

Measuring the parameters of an HNL model at FCC-ee

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ABSTRACT: Based on formalism of the recently proposed pSPSS model for heavy neutral leptons (HNLs), we study the possibility of measuring the oscillations of HNLs at the FCC-ee with a parametrised detector simulation. We further discuss strategies for discrimination of the Dirac or Majorana nature of the HNLs, and we assess how this strategy can be applied to constrain the parameters of the model in the parameter space in which the oscillation may not be observable.

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1 Introduction

The next generation of proposed e^+e^- circular colliders, such as the CERN FCC-ee [1] will provide access to a broad range of physics studies, from precision measurements of the Higgs boson and of Standard Model (SM) parameters, to direct searches for physics beyond the Standard Model (BSM).

The production of the HNL has been identified as one of the most promising new physics channels for FCC-ee at the Z pole in a seminal paper of 2014 [2]. Several different decay channels and lifetime scenarios are accessible at the FCC, yielding severe requirements on the performance of the detectors, which must be quantified through detailed studies taking into account realistic performance figures for the foreseen detectors. Detailed discussions of previous experimental studies targeting FCC-ee are contained in [3, 4].

The present study addresses the Z -pole run of the FCC-ee, at a centre of mass energy $\sqrt{s} = 91$ GeV and with target integrated luminosity $L_{int} = 2.05 \times 10^8 \text{ pb}^{-1}$, corresponding to 6×10^{12} Z bosons produced.

The sensitivity for HNL production of the FCC-ee has been recently studied in a benchmark model with a single Majorana HNL [5], and in a realistic model featuring two quasi-mass-degenerate Majorana HNLs mixing with all three lepton flavours [6].

In the case that the HNL is discovered at the FCC-ee the task of the experiments will be the investigation of the model yielding the observed signal.

We concentrate in this paper on a specific implementation of the HNL model, the "Symmetry Protected Seesaw Scenario" (pSPSS) defined in [7–9] featuring two pseudo-Dirac sterile neutrinos with a small mass splitting, which oscillate into each other as the

neutrino flies in the detector before decaying. Following the phenomenological work documented in [10, 11] we investigate the possibility of detecting the oscillations at the FCC-ee, on the basis of a parametrised simulation of the IDEA Detector.

To the purpose we study the production of HNL in Z decay through mixing with light neutrinos and its decay into μjj , where j are hadronic jets from $q\bar{q}'$ pairs coming from the charged vector boson coupling with HNL and the muon. This channel is the most copiously produced, with a total branching fraction of approximately 50% over the complete HNL mass range of interest. Moreover this channel, with a single neutrino in the final state allows the complete reconstruction of the 4-momenta of both the HNL and of the recoiling neutrino from the Z decay. With the full kinematics of the event available, variables in the rest frame of the HNL can be calculated which allow a very clean experimental determination of its decay properties.

2 Kinematics of HNL decays and HNL polarisation

As explained in [12], the observation of the spin-parity properties of the HNL at FCC-ee can be studied in two ways, either through its production in Z decays, or studying the kinematics of its decay products. A convenient reference system for collider physics is based on cylindrical coordinates oriented such that the z axis coincides with the beam direction, and the origin of the coordinate system is the centre of the detector. For the $2 \rightarrow 2$ process $e^+e^- \rightarrow \nu N$ and fixed center-of-mass energy, the kinematics an HNL of mass m_N is fully defined in the laboratory frame by its angle with respect to the beam direction θ_{vis} and by its angle in the plane transverse to the beam, ϕ_{vis} . The modulus of HNL momentum is fixed by energy and momentum conservation through the recoil formula

$$|p_{HNL}| = \frac{E_{cms}^2 - m_N^2}{2E_{cms}}. \quad (2.1)$$

With an unpolarised beam, physics is invariant for rotations around the z axis, thus all of the dynamic information for the HNL production is contained in the θ_{vis} variable.

In order to analyse the polarisation of the HNL through the orientation of its decay products, it is useful to boost the 4-momenta of the observed decay products and of the neutrino from the HNL decay in the HNL rest frame. Two variables in this frame have been found useful, the θ angle of the muon from the HNL decay, θ_μ^{HNL} , and the angular distance between the muon and the neutrino from the Z decay, $\alpha_{\mu\nu}^{HNL}$. The latter variable was proposed in studies of the HNL polarisation in W decays at the LHC [13], and used in [11] for the study of HNL oscillations. Formulas relating these variables to the HNL polarisation can be found in those papers.

3 The SPSS model and HNL oscillations

The benchmark model for the present analysis is the "Symmetry Protected Seesaw Scenario" proposed in [7–9] which adds two right-chiral neutrinos required to explain the observed light neutrino mass squared differences, provided with a protective lepton number-like symmetry

(LNLS) to ensure that the two heavy neutral leptons (HNLs) form an almost degenerate pseudo-Dirac pair. In total, to describe the collider phenomenology of a dominant pseudo-Dirac pair of HNLs, the required parameters are m_N , the average of the masses of the two neutrinos, Δm , the mass splitting between them, and θ_α ($\alpha=(1,2,3)$) the mixing with the three active neutrino flavours, and a damping factor λ which according to [10] can be put to zero for collider studies.

The main phenomenological signature of the model is the production of both lepton number violating (LNV) and lepton number conserving (LNC) final states, such that the probability of producing each final state oscillates with the propagation time τ from the production vertex according to the formula:

$$P_{\text{osc}}^{\text{LNC}}(\tau) = \frac{1 \pm \cos(\Delta m \tau)}{2}. \quad (3.1)$$

The truth-level distribution of the distance of the HNL decay vertex from the production vertex for LNV (blue) and LNC (red) decays are shown in Figure 1. The oscillation pattern is shown for different values of the neutrino decay time τ_{dec} , and of the oscillation period τ_{osc} .

The possibility of observing the oscillation in a FCC-ee detector depends on the interplay of the neutrino decay time τ_{dec} , and of the oscillation period τ_{osc} . This works for values of $\tau_{\text{dec}} \sim \tau_{\text{osc}}$, and the oscillation pattern is shown in Figure 1 for different combinations of τ_{dec} and τ_{osc} , and an assumed value of m_N of 20 GeV.

4 Signal and background simulation

The signal samples were generated with MG5aMC@NLO [14], based on the pSPSS package [9]. The authors provide a patch which must be run after generation to simulate the oscillating behaviour of the neutrino.

The associated production of a muon (anti)neutrino and the heavy neutrino N was generated, with the N directly decayed into a muon and two jets in the MG5 process card, so as to have the correct decay kinematics.

A scan was performed over the mass of the Majorana neutrino (m_{maj}) of the model between 10 and 35 GeV. The mixing angles with the SM electron and tau neutrino were set to zero, and a scan was performed on mixings with the muon neutrino for values yielding lifetimes between 1 mm and 10 cm. The generated points in the (m_N, U_μ^2) plane are shown in Figure 2, superimposed to the present limits from the CMS Collaboration at the LHC [15, 16]. Following [11] for each point four values of the mass splitting of the two pseudo-Dirac neutrinos were generated; the mass splitting values and the corresponding values of $c\tau_{\text{osc}}$ are shown in Table 1. For the oscillation study only the first three values of Δm were used, as the period for the fourth point is smaller than the expected experimental resolution on the measurement of the HNL decay path, discussed in [17]. For each sample 100k events were generated at a centre-of-mass energy of 91.2 GeV.

