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Statistical and experimental systematic uncertainties on EWPOs at the Z pole in FCC-ee

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Abstract. The FCC-ee Z run, with a total of 6 10^{12} Z produced, provides an extraordinary opportunity to test the existence of more particle or phenomena via precision electroweak measurements. This note gives an estimate of realistic precisions that could be achieved on the experimental side, and clarifies the assumptions. One assumption that is essential is the absence of any flavour changing neutral currents (FCNC), since double tags are routinely used throughout. Here, only the experimental precisions that are likely to be achieved are discussed, setting the target for the accuracy with which the theoretical calculations must be performed. Achieving these precisions in real data is conditional to a number of detector and accelerator requirements being fulfilled.

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1 Introduction

We have estimated the experimental errors, statistics and systematics, on di-fermion measurements based on the experience and tricks collected at LEP, and taking into account recent improvements obtaained at FCC-ee, namely:

- A factor 300'000 improvement in statistics w.r.t. LEP at the Z pole, with four experiments;
- Improvements in the determination of point-to-point centre-of-mass energies uncertainties, presently down to 20 keV;
- Improvements in determination of the luminosity, the luminosity increase making it possible to use the di-photon final state, with a potential precision of better than 2 10-5;
- Improvements in the design of the detectors towards high precision, with dedicated definition of fiducial volume and absence of trigger;
- Last but not least, large improvement in b/c/s tagging efficiency and resulting major improvement in the purity of the selected samples [1]. This capitalizes, with respect to the LEP state-of-the-art, on the much smaller beam size and smaller vacuum pipe radius, with great progress in vertex detector impact parameter accuracy; systematic hadron identification, with dE/dx and Time-of-flight, all combined with a machine learning tool. This is a major breakthrough over the FCC CDR studies and leads to improvement in the heavy flavor EW observables that potentially exceeds the purely statistical improvement.

As a consequence, one should be able to extract efficiencies for event selection, flavor tagging and charge assignment from 'double-tags' from the data themselves; this is also true for other required information contained in the data themselves, and redundant cross-checks. It is thus expected that experimental systematic errors will be statistically limited. The progress is particularly dramatic on the electroweak Heavy Flavour observables such as $\mathrm{R_{b,c,s}}$ and $\mathrm{A_{FB}^{b,c,s}}$

2 Assumptions ad caveats

Some assumptions and caveats are worth noting:

1 Double tag methods are used systematically, both for detection efficiency and for charge determination. This explicitly assumes that all di-fermion production arises from pairs of identical fermions, i.e. assumes NO FLAVOUR CHANGING NEUTRAL CURRENTS (FCNC). The search for FCNC is a very interesting project that has not been investigated at the Z so far.

2 These are projected experimental errors only. Not theoretical uncertainty has been discussed. It is understood that the theoretical community sets its target to match the statistical precision by the time of data taking (2045). This concerns mostly QED and more specifically initial state radiation; QCD effects and fragmentation are limited to the final state and are easier to constrain directly from the data.
3 No full simulation has been effected at the statistical level of the data. In practice, it is more important at this stage to understand what needs to be known precisely, in order to design the detectors and their construction quality accordingly. A number of experimental cross-checks are expected to be feasible, but have not actually been done. They absolutely will need to be performed before the detector designs are finalized. At the later time, after data are already taken, tests will be surely performed to check agreement between data and simulations, but if they exhibit discrepancies, either a data fault can be found, or simulations can be reasonably adjusted in a consistent way, or the systematic errors will need to be increased.
4 The analyses are not optimized: further optimization might yield better results. Most results below follow the methods and notations developed during the LEP era and summarized in [2], or specific ALEPH EW papers, which this author understands better for having actively contributed.

3 Dominant and common systematic uncertainties

3.1 Total cross-section

For the hadronic cross-section at the peak of the resonance, the following systematic errors can be considered.

