Analytical Emittance Estimates for the FCC-ee
Summer Student Project Report 2021

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In this report, a series of analytical methods were carried out to study the effect of specific Future Circular electron-positron Collider (FCC-ee) lattice errors on the vertical emittance estimates, specifically, from the quadrupoles. A study into the limitations of the Analytical formula used, along with a comparison with a corrected FCC-ee lattice was carried out for a single rotated quadrupole. The overall objective is to apply a range of analytical methods to study the alignment sensitivity of the FCC-ee optics. A basic understanding in accelerator optics is assumed when reading this report.

1 Introduction

1.1 The Future Circular Collider

The Future Circular Collider (FCC) study is a research development of a post-LHC particle accelerator. With the aim to reach energies much higher than any previous accelerator at CERN. The FCC project examines scenarios for three different types of particle collisions: hadron (proton–proton and heavy ion) collisions in a collider known as FCC-hh, electron–positron collisions in the FCC-ee, and proton–electron collisions in the FCC-eh.

This project area's overall purpose is to continue and exceed the research of the LHC after its closure in around 2040. The table below shows some comparison figures between the HL-LHC, FCC-hh and FCC-ee.

	HL-LHC	FCC-hh	FCC-ee ($t\bar{t}$)
Circumference	27 km	100 km	100 km
COM collision energy	14 TeV	100 TeV	365 GeV
No of particle bunches	2808	10600 - 55300	16640
Particles	Protons/Heavy Ions	Protons/Heavy Ions	Electrons/Protons
Luminosity 10^{34} cm^{-2} s^{-1}	5	5-30	1.55-1.8
Nominal transverse normalized emittance nm	2500	2200	1.34-1.46

Table 1: A table outlining some basic differences between the High Luminosity LHC, FCC-hh and FCC-ee, putting into perspective the scale of the proposed FCC [4].

The FCC-ee is a proposed circular e^+ e^- collider installed in the new 100 km tunnel, delivering high luminosity to four experiments at centre-of-mass energies ranging from 91 GeV (Z pole), over 160 GeV (W threshold), and 240 GeV (H production) to 350 GeV (t physics). The size upscale can be seen by the values shown in table 1 [3].

1.2 Accelerator Optics for the FCC-ee

To guide particles on a curved design orbit, bending elements are required. In an ideal ring, the vertical closed orbit is flat and does not depend on the momentum of the particles and discrete momentum changes should not excite the particles. However, in reality magnetic imperfections and alignment errors couple the vertical motion in the horizontal and longitudinal planes. This causes the vertical closed orbit also to be momentum dependant and subsequently the vertical emittance to be driven by quantum excitation [9].

A key challenge when designing an electron-positron collider is to predict the impact that magnet and alignment errors will have on the beam emittance and to develop strategies to minimise it [1].

1.3 Alignment Challenges

The alignment strategy of the FCC-ee could have a large impact on its luminosity. Looser alignment tolerances result in higher vertical emittance; tight alignment tolerances around the 100 km ring increases expenses [8]. Studying the results of permutative errors on the magnets, both with simulations and analytically, is crucial for the design development of the FCC-ee. To achieve ultra-low vertical emittance, a highly effective emittance tuning scheme is required. The effect of alignment errors on the vertical beam size in electron storage rings is laid out in [2].

Such methods can be used to help understand the impact of misalignments of certain magnet groups and to develop an efficient alignment strategy. The goal here is to directly quantify the impact that different errors in the quadrupoles around the ring have on the vertical emittance.

2 Methods

2.1 Analytical Methods

This paper is built upon the previous work of F. Zimmerman, L. Van Riesen-Haupt, T. Charles and R. Tomaś in [6]. In which similar analytical methods are compared to MADX simulations of vertical emittance estimates for the FCC-ee. In addition, analytical methods for studying the alignment tolerances in linear accelerators have been established, tested and are well described in the literature [7].

