Simulation Beyond the nuSTORM Production Straight

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Abstract: We extend the nuSTORM production straight simulation beyond the production straight to simulate the entire muon storage ring. The position and momentum coordinates of muon decays and the decay product momentum coordinates in the nuSTORM reference frame are calculated. The angular distributions of the decay product momentum are produced. The average detector background is estimated to be approximately 2%.

1 Introduction[1]

The nuSTORM ("Neutrinos from Stored Muons") facility is designed to produce neutrino beams from muon decays in a storage ring. The storage ring consists of two straight regions and two arcs. A detector is placed on the axis of one of the straight regions called the production straight. The muon beam is obtained from a pion beam injected into the production straight. The electron and muon neutrinos originated in the production straight create a pulse of neutrinos in the detector. A very small portion of the neutrinos originated in the arcs and the other straight section, the return straight produces a background in the detector. These neutrinos may contribute to the neutrino beam if they are directed along the beam. The neutrinos from the background have to be taken into account to specify the neutrino beam properties.

2 Muon Storage Ring

The muon decays in the production straight are already simulated [2]. In this report, we extend the simulation beyond the production straight.

The muon storage ring is assumed to be composed of a production straight, a semi-circular arc, a return straight, and another semi-circular arc. There is a detector placed along the axis of the production straight. The origin of the coordinate system adopted is at the beginning of the production straight. The Z-axis is directed along the production straight and Y-axis vertically upwards. The coordinate system is taken to be right-handed and X-axis is directed accordingly.

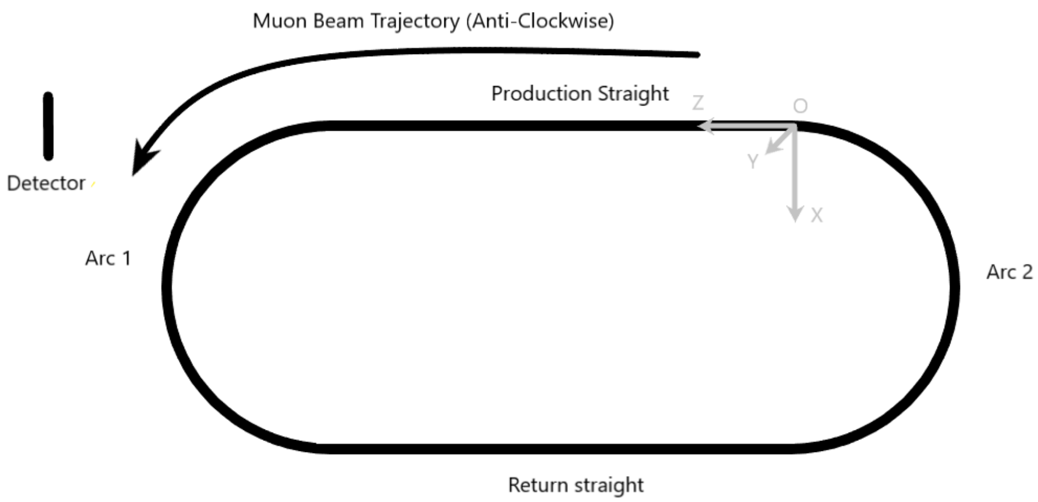

Figure 1: Muon Storage Ring

The muon beam originates at the beginning of the production straight. The muons in the beam decay exponentially as they go anti-clockwise around the ring and create electrons, electron neutrinos, and muon neutrinos. Our goal is to simulate the angular distribution of the decay products and calculate the background in the detector from muon decays occurring in the arcs and the return straight.

3 Assumptions

The following assumptions are made in the first approximation of muon decays in the storage ring.
1. The arcs are semi-circular.
2. Transverse momentum components of the muons due to beam divergence are negligible since the longitudinal momentum component along the storage ring axis is much greater than the transverse components.
3. Transverse position coordinates are taken into account.
4. Detector geometry is not defined; only the dimensions are specified. The detector is approximated as a 2-dimensional square.
5. Muon polarization is not taken into account.

