

Tuning of QCD Model Parameters Using LEP Data of Hadronic Z Decays

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To my parents

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Synopsis

Title of the Thesis : Tuning of QCD Model Parameters
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The initial state in e^+e^- interactions are well understood, and the cross-section for multi-hadron production is rather large at center of mass energies near Z boson mass. This makes the Large Electron Positron (LEP) Collider operating at the Z-peak an excellent laboratory for tests of Quantum Chromodynamics (QCD) [1], the theory of quark and gluon interactions. The initial state radiation (ISR) and the interference between initial and final state radiation are strongly suppressed at the Z-resonance allowing measurements of the Standard Model parameters with small systematic error.

However, tests of QCD differ in nature from other tests of the Standard model. Free quarks or gluons have not been observed in the detector so far, and a yet not completely understood non-perturbative phase sets in as the primary quarks resulting from the decay of the Z boson hadronize. Thus, in order to make any test of the predictions of perturbative QCD, one has to model the fragmentation and the subsequent decay process of primary partons into final state hadrons. Over the years several fragmentation models have been developed and based on them different event generating Monte Carlos programs have been written.

The perturbative and the non-perturbative phases are separated by an energy scale of the order of a few GeV. The Monte Carlo programs differ in their treatment to the perturbative phase, as well as the non-perturbative phase:

- Two approaches exist for modelling the *perturbative phase*. First is the Matrix Element method, in which Feynman diagrams are calculated order by order exactly. Due to technical difficulties, presently this is considered only upto second order. The more commonly used approach is based on the parton shower program in which an arbitrary number of partons are branched so as to yield a multi-jet topology. This has been carried out upto either Leading Logarithmic Approximation (LLA) or Next-to-Leading Logarithmic Approximation (NLLA) or Modified Leading Logarithmic Approximation (MLLA) in the different event generation programs. Parton shower cascades are developed using either the Altarelli-Parisi splitting functions governing the probability of emission of quarks and gluons, or treating the gluonic emission as from a quark-antiquark color dipole pair, which thereafter behaves as independent color dipole emitters.
- The *non-perturbative phase* is treated by modelling the fragmentation part by either string (SF) or cluster (CF) models. Decay length and rates of massive hadrons, suppression probability of higher spin states, Bose-Einstein correlation parameters, etc. are extracted from low energy experimental data of relevant processes. In the string fragmentation model, a string of color flux is stretched between the quark

and anti-quark pair, with gluons as kinks on the string, and the fragmentation proceeds in the formalism of the string breaking, whereas in the cluster fragmentation model, gluons are split into quark-antiquark pairs, which then form colorless clusters, depending on their masses or decay into lower mass clusters or directly into particles.

In this thesis, the following models are studied : ARIADNE [2], JETSET [3] with Parton Shower (PS) and Matrix Element (ME) option, and HERWIG [4]. The different options used by these models are listed in the table . Depending upon the scheme used in the perturbative phase of the models, different Monte Carlo programs use different values of the Λ , characterising the only free parameter α_s in the theory of QCD. The fragmentation models also involve several parameters which are eventually determined by data. For example, the string fragmentation model as used in JETSET or ARIADNE uses the parameters a , b for light quarks and ϵ_Q for heavy quarks.

$$\begin{aligned} f(z) &\sim z^{-1} (1-z)^a \exp\left(-\frac{bm_T^2}{z}\right) && \text{for light quarks} \\ &\sim z^{-1} \left(1 - \frac{1}{z} - \frac{\epsilon_Q}{1-z}\right)^{-2} && \text{for heavy quarks} \end{aligned}$$

where z is the light cone variable and m_T is the transverse mass. In addition, one smears the hadronic transverse momenta (p_T) with respect to the jet direction using a parameter σ_Q .

$$f(p_T) \sim \exp\left(-\frac{p_T^2}{\sigma_Q^2}\right)$$

Cluster fragmentation model as in the HERWIG event generator uses parameters like CLMAX, CLPOW which determine whether a cluster will split before hadronisation, or like B1LIM which decides whether a cluster with b-quark will undergo a two body decay etc.

These models serve as the bridge between the existing calculations of perturbative QCD and the experimental data. Determination of the model parameters are therefore the first step for any tests of perturbative QCD. The data used correspond to 250K hadronic events from Z decay collected at $\sqrt{s} \approx M_Z$ by the L3 detector [5] at LEP during 1991. The statistics is sufficiently large to tune the models. The observed distributions have been corrected [6] for detector effects – resolution and acceptance and also for initial and final state radiations.

The performance of different models can be studied by comparing, between data and Monte Carlo the global structures of the hadronic events, which are not only sensitive

QCD Model	Pre-Hadronization	Hadronization (Frag. & decay)
JETSET (PS)	Parton Shower (LLA)	String Model
JETSET (ME)	Matrix Element(NLLA)	String Model
ARIADNE	Color Dipole (LLA)	String Model
HERWIG	Parton Shower (MLLA)	Cluster Model
Table 1: The different options for the perturbative (Pre-Hadronization) and non-perturbative (Hadronization) phases as used by different QCD Monte Carlo programs.		

to perturbative QCD, but also to the fragmentation models. Such a comparison is done for eighteen event shape variables, out of which four are used to optimize the parameters of the fragmentation models. These four tuning variables, chosen to describe the lateral and longitudinal hadronic shower profiles, include jet resolution parameter in the JADE algorithm [8] which corresponds to transition from $2 \rightarrow 3$ jets (y_{23}^{JADE}), minor [9] calculated after dividing the event into two hemispheres by the thrust [10] axis and evaluated in the hemisphere corresponding to the narrow jet ($T_{\text{minor}}^{\text{NS}}$), the fourth Fox-Wolfram moment [11] [$H_4 = \sum_{a,b} \frac{\vec{p}_a \cdot \vec{p}_b}{s} P_4(\cos \theta_{ab})$ where P_4 is the fourth order Legendre polynomial], and the charge multiplicity.

Two methods are used for tuning of the parameters of the models: direct method and grid method. In the direct method one generates the Monte Carlo distribution of the event shape variables at each step of minimization, while in the grid approach one starts off with pre-generated Monte Carlo distribution over a grid on the parameter space, and the subsequent minimization proceeds by performing a varying degree non-linear multi-dimensional interpolation over a sub-grid about the point of interest in the parameter space. Systematic studies are done by varying the fit ranges for the four tuning variables, and by varying the initial choice of points on the grid. The final optimized parameters are chosen by performing cross checks, which gives best correspondence between data and Monte Carlo for all the event shape variables.

Prior to this work, the energy evolution [7] of the mean charge multiplicity was poorly described by ARIADNE using the previously tuned parameter set by the L3 experiment [6]. Furthermore, the ratio of expected to observed four jet rates as a function of the jet resolution parameter y_{cut} for jets reconstructed using the JADE [8] and the $k_{\perp}$ [12] algorithms showed a deficit at the level of 5-20% depending on the y_{cut} values by both the parton shower and the matrix element approach of JETSET event generator. This could give rise to a large systematic error on measuring cross section for the process $e^+e^- \rightarrow W^+W^-$, which is highly relevant at high energy runs at LEP II.

In this thesis, global event shape variables are used to tune JETSET 7.4 Parton Shower and Matrix Element models, ARIADNE 4.06 and HERWIG 5.8. The tuned parameter sets [13] are given in the tables [2-5].

Parameter	Fitted Value	Stat. Error	Syst. Error
Λ_{LLA} (GeV)	0.311	± 0.022	± 0.026
σ_Q (GeV)	0.411	± 0.019	± 0.028
b (GeV ⁻²)	0.886	± 0.060	± 0.104
Table 2: Tuned Parameters for the JETSET 7.4 (Parton Shower)			

Parameter	Fitted Value	Stat. Error	Syst. Error
Λ_{MLLA} (GeV)	0.166	± 0.002	± 0.015
CLMAX (GeV)	2.968	± 0.055	± 0.105
CLPOW	1.569	± 0.020	± 0.219
Table 3: Tuned Parameters for the HERWIG 5.8 (Parton Shower)			

Parameter	Fitted Value	Stat. Error	Syst. Error
Λ_{LLA} (GeV)	0.254	± 0.013	± 0.020
σ_Q (GeV)	0.384	± 0.017	± 0.018
b (GeV ⁻²)	0.772	± 0.034	± 0.067
Table 4: Tuned Parameters for the ARIADNE 4.06 (Parton Shower)			

Parameter	Fitted Value	Stat. Error	Syst. Error
Λ_{ME} (GeV)	0.152	± 0.005	± 0.005
σ_Q (GeV)	0.430	± 0.015	± 0.021
b (GeV ⁻²)	0.310	± 0.014	± 0.007
Table 5: Tuned Parameters for the JETSET 7.4 (Matrix Element)			

The retuned models give a somewhat better agreement with the data. For example, the evolution of mean charged multiplicity with CM energy is in better agreement with the predictions of ARIADNE 4.06. The ratios of expected to observed 4-jet rate as a function of the jet resolution parameter y_{cut} for JADE and $k_{\perp}$ algorithms shows improvement in the agreement with ARIADNE and HERWIG where the ratio is close to 1 within 5%. However, no improvement is observed in the prediction of the JETSET 7.4 PS model where the discrepancy still stays at the level of 5-20%.

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Declarations

I hereby state that the work presented in this thesis has not been submitted to this or any other university for M.Sc. or any other degree.

Statement Required Under Regulation 136

Statement No. 1 regarding new facts :

Charge multiplicity was not well described by ARIADNE, using the previous set of tuned parameters. In this tuning, charge multiplicity distribution has been also included in the list of tuning variables. ARIADNE now describes all the event shape variable distributions well.

In the previous analysis linear multi-dimensional interpolation was being used for estimating the Monte Carlo distributions between the grid. In course of this analysis, a non-linear multidimensional interpolation technique has been developed, and successfully used.

Statement No. 2 regarding author's contribution in joint work :

The L3 experiment is a huge collaborative effort comprising of around 500 physicists from about 40 institutes/laboratories all over the globe. Numerous individuals have contributed significantly towards the successful joint venture in the fields of detector design, building and maintenance as well as software development, both online and offline, without which this work would not have been realized. The scientists and engineers of the CERN accelerator divisions contributed towards the excellent performances of the LEP collider over the years.

I have carried out the analyses under the supervision of Prof. Sunanda Banerjee. The work where major responsibility towards the analyses rests on the author is described in this thesis.

References are properly cited at appropriate places whenever use has been made of earlier works. The principal aspects of the work of the author can be summarised as :

Inclusion of charge multiplicity in the list of tuning variables.

Use of non-linear multidimensional interpolation.

The charge multiplicity distribution is well described by ARIADNE.

Expected to observed ratio of jet-rates are within 5% for ARIADNE and HERWIG.

Signature of the Candidate

Signature of the Supervisor

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Chapter 1

Introduction

Electron-positron annihilation has been a very important testing ground for Quantum Chromodynamics (QCD) [1] as the theory behind the *strong* force describing the interaction between quarks: fractionally charged colored constituents of matter. However, free quarks have not been observed and QCD as a theory is not fully understood particularly in the low energy domain, where non-perturbative effects become dominant. Experimental verification of QCD has thus necessitated development of several phenomenological models to describe the hadronization phase in which the primary partons fragment into observable hadrons. Inputs from experimental observations have played a crucial role in the developments of these models, and they owe their success in describing today's experiments to the data from numerous experiments of the past.

e^+e^- annihilations, studied at center of mass energies upto 5 GeV, produced low multiplicity hadronic final states characterised by isotropic phase space distribution. In 1975, the MARK I experiment running at the SPEAR electron-positron collider found clear evidence of *jet* structure. A study of global event shape variables from the 6.2 – 7.4 GeV center of mass energy data revealed that the mean value of the sphericity variable ($= 1/0$ for spherical/2-jet events) decreases with increasing center of mass energy [2]: a feature which cannot be explained by an isotropic phase space model. In 1979, the MARK J collaboration, studying $e^+e^- \rightarrow \text{hadrons}$ at the PETRA electron-positron collider, furnished the first direct experimental evidence of the existence of gluons [3], the carrier of strong interaction.

$e^+e^- \rightarrow \text{hadrons}$ data have provided vital experimental support for QCD. Some of them are listed below :

- The ratio of the hadronic cross-section to that for $e^+e^- \rightarrow \mu^+\mu^-$ has shown step-wise jumps with increasing center of mass energy, each jump corresponding to the threshold for production of a new quark [4]. Agreement of the data with predicted rates from $e^+e^- \rightarrow q\bar{q}$ as the underlying process requires a color degeneracy factor

of 3 available to the quarks.

- In the MARK I data the jet axis of the events followed an angular distribution $d\sigma/d\Omega \propto (1 + \alpha \cos^2 \theta)$, with α near 1 at 7.4 GeV [2], demonstrating that in the underlying process spin-1/2 quarks were involved.
- In typical 2-jet events the charge distribution of the hadrons were not found to be stochastic in nature. PLUTO collaboration observed that each hemisphere had a small average total charge of $|Q_{\text{jet}}| = 0.55 \pm 0.25$ [5], in agreement with the average quark charge.
- Study of the angle between the thrust direction and the light jet direction (boosted to the rest frame of the 2 lighter jets) in a 3-jet decay system from e^+e^- annihilation study at TASSO, PLUTO and CELLO collaboration, gave experimental confirmation to the spin-1 property of gluons [6].

Numerous other experimental evidences from hadron spectroscopy, deep-inelastic scattering, etc. along with developments from the theoretical side [7] (like Yang-Mills's non-abelian gauge theory, concept of color, quark-parton model, scaling and its violation, renormalizability, asymptotic freedom, etc.) now lead us to believe that SU(3) based QCD describes the force of strong interaction. Along with the SU(2) $\otimes$ U(1) electro-weak theory of Glashow, Salam, and Weinberg [8], it forms the basis of our understanding of the theory of elementary particles, known as the Standard Model of particle physics.

LEP (Large Electron Positron collider) at CERN, has determined the parameters of Standard Model with a very high precision. Its initial mode of operation (LEP I) had been at center of mass energies around the mass of the Z boson, and presently it is running at center of mass energies above the threshold of W boson pair production (LEP II). In the LEP I data there is a relative abundance of hadronic events, characterised by high multiplicity and large energy deposited in the detectors. At LEP II, hadronic events pose as the most important background to the $e^+e^- \rightarrow W^+W^-$ events, approximately half of which decay hadronically.

The process $e^+e^- \rightarrow \text{hadrons}$ involves several steps (see figure 1.1) distinguished by the different length scales involved and the underlying nature of the interaction:

Quark-antiquark pair (and photon) production	$\sim 10^{-17}$ cm	Electro-weak
Gluon and quark radiation	$\sim 10^{-15}$ cm	Perturbative QCD
Fragmentation of quarks/gluons into hadrons	$\sim 10^{-13}$ cm	Non-perturbative QCD
Decays of unstable hadrons into stable particles	$> 10^{-13}$ cm	Electro-weak and QCD

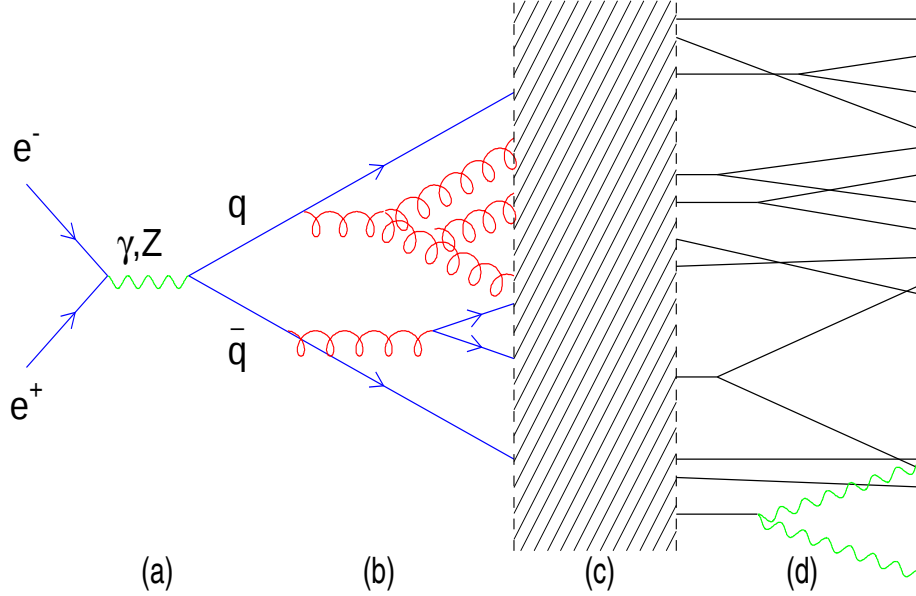


Figure 1.1: Schematic view of the distinct phases of the process $e^+e^- \rightarrow \text{hadrons}$: (a) e^+e^- annihilation and production of quark-antiquark pair in an electro-weak process, (b) perturbative QCD evolution, (c) non-perturbative regime representing transition from partons to hadrons and (d) decays of unstable hadrons into stable particles.

In order to understand the global event structure of hadronic events, we use QCD event generators, which are Monte Carlo simulation programs based on perturbative QCD calculations and phenomenological models describing the non-perturbative phase. These models serve as the bridge between the theoretical calculations and experimental observations, and thus determination of the parameters of these models is the first step in extracting any physics from the data. In this thesis, global event shape variables are exploited to tune JETSET 7.4 [9] Parton Shower and Matrix Element models, ARIADNE 4.06 [10] and HERWIG 5.8 [11] using $e^+e^- \rightarrow \text{hadrons}$ data from the L3 experiment [12].

A brief overview of the thesis is as follows:

Chapter 2 provides an overview of perturbative aspects of QCD, and their incorporation into different Monte Carlo programs.

Chapter 3 describes the various models of fragmentation that have phenomenologically evolved to bridge the gap between the theoretically understood part of strong interactions and the experimentally observed spectra of *stable* hadronic particles.

Chapter 4 presents a wide range of observables that are used to describe the *global shape* of an event. It is the study of these variables, which allows one to distinguish between the type, class and nature of the event being considered, and enables one to estimate crucial parameters of the Standard Model that govern, for example, the strength of the strong

interaction at the energy scale considered.

Chapter 5 explains the tuning procedure and methods used in optimising the parameter set of the different models, along with the need for present tuning, using hadronic data from Z decays collected by the L3 detector at LEP.

Chapter 6 presents the results of present tuning, and discusses the quality of fit obtained from previous and current tunings.

Chapter 7 compares the main results of the work with those as obtained previously. The models tuned using data from hadronic Z decays at LEP I are checked for their predictive powers at higher energies at LEP II as well.

A word about notation is in order: throughout this thesis, flags for different options, as available in the original code (e.g., PARJ, MSTJ, QCDLAM, etc.) of the different Monte Carlo programs, are referred to in parenthesis during the course of discussion.

Chapter 2

Perturbative Quantumchromodynamics

“May the Color Force be with you!”

... Yu.L.Dokshitzer, CERN-JINR school lectures (1995).

2.1 A brief preview of QCD

Quantum Chromodynamics (QCD) is a one parameter renormalizable gauge theory, with the free parameter (α_s) describing the strength of the *strong* force, developed in analogy with the relationship of fine coupling constant (α) of quantum electrodynamics (QED) to the charge of electron (q): $\alpha = \frac{q^2}{4\pi}$. Thus, ascribing a unit quanta of *color* charge (g_s) to quarks, the coupling constant of QCD is given by:

$$\alpha_s = \frac{g_s^2}{4\pi}.$$

One of the fundamental properties of QCD is that gluons, the carrier of strong force, also carry color charge as a consequence of the non-abelian nature of the internal symmetry group under which the QCD lagrangian is invariant. The choice of the color gauge group is constrained by the following requirements [13]:

- The total cross-section for $e^+e^- \rightarrow \text{hadrons}$ and decay rates of $\pi^0 \rightarrow \gamma\gamma$, are proportional to the number of colors and its square respectively. Agreement with data for these two processes require the number of color degeneracy to be *three*. Therefore we need a group with irreducible representations of dimension 3. The possible candidates are O(3), SU(2), U(2), U(3), SU(3), SO(3) and Sp(2).

2.1 A brief preview of QCD

- In order to solve the statistical puzzle for the spin $\frac{3}{2}$ baryon Δ^{++} in the symmetric **10** representation of the SU(3) flavour symmetry, there must be completely antisymmetric color singlet state: $\Delta^{++} = \frac{1}{\sqrt{6}} \sum_{\alpha\beta\gamma} \epsilon^{\alpha\beta\gamma} u_\alpha u_\beta u_\gamma$, with all the three u-quarks in spin $+\frac{1}{2}$ state and the greek indices denoting color degrees of freedom. Out of the above-mentioned seven groups, $\epsilon^{\alpha\beta\gamma}$ is not invariant under U(2), U(3) and O(3), which discards them.
- $q\bar{q}$ meson states are observed, but no similar qq bound state is seen. Therefore, the group must admit complex representation so that one can distinguish a quark from an antiquark. Out of the remaining four groups, Sp(2), SO(3) and SU(2) have (identical) *real* triplet representations, and would thus lead to diquark states.

Thus we identify SU(3) as the internal color symmetry group, with each flavour of the quarks in a fundamental representation of the group, and gluons in the adjoint representation of the group. With no experimental evidence of “free” quark found as yet, an important ingredient to the theory is that the color symmetry is exact.

2.1.1 Running of strong coupling constant

As in other renormalizable theories like QED, the coupling constant of QCD “scales” with the energy at which the interaction takes place. However, instead of only one vertex as in QED, the color charges of the quarks *and* gluons manifest themselves in three basic vertices in QCD (see figure 2.1). As a result, instead of only fermionic loop corrections to bosonic (photon) propagators as in QED, one has in addition bosonic (gluon) loop corrections to bosonic (gluon) propagators in QCD. The net effect is that in contrast to QED, the strong coupling constant decreases with increasing energy (or with decreasing distance): a phenomenon known as “asymptotic freedom” (ref. Politzer, Gross-Wilczek in [7]).

The *running* of the strong coupling constant is given by the Renormalization Group (RG) equations. The procedure of renormalization introduces a energy scale μ , which depends upon the renormalization scheme undertaken. For example, in the modified minimal subtraction ($\overline{\text{MS}}$) scheme, this represents the energy scale at which the ultraviolet divergences along with a constant term are subtracted. However, the concept of RG asserts that the observables of the theory remain independent of the choice of this scale μ . The RG equations of QCD are:

$$\mu^2 \frac{\partial \alpha_s}{\partial \mu^2} = -\alpha_s^2 \sum_{k=0} \beta_k \alpha_s^k$$

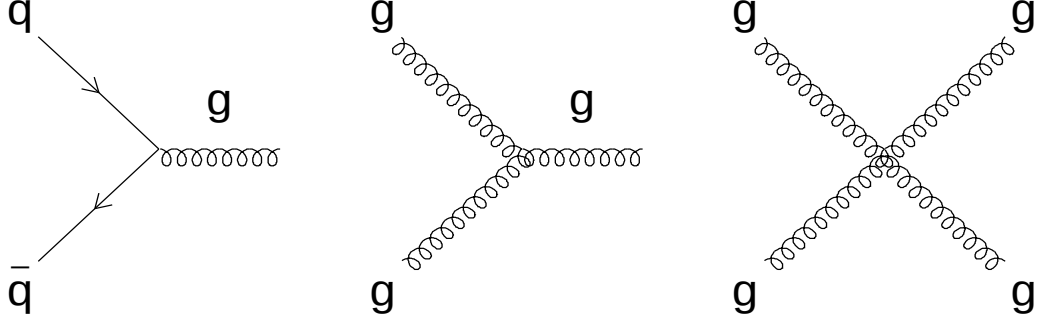


Figure 2.1: Basic vertices in QCD describing quark-gluon and gluon self couplings.

where the first three β -functions [14], in the $\overline{\text{MS}}$ scheme, in terms of n_f (the number of flavour degeneracy of quarks) are:

$$\begin{aligned}\beta_0 &= \frac{33 - 2n_f}{12\pi} \\ \beta_1 &= \frac{153 - 19n_f}{24\pi^2} \\ \beta_2 &= \frac{77139 - 15099n_f + 325n_f^2}{3456\pi^3}\end{aligned}$$

Note that the first two beta functions are scheme independent. At LEP energies, n_f is taken to be equal to *five*, since the sixth quark top with mass = 175.6 ± 5.5 GeV [15] cannot be pair produced at these center of mass energies.

