Higher Moments of Underlying Event Distribution*

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Abstract

We perform an Underlying Event analysis for real data sets from pp collisions at center of mass energy $\sqrt{s} = 5$ and 13 TeV and pPb collisions at $\sqrt{s} = 7$ TeV at the LHC, together with the Monte Carlo data sets generated with Pythia8 and EPOS in the same conditions. The analysis is focused on the transverse region which is more sensitive to the Underlying Event, and performed as a function of the leading track transverse - momentum p_t in each event. In our work, not only the average underlying event activity but also its fluctuation, namely its root mean square (RMS), Skewness and Kurtosis, are analyzed. We find that the particle density, energy density and their fluctuation magnitude (RMS) are suppressed at leading $p_t \approx 5$ GeV/c for all these cases, with EPOS having evident deviation of 10%-25%. The higher moments skewness and kurtosis decrease rapidly in low leading p_t region, and follow an interesting Gaussian-like peak centered at leading $p_t \approx 15$ GeV/c.

I. Introduction

The underlying event was first defined by the CDF collaboration as everything produced in the particle collision except for the two outgoing hard scattered jets, and was considered to consist of the beam-beam remnants (breakup of the colliding hadrons) and the contributions from the hard scattering from initial and final-state radiation [1]. And they found that none of the Monte Carlo models including HERWIG, ISAJET and PYTHIA produces the correct p_T dependence of charged particles in the transverse region. It was later pointed out that the main contribution to the UE comes from the color exchanges between beam particles, which is modeled by multiple parton interactions (MPIs) and significant in studying the QCD structure of the nucleus. Even though the effects of underlying event are often formally suppressed for inclusive cross sections, the study of underlying event is important also because a better understanding of UE properties is necessary for precision measurements of standard model processes and the search for new physics.

The UE measurement at the LHC was performed first by CMS collaboration [2, 3], and later also by ATLAS [4] and ALICE [5]. We perform a new ALICE underlying event analysis starting from the same strategy, namely analyzing charged particle and its energy density distributions in the transverse region as a function of leading track transverse momentum, for pp collisions at center of mass energy $\sqrt{s} = 5$ and 13 TeV and pPb collisions at $\sqrt{s} = 7$ TeV with comparison with MC data from generators Pythia8 and EPOS. To better understand the UE properties, we further analyze the fluctuations of the charged particle and its energy density distributions by studying their higher moments, i.e. studying the root mean square (RMS), skewness and kurtosis.

The central barrel of the ALICE detector covers the polar angle range 45°-135°, corresponding to pseudo-rapidity range $-1 < \eta < 1$, here we accept the Standard ITSTPC Track Cuts 2011 with $|\eta| < 0.8$ and track transverse momentum $p_t > 0.5$ GeV. The events generated with Pythia8 and EPOS passed through detector simulation based on GEANT4.

TABLE I. Track Selection Criteria

MinNClustersTPC	70
MaxDCAToVertexXY	2.4
MaxChi2PerClusterTPC	4
MaxDCAToVertexZ	2
DCAToVertex2D	True
MaxChi2TPCConstrainedGlobal	36
ClusterRequirementITS	SPD,Any
MaxFractionSharedTPCClusters	0.4
PtRange	0.5
EtaRange	(-0.8, 0.8)

We focus on the the transverse region with respect to the leading track and defined as following:

- Toward: $|\Delta\phi| < 1/3\pi$
- Transverse: $1/3\pi < |\Delta\phi| < 2/3\pi$
- Away: $|\Delta\phi| > 2/3\pi$

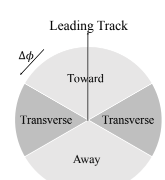

FIG. 1. Azimuthal topology of the collision.

We first characterize the underlying event activity by the average charged particle density $N_{ch}/(N_{ev} \cdot \Delta\eta \cdot \Delta\phi)$

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and the average summed p_t density $\Sigma p_t/(N_{ev} \cdot \Delta\eta \cdot \Delta\phi)$. Then we investigate their fluctuations by analyzing:

- Root Mean Square (definition of GetRMS in root):

$$RMS = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n}}$$

- Skewness:

$$Skew = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^3}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2}^{3}}$$

- Kurtosis:

$$Kurt = \frac{\frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^4}{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2}^{4}}$$

which represent the fluctuation magnitude, asymmetry and non-Gaussianity, respectively. More specifically, the larger skewness indicates the less symmetric the distribution is about its mean value, and the larger kurtosis ($Kurt > 3$) implies that the distribution has heavier tails and a higher peak comparing with the normal distribution, which has kurtosis 3.

