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Summer Student Project

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Summary

The aim of this project was to calculate the surface resistance of different materials and coatings as well as to design a multi-crystal multi-mode dielectric resonator. CST was used to design the dielectric resonator that can be used with four different dielectrics and to measure at seven different resonance frequencies. Additionally, the surface resistance of different coatings was measured at room and cryogenic temperatures, using VNA and a dielectric resonator of Hakki-Coleman type.

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1 Introduction

1.1 Scope of the project

Nowadays, there are more than 30000 accelerators in operation worldwide. All of them work at different frequencies and temperatures and are built from different materials. That is why it is beneficial to know the characteristics of different materials at different frequencies, in other words, to understand the frequency dependency of certain materials.

Each component of an accelerator has a specific resistivity, which leads to a surface resistance which then leads to a resistive wall impedance. Any change in terms of surface treatment on any component, ideally, should not change the resistive wall impedance.

This project aims to determine the surface resistance of several coatings, e.g., laser-treated copper, as well as accelerator components with the aid of a Hakki-Coleman based dielectric resonator [2]. In addition, we designed a cavity that could be used with four different dielectric crystals to measure at a minimum of four well-separated frequencies.

1.2 Resonator basics

A resonator is a device used to confine an electromagnetic field. This structure can either be hollow or filled with a dielectric material to lower its resonance frequency f_0 , often referred to as a dielectric resonator (DR). Resonator devices accommodate a number of different electromagnetic (EM) field configurations called modes. Depending on the field direction, they can be distinguished as follows:

- Transverse electromagnetic modes (TEM) - both electric and magnetic fields are perpendicular to the direction of propagation,
- Transverse electric modes (TE) - only electric field is perpendicular to the direction of propagation,
- Transverse magnetic modes (TM) - only magnetic field is perpendicular to the direction of propagation and
- Hybrid modes - neither electric nor magnetic fields are perpendicular to the direction of propagation.

Each mode results in a specific current flow on the resonator's wall and can be used, for example, to determine the surface resistance R_s of the material used in the resonator. R_s is a material property describing the RF losses of the material within the penetration depth. However, in order to determine the surface resistance of certain materials, it is necessary to know the quality factor of the specific mode in the resonator, which gives us information about the overall losses in the resonator. The unloaded quality factor Q_0 is defined as the ratio of energy stored U_{st} and power dissipated P_{diss} in the resonator [7]:

$$Q_0 = 2\pi f_0 \frac{U_{st}}{P_{diss}}, \quad (1)$$

where f_0 refers to the resonance frequency of the mode.

Generally, three main components contribute to the unloaded quality factor [7]:

$$\frac{1}{Q_0} = \frac{1}{Q_d} + \frac{1}{Q_\kappa} + \frac{1}{Q_r}, \quad (2)$$

where Q_d refers to the dielectric quality factor, Q_κ refers to the conductor quality factor, and Q_r to the radiation quality factor. Q_d and Q_κ can be expressed over the equations [6]:

$$Q_d = \frac{1}{p \cdot \tan(\delta)} \quad \text{and} \quad Q_\kappa = \frac{G}{R_s}, \quad (3)$$

where p is a ratio between energy stored within the dielectric and energy stored in the entire cavity, called *filling factor*, G is a parameter that depends only on the geometry of the resonating structure, called *geometry factor*. Radiation losses Q_r can usually be neglected in a closed cavity. Substituting Eq.(3) into Eq.(2) results in a new equation for Q_0 :

$$\frac{1}{Q_0} = \sum_i \frac{R_{si}}{G_i} + p_i \cdot \tan(\delta)_i. \quad (4)$$

To couple an electromagnetic field into the cavity, it is connected to a vector network analyzer (VNA), which is a test instrument used to validate the performance of specific components and devices. Besides the internal cavity losses described in Eq.(2), there are external losses in the transmission lines that must be taken into account, i.e., external power losses. The loaded quality factor is the total quality factor that includes both internal and external losses. In other words, it is the sum of the unloaded quality factor and external quality factor and the only actually experimentally measurable value [7]:

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}. \quad (5)$$

1.3 Dielectric resonator of Hakki-Coleman type

A DR of the Hakki-Coleman type is a resonator with metallic housing, and a cylinder-shaped dielectric crystal (dark blue) is set in the center. A schematic is shown in Fig.1. . On top of the metallic housing (yellow) is a plate that will be measured (pink).

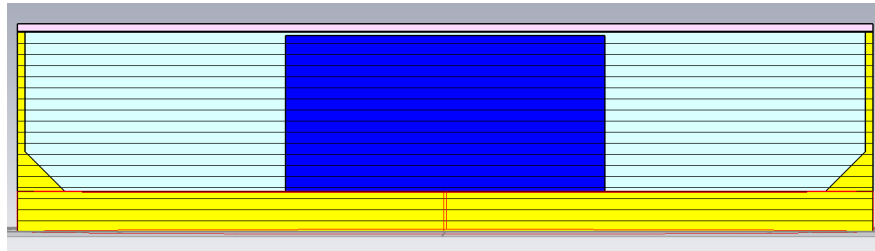


Figure 1: Dielectric resonator of Hakki-Coleman type

For the measurements, the TE_{011} mode is used, as shown in Fig.2,. The reason lies in the azimuthal surface current distribution; thus, insensitive to the electrical contact of the sample with the metallic enclosure.

