

Unfolding: $H \rightarrow WW \rightarrow l\nu l\nu$

Freya Haslam

Cern Summer Student Program 2021

Abstract

In this project three unfolding methods are tested on three different observables for the $H_{ggF} \rightarrow WW$ channel. Iterative Bayes Unfolding, Singular Value Decomposition, and Matrix Inversion are tested on the distributions of $M_{\ell\ell}$, p_T^H , and $p_T^{\ell\ell}$. These results are compared to research completed prior to the project to see if different methods give better results, and if statistical uncertainties are the main contributor to the accurate choice of regularisation strength in unfolding. It was found that regularisation is preferable for all observables, and some unfolding methods that had not been previously tested proved better than other methods. Statistical uncertainties showed to have less effect on the regularisation strength than systematic uncertainties.

1 Introduction

$H \rightarrow WW \rightarrow l\nu l\nu$

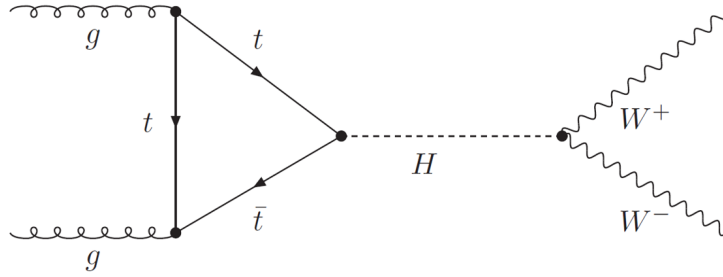


Figure 1: Feynmann diagram of the production of the Higgs Boson, and its decay to WW

The Higgs Boson is mainly produced at the LHC through the interaction of 2 gluons. Since gluons carry no mass, a closed loop of massive particles, such as quarks, must exist to allow the Higgs Boson to couple. There are many decay mechanisms available to the Higgs Boson; this channel produces many events with a signal over background ratio large enough to study. It first decays into two W bosons, then subsequently decays into two leptons and two neutrinos. The two leptons are most likely to be a muon and an electron due to the energy available. Their spin must be anti-aligned due to a net 0 spin from the W bosons.

Although this decay has a relatively high branching ratio, there are many difficulties with studying this channel. Despite the high numbers of events there are many background events too. Some of these background events are difficult to remove as the distribution is similar to the signal distribution. The main source of irremovable background events is the production of WW via other mechanisms, as well as Z+jets. Similarly, there is a large background from top quark production, however this background contribution can be reduced. [1]

Unfolding

The production of a Higgs Boson, and its decay, are now understood moderately well. The properties of the Higgs Boson, however, are not yet understood to the same level so it is necessary to continue to study the Higgs Boson to get a complete understanding of it. One method used to do this is unfolding. Unfolding is the process of taking measurements from a detector, then removing the effects of the detector to get an estimation of the true distribution. There are many unfolding methods, each with their own advantages and disadvantages. To find the optimum unfolding method, one must find the balance between the natural bias of the method, and

the variance. Bias occurs in the methods mostly due to the Monte Carlo model used to find the estimators.

Unfolding methods fall into two major categories; regularised or un-regularised. Regularised methods use a function to represent the smoothness of a distribution. This function is then minimised to smooth the distribution. If the distribution is smoothed in excess then valuable information may be lost. Prior to determining the most suitable unfolding method, the regularisation strength for each regularised method must be optimised so that the uncertainty is reduced without changing the distribution too much. The main regularised methods include: Iterative Bayes Unfolding (IBU); Singular Value Decomposition (SVD); TUnfold; Iterative Dynamically Stabilised (IDS); Gaussian Process (GP). In this project IBU and SVD are tested. The unregularised methods are Bin-by-bin and Matrix Inversion. [2]

Prior Research

For this Higgs Boson channel two unfolding methods have been tested up to the beginning of this project, Iterative Bayes Unfolding and a method comparable to SVD; In-Likelihood unfolding. Both these methods are regularised thus no un-regularised methods were tested. These two methods were tested on many different observables, in this project the observables were limited to $M_{\ell\ell}$, p_T^H , and $p_T^{\ell\ell}$, the main three observables with the highest sensitivity. This paper will compare the results of the previous study on expected data, this paper will compare the results on expected data with statistical uncertainties to examine the influences on regularisation. All observables studied here look at final states with only 0 or 1 jet to lower the chance of any events from other production modes.

2 Method

There were two main steps in the method for this project. All the data was provided at the start of the program with the relevant cuts and selections made. The first major step was to run the fit chain for each observable to prepare the data, then the data was unfolded using a chosen method.

FitChain

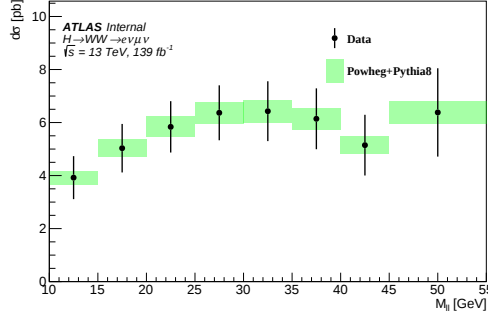
FitChain is a code repository used to complete several steps in this process. The first part of FitChain selects the appropriate events from the data. This means events with the correct final state that were measured accurately within the detector. From those selected events a subset is selected to improve the signal to background ratio. The remaining data is then fit in a low signal region that looks similar to the signal region to correct the Monte Carlo prediction in the signal region. The data within the signal region is then unfolded with different methods and different regularisation strengths. These results are then plotted and compared to the prediction of the interaction without taking into account the effects of the detector.

Unfolding

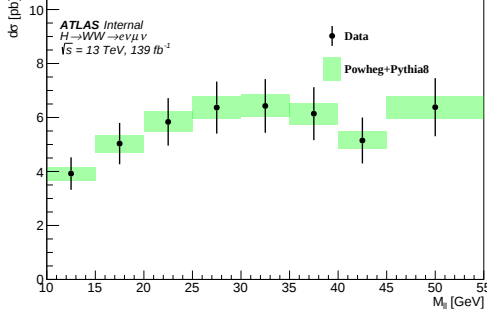
Three unfolding methods were tested in this project: Iterative Bayes Unfolding, Single Value Decomposition, and Matrix Inversion. IBU and SVD are regularised methods and matrix inversion is unregularised. The code for all three was written prior to this project. Details of the unfolding methods and their differences can be found in Ref [2]. Each unfolding method was run for the three observables. The regularised methods were run for several different choices of iterations. IBU was run for 1-10 iterations in steps of 1, and 10-100 in steps of 10. SVD was run from 2 iterations up to the number of bins.

3 Results and Discussion

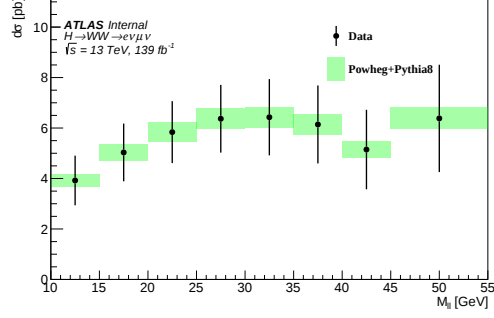
This project uses Run 2 expected data from ATLAS. Below only the graphs with the optimised regularisation will be shown, the rest of the results can be found in the appendix in section B. In conjunction with the graphs there are tables of the errors and uncertainties produced from the unfolding which can be found in Appendix A.



(a) $M_{\ell\ell}$ unfolded with IBU with 1 iteration



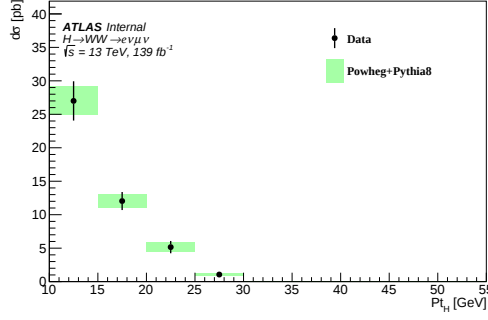
(b) $M_{\ell\ell}$ unfolded with SVD with 2 iterations



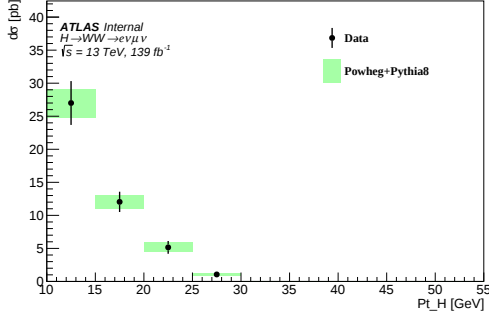
(c) $M_{\ell\ell}$ unfolded with Matrix Inversion

Figure 2: Unfolding results for $M_{\ell\ell}$, each with the regularisation optimised. The data is represented by the black dots with error bars showing the uncertainties, and the expected results from the model with uncertainties are shown as green boxes

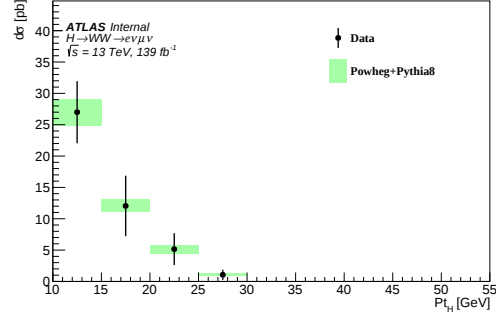
As seen from the graphs in figure 2 the three unfolding methods produce very similar results. None of the graph show significant migration between bins, however it is immediately clear that the matrix inversion method produces the highest uncertainties. Using the tables in appendix A the error on unfolding was smallest for svd with 2 iterations. Therefore, for this observable, despite being the lowest number of iterations possible, some regularisation may lead to better results. This little regularisation is expected since there are few events due large number of background events found in this channel. Many background contributions have been removed leaving a lower number of events, and thus less variation will be seen in the bins. The previous study found that the ideal number of iterations for the SVD inspired unfolding method was 7, suggesting that systematic uncertainties have a greater effect on the distribution than the statistical uncertainties.



