2021 Summer Research Placement Report

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Implementation of CRPA into GENIE and subsequent comparison to other models

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The Continuum Random Phase Approximation calculation for CCQE neutrino interaction cross sections was implemented into the GENIE Monte Carlo simulation. It's predictions were then compared to other versions of GENIE, as well as the NEUT Monte Carlo simulation used by the T2K experiment. It was also compared with T2K data for carbon and oxygen targets, as well as anti neutrino interactions, allowing a comparison of the models through the use of a χ^2 test.

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1 Introduction

1.1 Neutrino Interactions

Knowledge of neutrino interaction cross sections is vital for the study of neutrino oscillations, and CP violation in the neutrino sector.

The T2K experiment produces a neutrino beam near Tokai in Japan, which is sampled $280m$ from the source at the near detector, and then measured at the far detector $295km$ away at the Super-Kamiokande far detector. By measuring the ratio of muon and electron neutrinos at each detector allows for the measurement of neutrino oscillation probabilities.

Equation 1 shows how the event rate for neutrino-nucleus interactions at the far detector, N_μ, at a given neutrino every E_ν, is proportional to the product of the oscillation probability $P(\nu_\mu \to \nu_l)$, with the neutrino interaction cross section $\sigma(E_\nu)$, the neutrino flux $\Phi_\nu(E_\nu)$, and the detector efficiency $\epsilon(E_\nu)$.

$$N_\mu(E_\nu) = P(\nu_\mu \to \nu_l)\sigma(E_\nu)\ \Phi_\nu(E_\nu)\ \epsilon(E_\nu). \tag{1}$$

Given that the oscillation probability is the main interest of such studies, and the event rate is something that is measured, the uncertainty comes largely from the other factors. The interaction cross section is in fact the largest source of uncertainty, hence it is this that is trying to be improved.

The interaction measured in experiments is the CC0π interaction, which is where there is a charged current (W propagator) and the only final state hadrons are nucleons. To calculate the total CC0π neutrino cross section, cross sections from several contributing modes must be summed. For our study the largest contributing mode is the charged current quasi-elastic interaction (CCQE). In this interaction a neutrino interacts with a neutron (proton), and produces a (anti)muon and a proton (neutron). Since neutrino experiments use elements from carbon to argon in their detectors, interactions inside the nucleus play a major role.

The two other contributions to the total cross section are the two particle two hole interactions and pion absorption interactions (see figure 1). However the model investigated in this study (CRPA) models the CCQE interaction, hence this shall be the focus of the project.

1.2 The Models

There are various different Monte Carlo (MC) programs used to model neutrino interactions The two used in this placement are GENIE and NEUT.

The version of GENIE[1] used in this analyses employs a local Fermi gas model, to which an extension known as Random Phase Approximation (RPA) can optionally be applied. It can be seen in figure 2 that GENIE on its own tends to over

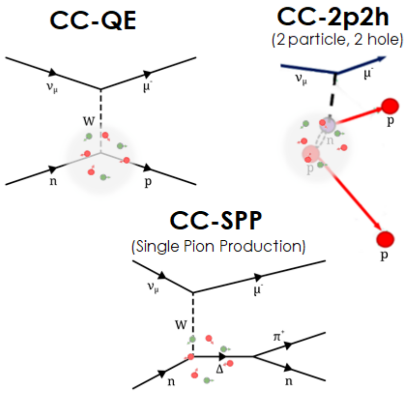


Figure 1: Feynman diagrams showing representative plots for the CCQE (top left), 2 particle 2 hole (top right) and pion production (bottom) interactions.

predict the cross section in the forward regions. It is for this reason that RPA was introduced to provide suppression to better match the experimental data.

RPA achieves this additional suppression by taking into account long range correlations between nucleons inside the nucleus. It accounts for the fact that at low energy transfers the neutrinos see more than one target nucleon such that the impulse approximation (the idea that neutrinos only interact once) breaks down. GENIE with RPA is similar to the model used by the NOvA experiment as an input to their oscillation analyses[2].

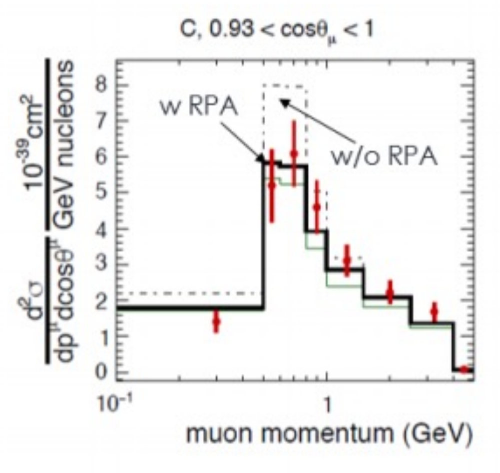
The input NEUT simulation uses the Spectral Function model [3] to simulate Charged-Current Quasi-Elastic (CCQE) interactions. This model constructs a semi-empirical model of the nucleus built using electron scattering data and is used by the T2K experiment in their analyses of neutrino oscillations.

There also exits the Hartree Fock model which is more accurate than a local Fermi gas model since it uses a more realistic ground state.

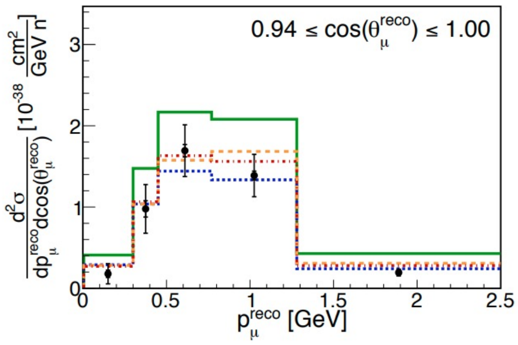
Continuum Random Phase Approximation (CRPA) is an extension to the Hartree Fock model in much the same way that RPA is to GENIE. CRPA therefore extends the Hartree Fock model to be valid beyond the impluse approximation. CRPA is specialised towards describing CCQE neutrino-nucleus interactions at low momentum and energy transfer, which is where other models are known to struggle. This corresponds to the forward region in which extra suppression (RPA) becomes important.

1.3 Project aims

Different models have different strengths and weaknesses. One model might make better predictions than another in a certain energy and $\cos\theta$ range, but worse in



(a) T2K



(b) μBooNE

Figure 2: Plots of neutrino interaction cross sections. a) showing T2K data in red, GENIE with RPA in bold, and GENIE without RPA as dashed lines [4]. b) showing MicroBooNE data in black, GENIE with RPA in blue, and GENIE without RPA in green [5]. $\cos\theta$ is the angle the outgoing muon makes to the neutrino beam line, hence forward angles ($\cos\theta \sim 1$) are aligned with the beam, whereas high angles ($\cos\theta \sim 0$) are perpendicular to the beam

another region. The initial aim of this project was therefore to implement CRPA into the GENIE Monte Carlo simulation, which before the start of this project had not been done before. The next aim was to see if CRPA provided any improvement over other models, particularly in the forward region.

