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Master Thesis

Simulation of Heavy Neutral Lepton production and decay with the Sherpa event generator

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Abstract

This thesis presents the validation of the implementation of heavy neutral leptons (HNLs) within the framework of the Type I Seesaw mechanism in the Monte Carlo event generator SHERPA [1]. The Seesaw mechanism is a widely studied extension of the Standard Model (SM). It explains the smallness of neutrino masses by introducing additional neutrino mass eigenstates, known as heavy neutral leptons. The observation of HNLs would provide clear evidence for physics beyond the SM. Accurate Monte Carlo simulations of their production and decay at collider experiments are therefore essential for experimental searches and the interpretation of their results. Previous studies [2, 3] have investigated this scenario using the matrix element generator MadGraph5_aMC@NLO [4] in combination with the parton shower PYTHIA8 [5]. In contrast, earlier studies of HNL simulations in SHERPA were limited and did not involve a systematic validation [6]. In this thesis, the implementation of HNL production and decay in SHERPA is validated by comparing its predictions to results obtained with MadGraph+PYTHIA. Good agreement between the two simulation frameworks is found for kinematic distributions in both production and decay processes. In particular, both approaches reproduce characteristic differences in angular distributions between lepton number conserving (LNC) and lepton number violating (LNV) HNL decays. Furthermore, this thesis represents a significant advancement in the simulation of HNLs compared to previous works, as it demonstrates several advantages of the SHERPA simulation which are not available in MadGraph+PYTHIA. These include the consistent treatment of spin correlations in the semileptonic decay channel into a ρ -meson, such as $HNL \rightarrow \rho^\pm \mu^\mp \rightarrow \pi^\pm \pi^0 \mu^\mp$, as well as the implementation of HNL decays into $\tau^\pm$ leptons.

Zusammenfassung

Diese Arbeit präsentiert die Validierung der Implementierung schwerer neutraler Leptonen (*Heavy Neutral Leptons*, HNLs) im Rahmen des Typ-I-Seesaw-Mechanismus im Monte-Carlo-Event Generator SHERPA [1]. Der Seesaw-Mechanismus ist eine weit verbreitete Erweiterung des Standardmodells (SM), die die kleinen Neutrinomassen durch die Einführung zusätzlicher Neutrinomasseneigenzustände erklärt. Diese werden als schwere neutrale Leptonen bezeichnet. Der experimentelle Nachweis von HNLs würde einen klaren Beweis für Physik jenseits des Standardmodells liefern. Zuverlässige Monte-Carlo-Simulationen ihrer Produktion und ihres Zerfalls an Kollisionsexperimenten sind daher für die experimentelle Suche und die Interpretation ihrer Ergebnisse von zentraler Bedeutung. Frühere Studien [2, 3] untersuchten dieses Szenario unter Verwendung des Matrixelement-Generators MadGraph5_aMC@NLO [4] in Kombination mit dem Parton-Shower-Programm PYTHIA8 [5]. Im Gegensatz dazu waren frühere Arbeiten zu HNL-Simulationen in SHERPA in ihrem Umfang begrenzt und beinhalteten keine systematische Validierung [6]. In dieser Arbeit wird die Implementierung der HNL-Produktion und -Zerfälle in SHERPA durch einen Vergleich mit Ergebnissen aus MadGraph+PYTHIA validiert. Für kinematische Verteilungen sowohl in der Produktion als auch im Zerfall wird eine gute Übereinstimmung zwischen beiden Simulationsprogrammen festgestellt. Insbesondere reproduzieren beide Ansätze charakteristische Unterschiede in den Winkelverteilungen zwischen leptonenzahlerhaltenden und leptonzahlverletzenden HNL-Zerfällen. Darüber hinaus demonstriert diese Arbeit mehrere Vorteile der SHERPA-Simulation. Dazu zählen die konsistente Behandlung von Spinkorrelationen in semileptonischen Zerfallskanälen, wie etwa $HNL \rightarrow \rho^\pm \mu^\mp \rightarrow \pi^\pm \pi^0 \mu^\mp$, sowie die Implementierung von HNL-Zerfällen in $\tau^\pm$ -Leptonen. Diese Eigenschaften stehen in den entsprechenden MadGraph+PYTHIA-Simulationen nicht zur Verfügung.

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CHAPTER 1

Introduction

The current state of knowledge in particle physics is summarised by the Standard Model (SM), which provides a well-established theoretical model for understanding the fundamental interactions between particles. It accurately describes the electromagnetic, weak, and strong interactions and has been able to predict a wide range of experimental observations with high precision. With the discovery of the Higgs boson in 2012 [7, 8], all particles predicted by the SM have been observed experimentally, highlighting the success of the SM as a quantum field theory.

Despite its success, the SM is known to be incomplete and leaves several fundamental questions unanswered. First, it does not include a description of gravity and therefore cannot be regarded as a complete theory of all fundamental interactions. A consistent unification of the SM with general relativity remains an open challenge in theoretical physics. Furthermore, there are phenomena that cannot be explained within the Standard Model. One prominent example is the existence of dark matter, whose gravitational effects are observed in astrophysical and cosmological measurements, while its fundamental nature remains unknown [9]. Another unresolved problem is the observed matter-antimatter asymmetry of the Universe. Observations indicate that the Universe consists predominantly of matter, whereas antimatter is almost entirely absent. No confirmed mechanism within the SM is capable of explaining this so-called baryon asymmetry [10]. In addition, the Standard Model contains a particle whose properties are not yet fully understood: the neutrino. In the original formulation of the SM, neutrinos were assumed to be massless [11]. However since the late 90s multiple experiments have observed phenomena, known as neutrino oscillations, that can only be explained if neutrinos possess small but non-zero masses [12]. While oscillation experiments provide precise measurements of neutrino mass splittings, the absolute neutrino mass scale is still unknown, and only upper limits exist [13]. Furthermore, the ordering of the neutrino mass eigenstates, known as the neutrino mass hierarchy, remains an open experimental question. Even more fundamentally, the mechanism responsible for generating neutrino masses is not known and thus not included in the SM.

A wide variety of theories Beyond the Standard Model (BSM) have been proposed to explain the origin of neutrino masses. The simplest extension introduces neutrino masses analogously to other SM fermions by adding right-chiral neutrino fields and generating Dirac masses through

Yukawa couplings to the Higgs field. Such right-handed neutrinos would be sterile, as they do not participate in SM gauge interactions. In this scenario, neutrinos would be Dirac particles, implying that neutrinos and antineutrinos are distinct states. However, since neutrinos are electrically neutral and do not carry colour charge, they may also be Majorana particles, meaning that their particle mass eigenstates are identical to their respective antiparticles. This possibility allows for fundamentally different mass-generation mechanisms that do not rely solely on the Higgs mechanism. Several BSM frameworks introduce Majorana mass terms for neutrinos. Among the most prominent is the Type I Seesaw mechanism [14]. In this theory, additional heavy Majorana mass eigenstates, referred to as heavy neutral leptons (HNLs), are introduced. The interplay between the heavy and light states naturally explains the smallness of the observed neutrino masses. Depending on the model realisation, Seesaw scenarios may also provide an explanation for the baryon asymmetry of the universe via leptogenesis, in which a lepton asymmetry generated in the early universe is converted into a baryon asymmetry [15]. Some extensions furthermore include viable dark matter candidates [16].

These aspects provide strong motivation for studying Seesaw scenarios further and searching for HNLs experimentally. To test and constrain BSM theories, experiments such as ATLAS [17] and CMS [18] at the Large Hadron Collider (LHC) [19] search for characteristic signatures predicted by these models. Due to the large number of particles in the final state, intrinsic quantum-mechanical uncertainties, and the complexity of hadronic collisions, purely analytical calculations are often not feasible. Instead, Monte Carlo (MC) event generators are used to simulate particle collisions numerically. The resulting simulated event samples can be compared to experimental data to validate or exclude BSM scenarios. While MC event generators are well established for Standard Model processes, extensions involving BSM particles require dedicated implementations and careful validation. To date, simulations of HNLs have primarily been performed using the matrix element generator MadGraph5_aMC@NLO [4] in combination with the parton shower program PYTHIA8 [5] [2]. In order to validate these results and exploit the complementary strengths of different generators, it is essential to implement HNL models in additional MC frameworks. In this thesis, the new implementation of HNLs into the Monte Carlo event generator SHERPA [1] is presented. Using simulations generated with MadGraph+PYTHIA as a benchmark, the implementation is validated and the additional features offered by SHERPA for HNL studies are explored.

The structure of this thesis is as follows: Chapter 2 introduces the Standard Model and discusses its limitations with respect to neutrino masses, followed by an overview of BSM theories with a focus on the Type I Seesaw mechanism and heavy neutral leptons. In Chapter 3, the principles of Monte Carlo event generation are reviewed and the implementation of HNL production and decay in SHERPA is described. Finally, Chapter 4 presents a comparison of MadGraph+PYTHIA and SHERPA simulation results, validating the SHERPA implementation, before the main conclusions of this thesis are summarised in Chapter 5.

Theoretical framework

This chapter, will give an introduction to the theoretical background needed to discuss the simulation of heavy neutral leptons (HNLs). As the HNL theory used in this thesis is an extension of the Standard Model (SM), I will start with a short overview of the SM in Section 2.1 with an emphasis on the weak interaction in Section 2.3. Afterwards, we will consider the SM neutrino in further detail in Section 2.5, before discussing different Beyond the Standard Model (BSM) theories which try to explain the open questions of the SM regarding neutrinos in Section 2.6.

2.1 Standard Model

2.1.1 Overview of the SM

The Standard Model of particle physics is currently the best description of the interactions of fundamental particles. Apart from gravity, it includes all the known fundamental interactions: the electromagnetic, weak, and strong interaction.

In the SM, there are 17 fundamental particles: 5 bosons and 12 fermions (see Figure 2.1). They are classified according to their spin, which is an integer for bosons and half-integer for fermions. Fermions can carry different kinds of charges, enabling them to participate in the respective interactions which are mediated by bosons. Fermions that carry colour charge (red, blue, or green) are called quarks. Due to their colour charge, they can interact via the strong interaction. As all quarks carry an electric charge of $\pm\frac{1}{3}$ or $\pm\frac{2}{3}$ in units of the elementary charge e_q , they can interact electromagnetically, too. Fermions without colour charge are called leptons. They can be electrically charged with charge $Q = \pm 1 e_q$ or electrically neutral, $Q = 0$. In the latter case, they are called neutrinos. Additionally, all fermions can have weak isospin, depending on whether they are left- or right-handed (see Section 2.3.1). If their weak isospin is non-zero, they can interact via the weak interaction.

Each of the 12 fermions has its own flavour. There are 6 different quark flavours (up, down, charm, strange, top and bottom) and 6 different lepton flavours (electron, muon, tau and their corresponding neutrinos). They are grouped into three generations, where each generation

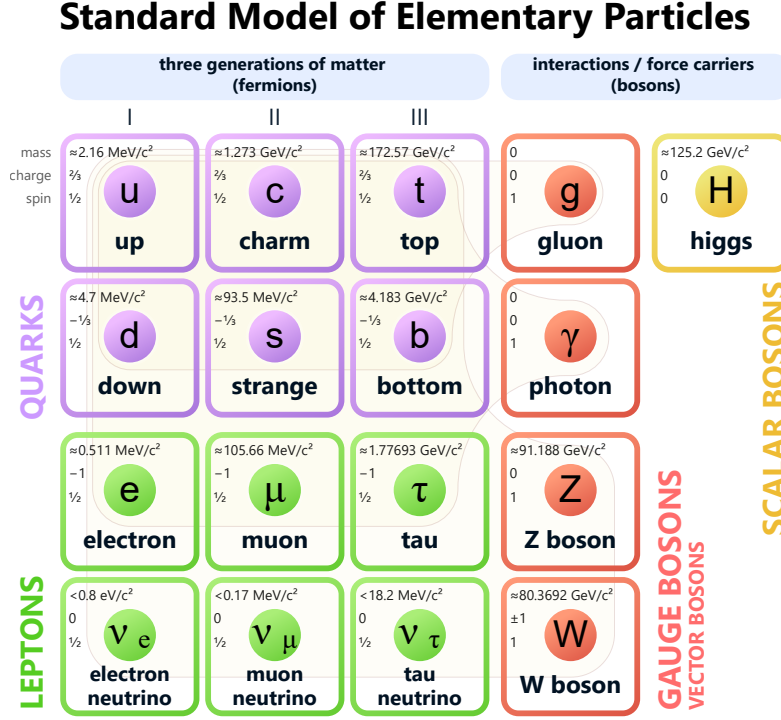


Figure 2.1: Fundamental particles of the Standard Model: The SM encompasses 12 fermions (6 quarks, in purple, and 6 leptons, in green) and 5 bosons (4 interaction-mediating particles, in red, and the Higgs boson, in yellow). Faint brown lines in the background indicate which particles interact with which of the fundamental forces. Figure taken from [20].

contains one up-type quark, one down-type quark, one charged lepton (electron e , muon μ or tau τ), and one neutrino (ν_e , ν_μ or ν_τ).

For every fermion, there exists a corresponding antifermion with the same mass and spin, but all its charges are inverted.

Bosons are the mediating particles for the three fundamental interactions of the SM. The electromagnetic interaction is mediated by photons and the strong interaction by gluons. $W^\pm$ bosons and Z bosons are the interacting particles of the weak interaction and are massive unlike the photons and gluons. The Higgs boson is the only elementary particle with spin zero. Quarks, charged leptons and the bosons of the weak interaction acquire their mass by coupling to the Higgs boson via the Higgs mechanism, as introduced in Section 2.4.

2.1.2 Mathematical description of the SM

The SM is a relativistic Quantum Field Theory (QFT). All particles, bosons, and fermions are described by quantum fields $\psi(x)$ spanning all of space-time, with particles being excitations of these fields.

Particle fields and their excitations

This section is based on [21].

The particle field $\psi(x)$ can be described by applying the field operator $\hat{\phi}(x)$ to the vacuum state $|\emptyset\rangle$.

In the case of fermions, $\hat{\phi}$ is defined as

$$\hat{\phi}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \sum_s \left(\hat{b}_s(\vec{p}) u_s(\vec{p}) e^{-ip \cdot x} + \hat{d}_s^\dagger(\vec{p}) v_s(\vec{p}) e^{+ip \cdot x} \right) \quad (2.1)$$

with $\vec{p}$ being the (three-)momentum, s the spin and $E_{\vec{p}}$ the energy of an excitation in the field, i.e. particle or antiparticle. u and v are the respective positive or negative energy spinor solutions to the Dirac equation

$$\begin{aligned} (\not{p} - m) u_s(\vec{p}) &= 0 \\ (\not{p} + m) v_s(\vec{p}) &= 0 \end{aligned} \quad (2.2)$$

and $e^{\pm ip \cdot x}$ ensures the evolution of the field in space-time.

The key point of Equation (2.1) is that $\hat{\phi}(x)$ is a sum of two terms: The first term contains the annihilation operator $\hat{b}_s(\vec{p})$ which annihilates a fermion with momentum $\vec{p}$ represented by the fermion spinor $u_s(\vec{p})$. The second term contains the creation operator $\hat{d}_s^\dagger(\vec{p})$ which creates an antifermion with momentum $\vec{p}$ represented by the antifermion spinor $v_s(\vec{p})$.

By applying $\hat{\phi}(x)$ to the vacuum state $|\emptyset\rangle$, an antifermion is created at the space-time coordinate x through the creation operator. Since no fermion is present in the vacuum state, the annihilation operator yields zero.

$$\hat{\phi}(x_1) |\emptyset\rangle = |1 \text{ antifermion at } x_1\rangle + 0. \quad (2.3)$$

A state with l fermions at positions $x_1, x_2, \dots, x_l$ and m antiparticles at positions $y_1, y_2, \dots, y_m$ can therefore be written as

$$|\psi(x)\rangle = \underbrace{\hat{\phi}^\dagger(x_1) \hat{\phi}^\dagger(x_2) \cdots \hat{\phi}^\dagger(x_l)}_{\text{creates } l \text{ fermions at } x_1, x_2, \dots, x_l} \underbrace{\hat{\phi}(y_1) \hat{\phi}(y_2) \cdots \hat{\phi}(y_m)}_{\text{creates } m \text{ antifermions at } y_1, y_2, \dots, y_m} |\emptyset\rangle$$

We see that fermions and antifermions are actually excitations of the same respective field. The same is true for bosons which are excitations of the respective boson fields.

SM Lagrangian

The mathematical description of the SM is given by the SM Lagrangian $\mathcal{L}_{SM}$. It contains all possible interactions between SM boson and fermion fields. Due to the symmetry of the SM, $\mathcal{L}_{SM}$

is invariant under local gauge transformations of the symmetry group

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

where each subgroup represents one of the described interactions. The $SU(3)_c$ corresponds to the strong interaction described by Quantum Chromodynamics (QCD). As further explained in Section 2.2, the subgroup $SU(2)_L \times U(1)_Y$ represents the electroweak interaction as a unification of the $SU(2)_L$ group of the weak interaction and the $U(1)_Y$ group of the electromagnetic interaction [22]. The latter is described by Quantum Electrodynamics (QED).

From Noether's theorem [23], we know that from every continuous symmetry follows a corresponding conserved current. In our case, these are the charge densities. Thus, the colour charge (QCD), the electric charge Q (QED), the weak isospin I_W^3 , and the weak hypercharge $Y_W = 2(Q - I_W^3)$ of the electroweak unification are conserved quantities in the SM.

Interaction probabilities

The following section is based on [24].

From the Lagrangian $\mathcal{L}_{SM}$, one can derive the probabilities of specific interaction processes between particles. When particles interact, they can transition from an initial state In to a final state Fi . The transition rate $\Gamma_{In,Fi}$ describes the probability of this transition per time interval. The cross-section $\sigma_{In,Fi}$ is independent of the flux ϕ_{In} of the incoming particles and defined as

$$\sigma_{In,Fi} = \frac{\Gamma_{In,Fi}}{\phi_{In}}.$$

The total cross-section σ now describes the probability for a certain particle to decay. It is determined by summing over all possible decay channels of the given particle:

$$\sigma = \sum_{Fi} \sigma_{In,Fi}.$$

The relative probability for the specific decay channel $In \rightarrow Fi$ is given by the branching ratio $\mathcal{B}$ with

$$\mathcal{B}(In \rightarrow Fi) = \frac{\sigma_{In,Fi}}{\sigma}.$$

The lifetime τ of the particle in its own rest frame, sometimes denoted by the proper lifetime, is defined using the total transition rate Γ

$$\tau = \frac{1}{\Gamma}.$$

In the context of decays, Γ is commonly referred to as the particle's decay width.