The solution at energy μ is related to the solution at energy μ_o (upto first order) by:

$$\alpha_s(\mu) = \frac{\alpha_s(\mu_o)}{1 + \beta_0 \alpha_s(\mu_o) \ln(\mu^2/\mu_o^2)},$$

For convenience, $\mu_o = M_Z$ is chosen to be the reference scale, and we write $\alpha_s \equiv \alpha_s(M_Z)$. A dimensional parameter Λ can also be used as the free parameter of QCD, interchangeably with α_s . This parameter Λ , marking the boundary between perturbative and non-perturbative energy domains of QCD, may be defined as :

$$\Lambda = \mu_o \exp\left(\frac{1}{-2\beta_0 \alpha_s(\mu_o)}\right)$$

Upto next-to-leading order, the energy (μ) dependence of the strong coupling constant is given by the following formula used extensively at LEP and related directly to the measurements at different energy scales:

$$\alpha_s(\mu) = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)} \left[1 - \frac{\beta_1 \ln \ln(\mu^2/\Lambda^2)}{\beta_0^2 \ln(\mu^2/\Lambda^2)} \right]$$

2.1.2 Jets in e^+e^- annihilation

Study and observation of jets has been a very important tool in understanding QCD at e^+e^- interactions. Two, three and four jet events are understood to arise out of $e^+e^- \rightarrow q\bar{q}$, $e^+e^- \rightarrow q\bar{q}g$ and $e^+e^- \rightarrow q\bar{q}q\bar{q}$ or $e^+e^- \rightarrow q\bar{q}gg$ type of processes at the underlying level (see figure 2.2). This widespread correspondence has resulted in associat-

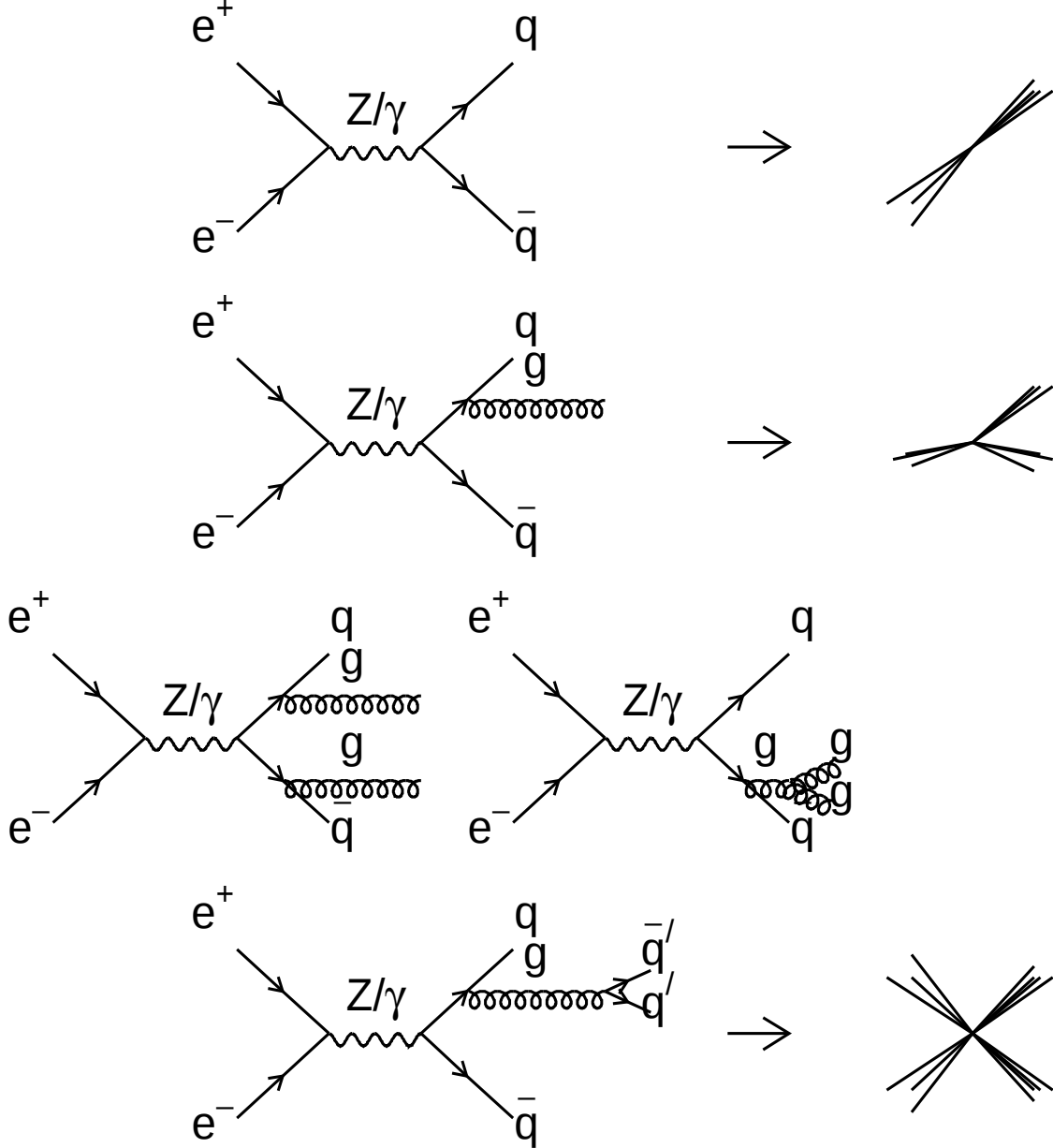


Figure 2.2: Feynman diagram representations of the underlying processes in two, three and four jet events in e^+e^- annihilation.

ing the word *jet* almost interchangeably with the quarks and gluons (generically termed *partons*) in the event description.

A “jet” is one or several nearby partons lumped together according to some jet reconstruction criterion - while the actual number of partons produced is an ill-defined concept, and might as well be infinite, the number of jets for a given jet definition is well defined. The jet characteristics is most intuitively studied using cone algorithms. Stermann and Weinberg [16] defined a two-jet event if $(1 - \varepsilon)$ fraction of the total center of mass energy lies inside two opposite cones (of half-angle δ). According to the Kinoshita-Lee-Nauenberg theorem (see section 2.2.1), the cross-section for such an event is free of infrared (collinear and soft) singularities. One may also define a single-jet inclusive cross-section by finding the distribution of the maximum amount of energy which lies in a cone of half-angle δ . Because the treatment adopted experimentally for overlapping jets has proved difficult to standardise, “Snowmass Accord” [17] on jet cone definitions has been set up, which helps to make more reliable comparison between the full next-to-leading order perturbative calculations for hadronic cross-section and experiments.

Alternatively, jets may be reconstructed using the JADE or the DURHAM jet algorithms. These algorithms cluster the final state particles into pseudoparticles by comparing their separation with the chosen resolution measure. This approach is both infrared and collinear safe, because the algorithms start off by combining the softest and most collinear particles, which thereafter do not affect the analysis at all. The measure of separation (y_{cut}) is related to the invariant mass in the JADE algorithm or transverse momentum in the DURHAM algorithm, both scaled with respect to the center-of-mass energy (see chapter 4 for details). Besides the fact that use of invariant mass related measure can lead to a unnatural assignments of particles (particularly back-to-back final state low momenta ones) to jets, theoretical criteria of good resolution parameters (like leading to smaller hadronization corrections, resummability of large logarithms at small values of y_{cut}) have favoured the DURHAM algorithm.

In the JADE algorithm, a fixed y_{cut} corresponds to a minimum mass between any two jets which grows linearly with the center of mass energy (see chapter 4): for example, at $y_{\text{cut}} = 0.01$ the mass separation between jets is 3.5 GeV at PETRA/PEP energies, while at LEP I it corresponds to 9 GeV, which is worth comparing to the 1 GeV scale below which the non-perturbative fragmentation process sets in, irrespective of center-of-mass energy. At this y_{cut} value, with $\alpha_s = 0.12$, second order QCD calculations give a 2 : 3 : 4 jet composition as approximately 11% : 77% : 12%, with the individual contributions from $e^+e^- \rightarrow q\bar{q}gg$ and $e^+e^- \rightarrow q\bar{q}q\bar{q}$ processes to 4 jet rate estimated to be 11.5% and 0.5% respectively.

2.1.3 Coherence effects

Given all the matrix elements ($\mathcal{M}_i$) of the relevant diagrams ($i = 1, n$) contributing to the cross-section of a particular process, taking *coherent* sum ($|\sum_i \mathcal{M}_i|^2$) refers to including all the interference terms, as opposed to taking the *incoherent* sum ($\sum_i |\mathcal{M}_i|^2$).

Beyond the leading logarithmic behaviour, the following two types of *coherence* effects have important bearing in describing the hadronic event topology in e^+e^- annihilations:

- *Intrajet coherence*, arising from destructive interference between the soft gluon emission within the jets, reduce the phase space available for further parton emission to an angular ordered region [18]. This effect is closely related to the Chudakov effect in electromagnetic processes in cosmic rays [19], where the electron and positron produced from photon conversion are unable to ionize independently of each other till they are separated by more than a typical atomic radius. In a $q \rightarrow q'/g$ process in QCD, the contribution from the emission from q and those from the q' and the g cancel each other outside an angular region given by the emission angle of secondary gluon g_1 (see figure 2.3).

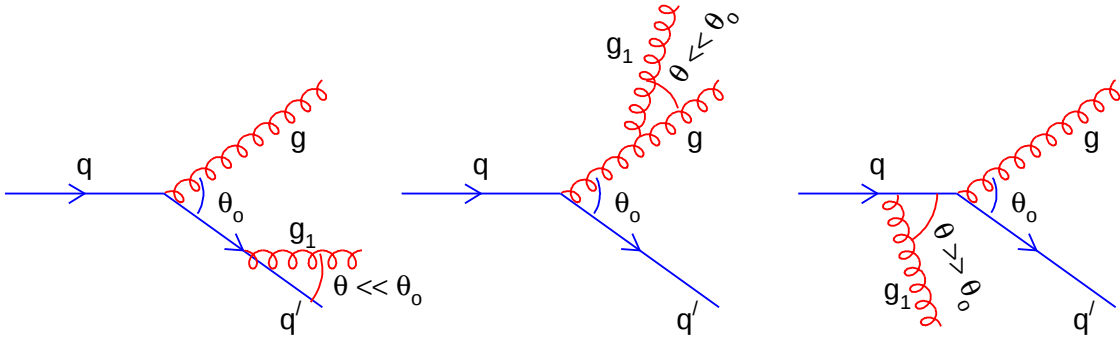


Figure 2.3: Regions of destructive interference in gluon emission from each of the three partons in a $q \rightarrow q'/g$ process.

Drawing the analogy from electromagnetism, one sees that a high energetic secondary gluon emitted under a small angle can resolve the color content of the gluon cloud, whereas a soft gluon emitted under a larger angle cannot, thereby decreasing its probability of interaction [20]. This dynamical suppression in the momenta in the infrared region leads to ordering (see figure 2.4) of the energy as well as emission angle of successive parton radiation in the jet formation stage. Direct consequences of this are reduced parton multiplicities and a dip in the parton momenta in the low momentum region, both of which have been observed in e^+e^- annihilation experiments, for example, in L3 detector at LEP [21].

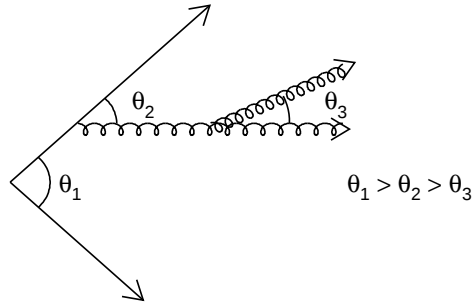


Figure 2.4: Schematic representation of angular ordering in consecutive parton radiations.

- *Interjet coherence* describe the interference between particles across the jets arising from the “color drag” phenomenon [22]. This implies ordering of azimuthal as well as polar emission angles. A prediction of this effect is that in 3 jet events, destructive interference between the q and the $\bar{q}$ jets results in a reduction of the number of observed hadrons as compared to the other two inter-jet regions. This so-called *string effect* has been observed in the L3 detector as well [23].

2.2 Modelling perturbative QCD in Monte Carlo simulations

The applicability of Monte Carlo techniques depend more on the user’s capability to formulate the problem such that random numbers may parametrise its solutions, than on the actual stochastic nature of the situation at hand. Thus, in addition to situations which are probabilistic or statistical in nature, processes of analytical or deterministic nature as well may be formulated by simulation with certain amount of imagination. For example, in a typical event in a present day high energy accelerator like LEP, e^+e^- annihilation results in tens to hundreds of particles in the final state. Analytical computation of scattering amplitude for $2 \rightarrow 100$ process, even with an explicitly solvable theory, would be quite a complicated task.

Depending upon the facet of the problem to be studied, two approaches have been traditionally developed in modelling the perturbative phase of QCD: the *matrix element* (ME) and the *parton shower* (PS) approach. Matrix element approach, resting on second order QCD calculations with a maximum of four primary partons generated, takes into account the exact kinematics and the full interference and helicity effects; whereas the parton shower approach using approximations derived by simplifying the kinematics, interference and helicity structures, includes modelling of multiple soft gluon emission.

While the first one is better suited for α_s determinations, the latter approach, albeit with less predictive power, gives a good description of the substructure of multi-jet event.

In this section, the implementation of both these approaches in the Monte Carlo programs: ARIADNE, HERWIG and JETSET will be discussed. JETSET offers options to use either the ME or PS [MSTJ(101)=5] approach, while ARIADNE and HERWIG are essentially PS models.

2.2.1 Matrix Element approach

In this approach Feynman diagrams are calculated order by order. However, due to technical reasons, presently calculations only upto second order [24] are implemented.

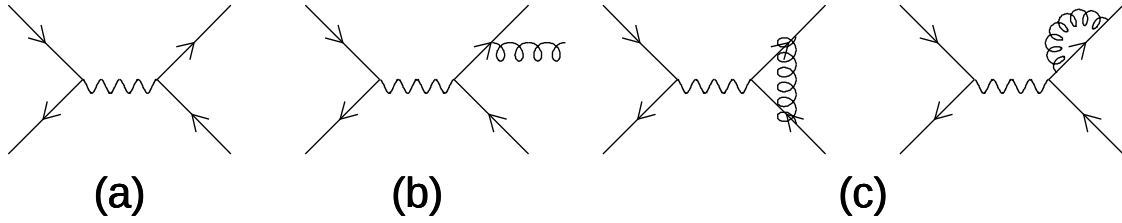


Figure 2.5: $\mathcal{O}(\alpha_s)$ corrections to the $e^+e^- \rightarrow \text{hadrons}$ process: (a) Born process, (b) and (c) are generic first order correction diagrams coming from single gluon emission (using symmetry more diagrams may be generated).

For *massless* quarks [25], the first order corrections to the Born level process (see figure 2.5a) arising from radiation of gluons from either one of the quark-antiquark pair (see figure 2.5b) are expressed in the center-of-mass frame in terms of the scaled energy variables x_q , $x_{\bar{q}}$ and x_g (scaled with respect to center-of-mass energy $\sqrt{s}$):

$$\frac{1}{\sigma_0} \frac{d^2\sigma}{dx_q dx_{\bar{q}}} = \frac{\alpha_s}{2\pi} C_F \frac{x_q^2 + x_{\bar{q}}^2}{(1-x_q)(1-x_{\bar{q}})}$$

where σ_0 is the born level cross section, $C_F = \frac{4}{3}$ is the appropriate color factor and the kinematically allowed region in Dalitz plot (see figure 2.6a) is $\{0 \leq x_i (\equiv 2E_i/\sqrt{s}) \leq 1, \sum_i x_i = 2, (i = 1, 2, 3)\}$. For *massive* quarks [26], the amount of correction is slightly smaller, with a corresponding reduction of phase space for emission by the requirement: $\frac{(1-x_q)(1-x_{\bar{q}})(1-x_g)}{x_g^2} \geq \frac{m_q^2}{s}$. At LEP energies, even for b quarks, the size of these corrections are fairly small.

This cross-section exhibits two kinds of singularities (see figure 2.6b):

- in the limit x_i and $x_j \rightarrow 1$ (which implies $x_k \rightarrow 0$ by momentum conservation), we have *infrared* singularity corresponding to soft gluon emission;

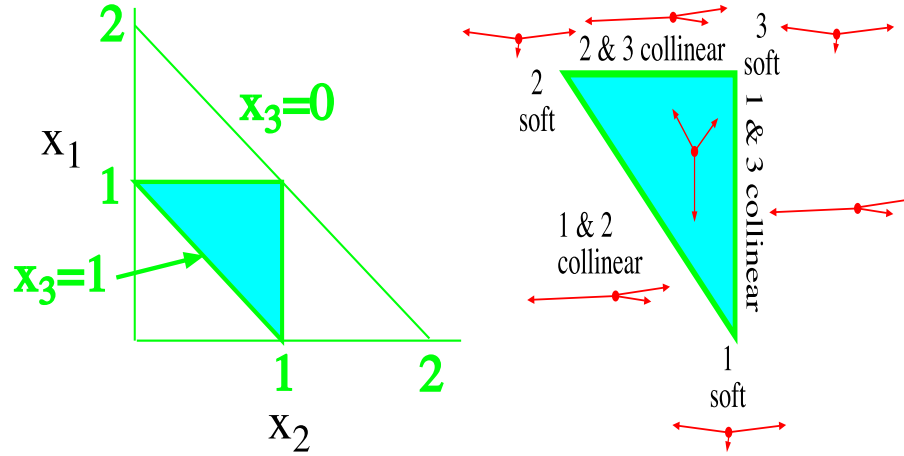


Figure 2.6: Dalitz Plot for available phase space: (a) shaded region in (x_1, x_2) plane, $x_3 = 2 - x_1 - x_2$; (b) insets around the shaded region show the corresponding physical configuration of the three partons.

- in the limit x_i or $x_j \rightarrow 1$ (which implies partons j and k or partons i and k are collinear), we have *collinear* singularity corresponding to collinear gluon emission.

Thus we see that in these two limits, the process in figure 2.5b becomes indistinguishable from the 2 jet process in figure 2.5a as far as any measurement is concerned. However the Born level diagram for 2 jet process also receives higher order vertex corrections from virtual diagrams (as in figure 2.5c), and their interference terms cancel the singular behaviour from the real gluon emission. Using suitable jet definitions, e.g., based on y_{cut} , these limiting three parton final states may be included in the 2 jet process, and the total first order hadronic cross-section remains finite:

$$\sigma_{\text{total}} = \sigma_0(1 + \alpha_s/\pi)$$

This kind of infrared and collinear singularity cancellation occurs order by order. The total cross-section is well behaved to all orders, as required by the Kinoshita-Lee-Nauenberg theorem [27], which states that the infrared singularities cancel each other if all the degenerate initial and final state diagrams are summed up. For example, the full second order cross-section involves both real parton emission terms and the vertex and propagator corrections (see figure 2.7), which modify the three and four jet cross-sections.

From three jet topology studies [28] at PETRA, there is evidence for the necessity of using a low y_{cut} , i.e., a larger 3 jet rate at higher energies where hard gluon emission becomes increasingly important. At LEP energies, the second order corrections to the three jet rate are large. One way to tackle this is to calculate the next full order, which is a formidable task. A more favoured approach is to use “optimised perturbation theory” [29],

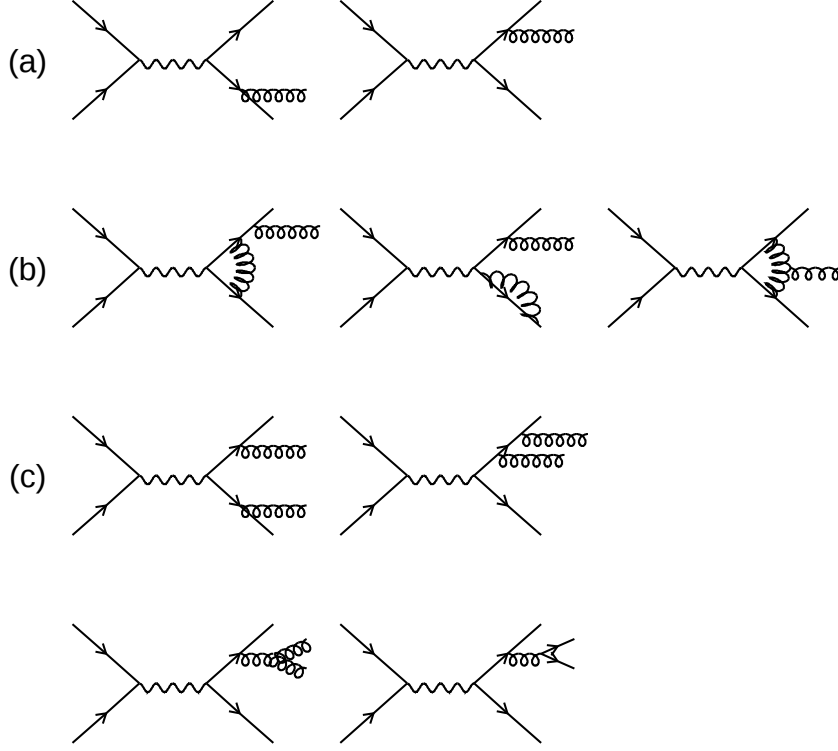


Figure 2.7: $\mathcal{O}(\alpha_s)$ & $\mathcal{O}(\alpha_s^2)$ diagrams for 3 and 4 jet cross-sections: (a) contribution to 3 jets in first order; (b) vertex and propagator corrections to 3 jet in second order; (c) second order contributions to 4-jet production (using symmetry more diagrams may be generated).

in which one tries to minimize the higher order contributions by a suitable choice of the renormalization scale Q^2 in α_s : $Q'^2 = f s$, with $f < 1$. In the use of JETSET Matrix Element option, f [PARJ(129)] is set to 0.003, the minimum y_{cut} [PARJ(125)] for the scaled invariant mass squared of any two partons in three or four jet events being set to 0.01. This scale of f corresponds to a Q^2 scale above the b quark mass, while the y_{cut} value still allows a positive two jet production rate.

2.2.2 Parton Shower approach

Given the limitations of matrix element approach in the maximum number of primary partons produced and the increase of available phase space for gluon emission with increasing energy, the *parton shower*(PS) approach derived within the framework of the Leading Logarithmic Approximation (LLA) [30] or Next-to-Leading Logarithmic Approximation (NLLA) [31] or Modified-Leading Logarithmic Approximation (MLLA) [32], is a much favoured approach, chiefly because of its simplicity and flexibility. There is indirect but

strong evidence that emission of multiple soft gluons plays an important role in the event topology at LEP, thus justifying the popularity of the PS approach, in which the primary quarks produced off-shell are branched into an arbitrary number of virtual partons (mostly gluons).

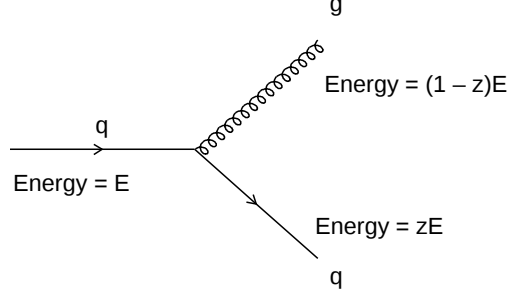


Figure 2.8: Four momentum sharing in the branching process $q \rightarrow qg$.

The parton shower approach based on an iterative branching of partons, involves numerous simplifications in the kinematic description, because of which the predictive power for hard, wide-angle parton emission is limited. Nevertheless, the probabilistic interpretation of the LLA being extremely suitable for event generations, has made this approach quite successful. To appreciate this interpretation, one notes that from the first order matrix element, the differential cross-section takes the form:

$$\frac{d^2\sigma}{dm^2 dz} \sim \alpha_s \cdot \frac{1}{m^2} \cdot \frac{1+z^2}{1-z},$$

where $z = \frac{x_q}{x_q + x_g}$ and m is the invariant mass of the qg system, the equality being exact only in the collinear kinematic limit. Integration over m^2 (for fixed z) gives:

$$\frac{d\sigma}{dz} \sim \alpha_s \cdot \ln(m^2) \cdot \frac{1+z^2}{1-z}$$

which governs the momenta dependence for the branching process: $q \rightarrow qg$ (see figure 2.8). Similar expressions may be derived for the parton splitting processes: $g \rightarrow gg$ and $g \rightarrow q\bar{q}$, which are collectively known as the Altarelli-Parisi (AP) splitting kernels [33] :

$$\begin{aligned} P_{q \rightarrow qg}(z) &= C_F \frac{1+z^2}{1-z} \\ P_{g \rightarrow gg}(z) &= N_C \frac{(1-z(1-z))^2}{z(1-z)} \\ P_{g \rightarrow q\bar{q}}(z) &= T_R(z^2 + (1-z)^2) \end{aligned}$$

with the co-efficients $C_F = 4/3$, $N_C = 3$ and $T_R = n_f/2$ arising from summation of all allowed color states for all the allowed n_f final state flavour degrees of freedom. In

Monte Carlo simulations, flavour and four-momentum conservation is incorporated in each branching.

The probability $d\mathcal{P}$ for a branching $a \rightarrow bc$ to take place for a particular value t of the “evolution parameter” is given by the naive probability of branching to occur at t , multiplied by the probability that a branching has not occurred between the starting value t_{max} and t , ie.

$$d\mathcal{P}_{a \rightarrow bc} = \frac{dt}{t} \frac{\alpha_s(Q^2)}{2\pi} P_{a \rightarrow bc}(z) dz \frac{\Delta_s(t_{max})}{\Delta_s(t)}$$

where Δ_s is known as a Sudakov-type form factor:

$$\Delta_s = \exp \left\{ - \int_{t_{min}}^t \frac{dt'}{t'} \int_{z_{min}}^{z_{max}} dz \frac{\alpha_s(Q'^2)}{2\pi} P_{a \rightarrow bc}(z) \right\}$$

(here q is the four momentum of the partons and $-q^2 = Q^2 < 0$ and Q'^2 is the appropriate Q^2 scale at t').