To do this, CRPA was compared to various models (GENIE with RPA, GENIE without RPA, and NEUT), as well as to T2K data for various targets. An additional investigation was undertaken to see how CRPA treated anti muon neutrinos compared to muon neutrinos, and again compare that to the other models and T2K data.

If more time were available, a comparison of how CRPA treats electron neutrinos with respect to muon neutrinos would have been carried out. Additionally it would have been desirable to extend CRPA to work at higher energy transfers.

2 Analysis

2.1 Theory Comparison

The first stage of the project was to produce cross sections using the CRPA theory code and compare them to current models (GENIE and NEUT).

Figure 3 shows how the different models compare to the CRPA theory calculation. Figure 3a shows the single differential cross section in $\cos\theta$, while the others show the double differential cross section for a given range of $\cos\theta$ values.

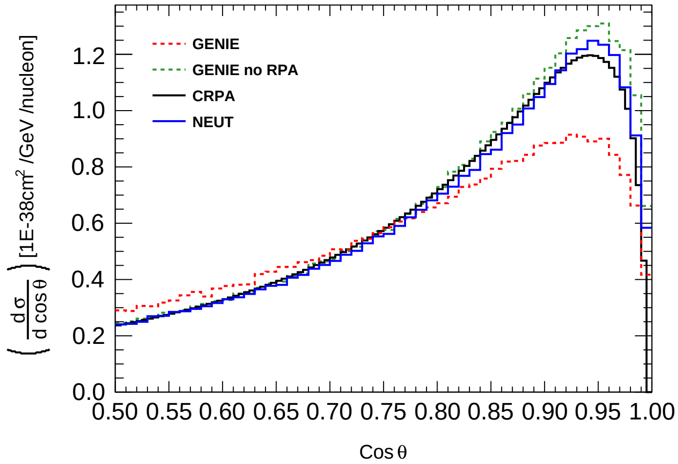
This was done by generating 500000 events for each model with a fixed neutrino energy, then selecting and plotting the appropriate events. The CRPA cross sections were calculated for a given neutrino energy and specified $\cos\theta$. Only one point was calculated in each $\cos\theta$ bin in the plots of figure 3.

The single differential cross section plot shows that NEUT and CRPA match fairly well over the whole range, with slight disagreement at forward angles. It suggests a very similar story for GENIE without RPA and CRPA, except that the disagreement at forward angles is larger. GENIE with RPA doesn't agree with CRPA except for a small region near $\cos\theta \sim 0.75$.

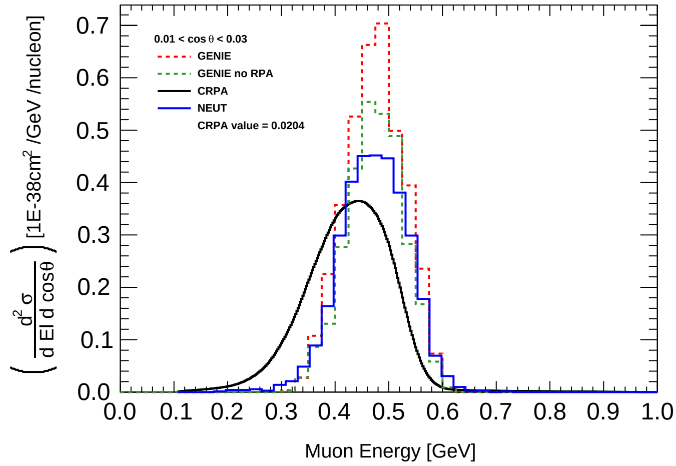
When considering the double differential cross sections it can be seen that at large angles none of the models agree particularly well, which is expected. NEUT and CRPA agree moderately well over the whole range, matching very closely at forward angles. However at the high angles where CRPA and NEUT's single differential cross sections matches, their kinematic distribution is quite different.

At extreme forward angles ($\cos\theta > 0.97$) the models all disagree. It is in this region that GENIE and NEUT are known to struggle since this is where low energy/momentum transfer occur. In this region it can be seen that CRPA exhibits sharp peaks in its distribution. This is due to the base Hartree Fock model using a more realistic ground state than the other models, which encapsulates low energy nuclear excitations.

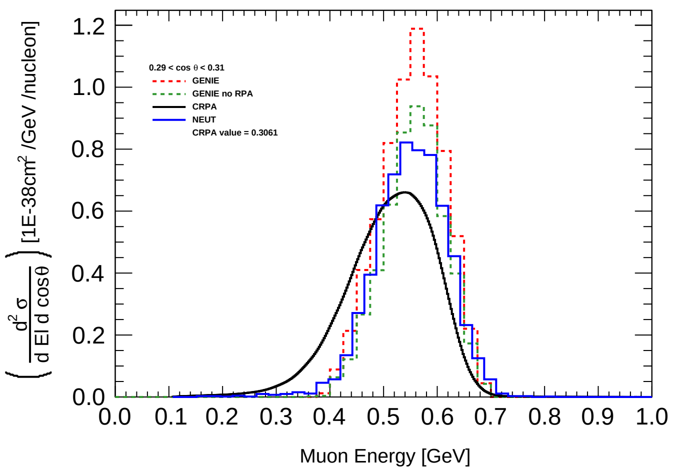
Once CRPA has been implemented into GENIE these regions can be compared to data. Currently this serves as an instructive comparison of how each model behaves in different regions for a fixed neutrino energy (1GeV).



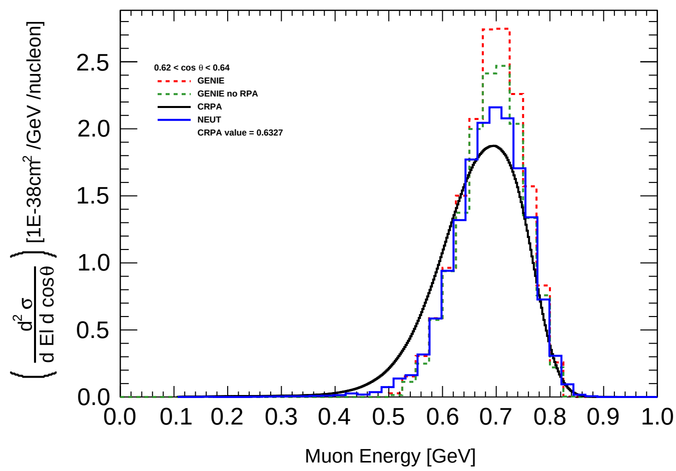
(a) Single dfferential cross section.



