Measurement of $Br(\Lambda_b^0 \to \Lambda_c^+ \pi^- \pi^+ \pi^-)$

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Abstract

In this report, the measurement of the branching ratio $Br(\Lambda_b^0 \to \Lambda_c^+ \pi^- \pi^+ \pi^-)$ is described. The measurement is done by comparing the closely related mesonic decay, $B^0 \to D^* 3\pi; D^* \to K^- \pi^+ \pi^+$. The result is $Br(\Lambda_b^0 \to \Lambda_c^+ \pi^- \pi^+ \pi^-) = (5.05 \pm 0.06(\text{stat.}) \pm 0.64(\text{syst.})) \times 10^{-3}$.

Contents

1 Introduction

The LHCb Collaboration performed the precise measurement of the baryon to meson production rate as function of p_T using 13 TeV data[1]. The knowledge of this ratio enables us to compute the branching ratio of any Λ_b^0 decay mode.

The best normalization channel for the decay $\Lambda_b^0 \to \Lambda_c^+ \tau^- \bar{\nu}_\tau$ when using the favorable $\tau^- \to 3\pi\nu_\tau$ hadronic decays is $\Lambda_b^0 \to \Lambda_c^+ \pi^- \pi^+ \pi^-$ because it leads to exactly the same final state particles. A precise knowledge of the $\Lambda_b^0 \to \Lambda_c^+ \pi^- \pi^+ \pi^-$ will then leads to the measurement of the ratio

$$K(\Lambda_c^+) = \frac{Br(\Lambda_b^0 \to \Lambda_c^+ \tau^- \bar{\nu}_\tau)}{Br(\Lambda_b^0 \to \Lambda_c^+ \pi^- \pi^+ \pi^-)}. \tag{1}$$

A measurement of the $Br(\Lambda_b^0 \to \Lambda_c^+ 3\pi)$ as precise as as possible is thus needed. We use the method developed for the measurement of the branching ratio $Br(\Lambda_b^0 \to \Lambda_c^+ \pi)$[2], using for the normalization channel $B^0 \to D^* 3\pi$ because they have quasi-identity final state.

2 Lepton Universality Violation

2.1 Lepton Universality

The coupling constants of leptons and gauge bosons seem to be flavor independent. Namely, the interaction between leptons and a gauge boson are the same for each lepton. This property is called lepton universality. This is not violated so far within the accuracy of any experiments and lepton universality has been tested in some experiments.

2.2 $\Lambda_b^0 \to \Lambda_c^+ l \bar{\nu}_l$

Final goal is to measure the ratio of the two branching ratio, $\Lambda_b^0 \to \Lambda_c^+ \tau^- \bar{\nu}_\tau$ and $\Lambda_b^0 \to \Lambda_c^+ \mu^- \bar{\nu}_\mu$. We define $R(\Lambda_c)$ as

$$R(\Lambda_c) = \frac{Br(\Lambda_b^0 \to \Lambda_c^+ \tau^- \bar{\nu}_\tau)}{Br(\Lambda_b^0 \to \Lambda_c^+ \mu^- \bar{\nu}_\mu)}. \tag{2}$$

If we neglect the dependence of the phase space which is derived from the mass difference of two charged leptons, $R(\Lambda_c)$ is equal to unity in the Standard Model. Measuring the ratio $R(\Lambda_c)$ is one of the way to find new physics related to the lepton universality.

However, very few absolute branching ratios are precisely known for Λ_b^0 decays. Experimentally, $Br(\Lambda_b^0 \to \Lambda_c^+ \tau^- \bar{\nu}_\tau; \tau^- \to 3\pi\nu_\tau)$ will be measured for numerator of $R(\Lambda_c)$. To reduce the uncertainty, $Br(\Lambda_b^0 \to \Lambda_c^+ 3\pi)$ needs to be measured because final state visible particles are the same.

3 How to measure $Br(\Lambda_b^0 \to \Lambda_c^+ 3\pi)$

We have two MCs, true MC and reconstructed MC. We also have Run2 2016 data. Combining them, "true data" of $\Lambda_b^0 \to \Lambda_c^+ 3\pi; \Lambda_c^+ \to pK\pi$ and $B^0 \to D^* 3\pi; D^* \to K\pi\pi$ are reconstructed.

$$\text{Yield observed in data} = \text{True yield} \times \epsilon \tag{3}$$

Here, ϵ is the efficiency related to signal selection and this efficiency can be calculated using true MC and reconstructed MC. The reconstructed MC is applied the same cuts as data.

