



Measuring physical properties of the W boson in 7 TeV p-p collisions

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ABSTRACT

The Large Hadron Collider (LHC) is a recently-activated proton-proton collider operating at a center-of-mass energy of $\sqrt{s} = 7$ TeV. The Compact Muon Solenoid (CMS) is one of two general-purpose collider-detectors centered at one of the interaction points of the LHC. A measurement of the W boson production cross-section and mass is presented to test the performance of the CMS detector in early ($\int dt \mathcal{L} = 70 \text{ nb}^{-1}$) LHC data and to test theoretical predictions at this never-before-experienced energy regime. The cross-section measurement uses standard event counting techniques, while the mass measurement is performed using fitting of the analytical differential cross-section formula to the m_T spectrum from selected $W \rightarrow \mu\nu_\mu$ events. The W production cross-section is measured to be

$$\sigma_{pp \rightarrow W} \times \text{BR}(W \rightarrow \mu\nu_\mu) = \\ 9013 \text{ pb} \pm 540 \text{ pb (stat.)} \pm 552 \text{ pb (sys.)} \pm 991 \text{ pb (lumi.)},$$

consistent with theoretical predictions at next-to-leading order in α_s . The mass of the W is measured to be

$$m_W = 80.07 \text{ GeV}/c^2 \pm 0.64 \text{ GeV}/c^2 \text{ (stat.)} \pm 0.48 \text{ GeV}/c^2 \text{ (sys.)},$$

consistent with previous experimental measurements.

DEDICATION

This work is dedicated to my wife Sasha, who, for many years, had to put up with incalculable schedules and indefinite plans. You encouraged me to be the dedicated student I needed to be. Thank you.

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Chapter 1

INTRODUCTION

The field of high-energy particle physics is entering a new realm of experimental reach with the long-awaited activation of the **L**arge **H**adron **C**ollider (LHC). The LHC collides bunches of protons together at a center-of-mass energy of $\sqrt{s} = 14$ TeV 40 million times per second, enabling physicists to probe the nature of subatomic particles at and above the weak energy scale (about 100 GeV)[1, 2]. Although the fundamental design of the LHC is similar to that of previous-generation hadron colliders (*e.g.* Fermilab’s Tevatron), its center-of-mass collision energy, bunch-crossing rate, and instantaneous luminosity are many times greater than what has formerly been achievable. In many cases, the experimental techniques used in previous-generation particle detectors had to be re-engineered to survive in the extreme collision environment at the LHC. Thus, before any search for new physics can be performed at the energy frontier, experimenters will have to validate the measurements from these detectors against predictions from a reliable theoretical framework.

The Standard Model[3, 4, 5] is one such reliable framework—it is a quantum field theory that describes the fundamental constituents of all matter and the interactions between those fundamental particles. The most successful accomplishment of the Standard Model theory was the prediction of the $W^\pm$ and Z bosons before their eventual discovery at the UA1 and UA2 experiments in 1983[6, 7, 8, 9, 10]. Many of

the measurable parameters of these particles, and of the Standard Model in general, have been shown to agree with experimental data to very high accuracies[11]. Because the Standard Model has been shown to be accurate and predictive, it is considered a reliable model of the fundamental structure of nature, at least at or below the energy scale of weak interactions.

The primary goal of this work is to validate data from these new experimental conditions at the LHC against theoretical predictions from the Standard Model and to provide as accurate as possible measurements of certain Standard Model parameters. The analyses in this dissertation use data taken at the **C**ompact **M**uon **S**olenoid (CMS), one of the two general-purpose particle detectors located on the LHC accelerator ring[12, 13]. This work presents measurements of the physical properties of the Standard Model $W^\pm$ weak gauge bosons as measured using 70 nb^{-1} of $\sqrt{s} = 7 \text{ TeV}$ proton-proton collision data taken at CMS in 2010. The paper focuses on $W^\pm$ bosons because they are well-studied in the body of theoretical work, they have had their properties measured in many previous experiments, and they are produced in ample numbers to allow for measurements of their properties in early LHC running. This analysis will present a measurement of the production cross-section resonant mass of the $W^\pm$ bosons in muonic-flavored lepton decays.

The layout of the remainder of this work is as follows: the second chapter introduces the theory of the Standard Model as developed by Glashow, Salam, and Weinberg[3, 4, 5], focusing on the theory underlying the production and decay of the $W^\pm$ bosons in hadron colliders such as the LHC; the third chapter describes the LHC and the CMS experiment; the fourth chapter discusses the strategy used to identify and measure the properties of the weak gauge bosons; the fifth chapter presents the results of the analysis, including discussions of the possible sources of error and uncer-

tainty in the measurements; and the final chapter concludes with a summary of the results in comparison to previous experimental results and theoretical predictions.

Chapter 2

THEORETICAL OVERVIEW

The Standard Model is a quantum field theory that unifies the electromagnetic and weak forces, describing them as gauge symmetries between the fundamental particles that make up all matter[3, 4, 5]. In this theory, the fermionic fields^a representing quarks and leptons are symmetric under local $SU(2)_L \times U(1)_Y$ gauge transformations. The theory also describes bosonic fields that correspond to these gauge transformations and represent the electroweak force-carrying particles: the photon (γ), representing the electromagnetic force; the twin $W^\pm$ bosons, representing the charge-current weak nuclear interaction; and Z^0 , representing the neutral-current weak nuclear force. All massive particles in this theory gain mass by coupling to a scalar bosonic field that acquires a vacuum expectation value through spontaneous symmetry breaking, a method known as the Higgs mechanism[14, 15].

This analysis is concerned with the bosons that mediate the weak nuclear force, or the apparent $SU(2)$ symmetry at low energies that remains after the fundamental $SU(2)_L \times U(1)_Y$ symmetry of nature is broken via the Higgs mechanism. The $W^\pm$ bosons, particle-antiparticle partners, are responsible for the charged-current interaction of the weak force, governing such interactions as nuclear β -decay[16]. The $W^\pm$

^a“Fermionic” particles/fields are those that obey Fermi-Dirac statistics, *i.e.* those that have angular momenta quantized in half-integer values. “Bosonic” particles/fields obey Bose-Einstein statistics, or have angular momenta quantized in integer values.

bosons obtain mass by coupling to the Higgs field, so accurate measurements of the $W^\pm$ mass could shed light on the properties of the yet-undiscovered Higgs boson[17].

2.1 The Standard Model

The Standard Model of particle physics describes the physical world as a set of fermionic fields obeying a local $SU(2)_L \times U(1)_Y$ symmetry that is broken at low energies into the apparent $U(1)$ symmetry of the electromagnetic force and the $SU(2)$ symmetry of the weak nuclear force. The fundamental fermions are divided into two groups. Quarks are those fields that have an added local $SU(3)$ symmetry to the Standard Model group and therefore experience a force called the strong nuclear force. Leptons, by contrast, are those fields that do not experience the strong nuclear force. The fermions are also organized into doublets and singlets of the $SU(2)_L$ symmetry: two species of chiral left-handed quarks (u_L and d_L) with electromagnetic charges $+2/3$ and $-1/3$ (respectively) form one $SU(2)$ doublet; two species of chiral left-handed leptons (ν_{eL} and e_L) with electromagnetic charges 0 and -1 (respectively) form a second $SU(2)$ doublet; and two right-handed quarks (u_R and d_R) with electromagnetic charges $+2/3$ and $-1/3$ (respectively), and one right-handed lepton (e_R) with electromagnetic charge -1 are each $SU(2)$ singlets (Table 2.1). The left-handed chirality components of the fields couple to the $SU(2)$ gauge fields, while the right-handed components do not.

The theory also describes three separate generations of fermions, with each generation identical in all ways to the previous generation except for a difference in mass. The quarks c and s and the leptons ν_μ and μ comprise the medium-mass generation, and the t and b quarks and the ν_τ and τ leptons comprise the high-mass generation. The mass eigenstates of the different generations of quarks (and possibly the leptons) are not identically the same as the weak-interaction eigenstates, a condition that al-

Particle	QED charge [e]	Particle type
$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$+\frac{2}{3}$ $-\frac{1}{3}$	quark
$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	0 -1	lepton
u_R	$+\frac{2}{3}$	quark
d_R	$-\frac{1}{3}$	quark
e_R	-1	lepton

Table 2.1: The first generation of fundamental fermions of the Standard Model. Particles in parentheses are a part of the same $SU(2)_L$ doublet. The subscripts L and R refer to the left- and right-handed chirality components, respectively, of the fermionic fields.

Particle	Mass [MeV/ c^2]	Particle	Mass [MeV/ c^2]	Particle	Mass [MeV/ c^2]
u	1.5–3.3	c	1.3×10^3	t	1.7×10^5
d	3.5–6.0	s	1.0×10^2	b	4.2×10^3
ν_e	$< 2.0 \times 10^{-6}$	ν_μ	< 0.19	ν_τ	< 18
e	5.1×10^{-1}	μ	1.1×10^2	τ	1.8×10^3
(a) First Generation		(b) Second Generation		(c) Third Generation	

Table 2.2: The three generations of Standard Model fermions and their experimentally-measured masses[11]. Uncertainties are not shown.

allows for cross-generational mixing through the weak interaction[18, 19, 20]. The three generations of particles and their respective masses are given in Table 2.2.

The physically observed gauge bosons of the Standard Model are mass-eigenstate mixings of the massless gauge fields with a fundamental $SU(2)$ doublet scalar field in a process known as the Higgs mechanism^b. In the simplest model of this mechanism, a

^bThis mechanism is required because, in quantum field theory calculations, a mass term for a gauge field in a Lagrangian density function cannot be symmetric under the gauge transformations that the gauge field introduces. Because the Lagrangian represents a physical quantity that is required to be gauge-invariant in a well-behaved theory, naïve gauge field mass terms are not allowed.

Gauge field	Associated symmetry
B	$U(1)_Y$
W_1	$\left. \begin{array}{l} \\ \\ \end{array} \right\} SU(2)_L$
W_2	
W_3	
g	$SU(3)_c$

Table 2.3: The massless gauge fields of the Standard Model (augmented with the $SU(3)$ local gauge symmetry of quantum chromodynamics).

single Higgs doublet has a scalar potential with a non-zero minimum value in the vacuum state. When coupled to the gauge fields of the Standard Model $SU(2)_L \times U(1)_Y$ group, this vacuum expectation value acts as a mass term for certain combinations of the gauge fields in the low-energy Lagrangian of the theory. Two of the gauge fields (corresponding to the τ^1 and τ^2 generators of the $SU(2)$ group) mix to form the massive $W^\pm$ bosons of the weak charge-current interaction (analogous to the $\tau^\pm$ raising and lowering operators of the $SU(2)$ group). The other two gauge fields (corresponding to the τ^3 generator of the $SU(2)$ group and the generator of the $U(1)$ group) mix to form the massive Z^0 boson, responsible for the weak neutral-current interaction, and the familiar massless photon of the electromagnetic force. The massless gauge fields of the Standard Model $SU(2)_L \times U(1)_Y$ gauge group (with the $SU(3)_c$ gauge bosons for the added strong nuclear force) are listed in Table 2.3, and the massive bosons that result from their Higgs mixing are listed in Table 2.4.

Relevance of the W mass

The mass of the W boson is predicted by the Standard Model to be related to other fundamental parameters. The W mass is strongly dependent on the Z mass and the aforementioned coupling constants g and g' but also weakly dependent on other parameters, such as the mass of the yet-undiscovered Higgs boson, through loop

Gauge boson	Constituent gauge fields	Associated force	Mass [MeV/ c^2]
A (γ , photon)	$\sqrt{\frac{1}{g^2+g'^2}}(g'W_3 + gB)$	Electromagnetic	0
$W^\pm$	$\sqrt{\frac{1}{2}}(W_1 \mp iW_2)$	Weak	80.395 ± 0.025
Z^0	$\sqrt{\frac{1}{g^2+g'^2}}(gW_3 - g'B)$	Weak	91.1876 ± 0.0021
g (gluon)	-	Strong	0

Table 2.4: The gauge bosons of the Standard Model (augmented with the SU(3) local gauge symmetry of quantum chromodynamics) after breaking SU(2)_L × U(1)_Y symmetry to the weak SU(2) and electromagnetic U(1) via the Higgs mechanism. The constants g and g' are the coupling constants between the fermions and the $W_{1...3}$ fields, and the fermions and the B field, respectively[11].

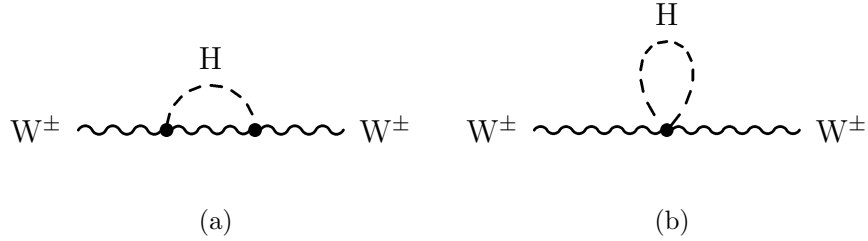


Figure 2.1: One-loop (vacuum polarization) corrections to the W propagator due to the existence of the Higgs boson.

corrections to the W propagator. The one-loop corrections to the W propagator^c depend logarithmically on the ratio of the Higgs and Z masses[17]. The mass of the Z boson has been measured to a very high precision because of its decay to two visible leptons (*c.f.* Table 2.4), so the uncertainty in the vacuum polarization is dominated by the uncertainty in the Higgs mass[11]. Figure 2.1 shows Feynman diagrams of the first-order corrections to the W propagator due to the existence of the Higgs boson.

The mass of the Higgs boson is the only unmeasured parameter of the Standard Model because no statistically significant signal of Higgs decay has been observed[17]. However, because of the logarithmic relationship between the Higgs and W masses,

^cThese single-loop corrections are sometimes known as “vacuum polarization corrections.”

the mass of the Higgs can be inferred from an accurate measurement of the W mass. The constraints placed on the Higgs mass from these measurements are useful in future searches for the Higgs because the mass determines how it is most likely to decay once created in high-energy particle collisions, as shown in Figure 2.2. Figure 2.3 demonstrates one of the problems facing the search for a Standard Model Higgs boson. Shown is the 68 % confidence surface for values of the W mass and t quark mass. If the Standard Model is correct, then diagonal lines on this plot are contours of constant Higgs mass. What this plot shows is that most of the Higgs masses encompassed by the 68 % confidence surface have been ruled out by direct or indirect Higgs searches. An increase in the accuracy of the W mass measurement could effectively rule out the existence of a light Standard Model Higgs boson, forcing researchers to focus on more exotic explanations for the apparent broken $SU(2)_L \times U(1)_Y$ symmetry in nature.

2.2 Physical properties of the $W^\pm$ bosons

The $W^\pm$ bosons, with which this analysis is chiefly concerned, couple lower-component fields of $SU(2)_L$ doublets to upper-component fields—for instance, the e to the ν_e , or the d to the u . The W^+ boson carries an electromagnetic charge^d of $+1$ (and likewise the W^- carries a charge of -1), so the coupling of the lower to upper components ensures conservation of electromagnetic charge. When produced in particle collisions, the relatively large mass of the $W^\pm$ bosons gives it many avenues for decay and a very short lifetime.

2.2.1 Differential production cross-section and muonic decay

Any unstable, massive particle will decay with an energy spectrum proportional to a relativistic Breit-Wigner function. This follows from the fact that an unstable particle

^dElectromagnetic charge is measured in units of proton charge, so the charge of an electron is -1 .

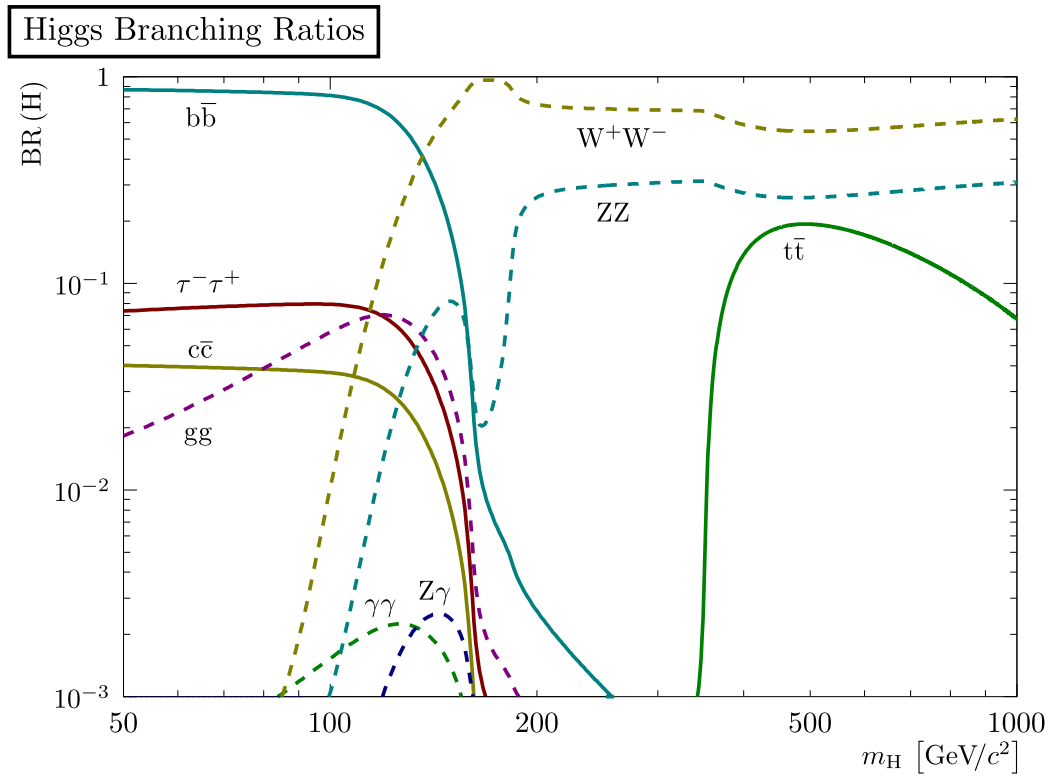


Figure 2.2: The branching ratios of the Higgs boson through various channels versus Higgs mass[11].

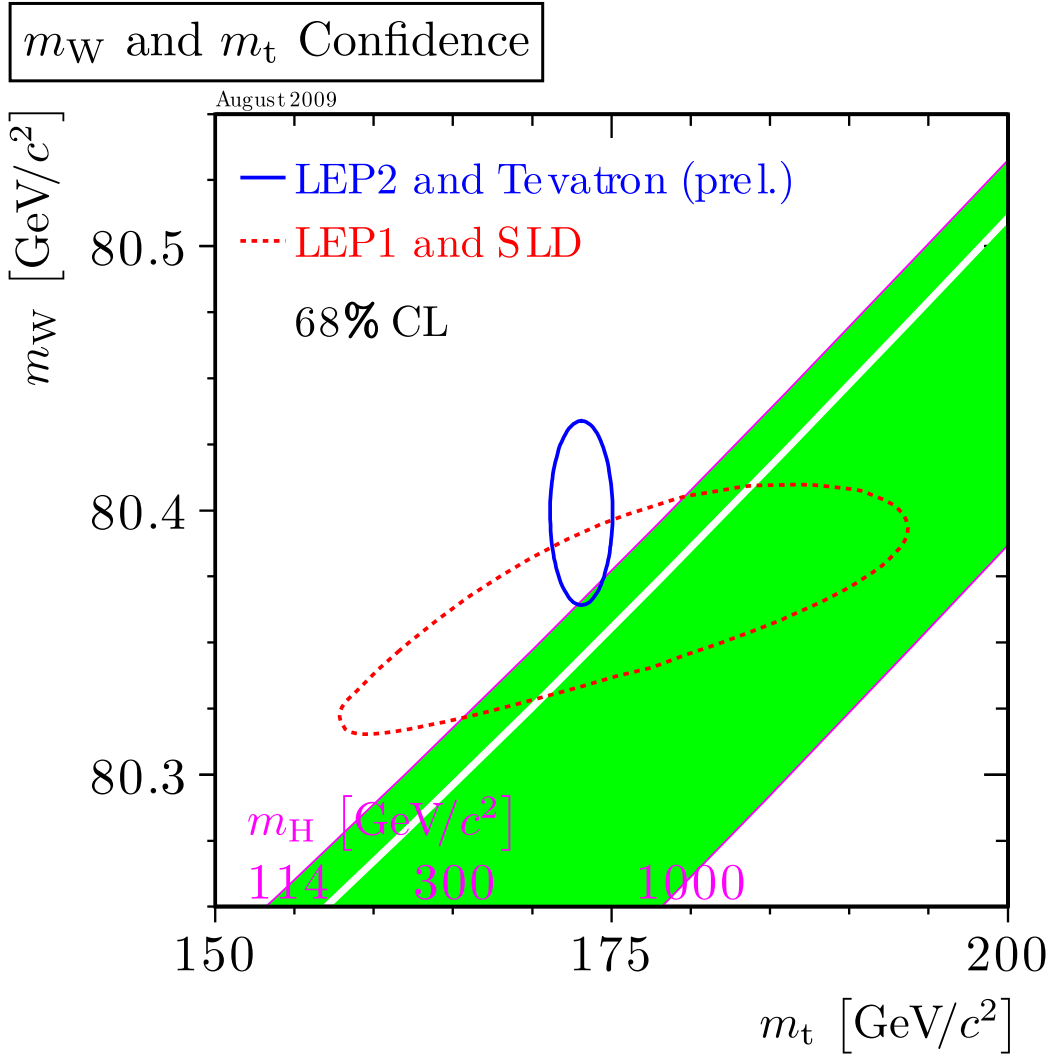


Figure 2.3: Contours describing the 68% confidence limit on the measurements of the W boson and t quark masses. The solid contour traces the area where data suggest the values of the parameters m_W and m_t lie with a 68% confidence level. The diagonal bands on the plot show the values of phase space that represent Higgs masses that have not been excluded by direct searches, indirect searches, or theoretical constraints. Notice that only a small amount of phase space is not excluded at the 68% confidence level. These data represent the combined results from the Large Electron-Positron (LEP) collider and Tevatron experiments[17].

of mass m with a typical lifetime T will have a quantum wave function that decays exponentially, like so:

$$\psi(t) \sim \psi(0)e^{-imt}e^{-\Gamma t}, \quad (2.1)$$

where the parameter $\Gamma = \frac{1}{T}$ is the particle's width. A Fourier transform of this wave function produces one that varies with the particle energy E :

$$\psi(E) = \int \psi(t)e^{iEt}dt \sim \frac{1}{E - m + i\Gamma}. \quad (2.2)$$

The probability of observing the particle with an energy E will follow the norm of this wave function squared:

$$\frac{\partial\sigma}{\partial E} \sim |\psi|^2 \sim \frac{1}{(E - m)^2 + \Gamma^2},$$

which is the non-relativistic Breit-Wigner (or Cauchy) distribution. The fully relativistic derivation of the cross-section for the W boson involves in the matrix-element calculation the propagator of a massive, unstable spin-1 particle, given by

$$\mathcal{M}(\text{pp} \rightarrow \text{W} \rightarrow \text{XX}') \sim \frac{1}{p^2 - m_{\text{W}}^2 + im_{\text{W}}\Gamma_{\text{W}}},$$

where $p^2 \equiv p_\mu p^\mu$ is the four-momentum squared of the W. This is similar in form to (2.2). The full cross-section has in it the norm of the matrix element:

$$|\mathcal{M}(\text{pp} \rightarrow \text{W} \rightarrow \text{XX}')|^2 \sim \frac{1}{(p^2 - m_{\text{W}}^2)^2 + m_{\text{W}}^2\Gamma_{\text{W}}^2},$$

The complete form of the cross-section (as a function of energy) of the W boson in the center-of-mass frame of the W is^e

$$\sigma_{\text{W} \rightarrow \text{XX}'} = \frac{12\pi\Gamma_{\text{W}}^2}{m_{\text{W}}^2} \frac{E^2}{(E^2 - m_{\text{W}}^2)^2 + E^4\Gamma_{\text{W}}^2/m_{\text{W}}^2} \text{BR}(\text{W} \rightarrow \text{XX}'), \quad (2.3)$$

^eThese formulæ are given in so-called natural units, where $c = \hbar = 1$

where E is the center-of-mass energy, $\text{BR}(W \rightarrow XX')$ is the branching-ratio of the W decay to the products X and X' (see equation (2.4) and the footnote on page 14), and the term m_W^2 multiplying the width has been replaced by E^4/m_W^2 to account for the scaling of the width as a function of energy (so that Γ_W here can be considered a constant term).

Using the principle of conservation of energy, the variable E in equation (2.3) can be interpreted as the total energy that is transferred to the decay products of the particle. In the case of the decay $W \rightarrow \mu\nu_\mu$, the energy of the decaying W is the same as the invariant mass of the μ and ν_μ . The invariant mass of two particles is defined as

$$\begin{aligned} m_{XX'}^2 &= (E_X + E_{X'})^2 - |\mathbf{p}_X + \mathbf{p}_{X'}|^2 \\ &= m_X^2 + m_{X'}^2 + 2(E_X E_{X'} - \mathbf{p}_X \cdot \mathbf{p}_{X'}). \end{aligned}$$

Thus if this quantity can be calculated for all decay product pairs created in a large number of W -decays, a histogram of the frequencies of each calculated energy value will reproduce the Breit-Wigner form of equation (2.3).

Because the $W^\pm$ bosons can decay into any $\text{SU}(2)_L$ doublet pair except the t - b quark pair^f, the complete way to measure the full cross-section of the W boson would be to calculate the invariant masses for all the observed decays of the boson, including all of the possible doublets. However, this thesis will focus solely on the decay of the W to μ - ν_μ pairs, primarily because the special experimental properties of the μ allow for easy identification of these events in the noisy environment of a hadron collider. Therefore, this analysis will measure the cross-section for the production of the W

^fBecause the rest mass of the t quark is much larger than the rest mass of the W , this decay is kinematically forbidden.

boson times the branching ratio for the process $W \rightarrow \mu\nu_\mu$, defined as

$$\text{BR}(W \rightarrow \mu\nu_\mu) = \frac{\Gamma(W \rightarrow \mu\nu_\mu)}{\Gamma(W \rightarrow \text{anything})}, \quad (2.4)$$

where $\Gamma(X \rightarrow Y)$ is the partial decay width for the process $X \rightarrow Y$ ^g. This branching ratio has been measured empirically from other (lower-energy) collider experiments to be[11]

$$\text{BR}(W \rightarrow \mu\nu_\mu) = (10.57 \pm 0.15) \%. \quad (2.5)$$

The branching ratios are invariant with respect to the frame in which the particle decays, so this value is not expected to be a function of the collision energy. However, the production cross-section of the W is expected to vary as a function of collision energy. Calculations using Standard Model parameters can predict what the cross-section should be at LHC energies, but this analysis will be one of the first to verify those theoretical predictions.

2.2.2 Transverse mass

The neutrino from the W decay poses a problem in particle detectors due to its experiencing only the weak nuclear force and not the electromagnetic or the strong nuclear forces. Modern particle detectors rely primarily on electromagnetic interactions in order to determine the momenta and energies of leptons and light hadrons^h and on strong nuclear interactions to break heavy hadrons into lighter electromagnetically interacting particles. The weakly-interacting neutrinos will interact with these detector elements only very rarely due to the incredibly short-ranged nature of the weak nuclear forceⁱ. Neutrinos produced in W decays are likely to “miss” everything in

^gThe partial decay width of a process is directly proportional to the probability of that decay occurring, so the branching ratio of a certain decay channel is, in essence, the overall probability of the particle decaying in that fashion.

^hHadrons are bound states of two or three quarks.

ⁱThis is due to the large masses of the W and Z bosons. The interaction length of a neutrino at 100 GeV is roughly 10^{13} m.

the detector and escape without ever interacting or leaving a measurable signature. The energy and momentum of the neutrino must therefore be inferred from the other particles detected in a collision event.

Collisions at the LHC are between two bunches of protons traveling in opposite directions with equal momentum and colliding head-on. To simplify further discussion, we will introduce a rectangular system of coordinates where the direction of travel of the colliding protons is called the z -axis (the beam axis), and the x - and y -axes are transverse to this axis. If the colliding protons conserve momentum, this would imply that the vector sum of all momentum measured in a collision event should be zero. A non-zero sum could be attributed to an undetected neutrino carrying momentum away from the collision. Thus, one strategy for reconstructing the momentum of the neutrino is to look for “missing” momentum in the detector.

However, these colliding protons are hadrons, composed of two u and one d “valence” quarks, plus some number of “sea” quarks and gluons^j. Each of these constituent “partons” carries some fraction of the proton’s total momentum. When two protons collide at energies above the proton binding energy scale (roughly the mass of the proton, or 938 MeV), it is the point-like partons that interact. Just because the sum of the two protons’ momenta is zero in the lab frame, it does not mean that the sum of the momenta of the colliding partons will be zero, as they could each be carrying a different fraction of the total momentum of the parent proton. To allow room for the incoming protons, practical experimental collider-detectors cannot have detector elements directly along the z -axis, thus a momentum imbalance in the z -direction between the two colliding partons can never be fully reconstructed from the momenta of the collision productions. This means the neutrino momentum

^jThe “sea” quarks are quark-antiquark pairs that are created and destroyed on very short time scales due to the incredibly powerful binding energies of the strong force and the turbulent nature of the SU(3) quantum vacuum.

cannot be completely reconstructed either. So while the strategy of calculating the neutrino momentum from the total missing momentum is appealing at first glance, it is ultimately infeasible.

But the calculation of the W cross-section using lepton pairs is not hopeless. The total transverse momentum of the colliding partons is negligible compared to the extreme energies parallel to the beam in the lab frame^k. The neutrino momentum in the transverse plane can therefore be reconstructed if the detector has fully hermetic coverage in the transverse directions. The missing transverse energy of a collision event is a vector quantity defined as

$$\cancel{E}_T = - \sum_i E_{Ti} \hat{\boldsymbol{p}}_{Ti}, \quad (2.6)$$

where E_{Ti} is the transverse energy of particle i and $\hat{\boldsymbol{p}}_{Ti}$ is the unit vector in the direction defined by the transverse momentum vector of the particle^l. The cross-section (2.3) can be written in terms of a new variable, the transverse mass of the decay products, defined as

$$\begin{aligned} m_{T,XX'}^2 &= (E_X + E_{X'})^2 - |\boldsymbol{p}_{T,X} + \boldsymbol{p}_{T,X'}|^2 \\ &\approx 2(E_{T,X}E_{T,X'} - \boldsymbol{p}_{T,X} \cdot \boldsymbol{p}_{T,X'}) \\ &\approx 2p_{T,X}p_{T,X'}(1 - \cos \phi_{XX'}). \end{aligned} \quad (2.7)$$

The second and third lines use the approximation $E_X \gg m_X$ and define $\phi_{XX'}$ as the angle between the particles X and X' in the transverse plane. With this as the

^kIf the proton is at rest in the transverse lab frame, then the individual partons can carry at most a momentum equivalent to the proton rest mass ($\sim 1 \text{ GeV}/c$ [11]). On top of this, non-tree-order radiative effects will also impart a p_T to the W, even if the p_T of the partons is zero. Later discussions will show that the effects of non-zero W p_T are negligible for this analysis.

^lThe typical convention of using the detected energy of the particle instead of the momentum is historical. The masses of the particles being measured are typically orders of magnitude smaller than their energies, so E_T and $|\boldsymbol{p}_T|$ are effectively interchangeable.

differentiation variable, the cross-section for the W boson decaying to a $\mu\nu_\mu$ pair is

$$\begin{aligned} \frac{1}{\sigma_{\text{pp} \rightarrow \text{W}}} \frac{\partial \sigma_{\text{pp} \rightarrow \text{W}}}{\partial m_{\text{T}}} \text{BR}(\text{W} \rightarrow \mu\nu_\mu) = \\ A(m_{\text{W}}, \Gamma_{\text{W}}) \times \int_{m_{\text{T}}}^{\infty} ds \frac{1}{(s - m_{\text{W}}^2)^2 + m_{\text{W}}^2 \Gamma_{\text{W}}^2} \\ \times \frac{m_{\text{T}}}{\sqrt{s(s - m_{\text{T}}^2)}} \left(2 - \frac{m_{\text{T}}^2}{s}\right). \end{aligned} \quad (2.8)$$

The normalization parameter $A(m_{\text{W}}, \Gamma_{\text{W}})$ ensures (2.8) is a proper probability distribution function (PDF) with an integral of unity:

$$\int \frac{1}{\sigma_{\text{pp} \rightarrow \text{W}}} \frac{\partial \sigma_{\text{pp} \rightarrow \text{W}}}{\partial m_{\text{T}}} dm_{\text{T}} = 1 \Rightarrow A(m_{\text{W}}, \Gamma_{\text{W}}) = \frac{3}{2\pi} m_{\text{W}}^2 \Gamma_{\text{W}}.$$

The value of this PDF is zero for $m_{\text{T}} = 0$, approximately linear for $m_{\text{T}} \lesssim m_{\text{W}}/2$, and strongly peaked at $m_{\text{T}} = m_{\text{W}}$. The width of the peak at $m_{\text{T}} = m_{\text{W}}$ is defined by the parameter Γ_{W} . For values of m_{T} greater than m_{W} , the PDF has a sharp decrease and a Breit-Wigner tail. The W cross section PDF is plotted as a function of m_{T} in Figure 2.4 using for the parameters m_{W} and Γ_{W} the world average of previous experimental measurements[11]:

$$m_{\text{W}} = 80.398 \pm 0.025 \text{ GeV}/c^2$$

$$\Gamma_{\text{W}} = 2.141 \pm 0.041 \text{ GeV}/c^2.$$

Because the peak in the PDF has a very sharp falling edge, the shape of the distribution is strongly dependent on the value of the parameter m_{W} . Figure 2.5a shows how the shape of the distribution is affected by a 10% increase or decrease in the value of the mass parameter. As m_{W} is increased, the distinct falling edge of the m_{T} distribution moves to larger values of m_{T} , and *vice versa* for decreasing m_{W} . The parameter Γ_{W} does not have as noticeable an effect on the shape of the distribution. As Γ_{W} is made smaller, the peak of the distribution becomes sharper and the tails

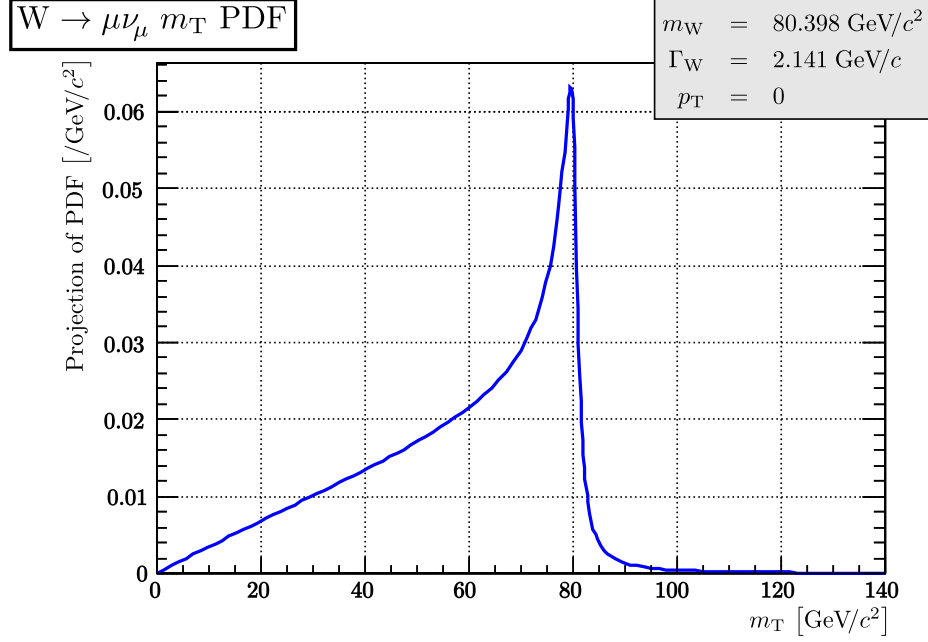
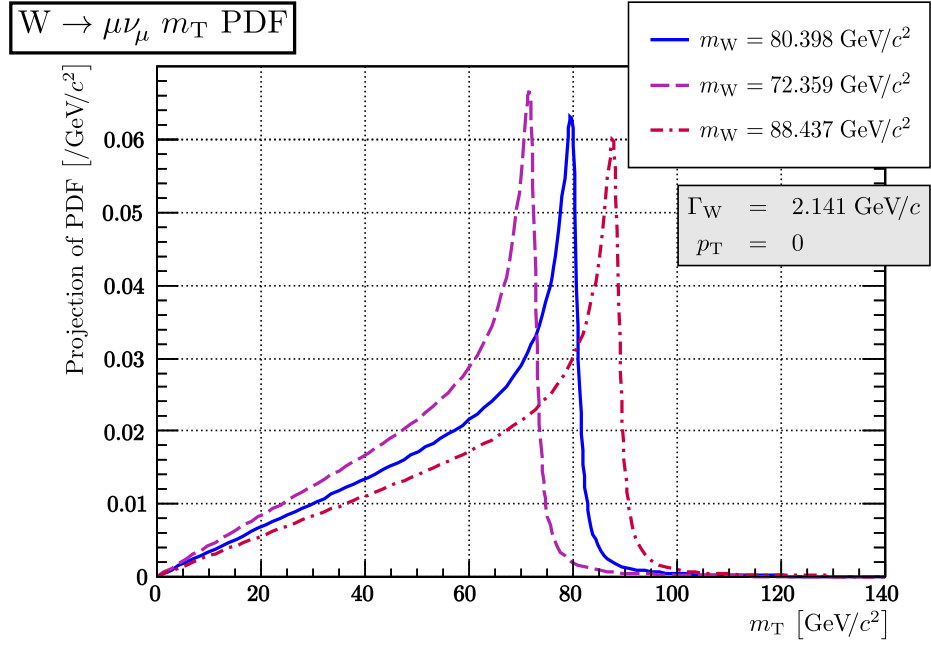


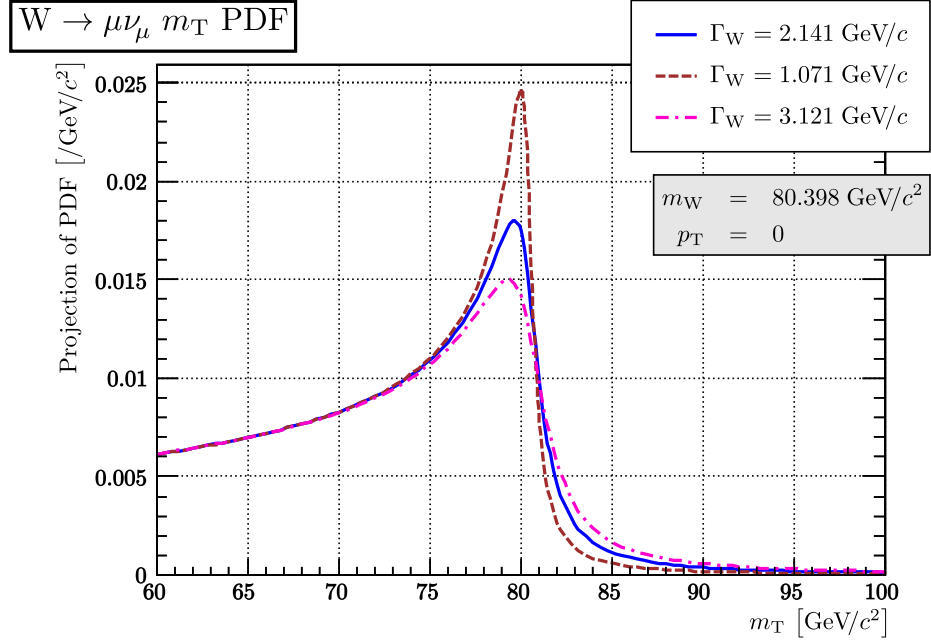
Figure 2.4: Ideal W cross section PDF as a function of m_T . The mass and width are fixed at the world-average values ($m_W = 80.398 \text{ GeV}/c^2$, $\Gamma_W = 2.141 \text{ GeV}/c$). The W is assumed to decay at rest in the transverse plane in the lab frame ($p_T = 0$).

diminish, and as Γ_W is made larger the peak becomes broader and more tail-heavy. Figure 2.5b shows how the shape of the distribution is affected by a 50% increase or decrease in the value of the width parameter. This suggests that the determination of the W mass can easily be made by observation of the $W \rightarrow \mu\nu_\mu m_T$ decay spectrum, but a measurement of the W width may not be feasible because of the very slight affect such large modifications have on the PDF.

