

Thermoelectric transport phenomena in
magnetized QCD matter and study of $\phi(1020)$
meson production at $\sqrt{s_{\text{NN}}} = 5.36$ TeV in Pb-Pb
collisions at forward rapidity with ALICE

Ph.D. Thesis

by

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Department of Physics

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Thermoelectric transport phenomena in
magnetized QCD matter and study of $\phi(1020)$
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Kamaljeet Singh



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INDIAN INSTITUTE OF TECHNOLOGY INDORE

I hereby certify that the work which is being presented in the thesis entitled **Thermoelectric transport phenomena in magnetized QCD matter and study of $\phi(1020)$ meson production at $\sqrt{s_{NN}} = 5.36$ TeV in Pb-Pb collisions at forward rapidity with ALICE** in the partial fulfillment of the requirements for the award of the degree of **DOCTOR OF PHILOSOPHY** and submitted in the **DEPARTMENT OF PHYSICS, Indian Institute of Technology Indore**, is an authentic record of my own work carried out during the time period from May 2021 to April 2026 under the supervision of Prof. Raghunath Sahoo (Institute Chair Professor), Department of Physics, Indian Institute of Technology Indore and Dr. Sumanta Samal (Associate Professor), co-supervisor, Department of Metallurgical Engineering and Materials Science (MEMS), Indian Institute of Technology Indore.

The matter presented in this thesis has not been submitted by me for the award of any other degree of this or any other institute.

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Signature of Thesis Co-Supervisor with Date

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Dedicated

to

My parents, My sisters, and My loved one

*“Family is the enduring fuel that powers your endless
journey of research.”*

(Kamaljeet Singh)

ਕਮਲਜੀਤ ਸਿੰਘ

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ABSTRACT

Understanding the behavior of strongly interacting matter under extreme conditions of temperature, density, and electromagnetic fields remains one of the central challenges in modern high-energy physics. Quantum Chromodynamics (QCD), the fundamental theory governing the strong interaction, predicts the existence of an intricate phase structure of matter, including the transition from hadronic matter to a deconfined state known as the quark–gluon plasma (QGP). Such conditions are believed to have prevailed in the early universe shortly after the Big Bang and can be recreated in laboratory environments through ultra-relativistic heavy-ion collisions at facilities such as the Large Hadron Collider (LHC). This thesis presents a comprehensive investigation of QCD matter by integrating theoretical, phenomenological, experimental, and applied perspectives. The work begins with a detailed overview of the Standard Model of particle physics, with particular emphasis on QCD as the underlying framework for strong interactions. The QCD phase diagram is introduced to illustrate the possible phases of strongly interacting matter as functions of temperature and baryon density, along with the expected phase transitions, including deconfinement and chiral symmetry restoration. Heavy-ion collisions are discussed as a unique tool to probe these extreme conditions, and the spacetime evolution of the collision system is described, highlighting key experimental signatures of QGP formation.

A significant part of this thesis is devoted to the study of transport phenomena in QCD matter under the influence of strong magnetic fields, which are known to be generated in non-central heavy-ion collisions. Within a kinetic theory framework extended to relativistic hydrodynamics, the transport properties of both confined and deconfined phases are systematically analyzed. In particular, transport coefficients such as electrical conductivity and thermal conductivity are evaluated, providing insights into the dissipative properties of the medium. Building upon this, the thesis further explores thermoelectric phenomena in QCD matter. The

thermoelectric response is investigated for both hadronic matter and quark–gluon plasma, considering scenarios with and without external magnetic fields. This study includes the computation of leading-order as well as higher-order thermoelectric coefficients, offering a deeper understanding of non-equilibrium processes in strongly interacting matter. These studies are particularly relevant in the context of heavy-ion collisions, where non-zero gradients and transient electromagnetic fields can give rise to novel transport effects.

In addition to the study of QCD matter, this thesis also addresses practical aspects of accelerator physics. An overview of particle accelerators and detectors, with particular focus on the LHC and the ALICE experiment, is presented. The design and optimization of novel materials for beampipe applications in accelerator technology are investigated, considering the demanding operational conditions involving high radiation length, thermal loads, and mechanical stresses. The proposed material design, basically Al and Mg-rich ternary alloys, aims to enhance the efficiency, durability, and cost-effectiveness of beampipe structures, with potential implications for both current and future high-energy physics experiments. The final part of the thesis is dedicated to an experimental analysis of $\phi(1020)$ meson production in Pb–Pb collisions at $\sqrt{s_{\text{NN}}} = 5.36$ TeV in forward rapidity using Run-3 data from the ALICE detector at the LHC. The ϕ meson, composed of a strange quark–antiquark pair ($s\bar{s}$), serves as an important probe of the medium due to its sensitivity to strangeness production and its relatively small hadronic interaction cross section. A detailed description of the detector setup, including standalone and global muon tracking, is provided along with the selection criteria and kinematic cuts applied in the analysis. The invariant mass spectra are reconstructed, and the ϕ meson yield is extracted after careful background subtraction and efficiency corrections using Monte Carlo simulations. The results contribute to the understanding of strangeness production and medium effects in heavy-ion collisions.

Overall, this thesis establishes a coherent link between the theoretical founda-

tions, phenomenological modeling, experimental investigations, and technological aspects of accelerator and detector systems used to study QCD matter. By bridging these diverse aspects, it provides a comprehensive and unified exploration of strongly interacting matter under extreme conditions.

ACRONYMS

- **ALICE** – A Large Ion Collider Experiment
- **ATLAS** – A Toroidal LHC Apparatus
- **CMS** – Compact Muon Solenoid
- **EMCAL** – Electromagnetic Calorimeter
- **FoM** – Figure of Merit
- **FT0** – Forward Time Zero detectors
- **FV0** – Forward VZERO Detector
- **GeV** – Giga Electron Volt
- **HRG** – Hadron Resonance Gas
- **ITS / ITS2** – Inner Tracking System / Upgraded ITS
- **LHC** – Large Hadron Collider
- **LINAC** – Linear Accelerators
- **MC** – Monte Carlo
- **MCH** – Muon Tracking Chambers
- **MID** – Muon Identifier
- **MFT** – Muon Forward Tracker
- **O2** – ALICE Online Offline computing framework
- **PHENIX** – Pioneering High Energy Nuclear Interaction eXperiment
- **PHOS** – Photon Spectrometer

- **PID** – Particle Identification
- **PS** – Proton Synchrotron
- **QGP** – Quark-Gluon Plasma
- **QCD** – Quantum Chromodynamics
- **QPM** – Quasi Particle Model
- **ROOT** – CERN’s data analysis framework
- **RHIC** – Relativistic Heavy-Ion Collider
- **SPS** – Super Proton Synchrotron
- **STAR** – Solenoidal Tracker at RHIC
- **TeV** – Tera Electron Volt
- **TPC** – Time Projection Chamber
- **TOF** – Time Of Flight detector
- **TRD** – Transition Radiation Detector
- **V0** – V Zero detector
- **ZDC** – Zero Degree Calorimeter

PUBLICATIONS

- **Publications included in this thesis**

- A. Published articles**

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- **ALICE Analysis notes**

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Chapter 1

Introduction

“The limits of the universe cannot be perceived. The more you say about them, the more there still remains to be said.”

— Guru Nanak

Our understanding of the universe has evolved from the discovery of atoms to uncovering the subatomic particles such as electrons, protons, and neutrons. Ultimately, probing into the nucleons opens the doors toward a new era of physics which reveals that quarks lie at the core of all known matter [1]. This journey reflects the human drive to answer some of the most basic questions: *What is matter made of? What holds it together?* Today, the Standard Model (SM) stands as a pillar of modern physics, a well-established theory that describes the fundamental building blocks of matter and their governing interactions. It includes a variety of elementary particles such as quarks, leptons, and force carriers of three of the four known fundamental interactions. Gluons (g) for strong interaction, photons (γ) for electromagnetic interaction and W-bosons ($W^\pm$), Z-bosons (Z^0) bosons for the weak interaction. Unlike the other three interactions, gravity is not yet unified into the SM, though it remains fundamental in governing the dynamics of the universe at astronomical scales. These fundamental interactions are responsible for everything from the stability of atoms to the fusion of stars and

the formation of galaxies. Understanding how these interactions behave across ordinary matter to extreme cosmic conditions remains a core focus of modern physics.

*“With visible light we see the matter around,
With X-rays, molecules and atoms are found,
With gamma rays, nuclei excite and glance,
In heavy-ion collisions, quarks and gluons dance.”*

1.1 The Standard Model: A map of the subatomic universe

The development of the Standard Model was driven by the fundamental need to understand the basic building blocks of matter and the forces governing their interactions within a unified framework. As experimental discoveries in the 20th century revealed a growing zoo of subatomic particles and complex phenomena, it became essential to establish a consistent and precise theory that could systematically classify these particles and explain their behavior. The SM emerged as a quantum field theoretic structure that not only unified the electromagnetic and weak forces, but also provided a robust description of the strong nuclear force. The SM of particle physics is a quantum field theory based on the gauge symmetry group $SU(3)_C \times SU(2)_L \times U(1)_Y$, which successfully describes three of the four known fundamental interactions: the strong, weak, and electromagnetic interactions. The $SU(3)_C$ symmetry governs Quantum Chromodynamics (QCD), the theory of the strong interaction, with gluons as the gauge bosons mediating color charge between quarks [2]. The electroweak interaction is unified under the $SU(2)_L \times U(1)_Y$ gauge group, where $SU(2)_L$ describes the weak isospin interaction and $U(1)_Y$ corresponds to weak hypercharge. This unification gives rise to four gauge bosons: W^+ , W^- , Z^0 , and the photon γ , with the weak and

1.1 The Standard Model: A map of the subatomic universe

electromagnetic sectors separated through spontaneous symmetry breaking via the Higgs mechanism [3]. Matter fields in the SM are spin-1/2 fermions: quarks and leptons, each arranged into three generations. The Higgs field, a scalar field with a non-zero vacuum expectation value, breaks the electroweak symmetry spontaneously [4], thereby generating masses for the W and Z bosons and the fermions through Yukawa interactions. Despite its success, the model's inability to incorporate gravity and explain phenomena such as dark matter and neutrino oscillations highlights the ongoing need for physics beyond the SM.

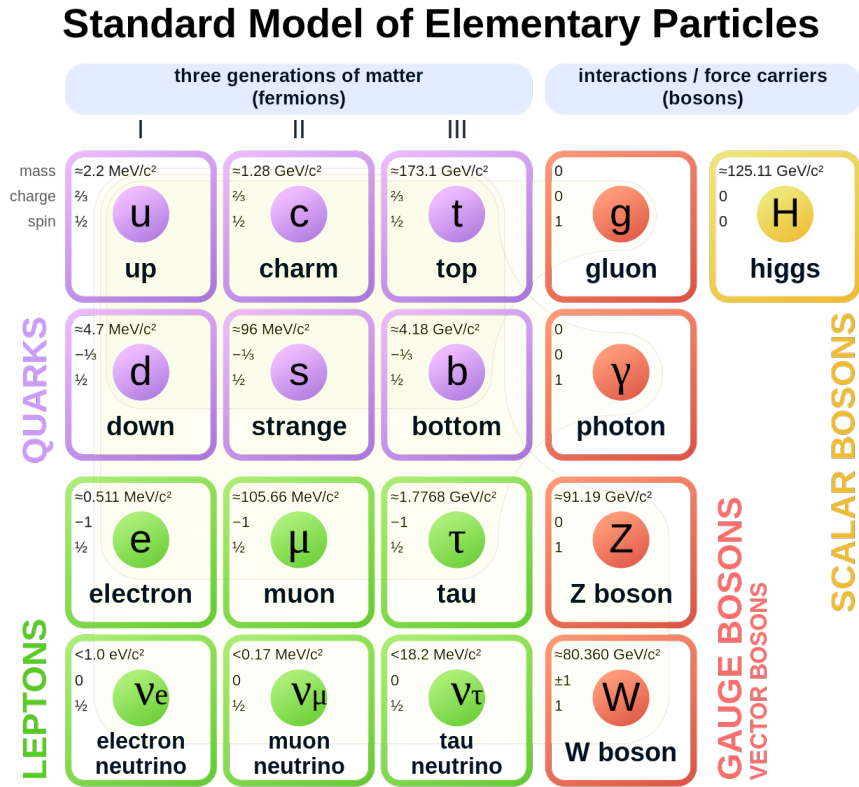


Figure 1.1: A schematic representation of the Standard Model of fundamental particles [5].

The SM of particle physics classifies all known fundamental particles into two broad categories: fermions and bosons. Fermions, which obey the Pauli exclusion principle and make up matter, are further divided into quarks and leptons. There

are six quarks—up (u), down (d), charm (c), strange (s), top (t), and bottom (b), arranged in three generations, with the up and down quarks forming the building blocks of protons and neutrons. Similarly, the six leptons include the electron (e), muon (μ), and tau (τ), along with their corresponding neutrinos (ν_e, ν_μ , and ν_τ). Interactions among these matter particles are mediated by gauge bosons, the force carriers of the SM. The photon mediates the electromagnetic force, the $W^\pm$ and Z^0 bosons mediate the weak force, and the gluon carries the strong nuclear force, binding quarks together inside hadrons. Additionally, the Higgs boson, discovered in 2012, is responsible for giving mass to other particles via the Higgs mechanism. Together, these particles form a remarkably successful framework that explains a vast range of physical phenomena at the subatomic level.

1.2 Theory of strong interactions: Quantum Chromodynamics

Quantum Chromodynamics is the quantum field theory that describes the strong interaction, one of the four fundamental forces in nature, responsible for binding quarks and gluons into protons, neutrons, and other hadrons. It is a non-Abelian gauge theory based on the $SU(3)$ color gauge group. In QCD, the force carriers, known as gluons, themselves carry color charge, which leads to gluon–gluon interactions in addition to quark–gluon interactions. This self-interaction of gluons gives rise to two key features of QCD: *asymptotic freedom* and *confinement* [2].

Asymptotic freedom refers to the phenomenon where the effective strong coupling between quarks becomes weaker at high energies or short distances. This behavior is described by the QCD *running coupling constant* $\alpha_s(Q^2)$, which depends on the characteristic momentum transfer (or energy scale) Q . In leading-order (LO) perturbative QCD, the running of the strong coupling constant is governed by the renormalization group equation and given by [33]:

1.2 Theory of strong interactions: Quantum Chromodynamics

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda_{\text{QCD}}^2)}, \quad (1.1)$$

where $\beta_0 = 11 - \frac{2}{3}n_f$ is the one-loop coefficient of the QCD beta function, n_f is the number of active quark flavors at the scale Q , and Λ_{QCD} is the QCD scale parameter, typically of the order of a few hundred MeV. This expression shows that as Q^2 increases, $\alpha_s(Q^2)$ decreases logarithmically, illustrating the weakening of the strong force at high energies. Beyond the leading order, the next-to-leading

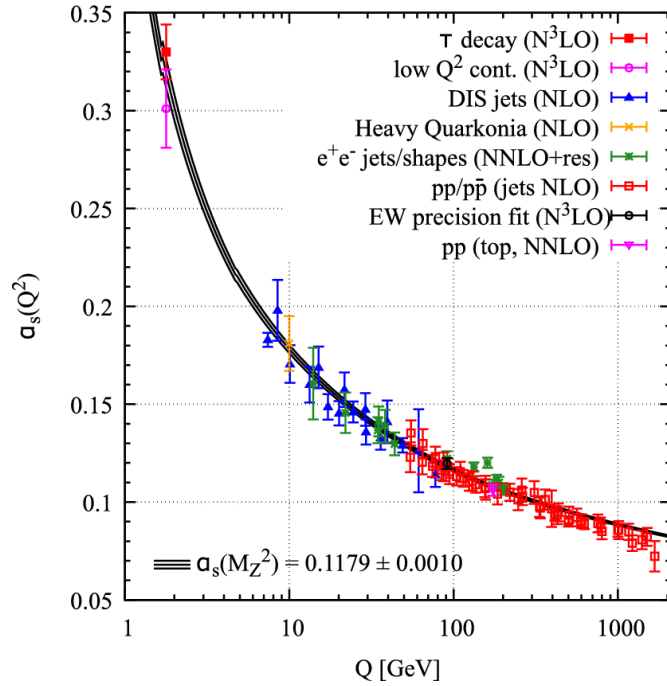


Figure 1.2: The Q dependence of running coupling constant α_s [7].

order (NLO) correction to the running coupling constant is given by [8, 9]:

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda_{\text{QCD}}^2)} \left[1 - \frac{\beta_1}{\beta_0^2} \frac{\ln[\ln(Q^2/\Lambda_{\text{QCD}}^2)]}{\ln(Q^2/\Lambda_{\text{QCD}}^2)} \right], \quad (1.2)$$

where $\beta_1 = 102 - \frac{38}{3}n_f$ is the two-loop coefficient of the QCD beta function. This higher-order correction further refines the running behavior of α_s and becomes important for precise QCD calculations at high energies. In contrast to asymptotic freedom, QCD exhibits *confinement* at low energies: the coupling becomes

strong as $Q^2 \rightarrow \Lambda_{\text{QCD}}^2$, and perturbation theory breaks down. This means quarks and gluons are permanently confined within hadrons and cannot be isolated under normal conditions.

Understanding the energy dependence of α_s is crucial in many areas of QCD, such as jet production in high-energy collisions, deep inelastic scattering, and the thermodynamics of the quark-gluon plasma (QGP). In particular, the running coupling plays an important role in computing QCD transport coefficients like shear viscosity, conductivity, and diffusion in both perturbative and semi-perturbative frameworks. Due to the non-perturbative nature of QCD at low energies, other theoretical tools like lattice QCD, effective models [10, 11] (e.g., Nambu–Jona-Lasinio or PNJL models), and holographic dualities are often employed to complement the perturbative approach and gain insight into confinement, chiral symmetry breaking, and the QCD phase diagram.

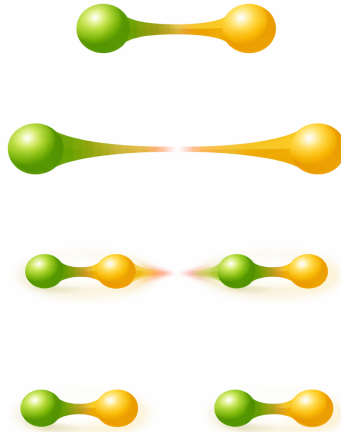


Figure 1.3: Illustration of QCD confinement and string breaking. The color flux tube between a quark–antiquark pair stretches with increasing separation and eventually breaks, producing a new quark–antiquark pair.

1.3 QCD Phase Diagram

An intuitive and instructive way to visualize the consequences of asymptotic freedom and confinement is through the QCD potential between a static quark–antiquark pair. In contrast to the Coulomb potential in electrodynamics, the QCD potential exhibits a two-component structure: a short-distance Coulombic term and a long-distance linear confinement term. This is often parametrized as [12]

$$V_{\text{QCD}}(r) = -\frac{4}{3} \frac{\alpha_s(r)}{r} + \sigma r, \quad (1.3)$$

where r is the separation between the quark and antiquark, $\alpha_s(r)$ is the strong coupling constant at distance r , and σ is the string tension, typically around 0.18 GeV^2 . The first term dominates at short distances, reflecting the weakly interacting nature of quarks at high momentum transfer (asymptotic freedom), while the second term dominates at larger distances, illustrating the linearly increasing energy required to separate quarks—an embodiment of confinement. This linearly rising potential implies that attempting to isolate a quark leads to the creation of new quark–antiquark pairs, preserving color neutrality and giving rise to hadronization. Lattice QCD simulations and phenomenological fits to heavy quarkonium spectra provide strong support for this potential form. Therefore, the QCD potential serves as a bridge connecting the perturbative and non-perturbative regimes of QCD, embedding within it the foundational dynamics of the strong interaction.

1.3 QCD Phase Diagram

Quantum Chromodynamics, the fundamental theory of the strong interaction, exhibits a rich phase structure when subjected to extreme conditions of temperature (T) and baryon chemical potential (μ_B). The QCD phase diagram is a schematic representation of different phases of strongly interacting matter in the (T, μ_B) plane. At low temperatures and chemical potentials, matter exists in a confined phase where quarks and gluons are bound into hadrons. As tempera-

ture increases, a transition occurs to a deconfined state known as the quark-gluon plasma, where quarks and gluons are no longer confined within hadrons and can move freely over larger distances. Lattice QCD [13] calculations at zero or small μ_B suggest that this transition is a smooth crossover, characterized by a rapid change in thermodynamic quantities without a true discontinuity. However, at large values of μ_B , model-based approaches predict a first-order phase transition, leading to the existence of a critical point that separates the crossover region from the first-order transition line. Locating this QCD critical point is a major goal of both theoretical and experimental studies, as it would provide profound insight into the behavior of matter under extreme conditions.

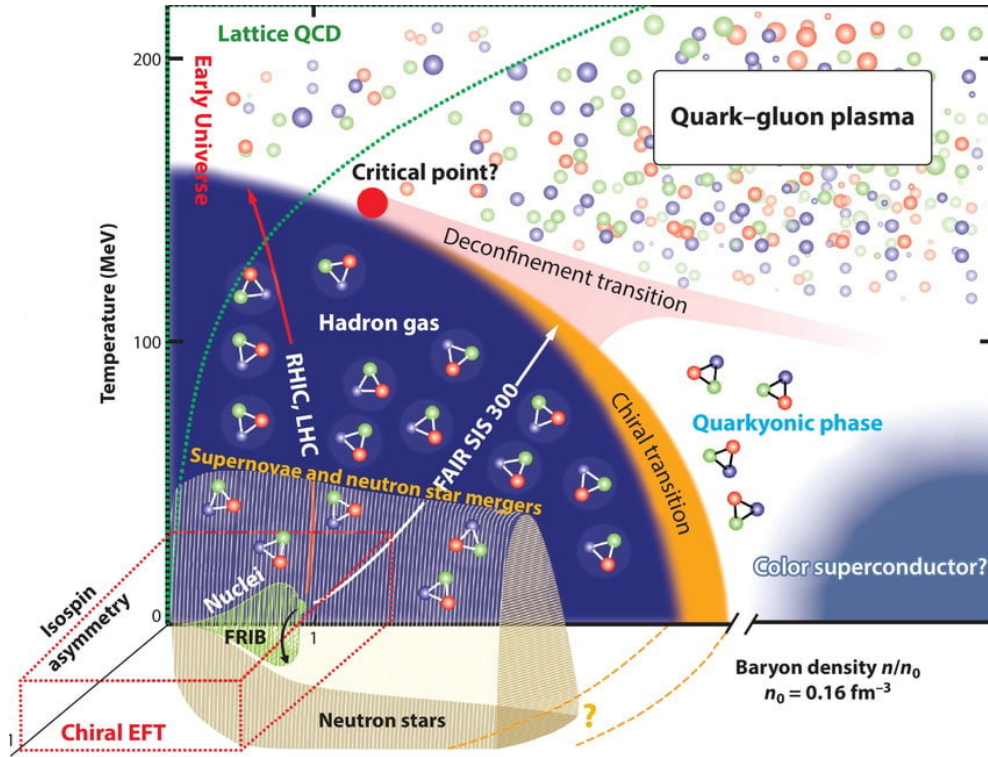


Figure 1.4: Schematic QCD phase diagram illustrating different phases of strongly interacting matter as a function of temperature T (MeV) and baryon density (n/n_0) [14].

Experimentally, different regions of the QCD phase diagram are explored through

1.4 Heavy-ion collisions as a little bang

ultra-relativistic heavy-ion collisions at varying beam energies. High-energy collisions at the LHC and RHIC probe the high-temperature, low- μ_B regime where QGP is expected to form and evolve rapidly. In contrast, lower beam energy experiments such as the Beam Energy Scan (BES) program at RHIC and upcoming facilities like FAIR [15] and NICA [16] are designed to access the high- μ_B , low-temperature region of the phase diagram. Observables related to fluctuations, correlations, and transport coefficients serve as important signatures for phase transitions and the possible existence of the critical point. Additionally, the phase diagram is not limited to just confinement and deconfinement; it also includes regions where chiral symmetry is restored, color superconductivity may occur, and quarkyonic matter might exist at asymptotically high densities. A complete understanding of the QCD phase diagram requires the synergy of first-principles calculations, effective models, and experimental measurements, making it one of the most compelling frontiers in modern nuclear and high-energy physics.

1.4 Heavy-ion collisions as a little bang

Ultra-relativistic heavy-ion collisions offer a remarkable parallel to the early Universe, providing a unique opportunity to recreate, on a microscale, the extreme conditions of temperature and energy density that prevailed shortly after the Big Bang. By colliding heavy nuclei such as gold or lead at nearly the speed of light, experimental facilities like RHIC and the LHC create fireballs of hot, dense matter that undergo rapid expansion and cooling, which is why these events are called “little bangs” [1]. Just as the Big Bang initiated the expansion and evolution of the Universe, a heavy-ion collision initiates the creation and evolution of a strongly interacting medium. In both cases, the system transitions from a hot, dense, deconfined phase to a cooler, confined state of matter. The spacetime evolution of these fireballs starting from the collision of Lorentz-contracted nuclei to the formation and cooling of quark–gluon plasma, followed by hadronization

and eventual freeze-out, mirrors, in a condensed timescale and volume, the key phases of the early Universe.

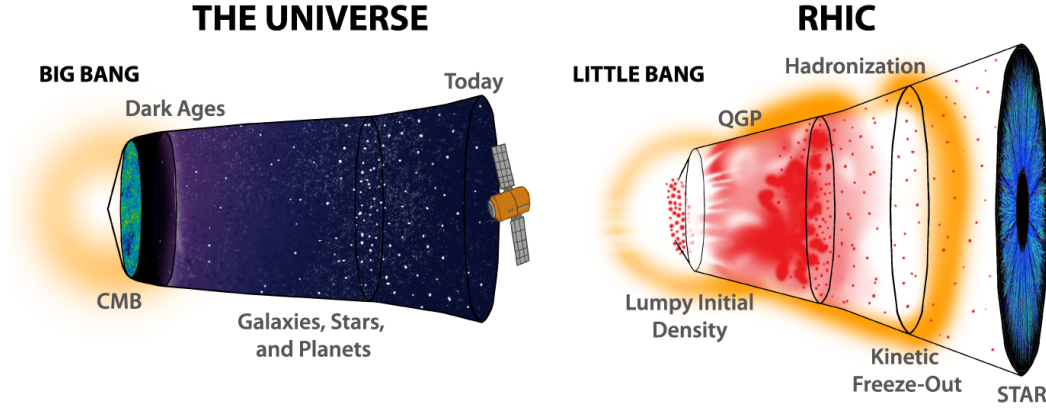


Figure 1.5: Schematic illustration of the Universe’s expansion after the Big Bang (left) compared with the fireball expansion in heavy-ion collisions, the so-called little bangs (right) [17].

Experimental observables from these little bangs, such as flow coefficients, particle spectra, electromagnetic radiation, and jet quenching, provide insights into the properties of QCD matter under extreme conditions. Through comparisons with hydrodynamic and transport models, along with lattice QCD, one can extract key thermodynamic and transport properties, study phase transitions, and explore the structure of the QCD phase diagram, including the possible critical point. Heavy-ion collisions thus serve as a bridge between nuclear physics and cosmology, enabling the study of the early Universe in a controlled laboratory setting. The “little bang” framework not only deepens our understanding of QCD matter but also offers a pedagogical analogy to the grandest explosion of all, the Big Bang.

1.5 Spacetime evolution of heavy-ion collisions

The study of relativistic heavy-ion collisions provides a unique opportunity to investigate the properties of strongly interacting matter under extreme conditions of temperature and energy density. When two heavy nuclei collide at ultra-relativistic speeds, the evolution of the system proceeds through several distinct stages, each characterized by different physical conditions and degrees of freedom. The complete spacetime evolution can be summarized as follows:

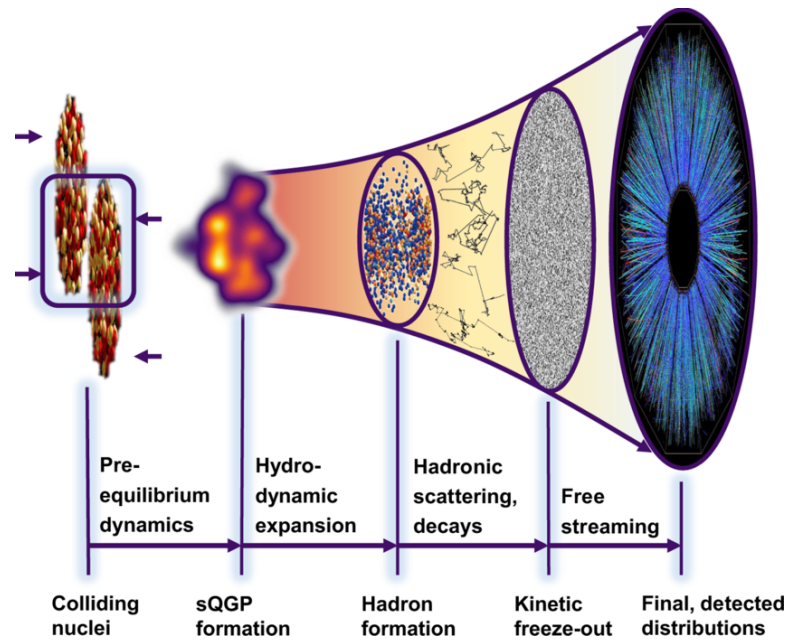


Figure 1.6: Space-time evolution of heavy-ion collisions [18].

- **Initial collision of Lorentz-contracted nuclei**

Prior to the collision, the nuclei are highly Lorentz-contracted along the beam axis due to their relativistic velocities, appearing as thin pancake-like objects in the laboratory frame. At the instant of collision ($t = 0$), the two nuclei pass through each other within a time scale of $\sim 0.1 \text{ fm}/c$. During this overlap, the large energy density deposited in the overlap region initiates the production of a dense system of gluons and quarks. This stage

is characterized by strong color fields and coherent gluon interactions, often described by models such as the Color Glass Condensate (CGC).

- **Pre-equilibrium dynamics**

Following the collision, the system enters a highly non-equilibrium phase where partons interact and scatter, redistributing energy and momentum. This stage, known as the pre-equilibrium phase, typically lasts for a few tenths of a femtosecond ($\sim 0.3 - 1.0 \text{ fm}/c$). While not yet in local thermal equilibrium, these interactions drive the system toward isotropization and thermalization. Theoretical frameworks such as effective kinetic theory or classical Yang–Mills dynamics are often used to describe this phase. The energy-momentum tensor evolves rapidly during this time, laying the foundation for the onset of hydrodynamic behavior.

- **Quark–Gluon Plasma phase** Once local thermal equilibrium is approxi-

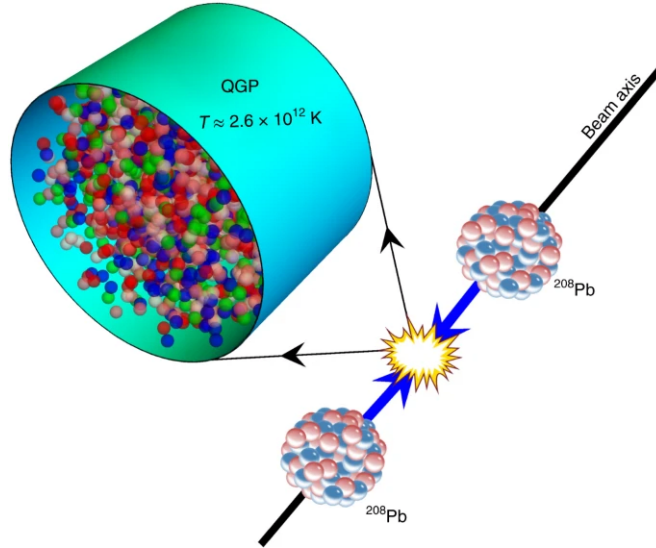


Figure 1.7: A high energy heavy-ion collision creates a quark–gluon plasma within an equivalent cylindrical volume, illustrated by colored quarks and gluons. [19].

mately achieved, the system enters a deconfined state known as the quark–gluon plasma, where quarks and gluons are no longer confined within hadrons.

1.5 Spacetime evolution of heavy-ion collisions

This phase exists at energy densities exceeding the QCD critical threshold ($\sim 1 \text{ GeV/fm}^3$), and the temperature reaches several hundred MeV ($T \sim 300 - 600 \text{ MeV}$). The QGP expands rapidly, primarily in the longitudinal direction, due to the pressure gradients. This stage is well-described by relativistic viscous hydrodynamics, which captures the collective flow behavior observed in experiments. During the QGP phase, partons interact strongly, leading to phenomena such as jet quenching and elliptic flow.

- **Hadronization and chemical freeze-out**

As the system expands and cools, it eventually reaches the critical temperature $T_c \sim 155 - 160 \text{ MeV}$, at which point quarks and gluons hadronize into color-neutral hadrons. This transition may occur via a smooth crossover, as suggested by lattice QCD calculations. The point at which inelastic collisions cease is known as the chemical freeze-out. After this point, the relative abundances of different hadron species become fixed. Statistical hadronization models are often employed to describe the particle ratios observed at this stage, assuming chemical equilibrium among hadrons.

- **Hadronic gas phase and kinetic freeze-out**

Following hadronization, the system enters the hadronic resonance gas (HRG) phase, consisting of interacting hadrons and resonances. Elastic collisions among hadrons continue to shape the momentum spectra and collective flow. As the system further dilutes and cools below $T \sim 100 - 120 \text{ MeV}$, elastic interactions become too infrequent to maintain kinetic equilibrium. This marks the kinetic freeze-out, after which particles no longer interact and their momentum distributions become fixed.

- **Free streaming and detection**

After kinetic freeze-out, particles stream freely to the detectors without further interactions. This final state, observed in experiments, carries im-

prints of the entire evolution history encoded in flow harmonics, transverse momentum spectra, particle yields, and correlations. From these observables, the properties of the QGP and the dynamics of the collision can be reconstructed using theoretical models.

Understanding the detailed evolution across these stages is crucial for probing the QCD phase diagram and extracting the transport and thermodynamic properties of the QGP and hadronic matter. The synergy between experimental data and theoretical modeling provides a comprehensive picture of the strongly interacting matter created in ultra-relativistic heavy-ion collisions.

1.6 Experimental signatures of QGP formation

The formation of a QGP in ultra-relativistic heavy-ion collisions is inferred through a variety of experimental observables. These signatures probe different aspects of the QGP's evolution, such as collective flow, energy loss, thermal radiation, and partonic interactions. Below, we summarize key signatures along with their corresponding mathematical formulations.

- **Anisotropic flow coefficients** (v_n): The azimuthal distribution of emitted particles relative to the reaction plane reflects the collective expansion of the QGP. The distribution is typically decomposed into a Fourier series [20]:

$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left[1 + 2 \sum_{n=1}^{\infty} v_n \cos(n(\tilde{\phi} - \Psi_n)) \right], \quad (1.4)$$

where $\tilde{\phi}$ is the azimuthal angle, Ψ_n is the event plane angle, and v_n are the flow coefficients. The second harmonic, v_2 (elliptic flow), is particularly sensitive to the initial spatial anisotropy and medium viscosity.

1.6 Experimental signatures of QGP formation

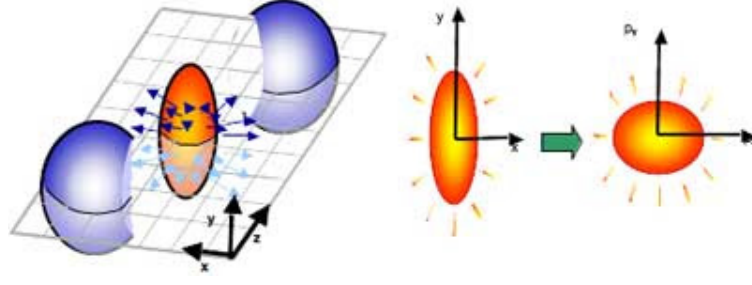


Figure 1.8: Left: Schematic of the collision zone between two incoming nuclei, with the $x - z$ plane representing the reaction plane. Right: Transformation of the initial spatial anisotropy in the overlap region into final-state elliptic flow, observed as anisotropy in particle momentum [21].

- **Jet quenching and nuclear modification factor (R_{AA}):** High-energy partons lose energy while traversing the QGP, resulting in a suppression of high- p_T hadrons. This suppression is quantified using the nuclear modification factor:

$$R_{AA}(p_T) = \frac{1}{\langle N_{\text{coll}} \rangle} \cdot \frac{d^2 N^{AA}/dp_T dy}{d^2 N^{pp}/dp_T dy}, \quad (1.5)$$

where $\langle N_{\text{coll}} \rangle$ is the average number of binary nucleon–nucleon collisions.

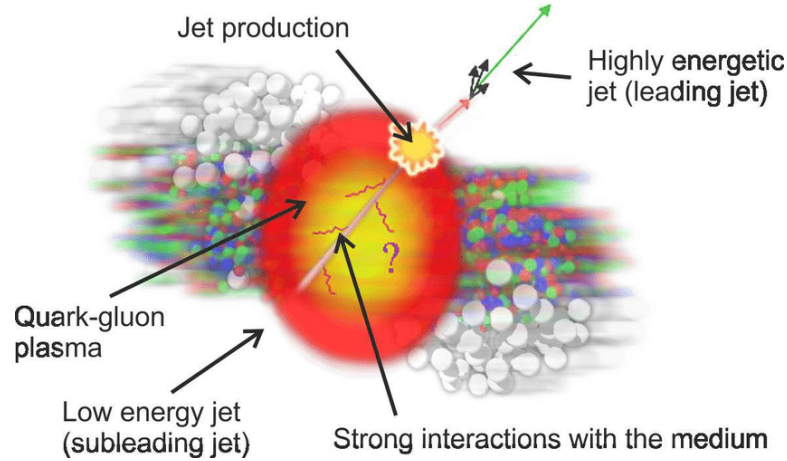


Figure 1.9: Schematic of the jet quenching mechanism during heavy ion collision [22].

A value $R_{AA} < 1$ indicates parton energy loss in the medium.

- **Strangeness enhancement:** The production of strange and multi-strange hadrons serves as an important signature of the formation of the quark–gluon plasma in heavy-ion collisions, a phenomenon commonly referred to as strangeness enhancement. Since the incoming nuclei are composed primarily of up and down quarks, strange quarks are not initially present and must be produced during the collision process. In the early stages,

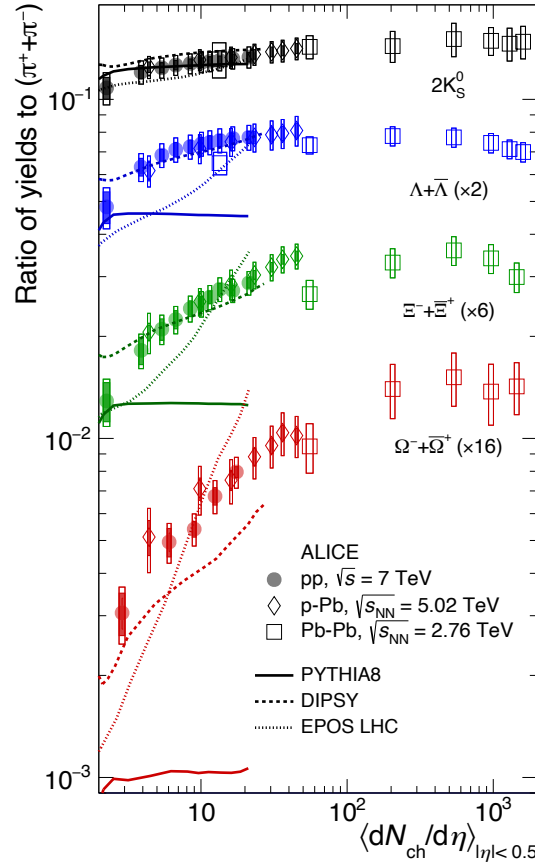


Figure 1.10: Transverse-momentum integrated yield ratios to pions ($\pi^+ + \pi^-$) as a function of the average charged-particle multiplicity density $\langle dN_{\text{ch}}/d\eta \rangle$, measured within $|y| < 0.5$ [23].

strange quarks are generated through perturbative Quantum Chromodynamics processes such as quark–antiquark annihilation ($q\bar{q} \rightarrow s\bar{s}$), flavor

1.6 Experimental signatures of QGP formation

excitation ($gs \rightarrow gs$, $qs \rightarrow qs$), and gluon fusion ($gg \rightarrow s\bar{s}$).

In the presence of a deconfined medium, gluon fusion becomes the dominant mechanism for strangeness production due to the high gluon density in the QGP. In contrast, in minimum-bias proton–proton (pp) collisions, strangeness production is primarily driven by flavor creation processes. The enhancement of strange hadron production in nucleus–nucleus (AA) collisions relative to pp collisions can be quantified through the enhancement factor, defined as [24]

$$E = \frac{2}{\langle N_{\text{part}} \rangle} \left[\frac{\left. \frac{dN^{\text{AA}}}{dy} \right|_{y=0}}{\left. \frac{dN^{\text{pp}}}{dy} \right|_{y=0}} \right], \quad (1.6)$$

where $\langle N_{\text{part}} \rangle$ denotes the average number of participating nucleons, and $dN/dy|_{y=0}$ represents the particle yield at midrapidity.

More recent studies, however, characterize strangeness enhancement by normalizing strange hadron yields to those of non-strange particles, such as pions. This approach is advantageous as it reduces the dependence on external baseline measurements from different collision systems. Measurements performed by the ALICE experiment show that the ratios of strange hadrons to pions, plotted as a function of charged-particle multiplicity density $\langle dN_{\text{ch}}/d\eta \rangle_{|\eta| < 0.5}$, exhibit a clear hierarchy: particles with higher strangeness content display stronger enhancement with increasing multiplicity as shown in Fig. 1.10. Interestingly, similar trends are observed not only in Pb–Pb collisions but also in high-multiplicity pp and p–Pb collisions, suggesting that QGP-like behavior may emerge even in small collision systems.

- **Quarkonium suppression (color screening):** The suppression of heavy quark bound states (e.g., J/ψ , Υ) arises due to color Debye screening in the QGP, which prevents the formation of quark–antiquark bound states.

The screening radius r_D is inversely proportional to the temperature:

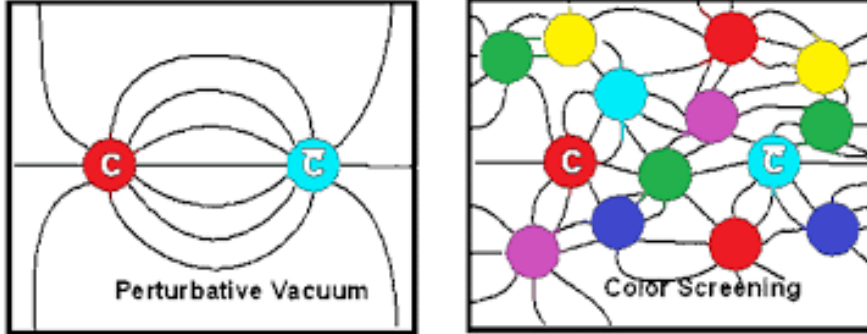


Figure 1.11: Schematic of suppression of quarkonia production by color screening [25].

$$r_D \sim \frac{1}{gT}, \quad (1.7)$$

where g is the strong coupling constant and T is the temperature. The suppression is often quantified using the nuclear modification factor [26]:

$$R_{AA}^{J/\psi} = \frac{(dN/dy)_{J/\psi}^{AA}}{\langle N_{\text{coll}} \rangle \cdot (dN/dy)_{J/\psi}^{pp}}, \quad (1.8)$$

where $\langle N_{\text{coll}} \rangle$ is the average number of binary nucleon–nucleon collisions. A value of $R_{AA} < 1$ indicates suppression relative to the scaled pp yield. The extent of suppression depends on the binding energy of the state, with more weakly bound states dissociating at lower temperatures, providing a QGP thermometer through the concept of “sequential melting.”

This hierarchical suppression pattern, referred to as sequential melting, arises from the color screening of the quark–antiquark potential in the medium. States with lower binding energy melt at lower temperatures, while more tightly bound states survive up to higher temperatures. This behavior provides direct evidence of deconfinement and allows quarkonium states to serve as probes of the thermodynamic properties of the quark–gluon plasma.

1.7 ϕ meson production and strangeness enhancement in heavy-ion collisions

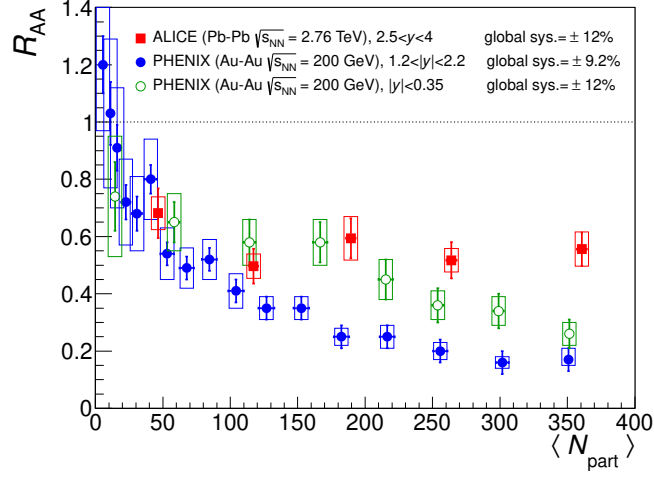


Figure 1.12: Inclusive J/ψ nuclear modification factor (R_{AA}) as a function of the number of participating nucleons $\langle N_{part} \rangle$ in Pb–Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, compared with PHENIX results in Au–Au collisions at $\sqrt{s_{NN}} = 200$ GeV [27].

Measurements have shown that the J/ψ production is more suppressed at RHIC energies compared to LHC energies, as shown in Fig. 1.12. This difference is attributed to the significantly larger charm quark production from decay of higher quarkonia states at LHC energies, which enhances the probability of $c\bar{c}$ recombination/regeneration in the medium. This regeneration mechanism leads to a comparatively weaker net suppression at LHC energies.

1.7 ϕ meson production and strangeness enhancement in heavy-ion collisions

The $\phi(1020)$ meson is a neutral vector meson composed predominantly of a strange quark and a strange antiquark ($s\bar{s}$). It belongs to the vector meson nonet with quantum numbers $J^{PC} = 1^{--}$ and has a mass of approximately 1.02 GeV/ c^2 . Due to its hidden strangeness content and relatively small hadronic interaction cross section, the ϕ meson serves as an important probe for studying the properties of the hot and dense medium created in relativistic heavy-ion

collisions.

In ultra-relativistic nucleus–nucleus collisions, such as those performed at the LHC, a strongly interacting state of matter known as the QGP is expected to be formed. Various experimental observables have been proposed as signatures of QGP formation. One of the most prominent signatures is the enhancement of strange and multi-strange hadron production in heavy-ion collisions compared to proton–proton collisions at similar energies. Since the ϕ meson consists of a strange quark pair, its production is closely related to the phenomenon of strangeness enhancement and provides valuable insight into the dynamics of strangeness production in the QGP medium.

The ϕ meson has several decay channels [22]. The dominant decay mode is the hadronic decay into a pair of charged kaons,

$$\phi \rightarrow K^+ K^-, \quad (1.9)$$

which has a branching ratio of approximately 49%. Other decay channels include neutral kaon decay modes such as

$$\phi \rightarrow K_L^0 K_S^0. \quad (1.10)$$

In addition to these hadronic decay modes, the ϕ meson can also decay electromagnetically into lepton pairs with a very small branching ratio (O^{-4}) as

$$\phi \rightarrow e^+ e^-, \quad (1.11)$$

and

$$\phi \rightarrow \mu^+ \mu^-. \quad (1.12)$$

Among these channels, the dilepton decay modes are of particular interest in heavy-ion experiments. Unlike hadrons, leptons interact only electromagnetically and therefore experience minimal final-state interactions with the surrounding medium. As a result, dileptons carry relatively undistorted information about the early stages of the collision and the properties of the produced medium.

1.7 ϕ meson production and strangeness enhancement in heavy-ion collisions

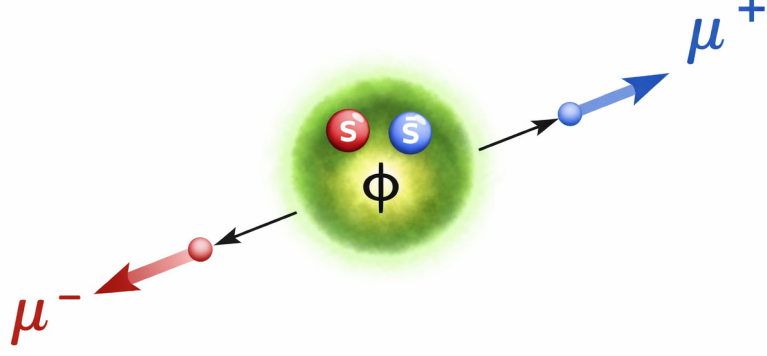


Figure 1.13: Schematic illustration of the $\phi(1020)$ meson, composed of a strange quark–antiquark pair ($s\bar{s}$), decaying into a dimuon pair ($\mu^+\mu^-$) at rest.

The dimuon decay channel,

$$\phi \rightarrow \mu^+\mu^-, \quad (1.13)$$

is especially advantageous for experimental measurements in the forward rapidity region of the ALICE detector [23]. Muons are highly penetrating particles that can traverse large amounts of material with relatively small energy loss. Consequently, they can be efficiently detected and identified using the muon spectrometer of ALICE. The ALICE experiment at the LHC is specifically designed to study the properties of strongly interacting matter created in heavy-ion collisions. The ALICE muon spectrometer, located at forward rapidity, enables precise measurements of vector mesons through their dimuon decay channels. By reconstructing the invariant mass distribution of opposite-sign muon pairs, the ϕ meson signal can be extracted and its production yield can be studied as a function of collision centrality, transverse momentum, and rapidity. Measurements of ϕ meson production provide important information about the mechanisms of strangeness

production and the evolution of the medium formed in heavy-ion collisions. In particular, comparisons between heavy-ion and proton–proton collisions allow the investigation of possible enhancements in strange particle production, which is considered one of the key signatures of QGP formation.

A detailed description of the analysis procedure used to reconstruct the ϕ meson via the dimuon decay channel, including event selection, signal extraction, background subtraction, and efficiency corrections, is presented in chapter 5 of this thesis.

1.8 Structure of the Thesis

This thesis is organized into five chapters that together provide a coherent investigation of strongly interacting matter under extreme conditions. Following the introductory overview of the Standard Model, QCD, and heavy-ion physics, the work proceeds to a phenomenological study of transport and thermoelectric properties of QCD matter in the presence of magnetic fields. The scope then broadens to include applied aspects through the design of novel materials for beampipe applications in accelerator technology. Finally, the thesis culminates with an experimental analysis of $\phi(1020)$ meson production in Pb–Pb collisions in Run-3 data using the ALICE detector at the LHC.

- **Chapter 1: Introduction**

This chapter provides the necessary theoretical background and motivation for the present work. It begins with a brief overview of the Standard Model of particle physics, emphasizing the role of Quantum Chromodynamics as the fundamental theory of the strong interaction. Subsequently, the QCD phase diagram is introduced, highlighting the possible phases of strongly interacting matter. The concept of heavy-ion collisions as a laboratory to recreate conditions similar to the early universe is then discussed. A detailed

1.8 Structure of the Thesis

description of the spacetime evolution of such collisions is presented, along with an overview of the experimental signatures of QGP formation. Finally, the motivation and scope of the thesis are summarized.

- **Chapter 2: Transport phenomena in QCD matter in the presence of a magnetic field**

In this chapter, the transport properties of hot and dense QCD matter subjected to strong magnetic fields are studied. The analysis begins with a kinetic theory framework and progresses to relativistic hydrodynamics, leading to a phenomenological description of QCD matter. Particular emphasis is given to the evaluation of transport coefficients, such as electrical and thermal conductivities, in both confined and deconfined phases. A dedicated analysis is devoted to the thermal conductivity of the evolving QGP, which plays a crucial role in characterizing the dynamics of the medium.

- **Chapter 3: Thermoelectric phenomena in QCD matter in the presence of a magnetic field**

This chapter explores the thermoelectric response of QCD matter, with and without the presence of external magnetic fields. Starting with a general introduction to the concept, the thermoelectric coefficients for hot and dense hadronic matter are computed. The discussion is then extended to the quark–gluon plasma, considering both static and dynamically evolving systems. The obtained results provide insights into leading-order and higher-order thermoelectric transport coefficients of the QCD matter in the context of heavy-ion collisions.

- **Chapter 4: Overview of particle accelerators, detectors and beam pipe design**

Moving beyond QCD matter itself, this chapter addresses the practical aspect of accelerator technology. An overview of the LHC and the ALICE

detector is also provided. It discusses the development and design of novel materials for use in accelerator beampipes, relevant for both low- and high-energy regimes. The aim is to improve the efficiency, cost and durability of beampipe structures under extreme radiation and thermal-stress conditions, with implications for current and future particle accelerator facilities.

• **Chapter 5: Analysis of $\phi(1020)$ meson production in Pb–Pb collisions at $\sqrt{s_{\text{NN}}} = 5.36$ TeV with ALICE at the Large Hadron Collider**

The final chapter presents an experimental analysis of $\phi(1020)$ meson production in Pb–Pb collisions at $\sqrt{s_{\text{NN}}} = 5.36$ TeV with ALICE at the LHC. The study is dedicated to the forward rapidity region of the ALICE detector, with a focus on the dimuon decay channel of the ϕ meson. The dimuon channel provides a very clean signal because of its minimal interactions with the strongly interacting medium. This chapter also discusses the detectors involved in standalone and global muon tracking, along with their kinematic cuts. The results of this analysis are then presented along with the complete details of the considered data and Monte-Carlo (MC) analysis.

Overall, the thesis thus bridges the theoretical, phenomenological, experimental, and applied aspects of QCD matter and accelerator physics, presenting a comprehensive exploration of the subject.

