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Impact of beam-beam effects and linear coupling resonance on the absolute luminosity calibration at the CERN Large Hadron Collider

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Thesis submitted for the degree of
MSci Physics with a Year Abroad

02 July 2024

Abstract

In colliders such as the CERN Large Hadron Collider (LHC), luminosity represents the ratio between the event rate at a particle detector and the cross section of the particle physics event being measured. Uncertainties in the luminosity directly propagate to uncertainties in the cross section of the event. Luminosity is defined by the accelerator properties and performance. Measurements of the luminosity are performed to define the absolute calibration of the detectors by applying the well known van der Meer (vdM) method. This is performed regularly to improve precision; the bias to such measurements should be corrected for in the detector data. One important bias is the electromagnetic interaction between the two charged beams, the so-called beam-beam effects. This effect causes a measurable deviation from the analytical expression, although recent simulations have proved that the effective bias on luminous region computation was previously overestimated. Moreover, recent studies have shown that the initial assumptions of the vdM method, stating that the particle distributions in the three spacial dimensions are factorizable, may no longer hold. This is recreated using a new beam simulation framework code called Xsuite. Further deviations from expected results may arise from other beam dynamics effects such as linear and non-linear resonances. One hypothesis is that linear coupling effects introduced by elements in the accelerator lattice create correlated distributions in the transverse plane, therefore producing non-factorizable distributions and distorting the luminosity in transverse scan measurements. This idea originates from the earlier finding that Landau damping is modified by linear coupling resonance. To investigate this effect, firstly numerical tools needed to reproduce the above effects were developed in Xsuite, and previous results obtained with COMBI were reproduced. Secondly, previous studies were extended by exploring the effects of linear coupling and beam-beam effects on luminosity and σ_{vis} as a possible source of the non-factorization problem observed in data analysis from the ATLAS and CMS detectors. This effect has been studied for different machine and beam configurations to create a comprehensive picture of the impact to luminosity.

Résumé

Dans les collisionneurs comme le Grand Collisionneur de Hadrons (LHC), la luminosité représente le rapport entre le taux d'événements d'un détecteur de particules et la section efficace de l'événement physique mesuré. Les incertitudes sur la luminosité se répercutent directement sur les incertitudes concernant la section efficace de l'événement physique. La luminosité est définie par les propriétés et les performances de l'accélérateur ; les mesures de luminosité sont effectuées pour définir l'étalonnage absolu des détecteurs en appliquant la méthode bien connue de van der Meer. Cette procédure est réalisée régulièrement pour améliorer la précision ; le biais de cette mesure doit être corrigé dans les données du détecteur. Un biais important est l'interaction électromagnétique entre les deux faisceaux chargés, les effets dits faisceau-faisceau. Cet effet provoque une déviation mesurable par rapport à l'expression analytique, bien que des simulations récentes aient prouvé que le biais effectif sur le calcul de la région lumineuse était précédemment surestimé. De plus, des études récentes ont montré que les hypothèses initiales de la méthode de van der Meer, stipulant que la distribution des particules dans les trois dimensions spatiales est factorisable, peuvent ne plus être valables. Cela sera recréé en utilisant un nouveau code de simulation de faisceau appelé Xsuite. D'autres écarts par rapport aux résultats attendus, pouvant provenir d'autres effets dynamiques des faisceaux tels que les résonances linéaires et non linéaires, seront également étudiés. Une des hypothèses est que les effets de couplage linéaire introduits par des éléments dans le réseau de l'accélérateur, créent des distributions corrélées dans le plan transverse, ce qui produit des distributions non factorisables. Cela déforme la luminosité lorsqu'on la mesure avec des balayages transversaux. Cette idée découle de la découverte antérieure selon laquelle l'amortissement de Landau est modifié en raison de la résonance de couplage linéaire. Premièrement, les outils numériques nécessaires pour reproduire les effets mentionnés ci-dessus ont été développés dans le code de simulation Xsuite, et les résultats précédemment obtenus avec COMBI ont été reproduits. Deuxièmement, les études précédentes ont été étendues en explorant l'effet du couplage linéaire et les effets faisceau-faisceau sur la luminosité et σ_{vis} , et en tant que source possible du problème de non-factorisation observé dans l'analyse des données des détecteurs ATLAS et CMS. L'effet a été étudié pour différentes configurations de machine et de faisceau afin d'avoir une image complète de l'impact sur la luminosité.

Preface

This research was conducted as part of the Laboratory of Particle Accelerator Physics of École Polytechnique Fédérale de Lausanne (EPFL) and CERN's Accelerator and Beam Physics Department. This Master's thesis was completed during an exchange year from Imperial College London to EPFL for the obtainment of the degree MSci Physics with a Year Abroad. Simulations were performed with CERN's Xsuite program, an open-source framework for beam dynamics studies. Gaussian luminosity studies and the analysis of beam-beam effects used code developed by Péter Kicsiny. The code developed in this project in Xsuite was inspired by Joanna Wańczyk's innovative work in COMBI, with additional contributions from Joanna and Tatiana Pieloni, who provided essential code for calculating analytical expressions. An additional feature in the simulation code was implemented in this study that stored the beam distributions turn after turn. The inspiration for this study came from an intriguing idea by Tatiana and Joanna: that accelerator resonances induced by lattice imperfections combine with beam-beam effects to have an important role in creating non-factorizable particle distributions. These interactions have implications for how luminosity is calibrated at the LHC, specifically using the van der Meer method, challenging the assumption that particle distributions are predictable and may be factorizable in luminosity calculations. Finally, the CERN HTCondor system was used to speed up computation.

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Chapter 1

Introduction and motivations

High-energy particle colliders accelerate charged particle beams to high energies and bring them into collision. At the LHC, two counter-rotating proton beams are accelerated to 6.8 TeV [1], colliding at four interaction points, two of which host the ATLAS and CMS experiments. In accelerator physics, luminosity represents the ratio between the rate at which a physics event is measured and the cross-section of that event [2]. Knowing the luminosity is therefore essential for determining the event cross-section in particle physics experiments. Luminosity is maximised in high-energy physics to be able to measure rare events with small cross sections. At the LHC, absolute luminosity calibration sessions are performed regularly to obtain accurate and precise values for the absolute luminosity, to calibrate the experimental luminometers, and to define the measurement uncertainties that will be propagated into the data analysis of particle physics results. Greater accuracy in luminosity readings leads to greater accuracy in event cross-sections.

Such luminosity measurements are obtained by the van der Meer method [3]. One beam is shifted transversely with respect to the other, and the interaction rate as a function of the beams' separation (expressed in units of the transverse beam size σ_{rms}) is recorded by the luminometers. During such transversal scans, the beams are affected by mutual electromagnetic interactions since they are constituted of charged particles, protons for the case of the LHC.

These so-called beam-beam effects [4] alter the motion and distributions of the beams. Therefore they must be evaluated and corrected for when deriving the absolute luminosity scale. Recent simulations and experimental studies have quantified such effects and a correction scheme has been proposed and applied to the luminosity analysis of on-axis scans [5]. However, no studies exist around off-axis and diagonal scans where recent observations show a deviation from the known bias.

The studies developed in this project are used to test the assumption of the factorizability of particle distributions. In luminosity calculations the assumption is made that particle distributions in the three directions (horizontal, vertical, and longitudinal) are independent of each other, and therefore the separate distribution densities may be factorized when calculating luminosity. However, it has been pointed out that the two transverse planes (horizontal and vertical) may not be independent, but instead coupled.

Linear coupling is an effect introduced by elements in the beam lattice whereby the motion of particles in the two transverse directions is coupled; the horizontal motion is influenced by the vertical motion and vice versa. Recent studies have quantified the combined effect of coupling with Landau octupoles and suggests further investigations with other tune-changing forces like beam-beam effects and electron-cloud effects. This project

explores the hypothesis that the coupling introduced by skew quadrupoles is responsible for the deviations in expected luminosity values, demonstrating the non-factorizability of beam distributions in luminosity calculations.

This thesis focuses on two aspects: the first is to reproduce the work done in [5] by J. Wańczyk and implementing the necessary tool (a numerical integrator to compute the luminosity) in the new computational framework Xsuite. The second aim is to extend the studies and evaluate the effect of linear coupling resonance in normal and special scan configurations, in order to highlight deviations from the findings in [5]. Whether the inclusion of beam-beam effects and linear coupling puts into question the assumption of factorizability in luminosity calculations is explored. Hence, the above studies attempt to shed light on the non-factorizable issues recently observed by the ATLAS and CMS experiments in their absolute luminosity analysis of the available LHC Run 2 and Run 3 van der Meer scan data.

Chapter 2

Luminosity and beam-beam effects

2.1 The Large Hadron Collider

Particle accelerators, in particular colliders, have been responsible for many discoveries in particle physics. This includes measurements that confirmed quantum electrodynamics (QED) [6] and quantum chromodynamics (QCD) theory [7], as well as confirming experimentally the full standard model [8].

The Large Hadron Collider (LHC) at CERN is a circular collider with a circumference of 27km that has been used to carry out discovery experiments with proton-proton beams since 2008. It is the largest accelerator and experiment in the world, with the most notable of all its discoveries being the confirmation of the existence of the Higgs Boson [8]. The LHC features two counter-rotating beams made of multiple bunches. It hosts eight straight insertion regions (IRs), with four IRs hosting experiments at the interaction points (IPs): CMS, ATLAS, LHCb, and ALICE (Fig. 2.1). CMS and ATLAS are high-luminosity general-purpose detectors, while ALICE and LHCb focus on specific experiments with lower luminosities. As of 2023, the LHC achieved a maximum 13.6 TeV centre of mass energy and an instantaneous luminosity of $2 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$ [9]. The future High-Luminosity upgrade will aim for a luminosity of $5 - 7.5 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$, to be operational starting in 2029 [9]. It is the aim of this higher luminosity to make further important physics discoveries.

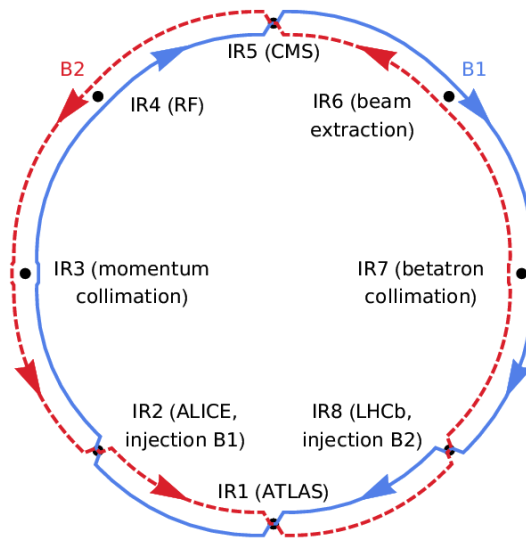


Figure 2.1: LHC Schematic diagram [10].

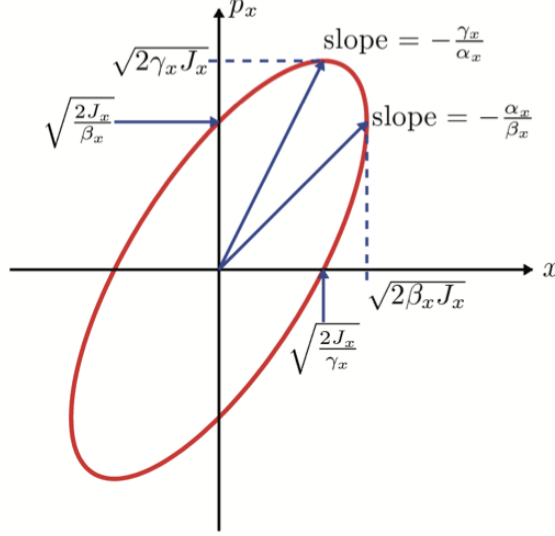


Figure 2.2: Transverse phase space ellipse [11].

2.2 Particle transverse dynamics

Particle accelerators, in their most basic definition, are a series of elements for guiding and accelerating charged particle beams. Dipoles change the beam trajectories, and quadrupoles focus and defocus the beams. Higher-order magnets are responsible for other corrections. For example, sextupoles can be used for correcting chromatic aberrations of quadrupoles.

The transverse motion in circular periodic accelerators can be derived from a general equation of motion. A curved coordinate system (x, y, s) is used where x is defined as the transverse horizontal direction pointing to the right in the direction of travel, y , the transverse vertical direction, pointing upwards and s , the longitudinal direction in the ring, pointing towards the direction of travel. As a particle traverses the circular accelerator, it travels across the phase space. This refers to the combined space of positions and momenta of particles in a given direction (x, p_x) and (y, p_y) . Along one turn, particles on the closed reference orbit rotate clockwise around the “phase space ellipse”, which changes depending on the optical beta, alpha, and gamma functions along the ring as illustrated in Fig. 2.2. The vertical phase space ellipse is similar to the horizontal version, so it is not displayed. The position and momenta of particles in the accelerator are locked on a regular motion following this ellipse. The area of the ellipse is an invariant of motion and its specific shape is obtained by solving Hill’s equation, Eq. 2.1. This equation describes particle dynamics on the reference circular orbit for linear fields, that is to say with the highest order magnets being quadrupoles:

$$x'' + K(s)x = 0. \quad (2.1)$$

$K(s)$ is the focusing term defined by the focal (magnet) system of the accelerator. For a periodic K : $K(s + C) = K(s)$, as is the case for circular accelerators, the periodic solution is obtained. In addition, if no coupling terms are present the two transverse planes, horizontal and vertical, are independent. The particle’s transverse position and momentum can be described by the following equations [12]:

$$\begin{aligned}
x(s) &= \sqrt{2\beta_x(s)J_x} \cos(\varphi_x(s) - \varphi_0), \\
p_x(s) &= -\sqrt{\frac{2J_x}{\beta_x(s)}} (\sin(\varphi_x(s)) + \alpha_x \cos(\varphi_x(s))).
\end{aligned} \tag{2.2}$$

The variable $\alpha_x(s)$ is the alpha function $\alpha_x(s) = -\frac{1}{2} \frac{d\beta_x}{ds}$ and $\beta_x(s)$ is the optical beta function defined in the Courant-Snyder Parametrization [13] of the particle motion and represents an envelope function of the oscillating sin and cos terms. J_x is the particle action in phase space and $\varphi(s)$ is the phase advance between s_0 and s , where $\varphi(s) = \int_{t=s_0}^s \frac{dt}{\beta(t)}$. The complete derivation is available in W.Hillert's lecture [14].

Each particle's action defines the area ($2\pi J$) of the phase space where a particle is confined if no dissipative forces are present. The beam emittance is the average action $\epsilon = \langle J \rangle$. Knowing the beam emittance and the optical beta function, one can derive the transverse beam size at any location s along the ring:

$$\sigma_{x,y}(s) = \sqrt{\epsilon_{x,y} \cdot \beta_{x,y}(s)}, \tag{2.3}$$

where $\sigma_{x,y}$ is the root mean square (r.m.s.) of the beam under the Gaussian approximation and $\epsilon_{x,y}$ is the emittance. In these studies for the LHC, the round beam approximation has been taken and so $\sigma_x = \sigma_y = \sigma$. Fig. 2.2 shows that the particle trajectories in phase space depend on the s coordinate. The ellipse changes shape and tilts depending on the accelerator magnet system along the s coordinate. That is to say, the ellipse shape will change following the optical functions, i.e. $\beta(s)$.

A more practical way to describe the particle motion in phase space can be obtained by normalising the coordinates. The normalisation eliminates the dependency on the focal system. The motion in normalised phase space of different particles under the influence of the accelerator magnet system is shown in Fig. 2.3. In normalised phase space, the motion is one of a pseudo-harmonic oscillator with amplitudes modulated by the envelope function $\beta(s)$ and confined on a circle with radius J . Particles with larger action J have trajectories with larger radii. In Fig. 2.3. Position and momenta are expressed in units of the respective r.m.s.. For position this is $\sigma_{x,y}$ and for momenta this is σ_{p_x,p_y} .

The betatron frequency Q , also known as the tune of the accelerator, is the number of oscillations per turn that particles perform under the influence of the focusing system, and is calculated by integrating the beta function:

$$Q_{x,y} = \frac{1}{2\pi} \oint \frac{ds}{\beta_{x,y}(s)}. \tag{2.4}$$

The integer and the fractional part of the betatron tunes are important and must be set in such a way as to avoid detrimental effects on the beams. The periodic nature of circular accelerators makes them very sensitive to perturbations that distort the particle motion turn after turn. In addition, resonant conditions exist that can lead to particle losses and unstable conditions. Such effects are driven by multipole imperfections in magnets, or from the beam collisions. In fact, if an imperfection is present, then depending on the tune values, this can either compound every turn or cancel out. In the first case, this will lead particles to larger actions and they will eventually become lost. In the second case, these imperfections will be cancelled out with no impact on the particle motion. When designing a circular accelerator one has to carefully choose the machine tunes in order to avoid resonant behaviour defined by the relation below:

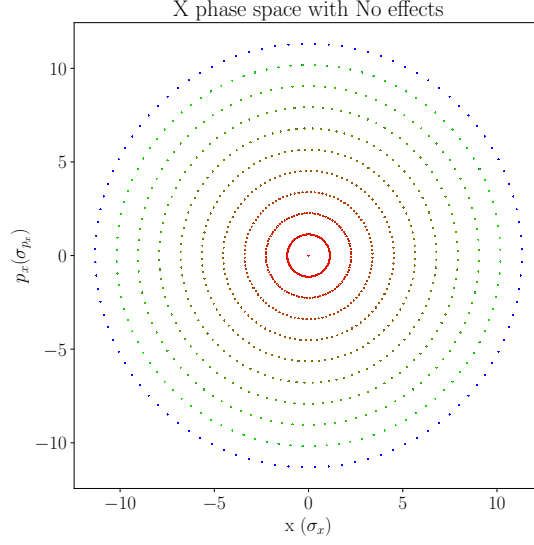


Figure 2.3: Motion of 10 particles in the normalised transverse phase space. Different colours represent particles with different actions J .

$$n \cdot Q_x + m \cdot Q_y = l, \quad (2.5)$$

where n , l and m are integers. The LHC nominal tunes have been set to (60.32, 62.31) as of 2024, although these values, particularly the integer part, have changed during the years of operation.

2.3 Luminosity and absolute calibration

In particle physics, the key concern is the energy available for the production of rare physics events, which can be quantified using the concept of luminosity. The instantaneous luminosity L_{inst} (units $\text{cm}^{-2}\text{s}^{-1}$ or number of events per second) defines the number of observable events at a given time. It is the proportionality factor between the rate of production of particles R and the production cross section for a given event σ_{ev} [2]:

$$R = L_{inst} \cdot \sigma_{ev}. \quad (2.6)$$

Therefore in high-energy accelerators like the LHC, the aim is to maximise luminosity in order to best measure rare events with small cross sections. This luminosity must also be measured with high accuracy as uncertainties in the luminosity have a direct impact on the uncertainties of production rates of rare particles, such as the Higgs boson.

To extend the concept of luminosity to data collection in real-time, the total integrated luminosity needs to be evaluated over total collision time T , as to define the total number of events observed:

$$L_{int} = \int_0^T L_{inst} dt. \quad (2.7)$$

To calculate the integrated luminosity, the time-dependent beam particle density distributions of both beams are integrated over in all three directions. Then, this overlap

integral is multiplied with the intensities of the two colliding beams, N_1 and N_2 , the revolution frequency, f_{rev} , and the number of bunches in one beam, N_b :

$$L = 2N_1N_2f_{rev}N_b \int \int \int \int_{-\infty}^{\infty} \rho_{1x}(x)\rho_{1y}(y)\rho_{1s}(s-s_0)\rho_{2x}(x)\rho_{2y}(y)\rho_{2s}(s+s_0) dx dy dx ds_0. \quad (2.8)$$

However, this expression is based on the assumption that the distribution $\rho(x, y, z, t)$ is independent in the three directions and that $\rho(x)\rho(y)\rho(z)$ are separable and may be multiplied together to produce $\rho(x, y, z)$. In this project, both beam-beam effects and linear coupling are considered, putting this hypothesis into question.

Sometimes, the distributions facilitate a simple analytical solution, for example, for Gaussian distributed particle profiles. In this case, the instantaneous luminosity is given by the formula derived in [2]:

$$L_{inst} = \frac{N_1N_2f_{rev}}{4\pi\sigma_x\sigma_y}. \quad (2.9)$$

However, in the case of separated beams there is a reduction factor, W , that is applied:

$$L = L_{inst} \cdot W, \quad W = e^{-\frac{\Delta_i^2}{4\sigma_i^2}}. \quad (2.10)$$

This reduction factor is relevant to the van der Meer (vdM) scans discussed shortly [2]. Further reduction factors in the calculation of luminosity apply where there is a crossing angle and an hourglass effect [2]. However, this is not relevant for vdM scans at the LHC as they only involve collisions with no crossing angle, and where the hourglass effect is negligible, in order to maximise luminosity [5]. These expressions for luminosity are for the case of single bunch crossings. During particle physics experiments at the LHC, the value of luminosity is multiplied by the number of bunches which make up the beam and is also enhanced through multiple bunch crossings.

2.3.1 Absolute luminosity scale calibration with van der Meer scans

The van der Meer method is the chosen method at the LHC for absolute luminosity calibration. It was pioneered by S. Van der Meer for the Intersecting Storage Rings (ISR) [3], the world's first hadron collider. At the LHC, absolute luminosity calibration is performed annually to minimize bias from long-term changes in the detector performance. The opposing beams are moved step-wise across each other in the two transverse planes and relative interaction rates are recorded as a function of transverse beam separation. Eq. 2.9 shows the result expected for Gaussian beams. Therefore by fitting the interaction rates as a function of beam separation, one can determine the effective beam size and maximum achievable collision rate.

Under the assumption of uncorrelated x and y planes, the transverse convolved bunch widths Σ_x, Σ_y can be extracted from the measured beam separation dependence of the collision rate R [15]:

$$\Sigma_x = \frac{1}{\sqrt{2\pi}} \frac{\int R(\Delta_x, 0) d\Delta_x}{R(0, 0)}, \quad \Sigma_y = \frac{1}{\sqrt{2\pi}} \frac{\int R(0, \Delta_y) d\Delta_y}{R(0, 0)}. \quad (2.11)$$

There is no assumption on the transverse particle distribution, instead "specific luminosity" is used where the measured collision rate is normalised to the colliding bunch-pair

intensities. The total rate R_{vis} measured by a detector is defined from Eq. 2.6, and is also dependent on features specific to the luminometer in the detector:

$$R^{vis} = L_{inst} \cdot \sigma_{vis}. \quad (2.12)$$

The combined information from the vertical and horizontal scans, bunch intensities and rate at peak $R^{vis}(0,0)$ allows for the calculation of instantaneous luminosity. The luminometer-specific visible cross section is obtained:

$$\sigma_{vis} = \frac{2\pi\Sigma_x\Sigma_y R^{vis}(0,0)}{N_1 N_2}. \quad (2.13)$$

In rearranging Eq. 2.12, the calibration constant is used to measure the luminosity directly from the detector rate, under any beam conditions:

$$L_{inst} = \frac{R}{\sigma_{vis}}, \quad (2.14)$$

where $R = \mu^{vis} f_{rev}$ is the observable rate of events.

