Tau Physics Prospects at FCC-ee

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Abstract

We estimate the FCC physics reach on Tau Physics measurements related to Lepton Flavour Universality tests and Lepton Flavour Violation searches.

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1 Introduction

We estimate the FCC-ee sensitivity for some Tau Physics measurements. We assume that FCC-ee will observe at the Z peak $N_Z^{\rm FCC} = 6{\cdot}10^{12}$ Z decays [1], corresponding to $2.0{\cdot}10^{11}$ tau pairs, and to an integrated luminosity of $\mathcal{L}_{\rm int}^{\rm FCC} = 210\,{\rm ab}^{-1}$.

2 Tau mass

Recently, the Belle II collaboration reported the most precise measurement of the tau mass, $1777.09 \pm 0.08 \pm 0.11\,{\rm MeV}/c^2$ [2]. The systematic uncertainties have been substantially reduced with respect to the previous B-factories' measurements, and the total uncertainty is smaller than the uncertainties of the measurements performed at the tau pair production threshold [3, 4]. The Belle II statistical uncertainty of $0.08\,{\rm MeV}/c^2$ (45 ppm) has been obtained with $175{\cdot}10^6$ tau pairs ($190\,{\rm fb}^{-1}$of intefrated luminosity), and could be improved to 1.3 ppm with $2.0{\cdot}10^{11}$ tau pairs at FCC-ee, without taking into account the larger efficiency that can be expected at FCC-ee from the comparison of LEP versus B-factories tau measurements. Rescaling the statistical uncerytainty of the OPAL tau mass measurement [5] to number of Z decays expected at FCC-ee gives an estimated statistical precision of 0.9 ppm. The Belle II leading systematic uncertainty of $0.07\,{\rm MeV}/c^2$ (39 ppm) is related to the knowledge of the beam energy, and is expected to be significantly smaller at FCC-ee, where the beam energy can be known with 1 ppm precision. The other Belle II leading systematic uncertainty of $0.06\,{\rm MeV}/c^2$ (34 ppm) is related to the understanding of the charged tracks reconstructed momentum scale, which can probably be calibrated with 2 ppm precision at FCC-ee by matching the measured J/ψ mass to its world average, presently known to 2 ppm. Belle II reports systematics related to the estimator bias ($0.03\,{\rm MeV}/c^2$), to the choice of the fit function of the pseudo-mass distribution (0.02,MeV$/c^2$), to the detector material ($0.03\,{\rm MeV}/c^2$), and to the modeling of ISR, FSR and tau decay ($0.02\,{\rm MeV}/c^2$), for a total of 29 ppm. We expect that these systematics may be reduced by a factor 3 to 10 ppm at FCC-ee, which we take as the estimated precision of the measurement of the tau mass at FCC-ee. Figure 1 reports the present and future expected experimental uncertainties on the tau mass.

3 Tau lifetime

With a sample of $6{\cdot}10^{12}$ Z decays, it is convenient to measure the tau lifetime on the relatively small sub-sample of tau pairs where both tau leptons decay into a 3-prong topology, like Belle did [6]. For these events, the two 3-prong vertices and the constraint of the very small luminous region precisely define the tau leptons' flight directions, significantly reducing systematics from Monte Carlo simulation of the effects of the undetected neutrinos on the reconstruction of the tau flight directions.

We consider as the baseline for extrapolating to the FCC sample the DELPHI tau lifetime measurement [7], which includes a measurement done on tau pairs both decaying to 3 charged tracks (3-3-prongs topology). The DELPHI measurement is performed on the 1991-1995 sample, corresponding to about $4.0{\cdot}10^6$ hadronic Z decays [8], hence about $N_Z^{\rm DELPHI\ 2004} = 4.0{\cdot}10^6/70\% = 5.7{\cdot}10^6$ Z decays. The statistical uncertainty on the tau lifetime with all 3-prong events is 2.4 fs, using $N_{1-3} = 15427$ 3-prong vs. 1-prong tau pairs, where only the 3-prong decay length is measured, and $N_{3-3} = 2101$ 3-prong vs. 3-prong tau pairs. By scaling we obtain the statistical uncertainty of just the 3-prong vs. 3-prong samples as:

$$\sigma(\tau_\tau, 3\text{–}3) = 2.4\,{\rm fs} \cdot \sqrt{\frac{N_{1-3} + 2\cdot N_{3-3}}{2\cdot N_{3-3}}} \simeq 5.19\,{\rm fs} \qquad (1)$$

$$\sigma(\tau_\tau, 3\text{–}3)\,[{\rm ppm}] = \sigma(\tau_\tau, 3-3)/\tau_\tau \cdot 1{\cdot}10^6\,{\rm ppm} \simeq 18000\,{\rm ppm}\ , \qquad (2)$$

