HFSS Simulation on Cavity Coupling for Axion Detecting Experiment

[1]Beomki Yeo, [1]Walter Wuensch
[1]*CERN, Geneva, Switzerland*

Abstract

In the resonant cavity experiment, it is vital maximize signal power at detector with the minimized reflection from source. Return loss is minimized when the impedance of source and cavity are matched to each other and this is called impedance matching. Establishing tunable antenna on source is required to get a impedance matching. Geometry and position of antenna is varied depending on the electromagnetic field of cavity. This research is dedicated to simulation to find such a proper design of coupling antenna, especially for axion dark matter detecting experiment. HFSS solver was used for the simulation.

1. Resonant Cavity for Axion Detection

Axion is the candidate of dark matter which interacts feebly with other particles. Despite of its feeble interaction, axion decays into two photons with long half life. The decaying rate can be enhanced under the magnetic field which is working as a catalyst. This is called Primakoff effect and the photon emitted from axion has a specific frequency depending on axion mass. (from 0.1 to 100 GHz). If photon from axion has a same frequency with one of resonant cavity, transition rate will be enhanced leading to detectable power. For more detail, Lagrangian for axion coupling to photons with classical electromagnetic field is given by

$$L_{\text{int}} = -\frac{g_{a\gamma\gamma}}{4\pi}\vec{E}\cdot\vec{B}\psi_a(t), \tag{1}$$

where $g_{a\gamma\gamma}$ is the coupling strength and $\psi_a(t)$ is the time dependent axion field. Assuming external magnetic field from magnet is applied, this Lagrangian can be solved into the following equation of motion.

$$\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} = \frac{g_{a\gamma\gamma}}{c^2}\vec{B}_{\text{ext}}\frac{\partial^2\psi_a(t)}{\partial t^2}. \tag{2}$$

With the boundary condition of resonant cavity the energy inside cavity of j-th mode is

$$U_j = g_{a\gamma\gamma}^2 G_j^2 V B_{\text{ext}}^2 \int \frac{|\psi_a(\omega)|^2\omega^4}{(\omega-\omega_j)^2+\omega^4/Q^2}\frac{d\omega}{4\pi}, \tag{3}$$

where Q and V is the quality factor and volume of the cavity. G_j is the form factor which shows how well resonant electric field is aligned with external magnetic field.

$$G_j^2 = \frac{1}{B_{\text{ext}}^2 V}\frac{|\int B_{\text{ext}}\cdot E dV|^2}{\int E^2 dV} \tag{4}$$

G_j^2 has a value from 0 to 1 according to mode of cavity and direction of magnetic field. One of the example is the cylindrical cavity excited at TM010 mode which has only an E_z component. If the cavity is put inside solenoid, external magnetic field is applied to the z-direction which makes it possible to get enough good form factor around 0.7. A rectangular cavity also can be exploited with the TE101 mode to excite only E_y. This shape of cavity can sit inside dipole magnet which generates magnetic field in the y-direction. This is important because superconducting magnets built for accelerators are commonly dipoles. Fig. 1 shows how electric field is generated inside the cavity for each mode. Both cases has the form factor (G_j^2) around 0.7.

Another important aspect of Eq. 3 is its dependence on resonant frequency. The term after integral symbol represents that the denominator should be small as much as possible to acquire more energy. To make it happen resonant frequency of cavity (ω_j) should be tunable so that it can be close to axion frequency (ω). High quality factor (Q) is also required to keep the value of denominator small. Hence tuning of resonant frequency should be done without decreasing quality factor of cavity. Dielectric or conducting material is used as a tuning material inside cavity to change the frequency of cavity while moving slightly. Fig. 2 shows how tuning mechanism works out for cylindrical and rectangular cavity.