From these definitions, we see that the cross-section σ (or transition rate Γ) is a useful parameter for describing interaction processes. The calculation of Γ follows Fermi's Golden Rule [25]. This states that the transition rate Γ for a decay process of a particle a into two particles $a \rightarrow b + c$ is given as

$$\Gamma = \frac{(2\pi)^4}{2E_a} \int |\mathcal{M}|^2 \delta(E_a - E_b - E_c) \delta^3(\vec{p}_a - \vec{p}_b - \vec{p}_c) \frac{d^3p_b}{(2\pi)^3 2E_b} \frac{d^3p_c}{(2\pi)^3 2E_c} \quad (2.4)$$

where E_i and $\vec{p}_i$ denote the energy and momenta of the respective particle i . The delta functions ensure energy and momentum conservation between the initial and final state.

In Equation (2.4), two ingredients are needed to calculate Γ , and thus σ of a given process. The first one is the phase space, which describes the kinematics of the specific process, and the second one is the matrix element $\mathcal{M}$ containing the physics behind the interaction. $\mathcal{M}$ can be calculated using the Feynman rules, which are derived from the Lagrangian $\mathcal{L}_{SM}$. They describe how to translate a Feynman diagram of the interacting (anti)particles into mathematical expressions. The Feynman rules of the SM are discussed in detail in common particle physics textbooks, see for example [24, 26]. BSM theories, however, often come with a new or modified set of Feynman rules, which have to be derived from the respective BSM Lagrangian. The Feynman rules for models with Majorana particles, such as the Seesaw mechanism Type I, will be presented in Section 3.4.2.

2.2 Electroweak unification and symmetry breaking

This section follows the descriptions from [24].

The $SU(2)_L$ symmetry of the SM Lagrangian requires the existence of three gauge fields W_μ^1 , W_μ^2 , and W_μ^3 with the weak gauge bosons W^1 , W^2 , and W^3 as their excitations. Analogously, the $U(1)_Y$ symmetry results in the weak hypercharge field B_μ with the weak hypercharge boson B as its excitation. Together, these fields mediate the electroweak interaction.

The fields W_μ^k only couple to particles with a non-zero third component of weak isospin I_W^3 (see Section 2.3), while the B_μ field couples to all particles with a non-zero weak hypercharge $Y_W = 2(Q - I_W^3)$ where Q is the electric charge of a particle.

Until now, all particles carry no mass. But especially the bosons of the weak interaction are observed to be massive in nature. These masses are introduced in the SM Lagrangian by spontaneous symmetry breaking (EWSB). Here, the $\mathcal{L}_{SM}$ without mass terms has a global gauge invariance that is broken spontaneously in the ground state. By this, fermions and bosons acquire their mass through the Brout-Englert-Higgs mechanism by coupling to the Higgs boson (see Section 2.4). The mass eigenstates Z_μ , $W_\mu^\pm$ and A_μ can be derived by diagonalizing the mass terms in the

constructed $\mathcal{L}_{SM}$ as:

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2), \\ Z_\mu &= W_\mu^3 \cos \theta_W - B_\mu \sin \theta_W, \\ A_\mu &= W_\mu^3 \sin \theta_W + B_\mu \cos \theta_W. \end{aligned}$$

where W_μ^1 , W_μ^2 , W_μ^3 , and B_μ are the boson fields of the $SU(2)_L$ and $U(1)_Y$. Here, θ_W is known as the weak mixing or Weinberg angle. Its value is not predicted by theory but has been measured experimentally to be $\sin^2(\theta_W) \approx 0.22$ [27]. $W^\pm$ represent the charged and Z the neutral weak boson field and A the electromagnetic (photon) field.

2.3 Weak interaction

In the following section, we will discuss the weak interaction, i.e. the interaction mediated by the $W^\pm$ bosons and Z boson, in more detail.

In 1957, the Wu experiment [28] showed that weak interaction processes are not parity invariant. The parity violation of the weak interaction implies that an inversion of the coordinate system affects the observed weak process. Thus, chirality was introduced as a new field property to describe this phenomenon.

2.3.1 Chirality and helicity

Chirality

The following description is based on [29].

Chirality is a property of a particle field. Chiral states are eigenstates of the chirality operator, the γ^5 -Matrix. In the Dirac notation, it reads as

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

The respective eigenvalues ± 1 determine whether a field is left-chiral (-1) or right-chiral ($+1$), also known as left- and right-handed.

The SM states that each fermion flavour field $\psi(x)$, apart from neutrinos, is composed of two fields, the left-chiral field $\psi_L(x)$ and the right-chiral field $\psi_R(x)$

$$\psi(x) = \psi_L(x) + \psi_R(x)$$

In order to project a field $\psi(x)$ onto the chiral eigenstates, one can use the left-/right-handed projection operators P_L and P_R

$$\begin{aligned}\psi_L(x) &= P_L \psi(x) \quad \text{with } P_L = \frac{1}{2} (1 - \gamma^5) \\ \psi_R(x) &= P_R \psi(x) \quad \text{with } P_R = \frac{1}{2} (1 + \gamma^5)\end{aligned}$$

From the Wu experiment we know that only left-chiral fermion fields ψ_L can couple to the W^k fields. Since the $W^\pm$ bosons are linear combinations of the W^1 and W^2 fields, $W^\pm$ boson mediated interactions involve exclusively left-chiral fermion fields. Consequently, the right-chiral fermion fields ψ_R do not couple to the $W^\pm$ bosons at all. The Z boson, on the other hand, is a linear combination of the W^3 and B fields. Thus, it can couple to a right-chiral fermion field ψ_R as well, if it has a non-zero electric charge Q . This phenomenon of differentiating between left- and right-chiral fields in weak interactions is a representation of the maximal parity violation of the weak interaction.

Chirality does not change under Lorentz transformations. Thus, it is a Lorentz invariant property of the fermion field. However, chirality is not a conserved quantity in general, due to mass terms mixing left- and right-chiral fields (see Section 2.4). When discussing Majorana masses later, one important property of the chiral fields is their behaviour under charge conjugation. Under charge conjugation, the chirality of a field is flipped (see Appendix A.2)

$$\begin{aligned}\psi_L(x) &\rightarrow (\psi_L(x))^c = (\psi^c)_R \\ \psi_R(x) &\rightarrow (\psi_R(x))^c = (\psi^c)_L.\end{aligned}\tag{2.5}$$

Helicity

This section follows [26].

While chirality provides the theoretical description of fermion fields participating in the weak interaction, it is not observable. In order to relate the chirality to a quantity that is reconstructable

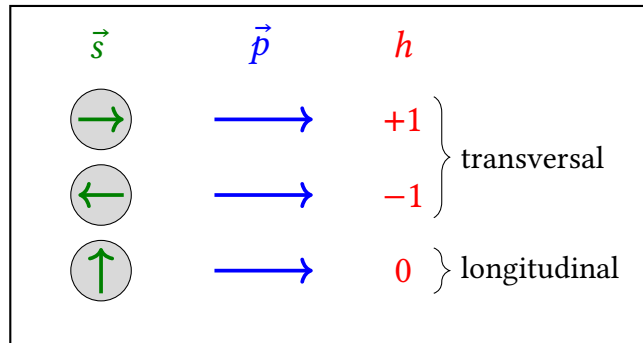


Figure 2.2: Helicity: The helicity h of a particle is described as the projection of its spin $\vec{s}$ onto its direction of momentum $\frac{\vec{p}}{|\vec{p}|}$. In this image, we assume $|s| = 1$.

in experiments, helicity is introduced. Contrary to chirality, helicity is not a property of the field but of its excitations, i.e. particles and antiparticles.

In the ultra-relativistic limit when a particle's energy is much larger than its mass, the helicity of a particle is directly linked to the chirality of its related field. In these cases, we technically still measure the particle's helicity, but can directly translate it to the respective field's chirality. Helicity h is defined as the eigenvalue of the operator

$$\hat{h} = \vec{s} \cdot \frac{\vec{p}}{|\vec{p}|}$$

where $\vec{s}$ is the spin operator and $\vec{p}$ denotes the momentum of the particle. Positive eigenvalues correspond to the parallel orientation of the particle spin to its direction of momentum (right-handed helicity) whereas negative eigenvalues correspond to the antiparallel orientation (left-handed helicity). These two modes are also referred to as the transverse polarisation states of a particle (cf. Figure 2.2).

Additionally, massive bosons can have an eigenvalue of $h = 0$. In this case, the spin is perpendicular to the momentum of the particle which corresponds to the longitudinal polarization state (cf. Figure 2.2). Since charge conjugation neither changes the direction of momentum nor the spin of a particle, helicity does not change under charge conjugation. Thus, positive helicity denotes parallel orientation of spin and the physical momentum for both particles and antiparticles [11]. In regard to the fermion line flow, however, the fermion line of an antiparticle is considered to be opposite to its physical momentum. Thus, positive helicity corresponds to spin and fermion line being parallel for particles but antiparallel for antiparticles (cf. Figure 2.3).

Boosting to another reference frame can change the sign of $\vec{p}$ if the considered particle has mass and thus flip an (anti)particle's helicity. Hence, helicity is not invariant under Lorentz transformations. In the absence of external forces, the helicity operator commutes with the Hamiltonian and thus, helicity is conserved [24].

Comparing a particle's chirality and helicity

Helicity and chirality are often used interchangeably, especially in the context of weak interactions. This is reasonable, since measurements are often made in the ultra-relativistic limit. However, one needs to be careful when doing so: A *particle's* helicity is the same as the chirality of its corresponding field in the ultra-relativistic limit. However, the helicity of an antiparticle is opposite to the chirality of its corresponding field in this limit, since chirality flips under charge conjugation (cf. Equation (2.5)) [29].

Thus, the left-chiral field $\psi_L(x)$ includes negative helicity particles and positive helicity antiparticles, while the right-chiral field $\psi_R(x)$ includes positive helicity particles and negative helicity antiparticles [29]. As only left-chiral fields can couple to the $W^\pm$ bosons, only negative helicity particles and positive helicity antiparticles can participate in $W^\pm$ boson mediated interactions in the ultra-relativistic limit.

In the case of massive particles, left- and right-handed fermions do not correspond to pure helicity states any more. Instead, they are a mixture of both helicity states, with the "wrong" helicity state being suppressed by a factor of $\frac{m}{E}$ [30]. This is due to mass mixing (see Section 2.4), where the left- and right-chiral fields are coupled via mass terms in the Lagrangian.

In order to avoid confusion as much as possible, this thesis uses the following naming scheme when referring to helicity and chirality: Helicity is referred to as positive or negative. When talking about the chirality of a field, we will use the terms right- and left-chiral. When comparing the chirality of a field to its charge conjugated field, i.e. how the two behave under the chirality operator γ^5 (cf. Equation (2.5)), we will refer to them as left- and right-handed. Thus, ψ_L and $(\psi_L)^c$ both contain the left-chiral field, but ψ_L is left-handed and $(\psi_L)^c$ is right-handed.

2.3.2 Coupling to fermion fields

This section follows the descriptions in [24].

The generators of the $SU(2)_L$ transformations, corresponding to the weak fields W_μ^k with $k = 1, 2, 3$, are the 2×2 Pauli matrices. Thus, we have to construct two-component fermion fields when wanting to couple fermions to W_μ^k . These two-component fields are called weak isospin doublets. From above, we know that the components of the doublets should only be left-chiral fields, since only these can couple to the W_μ^k fields. Hence, the doublets are given for the leptons $L_{\alpha L}$ and quarks Q_{iL} as

$$\begin{aligned} L_\alpha &= \begin{pmatrix} \nu_{\alpha L}(x) \\ \alpha_L(x) \end{pmatrix} \quad \text{with } \alpha = e, \mu, \tau \text{ lepton flavours} \\ Q_i &= \begin{pmatrix} u_{iL}(x) \\ d_{iL}(x) \end{pmatrix} \quad \text{with } i = 1, 2, 3 \text{ quark generations} \end{aligned} \quad (2.6)$$

The top entries of the doublets include fields with weak isospin $I_W^3 = +\frac{1}{2}$ and the bottom entries $I_W^3 = -\frac{1}{2}$. Since $W^\pm$ bosons do not contain any B_μ field component, they only couple to particles fields of a doublet.

Right-chiral fields are placed into isospin singlets

$$(\alpha(x))_R, (u_i(x))_R, (d'_i(x))_R. \quad (2.7)$$

They have a weak isospin of $I_W^3 = 0$ and thus cannot couple to the W_μ^k fields.

Due to the fact that Z bosons do contain a B_μ field component, they can couple to both doublet and singlet fields, as long as the weak hypercharge $Y_W \neq 0$.

Taking this together, we can already see that electrically neutral, right-chiral fields cannot couple to any of the electroweak interaction fields. Thus, the right-chiral neutrino field cannot interact with any of the forces described in the SM and it is therefore not included in the SM (see missing singlet $(\nu_\alpha(x))_R$).

Interaction Lagrangian

The term in the Lagrangian $\mathcal{L}_{SM}$ describing the interaction of the electroweak interaction fields with the lepton fields before symmetry breaking is given by [31]

$$\mathcal{L}_{EW} = \overline{L_{\alpha L}} \gamma^\mu D_\mu L_{\alpha L} + \overline{\alpha_R} \gamma^\mu D_\mu \alpha_R$$

using the above defined lepton doublets (2.6) and singlets (2.7). D_μ denotes the electroweak covariant derivative

$$D_\mu = \partial_\mu - ig \frac{\tau^a}{2} W_\mu^a - ig' Y B_\mu,$$

which ensures local invariance of the Lagrangian under $SU(2)_L \times U(1)_Y$ gauge transformations [31]. The $SU(2)_L$ generators $\tau^a/2$ act only on left-chiral fermion doublets, while right-chiral singlets couple through their hypercharge Y_W . γ^μ indicates the Dirac matrices $\gamma^0, \gamma^1, \gamma^2, \gamma^3$. g and g' are the coupling constants of the $SU(2)_L$ and $U(1)_Y$ interactions, respectively.

After symmetry breaking, the interaction terms of the $W^\pm$ bosons and Z boson with the lepton field α can be written as [31]

$$\begin{aligned} \mathcal{L}_{W^\pm, Z} = & \frac{g}{\sqrt{2}} \left[\overline{\nu_\alpha(x)} \gamma^\mu P_L \alpha W_\mu^+ + \overline{\alpha(x)} \gamma^\mu P_L \nu_\alpha W_\mu^- \right] \\ & + \frac{g}{\cos \theta_W} \overline{\alpha(x)} \gamma^\mu \left[(-I_W^3 + \sin^2 \theta_W) P_L + Q \sin^2 \theta_W P_R \right] \alpha Z_\mu \\ & + \frac{g}{2 \cos \theta_W} \overline{\nu_\alpha} \gamma^\mu P_L \nu_\alpha Z_\mu. \end{aligned} \quad (2.8)$$

The characteristic of the W_μ^k interactions of only coupling to left-chiral fields is reflected in the presence of the left-handed projection operator P_L .

Note, that in every term containing a projection operator $P_{L/R}$ the projection applies to both the fields on the left and right side of the operator. This becomes obvious using the following relations regarding two fermion fields $\zeta(x)$ and $\psi(x)$ are fermion fields.

$$\overline{\zeta(x)}_L = \overline{\zeta(x)} P_R \quad \text{and} \quad \gamma^\mu P_{L/R} = P_{R/L} \gamma^\mu$$

which lead to

$$\begin{aligned} \overline{\zeta_L(x)} \gamma^\mu \psi_R(x) &= \overline{\zeta(x)} P_R \gamma^\mu P_R \psi(x) \\ &= \overline{\zeta(x)} P_R P_L \gamma^\mu \psi(x) = 0 \end{aligned}$$

and thus

$$\begin{aligned}
\bar{\zeta}(x) \gamma^\mu P_L \psi(x) &= \left(\overline{\zeta(x)_L} + \overline{\zeta(x)_R} \right) \gamma^\mu P_L \left(\psi(x)_L + \psi(x)_R \right) \\
&= \overline{\zeta(x)_L} \gamma^\mu P_L \psi(x)_L + \overline{\zeta(x)_L} \gamma^\mu P_L \psi(x)_R + \dots \\
&= \overline{\zeta_L(x)} \gamma^\mu \psi_L(x) + 0 + 0 + 0.
\end{aligned}$$

Vector - Axial structure of the weak interaction

The Lagrangian in Equation (2.8) can be rewritten in terms of fermionic currents as

$$\begin{aligned}
\mathcal{L}_{W^\pm} &= -\frac{g}{\sqrt{2}} \left(J_W^\mu W_\mu^+ + J_W^{\mu\dagger} W_\mu^- \right) \\
\mathcal{L}_Z &= -\frac{g}{\cos \theta_W} \left(J_{Z\alpha}^\mu Z_\mu - J_{Z\nu_\alpha}^\mu Z_\mu \right)
\end{aligned}$$

where the currents are defined as

$$\begin{aligned}
J_W^\mu &= \overline{\nu_\alpha(x)} \gamma^\mu P_L \alpha(x) = \overline{\nu_\alpha(x)} \gamma^\mu c_{WL} P_L \alpha(x) \\
J_{Zf}^\mu &= \overline{f(x)} \gamma^\mu \left[(I_W^3 - Q_f \sin^2 \theta_W) P_L - Q_f \sin^2 \theta_W P_R \right] f(x) \\
&= \overline{f(x)} \gamma^\mu [c_{ZL} P_L + c_{ZR} P_R] f(x).
\end{aligned}$$

with f indicating the fermion flavour, i.e. either a charged lepton α or neutrino ν_α and Q_f the fermion's electric charge. For both currents, we introduced the effective left- and right-handed couplings c_L and c_R .

After expanding the projection operators $P_{L/R}$, the currents can be rewritten as

$$J_\mu^{W/Z} = \frac{g}{2} \overline{\nu_\alpha(x)} (c_V \gamma^\mu + c_A \gamma^\mu \gamma^5) \alpha(x) \quad (2.9)$$

with the vector and axial-vector coupling constants c_V and c_A given through the left- and right-chiral coupling constants $c_{L/R}$ as

$$\begin{aligned}
c_{ZV}^f &= c_{ZR} + c_{ZL} = I_W^{(3)} - 2 Q_f \sin^2(\theta_W), \\
c_{ZA}^f &= c_{ZR} - c_{ZL} = -I_W^{(3)}, \\
c_{WV} &= c_{WL} = 1, \\
c_{WA} &= -c_{WL} = -1.
\end{aligned}$$

From Equation (2.9), we see that the currents of the weak interaction can be regarded as a linear combination of two currents, which behave differently under parity transformation. Currents that change sign under parity transformation are called axial currents A_μ , while currents which are invariant under parity conversion are called vector currents V_μ . Thus, the weak interaction is

often referred to as a $V - A$ current since it is composed of an axial current being subtracted from a vector current.

2.4 Generating mass via the Higgs mechanism

This section follows the descriptions in [24].

The Higgs field is a scalar field doublet, i.e. is composed of two complex scalar fields h^+ and h^0 .

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$

Both components have weak hypercharge $Y_W = +1$, with the charged component h^+ having a weak isospin $I_W^{h^+} = +\frac{1}{2}$ and an electric charge of $Q_{h^+} = +1$, while the electrically neutral component h^0 has $I_W^{h^0} = -\frac{1}{2}$.

