# Statistical Thermal Studies of 5.02 TeV Pb-Pb Collisions

CERN Summer Student Report

Jihyeun Bang

supervised by

Boris Hippolyte

Alexander Philipp Kalweit

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**Abstract**

The statistical thermal model THERMUS is used to investigate, for the first time, freeze-out parameters obtained for central Pb-Pb collisions at $\sqrt{s_{\mathrm{NN}}} = 5.02$ TeV. Fits to the ALICE data are performed allowing for a systematic study of the freeze-out conditions of the Quark Gluon Plasma created in nucleus-nucleus at the highest LHC energy. The main results are a chemical freeze-out temperature of 151±2 MeV, a volume is $8350^{+590}_{-570}$ fm$^3$ with a large $\chi^2/N_{df}$ of 56.1/7. When trying to understand the origin of this significant $\chi^2/N_{df}$, we observe that proton yields are suppressed compared to the model expectations which could suggest the need for different values of chemical freeze-out temperature for different species. Thermal model parameters obtained at 5.02 TeV are compared with former results for Pb-Pb collisions at 2.76 TeV.

## 1 Introduction

Quantum chromodynamics (QCD) is a quantum field theory describing the strong interaction between quarks and gluons. Contrarily to Quantum electrodynamics (QED), QCD is a non abelian gauge theory due to the property that gluons, which are the mediators of the strong force, can interact together and not only with quarks. As a consequence the force between partons (quarks and gluons) increases with distance so partons are confined into hadrons (as it is the case for protons or neutrons). Inversely, the strength of the interaction decreases for very short distances and such specific behavior, called asymptotic freedom [1] means that the partons can move almost freely when they are very close within a small volume. Such a phase of deconfined quarks and gluons is called a Quark-Gluon Plasma (QGP) and a way to produce it in the laboratory is to collide heavy nuclei at very high energy.

[1] in 2004, the Nobel Prize was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek for this discovery.

In ultra-relativistic heavy ion collisions, a fireball at extremely high temperature and energy density is produced. Very soon after the collision, the fireball equilibrates to a QGP and then expands and cools down. Eventually the plasma goes through a phase transition when it reaches a critical temperature ($T_c$). After the transition, no more free quarks and gluons exist but form a gas of hadrons. At the end of the phase transition, the chemical composition of hadron gas is fixed, which means that the abundances of each hadron species is determined. This moment called chemical freeze-out is occurring for a temperature noted $T_{ch}$ (in fact, assuming that different species can freeze at different temperatures means that several values of $T_{ch}$ need to be considered). Just after chemical freeze-out, the hadron gas is still dense enough so hadrons can interact depending on their elastic cross-sections. These interactions stop at a moment called kinetic freeze-out. Once kinetic freeze-out happened, hadrons are flying without interacting until they reach the detectors assembled to measure precisely each of them. Figure 1 shows this process visually.

The production of hadrons can be very successfully described by statistical thermal models [1]. Such models rely on a thermodynamical description of the number of hadrons produced: the main parameters are the overall volume of the fireball, its temperature as well as chemical potentials when the formalism used is corresponding to grand canonical ensemble. By measuring the yields of hadrons (the average number of each species produced per event) and comparing them with the values predicted by the model, we infer an estimate of the chemical freeze-out temperature $T_{ch}$ [1]. It is important to note that the critical temperature calculated by lattice QCD is $155 \pm 9$ MeV and provides a baseline [2]. Temperatures already extracted with thermal statistical studies for central Pb-Pb collisions in the center of mass energy per nucleon pair of $\sqrt{s_{NN}} = 2.76$ TeV measured at the Large Hadron Collider are 156 MeV and they are very consistent with lattice QCD calculations [3].

In this paper, a thermal model called THERMUS [4] is applied for the first time to Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV which is the highest energy ever reached for heavy-ion