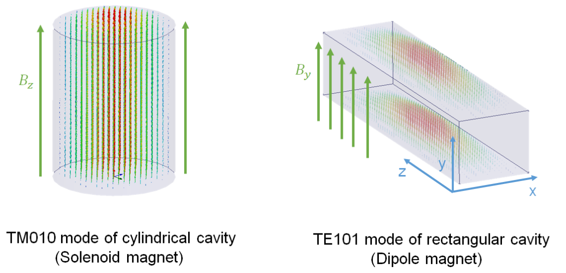

Figure 1: Cylindrical and rectangular cavity for axion detecting

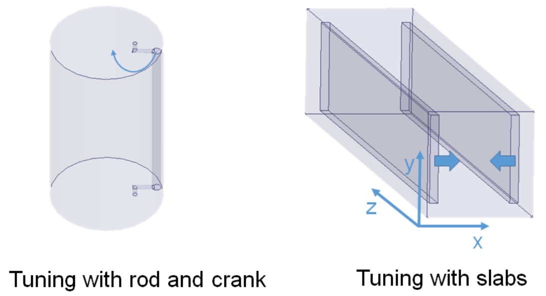

Figure 2: Tuning mechanism for cylindrical and rectangular cavity

2. Coupling to Transmit Power Every cavity has a finite quality factor which is represented by

$$Q = \omega \frac{U}{P}, \tag{5}$$

where U is the internal energy of cavity and P is the energy loss per one period. If it is not coupled to external source, it is called unloaded and the cavity has a distinct unloaded quality factor (Q_0). Since finite Q_0 means there is a energy loss during its oscillation it is vital to couple cavity to external source which transmits power in same frequency. External source also has a quality factor (Q_{ext}) like a cavity. It is said that cavity is loaded once the source are coupled. There are mainly two methods to transmit power along the source: coaxial cable and wave guide. Since electromagnetic field propagates inside coaxial cable at any frequency, coaxial cable is mainly used for the resonant cavity.

Coaxial Cable Let us say the resonant frequency of cavity is f_r for desirable mode. Then electromagnetic field of coaxial cable also oscillates at same frequency because TEM00 mode of coaxial cable can be excited for all frequencies. However, there must be a caution for selecting the size of coaxial cable so that its first higher order frequency (f_c) is higher than f_r. Otherwise, more than two modes are excited at the same time which can lead to the significant power loss if, for example, the cable is bent. First higher order frequency is determined by its inner and outer radius (r_a, r_b) and ϵ_r of dielectric filling between two conductors. (Fig. 3) Approximated value of first

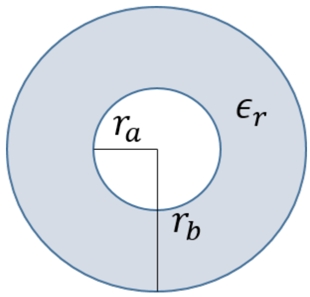

Figure 3: Geometry of coaxial cable

higher order frequency (f_c) is simply given by

$$f_c \approx \frac{c}{\pi \frac{r_a + r_b}{2} \sqrt{\epsilon_r}}, \tag{6}$$

where c is the velocity of light. In this paper UT-250C-LL semi-rigid coaxial cable was simulated. Specs of this cable is shown at Tab. 4.

	value
inner radius (r_a)	0.914mm
outer radius (r_b)	3.175mm
permittivity (ϵ_r)	2
1st higher order frequency (f_c)	20GHz

Figure 4: Spec of UT-250C-LL

Electric and Magnetic Coupling Coaxial cable should be coupled to the cavity considering the resonant electromagnetic field inside the cavity. The basic concept is transmitting the electromagnetic field of coaxial cable stably into the cavity satisfying the resonant mode and vice versa. One of the common antenna used is monopole coupler which is an extension of the inner conductor of the cable . Fig. 5 is the schematization of how this electric coupling works out for the TE101 mode of rectangular cavity. Same configuration is allowed for the cylindrical cavity as well.

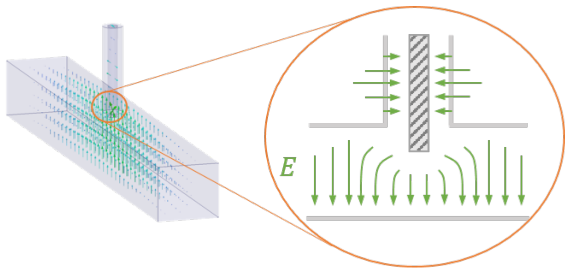

Figure 5: Electric coupling with monopole. Vectors in left model is representing the electric field.

