Implementation of Charge Exchange Processes in Geant4

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Contents

1 Abstract

We implement charge exchange processes in Geant4 and introduce a new *G4ChargeExchangeXS* class. We also update the *G4ChargeExchange* and *G4ChargeExchangePhysics* classes in Geant4. It is found that the parameterized experimental cross-section curve of π^0 production in charge exchange process $\pi^- + p \to \pi^0 + n$ aligns with the Bertini Cascade Model data and SAID data. We then extend this approach to calculate the eta and neutral kaons production in the corresponding charge exchange processes. We also introduce a new fitting formula to describe the charge exchange processes in nuclei. We expect our work will be useful in studying calorimeter reports in LHC experiments and simulating the background in the NA64 experiment at CERN.

2 Keywords

Geant4, Monte Carlo, Charge exchange reactions, ATLAS, CMS, NA64

3 Introduction

3.1 Geant4 simulation toolkit

Geant4 is a toolkit for the Monte Carlo simulation of the passage of particles through matter[1]. It provides applications in various fields, including high-energy physics, astrophysics, medical physics and radiation protection.

3.2 Motivations

There are two motivations for our work.

The first motivation of our work is to provide the simulation for the NA64 experiment at CERN[2]. A part of their experiment involves the charge exchange process $\pi^-(K^-)+p \to M^0+n$, where the mesons (M^0) decaying into $\nu\bar{\nu}$ would indicate new physics as the decay rate predicted by the standard model is very small[3]. Our implementation of the charge exchange process in Geant4 would serve as the background simulation of their experiment.

We also aim to evaluate the accuracy of the calorimeter responses of LHC detectors. Implementing the charge exchange processes in Geant4 would provide an option to simulate the effect of the charge exchange processes in calorimeters.

3.3 Charge Exchange Processes

Below are the charge exchange processes we have implemented,

$$\pi^- + p \to \pi^0 + n \tag{1}$$

$$\pi^- + p \to \eta + n \tag{2}$$

$$K^- + p \to \bar{K}^0 + n \tag{3}$$

$$\pi^+ + n \to \pi^0 + p \tag{4}$$

$$\pi^+ + n \to \eta + p \tag{5}$$

$$K^+ + n \to K^0 + p \tag{6}$$

For interaction (1), (2) and (3), the cross-section of π^0 , η and $\bar{K}^0$ can be obtained from the experimental data [4–9], while due to isospin symmetry, the cross-section of π^0 , η and K^0 production in interaction (4), (5) and (6) are the same as in interaction (1), (2) and (3) respectively.

Besides, we also implement the charge exchange process of K-long (K_L^0),

$$K^0 + p \rightarrow K^+ + n \tag{7}$$

$$\bar{K^0} + n \rightarrow K^- + p \tag{8}$$

As K_L^0 is the linear combination of K^0 and $\bar{K^0}$, the cross-section of secondary particles production would be

$$\sigma_{K_L^0} = \frac{1}{2}(\sigma_{K^+} + \sigma_{K^-}) \tag{9}$$

where σ_{K^+} and σ_{K^-} refer to the cross-section of secondary particles produced in interaction (3) and (6).

4 Cross section parameterization

4.1 Cross section parameterization formula

The total cross-section formula of π^0 and η production[4, 7]in interaction (1) and (2) are

$$\sigma(s)_{\pi^0} = (122 \pm 8)(\frac{s}{s_0})^{-1.23\pm0.02} \times 10^{-30} cm^2 \tag{10}$$

$$\sigma(s)_{\eta} = (31 \pm 3)(\frac{s}{s_0})^{-1.53\pm0.03} \times 10^{-30} cm^2 \tag{11}$$

where $s_0 = 10(GeV)^2$.

For $\bar{K^0}$ production, we introduce a new fitting formula with a similar form to the equation (10) and (11). Through fitting to the experimental data[8, 9], we obtain

$$\sigma(p)_{\bar{K^0}} = (56.3)(\frac{p}{p_0})^{-1.60} \times 10^{-30} cm^2 \tag{12}$$

where $p_0 = 10(GeV/c)$.