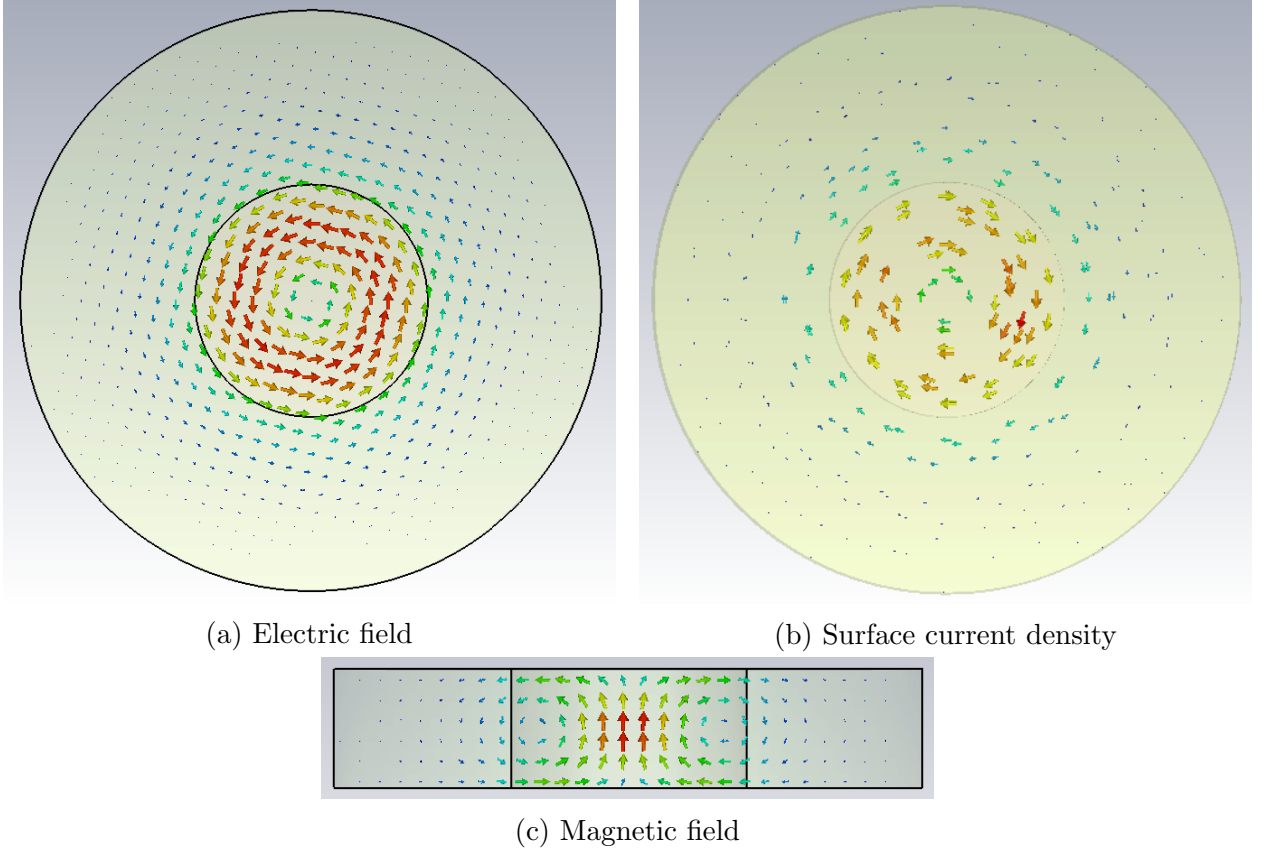


Figure 2: Electromagnetic field distribution of the TE_{011} mode.

2 Design of a multi-mode-multi-crystal dielectric resonator

Our goal was to find the dimensions of a cavity that would be capable of housing four different dielectrics, thus, making it possible to perform surface resistance measurements at a minimum of four fundamental resonance frequencies and their harmonics. The dielectrics that would be used in such a cavity are:

- sapphire (Al_2O_3)
- lanthanum aluminate ($LaAlO_3$)
- titanium dioxide (TiO_2)
- strontium titanate ($SrTiO_3$)

We approached the task in two steps. In the first step, we wanted to find the dimensions for a cavity that could be used to measure solely with the TE_{011} mode (see Section 1.3) for all dielectrics mentioned above, where all four crystals would preferably have the same dimensions, thus, a multi-crystal dielectric resonator. In the second step, we tried to determine if higher-order modes could be used in addition to the fundamental TE-mode. Hence,

a multi-mode-multi-crystal DR (M^3CDR). To accomplish this goal, we used the numerical computational tool CST STUDIO SUITE 2018 [1]; a software for designing and analyzing electromagnetic components.