The other coupling method is using loop (stretched from the inner conductor) which magnetic field is induced from. The schematization of magnetic loop for TE101 mode is described at Fig. 6.

3. Minimizing Return Loss Before detecting the axion signal, minimizing return loss from reflection is the most important task. When the return loss reaches almost 0 it is possible to see clear axion signal from the minimized background noise. (Return loss can be checked out with the network analyzer.) There are two main elements which cause undesirable return loss (Γ): mismatch loss ($\Gamma_{\Delta Z}$) and operating frequency deviated from

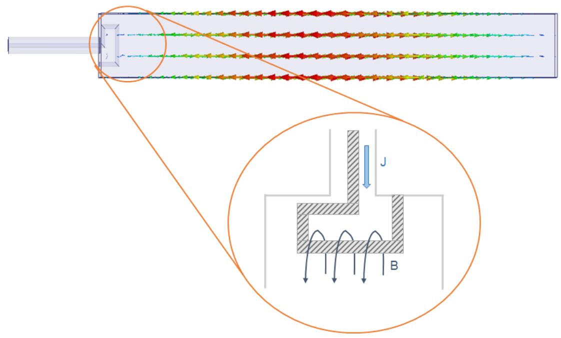

Figure 6: Magnetic coupling with loop. Vectors in above model is representing the magnetic field.

resonant mode frequency ($\Gamma_{\Delta f}$).

$$\Gamma = \Gamma_{\Delta Z} + \Gamma_{\Delta f}. \tag{7}$$

Correcting latter one is not a big problem because the point, where the peak is appeared on return loss vs. frequency graph, is the resonant mode frequency. Mismatch loss is repairable with impedance matching.

Impedance Matching When the source and the load (Cavity) is coupled to each other, signal reflection occurs from load unless their impedances are same with each other. As referred, power of reflected signal accounts for mismatch loss. This phenomena can be compared to incident electromagnetic field to the medium which has different ϵ or μ. If the ϵ, μ of both mediums is same there is no reflected field from the boundary. Similarly, there is also no reflection from the boundary if the impedance of source (Z_S) is matched to the one of load (Z_L). This is called impedance matching and summarized by

$$Z_S = Z_L^*, \tag{8}$$

where the asterisk means the complex conjugate. This impedance matching can be regarded as matching the quality factor (Q) of source and load. Revisiting the Eq. 5, Q can be also represented by reactance (X) and resistance (R).

$$Q = \omega \frac{U}{P} = \frac{X}{R}. \tag{9}$$

In the case of impedance matching, Q_0 and Q_{ext} have a same value since their reactance and resistance are same.

Loaded Quality Factor and Bandwidth The unloaded quality factor is the Q of cavity itself. To read out the signal in-and-out correctly, it is required to consider loaded quality factor (Q_L) which is the following:

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}}. \tag{10}$$

Then if there is a impedance matching at the resonant frequency ($Q_0 = Q_{\text{ext}}$) Q_L has a value of $\frac{Q_0}{2}$.

In practice it is not feasible to measure Q_0 directly but Q_L. Q_L can be measured by reading out 3dB bandwidth of return loss (Γ) under the condition of impedance matching. For this measurement it is normal to convert return loss (Γ) into dB unit like the following

$$RL(\text{dB}) = 20 \log_{10} \Gamma. \tag{11}$$

Q_L is the ratio of resonant frequency (f_0) and 3dB bandwidth ($\Delta f_{3\text{dB}}$)

$$Q_L = \frac{f_0}{\Delta f_{3\text{dB}}} \tag{12}$$

Following figure (Fig. 7) is one of the example.

