

The stability of electron-positron jets in laboratory plasmas



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Abstract

Relativistic electron-positron plasmas are found in extreme astrophysical environments such as black hole and neutron star magnetospheres, where accretion-powered jets and pulsar winds are expected to be enriched with electron-positron pairs. Their role in the dynamics of these environments is in many cases believed to be fundamental, but their collective plasma behaviour is expected to differ significantly from typical electron-ion plasmas due to the matter-antimatter symmetry of the charged components. Therefore, many attempts have been made over previous decades to generate electron-positron plasmas and study them in the laboratory. But so far, the challenge of producing large enough yields of positrons in quasi-neutral beams has restricted understanding to simple numerical and analytical studies. In this thesis, a novel scheme for generating high-density, quasi-neutral, relativistic electron-positron pair beams has been developed, using an ultra-relativistic proton beam at CERN's Super Proton Synchrotron (SPS). The achieved increase in electron-positron yield is orders of magnitude greater than neutral pair beams reported at high-power laser facilities, representing a key milestone, as the beams exceed characteristic scales necessary for collective plasma behaviour. This opens up the possibility of directly probing the microphysics of pair plasmas beyond quasi-linear evolution into regimes that are challenging to simulate or measure via astronomical observations. In the first application of this new experimental platform, pair beam-plasma instabilities are investigated by propagating the pair beam through a metre-length plasma. Due to their fast growth rates, pair beam-plasma instabilities are invoked to explain observations of γ -ray bursts and active galactic nuclei (AGN) jets, but theories expect the growth rates to be drastically reduced when non-idealized conditions are considered. The assertion that such instabilities can explain the lack of observed cascade γ -ray emission from TeV-blazars has been challenged using simplistic theoretical arguments, but in this work, theories are tested in the laboratory for the first time.

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This thesis is dedicated to the memories of Oscar Jakobsson and Rebecca Pilot.

Role of the author

The results presented in this thesis concern an experiment carried out at the CERN HiRadMat Facility (Switzerland/France) in May 2023, conceived by G. Gregori and R. Bingham in Spring 2020. The author's contributions are clarified here.

In Chapters 1 and 2, Monte-Carlo simulations are presented, performed by the author, simulating the generation of electron-positron pair beams using protons extracted from the Super Proton Synchrotron at CERN. These simulations culminated in a paper published in Physical Review Research describing the proposed experimental scheme. In this paper, initial two-dimensional particle-in-cell simulations of pair beam interactions with a plasma were performed by N. Shukla and L. O. Silva. Subsequently, the integration of the experimental design was performed by the author, with the assistance of G. Gregori, R. Bingham, N. Charitonidis and P. Simon. FLUKA simulations of the thermal loading on different target designs were performed by the author, and Ansys Mechanical finite-element simulations of the thermal stresses were performed by P. Simon. The radiological assessment of the experiment was carried out by the author. In Chapter 3, the design and construction of an inductively coupled plasma discharge is presented. The final design, construction, and characterization were the author's work, with guidance provided by A. Dyson, J. T. Gudmundsson and G. Gregori, published in the IOP Journal of Instrumentation. In Chapters 4 and 5, the results of the experiment carried out at HiRadMat in May 2023 are presented. The design of experimental diagnostics was performed by the author, with the assistance of P. Simon, S. Burger, G. Marshall, T. Hodge and G. Gregori. CAD drawings of the experiment integration were produced by V. Clerc and A. E. Rahmoun. The experimental setup was carried out by the author, along with P. Simon, N. Charitonidis, V. Stergiou, S. Burger, A. Goillot, A. E. Rahmoun, T. Hodge, R. Simpson, and S. Iaquinta. The experiment was carried out with the additional support of P. J. Bilbao, F. D. Cruz,

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First author publications

- Arrowsmith, C. D. et al., “Suppression of pair beam instabilities in a laboratory analogue of blazar pair cascades”, (in. prep.).
- Arrowsmith, C. D. et al., “Laboratory realization of relativistic pair-plasma beams”, Nature Communications **15**, 5029 (2024).
- Arrowsmith, C. D. et al., “Inductively coupled plasma discharge for use in high energy density science experiments”, Journal of Instrumentation **18**, P04008 (2023).
- Arrowsmith, C. D. et al., “Generating ultra-dense pair beams using 400 GeV/c protons”, Physical Review Research **3**, 023103 (2021).

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Chapter 1

Kinetic beam-plasma instabilities of electron-positron jets

1.1 Pair creation in the Universe

Dive into the heart of an active galaxy and you will find a supermassive black hole consuming matter from its surroundings [1]. In a fraction of these cases the accretion process leads to the launching of highly relativistic jets from the poles, for instance via the extraction of energy from magnetic fields in the accretion disk [2], or from the black hole rotation itself [3]. Though the supermassive black hole may have an event horizon comparable in size to the solar system (several light hours in size), emission from the jets may propagate for millions of years through the intergalactic medium (IGM), often through cosmic voids that exist between galaxies and galaxy clusters, corresponding to some of the lowest density regions in the Universe (less than one atom per cubic metre). Blazars are a class of powerful active galactic nuclei with relativistic jets pointed towards Earth [5]. Discovered in the early days of radio astronomy, their emission was deemed so bright that it was assumed to originate from within the galaxy, until measurement resolution improved and counterpart optical emission of extragalactic origin was identified [6]. From TeV-blazars, TeV-energy γ -rays can be detected using ground-based Cherenkov telescope arrays such as VERITAS, MAGIC and H.E.S.S. [7, 8], which image the flash of Cherenkov radiation emitted by showers of relativistic charged particles produced when very high energy γ -rays impact the atmosphere. But on

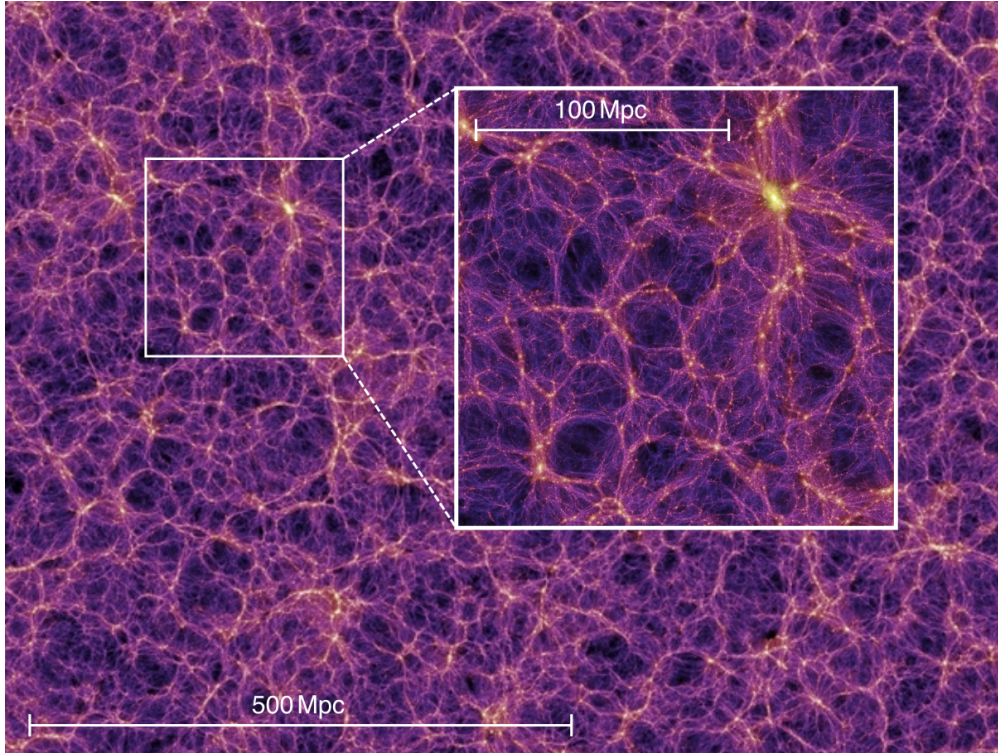


Figure 1.1: Large-scale structure in the Universe. (Adapted from Springel et al. [4].) An N -body simulation shows the emergence of large-scale structure in the Universe (the Millennium Simulation), with galaxies and galaxy clusters forming a filamentary structure, separated by cosmic voids. Supermassive black holes powering blazar jets can be found at the centre of active galaxies.

their journey through the IGM, the majority of TeV γ -rays are expected to scatter with diffuse extragalactic background light, triggering cascades of electron-positron ($e^\pm$) pairs, and subsequently a reprocessed spectrum of lower energy GeV γ -rays as pairs inverse-Compton scatter with Cosmic Microwave Background (CMB) photons [9]. But this prediction is at odds with measurements of the Fermi/LAT space observatory accumulated over more than a decade [7, 10, 11]. Understanding the fate of streaming, relativistic pair beams through ambient, collisionless plasmas, and whether plasma processes play a role in explaining such astronomical observations, is the key motivation of the work presented in this thesis.

Blazar jets are by no means the only place in the Universe where relativistic electron-positron pairs are produced. More generally, pairs can be created in environments of intense γ -ray flux, where scattering of sufficiently energetic γ -rays ($E_\gamma \geq 2m_e c^2 = 1.022 \text{ MeV}$) leads to pair production via the linear Breit-Wheeler process [12]. Alternatively, pairs can be produced via the strong-field Breit-Wheeler process when electromagnetic fields are comparable to the Schwinger field, the critical strengths of electric and magnetic fields for vacuum breakdown [13, 14] ($E_{\text{crit}} = m_e^2 c^3 / e \hbar = 1.3 \times 10^{18} \text{ V m}^{-1}$, $B_{\text{crit}} = m_e^2 c^2 / e \hbar = 4.4 \times 10^9 \text{ T}$). Such conditions can be found in the magnetospheres of black holes [15] and neutron stars [16].

Whether pair plasma processes play a role in energy dissipation and the observed radiative emission in these environments often concerns the timescales of kinetic plasma instabilities. Plasma systems with anisotropic features present a risk of instability, for instance those with temperature anisotropies and those with particles streaming through an ambient bulk (so-called bump-on-tail features). In these instances, free energy can be converted into electromagnetic fields via plasma instabilities as a means to restore momentum space isotropy. For a jet enriched with $e^\pm$ pairs propagating through an ambient plasma, this

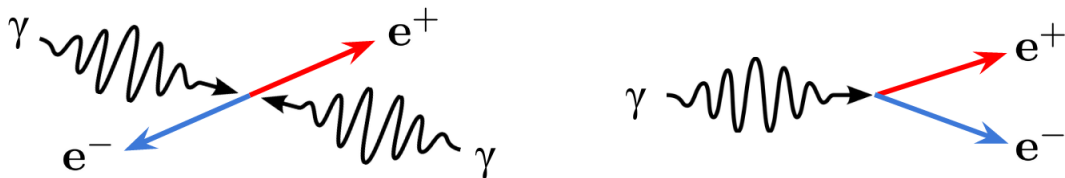


Figure 1.2: Linear and strong-field Breit-Wheeler processes.

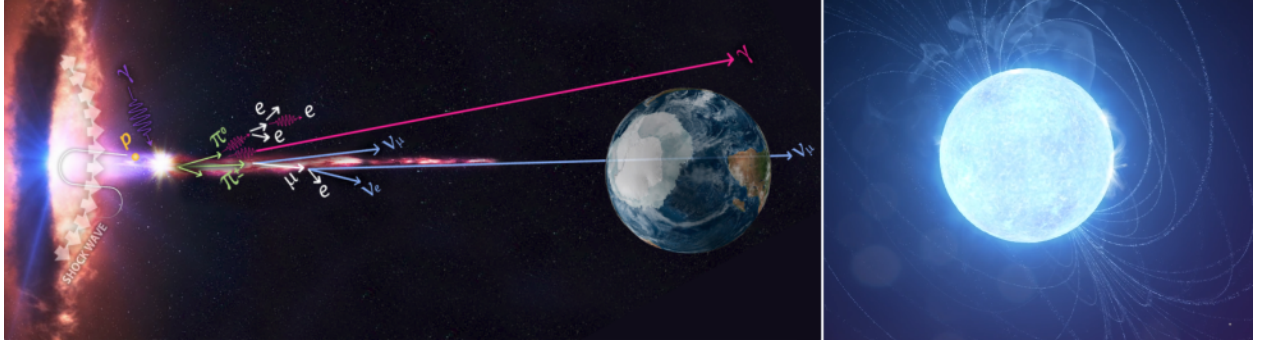


Figure 1.3: Pair creation in the Universe. Artist impressions of astrophysical environments where $e^\pm$ pair generation can occur: in a blazar jet (left), and in the magnetosphere of a magnetar, a neutron star with extremely strong magnetic fields (right). In the blazar jet, an electromagnetic cascade of electrons, positrons and γ -rays develops via photon-photon scattering and inverse-Compton scattering of pairs with cosmic microwave background photons. In the magnetar magnetosphere, magnetic fields are strong enough to exceed the critical field strength for vacuum breakdown. Image credits: IceCube/NASA, NASA's Goddard Space Flight Center/Chris Smith (USRA).

can manifest as the current separation of electrons and positrons (filamentation), which is depicted by a simulation shown in Figure 1.4. Spontaneous fluctuations of current due to thermal motion produces magnetic fields, which leads to amplification of the current fluctuations. Bulk kinetic energy of the pairs is dissipated, and electric and magnetic fields grow exponentially.

Due to the rapid theoretical growth rate of these instabilities, they are commonly invoked as a mechanism capable of generating strong magnetic fields that can explain various astrophysical emission signatures, such as those produced by γ -ray bursts and fast radio bursts [17–21]. But when non-idealized conditions are accounted for, such as finite divergence and energy spread, theoretical predictions are that the precise growth rates and development beyond quasi-linear phases of evolution can be dramatically affected. Notably, they can become significantly suppressed [22, 23]. This is particularly important in astrophysical

scenarios, where streaming pairs beams are neither perfectly collimated, nor monoenergetic.

In the remainder of this chapter, the basic framework of kinetic plasma theory is introduced, and the suppression of beam-plasma instabilities is illustrated by evaluating the stability of two simple example beam-plasma systems. Finally, the use of simulations and experiments to study non-linear plasma behaviour will be discussed.

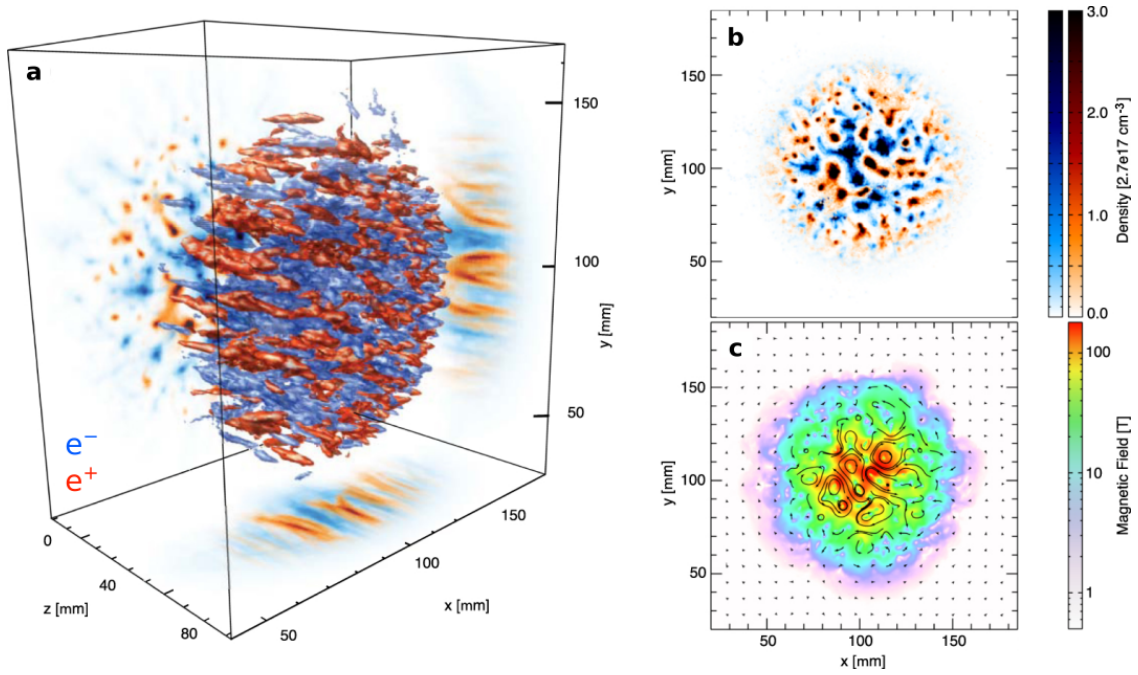


Figure 1.4: 3D particle-in-cell simulation showing filamentation of an e^+e^- beam. (Adapted from Shukla et al. [24].) (a) Isosurfaces show the density of electron (blue) and positron (red) filaments formed from an initially homogenous neutral pair beam after propagating through 10 cm of ambient plasma. The pair beam density ($n_{\pm}$) and plasma density (n_p) are equal: $n_{\pm}, n_p = 2.7 \times 10^{17} \text{ cm}^{-3}$. (b) A 2D central slice of the beam density. (c) A 2D central slice of the transverse magnetic fields, $B_{\perp} = \sqrt{B_x^2 + B_y^2}$.

1.2 The kinetic description of a plasma

First of all, we define a plasma as a quasi-neutral ionised gas (a fluid containing free charges), characterized by large-scale collective behaviour due to the mediation of long-range electric and magnetic fields. The repulsion of like charges and attraction of opposing charges leads to a screening effect of the Coulomb potential. If energy states are arranged according to a Boltzmann distribution in thermodynamic equilibrium, this screening length scale is called the Debye length. Summing over species α ,

$$\lambda_D \equiv \left(\sum_{\alpha} \frac{4\pi n_{\alpha} q_{\alpha}^2}{k_B T_{\alpha}} \right)^{-1/2}. \quad (1.1)$$

Provided the number of charges within a Debye screening volume is sufficiently large:

$$n\lambda_D^3 \gg 1, \quad (1.2)$$

then the motion of particles will be primarily decided by the collective influence of large numbers of charges; macroscopic electric and magnetic fields, as opposed to binary two-particle collisions, which is the case in a neutral gas. An important quantity is the plasma frequency:

$$\omega_{p\alpha} = \left(\frac{4\pi n_{\alpha} q_{\alpha}^2}{m_{\alpha}} \right)^{1/2}, \quad (1.3)$$

the natural frequency of plasma oscillations due to the electrostatic restoring force when a plasma is perturbed from quasi-neutrality. Much wave-like phenomena in collisionless

plasmas occurs on timescales comparable to the plasma period, ω_p^{-1} . The plasma skin depth: $\lambda_s \equiv c/\omega_p$ is another characteristic length scale, corresponding to the attenuation length of electromagnetic waves in a plasma. The physical size of a plasma must exceed the Debye screening length and the plasma skin depth ($\ell > \lambda_D, \lambda_s$) to exhibit collective wave-like phenomena.

The evolution of a plasma using kinetic theory concerns the behaviour of the Klimontovich distribution function, F_α , a function defining the state of the plasma according to the positions and momenta of all the constituent particles. For particles i of a species α at any given moment:

$$F_\alpha(\mathbf{r}, \mathbf{p}, t) = \sum_{i=1}^{N_\alpha} \delta^3(\mathbf{r} - \mathbf{r}_i^{(\alpha)}(t)) \delta^3(\mathbf{p} - \mathbf{p}_i^{(\alpha)}(t)), \quad (1.4)$$

where δ is a Dirac delta function. The damping or growth of waves in a plasma occurs primarily via their interaction with particles whose velocities coincide with the phase velocity of the wave (Landau resonance), whilst particles moving sufficiently far out-of-phase with the plasma wave experience no mean energy change over time. Hence, whether a wave is amplified depends intimately on the precise shape of the distribution function.

To eliminate the need to know the exact trajectories of particles due to microscopic fields (which fluctuate on scales $\ll \lambda_D$), a course-graining is performed by averaging the distribution function and the electromagnetic fields over some intermediate scale, l , which is

smaller than the Debye length but larger than the interparticle separation:

$$n^{-1/3} \ll l \ll \lambda_D, \quad (1.5)$$

and the distribution function is dealt with in terms of its average over microscales, f_α . The evolution of a plasma is described by the Vlasov-Landau equation:

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + q_\alpha (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_\alpha}{\partial \mathbf{p}} = \left(\frac{\partial f_\alpha}{\partial t} \right)_c, \quad (1.6)$$

where $\mathbf{E}$ and $\mathbf{B}$ are electric and magnetic fields (averaged over microscales) satisfying Maxwell's equations:

$$\nabla \cdot \mathbf{E} = 4\pi\rho, \quad \nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad \nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \mathbf{J}. \quad (1.7)$$

Current and charge density are written in terms of the distribution function as:

$$\mathbf{J} = \sum_\alpha q_\alpha n_\alpha \int \mathbf{v} f_\alpha d\mathbf{p}, \quad \rho = \sum_\alpha q_\alpha n_\alpha \int f_\alpha d\mathbf{p}. \quad (1.8)$$

The term $(\partial f_\alpha / \partial t)_c$ is the collision operator, which can be ignored for collisionless plasmas provided the frequency of collisions is much smaller than the plasma frequency. The Vlasov-Maxwell system of equations is highly non-linear, so analytical progress is restricted. The simplest treatment involves solving a closed set of linearized forms for the equations (the so-called linear theory). Perturbations of an initial distribution function, $f_\alpha^{(0)}$, are

considered of the form $f_\alpha^{(1)} \propto \exp(i\mathbf{k} \cdot \mathbf{x} - i\omega t)$, and the analysis is valid provided $f_\alpha^{(1)}$ remains a sufficiently small perturbation of $f_\alpha^{(0)}$. $f_\alpha^{(1)}$ is written in terms of $f_\alpha^{(0)}$ using the linearized Vlasov equation:

$$\begin{aligned} -i\omega f_\alpha^{(1)} + i\mathbf{v} \cdot \mathbf{k} f_\alpha^{(1)} + q_\alpha \left(\mathbf{E}^{(1)} + \mathbf{v} \times \mathbf{B}^{(1)} \right) \cdot \frac{\partial f_\alpha^{(0)}}{\partial \mathbf{p}} &= 0 \\ \longrightarrow f_\alpha^{(1)} &= \frac{iq_\alpha}{\mathbf{k} \cdot \mathbf{v} - \omega} \left(\mathbf{E}^{(1)} + \mathbf{v} \times \mathbf{B}^{(1)} \right) \cdot \frac{\partial f_\alpha^{(0)}}{\partial \mathbf{p}}. \end{aligned} \quad (1.9)$$

With the current density written $\mathbf{J}^{(1)} = \sum_\alpha q_\alpha n_\alpha \int \mathbf{v} f_\alpha^{(1)} d\mathbf{p}$, closure is achieved using linearized forms of Ampère's law and Faraday's law:

$$i\mathbf{k} \times \mathbf{B}^{(1)} = -i\frac{\omega}{c} \mathbf{E}^{(1)} + \frac{4\pi}{c} \mathbf{J}^{(1)}, \quad (1.10)$$

$$i\mathbf{k} \times \mathbf{E}^{(1)} = i\frac{\omega}{c} \mathbf{B}^{(1)}. \quad (1.11)$$

Combining these equations, finding solutions amounts to solving the equation:

$$\mathcal{D} \cdot \mathbf{E}^{(1)} = 0, \quad (1.12)$$

where all the terms have been combined into the tensor $\mathcal{D}$:

$$\mathcal{D}_{ij} = k_i k_j - k^2 \delta_{ij} + \omega^2 \varepsilon_{ij}, \quad (1.13)$$

and $\varepsilon_{ij}(\mathbf{k}, \omega)$ is the dielectric tensor with components:

$$\varepsilon_{ij}(\mathbf{k}, \omega) = \delta_{ij} + \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} \int d\mathbf{p} \frac{p_i}{\gamma} \cdot \frac{\partial f_{\alpha}^{(0)}}{\partial p_j} + \sum_{\alpha} \frac{\omega_{p\alpha}^2}{\omega^2} \int d\mathbf{p} \frac{p_i p_j}{\gamma} \frac{\mathbf{k} \cdot \partial f_{\alpha}^{(0)} / \partial \mathbf{p}}{m\omega\gamma - \mathbf{k} \cdot \mathbf{p}}, \quad (1.14)$$

where $\gamma = (1 - \beta)^{-1/2} = \sqrt{1 + (p/m_{\alpha}c)^2}$ and $\beta = v/c$ are the relativistic Lorentz factors.

Dispersion relations are obtained by evaluating the characteristic equation $\det \mathbf{D} = 0$ for a given initial distribution function, $f_{\alpha}^{(0)}$.

The form of the dispersion relations is simplified without loss of generality by restricting the examination to perturbations with wavevectors in the x - z plane, $\mathbf{k} = (k_x, 0, k_z)$:

$$\mathbf{D} = \begin{bmatrix} \frac{\omega^2}{c^2} \varepsilon_{xx} - k_z^2 & 0 & \frac{\omega^2}{c^2} \varepsilon_{zx} + k_x k_z \\ 0 & \frac{\omega^2}{c^2} \varepsilon_{yy} - k^2 & 0 \\ \frac{\omega^2}{c^2} \varepsilon_{xz} + k_x k_z & 0 & \frac{\omega^2}{c^2} \varepsilon_{zz} - k_x^2 \end{bmatrix}. \quad (1.15)$$

Evaluating the determinant gives the general solution:

$$\underbrace{\left(\omega^2 \varepsilon_{yy} - k^2 c^2 \right)}_{\text{Electromagnetic waves}} \underbrace{\left[\left(\omega^2 \varepsilon_{xx} - k_z^2 c^2 \right) \left(\omega^2 \varepsilon_{zz} - k_x^2 c^2 \right) - \left(\omega^2 \varepsilon_{xz} - k_x k_z c^2 \right)^2 \right]}_{\text{Growing/damped electromagnetic perturbations}} = 0. \quad (1.16)$$

In the literature, unstable wave solutions in beam-plasma (bump-on-tail) systems are categorized according to whether the wavevector is oriented parallel, perpendicular or oblique to the beam direction.

Choosing z as parallel to the beam propagation:

1. Longitudinal modes ($\mathbf{k} = k\hat{\mathbf{e}}_z$): $(\omega^2\varepsilon_{xx} - k^2c^2)\omega^2\varepsilon_{zz} = 0$

- Two-stream instability [25, 26] – an electrostatic instability. The growth of longitudinal plasma waves leads to a longitudinal electric field structure and bunching of the beam into space charges. It is sometimes referred to as ‘hydrodynamic beam instability’.

2. Transverse modes ($\mathbf{k} = k\hat{\mathbf{e}}_x$): $\varepsilon_{xx}(\omega^2\varepsilon_{zz} - k^2c^2) - \omega^2\varepsilon_{xz}^2 = 0$

- Weibel instability [27] – for a distribution function with a temperature anisotropy. Modulations grow transversely to the higher temperature axis of the distribution function. It can be present in a plasma that has no bulk flow.
- Current filamentation [28] – similar to Weibel instability, modulations grow transversely to the primary axis of anisotropy, but this case refers to when the distribution function has bump-on-tail features (such as injected particle beams). Electromagnetic fields grow convectively with the beam and the beam separates into current filaments.

3. Oblique modes ($\mathbf{k} = k_x\hat{\mathbf{e}}_x + k_z\hat{\mathbf{e}}_z$):

- Oblique instability [29] – The generalized version of current filamentation instability with arbitrarily pointing $\mathbf{k}$. For a dilute, collimated, relativistic, monoenergetic beam propagating through a cold ambient plasma the fastest growing modes are oblique to the beam propagation.

1.3 The stability of transverse modes in two simple beam-plasma systems

The stability of transverse modes is now considered for two simple beam-plasma systems with slightly different beam-particle distribution functions:

1. Cold beam - a neutral beam composed of an equal number of electrons and positrons with a negligible thermal spread.
2. Finite transverse beam temperature - identical, except the beam particles now have a finite transverse momentum spread.

In both systems the ambient plasma has a zero-order electron distribution function described in terms of Dirac delta functions:

$$f_p^{(0)} = \delta(p_x)\delta(p_y)\delta(p_z), \quad (1.17)$$

with a neutralizing population of ions rendered immobile for simplicity. The momentum space of the two systems is illustrated in Figure 1.5.

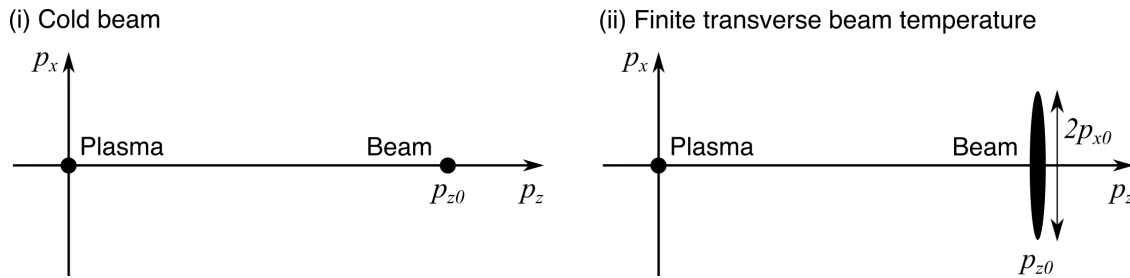


Figure 1.5: Linear stability in two simple cases. Kinetic theory is used to assess the linear stability of a monoenergetic, neutral beam propagating through a plasma. (i) The beam is cold (monoenergetic and collimated). (ii) The beam has a finite transverse momentum spread.

Case 1: Cold beam

Beam particles have a distribution function:

$$f_{\mathbf{b}}^{(0)} = \delta(p_x)\delta(p_y)\delta(p_z - p_{z0}), \quad (1.18)$$

where $\gamma = \sqrt{1 + u^2}$ and $u = \gamma\beta = p_{z0}/m_e c$. The dispersion relation for transversely growing modes is evaluated:

$$\omega^2 \varepsilon_{zz} - k^2 c^2 = 0 \quad \longrightarrow \quad \omega^2 - k^2 - 1 - \frac{\alpha}{\gamma} + \frac{\alpha}{\gamma} \beta^2 - \frac{\alpha k^2 \beta^2}{\gamma \omega^2} = 0, \quad (1.19)$$

where the frequency ω has been normalized to ω_p , the wavevector is expressed in units of inverse collisionless skin depth, $(c/\omega_p)^{-1}$, and the plasma frequencies have been replaced with the beam-plasma density ratio, $\alpha = n_b/n_p$. Solving this bi-quadratic equation gives:

$$\omega^2 = \left[k^2 + 1 + \frac{\alpha}{\gamma} (1 - \beta^2) \right] \pm \sqrt{\left[k^2 + 1 + \frac{\alpha}{\gamma} (1 - \beta^2) \right]^2 + 4 \left(\frac{\alpha}{\gamma} \beta^2 k^2 \right)}. \quad (1.20)$$

Unstable solutions correspond to when $\omega^2 < 0$, which is the case for all wavenumbers. The growth rate of the fastest growing mode, $\Gamma_{\max} = \text{Max} [\text{Im}(\omega)]$, is efficiently found by taking the partial derivative of Eq. 1.19, for instance $\partial(\omega^2)/\partial(k^2) = 0$:

$$\longrightarrow \quad \Gamma_{\max} = \beta \sqrt{\frac{\alpha}{\gamma}} \omega_p, \quad (1.21)$$

with a characteristic scale comparable to the skin depth of the ambient plasma: $k^{-1} \sim \lambda_s$.

Case 2: Finite transverse beam temperature

Following Ref. [23], a beam is considered which has a finite transverse momentum spread, represented by a waterbag distribution:

$$f_b^{(0)} = \left\{ \frac{1}{2p_{x0}} [\Theta(p_x - p_{x0}) - \Theta(p_x - p_{x0})] \right\} \delta(p_y) \delta(p_z - p_{z0}), \quad (1.22)$$

where p_{x0} is the beam temperature in the transverse x -direction, and $\Theta(p)$ is the Heaviside step function. The beam remains cold in the y -direction. In this case the dispersion relation for transversely growing modes is:

$$\omega^2 - k^2 - 1 - \frac{\alpha}{2\gamma\beta_{x0}} \ln \left(\frac{1 + \beta_{x0}}{1 - \beta_{x0}} \right) + \frac{\alpha}{\gamma} \frac{u_{z0}^2}{1 + u_{z0}^2} - \frac{\alpha}{\gamma} \frac{k^2 \beta_{z0}^2}{\omega^2 - k^2 \beta_{x0}^2} = 0, \quad (1.23)$$

where $\gamma = \sqrt{1 + u_{x0}^2 + u_{z0}^2}$, $u_{x0} = \gamma\beta_{x0} = p_{x0}/m_e c$, and $u_{z0} = \gamma\beta_{z0} = p_{z0}/m_e c$. The solution in this case is

$$\omega^2 = \frac{1}{2} \left[k^2(1 + \beta_{x0}^2) + \mathcal{F} \right] \pm \frac{1}{2} \sqrt{[k^2(1 + \beta_{x0}^2) + \mathcal{F}]^2 - 4k^2\beta_{x0}(k^2 + \mathcal{F} - \mathcal{G})}, \quad (1.24)$$

where

$$\mathcal{F} = 1 + \frac{\alpha}{\gamma} - \frac{\alpha}{\gamma} \frac{u_{z0}^2}{1 + u_{z0}^2}, \quad \mathcal{G} = \frac{\alpha}{\gamma} \frac{\beta_{z0}^2}{\beta_{x0}^2}. \quad (1.25)$$

Modes are no longer unstable for all wavenumbers. Now, they are only unstable provided k is below a certain threshold corresponding to $k^2 < \mathcal{G} - \mathcal{F}$. Equivalently, if

$$\frac{\alpha}{\gamma} \frac{\beta_{z0}^2}{\beta_{x0}^2} > 1 + \frac{\alpha}{\gamma} - \frac{\alpha}{\gamma} \frac{u_{z0}^2}{1 + u_{z0}^2}. \quad (1.26)$$

The physical significance of this becomes clearer in the limit that the beam is highly relativistic ($u_{z0} \gg 1$):

$$\frac{\alpha}{\gamma} \frac{\beta_{z0}^2}{\beta_{x0}^2} > 1 \quad \longrightarrow \quad \Delta\theta < \sqrt{\frac{\alpha}{\gamma}} = \frac{\Gamma_{\max}}{\omega_p}, \quad (1.27)$$

where the transverse thermal spread of the beam is characterized by $\Delta\theta \equiv \beta_{x0}/\beta_{z0}$, and $\Gamma_{\max} = (\alpha/\gamma)^{1/2} \omega_p$ is the maximum growth rate in the relativistic, cold beam case (Eq. 1.21).

Given $v_x = c\Delta\theta$, modes are unstable if $\Gamma_{\max} > v_x/\lambda_s$. Physically, if the transverse thermal spread of the beam is large enough that particle trajectories cross the characteristic scale of unstable modes before current fluctuations can be amplified, current fluctuations are damped rather than amplified.

The resulting spectrum of unstable modes is plotted for a range of $\Delta\theta$ in Figure 1.6, showing that as $\Delta\theta$ is increased, small-scale (large k) modes become completely suppressed, and the growth rate of the fastest-growing mode is reduced. In the case shown, $\Delta\theta$ of a few percent is enough to completely stabilize purely transverse modes for all wavenumbers.

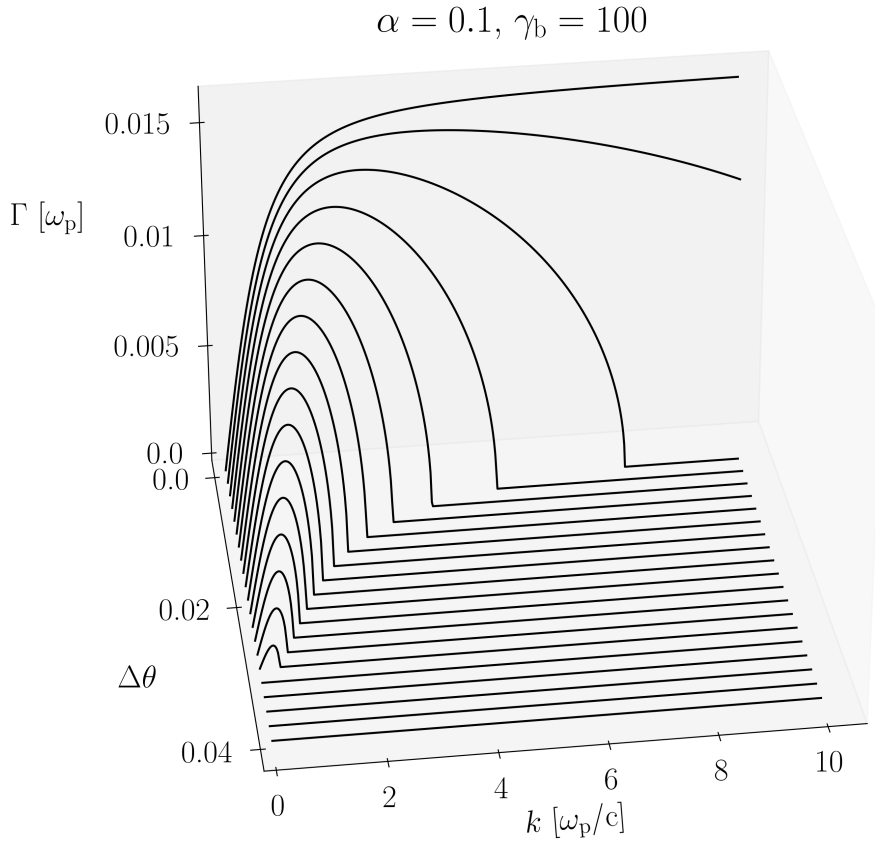


Figure 1.6: Suppression of transverse electromagnetic beam instability due to finite temperature. The growth rate (Γ) of transverse modes as a function of wavenumber (k) is plotted for a range of transverse momentum spreads of the beam ($\Delta\theta$), given a beam-to-plasma density ratio $\alpha = n_b/n_p = 0.1$ and monoenergetic beam with Lorentz factor $\gamma_b = 100$. With a non-zero $\Delta\theta$, small scale modes (large k) are stabilized, and the fastest growth rate is reduced.

1.4 Suppression of oblique and longitudinal modes

The growth rate of unstable oblique electromagnetic modes is given by Fainberg et. al. [29] in the case of a monoenergetic beam:

$$\Gamma = \frac{\sqrt{3}}{2^{4/3}} \omega_p \left(\frac{\alpha}{\gamma_b} \right)^{1/3} \left[\frac{k_{\perp}^{2/3}}{k_{\parallel}^{2/3}} - \frac{3k_{\perp}^2}{8k_{\parallel}^2} (\Delta\theta)^2 \left(\frac{2\gamma_b}{\alpha} \right)^{2/3} \right], \quad (1.28)$$

where $\Delta\theta \equiv \sigma_{p_{\perp}}/p_{\parallel}$, and $\sigma_{p_{\perp}}^2 = \int p_{\perp}^2 f_b^{(0)} d\mathbf{p}$, whilst $k_{\parallel}$ and $k_{\perp}$ are the parallel and perpendicular components of the wavevector relative to the beam propagation axis, such that the angle between the wavevector and the propagation axis is given by $\phi = \tan^{-1}(k_{\perp}/k_{\parallel})$. The factor in square brackets accounts for finite transverse thermal spread of the beam, tending to unity for a cold beam ($\Delta\theta = 0$). Eq. 1.28 is plotted as a function of ϕ for various $\Delta\theta$ in Figure 1.7, showing that a finite transverse thermal spread leads to tilting of the fastest-growing mode towards the longitudinal direction and a reduction of the fastest growth rate. This becomes important when $\Delta\theta \gtrsim (\alpha/\gamma_b)^{1/3}$. The growth rate of the fastest-growing mode is obtained by maximizing Eq. 1.28 with respect to angle:

$$\Gamma_{\text{fastest}} = \frac{\sqrt{3}}{2^{4/3}} \omega_p \left(\frac{\alpha}{\gamma_b} \right)^{1/3} \left[\frac{8}{3} \frac{1}{\Delta\theta} \left(\frac{\alpha}{2\gamma_b} \right)^{1/3} \cos^2 \phi_{\text{fastest}} - \frac{3}{8} (\Delta\theta)^2 \left(\frac{2\gamma_b}{\alpha} \right)^{2/3} \tan^2 \phi_{\text{fastest}} \right],$$

$$\longrightarrow \Gamma_{\text{fastest}} \propto \begin{cases} \left(\frac{\alpha}{\gamma_b} \right)^{1/3} \omega_p, & \text{for } \Delta\theta \ll (\alpha/\gamma_b)^{1/3}, \\ \frac{1}{\Delta\theta} \left(\frac{\alpha}{\gamma_b} \right)^{2/3} \omega_p, & \text{for } \Delta\theta \gtrsim (\alpha/\gamma_b)^{1/3}. \end{cases} \quad (1.29)$$

The linear phase of instability growth is expected to end when magnetic field amplitude

reaches the strength

$$B_{\perp,\text{sat}} \simeq \frac{mc\gamma_b\Gamma^2}{e\omega_p}, \quad (1.30)$$

corresponding to the particle-trapping condition: $\Gamma \sim v\sqrt{k/r_g}$, where r_g is the particle gyroradius, $k \sim \omega_p/c$, and $v \simeq c$ [30].

Longitudinal electrostatic (two-stream) modes may continue to grow (once electromagnetic modes reach saturation) at the quasi-linear rate [22, 31]:

$$\Gamma \simeq 0.85 \frac{\omega_p}{(\Delta\theta)^2} \frac{\alpha}{\gamma_b}. \quad (1.31)$$

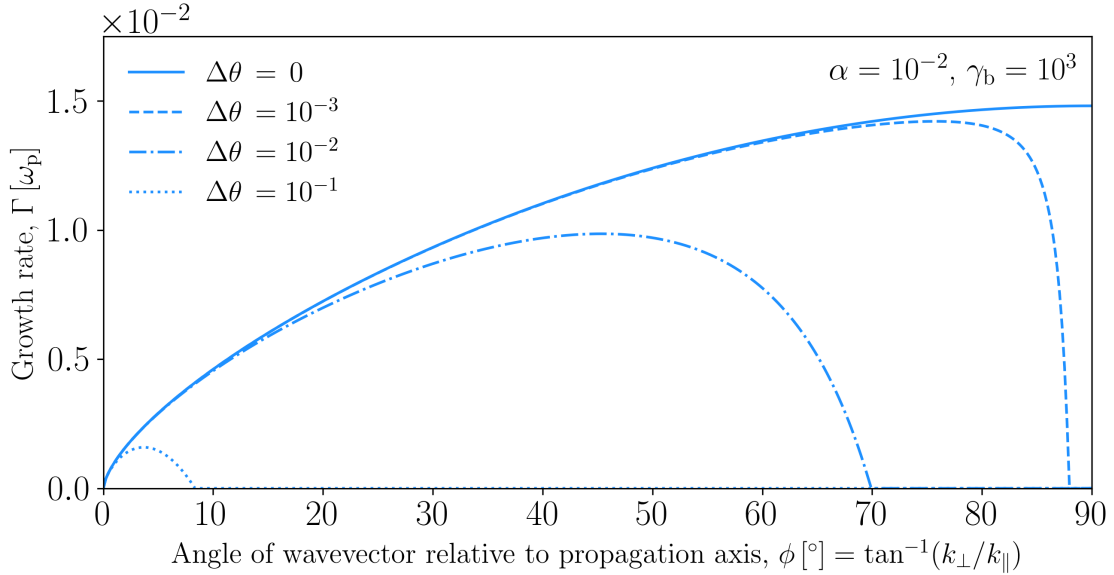


Figure 1.7: Suppression of oblique electromagnetic beam instability due to finite temperature. The growth rate (Γ) of oblique modes is plotted as a function of the angle between the wavevector and the propagation axis, $\phi = \tan^{-1}(k_{\perp}/k_{\parallel})$, for a range of transverse momentum spreads of the beam ($\Delta\theta$), given a beam-to-plasma density ratio $\alpha = n_b/n_p = 0.01$ and monoenergetic beam with Lorentz factor $\gamma_b = 10^3$. When the beam is perfectly collimated ($\Delta\theta = 0$), the fastest-growing oblique modes are oriented transverse to the beam ($\phi = 90^\circ$), and when $\Delta\theta \gtrsim (\alpha/\gamma_b)^{1/3}$, the fastest-growing mode becomes tilted relative to the beam propagation axis and in the most extreme case becomes quasi-parallel to the beam.

1.5 Particle-in-cell simulations

The theory discussed so far is only valid provided perturbations of the initial distribution function remain sufficiently small. When this is no longer the case, the non-linear behaviour must be considered, and analytical progress becomes limited. The particle-in-cell (PIC) method [32] is a numerical technique used to study the non-linear evolution of the Vlasov-Maxwell system of equations. The numerical scheme (illustrated in Figure 1.8) involves forcing groups of charged particles on a grid according to self-consistent electromagnetic fields calculated using a finite difference approximation. Simulation studies of kinetic beam-plasma instabilities evolving beyond linear growth have concluded that such instabilities may be capable of producing macroscopic magnetic fields required in models of γ -ray burst (GRB) emission [33, 34]. However, such numerical studies are limited in several

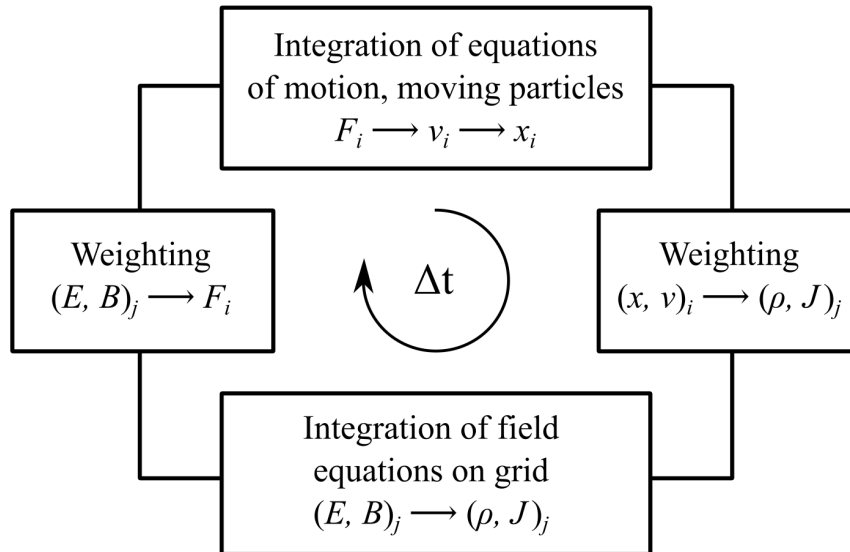


Figure 1.8: Particle-in-cell simulation typical cycle. (Adapted from Birdsall & Langdon [32].) Particles (labelled i) are pushed on the grid (indices j) according to the self-consistent electromagnetic fields.

ways, in particular the limitation on spatial and temporal range. Full three-dimensional dimensions can become very computationally expensive, for instance, several million CPU hours were required for the 3D simulations presented in the thesis. Typically, to explore evolution far beyond the linear phase, simulations are performed in two spatial dimensions. If laboratory experiments can be performed, they may be able to explore instabilities beyond the linear phase of growth, and there is potential to explore additional effects including scattering, radiative and atomic processes.

1.6 Laboratory astrophysics and pair beam experiments

Clearly, it is not possible to exactly recreate astrophysical systems exactly in the laboratory, but it may be possible to isolate aspects of the physics to verify microphysical models and even exploit scale invariance of the governing equations to produce scaled analogues of various phenomena. Such investigations can offer a lot of insight where the only available knowledge may be based upon remote astronomical observations and limited numerical simulations. In the earliest proposed ‘laboratory astrophysics’ experiments at high-intensity laser facilities, it was shown that aspects of the expansion of hydrodynamical shocks in supernovae can be studied in the laboratory [35–37]. Since, experiments have utilized a range of high power facilities such as particle accelerators, X-ray free electron lasers, and pulsed power facilities to explore an ever-expanding range of topics, including (but not limited to): magnetic reconnection [38], collisionless shocks [39, 40], particle acceleration [41] and turbulent dynamos [42].

Laboratory experiments involving relativistic electron-positron pair beams have until now not been possible due to the challenge of producing pair beams with the necessary characteristics [43], namely, high enough numbers and densities of a quasi-neutral population of pairs, maintained for a long enough period of time to observe collective plasma effects. Achieving these characteristics simultaneously has proven to be extremely difficult. Present high flux laboratory sources of positrons include: (i) nuclear reactors [44], (ii) electron accelerators [45, 46], and (iii) high-power lasers [47–52]. All involve producing pairs in the strong electric field of high- Z charged nuclei (the Bethe-Heitler process [53] depicted in Figure 1.9). Despite significant efforts, none of these approaches have so far been able to produce pair beams with the necessary properties.

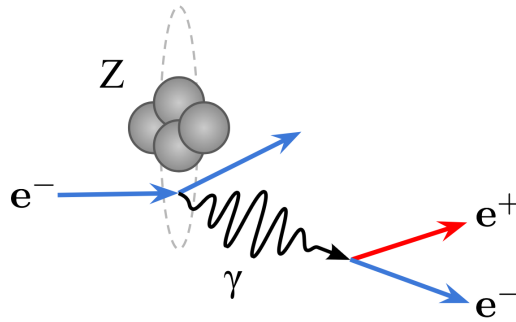


Figure 1.9: Bethe-Heitler process. An electron generates bremsstrahlung which can convert to an electron-positron pair in the nuclear field. The process is more efficient in high- Z materials.

1.7 Thesis layout

In this thesis, a novel approach to generating high-yield, quasi-neutral $e^\pm$ pair beams has been developed using ultra-relativistic proton beams at CERN, realizing the decades-long pursuit of performing experimental studies of pair beam-plasma instabilities. In Chapter 2, the scheme for generating electron-positron pairs is described, along with discussions of how the experiment is prepared at CERN. In Chapter 3, a plasma source is developed so that the stability of the pair beam as it propagates through a plasma can be explored experimentally. In Chapter 4, the generation of the pair beam is demonstrated, and in Chapter 5 the stability of the pair beam in the laboratory plasma is investigated. Specifically, the suppression of pair beam instability due to the beam's finite thermal spread will be compared with the predictions of the theory presented in this chapter. Finally, the implications will be discussed for blazar-induced pair beams propagating through cosmic voids.

Chapter 2

Generating electron-positron pair-plasma beams at CERN's HiRadMat facility

2.1 Introduction to CERN and the HiRadMat facility

Embedded in popular culture, CERN is the largest nuclear physics laboratory in the world, with notable achievements over the years including the discovery of the W and Z bosons, determination of the number of neutrino families, observation of direct CP violation, discovery of the Higgs Boson, the controlled creation of anti-matter elements, and the development of the World Wide Web [54]. The scale of experiments at CERN is mind-boggling. Charged particles are accelerated to ultra-relativistic energies using radio-frequency (RF) cavity technology, in which alternating electric fields in a string of cavities is timed so that particles are continuously accelerated. After initial acceleration takes place in a linear accelerator (LINAC), particles are injected into progressively larger rings as momentum is increased, and in the largest accelerator, the Large Hadron Collider (LHC), protons are accelerated up to a momentum of $6.8 \text{ TeV}/c$ ($\gamma = 7300$) after millions of cavity passes. Accelerated particles make over 10,000 laps of the 27 km circumference LHC every second, steered by over 1000 superconducting dipole magnets with extremely strong 8.3 T magnetic fields. At the four largest experiments of the LHC (ATLAS, ALICE, CMS and LHCb), counter-propagating beams are collided and the resulting train wreck is disentangled using the vast amounts of data collected by building-sized detectors.