Another parameter that affects the m_T distribution of $W \rightarrow \mu\nu_\mu$ decays is the transverse momentum of the W boson. Equation (2.8) was constructed assuming the decaying W had no transverse momentum in the lab frame. A non-zero W p_T will



(a)



(b)

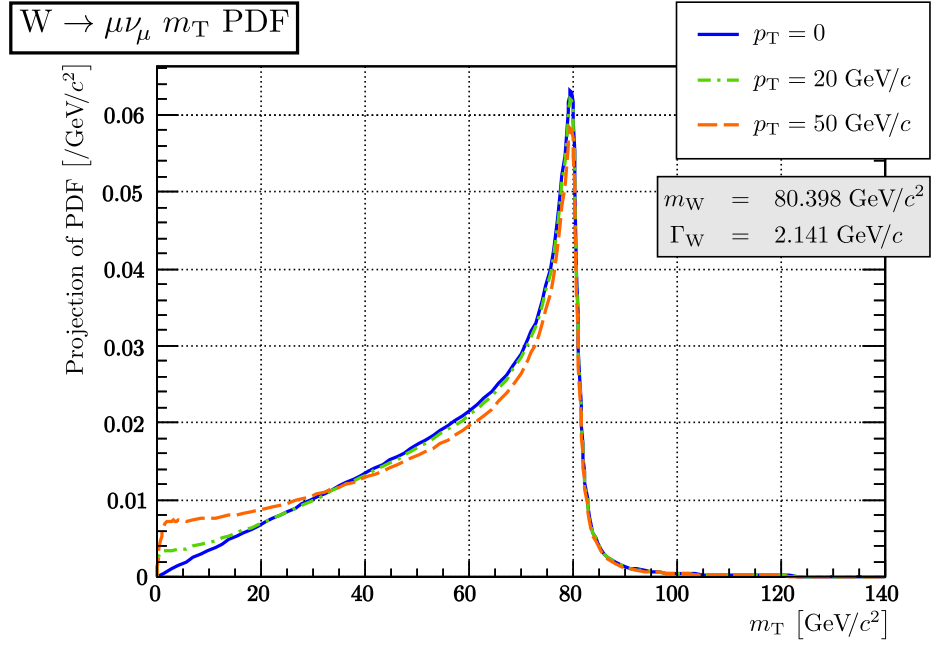
Figure 2.5: Variation of the ideal W cross section PDF as a function of m_T with changes in the (a) mass and (b) width parameters. The W is assumed to decay at rest in the transverse plane in the lab frame ($p_T = 0$).

alter the final term in the PDF to an integral over the azimuthal coordinate ϕ^{m} :

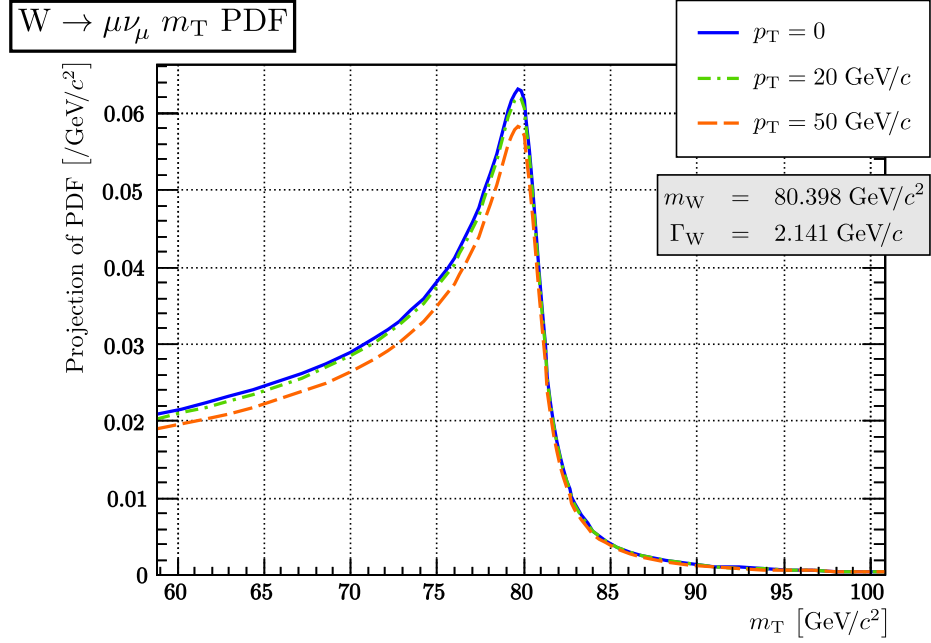
$$\begin{aligned}
& \frac{1}{\sigma_{\text{pp} \rightarrow \text{W}}} \frac{\partial \sigma_{\text{pp} \rightarrow \text{W}}}{\partial m_{\text{T}}} (p_{\text{T}} \neq 0) = \\
& A'(m_{\text{W}}, \Gamma_{\text{W}}) \text{BR}(\text{W} \rightarrow \mu \nu_{\mu}) \\
& \times \int_{m_{\text{T}}^2}^{\infty} ds \frac{1}{(s - m_{\text{W}}^2)^2 + m_{\text{W}}^2 \Gamma_{\text{W}}^2} \\
& \times \frac{m_{\text{T}}}{\sqrt{s(s - m_{\text{T}}^2)}} \int_0^{2\pi} d\phi (2 - F(m_{\text{T}}, s, m_{\text{W}}, \phi)) G(m_{\text{T}}, s, m_{\text{W}}, \phi). \quad (2.9)
\end{aligned}$$

Here, $F(m_{\text{T}}, s, m_{\text{W}}, \phi)$ is a function bounded on $[0, 1]$, and $G(m_{\text{T}}, s, m_{\text{W}}, \phi)$ is positive-definite for all parameter values[21]. The structure of the functions F and G are such that, for values $p_{\text{T}} \lesssim m_{\text{W}}$, the shape of the PDF hardly deviates from the form given in equation (2.8) and, even then, only in the low- m_{T} tail region of the distribution. Figure 2.6 shows the deviation in PDF shape from the ideal distribution for some moderate values of p_{T} . The effect on the location and width of the distribution peak is negligible at these values, and, as will be shown in Section 4.2.3, these values are much larger than the typical values of W p_{T} . This suggests that the p_{T} parameter of the distribution can be considered constant (and zero) for the purposes of measuring the W mass.

^mSee Section 3.2.1 for a detailed description of the coordinate systems used in this analysis.



(a)



(b)

Figure 2.6: Variation of the ideal W cross section PDF as a function of m_T with changes in the p_T of the W boson. The mass and width are fixed at the world-average values ($m_W = 80.398 \text{ GeV}/c^2$, $\Gamma_W = 2.141 \text{ GeV}/c$).

Chapter 3

EXPERIMENTAL SETUP

3.1 The Large Hadron Collider

The **L**arge **H**adron **C**ollider (LHC) is a state-of-the-art machine built at the European Organization for Nuclear Research (known by its French acronym CERN). It is designed to accelerate and collide protons and heavy ions in a controlled fashion for the current generation of high-energy particle physics experiments[2, 1]. It is located approximately 100 m under the surface of the Earth in a ring that crosses the French-Swiss border near the city of Geneva. The LHC accelerates protons around an underground ring of superconducting magnets 27 km in circumference and steers those protons to collide at four interaction points. Located at these interaction points are four major particle detector experiments—two general-purpose detectors, **A** **T**oroidal **L**HC **A**pparatu**S**[22] (ATLAS) and the **C**ompact **M**uon **S**oleno**i**d[12] (CMS), and two special-purpose detectors, **A** **L**arge **I**on **C**ollider **E**xperiment[23] (ALICE) and the **L**arge **H**adron **C**ollider **b**eauty[24] (LHCb) detector. The design specifications of the LHC boast a maximum center-of-mass p-p collision energy of $\sqrt{s} = 14$ TeV and a maximum instantaneous luminosity of $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$, though these figures are generously optimistic estimates of the possible future performance of the machine. Regardless, in the 2009 and 2010 physics runs, the LHC achieved center-of-mass collision

energies of $\sqrt{s} = 7$ TeV, making it the highest-energy particle collider in the world and a priceless tool for probing and understanding physics at the energy frontier.

3.1.1 Overview of the Accelerator Chain

The LHC is the final synchrotron accelerator in a chain of linear and synchrotron accelerators at the CERN complex near Geneva, Switzerland[25]. Protons are fed into a linear accelerator (Linac2) and accelerated to an energy of 50 MeV, then they are passed into the first of three pre-LHC synchrotron accelerators, the Proton Synchrotron Booster (PSB). The PSB accelerates the protons to 1.4 GeV and separates the beam into well-defined bunches spaced a minimum of 300 ns apart. These bunches are injected into the Proton Synchrotron (PS), where they are accelerated to 26 GeV and split into bunches spaced a minimum of 25 ns apart. This bunch configuration is maintained for the remainder of the acceleration chain. After the PS, the proton bunches are sent to the imaginatively named Super Proton Synchrotron (SPS) and accelerated to 450 GeV. At this point, the bunches contain approximately 10^{11} protons each in nominal running. The proton bunches are injected into the LHC via two transfer lines, one each for the clockwise and anticlockwise beams. Figure 3.1 diagrams the LHC accelerator chain.

The LHC employs 1 232 superconducting main dipole magnets (for steering and accelerating the beams) and 674 super- and regular-conducting multipole “corrector” magnets. The high-luminosity requirements of the LHC preclude the use of antiprotons as collision products due to the short lifetimes of antiproton beams and the lengthy antiproton production times. Both beams contain equally-charged particles, so a magnetic field in a single direction is insufficient to make the beams bend in opposite directions as they travel around the LHC ring. Because of this, the LHC uses two separate beam-pipes contained in the same superconducting cryostat—one

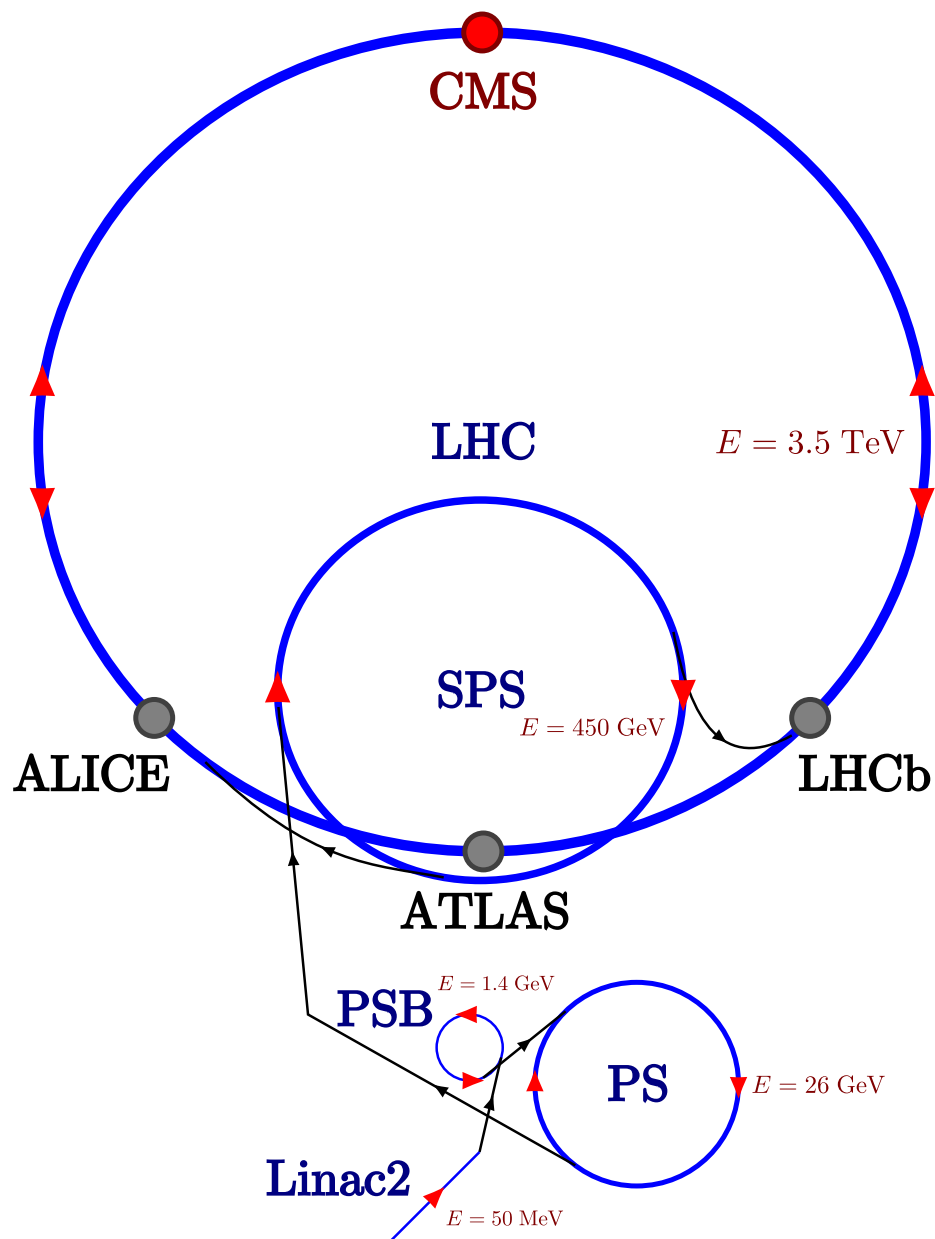


Figure 3.1: The accelerator chain at the CERN complex. The arrows show the direction of proton beams in the accelerators and the energy of the beam at these points. Filled circles show the locations of the four major detectors on the LHC. This diagram is not to scale.

for each beam—instead of sharing a single beam-pipe between both beams. The twin beam-pipe configuration of the LHC magnets steers the beams in opposite directions by using the return field from the dipole surrounding one beam-pipe to feed into the dipole surrounding the second. The dipole magnets are approximately 14.3 m long each, with a cold mass radius of approximately 57 cm. The magnetic field generated by the superconducting currents creates an approximately uniform magnetic field of up to 8.33 T (for 7 TeV beams) in the $+z$ -direction in one beam-pipe and in the $-z$ -direction in the other, with the magnetic field lines connecting through the cold mass of the magnet. The magnets are contained in a cryostat with a length of approximately 16 m each and of radius 91.4 cm. The magnets are kept at 1.9 K during LHC operations by cycling superfluid helium through the cryostats. Figure 3.2 shows a cross-sectional view of a standard LHC dipole and cryostat.

The multipole corrector magnets are dispersed around the LHC ring and perform beam focusing and “cleaning” operations. These magnets come in dipole, quadrupole, sextupole, octupole, and decapole varieties. They correct the beam orbits, focus or defocus the proton bunches, combine or split the beams for collision, and correct other beam anomalies that arise due to geometry effects, beam injection, and other specialized operations^a.

In order to collide at the center of the detectors on the LHC ring, the two beams are forced to share the same beam-pipe for 126 m to 140 m lengths through the experimental halls. In these stretches of beam-pipe, the beams are traveling sufficiently close to each other to collide at every bunch-crossing. With a minimum bunch-spacing of 25 ns, this means there can be up to 35 collision points in one of these combined-

^aMost of these magnets share a similar design to the main dipole magnets, with the obvious exceptions being the regular-conducting magnets used in the high-radiation areas of the ring near the collimators and the single-bore magnets, where the two beams share the same beam-pipe, used in the vicinity of the experiment detectors to allow collisions. The descriptions of the various multipole corrector magnets are detailed elsewhere[1, 2].

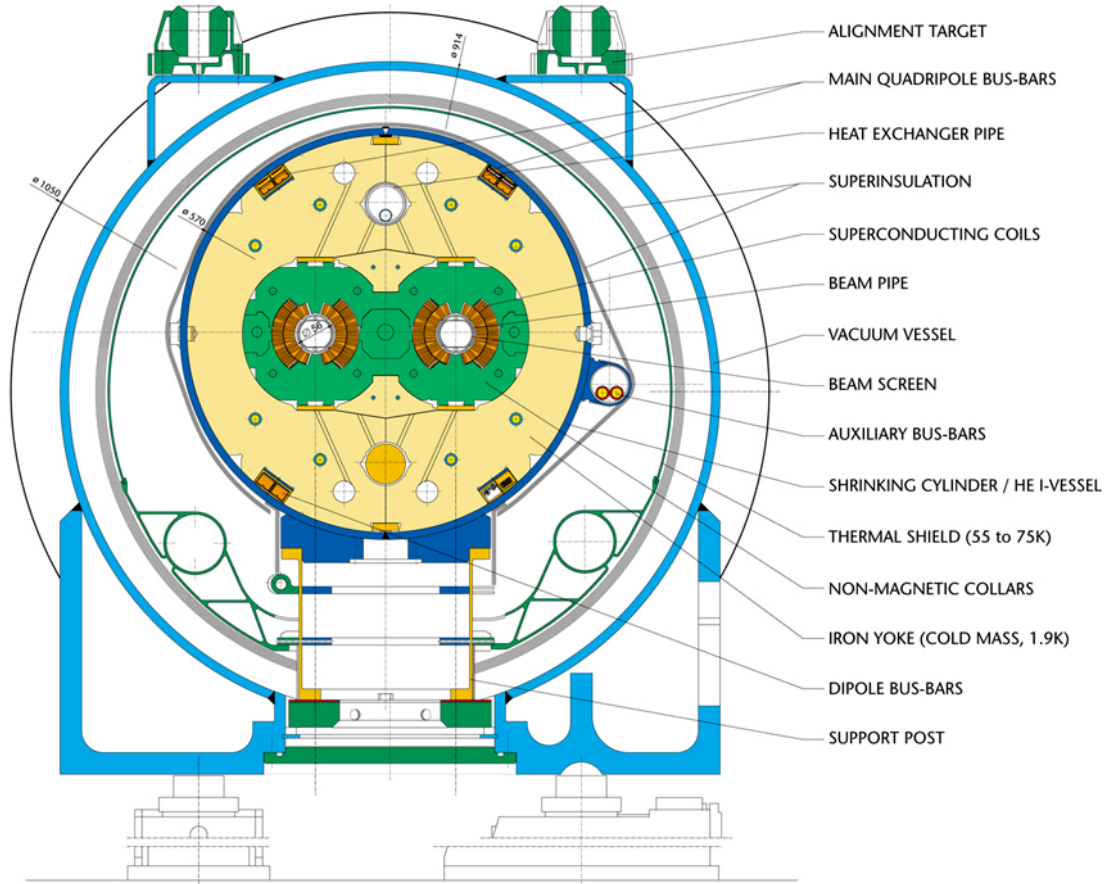


Figure 3.2: A cross-sectional view of a standard LHC dipole magnet and cryostat. Noted radial measurements are in millimeters[26].

beam sections. One of these collision points will be at the center of the experiment’s detector (the intended target), while the other 34 collision points—17 on either side of the central interaction point—are considered “parasitic” collisions. The experiments use triggering conditions to ensure that particles from these parasitic collisions do not interfere with the signal of particles created at the central collision[27].

Limitations in early LHC running

During the early data-taking runs of the LHC (with which this analysis is concerned), the machine was not yet operating at design specifications. The most striking deviation from the nominal operating values is the beam energy, which, due to defects discovered in some of the dipole magnet interconnect bus-bars, has been limited to a maximum of 3.5 TeV (a center-of-mass collision energy of $\sqrt{s} = 7$ TeV)[28]. The number of bunches stored and cycled in the LHC has been scaled back from 2808 to ~ 10 in order to limit possible damage to the magnets if, in the process of commissioning and tuning the machine, the beam is lost. Because of this, the instantaneous luminosity delivered by the machine to the experiments has been much lower than the peak design luminosity. If, in the process of colliding beams, the number of events of a certain type occur at a rate of R events per second, and that event has a cross-section of σ , then the instantaneous luminosity $\mathcal{L}$ of the colliding beams is defined by

$$R = \mathcal{L}\sigma. \tag{3.1}$$

Integrated luminosity is used to describe the total amount of data collected in an experiment. This is defined as

$$N = \int dt \mathcal{L}\sigma, \tag{3.2}$$

where N is the total number of events observed from the process described by the cross-section σ . The peak design luminosity of the LHC at the ATLAS and CMS collision points is $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$, but during early running instantaneous luminosities of $10^{28} - 10^{31} \text{ cm}^{-2} \text{ s}^{-1}$ are typical. Figure 3.3 shows the approximate total delivered luminosity at $\sqrt{s} = 7 \text{ TeV}$ as a function of time for the span of dates with which this analysis is concerned.

3.2 The Compact Muon Solenoid

The **Compact Muon Solenoid** (CMS) is a general-purpose particle detector attached to the LHC beam-pipe and centered at one of its four interaction points. The detector is approximately a cylinder 21.6 m in length and 7.3 m in radius[12, 13]. CMS is divided into four major detector components, as shown in Figure 3.4. Three of these detector components—the central tracker, the electromagnetic calorimeter, and the hadronic calorimeter—are contained within a 3.8 T, 3 m radius superconducting solenoid magnet, giving the detector as a whole its “compact” and “solenoid” names. The “muon” part of the name comes from the muon detection system on the outside of the magnet, interspersed within an iron magnetic flux return yoke. A brief description of these detector units and their functions in the identification and measurement of particles follows.

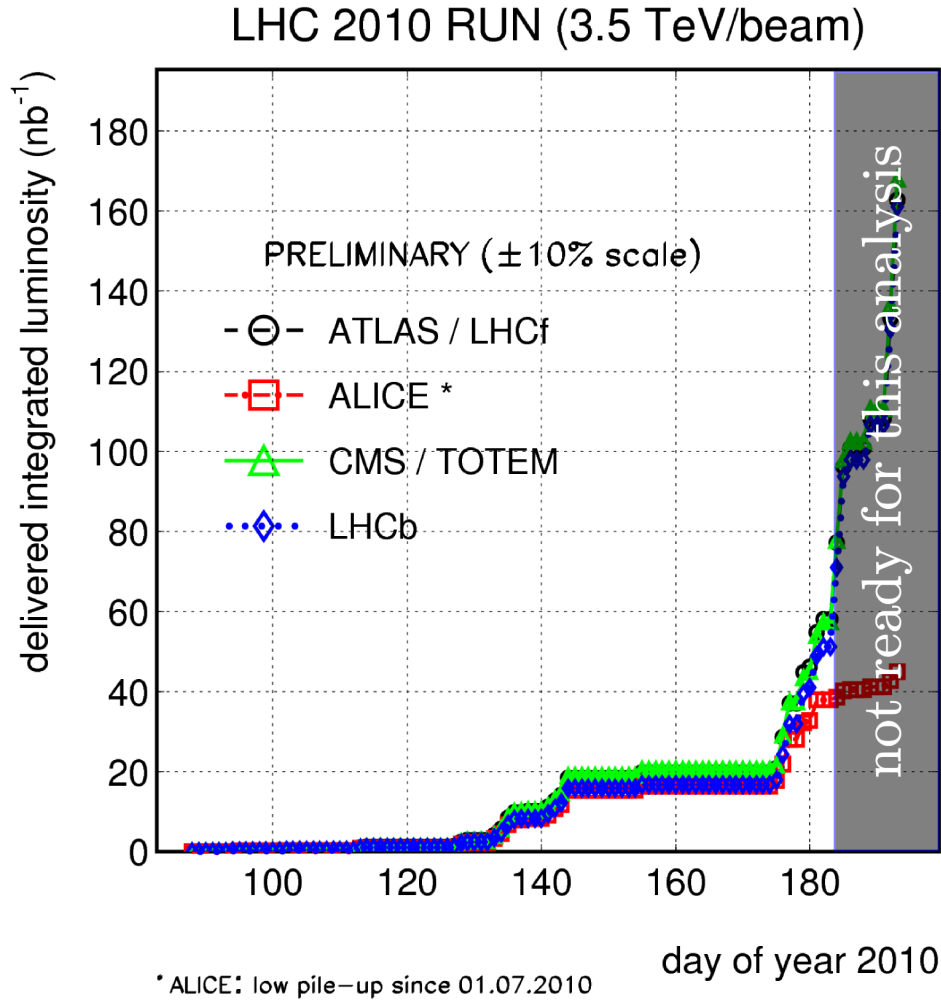


Figure 3.3: Approximate integrated luminosity as recorded by the various LHC experiments. The discrepancies between the delivered and recorded luminosities are due to data-taking inefficiencies of the individual experiments. An uncertainty of 10% is assumed on all luminosity measurements (not pictured)[29].

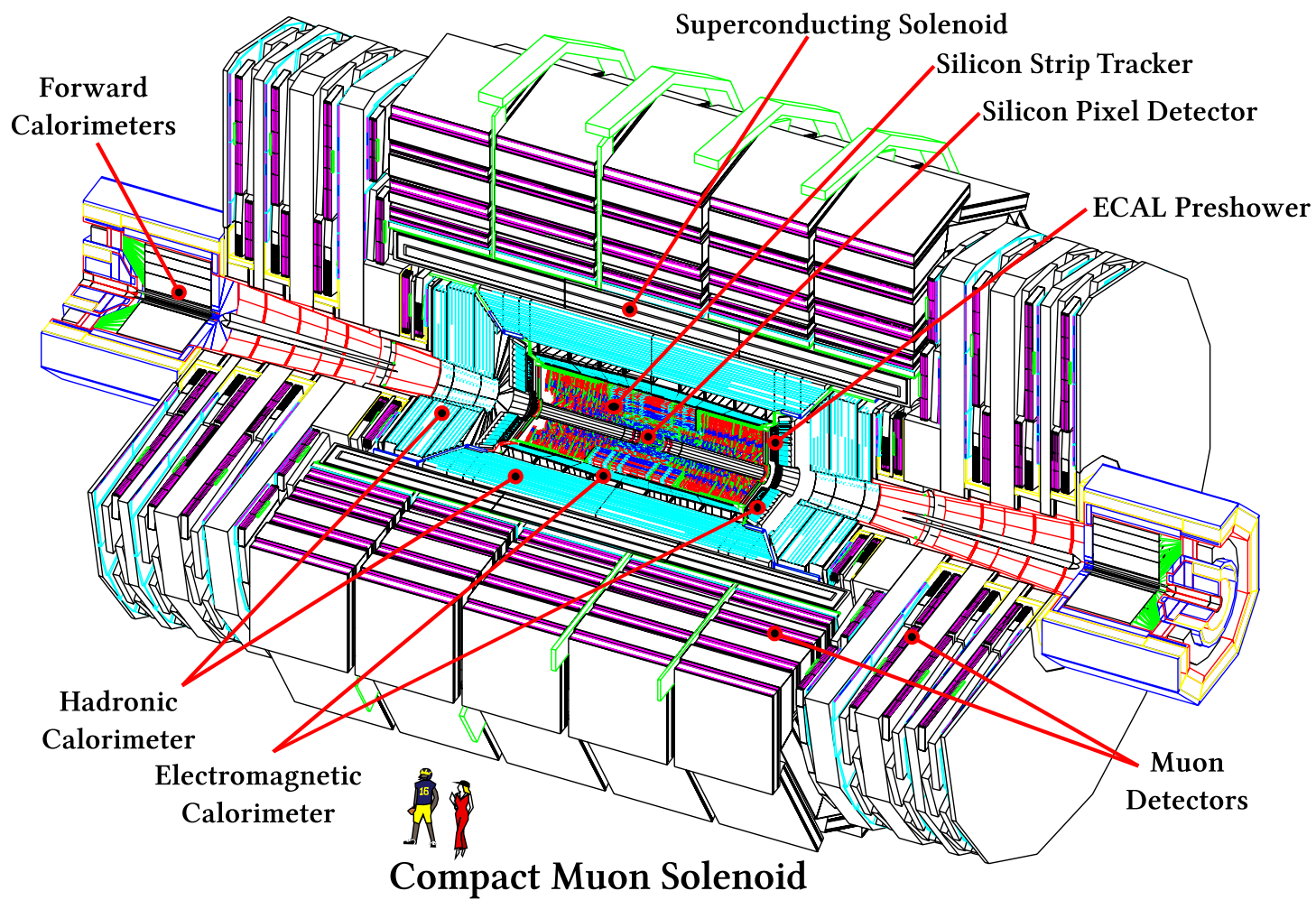


Figure 3.4: A cut-away view of the CMS detector with detector units labeled.

3.2.1 CMS Coordinate Systems

Before a description of the sub-detector components of CMS can take place, the coordinate convention of the experiment must be mentioned. Three different coordinate systems are used to describe positions and directions in the CMS detector. In all three coordinate systems, the origin of coordinates is taken to be the p-p collision point at the center of the detector. Figure 3.6 illustrates these three coordinate systems, and a detailed description of each follows.

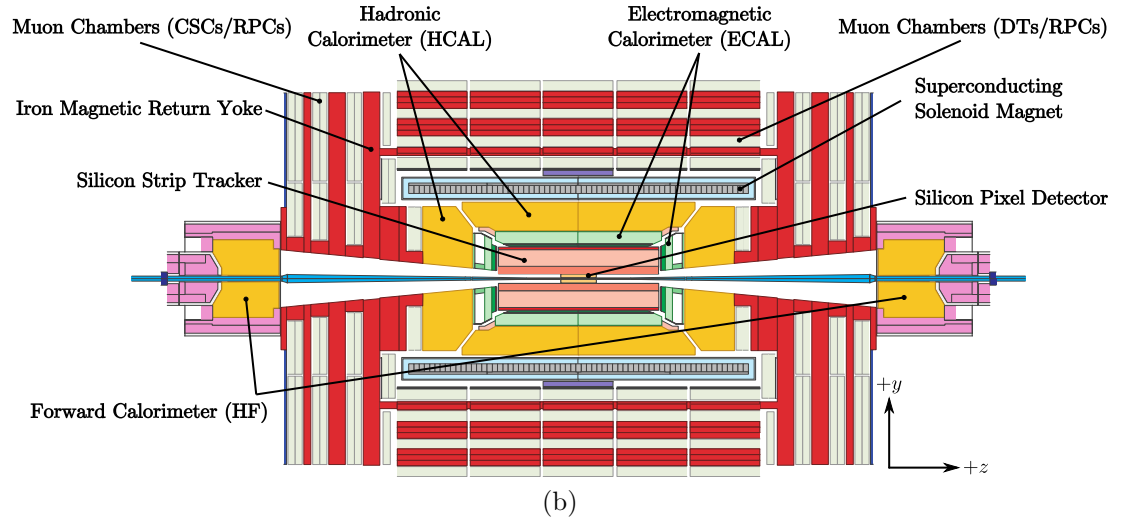
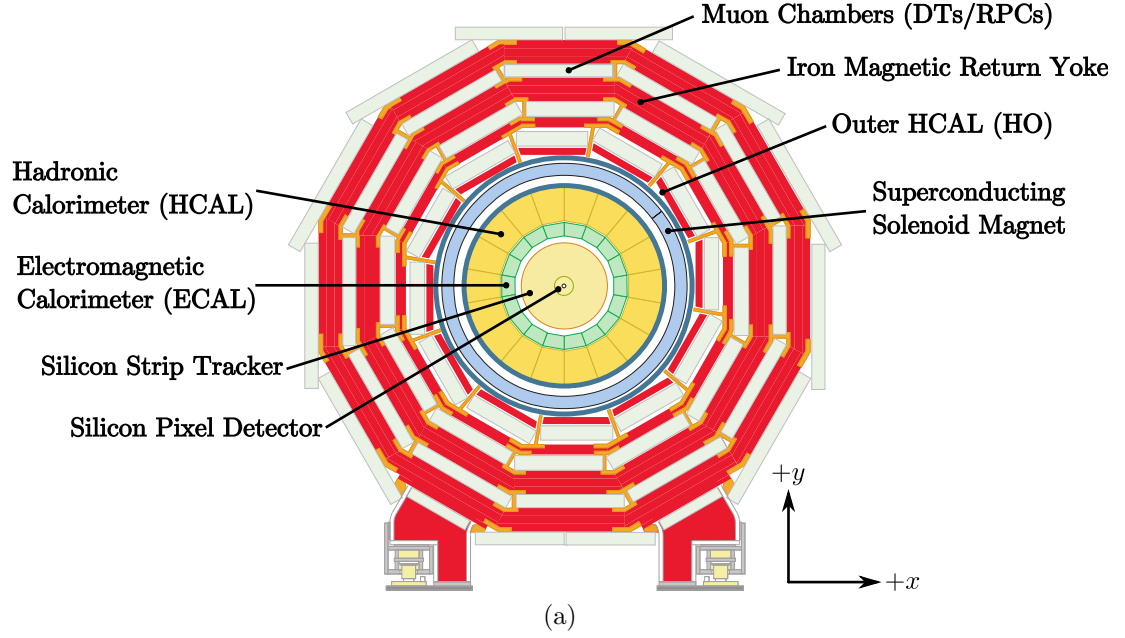


Figure 3.5: Projection drawings of the CMS detector in the planes (a) transverse and (b) parallel to the beam-pipe, with detector elements labeled.

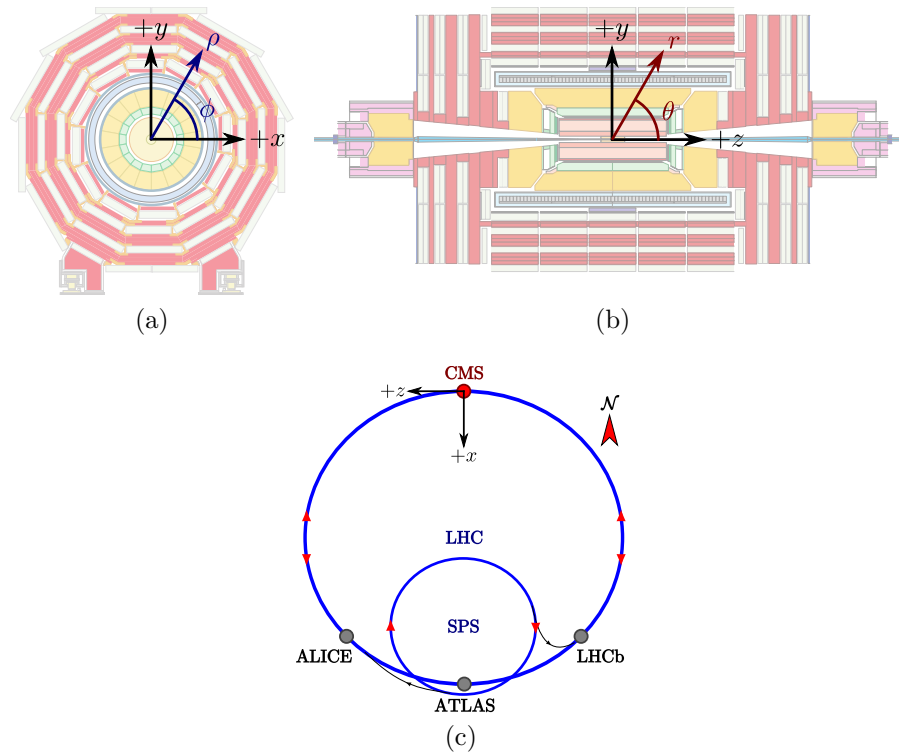


Figure 3.6: The three global coordinate systems used to describe CMS detector geometry. Figure (a) shows a cross-section of the CMS detector in the x - y plane, with the rectangular (in black) and cylindrical (in blue) coordinates labeled (the $+z$ axis is out of the page). Figure (b) shows a cross-section of the detector in the z - y plane with the rectangular (in black) and spherical (in red) coordinates labeled (the $+x$ axis is into the page). Figure (c) shows the orientation of the $+z$ and $+x$ directions relative to the geography of the LHC.

