Cosmological Consequences of Supersymmetric Flat Directions

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Abstract

In this work we analyze various implications of the presence of large field vacum expectation values (VEVs) along supersymmetric flat directions during the early universe.

First, we discuss supersymmetric leptogenesis and the gravitino bound. Supersymmetric thermal leptogenesis with a hierarchical right-handed neutrino mass spectrum normally requires the mass of the lightest right-handed neutrino to be heavier than about 10^9 GeV. This is in conflict with the upper bound on the reheating temperature which is found by imposing that the gravitinos generated during the reheating stage after inflation do not jeopardize successful nucleosynthesis. We show that a solution to this tension is actually already incorporated in the framework, because of the presence of flat directions in the supersymmetric scalar potential. Massive right-handed neutrinos are efficiently produced non-thermally and the observed baryon asymmetry can be explained even for a reheating temperature respecting the gravitino bound if two conditions are satisfied: the initial value of the flat direction must be close to Planckian values and the phase-dependent terms in the flat direction potential are either vanishing or sufficiently small.

We then show that flat directions also contribute to the total curvature perturbation. Such perturbation is generated at the first oscillation of the flat direction condensate when the latter relaxes to the minimum of its potential after the end of inflation. If the contribution to the total curvature perturbation from supersymmetric flat direction is the dominant one, then a significant level of non-Gaussianity in the cosmological perturbation is also naturally expected.

Finally, we argue that supersymmetric flat direction VEVs can decay non perturbatively via preheating even in the case where they undergo elliptic motion in the complex plane instead of radial motion through the origin. It has been generally argued that in this case adiabaticity is never violated and preheating is inefficient. Considering a toy $U(1)$ gauge theory, we explicitly calculate the scalar potential, in the unitary gauge, for excitations around several flat directions. We show that the mass matrix for the excitations has non-diagonal entries which vary with the phase of the flat direction vacuum expectation value. Furthermore, this mass matrix has zero eigenvalues whose eigenstates change with time. We show that these light degrees of freedom are produced copiously in the non-perturbative decay of the flat direction VEV.

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Contents

List of Figures

Chapter 1

Introduction

1.1 Scales and Scalars

Scalar fields are an essential component of our understanding of particle physics and cosmology. First of all they provide, via the Higgs mechanism, an explanation for the masses of fermions and gauge bosons. Indeed, in the Standard Model [1] of particle physics (SM), the Higgs mechanism is responsible for breaking the electro-weak force (described by the gauged $SU(2) \times U(1)_Y$ symmetry group) into a long range force - electromagnetism, mediated by the massless photon - and a short range force: the weak interaction, mediated by the massive W^+, W^- and Z bosons. The accuracy to which the SM describes Nature is frightening: from the most stringent precision tests carried at the earth-based Large Electron Positron (LEP) collider in CERN [2], the Stanford Linear Accelerator Centre (SLAC) Linear Collider (SLC), and at Tevatron, to a precise description of the generation of primordial light element abundances during the early universe (Big Bang Nucleonsynthesis [3]), in impressive accordance with experimental bounds from WMAP data and light element abundances [4, 5]. The simplest and most studied implementation of the Higgs mechanism [6] relies on the existence of a scalar field[1] - the Higgs field - charged under $SU(2) \times U(1)_Y$, which breaks the symmetry once it acquires a vacuum expectation value (VEV).

A scalar field is also possibly responsible for providing the initial conditions for Hot Big Bang cosmology: a flat, isotropic universe, free of unwanted relics, filled with homogeneous relativistic matter carrying the imprint of tiny perturbations which will eventually collapse and give rise to the complicate structures we observe today... stars, galaxies, clusters of galaxies. Such a scalar field - called *the inflaton* [9] - is supposed to have dominated the energy density of the whole baby-universe during at least 10^{-35} seconds. This short period of time was enough to cause the physical universe to expand exponentialy fast, faster than the speed of light, so fast that the scales that are coming back into the visible universe (the horizon) today are the ones that were inside the horizon shortly before inflation ended. And it expanded so much, that the quantum fluctuations of the initial vacuum were stretched and amplified into the classical density perturbations at the origin of large scale structures. This mechanism relies, again, on the existence of a scalar field.

Furthermore, a scalar field called *the axion* is thought to be at the origin of the smallness of the θ_{QCD} parameter in QCD - the strong CP problem in QCD [10]. In cosmology, the *K-essence* field [11] could be responsible for the cosmological constant and why it seems to dominate the energy density of the universe today. There is a plethora of other examples in which scalar fields play a major role in the understanding of Nature.

However, from the experimental point of view, there is a caveat to all this: no fundamental scalar field has ever been observed. The search of such fields is the biggest priority in experimental physics today, with the construction of the Large

[1] A number of alternatives have been proposed, which do not rely on the existence of scalar fields. In technicolor-like models [7] for example the Higgs scalar arises similarly to pions in QCD, from a composite state of fermions; in models with extra dimensions [8], on the other hand, electro-weak symmetry is broken by boundary conditions without the need for a Higgs field.

Hadron Collider (LHC) at CERN, searching for the Higgs particle, and the launch of the PLANCK satellite looking into the cosmic microwave background in search of signals from inflation. Independently of whether or not scalar fields are observed in these experiments, there is also a theoretical, fundamental problem with scalar fields in any field theory: quantum corrections to their masses inevitably drive these to the maximum allowed value, the scale at which that theory ceases to be valid which, for the case of the SM, is believed to be of order the (reduced) Planck scale, $M_p = 2.4 \times 10^{18}\,\mathrm{GeV}$. Scalar fields lighter than M_p are theoretically unacceptable in the SM. In the context of the Higgs field this is referred to as *The hierarchy problem*: through the exchange of virtual particles and through vacuum polarisation, the Planck scale and the weak scale, separated by 16 orders of magnitude, mix with each other. This large separation can only be maintained by repeatedly fine-tuning all orders in perturbation theory. We can quantify this problem by calculating the quantum corrections to the Higgs mass m_H originating, for example, from a coupling of the Higgs field H to fermion fields ψ of the form $-hH\overline{\psi}\psi$, at one loop

$$\left|\Delta m_H^2\right| = \frac{|h|^2}{8\pi^2} M_p^2 \gg m_H^2, \tag{1.1}$$

where we have assumed that the SM is valid up to the Planck scale and[2] that the Higgs mass $m_H \sim \mathcal{O}(100\,\mathrm{GeV})$.

It is important to stress that this plague affects only scalar fields. Fermion field masses are naturally [3] small thank to chiral symmetry. In fact, fermion mass terms have the form (here $\overline{\psi} = \psi^\dagger \gamma_0$)

$$m_\psi \overline{\psi}\psi, \tag{1.2}$$

[2]In the SM, from measurements of Gauge Boson masses, $\langle H\rangle = \sqrt{-2m_H^2/\lambda} = 174\,\mathrm{GeV}$, where λ is the Higgs self coupling (quartic). Therefore, without fine-tuning on the λ parameter, we expect the Higgs mass to be of order the Higgs VEV.

[3]*Natural* here meaning that the quantum corrections are proportional to the mass itself, not to some other (possibly large) energy scale involved in the theory, see the arguments by Weinberg, Susskind and 't Hooft [12].

and are incompatible with chiral symmetry

$$\psi \to e^{i\alpha\gamma_5}\psi, \tag{1.3}$$

where α is a real parameter while γ_5 and γ_0 are Dirac matrices which do not commute with each other. Since the contribution from eq. (1.2) is the only term in the fermion Lagrangian that violates chiral symmetry, we expect that in the limit $m_\psi \to 0$ this symmetry *must* be restored. It follows that also the contribution to the fermion mass coming from quantum corrections must be proportional to some power of m_ψ and, on dimensional grounds, it must be linear. A similar argument holds for gauge fields. In this case, mass terms are forbidden by gauge invariance and, as for fermions, quantum corrections cannot be proportional to any dimensionful quantity other than their own mass m_A. In both cases, for fermions and gauge bosons, the natural smallness of their mass is linked to a symmetry being restored when their mass goes to zero [12].

Is it possible to find such a symmetry for scalar fields? Let us note here that massive fermions have two more degrees of freedom than massless ones, while massive gauge bosons have an extra degree of freedom compared to massless ones: restoring symmetries kills degrees of freedom. Massive scalar fields have only one degree of freedom: is this the end of the story? Fortunately not, there are some ways out. The first solution to this problem, comes from noting that a mass term for a scalar ϕ is incompatible with the shift symmetry

$$\phi \to \phi + \alpha, \tag{1.4}$$

(where α is a constant parametrizing the symmetry) although the kinetic part of the scalar Lagrangian is invariant under this shift. This symmetry, characteristic of Nambu Goldstone degrees of freedom, could provide the kind of symmetry that we need to protect scalar masses. Little Higgs models [13], in which the Higgs field is the Nambu Goldstone mode originating from the breaking of a fundamental symmetry[4], are based on this idea. Another possibility would be to protect scalar masses by connecting them to fermion or vector masses which, in turn, are protected by chiral and gauge symmetries. This idea is exploited for instance in some models with extra dimensions where one embeds the Higgs scalar into a higher-dimensional vector multiplet [14], whose mass is protected by higher dimensional gauge invariance. Similarly, one can extend fermion multiplets to incorporate scalar fields, their mass will now be protected by chiral symmetry. This corresponds to introducing a new symmetry, called *Supersymmetry* (SUSY), relating bosons with fermions and protecting their mass difference from large quantum corrections. SUSY will be the main topic of this thesis.

In the simplest implementations of SUSY [15], every SM fermion is partnered with a supersymmetric scalar, and vice versa: SUSY is the *raison d'être* for light scalar fields. There are a number of other reasons, however, to believe that SUSY is the best extension of the Standard Model constructed so far. First of all, since SUSY transforms bosons into fermions, its generators must carry spin angular momentum 1/2 and must therefore represent a spacetime symmetry, just like the Poincaré group. It was shown by Haag, Lopuszanski and Sohnius in an extension of the Coleman-Mandula theorem [16] that SUSY is the *only* possible extension of the space-time symmetry group. In some sense then SUSY is the missing space-time symmetry of the SM. Another reason to believe in SUSY is related with Grand Unified Theories (GUT) [17]: SUSY could provide a perfect description of nature from low energies, up to the unification scale $M_{\rm GUT} = 10^{16}$ GeV. A study of the energy evolution of the gauge couplings (of the $SU(3)_c$ color, the $SU(2)_L$ weak and the $U(1)_Y$ hypercharge

[4]Note that a small amount of fine-tuning is still present in these theories. Indeed, gauge interactions break the shift symmetry of eq. (1.4) giving a contribution to the scalar mass proportional to the scale of new physics. On one hand, a light scalar is favored experimentally, on the other hand, this scale of new physics cannot be too low without being in conflict with electro-weak precision measurements. This tension is known as the *Little Hierarchy Problem*.

interactions) within SUSY, shows that these couplings could have a common origin at the scale M_{GUT}: the couplings unify almost exactly (unlike the SM). Furthermore, in the framework of cosmology, SUSY solves one of the biggest quests of astroparticle physics today: the nature of *dark matter*, needed to explain the formation of large scale structure in the universe and the anomalous rotation curves of galaxies. In SUSY, the Lightest Supersymmetric Particle (LSP) is protected from decaying by the same (R-type) symmetry that forbids proton decay[5]. If produced in the early universe, the LSP could survive till today and explain dark matter. The LSP has also the desirable feature that it can be produced at present colliders, probing with precision the viability of the dark matter model. Finally, the phenomenologically most attractive versions of String Theory, which possibly provides the consistent description of a quantum theory of gravity [18], naturally incorporate a form of supersymmetry. Some trace of this fundamental theory might then survive to low energy and can be described by SUSY.

All these theoretical motivations might look pointless in front of the experimental fact that supersymmetric particles have not been observed so far. But this shouldn't discourage believers in SUSY. In fact, all discovered fundamental particles have the common property that, without electro-weak symmetry breaking, they would be massless[6]. On the contrary, undiscovered SUSY particles can all have a mass term in the Lagrangian in agreement with $SU(2) \times U(1)_Y$ gauge invariance (see Section 1.3). It is therefore naturally expected that their mass be heavier than that of their SM partners.

[5] R-parity, is a multiplicatively conserved quantum number, defined for each SM field and its partner as

$$R_p = (-1)^{3(B-L)+2S} \tag{1.5}$$

where B and L are the lepton and baryon numbers carried by the field, S the spin. It is introduced in SUSY theories to avoid the appearance of dangerous B- or L- violating interactions at the renormalizable level. Such interactions can then appear only at the non-renormalizable level, and will be suppressed by powers of the (large) mass governing the scale of non-renormalizable interactions.

[6] This is not true for composite particles, such as baryons, which owe their mass to the strength of QCD interactions.

In SUSY, gauge symmetries and supersymmetry itself constrain the form of the Lagrangian density describing the dynamics of fields. In particular the potential of scalar fields is bound to include only certain combinations of interactions and the coupling constants multiplying these operators must respect certain rules. The most important consequence of this is that, along some particular directions in field space, the scalar potential vanishes exactly: these are called *flat directions*. A vanishing (renormalizable) potential[7] means that it is easy to excite these flat directions, and large VEVs can be produced. This has enormous implications for the cosmology of the early universe. Indeed during inflation large VEVs are produced along these directions and can take values as big as the Planck scale M_p. Large quantities of energy can be stored in these condensates and can change the history of the early universe when their energy is released once the condensate decays into the quanta of SM fields. The most dramatic example of this is provided by the baryogenesis model of Affleck and Dine [19]. They explain that a flat direction made up of a combination of fields with a non-vanishing total baryon (or $B - L$) number, once it decays, can fill the universe with enough matter (as opposed to antimatter) as is observed today.

In this thesis we analyse some major implications of flat directions for the history of the early universe, in particular in relation with their non-perturbative decay. Non-perturbative effects can lead to an explosively fast depletion of the energy carried by a scalar condensate. This is the theory of *preheating*, originally introduced by Kofman, Linde and Starobinsky [20] in the framework of inflation. We will show that, when applied to flat directions, preheating can for instance explain the baryonic (matter/antimatter) asymmetry or account for the generation of cosmological density perturbations responsible for the formation of large scale structures.

[7] As it will be discussed later, in the particular realisations of this scenario studied in this thesis, the scalar potential along the flat directions is not exactly zero but possesses a minimum which, depending on the parameters of the theory, can be as large as the Planck scale.

In the rest of this chapter we first give a short introduction to the early universe and then to supersymmetry in order to define flat directions and determine their dynamics. We also discuss how non-perturbative effects can influence flat directions. The remaining part of this thesis is organised as follows

- Chapter 2 is dedicated to the study of leptogenesis via supersymmetric flat directions and is based on [21].
- Chapter 3 analyses cosmological perturbations resulting from the decay of flat directions. This chapter is based on [22].
- Chapter 4 includes a detailed analysis of the non-perturbative decay of flat directions in the case relevant for Affleck-Dine baryogenesis. This chapter is based on [23] and [24] and is followed by the conclusions, two short appendices and the bibliography.

In support of this thesis, beside the studies published in [21], [22], [23] and [24], I also present the work published in [25] which, with a topic orthogonal to supersymmetry, didn't find a place in this thesis.

1.2 The Early Universe

Einstein's General Relativity relates the geometry of spacetime with its matter-energy content, describing the relationship between gravity (its geometrical origin) and its own sources. The geometry of an homogenous and isotropic universe is described in its most general form by the Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + R^2(t)\left(\frac{dx^2}{1-Kx^2} + x^2(d\theta^2 + \sin^2\theta d\phi^2)\right), \tag{1.6}$$

where $R(t)$, the *scale factor*, describes the physical size of the universe at time t and K is the *curvature* of our universe. Then, Einstein's equations provide a description of the evoultion of the scale factor via the Friedmann equation,

$$H^2(t) = \frac{\rho}{3M_p^2} + \frac{K}{3M_p^2 R^2(t)} + \frac{\Lambda}{3}, \tag{1.7}$$

where

$$H = \frac{\dot{R}}{R} \tag{1.8}$$

is the *Hubble parameter*, ρ is the total energy density of the universe and Λ is the *cosmological constant*, a term that can be added to Einstein's equation and can be interpreted as vacuum energy. The observational and theoretical values of Λ differ by about 120 orders of magnitude: this big puzzle is known as the *cosmological constant problem* and has no widely accepted solution at present (see [25] for example, where such a solution is discussed). For the present discussion we an assume $\Lambda \approx K \approx 0$, as suggested by observation. Eq. (1.6) together with the continuity equation

$$\dot{\rho} = -3H(\rho + P), \tag{1.9}$$

where P is the total pressure of the universe, provide a complete description of the evolution of our universe.

It is widely believed that right after the Big Bang, the universe underwent an extremely short period of extremely fast expansion: *inflation*. During this time the scale factor

$$R(t) \sim e^{H_I t} \tag{1.10}$$

grew exponentially fast (H_I is the nearly constant Hubble parameter during inflation). What caused this expansion is still a mystery (hints about inflation might soon be given by the Planck satellite mission), but the most accredited scenarios suppose that during this epoch the energy density of the universe was dominated by a scalar field, the inflaton. The particular equation of state of scalar fields is such that, when plugged into eq. (1.9) and eq. (1.6) one obtains the behavior of eq. (1.10). Inflation explains, for example, why the universe is so flat, homogenous and isotropic: before inflation, only small regions had time to become homogeneous, during inflation these regions where stretched to the size of the whole visible universe. For inflation to be effective and last long enough, the inflaton potential must be very flat. The inflaton vacuum expectation value (all space dependence of the inflaton is quickly wiped away by the expansion and can be treated as perturbation - see Chapter 3) then rolls slowly down this nearly flat potential, while the universe expands very fast. At some stage inflation must end (otherwise the universe will be empty today); this happens (the details are very much model-dependent) when the inflaton runs into a steeper part of the potential. The rapid expansion of the universe stops, and the energy stored in the inflaton is transferred to relativistic particles through a process known as *reheating*: this is the beginning of the Hot Big Bang cosmology, and the temperature of the universe can easily be of order 10^{10} GeV. The universe filled with highly relativistic particles continues its expansion and cooling (R increases, while H decreases) and, at some stage the Baryon asymmetry is generated (see Chapter 2). This happens certainly above a temperature of 100 GeV, when the electro-weak phase transition takes place.

The subsequent evolution of the universe can be studied very well within the Standard Model of particle physics. At a temperature of order 100 MeV the quarks bind into hadrons: protons and neutrons are formed. During the expansion, the density of particles is diluted and the interactions between particles become rarer and rarer. At some stage some interaction becomes too slow to keep on exchanging energy between a certain type of particle and the rest: these particles then decouple and are no longer in thermal equilibrium with the other particles. One can estimate when this takes place by comparing the total reaction of some particle with the expansion (Hubble) rate:

$$\Gamma \sim H. \tag{1.11}$$

This is what happens to neutrinos at $T \sim 1\,\mathrm{MeV}$ (the universe is only one second old). Weak reactions are too slow to keep the neutrinos in thermal equilibrium with the other SM particles; they decouple and will evolve undisturbed for ever. After about 3 minutes ($T \sim 0.1\,\mathrm{MeV}$) the primordial synthesis of nuclei takes place (Big Bang Nucleosynthesis), and all the neutrons of the universe bind subsequently into Deuterium, Helium 3 and 4 and a small quantity of Lithium. Since no stable nuclei with nuclear number 5 (which would result by the collision of Deuterium and Helium 3) or 8 (two nuclei of Helium 4 colliding together) exist in nature, this primordial chain production of nuclei stops here (heavier nuclei will form in the burning process inside stars). After this the universe evolves undisturbed for about $100'000$ years. At a temperature of 0.1 eV the amount of radiation equals the amount of non-relativistic matter in the universe; from then on the collapse of structures is no longer inhibited by the pressure of radiation and the formation of galaxies begins. Finally, after 10^9 years the first bound structures form. Today the universe is about 13×10^9 years old and its temperature, as measured by the WMAP experiment [4] probing the cosmic microwave background, is $2.728K$.

1.3 Supersymmetry

Having motivated supersymmetry in the first section, we shall now give it a short introduction, as this will be the main topic of this thesis. As argued in the previous section, supersymmetry transforms fermions into bosons and vice versa, schematically

$$Q|\text{fermion}\rangle = |\text{boson}\rangle \quad \text{and} \quad Q|\text{boson}\rangle = |\text{fermion}\rangle, \tag{1.12}$$

where Q denotes the quantum generator of supersymmetry. For consistency with eq. (1.12) and as a consequence of the extended Coleman-Mandula theorem [16], Q must be spinorial. The simplest (N=1) choice of SUSY generators is a 2-component Weyl spinor Q and its conjugate $Q^\dagger$. As discussed above, supersymmetry is a space-time transformation and therefore must have non-trivial (anti)commutation relations with the generators of the other space-time symmetries - the Poincaré group. In particular

$$\begin{aligned} \Big\{Q_\alpha, Q_\beta\Big\} &= \Big\{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\Big\} = 0 &(1.13)\\ \Big\{Q_\alpha, Q^\dagger_{\dot\beta}\Big\} &= 2\sigma^\mu_{\alpha\dot\beta} P_\mu &(1.14)\\ \Big[Q_\alpha, P_\mu\Big] &= \Big[Q^\dagger_{\dot\alpha}, P_\mu\Big] = 0 &(1.15) \end{aligned}$$

where the spinorial indices α, β of Q and $\dot\alpha$, $\dot\beta$ of $Q^\dagger$ take values 1 or 2, P_μ is the generator of space-time translations (the momentum) and $\sigma^\mu = (\mathbf{1}, \sigma_i)$ where σ_i are the Pauli matrices. Consider now the representation space of the SUSY algebra, divided into a bosonic subspace and a fermionic subspace; the generators Q and $Q^\dagger$ map one such subspace into the other. For the class of representations of interest to us, we know that acting on any subspace with the translation generator P_μ does not change the number of degrees of freedom (on quantum fields P_μ is represented by the partial derivative $i\partial_\mu$). This means, using eq. (1.14), that in the double mapping $QQ^\dagger + Q^\dagger Q$ no dimensions are lost. Since no dimensions can be created either in any of these mappings, we conclude that the fermionic and bosonic subspaces of the representation space of the SUSY algebra must have the same number of degrees of freedom:

$$n_B = n_F. \tag{1.16}$$

In the SM, left- and right-handed fermions have different transformation properties under the symmetry group $SU(2) \times U(1)_Y$. This makes it much simpler and more natural to use 2-component Weyl spinors rather than 4-component Dirac spinors to describe fields transforming differently under the symmetry group of electroweak interactions. This is why we are interested in the simplest representation of supersymmetry, the *chiral multiplet*, which contains a 2-component left-handed chiral spinor (right-handed spinors are obtained by taking the hermitian conjugate of these). The equality in eq. (1.16) tells us that we need a complex scalar with two degrees of freedom to partner this Weyl spinor inside the chiral multiplet. However, we know that a spinor fulfilling the equations of motion (on-shell) has only two degrees of freedom, while an off-shell spinor has four! Quantum computations are dominated by off-shell particles and since we introduced supersymmetry to cure quantum divergences, we must ensure that SUSY is preserved also off-shell. The solution is to introduce an *auxiliary field* F with the non-dynamical equation of motion (for a theory with no interactions)

$$F = F^* = 0. \tag{1.17}$$

The on-shell F will have zero degrees of freedom, while off-shell it will provide the two missing bosonic degrees of freedom to match the fermionic ones. The simplest SUSY multiplet thus consists of a complex scalar field ϕ, a 2-component left-handed Weyl spinor ψ_α and a complex auxiliary field F

$$(\phi_i, (\psi_i)_\alpha, F_i) \tag{1.18}$$

where i is a generic index denoting the type of field and $\alpha = 1, 2$ is the spinorial index. The Lagrangian describing the dynamics of a free scalar, a free spinor and the non-dynamical auxiliary field [8] is

$$\mathcal{L}_{\mathrm{WZ,\,kin}} = -\partial^\mu \phi^{*i} \partial_\mu \phi_i - i \left(\psi^{\dagger i}\right)_\alpha (\overline{\sigma}^\mu)^{\alpha\dot{\alpha}} \partial_\mu \left(\psi_i\right)_{\dot{\alpha}} + F^{*i} F_i, \tag{1.19}$$

where repeated field indices i are summed over, spinorial indices α and $\dot{\alpha}$ are also summed over using $\overline{\sigma}^\mu = (\mathbf{1}, -\sigma_\mathbf{i})$ and $\sigma^\mu = (\mathbf{1}, \sigma_\mathbf{i})$, while Lorentz indices are contracted as usual using the metric tensor. This is the non-interacting massless Wess-Zumino model [26]. The SUSY transformations that keep this Lagrangian invariant (up to a total derivative) can be worked out completely using the relations of eq. (1.13) - (1.15) (and imposing the constraint that the SUSY algebra closes on the chiral multiplet). Under the action of $\epsilon Q + \epsilon^\dagger Q^\dagger$,

$$\begin{array}{lcll} \delta\phi_i & = & \epsilon\psi_i & \text{(Boson} \rightarrow \text{Fermion)} \\ \delta(\psi_i)_\alpha & = & i(\sigma^\mu\epsilon^\dagger)_\alpha \partial_\mu \phi_i + \epsilon_\alpha F_i & \text{(Fermion} \rightarrow \text{Boson)} \\ \delta F_i & = & i\epsilon^\dagger_{\dot{\alpha}} \left(\overline{\sigma}^\mu\right)^{\alpha\dot{\alpha}} \partial_\mu (\psi_i)_\alpha & (F \rightarrow \text{Total derivative}), \end{array} \tag{1.20}$$

where, here and in what follows, inside products of spinors, spinorial indices are contracted using the antisymmetric tensor $((0, 1), (-1, 0))$. Here ϵ^α is an infinitesimal, anticommuting, two-component Weyl fermion object parametrising the SUSY transformation.