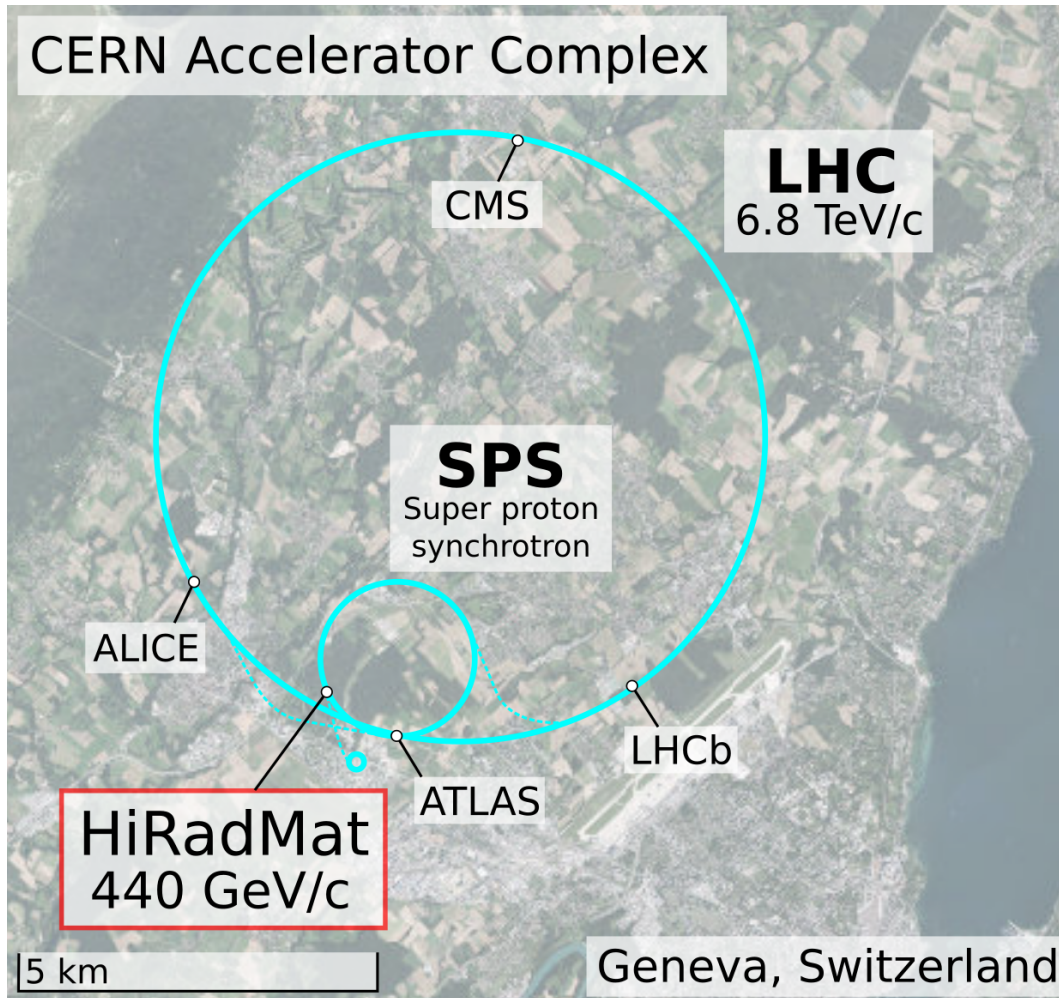


Figure 2.1: CERN accelerator complex, Geneva, Switzerland. The tunnels of the Proton Synchrotron (PS), Super Proton Synchrotron (SPS) and Large Hadron Collider (LHC) are marked in light blue (listed in order of increasing size). In the LHC, protons are accelerated to momentum 6.8 TeV/c, whilst in the SPS protons can be accelerated up to a momentum 450 GeV/c. At the High Radiation to Materials (HiRadMat) facility, protons are extracted from the SPS with 440 GeV/c.

But the frontiers of scientific discovery are not limited to the largest accelerator experiments; there are over 100 separate official experiments supplied with particle beams extracted at various points along the accelerator chain. The facility where the presented experiment was performed is called ‘High Radiation to Materials’ (HiRadMat) [55], marked on a map of the CERN accelerator complex in Figure 2.1.

At HiRadMat, 440 GeV/c protons are supplied to the facility in LHC-type nanosecond bunches extracted from the 7 km circumference Super Proton Synchrotron (SPS). In a single-bunch configuration, proton beams can contain over 3×10^{11} protons. Magnetic quadrupoles and sextupoles are used to deliver consistent, collimated beams with mm-scale beam waist [55] (beam parameters given in Table 2.1). Since its first experiment in 2011, HiRadMat has been a workhorse for ultra-high radiation testing of materials, including hardware to be installed in the LHC. Though until this work, the facility has never been used to perform plasma physics experiments.

A map of the experimental area is shown in Figure 2.2. In the tunnel referred to as ‘TNC’, the proton beam propagates approximately 11 m in air between the point of beam entry and the beam dump (inclined at 2° to follow the slope of the tunnel). Experiments must fit inside this gap, arranged on removable tables that are pre-aligned in a surface-level laboratory before being transported by crane and chariot 40 m underground into the tunnel. Any electronic equipment placed inside TNC will suffer radiation damage, and so equipment that requires shielding must be placed in the adjacent tunnel (TT61) and cable connections fed through the pipes connecting the tunnels.

Parameter	HiRadMat
Proton momenta, p (GeV/c)	440
Bunch intensity, N_{p+}	3×10^{11}
Bunch duration, σ_z/c (ps)	≥ 250
Beam radius, σ_r (mm)	$0.25 - 4$

Table 2.1: Parameters of the proton beams delivered to HiRadMat.

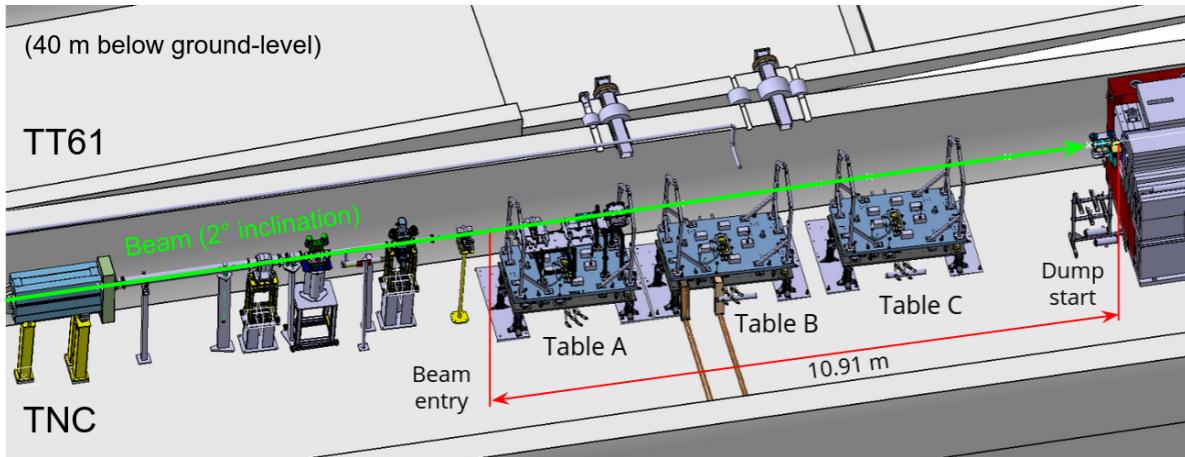


Figure 2.2: HiRadMat Experimental Area.

The prospect of radiation damage and the need to shield equipment is the single largest practical consideration in experiment design. Electronic equipment will require a power cycle any time that radiation significantly interferes with logic circuits, and accumulated radiation dose eventually leads to irreparable damage. Digital cameras must be carefully placed, which restricts the optical imaging lines and imaging performance, and power supplies for plasma sources must be positioned in a shielded bunker, the separation to the plasma limiting the efficiency that power that can be delivered.

HiRadMat has beam time for only a handful of weeks every year, to allow sufficient cool down time of the experimental area, so experiments must be prepared to work on the first attempt. Experiments are meticulously designed in a lengthy back-and-forth between investigators and facility staff and engineers, ensuring that all possible modes of failure are considered. HiRadMat has beam time for only a handful of weeks every year, to allow sufficient cool down time of the experimental area, so experiments must be prepared to work on the first attempt.

2.2 Particle scattering processes in matter

In the first calculations of the feasibility of performing electron-positron pair experiments at HiRadMat, different schemes for producing pairs using ultra-relativistic protons were considered. Here, the basic particle scattering processes are described, with a more detailed discussion included as Appendix A – “Particle scattering processes in matter”.

2.2.1 Hadronization of quarks and gluons

When ultra-relativistic protons directly irradiate a solid target and proton-nucleon interactions have a centre of mass energy, $\sqrt{s}$, in excess of GeV, inelastic hadronic interactions can lead to hadronic showers containing a large number of light hadrons such as pions: $\pi^\pm, \pi^0$ ($m_{\pi^\pm} = 139.6 \text{ MeV}/c^2$, $m_{\pi^0} = 134.5 \text{ MeV}/c^2$), for instance:

$$p^+ + n \rightarrow p^+ + n + \pi^+ + \pi^- + \pi^0.$$

Of these hadronic secondaries, neutral pions (π^0) are particularly important because they are composed of quark-antiquark pairs, so they can decay extremely quickly via the electromagnetic interaction (so no change in quark flavour is required). The dominant decay mode is into two photons, with a mean lifetime $\tau_{\pi^0 \rightarrow \gamma\gamma} = 8.5 \times 10^{-17} \text{ s}$ in the π^0 rest frame:

$$\pi^0 \rightarrow \gamma + \gamma \quad (\text{branching ratio} = 0.9882),$$

producing a highly collimated GeV-scale γ -ray beam.

The characteristic length scale associated with inelastic nuclear interactions is called the ‘nuclear interaction length’ (λ_I), and values are provided for an extensive range of elements and compounds by the Particle Data Group [56]. Materials with small atomic charge number (low- Z) generally have the shortest nuclear interaction lengths relative to the length scale of photon scattering, which makes them the preferred choice for generating γ -rays with minimal subsequent scattering.

2.2.2 Electromagnetic cascades

If γ -rays have an energy greater than the rest mass energy of an electron-positron pair ($E_\gamma \geq 2m_e c^2 = 1.022 \text{ MeV}$), they can interact with the Coulomb field of an atomic nucleus and be transformed into an electron-positron pair. In fact, electron-positron pair production is the dominant contribution to stopping power for MeV/GeV-scale photons in materials. An exact form of the pair production total cross section was first derived by Bethe and Heitler [53] for an unpolarized photon assuming the Born approximation applies and ignoring the screening of the nuclear field:

$$\sigma_{\text{pair}} = \frac{28}{9} Z^2 \alpha r_e^2 \left(\log 2k - \frac{109}{42} + f(k, Z) \right), \quad (2.1)$$

where α is the fine structure constant, r_e is the classical electron radius, k is the photon energy (normalized to $m_e c^2$), and $f(k, Z)$ represents higher order correction terms that must be included when the photon energy approaches $2m_e c^2$ (i.e. $E_\gamma \lesssim 10 \text{ MeV}$) [57, 58]. σ_{pair} increases logarithmically with photon energy, and is more efficient in high- Z materials.

The survival of positrons in a material depends on the probability of annihilation with atomic electrons. The total cross section was first given by Dirac [59]:

$$\sigma_{\text{ann}} = \pi r_{\text{e}}^2 \frac{1}{\gamma + 1} \left[\frac{\gamma^2 + 4\gamma + 1}{\gamma^2 - 1} \log \left(\gamma + \sqrt{\gamma^2 - 1} \right) - \frac{\gamma + 3}{\sqrt{\gamma^2 - 1}} \right], \quad (2.2)$$

where γ is the Lorentz factor of the positron. The annihilation mean free path depends on the total cross section and the number density of atomic electrons in the material:

$$\lambda_{\text{ann}}^{-1} [\text{cm}^{-1}] = \rho_0 \left(\frac{N_{\text{A}} Z}{A} \right) \sigma_{\text{ann}}, \quad (2.3)$$

where ρ_0 is the material mass density in g cm^{-3} , N_{A} is Avogadro's constant, A is the atomic mass number and σ_{ann} is the total annihilation cross section in cm^2 . Assuming $\sigma_{\text{ann}} \sim \pi r_{\text{e}}^2$, which is reasonable for positrons with kinetic energy keV-GeV, and taking $A \approx 2Z$, the annihilation mean free path of positrons is approximated: $\lambda_{\text{ann}} [\text{cm}] \approx 10 \rho_0^{-1}$, showing that relativistic positrons may propagate for hundreds of metres in air, and for several centimetres in solids before annihilation. However, in most solids, positrons tend to lose energy by other scattering processes at a faster rate, dominated at relativistic energies by the generation of bremsstrahlung ('braking radiation') produced when charged particles are deflected in the Coulomb field of other charged particles such as atomic nuclei. Stopping power due to bremsstrahlung is approximately proportional to particle energy:

$$\frac{1}{\rho_0} \left\langle - \frac{dE}{dx} \right\rangle_{\text{brem}} \approx \frac{E}{X_0}, \quad (2.4)$$

characterized by a length scale referred to as the ‘radiation length’ (X_0), also tabulated by the Particle Data Group for a range of materials [56]. Radiation length is shortest in high-density, high-Z materials. Subsequent pair production of bremsstrahlung photons can lead to electromagnetic cascades of electrons, positrons and γ -rays. The ‘Bethe-Heitler’ process refers to the two-step process of generating bremsstrahlung from relativistic electrons and subsequent conversion into an electron-positron pair.

2.2.3 Coulomb scattering

The divergence of experimental $e^\pm$ pair beams produced via electromagnetic cascades is largely determined by Coulomb scattering, the cumulative small-angle, elastic deflections of particles with atomic nuclei as described by the Rutherford cross section. Multiple scattering results in Gaussian-distributed angular spreads according to the central limit theorem, with an approximate formula for the half-angle is given by Lynch & Dahl [60]:

$$\sigma_\theta \text{ [rad]} = 26.6 \frac{z}{\beta^2 \gamma M} \sqrt{\frac{x}{X_0}} \left[1 + 0.038 \log \left(\frac{x z^2}{X_0 \beta^2} \right) \right], \quad (2.5)$$

for a material with thickness x , where z is the charge of the incident particle in units e , and M is the mass of the incident particle in units m_e . This formula tells us that MeV-scale electrons and positrons will be unavoidably scattered into relatively large opening angles ($10^2 - 10^3$ mrad) if they propagate over distances comparable to the radiation length in a material.

2.3 Monte-Carlo simulations

To account for all the possible scattering processes involved in high-energy particle interactions, numerical Monte-Carlo codes are used, programmed with scattering models and all the relevant probabilities. A large number of test particles are simulated, and assuming the models and probabilities are reasonable, the simulation results should converge towards experimental observations. In this thesis, the particle transport Monte-Carlo code FLUKA [61, 62] has been used extensively (in conjunction with the associated interface FLAIR [63]). Developed at CERN over the past decades, FLUKA uses a robustly bench-marked physics model and is widely used in the high-energy physics community. Using FLUKA it is simple to program very elaborate geometries and the entire experimental setup can be realized, so not only is it used to simulate the characteristics of secondary pair beams generated using the HiRadMat proton beam, but it has been vital for providing estimates of the beam-induced thermal stresses on the target, to help the design of a robust target, and to characterize the background radiation in the experimental area, guiding the design of diagnostics.

2.3.1 Modelling the secondary beams

Electron-positron beams are produced in the experiment in the following way: 440 GeV/ c protons are extracted from the SPS and delivered to the experimental area in a 1 mm radius, highly collimated, nanosecond-duration bunch containing 3×10^{11} protons. The protons irradiate a solid target composed of low-Z graphite rod and a high-Z tantalum converter.

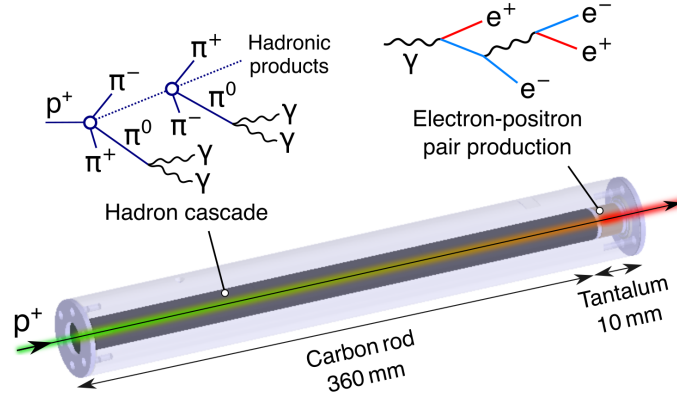


Figure 2.3: Solid target used to produce dense, quasi-neutral pair beams.

Material	Z	A	ρ_0 (g cm ⁻³)	λ_I (cm)	X_0 (cm)	Ratio (λ_I/X_0)
Graphite	6	12	1.84	46.6	23.2	2
Tantalum	73	180.9	16.7	11.5	0.41	28

Table 2.2: Nuclear interaction length and radiation length of graphite and tantalum.

A hadron cascade develops in the graphite rod, and neutral pion decays generate a highly collimated flux of GeV-scale γ -rays that propagate through the graphite with minimal subsequent scattering. The γ -rays trigger electromagnetic cascades in the tantalum converter, producing a secondary beam which contains a dominating fluence of $> 10^{13}$ electron-positron pairs and γ -rays, plus a much smaller number of secondaries such as charged pions and kaons. Graphite and tantalum are chosen as the target materials for practical reasons. Graphite is chosen for its high thermal conductivity and the fact that it is relatively safe compared with alternatives such as beryllium, whilst tantalum is chosen for its resilience to brittle fracture compared with tungsten, iridium and molybdenum [64]. The nuclear interaction length and radiation length of graphite and tantalum are provided in Table 2.2.

Species	Specified energy range	Yield (per primary)	Mean energy, $\langle E \rangle$ (GeV)	Beam radius, σ_r (mm)
e^-	≥ 1 MeV	56	0.20	2.0
e^+	≥ 1 MeV	46	0.27	1.9
p^+	≥ 100 keV, < 430 GeV	1.4	31	1.2
p^+	≥ 430 GeV	0.42	439	1.2
$\pi^\pm$	≥ 100 keV	6.2	6.6	1.7
$k^\pm$	≥ 100 keV	0.5	11	1.4
μ^-	≥ 100 keV	< 0.05	-	-
γ	≥ 10 keV	600	0.07	2.0
n	≥ 100 keV	12.6	3.1	2.2

Table 2.3: Detailed secondary beam characteristics from FLUKA simulations. The yield (per primary), mean energy ($\langle E \rangle = \langle \gamma \rangle Mc^2$), and beam radius (half-width-half-maximum) are provided for the most abundantly produced secondary species: electrons (e^-), positrons (e^+), protons (p^+), pions ($\pi^\pm$), kaons ($k^\pm$), muons (μ^-), γ -rays and neutrons (n). The specified energy range defines the range of energies that the table parameters refer to. In the case of electrons and positrons, for ease of comparison with other studies, the low-energy cutoff used is 1 MeV, though there are even more pairs (up to 10% more) if the energy cutoff of the simulation (10 keV) is considered. Multiplying the yield per primary by the primary beam intensity gives the actual yield, showing that $> 10^{13}$ $e^\pm$ are expected to be produced using a primary proton beam intensity of 3×10^{11} protons. Note that for the muon component, the produced number is too low for a reasonable estimate of mean energy and beam radius.

FLUKA simulations are performed to model the characteristics of the secondary beams. As the input for the simulations, protons are collimated with a Gaussian transverse spatial profile ($\sigma_r = 1$ mm) and an essentially monochromatic spectral profile peaked at 440 GeV/ c (with width corresponding to 0.03% of the central energy). At least 10^5 primary protons are simulated, and then the results are re-scaled to primary beam intensities of 3×10^{11} protons. Beam characteristics such as size, divergence and energy spectra were recorded for the different components of the generated beams escaping the tantalum. In all simulations, the low-energy cutoff for particle transport is set as 10 keV for $e^-/e^+/\gamma$ and 100 keV for hadrons. The simulated yield, mean energy and beam radius are given for the most significant beam components in Table 2.3, and the energy spectra and emittance plots

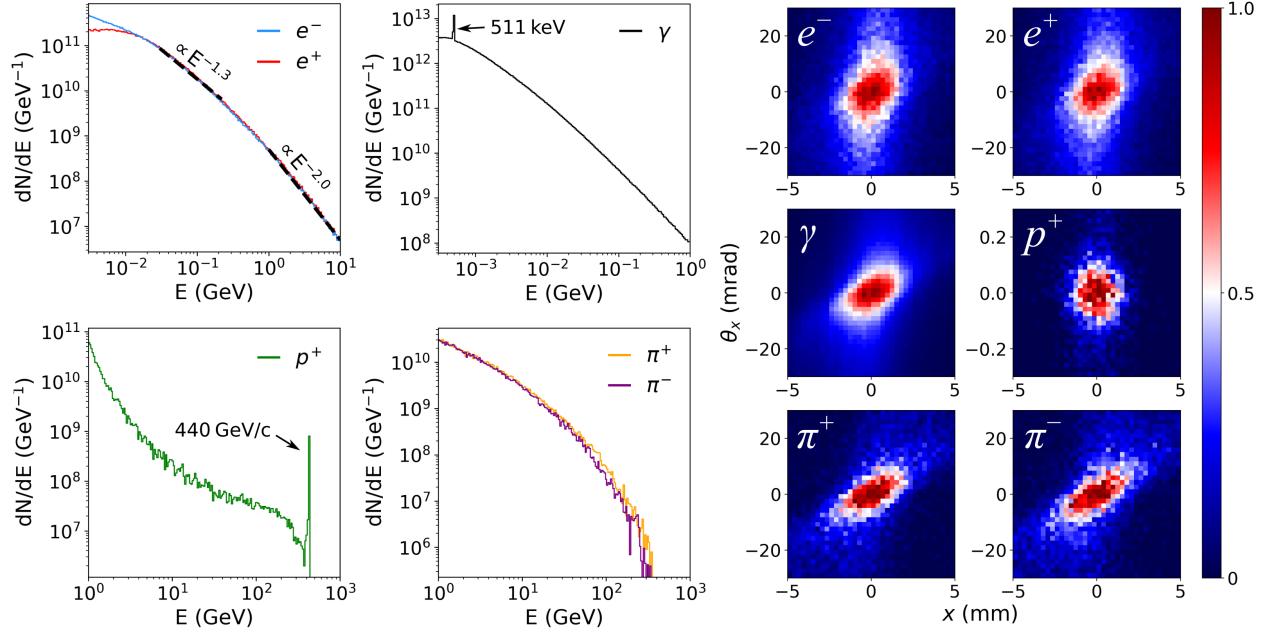


Figure 2.4: Secondary beam characteristics obtained from FLUKA simulations. Energy spectra (left) and angle-position phase space (emittance) plots (right) obtained at the immediate rear of the target. $e^\pm$ have a multi-power-law spectra, with spectral index $dN/dE \propto E^{-1.3}$ in the 30 – 220 MeV range, and $dN/dE \propto E^{-2.0}$ at $E > 1$ GeV. In the emittance plots, x is the direction transverse to beam propagation and $\theta_x = \arctan(v_x/v_z)$ is the pointing angle of the velocity vector relative to the beam axis (in the laboratory frame).

for electrons, positrons, protons, γ -rays and pions are shown in Figure 2.4. Secondary particles are generated in cascade processes, so their energy spectra are typically power-law (non-thermal) distributions extending over many decades in energy. The high-energy tail of the $e^\pm$ spectra extends up to tens of GeV, differing between electrons and positrons only at energies less than 10 MeV, where the stopping power is slightly different (see Appendix A, Section A.4.3). The difference between the electron and positron spectra at $\lesssim 10$ MeV limits the quasi-neutrality of the beam to $n_{e^+}/n_{e^-} \sim 0.83$ at the target rear, though downstream of the target, near-perfect quasi-neutrality is obtained as the higher divergence of lower-energy pairs causes them to preferentially escape the beam as it propagates. In the γ -ray spectrum,

a peak around 511 keV arises from annihilation of positrons with atomic electrons in the target. In the proton spectrum, a peak is observed at 440 GeV/ c which corresponds to primary protons that do not experience significant inelastic scattering. The emittance plots show that the transverse size of secondary components is not expected to differ significantly from the primary beam ($\sigma_r = 1$ mm), but the beam divergence is species dependent. Aside from the protons, the correlation between angle and position is lowest for the electrons and positrons as they are produced primarily in the tantalum at the end of the target.

2.3.2 Parameter scan of target material thicknesses

To decide the thickness of graphite and tantalum, an extensive parameter scan was performed using FLUKA. The thickness of graphite was simulated in steps of 100 mm up to 800 mm, whilst the thickness of tantalum was simulated up to 60 mm in steps of 10 mm. For each combination, the positron yield and peak density was obtained at the target rear. As before, a Gaussian beam profile ($\sigma_r = 1$ mm) was assumed for the primary protons. To obtain a smooth colourmap, the data points were linearly interpolated, and a Gaussian smoothing was applied with kernel size equal to half the scanning interval. The results are shown in Figure 2.5, indicating that pair yields of 5×10^{13} and densities $8 \times 10^{12} \text{ cm}^{-3}$ may be achievable using graphite thickness = 400 – 600 mm and tantalum thickness = 25 – 30 mm. The configuration of the final design (360 mm graphite and 10 mm tantalum) is not expected to produce the highest possible pair yields and densities, instead, achieving a low pair beam emittance was prioritized to maximize pair density maintained over a 1 m distance downstream of the target (where the plasma will be positioned). A thinner target

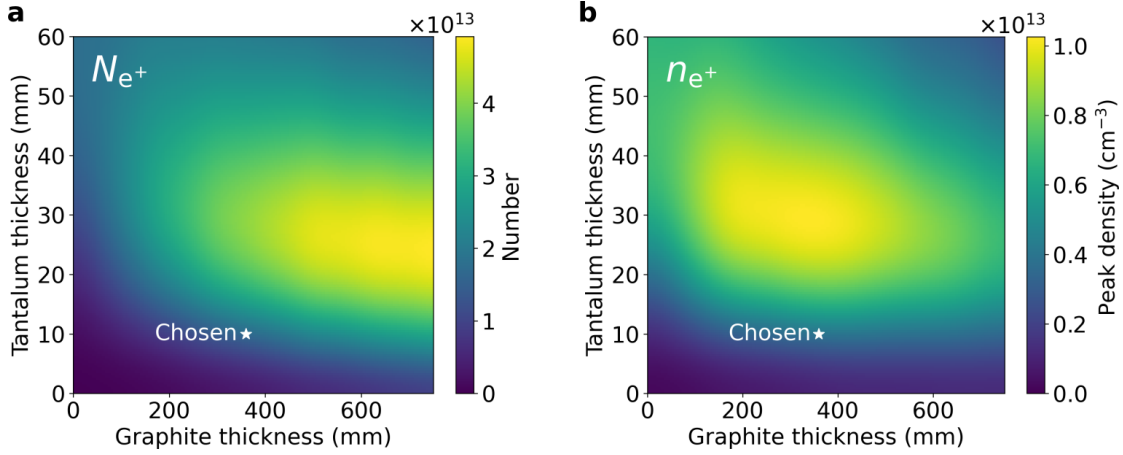


Figure 2.5: Positron yield and peak density in a parameter scan of material thicknesses. FLUKA Monte-carlo simulations of (a) the positron yield, and (b) the peak positron density, at the rear of the target when composed of a range of thicknesses of graphite (up to 800 mm) and tantalum (up to 60 mm). The primary beam is assumed to contain 3×10^{11} protons, and the results can be rescaled for different intensities. The yield and density presented corresponds to positrons with kinetic energy greater than 1 MeV. The combination of graphite and tantalum thicknesses chosen for the experiment is shown by the star.

was also preferable given the limited space in the experimental area. A dedicated study of the thickness of the tantalum was performed, showing that the highest densities can be maintained over 1 m if a tantalum thickness 10 mm is used (see Figure 2.6). Figure 2.6a shows the positron spectra for increasing thicknesses of tantalum. When the tantalum constitutes a larger number of radiation lengths, more generations of electromagnetic cascade develop and the spectrum is reprocessed, decreasing the mean energy. Meanwhile, the angular distribution as a function of energy is quite insensitive to tantalum thickness (Figure 2.6b). This is not totally unsurprising – electrons and positrons produced in the cascade have an energy-dependent mean free path in the tantalum, whilst the amount of Coulomb scattering is also energy dependent. So electrons/positrons escaping the target at a given energy may be Coulomb scattered by approximately the same amount. The overall divergence of the pair

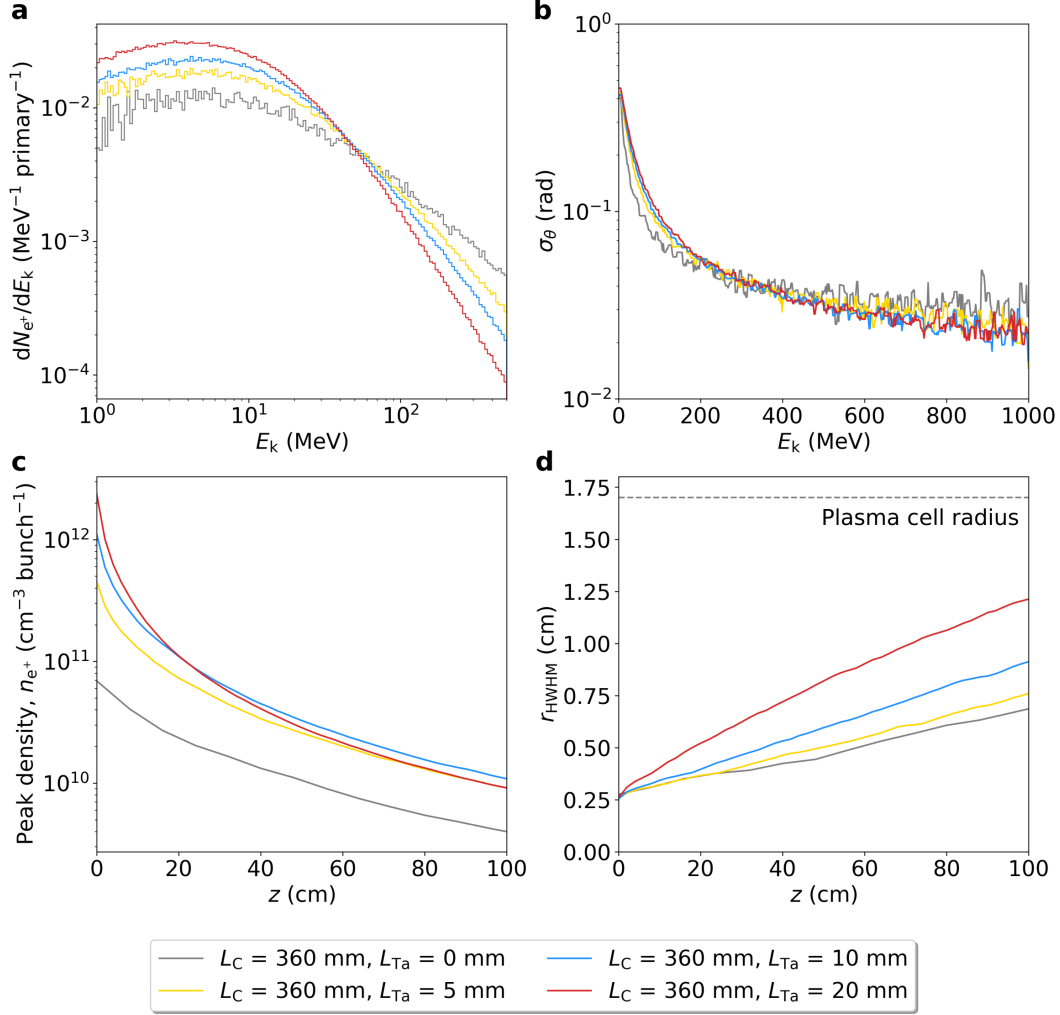


Figure 2.6: Sensitivity of pair beam properties to tantalum thickness. A dedicated FLUKA study is performed to optimize tantalum thickness to achieve a high density, low emittance pair beam. (a) Positron energy spectra per MeV, per primary, at the rear of the target. (b) Standard deviation of positron angle relative to the beam axis as a function of energy. (c) Peak positron density as a function of distance downstream of the target (z), per bunch containing 3×10^{11} primary protons, assuming a Gaussian pulse duration with $\sigma_t = 250$ ps (assuming ballistic motion of particles). (d) Transverse beam radius of positrons (half-width-half-maximum) as a function of distance downstream of the target. The dashed horizontal line represents the eventual inner radius of the designed plasma cell.

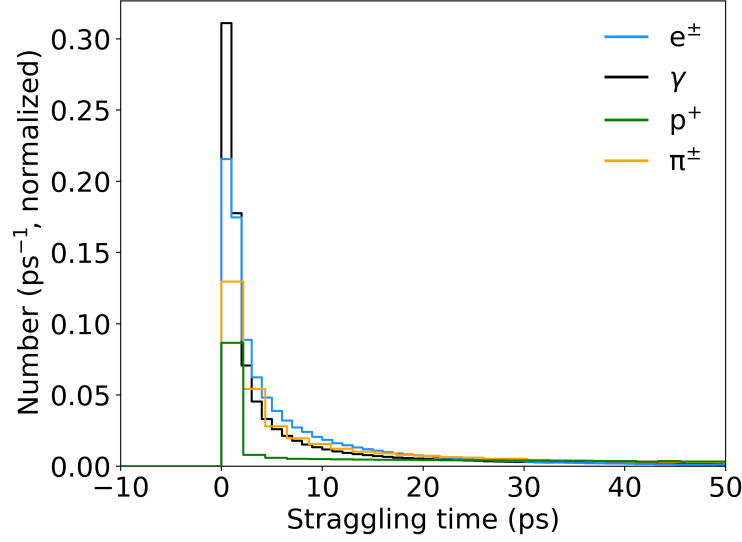


Figure 2.7: FLUKA simulations of the straggling time of secondary species. The distributions of times when particles breakthrough the rear surface of the target is obtained for different secondary species using a FLUKA simulation. The species plotted are: electrons/positrons ($e^\pm$), γ -rays, protons (p^+) and charged pions ($\pi^\pm$). The time $t = 0$ represents the earliest possible arrival time of particles at the rear of the target assuming they experience zero straggling. The straggling times ($\lesssim 5$ ps) are much smaller than the duration of the primary proton beam.

beam is largely determined by the extent to which the spectra is reprocessed. In Figures 2.6c and 2.6d, the peak density and beam radius are plotted at different downstream distances from the target, obtained using the velocity vectors of simulation particles at the target rear and assuming pairs propagate ballistically. Whilst the 20 mm tantalum thickness leads to initially higher densities, at distances 20 – 100 cm downstream of the target the pair beam produced with 10 mm thickness tantalum results in the highest pair beam density.

2.3.3 Beam elongation due to straggling and lagging

Figure 2.7 shows the spread of times that particles breakthrough the rear of the target (particle straggling) for different secondary species, calculated using FLUKA. This shows that the straggling times are generally much shorter than the primary beam duration and

the temporal profile of secondary beam components can be treated as identical to the primary proton beam. Downstream of the target, slower particles will start to lag behind ultra-relativistic particles (travelling effectively at c). But since the vast majority of pairs are calculated to arrive 1 m downstream of the target within $1\text{-}\sigma_t$ of the temporal profile of the primary protons (specifically, those with $\gamma \gtrsim 2.6$), it is assumed that the temporal profile of secondary beam is approximately constant throughout the experiment.

2.4 Simulations of the thermal loading on the target

The target is designed to be reusable for hundreds (or potentially thousands) of shots and specific features are included to ensure that beam-induced heating and thermal stresses are not excessive:

- The graphite and tantalum are housed inside a 400 mm length, 50 mm diameter cylinder of high-strength T9 aluminium alloy that acts as a confinement vessel and a heat sink.
- The tantalum is press-fit inside the aluminium to ensure maximal thermal contact.
- Expanded graphite pieces (2 mm thickness SGL Carbon Sigraflex, 1 g cm^{-3}) separate the target components to allow for thermal expansion and reduce contact stresses during irradiation.

In addition, 2 mm thickness Sigradur G glassy carbon beam windows are clamped onto both ends by aluminium flanges to hermetically seal the target materials and prevent any release of activated material.

Beam-induced thermal loading is deemed acceptable provided: (i) peak temperatures remain well-below melting points, (ii) cooling times are much shorter than the minimum shot separation ($t_{\text{cool}} \ll 40$ s), (iii) thermal stress is well-below the estimated tensile strength, and (iv) induced strain is well-below the plastic deformation limit, or that cumulative plastic deformation is less than a few percent of the beam radius. Simple estimates can be made using the energy deposition profile calculated using FLUKA when the target is irradiated with $N_{\text{p}+} = 3 \times 10^{11}$ protons (shown in Figure 2.8). The energy deposition is highest in the tantalum converter, peaking at $u_{\text{dep}} = 12 \text{ GeV cm}^{-3} \text{ primary}^{-1}$, in a region with size corresponding approximately to the beam waist ($\sigma_r = 1$ mm). It can be assumed that the target is heated adiabatically given that heat diffusion timescales are expected to be much longer than the beam duration.

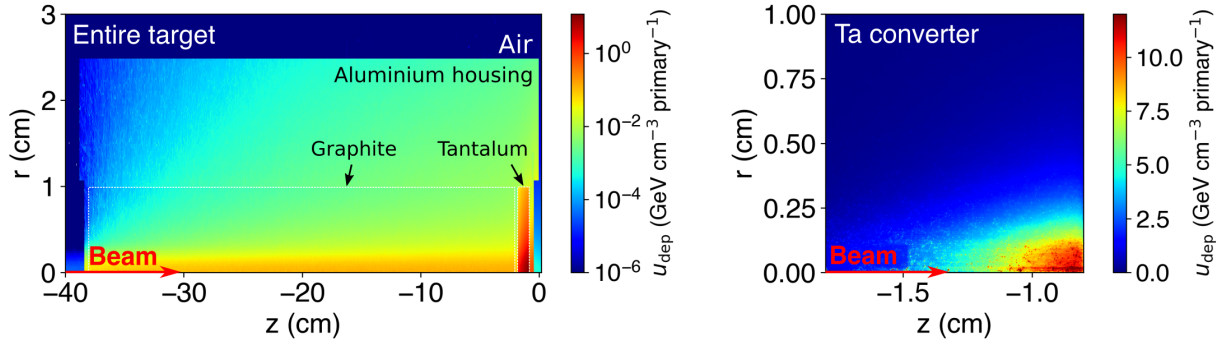


Figure 2.8: FLUKA simulations of the energy deposition into the target. The FLUKA card ‘USRBIN’ is used to obtain the energy deposition per unit volume, per primary proton, in the target capsule. Left: The entire target is simulated with bin size 0.25 mm. Right: A dedicated study of the tantalum converter is performed using a finer spatial resolution (0.025 mm). The results are consistent between the finer and coarser simulation, showing peak energy deposition $u_{\text{dep}} = 12 \text{ GeV cm}^{-3} \text{ primary}^{-1}$.

If a constant heat capacity for tantalum is assumed ($C = 0.14 \text{ J g}^{-1} \text{ K}^{-1}$) the peak instantaneous temperature increase can be estimated by

$$\Delta T \approx \frac{u_{\text{dep}} N_{\text{p}^+}}{\rho C}, \quad (2.6)$$

where $\rho = 16.7 \text{ g cm}^{-3}$ is the mass density of tantalum. Using a beam intensity $N_{\text{p}^+} = 3 \times 10^{11}$, the estimated peak temperature is $T_{\text{max}} \approx 270 \text{ K}$, much less than the melting temperature, $T_{\text{melt}} = 3290 \text{ K}$. The expected timescale for heat to dissipate via diffusion is estimated by

$$t_{\text{cool}} \approx \frac{\sigma_r^2}{4\alpha}, \quad (2.7)$$

which is obtained by solving the heat diffusion equation under spherical symmetry, with $\alpha = k/\rho C$ being the thermal diffusivity and k being the thermal conductivity. Using a thermal conductivity for tantalum $k = 57 \text{ W m}^{-1} \text{ K}^{-1}$, the cooling time is estimated to be: $t_{\text{cool}} \approx 10 \text{ ms} \ll t_{\text{sep}}$, much shorter than the time between shots, which will be no less than 40 s. The induced thermal stress is estimated by

$$\sigma \approx E \alpha_{\text{T}} \Delta T, \quad (2.8)$$

where $E = 186 \text{ GPa}$ is Young's modulus and $\alpha_{\text{T}} = 6.3 \times 10^{-6} \text{ K}^{-1}$ is the thermal expansion coefficient, giving an estimated maximum stress $\sigma \approx 290 \text{ MPa}$, which is much less than the ultimate tensile strength of tantalum ($\sigma_{\text{t.s.}} \approx 450 \text{ MPa}$).

Finally, the maximum strain is estimated by

$$\epsilon \approx \alpha_T \Delta T, \quad (2.9)$$

leading to $\epsilon \approx 1.6 \times 10^{-3} \text{ mm mm}^{-1}$, which is below the plastic deformation limit estimated by a Zerilli-Armstrong strength model (using parameters experimentally validated in Ref. [65]).

More accurately, thermo-mechanical simulations were performed using the commercial finite-element code Ansys Mechanical [67] using the FLUKA-simulated transient thermal field as an initial heat deposition map. The simulation results are summarized in Figure 2.9. The peak temperature agrees with the initial estimate and is well-below the melting point of

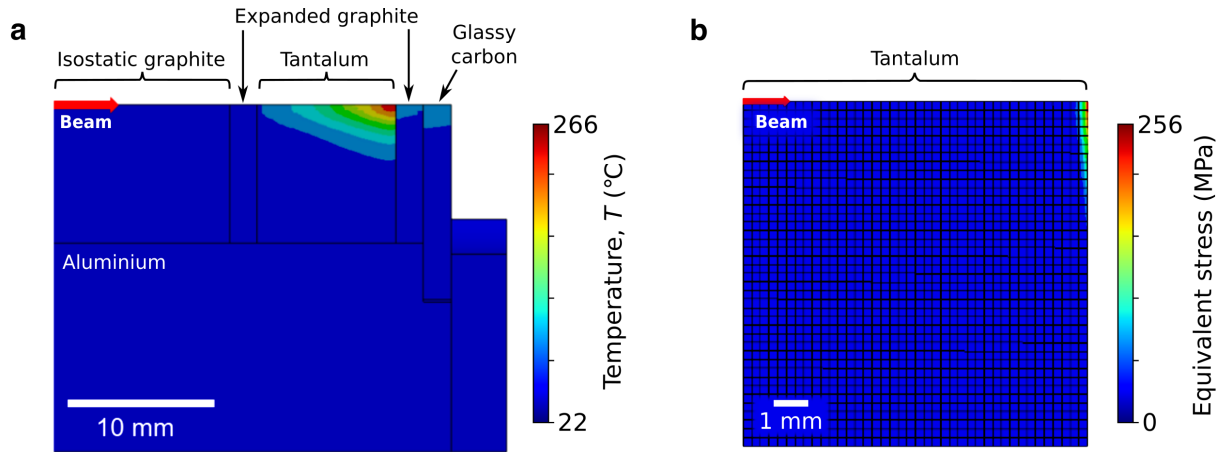


Figure 2.9: Ansys Mechanical finite-element simulations of the beam-induced thermal stresses. (a) The temperature increases in the target assuming a temperature-dependent model of heat capacity [66], and assuming the FLUKA-simulated transient thermal field as the initial heat deposition map. The peak temperature ($T_{\max} = 266^\circ\text{C}$) is well-below the melting temperature of tantalum. (b) Equivalent stress induced in the target, showing maximum equivalent Von-Mises (VM) stresses are around 256 MPa, below the estimated tensile strength of 445 MPa at this temperature. No plastic deformation is expected.

the tantalum. The maximum equivalent Von-Mises (VM) stresses are below the estimated tensile strength. No plastic deformation is expected, and the maximum elastic deformation in the axial direction is just a few microns, which can be compensated by the expanded graphite spacers. Cooling of the target is calculated assuming radiative losses via the glassy carbon beam windows (with emissivity of 0.3) and convective cooling via the outer surface of the aluminium cylinder (7 W m^{-2} film coefficient), estimating that the target returns to a few degrees above room temperature within a few seconds, so it can be reasonably assumed that there is no excessive heat build-up over multiple shots.

2.5 Summary

In this chapter, the considerations for designing a target capable of generating dense, quasi-neutral electron-positron beams have been described in detail. The resulting increase in electron-positron yield compared with previous experiments is predicted by FLUKA simulations to be impressive (several orders of magnitude higher than previously reported neutral beams [49, 51]). The result is presented in the context of previous laser-based approaches in Figure 2.10. In the next chapter, the design and construction of a suitable plasma device to study pair beam instabilities is presented.

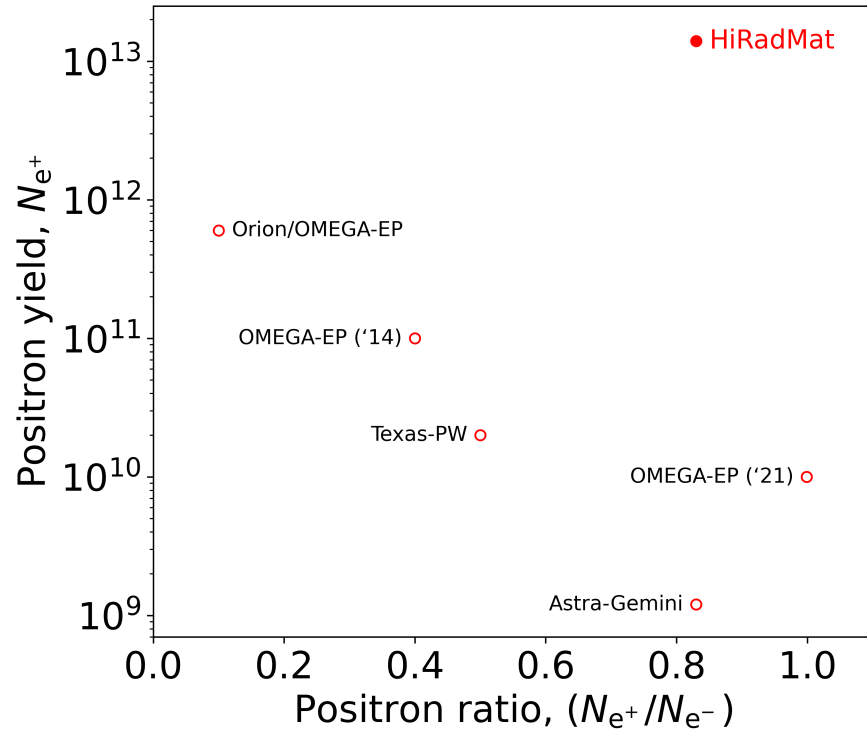


Figure 2.10: Comparison of pair beams generated at CERN’s HiRadMat facility with previous experiments at high-power laser facilities. The positron yield (N_{e^+}) and positron ratio (N_{e^+}/N_{e^-}) at HiRadMat (filled circle) is compared with previous experiments performed at high-power laser facilities (empty circles), where a positron ratio $N_{e^+}/N_{e^-} = 1$ corresponds to a purely quasi-neutral pair beam: Orion/OMEGA-EP [47], Texas-PW [48], Astra-Gemini [49], OMEGA-EP('21) [51], OMEGA-EP('14) [68]. The data are labelled by the facility where the experiment was performed. In the case of OMEGA-EP, the year of the experiment is also specified.

Chapter 3

An inductively coupled radio-frequency plasma discharge

In 1884, German physicist Johann Wilhelm Hittorf coiled several turns of wire around a jar of rarefied gas and discharged a Leyden jar through the wire, causing the gas inside to suddenly flash. The electrodeless plasma discharge was born, and the plasma device presented in this chapter is little more than a refinement of Hittorf's original design. The physical explanation of the plasma discharge was debated between J. J. Thomson and Nikolay Tesla in heated exchanges lasting nearly three decades [69, 70]:

“I did not, as Prof. J. J. Thomson seems to believe, misunderstand his position in regard to the cause of the phenomena considered, but I thought that in his experiments, as well as in my own, electrostatic effects were of great importance.”

– N. Tesla, 1891 [70]

Whether the plasma was formed due to the electrostatic potential between the ends of the coil, or whether Faraday's law of induction was responsible, was finally resolved in the late 1920s [71]. Both were correct, but whether the coupling between coil and gas is primarily electrostatic ('E' mode) or inductive ('H' mode) depends on the coil geometry and the power applied, with the inductive mode being dominant above a threshold absorbed power. The inductively coupled radio-frequency plasma discharge (ICP) is operation of the electrodeless

discharge in the H-mode, comprised of an inductive circuit element such as a helical coil or solenoid wrapped around a discharge region containing rarefied gas, separated from the element by a dielectric window such as glass [72, 73]. Analytical models treat the system as a transformer, with the plasma as the secondary coil (as in Figure 3.1).

Using an inductive discharge it is possible to generate large (litre-scale) volumes of plasma with densities $n_p = 10^{11} - 10^{12} \text{ cm}^{-3}$ using a commercially available kilo-Watt radio-frequency (RF) power source. The technology is mature, having been extensively developed over the past century for a range of applications including light sources [74], plasma propulsion [75], and within the semiconductor manufacturing industry for plasma-aided fabrication of integrated circuits [76, 77].

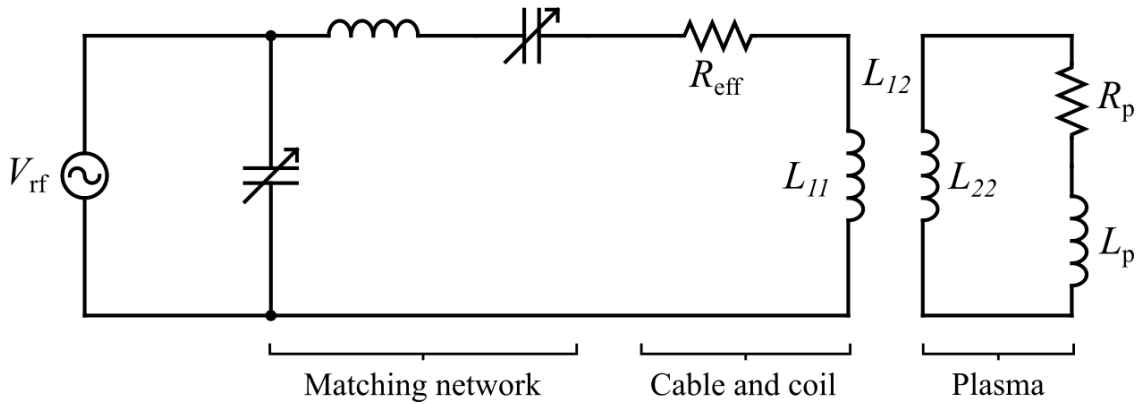


Figure 3.1: Transformer model of the inductively coupled plasma discharge. An equivalent circuit shows how the inductive discharge can be modelled as a transformer. A matching network, composed of variable capacitors, is used to ensure that the coupling of power into the plasma is maximized, by minimizing reflections due to impedance mismatch between the power supply and the combined load of the cable, coil and plasma. L_{11} is the inductance of the coil, L_{22} is the inductance of the induced current sheath in the plasma, and L_{12} is the mutual inductance. L_p and R_p are the inductance and resistance of the plasma arising from a combination of the finite inertia of the electrons and electron collisions.