4.2 Comparison of the cross-section parameterization formula and the data

The comparisons of the parameterized experimental cross-section curve and available published data are made in this section.

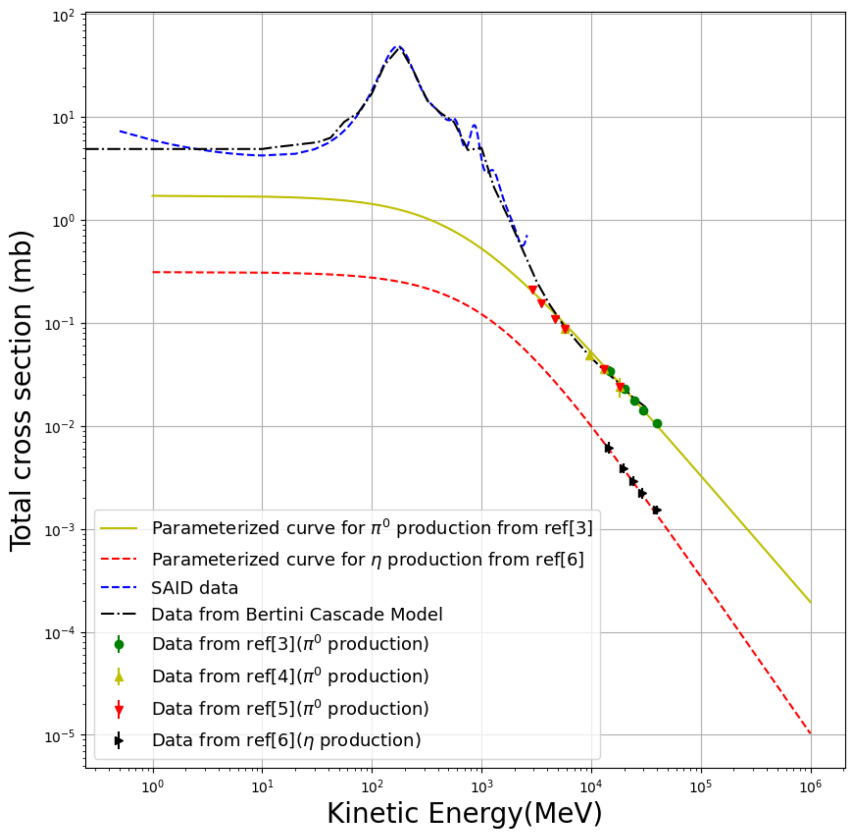

Figure 1: Plot of π^0 and η production in interaction (1) and (2)

We compare the π^0 and η cross-section parameterization formula (equation (10) and (11)) with Bertini Cascade Model data and SAID data of interaction(1) (see Fig.(1)). The Bertini Cascade data for interaction (1) agree well with the data from the SAID database. Also, the upper end of Bertini Cascade data aligns with the parameterization equation (10).