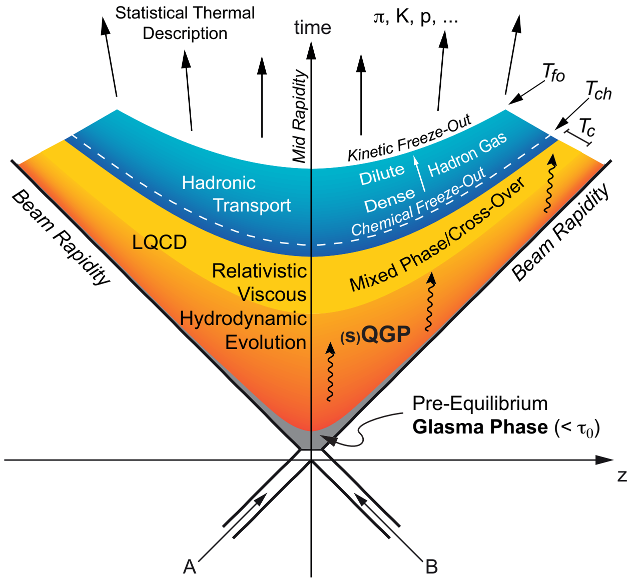



Figure 1: The evolution of QGP

collisions. The results of these studies, based on ALICE data, are presented and discussed. In section 2, brief but general description of thermal models is introduced. It covers basic equations to provide the main concepts of the model, as well as physical meaning of major parameters. Section 3 shows the fit result with various strategies to decide whether or not $\gamma_s$, volume corrections, and the nuclei yields should be applied to the fit. The results of the fits are presented in section 3.3. Further discussions are proposed in section 4. Proton and strange hadrons yields are tested separately to examine systematic variations of $T_{ch}$. The last section of the discussion provides more details on the temperature contraints coming from yields. Finally, all the results are summarized in section 5.

# 2 Thermal Model

The model starts with the assumption that the fireball is in thermal equilibrium state. If the system is large enough so if the quantum numbers can be treated statistically, then the fireball is described by grand canonical ensemble (GCE). Pb-Pb collisions at high energy are a good example of such a large system that has many degrees of freedom and quantum numbers including Q (charge), B (baryon number), and S (strangeness). Note that, for $pp$, $p\bar{p}$, and $e^+e^-$ collisions, a canonical ensemble formalism should be applied in order to conserve exactly quantum numbers.

Under this assumption, the partition function $Z$ (of GCE) is following :

$$\ln Z(T, V, \vec{\mu}) = \sum_{\text{species } i} \frac{g_i V}{2\pi^2} \int_0^\infty p^2 dp \ln\left(1 \pm \lambda_i e^{-\varepsilon_i/T}\right)^{\pm 1} \tag{1}$$

where $+$ is for fermions and $-$ is for bosons, and where

$$\begin{aligned} \vec{\mu} &= (\mu_B, \mu_S, \mu_Q) \\ \varepsilon_i &= \sqrt{p^2 + m_i^2} \\ \lambda_i(T, \vec{\mu}) &= \exp\left(\frac{B_i\mu_B + S_i\mu_S + Q_i\mu_Q}{T}\right) \end{aligned} \tag{2}$$

$T$ is temperature, $V$ is volume, $\mu_B, \mu_S, \mu_Q$ are chemical potentials related to baryon number, strangeness, and charge, respectively. The quantities $g_i$, $p$, $m_i$ and $\varepsilon_i$ are respectively the spin degeneracy, the momentum, the mass, and the energy associated to each species $i$.

For a defined partition function, particle multiplicities, entropy, pressure, and energy can be determined. After performing successively a Taylor expansion and an integral, particle multiplicities are given by

$$N_i(T, V, \vec{\mu}) = T\frac{\partial \ln Z}{\partial \mu_i} = \frac{g_i T V}{2\pi^2} \sum_{k=1}^{\infty} \frac{(\pm 1)^{k+1}}{k} \lambda_i^k m_i^2 K_2\left(\frac{k m_i}{T}\right) \tag{3}$$

where $K_2$ is the modified Bessel function. For the numerical calculations, we can apply Boltzmann approximation by considering only the first term of the summation.

In ideal conditions, Equation (1) is acceptable but in practical analysis, two parameters have to be introduced: an eigen volume correction and the strangeness saturation factor $\gamma_s$. In reality, hadrons and nuclei are not point-like so two of them can neither be located at the same position nor have overlap. Such volume correction is performed in a simplest way by assuming that all particles have the same radius of 0.3 fm, and exclude the volume of particle with Van der Waals-type correction.

The factor $\gamma_s$ represents the level of saturation of the strangeness flavour. It usually has value between 0 and 1. A value of $\gamma_s = 1$ means that the QGP lifetime is long enough so strangeness production be equilibrated. This is consistent with the assumption that the fireball is in equilibrium for light flavours. On the other hand, if $\gamma_s < 1$, then strangeness has no time to be equilibrated and it appears as suppression. In elementary particle collisions such as $e^+e^-$, $pp$, or $p\bar{p}$, such suppression of strangeness was observed [7]. The suppression is modelled by simply multiplying $\gamma_s^{|S_i|}$ to the elementary partition function of each particle, where $|S_i|$ is number of strange and anti-strange quarks of each hadron. As a result, the number of particles $N_i$ will be multiplied by same factor and becomes $\gamma_s^{|S_i|}N_i$.

