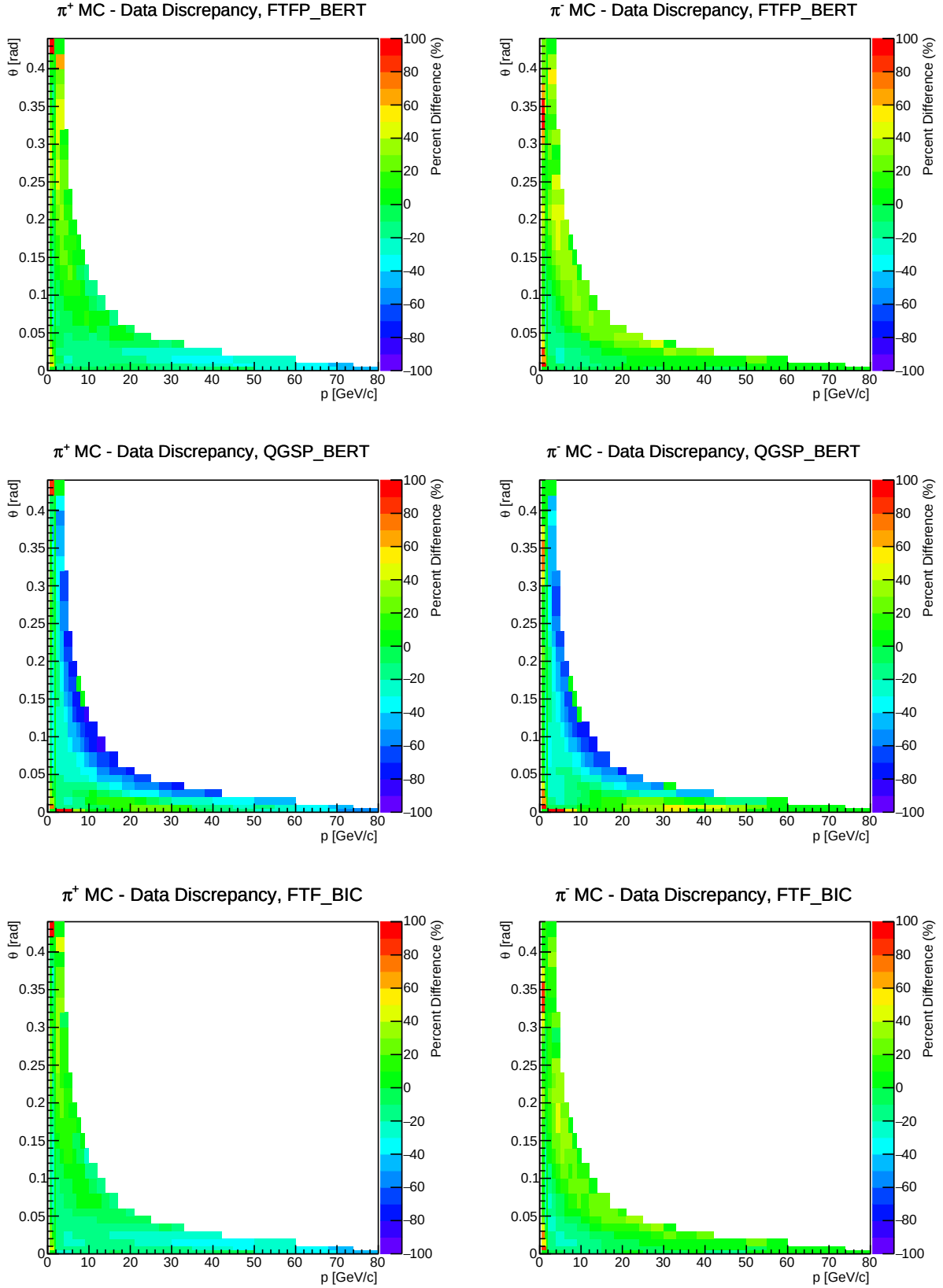


Figure 9.1: π^+ (left column) and π^- (right column) multiplicity measurement comparisons to three Monte Carlo models. Values plotted are $(m_{MC} - m_{Data})/m_{Data}$, where m is multiplicity.

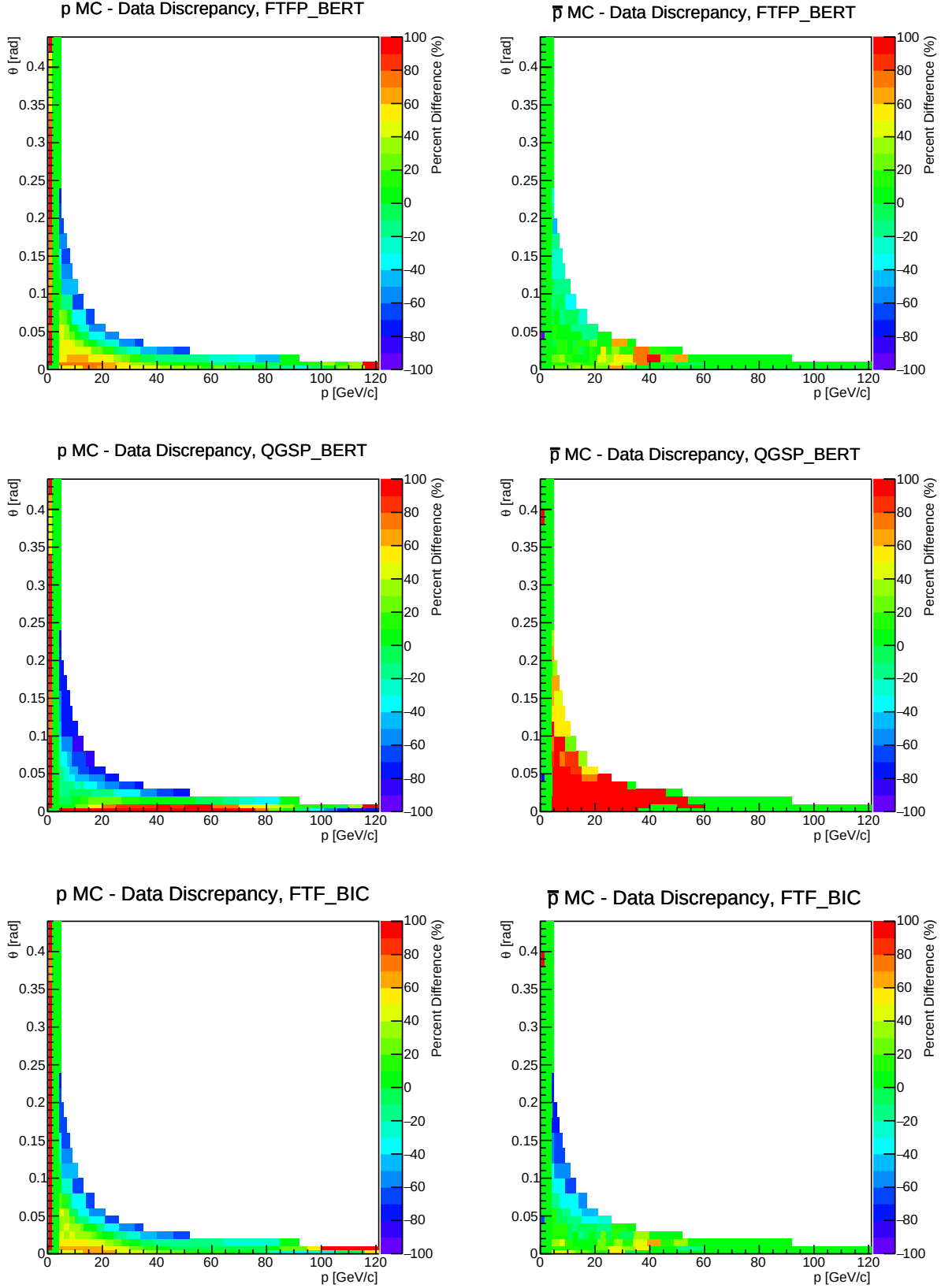


Figure 9.2: p (left column) and $\bar{p}$ (right column) multiplicity measurement comparisons to three Monte Carlo models. Values plotted are $(m_{MC} - m_{Data})/m_{Data}$, where m is multiplicity.

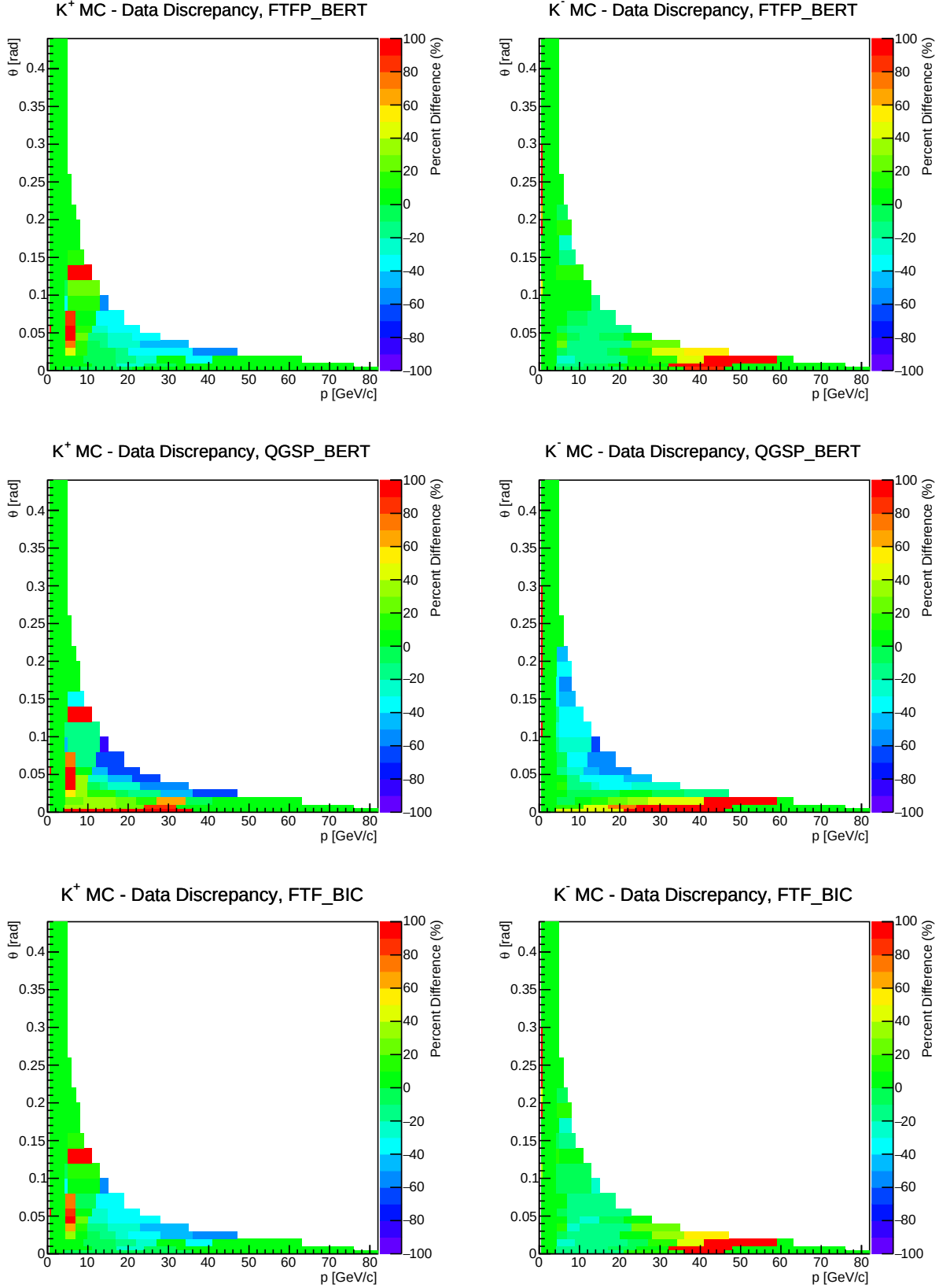


Figure 9.3: K^+ (left column) and K^- (right column) multiplicity measurement comparisons to three Monte Carlo models. Values plotted are $(m_{MC} - m_{Data})/m_{Data}$, where m is multiplicity.

Bibliography

- [1] S. M. Bilenky, “Neutrino. History of a unique particle,” **Eur. Phys. J. H**, vol. 38, pp. 345–404, 2013. DOI: 10.1140/epjh/e2012-20068-9. arXiv: 1210.3065 [hep-ph].
- [2] M. Kleesiek **et al.**, “ β -Decay Spectrum, Response Function and Statistical Model for Neutrino Mass Measurements with the KATRIN Experiment,” **Eur. Phys. J. C**, vol. 79, no. 3, p. 204, 2019. DOI: 10.1140/epjc/s10052-019-6686-7. arXiv: 1806.00369 [physics.data-an].
- [3] P. Zyla **et al.** [Particle Data Group], “Review of Particle Physics. Particle Data Group,” **Prog. Theor. Exp. Phys.**, 2020. [Online]. Available: <https://doi.org/10.1093/ptep/ptaa104>.
- [4] P. M. Watkins, “Discovery of the W and Z Bosons,” **Contemp. Phys.**, vol. 27, pp. 291–324, 1986. DOI: 10.1080/00107518608211015.
- [5] S. Abachi **et al.** [D0 Collaboration], “Observation of the top quark,” **Phys. Rev. Lett.**, vol. 74, pp. 2632–2637, 1995. DOI: 10.1103/PhysRevLett.74.2632. arXiv: hep-ex/9503003.
- [6] S. Chatrchyan **et al.** [CMS Collaboration], “Observation of a New Boson at a Mass of 125 GeV with the CMS Experiment at the LHC,” **Phys. Lett. B**, vol. 716, pp. 30–61, 2012. DOI: 10.1016/j.physletb.2012.08.021. arXiv: 1207.7235 [hep-ex].
- [7] Wikipedia contributors, **Standard model — Wikipedia, the free encyclopedia**, [Online; accessed 16-December-2020], 2020. [Online]. Available: https://en.wikipedia.org/w/index.php?title=Standard_Model&oldid=990507535.

- [8] S. F. King, “Neutrino mass models,” **Reports on Progress in Physics**, vol. 67, no. 2, pp. 107–157, Dec. 2003, ISSN: 1361-6633. DOI: 10.1088/0034-4885/67/2/r01. [Online]. Available: <http://dx.doi.org/10.1088/0034-4885/67/2/R01>.
- [9] F. Vissani, “Neutrino Sources and Properties,” **PoS**, vol. GSSI14, p. 001, 2015. DOI: 10.22323/1.229.0001. arXiv: 1412.8386 [hep-ph].
- [10] R. Wendell [Super-Kamiokande Collaboration], “Atmospheric Results from Super-Kamiokande,” **AIP Conf. Proc.**, vol. 1666, no. 1, E. Kearns, Ed., p. 100 001, 2015. DOI: 10.1063/1.4915569. arXiv: 1412.5234 [hep-ex].
- [11] K. Abe **et al.** [T2K Collaboration], “Measurement of Neutrino Oscillation Parameters from Muon Neutrino Disappearance with an Off-axis Beam,” **Phys. Rev. Lett.**, vol. 111, no. 21, p. 211 803, 2013. DOI: 10.1103/PhysRevLett.111.211803. arXiv: 1308.0465 [hep-ex].
- [12] D. Adey **et al.** [Daya Bay Collaboration], “Measurement of the Electron Antineutrino Oscillation with 1958 Days of Operation at Daya Bay,” **Phys. Rev. Lett.**, vol. 121, no. 24, p. 241 805, 2018. DOI: 10.1103/PhysRevLett.121.241805. arXiv: 1809.02261 [hep-ex].
- [13] J. W. F. Valle, “Neutrino physics overview,” **Journal of Physics: Conference Series**, vol. 53, pp. 473–505, Nov. 2006, ISSN: 1742-6596. DOI: 10.1088/1742-6596/53/1/031. [Online]. Available: <http://dx.doi.org/10.1088/1742-6596/53/1/031>.
- [14] I. Esteban, M. Gonzalez-Garcia, M. Maltoni, T. Schwetz, and A. Zhou, “The fate of hints: Updated global analysis of three-flavor neutrino oscillations,” **Journal of High Energy Physics**, vol. 2020, no. 9, Sep. 2020, ISSN: 1029-8479. DOI: 10.1007/jhep09(2020)178. [Online]. Available: [http://dx.doi.org/10.1007/JHEP09\(2020\)178](http://dx.doi.org/10.1007/JHEP09(2020)178).
- [15] W. C. Haxton, “The solar neutrino problem,” **Annual Review of Astronomy and Astrophysics**, vol. 33, no. 1, pp. 459–503, Sep. 1995, ISSN: 1545-4282. DOI: 10.1146/annurev.aa.33.090195.002331. [Online]. Available: <http://dx.doi.org/10.1146/annurev.aa.33.090195.002331>.