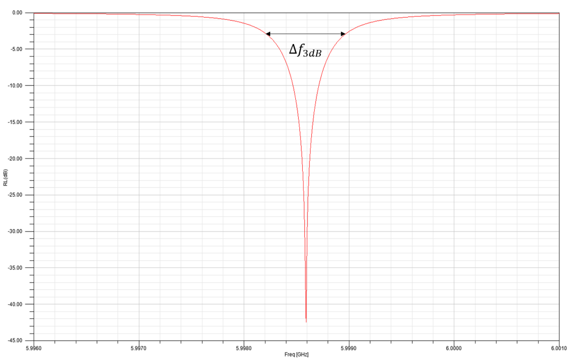

Figure 7: RL(dB) vs.frequency. $f_0 = 6GHz$ and $\Delta f_{3\text{dB}} = 0.74MHz$. Hence $Q_L = 8100$

However, Eq. 12 is only valid for impedance matching system. It is because that mismatch loss ($\Gamma_{\Delta Z}$) is not considered in that equation.

Coupling Coefficient β and Smith Chart Coupling coefficient (β) is a criterion to check the impedance matching. β is given by the ratio of Q_0 and Q_{ext}.

$$\beta = \frac{Q_0}{Q_{\text{ext}}}. \tag{13}$$

According to the value of β, it is called critically coupled, undercoupled and overcoupled. Critical coupling means impedance matching between the source and the load.

$$\begin{aligned} \beta = 1 &\quad : \quad \text{critically coupled} \\ \beta < 1 &\quad : \quad \text{undercoupled} \\ \beta > 1 &\quad : \quad \text{overcoupled} \end{aligned}$$

One of the ways to visualize β is using the Smith chart. Smith chart is the polar plot of complex Γ with the magnitude and phase as $\Gamma = |\Gamma| e^{j\theta}$. Minimized return loss occurs when the track of Γ crosses the origin where $\Gamma_{\Delta Z} = 0$ and $\Gamma_{\Delta f} = 0$. Fig. 8 shows three cases of coupling. Since critical

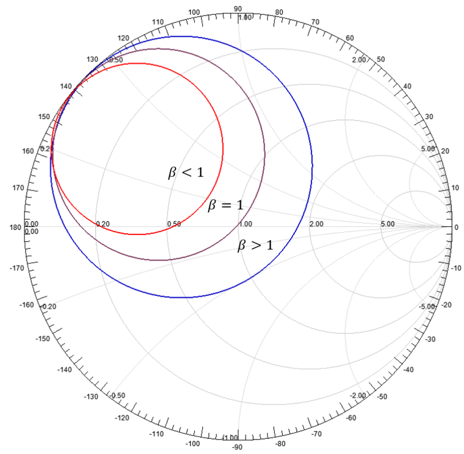

Figure 8: Smith chart for polar plot of Γ. Red, deep brown and blue lines indicate undercoupled, critically coupled and overcoupled respectively.

coupling is essential element for minimizing return loss it is desirable to make antenna tunable to find its optimal position. For example monopole antenna is to be moved longitudinally and loop antenna rotationable transversely.

4. HFSS Simulation for Loaded Cavity

Cylindrical and Rectangular cavity are investigated to find a critical coupling between the source (cable) and the cavity. Semi-rigid coaxial cable UT-250C-LL (Fig. 4) was realized in simulation. The size of cavity was selected to have 6GHz resonant frequency.

Cylindrical Cavity with Monopole To excite TM010 mode, radius of cavity is set to be 19.15 mm. Height of cavity is 50 mm and length of cable was tried at 40 mm. The variable is the length of inner conductor (l_z). If l_z is 0 mm, inner conductor is terminated at the position of top plate. Fig. 9 is describing how l_z is defined to avoid confusion.

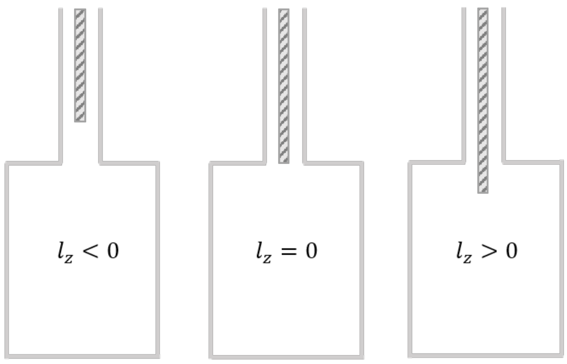

Figure 9: Schematization for the length of inner conductor (monopole)

The smith chart plot for each value of l_z is shown at Fig. 10. We can see the tendency of increasing diameter while monopole extends into the cavity. It is undercoupled at starting point ($l_z = -1mm$), critically coupled at $l_z = -0.8mm$ and overcoupled after that. This is because the accepted flux around monopole increases with the its length. Since there is a critical coupling at $l_z = -0.8mm$ we can get Q_L by measuring 3dB bandwidth and return loss. (Fig. 11 same with Fig. 7). Based on Eq. 12, Q_L has a value of 8100. When the cavity without cable is simulated its unloaded quality factor (Q_0) is 16180 which is double of Q_L.