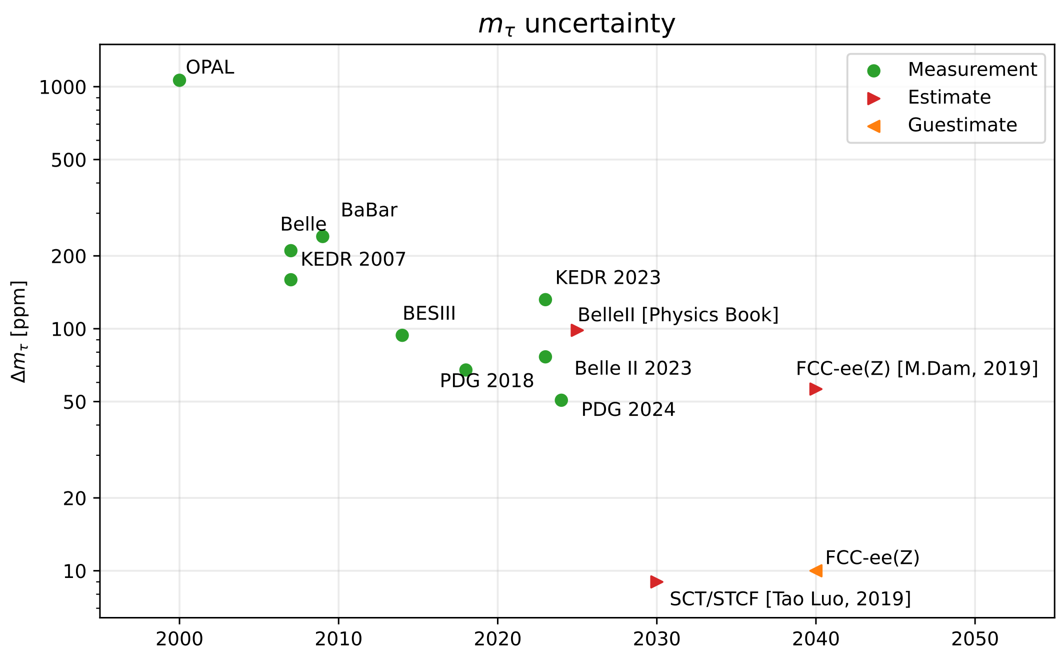

Figure 1: Present and future expected experimental uncertainties on the tau mass. The dates of the future measurements are speculative and mainly chosen for plotting purposes.

where τ_τ is the tau lifetime 2023 world average. The single event tau lifetime resolution for DELPHI is

$$\sigma(\tau_\tau, 3\text{–}3, 1\,\text{ev}) = \frac{\sigma(\tau_\tau, 3\text{–}3)}{\sqrt{2 \cdot N_{3-3}}} \simeq 336\,\text{fs}\ , \tag{3}$$

corresponding to a resolution of:

$$\sigma_\text{res}(\tau_\tau) = \sqrt{\sigma^2(\tau_\tau, 3\text{–}3, 1\,\text{ev}) - \tau_\tau^2} \simeq 172\,\text{fs}\ . \tag{4}$$

For LEP measurements, the tau lifetime single-event resolution consists of two contributions, one from the tracking resolution on the tracks transverse impact parameters, and a second one from the beam spot size in the transverse plane. The average signed transverse impact parameter for the tau decay is $\langle \hat{d}_0 \rangle \simeq 70\,\mu\text{m}$. The resolution on $\hat{d}_0$ is $\sigma_\text{res}(\hat{d}_0) = \sigma_\text{res}(\tau_\tau)/\tau_\tau \cdot \langle \hat{d}_0 \rangle \simeq 42\,\mu\text{m}$, which is consistent with the DELPHI tracking resolution and beam spot size, considering that 3 tracks are used for each event. We expect that the FCC tracking resolution and beam spot size in the transverse plane will be negligible with respect to $\langle \hat{d}_0 \rangle$, and therefore we compute the expected statistical uncertainty at FCC as:

$$\sigma(\tau_\tau, 3\text{–}3, \text{FCC})[\text{ppm}] = \sigma(\tau_\tau, 3\text{–}3)\,[\text{ppm}] \cdot \frac{\tau_\tau}{\sigma(\tau_\tau, 3\text{–}3, 1\,\text{ev})} \cdot \sqrt{\frac{N_Z^\text{DELPHI 2004}}{N_Z^\text{FCC}}} \tag{5}$$

$$\simeq 15.0\,[\text{ppm}]\ , \tag{6}$$

where τ_τ is the single-event intrinsic irreducible resolution assuming that that the beam spot size and the impact parameter resolution are negligible compared to the average tau-decay tracks' impact parameters.

Three of the DELPHI 2004 quoted systematics contributions can be optimistically expected to scale down according to statistics (i.e., with the square root of the number of events): background subtraction (for which the simulation can be tuned with data control samples), reconstruction bias (which can be studied with data prompt events), and vertex alignment (done with data events). Regarding alignment, past studies [9] indicate that the measurement of the tau lifetime is unaffected at first order by the vertex detector alignment, and in particular from the length scale of the radial positions of the vertex detector sensitive elements, provided that:

- the lifetime measurement relies on decay length measurements on the plane transverse to the beams,
- there is uniform and complete azimuthal acceptance.

Uniform azimuthal acceptance can also be obtained by weighting events. Assuming that the systematic uncertainties that can be studied with data samples will scale down according to the luminosity, their size will be reduced from 4500 ppm to 3.9 ppm for an FCC measurement.

While alignment using data tracks can precisely monitor the relative positions of the vertex detector elements, there is however an overall length scale uncertainty that cannot be fixed in this way, corresponding to the absolute size of the detector, or more precisely the absolute average spacing of the detector sensitive elements (strips, pixels). This uncertainty does not scale down with the luminosity.

For a typical LEP vertex detector, the knowledge of the vertex detector length scale can be assumed to be of order 1/10000 or 100 ppm. This limitation however can be reduced by using optical interferometry techniques. A study conducted for the MUonE future experiment [10] indicates that it is possible to monitor silicon vertex detectors longitudinal positions with about $2\,\mu$m precision over distances of order 1 m. The precision for monitoring the average spacing of the sensitive elements of a vertex detector at FCC will depend on the size of the modules and on the distance from the interferometric sensor [11]. We assume here that with optical interferometric techniques the vertex detector length scale systematic uncertainty can be limited to order of 5 ppm.

Furthermore, additional systematics contributions are not expected to scale with statistics. The tau lifetime is measured by reconstructing the average tau decay length and by dividing it by the average tau velocity. With the convention of setting $c = 1$:

$$\tau_\tau = \frac{\lambda_\tau}{\beta\gamma} = \frac{\lambda_\tau m_\tau}{\sqrt{E_\tau^2 - m_\tau^2}} = \frac{\lambda_\tau m_\tau}{\sqrt{(E_{\text{beam}} - E_{\text{rad}})^2 - m_\tau^2}} \; . \tag{7}$$

Therefore, there are systematic uncertainties from the knowledge of the beam energy E_{beam} or equivalently the e^+e^- center-of-mass energy, from the understanding of the average amount of energy radiated in the initial state (ISR) of the $e^+e^- \to Z^0 \to \tau^+\tau^-$ process, E_{rad}, and from the tau mass. The center-of-mass energy will be known with a 1 ppm precision at FCC [12], contributing to an uncertainty of the same size. The radiated energy E_{rad} is estimated with the $e^+e^- \to Z^0 \to \tau^+\tau^-$ Monte Carlo simulation, and it contributes a systematic uncertainty of 350 ppm to the DELPHI 2004 tau lifetime measurement. We optimistically speculate that an improvement of a factor 30 may be achieved, reducing the related uncertainty on the tau lifetime to 11.5 ppm at FCC. The tau mass 2024 PDG world average, which includes the 2023 measurements by KEDR and Belle II is ~50 ppm precise. Eventually, a $\tau^+\tau^-$ production threshold measurement at a future charm-tau factory may reduce the tau mass uncertainty to 9 ppm, and a measurement at FCC may attain a precision of 10 ppm. Furhermore, Belle II will improve its 2023 measurement using its entire dataset. We assume that a 10 ppm world average will set the FCC tau lifetime systematic uncertainty.

