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博士研究生学位论文

题目： LHCb 上奇特强子态的
实验研究

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摘要

奇特强子态（例如四夸克态和五夸克态）与普通强子的明显区别是，其内部夸克之间存在更多可能的组配方式，可以为理解强相互作用提供更加丰富的信息。本论文利用欧洲核子研究中心大型强子对撞机上的底夸克实验（LHCb）高统计量的质子-质子对撞数据，对含有正反粲夸克对（ $c\bar{c}$ ）的奇特强子态开展了系列研究。结合作者博士期间完成的工作，本论文汇报了三个物理分析结果。

首先，本论文对相关的物理背景和当前奇特强子态实验研究进展做了一个基本的介绍。粒子物理的标准模型描述了现在我们所有已知的基本粒子以及其电磁相互作用，弱相互作用和强相互作用。其中夸克和胶子之间的强相互作用是用量子色动力学（QCD）来描述的。QCD 作为强相互作用的基本理论在高能标下已经得到实验精确检验，但在低能标下的非微扰性质有待实验与理论的进一步研究。研究非微扰 QCD 是当代粒子物理学的重要前沿之一，一方面有助于深入理解强相互作用的本质，另一方面也有助于理解电弱过程中的强相互作用本底，提高寻找新物理的灵敏度。强子谱结构是强子内部复杂相互作用的外在表现，可以为研究非微扰 QCD 提供丰富的信息。重味强子中的重味夸克在强子内部的运动近似呈非相对论性，是建立低能有效模型的一个出发点，并且实验测量相对更容易。对强子谱进行研究，可以加深对非微扰 QCD 的理解。在夸克模型中，传统强子是指由两个或者三个价夸克组成的强子，超出三个价夸克组成的强子被称为奇特强子态，其中多夸克态（比如四夸克态，五夸克态）是最主要的一类奇特强子态。在实验上寻找和研究奇特强子态的性质为理解非微扰 QCD 提供了新的场所，并且可以检验很多现象学模型，是当前强子谱学研究的热点。实验上第一个被观测到的奇特强子态是 $\chi_{c1}(3872)$ ，它于 2012 年在 Belle 探测器上被发现。这个粒子态的质量在 $D^*\bar{D}$ 阈值附近，并且在势模型中，传统的粲夸克偶素谱中不包含这个粒子态，一直被认为是四夸克态的候选者。测量该粒子态的相关性质仍然是强子谱学当前研究的热点之一。自 $\chi_{c1}(3872)$ 被发现之后，目前实验上已经观察到了近三十个四夸克态候选者。除了像 $\chi_{c1}(3872)$ 这种中性的四夸克态候选者，带电的类粲夸克偶素结构是四夸克态存在更为直接的证据，这类带电类粲偶素四夸克态被称为 Z 粒子。实验上除了对四夸克态进行研究，也在一直寻找奇特重子态。2015 年，LHCb 首次报告了五夸克态的发现，进而激起五夸克态研究的热潮。目前实验上对五夸克的实验研究结果还不够丰富，对五夸克态理论上的理解需要更多实验结果的支持。从 QCD 第一性原理出发理解奇特强子态需要格点 QCD 的帮助，但是计算难度比较大。当前理论上对奇特强子态的理解更多是从夸克模型出发，使用不同的唯象学模型来解释实验发现或者预言

新的奇特强子及其性质。目前理论上对多夸克态的唯象学解释主要有三种。第一种模型是紧致多夸克结构，这种模型认为胶子传递多夸克之间的相互作用。第二种是分子态模型，该模型认为夸克云之间通过介子传递相互作用。第三种认为多夸克信号是运动学阈值效应造成的，并不是所有的多夸克候选者都是真正的强子态。

本论文的结果是通过分析 **LHCb** 探测器采集的数据得到的。**LHCb** 探测器是 **LHC** 上四个实验中专门用来研究重味物理的装置。不同于 **LHC** 上其它实验桶状的探测器结构，**LHCb** 是一个单臂前向的粒子探测谱仪，大致可以测量的粒子赝快度范围为 $1.8 < \eta < 4.9$ 。在对撞主顶点附近的探测器是顶点探测器，**LHCb** 顶点探测器可以准确地重建出对撞主顶点和衰变次级顶点的位置信息。粒子从顶点探测器飞出后，进入前端的切伦科夫探测器，它和径迹探测器系统后边的后端切伦科夫探测器一起为带电径迹提供了准确的粒子身份信息。接下来，粒子进入径迹探测器系统，在上游径迹探测器和下游径迹探测器之间是垂直地面的磁场，磁场强度大概为 4 Tm ，通过测量带电粒子在磁场前后的偏转，可以准确地重建出带电粒子的动量。最后，粒子进入量能器系统和缪子探测器。量能器系统用来区分光子，电子和强子的身份信息，这个系统包括了电磁量能器，强子量能器，闪烁体板和预簇射探测器。缪子探测器可以准确地测量出缪子的径迹。线上的事例筛选是基于量能器和缪子探测器的硬件触发。接下来在软件平台上对整个过触发的事例进行完全重建。**LHCb** 探测器优秀的径迹重建能力和卓越的粒子身份信息鉴别能力，为强子谱的研究提供了很好的保障。

正如前文提到的， $Z_c(3900)^+$ 和 $Z_c(4430)^+$ 等带电类粲偶素是存在四夸克态的直接证据，因为其最小夸克组分为 $c\bar{c}u\bar{d}$ ，除了 $c\bar{c}$ 之外，至少还包含一对不同味道的轻夸克 $u\bar{d}$ 。在本论文书写之前，在实验上还没有发现包含 s 夸克的带电四夸克态。为了寻找 Z_c^+ 对应的 $Z_{cs}^+(c\bar{c}u\bar{s})$ 态，本论文利用 **LHCb** 收集的 9 fb^{-1} 数据样本，在 $B^+ \rightarrow J/\psi \phi K^+$ 衰变道中对 Z_{cs}^+ 态进行了寻找。本论文首先对信号的重建和筛选进行了优化，比之于只采用 **Run 1** 数据进行的分析，引入了多变量分析方法来区分信号和本底，并且把末态强子的身份鉴别信息放入了多变量训练中。通过这个方法，使得重建效率提升了 15%，本底压低能力提升到了六倍。最后重建的信号个数为 24220 ± 170 。为了探究这个衰变道中可能存在的共振态结构，本论文对该衰变道开展了完整的六维分波分析。分波拟合模型主要包含两部分，其中 B^+ 衰变衰变概率密度函数通过螺旋度公式给出，本底概率密度函数通过非信号质量区间的实验数据样本给出。 B^+ 衰变包含了三种衰变链，分别为 $K^{*+}(\rightarrow \phi K^+)J/\psi$ ， $X(\rightarrow J/\psi \phi)K^+$ 和 $Z_{cs}^+(\rightarrow J/\psi K^+)\phi$ 。每个衰变链用一个质量参数和五个角度参数进行描述。比如 K^{*+} 衰变链中有六个可观测量： $\Phi \equiv (m_{\phi K}, \theta_{K^*}, \theta_{J/\psi}, \theta_{\phi}, \Delta\varphi_{K^*, J/\psi}, \Delta\varphi_{K^*, \phi})$ ，其中 θ 代表螺旋度角， $\Delta\varphi$ 是两个衰变平面的夹角。在该拟合模型中，共振态线型用 Breit-Weigner 公式描述，Flatté 和 K-Matrix 参数化模型用来进行系统误差的研究。本论文首先检验了 **Run 1** 分析中采用的拟合模型，发

现在更大统计量下，Run 1 拟合模型拟合不完善，所以对原拟合模型进行了优化。本论文首先优化了 K^{*+} 共振态模型，通过检验运动学空间允许下的所有 K^{*+} 贡献，发现数据无法被很好地描述。接下来研究了奇特强子对拟合结果的影响。本论文通过测试不同自旋宇称的奇特强子态对拟合结果的影响，经过多轮优化之后，找到了默认的拟合模型，该模型包含了九个 K^{*+} ，七个 X ，一个非共振态贡献和两个 Z_{cs}^+ 。如果再增加更多共振态的贡献，对最后的拟合没有明显的改善，通过这个方法验证了默认拟合模型的可靠性。此外本论文还研究了在默认的拟合模型下，加入更多 K^{*+} 的拟合结果，标记为扩展模型，用来保守地估计信号显著度。本论文首次在 $J/\psi K^+$ 末态中发现了两个含 s 夸克的带电四夸克态 $Z_{cs}(4000)^+$ 和 $Z_{cs}(4220)^+$ 。前者的质量和宽度分别为 $4003 \pm 6_{-14}^{+4} \text{ MeV}$ 和 $131 \pm 15 \pm 26 \text{ MeV}$ ，自旋宇称为 $J^P = 1^+$ ；后者的质量和宽度分别为 $4216 \pm 24_{-30}^{+43} \text{ MeV}$ 和 $233 \pm 52_{-73}^{+97} \text{ MeV}$ ，自旋宇称无法确定。其中第一项不确定度为统计不确定度，第二项为系统不确定度。此外，在 $J/\psi \phi$ 末态中，除了之前实验已经观测到的四个 X 态 $X(4140)$, $X(4274)$, $X(4500)$ 和 $X(4700)$ ，之外，还观测到两个新的 X 态 $X(4685)$ 和 $X(4630)$ ，并确定前者的自旋和宇称为 $J^{PC} = 1^{++}$ 。本论文还分析了测量结果的系统误差，其中系统误差的来源可以分为两类，第一种系统误差来源于数据样本，第二种系统误差来源于分波拟合模型，其中第二种系统误差占主导。对于 $Z_{cs}(4000)^+$ 质量和宽度的测量，其最大的系统误差来源于 $Z_{cs}(4220)^+$ 自旋宇称的假设。在这个分析中与四夸克态相关的拟合结果列于下表 (第一个误差为统计不确定度，第二个误差为系统不确定度，括号内的信号显著度为默认模型的结果)：

自旋宇称	成分	显著度 [$\times\sigma$]	质量 [MeV]	宽度 [MeV]	占比 [%]
2^-	$X(4150)$	4.8 (8.7)	$4146 \pm 18 \pm 33$	$135 \pm 28_{-30}^{+59}$	$2.0 \pm 0.5_{-1.0}^{+0.8}$
1^-	$X(4630)$	5.5 (5.7)	$4626 \pm 16_{-110}^{+18}$	$174 \pm 27_{-73}^{+134}$	$2.6 \pm 0.5_{-1.5}^{+2.9}$
0^+	$X(4500)$	20 (20)	$4474 \pm 3 \pm 3$	$77 \pm 6_{-8}^{+10}$	$5.6 \pm 0.7_{-0.6}^{+2.4}$
	$X(4700)$	17 (18)	$4694 \pm 4_{-3}^{+16}$	$87 \pm 8_{-6}^{+16}$	$8.9 \pm 1.2_{-1.4}^{+4.9}$
1^+	$X(4140)$	13 (16)	$4118 \pm 11_{-36}^{+19}$	$162 \pm 21_{-49}^{+24}$	$17 \pm 3_{-6}^{+19}$
	$X(4274)$	18 (18)	$4294 \pm 4_{-6}^{+3}$	$53 \pm 5 \pm 5$	$2.8 \pm 0.5_{-0.4}^{+0.8}$
	$X(4685)$	15 (15)	$4684 \pm 7_{-16}^{+13}$	$126 \pm 15_{-41}^{+37}$	$7.2 \pm 1.0_{-2.0}^{+4.0}$
1^+	$Z_{cs}(4000)$	15 (16)	$4003 \pm 6_{-14}^{+4}$	$131 \pm 15 \pm 26$	$9.4 \pm 2.1 \pm 3.4$
	$Z_{cs}(4220)$	5.9 (8.4)	$4216 \pm 24_{-30}^{+43}$	$233 \pm 52_{-73}^{+97}$	$10 \pm 4_{-7}^{+10}$

为了检验新的拟合模型相比于 Run 1 拟合模型对数据的描述更合理，本论文检查了不同阶勒让德正交矩下不变质量的分布，发现新的拟合模型相较于 Run 1 模型有明显的改善，这也是新的 X 态存在的证据。为了检验 $Z_{cs}(4000)^+$ 共振态的性质，本论文采用了共振态模型不依赖的方法，即允许 $Z_{cs}(4000)^+$ 在不同 $J/\psi K^+$ 质量区间里振幅任

意浮动,从而得到了 $Z_{cs}(4000)^+$ 的 Argand 图。该共振态在 Argand 图的轨迹近似为圆形,符合一般共振态的特征。BESIII 实验近期公布了 $Z_{cs}(3985)^-$ 的发现,测到的质量是 $3982.5^{+1.8}_{-2.6} \pm 2.1 \text{ MeV}$, 和本论文报告的 $Z_{cs}(4000)^+$ 质量非常接近,但是两个粒子态的宽度却相差一个量级。为了检查 $Z_{cs}(3985)^-$ 和 $Z_{cs}(4000)^+$ 是否是同一个粒子态,在分波分析中把 $Z_{cs}(4000)^+$ 的质量和宽度固定到 BESIII 的结果,最后拟合结果变得非常差。所以在当前的数据样本下,没有发现两个粒子态是同一个共振态的证据。此外,本论文尝试把 $Z_{cs}(3985)^-$ 添加到默认拟合模型中,发现新的共振态对最后拟合的改进非常小,说明目前的 $B^+ \rightarrow J/\psi \phi K^+$ 数据样本中没有 $Z_{cs}(3985)^-$ 存在的迹象。

2015 年, LHCb 使用 Run 1 的数据样本, 在 $\Lambda_b^0 \rightarrow J/\psi p K^-$ 衰变中首次发现了两个五夸克态, $P_c(4450)^+$ and $P_c(4380)^+$ 。但是五夸克的一些重要性质还没有测量, 比如自旋宇称。此外, 在 Run 1 的数据样本中, 从 $J/\psi p$ 不变质量谱中看到了更多五夸克态存在的迹象。为了寻找更多的五夸克态, 确认之前的发现, 本论文使用 Run 1 和 Run 2 所有的数据样本, 对 $\Lambda_b^0 \rightarrow J/\psi p K^-$ 衰变道中的五夸克态结构进行了研究。本论文优化了信号的重建和筛选策略, 放松了信号的预筛选条件, 并将末态粒子的身份鉴别信息放入了多变量分析的流程中, 从而大大提高了重建效率。在本底水平不变的情况下, Λ_b^0 信号的选择效率提高了一倍, 信号个数几乎提高了一个量级。在更大的信号统计量下, 在 $J/\psi p$ 不变质量分布上, 大概在 4312 MeV 的位置看到了明显的窄五夸克信号。本论文采用拟合一维 $J/\psi p$ 不变质量谱的方法测量的五夸克态的质量宽度, 而没有采用分波分析的方法, 这是因为目前研究的五夸克结构宽度很小, 一维拟合的方案不会影响对结果的测量。此外, 如果采用分波分析, 需要大量的 CPU 资源, 暂时无法实现。在一维拟合模型中, 使用了 Breit-Weigner 函数来描述五夸克的线型, 采用切比雪夫多项式来描述本底形状。由于信号比较窄, 质量分辨的影响以参数化的方式包含在拟合模型中。为了提高信号显著性, 压低 Λ^* 的影响, 验证五夸克态信号的稳定性, 本论文选用了三种不同的数据样本进行了拟合。1) 不做 m_{Kp} 筛选的数据样本。2) 加上 $m_{Kp} > 1.9 \text{ GeV}$ 筛选条件的数据样本。3) 加上五夸克态螺旋度角度的权重。其中第三种数据样本中五夸克态显著性最大, 用来测量五夸克态的质量和宽度信息, 其他样本用来研究系统误差。此外, 信号的显著性是通过第二种样本得到。接下来, 前面介绍的一维拟合模型分别应用到三种不同的数据样本上, 进而测量出了五夸克态的质量和宽度信息。测量结果列于下表 (第一个误差为统计不确定度, 第二个为系统不确定度):

粒子态	质量 [MeV]	宽度 [MeV]	(95% 置信水平)
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	(< 27)
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	(< 49)
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	(< 20)

表中系统误差来源主要有两种，第一种来源于数据样本，第二中来源于不同的拟合模型，其中不同拟合模型是系统误差的主要来源，对于质量和宽度测量，最大的系统误差来源于五夸克态之间的是否存在干涉。正如上表所列，在 Run 1 和 Run 2 的数据样本中，发现了一个新的窄五夸克态 $P_c(4312)^+$ ，其质量为 $4311.9 \pm 0.7^{+6.8}_{-0.6}$ MeV，宽度为 $9.8 \pm 2.7^{+3.7}_{-4.5}$ MeV；并观测到之前发现的 $P_c(4450)^+$ 劈裂成两个结构： $P_c(4440)^+$ 和 $P_c(4457)^+$ ，这两个五夸克态宽度也非常小。对于 Run 1 分析中观测到的 $P_c(4380)^+$ 在本研究中无法确认或者否认，由于 $P_c(4380)^+$ 宽度比较大，需要通过分波分析的方法来检验。此外， $P_c(4312)^+$ 和 $P_c(4457)^+$ 的质量分别在 $\Sigma_c \bar{D}$ 和 $\Sigma_c \bar{D}^*$ 的阈值附近，与分子态模型的预言较为一致。

目前，五夸克态只在 $\Lambda_b^0 \rightarrow J/\psi p K^-$ 衰变中被发现，在不同的衰变道中确认其存在是实验中的另一个重要任务。本论文尝试研究了 Cabbibo 压低衰变道 $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ ，已经完成了信号样本的重建和筛选。并且在达立兹分布图上观测到了五夸克态存在的迹象。接下来计划对这个衰变道进行完整的分波分析。

目前实验上只观察到了隐粲五夸克态，本论文尝试从 $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ 衰变道寻找显含粲味的五夸克态。这个分析利用 Run 1 采集到的 3 fb^{-1} 数据重建出了 $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ 衰变道，研究了 $\Lambda_c^+ K^+$ 的不变质量谱，但是从不变质量谱中没有看到明显的显含粲味的五夸克态结构。此外，由于 $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ 这种衰变模式是实验上首次发现，这个分析测量了这个衰变道与 $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ 衰变道之间的相对分支比。本论文通过拟合 $\Lambda_c^+ K^+ K^- \pi^-$ 和 $\Lambda_c^+ D_s^-$ 不变质量谱，从而估计出了两个衰变道的信号产额。然后通过模拟样本和刻度样本来估计了两个衰变道的信号总效率，其主要包含探测器接收度效率，信号触发效率，重建筛选效率，粒子身份信息鉴别效率。最后测得的相对分支比为 $\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-) / \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-) = (9.26 \pm 0.29 \pm 0.46 \pm 0.26) \times 10^{-2}$ 。其中第一项不确定度为统计不确定度，第二项为系统不确定度，第三项来源于 $D_s^- \rightarrow K^+ K^- \pi^-$ 衰变分支比的不确定度。

关键词：奇特强子态，四夸克态，五夸克态，量子色动力学，LHCb 实验

Experimental Study of Exotic Hadrons at LHCb

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ABSTRACT

The strong interaction is described by quantum chromodynamics (QCD), an $SU(4)$ gauge theory in the standard model (SM) of particle physics. While QCD at a high energy scale can be perturbatively calculated and has been well tested in experiments, its nonperturbative nature at a low energy scale is still to be understood. The study of the nature of nonperturbative QCD is one of the most important aspects of particle physics. It is on one side helpful to deeply understand the strong interaction, and on the other side useful to understand background due to strong interactions in electroweak processes, and hence to improve sensitivities of new physics searches. Hadron spectroscopy can provide rich information for the study of nonperturbative QCD, since it is the exhibition of the complex strong interactions inside hadrons, or a consequence of QCD. For heavy-flavor hadrons, nonrelativistic treatment is a good approximation for the heavy flavor quarks inside, which is a good starting point for effective models. In addition, measurements of heavy-flavor hadron decays are relatively easy. Compared with ordinary hadrons, exotic hadrons, such as tetraquark and pentaquark states, have richer structures and hence can provide more information to understand the strong interaction. With the large data sample of proton-proton collisions collected by the LHCb experiment at CERN, this dissertation studied a couple of tetraquark and pentaquark candidates containing at least one $c\bar{c}$ pair. These studies are expected to help to understand the properties and inner structures of these exotic hadrons.

Among more than twenty tetraquark candidates that contain at least one $c\bar{c}$ pair, the electrically charged states, *e.g.* $Z_c(3900)^+$ and $Z_c(4430)^+$, are smoking-gun evidence of tetraquark states, since their minimum-constituent-quark contents are $c\bar{c}u\bar{d}$. Apart from the $c\bar{c}$ pair, at least a pair of quarks with different flavors, $u\bar{d}$, exists. This dissertation used the full LHCb data sample corresponding to an integrated luminosity of 9 fb^{-1} to search for the strangeness partner ($c\bar{c}u\bar{s}$), *i.e.* Z_{cs}^+ . A six-dimensional amplitude analysis of the $B^+ \rightarrow J/\psi \phi K^+$ decay is performed, and two charged tetraquark states that contain an s quark in their quark contents, $Z_{cs}(4000)^+$ 和 $Z_{cs}(4220)^+$, are observed in the $J/\psi K^+$ final state. The mass and width of

the former are $4003 \pm 6_{-14}^{+4}$ MeV and $131 \pm 15 \pm 26$ MeV, respectively, and the spin-parity quantum numbers are $J^P = 1^+$; the mass and width of the latter are $4219 \pm 24_{-30}^{+43}$ MeV and $233 \pm 52_{-73}^{+97}$ MeV, respectively, and the spin and parity is not conclusive. The first uncertainties are statistical and the second systematic. In addition, apart from the four exotic states observed in the $J/\psi\phi$ combination in 2016, two new states, $X(4685)$ and $X(4630)$, are observed in this analysis, and their quantum numbers are determined to be $J^{PC} = 1^{++}$.

The same data sample was also used to study the $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay, in order to better determine the properties of the pentaquark states observed by the LHCb experiment in 2015. The event selection is significantly improved with respect to the 2015 analysis. The selection efficiency improves by a factor of two under the same purity, and the number of Λ_b^0 signals increases by a factor of nine. This great improvement leads to the observation of a new narrow pentaquark $P_c(4312)^+$ and the fine structure of the previously observed $P_c(4450)^+$, where two narrow peaks exist, $P_c(4440)^+$ and $P_c(4457)^+$. The masses and widths of these three new states are

$$\begin{aligned} m &= 4311.9 \pm 0.7_{-0.6}^{+6.8} \text{ MeV} & \Gamma &= 9.8 \pm 2.7_{-4.5}^{+3.7} \text{ MeV}, \\ m &= 4440.3 \pm 1.3_{-4.7}^{+4.1} \text{ MeV} & \Gamma &= 20.6 \pm 4.9_{-10.1}^{+8.7} \text{ MeV}, \\ m &= 4457.3 \pm 0.6_{-1.7}^{+4.1} \text{ MeV} & \Gamma &= 6.4 \pm 2.0_{-1.9}^{+5.7} \text{ MeV}, \end{aligned}$$

where the first uncertainties are statistical and the second systematic. The masses of $P_c(4312)^+$ and $P_c(4457)^+$ locate around the $\Sigma_c \bar{D}$ and $\Sigma_c \bar{D}^*$ thresholds respectively, and they are consistent with the hypotheses of molecular models. In addition, a preliminary study of the Cabibbo-suppressed decay $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ was also performed. An evidence of pentaquark states is observed.

A new decay $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ is observed using a data sample corresponding to an integrated luminosity of 3 fb^{-1} . The branching fraction ratio with respect to the $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ decay is measured to be $\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-) / \mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-) = (9.26 \pm 0.29 \pm 0.46 \pm 0.26) \times 10^{-2}$, where the first uncertainty is statistical, the second is systematic, and the third is due to the uncertainty of the branching fraction $\mathcal{B}(D_s^- \rightarrow K^+ K^- \pi^-)$. With a larger data sample, this decay can be used to search for open-charm pentaquark states in the $\Lambda_c^+ K^+$ final state.

KEYWORDS: Exotic hadrons, Tetraquark, Pentaquark, QCD, LHCb

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List of main acronyms

SM	The Standard Model
QM	The Quark Model
QED	Quantum electrodynamics
QCD	Quantum chromodynamics
CKM	Cabibbo–Kobayashi–Maskawa
LHC	The Large Hadron Collider
LHCb	The Large Hadron Collider beauty experiment
Exp	Experiment
PDG	Particle Data Group
MC	Monte Carlo
PHSP	Phase Space
MVA	The multivariate analysis
BDTG	The boost decision tree gradient
MLP	The multilayer perceptron
ROC	The receiver operating characteristic curve
CL	Confidence Level
FoM	Figure of Merit
PDF	Probability Density Function
PID	Particle Identification
PV	Primary Vertex
IP	Impact Parameter
ndf	Number of Degree of Freedom
L0	Level 0 Trigger
HLT	High Level Trigger
BW	Breit-Wigner
ECAL	Electromagnetic calorimeter
MIP	Minimum Ionizing Particle

Chapter 1 Introduction

There are three major approaches to understand fundamental laws of the Universe. The first one, which is represented by particle physics, explores the building blocks of matter and their interactions at the fundamental level, or at the smallest scale of size. The second one, which is represented by astronomy and cosmology, explores physics rules at the extremely large scale. The third one explores the statistical and/or collective properties of a system that contains a large amount of particles, with condensed matter physics as a representative. These three approaches are complementary to each other, and together with others, have shaped and are still shaping our understanding of the Universe.

The answer to the question what the matter is made of, owing to the exploration in the past more than one hundred years, is summarized in the Standard Model (SM) of particle physics. The SM has achieved tremendous success as a systematic theory^[1]. It classifies all known elementary particles and describes three of the four known fundamental forces: the electromagnetic, the weak, and the strong interactions. Only the gravitational force is not included. Figure 1.1 shows the elementary particles known so far. Among these fundamental particles, the twelve spin- $\frac{1}{2}$ fermions are the building blocks of matter, including six quarks and six leptons; the four spin-1 bosons are the gauge bosons that mediate interactions, including the massless photon for the electromagnetic interaction, the massive $W^\pm$ and Z bosons for the weak interaction, and the massless gluons for the strong interaction; the only spin-0 boson is the Higgs boson, which provides the mechanism by which all fundamental particles acquire mass.

The quarks and the leptons are classified by the interactions they involve. The former involves all three fundamental interactions that the SM describes, while the latter do not involve the strong interaction. The SM describes the three interactions in a unified picture following the principle of local gauge symmetry. The electromagnetic and the weak interactions are unified in the SM, referring to as the electroweak interaction. It is based on the gauge group $SU(2) \times U(1)$. The electroweak sector of the SM has been widely tested in experiments of enormously good accuracy. The theory of the strong interaction between the quarks and gluons is quantum chromodynamics (QCD). It is based on the gauge group $SU(3)$. The coupling constant of QCD is small at short distance, where perturbative calculations are therefore applicable, whereas at long distance a perturbative expansion is not admitted because the coupling constant

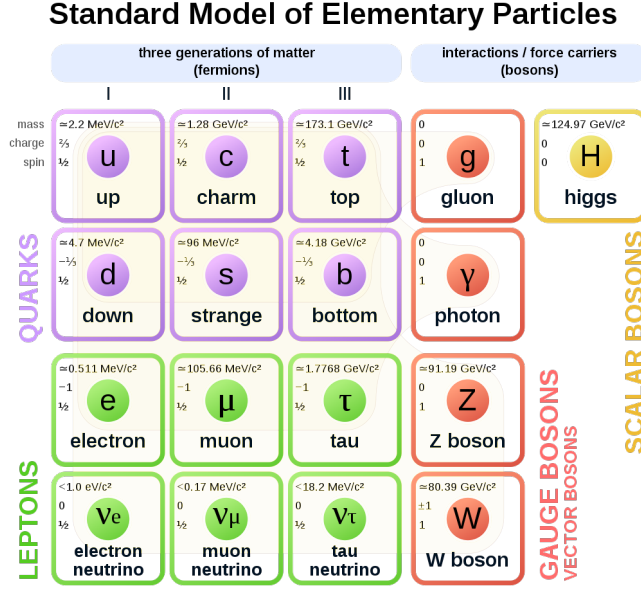


Figure 1.1 Elementary particles in the Standard Model of particle physics. Figure taken from Wikipedia.

becomes large here. While perturbative QCD (pQCD) has been well tested, QCD in non-perturbative regime is still not fully understood. Various theoretical models are developed to study phenomena involving long-distance effects. Many efforts have been made to solve this issue from both theoretical and experimental sides. The study of hadron spectroscopy, including exotic hadrons, can provide useful information to achieve a better understanding of the strong interaction and the theory that describes it, QCD.

This chapter gives a short introduction to the theoretical background and briefly summarizes the status of exotic hadrons from an experimental view.

1.1 QCD

The Lagrangian of QCD^[2], which describes the strong interaction between colored quarks and gluons, is given by

$$\mathcal{L} = \sum_q \bar{\psi}_{q,a} (i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t_{ab}^C \mathcal{A}_\mu^C - m_q \delta_{ab}) \psi_{q,b} - \frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu} \quad (1.1)$$

with repeated indices to be summed over. It is an SU(3) gauge theory. The matrices γ^μ are the Dirac γ -matrices. The $\psi_{q,a}$ are quark-field spinors for a quark of flavor q with a mass of m_q and a color-index a running from 1 to 3. This means that quarks have three different colors and are in the fundamental representation of the SU(3) color group. The $\mathcal{A}_\mu^C$ are the gluon fields, with the index C running from 1 to 8, corresponding to eight different gluons. The t_{ab}^C are eight

3×3 matrices representing the generators of the SU(3) group. It indicates that the interaction of a gluon with a quark rotates the color of the quark in the color space. The parameter g_s , or $\alpha_s \equiv g_s/4\pi$, is the QCD coupling constant. The coupling g_s and the quark masses m_q are the fundamental parameters of QCD, with the masses originating from the electroweak sector. The quantity $F_{\mu\nu}^A$ is the field tensor.

An interesting feature of QCD is asymptotic freedom, *i.e.* the strong coupling α_s is small enough for sufficiently large momentum transfer, which allows for perturbative calculations in order to provide quantitative predictions of physical processes. The energy-evolution of the QCD coupling constant α_s with is fundamentally different from that of the quantum electrodynamics (QED) coupling constant, because not only fermion loops but also gluon loops are included in vertex expansions, since gluons carry color charges as well. The gluon loops modify the QCD coupling strength α_s in a way that is opposite to the QED coupling strength: the QCD coupling constant α_s decreases at short distance (or large energy scale) and increases at long distance (or small energy scale), as shown in Fig. 1.2.

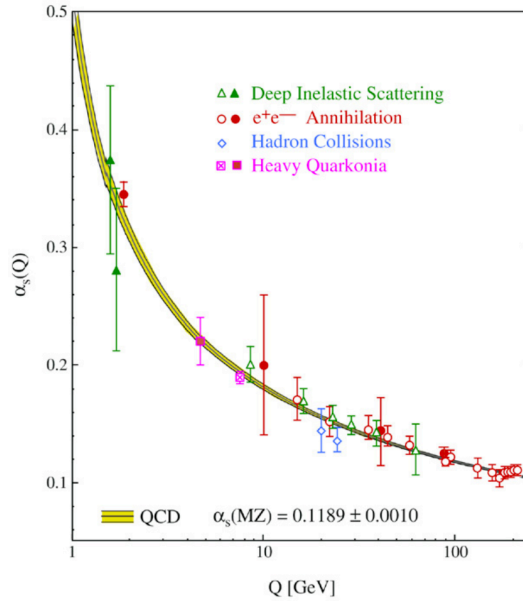


Figure 1.2 Summary of measurements of $\alpha_s(Q)$ versus momentum transfer Q . Figure taken from Ref. 3

1.1.1 The QCD dilemma

One of the most striking facts of the strong interaction is that quarks and gluons have never been observed as free particles, a phenomenon called “quark confinement” or “color confinement”. Experimentally observed particles of strong interactions are not quarks or gluons, they are always color-neutral hadrons, *i.e.* color-singlet combinations of quarks, antiquarks, and

gluons. The nature of the confinement is still to be understood. Although the QCD Lagrangian is expected to completely describe the spectrum of hadrons and all of their properties, there is no rigorous first-principle translation of it into useful mathematical expressions.

A fundamental process that can be computed with pQCD is quark-quark (or quark-gluon, gluon-gluon) elastic scatterings at high-momentum transfer, $qq \rightarrow qq$. This shows up in the high-energy proton-proton (pp) collisions as events with two high transverse momentum jets of hadrons that are almost back to back in the plane perpendicular to the beam direction. The momentum distribution of quarks inside the proton is governed by long-distance QCD and approximated by universal parton distribution functions that are taken from fits to data obtained from hadron-collider measurements at low center-of-mass energies, deep inelastic lepton-proton scattering experiments, *etc.*. The fundamental QCD $qq \rightarrow qq$ process cannot be directly measured. Instead, it has to be inferred from the jets of hadrons that the process produces. Thus, even processes that can be calculated via pQCD involve significant long-distance QCD effects in both initial and final states.

In practice, the hadrons that can be observed in experiments are almost entirely disconnected with the quarks and gluons that appear in the theory. This is a big problem in particle physics, and is referred to as the “QCD dilemma”^[4]. This is on one hand intellectual dissatisfaction, because the theory cannot be directly applied to experimentally detectable particles; on the other hand it is also a practical problem in many SM tests and searches for new physics (NP) beyond the SM, because of the involvement of hadrons in the initial and/or final states of the measurements, even for those experiments with no hadrons in the initial and final states, they are still subject to hadronic effects from virtual quantum fluctuations. Consequently, the sensitivities of many experiments aiming to search for NP will be limited by theoretical uncertainties that are related to QCD at some stage. Therefore, it will be more and more important to improve the understanding of nonperturbative QCD when the sensitivities of NP search experiments, *e.g.* the muon $g - 2$ experiment^[5,6], become better and better.

A first-principle approach to treat the nonperturbative calculations is lattice QCD^[7], which transcribes QCD on a lattice of discrete space-time points. This is a rigorous method to test the consistency between experimental discoveries and QCD principles. Many achievements have been made in lattice QCD in the past decades, however, realistic lattice QCD computations require extreme computational resources, which makes it very hard to perform exact computations and some trade-off is usually needed in practice. Besides, various exotic hadrons have masses very close to threshold. This phenomenon is more challenging for studies in lattice QCD because it is very difficult to compute scattering amplitudes or the properties of

resonances directly from finite-volume correction functions^[8]. Therefore, phenomenological models are needed to connect the hadrons observed in experiments and the quarks and gluons in QCD.

1.2 The quark model

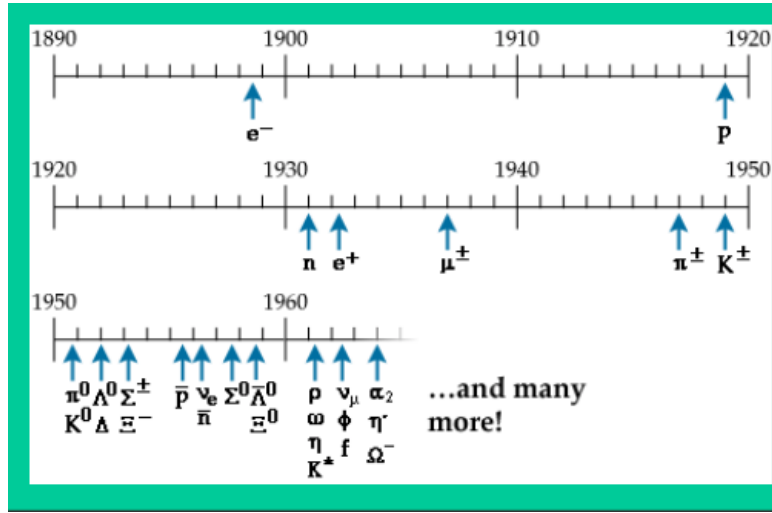


Figure 1.3 The particle zoo before 1964. Plot taken from Wikipedia.

The quark model was introduced to classify experimentally observed hadrons. In this model, quarks and antiquarks are regarded as the origins of the quantum numbers of hadrons.

Circa 1950, when particle accelerators were able to impart GeV-scale kinematic energy to a single particle, many new particles began to be discovered. Most of these particles are unstable and decay very quickly, hence were not seen in cosmic ray experiments. Figure 1.3 shows the particles that were discovered before 1964. With the proliferation of the particle "zoo" in 1950s and 1960s, people suspected that if all these particles are elementary. The prosperity of new particles observed in experiments promoted emergence of many new models, which were concocted to interpret the existence of these particles. For example, Fermi and Yang proposed that pions were composed of a nucleon and an anti-nucleon before the discovery of antiproton^[9]. The quark model was independently proposed by M. Gell-Mann and G. Zweig in 1964 to classify the hadrons^[10,11]. They suggested in their proposals that hadrons were made up of constituents, which were called quarks by Gell-Mann and aces by Zweig. In the quark model baryons consist of three quarks, and mesons consist of a quark and an antiquark. These hadrons are called "ordinary" hadrons. In the original proposal of the quark model, Gell-Mann and Zweig also introduced the concept of "exotic" hadrons. G. Zweig^[11] stated

in his paper: "In general, we would expect that baryons are built not only from product of these aces, AAA, but also from $\bar{A}AAAA$, $\bar{A}\bar{A}AAAA$ ", etc., where $\bar{A}$ denotes an anti-ace." M. Gell-Mann^[10] gave a similar expression: "Baryons can now be constructed from quarks by using the combinations (qqq) , $(qqqq\bar{q})$, etc., while mesons are made of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc.". According to this classification method, tetraquark states, containing four valence quarks, are exotic mesons; while the pentaquark states, consisting of five quarks, would be named exotic baryons. In general, color-neutral bound states of quarks and gluons that are different from ordinary hadrons are all referred to as exotic hadrons.

In the quark model, quarks are strongly interacting fermions with spin 1/2. By convention, quarks have positive parity, while antiquarks have negative parity. Table 1.1 lists some quantum numbers of all six kinds of quarks. These quantum numbers are related with each other through the generalized Gell-Mann-Nishijima formula^[12]:

$$Q = I_z + \frac{\mathcal{B} + S + C + B + T}{2} \quad (1.2)$$

The meaning of each symbol in this formula is expressed in Tab. 1.1. The convention used here implies that any flavor (I_z , S , C , B or T) carried by charged mesons has the same sign as their electric charge.

Table 1.1 Some quantum numbers of quarks.

	d	u	s	c	b	t
Electric charge (Q)	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{1}{3}$	$-\frac{1}{3}$	$+\frac{1}{3}$
Isospin (I)	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
Isospin z -component (I_z)	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	0	0
Strangeness (S)	0	0	-1	0	0	0
Charm (C)	0	0	0	+1	0	0
Bottomness (B)	0	0	0	0	-1	0
Topness (T)	0	0	0	0	0	+1
Baryon number ($\mathcal{B}$)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$

The original quark model proposed by Gell-Mann and Zweig is simply a classification scheme for hadrons, also known as the "Constituent Quark Model"^[13]. It has nothing related to dynamics. In a modern sense, the quark model can exist on many levels. Generally speaking, the quark model is a description of hadronic properties, in which the minimum-quark-content part of the hadron wave function plays an important role^[1]. The dynamics can be introduced by a potential V between the constituent quarks. If only heavy quarks (c and/or b) were involved,

nonrelativistic Schrödinger equation can be used to describe the motion of the quarks, and one can get the corresponding bound states, *i.e.* the hadron spectrum, by solving the equation.

1.3 Charmonium spectra

Many phenomenological models are proposed to solve hadron spectra. The potential model is successful to interpret charmonium spectroscopy^[14]. Since the discovery of the J/ψ meson, the hadrons with a $c\bar{c}$ pair sparked the interest of both theorists and experimentalists. Many charmonium states have been observed in experiments, as shown in Fig. 1.4 by black lines. These states can be classified according to the energy levels calculated by the potential models. In this model, the potential between the charmed quarks can be described as

$$V(r) = -\frac{k}{r} + \frac{r}{a^2} \quad (1.3)$$

where r is the distance between the quark and antiquark. The first term is the contribution of the gluons exchange between the quark and the antiquark. The second part is a linear potential component, where the parameter a^2 is determined by fits to experimental hadron spectra^[14–17]. Actually, the potential models are quite satisfactory for the description of conventional charmonium states^[1]. As shown in Tab. 1.2, the masses of vector charmonium states can be described by potential models pretty well.

Table 1.2 Masses (in MeV) of vector charmonium states from the experimental world averages^[1], and the calculations by the Godfrey-Isgur (GI) model^[15] and nonrelativistic (NR) potential model^[15].

State	World average	GI model	NR model
J/ψ (1^3S_1)	3096.9	3098	3090
$\psi(2S)$ (2^3S_1)	3686.1	3676	3672
$\psi(3770)$ (1^3D_1)	3773.1	3819	3785
$\psi(4040)$ (3^3S_1)	4039	4100	4072
$\psi(4160)$ (2^3D_1)	4191	4194	4142
$\psi(4415)$ (4^3S_1)	4421	4450	4406

In 2003, the Belle experiment reported the observation of $\chi_{c1}(3872)$ state above the $D\bar{D}^*$ mass threshold with a very narrow width^[18]. This state cannot be placed in the ordinary charmonium family. This discovery aroused studies of more charmonium-like exotic hadrons (*i.e.* exotic hadrons containing at least one $c\bar{c}$ pair), which will be introduced in the following sections.

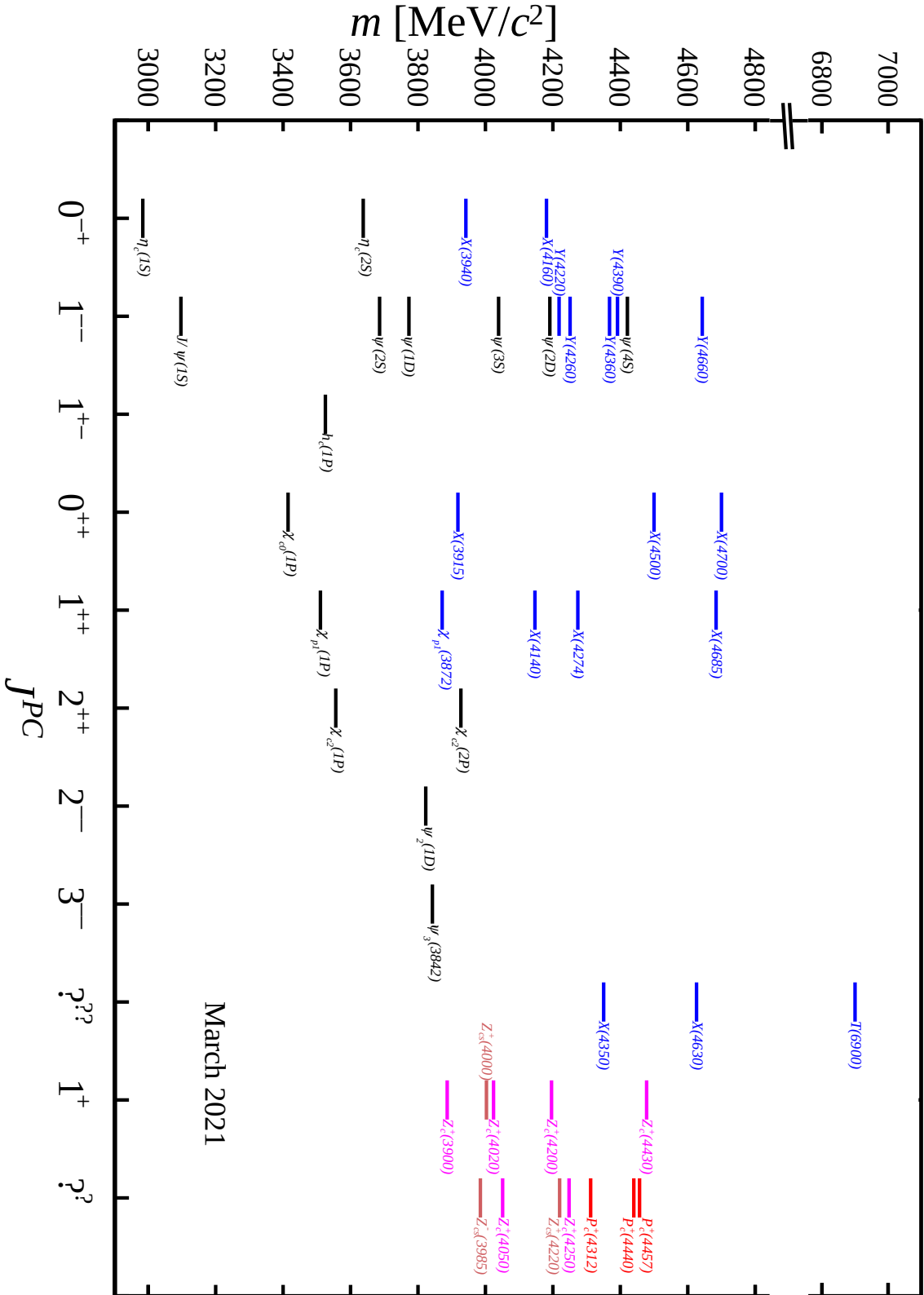


Figure 1.4 Mass spectrum of mesons (pentaquark states) containing a $c\bar{c}$ pair. Black lines present conventional charmonium states, blue lines show neutral tetraquark candidates, pink lines correspond to charged tetraquark candidates, orange lines indicate the charged tetraquark candidates with strangeness, and red lines represent pentaquark candidates. The masses are taken from PDG^[11].

1.4 Experimental evidences of exotic hadrons

Since the concept of “exotic” hadrons was proposed in the quark model, many experiments have tried to search for such kind of states. For example, The LEPS collaboration declared the observation of a narrow $\Theta^+(1540)$ particle, which was claimed to be a pentaquark candidate, then this state became a hotspot at that time^[19,20]. However, this structure was not confirmed by high precise experiments^[21], and it is not deemed as a genuine resonance now^[22]. The history of $\Theta^+(1540)$ is a combination of hard lessons and precious experience that we should not ignore the non-perturbative chromodynamics in spectroscopy study^[23]. Afterwards, some heavy exotic hadrons were observed in experiments.

In this section, experimental evidences of the charmonium-like exotic hadrons, which contain at least a $c\bar{c}$ pair, are introduced. As a hot topic in experimental particle physics, charmonium-like exotic hadrons are studied in many experiments. Figure 1.4 in the previous section summarizes the mass spectrum of observed charmonium-like states. The landscape of charmonium-like tetraquark candidates are also summarized in Tables 1.3 and 1.7.

1.4.1 Neutral tetraquark candidates

Since the observation of $\chi_{c1}(3872)$ ^① by the Belle experiment in 2003^[18], many tetraquark candidates have been discovered in experiments. $\chi_{c1}(3872)$ is one of neutral charmonium-like states. All observed neutral charmonium-like tetraquark candidates are summarized in Tab. 1.3.

1.4.1.1 First tetraquark candidate $\chi_{c1}(3872)$

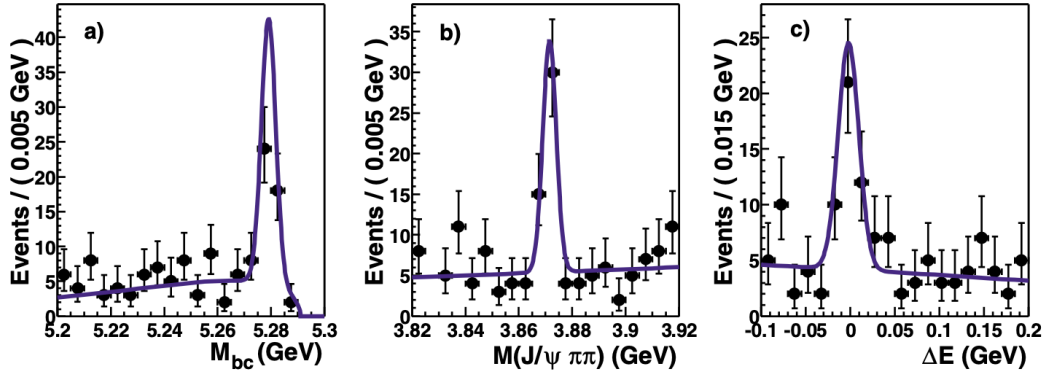
The first multiquark candidate is $\chi_{c1}(3872)$, which was observed by the Belle collaboration as a narrow peak in the $\pi^+\pi^-J/\psi$ invariant mass distribution in $B \rightarrow K\pi^+\pi^-J/\psi$ decays^[18]. Several invariant mass distributions related to $B \rightarrow K\pi^+\pi^-J/\psi$ decay are shown in Fig. 1.5. In the $\pi^+\pi^-J/\psi$ invariant mass distribution, a pretty narrow resonance close to the threshold of the $D\bar{D}^*$ appears. The $\chi_{c1}(3872)$ was popular to be assumed as a molecular state. However, the dynamic mechanism of this state is still under heated debate today^[4]. Note that it is already 18 years since it was discovered, and it has been confirmed by different experiments at different environments as listed in Tab. 1.3. The related studies are pretty fruitful in both experimental and theoretical sectors.

The spin and parity of this state were measured by the LHCb collaboration in 2013 with a data sample corresponding to an integrated luminosity of 3 fb^{-1} taken in 2011 and 2012^[51]. A

① It is referred to as $X(3872)$ in some literatures.

Table 1.3 Neutral charmonium-like mesons. The significance of those in parentheses are less than 5σ .

State	M (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment (σ)
$\chi_{c1}(3872)$	3871.69 ± 0.17	< 1.2	1^{++}	$B^+ \rightarrow [X \rightarrow \pi\pi J/\psi] K^+$ $B^0 \rightarrow [X \rightarrow \pi\pi J/\psi] K^0$ $B^0 \rightarrow [X \rightarrow \pi\pi J/\psi] K^*$ $B^+ \rightarrow [X \rightarrow \omega J/\psi] K^+$ $B^+ \rightarrow [X \rightarrow D^{*0} \bar{D}^0] K^+$ $B^+ \rightarrow [X \rightarrow D^{*0} \bar{D}^0] K^0$ $B^+ \rightarrow [X \rightarrow \gamma J/\psi] K^+$ $B^+ \rightarrow [X \rightarrow \gamma\psi(2S)] K^+$ $B^0 \rightarrow [X \rightarrow \gamma\psi(2S)] K^0$	BaBar ^[24] , Belle ^[25] BaBar ^[24] , Belle ^[25] Belle ^[26] BaBar ^[27] BaBar ^[28] BaBar ^[28] Belle ^[29] Belle ^[29] , LHCb ^[30] Belle ^[29]
$\chi_{c1}(3915)$	3918.4 ± 1.9	20 ± 5	0^{++}	$B \rightarrow K(\omega J/\psi)$ $e^+e^- \rightarrow e^+e^-(\omega J/\psi)$ $e^+e^- \rightarrow J/\psi(D\bar{D}^*)$	Belle ^[31] (8), BaBar ^[27,32] (19) Belle ^[33] (7.7), BaBar ^[34] (7.6) Belle ^[35] (6)
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	(0^{-+})	$e^+e^- \rightarrow J/\psi(D\bar{D}^*)$	Belle ^[35] (6)
$(Y(4008)?)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^- J/\psi)$	Belle ^[36,37] (7.4)
$X(4140)$	3813^{+62}_{-97}	477^{+78}_{-65}	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^- J/\psi)$	BESIII ^[38] (np)
$X(4160)$	$4146.5^{+6.4}_{-5.3}$	83^{+30}_{-25}	1^{++}	$B^+ \rightarrow (J/\psi\phi) K^+$	LHCb ^[39,40] (8.4), see Tab. 1.4
$Y(4220)$	4156^{+29}_{-25}	139^{+113}_{-65}	(0^{-+})	$e^+e^- \rightarrow J/\psi(D^*\bar{D}^*)$	Belle ^[35] (5.5)
$Y(4260)$	4218.4 ± 4.1	66.0 ± 9.0	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^- h_c)$	BESIII ^[38] (np)
	4251 ± 9	120 ± 12	1^{--}	$e^+e^- \rightarrow (\pi\pi J/\psi)$	BaBar ^[41,42] (8), CLEO ^[43,44] (11)
				$e^+e^- \rightarrow (f_0(980)J/\psi)$	Belle ^[36,37] (15), BESIII ^[45] (np)
				$e^+e^- \rightarrow (\pi^- Z_c(3900)^+)$	BaBar ^[42] (np), Belle ^[37] (np)
$X(4274)$	4273^{+19}_{-9}	56^{+13}_{-16}	1^{++}	$B^+ \rightarrow (J/\psi\phi) K^+$	BESIII ^[45] (8), Belle ^[37] (5.2)
$(X(4350)?)$	$4350.6^{+4.6}_{-5.1}$	13^{+18}_{-10}	$0/2^{++}$	$e^+e^- \rightarrow e^+e^-(\phi J/\psi)$	LHCb ^[39,40] (6.0), see Tab. 1.5
$Y(4360)$	4346.2 ± 6.3	102.3 ± 9.9	1^{--}	$e^+e^- \rightarrow e^+e^-(\phi J/\psi)$	Belle ^[46] (3.2)
$Y(4390)$	4391.6 ± 6.4	139.5 ± 16.1	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^- \psi(2S))$	Belle ^[47,48] (8), BaBar ^[49] (np)
$X(4500)$	4506^{+16}_{-19}	92^{+30}_{-29}	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^- \psi(2S))$	BESIII ^[38] (10)
$Y(4660)$	4657 ± 11	70 ± 11	0^{++}	$B^+ \rightarrow (J/\psi\phi) K^+$	LHCb ^[39,40] (6.1), see Tab. 1.5
$X(4700)$	4704^{+17}_{-26}	120^{+52}_{-45}	1^{--}	$e^+e^- \rightarrow (\pi^+\pi^- \psi(2S))$	Belle ^[47,48] (5.8), BaBar ^[49] (5)
$X(6900)$	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$	$?$	$B^+ \rightarrow (J/\psi\phi) K^+$	LHCb ^[39,40] (5.6), see Tab. 1.5
				$\rightarrow J/\psi J/\psi$	LHCb ^[50] (> 5)


 Figure 1.5 Signal-band projections of (a) M_{bc} , (b) $M(\pi^+\pi^- J/\psi)$, and (c) $\Delta E_{c.m.}$ signal region with the results of the unbinned fit^[18].

5-dimensional angular amplitude model was constructed to describe the data, and the likelihood ratio test is performed to discriminate between the $J^{PC} = 1^{++}$ and $J^{PC} = 2^{-+}$ assignments. Based on this technique, the quantum numbers of $\chi_{c1}(3872)$ are determined to be $J^{PC} = 1^{++}$, as shown in Fig. 1.6. From a pseudoexperiment method, the statistical significance of this

assignment reaches 16σ .

As the spin and parity quantum numbers of $\chi_{c1}(3872)$ are 1^+ , this hadron is also expected to be the $2^3P_1 c\bar{c}$ state. However, there are still several reasons to classify it as exotic state^[52], and one of the apparent evidences is isospin violation in the decay channel, $\chi_{c1}(3872) \rightarrow \rho J/\psi$. Since standard charmonium states contain no constituent u or d quark, their isospin number should be 0, as the ρ is isovector particle, the decay mode $\chi_{c1}(3872) \rightarrow \rho J/\psi$ should be strongly suppressed due to isospin violation. But the branching fraction of this decay is nearly equal to the $\chi_{c1}(3872) \rightarrow \omega J/\psi$ mode, which is an isospin non-violated channel. In consideration of a kinematic suppression to $\chi_{c1}(3872) \rightarrow \omega J/\psi$ by a factor of around 4, the $I = 0$ assignment is favored.

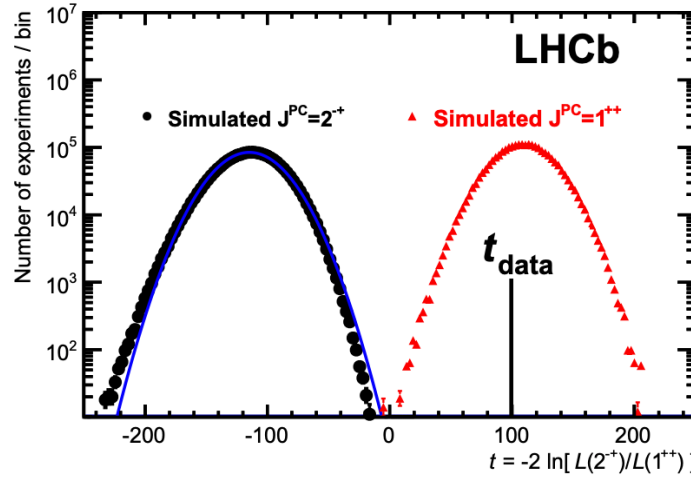


Figure 1.6 The log-likelihood ratio distribution between 1^{++} and 2^{+-} with a Gaussian fit to the 2^{+-} distribution^[51].

Many explorations to the nature of $\chi_{c1}(3872)$ were performed by different theoretical models^[53], and a good diagnosis to these predictions is to study the relative strength to the $\gamma\psi$ and $\gamma J/\psi$ decay modes. In the 1^{++} charmonium model, the χ_{c1}' and ψ' have the same radial wave function, and the $\chi_{c1}' \rightarrow \gamma\psi'$ transition is more advantageous than that for $\gamma J/\psi$. In contrast, in the molecular hypothesis, the decay to $\gamma\psi'$ is much suppressed^[54–56]. The evidence for the decay mode $\chi_{c1}' \rightarrow \gamma\psi'$ was found in 2014, and the measured value is consistent with expectations of pure charmonium interpretation of the $\chi_{c1}(3872)$ state and mixture of charmonium interpretations, it does not support a pure $D\bar{D}^*$ molecular interpretation^[30].

Another experimental test to distinguish different hypotheses is to measure the prompt production of $\chi_{c1}(3872)$ in high-energy hadron collisions. If the $\chi_{c1}(3872)$ is a $D\bar{D}^*$ state, its production in hadron collisions would be more like those of known nuclei stuff, rather than those of ψ' mesons^[57].

1.4.1.2 $\chi_{c1}(3915)$

After the $\chi_{c1}(3872)$ was observed, the Belle collaboration studied $B \rightarrow K\omega J/\psi$ decay, and a prominent enhancement appeared in the $\omega J/\psi$ invariant mass distribution near the threshold, as shown in Fig. 1.7 (a). Then, the BaBar collaboration confirmed this result with a larger sample, and the measured mass and width: $M = 3915 \pm 4 \text{ MeV}$ and $\Gamma = 31 \pm 11 \text{ MeV}$, which are lower values comparing with Belle's measurements^[58], as shown in Fig. 1.7 (b). Then, this state was also observed in the two-photon process $\gamma\gamma \rightarrow \omega J/\psi$, and the PDG table list the results from both channels^[1].

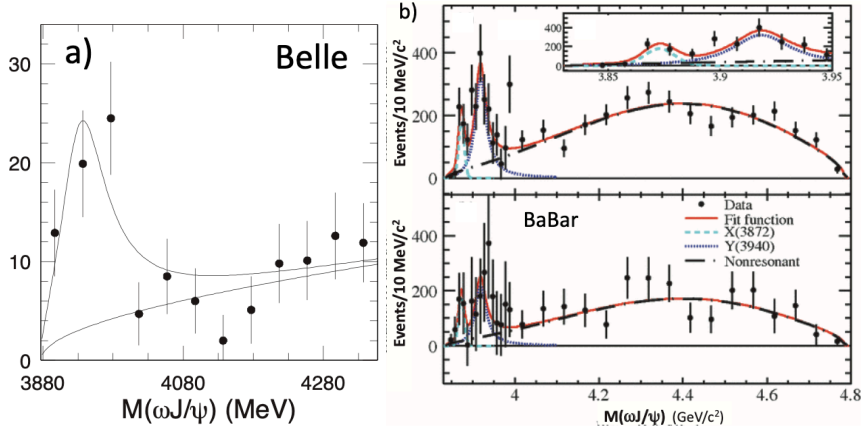


Figure 1.7 The $\chi_{c1}(3915) \rightarrow \omega J/\psi$ states in the $B \rightarrow K\omega J/\psi$ decay from Belle^[57] (left), and BaBar^[58] (right).

However, the spin-parity of this state has not been confirmed yet. With the $\gamma\gamma \rightarrow \omega J/\psi$ events, a spin-parity analysis was performed by BaBar, and their study demonstrated that the assignment $J^{PC} = 0^{++}$ is favoured. From this study, they thought the $\chi_{c1}(3915)$ state is a candidate for the 2^3P_0 charmonium state^[59]. However, this measured mass of $\chi_{c1}(3915)$ is too high for the 2^3P_0 charmonium state, which strongly contradicts with theoretical expectations^[60,61].

Besides, some people think the $J^{PC} = 2^{++}$ hypothesis cannot be ruled out by analysing the BaBar angular distributions^[62]. However, $\chi_{c1}(3915) = \chi'_{c2}$ is not in accordance with the consequence of $\chi_{c1}(3915)$ production in $B^+ \rightarrow K^+ \omega J/\psi$ decay, since the $\mathcal{B}(B^+ \rightarrow K^+ \chi_{c2}(1P)) = (1.1 \pm 0.4) \times 10^{-5}$ is much smaller than the branching fraction

$$\mathcal{B}(B^+ \rightarrow K^+ \chi_{c1}(3915)) \times \mathcal{B}(X \rightarrow \omega J/\psi) = 3.0^{+0.9}_{-0.7} \times 10^{-5}. \quad (1.4)$$

In the decay of B mesons to charmonium states, the widths are expected to be proportional to the square of the $c\bar{c}$ wave function at the origin, and decrease with increasing radial quantum number^[63]. The branching fraction measurement is obviously contrary to this model.

The mass of $\chi_{c1}(3915)$ is 18.2 MeV below the $2m_{D_s^+}$ threshold, and this might mean that it consists of $c\bar{c}s\bar{s}$ component, or even in a $D_s^+D_s^-$ molecule-like situation. In the molecular picture, the $D\bar{D}$ decay would violate the OZI-rule^[64]. In the $c\bar{c}s\bar{s}$ configuration, the decay mode least affected by OZI-suppression is $\chi_{c1}(3915) \rightarrow \eta\eta_c$, which is expected to be a dominant decay mode. However, Belle saw no significant signal from previous study^[65]. The absence of an $\eta\eta_c$ model would be fatal the $c\bar{c}s\bar{s}$ quark assignment, if its partial width was shown to be definitely much smaller than that for $\omega J/\psi$ decays.

The $\chi_{c1}(3915)$ is one of the most intriguing state of the XYZ exotic candidates, however, its underlying nature is not fully understood, and larger data sample of this state is required in future experiments. The J^P quantum numbers of this state are expected to be determined from Belle II, which have good ability to reconstruct ω mesons. Besides, the LHCb experiment also has potential capacity to probe the $\chi_{c1}(3915)$ quantum numbers, considering the ω meson has been detected in B^+ decays^[66].

1.4.1.3 $X(4140)$ and other $J/\psi\phi$ states

Among all known decays of b -hadrons, $B^+ \rightarrow J/\psi\phi K^+$ is unique, since conventional hadron contributions from kaon excitations (hereafter denoted as $K^{*+} \rightarrow \phi K^+$) are broad, and visible mass structures on the Dalitz plot are dominated by a number of relatively narrow $X \rightarrow J/\psi\phi$ states, which are candidates of tetraquark with hidden-charm and hidden-strangeness. Contributions from these exotic hadron components approach half of the entire rate in this decay channel. The first evidence for the $J/\psi\phi$ state was observed by CDF^[67], as shown in the left of Fig. 1.8. A very narrow ($\Gamma = 15.3_{-6.1}^{+10.4} \pm 2.5$ MeV), near-threshold ($M = 4143.4 \pm 3.0 \pm 0.6$ MeV) $X(4140)$ state was claimed. ^② Such narrow structure was not confirmed by the Belle^[68], BaBar^[69], and early low-statistics LHCb^[70] data. However, this near-threshold state was observed by the CMS collaboration^[71]. There was also an evidence for it from the D0 experiment^[72,73], as shown in Fig. 1.9.

An evidence for a second structure, $X(4274)$, in the CDF^[67] and CMS^[71] was also observed. The $J/\psi\phi$ mass distribution among 2480 ± 160 $B^+ \rightarrow J/\psi\phi K^+$ events detected by CMS is shown in right of Fig. 1.8, which was obtained after the subtraction of a very large combinatorial background. There is also hint of the second structure near 4274 MeV in the $J/\psi\phi$ mass distribution at D0^[73], as shown in the right of Fig. 1.9, with a small significance around 1.7σ . Besides, Belle reported a 3.2σ evidence of a narrow $J/\psi\phi$ peak around 4351 MeV,

^② The recent PDG^[1] naming convention calls this state $\chi_{c1}(4140)$. However, the generic X label for $J/\psi\phi$ states is used here, which is independent of their J^P assignments.

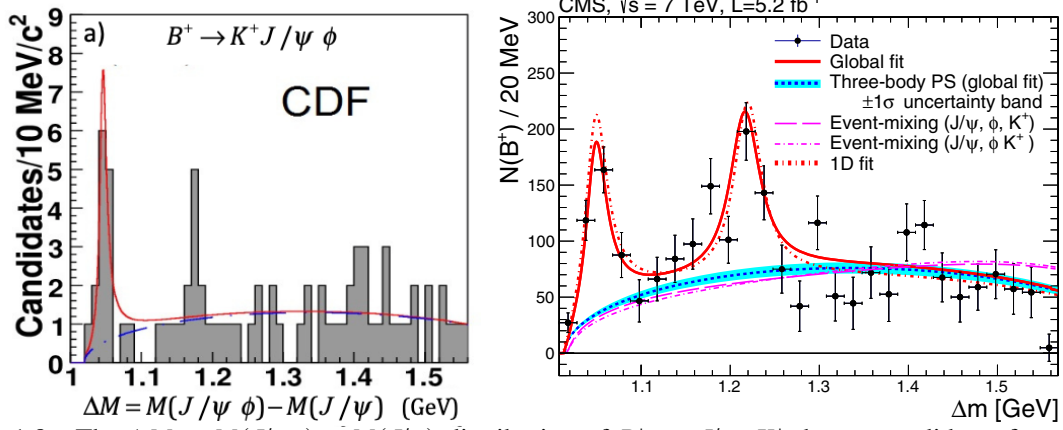


Figure 1.8 The $\Delta M = M(J/\psi\phi) - M(J/\psi)$ distribution of $B^+ \rightarrow J/\psi\phi K^+$ decays candidates from CDF experiment in left^[67]. The number of $B^+ \rightarrow J/\psi\phi K^+$ candidates as a function of $\Delta m = m(\mu^+\mu^-K^+K^-) - m(\mu^+\mu^-)$ at CMS in right^[71].

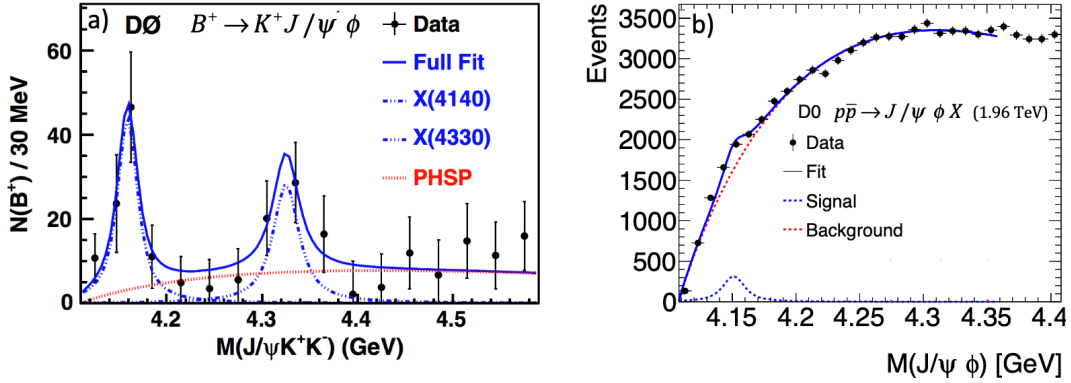


Figure 1.9 The $M(J/\psi K^+ K^-)$ distribution of $B^+ \rightarrow J/\psi\phi K^+$ decays candidates from D0 experiment in left^[72]. The distribution of invariant masses for prompt $J/\psi\phi$ combinations in $pp \rightarrow J/\psi\phi X$ process from D0 in right^[73].

and the corresponding J^P assignment is 0^{++} or 2^{++} . However, there is no obvious $X(4140)$ evidence in the same sample^[74].

LHCb performed the first amplitude analysis of $B^+ \rightarrow J/\psi\phi K^+$ decays, by investigating the $J/\psi\phi$ structures in Run 1 data^[39,40]. The number of $B^+ \rightarrow J/\psi\phi K^+$ signal events was 4289 ± 141 with a background fraction of 23%. The data can be described by including seven $K^{*+} \rightarrow \phi K^+$ resonances, which are determined by the fit and consistent with either known or predicted states. Besides, four $X \rightarrow J/\psi\phi$ structures and non-resonant ϕK^+ and $J/\psi\phi$ also contribute. The mass projections of ϕK^+ and $J/\psi\phi$ combinations are shown in Fig. 1.10. The four X structures and a non-resonant $J/\psi\phi$ contribution all had significance above 5σ . The existence of $X(4274)$ was established.

The J^{PC} quantum numbers of $X(4140)$ are determined to be 1^{++} . Besides, the $X(4274)$ state were identified as 1^{++} at greater than 5σ significance, while the $X(4500)$ and $X(4700)$

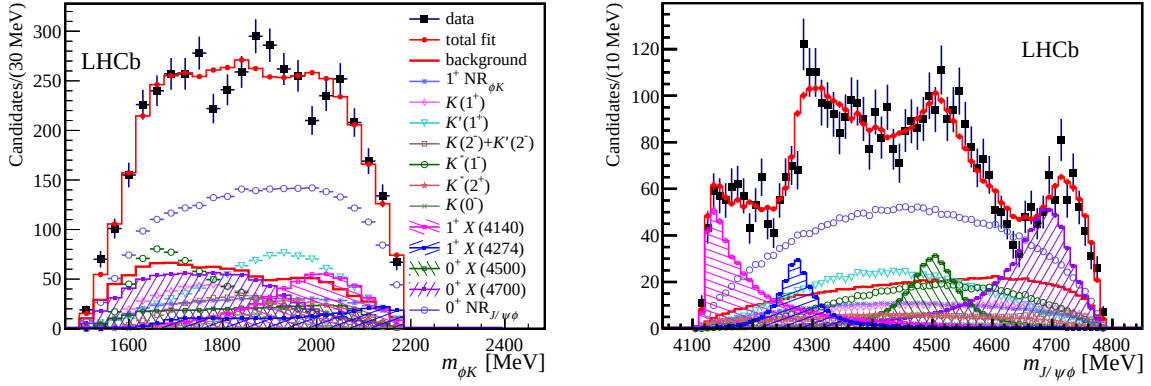


Figure 1.10 Distributions of ϕK^+ (left) and $J/\psi \phi$ (right) invariant masses for the $B^+ \rightarrow J/\psi \phi K^+$ candidates (black data points) compared with the results of the default amplitude fit to the Run 1 data^[39].

states as 0^{++} at greater than 4σ significance. Actually, the 1^{++} quantum number for $X(4140)$ and $X(4274)$ rule out the 0^{++} or 2^{++} $D_s^{*+} D_s^{*-}$ molecular models^[75–80]. Besides, some people suggested $X(4140)$ structure is kinematically induced cusp^[81,82]. No proposed molecular bound-state or cusp model can account for these $X(4274)$ J^{PC} values. A hybrid charmonium state in this mass region would have $J^{PC} = 1^{-+}$ ^[83]. An exception is a tetraquark model implemented by Stancu^[84], states that not only correctly assigned 1^{++} to the $X(4140)$, but also predicted a second 1^{++} state at a mass that is not much higher than that of the $X(4274)$. The $X(4500)$ and the $X(4700)$ have been suggested as candidates for the $\chi_{c0}(4P)$ and $\chi_{c0}(5P)$ states, since they lie in the predicted mass and width ranges for these states^[85,86]. However, none of the J/ψ structures observed in B decays are consistent with the state seen in two-photon collisions by the Belle collaboration.^[74]

Notably, the $X(4140)$ width was substantially larger than previously determined, while the mass is consistent with the $X(4140)$ values from CDF and CMS. In the spectroscopy study, a one-dimensional fit to a mass distribution for a resonance peak might lead to biased mass and width. Besides, a one-dimensional fit might underestimate systematic error, due to the given assumption about the background shape and its incoherence^[4].

The experimental results about $J/\psi \phi$ structures are summarized in Tab. 1.4 and Tab. 1.5. It is possible that the $J/\psi \phi$ structures are derived from molecular model or cusp effect, the mass thresholds of $D_s^+ D_s^-$, with S-wave J^P values, are shown in Tab. 1.6. It is found that only the $X(4140)$ is close to the thresholds and have the S-wave J^P consistent with the determinations from the amplitude fit. Higher values of angular momentum are expected to be suppressed for threshold effects.

Table 1.4 Results related to the $X(4140) \rightarrow J/\psi\phi$ mass peak, first observed in $B^+ \rightarrow J/\psi\phi K^+$ decays. The first (second) significance quoted for Ref. [73] is for the non-prompt (prompt) production components (the mass and width were determined from the non-prompt sample). The last column gives a fraction of $B^+ \rightarrow J/\psi\phi K^+$ rate attributed to the $X(4140)$ structure, however, the CDF and D0 results were normalized to a $B^+ \rightarrow J/\psi\phi K^+$ rate that excluded the high $J/\psi\phi$ mass range.

$B \rightarrow J/\psi\phi K$ Experiment	X(4140) peak				Significance	Fraction %
	yield	Mass [MeV]	Width [MeV]			
CDF 2.7 fb ⁻¹ [87]	58 ± 10	4143.0 ± 2.9 ± 1.2	11.7 ^{+8.3} _{-5.0} ± 3.7		3.8σ	
<i>Belle</i> ^[88]	325 ± 21	<i>4143.0 fixed</i>	<i>11.7 fixed</i>		1.9σ	
<i>CDF</i> 6.0 fb ⁻¹ [67]	115 ± 12	4143.4 ^{+2.9} _{-3.0} ± 0.6	15.3 ^{+10.4} _{-6.1} ± 2.5		5.0σ	14.9 ± 3.9 ± 2.4
LHCb 0.37 fb ⁻¹ [70]	346 ± 20	4143.4 fixed	15.3 fixed		1.4σ	< 7 @ 90%CL
CMS 5.2 fb ⁻¹ [71]	2480 ± 160	4148.0 ± 2.4 ± 6.3	28 ⁺¹⁵ ₋₁₁ ± 19		5.0σ	10 ± 3 (stat.)
D0 10.4 fb ⁻¹ [72]	215 ± 37	4159.0 ± 4.3 ± 6.6	19.9 ± 12.6 ^{+1.0} _{-8.0}		3.0σ	21 ± 8 ± 4
BaBar ^[69]	189 ± 14	4143.4 fixed	15.3 fixed		1.6σ	< 13.3 @ 90%CL
LHCb 3.0 fb ⁻¹ [39,40]	4289 ± 151	4146.5 ± 4.5 ^{+4.6} _{-2.8}	83 ± 21 ⁺²¹ ₋₁₄		8.4σ	13.0 ± 3.2 ^{+4.8} _{-2.0}
D0 10.4 fb ⁻¹ [73]	$p\bar{p} \rightarrow J/\psi\phi...$	4152.5 ± 1.7 ^{+6.2} _{-5.4}	16.3 ± 5.6 ± 11.4		5.7σ (4.7σ)	

Table 1.5 Results related to $J/\psi\phi$ mass structures heavier than the $X(4140)$ peak. The unpublished results are shown in italics. The last column gives a fraction of the total $B^+ \rightarrow J/\psi\phi K^+$ rate attributed to the given structure.

$B \rightarrow J/\psi\phi K$ Experiment	yield	X(4274 – 4351) peak(s)		Significance	Fraction [%]
		Mass [MeV]	Width [MeV]		
<i>CDF</i> 6.0 fb ⁻¹ [67]	115 ± 12	4274.4 ^{+8.4} _{-6.7} ± 1.9	32.3 ^{+21.9} _{-15.3} ± 7.6	3.1σ	
LHCb 0.37 fb ⁻¹ [70]	346 ± 20	4274.4 fixed	32.3 fixed		< 8 @ 90%CL
CMS 5.2 fb ⁻¹ [71]	2480 ± 160	4313.8 ± 5.3 ± 7.3	38 ⁺³⁰ ₋₁₅ ± 16		
D0 10.4 fb ⁻¹ [72]	215 ± 37	4328.5 ± 12.0	30 fixed		
BaBar ^[69]	189 ± 14	4274.4 fixed	32.3 fixed	1.2σ	< 18.1 @ 90%CL
LHCb 3.0 fb ⁻¹ [39,40]	4289 ± 151	4273.3 ± 8.3 ^{+17.2} _{-3.6}	56 ± 11 ⁺⁸ ₋₁₁	6.0σ	7.1 ± 2.5 ^{+3.5} _{-2.4}
		4506 ± 11 ⁺¹² ₋₁₅	92 ± 21 ⁺²¹ ₋₂₀	6.1σ	6.6 ± 2.4 ^{+3.5} _{-2.3}
		4704 ± 10 ⁺¹⁴ ₋₂₄	120 ± 31 ⁺⁴² ₋₃₃	5.6σ	12 ± 5 ⁺⁹ ₋₅
Belle ^[74]	$\gamma\gamma \rightarrow J/\psi\phi$	4350.6 ^{+4.6} _{-5.1} ± 0.7	13 ⁺¹⁸ ₋₉ ± 4	3.2σ	

1.4.1.4 X states in e^+e^- collision

Three X states, $X^*(3860)$, $X(3940)$ and $X(4160)$, are introduced in this section, which have not been observed in any B -meson decay channel. The first one is $X(3940)$, which was first seen at Belle in 2007^[89]. According to the distribution of masses recoiling against a J/ψ in inclusive $e^+e^- \rightarrow J/\psi X$ annihilation, four peaks appear clearly, as shown in Fig. 1.11 (top). The fourth peak is named as $X(3940)$, as is not associated with any known or expected charmonium. Besides, this peak was also observed in the $D\bar{D}^*$ invariant mass distribution for $e^+e^- \rightarrow J/\psi DX$ events^[35]. At the same energy point, Belle observed the $X(4160)$ structure in $D\bar{D}$ invariant mass distribution for exclusive $e^+e^- \rightarrow J/\psi D^*\bar{D}^*$ ^[35]. The J^{PC} of $X(3940)$

Table 1.6 Mass threshold and S-wave J^P of $D_s^+ D_s^-$ containing $c\bar{c}s\bar{s}$.

L=0	$0^- D_s^+$ (1968 MeV)	$1^- D_s^{**}$ (2112 MeV)	$0^+ D_{s0}^{**}$ (2318 MeV)	$1^+ D_{s1}^+$ (2460 MeV)	$1^+ D_{s1}^+$ (2536 MeV)	$2^+ D_{s2}^{**}$ (2573 MeV)
$0^- D_s^-$ (1968 MeV)	0^+ (3926 MeV)	1^+ (4080 MeV)	0^- (4286 MeV)	1^- (4428 MeV)	1^- (4504 MeV)	2^- (4541 MeV)
$1^- D_s^-$ (2112 MeV)	-	$(0, 1, 2)^+$ (4224 MeV)	1^- (4430 MeV)	$(0, 1, 2)^-$ (4572 MeV)	$(0, 1, 2)^-$ (4648 MeV)	$(1, 2, 3)^-$ (4685 MeV)
$0^+ D_{s0}^+$ (2318 MeV)	-	-	0^+ (4636 MeV)	1^+ (4778 MeV)	1^+ (4854 MeV)	2^+ (4891 MeV)
$1^+ D_{s1}^-$ (2460 MeV)	-	-	-	$(0, 1, 2)^+$ (4920 MeV)	$(0, 1, 2)^+$ (4996 MeV)	$(1, 2, 3)^+$ (5033 MeV)
$1^+ D_{s1}^-$ (2536 MeV)	-	-	-	-	$(0, 1, 2)^+$ (5072 MeV)	1^+ (5109 MeV)
$2^+ D_{s2}^{*-}$ (2573 MeV)	-	-	-	-	-	$(0, 1, 2, 3, 4)^+$ (5146 MeV)

and $X(4160)$ are 0^{-+} possibly according to these measurements by Belle. In consideration of the masses of these two states are far below expectations for the 0^{-+} charmonium, $X(3940)$ and $X(4160)$ are taken as tetraquark candidates. The bottom left plot of Fig. 1.11 shows the distribution of masses recoiling against a detected J/ψ and D meson in $e^+e^- \rightarrow J/\psi D^+ X$ annihilation events collected in Belle^[90], while the bottom right plot of Fig. 1.11 shows $D\bar{D}$ invariant mass distribution for the exclusive $e^+e^- \rightarrow J/\psi D\bar{D}$ events. From an amplitude fit, $X(3862)$ structure is very significant and $J^{PC} = 0^{++}$ quantum number assignment gives the best fit. This state is considered as $\chi_{c0}(2P)$ charmonium state now.

1.4.1.5 Y states with $J^{PC} = 1^{--}$

The BaBar group considered to search the possible 1^{--} state in the $e^+e^- \rightarrow \gamma\pi^+\pi^- J/\psi$ process after the $\chi_{c1}(3872)$ was discovered in Belle. No structure around 3872 MeV appeared. However, an unexpected strong accumulation of events in $J/\psi\pi^+\pi^-$ invariant masses distribution have a peak near 4260 MeV^[91], as shown in Fig. 1.12. This observation is confirmed very quickly by CLEO^[93] and Belle^[94].

Afterwards, BaBar observed another peak around 4320 MeV^[95], as shown in the left of Fig. 1.13. Then, Belle confirmed this state with a larger data sample^[92], as shown in the right of Fig. 1.13. As the mass peak is near 4360 MeV, finally, it is named as $Y(4360)$. Besides, from the right plot of Fig. 1.13, a second distinct $\psi\pi^+\pi^-$ invariant mass peak near 4660 MeV appeared, this new observation was named as $Y(4660)$. In addition, BESIII performed a data scan between $E_{cm} = 3882$ MeV and 4567 MeV later^[96], with "high luminosity" and "low luminosity" scans respectively, as shown in Fig. 1.14. It is obvious that the line shape cannot be well described by a single Breit-Wigner function, then the fit was performed using two BW amplitude by BESIII, and the corresponding masses of two resonances are 4222 ± 4 MeV and 4320 ± 13 MeV, which were explained as $Y(4220)$ and $Y(4260)$.

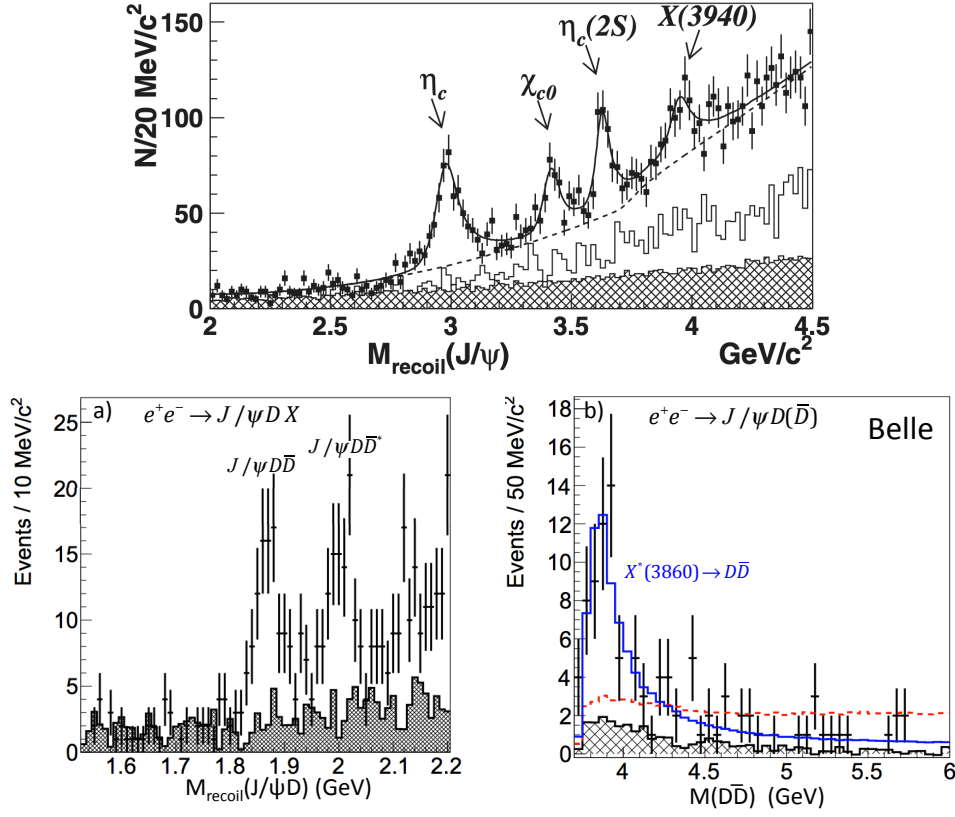


Figure 1.11 The distribution of masses recoiling from the J/ψ at Belle^[89] (above). The distribution of masses recoiling from a reconstructed J/ψ and D-meson in $e^+e^- \rightarrow J/\psi D X$ annihilation (bottom left) and $D\bar{D}$ invariant mass distribution (bottom right)^[35] at Belle.

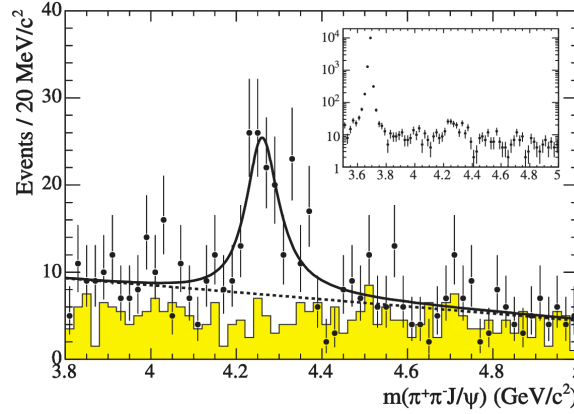


Figure 1.12 The $J/\psi\pi^+\pi^-$ invariant mass distribution in the process $e^+e^- \rightarrow \gamma\pi^+\pi^-J/\psi$ at BaBar^[91] (left) and at Belle^[92] (right).

The J^{PC} of Y states is 1^{--} , which is as same as photon and J/ψ . This is a strong evidence that it contains $c\bar{c}$ quark pair. However, all of the 1^{--} $c\bar{c}$ charmonium with mass below 4450 MeV have been seen in the e^+e^- collision between 2.6 and 4.6 GeV. In addition, there is no evidence it decaying to open-charmed mesons. Based on above considerations, many theorists

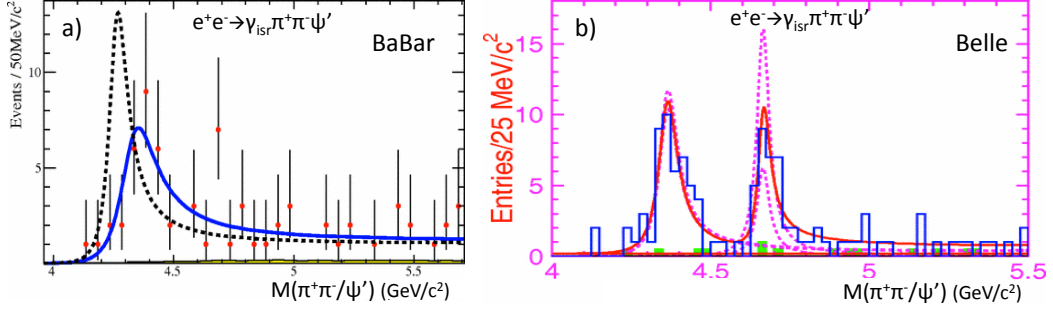


Figure 1.13 The $J/\psi\pi^+\pi^-$ invariant mass distribution in the process $e^+e^- \rightarrow \gamma\pi^+\pi^-J/\psi$ at BaBar^[91].

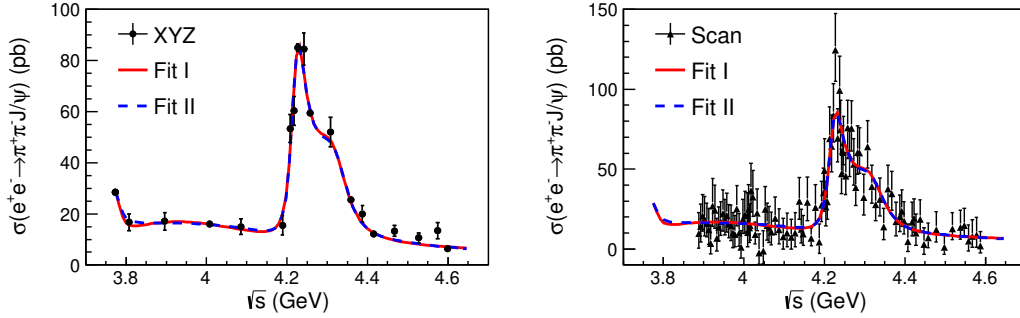


Figure 1.14 The cross section for $e^+e^- \rightarrow \gamma\pi^+\pi^-J/\psi$ at BESIII, right one for "high luminosity" scan, left one is "low luminosity" scan^[96].

infer that this state is multiquark meson or $c\bar{c}$ -gluon hybrid^[97,98].

1.4.1.6 Fully-charmed tetraquark

Recently, the LHCb collaboration studied the invariant mass spectrum of J/ψ pairs, a narrow structure around 6.9 GeV (referred to as $X(6900)$ or $T(6900)$) and a broad structure in the mass range 6.2-6.8 GeV were reported^[50]. The corresponding mass spectrum is shown in Fig. 1.15. The global significance for these two structures are larger than 5σ . Such distinct structures are expected to be with the $c\bar{c}c\bar{c}$ configuration. This important signal from the LHCb collaboration provides us a new ground to understand the nonperturbative behavior of QCD.

1.4.2 Charged tetraquark candidates

In this section, the charmonium-like charged tetraquark states are introduced. All observed charged charmonium-like tetraquark candidates are summarized in Tab. 1.7.

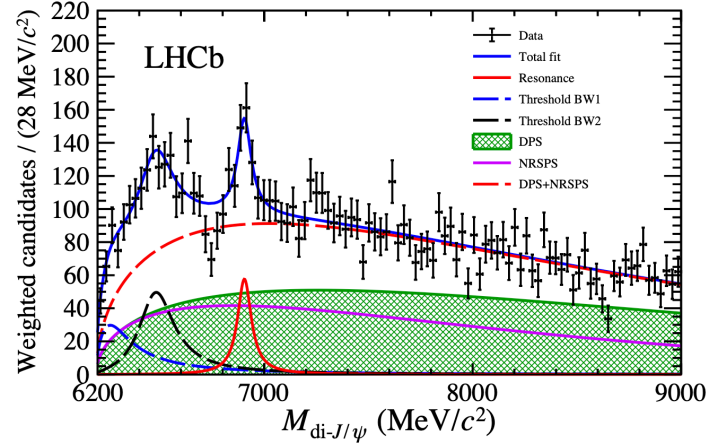

 Figure 1.15 Invariant mass spectra of weighted double J/ψ candidates^[50].

Table 1.7 Summary of charged charmonium-like mesons. The states in parentheses are mere evidence of states.

State	M (MeV)	Γ (MeV)	J^P	Process (mode)	Experiment (σ)
$Z_c(3900)^+$	3888.4 ± 1.6	27.9 ± 2.7	1^+	$Y(4260) \rightarrow \pi^-(D\bar{D}^*)^+$ $Y(4260) \rightarrow \pi^-(\pi^+ J/\psi)$	BESIII ^[99,100] (> 10) BESIII ^[45] (8), Belle ^[37] (5.2) NU group ^[101] (3.5)
$Z_c(3900)^0$	3893.6 ± 3.7	31 ± 10	1^+	$Y(4260) \rightarrow \pi^0(D\bar{D}^*)^0$ $Y(4260) \rightarrow \pi^0(\pi^0 J/\psi)$	BESIII ^[102] (10) BESIII ^[45] (10.4) NU group ^[101] (5.7)
$Z'_c(4020)^+$	4023.9 ± 2.4	10 ± 6	1^+	$e^+e^- \rightarrow \pi^-(\pi^+ h_c)$ $e^+e^- \rightarrow \pi^-(D^*\bar{D}^*)^+$	BESIII ^[103] (8.9) BESIII ^[104] (10)
$Z'_c(4020)^0$	4024.5 ± 3.1	$23 \pm 6 \pm 1$	1^+	$e^+e^- \rightarrow \pi^0(\pi^0 h_c)$ $e^+e^- \rightarrow \pi^0(D^*\bar{D}^*)^0$	BESIII ^[105] (5) BESIII ^[106] (5.9)
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^?$	$\bar{B}^0 \rightarrow K^-(\pi^+ \chi_{c1})$	Belle ^[107] (5.0), BaBar ^[108] (1.1)
$(Z(4050)^+?)$	4054 ± 3	45 ± 13	$?^?$	$e^+e^- \rightarrow (\psi(2S)\pi^+)\pi^-$	Belle ^[48] (3.5)
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^+	$\bar{B}^0 \rightarrow K^-(\pi^+ J/\psi)$	Belle ^[109] (7.2)
$Z(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^?$	$\bar{B}^0 \rightarrow K^-(\pi^+ \chi_{c1})$	Belle ^[107] (5.0), BaBar ^[108] (2.0)
$Z(4430)^+$	4478 ± 17	180 ± 31	1^+	$\bar{B}^0 \rightarrow K^-(\pi^+ \psi(2S))$	Belle ^[110,111] (6.4), BaBar ^[112] (2.4)
				$\bar{B}^0 \rightarrow K^-(\pi^+ J/\psi)$	LHCb ^[113] (13.9) Belle ^[109] (4.0)

1.4.2.1 Z_c in B decay

The first candidate of charged charmonium-like state is $Z_c(4430)$, which was observed at Belle in $\bar{B} \rightarrow \psi \pi^+ K^-$ decay in 2007^[114], and the signal structure is shown in the left of Fig. 1.16. Then the existence of $Z(4430)$ was independently confirmed in 2014 at LHCb experiment^[115], which was studied based on a 4-dimensional amplitude analysis in the $\bar{B} \rightarrow \psi \pi^+ K^-$ decay. According to these independent measurements, the average mass and width are determined to be $M = 4478^{+15}_{-81}$ MeV and $\Gamma = 181 \pm 31$ MeV for $Z_c(4430)$. Besides, the J^P assignment is ascertained to be 1^+ at 9.7σ level as well. The corresponding “Argand” plot is shown in the left of Fig. 1.17, revealing the character of Breit-Wigner resonances as a nearly circular in shape. In addition, a broad peak named as $Z_c(4240)$ is required for a satisfied fitting, and this additional structure appeared in $\psi \pi^+$ mass distribution as shown in right of Fig. 1.16.

Belle also performed an amplitude of $\bar{B} \rightarrow J/\psi \pi^+ K^-$ decays^[116], and a new very broad 1^+ $Z_c(4200)$ state is required for a satisfactory fit. The measured mass and width of this state are $M = 4196^{+31}_{-29} \pm^{+17}_{-13}$ MeV and $\Gamma = 370 \pm 70^{+70}_{-132}$ MeV. Besides, Belle also reported corresponding J^P is either 0^- or 1^+ , and “Argand” plot is shown in the right of Fig. 1.17, which displays a nearly circular phase motion.

The charged Z_c states are pretty broad, and the interference effect is very obvious, which can distort the resonance line shape. For example, the original $Z_c(4430)$ results were based on a one dimensional Breit-Wigner line-shape fit, so they obtained lower values of mass and width than that from LHCb.

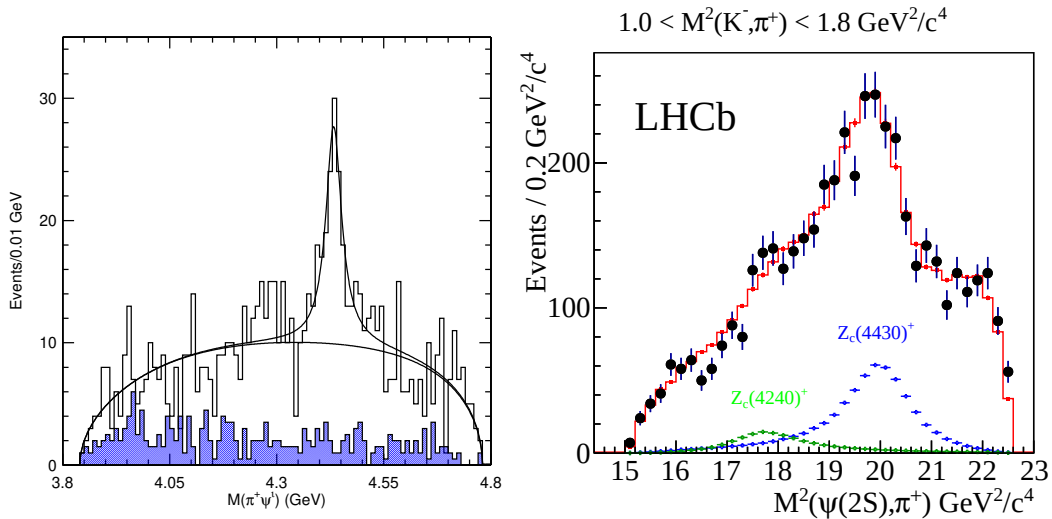


Figure 1.16 The $M(\psi\pi)$ distribution in $\bar{B} \rightarrow \psi \pi^+ K^-$ decay at Belle^[114], the $M^2(\psi\pi)$ in $\bar{B} \rightarrow \psi \pi^+ K^-$ from LHCb^[115].

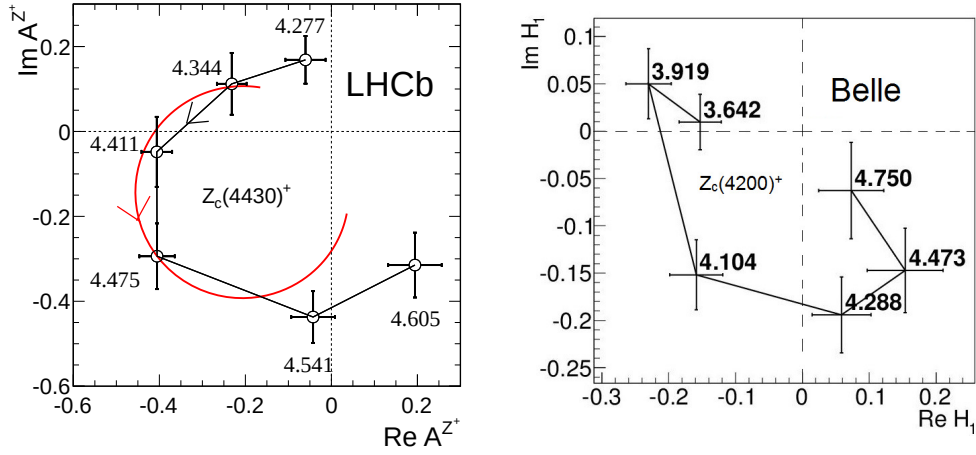


Figure 1.17 The real and imaginary parts of the amplitude for $Z_c(4430) \rightarrow \psi\pi^+$ decay at LHCb^[115], the $Z_c(4200) \rightarrow J/\psi\pi^+$ decay from Belle^[116].