Contrary to all the other fields in the SM, the Higgs field has a non-zero vacuum expectation value v , meaning even without any excitations (Higgs bosons) in the fields, it still interacts with other fields via this non-zero value. This mechanism is called the Higgs mechanism [32] and is responsible for generating masses of all the massive bosons and fermions in the SM. The Higgs field couples to the fermion fields via so-called Yukawa-couplings, defined by the respective Yukawa coupling constant y_f of a fermion flavour f .

Naively, one could try to couple the fields $\psi(x) = \psi_L(x) + \psi_R(x)$ via a term like

$$m_f \left(\overline{\psi(x)} \psi(x) \right)$$

These terms, however, are not gauge invariant under $SU(2)_L \times U(1)_Y$ transformations, since ψ_L and ψ_R transform differently. Instead, mass terms can only be generated by coupling left-chiral and right-chiral fermion fields through the Higgs field. For the lepton fields of generation α , this looks as follows:

$$\begin{aligned} \mathcal{L}_D(x) &= y_\alpha \bar{L}_\alpha(x) H \alpha_R(x) + h.c. = y_\alpha \begin{pmatrix} \overline{\nu_{\alpha L}(x)} & \overline{\alpha_L(x)} \end{pmatrix} \begin{pmatrix} h^+(x) \\ h^0(x) \end{pmatrix} \alpha_R(x) + h.c. \\ &= y_\alpha \begin{pmatrix} \overline{\nu_{\alpha L}(x)} & \overline{\alpha_L(x)} \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \alpha_R + h.c. \\ &= y_\alpha \frac{v}{\sqrt{2}} \left(\overline{\alpha_L(x)} \alpha_R(x) + \overline{\alpha_R(x)} \alpha_L(x) \right) \end{aligned} \quad (2.10)$$

The hermitian conjugate $h.c.$ was written out explicitly in the last line. From Equation (2.10) follows the so-called Dirac mass (see Section 2.6.1) of the charged leptons

$$m_\alpha = y_\alpha \frac{v}{\sqrt{2}}.$$

For three lepton generations, the Yukawa couplings are defined using a mass matrix m^{lep} and the Lagrangian term for the charged lepton masses can be written as

$$\begin{aligned}\mathcal{L}_D &= \sum_{\alpha\alpha'} \left(\overline{\alpha_L(x)} m_{\alpha\alpha'}^{lep} \alpha'_R(x) + h.c. \right) \\ &= \begin{pmatrix} \overline{e_L(x)} & \overline{\mu_L(x)} & \overline{\tau_L(x)} \end{pmatrix} m^{lep} \begin{pmatrix} e_R(x) \\ \mu_R(x) \\ \tau_R(x) \end{pmatrix} + h.c.\end{aligned}$$

In the case of charged leptons (and up-type quarks), the Yukawa coupling matrices can be chosen diagonally, e.g.

$$m^{lep} = v \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}.$$

As a consequence, the flavour eigenfields coincide with the mass eigenfields. However, one cannot diagonalise the Yukawa coupling matrices for both entries of the isospin doublets simultaneously. Thus, for massive neutrinos (and down-type quarks) flavour and mass eigenfields are not the same.

2.5 Neutrinos in the SM: description and limitations

In this section, we will discuss the properties of neutrinos in the SM as well as its inconsistencies regarding neutrino masses. In order to understand the BSM extensions that address these issues later on and make comparisons to the SM most simple, we will state the SM neutrino properties very explicitly, even if they are trivial.

2.5.1 Neutrino properties

There are three neutrino flavours α in the SM: the electron neutrino ν_e , the muon neutrino ν_μ and the tau neutrino ν_τ , each corresponding to their respective charged lepton. Neutrinos carry neither electric nor colour charge and are too light to participate in a measurable gravitational interaction. As a result, neutrinos can effectively only interact with other particles via the weak interaction. As explained above, the weak interaction only acts on left-chiral fermion fields for electrically neutral fermions. Therefore, we are only able to detect the left-chiral fields $\nu_{\alpha L}(x)$ and their excitations, the respective neutrinos and antineutrinos. As of now, it is unknown whether the corresponding right-chiral fields $\nu_{\alpha R}(x)$ exist. Hence, they are not part of the SM.

Neutrino masses

In the SM, masses are generated via the Higgs mechanism (see Section 2.4). For this to work, both left- and right-chiral fields need to exist so that they can mix and generate a mass term in the Lagrangian. As there are only the fields $\nu_{\alpha L}(x)$, neutrinos are massless in the SM.

Helicity and Chirality

The SM states that massive fermion fields $\psi(x)$ are composites of a left- and a right-chiral field

$$\psi(x) = \psi_L(x) + \psi_R(x).$$

For the neutrino, only left-chiral neutrino flavour fields $\nu_{\alpha L}(x)$ are present in the SM. Thus, with

$$\nu_{\alpha}(x) = \nu_{\alpha L}(x) + 0 = \nu_{\alpha L}(x)$$

the neutrino flavour field is purely left-chiral.

Because neutrinos are regarded as being massless, the neutrinos ν_{α} as the particle excitations of $\nu_{\alpha L}(x)$ always have negative helicity, while the antineutrinos $\bar{\nu}_{\alpha}$ as antiparticle excitations always have positive helicity

$$\nu_{\alpha} = \nu_{\alpha-} \quad \text{and} \quad \bar{\nu}_{\alpha} = \bar{\nu}_{\alpha+}.$$

Lepton number conservation

The number of leptons of a specific generation L_e , L_{μ} , and L_{τ} is defined as

$$L_{\alpha} = N_{\alpha-} + N_{\nu_{\alpha}} - N_{\alpha+} - N_{\bar{\nu}_{\alpha}}, \quad (2.11)$$

with N being the number of particles of the respective Type. The quantities L_e , L_{μ} , and L_{τ} are conserved separately in the SM [33]. Trivially, this means the total lepton number $L = L_e + L_{\mu} + L_{\tau}$ is also conserved, i.e. $\Delta L = 0$. This conservation arises because the Standard Model Lagrangian contains no renormalizable interaction terms that violate lepton number [33]. All gauge and Yukawa interactions automatically preserve the lepton number.

Dirac and Majorana Fields

A Dirac fermion is a mass eigenstate of a fermion which has different properties compared to its respective antifermion. The electron, for instance, is not identical to its antiparticle, the positron, since they carry opposite electric charges. If fermion and anti-fermion are the same, the particle is referred to as a Majorana fermion [34]. In the SM, there is no fermion known to be a Majorana fermion, since all fermions, apart from the neutrinos, carry electric or colour charge.

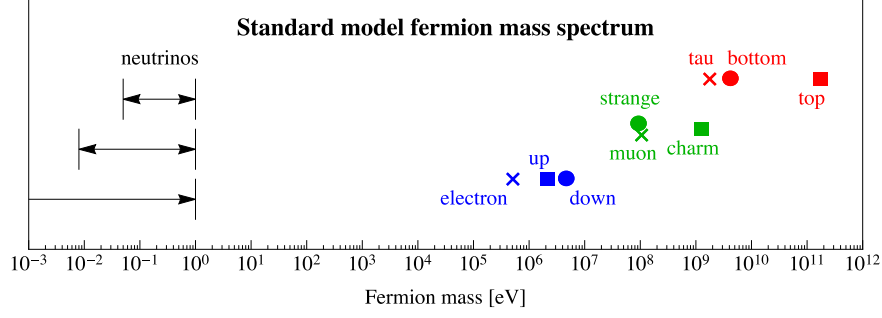


Figure 2.3: Comparison of fermion masses: Neutrinos are known to be lighter than 1 eV, while the other leptons have masses in the MeV to TeV scales. Figure taken from [35].

A more accurate description of Dirac and Majorana fermions is obtained by defining the Dirac/-Majorana property by the field's behaviour under charge conjugation, i.e. whether

$$\nu(x)^c \stackrel{?}{=} \nu(x)$$

holds true. The excitations of the field, the (anti)particles, then inherit the Dirac/Majorana property.

It is important to note that $\nu(x)^c$ is *not* equivalent to an antineutrino, since neutrinos and antineutrinos are described as excitations of the field $\nu(x)$. Instead, the conjugation c is a mathematical operation on the field itself that swaps all particles with antiparticles.

In the SM, neutrinos are massless and the "mass" eigenstates are equivalent to the flavour eigenstates. Since antineutrinos and neutrinos of the left-chiral field have opposite weak isospin I_W^3 they are not the same

$$\nu_{iL}(x)^c = \nu_{\alpha L}(x)^c \neq \nu_{\alpha L}(x) = \nu_{iL}(x).$$

The mass eigenstates are denoted with the index i and the flavour eigenstates with the index α .

2.5.2 Observation of neutrino masses

Up until the late 1990's, the assumption that (anti)neutrinos are massless was in accordance with experimental results [36]. Since then, however, multiple experiments have shown that neutrinos do have mass: When observing neutrinos on earth that originate from the sun and atmosphere, experiments detected less neutrinos of a specific flavour than theoretically predicted [37]. Instead, neutrinos with other flavours appeared, and together they matched the predicted amount of produced neutrinos. Thus, neutrinos changed their flavour on their way from the source to the detector. This effect can be explained with neutrinos having mass. It allows neutrinos to change their flavour while propagating through space-time as mass eigenstates, which is known as neutrino oscillation. Hence, two (tightly connected) assumptions of the SM have been shown to be false: Neutrinos are not massless, and the lepton numbers L_α are not conserved separately.

As of today, we still only have upper boundaries on the neutrino masses (effective mass of the antielectron neutrino: $m_{\bar{\nu}_e} < 0.8 \text{ eV}$ [13]), which is substantially smaller than for the other known fermions, which have masses in the MeV to TeV scales (cf. Figure 2.3).

Description of flavour and mass eigenstates

As discussed above, neutrinos have mass according to experimental observations. Thus, mass and flavour eigenstates of the neutrinos do not coincide. The flavour eigenfields, $\nu_{eL}(x)$, $\nu_{\mu L}(x)$, and $\nu_{\tau L}(x)$ are the ones which we know from the SM as described in Figure 2.1. They correspond to the three charged lepton fields, $e_L^-(x)$, $\mu_L^-(x)$, and $\tau_L^-(x)$, respectively, since they are part of the same weak isospin doublets (cf. Equation (2.6)). Consequently, only the flavour eigenstates can interact weakly, resulting in the switch from one entry of the doublet into the other one. Hence, neutrinos are produced and decay as flavour eigenstates.

Each mass eigenfield can be described as a superposition of the flavour eigenfields and vice versa

$$\begin{aligned} \nu_i(x) &= \sum_{\alpha} U_{\alpha i}^* \nu_{\alpha}(x), \\ \nu_{\alpha}(x) &= \sum_i U_{\alpha i} \nu_i(x), \end{aligned}$$

where α indicates the lepton flavours (e, τ, μ), and i indicates the mass eigenstates (1, 2, 3). U is the Pontecorvo-Maki-Nakagawa-Sakata-Matrix (PMNS-Matrix) [38], describing the mixing parameters of flavour and mass eigenstates

$$\begin{aligned} U &= \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \end{aligned}$$

with s_{ij} denoting $\sin(\theta_{ij})$ and c_{ij} denoting $\cos(\theta_{ij})$. U can be viewed as a rotation in the three-dimensional complex space by three angles θ_{ij} , where each angle describes the rotation angle on the plane spanned by the respective axes i and j (cf. Figure 2.4). In addition to the three mixing angles, the matrix contains a complex phase δ , representing a Dirac CP-violating phase. All of these values are not predicted by theory but measured experimentally [39].

In summary, neutrinos must have non-zero mass due to the observation of neutrino oscillation. However, the neutrino masses cannot be generated by the SM Lagrangian. Instead, there are multiple BSM theories trying to explain the origin of neutrino masses, some of which we will discuss in the next section.

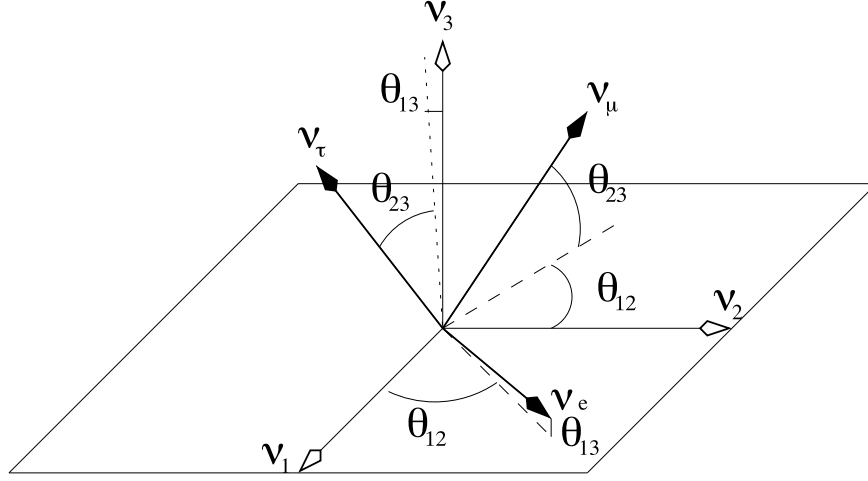


Figure 2.4: Mixing angles of neutrino eigenstates: θ_{ij} denotes the rotation angles on the plane spanned by the respective axes i and j . Figure taken from [40].

2.6 BSM mass of neutrinos

This section is based on [41].

In the following, we will discuss the Seesaw I mechanism which gives mass to the neutrinos and introduces a new particle: the heavy neutral lepton (HNL). This theory is effectively a mixture of two mass generation mechanisms for particles, the Dirac and the Majorana masses. Hence, we will first introduce them separately and afterwards bring them together to form the Seesaw I mechanism.

The common basis for all the following mass generations theories is to start with the SM Lagrangian and add new mass terms to it. These new mass terms need to couple left-handed and right-handed components of the neutrino fields in order to retain Lorentz invariance. This does *not* necessarily mean that the involved fields need to be left and right-chiral but that the terms in $\mathcal{L}$ need to behave as left- and right-handed. Overall, there are four key questions to consider when trying to understand where neutrino masses come from:

- (I) Does the right-chiral neutrino field exist?
- (II) Are the mass eigenstate neutrinos of Dirac or Majorana nature, i.e. are mass eigenstate neutrinos their own antifermions?
- (III) Why are neutrinos so much lighter than the other fermions?
- (IV) Is the total lepton number L conserved?

As explained in the following, each of the three theories answers these questions differently.

2.6.1 Dirac neutrino scenario

Lagrangian and eigenstates

An obvious strategy for generating neutrino masses is to employ the Higgs mechanism, in analogy with the mass generation of all other massive fermions in the Standard Model. Here, right- and left-chiral fields mix (cf. Section 2.4) and therefore the right-chiral field $\nu_{\alpha R}(x)$ is introduced (I). The excitations of this field are so-called sterile particles as they do not interact with any SM particles and thus are not detectable.

The new neutrino Dirac mass term of the Lagrangian reads as

$$\begin{aligned}
 \mathcal{L}_D &= \sum_{\alpha\alpha'} \left(\overline{\nu_{\alpha L}(x)} M_{\alpha\alpha'}^D \nu_{\alpha' R}(x) + \overline{\nu_{\alpha R}(x)} M_{\alpha\alpha'}^{D*} \nu_{\alpha' L}(x) \right) \\
 &= \sum_{\alpha\alpha'} \left(\overline{\nu_{\alpha L}(x)} M_{\alpha\alpha'}'^D \nu_{\alpha' R}(x) \right) + h.c. \\
 &= \begin{pmatrix} \overline{\nu_{eL}(x)} & \overline{\nu_{\mu L}(x)} & \overline{\nu_{\tau L}(x)} \end{pmatrix} M'^D \begin{pmatrix} \nu_{eR}(x) \\ \nu_{\mu R}(x) \\ \nu_{\tau R}(x) \end{pmatrix} + h.c. \\
 &= \overline{\nu'_L(x)} M'^D \nu'_R(x) + h.c.. \tag{2.12}
 \end{aligned}$$

In the last line, we defined new vectors $\nu'_L(x)$ and $\nu'_R(x)$, encapsulating each of the three flavour left-/right-chiral neutrino fields, respectively. The primed notation indicates the flavour basis. The Dirac mass matrix M'^D is not diagonal in the flavour basis. Hence, it not only couples the left- and right-chiral neutrino fields of the same ($\alpha = \alpha'$) flavours but also fields from different flavours ($\alpha \neq \alpha'$).

To diagonalise the M'^D , we use two unitary transformation matrices V_L and V_R [42]

$$M'^D = V_L M^D V_R^\dagger = V_L \text{diag} \begin{pmatrix} m_{D1}, & m_{D2}, & m_{D3} \end{pmatrix} V_R^\dagger.$$

Substituting this into Equation (2.12) yields

$$\begin{aligned}
 \mathcal{L}_D &= \overline{\nu'_L(x)} V_L M^D V_R^\dagger \nu'_R(x) + h.c. \\
 &= \overline{\nu_L(x)} M^D \nu_R(x) + h.c. \\
 &= \sum_i m_i \overline{\nu_i(x)} \nu_i(x)
 \end{aligned}$$

In the third line, the transformation matrices were absorbed into the fields, thus turning flavour (with primes) into mass eigenfields (without primes). Note, that by going from flavour to mass eigenstates, the additional term $h.c.$ is not present anymore as the constructed mass eigenfield is

a sum of both chiral flavour fields:

$$\begin{aligned} v(x) &= V_L^\dagger v'_L(x) + V_R^\dagger v'_R(x) \\ v_i(x) &= \sum_\alpha \left(V_L^\dagger \right)_{i\alpha} v_{\alpha L}(x) + \left(V_R^\dagger \right)_{i\alpha} v_{\alpha R}(x) \end{aligned} \quad (2.13)$$

Mass scale

We set the diagonal entries of M^D as the masses $m_{Di} = y_{vi} \frac{v}{\sqrt{2}}$ of the mass eigenfields $v_i(x)$. y_{vi} are the respective Yukawa coupling constants and v is the vacuum expectation value. The low value of the neutrino masses still requires extremely small Yukawa coupling constants y_{vi}

$$y_{vi} < 10^{-12} \quad \text{while} \quad y_e \approx 10^{-6}$$

The reason for small values of the Yukawa coupling constants y_{vi} is not known (III).

Consequently, the left-chiral flavour eigenfields predominantly create negative-helicity neutrino states and positive-helicity antineutrino states with admixtures of opposite helicity suppressed by $O(m_i/E)$. Analogously, the right-chiral flavour eigenfields predominantly create positive-helicity neutrino states and negative-helicity antineutrino states.

Dirac property

When applying charge conjugation to the mass eigenfields (Equation (2.13)), we see that

$$\begin{aligned} [v(x)]^c &= V_L^T (v'_L(x))^c + V_R^T (v'_R(x))^c \\ &\neq V_L^\dagger v'_L(x) + V_R^\dagger v'_R(x) = v(x). \end{aligned}$$

The mass eigenfields are not invariant under charge conjugation. Thus, they are of Dirac nature (II).