The LHE files are hadronised with PYTHIA8 [18] and then fed into the DELPHES [19] fast simulation of the IDEA Detector [20], based on the official datacards used for the

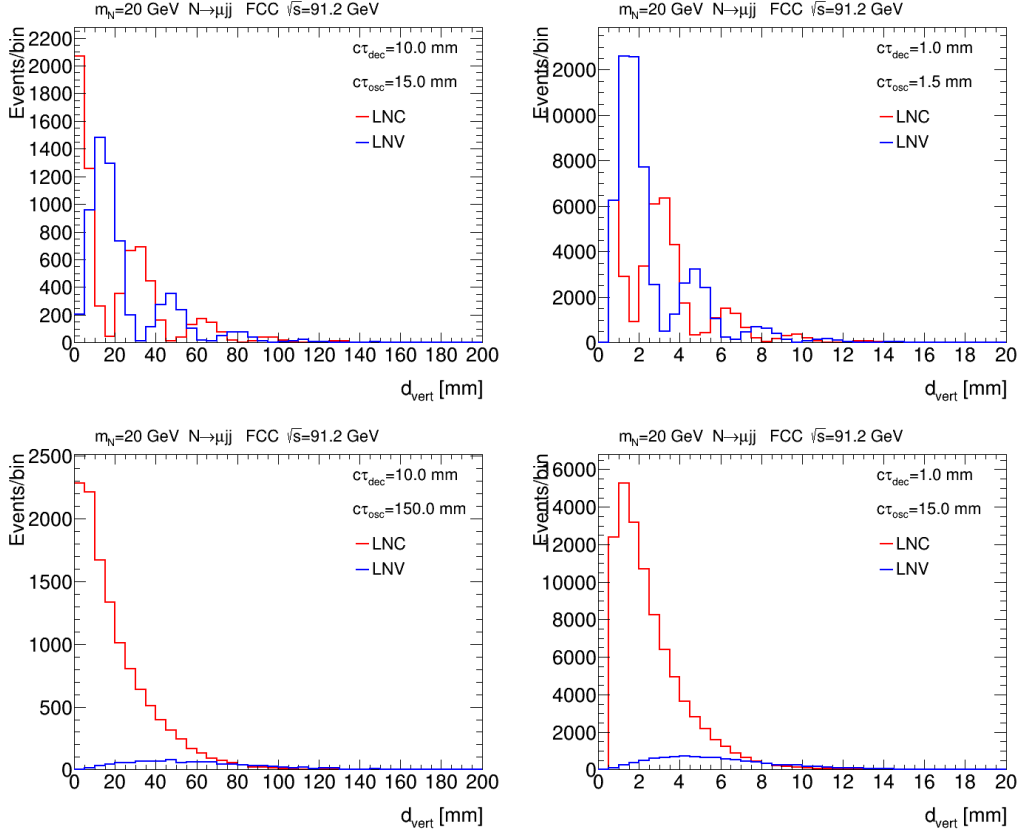


Figure 1. Distribution of the distance of the HNL decay vertex from the production vertex for LNV (blue) and LNC (red) decays. The oscillation pattern is shown for different values of the neutrino decay time τ_{dec} , and of the oscillation period τ_{osc} .

BM	$\Delta m/\mu\text{eV}$	$c\tau_{osc}/\text{mm}$
1	8.27	150
2	82.7	15.0
3	827	1.5
4	42300	0.0293

Table 1. Benchmark values of the mass difference of the two pseudo-Dirac HNL states in the pSPSS model.

"Winter2023" production of backgrounds, and the simulated events were reconstructed, resulting in final state analysis objects such as charged tracks, reconstructed electrons, muons and hadronic jets which are used for the analysis. In addition a global vertex and primary and secondary vertexes were built using the smeared reconstructed tracks with the package VertexFitterSimple, and were also made available for the analysis. This work relies on the data processing and background analysis of [5], and the main points are briefly recalled here.

For the backgrounds from Z decays, the official samples produced by the central soft-

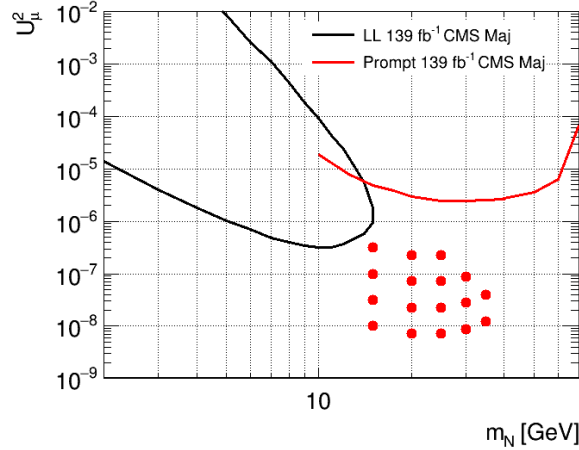


Figure 2. Position of the generated data samples in the plane defined by the HNL mass and the logarithm of the coupling squared (left). The present limits from the CMS Collaboration are also shown.

ware group for the FCC under the tag "Winter2023" were used.

The irreducible background from the four-fermion process

$$e^+e^- \rightarrow \mu\nu jj$$

was produced at LO with `MG5aMC@NLO`, including both the associated production of a real and a virtual W , and Z/γ production followed by the radiation of a virtual W off one of the decay legs of the Z . The events were then processed through the same `PYTHIA8-DELPHES` chain as the signal events.

A filtering step, is then applied needed to bring the background samples down to a manageable level, while retaining approximately 100% of the signal. It requires at least a muon with energy larger than 3 GeV, missing energy larger than 5 GeV and at least three tracks.

5 Oscillation analysis

The experimental analysis relies on the studies described in [5]. The final state of interest is isolated by selecting events with one and only one reconstructed muon, no other reconstructed lepton, at least three reconstructed tracks and missing momentum larger than 5 GeV. In order to guarantee that the vertex position is well reconstructed, the χ^2/ndf of the reconstructed vertex is required to be smaller than 10, and that at most 5 of the reconstructed tracks in the event are not used for the vertex reconstruction. As discussed in [5], this selection strongly reduces the background from the tails of $Z \rightarrow b\bar{b}$ production. The transverse position of the reconstructed vertex is required be larger of 0.5 mm. Additional minimal kinematic cuts are applied to guarantee that the surviving standard model background is negligible.

A relevant plane to understand where the oscillation may be observable is the $(\Gamma_N, \Delta m)$ plane. As discussed in [11], the integrated ratio of LNV and LNC events (R) is given by the formula:

$$R = \frac{\Delta m^2}{\Delta m^2 + 2\Gamma^2} \quad (5.1)$$

In Figure 3 the position of the generated points in the $(\Delta m, \Gamma_N)$ plane is shown, as well as the line corresponding to $R = 0.05$, i.e. where the LNV events are a factor 20 less than the LNC events. It is expected that for the points to the right of this line it will be difficult to observe the oscillation.