1.4.2.2 Z_c in e^+e^- collision

As mentioned above, the $Y(4260)$ was observed at BaBar. This discovery prompted BESIII group to collect data at $E_{cm} = 4260$ MeV to search possible charged resonance in this channel. Then, the $Z_c(3900)$ was observed in $J/\psi\pi^+$ and $J/\psi\pi^-$ invariant mass distribution, using a 525 pb^{-1} BESIII data sample accumulated at $E_{cm} = 4.26 \text{ GeV}$ ^[114], as shown in the left of Fig. 1.18. The measured mass is $3899.0 \pm 6.1 \text{ MeV}$ and the width is 46 ± 22 . Significantly, the mass of this state is around 24 MeV above the $m_{D^{*+}}m_{\bar{D}^{*0}}$ threshold. The $Z_c(3900)$ was observed by Belle at about the same time^[117].

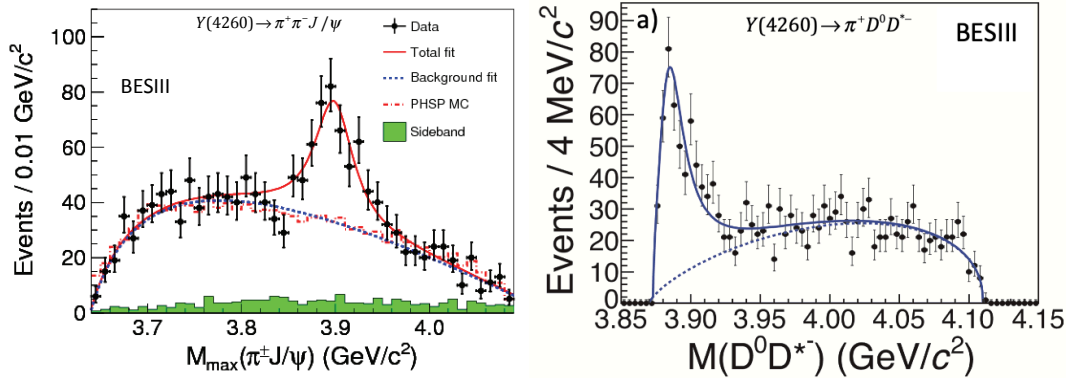


Figure 1.18 Distribution of the larger of two $J/\psi\pi^\pm$ masses in $e^+e^- \rightarrow J/\psi\pi^+\pi^-$ events collected in BESIII at $E_{cm} = 4260 \text{ MeV}$ ^[114] (left), The $D^0 D^{*-}$ invariant mass distribution in $e^+e^- \rightarrow D^0 D^{*-}\pi^+$ events collected in BESIII at $E_{cm} = 4260 \text{ MeV}$ ^[118] (right)

Subsequently, BESIII explored the $D^0 D^{*-}$ invariant mass distribution in $e^+e^- \rightarrow D^0 D^{*-}\pi^+$ events, and found the very strong near-threshold peak in this distribution, as shown in the right

of Fig. 1.18. Then the mass and width are determined as $M = 3883.9 \pm 4.5 \text{ MeV}$ and $\Gamma = 24.8 \pm 12 \text{ MeV}$ from a 1-dimensional fit, where the peak was described by a threshold-modified BW amplitude and background was represented by a incoherent phase-space-like function^[118]. Thus, this state was named as $Z_c(3885)$ by BESIII collaboration. From the $Z_c(3885) \rightarrow D\bar{D}^*$ strong decay, BESIII also determined the J^P quantum number to be 1^+ , which is preferred than the 0^- or 1^- assignments. BESIII also reported the observation of isospin partner of $Z_c(3900)$ in $e^+e^- \rightarrow J/\psi\pi^0\pi^0$ events^[119], as well as in $e^+e^- \rightarrow D^+D^{*-}\pi^0$ ^[118], with mass and width are consistent with the charged $Z_c(3900)$ state.

In addition to $Z_c(3900)$, BESIII collaboration observed the $Z_c(4020)$ in $h_c(1P)\pi^+\pi^-$ final states^[120], and the invariant mass distribution of $h_s\pi^\pm$ is shown in the left of Fig. 1.19. In this study, the mass of $Z_c(4020)$ was measured to be $4022.9 \pm 2.8 \text{ MeV}$, which is about 5 MeV above $m_{D^{*+}} + m_{\bar{D}^{*0}}$. Similar to the neutral $Z_c(3900)$ study, the isospin partner of $Z_c(4020)$ was searched in $e^+e^- \rightarrow h_c\pi^0\pi^0$ events^[121], as shown in the right of Fig. 1.19. Afterwards, $Z_c(4020)$ and its isospin partner were searched in $e^+e^- \rightarrow D^{*+}D^{*0}\pi^-$ ^[122] and $e^+e^- \rightarrow D^{*+}\bar{D}^{*0}\pi^0$ ^[123] events by BESIII. The measured mass and width from the double D invariant mass spectrum are agree with those measured for the $Z_c(4020) \rightarrow h_c\pi^+$ channel within errors.

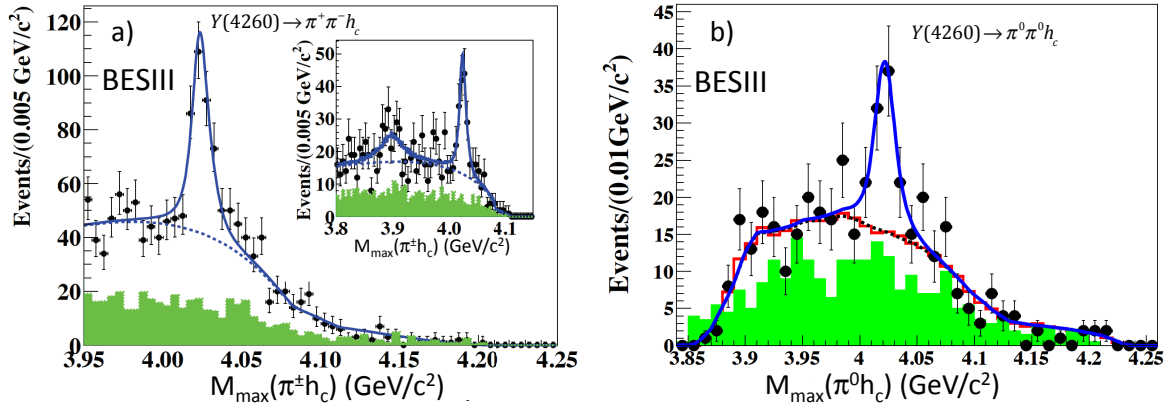


Figure 1.19 Distribution of the larger of two $h_c\pi^\pm$ masses in $e^+e^- \rightarrow h_c\pi^+\pi^-$ events collected in BESIII at $E_{cm} = 4260 \text{ MeV}$ and $E_{cm} = 4360 \text{ MeV}$ ^[120] (left), The $h_c\pi^0$ invariant mass distribution in $e^+e^- \rightarrow h_c\pi^0\pi^0$ events collected in BESIII at BESIII^[121] (right)

The theoretical pictures of $Z_c(3900)$ and $Z_c(4020)$ are not very clear yet. For example, the mass of these two states are in agreement with the molecular interpretation, and both Z_c states should copiously decay to $h_c\pi$ in a purely molecular interpretation. But the $Z_c(4020)$ is not observed in $J/\psi\pi$ and $Z_c(3900)$ is not observed in $h_c\pi$.

1.4.3 Pentaquark candidates

LHCb reported significant $J/\psi p$ mass structures in $\Lambda_b^0 \rightarrow J/\psi p K^-$ decays using Run 1 data sample, and the reflection from $\Lambda^* \rightarrow J/\psi K^-$ contributions have been excluded from full amplitude analysis^[124]. The Dalitz plot of $\Lambda_b^0 \rightarrow J/\psi p K^-$ is shown in the left of Fig. 1.20. From the 6-dimensional amplitude analysis, the $P_c(4450)^+$ structure was determined to peak at $4449.8 \pm 1.7 \pm 2.5$ MeV, with a width of $39 \pm 5 \pm 19$ MeV. The amplitude analysis also required a second broad $J/\psi p$ state for a reasonable description of the data, peaking at $4380 \pm 8 \pm 29$ MeV, with a width of $205 \pm 18 \pm 86$ MeV, even though it did not appear in the $m(J/\psi p)$ distribution. The fitting projection on $m_{J/\psi p}$ is shown in the right of Fig. 1.20. The spins of these structures were not determined, and the data preferred spin combinations involving 3/2 and 5/2 in either order. The spin-parity preferences exhibited significant Λ^* model dependence. Investigation of the “Argand” diagrams for the $P_c(4450)^+$ and $P_c(4380)^+$ structures was inconclusive because of the large statistical errors. Then, cross check was performed in a nearly model independent way^[125], where it was shown that the $J/\psi p$ structure near 4450 MeV was too narrow to be accounted by a collection of pK contributions.

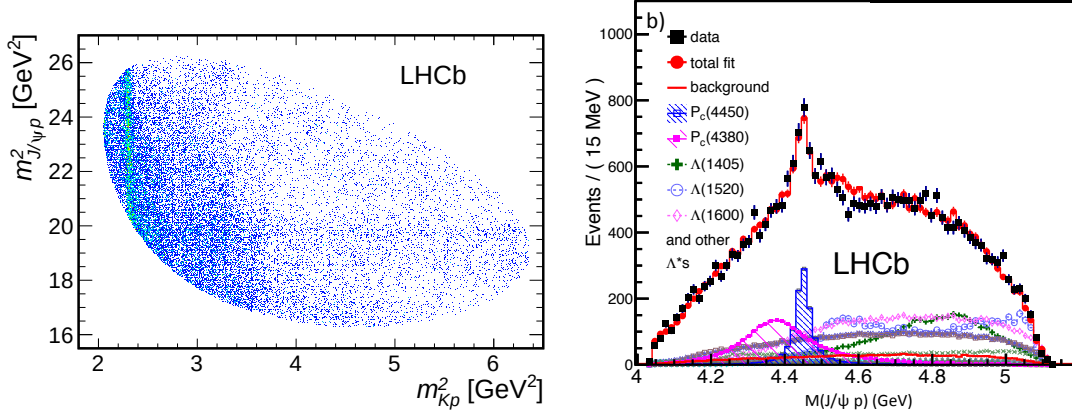


Figure 1.20 Dalitz plot of $\Lambda_b^0 \rightarrow J/\psi p K^-$ (left) and $m_{J/\psi p}$ mass distribution (right) with Run 1 data sample (left)^[124].

Since all three states are narrow and below the $\Sigma_c^+ \bar{D}^0$ and $\Sigma_c^+ \bar{D}^{*0}$ thresholds within plausible hadron-hadron binding energies, they provide the strongest experimental evidence to date for the existence of bound states of a baryon and a meson. According to some theoretical models^[126–130], several other P_c^+ states should exist. We expect more structures will be observed at LHCb with a larger data sample.

The Cabibbo-suppressed decay $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ was also inspected at LHCb with Run 1 sample^[131]. From a 6-dimensional amplitude analysis, 3.1σ evidence for summed presence of $P_c(4380)^+$, $P_c(445)^+$ and $Z_c(4200)^-$ is obtained. The $m_{p\pi}$ and $m_{J/\psi p}$ projections of the fit

are compared in Fig. 1.22. Due to the ambiguities between P_c^+ and Z_c^- terms, any individual exotic state cannot be confirmed from this study.

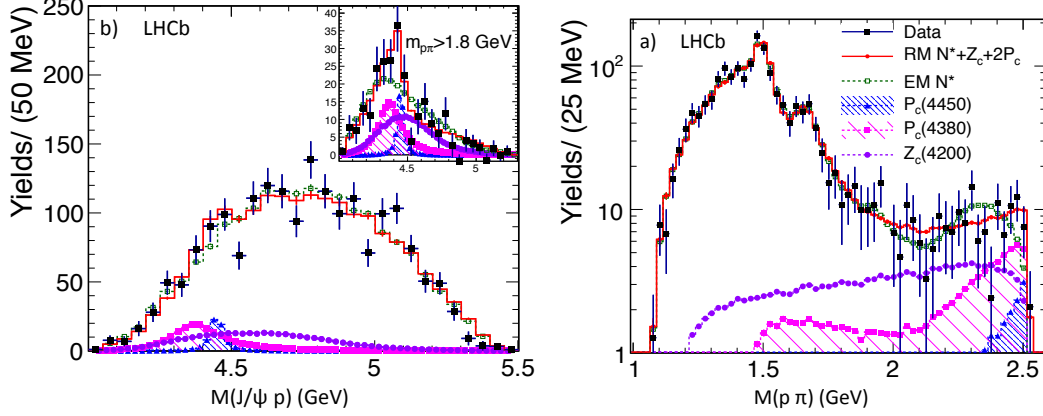


Figure 1.21 Projections of the amplitude fit onto $m_{p\pi}$ (left) and $m_{J/\psi p}$ (right) data distribution for $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decay in LHCb^[131].

Very recently, the open-strangeness pentaquark state was searched for in the $\Xi_b^- \rightarrow J/\psi \Lambda K^-$ decay at LHCb^[132], and the significance of a $J/\psi \Lambda$ structure exceeded 3σ , named as $P_{cs}(4459)^0$. The $P_{cs}(4459)^0 \rightarrow J/\psi \Lambda$ state has a mass only about 19 MeV below the $\Xi_c^0 \bar{D}^{*0}$ threshold and a narrow width. Motivated by this fact, the hypothesis of two resonances contributing to the enhancement is tested. The data cannot confirm or refute the two-peak hypothesis.

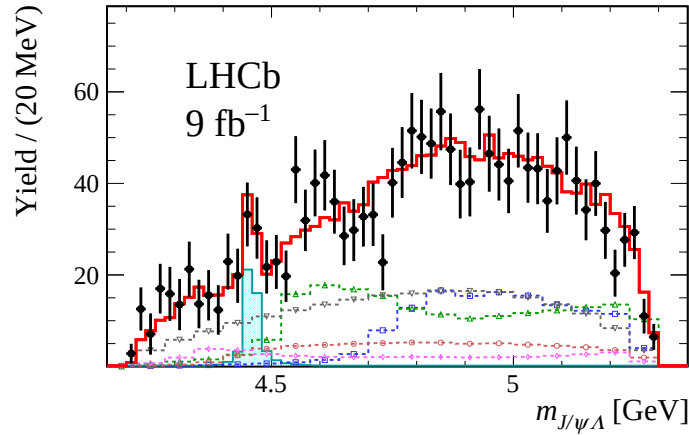


Figure 1.22 The $m_{J/\psi \Lambda}$ distribution of selected candidates compared to the fit with P_{cs}^0 state^[132].

1.5 Theoretical models of exotic hadrons

Along with many observations of exotic hadrons in experiments, the corresponding theories flourished over the past few decades. Because the low-energy QCD cannot be expressed with weak-coupling perturbative theory, many alternative approaches have been developed to explain the exotic hadrons observed in experiments^[8,53]. Effective field theory (EFT) is one kind of alternative tools that allow one to put generic observations into a rigorous framework. In some case, EFT can describe systems in several energy scales in terms of a few relevant degrees of freedom^[8]. For the exotic hadrons made of one heavy quark, the proper EFT is called Heavy Quark Effective Theory (HEFT). It can exploit the hierarchy of scales $m_h \gg \Lambda_{QCD}$, where m_h is the heavy quark mass and Λ_{QCD} is the hadronic scale. HEFT suits to describe the heavy-light hadrons, which are made of one heavy quark^[133]. Systems made of two or more heavy quarks are characterized by more energy scale than systems containing one heavy quark. This kinds of hadrons can be described by the non-relativistic QCD (NRQCD). However, the NRQCD is not renormalizable, it has been used first for lattice QCD calculations involving heavy quarks^[134].

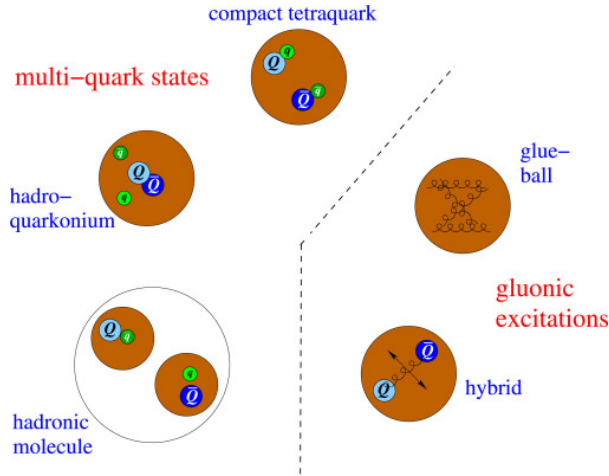


Figure 1.23 Graphic illustration of some proposals for the structure of exotic hadrons (tetraquark states)^[8].

As the theoretical calculations are pretty difficult, some phenomenological approaches were proposed to explain exotic hadrons, which are also often used to classify the exotic hadrons^[8]. Amongst the scenarios relevant for exotic hadrons, the phenomenological models give us some intuitive feelings. The graphical representation of some phenomenological models for exotic hadrons is shown in Fig. 1.23. In the following subsections, some models to interpret the exotic hadron candidates observed in experiments are introduced.

1.5.1 Phenomenological models

1.5.1.1 Compact multiquark states

In this picture, hadrons with a larger number of quarks (such as tetraquark or pentaquark states) are formed from the same quark-level interactions that bind conventional mesons or baryons. Due to the technical obstacles in extracting predictions from non-perturbative QCD, it is not recently possible to determine whether or not such states should exist from first principles. According to observations of experimental candidates, some phenomenological quark models have been used to explain the mass spectra, and production and decay characteristics, of compact multiquark states. The compact quark states involve quark or effective diquark degrees of freedom, with a long-range confining potential and short-range Coulomb contribution due to one-gluon exchange^[135]. A pair of quarks can be either in the $\bar{\mathbf{3}}$ or in the $\mathbf{6}$ representation of the color group, and the former configuration is more compact bound^[136]. Accordingly, the latter is neglected in many works, although the $\mathbf{6}$ can play an important role in studying the multiquark systems^[137]. The $\mathbf{6}$ representation is the basic assumption in the compact multiquark model. In this model, some parameters are fixed to the spectra of conventional hadrons, and the exotic hadron mass can be predicted with large uncertainties, but the spin-dependent mass splittings is better controlled.

A most obvious feature of the compact multiquark picture is that the existence of one such state typically implies a rich spectrum of partner states. A given set of constituents, even restricting to ground states (S-wave), can combine into many possible combinations of spin and isospin, and the multiplicity of states is enlarged further due to the non-trivial "hidden" color configurations. In the absence of experimental evidence for large families of multiquark states, a challenge for theory is to explain why certain states are special; one possibility is that restrictions on their allowed decays imply that some states are uniquely stable.

1.5.1.2 Hadronic molecules

The molecule model thinks the multiquark states are meson-meson or meson-baryon molecule-like systems that are bound via Yukawa-like nuclear forces, and bound states comprised of quarkonium cores surrounded by clouds of light quarks and gluons^[138]. Whereas compact multiquark states are built from quarks exchanging gluons, molecules are built from hadrons exchanging mesons. The closest analogy is the deuteron, which is well-understood in terms of proton-neutron degrees of freedom. Many of the tools of nuclear physics can be applied to the spectra of hadronic molecules: the only difference is that the proton and neutron

are replaced with other hadrons.

The pion exchange is the main contribution to binding in a molecule, as the same time, some models additionally include the exchange of heavier mesons. Exactly as the compact multiquark model, it is not possible to confirm definitively from theory whether such states should exist: the binding or otherwise tends to be sensitive to a poorly-constrained cut-off parameter in the hadron-hadron potential. Nevertheless, a feature common to most models is that its binding energy is quite small. Thus a key prediction of the molecular approach is that the masses of states are closely aligned to the relevant two-hadron thresholds. Weak binding also implies that hadronic molecules are extended objects, as distinct from multiquark states. Another key difference compared to the compact multiquark model is that the spectra of molecular states is restricted. Assuming the dominance of pion exchange, for example, many combinations of constituents can be ruled out on the basis of simple symmetry arguments, and even among the remaining combinations, only certain channels result in attractive binding potentials. In this sense, molecular models are more constrained, and can more readily be verified by experiment, compared to compact multiquark models, which can typically accommodate experimental candidates with a large variety of constituents and quantum numbers.

Within this model, the hidden-charm molecular type pentaquark states were predicted. In 2019, LHCb collaboration updated this analysis with a ten times larger data sample, which strongly supports the molecular pentaquark states, details will be presented in Chapter. 4. At present, the idea with one boson exchanged between meson-meson or meson-baryon is an popular way to treat many exotic hadronic interactions. Nevertheless, this model sometimes lacks the definite predictive power as too many unknown parameters such as the coupling constants and cutoff parameter^[53].

1.5.1.3 Hybrids

The exotic hadrons identified as quark-antiquark hadrons with excited gluonic degrees of freedom is named as “hybrids”. Unlike compact multiquark states and hadronic molecules, the distinguishing feature of hybrids is the main role of gluonic degrees of freedom. In this model, gluons bring a new type of excitation to the system, in addition to rotational and radial motion of the quarks in ordinary quark-antiquark systems. In conventional hadrons, the string-like flux tube is in its ground state, and the linear dependence of its energy with distance yields the familiar linear confining potential for quarks^[8,139]. In hybrids, the flux tube is in an excited state, with intrinsic angular momentum; the coupled quark-flux tube system has a wave-function analogous to a spinning top, with fast and slow rotational degrees of freedom.

An alternative picture of hybrids describes the gluonic excitation in terms of a point-like “constituent” gluon^[8], where the gluons carry both color and spin.

Although both hybrid mesons and hybrid baryons are possible, the meson sector will be most noticeable, as they have combinations of spin, parity and charge conjugation quantum numbers which are not possible for conventional mesons. Different from the other types of exotica discussed above, there are strong predictions from QCD for the existence of hybrids. Spectra of hybrid mesons have been unambiguously observed using lattice QCD, and the pattern of their quantum numbers confirms model expectations^[140].

However, there is not compelling experimental evidence for hybrids. A possible explanation is their unusual decay characteristics: both flux tube and constituent gluon models predict that hybrid mesons do not decay into pairs of ground state mesons, but prefer to decay into experimentally more challenging modes, like orbital-excited final states.

1.5.2 Threshold effects

While the classic signal for the presence of an unstable hadron resonance is a peak in the invariant mass distribution of its decay products, not all mass-spectrum peaks are genuine hadron states, since some peaks are produced by near-threshold kinematic effects. In these explanations, the exotic structures are usually generated by the branch points, that emerge whenever the energy crosses a threshold related to a new channel. These include threshold cusps and anomalous triangle singularities^[4,141].

In the two-body threshold cusps model, the threshold cusp may be observed when the intermediate two particles must rescatter into final states with a lower threshold, as shown in Fig. 1.24. In this situation, the cusp strength is controlled by the rescattering $B'C' \rightarrow BC$ at the threshold of the intermediate particles $B'C'$. On the contrary, a sharp cusp might hint at a near-threshold pole and thus the existence of a resonance.

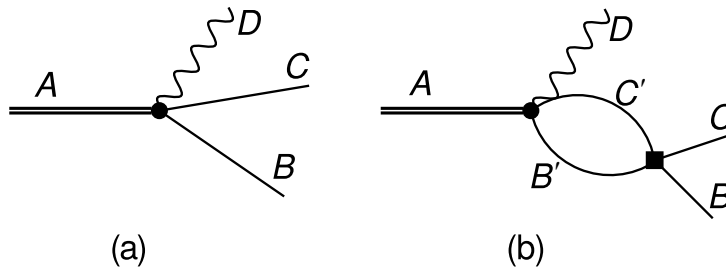


Figure 1.24 A decay process $A \rightarrow B C D$. (a): direct transition at tree level; (b): via final state interaction $B'C' \rightarrow BC$ ^[141].

In the view of triangle singularity model, some resonance-like structures may contain important triangle singularity effects, which arise from the rescattering processes with three particles in the intermediate state. The kinematic triangle singularities of the rescattering amplitudes will be performed as bumps in the invariant mass distributions, and usually these singularities are close to the thresholds. A graphic explanation of this phenomenon is shown in Fig. 4.12. In this case, the singularity effect appears in the mass distribution of C .

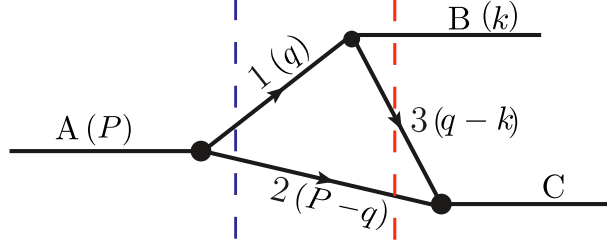


Figure 1.25 A triangle diagram for the reaction from A to B and C. The two vertical dashed lines denote the two relevant cuts^[141].

1.6 Motivation of this dissertation

As mentioned above, the concept of exotic hadrons were originated from the quark model. Searching for new exotic hadrons and measuring their properties in experiments can bring us closer to the nature of exotic hadrons, thereby to probe QCD in a unique window. Over the past two decades, very fruitful results related exotic hadrons spurted out both in experiments and theories. However, there is no unified theoretical model with ability to interpret all the exotic hadrons observed in experiments. Luckily, some properties of exotic hadrons predicted in different models are different. These provide us chances to distinguish different theoretical models according to the experimental measurements. Therefore, it is needed to search for more exotic hadrons and study their properties in experiments.

From an experimental view, there are some open issues to resolve, *e.g.* the exotic hadrons containing strange quark had not been found in experiments before 2020, the pentaquark states observed at LHCb had not been confirmed in different channels, and open-charm exotic hadrons have not been discovered in experiments. These issues can be explored at LHCb, and the corresponding answers will benefit the construction of theoretical models. This dissertation explores these issues using the pp collision data sample collected by LHCb. The structure of this dissertation is organized as follows. Chapter 2 provides a brief introduction to the LHCb

experiment. The observations of tetraquark states with strangeness and new neutral tetraquark candidates are reported in Chapter 3. The hidden-charm pentaquark studies are provided in Chapter 4. The search for open-charm pentaquark is presented in Chapter 5. Finally, Chapter 6 gives a summary and prospect. Furthermore, Appendix A gives some details related to physics analyses presented in the body of this dissertation; as the ECAL simulation study for LHCb Upgrade II is an important component of author's PhD project, some simulation studies for ECAL Upgrade II are introduced in Appendix B.

Chapter 2 The LHC and LHCb

The data of proton-proton collisions are used in this dissertation, which are collected by the Large Hadron Collider Beauty (LHCb) experiment at the Large Hadron Collider (LHC). A brief overview of the LHC operations and LHCb designs during Run 1 and Run 2 is presented in this chapter. During the years 2018-2021, the LHCb sub-detectors are being upgraded (Upgrade I) for higher luminosity conditions planned in the following Run 3 period, a brief introduction to the LHCb Upgrade I is also described, which is now in construction to meet the high luminosity condition from 2022.

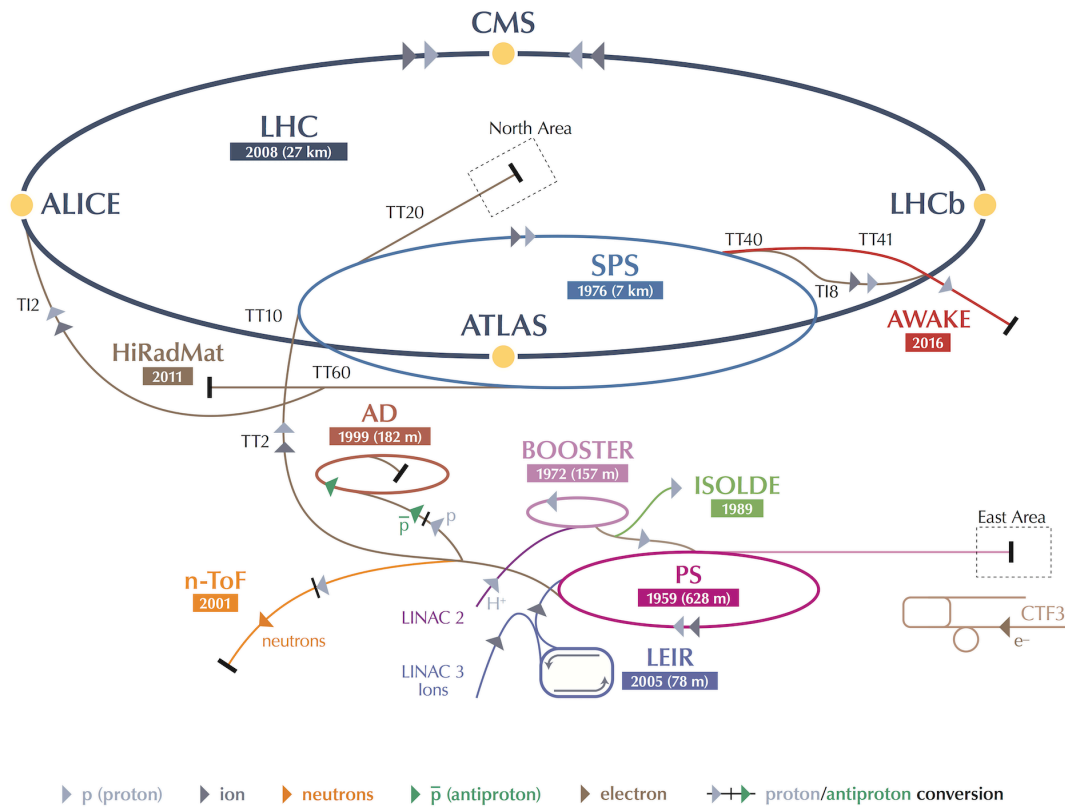
2.1 The Large Hadron Collider

The LHC is an international project of particle accelerator at the European Organization for Nuclear Research (CERN) in Geneva, Switzerland, and is the most powerful one ever constructed^[142,143]. It can accelerate particles to the world-highest energy ever, making it the foremost facility for research in high energy (particle) physics in the world. This superconducting particle accelerator, approximately 27 kilometers in circumference, provides two counter-rotating proton beams with colliding at a designed center-of-mass energy of 14 TeV (reached 13 TeV during Run 2), generating particles for physics studies. It can also provide beams of heavy ions, such as lead, for the studies of QCD matter under extreme conditions. During 2011 and 2012 (Run 1) the LHC was operated at 4 TeV per beam and was upgraded to 6.5 TeV per beam in the 2015-2018 (Run 2) period, reaching to a new level of energy which is not achieved in Run 1.

Actually, the LHC is the last facility of an accelerator complex at CERN, which is a succession of machines that accelerate particles to increasingly higher energies, as shown in Fig. 2.1. Each machine in the complex boosts the energy of a particle beam accepted from the previous setup, and injects the beam into the next machine in the sequence.

The proton source comes from a simple bottle of hydrogen gas. Then, electric field is used to strip the electrons in hydrogen atoms to create protons. Linac 2, the first accelerator in the complex, accelerates these protons to an energy of 50 MeV per particle, forming the primary proton beam. Subsequently, the beam is then injected into the Proton Synchrotron Booster (PSB), which accelerates the protons to 1.4 GeV per particle. Afterwards, the Proton Synchrotron (PS) pushes the beam to 25 GeV per particle. Then, protons are sent to the Super

CERN's Accelerator Complex



LHC Large Hadron Collider SPS Super Proton Synchrotron PS Proton Synchrotron
 AD Antiproton Decelerator CTF3 Clic Test Facility AWAKE Advanced WAKEfield Experiment ISOLDE Isotope Separator OnLine DEvice
 LEIR Low Energy Ion Ring LINAC LINear ACcelerator n-ToF Neutrons Time Of Flight HiRadMat High-Radiation to Materials

Figure 2.1 The CERN accelerator complex. In the complex, the LHC is the last accelerator (dark grey line). Particles are initially boosted in chains smaller machines before being injected to the LHC. Beams in the small machines are also used for physics research at dedicated experiments^[144].

Proton Synchrotron (SPS) which accelerates them to 450 GeV. Finally, the protons from the SPS are transferred to the two rings (beam pipes) of the LHC. The two beams circulate in the LHC either clockwise or anticlockwise respectively, and are boosted to targeting energies when they pass through boosters installed on the LHC rings. Usually, the synchronous protons get clumped in the space, as well as time, forming a bunch. The typical bunch spacing in the accelerator is 25 ns. It takes around 4 minutes to fill up the beam slots in each the LHC ring, and 20 minutes for the protons to reach their maximum energy, and are then operated for several hours under normal operating conditions. The two beams cross and collide inside four big particle detectors, ALICE, ATLAS, CMS and LHCb, leaving pp collisions of center-of-mass energy equal to 7 TeV in Run 1 and 13 TeV in Run 2. And one pp collision is called as one event for detectors. In addition to the protons, lead ions can be accelerated in the LHC, which start from a source of vaporized lead and enter Linac 3 before being collected, they then follow the same route to reach their targeting energy as the protons, and the lead-ion beams can be accelerated to 1.38 TeV per nucleon before the collision^[144].

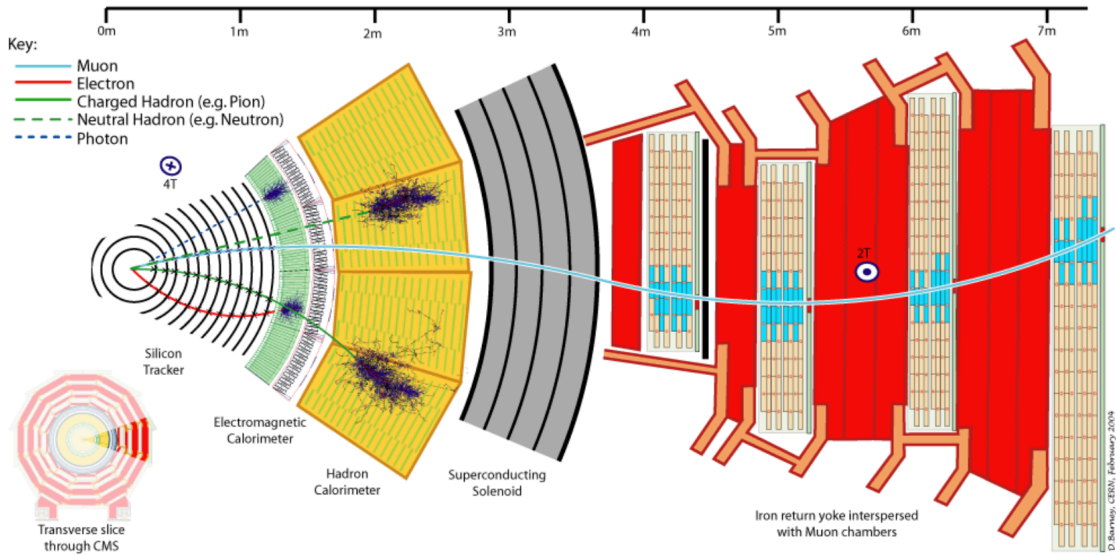


Figure 2.2 Transverse slice through the CMS detector, particle types are inferred by combining responses in different sub-detectors.^[145]

Four main experiments are placed at the LHC. Two of them are general purpose experiments, studying many aspects of particle productions and decays. These are “A Toroidal LHC Apparatus” (ATLAS)^[146] and “Compact Muon Solenoid experiment” (CMS)^[147], each involving more than 3000 scientists and engineers, in order to search for particles beyond the standard model directly. The other two experiments, each involving around 1000 scientists and engineers, have more specialized physics interests: “A Large Ion-Collider Experiment” (AL-

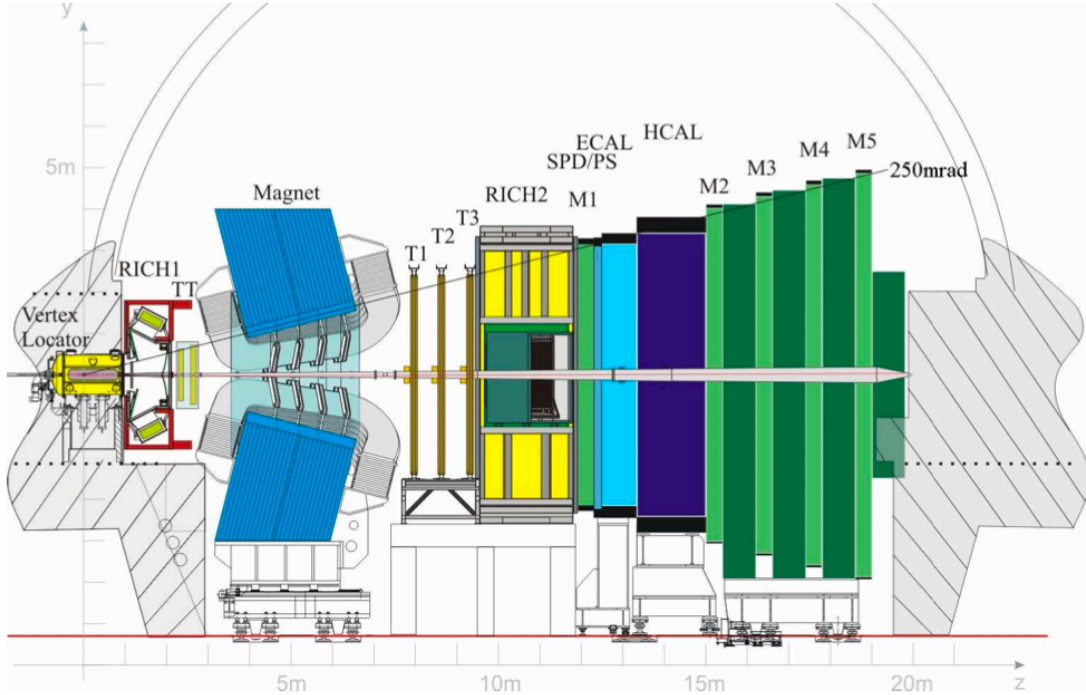
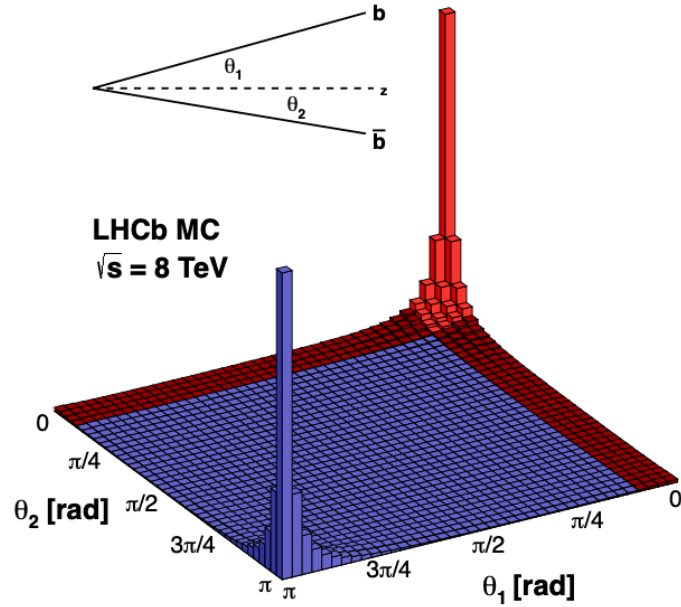
ICE)^[148] is concerned with heavy-ion collisions, and “LHC beauty experiment” (LHCb)^[149] focuses more on hadrons containing the bottom or charm quark.

The detectors of these experiments have been constructed as a series of sub-detectors, and all follow roughly the same scheme, as shown in Fig. 2.2, where the CMS is taken as an example. Starting from the inner layers, which are positioned closest to the collision region, each detector consists of: trackers to measure charged-particle trajectories; calorimeters to stop photons, electrons and hadrons, and measure their energies; muon detectors to record muons, the only charged particles that are able to reach the detector’s outermost layers. The ALICE, ATLAS and CMS detectors are approximately cylindrical in shape, so as to be sensitive to particles emerging in all directions from an interaction. The majority of the particles of interest in LHCb are emitted at small angles, and so the detector covers only a narrow cone around the direction of the incoming protons. In next sections, the detailed structure of LHCb will be reviewed.

2.2 The LHCb detector

LHCb is a single-arm spectrometer with a forward angular coverage from approximately 10 mrad to 300 mrad, corresponding to a pseudorapidity range of $1.8 < \eta < 4.9$ ^[149]. The layout of the LHCb spectrometer is shown in Fig. 2.3. The right-handed coordinate system is adopted with the z axis along the beam, and the y axis along the vertical direction. The particular geometry is dedicated to b and $\bar{b}$ quarks acceptance because the b and $\bar{b}$ quarks produced at LHC energies tend to have their directions along the same beam line. The polar angles of the b and $\bar{b}$ hadrons produced in $\sqrt{s} = 8$ TeV pp collisions are shown in Fig. 2.4, as predicted from PYTHIA simulations^[150], and similar results can be obtained for pp collisions with the center-of-mass energy ranging from 7 TeV to 14 TeV.

The LHCb detector during Run 1 and Run 2 was designed to operate at a peak luminosity of $\mathcal{L} = 2 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$. Considering the designed maximum luminosity of the LHC to be $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$, a lower luminosity is applied to collect for less “busy” events. Higher luminosities tend to produce a much larger number of pair-wise pp interactions per bunch crossing. In the bunch crossing, a spatial point where a proton-proton collision happens is named as a primary vertex (PV), and the identification of PVs is essential in many analyses in order to accurately reconstruct the tracks of decaying particles. A higher number of PVs make it more difficult to identify the originated vertexes of decaying particles. Events with higher track multiplicity as a result of larger number of collisions would also make event reconstruction


 Figure 2.3 Side view of the LHCb detector^[149].

 Figure 2.4 Display of $b\bar{b}$ production angles as simulated with PYTHIA 8. The LHCb acceptance is shown in red.

more difficult. Moreover, operating at higher luminosities also limits the radiation damage and reduce detector aging problems. A “luminosity leveling” method is used to achieve the lower luminosity, done by shifting the beams relative to each other in transverse direction^[149]. As the number of protons goes down due to collisions or losses in the beam pipe, the beams are moved closer in small increments so that the instantaneous luminosity is kept constant.

After Upgrade I, the targeting peak luminosity during the LHC Run 3 will be increased to $2 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$, which is around 5 times larger than the value in Run 2^[151]. After a further Upgrade II during 2031-2032, the peak luminosity could reach $1.5 \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$, enabling LHCb to collect a total integrated luminosity of 300 fb^{-1} after Run 5, which offers us opportunities to take full advantage of the flavor-physics at the HL-LHC^[152,153]. More aspects related to Upgrade II and the corresponding challenge to sub-detector upgrade will be discussed in Chapter B, especially about ECAL. Introductions to the sub-system of the LHCb detector will be described below.

2.2.1 Tracking

The tracking systems are used to reconstruct charged particles within the three components^[149]: the Vertex Locator (VELO), the Dipole Magnet and the four tracking stations, as shown in Fig. 2.5. Concerning the four tracking stations, one is placed in front of the Dipole Magnet, referred as Tracker Turicensis (TT) and is constructed using silicon microstrip sensors, and the other three are located behind the magnet, known as T1, T2 and T3 respectively. T1-T3 stations are divided into inner and outer parts, where the inner detector is built from silicon microstrip, called as Inner Tracker (IT), and the outer part is a drift-time straw tube detector, referred as Outer Tracker (OT). The following is some details about these tracking sub-detectors.

2.2.1.1 Vertex Locator

The VELO is used to reconstruct vertices of the pp collision point and the decays of long-lived particles (b and c -hadrons), which plays an important role in the online event selection^[155] ①. For precise position measurements, the VELO should be put as close to a vertex as possible, and make the material between the first measured point and the vertex as less as possible.

The current VELO is constructed by 25 stations placed perpendicular to the beam direction, each providing a measurement of the r (radial) and ϕ (polar) coordinates, as shown

① Details will be discussed in Sec. 2.2.3.

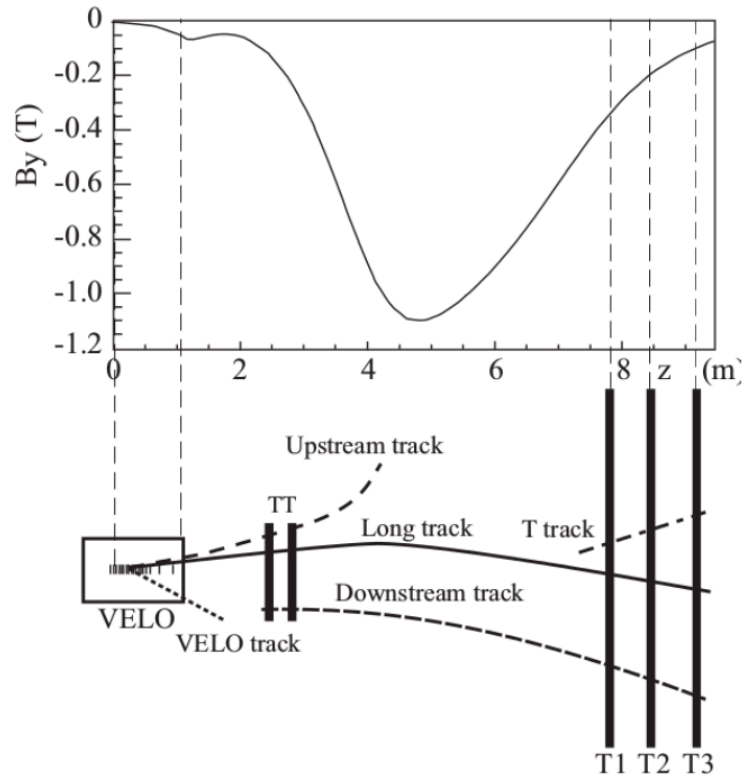


Figure 2.5 Schematic of the LHCb tracking system with different types of tracking definitions by the LHCb reconstruction algorithm^[154]. The long tracks are reconstructed with hits from at least the VELO and T-stations. The downstream tracks are reconstructed with hits from the TT and T-stations. The T tracks are reconstructed with hits only in T-stations. The upstream tracks are reconstructed based on VELO and TT. The VELO tracks are reconstructed only using the hits in VELO.

in Fig. 2.6. Each station consists of two retractable modules that can be closed or separated according to the running conditions. Every module has 2 silicon sensors. One is called ϕ -measuring sensor and provides information on the azimuthal coordinate around the beam. The other is called the r -measuring sensor and provides information on the radial distance from the beam axis. The thickness of strip is 300 μm . The total number of strips for each sensor type is about 180000 channels, counting all the stations.

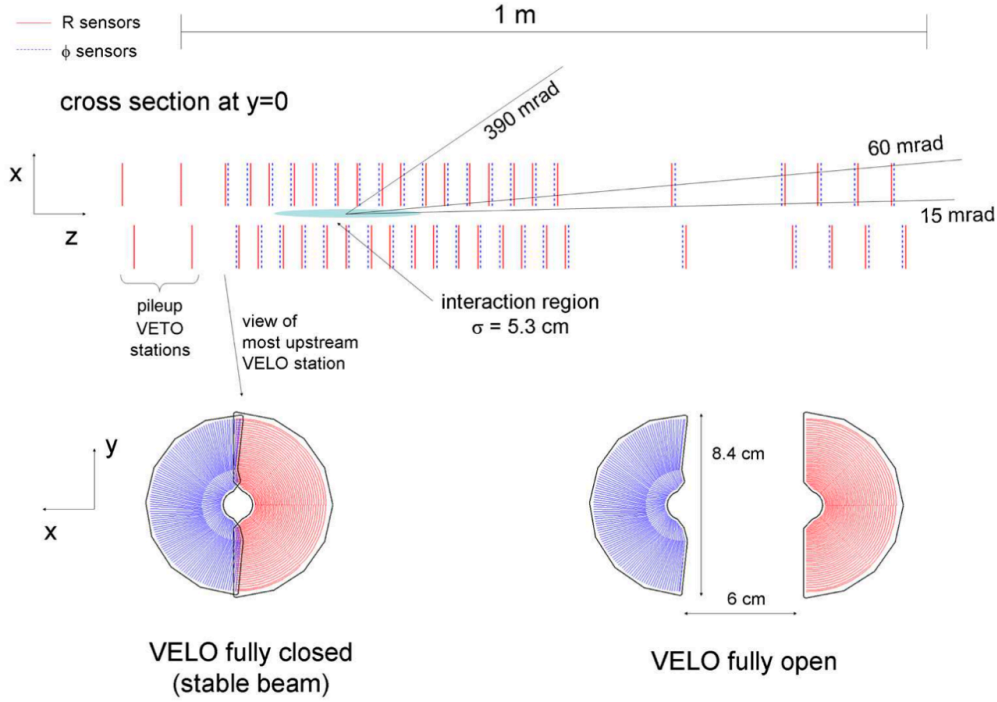


Figure 2.6 Top: Overview of the VELO as seen in the (x, z) plane, at $y = 0$. Bottom: The front face of the modules in the (x, y) plane, for both closed (left) and open (right) positions^[149].

Many auxiliary systems are designed to keep VELO running stably. As the VELO detector is running in extreme radiation environment with strongly non-uniform fluence, the sensors are required to maintaining at a temperature between -30 and -20 $^{\circ}\text{C}$ to counteract the radiation effects. Besides, the VELO detectors have also to be operated in vacuum. A movement system is connected to VELO, which is used to adjust the distance between VELO and colliding beams. Before the LHC ring is filled with beams, VELO has to move away to avoid damages due to fill-by-fill variations of beam positions; when the pp beams are in stable conditions, the VELO need to be placed into an optimized position that is about 8 mm to away from the measured coordinates of beam positions in each fill.

For LHCb Upgrade I, the VELO strip modules will be replaced by pixel detectors with pixel pitch of 55 μm , which will significantly reduce the fraction of fake tracks as seen by current detector^[156]. Besides, totally new front-end electronics are installed and a new thin

aluminum radio frequency foil is used to make the detector even closer to the interaction points with intensifier radiation^[157].

2.2.1.2 Dipole Magnets

The Dipole Magnet provides magnetic field allowing to measure the momentum of charged particles, which includes two separate aluminum coils, shaped like a saddle and mounted symmetrically in a window-frame magnetic yoke^[158]. Each coil has fifteen pancakes arranged in five triplets, and the conductor has a specific ohm resistance below $28 \Omega \text{ m}$ at 20°C . An overview of the magnet can be seen in Fig. 2.7. The magnetic field points vertically in the y -direction, and covers the polar angle acceptance of $\pm 250 \text{ mrad}$ vertically and $\pm 300 \text{ mrad}$ horizontally viewed from the interaction region. The integrated magnetic field for tracks of more than 10 m in length is 4 Tm. In order to achieve a good momentum resolution, the integrated magnetic field must be precisely determined for all trajectories, which has a precision on the order of 10^{-4} . This precision has been achieved using arrays of hall probes, with which the components of the field are measured in a fine grid spanning from the interaction point to the RICH2 detector. In order to study the left-right detection asymmetry for charged particles, the direction of the magnet field is also able to be reversed.

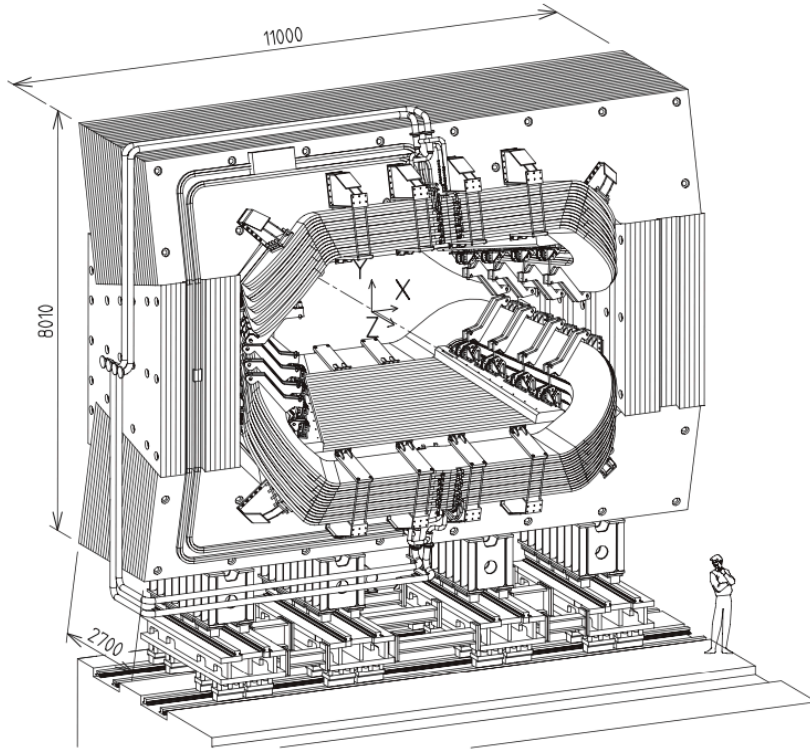


Figure 2.7 Perspective overview of the Dipole Magnet (the unit is mm)^[149].

2.2.1.3 Silicon Tracker

Silicon strip technology is used for TT and IT detectors, where the size of the strip pitch is about $200\text{ }\mu\text{m}$ ^[159]. The TT is a planar tracking station, which is located upstream of the LHCb Dipole Magnet and covers the full acceptance of the LHCb detector, as shown in the top of Fig. 2.8. The IT refers to the center region of the three tracking stations downstream of the magnet, which is about 120 cm wide and 40 cm tall, as shown in the bottom of Fig. 2.8.

The basic building blocks of the TT layers are half modules, which are joined together end-to-end in order to create the full module, as shown in the bottom of Fig. 2.8. The half modules are made up of seven silicon sensors, which are organized into either two or three readout sectors, depending on the proximity to the beam pipe. The read-out sectors have one, two, three, or four sensors bonded together, such that the sectors closer to the beam pipe, which encounter the higher particle flux, have the lower number of sensors bonded together. The space above and below the beam pipe are each covered by a half module, and the regions to the sides are covered by rows of seven full modules in the first two layers and eight full modules in the last two layers. To avoid acceptance gaps, adjacent modules are staggered by about 1 cm in z direction to allow overlap by a few millimeters in x . Layers of TT detectors are placed in a $(x-u-v-x)$ configuration (Fig. 2.8), and for the u and v layers, individual modules are rotated by the respective stereo angle, which would provide some information on the y -coordinate.

There are four detector boxes arranged on the left and right in each of IT stations. at the top and the bottom around the beam pipe. A detector box contains four detection layers which are arranged in the same $(x-u-v-x)$ configuration as the TT, as shown in the top of Fig. 2.8. Each of the detection layers in a box has seven silicon modules. Acceptance gaps are avoided by having adjacent modules staggered by 4 mm in the z direction and an overlap by 3 mm in the x direction. The modules in the top and bottom boxes consist of a single silicon sensor, while the modules in the side boxes consist of two silicon sensors.

2.2.1.4 Outer Tracker

The IT is enclosed by the Outer Tracker (OT) to record particles in the outer region^[160]. The OT is a large area straw-tube detector, and detects about 70% of the charged particles tracks inside the LHCb acceptance. It has a lower granularity than the IT due to the lower activity in the outer regions. Same as the IT, each OT station consists of four layers in an $(x-u-v-x)$ arrangement. The OT provides excellent momentum measurement which is necessary for the precise determination of the invariant mass of the reconstructed b -hadrons. The front-end

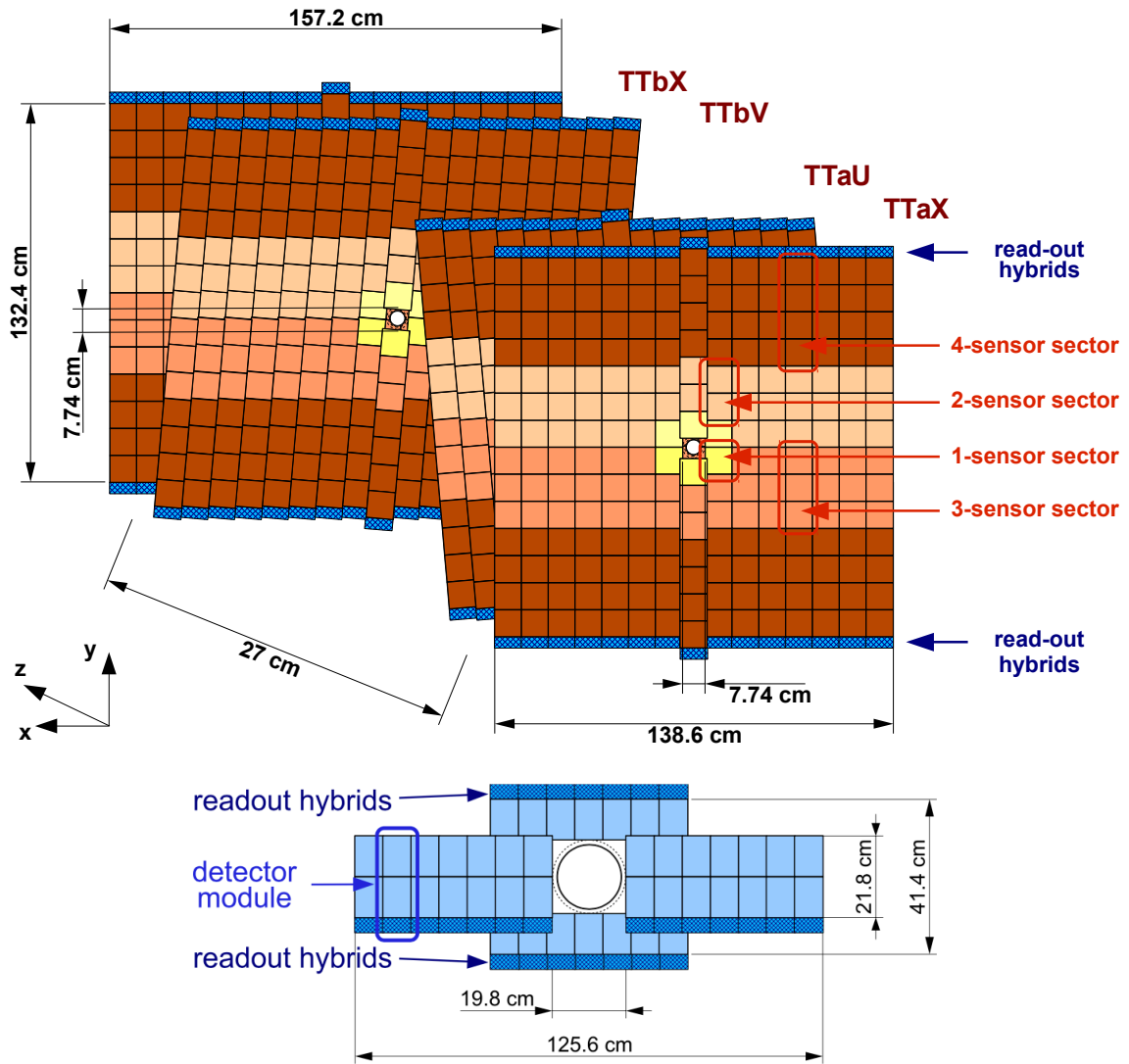


Figure 2.8 Overview of the TT (top) and IT (bottom)^[149].

electronics measure drift times of ionisation clusters produced by charged particles traversing the straw tubes, which are used to reconstruct hit position. Fig. 2.9 shows the three OT stations and the bridge carrying them. The straw tubes are filled with a mixture of Argon (70%), CO₂ (28.5%) and O₂ (1.5%) to ensure a fast drift time below 50 ns and a sufficient drift-coordinate resolution (200 μm).

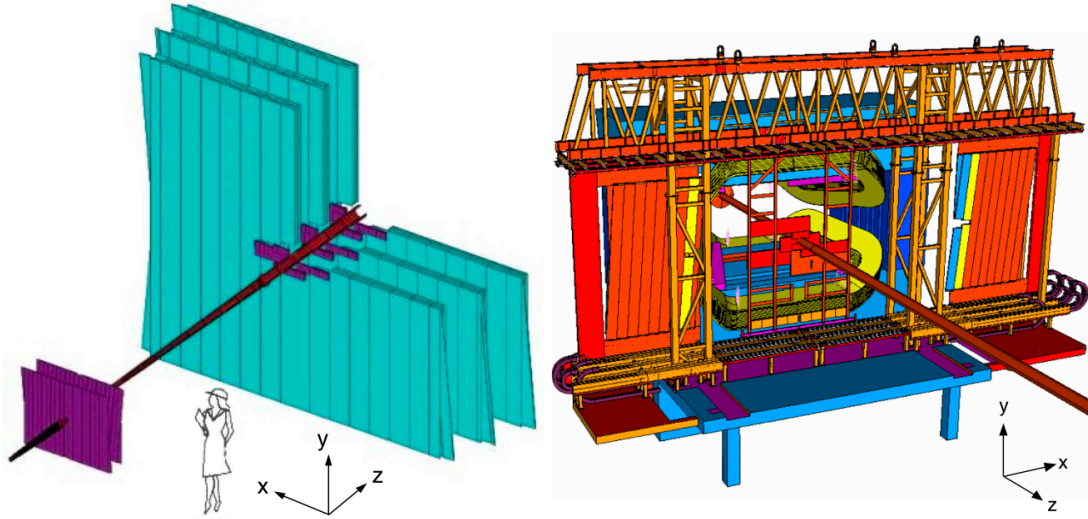


Figure 2.9 Left: The three OT stations surrounding the three IT stations (purple). Right: Overview of the OT bridge carrying the C-frames^[149].

All tracking detectors are being upgraded for Run 3 during the long shutdown 2^[161]. The upstream tracker (UT) will replace the current TT. It uses silicon strip technology with improved segmentation and acceptance, and the total structure is similar to TT. The sensors have finer granularity comparing to TT to cope with an increased particle density. Sensor strips are vertical, thereby having precise measurements in the horizontal position, which is affected by the magnetic bending due to the Dipole Magnet. Each UT plane is constructed from 16 columnar structure called “staves”, which mostly have 14 sensor modules, and mounted alternatively on the front and the back surfaces of its support structure^[161–163].

The tracking stations downstream of the magnet will be replaced with a scintillating fiber (SciFi) system during Upgrade I, which is a total different detector compared with OT. The SciFi tracker consists of three stations each with four detection planes. The detector is built from individual modules of dimension 0.5 m $\times$ 4.8 m, each comprising 8 fibre mats with a length of 2.4 m as active detector material. The fibre mats consist of 6 layers of densely packed blue-emitting scintillating fibres with a diameter of 250 μm . The scintillation light is recorded with arrays of state-of-the-art multi-channel silicon photomultipliers (SiPMs). A

custom application-specific integrated circuit (ASIC) is used to digitize the SiPM signals. Subsequent digital electronics perform clustering and data-compression before the data is sent via optical links to the DAQ (Data Acquisition) system. To reduce the thermal noise of the SiPM, in particular after being exposed to a neutron fluence of up to $10^{12} n_{\text{eq}}/\text{cm}^2$, expected for the lifetime of the detector, the SiPM arrays are mounted in so called cold-boxes and cooled down by 3D-printed titanium cold-bars to -40°C ^[161,164].

2.2.2 Particle identification

The purpose of the particle identification (PID) system is to identify the different particle species that are produced in the collisions^[149]. In pp collisions at the LHC, predominantly pions are produced, hence it is crucial to distinguish them from the other particle types. Information from the Ring Imaging CHerenkov (RICH) detectors is used to distinguish the charged final-state hadrons - kaons, pions, and protons. The calorimeters provide information to identify and measure the energy of photons, electrons and hadrons. The muon detectors are employed to detect and measure muons.

2.2.2.1 RICH

The RICH detectors at LHCb are based on the principle of Cherenkov radiation^[165]. Cherenkov radiation occurs when a charged particle travels through a medium faster than the speed of light in that medium. The opening angle θ_c of Cherenkov photon in a cone related to the particles momentum is

$$\cos(\theta_c) = \frac{1}{n\beta} \quad (2.1)$$

where n is the refractive index of the medium and $\beta = v/c$, where v is the velocity of the particle and c is the speed of light in vacuum.

The RICH detectors contains RICH1 and RICH2, aiming to identify charged tracks with low momentum and high momentum respectively. The RICH1 detector is positioned between the VELO and the TT. It covers the full LHCb angular acceptance and provides PID information for particles between $1 - 60 \text{ GeV}/c$. The photodetector planes are split into two halves, one above the beam line and one below the beam line. The RICH2 is placed between the third tracking station and the first muon station. It can successfully distinguish particles with momenta between 15 and $100 \text{ GeV}/c$. Since particles with high momentum are less bent by the magnet, the RICH2 detector covers a smaller angular acceptance of 15 mrad up to 100 (120) mrad in the vertical (horizontal) plane. The RICH2 photodetector plane is also made

up of two sections, placed at either horizontal side of the beam line. The layout of the RICH detectors is given in Fig. 2.10. The Cherenkov radiation is emitted and reflected out of the detector acceptance by a spherical mirror and a flat mirror. They are focused onto detection planes, where the Cherenkov photons are detected using Hybrid Photon Detectors (HPDs). Each half of RICH1 (RICH2) photodetector plane is equipped with 98 (144) HPDs. The particle velocity is reconstructed by identifying rings of Cherenkov photons. In combination with the momentum information from the tracking stations, the mass is determined, and the corresponding type can be identified.

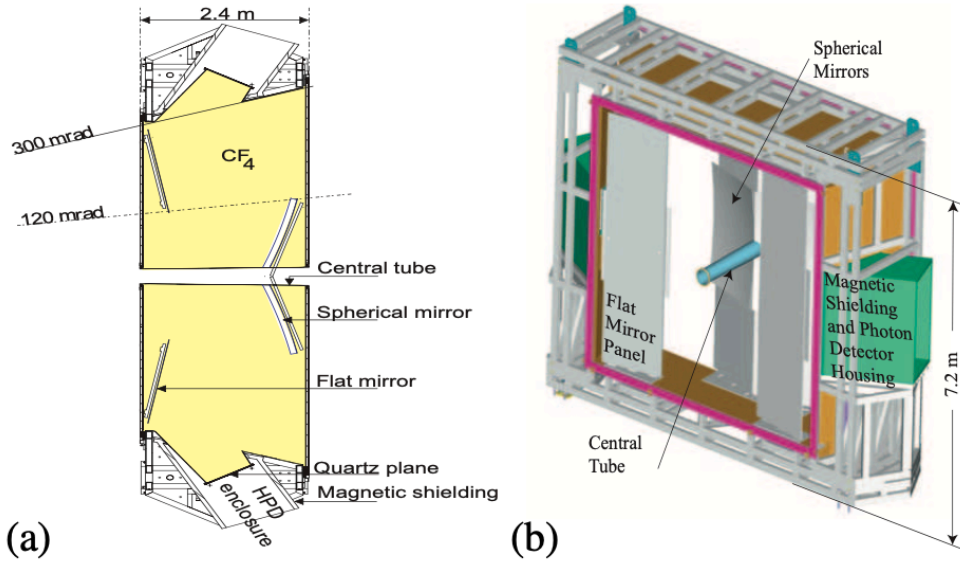


Figure 2.10 The RICH1 detector (left) side view and the RICH2 detector (right) seen from above^[149].

In order to allow operation of the RICH detector system at the Run 3 luminosity, the RICH detectors are being upgraded by installing new single-photon detectors (multianode photomultiplier tubes in place of hybrid photo-detectors) read out by 30 MHz capable electronics, and by modifying the upstream RICH optics and mechanics^[166]. The RICH1 optical system is redesigned, in particular the focal length of the spherical mirrors is increased from 2.7 m to 3.7 m to reduce the hit occupancy on the photodetectors^[167].

2.2.2.2 Calorimeters

The calorimeter system is used mainly to identify hadrons, electrons and photons, and to measure their positions and energies^[168]. It is important that calorimeters enable the reconstruction of B -decay channels containing a prompt photon or π^0 . The requirement of good background rejection capability and reasonable efficiency for these relevant physics channels imposes good calorimeter performance in terms of shower separation ability.

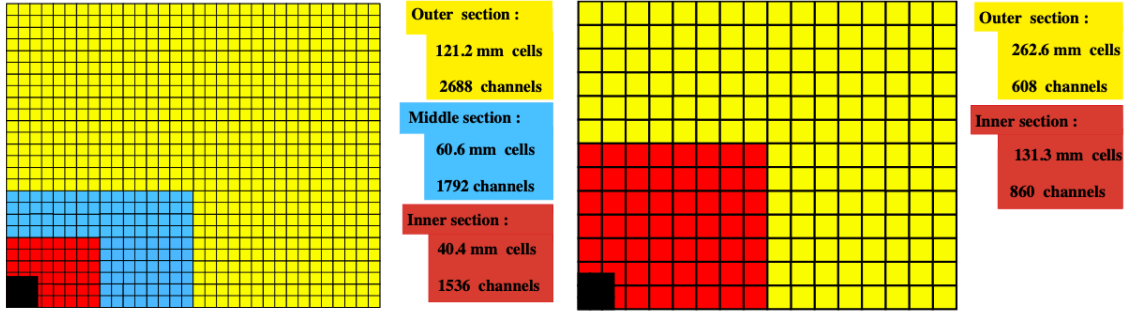


Figure 2.11 The side view of the SPD, PS and ECAL (left) and the HCAL (right) for a quarter of the detector^[149].

The general structure of a calorimeter system is an electromagnetic calorimeter (ECAL) followed by a hadron calorimeter (HCAL) as shown in Fig. 2.3. Besides, the scintillator pad detector (SPD) and preshower detector (PS) are set in front of ECAL, which help to separate electrons and photons for online event selections. The ECAL has three zones with different cell sizes, as shown in the left plot of Fig. 2.11. In the inner zone, the cell size is designed to be close to the corresponding Molière radius ^②. The right plot of Fig. 2.11 shows the lateral segmentation of HCAL, which is divided into two zone according to the cell size. Some properties to the calorimeter sub-detectors are summarized in Tab. 2.1.

Table 2.1 Properties of the calorimeter sub-detectors.

Sub-detectors	SPD /PS	ECAL	HCAL
Number of channels	2×5952	5952	1468
Overall lateral dimension in x,y	$6.2 \text{ m} \times 7.6 \text{ m}$	$6.3 \text{ m} \times 7.8 \text{ m}$	$6.8 \text{ m} \times 8.4 \text{ m}$
Depth in z	$2X_0$	$25X_0$	$5.6\lambda_{\text{int}}$
Basic requirements	20 – 30 photoelectrons per MIP	$\sigma(E)/E =$ $10\%/\sqrt{E} \oplus 1.5\%$	$\sigma(E)/E =$ $80\%/\sqrt{E} \oplus 10\%$
Dynamic range	0 – 100 MIP	0 – 10 GeV E_T	0 – 10 GeV E_T

The basic principle for current four calorimeter systems is that the light generated in scintillator is used to measure the energy deposition, which is transmitted by wavelength shifting (WLS) fibers. The ECAL will be taken as an example, and more details will be given here. Fig. 2.12 illustrates the structure of an ECAL module used in the inner zone. To set up this sampling calorimeter module, total 66 lead plates and 67 scintillator plates are placed alternatively, with plates thickness of Pb:Sc = 2 : 4 (mm), stringed by the WLS fibers. The WLS is used to transported the light to the photomultipliers (PM), which digitize the light

② The radius of a cylinder containing on average 90% of the shower's energy deposition.

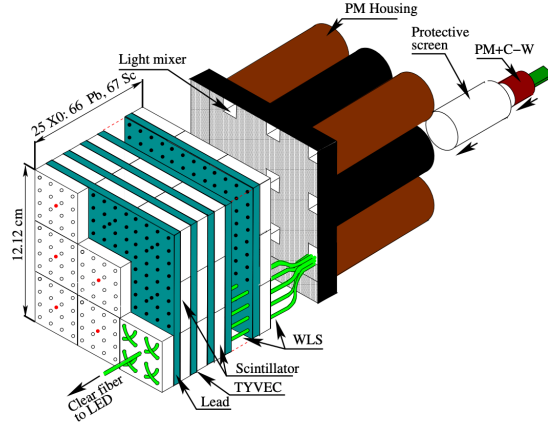


Figure 2.12 Sashlik construction of ECAL in the inner zone^[169].

signal. The digitized signal are processed at 30 MHz rate by the dead-timeless front-end electronics. This kind of module is called as a shashlik technology.

After the Upgrade I, the present ECAL and HCAL will be kept as they are, but the PS and SPD used for online trigger will be removed as all online selections will be performed based software algorithms^③. The ECAL and HCAL front-end electronics will be totally rebuilt, the PM gain will be reduced by factor of 5 to reduce the PM degradation^[166]. The HCAL will be removed after Upgrade II, and the ECAL will be rebuilt using a total new technology^[152,153]. The specific scheme of ECAL at LHCb is under consideration, and more details of this topic are discussed in Appendix B.

2.2.2.3 Moun system

The final part of the LHCb detector is the “Muon” system which provides the identification of muons^[170]. The muon system consists of five stations (M1 to M5), which cover an acceptance of ± 300 mrad in the horizontal plane and ± 200 mrad in the vertical plane, as shown in Fig. 2.13. This corresponds to a geometrical efficiency of approximately 46% for the detection of muons arising from b -hadrons. The first muon station, M1, is placed before the calorimeters in order to avoid possible multiple scattering effects that could modify the particle trajectory. The remaining stations, M2 to M5, are placed after the calorimeter system, at the end of the LHCb detector, and are separated by iron planes of 80 cm thick. Each muon station is divided into four regions (R1-R4) as shown in Fig. 2.13. The R1 region is the closest to the beam-pipe and has the most dense segmentation while the R4 region is placed far away. The segmentation defined per region is such that the charged particle occupancy is expected to be approximately

③ More details will be discussed in Sec. 2.2.3

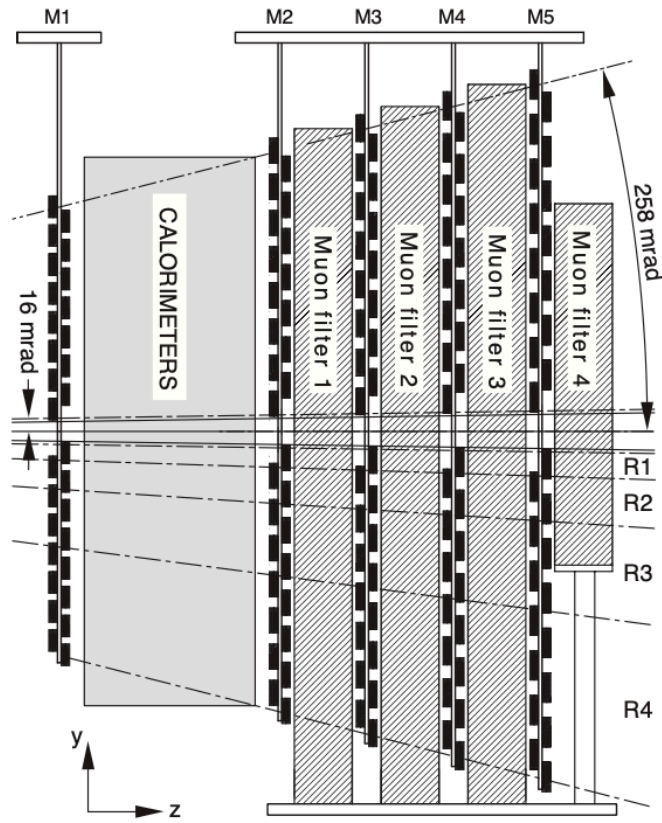


Figure 2.13 Sideview of the muon system^[149].

the same in each region. All the muon chambers are composed by Multi-Wire Proportional Chambers (MWPC), aside from the inner part of the M1 detector, which exploits three gas electron multiplier foils sandwiched between anode and cathode planes (GEM detectors). In total, the muon system consist of 1368 MWPC and 12 GEM detectors.

After Upgrade I, the muon off-detector readout electronics will be redesigned to allow a complete event readout at 30 MHz. Besides, the M1 station before the calorimeter system will be removed^[161,171].

2.2.3 Trigger

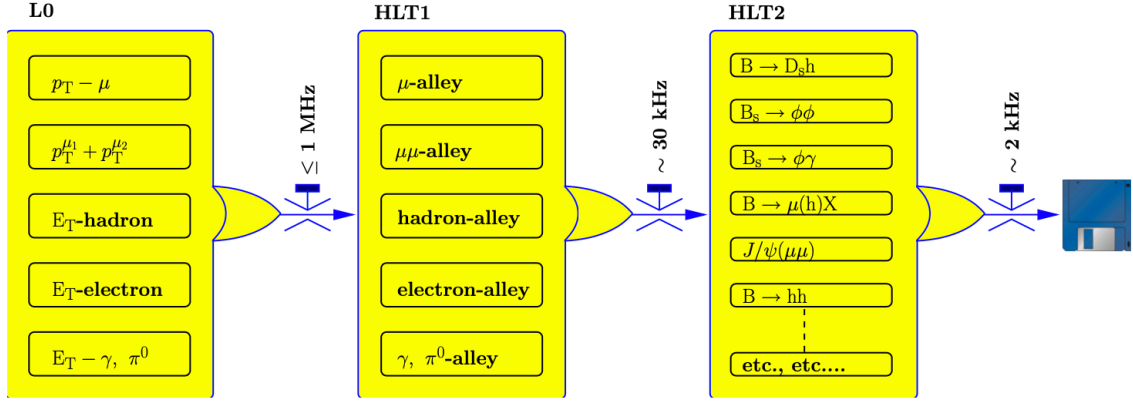


Figure 2.14 Flow-diagram of the different trigger sequences^[149].

The nominal instantaneous luminosity is around $2 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ during Run1 and Run2, and the rate of events is roughly 10 MHz for collisions that produce at least two charged particles with enough VELO and tracking hits to allow for their reconstruction^[172]. In fact, the rate to storage is about 2-5 kHz for Run 1^[173], and about 10-15 kHz for Run 2^[174], which is achieved by the trigger system that reduces the rate by selecting only interesting events. An initial reduction down to 1 MHz comes from the Level-0 (L0) hardware trigger, which uses custom electronics and runs synchronously with the 30 MHz bunch crossing frequency. High Level Trigger (HLT) follows the L0, which is a software-base trigger. The HLT is composed of two stage, which runs asynchronously on a processor farm and reduces the rate to the desired level. The LHCb trigger scheme is shown in Fig. 2.14. More details on each stage will be given in the following sections.

2.2.3.1 L0 trigger

The L0 trigger contains three components: the pile-up system, the Level-0 calorimeter trigger and the Level-0 muon trigger^[172]. All components are connected to the Level-0 decision unit (DU), which is used to evaluate the final decision by collecting all the information obtained from the trigger system, as shown in Fig. 2.15.

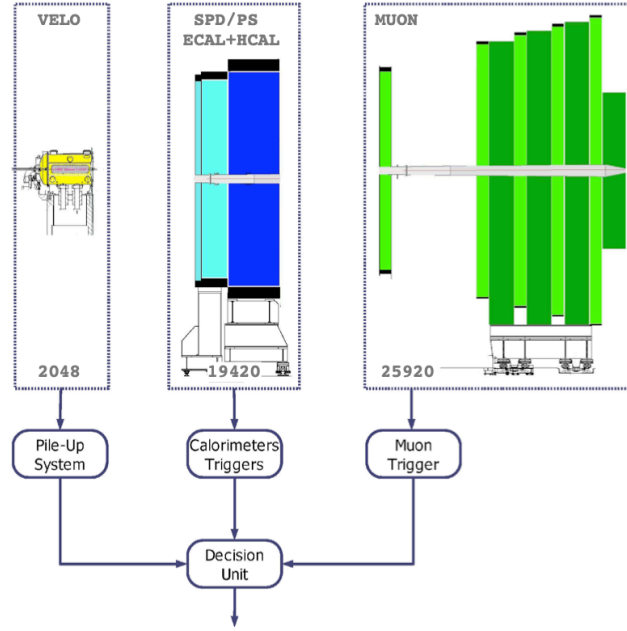


Figure 2.15 Overview of the L0 trigger^[149].

The pile-up system is used to distinguish the crossings with single and multiple visible interactions, which is based on the silicon sensors in the VELO to measure the radial position of tracks. The pile-up system has two functions, one to provide the position of the primary vertices candidates along the beam-line and the other one to measure of the total backward charged track multiplicity.

The calorimeter trigger is designed to search for particles with high transverse energy (E_T) by adding the E_T of 2×2 cells and selecting the clusters with the largest E_T , this step is performed on the Front-End card. Then the information from ECAL, PS and SPD are combined to identify the type of electromagnetic candidate, electron, γ or π^0 . Besides, the E_T of all HCAL cells is summed and a threshold is set to reject crossings without visible interactions. The total number of SPD cells with a hit are counted to provide a measure of the charged track multiplicity in the crossing.

A L0 muon processor is used to search for the two muon tracks with the largest and second largest p_T , which is performed on the logical pads. This trigger can look for hits regarded as

a straight line through the five muon stations and pointing to the interaction point. The track position in the first two stations allows the determination of its p_T with resolution of about 20%.

2.2.3.2 HLT1 trigger

The HLT1 trigger can reconstruct the trajectories of charged particles traversing the full LHCb tracking system. Besides, a precision primary vertex reconstruction is performed^[175]. Most particle-identification algorithms cannot be executed in this stage except the muon identification, thanks to the clean signature. At the same time, the electrons are identified by associating tracking to ECAL clusters, photons and neutral pions are also built from the clusters reconstructed in the L0 trigger.

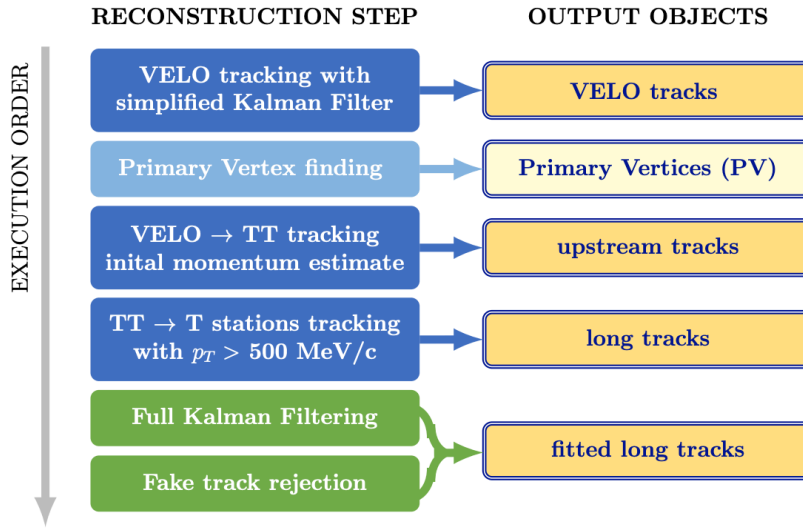


Figure 2.16 Track and vertex reconstruction in the HLT1 trigger^[175].

The steps of HLT1 algorithms are illustrated in Fig. 2.16. Firstly, the VELO tracks are reconstructed, as well as the finding of primary vertices^[176]. Then the VELO tracks are extrapolated to match the hits in the TT stations to form upstream tracks. By the way, this process can also reduce the number of fake VELO tracks. Finally, the long tracks are reconstructed by combining the information from downstream trackers, and the search window in the IT and OT is defined by the maximum possible intervals for charged particles with p_T larger than 500 MeV. The next step is to fit tracks with a Kalman fitter^[177] to obtain the optimal parameter estimate, as well as the fake track rejection to removed the tracks with poor quality.

The muon identification is performed in HLT1 with all fitted tracks. For the muon tracks, the corresponding hits are required in the four Muon stations, which is also used in HLT2 and offline.

2.2.3.3 HLT2 trigger

The full event reconstruction and identification are performed in the HLT2 stage, including the charged track reconstruction, neutral particle reconstruction and particle identification with the information from PID detectors^[172]. The corresponding steps of track and vertex reconstruction are shown in Fig. 2.17.

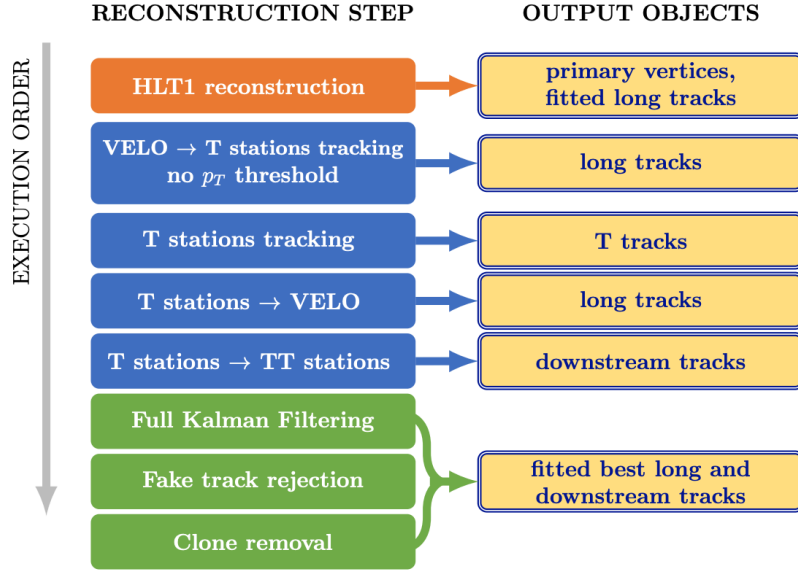


Figure 2.17 Track and vertex reconstruction in the HLT2 trigger^[175].

In the charged particle reconstruction process, the HLT1 algorithm is repeated in the first step. Then a second step is used to reconstruct the lower-momentum tracks that are overlooked in HLT1. The track multiplicity becomes larger as low- p_T tracks are included and no TT hits are required. Besides, the long-lived particles that decay outside the VELO are reconstructed using T-station segments, which are extrapolated to the magnetic field and combined with hits in the TT. Next step is to reject fake tracks from the random combination of hits or a mismatch between different tracks segments. This procedure is based on the Kalman filter^[177] and a multi-variable analysis tool (TMVA)^[178]. Then the remaining tracks are filtered to remove the clone tracks that share enough hits in each subdetector.

The RICH detectors provide the important discrimination between kaons, pions and protons, and this information is used to construct likelihoods for each particle hypothesis. The calorimeter system can offer PID information by testing the accordance between reconstructed tracks and calorimeter clusters. Finally, after combining the information from Muon system, mature PID values are assigned to the tracks.

2.2.3.4 Trigger system after Upgrade I

The most distinguishing feature of trigger after Upgrade I is a full software scheme, which can offer unprecedented flexibility in designing selections, and in particular allows efficient triggering on low-momentum signatures which would normally be out of the reach for hadron collider^[179]. The data flow after Upgrade I is illustrated in Fig. 2.18

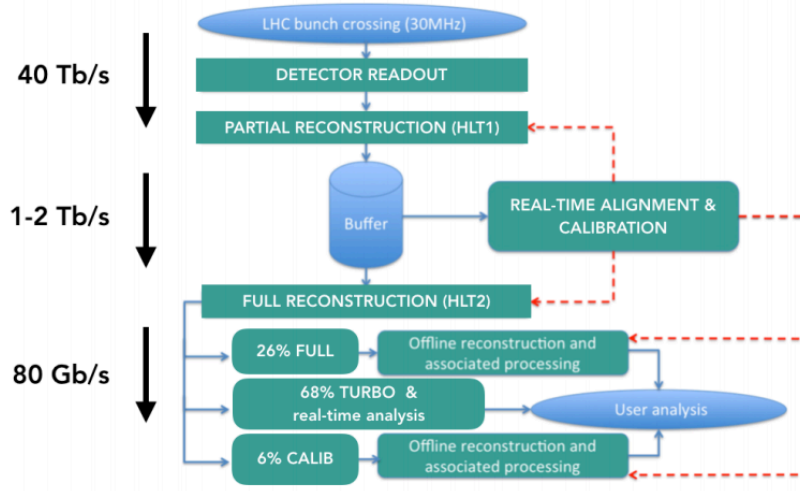


Figure 2.18 Dataflow in the upgraded LHCb detector^[180].

As the L0 trigger is removed, all the trigger decisions are based on the HLT algorithms. The HLT is split into two stages, as similar as the strategy used in Run 2. These two stages are separated by a disk buffer that saves data used for alignment and calibration. The HLT1 can reject events that are not interesting for any LHCb analysis, while HLT2 uses the full reconstruction and identification algorithms to obtain the high-level physics objects relevant to analysis. Many R&D projects are under way currently, and some algorithm optimization and code migration tasks have made great advances^[179–182].

Chapter 3 Observation of Z_{cs} states in the $B^+ \rightarrow J/\psi \phi K^+$ decay

3.1 Motivation

Since the discovery of the $X(3872)$ state by the Belle experiment in 2003^[18], more than twenty tetraquark candidates that contain $c\bar{c}$ quarks have been found and studied^[1], as mentioned in Chapter 1. Among these states, the charged states like $Z_c(3900)^{+}$ ^[114,117] and $Z_c(4430)^{+}$ ^[111,115,183] provide the smoking-gun evidence for tetraquark exotic states, because light quarks are required to account for the non-zero electric charge in addition to the heavy quarkonium. However, only the u or d quarks were observed to constitute the light quark content of such charged exotic states^①. The existence of a Z_{cs}^+ states ($c\bar{c}u\bar{s}$), as the strangeness-flavor partner of the Z_c^+ states ($c\bar{c}u\bar{d}$), have been predicted in several theories^[185–189], but have not been observed in experiment. Hence, searching for such states in experiment is important to test theoretical theories and models.

The $B^+ \rightarrow J/\psi \phi K^+$ decay is a potential channel to searching for Z_{cs}^+ , as the $Z_{cs}^+ \rightarrow J/\psi K^+$ mode has been predicted in some theories^[185–189]. Actually, $Z_{cs}^+ \rightarrow J/\psi K^+$ contributions were examined in the $B^+ \rightarrow J/\psi \phi K^+$ decay at LHCb using Run 1 data sample. The results were mentioned in Ref. [40] and detailed in Thomas Britton’s Ph.D. thesis^[190]. They are summarized in Tab. 3.1. The improvement for $J/\psi K^+$ mass distribution is shown in Fig. 3.1. Smaller than 3σ hint for a Z_{cs}^+ was found, but deemed insufficient to claim discovery of an exotic state with a new quark content. Therefore, this resonance was not included in the Run 1 nominal fit model. Totally around 9 fb^{-1} data sample has been collected at LHCb during Run 1 and Run 2 so far, the larger data sample makes it possible for us to search for Z_{cs}^+ states in the $B^+ \rightarrow J/\psi \phi K^+$ decay.

Table 3.1 Z_{cs}^+ examinations of LHCb Run 1 analysis^[190].

J^P	Mass [MeV]	Width [MeV]	Fit fraction	Significance
$2^+/0^-$	no inform	4-5	0.5-0.6%	$1.2\sigma/2.5\sigma$
$1^+/2^-$	no inform/ (3937 ± 7)	$47/(51 \pm 15)$	1.9/2.5%	$2.5\sigma/3.1\sigma$
1^- (fit did not converge)	~ 4220	~ 500	$\sim 10\%$	

① BESIII reported the 5.3σ observation of $Z_{cs}(3985)^-$ in the end of 2020, which is very close to our publication time^[184].

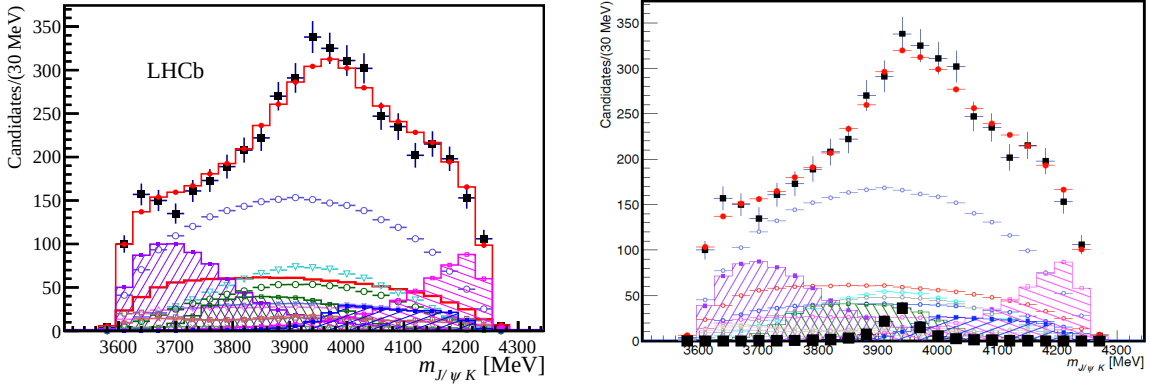


Figure 3.1 Distributions of $J/\psi K^+$ invariant masses for the $B^+ \rightarrow J/\psi \phi K^+$ candidates (black data points) compared with the results of the default amplitude fit (left) and amplitude fit with a Z_{cs}^+ (right) in Run 1 analysis^[190]. The Z_{cs}^+ contribution is shown by large black square point in the right plot.

In addition, four neutral tetraquark candidates decaying to $J/\psi \phi$ were observed in the $B^+ \rightarrow J/\psi \phi K^+$ decay at LHCb in 2016, including $X(4140)$, $X(4274)$, $X(4500)$ and $X(4700)$ ^[39]. Table 3.2 summarizes the results in the previous study^[39]. However, the width of $X(4140)$ measured at LHCb is significantly larger than the results from other experiments, which motivates the analysis using larger data sample with the Run 2 data included to test this result. Besides, the properties of $X(4274)$, $X(4500)$ and $X(4700)$ are also required to be confirmed with larger data sample.

In this analysis with full Run 1 and Run 2 data, and an improved selection, the B^+ signal yield is about 6 times larger than the yield used in the Run 1 analysis (20% more signal found in Run 1 data). At the same time, the background fraction is reduced to much smaller level, which is almost a factor of 6 smaller than the number in the previous analysis^[40]. The improved data sample offers better sensitivity to exotic hadron with smaller contributions.

3.2 Selection

3.2.1 Dataset

This analysis employs the full Run 1 and Run 2 data sample collected at a center-of-mass energy of 7, 8 and 13 TeV. This corresponds to around 9.1 fb^{-1} of total integrated pp luminosity collected by LHCb.

The simulation samples of the $B^+ \rightarrow J/\psi \phi K^+$ decay are generated by PYTHIA 8^[191] with pp collisions. Decays of unstable particles are described by EVTGEN^[192], in which final-state radiation is generated using PHOTOS^[193]. The interaction of the generated particles with the

Table 3.2 Results from previous publication based on Run 1 data at LHCb^[39].

Contribution		Significance (standard deviation)	M_0 [MeV]	Fit results Γ_0 [MeV]	FF%
All $K(1^+)$		8.0σ			$42 \pm 8_{-9}^{+5}$
NR $_{\phi K}$					$16 \pm 13_{-6}^{+35}$
2^1P_1	$K(1^+)$	7.6σ	$1793 \pm 59_{-101}^{+153}$	$365 \pm 157_{-215}^{+138}$	$12 \pm 10_{-6}^{+17}$
2^3P_1	$K'(1^+)$	1.9σ	$1968 \pm 65_{-172}^{+70}$	$396 \pm 170_{-178}^{+174}$	$23 \pm 20_{-29}^{+31}$
All $K(2^-)$		5.6σ			$11 \pm 3_{-5}^{+2}$
1^1D_2	$K_2(1770)$	5.0σ	$1777 \pm 35_{-77}^{+122}$	$217 \pm 116_{-154}^{+221}$	
1^3D_2	$K_2(1820)$	3.0σ	$1853 \pm 27_{-35}^{+18}$	$167 \pm 58_{-72}^{+83}$	
$K(1^-)$					
1^3D_1	$K^*(1680)$	8.5σ	$1722 \pm 20_{-109}^{+33}$	$354 \pm 75_{-181}^{+140}$	$6.7 \pm 1.9_{-3.9}^{+3.2}$
$K(2^+)$					
2^3P_2	$K_2^*(1980)$	5.4σ	$2073 \pm 94_{-240}^{+245}$	$678 \pm 311_{-559}^{+1153}$	$2.9 \pm 0.8_{-0.7}^{+1.7}$
$K(0^-)$					
3^1S_0	$K(1830)$	3.5σ	$1874 \pm 43_{-115}^{+59}$	$168 \pm 90_{-104}^{+280}$	$2.6 \pm 1.1_{-1.8}^{+2.3}$
All $X(1^+)$					$16 \pm 3_{-2}^{+6}$
	$X(4140)$	8.4σ	$4146.5 \pm 4.5_{-2.8}^{+4.6}$	$83 \pm 21_{-14}^{+21}$	$13.0 \pm 3.2_{-2.0}^{+4.8}$
	$X(4274)$	6.0σ	$4273.3 \pm 8.3_{-3.6}^{+17.2}$	$56 \pm 11_{-11}^{+8}$	$7.1 \pm 2.5_{-2.4}^{+3.5}$
All $X(0^+)$					$28 \pm 5 \pm 7$
NR $_{J/\psi \phi}$		6.4σ			$46 \pm 11_{-21}^{+11}$
	$X(4500)$	6.1σ	$4506 \pm 11_{-15}^{+12}$	$92 \pm 21_{-20}^{+21}$	$6.6 \pm 2.4_{-2.3}^{+3.5}$
	$X(4700)$	5.6σ	$4704 \pm 10_{-24}^{+14}$	$120 \pm 31_{-33}^{+42}$	$12 \pm 5_{-5}^{+9}$

detector, and its response, are implemented using the GEANT4 toolkit^[194]. All these simulation software are integrated in GAUSS^[195], as described in Ref. [196]. The simulation event type used in this analysis is 12145051^②, in which B^+ is forced to decay into the $J/\psi \phi K^+$ final state in the regular Dalitz plot^[197] (a so-called phase space, or PHSP, model).

The details on the sizes and GAUSS versions of used simulation samples are provided in Tab. 3.3, which contains the statistics with MagUp (MU) and MagDown (MD) situations^③.

Table 3.3 The statistics and simulation version of simulation samples.

Year	Statistics(MU/MD)	GAUSS version
2011	754500/766498	Sim08c
2012	1004812/1003052	Sim09b
2016	1147294/1013030	Sim09b
2017	1004122/1014701	Sim09f
2018	1008192/1000411	Sim09f

② The numerical event type identifications are used to mark different decay channels at LHCb.

③ MagUp and MagDown represent the directions of magnet in LHCb detector^[158], details in Chapter 2.

3.2.2 Stripping

Stripping is the first offline selection of candidates based on loose criteria that the reconstructed event has to satisfy^[198]. Stripping is ran on the full sample of data collected by LHCb, and brings the amount of events down to a manageable level. This shortens the computing time and saves valuable computing and storage resources. Each decay channel has a dedicated stripping line (collection of these selection criteria)^[174,198], which are often grouped into streams based on the intended physics goals.

In this analysis, the reconstruction of the B^+ candidate starts from the stripping line `FullDSTDiMuonJpsi2MuMuDetachedLine` in the `DiMuon` stream. The selection criteria in this stripping line do not vary in different years, and are summarized in Tab. 3.4. The J/ψ candidate is formed by two oppositely charged tracks consistent with muon hypotheses (`IsMuon`) with $p_T > 500$ MeV and good track fit quality $\chi^2/\text{ndf} < 4$. The J/ψ candidate is required to have an invariant mass within the ± 100 MeV window around the known J/ψ mass^[1], and to have a vertex fit quality $\chi^2_{\text{vtx}} < 20$. The J/ψ candidate is consistent with originating from b -hadron decays by requiring its decay length significance (DLS) to be greater than three.

Table 3.4 Stripping criteria of J/ψ candidates in the line `FullDSTDiMuonJpsi2MuMuDetachedLine`.

Quantity	Selections
<code>IsMuon(μ)</code>	<code>= TRUE</code>
<code>DLL$_{\mu\pi}$</code>	<code>> 0</code>
<code>p_T(μ)</code>	<code>> 500 MeV</code>
Track $\chi^2/\text{ndf}(\mu)$	<code>< 3</code>
$\Delta M(J/\psi)$	<code>< 100 MeV</code>
Vertex $\chi^2(J/\psi)$	<code>< 20</code>
<code>DLS(J/ψ)</code>	<code>> 3</code>

3.2.3 Trigger requirements

After stripping, the candidates are filtered according to the information coming from the trigger system described in Chapter 2. The final states for the B^+ candidates are required to have fired a selection of trigger lines at each stage of the trigger. These are chosen based on the particles in the final state and the decay topology.