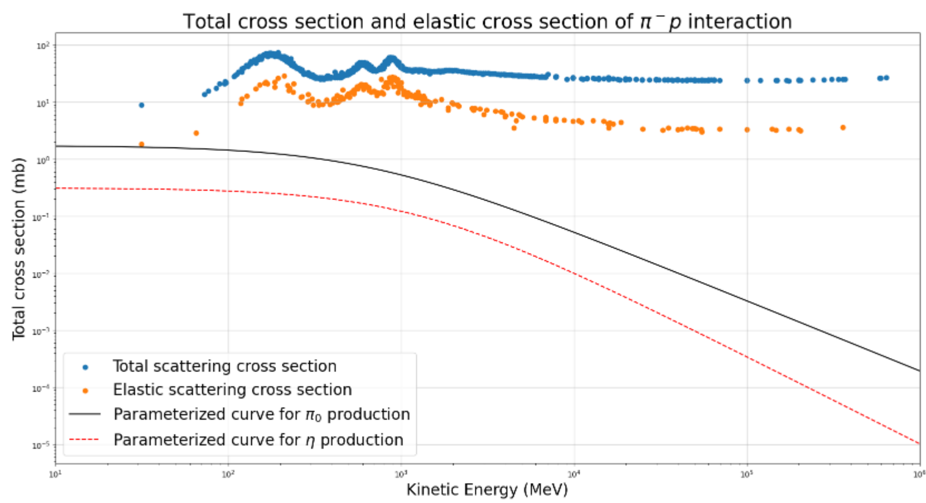

Figure 2: Plot of total and elastic cross-section data of $\pi^- p$ interaction [10] and the parameterization curves (10) and (11)

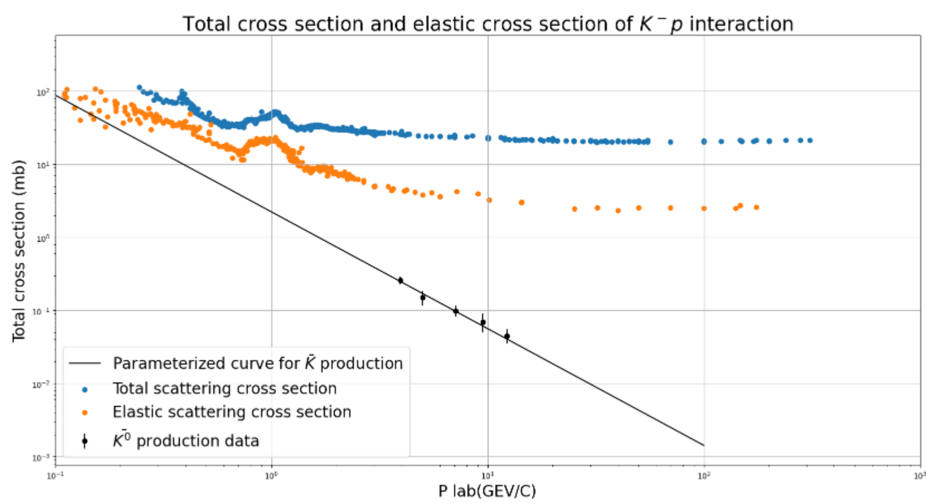

Figure 3: Plot of total and elastic cross-section data of $K^- p$ interaction [10] and the parameterization curves (12)

We also compare the π^0 and η cross-section parameterization curve (equation (10) and (11)) with total and elastic cross-section data of $\pi^- p$ [10](See Fig.(2) - (3)). We can see that the addition of the interaction (1) - (6) would be minor above 1 GeV.

5 Extension to interaction with nuclei

For the real application, it was necessary to extend the interaction interaction (1) to (6) to the interaction with nuclei. For π^- and π^+ interaction with nuclei, they would be

$$\pi^- + (A, Z) \rightarrow \pi^0 + (A, Z - 1) \tag{13}$$

$$\pi^- + (A, Z) \rightarrow \eta + (A, Z - 1) \tag{14}$$

$$\pi^+ + (A, Z) \rightarrow \pi^0 + (A, Z + 1) \tag{15}$$

$$\pi^+ + (A, Z) \rightarrow \eta + (A, Z + 1) \tag{16}$$

Where A and Z are the baryon number and atomic number, respectively.
A formula is extracted from the literature in [2],

$$\sigma(s) = \sigma(s)_{\pi^0} Z^{2/3} Z^{-ln(z)} \tag{17}$$

However, it does not apply to interactions (15) and (16). Therefore, we propose another formula. For the interaction (13), the formula would be

$$\sigma(s) = \sigma(s)_{\pi^0} Z^{2/3} A^{-\beta' \ln A} \tag{18}$$

where,

- A - atomic weight
- Z - the atomic number
- $\sigma(s)_{\pi_0}$ - total cross-section value obtained from equation (10)
- β' - the fitting parameter

The total cross section is calculated similarly for interaction (14).

Through fitting to the data (π^- beam with momentum 48 GeV/c) [11], we obtain that $\beta'_{\pi^0} = 0.04102$ and $\beta'_{\eta_0} = 0.04021$. (Both values are correct to 5 sig. fg.)