(a) PtH unfolded with IBU with 1 iteration



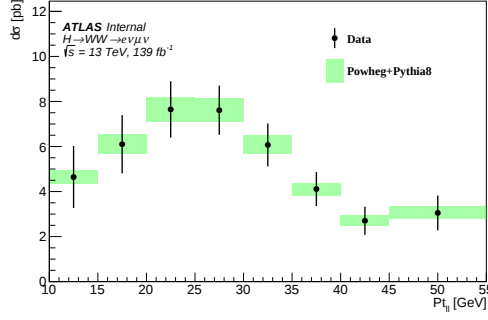
(b) PtH unfolded with SVD with 2 iterations



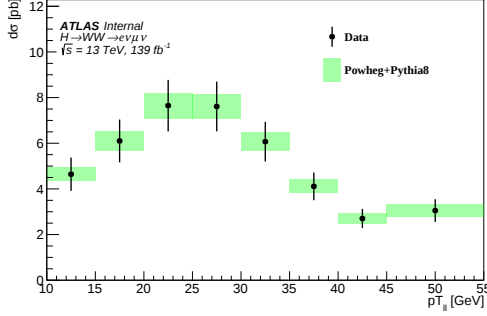
(c) PtH unfolded with Matrix Inversion

Figure 3: Unfolding results for PtH, each with the regularisation optimised. The data is represented by the black dots with error bars showing the uncertainties, and the expected results from the model with uncertainties are shown as green boxes

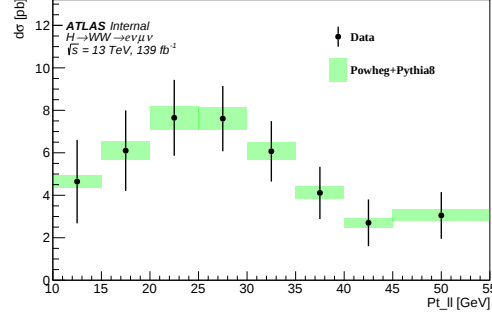
For PtH it is again true that regularisation seems to improve the results as the uncertainties produced from matrix inversion are the largest of the three methods. Unlike $M_{\ell\ell}$, the best method for PtH was the iterative bayes unfolding method with a regularisation strength of 1 iteration. These results can be seen in figure 3. The previous study concluded that 18 iterations were best. It is expected that PtH would favour a different method to $M_{\ell\ell}$. Most of the events in the PtH distribution are in the first bin, whereas the $M_{\ell\ell}$ distribution is more even, this means that bin migration will behave differently for each distribution and each unfolding method treats bin migration differently. This shows why it is important to investigate more than one observable.



(a) $P_T^{\ell\ell}$ unfolded with IBU with 1 iteration



(b) $P_T^{\ell\ell}$ unfolded with SVD with 2 iterations



(c) $P_T^{\ell\ell}$ unfolded with Matrix Inversion

Figure 4: Unfolding results for $P_T^{\ell\ell}$, each with the regularisation optimised. The data is represented by the black dots with error bars showing the uncertainties, and the expected results from the model with uncertainties are shown as green boxes

For $P_T^{\ell\ell}$ SVD proved to be the best method based on the uncertainties. The un-regularised matrix inversion method had significantly higher uncertainties than both SVD and IBU; SVD also had lower uncertainties than IBU in every bin. The best regularisation strength found for SVD was 2 iterations, again the lowest it could be. The previous study found the best regularisation for the comparable method was 8. There is a clear pattern that the systematic uncertainty distorts the distribution more than the statistical uncertainty regardless of the observable, thus when determining the best unfolding method systematics must be taken into account before any method can be considered the best. However, since systematic errors are the main contributors, increasing the regularisation significantly could lead to the statistical errors being smoothed out in excess, leading to a loss in information. This knowledge is important as it is common place to find the optimum unfolding method and regularisation strength using statistical errors, then applying that method to sys + stat data.

Despite matrix inversion consistently being the the worst method for each observable, it was important to look investigate whether regularisation has a significant effect when unfolding this data. Even just from a visual comparison of the graphs shown above, it is clear that some degree of regularisation can significantly reduce uncertainties on the unfolding method. A full conclusion about regularisation cannot be drawn since only one unregularised method was used. Similarly another regularised unfolding method may produce better results than the two studied here. Due to time constraints on this and other projects the code for other unfolding methods could not be produced in time, thus an investigation into them was not possible. Work has been done to study the observable Yll in this project, however due to technical troubles it could not be completed in time for the project deadline.

4 Conclusion

From this project it can be concluded that different unfolding methods are necessary for different observables, however regularisation is preferable for all. It is clear that systematic uncertainties have a larger effect on the regularisation strength than statistical uncertainties in this channel and therefore must be included in any unfolding tests. It cannot be concluded which unfolding method is best for each variable as not all methods could be tested, but from the two methods tested here that have not been investigated prior to this project SVD shows a lot of potential as a good method for some observables.

Appendix

In the appendix select results have been chosen to show why the optimum regularisation was selected.

A Tables

A.1 $M_{\ell\ell}$

a)

```

Now unfolding...
Iteration : 0
Chi^2 of change 2.45513e-28
Calculating covariances due to number of measured events
coverage probability: 68.25% 68.21% 68.25% 68.26% 68.26% 68.25% 68.20% 68.27%
=====
Bin   Train   Train   Test   Test Unfolded Error on   Diff   Pull
      Truth Measured Truth      Input      Output Unfolding
=====
1     643     288     643     288     643.4     133.0     0.0     0.0
2     826     278     826     278     825.6     158.1     -0.0    -0.0
3     958     326     958     326     957.6     158.9     0.0     0.0
4     1045     355     1045     355     1044.5     178.0     0.0     0.0
5     1054     363     1054     363     1054.5     185.1     0.0     0.0
6     1087     357     1087     357     1087.4     188.3     -0.0    -0.0
7     844     384     844     384     844.4     187.8     0.0     0.0
8     1047     397     1047     397     1046.8     273.3     0.0     0.0
=====
7424     2587     7424     2587     7424.1     146.0     0.0     0.0
=====
Bin-by-bin Chi^2/NDF=1.44735e-29/8
=====
Bin   Train   Train   Test   Test Unfolded Error on   Diff   Pull
      Truth Measured Truth      Input      Output Unfolding
=====
1     643     288     643     288     643.4     133.0     0.0     0.0
2     826     278     826     278     825.6     158.1     -0.0    -0.0
3     958     326     958     326     957.6     158.9     0.0     0.0
4     1045     355     1045     355     1044.5     178.0     0.0     0.0
5     1054     363     1054     363     1054.5     185.1     0.0     0.0
6     1087     357     1087     357     1087.4     188.3     -0.0    -0.0
7     844     384     844     384     844.4     187.8     0.0     0.0
8     1047     397     1047     397     1046.8     273.3     0.0     0.0
=====
7424     2587     7424     2587     7424.1     146.0     0.0     0.0
=====
Bin-by-bin Chi^2/NDF=1.44735e-29/8
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.259 [pb]

```

b)

```

Now unfolding...
Iteration : 0
Chi^2 of change 2.45513e-28
Iteration : 1
Chi^2 of change 6.06698e-30
Calculating covariances due to number of measured events
coverage probability: 68.25% 68.21% 68.25% 68.26% 68.26% 68.24% 68.17% 68.27%
=====
Bin   Train   Train   Test   Test Unfolded Error on   Diff   Pull
      Truth Measured Truth      Input      Output Unfolding
=====
1     643     288     643     288     643.4     148.8     0.0     0.0
2     826     278     826     278     825.6     168.4     -0.0    -0.0
3     958     326     958     326     957.6     177.8     0.0     0.0
4     1045     355     1045     355     1044.5     191.2     0.0     0.0
5     1054     363     1054     363     1054.5     211.3     0.0     0.0
6     1087     357     1087     357     1087.4     214.5     -0.0    -0.0
7     844     384     844     384     844.4     219.6     0.0     0.0
8     1047     397     1047     397     1046.8     308.8     0.0     0.0
=====
7424     2587     7424     2587     7424.1     146.0     0.0     0.0
=====
Bin-by-bin Chi^2/NDF=1.3507e-29/8
=====
Bin   Train   Train   Test   Test Unfolded Error on   Diff   Pull
      Truth Measured Truth      Input      Output Unfolding
=====
1     643     288     643     288     643.4     148.8     0.0     0.0
2     826     278     826     278     825.6     168.4     -0.0    -0.0
3     958     326     958     326     957.6     177.8     0.0     0.0
4     1045     355     1045     355     1044.5     191.2     0.0     0.0
5     1054     363     1054     363     1054.5     211.3     0.0     0.0
6     1087     357     1087     357     1087.4     214.5     -0.0    -0.0
7     844     384     844     384     844.4     219.6     0.0     0.0
8     1047     397     1047     397     1046.8     308.8     0.0     0.0
=====
7424     2587     7424     2587     7424.1     146.0     0.0     0.0
=====
Bin-by-bin Chi^2/NDF=1.3507e-29/8
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.259 [pb]