The reasons for developing an inductive plasma discharge for the experiment at HiRadMat instead of alternatives such as capillary discharges, helicon discharges or laser-ionized channels are that the plasma must satisfy the following criteria:

1. The plasma skin depth must be smaller than the beam size. For cm-scale beam diameters, this limits the plasma density to $n_p \gtrsim 10^{11} \text{cm}^{-3}$.
2. The plasma must be long enough for instabilities to develop, at least tens-of-centimetres based on the linear theory growth rates.
3. A diverging pair beam must be contained inside the plasma as it diverges, so the plasma diameter must be a few centimetres.

In addition, they are relatively cheap to develop.

3.1 Design of an inductively coupled argon plasma discharge

A schematic of the final design is shown in Figure 3.2, and a photograph during operation once installed at HiRadMat is shown in Figure 3.3. The discharge is composed of a vacuum chamber constructed from three six-way port crosses (non-ferrous, 304L-grade stainless steel), separated by two sections of glass tube (15 cm length Pyrex), terminated at each end by a 4 mm thickness glassy carbon beam window. The total length of the chamber is $L = 87 \text{ cm}$, with an inner diameter $d = 3.5 \text{ cm}$. Inductive coils wrap around the sections of glass tube (1 cm diameter copper pipe, with 8.5 turns, and coil-winding inner diameter 7.5 cm). The coils and surrounding metallic segments of the chamber are connected to a zero-ground potential. Vacuum pumps, gas lines, and diagnostics attach to the port crosses.

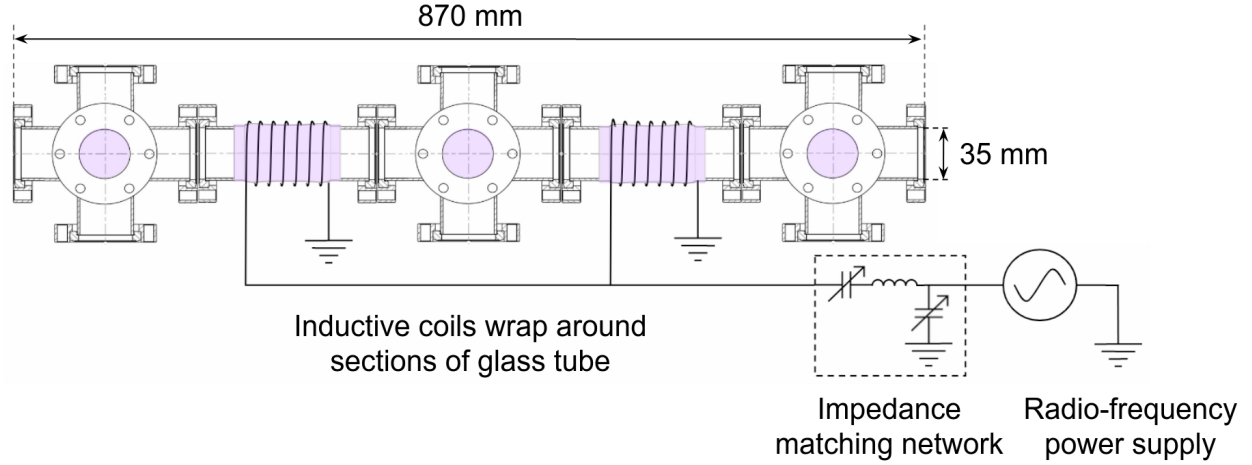


Figure 3.2: Schematic of the inductively coupled argon plasma discharge. Inductive coils made of copper pipe are wrapped around sections of glass tube of a vacuum vessel filled with argon gas to a pressure $p_g = 1 - 4$ Pa. Radio-frequency alternating current is supplied to the coils via an impedance matching network.

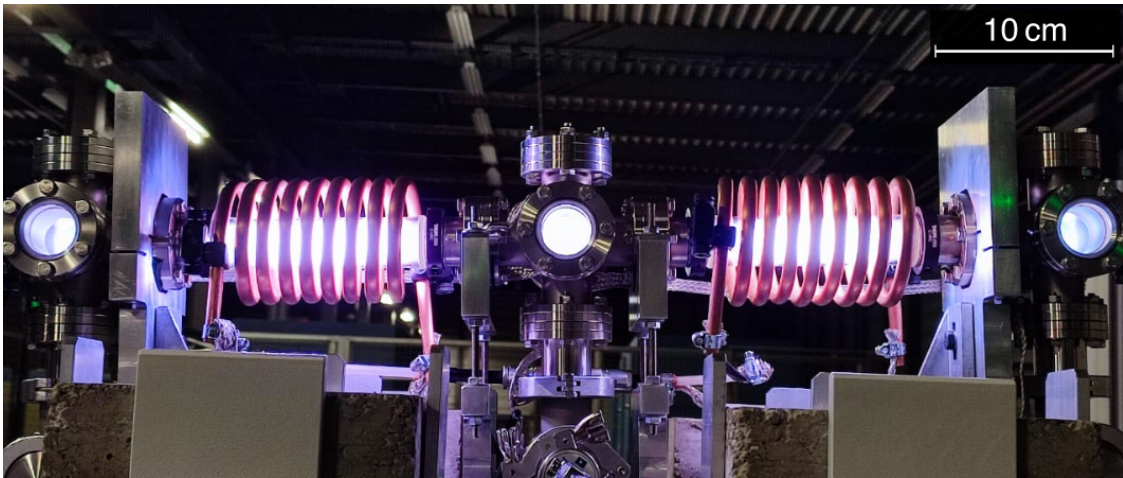


Figure 3.3: Photograph of the constructed plasma discharge. Photograph showing the plasma discharge in operation, installed on an experimental table at HiRadMat (imaged without the Faraday cage). The maximum plasma density obtained with gas pressure $p_g = 4$ Pa is $n_p = 2 \times 10^{12} \text{ cm}^{-3}$, with a corresponding plasma skin depth $\lambda_s = 4$ mm.

Before operation, the chamber is evacuated to a base pressure $p_g = 5$ mPa, before being filled with argon gas (purity 99.999%) to a pressure 1 – 4 Pa. The chamber is flushed several times to minimize impurity gases, as small amounts of electronegative gases such as O₂, N₂ and H₂O make it more difficult to sustain the discharge. Argon is used as an operating gas because it is inert, inexpensive, and has been well-studied for inductive discharges used in plasma materials processing. The use of higher-Z noble gases can create even higher electron densities due to the lower ionization potential [78]. For plasma experiments, the precise choice of operating gas matters little to the physics provided that ion/neutral dynamics aren't expected to be important, which is the case for the discussed experiment. RF power is supplied to the coils at a frequency $f = 13.56$ MHz using a commercially available kilo-Watt radio-frequency power generator (Advanced Energy CESAR 1310) via an impedance-matching network (Advanced Energy Navio). The matching network is tuned to minimize the reflected power due to impedance mismatch between the power supply and the load circuit. The length of cable connecting the power supply to the coils must be adjusted such that the combined reactance of the cable, coils and plasma can be compensated by the limited range of reactance of the matching network. An acceptable amount of reflected power is deemed to be $< 2\%$.

The plasma conditions in the discharge depend on the gas pressure and the power absorbed by the plasma: $P_{\text{abs}} = P_{\text{f}} - P_{\text{ref}} - P_{\text{loss}}$, where P_{f} is the input (forward) power, P_{ref} is the reflected power due to impedance mismatch, and $P_{\text{loss}} = I_{\text{rms}}^2 R_{\text{eff}}$ is the power lost due to ohmic heating in the coupling circuit. The effective resistance R_{eff}

is measured by recording $(P_{\text{inc}} - P_{\text{ref}})$ as a function of I_{rms} using a current transformer (Pearson 4100) when the vacuum chamber is completely evacuated so that no plasma can form ($P_{\text{abs}} = 0$). Power coupling efficiencies $\eta = P_{\text{abs}}/(P_{\text{inc}} - P_{\text{ref}}) \approx 20 - 30\%$ are found to be typical. When the fill pressure is in the range $p_g = 0.8 - 5 \text{ Pa}$ and $P_{\text{abs}} \gtrsim 150 \text{ W}$, power is coupled inductively into the gas. A high plasma density is produced inside the coils ($n_p \gtrsim 10^{12} \text{ cm}^{-3}$), and a lower density plasma ($n_p \sim 10^{10} - 10^{11} \text{ cm}^{-3}$) extends between, sustained by electrostatic/capacitive coupling.

3.2 Operating principle

Plasma is generated in an inductive discharge in the following way. When current passes through the inductive coil, an internal magnetic field is produced that is proportional to the current (I) and the number of coil windings per unit length (N). For an infinite-length coil, the magnetic field is axial: $\mathbf{B} = B_z \hat{\mathbf{z}}$, with

$$B_z = \mu_0 I N. \quad (3.1)$$

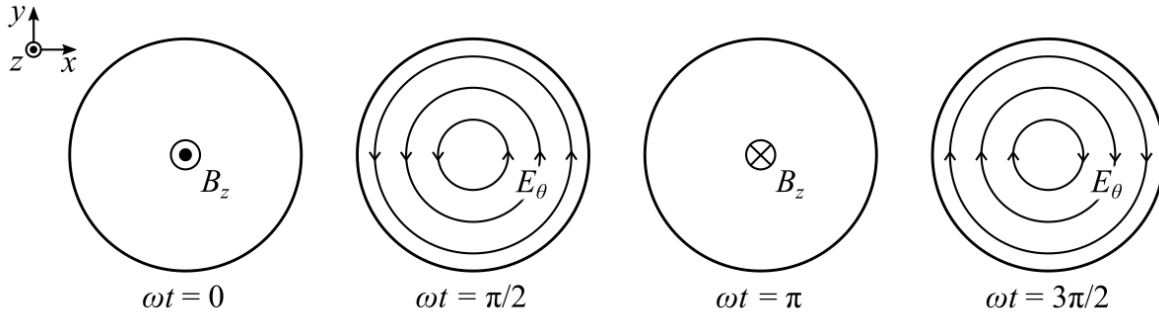


Figure 3.4: Electromagnetic fields in the cylindrical configuration ICP discharge.

For a finite-length coil of length $L = 15$ cm and diameter $d = 7.5$ cm, the peak magnetic field at the centre of the coil is only smaller by a factor $\sqrt{1 + (d/L)^2} = 1.1 \times$. If a time-varying current is supplied, for instance a sinusoidal current with a frequency $f = \omega/2\pi$: $I = I_0 \cos(\omega t)$, then an electric field is induced according to Faraday's law:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}. \quad (3.2)$$

Assuming the magnetic field is axial, this becomes:

$$\frac{1}{r} \frac{\partial(rE_\theta)}{\partial r} = -\frac{\partial B_z}{\partial t} = \mu_0 I_0 N \omega \sin(\omega t), \quad (3.3)$$

showing there is an electric field induced azimuthally to the coil axis:

$$E_\theta = \frac{1}{2} \mu_0 I_0 N r \omega \sin(\omega t). \quad (3.4)$$

The electric field is out of phase with the magnetic field by a quarter cycle. This is shown in Figure 3.4, where the black circle represents the circumference of the cylindrical discharge, and the axis of the surrounding inductive coil points out of the page. Free electrons in the discharge are accelerated by the induced azimuthal electric field. Calculating the speed of an electron after one half-cycle (starting from rest),

$$v_e = \frac{e}{m_e} \int_0^{\pi/\omega} E_\theta dt = \frac{e}{m_e} \mu_0 I_0 N r, \quad (3.5)$$

which is independent of the driving frequency. Using the design parameters (density of coil windings $N = 8.5/15 \text{ cm}^{-1}$, and expected maximum current in the coil $I_0 \sim 4 \text{ A}$), at a radius $r = 3 \text{ cm}$,

$$v_e = 1.6 \times 10^6 \text{ m s}^{-1} \left(\frac{I_0}{4 \text{ A}} \right) \left(\frac{N}{8.5/15 \text{ cm}^{-1}} \right) \left(\frac{r}{3 \text{ cm}} \right). \quad (3.6)$$

This compares with the mean speed of an electron in a Maxwellian distribution with temperature $T_e \sim \text{eV}$:

$$\bar{v}_e = \left(\frac{8k_B T_e}{\pi m_e} \right)^{1/2} = 6.7 \times 10^5 \text{ m s}^{-1} \left(\frac{T_e}{1 \text{ eV}} \right)^{1/2}. \quad (3.7)$$

Thermalization of electrons with the bulk gas via collisions forms a partially ionized plasma discharge, with the collision frequency of electrons with neutrals being approximately given by:

$$\nu_{\text{en}} = \frac{\bar{v}_e}{\lambda_{\text{en}}} \approx 8.5 \times 10^5 \text{ s}^{-1} \left(\frac{T_e}{1 \text{ eV}} \right)^{1/2} \left(\frac{p}{1 \text{ Pa}} \right) \left(\frac{T_g}{0.05 \text{ eV}} \right)^{-1}, \quad (3.8)$$

assuming an ideal gas law and the total cross section for room temperature neutrals $\sigma_n \approx 10^{-14} \text{ cm}^2$. Coupling of power into the plasma is most efficient if the timescale of thermalization is equivalent to the period of the driving fields, justifying a radio-frequency alternating current, $f \sim \mathcal{O}(10^6 \text{ Hz})$. By this process, ionization fractions $\chi = n_i/(n_i + n_n) \approx 0.1 - 5\%$ can be achieved.

3.3 An analytical model to estimate the plasma parameters

Analytical models can be used to estimate the plasma conditions, such as density and temperature, when the discharge is in a steady state of operation by considering the particle balance and power balance. Specifically, the rate of ionization is balanced with the rate at which ions drift into the walls, and power coupled into the plasma is balanced against the combined energy losses. In the ‘volume average global model’ [72, 73, 79] for a cylindrical discharge chamber of radius R and length ℓ , it is assumed that the bulk density is uniform (n_0) and there is a thin (skin-depth thickness) sheath where power is coupled into the electrons. Particle balance is written:

$$\underbrace{n_g n_0 k_{iz} R^2 \ell}_{\text{ionization in the bulk plasma}} = \underbrace{(2\pi R^2 h_\ell n_0 + 2\pi R \ell h_R n_0) u_B}_{\text{ion loss to walls}}, \quad (3.9)$$

where k_{iz} is the electron impact ionization cross section (given by Straub et al. [80]), $u_B = \sqrt{k_B T_e / m_i}$ is the Bohm velocity (approximately the ion loss velocity), and h_ℓ and h_R are expressions relating the bulk density to the sheath density [81]:

$$h_\ell \equiv \frac{n_{s\ell}}{n_0} \simeq 0.86 \left(3 + \frac{\ell}{2\lambda_i} \right)^{-1/2}, \quad (3.10)$$

$$h_R \equiv \frac{n_{sR}}{n_0} \simeq 0.80 \left(4 + \frac{R}{\lambda_i} \right)^{-1/2}, \quad (3.11)$$

where $\lambda_i = 1/n_g \sigma_i$ is the ion-neutral mean free path. These expressions are valid in the regime where $R, \ell \geq \lambda_i \geq T_i / T_e$.

Rearranging Eq. 3.9 allows the electron temperature in the bulk to be evaluated:

$$\frac{k_{iz}(T_e)}{u_B(T_e)} = \frac{1}{\frac{1}{2}R\ell/(Rh_\ell + \ell h_R)n_g}. \quad (3.12)$$

Power balance is written:

$$P_{\text{abs}} = en_0 u_B A_{\text{eff}} \mathcal{E}_T, \quad (3.13)$$

where A_{eff} is the effective area for particle loss, and $\mathcal{E}_T$ is the total energy loss per electron-ion pair created:

$$\mathcal{E}_T = \mathcal{E}_c + \mathcal{E}_e + \mathcal{E}_i. \quad (3.14)$$

For Maxwellian electrons, the mean kinetic energy lost per electron lost is $\mathcal{E}_e = 2T_e$ [73], and the ion kinetic energy lost to walls is $\mathcal{E}_i \approx \bar{V}_s$ (simply the d.c. potential across the sheath). The ‘collisional’ energy lost per electron-ion pair created, $\mathcal{E}_c$, includes the ionization energy and the excitation of atomic states and elastic collisions with neutrals

$$\mathcal{E}_c(T_e) = \mathcal{E}_{iz} + \sum_{i=1}^n \frac{k_{\text{ex},i}}{k_{iz}} \mathcal{E}_{\text{ex},i} + \frac{k_{\text{el}}}{k_{iz}} \frac{3m_e}{M} T_e, \quad (3.15)$$

where $k_{\text{ex},i}$ are cross sections for excitation of ground state electrons to the ‘ i ’-th state, and k_{el} is the cross section associated with elastic collisions of electrons with neutrals (given by Hayashi [82]). Eq. 3.13 is rearranged to solve for the centre particle density using the electron temperature obtained by the particle balance (Eq. 3.12).

In Figure 3.5, electron temperature and density have been estimated for the plasma discharge using this model, assuming a 35 mm diameter discharge region with a 300 mm length (the combined length of the inductive coils). These models show that with 100 – 300 W of coupled power, plasma densities exceeding $n_p \sim 10^{12} \text{ cm}^{-3}$ should be achievable, with electron temperatures of a few eV.

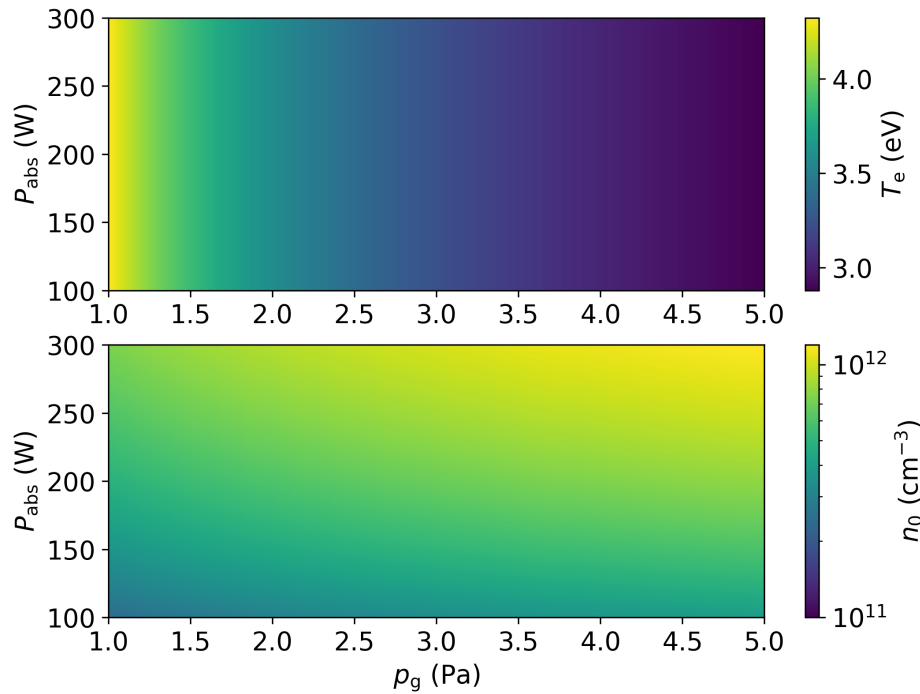


Figure 3.5: Analytical model of the inductive discharge. The electron temperature (upper) and plasma density (lower) are estimated using the volume-average global model, assuming a 35 mm diameter, 300 mm length discharge region. Electron temperature is sensitive to gas pressure (p_g) but is not sensitive to the absorbed power (P_{abs}). Plasma density increases with both gas pressure and absorbed power.

3.4 Measuring the plasma parameters using a Langmuir probe

To obtain the plasma characteristics experimentally, a Langmuir probe is used. In its most basic form, a Langmuir probe is simply a wire (probe tip) inserted into the plasma (schematic shown in Figure 3.6). When the probe tip is biased with a variable voltage, the collected current is used to obtain information about the plasma. The current measured as a function of the probe voltage is called the I-V characteristic, and careful analysis allows the electron density, ion density, electron temperature, and electron energy distribution function (EEDF) to be determined.

3.4.1 Langmuir probe theory

An example Langmuir probe I-V characteristic is shown in Figure 3.7. The I-V characteristic is divided into three regions by the floating potential (V_f) and the space potential (V_s). When the probe tip is at the floating potential, the current drawn by the probe has equal contributions from electrons and ions. The floating potential is positive because the electrons

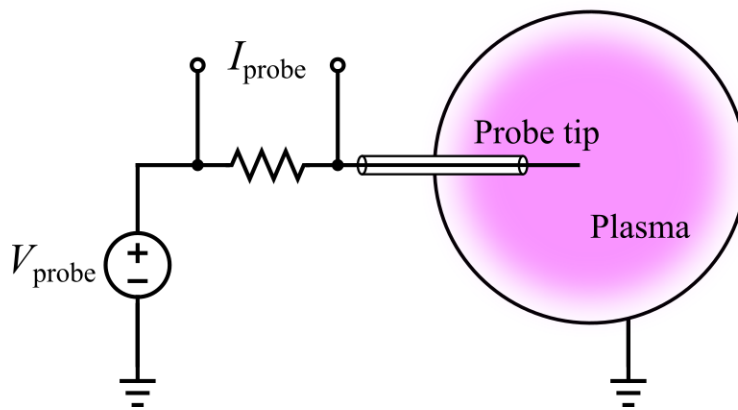


Figure 3.6: Langmuir probe schematic. A biased Langmuir probe tip is inserted into the plasma and the current drawn at the probe tip is measured to infer the plasma properties.

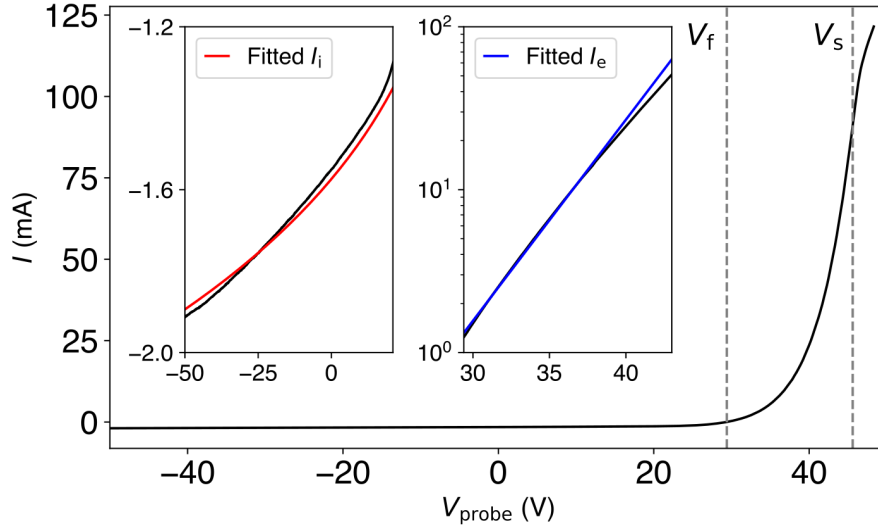


Figure 3.7: A typical Langmuir probe I-V characteristic. A Langmuir probe I-V characteristic obtained with the probe tip positioned at the centre of the inductive coil measuring the plasma generated with $p_g = 1$ Pa and $P_{abs} = 200$ W. The floating potential and space potential are labelled ($V_f = 29.4$ V, $V_s = 45.6$ V). The left-hand inset shows the fit to the ion-saturation region to obtain the ion density (red). The right-hand inset shows the fit of electron current in the transition region to obtain the electron density and temperature (blue).

have a higher temperature than the ions, so they can more easily cross the charged sheath surrounding the probe tip. At more negative voltages than the floating potential, fewer electrons can cross the sheath, until only positive ions are collected. This region is called the ion-saturation region. Here, theories of ion motion in the sheath potential are used to fit the ion current and obtain the ion number density [83–86]. When the probe tip is at the space potential, the plasma and probe potentials are equal (the plasma has a net positive potential because electrons more readily escape the bulk plasma than the ions). Above the space potential, ions are fully repelled and only electrons are collected (electron-saturation). In the transition region in-between, as the voltage is decreased from the space potential, fewer low energy electrons can cross the sheath, and so the rate of change of the I-V characteristic

allows the electron temperature to be measured. With the fitted ion current subtracted from the I-V characteristic, the remaining electron current is fitted to an exponential in the transition region to obtain the electron temperature:

$$I_e = I_s \exp \left(\frac{V - V_s}{T_e} \right), \quad (3.16)$$

where I_s is the current collected when the probe tip is at the space potential V_s . Once the electron temperature has been determined, the electron density is obtained using

$$n_e = \frac{I_s}{A_p} \left(\frac{2\pi m_e}{e^2 k_B T_e} \right)^{1/2}, \quad (3.17)$$

given the surface area of the probe tip, $A_p = 2\pi r_p l_p + \pi r_p^2$.

An exponential fit is valid provided the electron energy distribution function (g_e) is Maxwellian, which assumes a constant electron-neutral collision frequency ($\nu_{en} = \text{const}$). Other distributions may provide a more appropriate fit; for instance, a bi-Maxwellian distribution in which more energetic electrons are characterized by a second higher temperature, or a Druyvesteyn distribution in which a constant collision cross-section is assumed ($\nu_{en} = n_g \sigma_{en} v_e$), leading to a steeper decline of the distribution at higher energies [73]. The Maxwellian and Druyvesteyn distributions are written:

$$g_e(\mathcal{E}) = A \frac{n_e}{T_{e,\text{eff}}} \mathcal{E}^{1/2} \exp \left[-C \left(\frac{\mathcal{E}}{T_{e,\text{eff}}} \right)^x \right], \quad (3.18)$$

where $x = 1$ for a Maxwellian and $x = 2$ for a Druyvesteyn distribution, $\mathcal{E}$ is the electron energy in equivalent voltage units, $T_{e,\text{eff}} = \frac{2}{3}\langle\mathcal{E}\rangle = \frac{2}{3}\int_0^\infty g_e(\mathcal{E})\mathcal{E}d\mathcal{E}$ is the effective electron temperature, A is a normalization constant determined by $\int g_e(\mathcal{E})d\mathcal{E} = n_e$, and C is a constant which keeps the average energy per electron as $\bar{\mathcal{E}}_e = \frac{3}{2}T_{e,\text{eff}}$ ($C = 1$ for a Maxwellian, $C \approx 0.243$ for a Druyvesteyn) [73]. The electron energy distribution function (EEDF) can be obtained directly from the Langmuir probe trace using the Druyvesteyn formula [87]:

$$g_e(\mathcal{E}) = \frac{2m_e}{e^2 A_p} \left(\frac{2eV}{m_e} \right)^{1/2} \frac{d^2 I_e}{dV^2}. \quad (3.19)$$

Practically, the accuracy of this method is limited by the noise in the I-V characteristic, which becomes amplified by the two successive numerical differentiations. To reduce the noise, digital smoothing of the data is performed. A Blackman window is found to achieve the best smoothing with minimal distortion of the I-V characteristic when applied to datasets subjected to artificial noise [88].

In the measurements presented, the electron distribution is found to be well-approximated by a Maxwellian (as shown in the next section), so the exponential fit to the transition region is used as the primary measurement. The fractional difference between the temperature and density calculated using exponential fits to the I-V transition region (Eq. 3.16 and Eq. 3.17) versus integration of the EEDF obtained from the Druyvesteyn equation is found to be 10 – 15% ($\langle\Delta n_e/n_e\rangle = 0.12$, $\langle\Delta T_e/T_e\rangle = 0.11$). These fractional differences combine in quadrature with errors in the fitted parameters V_s , I_s and T_e to provide the error bars for the measured densities and temperatures.

3.4.2 Plasma measurements

A commercially available Langmuir probe unit was used (manufactured by Impedans, Ireland) to perform measurements of the plasma discharge. A single probe tip configuration is used with an RF compensation electrode. The probe tip is made of tungsten wire (4 mm length, 0.195 mm radius), held in position by an alumina tube (2 mm radius). The probe tip is connected to the data acquisition unit via a series of RF chokes to filter RF interference. The probe is inserted into the plasma at different positions along the axis to obtain the electron density, ion density, electron temperature and electron energy distribution function.

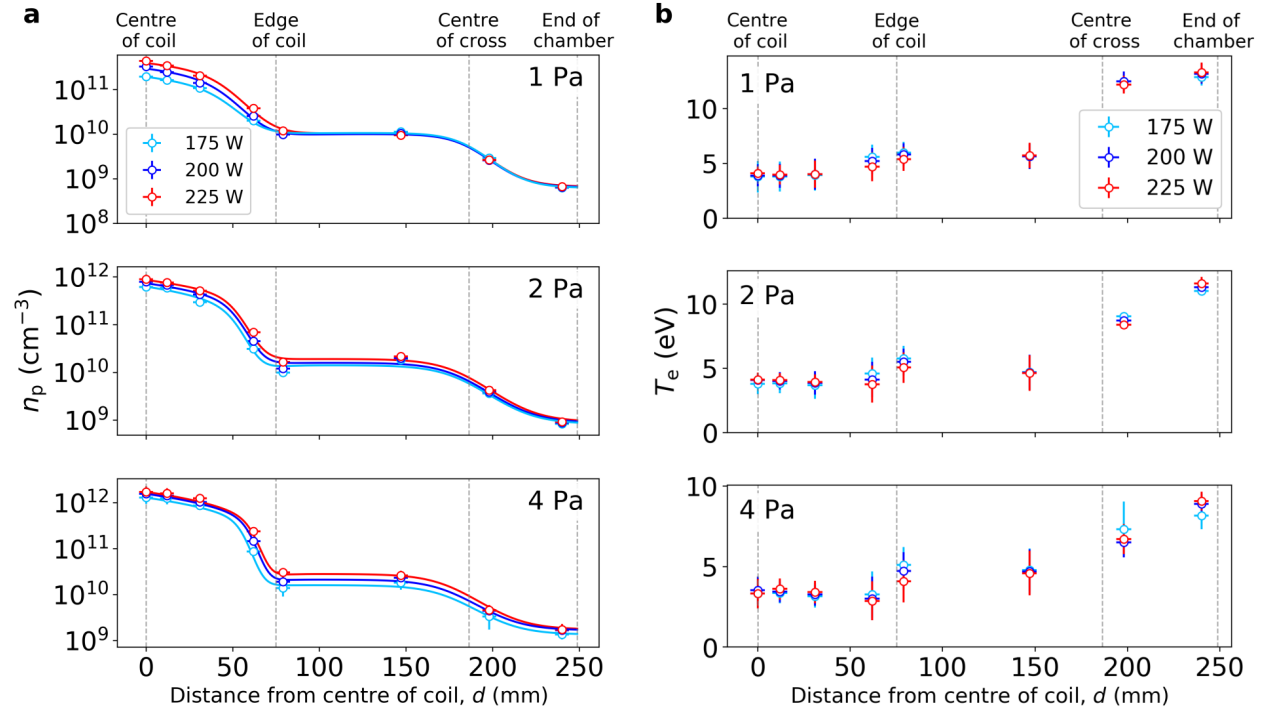


Figure 3.8: Langmuir probe measurements of the plasma parameters. (a) The plasma density, $n_p = (n_e + n_i)/2$ and (b) electron temperature, T_e , measured at different axial positions along the discharge relative to the centre of one of the inductive coils. Three power settings and pressure settings are measured: $P_{\text{abs}} = 175, 200, 225$ W and $p_g = 1, 2, 4$ Pa. The vertical dashed lines correspond to: the centre of the coil ($z = 0$ mm), the edge of the coil ($z = 75$ mm), the centre of the six-way cross ($z = 186$ mm), and the end of the chamber ($z = 245$ mm).

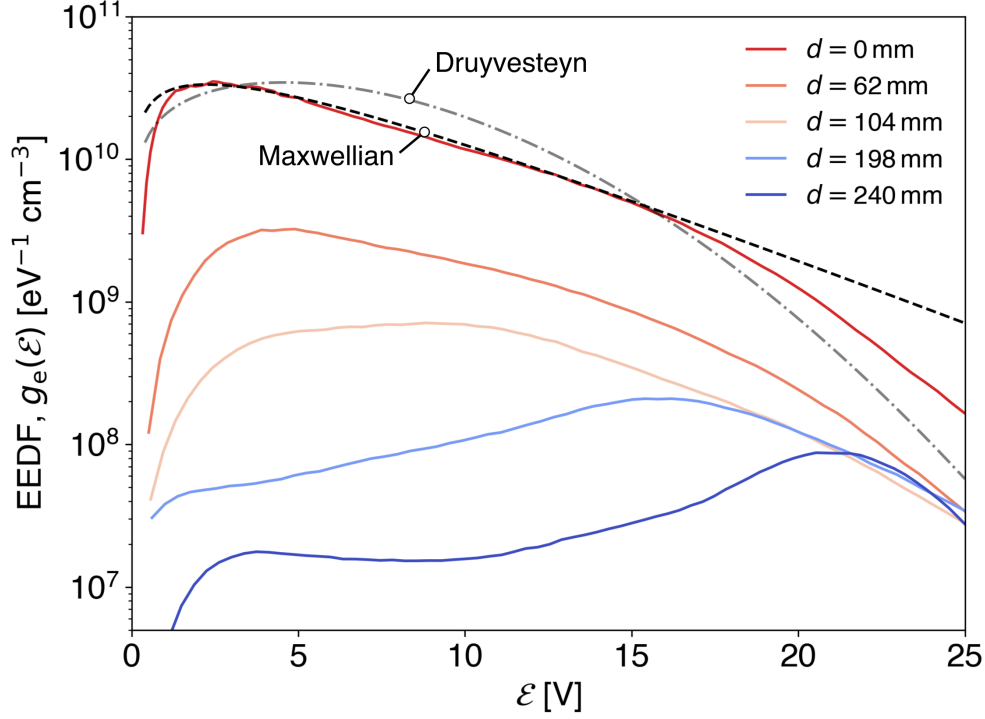


Figure 3.9: Electron distribution function at different positions along the discharge. The electron energy distribution function (EEDF) is obtained from measured Langmuir probe I-V characteristics using Eq. 3.19 at multiple positions longitudinally along the plasma (at a distance, d , from the centre of one of the inductive coils). Inside the coil ($d \leq 75$ mm), the distribution function is better fitted by a Maxwellian distribution (black, dashed) than alternatives such as a Druyvesteyn distribution (grey, dot-dashed), plotted assuming $T_e = 4.5$ eV and $n_e = 3.5 \times 10^{11} \text{ cm}^{-3}$, though the Maxwellian overestimates the higher energy portion of the spectra. Outside the coil, the distribution function becomes non-Maxwellian due to a hot electron population.

To account for the under-estimation of the electron density due to the presence of the probe holder, a correction factor is applied to the electron density of $(1 + r_h/\lambda_e)$, where r_h is the radius of the probe holder, and λ_e is the electron mean free path [89].

The density and temperature profiles measured from the centre of the coil to the end of the discharge are shown in Figure 3.8, for absorbed powers: $P_{\text{abs}} = 175, 200, 225$ W, and argon pressures: $p_g = 1, 2, 4$ Pa. The highest plasma densities are measured when the probe tip is positioned at the centre of the inductive coil, up to $n_p = 2 \times 10^{12} \text{ cm}^{-3}$ when the highest

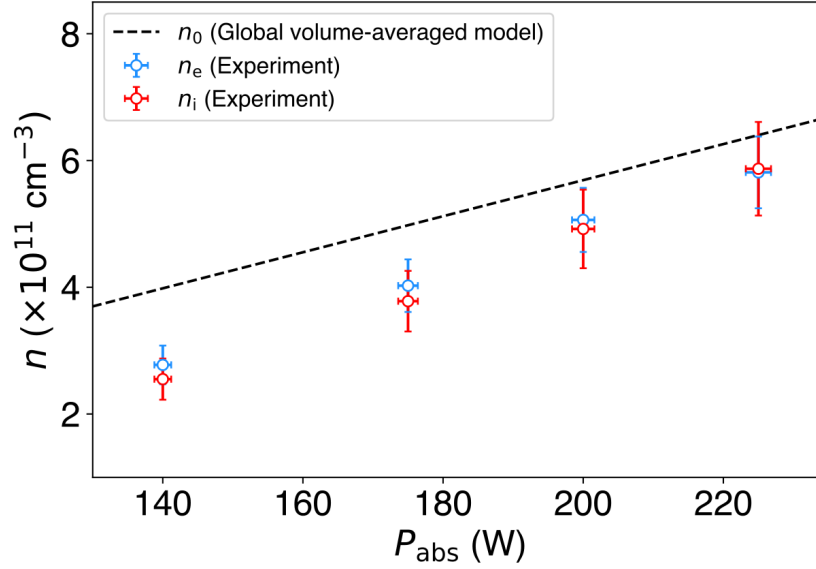


Figure 3.10: Langmuir probe measurements compared with the analytical model. The centre electron/ion density n_0 is obtained from the volume-average global model, comparing with the densities obtained in the centre of the coil from Langmuir probe measurements.

pressure and applied power are used: $P_{\text{abs}} = 225 \text{ W}$, $p_g = 4 \text{ Pa}$. At any given pressure, the peak plasma density scales approximately linearly with the absorbed power. Inside the discharge region, electron temperatures consistently lie in the range $T_e = 3 - 5 \text{ eV}$, although further from the coil, the measured electron temperature is higher, $T_e > 10 \text{ eV}$. Electron distribution functions obtained at different axial positions using the Druyvesteyn formula are plotted in Figure 3.9, showing that a hotter electron population is present further away from the coils. A possible explanation for this is that hot electrons escape from the discharge region inside the coils. There may be an additional accelerating effect at the ends of the coil: as the magnetic field diverges, the induced electric field has a component directed along the coil axis. This is how thrust is generated in some schemes of electrodeless plasma propulsion.

Comparing the measurements with the density and temperatures expected from the

analytical model of the previous section, the results show reasonable agreement (see Figure 3.10). There is better agreement when the absorbed power is higher, which may result from the fact that a higher proportion of the power is coupled inductively into the plasma when a higher power is applied, rather than the capacitive coupling which sustains the plasma between the coils.

3.5 Optical emission spectroscopy as a non-invasive diagnostic

A Langmuir probe is too invasive to perform in-situ measurements during the HiRadMat experiment, so optical emission spectroscopy is considered as a diagnostic to monitor the plasma conditions non-invasively, the principle being that optical line emission from the plasma (responsible for the purple glow) depends on the population of excited states in the plasma, which in turn depends on the temperature and density.

3.5.1 Einstein coefficients and line broadening

In systems where the radiation field is so weak that stimulated emission and absorption processes can be ignored, the optical emission is determined by the rate of collisional excitation to excited states (via electron collisions), $C_{ij}n_e$, and the rate of spontaneous emission, A_{ji} . This is depicted in Figure 3.11, which shows a simple two-level atomic system. If the number of electrons is conserved between the two levels (statistical equilibrium),

$$n_2(C_{21}n_e + A_{21}) = n_1C_{12}n_e \quad (3.20)$$

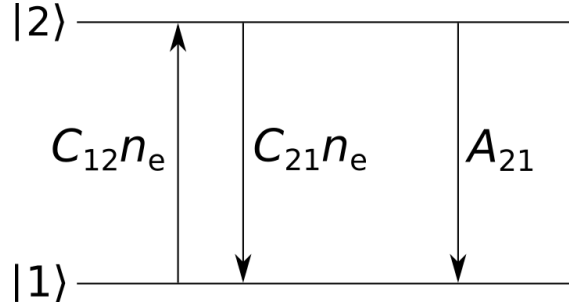


Figure 3.11: Two-level atomic model. Einstein coefficients for spontaneous emission (A_{21}) and collisional excitation/de-excitation (C_{12} , C_{21}) are shown for a two-level atomic system in which the radiation field is so weak that stimulated processes (B -coefficients) can be ignored.

where n_j is the number density of atoms in state ‘ j ’. A common approximation which applies when electron densities are low is $C_{21}n_e \ll A_{21}$ (the ‘coronal approximation’), and the ratio of the population of states becomes

$$\frac{n_2}{n_1} = \frac{C_{12}n_e}{A_{21}}. \quad (3.21)$$

The energy emitted is dependent on the free electron density, the density of atoms in the lower state, and the electron temperature:

$$dE_{21} = \frac{h\nu_{21}}{4\pi} (n_2 A_{21}) \phi(\nu) dV d\Omega d\nu dt = \frac{h\nu_{21}}{4\pi} (n_1 C_{12} n_e) \phi(\nu) dV d\Omega d\nu dt, \quad (3.22)$$

where $\phi(\nu)$ is the line profile. The rate equations for systems containing very few levels can be solved simply, but the algebra becomes impractical as soon as the number of atomic levels gets larger than about 3. To model realistic atoms, collisional radiative emission codes have been developed which include the complete atomic data. Codes such as PrismSPECT [90] can be programmed to consider non-steady state conditions, finite-size geometries (which

account for optical thickness), and where local thermodynamic equilibrium (LTE) may not apply (states are not populated according to Boltzmann statistics). In laboratory plasma experiments, pretty much all of these complications apply. Practically, finding atomic transitions that can give reliable information about the plasma conditions a priori becomes a dark art.

In certain cases, it is possible to infer plasma properties from spectral line broadening. Natural broadening (due to the Uncertainty Principle) is almost always negligible:

$$\left(\frac{\Delta\lambda}{\lambda}\right)_{\text{nat}} \sim \frac{\hbar}{E\Delta t} \sim \frac{\lambda}{2\pi c} A_{ji} \sim 5 \times 10^{-9} \left(\frac{\lambda}{1\text{ }\mu\text{m}}\right) \left(\frac{A_{ji}}{10^7\text{ s}^{-1}}\right). \quad (3.23)$$

Collisional broadening due to phase-destruction during emission via atomic collisions is also small, unless the density is far in excess of what it is in the discharge:

$$\left(\frac{\Delta\lambda}{\lambda}\right)_{\text{col}} \sim \frac{\lambda}{2\pi c} (2\nu_{\text{col}}) \sim 5 \times 10^{-8} \left(\frac{p}{1\text{ Pa}}\right) \left(\frac{T_{\text{a}}}{0.05\text{ eV}}\right)^{-1} \left(\frac{T_{\text{e}}}{1\text{ eV}}\right)^{1/2}, \quad (3.24)$$

considering electron-neutral collisions, though the result is similar for electron-ion collisions. Doppler broadening is negligible for ions and neutrals in the discharge because their temperature is comparable to room temperature:

$$\left(\frac{\Delta\lambda}{\lambda}\right)_{\text{Doppler}} \sim \frac{1}{c} \sqrt{\frac{2k_{\text{B}}T_{\text{a}}}{M_{\text{a}}}} \sim 10^{-5} \frac{1}{\sqrt{A}} \left(\frac{k_{\text{B}}T_{\text{a}}}{0.05\text{ eV}}\right)^{1/2}. \quad (3.25)$$

Stark and Zeeman broadening due to the electromagnetic field are also too weak. So the

width of spectral emission lines observed from the inductive plasma discharge is limited by the instrument function of the spectrometer ($\lambda/\Delta\lambda = 2200$, $\Delta\lambda \approx 0.34$ nm), and only the line ratios and intensities can be used. PrismSPECT modelling is able to generate synthetic spectra that can be compared with spectroscopic measurements.

3.5.2 Spectroscopic measurements

Spectroscopic measurements were performed using a Princeton Instruments Acton SpectraPro Czerny-Turner spectrometer coupled to a Hamamatsu Orca flash camera. Light is collected from a 2 mm area at the centre of the inductive coil using a collimating lens, which couples the light into an optical fibre and transports it to the slit of the spectrometer. The optical fibre is multimode with core diameter 105 μm and numerical aperture 0.1. The slit width was 70 μm and the grating had a line density 600 g/mm. A calibrated white light source is used to calibrate for any spectral dependence on transmission/reflection of the optics or spatial variation in the camera sensitivity. An argon-mercury lamp is used to calibrate the spectrum and obtain the instrument function. The spectra were measured in the spectral region $\lambda = 725 - 770$ nm, where emission lines are prominent and early indication showed variation between the line ratios for different plasma conditions. This can be seen in the

λ (nm)	Ionization state	Transition	E_i (eV)	E_j (eV)	$g_i - g_j$	A_{ji} (10^6 s^{-1})
738.40	Ar(I)	$2p_3$ (J=2) $\rightarrow$ $1s_4$ (J=1)	11.624	13.302	3-5	8.47
750.39	Ar(I)	$2p_1$ (J=0) $\rightarrow$ $1s_2$ (J=1)	11.828	13.480	3-1	44.5
751.47	Ar(I)	$2p_5$ (J=0) $\rightarrow$ $1s_4$ (J=1)	11.624	13.273	3-1	40.2
763.51	Ar(I)	$2p_6$ (J=2) $\rightarrow$ $1s_5$ (J=2)	11.548	13.172	5-5	24.5

Table 3.1: Spectroscopic data of argon lines used in optical emission spectroscopy [91].

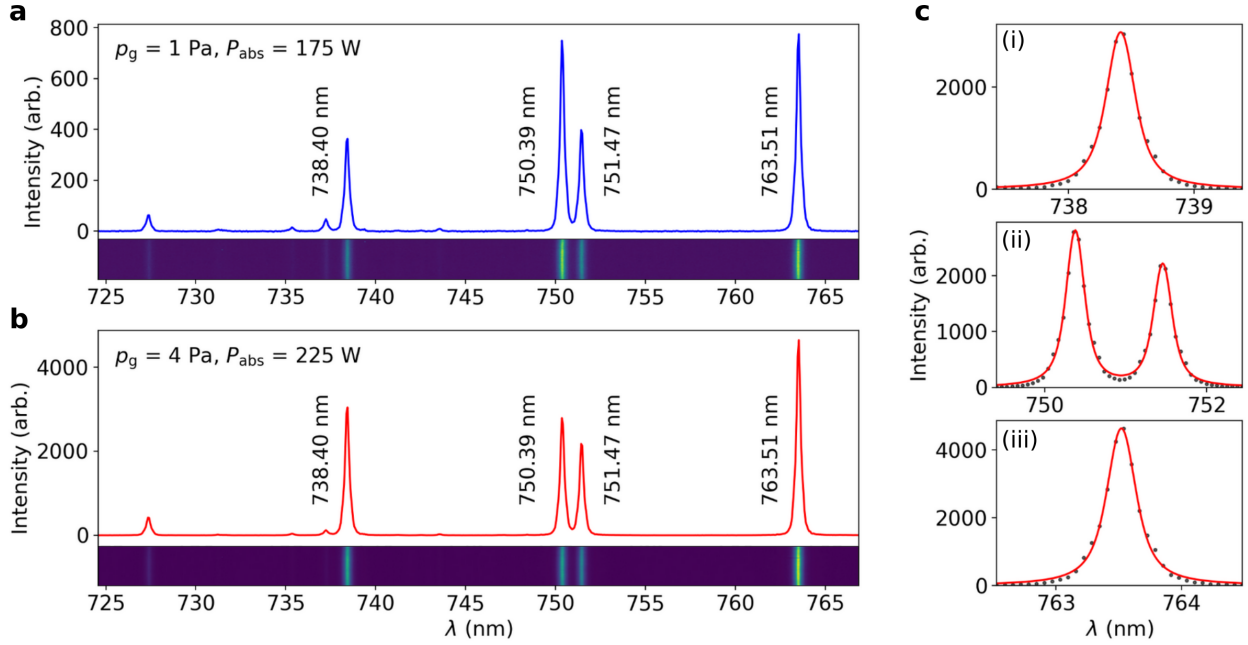


Figure 3.12: Raw data argon plasma spectrographs. Spectrographs obtained for two sets of operating conditions: (a) $p_g = 1 \text{ Pa}$, $P_{\text{abs}} = 175 \text{ W}$, corresponding to plasma conditions $n_e = 2 \times 10^{11} \text{ cm}^{-3}$, $T_e = 5.2 \text{ eV}$, and (b) $p_g = 4 \text{ Pa}$, $P_{\text{abs}} = 225 \text{ W}$, corresponding to plasma conditions $n_e = 2 \times 10^{12} \text{ cm}^{-3}$, $T_e = 4.2 \text{ eV}$. Fitting of Voigt profiles to the spectral lines in case (b) are shown in (c): (i) 738.4 nm, (ii) 750.4 nm, 751.5 nm, and (iii) 763.5 nm.

raw data and lineouts for two sets of plasma conditions in Figure 3.12. The emission lines are identified from atomic data (see Table 3.1). Emission from argon in the neutral state Ar(I) dominate the emission due to the low ionization fraction. Each emission line is fitted to a Voigt profile, defined as $V(x, \sigma, \gamma) = \int_{-\infty}^{+\infty} G(x', \sigma) L(x - x', \gamma) dx'$, the convolution of a Gaussian and a Lorentzian lineshape, where σ and γ correspond to the Gaussian standard deviation and the Lorentzian half-width. The complete set of line intensities and ratios for different plasma conditions is given in Figure 3.13. When plasma density is higher, the lines appear more intense, but the ratios are largely determined by the electron temperature.

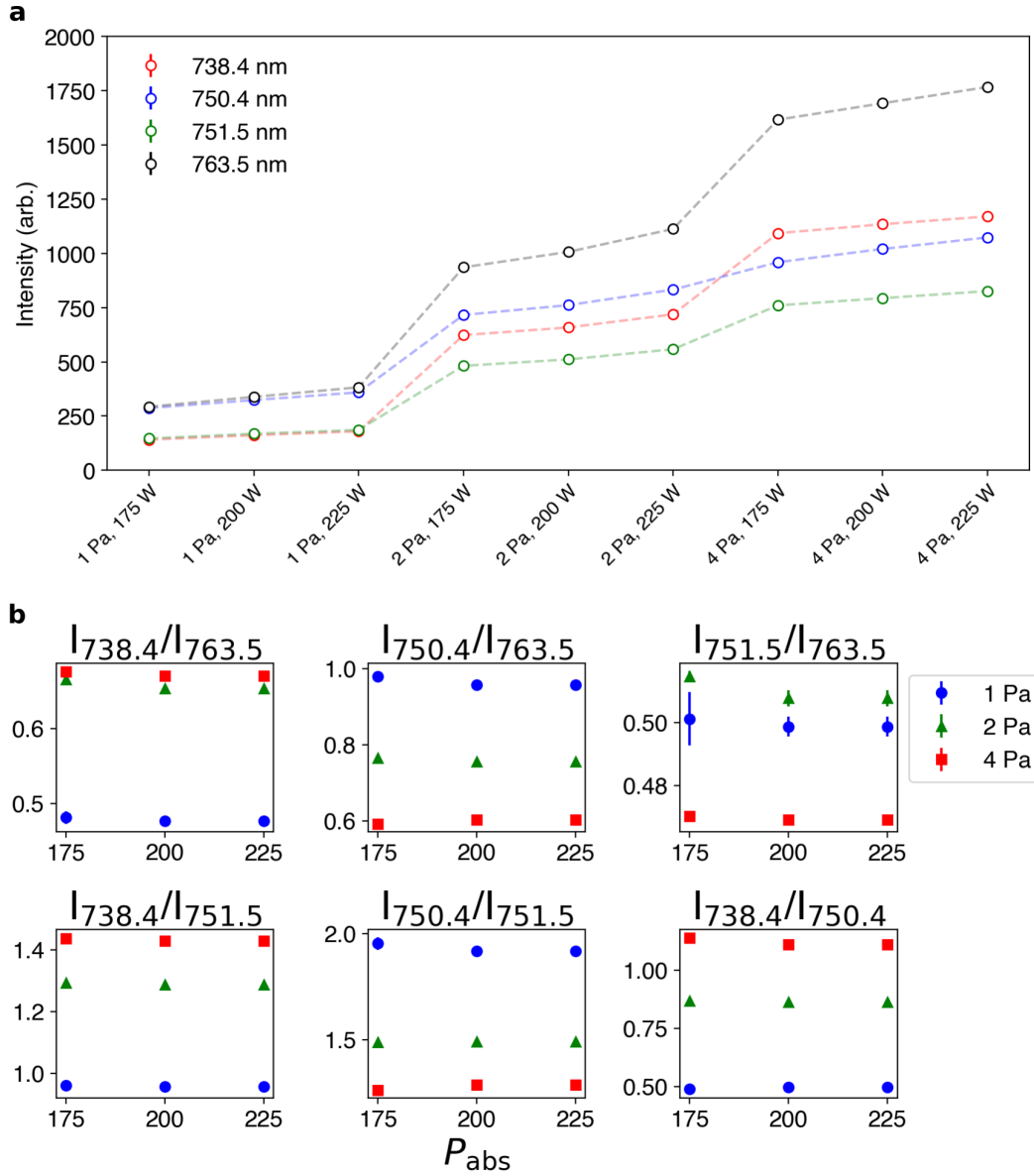


Figure 3.13: Measured argon emission line intensities and ratios. (a) The absolute intensity of emission lines for $p_g = 1, 2, 4$ Pa and $P_{abs} = 175, 200, 225$ W, ordered according to ascending plasma density measurement. (b) The corresponding line ratios.