For interaction (15) and (16), the same formula calculates the total cross section by replacing $Z^{2/3}$ by $(A - Z)^{2/3}$

$$\sigma(s) = \sigma(s)(A - Z)^{2/3} A^{-\beta' \ln A} \tag{19}$$

Through comparing equation (18) and (19), we can see that the terms $A^{-\beta' \ln A}$, which accounts for the absorption of secondary particles produces in nuclei, is the same in both equations. This is why we introduce the new fitting formula since we want the absorption of secondary particles to be the same when π^- and π^+ interact with the nuclei.

To validate our new fitting, we compare equation (17), (18) and the experimental data [11, 12] in the following plot (See Fig.(4) and (5)). We can see that equation (17) and (18) have similar total cross-section values along all Z. Both of the equations align with the experimental data [11, 12], except at the regime that Z is large and total cross-section data gets a large standard deviation.

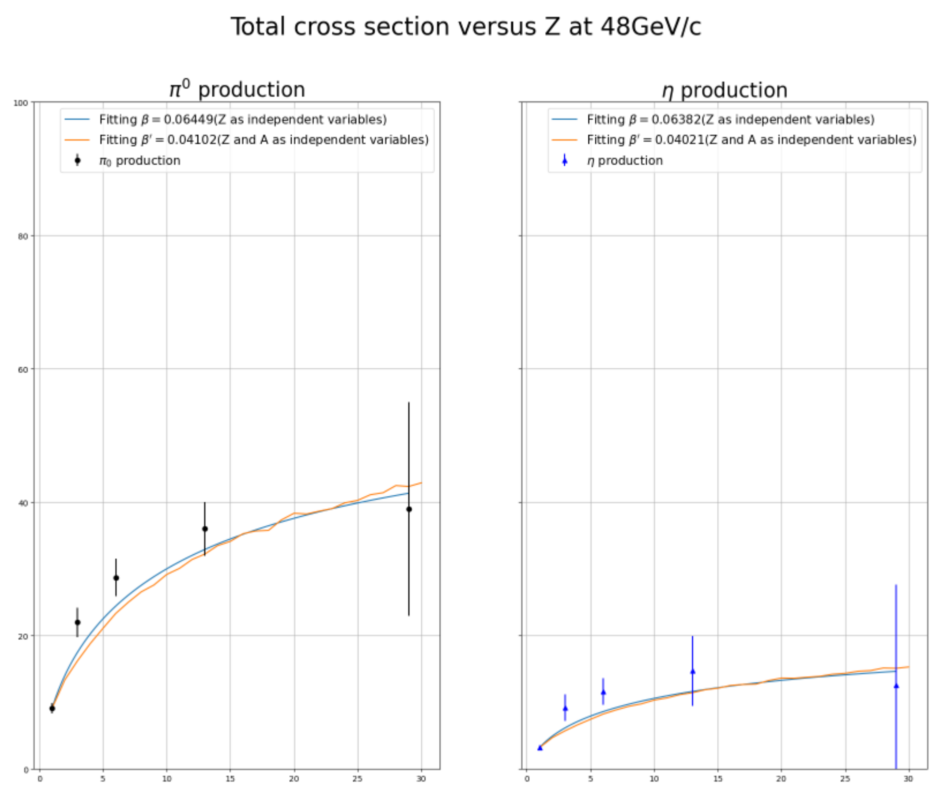

Figure 4: Plot of equation (17), (18) and the total cross-section of π^0 and η [11] at incident π^- beam with momentum 48 GeV/c