Table 1 summarizes the expected FCC tau lifetime measurement uncertainty contributions, which sum up to 21.5 ppm. Figure 2 reports the present and future expected experimental uncertainties on the tau lifetime.

Table 1: Tau lifetime uncertainties for a measurement using 3-prong tau vertices on a sample of tau pairs where both tau leptons decay with 3-prong topology, for the DELPHI 2004 measurement and as expected for a measurement at FCC-ee(Z) with $6{\cdot}10^{12}$ Z decays.

	DELPHI 2004 [fs]	DELPHI 2004 [ppm]	FCC-ee(Z) $6{\cdot}10^{12}$ Z [ppm]
statistical uncertainty	5.2	18000	15.0
luminosity-dependent systematics	1.3	4500	3.9
- background	0.2		
- reconstruction bias	0.8		
- vertex detector alignment	1.0		
luminosity-independent systematics			
- detector length scale	-	100	5.0
- average tau energy	-	-	1.0
- radiative energy loss	0.1	350	11.5
- tau mass	-	68	10.0
total systematics			15.9
total uncertainty			22.3

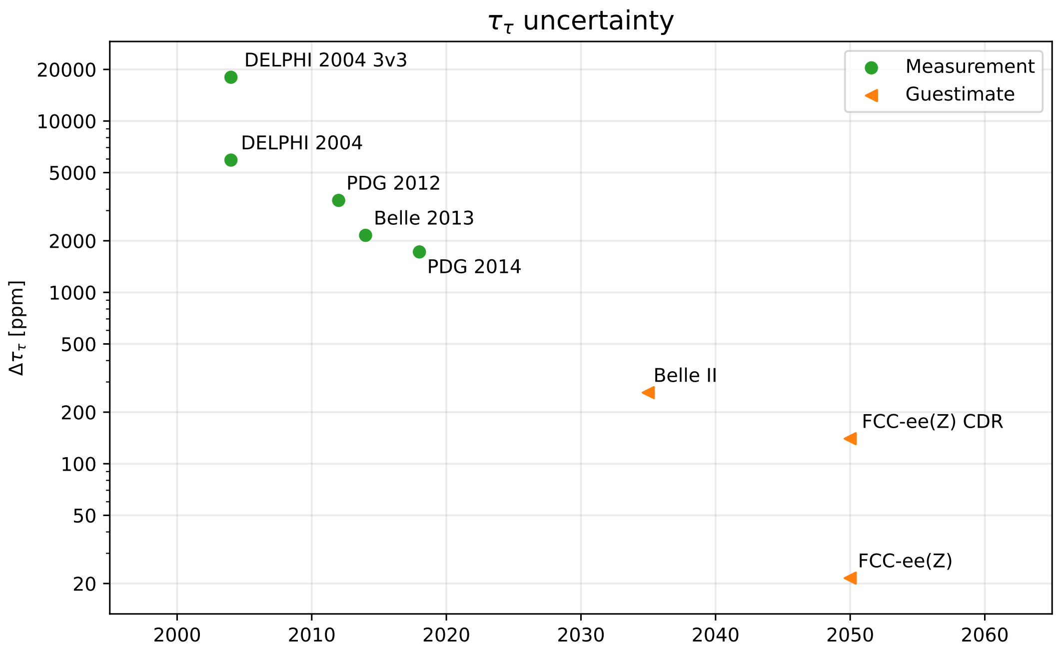

Figure 2: Present and future expected experimental uncertainties on the tau lifetime. The dates of the future measurements are speculative and mainly chosen for plotting purposes.

4 Tau leptonic branching fractions

ALEPH measured the tau leptonic branching fractions $\mathcal{B}(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)$ and $\mathcal{B}(\tau^- \to e^-\bar{\nu}_\mu\nu_\tau)$ with a precision of about 0.44% (0.40% statistical and 0.19% systematic), using $5.9{\cdot}10^6$ Z decays. The extrapolated statistical precision at FCC-ee with $6{\cdot}10^{12}$ Z decays amounts to 4.0 ppm. The measurement is complex and reducing the systematic uncertainties is challenging. We guess that at FCC-ee the ALEPH systematic uncertainty may be reduced by a factor 10 to 0.019%, 100% correlated between the muon and the electron tau decay modes, and that would set the