4 Parameters and Variables[3]

Parameter	Description	Value
Circumference	Storage ring circumference	616 m
PrdStrghtLngth	Length of production straight	180 m
pAcc	Momentum acceptance	15%
epsilon	Transverse acceptance (2D)	1 pi mm rad
beta	Transverse beta function	25000 mm
HallWallDist	Distance from end of straight to detector hall	50 m
DetHlfWdth	Detector half width	2 m
DetLngth	Detector length	4 m
Hall2Det	Distance from hall wall to detector entrance	5m
ArcLen	Arc length	128 m

The muon storage ring geometry is specified by three parameters; Circumference, PrdStrghtLngth and ArcLen. The circumference of the ring is 616 m, the production straight length is 180 m and the arc length is 128 m in this simulation.

The muon beam is specified by the parameters, pAcc, epsilon and beta. The beam momentum has a parabolic distribution centered at 1 Gev/c to 6 Gev/c (5Gev/c in this simulation) with momentum acceptance specified by the value of pAcc, 15%. The transverse acceptance, epsilon is 1 pi mm rad and the transverse beta function is 25000 mm.

The detector position is specified by the parameters HallWallDist, DetHlfWdth, DetLngth and Hall2Det. HallWallDist (50 m) specifies the distance of the detector hall from the end of the production straight. DetHlfWdth (2 m) is the half-width of the detector. DetLngth (4 m) is the detector length. The distance from the hall to the detector entrance is specified by Hall2Det (5 m).

Variable	Description
s	Longitudinal distance travelled by the muon along the storage ring axis
where	Longitudinal position of the muon decay along the storage ring axis measured anti-clockwise from the beginning of the production straight.
r	Radius of the arc
θ	Angle between the longitudinal direction along the storage ring axis and the Z axis of nuSTORM reference frame
Xb, Yb, Zb	Muon beam position at the instant of decay in nuSTORM reference frame
Pb	Longitudinal momentum of a muon in the muon beam (Beam Momentum)
Xt, Yt	Transverse displacement of muons from the beam position (center of the storage ring)
Xm, Ym, Zm	Position of muon decay
Xdmax, Xdmin, Ydmax, Ydmin	Coordinates specifying the dimensions of the detector
Xd, Yd Zd	Coordinates of the point at which a decay product hits the detector plane

The muons in the beam are originated at the beginning of production straight and go round the storage ring until they decay. The distance travelled by a muon is denoted as s. The number of turns around the ring is given by the quotient of the division of s by Circumference.

The variable 'where' is the remainder of the division of s by Circumference. It determines where the muon decay occurs.

The radius of the arc, r is determined by the parameter ArcLen.

The muon beam direction changes as it revolves around the storage ring. The direction is specified by the angle (θ) the beam makes with the Z-axis.

The muon position at the time of decay is specified by s, the distance travelled. This position in nuSTORM reference frame coordinates is given by (Xb, Yb, Zb).

The muon has a longitudinal momentum Pb from the parabolic distribution. This determines the velocity of the beam and relates the lifetime of the muon and the distance travelled.

The muons also have a transverse motion due to beam divergence. The transverse displacements from the center of the muon storage ring axis are given by Xt and Yt. The transverse displacements combined with the beam position give us the exact position of muon decay (Xm, Ym, Zm).

The detector is approximated as a 2- dimensional square. The square is in the Z = Zd plane and oriented along the Z direction. Zd is determined by the parameters HallWallDist, Hall2Det and PrdStrghtLngth.

The minimum and maximum values of the X and Y coordinates of the detector are given by Xdmax, Xdmin, Ydmax and Ydmin. These values are determined by the parameter DetHlfWdth.

The coordinates of the point at which the decay product hits the detector plane are given by (Xd, Yd, Zd).