II. Particle Density Analysis

The results for the charged particle multiplicity in the transverse region for p-p collision at 5.02 TeV and 13 TeV and p-Pb collision at 5.02 TeV are presented in FIG. 2.

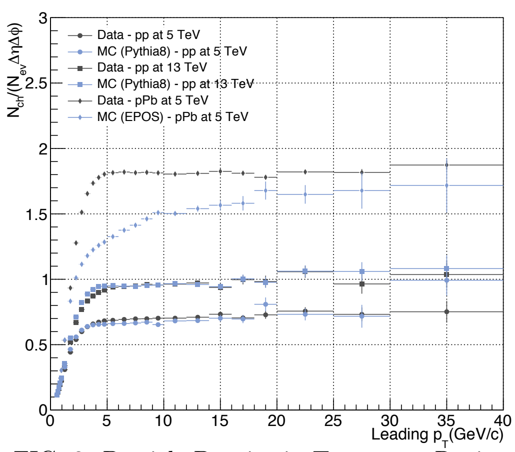
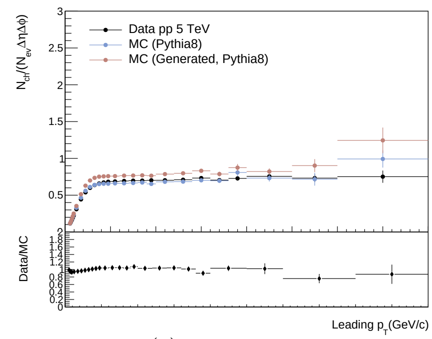


FIG. 2. Particle Density in Transverse Region.

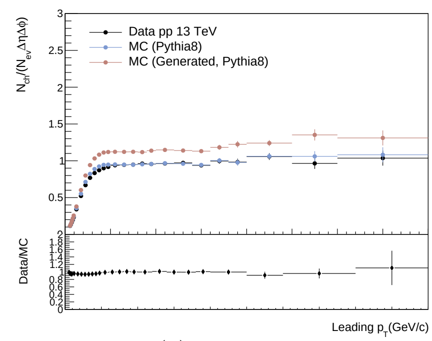
The particle density distributions for data from p-p collisions at 5.02 TeV and 13 TeV are well described by Pythia8 concerning both their shapes and values. While for the p-Pb at 5.02 TeV case, EPOS even though successfully describes the rapid increasing at low leading p_t (<3 GeV/c) region and the plateau at high leading p_t (>20 GeV/c) region, it fails to describe the trend in the intermediate region. In all cases, the data plots tend to a plateau when leading $p_t \approx 5$ TeV/c, which indicates the saturation of the MPIs.

1. *Particle Density – Average*

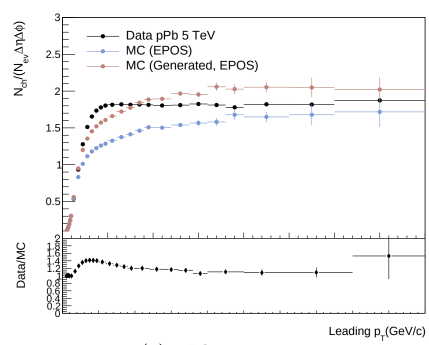
In this part, we present the average charged particle density distributions in transverse region with Data/MC ratio plots for p-p collision at 5.02 TeV and 13 TeV and p-Pb collision at 5.02 TeV in FIG. 3.