Rectangular Coordinates

In this system of coordinates, the z -axis is taken to be the LHC beam-pipe. When the LHC is viewed from above, the proton beam traveling with an anticlockwise rotation around the ring (“beam two”) is defined to be traveling in the positive- z direction through CMS. This means the positive- z axis points in a westerly direction. The positive- y axis points “up” on the surface of the Earth^b. To complete a proper right-handed coordinate system, the positive- x axis points toward the center of the LHC ring, or in a southerly direction^c.

Cylindrical Coordinates

The geometry of the CMS detector suggests the use of a cylindrical coordinate system is more useful than a rectangular one. In such a system, the z -axis is again taken to be the LHC beam-pipe, with the positive- z axis pointing in the westerly direction. The azimuthal angle is called ϕ and is defined such that $\phi = 0$ corresponds to the x -axis of the rectangular coordinate system, directed toward the center of the LHC ring. The cylindrical radial coordinate is designated ρ .

Spherical Coordinates

The kinematics of high-energy collisions make a spherical coordinate system a good choice to describe directions and positions of particle tracks produced in central collisions. The azimuthal angle is called ϕ , defined in the same way as in the cylindrical coordinate system. The polar angle is called θ and is defined such that $\theta = 0$ lies

^bThe $+y$ -direction is tilted $\sim 2^\circ$ with respect to true plumb in the CMS cavern, but because the effect of gravity on the particles in the detector is much smaller than the affect of the other fundamental forces, this discrepancy has no real consequences for physical measurements.

^cThe x - y plane of the CMS rectangular coordinate system is the same as the transverse plane used to defined transverse quantities in Section 2.2.2. The same can be said for the ρ - ϕ plane in the CMS cylindrical coordinate system.

along the beam-pipe in the positive- z axis of the rectangular and cylindrical systems. Polar coordinates are typically expressed in units of pseudo-rapidity, defined as

$$\eta \equiv -\ln \left(\tan \left(\frac{\theta}{2} \right) \right),$$

which approximates the rapidity angle of a particle relative to the beam axis,

$$y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right),$$

in the limit $E \gg m$. Pseudo-rapidity is a preferred choice for polar coordinates because the difference in pseudo-rapidity between two particles is approximately constant irrespective of Lorentz boosts along the axis of the beam-pipe. A pseudo-rapidity of zero represents a direction transverse to the beam-pipe, while $\eta = \pm\infty$ represents a direction along the $\pm z$ -axis. The spherical radial coordinate is designated r .

3.2.2 Central tracking sub-detectors

Closest to the interaction point at the heart of the detector is the central tracking detector unit. The tracker geometry is that of a 5.8 m-long by 1.3 m-radius cylinder [30, 31]. It is designed to measure with high resolution the momenta of charged particles as they propagate outward from the collision point while simultaneously minimizing the energy loss the particles experience from passing through matter. It does this by reconstructing charged particle tracks from points where the particles pass through various layers of thin silicon semiconductor material and measuring the bending of the tracks due to the 3.8 T solenoidal magnetic field. The central tracker is divided into a silicon pixel sub-detector at inner radii and a silicon strip sub-detector at outer radii. These two sub-detectors differ in their geometry, operation, and resolution, so they deserve separate mention.

Silicon pixel sub-detector

Closest to the interaction point, where particle flux from collisions is very high and high-precision position measurements are essential, lies the silicon pixel sub-detector[30, 32]. These silicon pixels are 270 μm - to 285 μm -thick, segmented n -on- n silicon sensors attached to integrated read-out chips by indium bump-bonds. The segments (pixels) are approximately $100\text{ }\mu\text{m} \times 150\text{ }\mu\text{m}$ in surface area and are arrayed in units of 53×80 pixels. These arrays are attached to modular detector units, and these units are mounted on a light carbon-fiber skeleton in the shape of three cylindrical barrels and four turbine-arranged disks.

The three layers of barrel pixels are mounted coaxially with the beam-pipe, centered at the interaction point, at radii $\rho = 4.4\text{ cm}$, 7.3 cm , and 10.2 cm . All three barrel cylinders are 53 cm in length and are comprised of modules containing eight to 16 arrays of 53×80 pixel units. The barrel section of the sub-detector contains in total 47 923 200 individual pixel channels and covers a range of $|\eta| < 1.6$.

The four disks are mounted as end-caps to the barrel cylinders, two on each side, at positions $z = \pm 34.5\text{ cm}$ and $\pm 46.5\text{ cm}$. The pixel end-cap disks extend the pixel coverage to $|\eta| < 2.5$. The disks are each divided into 24 trapezoidal blades, and each blade contains 1080 pixel arrays. Each blade is rotated 20° about the radial axis to enhance the Lorentz effect^d. The end-cap disks contain a total of 17 971 200 pixel channels, for a total of over 6×10^7 pixels in the entire sub-detector. At the luminosities of the early LHC running, this translates to an occupancy of approximately 10^{-4} hits per pixel per collision.

During operation, the silicon pixels are kept at a reverse-bias voltage of 100 V and 300 V for the barrel and pixel sub-detectors, respectively. As charged particles

^dThe Lorentz angle for drifting electrons in silicon in a 3.8 T magnetic field is approximately 24° [30]

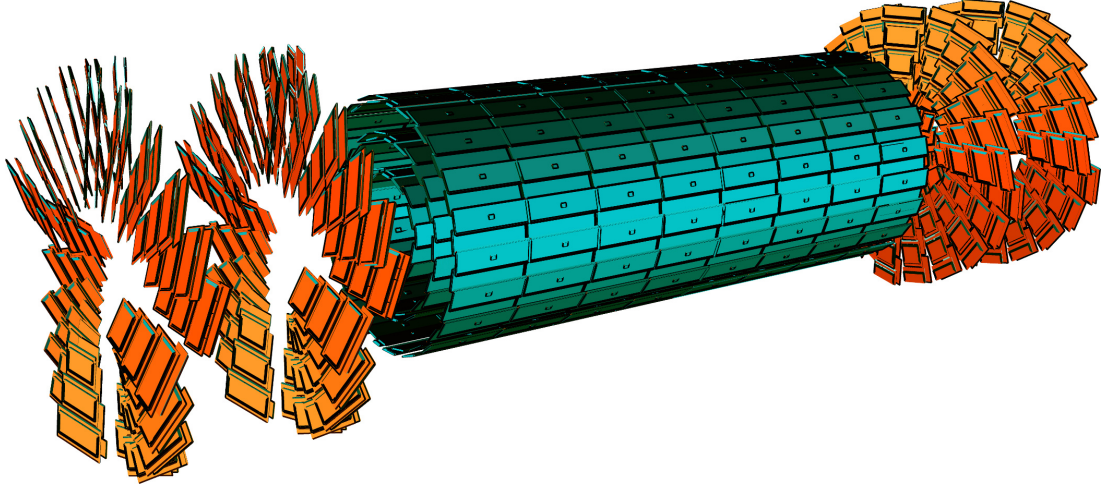


Figure 3.7: A perspective view of the CMS silicon pixel sub-detector geometry. In the middle are depicted the three layers of the barrel pixels, and on either end of the barrel are two end-cap disks[33].

pass through the pixels, they create small, detectable currents as the electrons in the silicon are excited into the conduction band of the semiconductor. These currents are amplified by the on-module electronics, and those above a programmable threshold value are stored for about $3.7\text{ }\mu\text{s}$, until either a trigger condition causes the data to be read out to the data acquisition (DAQ) system, or no trigger is received and the data are overwritten[34, 35]. Charge from one through-going charged particle is shared between several neighboring pixels and, because analog charge values are read from each pixel, a measurement of the charged particle’s track can be interpolated to an accuracy better than the $100\text{ }\mu\text{m} \times 150\text{ }\mu\text{m}$ partitioning of the pixels.

Only a small fraction of the pixel detector could not be used in the reconstruction of particle tracks due to hardware failure. In total, $\sim 1\%$ of the read-out boards in the barrel and $\sim 3\%$ of the read-out boards in the end-caps were considered unusable during the data-taking period with which this analysis is concerned[36]. Using the remaining “live” pixels, the hit reconstruction efficiency—the probability that a hit will be reconstructed in a pixel module given a charged track’s passage

through that module—is above 99 %, averaged over all modules. This efficiency is calculated by extrapolating (or interpolating) tracks from at least two sub-detector layers to another and looking for reconstructed hits in the region of the extrapolated track position[36, 37].

Using a technique of matching tracks between overlapping modules from data collected during cosmic-ray runs in October and November of 2008, the local position resolution for tracks is calculated to be 19 μm in the $\rho\text{-}\phi$ direction, and 31 μm in the z direction for tracks with momenta greater than 20 GeV/c[38]. This resolution estimate is in good agreement with Monte Carlo simulations. Recent unpublished studies using early LHC collision data show that the position resolutions are better than these earlier calculated values: $12.7 \pm 2.3 \mu\text{m}$ in the module’s x direction (corresponding to the $\rho\text{-}\phi$ direction in the barrel and the $r\text{-}\phi$ direction in the end-caps), and $24.1 \pm 0.5 \mu\text{m}$ in the module’s y direction (corresponding to the z direction in the barrel and the ρ direction in the end-caps)[36].

The pixel sub-detector is vitally important in the reconstruction of the primary vertex of the collision event. Due to the intrinsic size of each proton bunch, the collisions do not always occur at the exact center of the CMS detector. Where the hard-scattering collision occurs in the detector is known as the primary vertex of the event. In order to accurately reconstruct particle tracks and measure their momenta, one must know precisely whence the particle originated. This is especially important in the identification of b quark and τ lepton decays. Bottom quark hadrons and τ leptons have lifetimes sufficient to allow them to travel several millimeters away from their origin at the primary vertex before decaying[11]. Identification of these points of decay—so-called secondary vertices—which usually lie within the beam-pipe itself, requires high-resolution track measurements and good extrapolation to the center of the detector. Using a cosmic-ray track splitting and matching method detailed in

[38], the resolution on the point of closest approach to the primary vertex is found to be $23.3 \pm 0.3 \mu\text{m}$ in the plane transverse to the beam axis and $39.4 \pm 0.5 \mu\text{m}$ in the longitudinal plane^e.

Silicon strip sub-detector

On the outside of the silicon pixel sub-detector lies a second silicon-based tracking system. This system uses modules comprised of silicon strips instead of pixels[30, 31, 39]. The strip modules are arranged in a configuration similar to the pixel modules, but with ten coaxial cylinder layers in the barrel region and 24 disks of modules, 12 on each end of the barrel cylinders. The tracker is, in total, 5.5 m long and 2.4 m in diameter and contains 15 148 detector modules comprised of 9.3 million strips. The detector modules come in 15 different types, differing in various factors such as thickness, strip length, and strip pitch. The silicon strip detector is separated into four distinct sections based on the type of modules used in those sections: the four barrel layers of modules closest to the beam-pipe (two single, two stereo) are called the **Tracker Inner Barrel (TIB)**; the six outer-most barrel layers (four single, two stereo) make up the **Tracker Outer Barrel (TOB)**; the inner-most three endcap disks on either side of the TIB are called the **Tracker Inner Disks (TID)**, with three rings of modules per disk (two stereo rings, one single ring); and, finally, the outer nine end-cap disks on either side of the tracker, covering both the TIB and the TOB, are called the **Tracker End-Caps (TEC)**, with between seven and four rings of modules per disk (between one and three stereo rings and between three and four single rings).

Figure 3.8 shows a cross-section view of the pixel and strip tracker sub-detectors.

^eThe number is quoted for tracks with momenta greater than 4 GeV/c. For tracks with momenta greater than 20 GeV/c, the resolution is $18.3 \pm 0.4 \mu\text{m}$ in the plane transverse to the beam axis and $35.3 \pm 0.6 \mu\text{m}$ in the longitudinal plane.

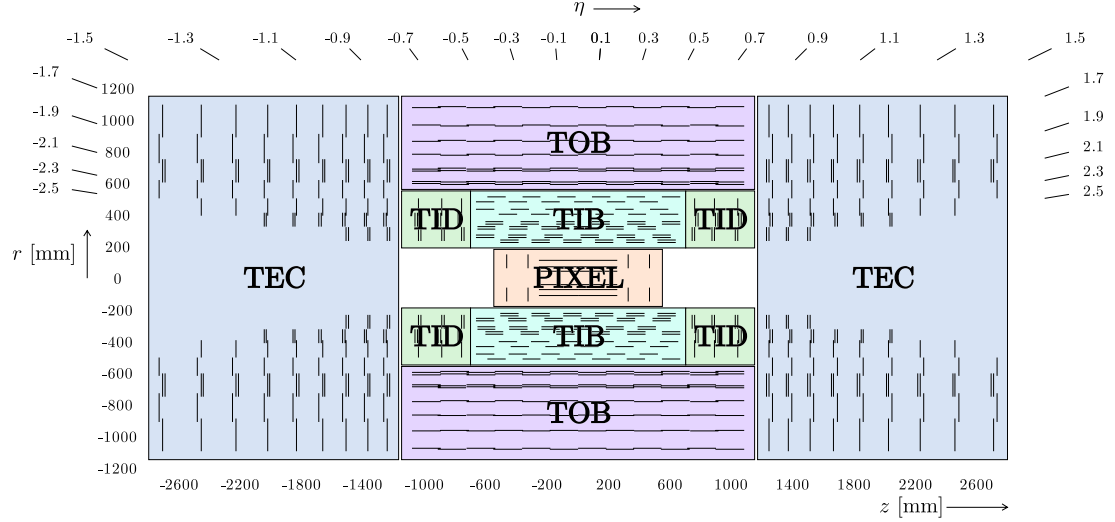


Figure 3.8: A cross-section of the inner tracker sub-detectors. Each line represents one tracker module. Double lines represent back-to-back modules with different strip pitches, used to produce stereo measurements of track positions[13].

Instead of being segmented into near-square pixels, the silicon of the strip detector is segmented into strips between 8.52 cm and 20.18 cm long with a pitch between 80 μm and 183 μm and a strip width roughly one quarter the pitch of the particular strip. Four of the barrel layers and three of the rings in the end-cap disks are double-sided silicon modules with strip pitches rotated 100 mrad relative to each other, used to produce stereo measurements of track positions. The tracking coverage of the silicon strip tracker extends to approximately $|\eta| < 2.5$.

The silicon strip detector modules work in a similar fashion to the pixel modules: a reverse-bias voltage below the breakdown voltage of the diode is applied to the strips, and any through-going charged particle will excite some electrons in the silicon into the conduction band of the semiconductor. On-module electronics read the analog ionization currents and store those values until a trigger condition instructs the modules to send their data to the DAq system.

During the data collection runs of the LHC with which this analysis is concerned, an average of 2.2% of the silicon strip tracker modules were inactive at any given time, but because of the many layers of the tracking system, the effect on the overall track reconstruction efficiency is negligible. The hit reconstruction efficiency due to causes other than known hardware problems has been calculated using an extrapolation method similar to the method used in the pixel sub-detector and found to be 99.8%, consistent with the 0.2% bad-channel production rate from the manufacturing process[36].

3.2.3 Calorimetry sub-detectors

Outside the central tracking sub-detectors lies the electromagnetic and hadronic calorimeters. These devices are designed to measure the energies lost by incident particles due to electromagnetic interactions with the detector material. They also serve to identify particles based on the depth of penetration into the calorimeters and the profile of the energy lost. CMS employs three distinct technologies to accomplish this—one calorimeter designed to measure the energies of electrons and photons, another designed to measure the energies of hadrons, and a third designed to detect the energies of particles in the very forward regions of the detector.

Electromagnetic calorimeter

The CMS electromagnetic calorimeter (ECAL) uses an array of lead tungstate crystals (PbWO_4) to detect particle energies[40, 13]. Lead tungstate is an ideal material for this purpose because of its stability in high-radiation environments, its relatively

fast scintillation response time, and its small radiation length^f and Molière radius^g[41]. When ionizing particles enter a crystal, it will ionize the atoms within the crystal structure, losing energy as it does so. As the crystal re-absorbs the ionized electrons, the atomic transitions result in a release of light that can be measured by attached photo-detectors. The incident particle is also likely to radiate photons in the material that photo-convert into light particle-antiparticle pairs. These particles photo-convert again, and the daughter particles from that process photo-convert again, and so on in a process known as electromagnetic showering. The total amount of light scintillated in this process is proportional to the amount of energy lost by the incident particle in the crystals. If the particle stops completely, then the total energy of the particle can be known.

The ECAL uses 75 848 crystals in a barrel/end-cap geometry similar to the central tracker's. The **ECAL Barrel** (EB) starts at a radius of 130 cm from the beam-pipe and extends to a pseudo-rapidity of $|\eta| < 1.48$. The EB crystals are approximately $2.2 \times 2.2 \text{ mm}^2$ in surface area with a length of approximately 23 cm each (corresponding to 25.8 radiation lengths). It uses 61 200 crystals arranged in 5×5 squares of crystals known as “trigger towers”. The **ECAL End-cap** detectors (EE) are two disks comprised of 7 324 crystals each. The end-cap extends the ECAL coverage to a pseudo-rapidity of approximately $|\eta| = 3.0$. The EE crystals are each approximately $2.68 \times 2.68 \text{ mm}^2$ in surface area with a length of approximately 22 cm (corresponding to 24.7 radiation lengths). Covering the end-caps is the **ECAL “Preshower”** detector (ES), extending in pseudo-rapidity from $1.65 < |\eta| < 2.61$. The preshower detector is

^fLight electromagnetically interacting particles lose energy in dense materials through *bremßstrahlung*. The amount of energy remaining in the particle falls exponentially as a function of distance traveled in a medium. The characteristic length of that exponential is defined as the radiation length.

^gAn electromagnetically interacting particle will lose energy in dense materials via chains of pair-produced particles (showering). The typical width of this shower in a given medium is called the Molière radius.

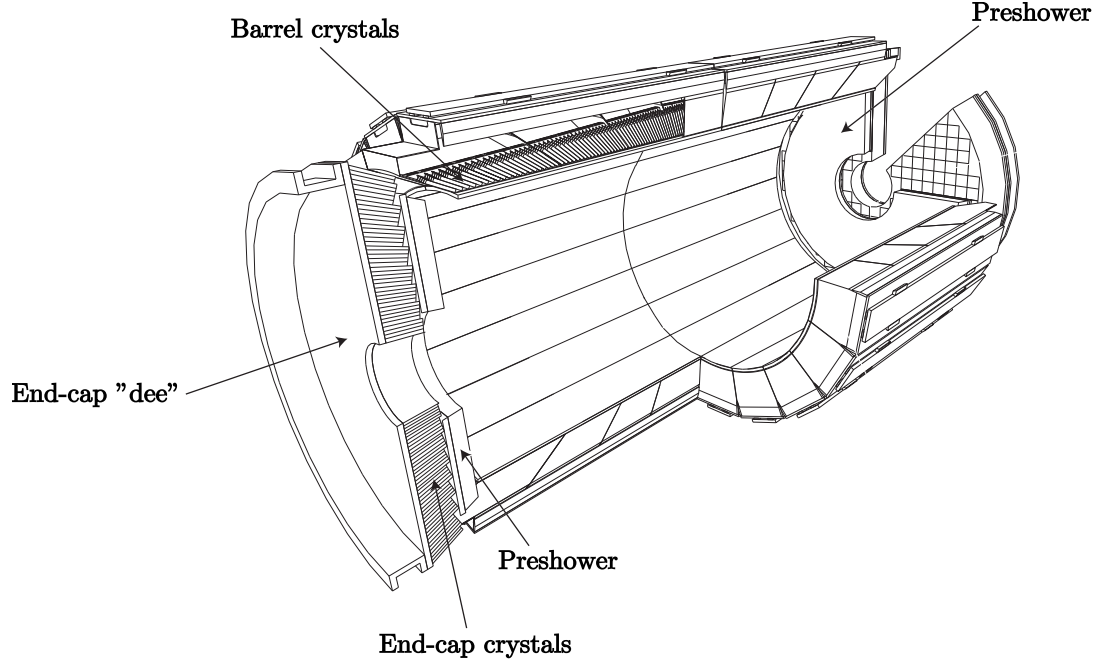


Figure 3.9: A perspective-view schematic of the CMS electromagnetic calorimeter with various components labeled.

composed of four layers of alternating lead and instrumented silicon strip detectors. The silicon strips in the two active detection planes are mounted orthogonally to each other to provide a two-dimensional reconstruction of the electromagnetic shower initiated in the lead plates. The total material added in front of the EE by the ES is approximately 3.0 radiation lengths, and approximately 95% of incident photons begin showering before reaching the second lead plate. The preshower detector's primary purpose is to measure the shower profile of incident particles in order to distinguish between photons and π^0 mesons. Figure 3.9 shows a perspective view of the ECAL geometry, and Figure 3.10 shows the material budget of the CMS detector in radiation lengths[42].

Avalanche photo-diodes are used in the EB for photo-detection, and vacuum photo-triodes are used in the EE. The purpose of these devices is to capture scin-

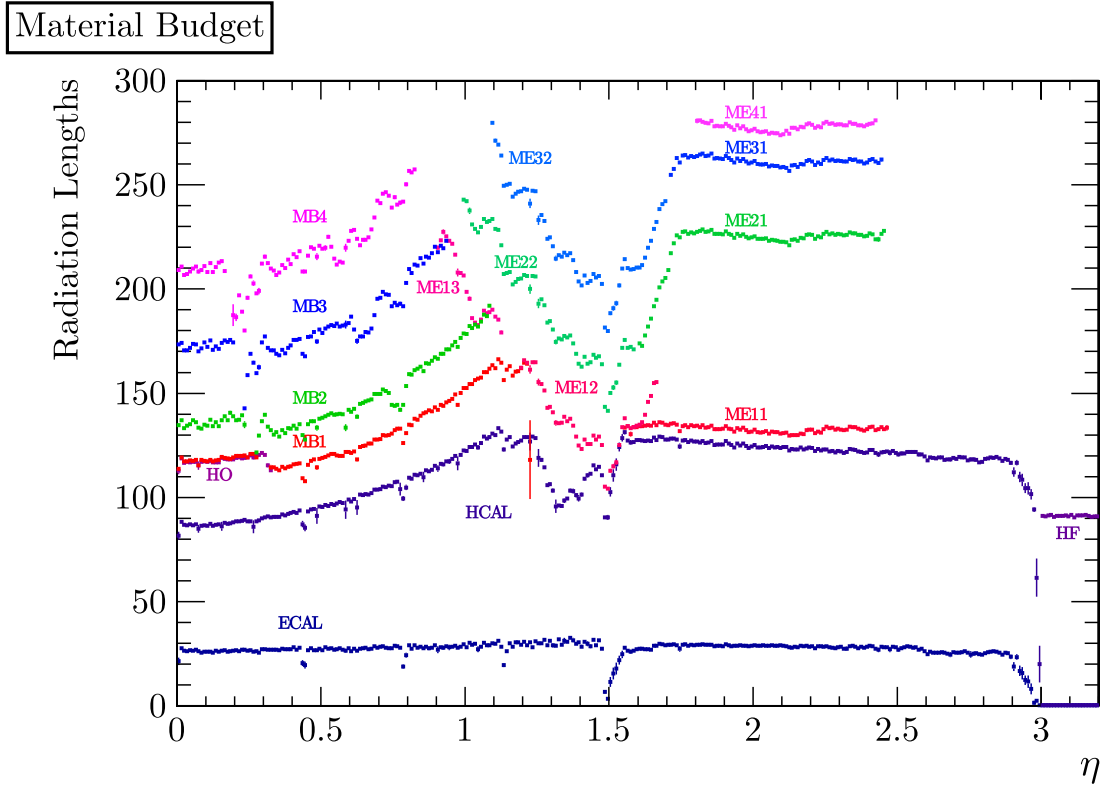


Figure 3.10: Material budget of the CMS detector (in electromagnetic radiation lengths) as a function of detector $|\eta|$ [42]. The plot is stacked by detector layer. Values are averaged over ϕ .

tillation photons and convert them into a detectable electronic signature. The characteristics of these devices and their responses are detailed elsewhere[13].

The energy resolution σ of the ECAL can be modeled with three terms:

$$\left(\frac{\sigma}{E}\right)^2 = \left(\frac{S}{\sqrt{E}}\right)^2 + \left(\frac{N}{E}\right)^2 + C^2,$$

where S is a stochastic term, N is a noise term, and C is a constant term[40]. The stochastic term is due to fluctuations in lateral shower size, the statistics of the photo-detectors, and the fluctuations in the amount of energy deposited in the preshower detector with respect to the measured energy (where applicable). The noise term has contributions from electronics noise, digitization noise, and pile-up^h noise. The constant term includes non-uniformity in the light collection of the individual crystals, improper calibration of the signal from the photo-detectors, and light leakage from the back of the crystals. Using an electron at the CERN accelerator facility, these terms were found to have typical values of $S \approx 0.028 \text{ GeV}^{1/2}$, $N \approx 0.12 \text{ GeV}$, and $C \approx 0.30 \%$, with E measured in GeV[43]. Figure 3.11 shows the resolution measurement for a typical 3×3 array of ECAL crystals in an electron test-beam of variable energy.

Hadronic calorimeter

Between the electromagnetic calorimeter and the CMS solenoid magnet (between $\rho = 1.77 \text{ m}$ and $\rho = 2.95 \text{ m}$) in the barrel, and outside the ECAL end-cap, is a second calorimeter system called the **hadronic calorimeter** (HCAL)[44]. It is designed to measure the energies of hadrons and other particles that escape the back-end of the ECAL crystals. The barrel and end-cap regions of the HCAL use towers of alternating

^h“Pile-up” is the term used to describe the interference of sequential collision events in the detector. Although the collisions themselves are independent events, sequential collisions occur in 25 ns spacings, and most of the active detector components have recovery times greater than this. If particles from sequential events enter the same detector component, there is a chance that the detector will produce faulty readings for the later particles.

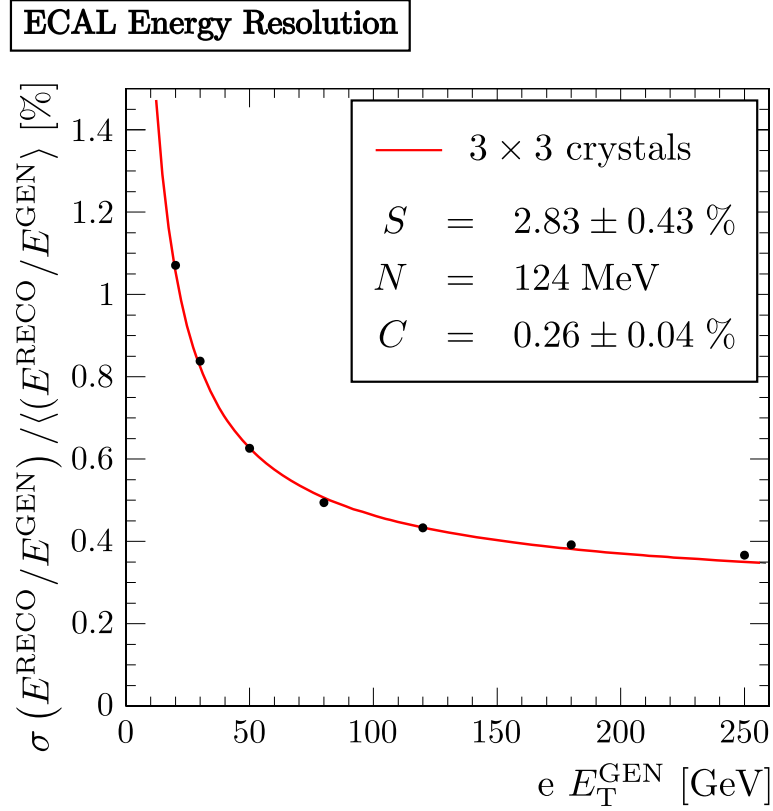


Figure 3.11: Typical energy resolution for a 3×3 cluster of ECAL crystals in an electron test beam[43]. Shown are the results of fitting the curve to the resolution function given in the text.

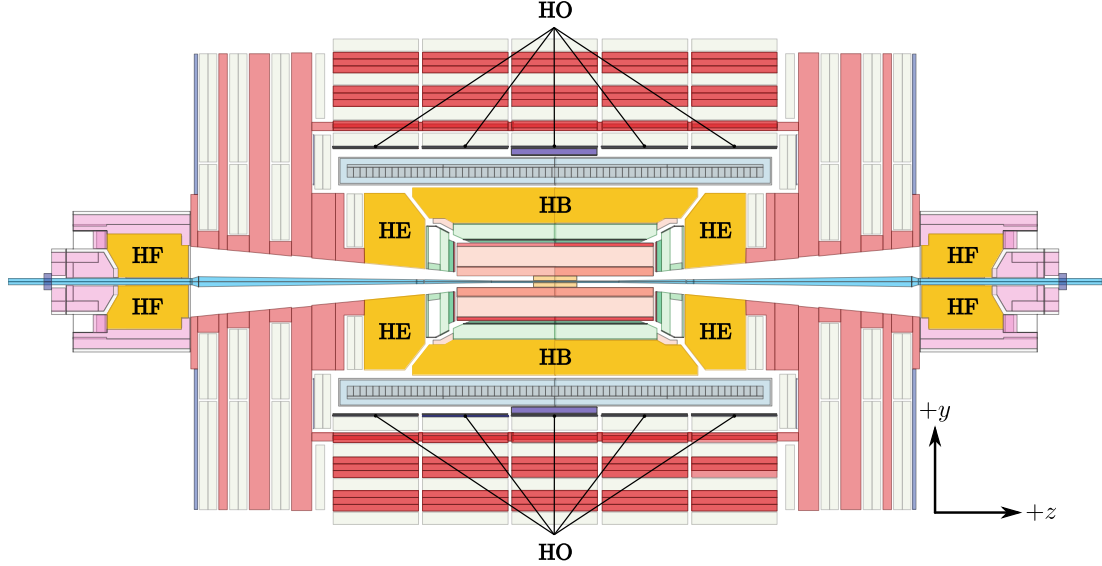


Figure 3.12: A view of the CMS detector in y - z projection with the components of the hadronic calorimeter labeled.

brass absorber plates and scintillator tiles to detect showers of ionizing particles. The HCAL detector has three distinct components—the **HCAL Barrel** detector (HB), the **HCAL End-cap** detector (HE), and the **HCAL Forward** detector (HF). Figure 3.12 shows the location of the CMS HCAL.

The HCAL barrel detector is a cylinder of brass/scintillator towers encompassing the entire ECAL detector. It extends in pseudo-rapidity to approximately $|\eta| = 1.3$. The barrel uses 70 000 scintillator tiles in total, read out using wavelength-shifting fibers connected to multi-pixel hybrid photo-detector devices. The towers of the calorimeter are arranged in 36 azimuthal wedges and 32 η sectors, for a total of 1 152 towers in uniform slices of $(\Delta\phi, \Delta\eta) = (0.087, 0.087)$. Each of these towers is aligned with a 5×5 group of ECAL crystals to form a complete trigger tower, used for fast identification and triggering of interesting physics objects (see Section 3.3.1). The

total thickness of the HB is between 5.82 hadronic interaction lengthsⁱ at $\eta = 0$ to 10.6 interaction lengths at $|\eta| = 1.3$. The ECAL adds an extra 1.1 interaction lengths to the front of the HB[44, 13].

Because of the very small number of interaction lengths in the low- η region, a layer of scintillation material is attached to the inner-most surface of the iron magnetic field yoke on the outside of the solenoid magnet. This portion of the barrel detector, called the **HCAL Outer calorimeter** (HO), uses the magnet material as an absorber to “catch” the shower particles that punch through the barrel. The geometry of the HO roughly follows that of the barrel, with a granularity of $(\Delta\phi, \Delta\eta) = (0.087, 0.087)$. Some constraints in the geometry are due to the requirements of the barrel muon system, also affixed to the magnetic return yoke, and the cryogenic chimneys, which supply cooling to the CMS magnet, tracker, and calorimeters. The scintillator layers of the HO lie at a radial distance of approximately $\rho = 4.07$ m, except for on the central barrel ring of the iron yoke (the so-called ring YB0, extending to ± 1.268 m along the z -axis), where a second scintillator tile is mounted at $\rho = 3.82$ m, with an iron plate mounted between the two layers of scintillator. In total, this extends the minimum thickness of the HCAL in the barrel to 11.8 interaction lengths[13, 45].

The HCAL end-cap detectors are plug-shaped devices that partially insert into the HB cylinder. They are similar in design to the HB—towers of layered brass and scintillator tiles. The end-caps cover a pseudo-rapidity range of $1.3 < |\eta| < 3.0$. Each end-cap uses 10 458 tiles in 576 towers, segmented into 16 η rings with 36 azimuthal segments per ring, and each tower is lined up with a group of 5×5 ECAL crystals to form a trigger tower. The total thickness of the HB, including the ECAL attached to the front of it, is approximately 10 interaction lengths[13].

ⁱInteraction lengths are defined similarly to electromagnetic radiation lengths, only concerning the strong nuclear interactions between hadrons instead of the electromagnetic interactions between charged particles. See the footnote on page 42.

The forward HCAL detector is mounted on the outside of the iron magnetic return yokes at a distance of ± 11.2 m along the z -axis, and its extent in pseudo-rapidity is roughly $3 < |\eta| < 5$. Being very close to the beam-pipe, each HF detector is exposed to roughly 380 GeV of energy per p-p collision due to low-angle scattering, compared to only 100 GeV of energy deposited in the rest of the detector. In this hostile and radioactive environment, a different type of detection system was chosen. The forward calorimeter uses over 1 000 km of quartz fiber as a passive detector, embedded in a steel absorber. The fibers channel the Čerenkov light from through-going charged particles to standard photomultiplier tubes, where the photons are converted into electrical pulses. The detector is segmented into 36 azimuthal wedges, with the fibers bundled into towers of $(\Delta\phi, \Delta\eta) = (0.175, 0.175)$. The total thickness of the HF is approximately 10 interaction lengths[13, 46]. Figure 3.13 shows the total number of hadronic interaction lengths for the CMS detector as a function of detector $|\eta|$ [42].

The energy resolution of the HCAL system is modeled as

$$\frac{\sigma}{E} = \frac{a}{\sqrt{S}} \oplus b, \quad (3.3)$$

where E is the energy of the incident particle, a corresponds to a stochastic term and b to a constant term, and the operator $\oplus$ represents addition in quadrature. Using test beams of electrons, pions, protons, and muons, characteristic values of these terms were measured to be $a = 0.847 \pm 0.016$ GeV^{1/2} and $b = 0.074 \pm 0.008$ for the HB and HE, and $a = 1.98$ GeV^{1/2} and $b = 0.09$ for the HF[45, 46, 47]. Figure 3.14 shows the simulated resolution of the HCAL system in response to hadronic jets of various energies.

Material Budget

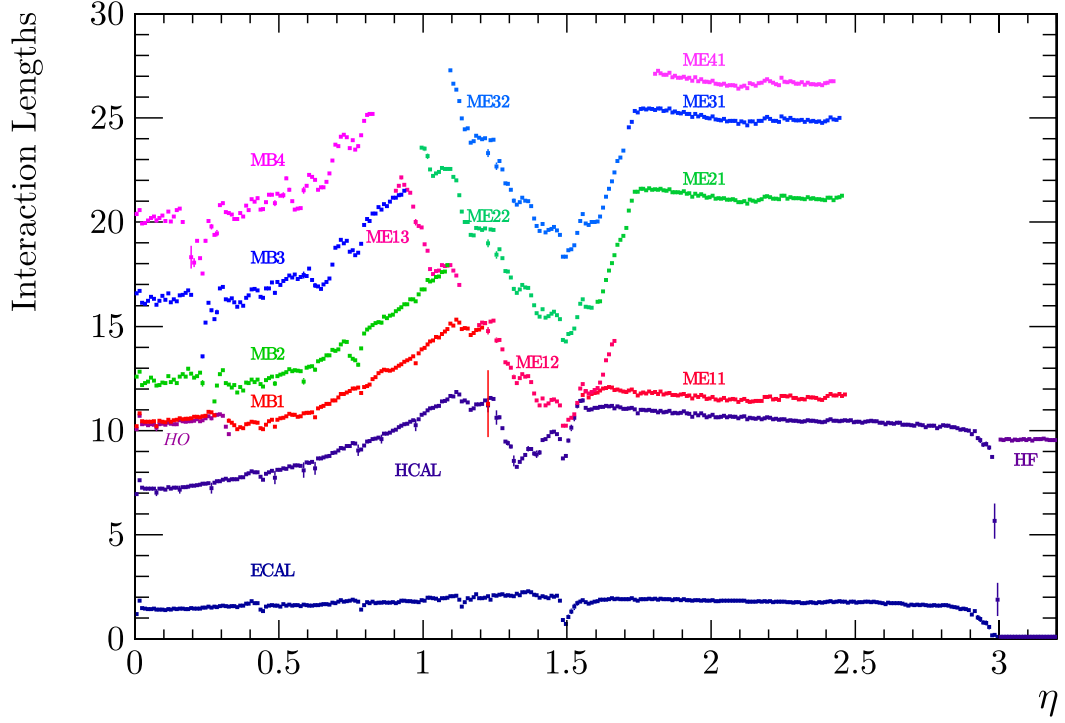


Figure 3.13: Material budget of the CMS detector (in hadronic interaction lengths) as a function of detector $|\eta|$ [42]. The plot is stacked by detector layer. Values are averaged over ϕ .

