Study of High Energy Positron Annihilation in GEANT4

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Abstract

A high energy positron may annihilate with an electron in atoms not only into two photons, but also into muon pairs or hadrons if the energy is over the energy threshold, 43.69 GeV in laboratory frame with electrons at rest. This report shows modification of high energy electromagnetic processes in GEANT4(version 10.01.b01) in order to include these annihilation processes properly, validation of GEANT4 cross-sections of these processes by theoretical calculation, and the results of simulation for high energy processes in a simple setup. As a results of simulation, both of annihilation to muon pairs and to hadrons happen by the probability of 10^{-6} to 10^{-5}.

1 Introduction

An incident positron usually annihilate with an electron into two photons by electromagnetic process. However, if the energy of positron becomes over a threshold, 43.69 GeV in the frame with the target electron at rest, positron may annihilate into muon pairs like Fig.1. This threshold is the energy enough to produce two muons at rest, $E_{th} = 2m_{\mu}^2/m_e - m_e \approx 43.69$ GeV. Annihilations to hadrons, such as to $(\pi^+, \pi^-), (\pi^+.\pi^-, \pi^0), (K^+, K^-), (K_L, K_S), (\eta, \gamma), (\pi^0, \gamma)$, may also happen, with higher energy than a threshold for each process.

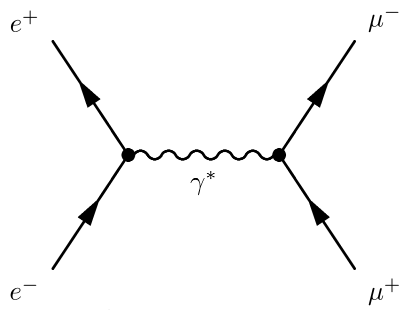

Fig. 1 Annihilation to muon pairs

These processes may provide a background to the interaction region of linear collider or to search for new physics at LHC[1] or CLIC[2]. Thus, understanding these high energy electromagnetic processes is needed in order to make a proper prediction for new experiments.

2 Modification of high energy electromagnetic process in GEANT4

In order to make a simulation about high energy process by GEANT4, cross sections should be properly calculated in GEANT4. Fig.2 shows the classes which give cross sections of these high energy processes in GEANT4. A helper class G4EmCalculator could extract cross sections of electromagnetic processes from GEANT4 kernel into UserAction. This class refers to G4VEmProcess in order to extract cross sections, however, a class G4AnnihiToMuPair(red box), which defines cross section of positron annihilation to muon pair, is not included in this G4VEmProcess. Thus, a new function is defined in G4EmCalculator to find the annihilation process to muon pairs.

On the other hand annihilation process to hadrons is included in G4VEmProcess but had several problems. This process has several sub-classes and each hadronic reaction channel class(see Table 1.)

These libraries have been cleaned up, in particular, a new function is defined to change from laboratory frame to center of mass frame and some initializations are done properly.

Every one of these updates passed all validation procedures in GEANT4, and now high-energy sub-library is included into private patch to CMS software.

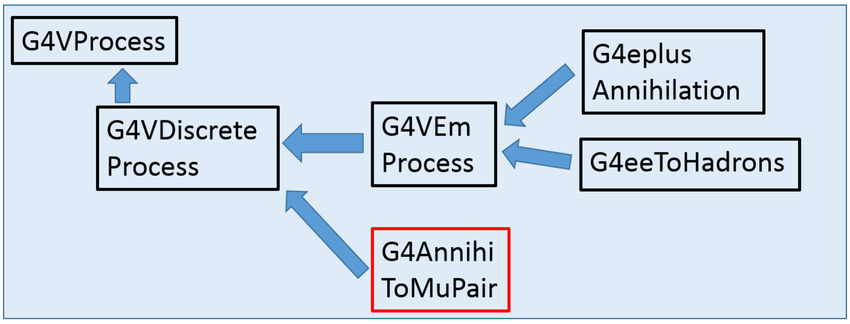

Fig. 2 classes for high energy electromagnetic process

Classes to define annihilation to hadrons	Classes for each processes
G4eeToHadrons	G4eeToTwoPiModel
G4eeToHadronsModel	G4eeTo3PiModel
G4eeToHadronsMultiModel	G4ee2KChargedModel
G4Vee2hadrons	G4ee2KNeutralModel
	G4eeToPGammaModel