At the LHC, vdM scans are typically carried out over a wide range of separations of $\pm 6\sigma$ between the two beams, typically with a total of 25 steps. Scans are carried in unique conditions of low-pileup and with low-intensity colliding bunches. These bunches are widely separated in time to reduce interference between adjacent crossings. The time for recording rates is adjustable from between a few seconds to a few minutes and the integrated luminosity is used to maximise measuring time [16].

Table. 2.1 shows the parameters used in 2023 for LHC vdM scans. All studies in this report use these parameters unless stated otherwise. Diagonal scans, scanning in both

	vdM reference parameter set
$E_B[\text{TeV}]$	6.5
N_p/bunch	7.825×10^{10}
$\beta^*[\text{m}]$	19.17
$\epsilon_n[\mu\text{mrad}]$	2.946
ζ	3.24×10^{-3}
Q_x, Q_y	62.31, 60.32

Table 2.1: Beam and machine parameters used for simulating vdM scans [5]

transversal planes simultaneously, are also explored in this project, as well as scans with an offset in the other transversal plane. These scans are important to explore the problem of non-factorization [16].

2.4 Beam-beam effects

Beam-beam interactions are a key limiting factor in colliders. Since the very first collider, these effects have represented the strongest non-linear force perturbing the particle motion. In addition, such electromagnetic force gets stronger while pushing for higher luminosity due to its proportionality to beam brightness. Beam-beam distortions can lead to detrimental

effects, such as particle losses, emittance blow-ups and beam instabilities [4]. When pushing for higher luminosity, all colliders face a limit represented by the maximum beam-beam force the beams can tolerate. Examples can be seen in both hadron and lepton colliders [17] [18].

2.4.1 The beam-beam force

Beams are inherently a collection of a large number of charges. Therefore they represent an electromagnetic potential for other charges and exert forces on the opposing beam in a collider. This is especially relevant in high luminosity colliders, which have small beam sizes and large intensities. To calculate the beam-beam force, the known electrostatic fields must be transferred from the rest frame of the beam to the lab frame, whilst taking into account special relativity as beams in high energy accelerators travel at relativistic speeds. With bi-Gaussian beam distributions, where beam r.m.s is σ_u , the densities may be written as:

$$\rho_u(u) = \frac{1}{\sigma_u \sqrt{2\pi}} \exp\left(-\frac{u^2}{2\sigma_u^2}\right), \quad \text{where } u = x, y. \quad (2.15)$$

Hence the two-dimensional potential can be given as:

$$U(x, y, \sigma_x, \sigma_y) = \frac{ne}{4\pi\epsilon_0} \int_0^\infty \frac{\exp\left(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q}\right)}{\sqrt{(2\sigma_x^2+q)(2\sigma_y^2+q)}} dq, \quad (2.16)$$

where n is the line density of particles in the circulating beam, e is the elementary charge and ϵ_0 is the permittivity of free space. By taking the gradient of the potential, the transverse fields are obtained:

$$\vec{E} = -\nabla U(x, y, \sigma_x, \sigma_y). \quad (2.17)$$

With a further simplification of round beams ($\sigma_x = \sigma_y = \sigma$), using the Lorentz force in cylindrical coordinates $\vec{F} = q(\vec{E}_r + \beta c \vec{B}_\Phi) \times \vec{r}$ and recognising that $r^2 = x^2 + y^2$, the electric field is given by:

$$E_r = -\frac{ne}{4\pi\epsilon_0} \frac{\partial}{\partial r} \int_0^\infty \frac{\exp\left(-\frac{r^2}{2\sigma^2+q}\right)}{2\sigma^2+q} dq, \quad (2.18)$$

and the magnetic field by:

$$B_\Phi = -\frac{ne\beta c\mu_0}{4\pi} \cdot \frac{\partial}{\partial r} \int_0^\infty \frac{\exp\left(-\frac{r^2}{2\sigma^2+q}\right)}{2\sigma^2+q} dq. \quad (2.19)$$

Finally, this gives the beam-beam force seen in Fig. 2.4 as:

$$F_r(r) = -\frac{ne^2(1+\beta^2)}{2\pi\epsilon_0} \cdot \frac{1}{r} \left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right) \right], \quad (2.20)$$

with β in this case being the relativistic beta. A more detailed derivation can be found in [4]. In Cartesian coordinates, this can be separated into:

$$F_x(r) = -\frac{ne^2(1+\beta^2)}{2\pi\epsilon_0} \cdot \frac{x}{r^2} \cdot [1 - \exp(-\frac{r^2}{2\sigma^2})] \quad (2.21)$$

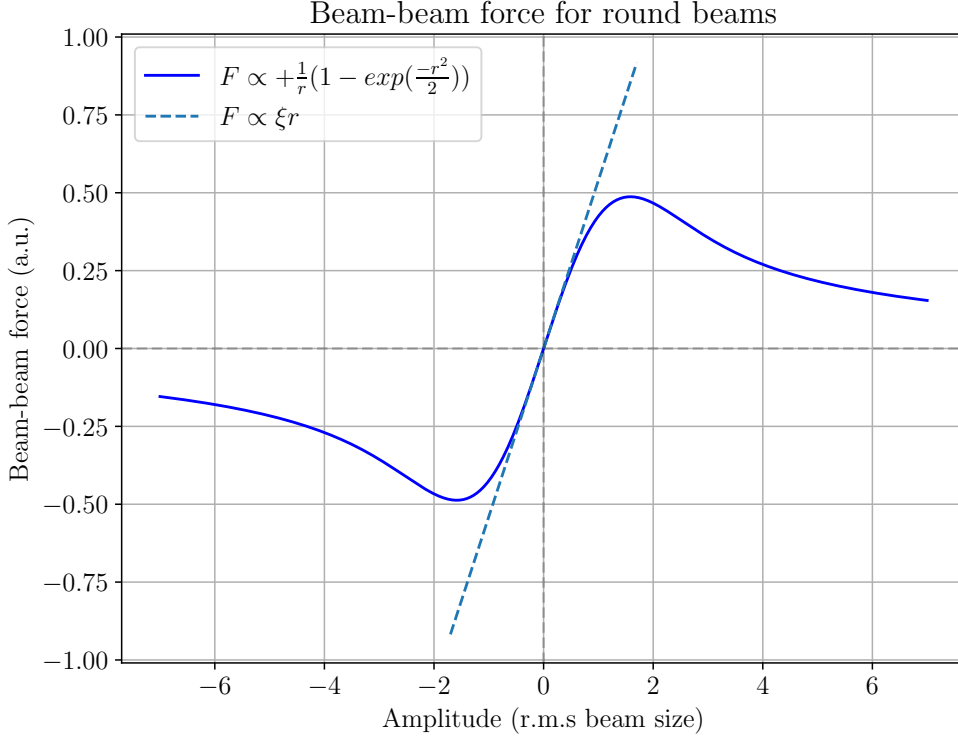


Figure 2.4: The beam-beam force in arbitrary units for equally charged beams as a function of distance to the centroid r in units of the r.m.s. beam size, Eq. 2.21.

$$F_y(r) = -\frac{ne^2(1 + \beta^2)}{2\pi\epsilon_0} \cdot \frac{y}{r^2} \cdot [1 - \exp(-\frac{r^2}{2\sigma^2})]. \quad (2.22)$$

To derive the beam-beam parameter, the exponential function in Eq. 2.20 is Taylor expanded at small amplitudes [15]:

$$F_r = \frac{Ne^2}{2\pi\epsilon_0 r} (1 - [1 + 0 \times r + \frac{-r}{w\sigma^2} \times r^2]) \approx \frac{Ne^2}{4\pi\epsilon_0 \sigma^2} r. \quad (2.23)$$

As the limit of r approaches 0 the gradient of the force is linearly dependent on r :

$$\lim_{r \rightarrow 0} \Delta r' = \frac{Nr_0}{\gamma\sigma^2} r. \quad (2.24)$$

Using a thin lens approximation, this gradient force is compared to a defocusing quadrupole magnet with a focal length corresponding to this proportionality factor, thereby defining the beam-beam parameter ξ :

$$\Delta Q \approx \xi = \frac{Nr_0\beta^*}{4\pi\gamma\sigma^2}. \quad (2.25)$$

Quadrupole magnets are known to modify the focal system of the accelerator, and therefore the oscillation tune. For this reason, the beam-beam force in its linear approximation will shift the collider tunes. However, at larger amplitudes, the force strongly deviates from this linear behaviour and the change in tune is highly dependent on the specific amplitude. Fig. 2.4 shows that the beam-beam force is very non-linear. Depending on the amplitude of the particle within the beam, different particles sample different regions of

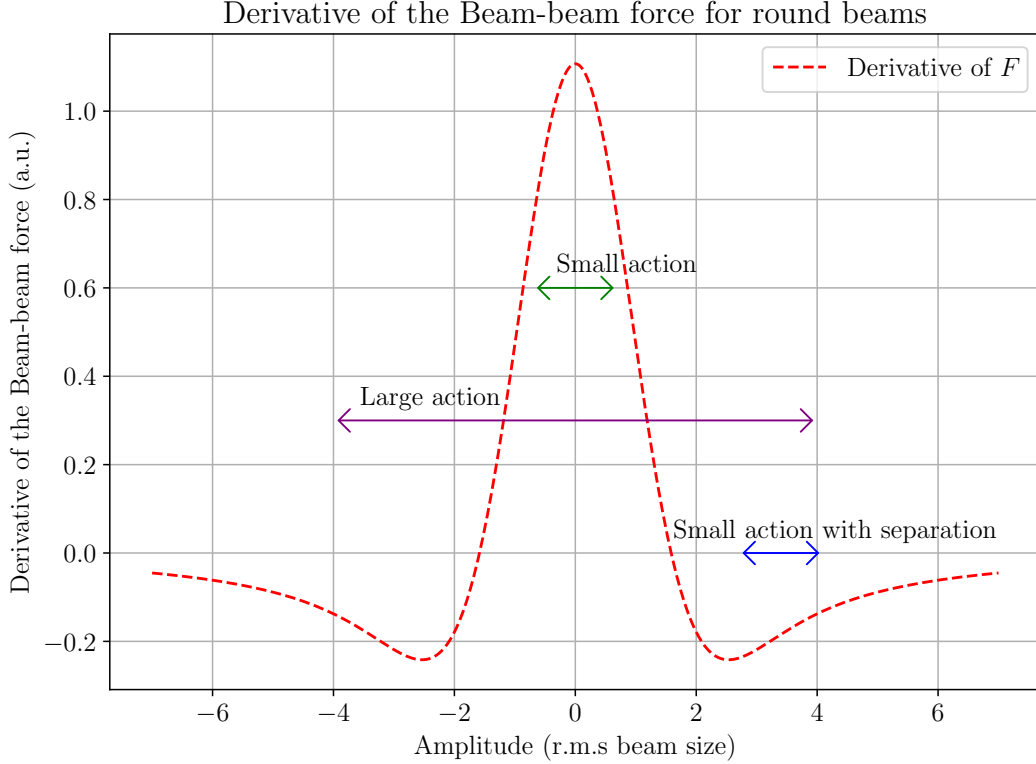


Figure 2.5: Derivative of the beam-beam force for equally charged particles (a.u.) as a function of distance to the centroid r . Small amplitude action particle oscillation path is shown in green versus a large action path in purple, for a head-on collision. The case of a separated collision with a small amplitude particle oscillation is shown in blue

the beam-beam force. For Gaussian distributions of particles and head-on collisions, the tune shift as a function of amplitude r can be calculated analytically [19]:

$$\Delta Q(r) = \frac{Nr_0\beta}{4\pi\gamma\sigma^2} \cdot \frac{1}{(\frac{r}{2})^2} \cdot (\exp - (\frac{r}{2})^2 I_0(\frac{r}{2})^2 - 1). \quad (2.26)$$

Fig. 2.5 shows the derivative of the beam-beam force, which is related to the instantaneous tune shift of the particle as it crosses the opposing beam. The tune shift is calculated for a particle with given amplitude oscillations by averaging the derivative of the force over the range of the oscillation amplitudes [4]. This will produce the detuning effect as shown in Fig. 2.6. This change in tune can be further visualised in tune footprints. This method involves mapping the amplitudes (x, y) of particles into the tune space (Q_x, Q_y) using Fourier transforms. Each pairing in this representation corresponds to different particle amplitudes, typically ranging from 0 to 6σ . Due to the beam-beam effect, all particles experience a shift away from the nominal working point $(0.31, 0.32)$, with the most significant shifts occurring in particles at the center of the beam [4]. By applying a Fourier transform to the average position of the particles, insights into all the underlying beam dynamics are obtained. As the bunches traverse each other, they not only experience the intrinsic harmonic motion of the accelerator but also the disruptive influence of beam-beam interactions, known as beam-beam kicks [20]. These interactions result in the coupling of the bunches within the beam. At an interaction point, the bunch oscillations may align perfectly in phase (0-mode), completely out of phase (π -mode), or exhibit a blend

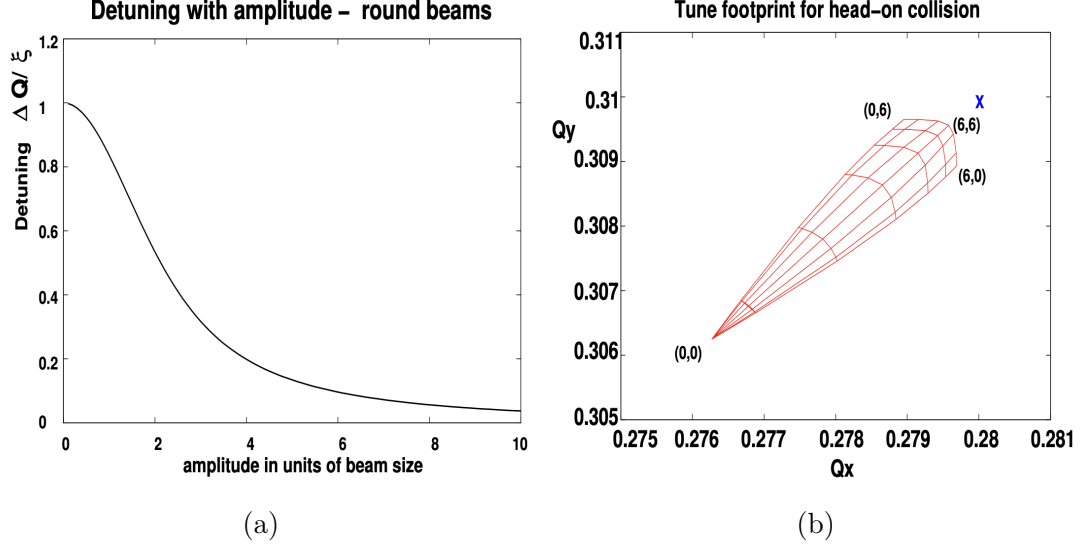


Figure 2.6: (a) Tune shift as a function of the amplitude of a test particle. (b) A 2-dimensional map (tune footprint) of the transverse tunes of particles with different amplitudes [4].

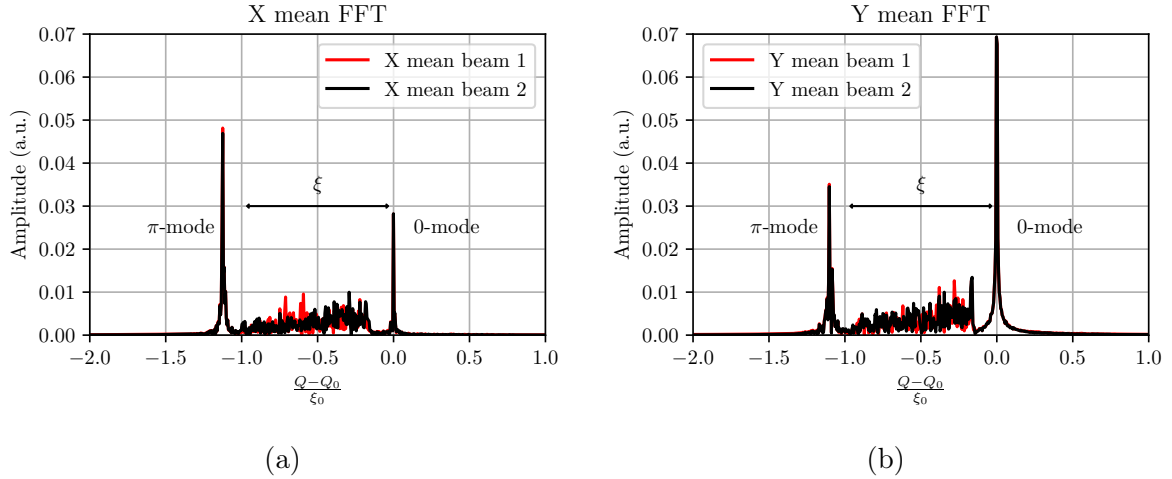


Figure 2.7: Frequency spectra illustrating the 0, π -mode and the beam-beam parameter for a head-on collision at 1 IP

of these two fundamental modes. The separation between these modes is defined by the beam-beam parameter, ξ . As illustrated in Fig. 2.7, the distinct behaviours of the 0-mode and π -mode can be observed, along with the measurable beam-beam parameter ξ . In this figure, the frequency spectra have been scaled by subtracting the nominal tunes and then divided by the beam-beam parameter.

2.5 Beam-beam effects on luminosity

During vdM scans, the two beams electromagnetically interact. Such interaction will result in three main effects on the beam properties with consequences for the luminosity [15]: a change in the closed orbit, a change in the transverse beam dimensions, and the modification of the particle distribution profiles deviating from a Gaussian distribution.

An example of the effect of beam-beam interactions on the luminosity calculations is shown in Fig. 2.8b. The deviations from beam-beam effects are very small (of the order of a few percent), but important for the precision goals that the LHC experiments have when calibrating the detectors. Also shown is the Σ_x from Eq. 2.13, which is calculated by numerically integrating the luminosity curve and scaling with the head-on collision luminosity.

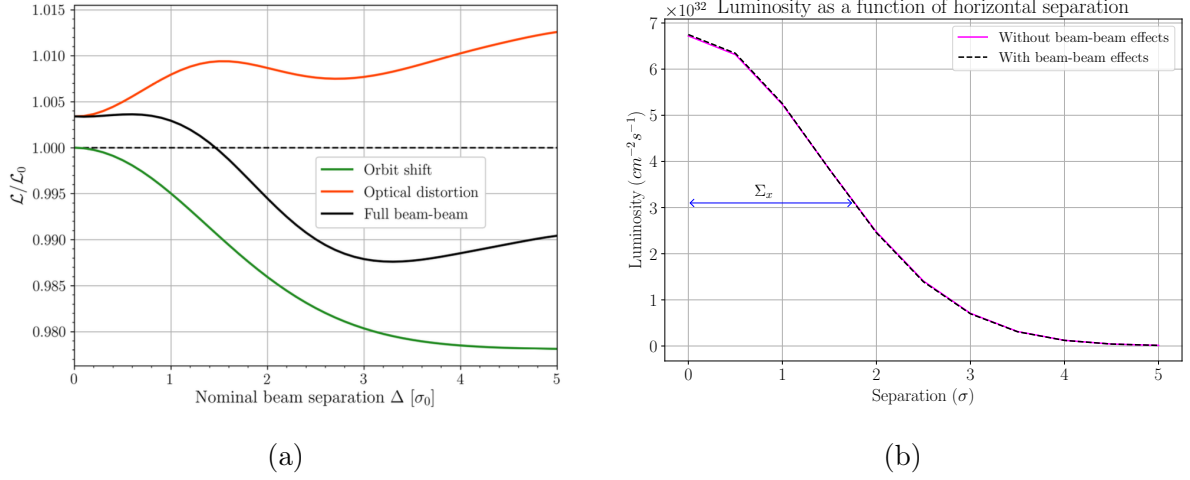


Figure 2.8: (a) Luminosity bias associated with the beam-beam orbit shift only (green), the optical distortions only (red), and their combination (black) computed with COMBI [15] (b) Luminosity as a function of separation in a horizontal vdM scan simulated with Xsuite.

2.5.1 Orbit deflection effect

When two beams do not collide exactly head-on the beam-beam force provides a constant contribution which deflects the orbit of the beam. The bunches of the beam have the same charge, therefore a repulsive kick is delivered. This can be seen when developing the kick $\Delta x'$ in a series:

$$\Delta x' = \frac{const}{d} \cdot \left[1 - \frac{x}{d} + O\left(\frac{x^2}{d^2}\right) + \dots \right] \quad (2.27)$$

During vdM scans, the two beams are pushed apart adding an extra separation (d_{bb}) to the purposely applied separation (d_{sep}). Luminosity therefore depends on the relative distance between the beam centroids $\Delta_i = d_{sep} + d_{bb}$ and the beam transverse dimensions $\sigma_{x,y}$; for the case of Gaussian profiles this corresponds to the standard deviation of the particle distributions.

The orbit deflection effect increases Δ_i in the W factor of Eq. 2.8. It may be measured by subtracting the total distance of separation of the beam centroids from the intended separation. This deflection can also be calculated analytically [15]. The deflection angle inferred by the opposing beam is:

$$\theta_x = \frac{2Nr_0}{\gamma} E_x. \quad (2.28)$$

The deflection-induced offset is therefore

$$\Delta_x^{BB} = \frac{\theta_x \beta_x^*}{2 \tan(\pi Q_x)}. \quad (2.29)$$

The orbit effect reduces the luminosity when such an extra separation is introduced. An example of such an effect is visible in the green curve of Fig.2.8a.