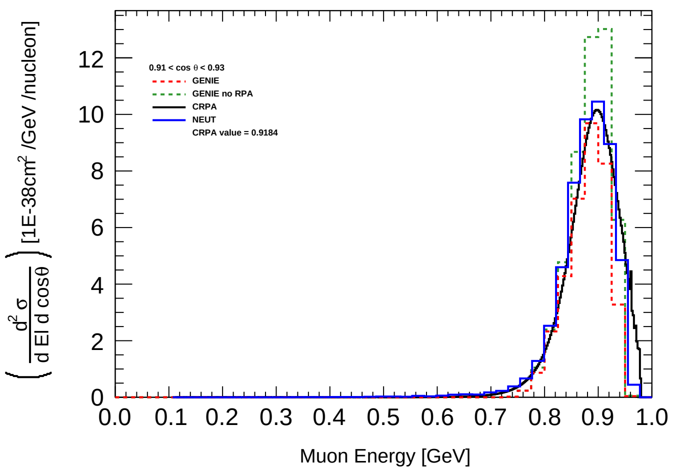
(b) Double differential cross section for $\cos\theta \in [0.01, 0.03]$



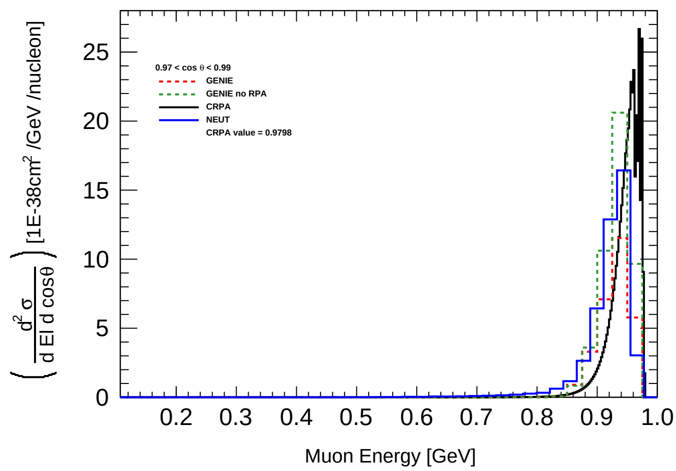
(c) Double differential cross section for $\cos\theta \in [0.29, 0.31]$



(d) Double differential cross section for $\cos\theta \in [0.62, 0.64]$



(e) Double differential cross section for $\cos\theta \in [0.91, 0.93]$



(f) Double differential cross section for $\cos\theta \in [0.97, 0.99]$

Figure 3: Plots comparing predicted cross sections from GENIE with RPA (red), GENIE without RPA (green), NEUT (blue), and the CRPA theory calculation (black). For a fixed neutrino energy of 1GeV, showing the CCQE interaction only.

2.2 CRPA Implementation

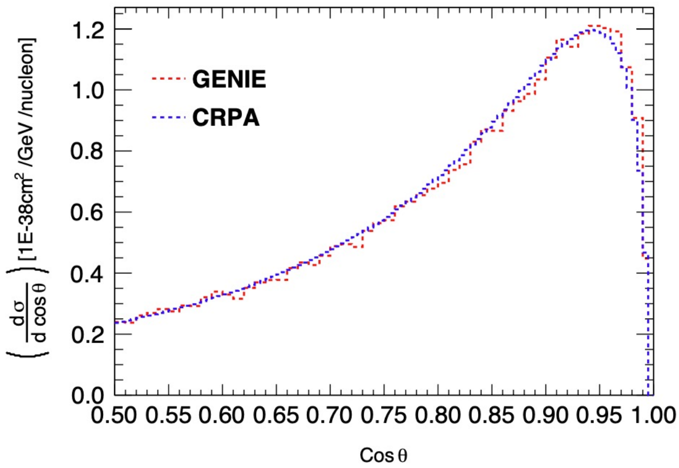
Equation 2 gives the double differential cross section, $\frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE'_l}$, in terms of lepton energy, E'_l, and solid angle, $\sigma_{\nu l}$. $L_{\mu\sigma}$ is the lepton tensor, $W^{\mu\sigma}$ is the hadron tensor, G the Fermi constant, and k' the outgoing lepton momentum vector. To implement CRPA into GENIE, tables containing CRPA's hadron tensor elements were produced and incorporated into GENIE's software. In terms of equation 2, we swapped GENIE's hadron tensor ($W^{\mu\sigma}$) with CRPA's hadron tensor.

$$\frac{d^2\sigma_{\nu l}}{d\Omega(\hat{k}')dE'_l} = \frac{|\vec{k}'|}{|\vec{k}|}\frac{G^2}{4\pi^2}L_{\mu\sigma}W^{\mu\sigma} \tag{2}$$

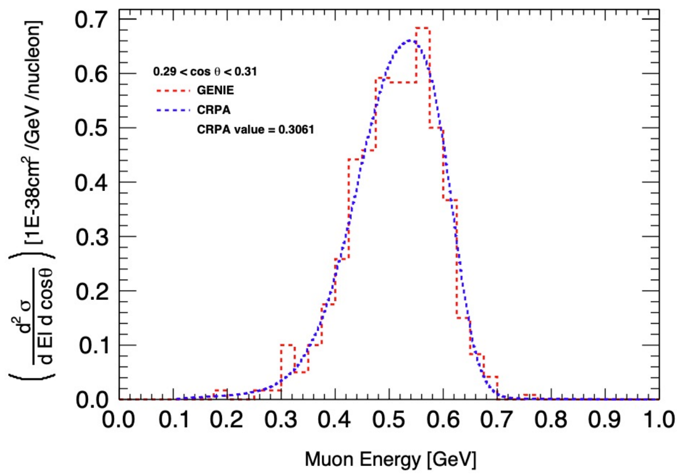
Different tables of hadron response elements must be produced for each target and ν_μ or $\bar{\nu_\mu}$ interaction.

To check that CRPA was implemented correctly, the predicted cross sections from the CRPA theory calculation and from CRPA GENIE were plotted together to verify that they were the same. This is presented in figure 4, where it can be seen that the predictions match, validating that fact that CRPA has been successfully implemented into GENIE.