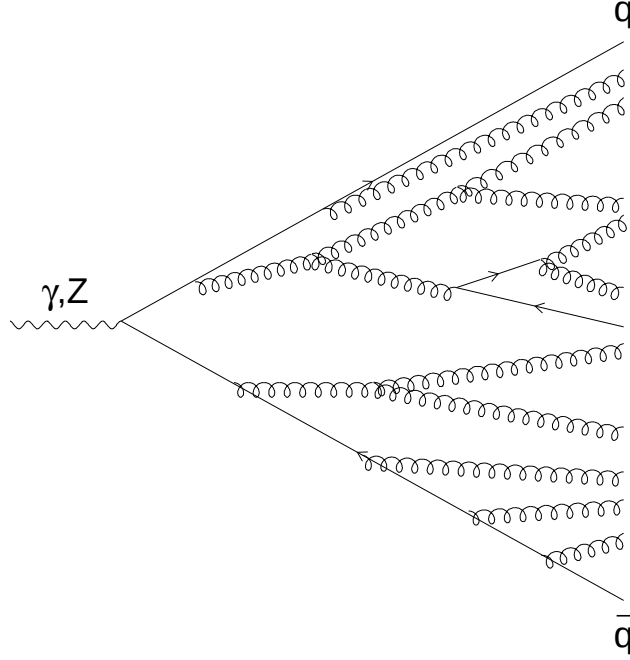


Figure 2.9: A typical parton shower evolution in e^+e^- interactions.

Thus the distribution of the four momenta of the partons during the shower evolution is given by the AP splitting equations, and the distribution of the parton virtualities are given by the Sudakov form factors. In this approach, partons lose their virtuality till they reach a certain cut-off mass Q_0 (a scale roughly of the order of few GeV marking the separation of the perturbative and fragmentation phases) through progressive branchings (see

figure 2.9), where Q_0 is so chosen such that perturbative expansion is still justified, i.e., $\frac{\alpha_s(Q_0^2)}{\pi} < 1$. In LLA, only leading terms in the perturbative expansion are retained, and the leading collinear singularities are resummed [34] to all orders using leading logarithmic terms such as $[\alpha_s(Q^2) \ln(Q^2/Q_0^2)]^n$ for the n^{th} order. In the NLLA, leading correction to LLA are considered, by including three body parton splitting in addition to two body splitting, ie. including $\mathcal{O}(\alpha_s)^2$ terms in the splitting function. However, subleading corrections involving higher order factors of $\ln Q^2$ or $\ln z(\ln(1-z))$ or powers of $\frac{1}{Q^2}$ are not taken uniformly into account.

Incorporation into the Monte Carlo programs

Depending upon the definition of the “evolution parameter” and implementation of the interference between the partons during shower evolution, different parton shower QCD cascade models developed. Some of their characteristic features are listed below:

- (a) In JETSET the evolution parameter is related to the mass squared of the branching parton 1:

$$t = \ln(Q^2/\Lambda^2); \quad Q^2 = m_1^2$$

In terms of successive energy branching fractions (z_i), mass (m_i) and opening angles (θ_i), the angular ordering requirement, for example $\theta_1 > \theta_2$, is imposed as an additional constraint on the successive branchings: $\frac{1-z_1}{1-z_2} < \frac{z_1 z_2 m_1^2}{m_2^2}$. The argument for α_s is taken to be $p_\perp^2 = (1-z)Q^2$, as this is indicated to be the proper choice of scale from loop calculations [35]. This $p_\perp$ corresponds exactly to the transverse momenta in the branching $1 \rightarrow 23$ only in the limit of partons 1 and 3 being massless, $Q^2 = -m_2^2$, and z being the fraction of the light-cone variable shared by parton 2.

- (b) HERWIG uses the following evolution variable:

$$\xi = \frac{q_2 q_3}{E_2 E_3}$$

where the daughter partons 2 and 3 of parent parton 1 have four momenta q_2 , q_3 and q_1 respectively. With θ the opening angle, the virtuality of the parent parton is $q_1^2 = q_2^2 + q_3^2 + 2E_1 E_2 \xi$, and if q_2^2 , q_3^2 are small then we have $\xi \sim (1 - \cos \theta)$. Thus ordering in terms of virtuality automatically guarantees angular ordering. In terms of the energy fraction z , carried by the daughter parton 2 for example, the scale of $\alpha_s(Q^2)$ is again related to the relative transverse momenta squared: $Q^2 = 2z^2(1-z)^2 E_1^2 \xi$. Although the shower formulated in terms of ξ is not manifestly Lorentz invariant, during the cascade the energies and the angular variables are expressed in terms of z and $E_1^2 \xi = \frac{q_2 q_3}{z(z-1)}$, and the final result is Lorentz invariant.

(b) ARIADNE using color dipole model (CDM) incorporates most QCD coherence effects “in a natural way” [36], by treating gluon bremsstrahlung as being radiated from color dipoles between partons instead of being radiated from the individual partons, the emitting dipole being converted into two or more independent dipoles for subsequent emission in the process. More specifically, the gluon emission is considered to arise out of three kinds of color dipoles: $q\bar{q}$, qg (or $\bar{q}g$) or gg dipoles (see figure 2.10), cross-sections of which reproduce the AP splitting kernels in the limit of small transverse momenta ($p_\perp$). In this explicitly Lorentz covariant formalism, angular ordering and non-trivial azimuthal effects are automatically included when the branchings, performed in the rest frame of the dipole, are boosted back to the e^+e^- annihilation center-of-mass frame. The evolution parameter for running of α_s as well as the ordering variable is chosen to be $p_\perp^2$, as CDM can be proven to be a good approximation in the limit of strongly ordered emission in transverse momenta: $p_{\perp 1}^2 \gg p_{\perp 3}^2 \gg p_{\perp 5}^2 \gg \dots$

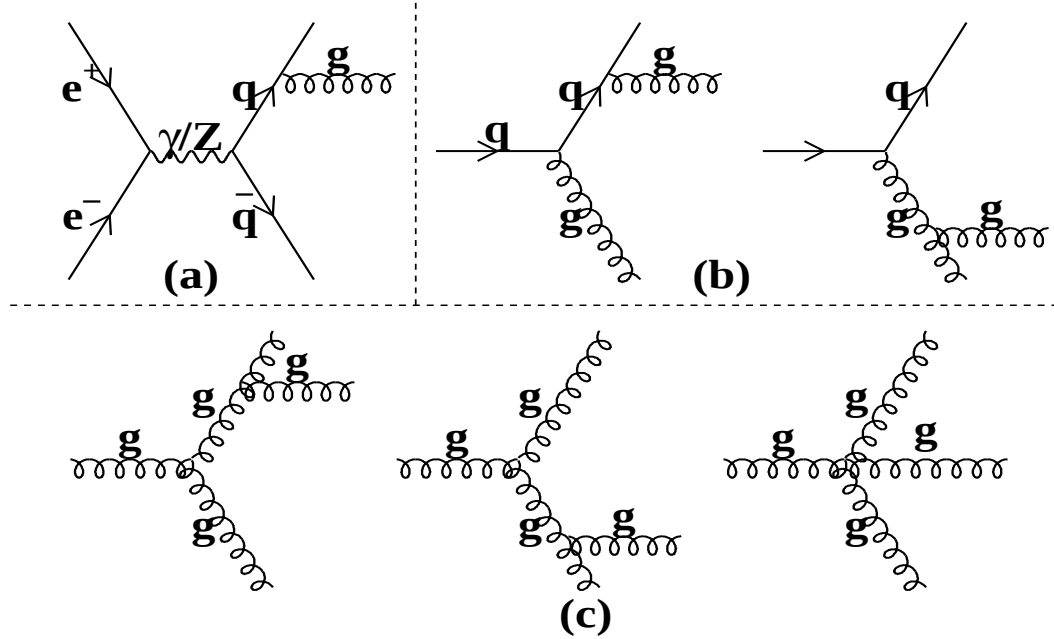


Figure 2.10: Dipoles for gluon emission: (a) $q\bar{q}$ -dipole, (b) qg -dipole and (c) gg -dipoles.

In CDM the transverse angular degree of freedom (the azimuthal angle of the emitted gluon being evenly distributed between 0 and 2π) of the recoiling dipole, the polar angle θ of parton 1, is chosen so as to minimize the color flow in neighbouring dipoles [36]. For $q\bar{q}$ dipole, a well defined prescription exists from spin consideration [37], where one of the quarks retains its direction after emission with

probability proportional to its energy squared. Since there isn't any neighbouring color correlated dipole during the emission from a qg dipole, the gluon always retains its direction. For a gg dipole, CDM postulates that the recoil is given as: $\theta = \frac{x_3^2}{x_1^2 + x_2^2}(\pi - \psi)$, where ψ is the opening angle between partons 1 and 3. (see figure 2.11)

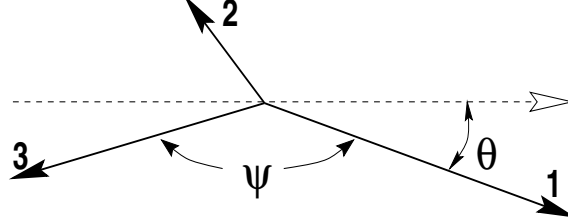


Figure 2.11: Angular orientation of recoiling dipole after emission.

It has been observed that the event description is determined to a large extent by the QCD scale Λ used in the models, which goes by different names in the different models, eg. Λ_{LLA} in JETSET and ARIADNE, and QCDLAM (Λ_{MLLA}) in HERWIG. However, in all these different Monte Carlos the Λ does not correspond to the fundamental QCD scale, but rather is a free parameter most sensitive to the parton shower or cascade development. Another crucial energy scale is Q_0 , where the hadronization phase sets in, and is set to 1 GeV as the L3 default value. In this thesis (see chapter 6), the values of these parameters determined for the different models are presented.

Chapter 3

Non-perturbative Aspects

3.1 Fragmentation processes

The non-abelian nature of QCD manifests itself in two characteristic features of the strong interaction: *asymptotic freedom* and *color confinement*. While the first one says that at small distances, quarks are *asymptotically* “free” and the strong coupling constant lies well within the domain of perturbative calculations, the latter postulates that only color singlet states exist independently in nature. Confinement is understood only qualitatively to arise out of the requirement that the color field between the quarks and gluons are *confined* to distances of the order of a fermi (10^{-15} m), since the strength of the strong interaction (α_s) increases with distance. This non-applicability of perturbation theory in this energy domain, leading to lack of our understanding of the dynamics of parton fragmentation into observable hadrons from first principles, has restricted precision tests of QCD to the level of 10%.

Attempt to understand the dynamic behaviour of quarks in color fields with growing distance scale has lead to the development of many phenomenological models for hadronization, the wide-spread ones being: *independent fragmentation*, *string fragmentation* and *cluster fragmentation*. These models use different types of “grouping blocks” of the primary partons produced in the perturbative QCD phase of e^+e^- annihilation (see part (c) in figure 1.1), succeeded by iterative fragmentations of these building blocks in terms of few underlying branchings as shown in table 3.1.

Independent fragmentation	jet $\rightarrow$ hadron + remainder-jet
String fragmentation	string $\rightarrow$ hadron + remainder-string
Cluster fragmentation	cluster $\rightarrow$ cluster + cluster, or cluster $\rightarrow$ hadron + hadron

Table 3.1: Basic branching processes in different fragmentation models.

Independent fragmentation (IF) model, the first attempt to parametrise the hadronization phase by Feynman and Field [38], dates back to the early seventies. IF, as the name suggests, is based on an incoherent summation of branchings starting from each of the individual primary partons. On the other hand, the string fragmentation (SF), developed by the Lund group [39], models hadronization in terms of a breaking of *strings* of color flux tubes stretched between $q\bar{q}$ pairs with finite string constant. Breaking of strings in the color field leads to the production of hadrons. Use of both these options are available in the JETSET [9] Monte Carlo event generator, while the ARIADNE [10] Monte Carlo uses the SF approach. In cluster fragmentation [40] approach as used in HERWIG [11] Monte Carlo program, the gluons from perturbative phase are all split into $q\bar{q}$ pairs, from which colorless clusters are formed. Depending upon their masses, these clusters are split into more clusters with reduced mass, or directly into hadrons.

It has been observed that parton shower interfaced with IF approach cannot explain the observed depletion of particles in between the $q\bar{q}$ jet region opposite to the gluon [41]. Although only about 0.5% of the particles contribute to this so-called “string effect” arising from QCD coherence phenomena (see section 2.1.3), the use of IF has been since then disfavoured. Both SF (interfaced with parton shower in JETSET) and CF (interfaced with the QCD cascade formulation in HERWIG) are able to reproduce this string effect, and are the popular choice for fragmentation of jets produced in e^+e^- annihilation at LEP energies.

Fragmentation functions

The aim of different fragmentation models has been to parametrise the sharing of the momentum or energy of the primary partons between the hadrons produced during the process.

Assuming that the quark-antiquark pair production arises out of a fluctuation of the QCD vacuum as described by a tunnelling effect from the negative energy continuum to the positive energy continuum solution of a linearly increasing external color potential [42], the transverse momenta distribution with respect to the jet direction is described by a gaussian parameterisation [43]:

$$f(p_T) \sim \exp\left(-\frac{p_T^2}{\sigma_Q^2}\right)$$

In IF and SF, the longitudinal component is studied in terms of the Lorentz invariant branching ratio z defined as:

$$z = \frac{(E + p_{\parallel})_{hadron}}{(E + p_{\parallel})_{quark}}$$

3.1 Fragmentation processes

where E and $p_{\parallel}$ are the energy and momentum along the initial quark direction respectively. Clearly, any boost along the quark direction keeps this z invariant, and so that the evolution of the fragmentation may be studied in terms of recursively defined branching ratios $z_i = \frac{(E+p_{\parallel})_{q_{i+1}}}{(E+p_{\parallel})_{q_i}}$, where q_i precedes or “ranks” higher than q_{i+1} in the iterative fragmentation process.

The inclusive cross-section for producing a primary hadron (h) in an e^+e^- annihilation may thus be expressed in terms of the probability density $D_q^h(z)$ of the initial q or $\bar{q}$ fragmenting into the hadron h produced anywhere in the jet with a fraction z of the light-cone variable of the initial quark or anti-quark as:

$$\frac{d\sigma}{dz}(e^+e^- \rightarrow hX) = \sum_q \sigma(e^+e^- \rightarrow q\bar{q}) [D_q^h(z) + D_{\bar{q}}^h(z)]$$

In terms of the probability density $f(z)$ of producing a hadron carrying a fraction z of the $(E + p_{\parallel})$ of initial quark formed first in the cascade, the probability density $D(z)$ of finding the primary hadron any time later with branching ratio between z and $z + dz$ may be calculated by assuming that the distributions scales with respect to this Lorentz invariant branching ratio. Thus, if h is a first rank hadron it has an associated probability $f(z)dz$, otherwise since the first rank hadron must have taken away a fraction $(1 - \eta)$ with probability $f(1 - \eta)d\eta$ (where η lies between z and 1 so that a higher rank hadron produced later in the chain has a fraction z of the initial quark), the probability of finding the hadron h with branching fraction between z and $z + dz$ from the remaining branching fraction η is $D(z/\eta)d(z/\eta)$. Thus, we have:

$$D(z)dz = f(z)dz + \int_z^1 f(1 - \eta)d\eta D\left(\frac{z}{\eta}\right)d\left(\frac{z}{\eta}\right)$$

Different parametric forms have phenomenologically evolved for $f(z)$, and form characteristic features of different models. In the IF, demanding that $D(z)$ approaches a constant in the limit $z \rightarrow 1$, Feynman and Field obtained the following parametrisation of $f(z)$:

$$f(z) = 1 - a + 3a(1 - z)^2.$$

In the SF, the Lund group obtained the following form for $f(z)$, called the Lund *left-right* symmetric function, by assuming that the branching of momenta for hadrons to be equivalent if treated from either the quark end or the antiquark end:

$$f(z) = \frac{(1 - z)^a}{z} \exp \{-bm_T^2/z\}.$$

It has been noted that the heavier quarks have much “harder” fragmentation function than lighter ones [44]. The following argument due to Peterson, et al. [45] based on

3.2 String Fragmentation model

kinematic considerations leads to mass dependent fragmentation function which is widely used. Upto first order in perturbation theory, the transition amplitude of a heavy quark Q with momentum p into a hadron H with momentum zp and a light quark q with momentum $(1-z)p$ is $\frac{\langle Hq|H'|Q\rangle}{\Delta E}$, where $\Delta E = E_Q - E_H - E_q$ and H' is the perturbing Hamiltonian. The fragmentation function obtained from the square of the amplitude is given by: $f(z) \sim \frac{1}{z(\Delta E)^2}$, where the $1/z$ factor comes from longitudinal phase space contribution. Taking $\frac{M_Q^2}{p^2}$ to be small, we have

$$\begin{aligned} E_Q &= \sqrt{M_Q^2 + p^2} \approx p + \frac{M_Q^2}{2p}, \\ E_H &= \sqrt{M_H^2 + z^2 p^2} \approx zp + \frac{M_Q^2}{2zp} \text{ and} \\ E_q &= \sqrt{M_Q^2 + (1-z)^2 p^2} \approx (1-z)p + \frac{M_Q^2}{2(1-z)p}. \end{aligned}$$

Denoting $\frac{M_Q^2}{M_Q^2}$ by ϵ_Q , thus we arrive at the Peterson fragmentation function:

$$f(z) \sim \frac{1}{z[1 - 1/z - \epsilon_Q/(1-z)]^2}$$

The leading $1/z$ behaviour that $D(z)$ inherits from these different fragmentation functions is very characteristic of the multiplicity behaviour of the observed hadrons. If $f(z)$ is normalized such that $\int dz f(z) = 1$, then $\int_{z_{min}}^1 dz D(z)$ denotes the multiplicity of the hadrons produced in the fragmentation process, and has a leading order logarithmic dependence on the center of mass energy as can be expected by integrating the $1/z$ term.

3.2 String Fragmentation model

The string fragmentation (SF) is based upon the idea that the linear confining potential property of color flux field may be illustrated by the dynamics of a one-dimensional massless relativistic string, with the string constant ($\kappa \sim 1 \text{ GeV}/fm$) [42] denoting the amount of energy stored per unit length. Actually the terminology “massless” string is somewhat of a misnomer, in the sense that κ effectively corresponds to a “mass density” along the string. String models, originally studied by Artu and Mennesier [46], have been applied as a Monte Carlo program by the Lund group [9] in JETSET by extending the one-dimensional string ideas to include discrete particle masses and flavours.

In this model, as two primary partons move apart a *string* of color flux tube stretches between them, and after a certain “breaking” distance, $q\bar{q}$ pairs *tunnel* out the color field leading to the formation of new color-singlet strings. The tunnelling probability, which

factors into the mass and transverse momenta terms [43], leads to a flavour-independent gaussian spectra for p_T of the $q\bar{q}$ pairs. This p_T is locally compensated between the quark-antiquark pair, since the string is assumed to have no transverse degrees of freedom. The final hadron produced from these pairs receives contribution from the p_T of its individual components. However, there are additional contributions to transverse momenta from hard scattering associated with gluon radiation, which results in somewhat higher $\langle p_T \rangle$ than what is expected from the tunnelling effect.

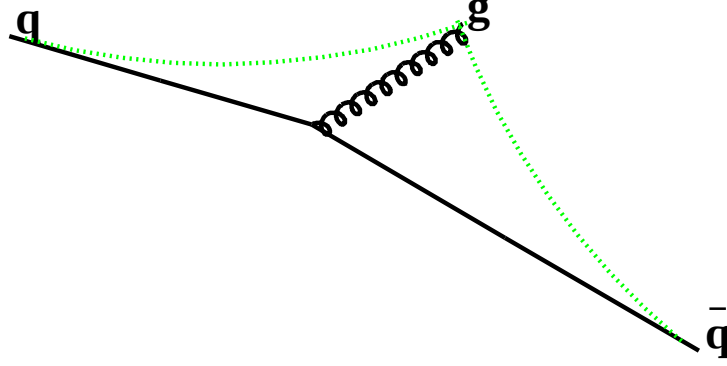


Figure 3.1: In a $q\bar{q}g$ system, the gluon is represented by a kink on the string pulled between the $q\bar{q}$ pair.

The Lund model, as SF is sometimes synonymously referred to, involves detailed string kinematics for multi-parton system subject to the constraint that the soft gluon exchange between partons will perturb least the original color assignment. An arbitrary number of gluons or closed gluon loops are treated in this model, with the color flow of the multi-gluon system being properly connected to each other. For example, in a $q\bar{q}g$ event the gluon is visualised to be a *kink* carrying momentum and energy on the string stretched between the quark-antiquark ends (see figure 3.1). In LLA where only two body branchings are considered, the color flow are given by the following rules:

- In a $q \rightarrow q/g$ branching, the gluon carries the original color of the quark, and a new color-anticolor pair is shared between the quark and the gluon.
- In a $g \rightarrow gg$ branching, the original color is ascribed to one gluon and the anti-color to the other, and again a new color-anticolor pair is shared between the gluons.
- In a $g \rightarrow q\bar{q}$ branching, the original string going through the gluon is split into two, the original color going to the quark and the anti-color going to the anti-quark.

Since the gluon has two string pieces attached to it, the ratio of the gluon to quark string force is two - as compared to the ratio of the color charge Casimir invariants:

$N_C/C_F = 2/(1 - 1/N_C^2) = 9/4$. This factor of two is independent of the kinematical configuration, because associated with a smaller opening angle between the two partons is not only a smaller string segment drawn out per unit time but also an increased transverse velocity to exactly compensate the boost factor in energy density per unit string length. In case of well-separated high energy gluon emission, the gluonic kink evolves into a third string like segment stretched from the two strings associated with the $q\bar{q}$ ends carrying the same momentum and energy as the gluon. If the gluon emitted is soft or collinear, then the two initial string segments collapse to a naught thus yielding an *infrared-safe* description of the fragmentation process.

String breaking continues until only on-mass-shell hadrons remain, with each hadron corresponding to a small piece of string. The flavour dependence of the $q\bar{q}$ pair production at each step in the fragmentation is given by the exponential dependence of the tunnelling probability [43] on the effective transverse mass squared for each flavour. Thus we expect a suppression of heavy quark production in soft fragmentation:

$$u : d : s : c \approx 1 : 1 : \gamma_s : 10^{-11}$$

with the s-quark suppression factor $\gamma_s = 0.3$ describing well the production rates of kaons, but is left as free parameter in the programs for varying flavour contents of different strange mesons eventually to be matched with the data.

The composition of pseudo-scalar (P) and vectors (V) spin states of mesons that can be formed from the quark and anti-quark from two adjacent strings breaking is naively expected to be in the ratio of 1:3 from spin-state counting. However, the wavefunction normalisation factor depending upon the flavour of the pair production at each step of fragmentation leaves scope for many free parameters like $V/(P + V)$, relative ratios of different angular momentum states corresponding to different flavour composition of the produced mesons in the model. Additional free parameters controlling further suppression factors for particular spin composition of different flavours, for example η or η' suppression factors, are necessary to reproduce a realistic description of the hadronization phase. Most of these parameters are eventually determined from the available experimental data for the corresponding processes.

In addition to meson production, baryons are also produced during fragmentation by considering diquark pair production (diquark model) [47] or successive quark pair productions in the transient color field of previous pair production (popcorn model) [42, 48]. Similar to the pure meson production scenario, the baryon production mechanism invokes a host of free parameters like the relative abundance of the spin states of the baryons, relative composition of octet-decuplet flavour states of the baryons, relative probability of separating the baryon-antibaryon pairs in rank by mesons, the strange

flavour content of the intermediate meson productions, etc. default values of which are all obtained from internal consistency of the model and experimental observations.

Default parameter settings

Even though the same string fragmentation is used in hadronization phase, depending upon the options chosen for modelling the perturbative phase, the default values may be chosen to be different in order to best match between the models and the data. Choice of the default values for the numerous parameters in the models has been a long process, and has been subject to change by the nature of experimental process to which the models are applied.

The default parameters for the models are usually chosen to be the default Monte Carlo settings. In L3, the SF is done in the hybrid scheme [MSTJ(11)=3] where the light quark (u,d,s) flavour generation is done using the Lund symmetric function and the heavy quarks (c and b) are generated using Peterson's function. The transverse momenta spectra is parametrised in terms of a gaussian with width σ_Q [PARJ(21)]. However, specific studies in L3 [49] has lead different choices of parameter settings than that in JETSET. For example, the JETSET default value ($= 0.3$) for parameter a [PARJ(41)] (in Lund symmetric function) is changed to a value of 0.5 in L3 default settings. The L3 default probabilities for formation of different spin states as compared to their default in JETSET are:

- probability of spin 0 meson production with $L = 1, J = 1$ [PARJ(14)] is changed to 0.1 from the default value 0;
- probability of spin 1 meson production with $L = 1, J = 0$ [PARJ(15)] is changed to 0.1 from the default value 0;
- probability of spin 1 meson production with $L = 1, J = 1$ [PARJ(16)] is changed to 0.1 from the default value 0;
- probability of spin 1 meson production with $L = 1, J = 2$ [PARJ(17)] is changed to 0.15 from the default value 0.

