Statistical Combination of Searches for $Z \rightarrow \mu\tau$ decays using CLs method

CERN Summer Student Project Report

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1 Introduction
1.1 Z Boson
Z boson, and W^+ and W^- bosons are all mediators of the weak force. The Z Boson is neutral. Z boson mass is 91.1876 ± 0.0021 GeV and the Z boson width is 2.4952 ± 0.0023 GeV. [2]

1.2 Decays of Z boson
Since Z is neutral the sum of the charges of its decay products must be 0.

Therefore Z must decay into a particle, antiparticle pair. The 100% probability of Z to decay is divided between groups of particles.

1. In 10% of the Z-decays, charged lepton-antilepton pairs are produced. The three possible charged lepton pair types are electron-positron, muon-antimuon, and tau-antitau pairs.
2. The Z boson decays in 20% of the cases into a neutrino-antineutrino pair.
3. In 70% of Z decays, a quark-antiquark pair is produced.

Precision measurements at the Z-boson resonance using electron–positron colliding beams began in 1989 at the SLC and at LEP. During 1989–95, the four LEP experiments (ALEPH, DELPHI, L3, OPAL) made high-statistics studies of the production and decay properties of the Z which may broadly be categorized as:

- The standard 'lineshape' parameters of the Z consisting of its mass, M_Z, its total width, Γ_Z, and its partial decay widths, Γ (hadrons), and $\Gamma(\ell,\bar{\ell})$ where ℓ=e,μ,τ,ν;
- Z asymmetries in leptonic decays and extraction of Z couplings to charged and neutral leptons;
- The b– and c-quark-related partial widths and charge asymmetries which require special techniques;
- Determination of Z decay modes and the search for modes that violate known conservation laws;
- Average particle multiplicities in hadronic Z decay;
- Z anomalous couplings. [1]

2. CLs Method
The following explanation of the CLs method is taken from [3]. The Cls method is This is a Technique for ATLAS and 4 LEP experiments. This is a framework to present search results by frequentist statics. CLs technique for setting limits is appropriate for determining exclusion intervals while the determination of confidence intervals advocated by Feldman and Cousins' method is more appropriate for treating established signals, i.e. going beyond discovery to measurement.
Scientist know from experience that frequentist confidence intervals and Bayesian credible intervals tend to converge when the statistics is large and the results are dominated by the signal, thus rendering misinterpretations harmless in practice.

However, search experiments tend to suffer from poor statistics and relatively large backgrounds, thus the misinterpretation of frequentist statements becomes a serious issue.
Feldman and Cousins advocate the determination of frequentist confidence intervals for physical constants also for search experiments. Because these confidence intervals make statements about a signal while neglecting the background, apparently unintuitive results maybe obtained. The most commonly known is the example of the two experiments with identical efficiencies and observations, but the experiment with the largest expected background quotes the strongest limit on the physical constant. What is even more disturbing is that the experiment with the largest expected background can show that its potential for excluding false signals reaches to lower signal rates than the superior experiment.
When researchers perform search experiments they are as interested in confirming new theories or discovering new phenomena as we are in excluding them. "Scientists ultimately put confidence in a hypothesis or a theory if it has been able to withstand empirical or observational attempts to falsify it". One of the preconditions for a discovery is that the theory or model under consideration must be falsifiable—it should be possible for the experiment to show that the theory or model is wrong. It is not a great leap to extend this thinking to measurements. An experiment must be sensitive to the parameters of the theory or model. Confidence intervals for insensitive experiments are uninformative. It is our duty to be sceptical, i.e. to try to falsify or exclude and this is not the same as, but rather complementary to, the determination of confidence intervals.

