Simulation of Particle Trajectories in the Mu2e Detector Solenoid

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This document summarizes my work simulating the magnetic field of the detector solenoid, and the trajectories of charged particles through the subsequent magnetic field on the Mu2e experiment through the 2021 CERN summer student program.

I. Introduction to Mu2e

The Muon to Electron (Mu2e) experiment at Fermilab was created with the goal of detecting the conversion of muons into electrons in the field of a nucleus. This is an example of a charged lepton flavor violation. This process is predicted under new physics models but was previously forbidden under the standard model. Mu2e is designed to detect conversion electrons with momentum near 105 MeV while rejecting backgrounds from cosmic rays and other conventional processes predicted by the standard model. Mu2e searches for the conversion of muons into electrons by measuring the ratio of rate of the neutrinoless, coherent conversion of muons into electrons in the field of a nucleus relative to the rate of ordinary muon capture on the nucleus; this value is called $R_{\mu e}$. The single-event sensitivity of Mu2e is $R_{\mu e} < 2.87 \times 10^{-17}$, four orders of magnitude greater than that of the SINDRUM II experiment, which previously had the best muon-to-electron conversion limit. [1]

a. The Mu2e Apparatus

The Mu2e apparatus is made up of three distinct solenoids (figure 1). Protons from the Fermilab

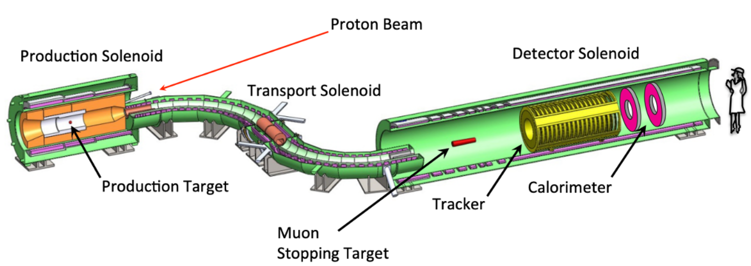

Figure 1: *The Mu2e Apparatus [1]*

particle accelerator are shot into the production solenoid where they collide with the production target, a tungsten rod about the size and shape of a pencil. This collision causes the protons to decay primarily into pions. The Production Solenoid (PS) contains three individual super conducting coils which produce a graded magnetic field that reverses the direction of the particles within and shoots them into the transport solenoid. The transport solenoid (TS) is an s- shaped solenoid with the purpose of transmitting low energy negatively charged muons from the PS to the Detector Solenoid while eliminating unwanted particles. The final solenoid in the Mu2e apparatus is the Detector Solenoid, (DS), which was the focus of my project and which will be described in further detail in the next section.

II. The Mu2e Detector Solenoid

As its name suggests, the Mu2e Detector Solenoid is where the decays of muons into electrons are detected. Conversion electrons from the stopping target housed within the DS are measured by the straw tube tracker, also housed within the DS.

The DS has a graded magnetic field designed to direct emitted conversion electrons towards the tracker. This magnetic field is achieved through eleven individual superconducting coils carrying a current of 6114 amps, centered around a shared z-axis in a Helmholtz configuration. [1]

There are four distinct sections of the DS, each with specific requirements that must be met for it to operate properly. The required magnetic field and location (represented as distance from the center of the apparatus) are listed below:

DS Requirments					
Section	Length (m)	s_{min} (m)	s_{max} (m)	$B_{initial}$ (Teslas)	B_{final} (Teslas)
DS 1 Gradient	3.0	6.58	9.58	2.00	1.18
DS 2 Transition	1.2	9.58	10.78	1.18	1.00
DS 3 Uniform	3.6	10.78	14.39	1.00	1.00
DS 4 Uniform	1.5	14.39	15.85	1.00	1.00

The configuration of these coils, called a Helmholtz configuration, is commonly used to create uniform magnetic fields. However, although Helmholtz configurations do create uniform magnetic fields along their z-axis, the field's uniformity decreases as distance from the z-axis increases.

The lack of true uniformity in the magnetic field within the DS is significant when considering how charged particles move through the B-field of the DS. Although the B-field along the z-axis has been plotted (see figure 2.1), there was no plot depicting the magnetic field in all space inside the detector solenoid. Knowing the true magnetic field over all space in the DS better allows us to understand why particles behave the way they do within the DS, and can show us why particles move in ways unexpected in a truly uniform magnetic field. This summer, I sought to model the magnetic field over all space in the Detector Solenoid, and to simulate the trajectory of charged particles through this magnetic field.