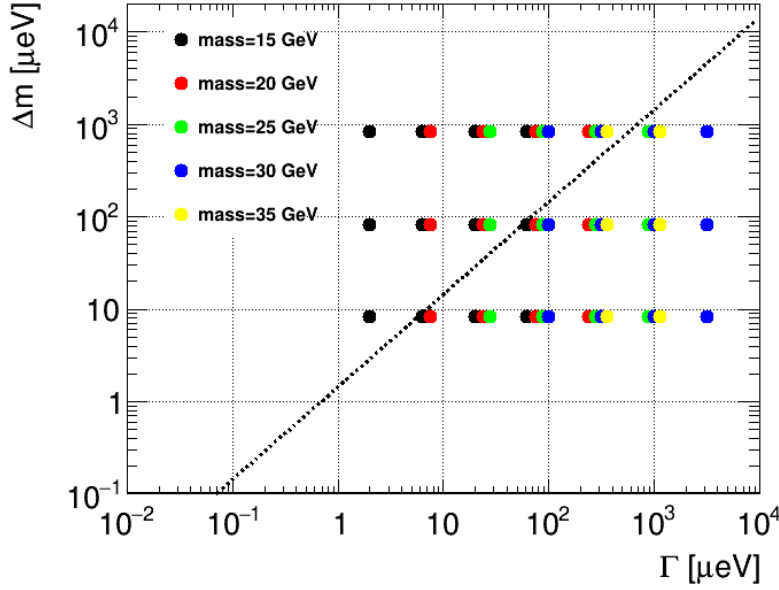


Figure 3. Position of the generated points in the $(\Gamma_N, \Delta m)$ plane. The points for the different masses are slightly displaced in x to avoid superposition. The dashed line corresponds to $R = 0.05$, where R is the ratio of LNV and LNC events.

Events are classified as LNV if the HNL recoils against a (anti) neutrino and decays into a (positive) negative muon. The nature of the neutrino from the Z decay recoiling against the HNL can not be experimentally detected, so it is not possible to experimentally separate pure samples of LNV and LNC events. It is necessary then to rely either on the small level of polarisation predicted by the SM for the Z or on the polarisation of the HNL decay.

Three variables sensitive to the flavour oscillation have been identified in [11]:

- A_{FB} : the difference of the forward-backward asymmetry of final states with a negative or positive muon as a function of d_{vert} , the distance of the reconstructed HNL decay vertex from the centre of the detector.

- The average value as a function of d_{vert} of $\cos \alpha_{\mu\nu}^{HNL}$, the angle between the muon and the reconstructed direction of the neutrino from the Z decay in the rest frame of the HNL. The distribution of this variable is shown in the left panel of Figure 4 for LNV (blue) and LNC (orange) decays.
- The average value as a function of d_{vert} of P_μ the momentum of the muon in the laboratory frame. The distribution of this variable is shown in the right panel of Figure 4 for LNV (blue) and LNC (orange) decays..

The first variable relies on the Z polarisation, and was analysed in [10]. The second and the third variables are sensitive to the polarisation of the HNL decay, and are expected to approximately encode the same information, and found in [11] to have a comparable analysing power.

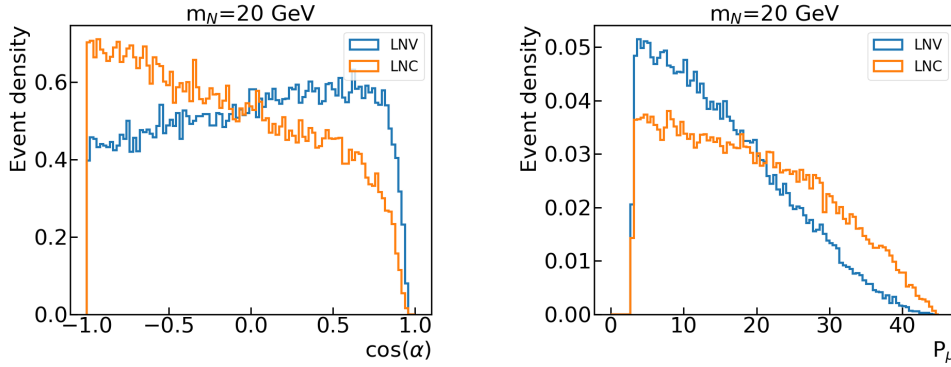


Figure 4. Left: distribution of the angle between the muon and the reconstructed direction of the neutrino from the Z decay in the rest frame of the HNL for LNV (blue) and LNC (orange) decays. Right: Distribution of P_μ , the momentum of the muon in the laboratory frame for LNV (blue) and LNC (orange) decays.

The distributions of the average value of three variables defined above as a function of d_{vert} is shown in Figures 5 and 6 for $m_N = 20$ GeV, and different values of τ_{osc} and τ_{dec} . For each value of τ_{dec} , and thence of the mixing with the SM neutrinos, the number of events expected for the total Z -pole run statistics after selections is used to build the plots. The error on the data points and the spread of the points around the periodic shape give a realistic representation of what one would observe in the real experiment.

The distributions of the three variables are fitted to a periodic shape, and the measured period is compared to τ_{osc} modified by the Lorentz dilation factor.

In Figure 5 we show the distributions for an oscillation length of 1.5 mm and two values of τ_{dec} , 1 mm (left) and 3.16 mm (right). For both $\tau_{dec} \sim \tau_{osc}$, and several oscillation periods can be fitted, yielding a precision below the percent level in the measurement of τ_{osc} . This is not the case for figure 6, where the oscillation length is 15 mm, larger than the two considered values of $c\tau_{dec}$, 1 mm (left) and 3.16 mm (right). In the case where τ_{osc} is 15 times τ_{dec} , only a fraction of the oscillation period is detectable, and no oscillation pattern