We can use a similar procedure to derive the form of the *vector multiplet*, the SUSY multiplet containing the SM vector bosons. A gauge boson A^a_μ has two degrees of freedom on-shell, three off-shell; its supersymmetric fermionic partner λ^a_α, with spinorial index α, will be a two component Weyl spinor with two (four) degrees of freedom on-(off-)shell. To fulfil the condition of eq. (1.16) and match the number of bosonic and fermionic degrees of freedom, we need a real auxiliary field D^a with only one degree of freedom off-shell. The vector multiplet is

$$(A^a_\mu, \lambda^a_\alpha, D^a) \tag{1.21}$$

[8] Once gauge interactions are included, partial derivatives have to be substituted with covariant derivatives, the same is true for the transformation of eq. (1.20).

and the kinetic Lagrangian is

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - i\lambda^{\dagger a}_{\alpha}\left(\overline{\sigma}^{\mu}\right)^{\alpha\dot{\alpha}} D_{\mu}\lambda^a_{\dot{\alpha}} + \frac{1}{2}D^a D^a \tag{1.22}$$

where the field strength is given by

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^a_\mu A^b_\nu \tag{1.23}$$

and the covariant derivative acting on the field λ^a is

$$D_\mu \lambda^a = \partial_\mu \lambda^a + g f^{abc} A^b_\mu \lambda^c. \tag{1.24}$$

Here g is the coupling of the gauge interaction and f^{abc} the structure constants of the symmetry group, with indices a, b, c running over its adjoint representation. Note from eq. (1.24) that the fields λ^a transform in the adjoint (real) representation of the gauge group. The SUSY transformation properties for the fields in the vector multiplet can be derived as above, from the algebra of eq. (1.13) - (1.15), by imposing that the algebra be closed on the vector multiplet; the Lagrangian of eq. (1.22) is automatically left invariant.

Supersymmetric multiplets are written more elegantly in the *Supefield* formalism. We shall briefly sketch how this works. We mentioned previously that in Supersymmetry the Coleman-Mandula theorem is avoided and a new spacetime symmetry is discovered. This requires the introduction of new space-time coordinates, which have the particularity of behaving as spinorial (anticommuting) degrees of freedom, θ_a and $\overline{\theta}_{\dot{a}} = (\theta_a)^*$, with $a = 1, 2$. The space made of standard spacetime (x^μ), extended with these spinorial coordinates is referred to as *Superspace*. Integration and differentiation in Superspace are defined as

$$\int d\theta_a \theta_a = \frac{\partial \theta_a}{\partial \theta_a} = 1 \quad \text{and} \quad \int d\theta_a = \frac{\partial}{\partial \theta_a} 1 = 0. \tag{1.25}$$

Fields which depend on all Superspace coordinates are called *Superfields*; every supersymmetric multiplet can be written in terms of its own single superfield (which we shall write, in what follows, using bold characters). For example, the chiral multiplet is defined as the superfield which is a function of θ_a and x_μ only (and not of $\overline{\theta}$):

$$\boldsymbol{\phi}_i(x,\theta) = \phi_i(x) + \theta\psi + \frac{1}{2}\theta\theta F(x), \tag{1.26}$$

where spinor indeces are contracted antisymmetrically, as usual. Note that this expression represents a Fourier expansion in the θ coordinate which, since anticommuting (so that $\theta_a^2 = 0$), stops after the third term in the expansion. The mass dimension of a chiral superfield is that of its scalar component, while the parameter θ itself carries mass-dimension 1/2. The vector multiplet can be written (in the so called Wess-Zumino gauge) as

$$\mathbf{A}(x,\theta,\overline{\theta}) = -\theta\sigma^\mu\overline{\theta}A_\mu(x) + i\overline{\theta}^2\theta\,\lambda(x) - i\theta^2\overline{\theta}\,\overline{\lambda}(x) - \frac{1}{2}\theta^2\overline{\theta}^2 D(x). \tag{1.27}$$

Similarly to the identification of $i\partial/\partial x^\mu$ with the generator of space-time translations, one can identify

$$Q_a = i\frac{\partial}{\partial\theta_a} \quad \text{and} \quad Q_a^\dagger = i\frac{\partial}{\partial\theta_a^*}. \tag{1.28}$$

In superfield notation it is then easy to understand how the supersymmetry transformations interchange the components of the chiral multiplet among each-other:

$$\delta_\epsilon\boldsymbol{\phi} = -i[\boldsymbol{\phi}, \epsilon Q] \quad \text{and} \quad \delta_{\epsilon^\dagger}\boldsymbol{\phi} = -i[\boldsymbol{\phi}, Q^\dagger\epsilon^\dagger] \tag{1.29}$$

give indeed eq. (1.20). Despite the elegance of superfield notation, in what follows we will mainly use component notation, unless we state otherwise.

This is a good moment to pause and introduce the field content of the Minimal Supersymmetric Standard Model (MSSM). In this model, every particle of the Standard Model is partnered with a supersymmetric particle to form a SUSY multiplet. The scalar partners ϕ_i of the fermions ψ_i are called the *sfermions* while the fermionic partners λ^a of the gauge bosons A_μ^a are called *gauginos*. Since the SUSY generator Q

Names		spin 0	spin 1/2	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
squarks, quarks	Q	$(\tilde{u}_L\ \tilde{d}_L)$	$(u_L\ d_L)$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$
(×3 families)	$\overline{u}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$
	$\overline{d}$	$\tilde{d}_R^*$	$d_R^\dagger$	$(\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3})$
sleptons, leptons	L	$(\tilde{\nu}\ \tilde{e}_L)$	$(\nu\ e_L)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$
(×3 families)	$\overline{e}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(\mathbf{1}, \mathbf{1}, 1)$
Higgs, higgsinos	H_U	$(H_U^+\ H_U^0)$	$(\tilde{H}_U^+\ \tilde{H}_U^0)$	$(\mathbf{1}, \mathbf{2}, +\frac{1}{2})$
	H_D	$(H_d^0\ H_D^-)$	$(\tilde{H}_D^0\ \tilde{H}_D^-)$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$

Table 1.1: Particle and sparticle content of the MSSM.

Names	spin 1/2	spin 1	$SU(3)_C$, $SU(2)_L$, $U(1)_Y$
gluino, gluon	$\tilde{g}$	g	$(\mathbf{8}, \mathbf{1}, 0)$
winos, W bosons	$\tilde{W}^\pm\ \tilde{W}^0$	$W^\pm\ W^0$	$(\mathbf{1}, \mathbf{3}, 0)$
bino, B boson	$\tilde{B}^0$	B^0	$(\mathbf{1}, \mathbf{1}, 0)$

Table 1.2: Gauge and gauginos in the MSSM.

commutes with all internal symmetries, particles and sparticles must share the same quantum numbers. In the MSSM, the quantum anomaly of electro-weak symmetry receives a non-vanishing contribution from the *higgsino* $\tilde{H}$, the fermionic partner of the Higgs scalar (the contribution to the anomaly from the other fermions exactly cancels like in the SM). Two higgs fields with opposite hypercharge are thus needed for the consistency of the electro-weak theory at the quantum level[9]. Tables 1.1 and 1.2 summarise schematically the particle content of the MSSM [15], where SUSY particles are generically distinguished using tildes.

For the present work, an important characteristic of SUSY is that at the renormalizable level all interactions between the fields in a chiral multiplet are fixed by a unique function: the *superpotential*, an analytic function of the superfields treated as complex variables, with dimension of $[\text{mass}]^3$. The superpotential that defines the MSSM is

$$W_{\text{MSSM}} = h_U \overline{\boldsymbol{u}} \boldsymbol{Q} \boldsymbol{H_U} - h_D \overline{\boldsymbol{d}} \boldsymbol{Q} \boldsymbol{H_D} - h_e \overline{\boldsymbol{e}} \boldsymbol{L} \boldsymbol{H_D} + \mu \boldsymbol{H_U} \boldsymbol{H_D}, \tag{1.30}$$

[9]Due to the constrained interactions of the MSSM - see below - two Higgs doublets are also needed to give masses to both up- and down-type quarks.

where we suppressed flavour indeces in the fields and the yukawa coupling matrices $h_{U,D,e}$; $SU(2)$ indices are contracted antisymmetrically. The dimensionful parameter $\mu \sim \mathcal{O}(100-1000)\,\mathrm{GeV}$ in order to properly break electro-weak symmetry giving a VEV $v = 174\,\mathrm{GeV}$ to the higgs field[10]. Given a superpotential, the interactions among the fields in the chiral multiplet take the form

$$\mathcal{L}_{\mathrm{WZ,\,int}} = \frac{1}{4}\int d\theta^2 W \tag{1.31}$$

in superfield notation, or

$$\mathcal{L}_{\mathrm{WZ,\,int}} = -\frac{1}{2}\left(\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_i\psi_j + \frac{\partial W}{\partial\phi_i}F_i\right) + \mathrm{c.c} \tag{1.32}$$

in component notation. In eq. (1.32) the superpotential W is treated as function of scalar components of superfields and derivatives are taken with respect to scalar fields. As we explained above, the auxiliary fields are not dynamical and the solution to the equations of motion derived from eq. (1.19) and eq. (1.32), in the presence of interactions, reads

$$F_i = -\frac{\partial W^*}{\partial\phi_i^*}. \tag{1.33}$$

Similarly, also the interactions between fields in the vector multiplet and fields in the chiral multiplet are constrained by supersymmetry and gauge symmetry: beside the terms already present in the covariant derivative we can also write the following terms

$$\mathcal{L}_{\mathrm{gauge,\,int}} = \left(-\sqrt{2}g\left(\phi^{*i}T^a_{ij}\psi^j\right)\lambda^a + \mathrm{h.c.}\right) + g\left(\phi^{*i}T^a_{ij}\phi^j\right)D^a \tag{1.34}$$

where T^a are the generators of the gauge symmetry[11] (we will omit to write family indices summed over in what follows). Note that the strength of these couplings in

[10] μ is a SUSY preserving quantity but is forced to have the same size as SUSY breaking quantities, in order to cancel the contribution from SUSY breaking masses at the miniumum of the Higgs potential. This is called the *μ-problem*; several solutions, all involving extensions of the MSSM have been proposed, see for example [27].

[11] The first two terms in eq. (1.34) resemble the couplings of gauge bosons and matter fields coming from the covariant derivative terms. This is in fact how they appear in the superfield formalism. It is interesting to note that, for this reason, gauginos are thought of as force-carriers as much as the gauge bosons, since they have similar interactions in the Lagrangian. The only difference is that, since fermions obey the Pauli exclusion principle, they will never conspire to form a coherent measurable potential, while gauge bosons do.

relation to the kinetic part of the Lagrangian eq. (1.19), is fixed by SUSY. Again, imposing the equation of motion gives an expression for the auxiliary fields

$$D^a = -g\left(\phi^* T^a \phi\right). \tag{1.35}$$

Putting all this together, eq. (1.19), eq. (1.22), eq. (1.32) and eq. (1.34), we find the full renormalizable supersymmetric Lagrangian. In particular, we identify the scalar potential with

$$V(\phi) = \sum_i |F_i|^2 + \frac{1}{2}\sum_a D^a D^a = \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2 + \frac{1}{2}\sum_a g_a^2 \left(\phi^* T^a \phi\right)^2. \tag{1.36}$$

We see that in SUSY, differently from the SM, gauge symmetries and supersymmetry strongly constrain the form of the scalar potential. The most striking consequence of this is that along some particular directions in field space the potential vanishes exactly: these are called *flat directions* [28]. Take for example the case in which a VEV develops along the L and H_U directions as

$$L = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \text{and} \quad H_U = \begin{pmatrix} v \\ 0 \end{pmatrix}, \tag{1.37}$$

where the upper (lower) field component carries weak isospin $T^3 = 1/2$ $(= -1/2)$. Since, as shown in Table 1.1, H_U carries opposite hypercharge than L and since the VEVs, with same value and phase, are assigned to different weak isospin charges, it is easy to show that

$$D_Y = D^3 = D^2 = D^1 = 0. \tag{1.38}$$

Similarly, since H_U and L never appear in the same term in the superpotential, we conclude that

$$F_i = 0 \tag{1.39}$$

for all fields i (assuming that no VEVs are developed along fields different from H_U and L). It follows that $V = 0$ for any value of v: no energy is required to produce a VEV along the flat direction.

In general, D-flat directions can be parametrised by the set of holomorphic gauge invariant polynomials [29] (F-flatness depends on the particular shape of the superpotential). This is an important result and we shall now briefly sketch the argument. By defining

$$\phi_\Lambda \equiv \exp\left(i\sum_a \Lambda_a T^a\right)\phi, \quad \Lambda_a \in \mathbb{C} \tag{1.40}$$

as the *complexified* gauge transformation $G^{\mathbb{C}}$ of the field configuration ϕ, the flatness condition $D^a = 0$ will be satisfied by ϕ_Λ if

$$\frac{\partial}{\partial(\mathrm{Im}\,\Lambda)}\left(\phi_\Lambda^\dagger\phi_\Lambda\right) = 0. \tag{1.41}$$

Since $\phi_\Lambda^\dagger\phi_\Lambda$ is real and bounded from below, it must have a minimum. This means that for every field configuration ϕ one can use a complexified gauge transformation to find a solution to the D-flatness condition, i.e. the space $\mathcal{M}$ of classical vacua is given by the space of all vacua $\mathcal{F}$, modulo complexified gauge transformations, $\mathcal{M} = \mathcal{F}/G^{\mathbb{C}}$. It follows that one can identify $\mathcal{M}$ with the space of holomorphic gauge invariant polynomials, since these polynomials are constant both under standard and complexified gauge transformation.

To conclude this section let us mention that, although these results were derived at the classical (tree-) level, they hold to every order in perturbation theory, thanks to the *supersymmetric non-renormalization theorem* [30]. In particular the flatness of flat directions is independent of quantum corrections.

1.4 Beyond Supersymmetry

The fact that the SUSY generator commutes with the translation generator, the momentum P^μ, means that particles and sparticles must share the same mass. But no supersymmetric particles have been ever observed so far: SUSY must be a badly broken symmetry. To describe broken SUSY in the most general way, a large number of terms must be added to the Lagrangians of eqs. (1.19), (1.22), (1.32) and (1.34). Moreover, once gravity is incorporated as a quantum theory[12] into a high-energy completion of SUSY, also non-renormalizable interactions will appear in the theory at low energies. These observations imply that the MSSM as outlined above can not be the end of the story. In this section we summarise the contributions coming from SUSY breaking and from non-renormalizable interactions to the low-energy effective Lagrangian of the MSSM and in particular to its scalar potential, eq. (1.36). As a by-product, we also mention the issue of gravitinos in the early universe.

From a theoretical point of view, we expect SUSY to be broken spontaneously, i.e. to be an exact symmetry with a symmetry breaking vacuum. This means that the vacuum is not annihilated by the action of the SUSY generators,

$$Q_\alpha|0\rangle \neq 0 \quad \text{and} \quad Q_\alpha^\dagger|0\rangle \neq 0. \tag{1.42}$$

On the other hand, from the algebra of eq. (1.14), we know that the energy operator can be expressed in terms of the SUSY generators as

$$H = P^0 = \frac{1}{4}\left(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2\right). \tag{1.43}$$

From the above expressions, we can evaluate the energy of the SUSY breaking vacuum:

$$\langle|H|\rangle = \frac{1}{4}\left(||Q_1^\dagger|0\rangle||^2 + ||Q_1|0\rangle||^2 + ||Q_2^\dagger|0\rangle||^2 + ||Q_2|0\rangle||^2\right) > 0. \tag{1.44}$$

That is, when (global) SUSY is spontaneously broken, the vacuum energy is necessarily positive. The expression for the scalar potential, eq. (1.36), tells us that this happens when some field acquires a non vanishing expectation value for its F- or D-component[13],

$$\langle F_i\rangle \neq 0 \quad \text{or} \quad \langle D^a\rangle \neq 0. \tag{1.45}$$

[12]Gravity must be quantized, otherwise we could imagine an experiment measuring exactly the position and the momentum of an electron to arbitrary precision, in contrast with Heisenberg principle.

[13]As we shall discuss later, in eq. (1.49), in the framework of *local* SUSY (Supergravity) negative contributions to the scalar potential generally arise and the vacuum energy can no longer be used as order parameter for SUSY breaking; on the other hand, non-vanishing expectation values for F- and D-terms always signal symmetry breaking.

There are several proposals for how this could take place in a phenomenological viable way [31, 32]. In this thesis we are not interested in the details of SUSY breaking, but only in how this breaking can be parametrised in the low energy Lagrangian by the inclusion of SUSY breaking interactions. We denote the SUSY breaking part of the Lagrangian as $\mathcal{L}_{\text{break}}$. First of all note that, for broken SUSY still to be able to solve the hierarchy problem, $\mathcal{L}_{\text{break}}$ can contain only terms with coupling constants having positive mass dimension (called *soft* SUSY breaking terms). Indeed, if this is the case, then all quantum corrections to the Higgs mass due to the SUSY breaking terms will be proportional to $\widetilde{m}$, the biggest scale associated with the SUSY breaking interactions (we will generically refer to this scale as *the SUSY breaking scale*), since they must vanish in the limit $\widetilde{m} \to 0$. On dimensional grounds then we can conclude that these quantum corrections cannot be proportional to a different scale possibly much bigger than $\widetilde{m}$, but at most to the logarithm of such a scale. Requiring that $\widetilde{m} < \mathcal{O}(\text{ TeV})$ makes the soft SUSY breaking terms compatible with a solution of the hierarchy problem. The most general renormalizable Lagrangian softly breaking supersymmetry in the MSSM can be written as

$$\mathcal{L}_{\text{break}} = -\left(\frac{1}{2}M_a\lambda^a\lambda^a + \frac{1}{6}A^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j\right) + c.c. - (m^2)^i_j\phi^{j*}\phi_i \qquad (1.46)$$

where the terms proportional to A^{ijk} and b^{ij} are trilinear and bilinear scalar interactions, while M_a and m^2 represent gaugino and scalar masses[14] respectively (SUSY is indeed broken, since particles within the same multiplet have now different masses). Generally,

$$A \sim M_a \sim \sqrt{|m^2|} \sim \sqrt{|b|} \sim \widetilde{m} \qquad (1.47)$$

Furthermore, note that part of the SUSY breaking terms have the same form as the terms appearing in the superpotential. This suggests that, if a source contributes in some form to the superpotential, then it can also generally contribute to the SUSY breaking part of the Lagrangian.

[14] Both gauginos (see eq. (1.24)) and Higgsinos are in a real representation of the gauge group, for this reason mass terms are compatible with gauge invariance.

During the early universe, quantum fluctuations can drive the VEVs along flat directions to very large values, even of order the Planck scale itself, where non-renormalizable interactions become important. Moreover, in SUSY, contributions originating from the unification with gravity, are particularly well motivated by *Supergravity*, local supersymmetric invariance. Once *global* SUSY is elevated to a *local* symmetry, gravity is naturally accounted for. This can be seen from the algebra of eq. (1.14): invariance under local SUSY transformations necessarily implies invariance under local coordinate shifts generated by P^μ. In turn, via the Poincaré algebra, this implies local invariance under the whole Poincaré group which is the underlying principle of Einstein's General Relativity. Supergravity, like all theories of gravity, is non-renormalizable as a quantum field theory. We will now say a few more words on supergravity, before characterising non-renormalizable interactions in the most general case. Similarly to the gauging of a gauge symmetry, which requires the introduction of gauge fields, promoting SUSY to a local symmetry requires the introduction of a new particle, the *gravitino*, the spin-3/2 partner of the graviton. Once SUSY is broken, this particle becomes massive by "eating" the degrees of freedom of the Nambu Goldstone boson of SUSY, the *goldstino*[15], via the *super-Higgs* mechanism. This gives it a mass $m_{3/2}$, whose magnitude depends on the details of SUSY breaking (if SUSY breaking is mediated by gravity, $m_{3/2} \sim \widetilde{m}$, while in gauge mediated scenarios $m_{3/2}$ can be much smaller: the gravitino is the LSP).

Gravitinos play a major role during the early universe. After inflation, when the energy density stored in the inflaton is converted into radiation at high temperatures (reheating), gravitinos can be produced copiously by scattering in the thermal bath - the production rate being bigger with bigger temperatures. If the gravitino is light $\mathrm{keV} \lesssim m_{3/2} \lesssim \widetilde{m}$ and stable, as in gauge-mediated SUSY breaking, then a large quantity of gravitinos could give too large a contribution to the total energy

[15] As shown above, SUSY breaking is equivalent to either $\langle |F| \rangle \neq 0$ or $\langle |D| \rangle \neq 0$. The goldstino is the fermionic component of the multiplet whose auxiliary field gets a VEV breaking SUSY.

density of the universe, over-closing it. On the other hand if the gravitino is heavier, $m_{3/2} \gtrsim \widetilde{m}$ as in gravity-mediated scenarios, then this poses a threat to Big Bang Nucleosynthesis (BBN). Indeed, at high energies, gravitinos couple with gravitational strength[16] to SM particles (and their supersymmetric partners), and therefore decay very late, jeopardizing the successful predictions of BBN. For gravitinos with $m_{3/2} \sim$ TeV decaying into a photon and an LSP, for example, we have

$$\tau_{3/2} \sim \frac{M_p^2}{m_{3/2}^3} \sim \mathcal{O}(10^5 \mathrm{sec}) \tag{1.48}$$

which is much longer than the time at which BBN starts, $t \sim \mathcal{O}(100\mathrm{sec})$. During BBN hydrogen nuclei are efficiently converted into heavier nuclei in almost perfect agreement with observations [5, 4] and gravitinos would compromise this picture because the highly energetic decay products of the gravitino would e.g. photodissociate helium nuclei and overproduce deuterium. This is the *gravitino problem* [33]; we shall discuss possible solutions in Chapter 2. For instance, if gravitinos are not efficiently produced during reheating, that is if the reheating temperature T_{RH} is small enough, this problem can be evaded. For gravitino masses in the natural range from 100 GeV to 1 TeV, within the minimal supergravity framework, the reheating temperature should be smaller than about 10^5–10^7 GeV [33], depending on the chosen values of the supersymmetric parameters and of the primordial element abundances. We will comment on this possibility in Chapter 2.

After this short parenthesis on gravitinos, let's come back to the problem of nonrenormalizable contributions to SUSY (this includes contributions from supergravity but applies in the more general case). It turns out that all nonrenormalizable corrections to SUSY (including terms with up to two spacetime derivatives) can be

[16]At low energy (relevant for collider phenomenology) it is the goldstino, helicity-1/2 component of the (now) massive gravitino which dominates its interactions, which are much faster than the interactions mediated by the helicity-3/2 component, suppressed by powers of M_P. At high energies, however, this is no longer true (for $m_{3/2}$ comparable to the other sparticle masses) and the gravitinos interact mostly gravitationally.

written in terms of just three functions: the superpotential, the *Kähler potential* and the *gauge kinetic function*. We have already met the superpotential $W(\phi_i)$ (here in terms of scalar components of superfields) in the previous section where, however, we have not included nonrenormalizable terms. $W(\phi_i)$ includes any gauge-invariant holomorphic combination of the fields such that its dimension is $[\text{mass}]^3$: for example nonrenormalizable terms of the form ϕ^n/M_p^{n-3} can be present in the superpotential if allowed by gauge symmetries. The Kähler potential $K(\phi,\phi^*)$ (this is generally written in terms of superfields although unless specified we shall refer to it as a function of the scalar component of superfields), on the other hand, is a real, gauge-invariant function of both ϕ_i and ϕ_i^* and has dimension $[\text{mass}]^2$, it obeys $K^\dagger = K$. In the MSSM, at tree level it's simply given by $\sum_i \phi^{i*}\phi_i$. The gauge kinetic function $f_{ab}(\phi_i)$ is a dimensionless analytic function of the fields, symmetric in its indices a and b, which determines the non renormalizable couplings between fields in the gauge multiplet. In the MSSM it's simply given by δ_{ab}/g_a^2. For example, the full supergravity potential for scalar interaction can be written uniquely in terms of these functions:

$$V = K_i^j F_j F^{*i} - 3e^{K/M_p^2}\frac{WW^*}{M_p^2} + \frac{1}{2}\,\mathrm{Re}\left[f_{ab}^{-1}\hat{D}^a\hat{D}^b\right] \tag{1.49}$$

where

$$K_j = \frac{\partial K}{\partial\phi^{*i}}, \quad (K^{-1})_i^j = \frac{\partial^2(K^{-1})}{\partial\phi_j\partial\phi^{*i}}, \tag{1.50}$$

and

$$F_i = -e^{K/2M_p^2}(K^{-1})_i^j\left(W_j^* + W^*K_j/M_p^2\right) \quad \text{and} \quad \hat{D}^a = -\phi^{*j}(T^a)_j^i K_i \tag{1.51}$$

are a generalization of the auxiliary fields of global SUSY. In superfield notation the supergravity Lagrangian part containing chiral fields only (without gauge interactions) can be written as

$$\mathcal{L} = \int d^2\theta d^2\overline{\theta} K(\boldsymbol{\phi}_i, \boldsymbol{\phi}^\dagger{}_j) + \left(\int d^2\theta W(\boldsymbol{\phi}_i) + h.c.\right). \tag{1.52}$$

The first two terms of eq. (1.49) can be easily derived from this expression, using eq. (1.26).