1 The hadronic event selection efficiency. The efficiency is very high, typically over 99%. This in principle is calculated by simulation and the efficiency will residually depend on the fragmentation model. A workaround was devised at LEP, see e.g. [3], which consists in using the data sample of then 3.10^6 large angle events and superposing for each event a family of simulated low angle acceptance cones. The method has very small uncertainties. Residual imperfections associated with the size and knowledge of the effective low angle cut can be reduced by design, with the implementation of a low angle tracker and calorimeters, resulting in considerably reduced inefficiency. Uncertainties associated with residual backgrounds will scale with available statistics. We have considered that the uncertainty on the efficiency evaluation, based on data themselves, will remain below the statistical precision.
2 The luminosity measurement. A novelty at FCC-ee comes with the high statistics available, making it possible to measure luminosity with $e^+e^- \rightarrow \gamma\gamma$ events, with a statistical accuracy of the order of 1.5 10^{-5} depending on the low angle cut used. The dominant experimental uncertainty is expected to stem from the precise knowledge of the low angle acceptance cut. This has been discussed in [4]. An acceptance cut at 15 degrees needs to be known with an accuracy of about 10 microns at a distance of 2.5 m (4 microradians), and will result in a systematic uncertainty of 1.5 10^{-5}. The full analysis of the theoretical accuracy of the $e^+e^- \rightarrow \gamma\gamma$ cross-section within acceptance remains to be performed but the prospects are promising at that level [5].
3 The peak hadronic cross-section is used to estimate the invisible Z partial width, from which the number of neutrinos is extracted (see [2]. This extraction requires also knowledge of the ratio of hadronic to leptonic partial widths, which will be discussed below to be directly measurable with sufficient precision.
4 Last but not least, the theoretical calculation of the measurable peak cross-section for a given value of the Z mass, total width and leptonic partial width, including the full effect of QED radiation, is also an essential input that will be required at the 10^{-5} level of precision.

When putting together the measurements of the peak cross-section, the Z width and the lepton to hadron partial width ratios, a precision on the number of neutrinos (as calculated in [2], section 7.2.2) is $\Delta N_\nu = \pm 0.00015$, an improvement of a factor 45 over the present uncertainty.

4 Di-lepton observables

Statistics of 1.8 10^{11} events on each channel will be available with around 95% acceptance for each of the three di-lepton final state, leading to a statistical accuracy of ± 2.3 10^{-6}. The electron channel, and to a smaller extent the tau channel, has to cope with t-channel subtraction [8], which worsens the statistical precision by 20% for the tau and 40% for the electron channel, respectively.

4.1 Leptonic branching ratios

1 The quantity that is needed to extract the number of neutrinos or the leptonic partial width from the total width and peak hadronic cross-section is the ratio $R_\ell \simeq \Gamma_{had}/\Gamma_\ell$. The normalization to hadrons involves practically no

Table 1. Z-pole line-shape measurements

Observable	present value	±	uncertainty	FCC-ee **Stat.**	FCC-ee Syst.	Comment and leading uncertainty
m_Z (keV)	91 186 700	±	2000	**4**	100	From Z line shape scan Beam energy calibration
Γ_Z (keV)	2 495 200	±	2300	**4**	12	From Z line shape scan Beam energy calibration
$1/\alpha_{\rm QED}(m_Z^2)$ ($\times 10^3$)	128 952	±	14	**3.9** **0.8**	small small, t.b.c.	From $A_{\rm FB}^{\mu}$ off peak [6] From $\mathcal{R}_{e^-/\ell^\pm}(\theta)$ on peak [7] QED&EW uncert. dominate
$\sigma^0_{\rm had}$ (pb)	41 541	±	37	**0.1**	0.8	Z Peak hadronic cross section luminosity measurement

systematic uncertainty compared to the di-lepton statistical precision. The two-lepton final state is a practically background-free and experimental tests based on the presence of two sides of the event reduce to a negligible level all issues of inefficiencies as well as particle and charge misidentification in a statistically limited way. The muon channel is the most efficient and accurate. The electron channel requires t-channel subtraction, which is greatly helped by the dominance of the Z pole and the abundance of t-channel events on the off-peak data taking points, but nevertheless leads to an increase of the statistical uncertainty. The di-tau channel is more complicated, and has a somewhat larger statistical uncertainty but benefits from the mutually independent decay modes of the two taus.