2.1 Multi-crystal dielectric resonator

The first step in finding the right dimensions for the DR was deciding the ratio between the diameter of the cavity and the diameter of dielectrics. We decided to vary the ratio in the interval of 2.5-3 because with this ratio, there will not be any influence of the lateral wall on the measurements [6]. Since TiO_2 cannot be grown as a single crystal beyond 16 mm in diameter and height, the maximum dimension of our cavity that can result is 48 mm in diameter and 16 mm in height. The main criteria that we set for the simulations were that the difference in resonance frequencies between our desired mode and its neighboring modes was at least 0.1 GHz and the resonance frequency for sapphire shall be less than 20 GHz.

The results of the CST simulations for the different dielectrics and mode separation are presented in Fig.3. As can be seen in the same figure, the separation between our desired mode and its neighboring modes indicated by the "error bars", at room temperature, for all four of the dielectrics is greater than our initial constraint of 0.1 GHz. This is as well valid at cryogenic temperatures, except for $SrTiO_3$. The exact values for this plot are summarised in the Tabs.7 to 9 in the appendix.

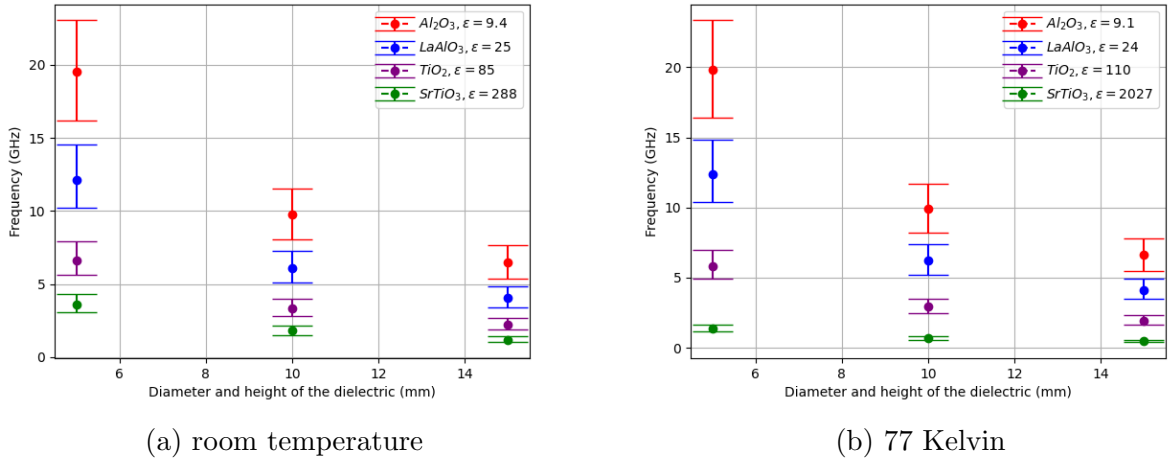


Figure 3: Separation of the TE_{011} -mode from its neighbouring modes at room temperature and cryogenic temperature for different dimensions of the dielectrics.

2.2 Multi-mode-multi-crystal dielectric resonator

Since our first goal was accomplished to a certain extent, we decided to go one step further and try to find dimensions for which the three modes: TE_{011} , TE_{021} and TE_{013} were well separated simultaneously; thus, making it possible to use one cavity to measure at 12 different frequencies. The TE_{021} and TE_{013} are shown in Fig.4. As can be seen from these figures,