Rectangular Cavity with Monopole The size of the rectangular cavity is also set to have TE101 mode at 6GHz. The width (W), height (H) and length (L) was given as 25, 24 and 200mm respectively. (Fig. 12 As a variable l_y is used instead of l_z because E_y is excited at TE101 mode.

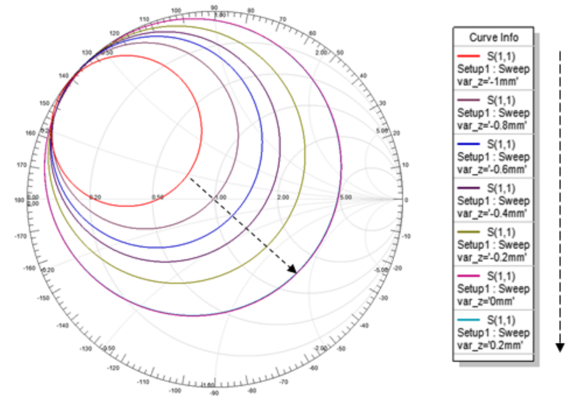

Figure 10: Smith chart for each value of l_z. var_z on the right table is equivalent to l_z.

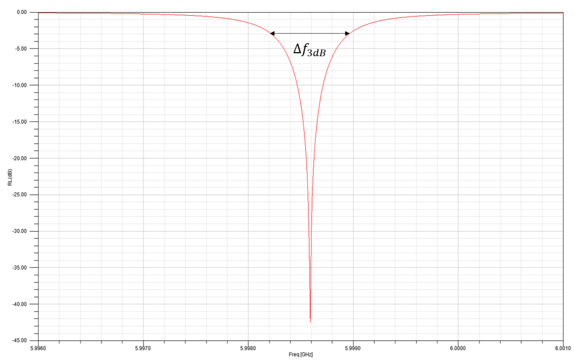

Figure 11: Return loss at critical matching. $f_0 = 6GHz$ and $\Delta f_{\text{3dB}} = 0.74MHz$. Hence $Q_L = 8100$

This case has the same tendency with previous one . The logner the antenna is, the stronger the coupling is. (Fig. 13)

Critical coupling occurs at $l_y = 0.1mm$ and the Fig. 14 is plot of the return loss when there is a critical coupling. Since Q_L is the ratio of f_0 and Δf_{3dB}, Q_L is calculated as $\frac{6GHz}{1.26MHz} = 4745$. Q_0 of Unloaded cavity was also simulated and it has a value of 9753 which is almost double of Q_L.

Rectangular Cavity with Loop Magnetic loop can be installed in the position where magnetic field pass through. Since there is only E_y in TE101 mode, magnetic field will make a curl around x-z plane so that the side wall of cavity is allowed for magnetic loop. In this section, small side (x-y plane) was used for loop simula-

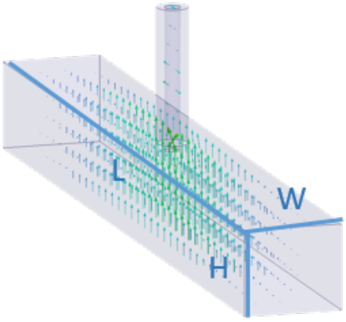

Figure 12: Geometry of rectangular cavity. W, H and L indicate the length in x,y and z direction.

