Optics measurement
Comparison between two algorithms

Written by: Hamda CHERIF

Supervisors:

Andy Langer
Rogelio Tomas

Contents

I. Fundamental concepts in accelerator physics:

1. Equation of motion:

The motion of particles in a storage ring is described by Hill's equation [1]:

$$\frac{d^2u}{ds^2} + k(s)u = 0$$

Where the variable u represents the transverse coordinates x and y. The function k(s) is periodic with the circumference C of the storage ring (k(s+C) =k(s)).

2. Twiss functions:

A solution which satisfies the Hill's equation is of the form [1]:

$$u(s) = a\sqrt{\beta(s)}\, cos[\mu(s) + b]$$

Where β and μ are periodic functions with periodicity C, while a and b are constants depending on initial conditions. The function β is called *beta function* (or amplitude function) and μ is called *phase function* or betatron phase which is related to the beta function by:

$$\mu(s_2) - \mu(s_1) = \int_{s_1}^{s_2} \frac{1}{\beta(s)} ds$$

The integral around the ring ($s_2 = s_1 + C$) is 2πQ, where Q is the betatron tune (number of betatron oscillations per revolution.

It's useful to define also the *alpha function* as the derivative:

$$\alpha(s) = -\frac{1}{2}\frac{d\beta(s)}{ds}$$

The beta function, the phase function and the alpha function are called the *Twiss function.*

The Twiss function describes the dynamics of the large number of particle which form the beam. We can see that the beta function is directly proportional to the beam size σ:

$$\sqrt{\beta\varepsilon} = \sigma$$

Where ε is a constant called *emittance*.

3. Beta function measurement at Beam Position Monitors BPMs:

a) Transport matrix:

A particle trajectory can be described in terms of a matrix in the phase space. Thus, the position of a particle at the longitudinal position s2 can be calculated as a function of its position at s1 using a transport matrix:

$$\begin{pmatrix} u_2 \\ u'_2 \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} u_1 \\ u'_1 \end{pmatrix}$$

The transport matrix can be expressed with Twiss functions. Then we have the relation between matrix elements of the first row and the second column as [2]:

$$\frac{m_{11}}{m_{12}} = \frac{1}{\beta_1}(cot\phi_{12} + \alpha_1)$$

$$\frac{m_{22}}{m_{12}} = \frac{1}{\beta_2}(cot\phi_{12} + \alpha_2)$$

Where $\beta_1 = \beta(s1), \beta_2 = \beta(s2)$, $\alpha_1 = \alpha(s1), \alpha_2 = \alpha(s2)$ and ϕ_{12} is the phase difference between s1 and s2: $\phi_{12} = \mu(s2) - \mu(s1)$.

Then we have [2]:

$$tan\phi_{12} = \frac{m_{12}}{m_{11}\beta_1 - m_{12}\alpha_1} = \frac{m_{12}}{m_{22}\beta_2 - m_{12}\alpha_2}$$

These expressions show the relationship between the Twiss parameters α and β at s1 and the phase difference ϕ_{12} between s1 and s2.

b) Beta function calculated from phase:

The beta function is calculated from the phase advances between at least three consecutive BPMs. We use an algorithm goes step by step through all available BPMs than for every BPM position beta function are calculated three times and averaged.

Figure 1 show a graphical representation for the phases of the 3-BPM method [2]:

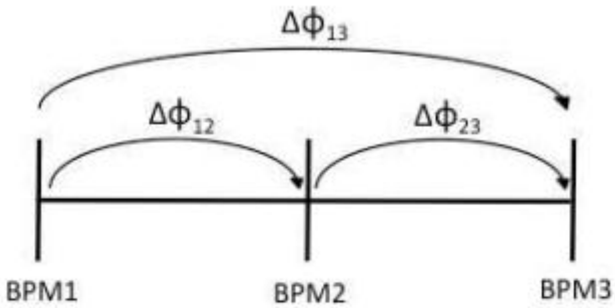


Figure 1 Graphical illustration of phases for the 3 BPM method

In this paper we will compare results between two algorithms, the first one using 3 BPMs set and the second using 15 combinations of BPM sets.