Parameter	ALEPH	DELPHI	L3 [49]	L3 [50]	OPAL
ϵ_c	-0.050	-0.038	-0.070	-0.030	-0.031
ϵ_b	-0.0045	-0.00284	-0.008	-0.0035	-0.0038

Table 3.2: ϵ_Q values for the JETSET 7.4 Parton Shower Program.

Parameter	L3 [49]	L3 [50]
ϵ_c	-0.180	-0.10
ϵ_b	-0.010	-0.004

Table 3.3: ϵ_Q values for the JETSET 7.4 Matrix Element Program.

Parameter	ALEPH	DELPHI	L3	OPAL
ϵ_c	-0.050	-0.0378	-0.030	-0.050
ϵ_b	-0.0060	-0.00255	-0.0035	-0.0050

Table 3.4: ϵ_Q values for the ARIADNE 4.06 Program.

Also, the suppression factors for η [PARJ(25)] and η' [PARJ(26)] have been changed to 0.6 and 0.3 respectively from their default values of 1 and 0.4 in JETSET. The mass [PARJ(123)] and width [PARJ(124)] of Z boson in Jetset are also changed from 91.187 GeV and 2.489 GeV to 91.185 GeV and 2.495 GeV respectively [51].

The different experiments at LEP running at same energies have their own set of parameters [52] which are also different. For example, the ϵ_Q values in the Peterson fragmentation function for c [PARJ(54)] and b [PARJ(55)] quarks as used by the different option in JETSET and ARIADNE (interfaced to the SF part of JETSET in the hadronization phase) are listed in tables 3.2, 3.3 and 3.4.

3.3 Cluster Fragmentation model

The cluster model, originally studied by Artu and Menessier [46], was first developed into dedicated programs in the CALTECH-II Monte Carlo [53], as pre-hadronic decay products in the fragmentation phase succeeding parton shower cascades. At LEP energies, the cluster fragmentation (CF) as developed in the QCD cascade program HERWIG by Marchesini and Weber [11], have been widely studied in parallel with the Lund string fragmentation model.

In this scheme (see figure 3.2), the parton showering continues upto a mass scale of few GeV, the typical fragmentation scale, till when the gluons at the end of the cascade are all forcibly split into quarks to form colorless clusters. These clusters which can be thought of as a superposition of short-lived resonances, then undergo isotropic two body decays into hadrons or more clusters depending upon the mass of the fragmenting cluster.

A *cluster*, unlike the *string*, does not possess any internal structure, and is characterised by its total mass, total spin and the total flavour content. The flavour evolution in CF is developed by forming new quark-antiquark or diquark-antidiquark pairs, with the

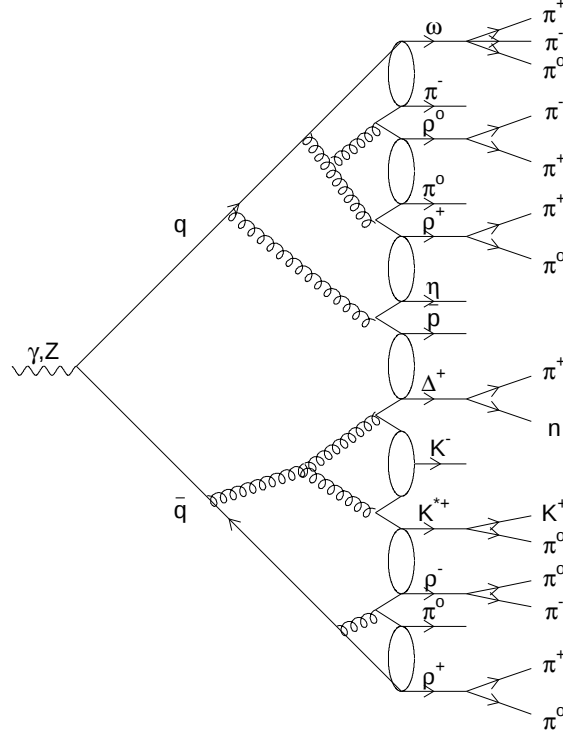


Figure 3.2: An illustrative fragmentation scheme in the cluster model.

“effective” mass assignments to the gluons, quarks and diquarks and the relative probability strengths are chosen to match the “observable” abundance of mesons and baryon production rate. Each allowed cluster decay channel is assigned a weight proportional to the density of states: $(2s_1 + 1)(2s_2 + 1)p^*/m$, where s_i ($i = 1, 2$) is the spin of the decay daughter cluster/hadron and p^* is the common momentum of the products in the rest frame of the decaying cluster with mass m . The probability of which flavour production is retained depends on this weight, a new flavour being considered in the event the former choice is rejected. Sufficiently light clusters are usually assumed to collapse into a single particle by four-momenta shuffling from nearby clusters (in accordance with total energy-momentum conservation), so that the probability of single particle production with large fraction of the total jet energy is not “severely underestimated”.

The number of free parameters in this model is much less as compared to the Lund model. The suppression of strange particles and baryons as well as the transverse momenta is determined by the average energy released and the mass of the decaying particle in the cluster decay and in resonance decays, in contrast to extra free parameters in SF. The crucial parameters to this model are the maximum cluster mass allowed (CLMAX) and the power (CLPOW) which, for example, determine whether a cluster of mass MCL made

3.4 Decay of unstable particles

of quarks of masses M_1, M_2 will split into lighters clusters before decaying if

$$MCL^{CLPOW} > CLMAX^{CLPOW} + (M_1 + M_2)^{CLPOW}$$

Smaller values of CLPOW tend to increase the yield of heavier cluster for heavier quarks without affecting the lighter quarks much, and hence increase the relative abundance of heavy flavour baryons. An additional parameter B1LIM (L3 default value = 0.35) governs the b-quark hadronization, with the probability of a b-cluster to decay into a single b-hadron given by

$$\mathcal{P} = \begin{cases} 1 & \text{if } MCL < MTH \\ 1 - \frac{(MCL-MTH)}{(B1LIM*MTH)} & \text{if } MTH \leq MCL \leq (1+B1LIM)*MTH \\ 0 & \text{if } MCL > (1+B1LIM)*MTH \end{cases}$$

where MTH is the threshold for the cluster to decay into 2 hadrons. Thus a non-zero B1LIM value gives a harder b-quark spectra.

Effective assignment of spins and cluster mass spectrum gives the spin and flavour content of the hadrons produced in this mass-dependent weight parametrization from phase-space consideration of cluster decays. The effective gluon mass (set to 0.75 GeV as L3 default) serves as the parton shower termination parameter. Inclusion of mass mixing (e.g., η_8/η_0 mixing angle ETAMIX with default value equal to -20°) produce relative abundance of the different flavour and spin dependence of observed hadrons. The individual rates are not very sensitive to their exact values, eg. the η - η' suppression is more dominated by the mass effects in CF.

3.4 Decay of unstable particles

As in any realistic model building, the characteristic decay length of the particle produced during fragmentation play an important role in describing the observed spectra in hadronic decays from e^+e^- annihilation (see part (d) in figure 1.1). A sizeable fraction of the hadrons produced in any of the above described fragmentation phase are in fact unstable, and eventually decay into stable observable hadrons. Thus most of the above mentioned models have a scheme for decaying the particles produced during the hadronization phase based upon the masses, life times and decay width of the particles obtained from numerous low energy experiments. For light hadrons, the branching fractions for their characteristic decay modes are well known, but for the heavier ones, for which not all exclusive branching fractions have been measured, statistical models are invoked.

The L3 standard default for *stable* particles are those which pass a certain cut-off on their decay length (e.g., in JETSET PARJ(71) is taken to 10 cm). Thus particles like

3.4 Decay of unstable particles

K_s^0 and Λ with proper lifetimes 2.675 cm and 7.89 cm respectively are considered to be unstable. This gives a mean charge multiplicity (see chapter 4) of 20.79 ± 0.52 [49] at the Z-peak. A typical list of some of the *stable* particles produced along with their relative abundance in an hadronic event at the Z-resonance are listed in table 3.5 (ref. Hebbeker in [1]).

Neutral		Charged	
particle	#/event	particle	#/event
γ	21.5	$\pi^\pm$	17.1
K_L^0	1.1	$K^\pm$	2.2
$n, \bar{n}$	1.1	$p, \bar{p}$	1.2
$\nu, \bar{\nu}$	0.3	$e^\pm, \mu^\pm$	0.4, 0.1

Table 3.5: Relative abundance of some of the *stable* particles produced in an typical hadronic event at the Z-peak.

Chapter 4

Observables

4.1 Event shape variables

The performance of different models can be studied by comparing, between data and Monte Carlo, the global structures of the hadronic events, which are not only sensitive to perturbative QCD, but also to the fragmentation models. These global *event shape* variables may be used to distinguish between the different types of the hadronic events that are frequently encountered in an e^+e^- annihilation experiment.

Analytical calculations based on second-order perturbative QCD predictions are available for many of these variables, and convolution of these perturbative components with the non-perturbative part (obtained from different hadronization models) has provided us with one of the most reliable method for determination of the strong coupling constant, α_s , from the $e^+e^- \rightarrow \text{hadrons}$ data available over the energy range $\sim 5 - 200$ GeV. The use of different renormalization schemes for the perturbative part and different combination schemes of the perturbative and non-perturbative parts (to take care of the common terms) presents a way for estimation of the theoretical errors, which are comparable with the associated experimental errors obtained from the measurement of the different event shape variables [54]. *Infrared safe* (soft and collinear) event shapes are particularly preferred because they are finite in perturbative QCD calculations, order by order. Furthermore, addition of soft particles or splitting of a particle into two daughter particles (obeying four momenta conservation) changes the measurement of such variables in a continuous way, thus allowing *calorimetric* measurements of these variables.

In this section the definition of these global event shape observables are presented.

Thrust, Major, Minor, Oblateness and Minor of Narrow Side

Thrust (T) [55] is an infrared-safe measure of the maximum energy flow in an event. It is defined to be a linear function of momenta by:

$$T = \max_a \frac{\sum_a |\vec{p}_a \cdot \hat{n}_T|}{\sum_a |\vec{p}_a|}$$

where the summation is taken over all the final state particles a with momentum vector $\vec{p}_a$. The thrust axis, $\hat{n}_T$, is taken to be the unit vector which maximises the above expression.

For massless two jet topology, the value of T is unity and the thrust axis coincides with the jet direction. However, finite mass effects and non-zero transverse momenta of the two jets, lead to a value smaller than one. For example, at LEP I energies ($s =$ square of center of mass energy $\sim 10000 \text{ GeV}^2$) for a two jet configuration with jet invariant mass squared $M^2 \sim 100 \text{ GeV}^2$, in terms of scaled momenta $x_i = 2|\vec{p}_i|/\sqrt{s}$, $\{i = 1, 2\}$, the thrust may be calculated to be: $T = x_1 = x_2 = 2 \cdot \sqrt{(\frac{\sqrt{s}}{2})^2 - M^2} \cdot \frac{1}{\sqrt{s}} \sim 1 - \frac{2M^2}{s} = 0.98$.

For three and four jet events, T lies between $[\frac{2}{3}, 1]$, and $[\frac{1}{\sqrt{3}}, 1]$ respectively, and for an isotropic distribution, $T = \langle \cos \theta_a \rangle = \frac{1}{2}$ (where θ_a is the angle between the thrust axis $\hat{n}_T$ and the momenta vector $\vec{p}_a$).

Major (T_{major}), another linear function of momenta, is defined by maximising the expression

$$T_{\text{major}} = \max_a \frac{\sum_a |\vec{p}_a \cdot \hat{n}_{\text{major}}|}{\sum_a |\vec{p}_a|}$$

in a plane perpendicular to the thrust axis. The resulting direction is known as the major axis, $\hat{n}_{\text{major}}$. An orthogonal system is constructed to describe the spatial distribution of the energy flow by defining the minor axis as

$$\hat{n}_{\text{minor}} = \hat{n}_T \times \hat{n}_{\text{major}}$$

A thrust like quantity describing the energy flow in this direction is called *minor* (T_{minor}). *Oblateness* (O) [56] is the difference of the major and the minor values,

$$O = T_{\text{major}} - T_{\text{minor}}$$

For narrow *two jet* events, *isotropic* events and *planar* events the variables T_{major} , O and T_{minor} respectively vanish.

4.1 Event shape variables

After dividing an event into two hemispheres by a plane perpendicular to the thrust axis, the transverse momentum fraction

$$f_T = \frac{\sum_a |\vec{p}_a \times \hat{n}_T|}{\sum_a |\vec{p}_a|}$$

is calculated for each hemisphere. The hemisphere with the smaller f_T is called the narrow side and the minor derived from the particles in this hemisphere is called the *minor of the narrow side*, ($T_{\text{minor}}^{\text{NS}}$) [57]. Thus this variable gives us information about the transverse momenta characteristics of the event. For *planar* events, this variable also vanishes.

Jet Masses

By dividing the event by a plane normal to the thrust axis ($\hat{n}_T$), the invariant masses ($M_{\pm}$) in the two hemispheres ($S_{\pm}$) are calculated:

$$M_{\pm}^2 = \left(\sum_{a \in S_{\pm}} p_a \right)^2,$$

where p_a is the four momentum of particle a .

The *heavy jet mass* (M_H) and the *light jet mass* (M_L) defined [58] to be:

$$M_H = \max[M_+(\hat{n}_T), M_-(\hat{n}_T)] \quad M_L = \min[M_+(\hat{n}_T), M_-(\hat{n}_T)]$$

are calculably finite in perturbation theory.

The scaled heavy jet mass (ρ_H) and the scaled light jet mass (ρ_L) are defined as

$$\rho_H = M_H^2/s \quad \rho_L = M_L^2/s$$

where $\sqrt{s}$ is the center of mass energy.

The ρ_H variable vanishes for narrow *two jet* configuration, and its range $[0, \frac{1}{3}]$ is independent [59] of the number of particles in the final state. To order α_s , the variable ρ_H is related to thrust as: $\rho_H = 1 - T$, and the variable ρ_L vanishes. The ρ_L variable predominantly receives contributions from events with four or more final states.

Fox Wolfram Moments

The l^{th} order ($l = 0, 1, 2 \dots$) *Fox-Wolfram moment* [60] is defined as

$$H_l = \sum_{a,b} \frac{|\vec{p}_a| |\vec{p}_b|}{s} P_l(\cos \theta_{ab})$$

where the summation is over all particles in the final state, p_a and p_b are the momenta of the particles a and b respectively, θ_{ab} is the angle between them, and P_l are the Legendre polynomials of order l :

$$\begin{aligned} P_0(x) &= 1, \\ P_1(x) &= x, \\ P_2(x) &= \frac{1}{2}(3x^2 - 1), \\ P_3(x) &= \frac{1}{2}(5x^3 - 3x), \\ P_4(x) &= \frac{1}{8}(35x^4 - 30x^2 + 2). \end{aligned}$$

These moments do not distinguish between the events differing by emission of soft and/or collinear particles, and thus are free of divergence in perturbation theory. The H_l 's form a complete set of rotationally invariant shape parameters, out of which the second (H_2), third (H_3) and fourth (H_4) Fox-Wolfram moments are the most commonly used ones. Energy-momentum conservation gives $H_0 \sim 1$ (to the extent that the particle masses may be neglected) and $H_1 = 0$. For *two jet* topology, $H_l \sim 1$ for even l and $H_l \sim 0$ for odd l .

3-jet resolution parameters

Jets are reconstructed using the JADE [61] or the DURHAM ($k_\perp$) [62] clustering options. In the JADE clustering algorithm, the separation (y_{ab}) between a pair of particles is measured using invariant mass of a pair of particles (ignoring particles masses):

$$y_{ab}^{\text{JADE}} = \frac{2E_a E_b}{\sqrt{s}}(1 - \cos\theta_{ab})$$

where E_a and E_b are the energies of the particles, θ_{ab} is the angle between them and $\sqrt{s}$ is the center of mass energy. In the DURHAM clustering algorithm jets are reconstructed using scaled transverse momenta between a pair of particles as the measure of separation between the particles (y_{ab}):

$$y_{ab}^{\text{DURHAM}} = 2 \frac{\min(E_a^2, E_b^2)}{\sqrt{s}}(1 - \cos\theta_{ab}).$$

The pair with the smallest value of the jet resolution variable is replaced by a pseudo-particle c with 4-momentum:

$$p_c = p_a + p_b.$$

This procedure is repeated until the resolution measure y_{ab}^{JADE} or y_{ab}^{DURHAM} for all the final state particles exceed a predefined jet resolution parameter y_{cut} . The remaining pseudo-particles at the end of this recombination procedure are called jets in the clustering algorithms. The 3-jet resolution parameter y_{23}^{JADE} (y_{23}^{DURHAM}) for JADE (DURHAM) algorithm is defined as that value of the maximum jet resolution parameter y_{cut} for which the event still has 3-jet structure.

Sphericity, Aplanarity and Planarity

For each event, one can define a bi-linear momentum tensor T_{ij} in its center-of-mass frame, in analogy with the moment of inertia of classical mechanics:

$$T_{ij} = \sum_a (\delta_{ij} p_a^2 - p_{ia} p_{ja})$$

where i and j run over three space dimensions and p_a is the momentum of particle a , summed over final state particles a in the event. The real symmetric matrix T_{ij} can be diagonalized by a principal axis transformation, and the eigenvalues ($\bar{\lambda}_i$) may be arranged as:

$$\bar{\lambda}_1 \geq \bar{\lambda}_2 \geq \bar{\lambda}_3; \quad \sum_{i=1}^3 \bar{\lambda}_i = \text{Trace}(T) = 2 \sum_a |\vec{p}_a|^2$$

Sphericity (S) is then defined in terms of this momentum ellipsoid as [63]:

$$S = \frac{3\bar{\lambda}_3}{\bar{\lambda}_1 + \bar{\lambda}_2 + \bar{\lambda}_3} = \frac{3}{2} \frac{(\sum_a |\vec{p}_{a\perp}|^2)}{(\sum_a |\vec{p}_a|^2)}_{\text{min}}$$

Similar to the case of rotating a cigar-shaped rigid body (along the cigar axis), where the momentum ellipsoid has a pan-cake shape, the sphericity is small for such a bounded transverse momenta description of 2 jet events. For an *isotropic* multi-particle production, the value of sphericity is close to unity. More detailed information about an event can be extracted by studying the relative values of the eigenvalues, which obey the triangle inequality: $\bar{\lambda}_i \leq \bar{\lambda}_j + \bar{\lambda}_3$ ($i \neq j \neq k$).

A more convenient approach for study is to use the the bi-linear momentum tensor s_{ij} defined as:

$$s_{ij} = \frac{\sum_a p_a^i p_a^j}{\sum_a p_a^2}$$

4.1 Event shape variables

which is normalized to have unit trace. This real symmetric matrix s_{ij} can be diagonalized by a related principal axis transformation, and the eigenvalues (Q_i) may be arranged as:

$$0 \leq Q_1 \leq Q_2 \leq Q_3 \quad Q_1 + Q_2 + Q_3 = 1$$

which are related to the eigenvalues ($\bar{\lambda}_i$) of T_{ij} by:

$$Q_i = \frac{\lambda_i}{\sum_i \lambda_i} \quad \text{where} \quad \lambda_i = \frac{1}{2} \sum_i \bar{\lambda}_i - \bar{\lambda}_i.$$

In terms of the eigenvectors $\hat{n}_i$ ($i = 1, 2, 3$), the corresponding eigenvalues tells us about the geometry of the momentum ellipsoid:

$$\begin{aligned} Q_1 &= \text{Min}_{\hat{n}} \frac{\sum_a (\vec{p}_a \cdot \hat{n})^2}{\sum_a |\vec{p}_a|^2} \Rightarrow \text{flatness of the event } (\hat{n} \equiv \hat{n}_1) \\ Q_2 &= \text{Min}_{\hat{n} \perp \hat{n}_1} \frac{\sum_a (\vec{p}_a \cdot \hat{n})^2}{\sum_a |\vec{p}_a|^2} \Rightarrow \text{width of the event } (\hat{n} \equiv \hat{n}_2) \\ Q_3 &= \text{Max}_{\hat{n}} \frac{\sum_a (\vec{p}_a \cdot \hat{n})^2}{\sum_a |\vec{p}_a|^2} \Rightarrow \text{length of the event } (\hat{n} \equiv \hat{n}_3) \end{aligned}$$

Sphericity (S), *aplanarity* (A) and *planarity* (P) are defined [63] in terms of the eigenvalues of this s_{ij} matrix as:

$$S = \frac{3}{2}(Q_1 + Q_2) \quad A = \frac{3}{2}Q_1 \quad P = \frac{2}{3}(S - 2A)$$

The sphericity axis is parallel to the eigenvector ($\hat{n}_3$) corresponding to largest eigenvalue (Q_3), and the sphericity variable lies in the range: $0 \leq S \leq 1$. Both aplanarity and planarity lies in the range: $0 \leq A(P) \leq \frac{1}{2}$. The variable A is small for *coplanar* events, and receives contribution mostly from events with four or more events. A triangular plot (with S as the abscissa and $\frac{\sqrt{3}}{2}(Q_2 - Q_1)$ as the ordinate) can be used to separate the two-jet, three-jet and non-planar events (see figure 4.1). *Collinear* and *coplanar* events are characterised by $Q_3 \gg Q_2$ and $Q_2 \gg Q_1$ respectively on the eigenvalue plane.

Sphericity tensor and C, D parameters

C and D parameters [64] are defined from the eigenvalues of the linearized momentum tensor (*sphericity tensor*):

$$\theta^{ij} = \frac{\sum_a p_a^i p_a^j / |p_a|}{\sum_a |p_a|}$$

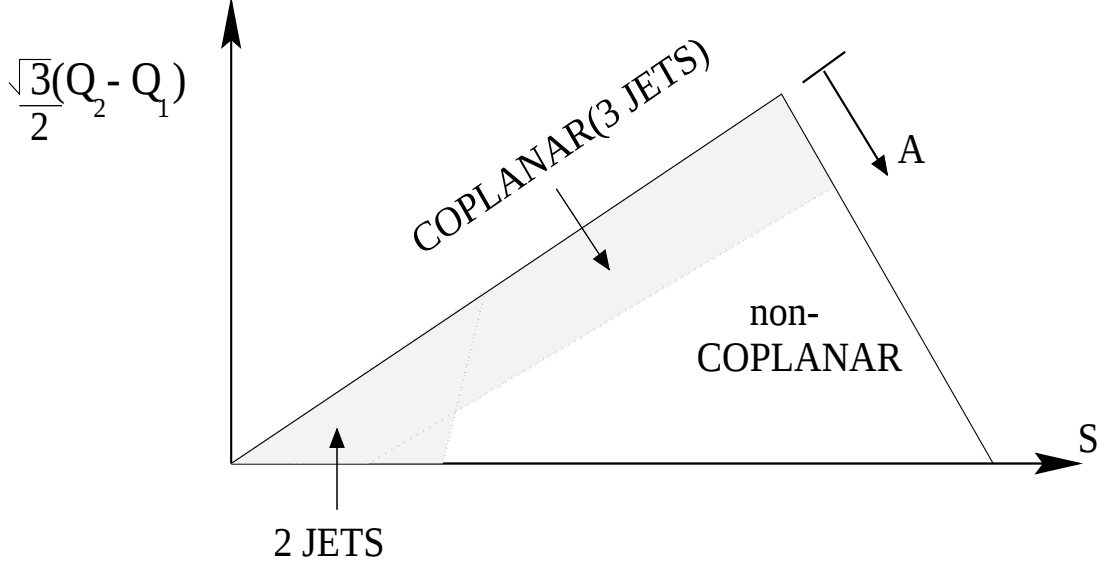


Figure 4.1: Sphericity-Aplanarity plot showing two jet, three jet and noncoplanar events.

where the sum on a runs over all the final state hadrons, and p_a^i is the i^{th} component of the three momentum of the hadron a in the center of mass system. The sphericity tensor θ is normalized to have unit trace, and being real symmetric may also be diagonalized. In terms of its eigenvalues $\lambda_{1,2,3}$, the C and D parameters are defined as

$$\begin{aligned} C &= 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1) \\ D &= 27\lambda_1\lambda_2\lambda_3 \end{aligned}$$

Note that if P is any point within an equilateral triangle ABC (of side l), then sum of the areas of the three triangles ($\triangle PBC + \triangle PCA + \triangle PAB = \frac{1}{2} \cdot (\sum_{i=1,3} \lambda_i) \cdot l$) equals that of the $\triangle ABC (= \frac{\sqrt{3}}{4} \cdot l^2)$, where λ_i is the perpendicular distance of the point P from each of the three bases of the triangle ABC. This geometric fact allows us to visualize the plots of C and D parameter as functions of the eigenvalues $\lambda_{1,2,3}$ of the unit trace sphericity tensor as contours inside an equilateral triangle of length $l = \frac{2}{\sqrt{3}}(\sum_{i=1,3} \lambda_i) = \frac{2}{\sqrt{3}}$ (see figure 4.2). For $\lambda_1 > \lambda_2 > \lambda_3$, only the top right-hand triangle in the top figure is populated.

For *two jet* final state events both C and D parameters vanish, while for *three jet* final state events with planar topology C ranges between 0 and 3/4 and $D = 0$. For large number of particles in the final state, both C and D are close to unity. Thus C provides a measure of the multi-jet structure of an event with special emphasis on planar events, while D measures the deviation from planarity of events by receiving major contribution from events with four or more jets.