- At the L0 level, the events are selected such that the B^+ candidate is on the global trigger (`L0Global_DecDecison`)^④.

④ The events pass all the trigger lines at L0 level, details in Sec. 2.2.3.

- At the HLT1 level, the signal candidates are required to be Trigger on Signal (TOS)^[172] type. The corresponding lines are the TrackMuon, DiMuonHighMass or TrackAllL0 for the Run 1 sample, and TrackMuon, DiMuonHighMass, TrackMVA or twoTrackMVA for the Run 2 sample. TrackMuon accepts events with B , D or τ decays with at least one muon in its final state by identifying and accepting events with muon candidates that have significant IP with respect to all PVs. DiMuonHighMass is complementary to TrackMuon in that it allows b -hadron decays to be selected without imposing lifetime related cuts. TrackAllL0 is the line that is executed for all events accepted by L0, and it is designed to select hadron decays which are significantly displaced from a PV. The last two Run 2 lines are built on a multi-variate approach, based on transverse momentum and impact parameter information, to select events with at least one(two) good-quality track(s).
- At the HLT2 level, the candidate is required to be TOS at least one of the three topological lines, Hlt2Topo2(3,4)BodyBBDT, DiMuonDetachedJPsi line or DiMuonDetachedHeavy line. Hlt2Topo2(3,4)BodyBBDT lines use a multivariate classifier to reduce combinatorial background. DiMuonDetachedJPsi line is the main trigger for low mass muon pairs, while the DiMuonDetachedHeavy line is an analogous trigger line for J/ψ and higher mass muon pairs, with relaxed lifetime selection criteria.

The trigger lines used in this analysis are summarized in Tab. 3.5.

Table 3.5 Tigger requirements on B^+ candidates.

	Run 1	Run 2
L0	Global_DecDecision	
Hlt1 TOS	TrackMuon	
	DiMuonHighMass	
	TrackAllL0	TrackMVA TwoTrackMVA
Hlt2 TOS	TopoMu2(3,4)Body	
	DiMuonDetachedJPsi	
	DiMuonDetachedHeavy	

3.2.4 Pre-selection

Then, more stringent criteria are performed to select $B^+ \rightarrow J/\psi \phi K^+$ candidates. To select good J/ψ candidates, vertex fit χ^2 is required to be less than 9, and the di-muon invariant mass window is reduced to $[-57, 43]$ MeV around the known J/ψ mass^[1]. The J/ψ mass resolution is around 13 MeV^[199]. The asymmetric window is used to take final-state radiation

of muons into account. To reconstruct the B^+ candidates, the J/ψ candidates are combined with two positively and one negatively charged tracks, all consistent with kaons. This is achieved by PID cuts on the ProbNN variables. The kaon candidate is required to have transverse momentum, $p_T > 250$ MeV, and detached from the primary vertex with impact parameter satisfying $\chi_{IP}^2 > 4$. The three kaon candidates are required to come from the same vertex with good vertex-fit quality, $\chi^2 < 50$ for $ndf = 3$. The B^+ candidate is required to have a displaced decay vertex, with flight distance (FD) more than 1.5 mm and form a good vertex with fit quality, $\chi^2/ndf < 9$. It should be consistent with being produced from PV by requiring $\chi_{IP}^2 < 20$ and cosine value of the angle with respect to PV, $DIRA > 0.99$. The B^+ candidates with mass $4.9 < M(J/\psi\phi K^+) < 5.7$ GeV are preselected. To reject clone tracks, any two tracks in B^+ candidate have an opening angle smaller than 0.5 mrad are removed.

Besides, at least one of the two K^+K^- combined mass to be within ± 15 MeV of the known ϕ mass is required^[1], and only one ϕ candidate is accepted for further analysis. To be more specific, candidates that have two K^+K^- combinations with mass falling into the ± 15 MeV window are abandoned. This reduces the sample by about 3%. To be consistent with quantities used in this amplitude analysis, the ϕ candidate mass is recalculated using four-momenta of kaons with J/ψ and B^+ masses constrained to the known values^[1]. All selection criteria are summarized in Tab. 3.6.

It should be mentioned that the $B^+ \rightarrow J/\psi\phi K^+$ decay chain is refitted with two sequential constraints in DecayTreeFitter^[200] tool to improve the mass resolution: A) the J/ψ candidate mass constrained to its known value^[1] and the B^+ candidate constrained to originating from the associated PV; B) the invariant mass of the B^+ candidate constrained to the know B^+ mass value^[1], with the momenta of the J/ψ and kaon daughters re-calculated according to this constraint. For amplitude fits discussed later, the updated four momentum of daughters are used to calculate invariant masses and angles.

Moreover, it should be noted that the PID requirements on the hadron tracks are not applied when reconstructing the B^+ candidates in the simulation sample. Although the simulation aims to represent the real data as well as possible, there are some variables which the simulation sample fails to accurately simulate. These are variables related to PID, such as ProbNN variables. To correct for this discrepancy, several different techniques are employed at LHCb. The one used in this analysis is based on unbinned re-sampling using kernel density estimation (KDE). This algorithm is based on PID re-sampling^[201], where the PID response is completely replaced by the one randomly generated from calibration probability density functions (PDFs). This sampling is done in an unbinned approach, where the 4-dimensional calibration PDFs are

Table 3.6 All pre-selections of $B^+ \rightarrow J/\psi \phi K^+$ candidates including stripping and offline levels.

Candidate	Quantity	Selections
All tracks	Track fit χ^2/ndf	< 3
All tracks	Ghost prob	< 0.47
μ	IsMuon	$= \text{TRUE}$
μ	$\text{DLL}_{\mu\pi}$	> 0
μ	p_T	$> 500 \text{ MeV}$
J/ψ	Mass window	$\in (-57, 43) \text{ MeV}$
J/ψ	Vertex χ^2	< 9
J/ψ	DLS	> 3
K	ProbNNk	> 0.1
K	p_T	$> 250 \text{ MeV}$
K	χ_{IP}^2	> 4
$3K$	Vertex χ^2/ndf	$< 50/3$
ϕ	mass window	$\pm 15 \text{ MeV}$
B^+	Vertex χ^2	< 9
B^+	FD	$> 1.5 \text{ mm}$
B^+	χ_{IP}^2	< 20
B^+	<i>DIRA</i>	> 0.99
B^+	M	$\in (4.9, 5.7) \text{ GeV}$
B^+	Opening angle of two tracks	$> 0.5 \text{ mrad}$
B^+	# of ϕ candidate	$= 1$

first sliced based on the transverse momentum, pseudorapidity, and event multiplicity of the track. The PID distributions in our simulation samples are corrected using this PID calibration method, which is performed using PIDCalib package^[201].

3.2.5 Multivariate analysis

The multivariate analysis (MVA)^[178] is used to further suppress the backgrounds. The signal training sample is from the $B^+ \rightarrow J/\psi \phi K^+$ simulation in the PHSP model, while the background sample is from the invariant-mass sidebands of $B^+ \rightarrow J/\psi \phi K^+$ data sample, with $M(J/\psi \phi K^+) \in [5.229, 5.254] \cup [5.304, 5.329] \text{ GeV}$. These samples are randomly split into two halves, one for the training and the other one for the test.

The following variables are used in the MVA: the minimum χ_{IP}^2 of kaons; the minimum kaon particle identification probability; the scalar sum of kaon momentum component transverse to the beam direction (p_T); the p_T of B^+ ; the decay vertex-fix quality, $\chi_{\text{vtx}}^2/\text{ndf}$, of B^+ ; the χ_{IP}^2 of B^+ ; the χ^2 of the flight distance (χ_{FD}^2) with respect to the associated PV; and the angle between the reconstructed momentum direction and the vector connecting the associated PV and the B^+ decay vertex (*DIRA*), where the associated PV is the one that gives the smallest χ_{IP}^2

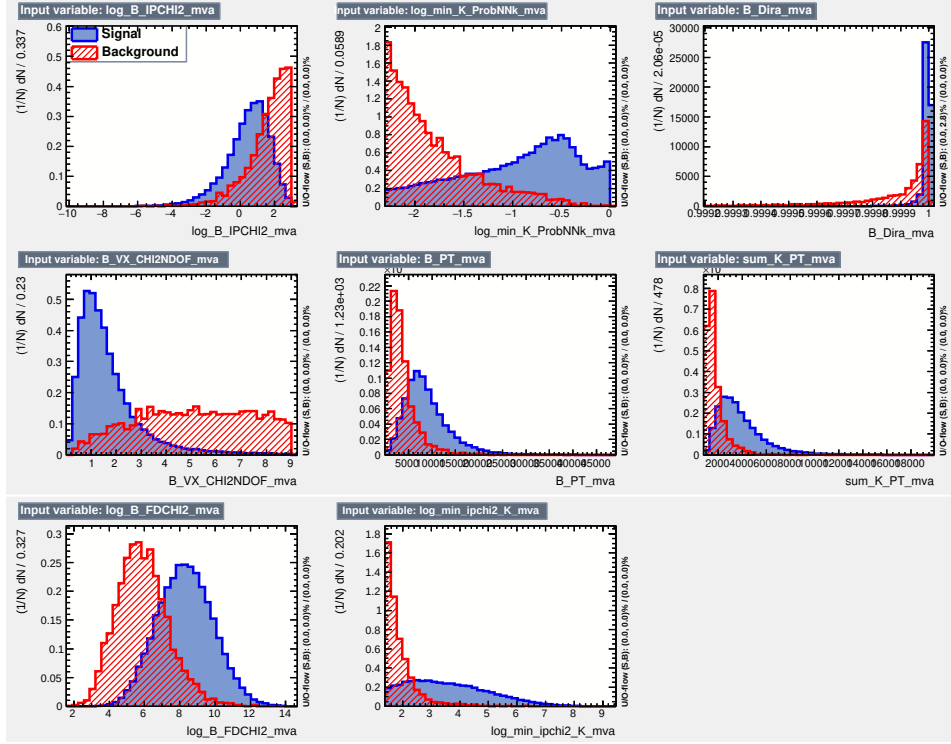


Figure 3.2 Discriminating variables used in the MVA training for Run 1 sample.

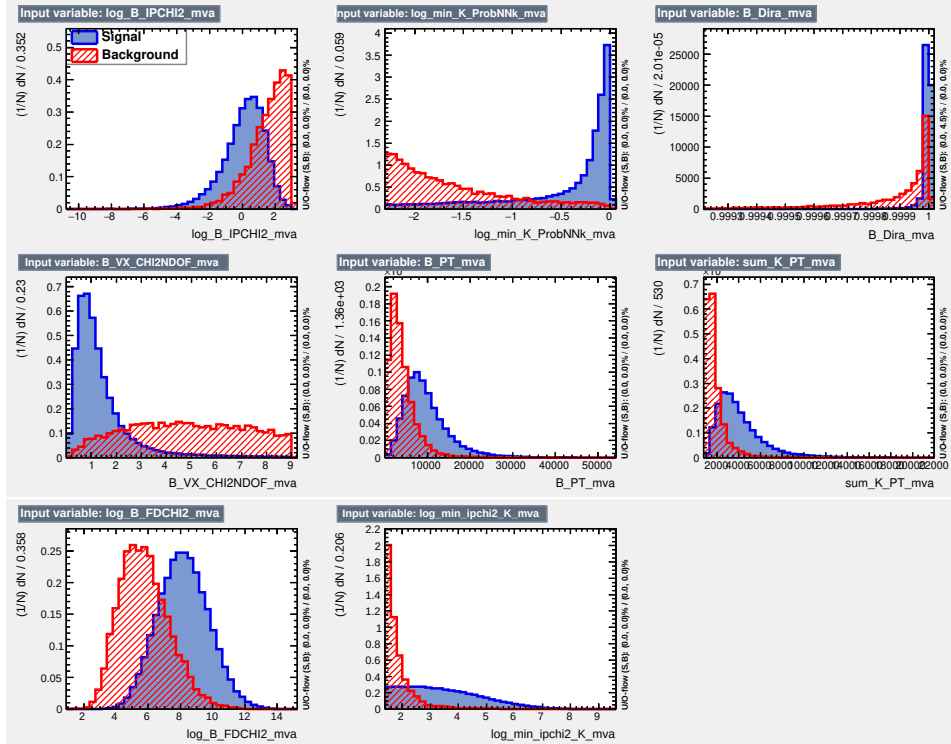


Figure 3.3 Discriminating variables used in the MVA training for Run 2 sample.

among all PVs. The distributions of these variables for the signal and for the background in the data B^+ sideband are shown in Fig. 3.2 (Run 1) and Fig. 3.3 (Run 2).

For Run 1, the receiver operating characteristic curves (ROC) of the results of different training methods are given in the left plot of Fig. 3.4. From this plot, boost decision tree (BDT), boost decision tree gradient (BDTG)^[202,203] and multilayer perceptron (MLP)^[204] methods all have good performances. We choose BDTG method as default method. The over-training test for the BDTG method is given for Run 1 in the right plot of Fig. 3.4, The BDTG discriminant distributions for the training samples are consistent with those in the test samples. No effect of over-training is observed. Similar results for Run 2 are given in Fig. 3.5.

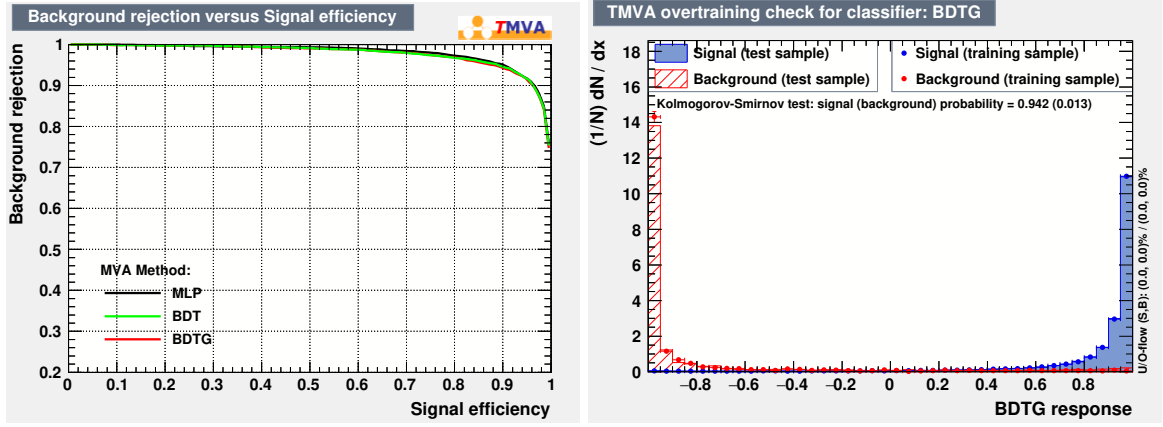


Figure 3.4 (left) ROC curves for different MVA methods and (right) the BDTG output distribution for signals and the background in the training sample and the test sample for Run 1.

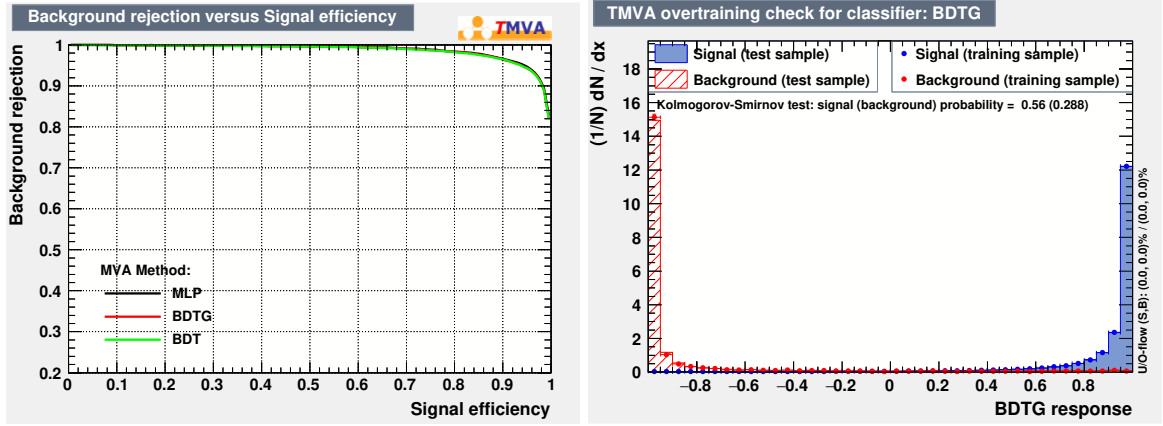


Figure 3.5 (left) ROC curves for different MVA methods and (right) the BDTG output distribution for signals and the background in the training sample and the test sample for Run 2.

A 2-dimensional scan of BDTG value and cut on track-ghost probability for the kaons is done to maximize the figure of merit (FoM) of the signal significance, defined as $\frac{S}{\sqrt{S+B}} \times \frac{S}{S+B}$. S (B) is the number of the signal (background) events in the $\pm 2.5\sigma$ region around the known B mass in the $J/\psi \phi K^+$ invariant mass distribution of the data, which is estimated from a preliminary fit to the pre-selected candidates before BDTG requirement. In Fig. 3.6, the

distribution of the FoM as a function of the BDTG cut value and the track-ghost probability for kaons are given for Run 1 (Run 2). A BDTG cut value of greater than 0.15 (−0.14) and the track-ghost probability for kaons of greater than 0.47 (0.40) are determined as the default working point for Run 1 (Run 2).

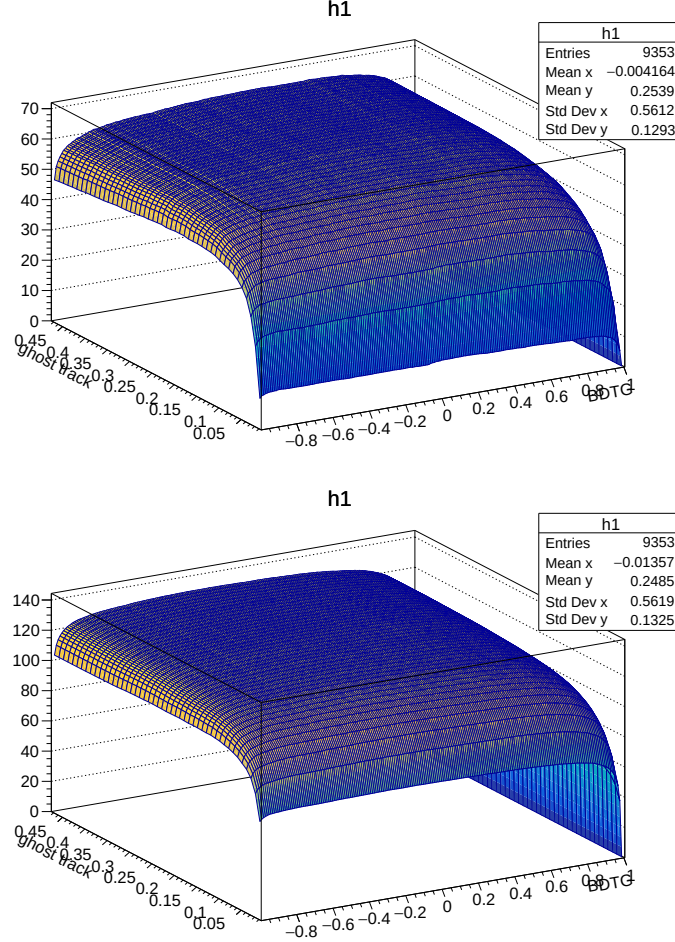


Figure 3.6 Signal significance (arbitrary units) for different cuts on BDTG and track-ghost probability for kaons for Run 1 (top) and Run 2 (bottom).

3.3 Invariant B mass fit and Dalitz-plot distributions

3.3.1 Signal yields

After the selections, the sample of $B^+ \rightarrow J/\psi \phi K^+$ candidates consists of the signal and of the combinatorial background. Figure 3.7 shows the fitted invariant mass distribution of the $B^+ \rightarrow J/\psi \phi K^+$ candidates, for Run 1 and Run 2 separately. The signal shape is described by an Hypatia function^[205], while the shape combinatorial background is described by a second

order Chebyshev polynomial function. In ± 15 MeV signal region around the B^+ peak, the signal yield is 5043 ± 77 (19176 ± 147), and the combinatorial background yield is 275 ± 10 (738 ± 17) for Run 1 (Run 2). Background fraction is 5.2% (3.7%) for Run 1 (Run 2), overall 4.0%. Due to the χ^2_{IP} selection of the kaon candidates from greater 9 to greater 4, 20% signal gain is achieved compared to the Run 1 publication. Also due to more input variables used in MVA (e.g. ProbNNk and χ^2_{FD}), the background fraction is significantly reduced from 27% to 5% (Run 1). To reject tracks that share the same segment in the VELO detector, any track pair with the same charge among the B^+ decay is required to have an opening angle larger than 0.5 mrad, the fraction of this kind of background is 0.9% in a B^+ mass range of 5220-5360 MeV.

3.3.2 Dalitz-plot distributions

Dalitz plots^[197] play an important role in the observation of new resonances, which can show a peak around the mass of the resonant decay. Figure 3.8 shows three Dalitz-plot distributions, with the background subtracted by *sPlot* technique^[206]. The Dalitz-plot distributions for the candidates in the B^+ signal region are also presented, as shown in Fig. 3.9. The most apparent features are four bands in $J/\psi \phi$ mass, corresponding to $X(4140)$, $X(4274)$, $X(4500)$ and $X(4700)$ claimed in the previous LHCb publication^[39]. No apparent structures in ϕK mass are seen, as many kaon excitations in this mass range are broad and overlapping. The increased data sample reveals a horizontal $J/\psi K$ band in the Dalitz-plot distributions.

3.3.3 Non- ϕ background fraction

The contribution of the four-body decay, $B^+ \rightarrow J/\psi K^+ K^- K^+$, is not considered in the 1-dimensional B^+ mass fit, which can be estimated from the ϕ mass distributions. To recover proper ϕ mass sideband, the one- ϕ selection requirement is removed. Later the efficiency of the one- ϕ cut using signal simulation is studied. A 2-dimensional (2D) fit to $m_{K^+ K^-}$ versus $m_{J/\psi K^+ K^- K^+}$ (2 entries per event) is done to Run 1 and Run 2 data sample. The fit result is shown in Fig. 3.10.

The yields of different fit components around the B^+ mass peak are listed in Tab.3.7. Without the one- ϕ cut, the non- ϕ contribution in $B^+ \rightarrow J/\psi K^+ K^- K^+$ decays is $1366/(22018 + 1366) = 5.8\%$ in the ± 15 MeV region around the ϕ and the B^+ mass peaks. According to the signal simulation, the real ϕ decays can generate combinatorial background in ± 15 MeV around the ϕ mass peak, via the second $K^+ K^-$ combination in the event, in a rate of 3.6 per 100 detected ϕ signal counts. This means 793 entries in the ϕ peak region are the reflections from ϕ signal. With the one- ϕ requirement, 94% of such non- ϕ combinations are rejected,

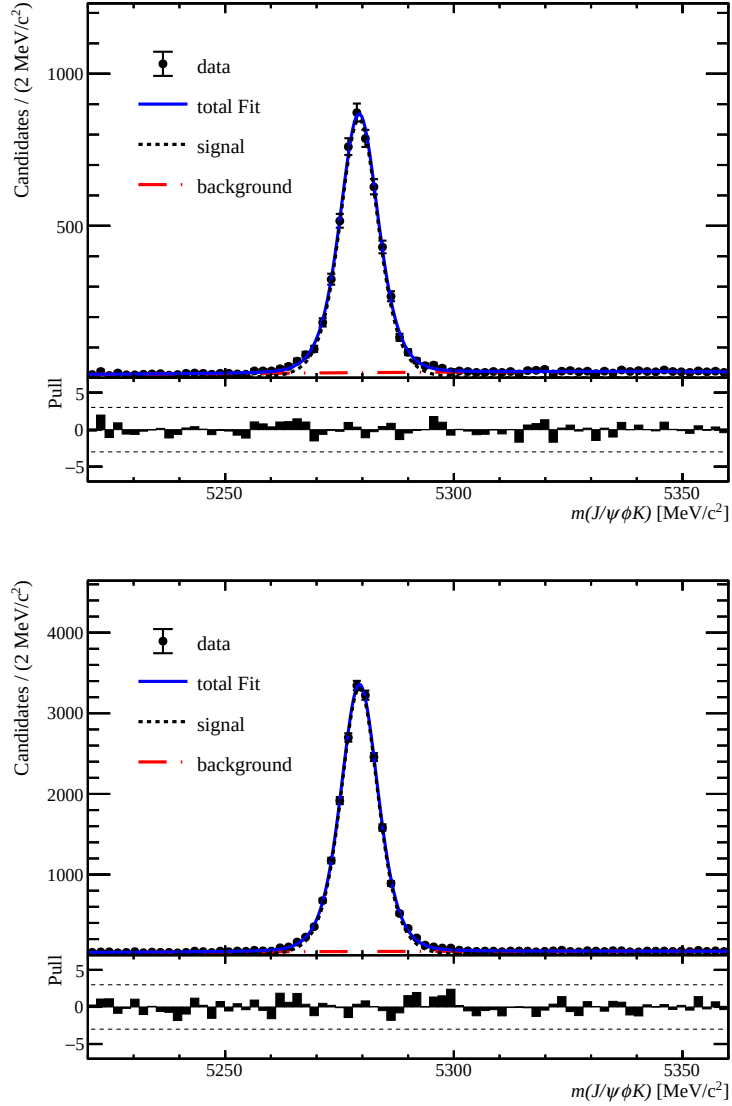


Figure 3.7 Invariant mass distribution of the $B^+ \rightarrow J/\psi \phi K^+$ decay and the fitted curve for Run 1 (top) and Run 2 (bottom). The blue solid line represents the total fit. The dashed blue line is the signal. The dashed red line is the combinatorial background.

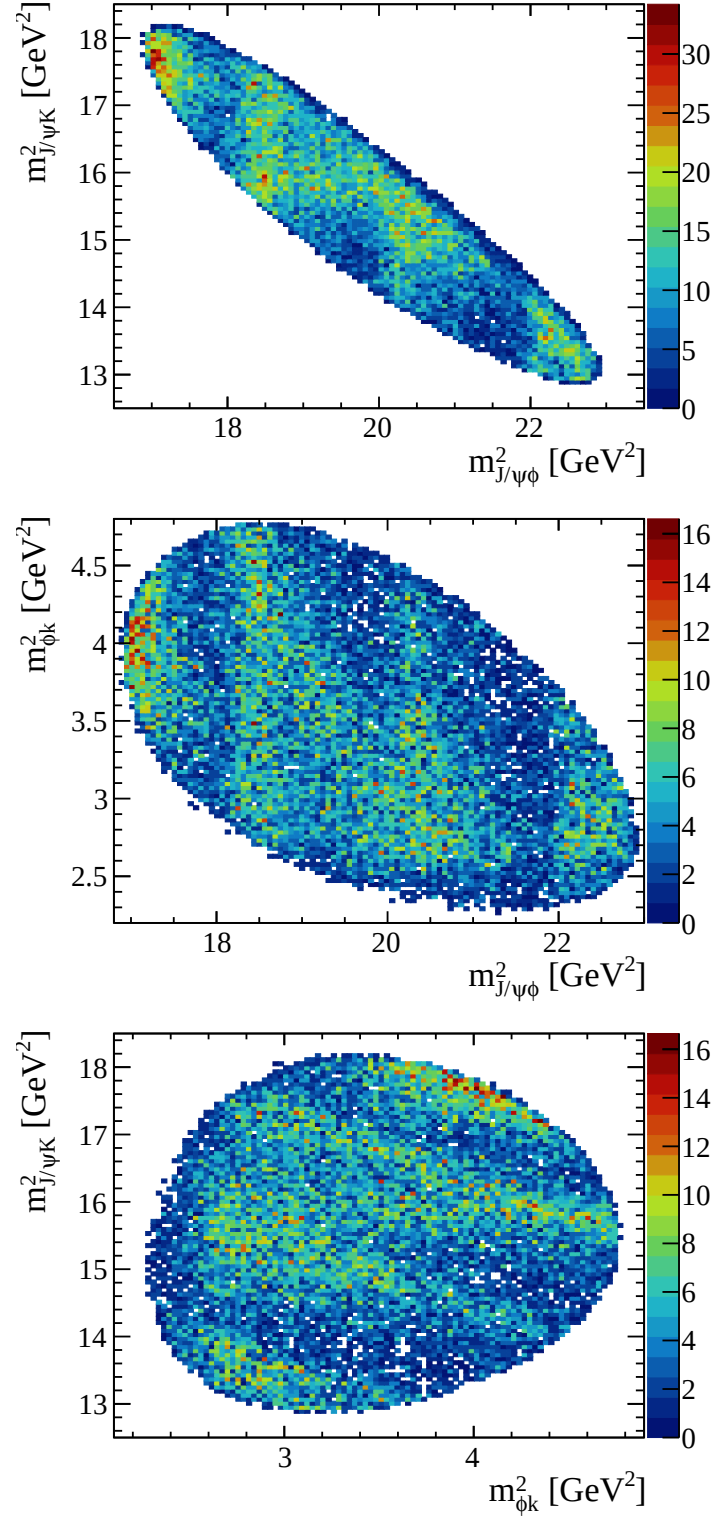


Figure 3.8 Dalitz-plot distributions of $J/\psi \phi$ vs $J/\psi K$ (top), $J/\psi \phi$ vs ϕK (middle) and ϕK vs $J/\psi K$ (bottom), where the background is subtracted by $sPlot$ technique.

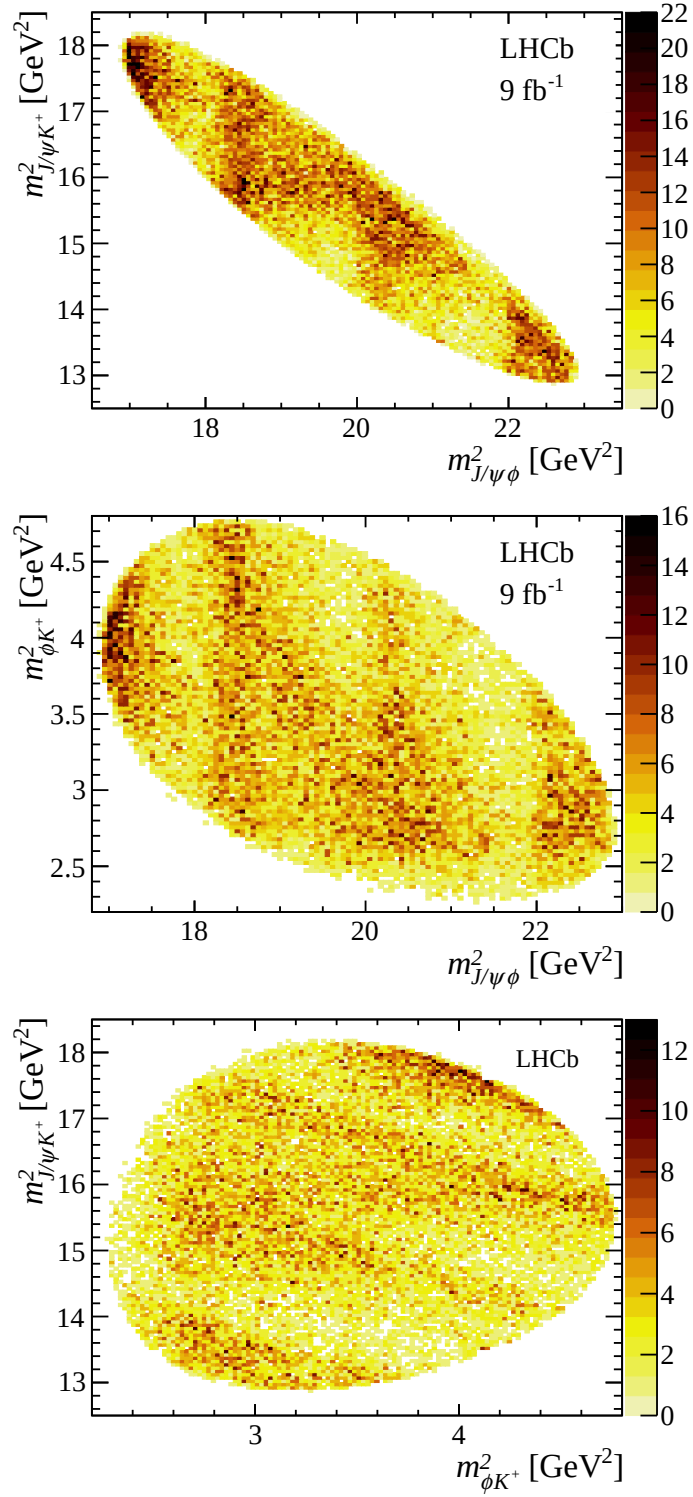


Figure 3.9 Dalitz-plot distributions of $J/\psi \phi$ vs $J/\psi K$ (top), $J/\psi \phi$ vs ϕK (middle) and ϕK vs $J/\psi K$ (bottom), where the $B^+ \rightarrow J/\psi \phi K^+$ candidates in a region ± 15 MeV around the B^+ mass peak.

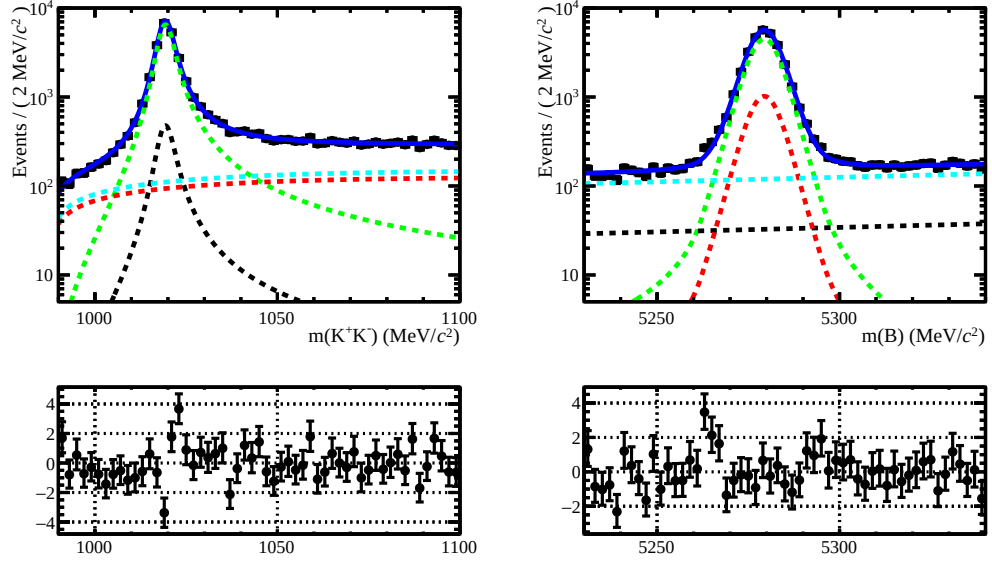


Figure 3.10 Projections of the 2D fit to m_ϕ vs. $m_{J/\psi K^+ K^-}$ in logarithm scale. The deep blue line is the total PDF. The red line is the component where ϕ is background (modelled by phase-space) but $B^\pm$ is signal (Hypathia function). The green line is the component where ϕ is signal (Breit-Wigner) and $B^\pm$ is signal as well. The black line is the component where ϕ is signal but $B^\pm$ is background (linear polynomial). The light blue line is the component where ϕ and $B^\pm$ are both background.

with a ϕ signal efficiency of 97%. The remaining ϕ signal is 21357, and the remaining non- ϕ background is 621, thus $621/(621 + 21357) = 2.8\%$ is the non- ϕ component in the $B^+ \rightarrow J/\psi K^+ K^- K^+$ signal sample after the one- ϕ requirement. We have assumed above that the non- ϕ background efficiency is 100% for the one- ϕ requirement. Otherwise, the estimate is even lower than 2.8%.

Table 3.7 The yields of 2D fit.

Yields	In fit region	In ± 15 MeV of ϕ and B^+ mass window
$\phi_{\text{signal}} \otimes B_{\text{signal}}$	24826 ± 185	22018 ± 164
non- $\phi \otimes B_{\text{signal}}$	5644 ± 124	1366 ± 30

3.4 The amplitude fitting formula

There are obvious structures in the Dalitz-plot distributions of $B^+ \rightarrow J/\psi \phi K^+$ decay, as shown in Fig. 3.8 and Fig. 3.9, which are possible resonances. To investigate the resonant structures, a full amplitude fit is performed using an unbinned maximum-likelihood method. In the following section, the general outline to construct helicity formula is introduced firstly, as

well as the corresponding notations. Then, the matrix element for $B^+ \rightarrow J/\psi \phi K^+$, $J/\psi \rightarrow \mu^+ \mu^-$, $\phi \rightarrow K^+ K^-$ is described in sequence. Assuming the selected events of $K^+ K^-$ are all from ϕ decays, the mass-dependent term of $m_{K^+ K^-}$ for ϕ lineshape is common for all decay chains. Subsequently, the method to combine all decay chains together is discussed. Next, a possible reduction of number of helicity couplings to determine from the data by their relation to LS couplings is introduced. Finally, the method to deal with the background contribution from the log-likelihood sum is provided, which is also named as "*cFit*" technique.

3.4.1 Helicity formalism and notation

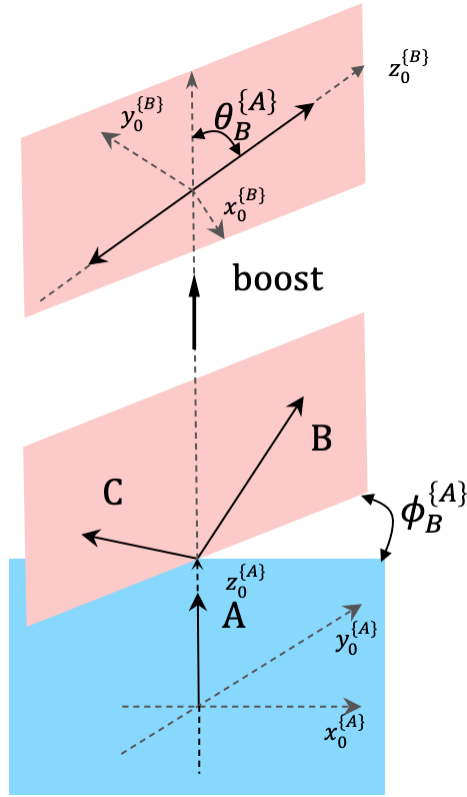


Figure 3.11 Coordinate axes for the spin quantization of particle A (bottom part), chosen to be the helicity frame of A ($\hat{z}_0 || \vec{p}_A$ in the rest frame of its mother particle or in the laboratory frame), together with the polar ($\theta_B^{(A)}$) and azimuthal ($\phi_B^{(A)}$) angles of the momentum of its daughter B in the B rest frame (top part). Notice that the directions of these coordinate axes, denoted as $(x_0^{(A)}, y_0^{(A)}$ and $z_0^{(A)})$, do not change when boosting from the helicity frame of A to its rest frame. After the Euler rotation $\mathcal{R}(\alpha = \phi_B^{(A)}, \beta = \theta_B^{(A)}, \gamma = 0)$ (see the text), the rotated z axis, $\hat{z}_2^{(A)}$, is aligned with the B momentum; thus the rotated coordinates become the helicity frame of B . If B has a sequential decay, then the same boost-rotation process is repeated to define the helicity frame for its daughters.

For each two-body decay $A \rightarrow BC$, a coordinate system is set up in the rest frame of A , with $\hat{z}$ being the direction of quantization for its spin. We denote this coordinate system

as $(x_0^{\{A\}}, y_0^{\{A\}}, z_0^{\{A\}})$, where the superscript “ $\{A\}$ ” means “in the rest frame of A ”, while the subscript “0” means the initial coordinates. For the first particle in the decay chain, the choice of these coordinates is arbitrary. However, once defined, these coordinates must be used consistently between all decay sequences described by the matrix element. For subsequent decays, *e.g.* $B \rightarrow DE$, the choice of these coordinates is already fixed by the transformation from the A to the B rest frames, as discussed below. Helicity is defined as the projection of the spin of the particle onto the direction of its momentum^[1]. When the z axis coincides with the particle momentum, we denote its spin projection onto it (*i.e.* the m_z quantum number) as λ . To use the helicity formalism, the initial coordinate system must be rotated to align the z axis with the direction of the momentum of one of the daughter particles, *e.g.* the B . A generalized rotation operator can be formulated in three-dimensional space, $\mathcal{R}(\alpha, \beta, \gamma)$, that uses Euler angles. Applying this operator results in a sequence of rotations: first by the angle α about the $\hat{z}_0$ axis, followed by the angle β about the rotated $\hat{y}_1$ axis and then finally by the angle γ about the rotated $\hat{z}_2$ axis. We use a subscript denoting the axes, to specify the rotations which have been already performed on the coordinates. The spin eigenstates of particle A , $|J_A, m_A\rangle^{\textcircled{5}}$, in the $(x_0^{\{A\}}, y_0^{\{A\}}, z_0^{\{A\}})$ coordinate system can be expressed in the basis of its spin eigenstates, $|J_A, m'_A\rangle$, in the rotated $(x_3^{\{A\}}, y_3^{\{A\}}, z_3^{\{A\}})$ coordinate system with the help of Wigner’s D -matrices^[207],

$$|J_A, m_A\rangle = \sum_{m'_A} D_{m_A, m'_A}^{J_A}(\alpha, \beta, \gamma)^* |J_A, m'_A\rangle \quad (3.1)$$

where

$$d_{m_A, m'_A}^{J_A}(\alpha, \beta, \gamma)^* = \langle J, m | \mathcal{R}(\alpha, \beta, \gamma) | J, m' \rangle^* = e^{im\alpha} d_{m, m'}^J e^{im'\gamma} \quad (3.2)$$

and where the small- d Wigner matrix^[207] contains known functions of β that depend on J, m, m' . To achieve the rotation of the original $z_0^{\{A\}}$ axis on to the B momentum ($\vec{p}_B^{\{A\}}$), it is sufficient to rotate by $\alpha = \phi_B^{\{A\}}, \beta = \theta_B^{\{A\}}$, where $\phi_B^{\{A\}}, \theta_B^{\{A\}}$ are the azimuthal and polar angles of the B momentum vector in the original coordinates *i.e.* $(x_0^{\{A\}}, y_0^{\{A\}}, z_0^{\{A\}})$, which is depicted in Fig. 3.11. For the case when the quantization axis for the spin of A is its momentum in some other reference frame. Since the third rotation is not necessary, we set it 0 here. The angle $\theta_B^{\{A\}}$ is usually called “the A helicity angle”, thus to simplify the B notation we will denote it

^⑤ J_A is the spin of particle A , m_A is the z -component of spin.

as θ_A . For compact notation, $\phi_B^{\{A\}}$ is denoted as ϕ_B . These angles can be determined as,

$$\begin{aligned}\phi_B &= \text{atan2} \left(p_B^{\{A\}}{}_y, p_B^{\{A\}}{}_x \right) \\ &= \text{atan2} \left(\hat{y}_0^{\{A\}} \cdot \vec{p}_B^{\{A\}}, \hat{x}_0^{\{A\}} \cdot \vec{p}_B^{\{A\}} \right) \\ &= \text{atan2} \left((\hat{z}_0^{\{A\}} \times \hat{x}_0^{\{A\}}) \cdot \vec{p}_B^{\{A\}}, \hat{x}_0^{\{A\}} \cdot \vec{p}_B^{\{A\}} \right),\end{aligned}\quad (3.3)$$

$$\cos \theta_A = \hat{z}_0^{\{A\}} \cdot \hat{p}_B^{\{A\}}. \quad (3.4)$$

Angular momentum conservation requires $m'_A = m'_B + m'_C = \lambda_B - \lambda_C$ (since $\vec{p}_C^{\{A\}}$ points in the opposite direction to $\hat{z}_3^{\{A\}}$, $m'_C = -\lambda_C$). Each two-body decay adds a multiplicative term to the matrix element

$$\mathcal{H}_{\lambda_B, \lambda_C}^{A \rightarrow B C} D_{m_A, \lambda_B - \lambda_C}^{J_A}(\phi_B, \theta_A, 0)^*. \quad (3.5)$$

The helicity couplings $\mathcal{H}_{\lambda_B, \lambda_C}^{A \rightarrow B C}$ are complex constants. Their products from subsequent decays are to be determined by the fit to the data (they represent the decay dynamics). If the decay is strong or electromagnetic, it conserves parity which reduces the number of independent helicity couplings via the relation

$$\mathcal{H}_{-\lambda_B, -\lambda_C}^{A \rightarrow B C} = P_A P_B P_C (-1)^{J_B + J_C - J_A} \mathcal{H}_{\lambda_B, \lambda_C}^{A \rightarrow B C}, \quad (3.6)$$

where P stands for the intrinsic parity of a particle.

After multiplying terms given by Eq. (3.5) for all decays in the decay sequence, they must be summed up coherently over the helicity states of intermediate particles, and incoherently over the helicity states of the initial and final-state particles. Possible helicity values of B and C particles are constrained by $|\lambda_B| \leq J_B$, $|\lambda_C| \leq J_C$ and $|\lambda_B - \lambda_C| \leq J_A$.

When dealing with the subsequent decay of the daughter, $B \rightarrow D E$, four-vectors of all particles must be first Lorentz boosted to the rest frame of B , along the $\vec{p}_B^{\{A\}}$ i.e. $\hat{z}_3^{\{A\}}$ direction (this is the z axis in the rest frame of A after the Euler rotations; the subscript “3” is used for the number of rotations performed on the coordinates, and the $\gamma = 0$, these coordinates are the same as after the first two rotations). This is visualized in Fig. 3.11, with $B \rightarrow D E$ particle labels replaced by $A \rightarrow B C$ labels. This transformation does not change vectors that are perpendicular to the boost direction. The transformed coordinates become the initial

coordinate system quantizing the spin of B in its rest frame,

$$\begin{aligned}\hat{x}_0^{\{B\}} &= \hat{x}_3^{\{A\}}, \\ \hat{y}_0^{\{B\}} &= \hat{y}_3^{\{A\}}, \\ \hat{z}_0^{\{B\}} &= \hat{z}_3^{\{A\}}.\end{aligned}\tag{3.7}$$

The processes of rotation and subsequent boosting can be repeated until the final-state particles are reached. In practice, there are two equivalent ways to determine the $\hat{z}_0^{\{B\}}$ direction. Using Eq. (3.7) can set it to the direction of the B momentum in the A rest frame

$$\hat{z}_0^{\{B\}} = \hat{z}_3^{\{A\}} = \hat{p}_B^{\{A\}}.\tag{3.8}$$

Alternatively, the fact that B and C are back-to-back in the rest frame of A can also be considered, $\vec{p}_C^{\{A\}} = -\vec{p}_B^{\{A\}}$. Since the momentum of C is antiparallel to the boost direction from the A to B rest frames, the C momentum in the B rest frame will be different, but it will still be antiparallel to this boost direction

$$\hat{z}_0^{\{B\}} = -\hat{p}_C^{\{B\}}.\tag{3.9}$$

To determine $\hat{x}_0^{\{B\}}$ from Eq. (3.7), the $\hat{x}_3^{\{A\}}$ should be calculated first. After the first rotation by ϕ_B about $\hat{z}_0^{\{A\}}$, the $\hat{x}_1^{\{A\}}$ axis is along the component of $\vec{p}_B^{\{A\}}$ which is perpendicular to the $\hat{z}_0^{\{A\}}$ axis

$$\begin{aligned}\vec{a}_{B\perp z_0}^{\{A\}} &\equiv (\vec{p}_B^{\{A\}})_{\perp \hat{z}_0^{\{A\}}} = \vec{p}_B^{\{A\}} - (\vec{p}_B^{\{A\}})_{\parallel \hat{z}_0^{\{A\}}}, \\ &= \vec{p}_B^{\{A\}} - (\vec{p}_B^{\{A\}} \cdot \hat{z}_0^{\{A\}}) \hat{z}_0^{\{A\}}, \\ \hat{x}_1^{\{A\}} &= \hat{a}_{B\perp z_0}^{\{A\}} = \frac{\vec{a}_{B\perp z_0}^{\{A\}}}{|\vec{a}_{B\perp z_0}^{\{A\}}|}.\end{aligned}\tag{3.10}$$

After the second rotation by θ_A about $\hat{y}_1^{\{A\}}$, $\hat{z}_2^{\{A\}} \equiv \hat{z}_3^{\{A\}} = \hat{p}_B^{\{A\}}$, and $\hat{x}_2^{\{A\}} = \hat{x}_3^{\{A\}}$ is antiparallel to the component of the $\hat{z}_0^{\{A\}}$ vector that is perpendicular to the new z axis *i.e.* $\hat{p}_B^{\{A\}}$. Thus

$$\begin{aligned}\vec{a}_{z_0\perp B}^{\{A\}} &\equiv (\hat{z}_0^{\{A\}})_{\perp \vec{p}_B^{\{A\}}} = \hat{z}_0^{\{A\}} - (\hat{z}_0^{\{A\}} \cdot \hat{p}_B^{\{A\}}) \hat{p}_B^{\{A\}}, \\ \hat{x}_0^{\{B\}} &= \hat{x}_3^{\{A\}} = -\hat{a}_{z_0\perp B}^{\{A\}} = -\frac{\vec{a}_{z_0\perp B}^{\{A\}}}{|\vec{a}_{z_0\perp B}^{\{A\}}|}.\end{aligned}\tag{3.11}$$

Then obtain $\hat{y}_0^{\{B\}} = \hat{z}_0^{\{B\}} \times \hat{x}_0^{\{B\}}$.

If C also decays, $C \rightarrow F G$, then the coordinates for the quantization of C spin in the C

rest frame are defined by

$$\hat{z}_0^{\{C\}} = -\hat{z}_3^{\{A\}} = \hat{p}_C^{\{A\}} = -\hat{p}_B^{\{C\}}, \quad (3.12)$$

$$\hat{x}_0^{\{C\}} = -\hat{x}_3^{\{A\}} = \hat{a}_{z_0 \perp B}^{\{A\}} = -\hat{a}_{z_0 \perp C}^{\{A\}}, \quad (3.13)$$

$$\hat{y}_0^{\{C\}} = \hat{y}_0^{\{B\}} = \hat{z}_0^{\{C\}} \times \hat{x}_0^{\{C\}}, \quad (3.14)$$

i.e. the z axis is reflected compared to the system used for the decay of particle B (it must point in the direction of C momentum in the A rest frame), but the y axis is kept the same, since particle B for the rotation used in Eq. (3.5) is chosen. This is to rotate 180 degrees about the y axis.

3.4.2 Matrix element for the $K^{*+} \rightarrow \phi K^+$ decay chain

For the $B^+ \rightarrow \psi K_n^*$, $K_n^* \rightarrow \phi K$, $\phi \rightarrow K^+ K^-$, $\psi \rightarrow \mu^+ \mu^-$, this chain is described by one mass and 5 helicity angles. The weak decay $B^+ \rightarrow \psi K_n^*$ is described only by helicity coupling $\mathcal{H}_{\lambda_\psi}^{B \rightarrow \psi K_n^*}$, with $\lambda_\psi = \lambda_{K^*}$, because B^+ has no spin. The strong decay $K_n^* \rightarrow \phi K$ is described by helicity couplings, angular and mass terms, as

$$\mathcal{H}_{\lambda_\phi}^{K_n^*} D_{\lambda_{K^*}, \lambda_\phi}^{J_{K_n^*}}(\phi_\phi, \theta_{K^*}, 0)^* R_{K_n^*}(m_{\phi K}), \quad (3.15)$$

where the two helicity angles ϕ_ϕ , θ_{K^*} of K^* decays are defined by the particle ϕ . Wigner-D function requires $|\lambda_\phi| \leq J_{K_n^*}$ and $|\lambda_\psi| \leq J_{K_n^*}$. Strong decay conserves parity, and gives the relation $\mathcal{H}_{-\lambda_\phi}^{K_n^*} = (-1)^{J_{K_n^*}} P_{K_n^*} \mathcal{H}_{\lambda_\phi}^{K_n^*}$, where $P_{K_n^*}$ is the parity of the K_n^* resonance.

The $\phi \rightarrow K^+ K^-$ decay provides one term

$$D_{\lambda_\phi, 0}^1(\phi_{K^+}, \theta_\phi, 0)^*, \quad (3.16)$$

where K^+ is used to define the two helicity angles of ϕ_{K^+} and θ_ϕ from ϕ decays. No independent helicity couplings are needed, as the daughters of ϕ are spineless.

The electromagnetic decay $\psi \rightarrow \mu^+ \mu^-$ is described by a term

$$D_{\lambda_\psi, \Delta\lambda_\mu}^1(\phi_\mu, \theta_\psi, 0)^*, \quad (3.17)$$

where $\Delta\lambda_\mu \equiv \lambda_{\mu^+} - \lambda_{\mu^-} = \pm 1$, and ϕ_μ , θ_ψ are the azimuthal and polar angles of μ^+ for B^+ in the ψ rest frame. There are no independent helicity couplings in Eq. (3.17), since they are all equal due to conservation of C and P parities. Therefore, this coupling can be set to unity as its magnitude and phase can be absorbed into the other helicity couplings which are left free in the fit.

Collecting all terms together, the matrix element is

$$\begin{aligned} \mathcal{M}_{\Delta\lambda_\mu}^{K^*}(\theta_{K^*}, \theta_\psi, \theta_\phi, \phi_{(\psi K^*)} \equiv \phi_\phi + \phi_\mu, \phi_{K^+}, m_{\phi K}) = \\ \sum_{\lambda_\psi} e^{i\lambda_\psi \phi_{(\psi K^*)}} d_{\lambda_\psi, \Delta\lambda_\mu}^1(\theta_\psi) \sum_{\lambda_\phi} e^{i\lambda_\phi \phi_{K^+}} d_{\lambda_\phi, 0}^1(\theta_\phi) \sum_n \mathcal{H}_{\lambda_\psi}^{B \rightarrow \psi K_n^*} \mathcal{H}_{\lambda_\phi}^{K_n^*} d_{\lambda_\psi, \lambda_\phi}^{J_{K_n^*}}(\theta_{K^*}) R_{K_n^*}(m_{\phi K}) \end{aligned} \quad (3.18)$$

Here, $\phi_{(\psi K^*)} \equiv \phi_\phi + \phi_\mu$ is the azimuthal angle between the K^* and ψ decay angles (ϕ_ϕ and ϕ_μ cannot be independently probed). The first two terms, summing over $\lambda_\psi = \pm 1, 0$ and $\lambda_\phi = \pm 1, 0$, are independent of the other terms, thus can be coded separately and cached. The last term depends on 3×3 matrix of λ_ψ and λ_ϕ values.

The term $R_{K_n^*}(m_{K\phi})$ describes the K_n^* resonance that appears in the invariant mass distribution of the $K\phi$ system,

$$R_{K_n^*}(m_{K\phi}) = B'_{L_B^{K_n^*}}(p, p_0, d) \left(\frac{p}{M_B} \right)^{L_B^{K_n^*}} \text{BW}(m_{K\phi} | M_0^{K_n^*}, \Gamma_0^{K_n^*}) B'_{L_{K_n^*}}(q, q_0, d) \left(\frac{q}{M_0^{K_n^*}} \right)^{L_{K_n^*}}. \quad (3.19)$$

Here, p is the K^* momentum in the B rest frame ($p = |\vec{p}_{K^*}^{(B)}|$). Similarly, q is the K momentum in the K^* rest frame ($q = |\vec{p}_K^{(K^*)}|$). The symbols p_0 and q_0 denote values of these quantities at the resonance peak ($m_{K\phi} = M_0^{K^*}$). The orbital angular momentum between the K^* and ψ particles in the B decay is denoted as $L_B^{K^*}$. Similarly, L_{K^*} is the orbital angular momentum between the K and ϕ in the K^* decay. For the orbital angular momentum barrier factors, $p^L B'_L(p, p_0, d)$, the Blatt-Weisskopf functions^[208] are used,

$$\begin{aligned} B'_0(p, p_0, d) &= 1, \\ B'_1(p, p_0, d) &= \sqrt{\frac{1 + (p_0 d)^2}{1 + (p d)^2}}, \\ B'_2(p, p_0, d) &= \sqrt{\frac{9 + 3(p_0 d)^2 + (p_0 d)^4}{9 + 3(p d)^2 + (p d)^4}}, \\ B'_3(p, p_0, d) &= \sqrt{\frac{225 + 45(p_0 d)^2 + 6(p_0 d)^4 + (p_0 d)^6}{225 + 45(p d)^2 + 6(p d)^4 + (p d)^6}}, \\ B'_4(p, p_0, d) &= \sqrt{\frac{11025 + 1575(p_0 d)^2 + 135(p_0 d)^4 + 10(p_0 d)^6 + (p_0 d)^8}{11025 + 1575(p d)^2 + 135(p d)^4 + 10(p d)^6 + (p d)^8}}, \\ B'_5(p, p_0, d) &= \sqrt{\frac{893025 + 99225(p_0 d)^2 + 6300(p_0 d)^4 + 315(p_0 d)^6 + 15(p_0 d)^8 + (p_0 d)^{10}}{893025 + 99225(p d)^2 + 6300(p d)^4 + 315(p d)^6 + 15(p d)^8 + (p d)^{10}}}, \end{aligned} \quad (3.20)$$

to account for the difficulty in creating the orbital angular momentum L , which depends on the momentum of the decay products p (in the rest frame of the decaying particle) and on the size of the decaying particle given by the constant d . We set $d = 3.0 \text{ GeV}^{-1} \sim 0.6 \text{ fm}$. The

relativistic Breit-Wigner amplitude is given by

$$\text{BW}(m|M_0, \Gamma_0) = \frac{1}{M_0^2 - m^2 - iM_0\Gamma(m)}, \quad (3.21)$$

where

$$\Gamma(m) = \Gamma_0 \left(\frac{q}{q_0} \right)^{2L_{K^*}+1} \frac{M_0}{m} B'_{L_{K^*}}(q, q_0, d)^2. \quad (3.22)$$

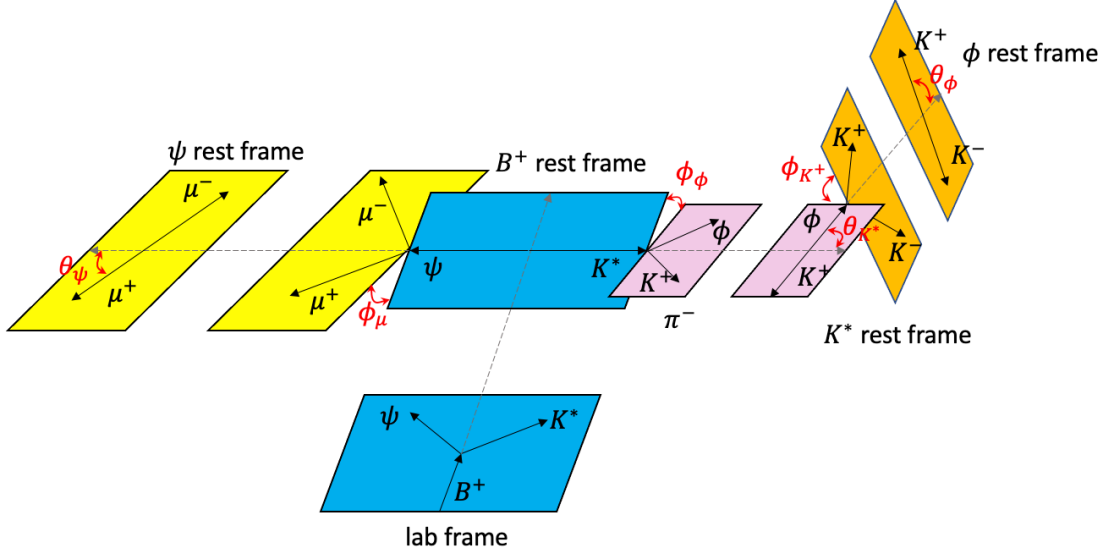


Figure 3.12 Defination of decay angles in the K^* chain.

The sketch of angles used here is shown in Fig. 3.12. The $z_0^{\{K^*\}}$ direction is defined by the boost direction from the B^+ rest frame, which coincides with the $-\hat{p}_\psi^{\{K^*\}}$ direction in this frame, details have been discussed in Sec. 3.4.1. Some expressions to calculate the angles are listed below. The axis of K^* rest frame is defined as:

$$\hat{x}_0^{\{K^*\}} = -\frac{\hat{p}_{B^+}^{\{LAB\}} - \left(\hat{p}_{B^+}^{\{LAB\}} \cdot \hat{p}_{K^*}^{\{B^+\}} \right) \hat{p}_{K^*}^{\{B^+\}}}{\left| \hat{p}_{B^+}^{\{LAB\}} - \left(\hat{p}_{B^+}^{\{LAB\}} \cdot \hat{p}_{K^*}^{\{B^+\}} \right) \hat{p}_{K^*}^{\{B^+\}} \right|} \quad (3.23)$$

$$y_0^{\{K^*\}} = -\hat{p}_\psi^{\{K^*\}} \times x_0^{\{K^*\}} \quad (3.24)$$

$$z_0^{\{K^*\}} = -\hat{p}_\psi^{\{K^*\}} \quad (3.25)$$

with this choice, the polar angle of ϕ in K^* rest frame can be calculated:

$$\cos \theta_{K^*} = -\hat{p}_\psi^{\{K^*\}} \cdot \hat{p}_\phi^{\{K^*\}} \quad (3.26)$$

and the azimuthal angle of ϕ in $K^*(\phi_\phi)$ rest frame can be determined from Eq. (3.3).

$$\phi_\phi = \text{atan2} \left(\hat{y}_0^{\{K^*\}} \cdot \hat{p}_\phi^{\{K^*\}}, \hat{x}_0^{\{K^*\}} \cdot \hat{p}_\phi^{\{K^*\}} \right) \quad (3.27)$$

For the rest frame of ψ , the definition of axis:

$$\hat{x}_0^{\{\psi\}} = -\hat{x}_0^{\{K^*\}} \quad (3.28)$$

$$y_0^{\{\psi\}} = y_0^{\{K^*\}} \quad (3.29)$$

$$z_0^{\{\psi\}} = -z_0^{\{K^*\}} \quad (3.30)$$

The polar angle of μ^+ in ψ rest frame (θ_ψ):

$$\cos \theta_\psi = -\hat{p}_{K^*}^{\{\psi\}} \cdot \hat{p}_{\mu^+}^{\{\psi\}} \quad (3.31)$$

and the corresponding azimuthal angle of μ^+ in ψ rest frame:

$$\phi_\mu = \text{atan2} \left(\hat{y}_0^{\{\psi\}} \cdot \hat{p}_{\mu^+}^{\{\psi\}}, \hat{x}_0^{\{\psi\}} \cdot \hat{p}_{\mu^+}^{\{\psi\}} \right) \quad (3.32)$$

The same calculation procedure is applied to $\phi \rightarrow K^+ K^-$ channel, the polar angle of K^+ in ϕ rest frame is defined as:

$$\cos \theta_\phi = -\hat{p}_{K^+}^{\{\phi\}} \cdot \hat{p}_{K^+}^{\{\phi\}} \quad (3.33)$$

and azimuthal angle of K^+ in ϕ rest frame:

$$\phi_K = \text{atan2} \left(\hat{y}_0^{\{\phi\}} \cdot \hat{p}_{K^+}^{\{\phi\}}, \hat{x}_0^{\{\phi\}} \cdot \hat{p}_{K^+}^{\{\phi\}} \right) \quad (3.34)$$

3.4.3 Matrix element for the $Z^+ \rightarrow \psi K^+$ chain

The chain of $B^+ \rightarrow Z_j^+ \phi$, $Z_j^+ \rightarrow \psi K^+$, $\phi \rightarrow K^+ K^-$, $\psi \rightarrow \mu^+ \mu^-$ also contains four terms. The $B_s^0 \rightarrow Z_j^+ \phi$ gives a term, $\mathcal{H}_{\lambda_\phi^Z}^{B \rightarrow Z_j \phi}$ and limits $\lambda_Z = \lambda_\phi^Z$. The superscript Z in λ_ϕ accounts for different helicity frame than that in the K^* chain.

The $Z_j^+ \rightarrow \psi K^+$ is described by one term

$$\mathcal{H}_{\lambda_\psi^Z}^{Z_j} D_{\lambda_Z, \lambda_\psi^Z}^{J_{Z_j}}(\phi_\psi^Z, \theta_Z, 0)^* R_{Z_j}(m_{\psi K}), \quad (3.35)$$

The helicity angles of Z decays ϕ_ψ^Z and θ_Z are defined by the ψ direction. The strong decay conserve parity, which gives the relation $\mathcal{H}_{-\lambda_\psi^Z}^{Z_j} = (-1)^{J_{Z_j}-1} P_{Z_j} \mathcal{H}_{\lambda_\psi^Z}^{Z_j}$.

The $\phi \rightarrow K^+ K^-$ provides one term

$$D_{\lambda_\phi^Z, 0}^1(\phi_{K^+}^Z, \theta_\phi^Z, 0)^*, \quad (3.36)$$

where K^+ is used to define the two helicity angles of $\phi_{K^+}^Z$ and θ_ϕ^Z from ϕ decays. We use the superscript Z to differentiate from the K^* chain.

The $\psi \rightarrow \mu^+ \mu^-$ decay is described by the term

$$D_{\lambda_\psi^Z, \Delta\lambda_\mu^Z}^1(\phi_\mu^Z, \theta_\psi^Z, 0)^*. \quad (3.37)$$

Since the muons are final-state particles, their helicity states in the Z decay chain, $|\lambda_\mu^Z\rangle$, need to be rotated to the muon helicity states in the f decay chain, $|\lambda_\mu\rangle$, before the Z matrix element terms can be coherently added to the f matrix element terms. The term aligning the muon helicity states between the two reference frames is given by

$$\sum_{\lambda_\mu^Z} D_{\lambda_\mu^Z, \lambda_\mu}^{J_\mu}(\alpha_\mu^Z, 0, 0)^* = \sum_{\lambda_\mu^Z} e^{i\lambda_\mu^Z \alpha_\mu^Z} \delta_{\lambda_\mu^Z, \lambda_\mu} = e^{i\lambda_\mu \alpha_\mu^Z}. \quad (3.38)$$

The transformation of μ^- states will be similar to that of the μ^+ states, except that since $\hat{z}_\psi$ will have the opposite direction, $\alpha_{\mu^+}^Z = -\alpha_{\mu^-}^Z$. The transformation of $|\lambda_{\mu^+}^Z\rangle|\lambda_{\mu^-}^Z\rangle$ to $|\lambda_{\mu^+}\rangle|\lambda_{\mu^-}\rangle$ states will require multiplying the terms for the Z decay chain by

$$e^{i\lambda_\mu \alpha_\mu^Z} e^{i\lambda_{\bar{\mu}} \alpha_{\bar{\mu}}^Z} = e^{i(\lambda_\mu - \lambda_{\bar{\mu}}) \alpha_\mu^Z} = e^{i\Delta\lambda_\mu \alpha_\mu^Z}. \quad (3.39)$$

The total matrix element for the Z^+ chain from B^+ decay is

$$\begin{aligned} \mathcal{M}_{\Delta\lambda_\mu}^Z(\theta_Z, \theta_\psi^Z, \theta_\phi^Z, \phi_{(Z\phi)} \equiv \phi_{K^+}^Z + \phi_\psi^Z, \alpha_\mu^Z, m_{\psi K^+}) &= e^{i\Delta\lambda_\mu \alpha_\mu^Z} \sum_{\lambda_\psi^Z} e^{i\lambda_\psi^Z \phi_\mu^Z} d_{\lambda_\psi^Z, \Delta\lambda_\mu}^1(\theta_\psi^Z) \\ &\times \sum_{\lambda_\phi^Z} e^{i\lambda_\phi^Z \phi_{(Z\phi)}} d_{\lambda_\phi^Z, 0}^1(\theta_\phi^Z) \sum_j \mathcal{H}_{\lambda_\phi^Z}^{B \rightarrow Z_j \phi} \mathcal{H}_{\lambda_\psi^Z}^{Z_j} d_{\lambda_\phi^Z, \lambda_\psi^Z}^{J_{Z_j}}(\theta_Z) R_{Z_j}(m_{\psi K}). \end{aligned} \quad (3.40)$$

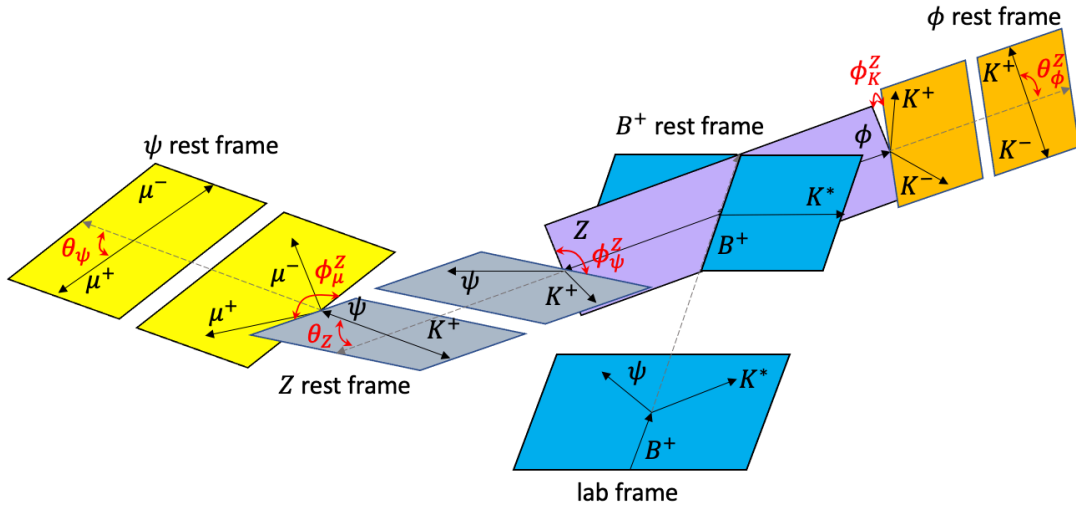


Figure 3.13 Definition of decay angles in the Z^+ chain.

The angles used in Z^+ chain are defined in Fig. 3.13. The polar angle of ϕ in Z rest frame

can be calculated:

$$\cos \theta_Z = -\hat{p}_\phi^{\{Z\}} \cdot \hat{p}_\psi^{\{Z\}} \quad (3.41)$$

The axis of Z rest frame is defined as:

$$\hat{x}_0^{\{Z\}} = -\frac{\hat{p}_{B^0}^{\{LAB\}} - \left(\hat{p}_{B^0}^{\{LAB\}} \cdot \hat{p}_Z^{\{B^0\}} \right) \hat{p}_Z^{\{B^0\}}}{\left| \hat{p}_{B^0}^{\{LAB\}} - \left(\hat{p}_{B^0}^{\{LAB\}} \cdot \hat{p}_Z^{\{B^0\}} \right) \hat{p}_Z^{\{B^0\}} \right|} \quad (3.42)$$

$$\hat{y}_0^{\{Z\}} = -\hat{p}_\phi^{\{Z\}} \times \hat{x}_0^{\{Z\}} \quad (3.43)$$

$$\hat{z}_0^{\{Z\}} = -\hat{p}_\phi^{\{Z\}} \quad (3.44)$$

The azimuthal angle of ψ in Z rest frame can be determined from:

$$\phi_\psi^Z = \text{atan2} \left(\hat{y}_0^{\{Z\}} \cdot \hat{p}_\psi^{\{Z\}}, \hat{x}_0^{\{Z\}} \cdot \hat{p}_\psi^{\{Z\}} \right) \quad (3.45)$$

For the rest frame of ϕ , the definition of axis:

$$\hat{x}_0^{\{\phi\}Z} = -\hat{x}_0^{\{Z\}} \quad (3.46)$$

$$\hat{y}_0^{\{\phi\}Z} = \hat{y}_0^{\{Z\}} \quad (3.47)$$

$$\hat{z}_0^{\{\phi\}Z} = -\hat{z}_0^{\{Z\}} \quad (3.48)$$

The polar angle of K^+ in ϕ rest frame:

$$\cos \theta_\phi^Z = -\hat{p}_Z^{\{\phi\}Z} \cdot \hat{p}_{K^+}^{\{\phi\}Z} \quad (3.49)$$

The azimuthal angle of K^+ in ϕ rest frame:

$$\phi_K^Z = \text{atan2} \left(\hat{y}_0^{\{\phi\}Z} \cdot \hat{p}_{K^+}^{\{\phi\}Z}, \hat{x}_0^{\{\phi\}Z} \cdot \hat{p}_{K^+}^{\{\phi\}Z} \right) \quad (3.50)$$

The definition of axis of ψ rest frame is shown:

$$\hat{x}_0^{\{\psi\}Z} = -\frac{-\hat{p}_\phi^{\{Z\}} + \left(\hat{p}_\phi^{\{Z\}} \cdot \hat{p}_\psi^{\{Z\}} \right) \hat{p}_\psi^{\{Z\}}}{\left| -\hat{p}_\phi^{\{Z\}} + \left(\hat{p}_\phi^{\{Z\}} \cdot \hat{p}_\psi^{\{Z\}} \right) \hat{p}_\psi^{\{Z\}} \right|} \quad (3.51)$$

$$\hat{y}_0^{\{\psi\}Z} = -\hat{p}_{K^+}^{\{\psi\}Z} \times \hat{x}_0^{\{\psi\}Z} \quad (3.52)$$

$$\hat{z}_0^{\{\psi\}Z} = -\hat{p}_{K^+}^{\{\psi\}Z} \quad (3.53)$$

The polar angle of μ^+ in ψ rest frame is defined as:

$$\cos \theta_\psi^Z = -\hat{p}_{K^+}^{\{\psi\}Z} \cdot \hat{p}_{\mu^+}^{\{\psi\}Z} \quad (3.54)$$

The azimuthal angle of μ^+ in ψ rest frame:

$$\phi_\mu^Z = \text{atan2} \left(\hat{y}_0^{\{\psi\}Z} \cdot \hat{p}_{\mu^+}^{\{\psi\}Z}, \hat{x}_0^{\{\psi\}Z} \cdot \hat{p}_{\mu^+}^{\{\psi\}Z} \right) \quad (3.55)$$

The angle α_μ^Z is defined as:

$$\alpha_\mu^Z = \text{atan2} \left(\left(\hat{z}_0^{\{\mu^+\}Z} \times \hat{x}_0^{\{\mu^+\}Z} \right) \cdot \hat{x}_0^{\{\mu^+\}K^*}, \hat{x}_0^{\{\mu^+\}Z} \cdot \hat{x}_0^{\{\mu^+\}K^*} \right) \quad (3.56)$$

$\hat{z}_0^{\{\mu^+\}Z}$ means the z axis of μ^+ rest frame in the Z chain. Other symbols are similar. Here, $\hat{x}_0^{\{\mu^+\}K^*}$, $\hat{x}_0^{\{\mu^+\}Z}$ and $\hat{x}_0^{\{\mu^+\}Z}$ are defined as:

$$\hat{x}_0^{\{\mu^+\}K^*} = - \frac{-\hat{p}_{K^*}^{\{\psi\}} + \left(\hat{p}_{K^*}^{\{\psi\}} \cdot \hat{p}_{\mu^+}^{\{\psi\}} \right) \hat{p}_{\mu^+}^{\{\psi\}}}{\left| -\hat{p}_{K^*}^{\{\psi\}} + \left(\hat{p}_{K^*}^{\{\psi\}} \cdot \hat{p}_{\mu^+}^{\{\psi\}} \right) \hat{p}_{\mu^+}^{\{\psi\}} \right|} \quad (3.57)$$

$$\hat{x}_0^{\{\mu^+\}Z} = - \frac{-\hat{p}_{K^+}^{\{\psi\}} + \left(\hat{p}_{K^+}^{\{\psi\}} \cdot \hat{p}_{\mu^+}^{\{\psi\}} \right) \hat{p}_{\mu^+}^{\{\psi\}}}{\left| -\hat{p}_{K^+}^{\{\psi\}} + \left(\hat{p}_{K^+}^{\{\psi\}} \cdot \hat{p}_{\mu^+}^{\{\psi\}} \right) \hat{p}_{\mu^+}^{\{\psi\}} \right|} \quad (3.58)$$

$$\hat{z}_0^{\{\mu^+\}Z} = \hat{p}_{\mu^+}^{\{\psi\}} \quad (3.59)$$

Keep in mind that the axes have to be calculated in the same ψ rest frame.

3.4.4 Matrix element for the $X \rightarrow \psi\phi$ chain

The decay chain of $B^+ \rightarrow X_i K^+$, $X_i \rightarrow \psi\phi$, $\phi \rightarrow K^+ K^-$, $\psi \rightarrow \mu^+ \mu^-$ also contains four terms. The $B_s^0 \rightarrow X_i K^+$ gives a constant term $\mathcal{H}^{B^+ \rightarrow X_i K^+}$ and limits $\lambda_X = \lambda_K^X = 0$.

The $X_i \rightarrow \psi\phi$ is described by the term

$$\mathcal{H}_{\lambda_\psi^X, \lambda_\phi^X}^{X_i} D_{0, \lambda_\psi^X - \lambda_\phi^X}^{J_{X_i}}(\phi_\psi^X, \theta_X, 0)^* R_{X_i}(m_{\psi\phi}), \quad (3.60)$$

The helicity angles of X decays ϕ_ψ^X and θ_X are defined by the ψ direction. The strong decay conserves parity, which gives the relation $\mathcal{H}_{-\lambda_\psi^X, -\lambda_\phi^X}^X = (-1)^{J_X} P_{X_j} \mathcal{H}_{\lambda_\psi^X, \lambda_\phi^X}^X$.

The $\phi \rightarrow K^+ K^-$ provides one term

$$D_{\lambda_\phi^X, 0}^1(\phi_{K^+}^X, \theta_\phi^X, 0)^*, \quad (3.61)$$

where K^+ is used to define the two helicity angles of $\phi_{K^+}^X$ and θ_ϕ^X from ϕ decays. We use the superscript X to differentiate from the other decay chains.

The $\psi \rightarrow \mu^+ \mu^-$ decay is described by the term

$$D_{\lambda_\psi^X, \Delta\lambda_\mu^X}^1(\phi_\mu^X, \theta_\psi^X, 0)^*. \quad (3.62)$$

The muon helicity states are aligned by α_μ^X rotation.

The total matrix element for the X chain from B^+ decay is

$$\begin{aligned} \mathcal{M}_{\Delta\lambda_\mu}^X(\theta_X, \theta_\psi^X, \theta_\phi^X, \phi_{K^+}^X, \phi_\mu^X, \alpha_\mu^X, m_{\psi\phi}) &= e^{i\Delta\lambda_\mu \alpha_\mu^X} \sum_{\lambda_\psi^X} e^{i\lambda_\psi^X \phi_\mu^X} d_{\lambda_\psi^X, \Delta\lambda_\mu}^1(\theta_\psi^X) \\ &\times \sum_{\lambda_\phi^X} e^{i\lambda_\phi^X \phi_{K^+}^X} d_{\lambda_\phi^X, 0}^1(\theta_\phi^X) \sum_i \mathcal{H}^{B^+ \rightarrow X_i K^+} \mathcal{H}_{\lambda_\psi^X, \lambda_\phi^X}^{X_i} d_{0, \lambda_\psi^X - \lambda_\phi^X}^{J_{X_i}}(\theta_X) R_{X_i}(m_{\psi\phi}). \end{aligned} \quad (3.63)$$

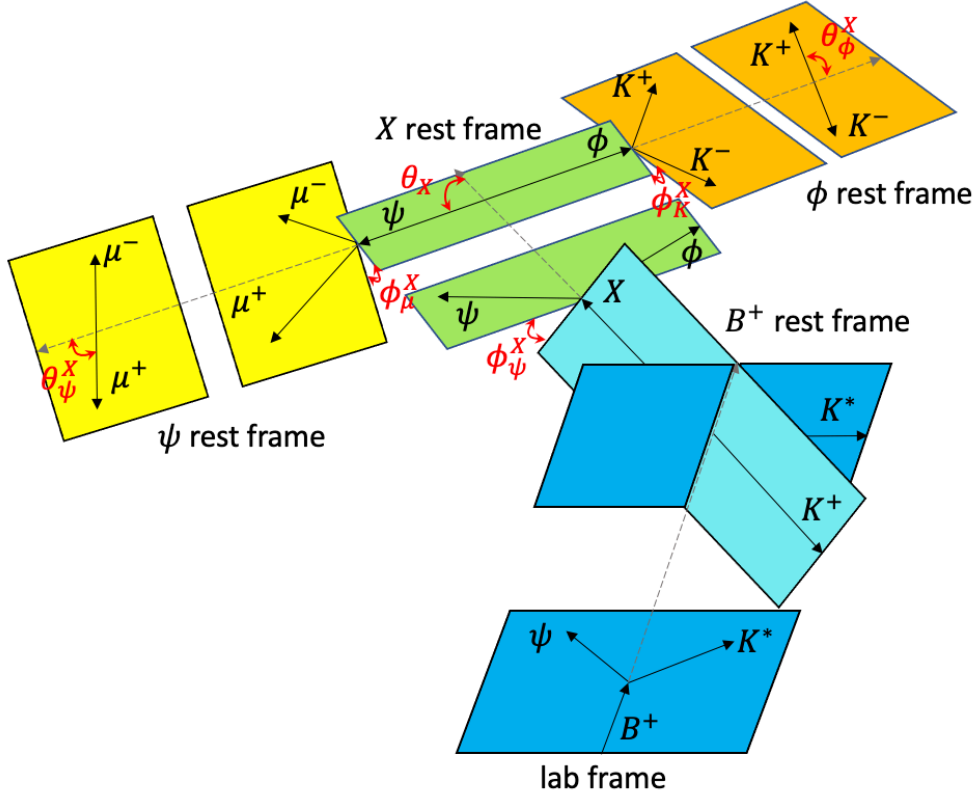


Figure 3.14 Defination of decay angles in the X chain.

The angles used in X chain are defined in Fig. 3.14. The polar angle of ψ in X rest frame can be calculated:

$$\cos \theta_X = -\hat{p}_{K^+}^{\{X\}} \cdot \hat{p}_\psi^{\{X\}} \quad (3.64)$$

The axis of X rest frame is defined as:

$$\hat{x}_0^{\{X\}} = -\frac{\hat{p}_{B^0}^{\{LAB\}} - \left(\hat{p}_{B^0}^{\{LAB\}} \cdot \hat{p}_X^{\{B^0\}} \right) \hat{p}_X^{\{B^0\}}}{\left| \hat{p}_{B^0}^{\{LAB\}} - \left(\hat{p}_{B^0}^{\{LAB\}} \cdot \hat{p}_X^{\{B^0\}} \right) \hat{p}_X^{\{B^0\}} \right|} \quad (3.65)$$

$$\hat{y}_0^{\{X\}} = -\hat{p}_{K^+}^{\{X\}} \times \hat{x}_0^{\{X\}} \quad (3.66)$$

$$\hat{z}_0^{\{X\}} = -\hat{p}_{K^+}^{\{X\}} \quad (3.67)$$

The azimuthal angle of ψ in X rest frame can be determined from:

$$\phi_\psi^X = \text{atan2} \left(\hat{y}_0^{\{X\}} \cdot \hat{p}_\psi^{\{X\}}, \hat{x}_0^{\{X\}} \cdot \hat{p}_\psi^{\{X\}} \right) \quad (3.68)$$

The definition of axis of ψ rest frame is shown:

$$\hat{x}_0^{\{\psi\}X} = - \frac{-\hat{p}_{K^+}^{\{X\}} + \left(\hat{p}_{K^+}^{\{X\}} \cdot \hat{p}_\psi^{\{X\}} \right) \hat{p}_\psi^{\{X\}}}{\left| -\hat{p}_{K^+}^{\{X\}} + \left(\hat{p}_{K^+}^{\{X\}} \cdot \hat{p}_\psi^{\{X\}} \right) \hat{p}_\psi^{\{X\}} \right|} \quad (3.69)$$

$$\hat{y}_0^{\{\psi\}X} = -\hat{p}_\phi^{\{\psi\}X} \times \hat{x}_0^{\{\psi\}X} \quad (3.70)$$

$$\hat{z}_0^{\{\psi\}X} = -\hat{p}_\phi^{\{\psi\}X} \quad (3.71)$$

The polar angle of μ^+ in ψ rest frame is defined as:

$$\cos \theta_\psi^X = -\hat{p}_\phi^{\{\psi\}X} \cdot \hat{p}_{\mu^+}^{\{\psi\}X} \quad (3.72)$$

The azimuthal angle of μ^+ in ψ rest frame:

$$\phi_\mu^X = \text{atan2} \left(\hat{y}_0^{\{\psi\}} \cdot \hat{p}_{\mu^+}^{\{\psi\}}, \hat{x}_0^{\{\psi\}} \cdot \hat{p}_{\mu^+}^{\{\psi\}} \right) \quad (3.73)$$

The angle α_μ^X is defined as:

$$\alpha_\mu^X = \text{atan2} \left(\left(\hat{z}_0^{\{\mu^+\}X} \times \hat{x}_0^{\{\mu^+\}X} \right) \cdot \hat{x}_0^{\{\mu^+\}K^*}, \hat{x}_0^{\{\mu^+\}X} \cdot \hat{x}_0^{\{\mu^+\}K^*} \right) \quad (3.74)$$

$\hat{z}_0^{\{\mu^+\}X}$ means the z axis of μ^+ rest frame in the X chain. Other symbols are similar. Here, $\hat{x}_0^{\{\mu^+\}K^*}$ is defined in Eq.(3.57), $\hat{x}_0^{\{\mu^+\}X}$ and $\hat{x}_0^{\{\mu^+\}X}$ are defined as:

$$\hat{x}_0^{\{\mu^+\}X} = - \frac{-\hat{p}_\phi^{\{\psi\}} + \left(\hat{p}_\phi^{\{\psi\}} \cdot \hat{p}_{\mu^+}^{\{\psi\}} \right) \hat{p}_{\mu^+}^{\{\psi\}}}{\left| -\hat{p}_\phi^{\{\psi\}} + \left(\hat{p}_\phi^{\{\psi\}} \cdot \hat{p}_{\mu^+}^{\{\psi\}} \right) \hat{p}_{\mu^+}^{\{\psi\}} \right|} \quad (3.75)$$

$$\hat{z}_0^{\{\mu^+\}X} = \hat{p}_{\mu^+}^{\{\psi\}} \quad (3.76)$$

For the rest frame of ϕ , the definition of axis:

$$\hat{x}_0^{\{\phi\}X} = -\hat{x}_0^{\{X\}} \quad (3.77)$$

$$\hat{y}_0^{\{\phi\}X} = \hat{y}_0^{\{X\}} \quad (3.78)$$

$$\hat{z}_0^{\{\phi\}X} = -\hat{z}_0^{\{X\}} \quad (3.79)$$

The polar angle of K^+ in ϕ rest frame:

$$\cos \theta_\phi^X = -\hat{p}_\psi^{\{\phi\}X} \cdot \hat{p}_{K^+}^{\{\phi\}X} \quad (3.80)$$

The azimuthal angle of K^+ in ϕ rest frame:

$$\phi_K^X = \text{atan2} \left(\hat{y}_0^{\{\phi\}X} \cdot \hat{p}_{K^+}^{\{\phi\}X}, \hat{x}_0^{\{\phi\}X} \cdot \hat{p}_{K^+}^{\{\phi\}X} \right) \quad (3.81)$$

Finally, total matrix element squared is given by

$$|\mathcal{M}|^2 = \sum_{\Delta\lambda_\mu=\pm 1} \left| \mathcal{M}_{\Delta\lambda_\mu}^{K^*} + \mathcal{M}_{\Delta\lambda_\mu}^Z + \mathcal{M}_{\Delta\lambda_\mu}^X \right|^2 \quad (3.82)$$

Assuming approximate CP symmetry, the helicity couplings for B^+ and B^- can be made equal, but the calculation of the angles need to be carefully considered, since parity (P) conservation does not change polar (i.e. helicity) angles, but does change azimuthal angles. Thus, not only must $\vec{p}_{\mu^-}$ be used instead of $\vec{p}_{\mu^+}$ for B^- candidates, as well as K^- instead of K^+ for the ϕ decay, but also all azimuthal angles must be reflected before entering the matrix element formula: all $\phi_i \rightarrow -\phi_i$, and two $\alpha_\mu \rightarrow -\alpha_\mu$ [111].

3.4.5 Usage of LS couplings

Since not all helicity couplings are independent because of parity conservation in strong decays, it is convenient to related them to the LS couplings ($B_{L,S}$) using Clebsch-Gordan coefficients

$$\mathcal{H}_{\lambda_B, \lambda_C}^{A \rightarrow B C} = \sum_L \sum_S \sqrt{\frac{2L+1}{2J_A+1}} B_{L,S} \begin{pmatrix} J_B & J_C \\ \lambda_B & -\lambda_C \end{pmatrix} \begin{pmatrix} S \\ \lambda_B - \lambda_C \end{pmatrix} \times \begin{pmatrix} L & S \\ 0 & \lambda_B - \lambda_C \end{pmatrix} \begin{pmatrix} J_A \\ \lambda_B - \lambda_C \end{pmatrix}. \quad (3.83)$$

Here L is the orbital angular momentum in the decay, and S is the total spin of the daughters, $\vec{S} = \vec{J}_B + \vec{J}_C$ ($|J_B - J_C| \leq S \leq J_B + J_C$). The LS couplings are all independent since they automatically satisfy the parity conservation relation in strong decays, by restricting L value to be odd or even using the selection $P_A = (-1)^L P_B \cdot P_C$, where P is parity of the particle.

Table 3.8 shows the allowed LS for various J^P of K excitations. The rule also applies to the $Z \rightarrow \psi K$ chain, because ψ and ϕ are vector particles, have the same $J^P = 1^-$. For $B^+ \rightarrow \psi K_n^*$ decays, $S = L_B^{K_n^*}$ can have up to three values, $J_{K_n^*} - 1$, $J_{K_n^*}$ and $J_{K_n^*} + 1$ (except for when $J_{K_n^*} = 0$, only one value $L_B^{K_n^*} = 1$ is possible), corresponding to three (or one) $B_{L,S}^{B^+ \rightarrow \psi K_n^*}$ couplings, respectively. For $K_n^* \rightarrow \phi K$ decay, $S = 1$, and $|J_{K_n^*} - 1| \leq L_{K_n^*} \leq J_{K_n^*} + 1$. Again when $J_{K_n^*} = 0$, $L_{K_n^*} = 1$. Parity conservation limits $L_{K_n^*}$ to be even or odd by $P_{K_n^*} = (-1)^{L_{K_n^*}}$, and natural parity ($1^-, 2^+, 0^+$ is forbidden) allows one $L_{K_n^*}$, while unnatural parity ($1^+, 2^- \dots$) allows two $L_{K_n^*}$. Since only the product of B^+ and K_n^* couplings can be determined, lowest $L_{\min}$ coupling of K^* decay $B_{L_{\min}, S}^{K_n^* \rightarrow \phi K}$ is defined as $(1, 0)$, which reduces one free coupling. To summarize whole K^* decay chain, $0^+ K_n^*$ is forbidden, 0^- is described by one coupling, $1^-, 2^+$

... by three couplings, and $1^+, 2^-$... by four couplings. The $0^- 2^1 S_0$ is chosen as the reference channel, and the only one coupling of B decay is fixed to be $(1, 0)$. This is a very good choice to make fit stable, since the state has large and stable fit fraction about 10%, and only-one coupling is directly proportional to its fit fraction.

For $B^+ \rightarrow X_i K^+$ decays, only one coupling is needed, the coupling $B^{B^+ \rightarrow X_i K^+}$ is equal to $(1, 0)$, again because only the product of B^+ and X_i couplings can be determined. For $X_i \rightarrow \psi \phi$ decays, $S = 0, 1, 2$ and parity conservation relation $P_{X_i} = (-1)^{L_{X_i}}$ reduces L_{X_i} to about half. Table 3.9 shows the allowed LS for various J^P of X_i . Each LS combination requires one coupling that are determined by the fit.

Table 3.8 Allowed LS for various J^P in the $K_n^* \rightarrow \phi K$ decay chain.

J^P	$K_n^* \rightarrow \phi K$	$B \rightarrow J/\psi K_n^*$
0^+	forbidden	-
0^-	11	11
1^+	01, 21	00, 11, 22
1^-	11	00, 11, 22
2^+	21	11, 22, 33
2^-	11, 31	11, 22, 33

Table 3.9 Allowed LS for various J^P in the $X \rightarrow J/\psi \phi$ decay chain.

J^P	$X_i \rightarrow J/\psi \phi$	$B \rightarrow X_i K$
0^+	00, 22	00
0^-	11	00
1^+	01, 21, 22	11
1^-	10, 11, 12, 32	11
2^+	02, 20, 21, 22, 42	22
2^-	11, 12, 31, 32	22

3.4.6 $cFit$ technique

We use $cFit$ technique to estimate the background contribution as same as the one used in previous publication^[39]. The signal PDF, $\mathcal{P}_{\text{sig}}$, is proportional to the matrix element squared, which is a function of six independent variables: $m_{\phi K}$ and the independent angular variables in the K^* decay chain Ω . The PDF also depends on the fit parameters, $\vec{\omega}$, which include the helicity couplings, and masses and widths of resonances. The two other invariant masses, $m_{\phi K}$ and $m_{J/\psi K}$, and the angular variables describing the X and $Z_{c s}^+$ decay chains depend on $m_{\phi K}$

and Ω , therefore they do not represent independent dimensions. The signal PDF is given by:

$$\frac{d\mathcal{P}}{dm_{\phi K} d\Omega} \equiv \mathcal{P}_{\text{sig}}(m_{\phi K}, \Omega | \vec{\omega}) = \frac{1}{I(\vec{\omega})} |\mathcal{M}(m_{\phi K}, \Omega | \vec{\omega})|^2 \Phi(m_{\phi K}) \epsilon(m_{\phi K}, \Omega), \quad (3.84)$$

where $\mathcal{M}(m_{\phi K}, \Omega | \vec{\omega})$ is the total matrix element given by Eq. (3.82) and discussed in detail above. $\Phi(m_{\phi K}) = p q$ is the phase space function, where p is the momentum of the ϕK^+ (*i.e.* K^*) system in the B^+ rest frame, and q is the K^+ momentum in the K^{*+} rest frame. The function $\epsilon(m_{\phi K}, \Omega)$ is the signal efficiency, and $I(\vec{\omega})$ is the normalization integral,

$$I(\vec{\omega}) \equiv \int \mathcal{P}_{\text{sig}}(m_{\phi K}, \Omega) dm_{\phi K} d\Omega \propto \frac{\sum_j w_j^{\text{MC}} |\mathcal{M}(m_{K^* p}, \Omega_j | \vec{\omega})|^2}{\sum_j w_j^{\text{MC}}}, \quad (3.85)$$

where the sum is over simulated events, which are generated uniformly in B^+ decay phase space and passed through the detector simulation^[196] and data selection. The weights w_j^{MC} introduced in Eq. (3.85) contain corrections to the B^+ production kinematics and multiplicity in the generation and to the detector response to bring the simulations into better agreement with the data. The simulation sample contains 390 000 events reconstructed and selected, approximately 15 times the signal size in data. Run 1 MC w_j^{MC} is multiplied by a constant scale (about 0.5) to let the sum of weights is proportional to the signal yield found in data, *i.e.* the ratio of MC sum weights is the same as the ratio of signal yields in Run 1 and Run 2 data. This procedure folds the detector response into the model and allows a direct determination of the parameters of interest from the uncorrected data.

In addition to the signal PDF, $\mathcal{P}_{\text{sig}}(m_{\phi K}, \Omega | \vec{\omega})$, the background PDF, $\mathcal{P}_{\text{bkg}}(m_{\phi K}, \Omega)$ determined from the B^+ mass peak sidebands, is included. We minimize the negative log-likelihood defined as

$$\begin{aligned} -\ln L(\vec{\omega}) &= -\sum_i \ln \left[(1 - \beta) \mathcal{P}_{\text{sig}}(m_{\phi K i}, \Omega_i | \vec{\omega}) + \beta \mathcal{P}_{\text{bkg}}(m_{\phi K i}, \Omega_i) \right] \\ &= -\sum_i \ln \left[(1 - \beta) \frac{|\mathcal{M}(m_{\phi K i}, \Omega_i | \vec{\omega})|^2 \Phi(m_{\phi K i}) \epsilon(m_{\phi K i}, \Omega_i)}{I(\vec{\omega})} + \beta \frac{\mathcal{P}_{\text{bkg}}^u(m_{\phi K i}, \Omega_i)}{I_{\text{bkg}}} \right] \\ &= -\sum_i \ln \left[|\mathcal{M}(m_{\phi K i}, \Omega_i | \vec{\omega})|^2 + \frac{\beta I(\vec{\omega})}{(1 - \beta) I_{\text{bkg}}} \frac{\mathcal{P}_{\text{bkg}}^u(m_{\phi K i}, \Omega_i)}{\Phi(m_{\phi K i}) \epsilon(m_{\phi K i}, \Omega_i)} \right] + N \ln I(\vec{\omega}) + \text{const.}, \end{aligned} \quad (3.86)$$

where β is the background fraction in the peak region determined from the fit to the $m_{J/\psi \phi K}$ distribution (Fig. 3.7), $\mathcal{P}_{\text{bkg}}^u(m_{\phi K}, \Omega)$ is the unnormalized background density proportional to

the density of sideband events, with its normalization determined by^⑥

$$I_{\text{bkg}} \equiv \int \mathcal{P}_{\text{bkg}}^u(m_{\phi K}) dm_{\phi K} d\Omega \propto \sum_j w_j^{\text{MC}} \frac{\mathcal{P}_{\text{bkg}}^u(m_{\phi K}, \Omega_j)}{\Phi(m_{\phi K}, \Omega_j)} / \sum_j w_j^{\text{MC}}. \quad (3.87)$$

The equation above implies that the background term is efficiency corrected, so it can be added to the efficiency-independent signal probability expressed by $|\mathcal{M}|^2$. This way the efficiency parameterization, $\epsilon(m_{\phi K}, \Omega)$, becomes a part of the background description which affects only a very small part of the total PDF.

The efficiency parameterization in the background term is assumed to factorize as

$$\begin{aligned} \epsilon(m_{\phi K}, \Omega) = & \epsilon_1(m_{\phi K}, \cos \theta_{K^*}) \epsilon_2(\cos \theta_{\phi} | m_{\phi K}) \times \\ & \epsilon_3(\cos \theta_{J/\psi} | m_{\phi K}) \epsilon_4(\Delta \phi_{K^*, \phi} | m_{\phi K}) \epsilon_5(\Delta \phi_{K^*, J/\psi} | m_{\phi K}). \end{aligned} \quad (3.88)$$

The $\epsilon_1(m_{\phi K}, \cos \theta_{K^*})$ term is obtained by binning a 2-dimensional (2D) histogram of the simulated signal events. Each event is given a $1/(pq)$ weight, since at the generator level the phase space is flat in $\cos \theta_{K^*}$ but has a pq dependence on $m_{\phi K}$. A bi-cubic function is used to interpolate between bin centers. The $\epsilon_1(m_{\phi K}, \cos \theta_{K^*})$ efficiency and its visualization across the normal Dalitz plane are shown in Fig. 3.15. The other terms are again built from 2D histograms, but with each bin divided by the number of simulated events in the corresponding $m_{\phi K}$ slice to remove the dependence on this mass (Fig. 3.16).