2 The dominant systematic uncertainty results from the knowledge of the low angle acceptance cut. This is a similar exercise than for the di-photon channel and is described in [4]. Remarkably the same accuracy that provides the precision for the di-photon channel provides an accuracy that matches the statistical precision for the di-leptons. This irreducible common experimental accuracy has been assumed for the di-lepton experimental error.

3 The ratio R_ℓ seems essentially free of other uncertainties at the Z pole. Two-photon processes are efficiently monitored, and if needed, subtracted using the off-peak data points, since they are not resonant. A high efficiency and trigger-free detector will be able to correct for imperfections and reach the quoted precision using the data themselves.

4.2 Forward backward asymmetries

The forward backward asymmetries using di-leptons have the same qualities given above. The asymmetry of interest is the value at the Z pole

$$A_{\rm FB}^{\rm f\bar{f}} = \frac{3}{4}\mathcal{A}_{\rm e}\mathcal{A}_{\rm f} \qquad (1)$$

Where $\mathcal{A}_{\rm f} \equiv \frac{g_L^{f\,2} - g_R^{f\,2}}{g_L^{f\,2} + g_R^{f\,2}} = \frac{2 g_V^f g_A^f}{g_V^{f\,2} + g_A^{f\,2}}$ is the left-right asymmetry of the chiral couplings of fermion f. The chiral couplings are related to the vector and axial-vector couplings by the relations $g_V = g_R - g_L$, $g_A = g_R + g_L$. As seen in the equation above, the measurement of $\mathcal{A}_{\rm e}$ for the electron is of particular importance: being the coupling from the beam particles to the Z in the initial state, it appears in all asymmetries for any final state. It must be divided out, if one wants to extract the same asymmetry for the various leptons or quarks from their forward backward asymmetries without universality assumption. The effective electroweak mixing angle is *defined* by the measurement of $\mathcal{A}_{\rm e}$ using the relation $\sin^2\theta_W^{\rm eff} \equiv \frac{1}{4}(1 - g_V^e/g_A^e)$ for the electron, or, assuming universality, for leptons.

The experimental sources of uncertainty on the forward backward asymmetries for leptons are expected to be very small, since any experimental effect would have to be both charge asymmetric and forward backward asymmetric to affect the result.

The main source of uncertainty arises from the rapid variation of the asymmetry around the resonance. Consequently, the knowledge of the energy at which the asymmetry is exactly as expected at the Z peak enters in two ways.

1 The point-to-point uncertainties on the centre-of-mass energies affect the peak energy point in a way which is different to the Z mass, which is determined from the off-peak energy points. This uncertainty is dominant and irreducible. It is quoted as the dominant experimental error on the asymmetries, as described in [9],[2]. Recent studies have shown that the point-to-point uncertainties in the Z line-shape scan can be reduced to a precision of the order of 20 keV.

2 The initial state radiation modifies the line-shape, in particular by moving the point of maximum cross-section with respect to the nominal Breit-Wigner Z mass. Of course the point where the asymmetry takes the Z-pole value is also moved in a similar way, but any residual difference between the two must be known with a great accuracy. This requires an initial state radiation simulation that provides a precision of better than about 10 keV on this difference. Similar precision is required for the Z width measurement from the Z line shape, and is probably more constraining.

4.3 Summary on the di-lepton measurements

The dominant systematic errors have been identified and turned into detector and accelerator requirements.

– for the lepton total cross-section, the low angle cut must be known with a precision of 4 microradian, which is also adequate to provide a measurement of the luminosity using the two photon final state with a precision of 210−5.