Lepton number conservation

Due to the neutrino fields being invariant under global phase transformations, the total lepton number L stays conserved (IV) [41]. This means, only lepton number conserving (LNC) processes would be observed.

2.6.2 Majorana neutrino scenario

In the case of pure Majorana masses, no new fields are introduced. Therefore, no right-chiral neutrino fields exist (I). Instead, a completely new kind of mass term is introduced in the Lagrangian that couples the existing left-chiral neutrino field to itself.

Lagrangian and eigenstates

In order to be Lorentz invariant, the mass term needs to couple a right-handed to a left-handed component. For the left-handed component we simply use the left-chiral field $\nu_{\alpha L}(x)$ itself. For the right-handed component, we use the same field with some mathematical operation applied to it, so that it behaves right-handed. Charge conjugation accomplishes exactly this. Sometimes this is denoted by

$$(\nu_{\alpha L}(x))^c = \nu_{\alpha R}^c(x).$$

We will refrain from using this notation however, in order to emphasise that *no new* field is introduced here. Instead, we have to keep in mind that charge conjugation switches the handedness of a term, i.e. $(\nu_L)^c$ is right-handed.

We write the Majorana mass term as

$$\begin{aligned}\mathcal{L}_{LM} &= \sum_{\alpha\alpha'} \frac{1}{2} \left(\overline{\nu_{\alpha L}(x)} M'_{\alpha\alpha'} (\nu_{\alpha' L}(x))^c \right) + h.c. . \\ &= \frac{1}{2} \begin{pmatrix} \overline{\nu_{eL}(x)} & \overline{\nu_{\mu L}(x)} & \overline{\nu_{\tau L}(x)} \end{pmatrix} M'^M \begin{pmatrix} (\nu_{eL})^c(x) \\ (\nu_{\mu L})^c(x) \\ (\nu_{\tau L})^c(x) \end{pmatrix} + h.c. \\ &= \frac{1}{2} \overline{\nu'_L(x)} M'^M (\nu'_L(x))^c + h.c.\end{aligned}$$

As above, primes indicate the flavour eigenstate representation. The factor $\frac{1}{2}$ avoids double counting. The mass matrix M'^M is again not diagonal in the flavour basis and needs to be diagonalised to find the mass eigenstates. Since M'^M is symmetric, we can do this using only one unitary matrix U [41]:

$$M'^M = U M^M U^T = U \text{diag} \begin{pmatrix} m_{M1}, & m_{M2}, & m_{M3} \end{pmatrix} U^T$$

Implementing this into the Lagrangian gives us

$$\begin{aligned}\mathcal{L}_{LM} &= \frac{1}{2} \overline{\nu'_L(x)} U M^M U^T (\nu'_L(x))^c + h.c. \\ &= \frac{1}{2} \overline{U^\dagger \nu'_L(x)} M^M \left(U^\dagger \nu'_L(x) \right)^c + h.c. \\ &= \frac{1}{2} \overline{\nu(x)} M^M (\nu(x))^c \\ &= \frac{1}{2} \sum_i m_{Mi} \left(\overline{\nu_i(x)} (\nu_i(x))^c \right)\end{aligned}$$

The corresponding mass eigenfields are thus defined as

$$\begin{aligned} \nu(x) &= U^\dagger \nu'_L(x) + \left(U^\dagger \nu'_L(x) \right)^c \\ \nu_i(x) &= \sum_{\alpha} \left(U^\dagger \right)_{i\alpha} \nu_{\alpha L}(x) + \left(\left(U^\dagger \right)_{i\alpha} \nu_{\alpha L}(x) \right)^c \end{aligned} \quad (2.14)$$

The entries of M^M are the masses m_{Mi} of the mass eigenfields $\nu_i(x)$. m_{Mi} are new parameters without any comparability to similar quantities in the SM. Thus, their smallness is more reasonable than in the Dirac case where we can compare the neutrino Yukawa couplings to those of other fermions (III).

Majorana property

Using $(\psi^c)^c = \psi$, one can easily see that the mass eigenfields in Equation (2.14) are the same as their charge conjugates:

$$\begin{aligned} [\nu(x)]^c &= \left[U^\dagger \nu'_L + \left(U^\dagger \nu'_L \right)^c \right]^c \\ &= \left(U^\dagger \nu'_L \right)^c + U^\dagger \nu'_L \\ &= \nu(x) \end{aligned}$$

Thus, the mass eigenfields are Majorana fields. This means that the neutrino mass eigenstates are the same as their respective antineutrino mass eigenstates (II).

Lepton number

Regarding (IV), we see that total lepton number L is not a reasonable quantity anymore, since we cannot state whether a neutrino should be counted as a lepton or an antilepton in Equation (2.11). This discrepancy persists even for processes in which only charged leptons appear in the initial and final states, despite their well-defined fermion or antifermion character: An electron neutrino, for instance, could either be turned into an electron or a positron by the weak interaction. Hence, if a neutrino was an intermediate state, the lepton number could be violated by two units and $L \neq 0$. These events are referred to as lepton number violating (LNV).

The theory of pure Majorana mass terms is forbidden in the SM because ν_L is part of an $SU(2)_L$ doublet and such a term would not be invariant under the $SU(2)_L \times U(1)_Y$ gauge symmetry. However, there are BSM theories that allow for purely Majorana neutrino masses, e.g. the Seesaw (II) [43].

2.6.3 Seesaw mechanism Type I

The following section is based on [42].

It is possible to combine the two mass generation theories explained in the previous sections. Most textbooks introduce this by first discussing the combination of Dirac and Majorana masses only regarding one flavour and two mass neutrino eigenfields. As we will use all three flavour fields, one should refer to [41, 42], in order to see the following considerations in simpler terms first.

First, right-chiral neutrino fields $\nu_R(x)$ are introduced as new fields to the SM, since this is the premise for Dirac masses (I). In the most general description of this theory, the number of new right-chiral neutrino fields is left open [42]. In accordance with the Seesaw Type I, we will introduce 3 right-chiral fields $\nu_{\alpha R}(x)$, one for each left-chiral flavour field $\nu_{\alpha L}(x)$ that already exists in the SM. Now, we introduce all possible mass terms to the Lagrangian using all possible combinations of the existing fields and their charge conjugates. The only restriction is that the mass terms need to couple left- and right-handed components in order to retain Lorentz invariance

$$\begin{aligned}\mathcal{L}_{D,LM,RM} &= \mathcal{L}_D + \mathcal{L}_{LM} + \mathcal{L}_{RM} \\ &= \sum_{\alpha,\beta=e,\mu,\tau} \overline{\nu_{\alpha R}(x)} M_{\alpha\beta}^D \nu_{\beta L}(x) \\ &\quad + \sum_{\alpha\alpha'} \frac{1}{2} \left(\overline{\nu_{\alpha L}(x)} M_{\alpha\alpha'}'^{LM} (\nu_{\alpha' L})^c(x) \right) \\ &\quad + \sum_{\alpha\alpha'} \frac{1}{2} \left(\overline{\nu_{\alpha R}(x)} M_{\alpha\alpha'}'^{RM} (\nu_{\alpha' R})^c(x) \right) + h.c..\end{aligned}$$

The "missing" combination of $(\nu_L(x))^c$ and $(\nu_R(x))^c$, which also fits the criteria stated above, is the same as the Dirac mass term that is already included (see Appendix A.1).

As already discussed above, any terms coupling $\nu_{\alpha L}(x)$ to its charge conjugate would violate SM gauge symmetries as they are both $SU(2)$ doublets. Thus, we set $M'^{LM} = 0$ for the Seesaw mechanism Type I. The remaining terms either couple singlets to doublets or singlets to singlets, which is both allowed. Furthermore, the Seesaw Type I model postulates the Dirac mass couplings to be much smaller than the Majorana couplings. This is denoted by using small letters m'^D for the Dirac mass matrix from now on ($m'^D \ll M'^{RM}$).

As before, our first course of action is to switch to the mass eigenfield representation. In order to

do this, we have to rewrite the $\mathcal{L}_{D,RM}$ so that it contains a mass matrix:

$$\begin{aligned}\mathcal{L}_{D,RM} &= -\frac{1}{2} \sum_{\alpha\alpha'} \left(\overline{\nu_{\alpha L}}(x) \quad \overline{(\nu_{\alpha R})^c}(x) \right) \begin{pmatrix} 0 & m'_{\alpha\alpha'}{}^D \\ m'_{\alpha\alpha'}{}^{D,T} & M'_{\alpha\alpha'}{}^{MR} \end{pmatrix} \begin{pmatrix} (\nu_{\alpha' L})^c(x) \\ \nu_{\alpha' R}(x) \end{pmatrix} + h.c. \\ &= -\sum_{\alpha\alpha'} \frac{1}{2} \overline{\omega'_{\alpha L}} M'^{D,RM} \omega'_{\alpha' R} + h.c. \\ &= -\frac{1}{2} \overline{\Omega'_L} M'^{D,RM} \Omega'_R + h.c.\end{aligned}$$

In the last two steps, we used the fact that all the components on the left behave left-handed, while on the right all components behave right-handed. They were combined into new, auxiliary vectors

$$\begin{aligned}\omega'_{\alpha L} &= \begin{pmatrix} \nu_{\alpha L} \\ (\nu_{\alpha R})^c \end{pmatrix} \quad \text{and} \quad \omega'_{\alpha R} = \begin{pmatrix} (\nu_{\alpha L})^c \\ \nu_{\alpha R} \end{pmatrix} \\ \Omega'_L &= \begin{pmatrix} \omega'_{eL} \\ \omega'_{\mu L} \\ \omega'_{\tau L} \end{pmatrix} \quad \text{and} \quad \Omega'_R = \begin{pmatrix} \omega'_{eR} \\ \omega'_{\mu R} \\ \omega'_{\tau R} \end{pmatrix}\end{aligned}$$

Using $m'^D \ll M'^{RM}$ the diagonalised mass matrix M^{M+D} can be approximated [44, 45]

$$M'^{RM+D} = W M'^{RM+D} W^T \approx W \begin{pmatrix} m & 0 \\ 0 & M \end{pmatrix} W^T$$

with m and M being 3×3 diagonal matrices. Using the diagonal matrices m^D and M^{MR} , we can approximate m and M by

$$m \approx \left(m^D\right)^T \left(M^{MR}\right)^{-1} m^D \Rightarrow m_j \approx \frac{(m_i^D)^2}{M_i^{MR}} \quad \text{and} \quad M \approx M^{MR} \Rightarrow M_k \approx M_i^{MR}$$

Here, j indicates the three light neutrino masses and k the three heavy neutrino masses. These relations are the reason for the name "Seesaw mechanism". The higher the masses M_k are, the lighter the masses $m_i \sim \frac{1}{M_k}$ get.

Eigenstates

The 3 eigenfields $n_j(x)$ corresponding to the masses m_j and the 3 eigenfields $N_k(x)$ corresponding to the heavy masses M_k are approximated by [46]

$$\begin{aligned} n_i(x) &\approx \frac{1}{\sqrt{2}} \left[\sum_{\alpha} (v_{\alpha L} + (v_{\alpha L})^c) + \Theta (v_{\alpha R} + (v_{\alpha R})^c) \right] \quad \text{with} \quad \Theta \approx m_d M_k^{-1} \\ N_k(x) &\approx \frac{1}{\sqrt{2}} \left[\sum_{\alpha} (v_{\alpha R} + (v_{\alpha R})^c) - \Theta (v_{\alpha L} + (v_{\alpha L})^c) \right] \end{aligned} \quad (2.15)$$

Due to the small share of $v_{\alpha R}(x)$ in $n_i(x)$, the excitations of the eigenfield $n_i(x)$ can be identified as a superposition of the Dirac neutrino and antineutrino excitations of the left-chiral field, we know from the SM [46]. The eigenfields N_i , however, are new, and their excitations are called Heavy Neutral Leptons (HNLs).

Mass scale

Depending on how high the Yukawa coupling is chosen to be, different masses of the HNLs are necessary. For neutrino Yukawa coupling constants of $y_{\nu i} \sim 10^{-6} \dots 10^{-7}$ (similar or slightly smaller scale than for other leptons) and neutrino masses of $m_i \approx 1$ eV, we get HNL masses of

$$M_i \approx \frac{m_i}{(m_i^D)^2} = \frac{m_i}{\left(y_{\nu i} \frac{v}{\sqrt{2}}\right)^2} \approx 1 \dots 10 \text{ GeV}$$

Thus, the smallness of the neutrino masses is explained by the introduced mass parameters $M^{MR} \approx M_i$ while the Yukawa coupling is similar compared to the charged leptons (III).

Majorana property

The definitions of $n_i(x)$ and $N_i(x)$ in Equation (2.15) show that both mass eigenfields are Majorana fields invariant under charge conjugation (II)

$$\begin{aligned} [n_i(x)]^c &= n_i(x) \\ [N_i(x)]^c &= N_i(x). \end{aligned}$$

The difference to the pure Majorana case is that we now have six mass eigenfields instead of three. Each of those six mass eigenfields is composed of both left- and right-chiral flavour fields (with charge conjugate) instead of only the left-chiral fields (with charge conjugate).

Lepton number

As already discussed in the pure Majorana case, the Majorana mass fields immediately result in the total lepton number not being conserved, generally (IV). If a neutrino participates in an

interaction, $\Delta L = 2$ and thus LNV processes can occur.

Helicity and Chirality

Due to both mass eigenstates in Equation (2.15) containing the left-chiral neutrino field $\nu_{\alpha L}(x)$ as well as its charge conjugated $(\nu_{\alpha L}(x))^c$, the helicity of their excitations does not have a preferred value anymore in the ultra-relativistic limit. Instead, the light mass neutrino fields $n_i(x)$ can be seen as superpositions of neutrino and antineutrino excitations of the left-chiral field, i.e. the SM Dirac neutrino and antineutrino states. The HNL fields $N_k(x)$ can be seen as superpositions of neutrino and antineutrino excitations of the right-chiral field $\nu_{\alpha R}(x)$ [46].

2.7 HNLs at the LHC

2.7.1 Production

For our analysis, we consider simulations of heavy neutral leptons (HNLs) produced at the Large Hadron Collider (LHC) at CERN (Conseil Européen pour la Recherche Nucléaire). At the LHC, high-energy protons are accelerated and collided. Dedicated experiments, such as ATLAS [17] and CMS [18], are located at the interaction points of the two beams and record the products of the collisions. In proton-proton collisions at the LHC, it is not the protons as composite objects that interact directly, but rather their constituent partons (quarks and gluons).

In Figure 2.5, the Feynman diagram of a possible HNL production is depicted. The HNL is produced promptly, meaning it is created at the primary interaction vertex, alongside a charged (prompt) lepton of flavour α via the decay of a $W^\pm$ boson. The $W^\pm$ boson, in turn, is produced in a Drell-Yan process, meaning by the collision of a quark and an antiquark stemming from the central colliding protons.

Although the HNL could also be produced via a Z boson, we will not consider this production channel, as it has a significantly lower cross-section. Furthermore, the displaced lepton would be a neutrino, which cannot be detected directly, complicating the detection of this process. In the comparisons and simulations performed in this thesis, we limited ourselves to the case where the prompt lepton produced alongside the HNL is a $\mu^\pm$, i.e. $\alpha = \mu$.

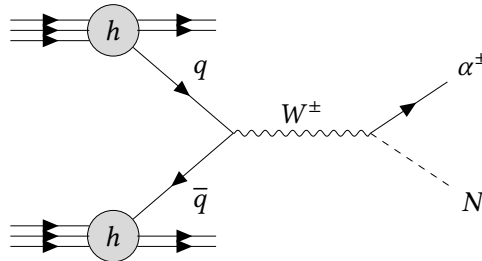


Figure 2.5: Production of the HNL at LHC via a $W^\pm$ boson : One quark q and one antiquark $\bar{q}$ from the colliding protons h interact and produce a $W^\pm$ boson which subsequently decays into a HNL N and a charged prompt lepton of flavour α .

2.7.2 Decay modes

There are multiple possible decay channels for the HNL, which are listed in Table 2.1. Due to its small mixing with the active neutrinos, the HNL is long-lived compared to other unstable particles of similar mass. Therefore, it travels a significant distance before decaying, leading to a displaced decay vertex.

First, the HNL can either decay into a so-called displaced charged lepton of flavour $\beta^\mp$ and an off-shell (denoted by $*$) $W^\pm$ boson of opposite charge or into a displaced neutrino of flavour β and an off-shell Z -boson. Depending on the final states, there are two additional categories which the HNL decays can be divided into: the leptonic and semileptonic decays.

Leptonic decays

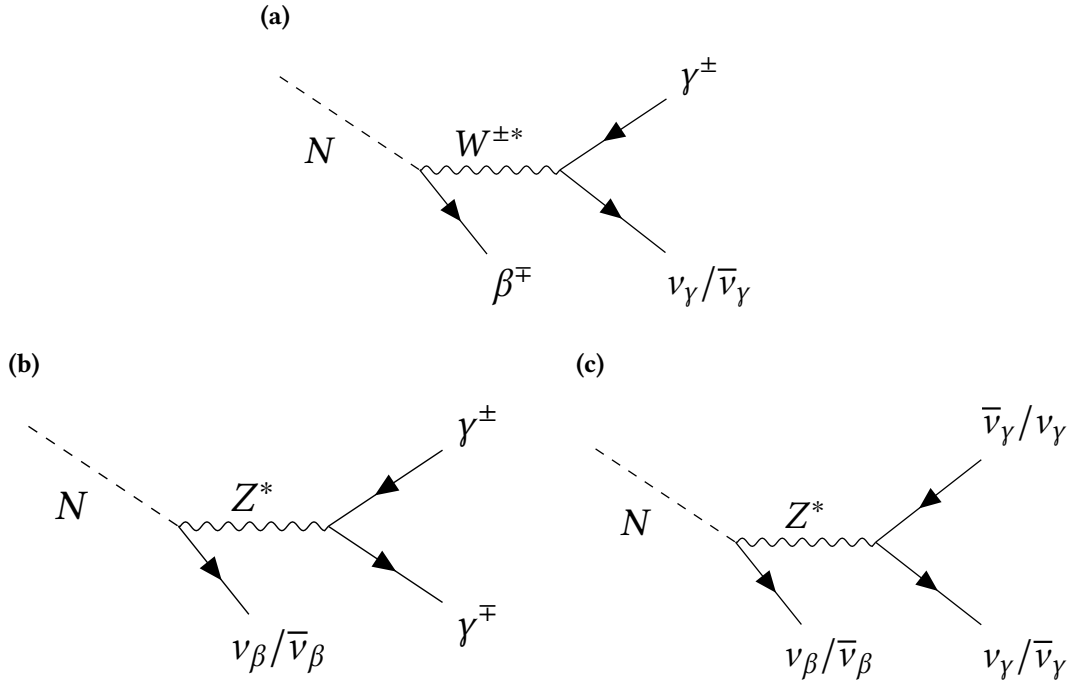


Figure 2.6: Leptonic decay channels of the HNL: Arrows indicate the particle/antiparticle property of the particles for the upper/left version. (a) The HNL decays into a displaced lepton $\beta^\mp$ and a $W^\pm$ boson, the $W^\pm$ boson decays into a charged lepton $\gamma^\pm$ and a neutrino/antineutrino $\nu_\gamma/\bar{\nu}_\gamma$. (b) The HNL decays into a displaced neutrino/antineutrino $\nu_\beta/\bar{\nu}_\beta$ and a Z boson, the Z boson decays into a charged lepton pair $\gamma^\pm$. (c) The HNL decays into a displaced neutrino/antineutrino $\nu_\beta/\bar{\nu}_\beta$ and a Z boson, the Z boson decays into a neutrino/antineutrino pair $\nu_\gamma/\bar{\nu}_\gamma$.