The analytical vertical emittance for a single quadrupole is said to be,

$$\frac{\epsilon_y}{\theta^2_{\text{quad}}} = \frac{J_z\sigma_\delta^2}{\sin^2(\pi Q_y)}\beta_y\eta_x^2(k_1L)^2 \ + \frac{J_x[1-\cos(2\pi Q_x)\cos(2\pi Q_y)]\epsilon_x}{J_y[\cos(2\pi Q_x)-\cos(2\pi Q_y)]^2}\beta_x\beta_y(k_1L)^2, \tag{1}$$

where J_x, J_y and J_z are the horizontal, vertical and longitudinal partition numbers respectively; Q_x and Q_y are the horizontal and vertical tunes; ϵ_x and σ_δ are the horizontal emittance and normalised momentum spread at equilibrium. The calculation is computed for a single turned quadrupole with β_x, β_y and η_x being the horizontal and vertical beta functions and the horizontal dispersion in the misaligned quadrupoles and k_1L being the normalised integrated strength of the quadrupole.

2.2 Simulations on MADX

In this paper, the results for the vertical emittance estimates from the analytical expression below in equation 1 are compared against results obtained from simulations done using MADX.

A single quadrupole is rolled by a series of angles and the below formula is applied to calculate respective the vertical emittance via a python script. The ratio between the vertical emittance MADX simulation, $\epsilon_y MADX$, and the analytical vertical emittance estimate of the rolled quadrupole,$\epsilon_y ANALYTICAL$, is noted and compared to the ideal value: 1.

$$ratio = \frac{\epsilon_{yMADX}}{\epsilon_{yANALYTICAL}} \tag{2}$$

A python script was set up to load and extract the sequence simulations from MADX. It then turns the quadrupole at an angle and calculates the vertical emittance using 'EMIT MODULES' [5]. It then extracts this simulation value and compares it to the value from the analytical formula. It takes a ratio of the MADX simulated vertical emittance to calculated analytical vertical emittance; if this ratio is close to 1 then the analytical formula is sufficient in calculating the vertical emittance.

Two studies were carried out, the first being an investigation into the effect of a rotation angle of the quadrupole on the vertical emittance ratio. This plot highlights the limitations of the analytical formula as the plot breaks down when the angle is sufficiently large. In this study, an ideal lattice is used where the initial vertical emittance is zero, meaning any vertical emittance seen is a result of the rotation in the quadrupole at hand.

The second study performed after applying a corrected lattice that contains a series of 'typical errors', which are then corrected. A single quadruple is once again rotated by a series of angles to study whether the formula can account for a 'non-ideal' lattice. This new formula accounts for the initial offset by subtracting the vertical emittance from the MADX simulation; this gives the increase in emittance as a result of the quadrupole being rotated after the errors have been accounted for.

$$\Delta\epsilon_y = \epsilon_{yMADX} - \epsilon_y 0 \tag{3}$$

where $\epsilon_y 0$ is the initial vertical emittances due to the errors from the lattice, ϵ_{MADX} is the MADX simulation emittance and $\Delta\epsilon_y$ is the increase in emittance due to the rotated quadrupole. The ratio in equation 2 is once again taken, with this new value $\Delta\epsilon_y$ against the analytical solution.

3 Results

3.1 Limitations of the Analytical Formula

A single quadruple rotated at angles 10 μrad, 100μrad and 1000 μrad was studied. The beam ring was simulated on MADX with this initial rotation error and the mean ratio was plotted against the rotation angle, it is clear from figure 1 that equation 1 breaks down after approximately 1000 μrad.

3.2 Corrected Lattices

The analytical formula was once again tested, this time after applying a corrected FCC-ee lattice. The figure 2 shows this method applied to the first four quadrupoles. A mean of ratio at each angle from 600-1000 μrad is plotted against the rotation angle. If the angle is made smaller than this, the formula breaks down. Figures 2 and shows the average ratio vs rotation angle as a linear increase, tending toward 1.