– for the forward backward asymmetries, the point to point errors on the centre-of-mass determination is assummed to be less than 20 keV. Furrther improvement on this number will directly improve all forward-backward leptonic asymmetries.

– for all measurements, dedicated efforts must be placed on QED corrections to the line shape and the luminosity monitoring processes.

The leptonic measurements provide Universality Tests by directly comparing the ratios of at the level of a few 10^{-6} for the leptonic partial widths (axial vector couplings), and better than 10^{-4} for the coupling asymmetries (vector couplings). This is best done by a direct evaluation of the ratios of the R_ℓ ratios and of the forward-backward asymmetries, as done in [2].

Table 2. Projected uncertainties on and partial-width ratios, forward-backward and polarization asymmetries obtained on peak at the Z-pole from FCC-ee. When two numbers are given, the first (second) corresponds to the projected statistical (systematic) uncertainty. .

fermion (f)	$R_f \equiv \frac{\Gamma_{had}}{\Gamma_f}$ relative (10^{-6})	$\mathcal{A}_f$ (10^{-6})	$A_{FB}^{0,f}$ (10^{-6})	comment
e	$\pm3.4 \pm 2.3$	$\pm14 \pm 11$	$\pm3.3 \pm 2.4$	From $e^+e^- \to e^+e^-$
e	-	$\mathbf{\pm7 \pm 20}$	-	from $A_{FB}^{pol(\tau)}$
e	-	±13.5	-	combined
μ	$\pm2.4 \pm 2.3$	$\pm25 \pm 25$	$\pm2.3 \pm 2.4$	from $e^+e^- \to \mu^+\mu^-$
τ	$\pm2.7 \pm 2.3$	$\pm28 \pm 25$	$\pm2.8 \pm 2.4$	from $e^+e^- \to \tau^+\tau^-$
τ	-	$\pm5 \pm 200$	-	from τ polarization
τ	-	±38	-	combined
All leptons	$\pm1.5 \pm 2.3$	$\pm1.4 \pm 2.4$		From $e^+e^- \to \ell^+\ell^-$
All leptons		±12.4		assuming Universality
b	$\pm1.2 \pm 1.6$	±98	$\pm4 \pm 4$	
c	$\pm1.4 \pm 2.2$	±66	$\pm5 \pm 5$	
s	$\pm2.5 \pm 11$	±130	$\pm7.4 \pm 7.4$	

5 Quark observables

5.1 Quark partial widths

At LEP, total rate measurements for specific observables were obtained for b and c quark pairs [2]. The fraction of $b\bar{b}$ in the total hadronic sample, $R_b \equiv \frac{\Gamma_b}{\Gamma_{had}}$ was measured with a relative precision of 3.10^{-3}, out of a total of 3.3 10^6 $b\bar{b}$ events produced. This loss of precision can be traced directly to the relatively low tagging efficiency (10 to 15 percent for a single hemisphere) and the significant contamination of lighter quarks (10-15% charm pair contamination), see [10,11, 11]. The inspection of systematic uncertainties reveals that most are reduced to experimentally accessible quantities, with two main scaling factors: the uncertainties related to tagging efficiency and hemisphere correlations scale as $1/\epsilon_b^2$, while the uncertainties related to the understanding of backgrounds are proportional to the contamination.

A breakthrough for the FCC arose when the actual experimental conditions were evaluated. An artificial intelligence-based algorithm investigated the flavour tagging, in the context of the Higgs analyses mostly, with the purpose of extracting the Higgs boson decay rate into pairs of b, c, s quarks or gluons. The results are given in [1]. The simulation takes into account the IDEA detector properties, including dE/dx measurement in the drift chamber, as well as a time-of-flight measurement with 30 ps resolution in the vertex detector and the silicon wrapper. The beam size, beam pipe radius and the position of the vertex tracker around it are also taken into account, as well as the few-micrometer resolution of the vertex detector. Each of these parameters leads to considerable improvement w.r.t. LEP. The choice of efficiency vs purity is somewhat arbitrary, and in the end should be optimized. We have chosen the working points as follows.