The background PDF, $\mathcal{P}_{\text{bkg}}^u(m_{\phi K}, \Omega)/\Phi(m_{\phi K})$, is built using the same approach,

$$\begin{aligned} \frac{\mathcal{P}_{\text{bkg}}^u(m_{\phi K}, \Omega)}{\Phi(m_{\phi K})} = & P_{\text{bkg}1}(m_{\phi K}, \cos \theta_{K^*}) P_{\text{bkg}2}(\cos \theta_{\phi} | m_{\phi K}) \times \\ & P_{\text{bkg}3}(\cos \theta_{J/\psi} | m_{\phi K}) P_{\text{bkg}4}(\Delta \phi_{K^*, \phi} | m_{\phi K}) P_{\text{bkg}5}(\Delta \phi_{K^*, J/\psi} | m_{\phi K}). \end{aligned} \quad (3.89)$$

The background function $P_{\text{bkg}1}(m_{\phi K}, \cos \theta_{K^*})$ is shown in Fig. 3.17 and the other terms are shown in Fig. 3.18.

The fit fraction (FF) of any component is an important information to understand the internal nature of the resonances (R) in this decay, which is defined as,

$$\text{FF} = \frac{\int |\mathcal{M}^R(m_{\phi K}, \Omega)|^2 \Phi(m_{\phi K}) dm_{\phi K} d\Omega}{\int |\mathcal{M}(m_{\phi K}, \Omega)|^2 \Phi(m_{\phi K}) dm_{\phi K} d\Omega}, \quad (3.90)$$

where in $\mathcal{M}^R$ all terms except those associated with the R amplitude are set to zero.

⑥ Notice that the distribution of MC events includes both the $\Phi(m_{\phi K})$ and $\epsilon(m_{\phi K}, \Omega)$ factors, which cancel their product in the numerator.

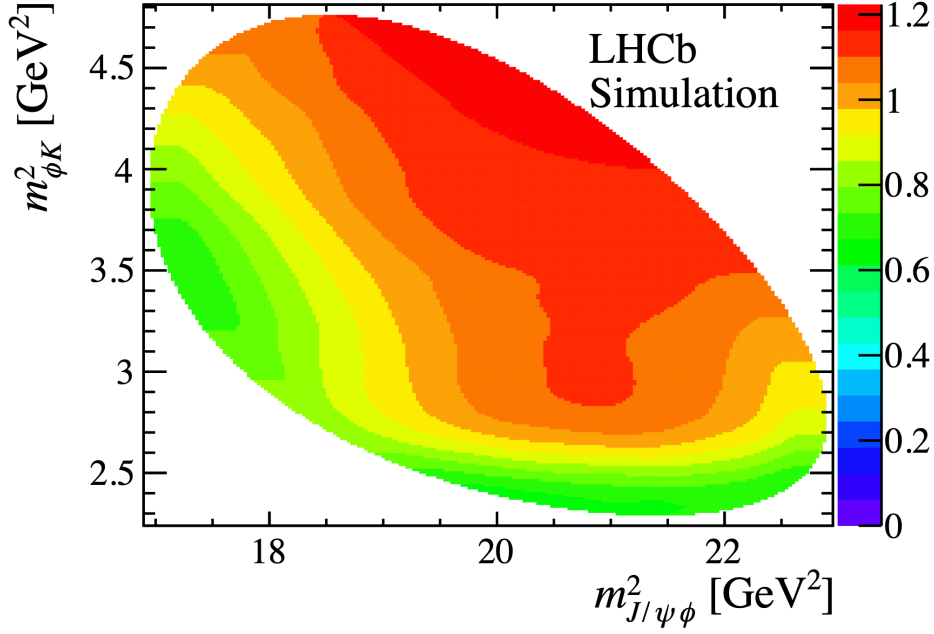


Figure 3.15 Parameterized efficiency representation in the Dalitz plane ($m_{J/\psi\phi}^2, m_{\phi K}^2$). Function values corresponding to the color encoding are given on the right. The normalization arbitrarily corresponds to unity when averaged over the phase space.

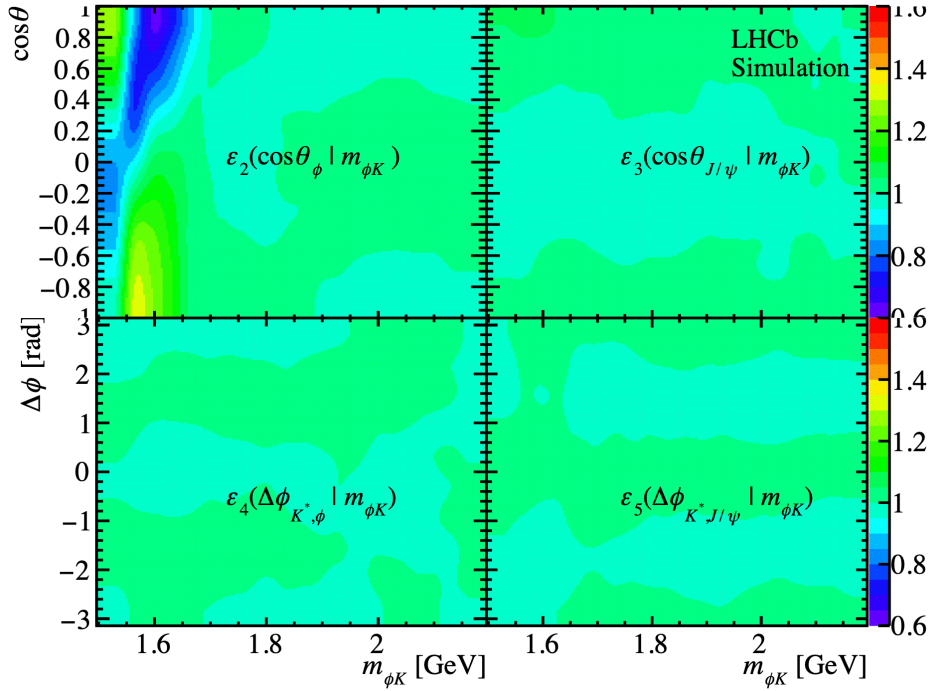


Figure 3.16 Parameterized efficiency $\epsilon_2(\cos\theta_\phi|m_{\phi K})$, $\epsilon_3(\cos\theta_{J/\psi}|m_{\phi K})$, $\epsilon_4(\Delta\phi_{K^*,\phi}|m_{\phi K})$, $\epsilon_5(\Delta\phi_{K^*J/\psi}|m_{\phi K})$ functions. Function values corresponding to the color encoding are given on the right. By construction each function integrates to unity at each $m_{\phi K}$ value. The structure in $\epsilon_2(\cos\theta_\phi|m_{\phi K})$ present between 1500 and 1600 MeV is an artifact of removing $B^+ \rightarrow J/\psi K^+ K^- K^+$ events in which both $K^+ K^-$ combinations pass the ϕ mass selection window.

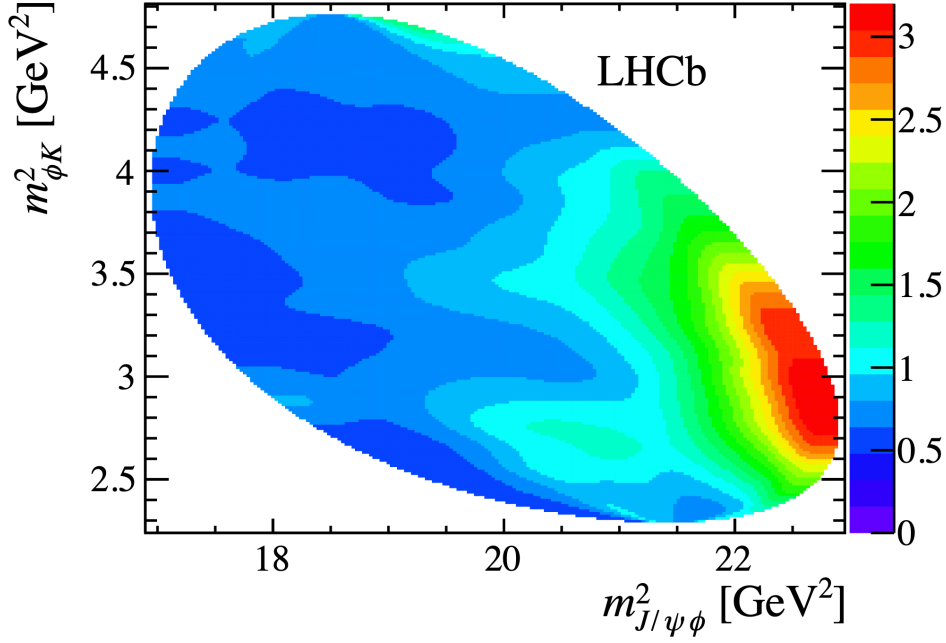


Figure 3.17 Parameterized background representation in the Dalitz plane ($m_{J/\psi\phi}^2, m_{\phi K}^2$). Function values corresponding to the color encoding are given on the right. The normalization arbitrarily corresponds to unity when averaged over the phase space.

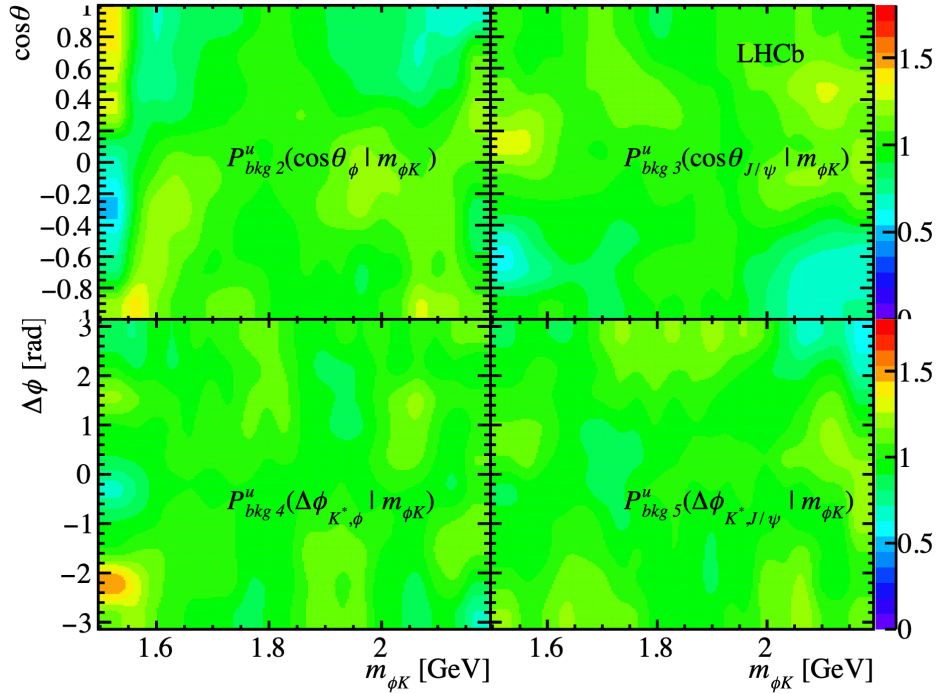


Figure 3.18 Parameterized background functions: $P_{bkg\ 2}^u(\cos\theta_\phi | m_{\phi K})$, $P_{bkg\ 3}^u(\cos\theta_{J/\psi} | m_{\phi K})$, $P_{bkg\ 4}^u(\Delta\phi_{K^*,\phi} | m_{\phi K})$, $P_{bkg\ 5}^u(\Delta\phi_{K^*,J/\psi} | m_{\phi K})$. Function values corresponding to the color encoding are given on the right. By construction each function integrates to unity at each $m_{\phi K}$ value.

3.5 The amplitude fit results

We fit the events in ± 15 MeV around B^+ known mass^[1]. For the background, its fraction is fixed to 4.0% and its PDF is obtained from $\pm(25 - 50)$ MeV B^+ sideband events as described above.

3.5.1 Kaon excitations in $B^+ \rightarrow J/\psi \phi K^+$

The kaon mass spectrum as predicted in the relativistic potential model by Godfrey–Isgur^[209] is shown in Fig. 3.19 together with the experimentally determined masses of both well-established and unconfirmed K^* resonances, with the status before our first amplitude analysis of $B^+ \rightarrow J/\psi \phi K^+$ ^[1]. Table 3.10 summarizes this status, with some more recent updates. Past experiments on K^* states decaying to ϕK ^[210–212] had limited precision, especially at high masses, gave somewhat inconsistent results, and provided evidence for only a few of the states expected from the quark model in the 1513–2182 MeV range probed in our data set. However, except for the $J^P = 0^+$ states which cannot decay to ϕK because of angular momentum and parity conservation, all other kaon excitations above the ϕK threshold are predicted to decay to this final state^[213]. In B^+ decays, production of high spin states, like the $K_3^*(1780)$ or $K_4^*(2045)$ resonances, is expected to be suppressed by the high orbital angular momentum required to produce them.

Like in the previous amplitude analysis^[40], the predictions of the Godfrey–Isgur model as a guide to the number and J^P of the K^{*+} states are included in the model. We also consider several states that are below the mass threshold. We fix the masses and widths of well measured states to improve fit stability, and let the others vary.

3.5.2 Changes to the previous amplitude model

The model used in Run 1 publication^[39,40], listed in Tab. 3.2 is tested first. Many fits are tested using random initial input values and the best solution is obtained with $\ln \mathcal{L} = 4367$. Figure 3.20 shows the fit projections onto the three masses. While gross mass peaking structures are reproduced, deficiencies in describing the $m_{J/\psi \phi}$ and $m_{J/\psi K}$ projections accurately are apparent. Therefore, improvements to the model are needed.

The fit results with the Run 1 model suggest several rather minor adjustments:

- $K(0^-)$: In Run 1 model, a candidate for 3^1S_0 state was found at $1874 \pm 43^{+59}_{-115}$ MeV with width of $168 \pm 90^{+280}_{-104}$ MeV, however, it was not very significant (3.5σ). In our fits, the width of such high-mass 0^- state reaches to the upper limit set in the fits

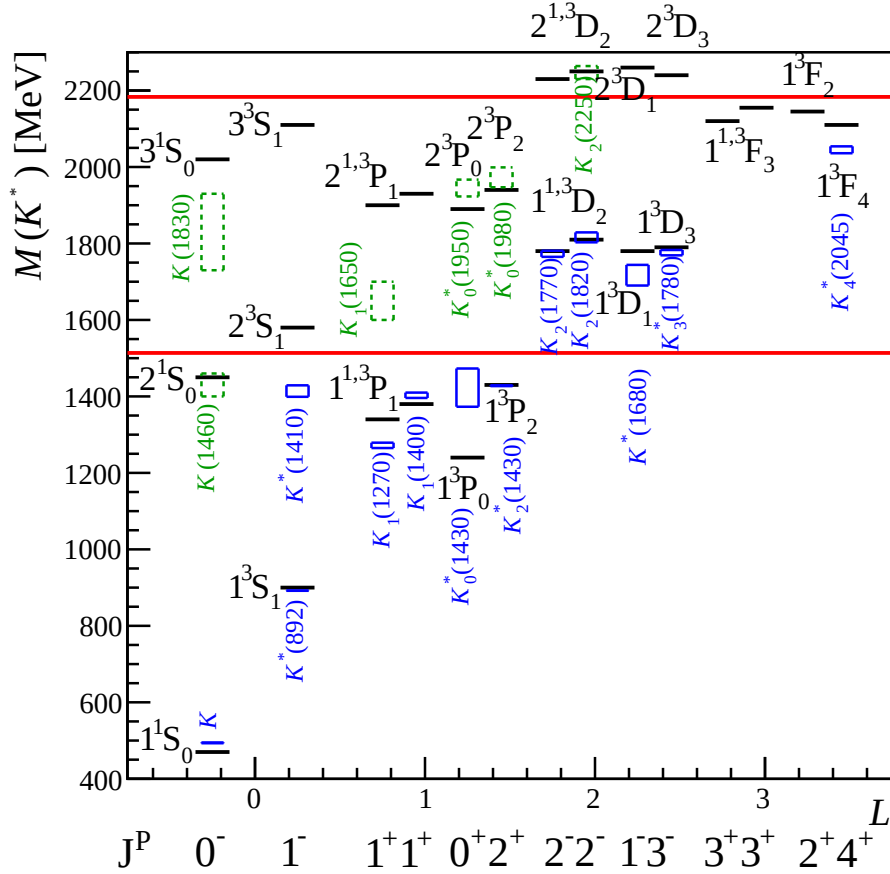


Figure 3.19 Kaon excitations predicted by Godfrey–Isgur^[209] (horizontal black lines) labeled with their intrinsic quantum numbers: $n^{2S+1}L_J$ (see the text). Well established states are shown with narrower solid blue boxes extending to $\pm 1\sigma$ in mass and labeled with their PDG names^[214]. Unconfirmed states are shown with dashed green boxes. The long horizontal red lines indicate the ϕK mass range probed in $B^+ \rightarrow J/\psi \phi K^+$ decays.

Table 3.10 Possible contribution of kaon excitations to $B^+ \rightarrow J/\psi \phi K^+$ decays. For those states in bold, their masses and widths are fixed to listed values, in which 2014 PDG values to remove the contribution from the previous LHCb publication is used. That of the other states are varied in the fit.

States	M_0 [MeV]	Γ_0 [MeV]	Ref.
$K(0^-)$			
2^1S_0 $\mathbf{K(1460)}$	1482.4 ± 15.6	335.6 ± 10.6	LHCb ^[215]
3^1S_0	Not observed		
$K(1^-)$			
2^3S_1 $\mathbf{K^*(1410)}$	1414 ± 15	232 ± 21	PDG ^[1]
3^3S_1	Not observed		
1^3D_1 $\mathbf{K^*(1680)}$	1717 ± 27	322 ± 110	PDG ^[214]
$K(1^+)$			
1^3P_1 $\mathbf{K_1(1400)}$	1403 ± 7	174 ± 13	PDG ^[1]
2^1P_1 $K(1650)$	1672 ± 50	158 ± 50	PDG ^[1]
2^3P_1	Not observed		
$K(2^+)$			
2^3P_2 $K_2^*(1980)$	1942 ± 50	307^{+50}_{-31}	PDG ^[1]
1^3F_2	Not observed		
$K(2^-)$			
1^1D_2 $\mathbf{K_2(1770)}$	1773 ± 8	186 ± 14	PDG ^[214]
1^3D_2 $\mathbf{K_2(1820)}$	1816 ± 13	276 ± 35	PDG ^[214]
$K(3^-)$			
1^3D_3 $\mathbf{K_3^*(1780)}$	1776 ± 7	159 ± 21	PDG ^[1]
$K(3^+)$			
1^1F_3	Not observed		
1^3F_3	Not observed		
$K(4^+)$			
1^3F_4 $\mathbf{K_4^*(2045)}$	2048^{+8}_{-9}	199^{+27}_{-19}	PDG ^[1]

(2 GeV). Recently, LHCb established a lower-mass 2^1S_0 $K(1460)$ state^[215], whose mass and width are larger than the previously measured values. We include this state with the fixed mass and width, as its tail can clearly contribute. We explore adding a higher-mass excitation to it in extended model.

- $K(1^+)$: This major contribution to the total amplitude was described in Run 1 model as two resonances (roughly speaking mixed 2^3P_1 and 2^1P_1 states) and large non-resonant contribution. In the fit to the new data, such representation makes the higher-mass state very broad (up to 1.2 GeV), thus similar to non-resonant contribution, and they interfere profoundly (-84%). We obtain a better model by replacing the non-resonant contribution by the tail of the 1^3P_1 $K_1(1400)$ resonance with the mass and width fixed to the previous measurements (see Tab. 3.10).
- $K(1^-)$: In addition to 1^3D_1 $K^*(1680)$ state, a tail of 2^3S_1 $K_1^*(1410)$ resonance is also included, whose mass and width are fixed to the values averaged over the previous measurements (Tab. 3.10).
- $K(2^-)$: The Run 1 model left the masses and widths of $K_2(1770)$ and $K_2(1820)$ free in the fit and demonstrated good consistency with the previous measurements, albeit with large errors. Since there are large interferences between these two close in mass and broad resonances, overall model stability by fixing them to the other measurements is improved.
- $X(4140)$: When fit with single-component mass-dependent width (Eq. (3.22)), we obtain $\Gamma_0 = 161 \pm 28$ MeV, which is about twice larger than Run 1 result (Tab. 3.2). Because M_0 is very close to the kinematic threshold, q_0 can be very small and strongly dependent on exact value of M_0 . This in turn, makes Γ_0 unstable too. Therefore, it is required to use of a constant width in the Breit-Wigner formula for $X(4140)$ ($\Gamma(m) = \Gamma_0$). As an alternative, two-component mass-dependent width formula is investigated later, in which $D_s^{*+}D_s^-$ is included as likely decay channel with the lower kinematic threshold (see Sec. 3.10.1).

To summarize these changes, fit quality and stability are improved by including a number of broad below-the-threshold K^* resonances (in Run 1 analysis such additional components were considered among systematic errors) and by fixing the masses and widths of several well established K^* resonances above the ϕK threshold to the other measurements. For K^* s which are below the ϕK decay threshold, and for X states that are close to the $J/\psi\phi$ threshold (*i.e.* $X(4140)$ and $X(4127)$ discussed later), a constant width in the Breit-Wigner formula is used. After these changes, with 9 K^* components and 5 X components the likelihood is slightly

improved to $\ln \mathcal{L} = 4424$ (i.e. $\Delta \ln \mathcal{L} = 56$), but the deficiencies in description of the mass projections persist.

Then, more drastic changes by adding new exotic hadron components are studied, X or Z_{cs}^+ states with different J^P hypotheses. To ensure that such components are not reflections of inadequacy of our K^* model, besides, more predicted higher-mass K^* resonances is also tried. The results from adding one component at a time are listed in Tab 3.11, which shows two states giving the largest likelihood improvements first ($1^+ Z_{cs}^+$ and $1^+ X$), and then more check, if any further X or Z_{cs}^+ state significantly improved the model, is applied. In the second iteration, additions of several states give large improvements; 1^- and $1^+ Z_{cs}^+$ improve $\ln \mathcal{L}$ by 110 and 104, while 1^- and $2^- X$ by 100 and 69. Since the 1^+ and $1^- Z_{cs}^+$ are at about the same mass (4.2 GeV), they might be the same state. Therefore the two X states and one Z_{cs}^+ state are chosen in the third iteration. In total, three more X and two Z components are added to the model. All new states have significances above 5σ , as evaluated by taking them out of the default model one-by-one. Further addition of an X state of any J^P gives components with significance below 5σ .

Larger improvement is from addition of the 3^3S_1 K^* candidate state, but the fit returns its mass at the upper kinematic limit and a width of more than 1 GeV. This state and $X(4140)$ state get very large fit fractions, each about 67%, and strong negative interference fraction between them of -93% . This is not physical, therefore, it is not considered in the nominal fit, but instead include it in extended K model.

Table 3.11 Improvement on $\ln \mathcal{L}$ by adding one X or Z or K with different J^P hypothesis.

J^P	0^+	0^-	1^+	1^-	2^+	2^-
X	25	25	133	98	64	81
Z_{cs}^+	forbidden	36	195	94	41	42
J^P	$3^1S_0(0^-)$	$1^3D_3(3^-)$	$1F_3(3^+)$	$1^3F_2(2^+)$	$3^3S_1(1^-)$	
K^*	5	18	5	37	70	

3.5.3 Nominal amplitude model

Based on the investigation described in the previous section, the nominal model is now built, which includes two Z_{cs}^+ and three new X components in the fit, called 9K+5X+3X+2Z model. Since additional K^* resonances are not significant, which are used to study systematic uncertainties. Inclusion of the second Z_{cs}^+ with $J^P = 1^+$ or 1^- gives similar fit likelihood. However the $1^- Z_{cs}^+$ fit is not very physical, because it has large interference fraction (-8.8%)

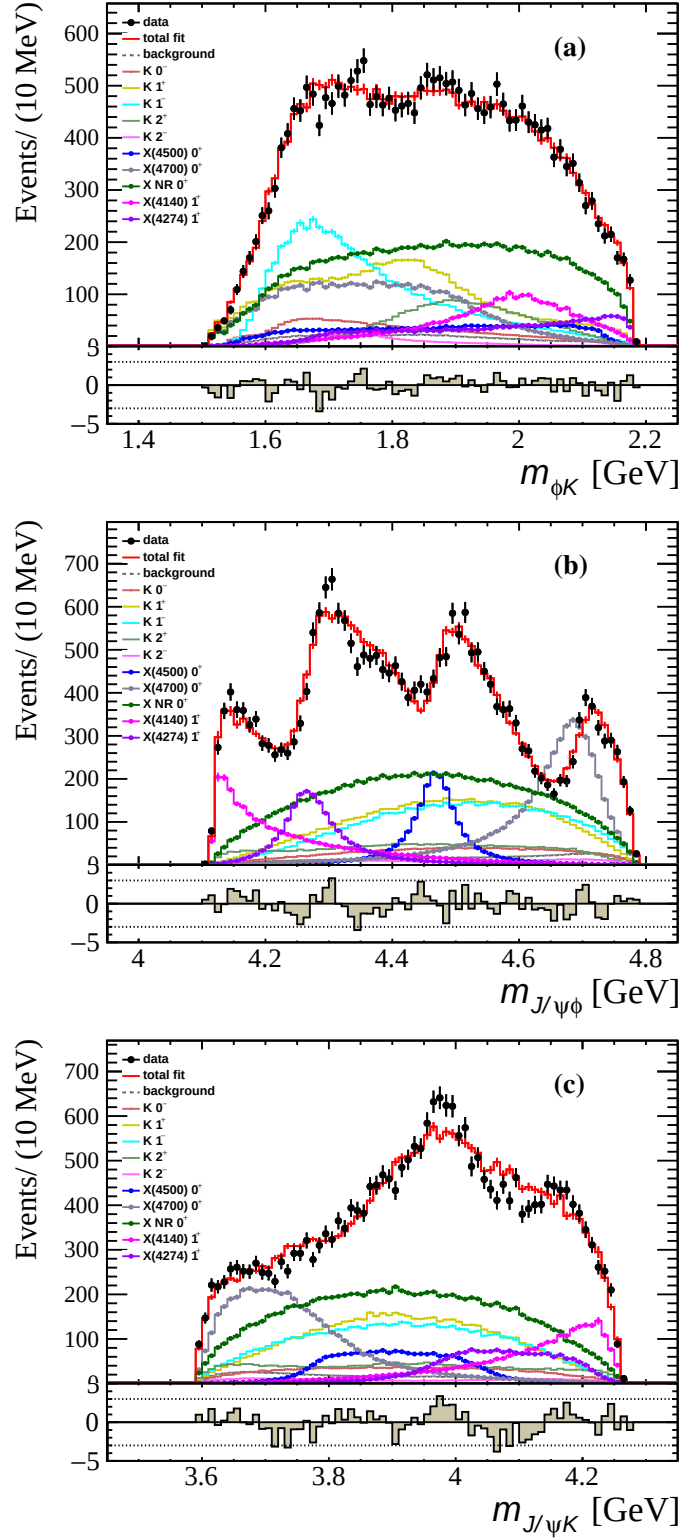


Figure 3.20 Fit projections of (a) $m_{\phi K}$, (b) $m_{J/\psi \phi}$, (c) $m_{J/\psi K}$ by fitting Run 1 model to the new Run 1+2 data sample.

with the $X(4140)$ state compared to the relative small FF (8.2%) and its mass is also at the high mass boundary. Instead, the $1^+ Z_{cs}^+$ looks more physical. Therefore, the $1^+ Z$ fit for the nominal model is chosen, and use the other one as model variation used to evaluate systematic uncertainties.

The nominal model gives $\ln \mathcal{L} = 5004$, about 580 improvement for 45 more free parameters with respect to the 9K+5X model which does not include new exotic hadron components. The improvement in $\ln \mathcal{L}$ is about equal to the sum of individual improvements in Tab 3.11, illustrating that the added five exotic states have independent effects on the fit model.

The mass projections are well modelled by the nominal fit as shown in Fig. 3.21. Two-dimensional Dalitz-plot distributions generated from the nominal amplitude model are shown in Fig. 3.22. The data structures in Fig. 3.8 and Fig. 3.9 are well reproduced. Figure 3.23 shows the $m_{J/\psi K}$ distributions in two slices of $m_{J/\psi \phi}$, as well as the overall $m_{J/\psi K}$ projection without the two $1^+ Z_{cs}^+$ states included in Fig. 3.24, both demonstrate the need for the narrower $Z_{cs}(4000)^+$ state. Including the $1^+ Z_{cs}(4000)^+$ states improves the χ^2/nbins from 84/35 to 43/35 (left slice), and from 79/37 to 32/37 (right slice), where nbins is the number of non-zero bins. The angular distributions in ϕK decay chain are shown in Fig. 3.25, they are also well described by the nominal fit. All fit results are shown in Tab. 3.12, including masses, widths, fit fractions, and significance of each component.

The significance of each component is evaluated assuming that the change of $2 \ln \mathcal{L}$ between the default fit and the fit without this component obeys the χ^2 distribution with number of degrees of freedom (ndf) equal to twice the reduction in number of free parameters, to take into account look elsewhere effect^[40]. The ndf is not multiplied by two for the K^* resonances whose mass and width are fixed. This is a common practice to use Wilks' theorem^[216], which states that for continuous hypotheses *i.e.* when H_0 (*e.g.* no-exotic) hypothesis is a special case of more general H_1 (H_0 is “nested” in H_1) asymptotically for infinite statistics ($N \rightarrow \infty$) $\Delta(-2 \ln \mathcal{L})$ has a χ^2 distribution with ndf equal to the number of constrained parameters in H_0 hypotheses relatively to the H_1 . This leads to the following formula for significance expressed in standard deviations:

$$n_\sigma(\Delta(-2 \ln \mathcal{L})) = \sqrt{2} \text{TMath::ErfcInverse}(\text{TMath::Prob}(\Delta(-2 \ln \mathcal{L}), \text{ndf})).^{(7)} \quad (3.91)$$

All the included states are above 5σ . The statistical significance for two Z_{cs}^+ states are 16 and 8σ , respectively. The significance of the three new X states are 15, 8.7 and 5.7σ , respectively.

⁽⁷⁾ `TMath::ErfcInverse` is the inversed error function, `TMath::Prob` is the cumulative distribution function of χ^2 distribution.

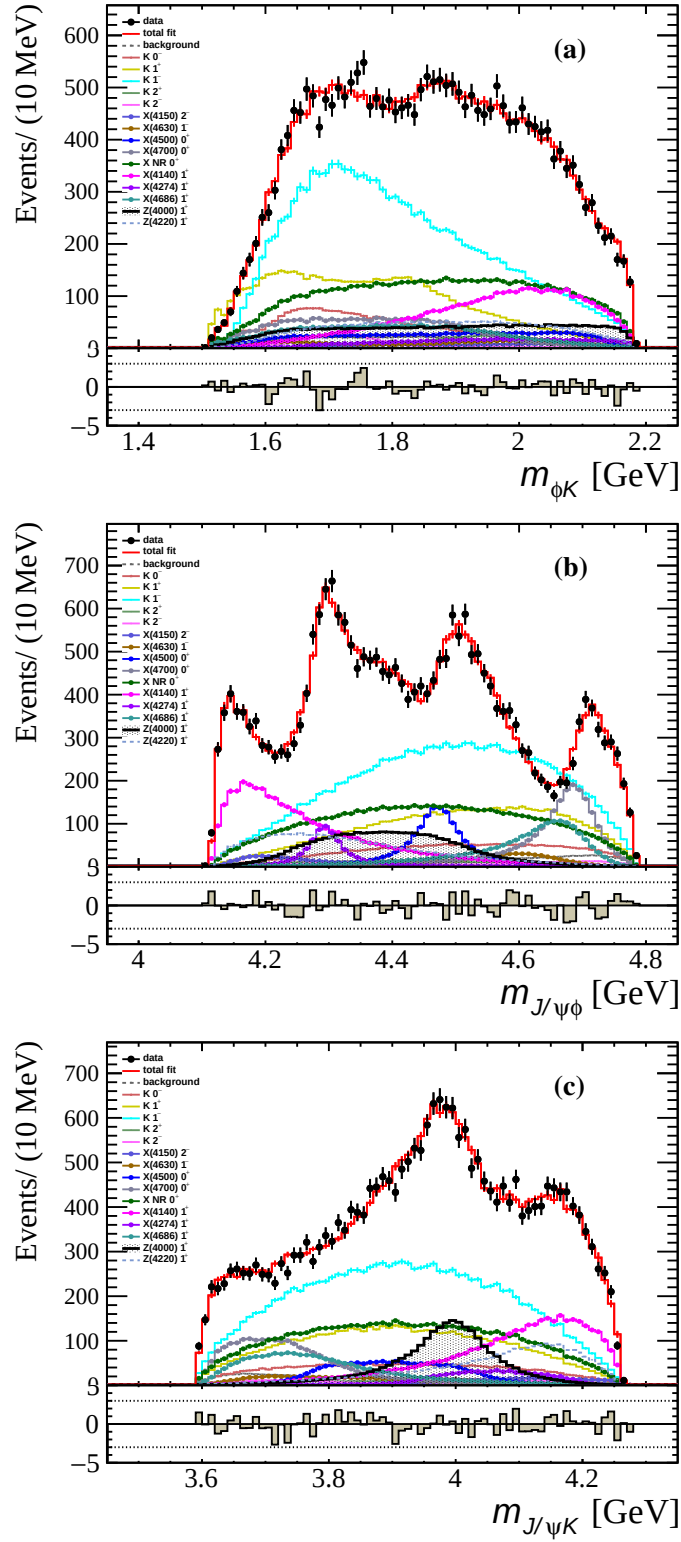


Figure 3.21 Fit projections of (a) $m_{\phi K}$, (b) $m_{J/\psi \phi}$, (c) $m_{J/\psi K}$ from the nominal model (9K+5X+2Z).

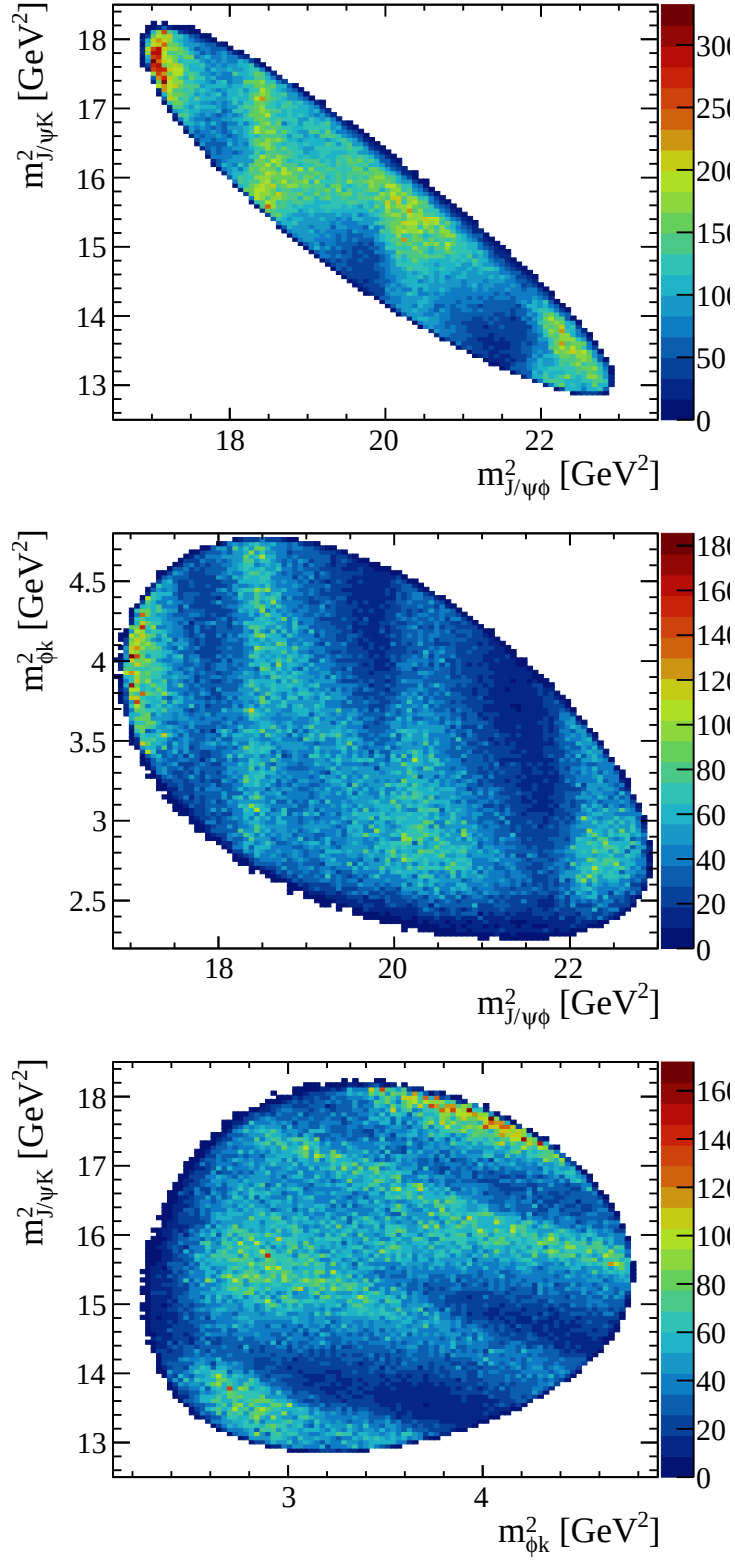


Figure 3.22 Dalitz-plot distributions of signal from the nominal fit model (toy sample).

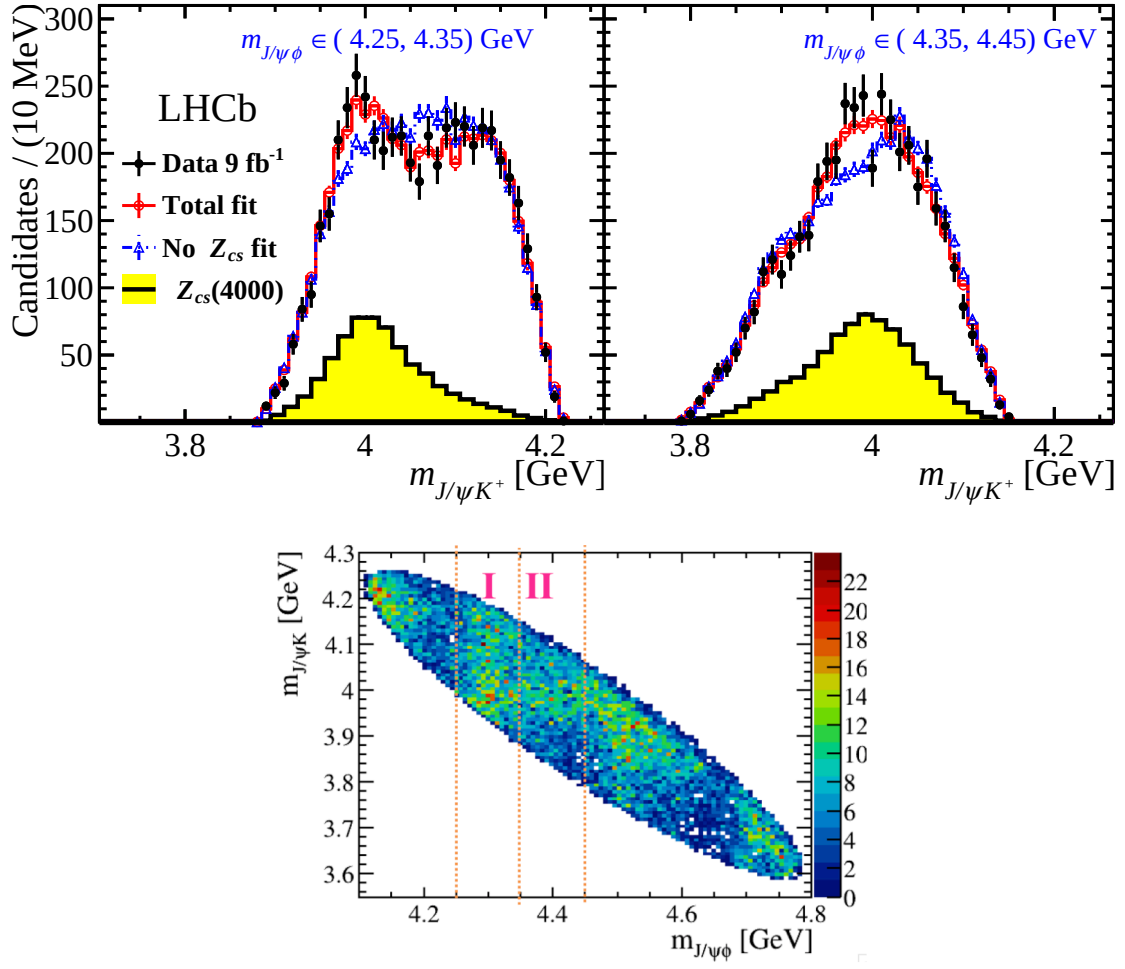


Figure 3.23 Fit projections of $m_{J/\psi K}$ in two slices of $m_{J/\psi \phi}$ from the nominal model and the nominal model without $1^+ Z_{cs}^+$ (top). The narrow Z_{cs}^+ is evident in the two $m_{J/\psi \phi}$ slices. Bottom figure shows the two slices of $m_{J/\psi \phi}$ regions.

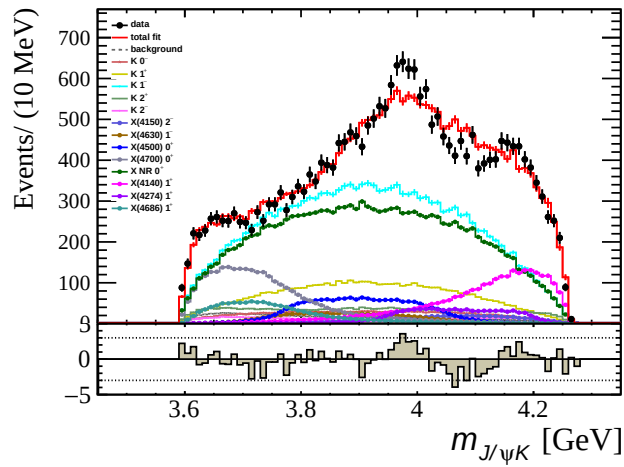


Figure 3.24 Fit projections of $m_{J/\psi K}$ from the nominal model without $1^+ Z_{cs}^+$.

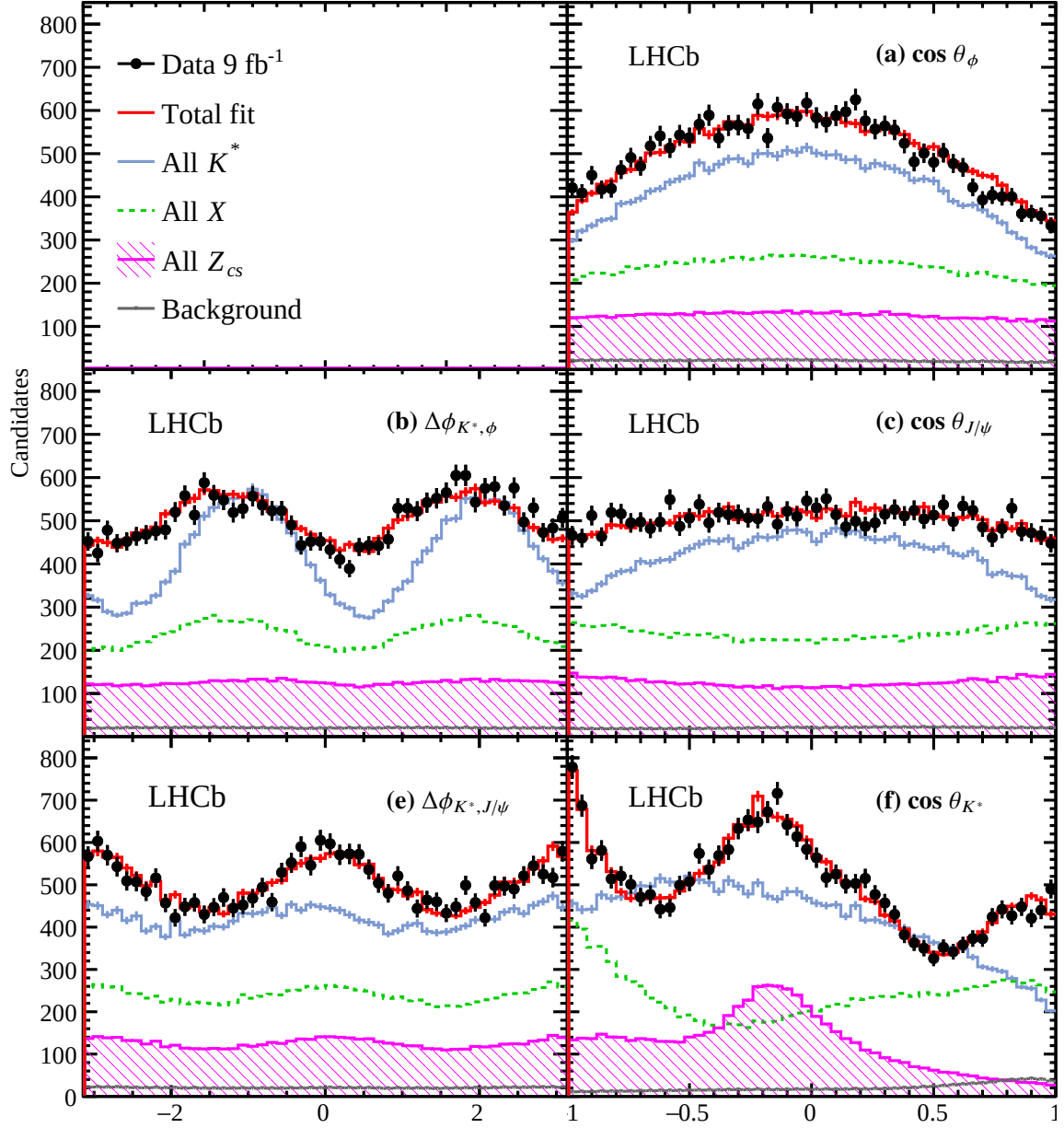

 Figure 3.25 Fit projections of angles in the ϕK decay chain.

Table 3.12 Fit results from the 9K+5X+3X+2Z model with $Z_{cs}(4220)^+ J^P = 1^+$. Significance is evaluated from the change of the $2 \ln \mathcal{L}$ assuming ndf to be **twice** the reduction in number of free parameters when removing the component in the fit. The ndf is not multiply by two for the K^* resonances whose mass and width are fixed.

Contribution		Significance	Fit results		
			M_0 [MeV]	Γ_0 [MeV]	FF%
2^1P_1 2^3P_1 1^3P_1	All $K(1^+)$				25.1 ± 3.6
	$K(1^+)$	4.5σ	1861 ± 10	149 ± 41	
	$K'(1^+)$	4.5σ	1911 ± 37	276 ± 50	
1^1D_2 1^3D_2	$K_1(1400)$	11σ			15.1 ± 2.7
	All $K(2^-)$				2.1 ± 0.4
	$K_2(1770)$	8.0σ			
	$K_2(1820)$	5.8σ			
1^3D_1 2^3S_1	All $K(1^-)$				50.3 ± 4.1
	$K^*(1680)$	13σ			13.5 ± 2.3
	$K^*(1410)$	15σ			37.9 ± 5.2
2^3P_2	$K(2^+)$ $K_2^*(1980)$	7.4σ	1988 ± 22	318 ± 82	2.3 ± 0.5
2^1S_0	$K(0^-)$				
	$K(1460)$	13σ			10.2 ± 1.2
	$X(2^-)$				
	$X(4150)$	8.7σ	4146 ± 18	135 ± 28	2.0 ± 0.5
	$X(1^-)$				
	$X(4630)$	5.7σ	4626 ± 16	174 ± 27	2.6 ± 0.5
	All $X(0^+)$				19.5 ± 4.8
	$X(4500)$	20σ	4474 ± 3	77 ± 6	5.6 ± 0.7
	$X(4700)$	18σ	4694 ± 4	87 ± 8	8.9 ± 1.2
	$NR_{J/\psi\phi}$	5.7σ			28.0 ± 7.5
	All $X(1^+)$				26.0 ± 3.4
	$X(4140)$	16σ	4118 ± 11	162 ± 21	17.2 ± 2.9
	$X(4274)$	18σ	4294 ± 4	53 ± 5	2.8 ± 0.49
	$X(4685)$	15σ	4684 ± 7	126 ± 15	7.2 ± 1.0
	All $Z(1^+)$				25.0 ± 4.9
	$Z_{cs}(4000)^+$	16σ	4003 ± 6	131 ± 15	9.4 ± 2.1
	$Z_{cs}(4220)^+$	8.4σ	4216 ± 24	233 ± 52	10.3 ± 3.8

3.5.4 Extended K^* model

We add more K^* resonances to test the significance of each of component, and evaluate systematic uncertainties. Seven more K resonances are predicted in the allowed ϕK mass range by the relativistic potential model by Godfrey-Isgur^[209]. They are $3^1S_0(0^-)$, $3^3S_1(1^-)$, $1^3D_3(3^-)$, $1^{1,3}F_3(3^+)$, $1^3F_2(2^+)$, and $1^3F_4(4^+)$. We add all of them except for the 4^+ resonance because $L_B = 3$ decays of B are expected to be much suppressed. We use only one BW to represent $1^{1,3}F_3(3^+)$ resonances to reduce possible large interference effects between these two states predicted to be very close-in-mass. The fit results are listed in Tab 3.13. All the new exotic components preserve large significances of more than 5σ . Among additional K^* components, a candidate 3^3S_1 state is 4.4σ significant, and has large mass and width, as expected for such high radial excitation of triplet S -wave. A candidate for higher radial excitation of the singlet S -wave 3^1S_0 has a significance of 4.6σ , but very small fit fraction. Other states are even less significant. The mass projections of this fit are shown in Fig. 3.26.

In all the models, the two new X states around 4650 MeV are found to have significance of more than 5σ , each. Together with the $X(4700)$ state, there are three X states around this mass region. We use χ^2 -test^⑧ on $m_{J/\psi\phi}^2$ vs. $m_{J/\psi K}^2$ plane to evaluate the fit quality in 2-dimensions. The nominal model has $\chi_{2D}^2/\text{nbin} = 284.5/256$, while removing the two new X states gives $341.5/256$. The contributions to the χ_{2D}^2 value for the nominal fit and for the fit without the two X states are compared in Fig. 3.27. The pull distributions are shown in Fig. 3.28.

3.6 Spin and parity study

To determine the quantum numbers of each X or Z_{cs}^+ state, fits are done under alternative J^P hypotheses. The likelihood-ratio test is used to quantify rejection of these hypotheses. Since different spin-parity assignments are represented by different functions in the angular part of the fit PDF, they represent separate hypotheses. For two models representing separate hypotheses, assuming a χ^2 distribution with one degree of freedom for $\Delta(-2 \ln \mathcal{L})$ under the disfavored J^P hypothesis gives a lower limit on the significance of its rejection^[115]. In practice, the significance for rejection of a J^P hypothesis is computed as $\sigma(J^P) = \sqrt{\Delta(-2 \ln \mathcal{L})}$, where $\Delta(-2 \ln \mathcal{L}) = 2 \ln \mathcal{L}(J^P) - 2 \ln \mathcal{L}(\text{preferred})$, *i.e.* the likelihood difference between the fits for given J^P hypothesis and for the preferred J^P for this state is used. The preferred J^P corresponds to the fit returning the smallest $-2 \ln \mathcal{L}$. In order to make sure the state is the same for different J^P hypotheses, mass and width are limited to be within $\pm 5\sigma$ range of the nominal

⑧ Similar method are used in pentaquark study at LHCb^[124].

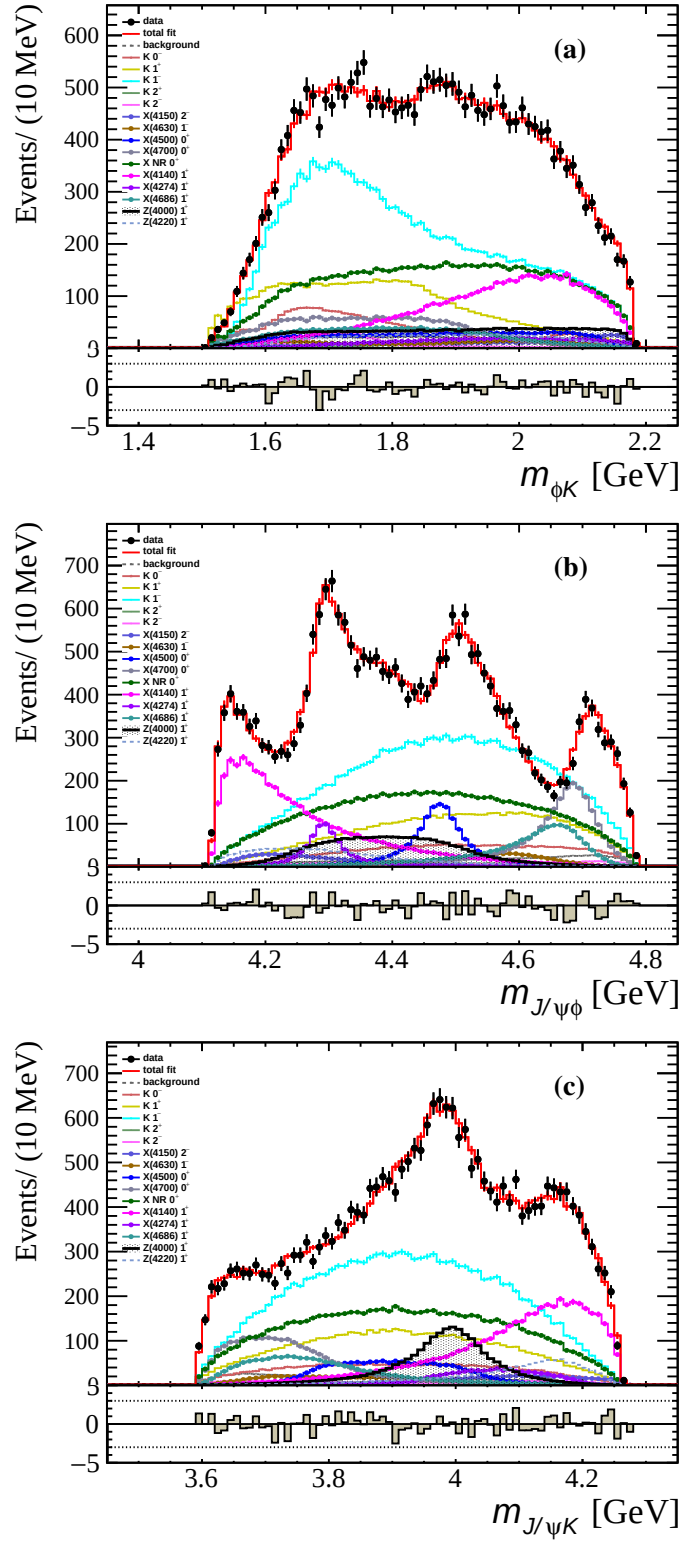

 Figure 3.26 Fit projections of (a) $m_{\phi K}$, (b) $m_{J/\psi \phi}$, (c) $m_{J/\psi K}$ from the Extended K+5X+2X+2Z model.

Table 3.13 Fit results from Extended K+5X+3X+2Z model for $X(4220)$ with $J^P = 1^+$. Significance is evaluated from the change of the $2 \ln \mathcal{L}$ assuming ndf to be **twice** the reduction in number of free parameters when removing the component in the fit. The factor of two is not applied when mass and width are fixed. The two F K states have almost zero fit fractions and significances, therefore, their masses and widths from the fit are meaningless and not shown.

Contribution		Significance	Fit results		
			M_0 [MeV]	Γ_0 [MeV]	FF%
	All $K(1^+)$				23.2
2^1P_1	$K(1^+)$	4.9σ	1821 ± 15	372 ± 63	
2^3P_1	$K'(1^+)$	4.9σ	1871 ± 20	154 ± 28	
1^3P_1	$K_1(1400)$	9.2σ			15.0
	All $K(2^-)$				2.1
1^1D_2	$K_2(1770)$	7.9σ			
1^3D_2	$K_2(1820)$	6.4σ			
	All $K(1^-)$				52.5
1^3D_1	$K^*(1680)$	4.7σ			6.4
2^3S_1	$K^*(1410)$	7.7σ			34.2
3^3S_1		4.4σ	2128 ± 36	448 ± 132	4.5
	All $K(2^+)$				1.6
2^3P_2	$K_2^*(1980)$	1.6σ	2182 ± 73	800 ± 230	2.4
1^3F_2		1.3σ	2076 ± 25	100 ± 74	0.4
	All $K(0^-)$				10.0
2^1S_0	$K(1460)$	12σ			11.0
3^1S_0		4.6σ	2064 ± 15	108 ± 46	0.5
1^1F_3	$K(3^+)$	0.3σ			0.2
1^3D_3	$K(3^-)$	2.3σ			0.2
	$X(2^-)$				
	$X(4150)$	6.3σ	4117 ± 13	193 ± 81	2.8
	$X(1^-)$				
	$X(4630)$	5.5σ	4611 ± 28	193 ± 25	2.7
	All $X(0^+)$				22.8
	$X(4500)$	20σ	4476 ± 3	73 ± 6	5.6
	$X(4700)$	17σ	4695 ± 4	89 ± 9	9.2
	$NR_{J/\psi\phi}$	4.8σ			33.0
	All $X(1^+)$				31.2
	$X(4140)$	13σ	4107 ± 12	138 ± 22	21.1
	$X(4274)$	18σ	4293 ± 3	53 ± 5	3.1
	$X(4685)$	15σ	4681 ± 9	120 ± 17	6.3
	All $Z(1^+)$				15.9
	$Z_{cs}(4000)^+$	15σ	4001 ± 5	124 ± 19	8.2
	$Z_{cs}(4220)^+$	5.9σ	4196 ± 22	176 ± 35	5.6

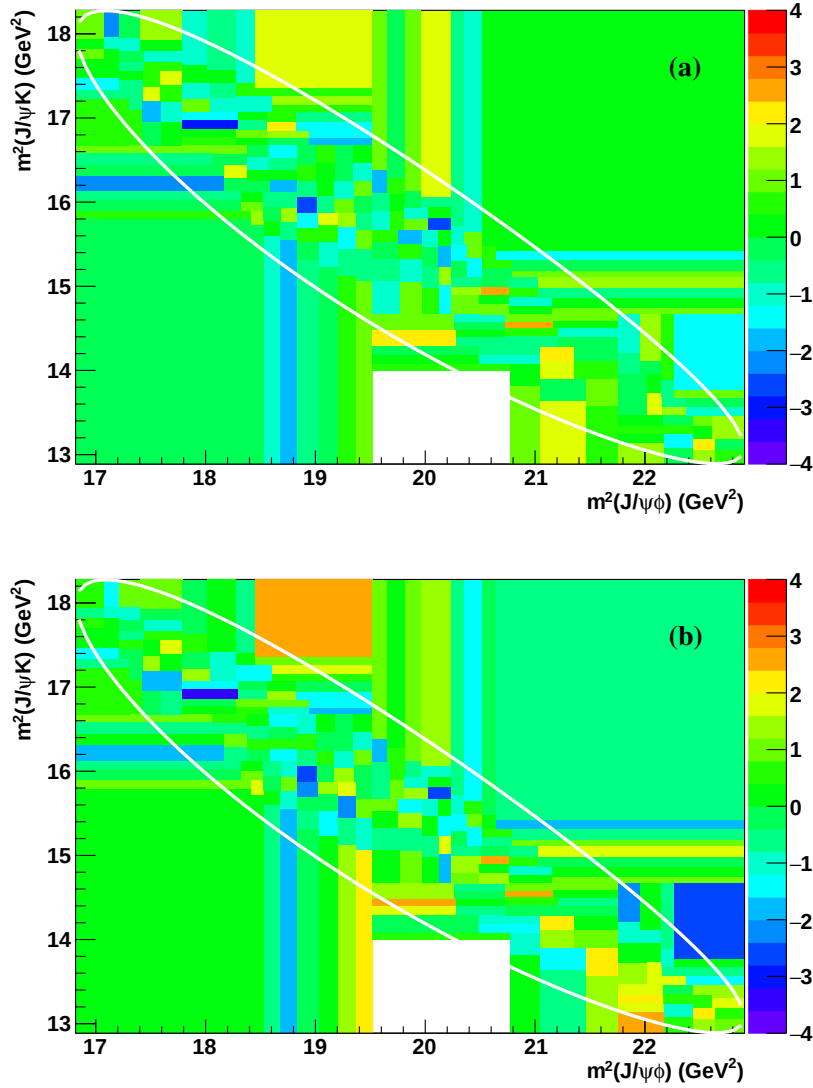


Figure 3.27 Comparison of the standard deviations between the data and the fit model, contributing to χ^2_{2D} value, for (a) the nominal fit, and (b) the fit without the two new X states. Some improvements are seen in (a) at $m^2(J/\psi\phi) = 22$ GeV², as compared to (b).

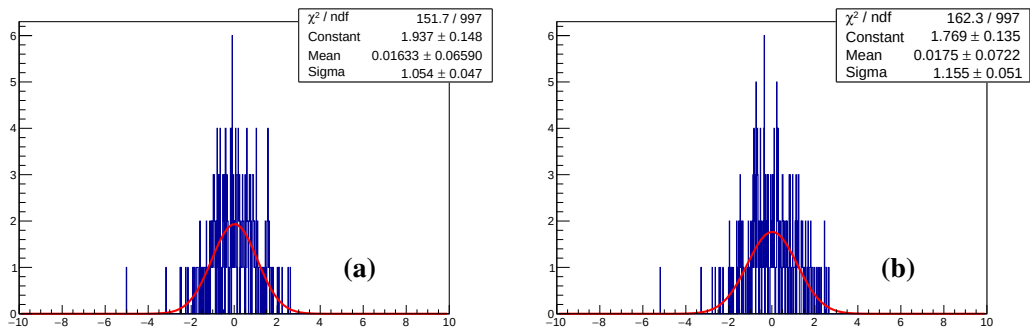


Figure 3.28 The pull distributions of χ^2_{2D} value in Fig. 3.27. The RMS of the pull distribution is smaller when additional X components are introduced.

fit results. The results for the default model are shown in Tab. 3.14, and for the extended- K^* model in Tab. 3.15. The J^P of previous reported four X states are confirmed to be correct. Two new exotic states $Z_{cs}(4000)^+$ and $X(4675)$ are determined to be both 1^+ for more than 15σ level. The J^P of the other new exotic states are not uniquely determined. The broad $Z_{cs}(4220)^+$ J^P hypothesis can be 1^+ or 1^- within about 2σ , therefore, both hypotheses for $Z_{cs}(4220)^+$ are used to test J^P values of other states. The $X(4175)$ state prefers 2^- by more than 4σ . The $X(4650)$ state prefers 1^- by 3σ over 2^- hypothesis. The other hypotheses are rejected by more than 5σ .

Table 3.14 Statistical significance of J^P preference for the X and Z_{cs}^+ states in the default model. For the states with two lines, the upper (lower) row corresponds to the tests with $Z_{cs}(4220)^+ J^P = 1^{-(+)}$. The bold numbers highlight J^P values which cannot be ruled out with at least 5σ significance.

J^P	0^+	0^-	1^+	1^-	2^+	2^-
$X(4150)$	4.7σ	6.1σ	5.9σ	5.5σ	4.6σ	prefer
$X(4150)$	9.7σ	12σ	9.6σ	5.4σ	9.2σ	prefer
$X(4630)$	12σ	11σ	9.6σ	prefer	8.5σ	9.1σ
$X(4630)$	8.3σ	6.1σ	7.3σ	prefer	6.3σ	4.1σ
$X(4500)$	prefer	19σ	19σ	18σ	21σ	20σ
$X(4500)$	prefer	21σ	20σ	19σ	21σ	20σ
$X(4700)$	prefer	19σ	18σ	19σ	15σ	18σ
$X(4700)$	prefer	19σ	19σ	19σ	16σ	18σ
$X(4140)$	16σ	15σ	prefer	16σ	14σ	15σ
$X(4140)$	16σ	17σ	prefer	15σ	17σ	15σ
$X(4274)$	18σ	19σ	prefer	18σ	18σ	19σ
$X(4274)$	18σ	18σ	prefer	18σ	18σ	18σ
$X(4685)$	16σ	16σ	prefer	16σ	15σ	16σ
$X(4685)$	17σ	17σ	prefer	16σ	16σ	16σ
$Z_{cs}(4000)^+$	-	19σ	prefer	19σ	18σ	19σ
$Z_{cs}(4000)^+$	-	18σ	prefer	16σ	16σ	16σ
$Z_{cs}(4220)^+$	-	11σ	prefer	2.4σ	9.5σ	9.1σ

We also show the nominal $9K+5X+3X+2Z$ model with a $1^- Z_{cs}(4220)^+$ in Tab. 3.16 for completeness. The $1^- Z_{cs}(4220)^+$ has large width and is at the high mass boundary allowed in the $B^+ \rightarrow J/\psi \phi K^+$ decay.

Table 3.15 Statistical significance of J^P preference for the X and Z_{cs}^+ states in the extended model. For the states with two lines, the upper (lower) row corresponds to the tests with $Z_{cs}(4220)^+ J^P = 1^{-(+)}$. The bold numbers highlight J^P values which cannot be ruled out with at least 5σ significance.

J^P	0^+	0^-	1^+	1^-	2^+	2^-
$X(4150)$	6.7σ	7.4σ	7.4σ	5.8σ	4.2σ	prefer
$X(4150)$	6.6σ	9.1σ	7.7σ	6.6σ	6.7σ	prefer
$X(4630)$	12σ	11σ	10.0σ	prefer	10σ	8.8σ
$X(4630)$	6.7σ	5.3σ	5.8σ	prefer	5.9σ	3.0σ
$X(4500)$	prefer	19σ	20σ	18σ	20σ	19σ
$X(4500)$	prefer	18σ	18σ	18σ	18σ	18σ
$X(4700)$	prefer	19σ	19σ	19σ	16σ	19σ
$X(4700)$	prefer	18σ	19σ	18σ	14σ	17σ
$X(4140)$	14σ	15σ	prefer	20σ	13σ	15σ
$X(4140)$	14σ	15σ	prefer	14σ	13σ	14σ
$X(4274)$	18σ	19σ	prefer	18σ	18σ	19σ
$X(4274)$	18σ	18σ	prefer	18σ	18σ	19σ
$X(4685)$	17σ	17σ	prefer	17σ	17σ	15σ
$X(4685)$	16σ	16σ	prefer	15σ	16σ	15σ
$Z_{cs}(4000)^+$	-	18σ	prefer	18σ	17σ	17σ
$Z_{cs}(4000)^+$	-	17σ	prefer	17σ	15σ	16σ
$Z_{cs}(4220)^+$	-	8.6σ	2.0σ	prefer	4.9σ	5.7σ

Table 3.16 Fit results from the 9K+5X+3X+2Z model with $Z_{cs}(4220)^+ J^P = 1^-$. Significance is evaluated from the change of the $2 \ln \mathcal{L}$ assuming ndf to be **twice** the reduction in number of free parameters when removing the component in the fit. The ndf is not multiply by two for the K^* resonance whose mass and width are fixed.

Contribution		Significance	Fit results		
			M_0 [MeV]	Γ_0 [MeV]	FF%
	All $K(1^+)$				10.8 ± 2.6
2^1P_1	$K(1^+)$	6.4σ	1867 ± 10	147 ± 27	
2^3P_1	$K'(1^+)$	6.4σ	1918 ± 14	438 ± 74	
1^3P_1	$K_1(1400)$	6.9σ			12.8 ± 6.5
	All $K(2^-)$				4.0 ± 0.4
1^1D_2	$K_2(1770)$	13.0σ			
1^3D_2	$K_2(1820)$	12.0σ			
	All $K(1^-)$				56.5 ± 3.3
1^3D_1	$K^*(1680)$	9.5σ			6.2 ± 1.3
2^3S_1	$K^*(1410)$	11.7σ			35.8 ± 3.9
	$K(2^+)$				
2^3P_2	$K_2^*(1980)$	7.3σ	1973 ± 18	301 ± 67	2.8 ± 0.5
	$K(0^-)$				
2^1S_0	$K(1460)$	14.3σ			9.4 ± 0.9
	$X(2^-)$				
	$X(4150)$	4.8σ	4176 ± 14	112 ± 32	1.0 ± 0.3
	$X(1^-)$				
	$X(4630)$	11.0σ	4650 ± 23	121 ± 15	5.5 ± 1.0
	All $X(0^+)$				24.6 ± 7.7
	$X(4500)$	18.0σ	4475 ± 3	70 ± 6	5.1 ± 0.5
	$X(4700)$	18.0σ	4693 ± 4	83 ± 8	7.8 ± 0.7
	$NR_{J/\psi\phi}$	6.9σ			18.7 ± 4.3
	All $X(1^+)$				25.0 ± 3.6
	$X(4140)$	14.8σ	4116 ± 10	130 ± 21	18.0 ± 6.8
	$X(4274)$	17.3σ	4295 ± 4	53 ± 5	2.7 ± 0.6
	$X(4685)$	14.9σ	4675 ± 9	121 ± 15	5.7 ± 1.2
	$Z_{cs}^+(1^+)$				
	$Z_{cs}(4000)^+$	18.2σ	3989 ± 4	109 ± 15	5.8 ± 0.4
	$Z_{cs}(1^-)$				
	$Z_{cs}(4220)^+$	9.2σ	4263 ± 39	468 ± 126	8.1 ± 3.4

3.7 Argand diagram

Further evidence for the resonant character of $Z_{cs}(4000)^+$ is shown in Fig. 3.29 by demonstrating the evolution of the complex amplitude on the Argand diagram, obtained with the same method as previously reported for the $Z_c(4430)^-$ state^[115]. The Argand diagram is obtained by splitting the data into 8 bins from -1.4Γ to 1.4Γ in $m_{J/\psi K^+}$ around the mass of $Z_{cs}(4000)^+$, and the corresponding mass points are 3839, 3898, 3938, 3982, 4024, 4068, 4108 and 4167 MeV in anti-clockwise direction. In each $m_{J/\psi K^+}$ bins, the complex amplitudes of $Z_{cs}(4000)^+$ are floated in the fit. The magnitude and phase have approximately circular evolution with $m_{J/\psi K^+}$ in the counter-clockwise direction, as expected for a resonance. However, the Argand diagram cannot prove that the $Z_{cs}(4000)^+$ is a real resonance, as the threshold effect would also create an approximately circular evolution in Argand diagram^[141].

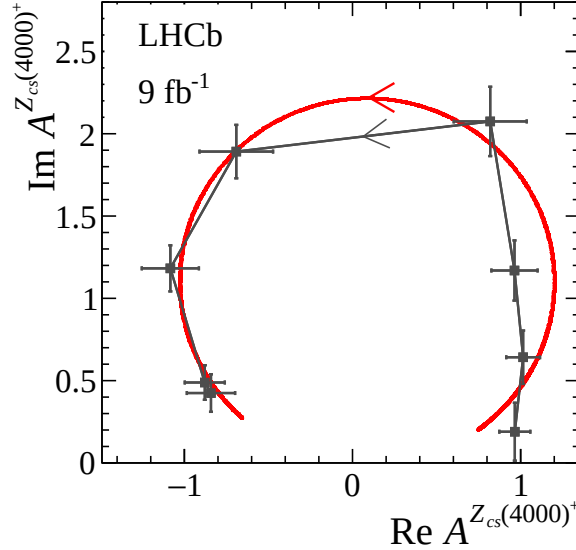


Figure 3.29 Fitted values of the $Z_{cs}(4000)^+$ amplitude in eight $m_{J/\psi K^+}$ bins, shown on an Argand diagram (black points). The red line represents the expected Breit-Wigner behaviour in between -1.4Γ to 1.4Γ around the $Z_{cs}(4000)^+$ mass.

3.8 Angular moments check

In order to check the improvements of the nominal model comparing with the old Run 1 model in different invariant mass regions, the distribution of Legendre moments^[125,217] of different helicity angle are also checked with different models. The unnormalized Legendre moments is defined as

$$\langle P_\ell^U \rangle = \sum_{i=1}^{N_{\text{events}}} \frac{1}{\epsilon_i} P_\ell(\cos \theta), \quad (3.92)$$

where $P_\ell(\cos \theta)$ is the Legendre polynomial of order ℓ and ϵ_i is the efficiency for each event, depending on its phase-space location. Structures related to a spin J resonance described by the helicity angle θ , can show up in the Legendre moments of rank up to $l = 2J$, or up to $l = J + J'$ in interferences of J and J' amplitudes. Mass dependence of the Legendre moments can provide hints about resonances present in the data, since their quantum numbers restrict J values. Unfortunately, reflections of decays from the other decay chain(s) can generate mass dependent variations of the moments at any rank, obscuring interpretation of the data.

The moments distribution is obtained by weighting each events by a weight $\frac{1}{\epsilon_i} P_\ell(\cos \theta)$. The background contribution in the data is subtracted, and the signal contribution is achieved as the moments distribution of events in the signal region minus the moments distribution of sideband events, the latter is scaled by the ratio of background yields between the signal and the sideband regions. In Fig. 3.30, we show the Legendre moments vs. $m_{J/\psi \phi}$ using cosine helicity angle of $J/\psi \phi$ as the argument of the polynomial. The nominal model describes the data reasonably well. Improvements over the old Run 1 model are clearly visible in several moments. In Fig. 3.31, the Legendre moments vs. $m_{\phi K}$ using cosine helicity angle of ϕK are shown. The agreement is reasonable, but the differences between the nominal and old Run 1 model are smaller. The Legendre moments vs $m_{J/\psi K}$ using cosine helicity angle of $J/\psi K$ are shown in Fig. 3.32. Some improvements of the nominal model as compared to the old Run 1 model are see.

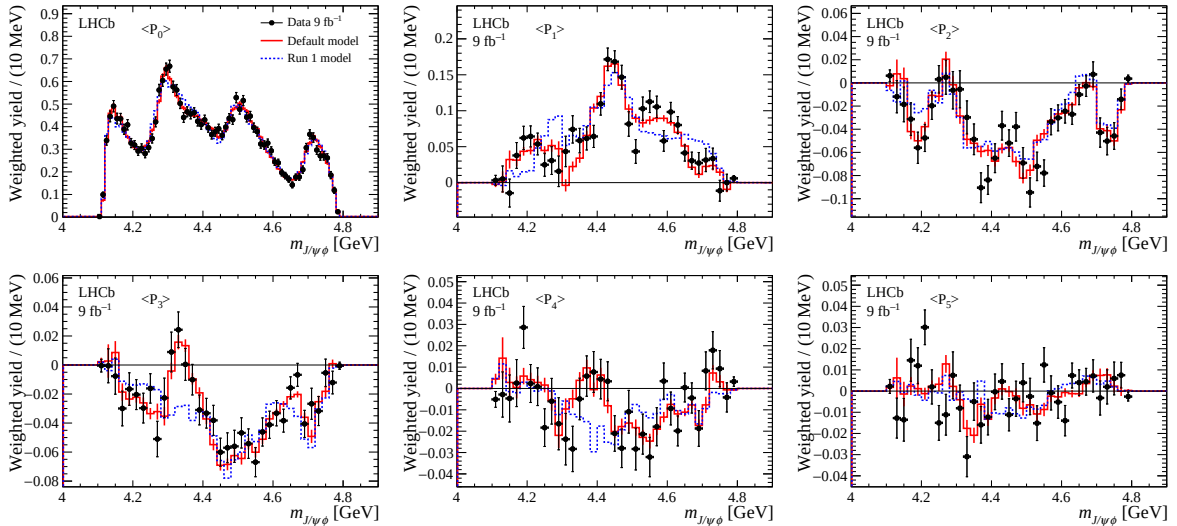


Figure 3.30 Angular moments of $J/\psi \phi$ helicity angle as a function of $m_{J/\psi \phi}$ compared between the background subtracted data, and PDFs from run1 model (blue dashed) and the nominal model (red solid). The χ^2/nbin for the fit of nominal model (run1 model) are 83(136)/69, 59(117)/35, 48(89)/35, 33(89)/35, 42(81)/35, 55(65)/35 for order from 0 to 5, respectively.

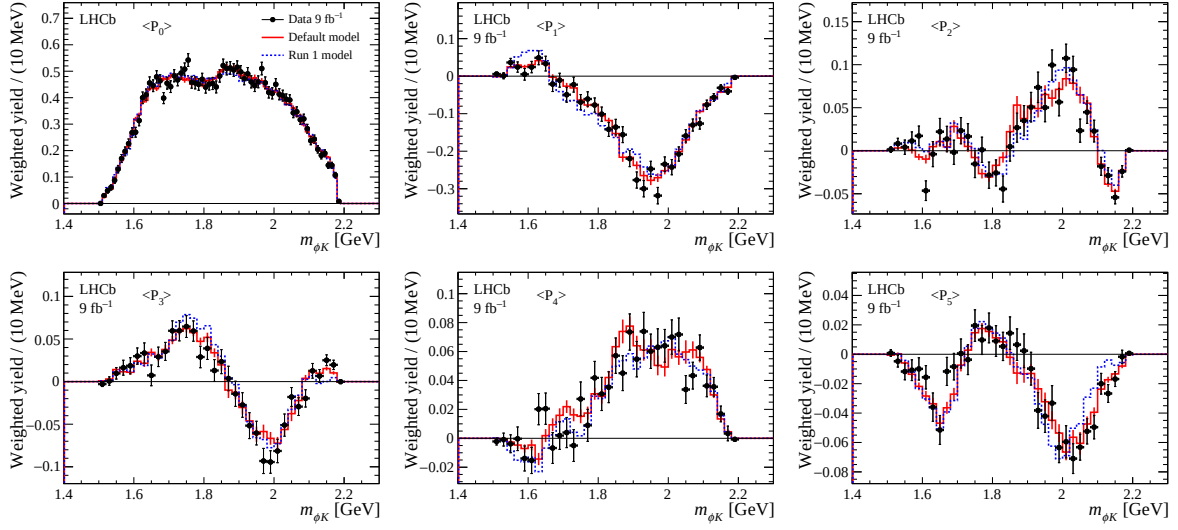


Figure 3.31 Signal angular moments of ϕK helicity angle as a function of $m_{\phi K}$ compared between the background subtracted data, and PDFs from run1 model (blue dashed) and the nominal model (red solid). The χ^2/nbin for the fit of nominal model (run1 model) are 66(79)/69, 37(86)/35, 49(48)/35, 35(59)/35, 46(44)/35, 36(74)/35 for order from 0 to 5, respectively.

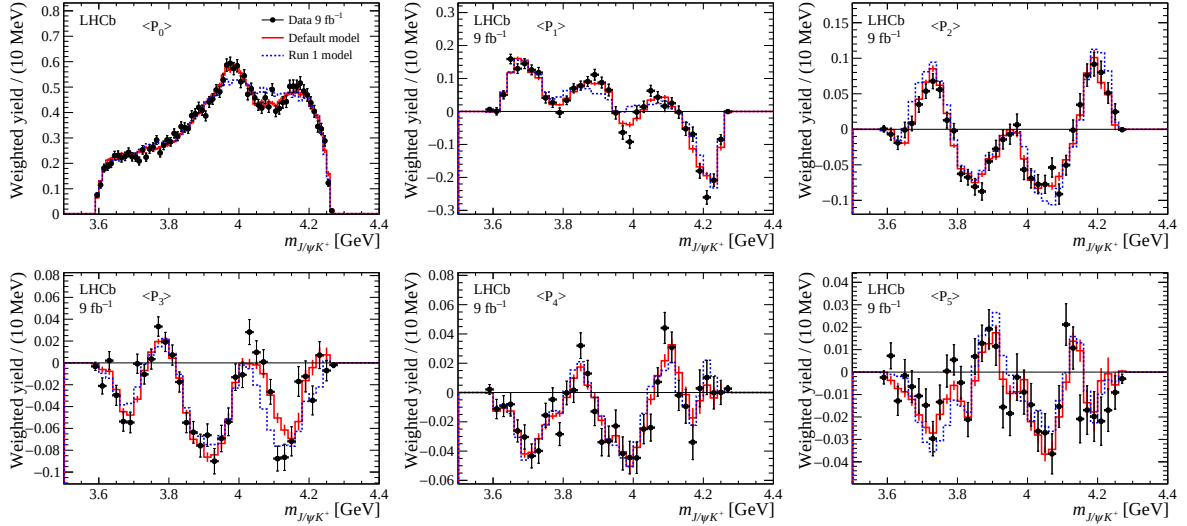


Figure 3.32 Angular moments of $J/\psi K$ helicity angle as a function of $m_{\phi K}$ compared between the background subtracted data, and PDFs from run1 model (blue dashed) and the nominal model (red solid). The χ^2/nbin for the fit of nominal model (run1 model) are 77(138)/69, 50(108)/35, 30(75)/35, 60(81)/35, 31(41)/35, 44(68)/35 for order from 0 to 5, respectively.

3.9 Comparison with BESIII

The BESIII experiment reported observation of the threshold structure in the $D_s^- D^{*0} + D_s^{*-} D^0$ mass distribution^[184]. When interpreted as a resonance, called $Z_{cs}(3985)^-$, its mass $3982.5^{+1.8}_{-2.6}(\text{stat}) \pm 2.1(\text{syst})$ MeV is consistent with the $1^+ Z_{cs}(4000)^+$ state observed in this analysis, but with significantly narrower width $12.8^{+5.3}_{-4.4}(\text{stat}) \pm 3.0(\text{syst})$ MeV. When fixing the mass and width of this state to the nominal BESIII result in the amplitude fit to our data, twice the log-likelihood is worse by 160 units. The narrower width is also not supported by an alternative Flatté model with parameters obtained from our data. Therefore, there is no evidence that the $Z_{cs}(4000)^+$ state observed here is as same as the $Z_{cs}(3985)^-$ state observed by BESIII.

3.10 Systematic uncertainties

Our main results are the properties and fit fractions of the resonances in the nominal model, as well as the significance of the resonances and of their J^P assignments. Systematic uncertainties are assessed for these quantities in our nominal model. All systematic uncertainties are summarized in Tabs. 3.17 to 3.23. For the significance values, smallest one among these systematic studies is used for the final result. We consider the following systematic sources:

- Uncertainties due to the fixed masses and widths of known K^* resonances. To evaluate them, we perform fit with these masses and widths as free parameters but with Gaussian constraints on them to the values listed in Tab. 3.10. (Label “Fixed M&W”).
- Simulation efficiency. The variable χ^2_{IP} of B is not well modelled in simulation, thus we smear it in the simulation sample to match the data, and repeat the nominal fit. This variation is labelled as “ χ^2_{IP} smear” in the tables of the systematic variations.
- We explore uncertainty in the background model by varying the B^+ mass sidebands used to derive the background PDF. The alternative choice like as left sideband of right sideband is used to parameterize the background PDF. The differences are taken as systematic uncertainties (Label “sideband”).
- The systematic uncertainty in the background fraction (β) is estimated by varying the background shape in the B mass fit, from the second order polynomial function to an exponential function: $\beta = 4.02 \pm 0.08 \pm 0.22\%$. We then vary it in the amplitude fit within its total uncertainty. (Label “ β ”).
- The L0 trigger efficiency is not modelled well in simulation sample. We change the

L0 trigger requirement from L0 Global Decision to TOS on L0 Muon or Dimuon Decision. The variations of fit results are taken as systematic uncertainties. (Label "L0 Trigger").

- We re-calibrated ProbNN k from PIDGen method, which is a set of scripts within PIDCalib package^[201] to generate PID variable that looks like in data. While the nominal calibration is performed using PIDcorr approach, which constructs a function that transforms simulated PID response such that it matches data, as mentioned in Sec. 3.2. The differences are taken as systematic uncertainties from the PID calibration process. (Label "PID corr").

Below are amplitude modelling systematic uncertainties, in which we take the largest variation as final uncertainties.

- Hadron-size parameter in the Blatt-Weisskopf barrier factor (d) is 3 GeV^{-1} in the nominal fit. We vary it between 1.5 and 4.5 GeV^{-1} to evaluate systematic errors. (Label "BW d").
- Uncertainty in selection of amplitude components is considered as follows: 1) the difference between the nominal and extended K^* models, labelled as "Extended"; 2) different parameterizations of the $X(4140)$ state, labelled as "Flatté"; 3) addition of one extra X state of different J from 0 to 2 into the extended model, labelled as "Additional X". None of them has significance of more than 5σ . The most significant one is a $2^- X$ with the mass of 4530 MeV and the width of 70 MeV , giving the largest $\ln \mathcal{L}$ improvement, corresponding to a significance of 4.3σ . 4) $Z(4420) 1^-$ hypothesis. (Label "1⁻ Z").
- Uncertainty due to choice of non-resonant (NR) component in the X sector. We use one NR component for 0^+ , which is set to constant with Blatt-Weisskopf barrier factors. Alternative parameterization is tried, such as 1st order polynomial in $m_{J/\psi\phi}^2$. The improvement is small. The other NR S-wave contributions in X decays, *i.e.* 1^+ or 2^+ , are tried. Their significance are less than 5σ . We take them as systematic uncertainty in modeling of the amplitude. (Label that containing "XNR").
- The smallest allowed orbital angular momentum L is used in Eq. (3.19). As an alternative, we associate the L dependent terms in the numerator (Blatt-Weisskopf factors, p^L, q^L) with each LS coupling and use its corresponding L value. The width term of Breit-Wigner function in Eq. (3.22), still uses the lowest allowed L value, because the width is expected to be dominated by it. From the fit, the higher L contribution(s) are less than 5% in most cases of the X and Z_{cs}^+ resonances. (Label "L

vars").

- The default ϕ mass selection window is ± 15 MeV. We remove candidates in which more than one $K^+ K^-$ pair fall within this window, which removes 3.2% of the events. After these cuts non- ϕ fraction of the selected $B^+ \rightarrow J/\psi K^+ K^- K^+$ events is 2.1% from the previous publication^[39,40]. In our amplitude fits, we neglect this component. To evaluate systematics of this approximation we narrow down the ϕ mass selection window to ± 7 MeV, which reduces the ϕ signal yield by 13%, while decreasing the non- ϕ fraction by a factor of 2. We repeat the fit with the nominal amplitude model. Labelled as “ ϕ win”. Alternately, we fit $K^+ K^-$ mass distribution for a range below 1050 with ϕ and no ϕ components, and generate $sWeights$ ^[218] to subtract no ϕ contribution. The amplitude fit is performed again, difference between this fit and the nominal fit is taken as systematic uncertainty. Labelled as “non ϕ ”.
- We also consider modification of the width in the denominator of the BW function for the K^* resonances. In fact, all K^* states are expected to have sizable widths to the other decay modes, $K\pi$, $K\rho$, $K(892)\pi$, etc. Since ratios of these partial widths to the ϕK partial width are unknown, it is not possible to implement such formulation exactly. However, as a systematic check we have performed a fit in which mass dependence of the width is driven by the lightest decay mode possible, which is $K\pi$ for natural spin-parity resonances, and $K\omega$ for the others. This also changes L_{K^*} value. (Label “ K^* BW”).
- K-Matrix model for overlapping K^* components, instead of the BW sum (see Sec. 3.10.2) (Label “K-Matrix”).
- Change the BW model to Flatté model to describe $Z_{cs}(4000)^+$ in order to investigate possible $D_s D^{*0}$ threshold effects. (Label “Z(4000) Flatté”).
- As an alternative to the nominal background parameterization in the fit, which assumes the factorization in six-dimensions, we use approach of the multi-dimensional moments described in details in Sec. 3.10.3. (Label “BKG factorization”).

We found the largest systematic uncertainty for all resonances comes from the amplitude modelling. For $Z_{cs}(4000)^+$ state, the dominant uncertainty is the $1^- Z_{cs}(4220)^+$ hypothesis in selection of amplitude components.

Table 3.17 Summary of the systematic errors on the parameters of the $K^{*+} \rightarrow \phi K^+$ states with $J^P = 1^+$. All numbers for masses and width are in MeV and fit fraction in %.

Systematic	$K(1^+)$	$K(2^1P_1)$		$K(2^3P_1)$		$K_1(1400)$
	FF	M_0	Γ_0	M_0	Γ_0	FF
Fixed M&W	0.59	0.56	-3.10	-0.31	5.54	0.53
$\chi^2_{\text{IP}} \text{ smear}$	-0.08	1.06	2.62	0.90	-2.80	0.01
Right sideband	-0.86	-0.14	-0.60	-1.92	4.14	-0.67
Left sideband	0.89	-1.31	6.29	2.42	-0.60	0.68
$\beta = 0.043$	-0.05	-0.38	0.75	0.15	-0.07	0.01
$\beta = 0.037$	0.09	-0.85	1.32	-0.06	1.83	0.00
L0 Trigger	0.54	0.59	-2.10	-3.10	0.08	0.63
PID corr	-2.59	-2.36	-10.44	6.44	16.02	-1.36
ϕ win 7	-5.19	-8.73	54.82	73.12	99.12	-3.96
non ϕ	-4.70	-2.24	51.09	63.94	118.36	-2.51
Poly $m(XNR)$	-2.29	-13.82	68.40	122.19	317.14	0.21
$XNR(1^+)$	-4.51	-9.56	49.29	-43.49	-84.37	0.74
$XNR(2^+)$	5.14	11.37	23.26	-33.74	51.54	0.50
BW d=1.5	2.87	1.24	18.21	-10.98	14.26	1.16
BW d=4.5	-1.42	-1.50	1.76	4.21	4.64	-0.77
L vars	-4.18	5.12	78.89	71.30	148.42	-0.87
Flatté	-0.27	-6.16	-0.33	10.02	6.83	0.47
Extended	-2.13	-45.36	230.20	-29.18	-118.00	-0.29
Additional X	-1.12	-4.67	4.32	11.51	3.43	-0.48
1^-Z	-14.50	1.72	4.73	18.76	165.40	-10.77
K^*BW	0.01	-10.69	-9.48	5.49	17.41	-1.15
K-Matrix	-6.45	14.53	30.27	80.71	-155.88	-7.33
Z(4000) Flatté	0.14	-6.02	4.81	10.16	-1.97	0.14
BKG factorization	-0.31	-7.27	2.42	8.91	-0.01	-0.08
Total Sys.	(-14.80 , +5.99)	(-45.47, +14.86)	(-16.17, +230.57)	(-44.19, +122.44)	(-156.89, +317.6)	(-10.93 , +2.19)

 Table 3.18 Summary of the systematic errors on the parameters of the $K^{*+} \rightarrow \phi K^+$ states with $J^P = 1^-$ and $J^P = 2^-$. All numbers for masses and width are in MeV and fit fraction in %.

Systematic	$K(1^-)$	$K^*(1680)$	$K^*(1410)$	$K(2^-)$
	FF	FF	FF	FF
Fixed M&W	4.69	2.70	1.62	0.16
$\chi^2_{\text{IP}} \text{ smear}$	0.07	-0.08	0.85	0.00
Right sideband	0.30	0.08	0.72	-0.03
Left sideband	-0.34	-0.17	-0.63	-0.58
$\beta = 0.043$	-0.13	-0.08	-0.24	0.01
$\beta = 0.037$	-0.03	-0.06	0.04	-0.04
L0 Trigger	-0.38	0.26	0.29	-0.04
PID corr	0.10	-0.26	0.39	0.10
ϕ win 7	0.77	-1.31	-3.80	-0.63
non ϕ	-6.70	-3.37	-9.67	-0.46
Poly $m(XNR)$	0.70	0.28	1.60	-0.44
$XNR(1^+)$	-18.79	-1.79	-14.20	-0.77
$XNR(2^+)$	0.48	-1.46	2.02	0.29
BW d=1.5	6.15	-4.10	10.68	-0.26
BW d=4.5	-2.75	4.54	-6.65	0.19
L vars	-2.24	-2.88	-2.56	-0.17
Flatté	-1.35	0.49	-0.98	-0.01
Extended	2.12 0	-7.34	-4.23	-0.02
1^-Z	-1.11	-5.82	-2.70	1.94
K^*BW	-3.36	1.50	-16.69	0.56
K-Matrix	8.27	34.35	-16.95	-0.91
Z(4000) Flatté	0.09	-0.11	-0.65	0.08
BKG factorization	-0.62	0.19	-1.70	0.01
Total Sys.	(-19.37, +9.53)	(-7.83 , +34.46)	(-17.08, +10.89)	(-1.10 , +2.03)

Table 3.19 Summary of the systematic errors on the parameters of the $K^{*+} \rightarrow \phi K^+$ states with $J^P = 2^+$ and $J^P = 0^-$. All numbers for masses and width are in MeV and fit fraction in %.

Systematic	$K_2^+(1980)$			$K(1460)$
	M_0	Γ_0	FF	FF
Fixed M&W	-1.09	20.69	0.06	0.04
$\chi_{\text{IP}}^2 \text{ smear}$	0.61	-2.50	-0.14	0.11
Right sideband	0.67	13.72	0.06	-0.06
Left sideband	-0.05	-16.17	-0.08	0.06
$\beta = 0.043$	0.10	1.16	0.00	-0.03
$\beta = 0.037$	0.69	-1.09	-0.01	0.06
L0 Trigger	-2.33	-7.87	-0.22	0.30
PID corr	-3.89	21.26	0.00	0.00
ϕ win 7	-14.94	-16.39	0.14	-2.43
non ϕ	-29.18	-93.12	0.01	-3.73
Poly $m(XNR)$	-11.65	40.03	0.39	-1.31
$XNR(1^+)$	-11.50	277.98	-0.51	-1.21
$XNR(2^+)$	18.72	3.83	-0.47	-0.22
BW d=1.5	55.25	196.17	0.01	0.45
BW d=4.5	-21.89	-43.92	0.30	-0.67
L vars	9.73	133.00	0.05	-0.02
Flatté	1.55	-21.41	-0.19	0.06
Extended	193.57	477.92	0.12	0.83
Additional X	-2.02	3.22	-0.02	-0.17
$1^- Z$	-15.04	-21.18	0.56	-0.79
$K^* BW$	-65.44	-9.86	1.01	0.41
K-Matrix	-25.84	5.03	0.11	-0.36
Z(4000) Flatté	-3.06	1.68	0.04	0.00
BKG factorization	-3.34	-3.33	0.02	-0.27
Total Sys.	(-29.57, +193.63)	(-100.35, +479.38)	(-0.58, +0.63)	(-3.75, +0.89)

 Table 3.20 Summary of the systematic errors on the parameters of the $X \rightarrow J/\psi \phi$ states with $J^P = 0^+$. All numbers for masses and width are in MeV and fit fraction in %.

Systematic	$X(0^+)$	$X(4500)$			$X(4700)$			$NR_{J/\psi \phi}$
	FF	M_0	Γ_0	FF	M_0	Γ_0	FF	FF
Fixed M&W	2.42	0.15	-0.03	0.05	-0.39	-0.60	-0.13	3.65
$\chi_{\text{IP}}^2 \text{ smear}$	-0.05	0.27	0.35	0.08	0.12	1.33	0.22	-0.15
Right sideband	-0.59	0.09	0.28	0.03	0.34	0.86	0.13	-0.85
Left sideband	0.71	-0.04	-0.15	0.00	-0.20	-0.98	-0.16	1.09
$\beta = 0.043$	0.18	0.08	0.02	0.04	0.20	-0.02	0.00	0.29
$\beta = 0.037$	-0.11	-0.03	-0.07	-0.04	-0.20	-0.23	-0.06	-0.19
L0 Trigger	-0.17	0.15	-1.83	-0.05	-0.88	-1.73	-0.47	-0.78
PID corr	0.91	-0.10	0.64	0.11	0.65	0.45	0.27	1.60
ϕ win 7	-2.33	0.91	4.77	1.27	8.49	5.96	2.56	-2.43
non ϕ	0.55	1.33	9.23	2.41	15.86	15.44	4.85	4.05
Poly $m(XNR)$	-3.79	-2.91	2.37	0.61	2.38	10.70	4.52	-1.59
$XNR(1^+)$	13.00	2.70	6.91	1.75	4.03	7.68	2.69	17.70
$XNR(2^+)$	-3.11	0.95	-1.06	0.26	-1.08	-0.30	-0.36	-4.63
BW d=1.5	-0.20	0.90	0.48	0.08	0.40	1.17	0.69	-0.53
BW d=4.5	-0.61	-0.62	0.30	0.04	0.68	-0.00	-0.27	-0.51
L vars	-1.03	0.38	3.53	1.22	7.89	12.24	4.18	-1.84
Flatté	0.51	-0.44	0.18	0.04	-0.18	0.26	0.25	0.66
Extended	3.38	2.50	-3.81	0.07	0.47	1.64	0.32	5.15
Additional X	1.10	-1.77	-2.18	-0.06	0.06	1.89	0.52	2.83
$1^- Z$	-5.43	1.41	-6.70	-0.49	-1.21	-4.52	-1.09	-9.09
$K^* BW$	0.76	0.40	-2.03	-0.17	-1.70	1.59	0.11	0.87
K-Matrix	-3.27	0.14	9.84	1.58	5.80	11.23	3.65	-4.03
Z(4000) Flatté	0.32	-0.14	-0.31	-0.07	-0.05	-0.01	0.06	0.37
BKG factorization	0.28	0.61	1.13	0.08	0.63	-2.06	-0.25	0.29
Total Sys.	(-6.09, +13.29)	(-2.93, +2.72)	(-6.99, +10.04)	(-0.52, +2.42)	(-2.10, +15.91)	(-5.27, +15.67)	(-1.26, +4.89)	(-10.06, +18.22)

Table 3.21 Summary of the systematic errors on the parameters of the $X \rightarrow J/\psi \phi$ states with $J^P = 1^+$. All numbers for masses and width are in MeV and fit fraction in %.

Systematic	$X(1^+)$	$X(4140)$			$X(4274)$			$X(4685)$		
	FF	M_0	Γ_0	FF	M_0	Γ_0	FF	M_0	Γ_0	FF
Fixed M&W	2.02	0.12	5.76	1.64	0.26	0.03	0.03	-0.14	2.72	0.25
$\chi^2_{\text{IP}} \text{ smear}$	0.13	2.05	-0.12	0.06	0.76	0.65	0.00	-0.53	1.11	0.12
Right sideband	-0.09	0.14	1.07	0.19	-0.13	-0.01	0.00	-0.13	1.07	-0.13
Left sideband	-0.02	-0.45	-3.51	-0.42	-0.01	-0.17	-0.01	-0.09	-2.21	0.09
$\beta = 0.043$	-0.16	-0.12	-1.45	-0.12	-0.11	-0.10	0.01	0.01	-0.70	-0.09
$\beta = 0.037$	0.00	-0.02	-0.24	-0.07	0.03	0.03	-0.01	-0.33	0.21	0.03
L0 Trigger	0.25	3.40	-3.78	0.01	0.66	0.18	0.04	-0.58	1.12	0.11
PID corr	-1.86	9.65	-5.07	-1.57	0.52	0.83	0.02	-0.82	-4.42	-0.26
ϕ win 7	-3.65	0.30	-22.51	-3.05	-2.07	-2.83	-0.11	8.60	-26.60	-1.17
non ϕ	-6.93	1.94	-45.90	-5.75	-5.18	-3.77	-0.22	12.40	-39.80	-1.80
Poly $m(XNR)$	-4.03	6.31	-10.17	-2.53	-2.06	-2.45	-0.27	4.26	-22.07	-1.28
$XNR(1^+)$	-9.51	-33.65	-3.63	18.32	0.15	4.36	0.78	-15.72	35.54	3.84
$XNR(2^+)$	3.31	-0.90	-6.99	2.43	0.75	-0.02	0.08	1.88	-6.87	-0.03
BW d=1.5	6.12	-5.67	16.37	3.55	1.06	1.51	0.14	0.29	1.55	2.14
BW d=4.5	-3.02	6.10	1.30	-1.05	-0.05	-0.46	-0.08	0.06	-3.53	-1.06
L vars	-3.79	0.30	-38.61	-3.00	-4.03	-3.38	-0.19	2.45	-24.33	-1.48
Flatté	-2.30			0.22	0.25	4.26	0.61	-3.77	15.14	1.37
Extended	5.15	-10.87	-26.02	3.55	-1.94	-0.15	0.20	-3.61	-6.53	-0.94
Additional X	-0.66	-0.64	-4.77	-0.74	-1.12	-0.07	-0.06	0.74	-3.11	-0.18
1^-Z	-0.48	-1.84	-33.39	0.50	0.63	0.48	-0.24	-9.41	-5.60	-1.52
K^*BW	-5.72	14.57	2.72	-3.30	-0.31	-1.44	-0.16	-0.06	-8.09	-0.82
K-Matrix	0.17	8.69	-7.91	-0.23	0.06	-3.07	-0.25	4.10	-11.95	-0.06
Z(4000) Flatté	0.03	1.27	3.82	0.19	0.09	0.29	0.00	-0.85	2.79	0.18
BKG factorization	-0.76	1.08	1.48	-0.26	-0.18	0.14	0.01	-1.04	-1.72	-0.15
Total Sys.	(-9.90 , +7.46)	(-35.23, +17.93)	(-46.86, +18.89)	(-6.20, +18.47)	(-5.31 , +1.58)	(-3.92 , +4.49)	(-0.28 , +0.78)	(-15.77, +12.46)	(-40.25, +36.04)	(-1.85 , +3.86)

 Table 3.22 Summary of the systematic errors on the parameters of the $X \rightarrow J/\psi \phi$ states with $J^P = 1^-$ and $J^P = 2^-$. All numbers for masses and width are in MeV and fit fraction in %.