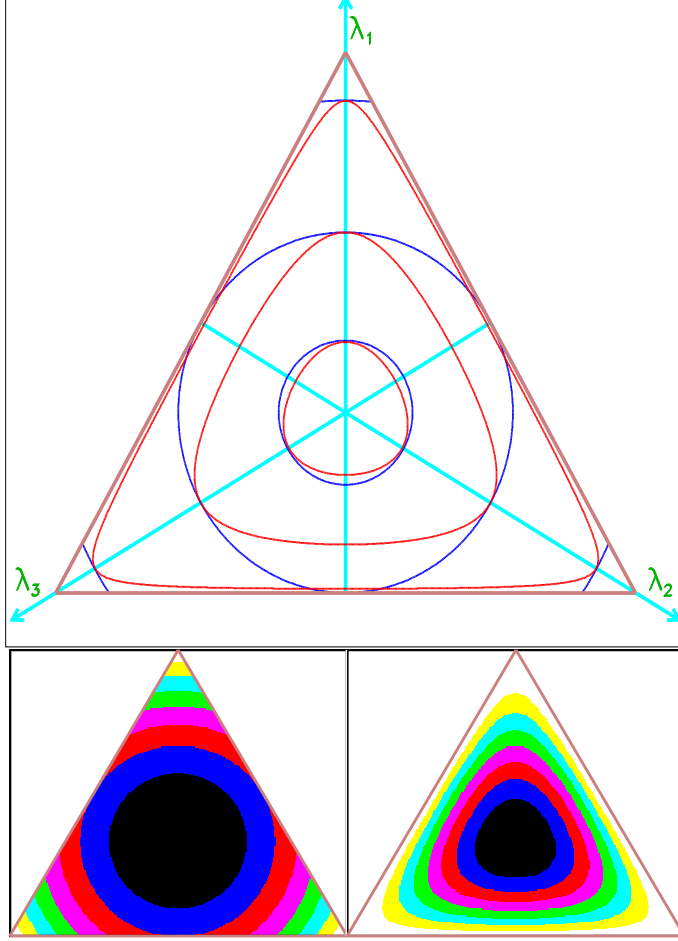


Figure 4.2: In λ_1 - λ_2 - λ_3 co-ordinate system, lines of constant C and D parameters (top figure) and contour plots of C parameter (bottom left) and D parameter (bottom right) are shown. For C values = 0.25, 0.75, 0.96 and D values = 0.05, 0.5, 0.9 respectively, the lines of constant C touch those of constant D on outside.

The quadratic momenta dependence (e.g., $\sum |p_i|^2$) of the numerator in both the sphericity and sphericity tensor has a different value for two massless collinear particles from the value it attains for one particle with the sum of the two momenta. Since the squares of momenta are not additive, in the quadratic sphericity tensor infrared singularity cancellation does not occur, and as a result it is not finite in perturbation theory calculations. Although the sphericity tensor is not linear for general momenta, it is linear when the non-soft particles are moving parallel to each other, where infrared problems might arise. Thus, the C and D parameters are infrared safe.

By expanding the sphericity tensor θ^{ij} in terms of spherical tensors [24], one arrives at the following identity relating the C parameter to the second Fox Wolfram moment: $C = 1 - H_2$. This identity not only furnishes correlated error estimations, but also provides two

different approaches to study an event: while reduction of the sphericity eigenvectors to the principal axes of a momentum ellipsoid helps to visualize the C parameter, the readily covariant form of H_2 yields a more convenient starting point in perturbative calculations.

Jet Broadenings

Jet Broadenings [65] are sensitive to the development of a jet transverse to the thrust axis. For e^+e^- interactions, these variables are defined by dividing the event in two hemispheres $S_\pm$ by a plane perpendicular to the thrust axis $\hat{n}_T$, and computing in each hemisphere the quantity:

$$B_\pm = \frac{\sum_{a \in S_\pm} |\vec{p}_a \times \hat{n}_T|}{2 \sum_a |\vec{p}_a|}$$

The sum in the denominator runs over all final state particles, while that in the numerator runs over particles in one hemisphere. The observables, *total* (B_T) and *wide* (B_W) *jet broadening*, are then defined respectively as:

$$\begin{aligned} B_T &= B_+ + B_- \\ B_W &= \max(B_+, B_-) \end{aligned}$$

The advantages of studying such variables are that the transverse momenta are defined with respect to the more intuitive direction of thrust axis, instead of being minimized with respect to the choice of axis, as e.g., in the case of *sphericity*. Although the thrust and the sphericity axes coincide to order α_s , the plane perpendicular to the thrust axis does not contain any final-state momentum vectors, while a similar plane defined with respect to the sphericity axis might, and thus, the latter choice does not divide all events unambiguously into hemispheres.

Both B_T and B_W tend to zero in the *two jet* region. To leading order in α_s , $B_T = B_W = \frac{1}{2} O$, where O is the oblateness, and the mean and the maximum value of B_T are $0.65\alpha_s$ [66] and $\frac{1}{2\sqrt{3}}$ respectively. However, in higher orders, jet broadenings and oblateness are not related, e.g., for a *spherical* event, $B_T = 2B_W = \frac{\pi}{8}$, while $O = 0$.

Charged Particle Multiplicity

The average multiplicity and other multiplicity moments in a hadronic event depend upon the coherence effects in the summation of large infrared logarithms to all orders [67]. For large momentum transfer Q , the leading logarithmic dependence ($\exp \sqrt{\ln(Q^2/\Lambda^2)}$) of the multiplicity exhibits a reduction of the exponent, relative to the incoherent approximation,

which arises from destructive interference of soft gluon emission in the region of disordered emission angles. The next-to-leading order corrections to the infrared-sensitive quantities like average multiplicity are particularly important, as they turn out to be of relative order $\sqrt{\alpha_s} \ln \alpha_s$ instead of α_s [67].

At LEP energies, *charge multiplicity* is taken to be the number of *stable* (mean lifetime $> 3.3 \times 10^{-10}$ sec.) charged particles observed. Thus, particles like K_S^0 and Λ (with mean proper lifetime of $(0.8922 \pm 0.0020) \times 10^{-10}$ sec. and $(2.632 \pm 0.020) \times 10^{-10}$ sec. respectively) [4] are considered to be unstable in this counting. The raw distribution is corrected for detector resolution and acceptance effects using a *matrix unfolding* method using Monte Carlo distributions, which complies with the charge conservation condition. The measurement of charge particle multiplicity gives us valuable insight to the underlying hadronization process in the non-perturbative phase of the time-evolution of an event.

4.2 Variable ranges chosen

Out the several global event shape variables described in the preceding section, *eighteen* variables were chosen to compare the characteristics of a hadronic event between data and different Monte Carlo models. A list of these variables along with their ranges and number of points per variable is given in table 4.1.

4.2 Variable ranges chosen

#	Observable	Range		# of points
		Low	High	
1	Thrust (T)	0.5800	1.0000	13
2	Heavy Jet mass (ρ_H)	0.0000	0.3000	13
3	Total Jet Broadening (B_T)	0.0000	0.4000	11
4	Wide Jet Broadening (B_W)	0.0000	0.3000	8
5	Y23 of JADE (y_{23}^{ADE})	0.0000	0.3000	15
6	Y23 of DURHAM ($y_{23}^{k_\perp}$)	0.0000	0.3000	16
7	Sphericity (S)	0.0000	0.8000	14
8	Aplanarity (A)	0.0000	0.2000	12
9	C Parameter	0.0000	1.0000	14
10	D Parameter	0.0000	0.8000	11
11	Major (T_{major})	0.0000	0.700	14
12	Minor (T_{minor})	0.0000	0.5000	12
13	Oblateness (O)	0.0000	0.5000	12
14	Minor Narrow Side ($T_{\text{minor}}^{\text{NS}}$)	0.0000	0.4000	9
15	3 rd Fox Wolfram moment (H_3)	0.0000	0.5400	12
16	4 th Fox Wolfram moment (H_4)	0.0000	1.0000	12
17	Light Jet Mass (ρ_L)	0.0000	0.1200	11
18	Mean charged particle multiplicity ($\langle n_{\text{ch}} \rangle$)	6.0000	50.0000	17
ALL VARIABLES				226

Table 4.1: Ranges chosen for comparison between Data and MC.

Chapter 5

Monte Carlo Tuning

5.1 Introduction

The term *tuning* in context of Monte Carlo models refers to the process of finding the optimal parameter set of the models for which the Monte Carlo agrees with the data best. At LEP for different QCD models this tuning is done at the reference energy scale chosen to be M_Z , where the statistical error on the measured distribution is negligible. The models are then tested for their predictive power at other energy scales where data are available. A substantial amount of input for numerous free parameters, governing the details of the decays of unstable and resonant hadrons formed in the hadronization phase, is taken from numerous low energy data.

The remarkable success of the QCD parton shower models including coherence effects in explaining the energy evolution of the mean values of the global event shape variables rests to a large extent on the degree of matching of these event shapes between data and Monte Carlo at a reference energy point (at which the tuning is done). However, the matrix element model, tuned at one energy scale, do not reproduce the data at a different energy scale, because for this approach the hadronization dependence is superseded by the second order perturbation theory calculations, thereby restraining the number of hadrons formed in the final state, for example.

5.2 Data from hadronic Z decays

The events used for tuning were collected by the L3 detector at the center-of-mass energy $\sqrt{s} = 91.2$ GeV, during the 1991 LEP running period, corresponding to an integrated luminosity of 8.3 pb^{-1} .

The L3 detector

The L3 detector [12] covers 99% of the 4π geometry (see figure 5.1). It consists of a central tracking chamber (TEC), a high resolution electromagnetic calorimeter composed of bismuth germanium oxide (BGO) crystals, a ring of scintillation counters, a uranium and brass hadron calorimeter with proportional wire chamber readout, and a high precision muon spectrometer. These detectors are located in a 12 m diameter magnet which provides a uniform field of 0.5 T along the beam direction. Forward BGO arrays, on either side of the detector, measure the luminosity by detecting small angle Bhabha events.

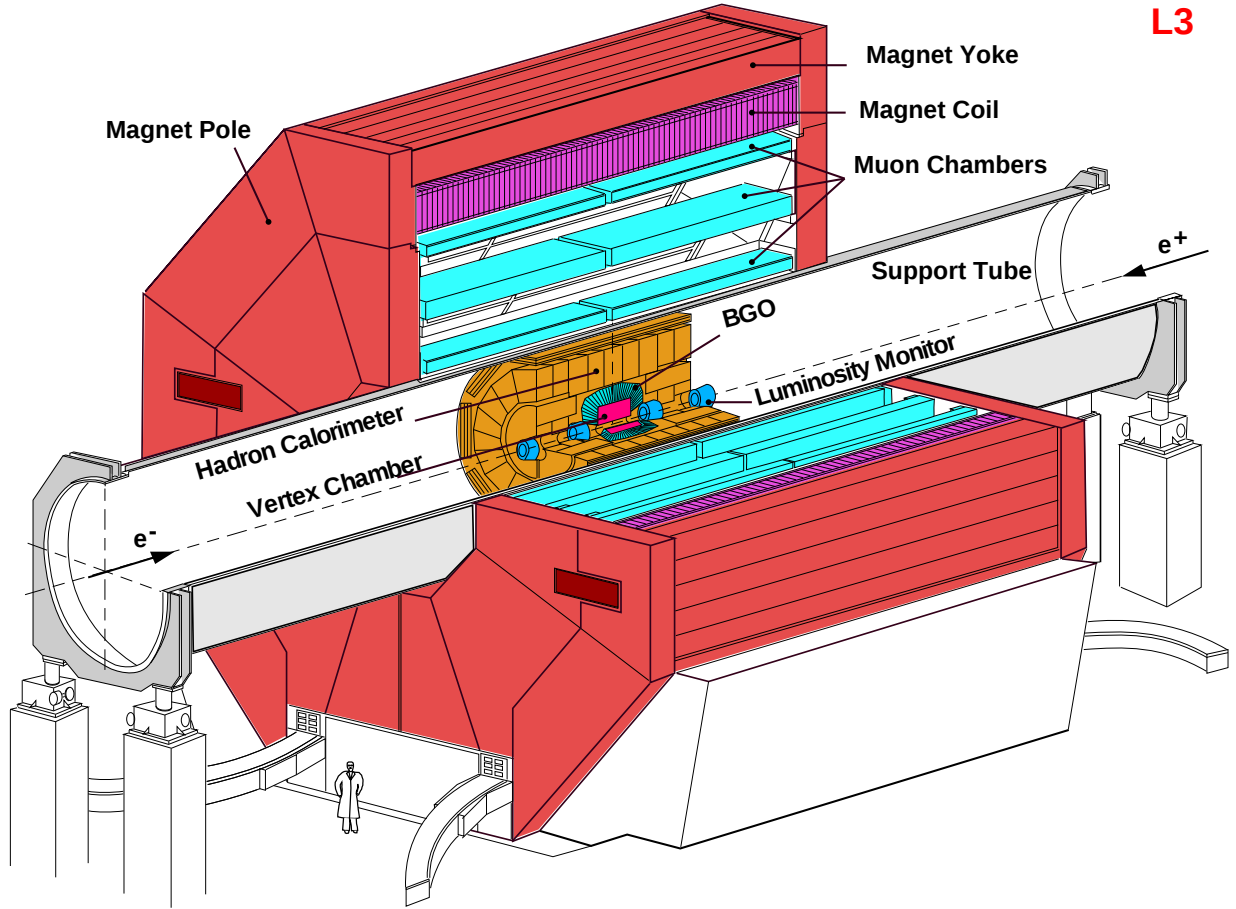


Figure 5.1: Perspective of the L3 detector.

For the present analysis, we use data collected in the following ranges of polar angle:

central tracking chamber:	$40^\circ \leq \theta \leq 140^\circ$,
electromagnetic calorimeter:	$11^\circ \leq \theta \leq 169^\circ$,
hadron calorimeter:	$5^\circ \leq \theta \leq 175^\circ$,
muon spectrometer:	$36^\circ \leq \theta \leq 144^\circ$,

where θ is defined with respect to the e^- beam direction.

Hadronic Trigger

The hadronic events are characterized by a large number of final state particles with most of the energy visible in the detector. Such events are identified by the logical OR of the level one [68] energy, TEC, and scintillation counter triggers. These three triggers have individual efficiencies of 99 %, 95 % and 95 % respectively. The energy trigger requires either a total energy of

- at least 25 GeV in the electromagnetic and hadron calorimeters, or
- a minimum energy of 15 GeV in the barrel or central region ($42^\circ < \theta < 138^\circ$) of calorimeters, or
- 8 GeV in the barrel of the electromagnetic calorimeter alone.

The TEC trigger requires at least two tracks identified with a maximum acolinearity of 60° . The scintillation counter trigger requirement is a coincidence of at least 5 hits, in the counters during a 30 ns interval about the beam crossing time, which must extend over an azimuthal angular region of at least 90° .

An event must be selected by at least one of the three above-mentioned *first level triggers* in order to be recorded as a hadronic event, thereby reducing the 45 kHz bunch crossing rate of LEP to about 8 Hz first level trigger rate. The first level trigger analyses the trigger data of an individual sub-detector and either initiates the digitisation of the main data or clears the front end electronics before the next beam crossing. A negative decision at the first level does not produce any dead time. However, a positive decision from any of the individual first level triggers, initiates the detector data to be digitised and stored in multi-event buffers which takes around $500\mu\text{s}$, thereby causing the dead time for the data acquisition.

The successive *second* [69] and *third* [70] levels of the trigger system further filter events arising due to cosmic rays, electronic noise, uranium noise, and beam-gas interactions. The *second level trigger* acts on the coarse digitized data from various sub-detectors, and additional informations from combined energy clusters in the calorimeters, loosely reconstructed tracks and the interaction point. It spends about 8 ms on an event without inducing any dead time. It has a rejection power of 20 to 30 % averaged over all first level triggers, and either passes all event information to an event builder memory or resets the event builder memories. The principle of the *third level trigger* (which is allowed ten times as much the time available to the previous levels) is similar to that of the second one, but accesses fully digitized data and bases its decision on recalculated and calibrated energies, fully reconstructed tracks and vertices which matched calorimetric clusters. On

a positive third level trigger the data is transferred to the main acquisition system which is subsequently written to tapes, which is typically of the order of 2-3 Hz. The overall efficiency for selecting hadronic Z decays by the online trigger is greater than 99.9%.

The L3 Event Reconstruction

The event reconstruction [71] proceeds in two steps: first signals from each sub-detector are analyzed and reconstructed locally, and then using a pattern recognition algorithm the event is globally reconstructed. In the first step, analog signals from *hits* in the calorimeter compatible with 2 MeV threshold in the BGO or 9 MeV threshold in the hadron calorimeter are studied, giving rise to energy clusters. Tracks are reconstructed in the central tracking chamber or the muon spectrometer by associating spatially adjacent hits. Energy clusters reconstructed from the calorimetric energy deposits are matched with the reconstructed tracks.

The global reconstruction algorithm starts from the most energetic cluster, taken as the “seed” of an “object”. Each *cluster* is characterised by its type, with its energy and direction being derived from the local maxima of the geometrical sum of the hits it contains, with the interaction vertex taken to be the origin. On the basis of the sub-detector informations, such a *smallest resolvable cluster* (ASRC) may be of electromagnetic or hadronic origin, depending upon its longitudinal and transverse profile. A cluster is defined such that it resembles particle cascade with very small contamination from random noise. For example, a 45 GeV hadronic jet from a Z decay contains about 70 *hits* in the hadronic calorimeter in a volume compatible with its one module, while the noise level for the same module corresponds to about 4 *hits* [72].

Electrons or photons deposit most of their energy in the electromagnetic calorimeter, the photons being differentiated from electrons by absence of tracks in the TEC. The BGO barrel calorimeter was calibrated in test beams using electrons of energies 0.18, 2, 10, and 50 GeV. The energy resolution obtained is $\approx 5\%$ at 100 MeV, less than 2% above 2 GeV and about 1.2% at 45 GeV as shown in figure 5.2. The linearity is better than 1%. The position resolution is determined by the centre of gravity method to be ~ 1 mm whereas the angular resolution is ~ 1 mrad for electromagnetic showers at 45 GeV.

Muons deposit small signals in the calorimeter compatible with that of a minimum ionizing particle (MIP) which are localized near its track reconstructed in the TEC and muon spectrometer, which respectively lie in the inner and outer ring of the detector.

Hadrons lose their energy mostly through nuclear interactions in both calorimeters, with large fluctuations corresponding to the diffuse deposits in BGO. The amount of material in hadron calorimeter traversed by such a particle originating at the interaction

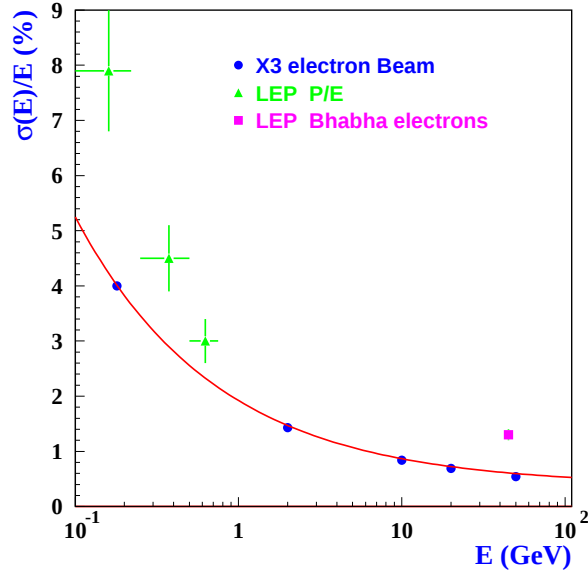


Figure 5.2: Energy resolution of the electromagnetic calorimeter.

point varies between 6 to 7 nuclear interaction length (λ_{int}). The hadron calorimeter provides 2.5° angular resolution for hadronic jets and the energy resolution is:

$$\frac{\Delta E}{E} = \left(5 + \frac{55}{\sqrt{E(\text{GeV})}} \right) \%$$

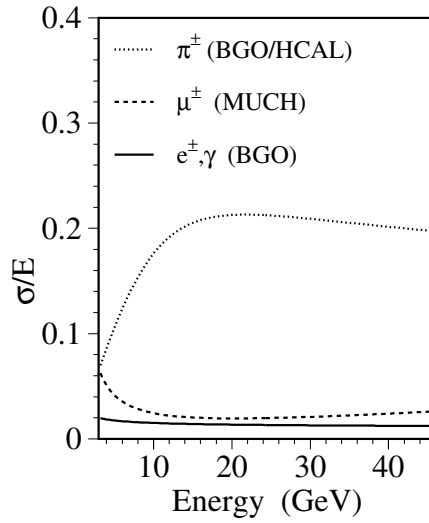


Figure 5.3: L3 detector resolution as a function of energy.

These characteristic features of photons, leptons and jets are exploited in the high resolution measurements at L3 to infer the properties of an event. The smallest resolvable

clusters, identified as electron, photon or hadrons, are combined into jets using an iterative procedure based on angular separation. Global energy resolution of few typical particles observed in the L3 detector are shown in figure 5.3 as a function of their energy.

Hadronic Event Selection Criteria

Events of the type $e^+e^- \rightarrow \text{hadrons}$ are selected in this analysis by measuring the energy deposited in the electromagnetic and hadronic calorimeters, which has a large fiducial volume coverage. Following an off-line global energy calibration, one divides the detector into 12 regions (see figure 5.4), and associates with each region a calibration factor to reweight its raw energy measured. These weight factors are collectively known as G-factors [73].

If $\bar{E}_i$ is the average energy in the geometric region i over a reasonably large number of hadronic events, then the average total energy and the variance are given by:

$$\begin{aligned}\bar{E} &= \sum_i G_i \bar{E}_i \\ \sigma^2 &= \overline{(E - \bar{E})^2} \\ &= \sum_{i,j} G_i G_j (\bar{E}_i \bar{E}_j - \bar{E}_i \bar{E}_j)\end{aligned}$$

The G-factors are fitted to minimise the function

$$M = \left(\frac{\sigma}{\bar{E}} \right)^2 + \lambda \left(\frac{\sqrt{s}}{\bar{E}} - 1 \right)^2$$

where the first term denotes the energy resolution and the second term imposes energy-momentum conservation at the center-of-mass energy $\sqrt{s}$. λ , the Lagrange's multiplier, is assigned a large value. This calibration gives good agreement between the hadronic data and realistic detector simulated [71] hadronic JETSET Monte Carlo, with a resolution of 12.4%.

For this calorimeter based hadronic event selection, one requires:

- $N_{cluster} > 12$
- $0.6 < E_{vis}/\sqrt{s} < 1.4$
- $|E_{||}|/E_{vis} < 0.4$
- $E_{\perp}/E_{vis} < 0.4$

where E_{vis} is the total energy observed in the calorimeters, $N_{cluster}$ is the total number of calorimetric clusters with energy greater than 100 MeV, $E_{||}$ is the energy imbalance along

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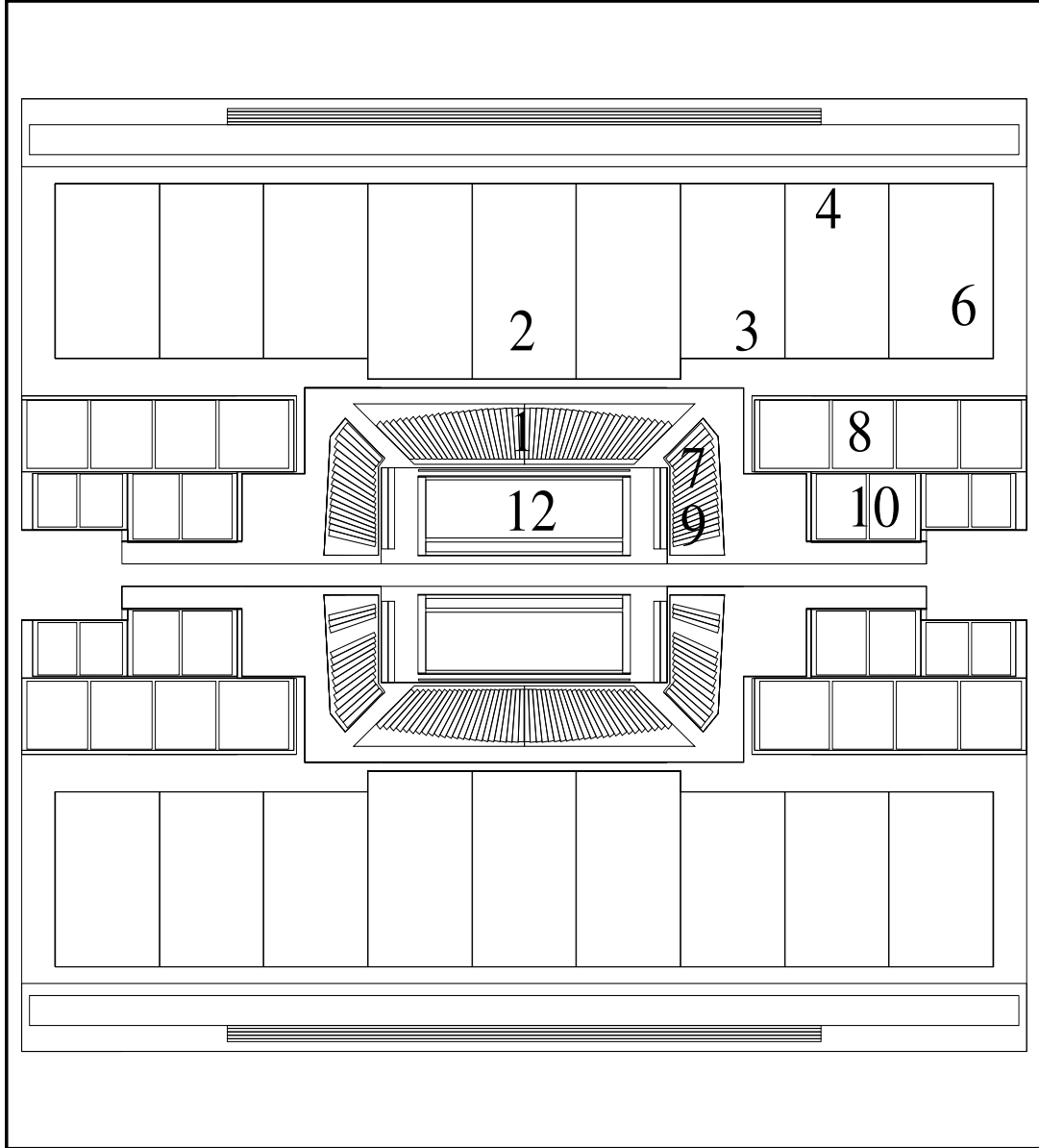


Figure 5.4: The different geometrical regions of the L3 Calorimeter system. Regions 1, 3, 7, and 9 are associated with the electromagnetic calorimeter whereas regions 2, 4, 6, 8 and 10 belong to the hadron calorimeter. Regions 11 and 12 correspond to the muon chambers and the tracking chamber respectively.