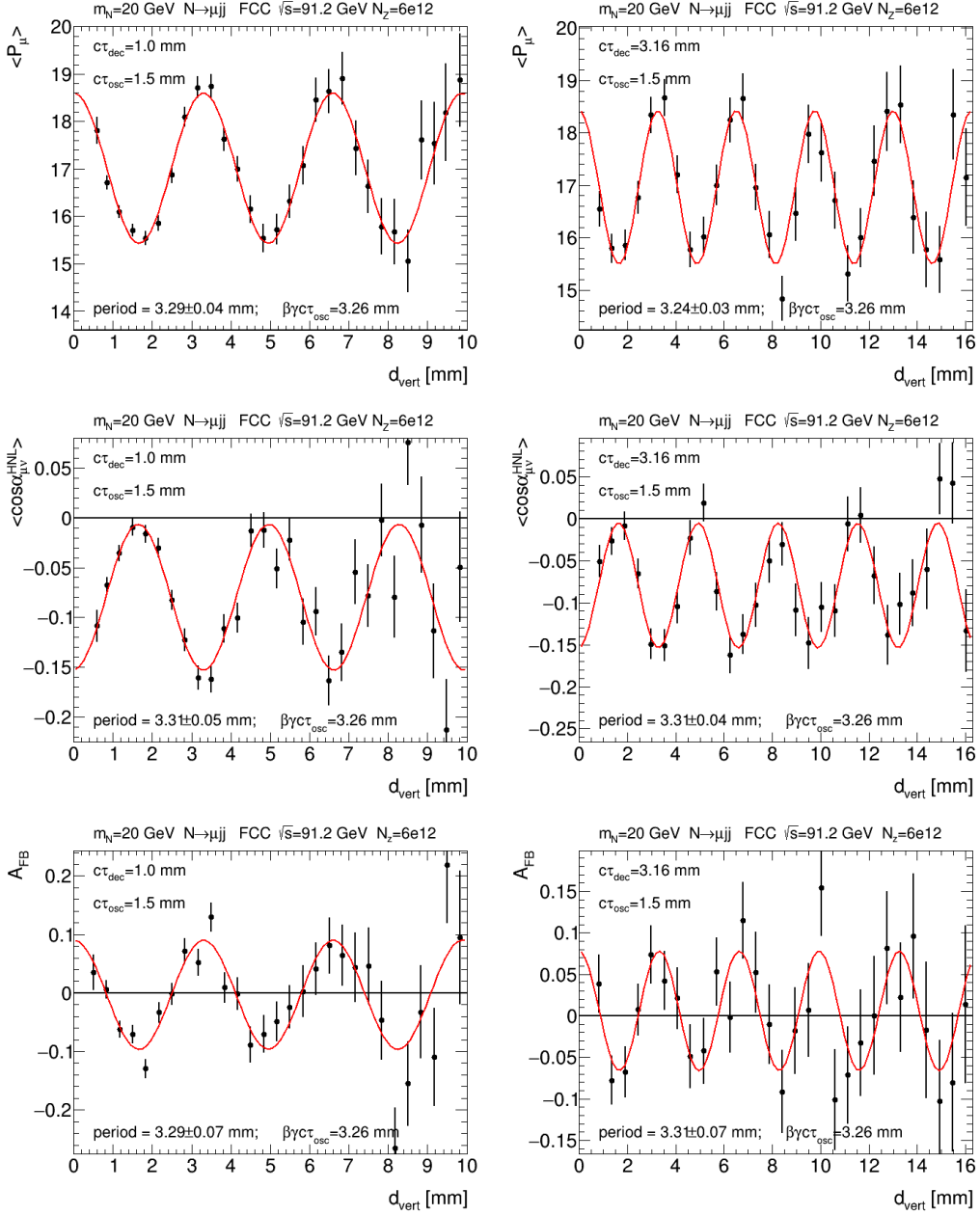


Figure 5. Distributions of the three variables sensitive to HNL oscillation as a function of the position of the reconstructed vertex for $m_N = 20$ GeV and an oscillation length of 1.5 mm. Top row: $\langle P_\mu \rangle$, Central row: $\langle \cos \alpha_{\mu\nu}^{HNL} \rangle$, Bottom row: A_{FB} . The distributions are shown for two different HNL decay lengths: 1 mm (left), 3.2 mm (right).

is observable. For the case where $\tau_{osc} \sim 3\tau_{dec}$, approximately half a period is observable, and the mass difference can still be measured with a reasonable precision.

As a conclusion, the present study, based on a detailed parametrised simulation of the IDEA tracking detector and on a realistic background analysis confirms the results of the phenomenological studies of [11]. The HNL oscillation between the two pseudo-Dirac states

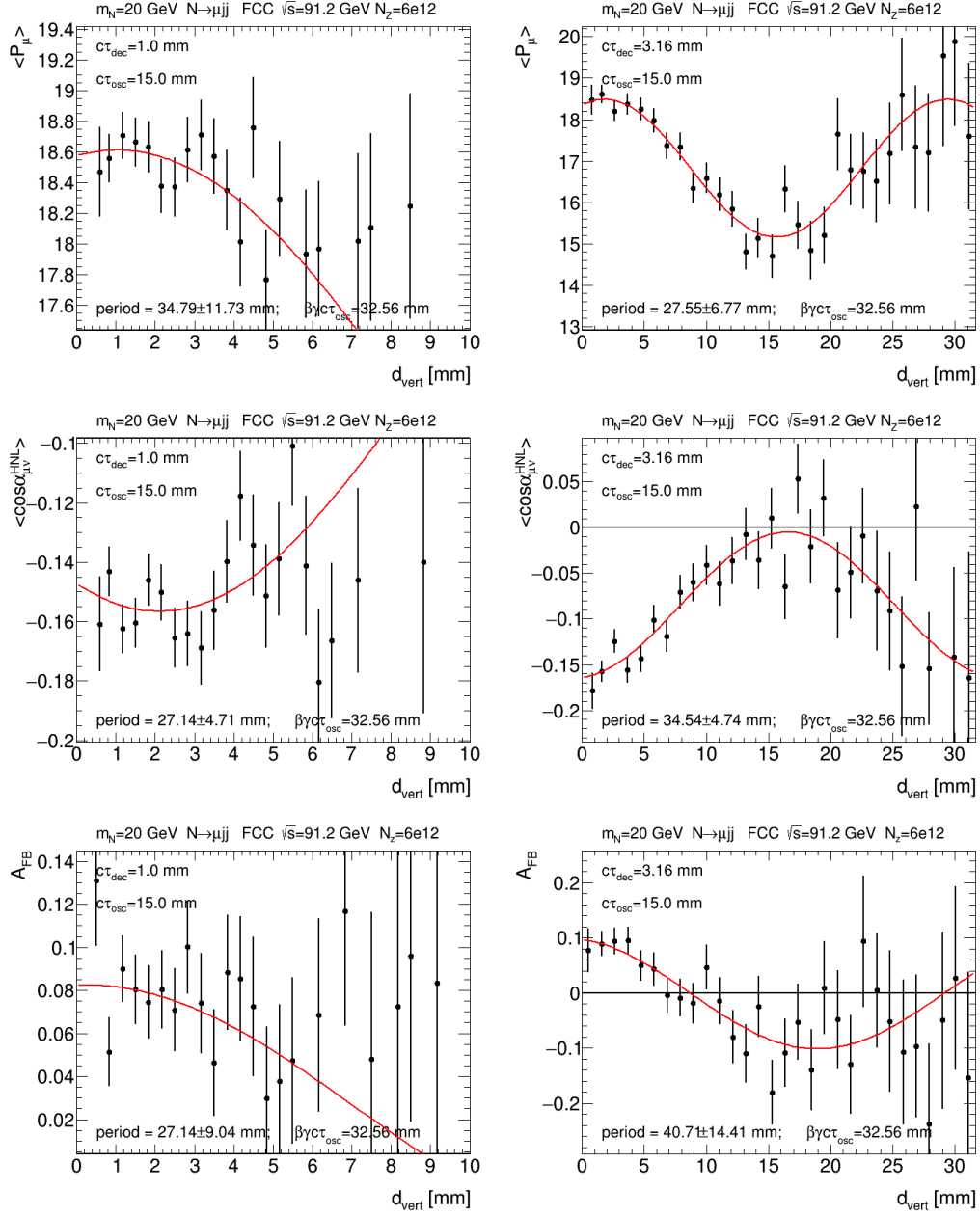


Figure 6. Distributions of the three variables sensitive to HNL oscillation as a function of the position of the reconstructed vertex for $m_N = 20$ GeV and an oscillation length of 15 mm. Top row: $\langle P_\mu \rangle$, Central row: $\langle \cos \alpha_{\mu\nu}^{HNL} \rangle$, Bottom row: A_{FB} . The distributions are shown for two different HNL decay lengths: 1 mm (left), 3.2 mm (right).

of the pSPSS model will be observable at the FCC-ee for a favourable range of parameters not yet excluded by present searches, allowing the measurement of the mass difference of the two states.