- for the b partial width: efficiency 75%, largest contamination (c) 2.10^{-4} of the b sample.
- for the c partial width: efficiency 60%, largest contamination (b) 3.10^{-4} of the c sample.
- for the s partial width: efficiency 10%, largest contamination (light quarks) 2.10^{-3} of the s sample.

To appreciate the improvement brought about by this performance, it suffice to remark that even the $s\bar{s}$ analysis will have a performance superior to the $b\bar{b}$ analysis in ALEPH. Ancillary quantities such as the gluon splitting into $b\bar{b}$ or $c\bar{c}$ will be determined with great precision by direct observation of millions of events with 3 or four heavy quarks in the data.

The dominant source of systematic uncertainties is contained in the statistics of double tags. The uncertainties in Table 2 reflect this considerable improvement. A factor greater than 1000 of precision is expected from LEP to FCC-ee for R_b, which is a consequence of the combination of more than 5 orders of magnitude in statistics, a factor 5 increase in single hemisphere efficiencies and a factor of nearly 1000 reduction in the flavour mistag. The possibility of a direct observation of $Z \to s\bar{s}$ is remarkable, (it was not event attempted at LEP) and should encourage an attempt to separate also $Z \to u\bar{u}$ and $Z \to d\bar{d}$.

5.2 Quark charge asymmetries

At LEP the charge of a hadronic system was initially determined from the lepton from semi-leptonic b or c decays. In later stages, a more reliable and self-calibrating method was devised using untagged hemisphere charges as described in detail in [12,13], later extended to samples of flavour-tagged events to extract e.g. the b forward-backward charge separation and the EW asymmetry [14]. See [15] for a pedagogical presentation on the topic.

A detailed description of the hemisphere charge analysis can be found in [12,13]. The analysis is self-calibrating: the long range anti-correlation of charges between the two hemispheres of a $Z \to q\bar{q}$ event is well measurable by observing the different widths of the sum $< Q >=< Q_F + Q_B >$ and difference $< Q_{FB} >=< Q_F - Q_B >$ of the two hemispheres of a given event. An effective hemisphere charge $\bar{\delta}^f$ is thus derived from the data themselves, which determines the sensitivity of the method. When applied to a tagged sample of events (either the inclusive vertex tagged or samples based on the selection of exclusive decays) the various systematics uncertainties can be extracted from data input, and mostly statistics limited, **especially when a high purity single flavour is selected**, as can be seen from the error breakdown tables in [14]. With a reduction of a factor more than 100 in the mistag rate for in FCC, the systematics will be determined mostly by the statistics of the charge separation $\bar{\delta}$ measurement.

Detector effects are controlled by the measurement of the integrated charge $< Q >$. Exclusive decays can be used to measure a possible difference of charge between quark and anti-quark separations, (although this is already included in $\bar{\delta}^f$), while the material asymmetry A_{mat} can be verified in data using pair conversions or secondary interactions – the error is the product of these two second order effects ($A_{mat} * < Q >$) and these uncertainties turn out to be very small (see appendix A of the 1996 ALEPH paper[[13]). The uncertainties are ultimately verified against a Monte-Carlo simulation tuned to reproduce a multitude of experimental inputs.

The main result is that, for a ¿ 99% pure sample, $\frac{\Delta A_{FB}}{A_{FB}} = \frac{\Delta < Q_{FB} >}{< Q_{FB} >} \oplus \frac{\Delta \bar{\delta}}{\bar{\delta}}$. The absolute uncertainty in $\bar{\delta}$ is 3 times larger than that in $< Q_{FB} >$, but $\bar{\delta}$ is ˜10 times larger than the largest charge asymmetry at the Z pole ($< QFB >= \delta * AFB$), so that the relative error should be about 3 times smaller. We have used here the charge separations determined in the ALEPH analysis, and considered, somewhat arbitrarily and conservatively, for the table2, that the systematics on AFB for high purity tags are not smaller than the statistical error at the Z pole pending further detailed analysis. The systematics at the other energy points are assumed to be controlled from the Z pole measurements and essentially negligible.