- [16] A. D. Sakharov, “Violation of CP invariance, c asymmetry, and baryon asymmetry of the universe,” **Soviet Physics Uspekhi**, vol. 34, no. 5, pp. 392–393, May 1991. DOI: 10.1070/pu1991v034n05abeh002497. [Online]. Available: <https://doi.org/10.1070/pu1991v034n05abeh002497>.
- [17] V. Papadimitriou [DUNE Collaboration], “Design of the LBNF Beamline,” in **38th International Conference on High Energy Physics**, 2017.
- [18] R. Acciarri **et al.** [DUNE Collaboration], “Long-Baseline Neutrino Facility (LBNF) and Deep Underground Neutrino Experiment (DUNE): Conceptual Design Report, Volume 2: The Physics Program for DUNE at LBNF,” Dec. 2015. arXiv: 1512.06148 [physics.ins-det].
- [19] R. Acciarri **et al.** [DUNE Collaboration], “Long-Baseline Neutrino Facility (LBNF) and Deep Underground Neutrino Experiment (DUNE): Conceptual Design Report, Volume 1: The LBNF and DUNE Projects,” Jan. 2016. arXiv: 1601.05471 [physics.ins-det].
- [20] A. Y. Smirnov, **The mikheyev-smirnov-wolfenstein (MSW) effect**, 2019. arXiv: 1901.11473 [hep-ph].
- [21] W. J. Marciano and Z. Parsa, “Intense neutrino beams and leptonic cp violation,” **Nuclear Physics B - Proceedings Supplements**, vol. 221, pp. 166–172, 2011, The Proceedings of the 22nd International Conference on Neutrino Physics and Astrophysics, ISSN: 0920-5632. DOI: <https://doi.org/10.1016/j.nuclphysbps.2011.03.114>. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0920563211001824>.
- [22] A. Abed Abud **et al.** [DUNE Collaboration], “Deep Underground Neutrino Experiment (DUNE) Near Detector Conceptual Design Report,” Mar. 2021. arXiv: 2103.13910 [physics.ins-det].
- [23] K. T. McDonald, “An Off-axis neutrino beam,” Nov. 2001. arXiv: hep-ex/0111033.
- [24] B. Abi **et al.** [DUNE Collaboration], “Deep Underground Neutrino Experiment (DUNE), Far Detector Technical Design Report, Volume IV: Far Detector Single-phase Technology,”

- JINST**, vol. 15, no. 08, T08010, 2020. DOI: 10.1088/1748-0221/15/08/T08010. arXiv: 2002.03010 [physics.ins-det].
- [25] B. Abi **et al.** [DUNE Collaboration], “First results on ProtoDUNE-SP liquid argon time projection chamber performance from a beam test at the CERN Neutrino Platform,” **JINST**, vol. 15, no. 12, P12004, 2020. DOI: 10.1088/1748-0221/15/12/P12004. arXiv: 2007.06722 [physics.ins-det].
- [26] T. Böhlen, F. Cerutti, M. Chin, A. Fassò, A. Ferrari, P. Ortega, A. Mairani, P. Sala, G. Smirnov, and V. Vlachoudis, “The fluka code: Developments and challenges for high energy and medical applications,” **Nuclear Data Sheets**, vol. 120, pp. 211–214, 2014, ISSN: 0090-3752. DOI: <https://doi.org/10.1016/j.nds.2014.07.049>. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0090375214005018>.
- [27] S. Agostinelli **et al.** [GEANT4 Collaboration], “GEANT4—a simulation toolkit,” **Nucl. Instrum. Meth. A**, vol. 506, pp. 250–303, 2003. DOI: 10.1016/S0168-9002(03)01368-8.
- [28] L. Aliaga Soplin, “Neutrino Flux Prediction for the NuMI Beamline,” PhD thesis, William-Mary Coll., 2016. DOI: 10.2172/1250884.
- [29] K. Abe **et al.** [T2K Collaboration], “Measurement of neutrino and antineutrino oscillations by the T2K experiment including a new additional sample of ν_e interactions at the far detector,” **Phys. Rev. D**, vol. 96, no. 9, p. 092006, 2017, [Erratum: Phys.Rev.D 98, 019902 (2018)]. DOI: 10.1103/PhysRevD.96.092006. arXiv: 1707.01048 [hep-ex].
- [30] M. G. Catanesi, “Hadron production measurements at CERN,” **Journal of Physics: Conference Series**, vol. 60, pp. 66–71, Mar. 2007. DOI: 10.1088/1742-6596/60/1/011. [Online]. Available: <https://doi.org/10.1088/1742-6596/60/1/011>.
- [31] T. Eichten, D. Haidt, J. Pattison, W. Venus, H. Wachsmuth, O. Wörz, T. Jones, B. Aubert, L. Chounet, P. Heusse, and C. Franzinetti, “Particle production in proton interactions in nuclei at 24 gev/c,” **Nuclear Physics B**, vol. 44, no. 2, pp. 333–343, 1972, ISSN: 0550-3213.

- DOI: [https://doi.org/10.1016/0550-3213\(72\)90120-4](https://doi.org/10.1016/0550-3213(72)90120-4). [Online]. Available: <https://www.sciencedirect.com/science/article/pii/0550321372901204>.
- [32] J. M. Paley **et al.** [MIPP Collaboration], “Measurement of charged pion production yields off the numi target,” **Phys. Rev. D**, vol. 90, p. 032001, 3 Aug. 2014. DOI: 10.1103/PhysRevD.90.032001. [Online]. Available: <https://link.aps.org/doi/10.1103/PhysRevD.90.032001>.
- [33] E. Radicioni **et al.** [HARP Collaboration], “HARP: A Hadron production experiment,” in **37th Rencontres de Moriond on Electroweak Interactions and Unified Theories**, Jun. 2002. arXiv: hep-ex/0206028.
- [34] C. Alt, B. Baatar, D. Barna, G. Barr, J. Bartke, L. Betev, H. Biakowska, C. Blume, B. Boimska, and et al., “Inclusive production of charged pions in p+c collisions at 158gev/c beam momentum,” **The European Physical Journal C**, vol. 49, no. 4, pp. 897–917, Feb. 2007, ISSN: 1434-6052. DOI: 10.1140/epjc/s10052-006-0165-7. [Online]. Available: <http://dx.doi.org/10.1140/epjc/s10052-006-0165-7>.
- [35] B. Baatar **et al.** [NA49 Collaboration], “Inclusive production of protons, anti-protons, neutrons, deuterons and tritons in p+C collisions at 158 GeV/c beam momentum,” **Eur. Phys. J. C**, vol. 73, no. 4, p. 2364, 2013. DOI: 10.1140/epjc/s10052-013-2364-3. arXiv: 1207.6520 [hep-ex].
- [36] J. R. Sanford and C. L. Wang, “Empirical formulas for particle production in p-Be collisions between 10 and 35 GeV/c, brookhaven national laboratory, AGS internal report,” unpublished, 1967.
- [37] M. Bonesini, A. Marchionni, F. Pietropaolo, and T. Tabarelli de Fatis, “On particle production for high energy neutrino beams,” **The European Physical Journal C**, vol. 20, no. 1, pp. 13–27, Apr. 2001, ISSN: 1434-6052. DOI: 10.1007/s100520100656. [Online]. Available: <http://dx.doi.org/10.1007/s100520100656>.

- [38] N. Abgrall **et al.** [NA61/SHINE Collaboration], “NA61/SHINE facility at the CERN SPS: beams and detector system,” **JINST**, vol. 9, P06005, 2014. DOI: 10.1088/1748-0221/9/06/P06005. arXiv: 1401.4699 [physics.ins-det].
- [39] A. László, E. Dénes, Z. Fodor, T. Kiss, S. Kleinfelder, C. Soós, D. Tefelski, T. Tölyhi, G. Vesztergombi, and O. Wyszyski, “Design and Performance of the Data Acquisition System for the NA61/SHINE Experiment at CERN,” **Nucl. Instrum. Meth. A**, vol. 798, pp. 1–11, 2015. DOI: 10.1016/j.nima.2015.07.011. arXiv: 1505.01004 [physics.ins-det].
- [40] B. Rumberger, A. Aduszkiewicz, J. Boissevain, M. Kuich, A. László, Y. Nagai, L. Oláh, P. Podlaski, D. Varga, M. Wensveen, and et al., “The Forward TPC system of the NA61/SHINE experiment at CERN: a tandem TPC concept,” **JINST**, vol. 15, no. 07, P07013, 2020. DOI: 10.1088/1748-0221/15/07/P07013. arXiv: 2004.11358 [physics.ins-det].
- [41] I. F. Kolpakov, “CAMAC High-Energy Physics Electronics Hardware,” **Comput. Phys. Commun.**, vol. 15, pp. 341–345, 1978. DOI: 10.1016/0010-4655(78)90064-4.
- [42] V. Blobel, “BOS: Bank Organization System. Dynamic Storage Organization with FORTRAN,” 1977.
- [43] R. Sipos, A. Laszlo, A. Marcinek, T. Paul, M. Szuba, M. Unger, D. Veberic, and O. Wyszynski, “The offline software framework of the NA61/SHINE experiment,” **J. Phys. Conf. Ser.**, vol. 396, M. Ernst, D. Düllmann, O. Rind, and T. Wong, Eds., p. 022 045, 2012. DOI: 10.1088/1742-6596/396/2/022045.
- [44] A. Lebedev, C. Hohne, I. Kisel, G. Ososkov, and et al. [CBM Collaboration], “Track reconstruction algorithms for the CBM experiment at FAIR,” **J. Phys. Conf. Ser.**, vol. 219, J. Gruntorad and M. Lokajicek, Eds., p. 032 048, 2010. DOI: 10.1088/1742-6596/219/3/032048.
- [45] M. Gardner, “The Fantastic Combinations of John Conways new solitaire game life by Martin Gardner,” **Scientific American**, vol. 223, pp. 120–123, 1970.

- [46] J. Stiller [ALICE Collaboration], “Gain calibration of the ALICE TRD using the decay of ^{83m}Kr by Internal Conversion,” **Nucl. Instrum. Meth. A**, vol. 706, P. Fusco, C. Favuzzi, N. Giglietto, M. N. Mazziotta, A. Rainò, and P. Spinelli, Eds., pp. 20–22, 2013. DOI: 10.1016/j.nima.2012.05.019.
- [47] M. Naskret, “Krypton calibration of time projection chambers of the NA61/SHINE experiment,” Master’s thesis, Wroclaw U., 2015.
- [48] A. Aduszkiewicz **et al.** [NA61/SHINE Collaboration], “Measurements of production and inelastic cross sections for p+C , p+Be , and p+Al at 60 GeV/c and p+C and p+Be at 120 GeV/c,” **Phys. Rev. D**, vol. 100, no. 11, p. 112001, 2019. DOI: 10.1103/PhysRevD.100.112001. arXiv: 1909.03351 [hep-ex].
- [49] H. Bichsel, “Approximation methods to calculate straggling functions,” **Nucl. Instrum. Meth. A**, vol. 566, R. Bernhard, J. Gassner, F. Lehner, M. Needham, O. Steinkamp, U. Straumann, and A. Vollhardt, Eds., pp. 1–4, 2006. DOI: 10.1016/j.nima.2006.05.022.
- [50] L. Rolandi, W. Riegler, and W. Blum, **Particle Detection with Drift Chambers**. Jan. 2008, vol. -1, ISBN: 978-3-540-76683-4. DOI: 10.1007/978-3-540-76684-1.
- [51] F. James, “MINUIT Function Minimization and Error Analysis: Reference Manual Version 94.1,” 1994.
- [52] S. Johnson, “Measurements of hadron production in pion nucleus collisions for neutrino oscillation experiments,” PhD thesis, 2019.
- [53] R. Barlow, “Asymmetric systematic errors,” Jun. 2003. arXiv: physics/0306138.
- [54] V. Uzhinsky, “Tuning of the GEANT4 FRITIOF (FTF) Model Using NA61/SHINE Experimental Data,” Sep. 2011. arXiv: 1109.6768 [hep-ph].
- [55] V. Uzhinsky and A. Galoyan, “Towards Simulation of Quark and Diquark Fragmentations in the Quark-Gluon String Model (QGSM),” **Phys. Part. Nucl. Lett.**, vol. 16, no. 6, pp. 985–988, 2019. DOI: 10.1134/S1547477119060335. arXiv: 1806.01534 [hep-ph].

Appendix A

Table of Measurements

This appendix presents numerical values for the measured differential $\pi^\pm$, $p/\bar{p}$, and $K^\pm$ multiplicities and their uncertainties in tables A.1–A.6. The uncertainties are reported as a percent of the measured multiplicity. In addition to the total uncertainty for each measurement, the correlated and uncorrelated uncertainties between the 2016 and 2017 analyses are reported.

Table A.1: π^+ Multiplicities and Uncertainties

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV \cdot c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[2.02,6]	[0,5]	0.136	± 1.53	$\pm_{9.54}^{13.5}$	$\pm_{10}^{14.2}$
[6,9]	[0,5]	0.164	± 1.57	$\pm_{6.86}^{14.6}$	$\pm_{7.05}^{14.7}$
[9,15]	[0,5]	0.185	± 1.08	$\pm_{4.94}^{13}$	$\pm_{5.15}^{13.3}$
[15,18]	[0,5]	0.191	± 1.63	$\pm_{4.89}^{13}$	$\pm_{5.19}^{13.2}$
[18,20]	[0,5]	0.206	± 2.53	$\pm_{5.09}^{12.7}$	$\pm_{5.68}^{12.9}$
[20,25]	[0,5]	0.196	± 2.14	$\pm_{3.87}^{12.7}$	$\pm_{4.42}^{12.8}$
[25,30]	[0,5]	0.2	± 2.67	$\pm_{3.69}^{12.6}$	$\pm_{4.55}^{12.9}$
[30,36]	[0,5]	0.188	± 3.41	$\pm_{3.83}^{12.6}$	$\pm_{5.13}^{13.1}$
[36,40]	[0,5]	0.162	± 2.37	$\pm_{5.28}^{12.7}$	$\pm_{5.81}^{13.3}$
[40,50]	[0,5]	0.112	± 2.18	$\pm_{5.62}^{12.6}$	$\pm_{6.39}^{13.7}$
[50,60]	[0,5]	0.0758	± 2.71	$\pm_{3.6}^{12.8}$	$\pm_{4.51}^{13.1}$
[60,70]	[0,5]	0.0495	± 8.4	$\pm_{4.59}^{12.7}$	$\pm_{9.57}^{15.2}$

Table A.1 π^+ Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[70,80]	[0,5]	0.0335	± 15.2	$\pm_{5.16}^{12.6}$	$\pm_{16.1}^{19.8}$
[0.5,1.64]	[5,10]	0.0979	± 2.79	$\pm_{43.1}^{15.1}$	$\pm_{44.4}^{15.6}$
[2.02,6]	[5,10]	0.404	± 0.925	$\pm_{10.5}^{13.5}$	$\pm_{10.8}^{14}$
[6,9]	[5,10]	0.506	± 0.919	$\pm_{5.54}^{13.5}$	$\pm_{5.71}^{13.8}$
[9,15]	[5,10]	0.562	± 0.652	$\pm_{4.59}^{13.1}$	$\pm_{4.75}^{13.5}$
[15,20]	[5,10]	0.571	± 0.934	$\pm_{3.91}^{12.7}$	$\pm_{4.06}^{12.9}$
[20,25]	[5,10]	0.543	± 1.13	$\pm_{3.84}^{12.7}$	$\pm_{4.08}^{13}$
[25,30]	[5,10]	0.483	± 1.37	$\pm_{3.68}^{12.7}$	$\pm_{4.01}^{13.1}$
[30,35]	[5,10]	0.382	± 2.44	$\pm_{3.66}^{12.6}$	$\pm_{4.4}^{12.9}$
[35,40]	[5,10]	0.325	± 3.07	$\pm_{3.61}^{12.7}$	$\pm_{4.74}^{13.1}$
[40,45]	[5,10]	0.235	± 3.74	$\pm_{3.63}^{12.8}$	$\pm_{5.21}^{13.3}$
[45,50]	[5,10]	0.182	± 3.3	$\pm_{3.61}^{13}$	$\pm_{4.89}^{13.4}$
[50,55]	[5,10]	0.12	± 2.53	$\pm_{3.93}^{13}$	$\pm_{4.95}^{14.6}$
[55,60]	[5,10]	0.094	± 3.21	$\pm_{3.74}^{12.7}$	$\pm_{4.95}^{13.2}$
[60,68]	[5,10]	0.0668	± 3.54	$\pm_{4.07}^{12.6}$	$\pm_{5.39}^{13.1}$
[68,74]	[5,10]	0.042	± 5.31	$\pm_{4.52}^{12.6}$	$\pm_{6.97}^{13.7}$
[0.5,1.64]	[10,20]	0.223	± 1.56	$\pm_{19.6}^{14.1}$	$\pm_{19.9}^{14.3}$
[2.02,4]	[10,20]	0.683	± 0.755	$\pm_{11.9}^{13.8}$	$\pm_{11.8}^{14.2}$
[4,6]	[10,20]	0.996	± 0.604	$\pm_{7.03}^{13.2}$	$\pm_{7.32}^{14.2}$
[6,9]	[10,20]	1.09	± 0.456	$\pm_{5.05}^{13}$	$\pm_{5.36}^{13.8}$
[9,12]	[10,20]	1.1	± 0.456	$\pm_{4.44}^{12.9}$	$\pm_{4.55}^{13.2}$
[12,15]	[10,20]	1.04	± 0.567	$\pm_{3.9}^{12.8}$	$\pm_{4.04}^{13.1}$
[15,18]	[10,20]	0.949	± 0.676	$\pm_{3.82}^{12.9}$	$\pm_{3.91}^{13}$
[18,21]	[10,20]	0.791	± 0.815	$\pm_{3.71}^{12.7}$	$\pm_{3.86}^{12.9}$
[21,24]	[10,20]	0.65	± 0.968	$\pm_{3.88}^{12.7}$	$\pm_{4.02}^{12.8}$