3.5.3 Comparison with collisional radiative atomic models

To infer the plasma conditions, the measured spectra are compared with synthetic spectra generated using the collisional-radiative atomic modelling code PrismSPECT in a parameter scan that encompasses the expected plasma conditions: Maxwellian (single temperature) electron energy distribution function, with $T_e = 0.5 - 4 \text{ eV}$ (intervals: 0.1 eV), and ion density $n_i = 10^{10} - 10^{13} \text{ cm}^{-3}$ (intervals: 4 datapoints per decade). Non-LTE conditions are assumed, with detailed configuration accounting, including collisional excitation/de-excitation, spontaneous emission, photoexcitation/stimulated emission, radiative recombination, collisional ionization/recombination, auto/dielectronic recombination and photoionization/stimulated recombination, in a steady state, zero-width geometry. Rate coefficient multipliers are set as standard.

A selection of the results is shown in Figure 3.14. The general behaviour that line ratios are more sensitive to electron temperature than density is observed. Many plasma conditions produce spectra that look similar to the measured spectra, but only when much lower temperatures are assumed than the measured conditions. Above a temperature $T_e = 2 \text{ eV}$, emission lines corresponding to ionization state Ar(II) start to become visible, indicating a much larger proportion of ionized argon atoms than present in the inductive discharge. The best fits of the synthetic spectra to the data are obtained by evaluating the goodness-of-fit using two-sample Kolmogorov-Smirnov (K-S) testing, which reports the maximum difference

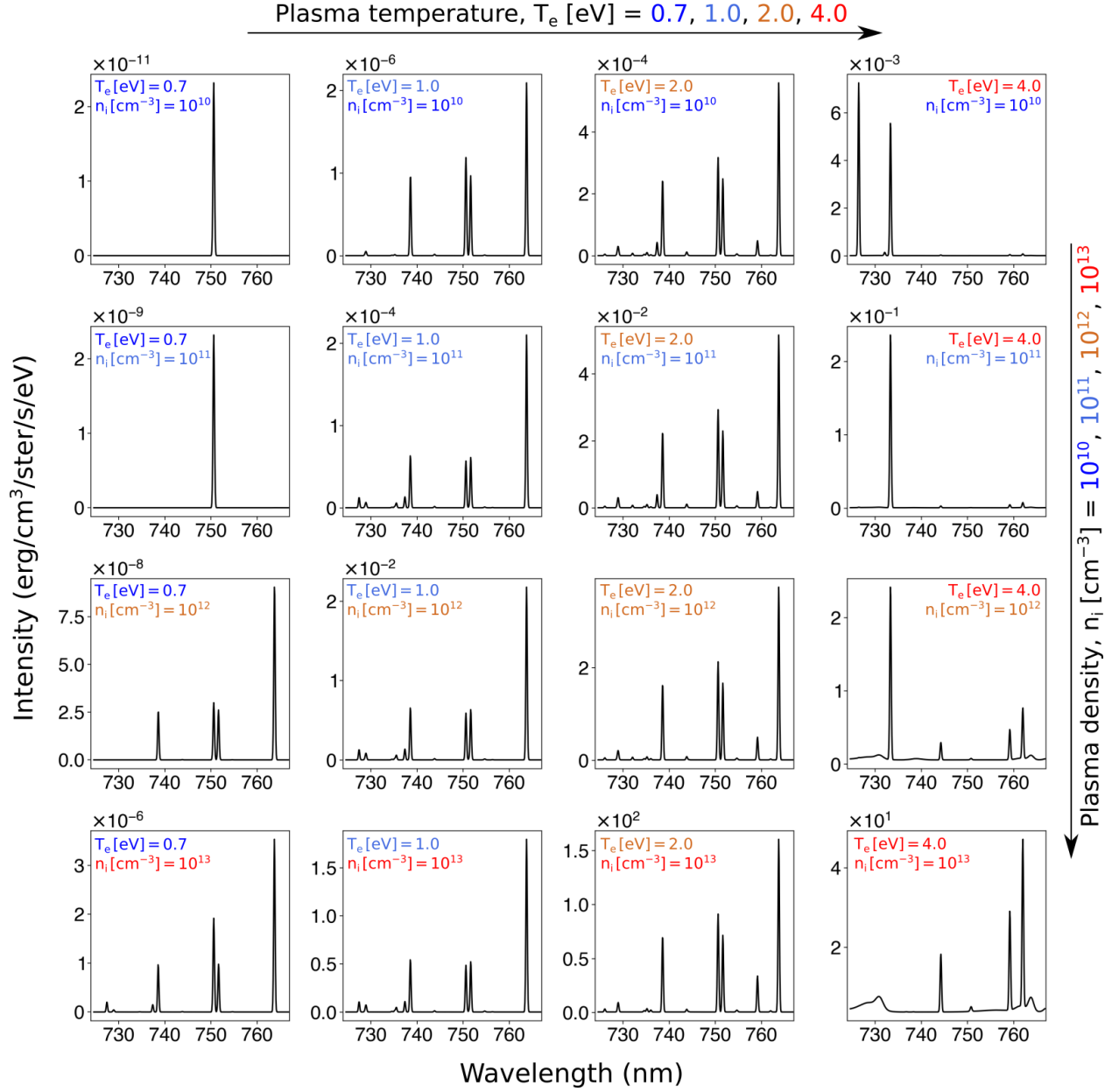


Figure 3.14: PrismSPECT Collisional-Radiative Atomic Modelling. Synthetic spectra are generated using collisional-radiative atomic modelling code PrismSPECT in a parameter scan of plasma conditions: $T_e = 0.5 - 4$ eV, $n_i = 10^{10} - 10^{13}$ cm⁻³. Many plasma conditions produce spectra that look similar to the measured spectra in Figure 3.12, although the best fits to the data correspond to much lower plasma density and temperature than measured.

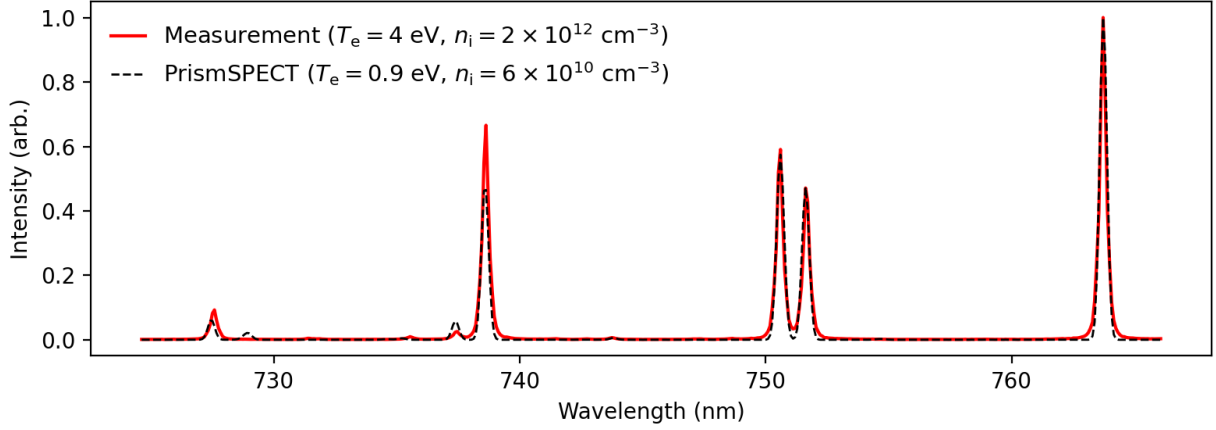


Figure 3.15: Best fit of PrismSPECT synthetic spectra to measured data.

between cumulative distribution functions of the data and the synthetic spectra [92]:

$$D = \sup_x |F_1(x) - F_2(x)|. \quad (3.26)$$

By minimizing the D-statistic, the best fit parameters for the measured conditions $T_e = 4 \text{ eV}$ and $n_i = 2 \times 10^{12} \text{ cm}^{-3}$ are found to be $T_e = 0.9 \text{ eV}$ and $n_i = 6 \times 10^{10} \text{ cm}^{-3}$ (see Figure 3.15) – substantially different. Possible reasons for the discrepancy are that the PrismSPECT modelling does not account for: acceleration/deceleration by the sheath potential, the loss of electrons and ions as they drift into the walls, the escape of metastable excited states before de-excitation, and the precise shape of the distribution function (which over-estimates the high-energy tail if a Maxwellian is assumed). Nevertheless, spectroscopy is still incredibly useful in our experiment as a non-invasive confirmation of the plasma conditions, given that spectra have been obtained simultaneously with Langmuir probe measurements.

Chapter 4

Laboratory realization of relativistic pair-plasma beams

In this chapter, the characteristics of the electron-positron pair beam are verified experimentally. It is not possible to measure certain properties of the pairs directly, such as the distribution function at the immediate rear of the target (where the density is highest), and immediately inside the plasma discharge (where the characteristics are needed to perform particle-in-cell simulations). In these cases, detailed properties are provided by FLUKA Monte-Carlo simulations, validated for the experimental geometry by measuring the transverse beam profile and the $e^\pm$ energy spectra downstream of the target (see Figure 4.1).

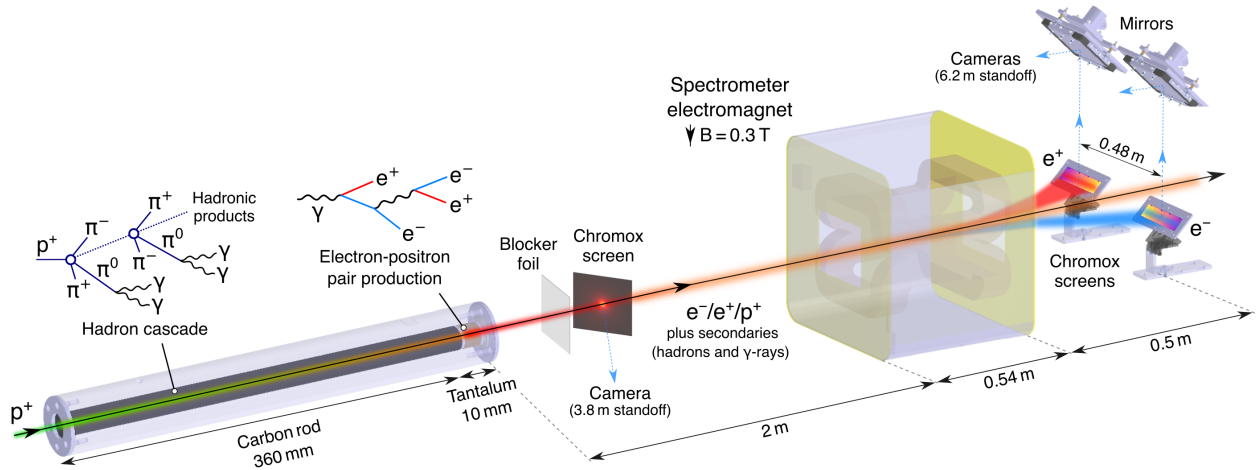


Figure 4.1: Experimental setup. The transverse beam profile is imaged using a 70 mm $\times$ 50 mm $\times$ 0.25 mm chromium-doped luminescence screen positioned 10 cm downstream of the target. A blocker foil (50 μ m aluminium) minimizes stray optical light. The screen is oriented at 45° to the beam path and viewed by a digital camera 3.8 m away. 2 m downstream of the target, $e^\pm$ are deflected away from the beam axis and spectrally resolved using a magnetic spectrometer comprised of an electromagnet and a pair of luminescence screens.

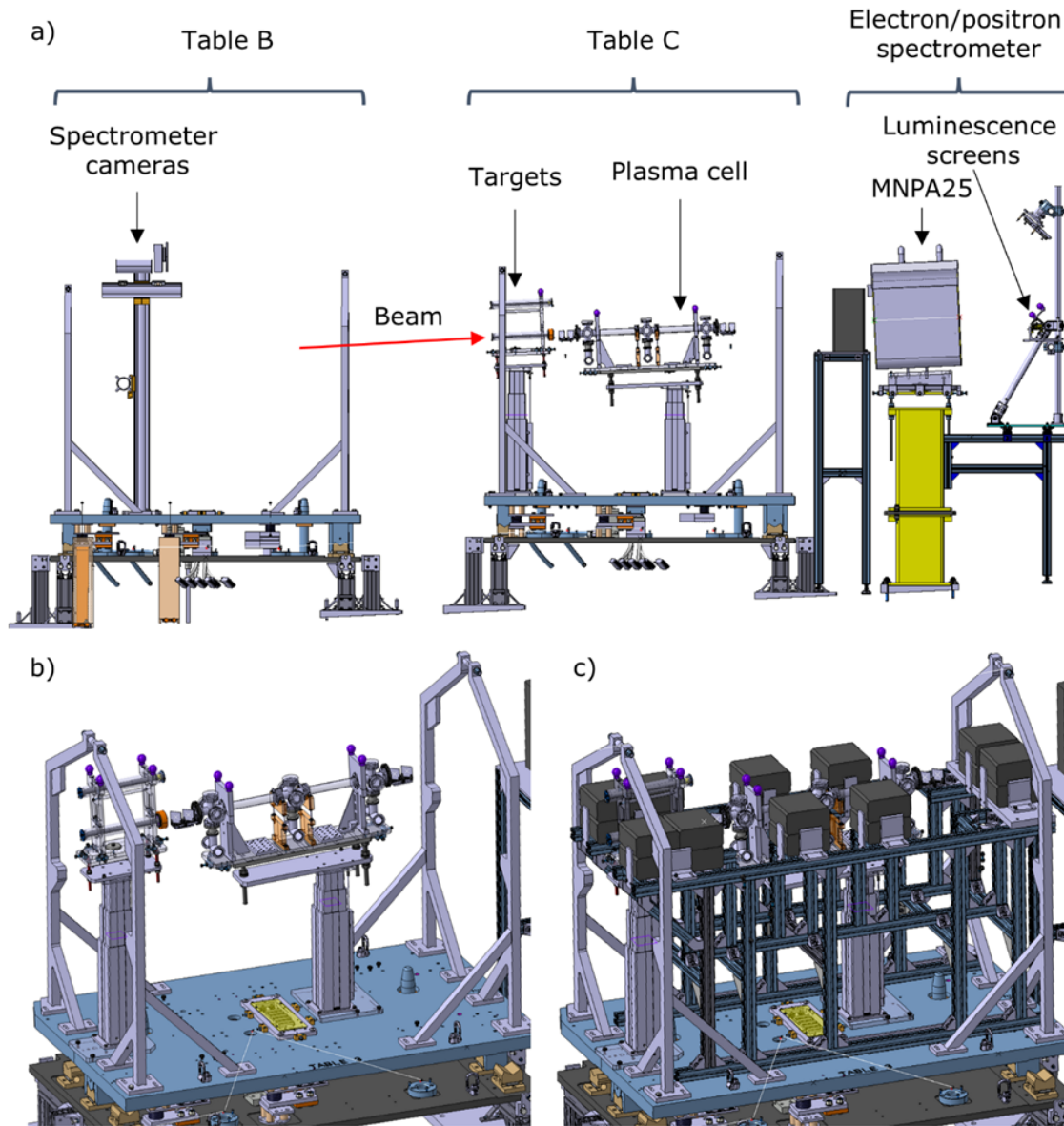


Figure 4.2: CAD designs of the complete experiment layout. (a) Side view of the implemented design. The target and plasma discharge are positioned on ‘Table C’, mounted on separate telescopic stages which enable each component to be removed entirely from the beam path. The electron-positron spectrometer is situated between Table C and the beam dump, with the dipole electromagnet labelled ‘MNPA25’ according to CERN’s magnet catalogue. The spectrometer luminescence screens positioned either side of the beam axis are viewed by cameras on ‘Table B’ via mirrors suspended above. (b) 3D view of Table C without the concrete shielding, (c) 3D view of Table C including the concrete shielding (designed to block the line of sight between the target and the luminescence screens and cameras).

4.1 Transverse beam profile imaged using a luminescence screen

4.1.1 Chromium-doped luminescence screens

In both measurements, pair fluence is measured using screens made of Chromox, a chromium-doped alumina-ceramic (Al_2O_3 : 99.5%, Cr_2O_3 : 0.5%) [93–95] that can survive direct irradiation of the primary proton beam. To image the beam profile, a screen with dimensions $70 \text{ mm} \times 50 \text{ mm} \times 0.25 \text{ mm}$ is used. Thicker screens ($200 \text{ mm} \times 50 \text{ mm} \times 1 \text{ mm}$) were used to capture the $e^\pm$ spectra. When ionizing particles or radiation are incident on the Chromox, the screen emits red visible light due to de-excitations of the lowest-excited state of Cr^{3+} (isotropically at principal wavelengths $\lambda_1 = 691 \text{ nm}$ and $\lambda_2 = 694 \text{ nm}$). Luminescence lasts several milliseconds (decay times 3–6 ms). At the intensities used in the experiment, the intensity of luminescence is directly proportional to the energy deposited

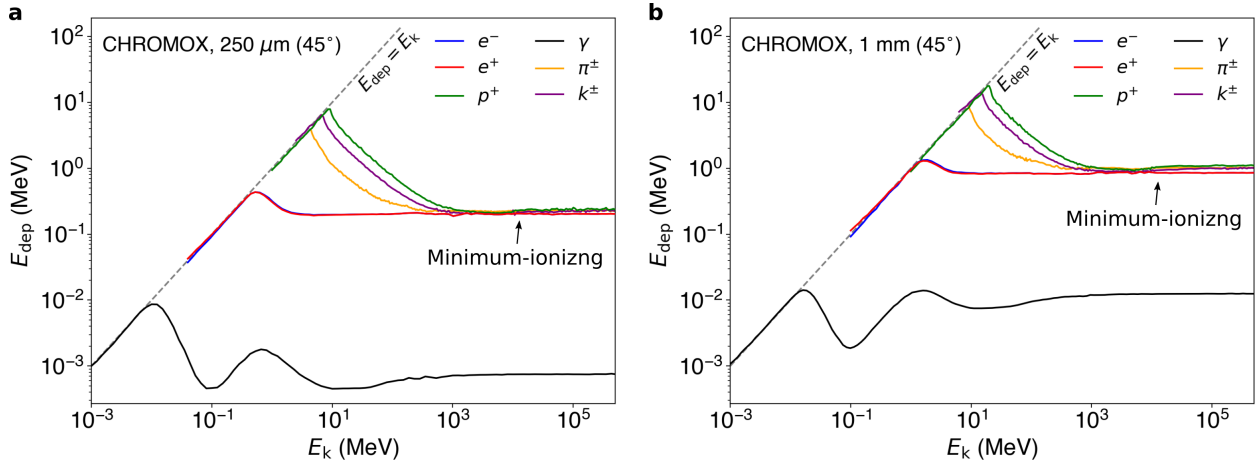


Figure 4.3: FLUKA simulations of the energy deposition in Chromox screens. The energy deposited per particle is obtained from FLUKA simulations of particles incident on Chromox screens at 45° with screen thickness: (a) $250 \mu\text{m}$, and (b) 1 mm . Energy deposited in the screen is plotted as a function of particle kinetic energy for electrons (e^- , blue), positrons (e^+ , red), protons (p^+ , green), charged pions ($\pi^\pm$, orange) and charged kaons ($k^\pm$, purple). The energy deposited by γ -rays is plotted as a function of total energy (black).

by ionizing particles/radiation. Although it is not possible to distinguish which kind of secondary particle is the cause of observed luminescence, at relativistic energies (Lorentz factor, $\gamma \gtrsim 2$) charged particles deposit energy in a ‘minimum-ionizing’ fashion (as described in Appendix A), where an almost identical amount of energy is deposited by each particle. This is confirmed by performing a dedicated FLUKA study, in which the energy deposited in Chromox is simulated when irradiated by a range of particle types and a range of energies (Figure 4.3). For particle Lorentz factors $2 \lesssim \gamma \lesssim 1000$, the deposited energy per particle is approximately the same and independent of the particle species. The energy deposited per γ -ray in the simulated range 50 keV – 1 TeV is also approximately constant, but it is $> 100\times$ smaller because significant ionization relies on the production of ionizing secondaries. So, the contribution of γ -rays in observed screen luminescence is negligible. It is observed that energy deposited in the 1 mm thickness screen is approximately $4\times$ greater than deposited in the 0.25 mm thickness screen. Given that the vast majority of secondary particles incident on the screen are relativistic, the observed brightness is assumed to be directly proportional to the fluence of incident particles.

4.1.2 Imaging the screens

The transverse beam profile is imaged 10 cm downstream of the target, oriented at 45° to the beam path, and viewed directly by a digital camera (Basler acA1920-40gm GigE camera with Sony IMX249 CMOS sensor and Canon EF 75-300 mm f/4-5.6 III lens). Appropriate camera placement is informed by a radiological study of the experiment modelling the radiation environment as the beam is delivered to the experimental area (see Appendix B). The

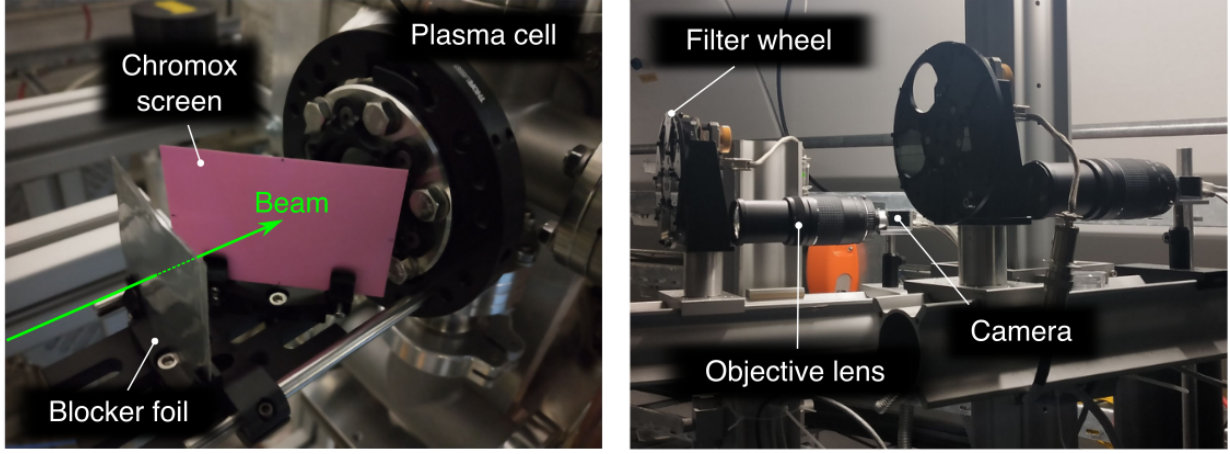


Figure 4.4: Photographs of the luminescence screen setup. Left: Between the target and the plasma cell, the beam propagates through a blocker foil and the Chromox screen. Right: A camera is positioned with a direct view of the screen, and a filter wheel adjusts the filtering level.

cameras are known to have a single-event effect cross section of $9.5 \times 10^{-10} \text{ cm}^2$, which means that placing the camera in a position where the prompt fluence of high-energy hadrons exceeds $1.05 \times 10^9 \text{ cm}^{-2}$ will (statistically) require a camera power cycle to restart it. FLUKA simulations are performed to determine how far away from the experiment table the camera needs to be placed in order for the expected high energy hadron fluence to be $< 10^8 \text{ cm}^{-2}$, and where the expected cumulative ionizing dose will not exceed the dose expected for damage beyond repair (100 Gy). A standoff distance of 3.8 m is decided as reasonable. At this distance, the cameras can resolve features as small as $50 \mu\text{m}$ (measured using a resolution test target), though the translucence of the Chromox screens (attenuation length, $\mu = 0.8 \text{ mm}^{-1}$) limits the spatial resolution to $\sim 100 \mu\text{m}$. The camera exposure time is 24 ms, which means that the full period of screen luminescence is captured. A timing signal linked to the extraction of the beam from the SPS triggers the image to be stored as a beam measurement. Photographs of the screen and camera setup are shown in Figure 4.4.

4.1.3 Sources of background

A common source of background in experiments producing energetic electromagnetic cascades is the large number of scattered sub-MeV e^- and γ -rays which can flood detectors. Given that the experiment is carried out in an air environment, a large fraction of low-energy particles and radiation are absorbed before they can reach the screens; for instance, $e^\pm$ with energy $\lesssim 100$ keV are mostly absorbed by a few centimetres of air. The air environment can provide additional sources of background in the form of stray light arising from Cherenkov emission, fluorescence of air molecules, and optical transition radiation (OTR), the latter produced by particles passing between materials of different dielectric constant (such as

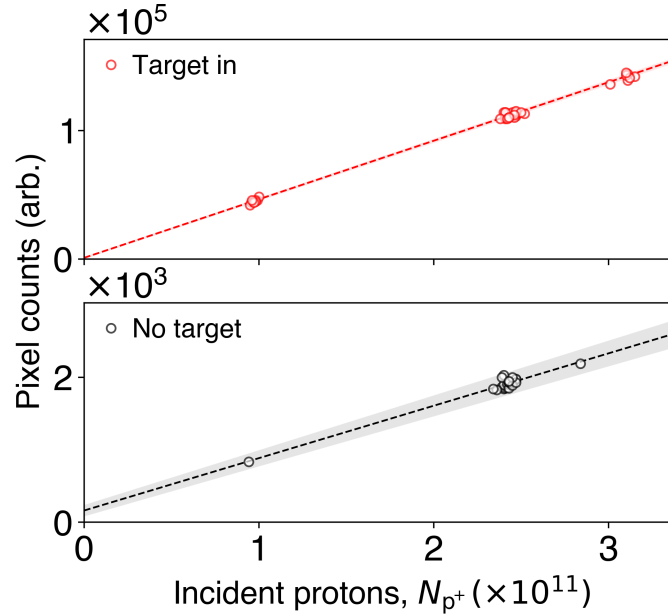


Figure 4.5: Background pixel counts providing a zero level in the luminescence screen images. The mean pixel count is calculated in the region of the data images outside of the luminescence screens to establish a zero level for image intensity. The top panel shows the mean pixel counts when the target is irradiated, and the bottom panel shows when the target is removed. In both cases, the background level is linearly proportional to the proton intensity. The shaded regions correspond to the standard errors of the linear fit.

the target-air interface). The contribution of all these sources is small compared with the luminescence light, but as a precaution a 50 μm -thick aluminium blocker foil is used. The most significant source of background in images of the screens is caused by the direct irradiation of the camera sensor with high-energy hadrons and other radiation scattered around the experimental area. This leads to a uniformly distributed speckle noise that scales linearly with primary proton intensity and is more pronounced when the target is irradiated. In the analysis of the data, an intensity-dependent zero-level is established for the images by fitting the average noise level in regions of the images outside of the luminescence screen (shown in Figure 4.5), then the images from multiple shots are combined. No median filtering is applied because combining multiple images naturally eliminates the noise without distorting the underlying distribution.

4.1.4 Comparison with FLUKA simulations

In total, 23 shots are collected with the target removed, and 47 shots are collected with target irradiated, covering a range of primary beam intensities between $1 - 3 \times 10^{11}$ protons. In Figure 4.6, the combined images obtained when the target is irradiated are compared with when the target is removed from the proton path, plotted alongside the expectation based on FLUKA Monte-Carlo simulations of the energy deposition in the screens. The image intensity is converted to an absolute particle fluence by normalizing to the known beam density profile of the primary proton beam (Gaussian width $\sigma_r = 1 \text{ mm}$). When the target is irradiated and the secondary beam is produced, a $5\times$ increase in peak brightness is observed and the total intensity increases by $80\times$. The transverse size of the beam broadens

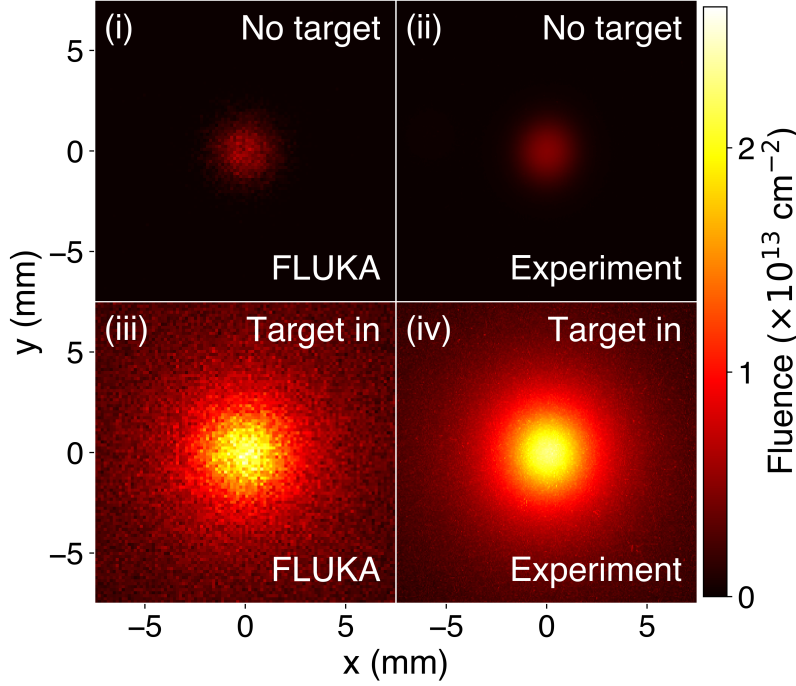


Figure 4.6: Transverse beam profile imaged using a luminescence screen. Direct comparison of FLUKA Monte-Carlo simulations with raw image data obtained when the target is irradiated and the secondary beam is produced (‘Target in’), versus when the target is removed and only the primary proton beam irradiates the screen (‘No target’). An absolute fluence calibration is obtained using the known density profile of the primary proton beam.

to a Lorentzian profile with half-width $\Sigma_r = 2.3$ mm due to its finite divergence. A radial lineout from the images is shown in Figure 4.7. The experimental and simulated cases are plotted for comparison, along with the contribution to the signal corresponding to $e^\pm$ pairs in the simulation. The integrated image intensities (total intensities) are shown in Figure 4.8 with the target irradiated (red circles) and the target removed (black diamonds). Again, the intensities have been converted to an absolute particle number. The total intensity scales with the number of protons in the primary beam. The number of $e^\pm$ pairs measured using the screen is $N_{\pm,\text{exp}} = (1.02 \pm 0.05) \times 10^{13}$. This result agrees with FLUKA predictions 10 cm downstream of the target: $N_{\pm,\text{FLUKA}} = 1.04 \times 10^{13}$. The corresponding peak pair

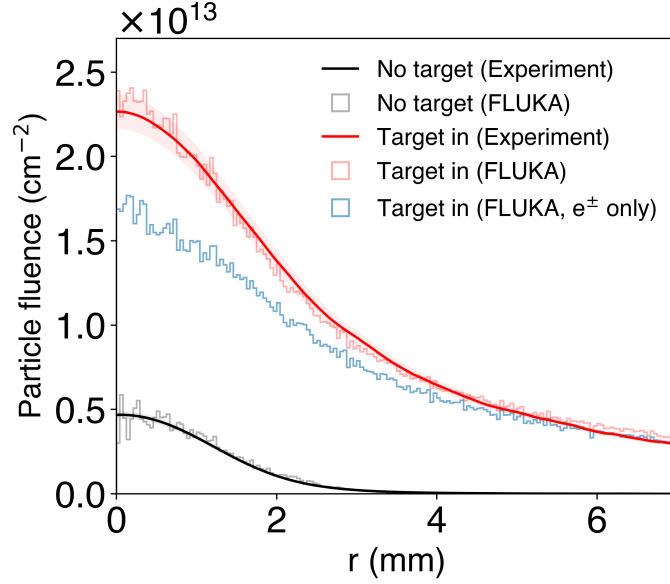


Figure 4.7: Radial lineouts of transverse beam profiles. The radial lineouts of the processed image data are plotted for the case where the target is irradiated (red), and when it is removed (black). Shaded regions correspond to the standard error on the fit of the absolute calibration. When the target is irradiated, an increase in peak image brightness of $5\times$ is observed, and the beam diameter increases to resemble a Lorentzian profile with half-width $\Sigma_r = 2.3$ mm. The experimental lineouts are compared with FLUKA simulations of the energy deposited in the screen for the two cases (light red and grey histograms). In addition, the contribution to the signal due to $e^\pm$ alone is shown separately, which is used to calculate the peak fluence of the pair beam.

density at the screen position is $n_{\pm,\text{exp}} = 4.5 \times 10^{11} \text{ cm}^{-3}$, assuming the longitudinal pair beam profile is identical to the primary beam (justified in Section 2.3.3).

4.2 Magnetic particle spectrometer

4.2.1 Spectrometer design

The $e^\pm$ pair spectrum resulting from the electromagnetic cascade is non-thermal, characterized by a power-law distribution spanning multiple decades in energy, so it is difficult to measure the spectrum in its entirety, and at lower energies ($E \lesssim 10 \text{ MeV}$), the beam divergence of $10^2 - 10^3 \text{ mrad}$ makes it difficult to collect the whole pair population

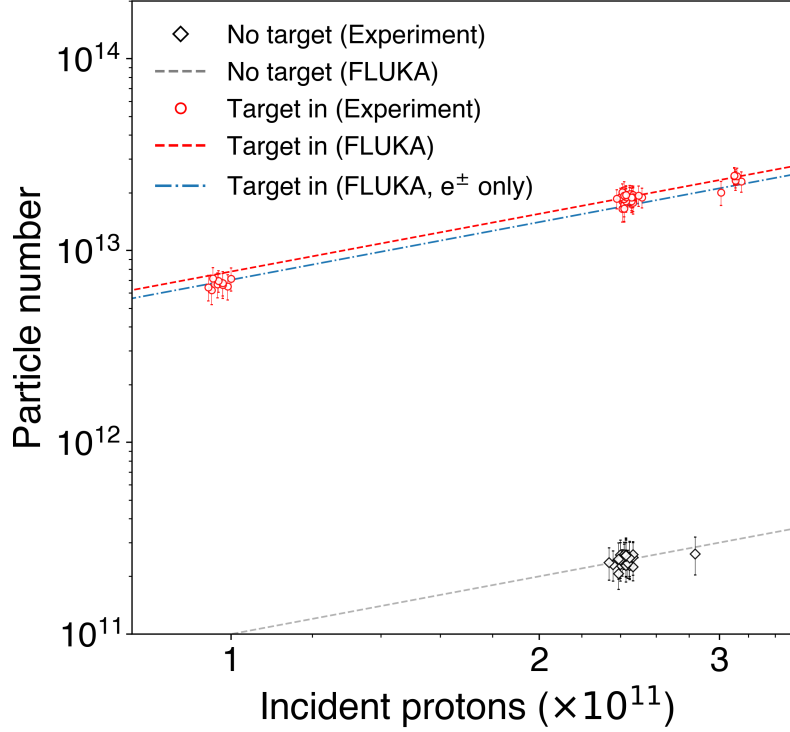


Figure 4.8: Comparison of Monte-Carlo simulations with experimental image data. Integrated image intensity (total intensity) from 70 shots is converted to an absolute particle number, showing the case where the target is irradiated (red circles, 47 shots), and when it is removed (black diamonds, 23 shots). The error bars reflect the standard errors of the fitted parameters for each shot. FLUKA Monte-Carlo simulations of the predicted light yield are shown for both cases (black-dashed and red-dashed lines), showing good agreement with the experimental data. The blue dot-dashed line indicates the contribution from $e^\pm$ in the FLUKA simulation, highlighting that this is the dominant contribution to the enhanced signal.

into a spectrometer and maintain a reasonable energy resolution. A practical design was chosen (shown in Figure 4.9), covering the 30 – 220 MeV range with an energy resolution $\Delta E/E = 10 - 20\%$. This corresponds to about 40% of the pair content of the spectrum (with the fraction of the spectrum at energy < 30 MeV being only 10%). The electromagnet is chosen for its large aspect ratio of width to length, meaning particles can exit the yoke at a large angle to the beam axis. This is required to obtain a reasonable energy resolution and keep the design compact (magnet specifications given in Table 4.1). Screens are imaged

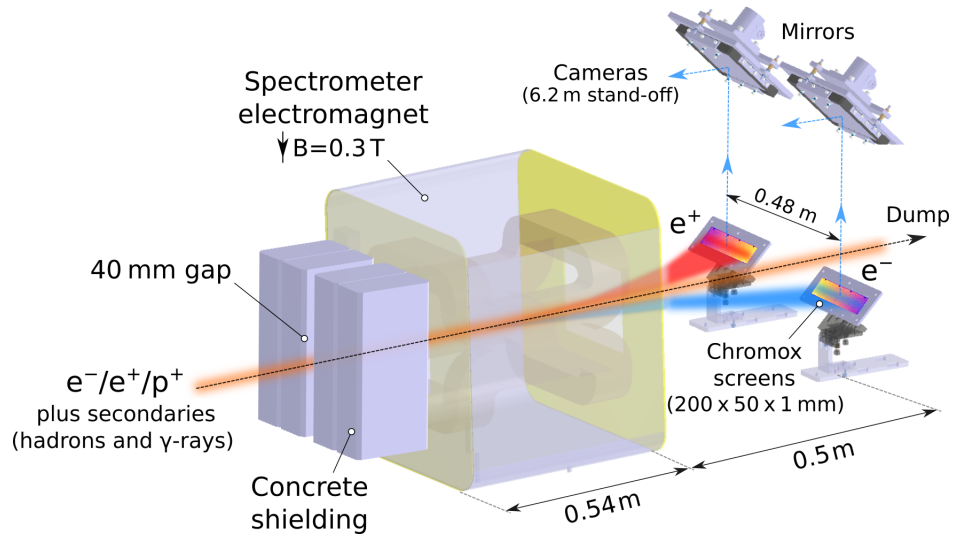


Figure 4.9: Electron-positron spectrometer design. The secondary beam passes through a 40 mm gap between 20 cm concrete blocks and through the aperture of an electromagnet (MNPA-type). When the electromagnet is supplied with current (up to $I = 400$ A) a dipole magnetic field can be produced in the aperture with peak strength $B = 0.34$ T. Sub-GeV/ c electrons and positrons are deflected off-axis onto Chromox screens (200 mm $\times$ 50 mm $\times$ 1 mm) at a distance 50 cm downstream of the magnet yoke, centred 240 mm off-axis on either side. The screens are viewed by digital cameras through a single mirror reflection at a standoff distance 6.2 m.

using the same camera setup used to image the transverse beam profile, except with an increased standoff distance of 6.2 m and through a single mirror reflection. Features as small as 120 μm can be resolved, though the spectral resolution is limited by the 20 cm thickness, 40 mm wide concrete aperture constructed at the entrance of the electromagnet. Concrete is chosen since high-Z collimators can cause unwanted shaping of the spectrum by inducing further scattering and conversion. The design of additional concrete shielding is informed by FLUKA simulations which model how different configurations affect the amount of background hadrons and γ -rays that can reach the screens. The designs which minimize the background block the line of sight between the target and the screens with as much concrete as possible. The effect of including the shielding is shown in Figure 4.10, where

a higher quality $e^\pm$ spectral measurement is indicated by the improved correlation between $e^\pm$ energy and deflected position. In addition, low-energy pair background is reduced. Equivalent plots showing the distribution of background hadrons and γ -rays (Figure 4.11) show that the majority pass between the screens (< 14 cm from the beam axis). The current in the magnet coils is varied for different shots to sample different portions of the e^- and e^+ energy spectra in the range $30 - 220$ MeV, which are pieced together to construct the full spectrum. The spectral range corresponding to each used setting is given in Table 4.2. The electromagnet can be switched off (leaving a negligible remanent field, $B \lesssim 0.3$ mT) to characterize the γ -ray and hadron background.

Electromagnet specifications	
Aperture width (mm)	260
Aperture height (mm)	202
Iron length (mm)	250
Total length (mm)	544
Total width (mm)	680
Total height (mm)	685
Weight (kg)	730
Maximum RMS current (A)	600
Dipole nominal field at peak current (T)	0.515
Dipole integrated field at peak current (Tm)	0.196
Maximum current available in experiment (A)	400

Table 4.1: Specifications of the electromagnet.

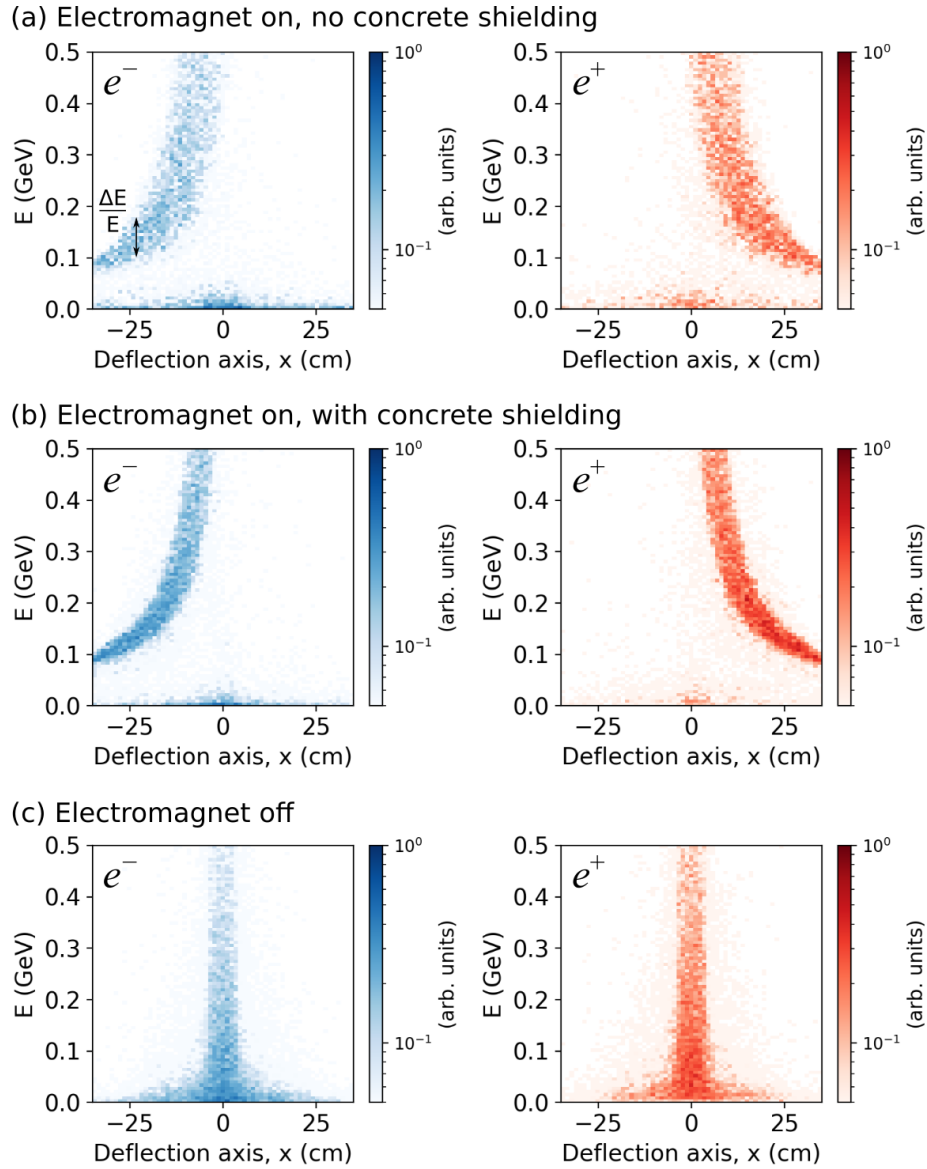


Figure 4.10: FLUKA simulations showing the effect of the concrete shielding on the dispersion of electrons and positrons along the deflection axis. The correlation between electron and positron energy and position along the deflection axis is obtained from FLUKA simulations of the complete experimental setup. Electrons/positrons are measured within an area corresponding to the screen positions (within a 2 cm vertical offset, at a distance 50 cm downstream of the magnet yoke). Plots (a) show when the electromagnet is on ($I = 400$ A), without any concrete shielding in place, whilst plots (b) show the same magnet conditions when the concrete shielding is included. With the shielding included, less background signal reaches the screens (positioned > 14 cm off-axis), and the energy resolution ($\Delta E/E$) is improved. Plots (c) show the case where the electromagnet is turned off and the electrons/positrons pass through undeflected.

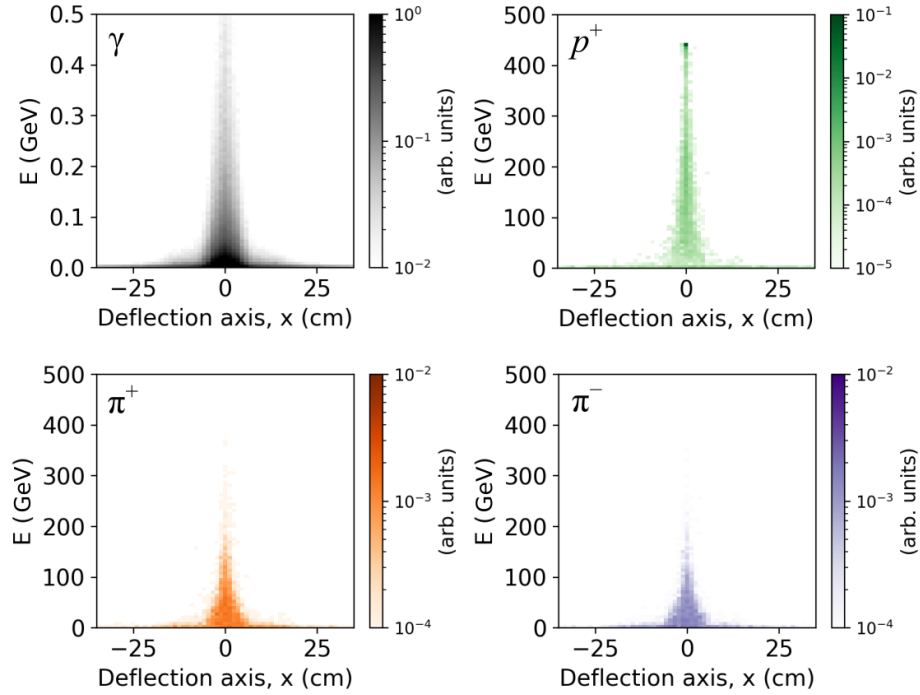


Figure 4.11: FLUKA simulations showing the dispersion of hadrons and γ -rays along the deflection axis. The distribution of γ -rays, protons (p^+) and charged pions $\pi^\pm$ are shown along the deflection axis in the plane corresponding to the screen positions. The results are shown in the case where the electromagnet is on ($I = 400$ A) and the concrete shielding is included, showing that the majority of these background species do not impinge on the screens (positioned > 14 cm off-axis) and are mostly absorbed by the beam dump.

Current (A)	Peak magnetic field (mT)	Spectral range (MeV)
100	85	30 – 55
200	170	55 – 80
300	255	75 – 110
400	340	90 – 220

Table 4.2: Spectrometer spectral range with different magnet settings.

4.2.2 Energy calibration

To obtain a precise energy calibration (the position on the screen corresponding to a particular electron/positron energy), the magnet geometry has been modelled using the finite-element code OPERA 3D [96]. The field map is calculated with sufficient resolution (10 mm) to capture the magnetic field gradients inside the magnet gap, given that magnetic field transitions from 10% – 90% peak field strength over a distance 200 mm. The field map is cross-checked against measurements using a Hall probe, consistent inside the magnet gap to within 2%. The energy calibration for each magnet setting is obtained using ray-tracing calculations, assuming a collimated, on-axis pair beam. Example trajectories are shown in Figure 4.12, and the resulting energy calibrations are shown in Figure 4.13.

4.2.3 Absolute calibration

For an optical setup collecting isotropically emitted photons produced in the screens, the number of pixel counts is related to the incident particle fluence by the following relation:

$$\frac{\text{Pixel counts}}{\text{Particle fluence}} = \underbrace{(\text{Q.E.})}_{\text{Quantum efficiency}} \times \underbrace{\left(\frac{dN_{\text{ph}}}{dN_{\text{p}}}\right)}_{\text{Luminescence photons per particle}} \times \underbrace{(G)}_{\text{Gain setting}} \times \underbrace{dA}_{\text{Screen area subtended per pixel}} \times \underbrace{\left(\frac{d\Omega}{4\pi}\right)}_{\text{Solid angle subtended by the collection optic}} . \quad (4.1)$$

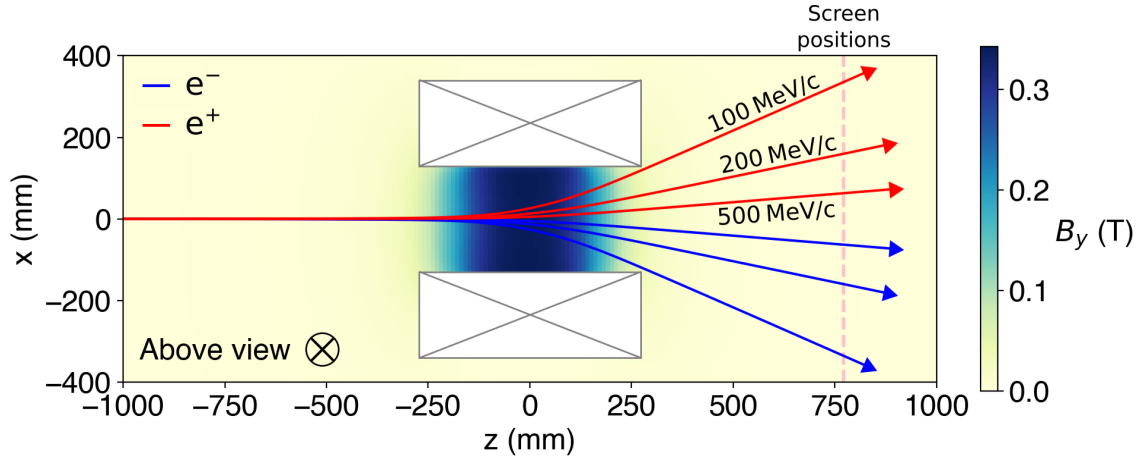


Figure 4.12: Ray tracing calculations of the electron-positron spectrometer. The energy calibration (energy as a function of position on the deflection axis) is calculated using magnetic field maps of the electromagnet obtained using finite-element code OPERA 3D, cross-checked against direct measurements of the magnet field. With the magnet supplied with current $I = 400$ A, the trajectories are plotted for on-axis electrons (blue) and positrons (red) with momenta 100, 200, 500 MeV/ c as they pass through the magnet yoke to the screen positions.