After the primordial calculation of yields with the grand canonical ensemble, particle decay is considered to reflect experimental measurements [1][4].

# 3 Fitting to Experimental Data

Once the formula of yields is known, by fitting it to experimental data, we can obtain estimated value of thermal variables; temperature ($T$), volume ($V$), and $\gamma_s$. In THERMUS, ROOT `TMinuit` class was used to minimize $\chi^2$ function, which is given by sum of square of standard deviation of species $i$,

$$\chi^2 = \sum_{\text{species } i} \left( \frac{N_i^{\text{mod}} - N_i^{\text{exp}}}{\sigma_i} \right)^2 \tag{4}$$

Here, for each hadron species $i$, $N_i^{\text{mod}}$ is yield predicted by model, and $N_i^{\text{exp}}$ and $\sigma_i$ are the yields and uncertainty respectively from experimental data [4]. Degree of freedom ($N_{df}$) is number of particles included in the fit subtracted by number of free parameters. From this, $\chi^2/N_{df}$ gives sense of how good the fit is.

Using the thermal model, a fit to the preliminary data of very central (0-10%) Pb-Pb collisions at 5.02 TeV from ALICE experiment has been performed with adopting several strategies. The data include yields of $\pi^{\pm}$, $K^{\pm}$, $K_s^0$, $p$, $\Lambda$, $\Xi$, $\Omega$, $K^*$, $\phi$, $d$, $^3He$. Resonances ($K^*$ and $\phi$) are not included in the fit. This is because the measurable yield of strongly decaying—thus short-lived[2]—hadronic resonances can be affected by hadronic interactions between chemical freeze-out and kinetic freeze-out.

## 3.1 $\gamma_s$ and nuclei

Figure 2 shows the comparison between the cases of fit with and without nuclei and fixed and free $\gamma_s$. Particles are arbitrarily assumed to be point-like (i.e. there is no volume correction.)

[2] lifetime is about 1-10 fm/c

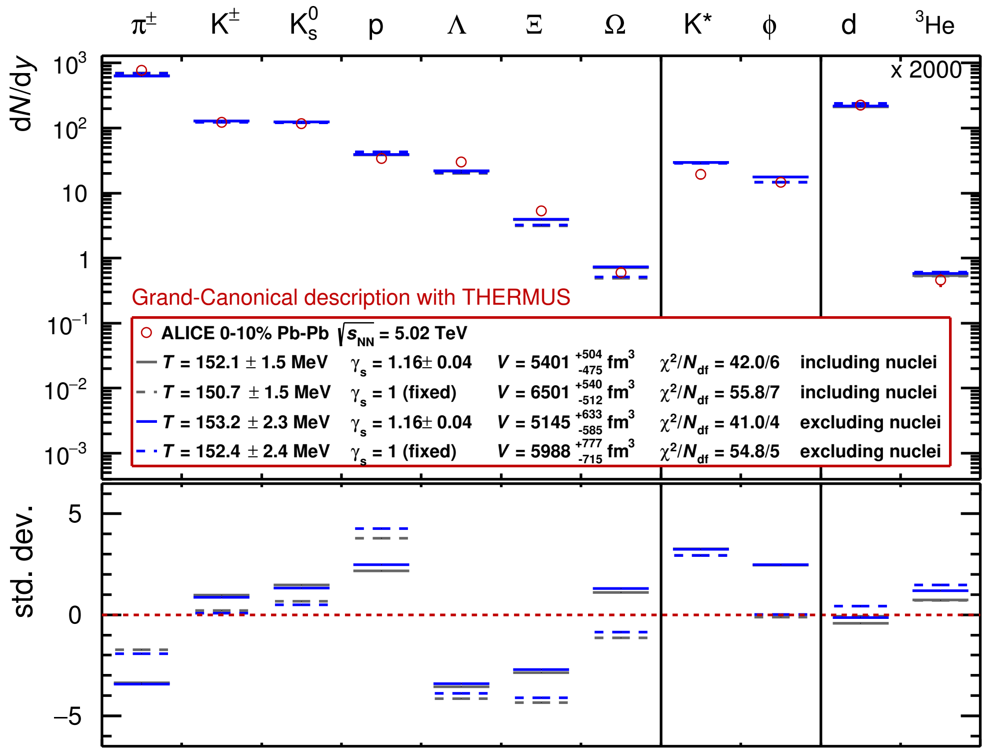



Figure 2: fit result without volume correction. Yields of nuclei ($d$ and $^3He$) are multiplied by 2000. Red circles are yields from ALICE experiment. Solid line represents fit with free $\gamma_s$ and dashed lines represents fixed $\gamma_s$ ($= 1$). Also, grey lines are fit to all particles except resonances and blue lines are fit without nuclei.