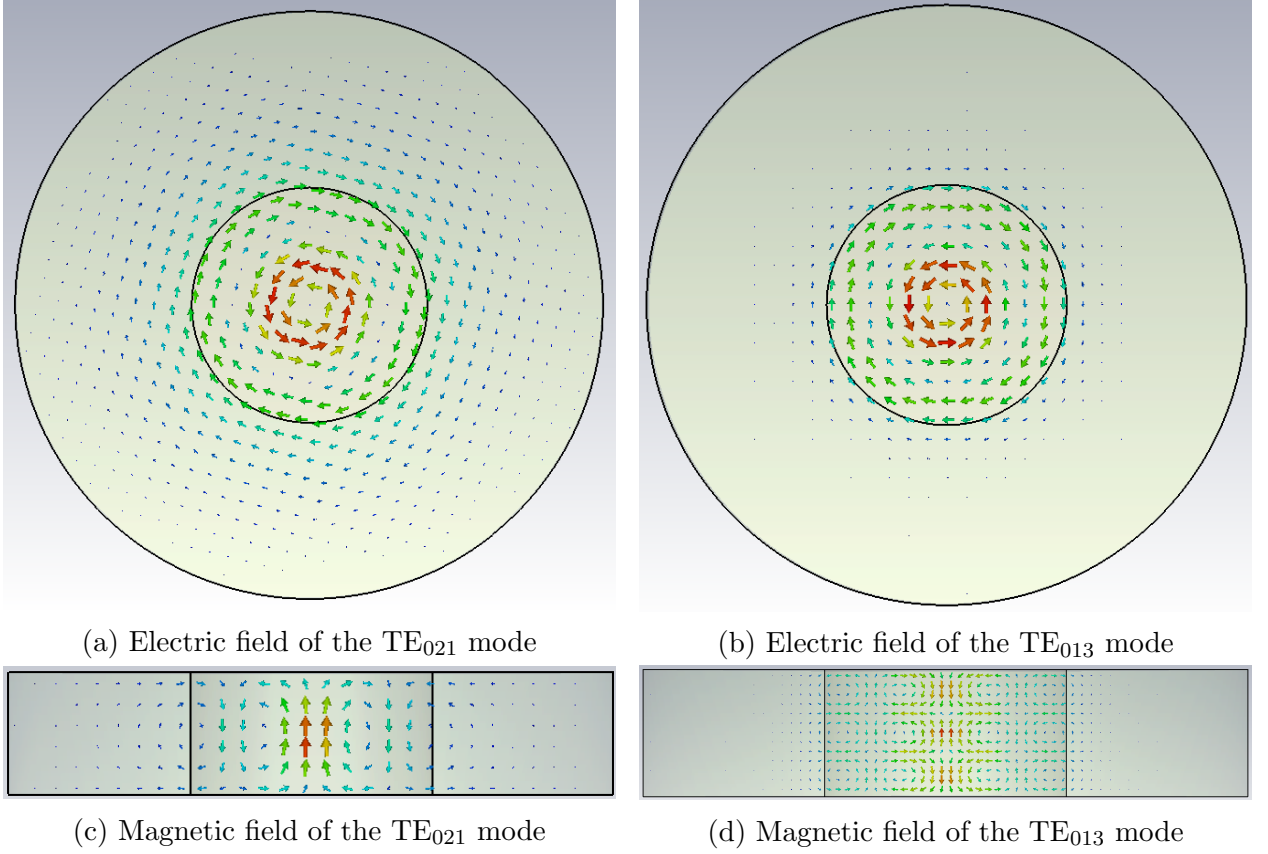


Figure 4: Electric and magnetic fields of modes TE_{012} and TE_{013}

the TE_{021} mode has two maxima in the azimuthal direction and one maximum across the z -component. The TE_{013} mode has one maximum in the azimuthal direction and three maxima across the z -component.

After running several simulations trying to find dimensions for which all three modes were well-separated according to the initial constraint of 0.1 GHz separation, it was evident that it is not possible to have all three of them well-separated from their neighboring modes. Consequently, we adjusted and decided to look for dimensions for which only the TE_{011} and TE_{021} modes were well-separated. The reason behind this is the fact that we can couple to TE_{021} similarly like to the TE_{011} mode.

The first simulations were of the dielectrics with equal diameter and height. It was not possible to find any dimensions for which TE_{021} or TE_{013} were well-separated for $SrTiO_3$, even after lowering the separation criteria. Because of that, we started looking for dimensions that would only work for the other three dielectrics. After more simulations, we concluded that keeping diameter and height equal would not give desired results, so we started varying the dielectric diameter and, thus, the cavity's diameter. Still, at least one dielectric never had enough separation between modes for every combination. Since TiO_2 has a high and stable permittivity we wanted to figure out the dimensions of that crystal first. The height of the crystal was kept constant, but its diameter was varied in order to find one case in which our desired modes would be well-separated. The TE_{021} mode has a lower resonance frequency than the TE_{013} mode when the radius of the dielectric and its height are close

in dimensions. After further simulations, we found that for the ratio between the height of the dielectric and its radius $\frac{h_{crystal}}{r_{crystal}} = 1.6$ both modes, TE₀₁₁ and TE₀₂₁, were well-separated from their neighboring modes. This ratio could be applied to LaAlO₃ and TiO₂ but did not work for Al₂O₃. However, Al₂O₃ had both modes well-separated in a few cases; one of them was when the height and radius of the dielectric were equal.

Since we want to use these dielectrics in the same cavity, their height must be the same, but their radii can differ. As aforementioned, TiO₂ cannot be grown beyond 16 mm neither in diameter nor in height, and as far as we know, all of the other three dielectrics can be grown beyond these dimensions, which means 16 mm was our limit for the height of the dielectrics. Two example configurations with resonance frequencies and separation between modes are given in Tabs. 1 and 2 for maximal and minimal cavity dimensions, given our limitations.