Table A.1 π^+ Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[24,27]	[10,20]	0.536	± 1.2	$\pm_{3.77}^{12.7}$	$\pm_{4.01}^{13}$
[27,30]	[10,20]	0.446	± 1.26	$\pm_{4.13}^{12.6}$	$\pm_{4.32}^{12.7}$
[30,35]	[10,20]	0.337	± 1.18	$\pm_{3.68}^{12.7}$	$\pm_{3.88}^{12.8}$
[35,40]	[10,20]	0.229	± 1.51	$\pm_{3.63}^{12.7}$	$\pm_{3.97}^{13}$
[40,45]	[10,20]	0.154	± 1.87	$\pm_{3.95}^{12.7}$	$\pm_{4.4}^{13}$
[45,50]	[10,20]	0.0903	± 2.08	$\pm_{3.71}^{12.7}$	$\pm_{4.81}^{14.9}$
[50,55]	[10,20]	0.0555	± 2.37	$\pm_{4.12}^{12.8}$	$\pm_{5.54}^{15.7}$
[55,60]	[10,20]	0.0327	± 2.95	$\pm_{3.62}^{13.9}$	$\pm_{5.08}^{15.8}$
[0.5,1.2]	[20,30]	0.281	± 1.47	$\pm_{14.1}^{15.4}$	$\pm_{14}^{15.6}$
[1.2,1.64]	[20,30]	0.56	± 3.15	$\pm_{15.6}^{14.7}$	$\pm_{17}^{16.1}$
[2.02,4]	[20,30]	1.17	± 0.629	$\pm_{6.78}^{13.5}$	$\pm_{6.82}^{13.5}$
[4,6]	[20,30]	1.59	± 0.502	$\pm_{4.75}^{13}$	$\pm_{5.08}^{13.8}$
[6,10]	[20,30]	1.61	± 0.318	$\pm_{4.3}^{12.8}$	$\pm_{4.63}^{13.9}$
[10,12]	[20,30]	1.35	± 0.608	$\pm_{3.89}^{12.8}$	$\pm_{4.22}^{13.8}$
[12,15]	[20,30]	1.09	± 0.541	$\pm_{3.91}^{12.8}$	$\pm_{4.1}^{13.4}$
[15,18]	[20,30]	0.831	± 0.728	$\pm_{3.84}^{12.7}$	$\pm_{3.92}^{12.8}$
[18,20]	[20,30]	0.65	± 1.09	$\pm_{3.68}^{12.8}$	$\pm_{3.89}^{13}$
[20,24]	[20,30]	0.476	± 0.974	$\pm_{3.69}^{12.8}$	$\pm_{3.94}^{13.3}$
[24,27]	[20,30]	0.326	± 1.3	$\pm_{3.78}^{12.7}$	$\pm_{4.07}^{13.1}$
[27,30]	[20,30]	0.235	± 1.6	$\pm_{5.23}^{13.5}$	$\pm_{5.48}^{13.6}$
[30,33]	[20,30]	0.16	± 1.91	$\pm_{4.27}^{13.6}$	$\pm_{4.71}^{13.9}$
[33,38]	[20,30]	0.0985	± 1.86	$\pm_{3.71}^{13.4}$	$\pm_{4.29}^{14}$
[38,42]	[20,30]	0.0552	± 2.4	$\pm_{3.71}^{13.3}$	$\pm_{4.5}^{13.9}$
[0.5,1.2]	[30,40]	0.431	± 1.47	$\pm_{14.8}^{14.5}$	$\pm_{15.2}^{14.8}$
[1.2,1.64]	[30,40]	0.827	± 3.12	$\pm_{12}^{13.9}$	$\pm_{12.4}^{15.3}$

Table A.1 π^+ Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[2.02,4]	[30,40]	1.51	± 0.662	$\pm_{6.64}^{13.3}$	$\pm_{7.02}^{14}$
[4,6]	[30,40]	1.92	± 0.502	$\pm_5^{12.9}$	$\pm_{5.13}^{13.5}$
[6,9]	[30,40]	1.7	± 0.418	$\pm_{4.13}^{12.7}$	$\pm_{4.51}^{13.9}$
[9,12]	[30,40]	1.24	± 0.496	$\pm_{3.83}^{12.7}$	$\pm_{4.23}^{14}$
[12,15]	[30,40]	0.832	± 0.66	$\pm_{4.08}^{12.7}$	$\pm_{4.54}^{14}$
[15,18]	[30,40]	0.528	± 0.908	$\pm_{3.69}^{12.6}$	$\pm_{3.91}^{13.1}$
[18,21]	[30,40]	0.335	± 1.09	$\pm_{4.14}^{12.7}$	$\pm_{4.31}^{12.8}$
[21,24]	[30,40]	0.205	± 1.47	$\pm_{4.68}^{12.8}$	$\pm_{4.93}^{13.2}$
[24,27]	[30,40]	0.128	± 1.76	$\pm_{5.15}^{13.5}$	$\pm_{5.44}^{13.6}$
[27,30]	[30,40]	0.0751	± 2.28	$\pm_{6.01}^{13.1}$	$\pm_{6.61}^{13.7}$
[30,33]	[30,40]	0.0425	± 3.78	$\pm_{4.95}^{12.7}$	$\pm_{6.23}^{13.2}$
[0.5,1.2]	[40,50]	0.616	± 1.54	$\pm_{12.9}^{14.2}$	$\pm_{12.9}^{14.4}$
[1.2,1.64]	[40,50]	1.15	± 3.31	$\pm_{10.7}^{13.7}$	$\pm_{11.7}^{14.8}$
[2.02,4]	[40,50]	1.82	± 0.749	$\pm_{5.48}^{13.3}$	$\pm_{5.61}^{13.6}$
[4,6]	[40,50]	2.07	± 0.59	$\pm_{4.77}^{12.9}$	$\pm_5^{13.8}$
[6,10]	[40,50]	1.44	± 0.484	$\pm_{3.95}^{12.7}$	$\pm_{4.31}^{13.8}$
[10,12]	[40,50]	0.919	± 0.825	$\pm_{3.74}^{12.9}$	$\pm_{4.45}^{15.2}$
[12,15]	[40,50]	0.552	± 0.898	$\pm_{3.71}^{12.8}$	$\pm_{4.2}^{14.1}$
[15,18]	[40,50]	0.288	± 1.12	$\pm_{4.41}^{12.7}$	$\pm_{5.03}^{14.2}$
[18,21]	[40,50]	0.156	± 1.55	$\pm_{3.63}^{13.2}$	$\pm_{4.17}^{14.2}$
[21,25]	[40,50]	0.0776	± 1.81	$\pm_{3.92}^{13.7}$	$\pm_{4.35}^{13.9}$
[0.5,1.2]	[50,60]	0.749	± 1.34	$\pm_{10.8}^{14.2}$	$\pm_{10.8}^{14.3}$
[1.2,1.64]	[50,60]	1.46	± 3.27	$\pm_{8.42}^{13.9}$	$\pm_{9.01}^{14.4}$
[2.02,4]	[50,60]	2.04	± 0.736	$\pm_5^{13}$	$\pm_{5.08}^{13.3}$
[4,6]	[50,60]	1.93	± 0.573	$\pm_{4.4}^{12.8}$	$\pm_{4.5}^{13.1}$

Table A.1 π^+ Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{\text{GeV}/c\text{-rad}}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[6,8]	[50,60]	1.38	± 0.631	$\pm_{3.9}^{12.9}$	$\pm_{4.18}^{13.6}$
[8,10]	[50,60]	0.921	± 0.759	$\pm_{3.96}^{12.8}$	$\pm_{4.31}^{13.7}$
[10,12]	[50,60]	0.581	± 1.01	$\pm_{3.83}^{12.8}$	$\pm_{4.14}^{13.6}$
[12,15]	[50,60]	0.346	± 1.16	$\pm_{3.69}^{12.7}$	$\pm_{4.17}^{13.9}$
[15,19]	[50,60]	0.145	± 1.31	$\pm_{4.84}^{12.6}$	$\pm_{5.07}^{13.2}$
[19,21]	[50,60]	0.0659	± 2.95	$\pm_{3.86}^{12.7}$	$\pm_{4.86}^{13}$
[0.5,1.2]	[60,80]	1.06	± 0.975	$\pm_{11.4}^{13.6}$	$\pm_{11.5}^{13.7}$
[1.2,1.64]	[60,80]	2.06	± 1.48	$\pm_{8.12}^{13.4}$	$\pm_{8.54}^{14}$
[2.02,3]	[60,80]	2.32	± 0.633	$\pm_{5.18}^{13}$	$\pm_{5.34}^{13.3}$
[3,4]	[60,80]	2.17	± 0.585	$\pm_{4.49}^{12.8}$	$\pm_{4.54}^{12.9}$
[4,5]	[60,80]	1.85	± 0.606	$\pm_{4.21}^{12.8}$	$\pm_{4.29}^{12.9}$
[5,6]	[60,80]	1.48	± 0.677	$\pm_{3.89}^{13.1}$	$\pm_{3.98}^{13.3}$
[6,7]	[60,80]	1.13	± 0.74	$\pm_{3.82}^{12.8}$	$\pm_{3.99}^{13.2}$
[7,8]	[60,80]	0.865	± 0.849	$\pm_{3.82}^{12.9}$	$\pm_{4.21}^{13.9}$
[8,9]	[60,80]	0.65	± 0.925	$\pm_{3.75}^{12.8}$	$\pm_{4.04}^{13.6}$
[9,10]	[60,80]	0.478	± 1.04	$\pm_4^{13}$	$\pm_{4.42}^{14}$
[10,11]	[60,80]	0.371	± 1.2	$\pm_{5.25}^{12.9}$	$\pm_{5.75}^{13.8}$
[11,12]	[60,80]	0.273	± 1.37	$\pm_{5.98}^{12.8}$	$\pm_{6.5}^{13.6}$
[12,13]	[60,80]	0.203	± 1.5	$\pm_{4.85}^{13.4}$	$\pm_{5.14}^{14}$
[13,15]	[60,80]	0.131	± 1.31	$\pm_{4.29}^{13.3}$	$\pm_{4.58}^{13.7}$
[15,17]	[60,80]	0.0762	± 1.68	$\pm_{5.37}^{12.8}$	$\pm_{5.69}^{13.1}$
[0.5,1.2]	[80,100]	1.34	± 0.841	$\pm_{8.01}^{13.5}$	$\pm_{8.04}^{13.8}$
[1.2,1.64]	[80,100]	2.45	± 1.21	$\pm_{6.61}^{13}$	$\pm_{6.87}^{13.5}$
[2.02,3]	[80,100]	2.37	± 0.643	$\pm_{5.19}^{12.8}$	$\pm_{5.36}^{13.5}$
[3,4]	[80,100]	1.9	± 0.575	$\pm_{4.11}^{12.8}$	$\pm_{4.31}^{13.4}$

Table A.1 π^+ Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[4,5]	[80,100]	1.47	± 0.658	$\pm_{3.94}^{12.7}$	$\pm_{4.2}^{13.4}$
[5,6]	[80,100]	1.07	± 0.786	$\pm_{3.81}^{12.7}$	$\pm_{4.28}^{14.1}$
[6,7]	[80,100]	0.747	± 0.956	$\pm_{3.96}^{13.1}$	$\pm_{4.29}^{14.1}$
[7,8]	[80,100]	0.519	± 1.11	$\pm_{4.44}^{12.7}$	$\pm_{4.85}^{13.5}$
[8,9]	[80,100]	0.364	± 1.4	$\pm_{5.57}^{12.7}$	$\pm_{5.75}^{12.8}$
[9,10]	[80,100]	0.242	± 1.64	$\pm_{4.18}^{12.8}$	$\pm_{4.57}^{13.3}$
[10,12]	[80,100]	0.146	± 1.71	$\pm_{7.48}^{12.6}$	$\pm_{8.04}^{13.4}$
[12,14]	[80,100]	0.0672	± 2.13	$\pm_5^{13.5}$	$\pm_{5.59}^{14}$
[0.5,1.2]	[100,120]	1.57	± 0.836	$\pm_{6.83}^{13.1}$	$\pm_{7.1}^{13.6}$
[1.2,1.64]	[100,120]	2.59	± 1.25	$\pm_{5.95}^{13.2}$	$\pm_{6.26}^{13.7}$
[2.02,3]	[100,120]	2.18	± 0.68	$\pm_{4.41}^{12.8}$	$\pm_{4.53}^{13}$
[3,4]	[100,120]	1.6	± 0.66	$\pm_{4.1}^{12.7}$	$\pm_{4.22}^{13}$
[4,5]	[100,120]	1.04	± 0.794	$\pm_{3.82}^{12.7}$	$\pm_{3.94}^{13}$
[5,6]	[100,120]	0.697	± 0.997	$\pm_{4.6}^{12.7}$	$\pm_{4.81}^{13.3}$
[6,7]	[100,120]	0.44	± 1.16	$\pm_{4.19}^{12.8}$	$\pm_{4.43}^{13.3}$
[7,8]	[100,120]	0.276	± 1.47	$\pm_{3.67}^{13.5}$	$\pm_{4.13}^{14.3}$
[8,9]	[100,120]	0.177	± 1.92	$\pm_{3.68}^{13.4}$	$\pm_{4.18}^{13.7}$
[9,12]	[100,120]	0.0823	± 1.64	$\pm_{4.72}^{13}$	$\pm_{5.03}^{13.5}$
[0.5,1.2]	[120,140]	1.78	± 0.846	$\pm_{6.9}^{13.6}$	$\pm_{6.98}^{13.8}$
[1.2,1.64]	[120,140]	2.68	± 1.3	$\pm_{7.72}^{13}$	$\pm_{8.24}^{13.7}$
[2.02,3]	[120,140]	1.89	± 0.922	$\pm_{4.15}^{12.8}$	$\pm_{4.26}^{12.9}$
[3,4]	[120,140]	1.21	± 0.785	$\pm_{3.9}^{12.7}$	$\pm_{4.18}^{13.4}$
[4,5]	[120,140]	0.743	± 1.01	$\pm_{4.23}^{12.7}$	$\pm_{4.63}^{13.6}$
[5,6]	[120,140]	0.427	± 1.31	$\pm_{4.07}^{12.8}$	$\pm_{4.38}^{13.2}$
[6,7]	[120,140]	0.266	± 1.63	$\pm_{3.86}^{12.9}$	$\pm_{4.3}^{13.5}$