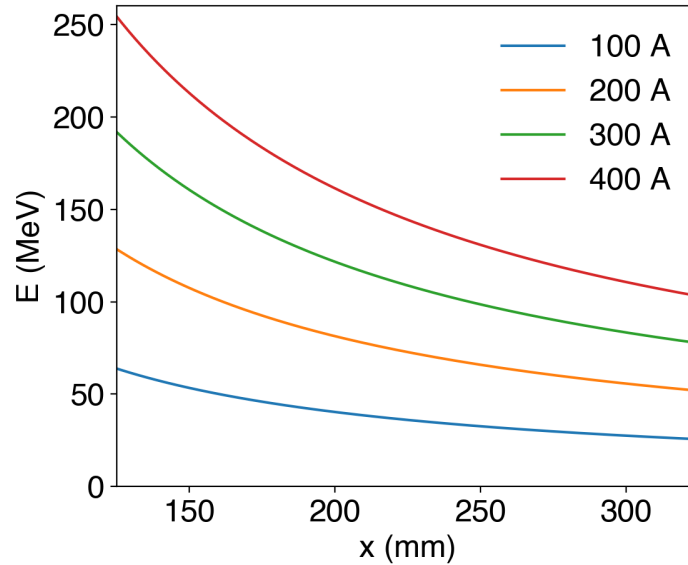


Figure 4.13: Electron-positron energy calibration from ray-tracing calculations. Assuming collimated, on-axis electron-positron pairs, the position that particles cross the plane 50 cm downstream of the magnet yoke is plotted as a function of energy when the electromagnet is supplied with different current ($I = 100, 200, 300, 400$ A).

The number of luminescence photons per incident particle ($dN_{\text{ph}}/dN_{\text{p}}$) is assumed to be a constant (approximately true for minimum-ionizing particles), obtained experimentally for a given particle fluence by measuring the pixel counts for a known fluence of primary protons. The absolute calibration for the electron-positron spectrometer setup is obtained by re-scaling the terms in Eq. 4.1:

- The quantum efficiency is the same, as the same type of camera sensor is used and the principal luminescence wavelength has not changed.
- The number of luminescence photons produced per particle has increased by a factor of $(4 \pm 0.5) \times$ as a result of the increased energy deposition in 1 mm thickness Chromox compared with 0.25 mm thickness.
- The camera gain setting is $G = 38.1 \pm 1.0$, compared with $G = 1.0$ in the cameras imaging the beam profile.
- The area subtended by a pixel is $120 \mu\text{m} \times 120 \mu\text{m}$, compared with $50 \mu\text{m} \times 60 \mu\text{m}$ area in the beam profile imaging.
- The increase in standoff distance from 3.8 m to 6.2 m means that the solid angle subtended by the collection optic is $(2.7 \pm 0.5) \times$ smaller.

Finally, a small $1/\cos\theta$ geometric correction is applied to the energy spectra to account for particles incident on the screen at a greater angle. At the edge closest to the beam axis, $\theta = 10^\circ$, and $\theta = 25^\circ$ at the furthest edge.

4.2.4 Comparison with FLUKA simulations

Raw image data of the luminescence screens in the magnetic particle spectrometer is shown in Figure 4.14, after subtraction of the speckle background. When the magnet is activated, the luminescence signal is enhanced on both screens, which can only be explained by large numbers of $e^\pm$ deflected onto the screens, and the collected segments of the spectrum overlap to form a smooth continuous spectrum (shown in Figure 4.15). The spectra agree with FLUKA simulations of the energy spectra of electrons and positrons within the error associated with the absolute calibration. Since the FLUKA modelling does not use the precise magnetic field map, the spectra shown corresponds to $e^\pm$ pairs that pass through a window $-35 \leq x \text{ [cm]} \leq 35$, $-1.25 \leq y \text{ [cm]} \leq 1.25$, 50 cm downstream of the magnet

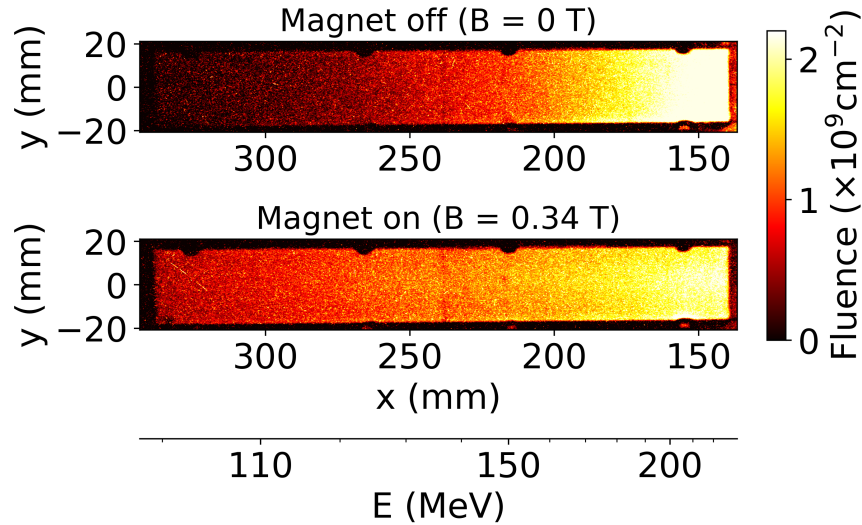


Figure 4.14: Magnetic particle spectrometer raw data. Raw image data of the luminescence screen onto which positrons are deflected is shown for a shot where the electromagnet is turned off (upper), and where the electromagnet is turned on with a peak magnetic field $B = 0.34 \text{ T}$ (lower). When the magnet is turned on, the observed luminescence is increased. The secondary x -axis shows the corresponding positron energy according to the energy calibration. To construct the spectra, a vertical lineout is taken from the central 25 mm of the image ($-1.25 \leq y \text{ [cm]} \leq 1.25$).

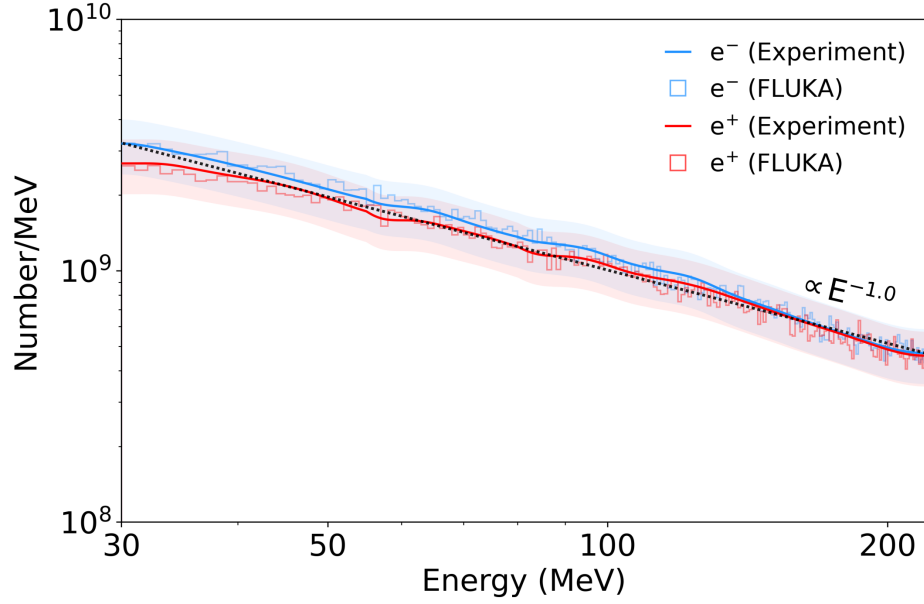


Figure 4.15: Electron-positron spectra measured using the magnetic particle spectrometer. The energy spectra of e^- (blue) and e^+ (red) are obtained from images of the luminescence screens positioned downstream of the electromagnet. The spectrum in the energy range $30 \leq E \text{ [MeV]} \leq 220$ is constructed by piecing together images from multiple shots using different magnetic field strengths ($B = 0.08\text{--}0.34 \text{ T}$). The inset shows the same plot obtained when the electromagnet is deactivated. The shaded regions correspond to the error associated with the absolute calibration. FLUKA simulations (histograms) accurately predict the shape and absolute intensity of the experimentally observed spectra.

yoke with the magnetic field deactivated. The measured $e^\pm$ spectra are characterized by a power-law with spectral index $dN/dE \propto E^{-1.0}$. This is slightly less steep than the same part of the spectrum observed at the target rear in simulations ($dN/dE \propto E^{-1.3}$) due to the spectrometer sampling bias towards the measurement of higher energy pairs which have a smaller divergence. FLUKA simulations accurately predict both the absolute number and the spectral shape of the $e^\pm$ pairs that can reach the screens. In this energy range, the number of pairs is measured to be $N_{\pm,\text{exp}} = (2.46 \pm 0.62) \times 10^{11}$ and positron fraction is $(N_{e^+}/N_{e^-})_{\text{exp}} = 0.92 \pm 0.05$, compared with the FLUKA simulation:

$N_{\pm, \text{FLUKA}} = 2.45 \times 10^{11}$ and $(N_{e+}/N_{e-})_{\text{FLUKA}} = 0.89$. The results of both luminescence screen diagnostics are summarized in Table 4.3.

Position	Pair yield, $N_{\pm}$		Positron fraction, (N_{e+}/N_{e-})	
	Simulation	Experiment	Simulation	Experiment
Target rear surface ($E \geq 1$ MeV)	1.53×10^{13}	-	0.82	-
Post-target screen ($E \geq 1$ MeV)	1.04×10^{13}	$(1.02 \pm 0.05) \times 10^{13}$	0.91	-
Spectrometer ($30 \leq E$ [MeV] ≤ 220)	2.45×10^{11}	$(2.46 \pm 0.62) \times 10^{11}$	0.89	0.92 ± 0.05

Table 4.3: Summary of the measured and simulated yields of $e^{\pm}$ pairs. The experimentally measured and FLUKA-simulated electron-positron pair yield ($N_{\pm}$) and positron fraction (N_{e+}/N_{e-}) are summarized for the following positions: (i) rear surface of the target, (ii) post-target luminescence screen, (iii) particle spectrometer luminescence screens.

4.3 Classification a pair plasma

Given the agreement between the measurements and the pair beam characteristics predicted by FLUKA, the simulations are validated and can be used to deduce the pair spectra and peak pair density at the rear of the target where the pair density is largest. The electron and positron radial density profiles at the immediate rear of the target are shown in Figure 4.16. For collective behaviour to be observed, the physical size of a plasma must exceed characteristic scale lengths of collective plasma processes, namely the Debye screening length, λ_D , and the collisionless plasma skin depth, λ_s . These quantities are traditionally defined for isotropic, thermal, non-relativistic plasmas, whereas the distribution function of produced $e^{\pm}$ pairs is highly anisotropic and non-thermal, with a relativistic thermal spread ($k_B T_{\pm} \gg m_e c^2$). The comparison is performed in the inertial frame co-moving with the bulk of the plasma (superscript ‘c.f.’), defined as the frame in which the net momentum of

the beam is zero. In the co-moving frame, the Lorentz-transformed distribution function becomes approximately isotropic, although high-energy tails are retained, characterized by power-law distributions: $f_\alpha(u) \propto \gamma^{-m}$, $m \approx 1 - 2$. A reasonable fit is obtained using a relativistic Cauchy distribution:

$$f_\alpha(u) = \frac{1}{\pi^2} \frac{\Theta}{(\Theta^2 + u^2)^2}, \quad (4.2)$$

where $\Theta = k_B T_\pm / m_e c^2$. The screening length specific to this distribution must be derived.

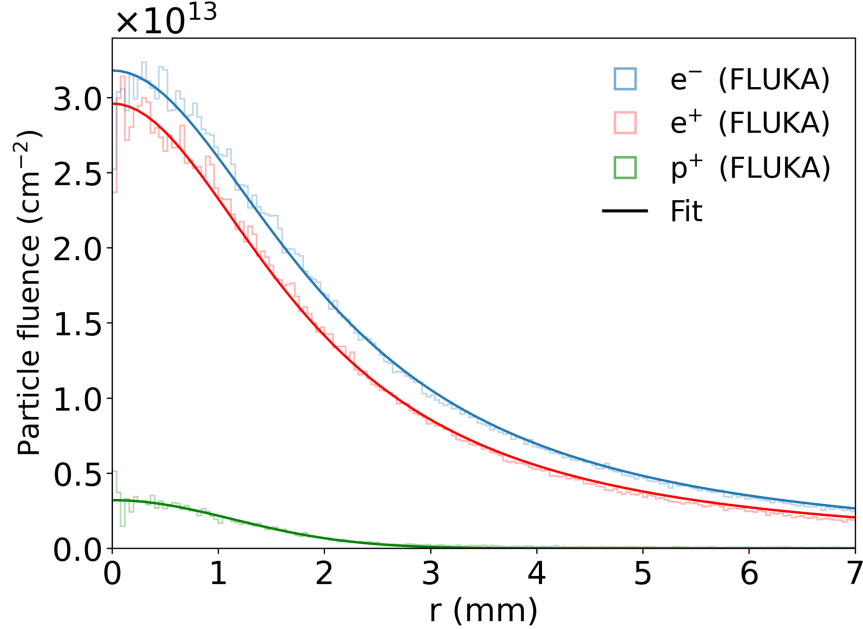


Figure 4.16: FLUKA simulation of the radial density distributions at the target rear. The fluence of electrons, positrons and protons is plotted where the pair density of the beam is the highest (at the immediate target rear). With the longitudinal profile being a Gaussian with standard deviation $\sigma_z = 7.5$ cm, the peak pair beam density is $n_\pm = 1.64 \times 10^{12} \text{ cm}^{-3}$.

4.3.1 Deriving the Debye screening length for relativistic plasmas

The plasma screening length, λ_D , is obtained by evaluating the static ($\omega \rightarrow 0$), long wavelength ($k \rightarrow 0$) limit of the dielectric tensor [97–99]:

$$\lambda_D^{-2} = \lim_{k \rightarrow 0} \lim_{\omega/k \rightarrow 0} k^2 (\varepsilon_\ell - 1), \quad (4.3)$$

where ε_ℓ is the longitudinal component of the dielectric tensor. Written in terms of the dimensionless momentum $\mathbf{u} = \mathbf{p}/m_\alpha c = \gamma \boldsymbol{\beta}$,

$$\varepsilon_\ell = 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{c^2 k^2} \int_{C_L} \frac{1}{\omega - c \mathbf{k} \cdot \boldsymbol{\beta}} c \mathbf{k} \cdot \frac{\partial f_\alpha(\mathbf{u})}{\partial \mathbf{u}} d^3 u, \quad (4.4)$$

integrated along the Landau contour, C_L , and summing the contribution from each species component of the plasma, α . If the distribution function is isotropic, the screening length is simply evaluated:

$$\lambda_D^{-2} = -4\pi \sum_\alpha \frac{\omega_{p\alpha}^2}{c^2} \int_0^\infty \gamma u \frac{\partial f_\alpha(u)}{\partial u} du. \quad (4.5)$$

This can be evaluated for a non-relativistic Maxwellian distribution to arrive at the well-known Debye screening length:

$$f_\alpha(u) = \frac{1}{(2\pi\Theta)^{3/2}} \exp\left(\frac{-u^2}{2\Theta}\right) \Rightarrow \lambda_D = \Theta \lambda_s, \quad (4.6)$$

where $\lambda_s = c/\omega_p = (8\pi n_\pm e^2/m_e c^2)^{-1/2}$ is the pair plasma skin depth. Assuming instead a plasma with a relativistic temperature ($\Theta \gtrsim 1$), a commonly used distribution function is

the relativistic Maxwellian (Jüttner-Synge) distribution. In this case:

$$f_\alpha(u) = \frac{\exp(-\gamma/\Theta)}{4\pi\Theta K_2(1/\Theta)} \Rightarrow \lambda_D = \sqrt{\Theta} \lambda_s, \quad (4.7)$$

where $K_2(1/\Theta)$ is a modified Bessel function of the second kind of order 2. For a relativistic Cauchy distribution (Eq. 4.2), the evaluated screening length takes the exact form

$$\lambda_D = \lambda_s \sqrt{\frac{\pi}{4}} \Theta \sqrt{\frac{\Theta^2 - 1}{\Theta^2 - \frac{1}{2}}} \left[1 - \frac{\sinh^{-1}(\sqrt{\Theta^2 - 1})}{\Theta \sqrt{\Theta^2 - 1} (2\Theta^2 - 1)} \right]^{-1/2}, \quad (4.8)$$

which approximates to $\lambda_D = \lambda_s \sqrt{\frac{\pi}{4}} \Theta$ when $\Theta \gtrsim 1$. As shown in Figure 4.17, this is only a slight modification from the screening length derived for a relativistic Maxwellian (Jüttner-Synge distribution), which is not unexpected since the two functions are approximately identical in the thermal bulk. It is reassuring that precise fitting of the extended tails is not critical to the accuracy of the screening length.

4.3.2 Fitting an isotropic Cauchy distribution in the co-moving frame

Since the screening length is derived assuming zero bulk motion, to obtain the appropriate pair temperature a Lorentz transformation is applied to the FLUKA-simulated momentum spectra to fit the data. The fitting of Cauchy and Jüttner-Synge distributions to the Lorentz-transformed momentum distributions is shown in Figure 4.18. The fitted longitudinal temperature is $\Theta_{\parallel} = 6.5$ and the transverse temperature is $\Theta_{\perp} = 3.5$. A temperature $\Theta = 5.0 \pm 1.5$ is assumed to use the screening length for an isotropic distribution.

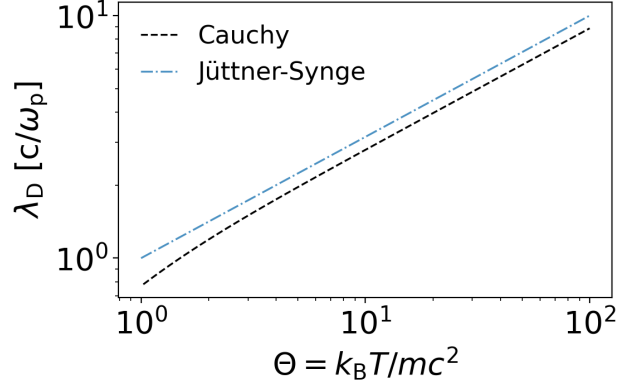


Figure 4.17: Debye screening lengths derived for relativistic Cauchy and Jüttner-Synge distribution functions. The screening lengths for Cauchy and Jüttner-Synge distribution are plotted in units of collisionless skin depth λ_s . The similarity is clear, differing by only 12% when the normalized temperature is $\Theta \gtrsim 1$.

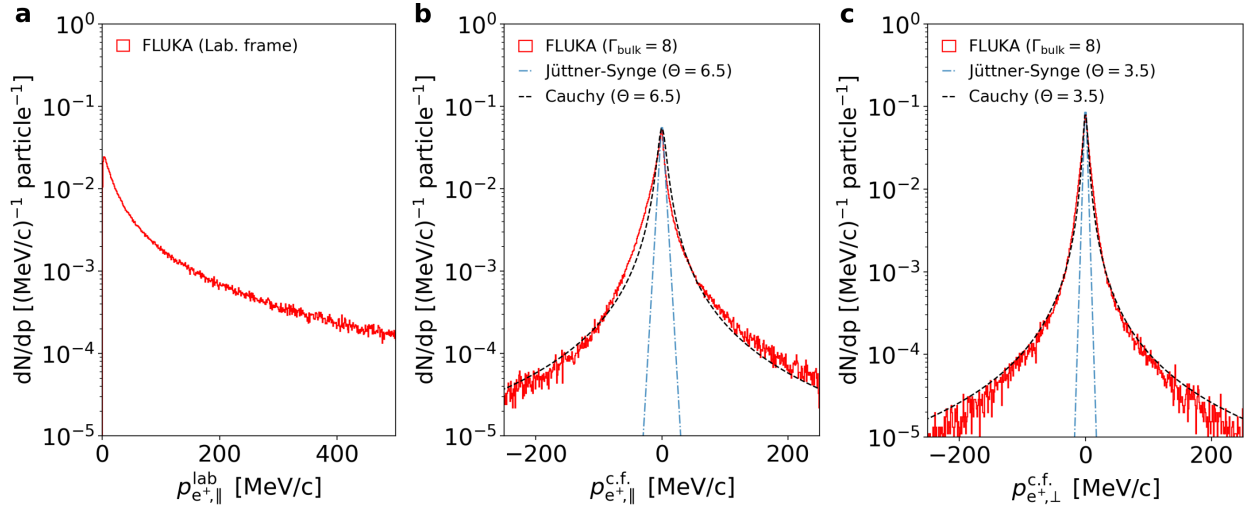


Figure 4.18: Fitting the momentum distributions in the frame co-moving with the beam. (a) The longitudinal momentum distribution of positrons in the laboratory frame, $p_{e+,||}^{\text{lab}}$ is obtained from FLUKA simulations (red). (b, c) A Lorentz transformation ($\Gamma_{\text{bulk}} = 8$) is applied to the particle momentum vectors to obtain the longitudinal and transverse momentum distributions in the frame co-moving with the beam, $p_{e+,||}^{\text{c.f.}}$ and $p_{e+,⊥}^{\text{c.f.}}$. The spectra shown in (b) and (c) correspond to 98.5% of the positrons. A Cauchy distribution provides a better fit to the spectrum than a relativistic Maxwellian (Jüttner-Synge) distribution. The obtained longitudinal temperature is $\Theta_{||} = 6.5$ and the transverse temperature is $\Theta_{\perp} = 3.5$.

4.3.3 Comparison with characteristic scales

At the rear of the target, the full-width-half-maximum of the beam length and beam width are $\ell_{\parallel} = 17.7 \text{ cm}$ and $\ell_{\perp} = 0.40 \text{ cm}$, respectively. It is accounted for that Lorentz transformation between the laboratory frame and the inertial frame co-moving with the bulk leads to an increase in beam length and reduction in pair density by a factor of Γ_{bulk} . Comparing the beam volume with the Debye volume,

$$\left(\frac{\ell_{\perp}^2 \ell_{\parallel}}{\lambda_{\text{D}}^3} \right)^{\text{c.f.}} = 5 \left(\frac{N_{\pm}}{1.5 \times 10^{13}} \right) \left(\frac{n_{\pm}}{1.6 \times 10^{12} \text{ cm}^{-3}} \right)^{1/2} \left(\frac{\Gamma_{\text{bulk}}}{8} \right)^{-1/2} \left(\frac{\Theta}{5} \right)^{-3/2}. \quad (4.9)$$

Making the same comparison with the collisionless plasma skin depth,

$$\left(\frac{\ell_{\perp}^2 \ell_{\parallel}}{\lambda_{\text{s}}^3} \right)^{\text{c.f.}} = 39 \left(\frac{N_{\pm}}{1.5 \times 10^{13}} \right) \left(\frac{n_{\pm}}{1.6 \times 10^{12} \text{ cm}^{-3}} \right)^{1/2} \left(\frac{\Gamma_{\text{bulk}}}{8} \right)^{-1/2}. \quad (4.10)$$

In addition, the number of particles per Debye screening volume greatly exceeds 1: $N_{\text{D}}^{\text{c.f.}} = (n_{\pm} \lambda_{\text{D}}^3)^{\text{c.f.}} = 7 \times 10^{12}$. Similarly for the collisionless skin depth, $N_{\text{s}}^{\text{c.f.}} = (n_{\pm} \lambda_{\text{s}}^3)^{\text{c.f.}} = 10^{12}$. Given that $(N_{\pm}/N_{\text{D}}) = (\ell_{\perp}^2 \ell_{\parallel} / \lambda_{\text{D}}^3) \gtrsim 1$, $(N_{\pm}/N_{\text{s}}) = (\ell_{\perp}^2 \ell_{\parallel} / \lambda_{\text{s}}^3) \gtrsim 1$ and $N_{\text{D}}, N_{\text{s}} \gg 1$ in the co-moving frame, it becomes possible for the first time to perform laboratory studies of the collective plasma behaviour of relativistic electron-positron pair plasmas. In Figure 4.19 the number, yield and beam neutrality of the $e^{\pm}$ pairs is compared with results reported in previous experiments at high-power laser facilities. It is clear that the presented scheme is able to achieve pair yields and densities in a quasi-neutral beam that will not yet be available at laser facilities without several orders of magnitude increase in laser energy.

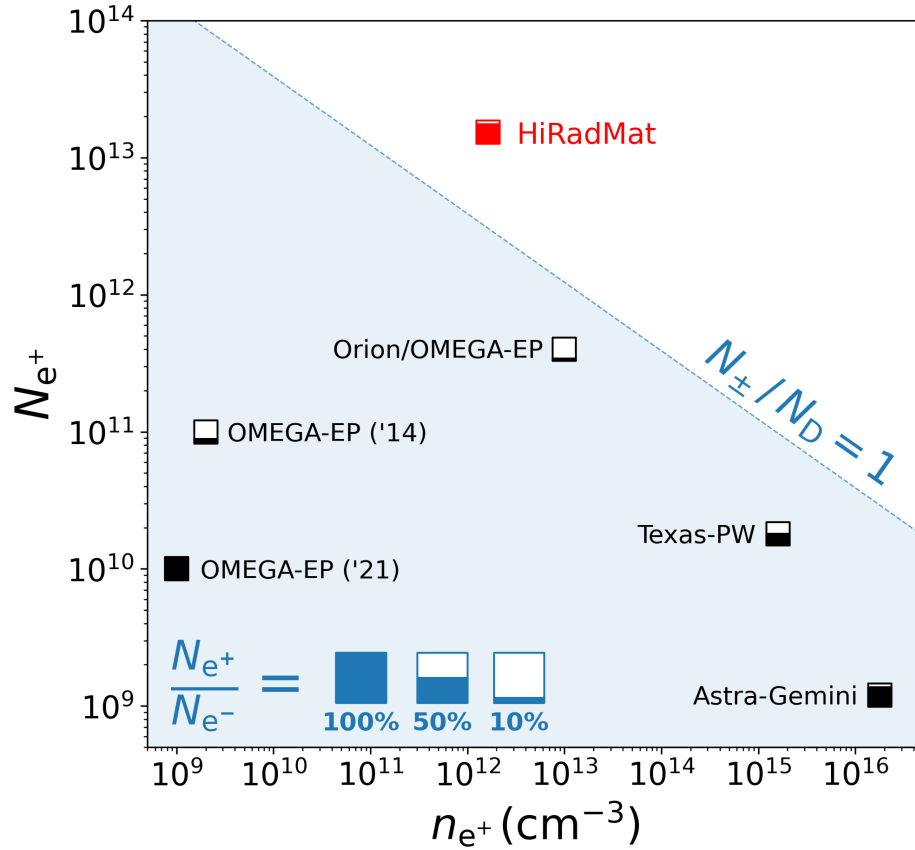


Figure 4.19: Comparison of laboratory-produced high-density pair beams. The peak number and density of pairs reported in this study (red square) compared with previous experiments performed at high-power laser facilities (black squares): Orion/OMEGA-EP [47], Texas-PW [48], Astra-Gemini [49], OMEGA-EP('21) [51], OMEGA-EP('14) [68]. The data are labelled by the facility where the experiment was performed, and the fill fraction of each marker corresponds to the fraction of positrons to electrons in the experiment, N_{e^+}/N_{e^-} ($\blacksquare = 100\%$, $\square = 0\%$, also see the key). The blue-shaded region corresponds to when $N_{\pm}/N_D < 1$ (assuming the screening length used in this work).

4.4 Optimizing the setup to maximize $(N_{\pm}/N_D)$

As was discussed in Section 2.3.2, the present target configuration is not optimized to produce the highest possible pair yields and densities. Instead, a relatively thin target was used, given the limited space in the experimental area. In addition, achieving a low pair beam emittance was preferable to maximize the measurable pair yield in downstream detectors and the pair density maintained in a metre-scale plasma downstream of the target. In the current setup a large fraction of the primary protons (40%) pass through the target without significant energy loss, suggesting that using a thicker target can increase the efficiency of pair conversion (provided it can be accommodated in the experimental area). If pair yields of 5×10^{13} and densities $8 \times 10^{12} \text{ cm}^{-3}$ can be achieved, as indicated by the parameter scan of target thicknesses, it may be possible to increase $(N_{\pm}/N_D)$ into the several tens, and $(N_{\pm}/N_s)$ into the several hundreds.

For the purposes of performing experiments of kinetic beam-plasma instabilities of electron-positron jets, the generated beams satisfy the conditions that high yields of pairs are achieved ($> 10^{13}$), with densities $n_{\pm} = 10^{10} - 10^{12} \text{ cm}^{-3}$ maintained over a 1 m distance downstream of the target. Using the inductive discharge plasma with a peak density $n_p = 2 \times 10^{12} \text{ cm}^{-3}$, the beam size will span multiple skin depths of the ambient plasma, and multiple e -folding lengths of instability are expected to develop, according to the growth rates derived from linear kinetic theory presented in Chapter 1.

Chapter 5

Pair jets propagating through an ambient electron-ion plasma

5.1 Blazar-induced pair beams in cosmic voids

As discussed in Chapter 1, TeV-blazars are a class of powerful active galactic nuclei (AGN) with relativistic jets pointed towards Earth. The jets propagate through the intergalactic medium (IGM), often through cosmic voids, where matter exists as a tenuous, collisionless plasma, corresponding to some of the lowest density regions of the Universe. The vast majority of TeV-energy γ -rays in the jet are expected to scatter with extragalactic background light in the void, triggering cascades of electron-positron ($e^\pm$) pairs which subsequently inverse-Compton scatter with Cosmic Microwave Background (CMB) photons to produce a spectrum of lower energy GeV γ -rays (see Figure 5.1). The mean free path of γ -rays with energy $E_{\gamma 0}$ scattering with extragalactic background light is given approximately by [10]:

$$\ell_{\gamma\gamma} \sim 80 \left(\frac{E_{\gamma 0}}{10 \text{ TeV}} \right)^{-1} \text{ Mpc}, \quad (5.1)$$

and generated electron-positron pairs inverse-Compton scatter with Cosmic Microwave Background (CMB) photons over a comparatively shorter distance:

$$\ell_{\text{IC}} \sim 73 \left(\frac{E_e}{5 \text{ TeV}} \right)^{-1} \text{ kpc}, \quad (5.2)$$

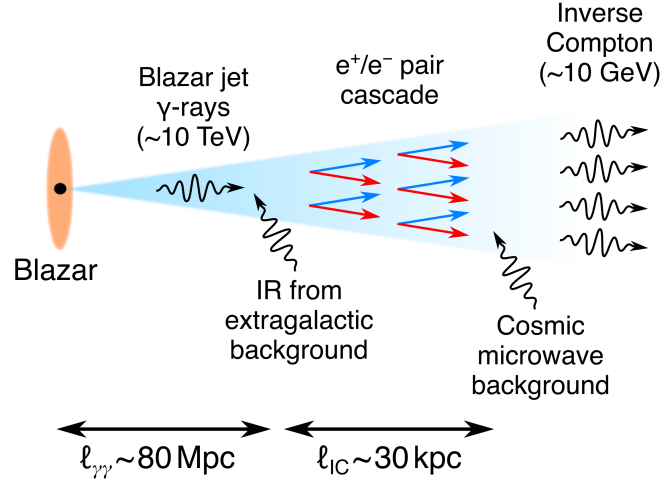


Figure 5.1: Blazar-induced pair beams in cosmic voids.

where E_e is the energy of the electron/positron. Pairs with energy E_e will inverse-Compton scatter with CMB photons to produce secondary cascade emission with typical energy:

$$E_{\gamma} = \frac{4}{3} \varepsilon_{\text{CMB}} \left(\frac{E_e}{m_e c^2} \right)^2 \sim 85 \left(\frac{E_e}{5 \text{ TeV}} \right)^2 \text{ GeV}, \quad (5.3)$$

where $\varepsilon_{\text{CMB}} = 6.6 \times 10^{-4} \text{ eV}$ is the typical energy of CMB photons. However, this prediction is at odds with astronomical observations of GeV emission from blazars accumulated over more than a decade [7, 10, 11] (see Figure 5.2). The leading hypothesis is that $e^{\pm}$ pairs are deflected by weak magnetic fields in the void, relics of fields generated in the early Universe [100–102], leading to the GeV emission being spread out into halos that are yet to be resolved by telescope measurements. An alternative explanation has been proposed that the pair cascade is disrupted by electromagnetic beam-plasma instabilities before pairs can inverse-Compton scatter with CMB photons [11, 103]. But it has been argued that

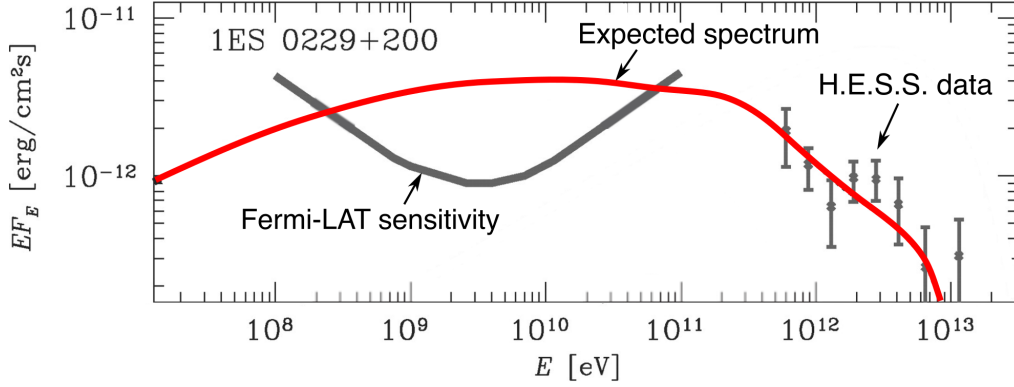


Figure 5.2: Blazar induced pair beams in cosmic voids. (Adapted from Neronov & Vovk [10].) The expected secondary cascade emission is modelled (red) for blazar source 1ES 0229+200 (redshift $z = 0.14$, $d \sim 600$ Mpc) based on measurements of TeV emission using the H.E.S.S. Cherenkov telescope (grey points). Given the sensitivity of the Fermi Large Area Telescope (grey line), cascade emission in the 1 – 100 GeV range should be observed.

such instabilities should be severely suppressed due to the finite thermal spread of the pair beam [31, 104], as discussed in Chapter 1. If collective plasma processes play a role, then it severely limits the validity of constraints on intergalactic magnetic fields based on blazar spectral measurements, but no consensus has been reached despite numerous theoretical and computational investigations (review provided in the introduction of Perry & Lyubarsky [31]), and theories of instability suppression have never been validated by experiments due to the challenges of producing an appropriate electron-positron beam. Using the scheme of pair beam generation in the previous chapters, an experimental setup has been designed to study this for the first time (setup shown in Figure 5.3). Filamentation of the pair beam is measured using a luminescence screen placed downstream of the plasma, detecting the emergence of density modulations in the transverse beam profile, and a Faraday rotation diagnostic is used to detect the emergence of magnetic fields.

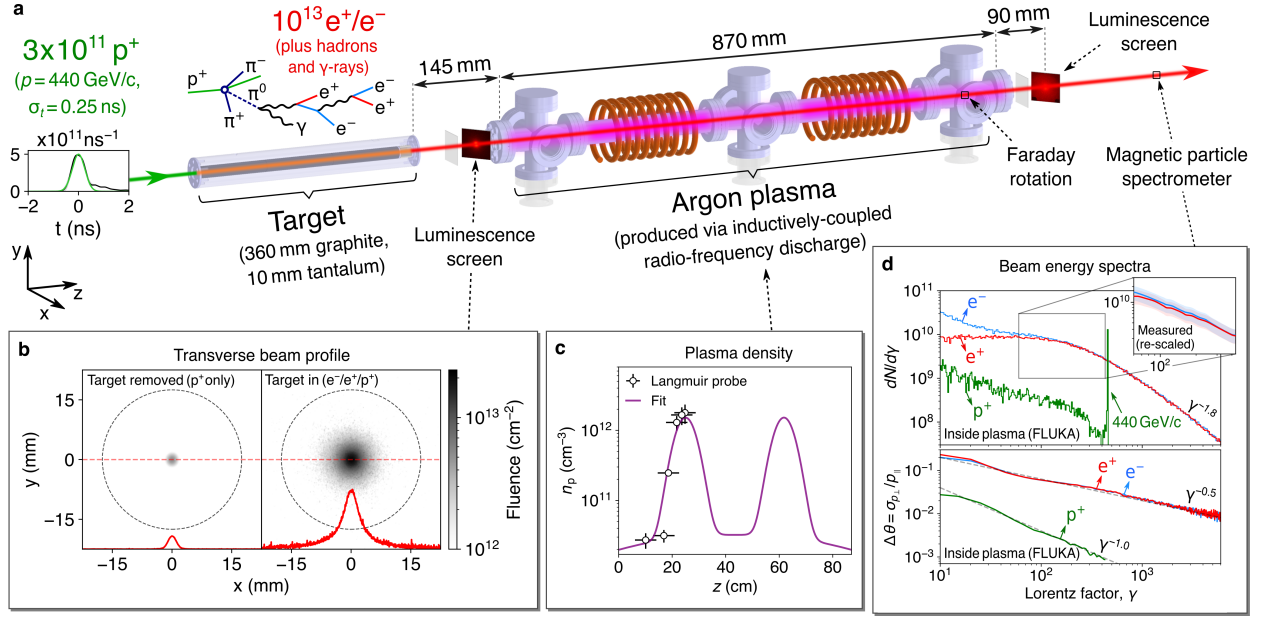


Figure 5.3: Experimental setup. (a) The electron-positron pair beam, produced by irradiating a solid target of graphite and tantalum with 440 GeV/c protons (with temporal profile shown in the inset), propagates through a metre-length argon plasma generated by an inductively coupled radio-frequency discharge. (b) The transverse beam profile is measured before the plasma using $70 \text{ mm} \times 50 \text{ mm} \times 0.25 \text{ mm}$ chromium-doped ceramic (Chromox) luminescence screens imaged by digital cameras, with a spatial resolution of $100 \mu\text{m}$. (c) The longitudinal plasma density profile is measured using a Langmuir probe and confirmed during the experiment using optical emission spectroscopy. Pair beam filamentation due to beam-plasma instability is measured using a luminescence screen placed downstream of the plasma, and a Faraday rotation diagnostic measures the growth of magnetic fields. (d) The electron and positron energy spectra are measured using a magnetic particle spectrometer (re-scaled according to the size of the collecting aperture).

5.2 Transverse beam profile measured downstream of the plasma

Density modulations in the transverse beam profile due to current filamentation of electrons and positrons can be detected using a luminescence screen setup identical to the one used to image the transverse beam profile downstream of the target (using a $70 \text{ mm} \times 50 \text{ mm} \times 0.25 \text{ mm}$ Chromox screen viewed by a digital camera at a standoff distance 3.9 m). If there is significant contrast in the current separation of electrons and positrons, then structural features should be observable in the profile. Images are collected when the pair beam passes through a range of plasma densities (discharge fill pressure $p_g = 1 \text{ Pa}$, 2 Pa , 4 Pa), with the discharge on and off. The plasma is ignited several seconds before the beam's arrival and is present continuously as the beam passes.

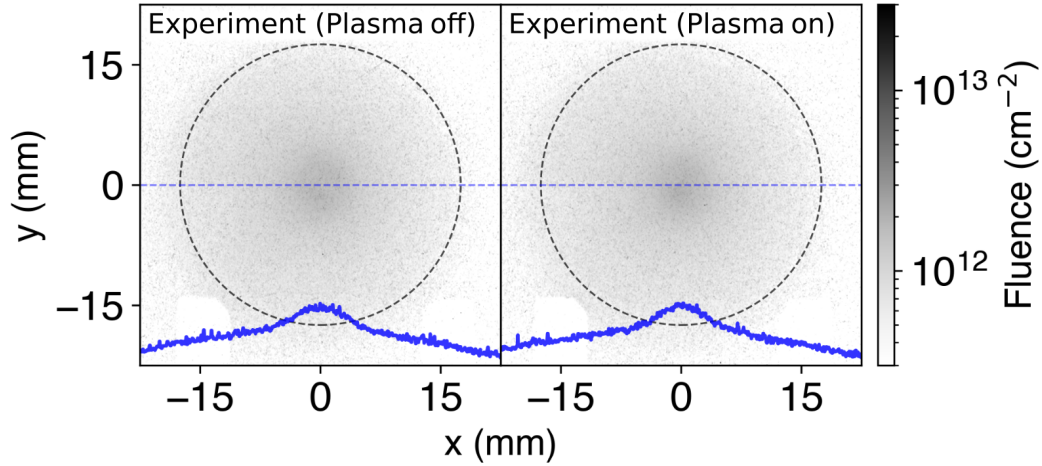


Figure 5.4: Transverse beam profile imaged downstream of the plasma. The transverse beam profile is imaged after the pair beam has passed through the plasma using a $70 \text{ mm} \times 50 \text{ mm} \times 0.25 \text{ mm}$ Chromox screen (shown after removal of the central $\sigma_r = 1 \text{ mm}$ peak due to the residual proton beam). Left: The image of the beam profile obtained when the plasma is off. Right: The profile observed when the plasma discharge is activated ($p_g = 4 \text{ Pa}$, $P_{\text{abs}} = 240 \text{ W}$). The plasma cell circumference is marked by the grey-dashed circle, and the profiles projected onto the x -axis (blue) correspond to the central lineouts (blue-dashed).

In all cases the result is the same: when the plasma discharge is activated, the density profile remains unchanged (Figure 5.4), showing no evidence of filamentation. There are a few possible explanations for this:

1. Modulations are a small enough scale ($\lesssim 100\mu\text{m}$) that they cannot be spatially resolved. But features are expected to emerge on the scale of the skin depth of the ambient plasma (several mm in size).
2. There is enough longitudinal variation in the structure that the filaments are not visible given the time-integrated nature of the measurement.
3. The screen is positioned far enough from the plasma that filaments diverge before the screen and structural features reduce in contrast.
4. Coulomb scattering of the pairs as they pass through the glassy carbon window at the rear of the plasma cell causes them to diverge. The amount of smoothing can be estimated. Given that the thickness of the window corresponds to 0.013 radiation lengths of glassy carbon, at the distance of the screen ($d_{\text{scr}} = 90\text{ mm}$), a point source of pairs with Lorentz factor $\gamma_{\pm}$ diverges to a transverse size:

$$\sigma_X [\text{mm}] = d_{\text{scr}} \tan(\sigma_{\theta}) \approx 0.3 \left(\frac{\gamma_{\pm}}{10^3} \right)^{-1}, \quad (5.4)$$

where $\langle \gamma_{\pm} \rangle \approx 10^3$ is the approximate mean Lorentz factor of pairs. So Coulomb scattering leads to smoothing of filaments on mm-scales only for pairs at a significantly lower energy than the mean.

The most plausible explanation is that the pair beam remains remarkably stable as it propagates through the plasma. This would mean an instability growth rate that is much lower than the theoretical growth rate assuming idealized beam conditions (monoenergetic, collimated pairs, Eq. 1.28).

5.3 Time-resolved Faraday rotation using a magneto-optic probe

The Faraday effect is a magneto-optic interaction caused by the difference between the dispersion of left-handed and right-handed circularly polarized waves in a medium with circular birefringence. Since linearly polarized electromagnetic waves are equivalent to a superposition of equal amplitude left and right-handed circularly polarized components, if a phase difference is introduced between the components due to travelling through the medium with different speeds, then the plane of polarization of a linearly polarized wave becomes rotated relative to its original orientation.

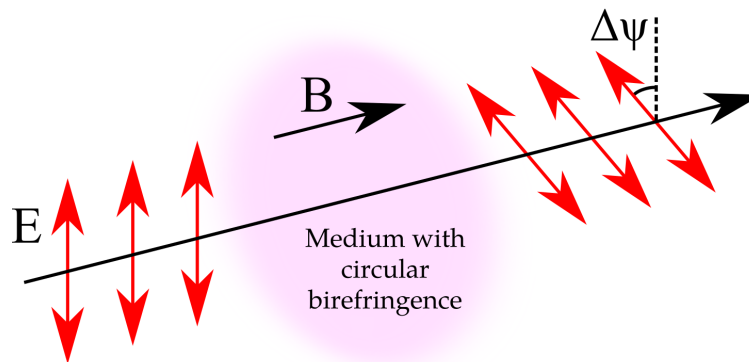


Figure 5.5: The Faraday effect. Circular birefringence causes a phase difference to arise between components of circular polarization of a linearly polarized electromagnetic wave, leading to rotation of the plane of polarization.

Circular birefringence arises naturally in plasmas exposed to a magnetic field due to the gyromotion of free electrons. In a cold unmagnetized plasma ($kv_{\text{th}} \ll \omega_{\text{pe}}$), the refractive indices are given by

$$n_{\pm}^2 = 1 - \frac{\omega_{\text{pe}}^2}{\omega(\omega \pm \omega_{\text{ce}})} \quad (5.5)$$

where $\omega_{\text{ce}} = eB/m_e c$ is the electron gyrofrequency, and the positive and negative signs correspond to left-handed and right-handed circularly polarized waves, respectively. The difference between the refractive indices depends on the free electron density (through the plasma frequency) and the strength of the magnetic field (through the electron gyrofrequency). In a plasma where the refractive index is only a slight deviation from unity ($\omega_{\text{pe}}, \omega_{\text{ce}} \ll \omega$), the difference between n_+ and n_- is given by: $\Delta n \approx \omega_{\text{pe}}^2 \omega_{\text{ce}} / \omega^3$, and the angle by which the plane of polarization is rotated ($\Delta\psi$) is directly proportional to the line-integrated product of the electron density and the component of the magnetic field along the direction of wave propagation:

$$\Delta\psi = \frac{e^3}{2\pi m_e^2 c^4} \lambda^2 \int n_e \mathbf{B} \cdot d\mathbf{s}, \quad (5.6)$$

where $e^3/2\pi m_e^2 c^4 = 2.62 \times 10^{-13} \text{ T}^{-1}$ and the factor of proportionality between $\Delta\psi$ and λ^2 is referred to as the ‘rotation measure’. Faraday rotation is used to probe magnetic fields in astrophysical plasmas, notably to determine the strength of magnetic fields in the interstellar medium of the Milky Way galaxy by observing radiation emitted from pulsars that has a degree of linear polarization [105, 106]. The magnetic fields measured ($> 10^{-10} \text{ T}$) are far

larger than can be explained by Biermann battery processes, and are believed to arise from the amplification of seed fields via a turbulent dynamo [107, 108], a process that has now been observed in laboratory experiments [42]. Faraday rotation is successfully used as a probe of magnetic fields [109–112], provided the combination of electron density, magnetic field strength and integrated path length are sufficiently high. To observe 1° of Faraday rotation using a laser probe with wavelength $\lambda = 1\ \mu\text{m}$ requires

$$n_e [\text{cm}^{-3}] \cdot B [\text{T}] \cdot L [\text{cm}] \geq 6.7 \times 10^{18} \text{ T cm}^{-2}, \quad (5.7)$$

meaning that the plasma densities in the inductive discharge ($n_e = 10^{11} - 10^{12} \text{ cm}^{-3}$) are far too small to measure sub-Tesla magnetic fields. Instead, a transparent solid material which exhibits a large degree of circular birefringence is suspended in the plasma and Faraday rotation is observed in a probe laser transmitted through the material. Rare-earth-doped garnet, such as terbium gallium garnet (TGG) is a suitable choice. Circular birefringence arises due to paramagnetism and TGG crystals are commonly used to build Faraday rotators for photonics applications. The rotation is usually parameterized by a single constant called the Verdet constant, $\mathcal{V}$:

$$\Delta\psi = \mathcal{V} \int \mathbf{B} \cdot d\mathbf{s}, \quad (5.8)$$

though Verdet constant is sensitive to temperature (Curie-Weiss law: $\Delta\psi \propto T^{-1}$) and probe wavelength ($\Delta\psi \propto \lambda^{-2}$) [113–115]. At room temperature, using a green probe laser ($\lambda = 532 \text{ nm}$), the Verdet constant of TGG is $\mathcal{V} \sim 190 - 230 \text{ rad T}^{-1} \text{ m}^{-1}$.

Using TGG, mT-scale magnetic fields can be measured in a centimetre-scale path length:

$$\langle B \rangle \equiv \frac{1}{L} \int_0^L \mathbf{B} \cdot d\mathbf{s} = \frac{\Delta\psi}{\mathcal{V}L} = 7 \left(\frac{\Delta\psi}{1^\circ} \right) \left(\frac{\mathcal{V}}{200 \text{ rad/T/m}} \right)^{-1} \left(\frac{L}{10 \text{ mm}} \right)^{-1} \text{ mT}. \quad (5.9)$$

Rotation angle is measured using a photodiode and oscilloscope by detecting changes in intensity of orthogonal components of linear polarization after passing the probe beam through a polarizing filter or polarizing beam splitter.

5.3.1 Design and performance

The probe is designed with the following requirements in mind:

- Fibre-optic-based design – The probe laser, photodiode and oscilloscope must be kept in the adjacent tunnel (TT61) to shield them from the prompt radiation. To ensure that optics remain aligned when the plasma discharge is moved on its telescopic stage, the use of optical fibres to transport the probe beam to and from the experiment is desired.
- Sub-nanosecond time resolution – This is determined by: (i) the time taken for the probe beam to propagate through the Verdet crystal, (ii) the bandwidth of the photodiode and oscilloscopes used, and (iii) the bandwidth of the optical fibres (which suffer from chromatic and modal dispersion). The rise time (10 – 90%) of Gaussian signal with $\sigma = 0.25 \text{ ns}$ is $t_r = 0.42 \text{ ns}$, so to resolve changes in magnetic field on the timescales of the beam passing, the bandwidth of components should exceed 0.8 GHz (given rise time is related to bandwidth $t_r \cong 0.35/f_{\text{BW}}$).

- Sensitivity to mT-scale magnetic fields – The magnetic field strength associated with magnetic filaments containing $\mathcal{O}(10^{12})$ electrons or positrons in a ns-scale bunch measured at a distance of a few mm is milli-Tesla scale:

$$B \sim \frac{\mu_0 I}{2\pi r} = 6 \left(\frac{N}{10^{12}} \right) \left(\frac{q}{e} \right) \left(\frac{\tau}{1 \text{ ns}} \right)^{-1} \left(\frac{r}{5 \text{ mm}} \right)^{-1} \text{ mT}. \quad (5.10)$$

The final design is shown in Figure 5.6. A Class 3B continuous-wave diode laser (Z-Laser ZM18, $\lambda = 532 \text{ nm}$, $P = 40 \text{ mW}$) is coupled into a 10 m length optical fibre (silica core, glass clad, step index with 0.22 NA, $\varnothing 105 \mu\text{m}$ core) and transported to the end of the plasma discharge, where the probe optics are arranged inside a compact Thorlabs 30 mm cage-rail system attached to the port-cross. The laser is coupled into and out of optical fibres using Thorlabs FiberPort collimator lenses, which can be finely adjusted to couple the laser into a large core, multimode fibre with almost complete efficiency. The cage-rail structure ensures robust alignment, with optical components locked in position. A photograph is shown in Figure 5.7.