Bibliography

- [1] W. Busza, K. Rajagopal and W. van der Schee, *Ann. Rev. Nucl. Part. Sci.* **68**, 339 (2018)
- [2] F. Gross, E. Klempt, S. J. Brodsky, A. J. Buras, V. D. Burkert, G. Heinrich, K. Jakobs, C. A. Meyer, K. Orginos and M. Strickland, *et al.* *Eur. Phys. J. C* **83**, 1125 (2023)
- [3] J. Paczos, S. Cedrowski, K. Turzyński and A. Dragan, *Nucl. Phys. B* **1024**, 117351 (2026)
- [4] C. Betancourt, *Nucl. Phys. B* **1017**, 116947 (2025)
- [5] https://upload.wikimedia.org/wikipedia/commons/thumb/0/00/Standard_Model_of_Elementary_Particles.svg/1254px-Standard_Model_of_Elementary_Particles.svg.png?20240130105357
- [6] K. Singh and R. Sahoo, *Phys. Lett. B* **872**, 140060 (2026)
- [7] M. Niiyama, *Few Body Syst.* **62**, 99 (2021)
- [8] A. Deur, S. J. Brodsky and G. F. de Teramond, *Nucl. Phys.* **90**, 1 (2016)
- [9] K. Singh, J. Dey and R. Sahoo, *Phys. Rev. D* **110**, 114051 (2024)
- [10] M. I. Gorenstein and S. N. Yang, *Phys. Rev. D* **52**, 5206 (1995)

- [11] K. Goswami, D. Sahu, J. Dey, R. Sahoo and R. Stock, Phys. Rev. D **109**, 074012 (2024)
- [12] D. J. Griffiths, “Introduction to elementary particles,” doi:10.1002/9783527618460
- [13] T. A. DeGrand, Lect. Notes Phys. **409**, 159 (1992)
- [14] R. X. Adhikari, L. A. Anchordoqui, *et al.* [arXiv:2209.11726 [hep-ph]]
- [15] P. Senger, Universe **7**, 171 (2021)
- [16] S. Afanasiev *et al.*, Nucl. Instrum. Methods Phys. Res., Sect. A **1065**, 169532 (2024)
- [17] A. Mocsy and P. Sorensen, [arXiv:1008.3381 [hep-ph]]
- [18] D. Kincses, M. Nagy and M. Csanád, Commun. Phys. **8**, 55 (2025)
- [19] F. G. Gardim, G. Giacalone, M. Luzum and J. Y. Ollitrault, Nature Phys. **16**, 615 (2020)
- [20] S. Prasad, N. Mallick, S. Tripathy and R. Sahoo, Phys. Rev. D **107**, 074011 (2023)
- [21] A. Saini, S. Bhardwaj, Journal of Nuclear and Particle Physics, **4**, 6 (2014)
- [22] P. Štefko, 18th Conference of Czech and Slovak Physicists, 93, (2015)
- [23] J. Adam *et al.* [ALICE], Nature Phys. **13**, 535 (2017)
- [24] S. Prasad, doi:10.17181/ts0ew-rb520
- [25] https://indico.cern.ch/event/703015/contributions/3095332/attachments/1712543/2761411/matonoha_hq_5.pdf

BIBLIOGRAPHY

- [26] D. Behera, S. Deb, C. R. Singh and R. Sahoo, Phys. Rev. C **109**, 014902 (2024)
- [27] B. Abelev *et al.* [ALICE], Phys. Rev. Lett. **109**, 072301 (2012)
- [28] E. A. R. Casula [ALICE], Nucl. Phys. A **967**, 425 (2017)
- [29] S. Acharya *et al.* [ALICE], Eur. Phys. J. C **78**, 559 (2018)

Chapter 2

Transport phenomenon in QCD matter in the presence of a magnetic field

“The entire universe is full of wonders.”

— Guru Gobind Singh

Transport phenomena have played a fundamental role in the development of physics since ancient times, long before their formal mathematical description. Early civilizations were already aware of transport processes through practical observations such as the flow of water in irrigation systems, the diffusion of heat in cooking and metallurgy, and the motion of air and fluids in natural environments. Greek philosophers like Aristotle discussed motion, flow, and resistance qualitatively, laying conceptual foundations for later scientific inquiry. During the classical era, the study of heat conduction, fluid flow, and momentum transfer gradually evolved through the works of scientists such as Galileo, Newton, and Fourier, leading to quantitative laws governing diffusion, viscosity, and thermal transport [1]. These processes are now understood as macroscopic manifestations of microscopic interactions among constituents of matter. Transport phenomena thus provide a unifying framework connecting microscopic dynamics to macroscopic behavior and remain central to diverse areas of physics, ranging from condensed matter systems and astrophysical plasmas to relativistic quantum field

theories such as quantum chromodynamics (QCD).

*“The gradient in temperature sets heat in flight,
The gradient in potential drives charges with might,
The gradient in concentration lets diffusion beat,
Thus flows arise where imbalances meet.”*

2.1 Kinetic theory framework

Transport phenomena in many-body systems arise from the response of microscopic constituents to external perturbations and thermodynamic gradients, leading to macroscopic flows of charge, energy, and momentum. In relativistic strongly interacting matter, such as hot and dense hadronic or partonic media, these processes are conveniently described within the framework of relativistic kinetic theory. The fundamental quantity in this approach is the single-particle phase-space distribution function $f_i(\vec{x}, \vec{k}, t)$ for the i th particle species, which encodes the statistical properties of the system at the microscopic level. In local thermodynamic equilibrium, the system is characterized by an equilibrium distribution function f_i^0 , while deviations from equilibrium are described by a small correction δf_i , such that $f_i = f_i^0 + \delta f_i$. These deviations are generated by spatial and temporal variations of thermodynamic variables, as well as by external force fields.

The time evolution of the distribution function is governed by the Boltzmann transport equation (BTE), which, in its most general form, can be written as

$$\frac{\partial f_i}{\partial t} + \vec{v}_i \cdot \vec{\nabla} f_i + \vec{F}_i \cdot \frac{\partial f_i}{\partial \vec{k}_i} = \mathcal{C}_i[f], \quad (2.1)$$

where $\vec{v}_i = \vec{k}_i/\omega_i$ is the particle velocity, $\vec{k}_i$ is the particle momentum, ω_i is particle energy, $\vec{F}_i$ denotes the external force acting on the i th particle, and $\mathcal{C}_i[f]$ represents the collision integral accounting for interactions among the constituents

2.2 Relativistic hydrodynamics

of the medium. The force term $\vec{F}_i$ is kept general at this stage; in the presence of electromagnetic fields, it reduces to the Lorentz force, $\vec{F}_i = q_i(\vec{E} + \vec{v}_i \times \vec{B})$, thereby allowing the systematic incorporation of magnetic field effects in the transport dynamics.

For systems close to local equilibrium, the collision term can be linearized using the relaxation time approximation (RTA), wherein $\mathcal{C}_i[f] \simeq -\delta f_i/\tau_R^i$. This approximation assumes that the system relaxes toward equilibrium over a characteristic time scale τ_R^i , specific to each particle species. Within this linear response regime, the deviations δf_i induced by external fields and thermodynamic gradients give rise to macroscopic currents. In kinetic theory, the electric current density of the system is defined as [2]

$$\vec{j} = \sum_i g_i \int \frac{d^3 k_i}{(2\pi)^3} q_i \frac{\vec{k}_i}{\omega_i} \delta f_i, \quad (2.2)$$

where g_i denotes the degeneracy factor and the summation runs over all particle species. Similarly, for a relativistic system with conserved baryon number, the heat current is defined with respect to the flow of baryonic enthalpy. The heat current $\vec{I}$ can be expressed as [2, 3]

$$\vec{I} = \sum_i g_i \int \frac{d^3 k_i}{(2\pi)^3} \frac{\vec{k}_i}{\omega_i} (\omega_i - b_i h) \delta f_i, \quad (2.3)$$

where $h = (\varepsilon + P)/n$ is the enthalpy per particle, b_i is the baryon number, and ε , P , and n denote the total energy density, pressure, and net baryon density of the medium, respectively. These expressions provide the microscopic foundation for evaluating transport coefficients such as electrical conductivity, thermal conductivity, and thermoelectric coefficients within the kinetic theory framework.

2.2 Relativistic hydrodynamics

Relativistic hydrodynamics provides a powerful macroscopic framework to describe the space-time evolution of strongly interacting matter created in ultra-

relativistic heavy-ion collisions, such as the quark–gluon plasma (QGP) formed at RHIC and the LHC [4, 5]. The underlying assumption of this approach is the existence of local thermodynamic equilibrium, allowing the complex microscopic dynamics of quantum chromodynamics to be encoded in a few hydrodynamic fields such as the energy density, pressure, and flow velocity. Once supplemented with an appropriate equation of state (EoS), relativistic hydrodynamics enables a quantitative description of the expansion dynamics, cooling behavior, and transport properties of the medium [6].

The fundamental object in relativistic hydrodynamics is the symmetric energy–momentum tensor $T^{\mu\nu}$, which can be decomposed in terms of hydrodynamic degrees of freedom as [7]

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + \pi^{\mu\nu} + W^\mu u^\nu + W^\nu u^\mu, \quad (2.4)$$

where ϵ denotes the energy density, p is the thermodynamic pressure, and u^μ is the four-velocity of the fluid satisfying $u^\mu u_\mu = 1$. The tensors $\pi^{\mu\nu}$ and W^μ represent dissipative contributions arising from shear viscosity and energy diffusion, respectively. The metric tensor of flat Minkowski spacetime is taken as $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$. The macroscopic evolution of the fluid is governed by conservation laws, namely the conservation of energy and momentum,

$$\partial_\nu T^{\mu\nu} = 0, \quad (2.5)$$

and relevant conserved charges such as baryon number. Projecting Eq. (2.5) along the fluid four-velocity leads to the energy conservation (continuity) equation,

$$u_\mu \partial_\nu T^{\mu\nu} = 0, \quad (2.6)$$

which plays a central role in determining the cooling rate and temperature evolution of the medium. Solving this equation requires specifying the flow profile and the equation of state of the system.

In the simplest formulation, dissipative effects are neglected, leading to ideal relativistic hydrodynamics. This approximation is justified at early times when the

2.2 Relativistic hydrodynamics

QGP behaves approximately as a near-perfect fluid with small viscosity. More refined descriptions incorporate viscous corrections; however, for analytical insight into the cooling dynamics, ideal hydrodynamic solutions provide a useful starting point.

2.2.1 Bjorken flow

Bjorken flow [8] represents one of the most widely used analytical solutions of relativistic hydrodynamics and describes a boost-invariant longitudinal expansion of the medium. In addition to boost invariance along the beam (z) direction, Bjorken flow assumes translational and rotational invariance in the transverse plane. These symmetries lead to the velocity profile

$$u^\mu = \gamma \left(1, 0, 0, \frac{z}{t} \right), \quad (2.7)$$

where $\gamma = t/\tau$ and $\tau = \sqrt{t^2 - z^2}$ is the proper time.

For an ideal fluid in the absence of external fields, the energy-momentum tensor from Eq. (2.2) reduces to

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}. \quad (2.8)$$

Substituting Eqs. (2.7) and (2.8) into the continuity equation (2.6), one obtains the Bjorken energy evolution equation,

$$\frac{d\epsilon}{d\tau} + \frac{\epsilon + p}{\tau} = 0. \quad (2.9)$$

Assuming a conformal equation of state, $p = \epsilon/3$, which is appropriate for a relativistic plasma at high temperature, the temperature evolution follows as

$$T(\tau) = T_0 \left(\frac{\tau_0}{\tau} \right)^{1/3}, \quad (2.10)$$

where T_0 is the initial temperature at the formation time τ_0 . This expression captures the characteristic power-law cooling of the QGP under longitudinal expansion.

2.2.2 Magnetized Bjorken flow

In non-central heavy-ion collisions, extremely strong magnetic fields are generated due to the relativistic motion of spectator protons. If the electrical conductivity of the medium is sufficiently large, these fields can persist long enough to influence the hydrodynamic evolution. The energy-momentum tensor for an ideal magnetized fluid can be written as [3, 9, 10]

$$T^{\mu\nu} = (\epsilon + p + B^2)u^\mu u^\nu - \left(p + \frac{B^2}{2}\right)g^{\mu\nu} - B^\mu B^\nu, \quad (2.11)$$

where B^μ is the magnetic field four-vector in the local rest frame of the fluid. The magnetic field is assumed to decay exponentially with proper time,

$$B(\tau) = B_0 \exp\left(-\frac{\tau}{\tau_B}\right), \quad (2.12)$$

with B_0 denoting the initial magnetic field strength and τ_B its decay time scale. Solving the hydrodynamic equations with a conformal equation of state leads to a modified cooling equation that can be cast into a Bernoulli-type differential equation for the temperature. The analytical solution yields a nontrivial temperature profile [6],

$$T(\tau) = \left[T_0^4 \left(\frac{\tau_0}{\tau}\right)^{4/3} + \frac{4\alpha}{(2\beta\tau)^{4/3}} \left\{ \Gamma\left(\frac{4}{3}, 2\beta\tau\right) - \Gamma\left(\frac{4}{3}, 2\beta\tau_0\right) \right\} - \frac{2\alpha}{(2\beta\tau)^{7/3}} \left\{ \Gamma\left(\frac{7}{3}, 2\beta\tau\right) - \Gamma\left(\frac{7}{3}, 2\beta\tau_0\right) \right\} \right]^{1/4}, \quad (2.13)$$

where $\alpha = B_0^2/(12a)$, $\beta = 1/\tau_B$, $a = (16 + \frac{21}{2}N_f)\pi^2/90$, and N_f is the number of degrees of freedom.

2.2.3 Gubser flow

While Bjorken flow accounts only for longitudinal expansion, Gubser flow [11] generalizes this framework by incorporating transverse dynamics while preserving

2.3 Phenomenological approach to QCD matter

conformal symmetry. Working in Milne coordinates (τ, η, r, ϕ) , the flow velocity consistent with boost invariance and transverse symmetry is given by

$$u^\tau = \cosh \kappa, \quad u^r = \sinh \kappa, \quad u^\eta = u^\phi = 0, \quad (2.14)$$

with the flow rapidity $\kappa(\tau, r)$ defined as

$$\kappa(\tau, r) = \operatorname{arctanh} \left(\frac{2q^2\tau r}{1 + q^2\tau^2 + q^2r^2} \right). \quad (2.15)$$

Assuming a conformal equation of state and shear viscosity proportional to $\epsilon^{3/4}$, the hydrodynamic equations can be solved analytically, yielding the temperature profile

$$T = \frac{1}{\tau f_*^{1/4}} \left[\frac{T_0}{(1 + g^2)^{1/3}} + \frac{H_0 g}{\sqrt{1 + g^2}} \left\{ 1 - (1 + g^2)^{1/6} {}_2F_1 \left(\frac{1}{2}, \frac{1}{6}; \frac{3}{2}; -g^2 \right) \right\} \right], \quad (2.16)$$

where $g = (1 - q^2\tau^2 + q^2r^2)/(2q\tau)$ and ${}_2F_1$ denotes the hypergeometric function. This solution provides a more realistic description of the spatiotemporal evolution of the QGP, particularly in the presence of transverse expansion.

2.3 Phenomenological approach to QCD matter

The phenomenological description of QCD matter relies on different effective models to characterize its behavior in distinct phases. In the deconfined phase, where quarks and gluons are the relevant degrees of freedom, quasiparticle models are commonly employed to incorporate medium effects via temperature and chemical-potential dependent masses, reproducing lattice QCD thermodynamics. In contrast, the confined hadronic phase is effectively described by the hadron resonance gas model, where the system is treated as an ensemble of hadrons and resonances in thermal equilibrium. Together, these approaches provide a consistent framework to study the thermodynamic and transport properties of QCD matter across the phase transition.

2.3.1 Quasi-particle model

For the numerical analysis of the QGP medium, we employ a quasiparticle model originally proposed by Gorenstein and Yang [12]. This phenomenological framework reproduces the lattice QCD equation of state for the quark–gluon plasma by incorporating temperature-dependent effective masses for partons. These thermal masses originate from medium-induced interactions among quarks and gluons. Thermodynamic consistency within the model is ensured through the inclusion of a vacuum energy contribution in the form of a bag constant.

Within this approach, the dispersion relation for a particle with momentum k_i and energy ω_i is given by $\omega_i^2(k_i, T) = k_i^2 + m_i^2(T)$, where $m_i(T)$ denotes the effective mass of the i th quark flavor. The effective mass can be decomposed into a bare and a thermal component as

$$m_i = m_{i0} + m_{iT}. \quad (2.17)$$

Here, m_{i0} represents the bare quark mass, while m_{iT} denotes the temperature-dependent thermal mass, expressed as

$$m_{iT}^2(T, \mu_B) = \frac{N_c^2 - 1}{8N_c} \left(T^2 + \frac{\mu_B^2}{9\pi^2} \right) g^2(T). \quad (2.18)$$

The effective gluon mass in this quasiparticle model is given by

$$m_g^2(T, \mu_B) = \frac{N_c}{6} g^2(T) T^2 \left(1 + \frac{N_f + \frac{\mu_B^2}{\pi^2 T^2}}{6} \right). \quad (2.19)$$

In the above expressions, N_c denotes the number of color degrees of freedom, and the temperature-dependent coupling is defined as $g^2(T) = 4\pi\alpha_s(T)$, where $\alpha_s(T)$ is the running strong coupling constant. The parametrization of the coupling follows the prescription adopted in Ref. [13], which successfully reproduces lattice QCD results obtained by the Wuppertal–Budapest Collaboration [14].

Further discussions and recent refinements of the quasiparticle model can be found in Refs. [15–19]. It is worth noting that, in the calculation of thermoelectric

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transport coefficients and the induced electric field, gluons contribute exclusively through their role in determining the total enthalpy of the system.

2.3.2 Hadron resonance gas model

We discuss the hadron resonance gas model and its extended versions, which we use to calculate the leading- and higher-order thermoelectric coefficients of a hot, dense hadron gas. Throughout the calculations, we have taken all the hadrons and their resonances up to the mass cut-off of 2.6 GeV from the particle data group [20].

The Hadron Resonance Gas (HRG) model provides a theoretical description of the thermodynamic behavior of strongly interacting matter in its hadronic phase. Within this approach, the medium is treated as an ensemble of hadrons and resonances in thermal equilibrium, such that macroscopic thermodynamic quantities, including temperature and chemical potential, are well defined across the system. In the following, we outline and discuss the different implementations of the HRG model.

2.3.2.1 Ideal hadron resonance gas model

The ideal HRG model describes a thermodynamic system of volume V composed of pointlike, non-interacting hadrons and resonances. Within this framework, the grand canonical partition function for each hadronic species can be expressed as [30],

$$\ln Z_i^{id} = \pm \frac{V g_i}{2\pi^2} \int_0^\infty k_i^2 dk_i \ln\{1 \pm \exp[-(\omega_i - \mu_i)/T]\}. \quad (2.20)$$

Here, g_i denotes the degeneracy factor of the i th hadron species. The quantities k_i , m_i , and $\omega_i = \sqrt{k_i^2 + m_i^2}$ correspond to the momentum, mass, and single-particle energy, respectively. The upper (lower) sign applies to fermionic (bosonic) degrees of freedom.

For simplicity, we consider vanishing electric charge and strangeness chemical potentials. Under this assumption, the chemical potential of the i th hadron species, μ_i , is determined solely by the baryon chemical potential μ_B and is given by

$$\mu_i = \beta_i \mu_B, \quad (2.21)$$

where β_i represents the baryon number of the corresponding hadron. Using the partition function, the thermodynamic observables such as pressure, energy density, and number density can be derived as [30]

$$P_i^{id}(T, \mu_i) = \pm \frac{T g_i}{2\pi^2} \int_0^\infty k_i^2 dk_i \ln\{1 \pm \exp[-(\omega_i - \mu_i)/T]\}, \quad (2.22)$$

$$\varepsilon_i^{id}(T, \mu_i) = \frac{g_i}{2\pi^2} \int_0^\infty \frac{\omega_i k_i^2 dk_i}{\exp[(\omega_i - \mu_i)/T] \pm 1}, \quad (2.23)$$

$$n_i^{id}(T, \mu_i) = \frac{g_i}{2\pi^2} \int_0^\infty \frac{k_i^2 dk_i}{\exp[(\omega_i - \mu_i)/T] \pm 1}. \quad (2.24)$$

2.3.2.2 Excluded volume hadron resonance gas model

The excluded volume HRG (EVHRG) model incorporates short-range repulsive interactions among hadrons by assigning them a finite hard-core size. As a consequence, the total system volume V is replaced by an effective available volume V_{avl} , which accounts for the reduction in accessible space due to repulsive interactions,

$$V_{avl} = V - \sum_i N_i b_i, \quad (2.25)$$

where N_i is the number of hadrons of species i and $b_i (= 16/3 \pi r_i^3)$ denotes the excluded volume associated with a hadron of hard-core radius r_i .

Within this framework, the pressure of the interacting system is obtained by relating it to the ideal HRG pressure evaluated at an effective chemical potential [22, 23],

$$P^{EV}(T, \mu) = P^{id}(T, \mu^*), \quad (2.26)$$

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where $\mu^* = \mu - bP^{EV}(T, \mu)$. Other thermodynamic quantities follow directly by taking appropriate derivatives of Eq. (2.26). In particular, the number density and energy density can be written as [22]

$$n^{EV}(T, \mu) = \sum_i \frac{n_i^{id}(T, \mu^*)}{1 + \sum_i b_i n_i^{id}(T, \mu^*)}, \quad (2.27)$$

$$\varepsilon^{EV}(T, \mu) = \sum_i \frac{\varepsilon_i^{id}(T, \mu^*)}{1 + \sum_i b_i n_i^{id}(T, \mu^*)}. \quad (2.28)$$

Since different hard-core radii are assigned to mesons and baryons, the excluded volume parameter b contributes differently to the thermodynamic observables.

2.3.2.3 van der Waals hadron resonance gas model

The van der Waals HRG (VDWHRG) model provides a more realistic description of hadronic matter by incorporating both repulsive interactions through excluded volume effects and attractive interactions among hadrons. The inclusion of these interactions leads to a liquid–gas phase transition terminating at a critical point [24]. In hadronic models, attractive interactions are typically assumed to act between baryon–baryon and antibaryon–antibaryon pairs [24–26]. Following Refs. [27, 30], repulsive interactions among mesons are also taken into account, while baryon–antibaryon and meson–(anti)baryon interactions are neglected.

In the canonical ensemble, the van der Waals equation of state is given by [26, 30]

$$P(T, n) = \frac{nT}{1 - bn} - an^2, \quad (2.29)$$

where a and b are positive parameters characterizing attractive and repulsive interactions, respectively. Here, P denotes the pressure and $n = N/V$ is the number density. The first term incorporates excluded volume effects through the replacement of the total volume by an effective volume, while the second term accounts for attractive interactions. The model reduces to the EVHRG limit when $a = 0$ and further simplifies to the ideal HRG when both interaction parameters vanish.

In the grand canonical ensemble, the corresponding equation of state takes the form [26, 28]

$$P(T, \mu) = P^{id}(T, \mu^*) - an^2(T, \mu), \quad (2.30)$$

with the particle number density given by

$$n(T, \mu) = \frac{\sum_i n_i^{id}(T, \mu_i^*)}{1 + b \sum_i n_i^{id}(T, \mu_i^*)}. \quad (2.31)$$

The effective chemical potential μ^* incorporates interaction effects and is defined as

$$\mu^* = \mu - bP(T, \mu) - abn^2(T, \mu) + 2an(T, \mu). \quad (2.32)$$

Owing to interactions, the total pressure in the VDWHRG model receives separate contributions from mesons, baryons, and antibaryons [24, 26],

$$P(T, \mu) = P_M(T, \mu) + P_B(T, \mu) + P_{\bar{B}}(T, \mu), \quad (2.33)$$

where the individual components are given by

$$P_M(T, \mu) = \sum_{i \in M} P_i^{id}(T, \mu^{*M}), \quad (2.34)$$

$$P_B(T, \mu) = \sum_{i \in B} P_i^{id}(T, \mu^{*B}) - an_B^2(T, \mu), \quad (2.35)$$

$$P_{\bar{B}}(T, \mu) = \sum_{i \in \bar{B}} P_i^{id}(T, \mu^{*\bar{B}}) - an_{\bar{B}}^2(T, \mu). \quad (2.36)$$

Here, M , B , and $\bar{B}$ denote mesons, baryons, and antibaryons, respectively. Due to excluded volume effects, the effective chemical potential for mesons is denoted by μ^{*M} , while μ^{*B} and $\mu^{*\bar{B}}$ correspond to baryons and antibaryons modified by van der Waals interactions [29]. These quantities follow from Eqs. (2.21) and (2.32) as

$$\mu^{*M} = -b_M P_M(T, \mu), \quad (2.37)$$

$$\mu^{*B(\bar{B})} = \mu^{B(\bar{B})} - b_B P_{B(\bar{B})}(T, \mu) - ab_B n_{B(\bar{B})}^2 + 2an_{B(\bar{B})}, \quad (2.38)$$

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where the modified number densities are given by

$$n_M(T, \mu) = \frac{\sum_{i \in M} n_i^{id}(T, \mu_i^{*M})}{1 + b_M \sum_{i \in M} n_i^{id}(T, \mu_i^{*M})}, \quad (2.39)$$

$$n_{B(\bar{B})}(T, \mu) = \frac{\sum_{i \in B(\bar{B})} n_i^{id}(T, \mu_i^{*B(\bar{B})})}{1 + b_B \sum_{i \in B(\bar{B})} n_i^{id}(T, \mu_i^{*B(\bar{B})})}. \quad (2.40)$$

The corresponding energy densities for mesons and (anti)baryons are obtained as

$$\varepsilon(T, \mu)_M = \frac{\sum_{i \in M} \varepsilon_i^{id}(T, \mu_i^*)}{1 + b_M \sum_{i \in M} n_i^{id}(T, \mu_i^*)}. \quad (2.41)$$

$$\varepsilon(T, \mu)_{B(\bar{B})} = \frac{\sum_{i \in B(\bar{B})} \varepsilon_i^{id}(T, \mu_i^*)}{1 + b_B \sum_{i \in B(\bar{B})} n_i^{id}(T, \mu_i^*)} - a n_{B(\bar{B})}^2(T, \mu). \quad (2.42)$$

2.3.2.4 Repulsive mean-field hadron resonance gas model

In the repulsive mean-field HRG (RMFHRG) model, short-range repulsive interactions among hadrons are incorporated within a mean-field framework by introducing a density-dependent shift in the single-particle energies. This shift is assumed to be proportional to the total hadron number density. Following the repulsive mean-field prescription adopted in Refs. [30–33], the modified single-particle dispersion relation in the presence of interactions is given by

$$\tilde{\omega}_i = \sqrt{k_i^2 + m_i^2} + U(n) = \omega_i + U(n). \quad (2.43)$$

Here, $U(n)$ denotes the mean-field potential energy arising from repulsive interactions and depends on the total hadron density n . For a given hadronic interaction potential $V(\mathbf{r})$, the corresponding mean-field contribution can be expressed as $U(n) = Kn$, where the phenomenological parameter K is obtained by integrating $V(\mathbf{r})$ over the spatial volume [30, 31].

In this analysis, different repulsive interaction strengths are assigned to baryons and mesons. The mean-field coupling for baryons (B) and antibaryons ($\bar{B}$) is

denoted by K_B , while mesons are characterized by a separate parameter K_M . Accordingly, the mean-field potentials take the form

$$U(n_{B\{\bar{B}\}}) = K_B n_{B\{\bar{B}\}}, \quad (2.44)$$

for (anti)baryons, and

$$U(n_M) = K_M n_M, \quad (2.45)$$

for mesons. The repulsive interaction strengths are chosen as $K_B = 0.450 \text{ GeV fm}^3$ for all (anti)baryons and $K_M = 0.050 \text{ GeV fm}^3$ for all mesons, consistent with Ref. [32].

The total hadron number density is then expressed as

$$n(T, \mu) = \sum_i n_i = n_B + n_{\bar{B}} + n_M, \quad (2.46)$$

where n_i denotes the number density of the i th hadronic species. Here, n_B , $n_{\bar{B}}$, and n_M represent the total baryon, antibaryon, and meson number densities, respectively. Explicitly, the (anti)baryon number density is given by

$$n_{B\{\bar{B}\}} = \sum_{i \in B\{\bar{B}\}} \frac{g_i}{2\pi^2} \int_0^\infty \frac{k_i^2 dk_i}{e^{\left(\frac{\omega_i - \mu_{\text{eff}}^i}{T}\right)} + 1}. \quad (2.47)$$

Here, the effective chemical potential for (anti)baryons is defined as $\mu_{\text{eff}}^i = \beta_i \mu_B - U(n_{B\{\bar{B}\}})$, where β_i is the baryon number of the i th species and μ_B denotes the baryon chemical potential. For mesons, since $\mu_B = 0$, the effective chemical potential reduces to

$$(\mu_{\text{eff}}^i)_M = -U(n_M) = -K_M n_M. \quad (2.48)$$

Consequently, the meson number density can be written as

$$n_M = \sum_{i \in M} \frac{g_i}{2\pi^2} \int_0^\infty \frac{k_i^2 dk_i}{e^{\left(\frac{\omega_i + K_M n_M}{T}\right)} - 1}, \quad (2.49)$$

where the summation runs over all mesonic states.

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The total energy density of (anti)baryons is given by

$$\begin{aligned} \varepsilon_{B\{\bar{B}\}} = & \sum_{i \in B\{\bar{B}\}} \frac{g_i}{2\pi^2} \int_0^\infty \frac{k_i^2 dk_i \tilde{\omega}_i}{e^{\left(\frac{\omega_i - \mu_{\text{eff}}^i}{T}\right)} + 1} \\ & + \phi_{B\{\bar{B}\}}(n_{B\{\bar{B}\}}), \end{aligned} \quad (2.50)$$

while for mesons it takes the form

$$\varepsilon_M = \sum_{i \in M} \frac{g_i}{2\pi^2} \int_0^\infty \frac{k_i^2 dk_i \tilde{\omega}_i}{e^{\left(\frac{\omega_i + K_M n_M}{T}\right)} - 1} + \phi_M(n_M). \quad (2.51)$$

Here, $\phi(n)$ denotes the correction to the energy density introduced to avoid double counting of the mean-field interaction. Taking the derivative of the (anti)baryon energy density with respect to the corresponding number density and using Eq. (2.44), one obtains

$$\frac{\partial \phi_{B\{\bar{B}\}}}{\partial n_{B\{\bar{B}\}}} = -K_B n_{B\{\bar{B}\}}, \quad (2.52)$$

which leads to

$$\phi_B(n_{B\{\bar{B}\}}) = -\frac{1}{2} K_B n_{B\{\bar{B}\}}^2. \quad (2.53)$$

Similarly, for mesons, the correction term becomes

$$\phi_M(n_M) = -\frac{1}{2} K_M n_M^2. \quad (2.54)$$

The pressure of the interacting hadron gas can now be obtained.

For (anti)baryons, it is given by

$$\begin{aligned} P_{B\{\bar{B}\}}(T, \mu) = & T \sum_{i \in B\{\bar{B}\}} \frac{g_i}{2\pi^2} \int_0^\infty k_i^2 dk_i \\ & \ln \left[1 + e^{-\left(\frac{\omega_i - \mu_{\text{eff}}^i}{T}\right)} \right] - \phi_{B\{\bar{B}\}}(n_{B\{\bar{B}\}}), \end{aligned} \quad (2.55)$$

while for mesons the pressure reads

$$\begin{aligned} P_M(T) = & -T \sum_{i \in M} \frac{g_i}{2\pi^2} \int_0^\infty k_i^2 dk_i \\ & \ln \left[1 - e^{-\left(\frac{\omega_i + K_M n_M}{T}\right)} \right] - \phi_M(n_M). \end{aligned} \quad (2.56)$$

2.4 Transport coefficients of hot and dense hadronic medium in the presence of time-varying magnetic field

Transport coefficients quantify the response of a many-body system to external perturbations and characterize the dissipative properties of the medium. In the context of a hot and dense hadronic medium, electrical and thermal conductivities describe, respectively, the transport of electric charge and energy under applied electromagnetic fields and temperature gradients. In this section, we compute these coefficients within the kinetic theory framework using the relativistic Boltzmann transport equation under the relaxation time approximation.

2.4.1 Electrical conductivity

In the presence of an external electromagnetic field, BTE under RTA can be expressed as [2, 3, 6]

$$\frac{\partial f_i}{\partial t} + \frac{\vec{k}_i}{\omega_i} \cdot \frac{\partial f_i}{\partial \vec{x}} + q_i \left(\vec{E} + \frac{\vec{k}_i}{\omega_i} \times \vec{B} \right) \cdot \frac{\partial f_i}{\partial \vec{k}_i} = -\frac{\delta f_i}{\tau_R^i}. \quad (2.57)$$

where τ_R^i denotes the relaxation time of the i th particle species. In order to obtain a solution of the above equation, an appropriate ansatz for the non-equilibrium correction δf_i is required. Since the departure from local equilibrium is induced by the presence of external electromagnetic fields, the leading-order contribution to δf_i can be constructed as a linear response to these fields [34],

$$\delta f_i = (\vec{k}_i \cdot \vec{\Omega}_\sigma) \frac{\partial f_i^0}{\partial \omega_i}. \quad (2.58)$$

Substitution of Eq. (2.58) into Eq. (2.57) leads to,

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$$\begin{aligned}
& (\vec{k}_i \cdot \dot{\vec{\Omega}}_\sigma) \frac{\partial f_i^0}{\partial \omega_i} + \frac{\vec{k}_i}{\omega_i} \cdot (\vec{k}_i \cdot \frac{\partial \vec{\Omega}_\sigma}{\partial \vec{x}}) \frac{\partial f_i^0}{\partial \omega_i} + q_i \left(\vec{E} + \frac{\vec{k}_i}{\omega_i} \times \vec{B} \right) \cdot \frac{\partial f_i^0}{\partial \omega_i} \frac{\partial \omega_i}{\partial \vec{k}_i} \\
& + q_i \left(\vec{E} + \frac{\vec{k}_i}{\omega_i} \times \vec{B} \right) \cdot (\vec{\Omega}_\sigma) \frac{\partial \vec{k}_i}{\partial \vec{k}_i} \frac{\partial f_i^0}{\partial \omega_i} = -\frac{1}{\tau_R^i} (\vec{k}_i \cdot \vec{\Omega}_\sigma) \frac{\partial f_i^0}{\partial \omega_i}. \tag{2.59}
\end{aligned}$$

The second term in Eq. (2.59) contributes only to the viscosity and thermal conductivity of the medium but it has a null contribution in the case of electrical conductivity.

The unknown $\vec{\Omega}_\sigma$ is,

$$\vec{\Omega}_\sigma = \alpha_1 \vec{E} + \alpha_2 \dot{\vec{E}} + \alpha_3 \vec{B} + \alpha_4 \dot{\vec{B}} + \alpha_5 (\vec{E} \times \vec{B}) + \alpha_6 (\dot{\vec{E}} \times \vec{B}) + \alpha_7 (\vec{E} \times \dot{\vec{B}}). \tag{2.60}$$

Substitution of Eq. (2.60) in Eq. (2.59) leads to,

$$\begin{aligned}
& \left\{ \vec{k}_i \cdot \left(\alpha_1 \vec{E} + \alpha_1 \dot{\vec{E}} + \alpha_2 \dot{\vec{E}} + \alpha_5 (\vec{E} \times \vec{B}) + \alpha_5 (\dot{\vec{E}} \times \vec{B}) + \alpha_5 (\vec{E} \times \dot{\vec{B}}) \right. \right. \\
& \left. \left. + \alpha_6 (\dot{\vec{E}} \times \vec{B}) + \alpha_6 (\vec{E} \times \dot{\vec{B}}) + \alpha_7 (\vec{E} \times \dot{\vec{B}}) + \alpha_7 (\dot{\vec{E}} \times \dot{\vec{B}}) \right) + q_i \vec{E} \cdot \vec{v}_i \cdot \frac{\partial f_i^0}{\partial \omega_i} \right. \\
& \left. + q_i \left(\vec{E} + \frac{\vec{k}_i}{\omega_i} \times \vec{B} \right) \cdot (\alpha_1 \vec{E} + \alpha_2 \dot{\vec{E}} + \alpha_5 (\vec{E} \times \vec{B}) + \alpha_6 (\dot{\vec{E}} \times \vec{B}) + \alpha_7 (\vec{E} \times \dot{\vec{B}})) \right\} \frac{\partial f_i^0}{\partial \omega_i} \\
& = -\frac{1}{\tau_R^i} \vec{k}_i \cdot \left\{ (\alpha_1 \vec{E} + \alpha_2 \dot{\vec{E}} + \alpha_5 (\vec{E} \times \vec{B}) + \alpha_6 (\dot{\vec{E}} \times \vec{B}) + \alpha_7 (\vec{E} \times \dot{\vec{B}})) \right\} \frac{\partial f_i^0}{\partial \omega_i}. \tag{2.61}
\end{aligned}$$

In this analysis, we consider only the terms with first-order derivatives of the fields and neglect terms with higher-order derivatives. Therefore, the terms with $\dot{\alpha}_2$, $\dot{\alpha}_6$, $\dot{\alpha}_7$ are ignored in the current analysis. Hence Eq. (2.61) turns to be,

$$\begin{aligned}
 & \left\{ \omega_i \vec{v}_i \cdot \left(\dot{\alpha}_1 \vec{E} + \alpha_1 \dot{\vec{E}} + \dot{\alpha}_5 (\vec{E} \times \vec{B}) + \alpha_5 (\dot{\vec{E}} \times \vec{B}) + \alpha_5 (\vec{E} \times \dot{\vec{B}}) + \alpha_6 (\dot{\vec{E}} \times \dot{\vec{B}}) \right. \right. \\
 & \left. \left. + \alpha_7 (\dot{\vec{E}} \times \dot{\vec{B}}) \right) + q_i \vec{v}_i \cdot \vec{E} \cdot \frac{\partial f_i^0}{\partial \omega_i} + q_i \left(\vec{E} \cdot \alpha_1 \vec{E} + \vec{E} \cdot \alpha_2 \dot{\vec{E}} \right) + q_i \alpha_1 (\vec{v}_i \times \vec{B} \cdot \vec{E}) \right. \\
 & \left. + q_i \alpha_2 (\vec{v}_i \times \vec{B} \cdot \dot{\vec{E}}) + q_i \alpha_5 (\vec{v}_i \times \vec{B} \cdot (\vec{E} \times \vec{B})) \right. \\
 & \left. + q_i \alpha_6 (\vec{v}_i \times \vec{B} \cdot (\dot{\vec{E}} \times \vec{B})) + q_i \alpha_7 (\vec{v}_i \times \vec{B} \cdot (\vec{E} \times \dot{\vec{B}})) \right\} \frac{\partial f_i^0}{\partial \omega_i} \\
 & = -\frac{1}{\tau_R^i} \vec{k}_i \cdot \left\{ \alpha_1 \vec{E} + \alpha_2 \dot{\vec{E}} + \alpha_5 (\vec{E} \times \vec{B}) + \alpha_6 (\dot{\vec{E}} \times \vec{B}) + \alpha_7 (\vec{E} \times \dot{\vec{B}}) \right\} \frac{\partial f_i^0}{\partial \omega_i}. \\
 \Rightarrow & \left\{ \omega_i \vec{v}_i \cdot \left(\dot{\alpha}_1 \vec{E} + \alpha_1 \dot{\vec{E}} + \dot{\alpha}_5 (\vec{E} \times \vec{B}) + \alpha_5 (\dot{\vec{E}} \times \vec{B}) + \alpha_5 (\vec{E} \times \dot{\vec{B}}) + \alpha_6 (\dot{\vec{E}} \times \dot{\vec{B}}) \right. \right. \\
 & \left. \left. + \alpha_7 (\dot{\vec{E}} \times \dot{\vec{B}}) \right) + q_i \vec{v}_i \cdot \vec{E} \cdot \frac{\partial f_i^0}{\partial \omega_i} + q_i \left(\vec{E} \cdot \alpha_1 \vec{E} + \vec{E} \cdot \alpha_2 \dot{\vec{E}} \right) - q_i \alpha_1 (\vec{v}_i \cdot (\vec{E} \times \vec{B})) \right. \\
 & \left. - q_i \alpha_2 (\vec{v}_i \cdot (\dot{\vec{E}} \times \vec{B})) + q_i \alpha_5 (-(\vec{v}_i \cdot \vec{B})(\vec{E} \cdot \vec{B}) + (\vec{v}_i \cdot \vec{E})(\vec{B} \cdot \vec{B})) + q_i \alpha_6 (-(\vec{v}_i \cdot \vec{B}) \right. \\
 & \left. (\dot{\vec{E}} \cdot \vec{B}) + (\vec{v}_i \cdot \dot{\vec{E}})(\vec{B} \cdot \vec{B})) + q_i \alpha_7 (-(\vec{v}_i \cdot \dot{\vec{B}})(\vec{E} \cdot \vec{B}) + (\vec{v}_i \cdot \vec{E})(\vec{B} \cdot \dot{\vec{B}})) \right\} \frac{\partial f_i^0}{\partial \omega_i} \\
 & = -\frac{\omega_i \vec{v}_i}{\tau_R^i} \cdot \left\{ \alpha_1 \vec{E} + \alpha_2 \dot{\vec{E}} + \alpha_5 (\vec{E} \times \vec{B}) + \alpha_6 (\dot{\vec{E}} \times \vec{B}) + \alpha_7 (\vec{E} \times \dot{\vec{B}}) \right\} \frac{\partial f_i^0}{\partial \omega_i}.
 \end{aligned} \tag{2.62}$$

Now, we compare the coefficients of same tensor structure on both sides of the above equation one by one. With the comparison of coefficient of $(\vec{v}_i \cdot \vec{E})$ on both sides, we get,

$$\omega_i \dot{\alpha}_1 + q_i + q_i \alpha_5 (\vec{B} \cdot \vec{B}) + q_i \alpha_7 (\vec{B} \cdot \dot{\vec{B}}) = -\frac{\omega_i}{\tau_R^i} \alpha_1,$$

$$\Rightarrow \dot{\alpha}_1 = -\left\{ \frac{1}{\tau_R^i} \alpha_1 + \frac{q_i \alpha_5 (\vec{B} \cdot \vec{B})}{\omega_i} + \frac{q_i \alpha_7 (\vec{B} \cdot \dot{\vec{B}})}{\omega_i} + \frac{q_i}{\omega_i} \right\}. \tag{2.63}$$

The comparison of coefficients of $(\vec{v}_i \cdot \dot{\vec{E}})$, $(\vec{v}_i \cdot \vec{E} \times \vec{B})$, $(\vec{v}_i \cdot \dot{\vec{E}} \times \vec{B})$ and $(\vec{v}_i \cdot \vec{E} \times \dot{\vec{B}})$

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gives us the values of α_2 , $\dot{\alpha}_5$, α_6 , and α_7 respectively as

$$\begin{aligned}\alpha_2 &= -\tau_R^i \left\{ \alpha_1 + \frac{q_i \alpha_6 (\vec{B} \cdot \vec{B})}{\omega_i} \right\}, \\ \dot{\alpha}_5 &= -\frac{1}{\tau_R^i} \alpha_5 + \frac{q_i \alpha_1}{\omega_i}, \\ \alpha_6 &= -\tau_R^i \left(\alpha_5 - \frac{q_i \alpha_2}{\omega_i} \right), \\ \alpha_7 &= -\tau_R^i \alpha_5.\end{aligned}\tag{2.64}$$

Substitution of α_7 from Eq. (2.64) into Eq. (2.63) leads to,

$$\dot{\alpha}_1 = -\left\{ \frac{1}{\tau_R^i} \alpha_1 + \frac{q_i}{\omega_i} \left((\vec{B} \cdot \vec{B}) - (\tau_R^i \vec{B} \cdot \dot{\vec{B}}) \right) \alpha_5 + \frac{q_i}{\omega_i} \right\}.\tag{2.65}$$

These coupled differential equations for α_1 and α_5 can be described in the following matrix equation as

$$\frac{dX}{dt} = AX + G.\tag{2.66}$$

where,

$$X = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix}, \quad A = \begin{pmatrix} -\frac{1}{\tau_R^i} & -\frac{q_i F^2}{\omega_i} \\ \frac{q_i}{\omega_i} & -\frac{1}{\tau_R^i} \end{pmatrix}, \quad G = \begin{pmatrix} -\frac{q_i}{\omega_i} \\ 0 \end{pmatrix},$$

with, $F = \sqrt{B(B - \tau_R^i \dot{B})}$.

Eq. (2.66) can be solved by diagonalizing the matrix A and using the method of the variation of constants. The eigen values corresponding to matrix A are,

$$\lambda_1 = -\frac{1}{\tau_R^i} - i\frac{q_i F}{\omega_i}, \quad \lambda_2 = -\frac{1}{\tau_R^i} + i\frac{q_i F}{\omega_i}.$$

The eigen vectors corresponding to these eigen values are,

$$v_1 = \begin{pmatrix} -iF \\ 1 \end{pmatrix}, \quad v_2 = \begin{pmatrix} iF \\ 1 \end{pmatrix}.$$

Hence, linear independent solutions corresponding to homogeneous part of differential equation Eq. (2.66) are,

$$H_1 = \begin{pmatrix} -iF e^{\eta_1} \\ e^{\eta_1} \end{pmatrix}, \quad H_2 = \begin{pmatrix} iF e^{\eta_2} \\ e^{\eta_2} \end{pmatrix}.$$

Where,

$$\eta_j = -\frac{t}{\tau_R^i} + a_j \frac{q_i}{\omega_i} \int F dt , \quad (2.67)$$

with $a_1 = -1$, $a_2 = 1$.

Therefore, the fundamental matrix for Eq. (2.66) is

$$Y = \begin{pmatrix} -iFe^{\eta_1} & iFe^{\eta_2} \\ e^{\eta_1} & e^{\eta_2} \end{pmatrix}. \quad (2.68)$$

We seek a particular solution of equation with given form of Eq. (2.66) is

$$Y_p = YU. \quad (2.69)$$

Where U is a column matrix of form,

$$U = \begin{pmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_n \end{pmatrix}.$$

We can see that Y_p is a column matrix with same coefficients as that of matrix X . The differentiation of Eq. (2.69) with respect to time gives us,

$$Y_p' = Y'U + YU'.$$

Here $Y' = AY$. Hence, $Y_p' = AY_p + YU'$.

Comparison of above equation with Eq. (2.66) shows

$$G = YU'.$$

The determinant of matrix Y is $-2iFe^\eta$. So

$$\begin{aligned} u_1' &= \frac{1}{-2iFe^\eta} \det \begin{pmatrix} -\frac{q_i}{\omega_i} & iFe^{\eta_2} \\ 0 & e^{\eta_2} \end{pmatrix}, \\ u_2' &= \frac{1}{-2iFe^\eta} \det \begin{pmatrix} -iFe^{\eta_2} & -\frac{q_i}{\omega_i} \\ e^{\eta_2} & 0 \end{pmatrix}. \end{aligned} \quad (2.70)$$

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After integrating both u'_1 and u'_2 with respect to time, we get the U matrix as

$$U = \frac{q_i}{2i\omega_i} \begin{pmatrix} \int \frac{e^{-\eta_1}}{F} dt \\ - \int \frac{e^{-\eta_2}}{F} dt \end{pmatrix}.$$

Substituting the value of U from above equation into Eq. (2.69)

$$Y_p = \frac{q_i}{2i\omega_i} \begin{pmatrix} -iFe^{\eta_1} & iFe^{\eta_2} \\ e^{\eta_1} & e^{\eta_2} \end{pmatrix} \begin{pmatrix} \int \frac{e^{-\eta_1}}{F} dt \\ - \int \frac{e^{-\eta_2}}{F} dt \end{pmatrix}.$$

With,

$$\begin{aligned} I_j &= \int \frac{e^{-\eta_j}}{F} dt, \\ k_j &= a_j \frac{iqI_j}{2\omega}. \end{aligned} \quad (2.71)$$

Hence, we get the particular solution,

$$Y_p = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \begin{pmatrix} -ik_1Fe^{\eta_1} + ik_2Fe^{\eta_2} \\ k_1e^{\eta_1} + k_2e^{\eta_2} \end{pmatrix}. \quad (2.72)$$

So,

$$\alpha_1 = -ik_1Fe^{\eta_1} + ik_2Fe^{\eta_2}, \quad \alpha_5 = k_1e^{\eta_1} + k_2e^{\eta_2}.$$

With, $\tilde{\Gamma}_i = \frac{q_i F}{\omega_i}$, finally we get,

$$\begin{aligned} \alpha_1 &= -\frac{\tilde{\Gamma}_i}{2}(I_1e^{\eta_1} + I_2e^{\eta_2}), \\ \alpha_5 &= -\frac{iq_i}{2\omega_i}(I_1e^{\eta_1} - I_2e^{\eta_2}). \end{aligned} \quad (2.73)$$

Substitution of Eq. (2.73) in α_7 of Eq. (2.64) leads to,

$$\alpha_7 = \frac{i\tau_R^i q_i}{2\omega_i}(I_1e^{\eta_1} - I_2e^{\eta_2}). \quad (2.74)$$

With use of Eq. (2.64) and Eq. (2.73), we obtain,

$$\begin{aligned}
 \alpha_1 &= -\frac{q_i}{\omega_i} \left(\frac{\frac{1}{\tau_R^i} + \frac{1}{\tau_B}}{(\frac{1}{\tau_R^i} + \frac{1}{\tau_B})^2 + (\frac{\sqrt{1+\frac{\tau_R^i}{\tau_B}}}{\tau_B})^2} \right), \\
 \alpha_2 &= \frac{(\frac{\tau_R^i \tilde{\Gamma}_i}{2} - \frac{i\tau_R^{i2} \Gamma_i^2}{2}) I_1 e^{\eta_1} + (\frac{\tau_R^i \tilde{\Gamma}_i}{2} + \frac{i\tau_R^{i2} \Gamma_i^2}{2}) I_2 e^{\eta_2}}{1 + \tau_R^{i2} \Gamma_i^2}, \\
 \alpha_5 &= -\frac{q_i^2}{\omega_i^2} \left(\frac{1}{(\frac{1}{\tau_R^i} + \frac{1}{\tau_B})^2 + (\frac{\sqrt{1+\frac{\tau_R^i}{\tau_B}}}{\tau_B})^2} \right), \\
 \alpha_6 &= \frac{(\frac{\tau_R^{i2} \tilde{\Gamma}_i^2}{2F} + \frac{iq_i \tau_R^i}{2\omega_i}) I_1 e^{\eta_1} + (\frac{\tau_R^{i2} \tilde{\Gamma}_i^2}{2F} - \frac{iq_i \tau_R^i}{2\omega_i}) I_2 e^{\eta_2}}{1 + \tau_R^{i2} \Gamma_i^2}, \\
 \alpha_7 &= \frac{q_i^2}{\omega_i^2} \left(\frac{\tau_R^i}{(\frac{1}{\tau_R^i} + \frac{1}{\tau_B})^2 + (\frac{\sqrt{1+\frac{\tau_R^i}{\tau_B}}}{\tau_B})^2} \right). \tag{2.75}
 \end{aligned}$$

In the presence of electromagnetic field the general form of electric current density can be expressed as,

$$\vec{j} = j_e \hat{e} + j_H (\hat{e} \times \hat{b}), \tag{2.76}$$

From Eq. (2.2),

$$\vec{j} = \sum_i q_i g_i \int \frac{d^3 |\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i}{\omega_i} f_i, \tag{2.77}$$

with $f_i = f_i^0 + \delta f_i$.

Substituting Eq. (2.58) into the definition of the electric current given in Eq. (2.77), the net current density can be expressed in terms of the non-equilibrium correction to the distribution function as

$$\vec{j} = \sum_i q_i g_i \int \frac{d^3 |\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i}{\omega_i}, (\vec{k}_i \cdot \vec{\Omega} \sigma) \frac{\partial f_i^0}{\partial \omega_i}.$$

2.4 Transport coefficients of hot and dense hadronic medium in the presence of time-varying magnetic field

Assuming isotropy in momentum space and using the identity $\langle k_i^l k_i^m \rangle = \frac{1}{3} \vec{k}_i^2 \delta^{lm}$, the above expression simplifies to

$$\vec{j} = \frac{1}{3} \sum_i q_i g_i \int \frac{d^3 |\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i} \left(\alpha_1 \vec{E} + \alpha_2 \dot{\vec{E}} + \alpha_5 (\vec{E} \times \vec{B}) \alpha_6 (\dot{\vec{E}} \times \vec{B}) + \alpha_7 (\vec{E} \times \dot{\vec{B}}) \right) \frac{\partial f_i^0}{\partial \omega_i}. \quad (2.78)$$

In the presence of a time-dependent electromagnetic field, the induced current naturally separates into longitudinal (Ohmic) and transverse (Hall-like) components. Accordingly, the total electric current can be decomposed as $j_e = j_e^{(0)} + j_e^{(1)}$ and $j_H = j_H^{(0)} + j_H^{(1)} + j_H^{(2)}$, consistent with the structure introduced in Eq. (2.76). The individual contributions to the net current density corresponding to different driving forces are then given by

$$\begin{aligned} j_e^{(0)} &= \frac{1}{3} \sum_i q_i g_i \int \frac{d^3 |\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i} (\alpha_1 \vec{E}) \frac{\partial f_i^0}{\partial \omega_i}, \\ j_e^{(1)} &= \frac{1}{3} \sum_i q_i g_i \int \frac{d^3 |\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i} (\alpha_2 \dot{\vec{E}}) \frac{\partial f_i^0}{\partial \omega_i}, \\ j_H^{(0)} &= \frac{1}{3} \sum_i q_i g_i \int \frac{d^3 |\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i} (\alpha_5 (\vec{E} \times \vec{B})) \frac{\partial f_i^0}{\partial \omega_i}, \\ j_H^{(1)} &= \frac{1}{3} \sum_i q_i g_i \int \frac{d^3 |\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i} (\alpha_6 (\dot{\vec{E}} \times \vec{B})) \frac{\partial f_i^0}{\partial \omega_i}, \\ j_H^{(2)} &= \frac{1}{3} \sum_i q_i g_i \int \frac{d^3 |\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i} (\alpha_7 (\vec{E} \times \dot{\vec{B}})) \frac{\partial f_i^0}{\partial \omega_i}. \end{aligned} \quad (2.79)$$

With use of Eq. (2.75), we get

$$\begin{aligned}
 j_e^{(0)} &= \frac{E}{3T} \sum_i g_i(q_i)^2 \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} f_i^0(1 \mp f_i^0) \tau_R^i \times \frac{1}{1 + \chi_i + \chi_i^2}, \\
 j_e^{(1)} &= -\frac{\dot{E}}{3T} \sum_i g_i(q_i)^2 \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} f_i^0(1 \mp f_i^0) \tau_R^{i^2} \times \frac{1 + \chi_i - \chi_i^2}{(1 + \chi_i)(1 + \chi_i^2)(1 + \chi_i + \chi_i^2)}, \\
 j_H^{(0)} &= \frac{EB}{3T} \sum_i g_i(q_i)^3 \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^3} f_i^0(1 \mp f_i^0) \tau_R^{i^2} \times \frac{1}{(1 + \chi_i)(1 + \chi_i + \chi_i^2)}, \\
 j_H^{(1)} &= -\frac{\dot{E}B}{3T} \sum_i g_i(q_i)^3 \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^3} f_i^0(1 \mp f_i^0) \tau_R^{i^3} \times \frac{\chi_i}{(1 + \chi_i)(1 + \chi_i^2)(1 + \chi_i + \chi_i^2)}, \\
 j_H^{(2)} &= -\frac{E\dot{B}}{3T} \sum_i g_i(q_i)^3 \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^3} f_i^0(1 \mp f_i^0) \tau_R^{i^3} \times \frac{1}{(1 + \chi_i)(1 + \chi_i + \chi_i^2)}.
 \end{aligned} \tag{2.80}$$

where $\chi_i = \frac{\tau_R^i}{\tau_B}$. Here, the Ohmic current density is $j_e = j_e^{(0)} + j_e^{(1)}$ and Hall current density is $j_H = j_H^{(0)} + j_H^{(1)} + j_H^{(2)}$.