$\Lambda_c^* \pi$ components in the $\Lambda_c^+ 3\pi$ sample are separated because $\tau \to 3\pi\nu$ has no intermediate states of Λ_c^*.

Calculating the ratio of data for two modes as function of p_T of b-hadrons.

$$\frac{\#(\Lambda_b^0 \to \Lambda_c^+ 3\pi; \Lambda_c^+ \to pK^-\pi^+)}{\#(B^0 \to D^* 3\pi; D^* \to K^-\pi^+)} \tag{4}$$

This ratio has p_T variation. This is equal to

$$\frac{Br(\Lambda_b^0 \to \Lambda_c^+ 3\pi)}{Br(B^0 \to D^* 3\pi)} \times \frac{f_{\Lambda_b^0}}{f_{B^0}} \times \frac{Br(\Lambda_c^+ \to pK^-\pi^+)}{Br(D^* \to K^-\pi^+)} \times \frac{\epsilon_{\Lambda_b^0}}{\epsilon_{B^0}}. \tag{5}$$

$Br(\Lambda_b^0 \to \Lambda_c^+ 3\pi)$ is what we want to know. $Br(B^0 \to D^* 3\pi)$ and $Br(D^* \to K^-\pi^+)$ are well known so we can use PDG values of them[3]. $f_{\Lambda_b^0}/f_{B^0}$ and $Br(\Lambda_c^+ \to pK^-\pi^+)$ are measured in LHCb-PAPER-2018-050.

$f_{\Lambda_b^0}/f_{B^0}$ should have the same p_T variation, so the ratio

$$\frac{\#(\Lambda_b^0 \to \Lambda_c^+ 3\pi; \Lambda_c^+ \to pK^-\pi^+)}{\#(B^0 \to D^* 3\pi; D^* \to K^-\pi^+)} \times \frac{f_{B^0}}{f_{\Lambda_b^0}} \tag{6}$$

has no p_T dependence anymore if everything is correct. This ratio is equal to

$$\frac{Br(\Lambda_b^0 \to \Lambda_c^+ 3\pi)}{Br(B^0 \to D^* 3\pi)} \times \frac{Br(\Lambda_c^+ \to pK^-\pi^+)}{Br(D^* \to K^-\pi^+)} \times \frac{\epsilon_{\Lambda_b^0}}{\epsilon_{B^0}} \tag{7}$$

so we can calculate $Br(\Lambda_b^0 \to \Lambda_c^+ 3\pi)$ from these equations.

4 Systematic Uncertainties

We need to estimate the systematic uncertainties related to calculating the branching ratio $Br(\Lambda_b^0 \to \Lambda_c^+ \pi^-\pi^+\pi^-)$. We look into them one by one below.

The result is listed in the table 1. Systematic uncertainties related to $B^0 \to D^* 3\pi$ is cited from LHCb-PAPER-2017-027[4]. Systematic uncertainty of $Br(B^- \to \mu D^0 \nu)$ is cited from LHCb-PAPER-2018-050.

Table 1: The various sources of systematic uncertainties

Source	$\Lambda_b^0 \to \Lambda_c^+ 3\pi$	$B^0 \to D^* 3\pi$	Ratio	$Br(\Lambda_b^0 \to \Lambda_c^+ 3\pi)$
Signal selection	0.9%	2.0%	2.2%	2.2%
Dalitz reweighting	2.1%	-	2.1%	2.1%
Λ_b^0 kinematics	4.5%	-	4.5%	4.5%
Λ_c^* feed-down removal	3.4%	-	3.4%	3.4%
Signal fit	4.0%	1.0%	4.1%	4.1%
MonteCarlo statistics	0.6%	0.4%	0.7%	0.7%
Trigger corrections	2.1%	1.6%	2.6%	2.6%
PID corrections	-	-	1.0%	1.0%
$f_{\Lambda_b^0}/f_d$	-	-	-	8.5%
$Br(B^- \to \mu D^0 \nu)$	-	-	-	1.8%
$Br(B^0 \to D^* 3\pi)$	-	-	-	4.0%
Total	-	-	-	12.6%

Table 2: The number of events with different impact parameters of pions.

least impact parameter	data	MC
> 15	18604 ± 212	47905 ± 290
> 20	18019 ± 165	46025 ± 238

4.1 Signal selection

Impact parameters of pions and Λ_c^+ are used to estimate the uncertainties of signal selection because these have the largest uncertainties. Usually, we use the cuts in which impact parameters are larger than 15. We check the number of signals changing this number to 20. First, we change impact parameter of pions and the result is listed in table 2.