Table 1: Separation of modes TE₀₁₁ and TE₀₂₁ from their neighbouring modes for different dielectrics for dimensions of $h_{crystal}=6$ mm, $d_{crystal-Al_2O_3}=12$ mm, $d_{crystal-LaAlO_3, TiO_2, SrTiO_3}=7.5$ mm, $d_{cavity}=30$ mm, which is minimum diameter of the cavity

Dielectric	Mode	f_0 (GHz)	Δf mode before (GHz)	Δf mode after (GHz)
Al ₂ O ₃	TE ₀₁₁	11.311	1.380	0.980
	TE ₀₂₁	17.175	0.441	0.115
LaAlO ₃	TE ₀₁₁	8.804	1.360	0.714
	TE ₀₂₁	15.410	0.146	0.648
TiO ₂	TE ₀₁₁	4.475	0.726	0.473
	TE ₀₂₁	8.413	0.241	0.307
SrTiO ₃	TE ₀₁₁	2.608	0.393	0.265

Table 2: Separation of modes TE₀₁₁ and TE₀₂₁ from their neighbouring modes for different dielectrics for dimensions of $h_{crystal}=12$ mm, $d_{crystal-Al_2O_3}=23$ mm, $d_{crystal-LaAlO_3, TiO_2, SrTiO_3}=15$ mm, $d_{cavity}=57.5$ mm, which is maximum diameter of the cavity

Dielectric	Mode	f_0 (GHz)	Δ mode before (GHz)	Δ mode after (GHz)
Al ₂ O ₃	TE ₀₁₁	5.753	0.572	0.538
	TE ₀₂₁	8.872	0.158	0.285
LaAlO ₃	TE ₀₁₁	4.402	0.679	0.386
	TE ₀₂₁	7.706	0.187	0.165
TiO ₂	TE ₀₁₁	2.397	0.362	0.237
	TE ₀₂₁	4.204	0.124	0.151
SrTiO ₃	TE ₀₁₁	1.304	0.132	0.259

Al₂O₃ does not have equal radius and height in Tab.2. But that is not an issue since it was stated before that there are more cases for which Al₂O₃ has well-separated modes. As can be seen from Tab.1 and 2, all four of the dielectrics have TE₀₁₁ mode well-separated

for given dimensions at room temperature. TE_{021} mode is also well-separated for dielectrics Al_2O_3 , $LaAlO_3$ and TiO_2 . However, $SrTiO_3$ does not meet the criteria of separation between modes, which we set for TE_{021} mode.

2.3 M^3 CDR conclusions

We found the cavity dimensions that could house four different dielectrics to measure surface resistance at seven different resonance frequencies. In order to have the TE_{021} mode well-separated from its neighboring modes ratio between the height and the radius of the dielectric must be 1.6 for TiO_2 . This ratio can be applied to $LaAlO_3$ as well. Al_2O_3 has this mode well-separated when its height and its radius are equal. To have $SrTiO_3$ dielectric with dimensions equal to the other three dielectrics, we would have to lower the criteria for well-separated modes for this dielectric.

3 Measurements of laser treated copper plates

Besides simulations, we performed measurements of the surface resistance of several copper plates with the aid of a previously developed dielectric resonator with a diameter of 105 mm and a height of 20 mm [4]. In the center of the cavity was a sapphire crystal with a diameter of 40 mm and a height of 19.5 mm. The DR was connected to a VNA, which gives the resonance frequency and quality factor from it. After that, the surface resistance was calculated using equations (2) and (3). We put the desired sample plate to close the DR, and on top of the sample plate was a ring that was screwed into the cavity, holding everything in place, as shown in Fig.5.



Figure 5: The set-up that we used in the lab, consisting of a copper plate on top of the dielectric resonator, connected to the VNA

3.1 Laser treated copper - LESS

Laser engineered surfaces were produced by the University of Dundee, with the main motivation of electron cloud mitigation without alternating the surface resistance [3]. Two sets of samples were created which should have been equally treated of which each sample pair consistst of one radial and one spiral laser pattern. In our measurements we wanted to see if the surface resistance varies for surface currents running perpendicular to the groves or along the length of them. Bear in mind, that the surface currents in the DR run azimuthally. A 10 ps pulsed laser with wavelength of 532 nm was used with a repetition rate of 200 kHz. An average fluence of 0.9 J/cm^2 was delivered on a $4\sigma = 52 \mu\text{m}$ surface, corresponding to an equivalent power of 4 W. The laser was swept along the radial or tangential direction with a speed of 15 mm/s, the distance between groves was set to $45 \mu\text{m}$. The resulting samples can be seen in Fig.6.

The first measurement was performed at room temperature and 3.4 GHz, to compare surface resistance of a simple copper plate, with the two laser engineered samples. The results are given in Tab.3. LESS I plates have surface resistance which is approximately equal to the surface resistance of the regular copper plate, while LESS II plates have a much larger surface resistance, especially the radially treated copper plate, which is to be expected considering the current flows azimuthally. The difference between LESS I and Cu for both, radial and spiral grooves, is rather small, which suggests that the main increase in surface resistance is caused by the general roughness of the surface and other treatment parameters than aforementioned as two years lie within the sample pairs. LESS II results seem more reasonable for the given Laser parameters and copper droplets, formed by the laser treatment, which could be the reason why the increase in surface resistance is so high.