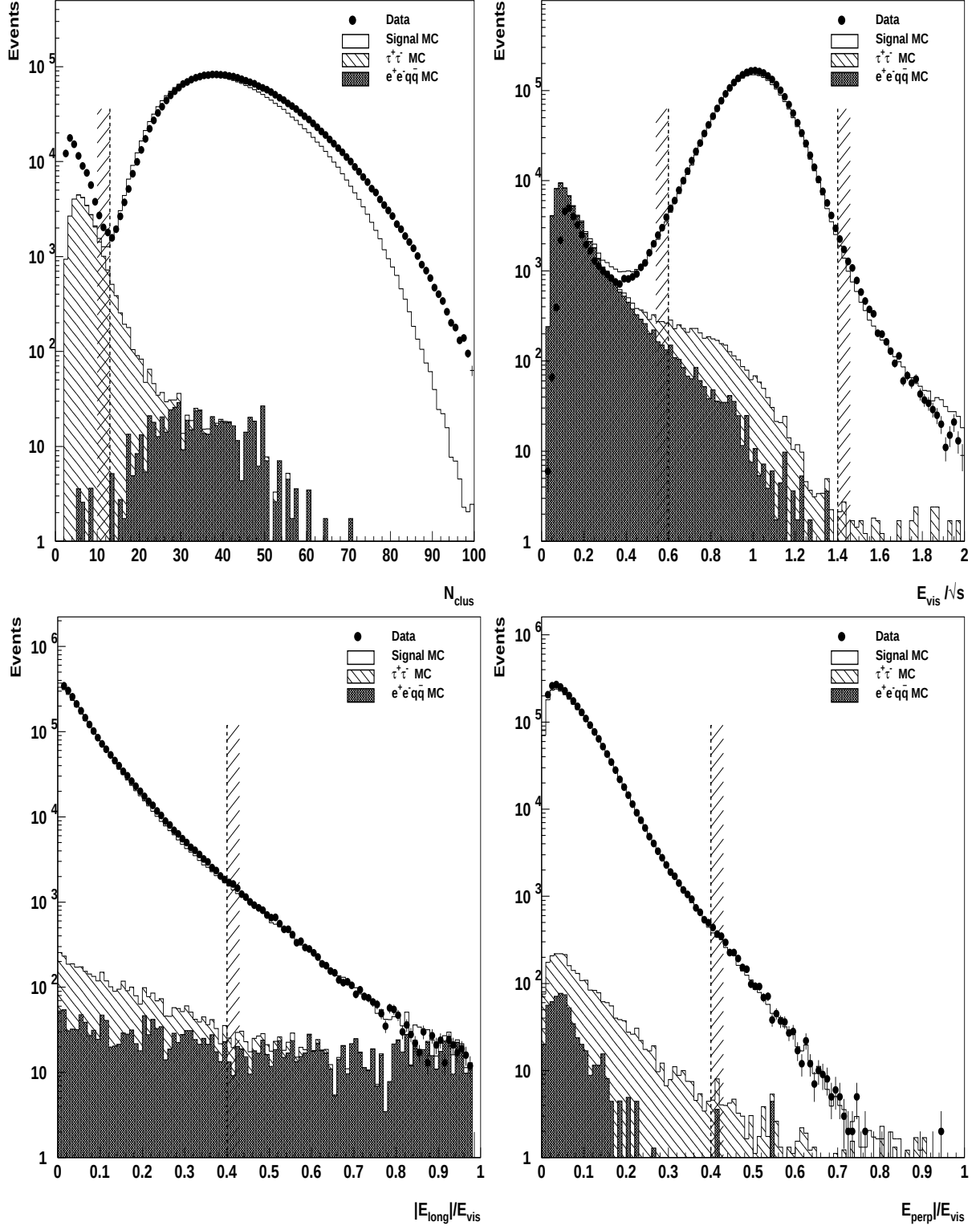


Figure 5.5: Plots of selection variables for Data and Monte Carlo with other cuts applied: $N_{cluster}$, $E_{vis}/\sqrt{s}$, $E_{\perp}/E_{vis}$, $|E_{\parallel}|/E_{vis}$ (clockwise from top left).

the beam direction, and $E_{\perp}$ is the energy imbalance in the plane perpendicular to the beam direction. In terms of the directed energy measurements $\vec{\mathcal{E}}_i$ of the i^{th} cluster, with the direction taken from the angle of the reconstructed *clusters*, these selection variables are defined as:

$$\begin{aligned} E_{\text{vis}} &= \sum_i |\vec{\mathcal{E}}_i| \\ \vec{\mathcal{E}}_{\text{miss}} &= -\sum_i \vec{\mathcal{E}}_i \\ E_{\parallel} &= \hat{z} \cdot \vec{\mathcal{E}}_{\text{miss}} \\ E_{\perp} &= \sqrt{(\hat{x} \cdot \vec{\mathcal{E}}_{\text{miss}})^2 + (\hat{y} \cdot \vec{\mathcal{E}}_{\text{miss}})^2}, \end{aligned}$$

where $\{\hat{x}, \hat{y}, \hat{z}\}$ are the three unit vectors in a right-handed orthogonal cartesian coordinate system, with origin at the interaction point and z-axis pointing along the direction of electron beam, and x-axis pointing towards the center of the LEP ring.

The cut on the number of calorimetric clusters with energy greater than 100 MeV rejects low multiplicity events such as $\tau^+\tau^-(\gamma)$, e^+e^- and $\mu^+\mu^-$ final state events, whereas the two photon events (characterized by low total visible energy) are removed by the cut on E_{vis} . Hadronic events are well balanced events, except for finite resolution and missing particles (falling in dead regions of the detector or dead materials). Cuts on $E_{\parallel}$ and $E_{\perp}$ select such well balanced events, compatible with the detector resolution.

Figures 5.5a-d show the distribution of each of the selection variables with all *other cuts* applied. The shaded regions shown are the contribution from the different backgrounds, whereas the empty region indicates the signal. The Monte Carlo predictions are shown one on top of the other so that the final histograms can be compared directly to data. All the Monte Carlo predictions are normalised by the number of selected hadron events to the observed luminosity.

These cuts select 248,100 hadronic events, with efficiency and purity estimated from JETSET [9] Monte Carlo studies to be 98.5% and 99.8% respectively. The observed distributions have been corrected [49] for detector effects – resolution and acceptance, by modelling the detector response [74, 75] and reconstructing the events [71] as if it were passed through the L3 detector. The data have also been corrected for initial and final state radiations.

5.3 QCD Monte Carlo parameters

The QCD Monte Carlo programs: JETSET 7.4 [9], HERWIG 5.8 [11] and ARIADNE 4.06 [10], differ in the description of the hard scattering process and also in the modelling

of the fragmentation process. For the hard scattering process, most of the programs follow leading log approximation with an arbitrary number of partons produced in the parton showers, whereas JETSET 7.4 also provides a user option to generate up to 4-parton events with a complete matrix element calculation to second order.

The fragmentation models involve several parameters which are eventually determined by data. The *tuning parameters* for JETSET and ARIADNE are taken to be Λ_{LLA} , σ_Q and the b parameter, while for HERWIG, the *tuning parameters* are Λ_{MLLA} , CLMAX and CLPOW.

The model parameters as determined by the previous tuning of the L3 experiment [49, 50] are summarised in tables 5.1, 5.2, 5.3 and 5.4. For comparison, similar parameters obtained by other LEP experiments [52] are also summarized in the same table.

Parameter	ALEPH	DELPHI	L3 [49]	L3 [50]	OPAL
$\Lambda_{LLA}(\text{GeV})$	0.320	0.297	0.30 ± 0.03	0.31 ± 0.03	0.250
$\sigma_Q(\text{GeV})$	0.360	0.408	0.39 ± 0.03	0.40 ± 0.06	0.400
$b(\text{GeV}^{-2})$	1.030	0.850	0.76 ± 0.08	0.85 ± 0.11	0.520

Table 5.1: Previous default parameters for the JETSET 7.4 Parton Shower Program.

Parameter	L3 [49]	L3 [50]
$\Lambda_{ME}(\text{GeV})$	0.17 ± 0.02	0.16 ± 0.02
$\sigma_q(\text{GeV})$	0.50 ± 0.05	0.48 ± 0.08
$b(\text{GeV}^{-2})$	0.42 ± 0.06	0.36 ± 0.07

Table 5.2: Previous default parameters for the JETSET 7.4 Matrix Element Program.

Parameter	ALEPH	DELPHI	L3	OPAL
$\Lambda_{LLA}(\text{GeV})$	0.218	0.237	0.22 ± 0.02	0.200
$\sigma_Q(\text{GeV})$	0.354	0.390	0.50 ± 0.04	0.377
$b(\text{GeV}^{-2})$	0.810	0.850	0.65 ± 0.07	0.340

Table 5.3: Previous default parameters for the ARIADNE 4.06 Program.

5.4 Tuning procedure

The tuning procedure for matching of data with Monte Carlo involves two steps of minimisation. First, a *few* (three or four) event shape variables are chosen to be *tuning*

Parameter	ALEPH	DELPHI	L3	OPAL
Λ_{MLLA} (GeV)	0.149	0.163	0.17 ± 0.02	0.160
CLMAX (GeV)	3.90	3.48	3.20 ± 0.05	3.40
CLPOW	2.00	1.49	1.45	1.30
Gluon mass (GeV)	0.726	0.650	0.75	0.750

Table 5.4: Previous default parameters for the HERWIG 5.8 Parton Shower Program.

variables for comparison between data and Monte Carlo. The *tuning* event shape variables chosen to describe the lateral and longitudinal hadronic shower profiles include jet resolution parameter in the JADE algorithm [61] which corresponds to transition from $2 \rightarrow 3$ jets (y_{23}^{JADE}), minor [57] calculated after dividing the event into two hemispheres by the thrust [55] axis and evaluated in the hemisphere corresponding to the narrow jet ($T_{\text{minor}}^{\text{NS}}$), the fourth Fox-Wolfram moment (H_4) [60] and charged particle multiplicity.

For different choices of values of the parameters ($\vec{p}_k$) to be tuned, the Monte Carlo distributions of the tuning variables are compared with the data distributions. This is done by minimising the χ^2 function defined as

$$\chi^2(\vec{p}_k) = \sum_{i=\text{tun. var}} \sum_{j=\text{bins}} \frac{[\text{Data}_i^j - \text{MC}_i^j(\vec{p}_k)]^2}{[\sigma_{\text{Data}_i^j}^{\text{stat}}]^2 + [\sigma_{\text{Data}_i^j}^{\text{syst}}]^2 + [\sigma_{\text{MC}_i^j(\vec{p}_k)}^{\text{stat}}]^2}$$

where the individual contributions to χ^2 are summed over all bins (j) of the distributions of the chosen tuning variables (i). Points with insignificant statistics are ignored in the fit. The optimal parameter set is taken to be one that minimizes the above χ^2 function, using the CERN program package MINUIT.

In the next step, these fits are repeated by varying the fit range of the tuning variables and also by changing the dependence of the fit procedure on the particular set of values chosen for tuning parameters at the beginning of the tuning procedure. Each of these systematic variations yield possible sets of optimal values for the tuning parameters. The Monte Carlo distributions for all the *eighteen* global event shape variables (see section 4.2) with these optimal sets of parameters are then compared with the data distributions of the corresponding variables. For this comparison, the χ^2 function with contributions from *all* the global event shape variables are used. The minimum value of this χ^2 function among these set of Monte Carlo predictions finally yeilds the *tuned* parameter set for the Monte Carlo model.

5.5 Tuning methods

The tuning parameters of a model span a continuous multi-dimensional space, and thus the χ^2 for tuning variables, as defined in the preceding section, is a real continuous function. However, in any realistic tuning procedure, one starts off with a finite set of guesses for the optimal parameter set, and generates Monte Carlo distributions for the event shape variables at these *discrete* points in the parameter space. In order to minimize the χ^2 function, one must have a knowledge of the *continuous* variation of the function in terms of the tuning parameters. Two methods are considered for tuning - *direct approach* and the *grid approach*, which differ in their technique for obtaining the Monte Carlo predictions at points outside the pre-guessed values of the tuning parameters. They are described in the following sub-sections.

5.5.1 Direct approach

In this first method, we use the CERN program package MINUIT to minimize the χ^2 function of tuning variables. The theoretical distributions are generated for the parameter ($\vec{p}_k$) values at each step of the minimization. This method is a direct approach but suffers due to insufficient Monte Carlo statistics at the chosen points resulting in discontinuous behaviour of the χ^2 function. If one tries to smoothen by increasing the statistics, then the CPU time necessary tends to become very large. For example, if there are about 1000 calls by MINUIT to generate Monte Carlo distributions, each with 100 K events, then assuming it takes 2 hours to generate the Monte Carlo distribution at one point in the parameter space (that is, 0.072 seconds per one event), it would take about 12 weeks of CPU time.

5.5.2 Grid approach

In the second approach, one generates events on several points on a grid in the parameters space with a large number of events (≥ 40 K). For a grid with k -parameters and n_p different values for a given parameter, one needs to generate events at $\prod_{p=1}^k n_p$ points. Subsequent minimization proceeds by estimating the theoretical prediction for a given bin in a given distribution in between the grid points using a local multidimensional interpolation. The fragmentation parameters ($\vec{p}$) are obtained from a fit which minimizes the χ^2 function defined in a similar way as in the previous method.

In this method one has a choice of using either *linear* or *non-linear* interpolation. The Monte Carlo distribution corresponding to j^{th} bin for i^{th} tuning variable (MC_i^j) for points

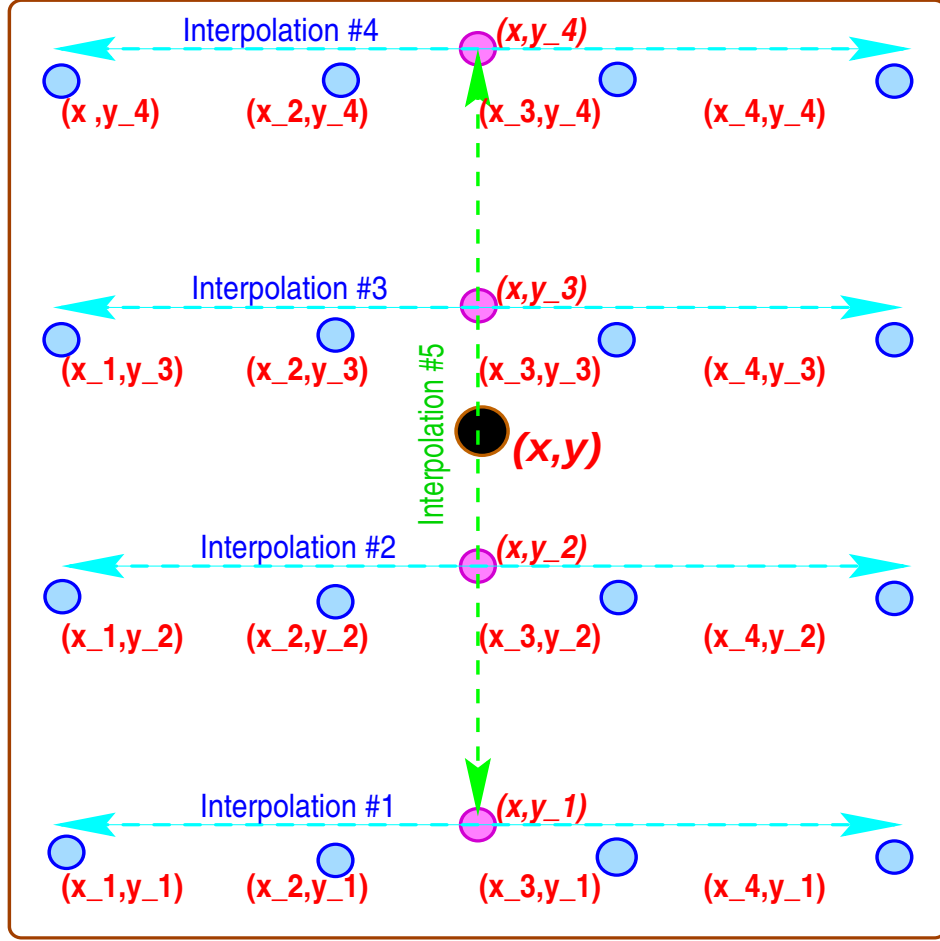


Figure 5.6: Illustration of local cubic interpolation for two variables on a 4×4 grid size lattice.

in parameter space inside the grid using a polynomial of given degree is given by ($\forall i, j$):

$$\text{MC}_i^j(\vec{p}_0 + \delta\vec{p}) = (a_0)_i^j + \sum_m ((a_1)_i^j)^m \delta p_m + \sum_{m,n} ((a_2)_i^j)^{mn} \delta p_m \delta p_n + \dots$$

For linear multidimensional interpolation a CERN program package FINT is available, and also available are routines for non-linear single dimensional interpolation (POLINT). For multidimensional non-linear interpolation, F2INT was developed in course of this study. An illustration of this algorithm of varying degree non-linear multi-dimensional interpolation over a sub-grid about the point of interest in the parameter space is shown in figure 5.6, where the degree of interpolation is taken to be three over a 4×4 grid of two variables. One dimensional cubic interpolation over the variable x of given set of values of the function at points (x_i, y_j) (keeping j fixed) yields a set of functions $f(x, y_j)$, over which a final interpolation over y gives the value of the interpolated function at any point

(x, y) in the grid spanned by the points $\{(x_i, y_j), (i, j = 1, 4)\}$.

5.6 Specifications of present tuning

Prior to this work, the energy evolution [76] of the mean charge multiplicity was poorly described by ARIADNE using the previously tuned parameter set by the L3 experiment [49] (see figure 5.7).

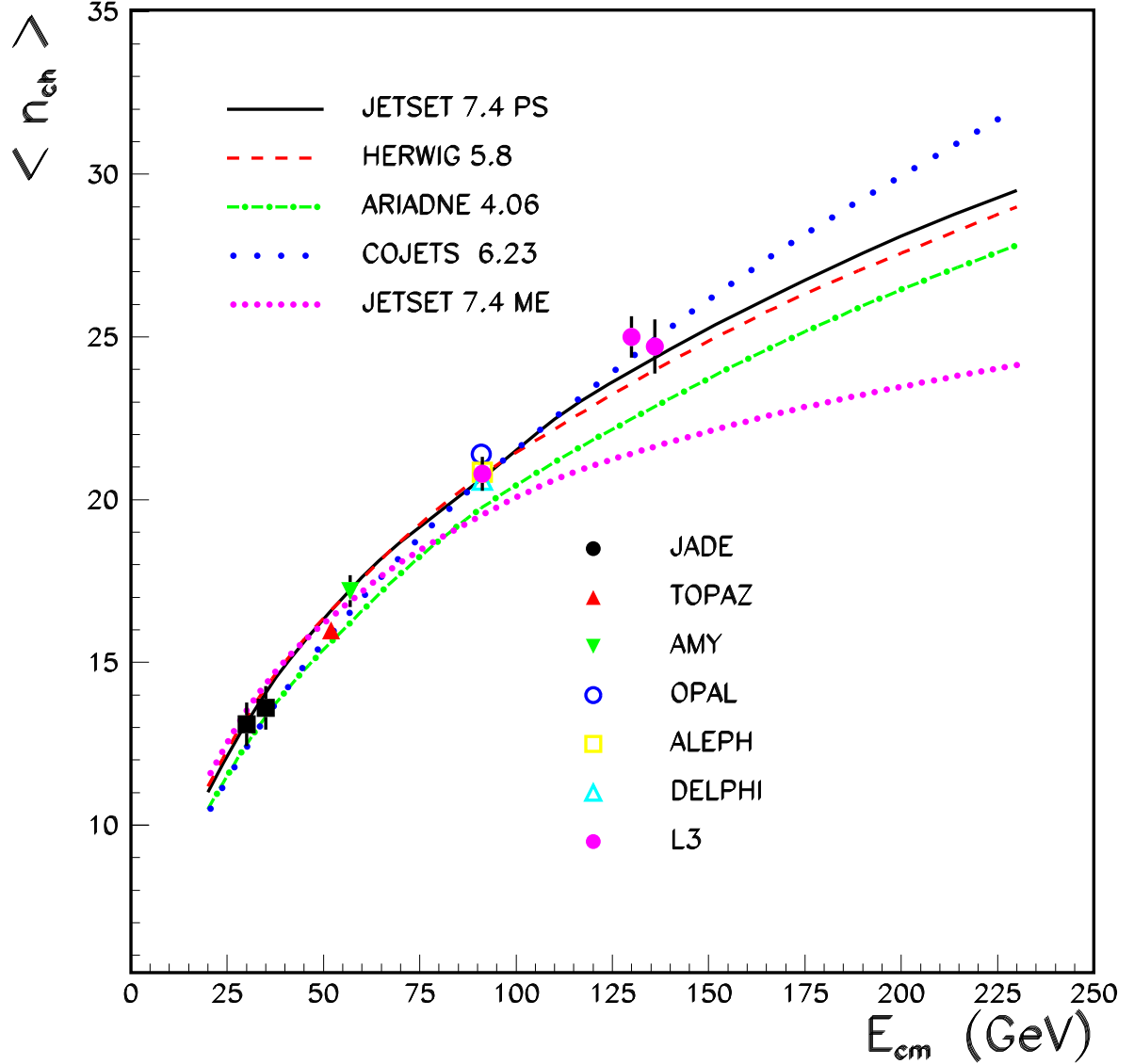


Figure 5.7: Center-of-mass energy evolution of measured mean charged multiplicity as compared to a number of QCD models with previous L3 defaults [49, 50].

Furthermore, the ratio of expected to observed four jet rates as a function of the jet

resolution parameter y_{cut} for jets reconstructed using the JADE [61] and the $k_{\perp}$ [62] algorithms showed a deficit at the level of 5-20% depending on the y_{cut} values by both the parton shower and the matrix element approach of JETSET event generator, as shown in figure 5.8. The expected rates from JETSET 7.4 PS and ME models for figure 5.8 are determined using the tuned values as in [50]. This could give rise to a large systematic error on measuring the cross-section for the process $e^+e^- \rightarrow W^+W^-$, which is highly relevant at high energy runs at LEP II.

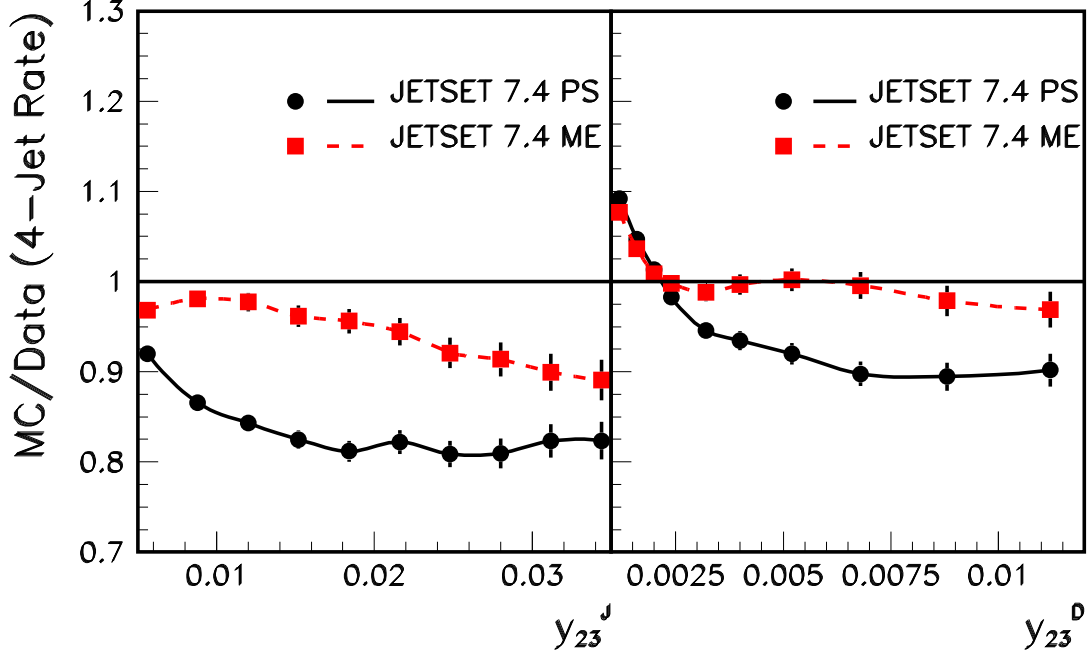


Figure 5.8: Ratio of 4-jet rate of data and MC with previously tuned parameters as function of jet resolution parameter for JADE and $k_{\perp}$ algorithms [50].