```

c)

```

Now unfolding...
Iteration : 0
Chi^2 of change 2.45513e-28
Iteration : 1
Chi^2 of change 6.06698e-30
Iteration : 2
Chi^2 of change 6.06698e-30
Calculating covariances due to number of measured events
coverage probability: 68.25% 68.20% 68.25% 68.25% 68.26% 68.23% 68.15% 68.27%
=====
Bin   Train   Train   Test   Test Unfolded Error on   Diff   Pull
      Truth Measured Truth      Input      Output Unfolding
=====
1     643     288     643     288     643.4     152.3     0.0     0.0
2     826     278     826     278     825.6     174.0     -0.0    -0.0
3     958     326     958     326     957.6     184.6     0.0     0.0
4     1045     355     1045     355     1044.5     200.0     0.0     0.0
5     1054     363     1054     363     1054.5     223.0     0.0     0.0
6     1087     357     1087     357     1087.4     227.1     -0.0    -0.0
7     844     384     844     384     844.4     234.4     0.0     0.0
8     1047     397     1047     397     1046.8     318.9     0.0     0.0
=====
7424     2587     7424     2587     7424.1     146.0     0.0     0.0
=====
Bin-by-bin Chi^2/NDF=1.51934e-29/8
=====
Bin   Train   Train   Test   Test Unfolded Error on   Diff   Pull
      Truth Measured Truth      Input      Output Unfolding
=====
1     643     288     643     288     643.4     152.3     0.0     0.0
2     826     278     826     278     825.6     174.0     -0.0    -0.0
3     958     326     958     326     957.6     184.6     0.0     0.0
4     1045     355     1045     355     1044.5     200.0     0.0     0.0
5     1054     363     1054     363     1054.5     223.0     0.0     0.0
6     1087     357     1087     357     1087.4     227.1     -0.0    -0.0
7     844     384     844     384     844.4     234.4     0.0     0.0
8     1047     397     1047     397     1046.8     318.9     0.0     0.0
=====
7424     2587     7424     2587     7424.1     146.0     0.0     0.0
=====
Bin-by-bin Chi^2/NDF=1.51934e-29/8
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.259 [pb]

```

d)

```

coverage probability: 68.25% 68.20% 68.25% 68.25% 68.26% 68.22% 68.15% 68.27%
=====
Bin   Train   Train   Test   Test Unfolded Error on   Diff   Pull
      Truth Measured Truth      Input      Output Unfolding
=====
1     643     288     643     288     643.4     153.3     0.0     0.0
2     826     278     826     278     825.6     176.4     -0.0    -0.0
3     958     326     958     326     957.6     188.4     0.0     0.0
4     1045     355     1045     355     1044.5     206.0     0.0     0.0
5     1054     363     1054     363     1054.5     232.6     0.0     0.0
6     1087     357     1087     357     1087.4     239.7     -0.0    -0.0
7     844     384     844     384     844.4     248.3     0.0     0.0
8     1047     397     1047     397     1046.8     324.1     0.0     0.0
=====
7424     2587     7424     2587     7424.1     146.0     0.0     0.0
=====
Bin-by-bin Chi^2/NDF=1.02197e-29/8
=====
Bin   Train   Train   Test   Test Unfolded Error on   Diff   Pull
      Truth Measured Truth      Input      Output Unfolding
=====
1     643     288     643     288     643.4     153.3     0.0     0.0
2     826     278     826     278     825.6     176.4     -0.0    -0.0
3     958     326     958     326     957.6     188.4     0.0     0.0
4     1045     355     1045     355     1044.5     206.0     0.0     0.0
5     1054     363     1054     363     1054.5     232.6     0.0     0.0
6     1087     357     1087     357     1087.4     239.7     -0.0    -0.0
7     844     384     844     384     844.4     248.3     0.0     0.0
8     1047     397     1047     397     1046.8     324.1     0.0     0.0
=====
7424     2587     7424     2587     7424.1     146.0     0.0     0.0
=====
Bin-by-bin Chi^2/NDF=1.02197e-29/8
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.259 [pb]

```

Table 1: a: Uncertainties from $M_{\ell\ell}$ unfolded with IBU with 1 iteration; b: Uncertainties from $M_{\ell\ell}$ unfolded with IBU with 2 iterations; c: Uncertainties from $M_{\ell\ell}$ unfolded with IBU with 3 iterations; d: Uncertainties from $M_{\ell\ell}$ unfolded with IBU with 100 iterations

SVD init 8 x 8 bins, kreg=2
Subtract 343.126 fakes from measured distribution
coverage probability: 68.26% 68.26% 68.26% 68.26% 68.26% 68.26% 68.26% 68.26%

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	643	288	643	288	643.4	98.8	-0.0	-0.0
2	826	278	826	278	825.6	125.5	-0.0	-0.0
3	958	326	958	326	957.6	144.4	-0.0	-0.0
4	1045	355	1045	355	1044.5	158.1	-0.0	-0.0
5	1054	363	1054	363	1054.5	163.1	-0.0	-0.0
6	1007	357	1007	357	1007.4	161.2	-0.0	-0.0
7	844	304	844	304	844.4	139.6	-0.0	-0.0
8	1047	397	1047	397	1046.8	176.6	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=1.13718\text{e}-13/8$

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	643	288	643	288	643.4	98.8	-0.0	-0.0
2	826	278	826	278	825.6	125.5	-0.0	-0.0
3	958	326	958	326	957.6	144.4	-0.0	-0.0
4	1045	355	1045	355	1044.5	158.1	-0.0	-0.0
5	1054	363	1054	363	1054.5	163.1	-0.0	-0.0
6	1007	357	1007	357	1007.4	161.2	-0.0	-0.0
7	844	304	844	304	844.4	139.6	-0.0	-0.0
8	1047	397	1047	397	1046.8	176.6	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=1.13718\text{e}-13/8$
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.259 [pb]

a)

SVD init 8 x 8 bins, kreg=3
Subtract 343.126 fakes from measured distribution
coverage probability: 68.26% 68.26% 68.26% 68.27% 68.27% 68.27% 68.26% 68.26%

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	643	288	643	288	643.4	110.8	-0.0	-0.0
2	826	278	826	278	825.6	132.0	-0.0	-0.0
3	958	326	958	326	957.6	145.6	-0.0	-0.0
4	1045	355	1045	355	1044.5	161.7	-0.0	-0.0
5	1054	363	1054	363	1054.5	174.2	-0.0	-0.0
6	1007	357	1007	357	1007.4	183.0	-0.0	-0.0
7	844	304	844	304	844.4	171.2	-0.0	-0.0
8	1047	397	1047	397	1046.8	230.4	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=7.86446\text{e}-16/8$

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	643	288	643	288	643.4	110.8	-0.0	-0.0
2	826	278	826	278	825.6	132.0	-0.0	-0.0
3	958	326	958	326	957.6	145.6	-0.0	-0.0
4	1045	355	1045	355	1044.5	161.7	-0.0	-0.0
5	1054	363	1054	363	1054.5	174.2	-0.0	-0.0
6	1007	357	1007	357	1007.4	183.0	-0.0	-0.0
7	844	304	844	304	844.4	171.2	-0.0	-0.0
8	1047	397	1047	397	1046.8	230.4	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=7.86446\text{e}-16/8$
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.259 [pb]

b)

SVD init 8 x 8 bins, kreg=4
Subtract 343.126 fakes from measured distribution
coverage probability: 68.27% 68.25% 68.25% 68.26% 68.27% 68.27% 68.26% 68.26%

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	643	288	643	288	643.4	126.4	-0.0	-0.0
2	826	278	826	278	825.6	139.7	-0.0	-0.0
3	958	326	958	326	957.6	152.0	-0.0	-0.0
4	1045	355	1045	355	1044.5	167.8	-0.0	-0.0
5	1054	363	1054	363	1054.5	180.2	-0.0	-0.0
6	1007	357	1007	357	1007.4	191.0	-0.0	-0.0
7	844	304	844	304	844.4	185.0	-0.0	-0.0
8	1047	397	1047	397	1046.8	266.7	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=3.52544\text{e}-17/8$

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	643	288	643	288	643.4	126.4	-0.0	-0.0
2	826	278	826	278	825.6	139.7	-0.0	-0.0
3	958	326	958	326	957.6	152.0	-0.0	-0.0
4	1045	355	1045	355	1044.5	167.8	-0.0	-0.0
5	1054	363	1054	363	1054.5	180.2	-0.0	-0.0
6	1007	357	1007	357	1007.4	191.0	-0.0	-0.0
7	844	304	844	304	844.4	185.0	-0.0	-0.0
8	1047	397	1047	397	1046.8	266.7	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=3.52544\text{e}-17/8$
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.259 [pb]

c)

SVD init 8 x 8 bins, kreg=5
Subtract 343.126 fakes from measured distribution
coverage probability: 68.27% 68.24% 68.24% 68.26% 68.26% 68.26% 68.24% 68.26%

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	643	288	643	288	643.4	137.0	-0.0	-0.0
2	826	278	826	278	825.6	146.6	-0.0	-0.0
3	958	326	958	326	957.6	161.7	-0.0	-0.0
4	1045	355	1045	355	1044.5	176.1	-0.0	-0.0
5	1054	363	1054	363	1054.5	188.3	-0.0	-0.0
6	1007	357	1007	357	1007.4	198.9	-0.0	-0.0
7	844	304	844	304	844.4	192.8	-0.0	-0.0
8	1047	397	1047	397	1046.8	290.3	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=4.55182\text{e}-18/8$