HCAL Jet Energy Resolution

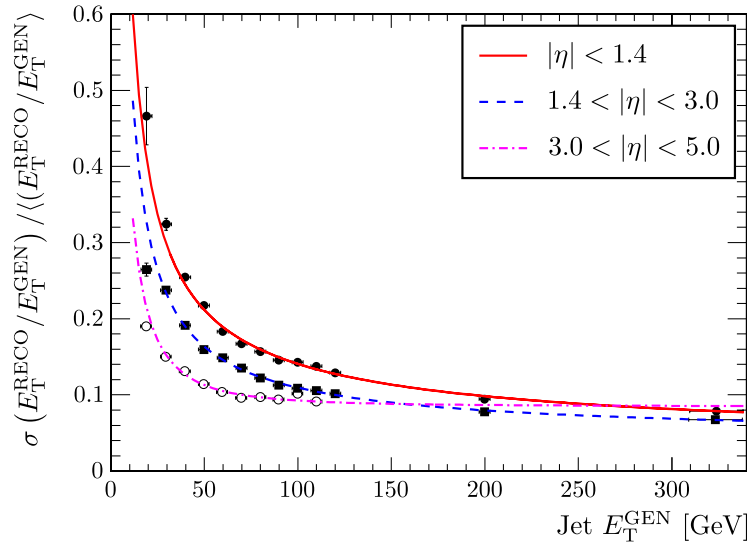


Figure 3.14: Typical energy resolution for the HCAL system in response to simulated hadronic jets[45, 46, 47].

Forward calorimeters

In regions of pseudo-rapidity greater than those covered by the HF ($|\eta| > 5.0$), CMS uses a series of calorimeters to detect and measure the very low-angle p-p elastic scattering cross-section. The **C**entauro **A**nd **S**Trange **O**bject **R**esearch detector (CASTOR) is a quartz-fiber sampling Čerenkov detector with a tungsten absorbing material that is located ± 14.38 m along the z -axis from the interaction point and subtends a pseudo-rapidity of $5.2 < |\eta| < 6.6$ [48]. The **Z**ero **D**egree **C**alorimeter (ZDC) is also a tungsten-quartz Čerenkov calorimeter, but is located at distances of ± 140 m on either side of the interaction point at pseudo-rapidities $|\eta| > 8.3$ [49]. The ZDC is designed to detect neutral particles from the p-p interactions. As these calorimeters are used primarily to detect the products from elastic collisions, low transverse-energy inelastic collisions, and heavy ion collisions, they do not concern this analysis.

Very near to the CMS detector, located between the HF and the CASTOR calorimeters, is the **T**OTAL **E**lastic and diffractive cross section **M**easurement experiment (TOTEM)[50]. This detector belongs to an independent experiment and is not used in the reconstruction of collision events in the CMS detector.

3.2.4 Muon sub-detectors

On the outside of the CMS detector lies a second set of tracking sub-systems for detecting and measuring the momenta of muons[51]. Muons are unique among the products of high-energy collisions in that they have a long lifetime (~ 7 μ s in the rest-frame of the muon, greatly extended in the lab frame due to relativistic time-dilation) and a large rest-mass (Table 2.2) and are not strongly interacting. The long lifetime enables the muons to survive to the outer layers of the detector and beyond. The large rest-mass means the muons will not be greatly affected by the *bremßstrahlung* radiation that is the largest source of energy loss for high-energy electrons in the detector

material^j. And because the muons do not experience the strong nuclear force, they do not lose energy due to inelastic nuclear collisions in the dense calorimeter material as hadrons do. Thus muons reach the outside of the solenoid magnet with relatively little energy lost.

The CMS detector is surrounded by a framework of solid iron to guide the intense return-field produced by the CMS solenoid magnet. The CMS muon sub-detectors are interspersed throughout this iron yoke in several layers, in both the barrel and end-cap regions. As the muons pass through the yoke, their paths are bent in the focused magnetic field proportional to their momenta, so their tracks can be further sampled in this region to increase momentum measurement resolution. The muon system uses three distinct technologies to perform these tracking measurements: gas drift-tube chambers in the barrel region, cathode strip chambers in the end-cap region, and resistive plate chambers in both the barrel and end-caps to supplement the other two technologies.

Barrel drift-tubes

In the outside central region of the CMS barrel ($|\eta| < 1.2$), **D**rift-**T**ube (DT) chambers are used to measure the positions of muon tracks. The active detector material is interspersed in gaps between solid iron plates that act as a return yoke for the solenoidal magnetic field. There are 250 DT chambers arranged in four concentric cylinders, coaxial with the beam-pipe and centered at the interaction point. Each cylinder is divided into five “wheels” along the z -axis, and the wheels are segmented into 12 straight segments so that their shape is an extruded dodecagon, as pictured

^jRelativistic muons are sometimes called “minimum-ionizing particles”, or “MIPs” because the amount of energy lost by the muon per unit distance traveled in a medium is at a minimum due to the falling nuclear ionization rate and the slowly-increasing *bremßstrahlung* rate. The energy loss rate of a muon increases only logarithmically between ~ 1 GeV/ c and ~ 100 GeV/ c , so that all muons in this momentum range are effectively minimum-ionizing.

in Figure 3.15. Each of the first three cylinders of DT chambers has 12 layers of drift-tube cells—eight of which measure the ρ - ϕ coordinate of the track, and four of which measure the z -coordinate[51]. The measurement of the track in the ρ - ϕ direction is redundant because this is the direction of charged particle bending in the magnetic field. The outermost cylinder has only eight layers of tubes, and only a ρ - ϕ coordinate is measured in these.

The tubes themselves are approximately 2.4 m in length each with a cross-section of $13 \times 42 \text{ mm}^2$ and are filled with a gas mixture of 85% argon and 15% CO_2 . A single cathode wire runs through the central axis of each tube, and it is kept at a potential of +3 600 V with respect to large plates on the tops and bottoms of the cells. The short walls of the tubes are plated with cathode strips at -1 200 V, and electrodes held at +1 800 V partially cover the long walls of the tubes (Figure 3.16). As a charged particle passes through the gas in the tube, it ionizes some of the gas, and the freed electrons “drift” toward the central anode. The electrons collected on the anode produce a current that is amplified and digitized by in-tube electronics and read out into off-chamber triggering and data acquisition electronics.

The maximum transverse measurement of each cell is 21 mm so that the maximum electron drift time is 380 ns. This is much longer than the 25 ns time between collisions at the full LHC luminosity, but the estimated muon flux in the barrel region will be small enough to make it unlikely that this long drift time will interfere with sequential events. The signal pulse from a single muon can be localized in time to a few nanoseconds, meaning the signal can be uniquely identified to a single LHC collision, and the DT system can be used as a stand-alone trigger for events containing muons.

During the early data run used for this analysis, an average of 99 % of the drift-tube sub-detector was operational. The efficiency to reconstruct the trajectory of a

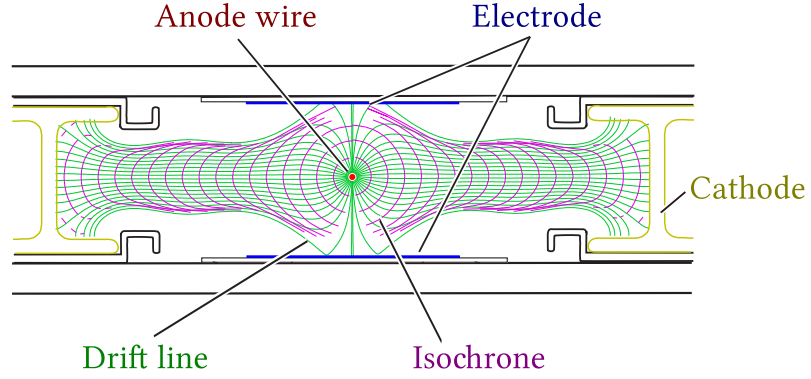


Figure 3.16: A cross-sectional view of a single drift-tube used in the barrel section of the muon sub-detector. Also pictured are electron drift lines and drift isochrones[51].

through-going muon in the barrel drift-tubes alone is expected to be nearly 100 % (calculated using **Monte Carlo** (MC) events reconstructed in a detector simulation[51]). This number was verified in cosmic ray data[52]. The resolution for measuring the position where a muon track passed through a drift-tube chamber is 260 μm in the ρ - ϕ direction, for a bending-angle resolution of 1.8 mrad, as measured by extrapolation of cosmic ray muon tracks from the inner tracking sub-detectors through to the drift-tube chambers[52]. This leads to a stand-alone DT muon momentum resolution of $\sim 10\%$ for muons with transverse momenta 10–1000 GeV/c in the barrel region (Figure 3.17a).

End-cap cathode strip chambers

Attached to three iron disks that constitute the magnetic return yoke in the detector end-caps are 473 multiwire proportional chambers known as **Cathode Strip Chambers** (CSCs)[51, 53]. These trapezoidal chambers are mounted in eight concentric rings per end-cap; the geometry and names of these rings are specified in Figure 3.18. There are 36 chambers in each of the rings $\text{ME}\pm 1/1$, $\text{ME}\pm 1/3$, $\text{ME}\pm 1/3$, $\text{ME}\pm 2/2$, and $\text{ME}\pm 3/2$,

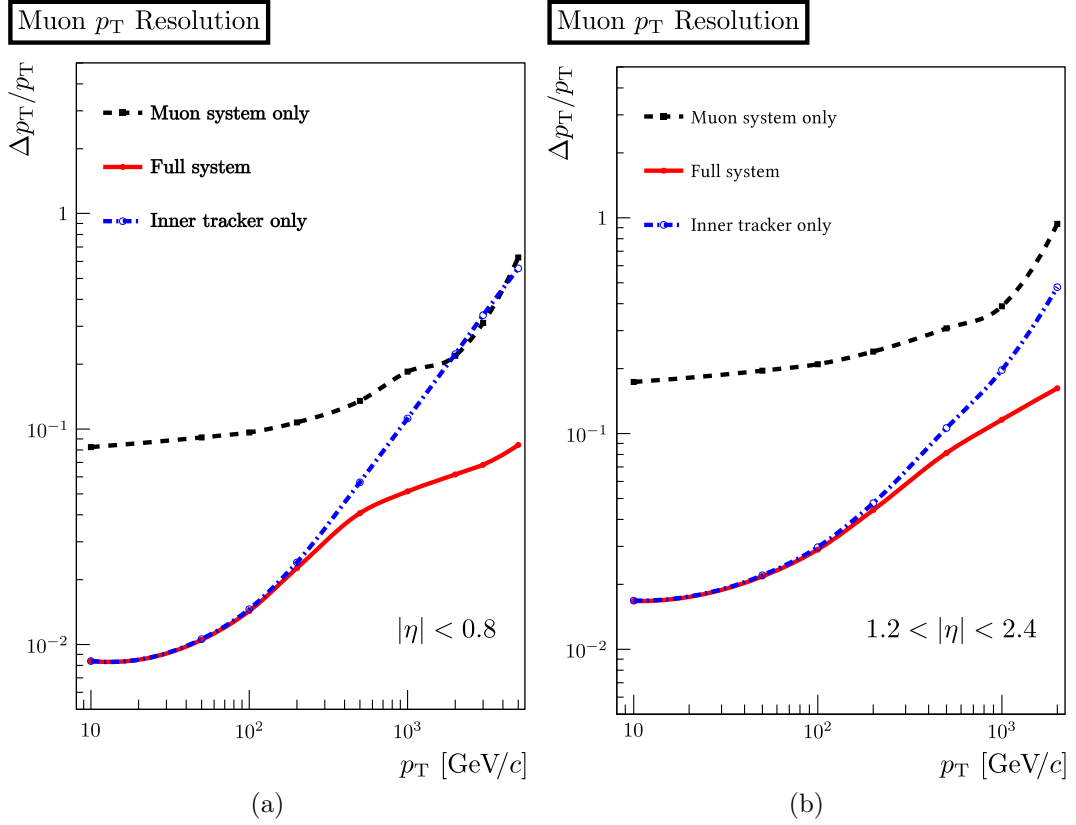


Figure 3.17: Simulated muon momentum measurement resolution for the (a) barrel and (b) end-cap regions of the combined central tracking and muon detectors as a function of muon energy. Resolution for low-momentum muons is dominated by multiple-scattering effects in the iron magnetic return yoke. Resolution for high-momentum muons is dominated by the decreasing sagitta of the curved track[51].

and 18 chambers in the each of the rings $\text{ME}\pm 2/1$, $\text{ME}\pm 3/1$, and $\text{ME}\pm 4/1$. The extra five chambers are mounted on the $\text{ME}+4/2$ ring^k. Each chamber (except for those in the $\text{ME}\pm 1/3$ rings) overlaps slightly with its neighbor to provide contiguous coverage in ϕ , and each successive station of chambers is positioned so that there is nearly complete coverage of active muon detection on both end-caps from $0.9 < |\eta| < 2.4$.

^kThe $\text{ME}\pm 4/2$ rings will be installed for higher LHC luminosity running. In the early LHC, the five chambers attached to the $\text{ME}+4/1$ ring are a fully operational test for the future upgrade, but are largely superfluous in the reconstruction of most muon tracks.

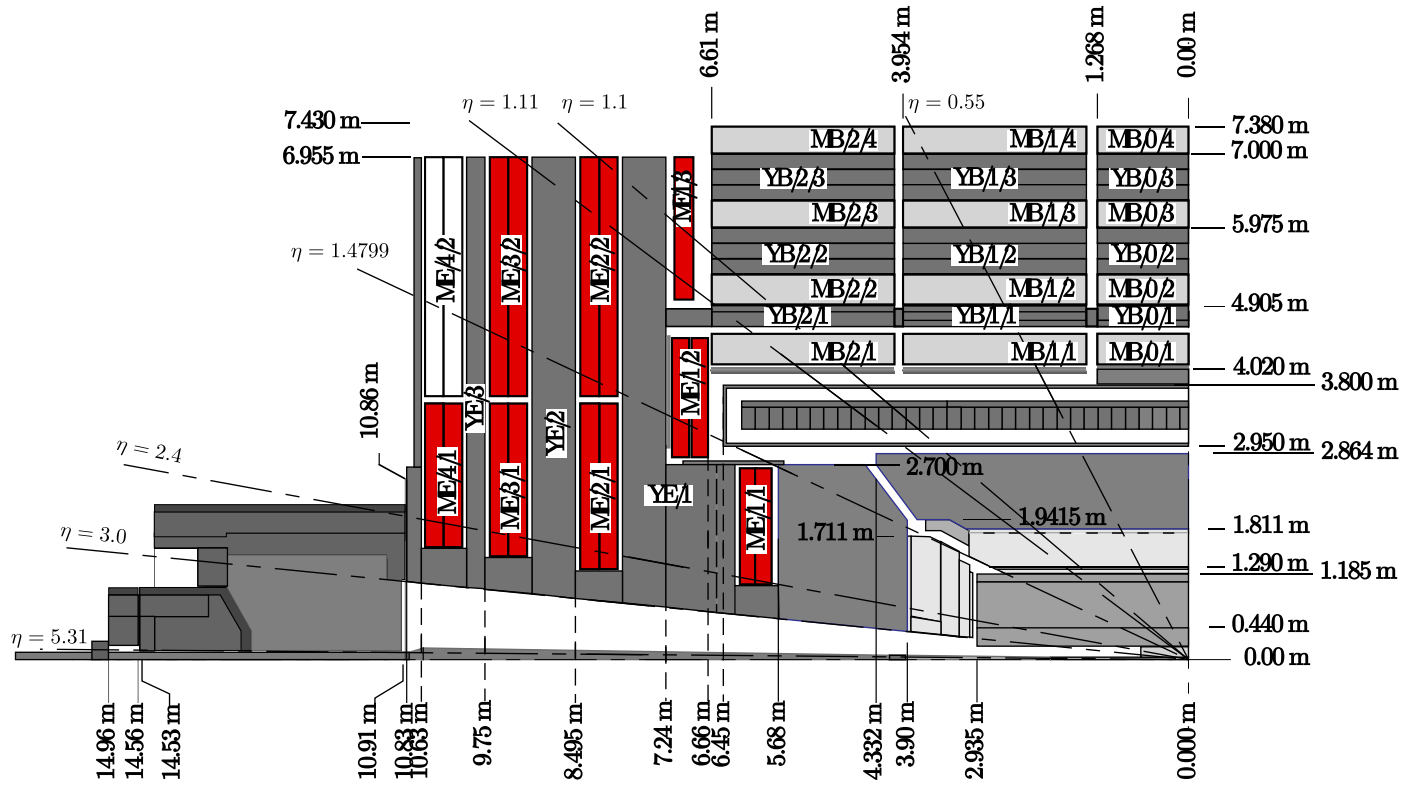


Figure 3.18: A cross-sectional view of one quarter of the CMS detector in the y - z plane showing the geometry and names of the end-cap cathode strip chambers. Only five of the ME+4/2 chambers and none of the ME-4/2 chambers have been installed[51].

Inside each CSC are twelve alternating planes of metal strips that run the length of the chamber (the global ρ -direction) and metal wires that run perpendicular to these strips (in the global ϕ -direction)¹. The chambers come in different sizes, the largest of which are roughly $3.4 \times 1.5 \text{ m}^2$ in surface area and have an internal volume of 200 L. The chamber is filled with a mixture of 40% argon, 50% CO_2 , and 10% CF_4 gas. As a muon passes through the chamber, it ionizes the gas. The wires are kept at a potential of +3 000 to +3 600 V with respect to the strips, so the freed electrons are strongly attracted to the anode wires. As the electrons concentrate at the very thin wires (50 μm in diameter each), they induce a current in the cathode strips, and this current is read out using on-chamber signal amplifiers and shapers. Each pair of wire and strip planes measures the ρ and ϕ coordinate of the through-going track (respectively), so each chamber samples the muon track six times.

The ρ - ϕ coordinate of the muon track is measured on the cathode strips by fitting the shape of the induced charge profile to the well-known Gatti distribution[54]. This allows for a position measurement resolution much better than the 8.4–16 mm width of each strip. No such interpolation is performed on the anode wires—in fact, the wires are ganged together in groups of five to sixteen for read-out purposes (to reduce the number of read-out channels required) and because the track measurement in the ρ direction is not as important as the measurement in the bending direction of the track. Studies of both collision and cosmic-ray data show a per-layer position resolution of $\sim 50 \mu\text{m}$ in the ρ - ϕ direction and $\sim 250 \mu\text{m}$ in the ρ -direction, in good agreement with simulations[55]. The overall muon momentum resolution using tracks exclusively reconstructed in the CSCs is expected to be 11–13 % for muons with transverse momenta 10–1 000 GeV/ c (Figure 3.17b).

¹The ME $\pm$ 1/1 chambers are mounted inside the solenoid magnet, so they are subject to a 3.8 T magnetic field normal to the plane of the chamber. To account for the drift of the electrons in this intense magnetic field, the wires are rotated by the Lorentz angle, roughly 29° , with respect to the long axis of the chambers.

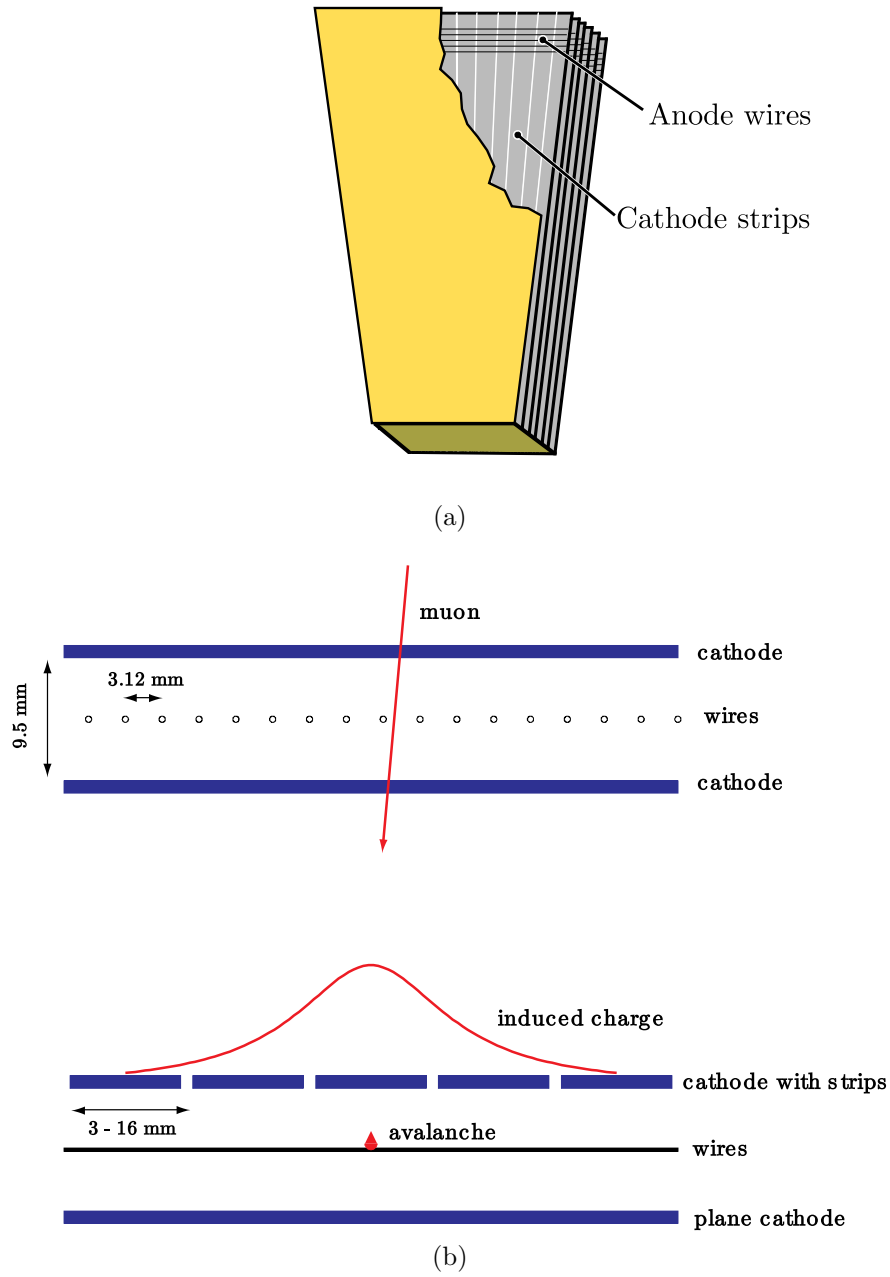


Figure 3.19: Diagrams of the internal workings of the end-cap cathode strip chambers. A single chamber has six independent planes of cathode strips and anode wires mounted orthogonally to each other (Figure (a)). Only a subset of the anode wires are pictured for clarity. As a muon passes through the chamber, the ionized electrons avalanche to the anode wires. This avalanche induces a charge on the cathode strips (Figure (b)). The spatial distribution of induced charge follows a Gatti distribution[51, 54].

Resistive plate chambers

In both the barrel and end-cap sections of the muon system, multi-gap **R**esistive **P**late **C**hambers (RPCs) are used to aid in fast triggering and, where required, arbitration of tracks projected between two DTs or a DT and a CSC[51]. The CMS RPCs use a multi-gap configuration with a series of metal strips sandwiched between two parallel-plate capacitors, each capacitor plate coated with 2 mm of Bakelite and separated by a 2 mm gas gap, as pictured in Figure 3.20[56]. There are 480 rectangular chambers in the barrel region, with strips aligned in the z -direction, and 216 trapezoidal chambers in the end-cap region, with strips aligned in the ρ -direction. This configuration allows for only coarse measurements of muon track η coordinates, but adequate measurements of the ρ - ϕ coordinate for triggering on muon transverse momentum. They are filled with a gas mixture of 96.2 % $\text{C}_2\text{H}_2\text{F}_4$, 3.5 % $i\text{C}_4\text{H}_{10}$, and 0.3 % SF_6 , and the strip-side of each capacitor is kept at a positive voltage of 9300 V relative to the outside plate so that the short ionization trail of a through-going muon will cause an avalanche of electrons traveling to the anode plate. This induces a charge on the strips, and the signal on those strips is digitized and read out. The RPC system has full ϕ coverage and extends to $|\eta| < 1.6$. The signals from the RPCs have a time resolution of 1–4 ns, allowing them to unambiguously determine bunch-crossing timing for detected muons.

The main function of the RPCs is fast muon identification for triggering, so efficiency was studied using cosmic muons and an independent scintillation detector to measure coincidences. The mean efficiency per chamber for triggering on these cosmic-ray muons was 97.2 %, with no chamber having an efficiency lower than 92 %[57].

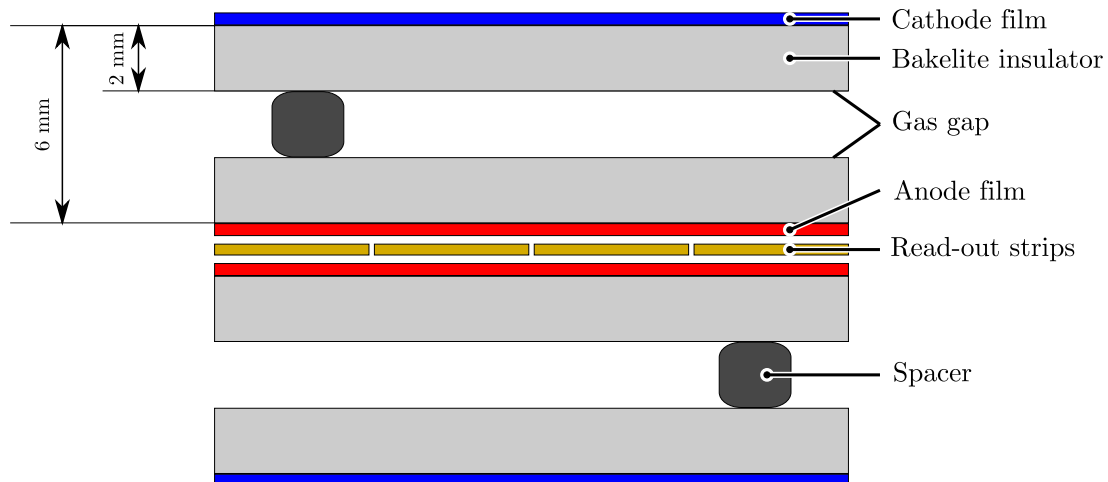


Figure 3.20: Drawing of a CMS double-gap RPC cell.

3.3 The CMS trigger and data acquisition system

At full intensity, the LHC will collide proton bunches at a rate of 40 MHz. Even with efficient zero-suppression of uninteresting data channels, it is still infeasible with present computing technologies to store every single event. The data coming from the CMS detector must therefore be sorted very rapidly into interesting events—to be stored for future off-line analysis—and uninteresting events—to be immediately discarded. CMS employs a two-tiered trigger system to make these decisions: a hardware-based “**Level-1**” (L1) trigger[27] and a software-based “**High-Level**” Trigger (HLT)[58]. The L1 trigger is designed to reduce the event rate by a factor of 10^3 and the HLT by another factor of 10^2 , for a combined output rate on the order of 100 Hz.

3.3.1 Level-1 trigger

The CMS Level-1 trigger is entirely hardware-based, using custom-built field programmable gate array chips and, where speed and radiation resistance are required,

application-specific integrated circuits[27]. Only the calorimeters and muon systems are used in the decisions of the L1 trigger—information from the central tracking subsystems is too complex to be analyzed on the short time-scales required for L1 decisions. The L1 trigger is divided into local, regional, and global components. At the local level, the individual sub-detectors use pattern-matching logic to find high-energy deposits in the calorimeter system, or high-momentum charged tracks in the muon system. The highest-quality primitives (assigned based on parameters like energy, momentum, and fit quality) from the local level are sent from each sub-system to one of two regional-level triggers: the Regional Calorimeter Trigger (RCT) combines the HCAL and ECAL trigger primitives, while the DT and CSC primitives are shared and analyzed at the DT and CSC track finders. The regional-level RCT and track finders feed information to the Global Calorimeter Trigger (GCT) and Global Muon Trigger (GMT), respectively, and, finally, the CMS Global Trigger (GT) compiles all the information from the GMT and GCT to make a trigger decision. If the collision event is deemed interesting by the GT, it sends a Level-1 Accept (L1A) message to the individual sub-detectors to begin the **Data Acquisition** (DAQ) read-out of the event. The architecture of the L1 system is diagrammed in Figure 3.21. The total time between a collision and an L1A decision is required to be less than 3.2 μs .

Calorimeter trigger

The Level-1 calorimeter trigger chain begins with trigger primitive generation. These primitives are the energy deposits constructed independently on every calorimeter “tower”—a stack of 25 ECAL crystals and an HCAL scintillator tower covering a region in (η, ϕ) of 0.087×0.087 for pseudo-rapidities of $|\eta| < 1.74$, and larger for higher pseudo-rapidities. The trigger primitives from the ECAL, HCAL, and HF are passed into the RCT, where 4×4 groups of trigger towers (or single HO towers) are ana-

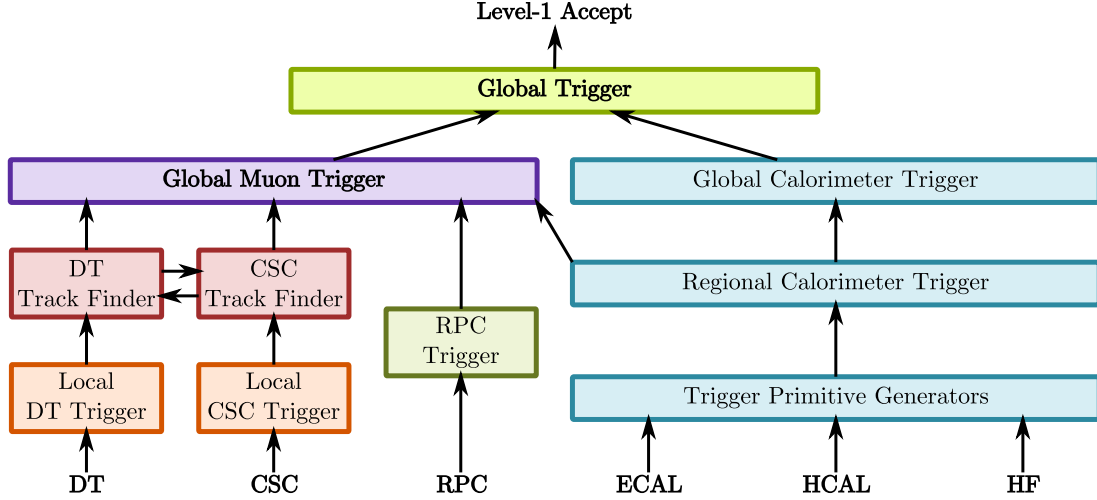


Figure 3.21: A diagram of the CMS Level-1 trigger.

lyzed for energy deposits and shower profiles that match high-transverse-momentum electrons and photons. The RCT also identifies and discriminates τ jets from the broader-profiled quark/gluon jets^m. The RCT passes isolation and minimum-ionizing particle primitives to the GMT to help the GMT further identify muons.

The GCT takes the e/γ and τ /jet candidates from the RCT and determines the total transverse energy of the candidates (and isolations of the e/γ candidates), along with global quantities like total missing transverse energy, jet counts, and the scalar sum of all jet energy (H_T). Jets are further analyzed using the combined regions of the RCT. The GCT forwards this information to the GT, where programmable conditions based on these variables are used to make ultimate L1A decisions.

Figure 3.22 shows the simulated trigger efficiency for generated single electron and photon particles. These plots assume a trigger threshold of $E_T \geq 20$ GeV. The

^mA “jet” is a collection of nearly collinear particles. When a high-momentum quark or gluon is produced, multiple hadrons and leptons can be created during the complicated hadronization process of that parton. These particles tend to have momenta clustered in a tight cone in the direction of the original parton and are called a jet. A “ τ jet” is simply a jet created by the hadronic decay of a τ lepton.

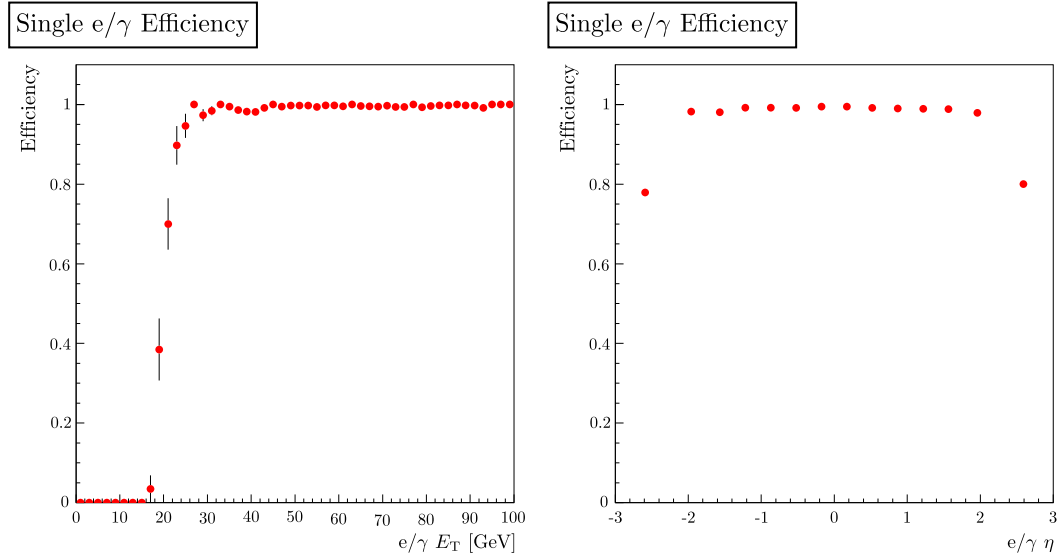


Figure 3.22: Simulated trigger efficiencies for the CMS Level-1 Global Calorimeter Trigger. Shown are the results for a trigger threshold of $E_T \geq 20$ GeV as a function of (a) generated $e/\gamma E_T$ and (b) generated η [27].

GCT for this particular threshold has a sharp turn-on curve at an electron/photon $E_T = 20$ GeV and has nearly perfect efficiency for $|\eta| < 2.0$.

Muon trigger

All three muon sub-systems, the DTs, CSCs, and RPCs, participate in the muon Level-1 trigger chain. Each separate DT chamber generates a ϕ -projection of a through-going muon (with a η -projection hit pattern), and each CSC chamber generates a three-dimensional track segment. The dedicated muon trigger hardware forwards to the muon Track Finders those track segments that match the pattern of a high-momentum muon coming from the interaction point. The DT and CSC Track Finders combine the segments from the individual chambers and make an approximate measurement of the transverse momenta of the muons. The Track Finders share information about track stubs to better identify and measure those muons that pass

through the region between the two sub-detectors ($1.0 < |\eta| < 1.2$). The best four muons (sorted by transverse momentum) from both the CSC and DT Track Finders are forwarded to the GMT.

The RPCs do not use on-detector electronics for trigger primitive generation, but forward all of their data to a specialized regional trigger system that detects muons based on pattern recognition between RPC chambers. The RPC **P**attern **C**omparator **T**rigger (PaCT) also uses data from the HO to identify and measure the transverse momenta of muons in the barrel portion of the detector. The PaCT forwards to the GMT the four best muon candidates that it identifies (sorted by transverse momentum) from the barrel and end-cap regions of the muon system separately.

The GMT receives information from the CSCTF, DTTF, and PaCT about muon candidate transverse momentum, charge, η - and ϕ -coordinates, and quality-of-fit codes from the pattern-matching algorithms. The GMT also uses information from the RCT to determine the isolation and minimum-ionizing status of the muon candidates. The GMT sends to the CMS global trigger this information after combining possibly duplicated muon information from the RPCs and Track Finders.

The trigger efficiencies of the CMS L1 muon triggers are plotted as a function of η in Figure 3.23[27]. These data are generated using Z^*/W^* Drell-Yan Monte Carlo and a hardware trigger simulator. Depicted on the plot are the trigger efficiencies of the various muon sub-systems and the GMT averaged over muon p_T and ϕ . The CMS L1 muon trigger is expected to be above 95 % efficient for triggering on these generic muons.