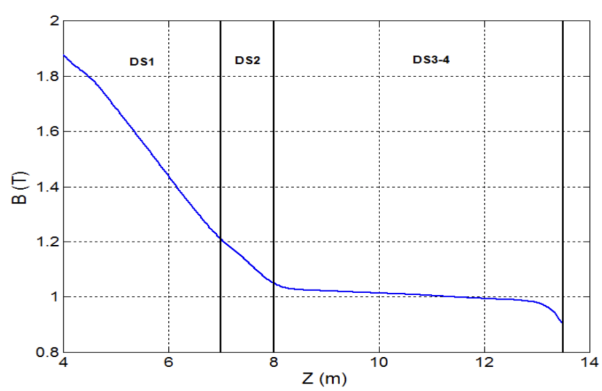

Figure 2: *Magnetic Field of the Detector Solenoid Along the z-axis [1]*

a. Modeling the Magnetic Field in the DS

To model the magnetic field over all space in the Mu2e DS, the Biot-Savart Law was used.

[1]It is worth noting that the overlap of the DS with the TS contributes to the magnetic field of the DS.

Definition (*Biot-Savart Law*).

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_c \frac{\vec{dl} \times \hat{\vec{r'}}}{|\vec{r'}|^3} dt \tag{1}$$

Where I is current, $\vec{l}$ is the total length of the coil, and $\vec{r'}$ is the point in space at which we are calculating $\vec{B}$, the magnetic field. A Python code was written to model the magnetic field over all space, using iteration to integrate a set number of dl over a three-dimensional array of field points $\vec{r'}$ throughout all space in the DS.

Below are the resulting plots of the magnetic field in the yz plane and the xy plane.

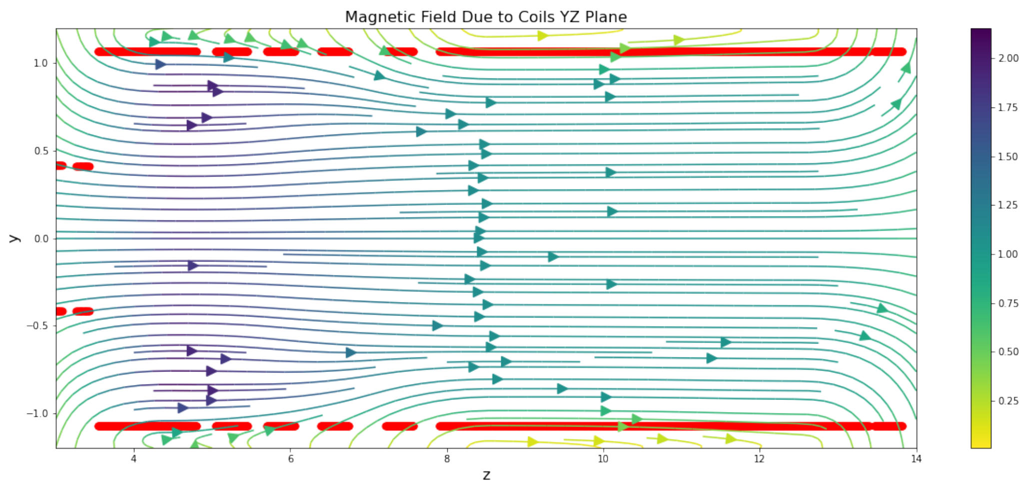

Figure 3: *Magnetic Field of the Detector Solenoid B-field in the yz plane (when x = 0)*

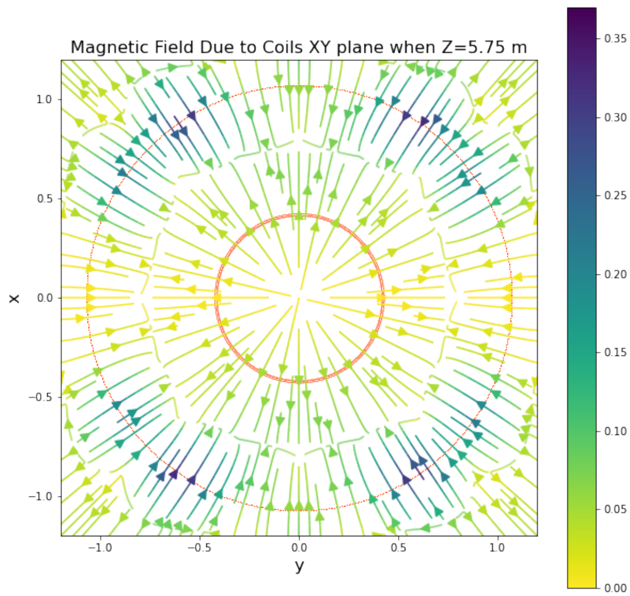

Figure 4: *Magnetic Field of the Detector Solenoid B-field in the xy plane when* $z = 5.75$ *m*