Now, from Ohm's law, we can write

$$j^i = \sigma^{ij} E_j, \tag{2.81}$$

where σ^{ij} is the conductivity tensor. In general, it contains contributions from all the first and higher-order derivatives of the fields. Now, we can write

$$\vec{j} = \sigma_e \vec{E} + \sigma_H (\vec{E} \times \hat{b}), \tag{2.82}$$

where Ohmic (σ_e) and Hall (σ_H) conductivity components are

$$\begin{aligned}
 \sigma_e &= \frac{j_e}{E} = \frac{j_e^{(0)} + j_e^{(1)}}{E} = \sigma_e^{(0)} + \sigma_e^{(1)}, \\
 \sigma_H &= \frac{j_H}{E} = \frac{j_H^{(0)} + j_H^{(1)} + j_H^{(2)}}{E} = \sigma_H^{(0)} + \sigma_H^{(1)} + \sigma_H^{(2)}.
 \end{aligned} \tag{2.83}$$

From Eq. (2.80) and Eq. (2.83), components of electrical conductivity can be

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found as

$$\begin{aligned}
\sigma_e^{(0)} &= \frac{1}{3T} \sum_i g_i(q_i)^2 \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} f_i^0 (1 \mp f_i^0) \tau_R^i \times \frac{1}{1 + \chi_i + \chi_i^2}, \\
\sigma_e^{(1)} &= \frac{1}{3T} \sum_i g_i(q_i)^2 \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} f_i^0 (1 \mp f_i^0) \frac{\tau_R^{i^2}}{\tau_E} \times \frac{1 + \chi_i - \chi_i^2}{(1 + \chi_i)(1 + \chi_i^2)(1 + \chi_i + \chi_i^2)}, \\
\sigma_H^{(0)} &= \frac{1}{3T} \sum_i g_i(q_i)^2 \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} f_i^0 (1 \mp f_i^0) \tau_R^{i^2} \Gamma_i \times \frac{1}{(1 + \chi_i)(1 + \chi_i + \chi_i^2)}, \\
\sigma_H^{(1)} &= \frac{1}{3T} \sum_i g_i(q_i)^2 \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} f_i^0 (1 \mp f_i^0) \frac{\tau_R^{i^3} \Gamma_i}{\tau_E} \times \frac{\chi_i}{(1 + \chi_i)(1 + \chi_i^2)(1 + \chi_i + \chi_i^2)}, \\
\sigma_H^{(2)} &= \frac{1}{3T} \sum_i g_i(q_i)^2 \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} f_i^0 (1 \mp f_i^0) \frac{\tau_R^{i^3} \Gamma_i}{\tau_B} \times \frac{1}{(1 + \chi_i)(1 + \chi_i + \chi_i^2)},
\end{aligned} \tag{2.84}$$

with $\Gamma_i = \frac{q_i B}{\omega_i}$. For the case of time-varying fields ($B(t)$), from Eqs.(2.83) and (2.84)

$$\begin{aligned}
\sigma_e &= \frac{1}{3T} \sum_{\text{baryon}} g_i(q_i)^2 \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} f_i^0 (1 - f_i^0) \tau_R^i \frac{2\chi_i^2 + 2\chi_i + 1}{(\chi_i + 1)(\chi_i^2 + 1)(\chi_i^2 + \chi_i + 1)} \\
&\quad + \frac{1}{3T} \sum_{\text{meson}} g_i(q_i)^2 \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} f_i^0 (1 + f_i^0) \tau_R^i \frac{2\chi_i^2 + 2\chi_i + 1}{(\chi_i + 1)(\chi_i^2 + 1)(\chi_i^2 + \chi_i + 1)}, \\
\sigma_H &= \frac{1}{3T} \sum_{\text{baryon}} g_i(q_i)^2 \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} f_i^0 (1 - f_i^0) \tau_R^i \frac{\chi_i(\chi_i^3 + \chi_i^2 + 2\chi_i + 1)}{(\chi_i + 1)(\chi_i^2 + 1)(\chi_i^2 + \chi_i + 1)} \\
&\quad + \frac{1}{3T} \sum_{\text{meson}} g_i(q_i)^2 \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} f_i^0 (1 + f_i^0) \tau_R^i \frac{\chi_i(\chi_i^3 + \chi_i^2 + 2\chi_i + 1)}{(\chi_i + 1)(\chi_i^2 + 1)(\chi_i^2 + \chi_i + 1)}.
\end{aligned} \tag{2.85}$$

Note that in $B(t)$ case, for simplicity we have considered $\tau_E = \tau_B$ and $\chi_i = \frac{\tau_R^i}{\tau_B} = \frac{\tau_R^i}{\tau_E}$. Also, Hall conductivity is negative (positive) for negatively (positively) charged particles and antiparticles.

In this work, we evaluate the relaxation time of a hadron arising from elastic $2 \rightarrow 2$ scattering processes with all other hadrons present in the medium, including both baryons and mesons. In general, the relaxation time τ_R depends on the particle

energy (or momentum). However, in order to ensure the conservation of energy and momentum within the relaxation time approximation (RTA), a momentum-independent relaxation time is required. Consequently, we compute the thermally averaged relaxation time for elastic scattering between the i th and j th hadronic species.

The inverse of the thermal average relaxation time for particle i is given by [14, 35]

$$(\tau_R^i)^{-1} = \sum_j n_j \langle \sigma_{ij} v_{ij} \rangle, \quad (2.86)$$

where the summation runs over all hadronic species j present in the medium, excluding particle i . The number density of species j is defined as

$$n_j = \int \frac{d^3 k_j}{(2\pi)^3} f_j^0, \quad (2.87)$$

and $\langle \sigma_{ij} v_{ij} \rangle$ denotes the thermally averaged cross-section in the center-of-mass frame, which can be written as

$$\begin{aligned} \langle \sigma_{ij} v_{ij} \rangle &= \frac{\sigma}{8T m_i^2 m_j^2 K_2(m_i/T) K_2(m_j/T)} \int_{(m_i+m_j)^2}^{\infty} ds \\ &\times \frac{s - (m_i - m_j)^2}{\sqrt{s}} [s - (m_i + m_j)^2] K_1(\sqrt{s}/T). \end{aligned} \quad (2.88)$$

Here, $\sqrt{s}$ represents the center-of-mass energy, while m_i and m_j denote the masses of the scattering particles. The functions K_1 and K_2 are the modified Bessel functions of the second kind of first and second order, respectively. The total elastic scattering cross-section is taken as $\sigma = \pi(r_i + r_j)^2$, corresponding to hard-sphere scattering with radii r_i and r_j . For numerical evaluations, we assume a common radius of 0.2 fm for all mesons and 0.62 fm for all baryons [29].

In the left plot of Fig. (2.1), the temperature dependence of the scaled electrical conductivity is shown, with the Ohmic component in the upper panel and the Hall component in the lower panel. For the Ohmic conductivity σ_e , results are presented at vanishing baryon chemical potential for three cases: zero magnetic field ($B = 0$), a constant magnetic field with strength $eB = 0.03 \text{ GeV}^2$, and a

2.4 Transport coefficients of hot and dense hadronic medium in the presence of time-varying magnetic field

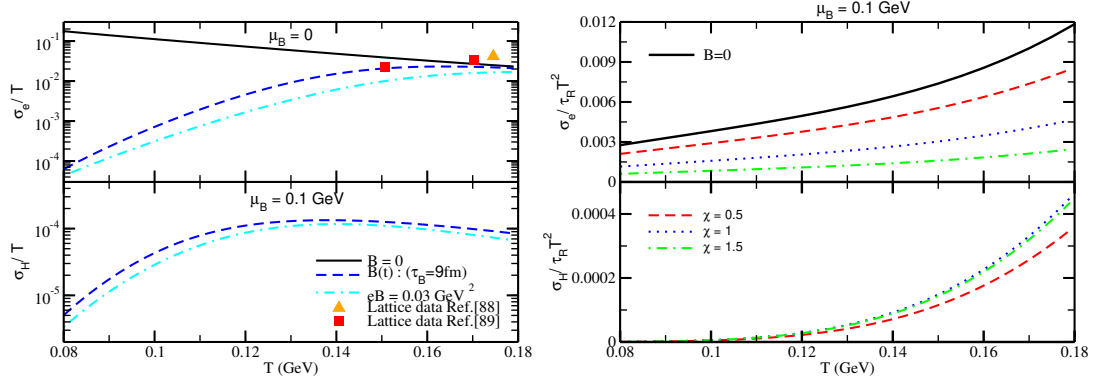


Figure 2.1: Scaled electrical conductivity as a function of temperature. The left panel displays the scaled electrical conductivity components (σ_e/T , σ_H/T) as functions of temperature (T) for different magnetic field scenarios: the black solid line corresponds to the zero magnetic field case ($B = 0$), the blue dashed line represents a time-dependent magnetic field $B(t)$, and the cyan dash-dotted line denotes a constant magnetic field with strength $eB = 0.03 \text{ GeV}^2$. The right panel shows the dependence of the scaled electrical conductivity on temperature for different values of $\chi = \tau_R/\tau_B$, where a larger χ corresponds to a faster decay of the magnetic field, leading to a suppression of the conductivity [2].

time-dependent magnetic field $B(t)$ with decay parameter $\tau_B = 9$ fm. In the absence of a magnetic field, σ_e decreases with temperature, reflecting its direct proportionality to the relaxation time τ_R , which decreases with increasing temperature in a hadron resonance gas (HRG) medium for fixed scattering cross sections. For quantitative estimates of τ_R , the hard-sphere radii of baryons and mesons are taken as 0.62 fm and 0.2 fm, respectively [29]. In contrast, in the presence of a magnetic field, σ_e increases with temperature and saturates below the $B = 0$ result at higher temperatures. This behavior arises because σ_e in a magnetized medium is approximately inversely proportional to τ_R , as seen from Eq. (2.85). The electrical conductivity for the time-dependent magnetic field case remains larger than that for a constant magnetic field, owing to the additional contribution $\sigma_e^{(1)}$ in Eq. (2.84) arising from the time-dependent electric field. For slowly varying fields, the temperature dependence of σ_e is similar to that in the constant field case, while its magnitude becomes independent of the field strength, in contrast to the constant magnetic field scenario, where it depends explicitly on the cyclotron frequency. The $B = 0$ results are compared with available lattice QCD calculations of σ_e/T from Refs. [37, 38], showing reasonable agreement in magnitude.

The lower panel of Fig. (2.1) displays the Hall conductivity for time-dependent and constant magnetic field cases. The Hall conductivity exists only at finite baryon chemical potential and vanishes at $\mu_B = 0$ due to the cancellation between particle and antiparticle contributions. Since σ_H depends on the sign of the particle charge, its magnitude is significantly smaller than that of σ_e in the HRG medium. Furthermore, σ_H is enhanced in the time-dependent magnetic field case due to the additional contributions $\sigma_H^{(1)}$ and $\sigma_H^{(2)}$ arising from the explicit time dependence of the electric and magnetic fields, respectively.

2.4 Transport coefficients of hot and dense hadronic medium in the presence of time-varying magnetic field

2.4.2 Thermal Conductivity

The presence of a temperature gradient in a medium induces a flow of heat, characterized by the thermal conductivity. In a charged medium subjected to a magnetic field, the thermal conductivity becomes anisotropic and, instead of a single transport coefficient, the heat current is described by multiple components. In the presence of a time-dependent magnetic field, the heat current in the local rest frame of the fluid can be written as [39]

$$\begin{aligned}\vec{I} &= \kappa_0 \vec{\nabla} T + \bar{\kappa}_1 (\vec{\nabla} T \times \vec{B}) + \bar{\kappa}_2 (\vec{\nabla} T \times \dot{\vec{B}}) \\ &= \kappa_0 \vec{\nabla} T + (\kappa_1 + \kappa_2) (\vec{\nabla} T \times \hat{b}) \\ &= \kappa_0 \vec{\nabla} T + \kappa_H (\vec{\nabla} T \times \hat{b}),\end{aligned}\tag{2.89}$$

where κ_0 denotes the longitudinal thermal conductivity along the temperature gradient, while the Hall-type components are defined as $\kappa_1 = \bar{\kappa}_1 B$ and $\kappa_2 = \bar{\kappa}_2 \dot{B}$, with $\hat{b}$ being the unit vector along the magnetic field direction.

The macroscopic properties of a relativistic medium are described by the energy–momentum tensor $T^{\mu\nu}$ and the conserved particle four-current N^μ . Within kinetic theory, these quantities can be expressed in terms of microscopic phase-space distributions as

$$\begin{aligned}T^{\mu\nu} &= \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{k_i^\mu k_i^\nu}{\omega_i} f_i, \\ N^\mu &= \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{k_i^\mu}{\omega_i} f_i,\end{aligned}\tag{2.90}$$

where the four-momentum of the i th particle species is $k_i^\mu = (\omega_i, \vec{k}_i)$ and g_i denotes the degeneracy factor.

The energy–momentum tensor and particle current can be decomposed into equilibrium and dissipative parts as

$$\begin{aligned}T^{\mu\nu} &= T_{\text{ideal}}^{\mu\nu} + \delta T^{\mu\nu}, \\ N^\mu &= N_{\text{ideal}}^\mu + \delta N^\mu.\end{aligned}\tag{2.91}$$

The ideal parts arise from the equilibrium distribution function f_i^0 , while the dissipative contributions originate from deviations from equilibrium δf_i . Since baryon number b_i is the conserved charge, the heat current is defined as the energy flow relative to the baryon current.

In first-order relativistic hydrodynamics [40], the heat current is proportional to the gradient of the thermal potential and is given by

$$I^\mu = -\kappa \frac{nT^2}{\epsilon + p} \nabla^\mu \left(\frac{\mu}{T} \right), \quad (2.92)$$

where κ is the thermal conductivity and $\nabla^\mu = \partial^\mu - u^\mu u^\nu \partial_\nu$. Working in the local rest frame (LRF), where $u^\mu = (1, \vec{0})$, and using the Gibbs–Duhem relation, the heat current reduces to

$$\vec{I} = -\kappa \vec{\nabla} T. \quad (2.93)$$

The velocity frame may be chosen as either the Landau or Eckart frame; however, the Landau matching condition is required when employing the relaxation time approximation [41]. In the Landau frame $T^{0j} = 0$, while in the Eckart frame $N^j = 0$. In both frames, the heat current is defined as [42]

$$I^j = \delta T^{0j} - h \delta N^j, \quad (2.94)$$

where $h = (\epsilon + P)/n$ is the enthalpy per particle, and ϵ , P , and n denote the total energy density, pressure, and net baryon density, respectively.

Substituting Eq. (2.90) into Eq. (2.94), the heat current for the i th particle species can be written as

$$\vec{I}_i = \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i}{\omega_i} (\omega_i - b_i h) \delta f_i. \quad (2.95)$$

To determine δf_i , we solve the Boltzmann transport equation in the presence of an external magnetic field under the relaxation time approximation in the local rest frame,

$$\frac{\partial f_i}{\partial t} + \frac{\vec{k}_i}{\omega_i} \cdot \frac{\partial f_i}{\partial \vec{x}} + q_i \left(\frac{\vec{k}_i}{\omega_i} \times \vec{B} \right) \cdot \frac{\partial f_i}{\partial \vec{k}_i} = -\frac{\delta f_i}{\tau_R^i}. \quad (2.96)$$

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The deviation from equilibrium relevant for thermal transport is assumed to be of the form

$$\delta f_i = (\vec{k}_i \cdot \vec{\Omega}_\kappa) \frac{\partial f_i^0}{\partial \omega_i}. \quad (2.97)$$

The general form of $\vec{\Omega}_\kappa$ is [42]

$$\begin{aligned} \vec{\Omega}_\kappa &= \alpha_1 \vec{B} + \alpha_2 \vec{\nabla} T + \alpha_3 (\vec{\nabla} T \times \vec{B}) + \alpha_4 \dot{\vec{B}} + \alpha_5 (\vec{\nabla} T \times \dot{\vec{B}}) \\ &= \alpha_1 \vec{B} + \alpha_2 \vec{\nabla} T + \alpha_4 \dot{\vec{B}} + (\alpha_3 |\vec{B}| + \alpha_5 |\dot{\vec{B}}|) (\vec{\nabla} T \times \hat{b}). \end{aligned} \quad (2.98)$$

The unknown coefficients α_i ($i = (1, 2, \dots, 5)$) can be obtained by substituting Eq. (2.98) in the Boltzmann equation. The Boltzmann equation under the relaxation time approximation

$$\frac{\partial f_i}{\partial t} + \frac{\vec{k}_i}{\omega_i} \cdot \frac{\partial f_i}{\partial \vec{x}} + q_i \left(\frac{\vec{k}_i}{\omega_i} \times \vec{B} \right) \cdot \frac{\partial f_i}{\partial \vec{k}_i} = -\frac{\delta f_i}{\tau_R^i}. \quad (2.99)$$

Employing Eq. (2.98) in Eq. (2.99), The first term in left hand side (lhs) of the equation becomes,

$$\begin{aligned} \omega_i \vec{v}_i \cdot \left\{ \dot{\alpha}_1 \vec{B} + \alpha_1 \dot{\vec{B}} + \dot{\alpha}_2 \vec{\nabla} T + \alpha_3 (\vec{\nabla} T \times \dot{\vec{B}}) + \dot{\alpha}_3 (\vec{\nabla} T \times \vec{B}) \right. \\ \left. + \dot{\alpha}_4 \dot{\vec{B}} + \alpha_4 \ddot{\vec{B}} + \dot{\alpha}_5 (\vec{\nabla} T \times \dot{\vec{B}}) + \alpha_5 (\vec{\nabla} T \times \ddot{\vec{B}}) \right\}. \end{aligned} \quad (2.100)$$

The second term in lhs of the Boltzmann equation leads to,

$$\begin{aligned} \frac{\partial f_i}{\partial x_i} &= \frac{\partial}{\partial x_i} (f_i^0 + \delta f_i) \\ &= \frac{\partial}{\partial x_i} (f_i^0) + \frac{\partial f_i}{\partial x_i} (\delta f_i) \\ &= \frac{\partial}{\partial x_i} \left\{ \frac{1}{1 \pm \exp(\beta(\omega_i - b_i \mu_i))} \right\} + 0 \\ &= -(\omega_i - b_i h) \frac{\vec{\nabla} T}{T} \frac{\partial f_i^0}{\partial \omega_i}. \end{aligned} \quad (2.101)$$

Here, in the above equation, we have ignored the contributions of shear and bulk viscosity components.

The third term in lhs leads to,

$$\begin{aligned}\frac{\partial f_i}{\partial k_i} &= \frac{\partial f_i^0}{\partial k_i} + \frac{\partial \delta f_i}{\partial k_i} \\ &= \vec{v}_i \frac{\partial f_i^0}{\partial \omega_i} + \vec{\Omega}_\kappa \frac{\partial f_i^0}{\partial \omega_i}.\end{aligned}\quad (2.102)$$

The identity $(\vec{v}_i \times \vec{B}) \cdot \vec{v}_i \frac{\partial f_i^0}{\partial \omega_i} = 0$. Thus we are left with only $\vec{\Omega}_\kappa \frac{\partial f_i^0}{\partial \omega_i}$ term of the above equation. Hence,

$$\begin{aligned}\frac{\partial f_i}{\partial k_i} &= -\alpha_2 q_i \vec{v}_i \cdot (\vec{\nabla} T \times \vec{B}) + \alpha_3 q_i \vec{v}_i \cdot \vec{\nabla} T (\vec{B} \cdot \vec{B}) - \alpha_3 q_i \\ &\vec{v}_i \cdot \vec{B} (\vec{B} \cdot \vec{\nabla} T) + \alpha_5 q_i \vec{v}_i \cdot \vec{\nabla} T (\vec{B} \cdot \vec{B}) - \alpha_5 q_i \vec{v}_i \cdot \vec{B} (\vec{B} \cdot \vec{\nabla} T).\end{aligned}\quad (2.103)$$

Finally, after the substitution of above results in both sides of Eq. (2.99), we get

$$\begin{aligned}&\omega_i \vec{v}_i \cdot \left\{ \dot{\alpha}_1 \vec{B} + \alpha_1 \dot{\vec{B}} + \dot{\alpha}_2 \vec{\nabla} T + \alpha_3 (\vec{\nabla} T \times \dot{\vec{B}}) + \dot{\alpha}_3 (\vec{\nabla} T \times \vec{B}) \right. \\ &\quad \left. + \dot{\alpha}_4 \vec{B} + \alpha_4 \ddot{\vec{B}} + \dot{\alpha}_5 (\vec{\nabla} T \times \dot{\vec{B}}) + \alpha_5 (\vec{\nabla} T \times \ddot{\vec{B}}) \right\} \\ &- (\omega_i - b_i h) \vec{v}_i \cdot \frac{\vec{\nabla} T}{T} - \alpha_2 q_i \vec{v}_i \cdot (\vec{\nabla} T \times \vec{B}) + \alpha_3 q_i \vec{v}_i \cdot \vec{\nabla} T \\ &(\vec{B} \cdot \vec{B}) - \alpha_3 q_i \vec{v}_i \cdot \vec{B} (\vec{B} \cdot \vec{\nabla} T) + \alpha_5 q_i \vec{v}_i \cdot \vec{\nabla} T (\vec{B} \cdot \vec{B}) - \alpha_5 q_i \\ &\vec{v}_i \cdot \dot{\vec{B}} (\vec{B} \cdot \vec{\nabla} T) = -\frac{\omega_i}{\tau_R^i} \left\{ \alpha_1 \vec{v}_i \cdot \vec{B} + \alpha_2 \vec{v}_i \cdot \vec{\nabla} T \right. \\ &\quad \left. + \alpha_3 \vec{v}_i \cdot (\vec{\nabla} T \times \vec{B}) + \alpha_4 \vec{v}_i \cdot \dot{\vec{B}} + \alpha_5 \vec{v}_i \cdot (\vec{\nabla} T \times \dot{\vec{B}}) \right\}.\end{aligned}\quad (2.104)$$

In current analysis, we consider only the terms with first-order derivatives of the fields and neglect higher-order derivative terms. Let us now compare the coefficients of same tensor structure on both sides of above equation one by one. The comparison of the coefficients of $\vec{v} \cdot \vec{B}$ on both sides leads to,

$$\dot{\alpha}_1 = -\frac{1}{\tau_R^i} \alpha_1 + \frac{q_i (\vec{B} \cdot \vec{\nabla} T)}{\omega_i} \alpha_3. \quad (2.105)$$

Similarly the comparison of coefficients of $\vec{v}_i \cdot \dot{\vec{B}}$, $\vec{v}_i \cdot \vec{\nabla} T$, $\vec{v}_i \cdot (\vec{\nabla} T \times \dot{\vec{B}})$, and

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$\vec{v}_i \cdot (\vec{\nabla}T \times \vec{B})$ gives us $\alpha_4, \dot{\alpha}_2, \alpha_5, \dot{\alpha}_3$ respectively as

$$\begin{aligned}\alpha_4 &= -\tau_R^i \left\{ \alpha_1 + \frac{\tau_R^i q_i (\vec{B} \cdot \vec{\nabla}T)}{\omega_i} \alpha_5 \right\}, \\ \dot{\alpha}_2 &= - \left\{ \frac{1}{\tau_R^i} \alpha_2 + \left(\frac{q_i (\vec{B} \cdot \vec{B} - \tau_R^i \vec{B} \cdot \dot{\vec{B}})}{\omega_i} \right) \alpha_3 - \frac{\omega_i - b_i h}{T \omega_i} \right\}, \\ \dot{\alpha}_3 &= -\frac{1}{\tau_R^i} \alpha_3 + \frac{q_i}{\omega_i} \alpha_2, \\ \alpha_5 &= -\tau_R^i \alpha_3.\end{aligned}\tag{2.106}$$

Here, α_1 from Eq. (2.105) and $\dot{\alpha}_2, \dot{\alpha}_3$ Eq. (2.106) can be expressed in terms of matrix equation as

$$\frac{dX}{dt} = AX + G,\tag{2.107}$$

where the matrices take the following forms,

$$X = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}, A = \begin{pmatrix} -\frac{1}{\tau_R^i} & 0 & \frac{q_i}{\omega_i} (\vec{B} \cdot \vec{\nabla}T) \\ 0 & -\frac{1}{\tau_R^i} & -\frac{q_i F^2}{\omega_i} \\ 0 & \frac{q_i}{\omega_i} & -\frac{1}{\tau_R^i} \end{pmatrix}, G = \begin{pmatrix} 0 \\ \frac{\omega_i - b_i h}{T \omega_i} \\ 0 \end{pmatrix},$$

with $F = \sqrt{B(B - \tau_R^i \dot{B})}$. Eq. (2.107) can be solved by diagonalizing the matrix A and using the method of the variation of constants. The eigen values corresponding to matrix A are,

$$\lambda_j = -\frac{1}{\tau_R^i} + a_j i \frac{q_i F}{\omega_i}, \quad \text{with } a_1 = 0, a_2 = -1, a_3 = 1.$$

The eigen vectors corresponding to these eigen values are,

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} \frac{-(\vec{B} \cdot \vec{\nabla}T)}{iF} \\ -iF \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} \frac{(\vec{B} \cdot \vec{\nabla}T)}{iF} \\ iF \\ 1 \end{pmatrix}.$$

Hence, the linear independent solutions corresponding to homogeneous part of differential Eq. (2.107)

$$y_1 = \begin{pmatrix} e^{\eta_1} \\ 0 \\ 0 \end{pmatrix}, y_2 = \begin{pmatrix} \zeta e^{\eta_2} \\ -iF e^{\eta_2} \\ e^{\eta_2} \end{pmatrix}, y_3 = \begin{pmatrix} -\zeta e^{\eta_3} \\ iF e^{\eta_3} \\ e^{\eta_3} \end{pmatrix}.$$

Where,

$$\zeta = \frac{i(\vec{B} \cdot \vec{\nabla} T)}{F}, \quad \eta_j = -\frac{t}{\tau_R^i} + a_j \frac{q_i i}{\omega_i} \int F dt. \quad (2.108)$$

Therefore, the fundamental matrix for Eq. (2.107) is

$$Y = \begin{pmatrix} e^{\eta_1} & \zeta e^{\eta_2} & -\zeta e^{\eta_3} \\ 0 & -iF e^{\eta_2} & iF e^{\eta_3} \\ 0 & e^{\eta_2} & e^{\eta_3} \end{pmatrix}, \quad (2.109)$$

We seek a particular solution of equation with given form of Eq. (2.107) is

$$Y_p = YU \quad (2.110)$$

where U is a column matrix of unknowns.

$$U = \begin{pmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_n \end{pmatrix}. \quad (2.111)$$

From Eq. (2.110) and Eq. (2.107) we can see that Y_p is a column matrix with same coefficients as that of matrix X . Further, the differentiation of Eq. (2.110) with respect to time gives us $Y_p' = Y'U + YU'$, where $Y' = AY$. Hence, $Y_p' = AY_p + YU'$. Comparison of above equation with Eq. (2.107) tells us

$$G = YU'. \quad (2.112)$$

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The determinant of matrix Y is $-2iFe^\eta$. Then,

$$\begin{aligned}
u'_1 &= \frac{1}{-2iFe^\eta} \det \begin{pmatrix} 0 & \zeta e^{\eta_2} & -\zeta e^{\eta_3} \\ \frac{\omega_i - b_i h}{T\omega_i} & -iFe^{\eta_2} & iFe^{\eta_3} \\ 0 & e^{\eta_2} & e^{\eta_3} \end{pmatrix}, \\
&= \frac{\zeta e^{-\eta_1}}{iF} \left(\frac{\omega_i - b_i h}{T\omega_i} \right), \\
u'_2 &= \frac{1}{-2iFe^\eta} \det \begin{pmatrix} e^{\eta_1} & 0 & -\zeta e^{\eta_3} \\ 0 & \frac{\omega_i - b_i h}{T\omega_i} & iFe^{\eta_3} \\ 0 & 0 & e^{\eta_3} \end{pmatrix}, \\
&= \frac{-e^{-\eta_2}}{iF} \left(\frac{\omega_i - b_i h}{2T\omega_i} \right), \\
u'_3 &= \frac{1}{-2iFe^\eta} \det \begin{pmatrix} e^{\eta_1} & \zeta e^{\eta_2} & 0 \\ 0 & -iFe^{\eta_2} & \frac{\omega_i - b_i h}{T\omega_i} \\ 0 & e^{\eta_2} & 0 \end{pmatrix}, \\
&= \frac{e^{-\eta_3}}{iF} \left(\frac{\omega_i - b_i h}{2T\omega_i} \right). \tag{2.113}
\end{aligned}$$

After integrating u'_1 , u'_2 and u'_3 with respect to time, we get the matrix U as

$$U = \begin{pmatrix} -\zeta i \frac{\omega_i - b_i h}{T\omega_i} \xi_1 \\ i \frac{\omega_i - b_i h}{2T\omega_i} \xi_2 \\ -i \frac{\omega_i - b_i h}{2T\omega_i} \xi_3 \end{pmatrix}. \tag{2.114}$$

Where $\xi_j = \int \frac{e^{-\eta_j}}{F} dt$.

After substitution the above value of U in Eq. (2.110) results in,

$$Y_p = \begin{pmatrix} e^{\eta_1} & \zeta e^{\eta_2} & -\zeta e^{\eta_3} \\ 0 & -iF e^{\eta_2} & iF e^{\eta_3} \\ 0 & e^{\eta_2} & e^{\eta_3} \end{pmatrix} \begin{pmatrix} -\zeta i \frac{\omega_i - b_i h}{T\omega_i} \xi_1 \\ i \frac{\omega_i - b_i h}{2T\omega_i} \xi_2 \\ -i \frac{\omega_i - b_i h}{2T\omega_i} \xi_3 \end{pmatrix},$$

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} \zeta e^{\eta_1} & \zeta e^{\eta_2} & -\zeta e^{\eta_3} \\ 0 & -iF e^{\eta_2} & iF e^{\eta_3} \\ 0 & e^{\eta_2} & e^{\eta_3} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}.$$

Hence,

$$\begin{aligned} \alpha_1 &= c_1 \zeta e^{\eta_1} + c_2 \zeta e^{\eta_2} - c_3 \zeta e^{\eta_3}, \\ \alpha_2 &= -c_2 iF e^{\eta_2} + c_3 iF e^{\eta_3}, \\ \alpha_3 &= c_2 e^{\eta_2} + c_3 e^{\eta_3}. \end{aligned} \tag{2.115}$$

The functions $c_1(t)$, $c_2(t)$ and $c_3(t)$ can be defined as $c_1 = -i \frac{(\omega_i - b_i h)}{T\omega_i} \xi_1$, $c_2 = i \frac{(\omega_i - b_i h)}{2T\omega_i} \xi_2$ and $c_3 = -i \frac{(\omega_i - b_i h)}{2T\omega_i} \xi_3$. Here, $B = B_0 e^{-\frac{t}{\tau_B}}$, and τ_B is used as a parameter. For time varying field we get

$$F = B \sqrt{1 + \frac{\tau_R^i}{\tau_B}}.$$

Hence, it leads to the given form of,

$$\begin{aligned} \eta_j &= -\frac{t}{\tau_R^i} + a_j i \frac{\sqrt{1 + \frac{\tau_R^i}{\tau_B}}}{\tau_B} t, \\ \xi_j &= \frac{1}{\sqrt{1 + \frac{\tau_R^i}{\tau_B}} B_0 \left(\frac{1}{\tau_R^i} + \frac{1}{\tau_B} - a_j i \frac{\sqrt{1 + \frac{\tau_R^i}{\tau_B}}}{\tau_B} \right)} e^{\left(\frac{1}{\tau_R^i} + \frac{1}{\tau_B} - a_j i \frac{\sqrt{1 + \frac{\tau_R^i}{\tau_B}}}{\tau_B} \right) t}. \end{aligned} \tag{2.116}$$

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The microscopic definition of heat flow can take from Eq. (2.95),

$$\begin{aligned}\vec{I}_i &= \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i}{\omega_i} (\omega_i - b_i h) \delta f_i. \\ &= \frac{1}{3} \sum_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3 \omega_i} \vec{k}_i^2 (\omega_i - b_i h) \left\{ \alpha_1 \vec{B} + \alpha_2 \vec{\nabla} T \right. \\ &\quad \left. + \alpha_3 (\vec{\nabla} T \times \vec{B}) + \alpha_4 \dot{\vec{B}} + \alpha_5 (\vec{\nabla} T \times \dot{\vec{B}}) \right\} \left(\frac{\partial f_i^0}{\partial \omega_i} \right). \end{aligned} \quad (2.117)$$

In the presence of time-varying magnetic field heat current in the fluid rest frame can be expressed as [39]

$$\begin{aligned}\vec{I} &= \kappa_0 \vec{\nabla} T + (\kappa_1 + \kappa_2) (\vec{\nabla} T \times \hat{b}) \\ &= \kappa_0 \vec{\nabla} T + \kappa_H (\vec{\nabla} T \times \hat{b}). \end{aligned} \quad (2.118)$$

Compare the coefficients of Eq. (2.118) and Eq. (2.117)

$$\begin{aligned}\kappa_0 &= \frac{1}{3} \sum_{\text{baryons}} g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3 \omega_i} k_i^2 (\omega_i - b_i h) \alpha_2 \left(\frac{\partial f_i^0}{\partial \omega_i} \right), \\ \kappa_1 &= \frac{1}{3} \sum_{\text{baryons}} g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3 \omega_i} k_i^2 (\omega_i - b_i h) \alpha_3 \left(\frac{\partial f_i^0}{\partial \omega_i} \right), \\ \kappa_2 &= \frac{1}{3} \sum_{\text{baryons}} g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3 \omega_i} k_i^2 (\omega_i - b_i h) (-\tau_R^i \alpha_3) \left(\frac{\partial f_i^0}{\partial \omega_i} \right). \end{aligned} \quad (2.119)$$

where,

$$\begin{aligned}\alpha_2 &= \frac{-(\omega_i - b_i h)}{T \omega_i} \frac{\frac{1}{\tau_R^i} + \frac{1}{\tau_B}}{\left(\frac{1}{\tau_R^i} + \frac{1}{\tau_B} \right)^2 + \left(\frac{\sqrt{1 + \frac{\tau_R^i}{\tau_B}}}{\tau_B} \right)^2}, \\ \alpha_3 &= \frac{(\omega_i - b_i h)}{T \omega_i B \tau_B} \frac{1}{\left(\frac{1}{\tau_R^i} + \frac{1}{\tau_B} \right)^2 + \left(\frac{\sqrt{1 + \frac{\tau_R^i}{\tau_B}}}{\tau_B} \right)^2}. \end{aligned} \quad (2.120)$$

Substitute value of α_2 and α_3 from Eq. (2.120) into Eq. (2.119)

$$\begin{aligned}\kappa_0 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^i \frac{1}{(1 + \chi_i + \chi_i^2)} f_i^0 (1 \mp f_i^0), \\ \kappa_1 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^i \frac{\chi_i}{(1 + \chi_i)(1 + \chi_i + \chi_i^2)} f_i^0 (1 \mp f_i^0), \\ \kappa_2 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^i \frac{\chi_i^2}{(1 + \chi_i)(1 + \chi_i + \chi_i^2)} f_i^0 (1 \mp f_i^0),\end{aligned}$$

or,

$$\kappa_H = \kappa_1 + \kappa_2 = \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^i \frac{\chi_i}{(1 + \chi_i + \chi_i^2)} f_i^0 (1 \mp f_i^0). \quad (2.121)$$

For a static and homogeneous magnetic field, the thermal conductivity tensor becomes anisotropic. Within the RTA, the longitudinal and Hall components of the thermal conductivity can be derived (see Ref. [14] for details) and are given by

$$\begin{aligned}\kappa_0 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^i \frac{1}{1 + (\tau_R^i \Gamma_i)^2} f_i^0 (1 \mp f_i^0), \\ \kappa_H &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^i \frac{\tau_R^i \Gamma_i}{1 + (\tau_R^i \Gamma_i)^2} f_i^0 (1 \mp f_i^0),\end{aligned} \quad (2.122)$$

where κ_0 denotes the longitudinal thermal conductivity and κ_H represents the thermal Hall conductivity.

In the absence of a magnetic field ($B = 0$), $\Gamma_i \rightarrow 0$ and the Hall component vanishes. The thermal conductivity then becomes isotropic and reduces to

$$\kappa_0 = \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^i f_i^0 (1 \mp f_i^0), \quad (2.123)$$

which coincides with the standard expression for thermal conductivity in an unmagnetized medium.

In the left plot of Fig. (2.2), the temperature dependence of the thermal conductivity components is shown for baryon chemical potentials $\mu_B = 0.1$ and 0.3 GeV.

2.4 Transport coefficients of hot and dense hadronic medium in the presence of time-varying magnetic field

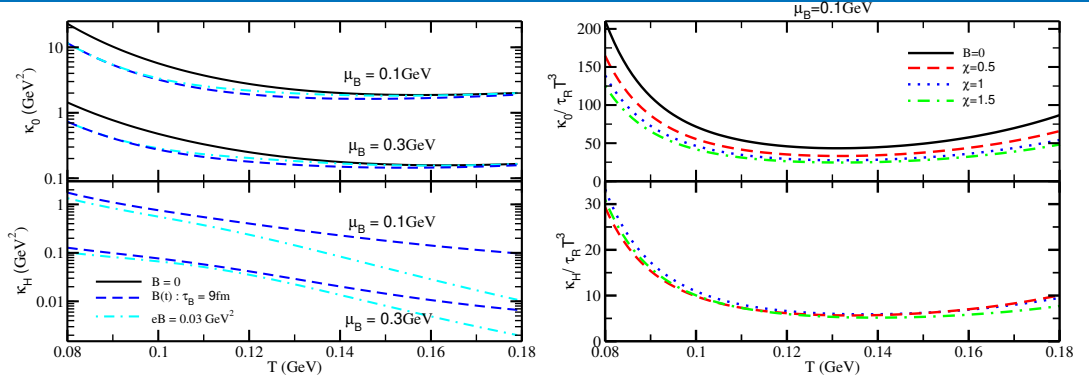


Figure 2.2: The left panel displays the components of thermal conductivity (κ) as a function of temperature (T). The right panel shows the scaled thermal conductivity as a function of temperature. Here, $\chi = \frac{\tau_R}{\tau_B}$. A higher value of χ represents faster decay of the field. The faster the decay, the lower the conductivity [2].

The upper panel displays the Ohmic-type thermal conductivity κ_0 for three different cases: zero magnetic field ($B = 0$), a constant magnetic field with strength $eB = 0.03$ GeV², and a time-dependent magnetic field $B(t)$ with decay parameter $\tau_B = 9$ fm. For all three cases, κ_0 decreases with increasing temperature. In the absence of a magnetic field, κ_0 is proportional to the relaxation time τ_R , and the temperature dependence directly reflects the decrease of τ_R with T . In contrast, for finite magnetic fields, κ_0 is approximately inversely proportional to τ_R , although this behavior is not clearly visible in the temperature profiles. This indicates the presence of additional contributions governing the temperature dependence. Unlike electrical conductivity, the integrand of thermal conductivity involves the enthalpy per net baryon density, which becomes large at low temperatures and leads to a decreasing $\kappa_0(T)$. Furthermore, neutral hadrons contribute to thermal conductivity but are unaffected by electromagnetic fields; consequently, their contribution to κ_0 remains identical to the $B = 0$ case and decreases with temperature. As the baryon chemical potential increases, κ_0 decreases due to the enhancement of enthalpy per particle with temperature, consistent with earlier findings [14]. It is also observed that κ_0 for the time-dependent magnetic field

is slightly smaller than that for the constant magnetic field case, which can be understood from their respective analytic expressions.

The lower panel of the left plot of Fig. (2.2) shows the Hall component of thermal conductivity κ_H for the time-dependent and constant magnetic field cases. For both scenarios, κ_H decreases with temperature. Although κ_H is approximately inversely proportional to the relaxation time, its temperature dependence is dominated by the thermodynamic phase-space factor involving the enthalpy per net baryon density, which decreases with increasing temperature. The magnitude of κ_H is roughly an order of magnitude smaller than that of κ_0 , primarily because neutral hadrons do not contribute to Hall transport and because particle and antiparticle contributions partially cancel for charged hadrons. The Hall thermal conductivity is larger for the time-dependent magnetic field case due to the additional contribution κ_2 , which is absent in the constant magnetic field case [Eq. (2.122)].

In the right plot of Fig. (2.2), the thermal conductivity components scaled by $\tau_R T^3$ are plotted as functions of temperature at $\mu_B = 0.1$ GeV for the time-dependent field case. The upper and lower panels correspond to the scaled κ_0 and κ_H , respectively. Results are shown for three values of the ratio $\chi = \tau_R/\tau_{B,E} = 0.5, 1$, and 1.5 , where $\chi > 1$ (< 1) corresponds to faster (slower) decay of the fields. To isolate the effect of the decay parameter, the same relaxation time is assumed for all hadrons. Although κ_0 is reduced in the presence of a magnetic field, it increases with decreasing χ , indicating that slower field decay enhances heat transport along the temperature gradient, independent of the field magnitude. In contrast, the Hall component exhibits a weaker dependence on χ and attains its maximum when the decay time is comparable to the relaxation time. Since the scaling removes the explicit temperature dependence arising from τ_R , the remaining temperature dependence originates solely from the thermodynamic phase-space factors, which decrease with temperature. The combined influence of relaxation dynamics and thermodynamic contributions to the temperature depen-

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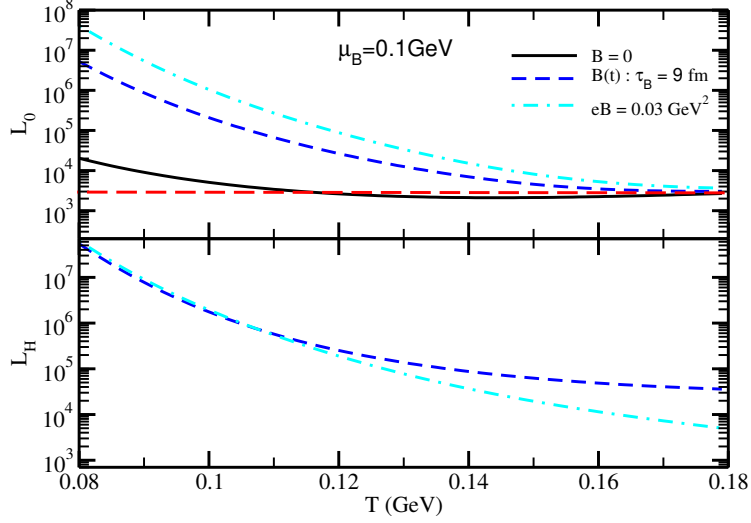


Figure 2.3: Lorenz number as a function of temperature. In the upper panel $L_0 = \frac{\kappa_0}{T \times \sigma_e}$ and in the lower panel $L_H = \frac{\kappa_H}{T \times \sigma_H}$. Red horizontal straight line (long dashed) is drawn for reference to show a violation of the WF law [2].

dence of the thermal conductivity is already evident in the left plot of Fig. (2.2). In Fig. (2.3), the temperature dependence of the Lorenz number, $L_0 = \kappa_0/(\sigma_e T)$ (upper panel), and the Hall-Lorenz number, $L_H = \kappa_H/(\sigma_H T)$ (lower panel), is shown at $\mu_B = 0.1$ GeV for $B = 0$, a time-dependent magnetic field $B(t)$ with $\tau_B = 9$ fm, and a constant magnetic field $eB = 0.03$ GeV² within the HRG model. The Lorenz number exhibits a clear temperature dependence in all cases, indicating a violation of the Wiedemann–Franz law for heat transport associated with the baryon current. For $B = 0$, L_0 varies more slowly with temperature compared to finite-field cases, while at higher temperatures it approaches an approximately constant value, suggesting a weak metal-like behavior. The saturation values differ between $B = 0$ and $B \neq 0$ cases, and the deviation from saturation follows the ordering: constant B case $>$ $B(t)$ case $>$ $B = 0$ case. This violation is further enhanced by the unequal microscopic contributions to thermal and electrical conductivities, since thermal conductivity receives contributions from both charged and neutral baryons, whereas electrical conductivity arises only from charged

baryons and mesons, consistent with earlier observations [30]. The Hall–Lorenz number L_H also shows significant deviation from saturation, mirroring behavior observed in condensed matter systems where violations of the Wiedemann–Franz law are well known [43].

2.5 Thermal conductivity of evolving QGP

In the evolving scenario, the space–time evolution of the QGP is incorporated through a time-dependent temperature profile governed by different hydrodynamic cooling rates. We consider three distinct cases—ideal hydrodynamics, ideal magnetohydrodynamics, and first-order dissipative hydrodynamics—which are discussed below.

For an evolving medium, the expression for heat current is modified compared to the static case. As the temperature decreases with time, the temperature gradient $\vec{\nabla}T$ acquires an additional time dependence, leading to a non-vanishing contribution from $\vec{\nabla}\dot{T}$. Consequently, the heat current in the local rest frame of the fluid takes the form

$$\vec{I} = - \left(\kappa_0 \vec{\nabla}T + \bar{\kappa}_1 \vec{\nabla}\dot{T} \right), \quad (2.124)$$

where $\bar{\kappa}_1$ is a new transport coefficient associated with the cooling rate of the medium.

In the presence of a time-dependent magnetic field, additional anisotropic contributions arise, and the heat current is generalized as

$$\vec{I} = - \left\{ \kappa_0 \vec{\nabla}T + \bar{\kappa}_1 (\vec{\nabla}T \times \vec{B}) + \bar{\kappa}_2 (\vec{\nabla}T \times \dot{\vec{B}}) + \bar{\kappa}_3 \vec{\nabla}\dot{T} + \bar{\kappa}_4 (\vec{\nabla}\dot{T} \times \vec{B}) \right\}, \quad (2.125)$$

where $\bar{\kappa}_3$ and $\bar{\kappa}_4$ arise solely due to the time evolution of temperature. These coefficients are evaluated for the three hydrodynamic cooling scenarios described below.

2.5 Thermal conductivity of evolving QGP

2.5.0.1 Case-I: Ideal hydrodynamics ($B = 0$)

For ideal hydrodynamic expansion, the temperature evolution is given by

$$T = T_0 \left(\frac{\tau_0}{\tau} \right)^{1/3}, \quad (2.126)$$

where T_0 and τ_0 denote the initial temperature and formation time, respectively. Although ideal hydrodynamics does not include dissipative effects, it is employed here only to obtain a reference cooling rate.

In this case, the heat current reduces to

$$\vec{I} = -\kappa^e \vec{\nabla} T, \quad (2.127)$$

with $\kappa^e = \kappa_0 + \kappa_1$ and $\kappa_1 = -\bar{\kappa}_1/(3\tau)$. The superscript e indicates the evolving medium.

The microscopic expression for κ^e is obtained by solving the Boltzmann transport equation in the relaxation-time approximation,

$$\frac{\partial f_i}{\partial \tau} + \frac{\vec{k}_i}{\omega_i} \cdot \frac{\partial f_i}{\partial \vec{x}} = -\frac{\delta f_i}{\tau_R^i}, \quad (2.128)$$

using the standard ansatz for thermal conductivity [42]

$$\delta f_i = \frac{\vec{k}_i \cdot \vec{\Omega}_\kappa}{T} \frac{\partial f_i^0}{\partial \omega_i}, \quad (2.129)$$

where

$$\vec{\Omega}_\kappa = \alpha_1 \vec{\nabla} T + \alpha_2 \vec{\nabla} \dot{T}. \quad (2.130)$$

The coefficients α_1 and α_2 are determined by substituting Eqs. (2.129) and (2.130) into Eq. (2.128). The resulting expression for the thermal conductivity in the

evolving medium are [3]

$$\begin{aligned}
 \kappa_0 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^i f_i^0 (1 \mp f_i^0) \\
 &\quad - \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i} (\omega_i - b_i h) \tau_R^i \exp(-\tau/\tau_R) f_i^0 (1 \mp f_i^0), \\
 \kappa_1 &= \frac{1}{9\tau T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^{i^2} f_i^0 (1 \mp f_i^0) \\
 &\quad - \frac{1}{9\tau T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i} (\omega_i - b_i h) \tau_R^{i^2} \exp(-\tau/\tau_R) f_i^0 (1 \mp f_i^0),
 \end{aligned}$$

or,

$$\begin{aligned}
 \kappa^e &= \kappa_0 + \kappa_1, \\
 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h) \tau_R^i \left(1 + \frac{\tau_R^i}{3\tau}\right) \{(\omega_i - b_i h) \\
 &\quad - \omega_i \exp(-\tau/\tau_R)\} f_i^0 (1 \mp f_i^0). \quad (2.131)
 \end{aligned}$$

2.5.0.2 Case-II: Ideal magnetohydrodynamics

In the presence of a time-dependent magnetic field, the cooling rate is modified and is given by [3]

$$\begin{aligned}
 T &= \left[T_0^4 \left(\frac{\tau_0}{\tau} \right)^{4/3} + \frac{4\alpha}{(2\beta\tau)^{4/3}} \{ \Gamma(4/3, 2\beta\tau) - \Gamma(4/3, 2\beta\tau_0) \} \right. \\
 &\quad \left. - \frac{2\alpha}{(2\beta\tau)^{7/3}} \{ \Gamma(7/3, 2\beta\tau) - \Gamma(7/3, 2\beta\tau_0) \} \right]^{1/4}, \quad (2.132)
 \end{aligned}$$

where $\alpha = B_0^2/(12a)$, $\beta = 1/\tau_B$, and $a = (16 + 21N_f/2)\pi^2/90$.

Using this cooling rate, the heat current can be rewritten as

$$\vec{I} = - \left\{ \kappa_0^e \vec{\nabla} T + \kappa_H^e (\vec{\nabla} T \times \hat{b}) \right\}, \quad (2.133)$$

where $\kappa_0^e = \kappa_0 + \kappa_3$ and $\kappa_H^e = \kappa_1 + \kappa_2 + \kappa_4$. The explicit expressions for these

2.5 Thermal conductivity of evolving QGP

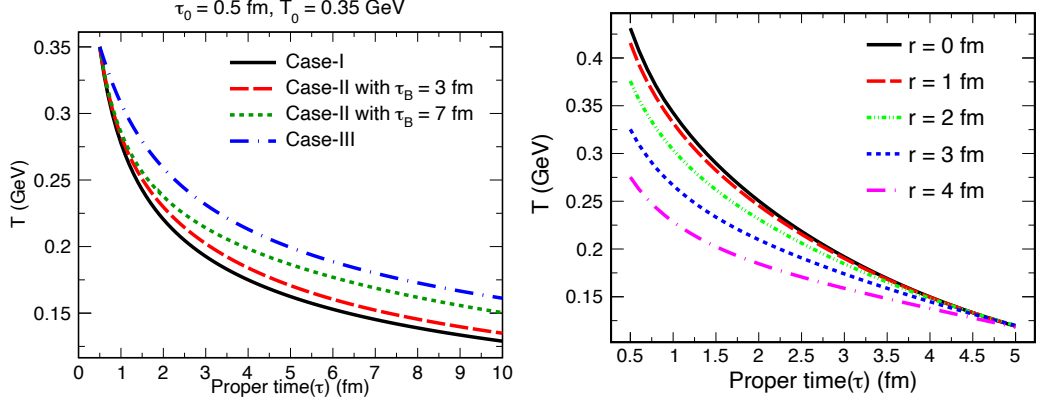


Figure 2.4: Left: cooling rate for three cases in Bjorken flow. Case-I: Ideal hydrodynamics, Case-II: Ideal magnetohydrodynamics, Case-III: First-order hydrodynamics. Right: cooling rate in Gubser flow for different values of $r = 0, 1, 2, 3, 4$ [3].

coefficients are presented as,

$$\begin{aligned}
 \kappa_0 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^i \frac{1}{(1 + \chi_i + \chi_i^2)} f_i^0 (1 \mp f_i^0), \\
 \kappa_1 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^i \frac{\chi_i}{(1 + \chi_i)(1 + \chi_i + \chi_i^2)} f_i^0 (1 \mp f_i^0), \\
 \kappa_2 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^i \frac{\chi_i^2}{(1 + \chi_i)(1 + \chi_i + \chi_i^2)} f_i^0 (1 \mp f_i^0), \\
 \kappa_3 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^{i2} \frac{((1 + \chi_i)(\chi_i^2 - 1)) + \chi_i^2}{(1 + \chi_i)(1 + \chi_i + \chi_i^2)(1 + \chi_i^2)} \\
 &\quad \left\{ \frac{-1}{3\tau} \left(1 - \frac{3B^2}{4aT^4} \right) - \frac{B^2}{4a\tau_B T^4} \right\} f_i^0 (1 \mp f_i^0), \\
 \kappa_4 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^{i2} \frac{\chi_i + \chi_i(1 + \chi_i)}{(1 + \chi_i)(1 + \chi_i + \chi_i^2)(1 + \chi_i^2)} \\
 &\quad \left\{ \frac{1}{3\tau} \left(1 - \frac{3B^2}{4aT^4} \right) + \frac{B^2}{4a\tau_B T^4} \right\} f_i^0 (1 \mp f_i^0),
 \end{aligned} \tag{2.134}$$

or,

$$\begin{aligned}
 \kappa_0^e &= \kappa_0 + \kappa_3, \\
 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \frac{\tau_R^i}{(1 + \chi_i + \chi_i^2)} \\
 &\quad \left[1 - \left\{ \tau_R^i \frac{(1 + \chi_i)(\chi_i^2 - 1) + \chi_i^2}{(1 + \chi_i)(1 + \chi_i^2)} \right\} \left\{ \frac{1}{3\tau} \left(1 - \frac{3B^2}{4aT^4} \right) + \frac{B^2}{4a\tau_B T^4} \right\} \right] \times f_i^0 (1 \mp f_i^0), \\
 \kappa_H^e &= \kappa_1 + \kappa_2 + \kappa_4, \\
 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \frac{\tau_R^i \chi_i}{(1 + \chi_i + \chi_i^2)} \\
 &\quad \left[1 + \left\{ \tau_R^i \frac{(2 + \chi_i)}{(1 + \chi_i)(1 + \chi_i^2)} \right\} \left\{ \frac{1}{3\tau} \left(1 - \frac{3B^2}{4aT^4} \right) + \frac{B^2}{4a\tau_B T^4} \right\} \right] \times f_i^0 (1 \mp f_i^0).
 \end{aligned} \tag{2.135}$$

Here $\chi_i = \tau_R^i / \tau_B$, and the results are valid in the limit of slowly varying magnetic fields.

2.5.0.3 Case-III: First-order dissipative hydrodynamics ($B = 0$)

In first-order dissipative hydrodynamics, the temperature evolution is given by [\[44\]](#)

$$T = T_0 \left(\frac{\tau_0}{\tau} \right)^{1/3} \left[1 + \frac{b}{6a} \frac{1}{\tau_0 T_0} \left(1 - \left(\frac{\tau_0}{\tau} \right)^{2/3} \right) \right], \tag{2.136}$$

where b depends on the strong coupling α_s and the number of quark flavors N_f .