2.1 From exclusion intervals to measurements

All the confidences, discovery and exclusion potentials, false exclusion rates, etc that make up the frequentist-motivated 'CLs method' are derived from the probability density functions (pdfs) of $-2\ln(Q)$ where $Q=L(s+b)/L(b)$ is the ratio of likelihoods for the two hypotheses of interest for the exclusion and discovery tests. The null hypothesis is the background hypothesis 'b', i.e. the data can be understood with existing physics explanations. The alternative hypothesis, which is favoured when the null hypothesis has been rejected to a sufficient degree, is that we need new physics to understand the data. Since the typical search is not free of background, we should call the alternative hypothesis the signal + background hypothesis 's+b'.

We can see an example of pdfs on the Fig. 1.0 of the combined Higgs search on LEP experiment.

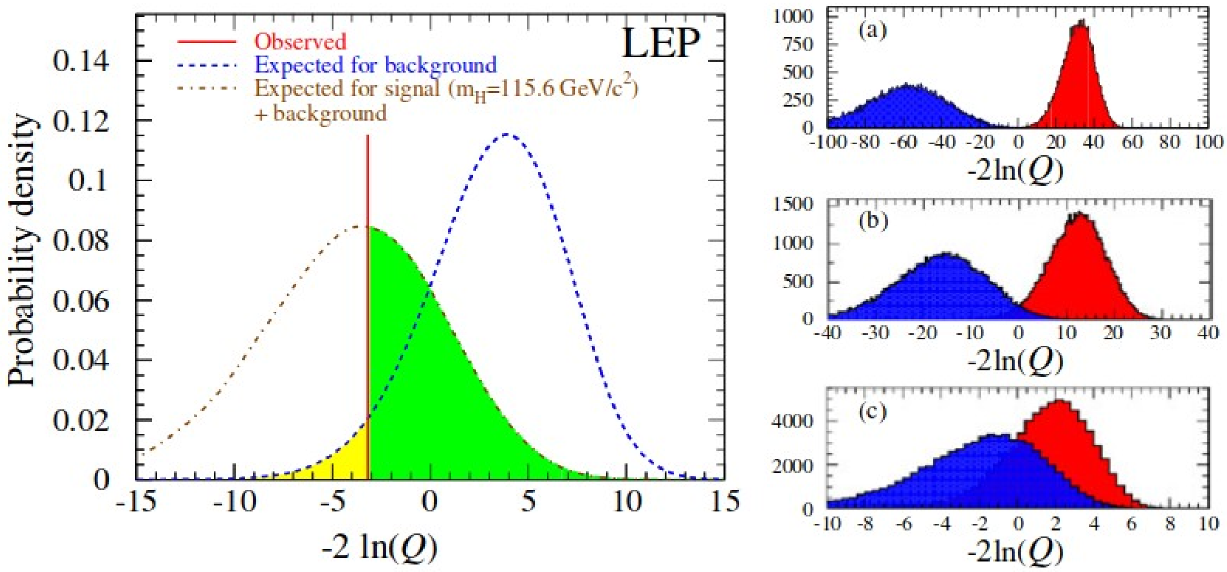


Fig.1.0

3. My Project

My project was the statistical combination of searches for Z boson decays to μτ (Z→μτ) using CLs method.

If we want to use CLs method we need probability density function known as pdf. In my case probability density function is the product of three Gaussian functions of three observables one. The first one is the observed number of Z boson decays to μτ (Z→μτ) the second is the observed number of Z boson decays to ττ (Z→ττ) and the one is the measured relative efficiency. Each of these observables are independent, so what I had to do is to multiply the single pdfs (simple Gaussian functions).

$$Pdf = G(N_Z \to \mu\tau) \times G(N_Z \to \tau\tau) \times G(\varepsilon^{rel}) \qquad \textbf{1.1}$$

The number of Z→μτ is equal to number of Z boson times branching ratio of Z→μτ, which is our parameter of interest, times efficiency of Z→μτ. As we can see in the formula:

$$(N_Z \to \mu\tau) = N_Z \times B_r^{Z\mu\tau} \times \varepsilon_{Z\mu\tau} \qquad \textbf{1.2}$$