Let us now comment on the relevance of SUSY breaking and nonrenormalizable terms in the scalar potential for the dynamics of flat directions. As argued before, a flat directions can be written as a gauge invariant polynomial $X = \phi^k$, where ϕ denotes the VEV of the k fields involved in the flat direction - we will refer to ϕ as the flat direction VEV. First of all there can be nonrenormalizable corrections to the superpotential of the form

$$W = \frac{\lambda}{nM^{n-3}}\phi^n \quad \text{or} \quad W = \frac{\lambda}{M^{n-3}}\psi\phi^{n-1}, \tag{1.53}$$

where ψ denotes a field not making up the flat direction (but such that $\psi\phi^{n-1}$ is a gauge singlet) and M denotes generically the scale where new physics becomes important. The exponent n is the order at which the the flat direction is lifted, i.e. the smallest possible power at which we can write a polynomial consistent with both gauge symmetries and R-symmetries. Both superpotential terms of eq. (1.53) result in the following term in the scalar potential[17]

$$V(\phi) \supset \frac{|\lambda|^2}{M^{2n-6}}|\phi|^{2n-2}. \tag{1.54}$$

SUSY breaking also gives a non-vanishing contribution to the scalar potential. In particular soft SUSY breaking masses of order $\widetilde{m}$ and soft A-type terms proportional to the superpotential of eq. (1.53) are generated:

$$V(\phi) \supset \widetilde{m}^2|\phi|^2 + \frac{\lambda A}{nM^{n-3}}\phi^n. \tag{1.55}$$

It is important to stress that the terms originating from the nonrenormalizable superpotential might vanish in the presence of a discrete R-symmetry. An R-symmetry is a non-trivial abelian symmetry acting on the components of a supermultiplet, such

[17] There are other, higher-order in M_p^{-1}, terms originating from these superpotential terms. Such terms become important if the lowest order terms are for some reason suppressed. We will discuss this possibility below and in the following chapters.

that different components have different charges. For this to be possible, also the spinorial parameter θ must carry charge (chosen to be unitary) and from expression eq. (1.31) and from the requirement that the Lagrangian be invariant, we see that the superpotnetial W must carry charge -2. For particular choices of such an R-symmetry, the flat direction is lifted by SUSY breaking terms only. In this case Planckian VEVs can develop and higher order terms coming from eq. (1.49) - (1.51) can become important; the scalar potential will have the form [35]

$$V = \widetilde{m}^2 M_p^2 f\left(\frac{|\phi|^2}{M_p^2}\right) + \widetilde{m}^2 M_p^2 g\left(\frac{\phi^n}{M_p^n}\right), \tag{1.56}$$

for some functions f and g.

During the early universe there are much bigger contributions [35] to the flat direction potential than the ones listed above. For example, during inflation, the finite positive energy density of the inflaton necessarily breaks supersymmetry (see the discussion below eq. (1.44)). For a Hubble parameter $H \gg \widetilde{m}$ the contribution from this type of breaking dominates the potential. Moreover, during the radiation dominated era, the different thermal occupation numbers of fermions and bosons also break SUSY. This SUSY breaking is then comunicated to the flat direction mainly by nonrenormalizable interactions (surprisingly, the effect of renormalizable terms is negligible, this is because the fields coupled to the flat direction via renormalizable terms acquire large masses of order ϕ, decoupling from the theory, and are not able to communicate the breaking). Nonrenormalizable interactions on the other hand give large contributions to the scalar potential. Consider, for example, a term in the Kähler potential of the form[18] (here K is written in terms of superfields)

$$\mathcal{L} \supset \int d\theta^2 d\overline{\theta}^2 K = \int d\theta^2 d\overline{\theta}^2 \frac{1}{M_p^2} \mathbf{\Phi}^* \mathbf{\Phi} \boldsymbol{\phi}^* \boldsymbol{\phi} \supset \frac{1}{M_p^2} F_\Phi^* F_\Phi \phi^* \phi \tag{1.57}$$

[18] This term, which contributes to the so called *η-problem* giving a large mass to the inflaton, can be avoided only in particular circumstances, see for example [36] .

where F_Φ is the auxiliary component of a field Φ dominating the energy of the universe (e.g. the inflaton). From [19] eq. (1.36),

$$F_\Phi^* F_\Phi = V(\Phi) = \rho, \tag{1.58}$$

where ρ is the energy density of the field. Since Φ dominates the energy density of the universe, we know from the Friedman equations, that $\rho = 3H^2 M_p^2$. Plugging all this into eq. (1.57) and into eq. (1.49) we obtain an effective mass term for ϕ of order the Hubble parameter H,

$$V(\phi) \supset H^2 \phi^2. \tag{1.59}$$

Similarly, one can show that A-type terms also arise from this kind of SUSY breaking [35],

$$V(\phi) \supset H M_p^3 \tilde{f}\left(\frac{\phi^n}{M_p^n}\right), \tag{1.60}$$

for some function $\tilde{f}$. The general message here is that the SUSY breaking caused by the finite energy density of the universe during inflation contributes to the scalar potential with terms similar to the soft SUSY breaking ones, but with $\widetilde{m}$ replaced by H. To summarize the results of this section, the scalar potential along SUSY flat directions has the general form

$$V(\phi) = \left(\widetilde{m}^2 - cH^2\right)|\phi|^2 + \left(\lambda \frac{A + aH}{nM^{n-3}}\phi^n + \text{h.c.}\right) + |\lambda|^2 \frac{|\phi|^{2n-2}}{M^{2n-6}}, \tag{1.61}$$

where c, a and λ are dimensionless model-dependent couplings, while A has dimensions of [mass] and is of order the SUSY breaking masses $\widetilde{m}$. Depending on the sign of c, see [35], this potential will have a minimum either at $\phi = 0$, for $c < 0$ or at large ϕ, for positive c; we shall consider the latter example (otherwise the flat direction VEV relaxes in the origin and has no consequences for cosmology). In the

[19] It can be shown that supergravity corrections to the Inflaton scalar potential are small, see [35].

case $c > 0$, during inflation, the flat direction amplitude will reach the minimum of the potential at

$$|\phi_0| = \left(\frac{\beta H M^{n-3}}{\lambda}\right)^{1/(n-2)}, \qquad (1.62)$$

where β is a numerical coefficient depending on a, c and n. Depending on the order at which the flat direction is lifted, ϕ_0 can take values up to the Planck scale. The later evolution of the flat direction depends on the details of the model and will be treated in more detail in the following chapters.

1.5 Preheating

As outlined in the previous section, large VEVs can develop along supersymmetric flat directions during the early universe. After inflation, the form of the flat direction potential eq. (1.61) will change with time (since the Hubble parameter is a function of time $H(t)$) and the homogeneous VEVs will evolve accordingly to their classical equations of motion in the expanding universe,

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + \Gamma_\phi \dot{\phi} = 0, \qquad (1.63)$$

dots standing for derivatives with respect to time. Eq 1.63 is obtained by varying the action

$$S = \int \sqrt{-g}\mathcal{L}_\phi \qquad (1.64)$$

with respect to the field ϕ. Here g is the determinant of the metric of eq. (1.6) and $\mathcal{L}_\phi$ is the standard Lagrangian for scalar fields. The term proportional to the decay rate Γ_ϕ is a friction-like term due to the decay rate of the VEV. Eventually the curvature of the potential around the origin will change (this will happen for example when SUSY breaking terms proportional to H become smaller than soft SUSY breaking terms proportional to $\widetilde{m}$) and the true minimum will shift to $\phi = 0$. At this stage, the flat direction VEV will start oscillating around the origin. This scenario is very similar to the inflationary scenario where the inflaton scalar field, after slow-roll inflation, starts oscillating coherently around the origin while the amplitude of its oscillations slowly decreases due to Hubble expansion. Eventually the inflaton decays, transferring its energy to matter (which is highly relativistic and behaves like radiation) and heats the universe giving birth to the Hot Big Bang. As discussed in section 1.2, the inflaton decays when its decay rate is comparable to the expansion rate

$$\Gamma_\varphi \simeq H \tag{1.65}$$

where the decay rate for an inflaton with VEV $\langle\phi\rangle$ coupled to scalar fields χ via the term

$$\frac{1}{2}h^2\varphi^2\chi^2 \tag{1.66}$$

is given by[20]

$$\Gamma_\varphi \sim \frac{h^4\langle\varphi^2\rangle}{8\pi m_\varphi}. \tag{1.67}$$

The amplitude squared, however, red-shifts like t^{-2} in the expanding universe, while $H \sim t^{-1}$ and the decay rate never keeps up with the expansion rate. Moreover, in many models of inflation H is believed to be quite large, which means that the inflaton may decay very late after inflation (leading to small reheat temperatures, since its energy has had time to be diluted by the expansion), when H has decreased enough that inflaton decays are faster than the expansion rate. The inflaton can, however, decay in a much faster, non-perturbative way, via the mechanism of *pre-heating* [20]. This happens because any field χ which couples to the inflaton with the coupling of eq. (1.66) will receive a time dependent contribution to its mass

$$m_\chi(t)^2 = m_{\chi 0}^2 + h^2\langle\varphi(t)\rangle^2, \tag{1.68}$$

with $m_{\chi 0}$ being the bare mass of the field χ, where $\langle\varphi\rangle(t)$ is the time-dependent inflaton VEV as it oscillates around the origin. Similarly to the Unruh effect in

[20] After inflation when the inflaton starts oscillating around the origin, we can still estimate its decay rate since inflaton oscillations can be taken to be very fast compared to H, in particular after a few oscilations, when H has had time to decrease.

particle physics, this time-dependence will have interesting consequences: quanta of the field χ will be created by the time varying background [20, 37] and, due to energy conservation, the inflaton condensate will be depleted. This non-perturbative decay of the inflaton takes place on time scales much shorter than the ones for perturbative decay. In what follows we will review some aspects of quantum field theory in a time-dependent background relevant for the theory of preheating.

The equations of motion for the scalar field χ coupled to the time varying background field $\varphi(t)$ (the oscillations of φ are treated classically, using eq. (1.63)) have the form

$$\ddot{\chi} + m_\chi(t)^2\chi = 0 \tag{1.69}$$

The decomposition (d is the number of space dimensions)

$$\chi(x,t) = \int \frac{dk^d}{(2\pi)^{d/2}} \left(a(k)\chi(k,t)e^{-ik.x} + a(k)^\dagger \chi^*(k,t)e^{+ik.x}\right) \tag{1.70}$$

leads to the following equation of motion for the functions $\chi(k,t)$:

$$\ddot{\chi}(k,t) + \omega(k,t)^2\chi(k,t) = 0. \tag{1.71}$$

Where $\omega(k,t)^2 = k^2 + m(t)^2$ incorporates all the information about the time-dependent background[21]. Exact solutions of (1.71) can be written as

$$\chi(k,t) = \frac{e^{-i\int^t dt' W(k,t')}}{\sqrt{2W(k,t)}}, \tag{1.72}$$

where $W(k,t)$ is solution of the non-linear equation

$$W(k,t)^2 = \underbrace{\omega(k,t)^2}_{0-\text{th order}} + \underbrace{\frac{1}{2}\left(\frac{3}{2}\frac{\dot{W}(k,t)^2}{W(k,t)^2} - \frac{\ddot{W}(k,t)}{W(k,t)}\right)}_{\text{higher order}}. \tag{1.73}$$

Once the field χ is quantized and the coefficients $a(k)$ and $a(k)^\dagger$ promoted to annihilation and creation operators, the expansion of eq. (1.70) also uniquely defines the vacuum state of the theory:

$$a(k)|0\rangle = 0. \tag{1.74}$$

[21] This formalism can also by used to study particle creation by a time-varying gravitational background, as during inflation, leading to the formation of density perturbations; the expression for ω will then include information about the changing metric.

Eventually, we want to compare the (particle content of the) vacuum state at late times, with the vacuum state at early times, before the background perturbation caused by the field φ had any effect. For this purpose we need to define a reference initial vacuum that evolves undisturbed by the dynamics of φ. We refer to this state as the *adiabatic* vacuum, $|0^{(0)}\rangle$. To find such an initial state (and its unperturbed evolution) let's note that the dynamics of the background $m_\chi(t)$ influence the quantum state of χ via the time-dependence of the frequencies $\omega(t,k)$ and, in particular, via the "higher order" terms of eq. (1.73). How much influence the dynamics of φ will have on χ depends on the size of $\dot{\omega}/\omega^2$. We can define unperturbed (adiabatic) states as the modes corresponding to 0-th order solutions of eq. (1.73),

$$\chi^{(0)}(k,t) = \frac{e^{-i\int^t dt'\omega(k,t')}}{\sqrt{2\omega(k,t)}}, \tag{1.75}$$

and we can equally well expand the quantum field χ along these modes:

$$\chi = \int \frac{dk^d}{(2\pi)^{d/2}} \left(a^{(0)}(k)\chi^{(0)}(k,t)e^{-ik.x} + a^{(0)\dagger}(k)\chi^{(0)*}(k,t)e^{+ik.x} \right). \tag{1.76}$$

This expansion defines the set of creation and annihilation operators $a^{(0)\dagger}(k)$ and $a^{(0)}(k)$ which, in turn, define the adiabatic vacuum, $a^{(0)}(k)|0^{(0)}\rangle = 0$. Equating eq. (1.76) with eq. (1.70) gives an expression for the perturbed creation and annihilation operators in terms of the adiabatic ones:

$$a(k) = \int \frac{dk'^d}{(2\pi)^{d/2}} (\alpha(k',k)a^{(0)}(k') + \beta^*(k',k)a^{(0)\dagger}(k')) \tag{1.77}$$

with

$$a(k)|0^{(0)}\rangle = \int \frac{dk'^d}{(2\pi)^{d/2}} \beta^*(k',k)|1^{(0)}_{k'}\rangle \neq 0, \tag{1.78}$$

where $\alpha(k',k) = (\chi^{(0)}(k')e^{-ik'x}, \chi(k)e^{-ikx})$ and $\beta(k',k) = -(\chi^{(0)}(k')e^{-ik'x}, \chi^*(k)e^{ikx})$ are the *Bogoliubov coefficients* between adiabatic and exact modes; the scalar product in a space-time with metric $g^{\mu\nu}$ is defined as

$$(\phi_1,\phi_2) = -i\int_{\mathrm{t=const}} \phi_1(x)(\overleftarrow{\partial_0} - \overrightarrow{\partial_0})\phi_2^* g^{00}\sqrt{-g}\, dx^3. \tag{1.79}$$

Expression eq. (1.78) shows that the adiabatic and perturbed vacua are different: particles must be created by the time-dependence of ω. The expectation value of the particle number operator $N(k) = a(k)^\dagger a(k)$ of the system at later times is indeed not vanishing when evaluated on the initial vacuum (after having evolved it adiabatically):

$$\langle 0^{(0)}|N(k)|0^{(0)}\rangle = \int \frac{dk'^n}{(2\pi)^{n/2}}|\beta(k',k)|^2. \tag{1.80}$$

One can see that creation of particles takes place only if the exact modes differ from the adiabatic ones (adiabaticity is violated), i.e. if the parameter

$$\frac{|\dot{\omega}|}{\omega^2} \gtrsim 1. \tag{1.81}$$

In Chapter 4 we shall extend this analysis to the more general case where the inflaton is coupled to a set of scalar fields χ_i via a time-dependent mass matrix - this will allow preheating even in the case where decay into the single fields seems not not be efficient. For the analysis of Chapters 2 and 3, however, we are mainly interested in order-of-magnitude estimates of the number density of particles produced by the decay of the oscillating background field into quanta of a single species. For this purpose, only the expression of eq. (1.81) is needed: when adiabaticity is violated, particles are created. Adiabaticity is maximally violated when the scalar field VEV passes close to the origin, since there the change in m_χ is maximal, while its absolute value is minimal. Let us then estimate the number of particles of a field χ created by a scalar field VEV ϕ passing close to the origin with maximal velocity $|\dot{\phi}_*|$, when the two fields are coupled with strength h. If we assume that the particles created have vanishing bare mass $m_{\chi 0} = 0$, then particle production occurs for

$$\frac{|\dot{m}_\chi|}{m_\chi^2} = \frac{h|\dot{\phi}|}{h^2|\phi|^2} \gtrsim 1 \quad \Rightarrow \quad |\phi| \lesssim \phi_* \equiv \sqrt{\frac{|\dot{\phi}_*|}{h}}, \tag{1.82}$$

where ϕ_* is the absolute value of the background field when creation of particles begins. Particle production occurs for a short period of time,

$$\Delta t_* = \frac{\phi_*}{|\dot{\phi}_*|} = (h|\dot{\phi}_*|)^{-1/2}. \tag{1.83}$$

The uncertainty principle implies that the particles created will have typical momentum

$$\overline{k} \approx (\Delta t_*)^{-1} = \sqrt{h|\dot{\phi}_*|}. \tag{1.84}$$

We can then assume the spectral density to be a Guassian distribution with average momentum given by eq. (1.84),

$$n_k = \exp\left(-\pi\frac{k^2}{\overline{k}^2}\right) = \exp\left(-\pi\frac{k^2}{h|\dot{\phi}_*|}\right) \tag{1.85}$$

which can be extended to the case $m_{\chi 0} \neq 0$ by a simple shift in the value of the momentum squared:

$$n_k = \exp\left(-\pi\frac{k^2 + m_{\chi 0}^2}{h|\dot{\phi}_*|}\right). \tag{1.86}$$

Momentum integration of the spectral density eq. (1.86) gives an estimate of the total number density of particles produced during one oscillation of the VEV through the origin:

$$n_\chi = \frac{1}{2\pi^2}\int_0^\infty k^2 dk n_k = \frac{\left(h|\dot{\phi}_*|\right)^{3/2}}{8\pi^3}\exp\left(-\frac{\pi m_{\chi 0}^2}{h|\dot{\phi}_*|}\right). \tag{1.87}$$

Chapter 2

Supersymmetric Leptogenesis and the Gravitino Bound

2.1 Thermal Leptogenesis

The observed baryon/antibaryon number asymmetry (normalized with respect to the entropy density s) of the Universe [4]

$$Y_B \equiv \frac{n_B - n_{\overline{B}}}{s} = (0.87 \pm 0.03) \times 10^{-10} \tag{2.1}$$

is one of the most intriguing puzzles of fundamental particle physics. During inflation, all the energy of the universe is stored in the 0-mode of the inflaton - there is no matter - and, even more importantly, the universe is perfectly baryon symmetric. But now, we do not observe any bodies of anti-matter within the solar system. Furthermore, the lack of highly energetic γ-rays coming from the annihilation of protons and anti-protons from nearby regions of matter and antimatter suggests that no antimatter is present within a radius of 10Mpc around the solar system. The universe looks completely baryon asymmetric to us. The mechanism responsible for matter/antimatter asymmetry is called *baryogenesis* [38] and cannot be accounted for within the Standard Model of particle physics. In fact, as pointed out by Sakharov [39] already in 1967, for the baryon asymmetry to be produced, three conditions need to be met:

1. Baryon number violation
2. C and CP violation
3. Out of equilibrium conditions.

In the Standard Model, surprisingly, condition 1 is met thanks to quantum anomalies that violate the $B+L$ - although the difference $B-L$ is still conserved. This implies the existence of a number of vacua differing by their B and L contents and separated by barriers of size of order the electro-weak scale. Therefore, for temperatures $10^{12}\,\mathrm{GeV} \gtrsim T \gtrsim 100\,\mathrm{GeV}$ the baryon and lepton numbers in the universe can change via *sphaleron* processes[1], as long as $B-L$ is kept constant. However, in the Standard Model, conditions 2 and 3 are not fulfilled. Out of equilibrium conditions are met only during the electro-weak phase transition, but this turns out to be too smooth, due to the present experimental bounds on the Higgs mass, $m_h > 115\,\mathrm{GeV}$. Since particles and antiparticles share the same mass and statistics, without out of equilibrium conditions they would appear in the same numbers. Anyway, even if the electro-weak phase transition had been strong enough, due to the smallness of CP violation in the SM it would have been impossible to explain the observed baryon asymmetry (without CP violation baryons and antibaryons behave in the same way).

One way out of this problem is the mechanism of thermal leptogenesis [40, 41], the simplest implementation of this mechanism being realised by adding to the Standard Model three heavy right-handed (RH) neutrinos N_1, N_2 and N_3. Heavy

[1]This common terminology refers to non-perturbative electro-weak processes interpolating between different vacuum configurations characterised by different $B+L$.

RH neutrinos can also provide an explanation for the smallness of neutrino masses, via the *see-saw* mechanism [42]. To illustrate how this idea works, we shall consider a simplified model with only one lepton flavour L. The part of the Lagrangian involving RH neutrinos is

$$\mathcal{L}_N = i\overline{N}_i \partial\!\!\!/ N_i + \lambda_i N_i H L + \frac{M_i}{2} N_i^2 + \text{h.c.} \tag{2.2}$$

where M_i is a Majorana-type mass for the RH neutrinos, while λ_i is a dimensionless coupling constant that gives a Dirac-type mass to the neutrinos once the Higgs field receives a VEV $\langle H \rangle = v = 174\,\text{GeV}$; $i = 1, 2, 3$. We take N_1 to be the lightest RH neutrino and work in the limit $M_{2,3} \gg M_1$. Diagonalising the mass matrix for the L, N_i fields, reveals three heavy states with masses $\approx M_i$ and one light state with mass

$$m_\nu \approx \frac{\lambda_1^2 v^2}{M_1}. \tag{2.3}$$

A large mass $M_1 \gg v$ explains the smallness of neutrino masses m_ν, which from atmospheric neutrino experiments [43] is bound to be of order $\Delta m_{\text{atm}} = 0.05\,\text{eV}$ (unless neutrinos are highly degenerate). On the other hand, the CP violation from the complex phases[2] in the couplings of the Lagrangian eq. (2.2) is responsible for an asymmetry in the decay rate of N_1 into leptons and anti-leptons [44]. From the diagrams of Fig.2.1,

$$\Gamma(N_1 \to LH) = \frac{M_1}{8\pi}\left|\lambda_1 + A\lambda_1^* \lambda_{2,3}^2\right|^2, \quad \Gamma(N_1 \to \overline{LH}) = \frac{M_1}{8\pi}\left|\lambda_1^* + A\lambda_1 \lambda_{2,3}^{*2}\right|^2, \tag{2.4}$$

where A is the loop factor, and has a complex part [44] which can be computed in the limit of hierarchical RH neutrino masses, $M_{2,3} \gg M_1$. The asymmetry is given, at leading order in $M_1/M_{2,3}$, by

$$\epsilon \equiv \frac{\Gamma(N_1 \to LH) - \Gamma(N_1 \to \overline{LH})}{\Gamma(N_1 \to LH) + \Gamma(N_1 \to \overline{LH})} \simeq \frac{1}{4\pi}\frac{M_1}{M_{2,3}} \text{Im}\,\lambda_{2,3}^2 \simeq \frac{3}{16\pi}\frac{M_1 \text{Im}\,\tilde{m}_{2,3}}{v^2} \tag{2.5}$$

where $\tilde{m}_{2,3} \equiv \lambda_{2,3}^2 v^2 / M_{2,3}$ is the part of the light neutrino mass mediated by the heavy $N_{2,3}$, see eq. (2.3).

[2] We can absorb all but two of the phases appearing in the parameters of eq. (2.2) into a redefinition of the fields N_i and L. We shall take λ_2 and λ_3 to be complex.

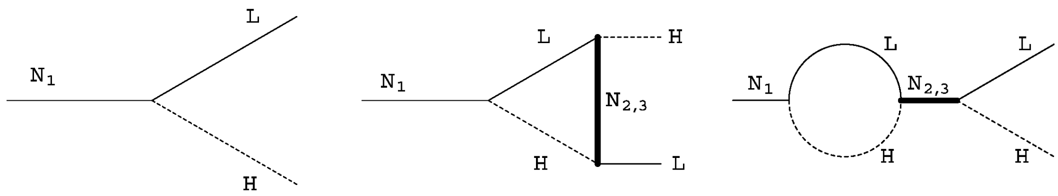

Figure 2.1: The Feynman diagrams describing the tree-level and one loop contributions to the decay of the lightest RH neutrino N_1.