Table 1 Classes for annihilation to hadrons

3 Validation of cross-section of high energy process

Validation of updated high energy electromagnetic process are done by comparison of cross section between that of GEANT4 and theoretical calculation. The formulas for cross section are the following two equations[3].

$$\sigma(e^+e^- \to \gamma\gamma) = \frac{Z\pi r_e^2}{\gamma+1}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right) - \frac{\gamma+3}{\sqrt{\gamma^2-1}}\right] \tag{1}$$

$$\begin{aligned} E &: \mathrm{totalenergyoftheincidentpositroninlaboratoryframe} \\ \gamma &= E/m_e \\ r_e &: \mathrm{classicalelectronradius} \end{aligned}$$

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{\pi r_\mu^2}{3}\xi\left(1+\frac{\xi}{2}\right)\sqrt{1-\xi} \tag{2}$$

$$\begin{aligned} r_\mu &= r_e m_e/m_\mu \\ \xi &= E_{th}/E \\ E_{th} &= 2m_\mu^2/m_e - m_e \end{aligned}$$

Fig.3 shows the comparison of three positron annihilation processes to two gammas(black), to muon pairs(red), and to hadrons(blue), and the cross section of GEANT4(lines) and that from theoretical calculation(triangle) are consistent for two processes of annihilation to double gamma and to muon pairs.

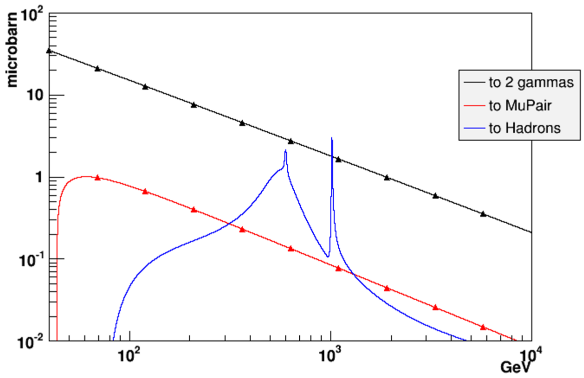

Fig. 3 Total cross section per atom for e^+ annihilation processes in GEANT4. X axis shows the energy of incident e^+ in the frame with e^- at rest, and Y axis shows total cross section per atom. Lines correspond to cross section in GEANT4 and triangles to the theoretical calculation.

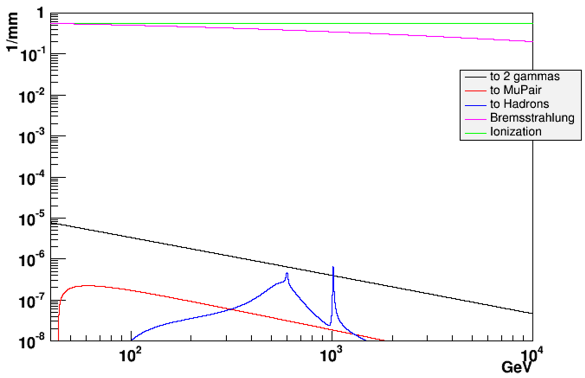

Fig. 4 Total cross section per volume for electromagnetic processes in GEANT4.X axis shows the energy of incident e^+ in the frame with e^- at rest, and Y axis shows total cross section per volume.

The cross section of annihilation to hadrons could be validated through R value, which is the ratio of total cross section of positron annihilation to hadrons to that to muon pairs. PDG gives data for this value(see Fig.5) and Fig.6 shows the R value which is calculated by GEANT4 cross sections. These two are also coincident at least up to around 1 TeV in laboratory frame. Here, processes with higher threshold are not included, such as annihilation to 4 pions, to ρ' mesons, and so on, but these processes are also needed to be included if higher energy region is needed.

Now high energy electromagnetic processes in GEANT4 are validated in the energy region needed in the experiment with LHC or linear collider such as CLIC.