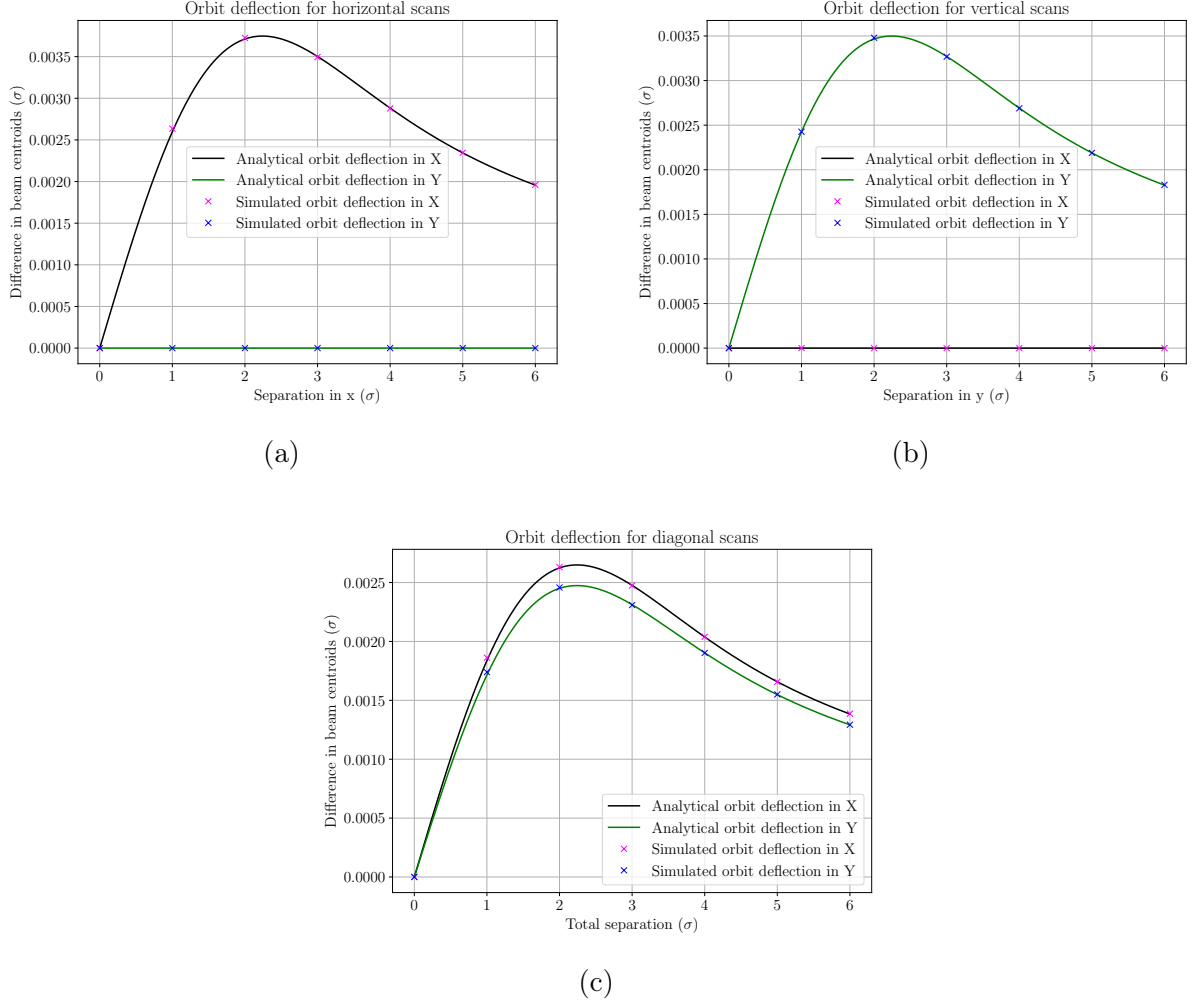


Figure 2.9: Orbit deflection graphs: comparison of results from simulation in Xsuite to analytical expressions for scans in three directions (a) Horizontal (b) Vertical and (c) Diagonal showing agreement of the orbit deflection effect.

The presence of this effect was tested and verified against the analytical expression in Xsuite simulations. Both of these curves for the three types of scans are shown in Fig. 2.9. The horizontal and vertical scans have different curves due to different machine tunes Q_x and Q_y . There is an orbit deflection only in the scan direction, and the other direction has no centroid separation. The diagonal scans have reduced orbit deflection for both coordinate planes compared to the one-directional horizontal and vertical scans because the separation is split across the two transverse directions. The effect on the orbit is small at the order of a few μm , however, there is a significant reduction of 1-2% on the observed luminosity [15].

2.5.2 Transverse beam sizes

The beam-beam force for the LHC case will act as a focusing lens modifying the transverse beam sizes $\sigma_{x,y}$. The change depends on the two beams' relative separation Δ_i of the distribution centroids, the betatron tunes Q , and the beam-beam parameter ξ . For the

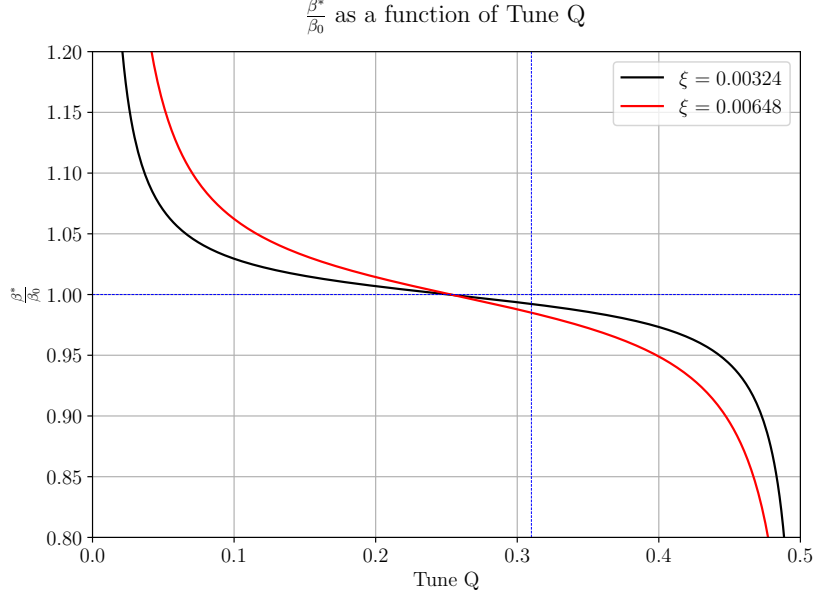


Figure 2.10: Change in optical function at the IP as a function of the machine unperturbed tune Q , for the vdM beam-beam parameter $\xi = 3.24 \times 10^{-3}$ (black) and double the vdM parameter $\xi = 6.48 \times 10^{-3}$ (red).

LHC vdM 1IP case, the integrated effect is an enhancement in the luminosity. This is visible in Fig. 2.8a, where the red curve shows a positive bias in luminosity. However, with multiple interaction points in different configurations, the effect could sum to produce a different effect.

In the first approximation with the linearized force, a change in dynamic- β is observed at the IP after propagation throughout the magnetic lattice:

$$\frac{\beta^*}{\beta_0} = \frac{1}{\sqrt{1 - 4\pi\xi \cot(2\pi Q) - 4\pi^2\xi^2}}. \quad (2.30)$$

Different transverse tunes result in different changes to the dynamic- β , the scale of which is different for different beam-beam parameters. In Fig. 2.10 the beta-beating is shown as a function of the unperturbed tune Q for two different beam-beam parameters $\xi = 3.24 \times 10^{-3}$ and $\xi = 6.48 \times 10^{-3}$. There is a focusing of the observed beam size at the LHC tunes as there is a decrease in β^* . However, this effect is small as tunes are close to $\frac{\pi}{2}$. The beating diverges when approaching the half-integer or integer values due to the cotangent in Eq. 2.30. If the tune was instead set as $Q < \frac{\pi}{2}$, the net effect would be an increase, resulting in a defocusing effect [15].

This only shows the effect on a single test particle. The effect on the entire beam is calculated in [15] and resulted in an effect 60% of that on a single particle. This effect enters the luminosity with a factor 1/2. When the beams have a transverse separation there is no analytical solution to compute this effect and numerical calculations are the only tools we can use to evaluate its impact.

2.5.3 Non-Gaussian particle profiles

The luminosity formula of Eq. 2.9 is computed in the assumption of Gaussian particle distributions. This assumption is not valid when beam-beam effects are introduced as the

Gaussian particle distributions are perturbed. For this reason, it is important to calculate the luminosity using the effective overlap integral of the two beams. A comparison of particle motion in the normalised phase space in the presence of beam-beam effects is shown. The motion of 20 particles of different actions, uniformly generated from -8 to $+8 \sigma$, through the horizontal normalised phase space is shown in Fig. 2.11. The figure on the left has conditions of a vdM scan (Table. 2.1 $\xi = 3.24 \times 10^{-3}$) and the figure on the right has a beam-beam parameter twice this. This has been included as in particle physics experiments the intensity is often greater.

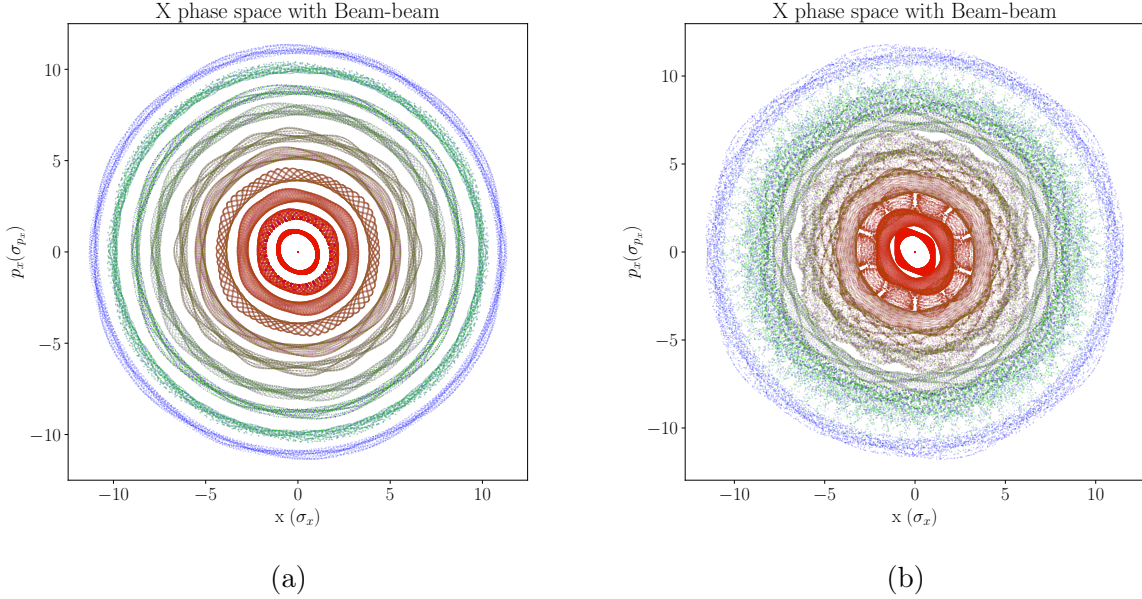


Figure 2.11: Horizontal plane particle phase space for different strengths of the beam-beam force (a) $\xi = 3.24 \times 10^{-3}$ (b) $\xi = 6.24 \times 10^{-3}$.

In comparison with the normalised phase space with no beam-beam effects in Fig. 2.3, the difference between ordinary betatron oscillations and the distorted orbits through the phase space can be observed and becomes increasingly evident. Without beam-beam effects, the particles return to their initial location with a frequency of $2\pi Q_x$. However, with the inclusion of beam-beam effects, the circular trajectory is deformed. Particles with very small amplitudes are less deformed, particles with slightly larger amplitudes are more strongly influenced, and then the impact on particles with the largest amplitudes is again smaller. This follows the shape of the beam-beam force of Eq. 2.20.

The above effects give a combined bias to the measured luminosity shown in Fig. 2.8a black curve [15].

2.6 Linear coupling

Lattice linear coupling is driven by errors and tilted installations of quadrupole magnets. It is controlled by means of skew quadrupoles (i.e. normal quadrupoles rotated 45 degrees). Circular accelerators in linear approximation can be represented by the one-turn transfer matrix:

$$\begin{pmatrix} M & m \\ n & N \end{pmatrix},$$

where M and N are the standard transfer matrices in the (x, x') and (y, y') phase spaces, respectively. These matrices are maps that transport the particle coordinates over one turn in the accelerator. The off-diagonal submatrices m and n represent the coupling terms between these two transverse planes x and y . In an ideal accelerator with no coupling, $m = n = 0$ [21]. Therefore the one-turn transfer matrix can be diagonalized as:

$$T = RUR^{-1},$$

where U represents the one-turn transfer matrix for the two decoupled transverse phase spaces (u, u', v, v') and:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = R \begin{pmatrix} u \\ u' \\ v \\ v' \end{pmatrix}.$$

Here, R is the transformation matrix between the coordinates (x, y, s) and (u, v, s) :

$$R = \begin{pmatrix} \gamma I & C \\ -C^+ & \gamma I \end{pmatrix},$$

I is the 2x2 identity matrix, and C^+ is the symplectic conjugate of C , where

$$C = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix},$$

and $\gamma^2 + |C| = 1$. The matrix C characterizes the strength of linear coupling between the two transverse planes; with $C = 0$, there is no coupling [21].

Previous studies using MAD-X, a CERN standard tool for beam simulations, measured coupling using the closest tune approach. If the horizontal tune is attempted to be brought closer to the vertical tune, this will not be possible due to the coupling term. The tunes will be pushed apart and equality, if coupling is present, cannot be reached. The coupling parameter $|C|$ represents the minimum tune separation achievable in the coupled system. It is defined according to [22] and [23] by:

$$C^- = \frac{1}{2\pi R} \int_0^{2\pi} \sqrt{\beta_x \beta_y} K e^{i[(\phi_x - \phi_y) - (Q_x - Q_y - p)\theta]} d\theta, \quad (2.31)$$

where R is the radius of the accelerator, K is the skew quadrupolar strength as defined in [22], $(Q_x - Q_y - p)$ is the fractional tune difference, $\phi_{x/y}$ are the horizontal and vertical phase advances, $\beta_{x/y}$ are the horizontal and vertical β -functions, and $|C|$ is the coupling stopband.

The coupling stopband is also ΔQ_{min} and is calculated using the closest tune approach as explained in Section. 3.4.1.

Previous research has also examined the impact of linear coupling on transverse beam instabilities in the LHC [24]. Landau damping is a process that suppresses collective oscillations of particles within the beam. It works by introducing a spread in the betatron tune of the particles. This spread in tune helps to damp coherent oscillations because

different particles oscillate at slightly different frequencies, thereby reducing the amplitude of the collective motion. This damping is introduced by special magnets called Landau octupoles which generate a non-linear magnetic field. This creates detuning with amplitude and stabilises the beams if it covers the frequency of the coherent motion [25].

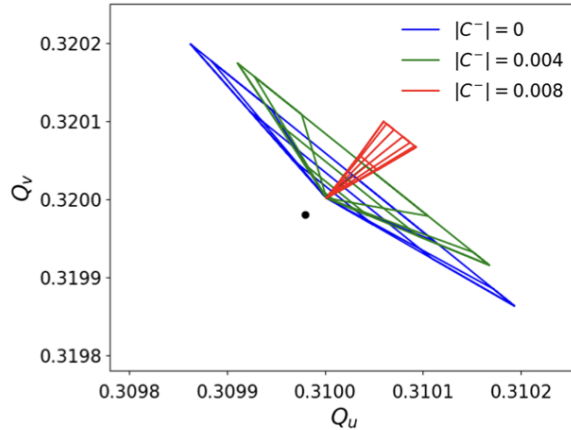


FIG. 1. MAD-X tracking results showing the tune footprint for different values of $|C^-|$ and constant tunes with $J_{\text{oct}} = 500$ A. The particle amplitude is extended to 4σ . A clear impact on the tune spread can be observed as the coupling approaches the coupled tune separation. This severely inhibits the ability of the LOs to stabilize against a typical unstable mode (black point).

Figure 2.12: Tune footprints for Landau octupoles with different values of coupling: without coupling (blue lines) and increasing strengths of coupling (green and red lines) [24]. There is a reduction in tune spread from coupling.

This study concluded that linear coupling can significantly affect the tune spread induced by Landau octupoles, visible in Fig. 2.12, where the footprints for a given set of octupole magnets are shown. This investigation focused solely on the influence of Landau octupoles, not considering other elements that cause tune spread. It was the effect of linear coupling on collective effects proven in this previous work that motivated this study. This project explores beam-beam effects associated with linear coupling, as these interactions are known to result in a significantly distorted tune spread, and have historically led to unexplained discrepancies in luminosity measurements.

Chapter 3

Studies and Results

The topic of beam-beam effects in colliders represents one of the limiting effects impacting luminosity. Increasing luminosity also strengthens the beam-beam distortion. Understanding and quantifying these effects is fundamental for defining the absolute calibration scale of the detectors, directly impacting the uncertainties associated with a physics event, such as the Higgs production cross section in the LHC. Analytical solutions do not apply when quantifying the impact of beam-beam effects on luminosity, as they are based on many simplified assumptions that do not hold in real collider operations, for example, Gaussian distributions. Additionally, there is the complexity of scanning in transversal planes with multiple collisions, as well as diagonal and offset scans. The level of precision required for uncertainties in luminosity poses a challenging problem even for numerical luminosity calculations, with biases to be studied at the level of tenths of a percent. This chapter describes the models available to explain beam-beam effects and the newly developed luminosity integrator, alongside the local linear coupling introduced in the LHC model. The new tools are benchmarked against past results. For the first time, the impact of beam-beam effects and linear coupling on beam frequency spectra and footprints, luminosity bias, and visible cross-section is described and quantified for different scenarios and beam-beam parameters.

3.1 Beam-beam models

In computational simulations of beam-beam effects, several methodologies can be applied, each offering different levels of efficiency and accuracy. There are two main models used in these studies, which are the weak-strong model and the strong-strong model [20].

The weak-strong model is one of the simplest and fastest methods. One beam is treated as "strong" and the other as "weak." The weak beam's influence on the strong beam's dynamics is negligible, allowing interactions to be calculated using a constant Gaussian transverse distribution of the opposing bunch. The strong bunch acts as a "frozen" non-linear electromagnetic lens, altering the paths of the weak beam's particles via the beam-beam force [26]. With this model, an individual particle's motion through phase space can be evaluated. The weak-strong is ideal for scenarios like the Tevatron collider at Fermilab, where a high-intensity proton beam collided with a lower-intensity antiproton beam [17]. The beam-beam effect on the antiproton beam was much stronger than the force on the proton beam because of the larger asymmetry in charges between the two beams [17]. This is evident in the force expression of Eq. 2.21 where the linear dependency of the force to the opposing beam charge N_p/σ is shown. In the LHC, this weak-strong

condition is not fulfilled as the two proton beams are equally charged and dense. Thus, more sophisticated models are needed. The strong-strong model calculates interactions for each particle distribution, considering their continuously evolving states at every turn. Two methods are used: the soft-Gaussian approximation or the full particle-in-cell (PIC) method. This study uses the soft-Gaussian approximation, as past studies have given sufficient accuracy [5]. The soft-Gaussian is also computationally faster and therefore represents a better option, particularly when the main goal is to understand the mechanism and not to quantify the impact.

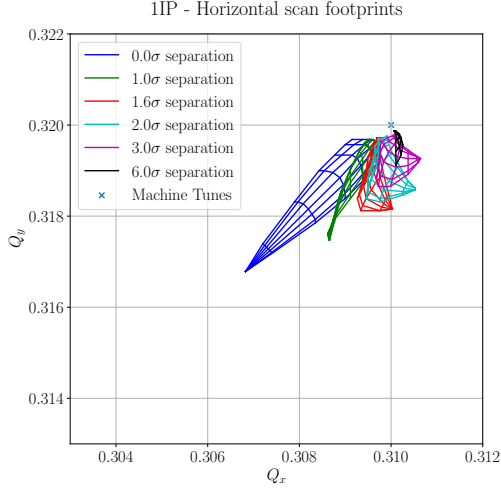
Although the strong-strong model has a higher computational cost, it provides a more realistic depiction. In simpler models of beam-beam effects like the weak-strong model, certain effects, such as relative orbit deviations, are not included. However, these effects are crucial for accurate luminosity calculations. Therefore, whilst the weak-strong model is good for indicating a trend in results and understanding the motion of single particles, the strong-strong model must be used to gain the full picture and realistic evaluation of the physical quantities being investigated.

3.2 Beams footprints and spectra during van der Meer scans

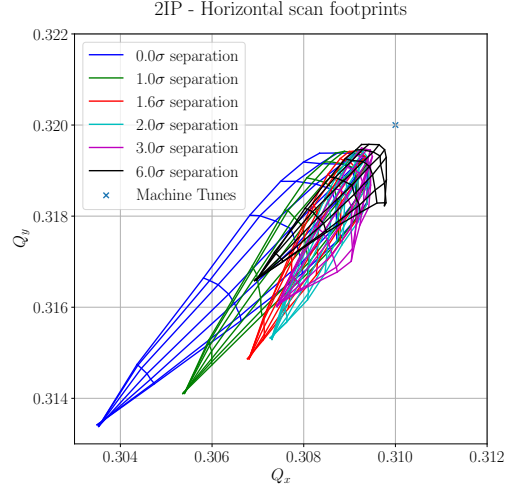
Using the weak-strong model, the analysis of beam frequency behaviours is facilitated through the examination of tune footprints as described in Section. 2.4.1. It is with tune footprints that vdM scans, distortions from beam-beam effects, and the impacts of resonances are first explored in simulation.

Particles at different amplitudes experience different sections of the beam-beam force and therefore their motion is altered in different ways. Furthermore, by applying separations between beams all particles sample a "shifted" section of the force, as illustrated in Fig. 2.4. In Fig. 3.1, the evolution of the beam footprint for one and two IPs is shown. Fig. 3.1a and 3.1b plots correspond to scans in the horizontal plane, 3.1c and 3.1d plots show results for vertical scans, and 3.1e and 3.1f the case when scans are performed in both planes with $d_{sep_x} = d_{sep_y}$, called diagonal scans. It can be observed that the horizontal scans cause the footprint to shift anticlockwise in the tune space and the vertical scans cause the footprint to shift clockwise. Diagonal scans produce a different effect by shrinking the tune space and causing the non centered particles to curl up back towards the working point. In all three directions, there is a reduction in total tune spread. This effect follows the relationship in Eq. 2.26 for the small amplitude particle and shows that for increasing separation the beam-beam force will asymptotically tend to zero.