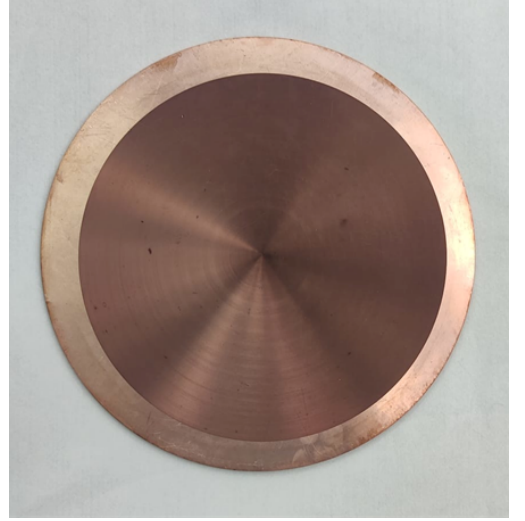
Table 3: Unloaded quality factor values and surface resistance for each of the plates, as well as surface resistance of LESS plates compared to the surface resistance of the regular copper plate.

Plate	Q_0	R_s	$R_{s, \text{LESS}}/R_{s, \text{Cu}}$
Copper plate	13563	0.0152	1
Radially treated, LESS I	13124	0.0163	1.0724
Azimuthally treated, LESS I	13191	0.0161	1.0592
Radially treated, LESS II	8176	0.0390	2.5658
Azimuthally treated, LESS II	10889	0.0234	1.5395

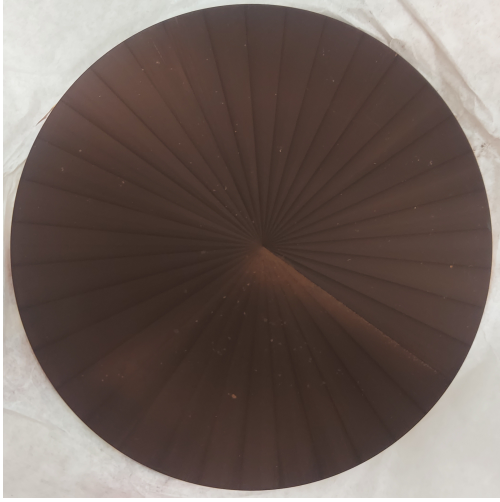
In CST it is possible to simulate the roughness of different materials. CST uses Hammerstad-Jensen model to describe effects of roughness on the conductivity [5]. The roughness that can be simulated in CST is of radially treated copper plates. For the azimuthally treated plates we should get the roughness that is more connected to the presence of nanoparticles while for the radial treated plates we should get the roughness of the grooves. Therefore, we compared results we would get based on this model and the ones we actually measured, to see if they would be approximately equal. Using optical imaging we measured roughness of LESS plates to be of nearly $30 \mu\text{m}$ and the roughness that we got from Hammerstad-Jensen formula was less by factor 10. The reason for this could be the presence of nanoparticles, but we cannot be sure at the moment.



(a) LESS I radially treated



(b) LESS I azimuthally treated



(c) LESS II radially treated



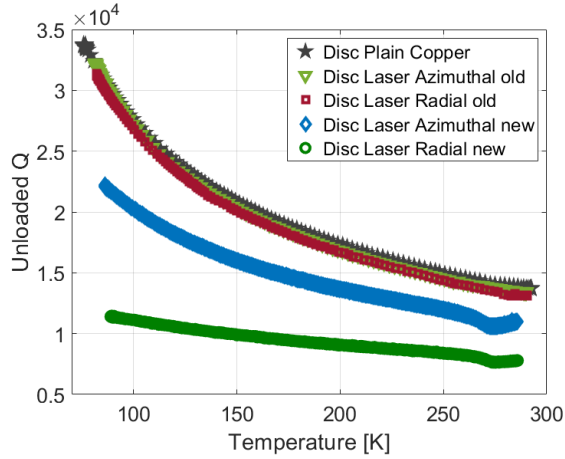
(d) LESS II azimuthally treated

Figure 6: Laser treated copper plates.

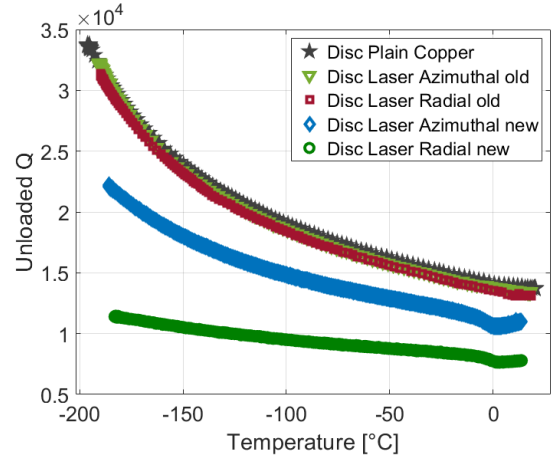
In addition, the quality factor of the LESS plates was measured as a function of temperature by cooling the DR down to cryogenic temperatures with liquid nitrogen and by natural heating back to room temperature. The data points were taken during the heat up and the results are given in Fig.7a. As can be seen in the same figure, the quality factor decreases with increase in temperature, which means that surface resistance increases. The behaviour seen at room temperature is as well seen at cryogenic temperatures. What is also visible in Fig.7b, is the fact that there was water in the system, which can be concluded from the fact that curve changes its behaviour around 0°C .