III. Trajectory of Charged Particles through the Mu2e DS

a. Cosmic Ray Background Mitigation

The most prevalent backgrounds in the Mu2e experiment are cosmic rays. Although a cosmic ray veto (CRV) (pictured in figure 5) surrounds the second half of the TS and the DS, some cosmic rays bypass the CRV and enter the DS.

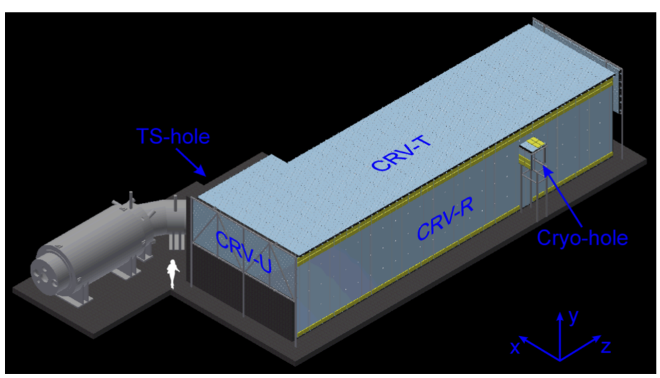

Figure 2.15. The cosmic ray veto covering the Detector Solenoid and half of the Transport Solenoid.

Figure 5: *The Mu2e Apparatus [1]*

When cosmic rays enter the DS, they can interact with the mass of the apparatus and kick out electrons. These electrons can be read as false positives in the DS.

Cosmic ray event reconstruction is essential in determining which events are false positives, and therefore in mitigating backgrounds. A plot of cosmic ray events – where a cosmic ray first interacts with the mass of the DS and kicks out an electron– is shown in figure 6.

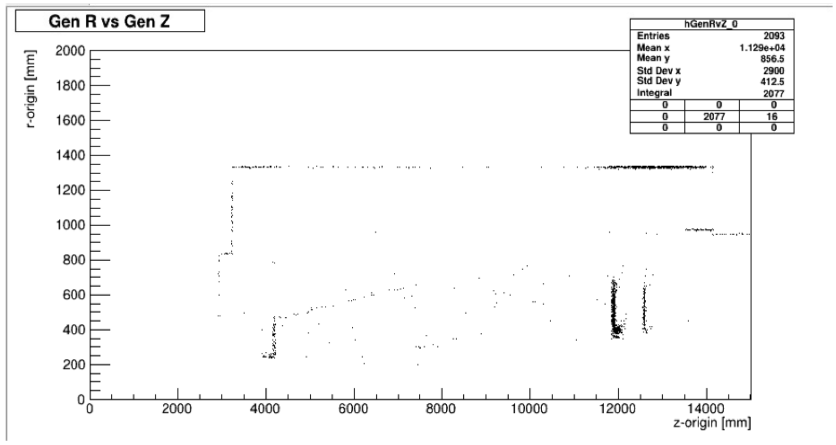

Figure 6: *Figure 1.2 Cosmic Ray Events Initial Positions [2]*

When an electron is kicked out, it travels along a downstream, upstream, or bouncing path. A downstream particle moves along the z-axis of the DS in the same direction as the magnetic field within the DS, an upstream particle moves in the opposite direction of the magnetic field, and a bouncing particle changes direction at some point along its path, moving both upstream and downstream.

b. Cosmic Ray Trajectory Simulation

Currently, the Mu2e software used to reconstruct cosmic ray events is under development. All particles are assumed to have a downstream initial momentum, even though it is known that some particles initially have an upstream momentum. This means that upstream and bouncing events can be misreconstructed.

I wrote a python simulation to model the path of an electron with a given initial position and momentum through the magnetic field in the DS. This was done by calculating the force on particle at a specific point in the DS using the Lorentz Force Law:

Definition (*Lorentz Force Law*).

$$F = q(\vec{E} + \vec{v} \times \vec{B}) \tag{2}$$

When F is the force from the magnetic field, q is the charge of the particle, $\vec{v}$ is the velocity of the particle, $\vec{B}$ is the magnetic field at the position of the particle, and $E = 0$ is the electric field inside the DS (and therefore at the position of the particle).