5 Coordinates of Muon Decay

We have to consider the longitudinal motion along the storage ring axis and the beam divergence to specify the exact position and momentum coordinates of muon decays. We do not take the transverse momentum coordinates into account in this approximation. The algorithm along with pseudocodes is given below.

Step 1: Determine where the muon decay occurs based on the longitudinal distance travelled.
where = s mod Circumference.
Case 1: PrdStrghtLngth >= where >=0 : The muon decay occurs in the production straight.
Case 2: PrdStrghtLngth + ArcLen >= where >= PrdStrghtLngth : The muon decay occurs in the first arc.
Case 3: 2 PrdStrghtLngth + ArcLen >= where >= PrdStrghtLngth + ArcLen : The muon decay occurs in the return straight.
Case 4: 2 PrdStrghtLngth + 2 ArcLen >= where >= 2 PrdStrghtLngth + ArcLen : The muon decay occurs in the second arc.

Step 2: Calculate the angle between the storage ring axis and Z axis of nuSTORM reference frame.

Production straight: $\theta = 0$

First Arc: Distance covered along the arc = where - PrdStrghtLngth,

θ = (where - PrdStrghtLngth)/r

Return Straight: $\theta = \pi$

Second Arc: Distance covered along the arc = where - 2 PrdStrghtLngth - ArcLen,

$\theta = \pi$ + (where - 2 PrdStrghtLngth - ArcLen)/r

Step 3: Calculate the corresponding nuSTORM reference frame position coordinates.

Production Straight: (Xb, Yb, Zb) = (0, 0, where)

Arc 1: (Xb, Yb, Zb) = (r- r $cos\theta$, 0, PrdStrghtLngth + r $sin\theta$)

Return straight: (Xb, Yb, Zb) = (2r, 0, PrdStrghtLngth - (where - ArcLen - PrdStrghtLngth))

Arc 2: (Xb, Yb, Zb) = (r - r $cos\theta$, 0, r $sin\theta$)

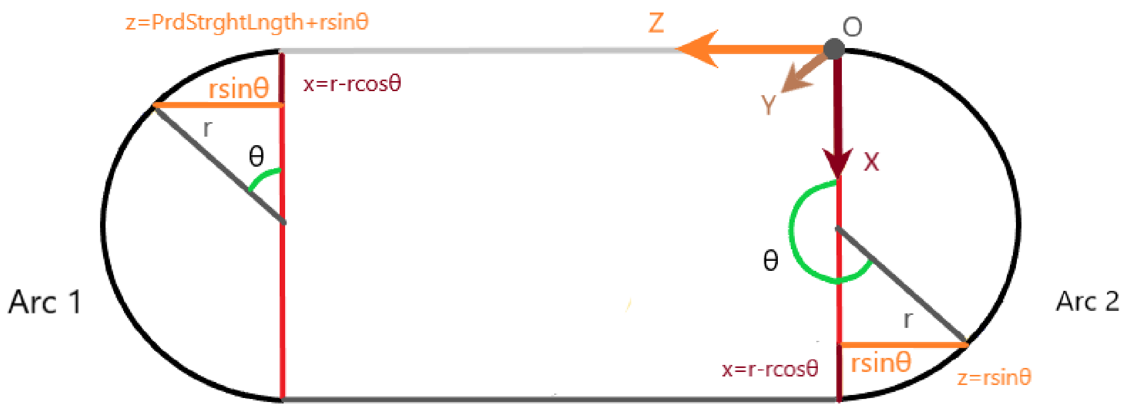

Figure 2: Position Coordinates of the Beam

Step 4: Calculate the corresponding nuSTORM reference frame momentum coordinates.
$(Pb_x, Pb_y, Pb_z) = (Pb\ cos\theta, 0, Pb\ sin\theta)$