3.3.2 Data acquisition system

The CMS Data Acquisition system is the set of hardware and software components responsible for reading stored data from the various detector elements, passing that

Muon Level-1 Trigger Efficiency

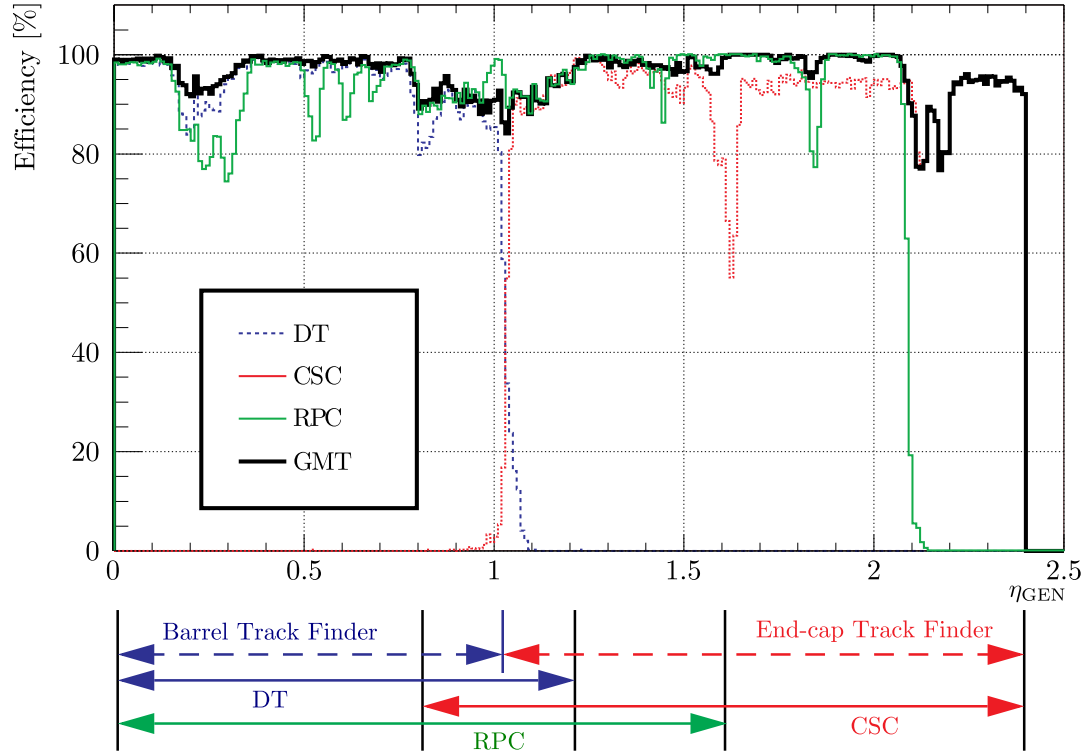


Figure 3.23: Efficiency of the CMS Level-1 muon triggers as a function of muon η . Efficiencies are averaged over muon p_T and ϕ . This plot assumes the final configuration of the RPC sub-detectors, which will extend to higher values of $|\eta|$ than in the present configuration[27].

information to the HLT, then finally storing that data to disk for off-line analysis[58]. The individual detector components are required to store data from each collision in a pipeline until the L1 trigger system has had a chance to make a decision about the event, or at least 3.7 μ s. When a **Level-1 Accept** (L1A) arrives at a detector component, it sends its data corresponding to the triggered event to specialized DAQ hardware. The data from the sub-detectors are heavily zero-suppressed (any data that do not meet certain minimum thresholds are simply not read out), but, even so, the amount of data per event is roughly 1 MB. This means that, at an L1 trigger rate of 100 kHz, the DAQ system must be able to read and process 100 GB/s of data.

The data from the detector components are combined at 626 **Front-End Driver** (FED) units into event “fragments” of approximately 2 kB per event (sometimes multiple detector components are combined to achieve this average). The FEDs send data to 458 **Front-End Read-Out Links** (FRLs), which merge FED data and pass it on to event **Builder Units** (BUs). The BUs combine all the FED data from one L1A into a single event. These events are finally passed to **Filter Units** (FUs), which reconstruct the event and apply high-level triggering algorithms.

Any build-up of queued events in the DAQ system (known as “back-pressure”) can propagate all the way down to the hardware components on the detector system, where limited memory space could mean buffer overflows and corrupted data. To prevent this, all the DAQ components communicate to the L1 trigger through the **Trigger-Throttling System** (TTS). Any DAQ component experiencing back-pressure can signal the TTS to halt L1As from being sent until the DAQ component has time to catch up. In the case of sudden, unexpected increases in events due to natural fluctuations of the beams, DAQ components could still end up with overflowed buffers, in spite of TTS throttling. In this case, the TTS can stop L1As and issue a signal

for all DAq components to clear their buffers and synchronize event counters. This system ensures minimal down-time and data corruption while the LHC is in operation.

3.3.3 High-Level Trigger

The CMS High-Level Trigger uses the full detector data to reconstruct the collision event and make a decision to write the event to tape or to discard it[58]. The HLT uses a version of the event reconstruction software that is similar to the full reconstruction suite used in off-line analyses, with certain routines simplified to satisfy time constraints. For the DAq builder units, CMS employs 720 off-the-shelf PCs, each with eight processing cores. In order to reduce the 100 kHz of L1As (at LHC maximum instantaneous luminosity) to the ~ 100 Hz rate allowed by the storage hardware, the HLT algorithm has to be applied with a mean time of 50 ms per event. To do this, the HLT only focuses on those areas of the detector “seeded” by the Level-1 trigger, or where the L1 algorithms identified interesting physics objects. The HLT is the first stage in which the central tracking data are used, so tracks in seeded regions of the detector are reconstructed using the tracking sub-detectors to achieve better momentum resolution of charged particles. The HLT can be programmed to make trigger decisions based on any information available to the off-line reconstruction, but time constraints typically limit the decisions to broad-stroke measurements like particle/jet p_T or E_T , particle/jet multiplicities, and event $\cancel{E}_T$.

If the event is triggered by one of the HLT algorithms, the reconstructed physics objects (formatted into C++ objects) are written along with the raw data in a ROOT-formattedⁿ file. The files are transferred to global data centers and reconstructed off-line with more detail, when time is not a constraint.

ⁿROOT is an object-oriented toolkit and analysis framework designed especially for high-energy physics analyses[59].

3.4 CMS analysis software

To aid in processing the enormous amounts of data that the CMS detector produces, a framework of dedicated analysis software packages called CMSSW was developed by the CMS community of physicists and software engineers[60]. It is designed around the abstract concept of the *event*, or a single triggered beam-crossing, and all the associated detector read-out and reconstructed physics data associated with the particles produced in that beam-crossing. The stored information in each event can include many different types of data, from raw detector output to reconstructed high-level analysis objects such as hypothetical particle candidates and mother-daughter mappings. CMSSW uses the ROOT analysis framework to store all the event data as object representations in C++[59]. Software “producer” modules can build and insert into events virtually any software object that can be represented in a C++ class. “Filter” modules can analyze and remove from data files those events that do not meet certain criteria, similar to an off-line trigger. “Analysis” modules allow a user to perform any arbitrary analysis on the events and produce summary output in a format of the users’ choosing. The analysis software also specifies ways to access distributed databases so that the detector conditions and configurations for every event can be looked up and used in analyses.

3.4.1 Event reconstruction

One of the most important functions of the analysis software is to reconstruct from the raw digitized detector data representations of candidate physics objects such as electrons, muons, photons, and jets. Once all the candidate objects in an event have been reconstructed, these **reconstructed** (RECO) data can be distributed to physicists separately from the raw detector data to save physical storage space and network transfer times. Many different producer modules are used to generate the

different RECO candidate objects used in analyses, and sometimes multiple producers are used to reconstruct the same candidate objects in different ways. The following will focus only on those reconstruction routines that pertain to this analysis.

Muon reconstruction

Muon reconstruction begins with track-building algorithms in both the central tracker and the muon detectors. The digitized raw data from these detectors, along with detector geometry and alignment information from the conditions database, are used to build 3-dimensional **Reconstructed Hits** (RecHits) in the detector where charged tracks were likely to have passed through the tracking elements. These RecHits are associated into tracks using Kalman filter techniques. A Kalman filter is a technique for fitting a parameterized functional form to data by iterating over the data points and taking advantage of known correlations between data points and measurement resolutions[61, 62].

Track candidates are constructed separately in the muon detectors (so-called “stand-alone” tracks) and the central tracker (so-called “tracker” tracks). Tracker tracks are then projected from the outer tracker layers through the mass of the calorimeters and magnet and into the muon sub-detectors. If the projected tracker track is close to a stand-alone track, the RecHits from the two are associated into one “global” track. A Kalman filter is used again to reconstruct the physical parameters of the global track (momentum direction, momentum magnitude, and particle charge), starting from the outermost RecHit and progressively iterating in the direction toward the beam-pipe. Once the global muon track has been completely reconstructed, the energy deposited in the calorimeters around the point where the track is projected to pass through them is associated with the track for isolation and compatibility checks.

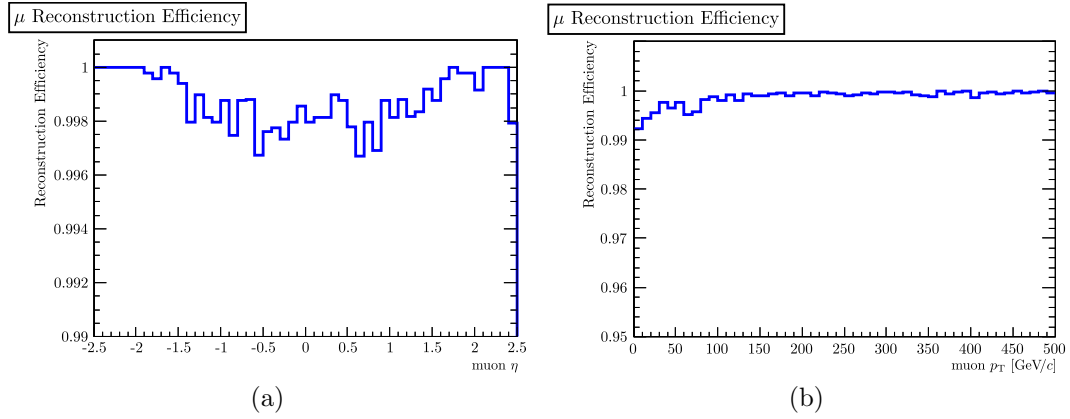


Figure 3.24: Simulated muon reconstruction efficiency as a function of (a) η and (b) p_T . Muon identification requires that the muon be reconstructed globally and independently identified as a muon by the central tracker.

The muon reconstruction algorithm is calculated to be over 99 % efficient in reconstructing global muons that have also been identified as muons by the central tracker^o. This is calculated using 500 000 Monte Carlo events of single muons with a flat p_T spectrum between 0 and 500 GeV/c and flat in $|\eta| < 2.5$ reconstructed in a full CMS detector simulation. The efficiency depends on the pseudo-rapidity of the muon track, though this variation is practically negligible. Figure 3.24a shows a plot of the reconstruction efficiency as a function of detector η . The efficiency is also dependent on the momentum of the muon. This is because lower-momentum muons are much more likely to be deflected by nuclei in the dense iron return yoke. Figure 3.24b shows a plot of the reconstruction efficiency as a function of muon transverse momentum, showing the slight drop-off in efficiency at low p_T .

Finally, the muon reconstruction algorithm may reconstruct as muons particles that are in fact not muons at all (a type-I error, as opposed to the type-II error of failing to reconstruct a real muon). Muon mis-reconstruction is typically due to $\pi^\pm$

^oThe tracker muon identification algorithms look at track bending and the amount of charge deposited in each layer of the central tracker to decide whether or not the track in question belongs to a minimum-ionizing particle. This identification is performed in off-line software reconstruction.

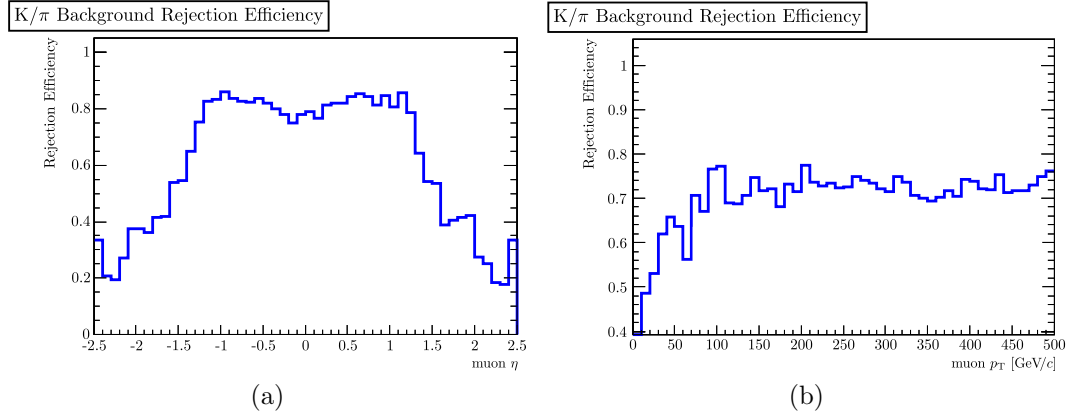


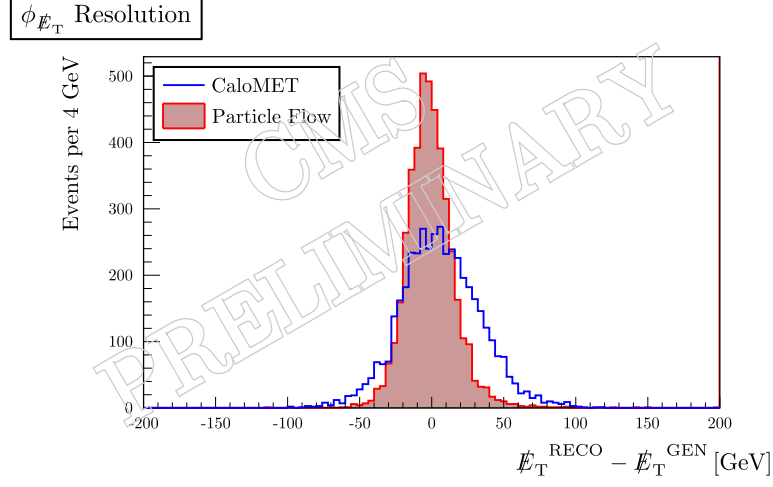
Figure 3.25: Simulated rejection efficiency for $\pi^\pm$ and $K^\pm$ mesons as a function of (a) detector η and (b) reconstructed (fake) muon momentum. Muon identification requires that the muon be reconstructed globally and separately identified as a muon by the central tracker.

and $K^\pm$ mesons punching through the calorimeters and leaving tracks in the muon system. A good muon reconstruction algorithm should reject these particles as not muons. The rejection efficiency (one minus the reconstruction efficiency) was calculated using a Monte Carlo sample of 1 000 000 single $\pi^\pm$ and $K^\pm$ mesons (500 000 each) with a flat p_T distribution between 0 and 500 GeV/ c and flat in $|\eta| < 2.5$. The rejection efficiency is approximately 70 % for these particles. Figure 3.25 shows the rejection efficiencies for $\pi^\pm$ and $K^\pm$ mesons as a function of detector η and reconstructed (fake) muon momentum. The fall-off at low p_T is due primarily to rejection by the tracker muon identification algorithm, while the fall-offs at high $|\eta|$ are due to the increased momentum these high- $|\eta|$ tracks must have (recalling that the MC are generated flat in p_T), allowing them to more easily penetrate to the deeper layers of the muon system.

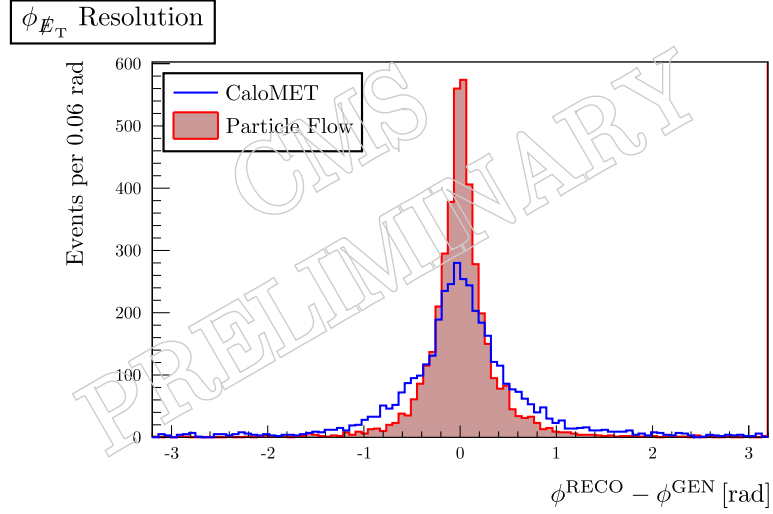
$\cancel{E}_T$ reconstruction

Missing transverse energy is a high-level value that can be compiled in many different ways. One way (called “CaloMET”) is to simply perform a vector sum on the calorimeter energies, then call the $\cancel{E}_T$ that vector that balances the sum in the transverse plane, as in equation 2.6. In this method, corrections must be applied to account for the different ionizing capabilities of different particles, especially minimum-ionizing particles that have very large energies but deposit very little of that energy in the calorimeters. This calculation can be performed very quickly because it does not rely on track information except for the few muons that might be present in the event. Because of this, it is the method used for reconstructing the $\cancel{E}_T$ at the HLT level.

Because $\cancel{E}_T$ plays a central role in the measurement of the W mass, the method of $\cancel{E}_T$ reconstruction chosen for this analysis had to be as accurate as possible with respect to the momentum of the ν_μ in $W \rightarrow \mu\nu_\mu$ decays. The method chosen is called “**P**article-**F**low” (PF), which attempts to use information from all sub-detectors to uniquely reconstruct and identify the particle sources of all detector signals[63]. The PF description of an event identifies muons, electrons, photons (including photons that convert to e^-e^+ pairs in the detector material and *bremßstrahlung* photons from electrons), and charged and neutral hadrons. The $\cancel{E}_T$ of the event is calculated by balancing the momenta of these more fundamental particles, and the result is a measure of the $\cancel{E}_T$ that is much more accurate to the ν_μ transverse momentum than the $\cancel{E}_T$ from the previously mentioned algorithm. Figure 3.26 shows a plot of the $\cancel{E}_T$ magnitude and direction (ϕ) resolution for simulated $W \rightarrow \mu\nu_\mu$ events, using different algorithms to reconstruct $\cancel{E}_T$. The PF algorithm shows a narrower peak and a mean closer to zero for both distributions as compared to the CaloMET algorithm.



(a)



(b)

Figure 3.26: $\cancel{E}_T$ (a) magnitude and (b) direction resolution comparing the performance of Particle Flow and CaloMET algorithms. These figures were made using $t\bar{t}$ Monte Carlo events with $20 \text{ GeV} < \cancel{E}_T < 200 \text{ GeV}$ [64].

Chapter 4

EXPERIMENTAL PROCEDURE AND ANALYSIS

In this analysis, two physical quantities of the W boson will be measured: the particle's mass and the total production cross-section in p-p collisions at $\sqrt{s} = 7$ TeV. In the process of selecting a sample of data events that are believed to contain a large concentration of $W \rightarrow \mu\nu_\mu$ decays in order to perform the measurement of m_W , a careful calculation of the selection efficiencies will produce a cross-section measurement almost incidentally. The measurement of the W mass will be accomplished by fitting the transverse mass distribution from selected $W \rightarrow \mu\nu_\mu$ events to the parameterized analytical form of the cross-section PDF from equation (2.8). From this fit, the mass parameter will be extracted. An accurate fit will require good event identification as well as a good understanding of background contamination, detector efficiencies, and measurement resolutions. This section will introduce the methods used in this analysis to ensure an accurate measurement of these quantities.

4.1 Properties of $W \rightarrow \mu\nu_\mu$ decays

The kinematics of a $W \rightarrow \mu\nu_\mu$ decay event in the CMS detector are uncomplicated. At tree-level, the decay is likely to produce one high- p_T muon and one high- p_T neutrino and no other particles with significant momentum in the transverse plane. If the W

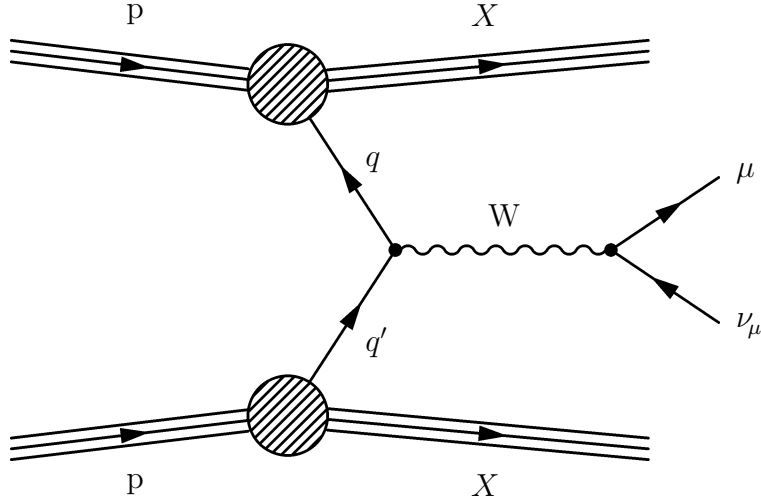


Figure 4.1: Feynman diagram for the tree-level process $pp \rightarrow W \rightarrow \mu\nu_\mu + X$. The X in the diagram represents hadronic activity due to the other interacting partons in the proton. Time, in this diagram, flows to the right.

decays at rest in the transverse plane of the lab frame, then the p_T of the muon and the p_T of the neutrino will be back-to-back. Boosts of the W in a transverse direction will decrease the separation angle between the muon and neutrino. Because the tree-level process does not have strongly-interacting partons, the event should be mostly free of hadronic activity in the transverse direction^a. This means track population density in the central tracker should be small. Because the only interacting particle produced in the decay is a minimum-ionizing muon, the energy deposited in the calorimeters should be small as well. Figure 4.1 shows the tree-level Feynman diagram for the process $pp \rightarrow W \rightarrow \mu\nu_\mu + X$ (where X represents the products in the underlying event), and Figure 4.2 shows an event display from a candidate $W \rightarrow \mu\nu_\mu$ event in the CMS detector, both showing pictorially the features described above.

^aSome hadronic activity will always be present due to soft scattering interaction between the beam protons, the so-called “underlying event” of the main hard scatter that produces the W .

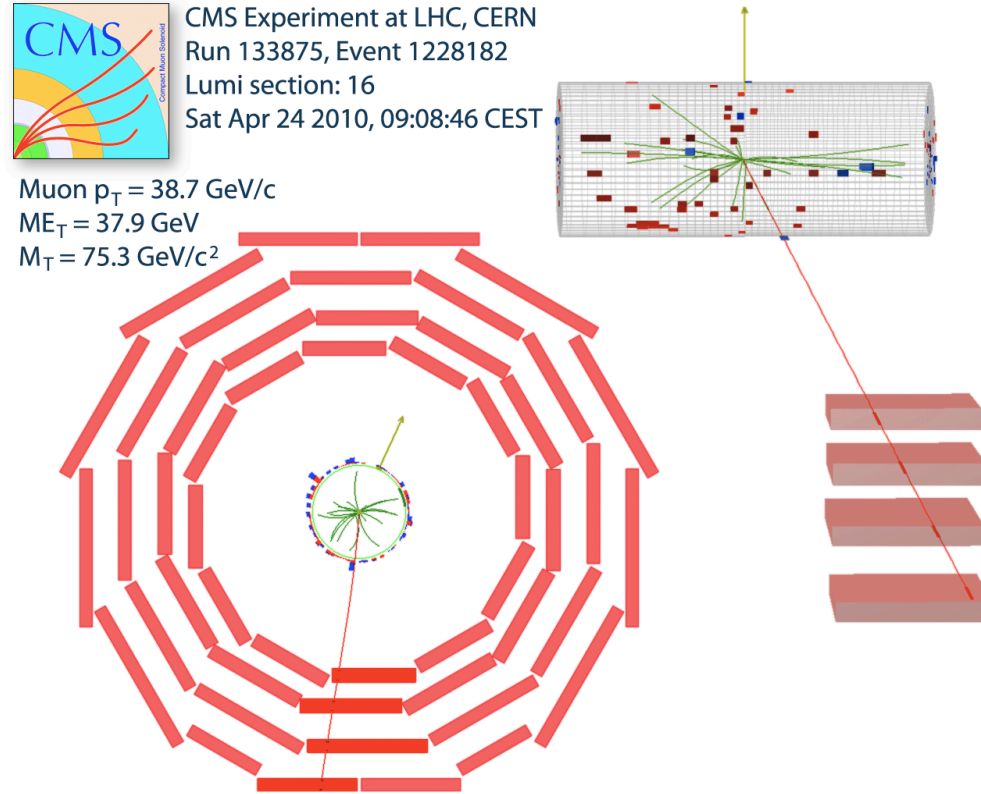


Figure 4.2: An event display for a candidate $W \rightarrow \mu\nu_\mu$ event in the CMS detector. The picture on the left shows an isometric transverse view of the detector. The central tracking subsystems are represented by the green circle in the center of the picture, and the barrel muon chambers are represented by the concentric rings of red blocks. The bars of different sizes and colors encircling the central tracker represent the amount of energy deposited in the calorimeters in that ϕ segment of the detector (red for electromagnetic, blue for hadronic). The green curves depict the tracks reconstructed in the central tracker. The single long red curve is the reconstructed track of a muon, and the yellow arrow shows the direction of the reconstructed $\cancel{E}_T$ in the event. The picture on the right shows the same event in a perspective view with line-of-sight along the x -axis. For clarity, The muon chambers not involved in the reconstruction of the muon are not pictured.

4.1.1 Background processes

A background event is any collision event in the CMS detector that mimics the primary features of the $W \rightarrow \mu\nu_\mu$ decay signal. The largest contribution of background events comes from QCD interactions: light quark (b and c) hadrons produced in the high-energy collisions of the LHC will decay through electroweak channels into μ/ν_μ and μ^-/μ^+ pairs, faking the muon and possibly the $\cancel{E}_T$ signal of the $W \rightarrow \mu\nu_\mu$ decay. These events tend to be easy to separate from $W \rightarrow \mu\nu_\mu$ decays because the muons are spawned from the decay chains of hadrons, so they will tend to be found in areas of high electromagnetic and hadronic activity. The other major source of background events are from the decays of W bosons to $\tau \nu_\tau$ pairs and the decays of Z bosons to both $\mu^- \mu^+$ and $\tau^- \tau^+$ pairs. The pair of charge leptons in the Z decays can be used as a handle to reduce those backgrounds, but the $W \rightarrow \tau \nu_\tau$ decays with the τ decaying to a muon and two neutrinos are indistinguishable from $W \rightarrow \mu\nu_\mu$ decays in every way. These backgrounds must be carefully studied using Monte Carlo simulations to understand in what ways they will contaminate the selection of $W \rightarrow \mu\nu_\mu$ events and how their presence can be accounted for in the determination of the W mass.

4.2 Signal and background Monte Carlo samples

Monte Carlo event samples in conjunction with a software simulation of the detector hardware are used to understand the response of the real CMS detector to complex collision events in a controlled manner. The MC samples used in this analysis were generated using the PYTHIA 6.4 high-energy physics MC software package[65]. The PYTHIA program simulates tree-level (no-loop) particle interactions using measured physical parameters of fundamental particles. The physical properties of the stable particles that PYTHIA generates are propagated through a detector simulation using

the geometry and material interaction simulator **GEANT4**[66, 67]. As the particles pass through the simulated detector elements, the detector electronics’ responses are simulated in software using information from a conditions database to account for known hardware inefficiencies such as broken components or un-powered electronics. The hardware simulation includes a simulation of the CMS L1 trigger algorithms. These simulated electronics signals are then passed through standard HLT and event reconstruction algorithms to produce events that are parallel in structure and directly comparable to real CMS data. Reconstruction of the events in this analysis use the **CMSSW** software package version 3_6_1_patch2.

4.2.1 $W \rightarrow \mu\nu_\mu$ signal sample

A $W \rightarrow \mu\nu_\mu$ MC sample of 46 032 generated events is used to simulate the target signal events. In order to reduce computer processing time and physical storage space, only events with a child muon pseudo-rapidity in the range $|\eta| < 2.5$ were generated. The CMS detector does not have tracking hardware in the region $|\eta| > 2.4$, so this is a reasonable cut. The theoretical cross-section (times $\mu\nu_\mu$ branching ratio) for the $W \rightarrow \mu\nu_\mu$ process at $\sqrt{s} = 7$ TeV is 10 400 pb^b. This number is calculated to **N**ext-to-**L**eading-**O**rders (NLO) in the strong coupling constant α_s using the software package **MCFM**[68] and the **MSTW2008** parton distribution functions[69]. Using equation (3.2), the $W \rightarrow \mu\nu_\mu$ MC sample used in this analysis has an apparent integrated luminosity of 6.5 pb⁻¹.

^bA *barn* (b) is a unit of area equal to 10^{-24} cm². One barn is approximately equal to the classical cross-sectional area of a uranium nucleus which, when compared to the nuclei of more common elements, is “as big as a barn.”

4.2.2 Background samples

The background samples selected for this analysis are those that mimic closely the signal signature in the detector. These samples are described below in detail. As with the signal sample, all next-to-leading-order production cross-sections in p-p collisions at $\sqrt{s} = 7$ TeV are calculated using MCFM and the MSTW2008 parton distribution functions.

Inclusive muon QCD

The primary background in the low- m_T region of phase-space comes from hadronic jet events where a muon is created in the showering process and a large $\cancel{E}_T$ is detected due to mis-reconstruction of the event. This background is represented with a MC sample of 461 120 hadronic jet (QCD) events. The inclusive QCD cross-section at these energies is several orders of magnitude larger than all other process cross-sections (roughly 1 b, calculated to leading-order using PYTHIA), so to save processing time and physical storage space, only events with at least one muon with $p_T > 15$ GeV/ c are considered. Approximately 0.03 % of all QCD events in CMS will produce such a muon. Factoring in the muon production efficiency, the QCD sample used in this analysis has an apparent integrated luminosity of 12 pb⁻¹.

$W \rightarrow \tau \nu_\tau$

The W boson is approximately as likely to decay to a τ - ν_τ pair as it is a μ - ν_μ pair[11]. The τ is unstable and will decay via the weak charged-current interaction to a μ , ν_μ , and ν_τ trio 17.4% of the time[11]. The existence of a muon and neutrinos in the decay products for this process mean this background is irreducible: there will always be some portion of this background that is indistinguishable from the true $W \rightarrow \mu \nu_\mu$ signal. The $W \rightarrow \tau \nu_\tau$ used in this analysis has 6 800 events. This process has a next-

to-leading-order cross-section (times branching ratio) of 10 312 pb, so the apparent integrated luminosity of this sample is 9.70 pb⁻¹.

$Z \rightarrow \mu^- \mu^+$

The process $Z \rightarrow \mu^- \mu^+$ produces two high-momentum muons. If one of these muons escapes detection in the CMS detector, or if the $\cancel{E}_T$ of the event is poorly reconstructed, the signal will mimic that of the $W \rightarrow \mu \nu_\mu$ process. The next-to-leading-order cross-section (times branching ratio) of this process is 1 667.3 pb. The MC sample of $Z \rightarrow \mu^- \mu^+$ used in this analysis contains 66,000 events, so the apparent integrated luminosity of the sample is 57 pb⁻¹.

$Z \rightarrow \tau^- \tau^+$

The process $Z \rightarrow \tau^- \tau^+$ is similar to the $Z \rightarrow \mu^- \mu^+$ process. If one of the τ particles decays in the process $\tau \rightarrow W \nu_\tau \rightarrow \mu \nu_\mu \nu_\tau$, the high-energy muon and neutrinos will mimic the signal of the $W \rightarrow \mu \nu_\mu$ process. The next-to-leading-order cross-section (times branching ratio) for the $Z \rightarrow \tau^- \tau^+$ process is 1 667.3 pb. The MC sample of $Z \rightarrow \tau^- \tau^+$ used in this analysis contains 7 400 events, so the apparent integrated luminosity of the sample is 57 pb⁻¹.

Table 4.1 lists summary statistics of the MC samples used in this analysis.

Process	Gen. events [$\times 1\,000$]	≥ 1 RECO muon [$\times 1\,000$]	Gen. efficiency [%]	NLO σ [nb]	$\int \mathcal{L} dt$ [pb^{-1}]
$W \rightarrow \mu \nu_\mu$	50	46	73.9	10.4	6.5
$W \rightarrow \tau \nu_\tau$	101	6.8	100	10.4	9.7
$Z \rightarrow \mu^- \mu^+$	95	66	100	1.67	57
$Z \rightarrow \tau^- \tau^+$	95	7.4	100	1.67	57
QCD	3 576	1 321	0.0268	1.1×10^9	12

Table 4.1: Summary statistics of the Monte Carlo samples used in this analysis. Cross-sections are calculated to next-to-leading order in the strong coupling constant α_s using the noted generator efficiency and (if pertinent) process branching ratio. The muon decay product in the $W \rightarrow \mu \nu_\mu$ sample is required to have a momentum with $|\eta| < 2.5$. QCD events are required to have at least one muon with $p_T > 20$ GeV/ c .

4.2.3 $W \rightarrow \mu\nu_\mu$ selection and background rejection

The ultimate goal of this analysis is to fit a known cross-section PDF to $W \rightarrow \mu\nu_\mu$ events from data. The distribution of m_T in background processes will not necessarily fit the m_T distribution from signal events. Although the background m_T distribution can be accounted for by fitting it to a different function, this background PDF cannot be calculated *a priori*, so the best one can do is fit the distribution to an empirical function derived from MC samples. This will add a systematic error to the fit of the $W \rightarrow \mu\nu_\mu$ PDF and will therefore decrease the accuracy of the mass and cross-section measurements. This is why it is important to minimize the background contamination in the data sample before the fit is attempted. The following gives details of how the $W \rightarrow \mu\nu_\mu$ events are selected from the detector data and how the background processes mentioned above are filtered.

Trigger

As explained in Section 3.3.3, the CMS High-Level trigger is the first event filter in any analysis. The algorithms chosen typically define the dataset on which further selection criteria will be applied off-line. Depending on the HLT algorithm chosen, it could be used as a very strict criterion, filtering out all but those events that are the most likely to contain interesting physics. However, the HLT's reconstruction of events is not complete, and not all the tools for accurately identifying and measuring particles are available at this level. Thus, HLT criteria are typically chosen to be broadly encompassing of the most obvious features of the target process.

The most obvious features of the $W \rightarrow \mu\nu_\mu$ decay are the single high- p_T muon and large $\cancel{E}_T$ from the neutrino. The time constraints at the HLT mean a full particle-flow reconstruction of the $\cancel{E}_T$ in an event cannot be performed, so the HLT uses the CaloMET algorithm. This is insufficient for all but the coarsest of $\cancel{E}_T$ measurements,

so it is not a good choice for a trigger criterion (see Figure 3.26). Muon identification at the HLT, however, is adequate: simulation studies of the HLT show that the HLT_Mu9 algorithm, which accepts events with at least one muon with a reconstructed $p_T > 9 \text{ GeV}/c$ within $|\eta| < 2.1$, is approximately 92 % efficient for all muons[70]. Figure 4.3 shows the “turn-on curve” of the HLT_Mu9 algorithm, or the efficiency to identify muons versus generated muon p_T and η . What is important for the calculation of the efficiency of the HLT algorithm is the efficiency to select any $W \rightarrow \mu\nu_\mu$ event, regardless of the η of the muon. Of the 46 032 $W \rightarrow \mu\nu_\mu$ MC events generated (with a muon within $|\eta| < 2.5$), 39 592 of them were accepted by the HLT_Mu9 algorithm, corresponding to a trigger efficiency of $86.0 \% \pm 0.5 \%$ (stat.). This efficiency does not include the 95 % efficiency of the L1 trigger muon algorithms as discussed in Chapter 3.

The HLT_Mu9 algorithm is chosen to define the starting dataset for this analysis. Other muon trigger algorithms exist with lower p_T thresholds, but algorithms with a p_T threshold lower than HLT_Mu9 are pre-scaled^c when the instantaneous luminosity rises to $10^{31} \text{ cm}^{-2} \text{ s}^{-1}$ to prevent overloading of the DAQ system. The 0.5 % statistical uncertainty in the trigger efficiency calculation is propagated as a systematic error for this study.

$W \rightarrow \mu\nu_\mu$ decay identification and background rejection

The goal of the event selection in this analysis is not only to eliminate as much background contamination from the sample as possible while retaining a large fraction of signal events, but also not to bias the shape of the m_T distribution away from the ideal form of Figure 2.4. Thus, the selection criteria are chosen to be as loose as

^cOnly a certain fraction of the events that pass these lower-threshold triggers are actually output to disk.