3.2 Collimator study

We wanted to measure the surface resistance of a collimator for the LHC, as visible in Fig.8a. For these measurements, the same DR was used that was described in the previous section,



(a) Q_0 dependancy on temperature of LESS I plates

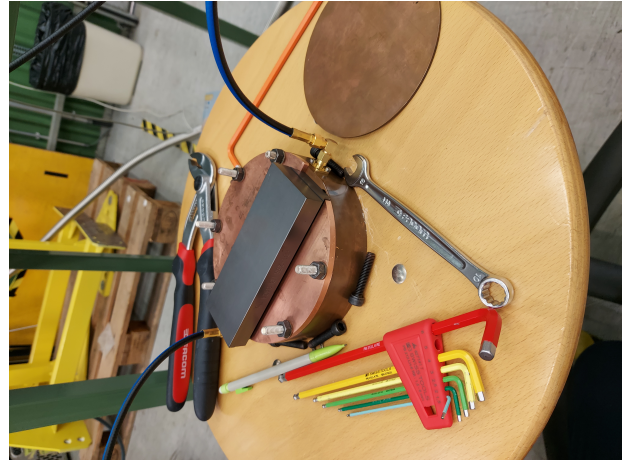


(b) Q_0 dependancy on temperature of LESS I plates

and since the collimator is smaller than the cavity (it cannot cover the entire area) we needed to construct a sample holder. The holder consisted of a separated copper plate with a cut-out for the collimator. The sample holder consists of two pieces, covering half of the dielectric. The assembled cavity is shown in Fig.8.



(a) Molybdenum collimator

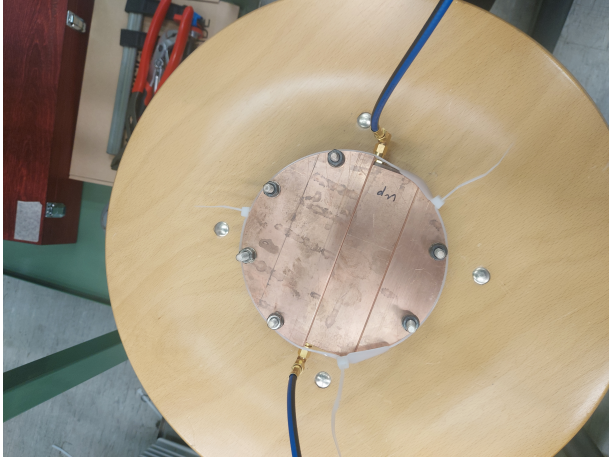


(b) Collimator DR assembled

Figure 8: Set-up with the collimator.

The first measurement, however, we performed with the copper piece that was initially cut-out of the copper disc, see Fig.9. In this way, we could measure the quality factor for a separated plate system with and without a gap, compare to an uncut copper samples, and see how big the radiation losses were. As can be seen in Tab.4, the radiation losses were enormous both, with and without the gap. Nevertheless, as expected, the smaller the gap, the higher the quality factor was and the best results were accomplished when lateral pressure was applied.

In order to check the contact between the separated plates, we performed a "light test". It is visible from Fig.10 that there is a large gap between the parts of the plate, meaning



(a) Separated plate without the gap



(b) Separated plate with the gap

Figure 9: Set-up with the separated plate

Table 4: Quality factors of separated plate in different cases

Plate	Regular Cu plate	No gap	1 mm gap	2 mm gap	With pressure
Q_0	13563	5742	3.49	297	10900

there are radiation losses and bad contact resistances, even when pressure is applied. The reason behind this are limitations in accuracy of manufacturing.



Figure 10: Limitations in accuracy of manufacturing on a separated plate

Nevertheless, after these measurements we replaced the middle part of the plate with the collimator made of Molybdenum. Tab.5 shows that the quality factor is very low when we use the collimator instead of the copper middle part of the plate. The resistivity of molybdenum is $0.056 \cdot 10^{-6} \Omega m$, but the resistivity we get when using the quality factor that we measured, 1246, is $34.3 \cdot 10^{-6} \Omega m$ which is by factor 100 more than the real value. These results could possibly be better if separated plate was made with more accuracy and if a way of applying more pressure on the sides of the plate was found.

Table 5: Values of quality factors when middle part of separated plate is replaced by the collimator

	Collimator	Collimator with weight on top	Pressure on sides of plate
Q_0	735	1041	1246

3.3 Strontium titanate dielectric for two cavities

The final goal was to find dimensions for a SrTiO_3 dielectric that could be used both in a new cavity we would construct and in the existing DR if possible. The best results, meaning the most significant separation between modes, were for the diameter of 11 mm and height of 8.8 mm. These dimensions were simulated for the cavity already available, as well as the one we could order with dimensions $h_{\text{cavity}} = 12 \text{ mm}$ and $d_{\text{cavity}} = 57.5 \text{ mm}$. Results are given in the table below. As can be seen from Tab.6, it is possible to use one SrTiO_3 dielectric for both cavities and get a separation that would be greater than 0.1 GHz.