Figure 2 show a graphical presentation for the two algorithms [3]:

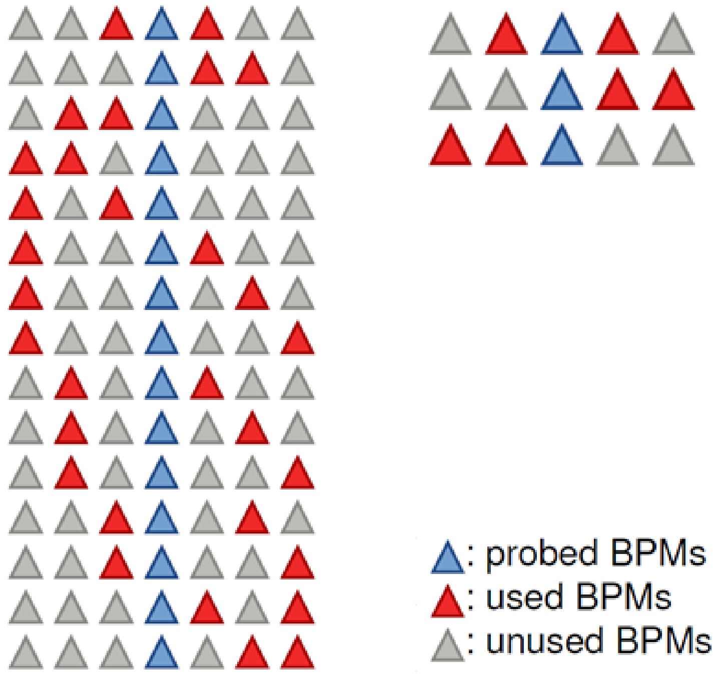


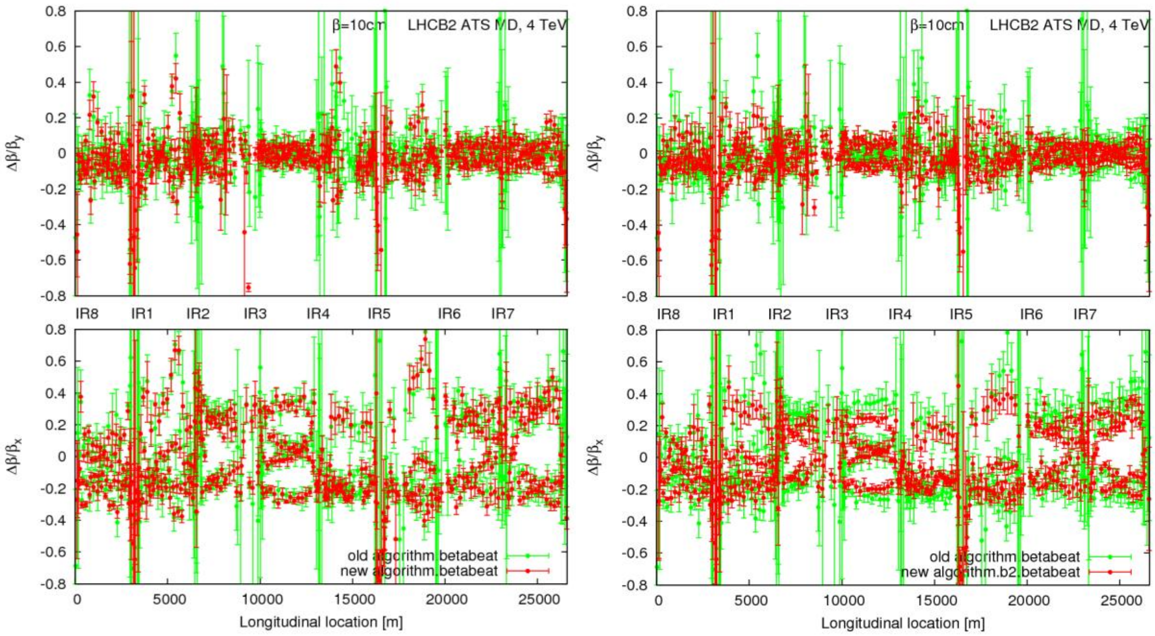
Figure 2 : Comparison between the old algorithm and the new on the left: the new algorithm,on the top right: the old algorithm