In thermal leptogenesis the heavy RH neutrinos are produced by thermal scatterings after inflation and, if their decay rate is small enough ($\Gamma \lesssim H(M_1)$, where $H(M)$ is the Hubble parameter when the temperature is of order M), they decay out-of-equilibrium generating the lepton asymmetry. This lepton asymmetry is then converted into a baryon asymmetry due to $(B+L)$-violating sphaleron interactions [45]. A complete study of baryogenesis via leptogenesis requires the use of Boltzmann equations which take into account sphaleron processes and processes that can potentially wash away the asymmetry. Eventually, one obtains [44] that

$$Y_B \approx \frac{8}{23}\epsilon \frac{n_{N_1}^{eq}}{s} \tag{2.6}$$

where $n_{N_1}^{eq} \approx 2T^3/\pi^2$ is the number density of RH neutrinos N_1 when they are still in thermal equilibrium at $T \gg M_1$ and $s = (2\pi^2/45)g_* T^3$ is the entropy density (g_* being the number of relativistic species). The factor of 8/23 in front of eq. (2.6) can be calculated [46] by an analysis of the chemical potentials of all particle species in the high temperature phase (where sphalerons are active) and takes into account the fact that the original $B-L$ asymmetry is converted into B asymmetry by sphaleron processes.

The same argument goes through even for supersymmetric theories, where now also the decays into and from scalar partners need to be taken into account, resulting in an asymmetry ϵ which is four times bigger[3].

[3]Note that generally, also wash-out effects are twice as big in SUSY than they are in the SM. So, effectively, the asymmetry is normally only two times bigger.

From eq. (2.6) and an estimate of the equilibrium abundance of RH neutrinos, we can estimate the produced baryon asymmetry which will put bounds on M_1, since $\varepsilon_1 \propto M_1$. Indeed, if RH neutrinos are hierarchical in mass successful leptogenesis requires that the mass M_1 is larger than 2×10^9 GeV, for vanishing initial N_1 density [47]. This lower limit on M_1 is reduced to 5×10^8 GeV when N_1 is initially in thermal equilibrium and to 2×10^7 GeV when N_1 initially dominates the energy density of the Universe [48]. Hence, in the standard framework of thermal leptogenesis, the required reheating temperature after inflation $T_{\rm RH}$ cannot be lower than about 2×10^9 GeV [48]. In supersymmetric scenarios this is in conflict with the upper bound on the reheating temperature necessary to avoid the overproduction of gravitinos during reheating [33] (see discussion below eq. (1.48)).

The severe bound on the reheating temperature makes the thermal generation of the RH neutrinos impossible, thus rendering the supersymmetric thermal leptogenesis scenario not viable if RH neutrinos are hierarchical. Of course, there are several ways out to this drawback. First of all, one can modify the usual assumptions on gravitinos. If the gravitino is stable, the nucleosynthesis limit depends on the nature of the next-to-lightest supersymmetric particle, but values of $T_{\rm RH}$ even larger than 10^9 GeV can be obtained [49]. Also, gravitinos lighter than 1 KeV (as possible in gauge mediated SUSY breaking [32]) or heavier than about 50 TeV (as possible in anomaly mediated SUSY breaking) avoid the stringent limits on $T_{\rm RH}$. Alternatively, one can modify the standard mechanism of leptogenesis, and rely on supersymmetric resonant leptogenesis [50] or soft leptogenesis [51]. Indeed, in resonant leptogenesis the RH neutrinos are nearly degenerate in mass and self-energy contributions to the CP asymmetries are enhanced, thus producing the correct baryon asymmetry even at temperatures as low as the TeV. Soft leptogenesis can be successful for values of the mass M_1 of the lightest RH neutrino as low as 10^6 GeV. Another interesting variation is the case in which the right-handed sneutrino develops a large amplitude, dominating the total energy density [52]. Then the sneutrino decay reheats the universe, producing a lepton asymmetry, where values of T_{RH} as low as 10^6 GeV do not cause a gravitino problem. Finally, one can modify the standard thermal production mechanism of N_1. The lightest RH neutrinos can be produced non-thermally either during the preheating stage [53], or from the inflaton decays [54] or from quantum fluctuations [55].

In this chapter, we would like to show that a solution to the tension between supersymmetric leptogenesis with hierarchical RH neutrinos and the gravitino bound is in fact already rooted in one of the basic properties of the supersymmetric theory, that is the presence of flat directions in the scalar potential [28]. No new ingredient has to be added to the theory. Let us briefly sketch how the solution works. As explained in the Introduction, the F- and D-term flat directions are lifted because of the presence of the soft supersymmetry breaking terms in our vacuum, of possible non-renormalizable terms in the superpotential and of finite energy density terms in the potential proportional to the Hubble rate H during inflation [35]. As a consequence, the field ϕ along the flat direction will acquire a large vacuum expectation value (VEV). After inflation, when the sign of the curvature of the potential changes, the condensate starts oscillating around the true minimum of the potential which resides at $\phi = 0$. If the condensate passes close enough to the origin, as discussed below, the particles coupled to the condensate are efficiently created at the first passage. The produced particles become massive once the condensate continues its oscillation leaving the origin and may efficiently decay into other massive states[4], in our case RH neutrinos. The latter will subsequently decay to generate the final baryon asymmetry. This process allowing the generation of very massive states is called *instant preheating* [56] and is a particular case of preheating [20] in which

[4]Notice that this prompt decay of the produced particles, effeciently removes them from the resonant band and no resonant preheating is expected from the oscillating flat direction. This is the reason why we consider particle production only at the first oscillation.

the produced particles are coupled to fermions. It represents a very efficient way of producing heavy states with only one oscillation of the condensate. In this sense, the solution we are proposing may be considered as a non-thermal production of RH neutrinos, but we stress that it does not involve any extra assumption such as a large coupling between the RH neutrinos and the inflaton field.

2.2 Flat Direction Leptogenesis

The generic potential for a supersymmetric flat direction ϕ is given by eq. (1.61) where we assume the scale M of non-renormalizable terms to be equal to the reduced Planck scale ($M = M_p = 2.4 \times 10^{18}$ GeV). For $c > 0$ and $H \gg \widetilde{m}$, the flat direction condensate acquires a VEV given by eq. (1.62). At the end of inflation, the inflaton field starts oscillating around the bottom of its potential and the Hubble rate decreases. As soon as $H \sim \widetilde{m}/3$, the condensate starts rolling down towards its minimum at $\phi = 0$. In this case, the field ϕ will oscillate coherently with frequency $\widetilde{m}$ around the origin.

Now, if in the potential of eq. (1.61) both terms proportional to A and aH are present and their relative phase $\theta_a - \theta_A$ does not vanish, the condensate $|\phi|\, e^{i\theta}$ will spiral around the origin at $\phi = 0$ with a nonvanishing $\dot{\theta}$ (possibly leading to a large baryon asymmetry through the Affleck-Dine mechanism [19, 35]). In this case instant preheating does not occur and no heavy states are produced [57], unless several flat direction phases are simultaneously excited [23, 58], see Chapter 4. We will focus on the opposite case, when the condensate passes through the origin (or sufficiently close to it). This is easy to achieve without any fine-tuning [35] as it is enough to consider a flat direction which is lifted only by a non-renormalizable superpotential term which contains a single field not in the flat direction and some number of fields which make up the flat direction [35],

$$W = \frac{\lambda}{M^{n-3}} \psi \phi^{n-1}. \tag{2.7}$$

For terms of this form, F_ψ is non-zero along the flat direction (see eq. (1.54)), but $W = 0$ along it. Examples of this type are represented by the direction with field content ue which is lifted by $W = (\lambda/M)uude$, since $F_d^* = (\lambda/M)uue$ is non-zero along the direction, and by the Que direction which is lifted by the $n = 9$ superpontial $W = (\lambda/M)QuQuQuH_Dee$ since $F_{H_D}^* = (\lambda/M)QuQuQuee$ does not vanish [59]. If $W = 0$ along the flat direction, no phase-dependent terms are induced. Alternatively, the superpotential may vanish along the flat direction because of a discrete R-symmetry. In such a case, when W exactly vanishes, the potential during inflation has the form [35]

$$V(\phi) = H_I^2 M_p^2 f(|\phi|^2 / M_p^2) + H_I^2 M_p^2 g(\phi^n / M_p^n), \tag{2.8}$$

(where H_I is the Hubble parameter during inflation) and the typical initial value ϕ_0 for the condensate is $\mathcal{O}(M_p)$, rather than eq. (1.62). For this reason we will treat ϕ_0 essentially as a free parameter in our analysis and not fixed by the relation eq. (1.62). Finally we remark that the coefficients A and a of eq. (1.61) depend on the specific form of the Kähler potential couplings and there are cases in which they are suppressed by inverse powers of M_p. In the case of D-term inflation [60] a vanishes identically and no phase-dependent terms are generated if along the flat direction $W = 0$.

From now on, we will consider a flat direction along which the induced phase dependent A-type terms are suppressed and therefore the corresponding condensate will oscillate passing very close to the origin. Furthermore, we will focus on the flat direction involving the third generation quark u_3. When the condensate passes through the origin, it can efficiently produce states which are coupled to it. Let us consider the scalar Higgs H_U which is relevant for leptogenesis although, of course, other states will be produced as well. If the third generation is involved in the flat direction, the up-Higgs is coupled to the condensate through the Lagrangian term $h_t^2 |\phi|^2 |H_U|^2$. Its effective mass is therefore given by $m_{H_U}^2 = \widetilde{m}_{H_U}^2 + h_t^2 |\phi|^2$, where $\widetilde{m}_{H_U}^2$ is the corresponding soft-breaking mass parameter. At the first passage through the origin, particle production takes place when adiabaticity is violated [56], $\dot{m}_{H_U}/m_{H_U}^2 \gtrsim 1$, as explained in Section 1.5. Following the discussion below eq. (1.82) and taking the velocity of the condensate at the origin to be

$$\dot{\phi}_* = \widetilde{m}\phi_0, \tag{2.9}$$

(where ϕ_0 is the amplitude of the flat direction vev when oscillations start) we can estimate the number density of up-Higgses generated during the first oscillation of the flat direction VEV (see eq. (1.87)):

$$n_{H_U} \sim \frac{\overline{k}^3}{8\pi^3} \sim \frac{(h_t \widetilde{m} |\phi_0|)^{3/2}}{8\pi^3}. \tag{2.10}$$

where we have assumed the SUSY breaking mass $\widetilde{m}_{H_U}$ of H_U (called m_χ eq. (1.87)) to be much smaller than its average momentum, $\overline{k} \sim (h_t \widetilde{m} |\phi_0|)^{1/2}$. After the condensate has passed through the origin continuing its motion, the up-Higgses become heavier and heavier, having an effective mass $\sim h_t |\phi|$. When this mass becomes larger than the lightest RH neutrino mass M_1, the up-Higgses will promptly decay into the lightest RH neutrinos N_1 through the superpotential coupling $h_{ij} N_i \ell_j H_U$, where ℓ_j stands for the lepton doublet of flavour j and $i, j = 1, 2, 3$ (an extension of the toy-model of eq. (2.2) to account for different lepton flavours). Indeed, the H_U decay is prompt because the decay rate into N_1 and leptons,

$$\Gamma_D \sim \sum_j \frac{|h_{1j}|^2 h_t \phi}{8\pi} \tag{2.11}$$

is faster than the oscillation rate $\Gamma_{\rm osc} \sim \dot{\phi}/\phi$ as long as

$$\phi^2 > \frac{8\pi \widetilde{m} \phi_0}{\sum_j |h_{1j}|^2 h_t}, \tag{2.12}$$

which is certainly satisfied during the first oscillation. Moreover, if one of the h_{1j} is not too small, and Q_3 is not involved in the flat direction[5], H_U will dominantly

[5] For the Que flat direction the $n = 9$ lifting superpotential contains Q_3 only if all the $n = 4$ lifting superpotentials $QQQL$, $QuQd$, $QuLe$ and $uude$ are present in the supersymmetric Lagrangian.

decay into $N_1\ell$, since any decay process occurring through top-Yukawa or gauge interaction is kinematically forbidden (or strongly suppressed) at large ϕ, due to the large masses obtained by the fields coupled to the flat direction.

To estimate the maximum value $M_1^{\max}$ that can be generated we have to compute the maximum value $\phi^{\max}$ achieved by the condensate during its first oscillation, after passing through the origin. The equation of motion for ϕ is [61]

$$\ddot{\phi} + \widetilde{m}^2\phi = -h_t\frac{|\phi|}{\phi}n_{H_U}. \tag{2.13}$$

The term on the right-hand side corresponds to the ϕ-dependent energy density $m_{H_U}(\phi)n_{H_U}$ generated by the H_U particles produced when ϕ crosses the origin. It acts as a friction term damping the ϕ oscillations. Solving eq. (2.13), we obtain

$$M_1^{\max} \simeq h_t\phi^{\max} = \frac{4\pi^3\widetilde{m}^{1/2}\phi_0^{1/2}}{h_t^{3/2}} = 4\times 10^{12}\ \text{GeV}\left(\frac{\phi_0}{M_p}\right)^{1/2}\left(\frac{\widetilde{m}}{100\,\text{GeV}}\right)^{1/2}, \tag{2.14}$$

where we have taken the top-Yukawa coupling $h_t \simeq 0.6$ at high-energy scales. Thus, very heavy RH neutrinos can be produced through this mechanism.

In first approximation, we can assume that all H_U decay into N_1 and the number density of the RH neutrinos is given by $n_{N_1} \sim n_{H_U} \sim (h_t\widetilde{m}|\phi_0|)^{3/2}/8\pi^3$. When the mass of the up-Higgses decreases because the condensate, after reaching its maximum value at the first oscillation, starts decreasing again, the RH neutrinos may efficiently decay into up-Higgses and leptons and produce a lepton asymmetry $n_L \sim \epsilon n_{N_1}$ where the usual CP asymmetry ϵ is generated by the complex phases in the Yukawa couplings h_{ij}.

During all these stages, the inflaton field continues to oscillate around the minimum of its potential and will eventually decay into SM degreees of freeedom giving rise to the reheating stage, leading to the Hot Big Bang. Before reheating, the universe is matter dominated because of the inflaton oscillations and the scale factor increases as $a \sim H^{-2/3}$. The lepton asymmetry $n_L \sim \epsilon n_{N_1}$ produced during the first oscillation at $H_{\rm osc} \sim \widetilde{m}/3$ is diluted at the time of reheating by the factor $a_{\rm osc}^3/a_{\rm RH}^3 = H_{\rm RH}^2/H_{\rm osc}^2$. Expressing n_{N_1} through eq. (2.10), we find that the baryon asymmetry $Y_B = (8/23)(n_L/s)(H_{\rm RH}^2/H_{\rm osc}^2)$ becomes

$$Y_B \sim \frac{9\,\epsilon\,h_t^{3/2}\,T_{\rm RH}\,|\phi_0|^{3/2}}{92\pi^3\widetilde{m}^{1/2}M_p^2} = 10^{-6}\,\epsilon\left(\frac{T_{\rm RH}}{10^7\,{\rm GeV}}\right)\left(\frac{|\phi_0|}{M_p}\right)^{3/2}\left(\frac{100\,{\rm GeV}}{\widetilde{m}}\right)^{1/2}. \quad (2.15)$$

Notice that in our estimate we have not inserted any wash-out factor, due to inverse processes that might erase the produced asymmetry. Indeed, as soon as the RH neutrinos decay, their energy density $\rho_{N_1} = M_1 n_{N_1}$ gets promptly converted into a "thermal" bath with an effective temperature

$$\widetilde{T} \sim \left(\frac{30\rho_{N_1}}{g_{*N}\pi^2}\right)^{1/4} \quad (2.16)$$

where g_{*N} is the corresponding number of relativistic degrees of freedom. We estimate that $\widetilde{T}$ is smaller than M_1 when

$$M_1 > 10^9\ {\rm GeV}\left(\frac{|\phi_0|}{M_p}\right)^{1/2}\left(\frac{\widetilde{m}}{100\,{\rm GeV}}\right)^{1/2}. \quad (2.17)$$

As much heavier RH neutrinos are generated through the preheating stage, we may safely conclude that $\Delta L = 1$ inverse decays are not taking place. Similarly, one can show that the $\Delta L = 2$ processes are out-of-equilibrium. Finally, flavour effects [62] play no role in determining the final baryon asymmetry as $\Delta L = 1$ inverse decays are out-of-equilibrium. The maximum CP asymmetry parameter for normal hierarchical light neutrinos, in the supersymmetric case, is given by

$$\epsilon = \frac{3M_1 m_3}{4\pi\langle H_U\rangle^2} \quad (2.18)$$

where $m_3 = (\Delta m_{\rm atm}^2)^{1/2}$ is the largest light neutrino mass, which we take from atmospheric neutrino experiments to be of order 0.05 eV; we assume maximal phases. From eq. (2.15), we therefore estimate that enough baryon asymmetry is generated if

$$M_1 \gtrsim 2\times 10^{11}\,{\rm GeV}\left(\frac{10^7\,{\rm GeV}}{T_{\rm RH}}\right)\left(\frac{M_p}{|\phi_0|}\right)^{3/2}\left(\frac{\widetilde{m}}{100\,{\rm GeV}}\right)^{1/2}. \quad (2.19)$$

This limit, together with the result in eq. (2.14), implies that a successful baryogenesis can occur only if $\phi_0 \gtrsim 0.2\, M_p\, (10^7\,\mathrm{GeV}/T_{\mathrm{RH}})^{1/2}$. The condensate of the flat direction has to start its oscillation from field values close to the reduced Planck mass. Notice that this limit on ϕ_0 is independent of h_t. However, the presence of the top Yukawa coupling is necessary to guarantee that the flat direction decays abundantly into H_U.

2.3 Remarks

We conclude with some remarks. First, gravitinos are produced also during the instant preheating phase by scatterings of the quanta generated at the first oscillation of the condensate; one might worry that this reintroduces the problem we set off to solve. It is easy to estimate that their abundance is

$$\frac{n_{3/2}}{s} \simeq 10^{-4} \left(\frac{T_{\mathrm{RH}}}{M_p}\right) \left(\frac{\phi_0}{M_p}\right)^3 \tag{2.20}$$

and therefore it is never larger than the gravitino abundance produced at reheating by thermal scatterings, given by

$$\frac{n_{3/2}}{s} \simeq 2 \times 10^{-12} \left(\frac{T_{\mathrm{RH}}}{10^{10}\ \mathrm{GeV}}\right). \tag{2.21}$$

Secondly, from eq. (2.19) we infer that large values of the lightest RH neutrino mass M_1 are needed for the generation of a sufficiently large baryon asymmetry. However, we would like to point out that our mechanism can work also in models with smaller values of M_1, since the baryon asymmetry could be generated by the decays of the heavier RH neutrinos. Indeed, the up-Higgs may decay into the RH neutrinos N_2 (or N_3) instead into the lightest RH neutrino N_1 if the condensate reaches the value $\phi = \phi_{N_2} \equiv M_2/h_t$ before the up-Higgs decays into N_1's plus leptons. The time needed for the condensate to reach the value M_2/h_t is

$$\Delta t_{N_2} \sim \phi_{N_2}/\dot{\phi} \sim (M_2/h_t \widetilde{m} \phi_0) \tag{2.22}$$

and is smaller than the decay time of the up-Higgs into N_1's plus leptons if

$$\phi \lesssim \left(\frac{8\pi \widetilde{m} \phi_0}{\sum_j |h_{1j}|^2 M_2} \right). \tag{2.23}$$

Imposing that this critical value is larger than M_2/h_t, we find that the up-Higgs will promptly decay into N_2's rather than N_1's if

$$M_2 \lesssim \left(\frac{8\pi h_t \widetilde{m} \phi_0}{\sum_j |h_{1j}|^2} \right)^{1/2}. \tag{2.24}$$

This condition can be satisfied if the Yukawas h_{ij} are hierarchical and $|h_{1j}| \ll 1$. If this is the case, one should replace M_1 with M_2 (or M_3) in eqs. (2.17) and (2.19).

Thirdly, in models of hybrid inflation, the flat direction might have different couplings to the inflaton and to the field that comes to dominate the energy density of the universe after slow-roll inflation. In this case, the curvature of the flat direction potential might change its sign promptly at the end of inflation, when $H \sim H_I \lesssim 10^{14}\,\mathrm{GeV}$ rather than $H \sim \widetilde{m}$. The flat direction will start oscillating around the origin with frequency of order the Hubble parameter during inflation, H_I and a much larger number of quanta of H_U can be produced when the flat direction VEV goes through the origin. This would relax the bound on ϕ_0.

In conclusion, the observed baryon asymmetry can be explained within the supersymmetric leptogenesis scenarios for low reheating temperatures and a RH hierarchical mass spectrum, thus avoiding the gravitino bound, if two conditions are met: the initial value of the flat direction is close to Planckian values, and the phase-dependent terms in the flat direction potential are either vanishing or sufficiently small for the particle production to happen efficiently.

Chapter 3

Cosmological Perturbations

3.1 Microcosmos to Macrocosmos

In our current understanding of cosmology, the universe was made *nearly* uniform (and flat) by a primordial stage of rapid expansion, inflation [9], driven by the energy density of a scalar field, the inflaton. During this period of time the physical universe expanded superluminally, although the visible universe always expanded at the speed of light. This explains why the universe looks so uniform, regions which were in causal contact before the expansion, went out of causal contact when the universe was stretched by inflation. The fast expansion is also responsible for the flatness of the visible universe.

The universe was not, however, perfectly uniform after inflation. Tiny quantum fluctuations in the inflating patch of the universe were stretched to classical sizes by the expansion[1], a link between the microcosmos of particle physics and the macrocosmos of galaxies. These *seed* perturbations excited metric (gravity) perturbations which, in turn, caused inhomogeneities in the baryon and photon (and dark matter) fluids.

[1]This process can be studied using the formalism introduced in Section 1.5: the time-dependent metric describing the expanding universe feeds into the equations of motion for the quantum modes populating the vacuum of the universe and (similarly to the theory of preheating) classical fluctuations (particles) are produced from this vacuum.

These inhomogeneities were then kept from collapsing by the expansion of the universe and by the pressure of the photon bath while the universe was still dominated by radiation. Eventually, when matter domination began, these primordial density inhomogeneities (of order $\delta\rho/\rho \sim 10^{-5}$) were amplified and then collapsed under the gravitational attraction, forming the structure we observe today and leaving an imprint in the Cosmic Microwave Background (CMB) radiation anisotropy. The latest confirmation of the inflationary paradigm has been recently provided by the five-year data from the Wilkinson Microwave Anisotropy Probe (WMAP) satellite [4].

Consider the fluctuations $\delta\chi(\mathbf{x}, \mathbf{t})$ of a massless scalar field χ during the early universe. The equation of motion for the Fourier transforms $\delta\chi_{\mathbf{k}}$ is

$$\delta\ddot{\chi}_{\mathbf{k}} + 3H\delta\dot{\chi}_{\mathbf{k}} + \frac{k^2}{R^2}\delta\chi_{\mathbf{k}} = 0, \tag{3.1}$$

where $k = |\mathbf{k}| = 2\pi R/\lambda$ and $R = e^{Ht}$ is the scale factor of the inflating universe (see eq. (1.10)). Consider a mode with physical wavelength much smaller than the visible universe (the horizon at time t has size H^{-1}): $\lambda \ll H^{-1}$. In this limit we can neglect the friction term in eq. (3.1) and write

$$\delta\ddot{\chi}_{\mathbf{k}} + \frac{k^2}{R^2}\delta\chi_{\mathbf{k}} = 0. \tag{3.2}$$

This is the equation of the harmonic oscillator (with time-dependent frequency): during inflation perturbations of scalar fields oscillate. When the universe expands, physical wavelengths are stretched proportionally to the scale factor and they soon become larger than the horizon, $\lambda \gg H^{-1}$. In this limit eq. (3.1) becomes

$$\delta\ddot{\chi}_{\mathbf{k}} + 3H\delta\dot{\chi}_{\mathbf{k}} = 0. \tag{3.3}$$

Only friction: on scales bigger than the horizon the perturbations $\delta\chi_{\mathbf{k}}$ are constant (frozen). An exact solution of eq. (3.1) interpolating between these limits shows that at the time perturbations freeze, when $k = RH$, their size is[2]

$$|\delta\chi_{\mathbf{k}}| \simeq \frac{H}{\sqrt{2k^3}}, \tag{3.4}$$

and it remains constant, while its wavelength grows exponentially. The perturbations of eq. (3.4) will, of course, produce inhomogeneities in the energy density of the field χ which, in turn, will be transmitted to the gravitational field, causing perturbations in the metric. To study perturbations to the metric it is convenient to introduce a gauge-invariant quantity which depends both on the density perturbation $\delta\rho$ and on the curvature perturbation ψ appearing in the perturbed metric:

$$\zeta_i \equiv \psi + H\frac{\delta\rho_i}{\dot{\rho}}, \tag{3.5}$$

where the index i reminds us that any field contributing to the total energy density ρ can induce perturbations. This quantity ζ is called 'curvature perturbation on slices of uniform energy density', but we shall refer to it simply as the *curvature perturbation*. The inhomogeneities of eq. (3.4) produces initially a non-vanishing contribution to the second term of eq. (3.5). When eventually, the field χ decays, the second term disappears, but the perturbations remain imprinted in the metric due to the first term in eq. (3.5). Poisson's equation (G_N is Newton's constant)

$$\nabla^2\psi = 4\pi G_N \delta\rho \tag{3.6}$$

will then communicate the metric perturbations to the energy density ρ of baryons, photons and dark matter, which populate the universe after inflation decay. Beside providing the seeds for structure formation, this mechanism also explains the tiny temperature anisotropies $\delta T/T \sim 10^{-5}$ observed in the CMB.