The formalism for computing the thermal conductivity remains identical to Case-I, with the cooling rate in Eq. (2.136) replacing Eq. (2.126). The resulting ex-

2.5 Thermal conductivity of evolving QGP

pressions for κ_0 , κ_1 , and κ^e are given as following

$$\begin{aligned}\kappa_0 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^i f_i^0 (1 \mp f_i^0) \\ &\quad - \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i} (\omega_i - b_i h) \tau_R^i \exp(-\tau/\tau_R) f_i^0 (1 \mp f_i^0), \\ \kappa_1 &= \frac{1}{3\tau T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \tau_R^{i^2} f_i^0 (1 \mp f_i^0) \\ &\quad - \frac{1}{3\tau T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i} (\omega_i - b_i h) \tau_R^{i^2} \exp(-\tau/\tau_R) f_i^0 (1 \mp f_i^0),\end{aligned}$$

or,

$$\begin{aligned}\kappa^e &= \kappa_0 + \kappa_1, \\ &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h) \tau_R^i \left(1 + \frac{\tau_R^i}{\tau}\right) \{(\omega_i - b_i h) - \omega_i \exp(-\tau/\tau_R)\} \\ &\quad f_i^0 (1 \mp f_i^0).\end{aligned}\tag{2.137}$$

To ensure consistency with energy-momentum conservation in the RTA, a momentum-independent relaxation time is employed [45],

$$\tau_R = \frac{1}{5.1 T \alpha_s^2 \ln(1/\alpha_s) [1 + 0.12(2N_f + 1)]},\tag{2.138}$$

with $\alpha_s = 0.5$ and $N_f = 3$. In Fig. 2.5 (left), the top panel represents κ_0 component of thermal conductivity with proper time (τ) at baryon chemical potential $\mu_B = 0.3$ GeV for all the three cases. This component is responsible for the heat current along the temperature gradient. When the effect of cooling is considered in the conductivity calculation, the corresponding result for each case is denoted by the term “evolving,” and where it is not considered is designated by “static.” Black lines represent case-I with a dotted line for the evolving picture and a solid line for the static one. Case-II represents a red dashed line for a static picture and a red dash-single dot line for the evolving one. The blue lines represent case-III with a dash-double dotted line for the evolving picture and a dash-dot line for the

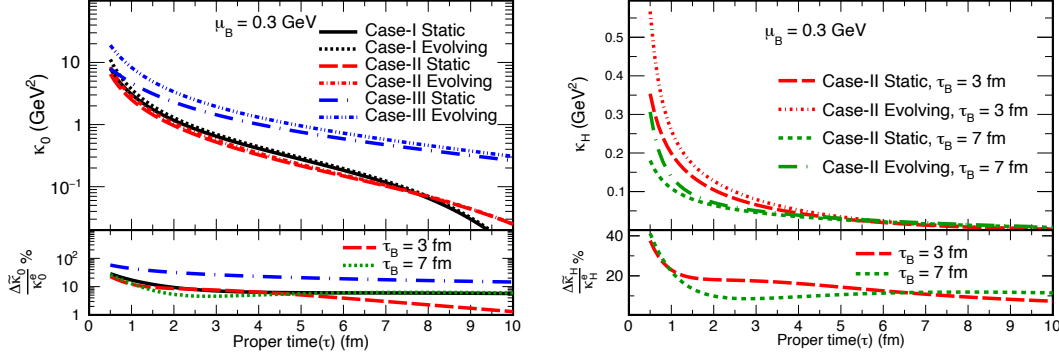


Figure 2.5: Component of thermal conductivity as a function of proper time (τ). Left: upper panel represents κ_0 component, bottom panel represents its percent-age deviation (where $\Delta\tilde{\kappa}_0 = \kappa_0^e - \kappa_0^s$) of static picture from evolving picture. Right: same as the left figure but for “Hall-like” component [3].

static one. In all three cases, we see an enhancement in the thermal conductivity in the evolving picture as compared to the static one. This comes from the new components κ_1 in case-I and case-III, and κ_3 in case-II arising due to $\vec{\nabla}\vec{T}$ term in heat current.

In static pictures, the expression of thermal conductivity (Eq. (2.123)) is the same for case-I and case-III except for their temperature evolution. Hence, we can see that the solid black line and blue dash-dotted line start from the same point, but later on follow different paths due to their respective cooling rates. For all three cases, we notice that conductivity decreases with time, mainly dominated by their thermodynamical phase space part for baryons as expected from the existing knowledge [42]. The effect of the magnetic field can be found in further suppression of conductivity, as introduced in case-II. We can quantify the effect of the cooling rate and magnetic field by looking at their respective expressions of conductivity. In the absence of magnetic field, comparing the static picture (Eq. 2.121) with evolving pictures (Eqs.(2.131) and (2.137)), we can argue that the thermodynamical phase space part ($\omega_i - b_i h$) as well as the relaxation time τ_R^i get modified in the evolving picture. However, in the presence of a magnetic

2.5 Thermal conductivity of evolving QGP

field, comparing the static picture (Eq.(2.121)) and the evolving picture (2.134), one observes that the effect of evolution is mainly taken care of by relaxation time or effective relaxation time. From the bottom panel of Fig. (2.5) (left), we can quantify the effect of cooling for all the cases. On average, 10% – 20% effects are there for all the cases. In the presence of a magnetic field, the effect is comparatively less and decreases with increasing decay parameters. The effect of evolution is greater in case-III than in the other cases, indicating that the evolution of thermal conductivity is highly sensitive to the system’s cooling rate. We can roughly state that the slower the cooling higher the effect of evolution in the conductivity, and the effect decreases with time evolution.

In Fig. 2.5 (right), the time evolution of the “Hall-like” component and the effect of evolution in this component is demonstrated at baryon chemical potential $\mu_B = 0.3$ GeV. This component arises only due to the magnetic field, and the heat current for this component is perpendicular to both the magnetic field and the temperature gradient. This figure represents κ_H for case-II at two decay parameters, $\tau_B = 3, 7$ fm. The absolute value of κ_H is almost ten times less than that of the κ_0 component because the contribution from particles and antiparticles are opposite, leading to a smaller net value. Similar to the κ_0 component, we see an enhancement in κ_H in the evolving picture as compared to the static one. In this case, enhancement comes from the new component κ_4 arising in the evolution picture due to $(\vec{\nabla}\dot{T} \times \vec{B})$ term. In the bottom panel of the figure, we notice that the effect of evolution is large in the early stages of evolution (around 40%) as compared to the later stages (around 10%). As the system evolves with time, the strength of the magnetic field also starts to weaken, and the κ_H component diminishes, leaving only the κ_0 component.

For the Gubser low profile, the components of thermal conductivity are given

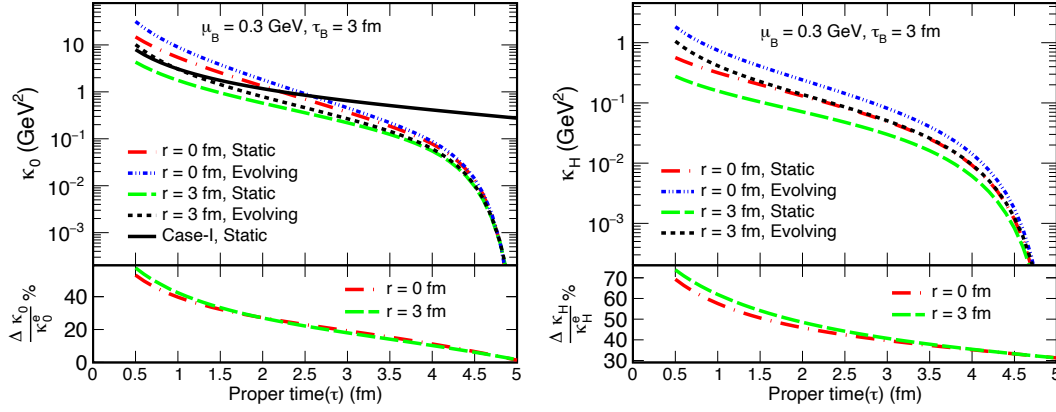


Figure 2.6: Component of thermal conductivity as a function of proper time (τ). Left: The upper panel represents κ_0 Component, and the bottom panel represents its percentage deviation (where $\Delta\tilde{\kappa}_0 = \kappa_0^e - \kappa_0^s$) of the static picture from evolving picture; Right: The same as the left figure but for “Hall-like” Component in Gubser flow.

as, [3]

$$\begin{aligned}
 \kappa_0^e &= \kappa_0 + \kappa_3, \\
 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \frac{\tau_R^i}{(1 + \chi_i + \chi_i^2)} \\
 &\quad \left[1 - \left\{ \tau_R^i \frac{(1 + \chi_i)(\chi_i^2 - 1) + \chi_i^2}{(1 + \chi_i)(1 + \chi_i^2)} \right\} \left\{ \frac{1}{\tau} \right\} \right] \times f_i^0(1 \mp f_i^0), \\
 \kappa_H^e &= \kappa_1 + \kappa_2 + \kappa_4, \\
 &= \frac{1}{3T^2} \sum_i g_i \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i h)^2 \frac{\tau_R^i \chi_i}{(1 + \chi_i + \chi_i^2)} \\
 &\quad \left[1 + \left\{ \tau_R^i \frac{(2 + \chi_i)}{(1 + \chi_i)(1 + \chi_i^2)} \right\} \left\{ \frac{1}{\tau} \right\} \right] \times f_i^0(1 \mp f_i^0).
 \end{aligned} \tag{2.139}$$

where $\kappa_1 = \bar{\kappa}_1 B$, $\kappa_2 = \bar{\kappa}_2 \dot{B}$, $\kappa_3 = \bar{\kappa}_3 \left\{ \frac{-1}{\tau} \right\}$, and $\kappa_4 = \bar{\kappa}_4 \left\{ \frac{-1}{\tau} \right\} B$

In the upper panels of Fig. 2.6, we present the proper-time (τ) dependence of the Ohmic (left) and Hall-like (right) components of thermal conductivity obtained with the Gubser cooling profile for two transverse positions, $r = 0$ and 3 fm.

2.6 Summary

The results are shown for a time-dependent magnetic field with decay parameter $\tau_B = 3$ fm. It is observed that both conductivity components decrease as r increases. The black solid curve in the upper-left panel corresponds to the static-limit result within ideal hydrodynamics. Compared to this reference, the Gubser evolution exhibits substantial deviations, reflecting the more rapid cooling of the medium relative to the Bjorken scenario, particularly at later proper times.

The lower panels quantify the relative difference between the evolving and static cases under Gubser flow. At early times ($\tau \sim 0.5$ fm), sizable deviations are found, amounting to about 50–60% for the Ohmic component and 60–70% for the Hall-like component. As the system evolves, these differences diminish, reaching approximately 0–10% for the Ohmic conductivity and 20–30% for the Hall-like conductivity at $\tau \sim 5$ fm.

2.6 Summary

This chapter presents a comprehensive theoretical framework for describing transport phenomena in hot and dense strongly interacting matter. It establishes a connection between microscopic dynamics and macroscopic observables by combining relativistic kinetic theory, hydrodynamic descriptions, and phenomenological models of QCD matter. These approaches provide the foundation for understanding non-equilibrium processes and transport properties relevant to heavy-ion collisions. A general overview of transport phenomena is provided, emphasizing the role of thermodynamic gradients and external fields in generating macroscopic currents of charge, heat, and particles.

In Sec. 2.1, the relativistic kinetic theory framework is introduced through the Boltzmann transport equation. The single-particle distribution function is decomposed into equilibrium and non-equilibrium components, and the relaxation time approximation is employed to simplify the collision integral. This allows for a tractable formulation of transport processes, enabling the derivation of micro-

scopic expressions for macroscopic quantities such as electric current, heat flow, and diffusion currents. The formalism provides a consistent way to calculate transport coefficients and understand their dependence on temperature, chemical potential, and interaction dynamics.

Section 2.2 focuses on relativistic hydrodynamics as an effective macroscopic theory describing the space-time evolution of the medium created in heavy-ion collisions. Starting from the conservation laws of energy and momentum, the ideal hydrodynamic framework is developed and its applicability to the nearly perfect fluid behavior of the QGP is discussed. Analytical solutions are presented to gain physical insight into the expansion dynamics:

- Bjorken flow (Sec. 2.2.1), which describes boost-invariant longitudinal expansion and predicts a characteristic cooling behavior of the system.
- Magnetized Bjorken flow (Sec. 2.2.2), which incorporates the influence of time-dependent electromagnetic fields and modifies the evolution of thermodynamic quantities.
- Gubser flow (Sec. 2.2.3), which extends the description to include transverse expansion under conformal symmetry, providing a more realistic description of the system geometry.

These solutions are essential for understanding the dynamical evolution and lifetime of the medium produced in relativistic heavy-ion collisions.

In Sec. 2.3, phenomenological models of QCD matter are discussed to describe both the deconfined and confined phases. The quasiparticle model (Sec. 2.3.1) is employed for the QGP phase, where medium effects are incorporated through temperature-dependent effective masses of quarks and gluons, allowing for a description consistent with lattice QCD thermodynamics. For the hadronic phase, the hadron resonance gas (HRG) model and its extensions (Sec. 2.3.2) are considered. These include the ideal HRG, excluded volume HRG (accounting for

2.6 Summary

repulsive interactions due to finite hardcore radius of hadrons), van der Waals HRG (including both attractive and repulsive interactions), and repulsive mean-field HRG models. These frameworks provide a unified description of thermodynamic properties across the QCD phase transition and are essential for evaluating transport coefficients in the hadronic medium.

Finally, Sec. 2.4 presents the derivation of transport coefficients in a hot and dense medium in the presence of time-dependent electromagnetic fields. Using the kinetic theory approach, the Boltzmann equation is solved with an appropriate ansatz for the non-equilibrium distribution function. The induced current is decomposed into longitudinal (Ohmic) and transverse (Hall-like) components, reflecting the anisotropic response of the system in the presence of magnetic fields. The analysis highlights how transport coefficients depend on key physical parameters such as relaxation time, magnetic field strength, and its temporal evolution, providing deeper insight into the behavior of strongly interacting matter under extreme conditions.

Overall, this chapter establishes a robust and self-consistent theoretical foundation for the study of heat and charge transport phenomena in QCD matter. The frameworks developed here serve as essential tools for the quantitative analysis of transport coefficients and their implications for the properties of the QGP and hadronic phases in heavy-ion collisions.

Bibliography

- [1] H.J. Goldsmid, Introduction to Thermoelectricity. Springer-Verlag, Berlin, Heidelberg (2009)
- [2] K. Singh, J. Dey, R. Sahoo and S. Ghosh, Phys. Rev. D **108**, 094007 (2023)
- [3] K. Singh, J. Dey and R. Sahoo, Phys. Rev. D **109**, 014018 (2024)
- [4] R. Sahoo and T. K. Nayak, “Possible early universe signals in proton collisions at the Large Hadron Collider,” Curr. Sci. **121**, 1403 (2021).
- [5] W. Busza, K. Rajagopal and W. van der Schee, “Heavy Ion Collisions: The Big Picture, and the Big Questions,” Ann. Rev. Nucl. Part. Sci. **68**, 339 (2018).
- [6] K. Singh, J. Dey and R. Sahoo, Phys. Rev. D **110**, 114051 (2024)
- [7] S. Mitra, Phys. Rev. C **103**, 014905 (2021)
- [8] J. D. Bjorken, Phys. Rev. D **27**, 140 (1983)
- [9] V. Roy, S. Pu, L. Rezzolla and D. Rischke, Phys. Lett. B **750**, 45 (2015)
- [10] A. K. Panda, A. Dash, R. Biswas and V. Roy, JHEP **03**, 216 (2021)
- [11] S. S. Gubser, Phys. Rev. D **82**, 085027 (2010)
- [12] M. I. Gorenstein and S. N. Yang, Phys. Rev. D **52**, 5206 (1995)
- [13] H. Berrehrah, E. Bratkovskaya, W. Cassing, P. B. Gossiaux, J. Aichelin and M. Bleicher, Phys. Rev. C **89**, 054901 (2014)

BIBLIOGRAPHY

- [14] S. Borsanyi, G. Endrodi, Z. Fodor, A. Jakovac, S. D. Katz, S. Krieg, C. Ratti and K. K. Szabo, JHEP **11**, 077 (2010)
- [15] P. K. Srivastava, S. K. Tiwari and C. P. Singh, Phys. Rev. D **82**, 014023 (2010)
- [16] V. M. Bannur, Phys. Lett. B **647**, 271-274 (2007)
- [17] A. Peshier, B. Kampfer, O. P. Pavlenko and G. Soff, Phys. Rev. D **54**, 2399 (1996)
- [18] A. Peshier, B. Kampfer and G. Soff, Phys. Rev. C **61**, 045203 (2000)
- [19] M. L. Sambaturo, V. Greco, G. Parisi and S. Plumari, Eur. Phys. J. C **84**, 881 (2024)
- [20] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667**, 1-1340 (2008)
- [21] K. K. Pradhan, D. Sahu, R. Scaria and R. Sahoo, Phys. Rev. C **107**, 014910 (2023)
- [22] D. H. Rischke, M. I. Gorenstein, H. Stoecker and W. Greiner, Z. Phys. C **51**, 485 (1991)
- [23] A. Bhattacharyya, S. Das, S. K. Ghosh, R. Ray and S. Samanta, Phys. Rev. C **90**, 034909 (2014)
- [24] V. Vovchenko, M. I. Gorenstein and H. Stoecker, Phys. Rev. Lett. **118**, 182301 (2017)
- [25] V. Vovchenko, Phys. Rev. C **96**, 015206 (2017)
- [26] S. Samanta and B. Mohanty, Phys. Rev. C **97**, 015201 (2018)
- [27] K. Singh, K. K. Pradhan, D. Sahu and R. Sahoo, Phys. Rev. D **111**, 074033 (2025)

- [28] V. Vovchenko, D. V. Anchishkin, M. I. Gorenstein and R. V. Poberezhnyuk, Phys. Rev. C **92**, 054901 (2015)
- [29] N. Sarkar and P. Ghosh, Phys. Rev. C **98**, 014907 (2018)
- [30] J. I. Kapusta and K. A. Olive, Nucl. Phys. A **408**, 478 (1983)
- [31] K. A. Olive, Nucl. Phys. B **190**, 483 (1981)
- [32] P. Huovinen and P. Petreczky, Phys. Lett. B **777**, 125 (2018)
- [33] G. Kadam and H. Mishra, Phys. Rev. D **100**, 074015 (2019)
- [34] G. K. K, M. Kurian and V. Chandra, Phys. Rev. D **104**, 094037 (2021)
- [35] P. Deb, G. P. Kadam and H. Mishra, Phys. Rev. D **94**, 094002 (2016)
- [36] A. Das, H. Mishra and R. K. Mohapatra, Phys. Rev. D **100**, 114004 (2019)
- [37] H. T. Ding, O. Kaczmarek and F. Meyer, Phys. Rev. D **94**, 034504 (2016)
- [38] A. Amato, G. Aarts, C. Allton, P. Giudice, S. Hands and J. I. Skullerud, Phys. Rev. Lett. **111**, 172001 (2013)
- [39] G. K. K, M. Kurian and V. Chandra, Phys. Rev. D **106**, 034008 (2022)
- [40] S. R. De Groot, W. A. Van Leeuwen and C. G. Van Weert (1980).
- [41] G. S. Rocha and G. S. Denicol, Phys. Rev. D **104**, 096016 (2021)
- [42] S. Gavin, Nucl. Phys. A **435**, 826 (1985)
- [43] Y. Onose, Y. Shiomi, and Y. Tokura, Phys. Rev. Lett. **100**, 016601 (2008).
- [44] A. Muronga, Phys. Rev. Lett. **88**, 062302 (2002)
- [45] A. Hosoya and K. Kajantie, Nucl. Phys. B **250**, 666 (1985)

Chapter 3

Thermoelectric phenomena in QCD matter in the presence of a magnetic field

“Change is the law of the universe.”

— Shri Krishna

Thermoelectricity captures the beauty of non-equilibrium physics, where gradients become the driving force for motion. A temperature or chemical-potential imbalance sets microscopic carriers into collective transport, giving rise to measurable currents. Its beauty lies in the fact that while gradients act as external driving forces, it is the intrinsic property of electric charge, acting as an additional degree of freedom, that enables the simultaneous transport of both energy and conserved charges. In conventional materials, electrons serve as the primary charge carriers, and their response to thermal gradients leads to familiar thermoelectric phenomena in metals, semiconductors, and complex quantum materials [1–4]. The interplay between electronic band structure, scattering mechanisms, and lattice dynamics governs the magnitude and efficiency of thermoelectric transport in such systems. Beyond conventional solids, thermoelectricity acquires an even richer character in electrodynamical plasmas, where charged particles, namely ions and electrons, interact collectively with electromagnetic fields and long-range forces shape transport behavior [5]. At the extreme end of

the energy spectrum, in the quark–gluon plasma formed under ultra-relativistic conditions, thermoelectric transport emerges from the motion of quarks carrying fractional electric charges, embedded in a strongly interacting medium governed by quantum chromodynamics. Despite the vast separation in energy scales, the underlying principles linking gradients, conserved charges, and collective dynamics remain universal, highlighting thermoelectricity as a unifying framework connecting condensed matter systems, electromagnetic plasmas, and the hottest forms of matter known in nature.

*“An electron behaves like a double-faced coin,
One face bears charge, the other heat in join,
Driven by gradients, it flows as one,
Binding current and warmth till transport is done.”*

3.1 Thermoelectric coefficients

The Seebeck coefficient [1, 6, 7] represents the leading-order thermoelectric response of a medium, quantifying the generation of an electric potential gradient in response to an applied temperature gradient. Physically, it measures the voltage difference established between the two extreme ends of a system when charge carriers diffuse from the hot region toward the cold region. In the presence of a magnetic field, this response becomes anisotropic, giving rise to the magneto-Seebeck effect [8, 9] in the longitudinal direction, and the Nernst effect in the transverse direction perpendicular to both the temperature gradient and the magnetic field. Beyond this leading-order behavior, when the Seebeck coefficient exhibits explicit temperature dependence, higher-order thermoelectric effects naturally emerge. The Thomson coefficient [10–12] characterizes the continuous absorption or release of heat along the length of a conducting medium carrying an electric current under a temperature gradient, reflecting the local coupling between charge flow

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and thermal variations. In magnetized systems, this effect generalizes further into magneto-Thomson and transverse Thomson phenomena, where the presence of a magnetic field induces anisotropic and transverse heat generation or absorption relative to the direction of the imposed gradients. These higher-order and transverse thermoelectric coefficients provide deeper insight into microscopic carrier dynamics, scattering mechanisms, and symmetry breaking induced by magnetic fields, thereby enriching the thermoelectric transport landscape.

3.1.1 Leading-order thermoelectric coefficients in QCD matter

To evaluate the leading-order thermoelectric transport coefficients in hot and dense QCD matter, we employ the relativistic Boltzmann transport equation (RBTE) within the relaxation time approximation (RTA). The kinetic-theory framework describes the evolution of the single-particle phase-space distribution and provides a systematic approach to investigate transport processes and equilibration dynamics in strongly interacting QCD matter. In the presence of an external magnetic field, the dynamics of charged constituents (quarks in the QGP phase and charged hadrons and mesons in the hadronic phase) are governed by the Lorentz force, which induces anisotropy in both electrical and thermal transport. Within the RTA framework under the kinetic theory approach [13–18], the RBTE is linearized by expanding the total single-particle distribution function (f_i) around its local equilibrium form (f_i^0). Accordingly, the distribution function is decomposed as $f_i = f_i^0 + \delta f_i$, where δf_i represents the non-equilibrium correction generated by gradients of thermodynamic variables. In the context of QCD matter, the total distribution function receives contributions from all relevant degrees of freedom, such as quarks, antiquarks, and hadronic resonances, depending on the phase under consideration.

For a particle species i with single-particle energy $\omega_i = \sqrt{\vec{k}_i^2 + m_i^2}$, baryon num-

ber b_i , and baryon chemical potential μ_B , the local equilibrium distribution function is given by

$$f_i^0 = \frac{1}{\exp\left(\frac{\omega_i - b_i \mu_B}{T}\right) \pm 1}, \quad (3.1)$$

where the upper (lower) sign corresponds to fermions (bosons). Under conditions of local thermodynamic equilibrium, the spatial dependence of the distribution function arises from gradients in temperature and baryon chemical potential.

The RBTE for an individual particle species in QCD matter, in the presence of an electromagnetic field and within the RTA, can therefore be written as [16, 19]

$$k^\mu \partial_\mu f_i + q_i F^{\mu\nu} k_\nu \frac{\partial f_i}{\partial k^\mu} = -\frac{k^\mu u_\mu}{\tau_R^i} \delta f_i, \quad (3.2)$$

where $k^\mu = (\omega_i, \vec{k}_i)$ is the particle four-momentum, u^μ is the fluid four-velocity, $F^{\mu\nu}$ is the electromagnetic field tensor, and τ_R^i is the relaxation time for species i .

To solve the above equation, we take an ansatz to express the deviation of the distribution function from the equilibrium, such that it should reflect the driving forces. The general form of ansatz is [13, 16]

$$\delta f_i = \frac{\partial f_i^0}{\partial(k^\mu u_\mu)} k^\mu \Omega_\mu, \quad (3.3)$$

where the vector Ω^μ reflects the response of the distribution function to external thermodynamic forces such as temperature gradients, electric fields, and magnetic fields. In the fluid rest frame ($u^\mu = (1, \vec{0})$), Ω^μ can be decomposed as

$$\begin{aligned} \Omega^\mu = & \alpha_1 E^\mu + \alpha_2 B^\mu + \alpha_3 \epsilon^{\mu\nu\rho\sigma} u_\nu E_\rho B_\sigma + \alpha_4 \nabla^\mu T \\ & + \alpha_5 \epsilon^{\mu\nu\rho\sigma} u_\nu \nabla_\rho T B_\sigma + \alpha_6 \epsilon^{\mu\nu\rho\sigma} u_\nu \nabla_\rho T E_\sigma, \end{aligned} \quad (3.4)$$

where the electric and magnetic fields in the fluid rest frame are

$$E^\mu = F^{\mu\nu} u_\nu, \quad B^\mu = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} u_\nu F_{\rho\sigma},$$

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and $\nabla^\mu = \Delta^{\mu\nu}\partial_\nu$ is the spatial gradient orthogonal to u^μ , with $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$. Here, $\epsilon^{\mu\nu\rho\sigma}$ denotes the Levi-Civita symbol, which is used to represent antisymmetric tensor components.

Using Eq.(3.3) into RBTE as given in Eq.(3.2) and projecting onto spatial components in the local rest frame yields [20],

$$\begin{aligned} \vec{v}_i \cdot \left[-\frac{\partial f_i^0}{\partial \omega_i} \left(\frac{\omega_i - b_i h}{T} \right) \vec{\nabla} T \right] + q_i (\vec{E} \cdot \vec{v}_i) \frac{\partial f_i^0}{\partial \omega_i} \\ - q_i \vec{v}_i \cdot (\vec{\Omega} \times \vec{B}) \frac{\partial f_i^0}{\partial \omega_i} = -\frac{\omega_i}{\tau_R^i} (\vec{v}_i \cdot \vec{\Omega}) \frac{\partial f_i^0}{\partial \omega_i}. \end{aligned} \quad (3.5)$$

Here, $\vec{v}_i = \vec{k}_i/\omega_i$ denotes the relativistic velocity of the particle, while $h = (\epsilon + P)/n$ represents the enthalpy per baryon. The quantities ϵ , P , and n correspond to the total energy density, total pressure, and net baryon number density of the system, respectively. The appearance of the enthalpy per baryon in the formalism follows naturally from the Gibbs–Duhem relation. By invoking this relation, the gradients of the inverse temperature and the baryon chemical potential can be combined in a consistent manner. As a result, the spatial derivative of the equilibrium distribution function can be expressed entirely in terms of the temperature gradient, with the enthalpy per baryon emerging as the appropriate thermodynamic variable governing the transport process [19, 21].

By substituting $\vec{\Omega}$ from Eq.(3.4) into Eq.(3.5), we can expand it analogously to the nonrelativistic case as,

$$\begin{aligned} q_i (\vec{E} \cdot \vec{v}_i) - \alpha_1 q_i \vec{v}_i \cdot (\vec{E} \times \vec{B}) - \alpha_3 q_i (\vec{E} \cdot \vec{B}) (\vec{v}_i \cdot \vec{B}) + \alpha_3 \\ q_i (\vec{v}_i \cdot \vec{E}) - \alpha_4 q_i \vec{v}_i \cdot (\vec{\nabla} T \times \vec{B}) - \alpha_5 q_i (\vec{\nabla} T \cdot \vec{B}) (\vec{v}_i \cdot \vec{B}) + \alpha_5 \\ q_i (\vec{v}_i \cdot \vec{\nabla} T) - \alpha_6 q_i (\vec{\nabla} T \cdot \vec{B}) (\vec{v}_i \cdot \vec{E}) + \alpha_6 q_i (\vec{E} \cdot \vec{B}) (\vec{v}_i \cdot \vec{\nabla} T) \\ - \left(\frac{\omega_i - b_i h}{T} \right) (\vec{v}_i \cdot \vec{\nabla} T) = -\frac{\omega_i}{\tau_R^i} \left[\alpha_1 (\vec{v}_i \cdot \vec{E}) + \alpha_2 (\vec{v}_i \cdot \vec{B}) \right. \\ + \alpha_3 \vec{v}_i \cdot (\vec{E} \times \vec{B}) + \alpha_4 (\vec{v}_i \cdot \vec{\nabla} T) + \alpha_5 \vec{v}_i \cdot (\vec{\nabla} T \times \vec{B}) + \\ \left. \alpha_6 \vec{v}_i \cdot (\vec{\nabla} T \times \vec{E}) \right]. \end{aligned} \quad (3.6)$$

Comparing the coefficients of same tensor structures on both sides of Eq.(3.6) we get,

$$\alpha_1 = -\frac{\tau_R^i}{\omega_i}(q_i E + \alpha_3 q_i B), \quad (3.7)$$

$$\alpha_2 = \alpha_3 \tau_R^i \Omega_{c_i} (\vec{E} \cdot \vec{B}) + b \tau_R^i \Omega_{c_i} (\vec{\nabla} T \cdot \vec{B}), \quad (3.8)$$

$$\alpha_3 = \alpha_1 \tau_R^i \Omega_{c_i}, \quad (3.9)$$

$$\alpha_4 = \frac{\tau_R^i}{\omega_i} \left(\frac{\omega_i - b_i h}{T} \right) - \alpha_5 \tau_R^i \Omega_{c_i}, \quad (3.10)$$

$$\alpha_5 = \Omega_{c_i} \tau_R^i \alpha_4, \quad (3.11)$$

$$\alpha_6 = 0. \quad (3.12)$$

where $\Omega_{c_i} = \frac{q_i B}{\omega_i}$ represents the cyclotron frequency of the particle with electric charge q_i . Rearranging the Eqs.(3.7) to (3.12), we get

$$\alpha_1 = -\frac{(q_i E)(\tau_R^i/\omega_i)}{1 + (\Omega_{c_i} \tau_R^i)^2}, \quad (3.13)$$

$$\alpha_4 = \frac{\tau_R^i}{\omega_i} \left(\frac{\omega_i - b_i h}{T} \right) \frac{1}{1 + (\Omega_{c_i} \tau_R^i)^2}. \quad (3.14)$$

Using α_1 and α_4 as given in Eq.(3.13) and Eq.(3.14), we can write deviation from the equilibrium distribution function as,

$$\begin{aligned} \delta f_i = & \frac{\tau_R^i}{1 + (\Omega_{c_i} \tau_R^i)^2} \left[q_i \left\{ (\vec{v}_i \cdot \vec{E}) + (\Omega_{c_i} \tau_R^i) \vec{v}_i \cdot (\vec{E} \times \vec{B}) + \right. \right. \\ & \left. \left. (\Omega_{c_i} \tau_R^i)^2 (\vec{E} \cdot \vec{B}) (\vec{v}_i \cdot \vec{B}) \right\} - \left(\frac{\omega_i - b_i h}{T} \right) \left\{ (\vec{v}_i \cdot \vec{\nabla} T) + \right. \right. \\ & \left. \left. (\Omega_{c_i} \tau_R^i) \vec{v}_i \cdot (\vec{\nabla} T \times \vec{B}) + (\Omega_{c_i} \tau_R^i)^2 (\vec{\nabla} T \cdot \vec{B}) (\vec{v}_i \cdot \vec{B}) \right\} \right] \left(-\frac{\partial f_i^0}{\partial \omega_i} \right). \end{aligned} \quad (3.15)$$

Now, we can express the electrical current and the heat current using δf_i as given

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in Eq.(3.15) as,

$$\begin{aligned}
j^l &= \sum_i g_i \int \frac{d^3 k_i}{(2\pi)^3} q_i v_i^l \delta f_i \\
&= \sum_i \frac{g_i q_i}{3} \int \frac{d^3 k_i}{(2\pi)^3} \frac{v_i^2 \tau_R^i}{1 + (\Omega_{c_i} \tau_R^i)^2} \left[q_i \delta^{lj} E^j + q_i (\Omega_{c_i} \tau_R^i) \right. \\
&\quad \left. \epsilon^{ljk} h^k E^j + q_i (\Omega_{c_i} \tau_R^i)^2 h^l h^j E^j - \left(\frac{\omega_i - b_i h}{T} \right) \left\{ \delta^{lj} \frac{\partial T}{\partial x^j} \right. \right. \\
&\quad \left. \left. + (\Omega_{c_i} \tau_R^i) \epsilon^{ljk} h^k \frac{\partial T}{\partial x^j} + (\Omega_{c_i} \tau_R^i)^2 h^l h^j \frac{\partial T}{\partial x^j} \right\} \right] \left(-\frac{\partial f_i^0}{\partial \omega_i} \right), \tag{3.16}
\end{aligned}$$

and,

$$\begin{aligned}
I^l &= \sum_i g_i \int \frac{d^3 k_i}{(2\pi)^3} v_i^l \left(\omega_i - b_i h \right) \delta f_i \\
&= \sum_i \frac{g_i}{3} \int \frac{d^3 k_i}{(2\pi)^3} \frac{v_i^2 \tau_R^i}{1 + (\Omega_{c_i} \tau_R^i)^2} \left(\omega_i - b_i h \right) \left[q_i \delta^{lj} E^j \right. \\
&\quad \left. + q_i (\Omega_{c_i} \tau_R^i) \epsilon^{ljk} h^k E^j + q_i (\Omega_{c_i} \tau_R^i)^2 h^l h^j E^j - \left(\frac{\omega_i - b_i h}{T} \right) \right. \\
&\quad \left. \left\{ \delta^{lj} \frac{\partial T}{\partial x^j} + (\Omega_{c_i} \tau_R^i) \epsilon^{ljk} h^k \frac{\partial T}{\partial x^j} + (\Omega_{c_i} \tau_R^i)^2 h^l h^j \frac{\partial T}{\partial x^j} \right\} \right] \\
&\quad \left(-\frac{\partial f_i^0}{\partial \omega_i} \right). \tag{3.17}
\end{aligned}$$

Here, to simplify further calculations, we can choose the magnetic field along the z direction. The direction of the electric field and the temperature gradient are perpendicular to the z axis, i.e., it is in the $x - y$ plane. Under these conditions, the components of the electric current in the $x - y$ plane are given as,

$$\begin{aligned}
j_x &= \sum_i \frac{g_i q_i}{3} \int \frac{d^3 k_i}{(2\pi)^3} \frac{v_i^2 q_i \tau_R^i}{1 + (\Omega_{c_i} \tau_R^i)^2} \left[E_x + (\Omega_{c_i} \tau_R^i) E_y \right] \\
&\quad (-) \frac{\partial f_i^0}{\partial \omega_i} - \sum_i \frac{g_i q_i}{3T} \int \frac{d^3 k_i}{(2\pi)^3} \frac{v_i^2 \tau_R^i (\omega_i - b_i h)}{1 + (\Omega_{c_i} \tau)^2} \\
&\quad \left[\frac{dT}{dx} + (\Omega_{c_i} \tau_R^i) \frac{dT}{dy} \right] \left(-\frac{\partial f_i^0}{\partial \omega_i} \right), \tag{3.18}
\end{aligned}$$

and,

$$\begin{aligned}
 j_y = & \sum_i \frac{g_i q_i}{3} \int \frac{d^3 k_i}{(2\pi)^3} \frac{v_i^2 q_i \tau_R^i}{1 + (\Omega_{c_i} \tau_R^i)^2} \left[E_y - (\Omega_{c_i} \tau_R^i) E_x \right] \\
 & (-) \frac{\partial f_i^0}{\partial \omega_i} - \sum_i \frac{g_i q_i}{3T} \int \frac{d^3 k_i}{(2\pi)^3} \frac{v_i^2 \tau_R^i (\omega_i - b_i h)}{1 + (\Omega_{c_i} \tau)^2} \\
 & \left[\frac{dT}{dy} - (\Omega_{c_i} \tau_R^i) \frac{dT}{dx} \right] \left(-\frac{\partial f_i^0}{\partial \omega_i} \right). \tag{3.19}
 \end{aligned}$$

Eqs.(3.18) and (3.19) can be written in a compact form by introducing the following integrals,

$$H_{1_i} = \frac{g_i}{3} \int \frac{d^3 k_i}{(2\pi)^3} \frac{\tau_R^i}{1 + (\Omega_{c_i} \tau_R^i)^2} \left(\frac{\vec{k}_i^2}{\omega_i^2} \right) \left(-\frac{\partial f_i^0}{\partial \omega_i} \right), \tag{3.20}$$

$$H_{2_i} = \frac{g_i}{3} \int \frac{d^3 k_i}{(2\pi)^3} \frac{\tau_R^i (\Omega_{c_i} \tau_R^i)}{1 + (\Omega_{c_i} \tau_R^i)^2} \left(\frac{\vec{k}_i^2}{\omega_i^2} \right) \left(-\frac{\partial f_i^0}{\partial \omega_i} \right), \tag{3.21}$$

$$H_{3_i} = \frac{g_i}{3} \int \frac{d^3 k_i}{(2\pi)^3} \frac{\tau_R^i \omega_i}{1 + (\Omega_{c_i} \tau_R^i)^2} \left(\frac{\vec{k}_i^2}{\omega_i^2} \right) \left(-\frac{\partial f_i^0}{\partial \omega_i} \right), \tag{3.22}$$

$$H_{4_i} = \frac{g_i}{3} \int \frac{d^3 k_i}{(2\pi)^3} \frac{\tau_R^i \omega_i (\Omega_{c_i} \tau_R^i)}{1 + (\Omega_{c_i} \tau_R^i)^2} \left(\frac{\vec{k}_i^2}{\omega_i^2} \right) \left(-\frac{\partial f_i^0}{\partial \omega_i} \right). \tag{3.23}$$

The integrals as given in Eqs.(3.20)-(3.23) allows us to write Eq.(3.18) and Eq.(3.19), respectively, as

$$\begin{aligned}
 j_x = & \sum_i q_i^2 H_{1_i} E_x + \sum_i q_i^2 H_{2_i} E_y - \frac{1}{T} \sum_a q_i \left(H_{3_i} - b_i h H_{1_i} \right) \frac{dT}{dx} \\
 & - \frac{1}{T} \sum_a q_i \left(H_{4_i} - b_i h H_{2_i} \right) \frac{dT}{dy}, \tag{3.24}
 \end{aligned}$$

and,

$$\begin{aligned}
 j_y = & \sum_i q_i^2 H_{1_i} E_y - \sum_i q_i^2 H_{2_i} E_x - \frac{1}{T} \sum_a q_i \left(H_{3_i} - b_i h H_{1_i} \right) \frac{dT}{dy} \\
 & + \frac{1}{T} \sum_a q_i \left(H_{4_i} - b_i h H_{2_i} \right) \frac{dT}{dx}. \tag{3.25}
 \end{aligned}$$

Here, in the presence of a magnetic field, the magneto-Seebeck coefficient (S_B) can be determined by setting $j_x = j_y = 0$ so that the electric field becomes

3.1 Thermoelectric coefficients

proportional to the temperature gradient. For $j_x = 0$ and $j_y = 0$ we can solve Eqs.(3.24) and (3.25) to get E_x and E_y in terms of temperature gradients $\frac{dT}{dx}$ and $\frac{dT}{dy}$ as,

$$E_x = \frac{\sum_i q_i^2 H_{1i} \sum_i q_i (H_{3i} - b_i h H_{1i}) + \sum_i q_i^2 H_{2i} \sum_i q_i (H_{4i} - b_i h H_{2i})}{T \left[\left(\sum_i q_i^2 H_{1i} \right)^2 + \left(\sum_i q_i^2 H_{2i} \right)^2 \right]} \frac{dT}{dx} + \frac{\sum_i q_i^2 H_{1i} \sum_i q_i (H_{4i} - b_i h H_{2i}) - \sum_i q_i^2 H_{2i} \sum_i q_i (H_{3i} - b_i h H_{1i})}{T \left[\left(\sum_i q_i^2 H_{1i} \right)^2 + \left(\sum_i q_i^2 H_{2i} \right)^2 \right]} \frac{dT}{dy}, \quad (3.26)$$

and,

$$E_y = \frac{\sum_i q_i^2 H_{2i} \sum_i q_i (H_{3i} - b_i h H_{1i}) - \sum_i q_i^2 H_{1i} \sum_i q_i (H_{4i} - b_i h H_{2i})}{T \left[\left(\sum_i q_i^2 H_{1i} \right)^2 + \left(\sum_i q_i^2 H_{2i} \right)^2 \right]} \frac{dT}{dx} + \frac{\sum_i q_i^2 H_{1i} \sum_i q_i (H_{3i} - b_i h H_{1i}) + \sum_i q_i^2 H_{2i} \sum_i q_i (H_{4i} - b_i h H_{2i})}{T \left[\left(\sum_i q_i^2 H_{1i} \right)^2 + \left(\sum_i q_i^2 H_{2i} \right)^2 \right]} \frac{dT}{dy}. \quad (3.27)$$

Eqs.(3.26) and (3.27) can be written in a compact form in the following way,

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} S_B & NB \\ -NB & S_B \end{pmatrix} \begin{pmatrix} \frac{dT}{dx} \\ \frac{dT}{dy} \end{pmatrix}, \quad (3.28)$$

here one can identify the magneto-Seebeck coefficient as,

$$S_B = \frac{\sum_i q_i^2 H_{1i} \sum_i q_i (H_{3i} - b_i h H_{1i}) + \sum_i q_i^2 H_{2i} \sum_i q_i (H_{4i} - b_i h H_{2i})}{T \left[\left(\sum_i q_i^2 H_{1i} \right)^2 + \left(\sum_i q_i^2 H_{2i} \right)^2 \right]} = \frac{(\sigma_{el}/T)(\mathcal{I}_{31}/T^2) + (\sigma_H/T)(\mathcal{I}_{42}/T^2)}{(\sigma_{el}/T)^2 + (\sigma_H/T)^2}, \quad (3.29)$$

and the normalized Nernst coefficient (NB) is given as,

$$NB = \frac{\sum_i q_i^2 H_{1i} \sum_i q_i (H_{4i} - b_i h H_{2i}) - \sum_i q_i^2 H_{2i} \sum_i q_i (H_{3i} - b_i h H_{1i})}{T \left[\left(\sum_i q_i^2 H_{1i} \right)^2 + \left(\sum_i q_i^2 H_{2i} \right)^2 \right]} = \frac{(\sigma_{el}/T)(\mathcal{I}_{42}/T^2) - (\sigma_H/T)(\mathcal{I}_{31}/T^2)}{(\sigma_{el}/T)^2 + (\sigma_H/T)^2}. \quad (3.30)$$

In this formulation, the Ohmic-type and Hall-type contributions to the electrical conductivity in the presence of an external magnetic field are identified as $\sigma_{el} = \sum_i q_i^2 H_{1i}$ and $\sigma_H = \sum_i q_i^2 H_{2i}$, respectively [18]. The quantities $\mathcal{I}_{31}$ and $\mathcal{I}_{42}$ appearing in Eqs. (3.29) and (3.30) are defined through $\mathcal{I}_{31} = \sum_i q_i (H_{3i} - b_i h H_{1i})$ and $\mathcal{I}_{42} = \sum_i q_i (H_{4i} - b_i h H_{2i})$.

It is important to note that, in the absence of a magnetic field, the integrals H_{2i} and H_{4i} vanish identically. Consequently, the normalized Nernst coefficient becomes zero when the magnetic field is switched off, while the magneto-Seebeck coefficient smoothly reduces to the conventional Seebeck coefficient in this limit [20].

$$\begin{aligned}
 S &= \frac{\sum_i \frac{g_i}{3} \int \frac{d^3 k_i}{(2\pi)^3} \tau_R^i q_i v_i^2 (\omega_i - b_i h) \left(-\frac{\partial f_i^0}{\partial \omega_i} \right)}{T \sum_i \frac{g_i}{3} \int \frac{d^3 k_i}{(2\pi)^3} \tau_R^i q_i^2 v_i^2 \left(-\frac{\partial f_i^0}{\partial \omega_i} \right)} \\
 &= \frac{\sum_i \frac{g_i}{3T} \int \frac{d^3 k_i}{(2\pi)^3} \tau_R^i q_i \left(\frac{\vec{k}_i}{\omega_i} \right)^2 (\omega_i - b_i h) f_i^0 (1 \mp f_i^0)}{T \sum_i \frac{g_i}{3T} \int \frac{d^3 k_i}{(2\pi)^3} \tau_R^i q_i^2 \left(\frac{\vec{k}_i}{\omega_i} \right)^2 f_i^0 (1 \mp f_i^0)} \\
 &= \frac{\mathcal{I}_1 / T^2}{\sigma_{el} / T}.
 \end{aligned} \tag{3.31}$$

where, the electrical conductivity σ_{el} can be identified as,

$$\sigma_{el} = \sum_i \frac{g_i}{3T} \int \frac{d^3 k_i}{(2\pi)^3} \tau_R^i q_i^2 \left(\frac{\vec{k}_i}{\omega_i} \right)^2 f_i^0 (1 \mp f_i^0), \tag{3.32}$$

and the integral $\mathcal{I}_1$ in Eq.(3.31) is,

$$\mathcal{I}_1 = \sum_i \frac{g_i}{3T} \int \frac{d^3 k_i}{(2\pi)^3} \tau_R^i q_i \left(\frac{\vec{k}_i}{\omega_i} \right)^2 (\omega_i - b_i h) f_i^0 (1 \mp f_i^0). \tag{3.33}$$

Here, the Seebeck coefficient can be both positive or negative because the numerator depends linearly on an electric charge while the integrand itself is not necessarily positive definite.

Furthermore, since the coefficients S_B and NB exhibit explicit temperature dependence, as evident from Eqs. (3.29) and (3.30), it becomes necessary to introduce higher-order thermoelectric coefficients to properly characterize the transport behavior of the medium.

3.1 Thermoelectric coefficients

3.1.2 Higher-order thermoelectric coefficients in QCD matter

Higher-order thermoelectric coefficients exhibit behavior that is qualitatively different from their leading-order counterparts. Leading-order thermoelectric effects, such as the Seebeck and Peltier effects, originate from linear responses of the system to small temperature or potential gradients and are therefore valid in the near-equilibrium regime. In contrast, higher-order thermoelectric effects emerge when the system responds nonlinearly to external perturbations.

In particular, the explicit temperature dependence of leading-order transport coefficients, such as the Seebeck coefficient, naturally generates higher-order thermoelectric contributions. These higher-order terms account for corrections beyond linear response and become relevant when gradients are sufficiently large. As a consequence, the presence of leading-order thermoelectric coefficients in a conducting medium modifies both the charge and heat currents. The electric current and heat current can be modified due to these thermoelectric coefficients as, [19, 20]

$$\begin{aligned}\vec{j} &= \sigma_{el} \vec{E} - \sigma_{el} S \vec{\nabla} T, \\ \vec{I} &= T \sigma_{el} S \vec{E} - \kappa_0 \vec{\nabla} T,\end{aligned}\tag{3.34}$$

where κ_0 is the thermal conductivity of the medium and is expressed as [20]

$$\kappa_0 = \sum_i \frac{g_i}{3T^2} \int \frac{d^3 k_i}{(2\pi)^3} \tau_R^i \left(\frac{\vec{k}_i}{\omega_i} \right)^2 (\omega_i - b_i h)^2 f_i^0 (1 \mp f_i^0).\tag{3.35}$$

Using Eq.(3.34), we can express the heat current $\vec{I}$ in terms of electric current $\vec{j}$ in the following way,

$$\vec{I} = T S \vec{j} - (\kappa_0 - T \sigma_{el} S^2) \vec{\nabla} T.\tag{3.36}$$

Here, $\kappa' = \kappa_0 - T \sigma_{el} S^2$ is defined as the modified thermal conductivity of the

medium. The Eq.(3.34) can be defined in matrix form as

$$\begin{pmatrix} \vec{j} \\ \vec{I} \end{pmatrix} = \begin{pmatrix} \sigma_{el} & \sigma_{el}ST \\ \sigma_{el}ST & \kappa_0 T \end{pmatrix} \begin{pmatrix} \vec{E} \\ -\vec{\nabla}T/T \end{pmatrix}. \quad (3.37)$$

The above matrix can be rearranged and expressed as follows

$$\begin{pmatrix} \vec{E} \\ \vec{I} \end{pmatrix} = \begin{pmatrix} \rho_{el} & S \\ ST & -\kappa' \end{pmatrix} \begin{pmatrix} \vec{j} \\ -\vec{\nabla}T/T \end{pmatrix}, \quad (3.38)$$

where resistivity of the medium is $\rho_{el} = 1/\sigma_{el}$. When a magnetic field is introduced in the z direction, electric field and temperature gradients in x-y plane, the above matrix in x-y plane can be expressed as

$$\begin{pmatrix} E_x \\ E_y \\ I_x \\ I_y \end{pmatrix} = \begin{pmatrix} \rho_{el} & \rho_H & S_B & NB \\ -\rho_H & \rho_{el} & -NB & S_B \\ S_B T & NB T & -\kappa'_{el} & -\kappa'_H \\ -NB T & S_B T & \kappa'_H & -\kappa'_{el} \end{pmatrix} \begin{pmatrix} j_x \\ j_y \\ \frac{dT}{dx} \\ \frac{dT}{dy} \end{pmatrix}, \quad (3.39)$$

The κ_{el} and κ_H are Ohmic-like and Hall-like components of thermal conductivity of the medium in the presence of a magnetic field [17], such that $\kappa'_{el} = \kappa_{el} - T\sigma_{el}S_B^2$ and $\kappa'_H = \kappa_H - T\sigma_H(NB)^2$. Also the $\rho_H = 1/\sigma_H$.

For any conducting medium having non-zero current density and temperature gradients along the x direction and an applied magnetic field along the z direction, the heat production rate per unit volume ($\dot{Q}$) can be expressed as [22]

$$\begin{aligned} \dot{Q} &= E_x j_x - \frac{dI_x}{dx} \\ &= (\rho_{el} j_x + S_B \frac{dT}{dx}) j_x - \frac{dT}{dx} (S_B T j_x - \kappa'_{el} \frac{dT}{dx}) \\ &= \rho_{el} j_x^2 + \frac{dT}{dx} (\kappa'_{el} \frac{dT}{dx}) - (T \frac{dS_B}{dT}) j_x \frac{dT}{dx}. \end{aligned} \quad (3.40)$$

3.1 Thermoelectric coefficients

The last term in the above equation is the heat generated by the combined action of j_x and $\frac{dT}{dx}$. The heat production rate per unit volume due to the Thomson effect in the presence of a magnetic field is given by [23]

$$\dot{Q} = -Th_B (\vec{j} \cdot \vec{\nabla} T) \quad (3.41)$$

Finally, from Eqs. (3.40) and (3.41) we can define the magneto-Thomson coefficient (Th_B) as

$$Th_B = T \frac{dS_B}{dT}. \quad (3.42)$$

In the absence of a magnetic field, the coefficient Th_B reduces to the Thomson coefficient Th [20, 21].

The transverse Thomson coefficient (Th_N) arises in the medium due to the existence of a nonvanishing normalized Nernst coefficient. In general, the coefficient Th_N becomes relevant in a conducting system when the charge current, the temperature gradient, and the external magnetic field are mutually orthogonal. In the present study, we examine the rate of heat generation per unit volume for a configuration in which the electric current flows along the x -direction, the temperature gradient is applied along the y -direction, and the magnetic field is oriented along the z -axis. Under these conditions, the corresponding heat production rate per unit volume can be expressed as

$$\begin{aligned} \dot{Q} &= E_x j_x - \frac{dI_y}{dy} \\ &= (\rho_{el} j_x + NB \frac{dT}{dy}) j_x - \frac{dT}{dy} (-NB T j_x - \kappa'_{el} \frac{dT}{dy}) \\ &= \rho_{el} j_x^2 + \frac{dT}{dy} (\kappa'_{el} \frac{dT}{dy}) + (T \frac{d(NB)}{dT} + 2NB) j_x \frac{dT}{dy}. \end{aligned} \quad (3.43)$$

Therefore, the transverse Thomson coefficient is defined as

$$Th_N = T \frac{d(NB)}{dT} + 2NB. \quad (3.44)$$

Unlike the Th_B , the coefficient Th_N not only depends on the temperature derivative of the leading thermoelectric coefficient (for this case NB) but also on its magnitude [22]. In the absence of a magnetic field, the coefficient Th_N vanishes because of the vanishing NB .

3.1.3 Thermoelectricity in deconfined phase

To obtain the leading and higher-order thermoelectric coefficients for deconfine phase, we have followed the quasi-particle model as mention in subsection 2.3.1. The medium induced thermal masses are used for the partons as mentioned in Eqs. (2.18) and (2.19). These masses modified the dispersion relations in the calculations of thermoelectric coefficients as detailed in previous section. Here, we consider a time-varying electromagnetic field of the form [24, 25]

$$B = B_0 \exp\left(-\frac{\tau}{\tau_B}\right), \quad E = E_0 \exp\left(-\frac{\tau}{\tau_E}\right), \quad (3.45)$$

where B_0 , E_0 are the magnitudes of the initial fields having decay parameters of τ_B and τ_E , respectively, and τ is the proper time. The exponential decay forms for the electric and magnetic fields are motivated by solutions to Maxwell's equations in a conducting medium, where finite electrical conductivity leads to time-dependent damping of the fields. In particular, such forms arise naturally when Ohm's law is coupled to Maxwell's equations. For simplicity, we focus on their time evolution in proper time τ , suppressing spatial dependence, which is assumed to be smooth.