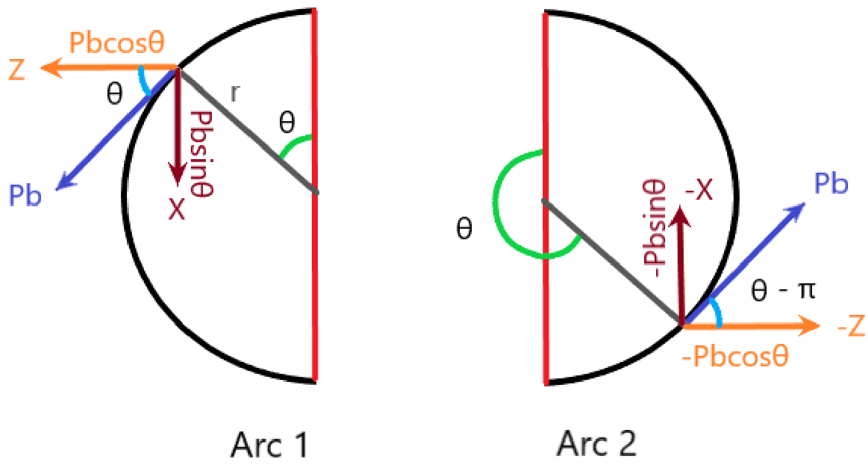

Figure 3: Beam Momentum Coordinates

Step 5: Obtain the transverse coordinates (Xt, Yt) from the storage ring axis due to beam divergence.

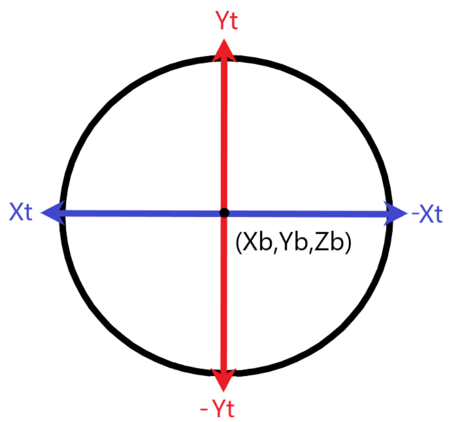

Figure 4: Cross Section View of the Storage Ring

Step 6: Combine the transverse position coordinates with beam position coordinates to obtain muon decay position coordinates.
Xm = Xb + Xt $cos\theta$, Ym = Yb + Yt, Zm = Zb - Xt $sin\theta$

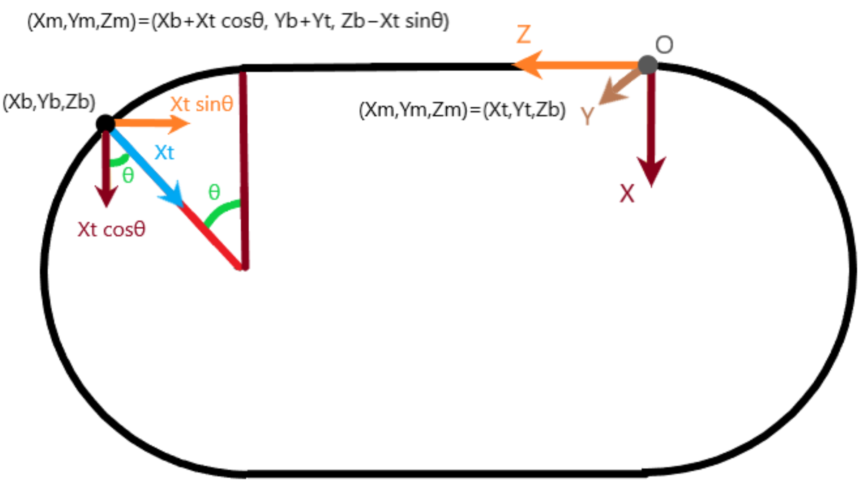

Figure 5: Muon Decay Position Coordinates

Step 7: Combine the transverse momentum coordinates with beam momentum coordinates to obtain muon decay momentum coordinates.
This step is not necessary for this approximation.