Systematic	$X(4630)$			$X(4150)$		
	M_0	Γ_0	FF	M_0	Γ_0	FF
Fixed M&W	0.72	-1.38	0.06	1.10	5.49	0.10
$\chi^2_{\text{IP}} \text{ smear}$	1.38	-0.25	-0.06	-3.86	2.81	0.09
Right sideband	-6.71	6.88	0.03	1.92	0.64	0.02
Left sideband	6.97	-11.30	-0.08	-2.95	-0.96	-0.02
$\beta = 0.043$	-0.62	0.62	0.00	-0.52	-0.04	0.00
$\beta = 0.037$	1.37	-1.79	0.00	-0.20	-0.24	-0.01
L0 Trigger	5.29	-11.86	-0.03	-10.69	15.05	0.14
PID corr	-8.33	6.88	0.08	5.87	-6.66	-0.10
ϕ win 7	-97.98	-49.06	-0.95	8.65	-9.15	-0.36
non ϕ	-101.91	-48.19	-0.66	-6.87	8.03	-0.28
Poly $m(XNR)$	-104.91	-63.91	-1.23	21.50	-12.76	-0.41
$XNR(1^+)$	-93.58	-33.65	-1.45	14.79	2.62	-0.05
$XNR(2^+)$	11.71	22.79	0.24	-20.93	47.18	0.34
BW d=1.5	11.47	-9.22	-0.46	-20.75	9.62	0.42
BW d=4.5	-10.96	4.57	0.24	19.55	-10.57	-0.32
L min sys	-108.92	-62.51	-1.29	-17.99	8.94	-0.04
Flatté	0.59	0.31	-0.15	10.54	-18.08	-0.29
Extended	-14.84	18.05	0.10	-30.11	56.48	0.81
Additional X	-3.75	2.36	-0.07	-0.83	-1.03	-0.02
1^-Z	24.01	132.82	2.94	29.42	-23.92	-0.97
K^*BW	0.15	17.14	0.28	27.38	-22.18	-0.63
K-Matrix	-104.00	-70.16	-1.29	-11.93	28.62	0.71
Z(4000) Flatté	-4.27	4.03	0.08	2.85	-1.41	-0.05
BKG factorization	-11.67	6.63	-0.23	3.70	-2.77	-0.14
Tot.	(-109.81, +18.23)	(-72.74, +134.20)	(-1.46 , +2.94)	(-32.93, +32.30)	(-29.71, +59.16)	(-0.99 , +0.84)

Table 3.23 Summary of the systematic errors on the parameters of the $Z \rightarrow J/\psi K^+$ states with $J^P = 1^+$. All numbers for masses and width are in MeV and fit fraction in %.

Systematic	$Z(1^+)$	$Z(4000)$			$Z(4220)$		
	FF	M_0	Γ_0	FF	M_0	Γ_0	FF
Fixed M&W	-3.34	-0.22	-3.60	-0.83	-2.28	-15.58	-1.67
$\chi^2_{\text{JP}} \text{ smear}$	1.71	0.21	1.01	0.09	5.15	4.49	1.05
Right sideband	-0.48	0.01	0.58	0.11	-2.25	-3.10	-0.28
Left sideband	0.41	-0.30	-1.16	-0.24	3.56	3.22	0.23
$\beta = 0.043$	-0.37	-0.06	-0.00	0.01	-1.14	-2.22	-0.25
$\beta = 0.037$	0.13	-0.02	0.26	0.02	0.41	0.96	0.05
L0 Trigger	1.77	0.45	0.58	0.19	0.64	13.05	1.20
PID corr	-2.75	-1.06	-1.82	-0.69	2.10	4.93	-1.67
ϕ win	-7.57	-4.71	-23.91	-2.75	6.10	-24.54	-3.38
non ϕ	-8.43	-2.87	-18.39	-1.79	-12.51	-25.80	-4.51
Poly $m(XNR)$	-3.66	-4.24	-16.36	-2.56	16.25	-5.14	-0.74
$XNR(1^+)$	10.10	1.49	-21.25	-2.53	38.13	89.32	9.20
$XNR(2^+)$	5.47	2.16	3.09	1.26	-2.85	62.58	5.16
BW d=1.5	2.22	-0.29	-5.27	-0.58	7.60	18.83	1.85
BW d=4.5	-1.07	0.08	1.81	0.04	-1.46	-8.52	-0.50
L vars	-2.47	2.75	-3.19	-1.18	-2.71	-55.64	-1.31
Flatté	2.29	0.52	-2.80	-0.45	11.99	33.00	2.01
Extended	-9.92	-2.35	-6.66	-1.16	-22.73	-61.76	-5.33
Additional X	-1.54	-0.68	2.07	0.30	-4.79	-11.37	-1.28
$1^- Z$		-14.00	-21.09	-3.46			
$K^* BW$	-5.98	0.08	-0.66	-0.32	-13.37	-29.68	-2.82
K-Matrix	-2.83	-3.75	-20.80	-2.85	8.10	-8.14	-0.46
$Z(4000)$ Flatté	-0.78	0.18	51.91	2.83	-17.81	-10.97	1.00
BKG factorization	-0.46	0.10	-0.32	-0.12	-0.91	0.36	-0.27
Tot.	(-11.11 , +11.28)	(-14.05 , +3.12)	(-24.31, +52.10)	(-3.07, +3.05)	(-23.92, +38.85)	(-65.55, +91.98)	(-6.06 , +9.64)

3.10.1 Flatté function for the $X(4140)$ and $Z_{cs}(4000)^+$ components

Nevertheless, only $X(4140)$ and $Z_{cs}(4000)$ peaks are not far from the coupled-channel thresholds of right J^P values in S-wave (higher angular momenta are suppressed for threshold effects). In this section, possible impact of these thresholds is investigated.

Since $X(4140)$ peaks near the $J/\psi\phi$ kinematic threshold, decays to lighter particle combinations can have a significant impact on its line shape. As a 1^+ state $X(4140)$ should easily decay via S-wave to $D_s^{*\pm}D_s^\mp$ which has the lower mass threshold than $J/\psi\phi$. As an alternative to our nominal fit, in which mass dependence of $X(4140)$ width is neglected, in this section, mass-dependent width with both channels included is explored. The Breit-Wigner formula $BW(m)$ (see Eq. (3.21)) is replaced by the Flatté model:

$$\text{Flatte}_X(m|M_0, g_{J/\psi\phi}, g_{D_s^*D_s}) = \frac{1}{M_0^2 - m^2 - iM_0(g_{J/\psi\phi}\rho_{J/\psi\phi} + g_{D_s^*D_s}\rho_{D_s^*D_s})}, \quad (3.93)$$

where the parameters g_i are the $X(4140)$ coupling constants to channel i , the phase-space factors $\rho_i = 2p_i/m$ are dependent on $m \equiv m_{J/\psi\phi}$ and on momentum p_i of one of the daughters in channel i . The $m_{J/\psi\phi}$ mass projections of the fits with $X(4140)$ in the Flatté parameterizations for different models of the other components are shown in Fig. 3.33. The fit parameters are given in Tab. 3.24. In term of fit likelihood, Flatté parameterization is better than the constant-width Breit-Wigner function by about $\Delta(2 \ln \mathcal{L}) \sim 20$ when the same resonance model is used. Since in the Flatté model p_i becomes imaginary below the kinematic threshold, the real part of the amplitude pole position is not necessarily equal to M_0 . The pole position $m_{\text{pole}} \equiv M_{\text{pole}} - i\Gamma_{\text{pole}}/2$ is obtained by solving the equation that the denominator in Eq. (3.93) becomes equal to zero when $m = m_{\text{pole}}$. The results are also reported in Tab. 3.24. The pole of Flatté fit corresponds to the 4th Riemann sheet. The $Z_{cs}(4220)$ with $J^P = 1^-$ and 1^+ hypotheses give similar fit likelihood and 2D χ^2 on Dalitz-plot $m_{J/\psi\phi}^2$ and $m_{J/\psi K}^2$ plane, and the fit results with $Z_{cs}(4220)$ in the 1^- hypothesis are also shown. In all Flatté fits, the coupling of $X(4140)$ to the coupled-channel is smaller than to the $J/\psi\phi$ and is insignificant.

The Flatté $X(4140)$ lineshape, moduli of Eq. (3.93) squared, are compared in Fig. 3.34. Without the phase-space factor of $X \rightarrow J/\psi\phi$ decays, the lineshapes peak at the threshold. Similar results are obtained with the Breit-Wigner parameterization using constant width.

Using Flatté for $Z_{cs}(4000)^+$ is also investigated since it peaks close to the mass threshold of $D_s^+\bar{D}^{*0}$ and $D_s^{*+}\bar{D}^0$, assuming the same couplings for the two open charm channels. Also note that when $m_{J/\psi K}$ is below the mass threshold of one of the coupled channels, the channel's phase space factor becomes imaginary. The fit is slightly better, $\Delta(-2 \ln \mathcal{L}) = 2.8$, but the improvement

Table 3.24 Comparison for $X(4140)$ resonance with Flatté and constant-width Breit-Wigner functions. The numbers between horizontal bands, the first (second), correspond to $Z(4220)$ with $J^P = 1^{-(+)}$. Also shown are the $\ln \mathcal{L}$ and 2D χ^2 of Dalitz-plot distribution in the $m_{J/\psi \phi}^2$ vs. $m_{J/\psi K}^2$ plane with 1018 adaptive bins.

Flatté	M_0 (MeV)	$g_{J/\psi \phi}$ (GeV)	$g_{D_s^* D_s} / g_{J/\psi \phi}$	Pole (MeV)	FF (%)	$\ln \mathcal{L}$	2D χ^2
Nominal	4202 ± 18	1.44 ± 0.36	$0.05^{+0.40}_{-0.05}$	$4092 - 90i$	19.1	5012.0	1022
Nominal	4223 ± 29	1.61 ± 0.48	$0.06^{+0.40}_{-0.06}$	$4082 - 113i$	17.7	5013.1	1039
Extended	4216 ± 23	1.46 ± 0.44	$0.10^{+0.40}_{-0.10}$	$4089 - 108i$	35.3	5080.1	1001
Extended	4187 ± 22	1.06 ± 0.49	$0.34^{+0.40}_{-0.28}$	$4084 - 83i$	21.0	5074.4	1027
BW	M_0 (MeV)	Γ (GeV)		Pole (MeV)	FF (%)	$\ln \mathcal{L}$	2D χ^2
Nominal	4116 ± 10	130 ± 21		$4116 - 65i$	18.0	5001.6	1028
Nominal	4118 ± 14	162 ± 22		$4118 - 81i$	17.4	5004.6	1044
Extended	4107 ± 12	168 ± 21		$4107 - 84i$	43.2	5070.4	1004
Extended	4108 ± 12	137 ± 22		$4108 - 69i$	21.0	5068.7	1029

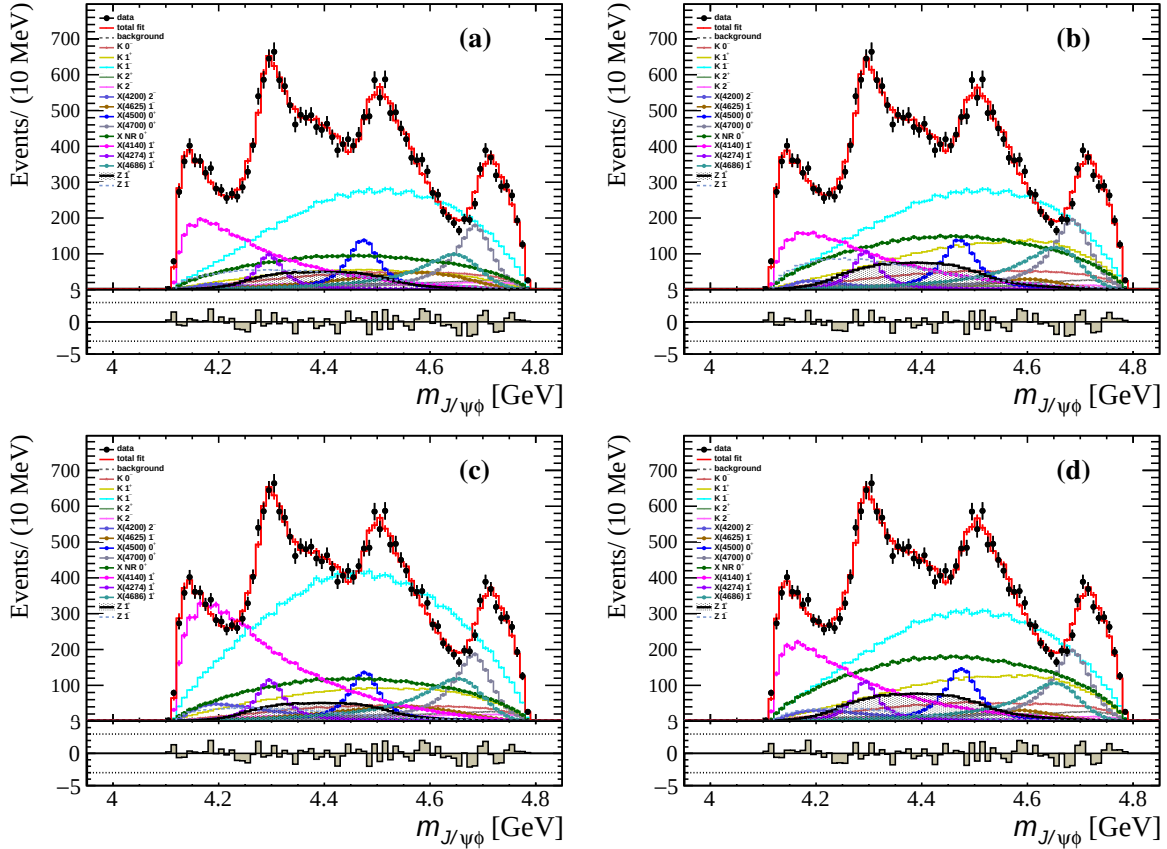
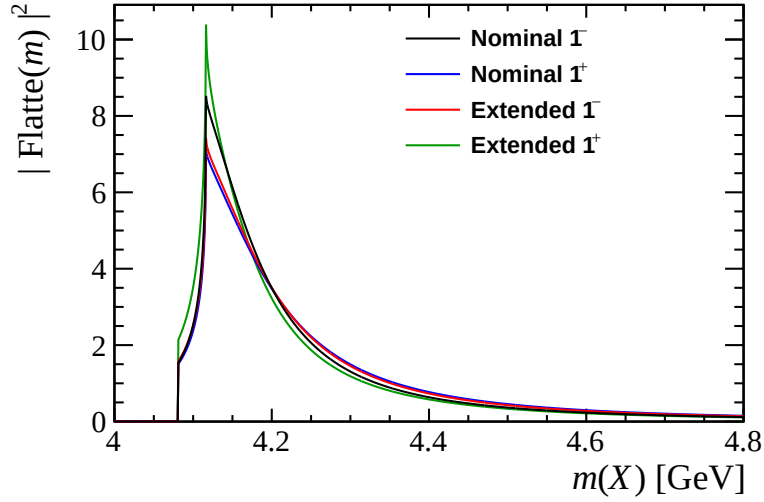
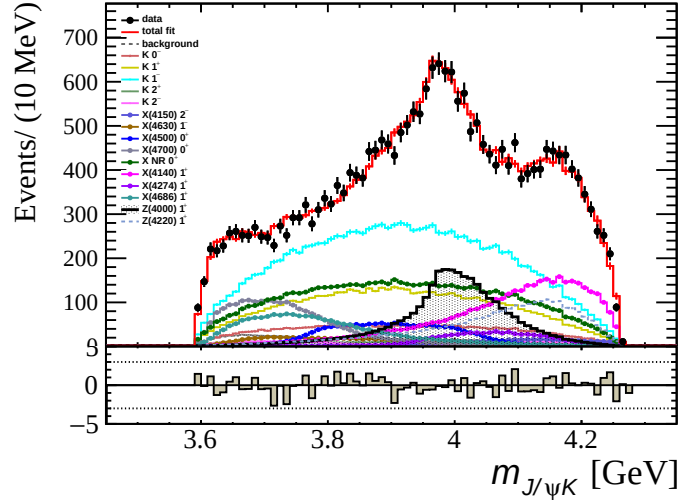


Figure 3.33 Projections of $m_{J/\psi \phi}$ from fits of Flatté function for describing $X(4140)$ in different resonance models (a) nominal K and $1^- Z(4220)$, (b) nominal K and $1^+ Z_{CS}(4220)$, (c) extended K and $1^- Z_{CS}(4220)$, (d) extended K and $1^+ Z(4220)$.


 Figure 3.34 Lineshapes of Flatté parameterization of the $X(4140)$ resonance.

is not significant given that there is one addition free parameter in this model. The fit results are $M_0^F = 4029 \pm 22$ MeV, $g_{J/\psi K^+} = 0.38 \pm 0.07$ GeV, $(g_{D_s^+ \bar{D}^{*0}} + g_{D_s^{*+} \bar{D}^0})/g_{J/\psi K^+} = 0.8 \pm 0.6$.


 Figure 3.35 Projections onto $m_{J/\psi K}$ from the fit with the Flatté function for describing $Z_{cs}(4000)^+$.

While the coupling to $J/\psi \phi$ is significant, the coupling to the coupled-channels is not. However, within the large errors it is possible that the threshold plays an important role. A pole for $Z_{cs}(4000)^+$ in Flatté representation is found to be $M - i\Gamma/2 = (4004 \pm 12 - i91 \pm 22)$ MeV. The statistical uncertainties are calculated by toy study where the correlations of the three parameters in Flatté function are taken into account. The projection of this fit onto $m_{J/\psi K}$ is shown in Fig. 3.35.

3.10.2 K-Matrix model for K^* components

Since many K^* resonances with the same J^P are overlapping (1^+ , 1^- and 2^- waves), the BW sum may not be their best representation. As an alternative, we use a simplified K-Matrix formula to describe them. The BW($m_{K\phi}|M_0^{K^*}, \Gamma_0^{K^*}$) term in Eq. (3.19) is replaced by:

$$RKM_n(m|M_{0n}, \Gamma_{0n}) = \frac{\frac{1}{M_{0n}^2 - m^2}}{1 - i(\sum_j \frac{M_{0j}\Gamma_{0j}(m)}{M_{0j}^2 - m^2} + f_{sc} \cdot \rho(m))}, \quad (3.94)$$

where the denominator sums over the same J^P K^* resonances, and f_{sc} accounts for a possible non-resonance contribution, phase space factor $\rho(m) = 2p/m$. For the non-resonance function, if including it, f_{sc} is a free parameter in the fit, and the numerator in Eq. (3.94) is 1 instead of $1/(M_{0n}^2 - m^2)$. Here we set $f_{sc} = 0$, as the non-resonances are modelled by tail of lower mass K^* s.

We use the same K^* resonant masses and widths as those used in the nominal fit. The conclusions for the exotic states are unchanged, and the results are taken as the systematic uncertainty. The projections of this fit onto masses are shown in Fig. 3.36.

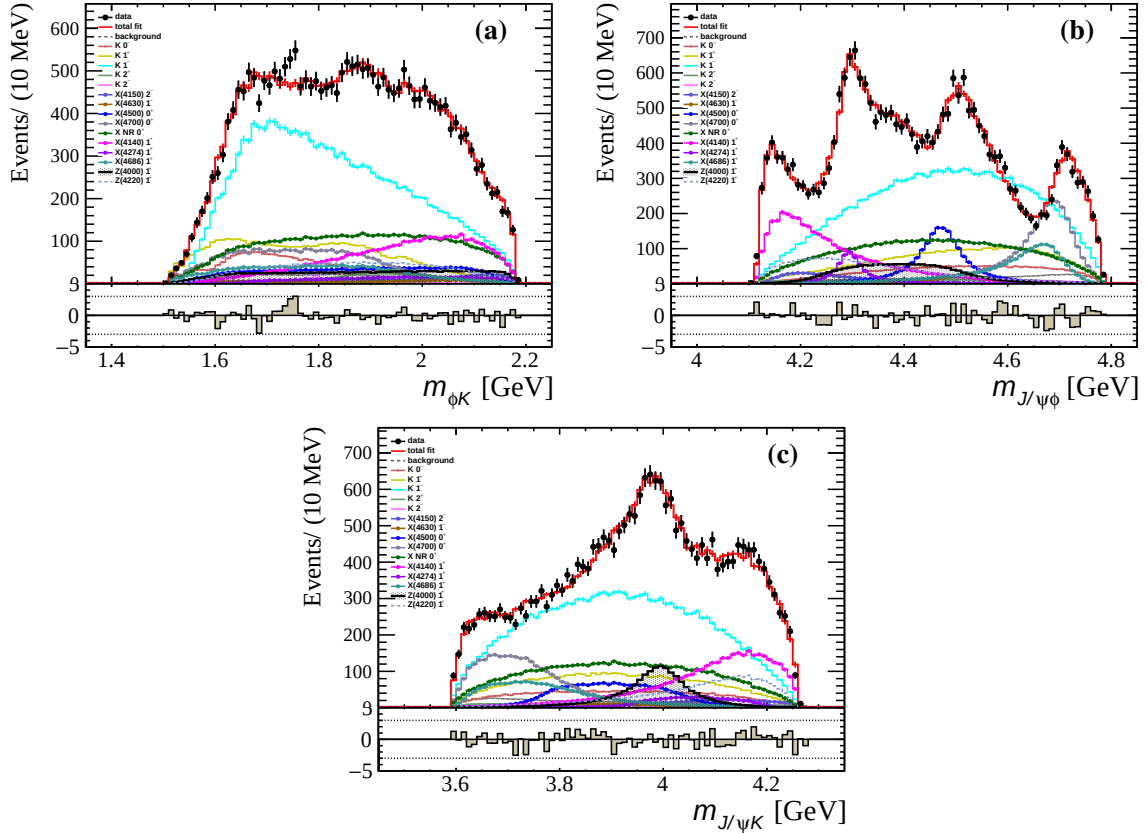


Figure 3.36 Fit projections of (a) $m_{\phi K}$, (b) $m_{J/\psi \phi}$, (c) $m_{J/\psi K}$ from the nominal K+5X+2X+2Z model with K-Matrix for describing the K^* resonances.

A more specific K-Matrix model is performed to check the possible peak in $m(\phi K^+)$ around 1.7 GeV. This K-Matrix model contains two coupling channels, which are used to describe the 2^1P_1 and 2^3P_1 K^* resonances, as the two states overlapping with each other^[1,219,220]. The 2×2 K-Matrix is parameterized as:

$$K_{ba}(s) = \sum_R \frac{g_b^R g_a^R}{M_R^2 - s} + \sum_{i=0}^{N_{b.g.}} b_{ba}^{(i)} s^i \quad (3.95)$$

where, R represents the number of pole, M_R refers to the bare mass of the resonance around 1.7 GeV, s is the square of ϕK^+ invariant mass, g_b^R (g_a^R) denotes the bare coupling of the resonance R to the channel b (a), and the $b_{ba}(i)$ is a matrix to parameterize the non-pole parts of the K-Matrix. We replace the BW to this model to describe 2^1P_1 and 2^3P_1 states, which overlapping with each other. The floating value in the fitting includes resonance pole mass, coupling and physical background parameters. We test this model in the nominal fitting, and the change of log-likelihood is around 10, which is pretty small. Therefore, we cannot prove the existence of a K^* resonance around 1.7 GeV in the $m(\phi K^+)$ distribution. The projections of this fit onto masses are shown in Fig. 3.37.

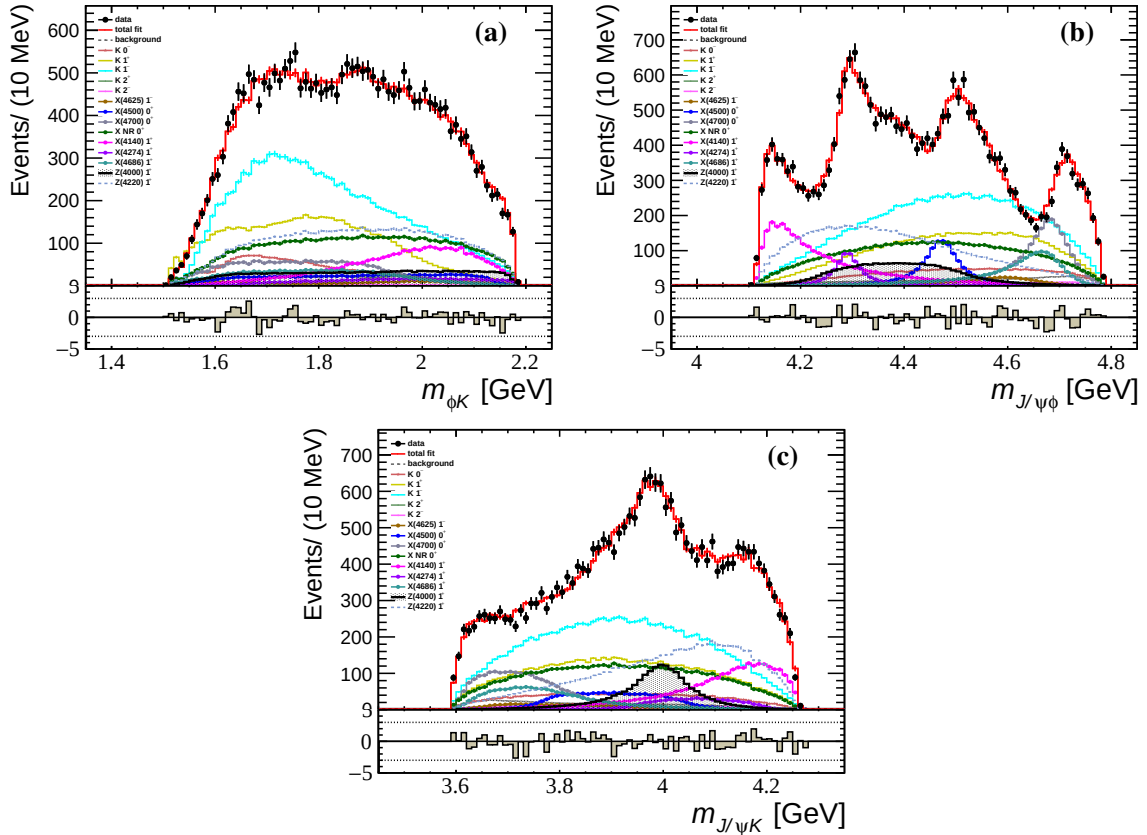


Figure 3.37 Fit projections of (a) $m_{\phi K}$, (b) $m_{J/\psi \phi}$, (c) $m_{J/\psi K}$ from the nominal 2K(K-Matrix)+7K+5X+2X+2Z model.

3.10.3 Check of the background PDF on B^+ mass sidebands

As described before, the 6D background PDF is constructed using the B^+ mass sideband events, assuming that it can be factorized into a product of 2D functions of $m_{\phi K}$ and decay angles in the K^* decay chain. Such constructed 6D background density is then divided by similarly constructed 6D selection efficiency parametrization, worked out using the simulated events, and then added to the signal matrix element squared in a proportion constrained by the background fraction in the real data sample. To cross-check how well such background PDF describes the sideband distributions, we show its projections onto the masses and decay angles from all three chains superimposed on the sideband data in Fig. 3.38. The projections are achieved by weighting the phase-space MC events, passed through the selection criteria, by the parameterized background PDF. Since the X and Z_{cs}^+ decay chain variables are not used in the parameterization of the background PDF, the cross-check is non-trivial. The cross-check also depends on the cancellation of the parameterized efficiency in the background PDF by the unparameterized efficiency encoded in the MC events, which is a part of our *cFit* approach. The agreement is fairly reasonable.

Among the systematic variations, we include an alternative way to parameterize the background PDF. Instead of assuming factorization, we use decomposition of the background density and of the efficiency into their multi-dimensional moments in the K^* decay chain variables. For examples, the efficiency parametrization becomes:

$$\begin{aligned} \epsilon(m_{\phi K}, \Omega) = & \sum_{a,b,c,d,e,f} \epsilon^{abcdef} \mathcal{P}_a(\cos \theta_{K^*}) Y_{bc}(\cos \theta_{\psi}, \phi_{(\psi K^*)}) \\ & \times \mathcal{P}_d \left(2 \frac{m_{\phi K} - m_{\phi K}^{\min}}{m_{\phi K}^{\max} - m_{\phi K}^{\min}} - 1 \right) \mathcal{P}_e(\cos \theta_{\phi}) \mathcal{P}_f \left(\frac{\phi_{K^+}}{\pi} \right), \end{aligned} \quad (3.96)$$

where $\mathcal{P}$ is Legendre polynomial, Y_{bc} are spherical harmonics, and $m_{\phi K}^{\min}$ and $m_{\phi K}^{\max}$ are the minimum and maximum allowed values for $m_{\phi K}$, respectively. The Y_{bc} functions are complex functions. To ensure that the efficiency function is real, we set $\epsilon^{abcdef} = -\epsilon^{ab(-c)def}$. The values of ϵ^{abcdef} are determined by summing over the fully simulated phase-space events or sideband events

$$\begin{aligned} \epsilon^{abcdef} = & \frac{1}{\sum_i w_i} \sum_i w_i \frac{2a+1}{2} \frac{2d+1}{2} \frac{2e+1}{2} \frac{2f+1}{2} \mathcal{P}_a(\cos \theta_{K^*,i}) Y_{bc}(\cos \theta_{\psi,i}, \phi_{(\psi K^*),i}) \\ & \times \mathcal{P}_d \left(2 \frac{m_{\phi K,i} - m_{\phi K}^{\min}}{m_{\phi K}^{\max} - m_{\phi K}^{\min}} - 1 \right) \mathcal{P}_e(\cos \theta_{\phi,i}) \mathcal{P}_f \left(\frac{\phi_{K^+,i}}{\pi} \right) \frac{1}{g_i}, \end{aligned} \quad (3.97)$$

where the weights w_i^{mc} account for simultaneous weights for p , p_T and multiplicity of B^+

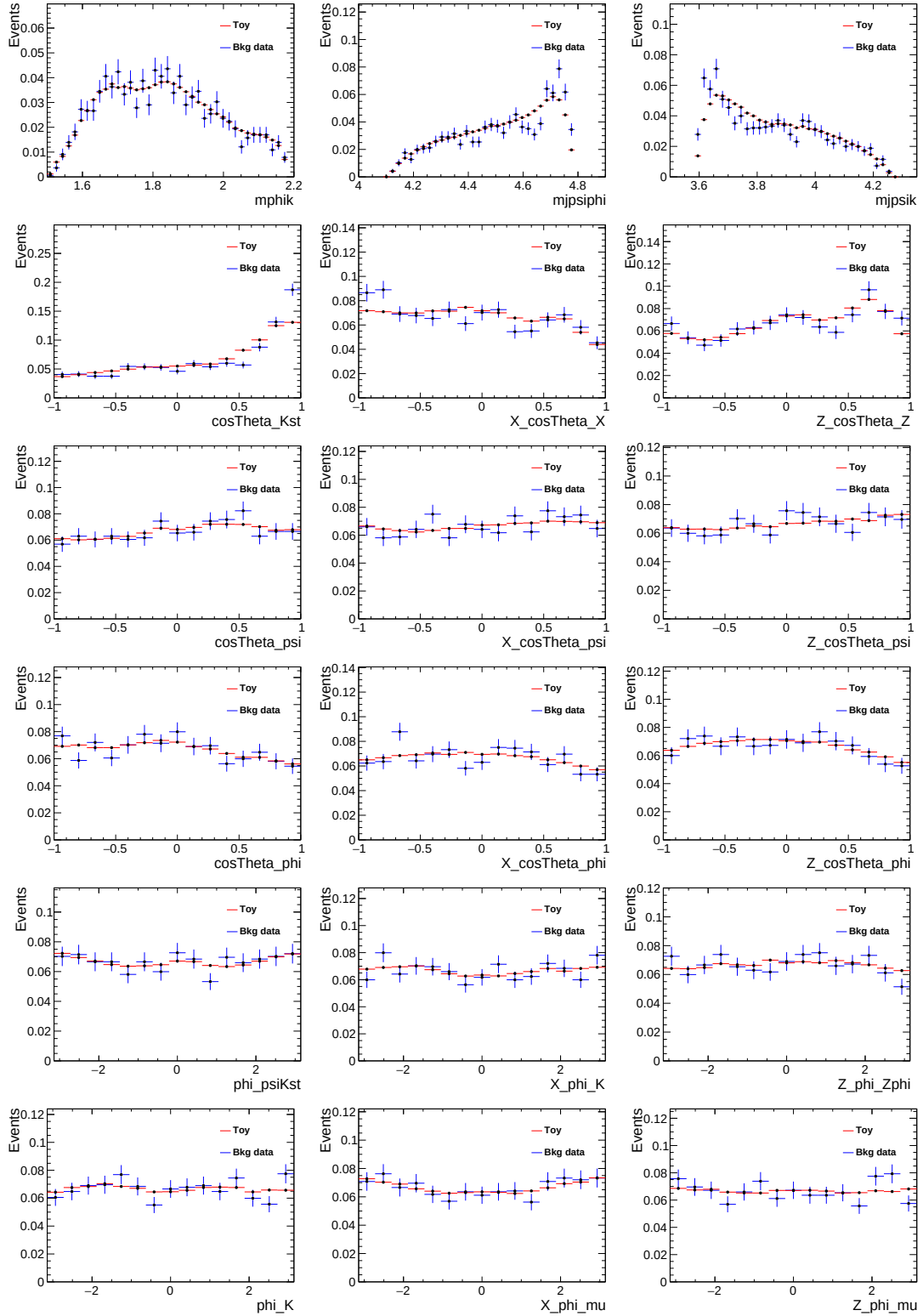


Figure 3.38 Comparison between the parameterized background PDF used in the default $cFit$ (red dots) with the B^+ sideband data (blue dots) for all masses and angles used in nominal fit, for K^* chain (left) X chain (middle) and Z_{cs}^+ chain (right).

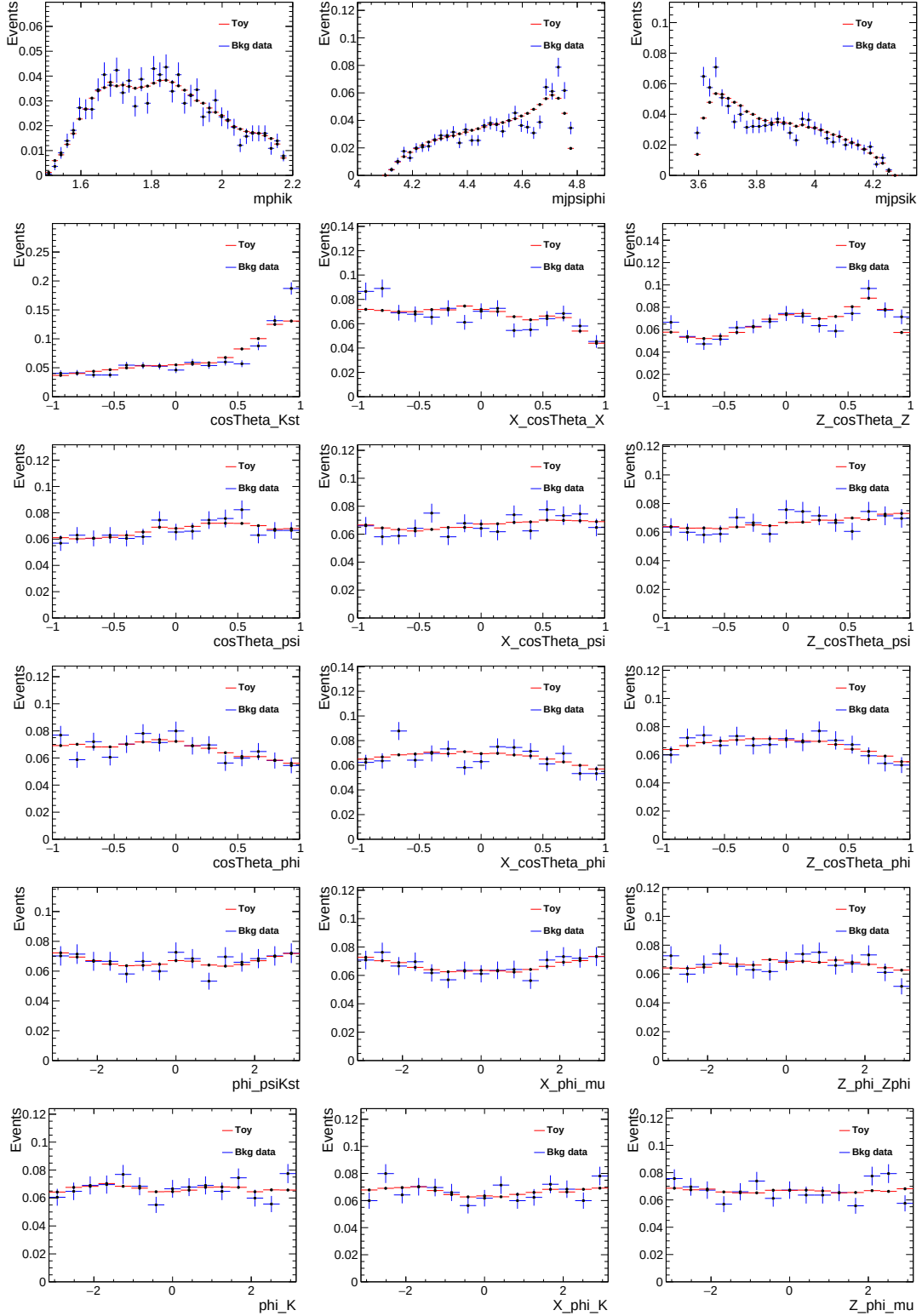


Figure 3.39 Comparison between the alternative parameterized background PDF (red dots) with the B^+ sideband data (blue dots) for all masses and angles used in nominal fit, for K^* chain (left) X chain (middle) and Z_{cs}^+ chain (right).

(for background $w_i^{\text{bkg}} = 1$, and g_i is the value of the phase-space probability density for event i). This approach allows the description of multi-dimensional correlations without assuming factorization. In practice, the sum is over a finite number of terms ($a \leq 10$, $b \leq 4$, $-2 \leq c \leq 2$, $d, e, f \leq 8$) and only coefficients with a statistical significance larger than three standard deviations (σ) from zero are retained. We also show comparison of this parameterization to the sideband data in Fig. 3.39.

3.11 Conclusions

All results obtained from this analysis are summarized in Tab. 3.25. By combining Run 1 and Run 2 data sample, and improving the data selection, we gained a factor of 6 in the $B^+ \rightarrow J/\psi \phi K^+$ signal statistics relatively to the previous amplitude analysis, while obtaining 6 smaller fractional non- B^+ background at the same time. The increased statistical power of the data revealed some inadequacies of the amplitude model developed to describe the previous Run 1 data sample.

The two $J^{PC} = 1^{++}$ ($X(4140)$ and $X(4274)$) and two $J^{PC} = 0^{++}$ ($X(4500)$ and $X(4700)$) states decaying to $J/\psi \phi$ are confirmed together with their quantum number assignments. The new mass and width determination supersede our previously published values^[40].

An additional 1^{++} $X(4685)$ state is observed with relatively narrow width ($\Gamma = 126 \pm 15_{-41}^{+37}$ MeV) at more than 15σ . New partial $J/\psi \phi$ wave $X(4630)$ state is required with at least 6σ significance with $J^{PC} = 1^{-+}$ 3σ better than the $J^{PC} = 2^{-+}$ hypothesis. Other J^{PC} hypotheses are rejected for more than 5σ .

Pattern of masses and quantum numbers is not the one aligning with the expectations for pure charmonium states, thus tetraquark $c\bar{c}s\bar{s}$ structures are likely contributing to the mass spectrum. Direct color binding is more likely to be responsible than meson-meson interactions, because of the rich spectrum of quantum numbers and the decay widths larger than for the narrow $c\bar{c}u\bar{d}$ and $c\bar{c}uud$ states observed near the constituent hadron-hadron mass thresholds.

While broader $c\bar{c}u\bar{d}$ states decaying to $J/\psi \pi^+$ or $\psi' \pi^+$ were observed in $B \rightarrow \psi \pi^+ K$ decays, our data requires for the first time $c\bar{c}u\bar{s}$ states decaying to $J/\psi K^+$. A relatively narrow, $\Gamma = 131 \pm 15_{-26}^{+26}$ MeV, $J^P = 1^+$ $Z_{cs}(4000)^+ \rightarrow J/\psi K^+$ state is observed at $4003 \pm 6_{-14}^{+4}$ MeV, with a significance of more than 15σ . A broad $J^P = 1^+$ $Z_{cs}(4220)^+$ state is also needed at 6σ significance.

Table 3.25 Fit results from the default amplitude model. The significance is evaluated accounting for total (statistical) uncertainties. The listed masses and widths without uncertainties are taken from PDG 1 and are fixed in the fit. The listed world averages of the two K_2 and $K^*(1680)$ resonances do not contain the contributions from the previous LHCb Run 1 results.

Contribution		Significance [$\times\sigma$]	M_0 [MeV]	Γ_0 [MeV]	FF [%]
All $K(1^+)$					$25 \pm 4^{+6}_{-15}$
2^1P_1	$K(1^+)$	4.5 (4.5)	$1861 \pm 10^{+16}_{-46}$	$149 \pm 41^{+231}_{-23}$	
2^3P_1	$K'(1^+)$	4.5 (4.5)	$1911 \pm 37^{+124}_{-48}$	$276 \pm 50^{+319}_{-159}$	
1^3P_1	$K_1(1400)$	9.2 (11)	1403	174	$15 \pm 3^{+3}_{-11}$
All $K(2^-)$					$2.1 \pm 0.4^{+2.0}_{-1.1}$
1^1D_2	$K_2(1770)$	7.9 (8.0)	1773	186	
1^3D_2	$K_2(1820)$	5.8 (5.8)	1816	276	
All $K(1^-)$					$50 \pm 4^{+10}_{-19}$
1^3D_1	$K^*(1680)$	4.7 (13)	1717	322	$14 \pm 2^{+35}_{-8}$
2^3S_1	$K^*(1410)$	7.7 (15)	1414	232	$38 \pm 5^{+11}_{-17}$
$K(2^+)$					
2^3P_2	$K_2^*(1980)$	1.6 (7.4)	$1988 \pm 22^{+194}_{-31}$	$318 \pm 82^{+481}_{-101}$	$2.3 \pm 0.5 \pm 0.7$
$K(0^-)$					
2^1S_0	$K(1460)$	12 (13)	1483	336	$10.2 \pm 1.2^{+1.0}_{-3.8}$
$X(2^-)$					
	$X(4150)$	4.8 (8.7)	$4146 \pm 18 \pm 33$	$135 \pm 28^{+59}_{-30}$	$2.0 \pm 0.5^{+0.8}_{-1.0}$
$X(1^-)$					
	$X(4630)$	5.5 (5.7)	$4626 \pm 16^{+18}_{-110}$	$174 \pm 27^{+134}_{-73}$	$2.6 \pm 0.5^{+2.9}_{-1.5}$
All $X(0^+)$					$20 \pm 5^{+14}_{-7}$
	$X(4500)$	20 (20)	$4474 \pm 3 \pm 3$	$77 \pm 6^{+10}_{-8}$	$5.6 \pm 0.7^{+2.4}_{-0.6}$
	$X(4700)$	17 (18)	$4694 \pm 4^{+16}_{-3}$	$87 \pm 8^{+16}_{-6}$	$8.9 \pm 1.2^{+4.9}_{-1.4}$
	$NR_{J/\psi\phi}$	4.8 (5.7)			$28 \pm 8^{+19}_{-11}$
All $X(1^+)$					$26 \pm 3^{+8}_{-10}$
	$X(4140)$	13 (16)	$4118 \pm 11^{+19}_{-36}$	$162 \pm 21^{+24}_{-49}$	$17 \pm 3^{+19}_{-6}$
	$X(4274)$	18 (18)	$4294 \pm 4^{+3}_{-6}$	$53 \pm 5 \pm 5$	$2.8 \pm 0.5^{+0.8}_{-0.4}$
	$X(4685)$	15 (15)	$4684 \pm 7^{+13}_{-16}$	$126 \pm 15^{+37}_{-41}$	$7.2 \pm 1.0^{+4.0}_{-2.0}$
All $Z_{cs}(1^+)$					$25 \pm 5^{+11}_{-12}$
	$Z_{cs}(4000)^+$	15 (16)	$4003 \pm 6^{+4}_{-14}$	$131 \pm 15 \pm 26$	$9.4 \pm 2.1 \pm 3.4$
	$Z_{cs}(4220)^+$	5.9 (8.4)	$4216 \pm 24^{+43}_{-30}$	$233 \pm 52^{+97}_{-73}$	$10 \pm 4^{+10}_{-7}$

Chapter 4 Studies of hidden-charm pentaquark states

4.1 Motivation

In 2015, LHCb reported significant structures in $J/\psi p$ mass spectrum in $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay, which were the first observations of pentaquark candidates^[124]. Some details have been introduced in Chapter 1. These observations were also checked by model-independent way^[125], where the $J/\psi p$ structure near 4450 MeV was too narrow to be described by $p K^-$ contributions. However, the previous $\Lambda_b^0 \rightarrow J/\psi p K^-$ study was based on the LHCb Run 1 data samples, some properties of the pentaquark states observed are still pending, *e.g.*, the spin and parity. Therefore, we need to revisit this decay to confirm the previous observations and investigate their properties. This analysis presented in this chapter is based on the complete LHCb Run 1 and Run 2 data set.

A full amplitude analysis was not performed in this study. First, the main goal of this analysis is to confirm the previous observations and to search for other possible pentaquark states in the mass spectrum of $J/\psi p$ in $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay, so a full amplitude analysis is not a must. Besides, we met some big computing challenges when tried to perform an amplitude analysis, analogous to the one utilized in previous analysis^[125]. The biggest obstacle is the huge data statistics, which would break the computational model of the fit. Moreover, if we want to examine the contributions from narrow pentaquark states, the mass resolution effects should be included in the fit model, which will lead to a huge amount of computation. Therefore, an amplitude analysis for this decay using Run 1 and Run 2 data sample is pretty difficult currently, we expect to achieve this study in future by performing the fits in GPUs^① platform.

4.2 Selection

This analysis uses all the pp collision data in Run 1 and Run 2 stage, corresponding to an integrated luminosity of 3 fb^{-1} (6 fb^{-1}) in Run 1 (Run 2). The simulation samples of the $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay are generated by PYTHIA^[221] with pp collisions at $\sqrt{s} = 7 \text{ TeV}$, 8 TeV and 13 TeV respectively, with the ratio of the numbers of events roughly one to two between Run 1 and Run 2. Both PYTHIA 6 and PYTHIA 8 generators are used. The Λ_b^0 is forced to decay into the $J/\psi p K^-$ final state with the PHSP model in the simulation sample.

① The graphics processing unit.

4.2.1 Reconstruction and pre-selection

In this analysis the reconstruction of the Λ_b^0 candidate starts from the stripping line `FullDSTDiMuonJpsi2MuMuDetachedLine`^[174] of the stripping reconstruction^②. The selection criteria of the stripping line are summarized in Tab. 3.4 in Chapter 3, which are the same as those used in the $B^+ \rightarrow J/\psi \phi K^+$ analysis.

Comparing with the previous study^[222], the selection criteria are optimized, all differences between the new selection criteria and the old one study are summarized in Tab. 4.1. To select good J/ψ candidates, a tightened PID cut $DLL_{\mu\pi} > 0$ is required for both muon tracks in the final state, and the di-muon invariant mass window is reduced to $[-48, 43]$ MeV around the known J/ψ mass^[1]. The J/ψ mass resolution is around 13 MeV. To reconstruct the Λ_b^0 candidate, the J/ψ candidate is combined with two opposite charged tracks, consistent with pions and protons respectively. This is achieved by PID cuts on the `ProbNNk` variables: `ProbNNk` > 0.2 and `ProbNNp` > 0.2. The two hadronic tracks are required to have large transverse momenta, $p_T > 250$ MeV, inconsistent with originating from the primary vertex (PV), $\chi_{IP}^2 > 9$. The Λ_b^0 candidate is required to have a good vertex fit quality, $\chi^2/ndf < 10$, with a displayed decay vertex, flight distance to primary vertex is larger than 1.5 mm. It should be consistent with being produced from PV by requiring $\chi_{IP}^2 < 25$ and cosine value of the direction angle, `DIRA` > 0.999. And the χ^2 of closet approach distance between p and K^- is smaller than 16. The Λ_b^0 candidates with mass $5.1 < M(J/\psi p K^-) < 6.5$ GeV are selected.

The $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay chain is refitted with two sequential constraints using the `DecayTreeFitter`^[200] to improve the mass resolution: A) the J/ψ candidate mass constrained to its known value^[1] and the Λ_b^0 candidate constrained to originating from the PV; B) the invariant mass of the Λ_b^0 candidate constrained to its know value^[1], with the momenta of the J/ψ , p and K^- daughters re-calculated according to this constraint.

It should be noted that the PID requirements on the hadron tracks are not applied when reconstructing the Λ_b^0 candidates in the simulation sample. The effect, instead, is considered using weights from the PID calibration (`PIDCalib`)^[201]. The kaon track is calibrated using kaon tracks from the D^* tagged $D^0 \rightarrow K^- \pi^+$ decay, while the proton track is calibrated using proton tracks from the $\Lambda_c^+ \rightarrow p K^- \pi^+$ decay.

Events with Λ_b^0 candidates in both the simulation and the data samples are required to fire the trigger sequences^③: at the HLT1 level by either of the `Hlt1DiMuonHighMass`, `Hlt1TrackMuon` and `Hlt1TrackAllL0` for Run 1 sample, while `Hlt1DiMuonHighMass`,

② Details of stripping line are introduced in Chapter 3.

③ Details of trigger line are introduced in Chapter 3.

Hlt1TrackMuon, Hlt1TwoTrackMVA and Hlt1TrackMVA for Run 2 sample; and at the HLT2 level by either of the Hlt2DiMuonDetachedJPsi lines.

Table 4.1 Pre-selections of $\Lambda_b^0 \rightarrow J/\psi p K^-$ candidates between old Run 1 analysis^[124] and this one.

	Quantity	Old Run 1	This analysis
1	All tracks χ^2/ndf	< 4	same
2	Clone track rejection	CloneDist cut	same
3	Ghose rejection	ghost probability < 0.2	same
2	Muon DLL(μ)	$> 0 \& \text{ISMUON}$	same
5	Vertex of J/ψ	$\chi^2 < 16$	same
6	Mass window of J/ψ	$[-48, 43] \text{ MeV}$	same
3	p_T of muon	$> 550 \text{ MeV}$	$> 500 \text{ MeV}$
4	p_T of hadron	$> 250 \text{ MeV}$	same
4	$p_T(p) + p_T(K^-)$	$> 900 \text{ MeV}$	same
7	χ_{VS}^2 of hadron	> 9	same
8	PID of K	$\text{DLL}(K - \pi) > 0$ and $\text{DLL}(p - K) < 3$	$\text{ProbNNk} > 0.2$
9	PID of p	$\text{DLL}(p - \pi) > 10$ and $\text{DLL}(p - K) > 3$	$\text{ProbNNp} > 0.2$
16	DOCA χ^2 of $p K^-$	< 16	same
11	Λ_b^0 vertex $\chi_{\text{vtx}}^2/\text{ndf}$	< 10	same
10	χ_{IP}^2 of Λ_b^0	< 25	same
12	DIRA of Λ_b^0	> 0.999	same
15	Flight distance of Λ_b^0	$> 1.5 \text{ mm}$	same

4.2.2 Reflection veto cuts

To suppress reflection from $B^0 \rightarrow J/\psi \pi^+ K^-$, with the π^+ faking a proton, we veto events in which $m(J/\psi \pi^+ K^-)$ mass is within $\pm 25 \text{ MeV}$ of the nominal B^0 mass^[1] and ProbNNpi for the proton candidate is > 0.2 . Only about 10% in total reflection background is observed when the latter variable is < 0.2 . Therefore, the use of hadron PID cut in the context of the reflection veto cut helps preserving the signal efficiency without compromising the veto effectiveness. This is an improvement over the 2015 data selection^[222].

An even larger source of reflection background is the $B_s^0 \rightarrow J/\psi K^+ K^-$ decay, with the K^+ faking a proton. We removed events with the $m(J/\psi K^+ K^-)$ within $\pm 22 \text{ MeV}$ around the nominal B_s^0 mass^[1] and ProbNNk for the proton candidate greater than 0.2. Since $B_s^0 \rightarrow J/\psi \phi$ decays are the dominant component in this category, and the ϕ meson width is small, we waived the ProbNNk requirement if the $m(K^+ K^-)$ mass is within $\pm 12 \text{ MeV}$ off the nominal ϕ meson mass.

Since our hadron PID pre-selection does not forbid the same J/ψ -track-track combination to pass as $J/\psi p K^-$ and as $J/\psi K^+ \bar{p}$ candidates, we also need to worry about self-reflection back-

ground due to double hadron misidentification. We suppress events if they are within ± 18 MeV off the Λ_b^0 mass^[1] under the swapped proton and kaon mass hypotheses, and $\text{ProbNNk} > 0.2$ ($\text{ProbNNp} > 0.2$) for the proton (kaon) hypotheses, respectively.

There are no obvious reflections from $B_{(s)}^0 \rightarrow J/\psi \pi^+ \pi^-$, $\Lambda_b^0 \rightarrow J/\psi p \pi^-$, and $B_{(s)}^0 \rightarrow J/\psi p p$ decays. The latter is because they would contribute to the region below the $B_{(s)}^0$ masses when one of the protons is assigned the kaon mass. After further background suppression via multivariate discriminant, discussed in the next section, the reflections constitute 16% of the remaining background, or equivalently only 1% of the entries in the signal region. The events removed by the reflection veto cuts do not show any structures in the $m(J/\psi p)$ distribution.

4.2.3 Multivariate analysis

The MVA^[178] is used to further suppress the background. In the MVA training, the signal sample is from the $\Lambda_b^0 \rightarrow J/\psi p K^-$ simulation using the PHSP model, while the background sample is from the upper sideband of the $\Lambda_b^0 \rightarrow J/\psi p K^-$ invariant mass distribution. These samples are randomly split into two halves, one for the training and the other for the test. Different from the previous study^[124], the PID information variables are included in the MVA classification. All variables used are listed in Tab. 4.2: The smaller value of the two muon PID variables ($\text{DLL}_{\mu\pi}$); the smaller value of the χ_{IP}^2 of the two hadron tracks; the arithmetic addition of the p_T of the two hadrons; the cosine of pointing angle to PV (θ_p), flight distance (FD), the p_T , the χ_{IP}^2 and the vertex fit quality ($\chi_{\text{vtx}}^2/\text{ndf}$) of the Λ_b^0 candidate; the PID variables of hadrons. In this procedure, the Run 1 and Run 2 samples are studied separately. The distributions of these variables of Run 2 sample for the signal in the simulation sample and the background in the far-sidebands (150-250 MeV) of the Λ_b^0 peak are shown in Fig. 4.1^④.

The BDTG^[202,203] is used as the default method to discriminate signal and background candidates. The over-training test for the BDTG method is given in the right plot of Fig. 4.2 for Run 2 samples. The BDTG discriminant distributions for the training samples are consistent with those in the test samples. No effect of over-training is observed. The working point of the BDTG value is optimized by maximizing the figure of merit (FoM) of the signal significance, defined as $S/\sqrt{S+B} \times \epsilon_S$. The ϵ_S is the signal efficiency for a specified BDTG cut and is determined from the simulation sample, while S (B) is the number of the signal (background) events in the $\pm 3\sigma$ region around the known Λ_b^0 mass^[1] in the $J/\psi p \pi$ invariant mass distribution of the data. The distribution of the FoM as a function of the BDTG cut value is given, and the value of 0 for Run 1 and -0.4 for Run 2 are determined as the default working points.

④ The distributions of these variables of Run 1 sample are similar.

Table 4.2 Variables used in TMA study.

	Variables
1	χ^2_{vtx} of Λ_b^0
2	$\log(1 - \cos\theta_p)$
3	χ^2_{IP} of Λ_b^0
4	Min χ^2_{IP} of hadron
5	FD of Λ_b^0
6	p_T of Λ_b^0
7	Sum p_T of hadrons
8	Min PID of muon
9	ProbNNp of p
10	ProbNNk of K^-
11	Momentum of p

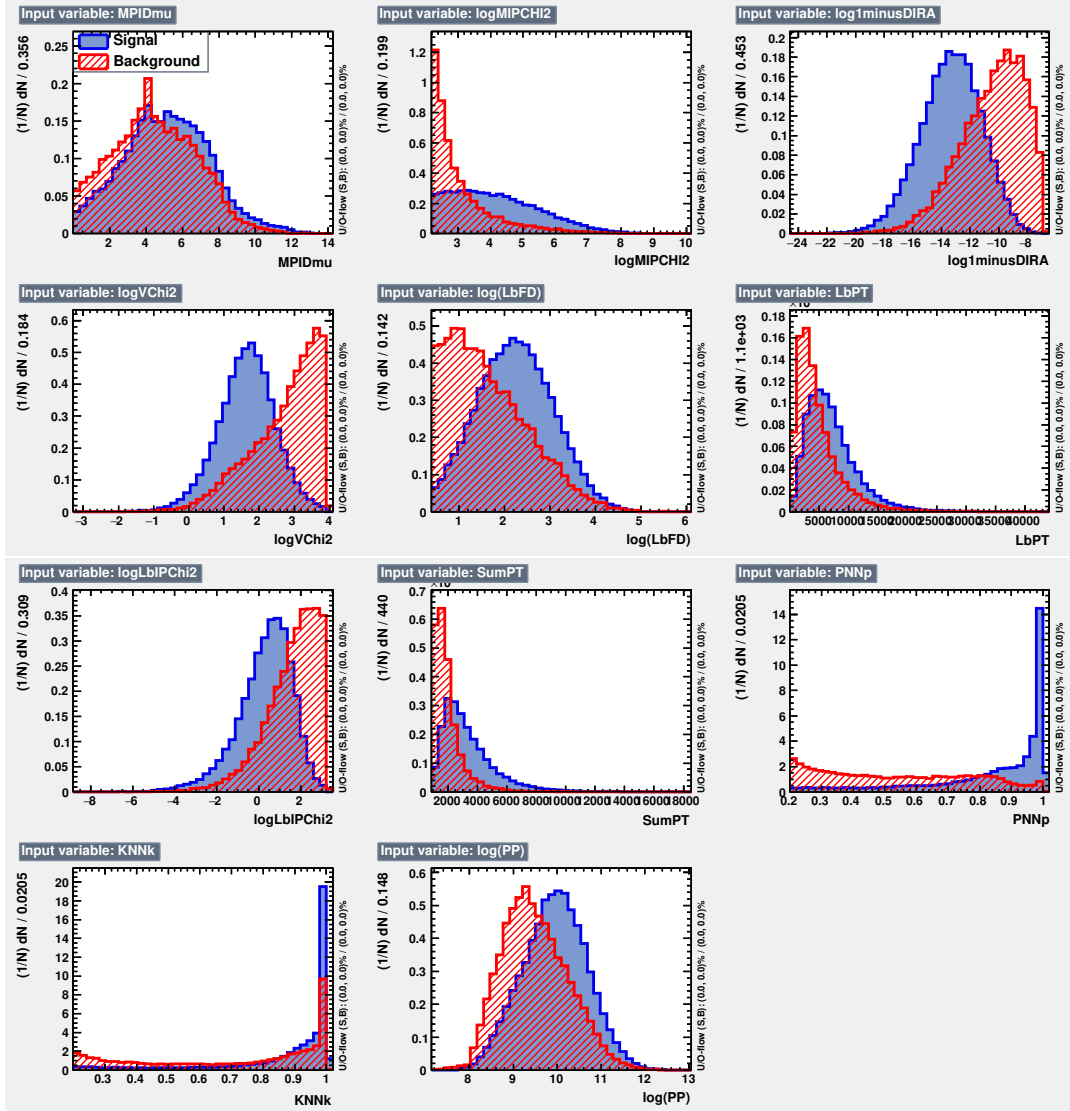


Figure 4.1 Discriminating variables used in the MVA training of Run 2 sample.

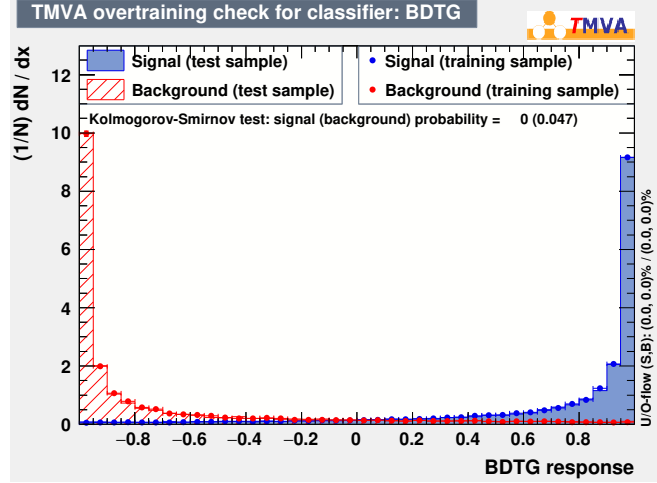


Figure 4.2 The BDTG output distributions for signal and the background in the training samples and the test samples of Run 2 sample.

The number of Λ_b^0 signal events is determined by a fit to $m(J/\psi p K^-)$ distribution, as shown in Fig. 4.3. In this Figure, the red line represents a fit of double-sided Crystal Ball function^[223] on top of the quadratic background, which we use to estimate the background in the signal-region. The fitted background under the peak is indicated with blue-dashed line. The fit gives a mass resolution of the Gaussian part to be 5.66 ± 0.02 MeV. We use 5605-5635 MeV range as signal-region for further analysis (indicated with vertical blue lines), the total number in this region is around 235 thousands. To subtract the backgrounds in efficiency corrected Λ_b^0 yield, we use the sideband-regions, given by 5480-5580 and 5560-5760 MeV ranges (indicated with vertical black lines). The combined data set has 9.1 larger Λ_b^0 yield than the number used in previous analysis in 2015^[222].

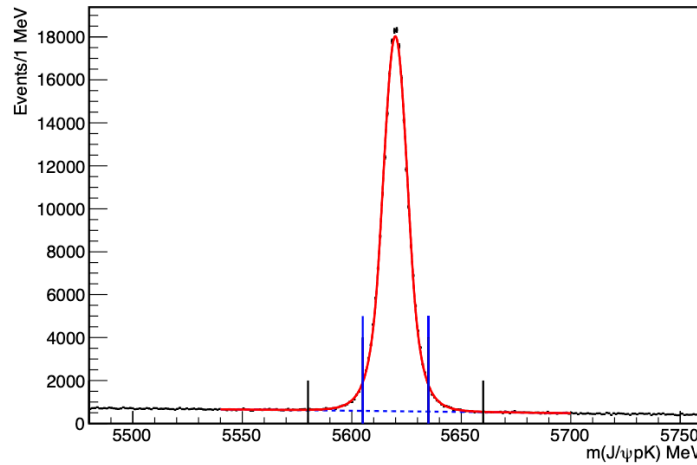


Figure 4.3 The distribution of $m(J/\psi p K^-)$ for the combined Run 1 and Run 2 data set.

4.2.4 Mass spectra

Invariant masses of subsystems in the $\Lambda_b^0 \rightarrow J/\psi p K^-$ decays are obtained after the J/ψ , Λ_b^0 mass and PV-pointing constraints. The $J/\psi p$ mass distributions with bin size equal to 2 MeV is shown in Fig. 4.4. It is apparent that there is a narrow, isolated peak at 4312 MeV. The present high statistics Λ_b^0 sample indicates also that the $P_c(4450)^+$ peak consists of two narrower and overlapping peaks. The $P_c(4312)^+$ peak is strong enough that is also visible in the Dalitz-plot distributions (Fig. 4.5) in addition to the $P_c(4450)^+$ band.

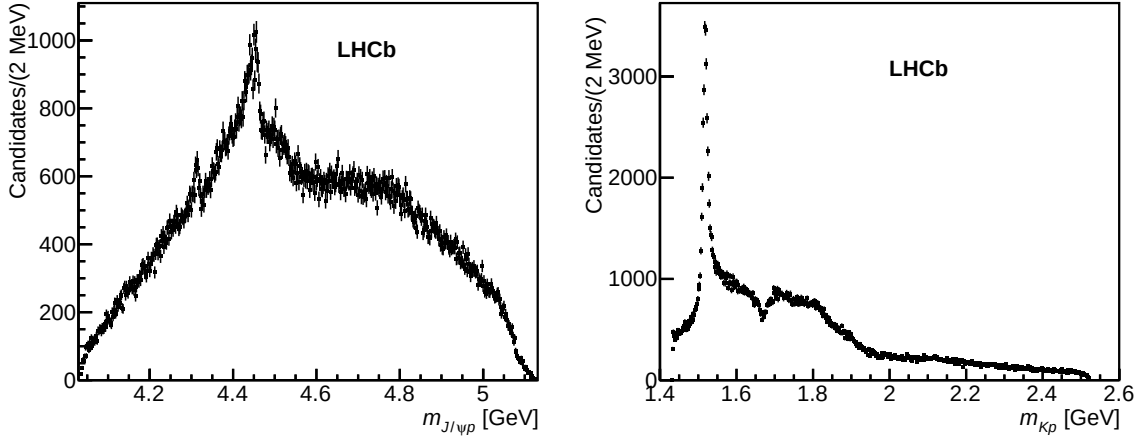


Figure 4.4 Distribution of (left) $m(J/\psi p)$ and (right) $m(K^- p)$ for $\Lambda_b^0 \rightarrow J/\psi p K^-$ candidates. The prominent peak in $m(K^- p)$ is due to the $\Lambda(1520)$ resonance.

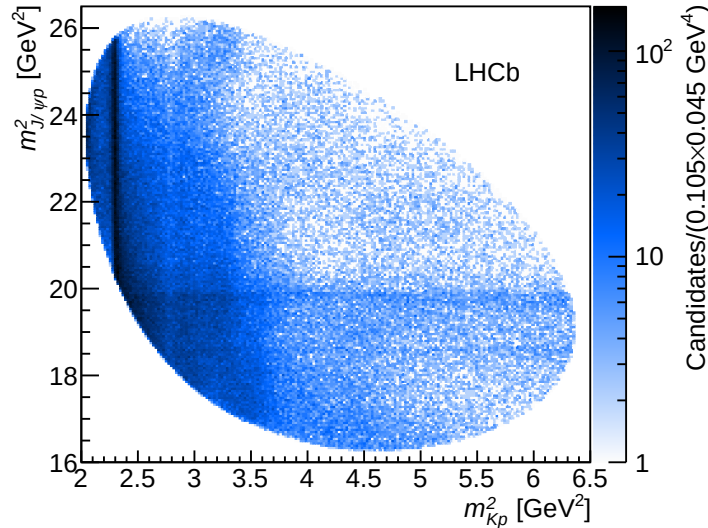


Figure 4.5 Dalitz plot for the Λ_b^0 signal region ($5605 < m(J/\psi p K^-) < 5635$ MeV). The background is not subtracted and constitutes $(6.3 \pm 0.1)\%$ of the displayed sample. The stronger horizontal band is due to the $P_c(4450)^+$ structure. The weaker horizontal band below it is due to $P_c(4312)^+$ peak.

4.3 One dimensional fit and mass resolution

While any broad $m(J/\psi)$ structure must be investigated via amplitude analysis to rule out reflections from $\Lambda_b^0 \rightarrow J/\psi \Lambda^*$ contributions, it is impossible for the latter to produce a very narrow $m(J/\psi)$ peak. Straightforward fit to the $m(J/\psi)$ distribution alone is a plausible approach. For the nominal results, the relativistic Breit-Wigner function (BW) of free pole mass (M) and width (Γ) is used for the signal shape:

$$\text{BW}(m|M, \Gamma) = \frac{B_p B_q}{\frac{M^2 - m^2}{M\Gamma} - i} \times \frac{1}{\sqrt{\pi I \Gamma/2}} \quad (4.1)$$

where B_p, B_q are optional Blatt-Weisskopf factors^[224] (the nominal fits assume S-wave production and decay, thus $B_p = B_q = 1$) and I is the normalization integral. The definition of I affects only the signal yield convention, and is given by:

$$I = \frac{1}{\pi} \left(\arctan\left(\frac{m_{\max} - M}{\Gamma/2}\right) - \arctan\left(\frac{m_{\min} - M}{\Gamma/2}\right) \right) \quad (4.2)$$

where $[m_{\min}, m_{\max}]$ is the fit range. The latter is an accurate integral of the magnitude ($|\text{BW}|^2$) of non-relativistic BW function, in which

$$\frac{M^2 - m^2}{M\Gamma} \rightarrow \frac{M - m}{\Gamma/2} \quad (4.3)$$

in Eq. (4.1). This formula is surprisingly accurate for the relativistic BW as well, especially when the fit boundaries are far from the pole mass as measured in units of the width. As seen from the equations above, we use mass-independent resonant width. We have checked that considering its mass dependence ($\Gamma \rightarrow \Gamma(m) = \Gamma \frac{q}{q_0} \frac{M}{m} B_q^2$) has negligible effect on our fits results, since the signal peaks are narrow. Besides, the Blatt-Weisskopf factor used here is similar to the one mentioned in $B^+ \rightarrow J/\psi \phi K^+$ amplitude analysis, as mentioned in Chapter 3.

We use polynomial functions (*pol*) of various orders to describe the smooth background under the P_c^+ peaks ($\Lambda_b^0 \rightarrow J/\psi \Lambda^*$ is the main background source). The polynomial coefficients are free fit parameters. Switching from the plain to Chebyshev polynomials improved numerical stability of the fits for high orders of the polynomial:

$$\begin{aligned} \text{pol}(m | \vec{a}) &= \sum_{n=0}^{N_{\text{pol}}} a_n T_n(x_m) \\ x_m &= 2 \frac{m - m_{\min}}{m_{\max} - m_{\min}} - 1, \\ T_0(x_m) &= 1, T_1(x_m) = x_m, T_{n+1}(x_m) = 2x_m T_n(x_m) - T_{n-1}(x_m) \end{aligned} \quad (4.4)$$

The Chebyshev polynomials (T_n) are orthogonal to each other, when the fit range of x_m variable is mapped to the $[-1, +1]$. This reduces correlations between the fitted polynomial coefficients, makes convergence faster and avoids round-off problems in the calculation of the covariance matrix.

The phase-space factor, pq , multiplies the signal and background functions added together, where $p(q)$ is the $P_c^+(J/\psi)$ momentum in the Λ_b^0 (P_c^+) rest frame. When overlapping signal peaks are fitted, we first add them incoherently when constructing the PDF function (P):

$$P(m) = pq \left(\sum_j Y_j |\text{BW}(m|M_j, \Gamma_j)|^2 + \text{pol}(m|\vec{a}) \right) \quad (4.5)$$

When allowing for their interference, a relative complex phase δ_j is added as a free parameter:

$$\left| \sum_{j=1} \sqrt{|Y_j|} e^{i\delta_j} \text{BW}(m|M_j, \Gamma_j) \right|^2 P(m) = pq \left(\sum_j Y_j |\text{BW}(m|M_j, \Gamma_j)|^2 + \text{pol}(m|\vec{a}) \right) \quad (4.6)$$

The mass resolution of $m_{J/\psi p}$ is parameterized with a quadratic function, which has strong dependence on the $m_{J/\psi p}$ mass itself, as shown in Fig. 4.6, and smear all fit function with a Gaussian of appropriate width:

$$P_{\text{smearred}}(m) = \sum_k P_k P(m + x_k \sigma_m(m)) \quad (4.7)$$

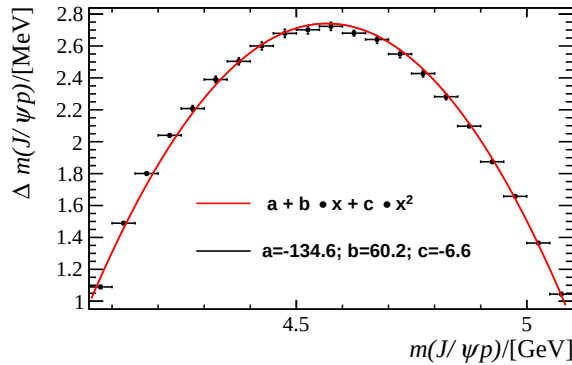


Figure 4.6 The $m_{J/\psi p}$ resolution dependent on the mass itself.

Besides, the mass resolution has very little dependence on Λ_b^0 and P_c^+ decay angles (both polar and azimuthal e.g., helicity-angles and angles between decay planes), as illustrated in Fig. 4.7. This effect on the signal shape is neglected, while polynomial background function automatically absorbs the effect from efficiency on the background shape.

To get a relative P_c^+ production rate ($\mathcal{R}$), the signal yields from the fits to the efficiency-

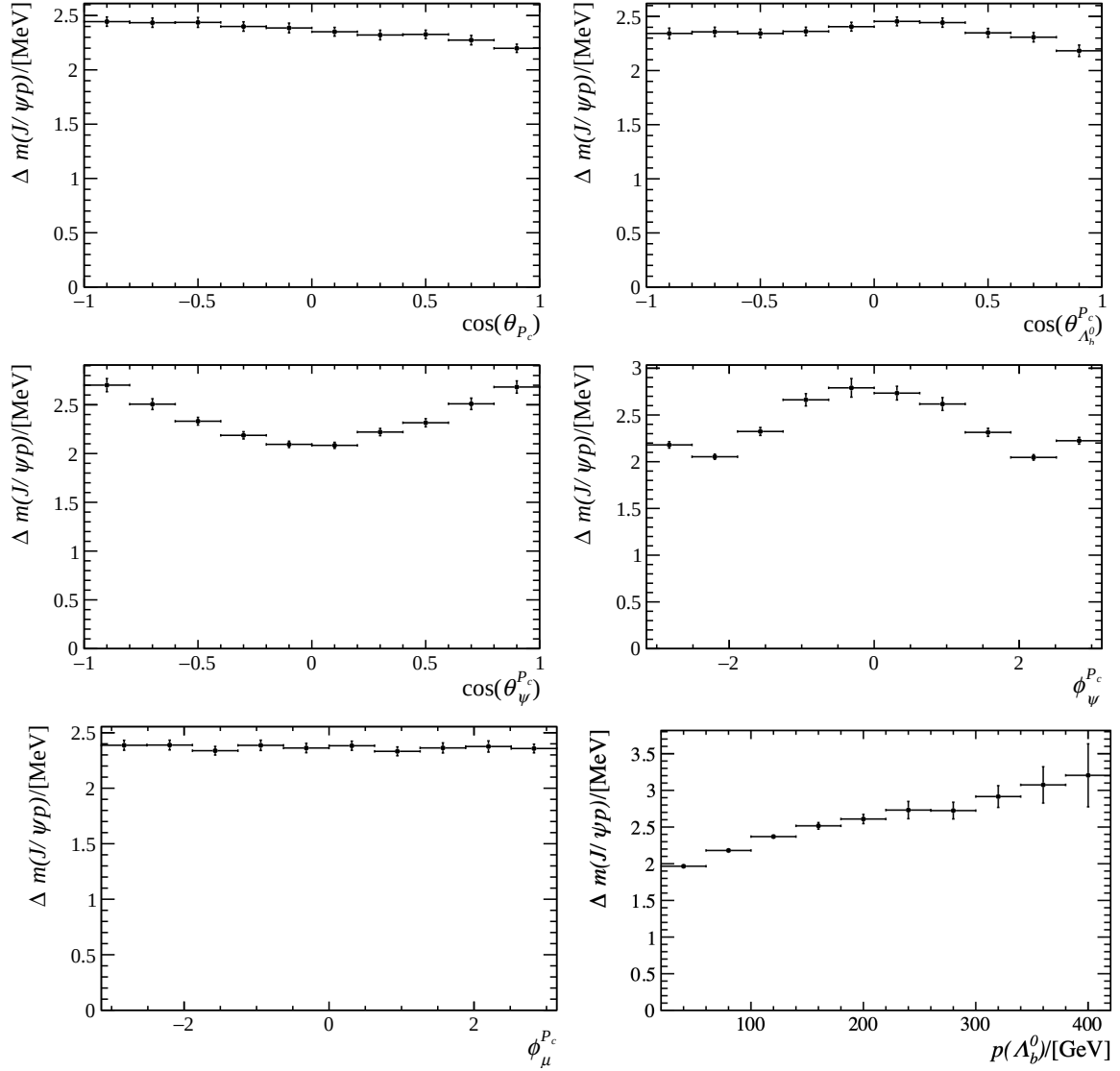


Figure 4.7 Dependence of the $m(J/\psi p)$ resolution (RMS) on: (top row) $\cos \theta_{P_c}$ (the P_c helicity angle in the P_c decay chain) and $\cos \theta_{\Lambda_b^0}^{P_c}$ (the Λ_b^0 helicity angle in the P_c decay chain), (middle row) $\cos \theta_\psi^{P_c}$ (the polar angle of the muon in the ψ rest frame) and $\phi_\psi^{P_c}$ (the azimuthal angle of ψ in the P_c decay chain), (bottom row) $\phi_\mu^{P_c}$ (the azimuthal angle of muon in the ψ rest frame) and $p_{\Lambda_b^0}$ (the momentum of Λ_b^0), in the $4.22 < m(J/\psi p) < 4.40$ GeV range. The definition of the angles can be found in Ref. [222].

corrected $m(J/\psi p)$ distributions (Y_{fit}) are divided by the total sum of the efficiency weights across the entire signal sample ($Y_{tot,sig}$), subtracting the background via scaled ($\lambda = 0.15$) sum of the efficiency weights from the sidebands of the Λ_b^0 mass peak ($Y_{tot,side}$):

$$\mathcal{R} = \frac{\mathcal{B}(\Lambda_b^0 \rightarrow P_c^+ K^-) \mathcal{B}(P_c^+ \rightarrow J/\psi p)}{\mathcal{B}(\Lambda_b^0 \rightarrow J/\psi p K^-)} = \frac{Y_{fit}}{Y_{tot}} \quad (4.8)$$

where $Y_{tot} = Y_{tot,sig} - \lambda Y_{tot,side}$.

4.4 Fit results

4.4.1 The $m(J/\psi p)$ histograms to fit

We perform fits to binned histograms, with bin width of 2 MeV which is small enough to probe signal widths dominated by the mass resolution in the relevant mass region. We use χ^2 -fits, since some of the histograms we fit are weighted. The bin content in all histograms is large enough for Gaussian fluctuations of bin content to be a well justified assumption.

While fitting all events projected onto $m(J/\psi p)$ is among options we employ, this cannot be an optimal way to probe for the P_c^+ signals, since such histogram is dominated by fluctuations of very large Λ^* contributions at low $m(pK^-)$ masses, as shown in Fig. 4.5, where contributions from the narrow P_c^+ states is extremely small. The Λ^* reflections in the total $m(J/\psi p)$ distribution can also produce broad bumps, due to prominent and narrow resonances. In particular, the $\Lambda(1520)$ resonance produces a broad bump just above the $m(pK^-)$ peak region, as shown in the right of Fig. 4.4, complicating the background subtraction via smooth polynomials. Suppression of low $m(pK^-)$ region is necessary to improve statistical and systematic uncertainties.

The simplest way to suppress the low $m(pK^-)$ region is to apply a cut on this mass. One of the slices of data from the 2015 data analysis, showing the cleanest P_c^+ signal peaks, was obtained with the $m(pK^-) > 2.0$ GeV requirement. However, this cut value was picked arbitrarily. A optimization using simulation sample was performed to optimize the significance of P_c^+ signal. According to this optimization, the data sets point to the $m(pK^-) > 1.9$ GeV requirement.

We can still do better than the fixed cut on $m(pK^-)$ value, by replacing the cut by an event weighting procedure. Just like a straight average $\bar{x} = \sum_i^N x_i/N$ is not the best estimator if errors on various measurements of x , σ_x , are not all equal, it is not optimal to mix events with equal weights in the $m(J/\psi p)$ distribution. A better estimator of x is a weighted average, $\bar{x} = \sum_i^N \omega_i x_i / \sum_i^N \omega_i$, where $\omega_i = 1/\sigma_{xi}^2$. Therefore, we need to weight events according to the inverse of the square of the expected error on the P_c^+ amplitude. When comparing with the background yield, the signal yield is pretty small, we weight events with $1/B$, where B is the event yield at given value of $m(pK^-)$. A better way is to weight the cosine the P_c^+ helicity angle, $\cos \theta_{P_c}$. For a fixed value of $m(J/\psi p)$ there is a one-to-one relation between the $m(pK^-)$ and $\cos \theta_{P_c}$ values. Low $m(pK^-)$ values (high B) correspond to $\cos \theta_{P_c} \rightarrow +1$, while high $m(pK^-)$ values (low B) to $\cos \theta_{P_c} \rightarrow -1$. Weighting in $\cos \theta_{P_c}$ is more convenient, since the range of this variable is independent of $m(J/\psi p)$, while the $m(pK^-)$ range changes.

This procedure again assumes that P_c^+ signal follows the phase-space distribution, thus the discussion from the previous paragraph applies here as well. To obtain $B(\cos \theta_{P_c})$, we project events onto $\cos \theta_{P_c}$ in a broad range around the $P_c(4312)^+$, $4200 < m(J/\psi p) < 4400$ MeV, as shown in Fig. 4.8. Including the $P_c(4312)^+$ peak region itself, can only bias its significance down. We chose not to exclude it, in a fear that this might bias its significance up. The weight function, $\omega(\cos \theta_{P_c}) \propto 1/B(\cos \theta_{P_c})$, is also shown in Fig. 4.8. We use a spline function to interpolate between the bin centers.

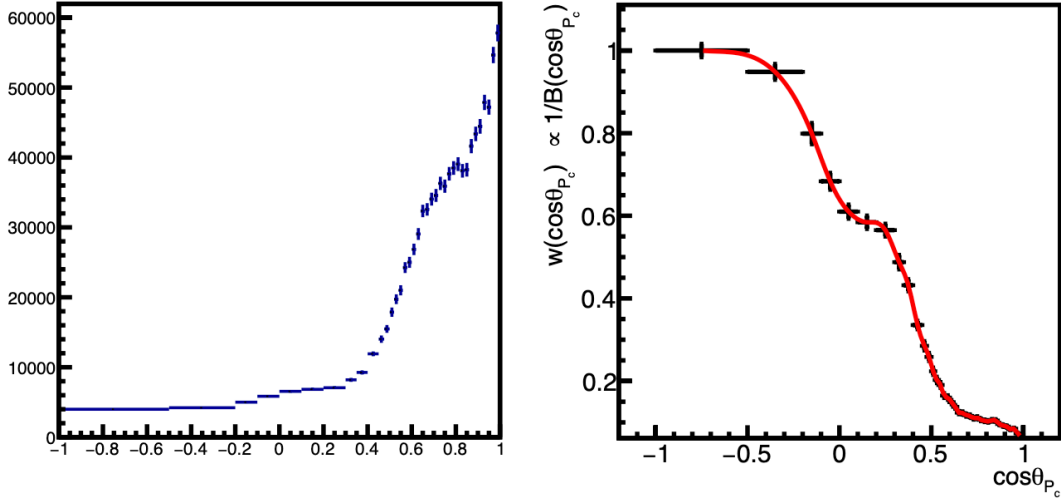


Figure 4.8 Density of events, from $4200 < m(J/\psi p) < 4400$ MeV region, in $\cos \theta_{P_c}$ (left) and its renormalized inverse (right). The red line shows the spline interpolation. The latter is used as a weight function to obtain weighted $m(J/\psi p)$ distribution.

4.4.2 The nominal fits

The fit with 3 Breit-Wigner resonances and 6th order polynomial background to the $\cos \theta_{P_c}$ -weighted nominal sample is used for the central values, as shown in the Fig. 4.9. To calculate the significance of P_c^+ peak in the data from the $\Delta\chi_{data}^2$ value, we can use the following formula:

$$n_\sigma = \sqrt{2} \text{TMath::ErfcInverse} \left(f \cdot \frac{\text{TMath::Prob}(\Delta_{2\ln \mathcal{L}}^{\text{data}}, \text{ndf})}{\text{TMath::Prob}(\Delta_{2\ln \mathcal{L}}^0, \text{ndf})} \right), \quad (4.9)$$

where f is the fraction of the simulated toy experiments which were used in the fit to the tail. The fitting results with one resonance excluded are shown in Fig. 4.9.

The masses and widths of the three observed states are compiled in Tab. 4.3. The systematic variations include 6 – 9th polynomial orders in the $m(pK^-) > 1.9$ GeV and $\cos \theta_{P_c}$ -weighted samples, and 8 – 9th polynomial order variation in the total sample. The $P_c(4440)^+$

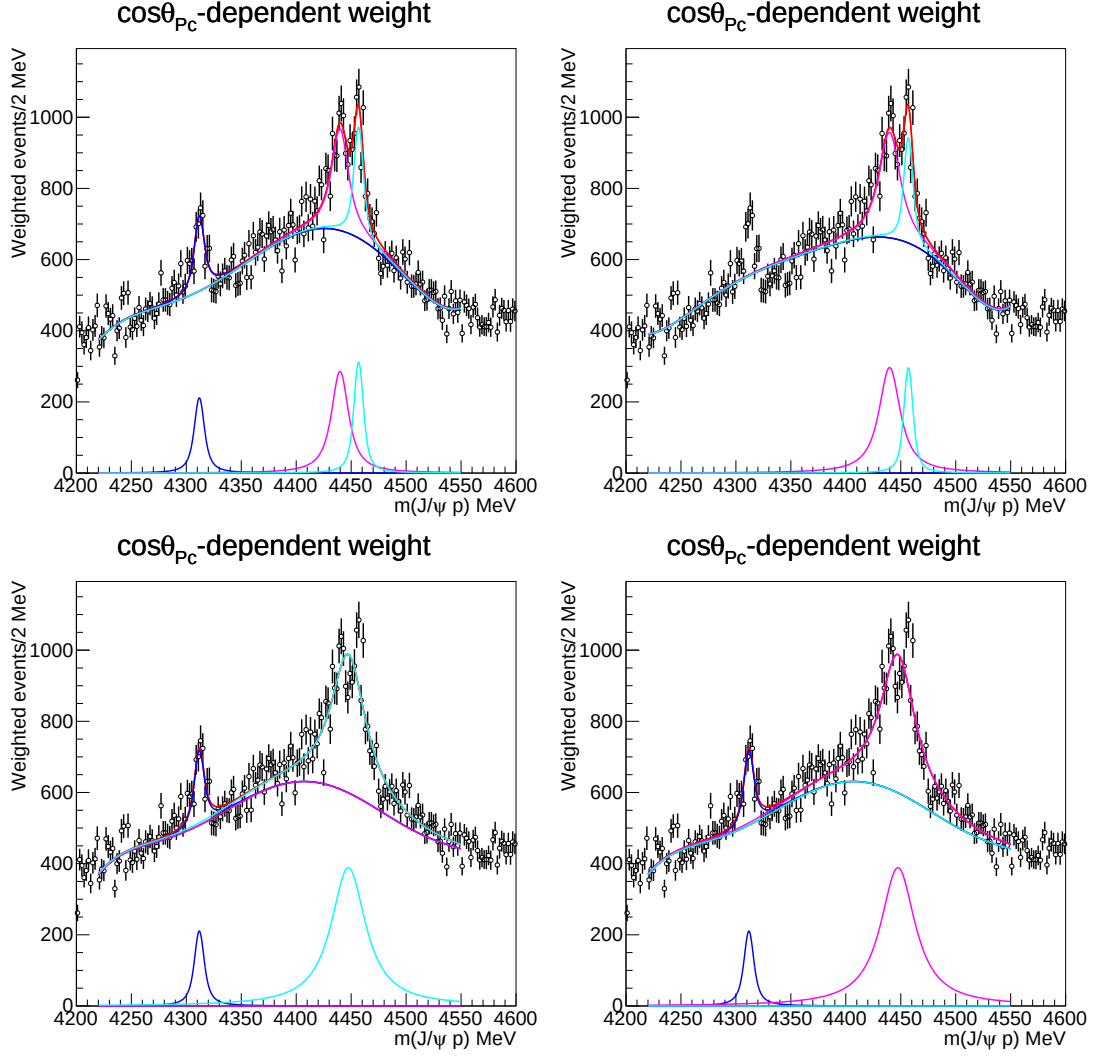


Figure 4.9 Fit of three relativistic BW to the $P_c(4312)^+$, $P_c(4440)^+$ and $P_c(4457)^+$ peaks on 6th polynomial background function to the $\cos\theta_{P_c}$ -weighted distribution (black points) in the data sample (top left). The fitting results with one resonance excluded are shown in other plots.

Table 4.3 Summary of the results obtained from the three BW fits in the 4.22 – 4.57 GeV mass range. The first error is statistical and the second systematic. The variations of polynomial order included here are 6 – 9 for $m(pK^-) > 1.9$ GeV

$P_c(4312)^+$	Mass	$4311.9 \pm 0.7^{+0.6}_{-0.6}$
	Width	$9.8^{+3.0+3.7}_{-2.5-0.7}$
$P_c(4440)^+$	Mass	$4440.3 \pm 1.3^{+0.1}_{-1.5}$
	Width	$20.6^{+5.6+3.0}_{-4.4-9.8}$
$P_c(4457)^+$	Mass	$4457.3 \pm 0.7^{+0.4}_{-1.3}$
	Width	$9.8^{+2.1+2.6}_{-2.0-0.4}$

and $P_c(4457)^+$ structure has a large significance of 6.2σ . Thus, the previous interpretation of the 4450 mass region as one $P_c(4450)^+$ resonance is ruled out. The $P_c(4457)^+$ is very narrow, while $P_c(4440)^+$ is broader in some fits. The systematic uncertainty on its width is considered,

and pretty large. Their production rates are of an order of a percent for the $P_c(4440)^+$ and half of a percent for the $P_c(4457)^+$.

4.4.3 Fits with four resonances

The 2015 amplitude analysis required a broad $P_c(4380)^+$ state in addition to the $P_c(4450)^+$ for a good description of the data, even though the former was not significant on the $m(J/\psi p)$ mass distribution, unlike the 4450 MeV peak. The amplitude fit, given a choice between Λ^* and additional P_c^+ state was overwhelmingly choosing the latter solution. The more advanced models of the Λ^* contributions, which we have tried since then, replacing the $P_c(4450)^+$ with the two narrow peaks and adding the $P_c(4312)^+$ signal to the model have not changed this. We still get the best description of the data when we include a broad state in the fits. Since we have not achieved satisfactory fits to the $m(J/\psi p)$ spectrum with all masses and widths free in the P_c^+ sector, the jury is still out if such broad state exists or not. We will not be able to answer such question from our mass fit analysis, since the broad structures cannot be distinguished from smoothly varying backgrounds. Nevertheless, motivation to include such a broad state in the fits is strong, especially that the data prefer high order polynomials when fitting with three narrow P_c^+ peaks. This serves as an alternative approach to subtracting the background under the narrow peaks when fitting the data in the broader 4.22-4.57 GeV range. The results from fitting different $m(J/\psi p)$ distributions are given in the right of Fig. 4.10. For convenience, the fitting results with only three resonances contributions are shown in the left of Fig. 4.10.

The masses and widths of all narrow states obtained here with four BWs and lower order polynomials are very close to the ones obtained from fitting the same histograms with the three signal peaks and higher order polynomials, as showed in Tab. 4.4. The masses and widths of all narrow states obtained here with four BWs and lower order polynomials are very close to the ones obtained from fitting the same histograms with the three signal peaks and higher order polynomials (compare Tab. 4.3).

Table 4.4 Summary of the results obtained from the three BW fits in the 4.22 – 4.57 GeV mass range. The first error is statistical and the second systematic. The variations of polynomial order included here are 1 – 3 for $m(pK^-) > 1.9$ GeV and $\cos \theta_{P_c}$ -weighted samples, and equal to 3 for the total sample.

$P_c(4312)^+$	Mass	$4312.1 \pm 0.7^{+0.6}_{-0.6}$
	Width	$9.1^{+3.0+1.9}_{-2.4-2.8}$
$P_c(4440)^+$	Mass	$4440.3 \pm 1.2^{+0.1}_{-1.9}$
	Width	$18.5^{+6.5+10.2}_{-6.7-5.8}$
$P_c(4457)^+$	Mass	$4457.3 \pm 0.6^{+0.4}_{-1.4}$
	Width	$7.2^{+2.3+2.8}_{-2.1-1.7}$

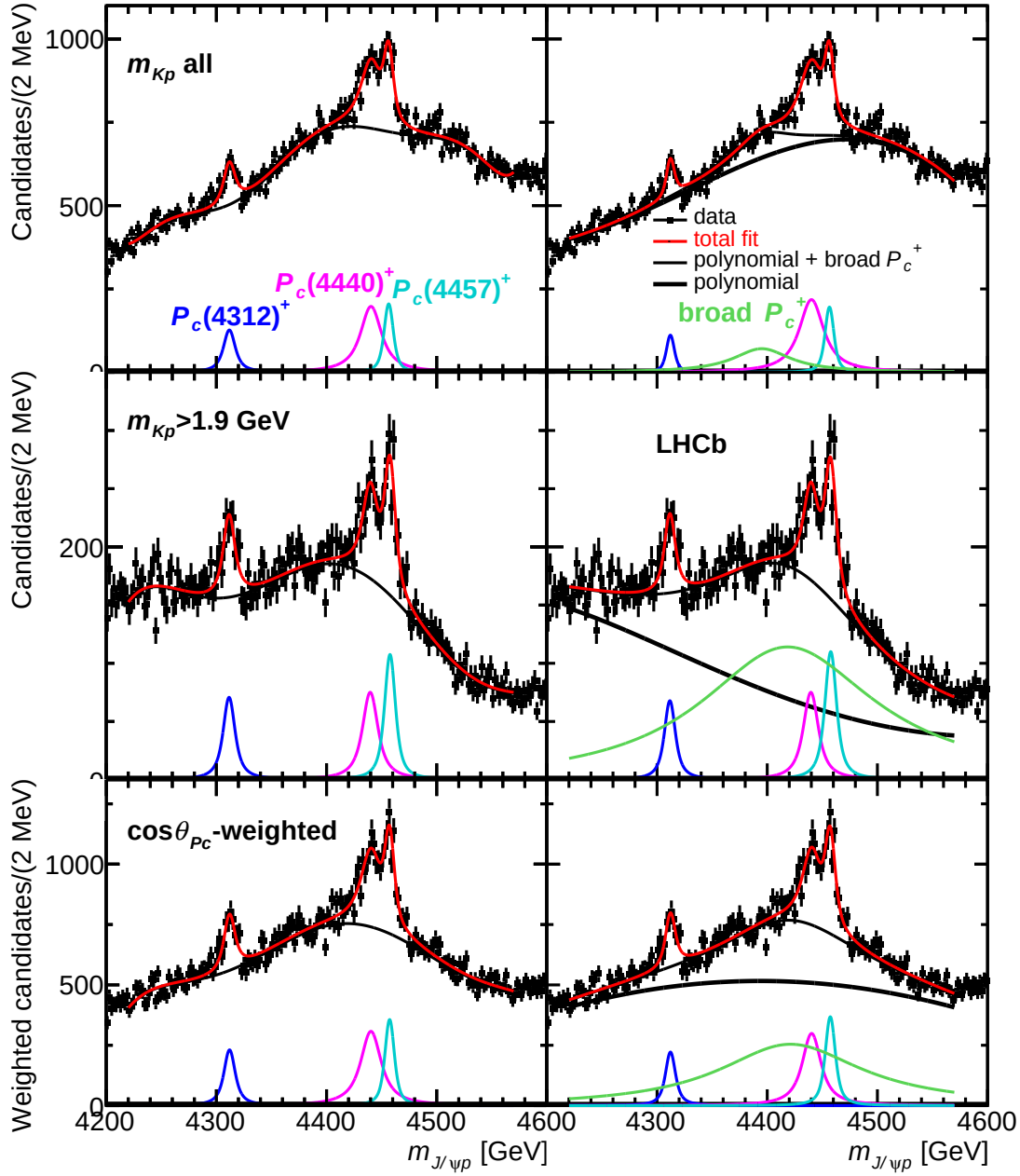


Figure 4.10 Fits to the $m(J/\psi p)$ distributions of the (top row) inclusive, (middle row) $m(pK^-) > 1.9$ GeV, and (bottom row) $\cos \theta_{P_c}$ -weighted samples with three incoherently summed BW amplitudes representing the narrow P_c^+ signals on top of a (left column) high-order polynomial function or (right column) lower-order polynomial plus a broad P_c^+ state represented by a fourth BW amplitude.

4.4.4 Fits with interference between resonances

As discussed in the previous sections (see Eq. (4.6)), fitting the amplitude toy data with incoherent sum of Breit-Wigners, gives a reasonable description of interfering resonances. Their fitted magnitudes reflect effective strength of resonances due to not only their own couplings, but possibly also due to interference with the other P_c^+ terms. In case of neglected

interference, small biases in mass and width were observed on the toy data, which generally were of the order, or smaller, of the expected statistical errors on fit results. However, since we do not yet have an amplitude model which reproduces all features of the data, we study in this section fits to the data which allow for interference of the P_c^+ states.

In fact, the interference between any two P_c^+ states is studied. The fitting formula is shown in Eq. (4.6). The mass and width differences were taken as systematic uncertainties, and the final results are summarized in Tab. 4.5. Besides, Fig. 4.11 gives examples for the fitting results with interference effects considered.