As the name already implies, in the leptonic case there are only leptons in the final state. The corresponding Feynman diagrams can be seen in Figure 2.6. The $W^\pm$ boson decays into a charged lepton and a neutrino (cf. 2.6a), whereas the Z -boson decays into a lepton antilepton pair of the same flavour γ , either having opposite charge, γ^- and γ^+ , (cf. 2.6b) or no charge, ν_γ and $\bar{\nu}_\gamma$, (cf. 2.6c). Although the Z bosonic decay is again hard to detect, we have to consider it, as in the final states where all displaced leptons are charged and of the same flavour, we cannot

differentiate between the two decays and thus their matrix elements can interfere. The decay into only neutrinos however, is undetectable and was thus not simulated.

Semileptonic decays

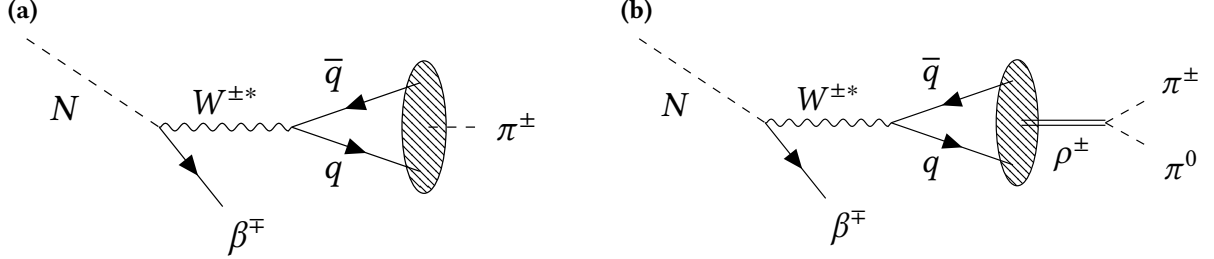


Figure 2.7: Semileptonic HNL decays via a $W^\pm$ boson: The Boson decays into a quark-antiquark pair. (a) The quarks hadronise and form a charged pion ($\pi^\pm$) (b) The quarks hadronise and form a $\rho^\pm$ -meson, which decays into a charged ($\pi^\pm$) and a neutral pion (π^0).

For the semileptonic decays, we will restrict ourselves to the decays via a $W^\pm$ boson, as the Z decays are again very unlikely, hard to detect and in this case well distinguishable from the $W^\pm$ boson decays. In Figure 2.7, the $W^\pm$ boson decays into a quark antiquark pair are depicted. Because the produced quark flavours need to fit the boson's charge, the $W^\pm$ boson decays into an up-type quark and a down-type antiquark or the other way around. The quarks subsequently hadronise, resulting in either a charged pion $\pi^\pm$ (cf. 2.7a) or a $\rho^\pm$ -meson, which in 99.95% of times decays further into a charged pion $\pi^\pm$ and a neutral pion π^0 (cf. 2.7b)[27]. Both pions and ρ -mesons consist of one up and one down quark.

2.7.3 Helicity states of the HNL and angular distributions

After introducing the HNLs and their production and decay, we will now take a closer look at the possible helicity states of the HNL and how these influence the angular distributions of its decay products. As we will see in the following, the helicity of the decaying HNL is tightly linked to the lepton number of the process and thus a valuable observable both in the validation of the simulations as well as in experimental searches for HNLs.

Angular distributions of the HNL's decay products

The following qualitative explanation closely follows the one given in [47], where the decay of a spin zero pseudoscalar meson into a charged lepton and an HNL and its subsequent decay are discussed.

For simplicity, we will restrict these considerations to the case of HNLs produced in the decay of

Category	HNL decay	Subsequent boson decay/ final state
Leptonic (via $W^{\pm*}$)	$N \rightarrow \beta^\mp W^{\pm*}$	$W^{\pm*} \rightarrow \gamma^\pm \nu_\gamma$ (or $\gamma^\pm \bar{\nu}_\gamma$) Final state: μ & $\beta^\mp \gamma^\pm \nu_\gamma$
Leptonic (via Z^*)	$N \rightarrow \nu_\beta Z^* / \bar{\nu}_\beta Z^*$	$Z^* \rightarrow \gamma^+ \gamma^-$ Final state: μ & $\nu_\beta \gamma^+ \gamma^-$
Semileptonic (via $W^{\pm*}, \pi^\pm$)	$N \rightarrow \beta^\mp W^{\pm*}$	$W^{\pm*} \rightarrow \pi^\pm$ Final state: μ & $\beta^\mp \pi^\pm$
Semileptonic (via $W^{\pm*}, \rho^\pm$)	$N \rightarrow \beta^\mp W^{\pm*}$	$W^{\pm*} \rightarrow \rho^\pm, \rho^\pm \rightarrow \pi^\pm \pi^0$ Final state: μ & $\beta^\mp \pi^\pm \pi^0$

Table 2.1: Summary of HNL decay channels simulated for this thesis. β denotes the lepton flavour of the displaced lepton stemming from the HNL decay, with $\beta \in \{e, \mu, \tau\}$. The lepton flavour of the subsequent boson decay is denoted by $\gamma \in \{e, \mu\}$. In the final state, the prompt lepton stemming from the HNL production is included. Depending on the LNC or LNV nature of the events, it can either be of the same or of opposite charge as the displaced lepton $\beta^\mp$. Without loss of generality, in this thesis, the prompt leptons flavour α is restricted to $\alpha = \mu$.

a $W^\pm$ boson and decaying semileptonically into a charged lepton of flavour μ and a pseudoscalar meson, as depicted in Figure 2.2. Analogous, but more complex, considerations can be made for the other decay channels [48].

The HNL's parent particle is a $W^\pm$ boson which is primarily produced in a transverse polarisation state. This is due to the fact that it is produced in the decay of a quark-antiquark pair. The quarks can be treated as ultra-relativistic particles due to their small mass compared to the $W^\pm$ boson's mass, i.e. $m_u \approx 2.2$ MeV and $m_d \approx 4.7$ MeV. Thus, we assume their helicity states to be directly linked to their chirality, i.e. the quark has negative helicity and the antiquark positive helicity. This means, the $W^\pm$ boson is primarily produced with spin $s = \pm 1$ along its direction of motion (see Figure 2.2) [49].

Thus, the helicity of the produced HNL is determined by the helicity of the prompt lepton and, in the ultra-relativistic limit, by the prompt lepton's charge, since negatively charged leptons are left-handed and positively charged antileptons are right-handed in weak interactions. Additionally, the HNL decays into a lepton of flavour μ and a $W^\pm$ boson. Again, we assume m_μ to be small compared to the HNL mass, such that the helicity of the lepton is determined by its charge. In order to conserve angular momentum, the spins of the decay products need to add up to the HNL's previous spin, which in turn is determined by the production.

Thus, the direction of emission of the charged lepton stemming from the HNL decay is determined by the lepton number of the process. In the LNC cases, the charged prompt and displaced lepton are of opposite charge and thus opposite helicity, therefore they need to be emitted in opposite directions with respect to the HNL's rest frame. For LNV events, however, both leptons have the same charge and thus helicity, such that they need to be emitted in the same direction to conserve angular momentum. This is schematically depicted in Figure 2.2 for the case of the semileptonic HNL decay into a pseudoscalar meson.

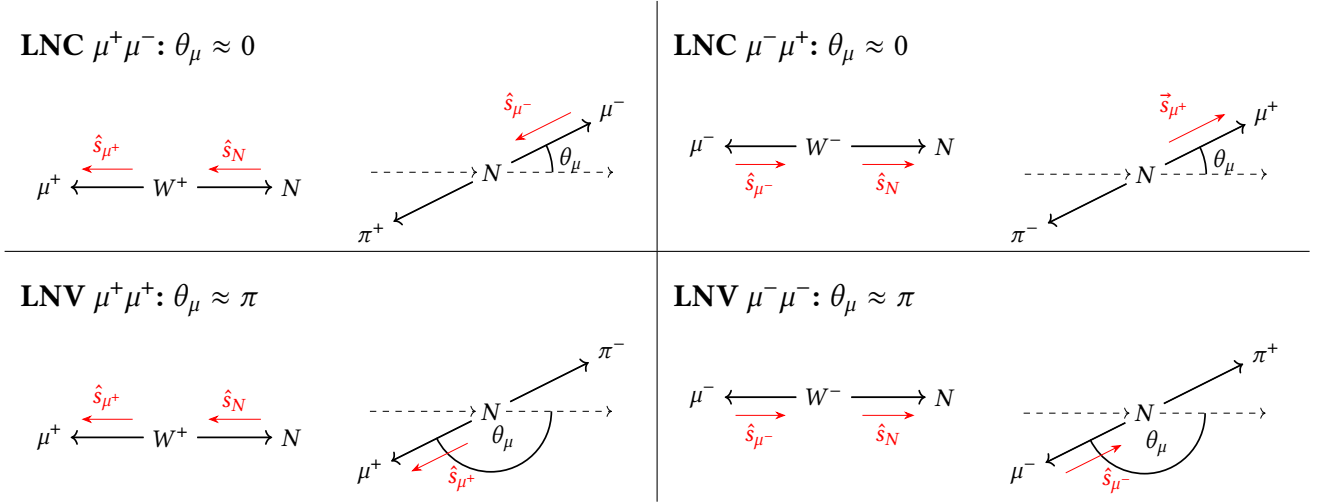


Table 2.2: Visualisation of decay angles and helicities in the semileptonic decay $HNL \rightarrow \pi^\pm, \mu^\mp$: In each box, the left diagrams are shown in the $W^\pm$ rest-frame and the right diagrams in the HNL rest frame. Black arrows indicate the particle's momentum, red arrows indicate particles' spin direction $\hat{s}$ in the respective rest frame. The direction of the HNL's momentum is indicated by the grey dashed line. The direction of motion for the prompt and displaced (anti)particles are determined by two factors: Due to the high energies of the particles involved, we assume helicity states to align with chirality states. Moreover, in order to conserve angular momentum, the total spin of the initial and final state need to be equal.

Energy distributions as an angular distribution identifier

When discussing angular distributions, we usually consider energy fractions of the decay products with respect to the parent's Energy

$$\mathcal{E}_{chi} = \frac{E_{chi}}{E_{par}}.$$

This is due to the fact, that the angle between two particles in the lab frame is not directly related to the angle θ in the parent's rest frame, as the boost of the parent particle needs to be taken into account. However, the energy fractions of the decay products are directly linked to the angles in the parent's rest frame, as we will see in the following.

$\mathcal{E}_{chi} \approx 1$ indicates that the child particle inherited most of the parent's energy and thus, has a momentum in the same direction as the parent in the parent's rest frame. On the other hand, a $\mathcal{E}_{chi} \approx 0$ indicates that the child particle inherited very little of the parent's energy and thus, was emitted in the opposite direction, meaning in the parent particle's rest frame, the child particle was emitted backwards compared to the parent particle's momentum ($\theta = \pi$)

$$\cos(\theta) \approx 2, \mathcal{E}_{chi} - 1.$$

$W^\pm$ production	HNL production	HNL propagator	HNL decay
$W^- \rightarrow \mu^- \text{ HNL}$			
$W^+ \rightarrow \mu^+ \text{ HNL}$			
$W^- \rightarrow \mu^- \text{ HNL}$			
$W^+ \rightarrow \mu^+ \text{ HNL}$			

Table 2.3: Spin correlations in the HNL event: The momentum directions of the particles are indicated by green arrows, while the red arrows indicate the spin directions in the respective rest frames. The black lines indicate the fermion flow, which is opposite to the momentum direction for anti-fermions. Labels L indicate whether the HNL is behaving as a particle ($L = +1$) or an antiparticle ($L = -1$) in the respective interaction. In order to conserve angular momentum, the spins and momenta of the particles have to align as shown in the diagrams. The helicity of the HNL is restricted by the prompt leptons charge and stays the same during the decay process. The particle and antiparticle nature of the HNL is conserved in the LNC events and flipped in LNV events.

In summary, we arrive at the following relations:

$$\mathcal{E}_{chi} \approx 1 \Rightarrow \langle \text{parent, child } \theta \approx 0 \Rightarrow \cos(\theta) = 1$$

$$\mathcal{E}_{chi} \approx 0 \Rightarrow \langle \text{parent, child } \theta \approx \pi \Rightarrow \cos(\theta) = -1$$

Since the energy of the HNL cannot be lost, the other decay product will have the complementary energy fraction, i.e. $\mathcal{E}_{chi2} = 1 - \mathcal{E}_{chi1}$.

HNL helicity states

Lastly, we can discuss the helicity states of the HNL in regard to the lepton number of the process in more detail. In Figure 2.3, the following considerations are schematically depicted for both LNC and LNV events.

The HNL's direction of momentum and the direction of its spin are determined in the production process. The HNL is either produced due to coupling to its $\nu_L(x)$ or its $(\nu_L)^c(x)$ component (see Equation (2.15)), depending on the charge of the prompt lepton. The same holds true for the decay. If the HNL in both cases couples to the same component, i.e. $\nu_L(x)$ or $(\nu_L)^c(x)$ we have a lepton number conserving (LNC) event. In these cases, the HNL behaves as either a negative helicity particle ($\nu_L(x)$) or a positive helicity antiparticle ($(\nu_L)^c(x)$).

When the HNL couples to different components in production and decay, we have a lepton number violating (LNV) event. In these cases, the HNL switches its "particle/antiparticle" state between production and decay, while maintaining the direction of spin and momentum. Thus, it retains its helicity state and decays as either a negative helicity antiparticle or a positive helicity particle [50].

The effects of these helicity states on the angular momentum distributions of the decay products were discussed phenomenologically in the previous section regarding the semileptonic decay channel into a pseudoscalar meson. Furthermore, we will compare the angular distributions of the decay products to the expected distributions stemming from these helicity states for all decay channels considered in order to validate the simulations. To this end, we will compare our distributions to those predicted for polarised $\tau^\pm$ decays in [48], in Chapter 4 where negative helicity τ^- and positive helicity τ^+ correspond to LNC events and positive helicity τ^- and negative helicity τ^+ correspond to LNV events. As the HNL is a majorana particle, we will refer to this behaviour as LNC cases having negative helicity HNLs and LNV cases having positive helicity HNLs, from now on.

CHAPTER 3

Simulation

The field of particle physics currently tries to explain the origin of neutrino masses, by adapting the SM to BSM theories. However, no real hints into which direction we should choose exist so far. As mentioned in the introduction, there are multiple questions in particle physics to which the SM does not provide answers. There are, however, many different theories *suggesting* answers. The most popular approach to test these theories are collider experiments, such as the proton-proton collisions at energies up to 13 TeV studied at the Large Hadron Collider (LHC). In order to make an assessment on whether a specific theory is more probable than another, we need to compare the experimental results to the predictions of different theories. This process appears to be rather complex and dominated by a lot of randomness, due to the quantum nature of particle interactions. Thus, we cannot predict experimental observations analytically, but have to model them using Monte Carlo event generators. The term *Monte Carlo* refers to a class of numerical methods based on the random sampling of kinematic variables according to their underlying probability distributions [51]. In particle physics, Monte Carlo techniques are used to generate large samples of simulated events, providing so-called truth-level information on a particle's properties, for example its charge, momentum, lifetime, etc. These truth-values are typically not directly measurable by detectors. Therefore, to compare event generator predictions with real observations, the simulation data is handed over to a detector simulation, modelling the experimental signatures the particles have in the detector interacting with its material. The resulting samples are then compared to real measurements, allowing us to evaluate SM and BSM predictions in order to confirm or reject them up to a certain probability.

Both for detector simulations and event generation, there are multiple tools to choose from. For the event generation, standard tools comprise PYTHIA [52], HERWIG [53, 54], and SHERPA [1]. This thesis focuses on the implementation of HNLs in SHERPA and therefore, after introducing the basic principles of Monte Carlo event generators, we will discuss SHERPA modules relevant for the HNL simulation in more detail.

3.1 Monte Carlo event generation

3.1.1 General overview of event generation

This section follows the descriptions of [55], [56] and [57].

A Monte Carlo event generator is a software tool which simulates high-energy particle collisions. Collision events are dominated by the effects of the strong interaction (QCD effects), whose coupling constant α_s depends on the energy scale, defined by the momentum transfer in the interaction. As a hadronic collision involves several distinct physical processes, the full event is characterised by multiple, widely separated energy scales, ranging from hard interactions at large momentum transfer to soft interactions at small momentum transfer. This separation of scales is particularly useful, as it allows the collision event to be described as a sequence of processes associated with different energy regimes. The individual phases can therefore be factorised, meaning that calculations can be performed separately and combined into a consistent event description.

In Figure 3.1, a typical MC simulated event of a collision at the LHC is depicted where the different event phases differentiated by colour. The event starts with a hard collision (central red dot), which is the collision of two high-energy partons. Partons are the constituents of hadrons, meaning a parton can either be a quark or a gluon. In the depicted example event, the hard collision results in two top quarks and a Higgs boson (smaller red dots). Since the coupling constant of the strong interaction, α_s , is inversely related to the energy scale (see Equation (3.6)), the probabilities of this hard interaction can be calculated by the matrix element generator using perturbation theory. This is usually done at leading order (LO), or next-to-leading order (NLO) in QCD. In this hard collision, the protons do not interact as a whole but rather their constituent partons, the quarks and gluons. First of all, this requires the usage of parton distribution functions (PDFs) to determine the probability of finding a parton with a certain flavour and momentum fraction of the original proton. Secondly, this means that the remaining partons not involved in the primary hard scattering, also interact and pollute the event and therefore have to be simulated as well. These interactions are called underlying event (purple circle and lines).