II. Results and comments:

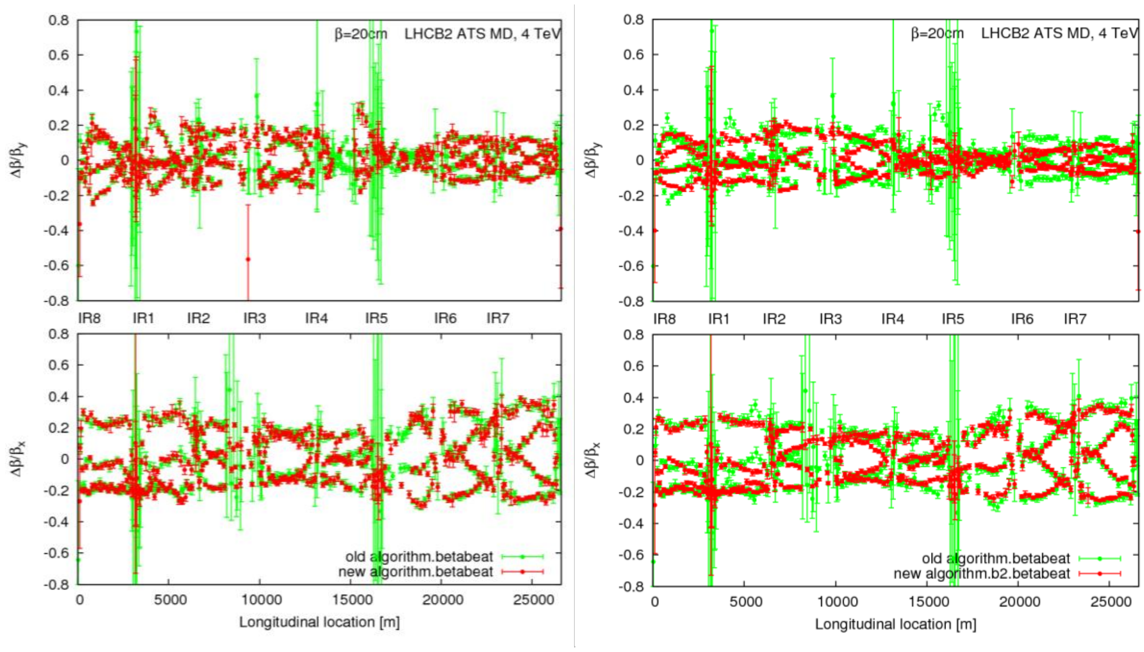
1) Comparison between β_beat for different values of β:

In this part we will show the different plots of the beta beat with different values of beta function, and we will compare in each case between the old and the new algorithm. We include also in the algorithm dipole b2 error and we compare between the two results.

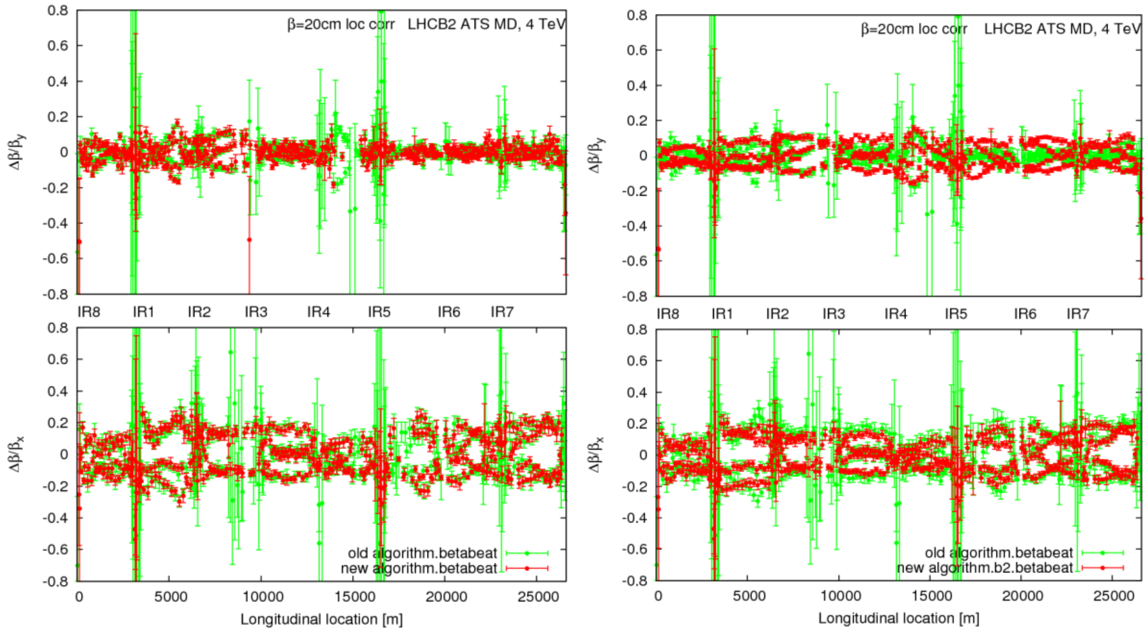
a. β = 10cm:



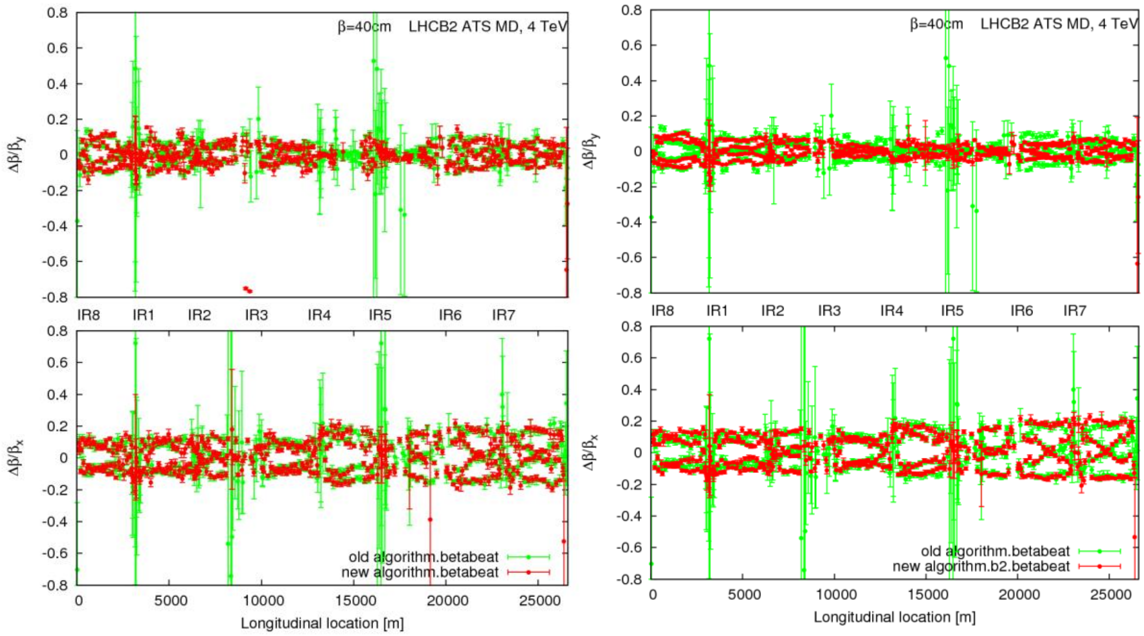
b. β = 20cm:



c. β = 20cm with local correction:



d. β = 40cm:



Figures above show the difference in the β-beat, for both planes and both beams between the new algorithm and the old algorithm (figures on the left) and between the old algorithm and the new algorithm including dipole b2 error (figures on the right).

We note that the new algorithm is more accurate than the old and the graphics of beta beat are more compact and précis.

2) Errors bars of measured betas:

In this part we show histograms with the relative error and we compare between the old algorithm and the new algorithm.