6 Discriminating the pure Dirac and double Majorana limits

As discussed in the previous sections, for an HNL mass range below 35 GeV, and favourable values of the parameters τ_{dec} and τ_{osc} , respectively determined by the HNL width and by the mass difference of the two states, it will be possible to observe the oscillation between the two HNL states of the pSPSS model. It is interesting to consider two extreme cases, where the oscillation may not be directly observable, i.e. the cases where $\tau_{dec} \ll \tau_{osc}$, and where $\tau_{dec} \gg \tau_{osc}$. As pointed out in [11], in the first case the HNL decays before it has had the time to oscillate. In this limit the processes are almost always LNC, and the two HNLs behave as a pure Dirac HNL. In the second case, the fraction of LNV decays is maximized to 50%, and the pseudo-Dirac HNLs behave as two non-interfering Majorana states.

If the Dirac case can be distinguished from the double-Majorana case, constraints can be obtained on the parameters of the pSPSS model.

This issue is the subject of a large literature see e.g. [21] and citations therein, mostly addressed by comparing a toy model with a single Dirac doublet with a model with a single Majorana HNL.

We approach the problem by developing a strategy allowing us to distinguish a single Dirac doublet from a pair of Majorana HNLs close enough in mass as to be experimentally indistinguishable. For this study samples of 100k events for the two models, are produced for different masses and submillimeter lifetime, and passed through the simulation chain described in Section 4. No attempt is done at this stage to apply selection cuts to the signal, except the initial filter, and the statistical errors on the figures only correspond to the generated Monte Carlo statistics.

We will then study the behaviour of the discriminant variables on a $(\Delta m, \Gamma_{HNL})$ grid of the realistic pSPSS model, based on the Monte Carlo samples already used for the oscillation studies, in order to verify whether they can be used to constrain the model parameters.

6.1 Dirac/Majorana discrimination on toy models

The two main handles which can be used for the Dirac/Majorana discrimination are [12]:

- the angular asymmetry in the HNL production process, resulting from the residual polarisation in Z production;
- kinematic distributions of the decay products of the HNL acting as analysers of its polarisation.

For the HNL decaying into a muon and two jets the HNL can be fully reconstructed, and angular variables both in the laboratory frame and in the HNL rest frame can be used to study the nature of the neutrino. Three different kinematic variables were found useful for the Dirac-Majorana separation:

- The forward-backward asymmetry distribution of θ_{vis} , the angle of the HNL with respect to the beam. As shown in the upper left panel of Figure 7, if the final state features a negative (positive) muon, because of the residual EWK Z polarisation, the HNL is preferentially produced in the forward (backward) direction with respect to

the flight direction of the beam electron for the LNC production and decay of a Dirac HNL. This is not the case for two Majorana HNLs which exhibit the same $\cos\theta_{vis}$ distribution independently of the charge of the muon from the HNL decay. This property can be exploited by building the asymmetry variable

$$A(\cos\theta_{vis}) = \frac{N_{\mu^-}(\cos\theta_{vis}) - N_{\mu^+}(\cos\theta_{vis})}{N_{\mu^-}(\cos\theta_{vis}) + N_{\mu^+}(\cos\theta_{vis})}$$

The distributions of $A(\cos\theta_{vis})$ are shown in the lower left (right) panel of Figure 7 for Dirac (Majorana) HNLs, exhibiting a clear slope for the Dirac case, whereas the Majorana is flat.

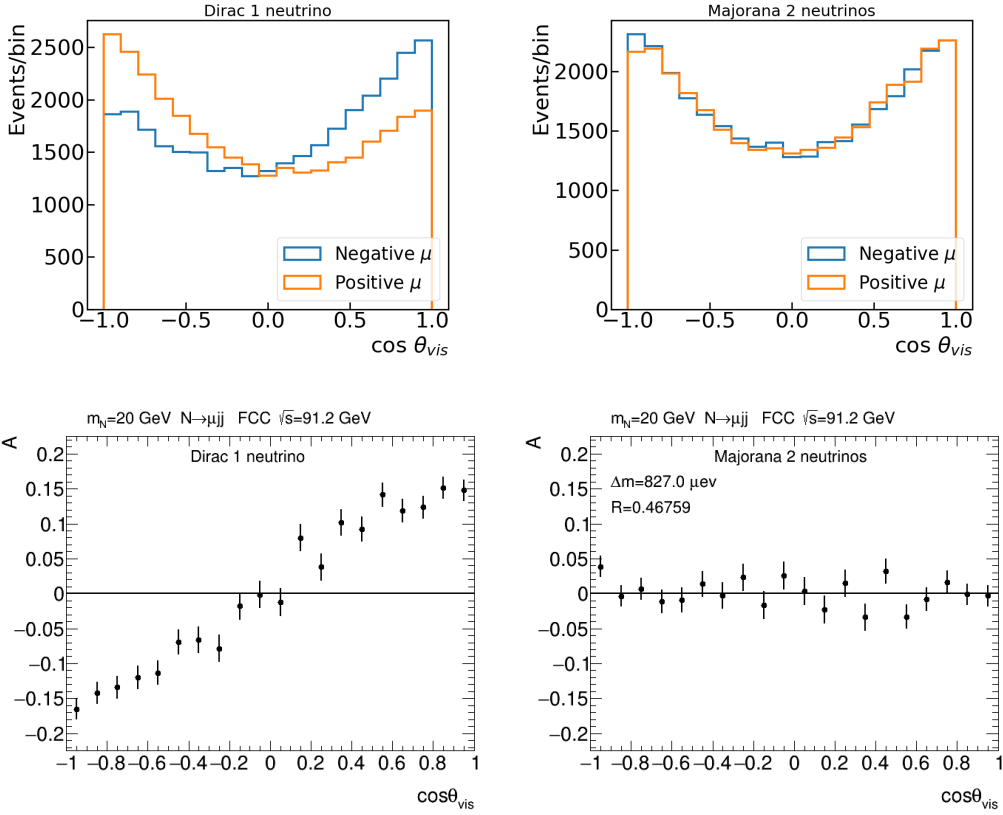


Figure 7. Top row: distribution of the variable $\cos\theta_{vis}$ for the Dirac (left) and Majorana (right) toy models respectively. The blue (orange) line corresponds to events containing a negative (positive) muon. Bottom row: value of $(N_{\mu^-} - N_{\mu^+})/(N_{\mu^-} + N_{\mu^+})$ for each bin in $\cos\theta_{vis}$ for the Dirac (left) and Majorana (right) toy models respectively. The error bars on the points correspond to the Monte Carlo statistics.

- The variable $\cos\alpha_{\mu\nu}^{HNL}$, as shown in Figure 4, for LNC HNL decays has a distribution with an downward slope, whereas for LNV violating decays it has an upward slope. Dirac HNLs only have LNC decays, whereas Majorana HNLs have the same number of LNV and LNC decays, resulting in an approximately flat distribution. This is

illustrated in Figure 8, where the distribution for $\cos \alpha_{\mu\nu}^{HNL}$ is shown on the left for Dirac HNLs and on the right for Majorana HNLs.