Next, we change impact parameter of Λ_c^+ and the result is listed in table 3.

From these tables, the systematic uncertainty related to signal selection is estimated to 0.9%.

Table 3: The number of events with different impact parameters of Λ_c^+.

impact parameter	data	MC
> 15	18604 ± 212	47905 ± 290
> 20	18505 ± 162	47225 ± 218

Table 4: The number of events before and after Dalitz reweighting.

-	MC before Dalitz reweighting	MC after Dalitz reweighting
no cut	121972 ± 76	150668 ± 399
normalisa2	47905 ± 290	60409 ± 312
ratio	39.28%	40.09%

Table 5: The number of events before and after kinematic reweighting.

-	MC before kinematic reweighting	MC after kinematic reweighting
no cut	150668 ± 399	143955 ± 380
normalisa2	60409 ± 312	55205 ± 265
ratio	40.09%	38.35%

4.2 Dalitz reweighting

Data and MC are different slightly. To match MC with data, Dalitz reweighting is used. Dalitz reweighting is made with Dalitz plot of $\Lambda_c^+ \to pK^-\pi^+$ and the difference between data and MC is the reweighting. The systematic uncertainty related to Dalitz reweighting can be estimated by comparing the number of events before and after this reweighting. The result is listed in table 4. From this table, systematic uncertainty related to Dalitz reweighting is estimated 2.1%. We compared before and after reweighting, so this is very conservative.

4.3 Λ_b^0 kinematics

Another reweighting are made using differences of data and MC of momentum p and transverse momentum p_T of Λ_b^0. This is a kinematic reweight. The result is listed in table 5. From this table, systematic uncertainty related to Λ_b^0 kinematics is estimated 4.5%.

4.4 Λ_c^* feed-down removal

Four intermediate states can be present in the decay $\Lambda_b^0 \to \Lambda_c^+ 3\pi$.

$$\Lambda_b^0 \to \Lambda_c(2595)^+\pi; \Lambda_c(2595)^+ \to \Lambda_c^+\pi\pi$$
$$\Lambda_b^0 \to \Lambda_c(2625)^+\pi; \Lambda_c(2625)^+ \to \Lambda_c^+\pi\pi$$
$$\Lambda_b^0 \to \Sigma_c(2455)^0\pi\pi; \Sigma_c(2455)^0 \to \Lambda_c^+\pi$$
$$\Lambda_b^0 \to \Sigma_c(2455)^{++}\pi\pi; \Sigma_c(2455)^{++} \to \Lambda_c^+\pi$$

We use Λ_c^* veto cut so we need to estimate the uncertainties related to it. All these transitions can be easily reconstructed for events in the $\Lambda_b^0 \to \Lambda_c^+ 3\pi$ peak by looking at the $\Lambda_c^+\pi\pi$ and $\Lambda_c^+\pi$ mass distribution. In order to gain precisely,

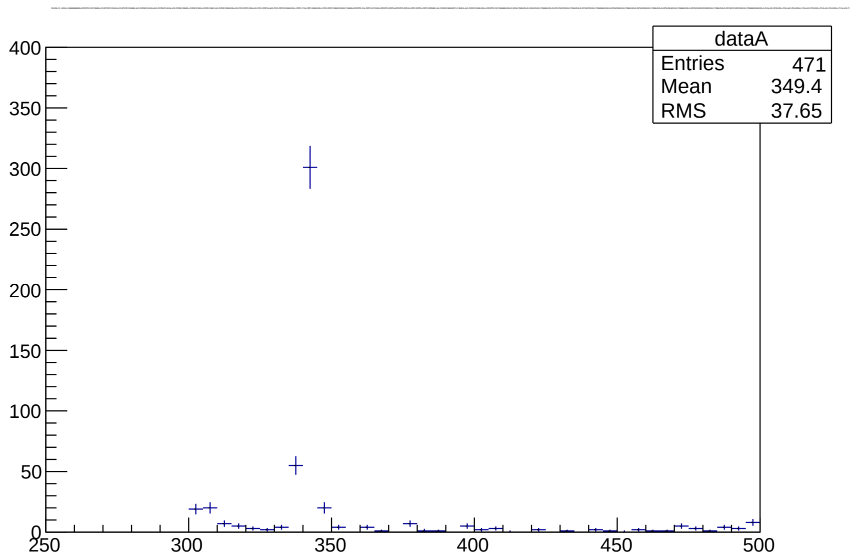

Figure 1: Distribution of $\mathrm{M}_{\Lambda_c^+\pi^+\pi^-} - M_{\Lambda_c^+}$ in MeV.