Also we have the same for the Z→ττ in **1.3** :

$$(N_Z \to \tau\tau) = N_Z \times B_r^{Z\tau\tau} \times \varepsilon_{Z\tau\tau} \qquad \textbf{1.3}$$

From these two formulas with some calculations we can figure it out the branching ratio of Z→μτ in **1.4**

$$\frac{N_Z \to \mu\tau}{N_Z \to \mu\tau} = \frac{B_r^{Z\mu\tau}}{B_r^{Z\tau\tau}} \times \varepsilon^{rel}$$

$$B_r^{Z\mu\tau} = \frac{N_Z \to \mu\tau}{N_Z \to \tau\tau} \times B_r^{Z\tau\tau} \times \varepsilon^{rel} \qquad \textbf{1.4}$$

For this work I described before I had to use a software toolbox in ROOT and C++ . In the program written in C++ I had two different sets of data Bin1 and Bin2 which were referring to the reconstructed τ lepton in the $Z \rightarrow \mu\tau$ decay as 1-prong and 3-prong. Where for both Bins I had to work on changing the values as I mentioned before multiplying by a scale factor . The values I put were from 0.2 to 2.0 as a scale factor, first for the first data set and for the three σ-s at the same time. Then changing the $\sigma_{\mu\tau}$ from 0.2 to 2.0 and keeping the others σ-s constant. Continuing the same with other σ-s with $\sigma_{\mu\tau}$ and σ_{ε}. For each of those I saw different CLs which I had to analyze. Also for each I had to see how the σ-s impact the upper limit. After this I made plots between scale factor, upper limit and the error of the upper limit.

From this study I concluded that the upper limit is more sensitive to the $\sigma_{\mu\tau}$, while a variation of $\sigma_{\tau\tau}$ and σ_{ε} has a much lower impact.

I did the same with the second data set of these σ-s which had different values and it gave me different CLs also, and still the upper limit depends mainly on $\sigma_{\mu\tau}$.

Finally I combined the two Bins together and analysed how the upper limit extracted with the CLs method improves with the combination

3.1 Results

With the definition of pdf I was able to run the CLs extraction of the upper limit. The CLs in this figure is from the first set of data Bin1 of where I have to change three σ-s at the same time the first is $\sigma_{\tau\tau}$, $\sigma_{\mu\tau}$ and the σ_{ε} one with the scale factor 0.2.