We have calculated the thermoelectric coefficients for the QGP medium for both the cases of zero and non-zero external magnetic fields using the QPM model. The left panel of Fig. 3.1 represents the Seebeck coefficient S as a function of temperature T for different values of baryon-chemical potential $\mu_B = 0.10, 0.30, 0.60$ GeV. The solid black line represents the case of $\mu_B = 0.10$ GeV. On the other hand, the red dotted line and the blue dashed line represent the case for $\mu_B = 0.30, 0.60$ GeV, respectively. The value of enthalpy per particle h plays a crucial

3.1 Thermoelectric coefficients

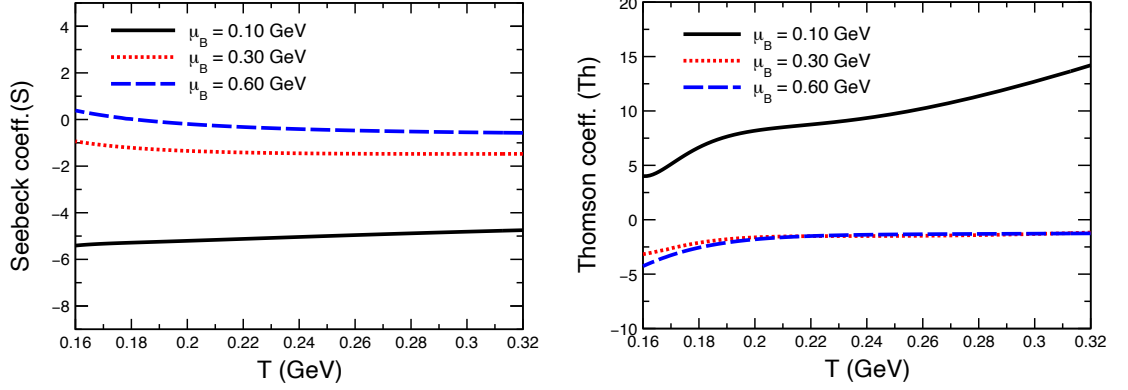


Figure 3.1: Left: Seebeck coefficient (S), right: Thomson coefficient (Th) as a function of temperature (T) for different values of $\mu_B = 0.10, 0.30$, and 0.60 GeV [21].

role in the flow of heat current $\mathcal{I}_1$. The gluons do not directly contribute to $\mathcal{I}_1$ due to their zero electric charge but enter through their contribution to the enthalpy of the medium. It is to be noted that h increases as the temperature T of the medium increases, but it decreases with the increase of the μ_B . This means that for the fixed value of T , h is higher for the lower value of μ_B . Therefore, for lower values of μ_B ($= 0.10, 0.30$ GeV), the S is negative throughout the temperature range, which is due to a larger value of enthalpy per particle h as compared to the single-particle energy ω_i ; hence the term $\omega_i - b_i h$ becomes negative in Eq. (3.29). On the other hand, for the higher value of μ_B ($= 0.60$ GeV), the S is slightly positive at lower values of T ; later on, it turns negative. For a particular medium, the sign of S describes the alignment of the induced electric field with respect to the temperature gradients present in the medium. Here, the temperature dependence of S gives rise to the non-zero Thomson coefficient in the medium. The right panel of Fig. 3.1 represents the Thomson coefficient Th as a function of temperature T for different values of baryon-chemical potential. The solid black line represents the case of $\mu_B = 0.10$ GeV. On the other hand, the red dotted line and the blue dashed line represent the case for $\mu_B = 0.30, 0.60$

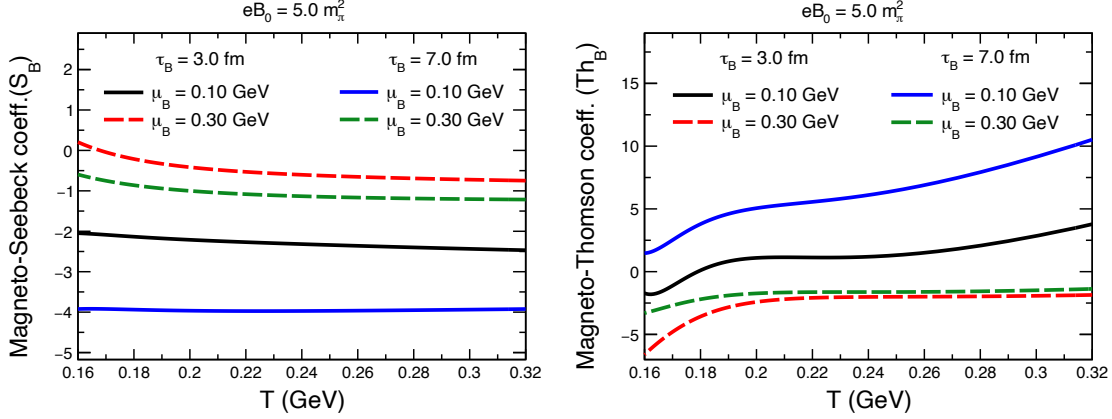


Figure 3.2: Left: magneto-Seebeck coefficient (S_B), right: magneto-Thomson coefficient (Th_B) as a function of temperature (T) for different values of $\mu_B = 0.10$ and 0.30 GeV and at magnetic field $(eB_0) = 5.0 m_\pi^2$ with decay parameter $(\tau_B) = 3.0, 7.0$ fm [21].

GeV, respectively. The Thomson coefficient describes the continuous absorption or release of heat in the charge-carrying medium in the presence of temperature gradients. In condensed matter systems, for a material having positive Th , the heat is absorbed when current flows from a region of lower temperature to a higher temperature, such as copper, silver, zinc, etc. On the contrary, the heat is released if Th is negative, such as cobalt, nickel, bismuth, etc. [1]. Here, we observe that for a QGP medium having a low value of μ_B ($= 0.10$ GeV), the Th is positive and increases as T increases. Whereas for other values of μ_B ($= 0.30, 0.60$ GeV), Th is negative for the whole temperature range. It slightly increases with T initially, then gets saturated at higher values of T . In a trivial sense, we can estimate that the higher the slope of S in the left panel, the higher the value of Th in the right panel of Fig. 3.1. The non-zero values of Th may affect the cooling of the medium during the evolution.

In Fig. 3.2, we represent the results for the thermoelectric coefficient of the medium for the case of nonzero magnetic fields. To account for the simultaneous evolution of the magnetic field and temperature, we adopt a mapping between

3.1 Thermoelectric coefficients

proper time and temperature based on an ideal magnetohydrodynamics (MHD) framework, as discussed in our earlier work [18]. In that study, the temperature and magnetic field were both expressed as functions of proper time for different decay parameters. For the present analysis, which focuses on higher-order thermoelectric coefficients in a static QGP scenario, we extract consistent pairs of magnetic field and temperature values corresponding to the same proper time using these decay profiles. This allows us to plot thermoelectric coefficients as a function of temperature while implicitly incorporating the effect of time-varying magnetic fields for different decay rates. The left panel represents the magneto-Seebeck coefficient S_B as a function of temperature T for two different values of baryon-chemical potential $\mu_B = 0.10, 0.30$ GeV. The initial value of the magnetic field (eB_0) is taken to be $5 m_\pi^2$ along with two different magnetic decay parameters τ_B at 3, 7 fm. The solid black and blue lines represent the case of $\mu_B = 0.10$ GeV for $\tau_B = 3, 7$ fm, respectively. On the other hand, the red dashed and green dashed lines represent the case for $\mu_B = 0.30$ for $\tau_B = 3, 7$ fm, respectively. Unlike the case of zero magnetic fields, we can see in Eq. (3.29) that, along with the Ohmic-like components of electrical conductivity and heat current, the Hall-like components also contribute. In the QGP medium, only quarks contribute to the Hall-like components as they experience the Lorentz force in the magnetic field. To study the Seebeck effect in the presence of a magnetic field, we now discuss the case of $\mu_B = 0.10$ GeV. For this value of μ_B , both components of electrical conductivity (σ_{el}, σ_H) contribute to the magneto-Seebeck coefficient, which is positive throughout the full temperature range. On the other hand, the Ohmic-like component of heat current ($\mathcal{I}_{31}$) is negative, but the Hall-like component ($\mathcal{I}_{42}$) is positive because of the multiplied factor χ_i . Hence, the product $(\sigma_{el}/T)(\mathcal{I}_{31}/T^2)$ is negative, but $(\sigma_H/T)(\mathcal{I}_{42}/T^2)$ is positive, and this results into decrease in value of S_B . In the absence of a magnetic field, all the Hall-like components vanish; hence, S_B turns the same as S . In the left panel of Fig. 3.2, we can see that the different magnetic decay parameters τ_B have different effects on S_B . For the case

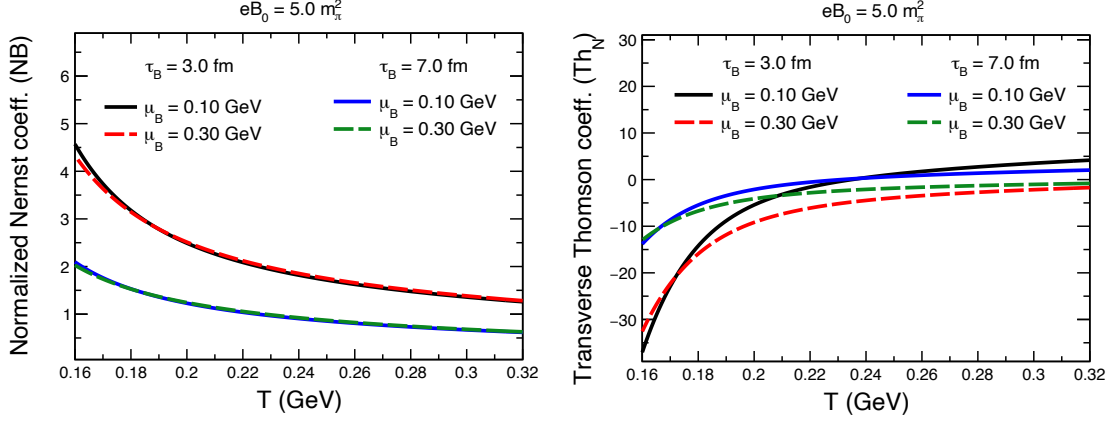


Figure 3.3: Left: normalized Nernst coefficient (NB), right: transverse Thomson coefficient (Th_N) as a function of temperature (T) for different values of $\mu_B = 0.10$ and 0.30 GeV and at magnetic field (eB_0) = $5.0 m_\pi^2$ with decay parameter (τ_B) = $3.0, 7.0$ fm [21].

of a fast decaying magnetic field ($\tau_B = 3$ fm), S_B reduces more as compared to the case of a slowly decaying magnetic field ($\tau_B = 7$ fm). This is because of the multiplied factor χ_i , which is higher in magnitude for the lower value of τ_B . Now, the right panel of Fig. 3.2 represents the magneto-Thomson coefficient Th_B as a function of temperature T for two different values of baryon-chemical potential $\mu_B = 0.10, 0.30$ GeV. As we mentioned above, the discussion for S_B , Th_B also decreases in the presence of a magnetic field. From Eq. (3.42), we can see that the higher the slope of S_B in the left panel, the larger the value of Th_B . The complete saturation of S_B over the temperature leads to the vanishing Th_B . The similar effect of τ_B is reflected in Th_B as it is for S_B . For a fast decaying magnetic field, the value of Th_B is lower than compared of a slowly decaying magnetic field.

The left panel of Fig. 3.3 represents the normalized Nernst coefficient NB as a function of temperature for two different values of $\mu_B = 0.10, 0.30$ GeV and $\tau_B = 3, 7$ fm. We observe that NB decreases as the temperature increases for all values of μ_B and τ_B . Here, both σ_{el}, σ_H contribute to the NB , which are positive throughout the temperature range discussed here. On the other hand, $\mathcal{I}_{31}$ is

3.1 Thermoelectric coefficients

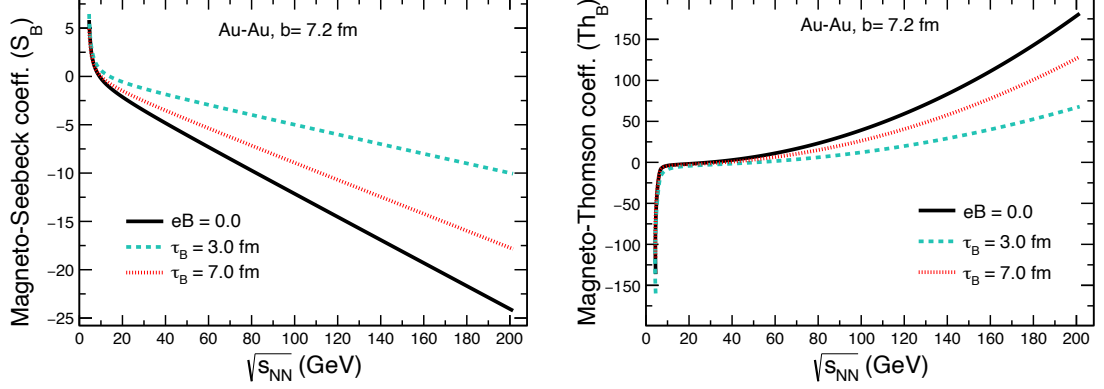


Figure 3.4: Left: magneto-Seebeck coefficient (S_B), right: magneto-Thomson coefficient (Th_B) as a function of center of mass energy ($\sqrt{s_{NN}}$) for vanishing magnetic field (eB) = 0.0 and different values of decay parameter (τ_B) = 3.0, 7.0 fm [21].

negative, but the $\mathcal{I}_{42}$ is positive because of the multiplication factor χ_i . Hence, the product $(\sigma_H/T)(\mathcal{I}_{31}/T^2)$ is positive, along with the product $(\sigma_H/T)(\mathcal{I}_{42}/T^2)$ which is also positive, and this results into positive values of NB . The values of NB are higher for the faster decaying magnetic field. This is because of the fact that χ_i is higher for the smaller value of τ_B . In the absence of a magnetic field, all the Hall-like component vanishes; hence, NB also vanishes. The right panel of Fig. 3.3 represents the transverse Thomson coefficient Th_N as a function of temperature for two different values of μ_B and τ_B . The results demonstrate that the coefficient Th_N is highly sensitive to both μ_B and τ_B , with more negative values observed at lower temperatures. For the higher values of temperatures, the coefficient Th_N seems to be saturated and approaches zero. In a trivial sense, this can be estimated from the coefficient NB that has the minimum slope towards the higher temperature range and the smaller values in magnitude. Therefore, for the low temperature region, the derivative term in Eq. (3.44) dominates over $2NB$, whereas at high temperatures, where NB seems to saturate, the term $2NB$ dominates. To express Th_B and Th_N as a function of center of mass energy $\sqrt{s_{NN}}$

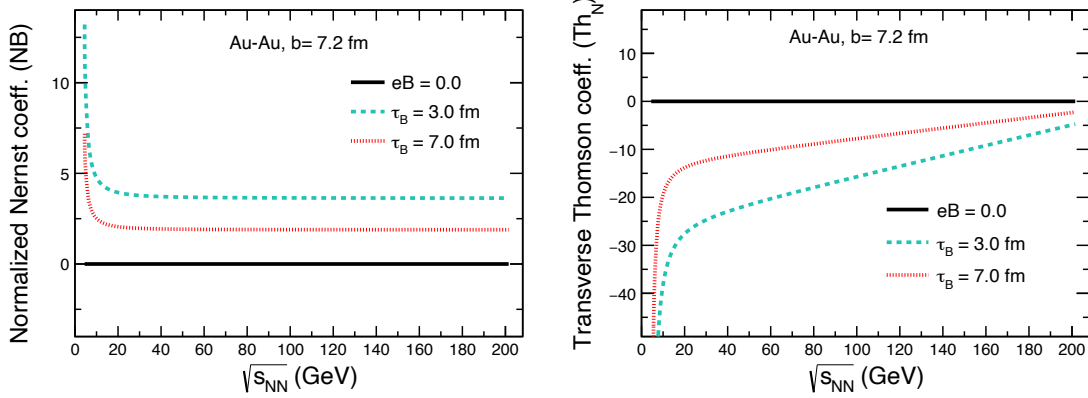


Figure 3.5: Left: normalized Nernst coefficient (NB), right: transverse Thomson coefficient (Th_N) as a function of center of mass energy ($\sqrt{s_{NN}}$) for vanishing magnetic field (eB) = 0.0 and different values of decay parameter (τ_B) = 3.0, 7.0 fm [21].

(in GeV) we follow the parameterization [26]

$$T(\mu_B) = 0.166 - 0.139\mu_B^2 - 0.053\mu_B^4, \\ \mu_B = \frac{1.308}{1 + 0.273\sqrt{s_{NN}}}. \quad (3.46)$$

Also, the magnetic field is parameterized as [27],

$$B = \frac{\sqrt{s_{NN}}}{8\pi m_N} Ze \frac{b}{R^3} \exp\left(-\frac{\tau}{\tau_B}\right). \quad (3.47)$$

Here, R is the radius of colliding ions with electric charge Ze , b is the impact parameter, and $m_N = 0.938$ GeV is the nucleon mass. For Au-Au collision at $\sqrt{s_{NN}} = 200$ GeV, with $R = 6.38$ fm, $b = 7.2$ fm, and $Z = 79$ we get $eB_0 \approx 3 m_\pi^2$ at $\tau = 0$, which is in accordance with the results for RHIC energies [28]. The left panel of Fig. 3.4 represents the S_B as a function of $\sqrt{s_{NN}}$. The black solid line represents the case of zero magnetic fields, whereas the cyan dashed line and red dotted line represent the case of the non-zero magnetic field for $\tau_B = 3, 7$ fm, respectively. We observe that for lower values of center of mass energies, nearly up to 10 GeV, the magnetic field does not show any significant effect on

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S_B . Later on, as the values of $\sqrt{s_{NN}}$ increase, the effect of the magnetic field, along with different decay parameters, also increases. The faster the decay of the magnetic field, the lower the value of S_B . Here, we also observe that the S_B is positive at lower energies; it vanishes nearly at $\sqrt{s_{NN}} \approx 20$ GeV and then starts increasing towards the negative direction. It indicates that the QGP medium created at lower values of $\sqrt{s_{NN}}$, which corresponds to a baryon-rich medium, has a positive Seebeck coefficient. The positive Seebeck coefficient means that the induced electric field in the medium is aligned in the same direction as the temperature gradient. For the QGP medium created at higher values of $\sqrt{s_{NN}}$, the Seebeck coefficient is negative and indicates the opposite alignment of the induced field with respect to temperature gradients in the medium. The right panel of Fig. 3.4 represents the Th_B as a function of $\sqrt{s_{NN}}$. Here, we can also see that for lower values of $\sqrt{s_{NN}}$, nearly up to 20 GeV, the Th_B is negative. It vanishes near $\sqrt{s_{NN}} \approx 20$ GeV and then increases towards the positive direction. For a baryon-rich QGP medium, having a negative Th_B , which may indicate that the heat is continuously released from the hotter region to the colder region when current flows from a region of lower temperature to a higher temperature. On the contrary, with the same condition of current flow and temperature gradient, the heat is continuously absorbed at the hotter region for a QGP medium created at higher values of $\sqrt{s_{NN}}$ as it has positive values of Th_B .

The left panel of Fig. 3.5 presents NB as a function of $\sqrt{s_{NN}}$ for Au+Au collisions at an impact parameter of $b = 7.2$ fm. The results are shown for different magnetic field decay times, $\tau_B = 3.0$ fm and $\tau_B = 7.0$ fm, along with a comparison with the zero magnetic field case ($eB = 0.0$). The coefficient NB , which quantifies the generation of an electric field perpendicular to both the magnetic field and temperature gradient, is found to decrease with collision energy and saturate at higher $\sqrt{s_{NN}}$. We observe that for the lower values of $\sqrt{s_{NN}}$, nearly up to 20 GeV, there is a sharp fall in values of NB . As expected, no Nernst effect appears in the absence of a magnetic field. This behavior of NB also reflects the parameterized

behavior of μ_B as a function of $\sqrt{s_{NN}}$. For lower values of $\sqrt{s_{NN}}$, the μ_B is high, but it decreases with increasing $\sqrt{s_{NN}}$. A higher number of baryons over the anti-baryons leads to significant Hall-like effects in the presence of a magnetic field. On the other hand, for an equal number of baryons and anti-baryons, the Hall-like effects get canceled due to equal and opposite Lorentz force. The sensitivity of the NB to both collision energy and the lifetime of the early-time magnetic field provides valuable insights into the transport properties of the magnetized QGP. The right panel of Fig. 3.5 presents Th_N as a function of the $\sqrt{s_{NN}}$ for the same parameters as the left panel. Here, we observe that the values of Th_N are decreasing in magnitude as $\sqrt{s_{NN}}$ increases. Below the values of $\sqrt{s_{NN}} = 20$ GeV, this fall is rapid, but for the higher values of $\sqrt{s_{NN}}$, this fall is slower. Nearly, at $\sqrt{s_{NN}} = 200$ GeV the coefficient Th_N approaches zero.

3.1.4 Thermoelectricity in confined phase

In this section, we discuss the results for higher-order thermoelectric coefficients of a hot and dense hadron gas in the presence of an external magnetic field. We also present a short description of the Thomson coefficient to show a comparative analysis between the magneto-Thomson coefficient and the Thomson coefficient. While the latter was estimated from the analysis of the Seebeck coefficient in our previous study [20], the results shown here are obtained from more complete and rigorous calculations. This allows for a qualitative understanding of how the presence of a magnetic field modifies the higher-order transport properties, particularly in relation to the temperature dependence and baryon chemical potential. We have considered four different versions of hadron resonance gas models, including a discrete particle spectrum having all the hadrons and their resonances up to the mass cutoff $\Lambda = 2.6$ GeV [20]. To estimate the relaxation time of the hadrons, we follow the hard-scattering approximation. A uniform radius, $r_m = 0.2$ fm for mesons and $r_{b,\bar{b}} = 0.62$ fm for baryons and antibaryons throughout the

3.1 Thermoelectric coefficients

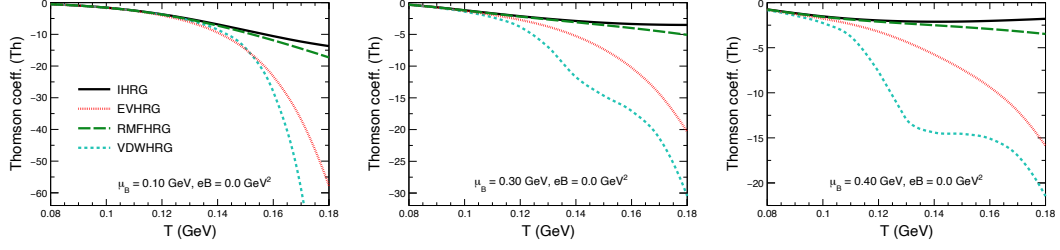


Figure 3.6: Thomson coefficient (Th) obtained in different hadronic models as a function of temperature at $\mu_B = 0.10$ GeV (left panel), 0.30 GeV (middle panel), and 0.40 GeV (right panel) [29].

calculations [30]. For the VDWHRG model, we take van der Waals parameters as $a = 0.926 \text{ GeV fm}^3$ and $b = (16/3)\pi r^3$ [30, 31].

Fig. 3.6 represents the Thomson coefficient Th as a function of temperature T for three different baryon chemical potentials, i.e., at $\mu_B = 0.10, 0.30$, and 0.40 GeV. The black solid line represents the Thomson coefficient obtained in the IHRG model, whereas the red dotted line represents the EVHRG model. The green and cyan dashed lines represent the same for the RMFHRG and VDWHRG models, respectively. It is observed that for the chosen range of temperature ($0.08 - 0.18$ GeV), Th is negative for a hot and dense hadron gas. The Th is universally decreasing as the temperature increases. However, the degree of decrease is different for different models. To define the net heat flow for a relativistic system, we need a conserved charge. For the study of the HRG system, the heat current corresponds to the net baryon current. It is observed that the baryons have a dominant contribution over the mesons to the Thomson coefficient. The mesons contribute to thermoelectricity only through the enthalpy of the system. In hadron gas, the entropy production for lighter mesons like pions, kaons, etc., is higher. Therefore, the enthalpy per baryon (h) exceeds the single-particle energy (ω_i), which results in the negative values of Th [20]. The positive (negative) values of the Thomson coefficient lead to the net absorption (release) of heat in

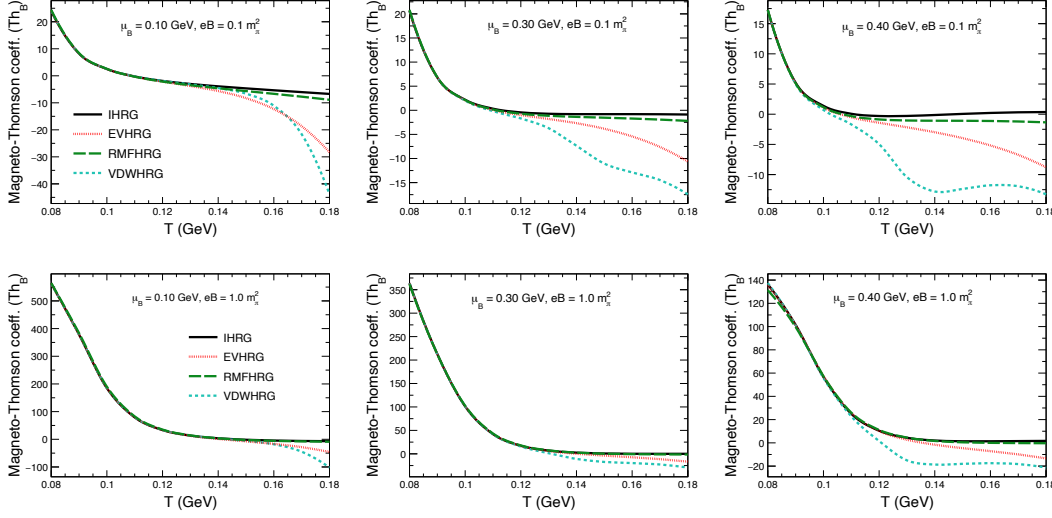


Figure 3.7: Magneto-Thomson coefficient (Th_B) as a function of temperature at magnetic field $eB = 0.1 m_\pi^2$ (upper panel) and $eB = 1.0 m_\pi^2$ (lower panel) for baryon chemical potential at $\mu_B = 0.10$ GeV (left), 0.30 GeV (middle), and 0.40 GeV (right) [29].

the system. Therefore, a non-zero Thomson coefficient can significantly affect the cooling dynamics of the medium. The Thomson coefficient can be calculated as the product of temperature and the rate of change of the Seebeck coefficient with respect to temperature [20]; therefore, the higher the rate of change of the Seebeck coefficient with temperature, the higher the value of Th in magnitude as a function of T . The coefficient Th shows overlap at lower values of temperature, nearly up to ≈ 0.14 GeV for the case of $\mu_B = 0.10$ GeV. As the value of μ_B increases, the models start deviating from each other even at low temperatures. Additionally, a noticeable flatness appears in the VDWHRG model in the temperature range around $T \approx 0.13$ - 0.16 GeV for $\mu_B = 0.40$ GeV, indicating a plateau-like behavior of the Thomson coefficient in this region. On the other hand, the constant decreasing rate of the Seebeck coefficient around the above-mentioned temperature range [20] leads to the constant slope. Hence, the Th shows a non-monotonic behavior at high values of μ_B . The qualitative

3.1 Thermoelectric coefficients

differences observed among the various HRG model results can be attributed to the manner in which different interaction mechanisms modify the fundamental thermodynamic inputs that enter transport phenomena. In this context, it is instructive to consider the behavior of the electrical conductivity scaled by temperature, σ_{el}/T , since electrical conductivity provides the leading contribution to thermoelectric transport coefficients and therefore serves as a useful baseline for understanding model-dependent trends. At higher temperatures, the number density of the medium increases rapidly, and the repulsive component present in interacting HRG models becomes increasingly significant, as discussed in the literature [30]. This repulsive interaction effectively reduces the number of particles per unit volume, which in turn enhances the relaxation time (2.86) relative to the IHRG scenario. As a consequence, a finite enhancement in the magnitude of σ_{el}/T and similarly in other transport coefficients is observed once hadronic interactions are incorporated. With increasing baryon chemical potential μ_B , the baryon number density further increases, causing the deviations from the IHRG behavior to shift toward lower temperatures. Since the Thomson coefficients are constructed from the same underlying thermodynamic quantities, such as number density and enthalpy, as well as the same relaxation times, their model-dependent behavior naturally follows the same physical mechanism. This explains why differences among HRG models become more pronounced in regimes of higher temperature and baryon density.

The upper panel of Fig. 3.7 represents Th_B as a function of T for three different values of μ_B at 0.10, 0.30, and 0.40 GeV at magnetic field $0.1 m_\pi^2$, whereas the lower panel represents those for the magnetic field $1.0 m_\pi^2$. For the case of $eB = 0.1 m_\pi^2$, we observe that Th_B decreases with the temperature up to 0.1 GeV, becomes zero at a particular T , and further, it starts to increase in the negative direction for higher values of temperature. Similar to what was observed in the case of a vanishing magnetic field, all the models agree up to a certain temperature and then start deviating from each other at higher temperatures.

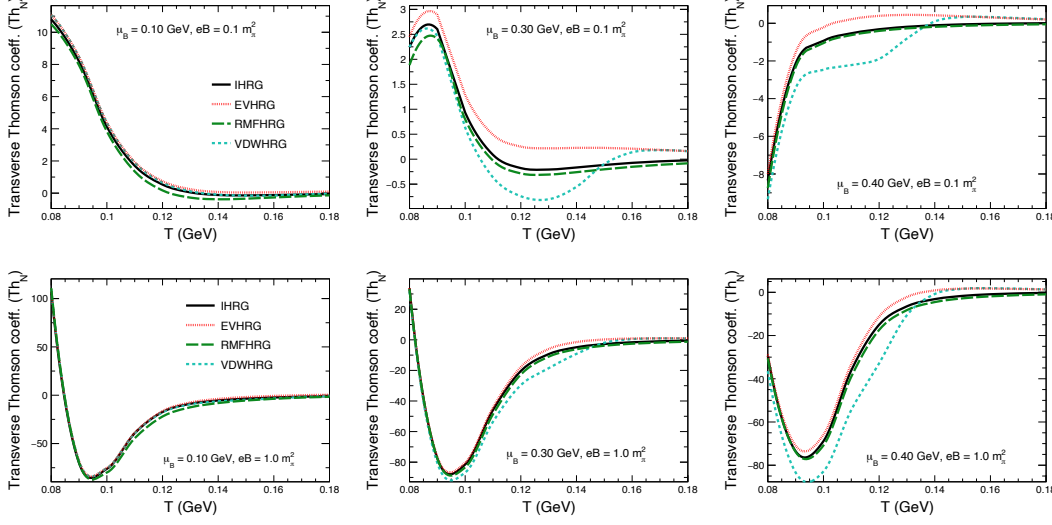


Figure 3.8: Transverse Thomson coefficient (Th_N) as a function of temperature at magnetic field $eB = 0.1 m_\pi^2$ (upper panel) and $eB = 1.0 m_\pi^2$ (lower panel) for baryon chemical potential at $\mu_B = 0.10$ GeV (left), 0.30 GeV (middle), and 0.40 GeV (right) [29].

Furthermore, we also observe that the values of Th_B increase in the negative direction with the increasing values of μ_B for higher T but decrease in the positive direction for the lower values of T . Unlike the case without a magnetic field, the magneto-Thomson coefficient Th_B exhibits an additional dependence on the Hall-like component of electrical conductivity and other integral terms that arise due to the presence of an external magnetic field. The Hall-like conductivity term, σ_H/T , increases with baryon chemical potential μ_B because of the growing net baryonic contributions. It is important to note that mesons do not contribute directly to the integrals $\mathcal{I}_{31}/T^2$ and $\mathcal{I}_{42}/T^2$, though their influence appears indirectly through the enthalpy per baryon h . The effect of the magnetic field is significant on $\mathcal{I}_{31}/T^2$ at low temperatures, but almost diminishes at higher temperatures due to a reduced relaxation time. On the other hand, $\mathcal{I}_{42}/T^2$ first increases with temperature and then starts to decrease. The higher the magnetic field, the lower the $\mathcal{I}_{42}/T^2$. Here, the magnitude of σ_{el}/T is significantly larger than that of σ_H/T ,

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the integrals $\mathcal{I}_{31}/T^2$ and $\mathcal{I}_{42}/T^2$ are of comparable scale [32]. Consequently, the magneto-Seebeck coefficient S_B can be approximately represented by the ratio $(\mathcal{I}_{31}/T^2)/(\sigma_{el}/T)$, and its temperature dependence is reflected in the behavior of the magneto-Thomson coefficient Th_B , as shown in Fig. 3.7. The T and μ_B dependence of σ_{el}/T in a magnetic field causes S_B to increase at low T and decrease at high T . This results in a positive Th_B at lower temperatures, which decreases with T and becomes negative at higher temperatures. At low T , a large value of $\Omega_{c_i}\tau_{R_i}$ suppresses the mesonic contribution to σ_{el}/T , and this reduction is compensated by the enhanced baryonic contribution as μ_B increases. However, at higher temperatures, the baryonic increase is insufficient to compensate for the mesonic suppression, leading to an overall decrease in σ_{el}/T with μ_B . For stronger magnetic fields, as shown in the lower panel of Fig. 3.7, Th_B shows large and positive values at low T , gradually approaching zero at high T . It is observed that for higher values of a magnetic field, results from all the models agree up to $T \approx 0.12$ GeV at $\mu_B = 0.10$ GeV. With increasing values of μ_B , the disagreement between the models starts from even lower values of T . This observation is in line with what was observed for the magneto-Seebeck coefficient in Ref. [20].

In the presence of an external magnetic field, we also observe a Hall-like component for the higher-order thermoelectric coefficients. The presence of a non-zero temperature gradient in a conducting medium directed perpendicular to the direction of a magnetic field gives rise to the Nernst coefficient. It measures the electric current perpendicular to both the magnetic field and the temperature gradient. In the absence of a magnetic field, this coefficient vanishes. The temperature dependence of the Nernst coefficient in the medium further gives rise to the transverse Thomson coefficient Th_N as shown in Eq. (3.44). Fig. 3.8 represents Th_N as a function of T for three different values of μ_B at 0.10, 0.30, and 0.40 GeV. The upper and lower panels present the results for the case of the magnetic field with values 0.1 and 1.0 m_π^2 , respectively. In the upper panel, we observe that Th_N is positive for lower values of T up to 0.10 GeV for $\mu_B = 0.10$ and 0.30 GeV.

The positive values of Th_N signify that in Eq. (3.44), the quantity $2NB$ is dominating over the dynamical term, $T \frac{dNB}{dT}$ at lower temperatures. The impact of this dynamic part slowly starts increasing with T , and hence the negative behavior of Th_N is also observed at higher T . For the higher values of μ_B , it is observed that the dynamical part of Eq. (3.44) is dominating almost for the whole temperature range considered here. As it is observed in Ref. [20], the coefficient NB vanishes near high temperatures; here, Th_N also starts disappearing at higher temperatures. For $eB = 1.0 \text{ } m_\pi^2$, results from different models are observed to be almost similar, i.e., no significant model dependence is observed. Even at higher μ_B , only a slight deviation in the models can be seen. As one goes to a higher value of the magnetic field ($eB = 1.0 \text{ } m_\pi^2$), Th_N is observed to be higher. Due to the higher order of magnitude of σ_{el}/T as compare to σ_H/T , NB can be approximated by ratio $(\mathcal{I}_{42}/T^2)/(\sigma_{el}/T)$. The sharp decrease of $\mathcal{I}_{42}/T^2$ as compare to σ_{el}/T give rise to decreasing trend of NB with μ_B . Though NB decreases with increasing μ_B but the dynamical rate of decrease of NB increases. Hence, the coefficient Th_N mostly has negative values throughout the whole temperature range at high values of μ_B . Here, a clear effect of magnetic field values can be observed in the upper and bottom panels of Fig. 3.8. For the case of a higher magnetic field, the effect of the Lorentz force is also high. Thus, the higher Lorentz force increases the thermoelectric effects in the transverse direction, which results in higher values of the transverse Thomson coefficient. Once this magnetic field vanishes, the coefficient Th_N drops to zero only.

3.2 Thermoelectric figure of merit of QCD matter

From Eq. (3.48), we can express the heat current $\vec{I}$ in terms of electric current $\vec{j}$ in the following way,

$$\begin{aligned}\vec{I} &= TS\vec{j} - (\kappa_0 - S^2\sigma_{el}T)\vec{\nabla}T, \\ &= TS\vec{j} - \kappa'_0\vec{\nabla}T, \\ &= TS\vec{j} - \left(1 - \frac{S^2\sigma_{el}T}{\kappa_0}\right)\kappa_0\vec{\nabla}T.\end{aligned}\tag{3.48}$$

Here, $\kappa'_0 = \kappa_0 - S^2\sigma_{el}T$ represents the coefficient of effective thermal conductivity in the presence of the electric field. The three transport coefficients, namely S , σ_{el} , and κ_0 , are closely related to each other because of the common factors such as mobility and concentration of medium constituents. The thermoelectric performance of any thermoelectric material can be measured by using the dimensionless quantity named as the figure of merit (ZT), given as [34]

$$ZT = \frac{S^2\sigma_{el}T}{\kappa_0}.\tag{3.49}$$

Here, the term $S^2\sigma_{el}$ in the numerator of the above equation is called the power factor. It reflects the ability of the thermoelectric medium to convert heat to electrical energy, excluding the effects of heat conduction. The first term in Eq. (3.48) represents the heat carried by charge carriers due to the Seebeck effect. When an electric current $\vec{j}$ flows in a material with a non-zero Seebeck coefficient, the carriers transport energy, thus producing a heat current proportional to $TS\vec{j}$. Whereas the second term describes heat flow due to the temperature gradient, similar to Fourier's law, but corrected by the factor $(1 - ZT)$. This correction shows that the presence of charge carriers affects the net heat flow. Hence, the systems having $ZT = 0$ behave like a regular thermal conductor, and heat flows in response to the temperature gradient with no thermoelectric effect. Whereas,

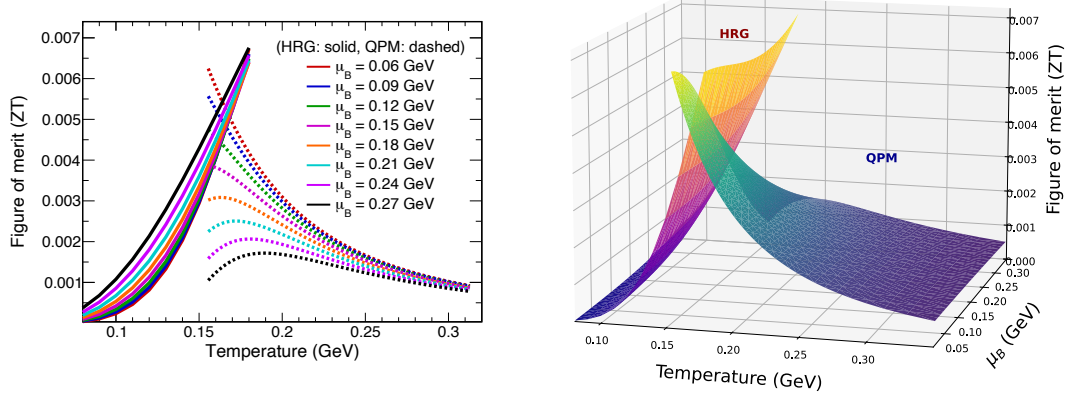


Figure 3.9: Left: ZT as a function of temperature for various values of μ_B in the HRG (solid lines) and QPM (dashed lines) phases. Right: 3D surface plot showing ZT across a thermodynamic phase space spanned by T and μ_B in both HRG and QPM phases [33].

for the systems having $ZT \neq 0$, a portion of the temperature-driven heat flow is converted into electric energy or compensated by the thermoelectric current. For the very first time, we have calculated the thermoelectric figure of merit ZT for QCD matter created in heavy-ion collisions. As mentioned in Eq.(3.49), ZT serves as a key dimensionless parameter to quantify the efficiency of a medium in converting a temperature gradient into electrical energy. The Seebeck coefficient S characterizes the voltage induced per unit temperature gradient across the medium and reflects the strength of the thermoelectric response. The electrical conductivity σ_{el} measures the ability of the medium to conduct electric current, while the thermal conductivity κ_0 , comprising both baryonic and mesonic contributions, governs the transport of heat. A high value of ZT indicates an efficient thermoelectric material, which ideally requires a large Seebeck coefficient, high electrical conductivity, and low thermal conductivity. In this study, we explore

3.2 Thermoelectric figure of merit of QCD matter

the temperature dependence of ZT for various values of baryon chemical potential in both the HRG medium and QGP medium. Below, without repeating all the details, we discuss the behavior of each transport coefficient first and then discuss their combined impact on ZT in detail.

The Seebeck coefficient S shows a strong temperature and μ_B dependence. In the HRG phase, S becomes increasingly negative with temperature as mentioned in our previous study [20]. This reflects enhanced baryonic transport and increasing charge asymmetry. In relativistic systems, defining heat flow requires a conserved charge. For the HRG medium, this role is played by the net baryon number, making the net heat current directly related to the baryon current. Here, baryons are found to contribute more significantly to the Seebeck coefficient than mesons. The contribution of mesons to thermoelectric properties arises solely through the enthalpy of the medium. Notably, light mesons such as pions and kaons contribute largely to entropy production, which in turn elevates the enthalpy per baryon h over the single-particle energy ω_i in Eq. (3.31). This imbalance is responsible for the emergence of negative Seebeck coefficients in the HRG medium. In contrast, the QGP phase exhibits charged quarks and neutral gluons. The quarks have the leading contribution to thermoelectric transport of the medium, whereas gluons contribute through the enthalpy. For this phase, the S is observed to be almost saturated in the higher temperature region [21]. This distinction in S behavior between hadronic and deconfined phases critically influences the temperature profile of ZT through the S^2 term. While the sign of the Seebeck coefficient indicates the relative alignment between the electric field and the temperature gradient, this directional information is lost in the ZT , where S appears squared and thus contributes only through its magnitude. The numerator of Eq. 3.31 also shows the dependence of an electrical conductivity σ_{el} of the medium. As it is proportional to electric charge squared, it is positive for both phases. In the hadronic phase, σ_{el} decreases with temperature and increasing values of μ_B mainly due to the decrease of the relaxation time of the mesons. However, in

the QGP phase, the σ_{el} increases with both T and μ_B because of an increase in the relaxation time of quarks. This turnaround in both phases, HRG and QPM, contributes to shaping the peak structure in ZT . In the denominator of Eq. (3.31), the thermal conductivity κ_0 decreases with T and μ_B in the HRG phase [16], but on the contrary, it increases in the QGP phase as T increases but decreases with μ_B [17]. As mentioned in Eq. 3.35, the κ_0 has a contribution of the square of $(\omega_i - b_i h)$, hence it is positively contributing to ZT . Therefore, a dip near the transition region contributes to ZT inversely. The combined effect of S , σ_{el} , and κ_0 results in a clear peak in ZT near the QCD crossover temperature region. This peak signifies the region where thermoelectric conversion is most efficient. In a nutshell, the thermoelectric figure of merit is a powerful combined measure of transport efficiency, capturing the interplay between electric and thermal responses in a single, dimensionless quantity. Unlike the individual transport coefficients, ZT integrates their effects to reflect the overall ability of a system to convert thermal gradients into electrical energy. Its strength lies in its sensitivity to collective medium properties, making it especially valuable near the phase transition, where rapid changes in transport dynamics occur. A peak in this combined quantity can signal critical behavior more effectively than any single transport coefficient on its own.

Figure 3.9 displays the behavior of the thermoelectric figure of merit ZT as a function of temperature T and baryon chemical potential μ_B , for the hadronic phase using the HRG model, and the QGP phase using the quasiparticle model. The left panel shows 2D profiles of ZT versus T for fixed values of μ_B , while the right panel presents the full 3D surface of ZT across a thermodynamic phase space spanned by both T and μ_B . In the left panel, solid lines correspond to the HRG model, and dashed lines represent the QPM. The ZT increases monotonically with temperature within the HRG model up to the transition region, after which it sharply drops for the QGP medium described by the quasi-particle model. For low μ_B values (e.g., $\mu_B = 0.06$ GeV), the peak in ZT is smooth around the transition

3.3 Coupled-transport phenomenon in QCD matter

temperature region. As μ_B increases, the discontinuity starts appearing in the peak of both phases. Notably, at high temperatures, ZT is nearly independent of μ_B in the QPM, which may be due to the existence of near-symmetry of quark and antiquark contributions and the negligible role of net baryon number in a weakly interacting quark-gluon plasma. The right panel of Fig. 3.9 further highlights the 3D surface plot of ZT with respect to T and μ_B . In the HRG model, ZT grows significantly with both T and μ_B up to the transition region, peaking sharply before dropping in the QGP region. These results collectively suggest that the thermoelectric figure of merit could serve as a sensitive probe of the QCD transition. The presence of a peak in ZT near the transition may provide a novel, transport-based phenomenological signature of the deconfinement transition and the evolution of charge-carrying degrees of freedom in hot and dense QCD matter. It is important to note here that the point of discontinuity in the temperature dependence of ZT occurs around $T \simeq 160$ MeV for lower values of μ_B . This corresponds to the lattice QCD predicted value of a deconfinement phase transition [35].

3.3 Coupled-transport phenomenon in QCD matter

In a medium with a non-zero net baryon number, the transport of baryons and energy (heat) can be described by linear response theory. The diffusion current $\vec{j}$ and the heat current $\vec{I}$ are expressed using Fick's law generalised for coupled-transport phenomena as

$$\begin{pmatrix} \vec{j} \\ \vec{I} \end{pmatrix} = - \begin{pmatrix} D & S_T \\ D_F & \kappa \end{pmatrix} \begin{pmatrix} \vec{\nabla} \mu_B \\ \vec{\nabla} T \end{pmatrix} \quad (3.50)$$

where D is the baryon diffusion coefficient, S_T is the Soret coefficient, κ is the coefficient of thermal conductivity, and D_F is the Dufour coefficient. These ex-

pressions capture the interplay between chemical potential and thermal gradients in driving both baryons and heat transport in the medium.

To evaluate the coupled-transport coefficients in QCD matter, we follow the kinetic theory framework. We begin by decomposing the single-particle distribution function for species i into its equilibrium part and a small deviation

$$f_i = f_i^0 + \delta f_i,$$

where f_i^0 represents the equilibrium distribution and δf_i accounts for slight deviation from equilibrium. At equilibrium, the distribution for the i th species is expressed as

$$f_i^0 = \frac{1}{\exp \left[\frac{\omega_i - b_i \mu_B}{T} \right] \pm 1}, \quad (3.51)$$

with ω_i being the energy of the particle, μ_B the baryon chemical potential, and b_i the baryon number (*e.g.*, $b_i = 1$ for baryons, $b_i = -1$ for antibaryons, and $b_i = 0$ for mesons). The plus and minus signs correspond to fermions and bosons, respectively.

In the absence of any external force, in the local rest frame, the Boltzmann transport equation under the relaxation time approximation for species i can be written as [16, 19]

$$\vec{v}_i \cdot \vec{\nabla} f_i = -\frac{\delta f_i(\vec{x}_i, \vec{k}_i)}{\tau_R^i}. \quad (3.52)$$

Here, τ_R^i denotes the relaxation time for species i , and $\vec{v}_i = \vec{k}_i/\omega_i$ is the particle velocity. Additionally, the spatial gradient of f_i^0 can be recast using

$$\vec{\nabla} f_i^0 = -\frac{\partial f_i^0}{\partial \omega_i} \left\{ \frac{(\omega_i - b_i \mu_B)}{T} \vec{\nabla} T + b_i \vec{\nabla} \mu_B \right\}, \quad (3.53)$$

At leading order, an ansatz for the nonequilibrium part can be chosen as [16, 33]

$$\delta f_i = (\vec{k}_i \cdot \vec{\Omega}) \frac{\partial f_i^0}{\partial \omega_i}, \quad (3.54)$$

where the vector $\vec{\Omega}$ encapsulates the influence of the driving forces. It is natural to express $\vec{\Omega}$ as a linear combination of the perturbed fields

$$\vec{\Omega} = \alpha_1 \vec{\nabla} T + \alpha_2 \vec{\nabla} \mu_B, \quad (3.55)$$

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with coefficients α_1 and α_2 that quantify the strength of the electric field and temperature gradient in pushing the system out of equilibrium.

Substituting Eq. (3.55) into (3.54), one obtains the following expression for the deviation δf_i

$$\delta f_i = \omega_i \left[\alpha_1 (\vec{v}_i \cdot \vec{\nabla} T) + \alpha_2 (\vec{v}_i \cdot \vec{\nabla} \mu_B) \right] \frac{\partial f_i^0}{\partial \omega_i}. \quad (3.56)$$

Hence, the relativistic Boltzmann equation in Eq. (3.52) can be expressed as

$$\begin{aligned} \frac{(\omega_i - b_i \mu_B)}{T} (\vec{v}_i \cdot \vec{\nabla} T) + b_i (\vec{v}_i \cdot \vec{\nabla} \mu_B) &= \frac{\omega_i}{\tau_R^i} [\alpha_1 (\vec{v}_i \cdot \vec{\nabla} T) \\ &\quad + \alpha_2 (\vec{v}_i \cdot \vec{\nabla} \mu_B)] \end{aligned} \quad (3.57)$$

After comparing the tensors of the same notations, we get

$$\alpha_1 = \frac{\tau_R^i (\omega_i - b_i \mu_B)}{T \omega_i}, \quad \alpha_2 = \frac{b_i \tau_R^i}{\omega_i} \quad (3.58)$$

Hence,

$$\delta f_i = \frac{\vec{k}_i}{\omega_i} \tau_R^i \left[\frac{(\omega_i - b_i \mu_B)}{T} \vec{\nabla} T + b_i \vec{\nabla} \mu_B \right] \frac{\partial f_i^0}{\partial \omega_i}. \quad (3.59)$$

Following the kinetic theory, one can write the particle diffusion current and heat current as

$$\begin{aligned} \vec{j} &= \sum_i g_i \int \frac{d^3 |\vec{k}_i| \vec{k}_i}{(2\pi)^3 \omega_i} \delta f_i, \\ \vec{I} &= \sum_i g_i \int \frac{d^3 |\vec{k}_i| \vec{k}_i}{(2\pi)^3 \omega_i} (\omega_i - b_i \mu_B) \delta f_i. \end{aligned} \quad (3.60)$$

Here, g_i is the degeneracy for the i th species of the medium. After substituting the δf_i from Eq. (3.59) in the above equation and comparing with the equations defined in the matrix. (3.50), we get

- Diffusion coefficient (baryon diffusion due to baryon chemical potential gradient):

$$D = \sum_i g_i \frac{1}{3T} \int \frac{d^3 |\vec{k}_i| \vec{k}_i^2}{(2\pi)^3 \omega_i^2} b_i \tau_R^i f_i^0 (1 \pm f_i^0). \quad (3.61)$$

- Soret coefficient (baryon diffusion due to temperature gradient):

$$S_T = \sum_i g_i \frac{1}{3T^2} \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i \mu_B) \tau_R^i f_i^0 (1 \pm f_i^0). \quad (3.62)$$

- Dufour coefficient (heat flow due to baryon chemical potential gradient):

$$D_F = \sum_i g_i \frac{1}{3T} \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i \mu_B) b_i \tau_R^i f_i^0 (1 \pm f_i^0). \quad (3.63)$$

- Thermal conductivity (heat flow due to temperature gradient):

$$\kappa = \sum_i g_i \frac{1}{3T^2} \int \frac{d^3|\vec{k}_i|}{(2\pi)^3} \frac{\vec{k}_i^2}{\omega_i^2} (\omega_i - b_i \mu_B)^2 \tau_R^i f_i^0 (1 \pm f_i^0). \quad (3.64)$$

These transport coefficients collectively describe the coupled flow of heat and baryon charge in the QCD medium. These coefficients incorporate the statistical weights, baryon numbers, effective energies, relaxation times, and equilibrium distributions, providing a comprehensive description of transport phenomena in the strongly interacting medium.

In this study, we have calculated the coupled-transport coefficients for the QCD matter using the QPM model for the deconfined phase and the HRG model for the confined phase. Figure 3.10 shows the temperature dependence of the scaled coupled-transport coefficients in the hadron resonance gas (solid lines) and quark-gluon plasma (dashed lines) phases for different values of the baryon chemical potential, $\mu_B = 0.05, 0.20, 0.40$, and 0.60 GeV. The four panels correspond to: (top left) the scaled diffusion coefficient (D/T), (top right) the scaled Soret coefficient (S_T/T), (bottom left) the scaled Dufour coefficient (D_F/T^2), and (bottom right) the scaled thermal conductivity (κ/T^2). The mathematical forms of these coefficients are mentioned in Eqs. (3.61)-(3.64). The integrands of these integrals contain several key terms with distinct physical meanings. The particle momentum squared, $\vec{k}_i^2$, scaled by the squared particle energy, ω_i^2 , represents the kinetic contribution of the particles to the transport processes. The relaxation time, τ_R^i , characterizes the interaction strength or mean free time between

3.3 Coupled-transport phenomenon in QCD matter

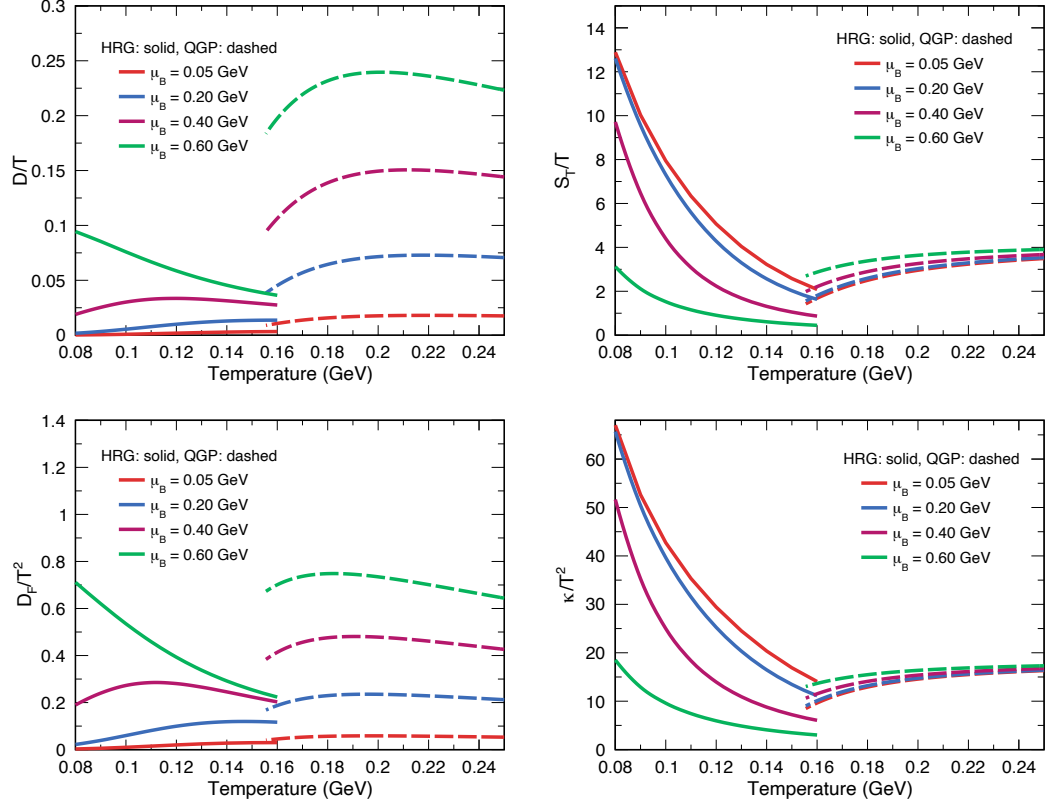


Figure 3.10: Temperature dependence of scaled coefficients of coupled-transport in the hadron resonance gas (HRG) and quark-gluon plasma (QGP) phases at different values of baryon chemical potential μ_B . Shown are: (top left) diffusion coefficient (D/T), (top right) Soret coefficient (S_T/T), (bottom left) Dufour coefficient (D_F/T^2), and (bottom right) thermal conductivity (κ/T^2). HRG results are plotted with solid lines and QGP results with dashed lines. Each colour corresponds to a fixed μ_B value [36].

successive collisions for the i th species, dictating how effectively each particle species participates in the transport phenomena. The equilibrium occupation factor, $f_i^0(1 \pm f_i^0)$, accounts for the quantum statistical effects, where the '+' sign corresponds to bosons and the '-' sign corresponds to fermions, thereby modifying the overall transport behavior according to the underlying quantum statistics of the medium. The Eqs. (2.86) and (2.138) represent the relaxation time of the particle species for the HRG and the QGP phases, respectively. For the QGP phase, the τ_R^i dependence only on the temperature, and it decreases as the temperature increases. Whereas for the HRG phase, the relaxation time τ_R exhibits a strong dependence on temperature and baryon chemical potential due to its inverse relation with the product of particle density, relative velocity, and interaction cross section. At low temperatures, the densities of most hadrons are exponentially suppressed, resulting in very large relaxation times, indicating infrequent collisions. As the temperature increases toward the crossover region, the hadronic densities grow rapidly, particularly for light mesons like pions, leading to a significant decrease in τ_R . The baryon chemical potential further decreases τ_R in a baryon-rich medium due to the higher number density. Consequently, in the HRG phase, τ_R evolves from a dilute, weakly interacting regime at low T to a more collisional, rapidly relaxing medium near the QCD crossover.