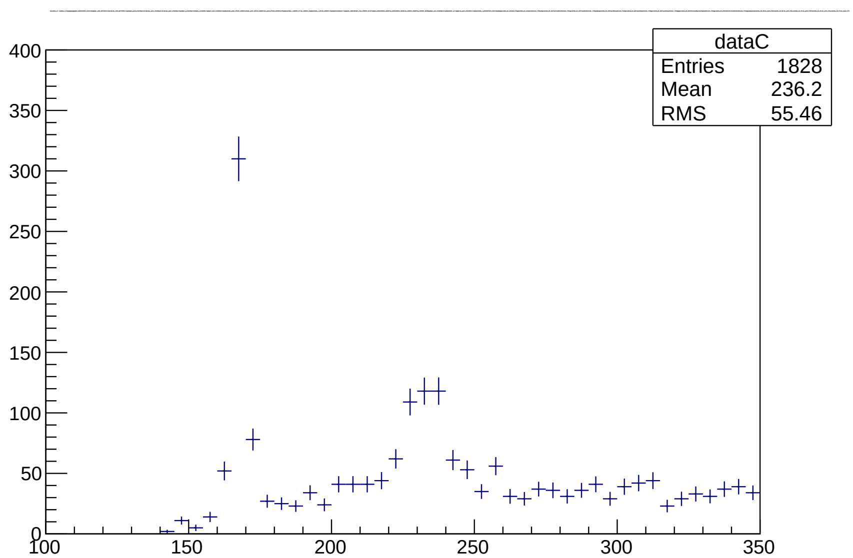

Figure 2: Distribution of $\mathrm{M}_{\Lambda_c^+\pi^+} - M_{\Lambda_c^+}$ in MeV.

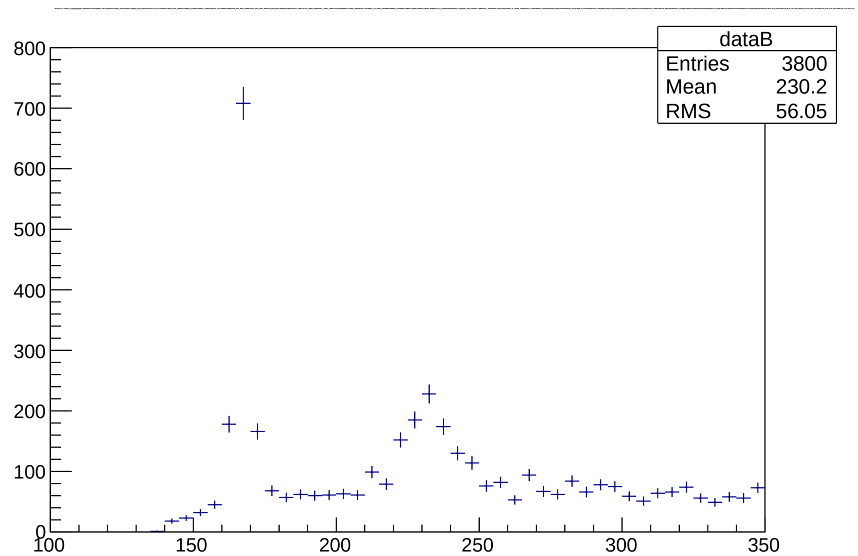

Figure 3: Distribution of $\mathrm{M}_{\Lambda_c^+\pi^-} - M_{\Lambda_c^+}$ in MeV.

Table 6: The number of events with different range

Range (in MeV)	data	MC
5500 - 5800	18424 ± 129	55236 ± 250
5500 - 5700	18604 ± 212	55205 ± 265
Difference	1%	0.6%

we plot the mass difference with respect to the measured Λ_c^+ mass. Figure 1 ,2 and 3 show these Δm distributions.

As for $\Lambda_c(2595)^+$ and $\Lambda_c(2625)^+$, two clear peaks can be seen then the signals can be removed easily so we do not need to pay attention to them. However, $\Sigma_c(2455)^0$ and $\Sigma_c(2455)^{++}$ peaks have non-negligible backgrounds. If we cut signals from 150 MeV to 260 MeV, some true signals also removed, so signals from 260 MeV to 370 MeV are added. This new data is compared to data using Λ_c^* veto cut. The difference of the two is 3.4%, and this is the estimated systematic uncertainties related to Λ_c^* removal. This is very conservative too because we compare no adding and adding.