As we saw in section II, the magnetic field throughout the DS is not uniform. Therefore, to accurately determine the motion of a particle through the DS, the magnetic force acting on it must be calculated at each of the particle's positions. This is done using the Biot-Savart Law $\vec{B} = \frac{\mu_0 I}{4\pi} \int_c \frac{\vec{dl} \times \hat{r'}}{|\vec{r'}|^3} dt$. The particle's path was calculated using iteration; the initial momentum and position were used to determine the initial force acting on the electron and resulting acceleration, velocity, and change in position and momentum over a set change in time dt. The resulting momentum and position were used to determine the force acting on the particle in its new location, and the process was repeated for N amount of dt.

The general equations used to determine motion are:

$$m(i) = \sqrt{\frac{\vec{p}^2}{c^2} - m_0}$$

$$\vec{v}(i) = \frac{\vec{p}(i)}{m(i)}$$

$$\vec{a}(i) = \frac{\vec{F}(i)}{m(i)}$$

$$s(i+1) = s(i) + \vec{v}(i)dt + \frac{1}{2}\vec{a}(i)dt^2$$

$$\vec{v}(i+1) = \frac{s(i+1) - s(i)}{dt}$$

$$m(i+1) = \frac{m_0}{\sqrt{1 - \frac{(\vec{v}(i+1))^2}{c^2}}}$$

Below are examples of simulated downstream, upstream, and bouncing particle trajectories in the DS.