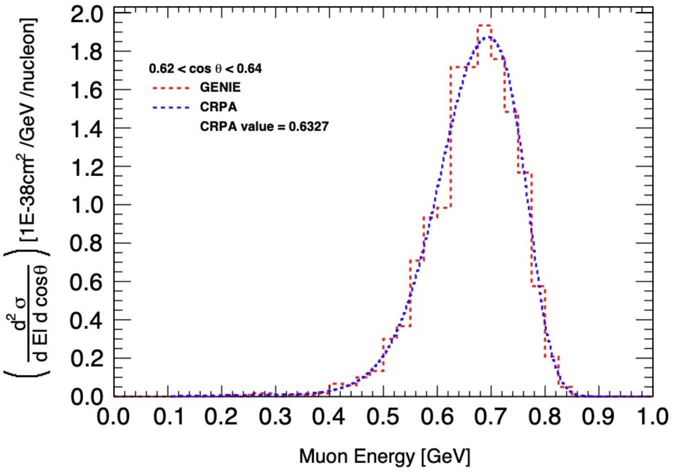
The total cross section and its components were plotted as a check that each mode was present in the expected proportions. This can be seen in figure 5. On the same figure was also plotted the total double differential cross section as measured by the T2K experiment. It can be seen that the CCQE mode is the major contribution, which is as expected.



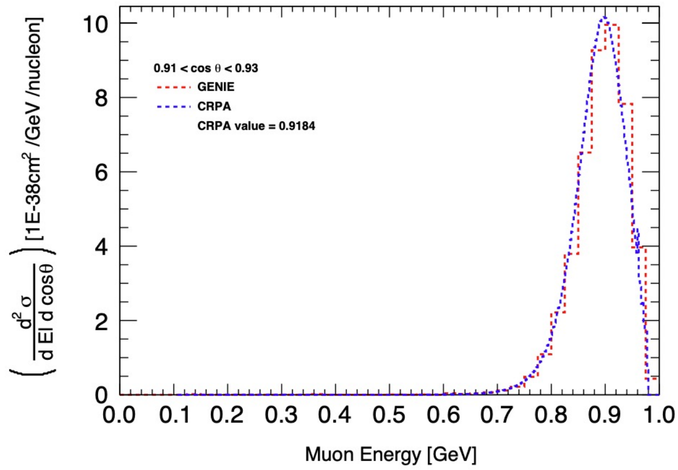
(a) Single differential cross section in $\cos\theta$



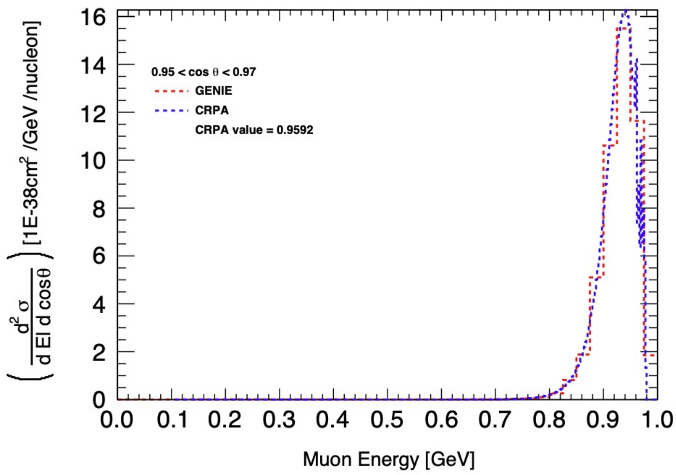
(b) $\cos\theta \in [0.29, 0.31]$



(c) $\cos\theta \in [0.62, 0.64]$

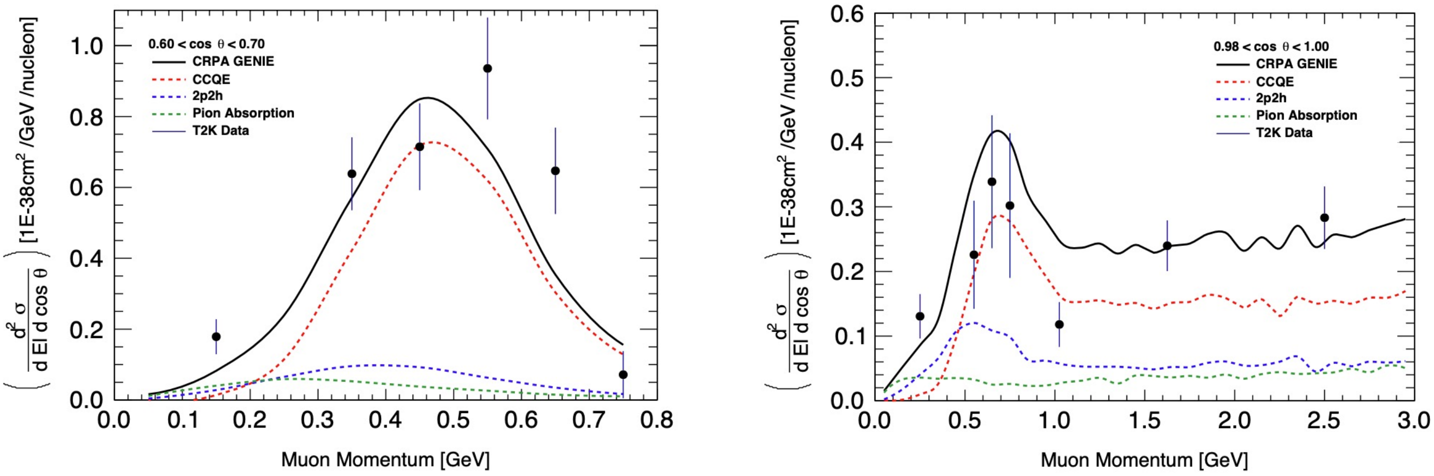


(d) $\cos\theta \in [0.91, 0.93]$



(e) $\cos\theta \in [0.95, 0.97]$

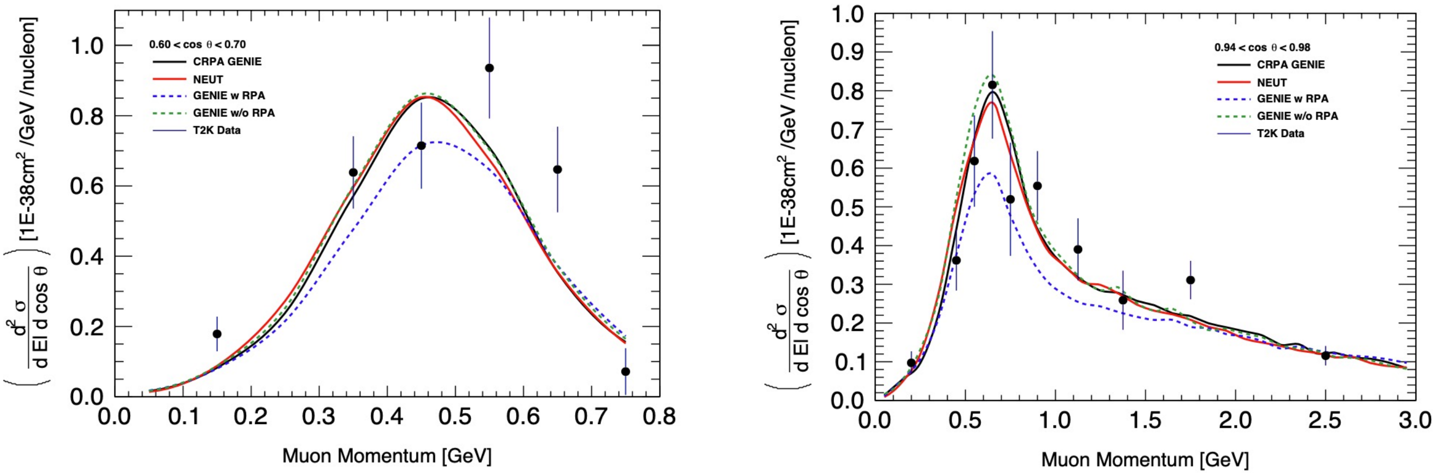
Figure 4: Plots showing the predicted CCQE double differential cross section for CRPA GENIE (red) and the CRPA theory (blue). CRPA GENIE matches the CRPA code, validating CRPA's implementation into the GENIE Monte Carlo software.