4.5 Signal fit

Figure 4 shows the Λ_b^0 mass distribution. First, the number of events is remeasured with different range. The result is listed in the table 6.

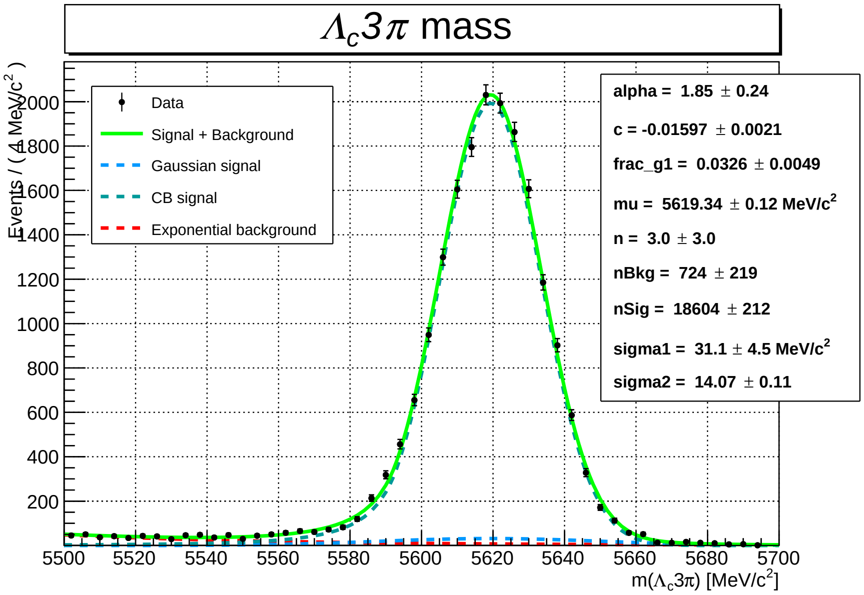

Figure 4: Λ_b^0 mass distribution with range from 5500 to 5700 in MeV for data

Next, the number of events is remeasured with some fitting parameters fixed to MC value. The number of events is shifted 1.6%.

The background is estimated as exponential above. If this is changed to polynomial, the number of events is shifted to 4.0% in data and 0.4% in MC.

The uncertainties related to signal fit can be estimated about 4.0%.

Here, statistical uncertainty is 1.1%.

4.6 MonteCarlo statistics

The number of data in true MC is 29419. We estimate the uncertainties related to MC statistics as $\sqrt{29419}/29419 = 0.6\%$.

4.7 Trigger corrections

Trigger efficiency is measured using trigger independent signal (TIS) and trigger on signal (TOS). The ratios between TOS and TIS for data and MC are shown in figure 5. The ratio between data and MC after kinematic reweighting is shown in figure 6. This ratio is new reweighting. Transverse momentum of Λ_b^0 is also used for horizontal axis so two reweightings are made. Table 7 shows the result of new reweightings. From this table, the difference between two TIS-TOS reweighting is about 2.1%. This is estimated uncertainties related to trigger corrections.

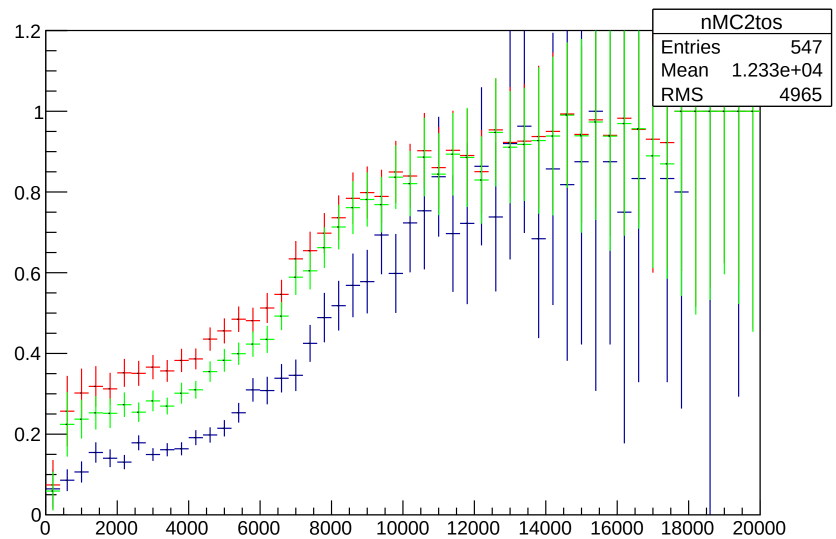

Figure 5: Ratios between TOS and TIS for data (blue) and MC. Two MCs are shown, before (red) and after (green) kinematic reweighting. Horizontal axis shows transverse momentum of three pions.