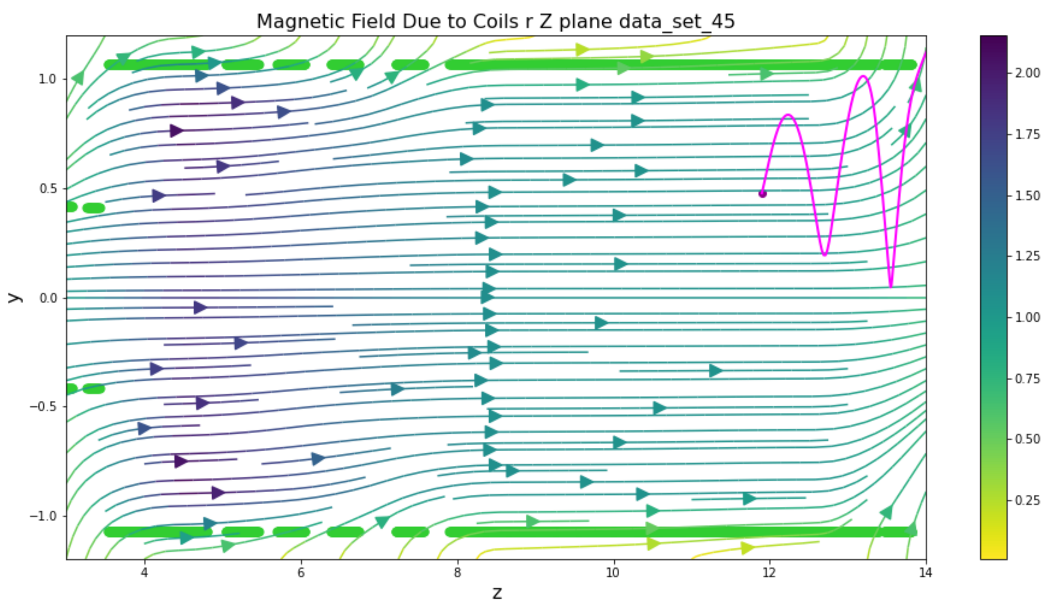

Figure 7: *Downstream electron in the yz plane (when x = 0)*

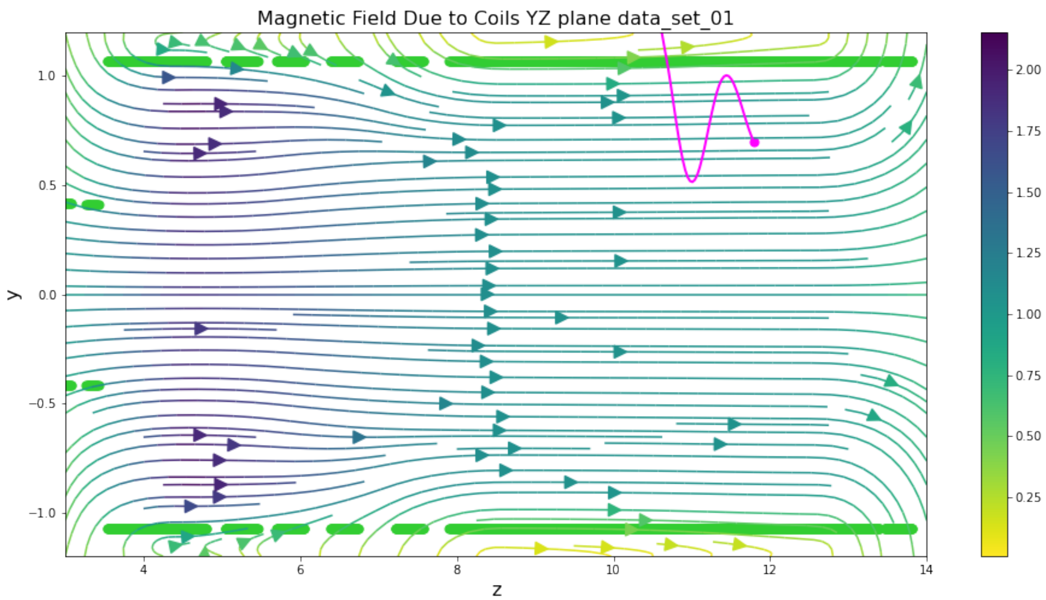

Figure 8: *Upstream electron in the yz plane (when x = 0)*

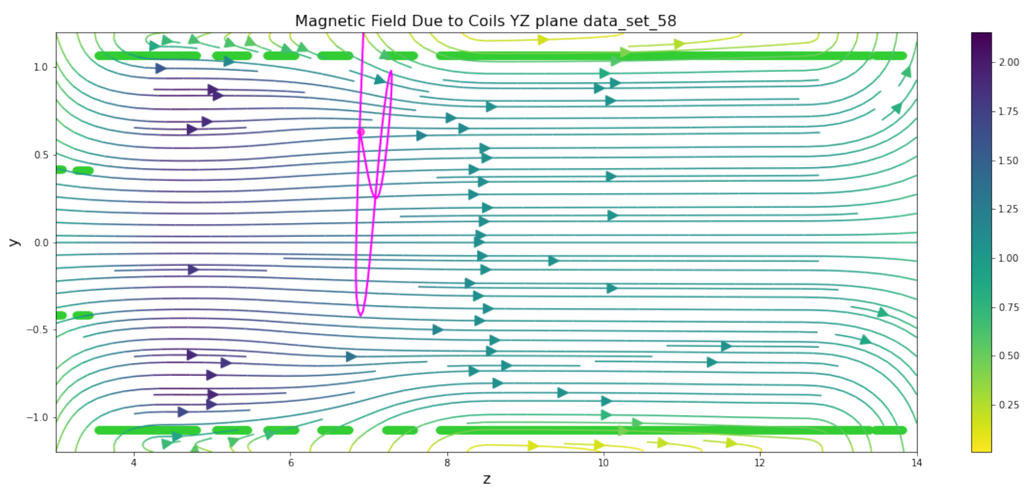

Figure 9: *Bouncing electron in the yz plane (when x = 0)*

Initial Electron Positions and Momentums						
Data Set	origin z	origin r	p total,	p x	p y	p z
45	11899.612305	476.729767	88.644436	17.044792	73.272186	46.888142
01	11791.866211	700.152954	65.403100	38.683067	27.739140	−44.852268
58	6888.726562	629.597961	105.239638	−42.909065	−87.229141	40.314644

b.1 Conservation of Energy and Further Research

The Conservation of Energy Law states that Energy must always be conserved, but the plots of particle trajectories show that some particles gain energy as they travel through the DS. (figure 10). The reason for this is unknown, but it could be due to an issue with the iterative integration and/or step size of dt in the code. Further research on this project will involve rectifying issues related to conservation of energy.