6 Treating the Decay Products

The decay product momentum 4-vectors are obtained in the muon rest frame. These four vectors need to be boosted to the nuSTORM reference frame. The algorithm is given below.

Step 1: Determine where the muon is based on the longitudinal position.

Step 2: Obtain the calculated muon momentum 3-vector and determine the beam direction

Step 3: Boost the decay product momentum vectors from muon rest frame to the beam direction in the nuSTORM reference frame.

The last task is to check whether the decay products are detected in the detector. The algorithm along with pseudocodes is given below.

Step 1: Calculate the coordinates of the detector.
Xdmax = DetHlfWdth; Xdmin= - DetHlfWdth;
Ydmax = DetHlfWdth; Ydmin = - DetHlfWdth;
Zd = PrdStrghtLngth + HallWallDist + Hall2Det

Step 2: Obtain the muon decay position coordinates (Xm, Ym, Zm) and the decay product 3-momentum (Px, Py, Pz).

Step 3: Find the equation of the straight line through the muon decay position and along the momentum vector direction.
X = Xm + l t
Y = Ym + m t
Z = Zm + n t
where l, m and n are the direction cosines corresponding to the vector (Px, Py, Pz)

Step 4: Obtain the value of the parameter t for the detector plane Zd.
t = (Zd - Zm)/n

Step 5: Calculate the coordinate of the point at which the decay product hits the detector plane.
Xd = Xm + l t
Yd = Ym + m t

Step 6: Check if the decay product is detected.
If t > 0 and Xdmax ≥ Xd ≥ Xdmin and Ydmax ≥ Yd ≥ Ydmin, the decay product is detected.

7 Testing the Simulation

Position Coordinates: The muon decay coordinates for 10000 neutrino events are plotted. We can see that the geometry of the storage ring is reproduced. This proves the validity of the position coordinate calculations.