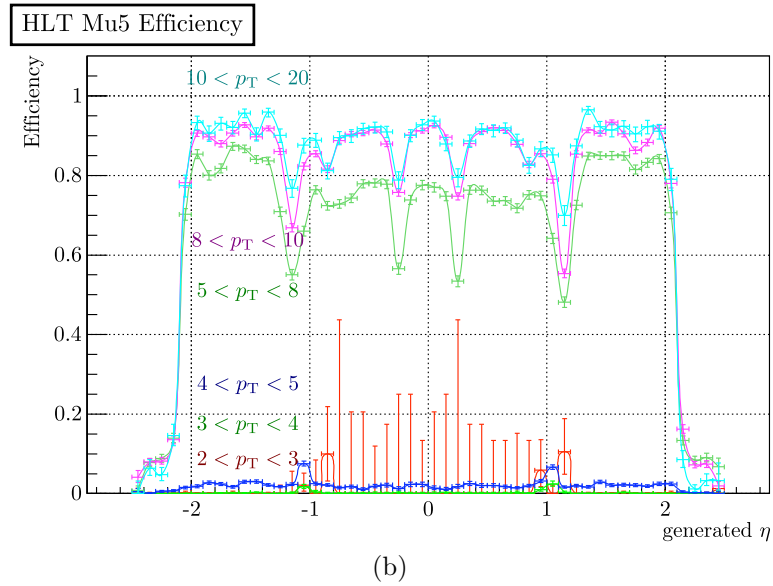
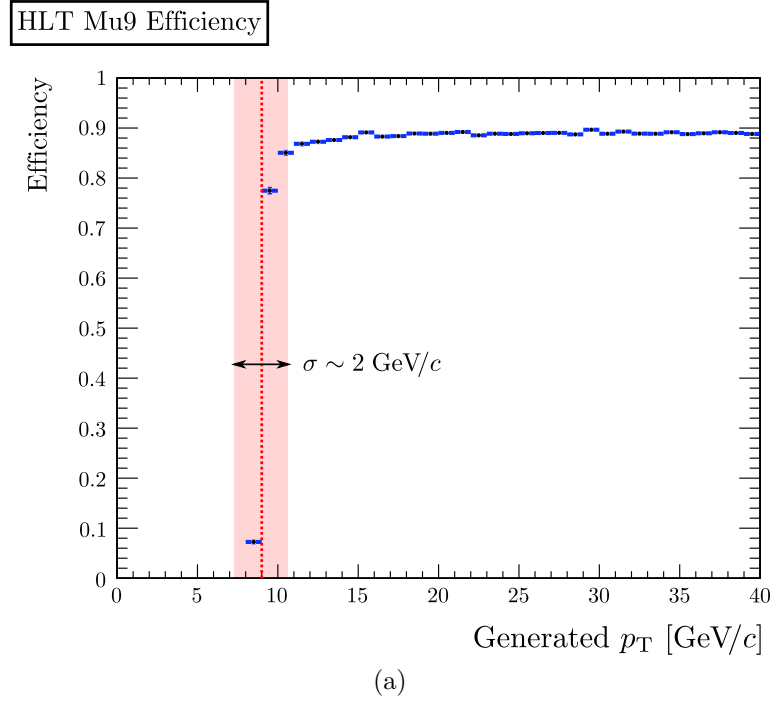


Figure 4.3: Turn-on curves for sample HLT muon algorithms in (a) p_T and (b) η . The p_T plot is generated using muons generated with a flat p_T spectrum and averaged over η , while the η plot is separated into different p_T bins to highlight the low- p_T response of the HLT. The η plot shows the HLT Mu5 algorithm response, but all muon algorithms show similar features[71].

possible on the kinematic observables that enter into the m_T formula (2.7) (muon p_T , $\vec{E}_T$, and $\Delta\phi_{\mu\nu_\mu}$).

The ability of the standard reconstruction software to globally reconstruct and identify muons (and to reject fakes) is already very good (see Section 3.4.1), but further identification methods are used to identify those muons from $W \rightarrow \mu\nu_\mu$ decays specifically. For instance, muons can be produced in γ conversions to $\mu^- \mu^+$ pairs, or decays of virtual Z or W bosons radiated by quarks or antiquarks. These are real muons, but they are not from W bosons produced in the initial p-p interaction, so they are considered background for this analysis. To be considered a muon from a $W \rightarrow \mu\nu_\mu$ decay, the reconstructed muon must meet the following criteria^d:

1. The muon must be identified as a muon by the central tracker alone and be reconstructed as a muon by the combined central tracking and muon detectors. This ensures the stand-alone muon is not spuriously matched to a random tracker track. Muons tracks in the tracker are distinguished from other particle tracks by their low deposits of charge in the silicon tracking elements. Of the 39 592 $W \rightarrow \mu\nu_\mu$ MC events that pass the trigger selection, 38 699 of them have at least one muon identified as both a tracker and global muon for a selection efficiency of $97.7\% \pm 0.5\%$.
2. The reconstructed global track of the muon must have $p_T > 16 \text{ GeV}/c$ and $|\eta| < 2.1$. This simply removes those muons that lie near the turn-on curves of the HLT Mu9 trigger algorithm as pictured in Figure 4.3. The cut in p_T also removes the low- p_T tail of the $W \rightarrow \mu\nu_\mu$ and QCD MC samples, where cuts on p_T during generation were applied. These cuts combined retain 33 838 of the 38 699 events to pass the tracker muon identification cut for a combined

^dThe following discussion speaks of these selections in a specific order only for convenience's sake. The ultimate selection efficiency, which is the product of these individual selection efficiencies, will be independent of the selection order.

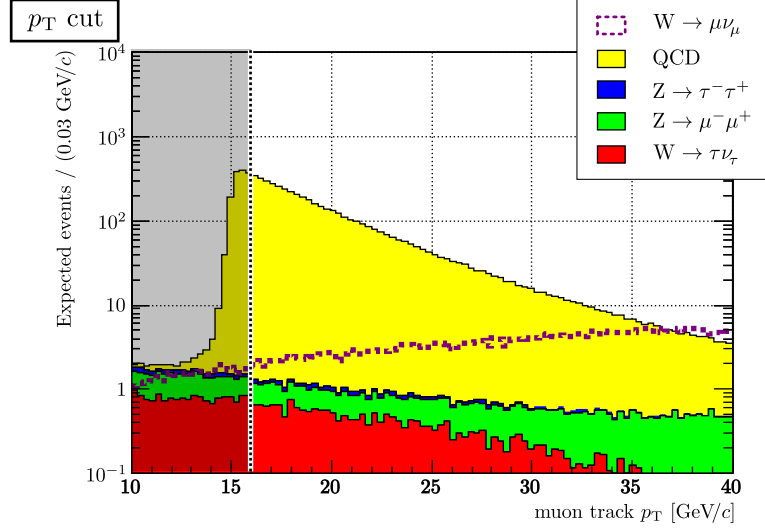
selection efficiency of $87.4\% \pm 0.8\%$. Figure 4.4 shows the stacked signal and background muon p_T and η distributions from events that passed the previous cuts.

3. The reconstructed global track of the muon must pass within 2 mm of the reconstructed primary vertex of the event ($|d_0| < 2\text{ mm}$). This cut ensures the muon is the product of a decay that occurs very close to the primary vertex of the event. The effective lifetime of the W boson is $3 \times 10^{-25}\text{ s}$ (calculated using Equation (2.1)), so even if the W is traveling relativistically in the lab frame, it will not decay more than a few attometers from its point of creation. This cut removes from the sample those muons that are created in interactions far from the primary vertex, likely from hadronic jets or decay-in-flight mesons. Of the 33 838 $W \rightarrow \mu\nu_\mu$ MC events that pass the previous selections, all of them also meet this criterion for a selection efficiency of $100\% \text{ }^{+0.0}_{-0.6}\%$.
4. The muon must be isolated from other jet or leptonic activity, with a sum total energy of the particles within a $\Delta R = 0.3$ cone^e around the muon track not exceeding 10 % of the reconstructed m_T in the event. This cut attempts to further remove those muons generated in jet activity. Such muons are likely to be associated with other tracks and calorimeter deposits nearby, while muons from prompt electroweak interactions at the primary vertex are likely to be isolated from such activity. QCD events are more likely to have low m_T than signal events, so normalization to the event m_T effectively scrutinizes those events that are more likely to be background in nature. Of the 33 838 $W \rightarrow \mu\nu_\mu$ MC events that pass the previous criteria, 32 611 of them also meet this criterion for a selection efficiency of $96.4\% \pm 0.6\%$.

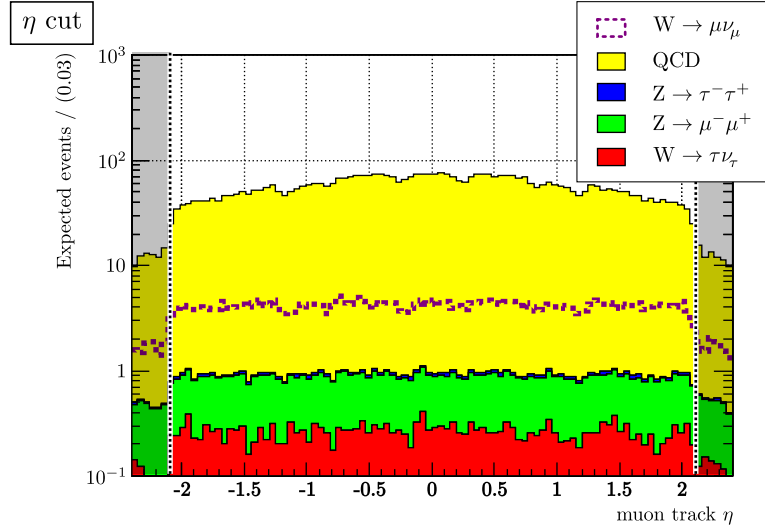
^e $(\Delta R)^2 \equiv (\Delta\phi)^2 + (\Delta\eta)^2$

5. If there is more than one muon reconstructed in an event, the invariant mass of the two muons must be less than $15 \text{ GeV}/c^2$. The largest remaining background after the above muon cuts is from $Z \rightarrow \mu^- \mu^+$ events. To eliminate this background contribution, a “Z-veto” is applied. The two muons from the Z resonance will have an invariant mass peaked very sharply at the Z mass of $91.1876 \text{ GeV}/c^2$ [11]. If a second muon is reconstructed in a $W \rightarrow \mu \nu_\mu$ decay, it will be uncorrelated with the muon from the W. The “Z-veto” then rejects events in which the invariant mass of the two leading- p_T momenta muons is greater than $15 \text{ GeV}/c^2$. No veto is applied to events with fewer than two muons reconstructed. Of the 32 611 $W \rightarrow \mu \nu_\mu$ MC events that pass the previous criteria, 32 169 of them also pass the Z-veto for a selection efficiency of $98.6 \% \pm 0.6 \%$.
6. Finally, the cut on muon p_T biases the low-end of the m_T spectrum (up to $m_T = 32 \text{ GeV}/c^2$, from equation (2.7)). To avoid problems associated with this bias and to remove a large fraction of background events with low m_T , the analytical cross-section PDF will only be fit in the range $m_T > 50 \text{ GeV}/c^2$. This effective cut retains 28 933 of the 32 169 events to pass the previous cuts for an efficiency of $89.9 \% \pm 0.6 \%$.

It should be noted that no attempt to control the p_T of the W is done in this analysis. The reconstructed p_T distribution of the W bosons from the combined Monte Carlo samples and from data is plotted in Figure 4.9. The W p_T is reconstructed as the vector sum of the reconstructed μp_T and $\vec{E}_T$. The average reconstructed W p_T in the $W \rightarrow \mu \nu_\mu$ Monte Carlo sample is below $20 \text{ GeV}/c$, and 90 % of all generated W bosons have $p_T < 40 \text{ GeV}/c$. Because the effect of the W p_T on the shape of the transverse mass PDF is negligible even for p_T up to $50 \text{ GeV}/c$, the W p_T assumed to be zero (see figure 2.6 in Section 2.2). This greatly simplifies the calculation of the



(a) $p_T > 16 \text{ GeV}/c$



(b) $|\eta| < 2.1$

Figure 4.4: The results of the muon (a) p_T and (b) η cuts on the combined Monte Carlo sample. The background samples are stacked and the $W \rightarrow \mu\nu_\mu$ sample is overlain for clarity. The cuts are at $p_T > 16 \text{ GeV}/c$ and $|\eta| < 2.1$, respectively. The darkened regions of the plots show the portion of the samples that are excluded from further selection.

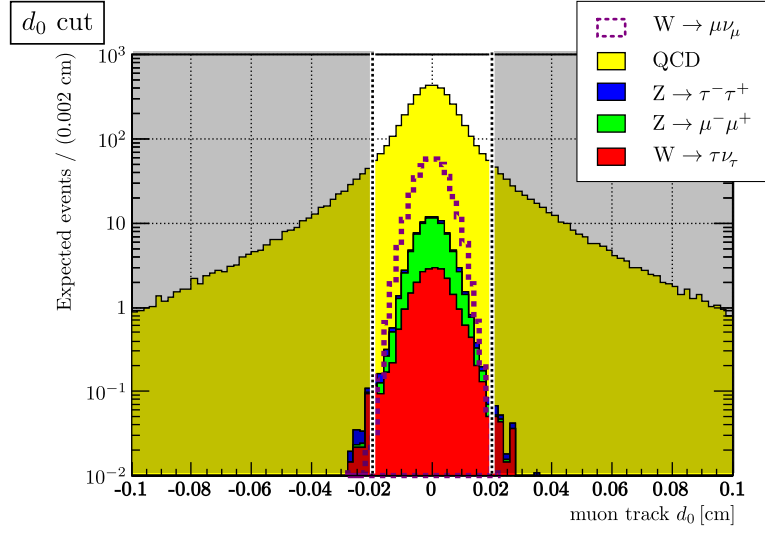


Figure 4.5: The results of the muon d_0 cut on the combined Monte Carlo sample. The background samples are stacked and the $W \rightarrow \mu\nu_\mu$ sample is overlain for clarity. The cut is at $|d_0| < 0.2$ cm. The darkened regions of the plot shows the portion of the samples that are excluded from further selection.

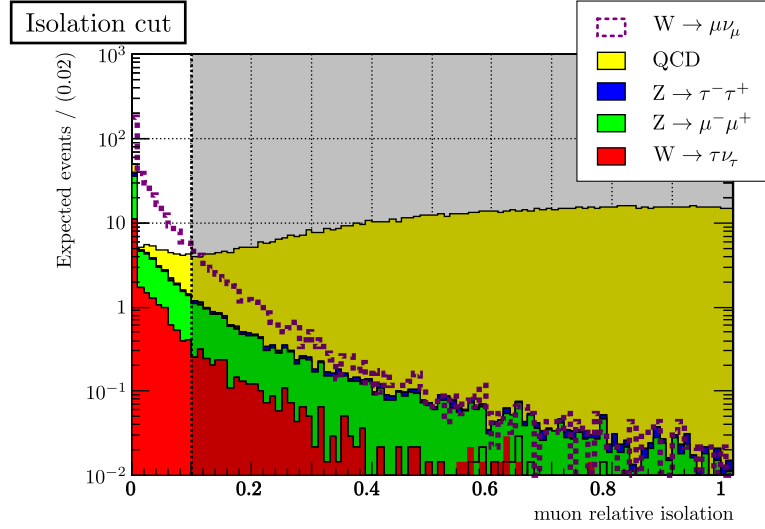


Figure 4.6: The results of the muon relative isolation on the combined Monte Carlo sample. The background samples are stacked and the $W \rightarrow \mu\nu_\mu$ sample is overlain for clarity. The cut is at $\text{iso}_\mu^{\Delta R=0.03}/m_T < 0.1$. The darkened regions of the plot shows the portion of the samples that are excluded from further selection.

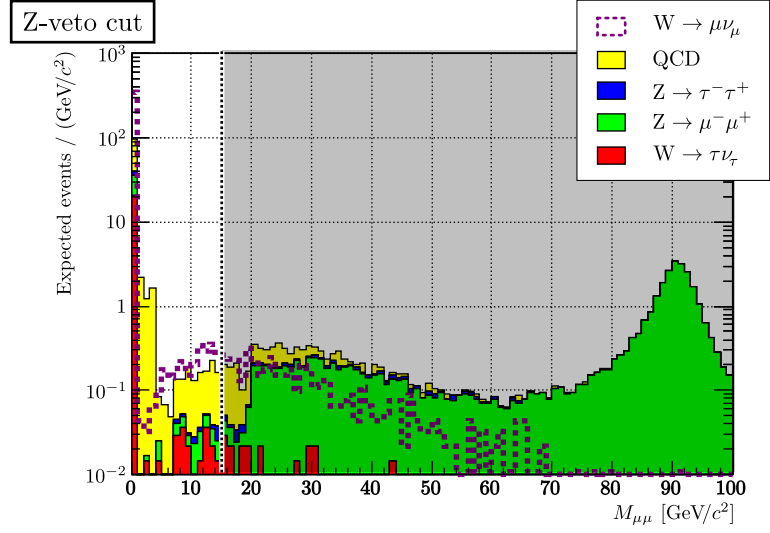


Figure 4.7: The results of the di-muon Z-veto ($M_{\mu\mu}$) cut on the combined Monte Carlo sample. The background samples are stacked and the $W \rightarrow \mu\nu_\mu$ sample is overlain for clarity. The cut is at $M_{\mu\mu} < 15 \text{ GeV}/c^2$. The darkened regions of the plot shows the portion of the samples that are excluded from further selection.

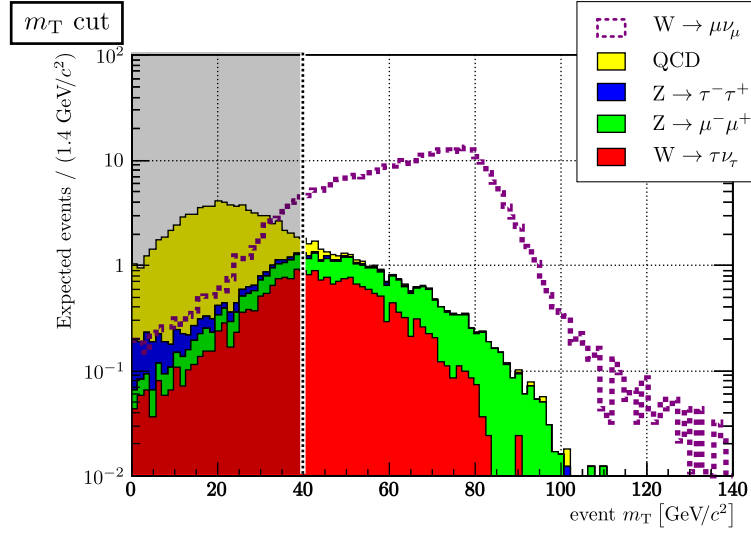


Figure 4.8: The results of the event m_T cut on the combined Monte Carlo sample. The background samples are stacked and the $W \rightarrow \mu\nu_\mu$ sample is overlain for clarity. The cut is at $m_T > 40 \text{ GeV}/c^2$. The darkened regions of the plot shows the portion of the samples that are excluded from the overall selection.

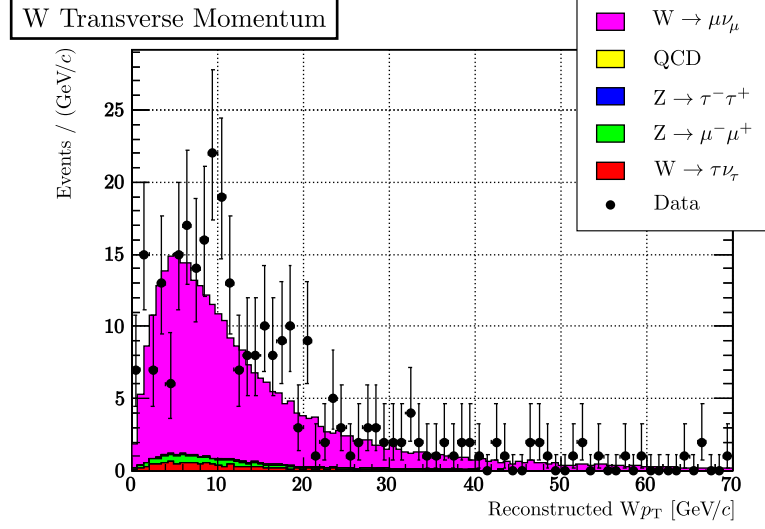


Figure 4.9: Reconstructed transverse momentum of the W boson from data and Monte Carlo after selection cuts. The Monte Carlo samples are stacked, while the data points are overlain for comparison.

m_T spectrum by eliminating the computation time of the second numerical integral in equation (2.9).

Combining these cuts and applying them to the $W \rightarrow \mu\nu_\mu$ MC gives an overall cut efficiency of $64.1\% \pm_{-2.2}^{+1.8}\%$ (stat.). The sequence of cuts and their effects on the individual MC samples are detailed in Table 4.2. Figure 4.10 shows the m_T distribution of the combined Monte Carlo events that survive the aforementioned selection criteria. Plotted over this distribution is the distribution of data events. The plot demonstrates the agreement between the data and detector simulation in this region of m_T .

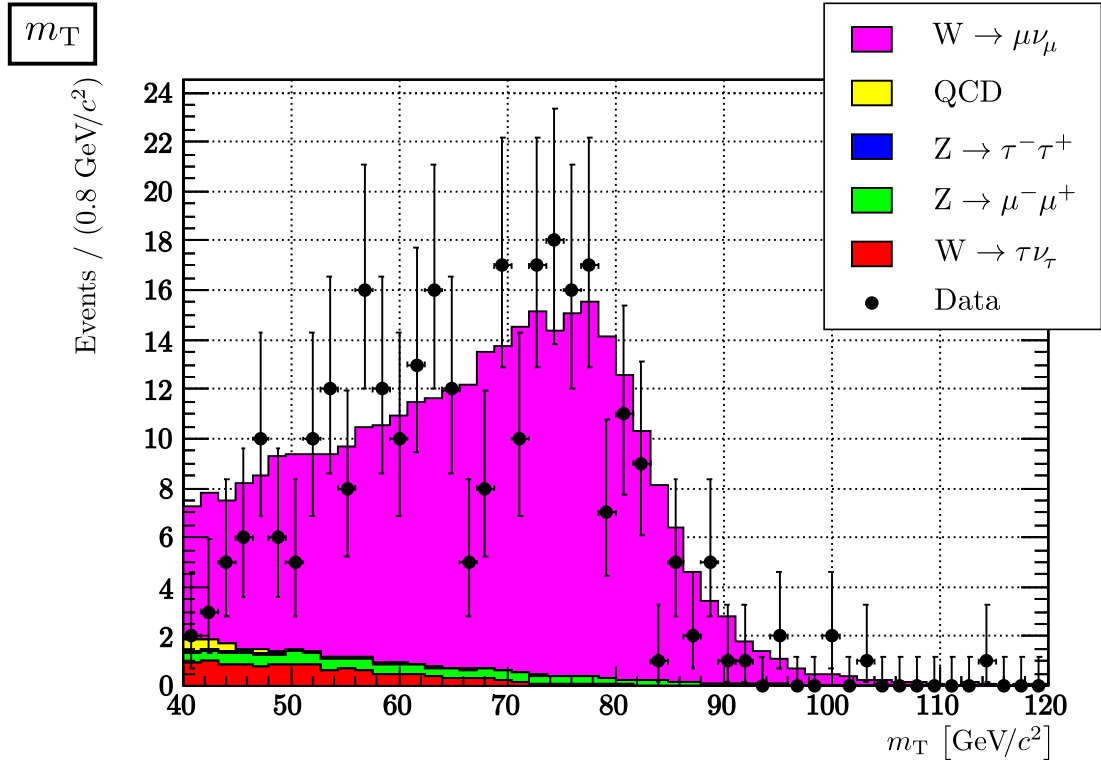


Figure 4.10: A comparison of the transverse mass distribution of the Monte Carlo and data samples for events that survive the selection criteria.

Sample	HLT_Mu9	Tracker muon	Events remaining after cut						Efficiency [%]
			p_T	$ \eta $	$ d_0 $	Muon isolation	Z-veto	m_T	
$W \rightarrow \tau \nu_\tau$	41.49	40.48	24.69	23.18	23.18	21.48	21.21	12.48	30.08
$Z \rightarrow \mu^- \mu^+$	73.86	73.26	59.31	54.29	54.29	47.35	15.62	11.61	15.71
$Z \rightarrow \tau^- \tau^+$	7.773	7.610	4.601	4.371	4.371	3.367	2.935	0.6118	7.871
QCD	6 660	6 448	5 046	4 892	4 890	66.06	63.62	1.947	0.02923
All Background	6 783	6 569	5 135	4 974	4 972	138.3	103.4	26.65	0.3928
$W \rightarrow \mu \nu_\mu$	426.6	417.0	386.4	364.6	364.6	351.4	346.6	311.7	73.08
Combined MC	7 210	6 986	5 521	5 338	5 337	489.6	450.0	338.4	4.693
Data	25 200	22 486	3 716	3 575	3 575	499	458	302	-

Table 4.2: Trigger and selection cut efficiencies for MC samples and data. Cuts are applied successively from left to right. Monte Carlo samples are scaled to match the integrated luminosity of the data (70 nb^{-1}). The overall efficiencies for each cut on the $W \rightarrow \mu \nu_\mu$ MC sample are tabulated in Table 5.1.

4.3 Luminosity measurement

An important part of an accurate cross-section measurement is an accurate accounting for the integrated luminosity of the LHC at the CMS detector. The integrated luminosity at the CMS detector will eventually be measured by counting the number of events seen that have a well-known cross-section and a signal that can be selected with high efficiency. The process especially targeted for this method is the $Z \rightarrow \mu^- \mu^+$ decay[42]. However, this method of calculating integrated luminosity requires a large number of $Z \rightarrow \mu^- \mu^+$ events to beat down statistical errors. Also, because the LHC collision energy is the highest ever recorded in experimental history, the cross-section of the $Z \rightarrow \mu^- \mu^+$ process must first be verified against theoretical predictions before it is used as a “standard candle” for other processes.

Off-line calculation of the integrated luminosity at CMS for early running uses so-called “minumum-bias” events—events that pass any of the HLT physics trigger bits. These events are counted and compared to Monte Carlo simulations to estimate the luminosity[72]. This method is only as accurate as the Monte Carlo simulation of the complex, low-energy QCD events that comprise most of the minimum-bias sample and so is not as reliable as the counting of $Z \rightarrow \mu^- \mu^+$ events.

The *instantaneous* luminosity of the LHC machine can be calculated at CMS with almost-live results. Two methods using the HF sub-detector and one method using trigger rates are employed for this measurement[72]. The first (the so-called “zero-counting” method) relies on the fact that the number of p-p interactions per bunch-crossing follows a Poisson distribution with a mean given by

$$\mu = \frac{\sigma \mathcal{L}}{f}.$$

In this equation, σ is the total inclusive p-p interaction cross-section, $\mathcal{L}$ is the instantaneous luminosity of the beams, and f is the frequency of bunch-crossings. Even at low

LHC luminosities, the mean is expected to be ~ 2.5 events per crossing, which means fewer than 8 % of the crossings will be collision-free. This number only decreases as the luminosity of the LHC increases, so counting non-colliding bunch-crossings is not statistically feasible. A solution to this problem is to count the number of HF towers that have zero occupancy per bunch-crossing. If there is some probability p that a given HF tower will be hit with products from a single collision, then, for a large number of towers, the number of non-hit “empty” towers will be binomially distributed with probability p . The fraction of towers not hit in some large number of bunch-crossings with a Poisson-distributed number of interactions μ will therefore be

$$\langle f_0 \rangle = \sum_{k=0}^{\infty} \frac{e^{-\mu} \mu^k}{k!} p^k = e^{\mu(p-1)}.$$

Thus, once p is measured from simulation, this equation can be used to calculate μ , and hence $\mathcal{L}$.

The second method to calculate the instantaneous luminosity also relies on μ from above, but relies on the assumption that the total amount of transverse energy deposited in the HF towers per bunch-crossing is linearly correlated with the number of collisions in the bunch-crossing[\[42\]](#). This method requires a better understanding of the E_T resolution and scaling of the HF than the previous method, but is neither affected by possible cut-off decisions for identifying empty towers, nor will it saturate at high luminosities.

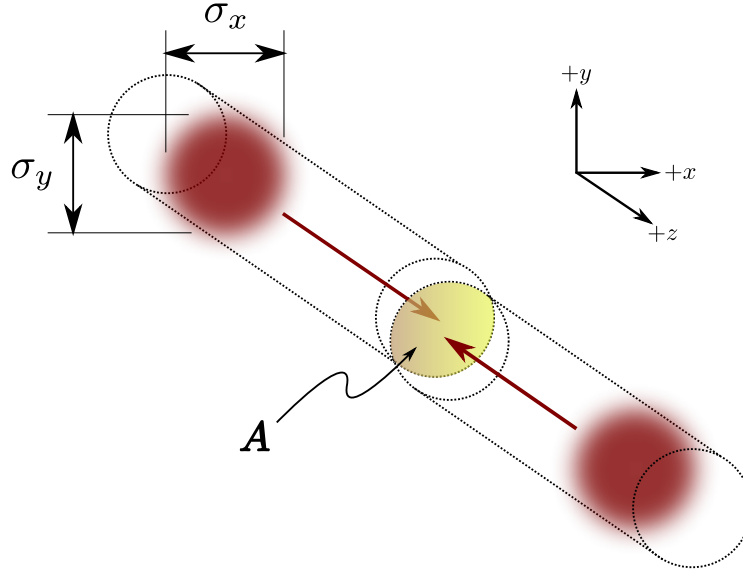
A third method for measuring the luminosity of the beams does not explicitly require counting using the CMS HF sub-detector. One can directly measure the physical properties of the beams by measuring their geometric parameters. The instantaneous luminosity of two bunches with N_1 and N_2 particles, each colliding at a rate of f , is given by

$$\mathcal{L} = \frac{N_1 N_2 f}{A}. \tag{4.1}$$

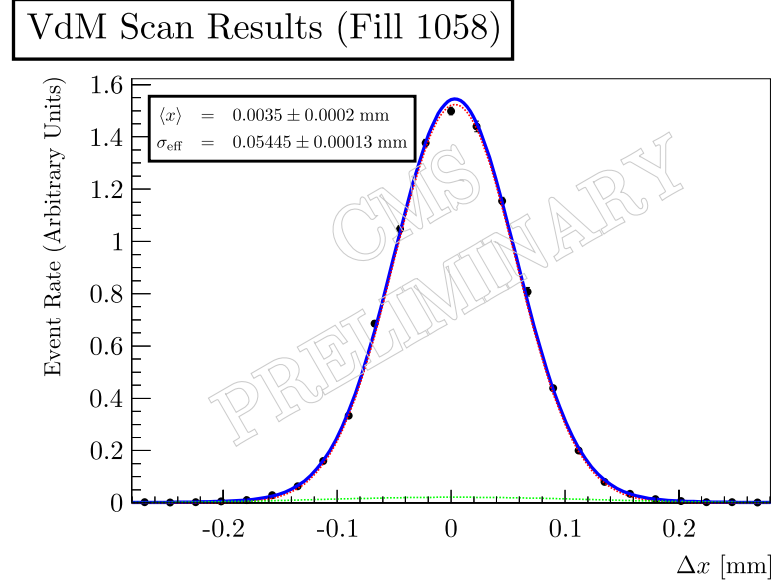
In this equation, A is the effective transverse area in which the collisions occur. If the beams have the same two-dimensional Gaussian transverse profile with widths σ_x and σ_y in the x and y dimensions respectively, and if they collide head-on, the effective transverse area is $4\pi\sigma_x\sigma_y$. The frequency of collisions f is set by the LHC hardware and is very accurately known. The number of particles in each bunch is measured by fast beam current transformers to within 1 %[\[2\]](#), so the main source of uncertainty comes from the geometric size of the beams.

To measure the transverse size of the beams, a method known as Van der Meer scanning is used[\[73\]](#). In the scanning process, the central transverse position of one beam is slowly shifted relative to the other along the two transverse axes sequentially. Events are observed in the CMS detector while the beams are being moved, and the width of the beams can be determined by plotting the event rate as a function of beam offset. [Figure 4.11](#) shows a sample Van der Meer scan along the x -axis taken during LHC running[\[72\]](#). The Gaussian shape of the beam along the x -axis is directly responsible for the Gaussian shape of the scan results.

Averaging these three methods to calculate the instantaneous luminosity per bunch-crossing and using the off-line counting of minimum-bias events gives a total integrated luminosity for the data used in this analysis of $\int \mathcal{L} dt = 70 \text{ nb}^{-1} \pm 7.7 \text{ nb}^{-1}$. A systematic uncertainty of 11 % is quoted as a conservative estimate of the performance of the three methods listed above[\[72\]](#).



(a)



(b)

Figure 4.11: Beam-profile diagram (a) and results (b) from a Van der Meer scan along the x -axis at CMS[72]. In Figure (a), the red clouds represent proton bunches, each with a Gaussian profile in the transverse plane. The area in the transverse plane where they intersect is the effective transverse area A in equation (4.1). The Gaussian shape of the beam in the x -direction is directly responsible for the Gaussian shape of the scan results in Figure (b). A double Gaussian shape is used in this particular fit, the blue curve representing the sum of the red and green curves.

Chapter 5

RESULTS

This section will present the results of the $\sigma_{\text{pp} \rightarrow \text{W}} \times \text{BR}(\text{W} \rightarrow \mu\nu_\mu)$ and m_{W} measurements using the sample of data and Monte Carlo events selected in the previous chapter. The cross-section will be measured using a simple counting strategy, while the m_{W} measurement will be obtained by performing a fit to the m_{T} distribution of data events in Figure 4.10.

5.1 $\sigma_{\text{pp} \rightarrow \text{W}} \times \text{BR}(\text{W} \rightarrow \mu\nu_\mu)$ calculation

Using the number of expected signal events from the fit in Table 4.2, the calculation of the cross-section $\sigma_{\text{pp} \rightarrow \text{W}} \times \text{BR}(\text{W} \rightarrow \mu\nu_\mu)$ is straight-forward. Solving for the cross-section in (3.2) gives

$$\sigma_{\text{pp} \rightarrow \text{W}} \times \text{BR}(\text{W} \rightarrow \mu\nu_\mu) = \frac{N_{\text{W} \rightarrow \mu\nu_\mu}}{\int dt \mathcal{L}}.$$

Here, $N_{\text{W} \rightarrow \mu\nu_\mu}$ represents the total number of $\text{W} \rightarrow \mu\nu_\mu$ decays occurring in the CMS detector for the integrated luminosity $\int dt \mathcal{L}$ and not simply the number reconstructed. This number can be estimated from the data by subtracting from the number of events that pass the selection criteria from Chapter 4 the fraction of background events expected from the Monte Carlo. Using the values in Table 4.2, the fraction of actual $\text{W} \rightarrow \mu\nu_\mu$ events in the data that survived the selection cuts is $92.1 \% \pm 0.8 \%$.

This suggests that the number of signal events in the 302 data events is 278 ± 2.4 . An estimate of the true number of $W \rightarrow \mu\nu_\mu$ decays that occurred in the detector can be written as

$$N_{W \rightarrow \mu\nu_\mu} = \frac{N_{\text{sig}}}{\epsilon},$$

where ϵ , the selection efficiency for $W \rightarrow \mu\nu_\mu$ events, has been studied in Chapter 4 using the Monte Carlo simulation and decomposed into individual components representing each cut:

$$\epsilon = \epsilon_{\text{trig}} \times \prod_i^{\text{all cuts}} \epsilon_i.$$

The results from the efficiency studies in Section 4.2.3 are summarized in Table 5.1. From the 302 data events that pass the the selection criteria, along with the luminosity measurement given in Section 4.3, the cross-section times muon decay branching ratio of the W boson is calculated to be

$$\begin{aligned} \sigma_{\text{pp} \rightarrow W} \times \text{BR}(W \rightarrow \mu\nu_\mu) \\ = 9\,013 \text{ pb} \pm 540 \text{ pb (stat.)} \pm 552 \text{ pb (sys.)} \pm 991 \text{ pb (lumi.)}. \end{aligned}$$

This value compares well to the theoretical value 10 400 pb as calculated to Next-to-Leading Order (NLO) in α_s using MCFM and the MSTW2008 parton distribution functions. The calculation of the statistical uncertainty simply assumes a Poisson distribution of the number of $W \rightarrow \mu\nu_\mu$ events generated in the 70 nb^{-1} of data collected. The calculation of the systematic uncertainties for this measurement follows.

5.1.1 Systematic uncertainties for the $\sigma_{\text{pp} \rightarrow W} \times \text{BR}(W \rightarrow \mu\nu_\mu)$ calculation

The accuracy of the detector simulation is an important source of systematic uncertainties, especially in this and all early studies at CMS. Many of the parameters

Cut	Efficiency [%]	Uncertainty [%]
Generator-level $ \eta < 2.5$	73.9	± 1 (conservative)
L1	95.0	± 1 (conservative)
HLT_Mu9	86.0	± 0.6
tracker + global muon	97.7	± 0.5
muon $p_T + \eta $	87.4	± 0.8
muon $ d_0 $	100	$^{+0.0}_{-0.6}$
muon isolation	96.4	± 0.6
Z-veto	98.6	± 0.6
m_T	89.9	± 0.6
Total	44.1	± 2.1

Table 5.1: Summary of cut efficiencies as described in Section 4.2.3. The statistical uncertainties from that section are propagated as systematic uncertainties on the cross-section measurement.

that enter into this calculation of m_W and $\sigma_{pp \rightarrow W} \times \text{BR}(W \rightarrow \mu\nu_\mu)$ are derived from simulation values. The selection efficiencies are all directly dependent on comparison to Monte Carlo events, as are the fitted shape of the background distribution and the detector m_T reconstruction resolution used to smear the transverse mass PDF. The process of tuning the CMS detector simulation to match the response of the real data involves carefully studying the finest details of the data. Everything from the overall simulated event reconstruction to the simulated signal timings of the detector hardware components is compared to real data as they are collected, and adjustments to the simulation parameters are made in response. This is an iterative process that requires data samples with very large statistics, so, as CMS continues to take data, the accuracy of the simulation will continue to improve.

Detector reconstruction resolution affects the selection efficiencies quoted in Section 4.2.3 as well as the measured m_T distribution. Several studies were performed using recent collision data to understand the discrepancies between the detector re-

sponse and simulation[70, 74]. The studies pertinent to this analysis are detailed here.

Trigger studies using complementary HLT algorithms (such as one- and two-muon triggers at the same threshold muon p_T) have been performed using $Z \rightarrow \mu^- \mu^+$ and $J/\psi \rightarrow \mu^- \mu^+$ events in both simulation and data to determine the efficiency of the muon triggers. By matching one trigger and looking for the complementary trigger acceptance in events reconstructed as $Z \rightarrow \mu^- \mu^+$ or $J/\psi \rightarrow \mu^- \mu^+$ decays, the HLT_Mu9 trigger algorithm shows a 3.2 % discrepancy between simulated response and the response in data.

The tracker muon tagging algorithm was studied using tracks that were well-reconstructed in the muon system. These tracks were propagated to the inner tracking sub-detectors to search for tracker muons. Muons that then matched a tracker muon were counted toward the reconstruction efficiency, while events wherein no tracker muon was found within a certain ΔR of the stand-alone muon were counted against the efficiency. A 3 % difference between the data and simulation response was found using this “tag-and-probe” method.