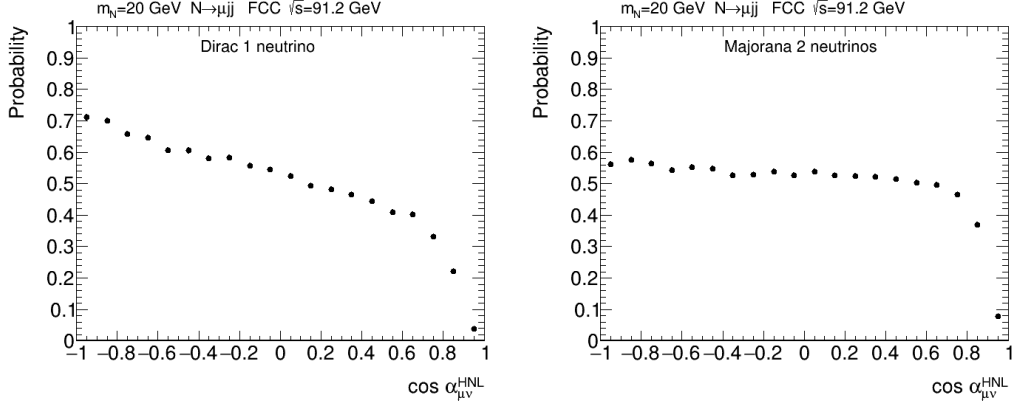


Figure 8. Distribution of the probability for the variable $\cos \alpha_{\mu\nu}^{HNL}$ for the Dirac (left) and Majorana (right) toy models respectively. The error bars on the points correspond to the Monte Carlo statistics.

- Another approach for the study of the HNL decay polarisation consists in studying the correlation of θ_{vis} , which, as seen above analyses the production polarisation of the HNL, and θ_{μ}^{HNL} which is sensitive to the decay polarisation. As shown in the upper panels of Figure 9, for a HNL produced in the forward (backward) direction θ_{μ}^{HNL} is preferentially positive (negative) for a Dirac HNL, whereas for double Majorana there is no correlation between the two angles. This property can be exploited by calculating the θ_{vis} asymmetry as a function of $\cos \theta_{\mu}^{HNL}$, or by calculating the average value of θ_{vis} in bins of $\cos \theta_{\mu}^{HNL}$. The latter approach results in an approximately linear distribution which can be fitted to a line passing through the origin, and the slope can be used to discriminate between the Dirac and Majorana case. This is shown in the lower two panels of Figure 9, where the result for the Dirac case is shown on the left and the one for the Majorana case on the right.

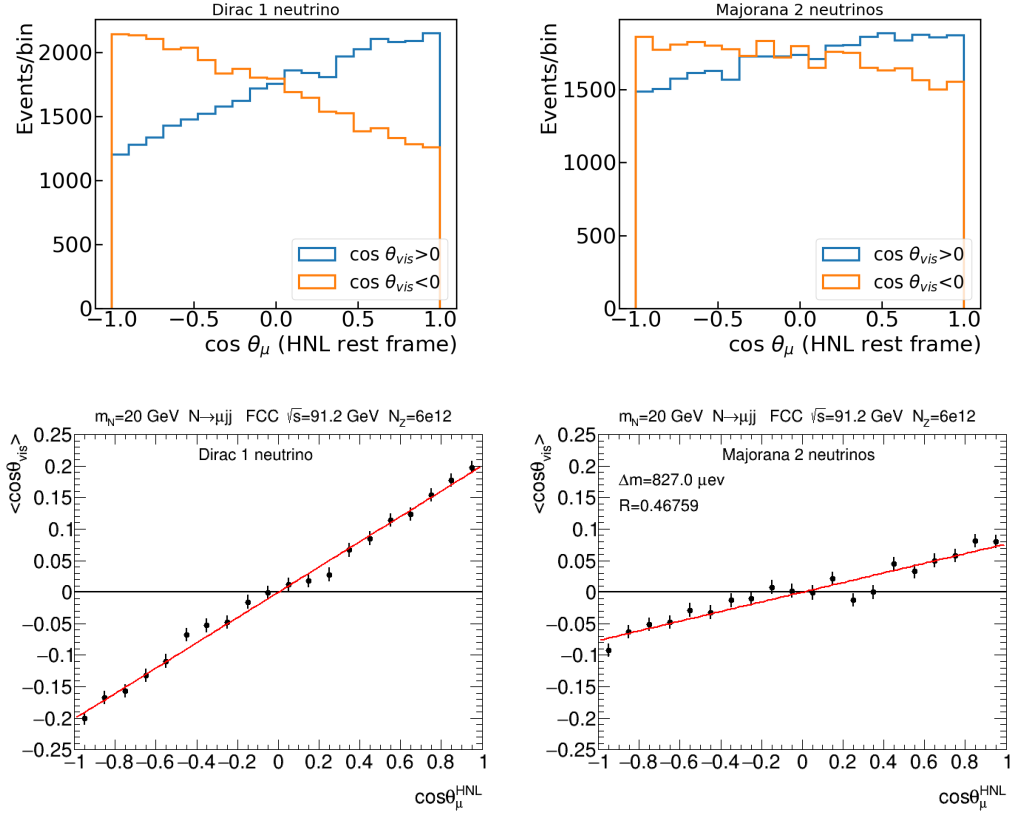


Figure 9. Top row: distribution of the variable $\cos \theta_{\mu}^{HNL}$ for the Dirac (left) and Majorana (right) toy models respectively. The blue (orange) line corresponds to events where the HNL is produced in the direction of the beam electron (positron). Bottom row: average value of $\cos \theta_{vis}$ for each bin in $\cos \theta_{\mu}^{HNL}$ for the Dirac (left) and Majorana (right) toy models respectively.

6.2 The Dirac/double Majorana limit in the pSPSS model

After having proposed three different variables sensitive to Dirac-Majorana separation on simplified toy model, we move to the study of the distributions of these variables as a function of the HNL decay width, and of the mass difference between the two pseudo-Dirac states of the pSPSS model.

In this case for each of the variables we compare the distribution of the four different values of Δm shown in Table 1 for $m_N = 20$ GeV and a width of $197.3 \mu\text{ev}$. The results are shown in Figures 10-12.

For all three considered discriminants it is found that it is possible to observe the difference between the case where the HNL behaves as a single Dirac doublet and the case where it behaves as two independent Majorana state. More work is needed to assess for each benchmark HNL mass the achievable bounds on Δm as a function of the measured value of the HNL decay width.