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	643	288	643	288	643.4	137.0	-0.0	-0.0
2	826	278	826	278	825.6	146.6	-0.0	-0.0
3	958	326	958	326	957.6	161.7	-0.0	-0.0
4	1045	355	1045	355	1044.5	176.1	-0.0	-0.0
5	1054	363	1054	363	1054.5	188.3	-0.0	-0.0
6	1007	357	1007	357	1007.4	198.9	-0.0	-0.0
7	844	304	844	304	844.4	192.8	-0.0	-0.0
8	1047	397	1047	397	1046.8	290.3	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=4.55182\text{e}-18/8$
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.259 [pb]

d)

Table 2: a: Uncertainties from $M_{\ell\ell}$ unfolded with SVD with 2 iteration; b: Uncertainties from $M_{\ell\ell}$ unfolded with SVD with 3 iterations; c: Uncertainties from $M_{\ell\ell}$ unfolded with SVD with 4 iterations; d: Uncertainties from $M_{\ell\ell}$ unfolded with SVD with 5 iterations

Subtract 343.126 fakes from measured distribution
coverage probability: 68.25% 68.21% 68.25% 68.26% 68.26% 68.23% 68.16% 68.27%

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	643	288	643	288	643.4	161.3	0.0	0.0
2	826	278	826	278	825.6	187.6	-0.0	-0.0
3	958	326	958	326	957.6	201.2	0.0	0.0
4	1045	355	1045	355	1044.5	220.0	-0.0	-0.0
5	1054	363	1054	363	1054.5	247.9	0.0	0.0
6	1007	357	1007	357	1007.4	253.5	0.0	0.0
7	844	304	844	304	844.4	258.3	-0.0	-0.0
8	1047	397	1047	397	1046.8	348.8	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	0.0	0.0	0.0

Bin-by-bin $\chi^2/\text{NDF}=3.01808\text{e}-29/8$

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	643	288	643	288	643.4	161.3	0.0	0.0
2	826	278	826	278	825.6	187.6	-0.0	-0.0
3	958	326	958	326	957.6	201.2	0.0	0.0
4	1045	355	1045	355	1044.5	220.0	-0.0	-0.0
5	1054	363	1054	363	1054.5	247.9	0.0	0.0
6	1007	357	1007	357	1007.4	253.5	0.0	0.0
7	844	304	844	304	844.4	258.3	-0.0	-0.0
8	1047	397	1047	397	1046.8	348.8	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	0.0	0.0	0.0

Bin-by-bin $\chi^2/\text{NDF}=3.01808\text{e}-29/8$
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.259 [pb]

Figure 5: Uncertainties from $M_{\ell\ell}$ unfolded with Matrix Inversion

A.2 P_T^H

Now unfolding...
Iteration : 0
Chi² of change 3.46887e-28
Calculating covariances due to number of measured events
coverage probability: 68.26% 68.25% 68.27% 68.17%

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	481.4	0.0	0.0
2	1975	620	1975	620	1975.4	221.0	0.0	0.0
3	846	264	846	264	845.8	150.9	0.0	0.0
4	174	62	174	62	173.8	64.5	0.0	0.0
7424	2587	7424	2587	7424.1	146.0	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=4.13656e-30/4

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	481.4	0.0	0.0
2	1975	620	1975	620	1975.4	221.0	0.0	0.0
3	846	264	846	264	845.8	150.9	0.0	0.0
4	174	62	174	62	173.8	64.5	0.0	0.0
7424	2587	7424	2587	7424.1	146.0	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=4.13656e-30/4
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230

a)

Now unfolding...
Iteration : 0
Chi² of change 3.46887e-28
Iteration : 1
Chi² of change 2.78572e-28
Calculating covariances due to number of measured events
coverage probability: 68.26% 68.24% 68.27% 68.15%

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	527.4	0.0	0.0
2	1975	620	1975	620	1975.4	254.7	0.0	0.0
3	846	264	846	264	845.0	207.5	-0.0	-0.0
4	174	62	174	62	173.8	93.2	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=1.25656e-29/4

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	527.4	0.0	0.0
2	1975	620	1975	620	1975.4	254.7	0.0	0.0
3	846	264	846	264	845.0	207.5	-0.0	-0.0
4	174	62	174	62	173.8	93.2	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=1.25656e-29/4
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.2589 [pb]

b)

Now unfolding...
Iteration : 0
Chi² of change 3.46887e-28
Iteration : 1
Chi² of change 2.78572e-28
Iteration : 2
Chi² of change 1.69314e-29
Calculating covariances due to number of measured events
coverage probability: 68.26% 68.24% 68.27% 68.14%

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	565.2	0.0	0.0
2	1975	620	1975	620	1975.4	294.1	0.0	0.0
3	846	264	846	264	845.8	244.8	-0.0	-0.0
4	174	62	174	62	173.8	187.9	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=1.08498e-29/4

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	565.2	0.0	0.0
2	1975	620	1975	620	1975.4	294.1	0.0	0.0
3	846	264	846	264	845.8	244.8	-0.0	-0.0
4	174	62	174	62	173.8	187.9	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=1.08498e-29/4
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.2589 [pb]

c)

coverage probability: 68.27% 68.25% 68.26% 68.15%

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	804.7	0.0	0.0
2	1975	620	1975	620	1975.4	777.9	-0.0	-0.0
3	846	264	846	264	845.8	384.5	-0.0	-0.0
4	174	62	174	62	173.8	127.6	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin Chi²/NDF=1.11649e-29/4

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	804.7	0.0	0.0
2	1975	620	1975	620	1975.4	777.9	-0.0	-0.0
3	846	264	846	264	845.8	384.5	-0.0	-0.0
4	174	62	174	62	173.8	127.6	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin Chi²/NDF=1.11649e-29/4
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.2589 [pb]

d)

Table 3: a: Uncertainties from P_T^H unfolded with IBU with 1 iteration; b: Uncertainties from P_T^H unfolded with IBU with 2 iterations; c: Uncertainties from P_T^H unfolded with IBU with 3 iterations; d: Uncertainties from P_T^H unfolded with IBU with 100 iterations

SVD init 4 x 4 bins, kreg=2
Subtract 343.126 fakes from measured distribution
coverage probability: 68.25% 68.27% 68.24% 68.24%

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	544.1	-0.0	-0.0
2	1975	620	1975	620	1975.4	251.7	-0.0	-0.0
3	846	264	846	264	845.8	150.6	-0.0	-0.0
4	174	62	174	62	173.8	61.3	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin Chi²/NDF=0.59115e-18/4

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	544.1	-0.0	-0.0
2	1975	620	1975	620	1975.4	251.7	-0.0	-0.0
3	846	264	846	264	845.8	150.6	-0.0	-0.0
4	174	62	174	62	173.8	61.3	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin Chi²/NDF=0.59115e-18/4
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.2589 [pb]

a)

SVD init 4 x 4 bins, kreg=3
Subtract 343.126 fakes from measured distribution
coverage probability: 68.26% 68.25% 68.25% 68.17%

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	636.4	-0.0	-0.0
2	1975	620	1975	620	1975.4	389.7	-0.0	-0.0
3	846	264	846	264	845.8	259.0	-0.0	-0.0
4	174	62	174	62	173.8	83.4	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin Chi²/NDF=2.79956e-20/4

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	636.4	-0.0	-0.0
2	1975	620	1975	620	1975.4	389.7	-0.0	-0.0
3	846	264	846	264	845.8	259.0	-0.0	-0.0
4	174	62	174	62	173.8	83.4	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin Chi²/NDF=2.79956e-20/4
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.2589 [pb]

b)

SVD init 4 x 4 bins, kreg=4
Subtract 343.126 fakes from measured distribution
coverage probability: 68.27% 68.26% 68.26% 68.16%

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	716.0	-0.0	-0.0
2	1975	620	1975	620	1975.4	567.6	-0.0	-0.0
3	846	264	846	264	845.8	380.2	-0.0	-0.0
4	174	62	174	62	173.8	180.1	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin Chi²/NDF=9.22911e-22/4

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	716.0	-0.0	-0.0
2	1975	620	1975	620	1975.4	567.6	-0.0	-0.0
3	846	264	846	264	845.8	380.2	-0.0	-0.0
4	174	62	174	62	173.8	180.1	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin Chi²/NDF=9.22911e-22/4
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.2589 [pb]

c)

SVD init 4 x 4 bins, kreg=5
Subtract 343.126 fakes from measured distribution
coverage probability: 68.27% 68.25% 68.27% 68.16%

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	813.0	0.0	0.0
2	1975	620	1975	620	1975.4	791.2	0.0	0.0
3	846	264	846	264	845.8	416.1	-0.0	-0.0
4	174	62	174	62	173.8	131.4	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=1.37282e-30/4

Bin	Train Truth	Train Measured	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1640	4429	1640	4429.2	813.0	0.0	0.0
2	1975	620	1975	620	1975.4	791.2	0.0	0.0
3	846	264	846	264	845.8	416.1	-0.0	-0.0
4	174	62	174	62	173.8	131.4	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=1.37282e-30/4
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.2589 [pb]

d)

Table 4: a: Uncertainties from P_T^H unfolded with SVD with 2 iteration; b: Uncertainties from P_T^H unfolded with SVD with 3 iterations; c: Uncertainties from P_T^H unfolded with SVD with 4 iterations; d: Uncertainties from P_T^H unfolded with SVD with 5 iterations