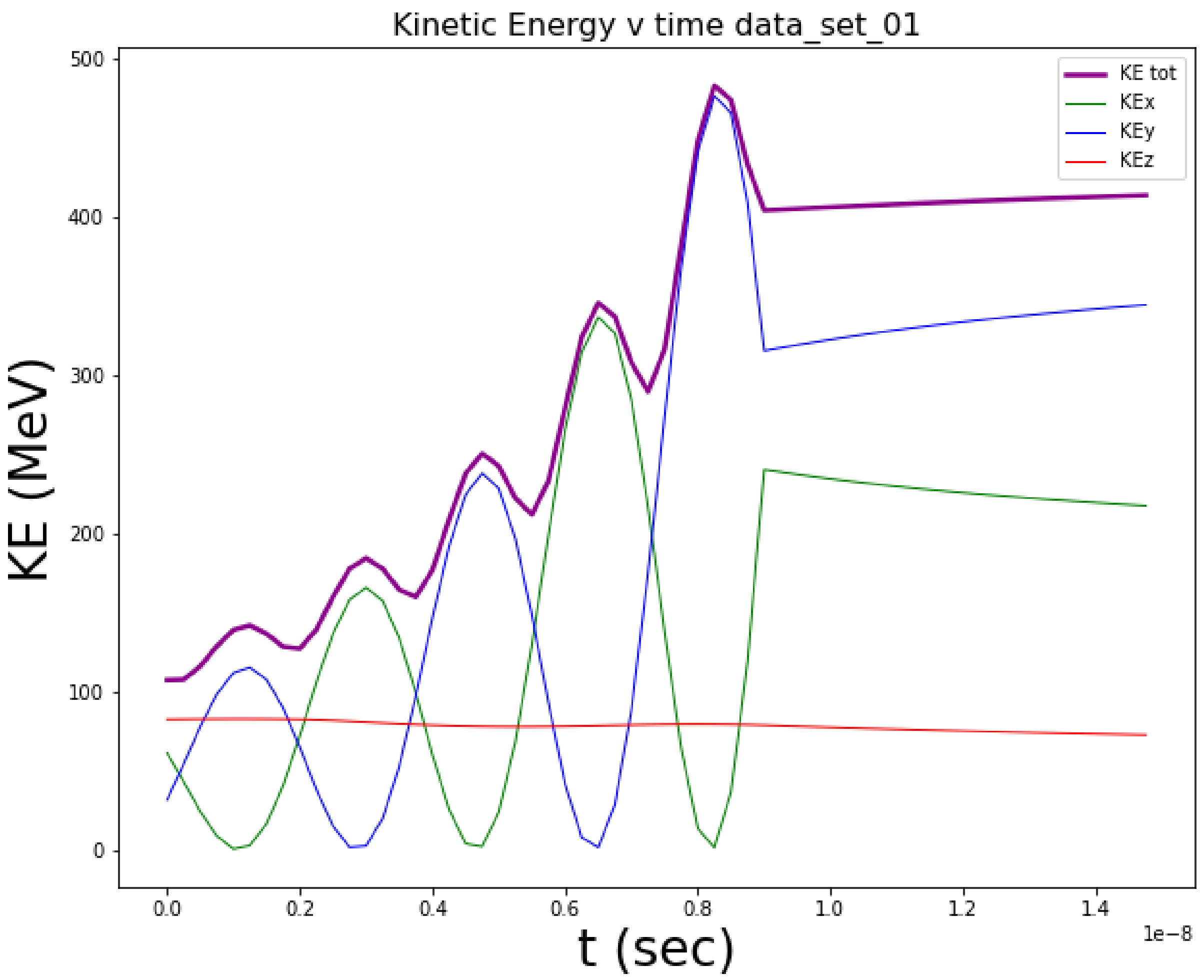

Figure 10: *Kinetic Energy of an Electron Over Time*

IV. Conclusion and Personal Growth

At the start of this program, I had an interest in particle physics and a limited knowledge of Python. Over the course of the summer, my understanding of particle physics vastly improved, and I grew far more comfortable using Python. Perhaps most importantly, this summer I gained problem-solving skills that are hard to learn in a classroom environment, but are essential for researching physicists. Much of my work this summer was spent problem-solving the code I was writing. Determining how to make my simulations do what I needed them to do was only the first step. Once I had my code, troubleshooting general errors that emerged during runs as well as making improvements to the code to minimize run time and improve accuracy helped teach me what to look for while coding, and how to discern between what is important and what is not.

V. Acknowledgments

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References

[1] Larry Bartoszek, E. Barnes, James Miller, J. Mott, A. Palladino, J. Quirk, B. Roberts, J. Crnkovic, V. Polychronakos, V. Tishchenko, P. Yamin, Chu-Hung Cheng, B. Echenard, K. Flood, David Hitlin, J. Kim, T. Miyashita, Frank Porter, M. Röhrken, and James Popp. Mu2e Technical Design Report. January 2015.

[2] MacKenzie Devilbliss. Cosmic Particle Initial Coordinates: (z_origin, r_origin, ptotal, px, py, pz):, August 2021.