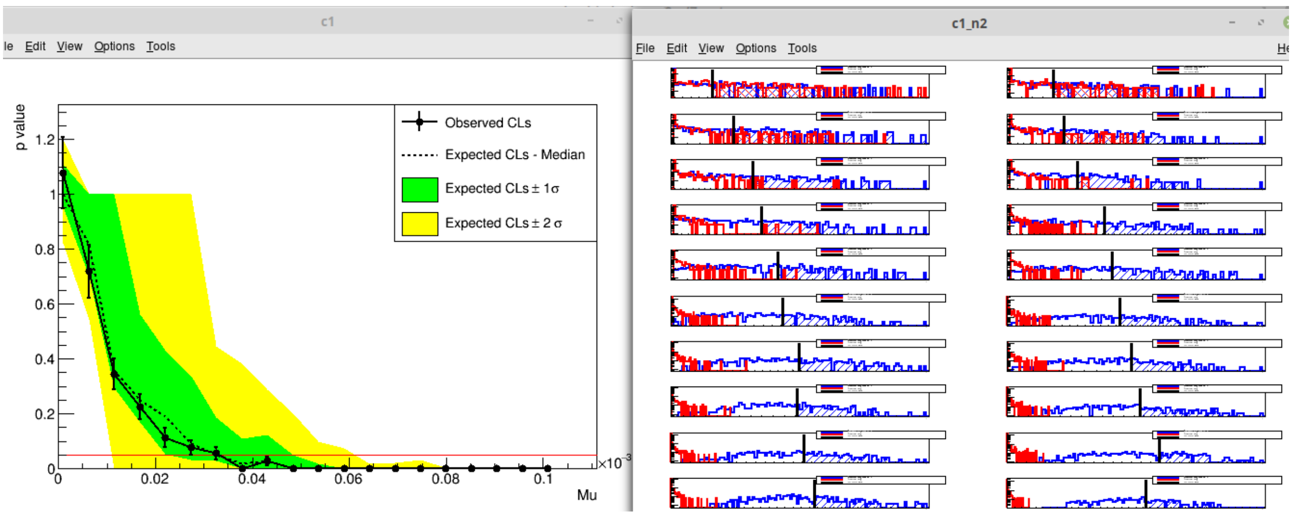


Fig. 1.1

Another CLs in Fig.1.2 of the same set of data the Bin1 and with the same value for σ-s with scale factor 2.0.

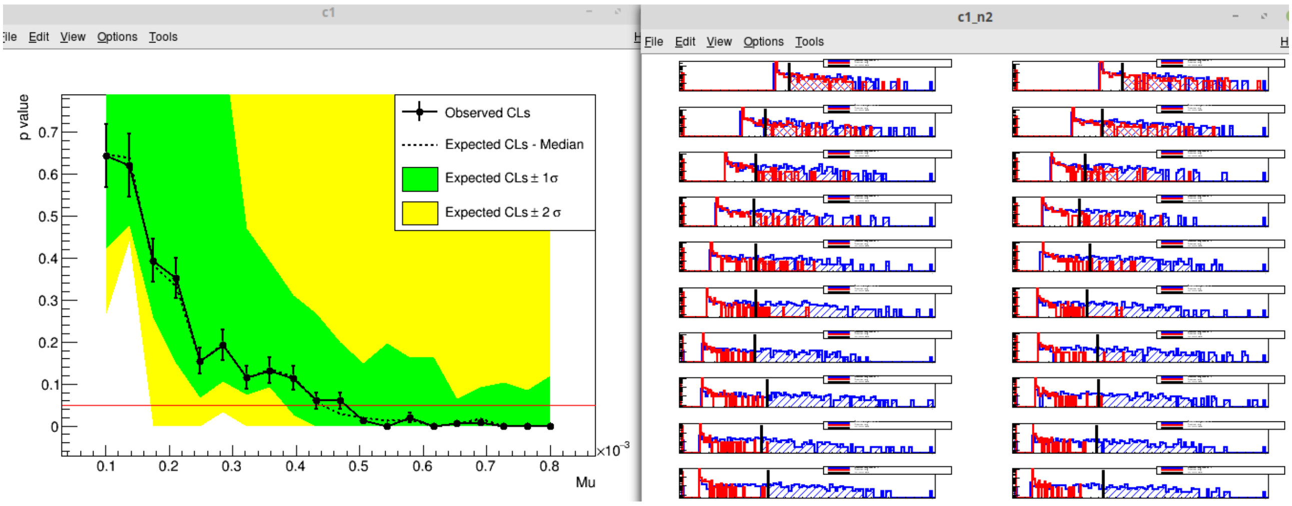


Fig. 1.2

So from all these CLs I had from scale factor 0.2 to 2.0 for each I had different upper limits and the final plot is reported in the Fig. 1.3.

	A	B	C
1	Scale Factor	Upper Limit (10-4)	Y errors (10-4)
2	0.2	0.33579	0.0499086
3	0.4	0.598948	0.0588082
4	0.6	1.00758	0.141651
5	0.8	1.36842	0.165547
6	1	2.06263	0.29419
7	1.2	2.20568	0.263212
8	1.4	2.90474	0.453794
9	1.6	3.48368	0.548419
10	1.8	3.55349	0.383313
11	2	4.32579	0.48545