The probe laser is linearly polarized using a nanoparticle linear film polarizer, chosen for its high extinction ratio (10,000:1), high transmission ratio at $\lambda = 532 \text{ nm}$ (73%) and high damage threshold. Then the beam is split using a 50:50 beamsplitter. The transmitted beam is coupled back into an optical fibre and used as a reference beam to monitor changes in intensity of the probe laser.

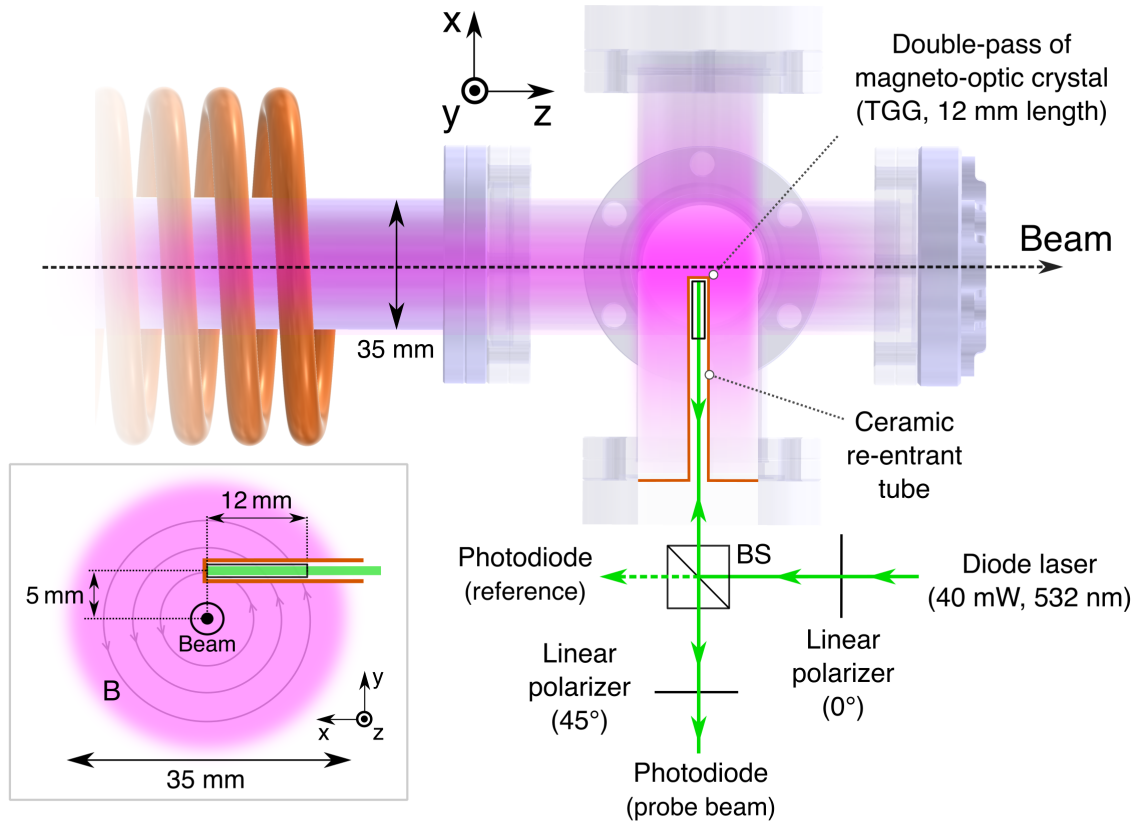


Figure 5.6: Design of the time-resolved magneto-optic Faraday probe. A continuous laser beam is transported via an optical fibre to the end of the plasma discharge, and linearly polarized before passing through a 50:50 beamsplitter (BS). The transmitted component acts as a reference of changes in laser intensity. The reflected component is directed along a ceramic re-entrant tube to a TGG magneto-optic crystal suspended in the plasma. The position of the crystal relative to the beam in the plane transverse to the beam axis is shown in the inset. The rear surface of the crystal has a highly reflective dielectric coating so the probe beam makes a double pass of the crystal, doubling the amount of Faraday rotation. Finally, the probe beam is passed through a second linear polarizer with zero-axis oriented at 45° to the first. Change in the transmitted intensity due to Faraday rotation is measured using a photodiode.

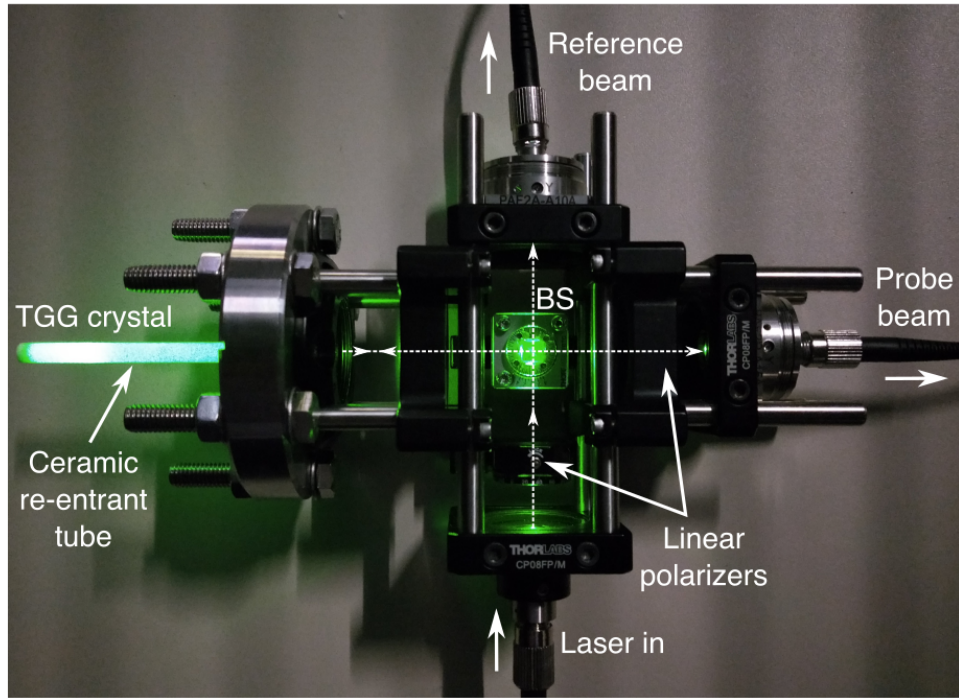


Figure 5.7: Photograph of the Faraday probe. Photograph of the Faraday probe setup prior to installation. The paths taken by the laser beams are shown by the white-dashed lines.

The reflected beam is directed towards a 12 mm length, 2 mm diameter TGG crystal suspended in the plasma at the end of a ceramic re-entrant tube (inner length 70 mm, outer diameter 5 mm). The crystal has an anti-reflective coating on the front surface and a highly reflective coating on the rear surface to reflect the probe beam so it makes two passes through the crystal. A two-pass design doubles the Faraday rotation for a given magnetic field because the Faraday effect is non-reciprocal – the circular birefringence depends on the orientation of the magnetic field, not the propagation direction of the light. The crystal length is as long as possible to maximize the magnetic field sensitivity, but not so long that mm-scale magnetic field structures might be averaged out and the time resolution is limited (the double pass takes $t_{\text{pass}} = nl/c = 0.16$ ns, where $n = 1.98$ is the refractive index of TGG

at $\lambda = 532 \text{ nm}$). A more involved consideration of retarded magnetic potentials affects the magnetic field measurements by $< 1\%$.

The position of the re-entrant tube relative to the plasma is fixed, but by steering the primary beam and adjusting the vertical position of the discharge, the probe crystal can be moved to sample azimuthal magnetic field components arising from axially aligned currents (as shown in the Figure 5.6 inset). With the crystal positioned 5 mm away from the beam axis, reduction in the Verdet constant due to crystal heating is not expected – the instantaneous temperature rise due to energy deposition of the beam is expected to be $\sim 1 \text{ K}$ (according to a FLUKA simulation), and the surrounding neutrals in the plasma are not expected to be heated significantly above room temperature [116]. The Verdet constant of the crystal is measured at the end of the beam time to ensure that no degradation of the sensitivity has occurred due to radiation damage.

The reflected probe beam passes back through the beamsplitter, and then through a second linear polarizer which has a zero-axis oriented at an angle $\Delta\phi$ to the first. The transmitted intensity is given by Malus's law:

$$I_t \propto \cos^2(\Delta\phi + \Delta\psi). \quad (5.11)$$

An offset angle $\Delta\phi = 45^\circ$ is chosen because it leads to the greatest sensitivity to small angle changes in Faraday rotation (the gradient $\partial I_t / \partial(\Delta\psi)$ evaluated for small $\Delta\psi$ is maximized for $\Delta\phi = 45^\circ$).

Small changes in intensity are directly proportional to $\Delta\psi = \mathcal{V}L\langle B \rangle$:

$$I_t \propto \frac{1}{2} [1 - \sin(2\Delta\psi)] \approx \frac{1}{2} (1 - 2\Delta\psi). \quad (5.12)$$

The probe beam is coupled into a 7 m length optical fibre (silica core, glass clad, step index with 0.22 NA, $\varnothing 200 \mu\text{m}$ core) and transported to a 2 GHz photodiode (Thorlabs DET025AFC/M) in TT61. A shielded coaxial cable connects the photodiode to a 3 GHz oscilloscope (LeCroy WavePro 7300A).

When selecting the optical fibres, modal and material dispersion must be considered [117], as they put limits on the transmissible bandwidth. Modal dispersion occurs in multimode fibres due to the difference in propagation time of signals transmitted through the cable. A ray travelling perfectly along the fibre axis will have a shorter path length through the fibre faster than a ray incident at the critical angle undergoing multiple reflections. The corresponding temporal spreading is given by

$$\frac{\delta\tau_{\text{modal}}}{L} = \frac{n_{\text{core}}}{cn_{\text{clad}}} (n_{\text{core}} - n_{\text{clad}}) \approx \frac{(\text{NA})^2}{2cn_{\text{core}}} \quad (\text{if } n_{\text{core}} \approx n_{\text{clad}}), \quad (5.13)$$

where L is the cable length, n_{core} and n_{clad} are the refractive indices of the core and cladding and $\text{NA} = \sqrt{n_{\text{core}}^2 - n_{\text{clad}}^2}$ is the numerical aperture. For the chosen fibre with a fused silica core ($\text{NA} = 0.22$, $n_{\text{core}} = 1.461$ at $\lambda = 532 \text{ nm}$), the bandwidth that can be maintained over 7 m of cable is at an acceptable level (1.3 GHz). The material dispersion (equivalently, the chromatic dispersion) arises due to the finite wavelength bandwidth of the laser ($\Delta\lambda$)

and the wavelength-dependence of the refractive of the core. The corresponding temporal spreading is given by

$$\frac{\delta\tau_{\text{mat}}}{L} = -\frac{\lambda}{c} \frac{d^2 n_{\text{core}}}{d\lambda^2} \Delta\lambda = -M \cdot \Delta\lambda, \quad (5.14)$$

where $M = 200 \text{ ps nm}^{-1} \text{ km}^{-1}$ at $\lambda = 532 \text{ nm}$ for the chosen fibres, so the bandwidth limitation assuming $\Delta\lambda = 1 \text{ nm}$ is 360 GHz , much less of a concern. Measurements of the Verdet constant, intrinsic noise and instrument response function are presented in Figure 5.8. To measure the Verdet constant, the change in probe beam intensity (ΔV) is recorded when the TGG crystal is exposed to a permanent magnetic field (neodymium ring

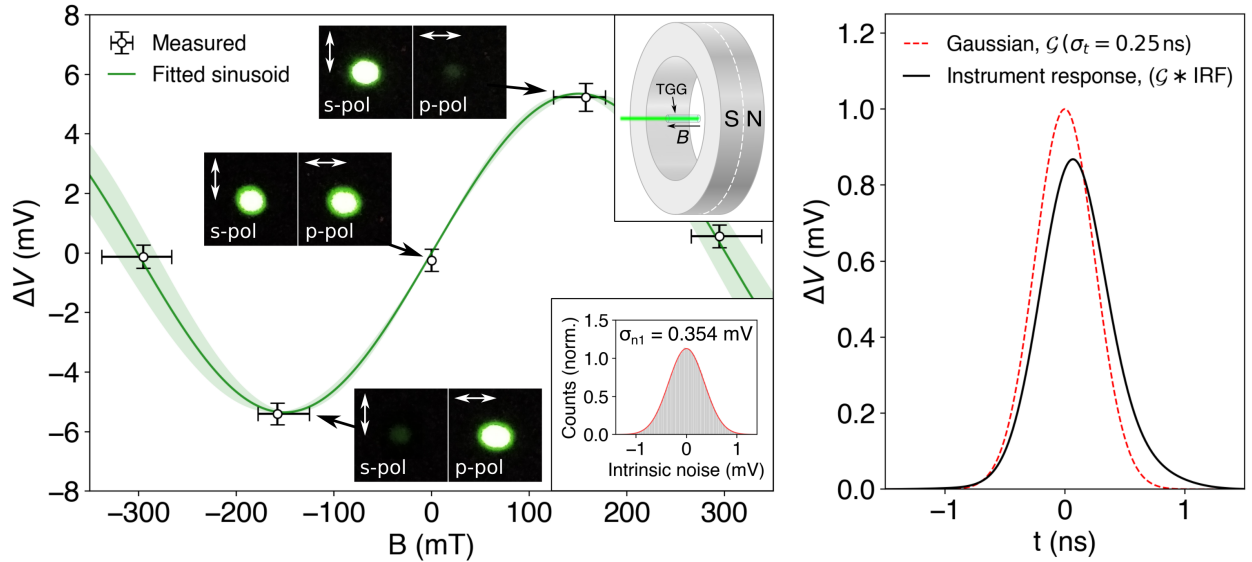


Figure 5.8: Performance of the Faraday probe. Left: The Verdet constant of the TGG crystal is measured by fitting a sinusoid to the change in probe beam intensity when exposed to a permanent magnet (as depicted in the upper left inset). The orthogonal components of linear polarization (labelled s-polarized and p-polarized) visibly change in intensity when the crystal is exposed to the magnetic field. The intrinsic electronic noise for a single measurement, which limits the probe sensitivity, is measured to be Gaussian with $\sigma_{n1} = 0.354 \text{ mV}$, as shown in the lower right inset. Right: The instrument response function obtained from impulse measurements with an ultra-fast femtosecond laser is convolved with a Gaussian $\sigma_t = 0.25 \text{ ns}$ to show that changes in intensity on timescales of the beam duration can be detected.

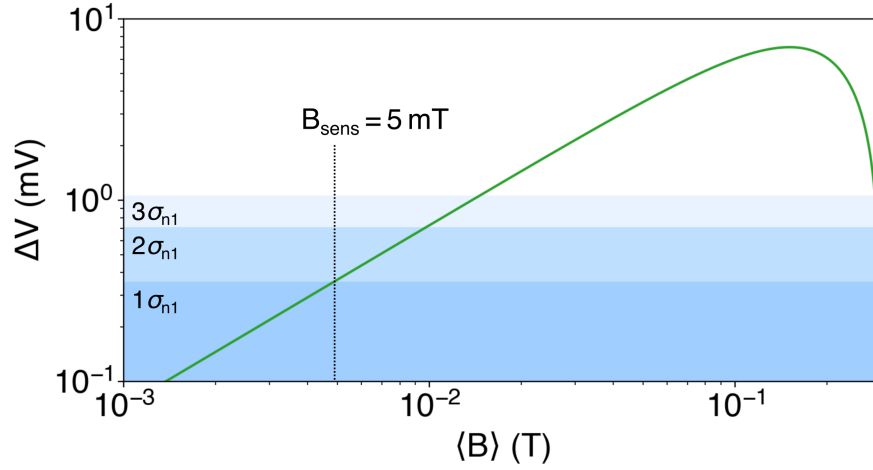


Figure 5.9: Faraday probe sensitivity to magnetic fields. The change in probe beam intensity (ΔV) is plotted with respect to the mean magnetic field aligned with path of the laser beam through the crystal ($\langle B \rangle$): $\Delta V = V_0 [\cos^2(\langle B \rangle \mathcal{V} L + 45^\circ) - \frac{1}{2}]$. The blue-shaded regions correspond to multiples of σ_{n1} , the intrinsic electronic noise for a single measurement.

magnet K&J Magnetics RZ0Y0X0, 3" outer diameter, 2" inner diameter, 1" thickness). The field strength of the magnet is measured using a Hall probe, in agreement with precise manufacturer specifications. The intensities of the orthogonal polarization components change sinusoidally with applied magnetic field strength according to a Verdet constant $\mathcal{V} = 217 \pm 15 \text{ rad T}^{-1} \text{ m}^{-1}$. The intrinsic electronic noise (of the photodiode and oscilloscope combination) is measured to be Gaussian distributed with $\sigma_n = 0.354 \text{ mV}$. The instrument response function is measured using mV-scale impulse signals produced by exposing the photodiode to an attenuated ultra-short femtosecond laser pulse. The measured rise time is 0.44 ns, and the instrument response function convolved with a $\sigma_t = 0.25 \text{ ns}$ Gaussian is shown in Figure 5.8, demonstrating that changes in intensity on timescales of the beam duration can be detected. The sensitivity of the probe to small magnetic fields depends on the signal-to-noise ratio that is achievable given the Verdet constant, the fraction of the laser

power which can reach back to the photodiode, and the intrinsic noise level. The magnetic field that corresponds to an observed change in probe beam intensity is shown in Figure 5.9. The $1\text{-}\sigma$ intrinsic noise results in an instrument sensitivity of $B_{\text{sens}} = 5\text{ mT}$.

5.3.2 Experimental measurements

The capture of a Faraday probe signal is triggered using the signal produced when the primary beam enters the experimental area, and the beam halo is detected by a biased diamond detector positioned 5 m upstream of the probe. The delay between the arrival of the two signals at the oscilloscope is used to determine the time in the recorded signal that represents when the pair beam passes the probe crystal ($t_{\text{rel}} = 0$). Since the timing is consistent between shots, the signal-to-noise ratio can be improved by combining signals from multiple shots. Five measurements of the pair beam are obtained with the plasma operating with maximum density conditions ($n_{\text{p,max}} = 2 \times 10^{12}\text{ cm}^{-3}$, $p_{\text{g}} = 4\text{ Pa}$, $P_{\text{abs}} = 240\text{ W}$), shown in Figure 5.10. The mean signal is plotted, with a maximum deviation corresponding to $\langle B \rangle = 5\text{ mT}$. A more precise measurement remains limited by the intrinsic electronic noise. The upper limit of the measured field is $\langle B \rangle_{\text{exp}} \leq 5\text{ mT}$, given that measurements consistently larger than 5 mT in five repeated measurements leads to a statistically significant detectable signal (at the $2.2\text{-}\sigma$ level). Positive measurement of magnetic field depends on the form it takes. A field growing in the early linear stage of amplification from a seed field produced by the net current of the residual protons will lead to a consistent measurement between shots, which will appear as a statistically significant deviation of the Faraday signal when multiple shots are combined. If instead the instability evolves to an advanced state,

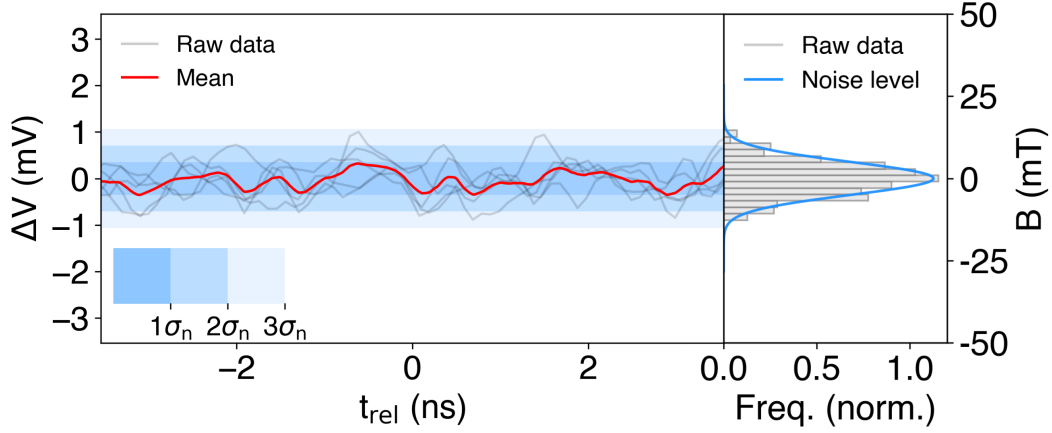


Figure 5.10: Faraday probe measurements. Left: Five Faraday probe measurements are plotted (‘raw data’), obtained when the pair beam propagates through identical plasma conditions (peak density $n_p = 2 \times 10^{12} \text{ cm}^{-3}$). The signals are plotted relative to a time $t_{\text{rel}} = 0$ that corresponds to when the pair beam passes the Faraday probe. The secondary y -axis gives the magnetic field strength that is equivalent to an observed change in signal ΔV . Plotting the mean of the five signals (red) does not reveal any underlying Faraday rotation corresponding to $\langle B \rangle > 5 \text{ mT}$. As in Figure 5.9, the blue-shaded regions show the intrinsic electronic noise level. Right: A histogram showing the distribution of the raw data combined from the five measurements, showing that it does not differ significantly from the Gaussian-distributed intrinsic noise.

the field may be much stronger but more random, in which case a noticeably enhanced standard deviation of the Faraday signal would result, accompanied by clear indications of beam filamentation in the transverse beam profile.

5.4 Particle-in-cell simulations

The effects of finite thermal spread of the $e^\pm$ pairs are explored further by performing three-dimensional (3D3V) particle-in-cell (PIC) simulations using the fully relativistic, massively parallel, PIC code OSIRIS [118]. In the simulations, a moving window follows an electron-positron-proton beam at the speed of light to model conditions closely resembling the experimental pair beam and the ambient plasma. The idealized case of a perfectly

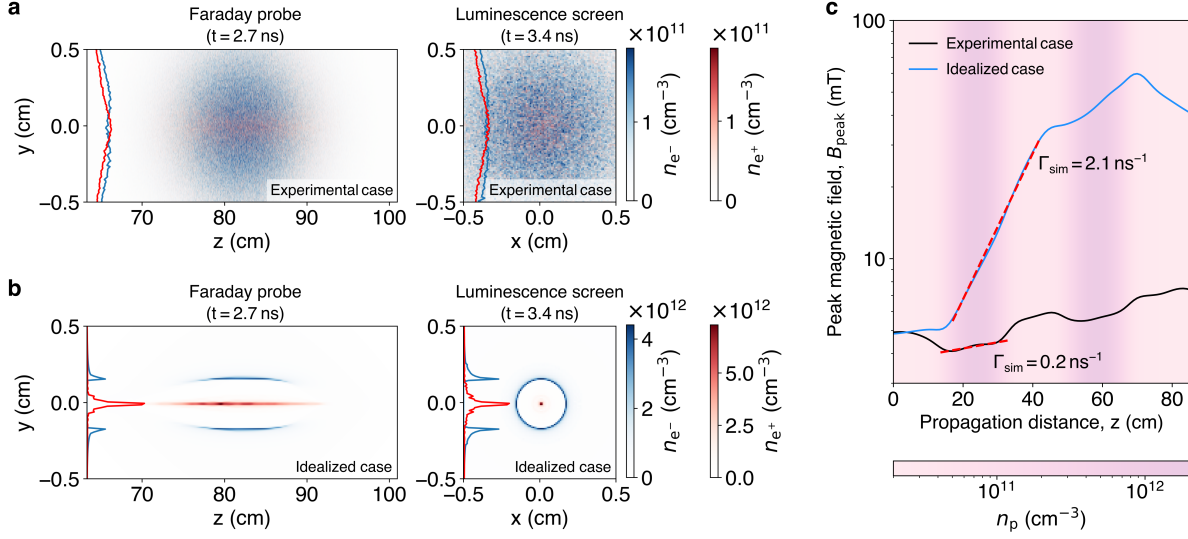


Figure 5.11: Three-dimensional particle-in-cell simulations. 3D simulations of the beam-plasma interaction are performed using the particle-in-cell code OSIRIS [118] in two cases: (a) with conditions closely resembling the experimental beam and ambient plasma (labelled ‘experimental’), and (b) an idealized case with a collimated, monoenergetic ($\gamma_{\pm} = 10^3$) pair beam (labelled ‘idealized’). In (a) and (b), the left panels show a central slice of the electron and positron densities in the longitudinal plane (x - z) at the time when the beam passes the Faraday probe ($t = 2.7 \text{ ns}$ after entering the plasma), whilst the right panels show a central slices of the transverse plane (x - y) when the beam passes the downstream luminescence screen ($t = 3.4 \text{ ns}$). (c) The peak magnetic field is plotted as a function of propagation distance through the plasma, with background shading showing the ambient plasma density. The maximum growth rate of the peak magnetic field is obtained by the shown fits (red-dashed). The anti-correlation between magnetic field and plasma density is an effect of the varying level of return current in the plasma.

collimated, monoenergetic pair beam is considered for comparison. The results are shown in Figure 5.11, with a time series included in Appendix D. In both simulations, current separation of electrons and positrons is observed, and the positron density increases on-axis leading to the exponential growth of an azimuthal magnetic field (expected to break into islands with further growth), confirming the role of the residual protons in providing a seed magnetic field for the instability. Otherwise, the protons have no effect due to their much larger momentum. In the case where the $e^{\pm}$ pairs are collimated and monoenergetic,

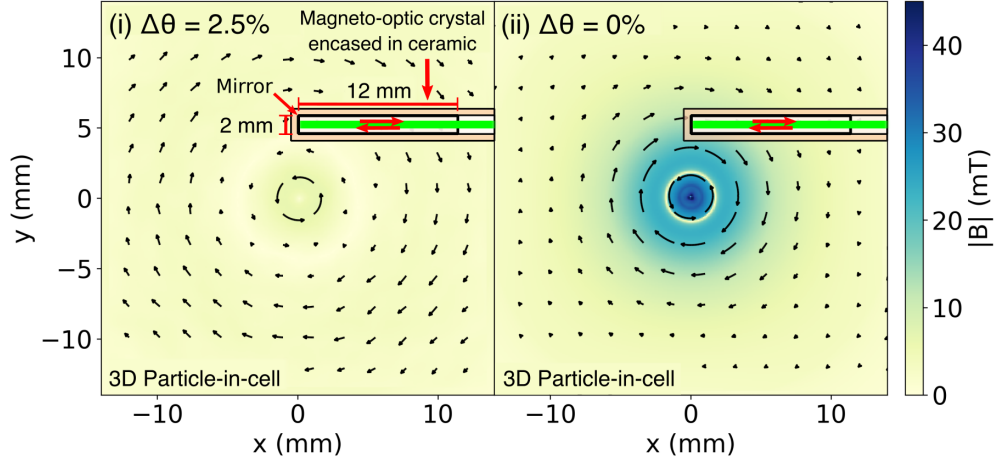


Figure 5.12: Simulated transverse magnetic fields at the Faraday probe position. The transverse magnetic fields in 3D-PIC simulations are shown at the position in the plasma corresponding to the Faraday probe for: (i) ‘experimental conditions’ (left), and (ii) ‘idealized conditions’ (right), as described in the caption of Figure 5.11. The relative position of the Faraday probe crystal is superimposed onto each.

filaments become completely separated and emerge on much smaller spatial scales. The peak magnetic field is vastly increased ($B_{\text{max}} = 60 \text{ mT}$) with a peak growth rate well-matched by the theoretical prediction of Eq. 1.28 ($\Gamma_{\text{sim}} = 2.1 \text{ ns}^{-1}$), as shown in Figure 5.11c. By contrast, when the finite thermal spread of pairs is accounted for, the small-scale modes are stabilized, and the peak magnetic field and growth rate are significantly reduced ($B_{\text{max}} = 7 \text{ mT}$, $\Gamma_{\text{sim}} = 0.2 \text{ ns}^{-1}$). The simulated magnetic fields provide a prediction of the Faraday rotation measurement of $\langle B \rangle_{\text{sim}} = 1.7 \text{ mT}$, leading to an estimated average growth rate $\langle \Gamma_{\text{sim}} \rangle = 0.3 \text{ ns}^{-1}$, consistent with the experimentally obtained bound $\langle \Gamma_{\text{exp}} \rangle \leq 0.7 \text{ ns}^{-1}$. The simulated transverse magnetic fields at the Faraday probe position are shown in Figure 5.12, and the combined density profile of electrons and positrons at the luminescence screen position is shown in Figure 5.13, accounting for the smoothing effect caused by Coulomb scattering of pairs as they pass through the glassy carbon by applying a Gaussian

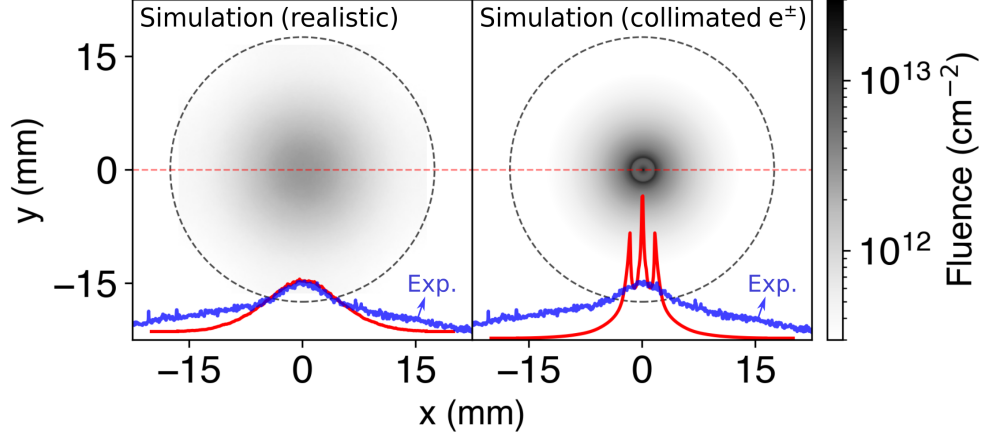


Figure 5.13: Synthetic images of the transverse beam profiles from simulations. Expected transverse beam profiles measured downstream of the plasma are obtained from the density profile of electrons and positrons in PIC simulations at the position corresponding to the luminescence screen. The transverse profiles are combined and time-integrated, then a Gaussian smoothing is applied according to the expected blurring arising from Coulomb scattering of $e^\pm$ pairs in the glassy carbon window between the plasma and the screen. The results are shown for the two simulated cases: (i) ‘experimental’ (left), and (ii) ‘idealized’ (right), as described in the caption of Figure 5.11. The grey-dashed circle represents the circumference of the plasma discharge. Lineouts taken along the red-dashed line are projected onto the x -axis (red) and compared with the experimental measurement (blue).

filter (G). The amount of smoothing (σ_X) corresponds to the projected size of a point source with divergence arising from window scattering (according to Eq. 5.4), weighted according to the momentum distribution of pairs (dN/dp):

$$I_{\text{smoothed}}(x, y) = \int [I(x, y) * G\{x, y, \sigma_X(p)\}] \left(\frac{1}{N} \frac{dN}{dp} \right) dp. \quad (5.15)$$

In the simulation where the pairs have zero thermal spread, beam filamentation is still visible when Coulomb scattering in the glassy carbon window is accounted for. In summary, similarly with the experimental results, the PIC simulations demonstrate a dramatic suppression of beam-plasma instability when the pairs have a finite momentum spread. In

the simulation modelling the experimental conditions, current separation does not lead to observable structural features in synthetic images of the transverse beam profile, and the expected strength of generated magnetic fields is weaker than the sensitivity of the Faraday probe.

5.5 An experimentally bounded instability growth rate

An upper bound of the instability growth rate is estimated given that the measured field is $\langle B \rangle_{\text{exp}} \leq 5 \text{ mT}$, and assuming the magnetic field grows exponentially from an azimuthally oriented seed field, B_0 , produced primarily by the net current of the residual proton beam propagating on-axis. The precise net current distribution is obtained from a FLUKA Monte-Carlo simulation modelling a simplified geometry of the experiment as shown in Figure 5.14. The simulated radial distributions of secondary components at the Faraday

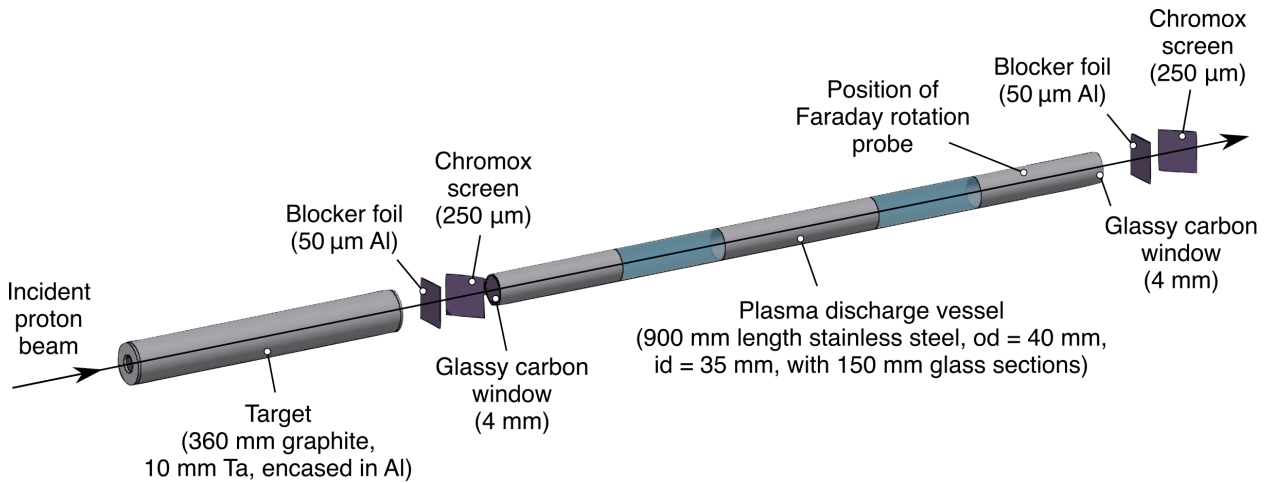


Figure 5.14: Simplified geometry of the experiment simulated using FLUKA. The plasma discharge vessel is modelled as a stainless steel tube with two glass sections where the inductive coils are located. The downstream beam dump, concrete walls and shielding are also included in the model (not shown). The exact material compositions are used in all cases where they can be obtained, which includes all the materials directly in the primary beam path.

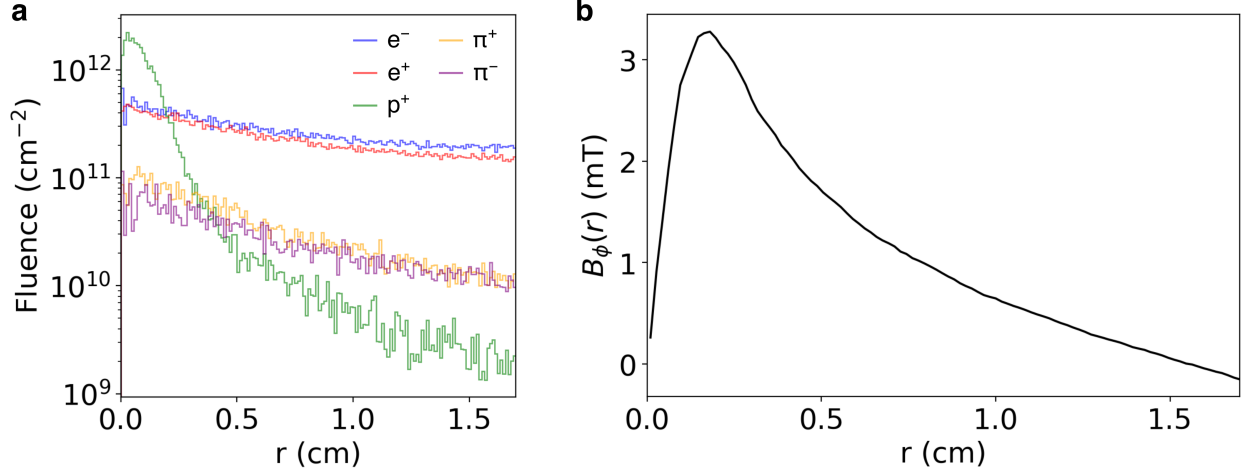


Figure 5.15: Obtaining the azimuthal magnetic field at the Faraday probe position in the absence of plasma using FLUKA simulations. (a) The particle fluence is plotted as a function of radius at the longitudinal position along the plasma discharge corresponding to the location of the Faraday probe (obtained from a FLUKA simulation) for electrons (blue), positrons (red), protons (green), positive pions (orange) and negative pions (purple). (b) Ampère’s circuital law is used to estimate the corresponding azimuthally oriented magnetic field as a function of radius. This calculation gives the most accurate expectation of the magnetic field in the absence of beam instability.

probe position are plotted in Figure 5.15a, and the magnetic field associated with the net current is shown in Figure 5.15b, calculated using Ampère’s circuital law:

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \iint_S \mathbf{J} \cdot d\mathbf{S} = \mu_0 I_{\text{enc}}, \quad (5.16)$$

where I_{enc} is the current passing through an enclosed surface, S , bounded by contour, C . It is assumed that all particles are relativistic (moving at almost the speed of light) and are aligned with the beam propagation axis. The magnetic field measurement of the probe in

the absence of amplification due to beam instability corresponds to:

$$B_{\text{probe}} = \langle \mathbf{B} \cdot \hat{\mathbf{x}} \rangle_{\text{probe}} = \frac{2}{l_c} \int_0^{l_c} \mathbf{B} \cdot \hat{\mathbf{x}}(y_0) dx = \frac{2}{l_c} \int_0^{l_c} B_\phi \frac{y_0}{\sqrt{x^2 + y_0^2}} dx, \quad (5.17)$$

where $l_c = 12 \text{ mm}$ is the length of the crystal, $y_0 = 5 \text{ mm}$ is the shortest perpendicular distance of the probe axis from the beam axis, and the prefactor of 2 accounts for a double pass of the crystal. Given the profile $B_\phi(r)$, the magnetic field measurement in the absence of beam-plasma interaction is $B_0 = 0.78 \pm 0.13 \text{ mT}$, where the uncertainty in B_0 accounts for a variation in the position of the probe crystal relative to the beam axis by 1 mm in each direction transverse to the beam axis (x and y). Using this value of B_0 , the upper bound of the mean growth rate is calculated to be:

$$\langle \Gamma_{\text{exp}} \rangle = t_{\text{prop}}^{-1} \log (\langle B \rangle_{\text{exp}} / B_0) \leq 0.7 \text{ ns}^{-1}, \quad (5.18)$$

where $t_{\text{prop}} = 2.7 \text{ ns}$ is the propagation time of the pair beam through plasma before it reaches the probe.

5.6 Application to the case of blazar-induced pair beams in cosmic voids

In the specific case of blazar-induced pair cascades propagating through cosmic voids, the density of pairs is estimated to be incredibly low ($n_{\pm, \text{blz}} \sim 10^{-23} \text{ cm}^{-3}$) [11, 104], such that $\Delta\theta \gg (n_{\pm}/n_p \gamma_{\pm})^{1/3}$. Under such conditions in the experiment, the thermal spread of

the pair beam resulted in suppression of beam-plasma instability, and this effect would be particularly severe for the blazar pair cascade. Maximizing the factor in square brackets in Eq. 1.28, in the limit of large $\Delta\theta$ the growth rate becomes

$$\Gamma = \sqrt{\frac{2}{3}} \frac{\omega_p}{\Delta\theta} \left(\frac{n_{\pm}}{2n_p\gamma_{\pm}} \right)^{2/3}, \quad (5.19)$$

which can be used to scale the experimentally obtained upper bound of the growth rate and estimate the maximum growth rate of suppressed pair beam instability in the blazar case. The relevant parameters for the blazar jet are shown alongside the experimental parameters in Table 5.1. Values for the blazar pair cascade are obtained using Monte-Carlo modelling described in Elyiv et al. [119] and applied for example in Neronov & Vovk [10] and Miniati & Elyiv [104]. The blazar spectral emission is initialized as a power law distribution, $dN_{\gamma}/dE_{\gamma} \propto E_{\gamma}^{-1.8}$ in the range $10^3 \leq E_{\gamma}/m_e c^2 \leq 10^8$, with equivalent isotropic luminosity $10^{45} \text{ erg s}^{-1}$, using a model for the extragalactic background light described by

	Background plasma		Pair beam		
	$n_p [\text{cm}^{-3}]$	ν_e/ω_p	$n_{\pm} [\text{cm}^{-3}]$	$\langle\gamma_{\pm}\rangle$	$\Delta\theta$
Laboratory	10^{12}	10^{-3}	5×10^{10}	10^3	0.025
Blazar jet	2×10^{-7}	10^{-13}	10^{-23}	10^5	10^{-4}

Table 5.1: Pair beam and plasma parameters in the experiment and blazar jet case. The background plasma density (n_p) and collisionality (ratio of electron-neutral/ion collisions to plasma frequency, ν_e/ω_p) are given for the experimental plasma and expected IGM conditions, as well as the pair density ($n_{\pm}$), mean Lorentz factor ($\langle\gamma_{\pm}\rangle$) and transverse momentum spread ($\Delta\theta = \sigma_{p\perp}/p_{\parallel}$) of the experimental pair beam and the modelled pair cascade of a typical blazar (luminosity $10^{45} \text{ erg s}^{-1}$) at a distance 300 Mpc from the source galaxy.

Aharonian [120]. Apart from the density, the experimental pair beam shares many qualitative features of the blazar counterpart, as both are produced in an electromagnetic cascade. In both cases the momentum distributions are relativistic and non-thermal, characterized by power-law distributions, and both have a momentum-dependent angular spread. The density of free electrons in the void is estimated by $n_{p,\text{IGM}} = \Omega_b f_{\text{IGM}} \rho_c / m_p$, where Ω_b is the cosmological baryon density parameter, f_{IGM} is the relative fraction of baryons in the IGM, m_p is the proton rest mass, and $\rho_c = 3H_0^2/8\pi G$ is the critical mass density of the Universe, where H_0 is the Hubble constant and G is the gravitational constant. The IGM is assumed to be fully ionized [121]. The fraction of baryons in the IGM, f_{IGM} , is constrained by recent measurements of the dispersion measure of extragalactic fast radio bursts, estimating present-day values of $f_{\text{IGM}} \approx 0.4 - 0.8$ [122, 123]. A void density $n_{p,\text{IGM}} = 2 \times 10^{-7} \text{ cm}^{-3}$ is obtained by assuming $f_{\text{IGM}} = 0.8$ and using cosmological parameters presented by the Planck collaboration [124]: $\Omega_b h^2 = 0.02237 \pm 0.00015$, where $h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$. Finally, in both the experiment and blazar cases, it is confirmed that the collisionless kinetic description is justified. Specifically, the collision operator can be ignored since the free electron collision frequencies with ions and neutrals are much smaller than the plasma frequency: $\nu_e \sim \frac{\lambda_D}{\lambda_{\text{mfp}}} \omega_p \ll \omega_p$. The ratio of the Debye length to the mean free path of electron collisions with ions (λ_{ei}) and neutrals (λ_{en}) can be approximated:

$$\frac{\lambda_D}{\lambda_{ei}} \sim 10^{-12} \left(\frac{T_e}{\text{eV}} \right)^{-3/2} \left(\frac{n_e}{\text{cm}^{-3}} \right)^{-1/2} \left(\frac{n_i}{\text{cm}^{-3}} \right), \quad (5.20)$$

$$\frac{\lambda_D}{\lambda_{\text{en}}} \sim 10^{-11} \left(\frac{T_e}{\text{eV}} \right)^{1/2} \left(\frac{n_e}{\text{cm}^{-3}} \right)^{-1/2} \left(\frac{n_n}{\text{cm}^{-3}} \right), \quad (5.21)$$

assuming Lennard-Jones potentials for the neutrals. In the laboratory: $\nu_{\text{ei}}/\omega_p \sim 10^{-6}$ and $\nu_{\text{en}}/\omega_p \sim 10^{-3}$, and in the void: $\nu_{\text{ei}}/\omega_p \sim 10^{-16}$ and $\nu_{\text{en}}/\omega_p \sim 10^{-13}$. The linear growth rate for the blazar scaled from the experimental growth rate is

$$\begin{aligned} \Gamma_{\text{blz}} &= \Gamma_{\text{exp}} \left(\frac{n_{\pm, \text{blz}}}{n_{\pm, \text{exp}}} \right)^{2/3} \left(\frac{n_{\text{p}, \text{blz}}}{n_{\text{p}, \text{exp}}} \right)^{-1/6} \left(\frac{\langle \gamma_{\pm, \text{blz}} \rangle}{\langle \gamma_{\pm, \text{exp}} \rangle} \right)^{-2/3} \left(\frac{\Delta \theta_{\text{blz}}}{\Delta \theta_{\text{exp}}} \right)^{-1} \\ &\leq 3 \times 10^{-11} \text{ s}^{-1} \left(\frac{\Gamma_{\text{exp}}}{0.7 \text{ ns}^{-1}} \right), \end{aligned} \quad (5.22)$$

where experimental parameters are subscripted ‘exp’ and blazar jet parameters subscripted ‘blz’. The experimentally scaled linear growth timescale, $\tau_{\text{ins}} \equiv 1/\Gamma_{\text{blz}} \geq 3 \times 10^{10} \text{ s}$, is plotted in Figure 5.16 alongside the theoretical estimates of Eq. 1.28 for a range of blazar jet pair densities. The growth rate is compared with the inverse-Compton cooling time of pairs with CMB photons:

$$\tau_{\text{IC}} \equiv \frac{E_e}{(dE_e/dt)_{\text{IC}}} = \frac{\gamma_{\pm} m_e c^2}{\frac{4}{3} c \sigma_T u_{\text{rad}} \beta^2 \gamma_{\pm}^2} = 3.8 \times 10^{13} \text{ s} \left(\frac{E_e}{1 \text{ TeV}} \right)^{-1}, \quad (5.23)$$

where σ_T is the Thomson cross section, $u_{\text{rad}} = 0.26 \text{ eV cm}^{-3}$ is the CMB energy density and $\beta = (1 - \gamma_{\pm}^{-2})^{1/2}$. Whilst severely suppressed compared to the growth rate assuming collimated, monoenergetic pairs, the scaled growth rate is still very fast compared to the inverse-Compton cooling time of pairs with the mean energy. Such a fast growth rate and tilting of modes towards a quasi-parallel direction to the jet axis has been used to propose

that significant dissipation of pairs' kinetic energy is possible due to advanced development of a quasi-electrostatic mode of the oblique instability [11, 103, 125–127]. However, the oblique electromagnetic beam instability is expected to saturate at $B_{\perp,\text{sat}}$ [mT] $\simeq 5.7 \times 10^{-9} \gamma_{\pm} (\Gamma [\text{s}^{-1}])^2 / \omega_p [\text{s}^{-1}]$ (see Eq. 1.30). Using the experimentally scaled growth rate (Eq. 5.22), the estimated saturated magnetic field amplitude in the blazar case is:

$$B_{\perp,\text{sat}} \lesssim 2 \times 10^{-26} \text{ mT} \left(\frac{\langle \gamma_{\pm,\text{blz}} \rangle}{10^5} \right) \left(\frac{\Gamma_{\text{blz}}}{3 \times 10^{-11} \text{ s}^{-1}} \right)^2 \left(\frac{n_{\text{p,blz}}}{2 \times 10^{-7} \text{ cm}^{-3}} \right)^{-1/2}. \quad (5.24)$$

Present constraints for the lower limit of intergalactic magnetic fields required to explain the lack of observed GeV cascade emission in analysis of Fermi/LAT and MAGIC telescope data imposes a lower bound of $B \gtrsim 1.8 \times 10^{-18} (\lambda_B / 0.2 \text{ Mpc})^{-1/2} \text{ mT}$, for a magnetic field coherence length $\lambda_B < 0.2 \text{ Mpc}$ [128]. Assuming a coherence length $\lambda_B \sim 4 \times 10^{-10} \text{ pc}$ (on the scale of the plasma skin depth in the IGM), leads to a bound $B \gtrsim 4 \times 10^{-11} \text{ mT} \gg B_{\perp,\text{sat}}$, far in excess of the saturated magnetic field amplitude. It is therefore possible to conclude that whilst the growth of electromagnetic beam instability may be sufficiently fast for many orders of magnitude of e -folding length to elapse during the inverse-Compton cooling time of pairs, the instability is expected to saturate at extremely small field strengths, far below current lower bounds of intergalactic magnetic field strengths.

Finally, other instabilities may develop after the oblique electromagnetic beam instability has saturated, such as the electrostatic two-stream instability [25, 26], with the broadened thermal spectra of the saturated oblique instability. In this case, longitudinal electrostatic oscillations will grow with the quasi-linear rate of Eq. 1.31, also plotted in Figure 5.16, though

this expression is derived assuming mono-energetic pairs and the growth timescale may be affected by considering a finite energy spread for the pairs. Though the growth timescale may still be shorter than the inverse-Compton cooling time, the velocity spread of pairs is too large for equipartition-scale coupling of the pair beam's kinetic energy into large-amplitude, resonantly driven plasma waves ($k \cdot \Delta v_{\pm} \gg \Gamma$). It is concluded that beam-plasma instabilities do not significantly affect TeV blazar pair cascades, and hence the present inferred lower bounds on intergalactic magnetic field strength are robust.