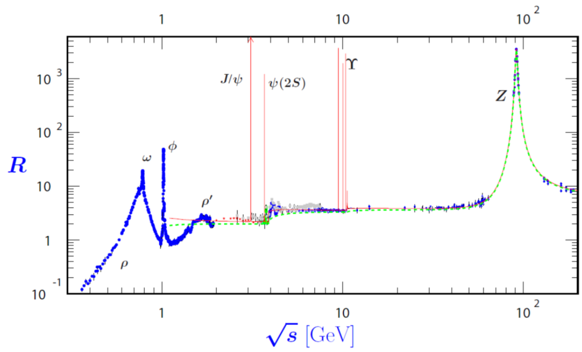

Fig. 5 PDG data for R value (the ratio: $\sigma(e^+e^- \to hadrons)/\sigma(e^+e^- \to \mu^+\mu^-)$. X axis shows the total energy in the center of mass frame.

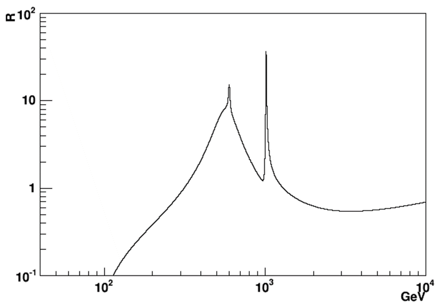

Fig. 6 R value in GEANT4. X axis shows the energy of incident e^+ in the frame with e^- at rest.

4 Simulation in a simple setup

In order to test high energy electromagnetic processes, a simple simulation is done, in which one large box($500 \times 500 \times 500\mathrm{m}^3$) is put in vacuum and 10^7 positrons are irradiated to the center of one of surfaces. Fig.7 shows this simple setup for simulation.According to Fig.4 the probability of annihilation processes is so smaller than that of bremsstrahlung and ionization that positrons lose their energy due to bremsstrahlung or ionization first and annihilation may happen at the end. Here, the number of positron annihilation processes, of annihilation to two gamma, to muon pairs, and to hadrons, is counted.

Fig.8 and Fig.9 show the probability of annihilation to muon pairs(blue) and annihilation to hadrons(red) as the results of this simulation. First, incident positron's energy is fixed as 1 TeV and the material of target is changed for each simulation(Fig.8). The fact that the probability of these process decreases as atomic number Z increases is confirmed. Second, the material of target is fixed as silicon and the energy of incident positron is changed for each simulation(Fig.9). Although statistics enough to make a proper confirmation are not available, the tendency that the probability increases as the positron energy increases is confirmed.

The point here is that both of annihilation processes to muon pairs and to hadrons may happen by the probability of 10^{-5} to 10^{-6}, and these probability may become a background in the experiment with high energy and high event rate, such as LHC.

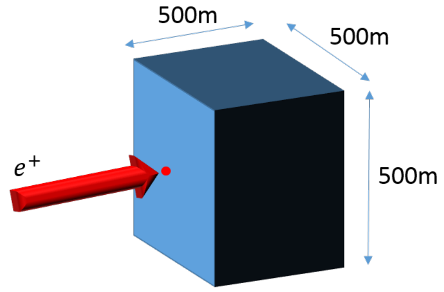

Fig. 7 setup for this simulation. Various materials are used for target(blue box).

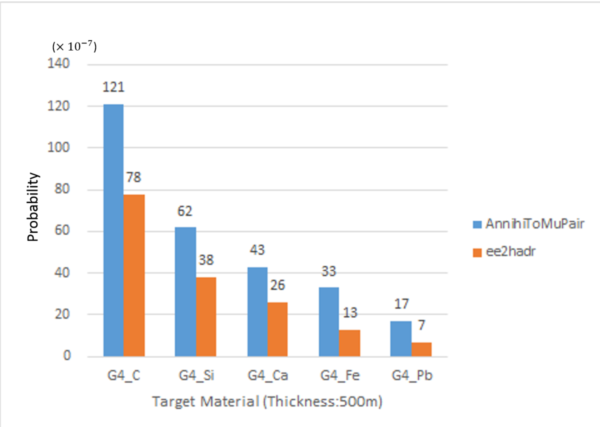

Fig. 8 Material dependency of the probabilities for e^+ annihilation to muon pair(blue) and to hadrons(red). Incident positron energy is 1 TeV.