(a) $\cos\theta \in [0.60, 0.70]$

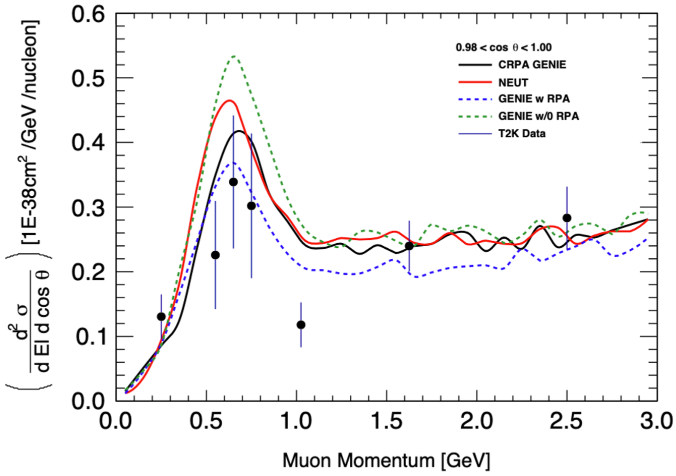
(b) $\cos\theta \in [0.98, 1.00]$

Figure 5: Plots showing total CC0π double differential cross section (black), and the contribution from each component; CCQE (red), 2 particle 2 hole (blue), and pion absorption (green). Data from [6].



(a) $\cos\theta \in [0.60, 0.70]$

(b) $\cos\theta \in [0.94, 0.98]$



(c) $\cos\theta \in [0.98, 1.00]$

Figure 6: Plots showing the predicted double differential cross section for CRPA GENIE (black), NEUT (red), GENIE with RPA (blue), and GENIE without RPA (green). On a target consisting of C and H with the fractions of 0.923 and 0.077 respectively. Data from [6].

Model on CH T2K	Regular χ^2 (67 degrees of freedom)	Shape Only χ^2 (Estimate)
CRPA GENIE	186	207
NEUT	228	248
GENIE + RPA	140	190
GENIE - RPA	295	298

Table 1: Table showing χ^2 fits for the models with T2K data on a CH target.

2.3 Model Comparison

Given that CRPA was correctly implemented into GENIE, the next goal was to obtain cross sections from each model (CRPA GENIE, GENIE with RPA, GENIE without RPA, and NEUT), and compare them to data from the T2K experiment.

This was done by generating 500k events from each model, then processing them using Nuisance to produce flat trees that could be read by Root. A macro was used that took the simulated data and plotted it against the T2K data.

Some representative plots are shown in figure 6. At moderately forward angles the models agree well, which is expected as that regime is well understood. The exception to this is GENIE with RPA which consistently under predicts the cross section, which is due to the suppression supplied by RPA.

As seen previously, at very forward angles the models show large disagreements. It is thought that CRPA should provide improvements in this region.

To quantify how well each model describes the data, a χ^2 test was used. This was achieved using the program Nuisance [7] to fit a Monte Carlo flat tree to a specified data set. For this study T2K data was used [6]. Table 1 shows the χ^2 values for the models' fit to T2K data on a CH target. It can be seen from the χ^2 values that no model accurately describes the data. There isn't any meaningful comparison that can be made, except to see that CRPA GENIE and GENIE with RPA are comparable and seemingly better than the other two models.

2.4 Peele's Pertinent Puzzle

It was noted that GENIE with RPA consistently under predicted the cross sections, due to the effects of RPA, yet has the best χ^2 value. This was thought to be due to a well known problem that results in the χ^2 being non representative of the how well the model actually fits the data. This is problem known as Peele's Pertinent Puzzle (PPP), and occurs when trying to use a χ^2 test on highly correlated data.

PPP is a consequence of using absolute uncertainties on measurements, then trying to fit the normalisation. This often occurs when the model used struggles to describe the data. Since the uncertainties on the shape of the measurement don't scale with the normalisation, the χ^2 test assigns a better match when the normalisation is minimised. This is due to the fact that while the uncertainties are absolute, they are larger relatively compared to the normalisation, hence the model 'fits' better.

Peele's pertinent puzzle isn't a mistake and its χ^2 value isn't objectively wrong, its just not the best validation of our models. There are a couple of ways to improve the tests such that the obtained χ^2 values are more useful. The simplest method is to fix the normalisation and instead use relative uncertainties. This is known as the shape only χ^2 test.

To estimate the shape only χ^2 of the models, the Monte Carlo histograms were scaled such that the total cross section matched that of the data histogram. A χ^2 value was then calculated with this re-normalised histogram through the regular calculation. Since the full covariance matrix was used instead of the shape only covariance matrix, the resultant χ^2 value only approximates the true shape only χ^2 value. However it still serves as a useful illustration of the effect of PPP. Note that for the comparison to T2K data on a carbon and oxygen target presented in section 2.5, a shape only covariance matrix was available, hence these χ^2 values are the true shape only χ^2 values.

2.5 Element Comparison

By this point CRPA had been implemented into GENIE, the implementation was verified, and the models compared to T2K data with a CH target. In order to obtain better comparisons, the models were compared against each other for other targets, and then again to T2K data.

The elements investigated were carbon, oxygen, and argon (no T2K data available). The plots and χ^2 test results for each model are shown in figure 7 for argon, figure 8 and table 2 for carbon, while the results for oxygen can be found in figure 9 and table 3. The T2K data for carbon and oxygen cross sections come from [4].

It can be seen that the χ^2 values for pure elements is significantly better than that of mixtures. This is because the CO analysis used coarser binnings than the pure element analyses.