Figure 3 shows of plot of the ratios obtained for the first 4 quadrupoles when the method above is applied. It can be seen that the ratio corresponding with 1000 μrad stays the closest to 1 at each point.

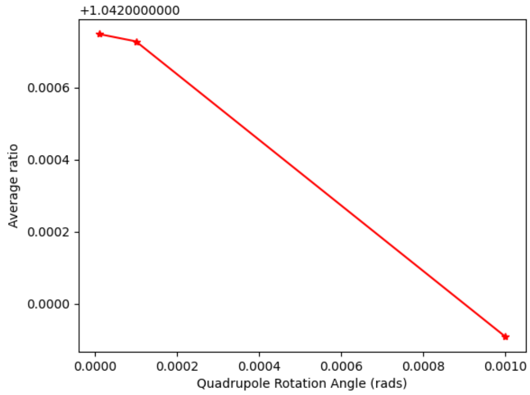

Figure 1: A plot showing how the average ratio deviates from 1.042 when the rotation angle is increased. The analytical formula breaks down completely a rotation angle higher than 1000 μrad.

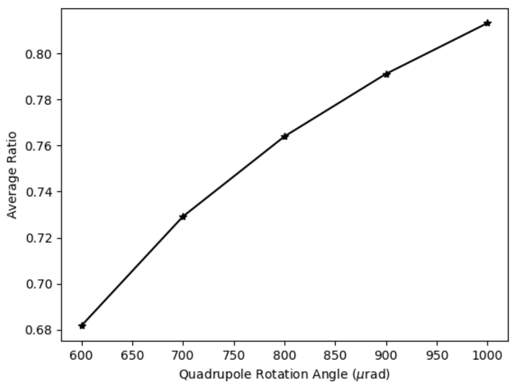

Figure 2: A plot showing the relationship between the average ratio and the rotation angle of the quadrupole. A clear trend that shows that the ratio tends to 1 as you approach 1000 μrad. The optimal angle was not found in this study.

4 Further Research and Conclusion

Overall, the analytical formula 1 is sufficient in predicting the vertical emittance for a single rotated quadrupole when compared with MADX simulations, granted that the rotation of the quadrupole is not too large or too small. A question to be asked is whether multiple quadrupole rotations applied simultaneously results in linear increase to the vertical emittance of the FCC-ee ring. In addition, exactly what angle of rotation creates the closest ratio to 1 in 'non-ideal' ring conditions. Or even to what extent can this formula simulate in 'worst case scenario' conditions.

In section 3.1, the plot shows to break down after approximately 1000 μrad. This is most likely due to higher order effects. Additionally, in section 3.2 a better ratio is established at approximately 1000 μrad, suggesting that perhaps the corrections to the lattice errors may have an impact on the

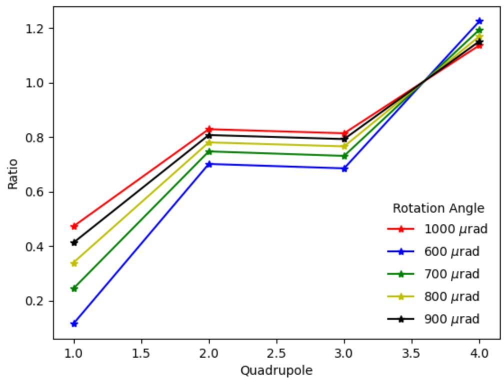

Figure 3: A plot showing the ratio obtained for each of the first four quadrupoles, each colour line represents a different rotation angle. The ratio gets closer to 1 by the fourth quadrupole, the this may be related to offset errors as the beam travels the ring.

vertical emittance in this case. It is unclear from the data whether this ratio would increase much past 1 as the beam continues interacting with dipoles around the ring, or whether the corrected lattice would maintain this ratio.

An extension to this study would be to attempt to compare the analytical formula to the MADX simulations for a higher number of quadrupoles. This would show whether the ratio would stay consistent as the beams continue through around the ring or not.

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