Table A.1 π^+ Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{\text{GeV}/c\text{-rad}}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[7,9]	[120,140]	0.121	± 1.68	$\pm_{6.31}^{13}$	$\pm_{6.59}^{14.1}$
[9,10]	[120,140]	0.0507	± 3.84	$\pm_{4.98}^{12.9}$	$\pm_{6.29}^{13.4}$
[0.5,1.2]	[140,160]	1.86	± 0.881	$\pm_{6.24}^{13.5}$	$\pm_{6.31}^{13.5}$
[1.2,1.64]	[140,160]	2.71	± 1.51	$\pm_{5.25}^{13}$	$\pm_{5.77}^{13.8}$
[2.02,3]	[140,160]	1.56	± 1.09	$\pm_{4.27}^{12.8}$	$\pm_{4.42}^{12.9}$
[3,4]	[140,160]	0.907	± 0.908	$\pm_{3.78}^{12.9}$	$\pm_{3.89}^{13}$
[4,5]	[140,160]	0.502	± 1.15	$\pm_{4.82}^{12.7}$	$\pm_{5.28}^{13.7}$
[5,6]	[140,160]	0.274	± 1.62	$\pm_{5.43}^{12.7}$	$\pm_{6.22}^{14.1}$
[6,8]	[140,160]	0.114	± 1.75	$\pm_{4.12}^{12.9}$	$\pm_{4.55}^{13.3}$
[0.5,1.2]	[160,180]	1.91	± 0.976	$\pm_{5.99}^{13}$	$\pm_{6.06}^{13.2}$
[1.2,1.64]	[160,180]	2.49	± 1.98	$\pm_{4.72}^{12.9}$	$\pm_{5.43}^{14.4}$
[2.02,3]	[160,180]	1.27	± 1.22	$\pm_{4.04}^{12.7}$	$\pm_{4.35}^{13.3}$
[3,4]	[160,180]	0.693	± 1.04	$\pm_{3.87}^{12.9}$	$\pm_{4.1}^{13.3}$
[4,5]	[160,180]	0.359	± 1.48	$\pm_{4.76}^{12.6}$	$\pm_{5.12}^{13.1}$
[5,7]	[160,180]	0.122	± 1.75	$\pm_{4.01}^{13.2}$	$\pm_{4.8}^{14.9}$
[0.5,1.2]	[180,200]	1.91	± 1.13	$\pm_{5.64}^{13.1}$	$\pm_{5.85}^{13.7}$
[1.2,1.64]	[180,200]	2.47	± 3.54	$\pm_{4.57}^{13}$	$\pm_{5.93}^{13.9}$
[2.02,3]	[180,200]	1.04	± 2.04	$\pm_{3.91}^{12.7}$	$\pm_{4.45}^{13.1}$
[3,4]	[180,200]	0.506	± 1.43	$\pm_{5.1}^{12.7}$	$\pm_{5.37}^{12.9}$
[4,5]	[180,200]	0.237	± 1.93	$\pm_{7.44}^{12.6}$	$\pm_{7.94}^{13.2}$
[5,7]	[180,200]	0.074	± 2.16	$\pm_{6.44}^{13.3}$	$\pm_{6.94}^{15.3}$
[0.5,1.2]	[200,220]	1.82	± 1.36	$\pm_{5.3}^{13}$	$\pm_{5.93}^{14.2}$
[1.2,1.64]	[200,220]	2.08	± 4.19	$\pm_{4.53}^{12.9}$	$\pm_{6.33}^{14.2}$
[2.02,3]	[200,220]	0.837	± 2.52	$\pm_{4.35}^{12.7}$	$\pm_{5.01}^{13}$
[3,4]	[200,220]	0.374	± 1.96	$\pm_{5.58}^{12.6}$	$\pm_{5.8}^{12.9}$

Table A.1 π^+ Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[4,6]	[200,220]	0.106	± 2.19	$\pm_{5.09}^{13.8}$	$\pm_{5.45}^{14}$
[0.5,1.2]	[220,240]	1.68	± 1.55	$\pm_{6.37}^{12.9}$	$\pm_{7.7}^{15.6}$
[1.2,1.64]	[220,240]	1.99	± 3.59	$\pm_{4.51}^{12.8}$	$\pm_{5.84}^{13.7}$
[2.02,3]	[220,240]	0.681	± 2.46	$\pm_{4.11}^{12.7}$	$\pm_{4.91}^{13.3}$
[3,4]	[220,240]	0.288	± 2.02	$\pm_{3.93}^{12.9}$	$\pm_{4.53}^{13.6}$
[4,6]	[220,240]	0.0737	± 2.29	$\pm_{5.42}^{13.1}$	$\pm_{5.93}^{13.8}$
[0.5,1.2]	[240,260]	1.62	± 1.79	$\pm_{5.38}^{12.9}$	$\pm_{6.88}^{16.2}$
[1.2,1.64]	[240,260]	1.49	± 4.62	$\pm_{4.29}^{12.8}$	$\pm_{6.45}^{14.2}$
[2.02,3]	[240,260]	0.525	± 3.37	$\pm_{4.46}^{12.7}$	$\pm_{5.61}^{13.4}$
[3,5]	[240,260]	0.138	± 2.48	$\pm_{5.4}^{12.7}$	$\pm_{6.14}^{14.4}$
[0.5,1.2]	[260,280]	1.56	± 2.02	$\pm_{4.92}^{12.9}$	$\pm_{6.62}^{16.7}$
[1.2,1.64]	[260,280]	1.38	± 5.87	$\pm_{4.22}^{13}$	$\pm_{7.65}^{16.5}$
[2.02,3]	[260,280]	0.42	± 4.69	$\pm_{4.98}^{12.7}$	$\pm_{6.87}^{13.6}$
[3,5]	[260,280]	0.0958	± 3.86	$\pm_{4.69}^{12.7}$	$\pm_{6.49}^{14.6}$
[0.5,1.2]	[280,300]	1.46	± 1.69	$\pm_{4.93}^{13.2}$	$\pm_{6.59}^{17.7}$
[1.2,1.64]	[280,300]	1.38	± 5.08	$\pm_{4.12}^{12.9}$	$\pm_{6.71}^{14.7}$
[2.02,3]	[280,300]	0.345	± 4.67	$\pm_{5.76}^{12.7}$	$\pm_{7.65}^{15.7}$
[3,5]	[280,300]	0.0671	± 2.96	$\pm_{7.55}^{13.1}$	$\pm_{9.43}^{15.6}$
[0.5,1.2]	[300,320]	1.56	± 1.96	$\pm_{4.36}^{12.9}$	$\pm_{5.6}^{16.5}$
[1.2,1.64]	[300,320]	1.07	± 6.07	$\pm_{5.38}^{12.9}$	$\pm_{9.39}^{18.3}$
[2.02,3]	[300,320]	0.262	± 5.34	$\pm_{5.02}^{13.1}$	$\pm_{8.24}^{19.8}$
[3,5]	[300,320]	0.0523	± 3.75	$\pm_{5.04}^{13.2}$	$\pm_{6.76}^{16.9}$
[0.5,1.2]	[320,340]	1.61	± 1.7	$\pm_{5.09}^{13.1}$	$\pm_{6.33}^{16.3}$
[1.2,1.64]	[320,340]	1.19	± 5.7	$\pm_{4.05}^{12.7}$	$\pm_{7.44}^{16.3}$
[2.02,4]	[320,340]	0.126	± 4.8	$\pm_{3.82}^{13.2}$	$\pm_{6.68}^{16.9}$

Table A.1 π^+ Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[0.5,1.2]	[340,360]	1.6	± 2.3	$\pm_{4.89}^{12.9}$	$\pm_{6.71}^{16.8}$
[1.2,1.64]	[340,360]	1.17	± 6.12	$\pm_{6.27}^{12.7}$	$\pm_{9.68}^{16.4}$
[2.02,4]	[340,360]	0.1	± 6.18	$\pm_{6.3}^{13}$	$\pm_{8.99}^{15.9}$
[0.5,1.2]	[360,380]	1.57	± 2.31	$\pm_{5.35}^{12.8}$	$\pm_{6.61}^{16.5}$
[1.2,1.64]	[360,380]	1.08	± 6.94	$\pm_{4.34}^{12.9}$	$\pm_{8.43}^{15.7}$
[2.02,4]	[360,380]	0.0786	± 7.88	$\pm_{5.27}^{12.7}$	$\pm_{12.1}^{23.7}$
[0.5,1.64]	[380,400]	1.31	± 2.93	$\pm_{4.27}^{12.9}$	$\pm_{5.33}^{13.9}$
[2.02,4]	[380,400]	0.0665	± 9.72	$\pm_{4.04}^{12.8}$	$\pm_{11.4}^{20.8}$
[0.5,1.64]	[400,420]	1.12	± 4	$\pm_4^{12.7}$	$\pm_{5.65}^{13.3}$
[2.02,4]	[400,420]	0.0371	± 14.9	$\pm_{4.4}^{13}$	$\pm_{16.6}^{29.7}$
[0.5,1.64]	[420,440]	0.671	± 11.4	$\pm_{3.75}^{13.1}$	$\pm_{12}^{17.3}$

Table A.2: π^- Multiplicities and Uncertainties

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[2.02,6]	[0,5]	0.112	± 1.66	$\pm_{14.9}^{20.9}$	$\pm_{15.3}^{21.6}$
[6,9]	[0,5]	0.147	± 1.69	$\pm_{11.6}^{17.6}$	$\pm_{11.7}^{17.7}$
[9,15]	[0,5]	0.15	± 1.22	$\pm_{7.68}^{14.1}$	$\pm_{7.71}^{14.3}$
[15,18]	[0,5]	0.142	± 1.87	$\pm_{5.25}^{14.1}$	$\pm_{5.61}^{14.5}$
[18,20]	[0,5]	0.132	± 3.18	$\pm_{3.95}^{13.2}$	$\pm_{5.07}^{13.5}$
[20,25]	[0,5]	0.12	± 2.32	$\pm_{3.79}^{13.1}$	$\pm_{4.45}^{13.3}$
[25,30]	[0,5]	0.106	± 2.62	$\pm_{3.72}^{12.7}$	$\pm_{4.55}^{12.9}$
[30,36]	[0,5]	0.0814	± 3.08	$\pm_{3.65}^{12.9}$	$\pm_{4.78}^{13.2}$
[40,50]	[0,5]	0.0456	± 3.33	$\pm_{5.02}^{13.8}$	$\pm_{6.08}^{14.4}$
[0.5,1.64]	[5,10]	0.0725	± 3.05	$\pm_{36.2}^{24.9}$	$\pm_{37.9}^{24.7}$

Table A.2 π^- Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[2.02,6]	[5,10]	0.345	± 0.934	$\pm_{17.6}^{18.2}$	$\pm_{18.3}^{18.9}$
[6,9]	[5,10]	0.439	± 0.981	$\pm_{12.2}^{16.3}$	$\pm_{12.6}^{16.8}$
[9,15]	[5,10]	0.452	± 0.749	$\pm_{6.17}^{13.6}$	$\pm_{6.31}^{13.8}$
[15,20]	[5,10]	0.391	± 1.07	$\pm_{4.6}^{12.7}$	$\pm_{4.82}^{13.1}$
[20,25]	[5,10]	0.317	± 1.31	$\pm_{3.79}^{13.3}$	$\pm_{4.04}^{13.5}$
[25,30]	[5,10]	0.254	± 1.64	$\pm_{3.68}^{12.7}$	$\pm_{4.18}^{13.4}$
[30,35]	[5,10]	0.181	± 2.69	$\pm_{4.4}^{12.7}$	$\pm_{5.16}^{12.9}$
[35,40]	[5,10]	0.131	± 3.26	$\pm_{4.43}^{12.6}$	$\pm_{5.5}^{13}$
[40,45]	[5,10]	0.0959	± 3.9	$\pm_{3.62}^{12.9}$	$\pm_{5.32}^{13.5}$
[45,50]	[5,10]	0.0704	± 4.67	$\pm_{4.22}^{14.8}$	$\pm_{6.29}^{15.5}$
[50,55]	[5,10]	0.0544	± 5.36	$\pm_{4.35}^{12.8}$	$\pm_{6.91}^{13.9}$
[0.5,1.64]	[10,20]	0.168	± 1.63	$\pm_{26.6}^{25.6}$	$\pm_{26.5}^{25.2}$
[2.02,4]	[10,20]	0.604	± 0.733	$\pm_{16.3}^{17.6}$	$\pm_{16.6}^{17.9}$
[4,6]	[10,20]	0.853	± 0.627	$\pm_{11.6}^{15.5}$	$\pm_{12.3}^{16.4}$
[6,9]	[10,20]	0.912	± 0.515	$\pm_{7.5}^{14}$	$\pm_{7.58}^{14.1}$
[9,12]	[10,20]	0.88	± 0.524	$\pm_{5.04}^{13}$	$\pm_{5.11}^{13.2}$
[12,15]	[10,20]	0.748	± 0.664	$\pm_{4.51}^{12.8}$	$\pm_{4.8}^{13.5}$
[15,18]	[10,20]	0.631	± 0.769	$\pm_{4.44}^{12.8}$	$\pm_{4.64}^{13.2}$
[18,21]	[10,20]	0.52	± 0.883	$\pm_{4.86}^{12.7}$	$\pm_{5.32}^{13.7}$
[21,24]	[10,20]	0.408	± 1.05	$\pm_{4.26}^{12.7}$	$\pm_{4.66}^{13.6}$
[24,27]	[10,20]	0.318	± 1.27	$\pm_{3.72}^{12.9}$	$\pm_{4.05}^{13.4}$
[27,30]	[10,20]	0.245	± 1.61	$\pm_{3.65}^{13.3}$	$\pm_{4.18}^{14.1}$
[30,35]	[10,20]	0.181	± 1.46	$\pm_{3.98}^{12.9}$	$\pm_{4.48}^{13.8}$
[35,40]	[10,20]	0.12	± 1.79	$\pm_{4.5}^{13.2}$	$\pm_{4.91}^{13.7}$
[40,45]	[10,20]	0.0783	± 2.25	$\pm_{3.65}^{13.2}$	$\pm_{4.44}^{14}$

Table A.2 π^- Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[45,50]	[10,20]	0.0498	± 2.75	$\pm_{7.84}^{13.2}$	$\pm_{9.62}^{15.5}$
[50,55]	[10,20]	0.0255	± 5.06	$\pm_{3.63}^{13.6}$	$\pm_{6.23}^{14.5}$
[0.5,1.2]	[20,30]	0.207	± 1.69	$\pm_{24.6}^{23.1}$	$\pm_{24.1}^{23}$
[1.2,1.64]	[20,30]	0.459	± 2.65	$\pm_{19}^{22.1}$	$\pm_{19}^{22.2}$
[2.02,4]	[20,30]	1.07	± 0.586	$\pm_{12.2}^{15.9}$	$\pm_{12.2}^{16}$
[4,6]	[20,30]	1.37	± 0.525	$\pm_{7.06}^{13.7}$	$\pm_{7.4}^{14.6}$
[6,10]	[20,30]	1.32	± 0.365	$\pm_{4.76}^{13}$	$\pm_{5.01}^{13.7}$
[10,12]	[20,30]	1.04	± 0.68	$\pm_{4.04}^{13.2}$	$\pm_{4.13}^{13.3}$
[12,15]	[20,30]	0.822	± 0.637	$\pm_{4.35}^{12.7}$	$\pm_{4.64}^{13.5}$
[15,18]	[20,30]	0.6	± 0.837	$\pm_{4.06}^{12.8}$	$\pm_{4.45}^{14.1}$
[18,20]	[20,30]	0.441	± 1.18	$\pm_{3.68}^{12.9}$	$\pm_{4.06}^{13.6}$
[20,24]	[20,30]	0.318	± 1.21	$\pm_{3.86}^{12.8}$	$\pm_{4.2}^{13.5}$
[24,27]	[20,30]	0.211	± 1.67	$\pm_{3.93}^{13}$	$\pm_{4.31}^{13.4}$
[27,30]	[20,30]	0.142	± 2	$\pm_{5.93}^{12.6}$	$\pm_{6.24}^{13.8}$
[30,33]	[20,30]	0.0884	± 2.51	$\pm_{5.1}^{12.7}$	$\pm_{5.78}^{13.2}$
[33,38]	[20,30]	0.0584	± 2.46	$\pm_{6.21}^{12.6}$	$\pm_{7.26}^{14.1}$
[38,42]	[20,30]	0.0295	± 5.05	$\pm_{5.65}^{12.6}$	$\pm_{7.58}^{13.6}$
[0.5,1.2]	[30,40]	0.34	± 1.56	$\pm_{21}^{22.3}$	$\pm_{20.5}^{22.3}$
[1.2,1.64]	[30,40]	0.846	± 2.78	$\pm_{16.9}^{20.1}$	$\pm_{18.6}^{22.4}$
[2.02,4]	[30,40]	1.46	± 0.619	$\pm_{8.35}^{14.5}$	$\pm_{8.48}^{14.7}$
[4,6]	[30,40]	1.68	± 0.52	$\pm_{5.91}^{13.1}$	$\pm_{6.1}^{13.7}$
[6,9]	[30,40]	1.43	± 0.449	$\pm_{4.31}^{12.8}$	$\pm_{4.61}^{13.7}$
[9,12]	[30,40]	0.982	± 0.543	$\pm_{3.87}^{12.8}$	$\pm_{4.05}^{13.2}$
[12,15]	[30,40]	0.634	± 0.719	$\pm_{4.17}^{12.8}$	$\pm_{4.39}^{13.3}$
[15,18]	[30,40]	0.393	± 1.12	$\pm_{3.82}^{13.8}$	$\pm_{4.15}^{14.5}$