A combined carbon and oxygen analysis was also computed, the χ^2 results can be found table 4. Much like with the CH target, the fits are found to be worse due to the coarser binning. It should be noted that GENIE with RPA has a much better χ^2 fit, but this isn't a very meaningful comparison given the χ^2 values.

Model on C T2K	**Regular χ^2 (29 degrees of freedom)**	**Shape Only χ^2 (Estimate)**
CRPA GENIE	43	48
GENIE + RPA	16	20
GENIE - RPA	49	49
Hartree Fock	69	58

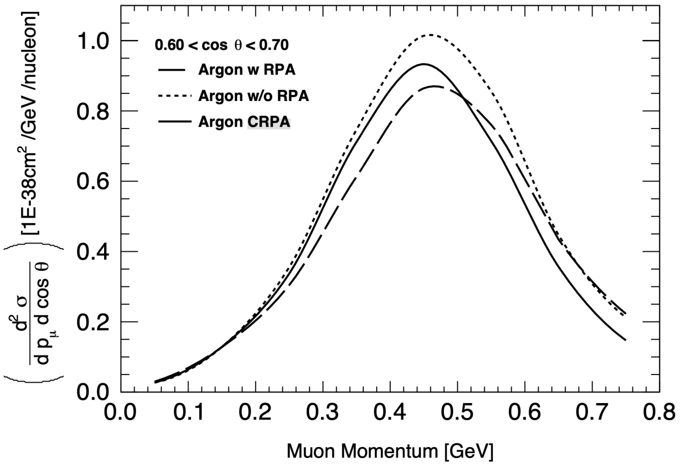
Table 2: Table showing χ^2 fits for the models to T2K data for C.

Model on O T2K	**Regular χ^2 (29 degrees of freedom)**	**Shape Only χ^2 (Estimate)**
CRPA GENIE	34	50
GENIE + RPA	26	35
GENIE - RPA	64	77
Hartree Fock	55	148

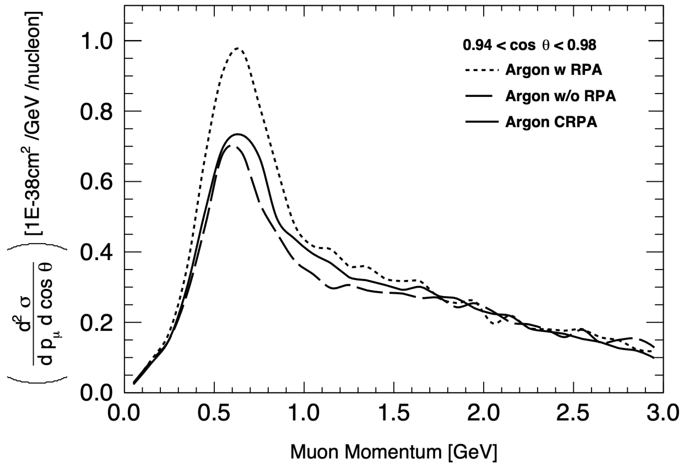
Table 3: Table showing χ^2 fits for the models to T2K data for O.

Model on CO T2K	**Regular χ^2 (58 degrees of freedom)**	**Shape Only χ^2 (True)**
CRPA GENIE	105	145
GENIE + RPA	45	63
GENIE - RPA	141	182
Hartree Fock	382	307

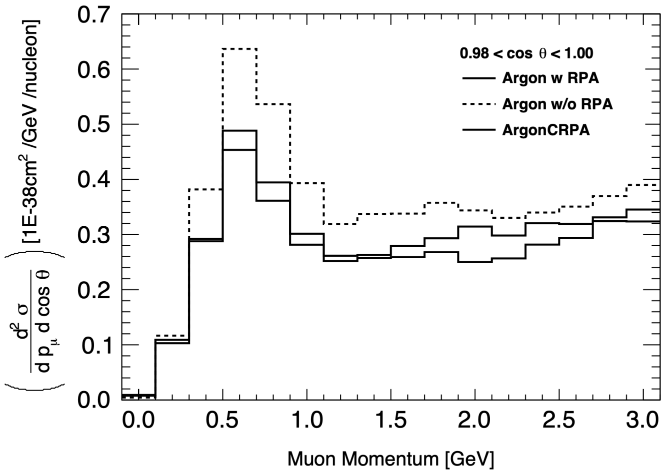
Table 4: Table showing combined χ^2 fits for the models to T2K data for simultaneous C & O analysis.



(a) $\cos\theta \in [0.60, 0.70]$

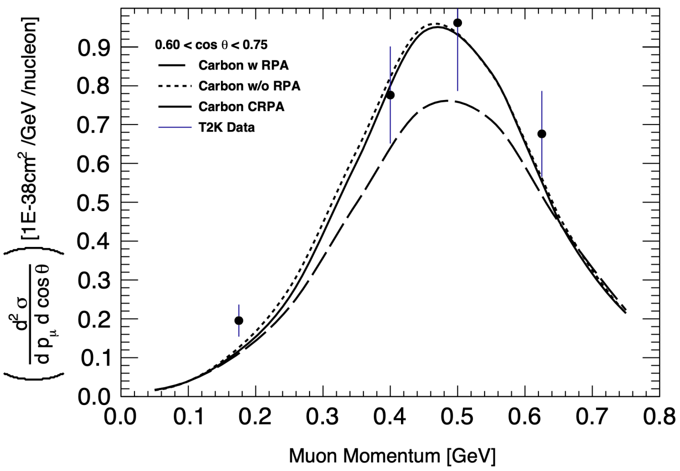


(b) $\cos\theta \in [0.94, 0.98]$

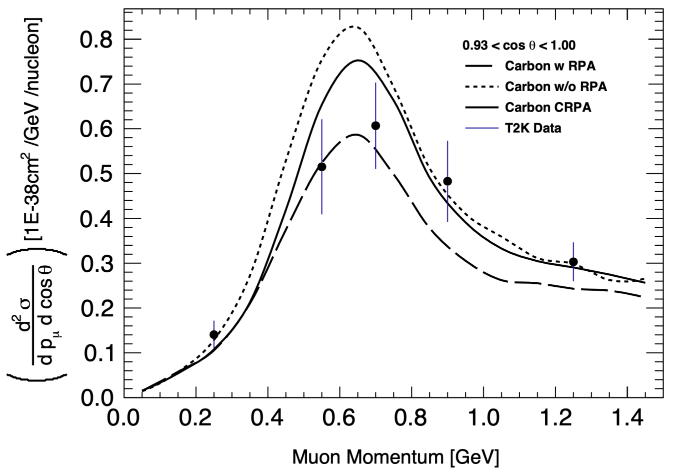


(c) $\cos\theta \in [0.98, 1.00]$

Figure 7: Plots showing the predicted CC0π double differential cross section for CRPA GENIE (solid), GENIE with RPA (small dash), and GENIE without RPA (big dash). On an argon target.