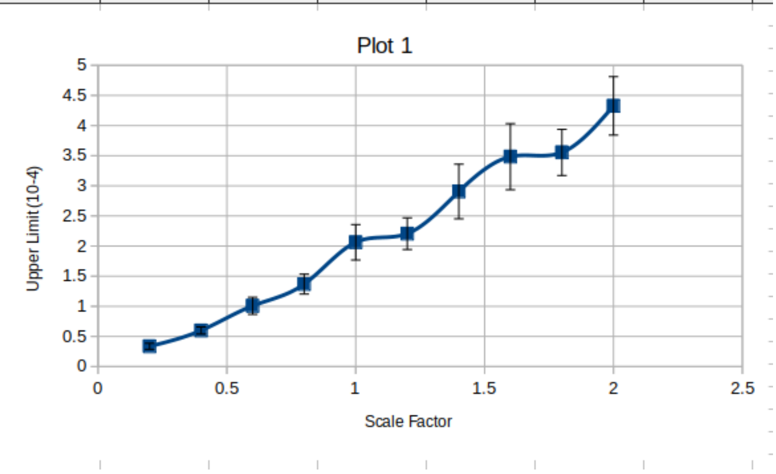
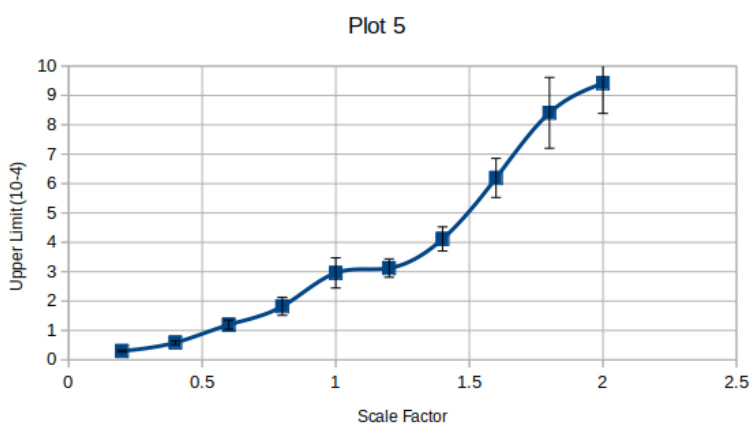


Fig. 1.3

So for the Bin1 with the inputs for $N_{Z\to\tau\tau}$=26000.0, $N_{Z\to\tau\tau}$=-20.0, ε^{rel}=0.1857, $\sigma_{\tau\tau}$=1000.0 $\sigma_{\mu\tau}$=360.0, σ_{ε}= 0.01857 keeping the scale factor constant the upper limit is 2.06263 $*10^{-4}$ +/- 0.29419$*10^{-4}$.

Also I'm gonna show you some other plots I have made with also with the second set of data known as Bin2.
So the first plot 5 in Fig. 1.4 is where the σ-s had the same scale factor and the plot 7 is where I only changed the $\sigma_{\mu\tau}$.

Scale Factor	Upper Limit (10^{-4})	Y errors (10^{-4})
0.2	0.299474	0.0344605
0.4	0.588947	0.0614311
0.6	1.19421	0.160108
0.8	1.82217	0.308988
1	2.95737	0.514332
1.2	3.12294	0.314574
1.4	4.11526	0.411701
1.6	6.19143	0.664973
1.8	8.40706	1.20549
2	9.41275	1.02641



Scale Factor	Upper Limit (10^{-4})	Y errors (10^{-4})
0.2	0.315686	0.0426005
0.4	0.666871	0.0788975
0.6	1.25317	0.194238
0.8	1.74684	0.22688
1	2.95737	0.514332
1.2	3.68421	0.470194
1.4	4.66879	0.519154
1.6	6.18914	0.743335
1.8	7.21114	0.739455
2	9.1638	1.00306