The initial and final state particles of the hard collision usually carry electric and colour charge and therefore, produce QED (yellow lines) and QCD radiation (blue lines). Since gluons themselves carry colour charge, this results in a cascade of additional partons being emitted. Initially, the partons coming out of this so-called parton shower have high enough energies and are thus unconfined. These high energies also allow us to describe this phase using perturbation theory. As the partons' energies decrease, at about 1 GeV they hadronise (light green), which means that they form bound states. At this energy scale, α_s is too large for perturbation theory to work because higher orders of QCD correction terms become too large to be neglected. Hence, processes have to be described phenomenologically, where the model parameters are tuned to fit experimental data. As a consequence of the parton shower, the resulting hadrons remain close in direction and are observed as jets, i.e. collimated streams of hadrons. Lastly, unstable hadrons

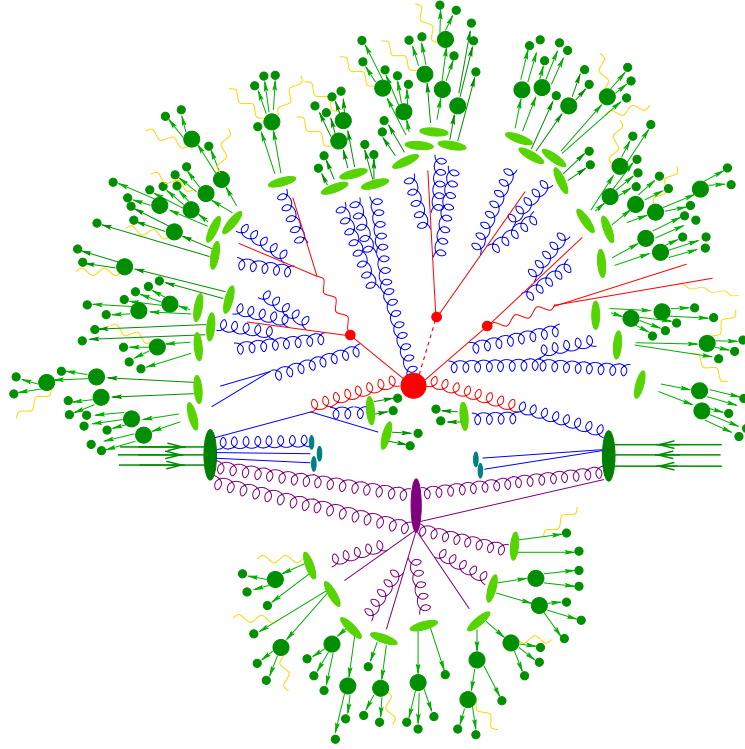


Figure 3.1: Exemplary Monte Carlo event of a proton-proton collision. The hard collision of two partons (big red circle) results in two top quarks and a Higgs boson (smaller red circles). Final and initial partons can produce QCD radiation (blue lines). After the resulting partons are hadronised (light green), the unstable hadrons decay further (dark green). The underlying event (purple) encompasses all the other partons of the initial protons which did not interact in the hard collision. Figure taken from [1].

decay too, until they form particles which are stable enough to not decay before being measured by the experiment's detectors (dark green).

Albeit differing in the specific implementations, all event generators are structured in this way. Sometimes, they are split into multiple programs, e.g. using MadGraph for the matrix element generation and PYTHIA for the subsequent event simulation (see Section 3.5).

In SHERPA, however, the *whole* event is generated within one program, with the subprocesses being handled by different modules. The event generation is steered by a run card (a .yaml file) written for a specific event type. Example run cards for the HNL events can be found in the Appendix B.

3.1.2 Multijet merging

This section is based on [57].

There are multiple ways to improve a simulation's accuracy. Besides considering higher orders of coupling constants for the matrix element generation, going, for instance, from an LO to an NLO calculation, one could also implement only some of the higher order effects, by applying multijet merging. If we take for instance the HNL production, it is reasonable to also consider our original

final state with up to n additional jets, as they also contribute to the total probability of the process. However, there are two parts of the calculation responsible for producing jets: the hard scattering matrix element generation and parton showers. While matrix element calculations are suited to describe few, hard and angularly well separated jets, parton showers mostly deal with multiple soft and collinear emissions. Nevertheless, there is always a small probability that a hard jet is produced by the parton shower. Thus, final states with additional hard jets are already included without multijet merging due to the parton shower generating them. However, this is less accurate, since the parton shower is not well suited to describe hard, well separated jets [58]. In order to better describe the final state with $i \leq n$ additional jets and avoid double counting, we turn to multijet merging using the CKKW algorithm, which consistently combines fixed-order matrix element calculations with parton showers [58, 59]. There, we define a new variable Q_{xy} , which is sensitive to the energies of any two partons (E_x and E_y) and their collinearity (angle θ_{xy} between them)

$$Q_{xy} = 2 \min [E_x^2, E_y^2] (1 - \cos \theta_{xy}) / Q_{cm}^2,$$

where Q_{cm} is the centre of mass energy of the initial hard parton scattering [58]. The higher Q_{xy} is, the worse the parton shower performs in the jets' accurate predictions. By assigning a threshold value Q_{CKKW} [58], we can separate the two regimes: Contributions corresponding to i additional hard jets with $Q_{xy} > Q_{CKKW}$ are described using fixed-order matrix element calculations. The parton shower still generates additional radiation, but is restricted to the region $Q_{xy} < Q_{CKKW}$. This way we avoid double counting, which could otherwise occur if the parton shower generated additional hard radiation in a region already described by matrix element calculations for $(i + 1)$ -jet final states. For the samples generated in this thesis using multijet merging, the merging scale was set to $Q_{CKKW} = 20 \text{ GeV}$.

3.2 Simulating HNLs with SHERPA

The HNL would be produced at the LHC in the central hard scattering (red in Figure 3.1) and thus its production is simulated in SHERPA using one of its matrix element generators, COMIX [60]. Before decaying, the HNL traverses through multiple stages of a SHERPA event as a stable particle, i.e. the parton shower and hadronisation. Lastly, the decay of the HNL is simulated in a module usually responsible for describing the decay of hadrons and $\tau^\pm$ -leptons, HADRONS++ (dark green in Figure 3.1) [61, 62]. Overall, each HNL appears four times in the record of a simulated event, representing each simulation stage in SHERPA.

In the next sections, we will review key aspects of SHERPA in more detail, which are crucial for the simulation of HNLs. The implementation of BSM models, the separation of matrix elements into terms corresponding to the production and decay of the HNL and lastly, the specific currents involved in its decays.

3.2.1 UFO Model

Many BSM models are published in the UFO format (Universal FeynRules Output) which was devised as a universal format to facilitate their implementation consistently into different event generators [63]. In [64], an interface for incorporating UFO models into SHERPA was introduced. At the moment, matrix element calculations in BSM are restricted to LO in SHERPA, however, multijet merging, as discussed in Section 3.1.2, can be used to introduce some higher-order QCD corrections.

We use the associated UFO model for the Seesaw type I mechanism, called SM_HeavyN_Meson_NLO. For this specific version, there is no publication available yet, however, the underlying model library, SM_HeavyN, was published in [63, 65] and used in [2].

The model expands the SM Lagrangian by the Type I Seesaw Lagrangian [50] and the Lagrangian of a low-energy effective field theory for mesons, which describes the couplings of heavy (Majorana or Dirac) neutrinos to the SM's electroweak sector and left-handed leptons via (effective) sterile mixing. It introduces three sterile heavy neutrino fields, which after mixing with the active neutrinos, result in three HNLs (see Section 2.6.3), with the particle identifiers (PID) 72, 73 and 74. Their respective interactions with the weak and Higgs bosons can be controlled in the run card by the mixing angles $V_{\alpha k}$ between flavour eigenstates $\alpha = e, \mu, \tau$ and HNL mass eigenstates $k = 1, 2, 3$.

As this thesis studies the basic implementation of HNLs in SHERPA, we will only consider the decay into one of the HNLs (PID 72) as it is often done in experimental studies [66].

3.2.2 Separation of production and decay of heavy resonances

Separating matrix elements into production and decay contributions is a fundamental part of the feasibility of Monte Carlo event generation. When simulating the production and subsequent decay of an unstable particle (in our case the HNL), the calculations of the complete processes prove to be too complex for practical computational applications. Fortunately, in many cases approximations can be made for the propagator of the unstable particle, resulting in the factorisation of production and decay matrix elements, allowing the independent calculation of the two. This significantly simplifies the simulation.

We start with an exemplary propagator Pr of the HNL with mass M , decay width Γ , and four-momentum p , using $\not{p} = \gamma^\mu p_\mu$

$$Pr_N = \frac{i(\not{p} + M)}{p^2 - M^2 + iM\Gamma}. \quad (3.1)$$

Here, the denominator has the form of a Breit-Wigner distribution [67]. Its form is analogous to the response of a damped harmonic oscillator and describes an intermediate unstable particle with a finite decay width [31]. Experimentally, this behaviour manifests itself as a resonant peak in the invariant mass distribution of the decay products [31]. In the following, we will discuss both the denominator and numerator of this propagator to justify the factorisation of production

and decay matrix elements.

Dealing with the denominator: Narrow Width Approximation

For the approximation of the propagator's denominator, we will apply the Narrow Width Approximation (NWA), a widely used method when considering processes involving an unstable intermediate particle. It is applicable when the decay width Γ of the resonant particle is small compared to its mass M , allowing us to treat it as on-shell ($p^2 = M^2$) [68]. With this, the denominator of the propagator (Equation (3.1)) corresponding to the unstable particle is approximated by a delta function [31]

$$\left| \frac{1}{p^2 - M^2 + iM\Gamma} \right|^2 \rightarrow \frac{\pi}{M\Gamma} \delta(p^2 - M^2).$$

Dealing with the numerator: Completeness relation and spin-correlations

The numerator of the HNL propagator in Equation (3.1) can be rewritten using the completeness relation for Dirac (anti)spinors

$$\sum_{\lambda=\pm\frac{1}{2}} u_\lambda(p) \bar{u}_\lambda(p) = \not{p} + M, \quad \sum_{\lambda=\pm\frac{1}{2}} v_\lambda(p) \bar{v}_\lambda(p) = \not{p} - M,$$

with $u_\lambda(p)$ as the Dirac spinor of a particle with momentum p and helicity λ and $v_\lambda(p)$ as the corresponding antiparticle spinor [24]. This results in the HNL contribution to the matrix element (exemplary for $W^\pm$ vertices)

$$\begin{aligned} \mathcal{M}_N &= \frac{\pi}{M\Gamma} \cdot \bar{u}_1(p_1) \gamma^\mu P_L (\not{p} + M) \gamma^\mu P_L u_2(p_2) \\ &= \frac{\pi}{M\Gamma} \cdot \bar{u}_1(p_1) \gamma^\mu P_L \left(\sum_{\lambda=\pm\frac{1}{2}} u_\lambda(p) \bar{u}_\lambda(p) \right) \gamma^\mu P_L u_2(p_2) \\ &= \frac{\pi}{M\Gamma} \cdot \sum_{\lambda=\pm\frac{1}{2}} \bar{u}_1(p_1) \gamma^\mu P_L u_\lambda(p) \bar{u}_\lambda(p) \gamma^\mu P_L u_2(p_2). \end{aligned} \quad (3.2)$$

At this stage the matrix element was simply rewritten. In order to calculate the production $\mathcal{M}_{\text{prod}}$ and decay $\mathcal{M}_{\text{decay}}$ independently, we move the sum over the helicities λ into each of the matrix elements:

$$\begin{aligned} \mathcal{M}_N &\approx \frac{\pi}{M\Gamma} \cdot \left(\sum_{\lambda=\pm\frac{1}{2}} \bar{u}_1(p_1) \gamma^\mu P_L u_\lambda(p) \right) \cdot \left(\sum_{\lambda=\pm\frac{1}{2}} \bar{u}'_\lambda(p) \gamma^\mu P_L u_2(p_2) \right) \\ &= \frac{\pi}{M\Gamma} \cdot \mathcal{M}_{\text{prod}} \cdot \mathcal{M}_{\text{decay}} \end{aligned} \quad (3.3)$$

Equation (3.3), however, does not equal the original matrix element from Equation (3.2) exactly, since we lose the correlation between the HNL's helicity λ in the production and decay. However,

the spin states of the decaying particles have significant effects on the angular distribution of their decay products. Therefore, SHERPA uses a dedicated spin correlation algorithm to account for spin correlations between production and decay [62].

3.3 HNL production in COMIX

The production of the HNLs is handled in SHERPA's default matrix element generator COMIX, introduced in [60]. COMIX is a tree-level matrix element generator, i.e. not including loops. The energy scale of the production of the HNL is set by the mass of the weak boson it was produced from ($m_{W/Z} \approx 80/91$ GeV [27]). Thus, we can use perturbation theory to describe the hard collision, since at this high energy scale, quarks are unconfined. Hence, they do not form colourless hadrons, but carry colour charges which consequently have to be accounted for.

For processes with many external particles, i.e. high multiplicity, the number of diagrams and possible colour configurations grows significantly, making a direct evaluation of all colour sums computationally expensive. To address this, COMIX performs the calculation on the amplitude level using colour sampling, rather than explicitly summing over all colour combinations. The individual Feynman diagrams and their corresponding amplitudes are constructed and evaluated using colour-dressed Berends–Giele recursion relations [69], described in depth in [70] and [60]. In this recursive approach, off-shell currents for a given set of external particles are built from currents with fewer external legs, multiplied by the appropriate scalar propagators. This allows for an efficient generation and evaluation of all contributing tree-level diagrams without explicitly enumerating each diagram independently.

As discussed above, helicity sums are taken within the production matrix element. Nevertheless, the calculation is performed on the amplitude level for fixed helicity states, ensuring that the helicity information is preserved for the matching to the decay matrix element. COMIX is able to handle both Dirac and Majorana fermions [64]. For an incoming or outgoing Majorana particle, the fermion number flow is not uniquely defined, since it is its own antiparticle (see Section 3.4.2). Consequently, the direction of the fermion flow has to be fixed manually and must be chosen consistently for both the production and decay matrix elements.

In COMIX, an incoming or outgoing Majorana particle can therefore be treated either as a particle or as an antiparticle by setting the parameter `WF_MODE` to 0 (particle, default) or 1 (antiparticle), respectively. This parameter determines the fermion flow direction in the corresponding Feynman diagrams and is thus essential for a consistent matching of production and decay amplitudes of specific helicities.

Although COMIX is also able to simulate the decay of Majorana particles, it is neither able to feature the needed vertex offset of the HNL due to its long lifetime, nor the semileptonic decay channels. Thus, we have to turn to the HADRONs++ module to deal with the decay of the HNLs.

3.4 HNL decay in HADRONS++

As discussed in Section 2.7, HNLs do not decay promptly after their production, but rather travel a measurable distance before decaying. Thus, their displaced decay vertex has to be taken into account in the simulation. Furthermore, semileptonic decays of HNLs into hadrons require a description of the hadronic currents. These kinds of decays are handled in the hadronic decay step in SHERPA in the module HADRONS++, introduced in [61] and [62]. Besides hadrons, it is also responsible for the decay of unstable leptons. In the SM, this only affects the $\tau^\pm$ -lepton, since the other leptons are stable enough to be considered as final state particles at the LHC. However, one can set other elementary particles to be decayed this way too, by including them in the list of UFO_HADRONS in the run card (see example run cards in Appendix B). Doing this for the HNL enables us to use the same framework as for the decays of the $\tau^\pm$ -lepton, which has very similar decay channels as the HNL (cf. [62]).

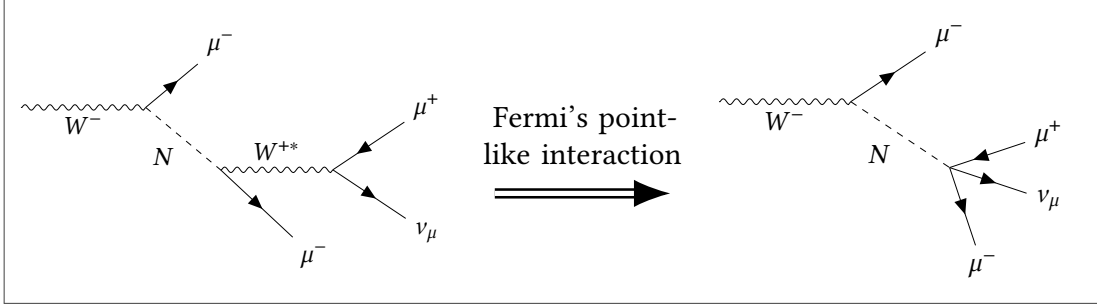


Figure 3.2: Example diagram for Fermi's point-like interaction: In the regime $q^2 \ll m_{W/Z}^2$, the momentum dependence of the weak propagator can be neglected, resulting in an effective local four-fermion (contact) interaction.

In order to describe the decay of an HNL, we first need to determine the associated matrix element

$$-i \mathcal{M} = L_{N\beta}'^\mu Pr_{W/Z} J^\nu \quad (3.4)$$

with $L_{N\beta}'^\mu$ as the current from HNL to the displaced lepton β , and J' as the current, describing the decay products of the HNL's child boson (Z or $W^\pm$). J' can either be of leptonic, L (leptonic decay), or hadronic, H , nature (semileptonic decay).

$Pr_{W/Z}$ is the weak propagator. Since both the $W^\pm$ and Z bosons have a relatively high mass, we can reasonably assume the squared four-momentum q^2 of the boson to be much smaller than its mass $q^2 \ll m_{W/Z}^2$ [26]

$$\begin{aligned} Pr_{W/Z} &= \frac{-i}{q^2 - m_{W/Z}^2} \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_{W/Z}^2} \right) \\ &\approx i \frac{g_{\mu\nu}}{m_{W/Z}^2}. \end{aligned}$$

In this limit, the momentum dependence of the propagator can be neglected, leading to an effective local four-fermion interaction (see Fig. 3.2) [24]. To make the structure of the resulting matrix element explicit, we now specify all relevant vertex and coupling factors. The vertex factors for the $W^\pm$ boson and Z -boson interactions are given by

$$\begin{aligned} V_W &= \gamma_\mu (c_{WV} + c_{WA}\gamma^5) \\ V_Z &= \gamma_\mu (c_{ZV} + c_{ZA}\gamma^5) \end{aligned}$$

The specific values of c_{WV} , c_{WA} , c_{ZV} , and c_{ZA} depend on the specific current and thus will be defined below. With this notation, the standard $V - A$ structure of the weak interaction (see Section 2.3.2) is achieved by setting $c_{WV} = 1$ and $c_{WA} = -1$.

All the remaining factors from the vertices and the propagator are combined into a single constant $K_{W/Z}$:

$$\begin{aligned} K_W &= \left(\frac{g}{2\sqrt{2}} \right)^2 \frac{1}{m_W^2} = \frac{G_F}{\sqrt{2}}, \\ K_Z &= \left(\frac{g}{2 \cos \theta_W} \right)^2 \frac{1}{m_Z^2} = \left(\frac{g}{2 \frac{m_W}{m_Z}} \right)^2 \frac{1}{m_Z^2} = \sqrt{2} G_F, \end{aligned}$$

with G_F as the Fermi coupling constant and θ_W as the Weinberg angle defined via $\cos(\theta_W) = \frac{m_W}{m_Z}$ [26]. Substituting these back into Equation (3.4), yields

$$\mathcal{M} = K_{W/Z} \cdot g_{\mu\nu} L_{N\beta}^\mu J^\nu = K_{W/Z} \cdot L_{N\beta}^\mu J_\mu, \quad (3.5)$$

where we have renamed $L'_{N\beta} \rightarrow L_{N\beta}$ and $J' \rightarrow J$ to indicate that all coupling and propagator prefactors are now included in $K_{W/Z}$.