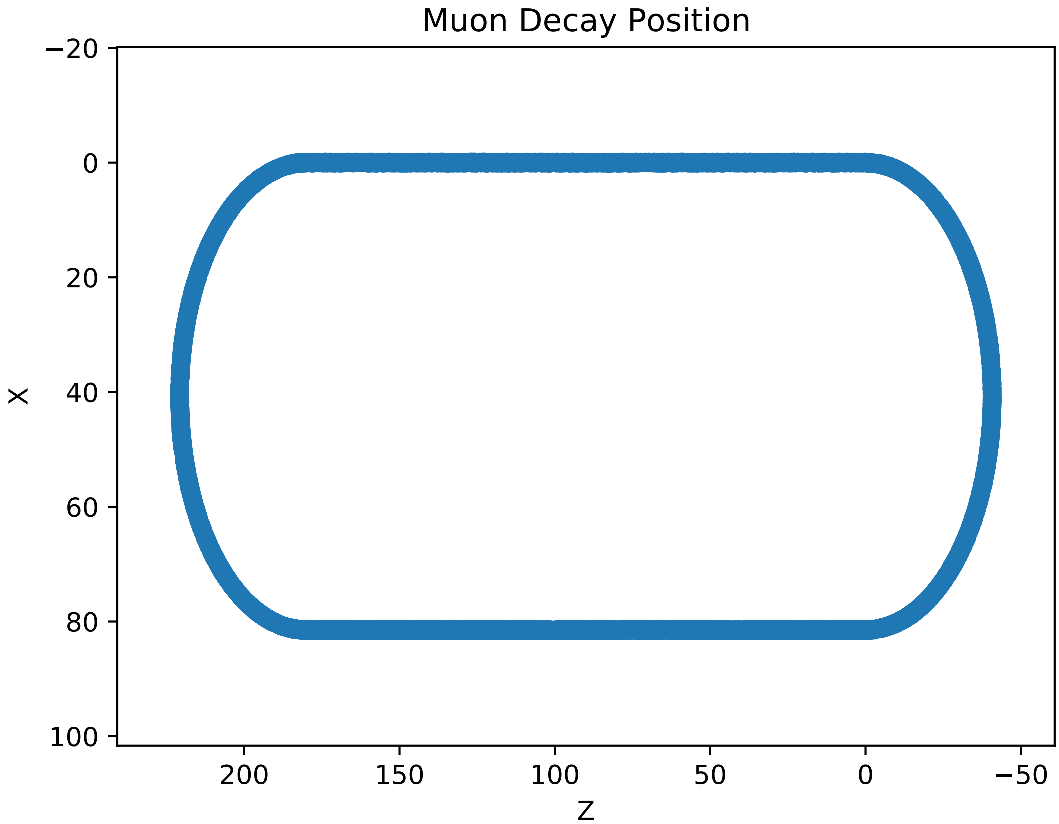

Figure 6: Scatter Plot of Decay Position Coordinates for 10000 events

Beam Momentum: The muon beam momentum vectors for 100 events are plotted. The beam momentum vectors are tangent to the muon storage ring and along the storage ring axis. This plot proves the validity of the beam momentum calculation.

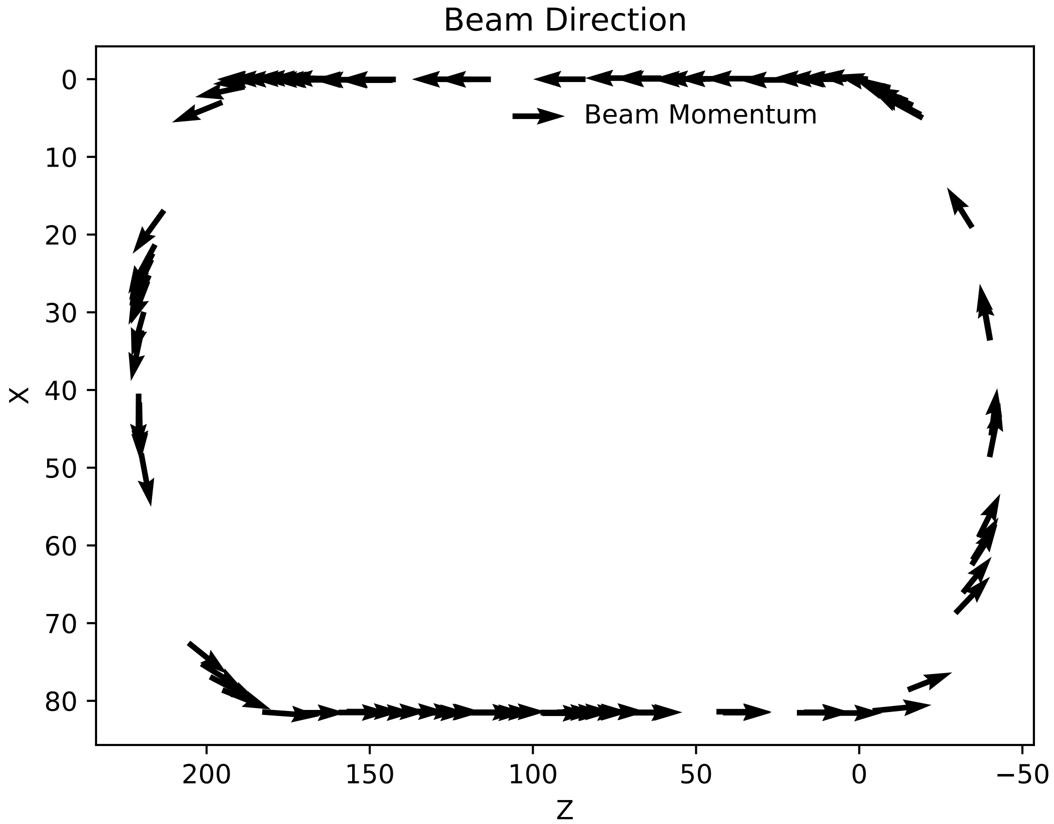

Figure 7: Beam Momentum Vectors for 100 Events

Decay Product Momentum Plots: The decay product momentum vectors along with the beam momentum vectors have been plotted for 100 events and the angular distribtution plots are produced for 10000 events.