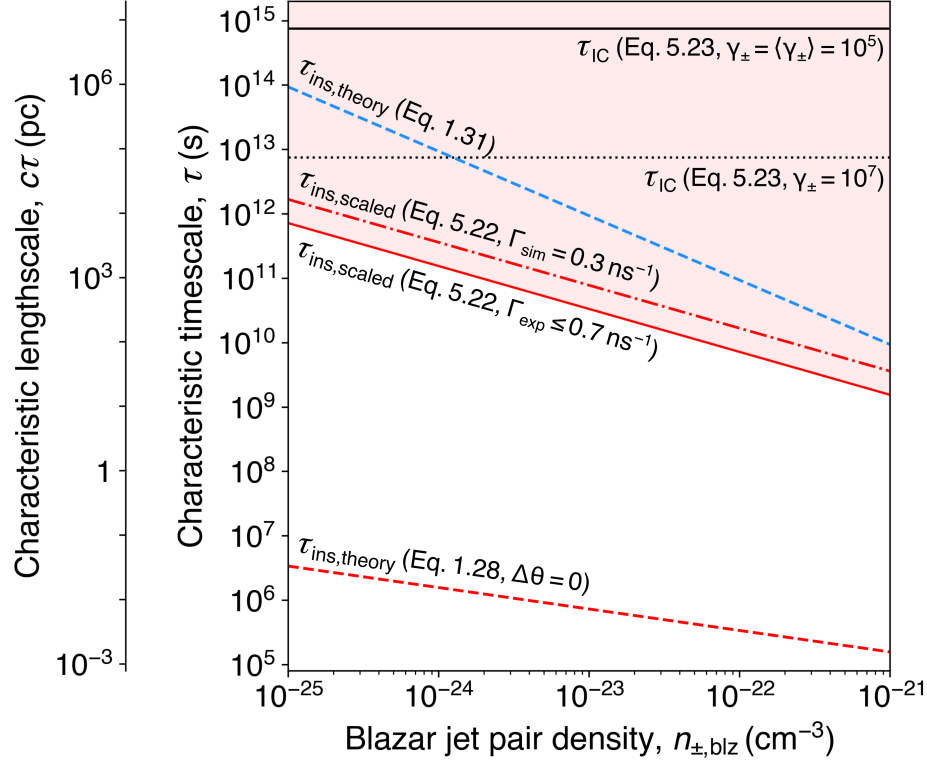


Figure 5.16: Characteristic timescales relevant to blazar-induced pair beams in cosmic voids. The experimentally obtained upper bound of the growth rate of electromagnetic pair beam instability ($\Gamma_{\text{exp}} \leq 0.7 \text{ ns}^{-1}$) is scaled using Eq. 5.22 to obtain a lower bound on the linear growth rate in the case of blazar-induced pair beams propagating through cosmic void (red-solid line and red-shaded region). The exact growth rate observed in 3D-PIC simulations ($\Gamma_{\text{sim}} = 0.3 \text{ ns}^{-1}$) is similarly scaled (red, dot-dashed). The growth timescales are drastically increased from the theoretical expectation assuming a collimated, monoenergetic pair beam (Eq. 1.28, $\Delta\theta = 0$, red dashed), but are still relatively small compared to the inverse-Compton cooling time of pairs with CMB photons (Eq. 5.23) for pairs with the mean energy in the cascade ($\gamma_{\pm} = \langle \gamma_{\pm} \rangle = 10^5$, black solid) and pairs with energy leading to inverse-Compton emission in the GeV range ($\gamma_{\pm} = 10^7$, black dotted). After saturation of electromagnetic beam instability, electrostatic two-stream instability may develop with longitudinal oscillations estimated to grow with the quasi-linear characteristic timescale, plotted assuming a monoenergetic pair beam (Eq. 1.31, blue dashed).

Chapter 6

Summary and closing remarks

In this thesis, a new laboratory astrophysics platform has been developed, in which extremely high yield, quasi-neutral electron-positron pair beams have been produced using ultra-relativistic protons extracted from CERN's Super Proton Synchrotron. The calculations performed to optimize the experimental scheme and develop a practical design have been presented in detail.

During the experiment performed at HiRadMat in May 2023, the beam profile and energy spectra of electron-positron pairs was measured, agreeing with Monte-Carlo simulations and confirming the generation of a beam with size exceeding characteristic scales necessary for collective plasma behaviour. This opens up the future possibility of directly probing the microphysics of relativistic pair plasmas. In addition, a suitable plasma discharge was designed, constructed and characterized to study kinetic instabilities of a quasi-neutral pair beam propagating through an ambient plasma. A Faraday rotation diagnostic was designed and implemented within the practical constraints of the experimental facility. Though the instrument was sensitive to magnetic fields as small as 5 mT, the instability did not generate sufficient magnetic fields or structural features to be detected. The most plausible explanation was that the instability was suppressed due to the finite transverse thermal spread of the pairs, and this was confirmed using three-dimensional particle-in-cell simulations modelling the experimental conditions. As a result, it was experimentally

demonstrated that finite-temperature effects can strongly suppress kinetic beam-plasma instabilities of relativistic electron-positron pair beams. This result is particularly relevant to astrophysical pair beams where pairs can have relatively large thermal spreads. Previously, numerical simulations were the only means to validate such theories, but the possibility to perform controlled laboratory experiments now becomes a reality.

The relevance to the specific case of blazar-induced pair cascades propagating through cosmic voids was investigated, with bounds on the linear growth rate and saturated magnetic field strength of electromagnetic beam instability obtained from the experimental observations. The results show that the growth rate of electromagnetic beam instability may be very fast compared with the inverse-Compton cooling time of pairs with cosmic microwave background photons, but the instability is expected to saturate at magnetic field amplitudes many orders of magnitude lower than bounds on the magnetic field provided by recent astronomical measurements of blazar GeV and TeV spectra. Therefore, beam-plasma instabilities are not expected to affect TeV blazar pair cascades and the inferred lower bounds on intergalactic magnetic field strength are robust.

With more sensitive measurements of blazar spectra in the GeV range it may be possible to resolve the halo of diffused GeV emission and constrain the extent of pair beam relaxation and expansion. But further laboratory investigations of kinetic beam plasma instabilities can also contribute to the discussion, particularly by attempting to validate theoretical models describing how beam instabilities evolve beyond the linear phase of growth. Now that the experimental concept has been demonstrated, higher sensitivity instrumentation

can be implemented, leading to statistically significant measurements of magnetic field and beam filamentation, making it possible to benchmark numerical simulations. Improvements in the Faraday rotation can be achieved by using a higher power laser, by using amplified diodes to improve the signal-to-noise ratio, and by using magneto-optic crystals with a higher Verdet constant [129]. Faster instability growth rates may be achieved by increasing the pair beam density and by reducing the pair beam divergence. In addition, using a longer plasma would allow a larger number of e -folding lengths to develop. Through a combination of these changes, it is expected that pair beam instabilities may be directly measured. It may even be possible to access regimes where the onset of collisionless shocks can be observed.

Meanwhile, efforts to study electron-positron plasma physics in the laboratory continue elsewhere, with electron-positron pairs confined in magnetic traps with ever-greater efficiency [130], and with plans to combine electrons and positrons into ultra-low emittance quasi-neutral pair jets using magnetic chicanes at FACET-II (SLAC) [131]. In addition, several tens-of-petawatt-class laser facilities will be completed in the coming decade [132], where it is expected to become possible to generate avalanche-type cascades of $e^\pm$ pairs and perform fundamental tests of non-perturbative strong-field quantum electrodynamics. The experimental field of electron-positron plasma physics is just beginning, and its future is an exciting one.

Appendices

Appendix A

Particle scattering processes in matter

A.1 The Bethe-Bloch formula

When a fast charged particle moves through matter, it gradually loses energy as it ionizes atoms along its path (ionization losses). For moderately relativistic, charged, heavy particles ($0.1 \lesssim \beta\gamma \lesssim 1000$, $M \gtrsim m_e$, where $\beta = v/c$ and $\gamma = 1/\sqrt{1 - \beta^2}$), ionization losses represent the dominant form of energy loss, and the loss per unit distance (linear stopping power) is well-approximated by the Bethe-Bloch formula [133, 134]:

$$\frac{1}{\rho_0} \left\langle -\frac{dE}{dx} \right\rangle = 2\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\log \left(\frac{2m_e c^2 \beta^2 \gamma^2 W_{\max}}{I^2} \right) - 2\beta^2 - \delta \right], \quad (\text{A.1})$$

where ρ_0 is the material density, Z is the material atomic charge number, A is the atomic mass number, I is the mean excitation energy of the material, z is the charge number of the incident particle, and M is the mass of the incident particle. $W_{\max}$ is the maximum energy that can be transferred from the incident particle to atomic electrons (conserving four-momentum):

$$W_{\max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}. \quad (\text{A.2})$$

The correction factor δ (introduced by Sternheimer [135]) accounts for relativistic extension of the particle's electric field at higher γ , leading to reduced ionization as the medium becomes locally polarized (density effect).

If $M \gg m_e$, then $W_{\max} \approx 2m_e c^2 \beta^2 \gamma^2$, and for a given material (fixed ρ_0 , I , Z , A) the Bethe-Bloch formula becomes a function of β alone, irrespective of incident particle type (assuming identical z), so different particles have approximately the same stopping power.

The main caveats of the Bethe-Bloch formula are:

1. The most probable energy loss is typically substantially different from the mean energy loss energy because the energy loss probability distribution (Landau-Vavilov distribution) is highly skewed by rare events with large single-collision energy losses.
2. While the Bethe-Bloch formula provides a good approximation to stopping power in the range when $M \gg m_e$, for electrons and positrons, bremsstrahlung losses dominate at $\beta\gamma \gtrsim 10$. Electron and positron scattering processes are discussed in Section A.4.
3. At energies where the Born approximation doesn't apply (the interaction between the electron and the nucleus cannot be considered a small perturbation), additional correction terms are required (e.g. 'Shell', 'Barkas', 'Bloch' terms).

The US National Institute of Standards and Technology (NIST) has published online databases [136] for stopping powers and ranges of particles. The databases for charged particles fit the functional form of the Bethe-Bloch formula to experimental data to obtain the model parameters which cannot be calculated accurately from first principles (such as the mean excitation value, I , although approximately $I \sim (10 \text{ eV}) \cdot Z$). Figure A.1 shows the NIST database stopping powers of protons and electrons for a low- Z material (graphite) and high- Z material (lead).

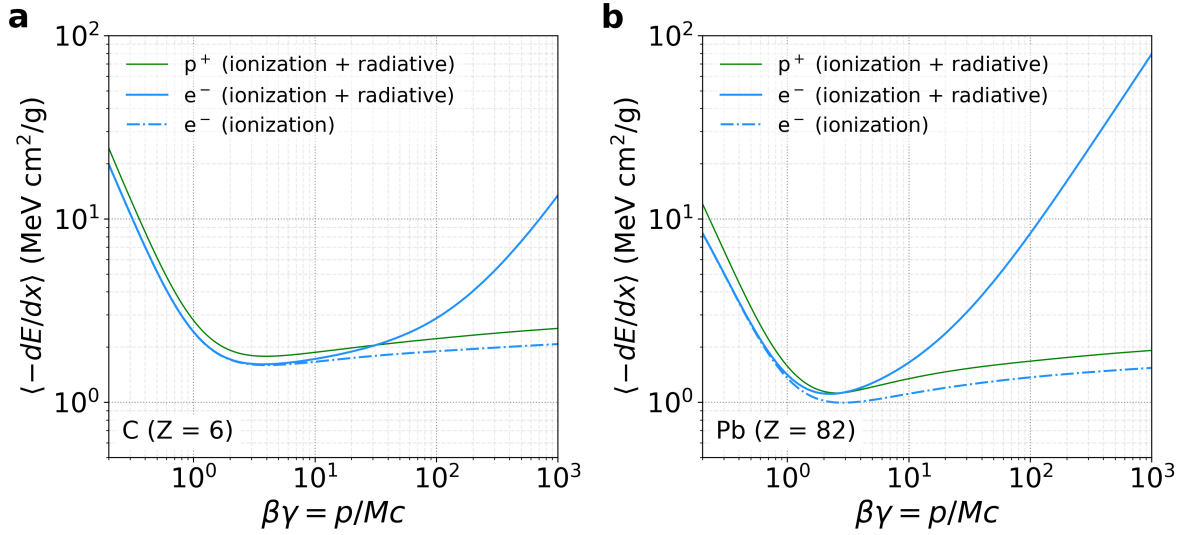


Figure A.1: Bethe-Bloch formula using parameters from the NIST database. The combined stopping power, $\langle -dE/dx \rangle$, is plotted for protons and electrons in graphite ($Z = 6$) and lead ($Z = 82$), using parameters provided by the NIST database [136], where the Bethe-Bloch parameterization is accurate to within a few percent. In lead, electronic radiative losses become important at a few $\beta\gamma$, which is not the case for protons. However, the stopping power still reaches a minimum point at a similar $\beta\gamma$. In low- Z materials (such as for graphite shown), the minimum-ionizing region extends multiple decades in $\beta\gamma$ for both electrons and protons.

Initially the stopping power falls as particle energy increases, but once the particle becomes relativistic, the stopping power reaches a minimum and the logarithmic term takes over. As γ increases, the density-effect correction term increases, whilst $W_{\max}$ increases less sharply due to the increasing denominator. This leads to a broad minimum region which extends until $\beta\gamma \sim 1000$, where radiative bremsstrahlung losses increase the stopping power. Broadly speaking, relativistic particles deposit approximately the same energy per unit distance in a material, regardless of particle type and momentum. This is referred to as the ‘minimum-ionizing’ property.

A.2 Hadronic inelastic nuclear interactions

As well as bremsstrahlung, at high energies, hadronization of quarks and gluons can become a significant contribution to stopping power of hadronic charged particles. Hadronization refers to the formation of hadrons out of quarks and gluons via inelastic hadronic interactions, for example during the hard collision of a sufficiently energetic proton ($\gtrsim$ GeV) with an atomic nucleus. In ‘hadronic showers’ a shower of secondary particles is produced which typically contains a large number of light hadrons, such as pions: $\pi^\pm, \pi^0$ ($m_{\pi^\pm} = 139.6 \text{ MeV}/c^2$, $m_{\pi^0} = 134.5 \text{ MeV}/c^2$), e.g.

$$p^+ + n \rightarrow p^+ + n + \pi^+ + \pi^- + \pi^0.$$

The primary decay mode of charged pions is leptonic decay into a muon and a muon neutrino via the weak interaction (mean lifetime $\tau_{\pi^+ \rightarrow \mu^+ \nu_\mu} = 2.6 \times 10^{-8} \text{ s}$ in the pion rest frame):

$$\pi^+ \rightarrow \mu^+ + \nu_\mu \quad (\text{branching ratio} = 0.9999).$$

Composed of quark-antiquark pairs, neutral pions can decay much faster via the electromagnetic interaction into two photons (mean lifetime $\tau_{\pi^0 \rightarrow \gamma\gamma} = 8.5 \times 10^{-17} \text{ s}$):

$$\pi^0 \rightarrow \gamma + \gamma \quad (\text{branching ratio} = 0.9882).$$

The photons have an energy 67.3 MeV in the π^0 rest frame. If the π^0 is relativistic, then they are also highly collimated. The characteristic lengthscale of hadronic shower development is the nuclear interaction length:

$$\lambda_{\text{I}} = \begin{cases} \text{Mean distance travelled by a hadronic particle before} \\ \text{undergoing an inelastic nuclear interaction.} \end{cases}$$

It's observed in simulations that the number of protons transmitted through a material without experiencing inelastic scattering is well-approximated by $N \approx N_0 \exp(-L/\lambda_{\text{I}})$.

Electrons and positrons are primarily generated through electromagnetic cascades initiated by γ -rays produced in the decay of neutral pions. Though, it is possible to produce electron-positron pairs directly from the decay of a neutral pion (Dalitz decay), albeit with a very small branching ratio:

$$\pi^0 \rightarrow e^+ + e^- \quad (\text{branching ratio} = 0.0117).$$

Relativistic electrons can also be produced from ionization ejection of atomic electrons (δ -rays). For heavy hadrons with $\gamma \gtrsim 10$, the maximum momentum transfer to electrons can exceed 100 MeV/c, though a transfer of this much momentum is statistically rare.

A.3 Photon scattering

A monoenergetic beam of photons with intensity, I_0 , penetrating a layer of material with thickness, x , and density, ρ , is attenuated in intensity by a factor:

$$I/I_0 = \exp[-(\mu/\rho)x], \quad (\text{A.3})$$

where $\mu/\rho = \sigma_{\text{tot}}/uA$ is the ‘mass attenuation coefficient’, u is the unified atomic mass unit, and σ_{tot} is the combined cross section (per atom) of photon scattering processes. Those relevant to photon energies $E_\gamma \lesssim 1 \text{ TeV}$ are:

$$\sigma_{\text{tot}} = \sigma_{\text{ph}} + \sigma_{\text{Comp}} + \sigma_{\text{pair}} + \sigma_{\text{trip}} + \sigma_{\text{pn}}, \quad (\text{A.4})$$

where: σ_{ph} = photoelectric effect, σ_{Comp} = Compton scattering, σ_{pair} = pair production of an e^+e^- pair in the nuclear field, σ_{trip} = pair production of an e^+e^- pair in the electronic field (triplet pair production), σ_{pn} = photonuclear interactions (e.g. nuclear resonances).

Figure A.2 shows the mass attenuation coefficient for tantalum, with the contributions from different scattering processes plotted separately (described in the following sub-sections). The attenuation length, $\lambda_{\text{att}} = 1/\mu$, represents the mean free path of photons, plotted for air, graphite, concrete and tantalum in Figure A.3.

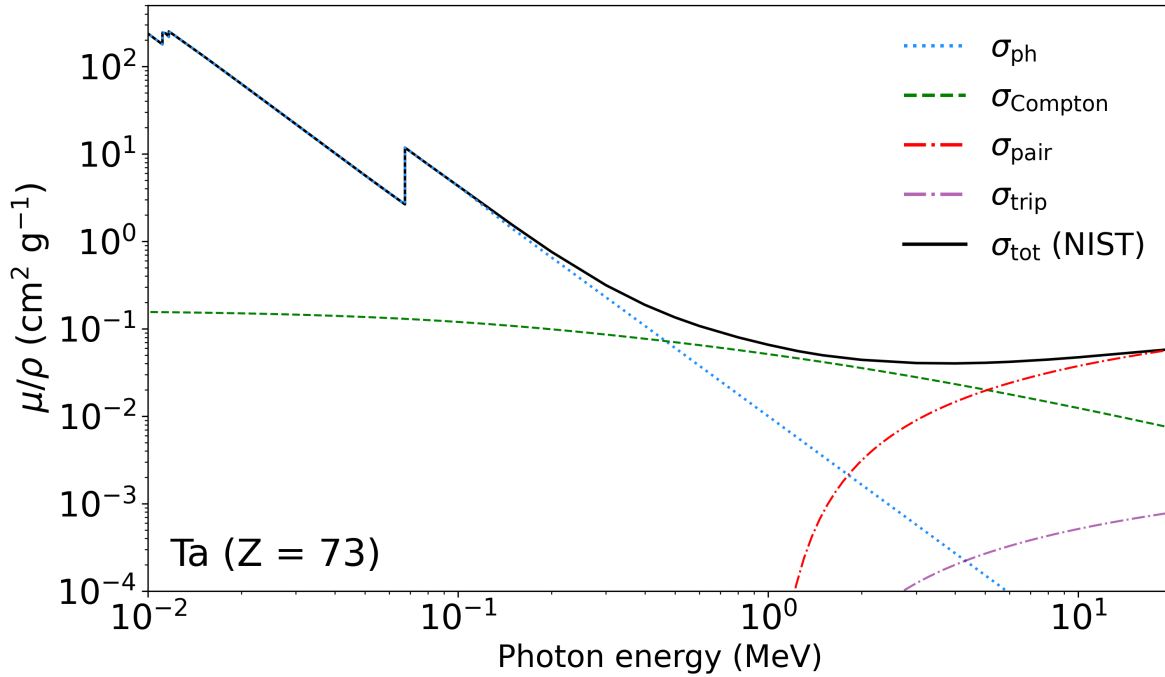


Figure A.2: Photon mass attenuation coefficient for tantalum. The mass attenuation coefficient (μ/ρ) is plotted for tantalum (black), using values provided in the NIST database [137]. The contributions from the cross sections for different photon scattering processes are also plotted: the photoelectric effect (σ_{ph} , blue-dotted), Compton scattering (σ_{Compton} , green-dashed), pair production in the nuclear field (σ_{pair} , red-dot-dashed), and pair production in the electronic field (σ_{trip} , purple-dot-dashed). The latter three are plotted using theoretical expressions for the cross sections presented in Sections A.3.2 and A.3.3.

A.3.1 Photoelectric effect

In the photoelectric effect a photon is absorbed by an atomic electron, which is ejected from the atom if the absorbed energy exceeds the binding energy. It is the dominant process of energy loss for X-rays up to a few 10's of keV (although it can still be dominant at hundreds of keV in high-Z materials).

The photoelectric cross section generally decreases with an increase in photon energy, though discontinuous jumps (absorption edges) are found where the thresholds for photoionization of atomic levels are exceeded. Theoretical treatments of the cross section are

difficult and tedious – the Dirac equation for electrons in bound atomic states must be solved (for instance, see Davisson [138]). The easiest way to obtain photoelectric cross sections at X-ray energies is to find them tabulated in the NIST database [137].

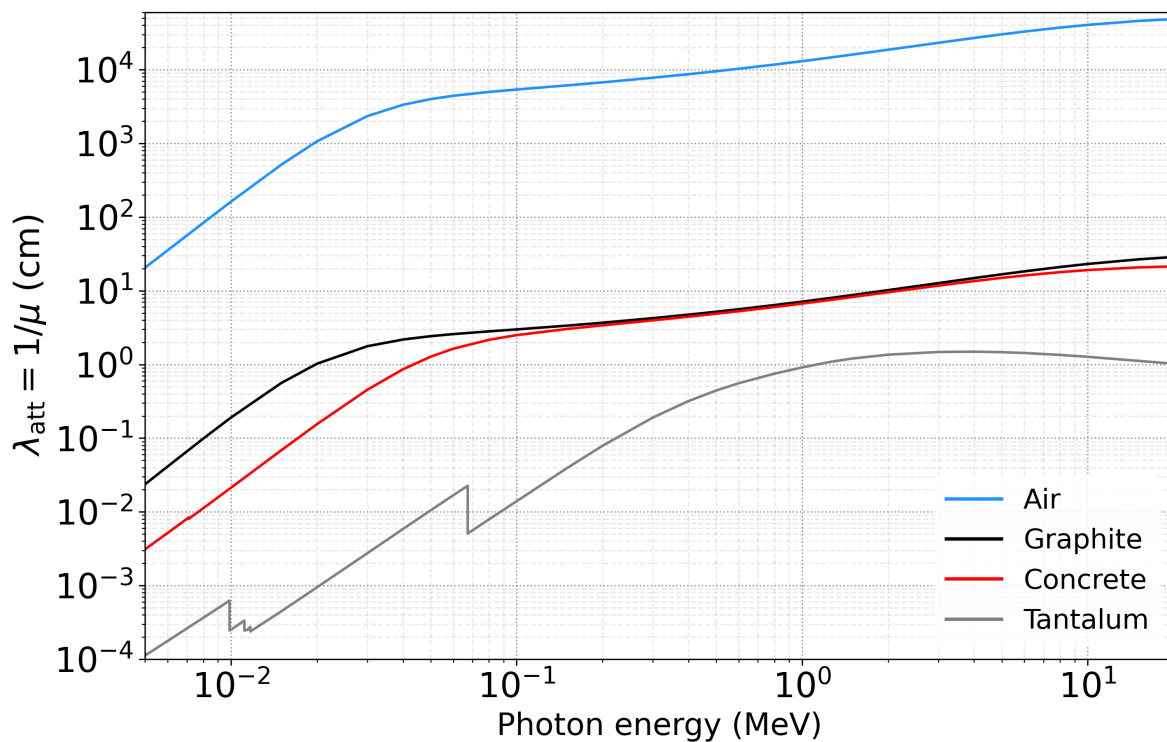


Figure A.3: Photon attenuation length for different materials. The attenuation length, $\lambda_{\text{att}} = 1/\mu$, is plotted for air (blue), graphite (black), concrete (red) and tantalum (gray).

A.3.2 Compton scattering

For photon energies of a few tens to hundreds of keV, Compton scattering dominates. The incident photon is scattered by an atomic electron, which recoils, and the photon energy and direction are changed. The new energy of the photon ($E_{\gamma'}$) is a function of the scattering angle θ :

$$E_{\gamma'} = \frac{E_{\gamma}}{1 + k(1 - \cos \theta)}, \quad (\text{A.5})$$

where $k = E_{\gamma}/m_e c^2$ is the photon energy in units of electron rest mass energy. The differential cross section giving the number of photons scattered per atom, per unit solid angle, in the direction θ is given by the following equation (Klein and Nishina [139]):

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{Comp}} = \frac{1}{2} r_e^2 Z \frac{1}{[1 + k(1 - \cos \theta)]^2} \left[1 + \cos^2 \theta + \frac{k^2(1 - \cos \theta)^2}{1 + k(1 - \cos \theta)} \right]. \quad (\text{A.6})$$

Integrating over all solid angles gives the total Compton cross section:

$$\sigma_{\text{Comp}} = 2\pi r_e^2 Z \left\{ \frac{1+k}{k^2} \left[\frac{2(1+k)}{1+2k} - \frac{1}{k} \log(1+2k) \right] + \frac{1}{2k} \log(1+2k) - \frac{1+3k}{(1+2k)^2} \right\}. \quad (\text{A.7})$$

Evaluating the limits of photon energy, when k is large, $\sigma_{\text{Comp}} \approx Z\pi r_e^2 \log(2k)/k$ and the Compton cross section slowly decreases. When photon energy is small and k tends to zero, the Compton cross section tends to a constant at the value of the Thomson cross section: $\sigma_{\text{Comp}} \approx (8/3)Z\pi r_e^2$.

A.3.3 Pair production

If photons have an energy greater than the rest mass energy of an electron-positron pair ($E_\gamma \geq 2m_e c^2 = 1.022 \text{ MeV}$), they can interact with the Coulomb field of an atomic nucleus and be transformed into an electron-positron pair.

The suggestion that increased photon absorption at high energies could be explained by production of electron-positron pairs has been attributed to Blackett and Occhialini, whilst Oppenheimer and Plesset presented the first theoretical estimates of the cross section [140], and an exact form of the cross section was first derived by Bethe and Heitler (assuming the Born approximation) [53]. Unfortunately, the Born approximation only works well when photon energy is sufficiently small, or sufficiently large ($k \gtrsim 10$), otherwise the interaction between the nucleus and at least one of the produced particles cannot be neglected. Things are further complicated by the fact that the nuclear field can be screened depending on the atomic form factor.

Ignoring the screening of the nuclear field and assuming the Born approximation applies, a simplified expression for the differential cross section for pair production can be derived for an unpolarized photon:

$$d\sigma_{\text{pair}} = 4Z^2 \alpha r_e^2 \frac{dE_+}{k^3} \left[E_+^2 + E_-^2 + \frac{2}{3} E_+ E_- \right] \left[\log \left(\frac{2E_+ E_-}{k} \right) - \frac{1}{2} \right], \quad (\text{A.8})$$

where E_+ , E_- are the total positron and electron energies in units of electron mass energy.

Integration over E_+ from 1 to k gives the total cross section:

$$\sigma_{\text{pair}} = \frac{28}{9} Z^2 \alpha r_e^2 \left(\log 2k - \frac{109}{42} + f(k, Z) \right), \quad f(k, Z) = \sum_{n=1}^{\infty} \left(\frac{2}{k} \right)^{2n} f_n(k, Z), \quad (\text{A.9})$$

where $f(k, Z)$ represents higher order terms in powers of $(2/k)^2$ which must be included when $k \rightarrow 2$ [57, 58]. Without the higher order terms, the expression is a good approximation of σ_{pair} for $k \gtrsim 20$ ($E_\gamma \gtrsim 10$ MeV).

As photon energy increases, pair production cross section increases logarithmically. The Z^2 dependence (compared with the Z dependence of the Compton cross section) explains why pair production begins to dominate at lower photon energies in materials with high- Z .

The cross section for the ‘triplet process’ (in which photons interact with the Coulomb field of atomic electrons to produce pairs) has a similar expression except it is smaller by a factor of Z due to atomic electrons being singly charged.

A.4 Electron and positron scattering

A.4.1 Møller and Bhabha scattering

Similarly with heavy charged particles, ionization losses dominate the energy loss of electrons and positrons propagating through matter at low energies ($\lesssim$ MeV). Assuming $M = m_e$ in the equation for maximum energy transfer: $W_{\text{max}} = m_e c^2 (\gamma - 1)$. This is the case for positrons, and the logarithmic term of the theoretical stopping power takes the same form as the Bethe-Bloch equation. For electrons, indistinguishability means the maximum energy transfer is accounted for as $W_{\text{max}}/2$. The proper treatment of electrons

and positrons is more complicated than for heavy charged particles as it can no longer be assumed that the projectile represents a perturbing potential moving with constant speed. Spin, kinematics and particle identity all have to be accounted for by additional terms.

The stopping power for electrons calculated from the Møller cross section:

$$\frac{1}{\rho_0} \left\langle -\frac{dE}{dx} \right\rangle = 2\pi N_A r_e^2 m_e c^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\log \left(\frac{m_e c^2 \beta^2 \gamma^2 \{m_e c^2 (\gamma - 1)/2\}}{I^2} \right) \right. \quad (\text{A.10})$$

$$\left. - \frac{2\gamma - 1}{\gamma^2} \log 2 + (1 - \beta^2) + \frac{1}{8} \left(\frac{\gamma - 1}{\gamma} \right)^2 - \delta \right], \quad (\text{A.11})$$

(given here is the first moment). Similarly for positrons, the Bhabha cross section:

$$\frac{1}{\rho_0} \left\langle -\frac{dE}{dx} \right\rangle = 2\pi N_A r_e^2 m_e c^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\log \left(\frac{2m_e c^2 \beta^2 \gamma^2 \{m_e c^2 (\gamma - 1)\}}{I^2} \right) \right. \quad (\text{A.12})$$

$$\left. - \frac{\beta^2}{12} \left(23 + \frac{14}{\gamma + 1} + \frac{10}{(\gamma + 1)^2} + \frac{4}{(\gamma + 1)^3} \right) - \delta \right]. \quad (\text{A.13})$$

A.4.2 Positron annihilation

The total cross section for annihilation of a positron with an atomic electron was first given by Dirac [59]:

$$\sigma_{\text{ann}} = \pi r_e^2 \frac{1}{\gamma + 1} \left[\frac{\gamma^2 + 4\gamma + 1}{\gamma^2 - 1} \log \left(\gamma + \sqrt{\gamma^2 - 1} \right) - \frac{\gamma + 3}{\sqrt{\gamma^2 - 1}} \right]. \quad (\text{A.14})$$

The mean free path for positron annihilation is given by:

$$\frac{1}{\lambda_{\text{ann}}} = \rho_0 \left(\frac{N_A Z}{A} \right) \sigma_{\text{ann}}. \quad (\text{A.15})$$

which is plotted for different materials in Figure A.4. The linear stopping power is simply

$\langle -dE/dx \rangle = \gamma m_e c^2 / \lambda_{\text{ann}}$, as all the positron energy is deposited in a single event.

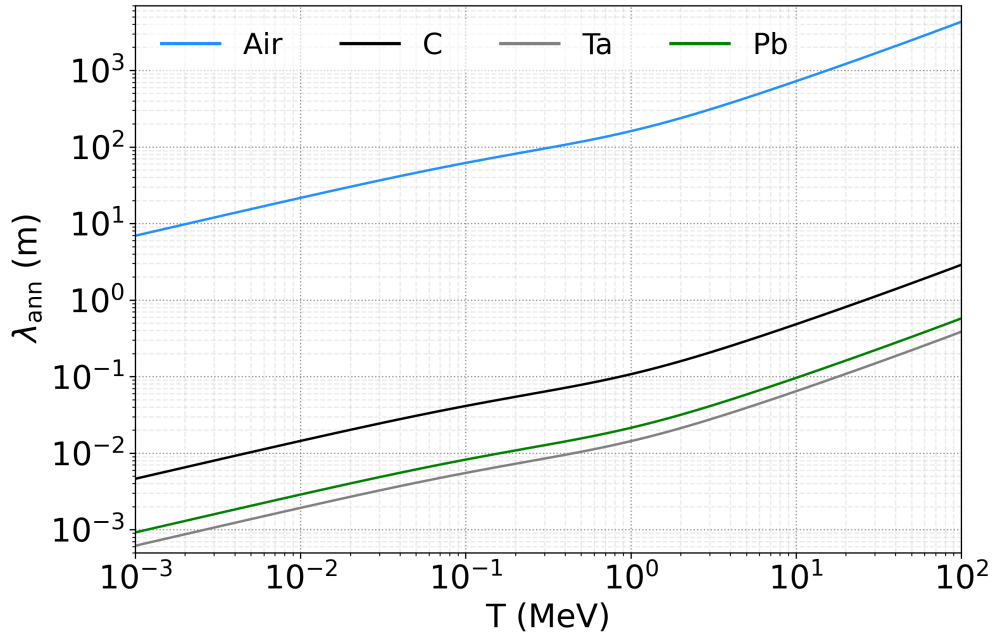


Figure A.4: Annihilation mean free path of positrons in different materials. The annihilation mean free path, λ_{ann} , is plotted as a function of positron kinetic energy, T , in air, graphite, tantalum and lead.

A.4.3 Bremsstrahlung

Bremsstrahlung (‘braking radiation’) is used to describe the emission of radiation by the deceleration of a charged particle when deflected by another charged particle. The theoretical treatment shares much with pair production from photons in a Coulomb potential, as both interaction vertices involve two leptons and a single photon. Even the cross section formulae are similar:

$$d\sigma_{\text{brem}} = \begin{cases} \frac{4Z^2\alpha r_e^2}{E_0^2} \frac{dk}{k} \left\{ \left[E^2 + E_0^2 + \frac{2}{3}EE_0 \right] \log(183Z^{-1/3}) + \frac{1}{9}EE_0 \right\}, & \text{complete screening,} \\ \frac{4Z^2\alpha r_e^2}{E_0^2} \frac{dk}{k} \left[E^2 + E_0^2 + \frac{2}{3}EE_0 \right] \left[\log\left(\frac{2EE_0}{k}\right) - \frac{1}{2} \right], & \text{unscreened,} \end{cases} \quad (\text{A.16})$$

where E_0, E are the initial and final total energy of the electron, and k is the energy of the emitted photon (all in $m_e c^2$ units). To arrive at these expressions, the Born approximation is assumed. Just like with the treatment of pair production, the true cross section depends on the validity of the Born approximation and the precise screening of the nuclear field. Expressions which include correction terms in the ‘intermediate screening’ case, and when the Born approximation is not valid are given by Koch and Motz [141].

Of the two expressions above, the complete screening case leads to the best approximation of the bremsstrahlung cross section at high energies where radiative losses dominate the electron stopping power.

To obtain the linear stopping power, the number of atoms per unit distance is multiplied by the average energy loss of an electron per scattering event:

$$\frac{1}{\rho_0} \left\langle -\frac{dE}{dx} \right\rangle = \frac{N_A}{A} \int_0^{E_0-1} k \left(\frac{d\sigma}{dk} \right)_{\text{brem}} dk \quad (\text{A.17})$$

$$= 4Z^2 \alpha r_e^2 \frac{N_A}{A} (E_0 - 1) \left[\log(183Z^{-1/3}) + \frac{1}{18} \right]. \quad (\text{A.18})$$

The result is identical for electrons and positrons under these approximations – an electron loses energy by bremsstrahlung at a rate approximately proportional to its energy. An approximate expression for the bremsstrahlung stopping power is:

$$\frac{1}{\rho_0} \left\langle -\frac{dE}{dx} \right\rangle_{\text{brem}} \approx \frac{E}{X_0} \quad (\text{A.19})$$

and a characteristic length scale is defined for the bremsstrahlung energy loss, the ‘radiation length’ (X_0):

$$X_0 = \begin{cases} \text{Mean distance over which a high-energy electron} \\ \text{loses all but } 1/e \text{ of its energy via bremsstrahlung.} \end{cases}$$

X_0 (often given in units g/cm^2) is smaller for high-Z materials. Mass density is commonly absorbed and X_0 given as an actual length. Dense, high-Z materials have the highest radiative losses. Values for X_0 are provided online by the Particle Data Group but can also be calculated using empirical formulae [142], plotted in Figure A.5.

The contribution of bremsstrahlung to stopping power of electrons and positrons is shown for tantalum in Figure A.6, relative to the stopping power due to ionization (Møller and

Bhabha scattering) and e^+ annihilation. At relativistic energies, bremsstrahlung dominates the stopping power because the ionization losses increase only logarithmically with the electron/positron energy. At energies $\lesssim 10$ MeV the combined stopping power of Bhabha and e^+ annihilation means that the stopping power is greater for positrons than electrons.

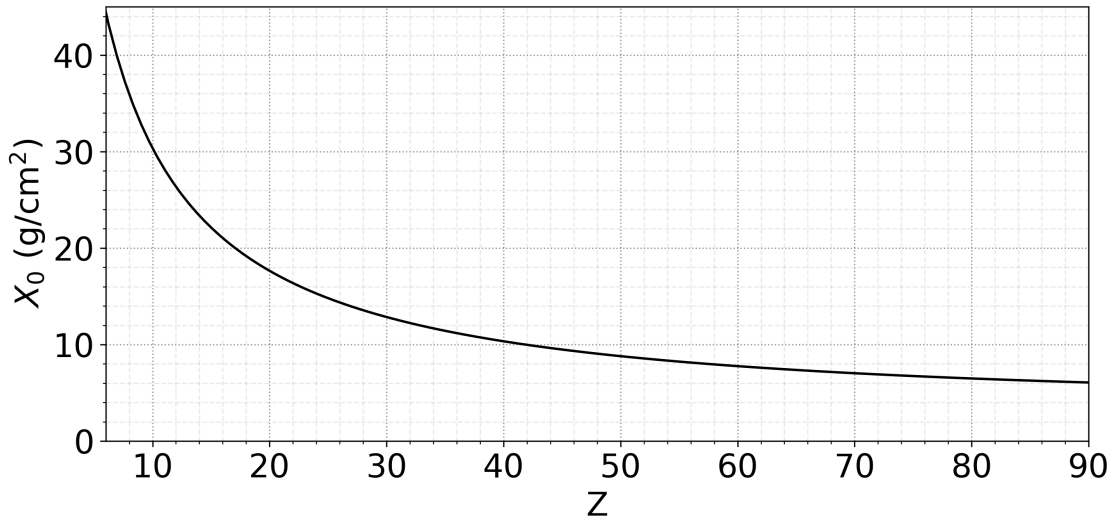


Figure A.5: Radiation length as a function of atomic charge number. The radiation length, X_0 , given here in units g/cm^2 , is plotted as function of atomic number, Z , using the empirical formula given by Tsai [142], and assuming a semi-empirical mass formulation of atomic mass as a function of Z . Dividing by mass density, ρ_0 , gives the characteristic scale length of electron or positron bremsstrahlung losses, e.g. for graphite ($\rho_0 = 1.80 \text{ g cm}^{-3}$), $X_0 = 23.7 \text{ cm}$, and for tantalum ($\rho_0 = 16.7 \text{ g cm}^{-3}$), $X_0 = 0.41 \text{ cm}$.

A.5 Bethe-Heitler and Trident pair production

The Bethe-Heitler refers to the two-step process of producing an electron-positron pair from a single electron or positron: (i) the electron/positron produces a photon via bremsstrahlung with energy exceeding $2m_e c^2$, (ii) the photon converts into a pair. An additional way in which electrons and positrons can produce pairs is by the Trident process, in which the real bremsstrahlung photon of the Bethe-Heitler process is replaced by a virtual photon.

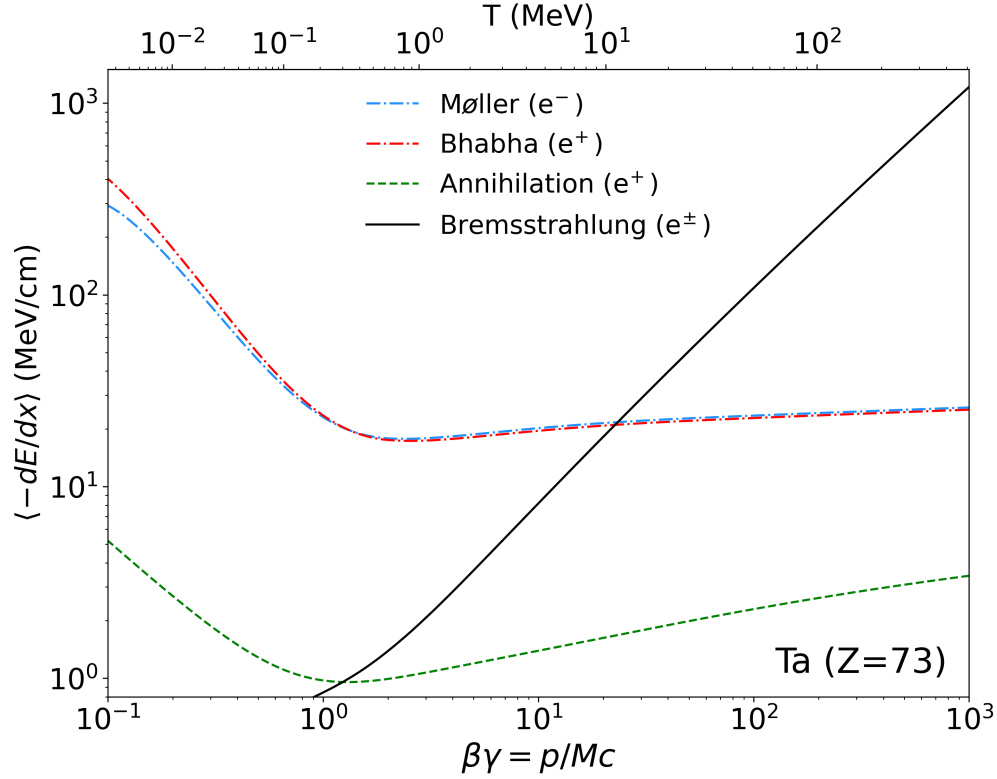


Figure A.6: Contributions to the linear stopping power of electrons and positrons in tantalum. The linear stopping power for ionization losses (Møller and Bhabha scattering), e^+ annihilation, and bremsstrahlung are plotted for tantalum ($Z = 73$).

The cross section of the Trident process can be calculated in the limit where the Born approximation is valid. For example, for sufficiently high energies of electrons and positrons:

$$\sigma_{\text{Tri}} = \frac{28\pi(\alpha r_e Z)^2}{27} [\log(\gamma - 1)]^3. \quad (\text{A.20})$$

However, it is found that the Bethe-Heitler process always dominates the Trident process, unless the thickness of matter is so much thinner than a radiation length that bremsstrahlung photons cannot be produced in large numbers [143]. In a high- Z material multiple radiation lengths thick, an electromagnetic cascade of pairs and photons can develop.

A.6 Coulomb scattering

As well as the inelastic scattering processes described above, charged particles moving through matter are subject to Coulomb scattering – small-angle, elastic deflections described by the Rutherford cross section. Multiple scattering through small angles introduces a beam divergence with Gaussian angular distribution according to the central limit theorem. For the experimentally produced electron-positron pair beams described in this thesis, Coulomb scattering in the high-Z converter material is the leading cause of beam divergence.

The full theory of cumulative deflections is described by Molière [144], but an approximate formula for the standard deviation of the half-angle is given by Lynch & Dahl [60]:

$$\sigma_\theta \text{ [rad]} = 26.6 \frac{z}{\beta^2 \gamma M} \sqrt{\frac{x}{X_0}} \left[1 + 0.038 \log \left(\frac{x z^2}{X_0 \beta^2} \right) \right], \quad (\text{A.21})$$

where z is the charge of the incident particle in units e , M is the mass of the incident particle in units m_e , and x is the thickness of the scattering material. The approximation works well when the fractional energy loss of the particle is small, and when $\sigma_\theta \lesssim 1$ rad. This formula is used to confirm that elements of the experiment placed in the path of electrons and positrons does not lead to excessive additional angular divergence.

Appendix B

Radiological Assessment of the HRMT-62 Experiment

B.1 FLUKA Monte-Carlo simulations

Monte-Carlo simulations have been performed using the code FLUKA to assess the radiation background environment of the experiment performed at HiRadMat in May 2023 (denoted ‘HRMT-62’). Calculation of prompt dose and hadron fluence is important to identify areas where survival of experimental instrumentation (such as cameras) can be ensured. Calculations of the residual dose rate are made by enabling activation of radionuclides in the FLUKA input deck, accounting for processes of evaporation, fragmentation and coalescence.

A simplified geometry of the experiment is used which includes all the material compositions and trace impurities of the materials directly in the beam path (Figure B.1). It is not practically possible to account for all pathways for activation, and the actual dose rates will need to be assessed during commissioning of the experiment. FLUKA simulations are used as guidance and to identify likely regions of high residual dose.

The scoring card `USRBIN → DOSE` is used to estimate the prompt dose, and `USRBIN → HEHAD-EQ` is used to estimate the weighted high-energy hadron equivalent fluence. To record the residual dose equivalent, the cards `PHYSICS → EVAPORAT`, `PHYSICS → COALESCENCE` and `RADDECAY` are activated.

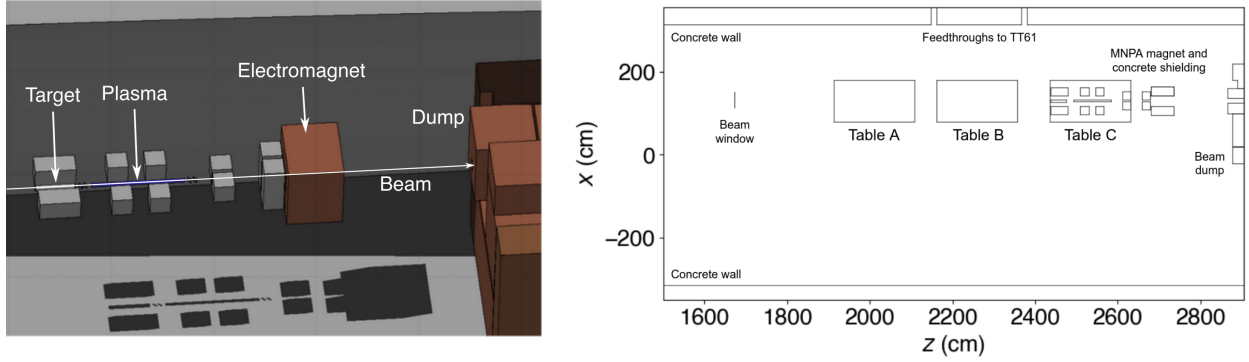


Figure B.1: FLUKA geometry of the HRMT-62 experiment. Left: 3D render of Table C setup and beam dump. Right: Above view of the experimental area.

B.2 Prompt dose and hadron fluence

The digital cameras used (Basler ACA1920-40gm) have a single-event effective cross-section of $9.5 \times 10^{-10} \text{ cm}^2$, which means that a camera will (statistically) require a camera power cycle to restart it if prompt high-energy hadron fluence exceeds $1.05 \times 10^9 \text{ cm}^{-2}$. A ‘reasonably safe’ operational limit is $< 10^8 \text{ cm}^{-2}$. The expected cumulative ionizing dose until a camera is damaged beyond repair is 100 Gy, so putting the cameras in a position where expected dose-per-extraction is $< 10^{-2} \text{ Gy}$ allows for 10,000 extractions.

Cameras are positioned in the horizontal plane of the beam and 55 cm above the horizontal plane of the beam. Figures B.2 and B.3 show the prompt dose and hadron fluence in the beam plane, and 55 cm above the beam plane. The proposed positions of cameras are marked by red boxes. In these locations the prompt doses are $< 10^{-2} \text{ Gy}$ and high-energy hadron fluences are approximately 10^7 cm^{-2} .

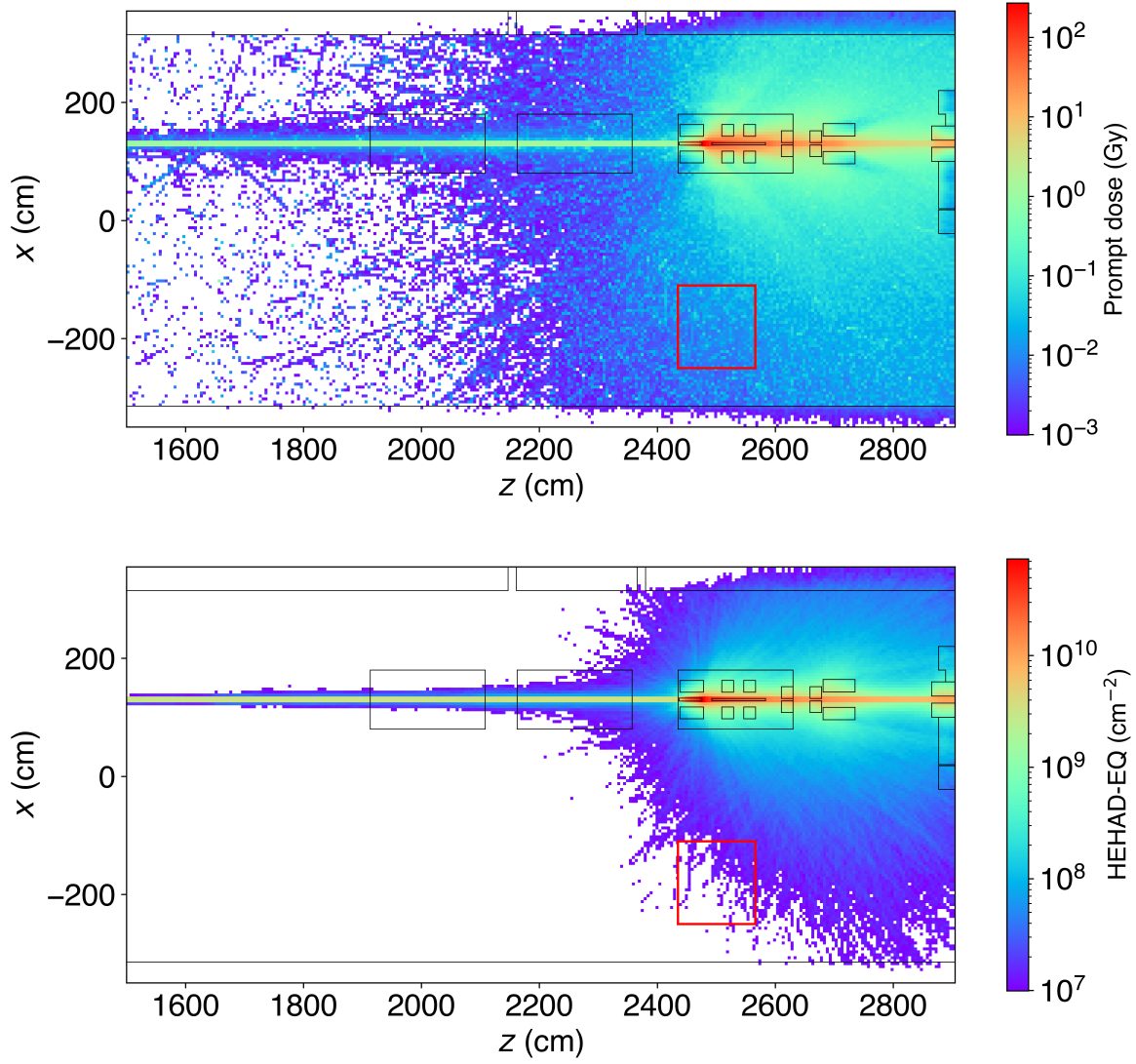


Figure B.2: The prompt dose and high-energy hadron fluence in the beam plane. The prompt dose (top) and high-energy hadron fluence (bottom) in TNC for a single-bunch extraction containing 3×10^{11} protons. The values are averaged over a region which extends 5 cm above and below the beam plane. Camera positioning is marked by a red box.