Despite the simplicity of the inflationary concept, the mechanism by which cosmological curvature (adiabatic) perturbations are generated is not yet fully established.

[2]These calculation can be extended the the case where the scalar field χ has a non-vanishing mass m_χ. In this case, a factor of $(k/R_k H)^{m_\chi^2/3H^2}$ needs to be included in eq. (3.4) (the subscript k is to remind us that the scale factor is evaluated when the scale k crosses the horizon).

In the standard slow-roll inflationary scenario associated with a single inflaton field, density perturbations are due to quantum fluctuations of the inflaton itself. While this possibility is in agreement with present CMB data [4, 63], it is not the only one. In the curvaton mechanism [64, 65], the final curvature perturbation ζ is produced from an initial isocurvature mode associated with quantum fluctuations of a light scalar (other than the inflaton), the curvaton, whose energy density is negligible during inflation and which decays much after the end of inflation. Further proposals invoke the inhomogeneity of the inflaton decay rate [66], inhomogeneous preheating [67] and the generation of the curvature perturbation at the end of inflation [68].

In this chapter we point out that the generation of a flat spectrum for the curvature perturbation may be the general consequence of the presence of flat directions in supersymmetric theories. Let us briefly sketch how this can happen. Similarly to the leptogenesis scenario described in the previous chapter, the large VEVs developed along flat directions will start oscillating around the origin sometime after the end of inflation. If the condensate passes close enough to the origin, it will quickly decay non-perturbatively into the particles coupled to it [56]. Subsequently, when the VEV continues to increase during its oscillations, the particles created acquire large masses (proportional to the VEV - note that this damps the oscillation of the VEV, see eq. (2.13)) and might decay into light degrees of freedom. Now, the key point is that the initial conditions for the flat direction when the oscillation starts may not be the same in separate horizon volumes. This happens if the degree of freedom associated to the phase of the flat direction is sufficiently light during inflation to be quantum mechanically excited. As a consequence, the condensate oscillates around the origin of its potential starting from slightly different values in different patches of the Universe. These different initial conditions give rise to fluctuations in the comoving number densities of the light relativistic states produced during the decay process after instant preheating and, ultimately, to CMB anisotropies. In this sense, supersymmetric flat direction provide a concrete and natural realization of the idea that the observed perturbations are associated to some underlying global symmetry which is slightly broken during inflation [67].

3.2 Initial Conditions

The generic potential for a supersymmetric flat direction ϕ during inflation is given, as usual, by eq. (1.61). For $\widetilde{m} \ll H_I$ we write it as

$$V = -c\frac{1}{2}H_I^2|\phi|^2 + \left(\lambda\frac{a_I H_I}{nM^{n-3}}\phi^n + h.c\right) + |\lambda|^2\frac{|\phi|^{2n-2}}{M^{2n-6}}. \tag{3.7}$$

As discussed above eq. (1.62), for $c > 0$ the flat direction condensate acquires a VEV given by

$$\phi_I = |\phi_I|e^{i\theta_I}, \ |\phi_I| = \left(\frac{\beta H_I M^{n-3}}{\lambda}\right)^{1/(n-2)}. \tag{3.8}$$

The phase $\theta_I(\mathbf{x}, t)$ is very likely to undergo quantum fluctuations during inflation. Indeed, its mass is given by

$$m_\theta^2 = n|a_I|\beta\cos(n\frac{\theta_I}{|\phi_I|} + \theta_{a_I} + \theta_\lambda)H_I^2, \tag{3.9}$$

where the factor of $1/|\phi_I|$ has been introduced to canonically normalize the field θ_I. Here θ_λ and θ_{a_I} are the phases of the coefficients λ and a_I; for the sake of simplicity we absorb these phases into θ_I in what follows. For small enough values of the a_I-parameter, m_θ is smaller than the Hubble rate H_I during inflation. In fact, as noted in Chapter 2, without any fine-tuning, the parameter a_I may be extremely small or even identically vanishing, showing that during inflation the phase $\theta(\mathbf{x}, t)$ may be an effectively massless degree of freedom.

In the post-inflationary era, the flat direction starts oscillating around $\phi = 0$. At which frequency these oscillations take place depends crucially on the post-inflationary inflaton dynamics. If the inflaton is very weakly coupled, it will undergo a long period of oscillations around the minimum of its potential and eventually decay into radiation. If this happens when the Hubble rate is smaller than $\widetilde{m}$, the flat direction will be anchored to the minimum of its potential till $H \sim \widetilde{m}/3$ when it will start oscillating with a frequency of order of $\widetilde{m}$.

On the other hand, the flat direction may oscillate around $\phi = 0$ with a much larger frequency. Indeed, the inflaton may release the energy stored in its potential very rapidly within a Hubble time. This is expected, for instance, if inflation ends through a rapid waterfall transition induced by a second field whose energy density dominates the energy density of the Universe in this phase, as in scenarios of hybrid inflation [9]. The potential of the flat direction will receive H-dependent corrections through the non-renormalizable couplings of the flat direction in the Kähler potential to the second field. In particular a mass squared term $c_{\rm af} H^2|\phi|^2$ may be induced. If $c_{\rm af}$ is positive, the flat direction starts oscillating around the minimum of its potential with a frequency of the order of H_I if the waterfall transition is fast[3]. Similarly, a phase-dependent term $(a_{\rm af} H_I \phi^n/M^{n-3})$ may be generated through the coupling of the flat direction to the second field driving the end of inflation.

In fact, the flat direction may oscillate around $\phi = 0$ with a frequency much larger than H_I. This happens if the inflaton energy is released through a preheating stage [70]. Fluctuations of the scalar fields produced at the stage of preheating after inflation are so large that they can break supersymmetry much strongly than the inflation itself. These fluctuations may lead to symmetry restoration along flat directions of the effective potential so that the ϕ condensate moves at the very early stage of the evolution of the Universe, during the preheating era, with a frequency

[3]Notice that a large positive mass squared of the order of H_I^2 may be induced even during the first stages of the radiation phase again by the non-renormalizable couplings of the flat direction in the Kähler potential to the light relativistic fields $\phi_{\rm light}$ [69]. Indeed, if there is a non-renormalizable coupling of the form $|\phi|^2|\phi_{\rm light}|^4/M^2$ and thermal effects generate the variance $\langle\phi^2_{\rm light}\rangle \sim T^2$, then the flat direction will acquire a mass squared $\sim T^4/M^2 \sim H_I^2$, where we have assumed that radiation is produced promptly after inflation and M is of the order of M_p.

which can be as high as $(m_\Phi M_p)^{1/2}$, where m_Φ is the inflaton mass. Furthermore, a phase-dependent term can be induced by the large fluctuations of the scalar degrees of freedom generated at preheating, $(m_\Phi \phi^n / M_p^{n-3})$ [70].

From now on, we will assume that the flat direction starts oscillating around the minimum of its potential right after the end of inflation with a frequency of the order of H_I (even though the reader should keep in mind that the frequency may be in fact larger), an initial amplitude ϕ_I and a phase dependent term of order of $(a_{\rm osc} H_I \phi^n / M^{n-3})$:

$$V = \frac{1}{2} H_I^2 |\phi|^2 + \left(\frac{\lambda}{\beta} \frac{a_{\rm osc} H_I}{M^{n-3}} \phi^n + h.c \right) + \lambda^2 \frac{|\phi|^{2n-2}}{M^{2n-6}}, \tag{3.10}$$

(where the factor of λ/β is introduced for later simplicity). All the considerations made so far lead us to treat ϕ_I and $a_{\rm osc}$ as basically free parameters. It is also important to point out that the power n of the non-renormalizable terms lifting the flat direction is not necessarily the same during and after inflation.

3.3 Perturbations

If the condensate passes sufficiently close to the origin, it can efficiently produce any state which is coupled to it. We generically call this state χ (it might be Higgses, squarks or sleptons) and suppose that it is coupled to the flat direction through the Lagrangian term $h^2 |\phi|^2 |\chi|^2$. Its effective mass is therefore given by $m_\chi^2 = \widetilde{m}_\chi^2 + h^2 |\phi|^2$, where $\widetilde{m}_\chi^2$ is the corresponding soft-breaking mass parameter. At the first passage through the origin, particle production takes place when adiabaticity is violated [56], $|\dot{m}_\chi| / m_\chi^2 \gtrsim 1$. The flat direction continues its classical oscillation, now giving a large mass to the produced quanta of order $m_\chi^2 \sim h^2 |\phi|^2$. If they couple to some other light fermionic degrees of freedom, they can efficiently decay into these light states.

The comoving number density of χ particles produced during the first flat direction oscillation is given by eq. (1.87),

$$n_\chi = \frac{\left(h|\dot{\phi}_*|\right)^{3/2}}{8\pi^3} \exp\left[-\frac{\pi h|\phi_*|^2}{|\dot{\phi}_*|}\right], \tag{3.11}$$

where $|\phi_*|$ and $|\dot{\phi}_*|$ are the minimum distance and maximum speed of the trajectory with respect to the origin

$$|\phi_*| \approx \pi a_{\rm osc}|\phi_I|\frac{\Gamma\left(\frac{1+n}{2}\right)}{\Gamma\left(1+\frac{n}{2}\right)}\sin(n\theta_I) \tag{3.12}$$

and

$$|\dot{\phi}_*| \approx H_I\,|\phi_I|(1+\beta^2)\,\sqrt{1+4(a_{\rm osc}/n)\cos(n\theta_I)}. \tag{3.13}$$

Eqs. (3.12) and (3.13) are obtained in Appendix A and comparison of these approximations with the numerical evaluations are shown in Fig. 3.1 and Fig. 3.2. The non-adiabaticity condition $|\dot{m}_\chi|/m_\chi^2 \gtrsim 1$ implies that $|\dot{\phi}|/(h|\phi|^2) \gtrsim 1$ or

$$a_{\rm osc} \lesssim a_{\rm osc}^{\rm max} \simeq \frac{1}{\sqrt{h}}\sqrt{\frac{H_I}{|\phi_I|}\frac{\Gamma\left(1+\frac{n}{2}\right)}{\pi\Gamma\left(\frac{1+n}{2}\right)\sin(n\theta_I)}} \approx \frac{1}{\sqrt{\pi h}}\sqrt{\frac{H_I}{|\phi_I|}\frac{1}{\sin(n\theta_I)}}. \tag{3.14}$$

Let us now compute the curvature perturbation associated to the light particles generated through the instant preheating stage. The presence of the phase-dependent term in the potential of the flat direction eq. (1.61) during inflation violates the $U(1)$ carried by the complex ϕ and gives n discrete minima for the phase of ϕ. The potential in the angular direction goes like $\cos(n\theta_I)$ during inflation (the reader should remember that for simplicity we have set to zero the phases of the parameters a_I and λ). If the phase is a light degree of freedom during inflation, the field θ does not sit at the minimum of its potential as it is quantum mechanically excited. It may acquire a random value, but constant over scales larger than the present horizon. Therefore, when the flat direction starts oscillating, its initial condition varies from patch to patch. When light particles are generated through the instant preheating phenomenon, their abundance will not be uniformly distributed. On the

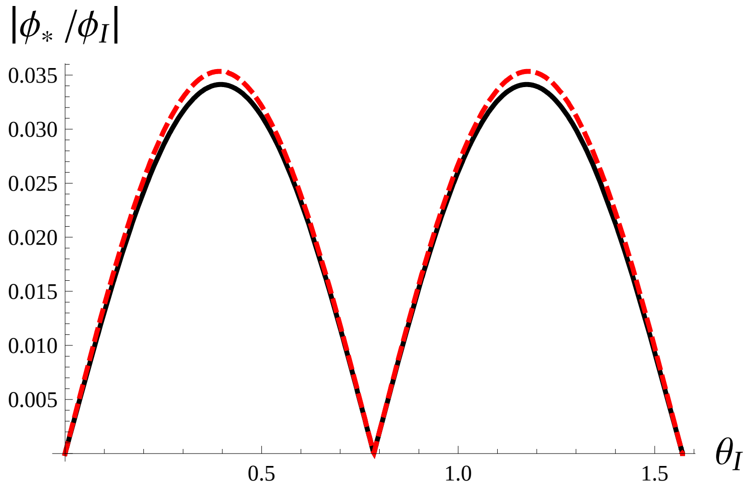

Figure 3.1: The angular dependence of $|\phi_*|$, the minimum distance from the origin plotted in units of its initial value $|\phi_I|$ for $a_{\rm osc} = 0.003$ and $n = 4$ (see Appendix A). The numerical evaluation (with $\beta = 0.1$ and $\lambda = 0.01$) is represented by the continuous line, while the analytical estimate of Eq. (3.12) is dashed.

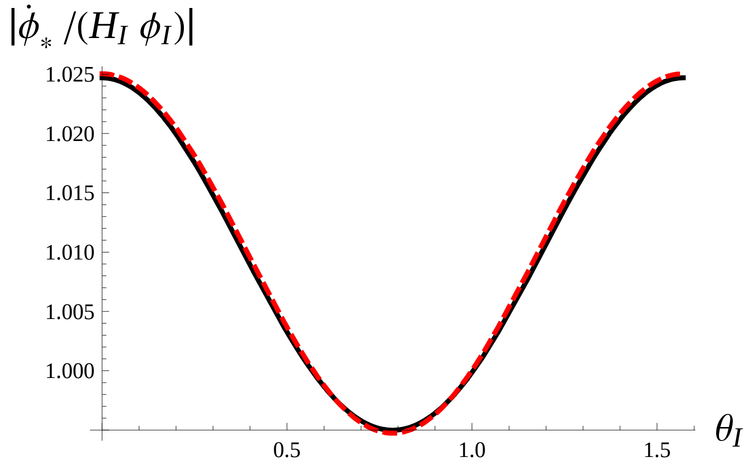

Figure 3.2: The angular dependence of the flat direction velocity $|\dot{\phi}_*|$ at ϕ_* (the minimum distance from the origin) plotted for $a_{\rm osc} = 0.003$ and $n = 4$ in units of $H_I|\phi_I|$ (see Appendix A) . The numerical evaluation (with $\beta = 0.1$ and $\lambda = 0.01$) is represented by the continuous line, while the analytical estimate of Eq. (3.13) is dashed.

contrary, isocurvature fluctuations of their number density are expected on super-horizon scales. These isocurvature fluctuations in the light fields are derived from eq. (3.11)

$$\frac{\delta n_{\rm light}}{n_{\rm light}} \simeq \frac{\delta n_\chi}{n_\chi} = \left(\frac{3}{2} + \frac{\pi h|\phi_*|^2}{|\dot{\phi}_*|}\right)\frac{\delta|\dot{\phi}_*|}{|\dot{\phi}_*|} - \frac{2\pi h|\phi_*|^2}{|\dot{\phi}_*|}\frac{\delta|\phi_*|}{|\phi_*|}, \tag{3.15}$$

where, from eqs. (3.12) and (3.13),

$$\frac{\delta|\phi_*|}{|\phi_*|} \simeq n\frac{\cos(n\theta_I)}{\sin(n\theta_I)}\delta\theta, \tag{3.16}$$

$$\frac{\delta|\dot{\phi}_*|}{|\dot{\phi}_*|} \simeq -2a_{\rm osc}\frac{\sin(n\theta_I)}{1+(4/n)a_{\rm osc}\cos(n\theta_I)}\delta\theta. \tag{3.17}$$

These expressions lead to

$$\begin{aligned}
\frac{\delta n_{\rm light}}{n_{\rm light}} &\simeq f\left(\theta_I\right)\delta\theta, \\
f\left(\theta_I\right) &\simeq -a_{\rm osc}\frac{\sin(n\theta_I)}{\sqrt{1+(4/n)a_{\rm osc}\cos(n\theta_I)}}\left\{\frac{3}{4\sqrt{1+(4/n)a_{\rm osc}\cos(n\theta_I)}}\right. \\
&+ \left. a_{\rm osc}\frac{\pi^3 h|\phi_I|}{2H_I(1+\beta^2)}\frac{\Gamma\left(\frac{1+n}{2}\right)^2}{\Gamma\left(1+\frac{n}{2}\right)^2}\left[\frac{a_{\rm osc}\sin^2(n\theta_I)}{\sqrt{1+(4/n)a_{\rm osc}\cos(n\theta_I)}} + n\cos(n\theta_I)\right]\right\}.
\end{aligned}$$

The fluctuation vanishes in the appropriate limit, $a_{\rm osc} = 0$: in this case there is no dependence on the phase in the potential of the flat direction and therefore the condensate starts oscillating around the origin from the same initial condition throughout the Universe. Supposing now that the inflaton field during inflation has generated a negligible amount of curvature perturbation during inflation, we can finally estimate the total curvature perturbation ζ generated by the supersymmetric flat direction

$$\zeta = H\frac{\delta\rho_{\rm light}}{\dot{\rho}_{\rm tot}} \simeq -\frac{1}{3}\frac{\delta\rho_{\rm light}}{\rho_{\rm tot}} = -\frac{1}{3}\frac{\rho_{\rm light}}{\rho_{\rm tot}}\frac{\delta n_{\rm light}}{n_{\rm light}}, \tag{3.18}$$

where

$$\rho_{\rm light}/\rho_{\rm tot} \approx \rho_\chi/\rho_{\rm tot} \approx m_\chi n_\chi/\rho_{\rm tot}, \tag{3.19}$$

and where we have used eq. (1.9). Notice that ρ_χ may not exceed the energy stored in the flat direction, $\rho_\chi \lesssim \rho_\phi$. The fluctuations in θ take the form

$$|\delta\theta(k)|^2 \approx \frac{H_I^2}{2k^3|\phi_I|^2}\left(\frac{k}{R_k H_I}\right)^{\frac{2}{3}na_I\beta\cos n\theta_I - 2\epsilon}, \tag{3.20}$$

where H_I is the Hubble parameter at the time when the scale k exits the horizon at the value of the scale factor R_k. The slow-roll parameter $\epsilon = -\dot{H}/H^2$ accounts for the fact that during inflation the Hubble rate is slowly decreasing with time [9]. The power spectrum of the phase therefore reads

$$\mathcal{P}_{\delta\theta}(k) \equiv \left(\frac{k^3}{2\pi^2}\right)|\delta\theta(k)|^2 = \left(\frac{H_I}{2\pi|\phi_I|}\right)^2 \left(\frac{k}{R_k H_I}\right)^{\frac{2}{3}na_I\beta\cos n\theta_I - 2\epsilon}. \tag{3.21}$$

We can therefore estimate the maximum value of the curvature perturbation by using the maximum allowed value eq. (3.14) of the $a_{\rm osc}$-term and assuming $\rho_\chi \simeq \rho_\phi$

$$\zeta_{\rm max} \simeq \left(\frac{|\phi_I|}{M_p}\right)^2 \cot(n\theta_I)\,\delta\theta, \tag{3.22}$$

corresponding to a maximum power spectrum of the curvature perturbation of the order of

$$\mathcal{P}_\zeta^{1/2}(k) \simeq \frac{\cot(n\theta_I)}{2\pi}\left(\frac{H_I^{n-1}}{\lambda M_p^{(n-1)}}\right)^{\frac{1}{n-2}}, \tag{3.23}$$

where we have chosen $M = M_p$. Barring possible fine-tuning over the angle θ_I, we find for instance that for $n = 4$ and $H_I \simeq 10^{-4}M_p$, the right amount of curvature perturbation $\mathcal{P}_\zeta^{1/2} \simeq 2.5 \times 10^{-5}$ is obtained for $\lambda \sim 10^{-2}$.

Our findings indicate that a large (and possibly dominating) curvature perturbation may be obtained through the interplay of the inflationary and post-inflationary dynamics. If during inflation the field parametrising the phase of the flat direction has a mass smaller than the Hubble rate, then the particles generated through the phenomenon of instant preheating, when the flat direction condensate oscillates around the minimum of its potential starting from slightly different initial conditions, will not be uniformly distributed on super-Hubble distances, thus leading to a nonvanishing curvature perturbation. This happens if particle production occurs. This requires that the flat direction passes sufficiently close to the origin, which imposes the bound eq. (3.14) of the $a_{\rm osc}$-term and, as a consequence, the upper bound eq. (3.22) on ζ.

Let us close with some comments. The fact that we can choose non-renormalizable operators lifting the flat direction at the $n = 4$ level is relevant because there are, among all the flat directions lifted by $n = 4$ non-renormalizable operators, two of them, $uude$ and $QQQL$, which carry no $(B - L)$ number [35] (the ones which are lifted by $n > 4$ non-renormalizable operators all carry a non-vanishing $(B - L)$-number). This implies that the oscillations of the ude and QL directions may be associated to the right amount of curvature perturbation without giving rise to any (eventually large) baryon isocurvature perturbation associated to the baryon number generated by the flat directions [28], which are bound by experiment to be subdominant. Indeed, being $uude$ and $QQQL$ neutral under $(B - L)$ any baryon asymmetry generated by their oscillations is promptly erased by the $(B+L)$-violating processes induced by sphalerons in the standard model.

3.4 Non-Gaussianities

We conclude this chapter with a brief discussion on non-gaussianities (NG). The fluctuations of a scalar field χ are Gaussian when their Fourier components (the $\delta\chi_{\mathbf{k}}$ introduced at the beginning of this chapter) have independent probability distributions and are, therefore, uncorrelated. When this is the case, all the statistical[4] properties of the system can be expressed via the 2-point correlation function, which is related to the spectrum of, for example, eq. (3.21),

$$\langle \delta\chi^*_{\mathbf{k_1}} \delta\chi_{\mathbf{k_2}} \rangle = \delta^{(3)}(\mathbf{k_1} - \mathbf{k_2}) \frac{2\pi^2}{k^3} \mathcal{P}_{\delta\chi}(k). \tag{3.24}$$

In particular, the 3-point correlation function, and all odd-point correlation functions, vanish in this case. For example, in single-field models of inflation (where both the density perturbations and inflation are caused by the same field) the slow roll

[4]One assumes that our universe belongs to an ensemble of typical universes over which statistical arguments make sense.

conditions constrain the self-interactions of the inflaton field to be very small. Then the equations for the Fourier components will be uncorrelated and they will behave like independent harmonic oscillators: the spectrum will be Gaussian to a very good approximation [71]. In models where the density perturbations are caused by a field different than the inflaton, self correlations can become important and give rise to NG. The subsequent evolution of these fluctuations and their collapse into structures will leave an imprint in the CMB anisotropies which can be observed today and offers an important discriminator between different theories of inflation (single-field versus multy-field). Phenomenologically, NG in the cosmological perturbations are parametrised by the non-linear parameter f_{NL} through Bardeen's gravitational potential [72]

$$\Phi = \Phi_G + f_{NL}\Phi_G^2, \tag{3.25}$$

where Φ_G represents the Gaussian part of the gravitational potential and, in our case we can express Φ in terms of the curvature perturbation [67]

$$\Phi = -\frac{3}{5}\zeta. \tag{3.26}$$

WMAP 5-year data [4] puts the bound

$$-9 < f_{NL} < 111, \tag{3.27}$$

which is consistent with vanishing NG.

If eq. (3.22) is the dominant component of the total curvature perturbation, the latter may have a sizeable non-Gaussian component which is easily found by expanding all the quantities obtained so far up to second-order in $\delta\theta$,

$$\Phi = f_1(\theta)\delta\theta + f_2(\theta)(\delta\theta)^2, \tag{3.28}$$

where f_1 and f_2 are the coefficients of the expansion and depend on θ. The non-linear parameter $f_{\rm NL}$ characterizing the level of NG then becomes

$$(3f_{\rm NL}/5) = \frac{3}{5}\frac{f_2}{f_1^2} \simeq -(M_p/\phi_I)^2(1/\cos^2\theta_I). \tag{3.29}$$

Thus, large (negative) values of NG are generically obtained unless the initial amplitude of the flat direction is close to Planckian values. Therefore, we conclude that, if the supersymmetric flat direction's dynamics produce a sizeable contribution to the total curvature perturbation, a large NG component in the CMB anisotropy is expected, as will be tested by the upcoming Planck satellite.

Chapter 4

Non-Perturbative Flat Direction Decay

4.1 Rotating Flat Directions and Delayed Thermalization

So far in this thesis, we have concentrated our attention on supersymmetric flat directions for which the phase dependent terms in the potential are relatively small, compared with the other terms in the Lagrangian. We have argued that this is naturally the case if the superpotential vanishes along the flat direction, or if inflation is driven by D-terms. Then, after inflation the flat direction VEV starts oscillating through the origin and decays non-perturbatively, providing, for example, a mechanism to describe the baryon asymmetry via leptogenesis (Chapter 2) or generating the density perturbations responsible for structure formation (Chapter 3).

