Physics monitor

25 years of the renormalization group

The technique of renormalization was introduced into physics through quantum electrodynamics, where it enabled theoreticians to handle the otherwise troublesome infinities which kept cropping up in their calculations. In this technique, the infinities are removed by a well-documented mathematical procedure, enabling highly accurate calculations to be made. The physical ideas behind this renormalization prescription in quantum electrodynamics are less clear, but the success of the method, which produces some of the most accurate predictions in the whole of physics, cannot be doubted.

In 1953, E. Stueckelberg and A. Peterman first drew attention to the freedom in 'renormalizable' field theories, such as quantum electrodynamics, of choosing any one of an infinite number of different renormalization prescriptions, each equally valid as the conventional one. This is known in the trade as 'convention independence'.

Each of these prescriptions defines its own coupling constants and other parameters, so that although individual calculations change, the final physical predictions remain independent of the renormalization scheme used.

Like other invariances found in physics (for example rotational or translational invariance), finite renormalizations can be expressed in terms of repeated infinitesimal operations, and described by the mathematical apparatus of group theory.

For some twenty years, this idea remained an intellectual curiosity, but recently it has found uses in new problems in field theory, such as in quantum chromodynamics, the candidate theory of inter-quark forces. There, renormalization group techniques for scale transformations introduce a 'running coupling constant' which gets smaller as the interacting quarks get closer together. (See November 1977 issue, p. 380.)

This produces the idea of 'asymptotic freedom', where quarks interact little when close together, but bind together more strongly as the distance between them increases. This apparently perverse phenomenon could result in the perpetual confinement of quarks and account for their consequent non-appearance as free particles in the laboratory.

The techniques of the renormalization group have also been used successfully in areas of physics which at first (or even second) glance might seem totally unrelated to field theory — for instance in the description of abrupt phase transitions in condensed matter.

In the attempt to cover more and more aspects of the behaviour of matter, the equations of physics have to contend with more and more variables. Luckily it is not necessary to write down all the variables in every case — by considering the behaviour of relatively few atoms, for example, the physics of gases can be understood without having to write down the quantum mechanics of every atom in a large gas sample.

Modern physics, as well as having to deal with continuous systems where quantities can change smoothly from one point to the next, also has to deal with more complex systems where variables, although continuous in principle, do not necessarily vary smoothly from point to point.

Such behaviour is found in the quantum fluctuations of the electromagnetic field in quantum electrodynamics. In this theory one cannot calculate the exact value of the electromagnetic field at every point. Instead one uses correlation functions to predict expectation values.

Similar difficulties arise in the physics of condensed matter, where one is faced with the problem of describing an obviously discrete system such as a lattice of atoms, in term of bulk properties. For instance, physicists try to arrive at a description of the onset of ferromagnetic behaviour of a sample at its Curie point without having to worry about individual interatomic effects.

At such critical points, the system becomes extremely (actually infinitely) sensitive, and small changes in the external conditions cause the whole system to change its configuration abruptly. The sample appears to ignore individual interatomic forces. Microscopic degrees of freedom are 'frozen out' to give a system which depends on just a few bulk variables. This is to be contrasted to the conventional theory of gases, for example, where the observed behaviour can be explained as an aggregate of smaller samples.

A sample at its critical point thus becomes invariant under scale transformations. In mathematical language, the critical point is a fixed point — no matter what group operations are carried out, the system stays the same.

The renormalization group has provided a tool to handle the study of critical phenomena. The technique involves the successive summation of a series of individual steps, each of which involves a successive reduction in the remaining number of degrees of freedom of the sample, until a stable solution — the fixed point — is reached.

Perhaps the main goal of physics is to describe a maximum of phenomena with a minimum of variables. The renormalization group appears to offer the mathematical apparatus to do just this. With important applications in field theory and in the study of critical phenomena already to its credit, its use could become more widespread in the future.