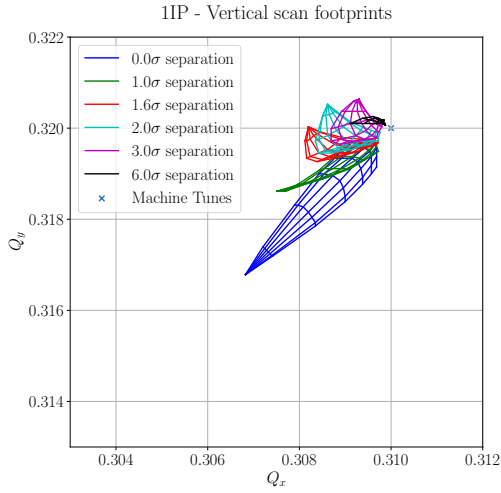
Depending on the scan direction, the beams are moving in different areas of the tune diagram, and therefore different resonances can get closer depending on the scan type. For the footprints in Fig. 3.1a and 3.1b, horizontal scans alter the tunes of the beams such that they closely approach the resonance line $Q_x = Q_y$. However, for vertical scans, the tunes are shifted further away, and diagonal scans are altered parallel to this resonance line. An example is given for illustration in Fig. 3.2 where a few resonance lines (magenta, cyan and yellow dashed lines) close to the footprint have been illustrated, not including the linear resonance line $Q_x = Q_y$. Depending on the direction of the scan, the total tune spread and consequently the final beam tune spread can get closer or further away from resonances, and therefore their behaviour can change drastically. For example, in Fig. 3.2b, a vertical scan will push particles in the direction of the cyan resonance line whilst moving them



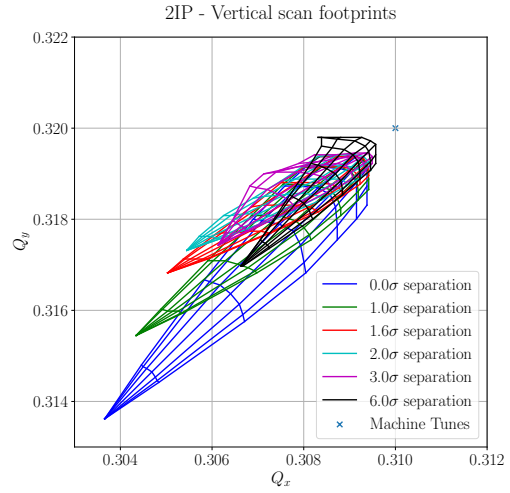
(a)



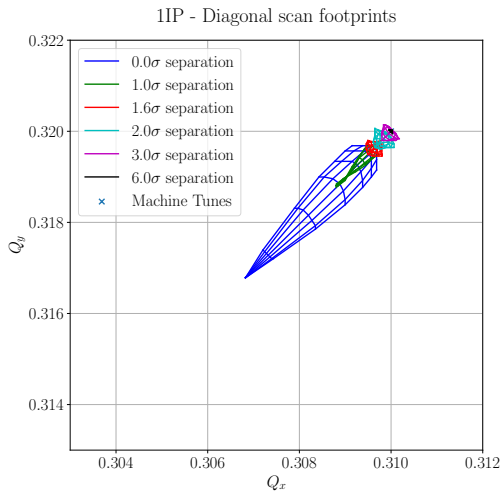
(b)



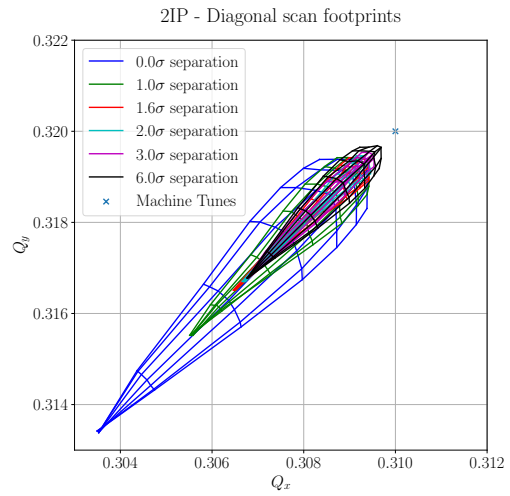
(c)



(d)



(e)



(f)

Figure 3.1: vdM scan footprints in three directions: (a)(b) horizontal, (c)(d) vertical and (e)(f) diagonal. The left column (a)(c)(e) shows the case with only 1 IP which is scanning while the right column (b)(d)(f) shows the 2 IP case where one IP is in a head-on collision and a second IP is scanning.

away from the magenta resonance line. The impact of resonances on the particle behaviour depends on mainly two things: the strength and nature of the resonance and the beam footprint in the frequency domain. Due to this, vdM scan procedures may hide the bias from resonances. In addition, the effect becomes quite complex because the beam-beam interaction during a scan modifies the footprints. This could be an indicator for later studies in non-factorization where asymmetric results for horizontal, vertical, diagonal or offset scans in vdM configurations are observed.

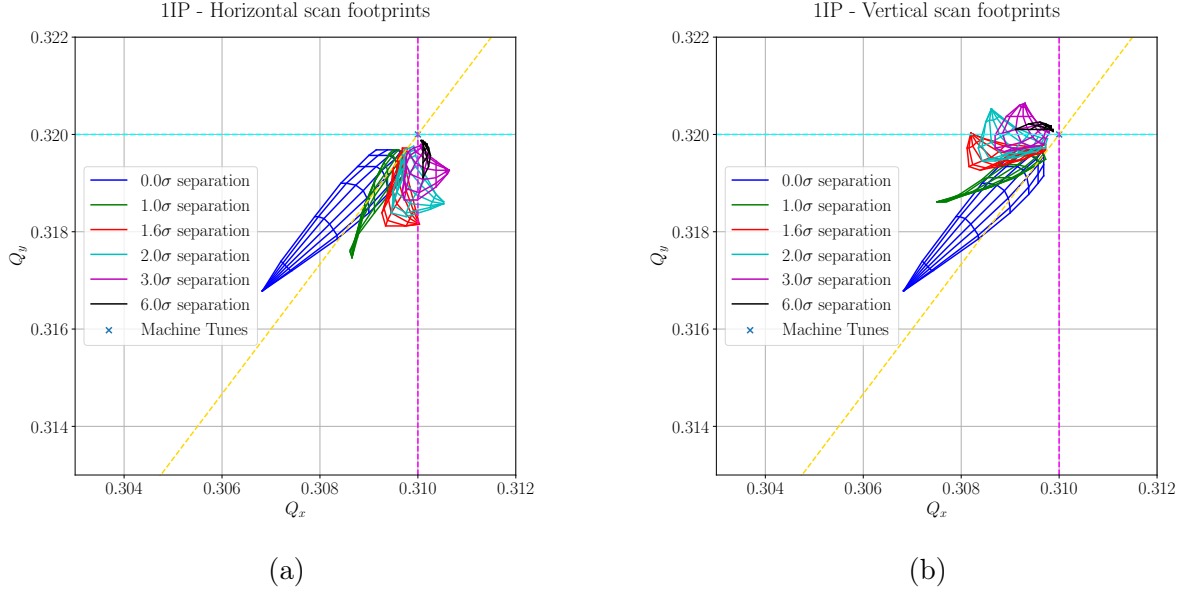


Figure 3.2: Footprint evolution plots during (a) horizontal and (b) vertical 1 IP vdM scans when beam-beam effects are included. Different colours of footprints indicate different separation steps. Straight lines illustrate hypothetical resonance lines close to the beam footprint.

In Fig. 3.1, it is observed in the column on the right that with more interaction points the tune spread increases. This shows the impact of beam-beam effects adding up from each interaction point. Similar to the plots on the left with one interaction point there are clockwise changes for vertical scans, anticlockwise changes for horizontal scans and changes along the center for diagonal scans. In simulations using configurations with more interaction points, further distortions will occur from more collisions, all impacted by beam-beam effects. As the footprints expand in the tune diagram, particles will sample different frequencies and be affected differently by the surrounding resonance lines. For this reason, their behaviour and dynamics can be strongly affected. Furthermore, with multiple IPs the beam-beam effects do not add up linearly but depend on the transport between different collisions. In this study, cases have been restricted to a maximum of 2 IPs in a fully symmetric configuration. This means that between the IPs the phase advance is exactly $\varphi_{x,y} = Q_{x,y}/2$. In reality, phase advances in the LHC are slightly different between the IPs and for the two beams. Nevertheless, the findings in this study still apply conceptually.

In offset scans, other shapes of tune spreads can be observed with combinations of the effects from both transverse and horizontal directions. For this type of scan, a detailed study of the beam-beam bias does not exist since studies in [5] only cover the on-axis cases. However, the imaged footprints highlight the need for a more complete study. This

would include luminosity bias and 3D σ_{vis} calculations for other scan configurations, but for which the techniques were not available during this thesis project.

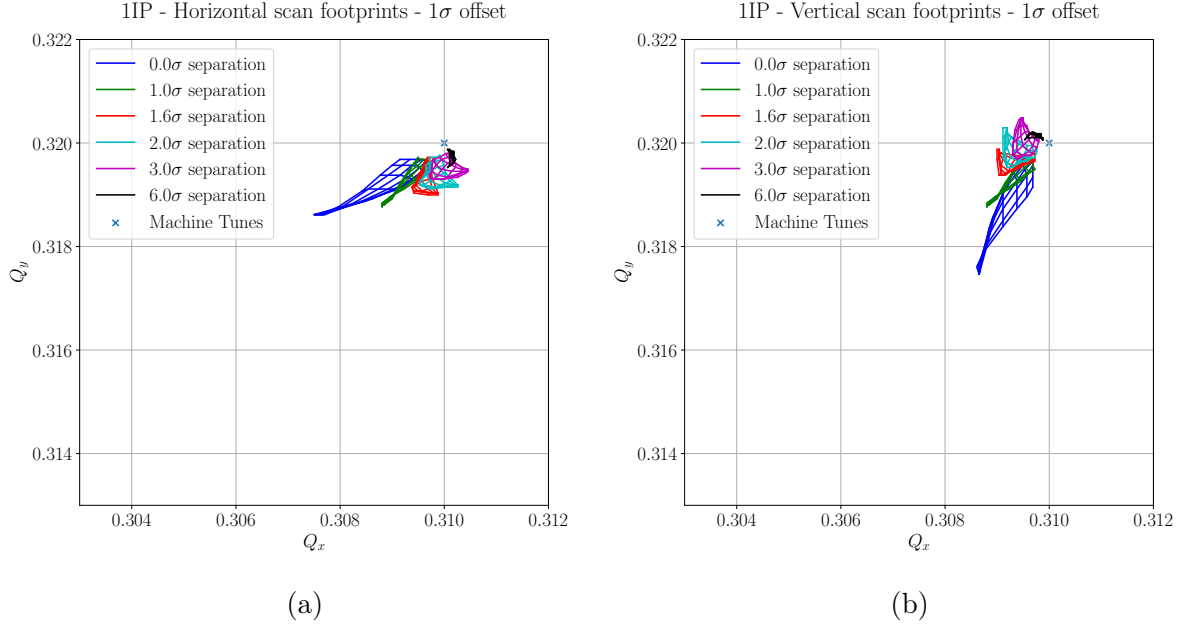


Figure 3.3: Footprint during offset vdM scans at a single IP. (a) 1σ offset in the vertical plane whilst scanning horizontally (b) 1σ offset in the horizontal plane whilst scanning vertically.

In the strong-strong interaction model, frequency analysis can also be performed, and unlike footprints, the frequency analysis of beam centroids can be measured in the real machine. The Fourier transform of the centroid motion over 32768 turns is plotted in Fig. 3.4 for different separations in a horizontal vdM scan. The horizontal axis represents the tune shift with respect to the unperturbed tunes, which are normalized to the beam-beam parameter. When performing vdM scans, the frequency spectrum changes depending on beam separation, as visible from the different shifts in frequency one has per separation. As observed in Fig. 2.5, there are different magnitudes of force exerted depending on the difference in beam centroids. Following the slope of the beam-beam force in Fig. 2.5, which reflects the tune shift, the difference in tunes between the 0 and π -mode reduces, increases in the other direction, then reduces again in the transverse frequency spectrum in Fig. 3.4. The vertical frequency spectrum in Fig. 3.4 shows a decrease in the maximum tune spread, approaching the origin. The spectra in between the two modes contains information on the other particle amplitudes including those imaged in the footprints.

By measuring the distance between the 0-mode and π -mode the relationship between beam separation and the maximum change in tune, which impacts the most central particle, is studied in Fig. 3.5 for the three scan directions. This was measured in the frequency spectra of both transverse directions. Results for the two beams were cross-compared and revealed to be identical.

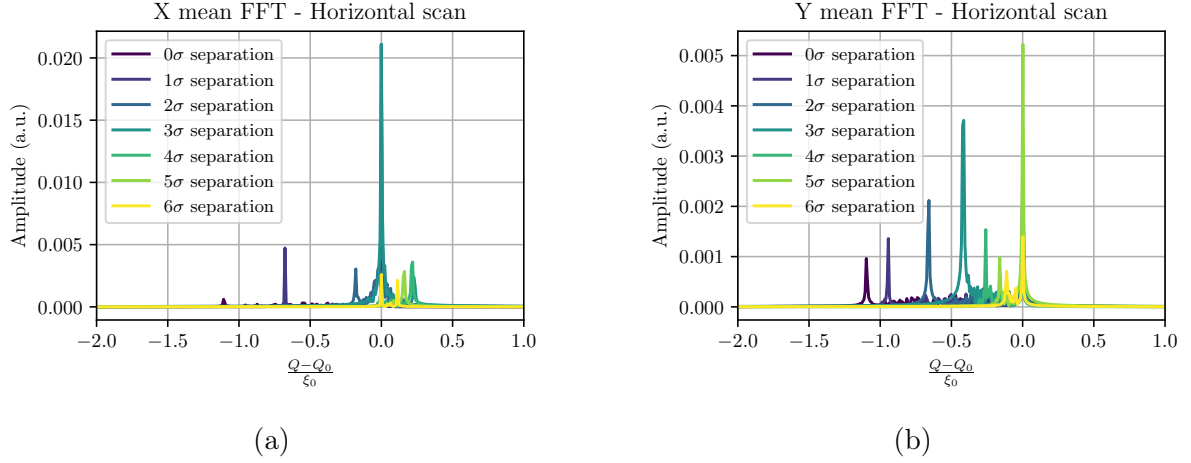


Figure 3.4: (a) X and (b) Y frequency spectra of a beam travelling through a collision point and a linear transfer in the arcs for different horizontal beam separations at the IP as in standard vdM procedure. The frequency is expressed in units of the beam-beam parameter ξ_0 , and 0 represents the unperturbed machine tunes.

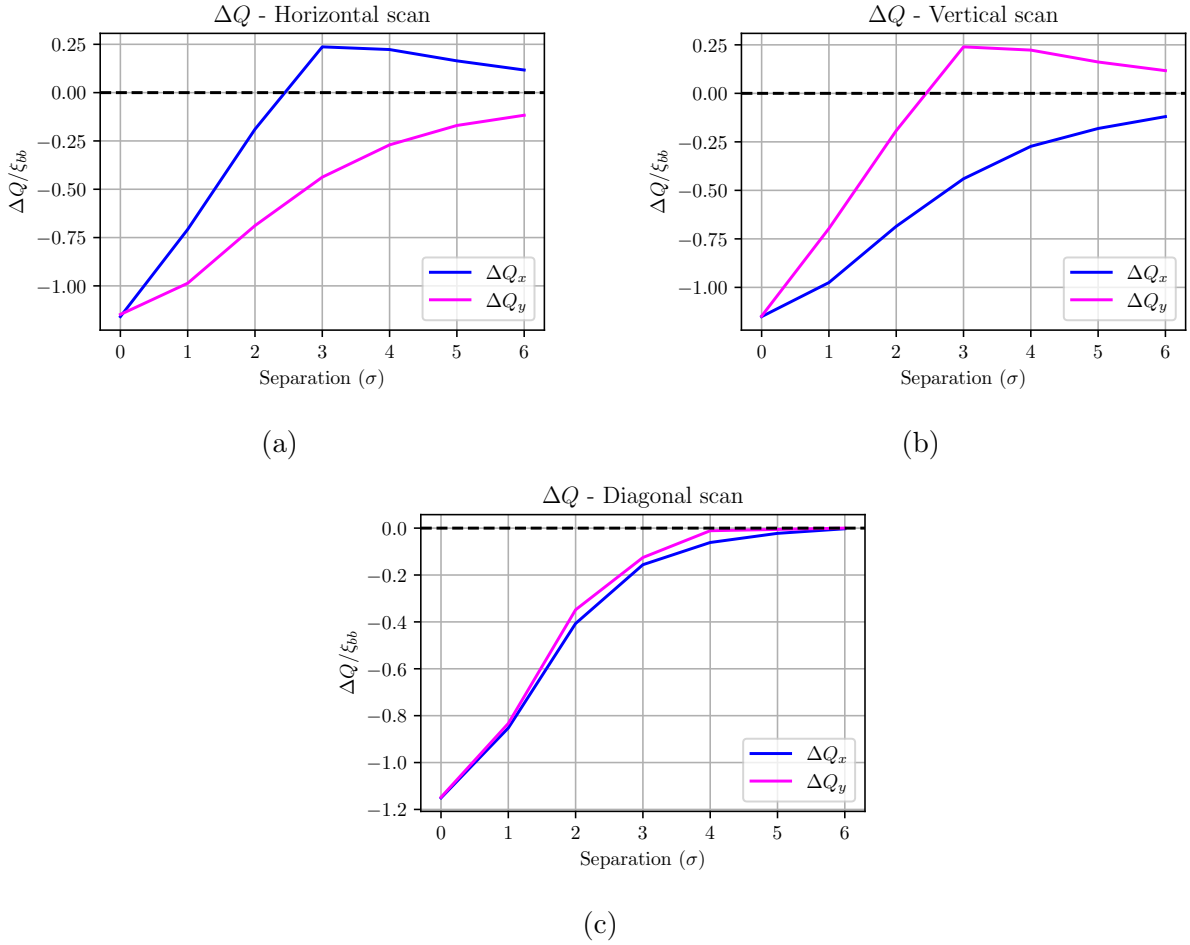


Figure 3.5: Maximum change in transverse tune per separation for (a) horizontal, (b) vertical and (c) diagonal scans.

3.3 Modelling the beam-beam bias to luminosity with Xsuite

Evaluation of the impact of beam-beam interactions on luminosity has already been examined using software for beam dynamics simulation called COMBI [15]. A new Python-based model for beam dynamics called Xsuite [27] is currently in development at CERN. Xsuite is used and extended for the studies in this project.

In order to simplify the analysis, the focus was primarily on configurations with one or two interaction points (1 IP and 2 IP(s)), aligning with the main detectors, CMS and ATLAS. Nevertheless, for a thorough examination, future studies should incorporate the full model of the LHC, which includes all four interaction points.

The vdM calibration observables can be compared using ratios [15]. Beam-beam effects are enabled and then disabled in simulations, facilitating estimates of their relative impact. For scenarios involving only one interaction point (1 IP), normalization can be against either an analytical expression or the calculated luminosity with the beam-beam element "turned off." However, the calculated luminosity is a more ideal choice to eliminate statistical noise for the randomly generated Gaussian beams. For consistency in simulations, the same random seed for random number generation is employed in normalizing luminosity scans. This removes statistical uncertainties from the particle distributions, as the luminosity under investigation is compared to a "standard" normalizing luminosity with exactly the same particle distribution. However, particle distributions are initialized and simulated separately for each separation. This ensures that simulations remain independent and unbiased towards any specific particle distribution.

In the two interaction point (2 IP) setup, one interaction point serves as an observer. This setup allows the measurement and quantification of the beam-beam effects at the active interaction point while also filtering out the numerical noise inherent in simulations, thereby enhancing the accuracy of findings.

3.3.1 Gaussian approximation

Luminosity can be first calculated in the Gaussian approximation using the previously designed code in Xsuite [28]. The luminosity in a vdM using a Gaussian approximation was benchmarked with the analytical expression in Fig. 3.6. In this calculation method, beam-beam effects were switched off by setting the beam charge in the beam-beam element to 0. The Gaussian approximation in the absence of beam-beam effects appropriately represents the analytical expression.

Luminosity calculated under the Gaussian approximation becomes inaccurate when considering separated beams under the influence of beam-beam effects as can be seen in Fig. 3.8. One of the main reasons is that the Gaussian distribution assumption no longer holds due to distortions from beam-beam effects. This becomes more apparent at increased separations as the luminous region, where the beams overlap, becomes increasingly composed of the beam edges, which are the most distorted by beam-beam effects. Nevertheless, the Gaussian approximation is a useful benchmark for numerical integrator results and as a trend indicator. The following studies were first carried out under the Gaussian approximation, which served as a comparison point.

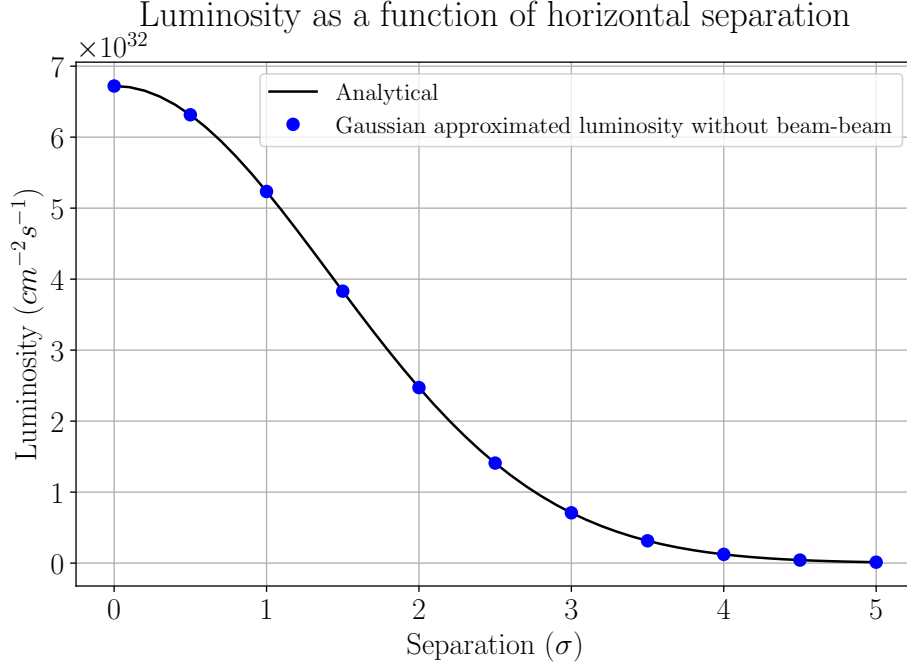


Figure 3.6: Luminosity comparison of the analytical expression and the Gaussian calculation in Xsuite without beam-beam effects.

3.3.2 Non-Gaussian extension

The Gaussian approximation is not sufficient to obtain the real bias to luminosity. The overlap integrals of the evolving distributions need to be evaluated. Hence a numerical integrator was developed in the Xsuite code and is benchmarked against the results from COMBI as published in [5]. Whilst COMBI is an effective and widely used tool for simulating beam-beam interactions, integrating this numerical integrator into Xsuite allows the simultaneous modelling of multiple effects and phenomena in beam dynamics. Further simulations were then carried out to study the impact on luminosity from beam-beam effects and linear coupling.