We will see the effect of volume correction in the next section. The temperature is around 152 MeV and it is slightly lower than 155 MeV, which is predicted by lattice QCD. Also $\chi^2/N_{df}$ is higher than 7. This is three times larger than the value of 2.76 TeV data [3]. If $\gamma_s$ is free, it becomes larger than unity. This means that larger yields of strange hadrons, which have strange valence quarks such as $K^{\pm}, K_s^0, \Lambda, \Xi, \Omega, K^*$, and $\phi$, make more sense. Note that $\gamma_s$ was originally introduced to explain suppression of strangeness in elementary particle collision so it was believed that $\gamma_s$ should be between 0 and 1. The value of $\gamma_s$ bigger then the unity is predicted by the other version of thermal model [6][8].

However, even if $\gamma_s$ is not fixed, the $\chi^2/N_{df}$ is still big and the fit is improved slightly. In following analyses, $\gamma_s$ will be fixed to 1 since there is no crucial reason $\gamma_s$ to be bigger than the unity

Comparing between fit with and without nuclei, there is no big differences and temperature and volume consist within the uncertainty. In following analyses, nuclei are always included in order to decrease $\chi^2/N_{df}$, except for the discussion about strangeness in section 4.2.

## 3.2 Volume correction

Figure 3 shows the effect of applying excluded volume corrections: instead of considering that particles are point-like, a radius of 0.3 fm is assumed for all species. The fitted values

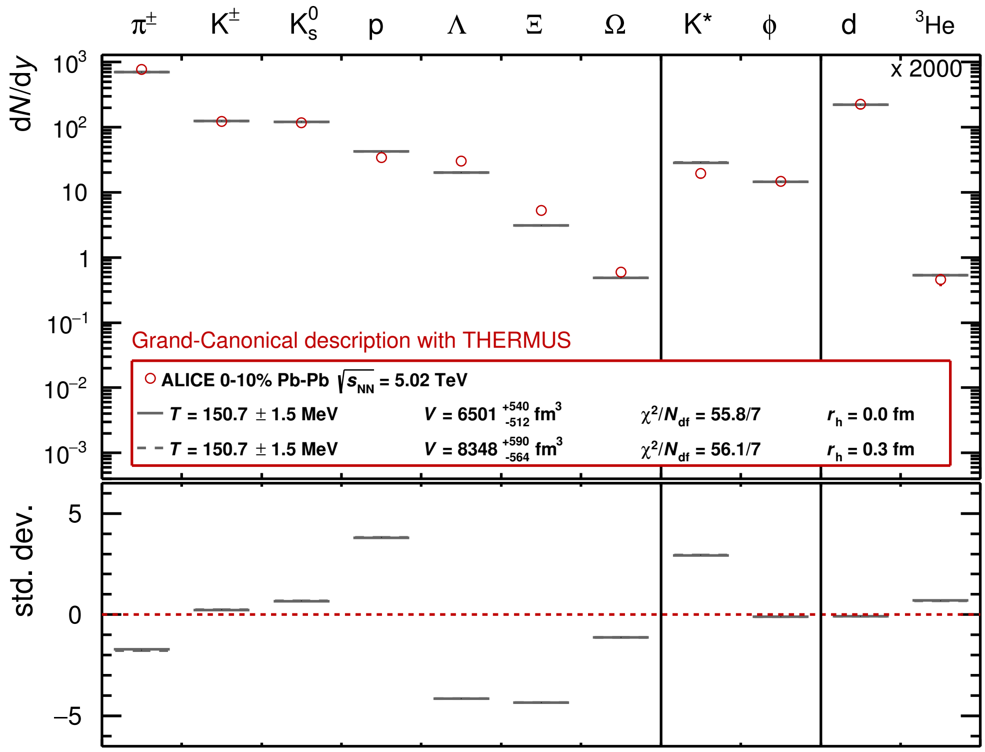



Figure 3: fit result with and without volume correction. Yields of nuclei ($d$ and $^3He$) are multiplied by 2000. Red circles are yields from ALICE experiment. Solid line represents fit without volume correction and dashed lines represents fit with volume correction.