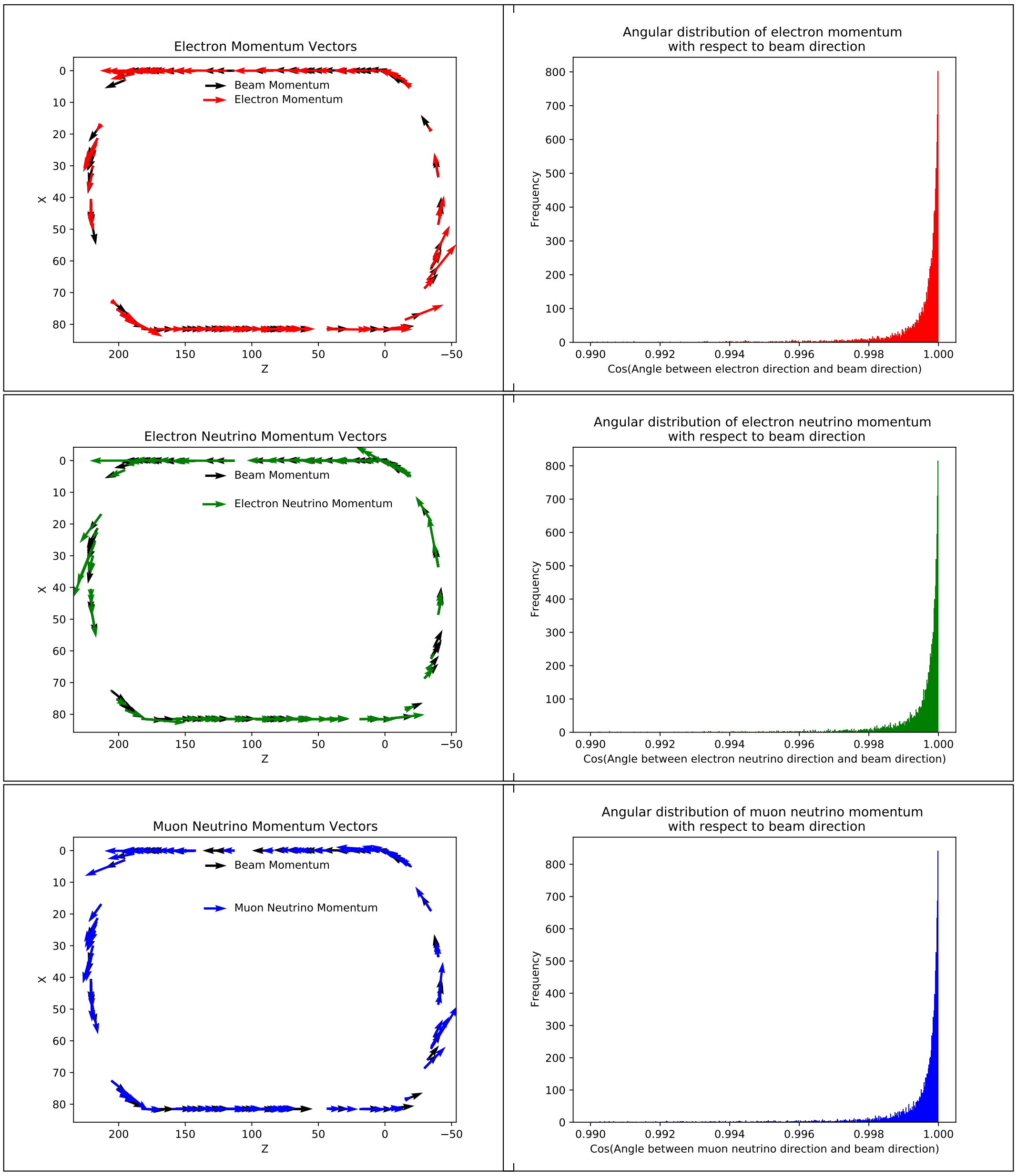

Figure 8: Decay Product Momentum Vectors and Angular Distributions

Most of the vectors are almost tangent to the beam direction. The decay products have arbitrary momentum in the muon rest frame. A boost along the muon beam direction results into a much larger component of momentum along that direction.The transverse momentum coordinates of the decay product produce a slight deviation.

8 Estimation of Background Radiation

We want to estimate the degree of detector background created by the muon decays in the arcs and the return straight. This is going to give us an idea of how many detected neutrinos are not from the beam in the production straight. The percentage of detected neutrinos that originate from the arcs and the return straight is a good measure of the background.

The percentage of the background neutrinos detected is simulated for different values of the number of events and maximum lifetime of the beam. The beam lifetime distribution can be truncated at a certain value. This value determines the maximum lifetime and the maximum distance travelled by the muon beams. We set the maximum lifetime such that the muon beams can complete 1 round trip, 10 round trips and unlimited number of round trips (no truncation) respectively.

Run No.	1 round trip	10 round trips	Unlimited round trips
1	1.97	2.63	2.05
2	2.09	1.80	2.47
3	1.31	1.97	1.96
4	1.86	1.59	1.41
5	2.74	2.09	2.28
6	2.34	1.99	1.82
7	2.42	2.03	2.31
8	1.69	1.86	2.25
9	2.56	2.07	1.92
10	2.01	2.02	2.23
Mean S.D.	2.101 ±0.429	2.005 ±0.267	2.070 ±0.309

Table 1: Percentage of Background Neutrinos for 10000 Events

Run No.	1 round trip	10 round trips	Unlimited round trips
1	1.85	2.03	2.08
2	1.87	2.00	1.61
3	1.94	1.69	1.89
4	1.97	2.03	2.16