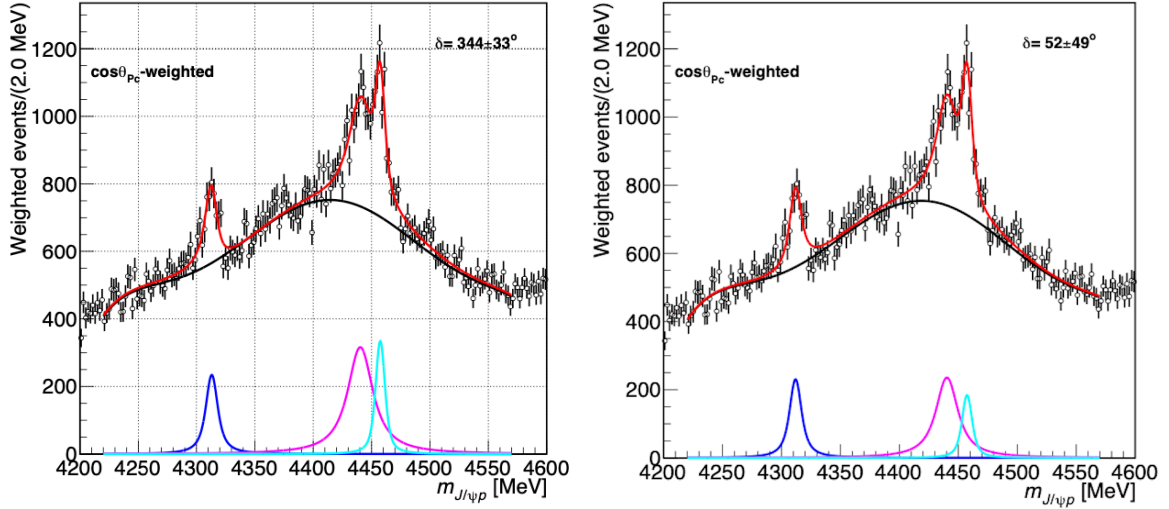


Figure 4.11 Fits to the $m(J/\psi p)$ distributions of in $\cos \theta_{P_c}$ -weighted samples. The $P_c(4312)^+$ and $P_c(4440)^+$ BWs are added coherently in the fit (left), and $P_c(4440)^+$ and $P_c(4457)^+$ BWs are added coherently in the fit (right).

Table 4.5 Summary of the results obtained from the three BW fits in the 4.22 – 4.57 GeV mass range. The first error is statistical and the second systematic. The variations of polynomial order included here are 1 – 3 for $m(pK^-) > 1.9$ GeV and $\cos \theta_{P_c}$ -weighted samples, and order equal to 3 for the total sample.

$P_c(4312)^+$	Mass	$4311.9 \pm 0.7^{+6.8}_{-0.6}$
	Width	$9.8 \pm 2.7^{+3.4}_{-4.5}$
$P_c(4440)^+$	Mass	$4440.3 \pm 1.3^{+4.1}_{-4.7}$
	Width	$20.6^{+4.9+10.2}_{-8.7-10.1}$
$P_c(4457)^+$	Mass	$4457.3 \pm 0.6^{+4.1}_{-1.7}$
	Width	$7.2^{+6.4+5.7}_{-2.0-1.9}$

4.5 Systematic uncertainties

The dominant systematic uncertainties on the width and mass measurements have been already summarized in Tab. 4.5, which included variations of fit range and number of the

narrow P_c^+ peaks included in the fit, variation of order of the polynomial background, addition of a fourth broad P_c^+ state, variation of the data selection cuts, variation of the level of Λ^* background suppression, and exploration of the possible effects from the P_c^+ interference. Except for the width of the $P_c(4312)^+$, the mass and width uncertainties are dominated by the possible interference effects. The systematic uncertainty on the $P_c(4312)^+$ width is dominated by the background subtraction via various fit approaches, including the order of polynomial background used in the fit, and the fit range (*e.g.* fitting $P_c(4312)^+$ locally or together with the other two narrow peaks).

Table 4.6 Additional systematic uncertainties on the mass and width.

	$P_c(4312)^+$		$P_c(4440)^+$		$P_c(4457)^+$	
	Mass	Width	Mass	Width	Mass	Width
Value $\pm$ statistical error	4311.9 ± 0.7	9.8 ± 2.7	4440.3 ± 1.3	20.6 ± 4.9	4457.3 ± 0.6	6.4 ± 2.0
Bkg.subtr. and cut variation	(0.6, +0.8)	(4.5, +3.7)	(-1.1, +0.1)	(8.2, +4.6)	(1.7, +0.4)	(0.9, +3.6)
Including interference	(0.6, +6.8)	(4.5, +3.7)	(4.7, +4.1)	(10.1, +8.7)	(1.7, +4.1)	(1.9, +5.7)
Mass resolution	< 0.1	(0.5, +0.3)	(0.0, +0.1)	± 0.2	(-0.1, +0.0)	(-0.8, +0.7)
Mass scale	< 0.2	–	< 0.2	–	< 0.2	–
Blatt-Weisskopf factors	< 0.1	(-0.1, +0.0)	< 0.1	< 0.1	< 0.1	< 0.1
Efficiency in fit function	< 0.1	(-0.1, +0.0)	< 0.1	(-0.2, +0.0)	< 0.1	< 0.1

The other systematic uncertainties are summarized in Tab. 4.6. Among them, the uncertainty in the $m(J/\psi p)$ mass resolution has the largest impact on the uncertainty of the width determinations. However, when combined in quadrature with the other contributions discussed above, it offers negligible contribution to the overall systematic errors. The mass resolution uncertainty does lower the estimates of the significance of the detection of the non-zero widths to 2.2σ for $P_c(4312)^+$, 1.8σ for $P_c(4440)^+$ and 1.6σ for $P_c(4457)^+$. These significance are not large enough for us to be able to claim definite observation of the non-zero widths. However, they do justify disclosing the central values of the widths with the error bars, in addition to the upper limits on them: $\Gamma(P_c(4312)^+) < 27$ MeV, $\Gamma(P_c(4440)^+) < 49$ MeV, and $\Gamma(P_c(4457)^+) < 20$ MeV (at 95% CL), respectively.

The absolute mass scale is set in our analysis by the J/ψ and Λ_b^0 mass constraints, which bracket the P_c^+ masses from below and above. The nominal data selection includes track momentum rescaling procedure. Compared with the dominant uncertainties in the mass determinations from possible interference effects, the uncertainty in the absolute mass scale is negligible.

Our default fits assume S-wave production and decay. We studied shaping Breit-Wigner amplitudes by angular momentum factors. The angular momentum factors have negligible impact on our fit results. Variation of the efficiency with $m(J/\psi p)$ is slow and neglected in default fit approach. Including it explicitly in the fits produces negligible changes in the result.

The relative production rates suffer from very large statistical errors, induced by fitting the $m(J/\psi p)$ spectrum without Λ^* suppression and by the efficiency weights applied to the events. Their systematic uncertainties due to the fit and data sample variations are also very large. All other systematic contributions are negligible.

The consistency of measurements over different running periods is very good as documented. The data are also consistent when split into Λ_b^0 and $\bar{\Lambda}_b^0$, or magnet-up and magnet-down sub-samples.

4.6 Triangle diagrams

The narrow widths of the P_c^+ peaks make a compelling case for the bound-state character of the observed states. However, it has been pointed out by many authors^[225–227] that peaking structures in this $J/\psi p$ mass range can also be generated by triangle diagrams (see Fig. 4.12). In these processes, the Λ_b^0 baryon (of mass m_1) decays into two nearly on-mass-shell hadrons, one of which (of mass $m_h \equiv \sqrt{t}$) is an excited strange meson or baryon (denoted here as h) that subsequently emits a kaon (of mass m_2) and a non-strange decay product (of mass m_4) that rescatters with the other Λ_b^0 child (of mass m_3) into the $J/\psi p$ system (of $m(J/\psi p) \equiv \sqrt{s}$). Such triangle-diagram processes are known to peak when all three hadrons in the triangle are nearly on their mass shells. Since the overall probability across coupled channels must be conserved, a peak in the $J/\psi p$ channel is only possible if there is a corresponding depletion in the final state composed of the particles that rescatter in Fig. 4.12 to form the $J/\psi p$ ^[228].

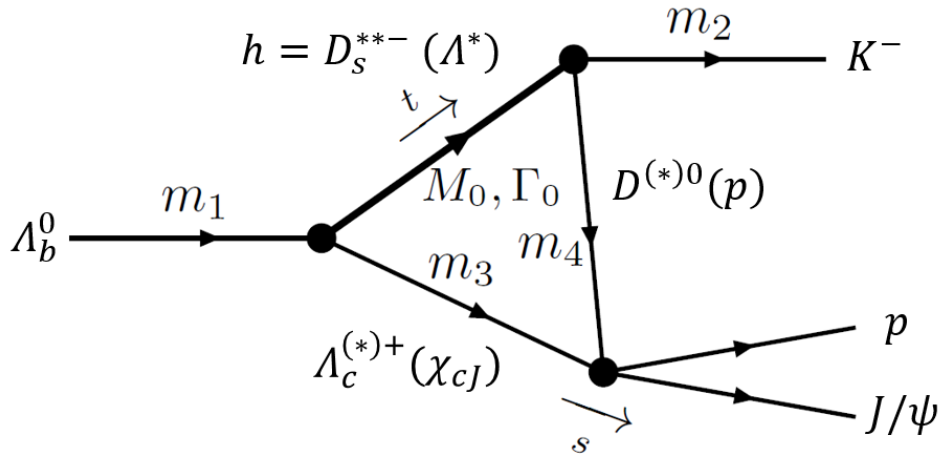


Figure 4.12 Triangle diagram for the $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay. The figure defines the symbols used in the formulae in the text.

The triangle-diagram contribution often peaks at a threshold, given by the sum of the masses of the rescattering hadrons ($m_3 + m_4$) creating a cusp. For a fine-tuned BW resonance

mass of the intermediate hadron h (M_0), the rate can peak above (but never below) the corresponding threshold. The amplitude for a triangle-diagram process, which incorporates a finite width for the exchanged particle (Γ_0), is given by

$$A(s) \propto \int_{(m_2+m_4)^2}^{\infty} dt |\text{BW}(t|M_0, \Gamma_0)|^2 F(t, s), \quad (4.10)$$

where all quantities are defined in Fig. 4.12. The BW term corresponds to the exchanged h hadron. The Feynman triangle-amplitude formula is expressed in terms of a one-dimensional integral over a single Feynman parameter x as follows:

$$F(t, s) \equiv 2 \int_0^1 \frac{dx}{y_- - y_+} \left[\log \left(1 - \frac{2sx}{y_+} \right) - \log \left(1 - \frac{2sx}{y_-} \right) \right], \quad (4.11)$$

where

$$\begin{aligned} y_{\pm} \equiv y_{\pm}(s, t, x) \equiv & (m_1^2 - m_2^2 + s)x - m_1^2 + m_2^2 + m_3^2 - m_4^2 \\ & \pm \{i\epsilon + x^2 \lambda(s, m_1^2, m_2^2) \\ & + 2x [(m_1^2 + m_2^2 - m_3^2 - m_4^2 + 2t)s \\ & - (m_1^2 - m_2^2)(m_1^2 - m_2^2 - m_3^2 + m_4^2)] \\ & - 4st + (m_1^2 - m_2^2 - m_3^2 + m_4^2)^2\}^{1/2}. \end{aligned} \quad (4.12)$$

Here, $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ and ϵ is a small real number.

The 4457 MeV structure peaks near the $\Lambda_c^+(2595)\bar{D}^0$ threshold ($J^P = 1/2^+$)^[227]. The best h candidate for the corresponding triangle diagram is the $D_{s1}(2860)^-$ meson, which has a mass of 2859 ± 27 MeV and a width of 159 ± 80 MeV^[1]. The $P_c(4312)^+$ is not far from the $\Lambda_c^+\bar{D}^{*0}$ threshold ($J^P = 1/2^-$ or $3/2^-$). Exchanging an excited $1^- D_s^{**}$ meson with $M_0 = 3288$ MeV produces the peak at 4312 MeV in the narrow width approximation ($\Gamma_0 \rightarrow 0$). The $P_c(4440)^+$ is well above the $\chi_{c0}p$ threshold ($1/2^+$). Exchanging an excited $1/2^+ \Lambda^*$ with $M_0 = 2153$ MeV produces a peak at the right mass when $\Gamma_0 \rightarrow 0$. In fact, a good quality fit to all three P_c^+ peaks is obtained when Γ_0 is small, as illustrated in Fig. 4.13 (top).

However, this interpretation is unrealistic for the $P_c(4312)^+$ and $P_c(4440)^+$ peaks. When more plausible widths for the excited hadrons are used, such as $\Gamma_0 = 50$ MeV, no narrow peaks can be created above the thresholds, as shown in Fig. 4.13 (bottom). The triangle-diagram hypothesis is more plausible for the $P_c(4457)^+$ state. An example fit using two BW terms and one triangle-diagram amplitude is shown in Fig. 4.14. The fit quality is lower than that obtained using three BW amplitudes. However, further investigation of this interpretation of the $P_c(4457)^+$ state is warranted within an amplitude analysis, which will provide greater

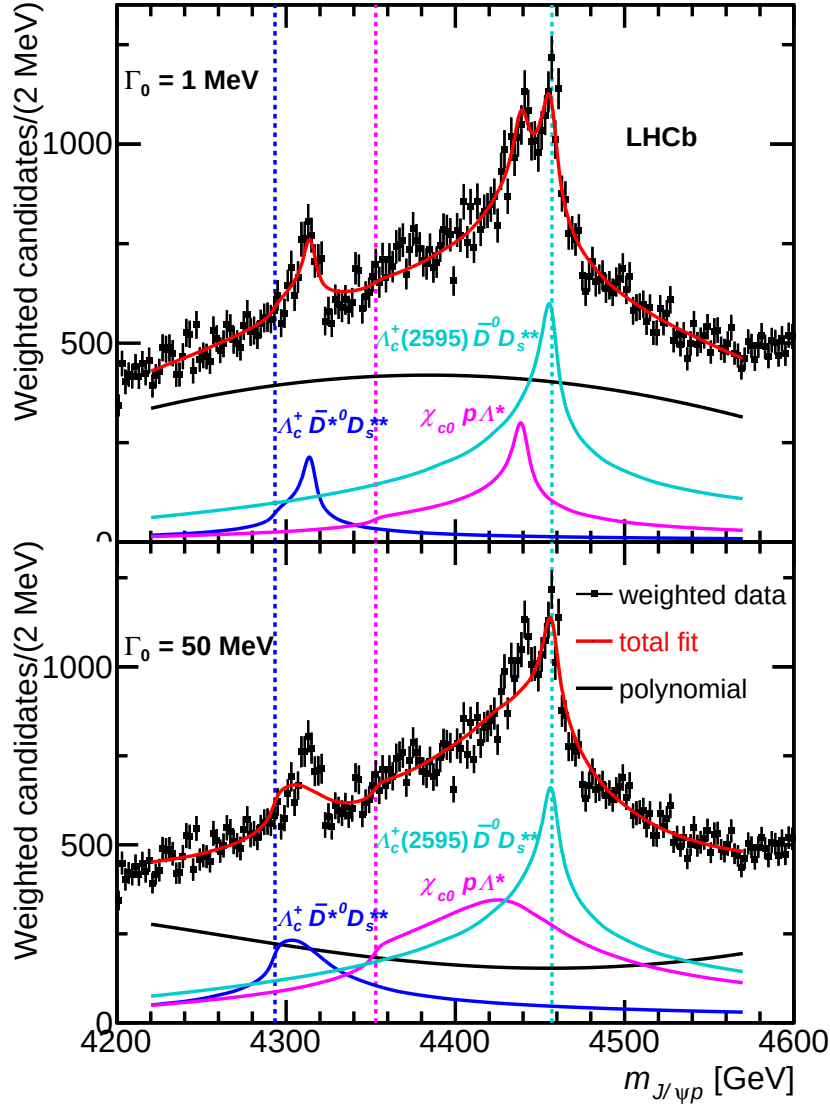


Figure 4.13 Fit of three triangle-diagram amplitudes and a quadratic background to the $\cos \theta_{Pc}$ -weighted distribution. The widths of the excited particles exchanged in the triangles is (top) an unrealistic value of 1 MeV or (bottom) a more plausible value of 50 MeV. Individual triangle diagram contributions are also shown. The dashed vertical lines are the $\Lambda_c^+ \bar{D}^{*0}$, $\chi_{c0} p$ and $\Lambda_c^+(2595) \bar{D}^0$ thresholds.

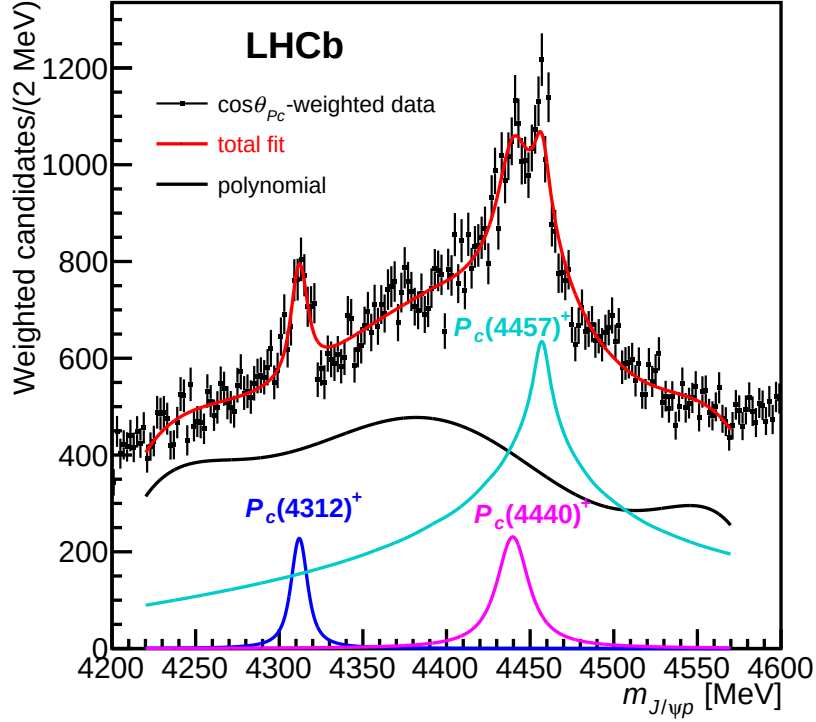


Figure 4.14 Fit of two BW amplitudes, one triangle-diagram amplitude, and a sixth-order polynomial background to the $\cos \theta_{P_c}$ -weighted distribution. The width of the excited D_s^* state exchanged in the triangle loop is set to $\Gamma(D_{s1}(2860)^-) = 159 \text{ MeV}^{[229]}$. The predicted width for this D_s^* state, interpreted as the 1^3D_1 $s\bar{c}$ excitation in the quark model, is $197 \text{ MeV}^{[230]}$.

discrimination between the triangle-diagram and BW amplitudes.

4.7 Revisit the Cabibbo-suppressed decay $\Lambda_b^0 \rightarrow J/\psi p \pi^-$

The Cabibbo-suppressed decay $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ offers a new possibility to study these two states, as well as the other type of exotic state $Z_c^- \rightarrow J/\psi \pi^-$. The study of the relative branching fraction $R_{\pi^-/K^-} \equiv \mathcal{B}(\Lambda_b^0 \rightarrow \pi^- P_c^+)/\mathcal{B}(\Lambda_b^0 \rightarrow K^- P_c^+)$ could be useful to discriminate among their possible interpretations. Starting from Λ_b^0 , $ud(b)$, the Cabibbo-suppressed $b \rightarrow c\bar{c}d$ transition combined with the creation of a light quark pair $u\bar{u}$ yields a final state with the quark content $ud(c\bar{c}d)u\bar{u}$. The d quark in the P_c^+ state can be either directly from the content in the Λ_b^0 baryon or from the $b \rightarrow c\bar{c}d$ transition. This results in two interfering processes, referred as the external W -emission and the internal W -emission diagrams^[231]. The two diagrams are shown in Fig. 4.15. While for the Cabibbo-favoured decay $\Lambda_b^0 \rightarrow K^- P_c^+$, only the external W -emission process exists, as the transition $b \rightarrow c\bar{c}s$ doesn't produce a d quark for the P_c^+ state. Assuming the internal W -emission process is zero, Ref. [231] predicted the relative

branching fraction $R_{\pi^-/K^-} = 0.07 \sim 0.08$, due to the differences of CKM elements and the phase-space factors. A sizable internal W -emission process could modify (either increase or reduce) the predicted value and even the relative production phase between the P_c states, resulting in different interference patterns between the two decays.

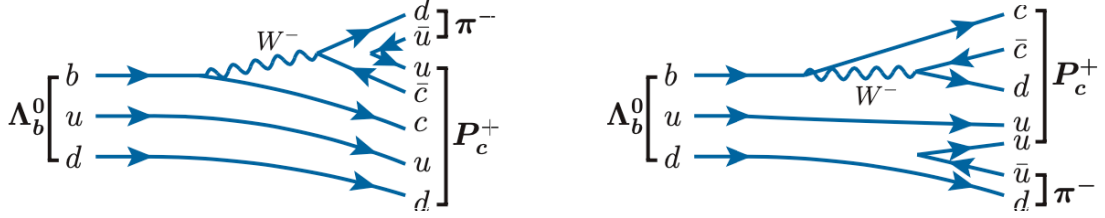


Figure 4.15 Feynman diagrams of the $\Lambda_b^0 \rightarrow P_c \pi$ decay via (left) the external and (right) the internal W -emissions.

A different production mechanism is proposed. Ref. [232] argues that the P_c state in the $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay is produced mainly by the charmless Λ_b^0 decay via the $b \rightarrow u\bar{u}s$ transition, in which the $c\bar{c}$ contents in P_c states arise from the intrinsic charms within the Λ_b^0 baryon. The Feynman diagram is shown in Fig. 4.16. A very large value $R_{\pi^-/K^-} = 0.58 \pm 0.05$ is predicted.

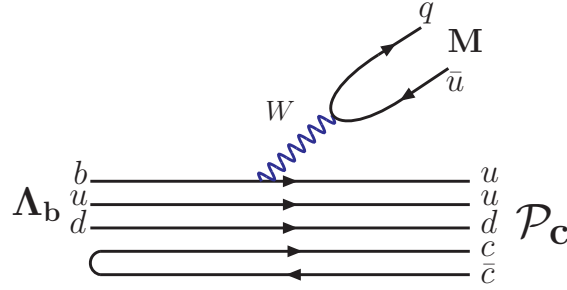


Figure 4.16 Feynman diagram of $\Lambda_b^0 \rightarrow P_c^+ M$ via the charmless $b \rightarrow u\bar{u}q$ transition^[232], where $M(q)$ can be $K^-(s)$ or $\pi^-(d)$.

The channel $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ is expected to be produced via the intermedia resonances K^* , $\Lambda_b^0 \rightarrow J/\psi N^*$ with $N^* \rightarrow p \pi^-$. The transition of $\Lambda_b^0 \rightarrow J/\psi N^*$ is dominated by the flavor changing decay of $b \rightarrow c + \bar{c}d$. This suggests that the u and d quark in the Λ_b^0 should be a spectator, hence their isospin quantum number $I = 0$ should be conserved. As a consequence, if the ud pair is to be combined with a d quark to form a baryon, it will favor N baryons instead of Δ ones.

LHCb has measured the branching fraction of the overall Cabibbo-suppressed decay relative to the Cabibbo-favoured decay to be $\mathcal{B}(\Lambda_b^0 \rightarrow J/\psi p \pi^-)/\mathcal{B}(\Lambda_b^0 \rightarrow J/\psi p K^-) = 0.0824 \pm$

$0.0024 \pm 0.0042^{[233]}$, where the first uncertainty is statistical and the second systematic.

Actually, a full amplitude analysis of $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decay has been performed using LHCb Run 1 data sample^[131]. If including both types of the exotic resonances report in 2015^[124], the total significance for them is 3.1σ using a conservative model of N^* resonances and 6.6σ when only well established N^* states are allowed. Within the large statistical and systematic errors, the data are consistent with the $P_c(4450)^+$ and $P_c(4380)^+$ production rates expected from their previous observation and Cabibbo suppression. However, as mentioned above, a new narrow $P_c(4312)^+$ state is discovered with a large statistical with Run 1 and Run 2 sample collected at LHCb, and the previously reported $P_c(4450)^+$ structure contains two peaks, $P_c(4440)^+$ and $P_c(4457)^+$. In this case, it is required to revisit this decay channel with both Run 1 and Run 2 data sample.

4.7.1 Invariant B mass fit and Dalitz-plot distributions

The data selection criteria are similar to those used in the $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay, the details of selection are introduced in Appendix A.2.

After the selections, the sample of $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decays consists of the signal, the combinatorial background and the $\Lambda_b^0 \rightarrow J/\psi p K^-$ reflections. The invariant mass distribution of the signal is described by a DSCB function^[223], an alternative fit with the Hypatia model^[205] is also performed; the shape of the combinatorial background is described by an exponential function; and the shapes of the $\Lambda_b^0 \rightarrow J/\psi p K^-$ reflections are determined from simulation. A simulation $\Lambda_b^0 \rightarrow J/\psi p K^-$ sample is used to determine the shapes of the reflections. The candidate is reconstructed as $\Lambda_b^0 \rightarrow J/\psi p [K^- \rightarrow \pi^-]$, and all selections except the PID requirements used to select $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ candidates in this analysis are applied to these candidates. Each event that passes the selections is assigned a weight to account for the PID efficiency, and the weight is determined from the PIDCalib package^[201,234]. By filling the invariant mass under $J/\psi p \pi^-$ hypotheses into histograms with the PID weight, the shapes are determined, as shown in Fig. 4.17. The histogram is converted to a histogram-PDF in the fit. It should be noted that the shape is found to be insensitive to the decay model ^⑤, so the shape determined from the simulation with the PHSP decay model is good enough to represent the complicated Dalitz-plot distributions seen in the data. In Fig. 4.18, the fitted invariant mass distributions of the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ candidate are shown for both Run 1 and Run 2 samples.

The invariant mass fit result is used to calculate the $sWeights$ ^[218] for both the signal and background components. In Fig. 4.19, the Dalitz-plot distributions using the invariant mass

⑤ The shape determined in different slices of $M(p\pi)$ is very similar.

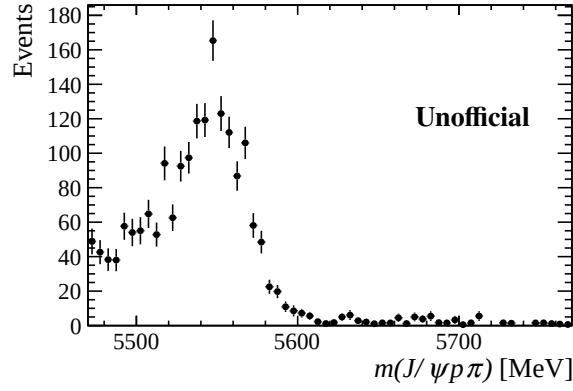


Figure 4.17 Distribution of the invariant mass of $\Lambda_b^0 \rightarrow J/\psi p K^-$ candidates reconstructed as $\Lambda_b^0 \rightarrow J/\psi p \pi^-$.

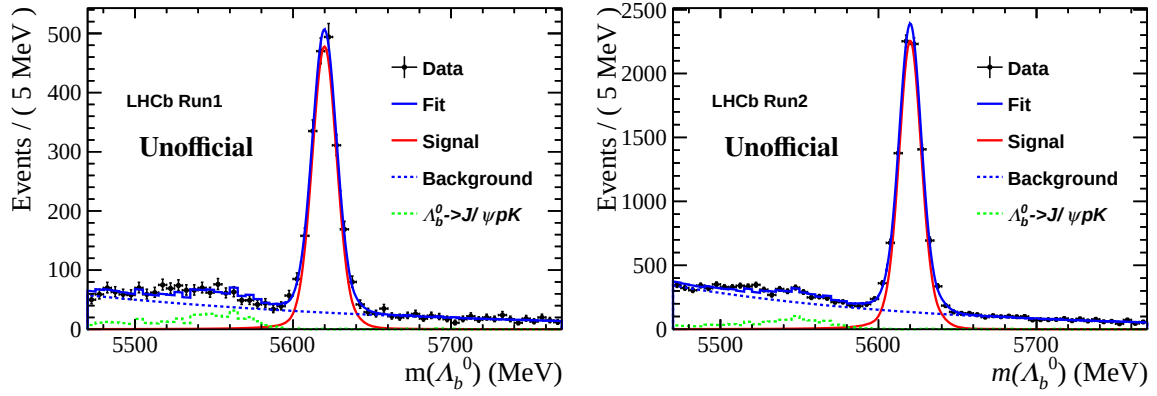


Figure 4.18 Invariant mass distribution of the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decay and the fitted curve for Run 1 (left) and Run 2 (right) samples.

squared are shown, $m_{p\pi^-}^2$ vs $m_{J/\psi p}^2$, and $m_{p\pi^-}^2$ vs $m_{J/\psi \pi}^2$ as independent variables. As expected, significant N^* contributions are seen, especially in the region around $m_{p\pi^-}^2 = 2 \text{ GeV}^2$. It seems that narrow belt around the $P_c(4450)^+$ region at about 20 GeV^2 in the $J/\psi p$ mass-squared is visible. While many events concentrate in a limited window around $m_{p\pi^-}^2 = 6 \text{ GeV}^2$ and $m_{J/\psi p}^2$ between 18 to 20 GeV^2 , which could be due to the P_c^+ contribution. These events are also presented at the region of $m_{J/\psi \pi^-}^2$ around 17 GeV^2 with $m_{p\pi^-}^2$ around 6 GeV^2 .

4.7.2 Further plan and *sFit* fitting model

Currently, we have finished the data selection using Run 1 and Run 2 data sample collected at LHCb. Besides, a full amplitude analysis model has been constructed to examine the contributions of the two P_c^+ states and possible Z_c^- state. Isobar model approach can be used in which each resonance is described by a Breit-Wigner.

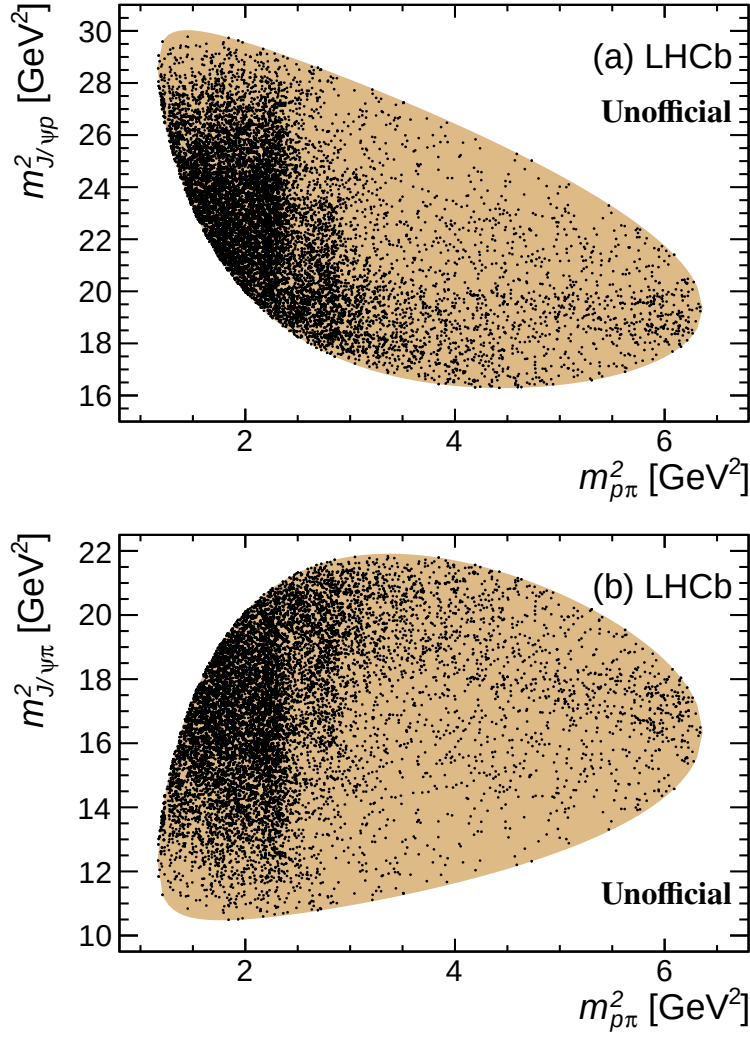


Figure 4.19 Invariant mass squared of $p\pi$ versus either $J/\psi p$ (top) or $J/\psi \pi$ (bottom) for candidates with $sWeights$ to subtract background contribution. The hatched area shows the kinematic allowed region.

4.7.2.1 Amplitude analysis model

The amplitude formalism has been described in the $B^+ \rightarrow J/\psi p K^+$ analysis in Sec. 3.4, the form corresponding to the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decay can be obtained by replacing variables with the particle K^+ by these of the π^- , the p by these of the ϕ , and B^+ by these of the Λ_b^0 . There are 6 fitting observed quantities: invariant mass of $p\pi$, $m(p\pi)$, and five decay angles denoted as Ω , including (i) cosine of helicity angle of the Λ_b^0 decays $\cos \theta_{\Lambda_b^0}$; (ii) cosine of helicity angle of the N^* decays $\cos \theta_{N^*}$; (iii) cosine of helicity angle of the J/ψ decays $\cos \theta_{J/\psi}$; (iv) angle between Λ_b^0 and N^* decay planes ϕ_π ; and (iv) angle between Λ_b^0 and J/ψ decay planes ϕ_μ .

Except for the exotic baryonic P_c^+ states, exotic mesonic states $Z_c^- \rightarrow J/\psi \pi^-$ could also contribute to this decay. The matrix elements for the $J/\psi N^*$, $P_c^+ \pi^-$ and $J/\psi \pi^-$ decay chains, are similar to that in the previous $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ study^[131]. Furthermore, the proton and muon

helicity states in each of the exotic decay chains must be expressed in the basis of helicities in the N^* decay chain, which is incorporated in the total decay matrix element as follows

$$|\mathcal{M}|^2 = \sum_{\lambda_{\Lambda_b^0}} \sum_{\lambda_p} \sum_{\Delta\lambda_\mu} \left| \mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p, \Delta\lambda_\mu}^{N^*} + e^{i\Delta\lambda_\mu \alpha_\mu} \sum_{\lambda_p^{P_c^+}} d_{\lambda_p^{P_c^+}, \lambda_p}^{\frac{1}{2}}(\theta_p) \mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p^{P_c^+}, \Delta\lambda_\mu}^{P_c^+} + e^{i\Delta\lambda_\mu \alpha_\mu^{Z_c^-}} \sum_{\lambda_p^{Z_c^-}} e^{i\lambda_p^{Z_c^-} \alpha_p^{Z_c^-}} d_{\lambda_p^{Z_c^-}, \lambda_p}^{\frac{1}{2}}(\theta_p^{Z_c^-}) \mathcal{M}_{\lambda_{\Lambda_b^0}, \lambda_p^{Z_c^-}, \Delta\lambda_\mu}^{Z_c^-} \right|^2, \quad (4.13)$$

where the matrix element for each of the decay chains is denoted by the superscripts, and θ_p ($\theta_p^{Z_c^-}$) is the polar angle in the p rest frame between the boost directions from the N^* and P_c^+ (Z_c^-) rest frames, and α_μ ($\alpha_\mu^{Z_c^-}$) is the azimuthal angle correcting for the difference between the muon helicity states in the N^* and P_c^+ (Z_c^-) decay chains. Unlike in the P_c^+ chain, the azimuthal angle ($\alpha_p^{Z_c^-}$) aligning the two proton helicity frames is not zero.

A so-called *sFit* approach can be used in this analysis. In such approach the total PDF is equal to the signal PDF, as the background is subtracted from the log-likelihood sum using the *sWeights*. The signal PDF is obtained by multiplying the decay matrix element $\mathcal{M}$ squared with the selection efficiency, $\epsilon(m_{p\pi}, \Omega)$,

$$\mathcal{P}_{\text{sig}}(m_{p\pi}, \Omega | \vec{\omega}) = \frac{1}{I(\vec{\omega})} |\mathcal{M}(m_{p\pi}, \Omega | \vec{\omega})|^2 \Phi(m_{p\pi}) \epsilon(m_{p\pi}, \Omega), \quad (4.14)$$

where $\Phi(m_{p\pi})$ is the phase space function equal to pq , with p being the momentum of the $p\pi$ system (*i.e.* N^*) in the Λ_b^0 rest frame, and q being the momentum of π^- in the N^* rest frame, and $I(\vec{\omega})$ is the normalization integral. The fitting parameters, $\vec{\omega}$, represent independent helicity or LS couplings, and masses and widths of resonances (or Flatté parameters).

This quasi-log-likelihood sum is scaled by a constant factor, $s_W \equiv \sum_i W_i / \sum_i W_i^2$ to account for the effect of the background subtraction on the statistical uncertainty. To determine $\vec{\omega}$,

$$\begin{aligned} -2 \ln \mathcal{L}(\vec{\omega}) &= -2s_W \sum_i W_i \ln \mathcal{P}_{\text{sig}}(m_{p\pi i}, \Omega_i | \vec{\omega}) \\ &= -2s_W \sum_i W_i \ln |\mathcal{M}(m_{p\pi i}, \Omega_i | \vec{\omega})|^2 + 2s_W \ln I(\vec{\omega}) \sum_i W_i \\ &\quad - 2s_W \sum_i W_i \ln [\Phi(m_{p\pi i}) \epsilon(m_{p\pi i}, \Omega_i)]. \end{aligned} \quad (4.15)$$

is minimized. The last term does not depend on the fitted parameters $\vec{\omega}$ and is therefore dropped. The efficiency still appears in the normalization integral. The integration is done without the need to parameterize the efficiency, by summing the matrix element squared over

the simulated events that are generated uniformly in phase space and pass through the detector modelling and the data selection procedure,

$$\begin{aligned}
 I(\vec{\omega}) &\equiv \int |\mathcal{M}(m_{p\pi}, \Omega|\vec{\omega})|^2 \Phi(m_{p\pi}) \epsilon(m_{p\pi}, \Omega) dm_{p\pi} d\Omega \\
 &\propto \sum_j^{N_{\text{MC}}} w_j^{\text{MC}} |\mathcal{M}(m_{p\pi j}, \Omega_j|\vec{\omega})|^2,
 \end{aligned} \tag{4.16}$$

where w_j^{MC} are the weights given to the simulated events to improve the agreement between data and simulations.

4.7.2.2 Current dilemma

As mentioned above, a 6-dimensional amplitude fitting model has been constructed. However, some coupling variables related to the P_c^+ states are expected to be constrained to the measurements obtained from the Cabibbo-favoured channel, $\Lambda_b^0 \rightarrow J/\psi p K^-$. This means that the fitting results will be published when the amplitude analysis for $\Lambda_b^0 \rightarrow J/\psi p K^-$ channel is finished. Moreover, the current statistics is not large enough to see obvious pentaquark structures in Dalitz-plot distributions, as shown in Fig. 4.19. Therefore, it is possible that the Run 3 data sample is required if we want to confirm the pentaquark states in this decay.

4.8 Conclusions

In this updated analysis of the $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay, the data selection criteria have been improved by using the PID information in MVA, nearly doubling the number of Λ_b^0 decays utilized in Run 1 data. Together with additional integrated luminosity accumulated in Run 2 and higher pp collision energies, the present Λ_b^0 data set is almost an order of magnitude larger than the data used in the previously published analysis. The results obtained in this analysis are consistent with the published. Besides, a clear new signal of a very narrow state at 4312 MeV was observed in the enlarged data sample in the mass spectrum of $J/\psi p$, with natural width insignificant compared with the mass resolution. The production rate of the $P_c(4312)^+$ is a fraction of a percent of all $\Lambda_b^0 \rightarrow J/\psi p K^-$ decays and was too small to be detectable in the original analysis of the Run 1 data. Furthermore, there is also significant evidence for the 4450 structure to be an overlap of two narrower peaks near 4440 and 4457 MeV. While the claim of significant $J/\psi p$ structure around 4450 MeV in the 2015 amplitude analysis is fully consistent with the new results, the interpretation of this relatively narrow peak at 4450 MeV as a single resonance was not correct, which formally puts in question the other claims according to this

Table 4.7 Summary of P_c^+ masses and widths (as well as the upper limits at 95% CL).

State	M [MeV]	Γ [MeV]	(95% CL)
$P_c(4312)^+$	$4311.9 \pm 0.7^{+6.8}_{-0.6}$	$9.8 \pm 2.7^{+3.7}_{-4.5}$	(< 27)
$P_c(4440)^+$	$4440.3 \pm 1.3^{+4.1}_{-4.7}$	$20.6 \pm 4.9^{+8.7}_{-10.1}$	(< 49)
$P_c(4457)^+$	$4457.3 \pm 0.6^{+4.1}_{-1.7}$	$6.4 \pm 2.0^{+5.7}_{-1.9}$	(< 20)

analysis, including the broader $P_c(4380)^+$ state. The newly observed structures are narrow enough to be investigated from the $J/\psi p$ mass distribution alone. The measured masses and widths of P_c^+ states are summarized in Tab. 4.7.

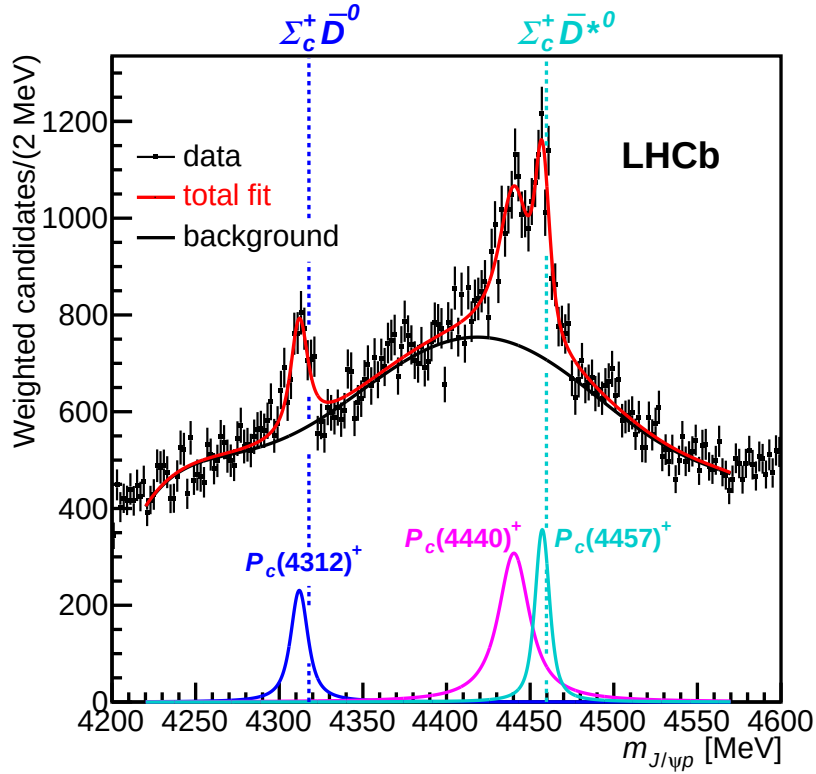


Figure 4.20 Fit to the $\cos \theta_{P_c}$ -weighted $m(J/\psi p)$ distribution with three BW amplitudes and a sixth-order polynomial background. This fit is used to determine the central values of the masses and widths of the P_c^+ states. The mass thresholds for the $\Sigma_c \bar{D}^0$ and $\Sigma_c \bar{D}^{*0}$ final states are superimposed.

The properties of the $P_c(4312)^+$ fit the molecular hypothesis, since it is a narrow state with the mass about 9 MeV below the $\Sigma_c \bar{D}$ threshold ($J^P = \frac{1}{2}^-$), as shown in Fig. 4.20. The $P_c(4457)^+$ could also be a bound system of $\Sigma_c \bar{D}^*$ ($J^P = \frac{1}{2}^-$ or $\frac{3}{2}^-$), with a weak 3 MeV binding. Several papers on hidden-charm states created dynamically by charmed meson-baryon interactions^[235] were published well before the first observation of the P_c^+ structures by LHCb. Some of them^[236,237] match the properties of the $P_c(4312)^+$ and of the $P_c(4457)^+$

states very well. $P_c(4440)^+$ is about 20 MeV below the $\Sigma_c \bar{D}^*$ threshold, and some will argue that such molecular binding is possible. In fact, many models predict two $\Sigma_c \bar{D}^*$ states, $\frac{1}{2}^-$ and $\frac{3}{2}^-$. The $P_c(4440)^+$ is 9 MeV below the $\chi_{c1} p$ threshold ($J^P = \frac{1}{2}^-$ or $\frac{3}{2}^-$), thus it could also be a bound state of $\chi_{c1} p$. However, there is no S-wave binding mechanism for this meson-baryon combination^[238], and therefore such molecular hypothesis is unlikely. Totally, the nature of these states deserves further investigation.

For the Cabibbo-suppressed decay $\Lambda_b^0 \rightarrow J/\psi p \pi^-$, there are also hints of pentaquark states in the mass spectrum of $J/\psi p$. Due to the limited statistic, further study with larger data sample is required.

Chapter 5 Searching for open-charm pentaquark state in the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ decay

5.1 Motivation

Over the last two decades, a wealth of information has been accumulated on the decays of b -hadrons. Measurements of their decays have been used to test the CKM mechanism^[239] for describing weak decay phenomena in the Standard Model, as well as provide measurements against which various theoretical approaches, such as heavy quark effective theory^[240] and the factorization hypothesis, can be compared. While many decays have been measured, a large number remain either unobserved or poorly measured, most notably decays of Λ_b^0 baryons. LHCb has observed many new Λ_b^0 decays to open charm modes, such as $\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^- \pi^+ \pi^-$ ^[241], $\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^- p \bar{p}$ ^[242], and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_{(s)}^-$ ^[243].

Besides, the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ decay also provides an opportunity to search for possible single charmed pentaquarks ($c\bar{s}uud$) that could decay strongly to $\Lambda_c^+ K^+$. The LHCb experiment observed two pentaquark states using $\Lambda_b^0 \rightarrow J/\psi p K^-$ decays in 2015^[124]. This discovery motivates us to search for more pentaquark states. Recently, Belle reported an observation of a new $\Xi_c(2930)^0$ resonance decaying to $\Lambda_c^+ K^-$ in $B^+ \rightarrow \Lambda_c^+ \bar{\Lambda}_c^- K^+$ decay^[244], and we can also examine the decays of resonances e.g. $\Xi_c^0 \rightarrow \Lambda_c^+ K^-$.

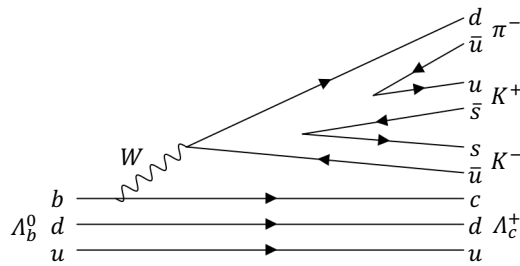


Figure 5.1 Possible Feynman diagrams contributing to the decay of $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$.

Actually, the decay of $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ has been shown as a background in the analysis of $\Lambda_b^0 \rightarrow \Lambda_c^+ p \bar{p} \pi^-$ ^[242]. Figure 5.1 shows possible contributing Feynman diagrams to the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ decay. As the knowledge about this channel is poor, the first step is the measurement of the branching ratio. The $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ ($D_s^- \rightarrow K^+ K^- \pi^-$) decay channel is taken as a normalization channel, which has identical final states as that of the signal channel

to cancel large systematic uncertainties. The branching fraction ratio is determined as

$$\begin{aligned}
 R &= \frac{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-)}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-; D_s^- \rightarrow K^+ K^- \pi^-)} \\
 &= \frac{N(\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-)}{N(\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-; D_s^- \rightarrow K^+ K^- \pi^-)} \frac{\epsilon(\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-; D_s^- \rightarrow K^+ K^- \pi^-)}{\epsilon(\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-)},
 \end{aligned} \tag{5.1}$$

which can be obtained by measuring the numbers (N) of candidates from signal and normalization modes and the relative efficiency (ϵ) between the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ ($D_s^- \rightarrow K^+ K^- \pi^-$) decays.

5.2 Selection

5.2.1 Reconstruction and pre-selection

In this analysis, the dataset, corresponding to an integrated luminosity of 3 fb^{-1} , collected by the LHCb experiment in 2011 and 2012 is used. In both data and Monte Carlo (MC) samples^①, the decay tree fitter^[200] algorithm is used to constrain the Λ_b^0 to originate from the primary vertex, and the invariant mass to the known value^[1] for each open charm candidate. For the study of the relevant resonance structures, further constraint of the mass of Λ_b^0 to the PDG value^[1] is also applied. The resulting four momenta of final states are used to compute the invariant masses of the intermediate resonances in $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ decays. In the simulation study, 2.2 million events for $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and 0.7 million for $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ ($D_s^- \rightarrow K^+ K^- \pi^-$) are generated in the simulated pp collisions at $\sqrt{s} = 7 \text{ TeV}$ and 8 TeV . Beyond that, the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ channel is also generated via intermediate states: 1.4 million with $K(892)^{*0} \rightarrow K^+ \pi^-$ resonance and 1.4 million with also $a_1(1260)^+ \rightarrow K(892)^{*0} K^-$ resonance.

Table 5.1 Trigger lines for the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ candidates.

L0	Λ_b^0	L0HadronDecision(TOS)
HLT1	Λ_b^0	TrackAllL0Decision(TOS)
HLT2	Λ_b^0	Topo2(3,4)BodyBBDTDecision(TOS)

The adopted trigger lines^[172] for $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ are the same and listed in Tab. 5.1 ^②. The same L0 trigger is chosen so that the systematic uncertainty of L0 efficiency could cancel a lot in the normalization ratio, and the possible resonance state has been considered in the four-body topological line. In these two decays,

^① Simulation samples.

^② Details of trigger line are introduced in Chapter 3.

the Λ_c^+ is reconstructed as a combination of $pK^-\pi^+$. Then Λ_c^+ is combined with D_s^- or $K^+K^-\pi^-$ for the reconstruction of Λ_b^0 . These decays through $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ pass the Lb2LcKKPiLc2PKPiBeauty2CharmLine stripping line^[198], while the $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ candidates pass the Lb2LcDD2HHHPIDBeauty2CharmLine^③. Actually, the selection criteria for Λ_c^+ and Λ_b^0 candidates are identical in the stripping, while some difference is used to select the $K^+K^-\pi^+$ candidates directly from the Λ_b^0 or the D_s^+ decays. The details of the stripping line is shown in Appendix A.3.

Table 5.2 The shared pre-selections on the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ candidates.

#	Selection variable	requirements
1	$p(p)$	$> 10 \text{ GeV}$
2	$ M_{rec}(\Lambda_c^+) - M_{pdg}(\Lambda_c^+) $	$< 15 \text{ MeV}$
3	$\text{ProbNNp}(p \text{ from } \Lambda_c^+)$	> 0.1
4	$\text{DLL}_{p\pi} (p \text{ from } \Lambda_c^+)$	> -10
5	$\text{ProbNNk}(K \text{ from } \Lambda_c^+)$	> 0.1
6	$\text{DLL}_{K\pi} (K \text{ from } \Lambda_c^+)$	> -10
7	$\text{DLL}_{K\pi} (\pi \text{ from } \Lambda_c^+)$	< 20
8	$\text{DLL}_{K\pi} (\pi \text{ from } \Lambda_b^0 \text{ or } D_s^+)$	< 10
9	$\text{DLL}_{K\pi} (K \text{ from } \Lambda_b^0 \text{ or } D_s^+)$	> -2
10	$\chi_{vtx}^2(\Lambda_b^0)/\text{ndf}$	< 10
11	$\chi_{FD}^2(\Lambda_b^0)$	> 64
12	$\chi_{IP}^2(\Lambda_b^0)$	< 20
13	$\cos \theta_p$	> 0.99994
14	$z_{endvtx}(\Lambda_c^+) - z_{endvtx}(\Lambda_b^0)$	$> -1 \text{ mm}$
15	All tracks η	$[1.9, 4.9]$
16	All tracks p	$[5, 201] \text{ GeV}$

In addition to the stripping selection, additional pre-selection criteria for both channels are used to suppress the background, which are given in Tab. 5.2. The proton candidates are required to have a large momentum to aim for a better particle identification. The Λ_b^0 particles come from the primary vertex, and decay after a short while, so the Λ_b^0 candidates are required to have a relatively large flight distance significance (χ_{FD}^2) and a small impact parameter significance (χ_{IP}^2). The reconstructed Λ_b^0 momentum should point back to the primary vertex, which leads to a small value of pointing angle θ_p . The Λ_c^+ particles decay after flying a short distance from Λ_b^0 decay vertex, so the z position of the decay vertex of Λ_c^+ should be larger than that of Λ_b^0 . However $z_{endvtx}(\Lambda_c^+) - z_{endvtx}(\Lambda_b^0)$ could also be slightly smaller than zero due to the resolution, thus $z(\Lambda_c^+) - z(\Lambda_b^0) > -1\text{mm}$ is required. The same PID requirements in stripping as $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ are required in the pre-selections of $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ candidates.

③ Details of stripping line are introduced in Chapter 3.

Besides, momentum and pseudorapidity cuts on final state particles are applied in the pre-selection because of the coverage of the tracking efficiency calibration provided by tracking group^[245].

In the channel $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$, there is a peak from D_s^- in the invariant mass spectrum of $K^+ K^- \pi^-$ that are directly produced from the Λ_b^0 particle. They are the $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ decays, which are not our signals, thus are vetoed by applying a ± 30 MeV veto mass window. Besides, we applied a veto on $K^+ \pi^-$ mass spectrum on D^0 mass window (± 30 MeV). A ± 40 MeV D_s^- mass window cut is applied for the normalization channel, which can help remove many background.

5.2.1.1 Reflection veto cuts

The Λ_b^0 candidate could originate from $\bar{B}^0 \rightarrow D^+ K^+ K^- \pi^-$ or $\bar{B}_s^0 \rightarrow D_s^+ K^+ K^- \pi^-$ decays, where a pion or kaon in $D^+ \rightarrow K^+ \pi^- \pi^+$ or $D_s^+(D^+) \rightarrow K^+ K^- \pi^+$ decays is misidentified as a proton. These background contributions are vetoed if the invariant masses of the Λ_c^+ and Λ_b^0 candidates, evaluated by replacing the proton by either the pion or kaon mass hypothesis, are within ± 15 MeV of the known $D^+(D_s^+)$ mass and ± 25 MeV of the known $\bar{B}^0(\bar{B}_s^0)$ mass^[1]. These vetoes are applied to both the signal and the normalisation channels. For the signal decay, additional vetoes are applied if the invariant mass of the $K^+ \pi^-$ or $K^+ K^- \pi^-$ companion tracks falls within ± 30 MeV of the $\bar{D}^0$ or D_s^- known mass, respectively^[1].

5.2.1.2 Removal of internal track clone candidates

There is a special case of background: among the 6 tracks that are used to reconstruct $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^- (D_s^- \rightarrow K^+ K^- \pi^-)$, at least one pair of the tracks that have the same charge are clones to each other, identified by the angle between the two tracks. The distribution of the angle between a clone track pair accumulates around zero, while non-clone track pair has wider angle distribution. By requiring all pairs of tracks with the same charge that are in one Λ_b^0 candidate to have an opening angle larger than 0.5 mrad, these background contributions are removed. This procedure is made with the data sample after all the event selections above, and it removes around 2.5% candidates for signal channel and around 2.5% candidates for normalization channel.

5.2.1.3 Multiple candidates

There are multiple candidates in the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ decays due to the exchange between the kaon from Λ_c^+ and the kaon directly from the Λ_b^0 decays or from the

D_s^- for normalization channel. In this case, we also kept one of the candidate in the event randomly. This procedure is made with the data sample after all the event selections above, and it removes around 0.56% candidates for signal channel. The same process is applied to the normalization channel, though the removed number is negligible. The MC sample are also checked, and this kind of background does not appear in the simulation sample.

5.2.2 Multivariate analysis

The $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ events satisfying these pre-selections are then further filtered using a multivariate analyzer^[178]. Gradient Boosted Decision Tree (BDTG) and Multilayer perceptron (MLP) techniques are compared^[202,203]. We optimized the choice of discriminating variables and use thirteen variables: p_T sum of the Λ_c^+ decay products, minimal χ_{IP}^2 among the Λ_c^+ decay products, minimal p_T and minimal χ_{IP}^2 among the kaons originating directly from the Λ_b^0 decay, p_T and χ_{IP}^2 of the π^- from the Λ_b^0 decay, p_T sum of all π and K originating directly from the Λ_b^0 decay, χ_{vtx}^2 of the Λ_c^+ candidate, χ^2 of the flight distance between the Λ_b^0 decay vertex and the associated PV, $\cos \theta_p$, χ^2 probability of the Λ_b^0 candidate vertex fit, and the difference of longitudinal position between the Λ_c^+ and the Λ_b^0 decay vertices.

The BDTG and MLP techniques both involve a “training” procedure for event selection. A simulated signal MC and sideband (Λ_b^0 mass is larger than 5750 MeV) data background samples, both after pre-selection, are used to train the selection. Then separate samples are used to test the performance. The training and testing samples have almost equal event number, randomly divided from the total samples. Both the signal samples and the background samples are from the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ decays.

The distributions of the BDTG and MLP classifier for signal and background are shown in Fig. 5.2. Both MVA classifiers do not have obvious over-training and using MLP technique can get more consistent distributions between the training and test samples. An optimal cut point on the BDTG and MLP response is determined by maximizing the signal significance, $S/\sqrt{S+B}$, where S (B) is the expected signal (background) yield in signal region, which is around $\pm 3\sigma$ (± 30 MeV) of the known Λ_b^0 mass^[1]. The initial yields for signal and background, S_0 and B_0 , are estimated by using the mass spectrum of Λ_b^0 without BDTG or MLP requirements,

To maximize the signal significance, the BDTG and MLP working points are evaluated independently. We found the MLP method is more powerful to discriminate the background and signal, with a larger significance and less over-training effect. Hence, we used the MLP as the default method. A value of 0.35 is determined as the default working point with a significance about 69.1 using MLP method.

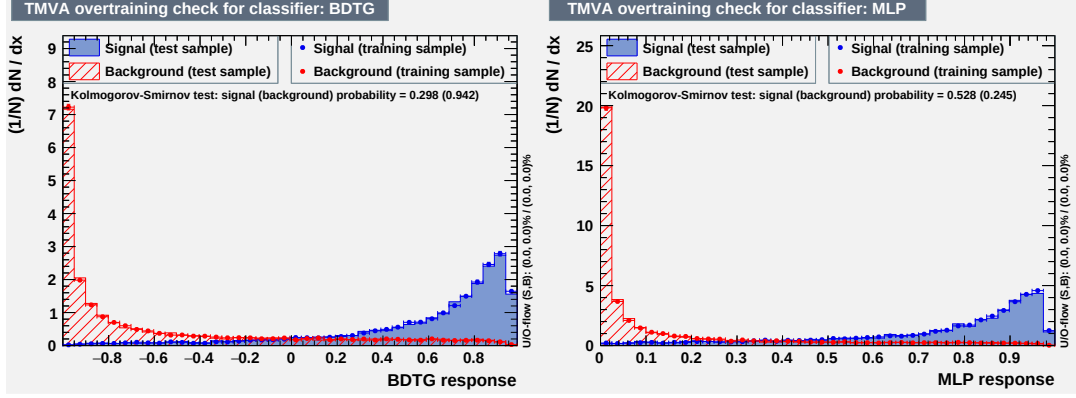


Figure 5.2 BDTG (left) and MLP (right) distributions for the signal and background for $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ decays.

For the normalization channel $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$, the BDTG and MLP weights obtained in the signal mode are used to evaluate the MVA outputs. To take into account the difference of number of degrees of freedom (ndf) between the signal mode and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$, χ^2 probability of the Λ_b^0 candidate vertex fit is adjusted to be ndf=5. Yields of signal and background are estimated and the MLP cut determined to be 0.09.

5.3 Signal extraction

5.3.1 Fit description

The signal yields of $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ ($D_s^- \rightarrow K^+ K^- \pi^-$) are extracted using unbinned extended maximum likelihood fits to the distributions of the PV-constrained and Λ_c^+ mass constrained invariant mass (m), while D_s^+ mass is not constrained to the known value^[200]. For the signal channel, the likelihood has the form

$$\mathcal{L} = \frac{e^{-(N_S + \sum_j N_{B_j})}}{N!} \times \prod_{i=1}^N [N_S P_S(m_i) + \sum_j N_{B_j} P_{B_j}(m_i)] \quad (5.2)$$

where N is the total number of events, index i run over all candidates. N_k ($P_k(m)$) is the yield (probability density function) for the category k as signal (S) and background (B_j) respectively. In addition to the combinatorial background, there is also physics background, thus index j accounts for different background sources.

5.3.1.1 Signal shape

The signal shape for both channels is modeled by the sum of two Crystal Ball (CB) functions^[223]. The pair of CB functions for each signal have a common mean. The general

form of the signal shapes is:

$$f_{sig} = f_{low} \times CB_-(m_0, \sigma_-, \alpha_-, n) + (1 - f_{low})CB_+(m_0, \sigma_+, \alpha_+, n) \quad (5.3)$$

where m_0 is the shared mean value of the CB functions, $\sigma_{\pm}$ are the width of the CB functions, $\alpha_{\pm}$ and n are parameters to describe the shape corresponding to the energy loss due to bremsstrahlung. And plus and minus signs suggest that the two CB have tails on opposite sides. With all parameters free, there are large correlations between the parameters. Fewer independent parameters are actually needed to describe the signal shapes well. $\alpha_{\pm}$ and n are determined using the simulated signal decays that pass all the selection as used for real data as well as the true particle matching. The fitted shapes are shown in Fig. 5.3, and the fit parameters are shown in Tab. 5.3, and these parameters will be fixed when we fit the data.

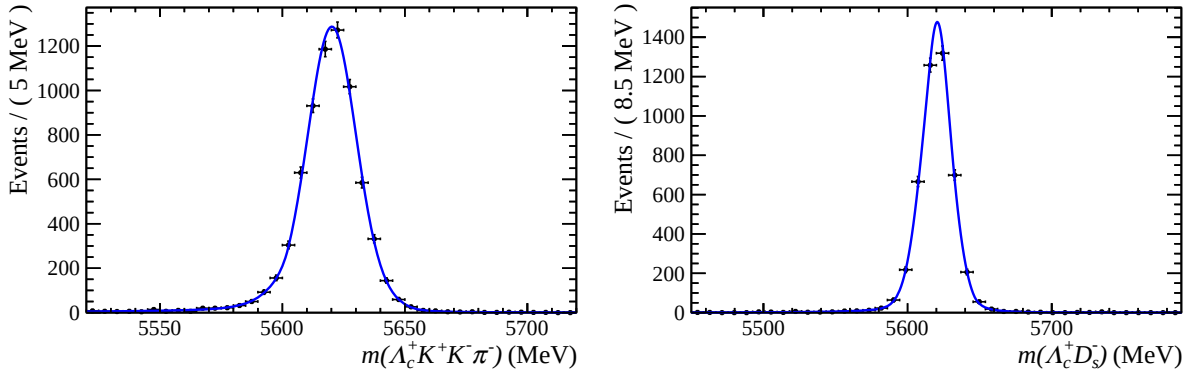


Figure 5.3 Signal shape for $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ (left) and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ (right) in the simulated events.

5.3.1.2 Background sources

The decay channel $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ may emit a photon and be reconstructed as $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$, the shape of this background is obtained from $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^*$ simulated decays, and this shape will be used in the mass spectrum fitting. The fitted shapes are shown in bottom Fig. 5.6.

For $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ channel, there is a clear presence of $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ in D_s^- sideband, where the value of $|m(K^+ K^- \pi^-) - m(D_s^-)|$ is in the range from 40 to 80 MeV, as shown in Fig. 5.4. The width of this sideband is the same as the D_s^- mass window, ± 40 MeV, making it

Table 5.3 Fit result of simulated $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$, $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ ($D_s^- \rightarrow K^+ K^- \pi^-$) mass spectrum

Parameter	$\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$	$\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ ($D_s^- \rightarrow K^+ K^- \pi^-$)
α_+	1.8893 ± 0.13594	1.5740 ± 0.26813
α_-	-2.37242 ± 0.16725	-2.11783 ± 0.31333
n	1.2957 ± 0.22705	2.4483 ± 0.61617

convenient to estimate the number of this background. The number of this kind of background and shapes would be fixed in the mass fit. For signal channel, there is a clear presence of $\Lambda_b^0 \rightarrow \Sigma_c^+ K^+ K^- \pi^-$ in $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ low sideband. The shape of this background is obtained from generator level simulated sample. Considering the mass resolution, we smeared the Λ_b^0 candidate mass with 27.4 MeV for the MC sample, which is the signal mass resolution estimated by fitting data sample. This kind of background distribution is shown in Fig. 5.5. An exponential function is used to describe the combinatorial background shape, with a shape parameter that is freely varied in the fit.

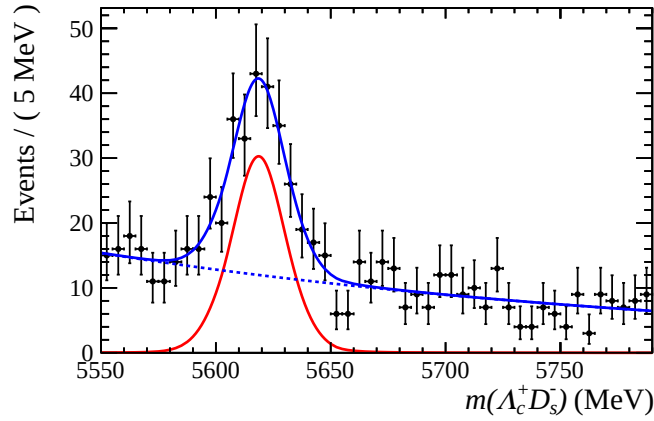


Figure 5.4 $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ where the $KK\pi$ mass is in the sideband of the D_s^- . The blue line is full fit, the blue dashed line is the combinatorial background, and the red line is the signal shape.

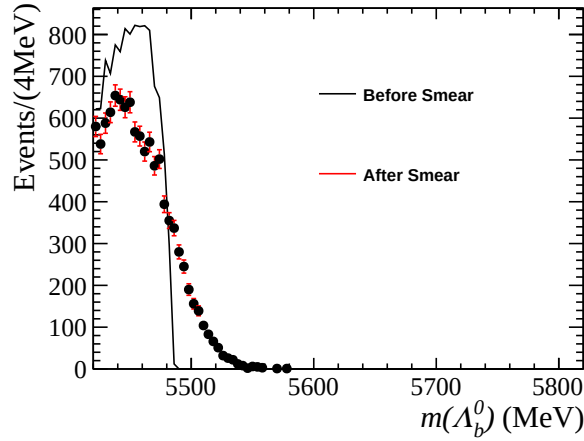


Figure 5.5 The distribution gained from $\Lambda_b^0 \rightarrow \Sigma_c^+ K^+ K^- \pi^-$ MC sample.

5.3.2 Fit results

The unbinned maximum likelihood fits were performed to the $\Lambda_c^+ K^+ K^- \pi^-$ and $\Lambda_c^+ D_s^-$ mass spectra, with some parameters fixed to the value obtained from MC. The fit results

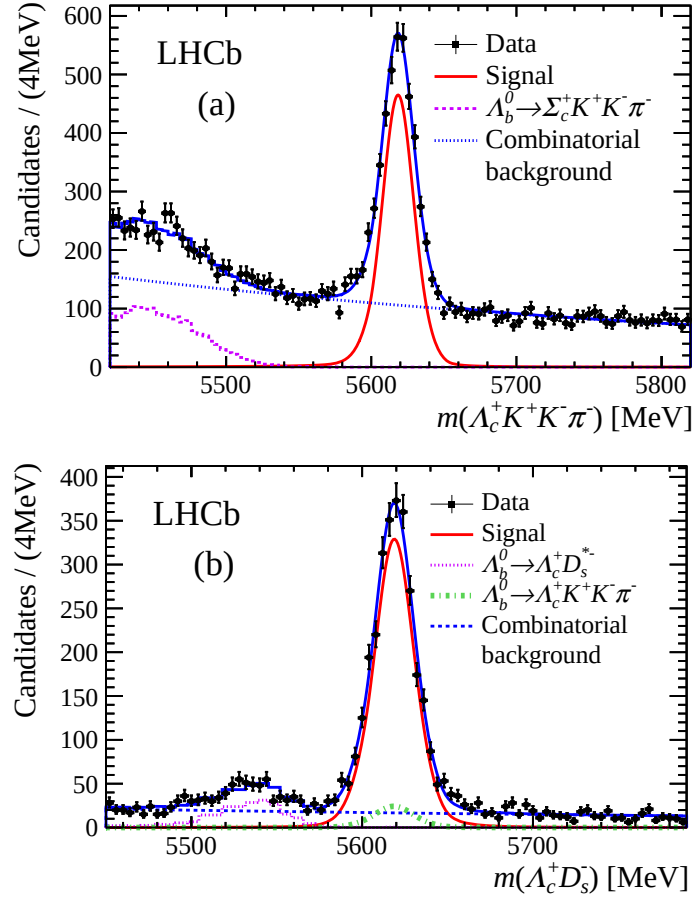


Figure 5.6 Invariant mass distribution of (a) $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and (b) $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ candidates. Fit projections are overlaid as a blue solid line. For (a), the red solid line represents the signal component, the blue dashed line is the background due to random combinations, and the violet dotted line is the contribution from $\Lambda_b^0 \rightarrow \Sigma_c^+ K^+ K^- \pi^-$ decays. For (b), the red solid line is the normalization channel component, the violet dotted line is the $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^{*-}$ background, the green dashed-dotted line is the contribution from $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ decays, and the blue dashed line represents combinatorial background.

Table 5.4 Fit results of $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ mass spectrum in data sample.

Parameter	Value
$\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$	
m_0 (MeV/ c^2)	5618.5 ± 0.3
n_{sig}	3411.0 ± 75.5
n_{phybkg}	1672.4 ± 26.3
$\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$	
m_0 (MeV/ c^2)	5618.8 ± 0.3
n_{sig}	2546.2 ± 57.1
n_{phybkg}	477.7 ± 14.8

for $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ are shown in Fig. 5.6 and Tab. 5.4. To reduce the number of free parameters, the ratio of σ^+ and σ_- in $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ signal shape is fixed to the

value obtained from the fit in MC sample.

5.4 Efficiency

5.4.1 Simulation sample reweights

To have a better estimation of the efficiency, the Monte Carlo (MC) sample is required to be tuned to match the data. This procedure including several steps is introduced below.

5.4.1.1 IP smearing

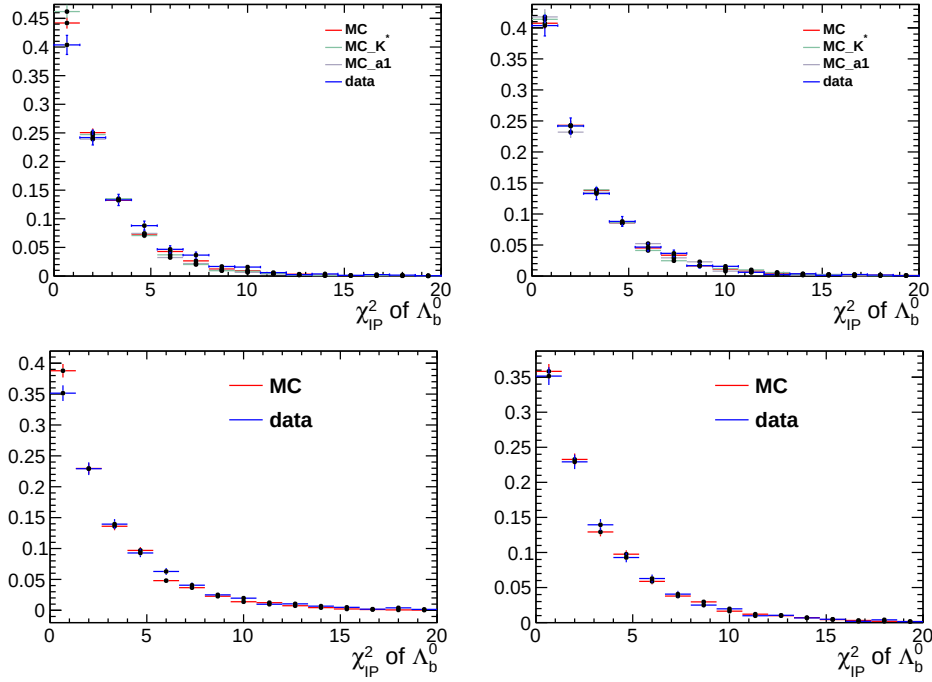


Figure 5.7 The comparison of background subtracted data and signal MC for and $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ (top) and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ (bottom), and the *sPlot* method is used to get the background subtracted data. The red line is the MC signal and blue is data. Left is the distribution before smearing and Right is after smearing.

The χ_{IP}^2 of the final state tracks and Λ_b^0 are different in the background subtracted data and signal MC in $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ and in $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ (Fig. 5.7). We smeared these variables by generating gaussian random numbers to get χ^2 distributed variables, and the distribution after smearing are shown in the right of Fig. 5.7. As for $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ decay, the same smear are applied to the MC sample. Besides, the variables used in TMVA, χ_{IP}^2 of the final tracks, are also tuned.

5.4.1.2 p_T , y and nTracks reweights

In order to take into account data-MC difference, especially the initial kinematic of the mother particle Λ_b^0 , the MC samples need to be reweighted. The p_T , rapidity y and nTracks^④ weights are obtained from the normalization channel $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$, we apply the weights to both $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ MC samples. Given that the p_T and y are feature variables of Λ_b^0 , which is the same mother particle for signal channel and normalization channel, it should be fine to apply the same weight to the signal channel. Besides, the track correction and PID efficiency correction are applied before applying p_T , y and nTracks weights. The p_T , y are weighted at the same time, the nTracks weight is applied next. The transverse momenta and rapidity of Λ_b^0 are compared between the data sample after background removed using the *sPlot*^[206] technique and the MC sample.

5.4.1.3 Λ_c^+ Dalitz reweight

The description of Λ_c^+ decays in simulated events can be different from the real situation. Since the MC have different Λ_c^+ decays in the signal and normalization channels, the weights are obtained separately. For normalization channel, the statistics of data and MC are both large, so we applied a 2-dimensional Dalitz weights to the MC sample according to the distributions of data. The comparison between MC and data for normalization channel is shown in Fig. 5.8.

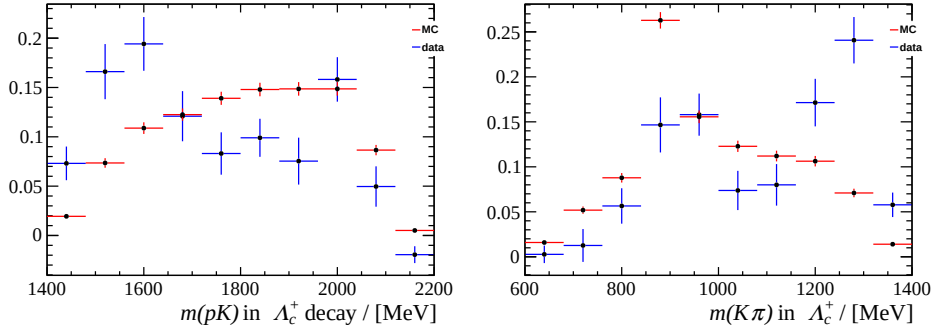


Figure 5.8 The distributions of invariant mass of combination of Λ_c^+ daughters in $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ channel before reweights.

For the signal channel, the same method is used. Because the statistics of the data is limited for signal channel, the bin width is larger than that used in the normalization channel. Figure 5.9 shows the comparison between data and MC for invariant mass distributions of pK and πK . 2-dimensional $(m(pK), m(\pi K))$ weights are obtained from MC and background subtracted data, and these weights are applied to both signal and normalization channels then.

④ The nTracks represents the multiplicity of the event, and this variate in the signal channel is similar to that in the normalization channel.

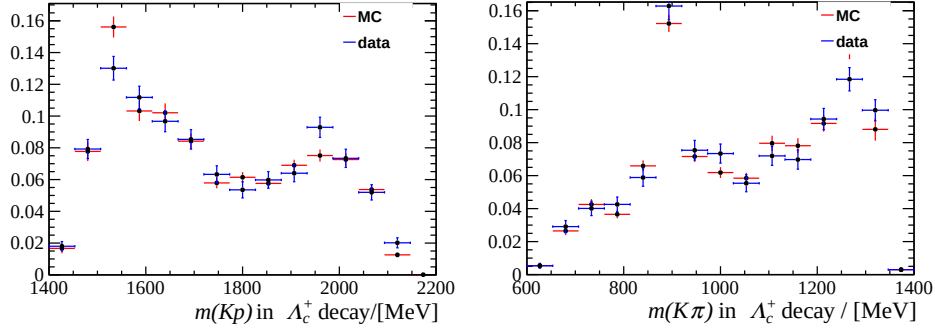


Figure 5.9 The distributions of invariant mass of combination of Λ_c^+ daughters in $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ channel before reweights.

5.4.1.4 Λ_c^+ polarization

The polarization of Λ_b^0 is consistent as no-polarization, however, Λ_c^+ could polarize because of weak decay of Λ_b^0 [242]. The MC simulation assumed no-polarization for both Λ_b^0 and Λ_c^+ . This assumption may lead to a different angular distribution from the Λ_c^+ decay in data and MC. As the polarization of Λ_c^+ in $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ may not be the same, so the signal channel and normalization channel are handled separately.

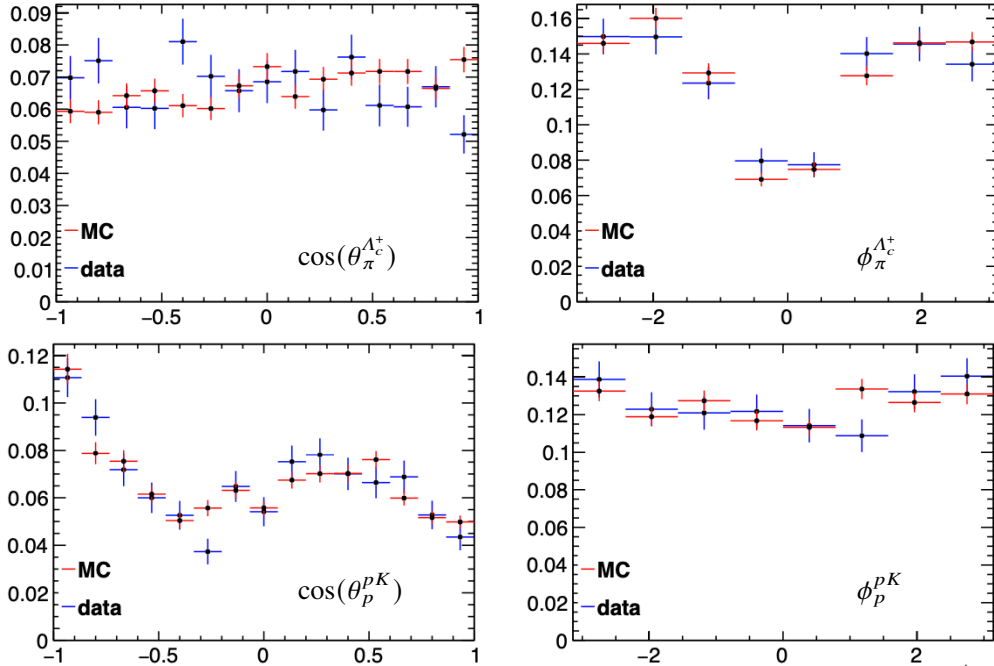


Figure 5.10 The distribution of some Λ_c^+ polarization angles in normalization channel ($\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$) before reweights.

We compare the distributions of $\theta_{\pi^+}^{\Lambda_c^+}$, $\phi_{\pi^+}^{\Lambda_c^+}$, θ_p^{pK} and ϕ_p^{pK} in data and MC of both signal channel and normalization channel, and all weights mentioned before had been applied to the MC sample. Here $\theta_{\pi^+}^{\Lambda_c^+}$, $\phi_{\pi^+}^{\Lambda_c^+}$ correspond to the polar angle and azimuthal angle, respectively,

of the π in the Λ_c^+ rest frame. And θ_p^{pK} and ϕ_p^{pK} correspond to the polar and azimuthal angle, respectively, of the proton in the pK rest frame. The method to obtain these decay angles are discussed in Chapter 3 and Ref. [124].

The results of these comparisons are shown in Fig. 5.10 for normalization channel and Fig. 5.11 for signal channel. There are not obvious inconformities for these distributions, therefore, these distributions are not weighted.

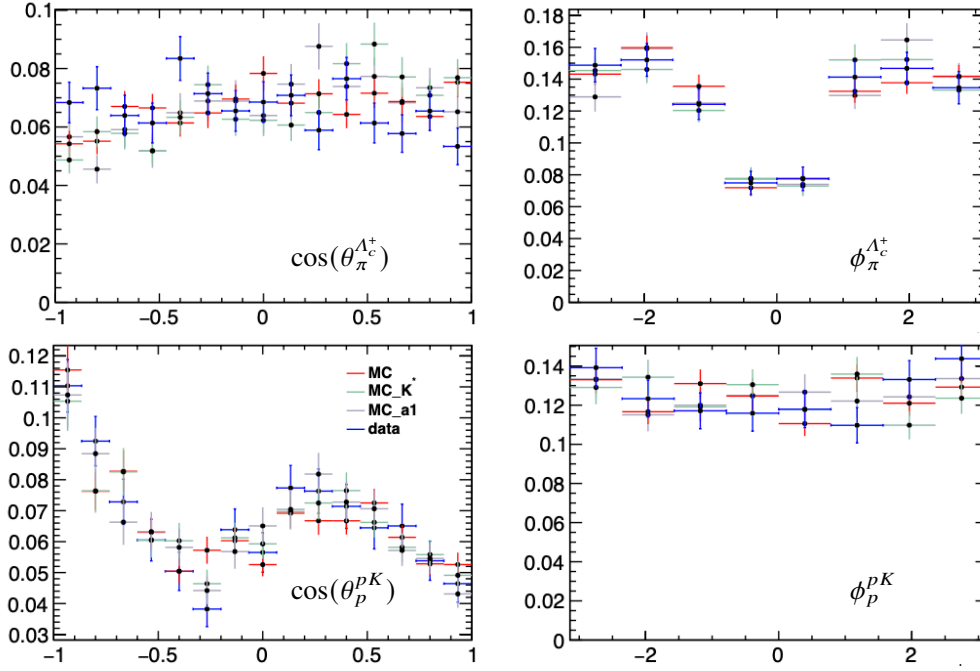


Figure 5.11 The distribution of some Λ_c^+ polarization angles in signal channel ($\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$) before reweights.

5.4.1.5 Resonances in $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$

The description of the intermediate states in the signal Λ_b^0 decays in simulated events can be different from the real situation, as many resonances are found in the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ from the data sample while PHSP model is used in the MC sample. Comparing the invariant mass distribution of two body combination or three body combination, we found the invariant mass of $K^+ \pi^-$ and $K^+ K^- \pi^-$ have large differences. To estimate the components of data sample, we fit it using three MC samples – PHSP $K^+ K^- \pi^-$ sample, with $K^*(892)^0 K^-$, and with $a_1(1260)^- \rightarrow K^*(892)^0 K^-$ resonance^[1] in two dimensional invariant masses of $K^+ \pi^-$ and $K^+ K^- \pi^-$. The fitting fractions are used to construct a blended MC sample to better describe data. The fitting results are listed in Tab. 5.5.

An open-charm pentaquark state could be revealed as a structure in the invariant mass

Table 5.5 Fit result of $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ using MC samples.

Parameter	Number
$n_{nonresonance}$	824.1 ± 36.7
n_{K^*}	275.2 ± 49.2
$n_{a_1(1260)^-}$	2059.8 ± 61.4

distribution of the $\Lambda_c^+ K^+$ system, shown in Fig. 5.12 for data and simulation. The data distribution is background subtracted through the *sPlot* weighting technique^[206], using the $\Lambda_c^+ K^+ K^- \pi^-$ invariant mass as discriminating variable. No structure is observed. A full amplitude analysis is needed to estimate the limit of the pentaquark contribution, which is beyond the scope of this analysis.

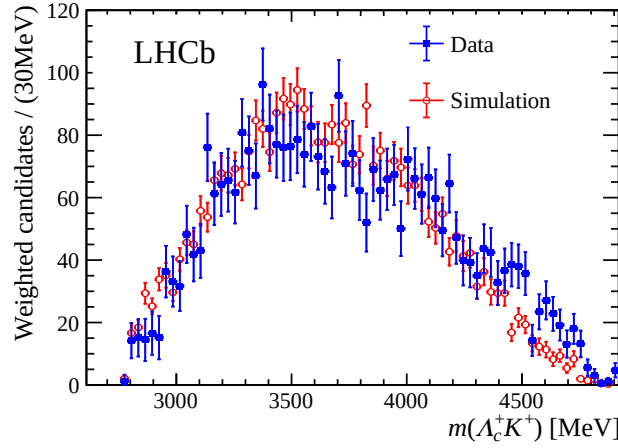


Figure 5.12 Invariant mass distributions of $\Lambda_c^+ K^+$ candidates in the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ signal channel for the simulation (red line) and the background-subtracted data (blue points with error bars).

Instead, a rich structure of known hadron contributions is visible in the background-subtracted invariant mass distributions of the $\Lambda_c^+ \pi^-$, $K^+ \pi^-$ and $K^+ K^- \pi^-$ systems, shown in Fig. 5.13. The $\Sigma_c(2455)^0$ and $\Sigma_c(2520)^0$ resonances are visible in the $\Lambda_c^+ \pi^-$ distribution. A large $K^*(892)^0$ resonance is observed in the $K^+ \pi^-$ projection. In the $K^+ K^- \pi^-$ system, a broad peaking structure at about 1.5 GeV is also observed. A similar structure is also seen in $\bar{B} \rightarrow D^{(*)} K^{*0} K^-$ decays by the Belle experiment^[246], and is explained as the tail contribution of the $a_1(1260)^-$ resonance.

5.4.2 Efficiency calculation

The total efficiency for $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ decays is defined as

$$\epsilon_{\text{tot}} = \epsilon_{\text{acc}} \cdot \epsilon_{\text{rec\&sel\&hlt|acc}} \cdot \epsilon_{\text{L0trig|sel\&hlt}}, \quad (5.4)$$

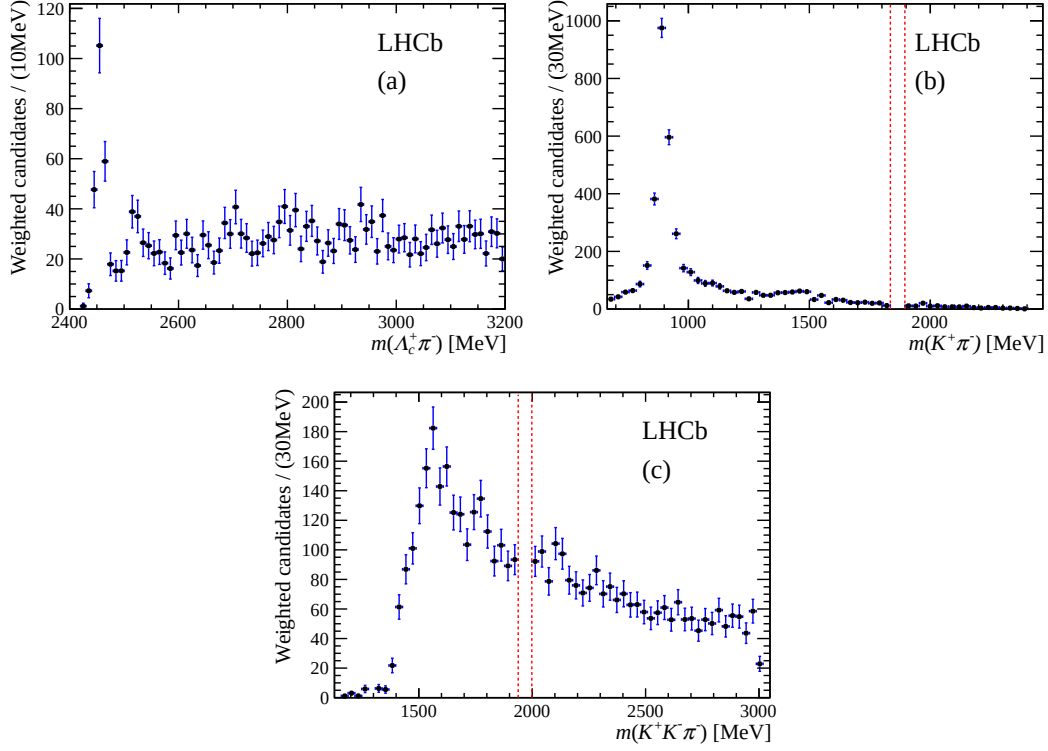


Figure 5.13 Invariant mass distributions of (a) the $\Lambda_c^+ \pi^-$, (b) $K^+ \pi^-$, and (c) $K^+ K^- \pi^-$ systems in the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ signal channel, for the background-subtracted data. The red dashed vertical lines indicate the veto mass intervals for $\bar{D}^0$ mesons in the $K^+ \pi^-$ distribution, and D_s^- in the $K^+ K^- \pi^-$ distribution.

where ϵ_{acc} is the geometrical acceptance of the LHCb detector, $\epsilon_{\text{rec\&sel|acc}}$ stands for the reconstruction and selection efficiency, $\epsilon_{\text{trig|sel}}$ is the trigger efficiency. The summary of efficiencies is shown in Tab. 5.6. The detailed approach to estimate these efficiencies is introduced below.

5.4.2.1 Geometrical acceptance

The geometrical acceptance requires all final state particles in $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ ($\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$) decays to be within a polar angle from 10 to 400 mrad. It is studied with generator level MC events for both $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ decays respectively, and these events are generated without the daughter in LHCb acceptance requirement. Then the requirement is applied to the MC samples, and the acceptance efficiency is calculated as $\epsilon_{\text{acc}} = N_{\text{acc}}/N_{\text{GL}}$, where N_{acc} is the number of events after the LHCb acceptance cuts and N_{GL} is the number of events with only the requirement of the fiducial cuts.

5.4.2.2 Reconstruction, selection and HLT efficiency

The reconstruction and selection efficiency is defined as $\epsilon_{\text{rec\&sel|acc}} = N_{\text{rec\&sel\&hlt}}/N_{\text{acc}}$, where $N_{\text{rec\&sel\&hlt}}$ is the signal yield of the full simulation sample after event selection and

Table 5.6 Relative efficiency values. Uncertainties listed are due to finite MC statistic, and are discussed in detail in Sec. 5.5

$\frac{\epsilon(\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-)}{\epsilon(\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-; D_s^- \rightarrow K^+ K^- \pi^-)}$	Value
Acceptance	0.9947 ± 0.0061
Rec. Sel. and Hlt (No PID)	0.8286 ± 0.0128
Tracking Correction	1.0003 ± 0.0031
PID	1.0032 ± 0.0064
L0 Trigger	0.9458 ± 0.0168
Total	0.7822 ± 0.0198

passing HLT1 and 2 requirements (without PID requirement), and N_{acc} is the corresponding number of generated events in the LHCb acceptance. The efficiency is estimated by candidate-by-candidate method using weighted MC samples, while HLT and MVA selections are also included.

5.4.2.3 Tracking efficiency

The tracking efficiency is estimated from simulation, and this value should be calibrated by factors of the MC efficiencies^[245]. The correction of MC efficiency is taken as a function of the momentum and pseudorapidity of the track, and the event-by-event correction factor is taken as the product of the correction factors of each final tracks. After the correction, we calculate the ratio tracking efficiencies with the weight.

5.4.2.4 L0 Trigger efficiency

The trigger efficiency tables are taken from calibration data, with 2011 and 2012 separately. These tables are used to compute L0 Hadron TOS decision, in bins of their real E_T ^⑤ when arriving to the HCAL surface, and separately for the inner and outer regions of the calorimeter^[247]. We reweight the MC samples using these tables. Then, the trigger efficiency of per event is computed by combining the efficiencies corresponding to each one of the final tracks.

5.4.2.5 PID selection efficiency

The PID variables are not well described by the simulation, therefore a data-driven method is used to calculate the PID efficiency. The PID efficiencies for the $p, \bar{p}, \pi^\pm, K^\pm$ are computed using the PIDCALIB package^[234], which offer 3-dimensional histograms, where the

⑤ E_T is the transverse energy of tracks.

efficiency of a certain PID requirement of a track are shown as a function of the momentum and pseudorapidity of the track, and the multiplicity of the event. The single event efficiency is taken from this method, and the total PID efficiency of $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ ($\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$) is derived by averaging the single event efficiencies of the candidates in the full simulated signal samples that pass all the event selections.

5.5 Systematic uncertainties

Table 5.7 Summary of systematic uncertainties on the ratio of branching fractions.

Source	Uncertainty (%)
Combinatorial background	0.9
Shape of $\Lambda_b^0 \rightarrow \Sigma_c^+ K^+ K^- \pi^-$ contribution	0.3
$\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ background in normalization channel	0.8
Signal fit model	0.5
Simulation sample size	2.5
PID efficiency	0.4
Trigger efficiency	0.1
(p_T, y) weight	0.8
Track multiplicity weight	0.8
Λ_c^+ Dalitz structure	1.4
Mixture fraction in simulation	0.2
Resonance structure	3.6
Multiple candidates	0.3
MVA selection	0.5
Total	4.9

All systematic uncertainties on the measurement of the ratio of branching fractions are listed in Tab. 5.7. The total uncertainty is determined from the sum of all contributions in quadrature. The dominant uncertainty is related to the resonance structure that is not perfectly modeled by the simulation.

Uncertainties due to the fit model are considered. For the background due to random combinations of final-state particles in both the signal and normalization channels, the exponential function is replaced by a second-order polynomial function. From the comparison to the default result, the relative uncertainty on the ratio of branching fractions is 0.9%. In the signal channel, the uncertainty due to the $\Lambda_b^0 \rightarrow \Sigma_c^+ K^+ K^- \pi^-$ background contribution is assessed by performing the fit with a widened mass region, resulting in a relative uncertainty of 0.3%. For the normalization channel, changing the yield of the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ contribution within its uncertainty results in a relative 0.8% variation.

The systematic uncertainty due to the model for both signal and normalization channels, is studied by changing to a single Hypatia function^[205], where the mean and width parameters are left free while all other parameters are taken from simulation. This results in a relative uncertainty of 0.5%.

The uncertainties on the ratio of efficiencies are evaluated. The uncertainty due to the finite simulation sample size is evaluated from the expected efficiency variation in bins of p_T and y of the Λ_b^0 candidate as

$$\sigma_\epsilon = \sqrt{\sum_i \epsilon_i (1 - \epsilon_i) N_i w_i} / \sum_i N_i w_i, \quad (5.5)$$

for each bin i , where N_i is the number of generated events, w_i is a correction weight, and ϵ_i is the candidate efficiency. The normalization of the weights is chosen such that the denominator is equal to total number of generated events without the weighting. The relative uncertainty is found to be 2.5%.

Pseudoexperiments are used to evaluate the systematic effects due to uncertainties on the weights or efficiencies in different bins. For a given source, many pseudoexperiments are generated, in which each produces a new set of weights or efficiencies according to the central values and uncertainties following Gaussian distributions. The efficiency ratio between the signal and normalization channels is recomputed. The resulting efficiency ratios from many pseudoexperiments of this source produce a Gaussian distribution centering at the baseline value. The standard deviation of the Gaussian distribution is taken as absolute uncertainty on the efficiency ratio for the given source. The procedure is applied to obtain the systematic uncertainty related to the PID and trigger efficiencies and to (p_T, y) and track multiplicity weighting.

The tracking efficiency returned by the simulation is calibrated using a data-driven method^[245]. The uncertainty on the calibration sample size is propagated to the efficiency ratio using pseudoexperiments, resulting in a systematic uncertainty of 0.8%. Because the final states for signal and normalization modes are identical, possible data-simulation differences in hadron interactions with the detector material are assumed to be negligible.

The agreement between data and simulation for the $\Lambda_c^+ \rightarrow pK^-\pi^+$ channel is tested by comparing the Dalitz structure. The signal simulation sample is weighted in the $m(pK^-)$ versus $m(K^-\pi^+)$ plane to match the distribution of the background-subtracted data. The uncertainty related to the limited sample size used for obtaining these weights is 1.1%, obtained from pseudoexperiments. The uncertainty related to the choice of binning is 0.8%, determined by

using an alternative binning. A total of 1.4% is assigned as systematic uncertainty.

The contributions of the Λ_b^0 decays through the mixture of the three decay modes are considered when generating the simulated events of the signal channel, and their fractions are obtained by fitting the two-dimensional distribution of the $K^+ K^- \pi^-$ and $K^+ \pi^-$ systems in the background-subtracted signal data. The fractions are changed according to the statistical uncertainty of the fit result, yielding 0.2% of relative uncertainty.

The simulation does not fully model the resonance structure, *e.g.* the contribution of Σ_c^0 resonances, which is clearly seen in the $\Lambda_c^+ \pi^-$ invariant mass distribution, as illustrated in Fig. 5.13. By weighting the simulation to match the $m(\Lambda_c^+ \pi^-)$ distribution in the data, a 1.3% variation of the ratio of branching fractions is found and assigned as systematic uncertainty. Besides, differences between background-subtracted data and simulated signal events are also observed in the invariant mass distributions of the $\Lambda_c^+ K^+ K^-$ and $K^+ K^-$ systems. To account for this discrepancy, the simulated sample is weighted according to the $\Lambda_c^+ K^+ K^-$ or $K^+ K^-$ mass distribution of background-subtracted data, and the ratio of branching fractions is reevaluated. The two procedures return changes of 2.6% and 2.0%, respectively. The three values are added in quadrature to account for the uncertainty due to resonance structure.

Simulation does not account well for multiple candidates, which is found to be about 0.6% of the data sample in the signal channel. Half of this fraction is assigned as systematic uncertainty due to the random choice to retain only one candidate.

The MVA selection criteria are optimized separately for the signal and normalization channels. As an alternative choice, the MVA selection of the normalization channel is fixed to be the same as that of the signal channel to test the robustness of the MVA selection. The relative variation of the branching fraction ratio is 0.5%, which is assigned as systematic uncertainty.

5.6 Conclusions

With the 3 fb^{-1} data collected in 2011 and 2012, the observation $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ decay channel has been performed. Taking $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ as the normalization channel, the ratio of products of branching fraction is measured to be

$$\frac{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-)}{\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-)} = (9.26 \pm 0.29 \pm 0.46 \pm 0.26) \times 10^{-2},$$

where the uncertainties are statistical, systematic and uncertainty of $\mathcal{B}(D_s^- \rightarrow K^+ K^- \pi^-)$. Using this ratio, the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ branching fraction is determined to be

$$\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-) = (1.02 \pm 0.03 \pm 0.05 \pm 0.10) \times 10^{-3},$$

where the third term is the uncertainty on the branching fraction of the $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ decay^[1].

The predicted pentaquark can be checked in the $\Lambda_c^+ K^+$ mass spectrum^[248]. Figure 5.12 shows the invariant mass distribution of $\Lambda_c^+ K^+$, obtained with the background subtracted data. There is no significant peak in this distribution.

Chapter 6 Summary and prospects

6.1 Summary

This dissertation reports a couple of studies of exotic hadrons using proton-proton collision data collected by the LHCb experiment in Run 1 and Run 2, aiming at a better understanding of the properties exotic hadrons. The first study is a full amplitude analysis of the $B^+ \rightarrow J/\psi \phi K^+$ decay. It results in the observation of two tetraquark states with strangeness in the $J/\psi K^+$ final state and two X states in the $J/\psi \phi$ combination. The second study is the analysis of the $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay, which leads to the observation a new narrow pentaquark state $P_c(4312)^+$ and the fine structure of the previously observed $P_c(4450)^+$. The third analysis is a search for open-charm pentaquark states, which leads to the observation of a new Λ_b^0 decay, $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$. No open-charm pentaquark state is observed, and a measurement of the branching fraction, $\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-)/\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-)$, is measured. Main results are summarized below.