obtained for the model (yields and temperature) are same except for the volume of the fireball at freeze-out: estimated volume is 6500 fm$^3$ for point-like and 8350 fm$^3$ for a radius of 0.3 fm (which is consistent with the increase coming from the sum of the eigen volumes of each produced particles). In the following fits, excluded volume corrections are applied in order to have a more realistic estimate of the total correlated volume of the fireball. It must be noted that the volume estimated from thermal fits to Pb–Pb collisions at 2.76 TeV, once excluded volume corrections are applied, is consistent with the homogeneity volume derived from Bose-Einstein two particle correlations (HBT) [5].

## 3.3 The result

From the different envisaged strategies, the following options are eventually retained for performing the fit: $\gamma_s$ is fixed to unity, excluded volume corrections are applied and all the yields except resonances are included. The result is shown in Fig. 3 with dashed lines. The extracted values are the following: $T_{ch} = 150.7 \pm 1.5$ MeV and $V = 8348^{+590}_{-564}$ fm$^3$ leading to $\chi^2/N_{df} = 56.1/7$. The value obtained for the temperature is significantly lower than $T_c = 155 \pm 9$ MeV, the critical temperature from Lattice QCD, although they still agree within uncertainties. Comparing the results with the ones extracted for collisions at 2.76 TeV, $T_{ch}$ is lower and the correlation volume is higher at 5.02 TeV. In addition, the

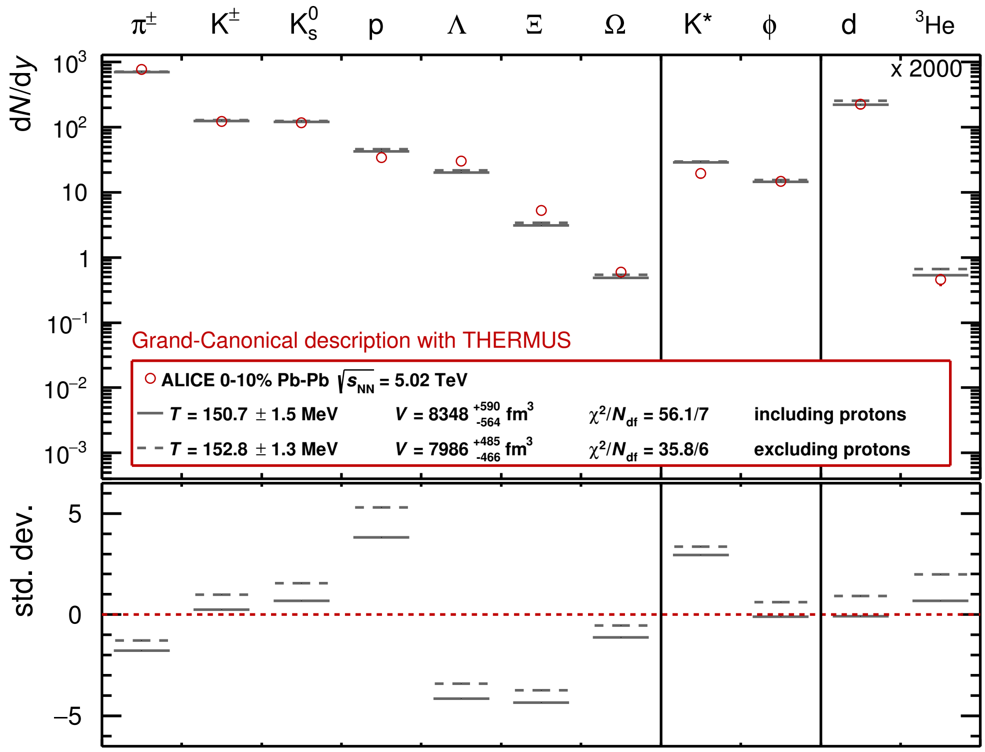



Figure 4: fit result with and without protons. Yields of nuclei ($d$ and $^3He$) are multiplied by 2000. Red circles are yields from ALICE experiment. Solid line represents fit with protons and dashed lines represents fit without protons.

deviations between the measured yields and the model values are within $\pm 5\sigma$ reflected in an overall $\chi^2/N_{df}$ which is larger than 8.