Table A.2 π^- Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{\text{GeV}/c \cdot \text{rad}}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[18,21]	[30,40]	0.246	± 1.47	$\pm_{4.47}^{12.7}$	$\pm_{4.78}^{13.2}$
[21,24]	[30,40]	0.148	± 1.87	$\pm_{5.31}^{12.6}$	$\pm_{5.71}^{13.6}$
[24,27]	[30,40]	0.0873	± 2.49	$\pm_{4.48}^{13.1}$	$\pm_{5.46}^{14.3}$
[27,30]	[30,40]	0.0467	± 3.3	$\pm_{4.46}^{12.8}$	$\pm_{5.98}^{14.7}$
[0.5,1.2]	[40,50]	0.521	± 1.85	$\pm_{19.3}^{19.4}$	$\pm_{19.5}^{20}$
[1.2,1.64]	[40,50]	1.27	± 2.84	$\pm_{16.1}^{19}$	$\pm_{16.7}^{20.1}$
[2.02,4]	[40,50]	1.82	± 0.652	$\pm_{6.93}^{13.5}$	$\pm_{7.04}^{13.7}$
[4,6]	[40,50]	1.8	± 0.612	$\pm_{5.58}^{12.9}$	$\pm_{5.83}^{13.6}$
[6,10]	[40,50]	1.22	± 0.543	$\pm_{4.4}^{12.8}$	$\pm_{4.71}^{13.9}$
[10,12]	[40,50]	0.695	± 0.987	$\pm_{4.03}^{12.7}$	$\pm_{4.37}^{13.7}$
[12,15]	[40,50]	0.411	± 1.07	$\pm_{3.76}^{12.7}$	$\pm_{4.09}^{13.4}$
[15,18]	[40,50]	0.227	± 1.44	$\pm_{3.78}^{12.9}$	$\pm_{4.18}^{13.5}$
[18,21]	[40,50]	0.117	± 2.02	$\pm_{7.59}^{12.8}$	$\pm_{8.27}^{13.7}$
[21,25]	[40,50]	0.0532	± 2.51	$\pm_{3.63}^{14.1}$	$\pm_{4.67}^{15.1}$
[0.5,1.2]	[50,60]	0.646	± 1.67	$\pm_{18.8}^{20.2}$	$\pm_{18.6}^{20.4}$
[1.2,1.64]	[50,60]	1.54	± 3.13	$\pm_{13.2}^{15.9}$	$\pm_{14.6}^{17.3}$
[2.02,4]	[50,60]	2.01	± 0.66	$\pm_{6.3}^{13.2}$	$\pm_{6.34}^{13.3}$
[4,6]	[50,60]	1.7	± 0.66	$\pm_{5.28}^{12.8}$	$\pm_{5.34}^{13.3}$
[6,8]	[50,60]	1.16	± 0.69	$\pm_{3.91}^{12.8}$	$\pm_{4.17}^{13.5}$
[8,10]	[50,60]	0.765	± 0.856	$\pm_{3.79}^{12.9}$	$\pm_{4.31}^{14.4}$
[10,12]	[50,60]	0.477	± 1.16	$\pm_{4.56}^{12.7}$	$\pm_{5.02}^{14.4}$
[12,15]	[50,60]	0.257	± 1.34	$\pm_{3.78}^{12.7}$	$\pm_{4.35}^{14.1}$
[15,19]	[50,60]	0.108	± 1.6	$\pm_{4.36}^{12.7}$	$\pm_{4.8}^{13.6}$
[19,21]	[50,60]	0.0502	± 4.08	$\pm_{5.17}^{12.6}$	$\pm_{6.59}^{13.3}$
[0.5,1.2]	[60,80]	0.873	± 1.13	$\pm_{13.2}^{17.1}$	$\pm_{14}^{18.7}$

Table A.2 π^- Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{\text{GeV}/c \cdot \text{rad}}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[1.2,1.64]	[60,80]	1.97	± 1.58	$\pm_{8.47}^{14.7}$	$\pm_{8.88}^{15.6}$
[2.02,3]	[60,80]	2.32	± 0.631	$\pm_{6.07}^{13.1}$	$\pm_{6.28}^{13.8}$
[3,4]	[60,80]	2	± 0.605	$\pm_{5.04}^{12.8}$	$\pm_{5.09}^{13}$
[4,5]	[60,80]	1.68	± 0.646	$\pm_{4.36}^{12.8}$	$\pm_{4.52}^{13.4}$
[5,6]	[60,80]	1.33	± 0.713	$\pm_{4.15}^{12.7}$	$\pm_{4.47}^{13.7}$
[6,7]	[60,80]	0.985	± 0.812	$\pm_{4.32}^{12.8}$	$\pm_{4.8}^{14}$
[7,8]	[60,80]	0.734	± 0.931	$\pm_{4.11}^{12.7}$	$\pm_{4.52}^{14}$
[8,9]	[60,80]	0.549	± 1.09	$\pm_{3.84}^{12.8}$	$\pm_{4.3}^{14}$
[9,10]	[60,80]	0.409	± 1.22	$\pm_{4.34}^{12.8}$	$\pm_{4.98}^{14.3}$
[10,11]	[60,80]	0.309	± 1.41	$\pm_{4.33}^{13}$	$\pm_{4.71}^{13.9}$
[11,12]	[60,80]	0.229	± 1.66	$\pm_{5.86}^{13.1}$	$\pm_{6.15}^{14.4}$
[12,13]	[60,80]	0.157	± 1.95	$\pm_{4.07}^{13}$	$\pm_{4.83}^{14.7}$
[13,15]	[60,80]	0.102	± 1.71	$\pm_{4.78}^{13.1}$	$\pm_{5.15}^{13.8}$
[15,17]	[60,80]	0.0525	± 2.39	$\pm_{3.61}^{14.1}$	$\pm_{4.69}^{15.9}$
[0.5,1.2]	[80,100]	1.04	± 0.971	$\pm_{11.7}^{17}$	$\pm_{13}^{19.6}$
[1.2,1.64]	[80,100]	2.29	± 1.33	$\pm_{6.64}^{13.7}$	$\pm_{6.94}^{14.7}$
[2.02,3]	[80,100]	2.29	± 0.602	$\pm_{5.06}^{13}$	$\pm_{5.23}^{13.8}$
[3,4]	[80,100]	1.81	± 0.627	$\pm_{4.45}^{12.8}$	$\pm_{4.79}^{13.8}$
[4,5]	[80,100]	1.3	± 0.706	$\pm_{4.35}^{13.1}$	$\pm_{4.54}^{13.6}$
[5,6]	[80,100]	0.907	± 0.872	$\pm_{4.31}^{12.7}$	$\pm_{4.64}^{13.7}$
[6,7]	[80,100]	0.633	± 1.06	$\pm_{4.14}^{12.8}$	$\pm_{4.66}^{14.1}$
[7,8]	[80,100]	0.429	± 1.25	$\pm_{6.05}^{12.8}$	$\pm_{6.85}^{14.3}$
[8,9]	[80,100]	0.281	± 1.54	$\pm_{6.17}^{13}$	$\pm_{6.85}^{14.2}$
[9,10]	[80,100]	0.192	± 1.91	$\pm_{4.63}^{12.7}$	$\pm_{5.3}^{14.5}$
[10,12]	[80,100]	0.107	± 1.89	$\pm_{3.84}^{14.6}$	$\pm_{4.65}^{16.3}$

Table A.2 π^- Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[12,14]	[80,100]	0.0495	± 2.61	$\pm_{4.56}^{12.8}$	$\pm_{5.92}^{15.1}$
[0.5,1.2]	[100,120]	1.19	± 0.934	$\pm_{10.1}^{15.3}$	$\pm_{11.7}^{17.7}$
[1.2,1.64]	[100,120]	2.5	± 1.21	$\pm_{5.61}^{13.3}$	$\pm_{5.96}^{14}$
[2.02,3]	[100,120]	2.12	± 0.624	$\pm_{4.81}^{12.8}$	$\pm_5^{13.5}$
[3,4]	[100,120]	1.44	± 0.689	$\pm_{4.19}^{12.7}$	$\pm_{4.49}^{13.7}$
[4,5]	[100,120]	0.952	± 0.85	$\pm_{4.02}^{12.7}$	$\pm_{4.39}^{13.6}$
[5,6]	[100,120]	0.594	± 1.09	$\pm_{3.84}^{12.8}$	$\pm_{4.27}^{13.9}$
[6,7]	[100,120]	0.383	± 1.3	$\pm_{5.28}^{12.7}$	$\pm_{5.78}^{13.6}$
[7,8]	[100,120]	0.237	± 1.66	$\pm_{4.83}^{12.6}$	$\pm_{5.22}^{13.1}$
[8,9]	[100,120]	0.137	± 2.14	$\pm_{3.66}^{14.7}$	$\pm_{4.59}^{16.4}$
[9,12]	[100,120]	0.056	± 1.95	$\pm_{4.57}^{13.5}$	$\pm_{5.68}^{15.5}$
[0.5,1.2]	[120,140]	1.37	± 1.01	$\pm_{8.99}^{14.6}$	$\pm_{8.97}^{15.6}$
[1.2,1.64]	[120,140]	2.48	± 1.71	$\pm_{7.26}^{13.1}$	$\pm_{7.7}^{14.1}$
[2.02,3]	[120,140]	1.76	± 0.78	$\pm_{4.5}^{12.9}$	$\pm_{4.97}^{14.1}$
[3,4]	[120,140]	1.12	± 0.891	$\pm_{4.11}^{12.8}$	$\pm_{4.37}^{13.4}$
[4,5]	[120,140]	0.652	± 1.14	$\pm_{4.36}^{12.7}$	$\pm_{4.91}^{14.1}$
[5,6]	[120,140]	0.361	± 1.49	$\pm_{4.19}^{13.1}$	$\pm_{4.64}^{13.8}$
[6,7]	[120,140]	0.209	± 1.91	$\pm_{3.71}^{13.7}$	$\pm_{4.78}^{15.8}$
[7,9]	[120,140]	0.0894	± 2.03	$\pm_{3.69}^{12.8}$	$\pm_{4.5}^{14.1}$
[0.5,1.2]	[140,160]	1.49	± 0.932	$\pm_{7.49}^{13.9}$	$\pm_{7.58}^{14.1}$
[1.2,1.64]	[140,160]	2.36	± 1.98	$\pm_{5.13}^{13.3}$	$\pm_{5.55}^{13.9}$
[2.02,3]	[140,160]	1.49	± 0.891	$\pm_{4.11}^{12.8}$	$\pm_{4.4}^{13.5}$
[3,4]	[140,160]	0.84	± 1.04	$\pm_{4.23}^{12.7}$	$\pm_{4.6}^{13.8}$
[4,5]	[140,160]	0.436	± 1.29	$\pm_{4.02}^{12.7}$	$\pm_{4.56}^{13.9}$
[5,6]	[140,160]	0.223	± 1.87	$\pm_{4.78}^{13}$	$\pm_{5.26}^{14}$

Table A.2 π^- Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{\text{GeV}/c\text{-rad}}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[6,8]	[140,160]	0.0841	± 2	$\pm_{3.65}^{13.2}$	$\pm_{4.78}^{15.6}$
[0.5,1.2]	[160,180]	1.58	± 1.04	$\pm_{6.6}^{13.8}$	$\pm_{6.88}^{14.3}$
[1.2,1.64]	[160,180]	2.12	± 2.22	$\pm_{5.55}^{13}$	$\pm_{6.12}^{13.5}$
[2.02,3]	[160,180]	1.2	± 1.1	$\pm_{3.92}^{12.9}$	$\pm_{4.35}^{14}$
[3,4]	[160,180]	0.612	± 1.19	$\pm_{4.05}^{12.7}$	$\pm_{4.76}^{14.6}$
[4,5]	[160,180]	0.279	± 1.62	$\pm_{4.39}^{13}$	$\pm_{5.38}^{15.1}$
[5,7]	[160,180]	0.0975	± 2	$\pm_{6.31}^{12.8}$	$\pm_{8.01}^{15.8}$
[0.5,1.2]	[180,200]	1.59	± 1.15	$\pm_{6.62}^{13.7}$	$\pm_{7.67}^{15.8}$
[1.2,1.64]	[180,200]	1.82	± 2.94	$\pm_{4.78}^{13}$	$\pm_{5.65}^{13.5}$
[2.02,3]	[180,200]	0.995	± 1.55	$\pm_{4.33}^{12.7}$	$\pm_{4.66}^{13}$
[3,4]	[180,200]	0.464	± 1.59	$\pm_{5.2}^{12.7}$	$\pm_{5.52}^{13.3}$
[4,5]	[180,200]	0.187	± 2.19	$\pm_{4.41}^{12.7}$	$\pm_{5.17}^{14.1}$
[5,7]	[180,200]	0.0584	± 2.68	$\pm_{3.63}^{13.2}$	$\pm_{5.01}^{15.6}$
[0.5,1.2]	[200,220]	1.54	± 1.38	$\pm_{6.44}^{13.3}$	$\pm_{7.71}^{16.1}$
[1.2,1.64]	[200,220]	1.66	± 3.67	$\pm_{4.33}^{12.9}$	$\pm_{5.8}^{14.2}$
[2.02,3]	[200,220]	0.779	± 1.92	$\pm_{4.2}^{12.8}$	$\pm_{4.84}^{13.7}$
[3,4]	[200,220]	0.326	± 2.04	$\pm_{3.87}^{13.2}$	$\pm_{4.69}^{14.6}$
[4,6]	[200,220]	0.0853	± 2.5	$\pm_{4.05}^{12.7}$	$\pm_{5.72}^{16.4}$
[0.5,1.2]	[220,240]	1.7	± 1.54	$\pm_{6.18}^{13.3}$	$\pm_{6.62}^{14.2}$
[1.2,1.64]	[220,240]	1.68	± 3.12	$\pm_{4.86}^{12.8}$	$\pm_{5.79}^{13.2}$
[2.02,3]	[220,240]	0.693	± 1.92	$\pm_{5.45}^{12.7}$	$\pm_{6.46}^{14.5}$
[3,4]	[220,240]	0.259	± 2.17	$\pm_{4.61}^{12.6}$	$\pm_{5.61}^{15.7}$
[4,6]	[220,240]	0.0566	± 2.77	$\pm_{4.29}^{13.5}$	$\pm_{6.71}^{20.7}$
[0.5,1.2]	[240,260]	1.7	± 1.84	$\pm_{5.82}^{13.5}$	$\pm_{6.22}^{14}$
[1.2,1.64]	[240,260]	1.61	± 3.76	$\pm_{5.98}^{12.9}$	$\pm_{7.06}^{13.6}$

Table A.2 π^- Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[2.02,3]	[240,260]	0.596	± 2.73	$\pm_{5.11}^{12.7}$	$\pm_{6.63}^{15.2}$
[3,5]	[240,260]	0.116	± 2.57	$\pm_{4.18}^{12.9}$	$\pm_{6.32}^{18}$
[0.5,1.2]	[260,280]	1.69	± 1.85	$\pm_{4.98}^{13.3}$	$\pm_{5.3}^{13.4}$
[1.2,1.64]	[260,280]	1.51	± 4.45	$\pm_{5.07}^{12.9}$	$\pm_{7.01}^{14.4}$
[2.02,3]	[260,280]	0.48	± 3.96	$\pm_{4.94}^{12.7}$	$\pm_{6.84}^{16.6}$
[3,5]	[260,280]	0.0987	± 3.54	$\pm_{7.08}^{12.7}$	$\pm_{9.83}^{21.5}$
[0.5,1.2]	[280,300]	1.46	± 1.9	$\pm_{6.48}^{13.2}$	$\pm_{6.79}^{13.6}$
[1.2,1.64]	[280,300]	1.32	± 3.33	$\pm_{4.57}^{12.7}$	$\pm_{5.71}^{13.6}$
[2.02,3]	[280,300]	0.364	± 3.14	$\pm_{5.7}^{12.7}$	$\pm_{6.79}^{16.4}$
[3,5]	[280,300]	0.0728	± 3.15	$\pm_{5.81}^{12.7}$	$\pm_{7.64}^{20.7}$
[0.5,1.2]	[300,320]	1.19	± 2.27	$\pm_{5.29}^{12.9}$	$\pm_{6.57}^{15.6}$
[1.2,1.64]	[300,320]	1.15	± 3.29	$\pm_{4.25}^{13}$	$\pm_{5.68}^{14.7}$
[2.02,3]	[300,320]	0.304	± 3.42	$\pm_{5.6}^{13}$	$\pm_{6.66}^{16.8}$
[3,5]	[300,320]	0.038	± 5.14	$\pm_{3.71}^{12.8}$	$\pm_{6.34}^{13.8}$
[0.5,1.2]	[320,340]	0.92	± 2.39	$\pm_{4.56}^{13}$	$\pm_{6.82}^{20}$
[1.2,1.64]	[320,340]	1.03	± 4.21	$\pm_{3.94}^{13}$	$\pm_{6.11}^{15.4}$
[2.02,4]	[320,340]	0.146	± 3.53	$\pm_{4.05}^{13}$	$\pm_{5.89}^{16.1}$
[0.5,1.2]	[340,360]	0.879	± 2.78	$\pm_{4.12}^{13.1}$	$\pm_{7.13}^{21.3}$
[1.2,1.64]	[340,360]	0.717	± 4.39	$\pm_{8.27}^{12.7}$	$\pm_{11.3}^{16.5}$
[2.02,4]	[340,360]	0.107	± 4.12	$\pm_{5.43}^{12.9}$	$\pm_{8.14}^{17.2}$
[0.5,1.2]	[360,380]	1.03	± 2.83	$\pm_{4.15}^{13.2}$	$\pm_{6.39}^{18.8}$
[1.2,1.64]	[360,380]	0.698	± 4.37	$\pm_{11.7}^{12.9}$	$\pm_{13.9}^{15.1}$
[2.02,4]	[360,380]	0.0797	± 5.06	$\pm_{3.72}^{14.2}$	$\pm_{7.52}^{21.9}$
[0.5,1.64]	[380,400]	0.991	± 3.47	$\pm_{4.33}^{13.9}$	$\pm_{5.9}^{15.8}$
[2.02,4]	[380,400]	0.049	± 5.31	$\pm_{3.65}^{14.3}$	$\pm_{9.1}^{26.8}$