For the purpose of efficiency, bunches of the beam were divided into macroparticles, which each represent a number of particles, as the real LHC beam is made of more than 100 billion particles. Together, the macroparticles reflect the charge of the bunch i.e. each macroparticle has an intensity equal to the total bunch intensity divided by the total number of macroparticles in a bunch.

In the calculation of luminosity with a numerical integrator, no assumption is made on the shape of the beam distributions. Instead, the transverse distributions of each beam are calculated term by term, populating a 2D histogram with the positions of the macroparticles. The grid can be defined such that it can encapsulate all of the particles in the beam, accommodating separations up to 6σ [15], which is used for vdM scans at the LHC. However, for time efficiency during this project, the focus was on separations up to 5σ where there is the highest impact on σ_{vis} . This was proven in [15] using Fig. 3.7 where the convergence of the σ_{vis} is shown at the 5σ separation.

For the purposes of efficiency whilst still encapsulating the luminous region, the grid dimensions of 15σ by 15σ were divided into 1200 by 1200 smaller grids. This histogram of macroparticles is denoted as $H_{x,y}^B$, and the number of cells in each grid as $N_{cells} = n \times m$,

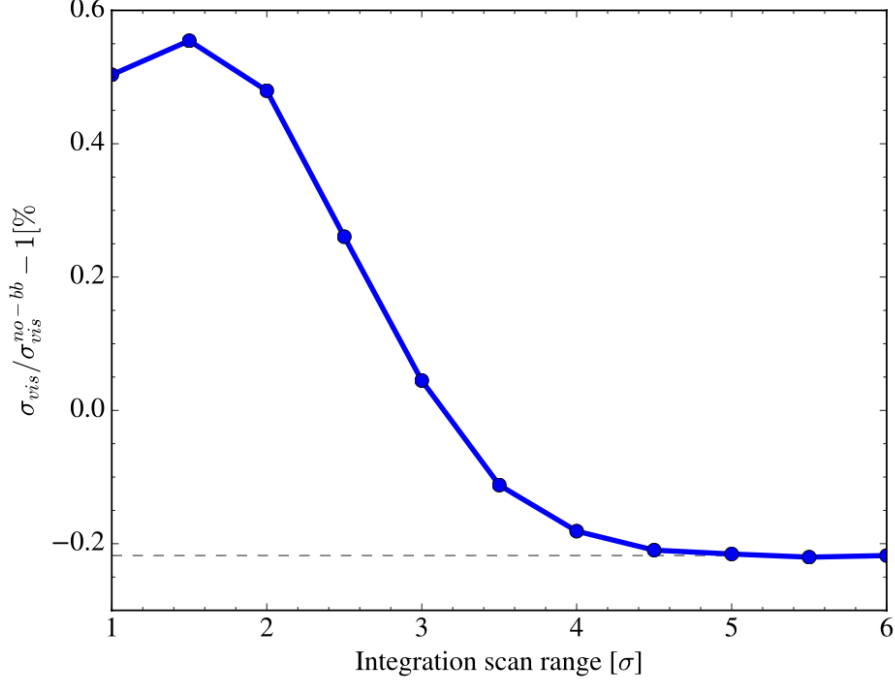


Figure 3.7: Convergence of σ_{vis} for a horizontal vdM scan with 1 IP, simulated in COMBI [15].

where n and m represent the number of bins in the x and y directions, respectively. The grid boundaries are set at a distance $k_i \times \sigma_i^0$ ($i = x, y$) from the grid center, where σ_i^0 is the initial, unperturbed nominal transverse beam size derived from the input emittance and β^* values in the i plane, with the scale factor k_i being 7.5 in both the positive and negative directions along x and y in these studies [15].

Consequently, the area of each cell is given by:

$$\Delta = \Delta x \times \Delta y = \frac{2k_x\sigma_x^0}{m} \times \frac{2k_y\sigma_y^0}{m}$$

The discretized macroparticle density distribution can then be integrated using the following 2D integral formula:

$$I_{2D} = \frac{1}{2} \left[\frac{1}{4} \Delta x \Delta y \left(\lambda_{0,0} + \lambda_{m,0} + \lambda_{0,n} + \lambda_{m,n} + 2 \sum_{i=1}^{m-1} \lambda_{i,0} + 2 \sum_{i=1}^{m-1} \lambda_{i,n} \right. \right. \\ \left. \left. + 2 \sum_{j=1}^{n-1} \lambda_{0,j} + 2 \sum_{j=1}^{n-1} \lambda_{m,j} + 4 \sum_{j=1}^{n-1} \sum_{i=1}^{m-1} \lambda_{i,j} \right) \right].$$

3.3.3 Comparison with results from COMBI

In previous studies [15] the numerical integrator in COMBI was compared first to the analytical expression, as well as results from simulations in MAD-X and from the COMBI Gaussian approximation calculation. The results are shown below in Fig. 3.8. There

is a clear distinction between using the numerical integrator and the Gaussian approximation. The bias is smaller at higher separations than previously supposed, and larger at smaller separations compared to the non-integrator case. With the same vdM beam-

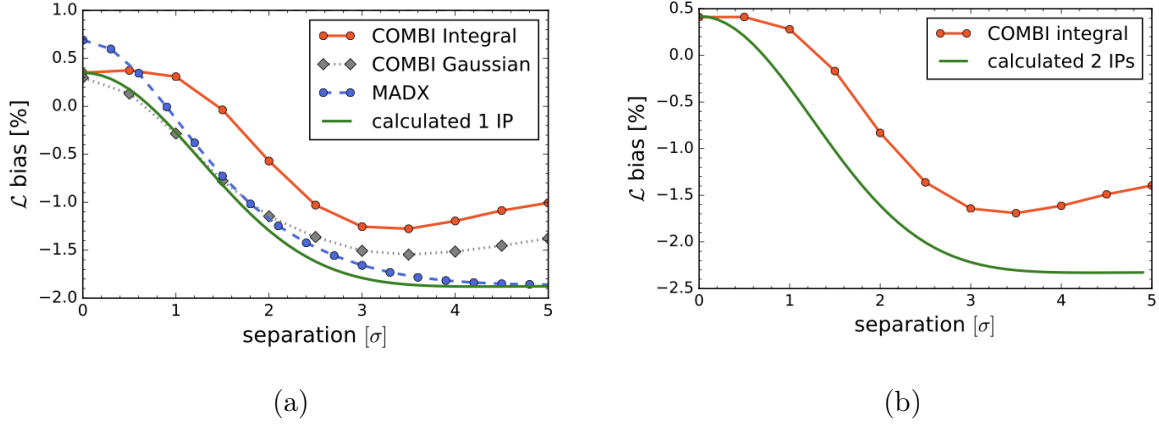


Figure 3.8: Luminosity bias comparison: state of the art results using COMBI for (a) 1 IP (b) 2 IPs.

beam parameter of 3.24×10^{-3} , simulations were run using the Gaussian and numerical integral calculations in Xsuite, then compared with the COMBI integrator in Fig. 3.9. By comparing the luminosity bias due to beam-beam effects, it can be seen that for the 1 IP case, the Xsuite integrator follows the COMBI integrator very closely. As a key indicator of reliability for head-on collisions, all results are at the $\approx 1.4\%$ level. However, for the 2 IP case there are deviations at the tails for the Xsuite integrator. This could be because simulations in COMBI used grids with finer resolution, allowing greater precision in recording tail particles.

However, this does not have a major impact on the σ_{vis} as seen in Fig. 3.10 where with 1 IP there is a 0.00333% effect on the σ_{vis} with beam-beam and 0.00608% effect without. In the 2 IP case, this discrepancy increases to 0.0397% and 0.0120%. The magnitude of the differences remains smaller than the 0.1% uncertainty defined with previous studies [15]. Therefore, the numerical integrator has been successfully implemented in Xsuite and benchmarked against COMBI results. Future studies should investigate whether tail deviations increase further with more interaction points than 2 IPs since the statistics might deprecate. In addition, the integrator will need to be expanded to the longitudinal dimension (3D) for studies of luminosity in the presence of a crossing angle.

3.4 Impact of linear coupling resonance

In the previous sections, the impact of beam-beam effects in vdM scans has been evaluated in the frequency domain with tune footprints, as well as the luminosity bias for different configurations using Xsuite. This represents a solid benchmark for the next step in this study where resonances are introduced. As set out in the introduction, the LHC has several resonances of different orders and strengths. This section will address the influence of linear coupling resonance inspired by observations of the non-factorizability of the $x - y$ distributions. Since this resonance introduces coupling, the hypothesis is that with beam-beam effects this can be a source of non-factorization. Nevertheless,

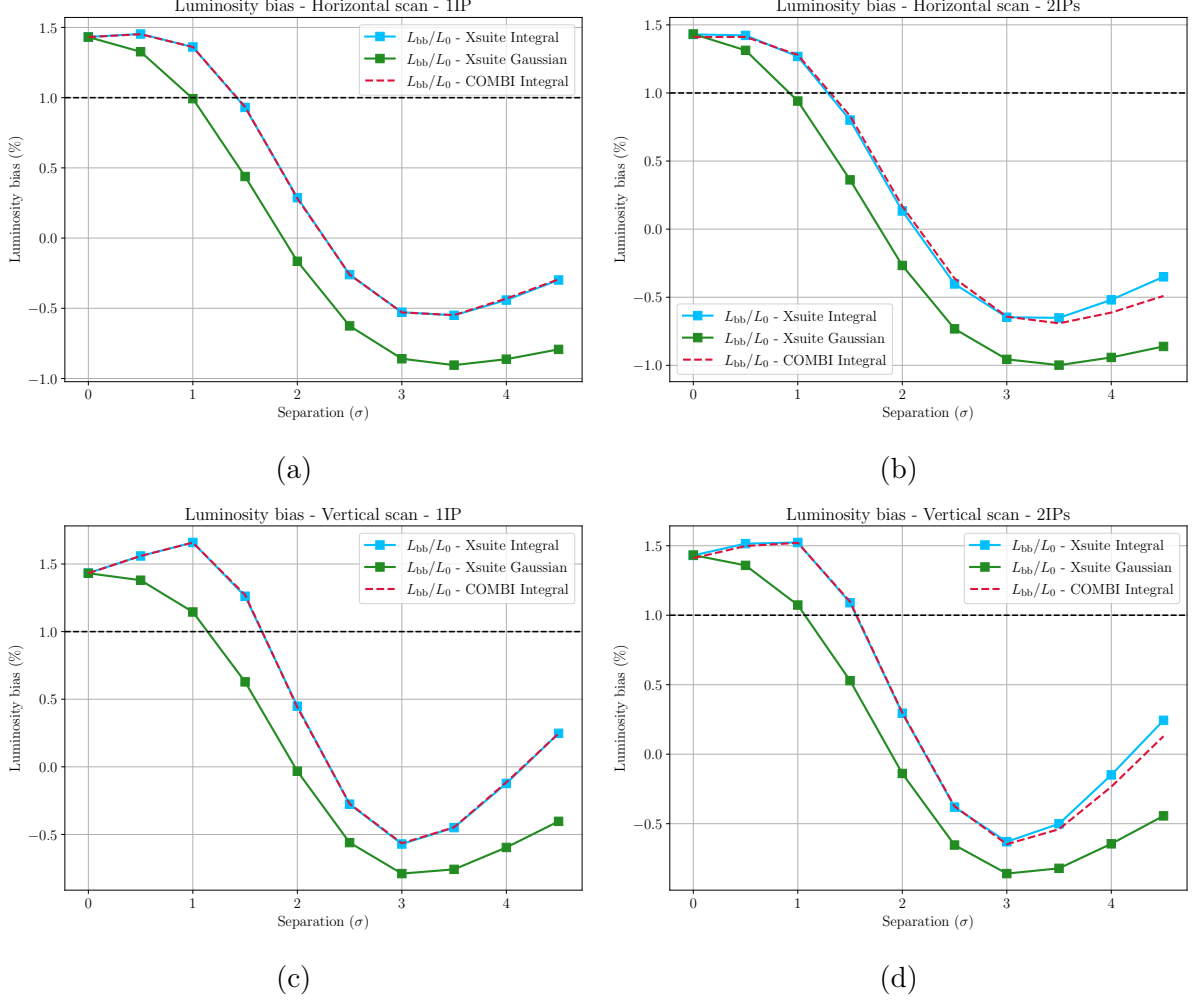


Figure 3.9: Luminosity bias for vdM parameters ($\xi = 3.24 \times 10^{-3}$), (a) 1 IP horizontal scan (b) 1 IP vertical scan (c) 2 IP horizontal scan (d) 2 IP vertical scan, comparing results from Xsuite and COMBI.

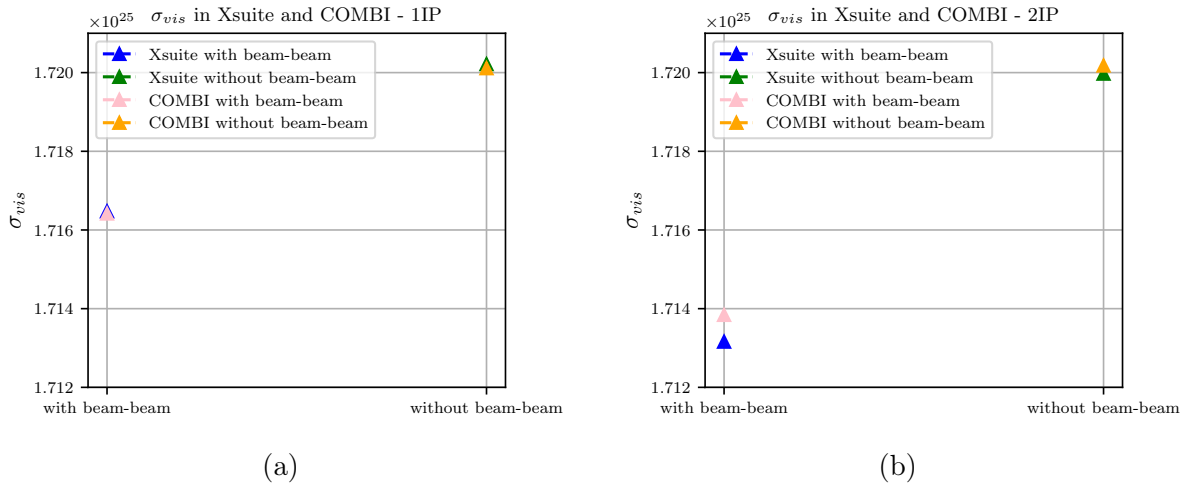


Figure 3.10: Comparison of σ_{vis} with and without beam-beam interactions in the two simulation codes for (a) 1 IP and (b) 2 IPs

higher-order resonances might also be contributing to the whole picture and therefore should be investigated in future studies.

3.4.1 Modelling local linear coupling

Global transverse coupling prohibits the fractional tunes Q_x and Q_y from being matched to the same value; instead, they have a minimal separation of $C^- = \Delta Q_{min}$. This value is called the closest tune approach and can be used to quantify transverse coupling in an accelerator [29].

By adding a skew quadrupole to the simulated machine, linear coupling is introduced between the horizontal and vertical directions [24]. A skew quadrupole does not focus or defocus the beam as with other quadrupoles but rather adds a linking term between the horizontal and transverse coordinates of beams. In the matrix representing these elements, the diagonal terms are zero and the off-diagonal terms will have some small but important contribution to coupling as seen in Section. 2.6. In Fig. 3.11, the difference in transverse tunes of the machine arc Q_x and Q_y was numerically minimized. Then, a single particle was tracked through this arc and a skew quadrupole. The particle's tunes are plotted against the mutual difference of the machine tunes for different values of coupling. The uncoupled beams are able to reach the point of equal tunes but coupled beams are pushed away from this point; the stronger the coupling, the larger the closest tune approach. Fig. 3.12 shows the linear relationship between ' K ' and C^- , the coupling variable referred to as the closest tune approach in literature [21]. ' K ' is the strength of the skew quadrupole component in m^{-2} as defined in the Xsuite element [27].

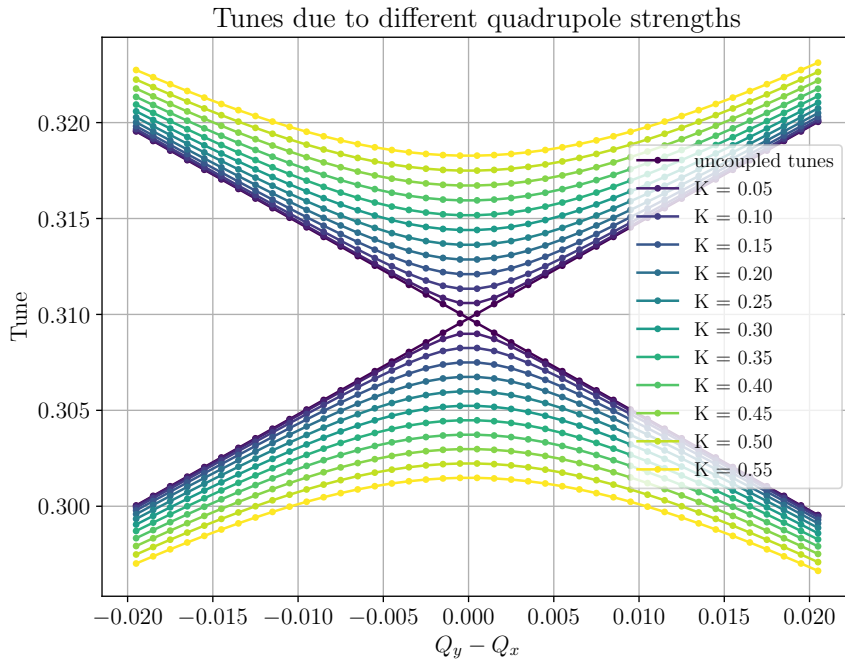


Figure 3.11: Closest tune approach to determine the coupling variable C^- . Change in particle tunes from linear coupling effects as a function of the unperturbed difference in machine tunes ($Q_y - Q_x$). The LHC tunes sit at the +0.010 value on the x-axis.

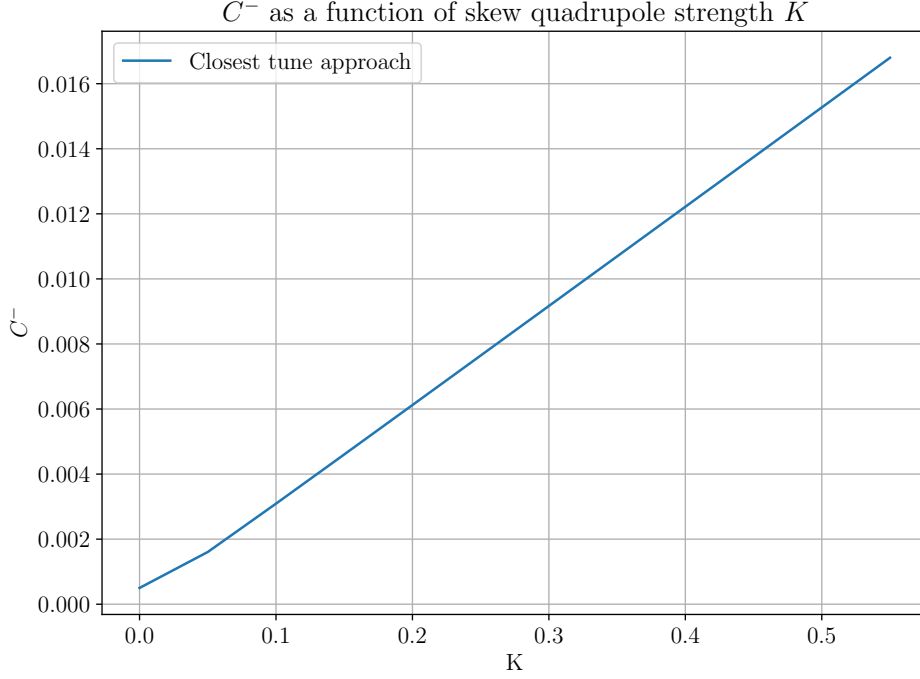


Figure 3.12: Relationship between skew quadrupole strength ‘ K ’ and C^- .

The effects of coupling can be observed in this normal phase space plot with both beam-beam effects and coupling, compared to without coupling. The reference normal phase space without any effects can be seen in Fig. 2.3

In Fig. 3.14, particles’ coordinates (x, y) are plotted turn after turn for different configurations with beam-beam effects and coupling. In an ideal case, with no collective beam-beam effects and no coupling, the (x, y) space would instead show rigid upright rectangles demonstrating only betatron oscillations with no distortions. If coupling is present, a tilt angle is applied on the $x - y$ trajectories as evident in Fig. 3.14a. In the case where beam-beam effects are present and there is no coupling, the $x - y$ space is modified as shown in Fig. 3.14b. The beam-beam force will change the particle coordinates and this distortion will depend on the amplitude of the particle, some trajectories are shrunk showing focusing, such as the core region, while others show a defocusing effect at larger amplitudes. This gives the known distortion of the particle distributions. In Fig. 3.14c, the effect of both can be seen whereby the $x - y$ trajectories are tilted and compounded by the beam-beam effect, causing further distortions and adding a dependency between the distributions in x and y plane. Linear coupling effects, when applied equally to both beams under the approximation of round beams ($\sigma_x = \sigma_y$), will not give any geometrical impact to the luminosity [30]. In this project, only the impact on tunes and tune spread in combination with beam-beam effects is explored as a possible source of deviations.