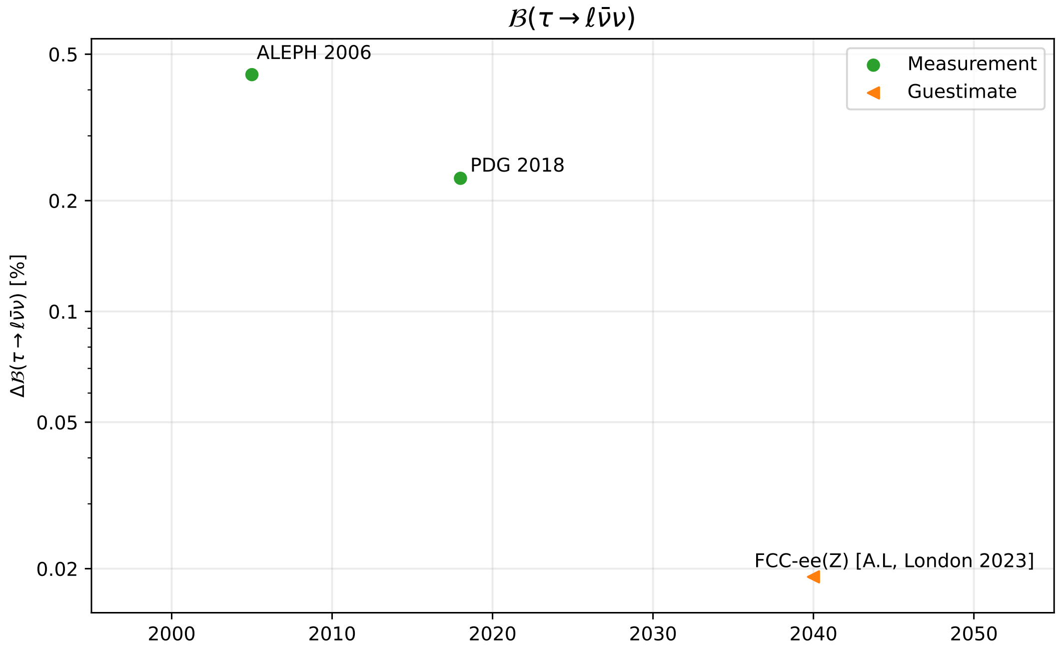

Figure 3: Present and future expected experimental uncertainties on the tau leptonic branching fractions. The dates of the future measurements are speculative and mainly chosen for plotting purposes.

estimated precision at FCC-ee. Figure 3 reports the present and future expected experimental uncertainties on the tau leptonic branching fractions.

5 Lepton universality test with the tau leptonic branching fractions

Figure 4 reports the precision of the lepton universality test using the PDG 2024 measurements compared with the test that will be possible at FCC-ee with the estimated precision of the measurements of the tau mass, tau lifetime and tau leptonic branching fractions.

6 Search for $\tau \to \mu\gamma$

A Monte Carlo simulation corresponding to $7{\cdot}10^{10}$ visible Z decays has been used to estimate how many $\tau \to \mu\gamma$ decay candidates from background sources are to be expected for $3{\cdot}10^{12}$ Z decays at an FCC-ee experiment [13, 14]. By assuming a reasonable reconstruction and selection efficiency, a sensitivity of $2{\cdot}10^{-9}$ has been estimated, corresponding to a signal equal to a double-sided 2σ fluctuation of the large number of expected background events, in the Gaussian approximation [15]. We recompute here the sensitivity in terms of the expected upper limit at 90% confidence level (CL) for a search of $\tau \to \mu\gamma$ on a sample of $6{\cdot}10^{12}$ Z decays at FCC-ee,

$$\mathcal{B}(\tau \to \mu\gamma) < 2.0{\cdot}10^{-9} \cdot \frac{qN\left[1 - \frac{1}{2}(1 - 90\%)\right]}{2} \cdot \frac{\sqrt{3{\cdot}10^{12}}}{\sqrt{6{\cdot}10^{12}}} \simeq 1.2{\cdot}10^{-9} \quad \text{at } 90\%\,\text{CL}, \tag{8}$$

where $qN(p)$ is the inverse of $pN(x) = \int_{-\,\mathrm{inf}}^{x} dN(x')dx'$, with $dN(x)$ being the Normal distribution. Figure 5 reports reports the present upper limits and the future expected upper limits for $\mathcal{B}(\tau \to \mu\gamma)$.