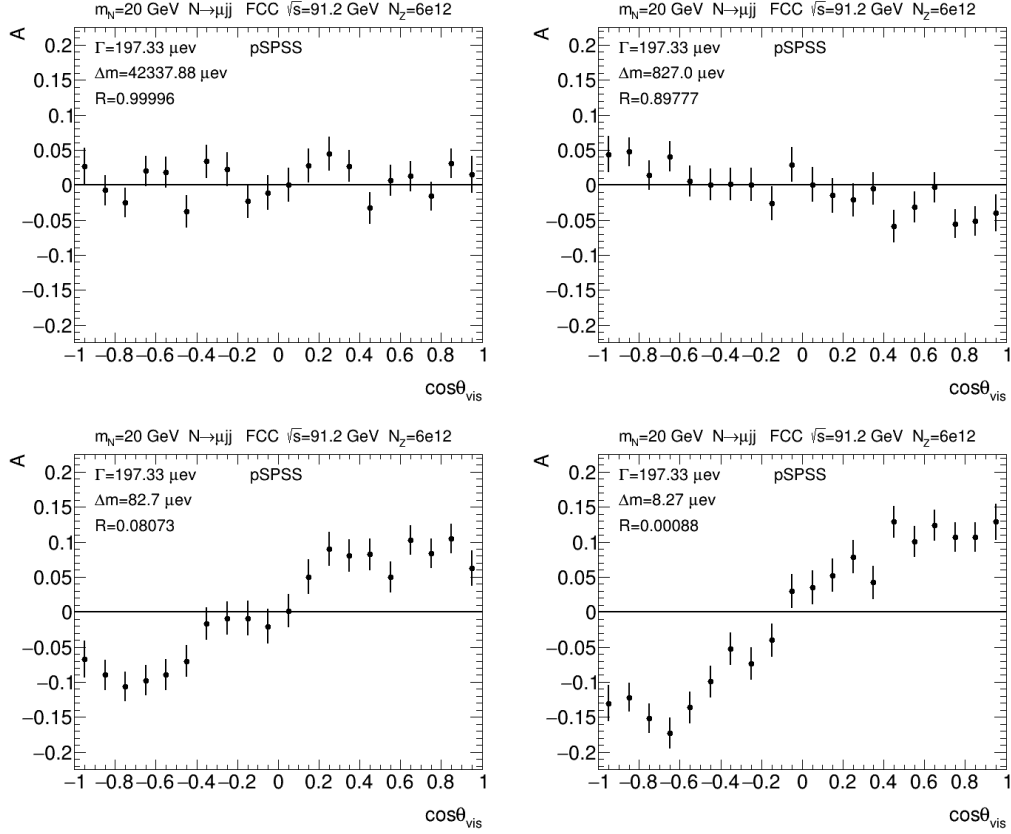


Figure 10. Distribution of the variable $A(\cos \theta_{vis})$ for $\Gamma_{HNL} = 197.3 \mu\text{ev}$, the four benchmark values of Δm , and $m_N = 20 \text{ GeV}$. The error bars on the points correspond to the expected statistical errors for the FCC-ee Z-pole run.

7 Conclusions

We have studied the possibility of detecting HNLs oscillations in the framework of the pSPSS model, featuring two pseudo-Dirac HNL states. The work assumes a statistics of 6×10^{12} Z bosons, and is based on a parametrised simulation of the proposed IDEA detector. The study is performed for the three variables proposed in [11].

In all cases it is found that for HNL masses between 10 and 35 GeV, and values of the HNL mixing yielding an adequate number of produced events, the oscillations can be clearly seen, and the mass different between the two states measures, if the oscillation period is larger than the experimental precision on the HNL decay path in the detector, and of the same order of the mean decay path of the HNL in the detector.

A further study addresses the possibility of constraining the parameter space of the pSPSS, independently from the observation of oscillation. This is based on the fact that in the pSPSS model for extreme values of the ratio between Δm and Γ_{HNL} the HNL behaves as a Dirac doublet, or as two Majorana states. The problem thus reduces to the classical one of the experimental discrimination of Dirac and Majorana HNLs. Based on the study of a Dirac and Majorana toy model, three variables with good discriminating power are

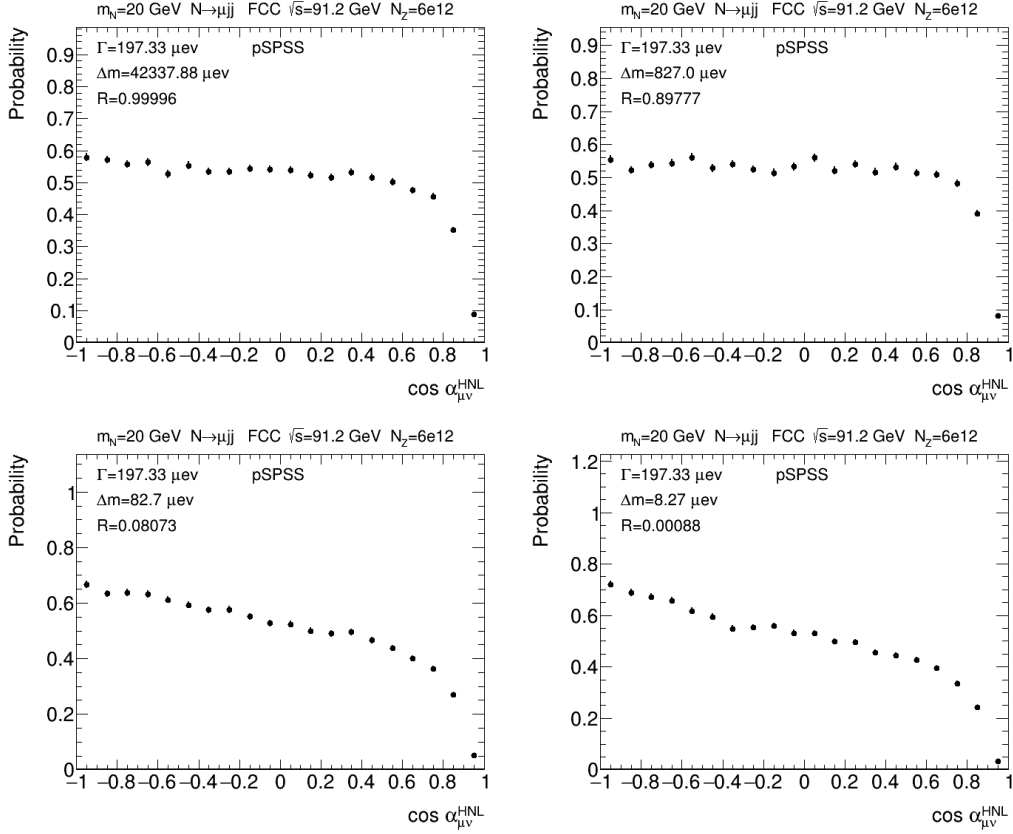


Figure 11. Distribution of $\cos \alpha_{\mu\nu}^{HNL}$ for $\Gamma_{HNL} = 197.3 \mu\text{ev}$, the four benchmark values of Δm , and $m_N = 20 \text{ GeV}$. The error bars on the points correspond to the expected statistical errors for the FCC-ee Z-pole run.

defined, based on a study of the angular orientation of the final state particles, both in the lab frame and in the HNL rest frame.

These variables are applied to benchmark models in the pSPSS parameter space, and the expected dependence from Δm for fixed Γ_{HNL} is found to be observable, paving the way for more detailed studies of the parameters of the HNL model.

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References

- [1] A. Abada et al. FCC-ee: The Lepton Collider: Future Circular Collider Conceptual Design Report Volume 2. *Eur. Phys. J. ST*, 228(2):261–623, 2019.