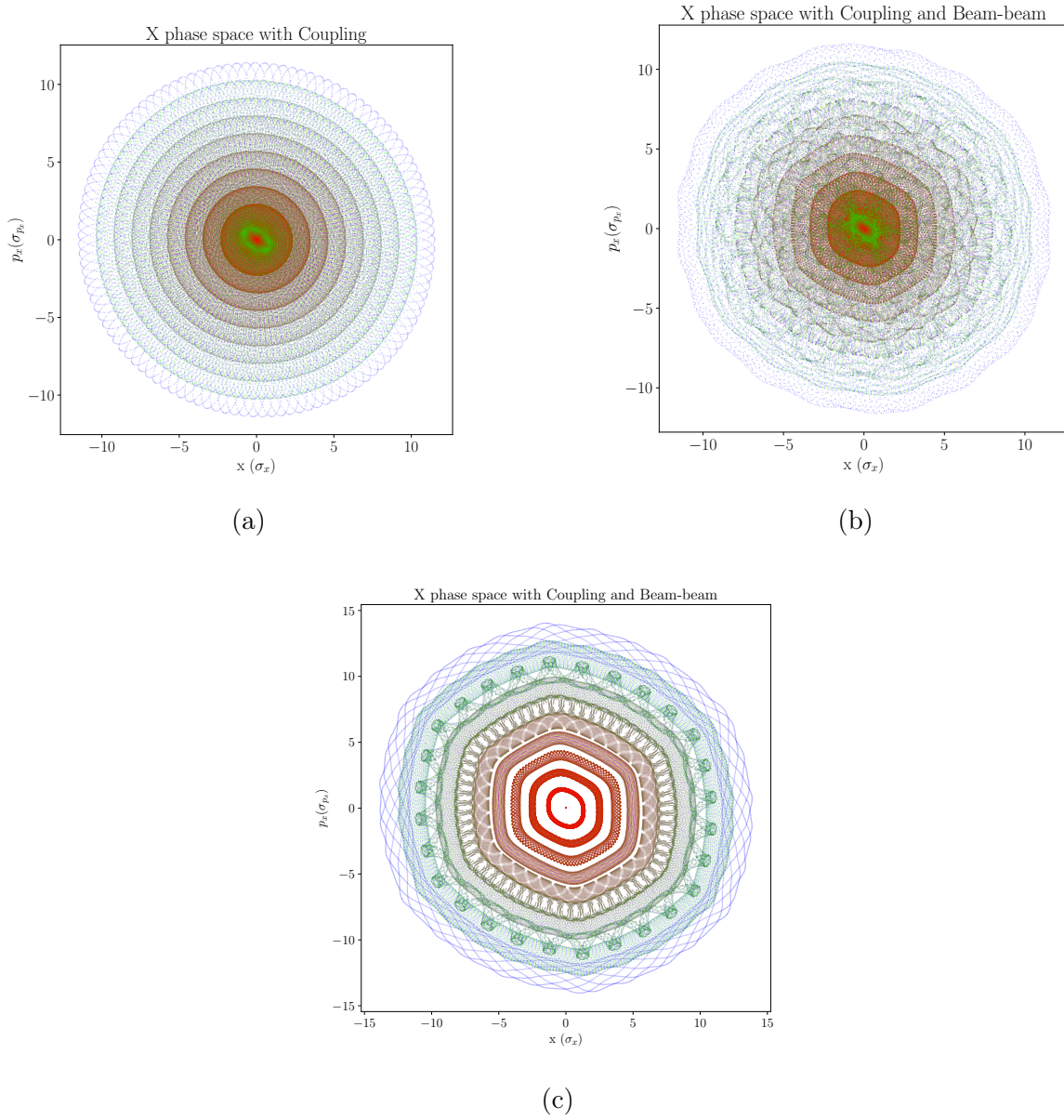


Figure 3.13: Comparative normalised phase space diagrams under different conditions of beam-beam interactions and coupling, modified for coupling strengths of (a), (b) $C^- = 8 \times 10^{-3}$ and (c) 16×10^{-3} . The inclusion of coupling distorts the particle trajectories.

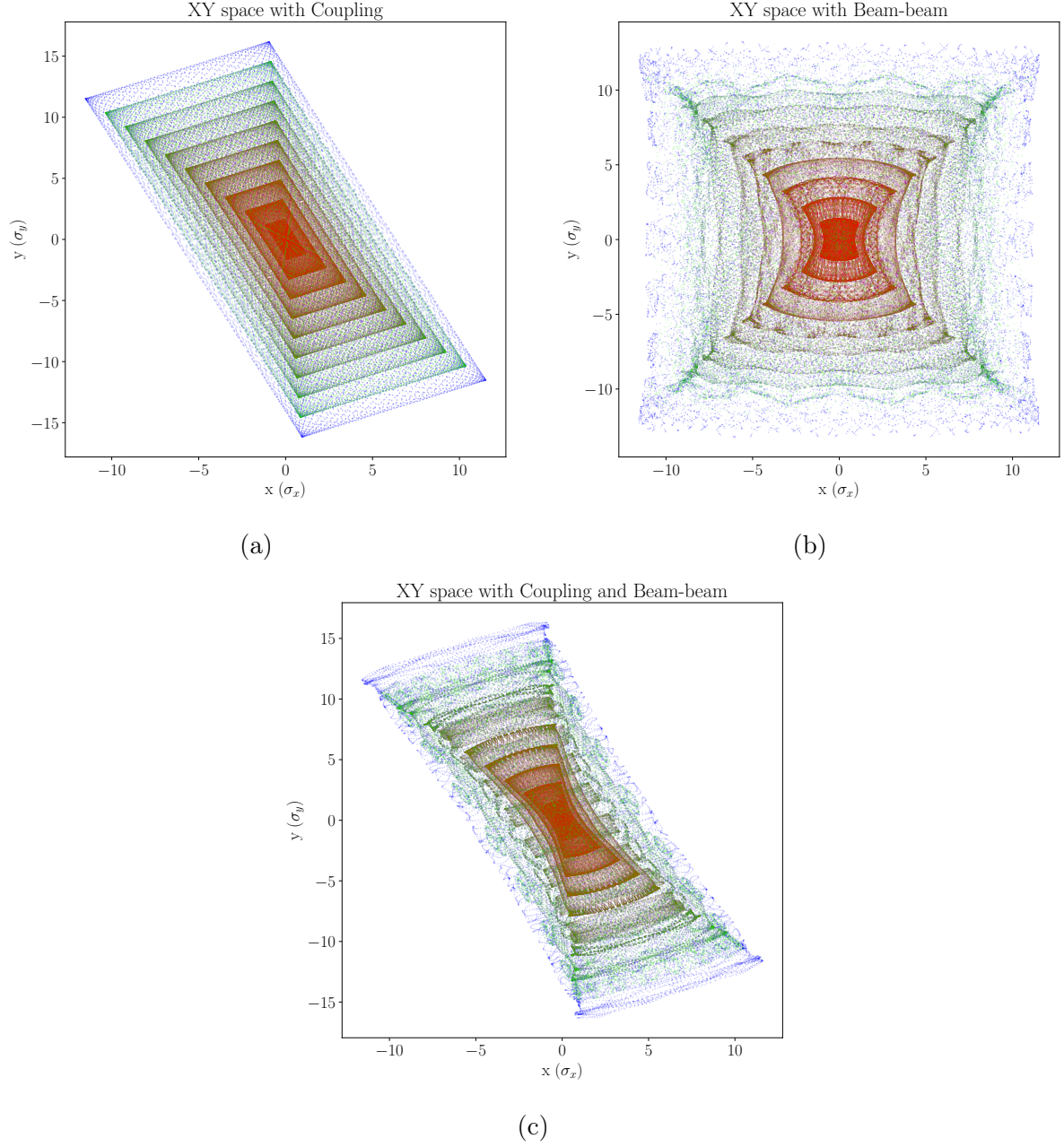


Figure 3.14: Particle trajectories in real $x - y$ space. Coordinates are expressed in units of the transverse beam sizes r.m.s $\sigma_{x,y}$ and σ_{p_x,p_y} . (a) case with coupling, (b) case with beam-beam effects, and (c) both effects. $C^- = 8 \times 10^{-3}$ and double the vdM ξ is used for demonstration

3.4.2 Deformation of footprints and tune shift

As a result of the introduction of linear coupling, tune spreads of the beams are 'pushed' away from the resonance line as seen in Fig. 3.11. This is further evident in Fig. 3.16a, which shows the tune spread of 1000 particles tracked through a lattice with a skew quadrupole and beam-beam effects from 1 IP. The footprint from beam-beam effects is evidently shifted away from the coupling resonance ($Q_x = Q_y$) as a function of the strength of the linear coupling C^- . For the largest coupling $C^- = 0.008$ the footprint in black is shifted by 6.92×10^{-3} in units of the tune.

Further to this, when the machine is in operation, this tune shift from coupling can be corrected or uncorrected depending on the operational observations. Changes in tunes of a few 10^{-3} may not be visible and therefore can remain uncorrected. In addition, if tune shifts are corrected this will reinforce the effect of resonance since the correction will shift the tunes in the direction of $Q_x = Q_y$. A picture of the footprints with corrected tune shifts is visible in Fig. 3.16b. If the tune shifts are not corrected for, then the change in tunes will still show an impact on the luminosity bias. In this study, both cases have been simulated and studied to evaluate the impact of both operational conditions.

The tune shift correction can be measured in simulation by visualising the frequency spectrum of a case with no beam-beam effects, and with different values of coupling applied. The difference in frequency spectra with and without correction can be seen in Fig. 3.15. The result after correcting the shift is visible in Fig. 3.15b, where pushing the unperturbed tunes closer to each other enhances resonance effects. This can be reduced if the coupling is corrected locally, correcting for a local source, or globally, by reducing the impact of many sources of coupling.

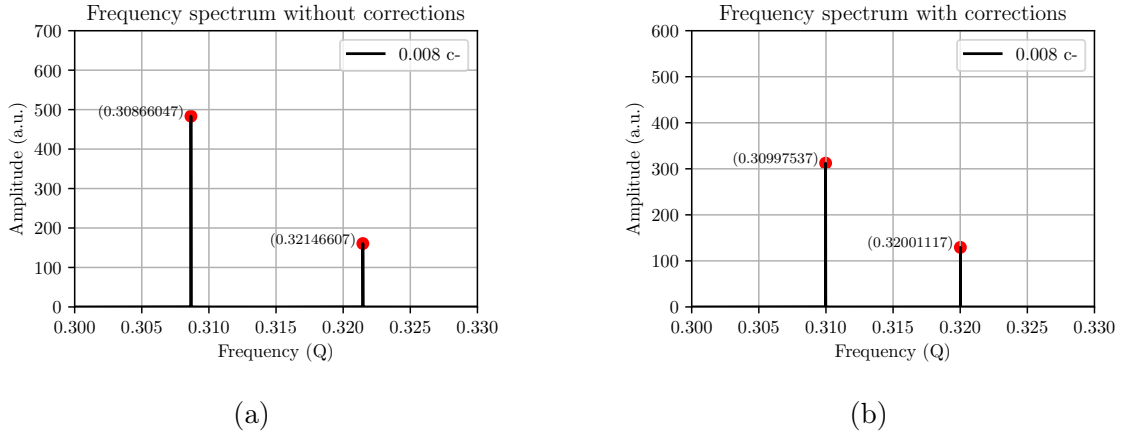


Figure 3.15: Comparison of frequency spectra of coupled beams (a) without and (b) with corrections for $C^- = 8 \times 10^{-3}$.

When correcting the tune of the arcs back to the working point, the coupling of the beams alters the tunes again, so this method does not fully correct the tunes back to the working point as seen in Fig. 3.15. However, this effect is small and becomes negligible in comparison to other sources of uncertainties.

The same considerations made for the head-on collision case of Fig. 3.16 apply to the footprints with separations and offsets during a vdM scan. This is evident in Fig. 3.17 where beam footprints are plotted for different strengths of the coupling resonance after the correction of the tune shift. The greater the strength of coupling, the greater the effect

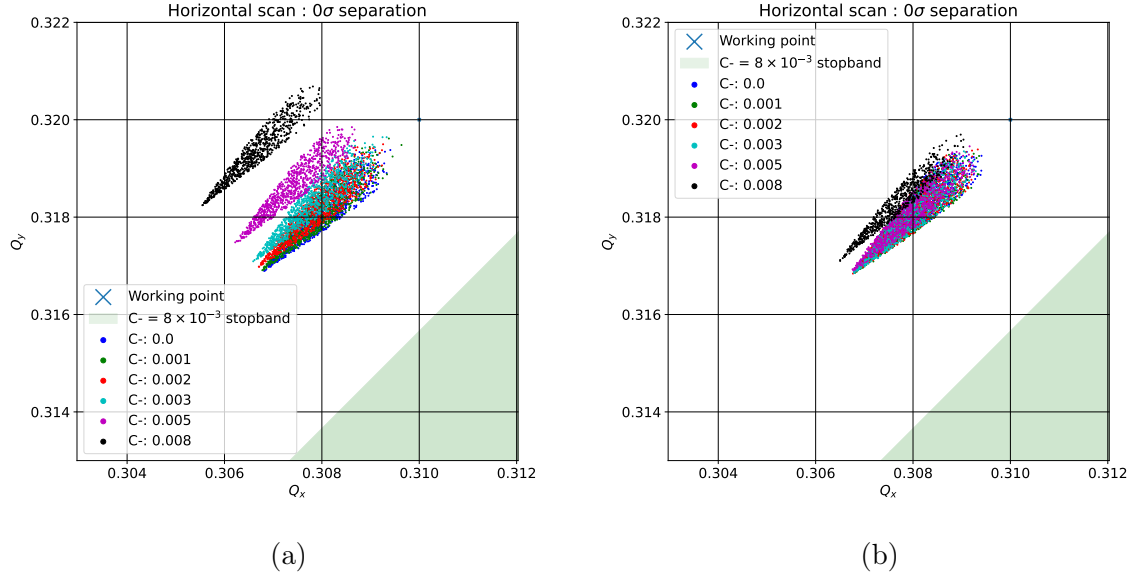


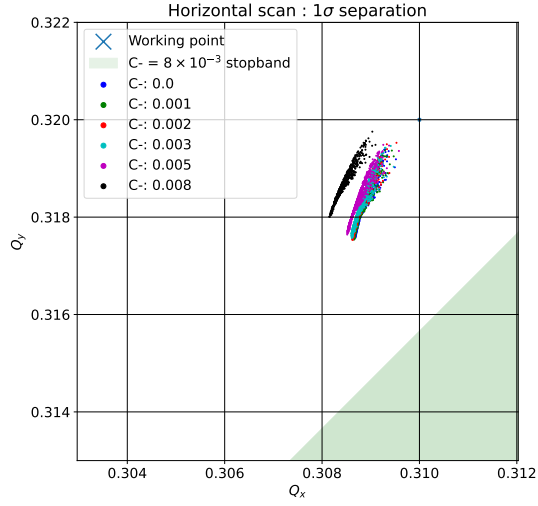
Figure 3.16: Comparison of frequency spectra of coupled beams with and without correction for different levels of coupling in a 1 IP head-on collision. 1000 particles were tracked for 1000 turns under the weak-strong model. (a) Footprints with tune shift as a function of coupling strength C^- (b) Footprints after the correction of the tune shifts.

of distortion of the tune space of the beam. These plots also show the impact on spread, where it is important to notice that depending on the specific separation and offset, the footprint shifts towards or away from the resonance bandwidth (green). Intuitively, the results show that the coupling effect is stronger or weaker depending on the separation and offset.

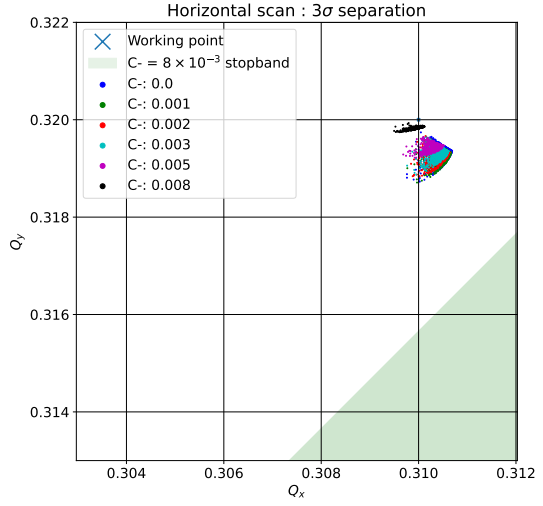
Certain footprints decrease in size, such as in Fig. 3.17a with a separation of 3σ , however with others such as Fig. 3.17c with a 1σ separation and a constant 1σ offset there is an increase in tune spread. Additionally, footprints from horizontal scans are more affected than vertical scans, as scans move the footprint towards the coupling bandwidth, as can be seen when comparing Fig. 3.17b and Fig. 3.17d. These pictures give a clear idea of what is happening to the beam particles in the frequency domain but it is difficult to exactly quantify the effect.

It is important to mention that as the strength of coupling increases, a greater number of particles are mapped onto the resonance line during the Fourier transform. The Fourier transform finds it difficult to distinguish between the x and y coordinates of coupled particles. Fig. 3.18 shows the number of particles per footprint, demonstrating that at higher levels of coupling, the full tune distortion will not be seen. Nevertheless, this effect can act as further proof that the skew quadrupole is introducing coupling between transverse and vertical motion.

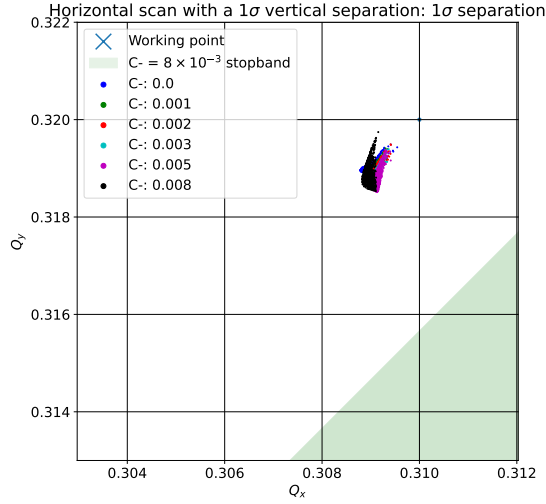
As mentioned previously, vdM scans occur with multiple IPs in collisions as witnesses. Although these IPs are not scanning, the extra collisions further modify the tune spread of the beam because extra beam-beam interactions occur. In Fig. 3.19, the footprints for 2 IPs are shown in different configurations and for different C^- . The shapes of the spreads differ and 1σ separation in the horizontal or vertical plane results in the largest shift towards or away from the coupling resonance, respectively. This is an important observation as it highlights that scans in the two directions give different results.



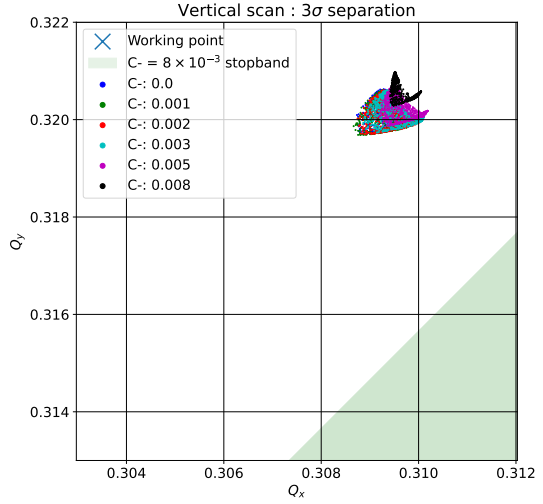
(a)



(b)



(c)



(d)

Figure 3.17: Comparison of tune spreads for 1 IP configuration with different levels of coupling, combinations of separations and offsets typical of vdM scans. a) 1 IP horizontal scan, 1σ separation b) 1 IP horizontal scan, 3σ separation c) 1 IP horizontal scan, 1σ separation, 1σ offset d) 1 IP vertical scan, 3σ separation.

In the case of 2 IPs, the tune footprint with strong coupling visibly narrows in relation to the cases without coupling. In addition to the tune shift, there is a clear effect on the spread, which is amplitude and separation-dependent, and representing a change in particle distribution. Therefore some change towards the luminosity is expected.

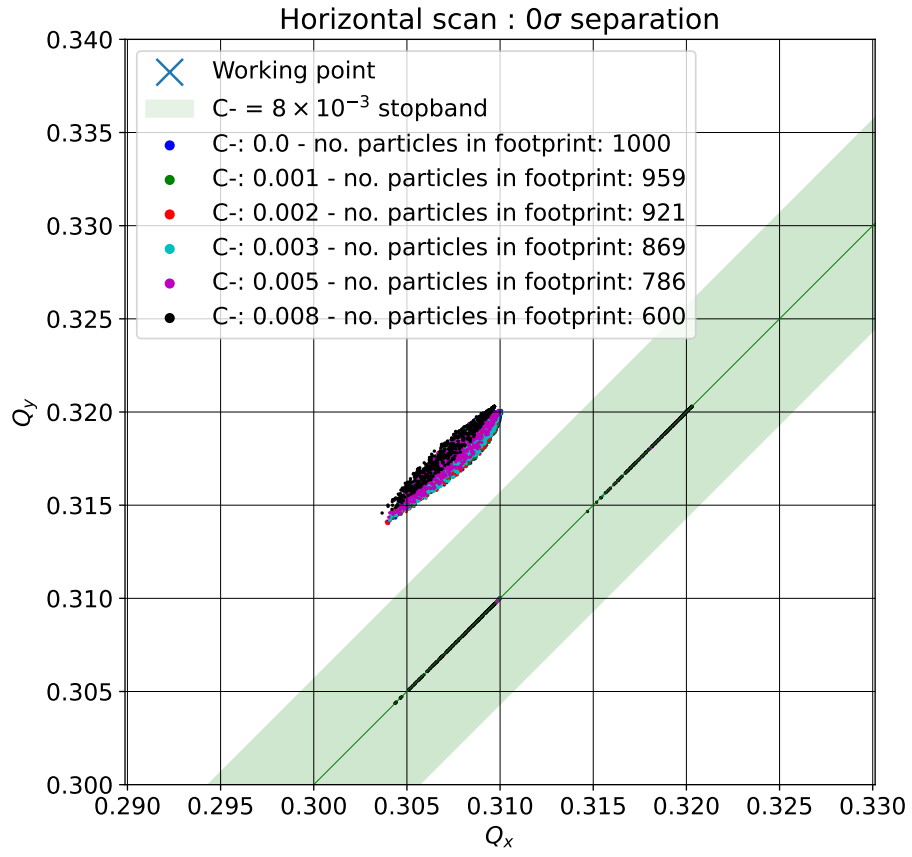
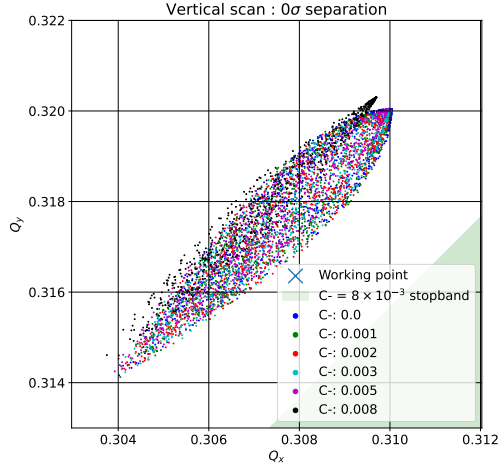
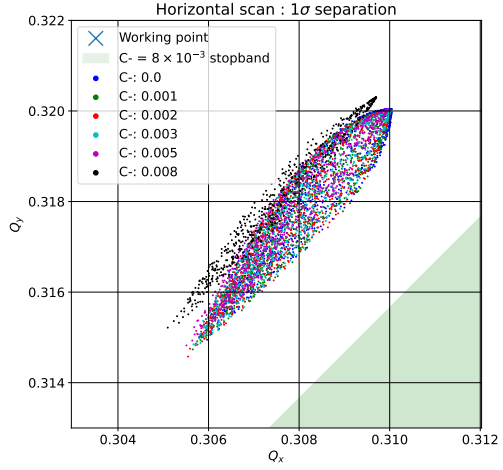


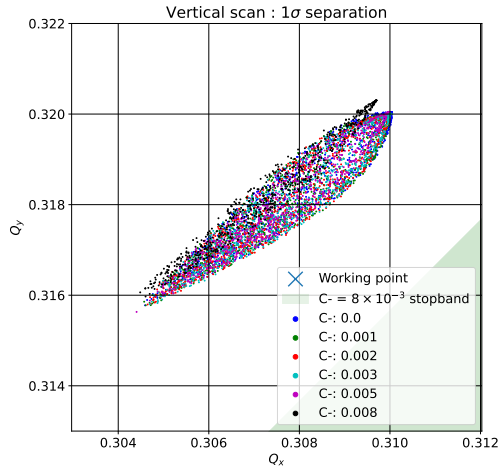
Figure 3.18: Number of particles in the footprint for different levels of coupling in a 2 IP head-on collision.