Table A.2 π^- Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{\text{GeV}/c \cdot \text{rad}}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[0.5,1.64]	[400,420]	0.995	± 4.05	$\pm_{5.08}^{15}$	$\pm_{6.88}^{17.1}$
[2.02,4]	[400,420]	0.0421	± 7.29	$\pm_{5.47}^{12.6}$	$\pm_{9.11}^{14.6}$
[0.5,1.64]	[420,440]	0.876	± 4.38	$\pm_{3.84}^{17.2}$	$\pm_{5.91}^{18.2}$

Table A.3: p Multiplicities and Uncertainties

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{\text{GeV}/c \cdot \text{rad}}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[4.32,13]	[0,5]	0.0198	± 2.75	$\pm_{13.5}^{14.8}$	$\pm_{13.6}^{15.1}$
[13,18]	[0,5]	0.0346	± 2.82	$\pm_{11.3}^{14.7}$	$\pm_{11.7}^{15}$
[18,25]	[0,5]	0.0582	± 3.57	$\pm_{9.47}^{12.9}$	$\pm_{10.1}^{13.4}$
[25,31]	[0,5]	0.0894	± 4.73	$\pm_{8.42}^{12.8}$	$\pm_{9.66}^{13.6}$
[31,36]	[0,5]	0.115	± 4.97	$\pm_{5.35}^{13.3}$	$\pm_{7.3}^{14.2}$
[36,41]	[0,5]	0.139	± 2.13	$\pm_{8.69}^{13.4}$	$\pm_{8.92}^{13.7}$
[41,46]	[0,5]	0.167	± 2.2	$\pm_{10.9}^{12.7}$	$\pm_{12.1}^{14}$
[46,50]	[0,5]	0.171	± 2.48	$\pm_{9.14}^{12.7}$	$\pm_{9.74}^{14}$
[50,56]	[0,5]	0.196	± 2.54	$\pm_{6.03}^{12.8}$	$\pm_{6.54}^{13.1}$
[56,61]	[0,5]	0.214	± 2.94	$\pm_{7.7}^{12.6}$	$\pm_{8.24}^{13}$
[61,66]	[0,5]	0.2	± 3.71	$\pm_{9.77}^{12.7}$	$\pm_{10.5}^{13.2}$
[66,71]	[0,5]	0.22	± 4	$\pm_{6.76}^{12.6}$	$\pm_{7.86}^{13.3}$
[71,76]	[0,5]	0.228	± 4.35	$\pm_{8.51}^{12.7}$	$\pm_{9.56}^{13.5}$
[76,81]	[0,5]	0.241	± 3.92	$\pm_{7.39}^{12.6}$	$\pm_{8.36}^{13.2}$
[81,86]	[0,5]	0.289	± 3.48	$\pm_{5.84}^{12.6}$	$\pm_{6.79}^{13.1}$
[86,91]	[0,5]	0.287	± 3.89	$\pm_{5.17}^{12.9}$	$\pm_{6.48}^{13.5}$
[91,96]	[0,5]	0.308	± 3.72	$\pm_{7.91}^{12.7}$	$\pm_{8.74}^{13.3}$
[96,101]	[0,5]	0.286	± 4.04	$\pm_{6.78}^{12.9}$	$\pm_{7.89}^{13.6}$

Table A.3 p Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/ c]	θ [mrad]	Mult. [$\frac{1}{\text{GeV}/c \cdot \text{rad}}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[101,106]	[0,5]	0.307	± 3.94	$\pm_{10.8}^{12.8}$	$\pm_{11.5}^{13.4}$
[106,111]	[0,5]	0.323	± 3.69	$\pm_{12.7}^{12.7}$	$\pm_{13.2}^{13.3}$
[111,116]	[0,5]	0.512	± 2.9	$\pm_{55}^{13.2}$	$\pm_{55.1}^{13.5}$
[116,121]	[0,5]	0.76	± 2.06	$\pm_{87.5}^{13.3}$	$\pm_{87.5}^{13.5}$
[0.5,1.64]	[5,10]	0.0116	± 10	$\pm_{41.7}^{13.2}$	$\pm_{49.4}^{18.4}$
[4.32,10]	[5,10]	0.0436	± 2.94	$\pm_{30}^{14.2}$	$\pm_{30.3}^{14.5}$
[10,15]	[5,10]	0.0771	± 2.28	$\pm_{21.6}^{13.5}$	$\pm_{22.5}^{14.1}$
[15,20]	[5,10]	0.113	± 1.78	$\pm_{15.5}^{13.2}$	$\pm_{15.6}^{13.5}$
[20,25]	[5,10]	0.161	± 1.88	$\pm_{11.3}^{13.3}$	$\pm_{11.4}^{13.7}$
[25,30]	[5,10]	0.217	± 2.3	$\pm_{9.76}^{12.9}$	$\pm_{10}^{13.1}$
[30,35]	[5,10]	0.282	± 3.25	$\pm_{6.13}^{12.8}$	$\pm_{6.94}^{13.2}$
[35,40]	[5,10]	0.328	± 3.34	$\pm_{6.17}^{12.7}$	$\pm_{7.02}^{13.1}$
[40,45]	[5,10]	0.358	± 3.3	$\pm_{5.63}^{12.7}$	$\pm_{6.53}^{13.1}$
[45,50]	[5,10]	0.387	± 3.43	$\pm_{5.93}^{12.6}$	$\pm_{6.86}^{13.1}$
[50,60]	[5,10]	0.36	± 1.41	$\pm_{7.29}^{12.7}$	$\pm_{7.51}^{13.8}$
[60,65]	[5,10]	0.352	± 2.07	$\pm_{6.72}^{12.6}$	$\pm_{7.03}^{12.8}$
[65,70]	[5,10]	0.306	± 2.3	$\pm_{4.89}^{12.6}$	$\pm_{5.4}^{12.8}$
[70,75]	[5,10]	0.268	± 2.62	$\pm_{5.84}^{12.6}$	$\pm_{6.4}^{12.9}$
[75,80]	[5,10]	0.226	± 2.97	$\pm_{7.63}^{12.7}$	$\pm_{8.19}^{13}$
[80,85]	[5,10]	0.185	± 3.29	$\pm_{7.32}^{12.7}$	$\pm_{8.02}^{13.1}$
[85,90]	[5,10]	0.144	± 3.8	$\pm_{3.89}^{15.3}$	$\pm_{5.44}^{15.8}$
[90,95]	[5,10]	0.124	± 4.8	$\pm_{8.28}^{12.7}$	$\pm_{9.57}^{13.6}$
[95,100]	[5,10]	0.0948	± 4.95	$\pm_{6.64}^{13.5}$	$\pm_{8.28}^{14.4}$
[100,105]	[5,10]	0.0794	± 6.24	$\pm_{8.19}^{12.6}$	$\pm_{10.3}^{14.1}$
[105,110]	[5,10]	0.0862	± 5.21	$\pm_{9.44}^{12.9}$	$\pm_{10.8}^{13.9}$

Table A.3 p Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/ c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[110,115]	[5,10]	0.0923	± 5.67	$\pm_{9.61}^{12.6}$	$\pm_{11.2}^{13.8}$
[115,121]	[5,10]	0.0958	± 5.08	$\pm_{64.3}^{13.3}$	$\pm_{64.5}^{14.3}$
[0.5,1.64]	[10,20]	0.0201	± 4.87	$\pm_{17}^{15.4}$	$\pm_{18.4}^{16.7}$
[4.32,7]	[10,20]	0.0786	± 3.33	$\pm_{29.7}^{15.7}$	$\pm_{31}^{16.8}$
[7,9]	[10,20]	0.0946	± 2.74	$\pm_{33}^{14.3}$	$\pm_{35}^{15.3}$
[9,12]	[10,20]	0.116	± 2.05	$\pm_{21.7}^{13.8}$	$\pm_{23.1}^{14.7}$
[12,15]	[10,20]	0.146	± 1.74	$\pm_{17.6}^{14.4}$	$\pm_{18.1}^{14.6}$
[15,18]	[10,20]	0.185	± 1.66	$\pm_{13.8}^{13.5}$	$\pm_{14.1}^{13.7}$
[18,21]	[10,20]	0.21	± 1.54	$\pm_{13.8}^{13.1}$	$\pm_{13.9}^{13.2}$
[21,24]	[10,20]	0.237	± 1.65	$\pm_{10.9}^{12.9}$	$\pm_{11.1}^{13.1}$
[24,27]	[10,20]	0.259	± 1.61	$\pm_{11.7}^{12.8}$	$\pm_{11.8}^{13}$
[27,30]	[10,20]	0.283	± 1.56	$\pm_{11.5}^{12.7}$	$\pm_{11.7}^{13.5}$
[30,34]	[10,20]	0.284	± 1.59	$\pm_{7.79}^{12.7}$	$\pm_{8.21}^{13.9}$
[34,39]	[10,20]	0.292	± 1.79	$\pm_{7.31}^{12.7}$	$\pm_{7.56}^{13.6}$
[39,44]	[10,20]	0.265	± 1.85	$\pm_{7.61}^{12.6}$	$\pm_{7.74}^{13.5}$
[44,49]	[10,20]	0.22	± 1.61	$\pm_{7.17}^{12.6}$	$\pm_{7.5}^{15}$
[49,54]	[10,20]	0.18	± 1.69	$\pm_{5.44}^{12.8}$	$\pm_{6.27}^{16.3}$
[54,59]	[10,20]	0.134	± 1.97	$\pm_{5.85}^{12.9}$	$\pm_{6.84}^{17.6}$
[59,64]	[10,20]	0.106	± 2.17	$\pm_{5.02}^{12.8}$	$\pm_{7.1}^{18.2}$
[64,70]	[10,20]	0.0819	± 2.43	$\pm_{8.17}^{12.7}$	$\pm_{8.88}^{18.9}$
[70,76]	[10,20]	0.0583	± 2.8	$\pm_{6.48}^{12.7}$	$\pm_{10.1}^{19.6}$
[76,85]	[10,20]	0.037	± 2.96	$\pm_{5.82}^{12.8}$	$\pm_{8.94}^{18.8}$
[0.5,1.64]	[20,30]	0.0368	± 3.82	$\pm_{21.8}^{16}$	$\pm_{20.7}^{19.5}$
[4.32,6]	[20,30]	0.124	± 3.42	$\pm_{31.9}^{15.3}$	$\pm_{32.7}^{16.5}$
[6,8]	[20,30]	0.146	± 2.3	$\pm_{29.6}^{14.9}$	$\pm_{32.2}^{15.7}$

Table A.3 p Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/ c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[8,10]	[20,30]	0.156	± 2.06	$\pm_{24.9}^{13.9}$	$\pm_{25.7}^{14.7}$
[10,12]	[20,30]	0.178	± 2.01	$\pm_{21.3}^{13.2}$	$\pm_{22.4}^{14.2}$
[12,14]	[20,30]	0.189	± 1.76	$\pm_{16.4}^{13.5}$	$\pm_{17.7}^{14.4}$
[14,16]	[20,30]	0.199	± 1.85	$\pm_{18.1}^{13.1}$	$\pm_{18.1}^{13.4}$
[16,18]	[20,30]	0.217	± 1.66	$\pm_{14.8}^{12.9}$	$\pm_{15.2}^{13.3}$
[18,20]	[20,30]	0.21	± 1.82	$\pm_{11.9}^{13.3}$	$\pm_{12}^{13.4}$
[20,22]	[20,30]	0.216	± 1.72	$\pm_{12.2}^{12.7}$	$\pm_{12.3}^{12.9}$
[22,24]	[20,30]	0.198	± 1.85	$\pm_{8.63}^{13.9}$	$\pm_{8.81}^{14.1}$
[24,26]	[20,30]	0.197	± 1.9	$\pm_{11.2}^{14.2}$	$\pm_{11.5}^{14.4}$
[26,29]	[20,30]	0.19	± 1.92	$\pm_{9.3}^{12.7}$	$\pm_{9.46}^{12.9}$
[29,34]	[20,30]	0.155	± 1.86	$\pm_{7.66}^{13.2}$	$\pm_{7.78}^{14}$
[34,40]	[20,30]	0.113	± 2.03	$\pm_{7.42}^{12.7}$	$\pm_{7.9}^{13.7}$
[40,46]	[20,30]	0.0728	± 2.71	$\pm_{11.1}^{13.4}$	$\pm_{11.3}^{13.8}$
[46,52]	[20,30]	0.0468	± 4.95	$\pm_{6.27}^{12.6}$	$\pm_{7.99}^{13.6}$
[0.5,1.64]	[30,40]	0.0708	± 3.65	$\pm_{22.5}^{14.2}$	$\pm_{22.6}^{15.2}$
[4.32,6]	[30,40]	0.169	± 5.19	$\pm_{38.7}^{14.8}$	$\pm_{40}^{15.8}$
[6,8]	[30,40]	0.165	± 3.21	$\pm_{28.6}^{14.4}$	$\pm_{34}^{16.5}$
[8,10]	[30,40]	0.18	± 2.73	$\pm_{26.7}^{14.3}$	$\pm_{30.4}^{15.9}$
[10,12]	[30,40]	0.185	± 2.41	$\pm_{22.3}^{13.5}$	$\pm_{25.6}^{15.3}$
[12,13]	[30,40]	0.177	± 2.32	$\pm_{15.7}^{13.8}$	$\pm_{16.7}^{14.5}$
[13,15]	[30,40]	0.197	± 1.61	$\pm_{20}^{12.9}$	$\pm_{21.8}^{14.2}$
[15,18]	[30,40]	0.176	± 1.53	$\pm_{13}^{12.7}$	$\pm_{14.4}^{14.1}$
[18,21]	[30,40]	0.157	± 1.58	$\pm_{11.8}^{12.7}$	$\pm_{12.3}^{13.2}$
[21,26]	[30,40]	0.116	± 1.61	$\pm_{9.01}^{12.8}$	$\pm_{9.17}^{12.9}$
[26,32]	[30,40]	0.0702	± 1.95	$\pm_{12.5}^{13.3}$	$\pm_{12.9}^{13.7}$

Table A.3 p Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/ c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[32,35]	[30,40]	0.0363	± 7.37	$\pm_{3.6}^{16.4}$	$\pm_{8.2}^{18}$
[0.5,1.64]	[40,50]	0.0868	± 3.3	$\pm_{18.1}^{15.1}$	$\pm_{18.3}^{16}$
[4.32,6]	[40,50]	0.196	± 3.63	$\pm_{30.1}^{15.7}$	$\pm_{31}^{16.2}$
[6,8]	[40,50]	0.197	± 2.55	$\pm_{26.3}^{13.9}$	$\pm_{28.9}^{15.5}$
[8,10]	[40,50]	0.196	± 2.39	$\pm_{22.5}^{13.4}$	$\pm_{23.6}^{14.2}$
[10,12]	[40,50]	0.199	± 1.87	$\pm_{19.7}^{13.1}$	$\pm_{21.8}^{14.3}$
[12,15]	[40,50]	0.172	± 1.63	$\pm_{17.2}^{13.3}$	$\pm_{18.8}^{14.3}$
[15,21]	[40,50]	0.109	± 1.45	$\pm_{12.2}^{12.6}$	$\pm_{13}^{13.5}$
[21,26]	[40,50]	0.0549	± 2.1	$\pm_{13.9}^{13.4}$	$\pm_{14.3}^{13.9}$
[0.5,1.64]	[50,60]	0.119	± 3.23	$\pm_{21.5}^{14.5}$	$\pm_{22.5}^{14.9}$
[4.32,6]	[50,60]	0.205	± 3.22	$\pm_{29.7}^{14.2}$	$\pm_{34.3}^{17.3}$
[6,8]	[50,60]	0.19	± 2.64	$\pm_{30.3}^{13.4}$	$\pm_{30.5}^{13.7}$
[8,11]	[50,60]	0.188	± 1.67	$\pm_{21.1}^{13.2}$	$\pm_{21.8}^{13.9}$
[11,15]	[50,60]	0.141	± 1.59	$\pm_{14.4}^{12.7}$	$\pm_{15.2}^{13.4}$
[15,21]	[50,60]	0.0709	± 1.63	$\pm_{12.7}^{12.7}$	$\pm_{12.8}^{13.5}$
[0.5,1.64]	[60,80]	0.132	± 1.45	$\pm_{12.9}^{17.2}$	$\pm_{12.9}^{17.3}$
[4.32,5]	[60,80]	0.243	± 3.63	$\pm_{34.6}^{17.9}$	$\pm_{32.1}^{20.3}$
[5,7]	[60,80]	0.22	± 2.18	$\pm_{29.9}^{13.1}$	$\pm_{30.8}^{13.7}$
[7,9]	[60,80]	0.179	± 1.65	$\pm_{23.8}^{13}$	$\pm_{24.7}^{13.6}$
[9,14]	[60,80]	0.114	± 1.23	$\pm_{18.7}^{13}$	$\pm_{20.4}^{14.1}$
[14,17]	[60,80]	0.0523	± 1.83	$\pm_{15.1}^{12.6}$	$\pm_{14.8}^{13.2}$
[0.5,1.64]	[80,100]	0.19	± 1.1	$\pm_{12.6}^{16.6}$	$\pm_{13.6}^{17.2}$
[4.32,5]	[80,100]	0.275	± 3.53	$\pm_{29.4}^{13.5}$	$\pm_{29.8}^{14}$
[5,9]	[80,100]	0.195	± 2.06	$\pm_{24.3}^{13.2}$	$\pm_{23.7}^{14.1}$
[9,13]	[80,100]	0.0765	± 1.97	$\pm_{18.3}^{12.6}$	$\pm_{17.6}^{13.3}$