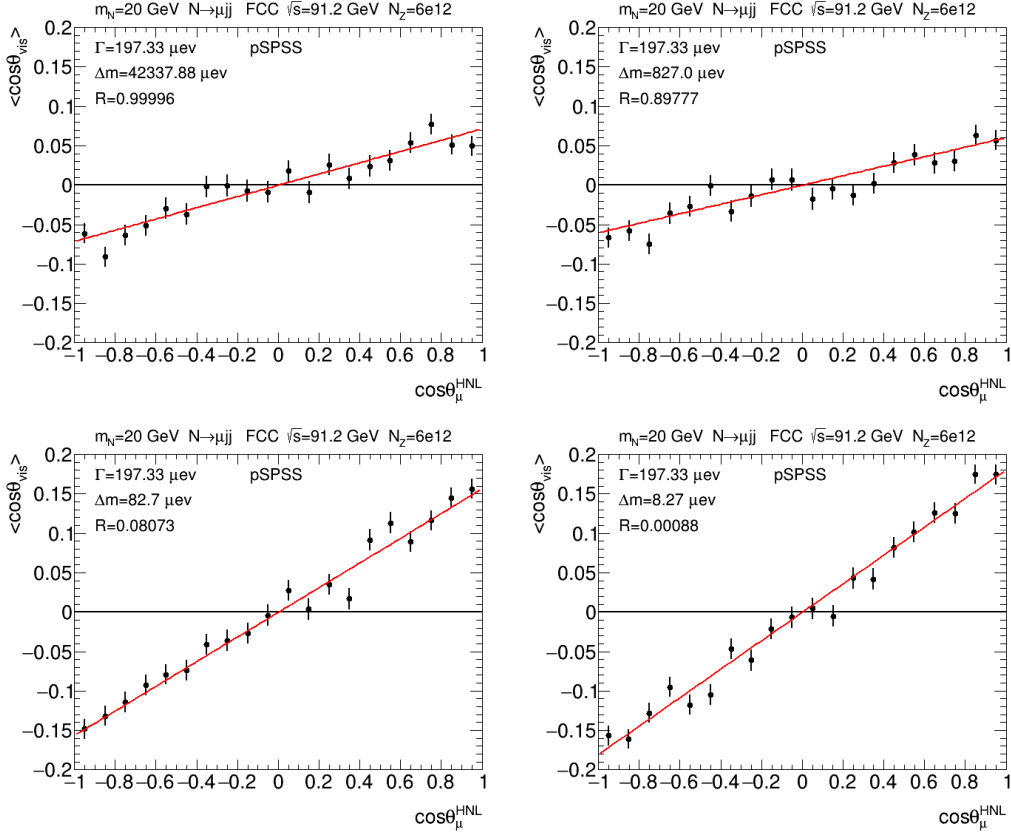


Figure 12. Distribution of the average value of $\cos \theta_{vis}$ in bins of $\cos \theta_{\mu}^{HNL}$ for $\Gamma_{HNL} = 197.3 \mu\text{ev}$, the four benchmark values of Δm , and $m_N = 20 \text{ GeV}$. The error bars on the points correspond to the expected statistical errors for the FCC-ee Z-pole run.

- [2] Alain Blondel, E. Graverini, N. Serra, and M. Shaposhnikov. Search for Heavy Right Handed Neutrinos at the FCC-ee. *Nucl. Part. Phys. Proc.*, 273-275:1883–1890, 2016.
- [3] A. Blondel et al. Searches for long-lived particles at the future FCC-ee. *Front. in Phys.*, 10:967881, 2022.
- [4] Asli M. Abdullahi et al. The present and future status of heavy neutral leptons. *J. Phys. G*, 50(2):020501, 2023.
- [5] L. Bellagamba, G. Polesello, and N. Valle. Searches for Heavy Neutral Leptons at FCC-ee in final states including a muon. *arXiv:2503.19464*, 3 2025.
- [6] Sehar Ajmal, Patrizia Azzi, Sofia Giappichini, Markus Klute, Orlando Panella, Matteo Presilla, and Xunwu Zuo. Searching for type I seesaw mechanism in a two Heavy Neutral Leptons scenario at FCC-ee. *arXiv:2410.03615*, 10 2024.
- [7] Stefan Antusch and Oliver Fischer. Testing sterile neutrino extensions of the standard model at future lepton colliders. *Journal of High Energy Physics*, 2015(5), May 2015.
- [8] Stefan Antusch, Eros Cazzato, and Oliver Fischer. Sterile neutrino searches at future e-e+, pp and e-p colliders. *International Journal of Modern Physics A*, 32(14):1750078, May 2017.
- [9] Stefan Antusch, Jan Hajer, and Johannes Roszkopp. Simulating lepton number violation

- induced by heavy neutrino-antineutrino oscillations at colliders. *JHEP*, 03:110, 2023.
- [10] Stefan Antusch, Jan Hajer, and Bruno M. S. Oliveira. Heavy neutrino-antineutrino oscillations at the FCC-ee. *JHEP*, 10:129, 2023.
 - [11] Stefan Antusch, Jan Hajer, and Bruno M. S. Oliveira. Discovering heavy neutrino-antineutrino oscillations at the Z-pole. *JHEP*, 11:102, 2024.
 - [12] Marco Drewes. Distinguishing Dirac and Majorana Heavy Neutrinos at Lepton Colliders. *PoS*, ICHEP2022:608, 2022.
 - [13] Tao Han, Ian Lewis, Richard Ruiz, and Zong-guo Si. Lepton Number Violation and W' Chiral Couplings at the LHC. *Phys. Rev. D*, 87(3):035011, 2013.
 - [14] J. Alwall, R. Frederix, S. Frixione, V. Hirschi, F. Maltoni, O. Mattelaer, H. S. Shao, T. Stelzer, P. Torrielli, and M. Zaro. The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations. *JHEP*, 07:079, 2014.
 - [15] Armen Tumasyan et al. Search for long-lived heavy neutral leptons with displaced vertices in proton-proton collisions at $\sqrt{s} = 13$ TeV. *JHEP*, 07:081, 2022.
 - [16] Aram Hayrapetyan et al. Search for heavy neutral leptons in final states with electrons, muons, and hadronically decaying tau leptons in proton-proton collisions at $\sqrt{s} = 13$ TeV. *JHEP*, 06:123, 2024.
 - [17] Roy Aleksan, Emmanuel Perez, Giacomo Polesello, and Nicolò Valle. Timing-based mass measurement of exotic long-lived particles at the FCC-ee. *Eur. Phys. J. C*, 85(1):14, 2025.
 - [18] Torbjörn Sjöstrand, Stefan Ask, Jesper R. Christiansen, Richard Corke, Nishita Desai, Philip Ilten, Stephen Mrenna, Stefan Prestel, Christine O. Rasmussen, and Peter Z. Skands. An introduction to PYTHIA 8.2. *Comput. Phys. Commun.*, 191:159–177, 2015.
 - [19] J. de Favereau, C. Delaere, P. Demin, A. Giammanco, V. Lemaître, A. Mertens, and M. Selvaggi. DELPHES 3, A modular framework for fast simulation of a generic collider experiment. *JHEP*, 02:057, 2014.
 - [20] M. Abbrescia et al. The IDEA detector concept for FCC-ee. *arXiv:2502.21223*, 2 2025.
 - [21] Alain Blondel, André de Gouvêa, and Boris Kayser. Z-boson decays into Majorana or Dirac heavy neutrinos. *Phys. Rev. D*, 104(5):055027, 2021.