(a) pp 5.02 TeV



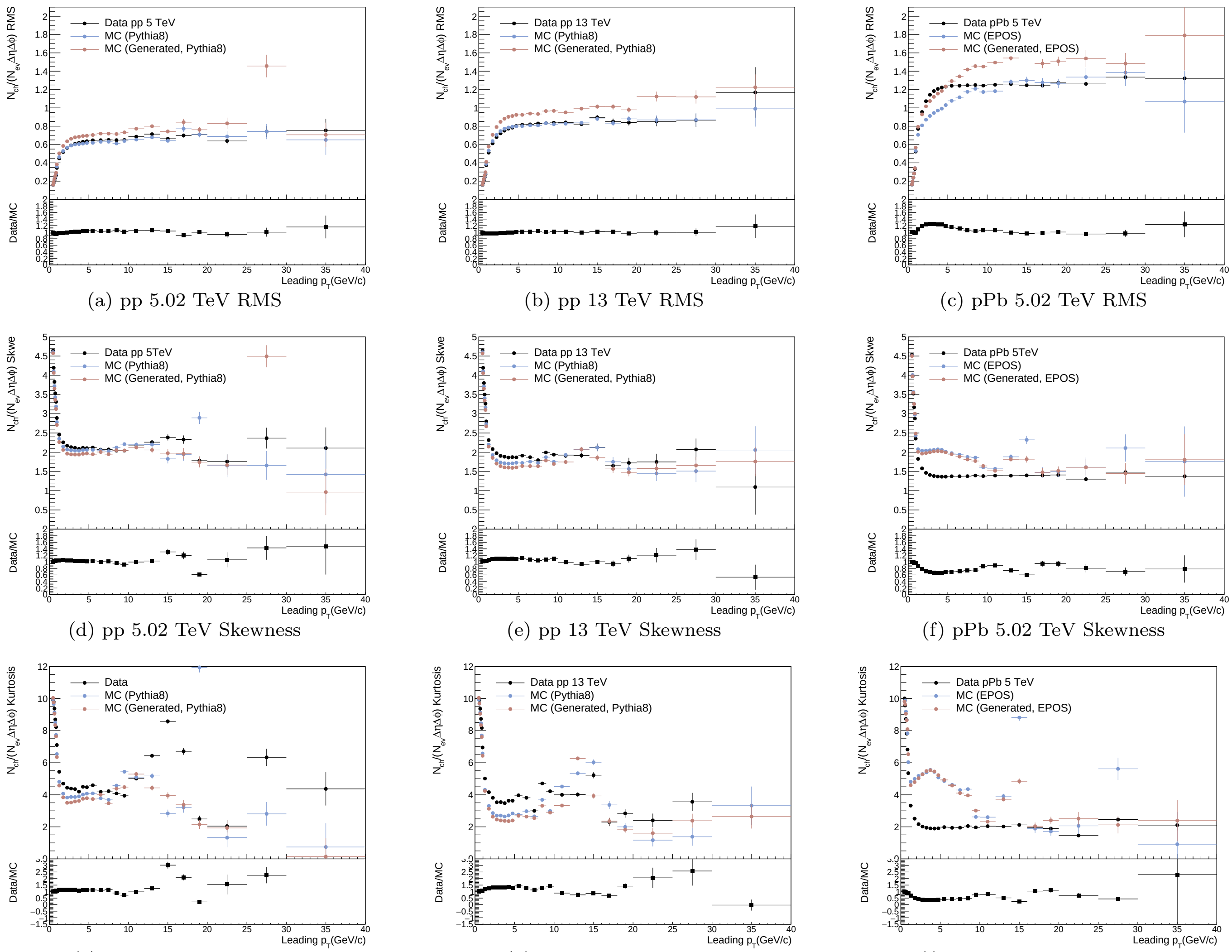
(b) pp 13 TeV



(c) pPb 5.02 TeV

FIG. 3. Average charged particle density in transverse region with Data/MC ratio plots. ALICE data (black) from pp at 5.02 TeV (top), pp at 13 TeV (middle) and pPb at 5.02 TeV (bottom). And correspondingly, the generated MC data (red) and reconstructed data (blue).

In the pp 5.02 TeV and pp 13 TeV cases, the particle densities of the generated MC data by Pythia8 exceed the experimental data, but the reconstructed MC data by Pythia8, which passed through the detector simulation, show significant accordance with the experimental data. EPOS fails to describe the UE average well, especially in low leading p_t region, having discrepancy $\sim 25\%$.