Table A.3 p Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/ c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[0.5,1.64]	[100,120]	0.238	± 0.976	$\pm_{14.8}^{14.9}$	$\pm_{16.5}^{16}$
[4.32,5]	[100,120]	0.254	± 3.7	$\pm_{22.4}^{13.5}$	$\pm_{22.3}^{14.2}$
[5,11]	[100,120]	0.13	± 1.88	$\pm_{22.3}^{12.7}$	$\pm_{23.3}^{13.4}$
[0.5,1.64]	[120,140]	0.263	± 1.01	$\pm_{15}^{15}$	$\pm_{15.5}^{15.4}$
[4.32,5]	[120,140]	0.25	± 3.1	$\pm_{22.5}^{13.3}$	$\pm_{23.6}^{14.7}$
[5,9]	[120,140]	0.127	± 2.17	$\pm_{18.9}^{13.1}$	$\pm_{21.5}^{14.8}$
[0.5,1.64]	[140,160]	0.3	± 0.968	$\pm_{12.2}^{15.6}$	$\pm_{12.4}^{15.8}$
[4.32,5]	[140,160]	0.214	± 2.91	$\pm_{23.1}^{13.2}$	$\pm_{25.9}^{14.9}$
[5,8]	[140,160]	0.103	± 2.47	$\pm_{25.2}^{12.8}$	$\pm_{30.3}^{15.4}$
[0.5,1.64]	[160,180]	0.318	± 1.3	$\pm_{12}^{15.5}$	$\pm_{12.7}^{16.1}$
[4.32,7]	[160,180]	0.115	± 2.78	$\pm_{16.5}^{12.9}$	$\pm_{16.8}^{14.5}$
[0.5,1.64]	[180,200]	0.315	± 1.69	$\pm_{12.3}^{14.4}$	$\pm_{11.8}^{15}$
[4.32,6]	[180,200]	0.117	± 3.62	$\pm_{18.8}^{13}$	$\pm_{22}^{15.5}$
[0.5,1.64]	[200,220]	0.31	± 1.97	$\pm_{16.5}^{14.4}$	$\pm_{14.6}^{15.8}$
[4.32,5]	[200,220]	0.107	± 5.55	$\pm_{11.3}^{14.3}$	$\pm_{13}^{18.8}$
[0.5,1.64]	[220,240]	0.31	± 1.68	$\pm_{11.2}^{14.2}$	$\pm_{11.3}^{16.9}$
[4.32,5]	[220,240]	0.0954	± 5.04	$\pm_{7.66}^{12.7}$	$\pm_{9.21}^{15.5}$
[0.5,1.64]	[240,260]	0.323	± 2.39	$\pm_{11.5}^{13.1}$	$\pm_{13.3}^{15}$
[0.5,1.64]	[260,280]	0.324	± 2.85	$\pm_{10.2}^{13.7}$	$\pm_{11.2}^{15.9}$
[0.5,1.64]	[280,300]	0.294	± 2.07	$\pm_{10.8}^{13.3}$	$\pm_{12.4}^{16.9}$
[0.5,1.64]	[300,320]	0.353	± 2.29	$\pm_{12.2}^{13.8}$	$\pm_{12.4}^{15.3}$
[0.5,1.64]	[320,340]	0.367	± 2.31	$\pm_{11.6}^{12.8}$	$\pm_{13.6}^{14.9}$
[0.5,1.64]	[340,360]	0.408	± 2.35	$\pm_{11.2}^{12.9}$	$\pm_{11.3}^{13.1}$
[0.5,1.64]	[360,380]	0.441	± 3.04	$\pm_{11.6}^{13.1}$	$\pm_{12.1}^{13.4}$
[0.5,1.64]	[380,400]	0.42	± 3.62	$\pm_{9.49}^{15}$	$\pm_{10.2}^{15}$

Table A.3 p Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/ c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[0.5,1.64]	[400,420]	0.358	± 5.8	$\pm_{12.5}^{13.4}$	$\pm_{12}^{14.6}$
[0.5,1.64]	[420,440]	0.356	± 12.3	$\pm_{29.7}^{20.6}$	$\pm_{25.5}^{21.5}$

Table A.4: $\bar{p}$ Multiplicities and Uncertainties

p [GeV/ c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[4.32,13]	[0,5]	0.00259	± 8.13	$\pm_{54.6}^{31.8}$	$\pm_{65}^{33.6}$
[13,18]	[0,5]	0.00353	± 8.87	$\pm_{39.6}^{18.2}$	$\pm_{40.4}^{20.2}$
[18,25]	[0,5]	0.00316	± 11.6	$\pm_{24.3}^{19.7}$	$\pm_{26.9}^{22.9}$
[25,31]	[0,5]	0.00197	± 20	$\pm_{4.03}^{30.9}$	$\pm_{20.4}^{36.8}$
[31,36]	[0,5]	0.00229	± 30.2	$\pm_{12.9}^{20.3}$	$\pm_{32.9}^{36.4}$
[4.32,10]	[5,10]	0.00836	± 6.62	$\pm_{52.4}^{21.8}$	$\pm_{60.4}^{25.9}$
[10,15]	[5,10]	0.0105	± 5.24	$\pm_{41.2}^{28.4}$	$\pm_{40.7}^{28.6}$
[15,20]	[5,10]	0.0108	± 5.72	$\pm_{39.5}^{13.3}$	$\pm_{42.8}^{16}$
[20,25]	[5,10]	0.00782	± 6.5	$\pm_{28.5}^{29.6}$	$\pm_{29.9}^{29.9}$
[25,30]	[5,10]	0.00593	± 8.87	$\pm_{28.3}^{21.8}$	$\pm_{29.1}^{26}$
[30,35]	[5,10]	0.00479	± 20	$\pm_{28}^{12.6}$	$\pm_{34.4}^{23.7}$
[35,40]	[5,10]	0.00267	± 25.6	$\pm_{15.8}^{21.7}$	$\pm_{30.1}^{33.5}$
[50,60]	[5,10]	0.00134	± 21.9	$\pm_{28.4}^{12.6}$	$\pm_{35.9}^{25.3}$
[4.32,7]	[10,20]	0.0118	± 6.94	$\pm_{52.7}^{29.3}$	$\pm_{62.3}^{27.5}$
[7,9]	[10,20]	0.0143	± 5.84	$\pm_{39.2}^{25.7}$	$\pm_{39.3}^{26.6}$
[9,12]	[10,20]	0.0187	± 3.96	$\pm_{42.3}^{24.2}$	$\pm_{43.2}^{24.4}$
[12,15]	[10,20]	0.0204	± 4.04	$\pm_{39.8}^{19.3}$	$\pm_{45.6}^{19.8}$
[15,18]	[10,20]	0.0186	± 4.21	$\pm_{42.8}^{16.7}$	$\pm_{47.6}^{17.5}$
[18,21]	[10,20]	0.0159	± 5.01	$\pm_{30.3}^{16.2}$	$\pm_{33.6}^{18.6}$

Table A.4 $\bar{p}$ Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[21,24]	[10,20]	0.0106	± 5.75	$\pm_{19.1}^{23.7}$	$\pm_{20.3}^{24.8}$
[24,27]	[10,20]	0.00958	± 6.48	$\pm_{33.1}^{14.4}$	$\pm_{33.6}^{15.8}$
[27,30]	[10,20]	0.00707	± 7.9	$\pm_{25.3}^{15.9}$	$\pm_{26.6}^{17.9}$
[30,34]	[10,20]	0.00591	± 7.56	$\pm_{26.9}^{28.1}$	$\pm_{26.8}^{30.5}$
[34,39]	[10,20]	0.00327	± 10.8	$\pm_{17.9}^{15.9}$	$\pm_{19.5}^{21}$
[39,44]	[10,20]	0.00199	± 16.4	$\pm_{7.33}^{25.4}$	$\pm_{18}^{30.3}$
[44,49]	[10,20]	0.00185	± 10.4	$\pm_{33.2}^{14.4}$	$\pm_{34}^{18.7}$
[49,54]	[10,20]	0.00102	± 29.4	$\pm_{20.9}^{14.1}$	$\pm_{36.1}^{32.6}$
[4,32,6]	[20,30]	0.0206	± 6.63	$\pm_{53.2}^{28.8}$	$\pm_{56.8}^{32.7}$
[6,8]	[20,30]	0.0235	± 5.85	$\pm_{44}^{27.8}$	$\pm_{64.4}^{31.7}$
[8,10]	[20,30]	0.0256	± 4.43	$\pm_{38.7}^{23.1}$	$\pm_{45.9}^{27.6}$
[10,12]	[20,30]	0.0289	± 4	$\pm_{30.9}^{18.4}$	$\pm_{30.3}^{20}$
[12,14]	[20,30]	0.025	± 4.12	$\pm_{36.8}^{21.2}$	$\pm_{37.6}^{22.5}$
[14,16]	[20,30]	0.0261	± 3.83	$\pm_{34.5}^{13.7}$	$\pm_{37.5}^{16}$
[16,18]	[20,30]	0.0216	± 4.78	$\pm_{25.8}^{16.1}$	$\pm_{27.4}^{16.8}$
[18,20]	[20,30]	0.0162	± 4.34	$\pm_{17.7}^{22.8}$	$\pm_{17.9}^{24.8}$
[20,22]	[20,30]	0.0153	± 5.17	$\pm_{26.9}^{15.1}$	$\pm_{27.2}^{16.1}$
[22,24]	[20,30]	0.00878	± 7.07	$\pm_{22.4}^{27.2}$	$\pm_{26.4}^{26.3}$
[24,26]	[20,30]	0.00927	± 7.51	$\pm_{16.2}^{22.1}$	$\pm_{17.6}^{22.8}$
[26,29]	[20,30]	0.00622	± 7.25	$\pm_{14.5}^{22.8}$	$\pm_{17.9}^{26.8}$
[29,34]	[20,30]	0.00467	± 6.88	$\pm_{10.5}^{23.7}$	$\pm_{12.5}^{27.1}$
[34,40]	[20,30]	0.00163	± 12.8	$\pm_{23.9}^{16.1}$	$\pm_{26.2}^{20.6}$
[40,46]	[20,30]	0.00119	± 23.6	$\pm_{23}^{15}$	$\pm_{33}^{27.9}$
[4,32,6]	[30,40]	0.0275	± 10.7	$\pm_{55.8}^{28}$	$\pm_{59.6}^{34.2}$
[6,8]	[30,40]	0.0295	± 6.05	$\pm_{49.2}^{18.7}$	$\pm_{59.6}^{23.9}$

Table A.4 $\bar{p}$ Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[8,10]	[30,40]	0.0285	± 5.73	$\pm_{30.9}^{21}$	$\pm_{35.3}^{22.2}$
[10,12]	[30,40]	0.0301	± 4.84	$\pm_{33.3}^{19.8}$	$\pm_{34.5}^{21.2}$
[12,13]	[30,40]	0.0277	± 4.79	$\pm_{32}^{20}$	$\pm_{34.6}^{23}$
[13,15]	[30,40]	0.0226	± 4.27	$\pm_{25.2}^{17.7}$	$\pm_{27.7}^{20.8}$
[15,18]	[30,40]	0.0163	± 4.7	$\pm_{20.4}^{21.1}$	$\pm_{25}^{22}$
[18,21]	[30,40]	0.0106	± 5.63	$\pm_{26.2}^{14}$	$\pm_{34.8}^{18.2}$
[21,26]	[30,40]	0.00665	± 6.04	$\pm_{25.4}^{15.2}$	$\pm_{29}^{18.4}$
[26,32]	[30,40]	0.00206	± 7.84	$\pm_{8.37}^{15.9}$	$\pm_{12.7}^{20.1}$
[0.5,1.64]	[40,50]	0.0283	± 8.48	$\pm_{53}^{54.1}$	$\pm_{53.6}^{54.7}$
[4.32,6]	[40,50]	0.0294	± 8.95	$\pm_{52.9}^{18.1}$	$\pm_{66.9}^{24.7}$
[6,8]	[40,50]	0.0336	± 6.09	$\pm_{40.6}^{20.1}$	$\pm_{49.5}^{25}$
[8,10]	[40,50]	0.0319	± 5.5	$\pm_{40}^{21.2}$	$\pm_{55.4}^{29.8}$
[10,12]	[40,50]	0.0264	± 4.37	$\pm_{28.9}^{15.4}$	$\pm_{32.3}^{16.7}$
[12,15]	[40,50]	0.0191	± 4.3	$\pm_{20.2}^{19}$	$\pm_{21.6}^{19.4}$
[15,21]	[40,50]	0.0113	± 3.66	$\pm_{21.4}^{13.3}$	$\pm_{25.4}^{16}$
[21,26]	[40,50]	0.00314	± 6.44	$\pm_{10.3}^{18.2}$	$\pm_{11.9}^{19.3}$
[4.32,6]	[50,60]	0.0307	± 8.93	$\pm_{56}^{24.6}$	$\pm_{56.7}^{30.5}$
[6,8]	[50,60]	0.0327	± 5.63	$\pm_{45.2}^{17.1}$	$\pm_{54.7}^{21.5}$
[8,11]	[50,60]	0.0266	± 4.03	$\pm_{22.7}^{16.9}$	$\pm_{27}^{19.7}$
[11,15]	[50,60]	0.0173	± 3.83	$\pm_{23.9}^{14.2}$	$\pm_{27.4}^{16}$
[15,21]	[50,60]	0.00649	± 4.61	$\pm_{18.5}^{13.7}$	$\pm_{21.9}^{16.1}$
[4.32,5]	[60,80]	0.0386	± 7.91	$\pm_{54.4}^{48.5}$	$\pm_{55.3}^{48.9}$
[5,7]	[60,80]	0.0336	± 3.95	$\pm_{41}^{23.6}$	$\pm_{55.4}^{24.7}$
[7,9]	[60,80]	0.0308	± 3.2	$\pm_{35.3}^{14.8}$	$\pm_{39.1}^{17.5}$
[9,14]	[60,80]	0.0138	± 3.05	$\pm_{17.4}^{17}$	$\pm_{21.8}^{19.7}$

Table A.4 $\bar{p}$ Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{\text{GeV}/c \cdot \text{rad}}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[14,17]	[60,80]	0.00529	± 4.65	$\pm_{15.5}^{13.3}$	$\pm_{16}^{14.4}$
[4.32,5]	[80,100]	0.038	± 8.1	$\pm_{51.4}^{20.6}$	$\pm_{52.9}^{21.8}$
[5,9]	[80,100]	0.0243	± 4.35	$\pm_{32.3}^{18.1}$	$\pm_{32.7}^{19.1}$
[9,13]	[80,100]	0.0101	± 4.04	$\pm_{22.7}^{13}$	$\pm_{25.5}^{14.8}$
[4.32,5]	[100,120]	0.0301	± 9.17	$\pm_{33.5}^{25.2}$	$\pm_{37.9}^{25.7}$
[5,11]	[100,120]	0.0144	± 4.21	$\pm_{28.8}^{16.6}$	$\pm_{28.6}^{17.6}$
[4.32,5]	[120,140]	0.0301	± 8.19	$\pm_{48.7}^{16.2}$	$\pm_{73.8}^{22.8}$
[5,9]	[120,140]	0.0121	± 6.73	$\pm_{28.2}^{17.9}$	$\pm_{27.8}^{20.6}$
[4.32,5]	[140,160]	0.0224	± 10	$\pm_{45.2}^{13.9}$	$\pm_{62.4}^{24.6}$
[5,8]	[140,160]	0.00977	± 8.12	$\pm_{30.1}^{16.9}$	$\pm_{39.2}^{20.7}$
[4.32,7]	[160,180]	0.00861	± 10.2	$\pm_{36.7}^{13}$	$\pm_{40.2}^{17.1}$
[4.32,6]	[180,200]	0.00959	± 12.7	$\pm_{8.86}^{21.5}$	$\pm_{15.5}^{25}$
[4.32,5]	[200,220]	0.00624	± 27.3	$\pm_{32.6}^{14.5}$	$\pm_{42.5}^{30.9}$
[4.32,5]	[220,240]	0.00526	± 23.8	$\pm_{8.16}^{36.2}$	$\pm_{25.1}^{43.3}$
[0.5,1.64]	[380,400]	0.00255	± 47.7	$\pm_{17.1}^{20.3}$	$\pm_{50.7}^{51.9}$