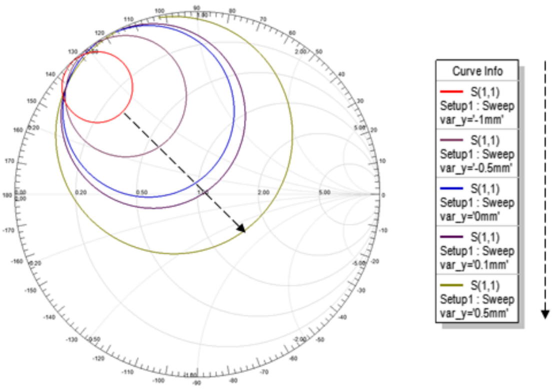

Figure 13: Smith chart for each value of l_y. var_y on the right table is equivalent to l_y.

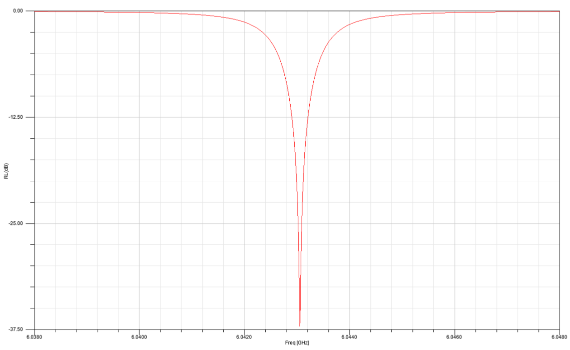

Figure 14: Return loss at critical matching. $f_0 = 6GHz$ and $\Delta f_{\mathrm{3dB}} = 1.26MHz$.

tion. (Fig. 15) The given variable is the angle of the loop. 0° means unperturbed magnetic field orthogonally pass through the loop with maximum flux. 90° is the opposite case where no magnetic field can make flux into loop.

Figure 15: Cross section of cavity on y-z plane

The strategy to find critical coupling was making a loop enough big to be overcoupled with enough flux. Then it was expected to have a critical coupling at specific angle between 0° and 90°. Fig. 16 says that 52.1° is the critical coupling point. One can see the tendency that the diameter of circle decreases as the loop is rotated. Calculated Q_L from 3dB bandwidth is 4808 which is half of Q_0 (9753).

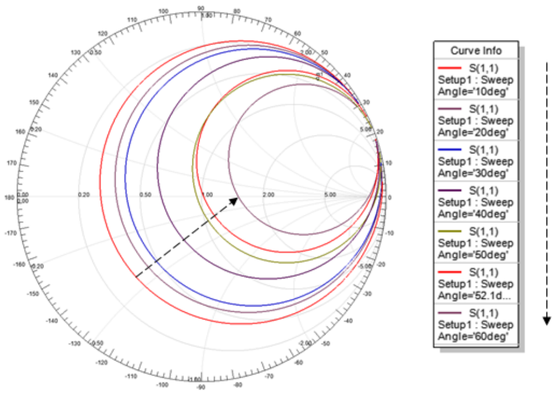

Figure 16: Smith chart for each rotation angle.

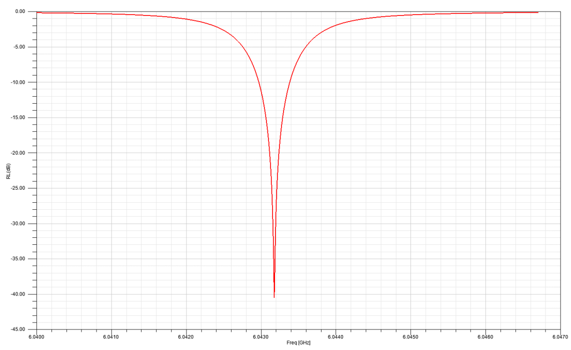

Figure 17: Return loss at critical matching. $f_0 = 6GHz$ and $\Delta f_{\mathrm{3dB}} = 1.25MHz$.

5. Conclusion It turned out that antennas made for the cavity operating at 6GHz has a critical coupling point with a clear tendency. Especially simulation of rectangular cavity coupled to loop bears a positive result for the future axion research in CAST (Cern Axion Solar Telescope)[3]. However, since the position of critical coupling had a dependence on lots of variables like length of cable or size of loop, those variables kept invariant for simplicity. Therefore further investigation on how those variables affect the result is required. Finally practical considerations such as standard coaxial cable dimensions and available space inside the magnet will affect the final design.

6. References

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