(a) pp 5.02 TeV RMS (b) pp 13 TeV RMS (c) pPb 5.02 TeV RMS

(d) pp 5.02 TeV Skewness (e) pp 13 TeV Skewness (f) pPb 5.02 TeV Skewness

(g) pp 5.02 TeV Kurtosis (h) pp 13 TeV Kurtosis (i) pPb 5.02 TeV Kurtosis

FIG. 4. The fluctuation–RMS (top), skewness (middle) and kurtosis (bottom) of charged particle density in transverse region with Data/MC ratio plots. ALICE data (black) from pp at 5.02 TeV (left), pp at 13 TeV (middle) and pPb at 5.02 TeV (right). And correspondingly, the generated MC data (red) and reconstructed data (blue).

2. Particle Density – Fluctuation

In this part, we present the fluctuation of the charged particle density distributions in transverse region with Data/MC ratio plots for p-p collision at 5.02 TeV and 13 TeV and p-Pb collision at 5.02 TeV in FIG. 4.

RMS. We see again from plots (a) and (b) that the reconstructed MC data by Pythia8 show significant accordance with the real data from LHC, while in plot (c) the EPOS MC data show similar shape with the real data with discrepancy in their values. This indicates that not only the average UE activities in p-p collisions (at 5.02 and 13 TeV) but also their fluctuation magnitudes are well described by the modeling of Pythia8, and the EPOS here is performing well to describe the fluctuation magnitude but still need to be improved to fit the data better. In each case, the RMS distribution increases rapidly at low leading p_t and tends to reach a plateau at leading $p_t \approx 5$ GeV/c as the particle density distribution does at the same energy scale.

Skewness. It can be seen from plots (d) and (e) that the shapes and values of these skewness distributions for both 5.02 and 13 TeV p-p collisions are well described by the reconstructed MC data generated with Pythia8, while they all fluctuate more in high leading p_t region and need more statistics to clarify. In plot (f), there shows up an evident protuberance near leading $p_t \approx 5$ TeV in the MC data, which may indicate some interesting physics. In all these skewness plots, their values decrease rapidly and turn to be flat. The main contribution to this effect may come from the fact that few charged particles are being able to be produced when the leading track p_t is low and the distribution is certainly asymmetric and becomes more symmetric and stable when increasing the leading p_t and producing more particles.

Kurtosis. In plots (g) and (h), we present the kurtosis distributions for 5.02 and 13 TeV p-p collisions with MC data by Pythia8, in which one can see that they share the same shape and the Data/MC ratio plots are slightly above the unit. In plot (i), we see again an evident protuberance for the MC data generated with EPOS for 5.02 TeV p-Pb collisions, in which a rising for 3 times shows up at leading $p_t \approx 5$ GeV/c. The rapid decrease at low leading p_t region may be interpreted as the charged particle distribution tends to be more Gaussian or have lighter tails comparing with Gaussian/normal distribution. In the cases of 5.02 and 13 TeV p-p collisions, their kurtosis values fluctuate around 3 at high leading p_t, indicating that their distributions have around the same peak height and tail heaviness as normal distribution. While in the 5.02 TeV p-Pb collision case, the kurtosis tends to be in a plateau of $Kurt \approx 2$, which indicates that its distribution has lighter tails and is flatter.

III. Energy Density Analysis

Apart from the particle density analysis, we perform the similar analysis for the energy density of charged particles in the transverse region by considering their average summed p_t distributions and the corresponding RMS, Skewness and Kurtosis distributions. The results for the summed p_t of charged particles in the transverse region for p-p collision at 5.02 TeV and 13 TeV and p-Pb collision at 5.02 TeV are presented in FIG. 5.

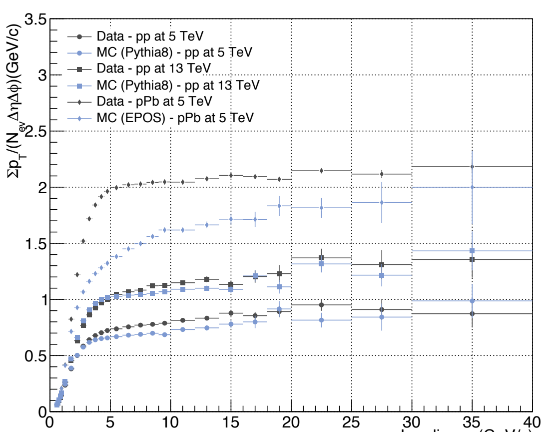
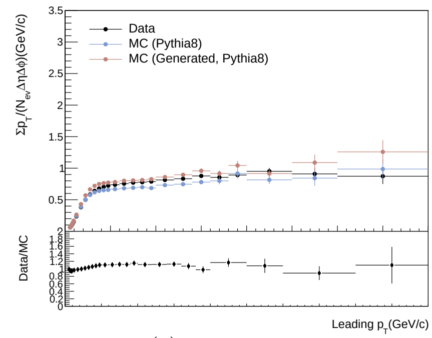


FIG. 5. Energy Density in Transverse Region.