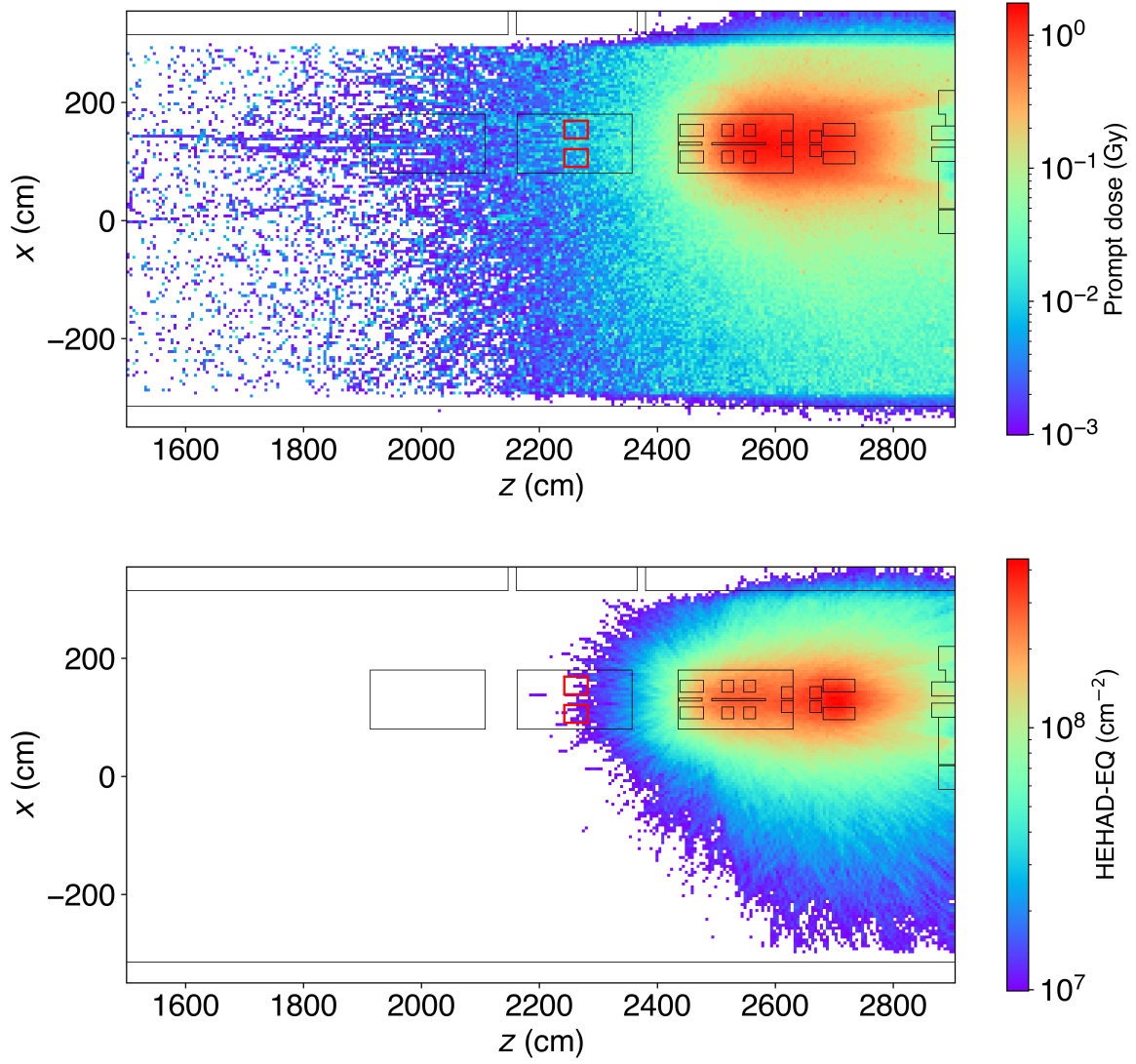


Figure B.3: The prompt dose and high-energy hadron fluence 55 cm above the beam plane. The prompt dose (top) and high-energy hadron fluence (bottom) in TNC for a single-bunch extraction containing 3×10^{11} protons. The values are averaged over a region which extends 45-65 cm above the beam plane. Camera positioning is marked by the red boxes.

B.3 Residual dose equivalent

Calculation of the residual dose equivalent is needed to evaluate the safety of accessing TNC and TT61 during and after the experiment. It is expected that some materials will become activated as the beam is delivered into TNC, particularly the high-Z tantalum which is directly irradiated by the primary beam. Activated material will accumulate over successive beam extractions, but after irradiation the radioactivity will cool down. To assess the safety of accessing TNC and TT61 after a period of cool down for different amounts of consumed beam quota, FLUKA simulations are performed with irradiation patterns containing multiple extractions separated by two-minute intervals.

Equivalent doses after cooling times of 1 hr, 2 hrs, 6 hrs and 1 day are investigated for the cases of (i) single-bunch, (ii) $10\times$ single-bunch extractions separated by two-minute intervals, and (iii) $50\times$ single-bunch extractions separated by two-minute intervals. A sequence of extractions is defined using IRRPROFI and decay times for different cool-down periods using DCYTIMES. In the simulations 10,000 primaries are run. The spatial distributions of residual dose equivalent are shown in Figures B.4, B.5 and B.6.

To evaluate the residual equivalent dose present after the entire beam quota has been consumed, cooling times of 1 week and 4 weeks are investigated, assuming the worst case (from a radioprotection point of view) that there are 7 consecutive days containing $50\times$ single-bunch extractions separated by two-minute intervals. The residual dose equivalent in these cases is shown in Figure B.7, where the values are averaged over a region which extends 5 cm above and below the beam plane.

Red crosses mark two positions on Table C chosen for a quantitative analysis of the residual dose equivalent. The first marks a position near to the target (20 cm from the tantalum), and the second marks a point on the edge of the table where the dose is high compared with the rest of the table edge. The residual equivalent dose is plotted against cooling time in Figure B.8. The tantalum target is identified as a hot spot of residual radiation dose. After several weeks of cool down, the tantalum poses a radiation hazard with dose equivalent exceeding 10's of $\mu\text{Sv/h}$ for direct contact.

At 20 cm from the tantalum the residual dose equivalent never exceeds a few mSv/h even after a cooling time as short as 1 hr and for the worst case of tens to hundreds of extractions. At the edge of the table the residual dose equivalent never exceeds a few tens of $\mu\text{Sv/h}$.

B.4 Summary

From these simulations it has been identified that the diagnostic cameras can be placed at Tables A and B with confidence that they will survive the experiment. Access to the TNC will pose a radiation hazard during beam operation, but the residual dose equivalent is not expected to exceed mSv/h unless closer than 20 cm to the target. In these simulations a simplified geometry was, including the material placed directly in the beam path. To assess the true radiation hazard, a radiation officer should use a dosimeter to check the radiation hazard after a cooling time of 1-2 hours after a single-bunch extraction.

In the case that the true residual dose equivalent is higher than simulations predict, the experiment is designed so access to TNC shouldn't be necessary once a significant fraction

of the beam quota has been consumed. Access to TT61 is not expected to pose a significant radiation hazard.

The simulations give an approximate idea of the residual dose equivalent present after several weeks of cooling time. After the experiment has been completed, Table C and the MNPA electromagnet will need to be removed from TNC within 3 weeks to accommodate the installation of the next scheduled experiment at HiRadMat. These simulations suggest that the radioactivity should be low enough for handling the table at the edges, but additional controls may be required if contact handling of the targets is necessary. The targets will need to be disposed of as radioactive waste.

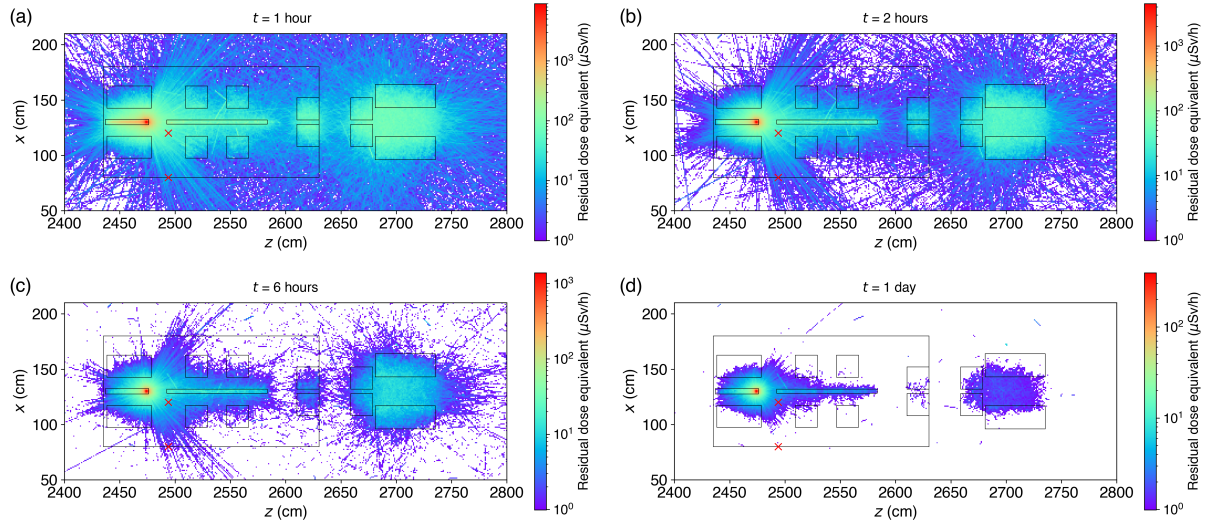


Figure B.4: Residual dose equivalent following a single-bunch extraction. The residual dose equivalent in TNC for cool down periods of (a) 1 hour, (b) 2 hours, (c) 6 hours, and (d) 1 day. A quantitative analysis is performed at the positions marked by the red crosses (shown in figure B.8).

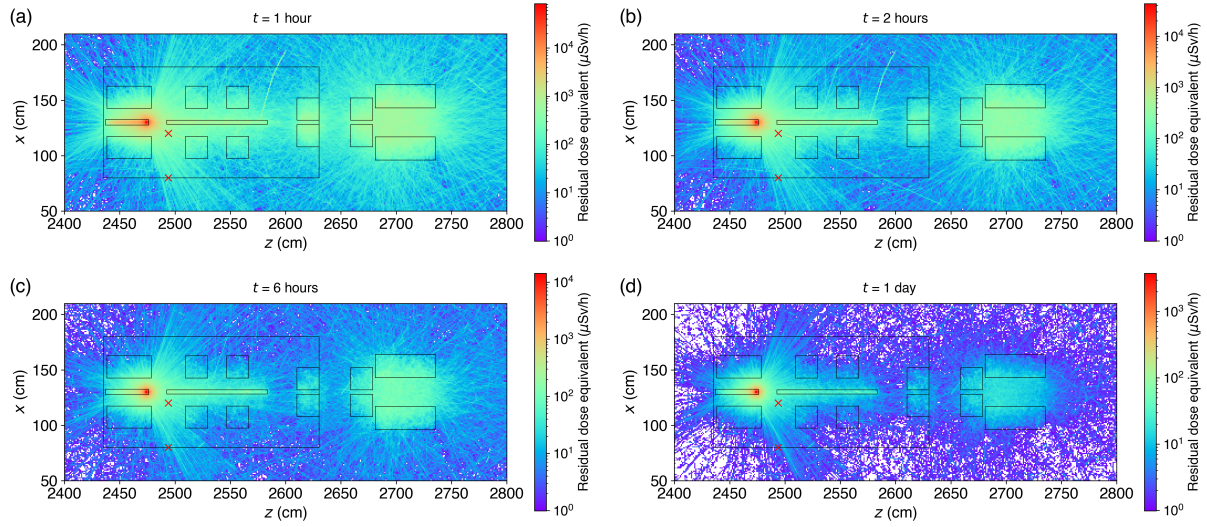


Figure B.5: Residual dose equivalent following 10 single-bunch extractions. The residual dose equivalent at Table C for cool down periods of (a) 1 hour, (b) 2 hours, (c) 6 hours, and (d) 1 day, following $10\times$ single-bunch extractions of separated by two-minute intervals.

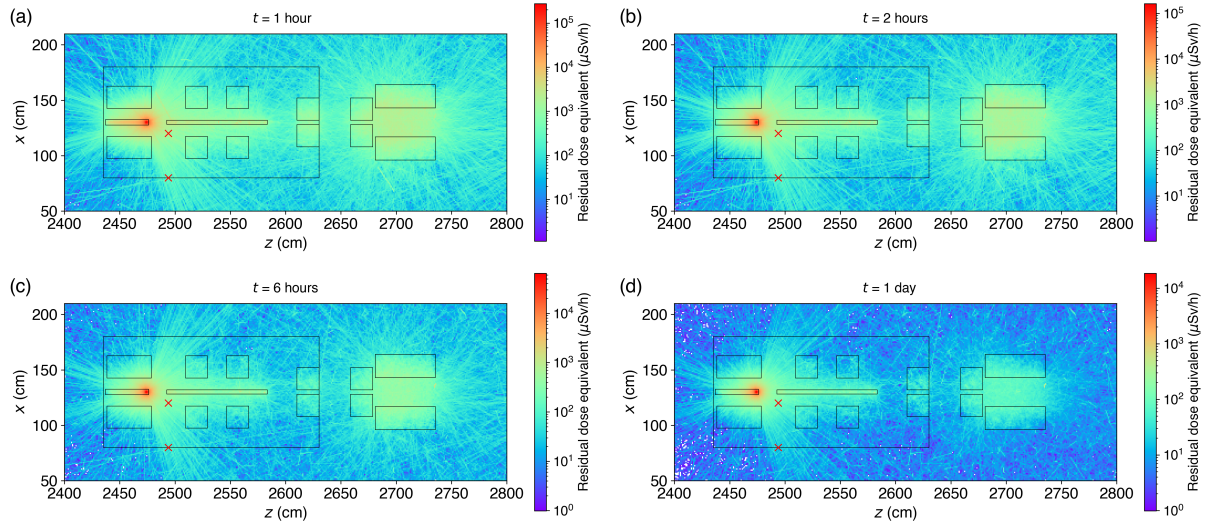


Figure B.6: Residual dose equivalent following 50 single-bunch extractions. The residual dose equivalent at Table C for cool down periods of (a) 1 hour, (b) 2 hours, (c) 6 hours, and (d) 1 day, following $50\times$ single-bunch extractions of separated by two-minute intervals.

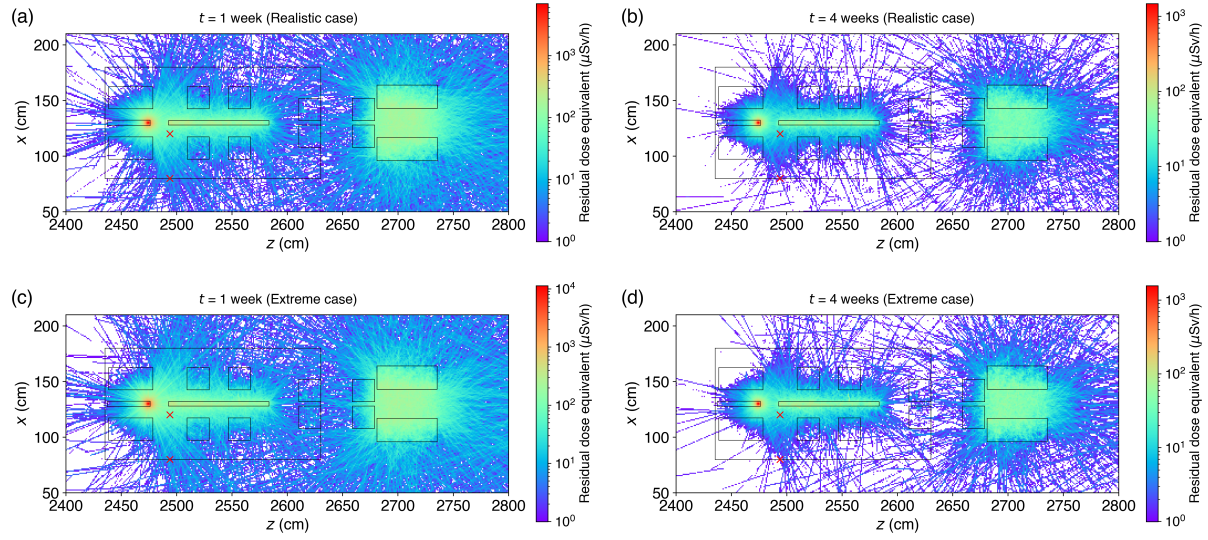


Figure B.7: Residual dose equivalent following 350 single-bunch extractions. The residual dose equivalent at Table C for cool down periods of (a) 1 week, and (b) 4 weeks, following 7 consecutive days containing $50\times$ single-bunch extractions separated by two-minute intervals. Plots (c) and (d) show the residual dose equivalent in the most-extreme case that all 350 extractions occur at the very end of the beam time.

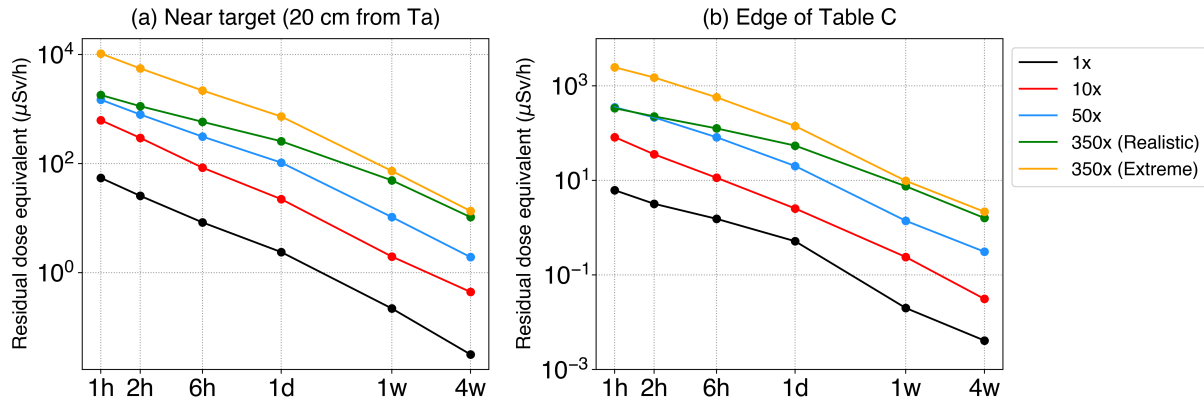


Figure B.8: Residual dose equivalent plotted against time for different extraction patterns. (a) At a position 20 cm from the target. (b) At the edge of the table.

Appendix C

The catastrophic melting of coaxial cables

In principle, plasmas generated by inductive discharge are sustained continuously as long as the RF power is supplied. In practice, the running time is limited by the ohmic heating in the cladding of the coaxial cable. Heating is localized to antinodes of the RF standing wave which forms in the cable from the sum of the forward wave and the wave reflected from the impedance mismatch between the cable and the inductive coils. The polyethylene cladding has a melting point of approximately $110 - 130^\circ\text{C}$, though it may soften at a lower temperature. Drifting of an insufficiently rigid core can lead to shorting between the core and the coaxial sheath, abruptly changing the impedance and leading to the complete reflection of kilowatts of power back towards the matching network and power supply.

Ideally, the cable is short enough that there are no standing wave antinodes in the cable, however the wavelength of a 13.56 MHz RF wave travelling at $0.82c$ in a coaxial cable is $\lambda = 18.1\text{ m}$. The cable going from the matching network in TT61 to the coils in TNC needs to be $> 7\text{ m}$, so it's likely an antinode will form. The amount of power coupled into the plasma is decreased as the number of antinodes increases because the ohmic losses are larger (shown in Figure C.1).

Using a standard RG213 commercial coaxial cable, the time taken for the cable to fail at peak forward power is approximately one minute. Alternative cables were tested to extend the time the discharge can be run without failure. The tests involved running the discharge

Cable type	RG213	C-50-6-1 (CC50)	C-50-11-1 (CK50)
Inner conductor	7 x 0.75 mm copper	7 x 0.75 mm copper	1 x 4.2 mm copper
Dielectric	Polyethylene (7.25 mm)	Polyethylene (6.10 mm)	Polyethylene (11.5 mm)
Outer conductor (shield)	192 x 0.18 mm copper braid	Al-PETP-Al foil + 144 x 0.18 mm tinned copper braid	Al-PETP-Al foil + 192 x 0.18 mm tinned copper braid
Characteristic impedance (Ω)	50	50	50
Velocity of propagation (/c)	0.66	0.82	0.82

Table C.1: Cable properties of the coaxial cables tested to failure.

in a high-current mode and timing until cable/connector shorting, or when the impedance matching became unstable. The cables tested are described in Table C.1, and the results of the tests are shown in Figure C.2. The most successful cable is the cable labelled ‘CK50’, which is the only cable that has a single solid copper core. It also has thicker dielectric cladding. The rigidity of the core makes it harder for the core to drift when the cladding is heated and softens. In failures of CK50, which occurred after 5 mins of operation, damage at the cable connection to the matching network was responsible, rather than damage of the cable itself (this was the cable connection closest to an antinode).

In the final setup, the shape of the RF standing wave was measured by moving a current probe (Pearson 4100 Current Transformer) along its 13.1 m length with a fixed forward power ($P_f = 30$ W). See Figure C.3. The data can be fitted well to a sinusoid. As an additional precaution a thermocouple is attached to the cable at an antinode position and a hardware interlock cuts power to the generator if the temperature rises by 30 °C.

At high forward powers, the range of sensitivity of the current probe ($I_{\text{rms,max}} = 5$ A) will be exceeded if it is placed near an antinode. Positions where the current can be measured without saturating the probe are calculated (given rms current scales with $P_f^{1/2}$ and the

maximum forward power is 1 kW). The probes can be positioned where $I_{\text{rms}} < \sqrt{30/1000} \cdot (5 \text{ A}) = 1.7 \text{ A}$ when $P_f = 30 \text{ W}$ is applied (region identified in green in Figure C.3, with lower cutoff provided so that at least 1/4 of the dynamic range of the current probe is utilized at full power).

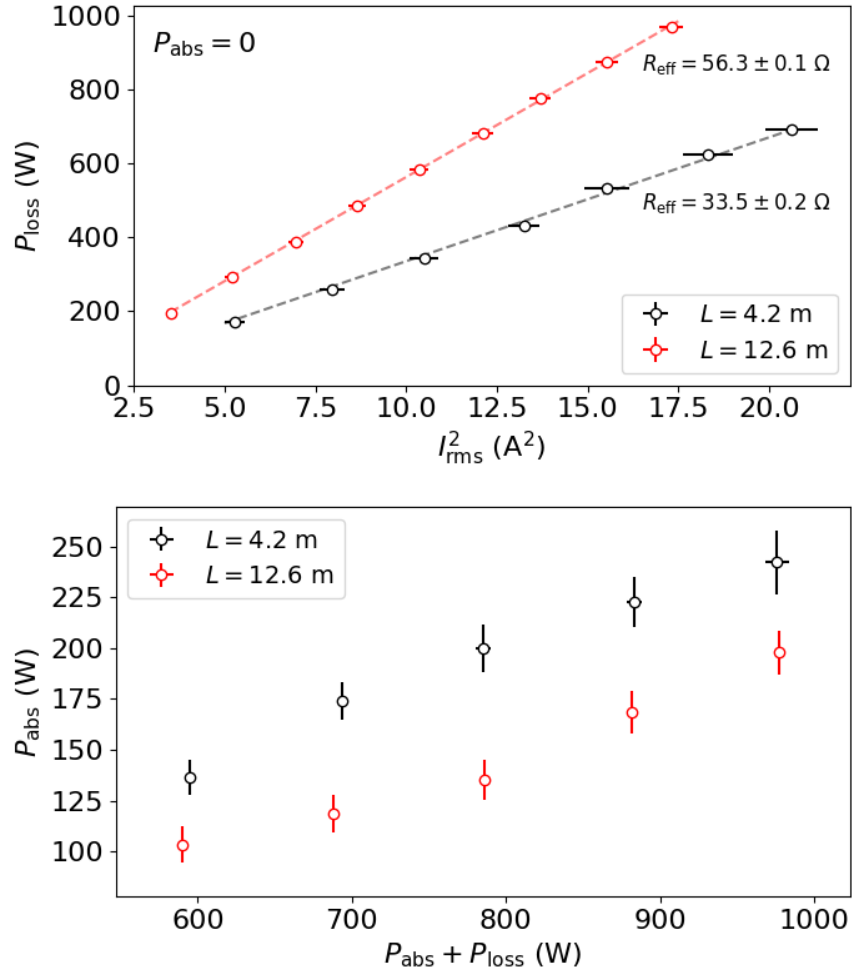


Figure C.1: The effect of multiple antinodes on the percentage power coupled into the plasma. Upper: The power consumed by the coupling circuit in the absence of a plasma is used to find the effective resistance which leads to ohmic losses. Increasing the cable length from 4.2 m to 12.6 m increases the number of antinodes from 1 to 2, so effective resistance is increased. Lower: The power coupled into the plasma is reduced when the cable length is increased.

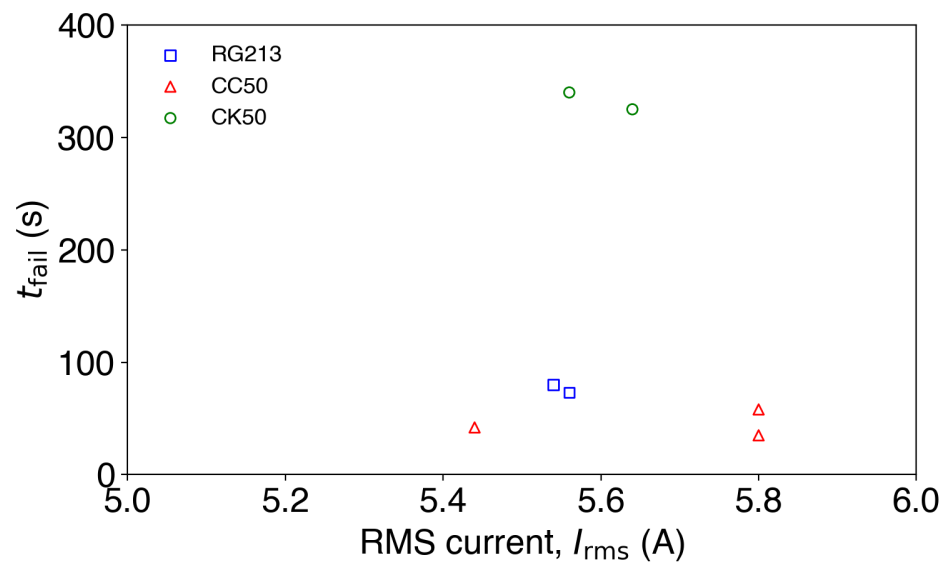


Figure C.2: Time taken for cable failure of different kinds of coaxial cable.

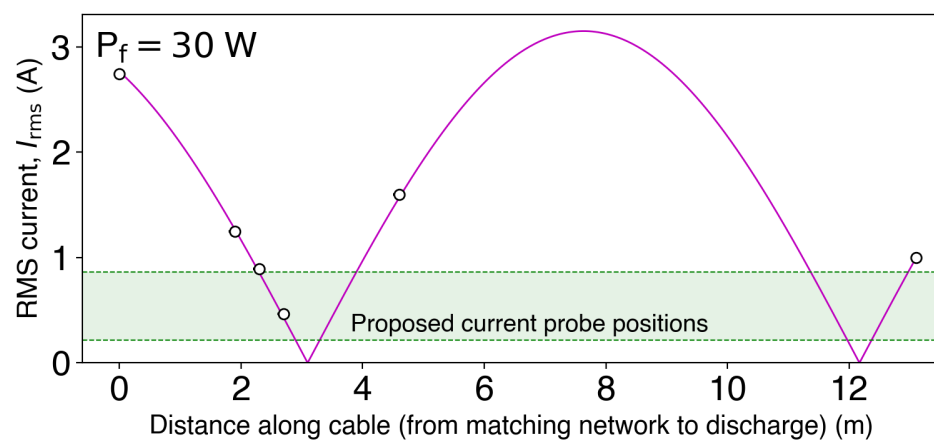


Figure C.3: Amplitude of standing wave along the coaxial cable.

Appendix D

Particle-in-cell simulations supplementary information

D.1 Setup parameters

OSIRIS PIC simulations were performed at the exascale LUMI supercomputer (Finland). In the simulations, all quantities are normalized to the peak plasma density $n_0 = 1.78 \times 10^{12} \text{ cm}^{-3}$ (associated plasma period $\omega_{\text{pe}}^{-1} = 13.29 \text{ ps}$, and plasma skin-depth $c/\omega_{\text{pe}} = 3.98 \text{ mm}$). 3D3V simulations have a moving window travelling at c along the z -direction with absorbing boundary conditions. The simulation box has dimensions $L_x \times L_y \times L_z = 3.5 \text{ cm} \times 3.5 \text{ cm} \times 40 \text{ cm}$, discretized into $704 \times 704 \times 4017$ cells. This yields a spatial resolution $\Delta x = 0.025 c/\omega_{\text{pe}} = 0.096 \text{ mm}$. The simulations employ a time resolution $\Delta t = 0.014 \omega_{\text{pe}}^{-1} = 186 \text{ fs}$, fulfilling the 3D Courant-Friedrichs-Lewy condition: $c\Delta t < \Delta x/\sqrt{3}$. The macro-particles employ cubic interpolation with first-order binomial current smoothing.

A background stationary plasma is initialized with a longitudinal profile given by

$$\rho(z) = \frac{n_0 \left[e^{-(z-z_1)^2/(2\sigma_c^2)} + e^{-(z-z_2)^2/(2\sigma_c^2)} + n_{\text{min}} \right]}{\left(1 + e^{-z/20} \right) \left(1 + e^{-(z+l)/20} \right)}, \quad (\text{D.1})$$

where $\sigma_c = 3.75 \text{ cm}$, $l = 87 \text{ cm}$, $n_{\text{min}} = 0.02$, and $z_1 = 24.88 \text{ cm}$ and $z_2 = 61.88 \text{ cm}$ correspond to the centres of the inductive coils. The profile is chosen to closely match the

Langmuir probe density measurements when the discharge is filled to a pressure $p = 4$ Pa and the power absorbed by the plasma is $P_{\text{abs}} = 240$ W. The plasma is modelled as homogeneous in the perpendicular direction.

The electron-positron-proton beam is initialized before entrance into the plasma, centred at $z = -50.21 c/\omega_{\text{pe}} = -20$ cm and $x = y = 0$. The density and momentum phase space distribution are modelled by fitting analytical forms to the distributions at the entrance of the plasma cell (after the glassy carbon window) obtained from a FLUKA simulation with the setup as shown in Figure 5.3 and number of primary protons $N_{\text{pri}} = 3 \times 10^{11}$.

The proton population is split into the population which undergoes no inelastic scattering (with number $N_{\text{p}^+} = 1.26 \times 10^{11}$, represented by a delta function in momentum space at $p = 440$ GeV/ c) and the population at lower momenta ($p = 1 - 440$ GeV/ c) which has a power-law spectrum:

$$\frac{dN}{dp} = N_{\text{p}^+} \cdot A p^{k_1}, \quad (\text{D.2})$$

where $N_{\text{p}^+} = 1.50 \times 10^{11}$, $k_1 = -0.81$, and prefactor $A = 0.087$ is fixed by the normalization condition $\int_{x_{\text{min}}}^{x_{\text{max}}} \left(\frac{dN}{dp} \right) dp = 1$. The proton transverse momentum spread is set as $\sigma_{p_{\perp}} = 0.25$ GeV/ c . The density profile follows a Gaussian distribution in all dimensions:

$$\rho(r, z) = \frac{1}{(2\pi)^{3/2} \sigma_r^2 \sigma_z} \exp \left[- \left(\frac{r^2}{2\sigma_r^2} + \frac{z^2}{2\sigma_z^2} \right) \right], \quad (\text{D.3})$$

with $\sigma_r = 0.11$ cm and $\sigma_z = 7.5$ cm.

Electrons and positrons are modelled in the spectral range $1 \text{ MeV} - 10 \text{ GeV}$ with a multi-power-law distribution. The electron momentum spectrum is modelled by a triple-index power-law distribution:

$$\frac{dN}{dp} = N_{e^-} \cdot A p^{k_1} (p + p_{1,2})^{k_2 - k_1} (p + p_{2,3})^{k_3 - k_2}, \quad (\text{D.4})$$

where $N_{p^-} = 5.48 \times 10^{12}$, $k_1 = -1.6$, $p_{1,2} = 0.012 \text{ GeV}$, $k_2 = 0.8$, $p_{2,3} = 0.25 \text{ GeV}$, $k_3 = -2.2$, and the prefactor $A = 0.16$ is fixed by the normalization condition $\int_{x_{\min}}^{x_{\max}} \left(\frac{dN}{dp} \right) dp = 1$:

$$A \left[(k_1 + 1)^{-1} p^{k_1+1} (p + p_{1,2})^{k_2 - k_1} \left(\frac{p + p_{1,2}}{p_{1,2}} \right)^{k_1 - k_2} (p + p_{2,3})^{k_1 - k_3} \left(\frac{p + p_{2,3}}{p_{2,3}} \right)^{k_3 - k_1} \right. \\ \left. F_1 \left(k_1 + 1, k_1 - k_2, k_3 - k_1; k_1 + 2; -\frac{p}{p_{1,2}}, -\frac{p}{p_{2,3}} \right) \right]_{p_{\min}}^{p_{\max}} = 1 \quad (\text{D.5})$$

where F_1 is the Appell hypergeometric function of two variables. The positron momentum spectrum is modelled by a double-index power-law distribution:

$$\frac{dN}{dp} = N_{e^+} \cdot A p^{k_1} (p + p_{1,2})^{k_2 - k_1}, \quad (\text{D.6})$$

where $N_{e^+} = 4.27 \times 10^{12}$, $k_1 = 0.17$, $p_{1,2} = 0.19 \text{ GeV}$, $k_2 = -2.2$, and $A = 0.20$, according to:

$$A \left[\left(\frac{p^{k_1+1} p_{1,2}^{k_2 - k_1}}{k_1 + 1} \right) {}_2F_1 \left(k_1 + 1, k_1 - k_2; k_1 + 2; -\frac{p}{p_{1,2}} \right) \right]_{p_{\min}}^{p_{\max}} = 1 \quad (\text{D.7})$$

where ${}_2F_1$ is the hypergeometric function.

The density distributions follow a Gaussian distribution longitudinally and a Cauchy distribution radially:

$$\rho(r, z) = \frac{1}{(2\pi)^{3/2}\sigma_z} \frac{\frac{1}{2}\Sigma_r}{(r^2 + (\frac{1}{2}\Sigma_r)^2)^{3/2}} \exp\left(-z^2/2\sigma_z^2\right), \quad (\text{D.8})$$

with $\sigma_z = 7.5$ cm and a radial half-width-half-maximum $\Sigma_r = 0.32$ cm. The transverse momenta are specified by a Gaussian with a momentum-dependent spread:

$$\frac{d^2N}{dp_\perp^2} = \frac{1}{2\pi\sigma_{p_\perp^2}} \exp\left(-p_\perp^2/2\sigma_{p_\perp^2}^2\right), \quad \sigma_{p_\perp} = p_\parallel \cdot \Delta\theta, \quad (\text{D.9})$$

where $\Delta\theta = 0.025$ provides an appropriate fit (shown in Figures D.1 and D.2).

D.2 Time series of the 3D simulations

Included here is a time series of the 3D-PIC simulations presented in Chapter 5 (Figures D.3 and D.4). Filamentation of the pair beam as it propagates through the plasma discharge is simulated in two cases: (i) ‘realistic thermal spread’, that considers the finite transverse momentum spread of the pair distribution function ($\Delta\theta = 2.5\%$), and (ii) ‘collimated pairs’, where the $e^\pm$ pairs are assumed to have zero divergence ($\Delta\theta = 0\%$). The left-hand panels show a central slice from the y - z plane and the right-hand panels show a central slice from the transverse x - y plane. The lineouts projected onto the vertical axis correspond to the central 1 cm of the beam profile. The coordinate $z = 0$ corresponds to the beginning of the plasma, and the time $t = 0$ corresponds to when the centre of the beam passes $z = 0$.

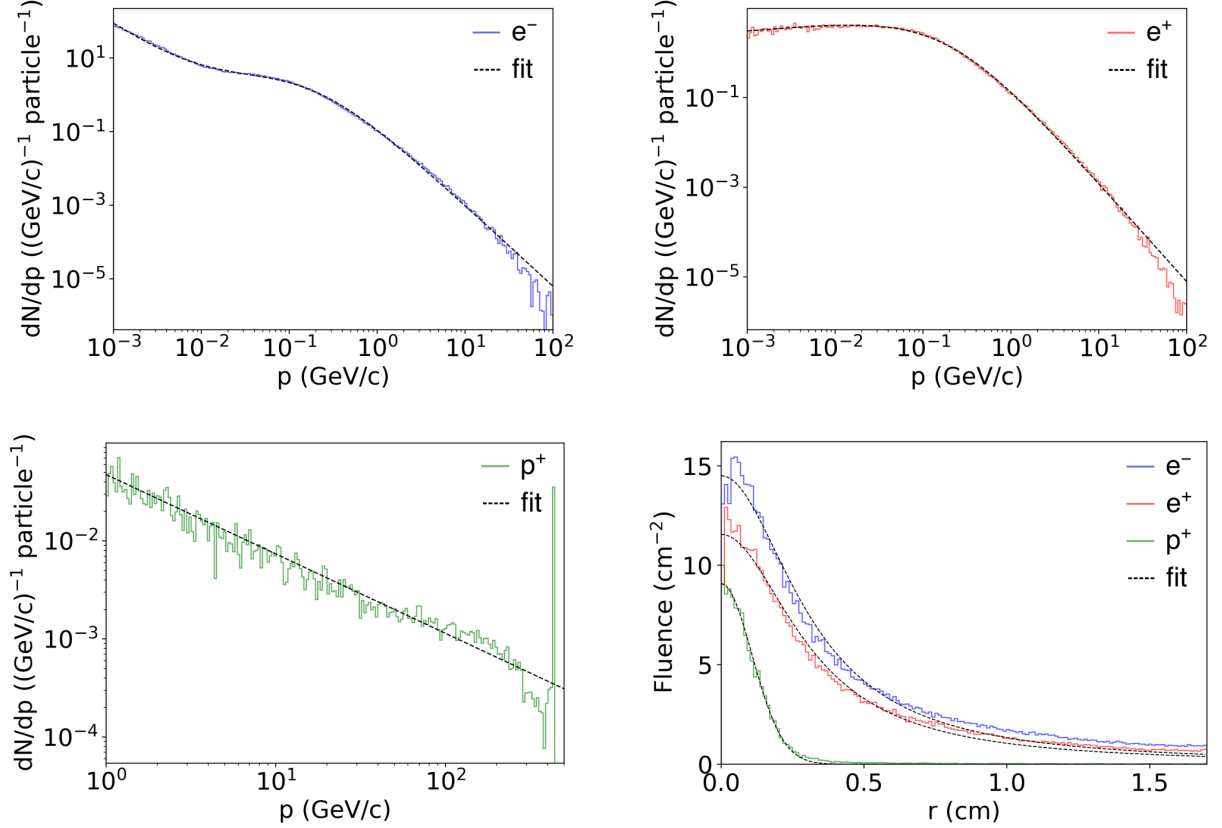


Figure D.1: Fitted momentum and density distributions of beam species. The momentum and density distribution of electrons (blue), positrons (red) and protons (green) obtained from a FLUKA simulation are fitted to analytical forms (black-dashed) and used as setup parameters for OSIRIS PIC simulations. The FLUKA simulation models the experimental setup shown in Figure 5.3, and conditions are sampled at the entrance of the plasma (after passing through the glassy carbon window).

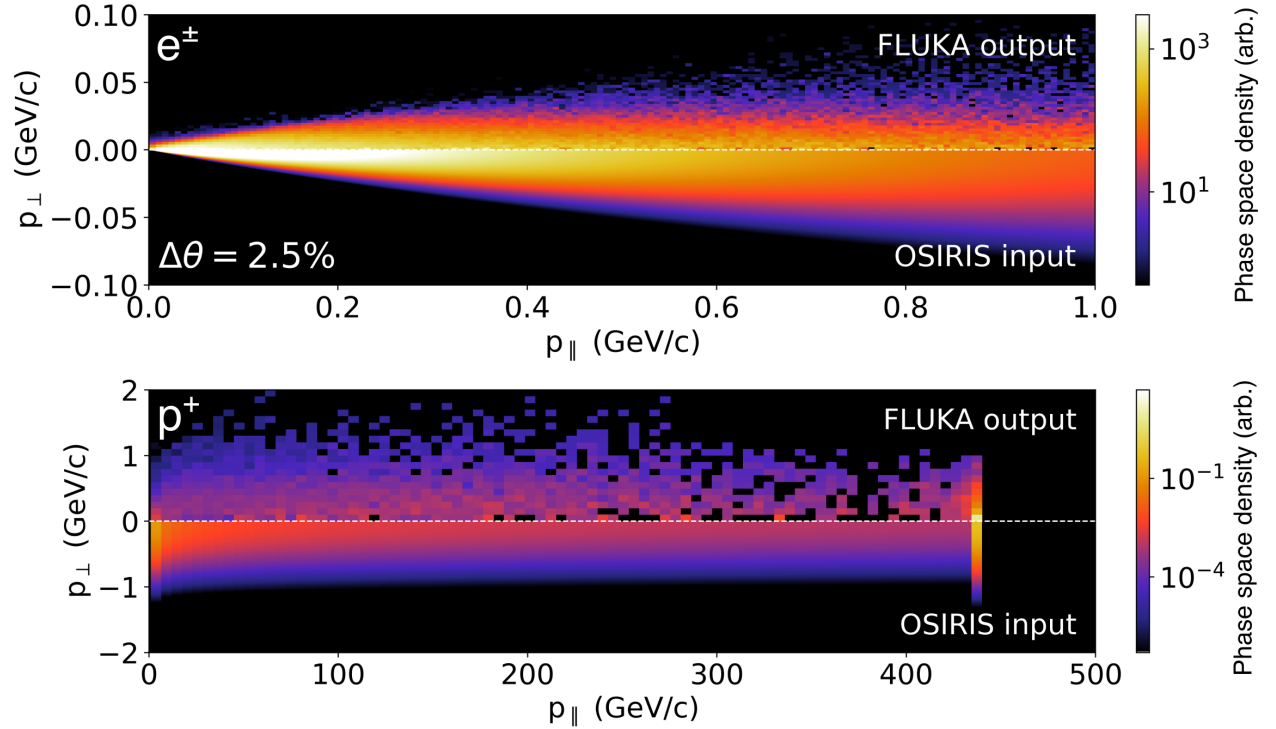


Figure D.2: Fitted momentum-space distributions of beam species. The fits to the full momentum space distribution are shown for $e^\pm$ (upper) and p^+ (lower). In each frame, the top half shows the momentum space obtained from a FLUKA simulation, and the lower half shows the analytical fitted function. For the $e^\pm$ pairs, a Gaussian-distributed transverse momentum with $\sigma_{p_\perp} = p_\parallel \cdot \Delta\theta$ and $\Delta\theta = 0.025$ provides an appropriate fit. For the p^+ , $\sigma_{p_\perp} = 0.25 \text{ GeV}/c$.

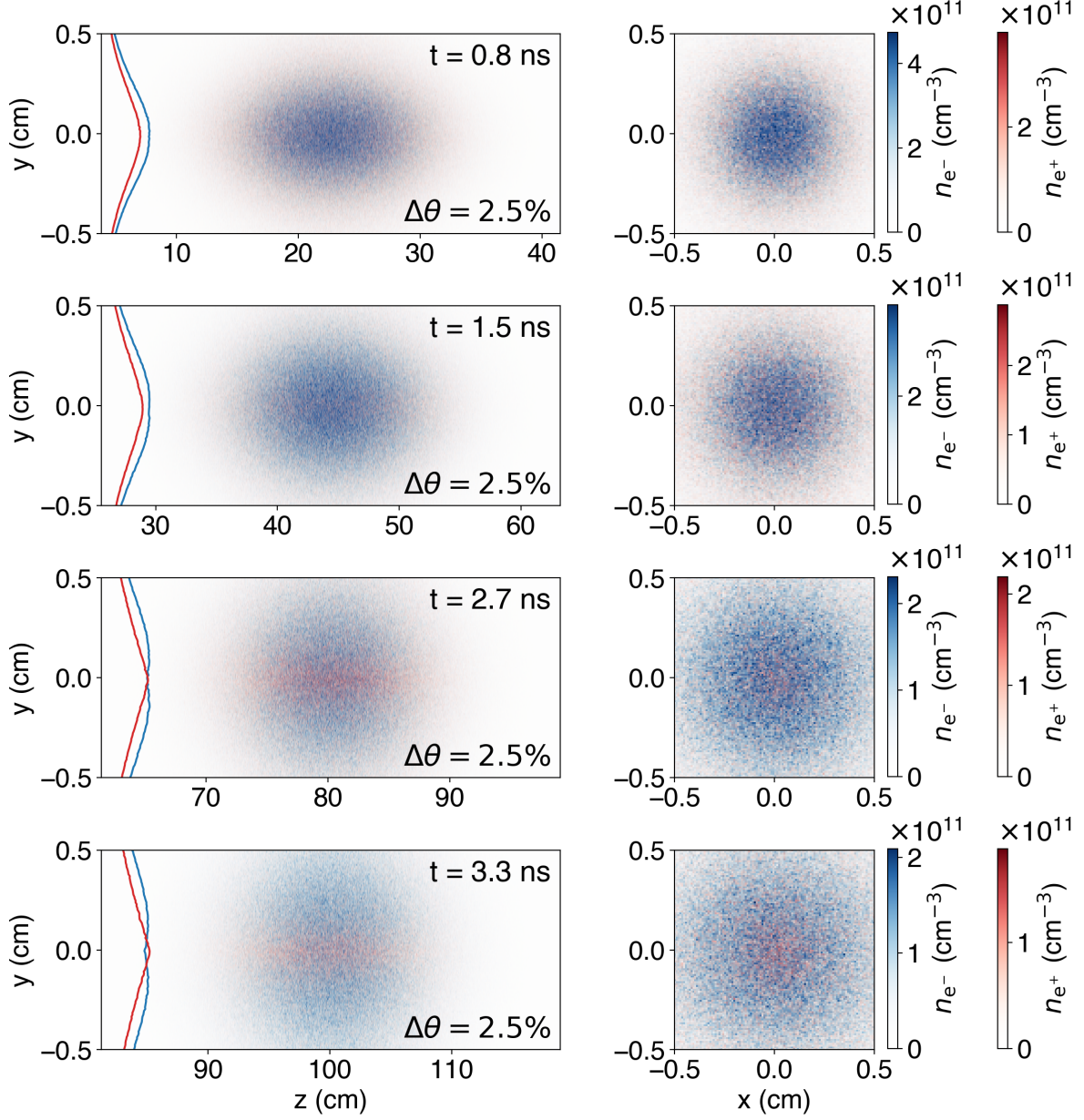


Figure D.3: Time series of 3D particle-in-cell simulations (realistic thermal spread).

The spatial distribution of electrons (blue) and positrons (red) is shown at times: (i) $t = 0.8$ ns, when the beam is passing through the centre of the first inductive coil, (ii) $t = 1.5$ ns, at the midpoint along the length of the discharge, (iii) $t = 2.7$ ns, at the position of the Faraday probe, after passing through the second coil, (iv) $t = 3.3$ ns, at the position of the luminescence screen positioned downstream of the plasma.

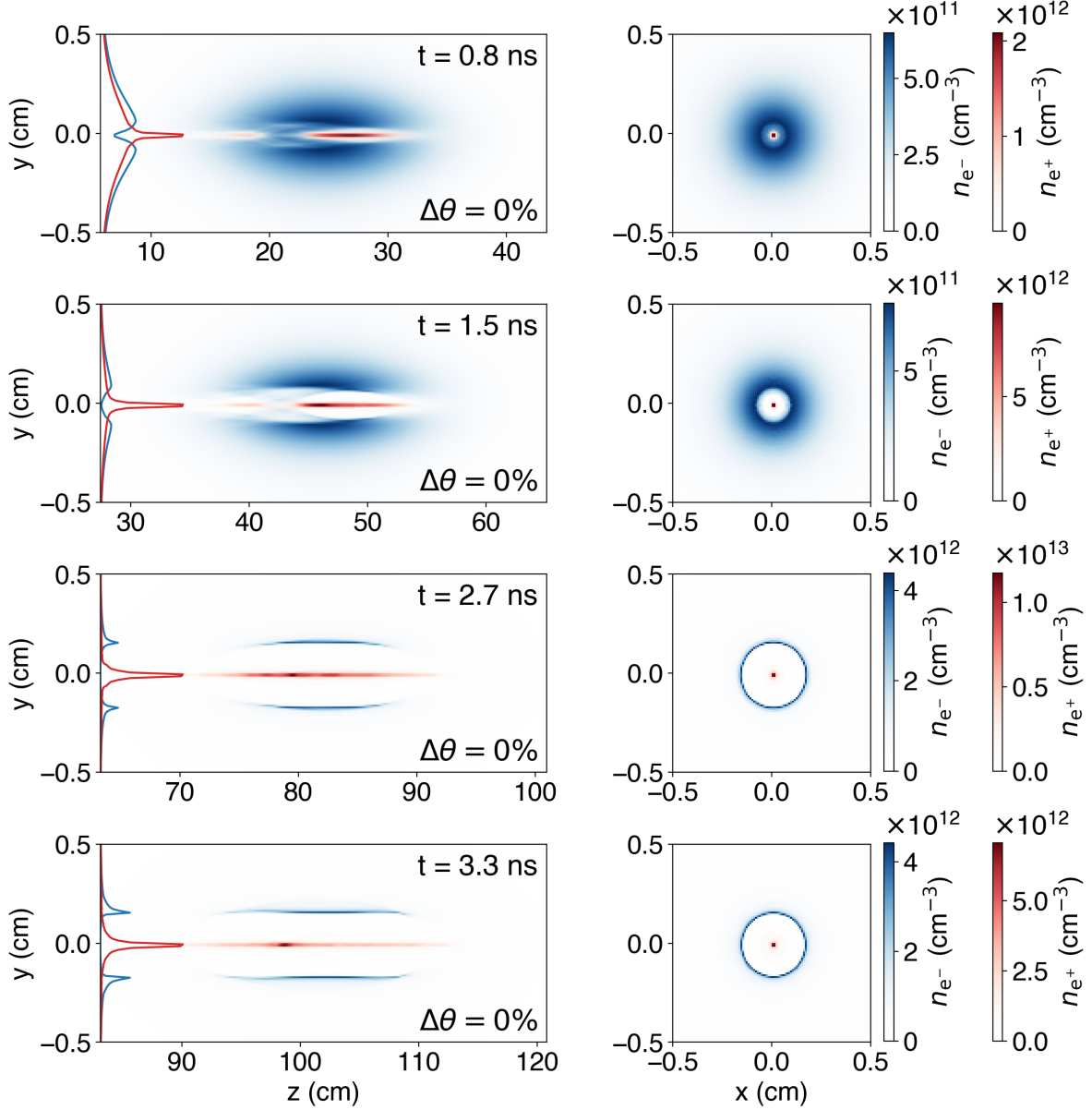


Figure D.4: Time series of 3D particle-in-cell simulations (collimated pairs). The spatial distribution of electrons (blue) and positrons (red) is shown at times: (i) $t = 0.8$ ns, when the beam is passing through the centre of the first inductive coil, (ii) $t = 1.5$ ns, at the midpoint along the length of the discharge, (iii) $t = 2.7$ ns, at the position of the Faraday probe, after passing through the second coil, (iv) $t = 3.3$ ns, at the position of the luminescence screen positioned downstream of the plasma.

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