In this study the discrete distribution of mean charge multiplicity ($\langle n_{\text{ch}} \rangle$) is also included in the list of *tuning variables*, along with the continuous distributions of three other tuning variables. Another feature of this analysis is the use of non-linear multidimensional interpolation and cross-checking the results of the fits by rerunning Monte Carlo for large number of events.

Chapter 6

Results

6.1 Quality of fit

The predictions of these QCD models to global event shape variables are obtained and compared with the measurements done at LEP I [49, 54]. The event shape variables studied here are:

- thrust (T) [55],
- major (T_{major}),
- minor (T_{minor}),
- oblateness (O) [56],
- sphericity (S) [63],
- aplanarity (A),
- scaled jet masses (ρ_{H} and ρ_{L}) [58],
- C and D parameters [64],
- jet broadening variables (B_{T} , B_{W}) [65],
- 3rd Fox-Wolfram moment (H_3) [60], and
- 3-jet resolution parameters in $k_{\perp}$ ($y_{23}^{k_{\perp}}$) [62] algorithm,

in addition to the four tuning variables:

6.1 Quality of fit

- 3-jet resolution parameters in JADE (y_{23}^{JADE}) [61] algorithm,
- minor of the narrow side ($T_{\text{minor}}^{\text{NS}}$) [57],
- 4th Fox-Wolfram moment (H_4) [60], and
- mean charge multiplicity ($\langle n_{\text{ch}} \rangle$).

	JETSET 7.4 PS		JETSET 7.4 ME		HERWIG 5.8	ARIADNE 4.06
	L3 [49]	L3 [50]	L3 [49]	L3 [50]	L3 [49]	L3 [49]
Thrust	8.1	5.8	33.8	32.7	16.6	22.5
ρ_{H}	24.7	25.9	60.2	61.7	14.3	30.8
B_{T}	20.3	24.9	20.3	25.5	14.8	10.3
B_{W}	27.8	29.7	32.4	27.0	44.3	27.8
y_{23}^{JADE}	9.9	7.4	15.1	17.1	10.2	6.6
$y_{23}^{\text{k}\perp}$	15.4	9.6	6.7	7.1	10.7	9.5
Sphericity	11.6	7.7	8.2	11.6	8.7	11.3
Aplanarity	10.9	8.6	25.7	46.3	32.5	20.2
C-parameter	12.1	8.6	41.1	52.0	31.2	21.6
D-parameter	28.4	28.1	45.9	51.8	15.4	24.7
T_{major}	3.5	4.9	11.7	14.3	9.5	6.2
T_{minor}	22.6	21.7	24.0	28.1	33.7	23.9
Oblateness	13.4	17.2	28.3	17.7	20.8	7.0
$T_{\text{minor}}^{\text{NS}}$	2.4	1.7	6.6	6.2	14.0	8.2
H_3	5.8	5.3	6.0	6.9	6.9	5.4
H_4	17.0	11.7	11.4	9.5	44.1	12.8
ρ_{L}	19.7	28.3	21.0	24.1	14.0	6.0
$\langle n_{\text{ch}} \rangle$	19.6	9.9	51.8	33.1	5.7	34.1
Overall	273.2	257.1	450.0	472.6	347.3	289.0
Tuning variables	(48.9)	(30.7)	(84.8)	(65.9)	(74.0)	(61.7)

Table 6.1: Goodness of matching between data and various QCD models with parameter sets from earlier tuning, as determined by the χ^2 to the global event shape variables.

The χ^2 obtained from the various distributions of the different Monte Carlo programs with for default parameter values as obtained in the previous tunings at L3 [49, 50] are summarised in table 6.1. The overall χ^2 refers to 226 data points for all the *eighteen* event shape variables, while for the tuning variables the χ^2 is taken for 53 data points corresponding to the default range of these variables (see table 4.1). With the exception of JETSET 7.4 matrix element model, all the QCD models describe the global event shape variables rather well. JETSET 7.4 Monte Carlo with matrix element option is not expected

6.1 Quality of fit

to describe all the detail of multi-jet production since it is restricted to complete second order in α_s only.

Variables	# points	JETSET PS	JETSET ME	HERWIG	ARIADNE
Thrust	13	7.0	31.9	15.6	9.4
ρ_H	13	26.0	64.0	14.7	22.5
B_T	11	19.3	26.4	12.4	12.7
B_W	8	28.2	25.6	32.0	29.5
y_{23}^{JADE}	15	7.8	11.5	11.4	7.6
$y_{23}^{K_{\perp}}$	16	8.6	9.4	7.2	5.3
Sphericity	14	10.2	15.7	9.7	9.0
Aplanarity	12	6.3	70.2	35.6	3.6
C-parameter	14	6.2	47.4	21.4	8.0
D-parameter	11	25.2	55.7	15.4	18.3
T_{major}	14	3.7	17.4	8.8	3.0
T_{minor}	12	17.8	31.8	30.6	11.5
Oblateness	12	17.0	19.3	16.3	20.0
$T_{\text{minor}}^{\text{NS}}$	9	1.6	6.2	12.3	3.5
H_3	12	4.4	8.0	7.1	2.4
H_4	12	10.3	9.9	37.6	8.0
ρ_L	11	26.7	20.0	15.9	7.1
$\langle n_{\text{ch}} \rangle$	17	10.5	12.4	2.9	6.7
Overall	226	236.8	482.6	306.9	188.2
Tuning variables	53	(30.3)	(40.0)	(64.2)	(25.9)

Table 6.2: Goodness of matching between data and various QCD models with parameter sets from the current tuning, as determined by the χ^2 to the global event shape variables.

In this study, an overall improvement in the description of the data from the currently tuned parameters than those from earlier tuned parameter sets is found. For a comparison, similar χ^2 table with the currently obtained set of parameters [77] are tabulated in table 6.2. The improvement in ARIADNE is rather substantial. For JETSET 7.4 PS and HERWIG 5.8, some improvement is observed. Though the χ^2 for the tuned variables improve significantly in case of JETSET 7.4 ME model, the overall χ^2 has increased.

Reasonable fits to the observed distributions are obtained. The measured distributions for the four fitted variables together with the theoretical predictions as obtained from the fit to the models are shown in figures 6.1, 6.2, 6.3 and 6.4. The final χ^2 for 53 measured points of tuning variables come out to be 30.3, 40.0, 64.2 and 25.9 for JETSET 7.4 PS, JETSET 7.4 ME, HERWIG 5.8 and ARIADNE 4.06 respectively.

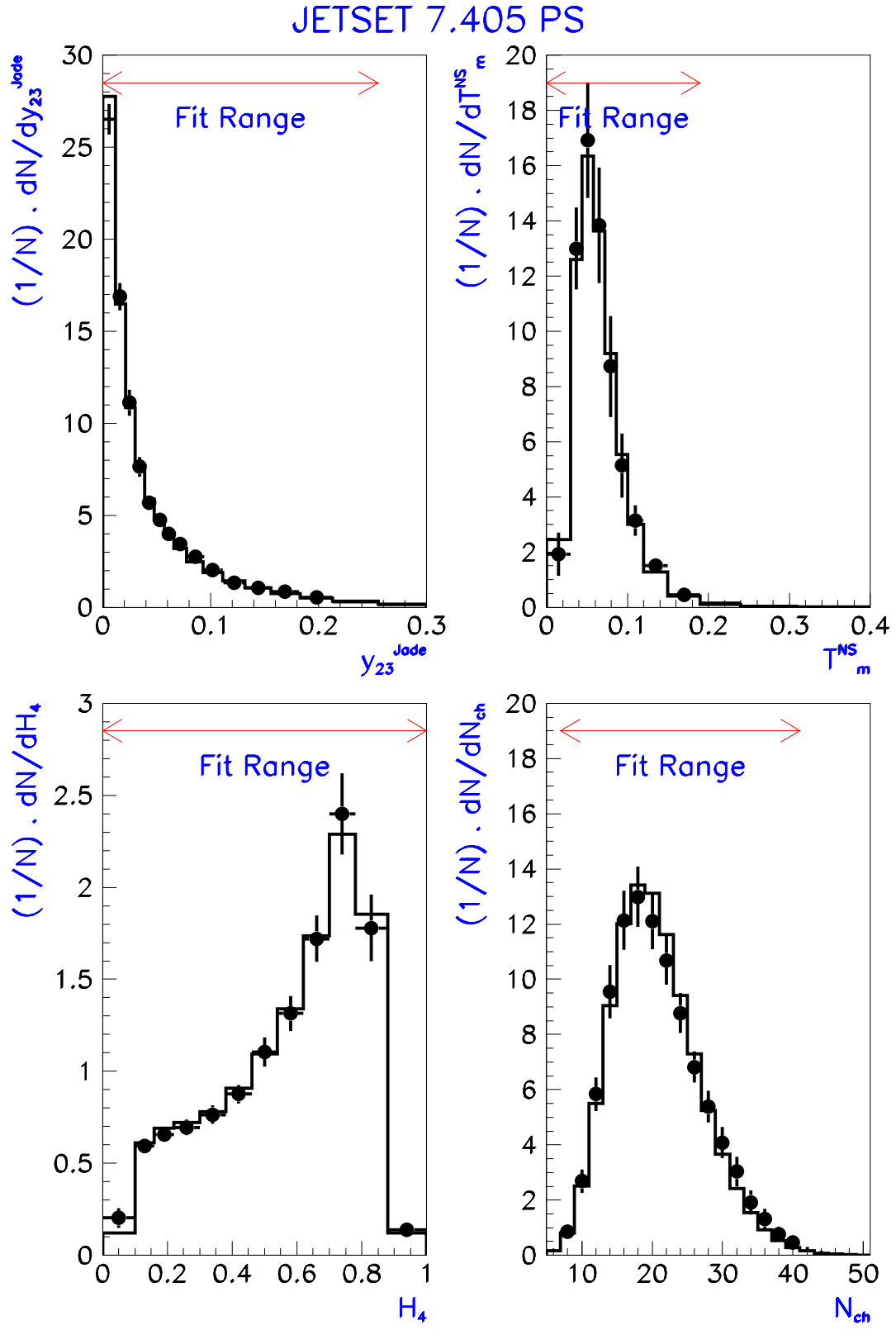


Figure 6.1: Measured distributions of the tuning variables together with the predictions of JETSET 7.4 Parton Shower Program using the tuned parameters.

JETSET 7.405 ME

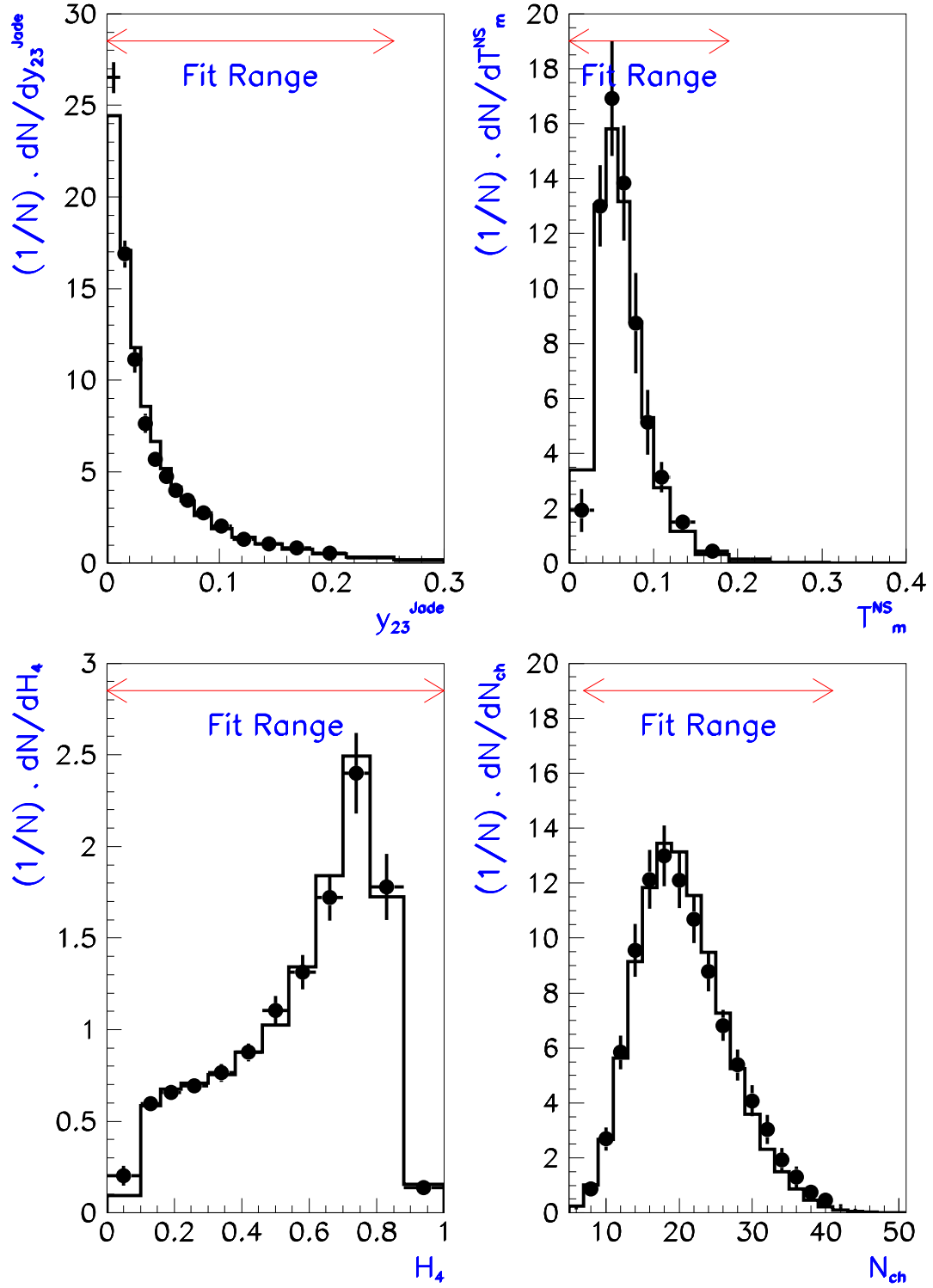


Figure 6.2: Measured distributions of the tuning variables together with the predictions of JETSET 7.4 Matrix Element Program using the tuned parameters.

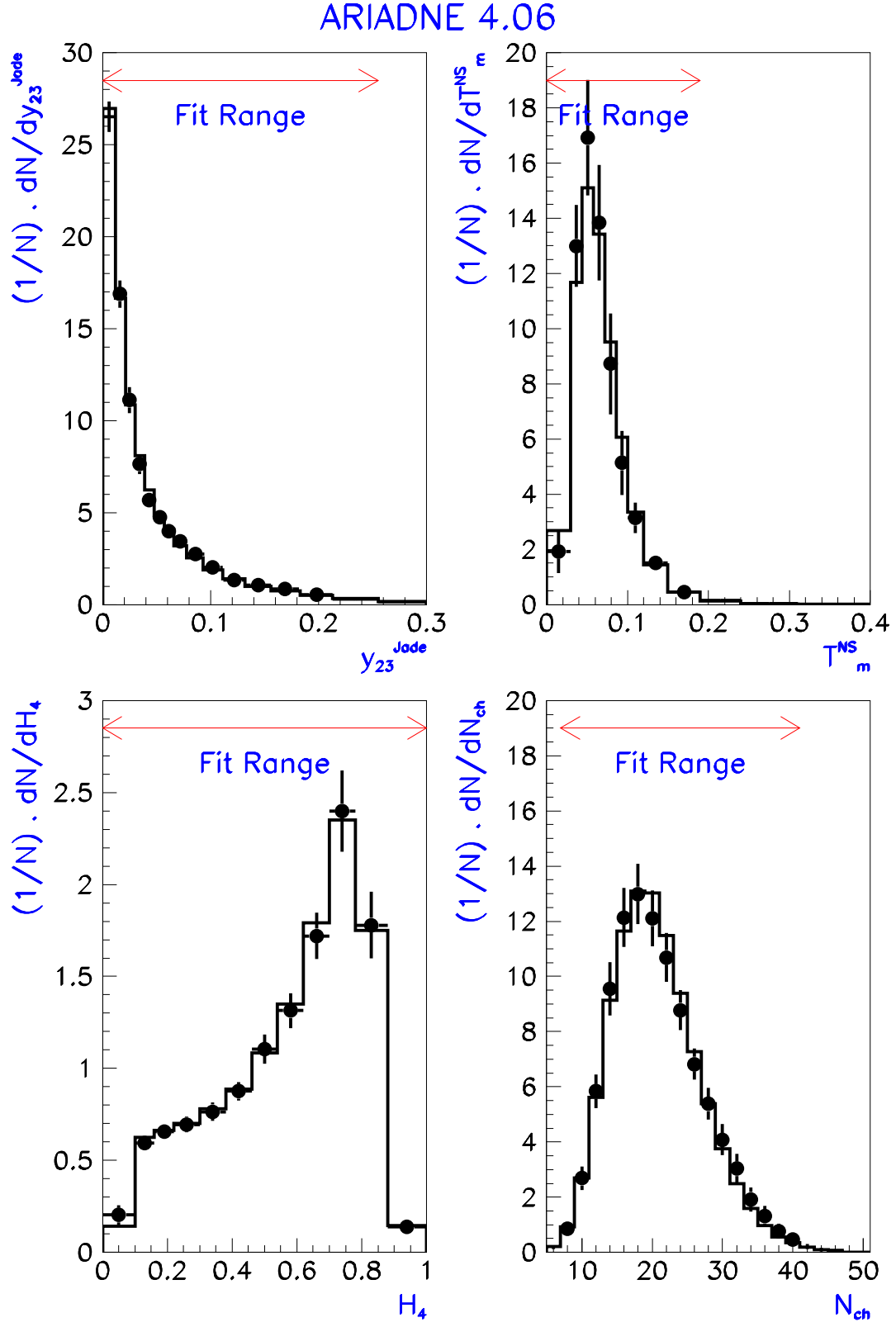


Figure 6.3: Measured distributions of the tuning variables together with the predictions of ARIADNE 4.06 Program using the tuned parameters.

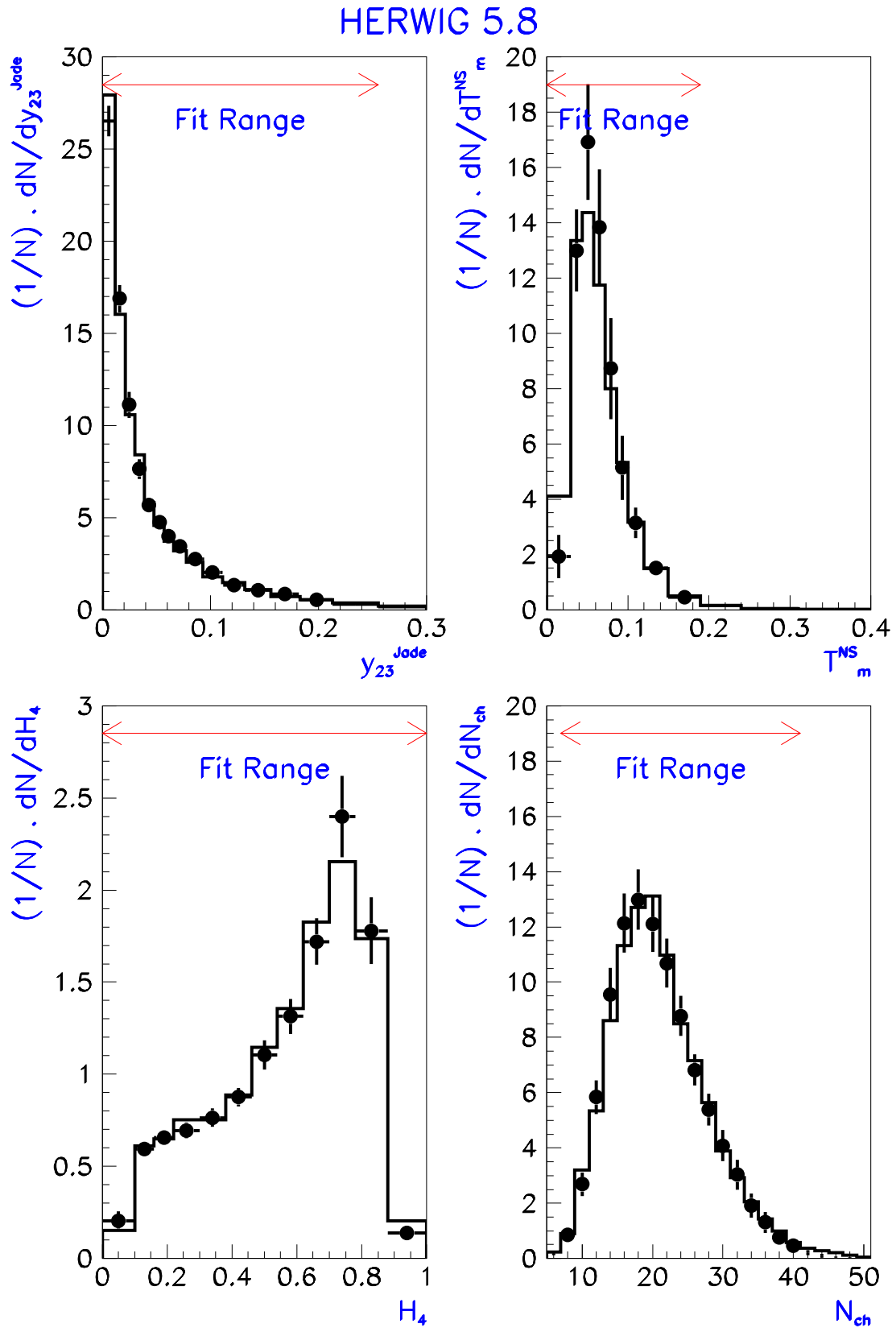


Figure 6.4: Measured distributions of the tuning variables together with the predictions of HERWIG 5.8 Program using the tuned parameters.

6.2 Systematic effects and tuned parameter values

For each of the *three* tuning parameters, grids were generated for *nine* values of these parameters, with a large number of events (≥ 40 K) for each of the $729 (= 9^3)$ choices of the parameter sets, from which the optimal parameter set is chosen corresponding to the minimum χ^2 value for the tuning variables. Initial and final state radiations are switched off in these event generations as the data used has been corrected for initial and final state radiations, in addition to detector (resolution and acceptance) corrections.

For systematic studies, we have repeated the fits by changing:

- (a) the degree of polynomial in the interpolation of the theoretical prediction;
- (b) the grid size by dropping a few grid points – by dropping the i^{th} array from the 729 points grid:

$$512 = \left(\begin{array}{c} 9 - [i]^{th} \\ \text{POINT for} \\ \text{parameter 1} \end{array} \right) \times \left(\begin{array}{c} 9 - [i]^{th} \\ \text{POINT for} \\ \text{parameter 2} \end{array} \right) \times \left(\begin{array}{c} 9 - [i]^{th} \\ \text{POINT for} \\ \text{parameter 3} \end{array} \right)$$

seven sub-grids of 512 points were considered retaining the end values of the original grid in each of these seven sub-grids;

- (c) the fit range of the tuning variables – a suitable choice of 11 cuts (as defined in table 6.3) were considered for comparison of the *tuning* χ^2 over the grid of 729 points.

From each of these possible optimal values of the parameter set, the central value and the statistical error on each parameter is obtained from a cross-fitting procedure (with large number of events $\sim \mathcal{O}(100$ K events) for Monte Carlo distributions) which gives the minimum value of the χ^2 function for all the eighteen event shape variables (226 points as given in table 4.1) during these systematic studies.

The cross-fitting procedures in systematic variations (a) and (c) serve to minimize the bias added by the interpolation algorithm during the fit to the tuning variables over the grid chosen. Additional possible bias due the choice of the grid points is taken care by the variation of the sub-grids chosen in the systematic study (b). For illustration, the systematic variation of the parameters for variation of the degree of interpolating polynomial (JETSET PS), sub-grid variation (ARIADNE) and fit-range variation (JETSET ME) are shown in figures 6.5, 6.6 and 6.7 respectively.

The half of the maximum spread for each parameter is attributed to its systematic error. To be conservative, the largest of the three different estimates is quoted.