The currents $L_{N\beta}$ and J are only connected by multiplication and are thus independent of each other. Hence, these currents can be defined generally and used whenever needed. In the following, we will discuss the hadronic and leptonic currents in more detail.

3.4.1 Hadronic currents

Even though the H currents are of the weak interaction, QCD still needs to be taken into account, since the involved quarks carry colour charge and therefore interact strongly. As a result, the calculation gets significantly more complicated compared to when only leptons (colourless particles) are involved. This becomes apparent when looking at the coupling constant of the strong interaction

$$\alpha_s(q^2) = \frac{12\pi}{(11N_C - 2n_f) \log \left(\frac{q^2}{\Lambda_{QCD}^2} \right)}, \quad (3.6)$$

where Λ_{QCD} is known as the QCD scale, $N_C = 3$ is the number of colour charges and $n_f = 3$ the number of active flavours, i.e. the quark flavours producible at our given energy scale. We see that the smaller the transferred momentum q^2 is, the higher α_s and thus stronger the interaction gets. As $|q|$ approaches $\Lambda_{QCD} \approx 0.3 \text{ GeV}$ [71], we see that $\alpha_s > 1$ and thus, perturbation theory fails, since higher order terms of α_s in the expansion do not vanish any more. Interestingly, if we decrease $|q|$ even further ($|q| < \Lambda_{QCD}$) α_s even seemingly becomes negative, which is not a physical observation but rather a result of the fact that Equation (3.6) was originally derived using perturbation theory, which does not apply here.

Hence, for $|q| \leq \Lambda_{QCD}$, we are in the non-perturbative regime of QCD and the strong interaction results in confined quarks, forming hadrons with a typical radius of $R \approx \frac{1}{\Lambda_{QCD}} = 0.67 \text{ fm}$.

For HNL masses of $m_N = 2 \text{ GeV}$ we are right at the border of this non-perturbative regime and therefore, we cannot consider the decay products as free quarks, but have to regard the hadronic final states when discussing currents involving quarks. This is represented in the description of the hadronic current H , where the hadronic states h_i (n hadrons in the final state, 0 in the initial state) are attached to the sides of the weak quark current

$$H_\mu = V_{CKM} \cdot \langle h_1 h_2 \dots h_n | V_\mu - A_\mu | 0 \rangle. \quad (3.7)$$

V_{CKM} represents the respective CKM-Matrix elements, in our case V_{ud} , and V_μ and A_μ are the vector and axial vector currents, typical for the weak interaction (see Section 2.3.2).

V_μ and A_μ are not set universally as it is the case for the leptonic current but rather depend on the specific hadron final state [61]. Thus, we need to specify H for each hadronic final state separately, which we will do below for the cases we are interested in (decay into one or two π -mesons).

Generally, there are multiple approaches to circumvent the difficulties arising from QCD not being perturbatively solvable at low energies, each with their own range of validity regarding energy scales, quark flavours and hadron types. In our case we will only deal with up and down quarks and π -mesons (pseudoscalar mesons) in the final state.

QCD is parity conserving. Firstly, this implies that only either the vector or axial current contributes, while the other one must vanish, since vector and axial currents do not conserve parity simultaneously [24]. Secondly, we know that pseudoscalar mesons and axial currents change sign under parity transformations (odd/negative parity), while vector currents do not (even/positive parity) [24]. Thus, an odd number of pseudoscalar mesons in the final state can only be achieved via an axial current, to cancel the negative parity of the meson ($(-1)_\pi \cdot (-1)_a = +1$), while an even number of pseudoscalar mesons require a vector current to conserve parity ($(-1)_\pi \cdot (-1)_\pi \cdot (+1)_v = +1$). Thus, we can rewrite Equation (3.7) as

$$\begin{aligned} \text{Decay into } \pi^\pm : \quad & H_\mu = V_{CKM} \cdot \langle \pi^\pm | A_\mu | 0 \rangle, \\ \text{Decay into } \pi^\pm \pi^0 : \quad & H_\mu = V_{CKM} \cdot \langle \pi^\pm \pi^0 | V_\mu | 0 \rangle. \end{aligned}$$

Independently of the used model, the hadronic current H can be expressed as [72]

$$H_\mu = V_{CKM} \cdot \sum_{i=1}^n F_i(q^2) S_i,$$

with the form factors F_i being scalar functions of the transferred momentum q and S_i being Lorentz structures, i.e. Lorentz invariant four vectors built from the available momenta and polarisation vectors. As π -mesons have spin zero, the only available four-vectors in this case are combinations of the momenta of the involved particles. Now, the task is to find the appropriate F_i and S_i for each hadronic final state.

Decay into $\pi^\pm$ or π^0

First we take a look at the semileptonic decay into one $\pi^\pm$ (via a $W^\pm$ boson) or π^0 (via a Z boson). A very popular method to describe strong interaction at low energies with pseudoscalar mesons is the Chiral Perturbation Theory (χ PT). This effective field theory was introduced in [73, 74] and utilises the fact that for massless quarks, the right- and left-handed terms in the Lagrangian decouple. It is intended for the description of processes involving the lightest pseudoscalar mesons, which consist of two of the three lightest quarks (up, down and strange).

It behaves well for very low energies [27]

$$E \ll M_\rho \approx 775 \text{ MeV},$$

with M_ρ being the mass of the ρ -meson [75]. This fits well to the regarded case of decays into one π -meson, which has a mass of $M_\pi \approx 140 \text{ MeV}$ [27]. Using χ PT, the hadronic current can be expressed as [76]

$$H^\mu = V_{ud} \cdot \langle \pi^\pm | A_\mu | 0 \rangle = V_{ud} i f_\pi p^\mu,$$

with f_π as the decay constant for the π -meson in SHERPA 3.1.x set to

$$f_\pi = 130.4 \text{ MeV},$$

$$V_{ud} = 0.97377.$$

The corresponding SHERPA current for this decay mode is called VA_0_P, as it describes a process with an initial state of zero hadrons and a final state of one pseudoscalar [61].

Decay into $\pi^\pm$ and π^0

The other important semileptonic decay channel for the HNL is the decay into two π -mesons, a $\pi^\pm$ and a π^0 . This decay is not immediate, but goes via an intermediate ρ -meson which almost exclusively decays into the two π -mesons [27]. Therefore, resonance effects have to be taken into

account. χ PT is not valid for energy scales $E \approx M_\rho$ and thus, we have to turn to other models describing the form factors.

There are two models implemented in SHERPA for calculating the specific form factors F_i when resonances are involved [62]. Firstly, there is the Kühn-Santamaria model (KSM) [77], which utilises a Breit-Wigner factor for each ρ -meson resonance, respectively. Secondly, there is the Chiral Resonance Theory (χ RT) [78], which effectively is an extension to χ PT including resonances. The default model used in SHERPA is the Kühn-Santamaria model, albeit, the model can be changed in the run card (see Appendix B.3).

Regardless of the used model, the hadronic current can be expressed as

$$H^\mu = V_{ud} \cdot \langle \pi^\pm, \pi^0 | V_\mu | 0 \rangle = V_{ud} C_\pi F_V(q_{\pi+\pi}^2) (p_{\pi^0} - p_{\pi^\pm})^\mu,$$

with $C_\pi = 1$ being the Clebsch-Gordan coefficient and $F_V(q_{\pi+\pi}^2)$ as the vector form factor, determined by the used model (KSM or χ RT), depending on the momentum of the two pions $q_{\pi+\pi} = p_{\pi^0} + p_{\pi^\pm}$ [62].

The corresponding SHERPA current is called VA_0_PP, as it describes a process with an initial state of zero and a final state of two pseudoscalars [61].

3.4.2 Leptonic currents L

The leptonic currents are used for the description of both the HNL vertex and the additional leptonic vertex in the purely leptonic decay. It is implemented in SHERPA as a VA_FF current, i.e. going from one fermion (x) to another (y) [61]. As described in Section 2.3.2 the respective current looks like this:

$$L_\mu^{x \rightarrow y}(p_x, p_z) = \bar{r}_x(p_x) (c_V \gamma_\mu + c_A \gamma_\mu \gamma_5) r_z(p_z),$$

where c_V and c_A are the coupling constants of the vector/axial contributions for the W and Z -decays and r represents either a spinor or antispinor. Both c_V and c_A , and the participating (anti)particles, are set in the HADRON_DECAYS block of the run card (see in the Appendix B).

In accordance with [79], the spinors are calculated in SHERPA [61] as follows: for a particle spinor u :

$$u(p, \lambda) = \frac{\not{p} + m}{\sqrt{2 p \cdot k_0}} w(k_0, -\lambda), \quad \bar{u}(p, \lambda) = \bar{w}(k_0, -\lambda) \frac{\not{p} + m}{\sqrt{2 p \cdot k_0}} \quad (3.8)$$

and for an antiparticle spinor v :

$$v(p, \lambda) = \frac{\not{p} - m}{\sqrt{2 p \cdot k_0}} w(k_0, -\lambda), \quad \bar{v}(p, \lambda) = \bar{w}(k_0, -\lambda) \frac{\not{p} - m}{\sqrt{2 p \cdot k_0}}. \quad (3.9)$$

The only difference between the particle and antiparticle spinors is the sign of the mass m which matches the fact that u is the positive energy solution to the Dirac equation, while v is the negative

energy one (see Equation (2.2)). The other parameters in the Equations (3.8) and (3.9) are the particles' helicity λ , their four-momentum p , and $k_{0,1}$ and w as auxiliary four-vectors, where w behaves as a massless spinor with momentum k_0 and helicity λ . They are defined through

$$k_0^2 = 0, \quad k_1^2 = -1, \quad k_0 \cdot k_1 = 0, \\ w(k_0, \lambda) \bar{w}(k_0, \lambda) = \frac{1 + \lambda \gamma_5}{2} \not{k}_0, \quad w(k_0, \lambda) = \lambda \not{k}_1 w(k_0, -\lambda).$$

There are multiple options for their exact definition in SHERPA, the only restriction being that they need to match between the production module COMIX and the decay module HADRONs++. For each helicity combination $\lambda_x, \lambda_y = \pm \frac{1}{2}$, the respective amplitude is calculated as

$$L_\mu(p_x, p_y, \lambda_x, \lambda_y) = \bar{r}_x(p_x, \lambda_x) [c_V \gamma_\mu + c_A \gamma_\mu \gamma_5] r_y(p_y, \lambda_y).$$

Both factors, c_V and c_A as well as the involved particles are set in the run card (see examples in Appendix B). In order to get the total decay matrix element, as used in Equation (3.5) the amplitudes for each helicity combination are summed. Nevertheless, the individual helicity amplitudes are stored too, for the spin correlation algorithm, matching the production and decay helicities [62, 80].

Leptonic currents containing only Dirac particles

If only Dirac fermions participate in the leptonic current, the information on whether the spinors $\bar{r}$ and r in Equation 3.4.2 are of particle (u) or antiparticle (v) nature, is determined automatically by SHERPA by looking at the particle's properties. If SHERPA registers the particle as being an antiparticle, it selects the v spinor description. Otherwise, the u formulation with $+m$ is chosen. Furthermore, in a regular leptonic current where only Dirac (anti)particles participate, c_V and c_A are set in the run card to be

$$c_{WV} = 1, \quad c_{WA} = -1, \\ c_{ZV} = I_3 - 2Q \sin^2(\theta_W) = -\frac{1}{2} + 2 \sin^2(\theta_W) \approx -0.037 [27], \quad c_{ZA} = -I_3 = +\frac{1}{2}. \quad (3.10)$$

In the case of the Z -boson couplings, we took into account that we only consider decay channels involving charged leptons. Consequently, the weak isospin and electric charge are fixed to $I_3 = -\frac{1}{2}$ and $Q = -1$, respectively.

Leptonic currents containing a Majorana particle

When having an HNL participating in the leptonic current, its Majorana property has to be taken into account. Since Majorana fermions are their own antiparticles, the common fermion flow going from particle to antiparticle cannot be defined in the case of lepton number violation (LNV). Thus, the regular Feynman rules of the SM do not apply any more and the currents involved in

the calculation of the matrix elements need to be revised. We will do exactly that following [81], where Feynman rules for processes involving Majorana fermions are derived and summarised. For further details and rules which do not have any application in this thesis, one should refer to the original paper [81].

It is important to note that these modified Feynman rules do not introduce any new interaction vertices or couplings. In particular, the $W^\pm$ bosons continue to couple exclusively to left-chiral fermion fields, as in the Standard Model. The purpose of the revised Feynman rules is to provide a consistent description of processes involving Majorana fermions, while reproducing the Standard Model Feynman rules in the absence of Majorana particles.

In order to have one consistent set of matrix elements, we will derive them for the HNL production and decay and only later factorise them for the decay implementation into HADRONS++.

Contrary to processes only involving Dirac fermions, the fermion flow is chosen arbitrarily in processes involving Majorana fermions. This results in the Feynman rules as described in Figures 3.3 to 3.5, where the appropriate expressions for spinors, propagators and vertex factors need to be chosen according to the particle types, the direction of momentum and the direction of fermion flow.

Due to the freedom of choosing the fermion flow direction, using these Feynman rules can result in two different expressions, depending on the selected fermion flow direction. However, these expressions are equivalent, apart from a global sign which can be determined by the number of permutations in the order that the spinors appear in the matrix element. An example calculation showcasing the equivalence can be found in Appendix A.3.

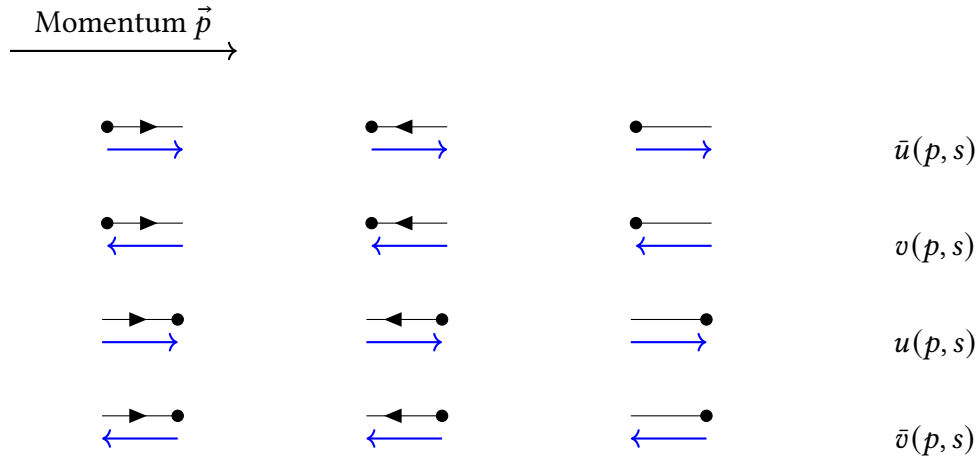


Figure 3.3: Spinors for processes involving Majorana Fermions: Momentum flows from left to right, indicated by the upperleft arrow. Blue arrows indicate the fermion flow direction, while black arrows indicate particles, antiparticles or Majorana properties. Figure adapted from [81]

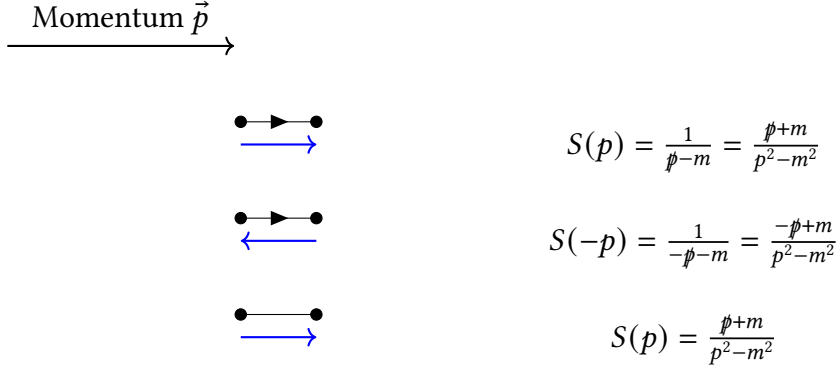


Figure 3.4: Fermion propagators for processes involving Majorana Fermions: Momentum flows from left to right. Blue arrows indicate the fermion flow direction, while black arrows indicate particle, antiparticle or Majorana properties. Figure adapted from [81]

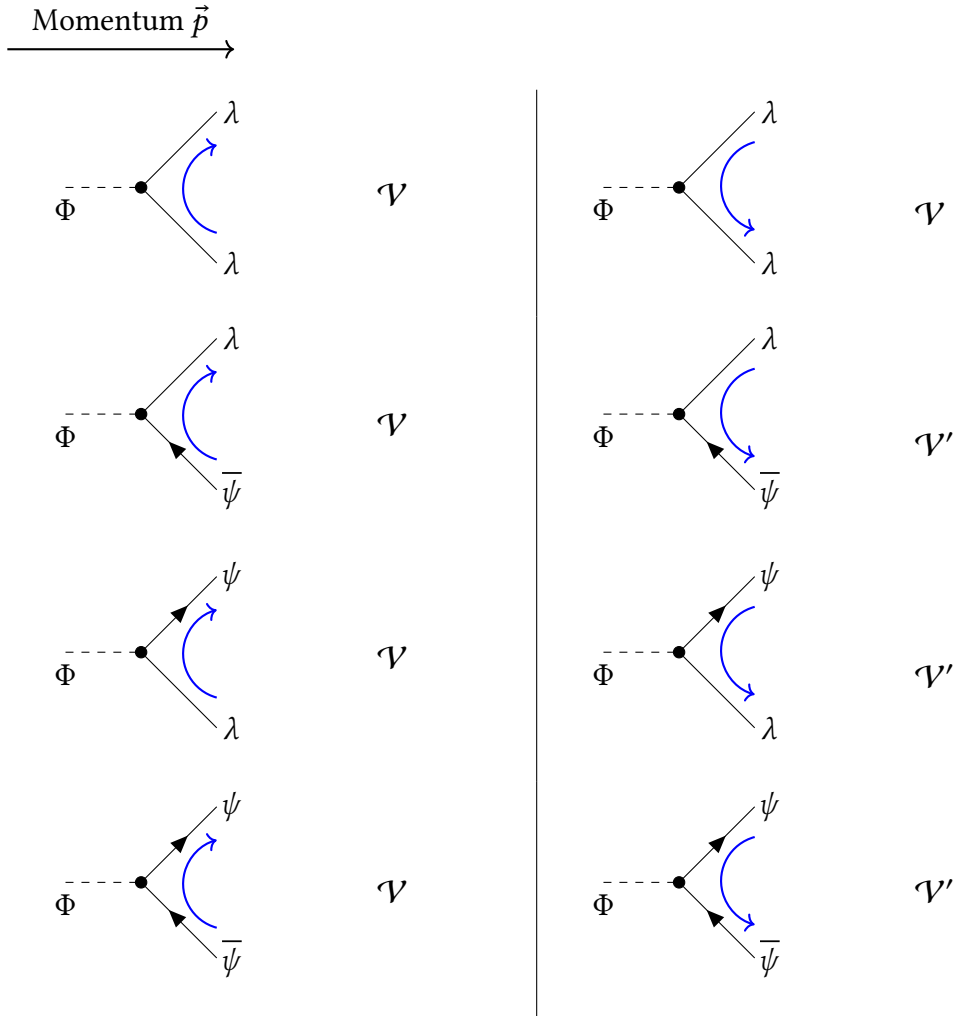


Figure 3.5: Fermion vertices for processes involving Majorana Fermions: Momentum flows from left to right. Blue arrows indicate the fermion flow direction, while black arrows indicate particle, antiparticle or Majorana properties. $\mathcal{V}$ is the regular vertex factor and $\mathcal{V}' = C\mathcal{V}^T C^{-1}$ with C being the charge-conjugation matrix. Figure adapted from [81]

In our case of the production and decay of an HNL, the interaction vertices are points of weak interaction. Therefore, the vertex factor $\mathcal{V}$ as seen in Figure 3.5 is given by

$$\mathcal{V}_{W/Z} = \gamma^\mu (c_V 1 + c_A \gamma^5),$$

where the γ^μ are the Gamma matrices and $\gamma^5 = i \gamma^0 \gamma^1 \gamma^2 \gamma^3$. Note that the proportionality factors have been omitted here, since they are already included in the general description of the decay (Equation (3.5)).