Muon isolation was studied using a “random-cone” technique, in which isolation cones were thrown in random directions in data and Monte Carlo events (being sure to avoid reconstructed muons). The isolation variables were then calculated for these cones, and rates of false positives were discovered (a false-positive being a random cone that appears to be non-isolated). A systematic discrepancy of 0.5 % was found between data and Monte Carlo for an isolation criterion similar to the one used in this analysis.

Another acceptance uncertainty is introduced by muon momentum scale resolution. The momentum scale difference between two muons is found in data by comparing the two daughter muons of low-mass resonance decays, as well as by splitting

cosmic ray muons at the point of closest approach to the detector interaction vertex and comparing the momenta of the two legs. The differences between Monte Carlo and data momentum scales lead to a selection uncertainty of 1 %.

The uncertainty in the reconstruction of the event $\cancel{E}_T$ leads to a selection uncertainty of 1 %, as studied using photon-plus-jet events in data[75]. This also leads to an uncertainty in the fit of the W mass, but this is folded into the uncertainties in the m_T resolution fitting.

Finally, systematic uncertainty can stem from the choice of event generators used to create the Monte Carlo event samples. Because physicists are only now beginning to understand what p-p collision events at $\sqrt{s} = 7$ TeV look like, the generated Monte Carlo has to rely on measurements of physical parameters at lower energy scales. These parameters may have much different values at higher collision energies. This is especially true for the proton parton distribution functions. These functions specify the probability that a given parton in a proton (a sea quark, valence quark, or gluon) carries a specific fraction of the proton's momentum. These functions are relied on heavily by Monte Carlo generators to produce the parton-level interactions at the origin of each collision event. These functions change as collision energies increase, mainly due to the increased resolution of the sea quarks to an incident probe parton.

Different collaborations have attempted parameterizations of the proton parton distribution functions. The specific parameterization used in this analysis to calculate cross-sections is MSTW2008[69]. A study was performed using scaling factors to adjust the physical parameters of the generator-level particles to simulate other parton distribution functions. This produced an over-all systematic uncertainty of 2 % in the acceptance calculation. Other simulation-based uncertainties such as initial- and final-state radiation effects and renormalization scale assumptions in theoretical

Source	Systematic uncertainty [%]
HLT_Mu9	3.2
Global/Tracker muon identification	3.0
Muon isolation	0.5
Muon momentum scale	1.0
$\cancel{E}_T$ resolution	1.0
Parton distribution functions	2.0
Other theoretical uncertainties	1.5
Total	5.7

Table 5.2: Systematic uncertainties affecting the W cross-section measurement[70, 74, 64].

calculations lead to an added systematic uncertainty in both the acceptance and m_T calculation of 1.5 %.

The details of the selection uncertainties are shown in Table 5.2. The statistical uncertainties in the selection efficiencies from Chapter 4 are also propagated to this measurement as systematic uncertainties. They are summarized in Table 5.1.

5.2 Fitting procedure for the measurement of m_W

The actual fitting of the m_T distribution in data is performed using the data modeling package RooFit for the statistics and analysis package ROOT[59, 76]. The parameterized m_T cross-section PDF for the $W \rightarrow \mu\nu_\mu$ decay is not a standard PDF built into the RooFit framework, so a customized version of the distribution (parameterized in m_W , Γ_W , and W p_T) had to be developed. The C++ code used to generate this PDF is included in Appendix B. All fits in this analysis are performed using unbinned

maximum likelihood methods^a calculated with the MINUIT minimization and error analysis toolkit[77].

5.2.1 Transverse mass PDF resolution smearing

The reconstruction of the muon and neutrino transverse momenta from $W \rightarrow \mu\nu_\mu$ decays in the CMS detector is not exact, so that the reconstructed m_T distribution in the detector, even from the pure $W \rightarrow \mu\nu_\mu$ MC sample, will not look exactly like the ideal distribution from Figure 2.4. To account for resolution effects, the PDF for the transverse mass distribution given in equation (2.8) is convoluted in m_T with a detector resolution function. The resolution function is measured directly from the $W \rightarrow \mu\nu_\mu$ Monte Carlo sample by subtracting the m_T calculated using generator-level muon and neutrino information from the m_T calculated using reconstructed values. The selection criteria of Section 4.2.3 are applied to the $W \rightarrow \mu\nu_\mu$ Monte Carlo sample before the resolution function is found to ensure any biases to the m_T distribution due to this selection are considered.

The resulting resolution histogram was used as a binned look-up table for numeric convolution^b with the “ideal” transverse mass PDF. Figure 5.1 shows the resolution histogram along with a comparison between the ideal transverse mass PDF and the convoluted PDF.

^aIf a distribution of measurements x_i are thought to come from a family of PDFs $f(x; \theta)$ parameterized by the set of parameters θ , then the likelihood function for some choice of θ called $\hat{\theta}$ is defined

$$L(\hat{\theta}; x) \equiv \prod_i f(x_i; \hat{\theta}),$$

assuming the set of measurements x are independent and identically distributed.

^bConvolution of two PDFs $f(x)$ and $g(x)$ is defined as the operation

$$\begin{aligned} (f * g)(x) &= \int_{-\infty}^{\infty} f(x')g(x - x')dx' \\ &= \int_{-\infty}^{\infty} f(x - x')g(x')dx'. \end{aligned}$$

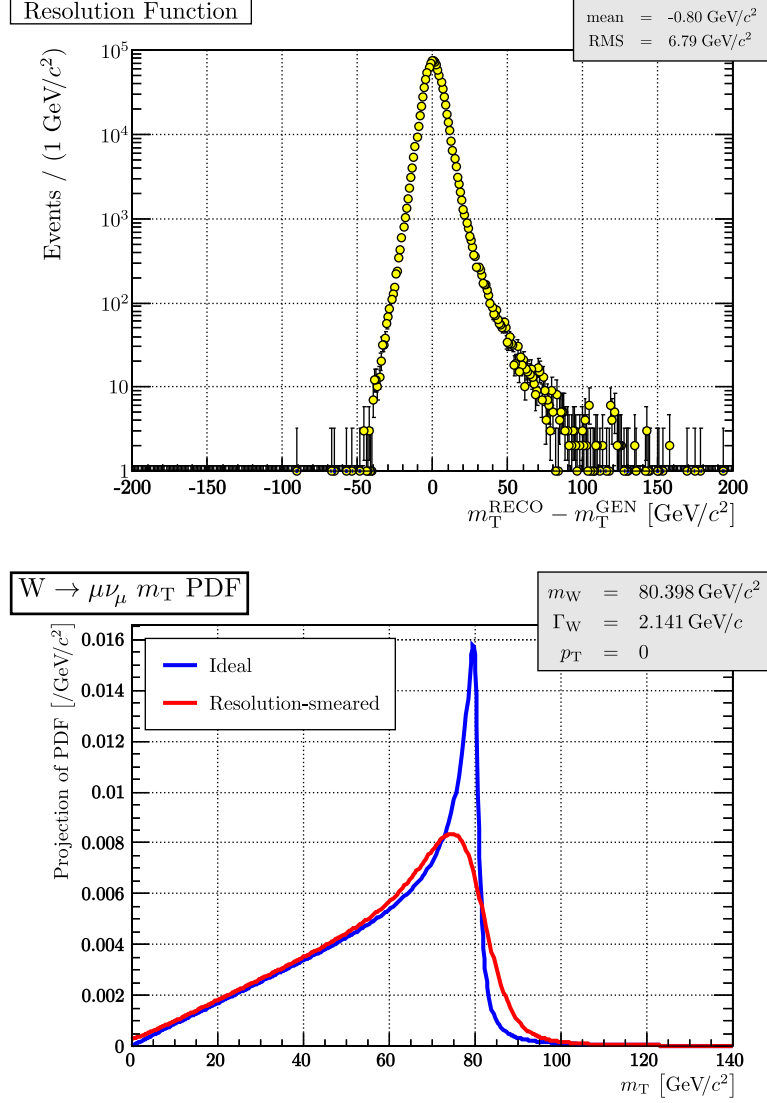


Figure 5.1: (a) m_T reconstruction resolution histogram (as calculated using $W \rightarrow \mu\nu_\mu$ Monte Carlo and the cuts described in Chapter 4) and (b) the convoluted transverse mass PDF. The transverse mass PDF in (b) uses the ideal parameters $m_W = 80.398$ GeV/c², $\Gamma_W = 2.141$ GeV/c, and $p_T = 0$.

To prove that the convoluted PDF accurately represents the reconstructed m_T resolution, it was fitted directly to the m_T distribution of the $W \rightarrow \mu\nu_\mu$ signal Monte Carlo. Figure 5.2a shows the *generated* m_T distribution from the full $W \rightarrow \mu\nu_\mu$ Monte Carlo sample (after all cuts on the reconstructed variables, scaled to 70 nb^{-1}) with the ideal transverse mass PDF fit to it. By contrast, Figure 5.2b shows the *reconstructed* m_T distribution from the same dataset and a fit of the resolution-smeared PDF. The result for fitting the mass parameter is $m_W^{\text{GEN}} = 80.5 \text{ GeV}/c^2 \pm 0.4 \%$ for the ideal distribution and $m_W^{\text{RECO}} = 80.4 \text{ GeV}/c^2 \pm 1.0 \%$ for the reconstructed distribution, comparing favorably to the true generated value of $m_W = 80.398 \text{ GeV}/c^2$. The width parameter Γ_W is also fit in the ideal m_T distribution as a consistency check. The fit result for the generated m_T distribution from the $W \rightarrow \mu\nu_\mu$ MC sample is $\Gamma_W^{\text{GEN}} = 1.92 \text{ GeV}/c \pm 22 \%$, in agreement with the value of $2.141 \text{ GeV}/c$ used in MC generation. The effective width of the resolution function is larger than the expected width of the W boson ($\sim 7 \text{ GeV}/c^2$ versus $2.141 \text{ GeV}/c$), so resolution effects will dwarf the widening of the m_T peak due to the Γ_W parameter (*cf.* Figures 2.5b and 5.1b). Therefore, in the following fits the Γ_W parameter is considered fixed at the experimentally-measured value of $2.141 \text{ GeV}/c$ [11].

5.2.2 Background fitting

After applying the selection cuts detailed in the previous sections, Figure 4.10 suggests that a non-negligible amount of low- m_T events in the fitting region $m_T > 40 \text{ GeV}/c^2$ come from background processes. To account for these background events in the data, a background PDF is added to the $W \rightarrow \mu\nu_\mu$ transverse mass PDF. This background PDF was chosen to be Gaussian in form based on inspection of the MC background m_T distribution. A single Gaussian was chosen instead of a more complicated function to avoid over-fitting to the MC distribution. The width (σ_{bkg})

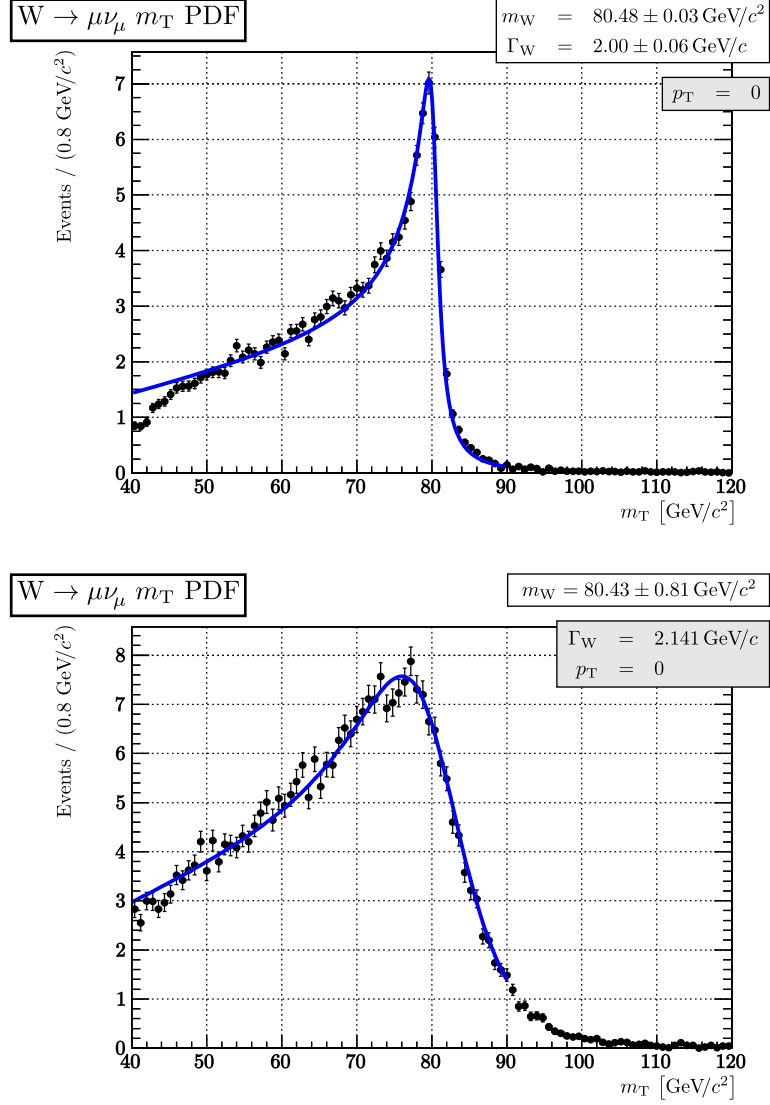


Figure 5.2: Fits of the (a) ideal (b) and resolution-smeared transverse mass PDFs to generated and reconstructed $W \rightarrow \mu\nu_\mu$ Monte Carlo m_T distributions (respectively). The m_T distributions shown are obtained after the cuts described in Chapter 4 are applied to the reconstructed variables. The Monte Carlo sample is scaled to the expected number of events at 70 nb^{-1} . The difference between the fitted result of the convoluted PDF and the generated value of $80.398 \text{ GeV}/c^2$ results in a systematic error in the fitting procedure due to the convolution process and the accuracy of the resolution histogram of less than 0.01 %.

and mean ($\langle m_T \rangle_{\text{bkg}}$) parameters of the Gaussian were fit to the m_T distribution from the full, appropriately weighted sample of combined background MC events using a maximum-likelihood method, and the fitting range is truncated to $m_T \in [40, 90]$ to reduce likelihood maximization calculation errors by eliminating from the fit those values of m_T that lie mostly in the extreme upper tail of the distribution^c. The results of the fit are:

$$\begin{aligned}\langle m_T \rangle_{\text{bkg}} &= 25.6 \text{ GeV}/c^2 \pm 51 \% \\ \sigma_{\text{bkg}} &= 27.1 \text{ GeV}/c^2 \pm 23 \%.\end{aligned}$$

The m_T distribution of the combined background MC is pictured in Figure 5.3 along with the fitted Gaussian PDF and the fit pulls^d. Figure 5.4 shows the negative-log-likelihood plots for the two fitted parameters.

5.2.3 Determination of m_W by combined signal and background model fitting

Finally, the background Gaussian PDF is added to the resolution-convoluted transverse mass PDF and fit to the m_T distribution from the 302 data events that passed the selection criteria of Chapter 4. The fitted parameter of the combined PDF is the W mass alone—the relative normalization of the signal PDF to the total signal plus background PDF is fixed to $92.1 \pm 7.9 \%$ as calculated from the Monte Carlo selection efficiency for the signal and combined background samples (Table 4.2). Including the

^cIt is not shown, but the resolution function shown in Figure 5.1a is also a function of the generated m_T , generally becoming wider as m_T^{GEN} increases. This truncation of the fitting range also removes from consideration the values of m_T^{GEN} in the upper tails where the statistics of the resolution function are limited.

^dIf the measured value of a distribution in x at a given x_i is $D(x_i)$, the error on that measurement is $\delta(x_i)$, and the value of the fitted distribution F at that point is $F(x_i)$, then the pull P of the fit at that point is defined as

$$P(x_i) = \frac{D(x_i) - F(x_i)}{\delta(x_i)}.$$

The pull of a point with zero measurement error is undefined, but is shown at zero in these plots.

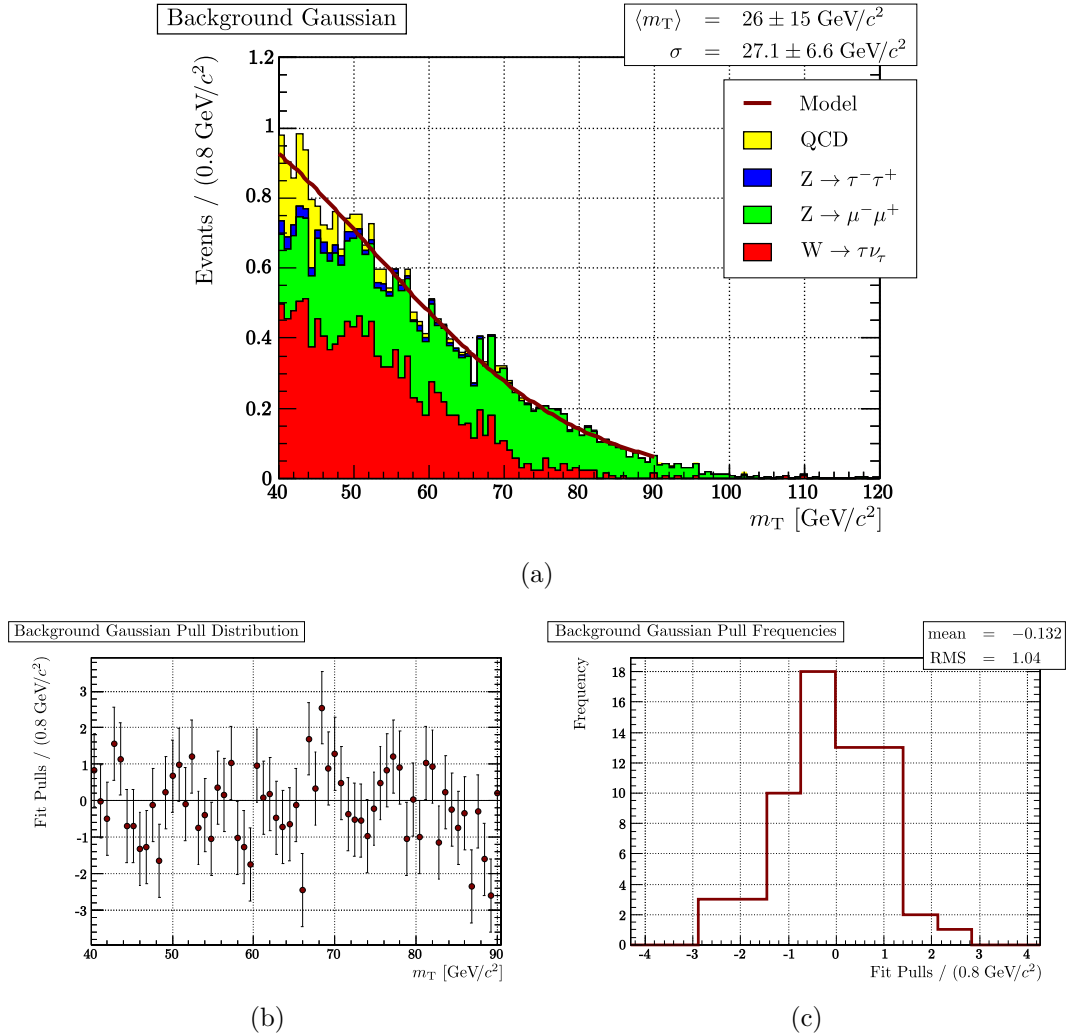


Figure 5.3: Gaussian PDF (a) fit results and (b) pull distribution for the combined background Monte Carlo sample. Also shown is (c) the pull frequency distribution. The small mean and RMS of the pull frequency distribution demonstrate the quality of the fit. The background MC samples are normalized to 70 nb⁻¹.

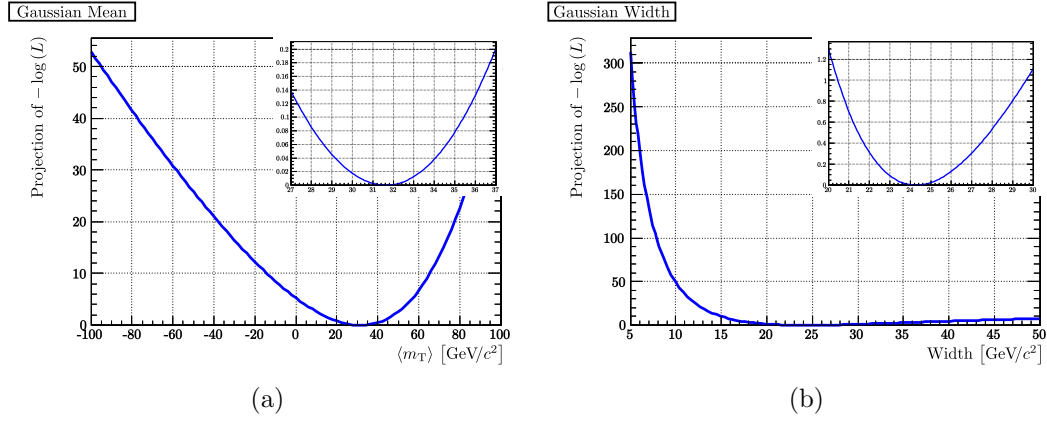


Figure 5.4: Negative-log-likelihood projections for Gaussian PDF fit results in (a) $\langle m_T \rangle_{\text{bkg}}$ and (b) σ_{bkg} . These plots are shifted so that the minima lie at zero.

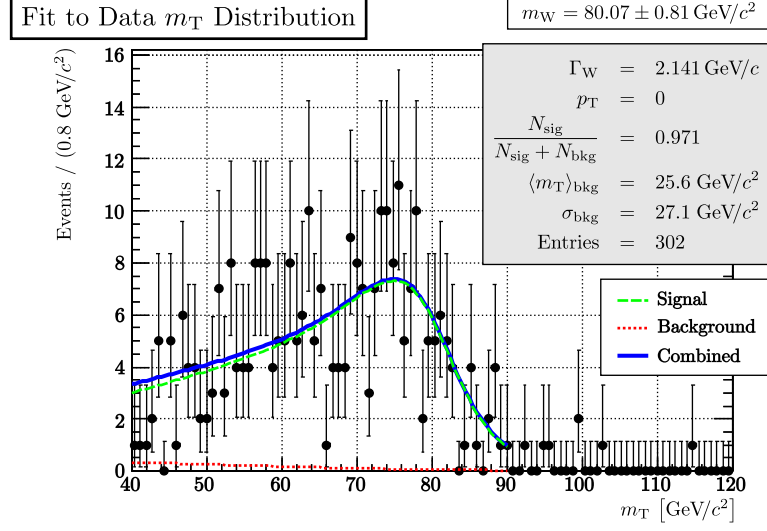
statistical and systematic errors to be discussed later, the results from the fit are:

$$m_W = 80.07 \text{ GeV}/c^2 \pm 0.64 \text{ GeV}/c^2 (\text{stat.}) \pm 0.48 \text{ GeV}/c^2 (\text{sys.}).$$

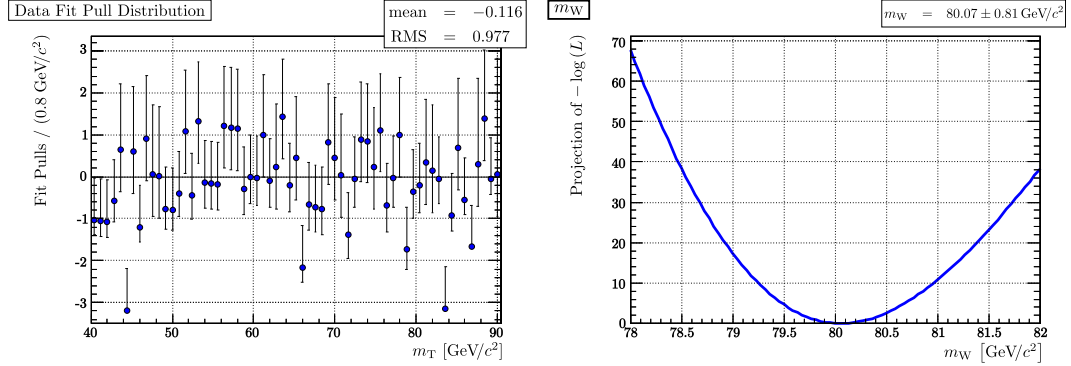
No uncertainty due to the luminosity measurement is quoted, as this is expected only to affect the normalization of the m_T spectrum (and hence the cross-section measurement), not the shape of the curve. The W mass result is consistent with the world-average experimentally-measured value of $80.398 \text{ GeV}/c^2 \pm 0.025 \text{ GeV}/c^2$. The fit to the data is shown in Figure 5.5, including the fit pull distribution and a negative-log-likelihood projection for the fitted parameters. The normalization of the PDF is fixed to the number of data points observed (302).

5.2.4 Model specification uncertainties in the m_W fit

The model used to fit the W transverse mass distribution has several components that each introduce an uncertainty into the W mass measurement. The `RooFit` fitting package has associated with it an uncertainty in the calculation of the mass parameter, part of which is due to the systematics of the fitting algorithms, and part



(a)



(b)

(c)

Figure 5.5: Fit results from (a) a resolution-convoluted transverse mass with background Gaussian PDF fit to data, including (b) fit pulls. Also shown is (c) the negative-log-likelihood distribution of the W mass parameter. The uncertainty shown in the plot is a convolution of the statistical and systematic uncertainty of the shape fitting. These uncertainties are untangled using the method described in Appendix A.

of which is due to the limited statistics of the sample. The decoupling of these two is done using the pseudo-experiment method described in Appendix A. Using this method, the statistical uncertainty in the W mass measurement is found to be 1 %, and the contribution to the systematic uncertainty by the fitting algorithm due to the convoluted analytical shape not exactly reproducing the shape of the m_T distribution is 0.5 %.

The specification of the background shape has an uncertainty associated with it due to both the choice of the Gaussian shape itself and the determination of the Gaussian parameters from the fit to the background MC sample. The systematic uncertainty due to the parameters of the Gaussian background shape was studied by varying these parameters according to the uncertainties in the fit results. The new background PDFs were individually added to the resolution-smeared transverse mass PDF and fit to the m_T distribution from the selected data. The effect on the measurement of m_W was observed to be 0.2 %.

The uncertainty due to the choice of the background model was estimated by attempting fits on the data sample using the next-best choices of background models. The two next-best background models (in terms of minimizing the RMS of the pull distributions) were a Landau^e function and a Breit-Wigner function. Figures 5.6 and 5.7 show the fit results and pulls for attempted fits of Landau and Breit-Wigner PDFs, respectively. Other simple PDF fits were attempted (exponential, order-3

^eThe Landau distribution[78] is defined by the integral

$$p(x) = \frac{1}{\pi} \int_0^\infty dt e^{-t \log t - xt} \sin(\pi t).$$

The distribution used in this analysis, with a parameterized most-probable value (MPV) and width, is in fact the more general stable distribution:

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^\infty dt \exp[it\mu - |ct|^\alpha (1 - i\beta \text{sgn}(t) \Phi)] \quad \Phi = \begin{cases} \tan(\pi\alpha/2) & (\alpha \neq 1) \\ -(2/\pi) \log|t| & (\alpha = 1) \end{cases},$$

with skewness $\beta = 1$.

polynomial, and order-3 Chebychev polynomial), but only the Gaussian, Landau, and Breit-Wigner PDFs produced stable, converging fits to the background.

The resulting measurements of m_W obtained by using these different models were nearly identical to the results from the Gaussian model (< 0.01 % effect), thus this choice can be disregarded as a source of significant systematic error.

The scale of the background PDF relative to the signal PDF was determined through a Monte Carlo selection study. The statistical uncertainty in the number of expected signal events to background events was used to vary the normalization fraction $N_{\text{sig}}/(N_{\text{sig}} + N_{\text{bkg}})$ and the fit was attempted again using the standard Gaussian background shape. The statistical fluctuations in the expected number of signal events resulted in an uncertainty in the measurement of m_W of less than 0.1 %.

The explicit choice to ignore the p_T of the W in the PDF fitting leads to a systematic uncertainty of no greater than 0.3 %. This was discovered by re-fitting the data using a transverse mass PDF with the p_T parameter fixed at 50 GeV/ c —an extreme value

Finally, the m_T resolution function is affected by both the muon momentum scale resolution and the $\cancel{E}_T$ reconstruction resolution. The systematic uncertainty of the fit due to these resolutions was tested by multiplying the entries in the m_T resolution histogram by constant values of 1.1 and 0.9 to either widen or narrow the function by 10 %, the refitting the convoluted PDF to the data. The mean of the resolution histogram was also shifted by 1 GeV/ c^2 in the positive- and negative-directions to test how a possible bias would affect the results of the fit. None of these modifications altered the results of the fit by more than 0.01 %.

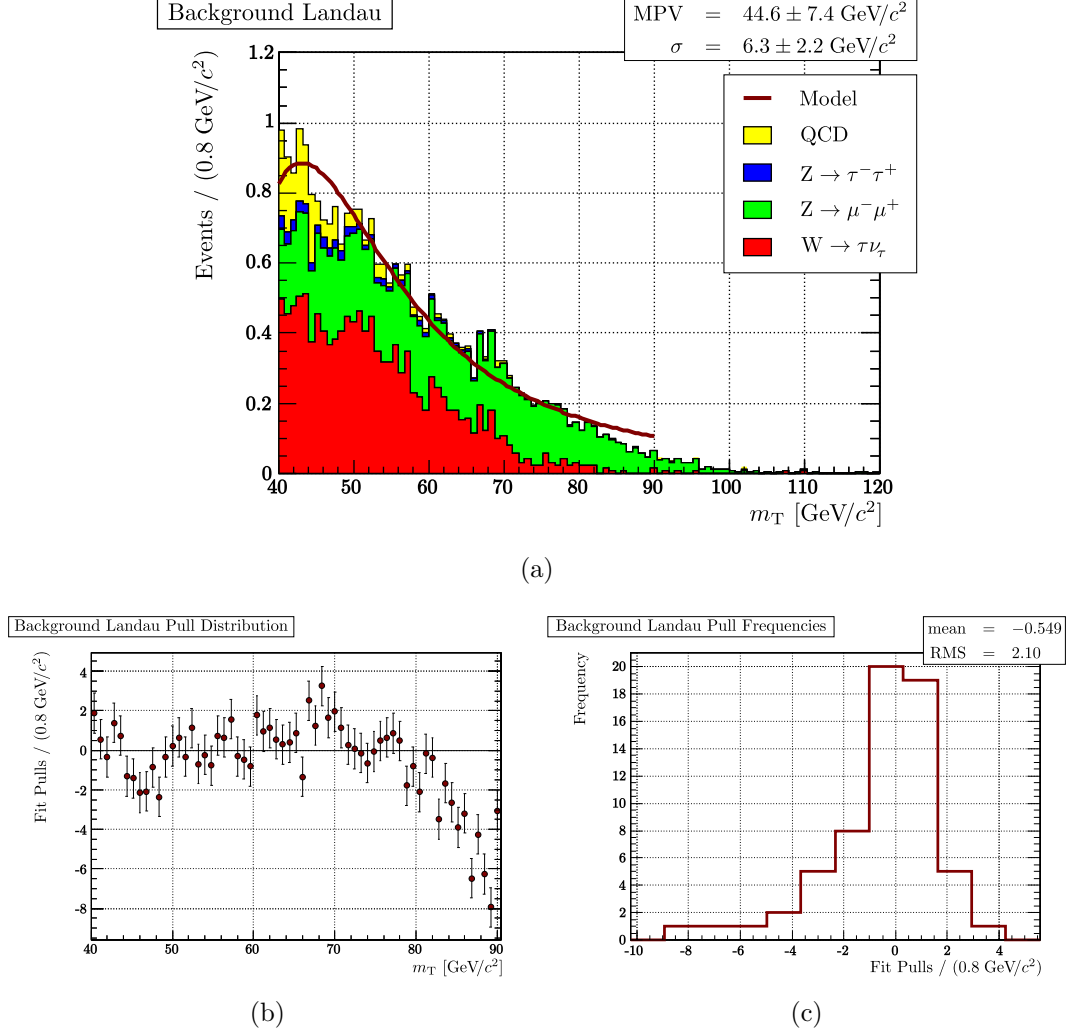
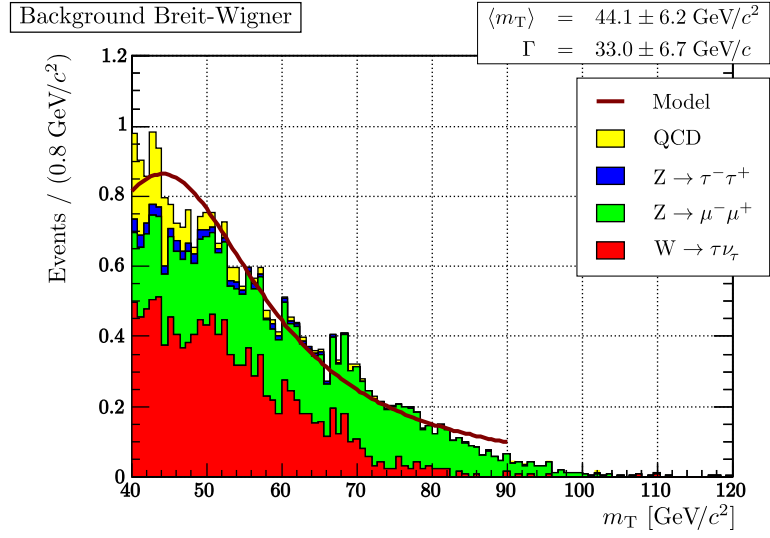


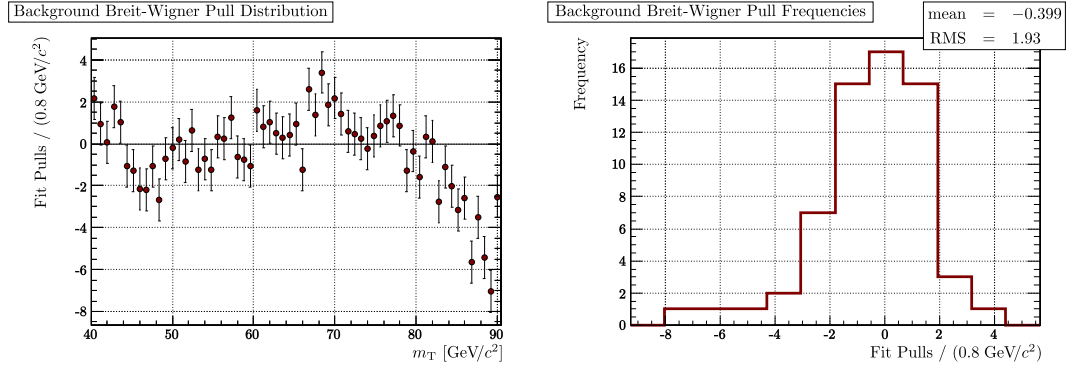
Figure 5.6: Landau PDF (a) fit results and (b) pull distribution for the combined background Monte Carlo sample. Also shown is (c) the pull frequency distribution (*cf.* Figure 5.3c). The background MC samples are normalized to 70 nb⁻¹.

Systematic	δm_W [%]
Fitting algorithm	0.5
Gaussian parameters	0.2
Background shape choice	0.1
$\frac{N_{\text{sig}}}{N_{\text{sig}} + N_{\text{bak}}}$	< 0.1
Resolution function	< 0.01
W p_T	0.3
Total	0.6

Table 5.3: Systematic uncertainties affecting the W mass measurement.



(a)



(b)

(c)

Figure 5.7: Breit-Wigner PDF (a) fit results and (b) pull distribution for the combined background Monte Carlo sample. Also shown is (c) the pull frequency distribution (*cf.* Figure 5.3c). The background MC samples are normalized to 70 nb⁻¹.

5.3 Comparison of W^+ and W^- results

The Standard Model predicts that the W^+ and W^- bosons are anti-particle partners, identical to one another in every way except for having opposite-signed quantum numbers (for instance, electromagnetic charge). This prediction can be verified in the data sample by performing separate fits for positively-charged and negatively-charged muons using the same apparatus developed for the combined $W^\pm$ sample used above. The only significant difference between the previous analyses and this one is the need to account for the muon charge identification efficiency in the cross-section measurements.

5.3.1 W^+ and W^- cross-section measurements

It is known that the CMS muon reconstruction algorithms tend to correctly identify muon charge better than 99.5 % of the time for muons with momenta $p_T < 500 \text{ GeV}/c$ [79]. Charge identification efficiency becomes worse as the muon momentum increases because charge is assigned based on the direction of track bending in the magnetic field. The amount of bending of the muon track (the sagitta) rapidly falls below detector spacial resolution for muon momenta greater than $\sim 500 \text{ GeV}/c$. The charge assignment efficiency drops to below 99 % for such muons, but, fortunately, all of the muons in the selected data sample have momenta below $200 \text{ GeV}/c$. This reconstruction efficiency is symmetric with respect to muon charge.

Though the selection criteria of Chapter 4 are agnostic to the muon charge, detector and reconstruction effects lead to different overall selection efficiencies. The selection efficiencies as calculated using the $W \rightarrow \mu\nu_\mu$ Monte Carlo sample split on reconstructed muon charge are summarized in Table 5.4.