The diffusion coefficient in Eq. (3.61) quantifies the baryon flux generated by a gradient in the baryon chemical potential $\vec{\nabla}\mu_B$, indicating how efficiently baryons redistribute in response to $\vec{\nabla}\mu_B$. It is directly dependent on the factor of $\tau_R^i f_i^0(1 \pm f_i^0)$. Hence, this product describes the overall behaviour of the diffusion coefficient. It is to be noted that the Eq. (3.61) also carries an explicit factor of baryon number, b_i , in the product to factor $\tau_R^i f_i^0(1 \pm f_i^0)$, hence for bosons, i.e., gluons in QGP phase and mesons in HRG phase, the diffusion coefficient vanishes. In the limit of vanishing baryon chemical potential, the net baryon density becomes zero because of the equal population of baryons and antibaryons carrying opposite baryon numbers, leading to the complete suppression of the

3.3 Coupled-transport phenomenon in QCD matter

diffusion coefficient. Consequently, with increasing μ_B , the baryon densities increase through the Boltzmann factor, $\exp(\mu_B/T)$, which leads to a significant rise in D/T with μ_B , especially in the moderate-temperature region. A larger diffusion coefficient indicates that baryons can diffuse more easily through the medium under a baryon chemical potential gradient. Similarly, the Dufour coefficient in Eq. (3.63) includes the same b_i dependence as the diffusion coefficient but is further weighted by the factor of $(\omega_i - b_i\mu_B)$, which significantly enhances its sensitivity to the baryon number density in the medium. This dependence makes the Dufour effect a predominantly baryon-driven transport phenomenon in the hadronic medium. At very low baryon chemical potential, the net baryon density is strongly suppressed, which results in negligible values of D_F/T^2 . However, as μ_B increases, the baryon density grows exponentially, and the Dufour coefficient rises sharply, showing the strongest enhancement among all the transport coefficients considered in this study. This behavior highlights the fact that the Dufour effect, which represents the heat flow driven by chemical potential gradients, becomes increasingly significant in baryon-rich environments, such as the lower-energy heavy-ion collisions. Moreover, the additional energy weighting $(\omega_i - b_i\mu_B)$ implies that heavier baryonic resonances contribute more effectively to this coefficient at moderate and high temperatures, further amplifying the growth of D_F/T^2 . Physically, this indicates that the chemical potential gradients can induce substantial heat transport when the system is baryon-rich, reflecting the strong coupling between baryon and heat currents in such media. Furthermore, in the hadronic medium, a strong dependence of temperature can be observed for both the Diffusion and Dufour coefficients. For $\mu_B = 0.20$ GeV, we observe an increasing trend which saturates around $T \simeq 0.13$ GeV. Meanwhile, for a slightly higher chemical potential of 0.40 GeV, we get competing effects from $f_i^0(1 \pm f_i^0)$ and τ_R^i , thus resulting in a slight dip at $T \simeq 0.15$ GeV. Moreover, the τ_R^i dominates for $\mu_B = 0.60$ GeV, and as a result, we observe a decreasing trend. However, in the QGP phase, the temperature dependence of D/T and D_F/T^2 is weak due

to the fact that $\tau_R^i \sim 1/T$, which can be seen from Eq. (2.138). Additionally, one can notice that the enhancing effect of baryon chemical potential on D/T and D_F/T^2 is still prominent in the QGP medium.

On the other hand, the Soret coefficient, expressed in Eq. (3.62), does not contain an explicit b_i weighting factor. Although baryonic species contribute through the $(\omega_i - b_i\mu_B)$ term, the overall magnitude and temperature dependence of S_T/T are largely governed by the light mesonic species ($b_i = 0$), such as pions, which remain abundant even at very small μ_B . With an increase in μ_B , we observe a decrease in S_T/T at low temperature. This is due to the presence of the factor of $(\omega_i - b_i\mu_B) \tau_R^i$, which decreases with increasing μ_B . As the system approaches the crossover region and transitions into the QGP phase, the Soret coefficient shows an increase with temperature due to the liberation of quark degrees of freedom, which carry baryon number but have much smaller effective masses. Furthermore, compared to the Dufour coefficient, the sensitivity of the Soret coefficient to μ_B remains relatively weak, indicating that temperature gradients, rather than baryon density, primarily drive its behavior in both phases. In contrast to the Diffusion and Dufour coefficients of the hadronic medium, we observe a large Soret coefficient at a lower baryon chemical potential. This hints towards the fact that the diffusion of baryons, due to the Soret effect, will be most affected by the temperature gradient in the nearly baryon-free region. A similar trend is observed for the thermal conductivity, κ/T^2 , as described by Eq. (3.64), due to the presence of the quadratic energy-weighting factor $(\omega_i - b_i\mu_B)^2$. We observe a similar temperature and baryon chemical potential dependence of κ/T^2 across the entire temperature range.

3.4 Summary

This chapter provides a detailed and systematic study of thermoelectric transport phenomena in strongly interacting QCD matter. It establishes a unified theoret-

3.4 Summary

ical framework to describe both leading-order and higher-order thermoelectric effects in the presence of temperature gradients, chemical potential, and external electromagnetic fields. The analysis spans both the deconfined quark–gluon plasma phase and the confined hadronic phase, highlighting the universal nature of thermoelectric transport across different forms of matter.

The chapter begins with a conceptual introduction to thermoelectricity, emphasizing its origin in non-equilibrium physics where gradients in temperature and chemical potential drive the transport of charge and heat. The fundamental idea that charged carriers simultaneously transport energy and matter is discussed, connecting thermoelectric behavior across condensed matter systems, electromagnetic plasmas, and QCD matter. This highlights thermoelectricity as a unifying phenomenon governed by the interplay of conserved charges and thermodynamic forces.

In Sec. 3.1, the thermoelectric coefficients are introduced, beginning with the Seebeck coefficient, which quantifies the generation of an electric field due to a temperature gradient. The role of magnetic fields is discussed in detail, leading to anisotropic transport phenomena such as the magneto-Seebeck and Nernst effects. The discussion is further extended to higher-order thermoelectric coefficients, including the Thomson effect, which arises due to the temperature dependence of the Seebeck coefficient and reflects the continuous absorption or release of heat in a conducting medium.

Section 3.1.1 presents the formal derivation of leading-order thermoelectric coefficients in QCD matter using the relativistic Boltzmann transport equation within the relaxation time approximation. The single-particle distribution function is expanded around equilibrium, and the non-equilibrium correction is constructed through a general ansatz incorporating electric fields, magnetic fields, and temperature gradients. The presence of an external magnetic field introduces anisotropy in transport, leading to distinct longitudinal (Ohmic-like) and transverse (Hall-like) contributions to both electrical and thermal currents. From

this formalism, compact expressions for the magneto-Seebeck coefficient and the normalized Nernst coefficient are obtained, demonstrating their dependence on microscopic dynamics such as relaxation time, cyclotron frequency, and thermodynamic quantities like enthalpy per baryon.

In Sec. 3.1.2, the framework is extended to higher-order thermoelectric effects. It is shown that the explicit temperature dependence of leading-order coefficients naturally generates higher-order contributions, modifying both electric and heat currents. The coupled transport equations are formulated in matrix form, revealing the interplay between electrical conductivity, thermal conductivity, and thermoelectric coefficients. In the presence of a magnetic field, these relations acquire tensorial structure with both diagonal (Ohmic-like) and off-diagonal (Hall-like) components. The magneto-Thomson coefficient is derived as a temperature derivative of the magneto-Seebeck coefficient, while the transverse Thomson coefficient is shown to arise from the combined effects of the Nernst response and its temperature dependence. These results provide deeper insight into non-linear transport behavior in strongly interacting systems.

Section 3.1.3 focuses on thermoelectric transport in the deconfined QGP phase. The quasiparticle model is employed to incorporate medium effects through temperature-dependent effective masses of quarks and gluons. The influence of time-dependent electromagnetic fields is explicitly considered, reflecting realistic conditions in heavy-ion collisions. The results demonstrate that thermoelectric coefficients are highly sensitive to temperature, baryon chemical potential, and magnetic field evolution. In particular, the sign and magnitude of the Seebeck coefficient are governed by the competition between single-particle energy and enthalpy per baryon, leading to distinct behaviors in baryon-rich and baryon-symmetric regimes. The presence of magnetic fields introduces Hall-type contributions, modifying both Seebeck and Thomson coefficients and giving rise to non-zero Nernst and transverse Thomson effects.

In Sec. 3.1.4, the thermoelectric properties of the confined hadronic phase are

3.4 Summary

investigated using various hadron resonance gas models, including ideal HRG, excluded volume HRG, repulsive mean-field HRG, and van der Waals HRG. The analysis reveals that baryonic contributions dominate thermoelectric transport, while mesons contribute indirectly through thermodynamic quantities such as enthalpy. The Thomson coefficient is found to be sensitive to both temperature and baryon chemical potential, with model-dependent deviations arising from different interaction mechanisms. The inclusion of repulsive interactions enhances relaxation times and modifies transport coefficients, particularly at high temperatures and baryon densities. The effects of magnetic fields further enrich the behavior of higher-order coefficients, leading to anisotropic and transverse transport phenomena in the hadronic medium.

In Sec. 3.2, the thermoelectric figure of merit ZT of QCD matter is introduced as a unified measure of thermoelectric efficiency. The figure of merit combines the Seebeck coefficient, electrical conductivity, and thermal conductivity into a single dimensionless quantity, thereby capturing the overall ability of the medium to convert thermal gradients into electrical energy. The analysis reveals that ZT is highly sensitive to the interplay of these transport coefficients and exhibits a pronounced peak near the QCD transition region. This peak arises due to competing behaviors of the individual transport coefficients in the hadronic and deconfined phases. In the hadronic phase, the dominance of baryonic contributions and the negative Seebeck coefficient lead to a steady increase in ZT with temperature, whereas in the QGP phase, the saturation of thermoelectric response and increasing thermal conductivity suppress ZT . The resulting non-monotonic behavior across the transition highlights ZT as a potential transport-based probe of the QCD phase transition. The three-dimensional dependence of ZT on temperature and baryon chemical potential further demonstrates its sensitivity to the thermodynamic state of the medium, suggesting its relevance in understanding the evolution of matter created in heavy-ion collisions.

In Sec. 3.3, the coupled-transport phenomena in QCD matter are investigated,

focusing on the interplay between baryon diffusion and heat transport. Within the kinetic theory framework, the diffusion current and heat current are expressed in terms of gradients of baryon chemical potential and temperature, leading to the definition of four key transport coefficients: the diffusion coefficient, Soret coefficient, Dufour coefficient, and thermal conductivity. These coefficients quantify, respectively, baryon transport driven by chemical potential gradients, baryon transport driven by temperature gradients, heat transport induced by chemical potential gradients, and heat conduction due to temperature gradients. The analysis shows that baryon-driven transport coefficients, such as diffusion and Dufour, are strongly sensitive to the baryon chemical potential and are dominated by baryonic degrees of freedom, while the Soret coefficient and thermal conductivity receive significant contributions from light mesonic species and thermal effects. A clear distinction between the HRG and QGP phases is observed, with strong temperature dependence in the hadronic phase due to rapidly varying relaxation times, and comparatively weaker dependence in the QGP phase. The results demonstrate that coupled-transport coefficients provide deeper insight into the mutual interaction between charge and heat flow, and are particularly important for understanding transport processes in baryon-rich environments created in relativistic heavy-ion collisions.

Overall, this chapter provides a comprehensive and unified description of thermoelectric and coupled transport phenomena in QCD matter across different phases and physical conditions. The interplay between microscopic dynamics, thermodynamic properties, and external electromagnetic fields is systematically explored, leading to a consistent understanding of both leading- and higher-order transport effects. In particular, the thermoelectric figure of merit ZT is introduced as a measure of thermoelectric transport efficiency, revealing a pronounced sensitivity to the QCD transition region. Furthermore, the coupled transport framework, incorporating diffusion, Soret, Dufour, and thermal conductivity coefficients, highlights the intricate interplay between baryon and heat transport in the medium.

3.4 Summary

Together, these results provide deeper insight into non-equilibrium processes in strongly interacting systems and establish a robust foundation for interpreting charge and heat transport phenomena in relativistic heavy-ion collisions.

Bibliography

- [1] F. Spathelf, B. Fauqué, and K. Behnia, Phys. Rev. B. **105**, 235116 (2022)
- [2] T. Geballe, and G. Hull Phys. Rev. **98**, 940 (1955)
- [3] K. Uchida, M. Murata, A. Miura, and R. Iguchi, Phys. Rev. Lett. **125**, 106601 (2020)
- [4] K. Morrison, and F. Dejene, Physics. **13** (2020)
- [5] Hall, S. Laurence. The Physics of Fluids **7**, 425 (1964)
- [6] Y. Wang, Y. Hu, S. Shang, B. Zhou, Z. Liu, and L. Chen. Phys. Rev. B. **98**, 224101 (2017)
- [7] J. Cai, and G. Mahan. Phys. Rev. B. **74**, 075201 (2006)
- [8] M. Kurian, Phys. Rev. D **103**, 054024 (2021)
- [9] H. X. Zhang, J. W. Kang and B. W. Zhang, Eur. Phys. J. C **81**, 623 (2021)
- [10] J. Young, Proc. Phys. Soc. London, **37**, 145 (1924)
- [11] D.K.C. MacDonald, Thermoelectricity: An Introduction to the Principles, Dover Publications, Inc., New York, (2006)
- [12] K. Morrison and F. K. Dejene, Physics, **13**, 137 (2020)
- [13] S. Gavin, Nucl. Phys. A **435**, 826 (1985)

BIBLIOGRAPHY

- [14] A. Das, H. Mishra and R. K. Mohapatra, Phys. Rev. D **100**, 114004 (2019)
- [15] A. Dash, S. Samanta, J. Dey, U. Gangopadhyaya, S. Ghosh and V. Roy, Phys. Rev. D **102**, 016016 (2020)
- [16] K. Singh, J. Dey, R. Sahoo and S. Ghosh, Phys. Rev. D **108**, 094007 (2023)
- [17] K. Singh, J. Dey and R. Sahoo, Phys. Rev. D **109**, 014018 (2024)
- [18] K. Singh, J. Dey and R. Sahoo, Phys. Rev. D **110**, 114051 (2024)
- [19] A. Das, H. Mishra and R. K. Mohapatra, Phys. Rev. D **102**, 014030 (2020)
- [20] K. Singh, K. K. Pradhan, D. Sahu and R. Sahoo, Phys. Rev. D **111**, 074033 (2025)
- [21] K. Singh and R. Sahoo, Phys. Rev. D **112**, 034032 (2025)
- [22] A.Takahagi, T.Hirai, A. Alasli, S. Park, H. Nagano, and K. Uchida, Nat. Phys. **21**, 1283 (2025)
- [23] A. F. Ioffe, Physics of semiconductors, Foreign Languages Publishing House, Moscow (1957)
- [24] D. Satow, Phys. Rev. D **90** 034018 (2014)
- [25] M. Hongo, Y. Hirono and T. Hirano, Phys. Lett. B **775**, 266 (2017)
- [26] J. Cleymans, H. Oeschler, K. Redlich and S. Wheaton, Phys. Rev. C **73**, 034905 (2006)
- [27] K. Tuchin, Adv. High Energy Phys. **2013**, 490495 (2013)
- [28] W. T. Deng and X. G. Huang, Phys. Rev. C **85**, 044907 (2012)
- [29] K. Singh, K. K. Pradhan and R. Sahoo, Phys. Rev. D **113**, 034021 (2026)

- [30] K. K. Pradhan, D. Sahu, R. Scaria and R. Sahoo, Phys. Rev. C **107**, 014910 (2023)
- [31] S. Samanta and B. Mohanty, Phys. Rev. C **97**, 015201 (2018)
- [32] A. Das, H. Mishra and R. K. Mohapatra, Phys. Rev. D **99**, 094031 (2019)
- [33] K. Singh and R. Sahoo, Phys. Lett. B **872**, 140060 (2026)
- [34] D. Nemir, and J. Beck, Journal Of Electronic Materials. **39**, 1897 (2010)
- [35] A. Bazavov *et al.* [HotQCD], Phys. Lett. B **795**, 15 (2019)
- [36] K. Singh, K. Goswami and R. Sahoo, Phys. Rev. D **113**, 014040 (2026)

Chapter 4

Overview of particle accelerators, detectors and beampipe design

“Science is a beautiful gift to humanity; we should not distort it. .”

— Dr. A. P. J. Abdul Kalam

Accelerator technology has become an essential tool in modern science and technology, serving a wide range of applications that directly benefit society. Particle accelerators are widely used in fundamental research to explore the structure of matter and the fundamental forces of the universe, particularly in high-energy physics experiments. Beyond basic research, accelerators play a crucial role in medical applications, such as cancer treatment via radiation therapy and the production of medical isotopes for diagnostic imaging. They are also employed in material science for probing the structural and electronic properties of materials, in semiconductor fabrication, in food sterilization, and in environmental and industrial processes. The versatility and precision of accelerator-based techniques have therefore made them indispensable across multiple scientific and technological domains. At the heart of every particle accelerator lies the beam transport system, whose central component is the beampipe. The beampipe provides a controlled pathway through which charged particle beams travel at high velocities. Maintaining an ultra-high vacuum inside the beampipe is essential to minimize

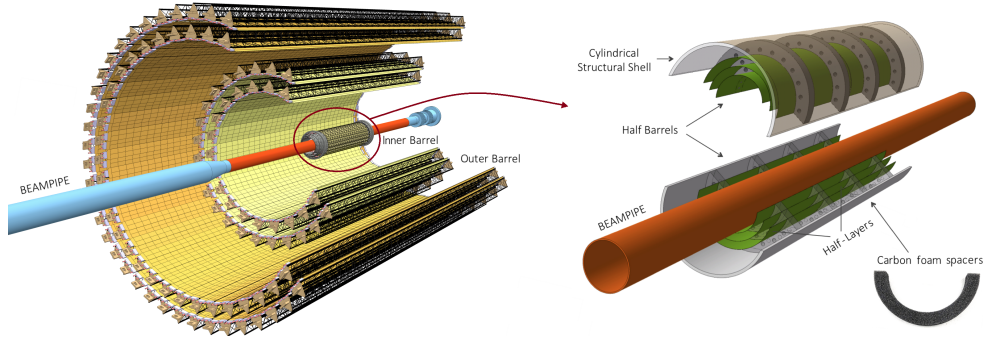


Figure 4.1: Schematic of beampipe at the core of ALICE detector [1].

interactions between the beam particles and residual gas molecules, which could otherwise lead to beam scattering, energy loss, and degradation of beam quality. Therefore, the beampipe is a critical element in accelerator design, directly influencing beam stability, operational efficiency, and the overall performance of the accelerator facility.

*“In the beampipe particles rain,
looks like the view of a moving train,
silent through the fields they glide,
Carrying the quest for hidden truth inside.”*

4.1 Particle accelerators

Particle accelerators are scientific instruments designed to accelerate charged particles to very high energies and guide them to collide with targets or with each other. These collisions allow physicists to probe the fundamental structure of matter and study the interactions among elementary particles. Over the past century, accelerator technology has evolved significantly, enabling experiments at progressively higher energies and luminosities.

The earliest electrostatic accelerators, such as the Van de Graaff generator developed in the 1930s [2], were capable of accelerating particles using high static

4.1 Particle accelerators

electric potentials. Although limited in achievable energies due to electrical breakdown, these devices laid the foundation for modern accelerator technology. The development of the cyclotron by Ernest O. Lawrence in 1932 [3] marked a major breakthrough. Cyclotrons used a combination of magnetic fields and alternating electric fields to accelerate charged particles along a spiral trajectory, allowing particles to reach much higher energies than electrostatic machines. However, relativistic effects at higher energies limited their applicability. To overcome these limitations, the synchrotron was developed [4]. In synchrotrons, both the magnetic field and the accelerating electric field are synchronized with the particle energy, allowing particles to be accelerated along a fixed circular path. This technology enabled the construction of very high-energy accelerators and remains the basis of many modern facilities. Another important accelerator type is the linear accelerator (linac) [5], where particles are accelerated in a straight line using a sequence of radio-frequency cavities. Linacs are widely used either as standalone accelerators or as injectors for larger circular machines. With advances in accelerator physics and technology, large-scale facilities such as the Relativistic Heavy Ion Collider (RHIC) [6] at Brookhaven National Laboratory were constructed to study strongly interacting matter under extreme conditions. RHIC enables collisions of heavy ions at relativistic energies, providing insights into the quark–gluon plasma (QGP).

The most powerful accelerator currently in operation is the Large Hadron Collider (LHC) at the European Organization for Nuclear Research (CERN) [7]. The LHC is a circular proton–proton and heavy-ion collider with a circumference of approximately 27 km, located underground near Geneva. It accelerates beams to multi-TeV energies and enables the study of fundamental particles and interactions at unprecedented energy scales. The LHC hosts several major experiments designed to explore different aspects of high-energy physics. The four primary detectors installed along the LHC ring are described below.

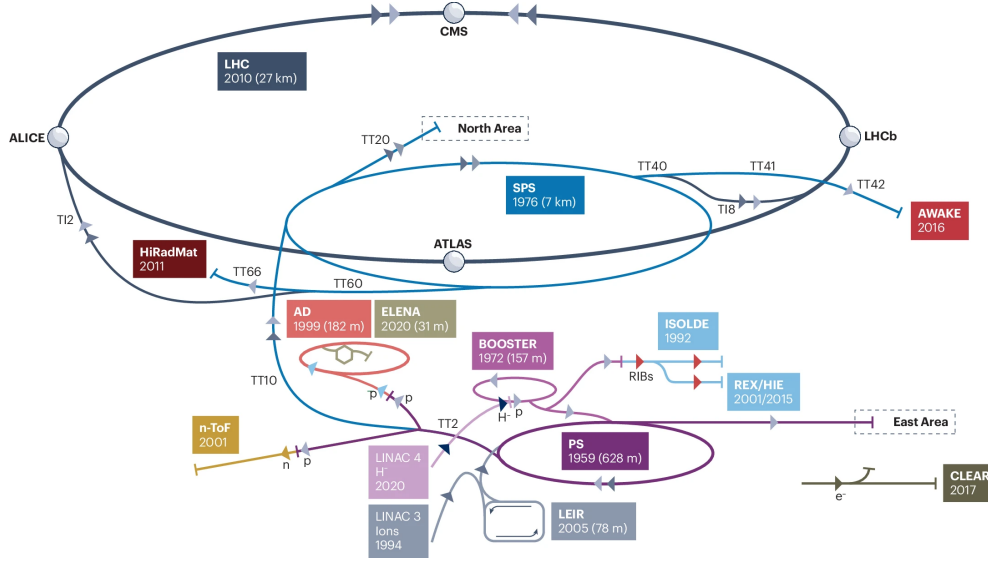


Figure 4.2: The accelerator complex at CERN consists of a series of machines designed to progressively increase the energy of particle beams. In this chain, each accelerator raises the energy of the particles before transferring the beam to the next stage of the acceleration sequence. [8].

4.1.1 ATLAS detector

The ATLAS (A Toroidal LHC ApparatuS) detector is a general-purpose detector designed to investigate a wide range of physics phenomena, including the search for new particles, precision measurements of Standard Model processes, and studies of the Higgs boson. It is one of the largest particle detectors ever constructed.

4.1.2 CMS detector

The CMS (Compact Muon Solenoid) detector is another general-purpose experiment at the LHC. Despite its relatively compact design compared to ATLAS, CMS incorporates a powerful superconducting solenoid magnet and highly precise tracking and calorimetry systems, enabling detailed measurements of particle interactions.

4.2 Beampipe - the heart of particle accelerators

4.1.3 LHCb detector

The LHCb (Large Hadron Collider beauty) experiment is specifically designed to study the properties and decays of particles containing bottom (beauty) quarks. Its primary objective is to investigate matter–antimatter asymmetry through precise measurements of CP violation and rare decays.

4.1.4 ALICE detector

The ALICE (A Large Ion Collider Experiment) detector is dedicated to the study of strongly interacting matter at extremely high energy densities, particularly the formation and properties of the quark–gluon plasma produced in heavy-ion collisions. ALICE is optimized to handle the very high particle multiplicities generated in nucleus–nucleus collisions. The experiment employs a variety of detector subsystems for tracking, particle identification, and calorimetry. Since our work is carried out within the ALICE collaboration, the subsequent sections of the next chapter will focus on the ALICE experiment and its detector systems in detail.

4.2 Beampipe - the heart of particle accelerators

Beampipes constitute an essential element of particle accelerators, acting as vacuum chambers through which accelerated particle beams propagate. The geometry and material choice for a beampipe depend on several parameters, such as the nature of the accelerated particles, their energies, and the intended purpose of the accelerator facility. In modern high-energy physics experiments, beampipes are employed in large accelerator complexes like the LHC and the RHIC, where protons and heavy ions are accelerated to velocities close to the speed of light

and subsequently collided to investigate the properties of the primordial matter believed to have existed shortly after the birth of the universe [9, 10]. The beampipes that traverse the four major experiments at the LHC form the closest physical interface between the accelerator machine and the detectors. Consequently, the beampipe represents a crucial structural and functional component located at the core of these experimental setups [11, 12].

In such experiments, the beampipe serves the fundamental role of guiding particle beams through the accelerator structure while preserving their trajectories. It also provides an ultra-high vacuum environment that minimizes interactions between the beam particles and residual gas molecules present in air. Ideally, a beampipe material should be highly transparent to radiation, capable of sustaining an extremely high vacuum to prevent collisions with residual gas atoms or molecules, and sufficiently thin in diameter to allow accurate tracking of particles produced near the interaction point [13, 14]. One of the primary criteria for selecting materials for accelerator beampipes is the minimization of interactions between traversing particles and the beampipe wall. This transparency can be quantified using the radiation length (X_0), defined as the mean distance in a material over which an electron loses a factor $1/e$ of its energy through electromagnetic processes such as bremsstrahlung and multiple scattering [15]. Since the radiation length decreases with increasing atomic number (Z), materials possessing large X_0 values are preferred for constructing beampipes in accelerators operating at various energy scales. The use of such materials reduces undesired interactions between beam or produced particles and the beampipe walls, thereby preserving beam quality and improving the signal-to-background ratio in measurements. For this reason, materials such as beryllium, which possess relatively large radiation lengths, are widely used in high-energy experiments, including the LHC. Conversely, materials with smaller radiation lengths can lead to increased background due to enhanced interactions with the detector environment [16]. In addition to radiation transparency, the mechanical stability of the beampipe must

4.2 Beampipe - the heart of particle accelerators

also be considered. The chamber wall must withstand ultra-high vacuum and mechanical stress, which are generally characterized by the elastic modulus (E) of the material [17]. Certain experimental programs also explore additional physics opportunities by installing compact detectors within the beampipe region at both small and large angles relative to the beam direction [18, 19]. In collider experiments, therefore, beampipes perform multiple roles beyond simply enclosing the particle beam. They function as primary vacuum barriers, maintaining ultra-high vacuum conditions that suppress particle scattering and energy loss caused by interactions with residual gases. The use of beampipes extends beyond accelerator physics and also finds applications in nuclear research facilities and high-power hadron beam systems. When selecting materials for beampipes in nuclear physics accelerators, both the atomic number Z and the mass number (A) are important parameters in minimizing interactions between the beam particles and the material. The ability of a material to slow down energetic particles traversing through it is described by its stopping power (S) [20]. Since stopping power scales with atomic number, materials with relatively low Z are generally favored because they provide greater transparency to energetic particles. During the early development of particle accelerators, beampipes were relatively simple cylindrical structures typically fabricated from materials such as stainless steel or copper. Their primary purpose was to guide and confine particle beams as they moved through the accelerator system. However, as accelerator technology advanced and high-energy collider experiments emerged, the requirements for beampipe design became significantly more demanding. With increasing beam energies, careful selection of materials became essential. The materials used must withstand ultra-high vacuum conditions, high radiation environments, and potential mechanical stresses arising from beam interactions. Commonly used materials include stainless steel, beryllium, aluminum alloys, and titanium, owing to their favorable mechanical strength, compatibility with ultra-high vacuum systems, and their influence on detector performance and signal-to-background conditions.

Several experiments at the LHC, including A Large Ion Collider Experiment ALICE [21] and A Toroidal LHC Apparatus ATLAS [22], employ beryllium beampipes because of their low material budget combined with adequate mechanical strength [11]. In the ALICE experiment, the current beampipe is approximately one meter long with a diameter of 36.4 mm, reduced from an earlier diameter of 50 mm, and a wall thickness of about 0.8 mm, which approaches the limits achievable with present manufacturing technology. Despite its favorable radiation transparency, beryllium presents several practical challenges. Its low elongation and inherent brittleness make machining difficult, and welding operations are often impractical because thermal stresses can induce cracks. In addition, beryllium is toxic; exposure to its dust or fumes during processing can be hazardous, thereby complicating handling, machining, and fabrication procedures. [13]. Although advanced welding techniques such as electron-beam welding and laser-beam welding can be applied, the brittleness of the material and the strict safety requirements associated with its toxicity significantly limit its widespread use in beam pipe fabrication. These safety precautions increase both the complexity and the cost associated with manufacturing and handling beryllium components. Furthermore, beryllium is considerably more expensive than alternative materials such as aluminum alloys or stainless steel [23]. Another practical limitation is that beryllium is typically produced in powder form and must be compressed at extremely high pressures to produce solid bars, which are then machined to form hollow structures. As a result, only a limited number of facilities worldwide possess the capability to manufacture such specialized components.

Apart from high-energy collider experiments, beampipes also play a crucial role in synchrotron radiation facilities and free-electron lasers. In these systems, they guide high-energy electron beams that are used to generate intense and coherent X-ray radiation for applications in materials science, biology, chemistry, and medical research. In such facilities, the design of the beampipe emphasizes minimizing

4.3 Calculations of radiation length and elastic modulus

beam impedance and ensuring the stability of the electron beam in order to produce high-quality photon beams. In India, synchrotron radiation facilities such as Indus-2 [24, 25], operating at beam energies of a few MeV, employ beampipes fabricated from aluminum alloy 5083 H321 with a thickness of approximately 0.2 mm. For this configuration, the figure of merit (FoM) defined as $X_0 E^{1/3}$ is approximately 0.374 [26].

4.3 Calculations of radiation length and elastic modulus

4.3.1 Radiation length

The radiation length is defined as the mean distance over which a high-energy electron loses $1/e$ of its energy through bremsstrahlung and multiple scattering processes. For an electron or a charged particle with initial energy ϵ_0 traversing a distance x in a material, the energy attenuation follows,

$$\epsilon = \epsilon_0 e^{-x/X_0}. \quad (4.1)$$

We employ the following approach to estimate X_0 for the alloy systems. For a given element, the radiation length constant $\bar{X}_0$ (expressed in kg/m^2) can be calculated using the semi-empirical relation [27, 28],

$$\bar{X}_0 = \frac{716.4A}{Z(Z+1) \ln\left(\frac{287}{\sqrt{Z}}\right)} \quad (kg/m^2), \quad (4.2)$$

where Z is the atomic number and A is the atomic mass number of the element. To obtain the radiation length X_0 in units of meters, the radiation length constant is divided by the material's mass density ρ (kg/m^3),

$$X_0 = \frac{\bar{X}_0}{\rho} \quad (m). \quad (4.3)$$

This method provides a quick and analytic estimate of X_0 for pure elements. Further, for multi-component systems such as Mg–Al–Li alloys, a more precise way to estimate X_0 is obtained using the weighted inverse sum of the radiation lengths of constituent elements [29], given as

$$\frac{1}{X_0} = \sum_{i=1}^n \frac{w_i}{X_{0,i}}, \quad (4.4)$$

where w_i is the weight fraction of the i th element and $X_{0,i}$ is the radiation length of the pure element i . This method accounts for compositional effects and is recommended for precise modeling of radiation transport in alloy systems.

4.3.2 Elastic modulus

In our study, we have measured elastic modulus both experimentally and computationally. Experimentally, the room-temperature nanoindentation test has been carried out to evaluate the mechanical properties of alloy-1 ($Al_{75.2}Ti_{22.8}V_2$). The elastic modulus (E) of alloy-1 is estimated using volume fraction of $(Al)_{ss}$ and Al_3Ti phases. The experimentally measured value of E is found to be 96.9 and 173.4 GPa for $(Al)_{ss}$ and Al_3Ti phases, respectively. Whereas, the computational detailing of the elastic modulus of the designed alloys is calculated using density functional theory as implemented in the VASP. The crystallographic input files (CIFs) for all phases considered in this work were obtained from the Materials Project database [30]. The Perdew-Burke-Ernzerhof (PBE) exchange-correlation functional, within the generalized gradient approximation (GGA), is employed, along with projector augmented-wave (PAW) pseudopotentials. A plane-wave cutoff energy of 520 eV is used, and k -point meshes $3 \times 3 \times 3$ are generated using the Monkhorst-Pack scheme with sufficient density to ensure energy convergence. An energy difference of 10^{-4} eV was used as the convergence criterion for electronic relaxation. The atomic positions were optimized until the force on each atom was less than 0.01 eV/Å. Spin polarization was not included in the present calculations.

4.3 Calculations of radiation length and elastic modulus

The elastic modulus, bulk modulus (K), and shear modulus (G) are derived from the full stiffness tensor using the Voigt–Reuss–Hill averaging method. These mechanical properties provide insight into the alloy’s structural stability and suitability for integration into accelerator beamlines where high vacuum and mechanical resilience are essential. We evaluated 21 independent elastic constants (C_{ij}) and averaged them to obtain the effective cubic elastic parameters: $\langle C_{11} \rangle = (C_{11} + C_{22} + C_{33})/3$, $\langle C_{12} \rangle = (C_{12} + C_{23} + C_{31})/3$, and $\langle C_{44} \rangle = (C_{44} + C_{55} + C_{66})/3$. Under the harmonic approximation and within the small-deformation limit, the change in total energy due to strain can be expressed as [31],

$$\Delta E(V, \{\varepsilon_i\}) = E(V, \{\varepsilon_i\}) - E(V_0, 0) = \frac{V_0}{2} \sum_{i,j=1}^6 C_{ij} \varepsilon_i \varepsilon_j, \quad (4.5)$$

where $E(V_0, 0)$ and $E(V, \{\varepsilon_i\})$ denote the total energies of the equilibrium and deformed structures with volumes V_0 and V , respectively. The quantities ε_i and ε_j represent the six independent components of the strain vector ($1 \leq i, j \leq 6$), and C_{ij} are the elements of the symmetric 6×6 elastic stiffness tensor. The elastic stiffness tensor was computed from the second derivative of the total energy with respect to strain, following the energy–strain relationship [32].

The bulk modulus and shear modulus were derived from the stiffness tensor using the Voigt (K_V and G_V) [33] and Reuss (K_R and G_R) [34] averaging schemes. The Voigt bounds are given by,

$$9K_V = (C_{11} + C_{22} + C_{33}) + 2(C_{12} + C_{23} + C_{31}), \quad (4.6)$$

$$15G_V = (C_{11} + C_{22} + C_{33}) - (C_{12} + C_{23} + C_{31}) + 3(C_{44} + C_{55} + C_{66}). \quad (4.7)$$

The corresponding Reuss bounds can be expressed as,

$$\frac{1}{K_R} = (S_{11} + S_{22} + S_{33}) + 2(S_{12} + S_{23} + S_{31}), \quad (4.8)$$

$$\frac{15}{G_R} = 4(S_{11} + S_{22} + S_{33}) - 4(S_{12} + S_{23} + S_{31}) + 3(S_{44} + S_{55} + S_{66}), \quad (4.9)$$

where S_{ij} are the components of the compliance tensor, defined as the inverse of the stiffness tensor ($[S_{ij}] = [C_{ij}]^{-1}$).

The arithmetic mean of the Voigt and Reuss bounds provides the Voigt–Reuss–Hill (VRH) average, representing the effective isotropic moduli of the polycrystalline material [35],

$$K_{\text{VRH}} = \frac{K_V + K_R}{2}, \quad (4.10)$$

$$G_{\text{VRH}} = \frac{G_V + G_R}{2}. \quad (4.11)$$

The Elastic modulus is then obtained using,

$$E = \frac{9K_{\text{VRH}}G_{\text{VRH}}}{3K_{\text{VRH}} + G_{\text{VRH}}}. \quad (4.12)$$

The designed Al-Ti-V alloys (alloy-1: $\text{Al}_{75.2}\text{Ti}_{22.8}\text{V}_2$ and alloy-2: $\text{Al}_{89}\text{Ti}_{10}\text{V}_1$) are synthesized by vacuum arc melting technique under a protective argon atmosphere in water-chilled Cu-mould using constituent elements having greater than 99.9 % mass purity. The alloy button is remelted at least 5 times to obtain a homogeneous chemical sample. The arc-melted sample is cut and mirror-polished for subsequent characterization of prepared samples. The structural characterization and microstructural characterization of the developed Al-Ti-V alloys are carried out using an X-ray diffractometer (Model No.:Panalytical Empyrean X-ray Diffractometer) and Scanning electron microscopy (SEM, model no.:JEOL JSM-7610 Fplus model). The XRD pattern of alloy-1 (as shown in Figure 4.3) shows the presence of an intense peak of the Al_3Ti phase and FCC $(\text{Al})_{ss}$ phase. SEM coupled with EDS mapping results are given in the same figure, highlighting the presence of bright contrast Al_3Ti phase and dark contrast Al-rich $(\text{Al})_{ss}$ phase as well as uniform distribution of each element in the microstructure of the developed Al-Ti-V alloy.

Figure 4.4 shows the XRD pattern of alloy-2, indicating the presence of FCC $(\text{Al})_{ss}$ and Al_3Ti phases. The diffraction peaks confirm the successful formation of a two-phase microstructure dominated by the intermetallic compound Al_3Ti ,

4.3 Calculations of radiation length and elastic modulus

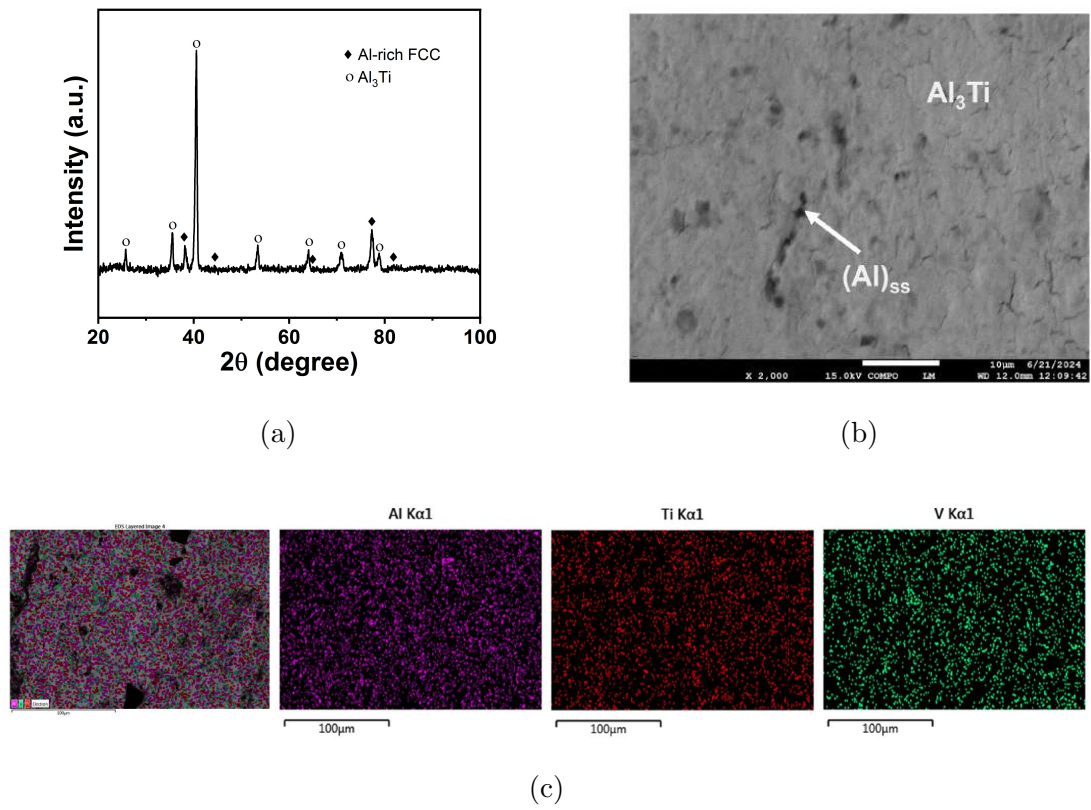


Figure 4.3: (a) XRD pattern, (b) SEM micrograph, (c) EDS mapping of alloy-1 [26].

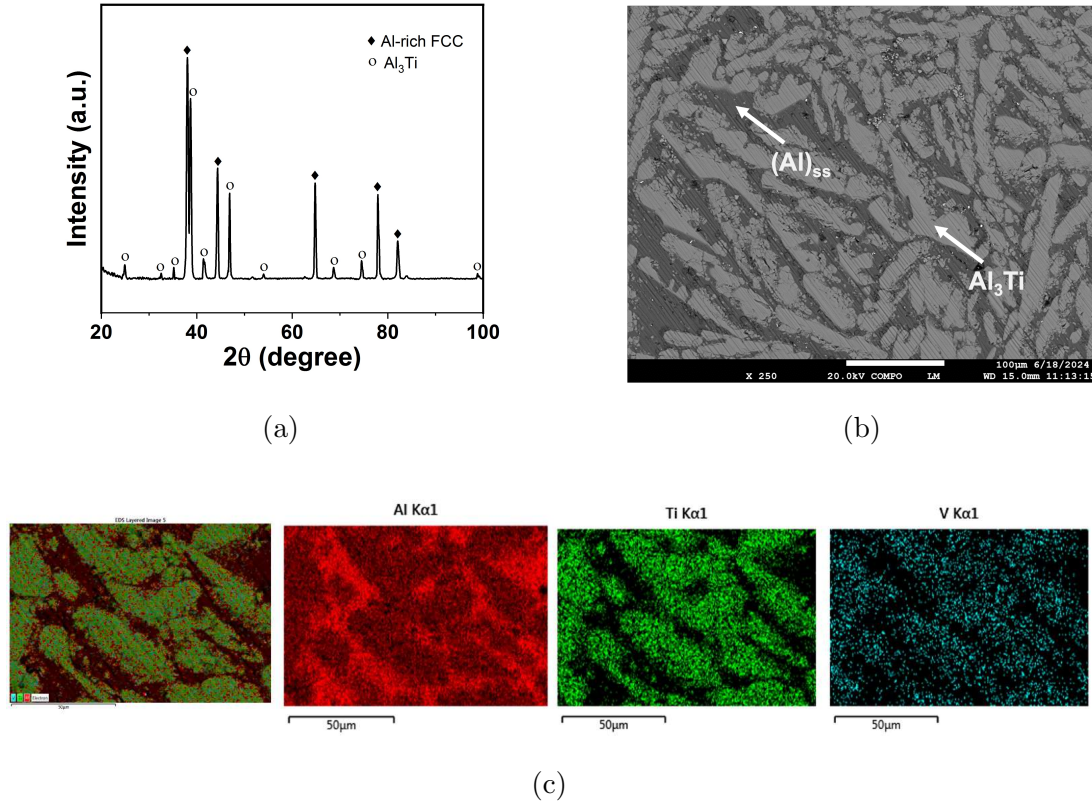


Figure 4.4: (a) XRD pattern, (b) SEM micrograph, (c) EDS mapping of alloy-2 [26].

along with a comparatively smaller fraction of the Al-rich solid solution phase. The figure also presents the SEM micrograph of alloy-2, which reveals a heterogeneous microstructure consisting of an Al-rich $(Al)_{ss}$ phase (dark contrast) and the Al_3Ti phase (bright contrast). The contrast difference clearly reflects the compositional and structural variation between the constituent phases. EDS elemental mapping further demonstrates a nearly uniform spatial distribution of each constituent element throughout the microstructure, suggesting effective alloy homogenization during processing.

The Al-Ti-V alloy containing a high volume fraction of Al_3Ti intermetallics can be considered a promising candidate material for structural applications, particularly

4.4 Thermodynamical simulation using Thermo-Calc software

for low-energy beampipe applications where high stiffness and low density are desirable. The presence of a large fraction of the intermetallic phase is expected to significantly enhance the mechanical rigidity and dimensional stability of the alloy. The elastic modulus of the individual phases of the developed alloy is 96.9 GPa for $(Al)_{ss}$ with a volume fraction of 9.4% and 173.4 GPa for Al_3Ti with a volume fraction of 90.6%, resulting in an overall elastic modulus of alloy-2 of approximately 166.2 GPa. Such a high modulus, combined with the lightweight nature of Al-based alloys, makes this material attractive for applications requiring a favorable stiffness-to-weight ratio and improved structural performance.

4.4 Thermodynamical simulation using Thermo-Calc software

4.4.1 For Al-Ti-V ternary alloy

4.4.1.1 Machine learning models

Machine learning excels in solving classification and regression problems due to its ability to learn from data and make predictions without being explicitly programmed. For classification problems, machine learning algorithms can efficiently categorize unseen data into predefined classes based on patterns learned from training data. On the other hand, for regression problems, machine learning can predict continuous outcomes by learning the relationship between input features and the target variable. The efficiency of machine learning in these areas originates from its ability to handle large datasets and identify correlations among the input and target variables, making it a powerful tool for both classification and regression tasks.

Hyperparameter	Default Value
fit_intercept	True
n_jobs	-1
positive	False

Table 4.1: Hyperparameters of the Linear Regression model [26].

Hyperparameter	Default Value
booster	gbtree
learning_rate	0.3
max_depth	6
subsample	1
objective	binary:logistic
n_estimators	100

Table 4.2: Hyperparameters of the XGBoost Classifier model [26].

Hyperparameter	Default Value
n_estimators	100
criterion	squared_error
min_samples_split	2
min_samples_leaf	1
max_features	sqrt

Table 4.3: Hyperparameters of the RandomForest Regressor model [26].

4.4.1.2 Input to the machine learning algorithms

In this study, we train the machine learning models with 3 input features, radiation length, density, and number of phases, obtained for a specific composition of alloy using Thermo-Calc. We apply two different models: Model A, which combines an XGBoost Classifier [36, 37] for phase prediction and Linear Regression for predicting density and radiation length, and Model B, which combines an XGBoost Classifier with a Random Forest Regressor [38] for the prediction of density and radiation length.

The input data from Thermo-Calc is split into 80 % for training and 20% for testing. We take a dataset of 2000 samples and their properties estimated using

4.4 Thermodynamical simulation using Thermo-Calc software

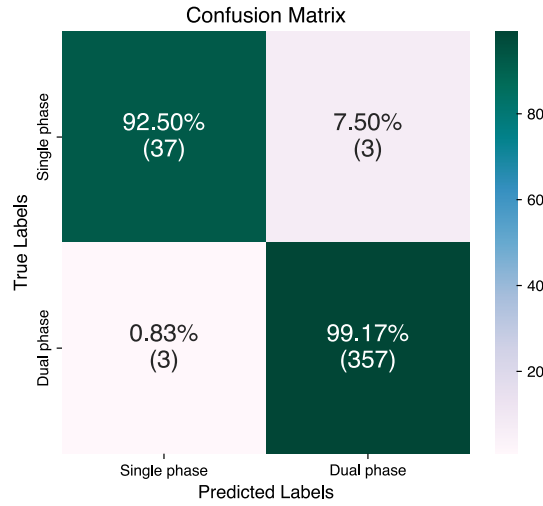


Figure 4.5: Confusion matrix for the classification of the phases using the XGBoost model [26].

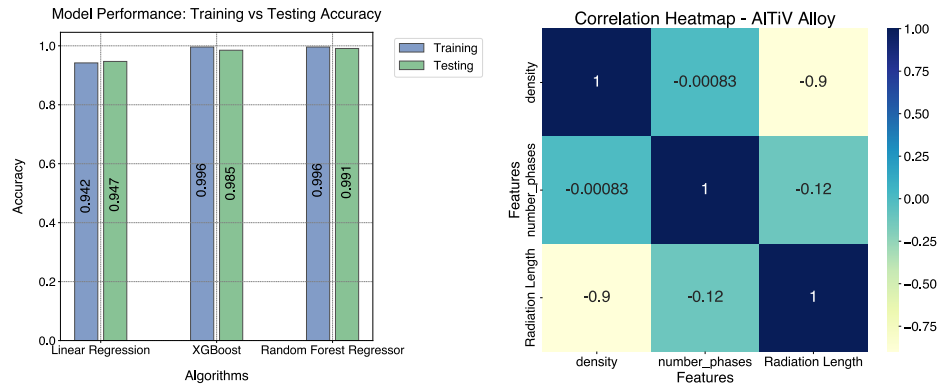


Figure 4.6: (Left Panel): Accuracies of three models. (Right Panel): Correlation matrix between the input features in machine-learning models for AlTiV [26].

Thermo-Calc. In Fig. 4.6, we plot the accuracies of the individual ML algorithms. We observe that the XGBoost Classifier and Random Forest regressor perform better than the linear regression model, which is expected as linear regression is a very simple algorithm. Due to the high accuracy achieved by the XGBoost Classifier and Random Forest Regressor models, we have chosen to use their default hyperparameters, tabulated in Table 4.1, 4.2, and 4.3, respectively. Moreover, changing the hyperparameters resulted in a very small improvement in the accuracy, but it resulted in substantially higher computational costs. Furthermore, to study the correlation among the input features, we plot the correlation matrix, shown in the right panel of Fig. 4.6. One can directly observe a strong negative correlation between density and radiation length for both of the alloys. However, there is a very small correlation between density and number of phases.

4.4.1.3 Machine learning prediction

We train two machine learning algorithms with different regressor models and use them to predict the properties of both single-phase and double-phase alloys composed of Al-Ti-V. These predictions are made using both Model A and Model B. To better understand the phase classification predictions made by the XGBoost model, we have plotted the confusion matrix in Fig. 4.5. We observe that the model successfully predicts the number of phases with higher accuracy; however, in our training and testing data, the number of single-phase alloys is much lower than the dual-phase alloys. Moreover, we have tabulated the Precision, Recall, and F1-score of the model for the respective class. For single-phase alloys, the values for all of them are around 0.93, while for dual-phase alloys, the values are near 0.99. In Table 4.5, we present the predicted composition of the Al-Ti-V alloy for a single-phase alloy, along with the corresponding number of phases, radiation length, and density. Similarly, Table 4.6 contains the predictions for the dual-phase Al-Ti-V. In Table 4.8, we compare the properties of the alloys predicted by

4.4 Thermodynamical simulation using Thermo-Calc software

the ML algorithm with Stainless steel 304, which is one of the preferred materials for the construction of beampipe for the low energy experiments.

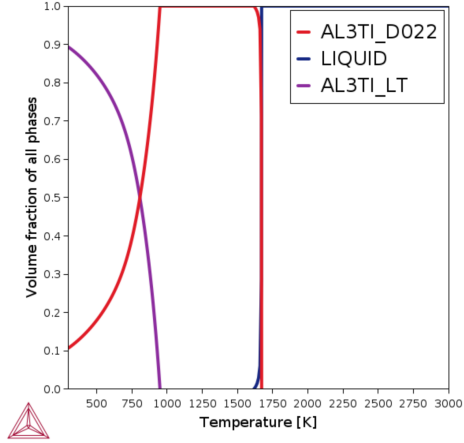
Class	Precision	Recall	F1-Score
Single phase	0.93	0.93	0.93
Dual phase	0.99	0.99	0.99

Table 4.4: Precision, Recall, and F1-Score for each class [26].