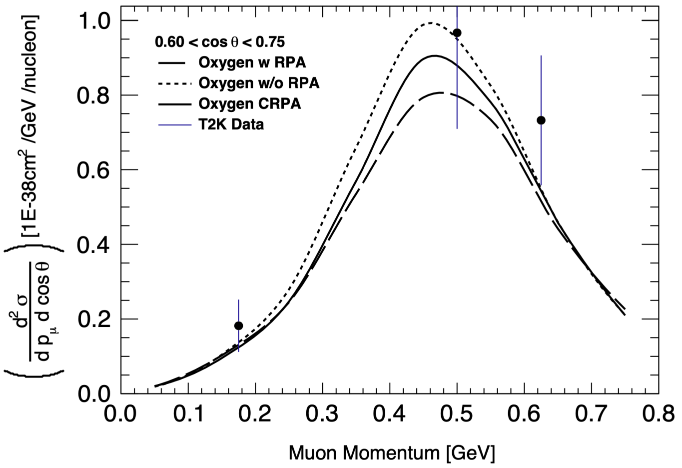


(a) $\cos\theta \in [0.60, 0.75]$

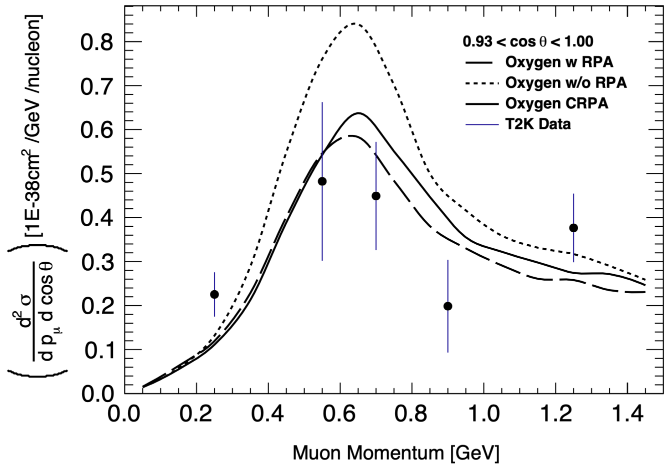


(b) $\cos\theta \in [0.93, 1.00]$

Figure 8: Plots showing the predicted CC0π double differential cross section for CRPA GENIE (solid), GENIE with RPA (small dash), and GENIE without RPA (big dash), T2K data with error bars. On a carbon target.

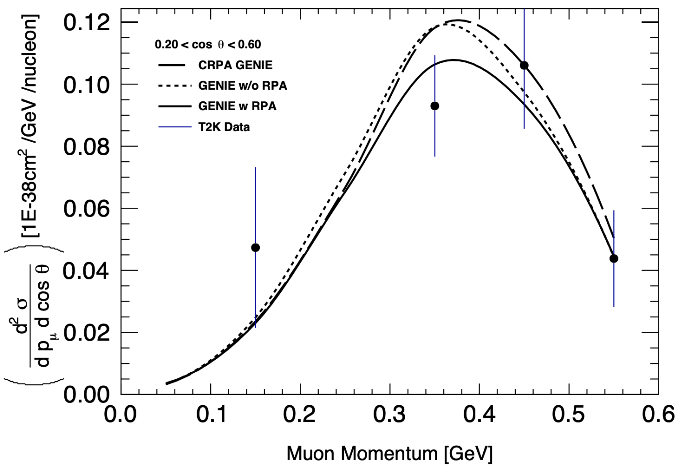


(a) $\cos\theta \in [0.60, 0.75]$

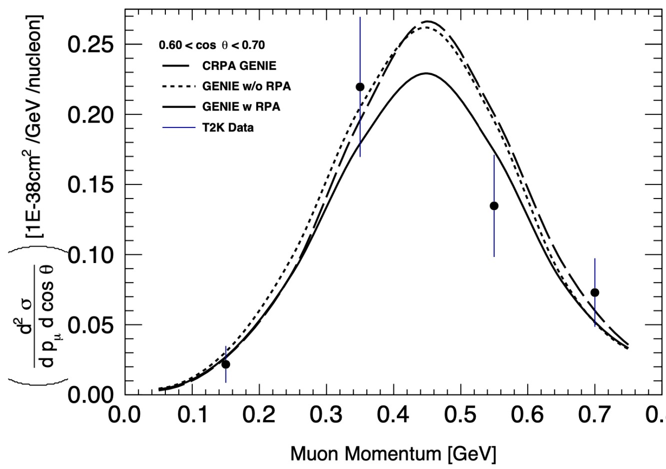


(b) $\cos\theta \in [0.93, 1.00]$

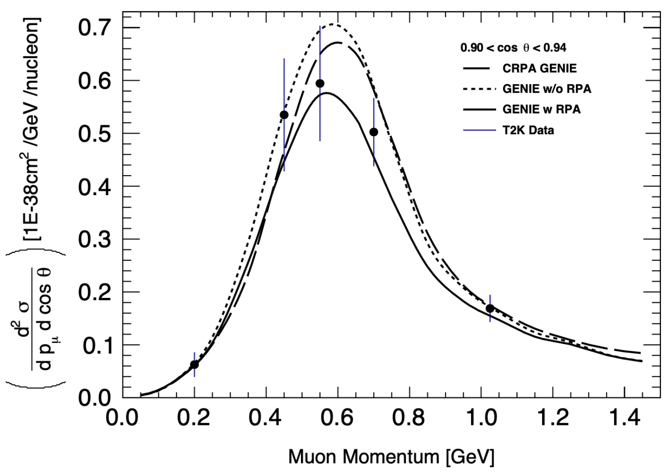
Figure 9: Plots showing the predicted CC0π double differential cross section for CRPA GENIE (solid), GENIE with RPA (small dash), and GENIE without RPA (big dash), T2K data with error bars. On an oxygen target.



(a) $\cos\theta \in [0.20, 0.60]$



(b) $\cos\theta \in [0.60, 0.70]$



(c) $\cos\theta \in [0.90, 0.94]$

Figure 10: Plots showing the predicted CC0π double differential cross section for CRPA GENIE (solid), GENIE with RPA (small dash), and GENIE without RPA (big dash), for anti muon neutrinos on CH. T2K data given with error bars.

2.6 $\bar{\nu_\mu}$ Comparison

Attempts to measure CP violation in the lepton sector require knowledge of both ν_μ and $\bar{\nu_\nu}$ cross sections, hence it is important to understand how models' treatment of each differ. Therefore the various models were also ran on CH using anti muon neutrinos. The χ^2 values were expected to be similar to regular muon neutrinos, and this is indeed the case as seen in table 5. Representative plots showing the comparison between models are shown in figure 10.