The data sample is split between those with a reconstructed μ^+ (200 events) and those with a reconstructed μ^- (102 events). From the Monte Carlo sample, the

Muon charge	Cut	Efficiency [%]	Uncertainty [%]
+	Generator-level $ \eta < 2.5$	73.9	± 1 (conservative)
	L1	95.0	± 1 (conservative)
	HLT_Mu9	85.8	± 0.6
	tracker + global muon	97.8	± 0.6
	muon $p_T + \eta $	86.4	± 0.7
	muon $ d_0 $	100	$^{+0.0}_{-0.7}$
	muon isolation	96.2	± 0.7
	Z-veto	98.7	± 0.7
	m_T	88.7	± 0.8
	charge identification	99.5	± 0.5
Total		42.6	$^{+2.3}_{-2.4}$
−	Generator-level $ \eta < 2.5$	73.9	± 1 (conservative)
	L1	95.0	± 1 (conservative)
	HLT_Mu9	86.3	± 0.8
	tracker + global muon	97.7	± 0.8
	muon $p_T + \eta $	89.3	± 0.9
	muon $ d_0 $	100	$^{+0.0}_{-0.9}$
	muon isolation	96.7	± 0.9
	Z-veto	98.5	± 0.9
	m_T	92.0	± 0.9
	charge identification	99.5	± 0.5
Total		46.1	$^{+2.6}_{-2.8}$

Table 5.4: Summary of cut efficiencies as described in Section 4.2.3 for the $W \rightarrow \mu\nu_\mu$ Monte Carlo sample, split by reconstructed muon charge. The statistical uncertainties from that section are propagated as systematic uncertainties on the cross-section measurement.

fraction of signal events expected in each sub-sample of data is $92.8\% \pm 1.0\%$ for the μ^+ sample and $91.1\% \pm 1.2\%$ for the μ^- sample. This translates to 185.6 ± 2.0 and 92.9 ± 1.2 events, respectively. Using the selection efficiencies detailed above, the measured cross-sections for the split samples are

$$\begin{aligned}\sigma_{\text{pp} \rightarrow \text{W}^+} \times \text{BR}(\text{W}^+ \rightarrow \mu^+ \nu_\mu) &= \\ 6\,224 \text{ pb} \pm 457 \text{ pb (stat.) } {}^{+388}_{-390} \text{ pb (sys.) } \pm 685 \text{ pb (lumi.)} \\ \sigma_{\text{pp} \rightarrow \text{W}^-} \times \text{BR}(\text{W}^- \rightarrow \mu^- \bar{\nu}_\mu) &= \\ 2\,823 \text{ pb} \pm 349 \text{ pb (stat.) } {}^{+180}_{-182} \text{ pb (sys.) } \pm 311 \text{ pb (lumi.)}.\end{aligned}$$

The theoretical values for these numbers, as calculated to NLO using **MC_{CFM}** with **MSTW2008** parton distribution functions, are $\sigma_{\text{pp} \rightarrow \text{W}^+} \times \text{BR}(\text{W}^+ \rightarrow \mu^+ \nu_\mu) = 6\,130 \text{ pb}$ and $\sigma_{\text{pp} \rightarrow \text{W}^-} \times \text{BR}(\text{W}^- \rightarrow \mu^- \bar{\nu}_\mu) = 4\,270 \text{ pb}$. The W^+ result is in good agreement with the theoretical prediction, while the W^- result is almost two standard deviations smaller than the theoretical predictions. This may be partially due to the nearly 4 % difference between the W^+ and W^- selection efficiencies, suggesting possible differential biases in the reconstruction of differently charged muons.

The W^+ and W^- cross-sections are not expected to be equal due to the parton compositions of the proton beams. In order to produce a W^+ boson in p-p collisions, a u-type quark (u, c, or t) within the proton must collide with a d-type antiquark ($\bar{\text{d}}$, $\bar{\text{s}}$, or $\bar{\text{b}}$) from the other proton (see Figure 4.1). Similarly, the W^- boson can only be produced by the collision of a d-type quark and u-type antiquark. The proton is a combination of two u quarks and one d quark in an $\text{SU}(3)_c$ singlet state (the so-called “valence” quarks), along with any number of virtual quark-antiquark pairs (the so-called “sea” quarks). The W^+ and W^- bosons can therefore be created in the collision of a valence and sea quark, or the collision of two sea quarks. The valence quarks tend to carry a much larger fraction of the over-all momentum of the proton,

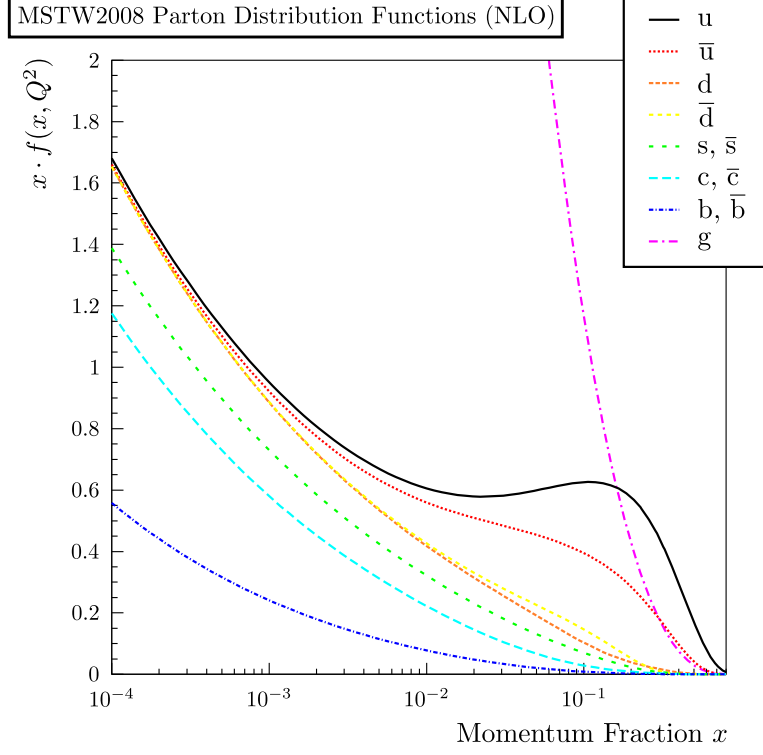


Figure 5.8: Theoretical parton distribution functions for protons at $Q^2 = 100 \text{ GeV}^2$ (roughly the W mass scale). Each curve shows the probability f that a given parton carries a given fraction x of the proton's longitudinal momentum times that fraction. These curves come from the MSTW2008 NLO calculations[69].

as the parton distribution function shown in Figure 5.8 demonstrates[69]. Because of this, the valence-sea interactions are more likely to have the momentum required to create W bosons. Because the proton has two u valence quarks, the valence-sea interaction $u\bar{d}$ is more likely than the $d\bar{u}$ interaction to have the energy required to generate a W boson. Therefore generation of the W^+ boson is more likely at the LHC than generation of the W^- boson.

5.3.2 m_{W^+} and m_{W^-} measurements

The fitting procedure for the separate μ^+ and μ^- samples follows the procedure for the combined sample step-by-step, treating each charge independently in both data

Muon charge	Gaussian parameter	Value [GeV/ c^2]	Error [%]
+	$\langle m_T \rangle_{\text{bkg}}^+$	22.6	70.8
	σ_{bkg}^+	27.7	30.4
-	$\langle m_T \rangle_{\text{bkg}}^-$	32.3	56.7
	σ_{bkg}^-	25.0	27.1

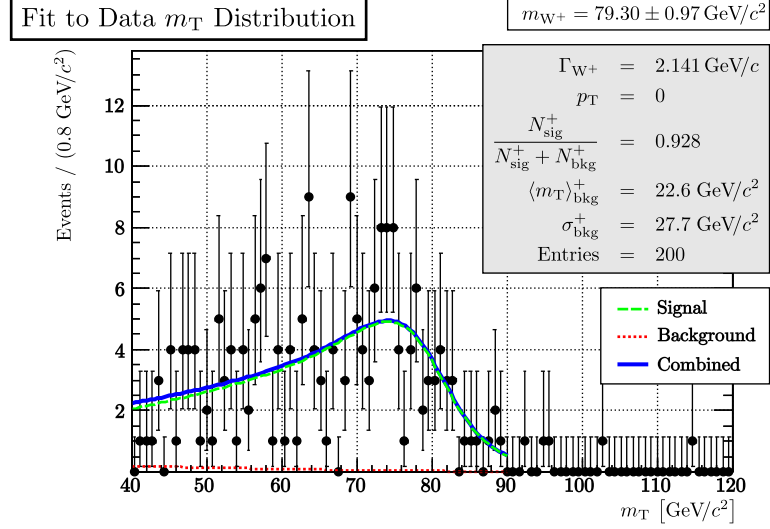
Table 5.5: Gaussian PDF fit results for the combined background Monte Carlo sample, separated by reconstructed muon charge.

and Monte Carlo. The resolution smearing histogram is assumed to be the same as the one used in the combined charge fit. The background distribution is still assumed to be Gaussian in shape, but fit independently for each charge. The results for the separate fitting of the background distributions for positively- and negatively-charged reconstructed muons are given in Table 5.5. The fit gives Gaussian means and widths for the distribution that are similar to the charge-combined sample, which is to be expected given that the background processes are mostly symmetric with respect to charge conjugation.

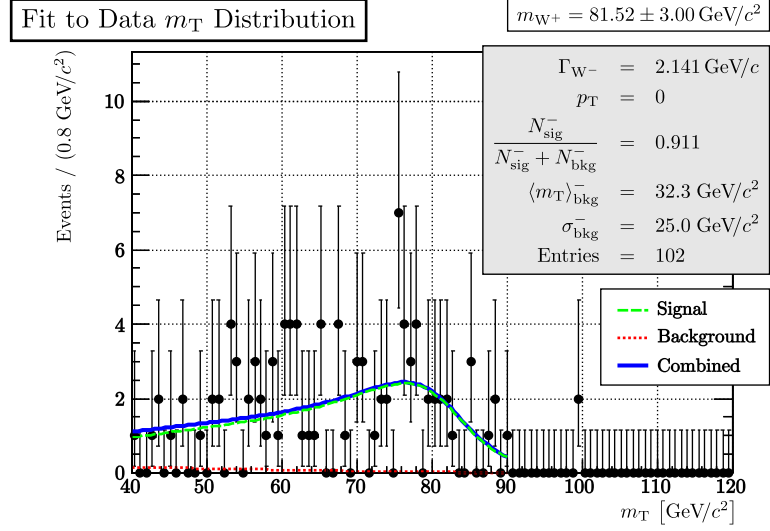
The mass measurements from the fit to the m_T spectra are

$$\begin{aligned}
m_{W^+} &= 79.30 \text{ GeV}/c^2 \pm 0.79 \text{ GeV}/c^2 \text{ (stat.)} \pm 0.48 \text{ GeV}/c^2 \text{ (sys.)} \\
m_{W^-} &= 81.52 \text{ GeV}/c^2 \pm 1.10 \text{ GeV}/c^2 \text{ (stat.)} \pm 0.49 \text{ GeV}/c^2 \text{ (sys.)}.
\end{aligned}$$

These results are plotted in Figures 5.9a and 5.9b. The ratio of the fitted m_W parameters is $m_{W^+}/m_{W^-} = 0.97 \pm 0.02 \text{ (stat.)} \pm 0.01 \text{ (sys.)}$, consistent with the Standard Model prediction of unity.



(a) $W^+ \rightarrow \mu^+ \nu_\mu$



(b) $W^- \rightarrow \mu^- \bar{\nu}_\mu$

Figure 5.9: Fit results from resolution-convoluted transverse mass with background Gaussian PDF fit to data for (a) μ^+ and (b) μ^- samples.

Chapter 6

CONCLUSIONS

In this analysis, the W production cross-section and mass were measured by analyzing LHC collision events with high- p_T muons and substantial $\cancel{E}_T$ in the CMS detector. By counting the number of events to pass a set of loose criteria and by calculating selection efficiencies and uncertainties using both Monte Carlo and data-driven methods, the W production cross-section was measured to be

$$\begin{aligned}\sigma_{pp \rightarrow W} \times \text{BR}(W \rightarrow \mu \nu_\mu) = \\ 9\,013 \text{ pb} \pm 540 \text{ pb (stat.)} \pm 552 \text{ pb (sys.)} \pm 991 \text{ pb (lumi.)},\end{aligned}$$

which is consistent with the NLO theoretical calculation of 10 400 pb (as calculated using MCFM and the MSTW2008 parton distribution functions). The cross-section was calculated for μ^+ and μ^- decay products independently, giving

$$\begin{aligned}\sigma_{pp \rightarrow W^+} \times \text{BR}(W^+ \rightarrow \mu^+ \nu_\mu) = \\ 6\,224 \text{ pb} \pm 457 \text{ pb (stat.)} {}^{+388}_{-390} \text{ pb (sys.)} \pm 685 \text{ pb (lumi.)} \\ \sigma_{pp \rightarrow W^-} \times \text{BR}(W^- \rightarrow \mu^- \bar{\nu}_\mu) = \\ 2\,823 \text{ pb} \pm 349 \text{ pb (stat.)} {}^{+180}_{-182} \text{ pb (sys.)} \pm 311 \text{ pb (lumi.)}.\end{aligned}$$

The NLO predictions for these numbers are $\sigma_{pp \rightarrow W^+} \times \text{BR}(W^+ \rightarrow \mu^+ \nu_\mu) = 6\,130 \text{ pb}$ and $\sigma_{pp \rightarrow W^-} \times \text{BR}(W^- \rightarrow \mu^- \bar{\nu}_\mu) = 4\,270 \text{ pb}$. An accurate measurement of the W

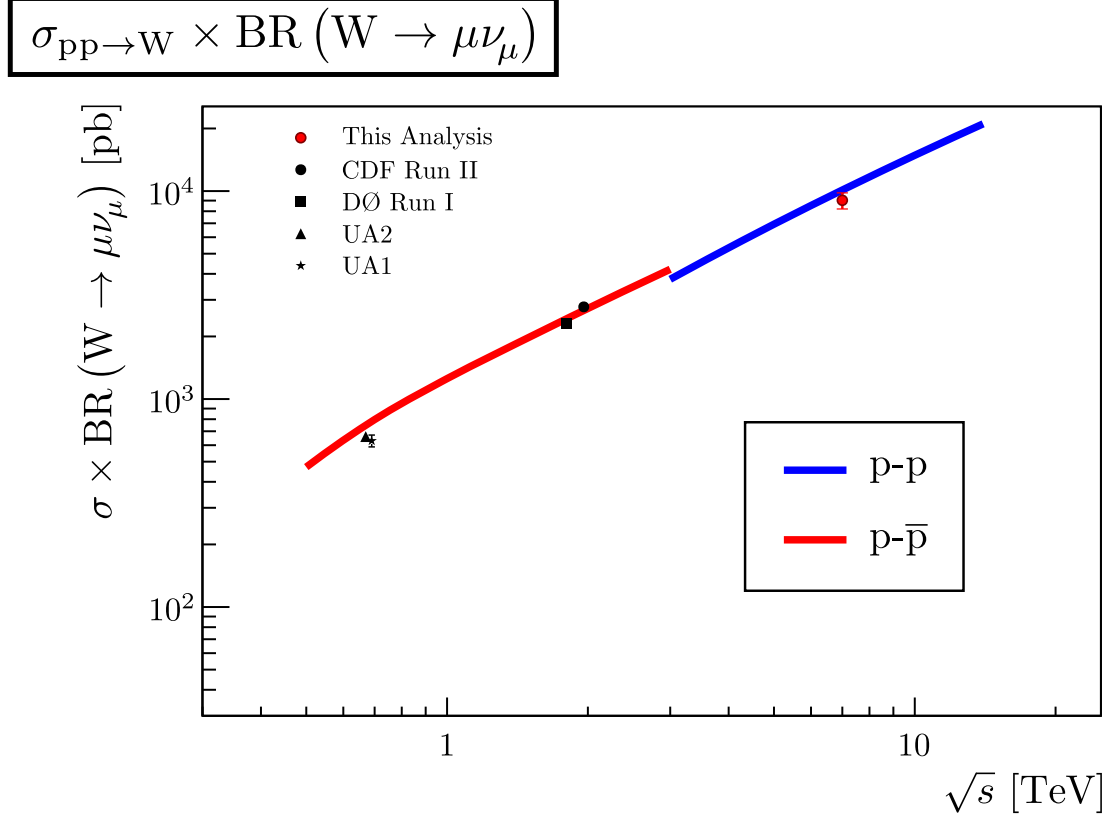


Figure 6.1: Previous measurements of the $\sigma_{pp \rightarrow W} \times \text{BR}(W \rightarrow \mu\nu_\mu)$ cross-section as compared to this analysis[11]. The error bars on the measurement from this analysis do not include the 11 % luminosity uncertainty. The theoretical projection is calculated at NNLO using FEWZ and the MSTW2008 parton distribution functions[80, 81].

cross-section at the LHC collision energy of $\sqrt{s} = 7$ TeV will play an important rôle in the accurate determination of LHC luminosities—an important factor in any new-physics discovery search. As seen in this analysis, the error in the determination of the LHC luminosity is and will remain the largest source of systematic uncertainties for measurements of this kind until the cross-sections of well-understood and easily identifiable processes such as $W \rightarrow \mu\nu_\mu$ can be verified against theoretical calculations. The result of this measurement is plotted in Figure 6.1, comparing to it the results from previous measurements and the theoretically predicted scaling of the W cross-section with collision energy.

The W mass was measured by fitting an analytical function to the m_T distribution of those events that pass the same selection criteria used in the cross-section measurement. After accounting for the uncertainties in the maximum-likelihood fitting procedure through functional form variations and pseudo-experiments, the W mass was measured to be

$$m_W = 80.07 \text{ GeV}/c^2 \pm 0.64 \text{ GeV}/c^2 \text{ (stat.)} \pm 0.48 \text{ GeV}/c^2 \text{ (sys.)},$$

consistent with the world average for previous experimental measurements of $80.398 \text{ GeV}/c^2 \pm 0.025 \text{ GeV}/c^2$. The measurement was also performed for μ^+ and μ^- decay products independently, resulting in a measurement of

$$\frac{m_{W^+}}{m_{W^-}} = 0.97 \pm 0.02 \text{ (stat.)} \pm 0.01 \text{ (sys.)},$$

consistent with the Standard Model prediction of unity. The W mass measurement is dominated by systematic errors that will need to be overcome in order to contribute significantly to a more accurate world average. Regardless, the measured result is the first of such results from the CMS experiment and serves to prove that the muon and $\cancel{E}_T$ reconstruction using early data is sensible. Figure 6.2 shows the results of the past major contributions toward the measurement of the W mass with this measurement noted at the bottom.

The analytical methods presented could also be used to measure the width of the W boson, should the resolution of the m_T reconstruction be improved, or the statistics become large enough to reduce both the resolution uncertainty and the statistical uncertainty in the m_T shape. In theory, the differential cross-section of the W with respect to its p_T could be measured using the same transverse mass PDF by simply fixing the mass and width parameters. This possible measurement

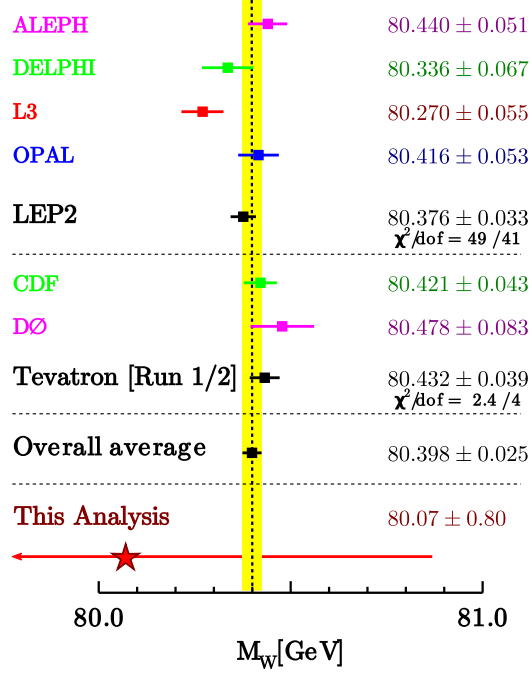


Figure 6.2: Previous measurements of the W mass as compared to this analysis[11]. The error bars on the measurement from this analysis are too large to fit on the scale of this plot.

is particularly interesting in that the W p_T reveals information about the underlying proton parton distribution functions and second-order QCD effects[82].

As statistics increase, the fit could also be performed using an extended maximum likelihood model to estimate the signal fraction of the selected events instead of relying on the Monte Carlo cut-and-count method to fix this ratio in the PDF[83]. This method could provide a more accurate estimate of the W mass, as well as provide an independent estimate of the underlying signal in the selected data events, thereby allowing a more accurate measurement of the W cross-section.

In summary, this analysis demonstrates the usefulness of analytical fitting methods for accurately measuring the mass of the W boson. This analysis also produces an estimate for the cross-section $\sigma_{pp \rightarrow W} \times \text{BR}(W \rightarrow \mu\nu_\mu)$ using standard event counting and cut efficiency calculations. The most popular method of measuring m_W in

past hadron collider experiments has been to generate several hundred independent samples of $W \rightarrow \mu\nu_\mu$ Monte Carlo events with varying W mass and width parameters, then use these to generate templates that are fitted to the data[84]. Not only is this method time-consuming, it is subject to additional systematic uncertainties due to simulation responses with possibly unphysical parameters. The analytical fitting method used here lacks these complicated systematic issues and can be applied to data within minutes of the data being recorded. The future use of this method will prove valuable in better understanding the mass, width, and p_T distribution of the W bosons produced in the ever-increasing number of collisions at the LHC.

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Appendix A

PSEUDO-EXPERIMENT METHOD FOR THE CALCULATION OF SYSTEMATIC UNCERTAINTIES

Pseudo-experiments are a useful method for understanding the systematics of a model due to possible statistical fluctuations in the observed sample, especially in the field of high-energy physics where models are typically complex. The method relies on a frequentist assumption of the nature of measurements in an experiment: namely that the actual result measured is drawn (by random chance) from a set of possible results that are dictated by Nature. Pseudo-experiments offer a way to understand that set of possible results (which cannot be directly measured) so that systematic uncertainties can be discovered and so that statistical confidence intervals can be assigned to the measurement made.

In our case, the frequentist assumptions are that there is some true value of the W mass and that the results of our attempted fit to the data is affected by both the systematic uncertainty in our model due to unaccounted-for parameters (so-called *nuisance parameters*) and by the statistics of the sample of real $W \rightarrow \mu\nu_\mu$ decays that nature happened to choose for us. The nuisance parameter of the detector m_T resolution has been organically incorporated into the model PDF by convolution of the observable with a resolution function[85], but other sources of systematic error, like the acceptances of signal and background events, are not incorporated so easily. These systematics would have to be treated very carefully to be convoluted into the model PDF, and such a model is likely to become computationally unwieldy, so these uncertainties are handled by propagation of the standard errors on the parameters to the ultimate measurement of m_W .

To understand how the statistics of the data sample affect the fitted measurement of m_W , we have to understand the set of possible $W \rightarrow \mu\nu_\mu$ datasets that Nature could have given to us. This sample can be approximated using the Monte Carlo simulation tools at our disposal. If we believe that the Monte Carlo generators produce events that are similar to those generated by Nature—that the randomness of the physical parameters of the MC samples are generated by throwing the same “dice” as those

thrown by Nature—then a large enough sample of MC events should be sufficient to understand the distribution of possible data samples.

A sample of 500 MC event samples with a simulated integrated luminosity similar to the collected data sample (70 nb^{-1}) was generated by *bootstrapping*. This involved first binning the full combined Monte Carlo event sample (weighted and normalized to 1 pb^{-1}) that passed the selection criteria of Chapter 4 into m_T bins of $0.8 \text{ GeV}/c^2$ (chosen arbitrarily), and then selecting from the full sample ~ 338 events, with the probability of selecting each event weighted by the frequency of events in the particular m_T bin the event occupies. The number 338 is the expected number of combined signal and background events from the Monte Carlo selection study at 70 nb^{-1} (see Table 4.2). It is taken to be the mean of a Poisson distribution so that the total number of events in each simulated 70 nb^{-1} sample can fluctuate within expected statistical sampling uncertainties. This process is repeated 500 times, and the W transverse mass PDF model (smeared and background included) is fit to each of these simulated datasets. Figure A.1 shows four sample 70 nb^{-1} pseudo-experiments and fits to those samples.

This process results in 500 measurements of m_W , each with an associated error due to the fitting algorithm. These results are plotted in histograms in Figure A.2. The width of the m_W measurement distribution around the central value of $80.4 \text{ GeV}/c^2$ is due primarily to statistical fluctuations of the sample of ~ 338 events on which the fit was performed. The width of the m_W distribution should be interpreted as a statistical error on the fitting procedure due to the sample handed to us by Nature. The RMS of the distribution is used as an estimate of the width, giving a statistical error of $\sim 0.8 \%$. The distribution of the fit errors on m_W has a mean of $0.816 \text{ GeV}/c^2$, which is understood to be the combined statistical and systematic error on the fit procedure. The systematic error due to the convoluted PDF used not exactly reproducing the shape of the reconstructed m_T distribution is found to be 0.5% by subtracting in quadrature. The fit error also fluctuates due to the statistics of the pseudo-experiment sample, but this fluctuation is small compared to that of the measurement of m_W , so it is ignored.

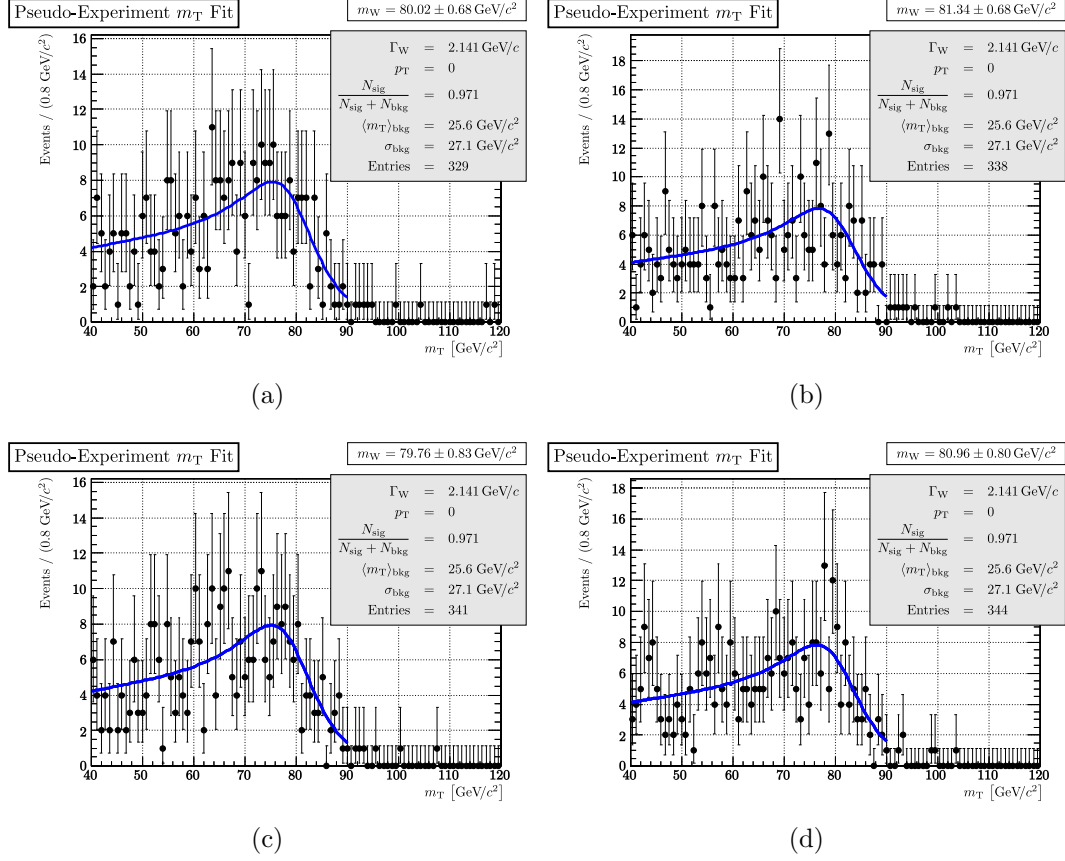
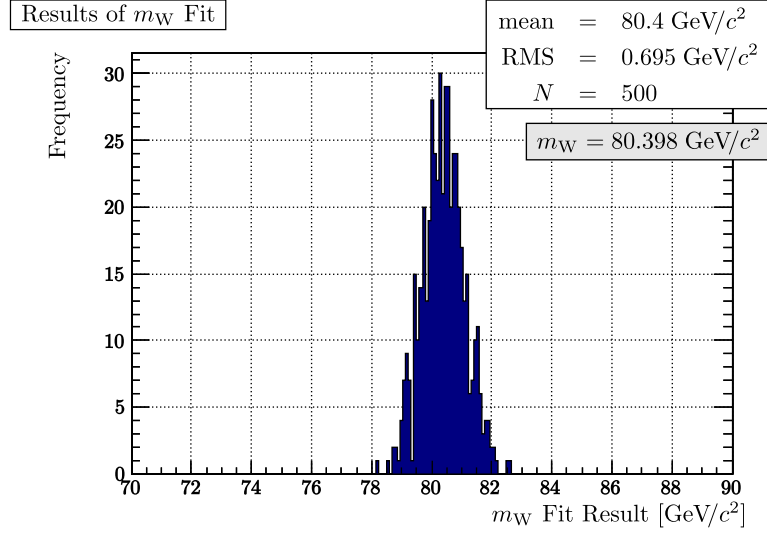
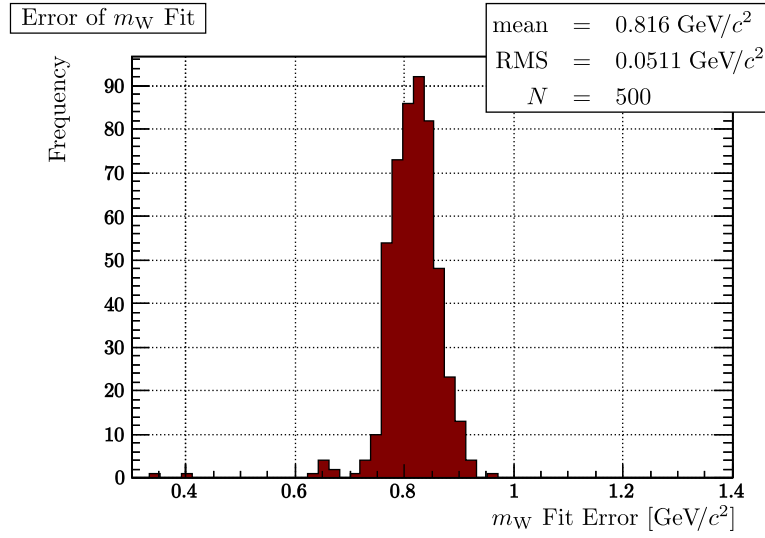


Figure A.1: Distributions of m_T from four sample 70 nb^{-1} pseudo-experiments. Fit results are also shown (*cf.* Figure 5.5).



(a)



(b)

Figure A.2: Histograms of the (a) m_W measurements and (b) fit errors from 500 pseudo-experiments at 70 nb^{-1} .

Appendix B

CODE LISTING FOR TRANSVERSE MASS P.D.F.

B.1 RooTransverseMass2Pdf.h

```
1  #ifndef ROOTTRANSVERSEMASS2PDF
2  #define ROOTTRANSVERSEMASS2PDF
3
4  #include "RooAbsPdf.h"
5  #include "RooRealProxy.h"
6  #include "RooCategoryProxy.h"
7  #include "RooAbsReal.h"
8  #include "RooAbsCategory.h"
9
10 class RooTransverseMass2Pdf : public RooAbsPdf
11 {
12 public:
13     RooTransverseMass2Pdf() {} ;
14     RooTransverseMass2Pdf(const char *name, const char *title, ←
        RooAbsReal &_mt, RooAbsReal &_M, RooAbsReal &_Gamma, ←
        RooAbsReal &_pt);
15     RooTransverseMass2Pdf(const RooTransverseMass2Pdf &other, ←
        const char *name = 0);
16     virtual TObject *clone(const char *newname) const { return ←
        new RooTransverseMass2Pdf(*this, newname); }
17     inline virtual ~RooTransverseMass2Pdf() { }
18
19 protected:
20
21     RooRealProxy mt;
22     RooRealProxy M;
23     RooRealProxy Gamma;
```

```

24     RooRealProxy pt;
25
26     Double_t evaluate() const;
27
28 private:
29
30 #ifdef __CINT__
31     ClassDef(RooTransverseMass2Pdf,1)
32 #endif
33 };
34
35 #endif

```

B.2 RooTransverseMass2Pdf.cc

```

1  #include "RooTransverseMass2Pdf.h"
2
3  #ifndef __CINT__
4
5  #include "Riostream.h"
6  #include "RooAbsReal.h"
7  #include "RooConstVar.h"
8  #include "RooAbsCategory.h"
9  #include <math.h>
10 #include "TMath.h"
11 #include <iostream>
12
13 #endif
14
15 #ifdef __CINT__
16 ClassImp(RooTransverseMass2Pdf)
17 #endif
18
19 RooTransverseMass2Pdf::RooTransverseMass2Pdf(const char *name, ↵
    const char *title, RooAbsReal &_mt, RooAbsReal &_M, ↵
    RooAbsReal &_Gamma, RooAbsReal &_pt) :
20 RooAbsPdf(name, title),
21 mt("mt", "Transverse Mass", this, _mt),
22 M("M", "Mass", this, _M),
23 Gamma("Gamma", "Width", this, _Gamma),
24 pt("pt", "Transverse Momentum", this, _pt)
25 {
26 }
27

```

```

28 RooTransverseMass2Pdf::RooTransverseMass2Pdf(const ↵
    RooTransverseMass2Pdf &other, const char *name) :
29 RooAbsPdf(other, name),
30 mt("mt", this, other.mt),
31 M("M", this, other.M),
32 Gamma("Gamma", this, other.Gamma),
33 pt("pt", this, other.pt)
34 {
35 }
36
37 Double_t RooTransverseMass2Pdf::evaluate() const
38 {
39
40     if (mt <= 0 || Gamma <= 0 || M <= 0 || pt < 0) return 0;
41
42     // The width of the integration bins will be variable in s.
43     // A smaller number here means a more accurate (but more ↵
        time-consuming) integral
44     const double accuracy = 0.1;
45
46     // Small numbers here makes the integral more accurate at the ↵
        expense of time.
47     const double minWidth = 0.01;
48     const double maxWidth = 1;
49
50     // We do not want to go to infinity. That would take a long ↵
        time.
51     const double maxS = 300;
52
53     // Some constants
54     const double M2 = M * M;
55     const double M2G2 = M2 * Gamma * Gamma;
56     const double mt2 = mt * mt;
57     const double denom = accuracy / (M2 * M2 * mt2 * mt2);
58
59     // The current bin width. Begins at 0.
60     double binWidth = 0;
61
62     // A cache from the previous iteration
63     double cache = 0;
64
65     // The return value
66     double ret = 0;
67
68     for (double s = mt + minWidth; s < maxS; s += binWidth) {
69

```

```

70     double s2 = s * s;
71
72     // Calculate the value of the integrand at this s
73     const double smM2 = s2 - M2;
74     const double smmt2 = s2 - mt2;
75     double value = s / (smM2 * smM2 + M2G2) * ←
        RooTransverseMass2Pdf::fastInvSqrt(s2 * (s2 - mt2));
76
77     // Begin second integral for non-0 pt
78     if (pt == 0) {
79         value *= (2 - mt2 / s2);
80     } else {
81         const double a2 = pt * pt / M2;
82         const double a4 = a2 * a2;
83         const double mu2 = mt2 / s2;
84         const double mu4 = mu2 * mu2;
85         double i2 = 0;
86         const double pi2 = 2.0 * TMath::Pi();
87         for (double phi = 0; phi < pi2; phi += 0.1) {
88             const double cosphi = TMath::Cos(phi);
89             const double cos2phi = cosphi * cosphi;
90             const double sinphi = TMath::Sin(phi);
91             const double sin2phi = sinphi * sinphi;
92             const double sin2theta = mu2 * (mu2 + a2) / (mu2 + mu2 ←
                * a2 * cos2phi + a2 * sin2phi);
93             const double denom2 = ←
                RooTransverseMass2Pdf::fastInvSqrt(mu2 + mu2 * a2 ←
                * cos2phi + a2 * sin2phi);
94             const double denom3 = denom2 * denom2 * denom2;
95             const double I = (mu4 + mu4 * a2 * cos2phi + 2 * mu2 * ←
                a2 * sin2phi + a4 * sin2phi) * ←
                RooTransverseMass2Pdf::fastInvSqrt(mu2 + a2 * ←
                sin2phi) * denom3;
96             i2 += (2 - sin2theta) * I * 0.1;
97         }
98
99         value *= i2;
100     }
101
102     // Add the trapezoid using the previous iteration value
103     if (value < 0) value = 0;
104     ret += 0.5 * binWidth * (value + cache);
105     // Cache the calculated value
106     cache = value;
107     // Update the new bin width to focus on points where the ←
        distribution blows up

```

```

108     binWidth = smM2 * smM2 * smmt2 * smmt2 * denom;
109     if (binWidth < minWidth) binWidth = minWidth;
110     else if (binWidth > maxWidth) binWidth = maxWidth;
111
112 }
113
114 return mt * ret;
115
116 }
117
118 float RooTransverseMass2Pdf::fastInvSqrt(float x)
119 {
120     union {
121         float f;
122         int i;
123     } tmp;
124     tmp.f = x;
125     tmp.i = 0x5f3759df - (tmp.i >> 1);
126     float y = tmp.f;
127     return y * (1.5f - 0.5f * x * y * y);
128 }

```