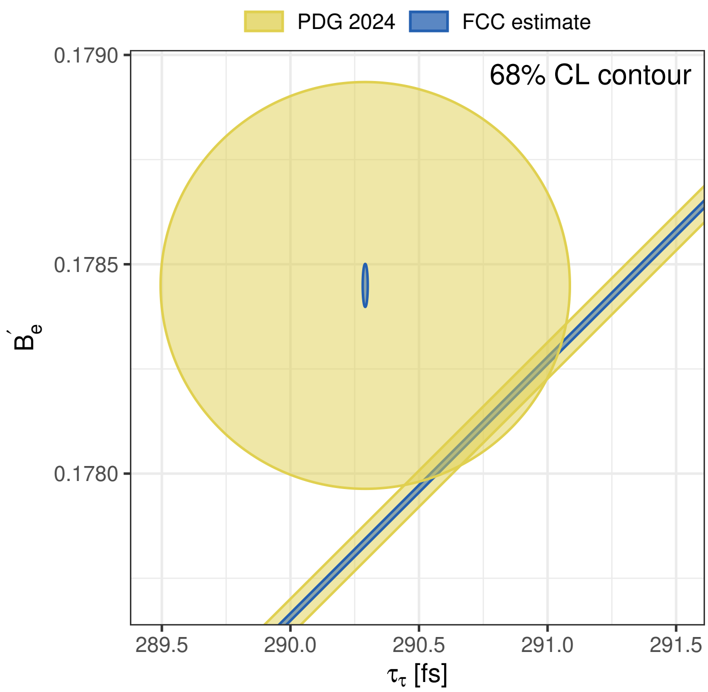

Figure 4: Lepton universality test using the tau mass, lifetime and leptonic branching fractions measurements. The test using the measurements reported in PDG 2024 is reported in yellow (lighter), and the estimated test at FCC-ee is reported in blue (darker). B_e' denotes the average between the measured branching fraction $\mathcal{B}(\tau^- \to e^-\bar{\nu}_\mu\nu_\tau)$ and its Standard Model prediction using the measured branching fraction $\mathcal{B}(\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau)$.

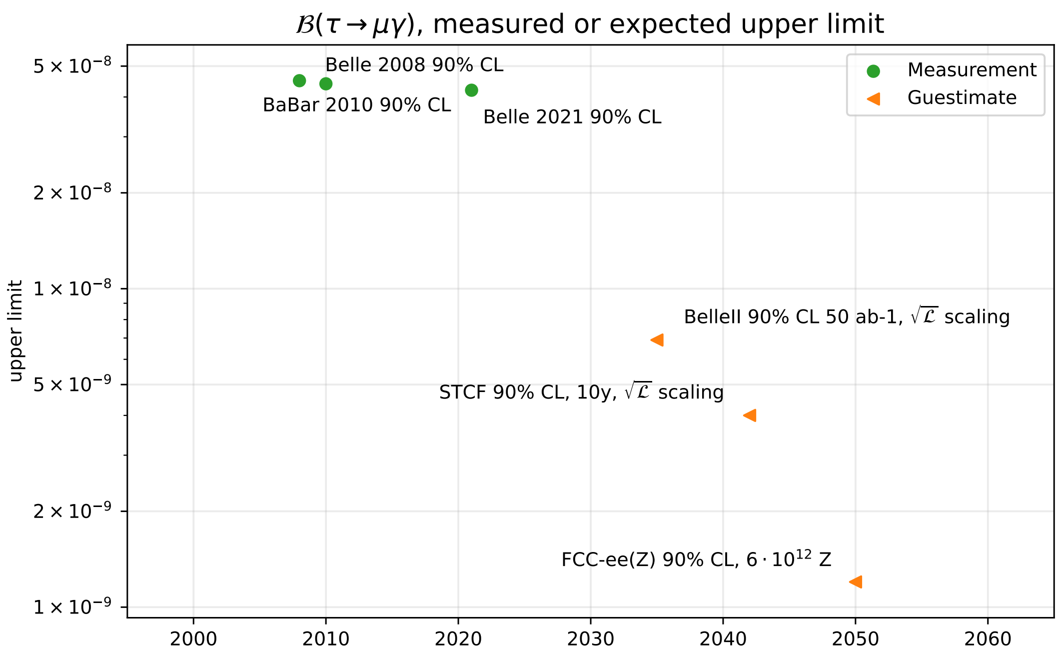

Figure 5: Present and future expected upper limits for $\mathcal{B}(\tau \to \mu\gamma)$. The dates of the future measurements are speculative and mainly chosen for plotting purposes. The expected limits for Belle II and the Super Charm-Tau factories are personal conservative estimates based on the assumption that those searches will be mostly background-constrained from the statistics that has been simulated for both facilities.