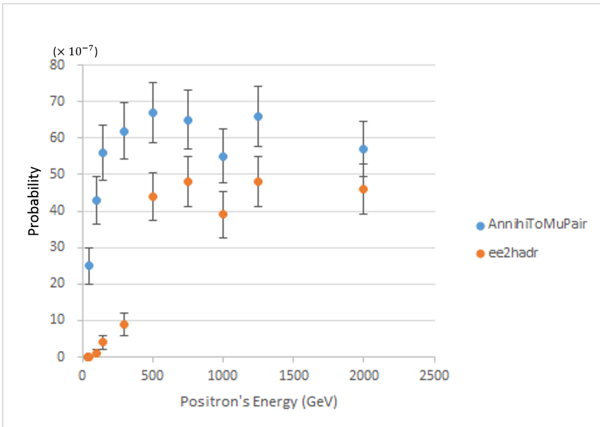

Fig. 9 Energy dependency of the probabilities for e^+ annihilation to muon pair(blue) and to hadrons(red). Target's material is Si.

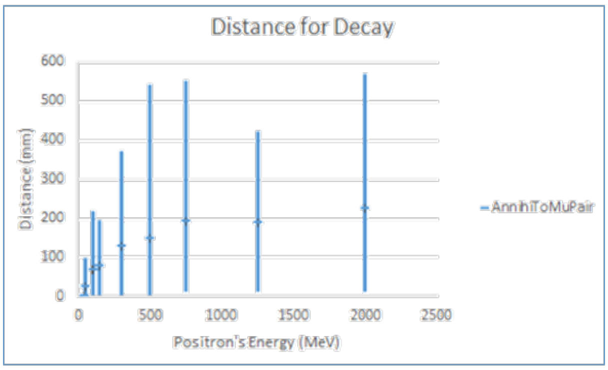

Fig. 10 Distance from the target's surface to the annihilation point to muon pairs. The top of bar is the maximum value and bottom is the minimum. The middle point is the average distance.

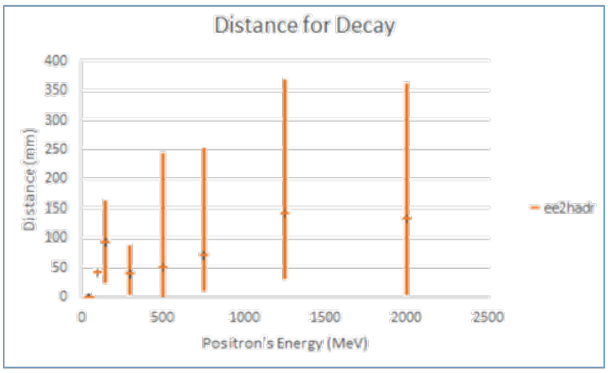

Fig. 11 Distance from the target's surface to the annihilation point to hadrons. The top of bar is the maximum value and bottom is the minimum. The middle point is the average distance.

The distances for which positron passes in the target to annihilate into muon pairs or hadrons are also measured in this simulation(Fig.10 and Fig.11). In such a high energy as around 1 TeV, positron passes almost straight through materials. Here again, although statistics are not enough the tendency that the distance becomes longer as the positron energy increases. And the important fact here is that both of these high energy annihilation processes to muon pairs and hadrons may happen for the distance of around 10 cm in silicon. This length corresponds to the tracker size in CMS, thus these processes may become a background for muon event or hadron event in the silicon tracker.

5 Conclusion

Through this work for GEANT4(version 10.01.b01), electromagnetic/high-energy sub-library has been improved. At the same time, an example codes, TestEm6, and a class G4EmCalculator has been updated. These improvement and update pass all validation procedures in GEANT4. Next GEANT4 cross sections with these high energy electromagnetic processes are validated up to around 1 TeV and now high-energy sub-library is included into private patch to CMS software.

According to the results of a simple simulation, high energy positron annihilation processes to muon pairs or hadrons may happen by the probability of 10^{-6} to 10^{-5} and for the distance of around 10 cm in silicon. This fact shows that these high energy processes may become a background in a tracker of CMS in LHC experiment with 13 TeV or a background in linear collider experiment[2].

6 Acknowledgement

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References

[1] Geant4 simulation of production and interaction of muons. A. G. Bogdanov, H. Burkhardt, V. N. Ivanchenko et al. 2006. 7pp. Published in IEEE Trans. Nucl. Sci. 53 (2):513-519, 2006.

[2] CLIC Conceptual Design Report. M. Aicheler, et al. CERN, 2012. -841 p.

[3] Physics Reference Manual, Version: geant4 10.0 (6 December 2013)