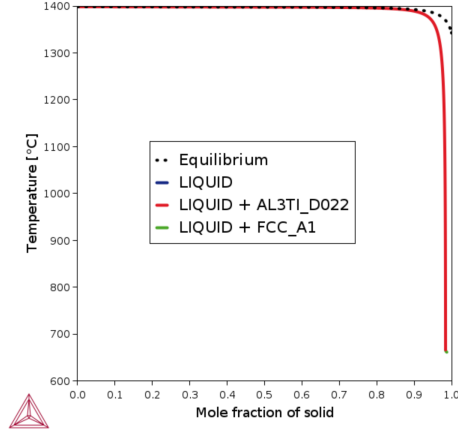
The predicted phases of the designed Al-Ti-V alloys by ML technique are cross-validated with the thermodynamic simulation results. The thermodynamic simulation of alloy-1 has been performed using Thermo-Calc software (Version: Al database TCAL8). The volume fraction of phases Vs. temperature plot of alloy-1 is given in Figure 4.7a. Firstly, both FCC_A1 and Al_3Ti_D022 phases form from the liquid at high temperature, followed by phase transformation of FCC_A1 to Al2V2 phase as well as Al_3Ti_D022 to Al3Ti_LT phase during solid-state transformation. Temperature Vs. mole fraction plot of Al-Ti-V alloy is given in Figure 4.7b, which shows the formation of Al_3Ti_D022 phase (i.e. $L \rightarrow L + Al_3Ti_D022$) from the liquid, followed by complete solidification of the remaining liquid to form FCC_A1 phase. It is found that the designed alloy exhibits almost a single-phase alloy with a very high-volume fraction of Al_3Ti_D022 phase (approx. 97-98 %) and 2-3 volume fraction % of FCC_A1 phase. Figure 4.7c shows the volume fraction of phases Vs. temperature plot

	Model A	Model B
Al	0.752	0.752
Ti	0.228	0.159
V	0.020	0.089
No. of phases	1	1
X_0 (m)	0.0756	0.0749
Density (kg/m^3)	3255.71	3361.21

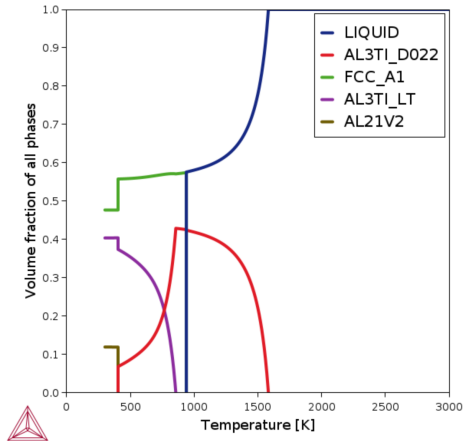
Table 4.5: Model predictions for the compound Al-Ti-V and their properties for a single-phase alloy [26].



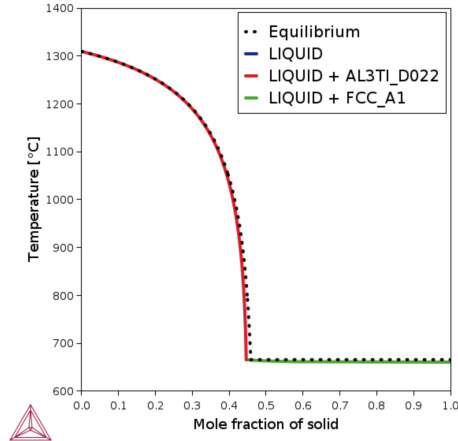
(a)



(b)



(c)



(d)

Figure 4.7: Machine learning predictions for (a) Volume fraction of solids with a single-phase alloy. (b) Mole fraction of solids with a single-phase alloy. (c) Volume fraction of solids with dual-phase alloy. (d) Mole fraction of solids with a dual-phase alloy [26].

4.4 Thermodynamical simulation using Thermo-Calc software

	Model A	Model B
Al	0.890	0.888
Ti	0.100	0.101
V	0.010	0.011
No. of phases	2	2
X_0 (m)	0.0830	0.0828
Density (kg/m^3)	2712.47	2744.98

Table 4.6: Model predictions for the compound Al-Ti-V and their properties for a dual-phase alloy [26].

Material	Radiation Length (X_0) [m]	Elastic Modulus (E) [GPa]	$X_0 E^{1/3}$	Density [kg/m^3]	Melting Temperature [$^{\circ}C$]
Stainless Steel 304	0.0176	193	0.1017	7930	1400-1450
Indus-2 [24]	0.09	72	0.374	2650	570
Designed alloy-1	0.0756	166.2	0.416	3259	1402

Table 4.7: Comparison of properties of different materials used for low-energy beampipes [26].

of alloy-2 ($Al_{89}Ti_{10}V_1$), highlighting the two-phase domain of Al_3Ti _D022 is very large with liquidus temperature of (~ 1675 K) and forms from the liquid at high temperature first. Subsequently, Al_3Ti _D022 is transformed to Al_3Ti _LT phase during solid-state transformation. Figure 4.7d represents temperature Vs. mole fraction plot of $Al_{89}Ti_{10}V_1$ alloy, revealing the evolution of various phases during solidification. Firstly, Al_3Ti _D022 phase is formed from the liquid as primary phase (i.e., $L \rightarrow L + Al_3Ti_D022$), followed by the formation of FCC_A1 phase from the remaining liquid (i.e., $L \rightarrow L + FCC_A1 \rightarrow FCC_A1$). The Al-Ti-V alloys consist of composite phases having both Al_3Ti _D022 (approx. 57 %) and FCC_A1 ($\simeq 43\%$). The liquidus temperature for the studied alloy is calculated to be around 1590 K. It is worth mentioning that the predicted results using ML techniques corroborate with the thermodynamic simulation outcomes.

4.4.2 For Mg-Al-Li ternary alloy

We employ the Thermo-Calc software suite to perform equilibrium and non-equilibrium phase calculations for various compositions of Mg–Al–Li alloys. The thermodynamic simulations are carried out using the TCAL8 database for alloys A1 ($\text{Al}_{61.5}\text{Li}_{10.8}\text{Mg}_{27.7}$) and A2 ($\text{Al}_{66}\text{Li}_{19.4}\text{Mg}_{14.6}$), which is specifically tailored for aluminum-based systems and includes relevant thermodynamic descriptions for binary and ternary phases involving Al, Li, and Mg. The TCMG6 (v6.3) dataset is used for thermodynamic simulations of M1 ($\text{Al}_{23.9}\text{Li}_{29.3}\text{Mg}_{46.8}$), M2 ($\text{Al}_{19}\text{Li}_{20.6}\text{Mg}_{60.4}$), and M3 ($\text{Al}_{39.8}\text{Li}_{20.1}\text{Mg}_{40.1}$) alloys, which are specifically tailored for magnesium-based systems.

The simulations are conducted via TC-Python, an integrated scripting environment that enables automated high-throughput exploration of compositional design space. In each simulation, the components and phases of interest are specified, and the equilibrium phase diagrams are studied over a temperature range relevant for alloy solidification and processing. To more realistically capture the solidification behavior observed in industrially processed alloys, we consider non-equilibrium solidification pathways using the Gulliver–Scheil model [39, 40]. This model assumes complete mixing in the liquid phase and no diffusion in the solid phase, making it suitable for representing solute segregation during rapid solidification. Under these assumptions, the liquid concentration at the solid–liquid interface is given by:

$$C_L = C_0(1 - f_s)^{k-1}, \quad (4.13)$$

where C_0 is the initial liquid concentration, k is the equilibrium partition coefficient, and f_s is the solid fraction. The Gulliver–Scheil simulations provide insight into phase formation sequences and solidification routes, which are critical for predicting microstructural evolution of the alloy. The phase formation and solidification behavior of the five designed Mg–Al–Li alloys can also be interpreted

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using the invariant reactions listed in Table 3 of Ref. [41].

4.4.3 Solidification behavior of Mg–Al–Li alloys

The equilibrium solidification behavior of the three magnesium-rich alloys M1, M2, and M3 was investigated using Thermo-Calc with the TCMG6 (v6.3) thermodynamic database. The evolution of solid phases during cooling is quantified in terms of both mole fraction and volume fraction as a function of temperature, as shown in Figs. 4.8–4.10. These representations provide complementary insights into phase formation and the resulting microstructural constitution relevant for beampipe applications.

4.4.3.1 Alloy M1

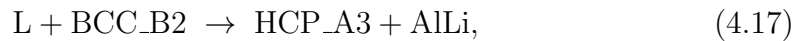
Figure 4.8 presents the solidification path of alloy M1. The left panel shows the mole fraction of solids as a function of temperature, indicating that solidification initiates slightly above ~ 720 K with the precipitation of the primary AlLi1 [i.e., (AlLi)] phase from the initial liquid.



As cooling proceeds, additional solid phases appear through successive monovariant reactions involving (L, AlLi1, BCC_B2) and (L, AlLi1, HCP_A3) phases.



where BCC_B2 is (Li)_{ss} and HCP_A3 is (Mg)_{ss} phases. This leads to a multiphase solid mixture at lower temperatures. Based on the above two monovariant reactions, alloy M1 undergoes an invariant reaction, as



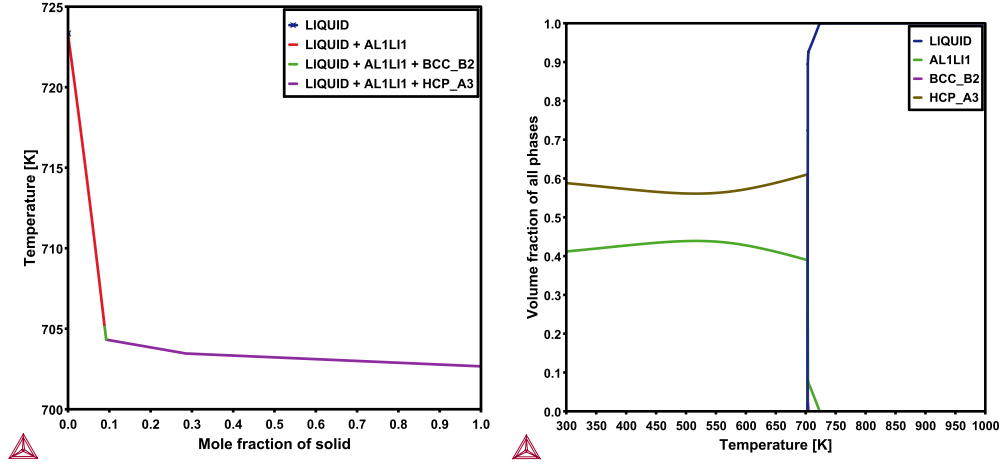


Figure 4.8: For alloy-M1, (left panel) mole fraction of solids and (right panel) volume fraction of solids [42].

This studied alloy shows a unique phase equilibrium with the formation of multiple stable phases. The right panel of Fig. 4.8 illustrates the corresponding volume fraction of each phase with respect to temperature during solidification. Near the liquidus, the HCP_A3 phase dominates the solid fraction, reflecting the Mg-rich nature of alloy M1. With decreasing temperature, the volume fraction of AlLi1 increases steadily, while the BCC_B2 phase remains comparatively minor. The coexistence of HCP_A3 and AlLi1 over a broad temperature range. This alloy shows a stable multiple-phase mixture, exhibiting low density due to the presence of low- Z lithium-containing phases.

4.4.3.2 Alloy M2

The solidification behavior of alloy M2 is shown in Fig. 4.9. Compared to M1, alloy M2 exhibits a higher liquidus temperature, reflecting its increased aluminum content. The left panel indicates that solidification begins with the formation of the HCP_A3 phase,



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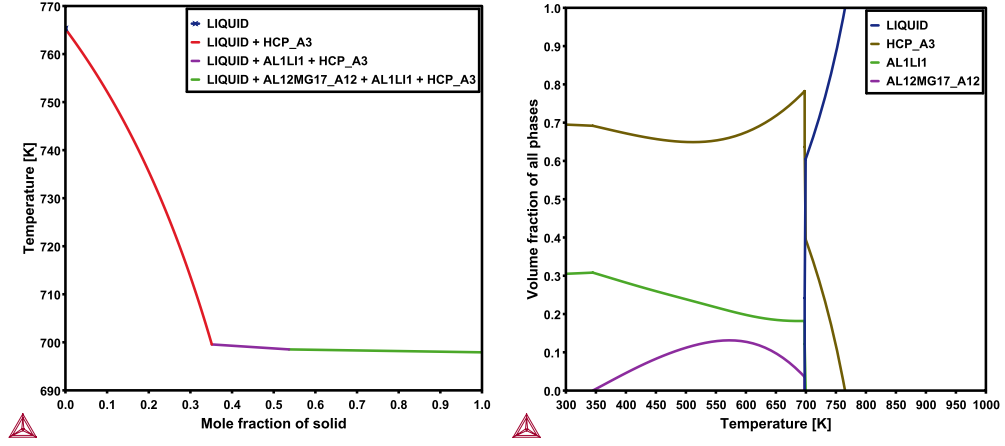
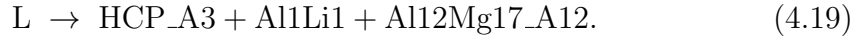


Figure 4.9: For alloy-M2, (left panel) mole fraction of solids and (right panel) volume fraction of solids [42].

followed by monovariant reaction involving (L, Al1Li1, HCP_A3). Finally, formation of a multi-phase mixture via an invariant reaction involving (L, Al12Mg17_A12, Al1Li1, HCP_A3). The invariant reaction is given as,



The volume fraction evolution shown in the right panel reveals that the HCP_A3 phase remains the dominant constituent across most of the solidification range. However, the contribution of Al12Mg17_A12 becomes increasingly significant at lower temperatures, while Al1Li1 contributes a moderate but non-negligible fraction.

4.4.3.3 Alloy M3

Figure 4.10 shows the solidification characteristics of alloy M3, which possesses a comparatively higher lithium content. The left panel indicates a more complex solidification path, with multiple phase fields encountered as the system cools from the liquid state. Primary solidification initiates with the formation of the intermetallic AlLiMg-T phase.



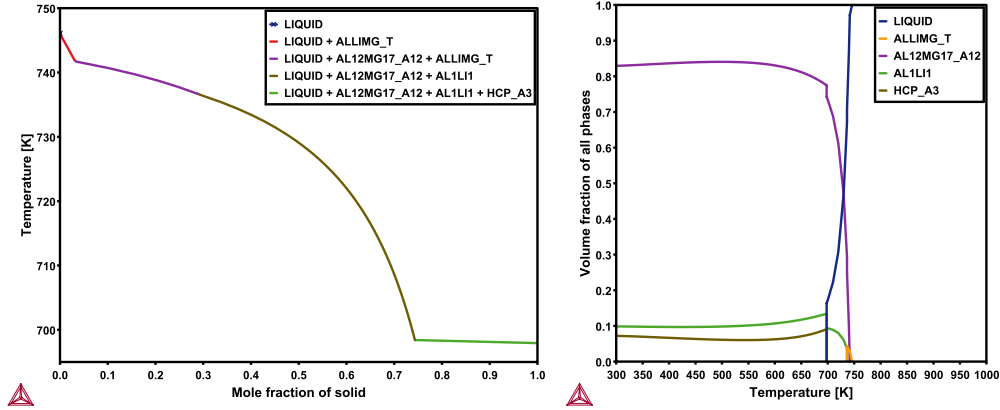


Figure 4.10: For alloy-M3, (left panel) mole fraction of solids and (right panel) volume fraction of solids [42].

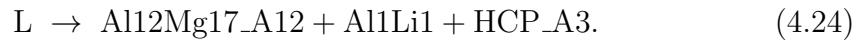
As cooling proceeds, additional solid phases appear through successive monovariant reactions involving (L, Al12Mg17_A12, AlLiMg_T) and (L, Al12Mg17_A12, Al1Li1) phases.



Based upon the above two monovariant reaction, we propose the following invariant reaction,



At lower temperatures, the residual liquid undergoes a final invariant eutectic reaction producing additional HCP_A3 phase with the following invariant reaction,



The corresponding volume fraction plot (right panel of Fig. 4.10) highlights the strong dominance of lithium-containing intermetallics over a wide temperature range. In particular, Al12Mg17_A12 occupies a substantial volume fraction prior to complete solidification, while Al1Li1 and HCP_A3 contribute smaller but stable fractions. It is important to note that the formation of multiple intermetallic

4.4 Thermodynamical simulation using Thermo-Calc software

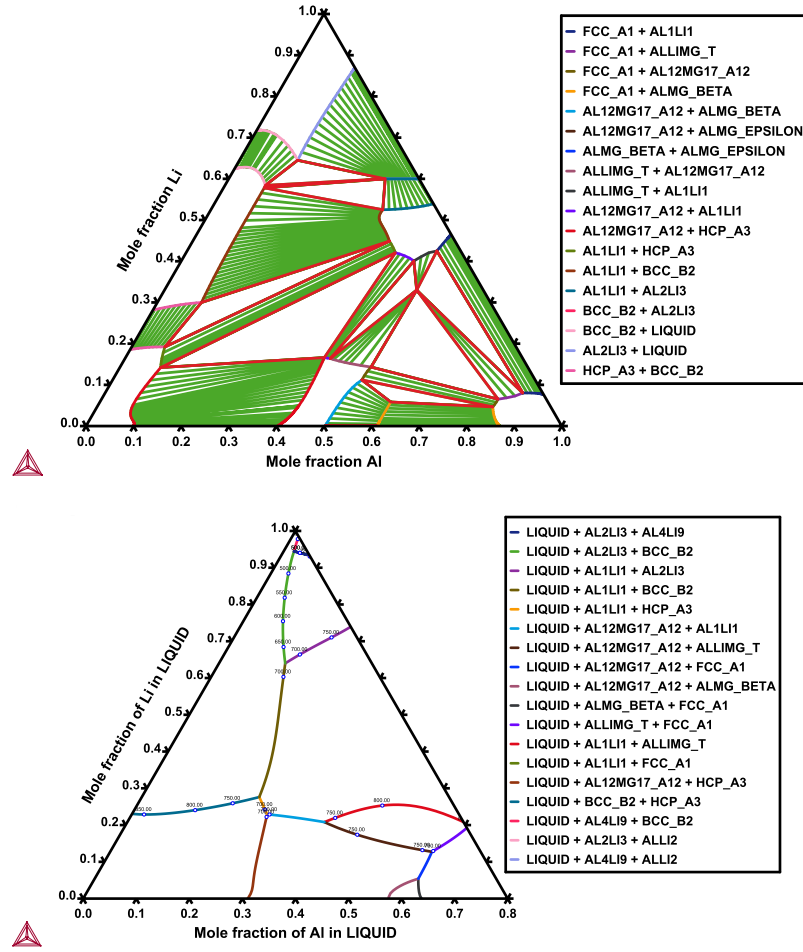


Figure 4.11: Top: Ternary plot, bottom: liquidus projection for Mg-Al-Li based alloy at 673 K [42].

phases in the studied alloy system is a good candidate for achieving good mechanical strength.

The overall phase stability and solidification trends of Mg–Al–Li alloys are summarized in Fig. 4.11. The top panel shows the equilibrium ternary phase diagram at 673 K, revealing extensive multiphase regions involving combinations of HCP_A3, Al1Li1, and Al12Mg17_A12. The broad stability fields observed in this diagram underscore the thermodynamic feasibility of tailoring alloy compositions to achieve desired phase fractions.

The bottom panel of Fig. 4.11 illustrates the monovariant reaction paths in the

Mg–Al–Li ternary system, represented as liquidus projection trajectories on the Gibbs composition triangle. Each curve corresponds to a two-phase equilibrium between the liquid and a single solid phase, evolving continuously with temperature during solidification. The liquidus surface is partitioned into regions corresponding to primary crystallization of HCP_A3, AlLi1, Al12Mg17_A12, and related phases. Along a monovariant path, the system satisfies the Gibbs phase rule with one degree of freedom, such that the liquid composition changes progressively as two solid precipitates. The direction of each trajectory indicates the evolution of the liquid composition as cooling proceeds, while the associated solid phase remains in equilibrium with the liquid over a finite temperature interval. The monovariant lines connect primary phase fields to terminal invariant points resulting three solid phases equilibrium with liquid. Transitions between different monovariant paths reflect changes in the stable solid phases as the liquid composition crosses phase boundaries in the ternary diagram. Such monovariant trajectories provide a thermodynamic map of solidification routes, which is essential for understanding how small variations in alloy composition lead to distinct solidification sequences. The Fig. 4.11, therefore, serves as a fundamental framework for interpreting phase evolution in multicomponent Mg–Al–Li alloys under near-equilibrium cooling conditions. The M1, M2, and M3 ternary alloys with selected composition are consistent with the solidification paths observed in Figs. 4.8–4.10, validating the internal consistency of the thermodynamic predictions. Table I presents a comparative summary of radiation length X_0 , elastic modulus E , figure of merit $X_0E^{1/3}$, density, and melting temperature for the designed Mg–Al–Li alloys together with commonly used benchmark beampipe materials. The results clearly demonstrate that targeted compositional tuning within the Mg–Al–Li system leads to a substantial enhancement in radiation length while maintaining competitive mechanical performance. Among the Al-rich compositions, alloys A1 and A2 exhibit radiation lengths of $X_0 = 0.1178$ m and 0.1240 m, respectively, both exceeding that of pure aluminum ($X_0 = 0.089$ m). This improvement arises

4.4 Thermodynamical simulation using Thermo-Calc software

Material	Radiation Length (X_0) [m]	Elastic Modulus (E) [GPa]	$X_0 E^{1/3}$	Density [kg/m ³]
Al-rich alloys				
A1 (Al _{61.5} Li _{10.8} Mg _{27.7})	0.1178	99.66	0.5460	2108
A2 (Al ₆₆ Li _{19.4} Mg _{14.6})	0.1240	117.43	0.6072	2035
Mg-rich alloys				
M1 (Al _{23.9} Li _{29.3} Mg _{46.8})	0.1703	27.538	0.5140	1559
M2 (Al ₁₉ Li _{20.6} Mg _{60.4})	0.1592	66.07	0.6436	1619
M3 (Al _{39.8} Li _{20.1} Mg _{40.1})	0.1428	104.64	0.6729	1795
Benchmark and reference materials				
Stainless Steel 304	0.0176	193	0.1017	7930
Aluminum (Al)	0.089	70	0.37	2699
Al-Ti-V alloy (Ref. [26])	0.0756	166.2	0.416	3259

Table 4.8: Comparison of radiation length, elastic modulus, figure of merit, and density for the designed Mg–Al–Li alloys and benchmark beampipe materials [42].

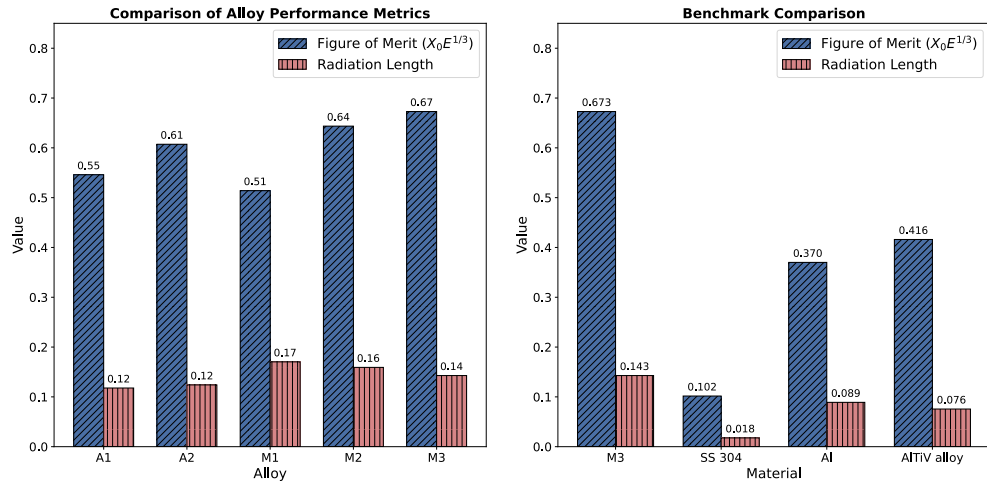


Figure 4.12: Left: A comparison of lightweight alloys (A1, A2, M1, M2, and M3) in terms of radiation length and figure of merit. Right: Benchmark comparison of M3 alloy with Stainless Steel 304, Aluminum, and Aluminum-rich alloy [42].

from the incorporation of low- Z lithium and magnesium, which effectively reduces the effective atomic number and density of the alloy. The elastic moduli of A1 and A2 remain in the moderate range of $E \approx 100$ – 120 GPa, resulting in figures of merit $X_0 E^{1/3} = 0.546$ and 0.607 , respectively. These values represent a clear improvement over conventional aluminum alloys and are significantly higher than those of stainless steel. A more pronounced enhancement in radiation length is observed for the Mg-rich alloys. Alloy M1 exhibits $X_0 = 0.1703$ m, nearly twice that of aluminum, albeit with a relatively low elastic modulus of $E = 27.5$ GPa. In contrast, alloys M2 and M3 provide a more favorable balance between radiation transparency and mechanical stiffness. Alloy M2 combines a high radiation length of $X_0 = 0.1592$ m with an elastic modulus of $E = 66.1$ GPa, yielding a figure of merit of $X_0 E^{1/3} = 0.644$. Alloy M3 further improves this balance, achieving $X_0 = 0.1428$ m and $E = 104.6$ GPa, corresponding to the highest figure of merit among all investigated compositions, $X_0 E^{1/3} = 0.673$.

For comparison, stainless steel 304, despite its high elastic modulus ($E = 193$ GPa), exhibits a very low radiation length ($X_0 = 0.0176$ m), resulting in a small figure of merit of $X_0 E^{1/3} = 0.102$. Pure aluminum shows a moderate improvement ($X_0 E^{1/3} = 0.37$), while the Al–Ti–V alloy offers increased stiffness but remains limited by its radiation transparency, yielding $X_0 E^{1/3} = 0.416$. In contrast, all designed Mg–Al–Li alloys outperform these conventional materials, with figures of merit that are approximately 1.5–2 times higher than aluminum and Al–Ti–V alloys, and up to 5–6 times higher than stainless steel. Overall, the results establish that Mg–Al–Li alloys constitute a promising materials platform for accelerator beampipe applications. By exploiting the low atomic numbers of lithium and magnesium while retaining aluminum as a stabilizing matrix element, these alloys achieve a significantly improved balance between radiation transparency, density, and mechanical performance.

Figure 4.12 provides a visual comparison of the radiation length and figure of merit for the designed Mg–Al–Li alloys and selected benchmark materials. The

4.4 Thermodynamical simulation using Thermo-Calc software

left panel highlights the systematic variation of these metrics across the alloy series A1, A2, M1, M2, and M3. While the radiation length increases significantly with higher Mg and Li content, the figure of merit reflects the competing influence of elastic stiffness. In particular, alloy M3 exhibits the highest $X_0 E^{1/3}$ among all investigated compositions, confirming that it offers the most favorable balance between radiation transparency and mechanical performance within the ternary system. The right panel places the best-performing alloy (M3) in direct comparison with conventional beampipe materials. Despite stainless steel and Al–Ti–V alloys possessing higher elastic moduli, their limited radiation lengths result in substantially lower figures of merit. Pure aluminum performs better than stainless steel but remains clearly inferior to the optimized Mg–Al–Li alloy. This comparison highlights that improvements in radiation length significantly enhance the overall performance metric relevant for beampipe applications, and that appropriately designed low- Z ternary alloys can outperform conventional materials by a substantial margin.

Summary

This chapter presents a comprehensive study of particle accelerator technology, with a particular focus on beampipe design, material optimization, and advanced computational approaches for developing next-generation beampipe materials. It integrates concepts from accelerator physics, materials science, thermodynamics, and machine learning to propose optimized alloy systems suitable for high-performance applications.

Section 4.1 introduces the fundamental principles of particle accelerators, highlighting their evolution from early electrostatic devices to modern high-energy facilities such as the Large Hadron Collider. The roles of major experiments, including ATLAS, CMS, LHCb, and ALICE, are briefly discussed to establish the broader experimental context. The importance of beam transport systems,

particularly the beampipe, is emphasized as a critical component governing beam stability, vacuum quality, and detector performance.

Section 4.2 focuses on the beampipe as a central element of accelerator infrastructure. The physical and functional requirements of beampipes are discussed in detail, including ultra-high vacuum maintenance, radiation transparency, and mechanical stability. Key material properties such as radiation length, atomic number dependence, stopping power, and elastic modulus are analyzed in the context of minimizing beam–material interactions. The advantages and limitations of commonly used materials, particularly beryllium, are critically examined, motivating the need for alternative materials with improved performance and reduced toxicity.

Section 4.3 presents the theoretical and computational framework for evaluating material properties. The radiation length is calculated using both semi-empirical relations for pure elements and compositional models for multi-component alloys. In parallel, the elastic properties are computed using density functional theory within the VASP framework as well as experimentally. The methodology includes structural optimization, calculation of elastic stiffness tensors, and evaluation of bulk, shear, and Young’s moduli using the Voigt–Reuss–Hill averaging scheme. These calculations provide a robust basis for assessing the mechanical suitability of candidate materials for beampipe applications.

This section also describes the synthesis and experimental characterization of Al–Ti–V alloys. The alloys are fabricated using vacuum arc melting and characterized using X-ray diffraction, scanning electron microscopy, and energy-dispersive spectroscopy. The microstructural analysis reveals the coexistence of intermetallic and solid-solution phases, with a high volume fraction of the Al_3Ti phase contributing to enhanced mechanical strength. The experimentally obtained elastic properties are shown to be consistent with theoretical predictions, validating the computational approach.

Section 4.4 introduces thermodynamic simulations using Thermo-Calc to study

4.4 Thermodynamical simulation using Thermo-Calc software

phase stability and solidification behavior in multi-component alloys. Both equilibrium and non-equilibrium (Gulliver–Scheil) models are employed to understand phase evolution during solidification. These simulations provide detailed insights into phase formation sequences, liquidus temperatures, and compositional effects, enabling systematic exploration of alloy design space.

Section 4.4.1 presents the integration of machine learning techniques for predictive materials design in Al–Ti–V alloys. Two hybrid models combining classification and regression algorithms are developed to predict phase formation, density, and radiation length. The models are trained on Thermo-Calc-generated datasets and demonstrate high predictive accuracy, particularly for phase classification using XGBoost. The results highlight the effectiveness of machine learning in accelerating materials discovery and optimization.

Section 4.4.2 extends the study to Mg–Al–Li ternary alloys, where detailed thermodynamic and solidification analyses are performed. The phase evolution of multiple alloy compositions is investigated, revealing complex multi-phase behavior involving lithium-rich and magnesium-rich phases. The interplay between composition, phase stability, and solidification pathways is systematically analyzed using phase diagrams, monovariant reactions, and invariant transformations.

Table 4.8 presents a comprehensive comparison of material performance metrics, including radiation length, elastic modulus, density, and the figure of merit $X_0 E^{1/3}$. The results demonstrate that the designed Mg–Al–Li alloys significantly outperform conventional materials such as stainless steel and aluminum, achieving superior radiation transparency while maintaining adequate mechanical strength. In particular, optimized compositions exhibit a remarkable balance between low density, high radiation length, and competitive stiffness, making them promising candidates for next-generation beampipe applications.

Overall, this chapter establishes an interdisciplinary framework for the design and optimization of advanced materials for accelerator beampipes. By combining

first-principles calculations, thermodynamic modeling, experimental validation, and machine learning techniques, it provides a systematic pathway for developing materials with enhanced performance. The findings offer valuable insights into the role of low- Z alloy systems in improving beam transparency and detector performance, and they open new avenues for accelerator physics.

Bibliography

- [1] B. M. Blidaru [ALICE], JINST **17**, C09006 (2022).
- [2] T. Lauritsen, <https://resolver.caltech.edu/CaltechETD:etd-10072004-162516>.
- [3] V. Smirnov, Phys. Part. Nucl. **52**, 913 (2021).
- [4] H. Winick and A. Bienenstock, Annu. Rev. Nucl. Part. Sci. **28**, 33 (2003).
- [5] Y. Iwata and K. Noda, Comprehensive Biomedical Physics **8**, 153 (2014).
- [6] W. Busza, J. W. Harris and S. Nagamiya, [arXiv:2504.08406 [nucl-ex]].
- [7] L. Evans and P. Bryant, JINST **3**, S08001 (2008).
- [8] O. Brüning, M. Klein, S. Myers and L. Rossi, Nature Rev. Phys. **6**, 628 (2024).
- [9] R. Sahoo and T. K. Nayak, “Possible early universe signals in proton collisions at the Large Hadron Collider,” Curr. Sci. **121**, 1403 (2021).
- [10] W. Busza, K. Rajagopal and W. van der Schee, “Heavy Ion Collisions: The Big Picture, and the Big Questions,” Ann. Rev. Nucl. Part. Sci. **68**, 339 (2018).
- [11] R. Veness, G. Simmons, and C. Dorn, Development of Beryllium Vacuum Chamber Technology for the LHC. Proceedings of IPAC2011, San Sebastian, Spain (2011).

- [12] H. Burkhardt, Accelerators for Particle Physics. Handbook of Particle Detection And Imaging 139 (2012).
- [13] J. Knaster, The vacuum chamber in the interaction region of particle colliders: a historical study and developments implementations in the LHCb experiment at CERN, Ph.D. Thesis, CERN-THESIS-2005-009 (2004).
- [14] O. Bruning, and L. Rossi, High Luminosity Large Hadron Collider, The: The New Machine For Illuminating The Mysteries Of Universe. (World Scientific, 2015).
- [15] M. Gupta, Calculation of radiation length in materials. PH-EP-Tech-Note-2010-013 (2010).
- [16] R. Veness, S. Karppinen, J. Knaster, P. Lepeule and G. Schneider, “Design of beampipes for LHC experiments,” Vacuum **64**, 467 (2002).
- [17] R. Veness, C. Dorn and G. Simmons, “Development of Beryllium Vacuum Chamber Technology for the LHC,” Conf. Proc. C **110904**, 1578 (2011), IPAC-2011-TUPS024.
- [18] D. Ayres, R. Diebold, *et al.*, FERMILAB-DESIGN-1981-02. “Design Report of the Fermilab Collider Detector Facility (CDF), August, 1981,”
- [19] G. Zunica [LHCb], Nucl. Instrum. Meth. A **1047**, 167804 (2023).
- [20] A. Correa, Calculating electronic stopping power in materials from first principles. Computational Materials Science. **150**, 291 (2018).
- [21] ALICE Collaboration, Letter of intent for ALICE 3: A next-generation heavy-ion experiment at the LHC [arXiv:2211.02491 [physics.ins-det]].
- [22] G. Aad *et al.* [ATLAS], “The ATLAS Experiment at the CERN Large Hadron Collider: A Description of the Detector Configuration for Run 3,” [arXiv:2305.16623 [physics.ins-det]].

BIBLIOGRAPHY

- [23] J. C. Brunet, J. C. Godot, *et al.*, "The design and construction of transparent vacuum chambers for interaction areas of colliding beam machines," IEEE Trans. Nucl. Sci. **26**, 4083 (1979).
- [24] S. Deb, G. Singh, and P. Gupta, Indus-2 Synchrotron Radiation Source: current status and utilization. Journal Of Physics: Conference Series. **425**, 072009 (2013).
- [25] D. Bhawalkar, G. Singh, and R. Nandedkar, Synchrotron radiation sources INDUS-1 and INDUS-2. Pramana. **50**, 467 (1998).
- [26] K. Singh, K. Goswami, R. Sahoo and S. Samal, Phys. Rev. Accel. Beams **28**, 043101 (2025).
- [27] U. H. Stolzenberg, Radiation length measurements with high-resolution telescopes. Ph.D Thesis (2019), doi:10.53846/goediss-7624.
- [28] L. Notari, M. Pasquali, F. Carra, *et al.*, Materials adopted for particle beam windows in relevant experimental facilities. Phys. Rev. Accel. Beams. **27**, 024801 (2024).
- [29] R. Jose Da Silva, CERN Technical Report, Geneva, 2025. Available at: <https://cds.cern.ch/record/2951644>.
- [30] A. Jain *et al.*, APL Mater. **1**, 011002 (2013).
- [31] G. M. Nishibuchi *et al.*, J. Appl. Phys. **134**, 173101 (2023).
- [32] Y. Le Page and P. Saxe, Phys. Rev. B **63**, 174103 (2001).
- [33] W. Voigt, Lehrbuch der Kristallphysik (B. G. Teubner, Leipzig, 1910).
- [34] A. Reuss, Z. Angew. Math. Mech. **10**, 266 (1930).
- [35] R. Hill, Proc. Phys. Soc. A **65**, 349 (1952).

- [36] T. Chen and C. Guestrin XGBoost: A Scalable Tree Boosting System [arXiv:1603.02754].
- [37] Available Online: <https://xgboost.readthedocs.io/en/stable/> (Accessed on: 05 Apr 2024).
- [38] G. Louppe, Understanding Random Forests: From Theory to Practice [arXiv:1407.7502].
- [39] S. Samal, K. Biswas, and G. Phanikumar Solidification behavior in newly designed Ni-rich Ni-Ti-based alloys. Metallurgical And Materials Transactions A. **47**, 6214 (2016).
- [40] Q.Zuo, F. Liu, L. Wang and C. Chen, Evolution of secondary phases formed upon solidification of a Ni-based alloy. Metallurgical And Materials Transactions A. **44**, 3014 (2013).
- [41] P. Wang, Y. Du, and S. Liu, Calphad **35**, 523 (2011).
- [42] K. Singh, K. Goswami, R. Sahoo and S. Samal, [arXiv:2601.12836 [physics.acc-ph]].

Chapter 5

Analysis of $\phi(1020)$ meson production at $\sqrt{s_{\text{NN}}} = 5.36$ TeV in Pb–Pb collisions with ALICE at the Large Hadron Collider

“In the history of science, we often find that the study of some natural phenomenon has been the starting point in the development of a new branch of knowledge.”

— Dr. C.V. Raman

The quest to understand the fundamental nature of matter has been a central theme in the development of modern physics. Beginning with the discovery of the atom and the realization that matter is composed of discrete building blocks, scientific progress has continually pushed the boundaries of knowledge toward ever smaller length scales. The subsequent identification of subatomic constituents, such as electrons, protons, and neutrons, marked a significant milestone in unveiling the internal structure of matter. With the advent of high-energy physics, these nucleons were further resolved into more fundamental particles, quarks and gluons, governed by the theory of Quantum Chromodynamics (QCD). As the established framework for the strong interaction, QCD describes how quarks and gluons interact and bind together to form hadrons, while also predicting

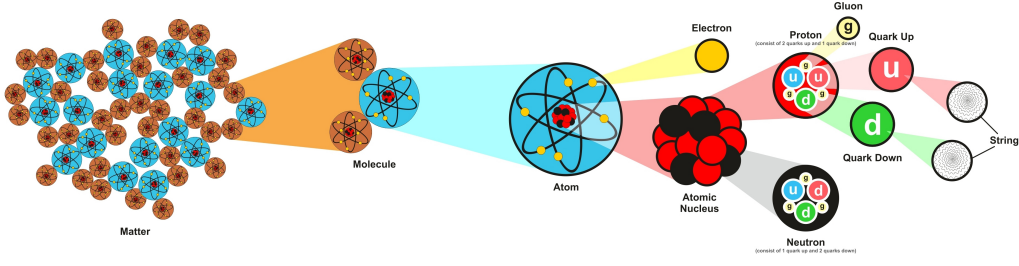


Figure 5.1: Hierarchical structure of matter illustrating the progression from macroscopic matter to molecules, atoms, and atomic nuclei, and further to subatomic constituents such as quarks and gluons governed by Quantum Chromodynamics [1].

the existence of a deconfined state of matter, known as the quark–gluon plasma (QGP), under extreme conditions of temperature and energy density. Relativistic heavy-ion collisions provide a unique experimental avenue to recreate such extreme conditions in the laboratory, thereby enabling the study of QCD matter beyond the confines of ordinary nuclear matter. Experiments at the Large Hadron Collider (LHC), particularly those conducted with the A Large Ion Collider Experiment (ALICE) detector, are specifically designed to investigate the properties of the QGP formed in these collisions. In this context, the study of hadronic resonances serves as an essential tool for probing the medium, as their production and propagation are sensitive to the underlying dynamics of the strongly interacting system. Among these probes, the $\phi(1020)$ meson holds special significance due to its hidden strangeness content ($s\bar{s}$) and its relatively small interaction cross section with the hadronic medium [2–5]. These properties make the ϕ meson an effective messenger of the early stages of the collision, carrying valuable information about strangeness production and medium modification effects. Furthermore, its decay into dilepton channels, particularly the dimuon channel, provides a clean experimental signal with minimal final-state interactions, allowing for a more direct investigation of the properties of the produced medium. In this chapter, a detailed analysis of $\phi(1020)$ meson production in Pb–

5.1 A Large Ion Collider Experiment

Pb collisions at $\sqrt{s_{\text{NN}}} = 5.36$ TeV is presented using Run-3 data collected with the ALICE detector at the LHC. The study focuses on the forward rapidity region, utilizing the dimuon decay channel to reconstruct the ϕ meson and extract its production characteristics. Through this analysis, insights into the behavior of strange quark production and the properties of the QCD medium formed in heavy-ion collisions are obtained, contributing to a deeper understanding of strongly interacting matter under extreme conditions.

*“Tiny quarks by strong force confined,
Electrons by electromagnetic fields aligned,
Neutrinos through weak interactions slide,
While gravity holds the universe combined.”*

5.1 A Large Ion Collider Experiment

The A Large Ion Collider Experiment is one of the four major experiments at the Large Hadron Collider at CERN, specifically designed to study the properties of strongly interacting matter at extreme energy densities. Its primary objective is to investigate the formation and characteristics of the quark–gluon plasma created in ultra-relativistic heavy-ion collisions. In addition to Pb–Pb collisions, ALICE also studies proton–proton (pp) and proton–nucleus (p–A) interactions to provide essential baseline measurements.

5.1.1 Detector overview

ALICE consists of a central barrel covering mid-rapidity and a forward muon spectrometer (upgraded to Muon Forward Tracker system) optimized for muon detection. The detector is embedded in a large solenoidal magnet providing a uniform magnetic field of $B = 0.5$ T. The coordinate system used in ALICE is defined with the z -axis along the beam direction, the x -axis pointing towards the

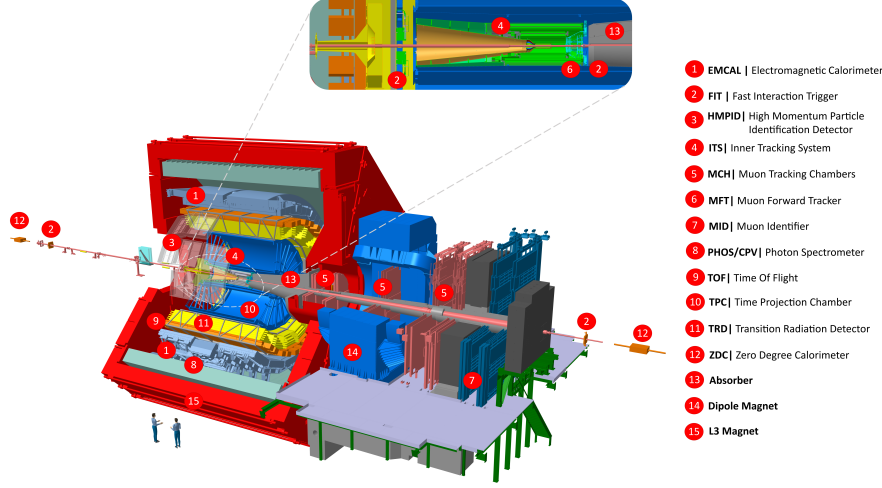


Figure 5.2: Schematic layout of ALICE 2 detector during Run 3 data-taking of LHC [6].

center of the LHC ring, and the y -axis pointing upwards. The kinematic variables commonly used are transverse momentum (p_T), rapidity (y), and pseudorapidity (η), defined as [7]:

$$\eta = -\ln \left[\tan \left(\frac{\theta}{2} \right) \right], \quad (5.1)$$

where θ is the polar angle with respect to the beam axis.

The central barrel covers the pseudorapidity range $|\eta| < 0.9$, while the forward muon spectrometer covers the range $-4.0 < \eta < -2.5$.

5.1.2 Central barrel detectors

The central barrel detectors of the ALICE experiment cover the mid-pseudorapidity region ($|\eta| < 0.9$) with full azimuthal coverage (2π). These detectors play a crucial role in charged particle tracking, primary and secondary vertex reconstruction, and particle identification over a broad momentum range. The central barrel is

5.1 A Large Ion Collider Experiment

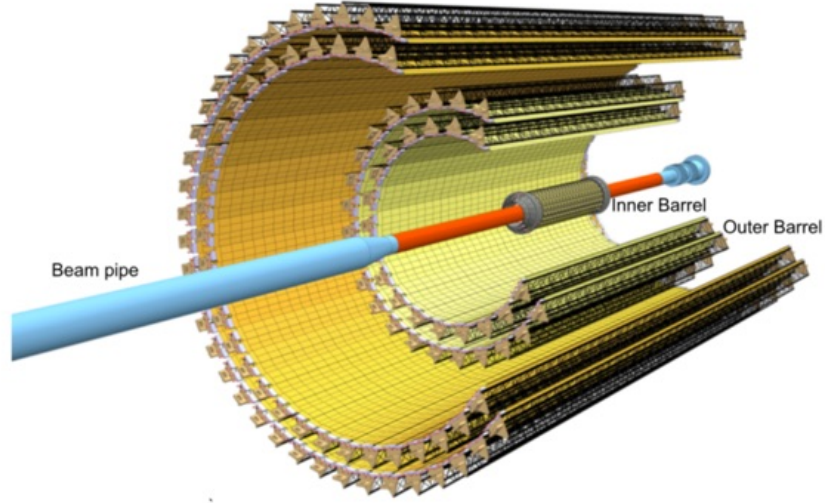


Figure 5.3: Schematic layout of ITS detector in ALICE 2 [8].

composed of several subsystems, including the Inner Tracking System (ITS) [9], the Time Projection Chamber (TPC) [10], the Transition Radiation Detector (TRD) [11], the Time-of-Flight (TOF) detector [12], the High Momentum Particle Identification Detector (HMPID) [13], the Photon Spectrometer (PHOS) [14], and the Electromagnetic Calorimeter (EMCal) [15]. The spatial arrangement and geometrical configuration of these detectors within the ALICE setup are illustrated in Fig. 5.2.

- **Inner Tracking System :**

The Inner Tracking System is the detector closest to the beampipe and plays a fundamental role in the precise reconstruction of primary and secondary vertices, as well as in the tracking and identification of low-momentum charged particles [8]. The accurate determination of primary and secondary vertices is essential for the reconstruction of short-lived particles, including light- and heavy-flavor hadrons and resonances. In addition, the ITS significantly improves the overall tracking performance of the ALICE central barrel by providing high-resolution space points close to the interaction region,

thereby complementing the measurements of the Time Projection Chamber. In Run 3, the upgraded ITS (ITS2) is based entirely on Monolithic Active Pixel Sensor (MAPS) technology and consists of seven concentric layers of silicon pixel detectors, arranged into an Inner Barrel and an Outer Barrel configuration.

- **Time Projection Chamber:**

The Time Projection Chamber is the primary detector of the ALICE central barrel for charged particle tracking and particle identification. It covers the mid-pseudorapidity region ($|\eta| < 0.9$) with full azimuthal acceptance (2π). The TPC is capable of reconstructing primary charged particle tracks with transverse momentum in the range 0.1 GeV/ c to 100 GeV/ c with excellent momentum resolution [16]. Combined with the Inner Tracking System, the TPC enables precise momentum determination through the measurement of track curvature in the solenoidal magnetic field. The momentum resolution is better than 1% at low transverse momentum and gradually degrades to about 20% at $p_T \sim 100$ GeV/ c .

The TPC simultaneously measures the momentum and specific energy loss (dE/dx) of charged particles by collecting the ionization charge on the read-out pads. This allows for efficient particle identification (PID), particularly in the low- p_T region. The energy loss of charged particles traversing the active volume of TPC is described by the Bethe–Bloch formula:

$$-\frac{dE}{dx} = \frac{4\pi N z^2 e^4}{mc^2 \beta^2} \left[\ln \left(\frac{2mc^2 \beta^2 \gamma^2}{I} \right) - \beta^2 - \frac{\delta(\beta)}{2} \right], \quad (5.2)$$

where z , β , and γ represent the charge, velocity, and Lorentz factor of the particle, respectively, I is the mean excitation energy of the medium, and $\delta(\beta)$ accounts for density effect corrections.

Overall, the TPC serves as the backbone of tracking and particle iden-

5.1 A Large Ion Collider Experiment

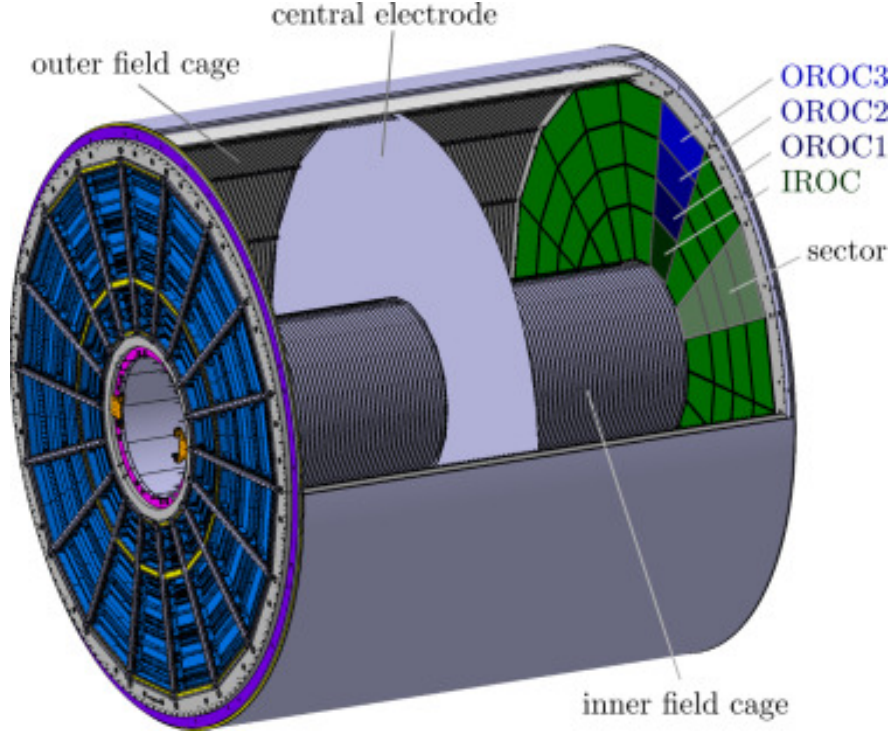


Figure 5.4: Schematic layout of TPC detector in ALICE 2 [16].

tification in ALICE, providing high-precision measurements essential for understanding the dynamics of heavy-ion collisions.

- **Time-of-Flight:**

The Time-of-Flight detector is a key subsystem of the ALICE central barrel dedicated to particle identification over an intermediate momentum range. It is based on Multi-gap Resistive Plate Chamber (MRPC) technology, which provides excellent time resolution of about $\sim 50\text{--}80$ ps. The TOF detector is arranged in a cylindrical geometry around the TPC and covers the pseudorapidity region $|\eta| < 0.9$ with full azimuthal acceptance. The TOF measures the flight time of charged particles from the interaction point to the detector surface. By combining this information with the track length and momentum obtained from the ITS and TPC, the particle velocity (β) can be determined. This allows for the calculation of the particle mass

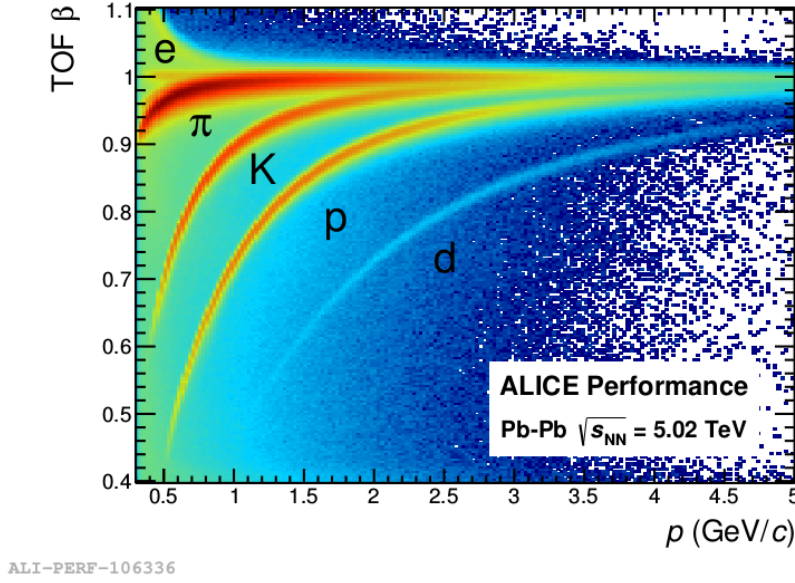


Figure 5.5: The particle identification by TOF is shown [17].

using the relation:

$$m = \frac{p}{c} \sqrt{\frac{1}{\beta^2} - 1}, \quad (5.3)$$

where p is the particle momentum. The TOF enables clear separation of charged hadrons such as pions, kaons, and protons in the intermediate transverse momentum range ($p_T \sim 0.5\text{--}3 \text{ GeV}/c$). It also plays an important role in event time determination and improves the overall PID performance when combined with other detectors in ALICE.

- **Electromagnetic Calorimeter:**

The Electromagnetic Calorimeter is designed to measure the energy of electrons and photons produced in high-energy collisions. It is a sampling calorimeter composed of alternating layers of absorber material (lead) and scintillator, providing a compact and efficient detection system [15]. The EMCal covers a pseudorapidity range of $|\eta| < 0.7$ and a partial azimuthal acceptance. When high-energy electrons or photons enter the EMCal, they initiate electromagnetic showers through processes such as bremsstrahlung

5.1 A Large Ion Collider Experiment

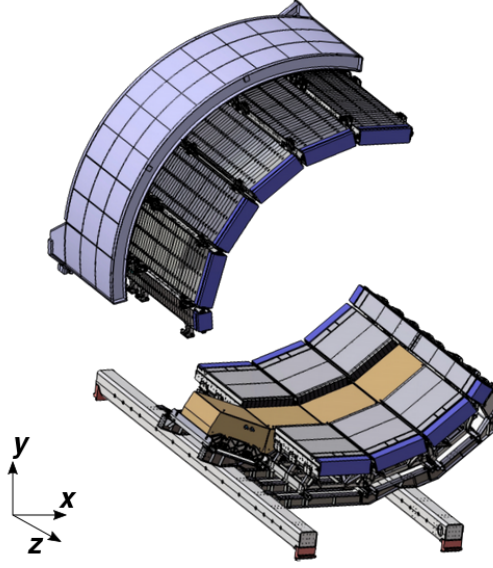


Figure 5.6: Schematic view of the EMCal illustrating the module position on two approximately opposite locations in azimuth [15].

and pair production. The deposited energy is sampled by the scintillator layers and converted into light signals, which are then read out by photodetectors. The EMCal is essential for the reconstruction of jets, direct photons, and high- p_T electrons, making it a crucial tool for studying parton energy loss and jet quenching phenomena in the QGP.

5.1.3 Forward muon detectors

The forward region of ALICE is dedicated to the detection of muons, which are key probes of heavy-ion collisions due to their minimal interaction with the strongly interacting medium.

The ALICE experiment is equipped with a dedicated forward muon detection system designed to study heavy-flavour production, quarkonia, and low-mass vector mesons via their dimuon decay channels. This system covers the pseudorapidity range $-4.0 < \eta < -2.5$ and consists of three main subsystems: the Muon Chambers (MCH), the Muon Identifier (MID), and the Muon Forward Tracker (MFT).

Together, these detectors provide precise tracking, triggering, and vertexing capabilities for muons in the forward region.

- **Muon Chambers:** The Muon Chambers form the core tracking system of the muon spectrometer and are responsible for reconstructing muon trajectories and determining their momentum. The MCH is located downstream of a thick hadron absorber, which suppresses hadrons, photons, and electrons, allowing predominantly muons to reach the tracking system [18, 19].