Subtract 363.126 fakes from measured distribution
coverage probability: 68.27% 68.25% 68.27% 68.16%

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1648	4429	1648	4429.2	813.8	-0.0	-0.0
2	1975	628	1975	628	1975.4	791.2	-0.0	-0.0
3	846	264	846	264	845.8	416.1	0.0	0.0
4	174	62	174	62	173.8	131.4	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.8	-0.0	-0.0	-0.0

Bin-by-bin Chi²/NDF=4.33662e-29/4

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	4429	1648	4429	1648	4429.2	813.8	-0.0	-0.0
2	1975	628	1975	628	1975.4	791.2	-0.0	-0.0
3	846	264	846	264	845.8	416.1	0.0	0.0
4	174	62	174	62	173.8	131.4	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.8	-0.0	-0.0	-0.0

Bin-by-bin Chi²/NDF=4.33662e-29/4
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0238
total XS: 45.2589 [pb]

Figure 6: Uncertainties from PtH unfolded with Matrix Inversion

A.3 $P_T^{\ell\ell}$

Now unfolding...
Iteration : 0
Chi² of change 9.78912e-29
Calculating covariances due to number of measured events
coverage probability: 68.25% 68.22% 68.25% 68.18% 68.28% 68.19% 68.26% 68.28% 68.27%

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	225.5	0.0	0.0
2	1081	286	1081	286	1080.9	212.3	-0.0	-0.0
3	1255	485	1255	485	1254.6	204.9	0.0	0.0
4	1248	427	1248	427	1248.4	179.4	0.0	0.0
5	996	364	996	364	995.7	155.3	0.0	0.0
6	674	262	674	262	674.4	124.3	0.0	0.0
7	443	177	443	177	443.5	102.8	-0.0	-0.0
8	580	213	580	213	580.4	126.7	-0.0	-0.0
9	544	272	544	272	544.5	174.6	0.0	0.0
7424	2587	7424	2587	7424.1	146.8	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=5.30553e-30/9

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	225.5	0.0	0.0
2	1081	286	1081	286	1080.9	212.3	-0.0	-0.0
3	1255	485	1255	485	1254.6	204.9	0.0	0.0
4	1248	427	1248	427	1248.4	179.4	0.0	0.0
5	996	364	996	364	995.7	155.3	0.0	0.0
6	674	262	674	262	674.4	124.3	0.0	0.0
7	443	177	443	177	443.5	102.8	-0.0	-0.0
8	580	213	580	213	580.4	126.7	-0.0	-0.0
9	544	272	544	272	544.5	174.6	0.0	0.0
7424	2587	7424	2587	7424.1	146.8	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=5.30553e-30/9
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0238
total XS: 45.259 [pb]

a)

Now unfolding...
Iteration : 0
Chi² of change 9.78912e-29
Iteration : 1
Chi² of change 7.13988e-29
Calculating covariances due to number of measured events
coverage probability: 68.26% 68.21% 68.25% 68.14% 68.16% 68.13% 68.25% 68.19% 68.27%

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	276.2	0.0	0.0
2	1081	286	1081	286	1080.9	254.2	-0.0	-0.0
3	1255	485	1255	485	1254.6	237.0	0.0	0.0
4	1248	427	1248	427	1248.4	202.3	0.0	0.0
5	996	364	996	364	995.7	182.1	0.0	0.0
6	674	262	674	262	674.4	158.2	0.0	0.0
7	443	177	443	177	443.5	138.5	-0.0	-0.0
8	580	213	580	213	580.4	151.3	-0.0	-0.0
9	544	272	544	272	544.5	212.6	0.0	0.0
7424	2587	7424	2587	7424.1	146.8	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=8.89224e-30/9

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	276.2	0.0	0.0
2	1081	286	1081	286	1080.9	254.2	-0.0	-0.0
3	1255	485	1255	485	1254.6	237.0	0.0	0.0
4	1248	427	1248	427	1248.4	202.3	0.0	0.0
5	996	364	996	364	995.7	182.1	0.0	0.0
6	674	262	674	262	674.4	158.2	0.0	0.0
7	443	177	443	177	443.5	138.5	-0.0	-0.0
8	580	213	580	213	580.4	151.3	-0.0	-0.0
9	544	272	544	272	544.5	212.6	0.0	0.0
7424	2587	7424	2587	7424.1	146.8	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=8.89224e-30/9
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0238
total XS: 45.259 [pb]

b)

Now unfolding...
Iteration : 1
Chi² of change 9.78912e-29
Iteration : 2
Chi² of change 7.13988e-29
Iteration : 3
Chi² of change 1.51892e-28
Calculating covariances due to number of measured events
coverage probability: 68.26% 68.22% 68.24% 68.14% 68.15% 68.09% 68.23% 68.18% 68.27%

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	294.5	0.0	0.0
2	1081	286	1081	286	1080.9	274.5	-0.0	-0.0
3	1255	485	1255	485	1254.6	253.2	-0.0	-0.0
4	1248	427	1248	427	1248.4	217.2	0.0	0.0
5	996	364	996	364	995.7	198.0	-0.0	-0.0
6	674	262	674	262	674.4	165.7	-0.0	-0.0
7	443	177	443	177	443.5	144.9	-0.0	-0.0
8	580	213	580	213	580.4	161.5	-0.0	-0.0
9	544	272	544	272	544.5	224.9	0.0	0.0
7424	2587	7424	2587	7424.1	146.8	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=1.48045e-29/9

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	294.5	0.0	0.0
2	1081	286	1081	286	1080.9	274.5	-0.0	-0.0
3	1255	485	1255	485	1254.6	253.2	-0.0	-0.0
4	1248	427	1248	427	1248.4	217.2	0.0	0.0
5	996	364	996	364	995.7	198.0	-0.0	-0.0
6	674	262	674	262	674.4	165.7	-0.0	-0.0
7	443	177	443	177	443.5	144.9	-0.0	-0.0
8	580	213	580	213	580.4	161.5	-0.0	-0.0
9	544	272	544	272	544.5	224.9	0.0	0.0
7424	2587	7424	2587	7424.1	146.8	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=1.48045e-29/9
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0238
total XS: 45.259 [pb]

c)

Calculating covariances due to number of measured events
coverage probability: 68.26% 68.22% 68.24% 68.16% 68.13% 68.03% 68.19% 68.15% 68.27%

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	307.3	0.0	0.0
2	1081	286	1081	286	1080.9	290.7	-0.0	-0.0
3	1255	485	1255	485	1254.6	282.8	-0.0	-0.0
4	1248	427	1248	427	1248.4	243.3	0.0	0.0
5	996	364	996	364	995.7	226.8	-0.0	-0.0
6	674	262	674	262	674.4	196.5	0.0	0.0
7	443	177	443	177	443.5	176.8	-0.0	-0.0
8	580	213	580	213	580.4	172.9	-0.0	-0.0
9	544	272	544	272	544.5	231.6	0.0	0.0
7424	2587	7424	2587	7424.1	146.8	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=1.75655e-29/9

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	307.3	0.0	0.0
2	1081	286	1081	286	1080.9	290.7	-0.0	-0.0
3	1255	485	1255	485	1254.6	282.8	-0.0	-0.0
4	1248	427	1248	427	1248.4	243.3	0.0	0.0
5	996	364	996	364	995.7	226.8	-0.0	-0.0
6	674	262	674	262	674.4	196.5	0.0	0.0
7	443	177	443	177	443.5	176.8	-0.0	-0.0
8	580	213	580	213	580.4	172.9	-0.0	-0.0
9	544	272	544	272	544.5	231.6	0.0	0.0
7424	2587	7424	2587	7424.1	146.8	0.0	0.0	0.0

Bin-by-bin Chi²/NDF=1.75655e-29/9
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0238
total XS: 45.259 [pb]

d)

Table 5: a: Uncertainties from $P_T^{\ell\ell}$ unfolded with IBU with 1 iteration; b: Uncertainties from $P_T^{\ell\ell}$ unfolded with IBU with 2 iterations; c: Uncertainties from $P_T^{\ell\ell}$ unfolded with IBU with 3 iterations; d: Uncertainties from $P_T^{\ell\ell}$ unfolded with IBU with 100 iterations

SVD init 9 x 9 bins, kreg=2
Subtract 343.126 fakes from measured distribution
coverage probability: 68.26% 68.27% 68.27% 68.27% 68.27% 68.27% 68.24% 68.26% 68.26%

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	120.0	-0.0	-0.0
2	1801	286	1801	286	1800.9	153.7	-0.0	-0.0
3	1255	485	1255	485	1254.6	185.2	-0.0	-0.0
4	1248	427	1248	427	1248.4	178.7	-0.0	-0.0
5	996	364	996	364	995.7	142.4	-0.0	-0.0
6	674	262	674	262	674.4	99.7	-0.0	-0.0
7	443	177	443	177	443.5	69.0	-0.0	-0.0
8	580	213	580	213	580.4	81.6	-0.0	-0.0
9	544	272	544	272	544.5	91.3	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=5.76797\text{e}-14/9$

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	120.0	-0.0	-0.0
2	1801	286	1801	286	1800.9	153.7	-0.0	-0.0
3	1255	485	1255	485	1254.6	185.2	-0.0	-0.0
4	1248	427	1248	427	1248.4	178.7	-0.0	-0.0
5	996	364	996	364	995.7	142.4	-0.0	-0.0
6	674	262	674	262	674.4	99.7	-0.0	-0.0
7	443	177	443	177	443.5	69.0	-0.0	-0.0
8	580	213	580	213	580.4	81.6	-0.0	-0.0
9	544	272	544	272	544.5	91.3	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=5.76797\text{e}-14/9$
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.259 [pb]

a)