CUT #::	y_{23}^{JADE}	$T_{\text{minor}}^{\text{NS}}$	H_4	$\langle n_{\text{ch}} \rangle$	TOTAL
0 :: RANGE :: # OF BINS	0.0 - 0.255 15	0.0 - 0.19 9	0.0 - 1.0 12	7 - 41 17	53
1 :: RANGE :: # OF BINS	0.0 - 0.213 14	0.0 - 0.19 9	0.0 - 1.0 12	7 - 41 17	52
2 :: RANGE :: # OF BINS	0.0 - 0.255 15	0.03 - 0.19 8	0.0 - 1.0 12	7 - 41 17	52
3 :: RANGE :: # OF BINS	0.0 - 0.255 15	0.0 - 0.19 9	0.0 - .88 11	7 - 41 17	52
4 :: RANGE :: # OF BINS	0.0 - 0.255 15	0.0 - 0.19 9	0.0 - 1.0 12	7 - 39 16	52
5 :: RANGE :: # OF BINS	0.0 - 0.255 15	0.03 - 0.15 7	0.0 - 1.0 12	7 - 41 17	51
6 :: RANGE :: # OF BINS	0.0 - 0.255 15	0.0 - 0.19 9	0.1 - .88 10	7 - 41 17	51
7 :: RANGE :: # OF BINS	0.0 - 0.255 15	0.03 - 0.19 8	0.0 - .88 11	7 - 41 17	51
8 :: RANGE :: # OF BINS	0.0 - 0.213 14	0.03 - 0.19 8	0.0 - 1.0 12	7 - 41 17	51
9 :: RANGE :: # OF BINS	0.0 - 0.255 15	0.03 - 0.19 8	0.1 - .88 10	7 - 41 17	50
10 :: RANGE :: # OF BINS	0.0 - 0.213 14	0.03 - 0.19 8	0.1 - .88 10	7 - 41 17	49

Table 6.3: Convention for varying fitting ranges.

Parameter	Fitted Value	Stat. Error	Syst. Error
Λ_{LLA} (GeV)	0.311	± 0.022	± 0.026
σ_{Q} (GeV)	0.411	± 0.019	± 0.028
b (GeV ⁻²)	0.886	± 0.060	± 0.104

Table 6.4: Tuned Parameters for the JETSET 7.4 Parton Shower Program.

Parameter	Fitted Value	Stat. Error	Syst. Error
Λ_{ME} (GeV)	0.152	± 0.005	± 0.005
σ_{Q} (GeV)	0.430	± 0.015	± 0.021
b (GeV ⁻²)	0.310	± 0.014	± 0.007

Table 6.5: Tuned Parameters for the JETSET 7.4 Matrix Element Program.

Parameter	Fitted Value	Stat. Error	Syst. Error
Λ_{LLA} (GeV)	0.254	± 0.013	± 0.020
σ_{Q} (GeV)	0.384	± 0.017	± 0.018
b (GeV ⁻²)	0.772	± 0.034	± 0.067

Table 6.6: Tuned Parameters for the ARIADNE 4.06 Program.

Parameter	Fitted Value	Stat. Error	Syst. Error
Λ_{MLLA} (GeV)	0.166	± 0.002	± 0.015
CLMAX (GeV)	2.968	± 0.055	± 0.105
CLPOW	1.569	± 0.020	± 0.219

Table 6.7: Tuned Parameters for the HERWIG 5.8 Program.

The tuned parameter sets [77] of JETSET 7.4 (Parton Shower and Matrix Element), ARIADNE 4.06 and HERWIG 5.8 models are presented in tables 6.4, 6.5, 6.6 and 6.7.

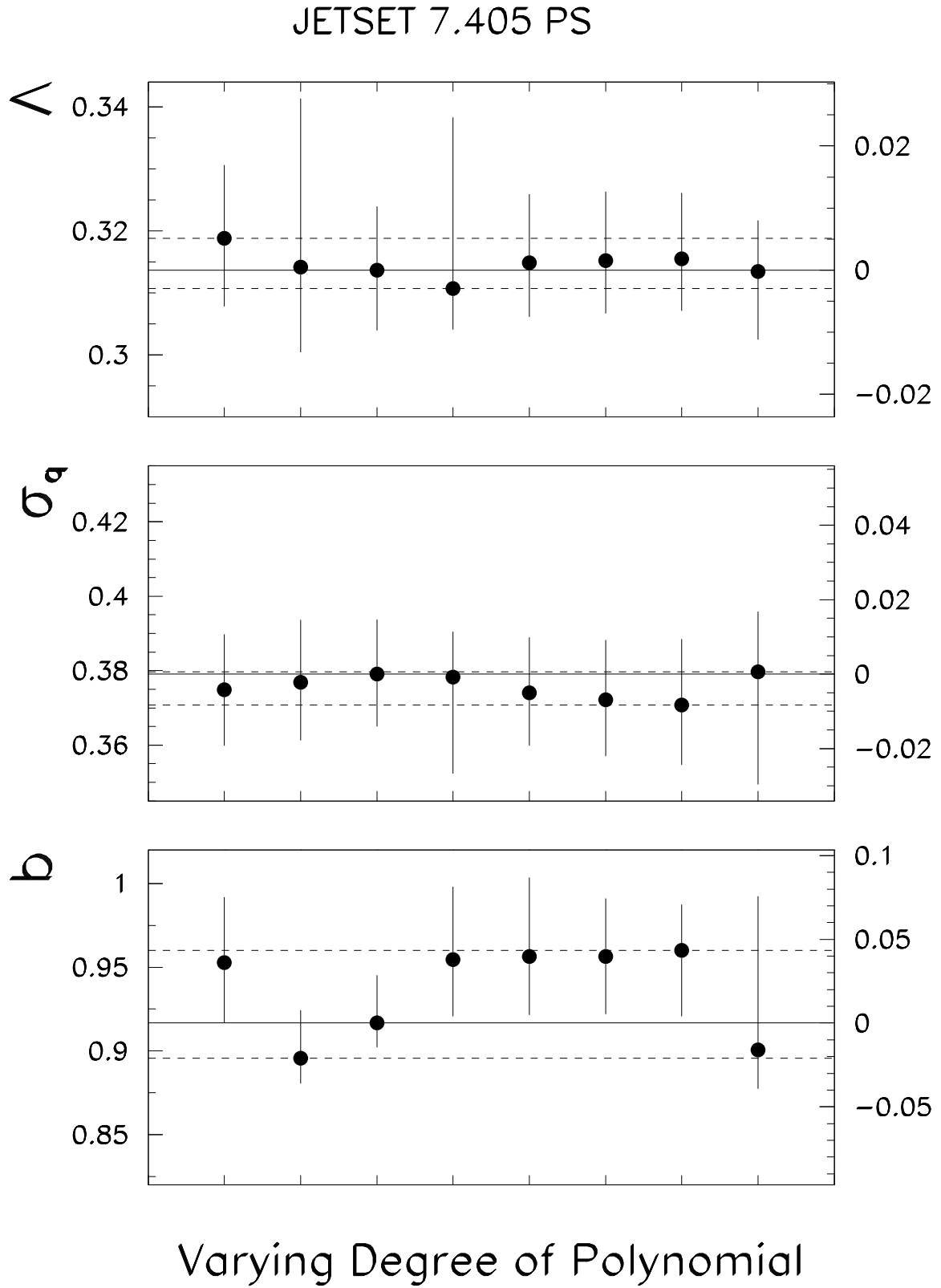


Figure 6.5: Degree of the interpolating polynomial variations of parameter values of the JETSET 7.4 Parton Shower Program.

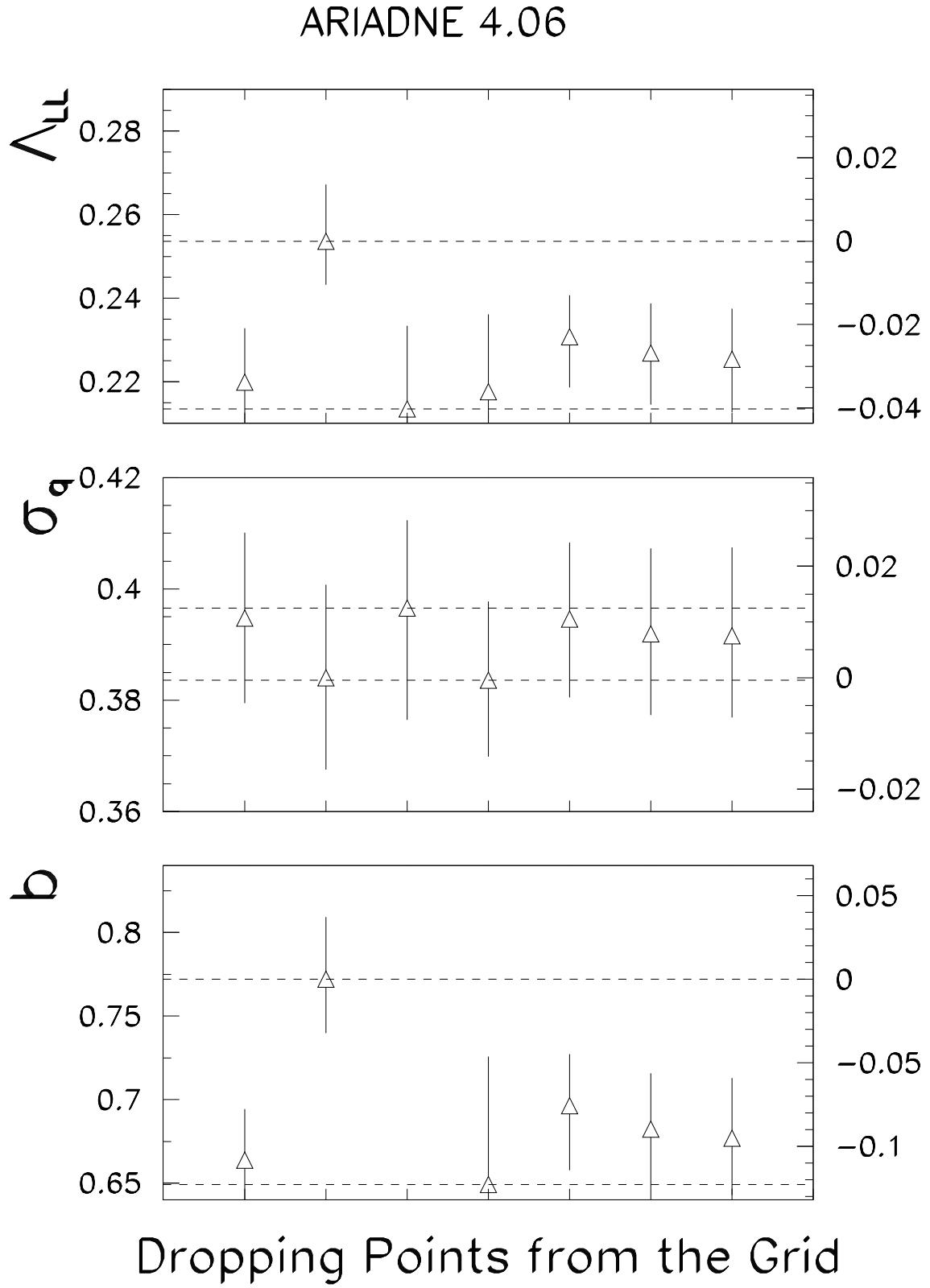


Figure 6.6: Sub-grid variations of parameter values of the ARIADNE 4.6 Program.

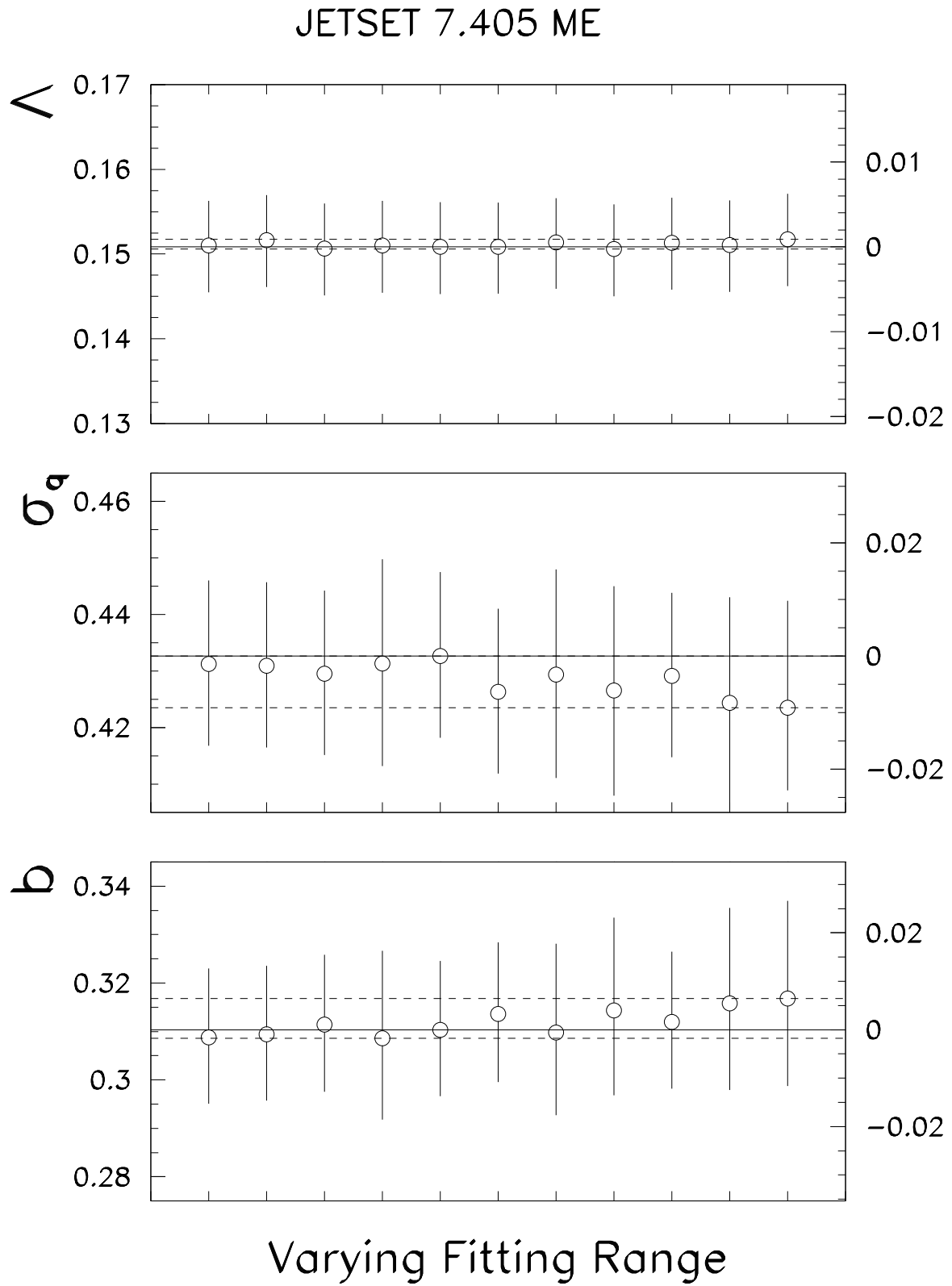


Figure 6.7: Range of tuning variables variations of parameter values of the JETSET 7.4 Matrix Element Program.

Chapter 7

Summary

The parameters of several QCD models have been tuned with the hadronic decays of Z measured by the L3 experiment at LEP. Global event shape variables were used to tune JETSET 7.4 Parton Shower and Matrix Element models, ARIADNE 4.06 and HERWIG 5.8.

This study [77] differs from the previous tunings [49, 50] in the following way :

- use of charge multiplicity distribution in addition to y_{23}^{JADE} , $T_{\text{minor}}^{\text{NS}}$ and H_4 ;
- use of multi-dimensional polynomial interpolation of arbitrary degree instead of linear interpolation;
- performing cross checks with ‘direct’ fitting method [rerun MC with $\approx \mathcal{O}(100 \text{ K})$ events];
- use of a very large grid (729 points).

The retuned models give a somewhat better agreement with the data. For example, the evolution of measured mean charged multiplicity with CM energy ($\sqrt{s}$) is in better agreement with the predictions of ARIADNE 4.06, as may be seen from figure 7.1. This agreement gives a better description of the non-perturbative phase in the energy evolution of an event as observed in the data in terms of the retuned Monte Carlo. No significant improvement is observed in other variables, as can be seen by comparing the energy evolution of the mean value of $(1-\text{Thrust})$ for example (see figure 7.2). The low energy measurements of the mean values are taken from [78], and the LEP I and LEP II data are taken from [49, 79] for L3 and from [80] for other LEP experiments.

The ratios of expected to observed 4-jet rate with previous and currently tuned parameter sets as a function of the jet resolution parameter y_{cut} for JADE and $k_{\perp}$ algorithms are shown in figures 5.8 and 7.3 respectively. One observes an improvement in the agreement with ARIADNE and HERWIG where the ratio is close to unity within 5%. However,

no improvement is observed in the prediction of the JETSET 7.4 PS model where the discrepancy still stays at the level of 5-20%.

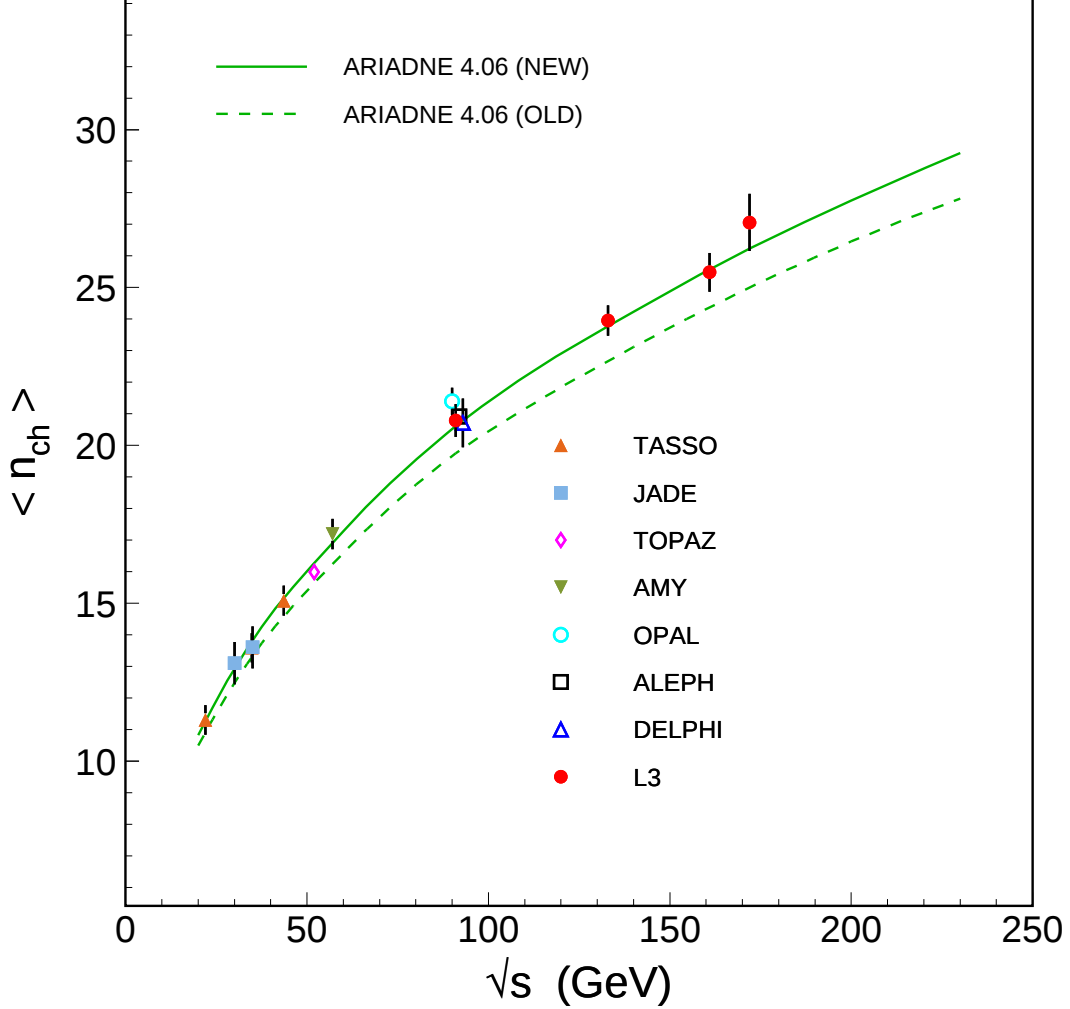


Figure 7.1: Center-of-mass evolution of measured mean charge multiplicity as compared to the ARIADNE Monte Carlo with previously [49] (OLD: dashed line) and currently [77] (NEW: solid line) tuned parameters.

In conclusion, one can note that all QCD parton shower models with coherence effect accounted for describe the data quite well, whereas the matrix element model, presently being restricted to only second order perturbation theory calculations with a provision of producing upto a maximum of four partons in the perturbative phase, fails to describe the data over a wide range of energy.

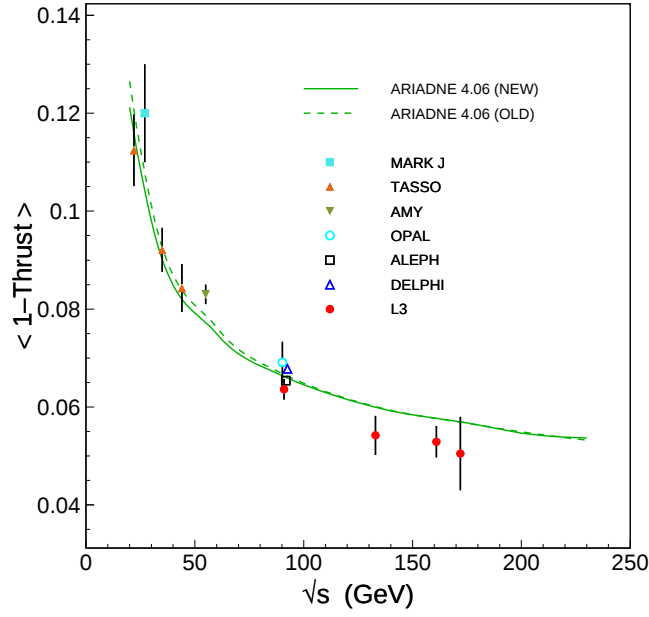


Figure 7.2: Center-of-mass evolution of measured $\langle 1-\text{Thrust} \rangle$ as compared to the ARIADNE Monte Carlo with previously [49] (OLD: dashed line) and currently [77] (NEW: solid line) tuned parameters.

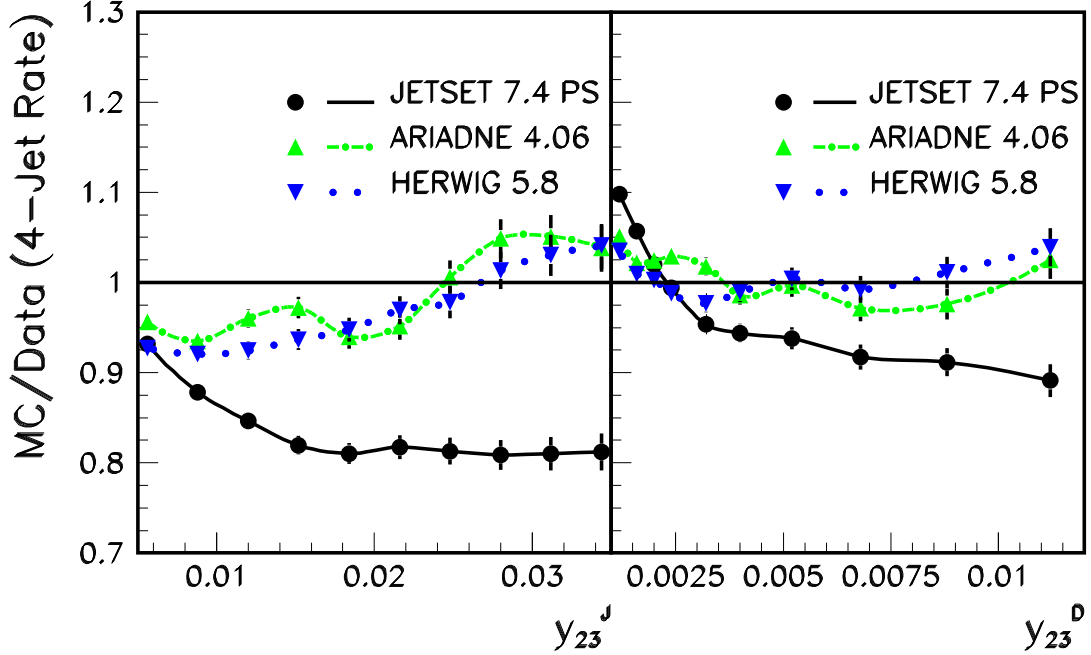


Figure 7.3: Ratio of 4-jet rate of data and MC with currently tuned parameters as function of jet resolution parameter for JADE and $k_{\perp}$ algorithms.

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- [1] Tuning of QCD Model Parameters using Hadronic Z decay data from LEP
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Talks/Seminars given

- Tuning of QCD model parameters using the LEP data of Hadronic Z decays.
– Presented at XII DAE Symposium on High Energy Physics, Guwahati, Dec. 1996.

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