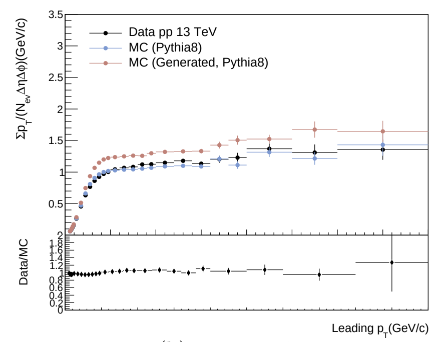
Similarly to the particle density distributions, the energy density distributions increase at low p_t region and tend to be stable at leading $p_t \approx 5$ GeV/c, corresponding to the saturation of the MPIs. The energy density distributions show slightly larger deviations between the real data and MC data comparing with the particle density distributions.

1. *Energy Density – Average*

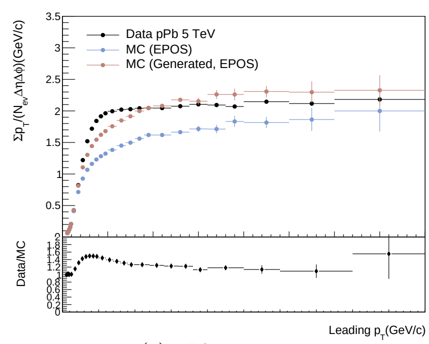
In this part, we present the energy density distributions of charged particle in transverse region with Data/MC ratio plots for p-p collision at 5.02 TeV and 13 TeV and p-Pb collision at 5.02 TeV in FIG. 6.