# 4 Discussion

## 4.1 $K^*$ and Protons

In the previous section, the fit results presented in Figure 3 (dashed lines) were discussed. The number of standard deviations between the measurements and the model values for $p, \Lambda, \Xi$ and $K^*$ were found to be large. This is expected in the case of the $K^*$ because such resonance has a short lifetime compared to the fireball one: its decay daughters can then scatter during the hadronic phase which means the $K^*$ cannot be reconstructed via an invariant mass analysis. Consequently the $K^*$ measurements do not reflect anymore the production yields which is the reason why they are generally excluded from the fits.

A significant discrepancy between the measured proton yields and the model value has already observed for Pb–Pb collisions at $\sqrt{s_{\mathrm{NN}}} = 2.76$ TeV and the observation of lower proton yields than expected at higher energy has been reported [9].

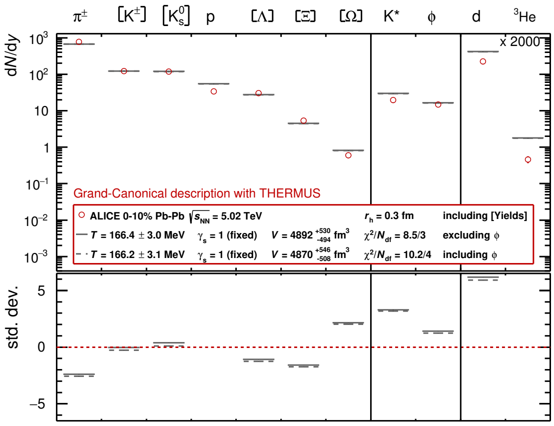
Figure 4 illustrates the results obtained when protons are not included in the fit. The temperature increases and consequently, all model yields are higher (i.e. closer to the measurements) and lead to a smaller $\chi^2/N_{df}$. Several hypotheses were proposed for explaining the measurement of proton yields lower than expected. The production of protons could be lower if the chemical freeze-out temperature is not the same for all particles. The possibility of different $T_{ch}$ will be discussed in the next section (which means a lower value for the protons). The main other explanation is that yields of protons could be suppressed because of annihilation with anti-protons in the hadronic phase. However the reason why the protons (and their antiparticle) would be significantly more affected than $\Lambda$ (as well as $\Xi$ and $\Omega$) still needs to be further justified.

## 4.2 Strangeness

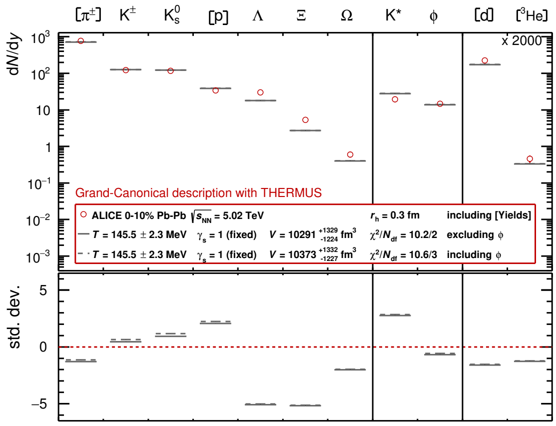
Checking if strange and non-strange hadrons are likely to freeze out at the same temperature or not is an important point. Figure 5a shows the fit results with only hadrons that contains strange valence quarks ($K^{\pm}$, $K_s^0$, $\Lambda$, $\Xi$, $\Omega$) whereas Figure 5b contains the fit with the other particles ($\pi^{\pm}$, $p$, $d$, $^3He$).

In both plots in Figure 5, including and excluding $\phi$ were treated separately since the $\phi$ contains strange and anti-strange valence quarks but its global strangeness is 0 (hidden strangeness). The $\phi$ could behave more like strange particles and lead to a relatively lower value of $\chi^2/N_{df}$ in Figure 5a and vice-versa for Figure 5b. However, $\phi$ does not make a significant difference for any of the plots. Consequently, fits discussed in the next paragraph are performed without the $\phi$ yields.

The estimated freeze-out temperature and volume obtained with strange particles are $166 \pm 3$ MeV and $4892^{+530}_{-494}$ fm$^3$ respectively. On the other hand, freeze-out temperature and volume obtained for non-strange particles are $146\pm2$ MeV and $10291^{+1329}_{-1224}$ fm$^3$. Temperatures differ significantly and volume obtained with non-strange hadrons is twice larger than volume from strange hadrons. Such results suggest that all particles freeze-out do not freeze-out simultaneously and more specifically that strange particles could freeze-out earlier.




(a) Strange




(b) Non-strange

Figure 5: Comparison of thermal variables between strange and non-strange particles. Particles in bracket [ ] are used to fit. Figure 5a is fit only with strange hadrons. Standard deviation is 9 for proton and 13 for $^3He$. Figure 5b is fit only with non-strange particles.