6 Conclusions

The statistical and systematic uncertainties of electroweak precision observables have been revisited, taking into account the recent progress regarding the dominant systematic uncertainties. For leptonic quantities, this confirms he estimates given in [16] in 2022. (The tau observables remain to be revisited). For the quark final states, the emergence of a high efficiency, high purity tagging algorithm leads to much reduction of the expected uncertainties. The possibility of producing precise Electroweak observables using s-quarks has been demonstrated. The resulting requirements on detector and accelerator have been specified. An intense programme of improved QED and Electroweak calculations down to the statistical precision of 10^{13} Z decays or more is fully motivated.

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summary table of uncertainties								
√s (GeV)	87.9	91.2	94.3	157.5	162.5	240	345	365
total hadronic cross-section uncertainties								
Statistics	2.62E-06	5.17E-07	1.82E-06	5.59E-05	5.84E-05	9.25E-05	7.11E-04	2.97E-04
Systematics (luminosity)	2.00E-05	1.60E-05	2.00E-05	1.00E-04	1.00E-04	1.00E-04	1.00E-04	1.00E-04
Total error (relative)	2.02E-05	1.60E-05	2.01E-05	1.15E-04	1.16E-04	1.36E-04	7.18E-04	3.13E-04
muon and tau asymmetries								
$A_{FB}^{\mu\mu}$ (statistics)	1.11E-05	2.35E-06	7.90E-06	1.23E-04	1.29E-04	2.00E-04	1.53E-03	6.37E-04
Systematic error (ΔE_{cm})	2.40E-06	2.40E-06	2.40E-06	2.40E-06	2.40E-06	2.40E-06	2.40E-06	2.40E-06
Total error (absolute)	1.14E-05	3.36E-06	8.25E-06	1.23E-04	1.29E-04	2.00E-04	1.53E-03	6.37E-04
√s (GeV)	87.9	91.2	94.3	157.5	162.5	240	345	365
$A_{FB}^{\tau\tau}$ (statistics)	1.34E-05	2.82E-06	9.48E-06	1.47E-04	1.54E-04	2.40E-04	1.83E-03	7.65E-04
Systematic error (ΔE_{cm})	2.40E-06	2.40E-06	2.40E-06	2.40E-06	2.40E-06	2.40E-06	2.40E-06	2.40E-06
Total error (absolute)	1.36E-05	3.71E-06	9.77E-06	1.47E-04	1.54E-04	2.40E-04	1.83E-03	7.65E-04
muon and tau branching ratios $\Gamma_{had}/\Gamma_{Lept}$								
R_{μ} (muon statistics)	1.16E-05	2.35E-06	8.22E-06	1.56E-04	1.62E-04	2.35E-04	1.75E-03	7.31E-04
Hadron statistics	2.62E-06	5.17E-07	1.82E-06	5.59E-05	5.84E-05	9.25E-05	7.11E-04	2.97E-04
Systematic error (detector)	2.20E-06	2.20E-06	2.20E-06	2.20E-06	2.20E-06	2.20E-06	2.20E-06	2.20E-06
Total error (relative)	1.21E-05	3.26E-06	8.70E-06	1.66E-04	1.72E-04	2.52E-04	1.89E-03	7.89E-04
√s (GeV)	87.9	91.2	94.3	157.5	162.5	240	345	365