(a) pp 5.02 TeV



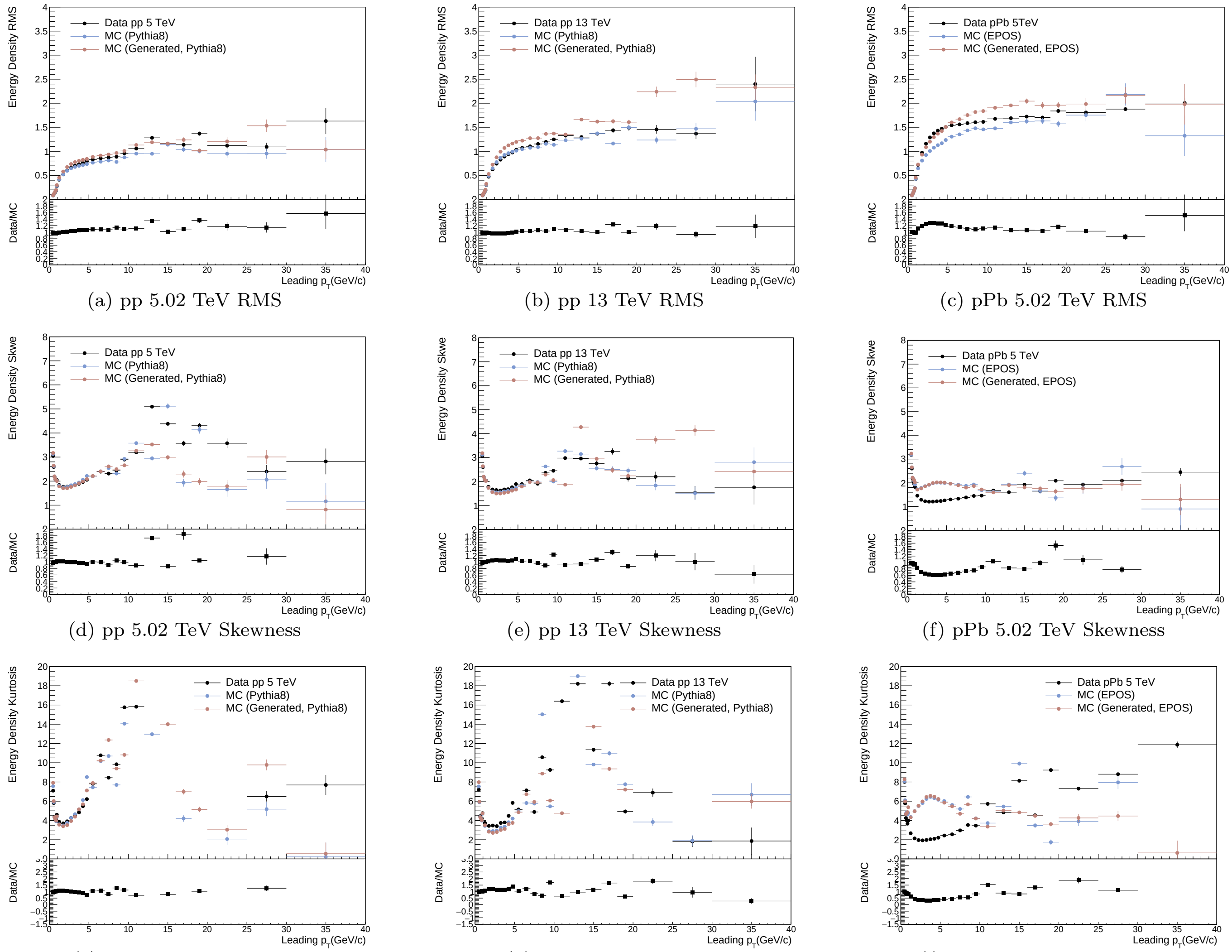
(b) pp 13 TeV



(c) pPb 5.02 TeV

FIG. 6. Average energy density of charged-particle in transverse region with Data/MC ratio plots. ALICE data (black) from pp at 5.02 TeV (top), pp at 13 TeV (middle) and pPb at 5.02 TeV (bottom). And correspondingly, the generated MC data (red) and reconstructed data (blue) by Pythia8 (top, middle) and EPOS(bottom).

The plots (a) and (b) are showing good accordance between the real data and reconstructed MC data by Pythia8 for p-p collision at 5.02 TeV and 13 TeV. In plot (c), the shape of real data is basically described by the MC data generated with EPOS and we see clearly the deviation of order 10%-25% from the Data/MC ratio distribution.



(a) pp 5.02 TeV RMS
(b) pp 13 TeV RMS
(c) pPb 5.02 TeV RMS
(d) pp 5.02 TeV Skewness
(e) pp 13 TeV Skewness
(f) pPb 5.02 TeV Skewness
(g) pp 5.02 TeV Kurtosis
(h) pp 13 TeV Kurtosis
(i) pPb 5.02 TeV Kurtosis

FIG. 7. The fluctuation–RMS (top), skewness (middle) and kurtosis (bottom) of energy density in transverse region with Data/MC ratio plots. ALICE data (black) from pp at 5.02 TeV (left), pp at 13 TeV (middle) and pPb at 5.02 TeV (right). And correspondingly, the generated MC data (red) and reconstructed data (blue) by Pythia8 (top, middle) and EPOS(bottom).

2. *Energy Density – Fluctuation*

Here in FIG. 7, we present the fluctuation information for the energy density distribution of charged particles in transverse region for p-p collision at 5.02 TeV and 13 TeV and p-Pb collision at 5.02 TeV.

RMS. Similar to the particle density distributions, from the energy density RMS plots (a) and (b) for the p-p collisions at two different colliding energy scales 5.02 and 13 TeV, we see a good accordance between the real data and MC data by Pythia8, especially for the main feature of a rapid increase following by a plateau, as well as a small discrepancy between the real data and EPOS MC data for p-Pb collision at 5.02 TeV. These similarities and accordances provide us more confidence that these MC models are performing well in describing the average UE activities and their fluctuation magnitudes, which ensures the validity of these models in describing many other processes and phenomena.

Skewness. In plots (d) and (e), the skewness of energy density distributions fluctuates more in high leading p_t region, but we see clearly the rapid decrease of skewness in low leading p_t region (leading $p_t < 2$ GeV), and interestingly, following by an increasing phase and then again a decreasing phase, which implement a Gaussian-like peak centered at leading $p_t \approx 15$ GeV/c. While in plot (f), we see again the protuberance of the MC data generated by EPOS at leading $p_t \approx 5$ GeV/c, but there doesn't show up the Gaussian-like peak centered at leading $p_t \approx 15$ GeV/c in the p-p collision cases at 5.02 TeV and 13 TeV, which may indicate some interesting effects. The decreasing phase in the low leading p_t region may be interpreted as in the particle density skewness distribution case, while the Gaussian-like peak needs to be addressed properly.