Model on $\bar{\nu_\mu}$ CH T2K	**Regular χ^2 (58 degrees of freedom)**	**Shape Only χ^2 (Estimate)**
CRPA GENIE	180	145
GENIE + RPA	187	202
GENIE - RPA	275	228

Table 5: Table showing χ^2 fits for the models to T2K data for CH using anti muon neutrinos.

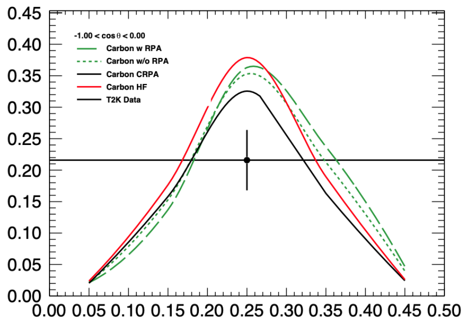
A combined ν_μ and $\bar{\nu_\mu}$ analysis was also carried out on CH and compared to T2K data[8]. The χ^2 values are shown in table 6. It is worth noting that in the $\bar{\nu_\mu}$ analyses GENIE with RPA's χ^2 is comparable to CRPA GENIE's, and for the combined analysis it apppears better. However when the shape only χ^2 is used GENIE with RPA's value becomes worse than CRPA GENIE's. This supports the idea that PPP applies in our analyses.

Combined ν_μ and $\bar{\nu_\mu}$ on T2K CH target	**Regular χ^2 (116 degrees of freedom)**	**Shape Only χ^2 (Estimate)**
CRPA GENIE	557	543
GENIE + RPA	367	582
GENIE - RPA	874	834

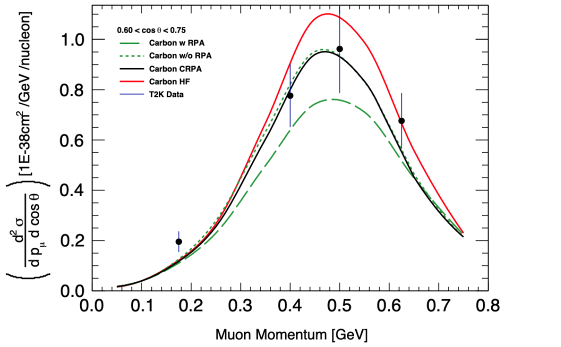
Table 6: Table showing combined χ^2 fits for the models using both ν_μ and $\bar{\nu_\mu}$ on a CH target. Using T2K data.

2.7 Hartree Fock Comparison

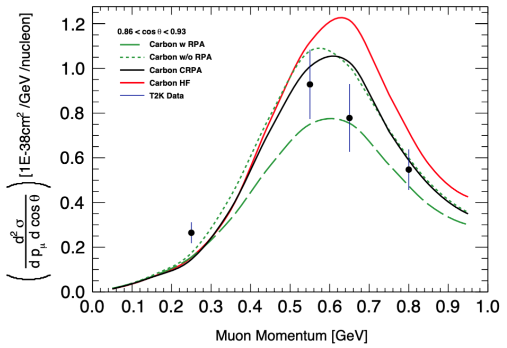
To see how CRPA varies from its base Hartree Fock model, the Hartree Fock model was also implemented into GENIE, ran on several data sets (carbon, oxygen, combined CO), and compared to T2K data. The χ^2 values can be found in tables 2, 3, and 4. It is worth noting that in all cases the Hartree Fock's χ^2 value is worse than that of CRPA. Figure 11 shows the predicted CCQE cross section on a carbon target compared to T2K data. It can be seen that the cross sections predicted by Hartree Fock are larger than tjose predicted by CRPA, which is due to the suppression supplied. The additional suppression supplied by CRPA visually matches the data better than the base Hartree Fock model, which suggests that CRPA does indeed provide some improvements.



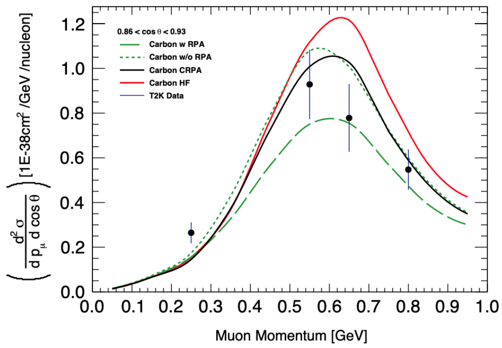
(a) $\cos\theta \in [-1.00, 0.00]$



(b) $\cos\theta \in [0.60, 0.75]$



(c) $\cos\theta \in [0.86, 0.93]$



(d) $\cos\theta \in [0.93, 1.00]$

Figure 11: Plots showing the predicted CCQE double differential cross section for CRPA GENIE (black), Hartree Fock (red), GENIE with RPA (green large dashed), GENIE without RPA (green small dashed). On a carbon target, T2K data from [6].

3 Discussion

It can be seen from the comparisons that CRPA provides suppression not present in regular GENIE, but less than GENIE with RPA. It also matches the data over a larger $\cos\theta$ range than GENIE with RPA.

It is very difficult to draw a definite conclusion from the data obtained. Ideally we would be able to produce a quantitative measure of how well each model fits the data in specified $\cos\theta$ and energy transfer ranges. This would allow us to determine which model provides a better description in each region.

There are of course lots of other aspects that could have been improved upon. For example increased Monte Carlo statistics could have been used to reduce statistical components in the comparisons. Additionally shape only covariance matrices could have been used for more of analyses instead of approximating the shape only χ^2.However the sample sizes used and the the estimated shape only χ^2 tests are accurate enough to allow comparison between the models and conduct initial analyses.

The next steps for this investigation (beyond my stay at CERN) will be to continue understanding why CRPA and RPA are so different at forward angles. The implementation will also be modified to allow it to be used for electron and neutrino scattering analyses. Some parameters in the model could be changed and tuned to data.

4 Conclusion

During my 2021 summer research placement under the supervision of Stephen Dolan in the EP NU group, I implemented CRPA into the GENIE Monte Carlo simulation. I then preceded to verify this implementation and carry out analyses on different targets to compare CRPA to other advanced models (GENIE and NEUT) as well as to T2K data. Further study was done into how CRPA affects anti neutrino predictions, as well how it compares to the Hartree Fock model upon which it builds.

Whilst it is difficult to say quantitatively that CRPA improves on previous predictions, its more sophisticated description of the nuclear physics that must be understood for future analyses of neutrino oscillations makes it a valuable addition to GENIE. Following the implementation made in this project, experimental collaborations will now be able to use CRPA as an alternative model to evaluate systematic uncertainties in their analyses.

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