In this chapter we concentrate on the opposite limit, in which the phase dependent terms in the potential eq.(1.61) are of the same order as the other terms in the scalar potential. During inflation, the flat direction develops a VEV given by eq.(1.62) as before (the parameter β will now be determined both by the quadratic and the phase-dependent terms in the potential). The phase dependent part of the potential, for

$H \gg \widetilde{m}$, will be proportional to $\cos(n\theta + \theta_a + \theta_\lambda)$; this determines the value of the phase during inflation (from now one we neglect fluctuations of the phase). After inflation, the Hubble parameter decreases and the flat direction VEV tracks the minimum of the potential, decreasing with H, with constant phase, until $H \sim \widetilde{m}$. At this moment its VEV is given by

$$|\phi_0| = \left(\frac{\beta \widetilde{m} M^{n-3}}{\lambda}\right)^{1/(n-2)} . \tag{4.1}$$

For $H \lesssim \widetilde{m}$, the SUSY breaking soft phase-dependent term in the potential becomes bigger than the term proportional to H: the potential for the flat direction phase changes and the condensate starts moving along the angular direction in its complex plane. For $H \ll \widetilde{m}$ the potential is well approximated by $V \approx (1/2)\widetilde{m}^2\phi^2$ and the flat direction VEV simply spirals slowly towards the origin red-shifted by Hubble expansion. The motion of the flat direction VEV described here is showed schematically in Fig.4.1. Affleck-Dine baryogenesis [19] is based on this scenario to produce the observed baryon asymmetry via the out of equilibrium decay of the rotating flat direction.

Recently, much interest has focused on the cosmological fate of these flat direction VEVs. In particular, it has been argued [75, 76] that in realistic supersymmetric models, large flat direction VEVs can persist long enough to delay thermalization after inflation, leading to low reheat temperatures, or also preventing preheating after inflation. Indeed, due to the presence of these large VEV amplitudes, all the fields coupled to the flat direction (including gauge bosons) become very massive, thus kinematically forbidding the decay of the inflaton into these heavy modes. These arguments hold so long as the flat direction VEVs do not rapidly decay – they must persist long enough so that they can delay thermalization and block inflaton preheating. Since now the flat direction undergoes a circular motion in the complex plane, we would expect that preheating of the flat direction is very

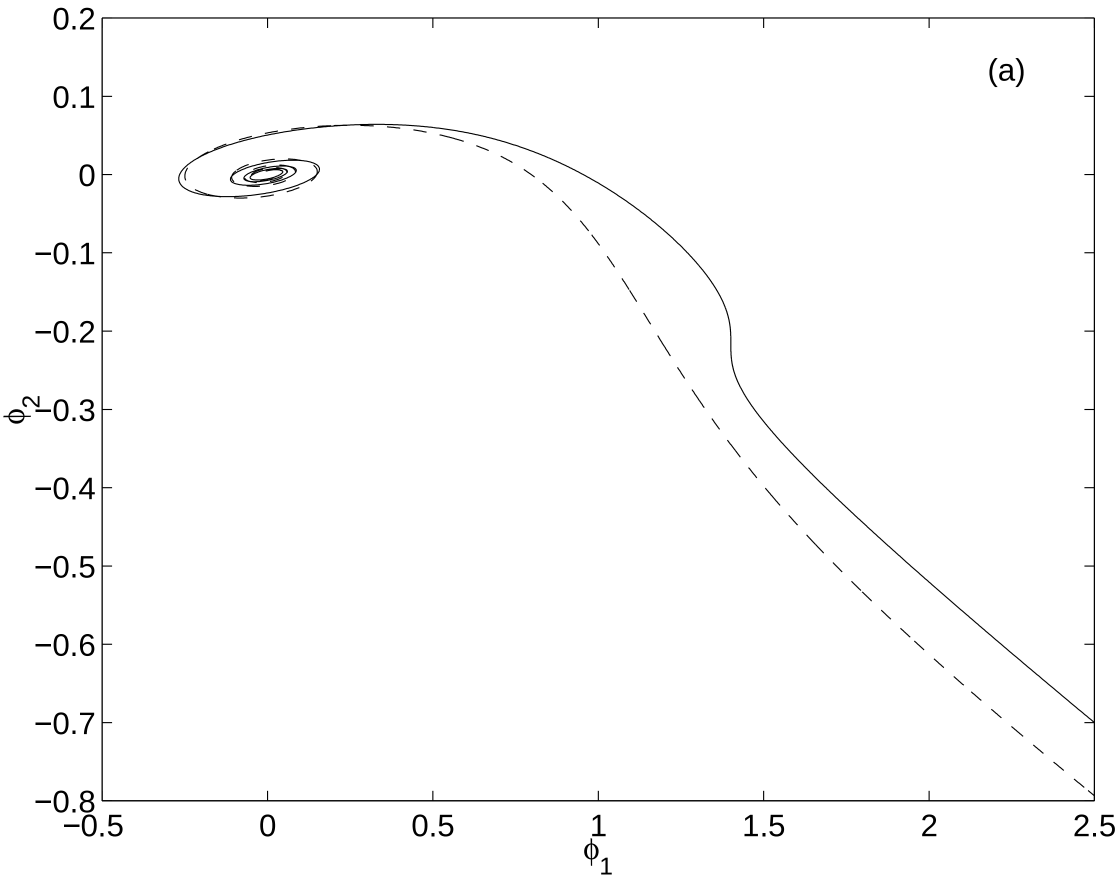

Figure 4.1: Motion of the flat direction VEV in the complex plane for $n = 4$ (solid) and $n = 6$ (dashed) with initial condition $\theta = -\pi/10$. Taken from [74].

inefficient. Indeed, for fields obtaining a mass m from couplings of strength h to the flat direction as in eq.(1.68),

$$\frac{\dot{m}}{m^2} \sim \frac{\widetilde{m}|\phi_0|}{h|\phi_0|^2} = \frac{\widetilde{m}}{h|\phi_0|} \ll 1, \tag{4.2}$$

which means that adiabaticity is never violated and particles are not produced. In [58], however, it was claimed that even in this case non-perturbative decay can lead to a rapid depletion of the flat direction condensate and thus precludes the delay of thermalization after inflation. But was also concluded that in order for the flat direction to decay non-perturbatively the system requires more than one flat direction [58, 77]. Finally, in [77] it was pointed out that even in the presence of multiple flat directions, some degree of fine-tuning was necessary to achieve flat direction decay.

An important aspect of this discussion centers on the issue of Nambu-Goldstone (NGo) bosons. In general, supersymmetric flat directions are charged under the (local) gauge group of the MSSM, $SU(3) \times SU(2) \times U(1)_Y$. Consequently, the flat direction VEV will break some or all of the gauge symmetries of the theory and thus we expect the presence of the associated NGo bosons which, via the Higgs mechanism, are absorbed as longitudinal degrees of freedom of gauge bosons. In calculating non-perturbative flat direction decays, [58] considers a gauged $U(1)$ model (as a simplified version of the MSSM) and constructs the mixing matrix for the excitations around the flat direction VEV. Similarly to preheating, discussed in Section 1.5, the time-dependence of the mass-matrix can cause the non-perturbative decay of the flat direction. The results in [58] show, however, that in the single flat direction case, non-perturbative decay proceeds solely via a massless NGo mode as only the NGo mode mixes with the Higgs and all other massless moduli remain decoupled. Since the NGo boson represents an unphysical gauge degree of freedom[1], it was concluded [58, 77] that no preheating occurs in the single flat direction case. As the appearance of a massless NGo boson in the spectrum is a gauge dependent artifact, it remains unclear if the conclusions drawn about the system hold in the unitary gauge, where NGo degrees of freedom are explicitly eliminated in favor of massive gauge bosons. In order to determine if flat direction VEVs decay non-perturbatively into scalar degrees of freedom, the effect of the NGo boson must first be removed from the low energy spectrum. The process of removing the NGo modes by switching to the unitary gauge changes the form of the mixing matrix among the left over scalar degrees of freedom.

In this chapter we consider toy models to demonstrate that, in the unitary gauge, the mixing matrix of the excitations around a flat direction VEV permits preheating.

[1]NGo bosons are the longitudinal component of the gauge bosons associated with the broken symmetry. As such, they become very massive and decouple from the low-energy theory.

Moreover, we find that flat direction decay depends on the *number of dynamical, physical phases* appearing in the flat direction VEV. Specifically, a physical phase difference between two of the individual field VEVs making up the flat direction is needed.

The outline of the rest of this chapter proceeds as follows: first we explicitly construct – in the unitary gauge – the mass squared matrix arising from the D-terms of a toy gauged $U(1)$ model with three charged chiral superfields. We then present the formalism of preheating with multi-component fields (following the discussion of Section 1.5) and show that preheating occurs for the light moduli associated with the flat direction. We then analyze the specific dynamics of the background field equations for the toy models examined. Finally we evolve the background field equations for one of the toy models to obtain quantitative results. Note that back-reaction effects are not taken into account in this analysis, which concentrates on a qualitative, rather than quantitative, discussion of the issue of flat direction decay.

4.2 A Toy Model

As an example, we examine a toy model which demonstrates the important features of supersymmetric flat direction VEV decay. We introduce three complex scalar fields (belonging to SUSY chiral multiplets), Φ_1, Φ_2 and Φ_3 charged under a $U(1)$ gauge group with charges $q_1 = +1/2$, $q_2 = -1$ and $q_3 = +1/2$ respectively. The Lagrangian reads

$$\mathcal{L} = \sum_{i=1}^{3} \frac{1}{2}|D_\mu \Phi_i|^2 - V - \frac{1}{4}F_{\mu\nu}^2, \tag{4.3}$$

where $D_\mu = \partial_\mu - iq_i A_\mu$ denotes the covariant derivative. The potential we consider arises from the supersymmetric D-terms and has the form

$$V = \frac{g^2}{2}(q_1\,|\Phi_1|^2 + q_2\,|\Phi_2|^2 + q_3\,|\Phi_3|^2)^2, \tag{4.4}$$

where g is the gauge coupling of the $U(1)$ gauge symmetry. In the above, we have neglected contributions from supersymmetry (SUSY) breaking and from any non-renormalisable terms arising from the superpotential. These contributions play the dominant role in the analysis of the evolution of the flat direction VEV, as discussed in the previous section. However, the effects we investigate here do not depend on their inclusion in the quadratic part of the potential and so for clarity we neglect them.

The potential in eq. (4.4) admits several flat directions. Choosing one particular direction and including excitations around the VEV we can write

$$\begin{aligned} \Phi_1 &= (\varphi + \xi_1) e^{i(\sigma_1 + \frac{\theta_1}{\varphi})}, \\ \Phi_2 &= (\varphi + \xi_2) e^{i(\sigma_2 + \frac{\theta_2}{\varphi})}, \\ \Phi_3 &= (\varphi + \xi_3) e^{i(\sigma_3 + \frac{\theta_3}{\varphi})}, \end{aligned} \tag{4.5}$$

where $\sigma_{1,2,3}$ represent time dependent (classical) phases of the VEVs, φ denotes the VEV's time dependent amplitude[2] while $\xi_{1,2,3}$ and $\theta_{1,2,3}$ parameterize the six real scalar (quantum) degrees of freedom corresponding to the excitations around the VEVs. Note that the flat direction VEV breaks the $U(1)$ gauge symmetry. Thus, out of the six real scalar degrees of freedom we expect one massive Higgs field and one massless NGo boson, leaving four massless scalar degrees of freedom.

The kinetic terms for the scalar fields play an important role in this analysis. Their expansion in eq. (4.3) includes the term

$$\mathcal{L} \supset -\varphi^2 A_0 (\dot{\sigma_1} - 2\dot{\sigma_2} + \dot{\sigma_3}) \tag{4.6}$$

which has the form of a coupling between the gauge field and the background condensate. Terms of this type will feed into the equations of motion for the gauge

[2]Throughout this analysis we limit ourselves to the conservative case $\dot{\varphi} \ll \varphi\dot{\sigma}$. In the opposite case the effect is enhanced.

field which, in turn, will have an effect on the equations of motion for the scalar excitations. By making a $U(1)$ gauge transformation on the VEV of the form

$$\langle \Phi_i \rangle \rightarrow \langle \Phi_i' \rangle = e^{iq_i\lambda} \langle \Phi_i \rangle \tag{4.7}$$

with

$$\lambda = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{3}, \tag{4.8}$$

we can gauge this term away and avoid a complicated analysis of the kinetic terms. The resulting form of the VEV reads

$$\begin{aligned} \langle \Phi_1 \rangle &= \varphi e^{i(\sigma+\gamma)}, \\ \langle \Phi_2 \rangle &= \varphi e^{i\sigma}, \\ \langle \Phi_3 \rangle &= \varphi e^{i(\sigma-\gamma)}, \end{aligned} \tag{4.9}$$

where $\gamma = (\sigma_1 - \sigma_3)/2$ and $\sigma = (\sigma_1 + \sigma_2 + \sigma_3)/3$ represent the two remaining independent physical phases. Following Kibble [78], we can write the fields in the unitary gauge as

$$\begin{aligned} \Phi_1 &= (\varphi + \xi_1) e^{i(\sigma+\gamma+\frac{\theta}{\varphi\sqrt{2}}+\frac{\theta'}{\varphi\sqrt{3}})}, \\ \Phi_2 &= (\varphi + \xi_2) e^{i(\sigma+\frac{\theta'}{\varphi\sqrt{3}})}, \\ \Phi_3 &= (\varphi + \xi_3) e^{i(\sigma-\gamma-\frac{\theta}{\varphi\sqrt{2}}+\frac{\theta'}{\varphi\sqrt{3}})}, \end{aligned} \tag{4.10}$$

where $\xi_{1,2,3}$, θ and θ' denote the physical excitations – the NGo boson has been removed[3]. We choose the particular combination of field excitations appearing in eq. (4.10) (the exponent in particular) in order to retain canonically normalised kinetic terms.

On substituting the fields of eq. (4.10) into the Lagrangian given in eq. (4.3) and defining the vector $\Xi \equiv (\xi_1, \xi_2, \xi_3, \theta, \theta')^T$, we find the quadratic terms

$$\mathcal{L} \supset \frac{1}{2}|\partial_\mu \Xi|^2 - \frac{1}{2}\Xi^T \mathcal{M}^2 \Xi + \frac{1}{2}\dot{\Xi}^T U \Xi + ... \tag{4.11}$$

[3]This can be verified by expanding out the scalar kinetic terms which reveals the absence of terms of the form $A_\mu \partial^\mu(...)$.

where the ellipses denote higher order terms and interactions. The matrix U given in the last term in eq. (4.11) reads

$$U = \begin{pmatrix} 0 & 0 & 0 & \frac{\dot{\sigma}+\dot{\gamma}}{\sqrt{2}} & \frac{\dot{\sigma}+\dot{\gamma}}{\sqrt{3}} \\ 0 & 0 & 0 & 0 & \frac{\dot{\sigma}}{\sqrt{3}} \\ 0 & 0 & 0 & \frac{-\dot{\sigma}+\dot{\gamma}}{\sqrt{2}} & \frac{\dot{\sigma}-\dot{\gamma}}{\sqrt{3}} \\ -\frac{\dot{\sigma}+\dot{\gamma}}{\sqrt{2}} & 0 & \frac{\dot{\sigma}-\dot{\gamma}}{\sqrt{2}} & 0 & 0 \\ -\frac{\dot{\sigma}+\dot{\gamma}}{\sqrt{3}} & -\frac{\dot{\sigma}}{\sqrt{3}} & \frac{-\dot{\sigma}+\dot{\gamma}}{\sqrt{2}} & 0 & 0 \end{pmatrix} \tag{4.12}$$

while the mass matrix for the physical excitations appears as

$$\mathcal{M}^2 = (g\varphi)^2 \begin{pmatrix} 1 & -2 & 1 & 0 & 0 \\ -2 & 4 & -2 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} = B\mathcal{M}_d^2 B^T \tag{4.13}$$

with eigenvalues $M_1^2 = 6(g\varphi)^2$, $M_2^2 = M_3^2 = M_4^2 = M_5^2 = 0$ (the entries of the diagonal matrix $\mathcal{M}_d$). B is an orthogonal matrix which diagonalizes $\mathcal{M}^2$ and M_1 corresponds to the mass of the physical Higgs field associated with the spontaneous breaking of the $U(1)$ symmetry. The four zero eigenvalues correspond to the massless excitations around the flat direction VEV.

The last term in eq. (4.11) appears as a consequence of the time-dependence of the background – it represents a mixing between the fields $\xi_{1,2,3}$, θ, θ' and their time-derivatives. The effect of these terms on the system becomes clear if we make field redefinitions that remove the mixed derivative terms. The resulting transformation leaves the system in an inertial frame in field space and leads to a time-dependent mass matrix. Defining $\Xi' = A\Xi$ (A is orthogonal), we find the condition that A must satisfy, in order for all the mixed derivative terms to cancel, to be

$$\dot{A}^T A = U. \tag{4.14}$$

The Lagrangian for the Ξ' system now reads

$$\mathcal{L} \supset \frac{1}{2}|\partial_\mu \Xi'|^2 - \frac{1}{2}\Xi'^T \mathcal{M}'^2 \Xi' \tag{4.15}$$

where $\mathcal{M}'^2 = A\mathcal{M}^2A^T = AB\mathcal{M}_d^2B^TA^T = C\mathcal{M}_d^2C^T$ and $C = AB$. The matrix C is an orthogonal time-dependent matrix, with columns corresponding to the eigenvectors of $\mathcal{M}'^2$. We now have a system of scalar fields with canonically normalized kinetic terms and time dependent eigenvectors.

The central point of this discussion centers precisely on the appearance of the *time dependent* eigenvectors for the five scalar fields. This satisfies a necessary but not sufficient condition for preheating. In what follows, we investigate the details of the non-perturbative production of the light scalar fields following the analysis of Section 1.5, extended to the particular case where preheating is shared among multiple degrees of freedom [79].

Including gravity, the dynamics of the re-scaled conformally coupled scalar fields, $\chi_i = R\Xi'_i$ (where R denotes, as usual, the scale factor and Ξ'_i the i-th component of the vector Ξ'), are governed by the following equations of motion (sum over repeated indices is implied),

$$\ddot{\chi}_i + \Omega^2_{ij}(t)\chi_j = 0 \tag{4.16}$$

where dots represent derivatives with respect to conformal time t, and

$$\Omega^2_{ij} = R^2\mathcal{M}'^2_{ij} + k^2\delta_{ij}, \tag{4.17}$$

where k labels the comoving momentum. This is very similar to eq. (1.69), but where the background time-dependance is included not only in the eigenvalues of the matrix Ω, but also possibly in its eigenvectors. Using an orthogonal time-dependent matrix $C(t)$, we can diagonalize Ω_{ij} via

$$C^T(t)\Omega^2(t)C(t) = \omega^2(t) \tag{4.18}$$

giving the diagonal entries $\omega_j^2(t)$. Terms of the form $\sim \varphi\dot{\sigma}\dot{\chi}$ arising from the kinetic terms do not affect the evolution of the non-zero k quantum modes [80].

Once we have identified the basis in which the Hamiltonian

$$H = 1/2 \int d^3x(\Pi_i\Pi_i + \chi_i\Omega^2_{ij}(t)\chi_j) \tag{4.19}$$

appears diagonal, $\tilde{\chi}_i = C^T\chi_i$, the study of particle creation by the time-varying background proceeds as in [81, 20, 79], which extends the results of [37]. Following [79], we assume that Ω_{ij} initially evolves adiabatically by assuming that the initial angular motion of the flat-direction varies slowly. This assumption allows us to define adiabatically evolving mode functions with positive and negative frequency and rewrite the quantum fields as mode expansions in terms of the mode functions and their associated creation/annihilation operators

$$\tilde{\chi}^{(0)}_j = \int \frac{dk^3}{(2\pi)^{3/2}}\left(\tilde{a}^{(0)}_j(k)\tilde{\chi}^{(0)}_j(k,t)e^{-ik.x} + \tilde{a}^{(0)\dagger}_j(k)\tilde{\chi}^{(0)*}_j(k,t)e^{+ik.x}\right) \tag{4.20}$$

where

$$\tilde{\chi}^{(0)}_j(k,t) = \frac{e^{-i\int^t dt'\omega_j(k,t')}}{\sqrt{2\omega_j(k,t)}} \tag{4.21}$$

and $\omega_j(k,t)^2 = \omega^2_j(t) + k^2$. This allows us to define the initial vacuum, $\tilde{a}^{(0)}_j(k)|0\rangle$. During the evolution, the entries of Ω_{ij} do not necessarily change adiabatically and consequently we must find new mode functions $\tilde{\chi}_l$ that satisfy eq. (4.16). The new set of creation/annihilation operators appearing in the expansion along $\tilde{\chi}_l$ (they define the true vacuum) can be related to the initial set using a Bogolyubov transformation with Bogolyubov coefficients α and β (which denote matrices in the multi-field case). Due to difficulties in finding exact solutions of eq. (4.16) (since C is time-dependent, it is not enough to diagonalize Ω and then write the eigenfunctions as $C^Te^{\pm i\omega_j t}$), it is preferable to translate this equation into an equation of motion for the Bogolubov coefficients directly. These equations of motion governing the system's time evolution are (matrix multiplication implied)

$$\begin{aligned} \dot{\alpha} &= -i\omega\alpha + \frac{\dot{\omega}}{2\omega}\beta - I\alpha - J\beta \\ \dot{\beta} &= \frac{\dot{\omega}}{2\omega}\alpha + i\omega\beta - J\alpha - I\beta \end{aligned} \tag{4.22}$$

where initially $\alpha = \mathbb{I}$ and $\beta = 0$ (since initially adiabatic and non-adiabatic modes coincide) while the matrices I and J are given by

$$I = \frac{1}{2}\left(\sqrt{\omega}\,C^T\dot{C}\frac{1}{\sqrt{\omega}} + \frac{1}{\sqrt{\omega}}\,C^T\dot{C}\sqrt{\omega}\right), \tag{4.23}$$

$$J = \frac{1}{2}\left(\sqrt{\omega}\,C^T\dot{C}\frac{1}{\sqrt{\omega}} - \frac{1}{\sqrt{\omega}}\,C^T\dot{C}\sqrt{\omega}\right). \tag{4.24}$$

Similarly to the single-field case it can be shown [79] that at any generic time the occupation number of the ith bosonic eigenstate reads

$$n_i(t) = (\beta^* \beta^T)_{ii}. \tag{4.25}$$

As pointed out in [79, 58], there exists two sources of non-adiabaticity in the multi-field scenario. The first source arises from the individual frequency time dependence and appears as the only source of non-adiabaticity in the single field case. The second source appears from the time dependence of the frequency matrix Ω_{ij} giving rise to terms in eq. (4.22) proportional to I and J. This second source provides the most important contribution in our analysis and gives rise to non-perturbative particle production.

Since initially $\alpha = \mathbb{I}$ and $\beta = 0$, eq. (4.22) shows that a non-vanishing matrix J is a necessary condition to obtain $\dot{\beta} \neq 0$ and hence $n_i(t) \neq 0$. In general, we have

$$C^T\dot{C} = B^T A^T \dot{A} B = -B^T U B \tag{4.26}$$

where A, B and U were defined in eqs.(4.12), (4.13) and (4.14). For the toy $U(1)$ example outlined above, J is a 5×5 matrix in the χ_i basis with non-vanishing components

$$J_{1,2} = J_{2,1} = \frac{k - \sqrt{k^2 + M_1^2}}{2\sqrt{3k}(k^2 + M_1^2)^{1/4}}\dot{\gamma}, \tag{4.27}$$

where M_1 denotes the mass of the heavy Higgs field. These entries in the matrix J link the eigenstate of the Higgs ($i = 1$) with one of the light eigenstates ($i = 2$). We see that in the toy $U(1)$ model, preheating can occur provided that $\dot{\gamma} \neq 0$.

We now demonstrate that both physical phases σ and γ are in general dynamical, i.e $\dot{\sigma}, \dot{\gamma} \neq 0$. In our particular toy example the cancellation of $U(1)^3$ and mixed $U(1)$-gravitational anomalies requires that we extend the field content of our model by including three additional complex superfields, Φ_4, Φ_5 and Φ_6. We assign the $U(1)$ charges and R-Parity (R_p) as follows:

	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6
$U(1)$	1/2	-1	1/2	-1/2	1	-1/2
R_p	+	-	+	-	+	-

This choice of R_p assignments forbids the superpotential term $\Phi_1\Phi_2\Phi_3$, thus preserving F-flatness. There exist several possible flat directions for this particular field content. We assume that VEVs for only Φ_1, Φ_2 and Φ_3 are turned on, leaving Φ_4, Φ_5 and Φ_6 with no VEVs. With this assumption, the lowest dimension terms appearing in the scalar potential, which are R_p and gauge invariant arise as soft SUSY breaking A-terms and appear as

$$\begin{aligned} V \supset \quad & a_1 \tfrac{\widetilde{m}}{M^3}\Phi_1^2\Phi_2^2\Phi_3^2 + a_2\tfrac{\widetilde{m}}{M^3}\Phi_1^3\Phi_2^2\Phi_3 + a_3\tfrac{\widetilde{m}}{M^3}\Phi_1\Phi_2^2\Phi_3^3 \\ & + a_4\tfrac{\widetilde{m}}{M^3}\Phi_1^4\Phi_2^2 + a_5\tfrac{\widetilde{m}}{M^3}\Phi_2^2\Phi_3^4 + h.c., \end{aligned} \tag{4.28}$$

where M denotes the cut-off scale of the theory (e.g. the Planck mass or GUT scale), $\widetilde{m}$ represents the scale of the SUSY breaking, and a_i label dimensionless coefficients of order one. Lower order phase-independent interactions will also contribute to the lifting of the flat direction and have the generic forms

$$V \supset \sum_i \frac{\widetilde{m}_i^2}{2}|\Phi_i|^2 + \sum_{i,j}\frac{\lambda_{ij}}{8}|\Phi_i|^2|\Phi_j|^2, \tag{4.29}$$

where $\widetilde{m}_i^2$ denote the soft SUSY breaking masses, and the second terms arise from loop corrections with $\lambda_{i,j} \sim g^4\widetilde{m}^2/\varphi^2$ (see for example [19] for similar loop induced terms). The potential from eq. (4.28) and eq. (4.29), for the single flat direction amplitude case considered in Section 4.2, using the VEV form shown in eq. (4.9), becomes

$$\begin{aligned} V \quad \supset \quad & \frac{\widetilde{m}_1^2 + \widetilde{m}_2^2 + \widetilde{m}_3^2}{2}\varphi^2 + \frac{\lambda'}{4}\varphi^4 \\ & + \varphi^6\big(A'_{11}e^{i6\sigma} + A'_{12}e^{i(6\sigma+2\gamma)} + A'_{13}e^{i(6\sigma-2\gamma)} + A'_{22}e^{i(6\sigma+4\gamma)} + A'_{33}e^{i(6\sigma-4\gamma)}\big), \end{aligned}$$

where A'_{ij} and λ' denote combinations of the couplings discussed above. The phase-dependent terms in the potential provide non-trivial dynamics for the phases σ and γ and will in general lead to $\dot{\gamma} \neq 0$ and therefore a non-vanishing J matrix. As discussed above, the appearance of a non-vanishing J matrix can lead to the non-perturbative production of particles by the rotating flat direction: the condensate can decay via preheating.