SVD init 9 x 9 bins, kreg=3
Subtract 343.126 fakes from measured distribution
coverage probability: 68.24% 68.25% 68.27% 68.26% 68.27% 68.27% 68.26% 68.26% 68.25%

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	158.8	-0.0	-0.0
2	1801	286	1801	286	1800.9	187.5	-0.0	-0.0
3	1255	485	1255	485	1254.6	202.0	-0.0	-0.0
4	1248	427	1248	427	1248.4	181.5	-0.0	-0.0
5	996	364	996	364	995.7	145.3	-0.0	-0.0
6	674	262	674	262	674.4	108.8	-0.0	-0.0
7	443	177	443	177	443.5	83.7	-0.0	-0.0
8	580	213	580	213	580.4	119.5	-0.0	-0.0
9	544	272	544	272	544.5	132.2	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=6.82646\text{e}-16/9$

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	158.8	-0.0	-0.0
2	1801	286	1801	286	1800.9	187.5	-0.0	-0.0
3	1255	485	1255	485	1254.6	202.0	-0.0	-0.0
4	1248	427	1248	427	1248.4	181.5	-0.0	-0.0
5	996	364	996	364	995.7	145.3	-0.0	-0.0
6	674	262	674	262	674.4	108.8	-0.0	-0.0
7	443	177	443	177	443.5	83.7	-0.0	-0.0
8	580	213	580	213	580.4	119.5	-0.0	-0.0
9	544	272	544	272	544.5	132.2	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=6.82646\text{e}-16/9$
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.259 [pb]

b)

SVD init 9 x 9 bins, kreg=4
Subtract 343.126 fakes from measured distribution
coverage probability: 68.22% 68.23% 68.27% 68.23% 68.24% 68.27% 68.26% 68.25% 68.26%

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	196.4	-0.0	-0.0
2	1801	286	1801	286	1800.9	212.2	-0.0	-0.0
3	1255	485	1255	485	1254.6	211.4	-0.0	-0.0
4	1248	427	1248	427	1248.4	185.4	-0.0	-0.0
5	996	364	996	364	995.7	151.1	-0.0	-0.0
6	674	262	674	262	674.4	116.7	-0.0	-0.0
7	443	177	443	177	443.5	92.1	-0.0	-0.0
8	580	213	580	213	580.4	128.8	-0.0	-0.0
9	544	272	544	272	544.5	164.6	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=3.73962\text{e}-17/9$

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	196.4	-0.0	-0.0
2	1801	286	1801	286	1800.9	212.2	-0.0	-0.0
3	1255	485	1255	485	1254.6	211.4	-0.0	-0.0
4	1248	427	1248	427	1248.4	185.4	-0.0	-0.0
5	996	364	996	364	995.7	151.1	-0.0	-0.0
6	674	262	674	262	674.4	116.7	-0.0	-0.0
7	443	177	443	177	443.5	92.1	-0.0	-0.0
8	580	213	580	213	580.4	128.8	-0.0	-0.0
9	544	272	544	272	544.5	164.6	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=3.73962\text{e}-17/9$
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.259 [pb]

c)

SVD init 9 x 9 bins, kreg=5
Subtract 343.126 fakes from measured distribution
coverage probability: 68.23% 68.22% 68.26% 68.19% 68.22% 68.25% 68.25% 68.24% 68.26%

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	238.3	-0.0	-0.0
2	1801	286	1801	286	1800.9	229.0	-0.0	-0.0
3	1255	485	1255	485	1254.6	222.3	-0.0	-0.0
4	1248	427	1248	427	1248.4	193.7	-0.0	-0.0
5	996	364	996	364	995.7	162.4	-0.0	-0.0
6	674	262	674	262	674.4	127.5	-0.0	-0.0
7	443	177	443	177	443.5	99.3	-0.0	-0.0
8	580	213	580	213	580.4	138.8	-0.0	-0.0
9	544	272	544	272	544.5	190.1	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=3.80409\text{e}-18/9$

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	238.3	-0.0	-0.0
2	1801	286	1801	286	1800.9	229.0	-0.0	-0.0
3	1255	485	1255	485	1254.6	222.3	-0.0	-0.0
4	1248	427	1248	427	1248.4	193.7	-0.0	-0.0
5	996	364	996	364	995.7	162.4	-0.0	-0.0
6	674	262	674	262	674.4	127.5	-0.0	-0.0
7	443	177	443	177	443.5	99.3	-0.0	-0.0
8	580	213	580	213	580.4	138.8	-0.0	-0.0
9	544	272	544	272	544.5	190.1	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	-0.0	-0.0	-0.0

Bin-by-bin $\chi^2/\text{NDF}=3.80409\text{e}-18/9$
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.259 [pb]

d)

Table 6: a: Uncertainties from $P_T^{\ell\ell}$ unfolded with SVD with 2 iteration; b: Uncertainties from $P_T^{\ell\ell}$ unfolded with SVD with 3 iterations; c: Uncertainties from $P_T^{\ell\ell}$ unfolded with SVD with 4 iterations; d: Uncertainties from $P_T^{\ell\ell}$ unfolded with SVD with 5 iterations

Subtract 343.126 fakes from measured distribution
coverage probability: 68.26% 68.23% 68.24% 68.17% 68.16% 68.04% 68.19% 68.16% 68.27%

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	322.4	0.0	0.0
2	1801	286	1801	286	1800.9	318.9	0.0	0.0
3	1255	485	1255	485	1254.6	293.0	-0.0	-0.0
4	1248	427	1248	427	1248.4	252.4	0.0	0.0
5	996	364	996	364	995.7	233.4	0.0	0.0
6	674	262	674	262	674.4	201.7	-0.0	-0.0
7	443	177	443	177	443.5	180.4	0.0	0.0
8	580	213	580	213	580.4	188.1	-0.0	-0.0
9	544	272	544	272	544.5	261.0	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	0.0	0.0	0.0

Bin-by-bin $\chi^2/\text{NDF}=1.20271\text{e}-28/9$

Bin	Train Truth Measured	Train Truth	Test Truth	Test Input	Test Unfolded Output	Error on Unfolding	Diff	Pull
1	762	182	762	182	761.8	322.4	0.0	0.0
2	1801	286	1801	286	1800.9	318.9	0.0	0.0
3	1255	485	1255	485	1254.6	293.0	-0.0	-0.0
4	1248	427	1248	427	1248.4	252.4	0.0	0.0
5	996	364	996	364	995.7	233.4	0.0	0.0
6	674	262	674	262	674.4	201.7	-0.0	-0.0
7	443	177	443	177	443.5	180.4	0.0	0.0
8	580	213	580	213	580.4	188.1	-0.0	-0.0
9	544	272	544	272	544.5	261.0	-0.0	-0.0
7424	2587	7424	2587	7424.1	146.0	0.0	0.0	0.0

Bin-by-bin $\chi^2/\text{NDF}=1.20271\text{e}-28/9$
scaling to lumi 139.0, acceptance 0.0513 and branching ratio 0.0230
total XS: 45.259 [pb]

Figure 7: Uncertainties from $P_T^{\ell\ell}$ unfolded with Matrix Inversion

B Graphs

B.1 $M_{\ell\ell}$

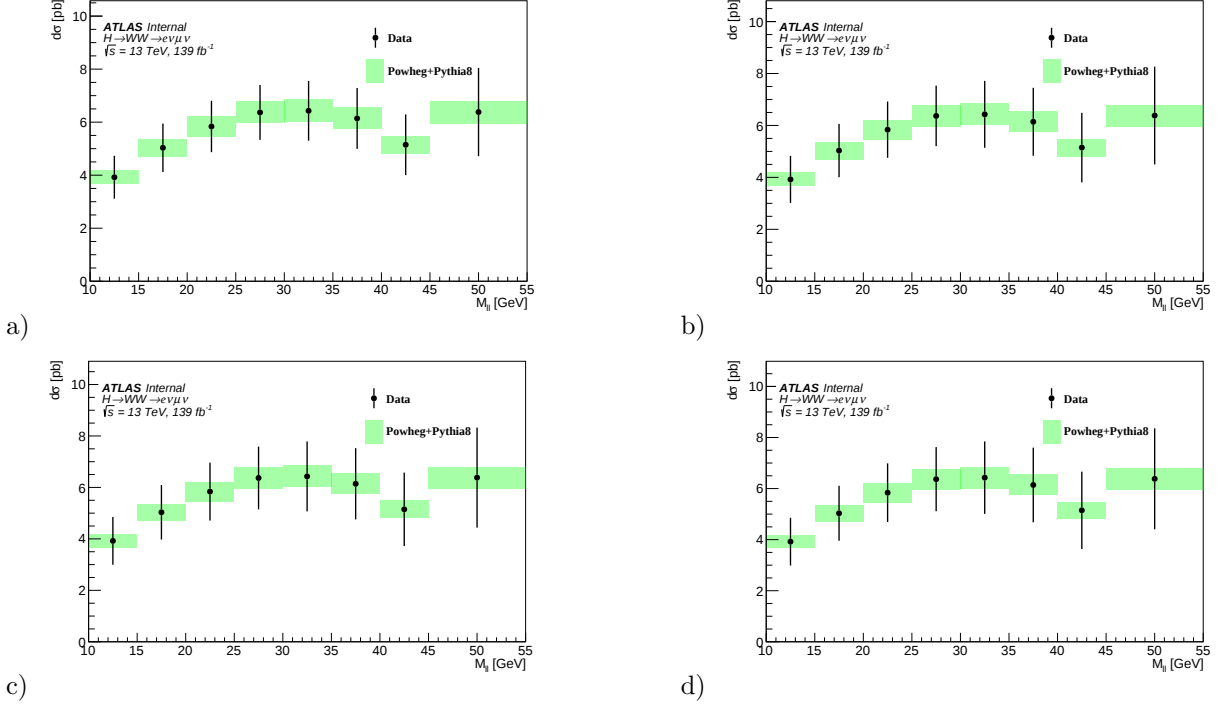


Table 7: a: $M_{\ell\ell}$ unfolded with IBU with 1 iteration; b: $M_{\ell\ell}$ unfolded with IBU with 2 iterations; c: $M_{\ell\ell}$ unfolded with IBU with 3 iterations; d: $M_{\ell\ell}$ unfolded with IBU with 100 iterations