## 4.3 Freeze-out temperature for each species

In order to complement the study described in section 4.2, the freeze-out temperature parameters of each hadrons are examined separately: the temperature of each species $i$ is obtained

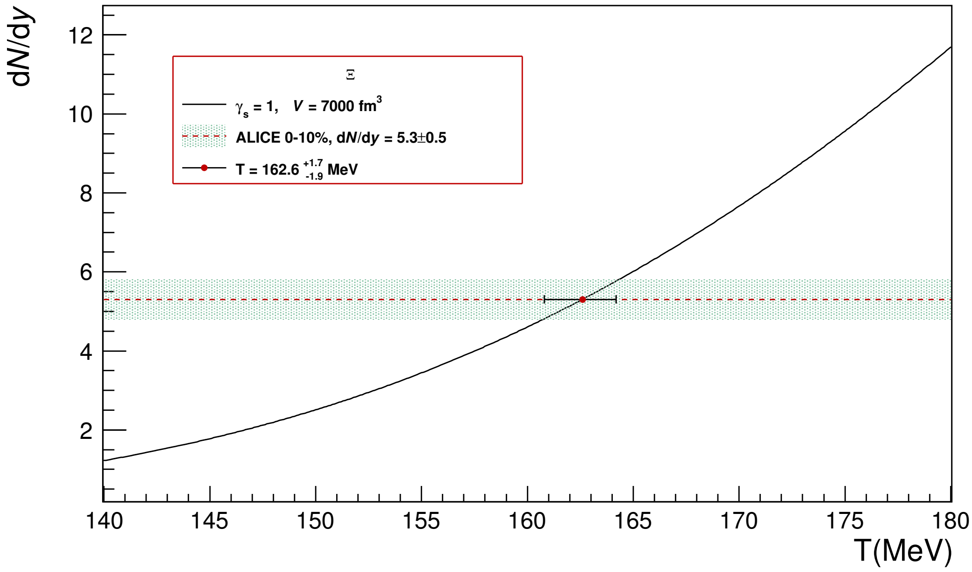



Figure 6: Yield of $\Xi$ with respect to temperature. Black line is model value at each temperature. Dashed red line shows experimental value and green region represents uncertainty. Freeze-out temperature and its uncertainty is marked with red dot.

| Species | Strangeness $\lvert S\rvert$ | Mass [MeV/$c^2$] | Temperature [MeV] |
|---|---|---|---|
| $\Xi^-$ | 2 | 1535 | $162.6^{+1.7}_{-1.9}$ |
| $\Lambda$ | 1 | 1116 | $160.9^{+1.6}_{-1.6}$ |
| $\pi^+$ | 0 | 140 | $158.9^{+1.6}_{-1.7}$ |
| $\Omega^-$ | 3 | 1672 | $155.9^{+2.3}_{-2.5}$ |
| $K^+$ | 1 | 494 | $155.2^{+1.4}_{-1.3}$ |
| $K^0_s$ | 1 | 498 | $154.8^{+1.1}_{-1.2}$ |
| $\phi$ | 0 | 1019 | $154.6^{+1.8}_{-1.8}$ |
| $d$ | 0 | 1875 | $152.9^{+1.7}_{-1.9}$ |
| $^3He$ | 0 | 2814 | $150.9^{+1.6}_{-1.9}$ |
| $p$ | 0 | 938 | $149.9^{+1.1}_{-1.2}$ |
| $K^*$ (892) | 1 | 892 | $146.0^{+3.3}_{-3.6}$ |

Table 1: Freeze-out temperature at $V$=7000 fm$^3$. $\lvert S\rvert$ is to the absolute value of strangeness for the valence quarks.

from the comparison to each yield with all the other ones excluded. When considering only one single data point (i.e. the yields of species $i$), only one parameter is left free. So the evolution of temperature is given while the volume and $\gamma_s$ remain fixed (with $V = 7000$ fm$^3$ and $\gamma_s = 1$ respectively). The value of fixed volume is arbitrary chosen but it is based on the volume that we got from various fits.

As an example, Figure 6 illustrates the increase of $\Xi$ yields with temperature. The corresponding freeze-out temperature is represented as the cross-point of the model and experimental value. Other particles behave similarly.

Table 1 provides the ordered values of the freeze-out temperature for each species. The temperature varies in wide range from 146 MeV to 163 MeV. No apparent ordering with respect to strangeness or mass is noticeable.