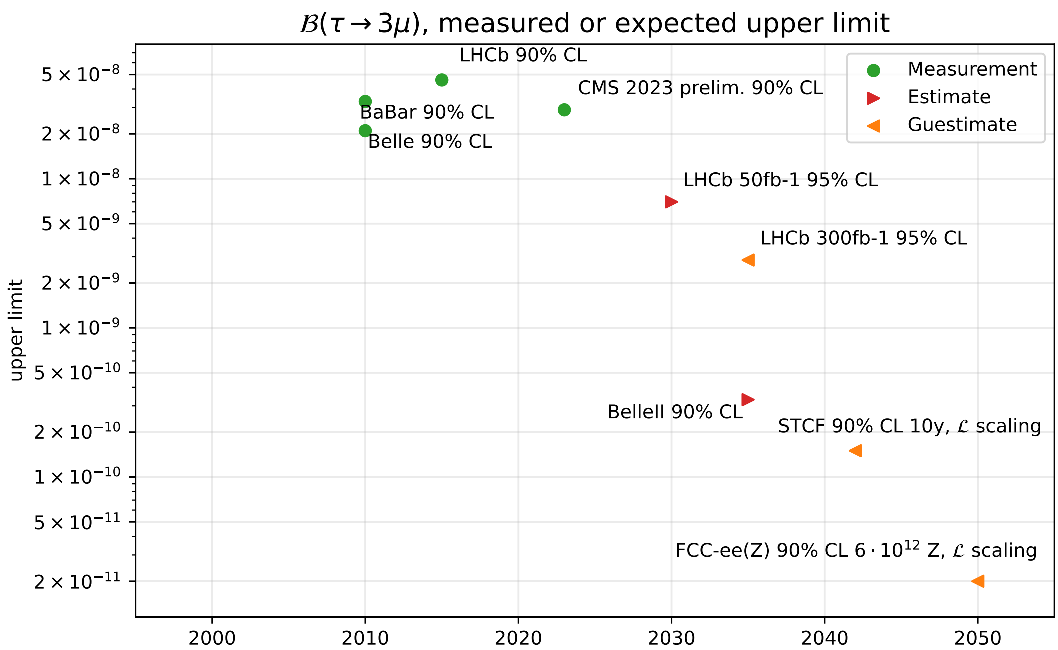

Figure 6: Present and future expected upper limits for $\mathcal{B}(\tau \to \mu\mu\mu)$. The dates of the future measurements are speculative and mainly chosen for plotting purposes.

7 Search for $\tau \to \mu\mu\mu$

The Belle II collaboration reported a 90% CL upper limit of $1.9{\cdot}10^{-8}$ for the lepton-flavour-violating branching fraction $\mathcal{B}(\tau \to \mu\mu\mu)$ [16], using $390{\cdot}10^6$ tau pairs (corresponding to 424 invfb of integrated luminosity). The estimated selection efficiency is about 20.4%, significantly larger than the one attained by the previous Belle search [17], 7.6%, and is now comparable to the efficiency that has been reported at LEP 1 for the DELPHI $\tau \to \mu\gamma$ search [18], 24.5%, which is about 4 times the efficiency reported by the same $\tau \to \mu\gamma$ search by *BABAR* [19]. When assuming that for the $\tau \to \mu\mu\mu$ search an efficiency of 35.0% may be obtained at FCC-ee, we estimate that the expected upper limit at FCC-ee will be:

$$\mathrm{UL}_{\mathrm{exp}}^{90} = 1.8{\cdot}10^{-8} \frac{20.4\%}{35.0\%} \frac{390{\cdot}10^6}{2.0{\cdot}10^{11}} = 2.0{\cdot}10^{-11} , \tag{9}$$

by linearly scaling the reported expected upper limit at 90% CL by Belle II, $1.8{\cdot}10^{-8}$, to the FCC-ee number of tau pairs, $2.0{\cdot}10^{11}$, while also assuming that the search will not be background-limited, exploiting the highly efficient and pure muon selection at the Z peak energies. Figure 6 reports the present upper limits and the future expected upper limits for $\mathcal{B}(\tau \to \mu\mu\mu)$.

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