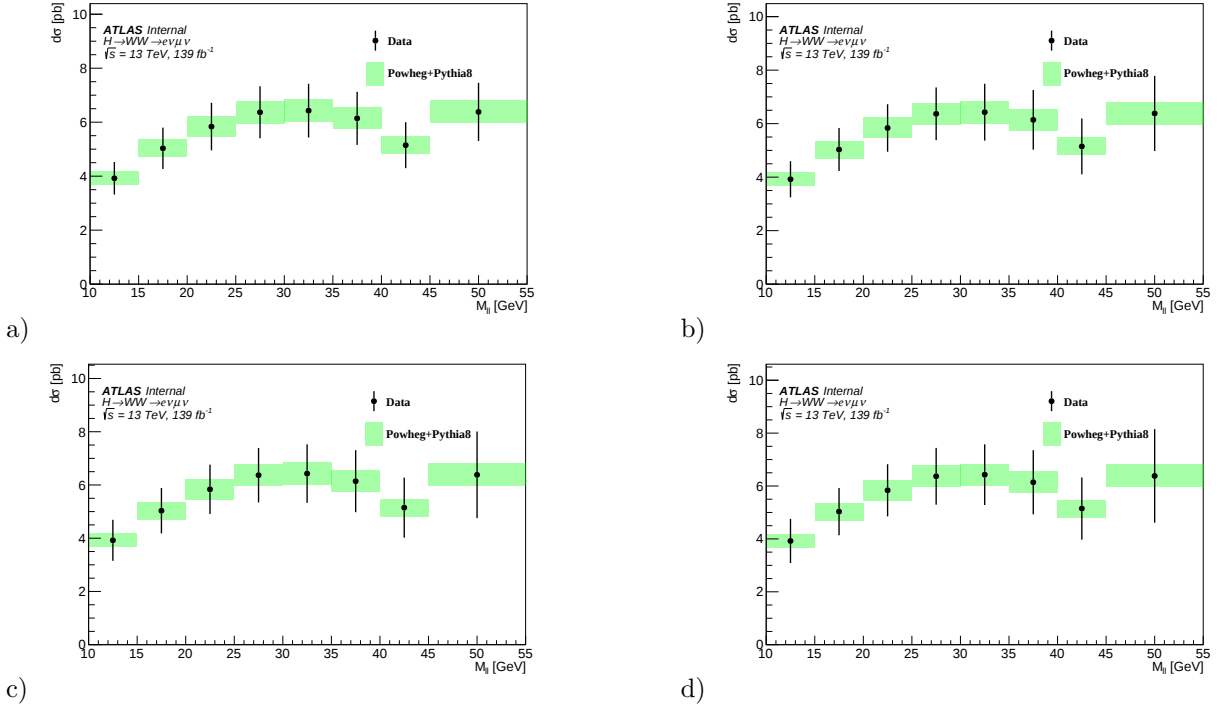


Table 8: a: $M_{\ell\ell}$ unfolded with SVD with 2 iteration; b: $M_{\ell\ell}$ unfolded with SVD with 3 iterations; c: $M_{\ell\ell}$ unfolded with SVD with 4 iterations; d: $M_{\ell\ell}$ unfolded with SVD with 5 iterations

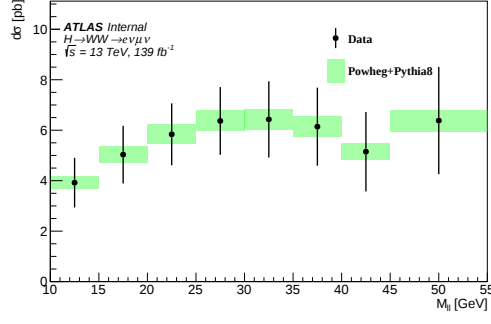


Figure 8: $M_{\ell\ell}$ unfolded with Matrix Inversion

B.2 P_T^H

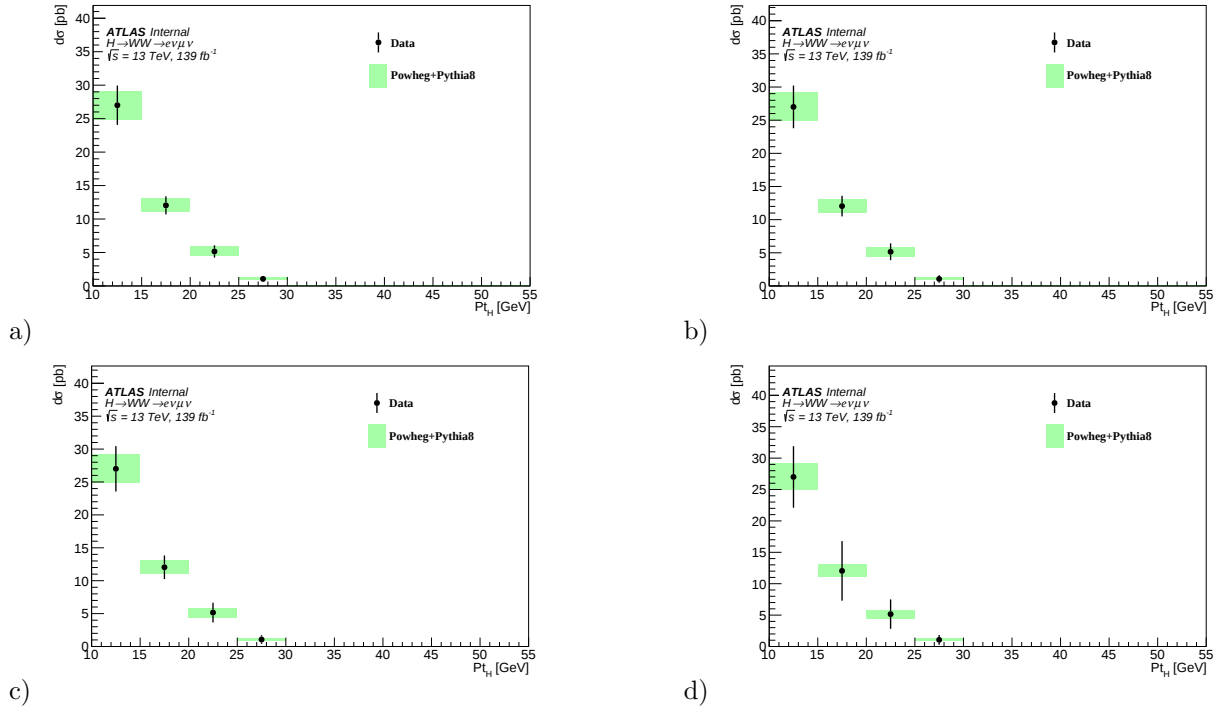


Table 9: a: P_T^H unfolded with IBU with 1 iteration; b: P_T^H unfolded with IBU with 2 iterations; c: P_T^H unfolded with IBU with 3 iterations; d: P_T^H unfolded with IBU with 100 iterations

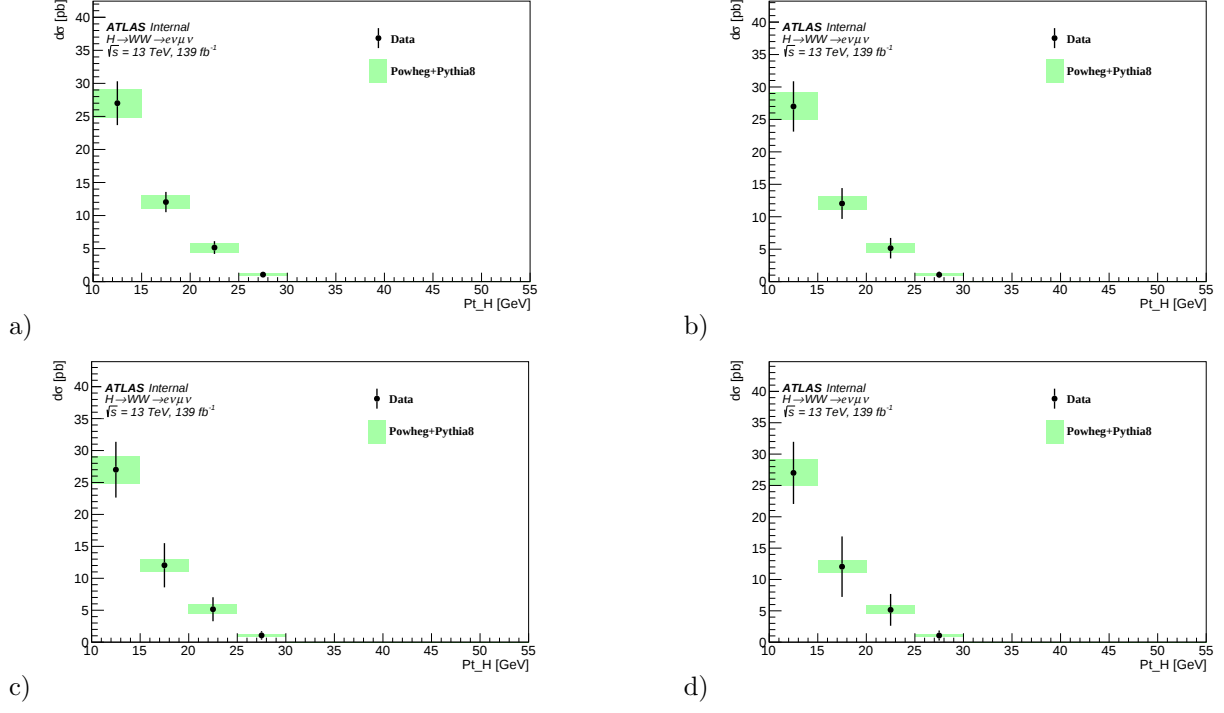


Table 10: a: P_T^H unfolded with SVD with 2 iteration; b: P_T^H unfolded with SVD with 3 iterations; c: P_T^H unfolded with SVD with 4 iterations; d: P_T^H unfolded with SVD with 5 iterations