4.3 Multiple Flat Directions

We can extend the analysis of the previous sections by allowing the magnitudes of the individual field VEVs to differ from one another. As above, we consider the case with three complex superfields charged under a $U(1)$ gauge group with charges $q_1 = +1/2$, $q_2 = -1$ and $q_3 = +1/2$ respectively, and with the scalar potential given in eq. (4.4). We can write the flat direction with the following VEV

$$\begin{aligned} \langle \Phi_1 \rangle &= \varphi_1 e^{i\sigma_1}, \\ \langle \Phi_2 \rangle &= \frac{1}{\sqrt{2}}(\varphi_1^2 + \varphi_2^2)^{1/2} e^{i\sigma_2}, \\ \langle \Phi_3 \rangle &= \varphi_2 e^{i\sigma_3}. \end{aligned} \tag{4.30}$$

By substituting the above into the potential given in eq. (4.4), it can readily be shown that the configuration satisfies D-flatness. Expanding around this VEV we have

$$\begin{aligned}
\Phi_1 &= (\varphi_1+\xi_1)e^{i\left(\sigma_1+\frac{\theta_1}{\varphi_1}\right)},\\
\Phi_2 &= \left(\frac{1}{\sqrt{2}}(\varphi_1^2+\varphi_2^2)^{1/2}+\xi_2\right)e^{i\left(\sigma_2+\frac{\sqrt{2}\theta_2}{\left(\varphi_1^2+\varphi_2^2\right)^{1/2}}\right)},\\
\Phi_3 &= (\varphi_2+\xi_3)e^{i\left(\sigma_3+\frac{\theta_3}{\varphi_2}\right)},
\end{aligned}\tag{4.31}$$

where the fields $\xi_{1,2,3}$ and $\theta_{1,2,3}$ represent the excitations around the VEVs. As in the previous case, we can use a gauge transformation to remove a phase from the VEV structure that ensures the absence of terms of the form appearing in eq. (4.6). The form of the VEV in this case becomes,

$$\begin{aligned}
\langle\Phi_1\rangle &= \varphi_1 e^{i\left(\sigma+\frac{\varphi_2}{\varphi_1}\gamma\right)},\\
\langle\Phi_2\rangle &= \frac{1}{\sqrt{2}}(\varphi_1^2+\varphi_2^2)^{1/2}e^{i\sigma},\\
\langle\Phi_3\rangle &= \varphi_2 e^{i\left(\sigma-\frac{\varphi_1}{\varphi_2}\gamma\right)},
\end{aligned}\tag{4.32}$$

where σ and γ represent two independent phases[4].

In the unitary gauge, a form that preserves the canonically normalized kinetic terms reads,

$$\begin{aligned}
\Phi_1 &= (\varphi_1+\xi_1)e^{i\left(\sigma+\frac{\varphi_2}{\varphi_1}\gamma+\frac{\theta\varphi_2+\theta'\varphi_1(2/3)^{1/2}}{\varphi_1(\varphi_1^2+\varphi_2^2)^{1/2}}\right)},\\
\Phi_2 &= \left(\frac{1}{\sqrt{2}}(\varphi_1^2+\varphi_2^2)^{1/2}+\xi_2\right)e^{i\left(\sigma+\frac{\theta'(2/3)^{1/2}}{(\varphi_1^2+\varphi_2^2)^{1/2}}\right)},\\
\Phi_3 &= (\varphi_2+\xi_3)e^{i\left(\sigma-\frac{\varphi_1}{\varphi_2}\gamma-\frac{\theta\varphi_1-\theta'\varphi_2(2/3)^{1/2}}{\varphi_2(\varphi_1^2+\varphi_2^2)^{1/2}}\right)},
\end{aligned}\tag{4.33}$$

where $\xi_{1,2,3}$, θ and θ' label the physical excitations around the VEV once the NGo boson has been gauged away. The resulting spectrum consists of one Higgs field with mass $M_1^2 = 3g^2(\varphi_1^2+\varphi_2^2)$, and four massless scalar fields.

[4]Again we have applied the limit $\dot{\varphi} \ll \varphi\dot{\sigma}, \varphi\dot{\gamma}$. If we do not apply this limit the gauge transformation parameter (λ) needed to remove the linear term in A_0 can be found by integrating the coefficient of the A_0 term with respect to time. This is in general complicated and we choose to assume that φ is varying very slowly with time.

We proceed, as before, by diagonalizing the kinetic terms and evaluating the J matrix given in eq. (4.24). The non-vanishing entries of the J-matrix are

$$J_{1,2} = J_{2,1} = \frac{k - \sqrt{k^2 + M_1^2}}{2\sqrt{3k}(k^2 + M_1^2)^{1/4}}\dot{\gamma}, \tag{4.34}$$

which demonstrates that also in this case preheating can take place provided that $\dot{\gamma} \neq 0$.

It is instructive to compare the two cases considered thus far. The first flat direction contained a single VEV amplitude, eq. (4.5), the second contained two independent VEV amplitudes, eq. (4.31). The final result, however, is the same for both cases. This demonstrates a simple property of flat direction VEV decay: the determining factor is not the *number of flat directions* present in the system, but the *number of fields* that have VEVs. In particular, a necessary condition for non-perturbative production of particles is the existence of at least one relative physical and dynamical phase between the field VEVs that constitute the flat direction. As shown above this is generally the case unless all phase dependent terms in the scalar potential are suppressed.

A further instructive toy model consists of two independent flat directions existing simultaneously. We consider four scalar fields Φ_1, Φ_2, Φ_3, and Φ_4 charged under a gauged $U(1)$ symmetry with charges $\pm q_1$ and $\pm q_2$ respectively. The potential arising from the D-terms reads

$$V = \frac{g^2}{8}(q_1 |\Phi_1|^2 - q_1 |\Phi_2|^2 + q_2 |\Phi_3|^2 - q_2 |\Phi_4|^2)^2. \tag{4.35}$$

Although this toy model has been examined previously in [58], applying the methods outlined above helps establish the important properties of the model. The potential in eq. (4.35) admits flat direction VEVs of the following forms

$$\begin{aligned}
\langle\Phi_1\rangle &= \varphi_1 e^{i\tilde{\sigma}_1}, \\
\langle\Phi_2\rangle &= \varphi_1 e^{i\tilde{\sigma}_2}, \\
\langle\Phi_3\rangle &= \varphi_2 e^{i\tilde{\sigma}_3}, \\
\langle\Phi_4\rangle &= \varphi_2 e^{i\tilde{\sigma}_4}.
\end{aligned} \tag{4.36}$$

We can write the excitations around the VEVs as

$$\begin{aligned}
\Phi_1 &= (\varphi_1 + \xi_1) e^{i(\tilde{\sigma}_1 + \frac{\theta_1}{\varphi_1})}, \\
\Phi_2 &= (\varphi_1 + \xi_2) e^{i(\tilde{\sigma}_2 + \frac{\theta_2}{\varphi_1})}, \\
\Phi_3 &= (\varphi_2 + \xi_3) e^{i(\tilde{\sigma}_3 + \frac{\theta_3}{\varphi_2})}, \\
\Phi_4 &= (\varphi_2 + \xi_4) e^{i(\tilde{\sigma}_4 + \frac{\theta_4}{\varphi_2})}.
\end{aligned} \tag{4.37}$$

As before we can make a gauge transformation and remove one phase in such a way that terms of the form shown in eq. (4.6) vanish. The final form appears as

$$\begin{aligned}
\langle\Phi_1\rangle &= \varphi_1 e^{i(\sigma_1 + \gamma\frac{\varphi_2}{q_1\varphi_1})}, \\
\langle\Phi_2\rangle &= \varphi_1 e^{i(\sigma_1 - \gamma\frac{\varphi_2}{q_1\varphi_1})}, \\
\langle\Phi_3\rangle &= \varphi_2 e^{i(\sigma_2 + \gamma\frac{\varphi_1}{q_2\varphi_2})}, \\
\langle\Phi_4\rangle &= \varphi_2 e^{i(\sigma_2 - \gamma\frac{\varphi_1}{q_2\varphi_2})},
\end{aligned} \tag{4.38}$$

demonstrating the existence of three physical phases. Transforming in to the unitary gauge, we can write the excitations around the VEVs as

$$\begin{aligned}
\Phi_1 &= (\varphi_1 + \xi_1) e^{i(\sigma_1 + \gamma\frac{\varphi_2}{q_1\varphi_1} + \theta\frac{1}{\sqrt{2}\varphi_1} + \theta''\frac{q_2\varphi_2}{\varphi'\varphi_1})}, \\
\Phi_2 &= (\varphi_1 + \xi_2) e^{i(\sigma_1 - \gamma\frac{\varphi_2}{q_1\varphi_1} + \theta\frac{1}{\sqrt{2}\varphi_1} - \theta''\frac{q_2\varphi_2}{\varphi'\varphi_1})}, \\
\Phi_3 &= (\varphi_2 + \xi_3) e^{i(\sigma_2 - \gamma\frac{\varphi_1}{q_2\varphi_2} + \theta'\frac{1}{\sqrt{2}\varphi_2} - \theta''\frac{q_1\varphi_1}{\varphi'\varphi_2})}, \\
\Phi_4 &= (\varphi_2 + \xi_4) e^{i(\sigma_2 + \gamma\frac{\varphi_1}{q_2\varphi_2} + \theta'\frac{1}{\sqrt{2}\varphi_2} + \theta''\frac{q_1\varphi_1}{\varphi'\varphi_2})},
\end{aligned} \tag{4.39}$$

where $\varphi' = \sqrt{2}(q_1\varphi_1^2 + q_2\varphi_2^2)^{1/2}$. The spectrum in this case consists of one massive Higgs particle and six massless scalar fields (the NG has been gauged away). Again, we must diagonalize the kinetic terms. Applying the necessary field redefinitions we are able to evaluate the J matrix. The non-vanishing J matrix elements read

$$
\begin{aligned}
J_{1,2} = J_{2,1} &= \frac{k - \sqrt{k^2 + M_1^2}}{\sqrt{k}(k^2 + M_1^2)^{1/4}} \frac{q_1 q_2}{\varphi'^2}(\dot{\sigma}_1 - \dot{\sigma}_2)\varphi_1\varphi_2 \\
J_{1,3} = J_{3,1} &= \frac{k - \sqrt{k^2 + M_1^2}}{\sqrt{2k}(k^2 + M_1^2)^{1/4}} \frac{\dot{\gamma}\varphi_2}{\varphi'} \\
J_{1,4} = J_{4,1} &= \frac{k - \sqrt{k^2 + M_1^2}}{\sqrt{2k}(k^2 + M_1^2)^{1/4}} \frac{\dot{\gamma}\varphi_1}{\varphi'},
\end{aligned}
\tag{4.40}
$$

which depend on the relative phases between the field VEVs. We should point out that only the Higgs eigenstate ($i = 1$) is distinguishable. The other indices label the light fields which at this level are all massless. Preheating is again possible provided two of the phases have non-zero time derivatives. Using the particular case with $q_1 = q_2$, we can write the scalar potential (see Appendix B for details) yielding the terms

$$
\begin{aligned}
V = \frac{1}{2}(\widetilde{m}_1^2 + \widetilde{m}_2^2)\varphi_1^2 + \frac{1}{2}(\widetilde{m}_3^2 + \widetilde{m}_4^2)\varphi_2^2 + \frac{a_1}{8}\frac{\widetilde{m}}{M}\varphi_1^4 e^{i4\sigma_1} \\
+ \frac{a_2}{8}\frac{\widetilde{m}}{M}\varphi_2^4 e^{i4\sigma_2} + \frac{a_3}{8}\frac{\widetilde{m}}{M}\varphi_1^2\varphi_2^2 e^{i2(\sigma_1+\sigma_2+\gamma\frac{\varphi_2^2-\varphi_1^2}{\varphi_2\varphi_1})} \\
+ \frac{a_4}{8}\frac{\widetilde{m}}{M}\varphi_1^2\varphi_2^2 e^{2i(\sigma_1+\sigma_2-\gamma\frac{\varphi_2^2-\varphi_1^2}{\varphi_2\varphi_1})} + ...
\end{aligned}
\tag{4.41}
$$

Clearly, non-trivial dynamics exist for the phases γ, σ_1 and σ_2.

4.4 Numerical analysis

As a proof-of-principle that achieves quantitative results, we numerically analyse the second model described in Section 4.3. We use a simplified version of the potential appearing in eq. (4.41), confining ourselves to the potential

$$
V = \frac{1}{2}\widetilde{m}_{\varphi_1}^2\varphi_1^2 + \frac{1}{2}\widetilde{m}_{\varphi_2}^2\varphi_2^2 + \frac{a_1}{8}\frac{\widetilde{m}}{M}\varphi_1^4 e^{i4\sigma_1} + \frac{a_2}{8}\frac{\widetilde{m}}{M}\varphi_2^4 e^{i4\sigma_2} + \text{h.c} \tag{4.42}
$$

where $\widetilde{m}^2_{\varphi_1} = \widetilde{m}^2_1 + \widetilde{m}^2_2$, $\widetilde{m}^2_{\varphi_2} = \widetilde{m}^2_3 + \widetilde{m}^2_4$. This potential decouples the equations of motion for γ, σ_1 and σ_2. The equation of motion for γ reduce to $\ddot{\gamma} = 0$, and with the choice of initial conditions, $\dot{\gamma} = 0$, the effects of γ on preheating are removed. Our simplified potential allows us to numerically evolve the classical evolution of the flat direction VEVs and analyze particle production in a self-consistent background. We also make the simplifying assumption setting $a_1 = a_2 = \lambda \frac{M}{\widetilde{m}}$ in eq. (4.41). Again, we stress that we use this grossly simplified potential simply to demonstrate the quantitative behaviour of the toy model class.

Measuring the conformal time in units of $\tau \rightarrow ft$ with $f = g\varphi_{1_0}$ and using the re-scaled flat-direction VEV amplitudes

$$\begin{aligned} \varphi_1 &= \frac{\varphi_{1_0}}{R} F_1 \\ \varphi_2 &= \frac{\varphi_{2_0}}{R} F_2 \end{aligned} \tag{4.43}$$

we find the background equations,

$$\begin{aligned} F_1'' + \left[\frac{\mu_1^2 R^2}{2} - \sigma_1'^2 - \frac{R''}{R}\right] F_1 + \frac{\lambda F_1^3}{2g^2} \cos(4\sigma_1) = 0 \\ F_2'' + \left[\frac{\mu_2^2 R^2}{2} - \sigma_2'^2 - \frac{R''}{R}\right] F_2 + \frac{\lambda F_2^3}{2g^2} \left(\frac{\varphi_{2_0}}{\varphi_{1_0}}\right)^2 \cos(4\sigma_2) = 0 \end{aligned} \tag{4.44}$$

where a prime represents a derivative with respect to τ and

$$\begin{aligned} \sigma_1'' + 2\sigma_1' \frac{F_1'}{F_1} - \frac{\lambda}{2g^2} F_1^2 \sin(4\sigma_1) = 0 \\ \sigma_2'' + 2\sigma_2' \frac{F_2'}{F_2} - \frac{\lambda}{2g^2} \left(\frac{\varphi_{2_0}}{\varphi_{1_0}}\right)^2 F_2^2 \sin(4\sigma_2) = 0 \end{aligned} \tag{4.45}$$

describes the motion of the flat direction VEVs; $\mu_1 = \widetilde{m}_{\varphi_1}/f, \mu_2 = \widetilde{m}_{\varphi_2}/f$. The scale factor evolves as,

$$\begin{aligned} \frac{R''}{R} = -\frac{R'^2}{R^2} + \frac{1}{2}\Bigg[f_p^2 \Bigg\{ \mu_1^2 F_1^2 + \frac{\lambda}{2g^2} \frac{F_1^4}{R^2} \cos(4\sigma_1) \\ + \mu_2^2 \left(\frac{\varphi_{2_0}}{\varphi_{1_0}}\right)^2 F_2^2 + \frac{\lambda}{2g^2} \left(\frac{\varphi_{2_0}}{\varphi_{1_0}}\right)^4 \frac{F_2^4}{R^2} \cos(4\sigma_2) \Bigg\} + \frac{R^2 \rho_\psi}{M_{pl}^2 f^2} \Bigg] \end{aligned} \tag{4.46}$$

where ρ_ψ is the energy density of the inflaton field and $f_p = \varphi_{1_0}/M_{pl}$ is set to $f_p = 0.1$ in our numerics. We also take $\mu_1 = 10^{-2}, \mu_2 = 10^{-2}/2$, and $\lambda = \mu_1^2$ for computational ease. As initial conditions, we start the flat direction at rest, such that $(\varphi_{1,2}\exp(i\sigma_{1,2}))' = 0$. We choose to set initially $F_{1,2} = 1$, $\sigma_{1,2} = 0.05$, $\sigma'_{1,2} = 0$, $R = 1$ and $R' = \mu_1$, which implies $F'_{1,2} = R' = \mu_1$. While these initial conditions do not present a realistic case (where $\mu \sim 10^{-14}$ and $F'_{1,2} \gg \mu_1$), they do provide a numerical proof-of-principle similar to [58].

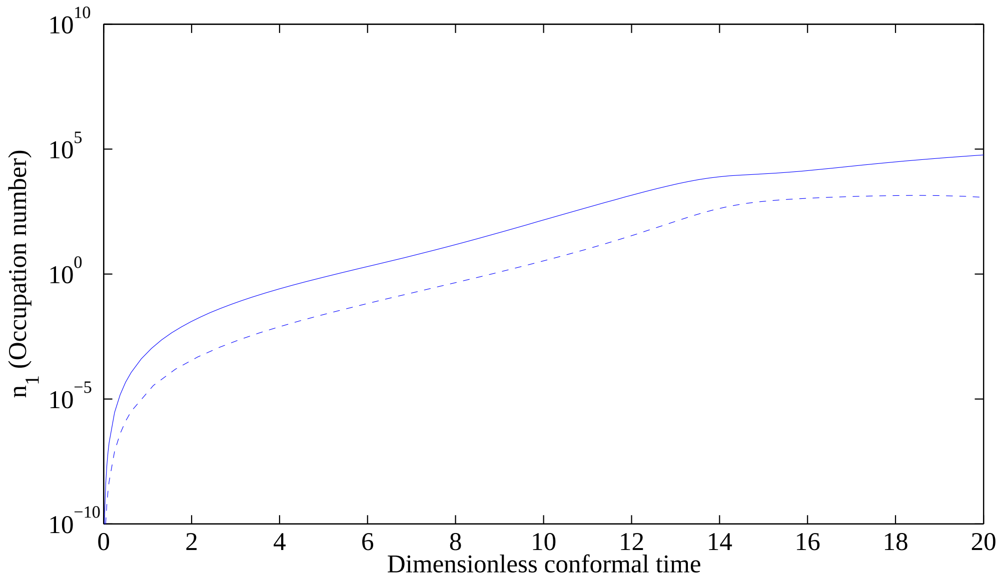

Figure 4.2: Occupation number for one of the excited fields as a function of dimensionless conformal time, obtained using eq. (4.25) after numerically integrating the background field equations and Bogolyubov matrices; $k = \mu_1/3 \times 10^5$, other parameters as explained in the text. The solid lines represents preheated fields with $\varphi_{2_0}/\varphi_{1_0} = 1$ while the dashed lines indicate the preheated fields with $\varphi_{2_0}/\varphi_{1_0} = 0.1$

Initially the flat direction VEVs correspond to a condensate of coherent particles with vanishing momentum. The motion of these VEVs, described by eq. (4.44) and eq. (4.45), and the interactions described in the previous section, cause the rapid decay of this condensate into a decoherent state of particles. FIG. 4.2 shows the occupation numbers, $n_i(t)$, of these light particles as a function of conformal time: the exponential growth of these functions signals the exponentially fast decay of the flat-direction VEV. The two line types, solid and dashed, represent the ratios $\varphi_{2_0}/\varphi_{1_0} = 1, 0.1$ respectively. We see that preheating occurs over a wide range of the ratio $\varphi_{2_0}/\varphi_{1_0}$. In this numerical example, preheating effects do not vanish until $\varphi_{2_0}/\varphi_{1_0} \lesssim 10^{-2}$. FIG 4.3 displays the resulting spectrum for one of the light fields, we see that production of higher momentum modes becomes kinematically suppressed.

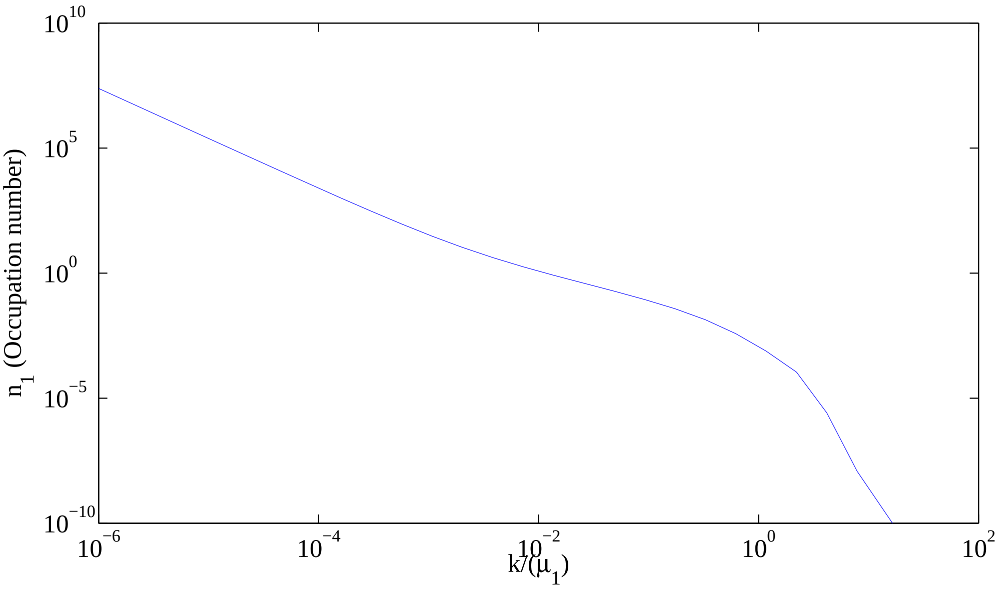

Figure 4.3: Occupation number as a function of comoving momentum obtained as for FIG. 4.2, with $\varphi_{2_0}/\varphi_{1_0} = 0.5$ at time $t = 20$.

We must stress that effects of SUSY breaking terms in the Lagrangian eq. (4.3) will significantly affect the amount of particle production produced by the rotating condensate. A mass term for the light fields translates into a momentum shift in eq. (4.17) and this corresponds to a kinematic suppression of the modes [58]. On the other hand, in general, the flat direction will undergo *elliptical* motion in the complex plane, with angular velocities at the origin much bigger than $\dot{\sigma} = \widetilde{m}$. A realistic model involving MSSM flat directions will in general contain many SUSY breaking terms and non-renormalisable operators, creating large model dependencies in the precise determination of the momentum shift. Consequently, we leave such a study to future work.

4.5 Discussion

The cosmological fate of flat directions provides a major ingredient for the history of the early Universe. Flat directions can provide mechanisms for generating the baryon asymmetry of the Universe and can play an important role in reheating after inflation. Our analysis stresses the use of the unitary gauge in which the physical content of the theory becomes manifest. By transforming to the unitary gauge, complications arising from massless NGo modes in the mixing of the excitations around the flat direction VEV are removed. The mixing matrix in this gauge defines the mass eigenstates of the physical scalar fields and determines if non-perturbative decay is possible. Since the mass matrix in the unitary gauge can contain time dependent mixing among all fields, one of the necessary conditions for preheating can be satisfied.