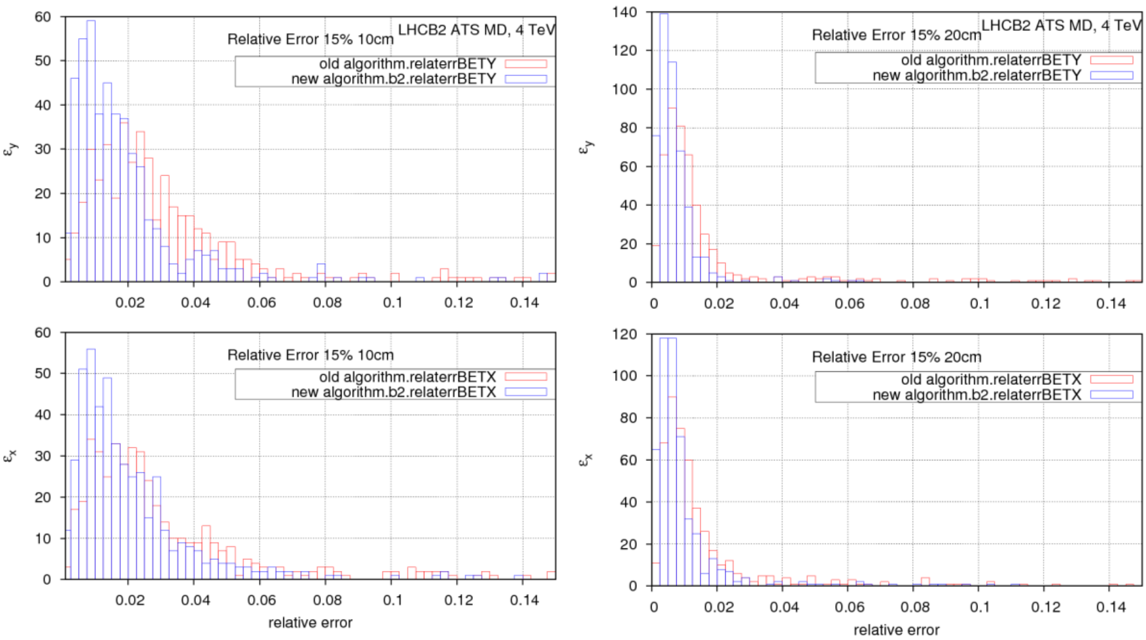


Figure 3 relative error for Beta=10cm (on the left) and Beta=20cm (on the right), comparison between the old algorithm and the new algorithm for error < 15%

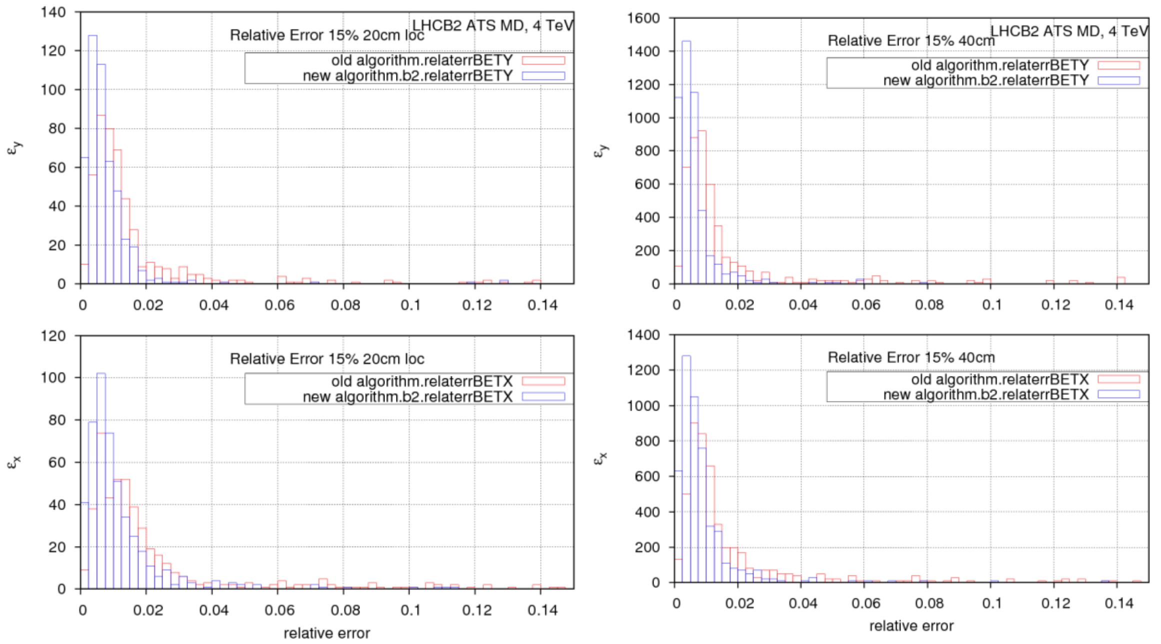


Figure 4: relative error for Beta=20cm with local correction (on the left) and Beta=40cm (on the right), comparison between the old algorithm and the new algorithm for error < 15%

We note that the b2 dipole errors increase precision of the measurement, and then the new algorithm is more accurate.

III. Conclusion and outlook:

In this short report, we showed the method followed to measure the beta function and we also presented the figures of β-beat to compare the new algorithm and the old one with different values of beta.

We thus find that:

- ✓ the new algorithm is more accurate than the old and the graphics of beta beat are more compact and précis
- ✓ the b2 dipole errors increase precision of the measurement

This paper could be more interesting in studying the luminosity with the new energy of the LHC beam,7 TeV.

IV. References:

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[3] A. Langner, R. Tomas "Improvements in optics measurement resolution and error reconstruction" ,LHC Optics Measurement and Corrections review, 18.06.13