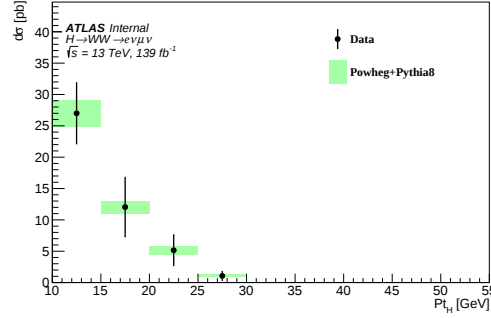


Figure 9: Uncertainties from P_T^H unfolded with Matrix Inversion

B.3 $P_T^{\ell\ell}$

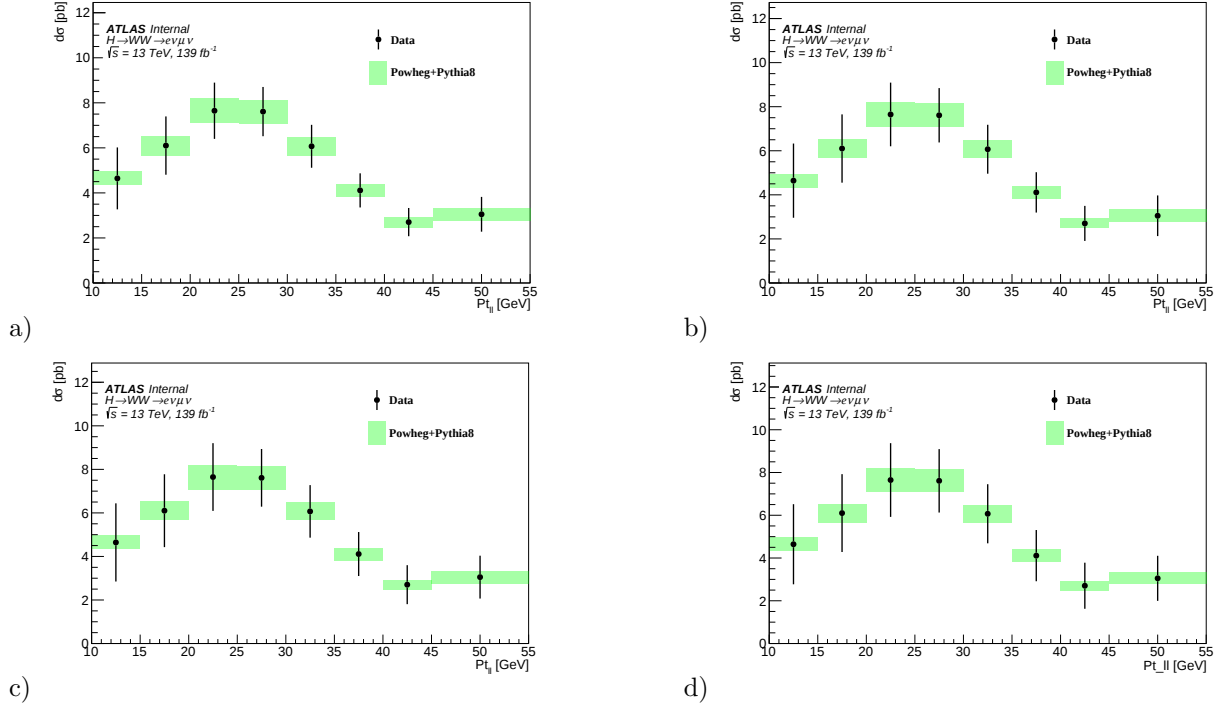


Table 11: a: $P_T^{\ell\ell}$ unfolded with IBU with 1 iteration; b: $P_T^{\ell\ell}$ unfolded with IBU with 2 iterations; c: $P_T^{\ell\ell}$ unfolded with IBU with 3 iterations; d: $P_T^{\ell\ell}$ unfolded with IBU with 100 iterations

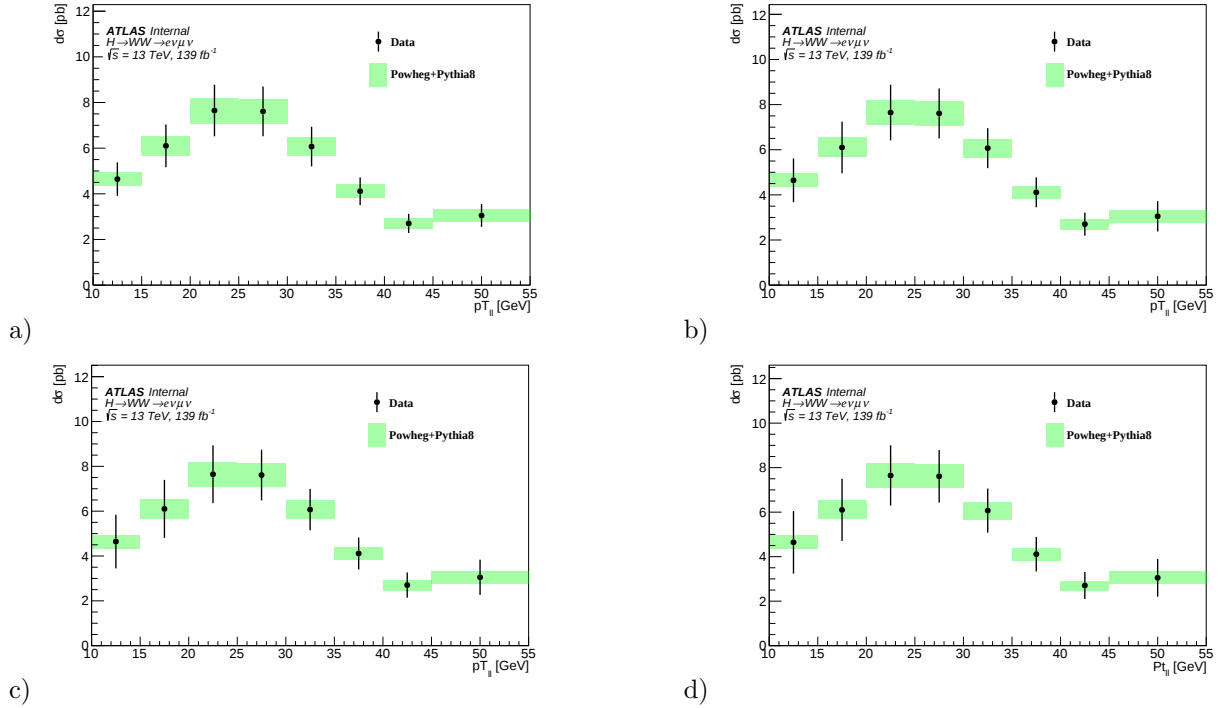


Table 12: a: $P_T^{\ell\ell}$ unfolded with SVD with 2 iteration; b: $P_T^{\ell\ell}$ unfolded with SVD with 3 iterations; c: $P_T^{\ell\ell}$ unfolded with SVD with 4 iterations; d: $P_T^{\ell\ell}$ unfolded with SVD with 5 iterations

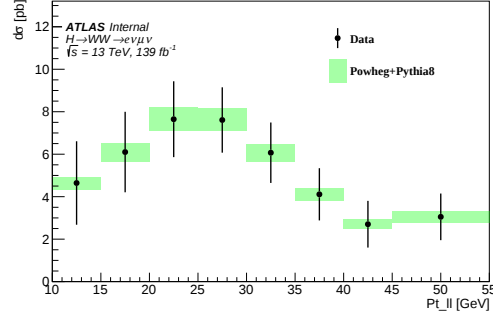


Figure 10: $P_T^{\ell\ell}$ unfolded with Matrix Inversion

References

- [1] Alan J Barr, Ben Gripaios, and Christopher G Lester. Measuring the higgs boson mass in dileptonic W-boson decays at hadron colliders. *Journal of High Energy Physics*, 2009(07):072–072, jul 2009.
- [2] Lydia Brenner, Pim Verschuuren, Rahul Balasubramanian, Carsten Burgard, Vincent Croft, Glen Cowan, and Wouter Verkerke. Comparison of unfolding methods using roofitunfold. *International Journal of Modern Physics A*, 35(24), 2020.