# 5 Summary

Chemical freeze-out conditions of the fireball produced at the highest LHC energy were investigated for the first time using the thermal model THERMUS. We obtained chemical freeze-out temperature ($T_{ch}$) and volume as well as a strangeness saturation factor $\gamma_s$ from fits to the yields of central Pb-Pb collisions at $\sqrt{s_{\rm NN}}$ =5.02 TeV. Fits were performed with various options for systematics.

Resonance yields (K$^*$ and $\phi$) were purposely not included in the fit because resonances can decay before kinetic freeze-out and have consequently production yields affected during the hadronic phase. Both options of having $\gamma_s$ free or fixed to unity were also investigated. When fitted, the value of $\gamma_s$ came out larger than unity in order to match the yields of strange hadrons. This also lead to a lower $\chi^2/N_{df}$ although it remained large. We also looked into the consequence of including or not nuclei yields and concluded this does not change significantly the fit parameters. The last option investigated was to take into account volume corrections related to eigen volumes of hadron species: this increased the extracted volume of fireball but did not affect other parameters.

The result of fit with volume correction and fixed $\gamma_s = 1$ to the yields of $\pi^{\pm}$, $K^{\pm}$, $K^0_s$, $p$, $\Lambda$, $\Xi$, $\Omega$, $d$, $^3He$ is shown in Figure 3 with black dashed-dotted lines: chemical freeze-out temperature is 150.7±1.5 MeV, volume is $8348^{+590}_{-564}$ fm$^3$, and $\chi^2/N_{df}$ is 56.1/7. $T_{ch}$ is slightly lower than what is expected from lattice QCD calculation and than what was obtained from Pb-Pb collisions at 2.76 TeV. Also, the $\chi^2/N_{df}$ is larger.

For several species, a number of standard deviations between the measurements and the model values were larger than $2\sigma$. The specific case of the proton yields was discussed. Proton yields could be lower than expected because of a lower production or modified during the hadronic phase. However these hypotheses are difficult to test and the exact reason of the discrepancy remains unclear.

We also divided particles into two groups, strange or non-strange, and performed fits for each group independently. We observed that chemical freeze-out temperatures and volumes differ significantly. The extracted value for $T_{ch}$ is 10% lower and the volume is twice larger for non-strange than for strange species. This observation suggests that different values of $T_{ch}$ may be needed depending on the particle flavour.

Expected yields as a function of temperature were extracted in the range of $T$=140–

180 MeV. Freeze-out temperatures for each particles were also independently calculated, by fitting or finding the crosspoint. The obtained range of $T_{ch}$ is wide. In addition, the corresponding temperatures do not seem to follow any mass or strangeness order.

As final notes, we remark that a possible improvement in this study would be adjusting the size of each species independently instead of assuming an identical radius for all of them. However, volume correction made no significant change of the yields and temperature as seen in section 3.2. Nevertheless, it would allow to extract a more realistic correlation volume. Also, in this work, only the most central collisions were considered: more insight could clearly be gained with studying peripheral collisions (for instance, depending on the evolution of the proton yields with centrality, the hypothesis that the hadronic phase has a significant role could be checked). In addition, in section 4.3, we only considered a fixed volume of 7000 fm$^3$ and no dependence on $T_{ch}$ were investigated.

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# References

[1] P. Braun-Munzinger, K. Redlich, J. Stachel, published in Hwa, R.C. (ed.) et al.: Quark-Gluon. Plasma 491-599, 2003.

[2] A. Bazavov et al. Phys. Rev. D85 (2012) 054503.

[3] B. Guerzoni for the ALICE Collaboration, J.Phys.Conf.Ser. 668 (2016) 012058

[4] S. Wheaton, J. Cleymans, M. Hauer, THERMUS – A Thermal Model Package for ROOT, Comput.Phys.Commun. 180 (2009) 84-106

[5] Dariusz Miskowiec for the ALICE Collaboration, PoS WPCF2011 (2011) 001

[6] R. Bellwied, Journal of Physics: Conference Series 736 (2016) 012018

[7] F. Beutler, arXiv:0904.2885 [nucl-th].

[8] J. Letessier and J. Rafelski, Eur. Phys. J. A **35**, 221 (2008) doi:10.1140/epja/i2007-10546-7 [nucl-th/0504028].

[9] M. Floris, Nucl. Phys. A **931**, 103 (2014) doi:10.1016/j.nuclphysa.2014.09.002 [arXiv:1408.6403 [nucl-ex]].