• Observation of Z_{cs}^+ and X states in the $B^+ \rightarrow J/\psi \phi K^+$ decay

The observation of two Z_{cs}^+ states, $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$, with a new quark content ($c\bar{c}u\bar{s}$) decaying to the $J/\psi K^+$ final state is reported with high significance from an amplitude analysis of the $B^+ \rightarrow J/\psi \phi K^+$ decay in Chapter 3. This analysis is carried out using proton-proton collision data corresponding to an integrated luminosity of 9 fb^{-1} collected by the LHCb experiment at center-of-mass energies of 7, 8 and 13 TeV. The most significant state, $Z_{cs}(4000)^+$, has a mass of $4003 \pm 6^{+4}_{-14} \text{ MeV}$, a width of $131 \pm 15 \pm 26 \text{ MeV}$, and spin-parity 1^+ , where the quoted uncertainties are statistical and systematic, respectively. In addition, The significance of the $Z_{cs}(4220)^+$ state also exceeds five standard deviations, while its spin-parity quantum number is not conclusive.

The $Z_{cs}(3985)^+$ state observed by BESIII^[184] is also explored in the $B^+ \rightarrow J/\psi \phi K^+$ sample, there is no evidence that the $Z_{cs}(4000)^+$ state observed is the same as the $Z_{cs}(3985)^+$ state discovered by BESIII. More studies are needed to clarify the situation from both experimental and theoretical sides.

The four exotic states, $X(4140)$, $X(4274)$, $X(4500)$ and $X(4700)$, which were previously observed in the $J/\psi \phi$ final state previously in 2016 using the Run 1 data, are confirmed with larger significance in this study. Apart from these four states, two new states, $X(4685)$ and

$X(4630)$, are observed with high significance. The J^P of the $X(4685)$ state is determined to be 1^+ , while that of the $X(4630)$ state is not conclusive. For the $X(4140)$ state, the width determined in the 2016 analysis is significantly larger than that from other experiments, which is confirmed by this study. Therefore, the tension between the LHCb results and those from other experiments still exists. Updated results from full amplitude analysis with data samples of much higher statistics from CMS, and probably Belle II, are expected.

• Studies of hidden-charm pentaquark states

The studies of hidden-charm pentaquark states using the full Run 1 and Run 2 data sample are presented in Chapter 4. With significantly improved event selection, and owing to the increased luminosity of the collected data, the number of the $\Lambda_b^0 \rightarrow J/\psi p K^-$ signals is larger than that of the 2015 analysis by a factor of 9, while the purity remains the same. With this pure and large data sample, and thanks to the excellent mass resolution of the LHCb detector, a new narrow pentaquark state, $P_c(4312)^+$, is observed in the $J/\psi p$ mass spectrum. In addition, the previously reported $P_c(4450)^+$ structure is resolved into two narrow structures whose masses are close to each other. These two states are referred to as $P_c(4440)^+$ and $P_c(4457)^+$. The $P_c(4312)^+$ and $P_c(4457)^+$ states match the predictions for loosely bound and narrow $\Sigma_c^+ \bar{D}^0$ and $\Sigma_c^+ \bar{D}^{*0}$ molecular states, respectively.

• Searching for open-charm pentaquark states in the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ decay

A new Λ_b^0 decay mode, $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$, is observed using the Run 1 data, and open-charm pentaquark states are searched for using this decay. This study is described in Chapter 5. No obvious open-charm pentaquark state is observed in the $\Lambda_c^+ K^+$ mass distribution. The ratio of the branching fractions between this decay and the $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ decay is measured to be $(9.26 \pm 0.29 \pm 0.46 \pm 0.26) \times 10^{-2}$.

6.2 Prospects

All the observations made in this dissertation indicate that LHCb is a good experiment to study exotic hadrons. The LHCb Upgrade is now under construction, and it is expected to start data taking in 2022, as illustrated in Fig. 6.1. The designed instantaneous luminosity for the data taking is five times higher than that in Run 1 and Run 2. With the almost fully new detector and fully new software-based triggers, the efficiency of the data taking will be significantly improved. The data sample size of hadronic modes is expected to be increased

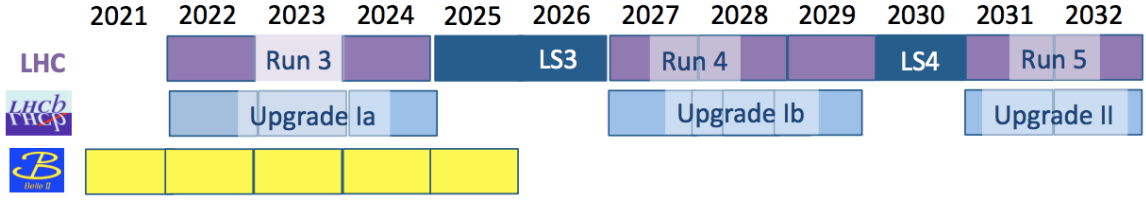


Figure 6.1 A time-line showing the operations of the LHC and the HL-LHC over the next decade, including long shutdown (LS) periods, as can be estimated today. The operational periods of LHCb and Belle II are shown^[249].

by a factor of 10 to 20, allowing for more precise measurements and new searches for exotic hadrons.

With a larger data sample collected after Run 3, the J^P quantum numbers of the $Z_{cs}(4220)^+$ and $X(4630)$ states are expected to be determined. The neutral isospin particle of $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ can be searched for in an akin decay channel, $B^0 \rightarrow J/\psi \phi K_S^0$. When it comes to the hidden-charm pentaquark sector, the J^P of observed pentaquark states have not been determined yet, and they are expected to be explored after Run 3. Considering the fact that the pentaquark states are currently only observed in the $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay at LHCb, it is greatly helpful if they could be confirmed in different decay channels and/or in different experiments. As mentioned in Chapter 4, there is a great hope to confirm the observed P_c^+ states in $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ channel after Run 3 at LHCb. As for open-charm pentaquark states, some analogous decay channels can be explored to search for such states. It is promising that the open-charm pentaquark states can be observed in further studies.

In addition, the LHCb Upgrade II is proposed to fully exploit the opportunity provided by the HL-LHC. It is expected to start data taking after 2030, and to accumulate a data sample of 300 fb^{-1} . With this expected data sample, the study of exotic hadrons might lead us to a new understanding of the strong interaction.

Appendix A Additions related to physics analyses

A.1 More checks on the $B^+ \rightarrow J/\psi \phi K^+$ decay

A.1.1 Check on the non- ϕ sideband

The B^+ candidates with $1050 < m_{KK} < 1100$ MeV are selected, and referred to as the ϕ sideband sample. The yields of $B^+ \rightarrow J/\psi K^+ K^- K^+$ signal and of combinatorial background in the ± 15 MeV mass range around the peak are summarized in Tab. A.1.

Table A.1 The signal and combinatorial background yields in the ± 15 MeV range around the B mass peak for $B^+ \rightarrow J/\psi K^+ K^- K^+$ candidates in the ϕ sideband (see the text).

Type	Yields
Signal	3766 ± 73
Combinatorial background	2609 ± 65

The $m_{J/\psi K}$, $m_{J/\psi \phi}$ and $m_{\phi K}$ distributions are shown in Fig. A.1. The $m_{J/\psi K}$ distribution for the $m_{J/\psi \phi} \in (4.25, 4.35)$ MeV slice is shown in Fig. A.2. There is no sign of any mass peaking, in particular of $Z_{cs}(4000)^+$. The Dalitz plots in the ϕ sideband sample are shown in Fig. A.3. The $J/\psi \phi$ mass structures, which are prominent in the $B \rightarrow J/\psi \phi K$ sample, are not present when the m_{KK} is selected in the ϕ sideband.

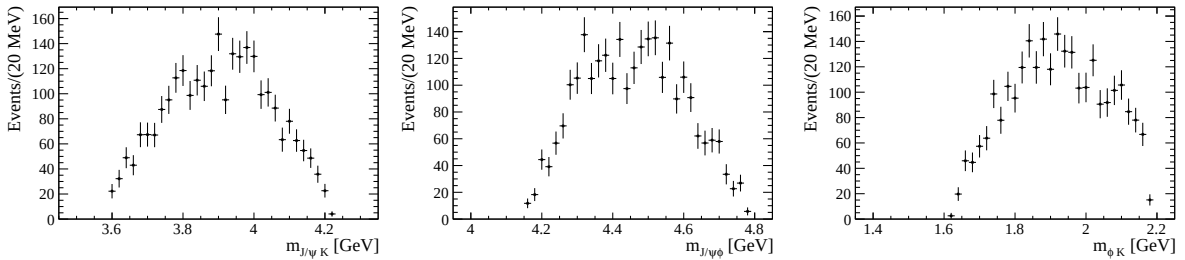


Figure A.1 The mass distribution of $m_{J/\psi K}$, $m_{J/\psi \phi}$ and $m_{\phi K}$ with phi sideband sample.

A.1.2 Reflection checks

We did several checks to show there is no reflection background for the bachelor kaon. In order to make sure that the Z_{cs}^+ peak in $m(J/\psi K)$ spectrum is not caused by wrongly reconstructing π as K , a tighter ProbNNk cut > 0.6 is applied. The spectrum of $m(J/\psi K)$ is shown in Fig. A.4. The $Z_{cs}(4000)^+$ peak remains.

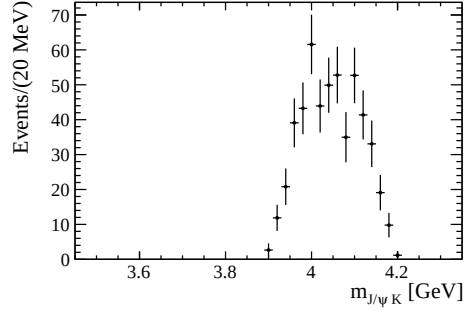


Figure A.2 The mass distribution of $m_{J/\psi K}$ with $m_{J/\psi\phi} \in (4.25, 4.35)$ MeV.

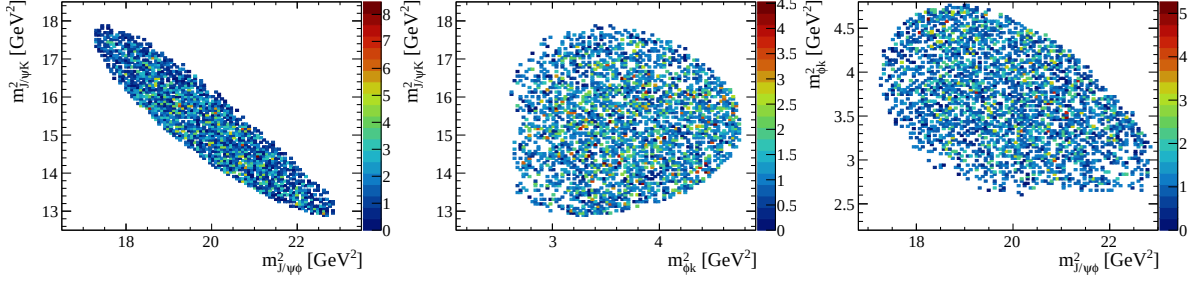


Figure A.3 The Dalitz plots with ϕ -sideband sample.

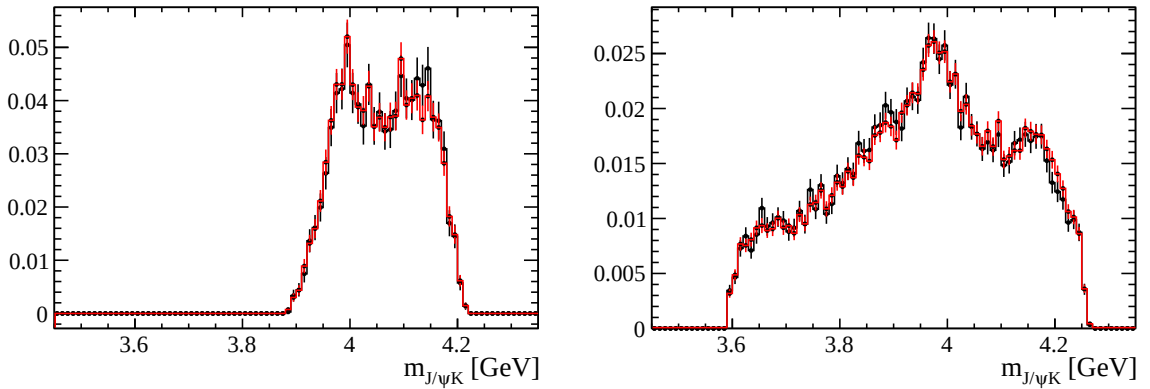


Figure A.4 The distribution of $m(J/\psi K)$. The left plot is obtained with $m(J/\psi\phi) \in [4.25, 4.35]$ GeV range, the right plot is the full range. The black histogram is obtained with tight ProbNNk cut, the red line with the nominal one.

We also showed the $J/\psi K^+$ vs B^+ masses in Fig. A.5. No sideband background connects with the $Z_{cs}(4000)^+$ peak region.

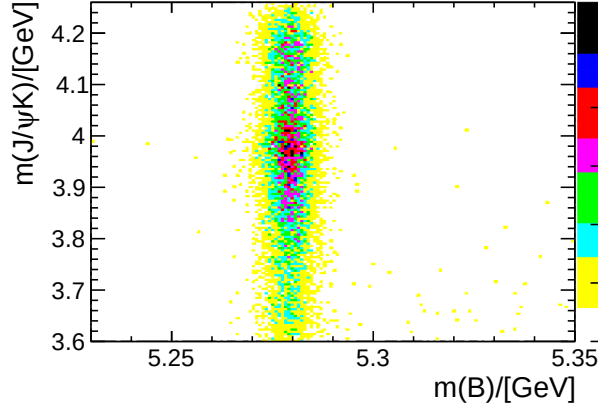


Figure A.5 Distribution of $m(J/\psi K)$ versus $m(J/\psi \phi K)$.

As an additional check, we look at $\beta = [p(J/\psi) + p(\phi) - p(K)]/[p(J/\psi) + p(\phi) + p(K)]$ vs. B^+ mass. In order to enhance the sideband visibility, we vetoed the B^+ peak region. If there was a reflection contribution from mis-ID of the bachelor K , we would have seen a slanted bend across the B^+ mass, but we don't find such bends. The plot is shown in Fig. A.6.

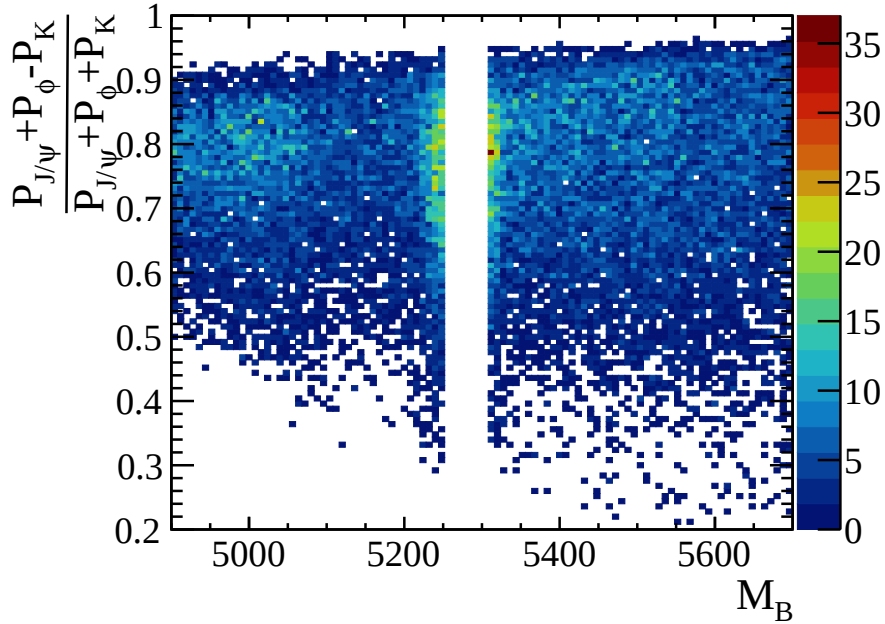


Figure A.6 β vs $m(J/\psi \phi K)$.

The distribution of $m_{J/\psi \pi}$ or $m_{J/\psi p}$ with the hypothesis that K wrongly reconstructed as π or p is shown in A.7. The $m_B \in [5249.64, 5309.64]$ MeV candidates are vetoed. The plots will have a narrow peak if there are reflections.

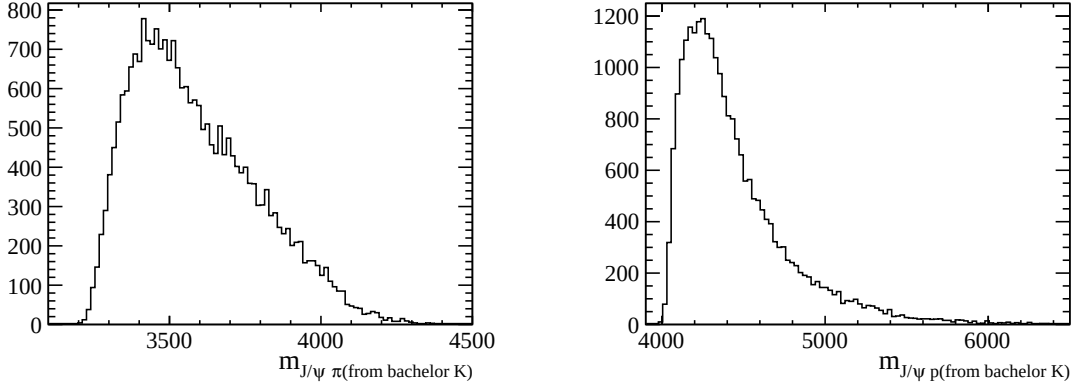


Figure A.7 The left plot is $m_{J/\psi \pi}$ distribution where K is wrongly constructed as π , the right plot is $m_{J/\psi p}$ distribution where K is wrongly constructed as p .

The distribution of $m_{J/\psi \phi K}$ with the hypothesis where K is wrongly reconstructed as π or p is shown in A.8.

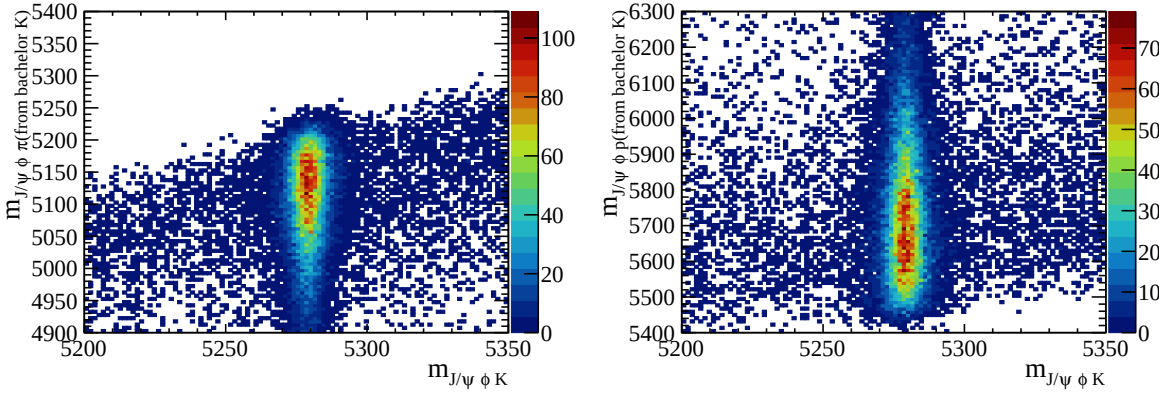


Figure A.8 The left plot is the $m_{J/\psi \phi \pi}$ distribution where K is wrongly constructed as π versus $m_{J/\psi \phi K}$, the right plot is the $m_{J/\psi \phi p}$ distribution where K is wrongly constructed as p versus $m_{J/\psi \phi K}$.

A.2 Selection of the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decay

A.2.1 Dataset

This analysis uses all the pp collision data in Run 1 and Run 2 stage, corresponding to an integrated luminosity of 3 fb^{-1} (6 fb^{-1}) in Run 1 (Run 2). The simulation samples of the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decay are generated by PYTHIA with pp collisions at $\sqrt{s} = 7 \text{ TeV}$, 8 TeV and 13 TeV respectively, with the ratio of the numbers of events roughly one to two between Run 1 and Run 2. Both PYTHIA 6 and PYTHIA 8 generators are used. The Monte Carlo event type is 15144021, in which Λ_b^0 is forced to decay into the $J/\psi p \pi^-$ final state with the PHSP model.

A.2.2 Reconstruction and pre-selection

In this analysis the reconstruction of the Λ_b^0 candidate starts from the stripping line `FullDSTDiMuonJpsi2MuMuDetachedLine` in the `DiMuon` stream of the stripping reconstruction, which different version, S21 for 2011, S21r1 for 2012, S24 for 2015, S28 for 2016, S29r2 for 2017, and S34 for 2018. The J/ψ candidate is formed by two oppositely charged tracks consistent with muon hypotheses (`IsMuon`) with $p_T > 550$ MeV and good track fit quality $\chi^2/\text{ndf} < 4$. The J/ψ candidate is required to have an invariant mass within the ± 100 MeV window around the known J/ψ mass^[214], and to have a vertex fit quality $\chi_{\text{vtx}}^2 < 20$. The J/ψ candidate is consistent with originating from b -hadron decays by requiring its decay length significance (DLS) to be greater than three.

Comparing to the previous study^[131], the selection criteria is optimized, all differences between the new selection criteria and old study are summarized in Tab. A.2. To select good J/ψ candidates, a tightened PID cut $\text{DLL}_{\mu\pi} > 0$ is required for both muon tracks in the final state, and the di-muon invariant mass window is reduced to $[-48, 43]$ MeV around the known J/ψ mass^[1]. The J/ψ mass resolution is around 13 MeV. To reconstruction the Λ_b^0 candidate, the J/ψ candidate is combined with two opposite charged tracks, consistent with pions and protons respectively. This is achieved by PID cuts on the `ProbNN` variables: $\text{ProbNN}_\pi(\pi) > 0.2$ and $\text{ProbNN}_p(p) > 0.2$. The two hadronic tracks are required to have large transverse momenta, $p_T > 250$ MeV, inconsistent with originating from the primary vertex (PV), $\chi_{\text{IP}}^2 > 9$. The Λ_b^0 candidate is required to have a good vertex fit quality, $\chi^2/\text{ndf} < 10$, with a displayed decay vertex, flight distance to primary vertex is larger than 1.5 mm. It should be consistent with being produced from PV by requiring $\chi_{\text{IP}}^2 < 25$ and cosine value of the direction angle, $\text{DIRA} > 0.999$. And the χ^2 of closet approach distance between p and π^- is smaller than 20. The Λ_b^0 candidates with mass $5.1 < M(J/\psi p \pi^-) < 6.5 \text{ GeV}/c^2$ are selected.

The $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decay chain is refitted with two sequential constraints using the `DecayTreeFitter` to improve the mass resolution: A) the J/ψ candidate mass constrained to its known value^[1] and the Λ_b^0 candidate constrained to originating from the PV; B) the invariant mass of the Λ_b^0 candidate constrained to its know value^[1], with the momenta of the J/ψ , p and π^- daughters re-calculated according to this constraint.

It should be noted that the PID requirements on the hadron tracks are not applied when reconstructing the Λ_b^0 candidates in the simulation sample. The effect, instead, is considered using weights from the PID calibration (`PIDCalib`). The pion track is calibrated using pion tracks from the D^* tagged $D^0 \rightarrow K^- \pi^+$ decay, while the proton track is calibrated using proton

tracks from the $\Lambda_c^+ \rightarrow pK^-\pi^+$ decay.

Events with Λ_b^0 candidates in both the simulation and the data samples are required to fire (TOS) the trigger sequences: at the HLT1 level by either of the `Hlt1DiMuonHighMass`, `Hlt1TrackMuon`, `Hlt1TrackMVADecision` and `Hlt1TwoTrackMVADecision` lines; and at the HLT2 level by either of the `Hlt2DiMuonDetachedJPsi` and `Hlt2TopoMu(2,3,4)BodyBBDT` lines.

Table A.2 Pre-selections of $\Lambda_b^0 \rightarrow J/\psi p\pi^-$ candidates between old Run 1 analysis and this one.

	Quantity	Old Run 1	This analysis
1	All tracks χ^2/ndf	< 3	same
2	muon DLL(μ)	$> 0\&\text{ISMUON}$	same
3	p_T of muon	$> 550 \text{ MeV}$	same
4	p_T of hadron	$> 250 \text{ MeV}$	same
5	vertex of J/ψ	$\chi^2 < 16$	same
6	mass window of J/ψ	$[-48, 43] \text{ MeV}$	same
7	χ_{VS}^2 of hadron	> 9	same
8	PID of π	$\text{ProbNN}_\pi \times (1 - \text{ProbNN}_\pi) > 0.5$	$\text{ProbNN}_\pi > 0.2$
9	PID of p	$\text{ProbNN}_\pi > 0.1$	$\text{ProbNN}_\pi > 0.2$
10	χ_{IP}^2 of Λ_b^0	< 25	same
11	Λ_b^0 vertex	$\chi_{\text{vtx}}^2/\text{ndf} < 10$	same
12	DIRA of Λ_b^0	> 0.9995	> 0.999
13	χ_τ^2 of Λ_b^0	> 9	-
14	p (p)	$> 10 \text{ GeV}$	-
15	Flight distance of Λ_b^0	-	$> 1.5 \text{ mm}$
16	DOCA χ^2 of $p\pi^-$	-	< 20

A.2.3 Reflections veto

Since the decays of $B^0 \rightarrow J/\psi K\pi$, $B_s^0 \rightarrow J/\psi K^+K^-$, $\Lambda_b^0 \rightarrow J/\psi pK^-$ have much larger branching fractions than the $\Lambda_b^0 \rightarrow J/\psi p\pi^-$ decay, even after the PID requirements on the hadron tracks, the $\Lambda_b^0 \rightarrow J/\psi p\pi^-$ sample is still potentially contaminated by these decays. To investigate the effects of the reflections from these decays, the invariant mass of the Λ_b^0 candidate in the $\Lambda_b^0 \rightarrow J/\psi p\pi^-$ sample is re-calculated with the proton assumed as a pion or kaon and/or the pion assumed as a kaon or proton. Figure A.9 shows these re-calculated invariant mass distributions for those $\Lambda_b^0 \rightarrow J/\psi p\pi^-$ candidates within the $\pm 20 \text{ MeV}$ mass window ($\approx 3\sigma$) around the known Λ_b^0 mass^[1]. Clear contamination with the $B^0 \rightarrow J/\psi K\pi$, $B_s^0 \rightarrow J/\psi KK$ and $\Lambda_b^0 \rightarrow J/\psi pK^-$ decays are observed, as shown by the narrow peaks around the known B^0 , B_s^0 and Λ_b^0 masses in the top right, bottom left and bottom middle plots in Fig. A.9. The contribution of $B_{(s)}^0 \rightarrow J/\psi \pi\pi$ decays is insignificant. The contamination from

$B^0 \rightarrow J/\psi K\pi$ is vetoed if: $M(J/\psi[p \rightarrow K]\pi)$ sits in the ± 22 MeV window around the known B^0 mass^[1], or $M(J/\psi[p \rightarrow \pi][\pi \rightarrow K])$ sits in the ± 22 MeV window around the known B^0 mass and $\text{ProbNN}_\pi(p) > 0.1$ & $\text{ProbNN}_K(\pi) > 0.1$. The contamination with the $\Lambda_b^0 \rightarrow J/\psi p K^-$ decay is vetoed if: $M(J/\psi p[\pi^- \rightarrow K])$ sits in the ± 22 MeV window around the known Λ_b^0 mass^[1] and $\text{ProbNN}_K(\pi) > 0.2$, or $M(J/\psi[p \rightarrow K][\pi \rightarrow p])$ sits in the ± 22 MeV window around the known B^0 mass and $\text{ProbNN}_K(p) > 0.1$ & $\text{ProbNN}_p(\pi) > 0.1$. Besides, the reflection of $B_s^0 \rightarrow J/\psi K^+ K^-$ decay is removed in this case: $M(J/\psi[p \rightarrow K][\pi^- \rightarrow K])$ sits in the ± 22 MeV window around the known B_s^0 mass^[1].

A large fraction of $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ candidates come from the decay $\Lambda_b^0 \rightarrow J/\psi \Lambda$ with $\Lambda \rightarrow p \pi^-$, which is not interesting in this analysis. These events are removed if the invariant mass $M(p \pi^-)$ locates in the ± 5 MeV/ c^2 window around the known Λ mass^[1].

During the invariant mass vetoing, the mass resolutions are determined by fitting the invariant mass distribution in a narrow region around the signal peak; the signal mass shape is described by double-sided crystal ball function (DSCB) with tail parameters fixed to the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ simulation, while the background is simply described by a linear function.

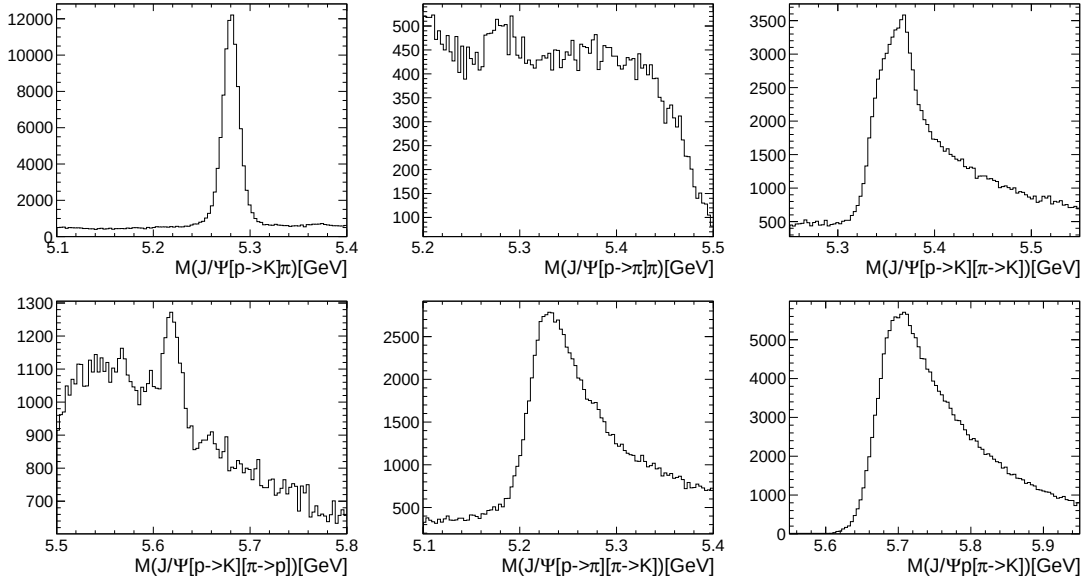


Figure A.9 Invariant mass distributions of the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ candidate with the p and the π^- assumed to be other hadrons for those Λ_b^0 candidates in the ± 20 MeV mass window around the known Λ_b^0 mass. The labels $h \rightarrow h'$ indicate that the h hadron in the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decay is assumed as a h' hadron.

A.2.4 Multivariate analysis

The multivariate analysis (MVA) is used to further suppress the background. In the MVA training the signal sample is from the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ simulation using the PHSP model,

while the background sample is from the upper sideband of the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ invariant mass distribution, $M(J/\psi p \pi) \in [5.67, 5.80] \text{ GeV}/c^2$. These samples are randomly split into two halves, one for the training and the other for the test. Different from the previous study, the PID information variables are included in the MVA classification. All used variables are listed in Tab. A.3. The smaller value of the two muon PID variables ($DLL_{\mu\pi}$); the smaller value of the χ_{IP}^2 of the two muon tracks; the smaller value of the χ_{IP}^2 of the two hadron tracks; the arithmetic addition of the p_T of the two hadrons; the DIRA, the flight distance (FD), the p_T , the χ_{IP}^2 and the vertex fit quality ($\chi_{\text{vtx}}^2/\text{ndf}$) of the Λ_b^0 candidate. the PID variables of hadrons. Besides, the Run 1 and Run 2 samples are studied separately. The distributions of these variables of Run 1 sample for the signal in the simulation sample and the background in the data sideband are shown in Fig. A.10.

Table A.3 Variables used in TMA between previous Run1 and this study.

	Previous Run 1	This analysis
1	χ_{vtx}^2 of Λ_b^0	same
2	sum p_T of hadrons	same
3	χ_{IP}^2 of Λ_b^0	same
4	DIRA of Λ_b^0	same
5	min χ_{IP}^2 of hadrons	same
6	min χ_{IP}^2 of muon	same
7	FD of Λ_b^0	same
8	p_T of Λ_b^0	same
9	min PID of muon	same
10	-	ProbNN _p of p
11	-	ProbNN _{π} of π^-
12	-	momentum of p

The BDTG method is used as the default method, and the over-training test for the BDTG method is given in the right plot of Fig. A.11 for both Run 1 and Run2 samples. The BDTG discriminant distributions for the training samples are consistent with those in the test samples. No effect of over-training is observed. The working point of the BDTG value is optimized by maximizing the figure of merit (FoM) of the signal significance, defined as $\epsilon_S/\sqrt{S+B}$. The ϵ_S is the signal efficiency for a specified BDTG cut and is determined from the simulation sample, while S (B) is the number of the signal (background) events in the $\pm 3\sigma$ region around the known Λ_b^0 mass in the $J/\psi p \pi$ invariant mass distribution of the data. In Fig. A.12, the distribution of the FoM as a function of the BDTG cut value is given, and the value of 0.88 for Run 1 and 0.9 for Run 2 are determined as the default working points.

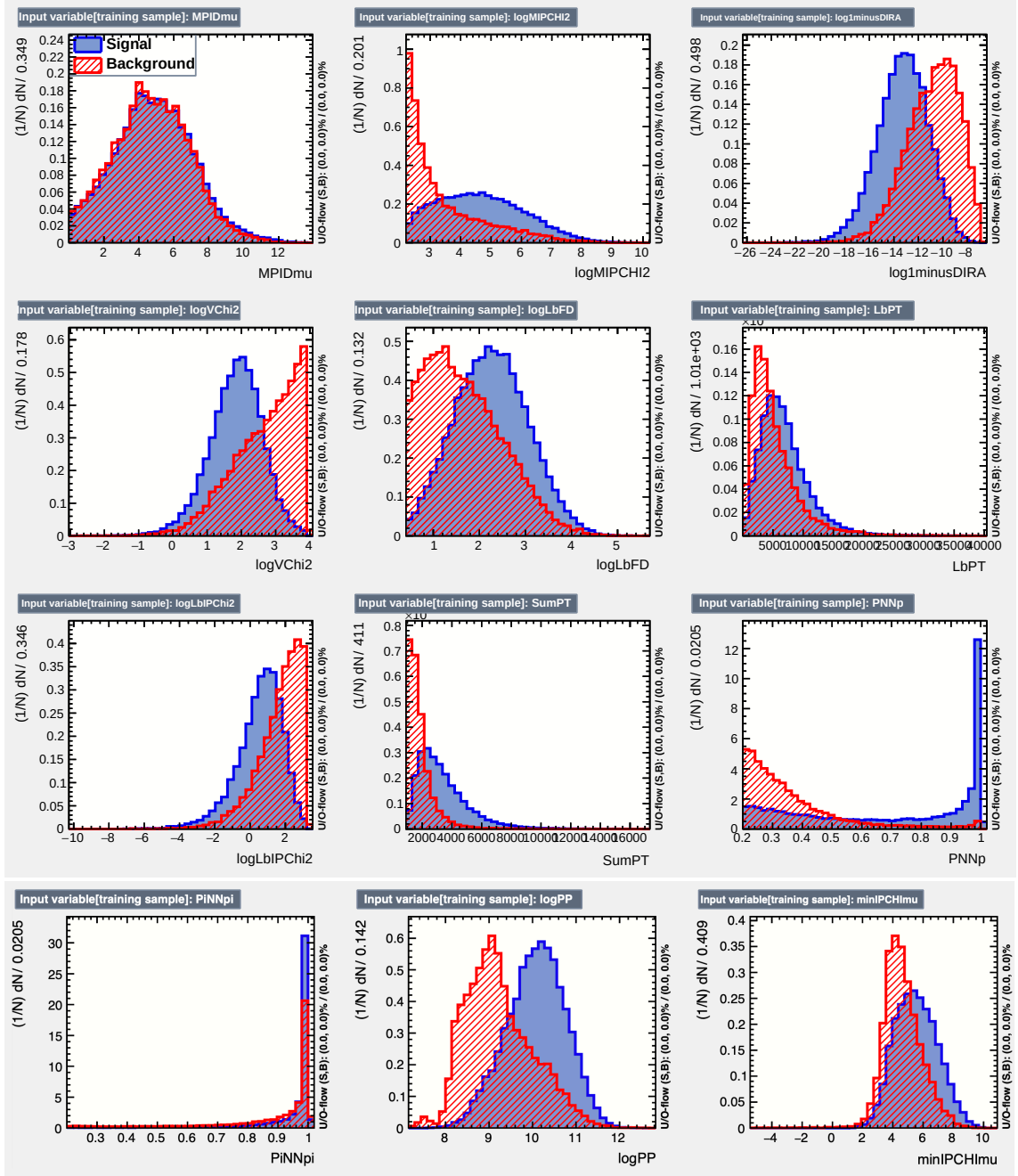


Figure A.10 Discriminating variables used in the MVA training of Run 1 sample.

A.3 Stripping selections in the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ decay study

The comparison of stripping selection for the two decay channels is shown in Tab. A.4.

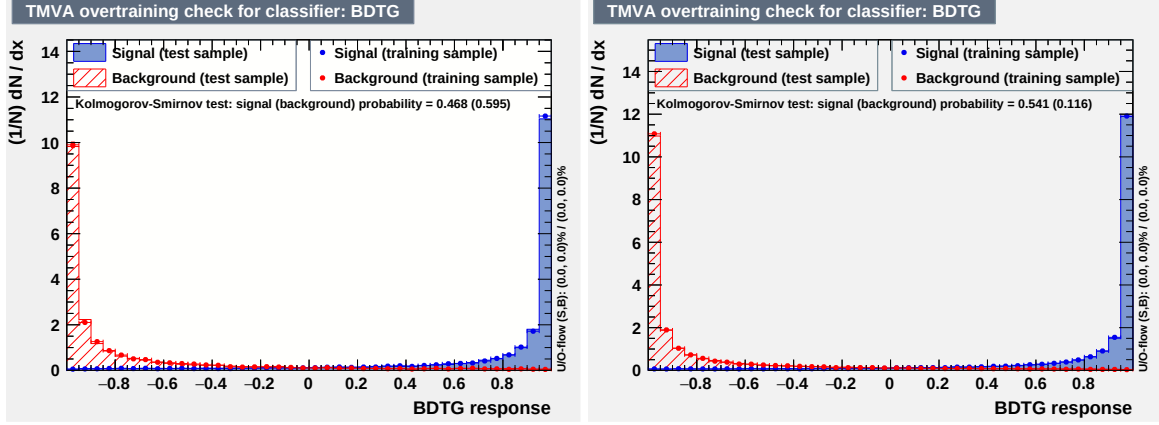


Figure A.11 The BDTG output distribution for signals and the background in the training sample and the test sample of Run 1(left) and Run 2(right) samples.

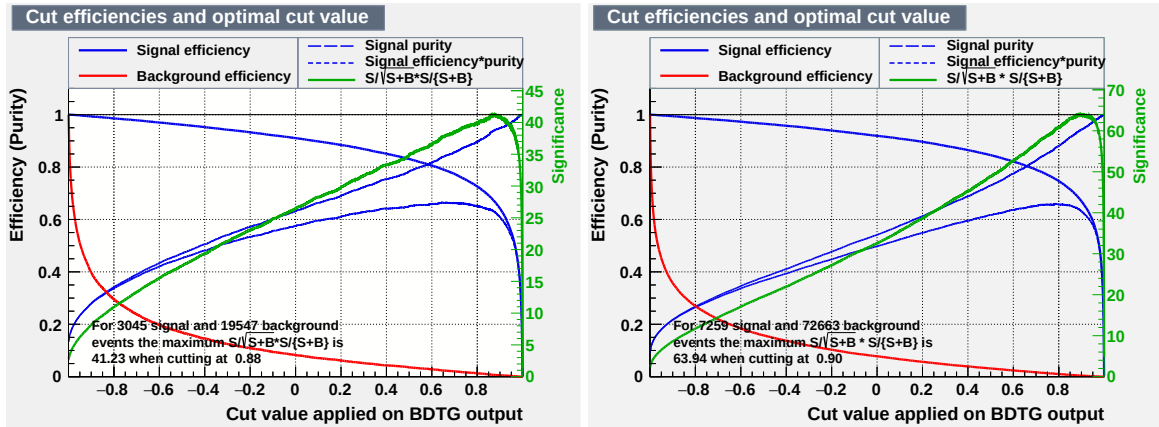


Figure A.12 Signal significance (arbitrary unit) for different BDTG cut values of Run 1(left) and Run 2(right) samples.

Table A.4 Stripping selections on the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$ candidates.

#	Selection variable	$\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$	$\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$
all basic particles			
1	χ_{irk}^2/ndf		< 3
2	$P(\text{ghost})$		< 0.4
3	p_T		$> 100 \text{ MeV}$
4	p		$> 1000 \text{ MeV}$
5	χ_{IP}^2		> 4
Λ_c^+ selection			
6	$p_T(K) + p_T(p) + p_T(\pi)$		$> 1800 \text{ MeV}$
7	χ_{vtx}^2/ndf		< 10
8	χ_{FD}^2		> 36
9	$ M_{rec} - M_{pdg} $		$< 100 \text{ MeV}$
10	One track with	$p_T > 500 \text{ MeV}, p > 5000 \text{ MeV}$	
11	$\text{PID}_p(p)$		> -10
12	$\text{PID}_K(K^-)$		> -10
13	$\text{PID}_K(\pi^+)$		< 20
K^+, K^-, π^- from Λ_b^0 or D_s^+			
14	p	$> 2000 \text{ MeV}$	$> 1000 \text{ MeV}$
15	$\text{PID}_K(K^+, K^-)$	> -2	> -10
16	$\text{PID}_K(\pi^-)$	< 10	< 20
$K^+ K^- \pi^-$ vertex cut			
17	χ_{vtx}^2/ndf	< 8	< 10
18	χ_{FD}^2	> 16	> 36
19	$\text{BPV} \cos(\theta_p)$	> 0.98	> 0
20	$\text{BPVVD } rho$	$> 0.1 \text{ mm}$	—
21	$\text{BPVVD } z$	$> 2.0 \text{ mm}$	—
22	$p_T(\text{daughter sum})$	$> 1250 \text{ MeV}$	$> 1800 \text{ MeV}$
23	Invariant mass	$< 3000 \text{ MeV}$	$(1769.62, 2068.49) \text{ MeV}$
24	Two track with	$p_T > 300 \text{ MeV}$	—
25	One track with	$p_T > 500 \text{ MeV}, p > 5000 \text{ MeV}$	
Λ_b^0 selections			
26	χ_{vtx}^2/ndf		< 10
27	τ		$> 0.2 \text{ ps}$
28	χ_{IP}^2		< 25
29	$\text{BPV} \cos(\theta_p)$		> 0.999
30	One track with	$p_T > 1.7 \text{ GeV}, p > 10 \text{ GeV}, \chi_{minIP}^2 > 16, \text{minIP} > 0.1 \text{ mm}$	
31	$p_T(\text{daughter sum})$		$> 5 \text{ GeV}$

Appendix B ECAL simulation study for Upgrade II

B.1 Introduction

It is generally agreed within the LHCb collaboration and the community of flavour physics that an upgrade of the LHCb detector to make full exploitation of the High-Luminosity LHC (HLLHC), which provides an opportunity to investigate the open questions in flavour physics^[152,153,250]. This proposed upgrade is referred to as the LHCb Upgrade II, which should be capable of taking data at an instantaneous luminosity of up to $1.5 \times 10^{34} \text{ cm}^{-2} \cdot \text{sec}^{-1}$. The goal is to collect a data sample of an integrated luminosity of 300 fb^{-1} after Run 6^[152]. Among the biggest challenges for the Upgrade are the large number of proton-proton interactions in a single bunch crossing (pile-up) and the large amount of radiation that the detector has to tolerate.

The present ECAL has good performance with run1 and run2 luminosity. There is not a certain answer if it can handle a very high luminosity, it is necessary to take a look at the performance of current ECAL with high luminosity in the parameterized simulation first. From the parameterized simulation, the possible methods are studied to optimize the detector and let it conquer the pile-up effect. There are many factors that could affect the ECAL performance, like intrinsic energy resolution and so on. By improving these factors, the performance of ECAL might be promoted. Besides, it is commonly agreed that high precision time information is needed to deal with the pile-up, which is the major item studied in the simulation. And high granularity is another requirement for the Upgrade ECAL, which can greatly help reducing the occupancy due to the high luminosity.

For the calorimeter, the main idea is that it should allow for the reconstruction and analyses of the decay modes involving neutral particles. This chapter will discuss the requirements to ECAL for Upgrade II in the parameterized simulation first. A detailed silicon-tungsten (SiW) ECAL is discussed then, *i.e.* a sampling ECAL with silicon as the active material and tungsten as the absorbers.

B.2 Parameterized simulation

The parameterized simulation tool is developed to study the requirements to ECAL for Upgrade II with pretty high instantaneous luminosity. This tool is based on GAUSS^[196] and

DELPHES^[251], GAUSS is used to produce generated level sample with different event types^① and DELPHES is used to run detector simulation by parameterized way. Actually, the DELPHES does not have appropriate module fittingly used for ECAL simulation for LHCb, since the default geometry of calorimeter in DELPHES is bucket, while the geometry of ECAL at LHCb is a plane. Therefore, new parameterized simulation modules for LHCb is developed first. Then, the specific decay channels are reconstructed using the output from parameterized simulation. Finally, these simulation samples are analyzed. The frame to achieve the parameterized simulation for ECAL is shown in Fig. B.1.

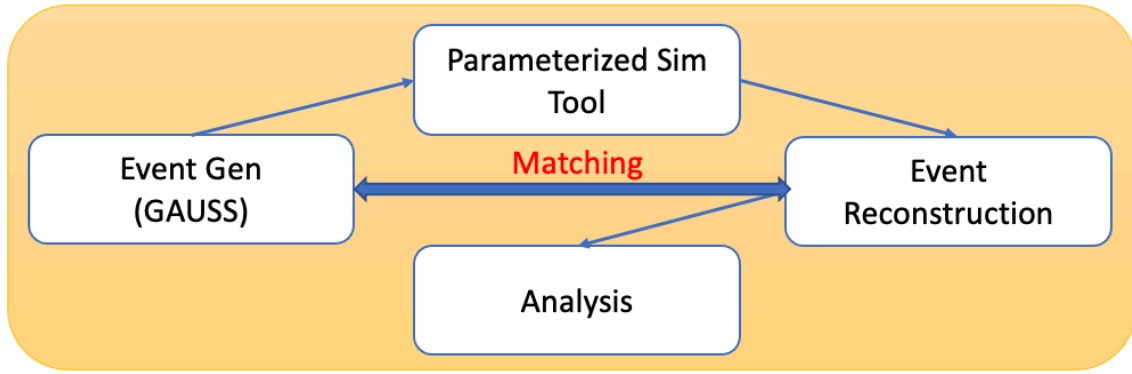


Figure B.1 The simulation procedure in the parameterized simulation.

B.2.1 Parameterization of shower

A method to achieve parameterized simulation is introduced below briefly. The particles like photon or electron will be transformed into electromagnetic showers in ECAL, which consist of many electrons or photons. Two processes, photon pair production and electron bremsstrahlung, continue with the shower development until photons fall below the pair production threshold. The transverse profile of the energy deposition in electromagnetic cascade can be described by this formula obtained from beam test^[252]:

$$F(x, y, f, \sigma, R) = \frac{f}{2\pi\sigma} e^{-\frac{x^2+y^2}{2\sigma^2}} + \frac{1-f}{2\pi} \frac{2R^2}{(x^2 + y^2 + z^2)^2} \quad (\text{B.1})$$

where the first part is used to describe the tail of shower, and the second part to describe the main section. The parameters used can be obtained from the beam test, like σ , f and R . Above formula is utilized to distribute the energy of entering particle to the corresponding cells. In the parameterized simulation tool, the length of the cascade shape is not considered.

Next step is to smear the deposited energy in the cell, the parameterized energy resolution

① Different event types present different decay channels in LHCb.

formula is used:

$$\frac{\sigma}{E} = \frac{\alpha}{\sqrt{E}} \oplus \beta \quad (\text{B.2})$$

where α is roughly equal to 10% and β is approximately equal to 1‰ for the current shashlik ECAL, as mentioned in Sec. 2.2.2.2. After Upgrade II at LHCb, the measuring unit of ECAL will have ability to record precision time information, the time resolution is a given value in this simulation tool, it is convenient to set this value in the configuration file.

B.2.2 Reconstruction and calibration

The reconstruction method in the parameterized simulation tool follows the one utilized in the current shashlik ECAL, which is named as cellular automaton algorithm^[253,254]. This method is processed based on the seed cell, which has a local maximum energy. In this tool, the clusters are determined by the algorithm mentioned above, and reconstructed using 3×3 cells, the energy and position are calculated as:

$$\begin{aligned} E &= \sum_i^{cells} E_i \\ x_c &= \left(\sum_i^{cells} x_i \times E_i \right) / E \\ y_c &= \left(\sum_i^{cells} y_i \times E_i \right) / E \\ z_c &= z_i \\ t_c &= \left(\sum_i^{cells} t_i \times E_i \right) / E \end{aligned} \quad (\text{B.3})$$

where the subscript i represents index the cell belonging to the corresponding cluster.

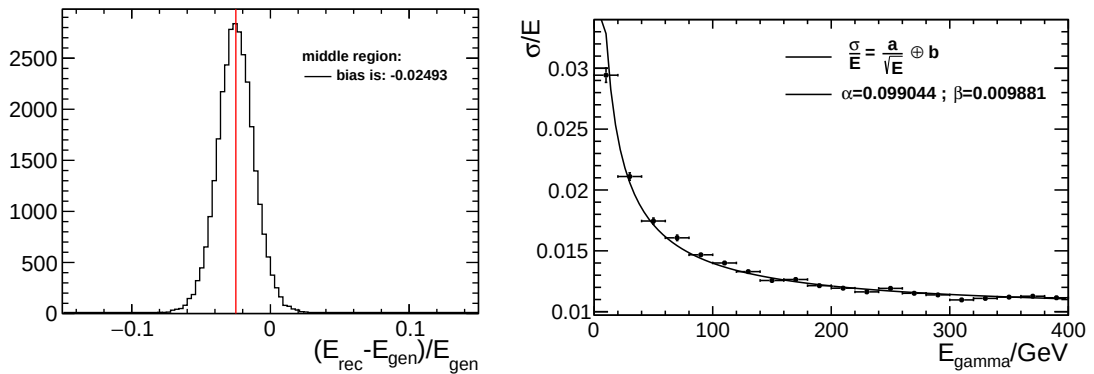


Figure B.2 The difference between the reconstructed energy and generated level energy (left), and the relation between the energy resolution and particle energy after calibration (right).

In this tool, the calibration to the reconstructed variables are also imported. The energy need to be recalculated by multiplying with a factor due the energy leaking effect. The bias between the reconstructed energy and generated energy is shown in left of Fig. B.2. In addition, the calibration to the position is achieved by using a correction formula:

$$x_{cali} = f \times b \times asinh(\frac{x_{rec}}{\Delta} cosh \frac{\Delta}{b}) \quad (B.4)$$

where the x_{cali} is the x -coordinate in the frame of seed cell, the x_{rec} is the position obtained from Eq. (B.3), and other variables are calibration parameters. By the way, the calibration need to be applied independently to the sample using different cell size.

To verify the credibility of this method, the energy resolution is checked by γ gun events, as shown in the right of Fig. B.2, which is consistent to the performance of current ECAL.

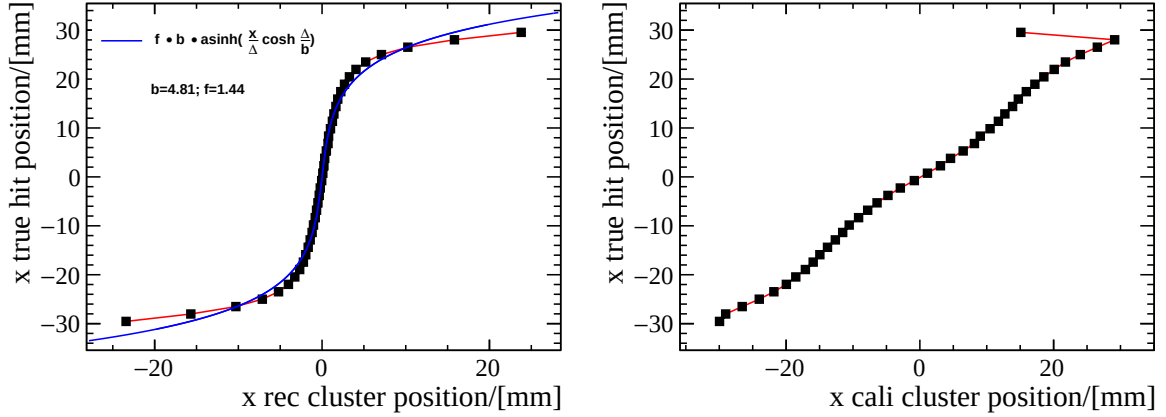


Figure B.3 The relation between the true position and reconstructed position before (left) and after (right) calibration.

B.2.3 Neutral particle identification

B.2.3.1 Removal of charged particles

Some charged particles can also deposit energy in ECAL, we can removed this effect by an anti-coincidence between the cluster ($E_T > 250$ MeV) position and the extrapolation of the reconstructed tracks ($p_T > 150$ MeV) up to the calorimeter. The distance between the reconstructed cluster and the nearest tracks is calculated, if the position resolution of tracks is overlooked, the required distance is larger than half of the cell size. If the further trackers and ECAL both have ability to record precision time, the time matching between track arrival time and shower reconstructed time can also help to identify the cluster caused from charged particle.

B.2.3.2 Characteristics of γ , π^0 and overlapping cluster

γ/π^0 separation is an important prerequisite for many physics analysis^[255]. Many photons detected by the ECAL are originated from π^0 decays. Two photons of the π^0 has possibility to be reconstructed as one cluster due to the granularity, especially for the π^0 with pretty large momentum, as shown in the Fig. B.4. If the distance between 2 γ decayed from the π^0 in the ECAL plane is large enough, the π^0 can be reconstructed as resolved one, otherwise, it will be reconstructed as merged particle, which means the showers from the 2 γ are overlapping together.

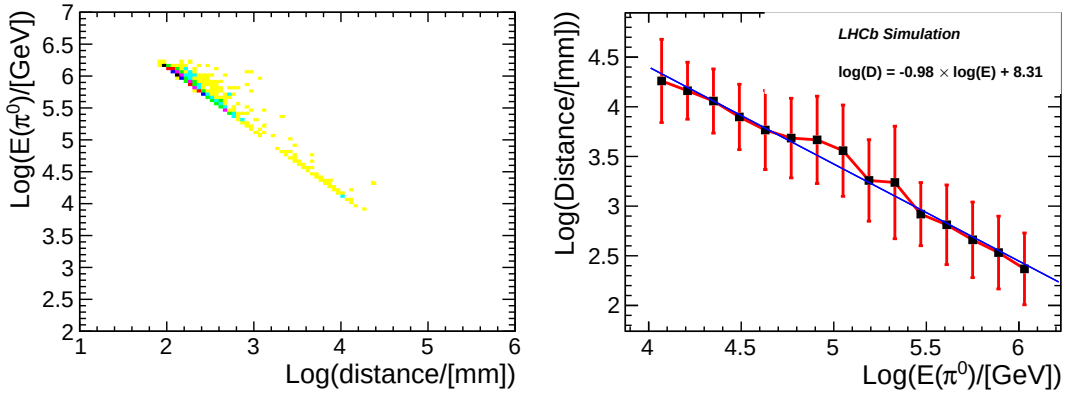


Figure B.4 The relation between the π^0 energy and the distance of two γ on the ECAL plane.

As the luminosity becomes pretty high after Upgrade II, it will increase the chances of overlapping clusters. And beyond that, Identifying the random overlapping cluster is another important task.

The method for γ , π^0 and overlapping cluster separation is based on the shape of cluster. As the timing information will be recorded after Upgrade II, it also has possibility to separate the clusters from different sources. The position spread matrix elements are defined as:

$$\begin{aligned}
 S_{XX} &= \frac{\sum_{i=1}^N E_i (x_i - x_c)^2}{\sum_{i=1}^N E_i} \\
 S_{YY} &= \frac{\sum_{i=1}^N E_i (y_i - y_c)^2}{\sum_{i=1}^N E_i} \\
 S_{XY} &= \frac{\sum_{i=1}^N E_i (x_i - x_c)(y_i - y_c)}{\sum_{i=1}^N E_i}
 \end{aligned} \tag{B.5}$$

where (x_c, y_c) is the position of the cluster, as defined in Eq. (B.3), and (x_i, y_i) is the position of used cell for a given cluster of N cells. In this simulation, seven separation variables are constructed including timing information:

- The size of the cluster, also explained as second order of moments, referred to as $r2$:

$$r2 = \langle r^2 \rangle = S_{XX} + S_{YY} \quad (B.6)$$

- The importance of the tails, referred to as $r2r4$:

$$r2r4 = 1 - \frac{\langle r^2 \rangle^2}{\langle r^4 \rangle} \quad (B.7)$$

- Ratio of the eigenvalues of the matrix S :

$$\kappa = \sqrt{1 - 4 \frac{S_{XX}S_{YY} - S_{XY}^2}{(S_{XX} + S_{YY})^2}} \quad (B.8)$$

- The orientation of the correlation between X and Y coordinates, referred to as $asym$:

$$asym = \frac{S_{XY}}{\sqrt{S_{XX}S_{YY}}} \quad (B.9)$$

- The energy fraction of the seed cell in the cluster:

$$E_{seed}/E_{cluster} \quad (B.10)$$

- The second order of moments about timing information, referred to as $t2$:

$$t2 = \langle t^2 \rangle = \frac{\sum_{i=1}^N E_i (t_i - t_c)^2}{\sum_{i=1}^N E_i} \quad (B.11)$$

- The importance of the tails of timing, referred to as $t2t4$:

$$t2t4 = 1 - \frac{\langle t^2 \rangle^2}{\langle t^4 \rangle} \quad (B.12)$$

The variables constructed above are taken as input for the multi-variable analysis tool, several methods are compared, and the BDTG is taken as the default training algorithm.

B.2.3.3 γ, π^0 and overlapping cluster separation

The discriminant variables introduced above are taken as input for Multivariate Analysis (TMVA)^[204]. Photons in LHCb acceptance with $E \in [10, 400]$ GeV events are taken as signal sample (EventType=56000400). The π^0 particle gun decaying to two γ events are used for the merged π^0 sample (EventType==56200100). Furthermore, the random overlapping cluster sample is created base on the gamma gun events, with a gamma particle is inserted in the generated level. The energy of accompanying gamma with the exponential distribution, with the mean value equal to 10 GeV, which is estimated from the minimum bias sample. The generated position and time of this accompanying gamma follow the character of primary vertex distribution. Besides, $E_T > 250$ MeV is applied to all γ candidates before being imported in

the TMVA tool. Considering the energy scale of γ and π^0 is different, $E < 200$ GeV is also applied to γ and random overlapping samples.

Discrimination between γ and merged π^0 is studied first. Figure B.5 shows these discriminant variables used in the multivariable analysis, while Fig. B.6 shows the distribution of BDTG, and no overtaining effect appears from this test. The receiver operating characteristic curve (ROC) is shown in Fig. B.7, the left one is obtained with 8 discriminant variables are included, while the one shows the performance with only 6 variables (no $t2$ and $t2t4$). According to these plots, the timing information does not contribute much on the γ and merged π^0 separation.

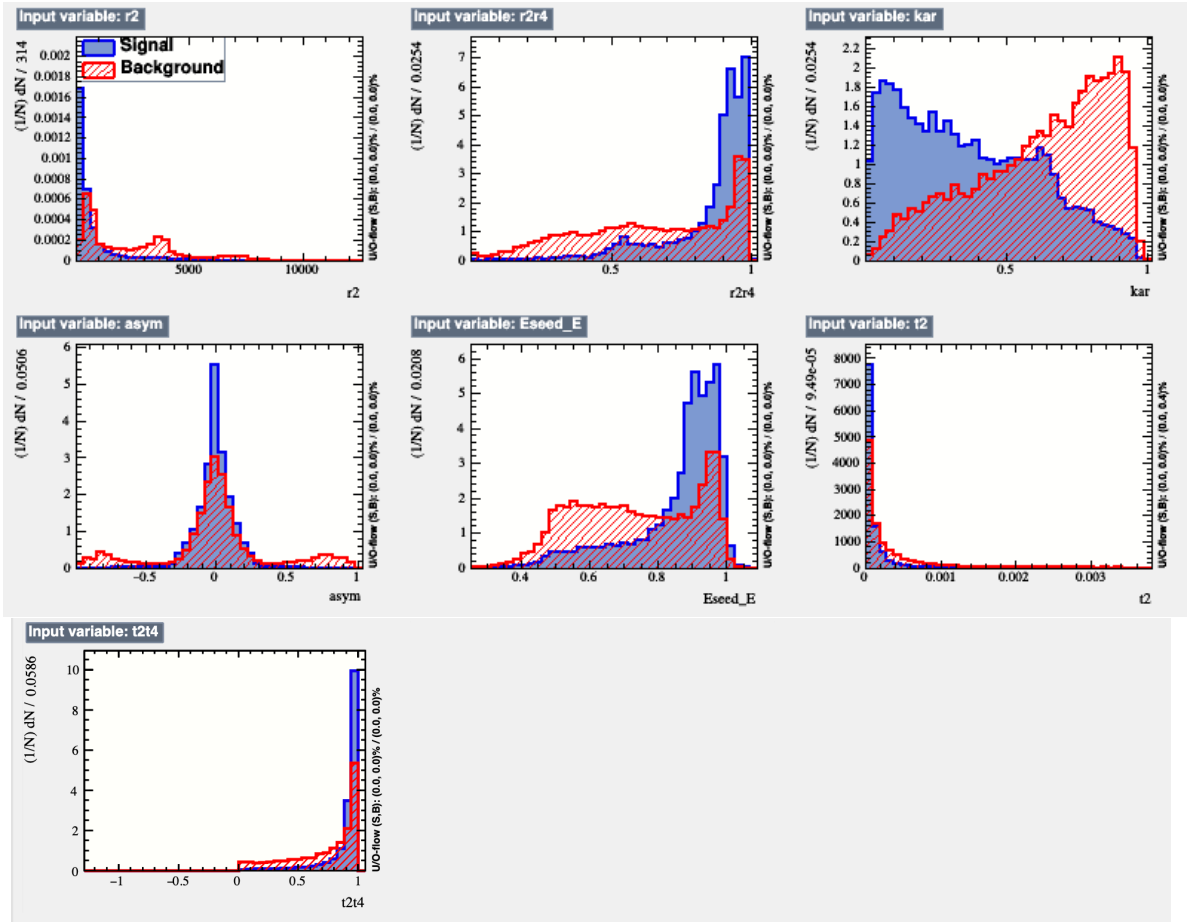


Figure B.5 The distribution of discriminant variables used for cluster identification.

Another tests are performed to identify the single γ and merged π^0 from random overlapping clusters. Fig. B.8 shows the identification ability between the pure γ and contaminated one, from this plot, it is found that the timing information plays a important role for this separation. Similar check is also performed on the merged π^0 and random overlapping cluster samples, the result is shown in the Fig. B.9. It is obvious that BDTG method has a good

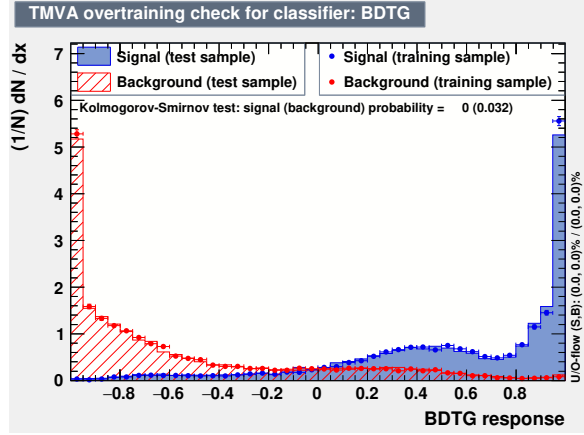


Figure B.6 The distribution of BDTG of the signal sample (γ) and background sample (merged π^0).

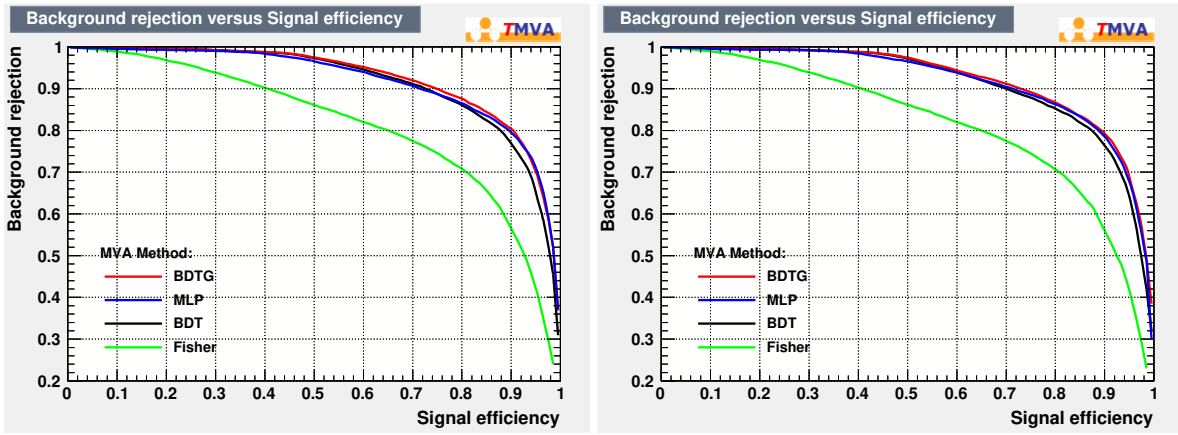


Figure B.7 Performance of different methods used on the simulation samples. The left one presents the performance of γ and π^0 separation with the timing information included, right one with the timing information excluded.

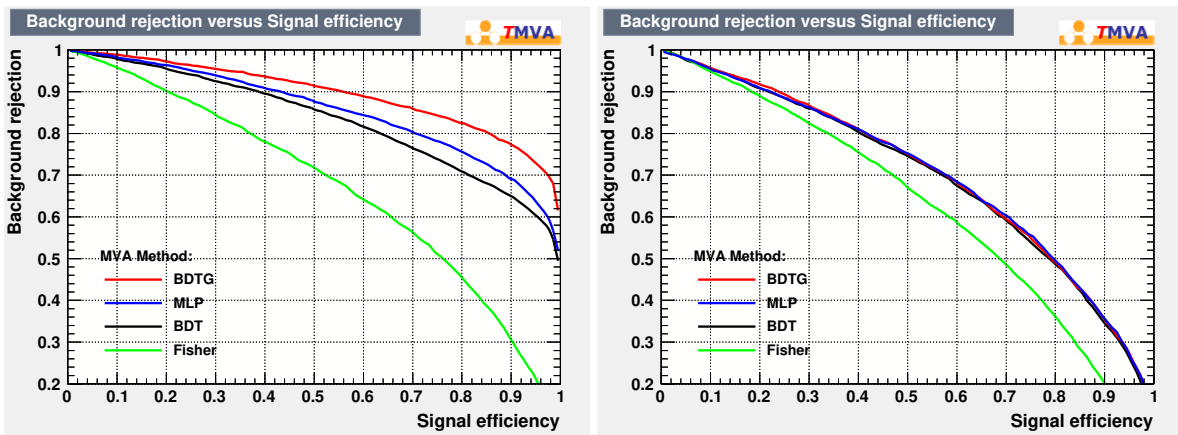


Figure B.8 Performance of different methods used on the simulation samples. The left one presents the performance of γ and random overlapping cluster separation with the timing information included, right one with the timing information excluded.

performance, three different variables are constructed to identify reconstructed cluster based

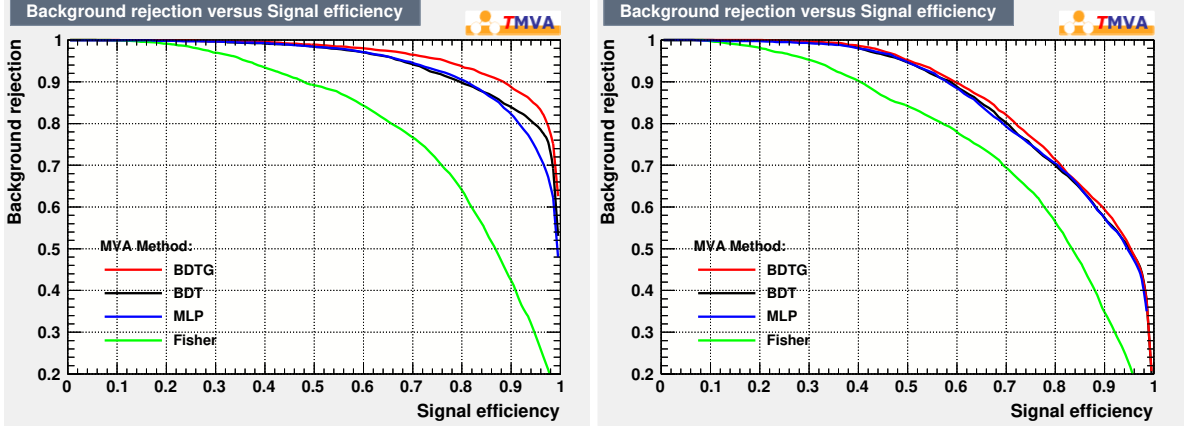


Figure B.9 Performance of different methods used on the simulation samples. The left one presents the performance of π^0 and random overlapping cluster separation with the timing information included, right one with the timing information excluded.

Table B.1 Identification variables and corresponding functions.

Variables	function
PID_g_piz	Is a pure γ rather than a merged π^0
PID_g_conta	Is a pure γ without contamination
PID_piz_conta	Is a merged π^0 without contamination
Dis_charge	Is a neutral particle rather than coming from charged track

on BDTG, as listed in Tab. B.1

B.2.4 Performance in $B_s^0 \rightarrow \phi\gamma$ decay

The decay channel of $B_s^0 \rightarrow \phi\gamma$ is studied with different ECAL cell size configuration or luminosity condition. In the reconstruction, the ϕ particle information is obtained from generated samples. Thus, the final performance is only related to the different configuration of ECAL in simulation. Table B.2 lists all selections used in this channel reconstruction, the transverse energy of reconstructed gamma is larger than 2600 MeV and the transverse momentum of the B_s^0 candidates greater than 3000 MeV. Table B.2 lists the selection applied to the simulation samples, we name them basic selections here.

Table B.2 Selection criteria in $B_s^0 \rightarrow \phi\gamma$ channel.

Quantity	Selections
E_T of γ	$> 2.6 \text{ GeV}$
p_T of B_s^0	$> 3.0 \text{ GeV}$

B.2.4.1 Luminosity effect

As mentioned above, the luminosity will increase markedly after Upgrade II, the physical performances in this channel with different luminosity are studied first. 20000 simulation samples for each luminosities were generated by this parameterized simulation tool, and by the way, the current ECAL cell segmentation is used for these samples. Figure B.10 shows the $\phi\gamma$ invariant mass distribution with different luminosities. $\mathcal{L} = 4 \times 10^{32} \text{ cm}^{-2} \cdot \text{s}^{-2}$ is the instantaneous luminosity during Run 1 and Run 2 at LHCb, and this value can reach up to $\mathcal{L} = 1.5 \times 10^{34} \text{ cm}^{-2} \cdot \text{s}^{-1}$ after Upgrade II. Two other luminosity cases are also studied, since the instantaneous luminosity descends as time goes by in one beam injection,

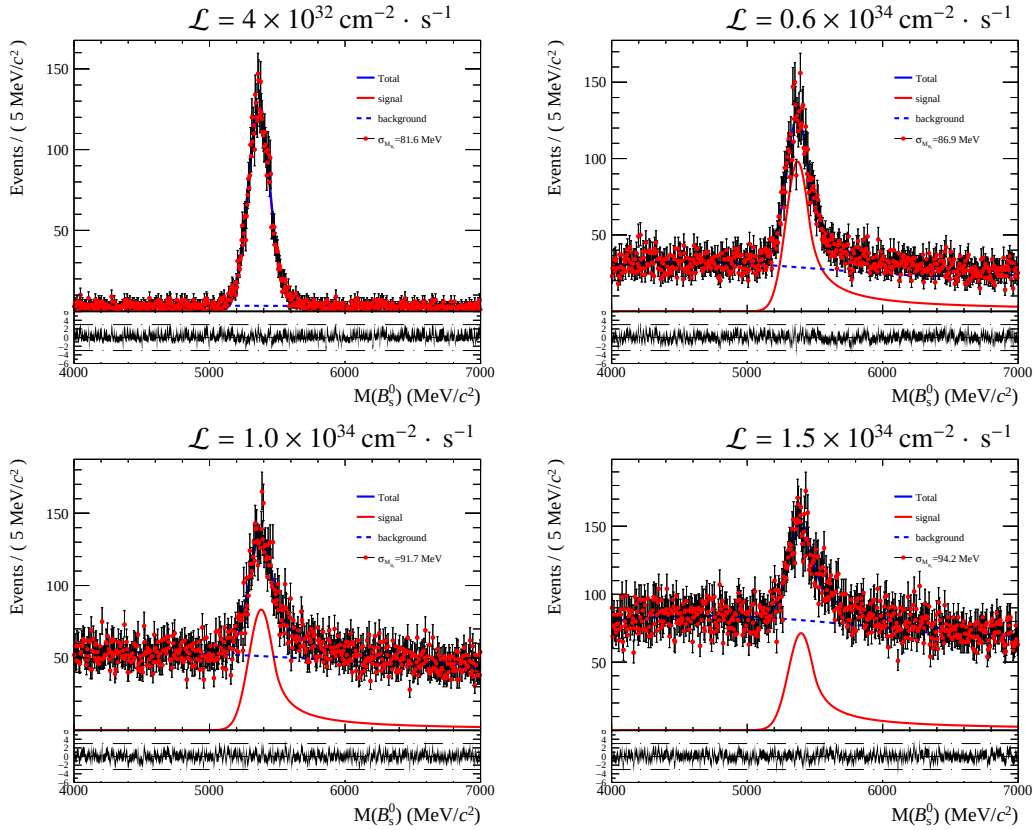


Figure B.10 The mass distribution of B_s^0 in $B_s^0 \rightarrow \phi\gamma$ decay with different luminosities.

The yields of signal and background are estimated by fitting the mass distribution for each sample, where the crystal ball function is used to describe the shape of signal and the background is described by 3 order Chebyshev polynomial function. The fitting results are listed in Tab. B.3. Comparing these four plots, the distinguishing feature is that the yields of combinatorial background increases rapidly along with the increase of luminosity, and we found that this increase is linear roughly. In addition, the significance is dominated by the high-background. By the way, the significance is defined as $S/\sqrt{S + 5B}$ here. The background

yield is multiplied with a factor of 5 as this value is underestimated from simulation sample, and this factor was obtained by comparing to previous study^[256].

Besides, the mass resolution of B_s^0 becomes larger with luminosity increased. Most clearly, a long tail appears in B_s^0 mass distribution in high luminosity case due to overlapping effect. Thus, we need to improve the properties of ECAL to meet these challenges.

B.2.4.2 Smaller cell size to remove overlapping effect

In order to remove the overlapping clusters, two other cell size segmentations are studied in parameterized simulation. The first one is designed according to the radiation^②, as shown in Fig. B.11. The other one is very high granularity silicon-tungsten ECAL, all cells are squares with width equal to 1 cm, as shown in Fig. B.18.

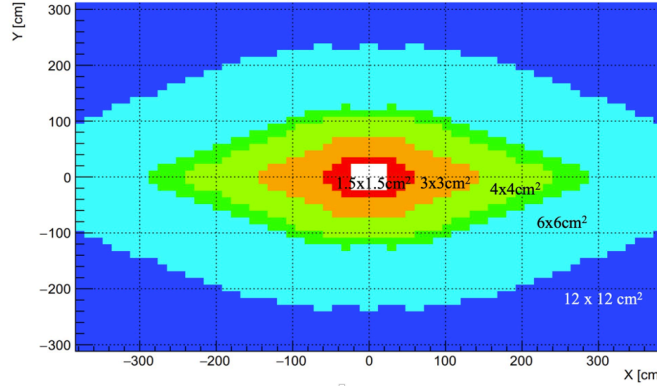


Figure B.11 The diagram of ECAL cell size segmentation proposed by Phillip Roloff for LHCb.

In these parameterized simulations, the energy resolution of Phillip's segmentation is set to be $\sigma(E)/E = 0.1/\sqrt{E} \oplus 0.024$ ^③, while the energy resolution of Si-W ECAL is $\sigma(E)/E = 0.2/\sqrt{E} \oplus 0.007$ ^④. All results with different simulation configurations are summarized in Tab. B.3. According to these simulation samples, we find that the reconstruction efficiencies do not change significantly. The significance of smaller cell size is a little better than the value obtained from current segmentation. Most obviously, the number of background candidates becomes larger with higher luminosity.

B.2.4.3 Precision timing to remove pile-up effect

The surge of combinatorial background comes from pile-up effect in high luminosity situation. In order to remove this kind of background candidates, the time information is

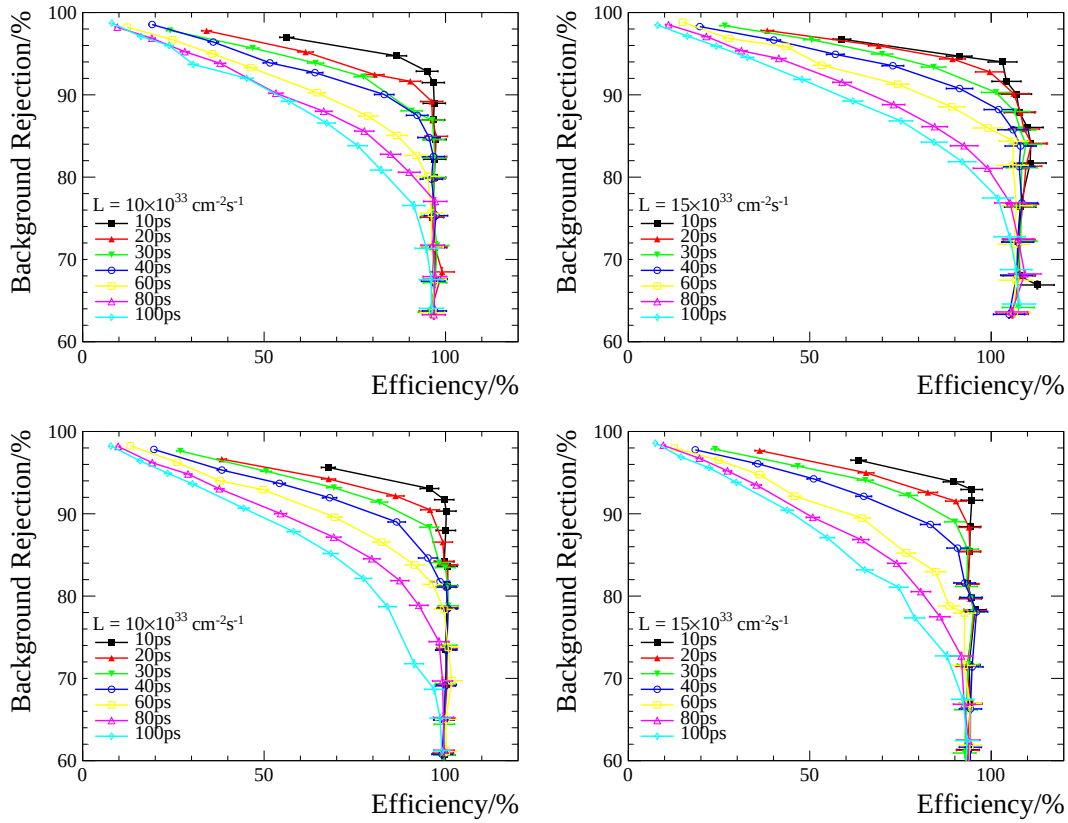
② This configuration was proposed by Philipp Roloff for LHCb.

③ According to GEANT4 simulation of spaghetti ECAL from other colleagues in LHCb.

④ Obtained from GEANT4 simulation of Si-W, details are shown in next section.

Table B.3 The physical performance in different luminosities with different ECAL cell segmentation.

Segmentation	Luminosity ($\text{cm}^{-2} \cdot \text{s}^{-1}$)	Efficiency	Bkg ($\times 1000$)	Significance	Mass resolution (MeV)
Current	4×10^{32}	0.25	1.9	62	82
	0.6×10^{34}	0.23	16.6	33	87
	1.0×10^{34}	0.21	29.8	23	92
	1.5×10^{34}	0.19	46.7	17	94
Phillip's	4×10^{32}	0.25	1.6	61	109
	0.6×10^{34}	0.23	13.9	33	104
	1.0×10^{34}	0.21	22.4	24	105
	1.5×10^{34}	0.21	31.1	22	105
Si-W	4×10^{32}	0.28	1.8	62	89
	0.6×10^{34}	0.27	15.5	38	90
	1.0×10^{34}	0.27	22.8	35	85
	1.5×10^{34}	0.24	30.2	27	88


 Figure B.12 The relation between signal efficiency and combinatorial background rejection fraction with different Δt cuts applied, in Phillip's segmentation (top) and Si-W segmentation (bottom) respectively.

used to distinguish the primary vertices. Considering the ECAL can record the precision time information, the shower time reconstructed from ECAL is named as t_1 . Assuming the further VELO at LHCb has ability to detect the vertex time, named as t_2 . The time difference between

the shower and vertex can be obtained as $\Delta t = t_1 - t_2 - l/c$, where l/c represents the γ flight time. In current simulation, the time resolution of vertex is overlooked and the ECAL time resolution is given value. In this way, we can study the requirement of ECAL time resolution.

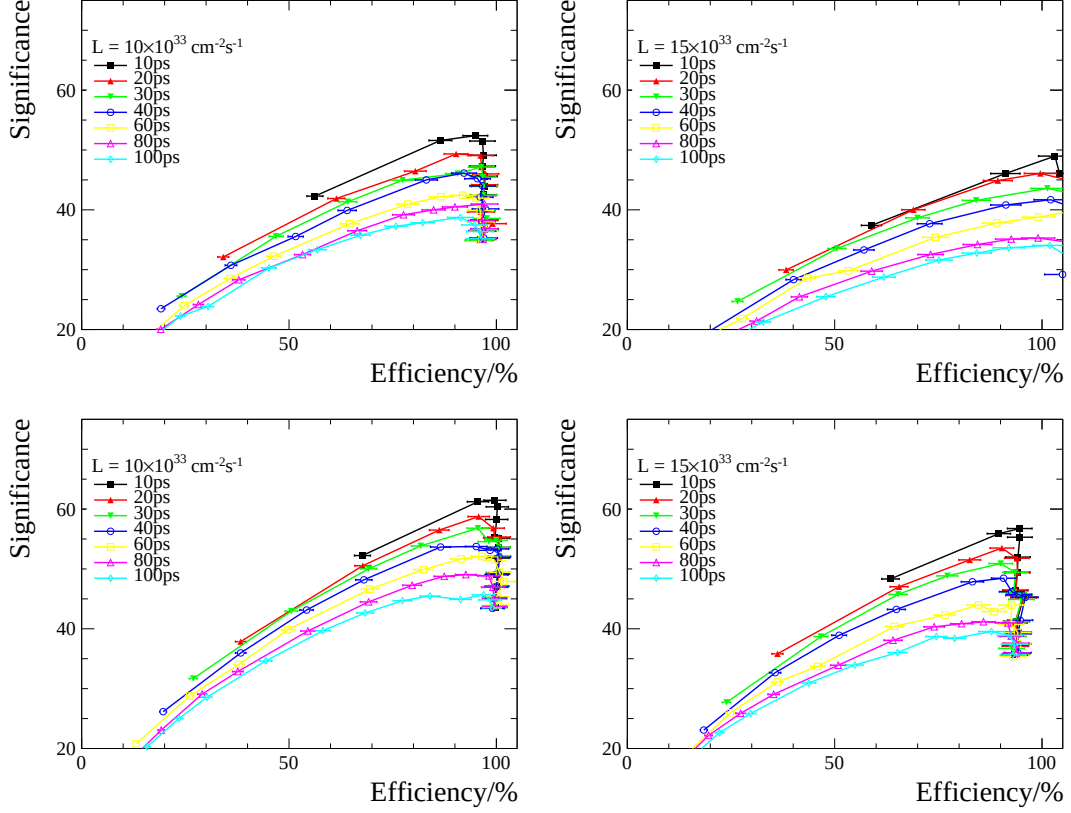


Figure B.13 The relation between signal efficiency and significance with different Δt cuts applied, in Phillip's segmentation (top) and Si-W segmentation (bottom) respectively.

The performances of this additional selection value Δt in this decay channel is studied then. The relation between selection efficiency and background rejection fraction after time matching applied with different time resolutions is shown in Fig. B.12. It is clearly that most of combinatorial background candidates can be removed with the time resolution equal to tens of picosecond for both cell size segmentation. The signal significance of 20000 events simulation sample was also studied, as shown in Fig. B.13. Based on the research described above, there is not a big challenge for this channel reconstruction in high luminosity situation, tens of picosecond can help to reject most of combinatorial background candidates due to high pile-up effect. Both Phillip's segmentation and Si-W segmentation have good performances. Nevertheless, we need to keep it in mind that this channel can be studied during Run 1 and Run 2 at LHCb, the mentioned requirements to ECAL for this channel cannot expand study scope at LHCb.

B.2.5 Performance in $B_s^0 \rightarrow J/\psi\pi^0$ decay

Another interesting decay channel is $B_s^0 \rightarrow J/\psi\pi^0$, which has not been observed yet. Besides, the channel $B^0 \rightarrow J/\psi\pi^0$ has similar topology to this one, we can estimate the both channels together using same simulation samples for this pre-study. The decay channel of $B_s^0 \rightarrow J/\psi\pi^0$ is studied with different ECAL cell size segmentations and luminosity conditions. Similar to the $B_s^0 \rightarrow \phi\gamma$ decay study, J/ψ information is obtained from generated samples. Table B.4 lists all selections used in this channel reconstruction, the transverse energy of reconstructed gamma is larger than 0.5 GeV and the transverse momentum of the π^0 candidates greater than 2 GeV. Besides, the π^0 candidates with mass in 2.5σ of π^0 mass resolution are used to reconstruct B_s^0 . In addition, the transverse energy of B_s^0 is required to be larger than 2 GeV.

Table B.4 Selection criteria in $B_s^0 \rightarrow J/\psi\pi^0$ channel.

Quantity	Selections
p_T of γ	$> 0.5 \text{ GeV}$
p_T of π^0	$> 2 \text{ GeV}$
mass window of π^0	$2.5\sigma_M$
p_T of B_s^0	$> 2.0 \text{ GeV}$

B.2.5.1 Luminosity effect

The physical performances in this channel with different luminosities are studied first. 20000 simulation samples for each luminosities were generated by this parameterized simulation tool. Figure B.14 shows the $J/\psi\pi^0$ invariant mass distribution with different luminosities. The yields of signal and background are estimated by fitting the mass distribution for each sample, where the gaussian function is used to describe the shape of signal and the background is described by 2 order Chebyshev polynomial function. The fitting results are listed in Tab. B.5. This channel is different from $B_s^0 \rightarrow \phi\gamma$, the number of background candidates grows even faster as luminosity increases, specially in the $\mathcal{L} = 1.5 \times 10^{34} \text{ cm}^{-2} \cdot \text{s}^{-1}$ case. The reason is that two γ candidates are used in reconstruction for this channel, and average energy of γ in this channel is usually smaller than the value in $B_s^0 \rightarrow \phi\gamma$ decay. By the way, the significance is defined as $S/\sqrt{S+20\bar{B}}$ here. The background yield is multiplied with a factor of 20^⑤ as this value is underestimated from signal simulation sample.

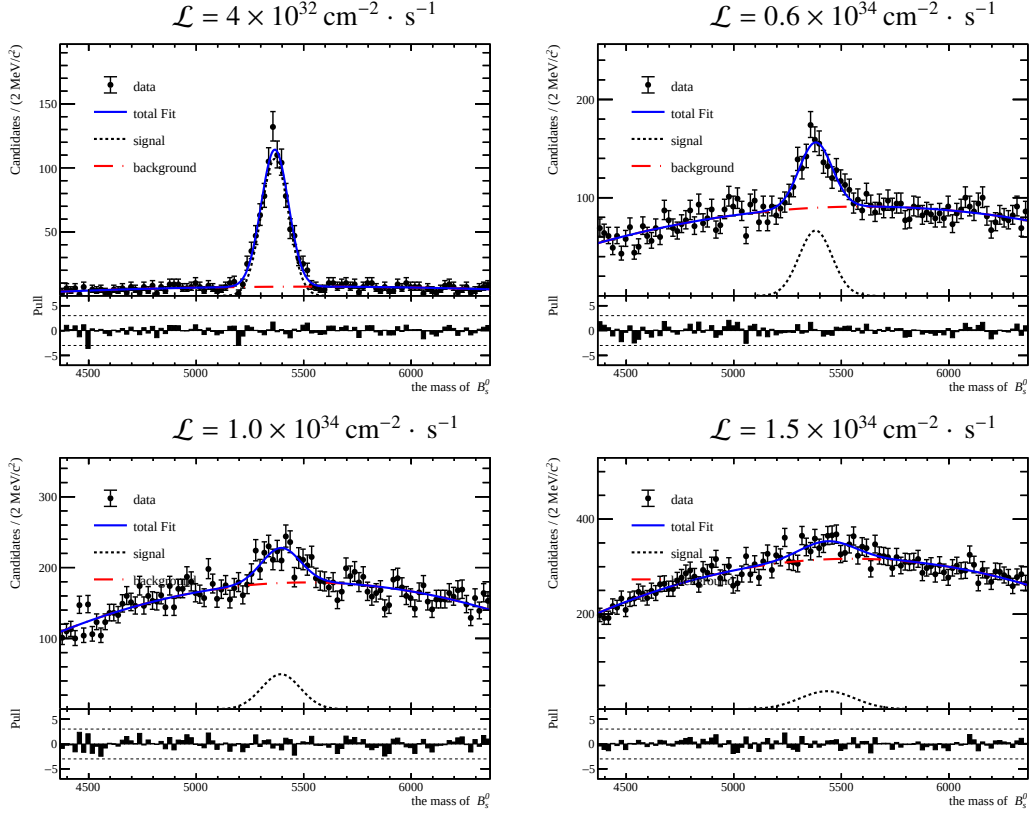

 Figure B.14 The mass distribution of B_s^0 in $B_s^0 \rightarrow J/\psi \pi^0$ decay with different luminosities.

Table B.5 The physical performance in different luminosities with different ECAL cell segmentation.

Segmentation	Luminosity ($\text{cm}^{-2} \cdot \text{s}^{-1}$)	Efficiency	Bkg ($\times 1000$)	Significance	Mass resolution (MeV)
Current	4×10^{32}	0.042	0.6	15	63
	0.6×10^{34}	0.032	8.1	3	77
	1.0×10^{34}	0.028	16.0	2	90
	1.5×10^{34}	—	—	—	—
Phillip's	4×10^{32}	0.071	0.7	21	65
	0.6×10^{34}	0.059	7.5	7	68
	1.0×10^{34}	0.059	13.0	5	70
	1.5×10^{34}	0.039	21.7	3	64
Si-W	4×10^{32}	0.168	1.3	34	82
	0.6×10^{34}	0.153	12.4	13	77
	1.0×10^{34}	0.147	21.1	9	79
	1.5×10^{34}	0.137	33.7	7	77

B.2.5.2 Smaller cell size to improve the reconstruction efficiency

Similar to the study of $B_s^0 \rightarrow \phi \gamma$ decay, all results with different simulation configurations are summarized in Tab. B.5. According to these simulation samples, we find that the recon-

⑤ The factor of 20 is obtained from internal $B^0 \rightarrow J/\psi \pi^0$ analysis at LHCb.

struction efficiencies change significantly, which is much different from the decay of $B_s^0 \rightarrow \phi\gamma$. The signal reconstruction efficiency of Si-W small cell size segmentation is around 4 times larger than the number of current ECAL segmentation. Besides, the number of background candidates are pretty large in high luminosity situation for each case. There is not chance to study this channel if the combinatorial background candidates cannot be rejected.

B.2.5.3 Precision timing to remove pile-up effect

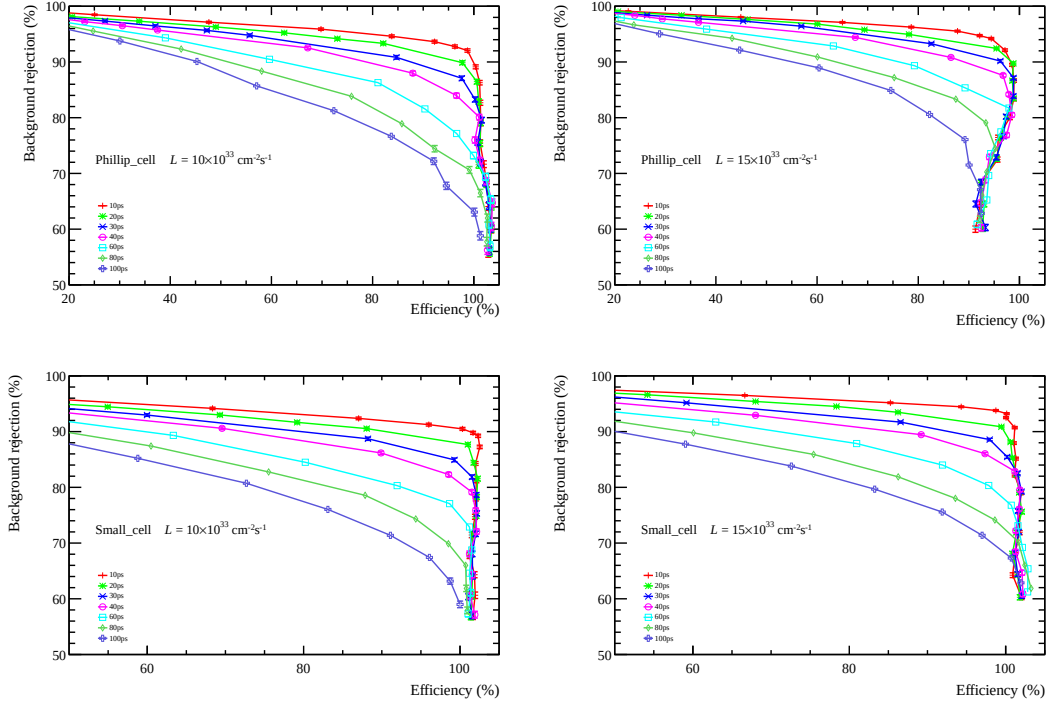


Figure B.15 The relation between signal efficiency and combinatorial background rejection fraction with different $R(t)$ cuts applied, in Phillip's segmentation (top) and Si-W segmentation (bottom) respectively.

High luminosity leads to a pretty high combinatorial background level. In order to remove this kind of background candidates, the time information is used to distinguish the primary vertices. Unlike the $B_s^0 \rightarrow \phi\gamma$, two γ are used in this channel reconstruction. A new variable $R(t)$ is used to reject background candidates, which is defined as $R(t) = \sqrt{\Delta(t_1)^2 + \Delta(t_2)^2}$, where the $\Delta(t_1)$ is the difference between the J/ψ vertex time and expected γ generated time measured by ECAL, $\Delta(t_2)$ the denotes the time difference of the other γ .

The performances of this additional selection value $R(t)$ in this decay channel is studied then. The relation between selection efficiency and background rejection fraction after time matching applied with different time resolutions is shown in Fig. B.15. It is clearly that most of combinatorial background candidates can removed with the time resolution equal to

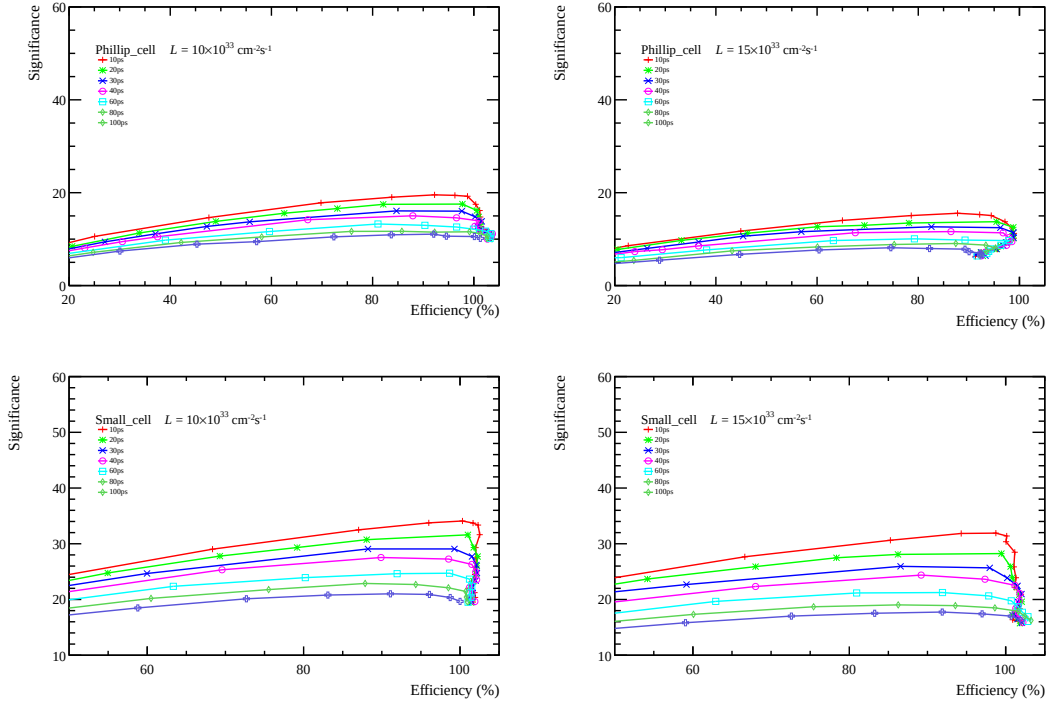


Figure B.16 The relation between signal efficiency and significance with different $R(t)$ cuts applied, in Phillip's segmentation (top) and Si-W segmentation (bottom) respectively.

tens of picosecond for both cell size segmentation. The signal significance of 20000 events simulation sample was also studied, as shown in Fig. B.16. When comparing the performances with different cell size segmentation, the signal significance of Si-W ECAL improves a lot comparing to the Phillip's segmentation due to larger reconstruction efficiency. It also means that smaller cell configuration can help improve the reconstruction efficiency of π^0 .

B.2.6 From parameterized simulation to GEANT4 simulation

From parameterized simulation, we find that overlapping effect is the biggest challenge after Upgrade II as high pile-up effect. This causes a bad performance in the cluster reconstruction and particle identification. Smaller cell size ECAL can recede this effect, which leads to a better reconstruction efficiency and mass resolution in some physical channels. Besides, smaller cell size configuration can improve the reconstruction efficiency of resolved π^0 , which also results in a better mass resolution in the physical channel containing π^0 .