R_{τ} (muon statistics)	1.39E-05	2.82E-06	9.86E-06	1.87E-04	1.94E-04	2.82E-04	2.10E-03	8.77E-04
Hadron statistics	2.62E-06	5.17E-07	1.82E-06	5.59E-05	5.84E-05	9.25E-05	7.11E-04	2.97E-04
Systematic error (detector)	2.64E-06	2.64E-06	2.64E-06	2.64E-06	2.64E-06	2.64E-06	2.64E-06	2.64E-06
Total error (relative)	1.44E-05	3.90E-06	1.04E-05	1.95E-04	2.03E-04	2.97E-04	2.22E-03	9.26E-04
b asymmetries and R_b								
A_{FB}^{bb} (statistics)	2.01E-05	3.96E-06	1.39E-05	4.86E-04	5.14E-04	8.44E-04	6.61E-03	2.77E-03
Systematics (jet charge)	3.96E-06	3.96E-06	3.96E-06	3.96E-06	3.96E-06	3.96E-06	3.96E-06	3.96E-06
Total error (absolute)	2.05E-05	5.75E-06	1.45E-05	4.86E-04	5.14E-04	8.44E-04	6.61E-03	2.77E-03
√s (GeV)	87.9	91.2	94.3	157.5	162.5	240	345	365
R_b (statistics)	6.47E-06	1.27E-06	4.49E-06	1.57E-04	1.66E-04	2.72E-04	2.13E-03	8.91E-04
Systematics (tag efficiency)	7.47E-06	1.47E-06	5.18E-06	1.81E-04	1.91E-04	3.14E-04	2.46E-03	1.03E-03
Total error (relative)	9.88E-06	1.95E-06	6.86E-06	2.39E-04	2.53E-04	4.15E-04	3.25E-03	1.36E-03
c asymmetries and R_c								
A_{FB}^{cc} (statistics)	2.49E-05	4.95E-06	1.74E-05	4.46E-04	4.60E-04	7.05E-04	5.33E-03	2.22E-03
Systematics (jet charge)	4.95E-06	4.95E-06	4.95E-06	4.95E-06	4.95E-06	4.95E-06	4.95E-06	4.95E-06
Total error (absolute)	2.55E-05	7.19E-06	1.81E-05	4.46E-04	4.60E-04	7.05E-04	5.33E-03	2.22E-03
√s (GeV)	87.9	91.2	94.3	157.5	162.5	240	345	365
R_c (statistics)	8.14E-06	1.62E-06	5.67E-06	1.46E-04	1.50E-04	2.30E-04	1.74E-03	7.27E-04
Systematics (tag efficiency)	1.05E-05	2.09E-06	7.32E-06	1.88E-04	1.94E-04	2.97E-04	2.25E-03	9.39E-04
Total error (relative)	1.33E-05	2.64E-06	9.26E-06	2.38E-04	2.46E-04	3.76E-04	2.84E-03	1.19E-03
s asymmetries and R_s								
A_{FB}^{ss} (statistics)	3.84E-05	7.37E-06	2.66E-05	1.10E-03	1.17E-03	1.95E-03	1.54E-02	6.44E-03
Systematics (jet charge)	7.37E-06	7.37E-06	7.37E-06	7.37E-06	7.37E-06	7.37E-06	7.37E-06	7.37E-06
Total error (absolute)	3.92E-05	1.07E-05	2.77E-05	1.10E-03	1.17E-03	1.95E-03	1.54E-02	6.44E-03
√s (GeV)	87.9	91.2	94.3	157.5	162.5	240	345	365
R_s (statistics)	1.77E-05	3.49E-06	1.23E-05	4.29E-04	4.53E-04	7.45E-04	5.83E-03	2.44E-03
Systematics (tag efficiency)	5.60E-05	1.10E-05	3.89E-05	1.36E-03	1.43E-03	2.35E-03	1.84E-02	7.71E-03
Total error (relative)	5.87E-05	1.16E-05	4.08E-05	1.42E-03	1.50E-03	2.47E-03	1.93E-02	8.09E-03

Fig. 1. Table of uncertainties for EWPOs as a function of center of mass energy. Experimental systematics as explained in the text. All systematics are taken from the high statistics Z pole data.