Table A.5: K^+ Multiplicities and Uncertainties

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{\text{GeV}/c \cdot \text{rad}}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[4.32,21]	[0,5]	0.0167	± 5.41	$\pm_{5.04}^{13}$	$\pm_{7.62}^{14.7}$
[21,36]	[0,5]	0.0129	± 28.1	$\pm_{7.93}^{12.6}$	$\pm_{29.2}^{30.8}$
[4.32,11]	[5,10]	0.0398	± 4.24	$\pm_{4.87}^{12.6}$	$\pm_{6.77}^{14.3}$
[11,17]	[5,10]	0.0513	± 5.02	$\pm_{3.67}^{13.2}$	$\pm_{6.31}^{14.5}$
[17,24]	[5,10]	0.0569	± 5.68	$\pm_{4.42}^{13.3}$	$\pm_{7.45}^{15.2}$
[24,32]	[5,10]	0.035	± 17.8	$\pm_{3.72}^{12.6}$	$\pm_{18.3}^{23}$

Table A.5 K^+ Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[4,32,9]	[10,20]	0.0721	± 3.41	$\pm_{3.77}^{12.6}$	$\pm_{5.13}^{13.3}$
[9,13]	[10,20]	0.0849	± 3.12	$\pm_{3.95}^{12.6}$	$\pm_{5.12}^{13.4}$
[13,17]	[10,20]	0.0842	± 3.64	$\pm_{4.1}^{12.6}$	$\pm_{5.61}^{13.7}$
[17,22]	[10,20]	0.0808	± 4.23	$\pm_{4.3}^{12.6}$	$\pm_{6.1}^{13.5}$
[22,27]	[10,20]	0.0666	± 6.58	$\pm_{4.03}^{13.6}$	$\pm_{7.84}^{15.5}$
[27,34.3]	[10,20]	0.0327	± 10.7	$\pm_{4.49}^{30}$	$\pm_{12.6}^{28.5}$
[34.3,41]	[10,20]	0.0336	± 25.4	$\pm_{6.92}^{12.6}$	$\pm_{26.4}^{28.4}$
[4,32,7]	[20,30]	0.0761	± 6.57	$\pm_{4.84}^{12.6}$	$\pm_{8.7}^{17.6}$
[7,10]	[20,30]	0.103	± 3.5	$\pm_{5.94}^{12.6}$	$\pm_{7.48}^{14.7}$
[10,13]	[20,30]	0.113	± 3.46	$\pm_{4.42}^{13}$	$\pm_{5.62}^{13.5}$
[13,17]	[20,30]	0.098	± 3.7	$\pm_{5.46}^{12.9}$	$\pm_{6.6}^{13.5}$
[17,20]	[20,30]	0.0865	± 5.35	$\pm_{4.2}^{12.8}$	$\pm_{6.85}^{14.2}$
[20,28]	[20,30]	0.0619	± 4.61	$\pm_{4.21}^{15.2}$	$\pm_{6.64}^{16.5}$
[28,36]	[20,30]	0.0306	± 11.6	$\pm_{3.6}^{13.9}$	$\pm_{12.2}^{18.1}$
[36,47]	[20,30]	0.0136	± 17.1	$\pm_{3.6}^{13.8}$	$\pm_{17.5}^{22}$
[4,32,7]	[30,40]	0.0769	± 5.39	$\pm_{6.45}^{12.6}$	$\pm_{8.77}^{18.7}$
[7,10]	[30,40]	0.111	± 3.55	$\pm_{4.98}^{12.6}$	$\pm_{6.8}^{15.1}$
[10,13]	[30,40]	0.121	± 3.29	$\pm_{4.85}^{12.6}$	$\pm_{6.03}^{13.6}$
[13,15]	[30,40]	0.103	± 3.83	$\pm_{7.47}^{12.6}$	$\pm_{8.94}^{14.2}$
[15,23]	[30,40]	0.0633	± 3.22	$\pm_{5.88}^{12.6}$	$\pm_{6.74}^{15}$
[23,35]	[30,40]	0.0198	± 5.28	$\pm_{3.72}^{17.6}$	$\pm_{6.96}^{19.4}$
[4,32,7]	[40,50]	0.0707	± 9.73	$\pm_{5.23}^{12.6}$	$\pm_{12.5}^{21.5}$
[7,10]	[40,50]	0.101	± 5.83	$\pm_{6.92}^{12.6}$	$\pm_{8.62}^{15.2}$
[10,15]	[40,50]	0.0992	± 2.5	$\pm_{4.8}^{12.6}$	$\pm_{5.57}^{13.3}$
[15,21]	[40,50]	0.0462	± 2.86	$\pm_{5.42}^{12.6}$	$\pm_{7.1}^{15.4}$

Table A.5 K^+ Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[21,28]	[40,50]	0.0149	± 6.22	$\pm_{8.11}^{15.2}$	$\pm_{13.9}^{21.3}$
[0.5,0.95]	[50,60]	0.00924	± 1.77	$\pm_{100}^{12.6}$	$\pm_{100}^{12.7}$
[4.32,7]	[50,60]	0.0771	± 13.4	$\pm_{5.58}^{16.8}$	$\pm_{15.8}^{25.5}$
[7,11]	[50,60]	0.0958	± 7.07	$\pm_{4.18}^{12.6}$	$\pm_{8.25}^{14.6}$
[11,15]	[50,60]	0.0708	± 4.17	$\pm_{4.62}^{12.6}$	$\pm_{6.12}^{13.7}$
[15,23]	[50,60]	0.0229	± 3.64	$\pm_{5.37}^{12.6}$	$\pm_{7.75}^{16.7}$
[4.32,7]	[60,80]	0.0782	± 9.43	$\pm_{3.97}^{22.2}$	$\pm_{11.3}^{27.8}$
[7,12]	[60,80]	0.0704	± 4.63	$\pm_{5.61}^{12.6}$	$\pm_{7.45}^{13.9}$
[12,19]	[60,80]	0.0222	± 4.42	$\pm_{5.53}^{12.7}$	$\pm_{7.96}^{15.8}$
[4.32,5]	[80,100]	0.232	± 15.1	$\pm_{4.29}^{12.6}$	$\pm_{15.7}^{19.7}$
[5,13]	[80,100]	0.0482	± 5.94	$\pm_{4.78}^{14.5}$	$\pm_{8.43}^{17.9}$
[13,15]	[80,100]	0.0107	± 14.5	$\pm_{13.2}^{12.6}$	$\pm_{19.6}^{19.2}$
[4.32,5]	[100,120]	0.119	± 18.7	$\pm_{3.62}^{12.8}$	$\pm_{19.1}^{22.7}$
[5,13]	[100,120]	0.0261	± 6.98	$\pm_{3.6}^{13.5}$	$\pm_{9.22}^{22.5}$
[5,11]	[120,140]	0.00221	± 5.25	$\pm_{9.28}^{12.7}$	$\pm_{71}^{96.4}$
[5,9]	[140,160]	0.0175	± 20.6	$\pm_{6.15}^{12.6}$	$\pm_{21.5}^{24.1}$

Table A.6: K^- Multiplicities and Uncertainties

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[4.32,21]	[0,5]	0.0101	± 3.85	$\pm_{3.67}^{12.7}$	$\pm_{5.32}^{13.3}$
[21,36]	[0,5]	0.00631	± 12.2	$\pm_{3.62}^{12.6}$	$\pm_{12.7}^{17.5}$
[36,46]	[0,5]	0.00249	± 21.2	$\pm_{3.6}^{34.7}$	$\pm_{21.5}^{40.7}$
[4.32,11]	[5,10]	0.0277	± 3.67	$\pm_{4.06}^{12.6}$	$\pm_{5.6}^{13.9}$
[11,17]	[5,10]	0.0297	± 4.41	$\pm_{3.62}^{13.4}$	$\pm_{5.78}^{14.5}$

Table A.6 K^- Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[17,24]	[5,10]	0.023	± 5.65	$\pm_{6.39}^{13.1}$	$\pm_{9.02}^{15.4}$
[24,32]	[5,10]	0.0173	± 9.31	$\pm_{4.22}^{12.6}$	$\pm_{10.3}^{16}$
[32,48]	[5,10]	0.00421	± 17.6	$\pm_{3.64}^{13.1}$	$\pm_{18}^{21.9}$
[4.32,9]	[10,20]	0.0539	± 2.19	$\pm_{4.29}^{12.6}$	$\pm_{4.91}^{13.5}$
[9,13]	[10,20]	0.0569	± 2.46	$\pm_{4.04}^{12.7}$	$\pm_{4.86}^{13.4}$
[13,17]	[10,20]	0.0535	± 2.55	$\pm_{7.18}^{12.9}$	$\pm_{7.87}^{13.5}$
[17,22]	[10,20]	0.0416	± 3.49	$\pm_{5.36}^{12.7}$	$\pm_{6.73}^{14.1}$
[22,27]	[10,20]	0.0275	± 5.13	$\pm_{5.66}^{12.6}$	$\pm_{7.79}^{14}$
[27,34.3]	[10,20]	0.0163	± 6.2	$\pm_{3.69}^{13}$	$\pm_{7.4}^{15.4}$
[34.3,41]	[10,20]	0.0077	± 8.72	$\pm_{3.65}^{17.1}$	$\pm_{9.78}^{20.5}$
[41,59]	[10,20]	0.00141	± 14.9	$\pm_{3.6}^{15.2}$	$\pm_{16.3}^{28.8}$
[4.32,7]	[20,30]	0.063	± 3.48	$\pm_{6.92}^{12.7}$	$\pm_{9.23}^{16}$
[7,10]	[20,30]	0.078	± 2.19	$\pm_{5.56}^{12.6}$	$\pm_{6.26}^{13.5}$
[10,13]	[20,30]	0.069	± 2.27	$\pm_{4.21}^{12.7}$	$\pm_{4.86}^{13.2}$
[13,17]	[20,30]	0.0557	± 2.91	$\pm_{4.8}^{12.6}$	$\pm_{5.76}^{13.4}$
[17,20]	[20,30]	0.0424	± 4.18	$\pm_{3.85}^{12.7}$	$\pm_{5.74}^{13.7}$
[20,28]	[20,30]	0.0206	± 4.25	$\pm_{3.65}^{12.9}$	$\pm_{6.08}^{15.9}$
[28,36]	[20,30]	0.00723	± 7.15	$\pm_{5.13}^{17.7}$	$\pm_{9.95}^{21.5}$
[36,47]	[20,30]	0.00221	± 24.1	$\pm_{3.67}^{12.6}$	$\pm_{24.4}^{27.2}$
[4.32,7]	[30,40]	0.0663	± 3.26	$\pm_{6.3}^{12.6}$	$\pm_{8.89}^{16.9}$
[7,10]	[30,40]	0.0927	± 1.9	$\pm_{5.2}^{12.6}$	$\pm_{5.46}^{13.4}$
[10,13]	[30,40]	0.0691	± 2.23	$\pm_{5.54}^{12.6}$	$\pm_{6.13}^{13.2}$
[13,15]	[30,40]	0.056	± 3.36	$\pm_{5.37}^{12.6}$	$\pm_{6.46}^{13.4}$
[15,23]	[30,40]	0.031	± 2.65	$\pm_{5.04}^{12.7}$	$\pm_{5.74}^{14.2}$
[23,35]	[30,40]	0.00579	± 5.44	$\pm_{8.84}^{13.5}$	$\pm_{12.9}^{18}$

Table A.6 K^- Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{\text{GeV}/c\text{-rad}}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[4.32,7]	[40,50]	0.0767	± 5.44	$\pm_{5.72}^{12.6}$	$\pm_{9.05}^{16.9}$
[7,10]	[40,50]	0.0821	± 3.32	$\pm_{7.9}^{12.6}$	$\pm_{7.88}^{13.9}$
[10,15]	[40,50]	0.0561	± 2.12	$\pm_{5.95}^{12.6}$	$\pm_{6.12}^{13.2}$
[15,21]	[40,50]	0.0203	± 2.95	$\pm_{6.99}^{12.8}$	$\pm_{8.9}^{15.6}$
[21,28]	[40,50]	0.00548	± 5.68	$\pm_{4.06}^{16.1}$	$\pm_{9.11}^{24}$
[4.32,7]	[50,60]	0.0945	± 6.35	$\pm_{5.61}^{12.6}$	$\pm_{8.66}^{14.7}$
[7,11]	[50,60]	0.0709	± 4.52	$\pm_{6.18}^{12.6}$	$\pm_{7.41}^{13.5}$
[11,15]	[50,60]	0.0375	± 3.11	$\pm_{5.38}^{13.1}$	$\pm_{6.35}^{13.7}$
[15,23]	[50,60]	0.0108	± 3.49	$\pm_{7.2}^{12.6}$	$\pm_{10.1}^{16.9}$
[4.32,7]	[60,80]	0.0777	± 3.74	$\pm_{4.86}^{12.6}$	$\pm_{6.19}^{13.5}$
[7,12]	[60,80]	0.0461	± 2.67	$\pm_{4.84}^{12.6}$	$\pm_{5.54}^{12.9}$
[12,19]	[60,80]	0.0112	± 2.55	$\pm_{5.33}^{12.8}$	$\pm_{7.13}^{16.1}$
[4.32,5]	[80,100]	0.0852	± 7.09	$\pm_{5.52}^{12.7}$	$\pm_{11.5}^{21.9}$
[5,13]	[80,100]	0.0322	± 3.28	$\pm_{5.46}^{12.8}$	$\pm_{7.17}^{15.2}$
[13,15]	[80,100]	0.00531	± 23.7	$\pm_{3.6}^{14.1}$	$\pm_{24}^{27.6}$
[0.5,0.95]	[100,120]	0.0146	± 1.49	$\pm_{3.6}^{17.3}$	$\pm_{3.9}^{17.4}$
[4.32,5]	[100,120]	0.0693	± 7.62	$\pm_{6.45}^{12.8}$	$\pm_{11.1}^{17.7}$
[5,13]	[100,120]	0.0203	± 3.76	$\pm_{4.72}^{13.4}$	$\pm_{7.28}^{17.5}$
[4.32,5]	[120,140]	0.0474	± 12.4	$\pm_{4.09}^{12.6}$	$\pm_{13.1}^{17.7}$
[5,11]	[120,140]	0.0145	± 5.25	$\pm_{6.02}^{13.3}$	$\pm_{9.54}^{17.9}$
[4.32,5]	[140,160]	0.0441	± 9.71	$\pm_{3.6}^{15.2}$	$\pm_{10.4}^{18.1}$
[5,9]	[140,160]	0.015	± 6.99	$\pm_{11.1}^{12.6}$	$\pm_{15.9}^{17.7}$
[4.32,5]	[160,180]	0.0298	± 12.3	$\pm_{9.56}^{14.2}$	$\pm_{20.8}^{26.1}$
[5,8]	[160,180]	0.013	± 10	$\pm_{12.6}^{12.6}$	$\pm_{15.6}^{19.1}$
[0.5,0.95]	[180,200]	0.00706	± 2.54	$\pm_{3.6}^{50.2}$	$\pm_{4.4}^{50.2}$

Table A.6 K^- Multiplicities and Uncertainties – *Continued from previous page*

p [GeV/c]	θ [mrad]	Mult. [$\frac{1}{GeV/c \cdot rad}$]	Uncorr. Unc. [%]	Corr. Unc. [%]	Total Unc. [%]
[4.32,8]	[180,200]	0.00752	± 15.9	$\pm_{6.8}^{15.2}$	$\pm_{22.4}^{34.1}$
[0.5,0.95]	[200,220]	0.0238	± 42.1	$\pm_{4.69}^{12.6}$	$\pm_{42.3}^{43.9}$
[4.32,7]	[200,220]	0.00715	± 16.2	$\pm_{4.32}^{14.7}$	$\pm_{17.4}^{28.9}$
[0.5,0.95]	[240,260]	0.0105	± 7.91	$\pm_{8.18}^{12.6}$	$\pm_{11.4}^{14.9}$
[0.5,0.95]	[260,280]	0.0122	± 15.2	$\pm_{17.3}^{12.6}$	$\pm_{23.1}^{19.8}$
[0.5,0.95]	[280,300]	0.0153	± 20.6	$\pm_{3.6}^{21.9}$	$\pm_{20.9}^{30.1}$