The MCH consists of five tracking stations, each composed of two detection planes equipped with Cathode Pad Chambers (CPCs). Stations 1 and 2 are positioned upstream of the dipole magnet, station 3 is located inside the magnetic field, and stations 4 and 5 are placed downstream. The dipole magnet provides a field integral of approximately $3.5 \text{ T} \cdot \text{m}$, enabling precise momentum measurement through track curvature. The spatial resolution of the MCH is of the order of $\sim 100 \mu\text{m}$ in the bending plane, which is crucial for accurate invariant mass reconstruction of dimuon pairs. The detector operates with a gas mixture optimized for stable charge amplification and fast signal response. The MCH plays a central role in reconstructing dimuon invariant mass spectra and identifying resonances such as the $\phi(1020)$ meson.

- **Muon Identifier:** The Muon Identifier is located downstream of a thick iron muon filter (also referred to as the absorber wall), which further suppresses residual hadrons that survive the front absorber. The primary purpose of the MID is to provide fast triggering and to discriminate between muons and hadrons [18–20].

The MID consists of two stations, each comprising two detection planes based on Resistive Plate Chamber (RPC) technology [20]. These detectors offer excellent timing resolution and high detection efficiency, making them

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suitable for trigger generation in high-rate environments. The MID is segmented into multiple strips that allow for coarse position determination and track matching with the MCH.

In Run 3, the MID has been upgraded to operate in a continuous read-out mode, replacing the previous trigger-only configuration. This upgrade enables improved matching between tracking and trigger information and enhances the overall muon identification capability. The MID is essential for reducing background contamination and ensuring high-purity muon samples for physics analysis.

- **Muon Forward Tracker:** The Muon Forward Tracker is a major upgrade introduced in Run 3 to significantly enhance the vertexing and tracking capabilities of the muon system. It is positioned upstream of the hadron absorber and is based on silicon pixel technology similar to the upgraded ITS.

The MFT consists of five disks of Monolithic Active Pixel Sensors (MAPS), arranged perpendicular to the beam direction. It covers the pseudorapidity range $-3.6 < \eta < -2.5$ and provides high-resolution spatial measurements with a precision of the order of a few micrometers. The low material budget of the MFT minimizes multiple scattering effects, which is critical for accurate track extrapolation [18, 19, 21]. By combining MFT information with tracks reconstructed in the MCH, it is possible to significantly improve the pointing resolution of muon tracks to the primary vertex. This allows for the separation of prompt muons (originating from the primary interaction) from non-prompt muons (e.g., from heavy-flavour decays). The MFT thus plays a crucial role in heavy-flavour physics and enhances the sensitivity of measurements involving dimuon final states.

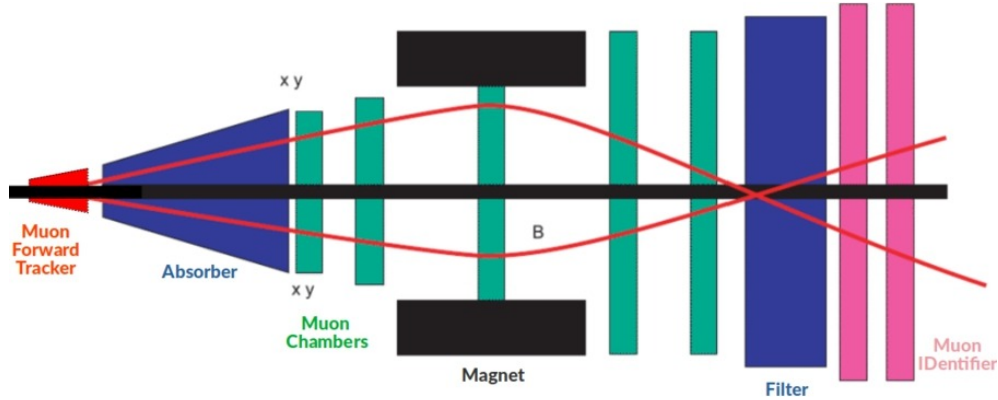


Figure 5.7: Sketch of the ALICE global muon tracking from LHC Run 3 in ALICE [18].

5.1.4 Combined performance for dimuon analysis

The combined operation of the MFT, MCH, and MID provides a powerful system for dimuon reconstruction in the forward region. The MCH ensures precise momentum measurement, the MID provides efficient muon identification and triggering, and the MFT enables accurate vertex determination. This synergy results in improved invariant mass resolution, better background suppression, and enhanced separation of signal and background contributions. Such capabilities are particularly important for the reconstruction of low-mass vector mesons like the $\phi(1020)$ via their dimuon decay channel, where clean signal extraction and precise kinematic measurements are essential for studying the properties of the QCD medium created in heavy-ion collisions.

- **Standalone muon tracks:** Standalone muon tracks are reconstructed using information solely from the Muon Chambers and the Muon Identifier, located downstream of the hadron absorber. In this approach, the track reconstruction relies on the tracking stations of the MCH, where the muon trajectory is determined from the measured space points and its momentum is obtained from the curvature induced by the dipole magnetic field. The

5.1 A Large Ion Collider Experiment

MID provides additional confirmation for muon identification by rejecting residual hadronic background.

Due to the presence of the thick front absorber, standalone muon tracks are affected by multiple scattering and energy loss fluctuations, which degrade the track pointing resolution at the interaction vertex. As a result, standalone tracks have limited capability in distinguishing prompt and non-prompt muons. The kinematic acceptance for standalone muons is typically defined in the forward pseudorapidity range $-4.0 < \eta < -2.5$ to ensure reliable reconstruction. Despite their limitations, standalone muon tracks provide a robust and efficient method for dimuon reconstruction.

- **Global muon tracks:** Global muon tracks are reconstructed by combining track information from the Muon Forward Tracker with the standalone muon tracks measured. This matching procedure significantly improves the track parameters by incorporating precise vertexing information from the MFT, which is located upstream of the hadron absorber. The inclusion of MFT hits allows for accurate extrapolation of muon tracks back to the primary vertex, resulting in enhanced pointing resolution and reduced uncertainties arising from multiple scattering.

The kinematic coverage for global muons is slightly reduced compared to standalone tracks, typically within $-3.6 < \eta < -2.5$, corresponding to the MFT acceptance, with similar transverse momentum thresholds. The combination of improved spatial resolution, vertexing capability, and background rejection makes global muon tracks particularly suitable for precision dimuon analyses. Invariant mass reconstruction using global muons leads to better mass resolution and cleaner signal extraction, which is essential for studying low-mass vector mesons such as the $\phi(1020)$ in heavy-ion collisions.

5.2 Analysis of $\phi(1020)$ meson with ALICE

The ϕ meson, with a mass of approximately $1.02 \text{ GeV}/c^2$, is a key probe of strangeness production and possible in-medium modifications of vector mesons in ultra-relativistic heavy-ion collisions. Owing to its $s\bar{s}$ quark content, the ϕ meson is directly related to strangeness enhancement, one of the classical signatures of the formation of a deconfined QCD medium. Because the ϕ interacts only weakly with non-strange hadrons, it is expected to retain information from the early and intermediate stages of the collision.

The ϕ meson decays predominantly into kaons:

$$\phi \rightarrow K^+ K^- \quad (49\%), \quad \phi \rightarrow K_L^0 K_S^0 \quad (34\%),$$

while the dilepton channels

$$\phi \rightarrow e^+ e^-, \quad \phi \rightarrow \mu^+ \mu^-,$$

have branching ratios of $\mathcal{O}(10^{-4})$ but offer unique advantages. In particular, muons suffer negligible final-state interactions and traverse the medium with minimal attenuation, making the dimuon channel a clean probe of the underlying spectral shape, mass, and width. The forward-rapidity region accessed by the ALICE muon spectrometer provides a favourable environment for low-mass dimuon studies. Moreover, forward rapidity probes a different region of the space-time evolution dominated by longitudinal expansion, providing complementary information on strangeness production mechanisms and collective behavior.

Our study presents the analysis strategy for extracting the ϕ meson in the dimuon decay channel in Pb–Pb collisions at $\sqrt{s_{\text{NN}}} = 5.36 \text{ TeV}$ recorded during Run 3 and compared to previous measurements at 5.02 TeV [22] and 2.76 TeV [23].

5.3 Selection criteria

5.2.1 Pb–Pb analysis Dataset at $\sqrt{s_{\text{NN}}} = 5.36$ TeV

This analysis is based on the dataset Dataset used: [EM_LHC23_PbPb_pass5_Dimuon](#) which corresponds to the derived data dedicated to dimuon analysis from pass5 for the 2023 Pb–Pb data-taking campaign at $\sqrt{s_{\text{NN}}} = 5.36$ TeV. The following 92 runs are used in our analysis:

544013, 544028, 544032, 544091, 544095, 544098, 544116, 544121, 544122, 544123, 544124, 544184, 544185, 544389, 544390, 544391, 544392, 544451, 544454, 544474, 544475, 544476, 544477, 544490, 544491, 544492, 544508, 544510, 544511, 544512, 544514, 544515, 544518, 544548, 544549, 544550, 544551, 544564, 544565, 544567, 544568, 544580, 544582, 544583, 544585, 544614, 544640, 544652, 544653, 544672, 544674, 544692, 544693, 544694, 544696, 544739, 544742, 544754, 544767, 544794, 544795, 544797, 544813, 544868, 544886, 544887, 544896, 544913, 544914, 544917, 544931, 544947, 544961, 544963, 544964, 544968, 544992, 545009, 545044, 545047, 545063, 545064, 545066, 545185, 545210, 545223, 545249, 545291, 545294, 545295, 545296, 545312.

5.3 Selection criteria

5.3.1 Event selection

Event selection in Run 3 Pb–Pb collisions at $\sqrt{s_{\text{NN}}} = 5.36$ TeV is performed by applying standard event-level selection criteria. The runs flagged as having good detector conditions in the run condition table are included in this analysis. The RCT label [CBT_muon](#) is used, which takes care that the following detectors were flagged with a good detector status during the data-taking period : FT0, ITS, TPC, MCH, and MID. The events having a vertex position within ± 10 cm of the

nominal interaction point, with a centrality range of (0-90)%, are selected. FT0C is specifically used for the centrality estimator. Furthermore, events passing the Sel8 trigger selection provided by the `o2-analysis-event-selection` framework are considered. The FT0AND (FT0A and FT0C) is used for event selection provided by `cfgRequireFT0AND`. For this analysis, a depth of 1000 is used for event mixing. The number of events before selection cuts is 1.3×10^9 , and after applying all event-level cuts, it is 1.29×10^9 .

Selection criterion	Applied requirement
Run Condition Table (RCT) label	CBT_muon
Good detector status	cfgRequireGoodRCT
Minimum-bias trigger selection	cfgRequireSel8
Forward trigger condition	cfgRequireFT0AND
ITS readout frame border rejection	cfgRequireNoITSROFB
Same bunch pileup rejection	cfgRequireNoSameBunchPileup
Time-frame border rejection	cfgRequireNoTFB
Primary vertex position	$ z_{\text{vtx}} < 10 \text{ cm}$

Table 5.1: Event-level selection criteria applied in this analysis [24].

5.3.2 Muon track selection

Currently, the muon-track selection for this analysis is based on standalone muon tracks reconstructed by matching tracks from the MCH and the MID. In the analysis we chose intermediate p_T range specifically $1.5 \leq p_T^{\mu\mu} \leq 7 \text{ GeV}/c$. Single-

5.4 Background subtraction

muon transverse momentum is required to satisfy $p_T^\mu > 0.8 \text{ GeV}/c$.

Observable	Applied cuts
Track transverse momentum	$p_T^{\text{track}} > 0.8 \text{ GeV}/c$
Pseudorapidity range	$-4.0 < \eta < -2.5$
Track type	MCH–MID (Standalone tracks)
MCH–MID matching quality	$\chi_{\text{MFTMCH}}^2 < 1e^{10}$
Number of MCH clusters	$N_{\text{cluster}}^{\text{MCH}} > 5$
Radial coordinate at absorber exit	$17.6 < R_{\text{Abs}} < 89.5 \text{ cm}$
$p \times \text{DCA}_{xy}$	$594 \text{ GeV}/c \cdot \text{cm}; R_{\text{Abs}} < 26.5 \text{ cm},$ $324 \text{ GeV}/c \cdot \text{cm}; R_{\text{Abs}} \geq 26.5 \text{ cm}$

Table 5.2: Selection criteria applied to single muon tracks used for dimuon reconstruction [24].

5.4 Background subtraction

The total dimuon invariant mass spectrum observed in the data can be decomposed into three main components:

$$N_{\text{OS}} = S_\phi + B_{\text{comb}},$$

where $S_\phi(m)$ represents the ϕ meson signal, $B_{\text{comb}}(m)$ denotes the combinatorial background from uncorrelated muons.

5.4.1 Combinatorial Background

The combinatorial background is extracted using the geometrical mean of the like-sign pairs from the same-event pairing as

$$B_{\text{LS}} = 2\sqrt{N_{\text{same}}^{\mu^+\mu^+} N_{\text{same}}^{\mu^-\mu^-}}. \quad (5.4)$$

5.4.1.1 Mixed-event method

To further improve statistical precision, the mixed-event (ME) technique is employed. Muons from different events with similar properties, centrality, primary vertex z position, event-plane orientation, and time-frame occupancy, are combined to form uncorrelated pairs. To account for acceptance differences between unlike-sign (US) and like-sign (LS) pairs, the mixed-event spectrum is scaled by the R factor:

$$R = \frac{N_{\text{mixed}}^{\mu^+\mu^-}}{2\sqrt{N_{\text{mixed}}^{\mu^+\mu^+} N_{\text{mixed}}^{\mu^-\mu^-}}}, \quad (5.5)$$

where $N_{\text{mixed}}^{\mu^+\mu^-}$, $N_{\text{mixed}}^{\mu^+\mu^+}$, and $N_{\text{mixed}}^{\mu^-\mu^-}$ are the numbers of mixed-event pairs for the respective charge combinations.

The combination of these three approaches, like-sign, unlike-sign (with R factor), and mixed-event, ensures a precise and reliable determination of the combinatorial background, enabling accurate extraction of the ϕ meson signal. The signal obtained is then

$$N_{\text{sig}} = N_{\text{same}}^{\mu^+\mu^-} - B_{\text{LS}}R. \quad (5.6)$$

5.4.2 Fit model

The like-sign-subtracted distribution is fitted with a combination of a Double Crystal Ball (DCB) function for the ϕ signal and an analytical function for the correlated background.

5.4 Background subtraction

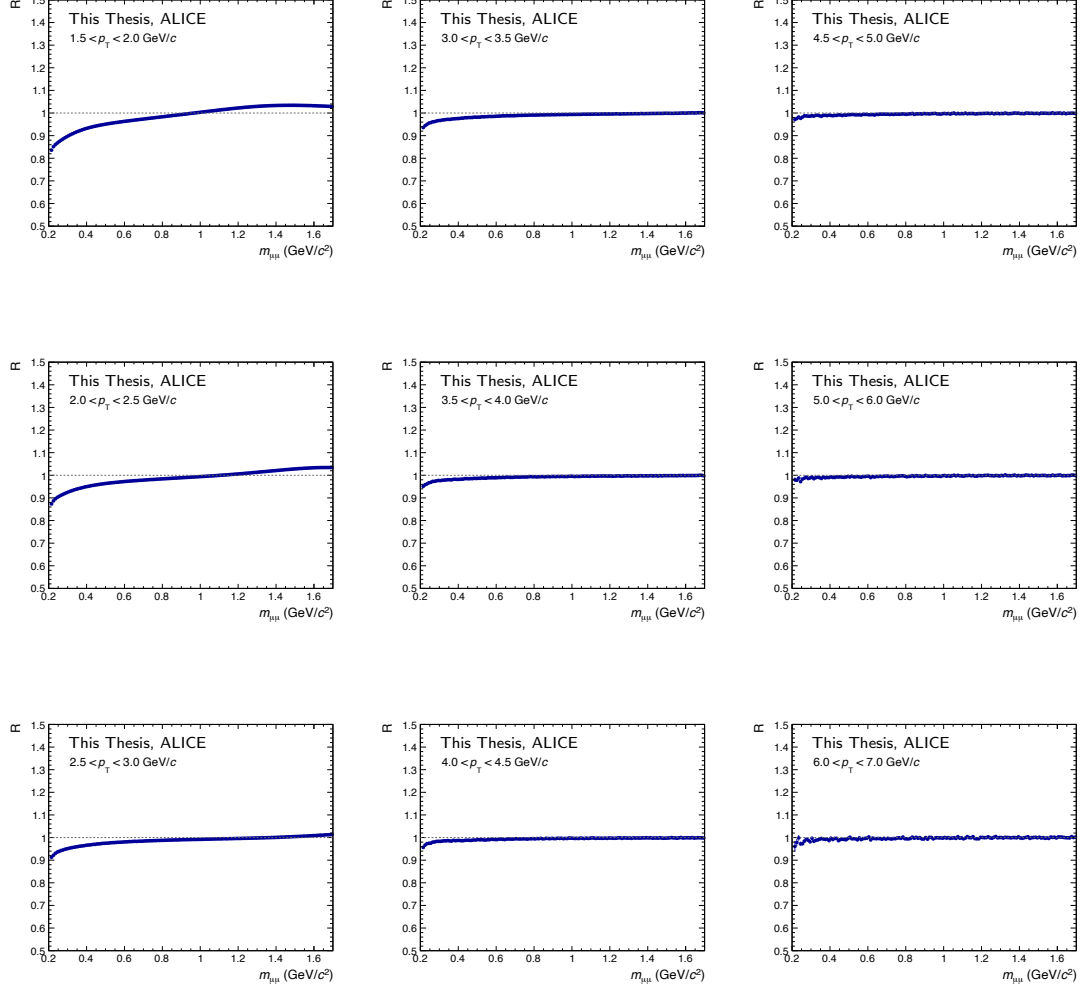


Figure 5.8: R -factor distributions for unlike-sign dimuon pairs in eight transverse-momentum intervals for the case of standalone muons. The R factor quantifies the residual acceptance difference between LS and US muon pairs and is used to correct the like-sign background estimate in the signal extraction [24].

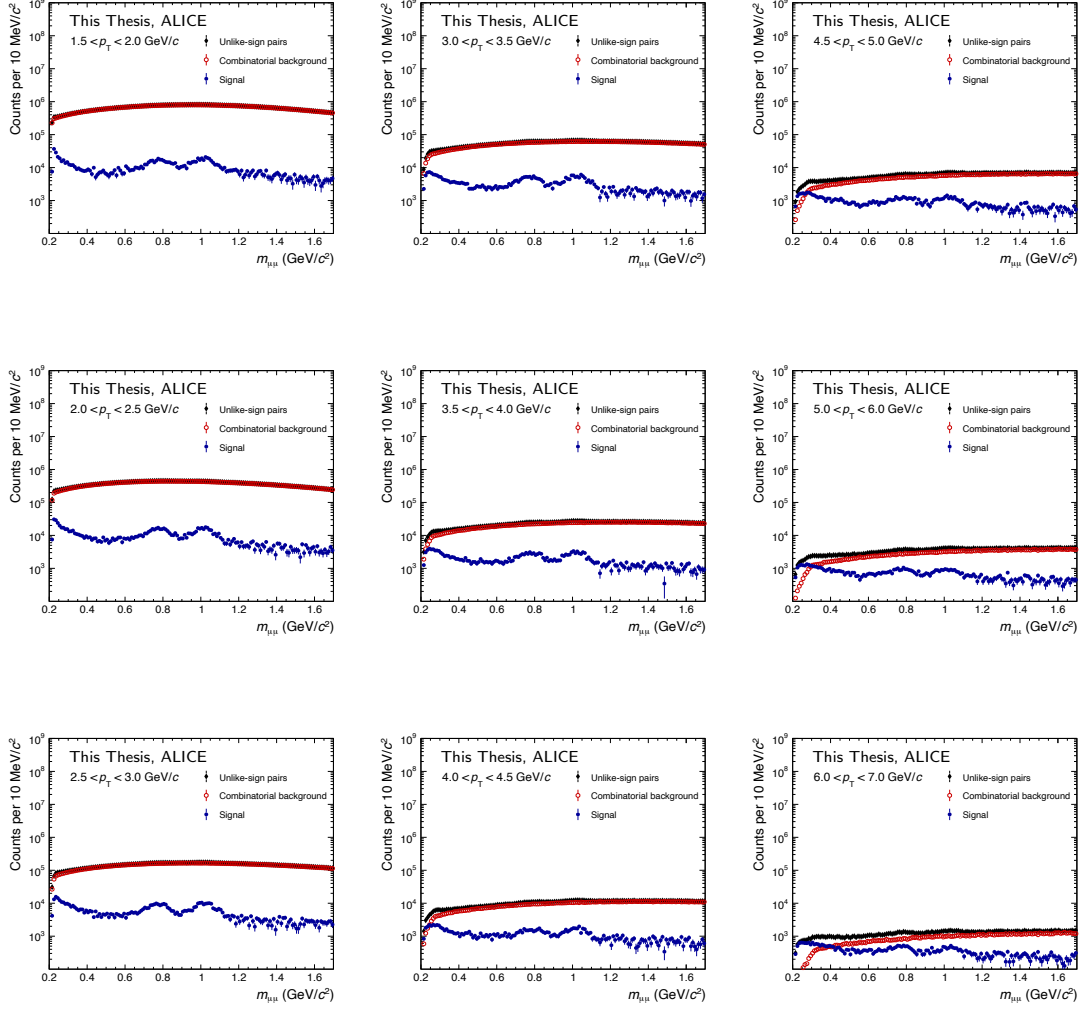


Figure 5.9: Illustration of the signal extraction procedure, where the signal yield N_{sig} is obtained by subtracting the like-sign background $B_{\text{LS}}^{\text{SR}}$ from the same-event unlike-sign dimuon yield [24].

5.4 Background subtraction

5.4.2.1 Signal fitting: Double Crystal Ball function

The ϕ meson line shape is modeled using a DCB function, which accounts for non-Gaussian tails on both the low- and high-mass sides:

$$f(x) = \begin{cases} N A_L \left(B_L - \frac{x - \bar{x}}{\sigma} \right)^{-n_L}, & \text{if } \frac{x - \bar{x}}{\sigma} \leq -\alpha_L, \\ N \exp\left(-\frac{(x - \bar{x})^2}{2\sigma^2}\right), & \text{if } -\alpha_L < \frac{x - \bar{x}}{\sigma} < \alpha_R, \\ N A_R \left(B_R + \frac{x - \bar{x}}{\sigma} \right)^{-n_R}, & \text{if } \frac{x - \bar{x}}{\sigma} \geq \alpha_R. \end{cases} \quad (5.7)$$

The coefficients ensuring continuity are defined as:

$$A_L = \left(\frac{n_L}{|\alpha_L|} \right)^{n_L} \exp\left(-\frac{\alpha_L^2}{2}\right), \quad B_L = \frac{n_L}{|\alpha_L|} - |\alpha_L|, \quad (5.8)$$

$$A_R = \left(\frac{n_R}{|\alpha_R|} \right)^{n_R} \exp\left(-\frac{\alpha_R^2}{2}\right), \quad B_R = \frac{n_R}{|\alpha_R|} - |\alpha_R|. \quad (5.9)$$

A second DCB function is included to model the nearby ω resonance. The results for the integrated p_T range are shown in Fig. 5.11.

5.4.2.2 Background fitting

The correlated background is described by two components:

- Exponential term:

$$N_1 e^{\gamma m}, \quad (5.10)$$

- Second-order polynomial:

$$a_0 + a_1 m + a_2 m^2. \quad (5.11)$$

5.4.2.3 Fit degrees of freedom

The DCB function introduces seven parameters for each resonance:

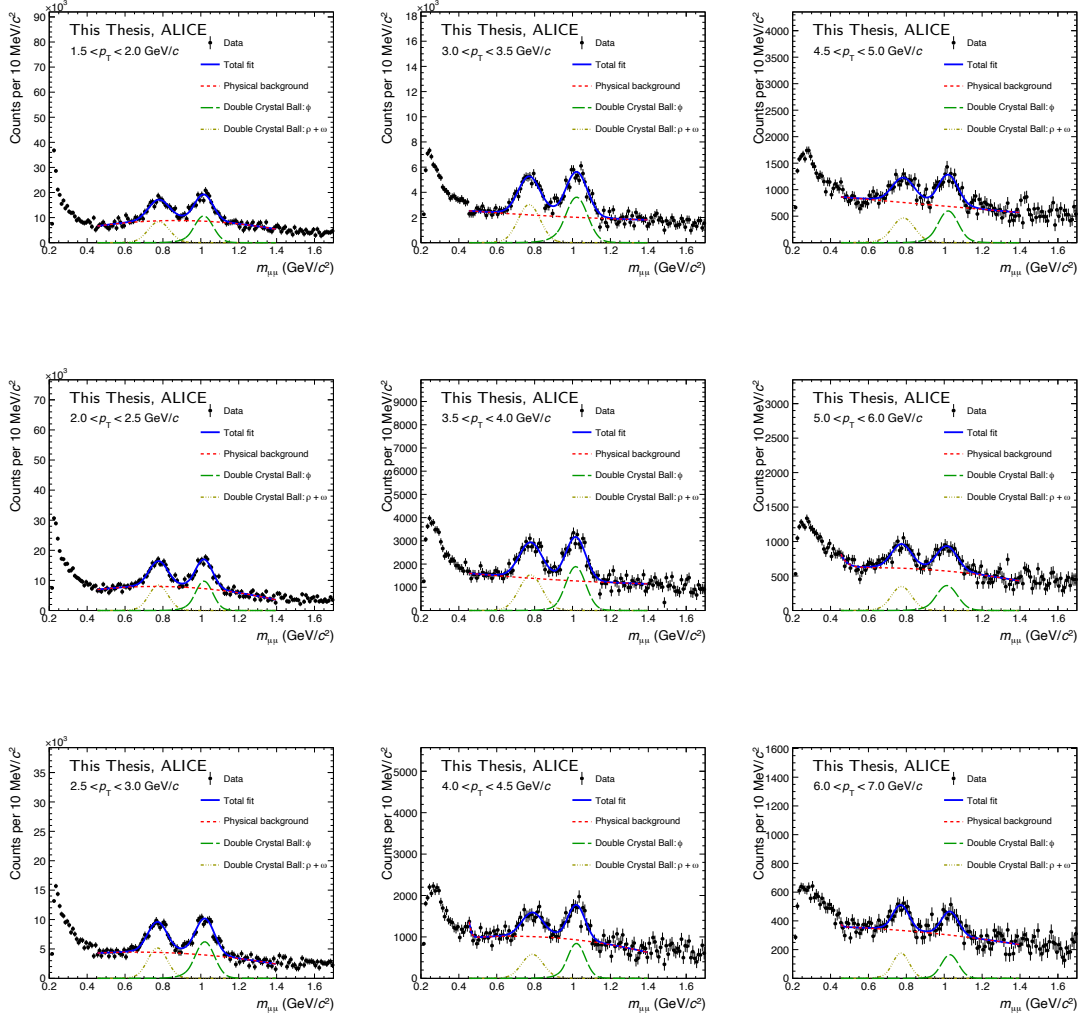


Figure 5.10: Invariant-mass fit results for US dimuon pairs in nine transverse-momentum intervals for standalone muons. Each panel displays the spectrum together with the full fit function with the DCB function and the analytical background parameterization by an exponential function combined with a second-order polynomial, used to extract the ϕ meson raw yield in each p_T interval [24].

5.4 Background subtraction

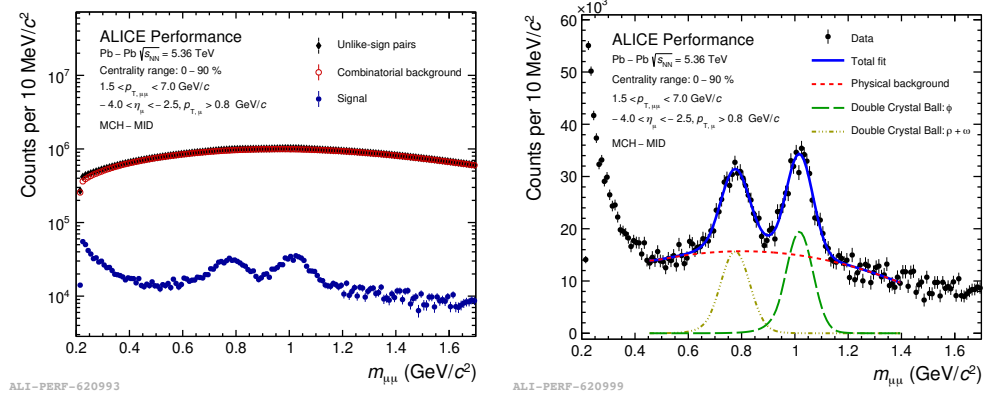


Figure 5.11: The left panel shows the invariant mass distribution of unlike-sign dimuon pairs measured in Pb–Pb collisions at $\sqrt{s_{\text{NN}}} = 5.36$ TeV with the ALICE detector for the centrality range 0 – 90%. The right panel shows that the signal contributions from the ϕ and $\rho + \omega$ mesons are described by Double Crystal Ball functions, while the residual background is parameterized by an exponential function combined with a second-order polynomial [24].

- N : normalization,
- $\bar{x}$: peak position (mean mass),
- σ : Gaussian width,
- α_L, n_L : parameters describing the low-mass tail,
- α_R, n_R : parameters describing the high-mass tail.

The full fit contains:

- 7 parameters for DCB_ϕ ,
- 7 parameters for DCB_ω ,
- 2 parameters for the exponential background,
- 3 parameters for the second-order polynomial,

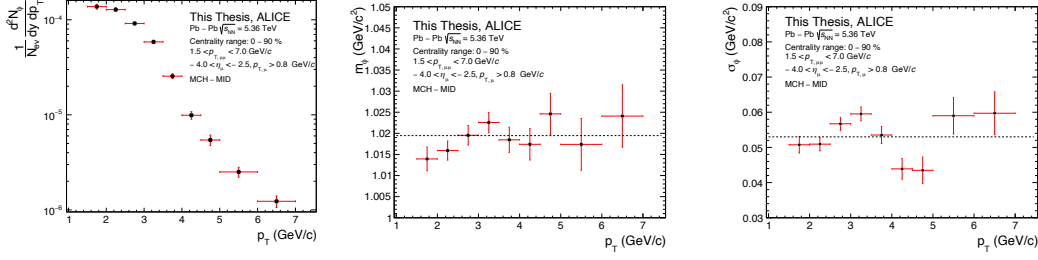


Figure 5.12: The left panel shows the transverse-momentum (p_T) spectrum of the raw ϕ yields obtained from the invariant-mass fits in each p_T interval. The central panel shows the extracted peak positions from the DCB, illustrating the stability of the reconstructed ϕ -meson mass as a function of transverse momentum. The right panel presents the corresponding width (σ) parameters of the DCB function as a function of p_T , reflecting the detector mass-resolution performance in Run 3 conditions [24].

for a total of 19 fit parameters. The total 8 parameters - α_L , n_L , α_R , and, n_R are taken fix for both ϕ , ω DCB functions. These parameters are extracted from DCB fits to Monte Carlo data for both ϕ , and ω peaks, as shown in Sec 5.5. The remaining 11 parameters are treated as free within specified limits.

The raw ϕ yield is obtained by integrating the fitted signal component:

$$N_{\phi}^{\text{raw}} = \int_{m_{\text{min}}}^{m_{\text{max}}} S_{\phi}(m) dm, \quad (5.12)$$

5.5 Monte Carlo analysis

For this analysis, the dataset used is: [EM_LHC26b13_Dimuon](#)

which is a derived dataset and dedicated to studying the enhanced low-mass vector meson via dimuon decay. The following 91 runs are considered:

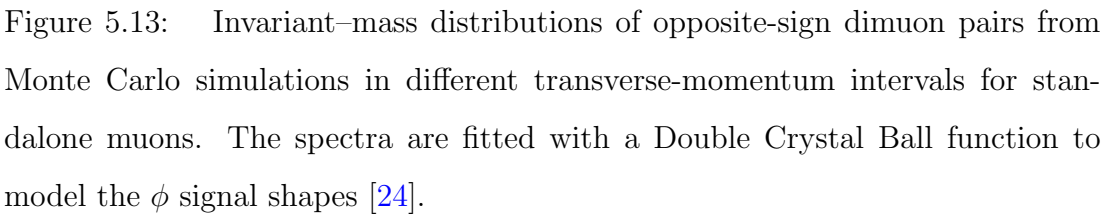
5.5 Monte Carlo analysis

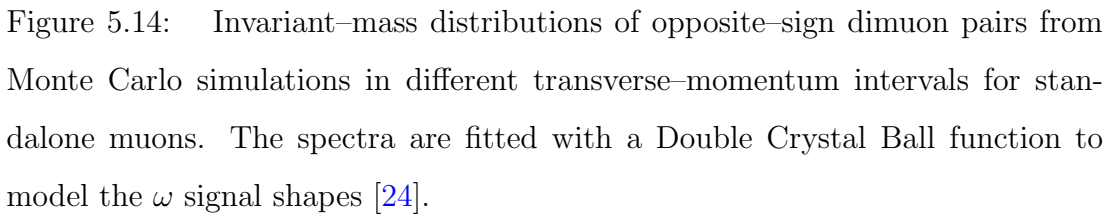
544013, 544028, 544032, 544091, 544095, 544098, 544116, 544121, 544122, 544123, 544124, 544184, 544185, 544389, 544390, 544391, 544392, 544451, 544454, 544474, 544475, 544476, 544477, 544490, 544491, 544492, 544508, 544510, 544511, 544512, 544514, 544515, 544518, 544548, 544549, 544550, 544551, 544564, 544565, 544567, 544568, 544580, 544582, 544583, 544585, 544614, 544640, 544652, 544653, 544672, 544674, 544692, 544693, 544694, 544696, 544739, 544742, 544754, 544767, 544794, 544795, 544797, 544813, 544868, 544886, 544887, 544896, 544913, 544914, 544917, 544931, 544947, 544961, 544963, 544964, 544968, 544992, 545009, 545044, 545047, 545063, 545064, 545185, 545210, 545223, 545249, 545291, 545294, 545295, 545296, 545312.

5.5.1 Signal extraction

The invariant-mass distributions of the ϕ and ω mesons in Monte Carlo (MC) simulations are fitted using a Double Crystal Ball function. Owing to the absence of a combinatorial background and the high-statistics nature of the MC samples, the signal shapes can be determined with high precision. This allows for a reliable extraction of the non-Gaussian tail parameters that characterize the detector response. In particular, the tail parameters, α_L , n_L and α_R , n_R , are independently determined for both the ϕ (as shown in Fig. 5.13) and ω (as shown in Fig. 5.14) resonances. These parameters encapsulate the asymmetric deviations from a purely Gaussian line shape arising from detector effects and radiative contributions.

The extracted tail parameters are then used as fixed inputs in the signal extraction procedure applied to real data. By constraining these shape parameters to





their MC-determined values, the number of free parameters in the data fit is significantly reduced, thereby improving the stability and robustness of the fitting procedure, particularly in low-statistics regions.

5.5.2 Acceptance and efficiency corrections

The acceptance and efficiency corrections are evaluated using Monte Carlo simulations in order to account for the finite geometrical coverage of the detector and the inefficiencies in the reconstruction and selection of muon tracks.

The detector acceptance and reconstruction efficiency, denoted as $A \times \varepsilon$, are determined using embedding studies, in which simulated $\phi \rightarrow \mu^+\mu^-$ decays are propagated through a realistic detector response and reconstructed using the same analysis chain as applied to real data [22, 23]. This ensures that the effects of detector resolution, tracking efficiency, and selection criteria are consistently incorporated. The generated particles are typically required to lie within a predefined kinematic phase space, whereas the reconstructed candidates must satisfy all track-quality and kinematic selection criteria used in the data analysis. This includes requirements on muon identification, track matching, and geometrical acceptance of the detector.

The evaluation of $(A \times \varepsilon)$ is performed differentially in p_T (and, where relevant, in rapidity), to properly account for the dependence of detector performance on the kinematics of the decay muons. The correction factor is defined as:

$$(A \times \varepsilon)(p_T, y) = \frac{N_{\text{reconstructed}}(p_T, y)}{N_{\text{generated}}(p_T, y)}. \quad (5.13)$$

Here, $N_{\text{generated}}$ represents the number of generated ϕ mesons within a given transverse momentum p_T and rapidity y interval at the particle level, while $N_{\text{reconstructed}}$ corresponds to those successfully reconstructed after full detector simulation and analysis selection cuts. The resulting correction factors are subsequently applied to the raw ϕ meson yields extracted from invariant-mass fits in data, thereby enabling the determination of corrected yields. Figure 5.16 presents the corrected

5.5 Monte Carlo analysis

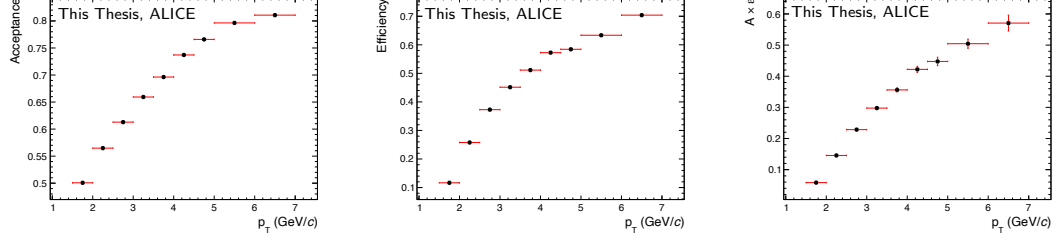


Figure 5.15: Acceptance (left), efficiency (middle), and acceptance $\times$ efficiency (right) versus p_T for $\phi \rightarrow \mu^+\mu^-$ decays from the `EM_LHC26b13_Dimuon` Monte Carlo data [24].

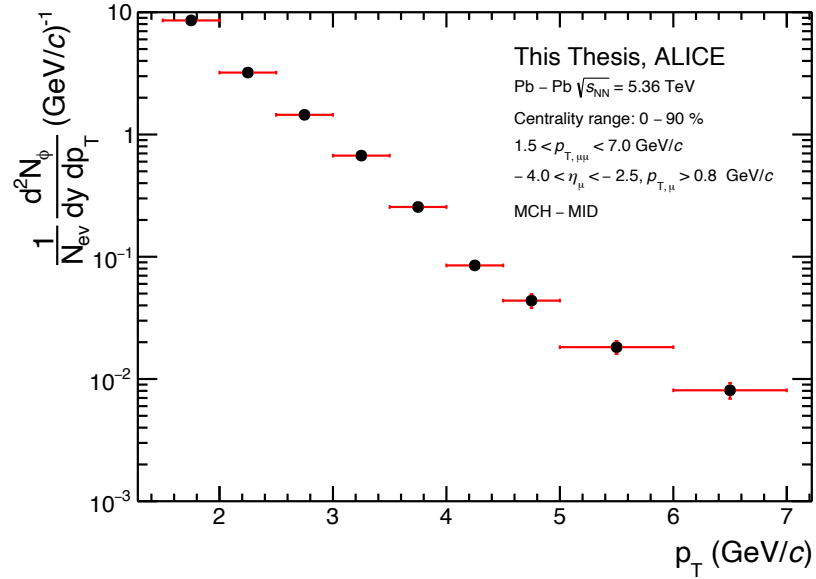


Figure 5.16: Corrected transverse-momentum (p_T) spectra of ϕ mesons in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV at forward rapidity, obtained after acceptance, efficiency, and branching-ratio corrections for the dimuon decay channel [24].

transverse-momentum (p_T) spectra of ϕ mesons measured in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV at forward rapidity, after applying acceptance, efficiency, and branching-ratio corrections for the dimuon decay channel. The corrected spectra serve as an essential observable for investigating the production mechanisms and medium effects in heavy-ion collisions. Further extensions of this analysis, including detailed systematic uncertainties and physics interpretations, will be reported in ALICE Analysis Note (ANA-1690) [24].

Summary

This chapter presents a detailed study of $\phi(1020)$ meson production in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV using Run 3 data collected with the ALICE detector at the Large Hadron Collider. The analysis focuses on the dimuon decay channel at forward rapidity, providing a clean probe of the quark–gluon plasma and strangeness production in heavy-ion collisions. The role of the ϕ meson as a penetrating probe, due to its hidden strangeness content and weak interaction with the hadronic medium, is emphasized.

Section 5.1 provides an overview of the ALICE experiment and its detector subsystems. The central barrel detectors, including the Inner Tracking System, Time Projection Chamber, Time-of-Flight detector, and Electromagnetic Calorimeter, are discussed in the context of tracking and particle identification at mid-rapidity. Particular emphasis is placed on the forward muon detector system, consisting of the Muon Chambers, Muon Identifier, and Muon Forward Tracker, which together enable precise reconstruction of dimuon pairs. The advantages of standalone and global muon reconstruction techniques are also discussed, highlighting their respective roles in invariant-mass analysis and precision measurements.

Section 5.2 introduces the physics motivation and analysis framework for studying the $\phi(1020)$ meson with ALICE in the dimuon decay channel. Owing to its hidden strangeness content ($s\bar{s}$), relatively long lifetime, and weak interac-

5.5 Monte Carlo analysis

tion with the hadronic medium, the ϕ meson serves as an important probe of strangeness enhancement and possible in-medium modifications in heavy-ion collisions. Although the dilepton branching ratios are small, the $\mu^+\mu^-$ channel offers a particularly clean signal because muons undergo negligible final-state interactions while traversing the produced medium.

Section 5.3 describes the selection criteria adopted for this analysis. Event selection is performed using standard Run 3 Pb–Pb quality requirements, including the CBT_muon Run Condition Table label, good detector status flags for FT0, ITS, TPC, MCH, and MID, Sel8 minimum-bias trigger conditions, FT0AND coincidence requirements, pileup rejection, and a primary-vertex constraint of $|z_{\text{vtx}}| < 10$ cm. Events within the centrality interval 0–90%, determined using the FT0C estimator, are retained, resulting in a high-quality dataset for physics analysis. The section further presents the standalone muon track selection used for dimuon reconstruction. Muon candidates are required to satisfy acceptance cuts in pseudorapidity ($-4.0 < \eta < -2.5$), a minimum transverse momentum threshold, MCH–MID track matching conditions, sufficient MCH cluster multiplicity, absorber radial-coordinate constraints, and $p \times \text{DCA}_{xy}$ selections for background suppression. In addition, dimuon candidates are selected in the intermediate transverse-momentum interval $1.5 \leq p_T^{\mu\mu} \leq 7$ GeV/ c for this analysis. Section 5.4 presents the methodology for background subtraction. The combinatorial background arising from uncorrelated muon pairs is estimated using the like-sign technique and further refined using the mixed-event method. The introduction of the R -factor corrects for acceptance differences between unlike-sign and like-sign pairs, enabling an accurate subtraction of the background contribution. The robustness of the background estimation is ensured through the combined use of same-event and mixed-event approaches. The invariant-mass spectra, after background subtraction, are fitted using a Double Crystal Ball function to model the ϕ and ω resonances, along with an analytical function describing the residual correlated background. The fitting strategy incorporates constraints on tail

parameters derived from Monte Carlo simulations, reducing the number of free parameters and improving the stability of the fits. The raw ϕ yields are obtained by integrating the fitted signal component over the chosen mass range. In addition, the extracted mass positions, widths, and transverse-momentum-dependent raw yields are studied to evaluate the overall fit quality and detector performance. Section 5.5 describes the Monte Carlo analysis used to determine detector acceptance and efficiency corrections. High-statistics MC simulations are employed to model the detector response and extract the signal shapes for both ϕ and ω mesons using Double Crystal Ball functions. The non-Gaussian tail parameters are extracted from these fits and subsequently fixed in the data analysis to ensure a consistent description of the signal shape. This procedure significantly improves the robustness of the fitting method, especially in limited-statistics intervals. The correction factor ($A \times \varepsilon$) is calculated as the ratio of reconstructed to generated ϕ mesons in a given kinematic interval. This procedure accounts for detector geometry, reconstruction efficiency, and selection effects. The obtained correction factors are then applied to the raw ϕ meson yields extracted from invariant-mass fits to the data, thereby allowing the determination of fully corrected yields.

Overall, this chapter provides a comprehensive framework for the extraction of ϕ meson yields in the dimuon channel at forward rapidity. By combining robust background subtraction techniques, precise signal modeling, Monte Carlo constrained fitting, and detailed detector corrections, the analysis enables a reliable investigation of strangeness production and medium effects in heavy-ion collisions. The methodology developed here establishes a strong foundation for future advanced studies of low-mass vector mesons in the context of heavy-ion collisions.

Bibliography

- [1] <https://www.livescience.com/65427-fundamental-elementary-particles.html>.
- [2] D. Adamova *et al.* [CERES], Phys. Rev. Lett. **96**, 152301 (2006).
- [3] R. Arnaldi *et al.* [NA60], Phys. Lett. B **699**, 325 (2011).
- [4] B. I. Abelev *et al.* [STAR], Phys. Rev. C **79**, 064903 (2009).
- [5] S. S. Adler *et al.* [PHENIX], Phys. Rev. C **72**, 014903 (2005).
- [6] S. Acharya *et al.* [ALICE], JINST **19**, P05062 (2024).
- [7] R. Sahoo, *Relativistic Kinematics: A Journey in Spacetime*, Lecture Notes in Physics, Vol. 1046, Springer, Cham, 2025. <https://doi.org/10.1007/978-3-032-09510-7>.
- [8] J. Kim [ALICE], PoS **VERTEX2025**, 006 (2025).
- [9] L. Musa, CERN-LHCC-2012-013, LHCC-P-005 (2012).
- [10] A. L. Gera, PoS **EPS-HEP2019**, 102 (2020).
- [11] A. Andronic [ALICE TRD], Nucl. Instrum. Meth. A **522**, 40 (2004).
- [12] A. Alici [ALICE], Nucl. Instrum. Meth. A **766**, 288 (2014).
- [13] S. Beole *et al.* [ALICE], CERN-LHCC-98-19.

- [14] G. Dellacasa *et al.* [ALICE], CERN-LHCC-99-04.
- [15] S. Acharya *et al.* [ALICE], JINST **18**, P08007 (2023).
- [16] P. Hauer [ALICE], Nucl. Instrum. Meth. A **1039**, 167023 (2022).
- [17] R. Schicker [ALICE], EPJ Web Conf. **138**, 01021 (2017).
- [18] L. Quaglia [ALICE], Nucl. Part. Phys. Proc. **347**, 75 (2024).
- [19] [ALICE], CERN-LHCC-2000-046.
- [20] L. Terlizzi [ALICE], Nucl. Instrum. Meth. A **1077**, 170518 (2025).
- [21] D. Krupova [ALICE], Nucl. Instrum. Meth. A **1064**, 169388 (2024).
- [22] E. A. R. Casula [ALICE], Nucl. Phys. A **967**, 425 (2017).
- [23] S. Acharya *et al.* [ALICE], Eur. Phys. J. C **78**, 559 (2018).
- [24] K. Singh, K. Goswami, and R. Sahoo, Analysis Note ID number: ANA-1690 (ALICE Internal), <https://alice-notes.web.cern.ch/node/1690>.

Chapter 6

Summary

“I don’t believe in taking the right decisions. I take decisions and then make them right.”

— Ratan Tata

This thesis presents a comprehensive investigation of strongly interacting matter across multiple scales, combining theoretical foundations, transport phenomena, applied detector physics, and experimental analysis of heavy-ion collisions. The work is structured to provide a coherent progression from the fundamental principles of Quantum Chromodynamics (QCD) to their manifestation in experimental observables.

*“The details of introduction in Chapter 1 extend,
The transport phenomena in Chapter 2 append,
The thermoelectricity in Chapter 3 defend,
The beampipe in Chapter 4 is designed and bend,
The ALICE data in Chapter 5 comprehend,
The details of the summary in Chapter 6 end.”*

Summary of Chapter 1

Chapter 1 establishes the theoretical and conceptual foundation of the thesis by introducing the Standard Model of particle physics and the role of Quantum Chromodynamics as the fundamental theory of strong interactions. The essential properties of QCD, such as asymptotic freedom and confinement, are discussed, highlighting how the behavior of the strong coupling governs the dynamics of quarks and gluons across different energy scales. The QCD phase diagram is presented as a central framework for understanding the different states of strongly interacting matter under extreme conditions of temperature and baryon density. Particular emphasis is placed on the transition from hadronic matter to the quark–gluon plasma (QGP), along with the possible existence of a critical point. The chapter further describes ultra-relativistic heavy-ion collisions as a laboratory for recreating early-universe conditions, outlining the spacetime evolution of the collision system from initial impact to freeze-out. Key experimental signatures of QGP formation, including anisotropic flow, jet quenching, strangeness enhancement, and quarkonium suppression, are discussed in detail. Finally, the motivation for studying the $\phi(1020)$ meson is established, emphasizing its role as a clean probe of the medium due to its hidden strangeness content and weak final-state interactions in the dilepton decay channel.

Summary of Chapter 2

Chapter 2 focuses on the transport properties of QCD matter in the presence of dynamic magnetic fields, which are naturally generated in non-central heavy-ion collisions. Using kinetic theory and relativistic frameworks, the study investigates how external electromagnetic fields influence the transport behavior of strongly interacting systems. Transport coefficients such as electrical conductivity and thermal conductivity are evaluated for both hadronic matter and the quark–

gluon plasma phase. The analysis highlights the sensitivity of these coefficients to temperature, baryon-chemical potential, and magnetic field strength. Special attention is given to the anisotropic nature of transport processes induced by magnetic fields, leading to distinct longitudinal and transverse responses.

The results provide valuable insights into the microscopic dynamics of QCD matter and establish a connection between theoretical transport and hydrodynamic models for the study of the evolving QGP medium in the context of heavy-ion collisions.

Summary of Chapter 3

Chapter 3 extends the study of transport phenomena to thermoelectric effects in QCD matter, exploring the coupling between heat and charge transport in both hadronic and deconfined phases. The formalism is developed within the framework of kinetic theory and linear response theory, allowing for a systematic evaluation of thermoelectric coefficients such as the Seebeck coefficient and the coupled transport coefficients such as Dufour and Soret coefficients. The analysis is performed for systems with and without external magnetic fields, revealing the significant role of electromagnetic fields in modifying thermoelectric behavior. Both leading-order and higher-order thermoelectric coefficients are systematically investigated, demonstrating that thermoelectric transport is strongly influenced by the interplay between particle interactions, medium properties, and external fields.

In particular, this study provides, for the first time, a detailed extraction of higher-order thermoelectric coefficients in QCD matter, including the Thomson, magneto-Thomson, and transverse Thomson coefficients. These coefficients arise from the temperature dependence of the leading-order response and capture the non-linear coupling between heat and charge transport in the presence of magnetic fields, thereby offering deeper insight into the non-equilibrium dynamics of the

medium.

Furthermore, the thermoelectric figure of merit (ZT) is evaluated as a global measure of thermoelectric efficiency. The results reveal a strong sensitivity of ZT to the deconfinement transition, exhibiting a non-trivial behavior across the hadronic and QGP phases. This highlights its potential as a novel transport-based probe of the QCD phase transition. These findings significantly advance the understanding of non-equilibrium processes in strongly interacting matter and open new avenues for connecting transport theory with experimentally accessible observables.

Summary of Chapter 4

Chapter 4 shifts the focus toward applied aspects of high-energy physics, providing an overview of particle accelerators, detector systems, and beampipe design. The Large Hadron Collider and the ALICE detector are discussed to establish the experimental context of the thesis.

A significant portion of the chapter is devoted to the development and optimization of materials for accelerator beampipes. The study addresses key challenges such as radiation hardness, thermal stability, mechanical strength, and cost efficiency. By exploring innovative material configurations and design strategies, the work aims to improve the performance and longevity of accelerator components under extreme operating conditions.

In this context, advanced ternary alloy systems, including Al–Ti–V and Mg–Al–Li, are investigated as promising alternatives to conventional materials. These low- Z alloys are systematically studied using a combination of first-principles calculations, thermodynamic simulations, and experimental characterization. The results demonstrate that such ternary systems exhibit superior performance compared to classical materials like stainless steel and pure aluminum. In particular, they offer enhanced radiation transparency (larger radiation length), reduced

density, and improved tunability of mechanical properties while maintaining sufficient structural strength. These advantages make them highly suitable for minimizing beam–material interactions and improving detector performance, thereby providing a viable pathway for next-generation beampipe design in high-energy accelerators.

This chapter bridges fundamental physics with engineering applications, demonstrating how advances in material science can directly impact experimental capabilities in high-energy physics.

Summary of Chapter 5

Chapter 5 presents the experimental analysis of $\phi(1020)$ meson production in Pb–Pb collisions at $\sqrt{s_{\text{NN}}} = 5.36$ TeV using Run 3 data from the ALICE detector. The study focuses on the forward rapidity region and utilizes the dimuon decay channel, which provides a clean and robust signal due to the minimal interaction of muons with the surrounding medium.

A detailed analysis framework is developed, including event selection, muon track reconstruction, and background subtraction using like-sign and mixed-event techniques. The combinatorial background arising from uncorrelated muon pairs is first estimated using the like-sign method and further refined using mixed-event distributions to account for acceptance effects. An R -factor correction is applied to properly normalize the background and ensure an accurate subtraction in the invariant-mass spectra. The signal extraction is performed by fitting the background-subtracted invariant-mass distributions with a Double Crystal Ball (DCB) function, which effectively describes the ϕ and ω resonance peaks, including non-Gaussian tails arising from detector resolution and radiative effects. The tail parameters of the DCB function are constrained using Monte Carlo (MC) simulations, thereby improving the stability and reliability of the fit, particularly in low-statistics regions.

The residual correlated background is modeled using a combination of an exponential function and a second-order polynomial. This combined approach ensures a robust separation of signal and background contributions, enabling a precise extraction of the ϕ meson raw yields across different transverse momentum intervals. The use of MC-derived tail parameters in the data fits significantly improves the stability and reliability of the signal extraction procedure. The corrected ϕ meson spectra, obtained as a function of transverse momentum after acceptance and efficiency ($A \times \varepsilon$) corrections, provide insight into strangeness production and the properties of the medium created in heavy-ion collisions.

Overall conclusions

This thesis provides a unified exploration of strongly interacting matter, connecting theoretical developments, thermoelectric transport phenomena, applications in accelerator technology, and real data analysis. The study demonstrates how fundamental QCD concepts manifest across different domains, from microscopic transport processes to macroscopic observables in heavy-ion collisions. The combined investigation of transport and thermoelectric properties enhances our understanding of non-equilibrium dynamics in QCD matter, while the experimental analysis of ϕ meson production offers direct insight into the behavior of the quark–gluon plasma in light-ion and heavy-ion collisions.

Overall, this work advances the understanding of strongly interacting matter under extreme conditions, providing a unified framework across different physical regimes. It establishes a solid foundation for future studies in high-energy nuclear physics and astrophysical systems, including dense stellar objects such as neutron stars. This thesis provides a unique guide to design the novel materials for beam pipe in future accelerator technology. The production analysis of $\phi(1020)$ serve as a baseline study for the future enhancement in terms of polarization, flow and system dependent studies.