Table 6: Separation of mode TE_{011} from its neighbouring modes for SrTiO_3 for two different cavities

Cavity	Temperature	f_0 (GHz)	Δ mode before (GHz)	Δ mode after (GHz)
Lab	RT	1.62077	0.329	0.287
	77 K	0.611534	0.123	0.108
To order	RT	1.64717	0.335	0.267
	77 K	0.621232	0.125	0.101

4 Conclusion

We demonstrated the possibility of constructing a multi-mode-multi-crystal dielectric resonator. After running simulations in CST, we found the cavity dimensions needed for it to house four different crystals. We found ratios of height and diameter of three different dielectrics, for which the TE_{011} and TE_{021} modes are well-separated from their neighboring modes. This cavity could be used to measure at seven different resonance frequencies because we also found dimensions of a fourth dielectric for which the TE_{011} mode was well-separated.

We performed room and cryogenic temperature surface resistance measurements of different laser-treated copper plates. We proved that the laser treatment direction compared to the current density flow correlates. Furthermore, we showed that the Hammerstad-Jensen model could not be used to describe the roughness of these plates.

The measurement of a collimator did not work out due to high radiation losses. The reason for this is the limitation in the accuracy of manufacturing.

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A Appendix

Table 7: Difference in frequencies between mode TE_{011} and its neighbouring modes at room temperature for four different dielectrics when ratio between diameter of the cavity and diameter of the crystal is $\frac{d_{cavity}}{d_{crystal}} = 2.5$ for dimensions of the crystal $d_{crystal} = 5\text{ mm}$ and $h_{crystal} = 5\text{ mm}$

Dielectric	Resonance frequency (GHz)	δ mode before (MHz)	δ mode after (MHz)
Al_2O_3	19.532	3373	3517
$LaAlO_3$	12.149	1939	2384
TiO_2	6.625	1022	1335
$SrTiO_3$	3.605	550	723

Table 8: Difference in frequencies between mode TE_{011} and its neighbouring modes at room temperature for four different dielectrics when ratio between diameter of the cavity and diameter of the crystal is $\frac{d_{cavity}}{d_{crystal}} = 2.5$ for dimensions of the crystal $d_{crystal} = 10\text{ mm}$ and $h_{crystal} = 10\text{ mm}$

Dielectric	Resonance frequency (GHz)	δ mode before (MHz)	δ mode after (MHz)
Al_2O_3	9.766	1686	1758
$LaAlO_3$	6.075	970	1192
TiO_2	3.313	511	667
$SrTiO_3$	1.801	275	361

Table 9: Difference in frequencies between mode TE_{011} and its neighbouring modes at room temperature for four different dielectrics when ratio between diameter of the cavity and diameter of the crystal is $\frac{d_{cavity}}{d_{crystal}} = 2.5$ for dimensions of the crystal $d_{crystal} = 15\text{ mm}$ and $h_{crystal} = 15\text{ mm}$

Dielectric	Resonance frequency (GHz)	δ mode before (MHz)	δ mode after (MHz)
Al_2O_3	6.511	1124	1172
$LaAlO_3$	4.049	646	795
TiO_2	2.208	340	445
$SrTiO_3$	1.201	183	240

Table 10: Difference in frequencies between mode TE_{011} and its neighbouring modes at 77 K for four different dielectrics when ratio between diameter of the cavity and diameter of the crystal is $\frac{d_{cavity}}{d_{crystal}} = 2.5$ for dimensions of the crystal $d_{crystal} = 5\text{ mm}$ and $h_{crystal} = 5\text{ mm}$

Dielectric	Resonance frequency (GHz)	δ mode before (MHz)	δ mode after (MHz)
Al_2O_3	19.835	3439	3517
$LaAlO_3$	12.396	1983	2428
TiO_2	5.827	896	1172
$SrTiO_3$	1.359	207	272

Table 11: Difference in frequencies between mode TE_{011} and its neighbouring modes at 77 K for four different dielectrics when ratio between diameter of the cavity and diameter of the crystal is $\frac{d_{cavity}}{d_{crystal}} = 2.5$ for dimensions of the crystal $d_{crystal} = 10\text{ mm}$ and $h_{crystal} = 10\text{ mm}$

Dielectric	Resonance frequency (GHz)	δ mode before (MHz)	δ mode after (MHz)
Al_2O_3	9.917	1719	1758
$LaAlO_3$	6.198	992	1214
TiO_2	2.913	448	586
$SrTiO_3$	0.679	103	136

Table 12: Difference in frequencies between mode TE_{011} and its neighbouring modes at 77 K for four different dielectrics when ratio between diameter of the cavity and diameter of the crystal is $\frac{d_{cavity}}{d_{crystal}} = 2.5$ for dimensions of the crystal $d_{crystal} = 15\text{ mm}$ and $h_{crystal} = 15\text{ mm}$

Dielectric	Resonance frequency (GHz)	δ mode before (MHz)	δ mode after (MHz)
Al_2O_3	6.612	1146	1172
$LaAlO_3$	4.132	661	809
TiO_2	1.942	299	390
$SrTiO_3$	0.453	69	91