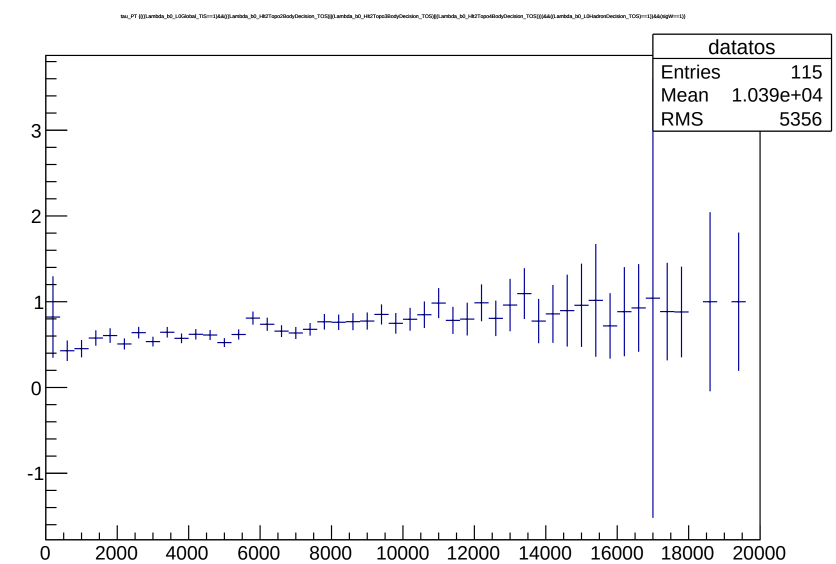

Figure 6: Ratios between data and MC after kinematic reweighting. Horizontal axis shows transverse momentum of three pions.

Table 7: Result of TIS-TOS reweighting.

cut	before reweighting	p_T of 3π	p_T of Λ_b^0
no cut	35103 ± 229	23474 ± 187	22711 ± 31
normalisa2	13841 ± 133	9285 ± 98	9172 ± 109
Ratio	39.43%	39.55%	40.39%

Table 8: Result of PID corrections.

cut	1d	2d	3d
no cut	113529 ± 345	111809 ± 390	103285 ± 321
normalisa2	45944 ± 215	45721 ± 215	41992 ± 204
Ratio	40.47%	40.89%	40.66%

4.8 PID corrections

Regarding the ratio $Br(\Lambda_b^0 \to \Lambda_c^+ 3\pi)/Br(B^0 \to D^* 3\pi)$, the final state particles are nearly the same but only proton is different in the numerator and denominator. All we have to take care is about this proton PID.

The numbers of data and MC are measured to make new reweighting. One is using only the momentum of protons. This is called 1d_weight.

The momentum of protons and the pseudo-rapidity of protons are also used to make 2d_weight. The momentum of protons, the pseudo-rapidity of protons and the number of tracks are also used to make 3d_weight.

The difference between these three weighting is related to uncertainties of PID on protons. The result is shown in table 8. From this systematic uncertainties related to PID corrections is estimated to 1.0%.

4.9 $f_{\Lambda_b^0}/f_d$

In the paper LHCb-PAPER-2018-050, the ratio between the number of produced Λ_b^0 and u quark plus d quark. It has p_T dependence and result is

$$\frac{f_{\Lambda_b^0}}{f_u + f_d} = A \times [p_1 + \exp(p_2 + p_3 \times p_T)] \tag{8}$$

$$p_1 = (7.93 \pm 1.41) \times 10^{-2} \tag{9}$$

$$p_2 = -1.022 \pm 0.047 \tag{10}$$

$$p_3 = -0.107 \pm 0.002 \tag{11}$$

$$A = 1 \pm 0.061 \tag{12}$$

Table 9: Result of changing fit parameters.