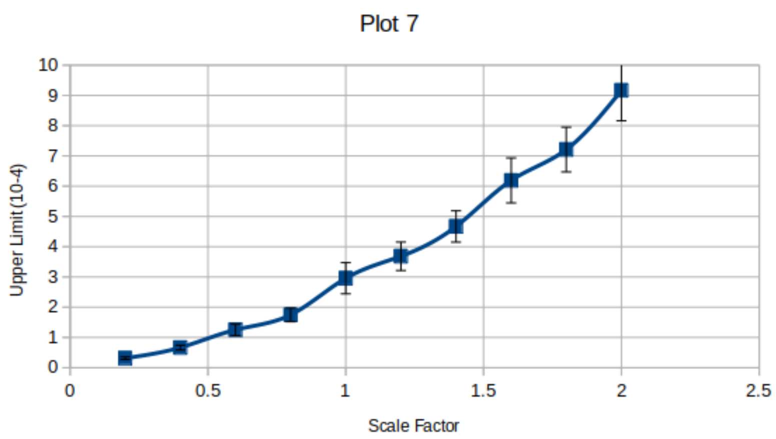


Fig 1.4

For the Bin2 with the inputs for $N_{Z\to\tau\tau}$=10300.0, $N_{Z\to\tau\tau}$=-10.0, ε^{rel}=0.1857, $\sigma_{\tau\tau}$=300.0 $\sigma_{\mu\tau}$=140.0, σ_{ε}= 0.01857 keeping the scale factor constant the upper limit is 2.95737*10^{-4} +/- 0.519154*10^{-4}.

Now this plot in the Fig. 1.5 is the combination of two Bins Bin1 plus Bin2, keeping σ-s constant for each Bin.

Scale Factor	Upper Limit (10-4)	Y errors (10-4)
0.2	0.184159	0.0251643
0.4	0.42095	0.0510182
0.6	0.799474	0.115714
0.8	0.957368	0.100264
1	1.38879	0.136678
1.2	1.71042	0.188151
1.4	2.2779	0.238525
1.6	2.37842	0.229815
1.8	3.26243	0.372518
2	4.09766	0.480575

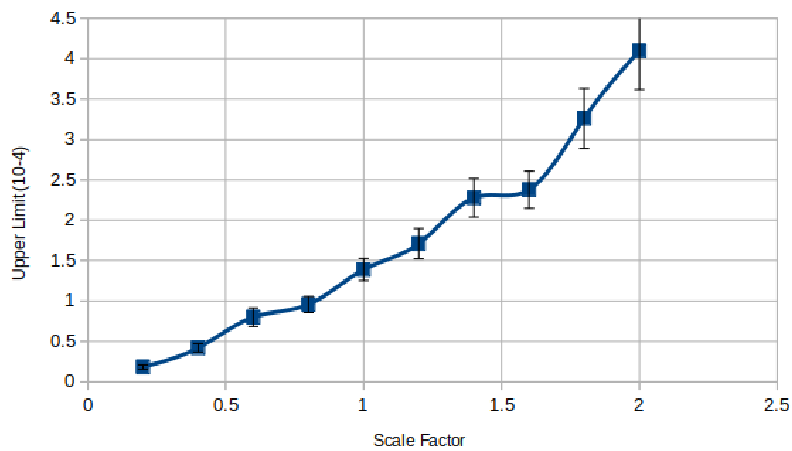


Fig.1.5

3.2 Conclusion

From this study I concluded that the upper limit is improved when we combined 2 Bins together. As we can see from the figures the upper limit of the Bin1 is 2.06263 $*10^{-4}$, Bin2 is 2.95737$*10^{-4}$ and for two Bins is 1.38879$*10^{-4}$.

4. Acknowledgments

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5. References

[1] http://pdg.lbl.gov/2019/reviews/rpp2018-rev-z-boson.pdf
[2] http://pdglive.lbl.gov/Particle.action?node=S044
[3] https://indico.cern.ch/event/398949/attachments/799330/1095613/The_CLs_Technique.pdf