5	2.31	1.98	2.08
6	2.04	2.15	2.07
7	2.16	1.76	1.65
8	2.07	2.24	2.73
9	1.99	2.05	2.17
10	2.05	1.71	1.95
Mean S.D.	2.025 ±0.137	1.964 ±0.184	2.038 ±0.313

Table 2: Percentage of Background Neutrinos for 20000 Events

The average percentage of background neutrinos does not vary significantly with the number of events and maximum lifetime of the muons in the beam. This is satisfactory because the background noise should only depend on storage ring geometry and muon lifetime distribution. These factors are kept unchanged and we get similar degree of background noise.

The pooled mean and standard deviation of the collected data are 2.034%±0.289%. The maximum and minimum values are 2.74% and 1.31%.

We can conclude that the background noise in the detector lies between 1% and 3% and the average is approximately 2%.

9 Conclusion

We have simulated the geometry, beam dynamics and muon decays beyond the nuSTORM production straight. The calculated detector background is negligible which is satisfactory. We still have not taken the transverse momentum of the muons into account. In the next approximation, the transverse muon momentum will give rise to a broader angular distribution of the decay products. This will result into a higher percentage of the detector background.

The codes can be found in the nuSTORM github repository [3].

References

[1] K. Long, "The nuSTORM Experiment," in *Journal of Physics: Conference Series*, vol. 1056, p. 012033, IOP Publishing, 2018.

[2] P. Kyberd and K. Long, "nuSIM: Parameters for first simulation of neutrino spectra." `https://github.com/ImperialCollegeLondon/nuSTORM/raw/main/00-Documentation/2021-02-20-FirstSimu/nuSIM-doc-01.pdf`.

[3] "nuSTORM GitHub Repository." `https://github.com/ImperialCollegeLondon/nuSTORM`.