Kurtosis. In plots (g) and (h), in which the kurtosis distributions of energy density for 5.02 and 13 TeV p-p collisions with MC data by Pythia8 are presented, again we see the data are well described by the MC data especially in the low leading p_t region and they all fluctuate more in high leading p_t region. They decrease rapidly in the low leading p_t region as expected and follow an interesting Gaussian-like peak centered at leading $p_t \approx$ 15 GeV/c. Here the transition from the decreasing phase and the increasing phase is sharp, making a deep valley with its bottom reaching 3 (the kurtosis of a Gaussian distribution), which indicates the energy density transits from a very peaked shape to a Gaussian-like peaked shape (at leading $p_t \approx 2$ GeV/c) and then increases its peakedness until leading $p_t \approx 15$ GeV/c and decreases again. The shape of the kurtosis distributions itself as well as the differences between the distributions in the p-p collision case and the p-Pb collision case may indicate some interesting physics.

IV. Poisson Distribution Approximation Analysis

Here we have been analyzing the underlying event as a function of the leading track p_t. In each leading p_t bin, we have the distributions of the number of charged particles and its energy density. We may expect at least for the particle density distribution within each leading p_t bin is a Poisson like distribution to some extent. We find that one can start with the well known particle density distribution and employ the characteristics of Poisson distribution to catch the main features of the higher moments of the distribution.

The following properties of the Poisson distribution are well known:

- Probability density function: $f(x) = \frac{\mu^x e^{-\mu}}{x!}$
- Expectation value: $E[x] = \mu$
- Standard deviation: $\sigma = \mu$
- $Skew[x] = \mu^{-1/2}$
- $Kurt[x] = 3 + \mu^{-1}$

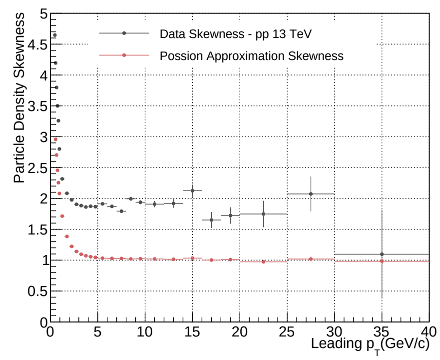


FIG. 8. Skewness estimated from a Poisson distribution and real data skewness.

In FIG. 8, we plot the skewness for real data from 13 TeV p-p collisions and the skewness as $\mu^{-1/2}$ calculated from the mean value by assuming the Poisson distribution, from which we can see that when input the μ (the particle density) the Poisson distribution successfully describes the rapid decreasing phase in low leading p_t region and the plateau starting at leading $p_t \approx 5$ GeV/c.

V. Summary and Acknowledgement

It has already been found that the average underlying event activity tend to be a plateau at leading $p_t \approx 5$ GeV/c in Ref.[2-5], which can also be seen in FIG. 2 and FIG. 5 above. One may expect that the underlying event activities are getting stable from leading $p_t \approx 5$ GeV/c, but we have shown that they are varying actively and actually very interesting by studying higher moments of the changed particle density its energy density distributions.

The author wish to thank all the people involved in this CERN Summer Student Programme, especially for the supervision of Prof. Paolo Bartalini and Dr. Prabhakar Palni for the helpful discussions and beneficial interactions (also with many other people at CERN). At the last, I wish to mention that this report is only a preliminary version that showing our work, and due to the very limited time for the project and writing the report, it may contain (probably) some typos and even significant mistakes. The statistics (number of events selected and analyzed) for each case is larger than 12 million, but still the results and conclusions may be inaccurate due to not enough statistics. If you have found any mistakes or have any suggestions, please contact me via the email address indicated above.

[1] T. Affolder et al. (CDF Collaboration). Phys. Rev. D 65, 092002 (2002).
[2] CMS Collaboration. Eur. Phys. J. C (2010) 70: 555572.
[3] CMS Collaboration. JHEP09(2011)109.
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