Two further crucial conditions for preheating in our analysis center on the existence of physical relative phases between the field VEVs that make up the flat direction(s) and that these phases possess non-trivial dynamics during the early universe. The first of these conditions generally becomes satisfied if the difference between the number of fields that acquire a VEV and the number of broken diagonal generators is larger than one – every diagonal generator removes one unphysical phase. The second condition generally becomes satisfied if terms which explicitly depend on the phase differences appear in the scalar potential. The existence of gauge invariant products of background fields exhibiting this phase dependence represents the crucial ingredient and determines the phase dependence in the scalar potential. Once these conditions are satisfied, the flat direction condensate can decay non-perturbatively via preheating.

Chapter 5

Conclusions

Our present understanding of Nature, as summarized in the Standard Model of particle physics is still incomplete. One of the major issues being the hierarchy problem: scalar masses (in particular the mass of the Higgs field) are driven by quantum corrections to very large values, requiring an unacceptable degree of fine-tuning to make the theory consistent. Solutions to this problem require the addition of new structure to the Standard Model of particle physics. One possibility is to introduce supersymmetry, a symmetry relating scalars with fermions, which protects the mass of scalar fields by connecting it to the mass of fermion fields which, in turn, is protected by chiral symmetry. Supersymmetry, besides solving the problem of diverging scalar masses, has other interesting features, such as the unification of gauge couplings at the GUT scale, a natural explanation for the negative Higgs mass needed for electro-weak symmetry breaking and, finally, SUSY incorporates a candidate for dark matter: the Lightest Supersymmetric Particle. The new structure needed to support supersymmetry has also other interesting consequences, among which, the existence of flat directions along the supersymmetric scalar potential.

Flat directions play a major role during the early universe. Indeed, during inflation, large field VEVs can develop along these directions providing a large source of energy that can have important consequences when it eventually decays. One example is Affleck-Dine baryogenesis, where the out-of-equilibrium decay of these VEVs can explain the observed baryon/anti-baryon asymmetry.

In this thesis we have analysed several implications of the presence of these large VEVs along supersymmetric flat directions and, in particular, in relation to their non-perturbative decay via preheating. In Chapter 2 we have shown that the non-perturbative decay of flat directions can generate a large lepton asymmetry even if the temperature at which the universe is reheated is low enough to avoid over-production of gravitinos. Normally, in scenarios of thermal leptogenesis, in order to produce enough right handed neutrinos, the reheat temperature must be bigger than about 10^9 GeV. At such high temperatures, large quantities of gravitinos are produced by scattering in the thermal bath. These gravitinos decay after Big Bang Nucleosynthesis, destroying its products and compromising its success. The solutions to this problem proposed so far, all involve adding new structure to the Minimal Supersymmetric Standard Model. We showed that the non-perturbative decay of flat directions is able to produce a large number of up-Higgses during its decay. These particles acquire a large mass when the flat direction continues its oscillation and can decay into the heavy right handed neutrinos, responsible for the production of the lepton asymmetry. The leptons produced with this mechanism are then converted into baryons by sphaleron processes. If the VEV along the flat directions are initially of order the Planck scale and if the phase dependent terms in the potential are small enough, the observed baryon asymmetry can be reproduced.

In Chapter 3 we studied a similar mechanism which produces cosmological perturbations of the right size to explain the temperature anisotropies in the CMB and to seed structure formation. Such perturbations are produced during inflation, when quantum fluctuations of scalar fields are stretched to macroscopic size by the rapid expansion of the universe and become classical. Once generated, these perturbations in the energy density of this primordial scalar field are transmitted to the metric which, in turn, creates perturbations in the baryon, photon and dark matter fluids. As soon as the pressure from photons is diluted enough by the expansion of the universe, these tiny perturbations collapse under the gravitational attraction and give birth to the large scale structure we observe today, galaxies, cluster of galaxies etc... It is generally assumed that the primordial field responsible for the fluctuations is the inflaton itself. In our study we showed that curvature perturbations can originate from quantum fluctuations in the phase of flat direction VEVs. The curvature of the scalar potential along flat directions can change sign soon after inflation (this is the case for instance in models of hybrid inflation) so that the flat direction VEV oscillates around the origin with frequencies comparable to the Hubble parameter during inflation. In this case, fluctuations in the initial conditions during inflation are transformed into fluctuations in the efficiency of the mechanism that converts the energy of the flat direction into the energy of its decay products. These fluctuations provide the right amount of curvature perturbations. If this is the dominant mechanism that generates the curvature perturbations, then large non-Gaussianities in the power spectrum are expected.

Finally in Chapter 4 we analysed the case where the phase dependent terms in the scalar potential along flat directions are of the same order as the other terms in the Lagrangian. In this case the flat direction VEV undergoes an elliptic motion in the complex plane, without passing through the origin. Since non-perturbative decay generally takes place when the VEV goes through the origin, it has generally been assumed that the flat direction would decay only perturbatively, after a long period of time. If these large VEVs persist long enough, they can delay thermalization (and block inflaton preheating), leading to low reheat temperatures. We have shown that this is not true: flat directions decay non-perturbatively, due to the time-dependence of the mass-matrix (from D-terms) for the excitations around the flat direction VEV. For this decay to be efficient, two conditions have to be met: the flat direction VEV must contain two physical phases (that one can not remove using a gauge transformation) and these phases need to have non-trivial dynamics during the early universe (which is generally the case if the phase dependent terms in the scalar potential for the flat direction are not suppressed).

Much attention has been dedicated to the study of SUSY particle phenomenology, in particular in view of the upcoming experiment at the Large Hadron Collider at CERN, partially devoted to the discovery of low-energy SUSY. Indeed, as motivated above, SUSY is a very attractive theory from the point of view of particle physics, and certainly the most compelling extension of the SM constructed so far. SUSY adds just enough structure to the SM in order to solve its major problems (hierarchy problem, dark matter, unification), without compromising its successes (electro-weak precision data, ...). In the forthcoming years the LHC will effectively probe energies up to a few TeV, where the SUSY partners of SM particles are believed to live, if SUSY has to provide a natural solution to the hierarchy problem. Therefore, SUSY should be soon proved either right or wrong. Despite the gigantic efforts invested in collider searches, it is important to analyse the astrophysical and cosmological implications of SUSY. For instance, direct or indirect searches of dark matter could lead to valuable information on the LSP, information which is hard to extract at colliders, where the LSP is invisible. Although cosmological tests generally do not provide stringent constraints on the parameters of SUSY, in specific models allowed regions in parameter space, derived from negative collider searches, can be tightened using cosmological inputs. In this thesis we have taken a positive approach to SUSY and, confident that it will be discovered at the LHC, we have tried to show that cosmological bounds can be evaded and solutions found. Flat directions are not present in the SM and represent the main difference between standard and supersymmetric cosmology. In this sense, they provide a valuable extra tool to address cosmological issues.

Appendix A

An Estimate for ϕ_*

In this short Appendix we will give an explicit derivation of the expressions of Eq. (3.12) and Eq. (3.13). Starting from the potential of eq.(3.10), we rescale the fields in units of $|\phi_I|$ and time in units of the inverse frequency H_I^{-1}; the potential in units of $H_I^2|\phi_I|^2$ becomes

$$\hat{V} = \frac{1}{2}|\hat{\phi}|^2 + \left(\frac{a_{\rm osc}}{n}\hat{\phi}^n + h.c.\right) + \beta^2|\hat{\phi}|^{2n-2}, \tag{A.1}$$

where $\hat{\phi} \equiv \phi/|\phi_I|$. Since we work in the limit of small $a_{\rm osc}$, we can write the real and imaginary part of the trajectories as

$$\begin{cases} \hat{\phi}_r &= \hat{\phi}_r^{(0)} + a_{\rm osc}\hat{\phi}_r^{(1)} + \mathcal{O}(a_{\rm osc}^2), \\ \hat{\phi}_i &= \hat{\phi}_i^{(0)} + a_{\rm osc}\hat{\phi}_i^{(1)} + \mathcal{O}(a_{\rm osc}^2), \end{cases} \tag{A.2}$$

where

$$\begin{cases} \hat{\phi}_r^{(0)} &\approx \cos(\theta_0)\cos\tilde{t}, \\ \hat{\phi}_i^{(0)} &\approx \sin(\theta_0)\cos\tilde{t}, \end{cases} \tag{A.3}$$

are the 0-th order (in $a_{\rm osc}$) solutions of the equations of motion (approximated for small β) and $\tilde{t} = (1+\beta^2)t$. Solving the first order equations

$$\ddot{\hat{\phi}} + 3H\dot{\hat{\phi}} = -\frac{\partial V}{\partial\hat{\phi}} \tag{A.4}$$

one finds

$$\begin{cases} \hat{\phi}_r^{(1)} &= 2\cos\left((n-1)\theta_0\right) f_n(\tilde{t}) + \mathcal{O}(a_{\rm osc}), \\ \hat{\phi}_i^{(1)} &= -2\sin\left((n-1)\theta_0\right) f_n(\tilde{t}) + \mathcal{O}(a_{\rm osc}), \end{cases} \tag{A.5}$$

where $f_n(\tilde{t})$ is a complicated n-dependent function of time. As we shall see below, for the present purpose we are only interested in its value and the value of its derivative at $\tilde{t} = \pi/2$,

$$f_n\left(\frac{\pi}{2}\right) = -\frac{\sqrt{\pi}}{2}\frac{\Gamma\left(\frac{1+n}{2}\right)^2}{\Gamma\left(1+\frac{n}{2}\right)^2} \quad \text{and} \quad f_n'\left(\frac{\pi}{2}\right) = -\frac{1}{n}. \tag{A.6}$$

We can now analyse the approximated trajectories as they pass close to the origin, expanding the time-parameter around $\pi/2$ (the time when the 0-th order solution crosses the origin) as $\tilde{t} = \pi/2 + \delta\tilde{t}$. We then find that the minimum of the distance $|\phi|$ from the origin lies at

$$\delta\tilde{t} = -2a_{\rm osc} f_n\left(\frac{\pi}{2}\right)\cos(n\theta_0) + \mathcal{O}(a_{\rm osc}^2). \tag{A.7}$$

Inserting this into the expressions of Eqs. (A.2)-(A.5), we can evaluate the minimum distance from the origin

$$|\hat{\phi}_*| = 2a_{\rm osc}\left|f_n\left(\frac{\pi}{2}\right)\right|\sin(n\theta_0) + \mathcal{O}(a_{\rm osc}^2), \tag{A.8}$$

which, when the proper dimensionful units are reintroduced, gives Eq.(3.12). Similarly one can evaluate the velocity taking the derivative of Eqs. (A.2)-(A.5), and obtains Eq. (3.13).

Appendix B

A Simple Model of Multiple Flat Directions

In this appendix we specify a simple model to justify the form of the potential given in eq.4.41. We have the following field content with $U(1)$ charges and R_p assignments,

	Φ_1	Φ_2	Φ_3	Φ_4
$U(1)$	1	-1	1	-1
R_p	-	+	-	+

The lowest dimension gauge invariant operators are

$$\begin{aligned} \mathcal{O}_1 &= \Phi_1\Phi_2, \quad \mathcal{O}_2 &= \Phi_3\Phi_4, \\ \mathcal{O}_3 &= \Phi_1\Phi_4, \quad \mathcal{O}_4 &= \Phi_2\Phi_3. \end{aligned} \tag{B.1}$$

The lowest dimension terms which are R_p invariant and phase-dependent arise as soft SUSY breaking A-terms and appear in the scalar potential as

$$V = \sum_{i=1}^{4} \frac{\widetilde{m}_i^2}{2}|\Phi_i|^2 + \frac{a_1}{8}\frac{\widetilde{m}}{M}\mathcal{O}_1^2 + \frac{a_2}{8}\frac{\widetilde{m}}{M}\mathcal{O}_2^2 + \frac{a_3}{8}\frac{\widetilde{m}}{M}\mathcal{O}_3^2 + \frac{a_4}{8}\frac{\widetilde{m}}{M}\mathcal{O}_4^2 + ... \tag{B.2}$$

where the ellipses stand for other terms of the same order with different products of the gauge invariant operators, loop induced contributions and higher order terms in the $(1/M)$ expansion. Substituting the vevs given in eq.4.38 we generate the potential terms given in eq.4.41.

References

[1] For reviews of the standard model measurements and precision tests, see for example, P. Langacker, arXiv:hep-ph/0503068; G. Altarelli, CERN-PH-TH-2005-051; G. Altarelli, arXiv:hep-ph/0406270; G. Altarelli and M. W. Grunewald, Phys. Rept. **403-404** (2004) 189, and references therein

[2] [LEP Collaborations], arXiv:hep-ex/0412015.

[3] For a Review of the status of BBN, see for example: G. Steigman, Int. J. Mod. Phys. E **15** (2006) 1.

[4] J. Dunkley *et al.* [WMAP Collaboration], arXiv:0803.0586 [astro-ph].

[5] K. A. Olive, E. Skillman and G. Steigman, Astrophys. J. **483** (1997) 788.

[6] P. W. Higgs, Phys. Rev. **145** (1966) 1156; P. W. Higgs, Phys. Rev. Lett. **13** (1964) 508.

[7] E. Farhi and L. Susskind, Phys. Rept. **74** (1981) 277.

[8] C. Csaki, C. Grojean, L. Pilo and J. Terning, Phys. Rev. Lett. **92** (2004) 101802.

[9] A. H. Guth, Phys. Rev. D **23** (1981) 347. For a review, see D. H. Lyth and A. Riotto, Phys. Rep. **314** 1 (1999); A. Riotto, `hep-ph/0210162`; W. H. Kinney, `astro-ph/0301448`.

[10] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38** (1977) 1440; S. Weinberg, Phys. Rev. Lett. **40** (1978) 223; F. Wilczek, Phys. Rev. Lett. **40** (1978) 279.

[11] C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. **85** (2000) 4438.

[12] S. Weinberg, Phys. Rev. D **13** (1976) 974; *ibid.* Phys. Rev. D **19** (1979) 1277. L. Susskind, Phys. Rev. D **20** (1979) 2619. G. 't Hooft, in "Recent Developments In Gauge Theories. Proceedings, Nato Advanced Study Institute, Cargese, France, August 26 - September 8, 1979," *New York, Usa: Plenum (1980) 438 P. (Nato Advanced Study Institutes Series: Series B, Physics, 59)*

[13] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B **513** (2001) 232 [arXiv:hep-ph/0105239]; for a review see: M. Schmaltz and D. Tucker-Smith, Ann. Rev. Nucl. Part. Sci. **55** (2005) 229.

[14] K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B **719** (2005) 165.

[15] For a review on SUSY see, for example: M. F. Sohnius, Phys. Rept. **128** (1985) 39; S. P. Martin, arXiv:hep-ph/9709356; I. J. R. Aitchison, arXiv:hep-ph/0505105.

[16] S. R. Coleman and J. Mandula, Phys. Rev. **159** (1967) 1251. R. Haag, J. T. Lopuszanski and M. Sohnius, Nucl. Phys. B **88** (1975) 257.

[17] G. G. Ross, *Reading, Usa: Benjamin/cummings (1984) 497 P. (Frontiers In Physics, 60)*

[18] See for example: J. Polchinski, "String theory. Vol. 1: An introduction to the bosonic string," *Cambridge, UK: Univ. Pr. (1998) 402 p*; and "String theory. Vol. 2: Superstring theory and beyond," *Cambridge, UK: Univ. Pr. (1998) 531 p*

[19] I. Affleck and M. Dine, Nucl. Phys. B **249**, 361 (1985).

[20] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D **56** (1997) 3258.

[21] G. F. Giudice, L. Mether, A. Riotto and F. Riva, Phys. Lett. B **664** (2008) 21.

[22] A. Riotto and F. Riva, arXiv:0806.3382 [hep-ph].

[23] A. Basboll, D. Maybury, F. Riva and S. M. West, Phys. Rev. D **76** (2007) 065005.

[24] F. Riva, Talk given at the Cargese Summer School 2007, to be published in Proceedings Of Science.

[25] J. March-Russell and F. Riva, JHEP **0607** (2006) 033.

[26] J. Wess and B. Zumino, Nucl. Phys. B **70** (1974) 39.

[27] G. F. Giudice and A. Masiero, Phys. Lett. B **206** (1988) 480.

[28] For a review, see K. Enqvist and A. Mazumdar, Phys. Rept. **380**, 99 (2003).

[29] M. A. Luty and W. Taylor, Phys. Rev. D **53** (1996) 3399.

[30] M. T. Grisaru, W. Siegel and M. Rocek, Nucl. Phys. B **159** (1979) 429.

[31] See for example: P. Fayet and J. Iliopoulos, Phys. Lett. B **51** (1974) 461. L. O'Raifeartaigh, Nucl. Phys. B **96** (1975) 331.

[32] M. Dine, A. E. Nelson and Y. Shirman, Phys. Rev. D **51** (1995) 1362.

M. Dine, A. E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D **53** (1996) 2658. For a review, see: G. F. Giudice and R. Rattazzi, Phys. Rept. **322** (1999) 419.

[33] M. Y. Khlopov and A. D. Linde, Phys. Lett. B **138**, 265 (1984); J. R. Ellis, D. V. Nanopoulos and S. Sarkar, Nucl. Phys. B **259** (1985) 175; M. Kawasaki, K. Kohri and T. Moroi, Phys. Lett. B **625**, 7 (2005);

[34] T. Moroi, H. Murayama and M. Yamaguchi, Phys. Lett. B **303** (1993) 289; for a recent review, see T. Moroi, AIP Conf. Proc. **805**, 37 (2006).

[35] M. Dine, L. Randall and S. D. Thomas, Phys. Rev. Lett. **75** (1995) 398. M. Dine, L. Randall and S. D. Thomas, Nucl. Phys. B **458**, 291 (1996).

[36] G. G. Ross and S. Sarkar, Nucl. Phys. B **461** (1996) 597

[37] Y. B. Zeldovich and A. A. Starobinsky, Sov. Phys. JETP **34** (1972) 1159 [Zh. Eksp. Teor. Fiz. **61** (1971) 2161].

[38] For a review on baryogenesis, see for example: A. Riotto, arXiv:hep-ph/9807454.

[39] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5** (1967) 32 [JETP Lett. **5** (1967 SOPUA,34,392-393.1991 UFNAA,161,61-64.1991) 24].

[40] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).

[41] W. Buchmuller, R. D. Peccei and T. Yanagida, Ann. Rev. Nucl. Part. Sci. **55**, 311 (2005);

[42] M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman, (North-Holland, 1979), p. 315; S.L. Glashow, in Quarks and Leptons, Cargèse, eds. M. Lévy et al., (Plenum, 1980, New-York), p. 707; T. Yanagida, in *Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, 1979), p. 95; R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, (1980) 912.

[43] See for example, Y. Ashie *et al.* [Super-Kamiokande Collaboration], Phys. Rev. D **71** (2005) 112005.

[44] For a review, see: S. Davidson, E. Nardi and Y. Nir, arXiv:0802.2962 [hep-ph], or A. Strumia, arXiv:hep-ph/0608347.

[45] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155**, 36 (1985).

[46] J. A. Harvey and M. S. Turner, Phys. Rev. D **42** (1990) 3344.

[47] S. Davidson and A. Ibarra, Phys. Lett. B **535**, 25 (2002).

[48] G. F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, Nucl. Phys. B **685**, 89 (2004).

[49] L. Roszkowski, R. Ruiz de Austri and K. Y. Choi, JHEP **0508**, 080 (2005); D. G. Cerdeno, K. Y. Choi, K. Jedamzik, L. Roszkowski and R. Ruiz de Austri, JCAP **0606**, 005 (2006); J. Pradler and F. D. Steffen, arXiv:0710.2213 [hep-ph].

[50] M. Flanz, E. A. Paschos and U. Sarkar, Phys. Lett. B **345**, 248 (1995) [Erratum-ibid. B **382**, 447 (1996)]; L. Covi and E. Roulet, Phys. Lett. B **399**, 113 (1997); A. Pilaftsis, Phys. Rev. D **56**, 5431 (1997);

T. Hambye, J. March-Russell and S. M. West, JHEP **0407** (2004) 070; A. Pilaftsis and T. E. J. Underwood, Phys. Rev. D **72** (2005) 113001.

[51] Y. Grossman, T. Kashti, Y. Nir and E. Roulet, Phys. Rev. Lett. **91**, 251801 (2003); G. D'Ambrosio, G. F. Giudice and M. Raidal, Phys. Lett. B **575**, 75 (2003).

[52] H. Murayama and T. Yanagida, Phys. Lett. B **322**, 349 (1994); K. Hamaguchi, H. Murayama and T. Yanagida, Phys. Rev. D **65**, 043512 (2002).

[53] G. F. Giudice, M. Peloso, A. Riotto and I. Tkachev, JHEP **9908**, 014 (1999).

[54] T. Asaka, K. Hamaguchi, M. Kawasaki and T. Yanagida, Phys. Rev. D **61**, 083512 (2000); T. Asaka, K. Hamaguchi, M. Kawasaki and T. Yanagida, Phys. Lett. B **464**, 12 (1999); F. Hahn-Woernle and M. Plumacher, arXiv:0801.3972 [hep-ph].

[55] G. F. Giudice, A. Riotto and A. Zaffaroni, Nucl. Phys. B **710**, 511 (2005).

[56] G. N. Felder, L. Kofman and A. D. Linde, Phys. Rev. D **59**, 123523 (1999)

[57] R. Allahverdi, R. H. A. Shaw and B. A. Campbell, Phys. Lett. B **473**, 246 (2000); Z. Chacko, H. Murayama and M. Perelstein, Phys. Rev. D **68**, 063515 (2003); M. Postma and A. Mazumdar, JCAP **0401**, 005 (2004); R. Allahverdi and A. Mazumdar, JCAP **0708**, 023 (2007).

[58] K. A. Olive and M. Peloso, Phys. Rev. D **74**, 103514 (2006).

[59] T. Gherghetta, C. F. Kolda and S. P. Martin, Nucl. Phys. B **468**, 37 (1996).

[60] P. Binetruy and G. R. Dvali, Phys. Lett. B **388**, 241 (1996).

[61] G. N. Felder, L. Kofman and A. D. Linde, Phys. Rev. D **60** (1999) 103505.

[62] R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, Nucl. Phys. B **575**, 61 (2000); T. Endoh, T. Morozumi and Z. h. Xiong, Prog. Theor. Phys. **111**, 123 (2004); A. Abada, S. Davidson, F. X. Josse-Michaux, M. Losada and A. Riotto, JCAP **0604**, 004 (2006); E. Nardi, Y. Nir, E. Roulet and J. Racker, JHEP **0601**, 164 (2006); A. Abada, S. Davidson, A. Ibarra, F. X. Josse-Michaux, M. Losada and A. Riotto, JHEP **0609**, 010 (2006).

[63] W. H. Kinney, E. W. Kolb, A. Melchiorri and A. Riotto, arXiv:0805.2966 [astro-ph].

[64] K. Enqvist and M. S. Sloth, *Nucl. Phys.* **B626**, 395 (2002); D. Lyth and D. Wands, *Phys. Lett. B* **524**, 5 (2002); T. Moroi and T. Takahashi, *Phys. Lett. B* **522**, 215 (2001) [Erratum-ibid. **539**, 303 (2002)].

[65] J. McDonald, Phys. Rev. D **69** (2004) 103511 [arXiv:hep-ph/0310126].

[66] G. Dvali, A. Gruzinov, and M. Zaldarriaga, *Phys. Rev. D* **69**, 023505 (2004).

[67] E. W. Kolb, A. Riotto and A. Vallinotto, Phys. Rev. D **71**, 043513 (2005); E. W. Kolb, A. Riotto and A. Vallinotto, Phys. Rev. D **73**, 023522 (2006).

[68] D. H. Lyth, JCAP **0511**, 006 (2005); D. H. Lyth and A. Riotto, Phys. Rev. Lett. **97**, 121301 (2006).

[69] D. H. Lyth and T. Moroi, JHEP **0405**, 004 (2004).

[70] G. W. Anderson, A. D. Linde and A. Riotto, Phys. Rev. Lett. **77**, 3716 (1996).

[71] J. M. Maldacena, JHEP **0305** (2003) 013.

[72] N. Bartolo, E. Komatsu, S. Matarrese and A. Riotto, Phys. Rept. **402**, 103 (2004).

[73] A. P. S. Yadav and B. D. Wandelt, Phys. Rev. Lett. **100** (2008) 181301; J. D. McEwen, M. P. Hobson, A. N. Lasenby and D. J. Mortlock, arXiv:0803.2157 [astro-ph].

[74] A. Jokinen, arXiv:hep-ph/0204086.

[75] See eg., R. Allahverdi, K. Enqvist, J. Garcia-Bellido and A. Mazumdar, Phys. Rev. Lett. **97** (2006) 191304; J. C. Bueno Sanchez, K. Dimopoulos and D. H. Lyth, JCAP **0701** (2007) 015.

[76] R. Allahverdi, A. Mazumdar, [arXiv:hep-ph/0603244].

[77] R. Allahverdi, A. Mazumdar, [arXiv:hep-ph/0608296].

[78] T. W. B. Kibble Phys. Rev. **155** (1967) 1554.

[79] H. P. Nilles, M. Peloso and L. Sorbo, JHEP **0104**, 004 (2001)

[80] R. Casadio, P. L. Iafelice and G. P. Vacca, Nucl. Phys. B **783** (2007) 1.

[81] J. H. Traschen and R. H. Brandenberger, Phys. Rev. D **42**, 2491 (1990).