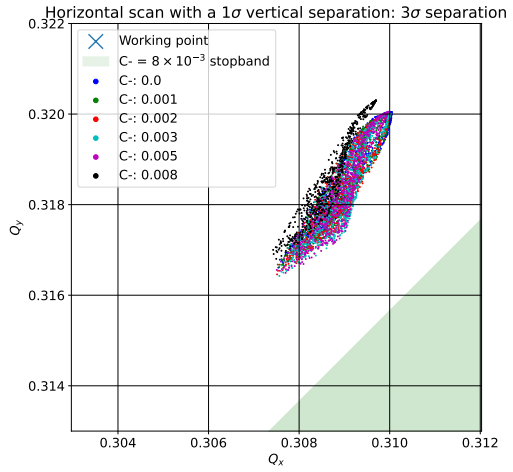
(a)



(b)



(c)



(d)

Figure 3.19: Comparison of tune spreads for the 2 IP configuration with different levels of coupling, combinations of separations, and offsets typical of vdM scans. (a) 2 IP vertical scan, head-on collision (b) 2 IP horizontal scan, 1σ (c) 2 IP vertical scan, 1σ separation (d) 2 IP horizontal scan, 3σ separation, 1σ offset.

3.4.3 Impact on luminosity and the visible cross section σ_{vis}

With the numerical integrator and linear coupling implemented in the LHC vdM model, the bias to luminosity measurements can now be evaluated, and the associated visible cross section quantified. To highlight the different effects, the luminosity will be shown normalized to different conditions. In Table. 3.1 the nomenclature used for the different luminosity calculations is defined for clarity. Bias in this thesis listed as $\frac{A}{B}$ is equal to $\frac{|A-B|}{B}$. Similar to simulations in Section. 3.3.3, the luminosity bias is evaluated, this time

Abbreviation	Explanation
L_0	no added effects
L_{bb}	beam-beam turned on, no coupling
L_c	coupling turned on, no beam-beam
L_{bbc}	beam-beam and coupling on

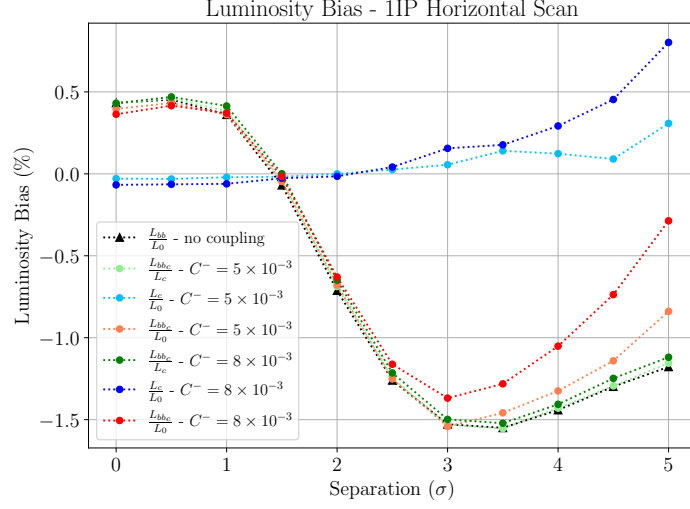
Table 3.1: A dictionary of abbreviations for luminosity terms.

with the inclusion of a skew quadrupole in the transfer matrix to introduce local coupling. Four cases of coupling were investigated: no coupling and coupling with $C^- = 5 \times 10^{-3}$, 8×10^{-3} and 16×10^{-3} . At the LHC, the level of coupling is estimated at around $3\text{-}5 \times 10^{-3}$. However, this value has never been properly measured during vdM sessions. For simulation purposes, the values of C^- were chosen as above: the first as a realistic case with current estimates, the second as an upper-limit case so effects are distinguishable from noise, and the third to establish a trend. The goal of the study is to show and characterize the impact of this coupling together with beam-beam effects on luminosity as a possible source of bias.

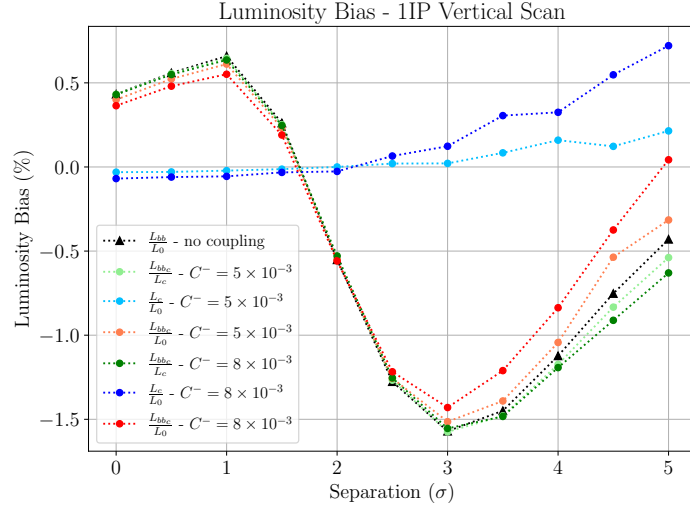
The luminosity was calculated with and without beam-beam effects over the last 500 of 1000 turns with 10 million particles, determined using convergence tests detailed in Appendix. 4. As before, the particle distributions were randomized to a Gaussian and tracked through the lattice. Fig. 3.20 shows the bias of both beam-beam effects and linear coupling for 1 IP, with different normalisations in order to pinpoint the contributions from the two different effects.

Observing the blue lines shows that by including coupling there is an initial small negative bias which then increases to a positive bias at larger separations. Stronger coupling gives a stronger effect. By comparing the red and black lines, the bias from the combined beam-beam effects and coupling is compared to the case with no coupling that has bias only from beam-beam effects. The red lines start with a smaller positive bias than the black but at 2σ the red lines have a smaller negative bias. The green lines show the bias from beam-beam effects and coupling normalized to results from a lattice that only has coupling. These green lines follow the previous trend for horizontal and diagonal scans, but in vertical scans at high separations, there is a slightly greater negative bias instead, demonstrating asymmetric results in different scan directions. The difference between the green and black lines should demonstrate the impact on luminosity from the interplay of beam-beam and coupling effects.

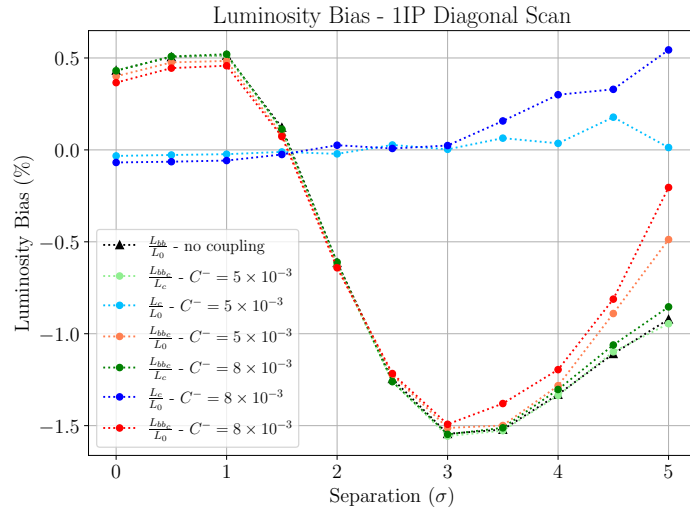
The same studies were carried out in the 2 IP case and shown in Fig. 3.21. The same trend in results is also seen as above, but with the main difference of coupling creating a noticeable positive bias at higher separations than seen with 1 IP.



(a)

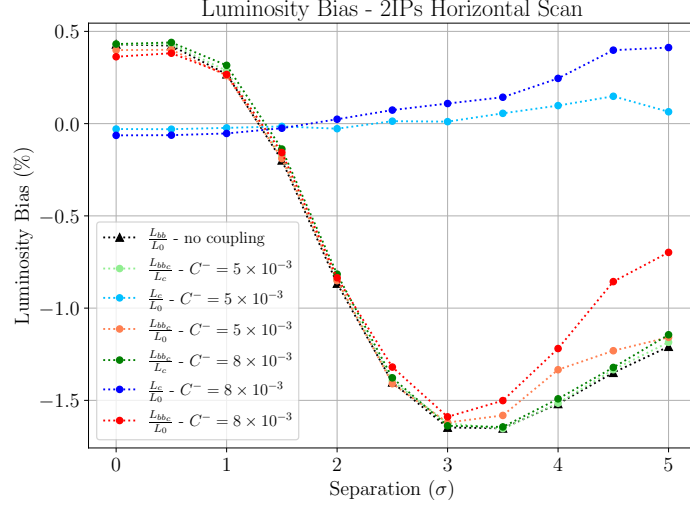


(b)

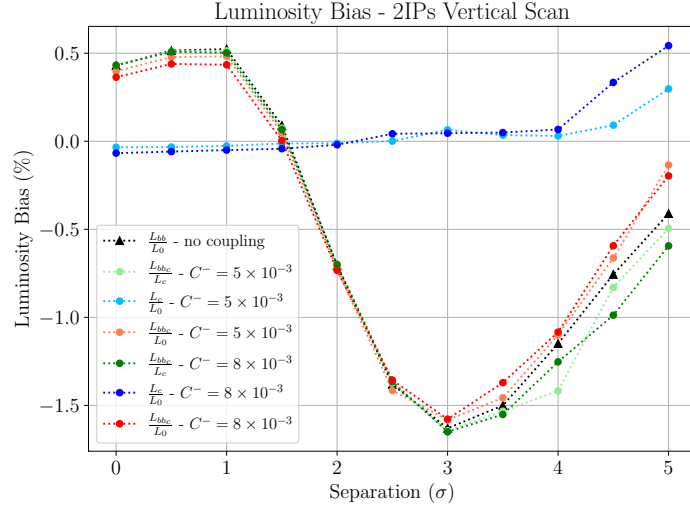


(c)

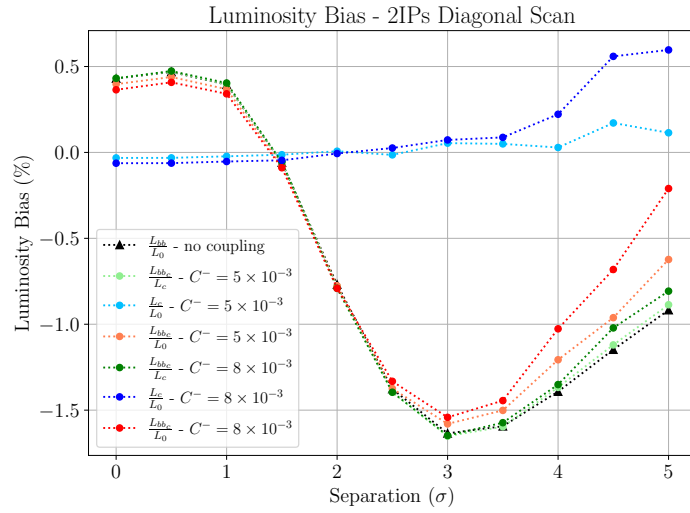
Figure 3.20: Comparison of luminosity bias for 1 IP vdM scans: (a) horizontal, (b) vertical, and (c) diagonal, with coupling and beam-beam effects.



(a)



(b)



(c)

Figure 3.21: Comparison of luminosity bias for 2 IP vdM scans: (a) horizontal, (b) vertical, and (c) diagonal, with coupling and beam-beam effects.

Furthermore, σ_{vis} was calculated for different levels of coupling utilising Eq. 2.13. The luminosity bias as a function of separation in the horizontal and vertical planes was used. The results are shown in Fig. 3.22a. As with luminosity, different normalization methods were utilised and are labelled in the legend following the conventions defined in Table. 3.1

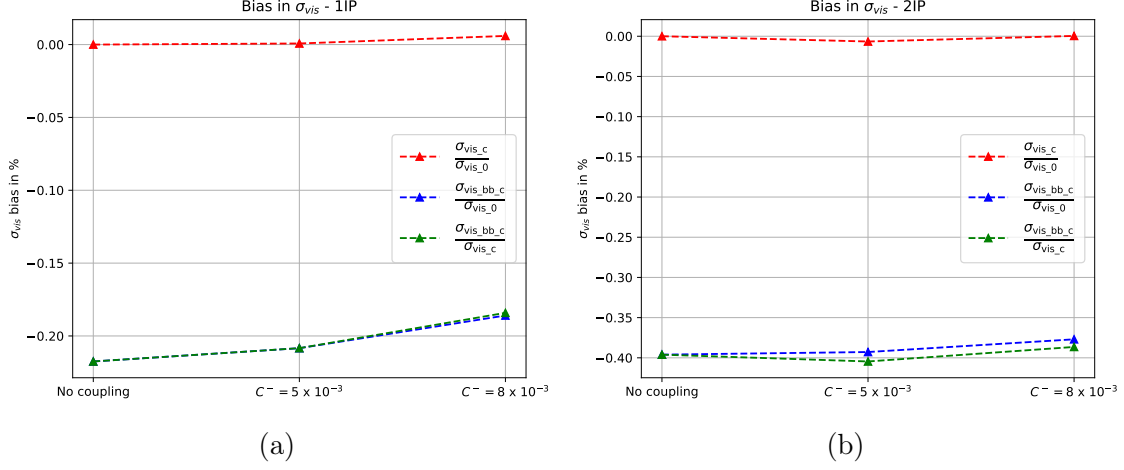


Figure 3.22: Bias in σ_{vis} for vdM parameters with (a) 1 IP and (b) 2 IPs for different values of coupling, $C^- = 5 \times 10^{-3}$ and $C^- = 8 \times 10^{-3}$.

In Fig. 3.22a there is a beam-beam enhancement to the effect of coupling. Whilst the differences are small as seen in Table. 3.2, there is a noticeable reduction in negative bias by combining beam-beam effects and linear coupling compared to the case with no coupling. Additionally, the biases from isolated beam-beam effects removing coupling (green in Fig. 3.22a) do not add linearly with the biases from just coupling (red) to produce the combined biases from both beam-beam and coupling (blue). However as the differences are small, it is difficult to state this with absolute conviction. For the case of 2 IPs, the

$\Delta\sigma_{vis}$ bias	$C^- = 5 \times 10^{-3}$	$C^- = 8 \times 10^{-3}$
$\Delta \frac{\sigma_{visc}}{\sigma_{vis0}}$	0.00075%	0.0059%
$\Delta \frac{\sigma_{visbbc}}{\sigma_{vis0}}$	0.00911%	0.0316%
$\Delta \frac{\sigma_{visbbc}}{\sigma_{visc}}$	0.00913%	0.0334%

Table 3.2: Difference in σ_{vis} biases under different conditions of coupling for 1 IP.

data is presented in Fig. 3.22b and Table. 3.3. There is a different trend with 2 IPs, with a further reduction in σ_{vis} bias for $C^- = 5 \times 10^{-3}$ in the case of isolating the beam-beam effects from coupling. Therefore, the application of local coupling influences different configurations of IPs in different ways, and it remains important for subsequent studies to model the full LHC realistically.

A possible hypothesis for this small impact could be that the deviations in luminosity bias from smaller separations are balanced out by those at larger separations, visible in Fig. 3.20. Although the smaller separations have more of an impact when calculating σ_{vis} , as seen in Fig. 3.7, the deviations in the opposite direction are larger at higher separations. Conversely, the asymmetry in horizontal and vertical results could also be cancelling out the effect.

$\Delta\sigma_{vis}$ bias	$C^- = 5 \times 10^{-3}$	$C^- = 8 \times 10^{-3}$
$\Delta \frac{\sigma_{vis_c}}{\sigma_{vis_0}}$	0.0066%	0.0004%
$\Delta \frac{\sigma_{vis_{bb_c}}}{\sigma_{vis_0}}$	0.0033%	0.0191%
$\Delta \frac{\sigma_{vis_{bb_c}}}{\sigma_{vis_c}}$	-0.0085%	0.0096%

Table 3.3: Difference in σ_{vis} biases under different conditions of coupling for 2 IPs.

As the impact on σ_{vis} is small for vdM-specific simulation parameters, other parameters were simulated for further investigation. The case of a larger beam-beam parameter of $\xi = 9.11 \times 10^{-3}$, facilitated by an increased bunch intensity of 2.2×10^{11} has been studied. This higher beam-beam parameter is still relevant for physics experiments at the LHC, but also as a full 4 IP configuration will have more compounded beam-beam effects than modelled in this study. A more significant impact (change of -0.03% for $C^- = 8 \times 10^{-3}$) total bias from beam-beam and coupling (blue) on the difference from coupling is visible in Fig. 3.23a and it is key to note that the bias in σ_{vis} now decreases in the case of 2 IPs.

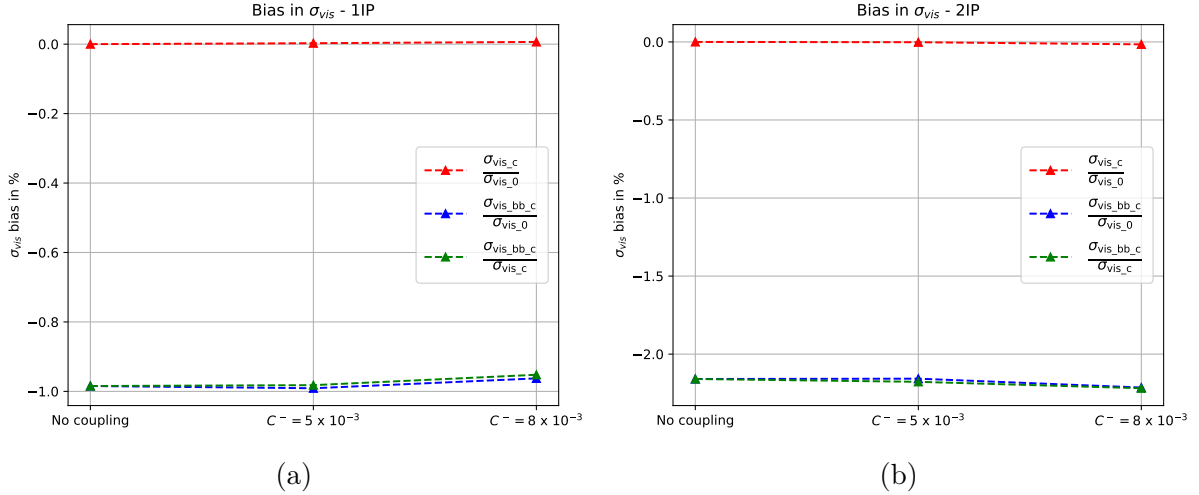


Figure 3.23: The σ_{vis} bias is plotted versus different strengths of the coupling resonance (C^-) for the case with (a) 1 IP and (b) 2 IPs. This case shows the impact of a beam-beam interaction with $\xi = 9.11 \times 10^{-3}$.

The hypothesis that the main impact of linear coupling comes from the shifted tunes and not the distribution deformation is also explored. Indeed, this is a possibility for the real machine as tune perturbations of the order of 10^{-3} cannot be detected and corrected for. This is the order of magnitude of the correction for $C^- = 5 \times 10^{-3}$. Therefore this aspect of coupling may not be corrected for in real life operation. Coupling is applied but none of the tune shifts are corrected for (for 8×10^{-3} the change in tune is 2×10^{-3}). This is carried out for vdM parameters for both one and two IPs as before, with results shown below in Fig. 3.24. With the tunes being uncorrected there is a significant impact from introducing coupling (-0.0743% for 1 IP and -0.0485% for 2 IPs for $C^- = 8 \times 10^{-3}$). From these results, it can be seen that coupling does in fact have an effect on luminosity when combined with beam-beam effects. In addition, it seems to be at first approximation linked to the associated tune shift and spread. However, in this first model, we have not considered cases where there are asymmetries in the x and y beam sizes or different

coupling strengths for the two beams. In addition, the presence of Landau octupoles on tail particles could also be a source of change to the global picture. For these reasons, they also need to be studied in combination to draw a final conclusion on the impact of linear coupling on luminosity.