Conditions	p_1	p_2	p_3	χ^2	Degree of freedom	Ratio
Center	7.93×10^{-2}	-1.022	-0.107	28.4749	13	3.009 ± 0.060
$p_1 \to +1\sigma$	9.34×10^{-2}	-1.003	-0.109	30.7654	13	2.783 ± 0.056
$p_1 \to -1\sigma$	6.52×10^{-2}	-1.041	-0.105	25.5716	13	3.264 ± 0.065
$p_2 \to +1\sigma$	8.49×10^{-2}	-0.975	-0.108	28.6828	13	2.860 ± 0.057
$p_2 \to -1\sigma$	7.37×10^{-2}	-1.069	-0.106	27.285	13	3.168 ± 0.063
$p_3 \to +1\sigma$	6.59×10^{-2}	-1.052	-0.105	25.9234	13	3.271 ± 0.065
$p_3 \to -1\sigma$	9.27×10^{-2}	-0.992	-0.109	30.182	13	2.779 ± 0.056

$$\rho_{12} = 0.40 \tag{13}$$
$$\rho_{13} = -0.95 \tag{14}$$
$$\rho_{23} = -0.63. \tag{15}$$

Here, ρ_{12}, ρ_{13} and ρ_{23} are correlation coefficients among the fit parameters.

The ratio of $\Lambda_b^0 \to \Lambda_c^+ 3\pi$ and $B^0 \to D^* 3\pi$ from our analysis is divided by this ratio to obtain the ratio

$$\frac{Br(\Lambda_b^0 \to \Lambda_c^+ 3\pi; \Lambda_c^+ \to pK^-\pi^+)}{Br(B^0 \to D^* 3\pi; D^* \to K^-\pi^+\pi^+)}.$$

This should not have p_T variance.

Systematic uncertainty related to equation 8 is estimated by changing the fit parameters with 1σ. The result is listed in table 9. Condition $p_1 \to -1\sigma$ is the most suitable fit in this table because χ^2 is the least. So we use value of $p_1 \to -1\sigma$. This result is shown in 7

The difference of ratio between center and $p_1 \to -1\sigma$ is 8.5% and this is estimated uncertainty related to $f_{\Lambda_b^0}/f_d$.

5 Result

The efficiency corrected yield ratio of the $\Lambda_b^0 \to \Lambda_c^+ 3\pi$ and $B^0 \to D^* 3\pi$ is equal to

$$\frac{N(\Lambda_b^0 \to \Lambda_c^+ 3\pi)}{N(B^0 \to D^* 3\pi)} = \frac{Br(\Lambda_b^0 \to \Lambda_c^+ 3\pi)}{Br(B^0 \to D^* 3\pi)} \times \frac{f_{\Lambda_b^0}}{f_{B^0}} \times \frac{Br(\Lambda_c^+ \to pK\pi)}{Br(D^* \to K\pi\pi)}. \tag{16}$$

From this equation, $Br(\Lambda_b^0 \to \Lambda_c^+ 3\pi)$ can be deduced.

$Br(B^0 \to D^* 3\pi)$ is given by the Particle Data Book.

$$Br(B^0 \to D^* 3\pi) = (7.21 \pm 0.29) \times 10^{-3} \tag{17}$$

The ratio $f_{\Lambda_b^0}/f_{B^0}$ can be calculated by the paper LHCb-PAPER-2018-050. In this paper, $f_{\Lambda_b^0}/(f_u + f_d)$ is given. Considering isospin symmetry $f_u = f_d$,

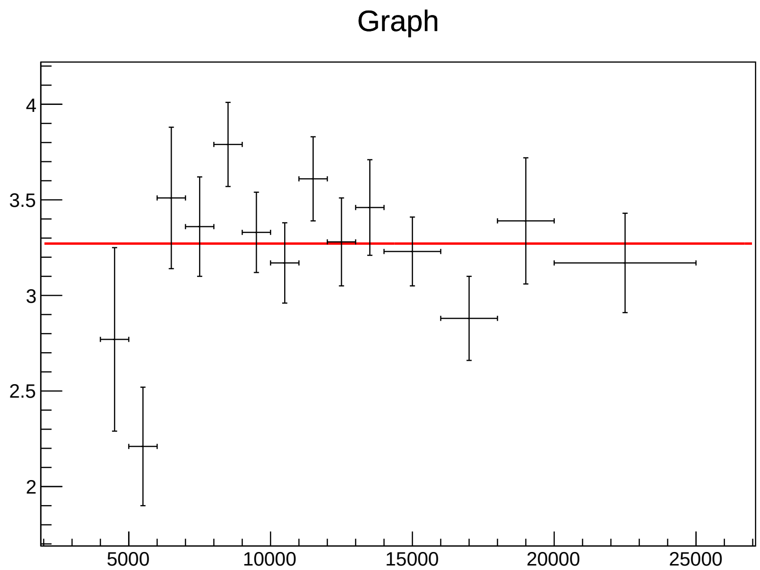

Figure 7: Ratio of the p_T distribution, as the function of p_T in MeV after efficiency correction. This is the result of the most suitable fit $p_1 \to -1\sigma$.