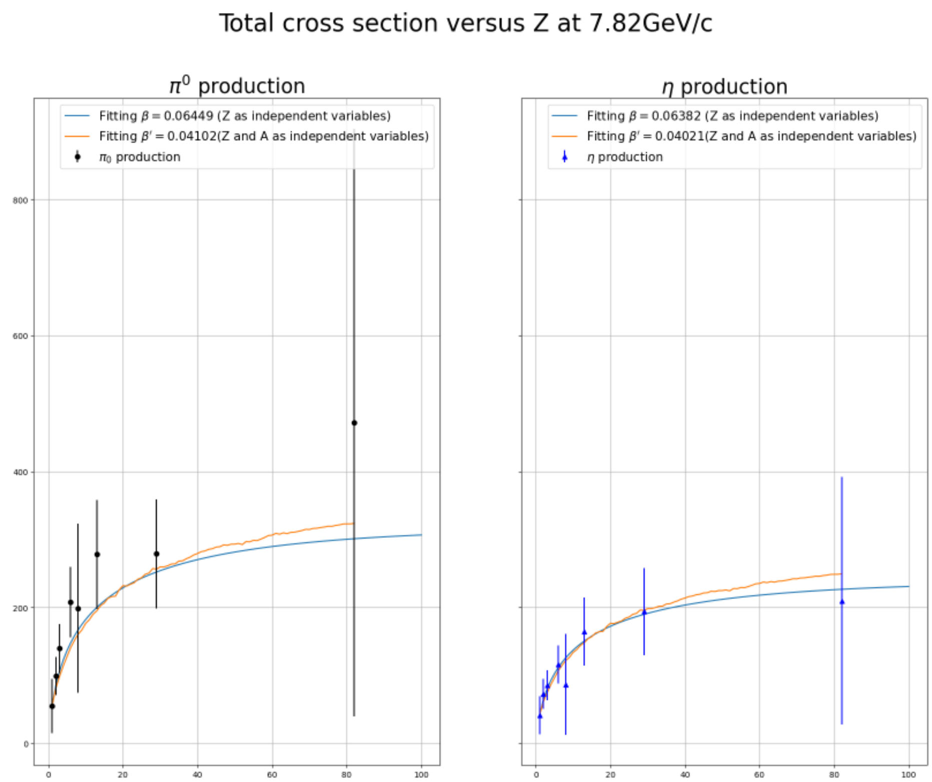

Figure 5: Plot of equation (17), (18) and the total cross-section of π^0 and η [12] at incident π^- beam with momentum 7.82 GeV/c

For interaction K^- and K^+ with nuclei,

$$K^- + (A, Z) \rightarrow \bar{K^0} + (A, Z - 1) \tag{20}$$

$$K^+ + (A, Z) \rightarrow K^0 + (A, Z + 1) \tag{21}$$

the total cross-section would be,

$$\sigma(p)_{K^+} = \sigma(p)_{\bar{K^0}}(Z)^{2/3} \tag{22}$$

$$\sigma(p)_{K^-} = \sigma(p)_{K^0}(A - Z)^{2/3} \tag{23}$$

We neglect the absorption terms in equation (22) and (23) since the absorption cross-section of the $\bar{K^0}$ are lower than that of π^0 and η.

For the cross-section of secondary particles produced by the interaction of K-long(K_L^0) and nuclei, the cross-section would be equal to $\frac{1}{2}(\sigma(p)_{K^+} + \sigma(p)_{K^-})$.

All of the equations mentioned in this section (equation (18),(19),(22),(23) and that of K_L^0) are all implemented in the Geant4 *G4ChargeExchangeXS* class. We also update the *G4ChargeExchange* and *G4ChargeExchangePhysics* classes in Geant4.

6 Conclusion

The charge exchange processes are implemented in Geant4. The parameterized experimental cross-section curve aligns with the Bertini Cascade and SAID model data for the charge exchange process (1). A new fitting formula is introduced for calculating the total cross-section of secondary particles produced in interaction (13) - (16), (20), (21) and they are now implemented in Geant4 *G4ChargeExchangeXS* class. Besides, the *G4ChargeExchange* and *G4ChargeExchangePhysics* classes in Geant4 are updated to implement the charge exchange process. We expect the *G4ChargeExchangeXS* class could be used to study calorimeter reports in ATLAS and CMS experiments and simulate the background in the NA64 experiment. Besides, η' production in charge exchange process might also be added in a similar approach.

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