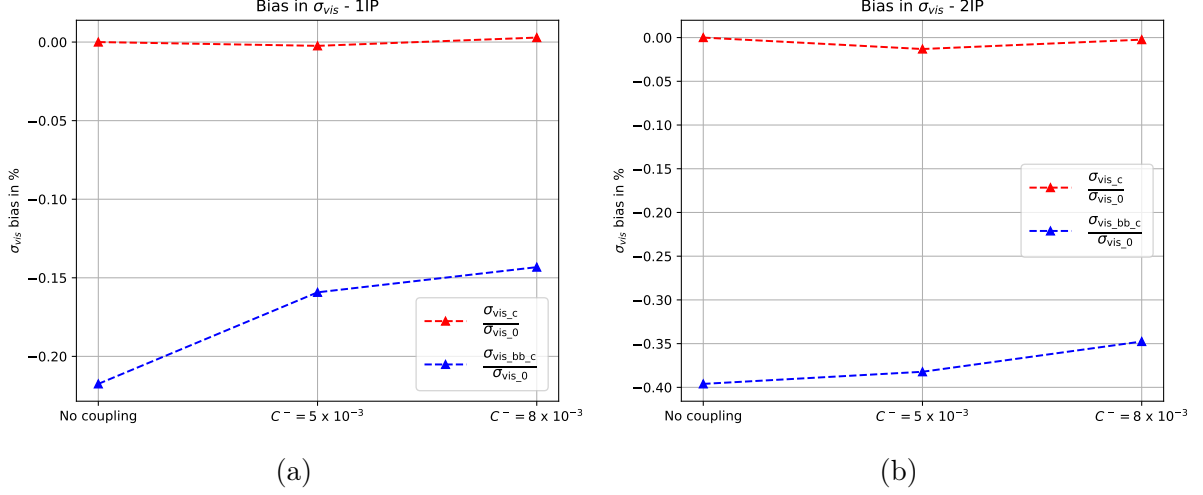
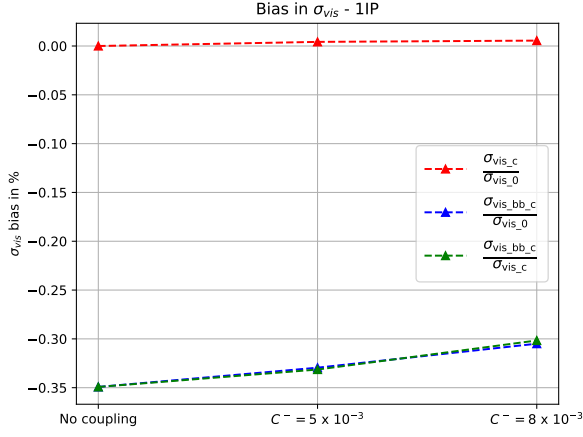


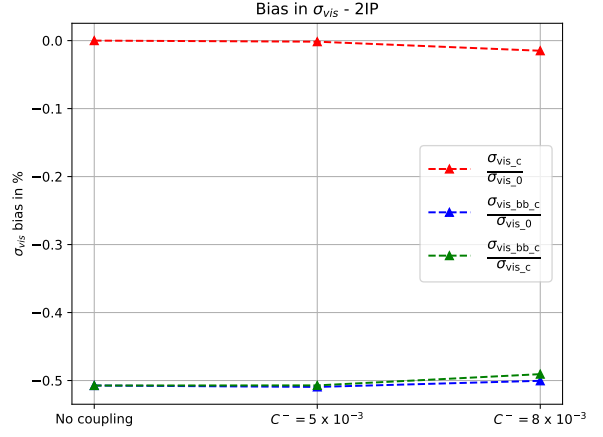
Figure 3.24: σ_{vis} bias plotted versus different strengths of the coupling resonance (C^-) and uncorrected tune shift induced by the coupling for the case with (a) 1 IP (b) 2 IPs.

Fig. 3.25 shows the difference in σ_{vis} where the tunes have been shifted closer to resonance: $Q_x = 60.312$ and $Q_y = 62.316$. The change in σ_{vis} bias is larger and shows the impact on the tune spread since the tune shifts are compensated for. Moving towards the diagonal in the case of uncorrected coupling can lead to relevant deviations of the σ_{vis} . For the 2 IP case, the impact is negligible. The difference lies in the very different frequency space that the core particles occupy.

Finally, these studies were repeated with an even stronger coupling of 16×10^{-3} to establish a trend in results. In this case, the impact is larger than the beam-beam effect itself. In addition, this configuration assumes corrected tune shifts which normally mitigate part of the effects. In a global view one can certainly state that linear coupling does have a relevant impact on the σ_{vis} cross section and it depends on several parameters.

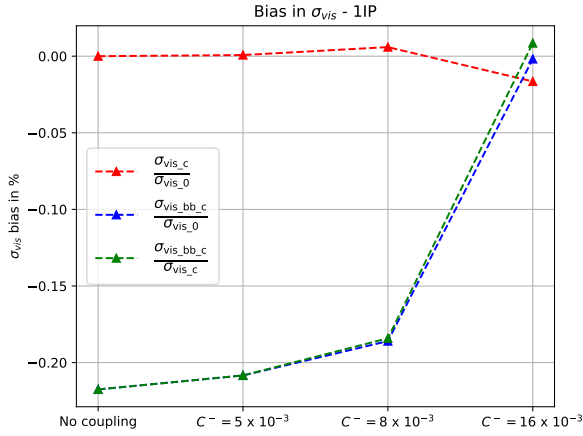


(a)

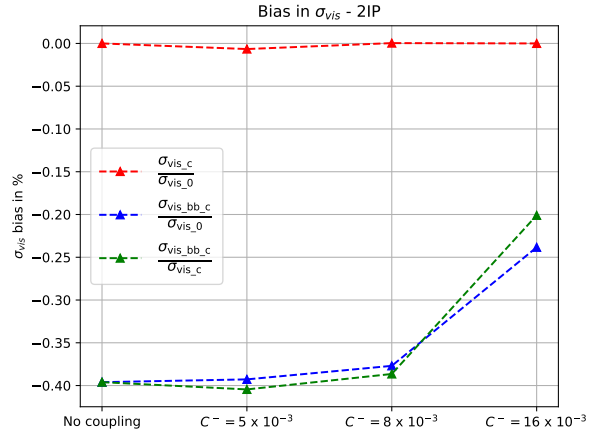


(b)

Figure 3.25: σ_{vis} bias plotted versus different strengths of the coupling resonance (C^-) for the case with (a) 1 IP and (b) 2 IPs. Both cases show the effect of coupling for unperturbed tunes of $Q_x = 0.312$ and $Q_y = 0.316$.



(a)



(b)

Figure 3.26: σ_{vis} bias plotted versus different strengths of the coupling resonance (C^-), including 16×10^{-3} with (a) 1 IP and (b) 2 IPs.

Chapter 4

Conclusions and outlook

Motivated by the observations in the luminosity calibration of ATLAS and CMS of correlated distributions in the different transverse planes, this study aimed to shed light on the impact of linear coupling and beam-beam effects on luminosity in the Large Hadron Collider during van der Meer scans.

The first step in this study was to develop the necessary numerical tools to compute the luminosity via a numerical integrator in Xsuite. A new numerical luminosity integrator for arbitrary particle distributions has been developed and tested to guarantee the required precision. Beam-beam interactions during vdM scans for absolute luminosity calibration were successfully simulated in the new simulation framework and have been compared with previous state-of-the-art results. In addition, the σ_{vis} cross section has been evaluated on both sets of simulations showing they agree at a level better than 0.04%. This value is smaller than the uncertainty on the σ_{vis} from other sources of distortions of 0.1%.

With this model developed, the second goal was to understand and quantify the impact of linear coupling and beam-beam effects on luminosity measurements, and in particular understand if these effects have a role in the non-factorization problem, which is an open question and experimental limitation of vdM scans. To address this question, a skew quadrupole was added to the LHC model. The gradient of the quadrupole was calibrated with the minimum tune approach to characterize the strength of linear coupling in terms of C^- . The linear coupling has been modelled and characterized for C^- in the range of 3×10^{-3} and 16×10^{-3} . For the first time, the effects of linear coupling intertwined with beam-beam effects were explored. Different configurations of the collisions were studied and diagonal scans have been explored. The impact of linear coupling and beam-beam effects on luminosity was characterized and a clear effect was shown on footprints, luminosity bias and the visible cross section bias.

The evolution of beam footprints during van der Meer scans under the presence of linear coupling has been studied. It has been shown that the beams will experience a tune shift and tune spread modification, which should impact the final luminosity. The study has been performed for the case of one single experiment scanning and also when an additional witness experiment stays in a head-on collision as typical during van der Meer measurements. The impact on the luminosity as well on the σ_{vis} bias from these results were evaluated. For reasonable values of C^- , the impact on σ_{vis} is very modest if the tune shift associated with the linear coupling is corrected for. However, during operation the tune shift due to the coupling effect may not be measurable due to its small amplitude (a few 10^{-3} in units of the tunes); for this reason, the impact of uncorrected tunes was also explored. In this configuration, a tangible effect on the luminosity was observed and

consequently on the σ_{vis} , causing changes in the σ_{vis} bias of around 0.074% to an original bias of 0.22% with $C^- = 8 \times 10^{-3}$ and one interaction point, showing a clear effect due to this distortion. Larger coupling strengths ($C^- = 16 \times 10^{-3}$) show an enormous effect at the same level of the beam-beam force. For this reason, a full detailed study should be pursued to address the realistic impact of such effects demonstrated in this thesis.

During the study, cumulative luminous region data was produced in order to evaluate the luminous region characteristics. The data could not be analysed and compared to available observations from CMS analysis due to time constraints. An important step would be to compare the expected effects to the experimental observations, to understand if the amplitude of the effect on the σ_{vis} shown is compatible with what is observed by the detectors. The study described in this thesis is a proof of concept of the impact of linear coupling combined with beam-beam effects on the absolute luminosity and σ_{vis} calculations. From the results presented, it is clear that further studies must take place as linear coupling is seen to produce a distortion on the particle distributions, introducing a $x - y$ correlation that changes as the beam-beam interaction evolves during the van der Meer scans. Future studies must include a more realistic model of the LHC coupling since the simplified model used in this work does not fully represent the real coupling effect in the collider lattice. Additionally, the effect of asymmetric configurations between the beams, non-round beams and higher-order resonances such as those introduced by sextupoles and octupoles should be evaluated, since they are known to introduce strong correlations between the horizontal and vertical particle distributions.

Acknowledgements

This project could not have been completed without the encouraging support and love from my supervisors, teachers, PhD colleagues, friends, and family. I would first like to thank my supervisor Tatiana Pieloni for setting up this opportunity for me, and for her continued guidance, mentorship, and encouragement. I could not have asked for a better supervisor in any way. I would also like to thank my co-supervisor Giovanni Iadarola for his help and guidance in all matters to do with coding and computation. Thank you to Joanna Wańczyk for all of your guidance, help, and time invested in helping me. I really appreciate it and this work could not be done without you and your previous efforts. Thank you as well to Péter Kicsiny for bearing with all of my impromptu office visits for help and support in all matters beam-beam and perseverance. I would also like to thank Carlo Emilio Montanari for his extremely helpful guidance on how to navigate the tricky HTCondor landscape, as well as Szymon Lopaciuk, Rebecca Taylor, and Fredrik Grønvold. Thanks to Abigail Levison, Yi Wu, Leon van Riesen-Haupt, and Tiziana Von Litzeben and all of my other friends for your much-needed moral support. Thank you to Zichen He for your love and support throughout this year abroad. Thank you to my family for all the opportunities you have provided me as well as the constant love and belief in me. Finally, thank you to Imperial College London, EPFL, and the exchange coordinators Gavin Davies and Marina Galand for providing support for this exchange experience.

Appendix: Convergence Studies

Convergence studies were performed to determine the range of turns and number of macroparticles needed for luminosity and σ_{vis} bias calculations. Beam-beam simulations are computationally expensive, so it is important to balance the accuracy of the computed result with the computational complexity.

Results varied for different separations, the Gaussian approximation and numerical integrator, and whether beam-beam effects were on or not, so convergence tests were carried out for all situations. Simulations ran for 3000 turns, with different numbers of macroparticle as indicated by the colour wheels in Fig. 3 and Fig. 4. Moving averages were taken across 100, 300 and 500 turns as luminosity will oscillate from betatron motion. Fig. 4 shows luminosity without beam-beam effects converging faster than when beam-beam effects are included, as well as a closer approximation by the analytical expression.

Two examples of analysis of a head-on collision simulated with 5 million particles are shown below. The moving average taken every 100 turns is plotted on top in Fig. 1, and the moving average of luminosity bias is shown in Fig. 2. A few convergence study results are shown as examples in Fig. 3 and Fig. 4, with a range in convergence rates. Vertical and diagonal scans showed similar results to horizontal.

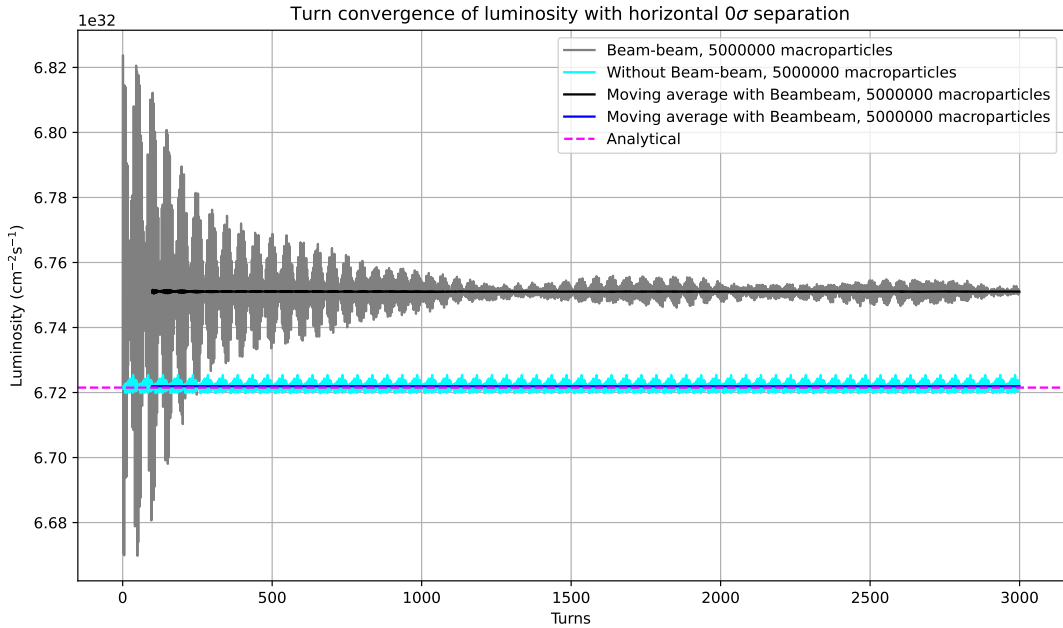


Figure 1: Luminosity with 5 million particles with moving average taken every 100 turns

As a result of the convergence studies it was decided that 10 million particles would be tracked for 1000 turns, with the final 500 turns being taken to obtain an average luminosity. On the graphs this can be seen as the value "at 1000 turns" as a running average of 500 takes a mean of the 500 values before that point.

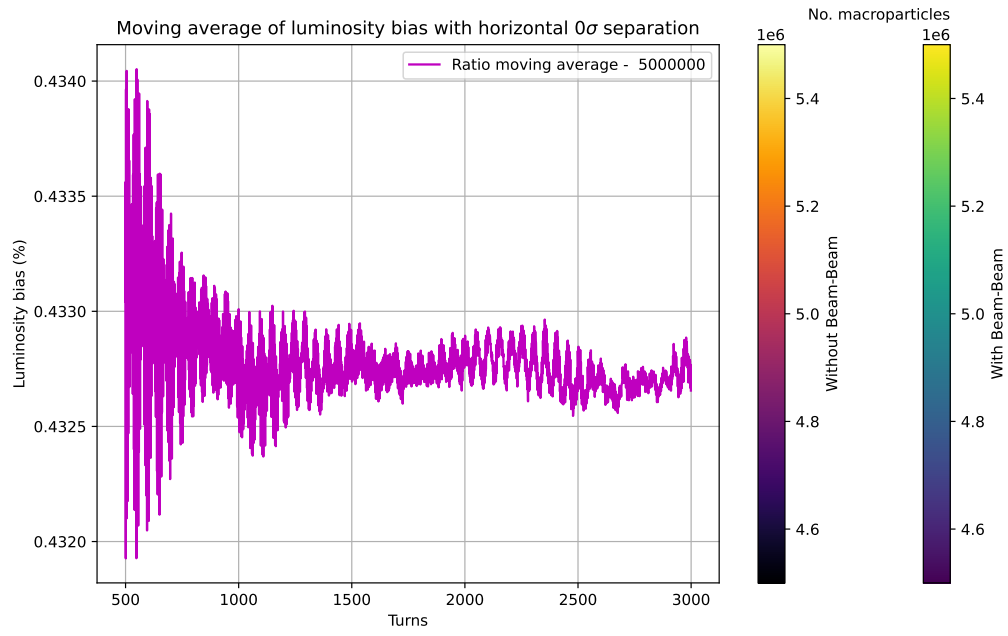
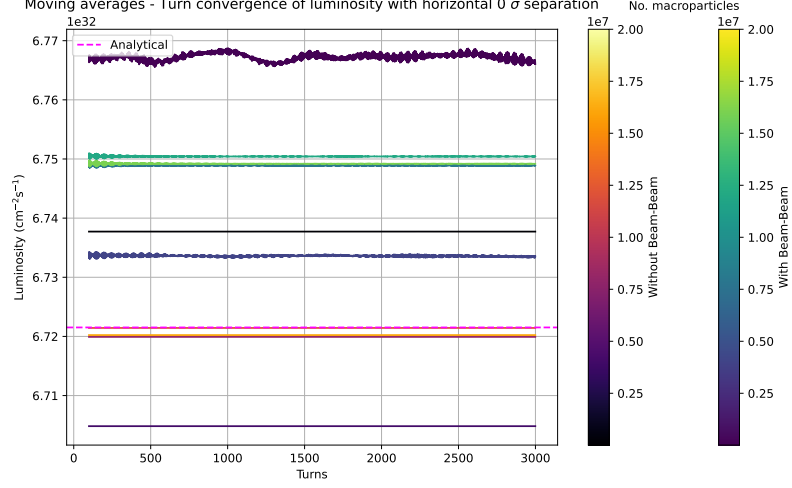
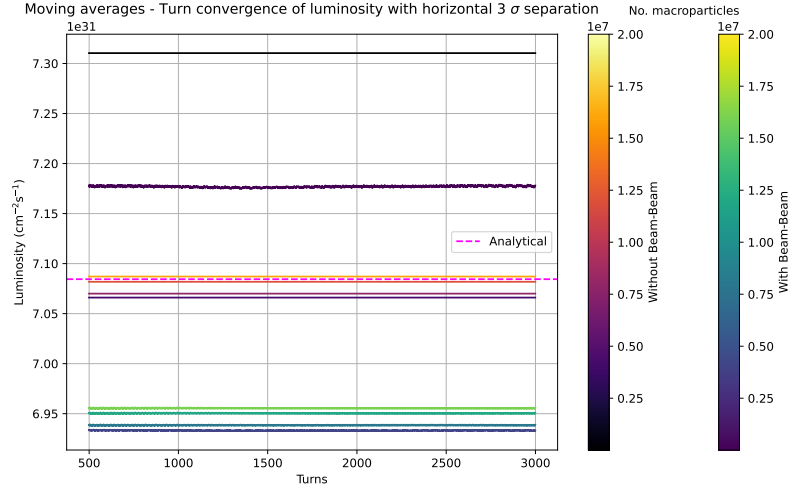


Figure 2: Moving average taken every 500 turns of luminosity bias with 5 million particles

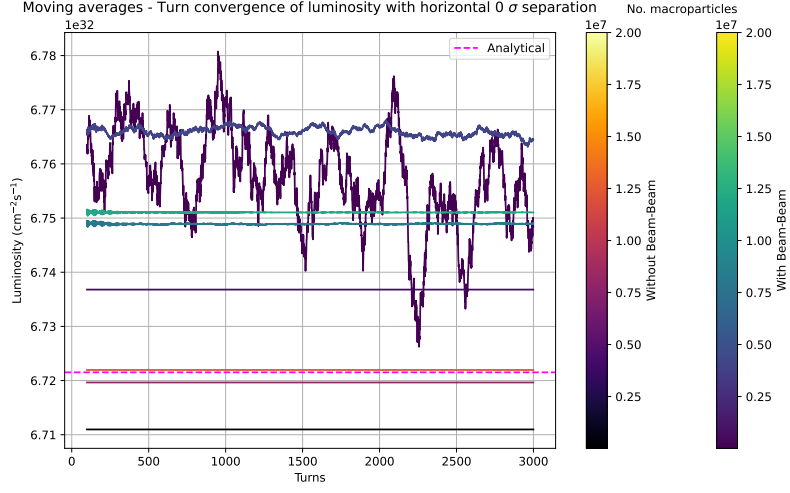


(a)

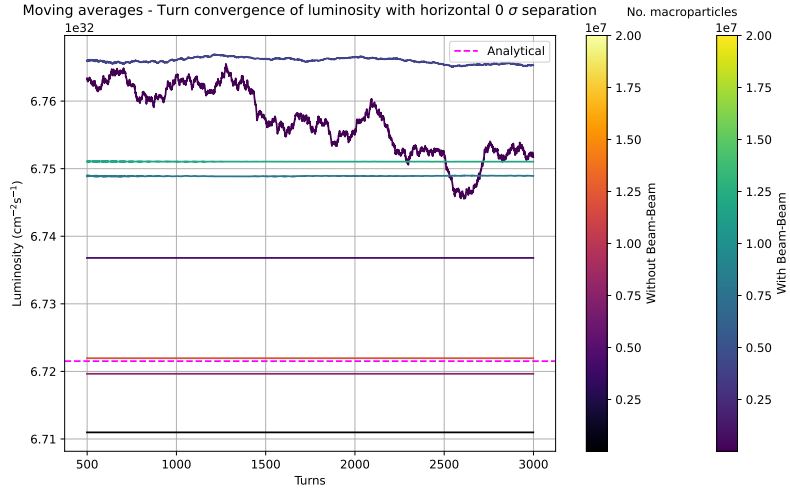


(b)

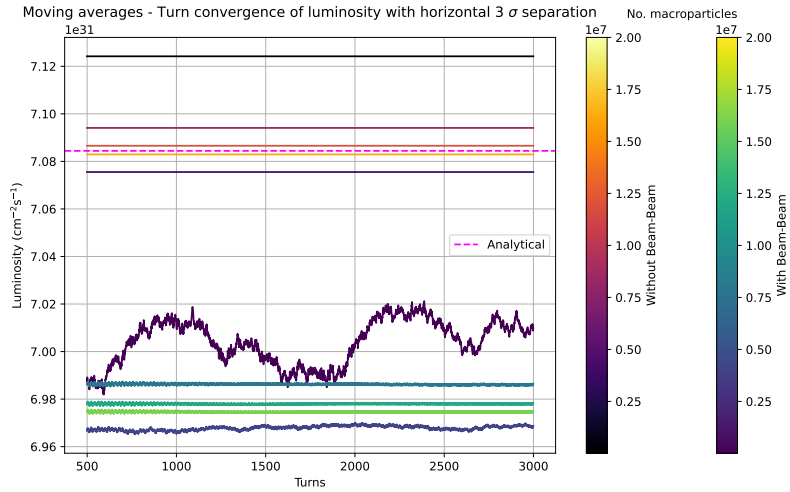
Figure 3: Moving averages for convergence tests for transverse scans in the Gaussian approximation: (a) taking a moving average over 100 turns for a head-on collision (b) taking a moving average over 500 turns for luminosity with a 3σ separation



(a)



(b)



(c)

Figure 4: Moving averages for convergence tests for transverse scans using a numerical integrator: (a) taking a moving average over 100 turns for a head-on collision (b) taking a moving average over 500 turns for a head-on collision and (c) taking a moving average over 500 turns for luminosity with a 3σ separation

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