the ratio $f_{\Lambda_b^0}/f_{B^0}$ is given by,

$$\begin{aligned}\frac{f_{\Lambda_b^0}}{f_{B^0}} &= 2 \times \frac{f_{\Lambda_b^0}}{f_u + f_d} \\ &= 2 \times A \times [p_1 + \exp(p_2 + p_3 \times p_T)]\end{aligned} \tag{18}$$

where p_1, p_2 and p_3 are given by equations 9, 10 and 11.

$Br(\Lambda_c^+ \to pK\pi)$ is measured in the paper LHCb-PAPER-2018-050.

$$Br(\Lambda_c^+ \to pK^-\pi^+) = (6.23 \pm 0.33)\% \tag{19}$$

This error is irrelevant because it will cancel.

$Br(D^* \to D^0\pi^+)$ and $Br(D^0 \to K^-\pi^+)$ are also given in the Particle Data Book.

$$Br(D^* \to D^0\pi^+) = (67.7 \pm 0.5)\% \tag{20}$$

$$Br(D^0 \to K^-\pi^+) = (3.946 \pm 0.030)\% \tag{21}$$

From table 9,

$$\frac{N(\Lambda_b^0 \to \Lambda_c^+ 3\pi)}{N(B^0 \to D^* 3\pi)} \times \frac{f_{B^0}}{f_{\Lambda_b^0}} = 1.632 \tag{22}$$

is obtained. Here, we do not consider uncertainty and added later.

Using the equation 16 and 22,

$$Br(\Lambda_b^0 \to \Lambda_c^+ 3\pi) = 1.632 \times Br(B^0 \to D^* 3\pi) \times \frac{Br(D^* \to K\pi\pi)}{Br(\Lambda_c^+ \to pK\pi)} \tag{23}$$

Using the values and the result of systematic uncertainties described in previous section,

$$Br(\Lambda_b^0 \to \Lambda_c^+ \pi^- \pi^+ \pi^-) = (5.05 \pm 0.06(\text{stat.}) \pm 0.64(\text{syst.})) \times 10^{-3} \tag{24}$$

is obtained.

In PDG Book,

$$Br(\Lambda_b^0 \to \Lambda_c^+ \pi^+ \pi^- \pi^-) = (7.7 \pm 1.1) \times 10^{-3} \tag{25}$$

$$Br(\Lambda_b^0 \to \Lambda_c(2595)^+ \pi^-; \Lambda_c(2595)^+ \to \Lambda_c^+ \pi^+ \pi^-) = (3.4 \pm 1.5) \times 10^{-4} \tag{26}$$

$$Br(\Lambda_b^0 \to \Lambda_c(2625)^+ \pi^-; \Lambda_c(2625)^+ \to \Lambda_c^+ \pi^+ \pi^-) = (3.3 \pm 1.3) \times 10^{-4} \tag{27}$$

$$Br(\Lambda_b^0 \to \Sigma_c(2455)^0 \pi^+ \pi^-; \Sigma_c(2455)^0 \to \Lambda_c^+ \pi^-) = (5.7 \pm 2.2) \times 10^{-4} \tag{28}$$

$$Br(\Lambda_b^0 \to \Sigma_c(2455)^{++} \pi^- \pi^-; \Sigma_c(2455)^{++} \to \Lambda_c^+ \pi^+) = (3.2 \pm 1.6) \times 10^{-4} \tag{29}$$

are listed. Our result does not include the data via intermediate states. From these, Λ_c^* fraction is $(20.3 \pm 4.4)\%$, which means that without Λ_c^* fraction is $(79.7 \pm 4.4)\%$. Therefore, this number can be calculated.

$$Br(\Lambda_b^0 \to \Lambda_c^+ \pi^- \pi^+ \pi^-; \text{without } \Lambda_c^*, \text{ PDG}) = (6.14 \pm 0.94) \times 10^{-3}. \tag{30}$$

Comparing our result with PDG value, we find our result is consistent with PDG value and precision is improved.

References

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