In addition to smaller cell size, timing information can also help eliminate high pile-up effect significantly. On the one hand, timing information can be used in the physical channel reconstruction, the shower time should be coincident to the vertex time that the neutral particle decaying from, a lot of combinatorial background candidates are removed by the

timing matching. On the other hand, timing information can also be used in the cluster reconstruction and identification level, helping to identify the neutral particles from charged tracks and remove random overlapping cluster effect according to the cell's time distribution in one cluster. By studying the physical performance with different time resolution, the timing resolution requirement for further ECAL is around 20 ps for the particle with energy larger than 10 GeV.

The technical proposal is not studied in the parameterized simulation, a possible technology choice is using silicon sensor to measure shower time out of the experience of CMS, which can meet the requirement to time resolution. In this next section, we will discuss this performance of silicon-tungsten ECAL at LHCb. In the GEANT4 model, the idea by inserting several silicon layers to measure time in the shashlik or crystal ECAL is also discussed.

B.3 Silicon-tungsten ECAL simulation

A standalone GEANT4 package is developed to study the performance of the SiW ECAL. Figure B.17 shows a flowchart of the simulation, reconstruction, and analysis. For the study of energy resolution and the determination of the calibration parameters, the input of the GEANT4 simulation is simply provided by G4ParticleGun. For the simulation of the benchmark decay channels, the Gauss package (v51r2) is used to produce the input for the full simulation of the SiW ECAL. In the Gauss simulation the beam condition of Upgrade II is used, and the information of all particles before entering the calorimeter is recorded and is used later as the input of the standalone ECAL simulation. In this case, apart from the SiW ECAL nothing else exists in the standalone simulation. In order to study the Bremsstrahlung effect of electrons, a simplified tracking system with the magnetic field before the ECAL is implemented in the simulation. The amount of material is around $0.6X_0$. In this case the input of the standalone simulation is the output of Gauss at generator level with the Upgrade II beam condition.

B.3.1 Simulation model in GEANT4

The multi-layer SiW detector model is constructed in a standalone GEANT4 framework. Details of the detector are described below.

B.3.1.1 Geometry

The default geometry of the SiW ECAL is as follows. It contains 26 layers of tungsten as the absorbers interleaved with 26 silicon layers as the active material. The thickness of each

silicon (tungsten) layer is 0.4 mm (3.5 mm). The cell size of the Si layers is $1.01 \text{ cm} \times 1.01 \text{ cm}$. Figure B.18 shows a side-view of the SiW ECAL with a shower of a 50 GeV photon. When studying the energy resolution, the thickness of each layer and the cell size are varied.

B.3.1.2 Detection efficiency of sensors

During the development of a shower some energies will deposit in the cells. It is assumed here that the sensor will not be triggered if the deposited energy in one cell is less than 0.5 MeV. We define the detection efficiency of sensors as the fraction of the cells with deposited energy greater than 0.5 MeV. One can imagine that this efficiency should be close to unit in the first stage of the shower development, while it will be small in the far-end part of the shower. Figure B.19 presents the efficiency varies with the energy and the direction of the incident particle. It shows that the dependence of the efficiency on the incident angle is almost negligible, while the efficiencies for the layers in the end part of the ECAL strongly depend on the energy of the incident photon. One can conclude from these plots that if only a few silicon layers will be used to obtain time information of the shower, they should not be placed in the end part of the ECAL;

B.3.1.3 Parameterized silicon sensor time resolution

The silicon sensor provides the information of time-of-arrival (ToA). From the CMS TDR (CERN-LHCC-2017-023)^[257], the precision of the overall timing measurement depends on the intrinsic performance of the silicon sensor, together with the preamplifier and the discriminator. Results from the CMS beam tests^[258] have shown that the timing resolution obtained with silicon sensors does not vary significantly with sensor thickness when the

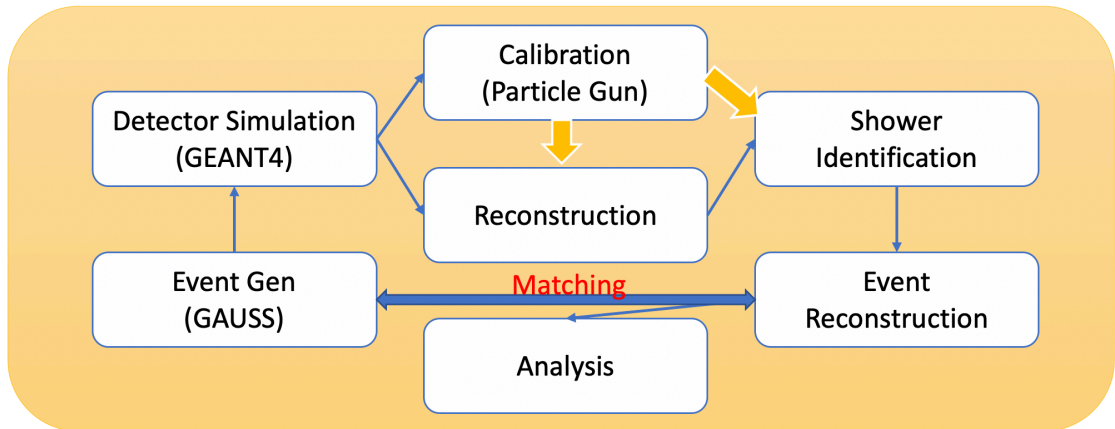


Figure B.17 The simulation procedure with Si-W ECAL model in GEANT4.

resolution is measured as a function of the signal-to-noise ratio, S/N .

The timing resolution can be expressed as

$$\sigma_t = \sigma_{jitter} \oplus \sigma_{floor}, \sigma_{jitter} = \frac{A}{S/N}, \quad (\text{B.13})$$

where σ_{floor} is a constant term, and the symbol $\oplus$ denotes quadratic summation. The constant

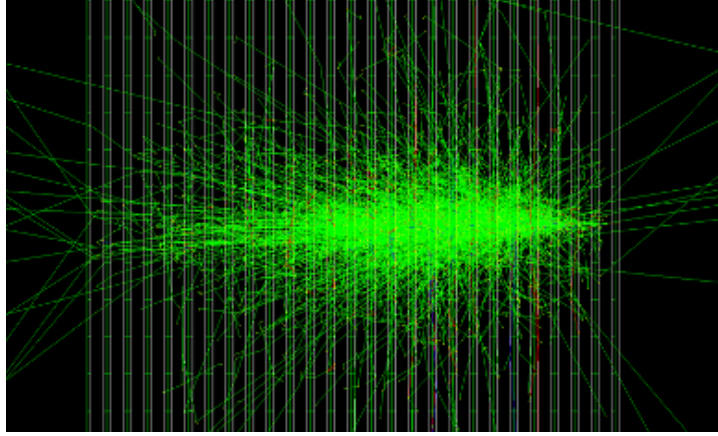


Figure B.18 Geometry of the SiW ECAL used in the GEANT4 simulation. Displayed is a shower of a 50 GeV photon.

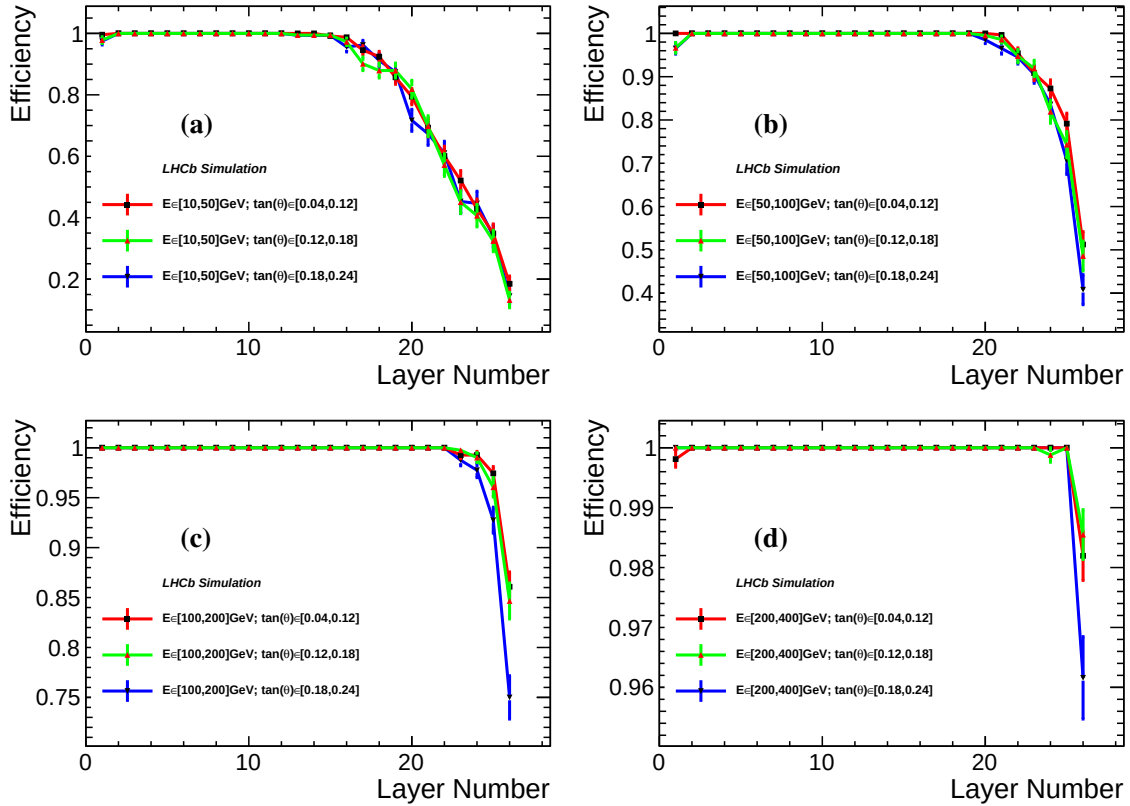


Figure B.19 The detection efficiency of sensors for γ with different energy and incident directions to the ECAL plane.

A is fixed by the response time and noise characteristics of the sensor and the preamplifier. According to the test by CMS, the expected parameters are: $A = 5$ ns, $\sigma_{floor} = 20$ ps and the S/N for MIP is equal to 12. The deposited energy MIP in a 0.4 mm-thick silicon for is around 0.16 MeV, so the parameterized function can be written as

$$\sigma_t = \frac{5 \text{ ns}}{12 \times \sqrt{E/0.16}} \oplus 0.02 \text{ ns}, \quad (\text{B.14})$$

where E is the deposited energy in MeV. For safety, the parameterized time resolution used in this simulation is

$$\sigma_t = \frac{5 \text{ ns}}{12 \times \sqrt{E/0.2}} \oplus 0.02 \text{ ns}. \quad (\text{B.15})$$

B.3.2 Reconstruction and calibration

The cell energy smaller than 0.5 MeV is neglected in the clustering. There are many different clustering algorithms to reconstruct the cluster for different kinds of calorimeters, *e.g.* Arbor, ML, APRIL, *et al.*^[259–261]. The main idea is to fully exploit the advantage of the shower characteristics for a given calorimeter. A simple but effective algorithm is used for preliminary analyses. The algorithm used this study is similar to the method used in the current LHCb calorimeter with an extension to multi-layer configuration. It is simple and effective for preliminary analyses. In this algorithm, the seed cell for each layer (26 layers in total), which is used as a starting point for the aggregation, is first searched for according to the energy distribution. The cell energies around the seed cell are then added for each layer. Finally, these energies in all layers are summed to form a shower if they are in a similar position in the x - y plane.

B.3.2.1 Shower reconstruction

The cluster layer energy is obtained by adding all cell's energy together:

$$E_{layer} = \sum_i^{cells} E_i \quad (\text{B.16})$$

and the layer cluster position are estimated:

$$\begin{aligned} x_{layer} &= \left(\sum_i^{cells} x_i \times E_i \right) / E_{layer} \\ y_{layer} &= \left(\sum_i^{cells} y_i \times E_i \right) / E_{layer} \end{aligned} \quad (\text{B.17})$$

$$z_{layer} = z_i$$

As the thickness of silicon sensor is very small, 4 μm here, the silicon sensor z position is taken as the layer cluster position.

The layer time is also obtained from the cell in the same layer:

$$T_{layer} = (\sum_i^{cells} T_{i,cali} \times E_i) / E_{layer} \quad (\text{B.18})$$

considering the developing time of the shower, usually, the measured time from the cells around seed cell is larger than the seed cell time, therefore, the used time in the above formula is calibrated, the calibration method will be addressed later.

The shower information is estimated from the each layer cluster calibrated, with the layer energy weight. In the current reconstruction algorithm, the layer cluster energy smaller then 3 MeV is overlooked, considering the random effect.

B.3.2.2 Energy calibration

The method to calibrate the reconstruct shower energy is similar to the previous study, by a linear function:

$$E_{cali} = \alpha \times E_{rec} + \beta \quad (\text{B.19})$$

In the real experiment, the calibrated parameters are obtained from beam test or real running, we estimate these numbers in from particle gun simulation here. From the γ event, we get the relation between the true particle energy and the reconstructed one, as shown in Fig. B.20, and fitted by a linear function. The obtained values will be taken as the calibration parameters. After the calibration is finished, the energy resolution is studied, as shown in Fig. B.21, where the thicknesses of silicon is 4 mm.

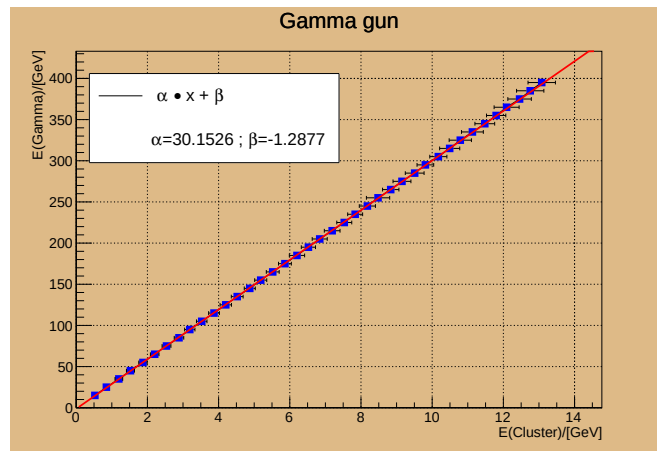


Figure B.20 The relation between particle true energy and reconstructed cluster shower energy.

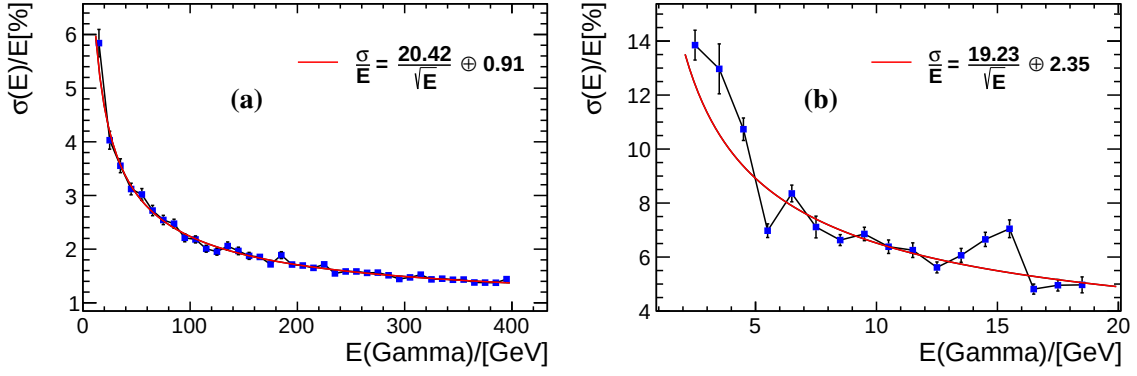


Figure B.21 The energy resolution in different particle energy regions.

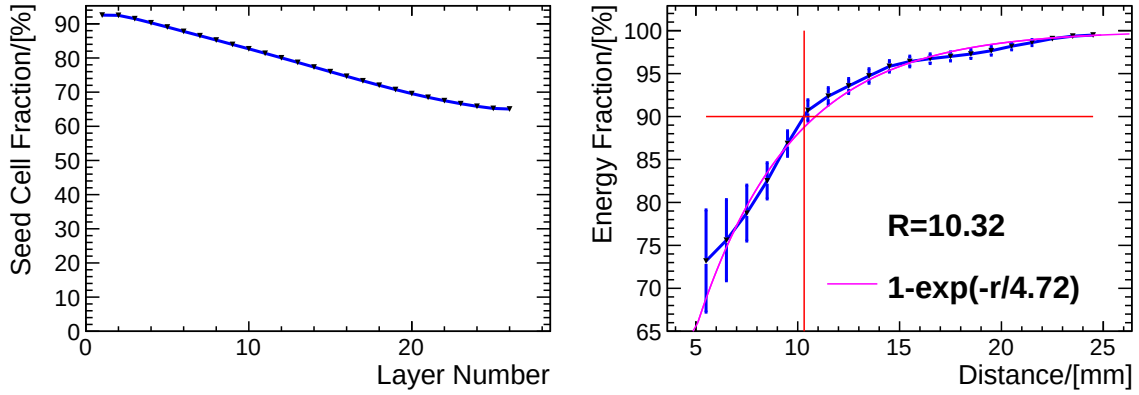


Figure B.22 The seed cell energy fraction for each layer (left). The estimated Moliere radius (right).

The seed cell energy fraction in one layer cluster for each layer is shown in left of Fig. B.22, according to this plot, we know that most of shower energy deposit in the seed cell, which is also the reason why we use current method to cluster. Besides, the seed energy fraction becomes smaller as the shower development.

A check to the Moliere radius is also performed, as shown in right of Fig. B.22, which is obtained from the reconstructed shown energy from the silicon sensors. The estimated Moliere radius is around 10.3 mm, which is a little larger than the value of tungsten from PDG^[1] as the silicon and gap in the current Multi-layer SiW ECAL model. Besides, these plots also show the lateral development of shower in SiW ECAL, the distribution is parameterized by this function:

$$f = 100(1 - e^{r/A}) \quad (\text{B.20})$$

where r is the cell distance to the shower energy baryon axis, $A = 4.73$ is obtained from fitting, which is a free parameter in the fit. Actually, the relations between energy fraction and distance in each layers are little different, not go too far into this in the preliminary study.

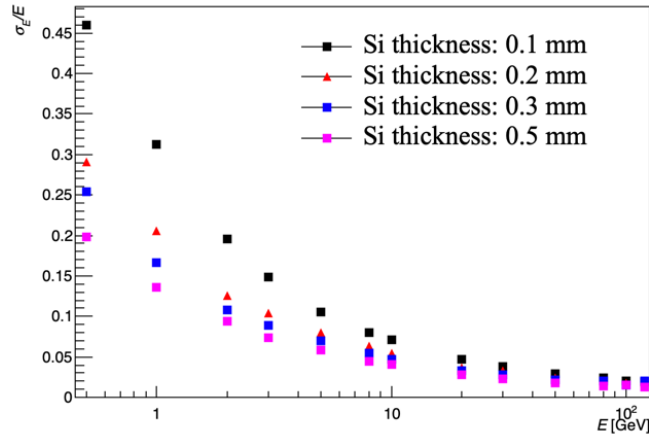


Figure B.23 Energy resolution of electron showers as a function of the incident electron energy for different thicknesses of the silicon layers. The cell size is $1.0 \times 1.0 \text{ cm}^2$ and the thickness of the tungsten layers is 3.5 mm.

Figure B.23 shows the energy resolution as a function of the incident electron energy for different thicknesses of the silicon layers with the cell size of $1.0 \times 1.0 \text{ cm}^2$ and the thickness of 3.5 mm for the tungsten layers. It shows that the energy resolution improves as the Si thickness increases. The dependence of the energy resolution on the cell size of the silicon layer and the thickness of the tungsten layer is almost negligible. The energy resolution is around 15% (5%) when the energy of the electron is 1 GeV (10 GeV).

B.3.2.3 Position calibration

The method to calibrate the layer cluster position is similar to the one used in current ECAL. And the position calibration is performed according to the relative position in one cell. As shown in the left of Fig. B.24, the reconstructed position is not exactly equal to the true position, the relation of these values is estimated by Function. B.4, also used in parameterized simulation. the left plot of Fig. B.24 is a example to the 15th layer, actually, every layer is studied separately, as shower shape changes with depth of the incident.

We also take a look at the position resolution with the different particle energy in each layer. As shown in the right Fig. B.24, the 5-17 layer performed better than other layers within a large particle gun energy region.

B.3.2.4 Timing calibration

The method to calibrate time information is also studied. As mentioned in the Sec. B.3.1.3, the parameterized method is used to simulate the silicon sensor time, which also rest with the

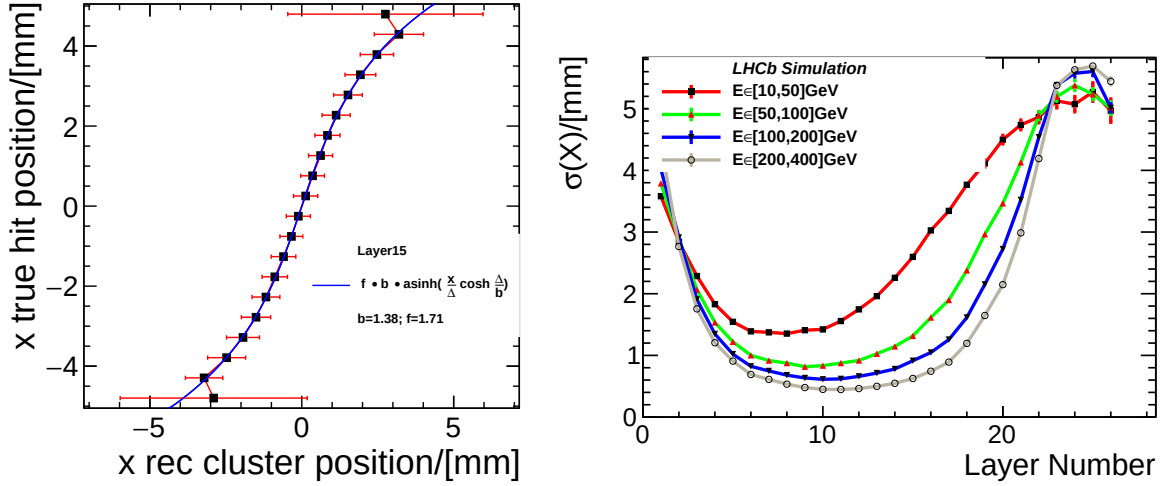


Figure B.24 The relative position to the cell between true layer cluster and true layer cluster of the 15th layer (left). The position resolution (right).

development characters of shower in SiW ECAL. Usually, the measured time around the seed cell perform a "time-lag" effect comparing to the value obtained from the seed cell. As shown in Fig. B.25, the time difference measured from the seed cell and around cells is related to the distance, in this case, we calibrate the cell time using a linear function:

$$\Delta(T_{cell}) = a \times d + b \quad (\text{B.21})$$

where the $\Delta(T_{cell})$ is the time difference to the seed cell, d is the distance, and a, b are the calibration parameters. We study the time calibration for each layer separately.

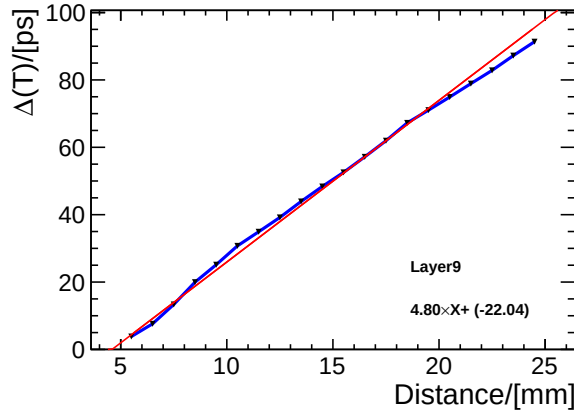


Figure B.25 The time difference between the seed cell time with the measured value around cells in 9th layer

The time resolutions of every layer are shown in Fig. B.26, which are different with each other as the shower deposited energy distribution in the z -direction. According to the energy resolutions of every layer mentioned above, we find that the layer with larger deposited energy

leads to better energy and time resolution.

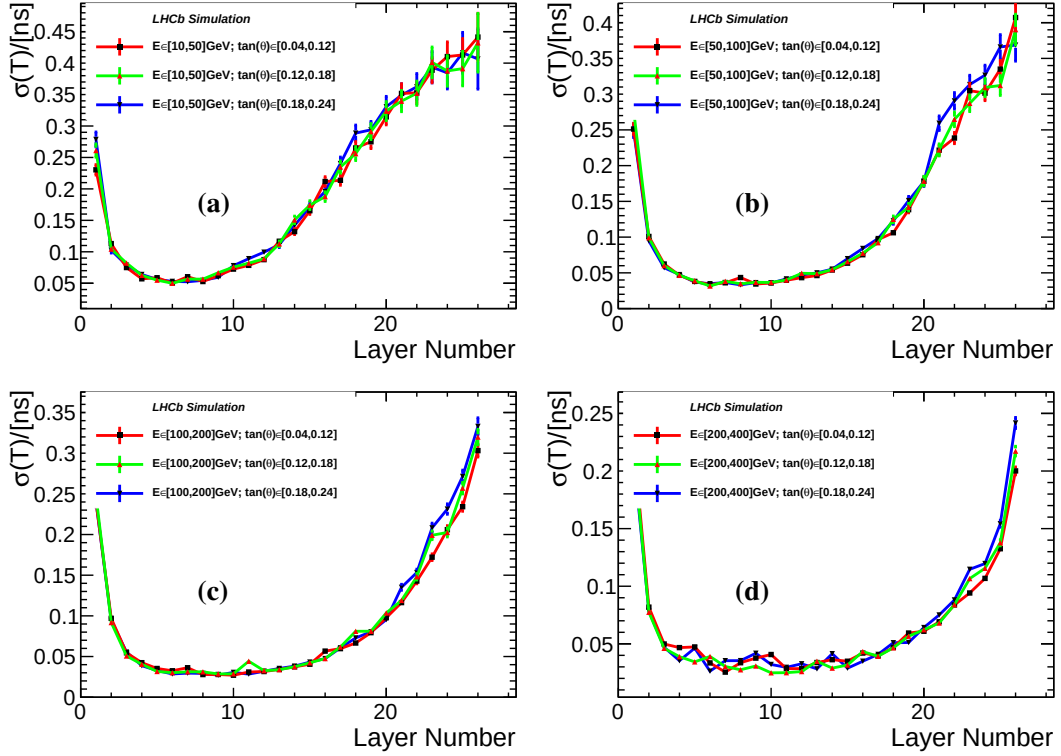


Figure B.26 The time resolution of each layer with different γ energy and incident angle.

Figure B.27 shows the time resolution in layer 4-14 before and after the cell time calibration, the time distribution peak becomes narrow, especially for the rear layers.

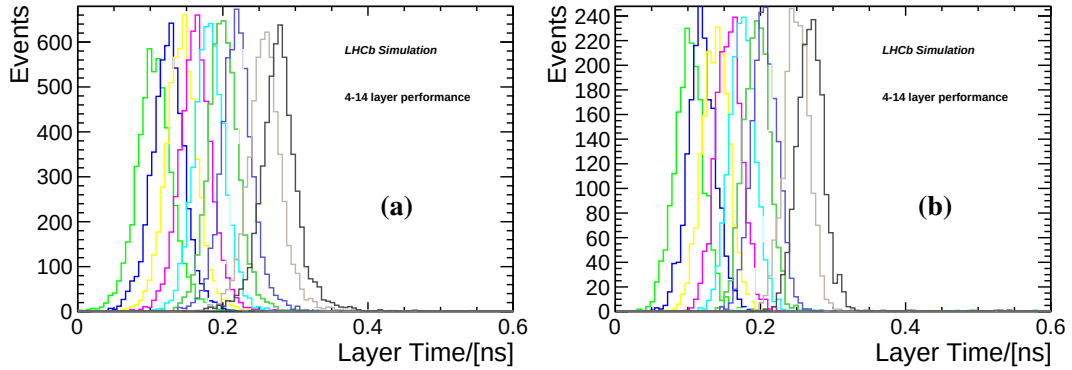


Figure B.27 The time distribution for each layer before(a) and after(b) calibration.

The shower time is reconstructed based on the measured time from each layer, the time recorded in each layer is different as shower developing in the longitudinal direction. As shown in Fig. B.28, the slope of this linear function obtained from fitting is around equal the value if assume the shower development speed is exactly equal to light speed, from this study, which

is around 2% smaller than the light speed. Besides, we use this parameter to calibrate the measured layer time when reconstruct the shower time.

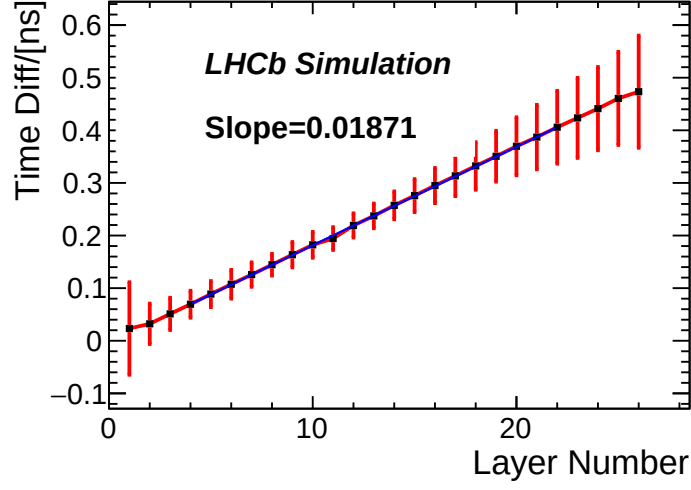


Figure B.28 The time difference measured in each layer.

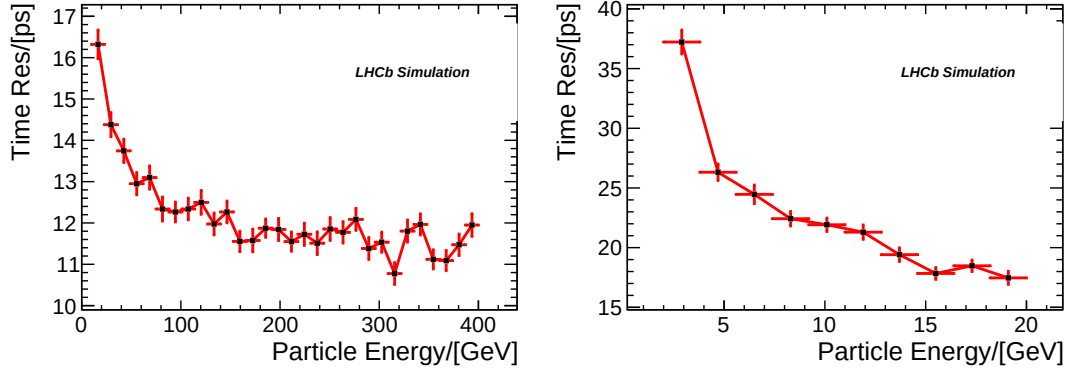


Figure B.29 The shower time resolution with three layers to measure shower time, the right plot shows the low energy result.

After time calibration, the shower time resolution is shown in Fig. B.29, which is related to the particle energy, and the shower time resolution is constructed from 4th, 5th and 6th layers. If only three layers have the ability to record the time information, the shower time resolution will reach to 15 ps. This number can meet the requirement obtained from parameterized simulation.

B.3.3 Physical performance with silicon of different thickness

As the known, the silicon sensors have to work at around -30°C , to reliably operate silicon sensors after irradiation, and to keep low electronic noise causing from leakage current. The

evaporative CO₂ is popularly used to cool the silicon sensors, which go through the pipes in the copper cooling plane as the copper can provide excellent thermal conductivity. In order to study the effect to the performance of copper plane cryostat, we change the geometry to each layer, with 6 mm copper plane added, which is attached to the silicon sensors. This is a safe estimation, usually 26 cooling layers are not necessary, according to the study from CMS HGCal group, five plants are employed to cool the system totally. The physical performances with copper cryostat are also studied.

B.3.3.1 $B_s^0 \rightarrow \phi\gamma$ performance

The simulation sample of this decay channel was generated in $\mathcal{L} = 4 \times 10^{32} \text{ cm}^{-2} \cdot \text{s}^{-1}$ situation, details procedure is shown in Fig. B.17. The reconstruction method for this channel is as same as the one performed in parameterized simulation. Figure B.30 presents the mass distributions of B_s^0 mesons which are constructed in $B_s^0 \rightarrow \phi\gamma$ channel with different thickness of silicon sensor. It is obvious that the Si-W with copper cooling included leads to a worse B_s^0 mass resolution. Besides, these results are consistent to these obtained from parameterized simulation.

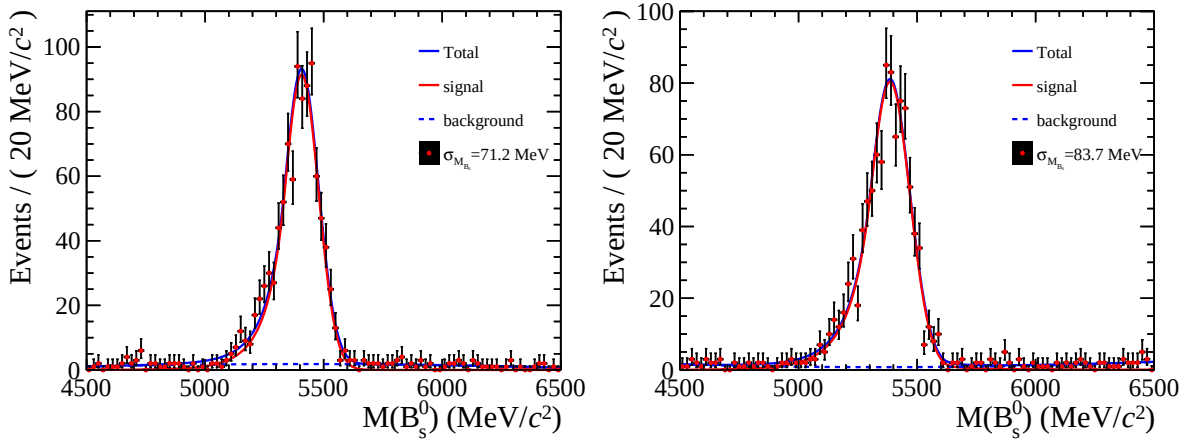


Figure B.30 Mass distributions of the B_s^0 mesons from $B_s^0 \rightarrow \phi\gamma$ decays. The thickness of silicon sensor is 2 mm, no copper cryostat applied (left). The thickness of silicon sensor is 2 mm, 26 cooling copper plane added against silicon layers.

B.3.3.2 $B_s^0 \rightarrow J/\psi\pi^0$ performance

The decay channel $B_s^0 \rightarrow J/\psi\pi^0$ is used to study the performance for π^0 mesons. Figure. B.31 shows the mass distributions of the π^0 mesons from $B_s^0 \rightarrow J/\psi\pi^0$ decays. Besides, the B_s^0 mass resolution is also studied. Figure B.32 shows the B_s^0 mass distributions with $\mathcal{L} = 4 \times 10^{32} \text{ cm}^{-2} \cdot \text{s}^{-1}$. The left one is the result with the thickness of silicon sensor equal to

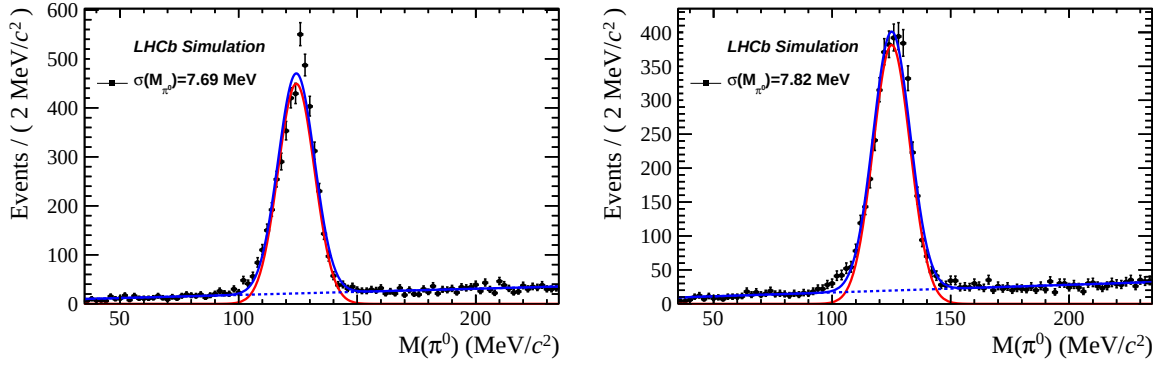


Figure B.31 Mass distributions of the π^0 mesons from $B_s^0 \rightarrow J/\psi \pi^0$ decays. The thickness of silicon sensor is 2 mm, no copper cryostat applied (left). The thickness of silicon sensor is 2 mm, 26 cooling copper plane added against silicon layers.

2 mm, while the right one shows the result with copper cooling plane applied. Similarly, the results from GEANT4 simulation are consistent to the parameterized simulation conclusion.

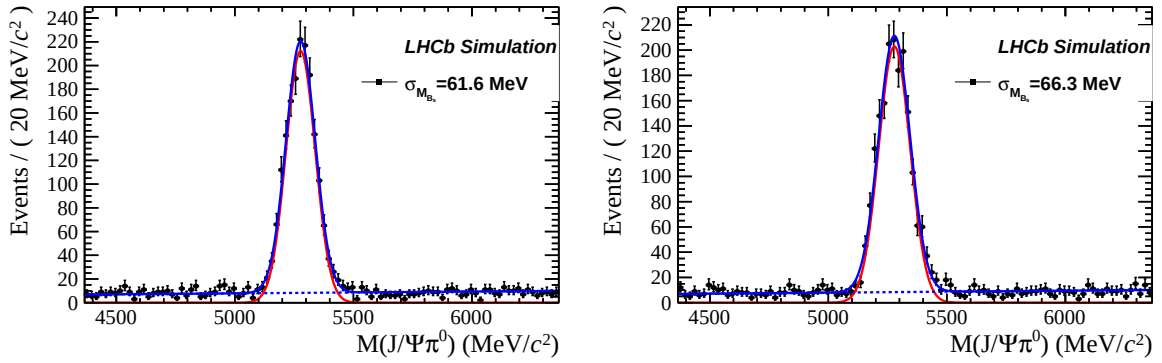


Figure B.32 The B_s^0 mass distribution reconstructed from $B_s^0 \rightarrow J/\psi \pi^0$ decay. The thickness of silicon sensor is 2 mm, no copper cryostat applied (left). The thickness of silicon sensor is 2 mm, 26 cooling copper plane added against silicon layers.

B.3.4 Bremsstrahlung recovery and $B^0 \rightarrow K^* e^- e^+$ performance

In consideration of the detector material budget before the ECAL, the electron has possibility to interact with trackers from bremsstrahlung, which cause the energy measurement to electron is not accurate, especially the reaction happening before the magnet. In this case, we want to study the method to recover the electron energy by adding the radiated γ .

The decay channel $B^0 \rightarrow K^* e^- e^+$ is taken as a benchmark channel, which can be used to study the bremsstrahlung effect. The electron energy and transverse momentum distribution are shown in Fig. B.33.

We take a look at the distance between the electron and the bremsstrahlung photon on the ECAL plane, as shown in the Fig. B.34.

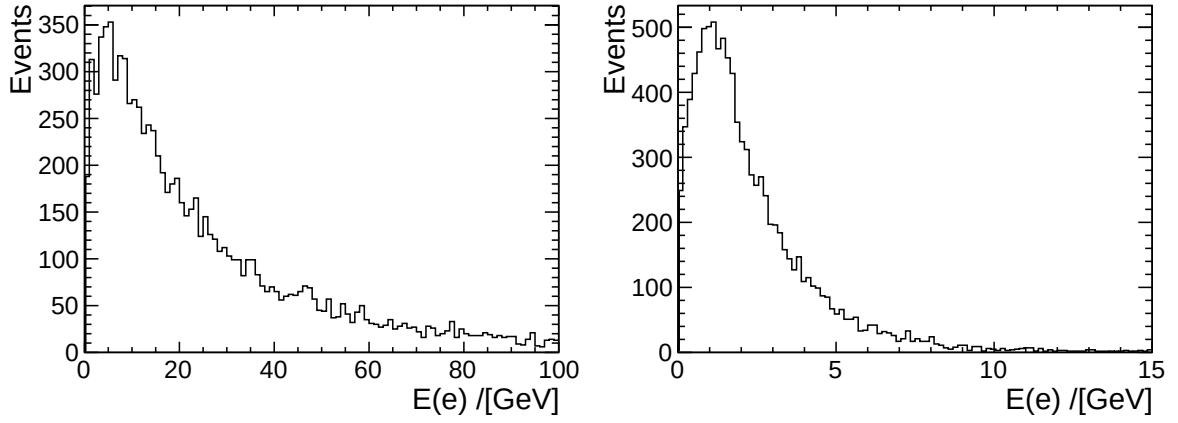


Figure B.33 The electron energy(left) and transverse momentum(right) distribution in $B^0 \rightarrow K^* e^- e^+$ decay.

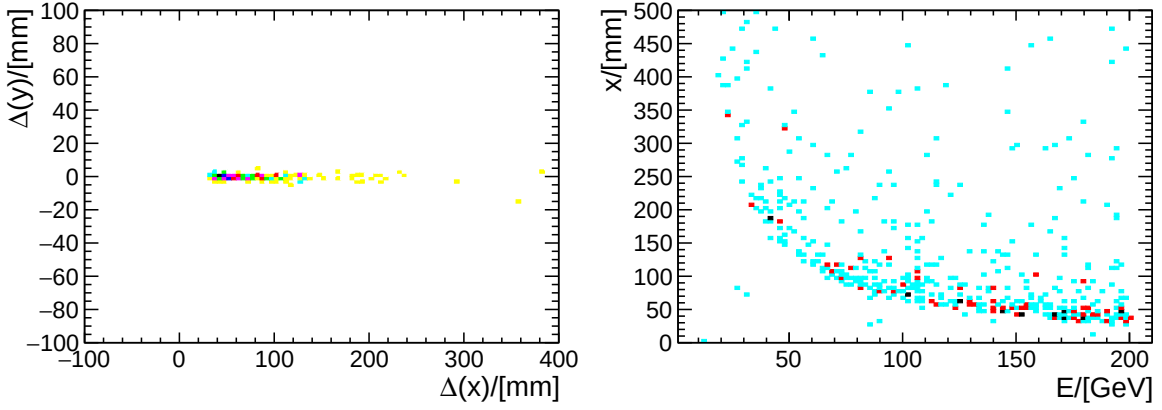


Figure B.34 Left: the distance between the electron and bremsstrahlung photon on the ECAL plane; Right: the relation between the electron energy and the x distance on the ECAL plane.

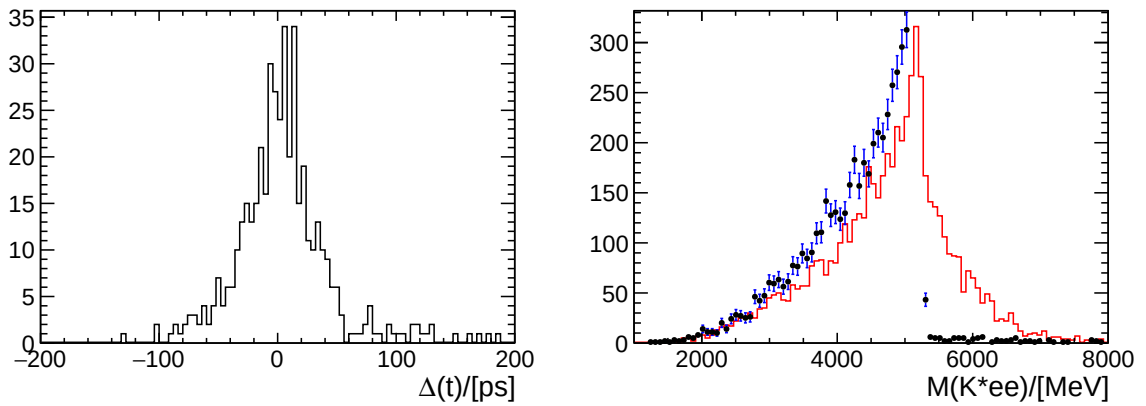


Figure B.35 Left: the time difference between the electron and bremsstrahlung photon. Right: the B^0 mass distribution before(blue point) or after bremsstrahlung recovery with time-matching included(red line).

The time difference between the electron and bremsstrahlung photon is shown in the left plot of Figure. B.35, we hope the time information can help improve the mass resolution of B^0 reconstructed from $B^0 \rightarrow K^* e^- e^+$ decay, the mass distribution is shown in the right plot of Fig. B.35. However, there is no significant improvement for the invariant mass resolution of $K^* e^+ e^-$ when time information is used.

B.4 Conclusions

The parameterized simulation tool for ECAL upgrade was developed based on GAUSS and DELPHES. By studying two decay channels, we found the high luminosity leads to an immense level of combinatorial background. The precision timing can help to remove many background candidates, and the required time resolution is around several tens of picosecond. Besides, smaller cell size is necessary for Upgrade II, as it has a great impact to reconstruction efficiency.

Then, a detailed Si-W ECAL simulation is performed with a stand-alone GEANT4 package. The energy resolution and time resolution are studied in this simulation tool, and these parameters can meet the requirements for ECAL Upgrade II at LHCb. The performances of baseline decay channels were also studied in this GEANT4 simulation, and these results fall in line with parameterized simulation findings.

References

- [1] P. A. Zyla *et al.* “Review of particle physics”. to be published in *Prog. Theor. Exp. Phys.* **2020**, 6: 083C01.
- [2] P. Pascual and R. Tarrach. *QCD: RENORMALIZATION FOR THE PRACTITIONER*. **1984**.
- [3] S. Bethke. “Experimental tests of asymptotic freedom”. *Progress in Particle and Nuclear Physics*, **2007**, 58(2): 351–386.
- [4] Stephen Lars Olsen, Tomasz Skwarnicki and Daria Zieminska. “Nonstandard heavy mesons and baryons: Experimental evidence”. *Rev. Mod. Phys.* 2018-02, 90: 015003.
- [5] B. Abi *et al.* “Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm”. *Phys. Rev. Lett.* **2021**, 126(14): 141801.
- [6] Sz. Borsanyi *et al.* “Leading hadronic contribution to the muon 2 magnetic moment from lattice QCD”. 2020-02.
- [7] J. Beringer *et al.* “Review of Particle Physics”. *Phys.Rev.* **2012**, D86: 010001.
- [8] Nora Brambilla, Simon Eidelman, Christoph Hanhart *et al.* “The XYZ states: Experimental and theoretical status and perspectives”. *Physics Reports*, **2020**, 873, The XYZ states: experimental and theoretical status and perspectives: 1–154.
- [9] E. Fermi and C. N. Yang. “Are Mesons Elementary Particles?” *Phys. Rev.* 1949-12, 76: 1739–1743.
- [10] Murray Gell-Mann. “A schematic model of baryons and mesons”. *Phys.Lett.* **1964**, 8: 214–215.
- [11] G Zweig. “An SU_3 model for strong interaction symmetry and its breaking; Version 2”. **1964**(CERN-TH-412).
- [12] T. Nakano and K. Nishijima. “Charge Independence for V-particles”. *Prog. Theor. Phys.* **1953**, 10: 581–582.
- [13] Stephen Godfrey and Stephen L. Olsen. “The Exotic XYZ Charmonium-Like Mesons”. *Annual Review of Nuclear and Particle Science*, **2008**, 58(1): 51–73.
- [14] E. Eichten, K. Gottfried, T. Kinoshita *et al.* “Charmonium: Comparison with Experiment”. *Phys. Rev. D*, **1980**, 21: 203.
- [15] T. Barnes, S. Godfrey and E.S. Swanson. “Higher charmonia”. *Phys. Rev. D*, **2005**, 72: 054026.
- [16] Stanley F. Radford and Wayne W. Repko. “Potential model calculations and predictions for heavy quarkonium”. *Phys. Rev. D*, **2007**, 75: 074031.
- [17] M. Atif Sultan, Nosheen Akbar, Bilal Masud *et al.* “Higher Hybrid Charmonia in an Extended Potential Model”. *Phys. Rev. D*, **2014**, 90(5): 054001.
- [18] S.-K. Choi, S. L. Olsen, K. Abe *et al.* “Observation of a Narrow Charmoniumlike State in Exclusive $B^\pm \rightarrow K^\pm \pi^+ \pi^- J/\psi$ Decays”. *Phys. Rev. Lett.* 2003-12, 91: 262001.
- [19] T. Nakano, D. S. Ahn, J. K. Ahn *et al.* “Evidence for a Narrow $S = +1$ Baryon Resonance in Photoproduction from the Neutron”. *Phys. Rev. Lett.* 2003-07, 91: 012002.

- [20] SHI-LIN ZHU. “PENTAQUARKS”. *International Journal of Modern Physics A*, **2004**, 19(21): 3439–3469.
- [21] Tianbo Liu, Yajun Mao and Bo-Qiang Ma. “Present status on experimental search for pentaquarks”. *International Journal of Modern Physics A*, **2014**, 29(13): 1430020.
- [22] A. Martínez Torres and E. Oset. “Novel Interpretation of the “ $\Theta^+(1540)$ Pentaquark” Peak”. *Phys. Rev. Lett.* 2010-08, 105: 092001.
- [23] Yan-Rui Liu, Hua-Xing Chen, Wei Chen *et al.* “Pentaquark and Tetraquark states”. *Prog. Part. Nucl. Phys.* **2019**, 107: 237–320.
- [24] Bernard Aubert *et al.* “A Study of $B \rightarrow X(3872)K$, with $X(3872) \rightarrow J/\psi\pi^+\pi^-$ ”. *Phys. Rev. D*, **2008**, 77: 111101.
- [25] S. -K. Choi *et al.* “Bounds on the width, mass difference and other properties of $X(3872) \rightarrow \pi^+\pi^-J/\psi$ decays”. *Phys. Rev. D*, **2011**, 84: 052004.
- [26] A. Bala *et al.* “Observation of $X(3872)$ in $B \rightarrow X(3872)K\pi$ decays”. *Phys. Rev. D*, **2015**, 91(5): 051101.
- [27] P. del Amo Sanchez *et al.* “Evidence for the decay $X(3872) \rightarrow J/\psi\omega$ ”. *Phys. Rev. D*, **2010**, 82: 011101.
- [28] Bernard Aubert *et al.* “Study of Resonances in Exclusive B Decays to $\bar{D}^*D^*K$ ”. *Phys. Rev. D*, **2008**, 77: 011102.
- [29] V. Bhardwaj *et al.* “Observation of $X(3872) \rightarrow J/\psi\gamma$ and search for $X(3872) \rightarrow \psi'\gamma$ in B decays”. *Phys. Rev. Lett.* **2011**, 107: 091803.
- [30] R. Aaij *et al.* “Evidence for the decay $X(3872) \rightarrow \psi(2S)\gamma$ ”. *Nucl. Phys.* **2014**, B886: 665.
- [31] Kazuo Abe *et al.* “Observation of a near-threshold omega J/ψ mass enhancement in exclusive $B \rightarrow K\omega J/\psi$ decays”. Ed. by He-Sheng Chen, Dong-Sheng Du, Wei-Guo Li *et al.* *Phys. Rev. Lett.* **2005**, 94: 182002.
- [32] Bernard Aubert *et al.* “Observation of $Y(3940) \rightarrow J/\psi\omega$ in $B \rightarrow J/\psi\omega K$ at BABAR”. *Phys. Rev. Lett.* **2008**, 101: 082001.
- [33] S. Uehara *et al.* “Observation of a charmonium-like enhancement in the $\gamma\gamma \rightarrow \omega J/\psi$ process”. *Phys. Rev. Lett.* **2010**, 104: 092001.
- [34] J. P. Lees *et al.* “Study of $X(3915) \rightarrow J/\psi\omega$ in two-photon collisions”. *Phys. Rev. D*, **2012**, 86: 072002.
- [35] P. Pakhlov *et al.* “Production of New Charmoniumlike States in $e^+e^- \rightarrow J/\psi D^*\bar{D}^*$ at $\sqrt{s} \approx 10.6$ GeV”. *Phys. Rev. Lett.* **2008**, 100: 202001.
- [36] C. Z. Yuan *et al.* “Measurement of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ cross-section via initial state radiation at Belle”. *Phys. Rev. Lett.* **2007**, 99: 182004.
- [37] Z. Q. Liu *et al.* “Study of $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ and Observation of a Charged Charmoniumlike State at Belle”. *Phys. Rev. Lett.* **2013**, 110, [Erratum: *Phys. Rev. Lett.* 111, 019901 (2013)]: 252002.
- [38] Medina Ablikim *et al.* “Precise measurement of the $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ cross section at center-of-mass energies from 3.77 to 4.60 GeV”. *Phys. Rev. Lett.* **2017**, 118(9): 092001.

-
- [39] R. Aaij *et al.* “Observation of exotic $J/\psi\phi$ structures from amplitude analysis of $B^+ \rightarrow J/\psi\phi K^+$ decays”. *Phys. Rev. Lett.* **2017**, 118: 022003.
 - [40] R. Aaij *et al.* “Amplitude analysis of $B^+ \rightarrow J/\psi\phi K^+$ decays”. *Phys. Rev.* **2017**, D95: 012002.
 - [41] Bernard Aubert *et al.* “Observation of a broad structure in the $\pi^+\pi^-J/\psi$ mass spectrum around $4.26\text{-GeV}/c^2$ ”. *Phys. Rev. Lett.* **2005**, 95: 142001.
 - [42] J. P. Lees *et al.* “Study of the reaction $e^+e^- \rightarrow J/\psi\pi^+\pi^-$ via initial-state radiation at BaBar”. *Phys. Rev. D*, **2012**, 86: 051102.
 - [43] T. E. Coan *et al.* “Charmonium decays of $Y(4260)$, $\psi(4160)$ and $\psi(4040)$ ”. *Phys. Rev. Lett.* **2006**, 96: 162003.
 - [44] Q. He *et al.* “Confirmation of the $Y(4260)$ resonance production in ISR”. *Phys. Rev. D*, **2006**, 74: 091104.
 - [45] M. Ablikim *et al.* “Observation of a Charged Charmoniumlike Structure in $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ at $\sqrt{s} = 4.26\text{ GeV}$ ”. *Phys. Rev. Lett.* **2013**, 110: 252001.
 - [46] C. P. Shen *et al.* “Evidence for a new resonance and search for the $Y(4140)$ in the $\gamma\gamma \rightarrow \phi J/\psi$ process”. *Phys. Rev. Lett.* **2010**, 104: 112004.
 - [47] X. L. Wang *et al.* “Observation of Two Resonant Structures in $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$ via Initial State Radiation at Belle”. *Phys. Rev. Lett.* **2007**, 99: 142002.
 - [48] X. L. Wang *et al.* “Measurement of $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$ via Initial State Radiation at Belle”. *Phys. Rev. D*, **2015**, 91: 112007.
 - [49] J. P. Lees *et al.* “Study of the reaction $e^+e^- \rightarrow \psi(2S)\pi^+\pi^-$ via initial-state radiation at BaBar”. *Phys. Rev. D*, **2014**, 89(11): 111103.
 - [50] R. Aaij *et al.* “Observation of structure in the J/ψ -pair mass spectrum”. *Science Bulletin*, **2020**, 65(23): 1983.
 - [51] R. Aaij *et al.* “Determination of the $X(3872)$ meson quantum numbers”. *Phys. Rev. Lett.* **2013**, 110: 222001.
 - [52] Stephen Lars Olsen, Tomasz Skwarnicki and Daria Zieminska. “Nonstandard heavy mesons and baryons: Experimental evidence”. *Rev. Mod. Phys.* **2018**, 90(1): 015003.
 - [53] Yan-Rui Liu, Hua-Xing Chen, Wei Chen *et al.* “Pentaquark and Tetraquark States”. *Progress in Particle and Nuclear Physics*, **2019**, 107: 237–320.
 - [54] Eric S. Swanson. “Short range structure in the $X(3872)$ ”. *Physics Letters B*, **2004**, 588(3): 189–195.
 - [55] Eric S. Swanson. “Diagnostic decays of the $X(3872)$ ”. *Physics Letters B*, **2004**, 598(3): 197–202.
 - [56] Yubing Dong, Amand Faessler, Thomas Gutsche *et al.* “ $J/\psi\gamma$ and $\psi(2S)\gamma$ decay modes of the $X(3872)$ ”. *Journal of Physics G: Nuclear and Particle Physics*, 2010-11, 38(1): 015001.
 - [57] C. Bignamini, B. Grinstein, F. Piccinini *et al.* “Is the $X(3872)$ Production Cross Section at $\sqrt{s}=1.96\text{ TeV}$ Compatible with a Hadron Molecule Interpretation?” *Phys. Rev. Lett.* 2009-10, 103: 162001.
 - [58] P. del Amo Sanchez, J. P. Lees, V. Poireau *et al.* “Evidence for the decay $X(3872) \rightarrow J/\psi\omega$ ”. *Phys. Rev. D*, 2010-07, 82: 011101.

- [59] S. Uehara, T. Aushev, A. M. Bakich *et al.* “*Observation of a Charmoniumlike Enhancement in the $\gamma\gamma \rightarrow \omega J/\psi$ Process*”. *Phys. Rev. Lett.* 2010-03, 104: 092001.
- [60] Estia J. Eichten, Kenneth Lane and Chris Quigg. “*Charmonium levels near threshold and the narrow state $X(3872) \rightarrow \pi^+\pi^- J/\psi$* ”. *Phys. Rev. D*, 2004-05, 69: 094019.
- [61] Bai-Qing Li and Kuang-Ta Chao. “*Higher charmonia and X, Y, Z states with screened potential*”. *Phys. Rev. D*, 2009-05, 79: 094004.
- [62] Zhi-Yong Zhou, Zhiguang Xiao and Hai-Qing Zhou. “*Could the $X(3915)$ and the $X(3930)$ Be the Same Tensor State?*” *Phys. Rev. Lett.* 2015-07, 115: 022001.
- [63] Geoffrey T. Bodwin, Eric Braaten, Tzu Chiang Yuan *et al.* “*P-wave charmonium production in B-meson decays*”. *Phys. Rev. D*, 1992-11, 46: R3703–R3707.
- [64] Tetsuo Hatsuda and Teiji Kunihiro. “*QCD phenomenology based on a chiral effective Lagrangian*”. *Phys. Rept.* **1994**, 247: 221–367.
- [65] A. Vinokurova *et al.* “*Search for B decays to final states with the η_c meson*”. *JHEP*, **2015**, 06, [Erratum: JHEP 02, 088 (2017)]: 132.
- [66] R. Aaij *et al.* “*Evidence for the decay $B^0 \rightarrow J/\psi\omega$ and measurement of the relative branching fractions of B_s^0 meson decays to $J/\psi\eta$ and $J/\psi\eta'$* ”. *Nucl. Phys.* **2013**, B867: 547.
- [67] T. Aaltonen *et al.* “*Observation of the $Y(4140)$ structure in the $J/\psi \phi$ Mass Spectrum in $B^\pm \rightarrow J/\psi \phi K$ decays*”. **2011**.
- [68] Shen Cheng-Ping. “*XYZ particles at Belle*”. *Chin. Phys.* **2010**, C34: 615–620.
- [69] J. P. Lees *et al.* “*Study of $B^{\pm,0} \rightarrow J/\psi K^+ K^- K^{\pm,0}$ and search for $B^0 \rightarrow J/\psi \phi$ at BABAR*”. *Phys. Rev.* **2015**, D91: 012003.
- [70] R. Aaij *et al.* “*Search for the $X(4140)$ state in $B^+ \rightarrow J/\psi \phi K^+$ decays*”. *Phys. Rev.* **2012**, D85: 091103(R).
- [71] Serguei Chatrchyan *et al.* “*Observation of a peaking structure in the $J/\psi \phi$ mass spectrum from $B^\pm \rightarrow J/\psi \phi K^\pm$ decays*”. *Phys. Lett.* **2014**, B734: 261–281.
- [72] Victor Mukhamedovich Abazov *et al.* “*Search for the $X(4140)$ state in $B^+ \rightarrow J/\psi \phi K^+$ decays with the D0 detector*”. *Phys. Rev.* **2014**, D89: 012004.
- [73] Victor Mukhamedovich Abazov *et al.* “*Inclusive Production of the $X(4140)$ State in $p\bar{p}$ Collisions at D0*”. *Phys. Rev. Lett.* **2015**, 115: 232001.
- [74] C. P. Shen, C. Z. Yuan, H. Aihara *et al.* “*Evidence for a New Resonance and Search for the $Y(4140)$ in the $\gamma\gamma \rightarrow \phi J/\psi$ Process*”. *Phys. Rev. Lett.* 2010-03, 104: 112004.
- [75] Xiang Liu and Shi-Lin Zhu. “ *$Y(4143)$ is probably a molecular partner of $Y(3930)$* ”. *Phys. Rev. D*, 2009-07, 80: 017502.
- [76] Tanja Branz, Thomas Gutsche and Valery E. Lyubovitskij. “*Hadronic molecule structure of the $Y(3940)$ and $Y(4140)$* ”. *Phys. Rev. D*, 2009-09, 80: 054019.
- [77] R. Molina and E. Oset. “ *$Y(3940)$, $Z(3930)$, and the $X(4160)$ as dynamically generated resonances from the vector-vector interaction*”. *Phys. Rev. D*, 2009-12, 80: 114013.

-
- [78] Raphael M. Albuquerque, Mirian E. Bracco and Marina Nielsen. “A QCD sum rule calculation for the $Y(4140)$ narrow structure”. *Physics Letters B*, **2009**, 678(2): 186–190.
 - [79] Zhi-Gang Wang, Zhi-Cheng Liu and Xiao-Hong Zhang. “Analysis of the $Y(4140)$ and related molecular states with QCD sum rules”. *Eur. Phys. J. C*, **2009**, 64: 373–386.
 - [80] Gui-Jun Ding. “Possible Molecular States of $D_s^* \bar{D}_s^*$ System and $Y(4140)$ ”. *Eur. Phys. J. C*, **2009**, 64: 297–308.
 - [81] E. S. Swanson. “ Z_b and Z_c exotic states as coupled channel cusps”. *Phys. Rev. D*, 2015-02, 91: 034009.
 - [82] Marek Karliner and Jonathan L. Rosner. “Exotic resonances due to exchange”. *Nuclear Physics A*, **2016**, 954, Recent Progress in Strangeness and Charm Hadronic and Nuclear Physics: 365–370.
 - [83] Namit Mahajan. “ $Y(4140)$: Possible options”. *Physics Letters B*, **2009**, 679(3): 228–230.
 - [84] Fl Stancu. “Can $Y(4140)$ be a $c\bar{c}s\bar{s}$ tetraquark?”. *Journal of Physics G: Nuclear and Particle Physics*, 2010-06, 37(7): 075017.
 - [85] Qi-Fang Lü and Yu-Bing Dong. “ $X(4140)$, $X(4274)$, $X(4500)$, and $X(4700)$ in the relativized quark model”. *Phys. Rev. D*, 2016-10, 94: 074007.
 - [86] Pablo G. Ortega, Jorge Segovia, David R. Entem *et al.* “Canonical description of the new $LHCb$ resonances”. *Phys. Rev. D*, 2016-12, 94: 114018.
 - [87] T. Aaltonen *et al.* “Evidence for a Narrow Near-Threshold Structure in the $J/\psi\phi$ Mass Spectrum in $B^+ \rightarrow J/\psi\phi K^+$ Decays”. *Phys. Rev. Lett.* **2009**, 102: 242002.
 - [88] J. Brodzicka. “Heavy flavour spectroscopy”. In: *24th International Symposium on Lepton-Photon Interactions at High Energy (LP09)*, 2009. **2009**.
 - [89] K. Abe, I. Adachi, H. Aihara *et al.* “Observation of a Charmoniumlike State Produced in Association with a J/ψ in e^+e^- Annihilation at $\sqrt{s} \approx 10.6$ GeV”. *Phys. Rev. Lett.* 2007-02, 98: 082001.
 - [90] K. Chilikin, I. Adachi, H. Aihara *et al.* “Observation of an alternative $\chi_{c0}(2P)$ candidate in $e^+e^- \rightarrow J/\psi D\bar{D}$ ”. *Phys. Rev. D*, 2017-06, 95: 112003.
 - [91] B. Aubert, R. Barate, D. Boutigny *et al.* “ $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$, $K^+K^-\pi^+\pi^-$, and $K^+K^-K^+K^-$ cross sections at center-of-mass energies 0.5–4.5 GeV measured with initial-state radiation”. *Phys. Rev. D*, 2005-03, 71: 052001.
 - [92] X. L. Wang, C. Z. Yuan, C. P. Shen *et al.* “Observation of Two Resonant Structures in $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$ via Initial-State Radiation at Belle”. *Phys. Rev. Lett.* 2007-10, 99: 142002.
 - [93] T. E. Coan, Y. S. Gao, F. Liu *et al.* “Charmonium Decays of $Y(4260)$, $\psi(4160)$, and $\psi(4040)$ ”. *Phys. Rev. Lett.* 2006-04, 96: 162003.
 - [94] C. Z. Yuan, C. P. Shen, P. Wang *et al.* “Measurement of the $e^+e^- \rightarrow \pi^+\pi^-J/\psi$ Cross Section Via Initial-State Radiation at Belle”. *Phys. Rev. Lett.* 2007-11, 99: 182004.
 - [95] B. Aubert, R. Barate, M. Bona *et al.* “Evidence of a Broad Structure at an Invariant Mass of 4.32 GeV/ c^2 in the Reaction $e^+e^- \rightarrow \pi^+\pi^-\psi(2S)$ Measured at BABAR”. *Phys. Rev. Lett.* 2007-05, 98: 212001.

- [96] M. Ablikim, M. N. Achasov, S. Ahmed *et al.* “Precise Measurement of the $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ Cross Section at Center-of-Mass Energies from 3.77 to 4.60 GeV”. *Phys. Rev. Lett.* 2017-03, 118: 092001.
- [97] L. Maiani, F. Piccinini, A. D. Polosa *et al.* “Z(4430) and a new paradigm for spin interactions in tetraquarks”. *Phys. Rev. D*, 2014-06, 89: 114010.
- [98] Peng Guo, Tochtli Yépez-Martínez and Adam P. Szczepaniak. “Charmonium meson and hybrid radiative transitions”. *Phys. Rev. D*, 2014-06, 89: 116005.
- [99] M. Ablikim *et al.* “Observation of a charged $(D\bar{D}^*)^\pm$ mass peak in $e^+e^- \rightarrow \pi D\bar{D}^*$ at $\sqrt{s} = 4.26$ GeV”. *Phys. Rev. Lett.* **2014**, 112(2): 022001.
- [100] M. Ablikim *et al.* “Confirmation of a charged charmoniumlike state $Z_c(3885)^\mp$ in $e^+e^- \rightarrow \pi^\pm(D\bar{D}^*)^\mp$ with double D tag”. *Phys. Rev. D*, **2015**, 92(9): 092006.
- [101] T. Xiao, S. Dobbs, A. Tomaradze *et al.* “Observation of the Charged Hadron $Z_c^\pm(3900)$ and Evidence for the Neutral $Z_c^0(3900)$ in $e^+e^- \rightarrow \pi\pi J/\psi$ at $\sqrt{s} = 4170$ MeV”. *Phys. Lett. B*, **2013**, 727: 366–370.
- [102] M. Ablikim *et al.* “Observation of a Neutral Structure near the $D\bar{D}^*$ Mass Threshold in $e^+e^- \rightarrow (D\bar{D}^*)^0\pi^0$ at $\sqrt{s} = 4.226$ and 4.257 GeV”. *Phys. Rev. Lett.* **2015**, 115(22): 222002.
- [103] M. Ablikim *et al.* “Observation of a Charged Charmoniumlike Structure $Z_c(4020)$ and Search for the $Z_c(3900)$ in $e^+e^- \rightarrow \pi^+\pi^- h_c$ ”. *Phys. Rev. Lett.* **2013**, 111(24): 242001.
- [104] M. Ablikim *et al.* “Observation of a charged charmoniumlike structure in $e^+e^- \rightarrow (D^*\bar{D}^*)^\pm\pi^\mp$ at $\sqrt{s} = 4.26$ GeV”. *Phys. Rev. Lett.* **2014**, 112(13): 132001.
- [105] M. Ablikim *et al.* “Observation of $e^+e^- \rightarrow \pi^0\pi^0 h_c$ and a Neutral Charmoniumlike Structure $Z_c(4020)^0$ ”. *Phys. Rev. Lett.* **2014**, 113(21): 212002.
- [106] M. Ablikim *et al.* “Observation of a neutral charmoniumlike state $Z_c(4025)^0$ in $e^+e^- \rightarrow (D^*\bar{D}^*)^0\pi^0$ ”. *Phys. Rev. Lett.* **2015**, 115(18): 182002.
- [107] R. Mizuk *et al.* “Observation of two resonance-like structures in the $\pi^+\chi_{c1}$ mass distribution in exclusive $\bar{B}^0 \rightarrow K^-\pi^+\chi_{c1}$ decays”. *Phys.Rev.* **2008**, D78: 072004.
- [108] J. P. Lees *et al.* “Search for the $Z_1(4050)^+$ and $Z_2(4250)^+$ states in $\bar{B}^0 \rightarrow \chi_{c1}K^-\pi^+$ and $B^+ \rightarrow \chi_{c1}K_S^0\pi^+$ ”. *Phys. Rev. D*, **2012**, 85: 052003.
- [109] K. Chilikin *et al.* “Observation of a new charged charmoniumlike state in $\bar{B}^0 \rightarrow J/\psi K^-\pi^+$ decays”. *Phys. Rev.* **2014**, D90: 112009.
- [110] R. Mizuk *et al.* “Dalitz analysis of $B \rightarrow K\pi^+\psi$ decays and the $Z(4430)^+$ ”. *Phys. Rev. D*, **2009**, 80: 031104.
- [111] K. Chilikin *et al.* “Experimental constraints on the spin and parity of the $Z(4430)^+$ ”. *Phys.Rev.* **2013**, D88: 074026.
- [112] Bernard Aubert *et al.* “Search for the $Z(4430)^-$ at BABAR”. *Phys. Rev. D*, **2009**, 79: 112001.
- [113] Roel Aaij *et al.* “Observation of the resonant character of the $Z(4430)^-$ state”. *Phys.Rev.Lett.* **2014**, 112: 222002.
- [114] M. Ablikim, M. N. Achasov, X. C. Ai *et al.* “Observation of a Charged Charmoniumlike Structure in $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ at $\sqrt{s}=4.26$ GeV”. *Phys. Rev. Lett.* 2013-06, 110: 252001.

-
- [115] R. Aaij *et al.* “Observation of the resonant character of the $Z(4430)^-$ state”. *Phys. Rev. Lett.* **2014**, *112*: 222002.
 - [116] K. Chilikin, R. Mizuk, I. Adachi *et al.* “Observation of a new charged charmoniumlike state in $\bar{B}^0 \rightarrow J/\psi K^- \pi^+$ decays”. *Phys. Rev. D*, 2014-12, *90*: 112009.
 - [117] Z. Q. Liu, C. P. Shen, C. Z. Yuan *et al.* “Study of $e^+e^- \rightarrow \pi^+\pi^- J/\psi$ and Observation of a Charged Charmoniumlike State at Belle”. *Phys. Rev. Lett.* 2013-06, *110*: 252002.
 - [118] M. Ablikim, M. N. Achasov, O. Albayrak *et al.* “Observation of a Charged $(D\bar{D}^*)^\pm$ Mass Peak in $e^+e^- \rightarrow \pi D\bar{D}^*$ at $\sqrt{s} = 4.26$ GeV”. *Phys. Rev. Lett.* 2014-01, *112*: 022001.
 - [119] M. Ablikim, M. N. Achasov, X. C. Ai *et al.* “Observation of $Z_c(3900)^0$ in $e^+e^- \rightarrow \pi^0\pi^0 J/\psi$ ”. *Phys. Rev. Lett.* 2015-09, *115*: 112003.
 - [120] M. Ablikim, M. N. Achasov, O. Albayrak *et al.* “Observation of a Charged Charmoniumlike Structure $Z_c(4020)$ and Search for the $Z_c(3900)$ in $e^+e^- \rightarrow \pi^+\pi^- h_c$ ”. *Phys. Rev. Lett.* 2013-12, *111*: 242001.
 - [121] M. Ablikim, M. N. Achasov, X. C. Ai *et al.* “Observation of $e^+e^- \rightarrow \pi^0\pi^0 h_c$ and a Neutral Charmoniumlike Structure $Z_c(4020)^0$ ”. *Phys. Rev. Lett.* 2014-11, *113*: 212002.
 - [122] M. Ablikim, M. N. Achasov, O. Albayrak *et al.* “Observation of a Charged Charmoniumlike Structure in $e^+e^- \rightarrow (D^*\bar{D}^*)^\pm \pi^\mp$ at $\sqrt{s} = 4.26$ GeV”. *Phys. Rev. Lett.* 2014-04, *112*: 132001.
 - [123] M. Ablikim, M. N. Achasov, X. C. Ai *et al.* “Observation of a Neutral Charmoniumlike State $Z_c(4025)^0$ in $e^+e^- \rightarrow (D^*\bar{D}^*)^0 \pi^0$ ”. *Phys. Rev. Lett.* 2015-10, *115*: 182002.
 - [124] R. Aaij *et al.* “Observation of $J/\psi p$ resonances consistent with pentaquark states in $\Lambda_b^0 \rightarrow J/\psi p K^-$ decays”. *Phys. Rev. Lett.* **2015**, *115*: 072001.
 - [125] R. Aaij *et al.* “Model-independent evidence for $J/\psi p$ contributions to $\Lambda_b^0 \rightarrow J/\psi p K^-$ decays”. *Phys. Rev. Lett.* **2016**, *117*: 082002.
 - [126] William R. Frazer and Archibald W. Hendry. “S-Matrix Poles Close to Threshold”. *Phys. Rev.* 1964-06, *134*: B1307–B1314.
 - [127] Emiko Hiyama, Atsushi Hosaka, Makoto Oka *et al.* “Quark model estimate of hidden-charm pentaquark resonances”. *Phys. Rev. C*, 2018-10, *98*: 045208.
 - [128] Luciano Maiani, Antonio D. Polosa and Veronica Riquer. “A theory of X and Z multiquark resonances”. *Physics Letters B*, **2018**, *778*: 247–251.
 - [129] D V Bugg. “How resonances can synchronize with thresholds”. *Journal of Physics G: Nuclear and Particle Physics*, 2008-05, *35*(7): 075005.
 - [130] Feng-Kun Guo, Christoph Hanhart, Qian Wang *et al.* “Could the near-threshold XYZ states be simply kinematic effects?” *Phys. Rev. D*, 2015-03, *91*: 051504.
 - [131] R. Aaij *et al.* “Evidence for exotic hadron contributions to $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decays”. *Phys. Rev. Lett.* **2016**, *117*: 082003.
 - [132] R. Aaij *et al.* “Evidence of a $J/\psi \Lambda$ structure and observation of excited Ξ^- states in the $\Xi_b^- \rightarrow J/\psi \Lambda K^-$ decay”. **2020**, to appear in Science Bulletin.

- [133] Nathan Isgur and Mark B. Wise. “Weak decays of heavy mesons in the static quark approximation”. *Physics Letters B*, **1989**, 232(1): 113–117.
- [134] B. A. Thacker and G. Peter Lepage. “Heavy-quark bound states in lattice QCD”. *Phys. Rev. D*, 1991-01, 43: 196–208.
- [135] Ahmed Ali, Luciano Maiani and Antonio D. Polosa. *Multiquark Hadrons*. Cambridge University Press, 2019-06.
- [136] S. Dubynskiy, A. Gorsky and M.B. Voloshin. “Holographic hadro-quarkonium”. *Physics Letters B*, **2009**, 671(1): 82–86.
- [137] J. -P. Ader, J. -M. Richard and P. Taxil. “Do narrow heavy multiquark states exist?” *Phys. Rev. D*, 1982-05, 25: 2370–2382.
- [138] Feng-Kun Guo, Christoph Hanhart, Ulf-G. Meißner *et al.* “Hadronic molecules”. *Rev. Mod. Phys.* **2018**, 90(1): 015004.
- [139] N. Brambilla *et al.* “Heavy Quarkonium: Progress, Puzzles, and Opportunities”. *Eur. Phys. J. C*, **2011**, 71: 1534.
- [140] A. Esposito, A. Pilloni and A.D. Polosa. “Multiquark resonances”. *Physics Reports*, **2017**, 668, Multiquark Resonances: 1–97.
- [141] Feng-Kun Guo, Xiao-Hai Liu and Shuntaro Sakai. “Threshold cusps and triangle singularities in hadronic reactions”. *Progress in Particle and Nuclear Physics*, **2020**, 112: 103757.
- [142] O. Brüning, H. Burkhardt and S. Myers. “The Large Hadron Collider”. *Prog. Part. Nucl. Phys.* **2012**, 67: 705–734.
- [143] Oliver Sim Brüning, Paul Collier, P Lebrun *et al.* *LHC Design Report*. Geneva: CERN, **2004**.
- [144] Julie Haffner. “The CERN accelerator complex. Complexe des accélérateurs du CERN”. 2013-10, General Photo.
- [145] Jason Nielsen. “Fundamentals of LHC Experiments”. *String Theory and Its Applications*, 2011-11.
- [146] G. Aad *et al.* “The ATLAS Experiment at the CERN Large Hadron Collider”. *JINST*, **2008**, 3: S08003.
- [147] S. Chatrchyan *et al.* “The CMS Experiment at the CERN LHC”. *JINST*, **2008**, 3: S08004.
- [148] K. Aamodt *et al.* “The ALICE experiment at the CERN LHC”. *JINST*, **2008**, 3: S08002.
- [149] A. A. Alves Jr. *et al.* “The LHCb detector at the LHC”. *JINST*, **2008**, 3(LHCb-DP-2008-001): S08005.
- [150] Torbjörn Sjöstrand, Stephen Mrenna and Peter Skands. “A brief introduction to PYTHIA 8.1”. *Computer Physics Communications*, **2008**, 178(11): 852–867.
- [151] *Framework TDR for the LHCb Upgrade: Technical Design Report*. Geneva, **2012**.
- [152] *Expression of Interest for a Phase-II LHCb Upgrade: Opportunities in flavour physics, and beyond, in the HL-LHC era*. Geneva, **2017**.
- [153] “Physics case for an LHCb Upgrade II — Opportunities in flavour physics, and beyond, in the HL-LHC era”. **2018**(CERN-LHCC-2018-027 LHCb-PUB-2018-009).
- [154] R. Aaij *et al.* “LHCb detector performance”. *Int. J. Mod. Phys.* **2015**, A30: 1530022.

-
- [155] *LHCb VELO (Vertex Locator): Technical Design Report*. Geneva, **2001**.
- [156] *LHCb VELO Upgrade Technical Design Report*. Geneva, **2013**.
- [157] Peter Svihra. “The Design and Construction of LHCb VELO Upgrades Modules”. 2020-08, Poster-2020-1041.
- [158] *LHCb magnet: Technical Design Report*. Geneva, **2000**.
- [159] *LHCb inner tracker: Technical Design Report*. Geneva, **2002**.
- [160] *LHCb outer tracker: Technical Design Report*. Geneva, **2001**.
- [161] *LHCb Tracker Upgrade Technical Design Report*. Geneva, **2014**.
- [162] *Proceedings, 24th International Workshop on Vertex Detectors (Vertex 2015): Santa Fe, New Mexico, USA, June 1-5, 2015*. SISSA, **2015**.
- [163] Matthew Scott Rudolph. “The LHCb Upstream Tracker Upgrade”. *PoS*, **2020**, Vertex2019: 013.
- [164] A. Massafferri. “SciFi, the new Tracker of the LHCb experiment”. *JINST*, **2020**, 15(08): C08006.
- [165] *LHCb RICH: Technical Design Report*. Geneva, **2000**.
- [166] *LHCb PID Upgrade Technical Design Report*. Geneva, **2013**.
- [167] Massimiliano Fiorini. “The upgrade of the LHCb RICH detectors”. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, **2020**, 952, 10th International Workshop on Ring Imaging Cherenkov Detectors (RICH 2018): 161688.
- [168] *LHCb calorimeters: Technical Design Report*. Geneva, **2000**.
- [169] Irina Machikhiliyan and. “The LHCb electromagnetic calorimeter”. *Journal of Physics: Conference Series*, 2009-04, 160: 012047.
- [170] *LHCb muon system: Technical Design Report*. Geneva, **2001**.
- [171] A Cardini. “The LHCb Muon Upgrade”. *Journal of Instrumentation*, 2014-02, 9(02): C02014–C02014.
- [172] *LHCb trigger system: Technical Design Report*. Geneva, **2003**.
- [173] R. Aaij *et al.* “The LHCb trigger and its performance in 2011”. *JINST*, **2013**, 8: P04022.
- [174] Roel Aaij *et al.* “Performance of the LHCb trigger and full real-time reconstruction in Run 2 of the LHC”. *JINST*, **2019**, 14(LHCb-DP-2019-001): P04013.
- [175] R. Aaij, S. Akar, J. Albrecht *et al.* “Design and performance of the LHCb trigger and full real-time reconstruction in Run 2 of the LHC”. *Journal of Instrumentation*, 2019-04, 14(04): P04013–P04013.
- [176] Marcin Kucharczyk, Piotr Morawski and Mariusz Witek. *Primary Vertex Reconstruction at LHCb*. Geneva, 2014-09. <http://cds.cern.ch/record/1756296>.
- [177] Jeroen Ashwin Niels van Tilburg. *Track Simulation and Reconstruction in LHCb*. 2005-09.
- [178] Helge Voss, Andreas Hoecker, Jörg Stelzer *et al.* “TMVA - Toolkit for Multivariate Data Analysis with ROOT”. *PoS*, **2007**, ACAT: 040.
- [179] *LHCb Trigger and Online Technical Design Report*. Geneva, **2014**.
- [180] *LHCb Upgrade GPU High Level Trigger Technical Design Report*. Geneva, **2020**.

- [181] *Computing Model of the Upgrade LHCb experiment*. Geneva, **2018**.
- [182] *LHCb Upgrade Software and Computing*. Geneva, **2018**.
- [183] S.K. Choi *et al.* “Observation of a resonance-like structure in the $\pi^\pm\psi'$ mass distribution in exclusive $B \rightarrow K\pi^\pm\psi'$ decays”. *Phys.Rev.Lett.* **2008**, 100: 142001.
- [184] Medina Ablikim *et al.* “Observation of a near-threshold structure in the K^+ recoil-mass spectra in $e^+e^- \rightarrow K^+(D_s^-D^{*0} + D_s^{*-}D^0)$ ”. *Phys. Rev. Lett.* **2021**, 126(10): 102001.
- [185] M.B. Voloshin. “Strange hadrocharmonium”. *Phys. Lett. B*, **2019**, 798: 135022.
- [186] Jorgivan M. Dias, Xiang Liu and Marina Nielsen. “Prediction for the decay width of a charged state near the $D_s\bar{D}^*/D_s^*\bar{D}$ threshold”. *Phys. Rev. D*, **2013**, 88(9): 096014.
- [187] Dian-Yong Chen, Xiang Liu and Takayuki Matsuki. “Predictions of Charged Charmoniumlike Structures with Hidden-Charm and Open-Strange Channels”. *Phys. Rev. Lett.* **2013**, 110(23): 232001.
- [188] J. Ferretti and E. Santopinto. “Hidden-charm and bottom tetra- and pentaquarks with strangeness in the hadro-quarkonium and compact tetraquark models”. *JHEP*, **2020**, 04: 119.
- [189] Su Hounq Lee, Marina Nielsen and Ulrich Wiedner. “ $D(s)D^*$ molecule as an axial meson”. *J. Korean Phys. Soc.* **2009**, 55: 424.
- [190] Thomas Britton. *First amplitude analysis of $B^+ \rightarrow J/\psi\phi K^+$ decays* [phdthesis]. **2016**.
- [191] Torbjörn Sjöstrand, Stephen Mrenna and Peter Skands. “PYTHIA 6.4 physics and manual”. *JHEP*, **2006**, 05: 026.
- [192] D. J. Lange. “The EvtGen particle decay simulation package”. *Nucl. Instrum. Meth.* **2001**, A462: 152–155.
- [193] Piotr Golonka and Zbigniew Was. “PHOTOS Monte Carlo: A precision tool for QED corrections in Z and W decays”. *Eur.Phys.J.* **2006**, C45: 97–107.
- [194] John Allison, K. Amako, J. Apostolakis *et al.* “Geant4 developments and applications”. *IEEE Trans.Nucl.Sci.* **2006**, 53: 270.
- [195] I. Belyaev *et al.* “Handling of the generation of primary events in Gauss, the LHCb simulation framework”. *J. Phys. Conf. Ser.* **2011**, 331: 032047.
- [196] M Clemencic *et al.* “The LHCb simulation application, Gauss: Design, evolution and experience”. *J. Phys. Conf. Ser.* **2011**, 331: 032023.
- [197] R.H. Dalitz. “On the analysis of τ -meson data and the nature of the τ -meson”. *Phil.Mag.* **1953**, 44: 1068–1080.
- [198] *LHCb computing: Technical Design Report*. Geneva, **2005**.
- [199] R. Aaij *et al.* “Measurement of J/ψ production in pp collisions at $\sqrt{s}=7\text{ TeV}$ ”. *Eur. Phys. J.* **2011**, C71: 1645.
- [200] Wouter D. Hulsbergen. “Decay chain fitting with a Kalman filter”. *Nucl.Instrum.Meth.* **2005**, A552: 566–575.
- [201] R. Aaij *et al.* “Selection and processing of calibration samples to measure the particle identification performance of the LHCb experiment in Run 2”. *Eur. Phys. J. Tech. Instr.* **2018**, 6: 1.

-
- [202] L. Breiman, J. H. Friedman, R. A. Olshen *et al.* *Classification and regression trees*. Belmont, California, USA: Wadsworth international group, **1984**.
 - [203] Robert E. Schapire and Yoav Freund. “A decision-theoretic generalization of on-line learning and an application to boosting”. *Jour. Comp. and Syst. Sc.* **1997**, 55: 119.
 - [204] Andreas Hoecker *et al.* “TMVA 4 — Toolkit for Multivariate Data Analysis with ROOT. Users Guide.” **2009**.
 - [205] Diego Martínez Santos and Frédéric Dupertuis. “Mass distributions marginalized over per-event errors”. *Nucl. Instrum. Meth.* **2014**, A764: 150–155.
 - [206] Muriel Pivk and Francois R. Le Diberder. “sPlot: A statistical tool to unfold data distributions”. *Nucl.Instrum.Meth.* **2005**, A555: 356–369.
 - [207] Eugene Paul Wigner. *Group theory and its application to the quantum mechanics of atomic spectra*. New York, NY: Academic Press, **1959**, Trans. from the German.
 - [208] J.M. Blatt and V.F. Weisskopf. *Theoretical Nuclear Physics*. Springer-Verlag, **1979**.
 - [209] S. Godfrey and Nathan Isgur. “Mesons in a relativized quark model with chromodynamics”. *Phys.Rev.* **1985**, D32: 189–231.
 - [210] T. Armstrong *et al.* “A Partial Wave Analysis of the $K^-\phi$ System Produced in the Reaction $K^-p \rightarrow K^+K^-K^-p$ at 18.5-GeV/c”. *Nucl. Phys.* **1983**, B221: 1.
 - [211] D. Frame, I. S. Hughes, J. G. Lynch *et al.* “A Spin Parity Analysis of the ϕK^+ System Produced in the Reaction $K^+p \rightarrow \phi K^+p$, $\phi \rightarrow K^+K^-$ at 13 GeV/c”. *Nucl. Phys.* **1986**, B276: 667.
 - [212] Young-joon Kwon. *Strange meson spectroscopy in K omega and K phi at 11-GeV/c and Cherenkov ring imaging at SLD* [phdthesis]. **1993**.
 - [213] Richard Kokoski and Nathan Isgur. “Meson Decays by Flux Tube Breaking”. *Phys. Rev.* **1987**, D35: 907.
 - [214] K. A. Olive *et al.* “Review of particle physics”. *Chin. Phys.* **2014**, C38, and 2015 update: 090001.
 - [215] R. Aaij *et al.* “Studies of the resonance structure in $D^0 \rightarrow K^\mp \pi^\pm \pi^+ \pi^-$ decays”. *Eur. Phys. J.* **2018**, C78: 443.
 - [216] S. S. Wilks. “The large-sample distribution of the likelihood ratio for testing composite hypotheses”. *Annals Math.Statist.* **1938**, 9: 60–62.
 - [217] R. Aaij *et al.* “Production of associated Υ and open charm hadrons in pp collisions at $\sqrt{s}=7$ and 8 TeV via double parton scattering”. *JHEP*, **2016**, 07: 052.
 - [218] Yuehong Xie. “sFit: a method for background subtraction in maximum likelihood fit”. **2009**.
 - [219] V.K. Henner and T.S. Belozeroва. “Analysis of Overlapping Resonances Using the K-Matrix and Breit–Wigner Methods”. *Phys. Part. Nucl.* **2020**, 51(4): 673–677.
 - [220] M. Nauenberg and A. Pais. “Woolly Cusps”. *Phys. Rev.* 1962-04, 126: 360–364.
 - [221] Torbjörn Sjöstrand, Stephen Mrenna and Peter Skands. “A brief introduction to PYTHIA 8.1”. *Comput.Phys.Commun.* **2008**, 178: 852–867.
 - [222] R. Aaij *et al.* “Measurement of the branching fraction ratio $\mathcal{B}(B_c^+ \rightarrow \psi(2S)\pi^+)/\mathcal{B}(B_c^+ \rightarrow J/\psi\pi^+)$ ”. *Phys. Rev.* **2015**, D92: 057007.

- [223] Tomasz Skwarnicki. *A study of the radiative cascade transitions between the Upsilon-prime and Upsilon resonances* [phdthesis]. **1986**, DESY-F31-86-02.
- [224] John Markus Blatt and Victor Frederick Weisskopf. *Theoretical nuclear physics*. New York: Springer, **1952**.
- [225] Feng-Kun Guo, Ulf-G. Meißner, Wei Wang *et al.* “How to reveal the exotic nature of the $P_c(4450)$ ”. *Phys. Rev. D*, **2015**, 92(7): 071502.
- [226] Ulf-G. Meißner and José A. Oller. “Testing the $\chi_{c1} p$ composite nature of the $P_c(4450)$ ”. *Phys. Lett. B*, **2015**, 751: 59–62.
- [227] Xiao-Hai Liu, Qian Wang and Qiang Zhao. “Understanding the newly observed heavy pentaquark candidates”. *Phys. Lett. B*, **2016**, 757: 231–236.
- [228] Adam P. Szczepaniak. “Dalitz plot distributions in presence of triangle singularities”. *Phys. Lett. B*, **2016**, 757: 61–64.
- [229] R. Aaij *et al.* “Observation of overlapping spin-1 and spin-3 $\bar{D}^0 K^-$ resonances at mass 2.86 GeV/c²”. *Phys. Rev. Lett.* **2014**, 113: 162001.
- [230] Stephen Godfrey and Kenneth Moats. “Properties of Excited Charm and Charm-Strange Mesons”. *Phys. Rev. D*, **2016**, 93(3): 034035.
- [231] Hai-Yang Cheng and Chun-Khiang Chua. “Bottom baryon decays to pseudoscalar meson and pentaquark”. *Phys. Rev.* **2015**, D92: 096009.
- [232] Y. K. Hsiao and C. Q. Geng. “Pentaquarks from intrinsic charms in Λ_b^0 decays”. *Phys. Lett.* **2015**, B751: 572–575.
- [233] R. Aaij *et al.* “Observation of the $\Lambda_b^0 \rightarrow J/\psi p \pi^-$ decay”. *JHEP*, **2014**, 07: 103.
- [234] Lucio Anderlini, Andrea Contu, Christopher Rob Jones *et al.* *The PIDCalib package*. **2016**.
- [235] Jia-Jun Wu, R. Molina, E. Oset *et al.* “Prediction of narrow N^* and Λ^* resonances with hidden charm above 4 GeV”. *Phys.Rev.Lett.* **2010**, 105: 232001.
- [236] W. L. Wang, F. Huang, Z. Y. Zhang *et al.* “ $\Sigma_c \bar{D}$ and $\Lambda_c \bar{D}$ states in a chiral quark model”. *Phys. Rev.* **2011**, C84: 015203.
- [237] Zhong-Cheng Yang, Zhi-Feng Sun, Jun He *et al.* “The possible hidden-charm molecular baryons composed of anti-charmed meson and charmed baryon”. *Chin. Phys.* **2012**, C36: 6–13.
- [238] Qian Wang, Xiao-Hai Liu and Qiang Zhao. “Photoproduction of hidden charm pentaquark states $P_c^+(4380)$ and $P_c^+(4450)$ ”. *Phys. Rev. D*, **2015**, 92: 034022.
- [239] Nicola Cabibbo. “Unitary Symmetry and Leptonic Decays”. *Phys. Rev. Lett.* 1963-06, 10: 531–533.
- [240] Estia Eichten and Brian Hill. “An effective field theory for the calculation of matrix elements involving heavy quarks”. *Physics Letters B*, **1990**, 234(4): 511–516.
- [241] R. Aaij *et al.* “Measurements of the branching fractions for $B_{(s)} \rightarrow D_{(s)} \pi \pi \pi$ and $\Lambda_b^0 \rightarrow \Lambda_c^+ \pi \pi \pi$ ”. *Phys. Rev.* **2011**, D84: 092001.
- [242] R. Aaij *et al.* “Observation of the decay $\Lambda_b^0 \rightarrow \Lambda_c^+ p \bar{p} \pi^-$ ”. *Phys. Lett.* **2018**, B784: 101.
- [243] R. Aaij *et al.* “Study of beauty hadron decays into pairs of charm hadrons”. *Phys. Rev. Lett.* **2014**, 112: 202001.

-
- [244] Y. B. Li *et al.* “Observation of $\Xi_c(2930)^0$ and updated measurement of $B^- \rightarrow K^- \Lambda_c^+ \bar{\Lambda}_c^-$ at Belle”. *Eur. Phys. J. C*, **2018**, 78(3): 252.
 - [245] R. Aaij *et al.* “Measurement of the track reconstruction efficiency at LHCb”. *JINST*, **2015**, 10: P02007.
 - [246] A. Drutskoy *et al.* “Observation of $B \rightarrow D^* K^- K^{0*}$ decays”. *Phys. Lett. B*, **2002**, 542: 171–182.
 - [247] Alexandra Martin Sanchez, Patrick Robbe and Marie-Hélène Schune. *Performances of the LHCb L0 Calorimeter Trigger*. Geneva, 2012-06. <http://cds.cern.ch/record/1407893>.
 - [248] P. Z. Huang, Y. R. Liu, W. Z. Deng *et al.* “Heavy pentaquarks”. *Phys. Rev.* **2004**, D70: 034003.
 - [249] C. Matteuzzi, I. Belyaev, G. Carboni *et al.* “The history of LHCb”. 2021-01.
 - [250] Richard Keith Ellis *et al.* “Physics Briefing Book: Input for the European Strategy for Particle Physics Update 2020”. 2019-10.
 - [251] Michele Selvaggi, J. Favereau, C. Delaere *et al.* “DELPHES 3: A modular framework for fast-simulation of generic collider experiments”. *Journal of Physics: Conference Series*, 2014-06, 523: 012033.
 - [252] Itamar Levy, “Measurement of shower development and its Molière radius with a four-plane LumiCal test set-up”. Ed. by Paolo Checchia *et al.* *PoS*, **2017**, EPS-HEP2017: 502.
 - [253] V. Breton, N. Brun and P. Perret. “A clustering algorithm for the LHCb electromagnetic calorimeter using a cellular automaton”. 2001-09.
 - [254] O Deschamps, F P Machefert, M H Schune *et al.* *Photon and neutral pion reconstruction* [techreport]. Geneva, 2003-09.
 - [255] Miriam Calvo Gomez, Eric Cogneras, Olivier Deschamps *et al.* *A tool for γ/π^0 separation at high energies* [techreport]. Geneva, 2015-08.
 - [256] R. Aaij *et al.* “Measurement of CP observables in $B^0 \rightarrow DK^{*0}$ with $D \rightarrow K^+ K^-$ ”. *JHEP*, **2013**, 03: 067.
 - [257] *The Phase-2 Upgrade of the CMS Endcap Calorimeter* [techreport]. Geneva, 2017-11.
 - [258] N. Akchurin *et al.* “First beam tests of prototype silicon modules for the CMS High Granularity Endcap Calorimeter”. *JINST*, **2018**, 13(10): P10023.
 - [259] Manqi Ruan. *Arbor, a new approach of the Particle Flow Algorithm*. **2014**.
 - [260] D. Belayneh, F. Carminati, A. Farbin *et al.* “Calorimetry with Deep Learning: Particle Simulation and Reconstruction for Collider Physics”. *ArXiv*, **2019**, abs/1912.06794.
 - [261] B. Li, R. Été, G. Grenier *et al.* “APRIL : a novel Algorithm for Particle Reconstruction at ILC”. *JINST*, **2020**, 15(05): C05016.

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Resume

Ph.D. student in Physics, Peking University, since October, 2019:

Concentration: Experimental High Energy Physics

Dissertation: Experimental Study of Exotic Hadrons at LHCb

Adivisors: Prof. Yuanning Gao, Prof. Liming Zhang (Associate)

Publications

A full publication list can be retrieved at the INSPIRE-HEP database. I have been on the LHCb Collaboration author list since 2017, and am listed as an author on all physics and upgrade publications since then. Listed here are the publications I have been directly involved in:

- R. Aaij *et al.* (LHCb Collaboration), "Observation of new resonances decaying to $J/\psi K^+$ and $J/\psi \phi$ ", LHCb-PAPER-2020-044, arXiv:2103.01803 Phys. Rev. Lett. 127, 082001 (2021)
- R. Aaij *et al.* (LHCb Collaboration), "Observation of the $\Lambda_b^0 \rightarrow \Lambda_c^+ K^+ K^- \pi^-$ decay", LHCb-PAPER-2020-028, arXiv:2011.13738, Phys. Lett. B. **815**, 136172 (2021)
- R. Aaij *et al.* (LHCb Collaboration), "Observation of a narrow pentaquark state, $P_c(4312)^+$, and of the two-peak structure of the $P_c(4450)^+$ ", LHCb-PAPER-2019-014, arXiv:2103.01803, Phys. Rev. Lett. **122**, 222001 (2019)

Selected Talks

- "Recent exotic hadron results at LHCb", 7th workshop on the XYZ particles, in Shandong University, Qingdao, China, May 15-18, 2021
- "Observation of Z_{cs} states at LHCb", Experimental and theoretical status of and perspectives for XYZ states, in GSI, Darmstadt, Germany, April 12-15, 2021
- "Exotic hadrons: recent LHCb discoveries", Rencontres de Physique de la Vallée d'Aoste, in La Thuile, Aosta Valley, Italy, March 8-12, 2021
- "Recent XYZ states at LHCb", 6th workshop on the XYZ particles, in Fudan University, Shanghai, China, January 11-13, 2020

- "Fast simulation of ECAL for LHCb Upgrade II", The 5th China LHC Physics Workshop, in Dalian University of Technology, Dalian, China, October 23-27, 2019
- "LHCb results on exotic hadrons", Implications of LHCb measurements and future prospects, in CERN, Geneva, Switzerland, October 16-18, 2019
- "ECAL - requirements from simulation", 4th Workshop on LHCb Upgrade II, in Vondelkerk, Amsterdam, Netherland, April 8-10, 2019