We apply the following relations regarding the charge conjugation C of the Gamma matrices

$$C (\gamma^\mu)^T C^{-1} = -\gamma^\mu, \quad C (\gamma^\mu \gamma^5)^T C^{-1} = +\gamma^\mu \gamma^5,$$

to obtain the modified vertex factor $\mathcal{V}'_{W/Z}$ as

$$\begin{aligned} \mathcal{V}'_{W/Z} &= C \mathcal{V}^T C^{-1} = C (c_V \gamma^\mu + c_A \gamma^\mu \gamma^5)^T C^{-1} \\ &= c_V C (\gamma^\mu)^T C^{-1} - c_V C (\gamma^\mu \gamma^5)^T C^{-1} \\ &= -c_A \gamma^\mu + c_V \gamma^\mu \gamma^5. \end{aligned}$$

The vector and axial factors are given as previously in Equation (3.11), now however, the Z decay couples together two neutrinos with, $Q = 0$ and with weak isospin $I_3 = +\frac{1}{2}$.

$$\begin{aligned} c_{WV} &= 1, \quad c_{WA} = -1, \\ c_{ZV} &= I_3 - 2Q \sin^2(\theta_W) = \frac{1}{2}, \quad c_{ZA} = -I_3 = -\frac{1}{2} \end{aligned} \tag{3.11}$$

Substituting the coupling constants c_V and c_A for the W and Z bosons (see Equation (3.11)), we find

$$\begin{aligned} \mathcal{V}_W &= \gamma^\mu (1 - \gamma^5) = \gamma^\mu P_L \quad \Rightarrow \quad \mathcal{V}'_W = -\gamma^\mu (1 + \gamma^5) = -\gamma^\mu P_R, \\ \mathcal{V}_Z &= \frac{1}{2} \gamma^\mu (1 - \gamma^5) = \frac{1}{2} \gamma^\mu P_L \quad \Rightarrow \quad \mathcal{V}'_Z = -\frac{1}{2} \gamma^\mu (1 + \gamma^5) = -\frac{1}{2} \gamma^\mu P_R. \end{aligned}$$

Knowing this, we can now build our matrix elements for the production and decay at the LHC. This is shown in Tables 3.1 and 3.2, where each table chooses an opposite fermion flow direction. This results in two matrix elements for each process, one for each fermion flow direction. However, they are equal (up to a minus sign), as is shown in Appendix A.3.

In order to make the matrix element ready to be used in HADRONs++, we factorise it into production and decay elements, in accordance with Section 3.2.2, summarised in Table 3.3. These matrix elements can also be found by applying the Majorana Feynman rules directly to the production and decay processes, respectively, one only needs to be careful to choose the fermion flow direction consistently.

Momentum $\vec{p}$				
Diagram	Spinors	Vertex factor	Matrix element	
LNC part-anti 	$v(p_1)$ $\bar{u}(p_2)$	$V1 : \mathcal{V}'$ $V2 : \mathcal{V}'$	$\bar{u}(p_2) \mathcal{V} S(p_{12}) \mathcal{V} v(p_1)$ $\propto \bar{u}(p_2) \gamma^\mu P_R (\not{p}_{12} + m) \gamma^\mu P_R v(p_1)$	
LNC anti-part 	$v(p_1)$ $\bar{u}(p_2)$	$V1 : \mathcal{V}$ $V2 : \mathcal{V}$	$\bar{u}(p_2) \mathcal{V} S(p_{12}) \mathcal{V} u(p_1)$ $\propto \bar{u}(p_2) \gamma^\mu P_L (\not{p}_{12} + m) \gamma^\mu P_L v(p_1)$	
LNV part-part 	$v(p_1)$ $\bar{u}(p_2)$	$V1 : \mathcal{V}'$ $V2 : \mathcal{V}$	$\bar{u}(p_2) \mathcal{V} S(p_{12}) \mathcal{V} v(p_1)$ $\propto -\bar{u}(p_2) \gamma^\mu P_L (\not{p}_{12} + m) \gamma^\mu P_R v(p_1)$	
LNV anti-anti 	$v(p_1)$ $\bar{u}(p_2)$	$V1 : \mathcal{V}$ $V2 : \mathcal{V}'$	$\bar{u}(p_2) \mathcal{V}' S(p_{12}) \mathcal{V} v(p_1)$ $\propto -\bar{u}(p_2) \gamma^\mu P_R (\not{p}_{12} + m) \gamma^\mu P_L v(p_1)$	

Table 3.1: Matrix Elements for the HNL productions and decays with fermion flow downwards (Majoranas are treated as particles): Blue arrows indicate the fermion flow direction, while black arrows indicate particle, antiparticle or Majorana properties. p_1 and p_2 are the momenta of the outgoing leptons ℓ_1 and ℓ_2 and $p_{12} = (p_1 - p_2)$ the HNL momentum with $\not{p}_{12} = \gamma^\mu p_{12,\mu}$

Momentum $\vec{p}$ Diagram	Spinors	Vertex factor	Matrix element
LNC part-anti 	$\bar{u}(p_1)$ $v(p_2)$	$V1 : \mathcal{V}$ $V2 : \mathcal{V}$	$\bar{u}(p_1) \mathcal{V} S(p_{21}) \mathcal{V} v(p_2)$ $\propto \bar{u}(p_1) \gamma^\mu P_L (\not{p}_{21} + m) \gamma^\mu P_L v(p_2)$
LNC anti-part 	$\bar{u}(p_1)$ $v(p_2)$	$V1 : \mathcal{V}'$ $V2 : \mathcal{V}'$	$\bar{u}(p_1) \mathcal{V}' S(p_{21}) \mathcal{V}' v(p_2)$ $\propto \bar{u}(p_1) \gamma^\mu P_R (\not{p}_{21} + m) \gamma^\mu P_R v(p_2)$
LNV part-part 	$\bar{u}(p_1)$ $v(p_2)$	$V1 : \mathcal{V}$ $V2 : \mathcal{V}'$	$-\bar{u}(p_1) \mathcal{V} S(p_{21}) \mathcal{V}' v(p_2)$ $\propto -\bar{u}(p_1) \gamma^\mu P_L (\not{p}_{21} + m) \gamma^\mu P_R v(p_2)$
LNV anti-anti 	$\bar{u}(p_1)$ $v(p_2)$	$V1 : \mathcal{V}'$ $V2 : \mathcal{V}$	$-\bar{u}(p_1) \mathcal{V}' S(p_{21}) \mathcal{V} v(p_2)$ $\propto -\bar{u}(p_1) \gamma^\mu P_R (\not{p}_{21} + m) \gamma^\mu P_L v(p_2)$

Table 3.2: Matrix Elements for the HNL productions and decays with fermion flow upwards (Majoranas are treated as antiparticles): Blue arrows indicate the fermion flow direction, while black arrows indicate particle, antiparticle or Majorana properties. p_1 and p_2 are the momenta of the outgoing leptons ℓ_1 and ℓ_2 and $p_{21} = (p_2 - p_1)$ the HNL momentum with $\not{p}_{21} = \gamma^\mu p_{21,\mu}$

Process type	$\mathcal{M}_{\text{prod}}$	$\mathcal{M}_{\text{dec}}$
LNC (part-anti)	$-\bar{u}(p_N) \gamma^\mu P_R v(p_1)$	$-\bar{u}(p_2) P_R u(p_N)$
LNC (anti-part)	$\bar{u}(p_N) P_L v(p_1)$	$\bar{u}(p_2) P_L u(p_N)$
LNV (part-part)	$-\bar{u}(p_N) P_R v(p_1)$	$\bar{u}(p_2) \gamma^\mu P_L u(p_N)$
LNV (anti-anti)	$\bar{u}(p_N) P_L v(p_1)$	$-\bar{u}(p_2) P_R u(p_N)$

Table 3.3: Matrix element of leptonic decays, involving Majorana particles: For these, the direction of the fermion flow was set to be downwards, i.e. WF_MODE= 0 (see Table 3.1). If the process is mediated by a Z boson, an additional factor of $\frac{1}{2}$ needs to be applied.

Finally, we use the $\mathcal{M}_{\text{dec}}$ for the calculations of the decay in HADRONs++ and need to change two things in the currents when having Majorana particles in a specific vertex: For one, we have to adjust the run card to contain the correct coupling constants c_V and c_A , according to whether we have a $V - A$ or $V + A$ structure and according to the mediating boson type (W or Z). The values are summarised in Table 3.3. This is done in the run card (see Appendix B).

Secondly, the spinors' particle/antiparticle property do not match the u and v allocation any more. Rather, u or v are now assigned depending on the direction of fermion flow in relation to the particle's momentum and whether it is produced or is decaying at the vertex (c.f. Figure 3.3). This is easily rectified by taking the parameter WF_MODE (see Section 3.3) into account. Per default the HNL is to be treated as a particle (u spinor) which matches the matrix elements as described in Table 3.1. In this case, *both* spinors of the HNL decay current are supposed to be of the u -type, regardless of their particle/antiparticle property and therefore we need to use $+m$ in Equation (3.8) for both spinors. If WF_MODE= 1 on the other hand, both spinors in the HNL decay are of the v -type ($-m$).

Thus, when calculating the matrix element of a leptonic current using the newly developed VA_F_F_Majorana, the calculation is automatically done using the WF_MODE information, instead of the particle's particle/antiparticle property.

3.5 MadGraph+PYTHIA

To verify the SHERPA simulations, we compare the results to a different event generator setup, where the HNLs have already been successfully implemented. These simulations were performed using MadGraph5_aMC@NLO [4] and PYTHIA 8 [5] with the NNPDF30NLO [82] parton distributions. They were provided by Marzieh Bahmani from the Humboldt University of Berlin, Germany. These simulations have been analysed previously in [3] and [83].

The matrix element generator MadGraph is responsible for the hard-scattering matrix elements, while PYTHIA handles the subsequent parton showering, hadronisation, and resonant decays, in particular those of the ρ -meson. In order to simulate the ρ -meson-decay, it is first produced on-shell in MadGraph, without decay width, which is added manually by applying a Breit-Wigner distribution on top of the ρ -meson. Subsequently, the ρ -meson is handed over to PYTHIA, where

its resonant decay is simulated. There, the spin information of the ρ -meson is not retained. For the leptonic channel, the hard QCD processes were calculated with MadGraph at next-to-leading order accuracy. For the semileptonic channels, only leading-order calculations were performed, since the newly implemented meson UFO model rendered NLO calculations impossible.

3.6 Analysis

The analysis is performed using Rivet 4 [84], a popular analysis tool for the validation of Monte Carlo event generators in high energy physics.

If one wants to analyse these simulations themselves, it is important to note that some particles appear multiple times in an event, although only one actual particle occurs (see introduction of Section 3.2). This is due to the fact that the HNL passes through multiple decay steps during which it does not decay, i.e. seemingly decays into itself. Thus, one needs to be careful to analyse the actually decaying particle instead of any one of the instances where the particle decays into itself. This can be ensured by requiring the HNL to have decay children unequal to itself in the analysis.

Cuts

For the MadGraph+PYTHIA simulation data, selection cuts are applied to restrict the simulated events to the phase space region that is experimentally accessible. For the prompt lepton, the transverse momentum p_T is required to satisfy $p_T > 10$ GeV and the absolute pseudorapidity $|\eta|$ must fulfil $|\eta| < 2.5$.

For the displaced lepton, only the pseudorapidity requirement of $|\eta| < 2.5$ is applied.

In order to compare both of the event generators, we manually implemented the mentioned cuts on the data generated by both, MadGraph+PYTHIA and SHERPA, using Rivet.

CHAPTER 4

Results

This chapter presents the validation of the implementation of HNLs in SHERPA.

For this purpose, we compare kinematic distributions to samples generated with established simulations of MadGraph+PYTHIA (MG+P) [2]. After comparing the kinematics of the HNL production process in both simulation types (Section 4.1), we will analyse and compare various parameters in the different HNL decay channels (Sections 4.2 to 4.4).

In particular, we will study the effects of the two possible polarisation states of the decaying HNL: Due to the Majorana property of the HNL, it can act as a particle or antiparticle in the production and decay processes, leading to both lepton number conserving (LNC) and lepton number violating (LNV) events. While both event types are allowed, the polarisation of the decaying HNL differs in the two (see Sections 2.6.3 and 2.7.3) and thus, spin correlation effects lead to different angular distributions of the decay products. Hence, in order to validate the SHERPA simulation correctly, it is crucial to take these differences into account and split both MG+P and SHERPA samples into these two categories in the analysis. As the production matrix elements are calculated independently of the decay of the HNL, the LNC / LNV character of an event is still unknown at this stage and thus, only influences the decay observables.

Furthermore, this thesis showcases the novel and unique simulation of HNLs decaying into $\tau^\pm$ leptons as well as the retention of spin correlations in the semileptonic HNL decays into ρ -mesons and their subsequent decays into two π mesons.

We will start with the validation of the HNLs production (in Section 4.1). Afterwards, we will study decay kinematics, common to all decay channels (Section 4.2), before considering the leptonic and semileptonic decay channels in further detail (Sections 4.3.1 and 4.4).

The visualisations of the HNL kinematics shown in this chapter are all structured in a similar manner: The x-axis represents an observable x , while on the y-axis the relative probability of a certain x value occurring is plotted.

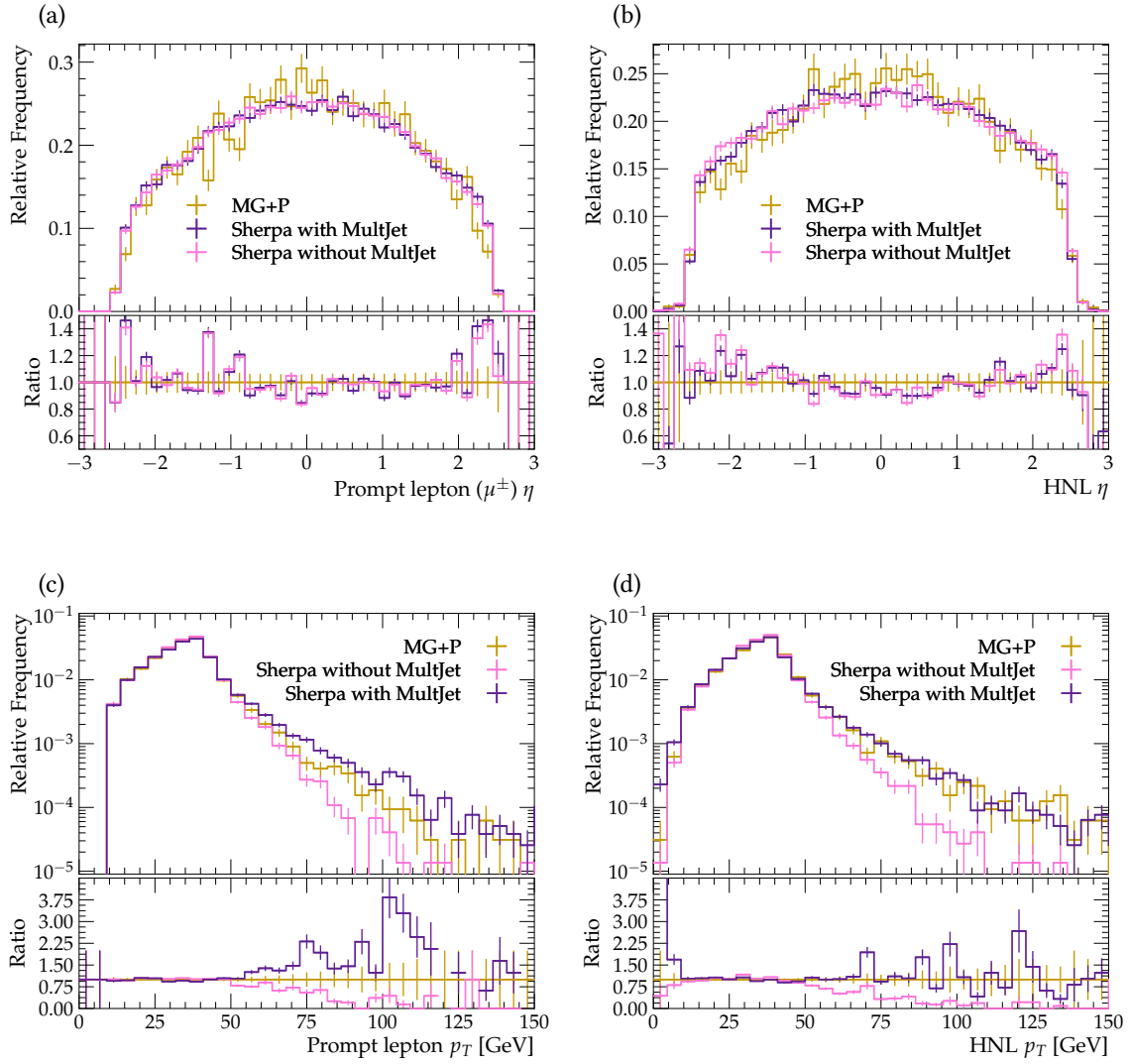


Figure 4.1: HNL production kinematics: The first row depicts the pseudorapidity η for the (a) prompt lepton and (b) HNL. The second row shows the transverse momentum p_T for the (c) prompt lepton and (d) HNL.

4.1 Validation of HNL production

The HNL is produced via a Drell-Yan process, through the decay of a $W^\pm$ boson into a charged lepton and the HNL (see Section 2.7.1). The charged lepton is called prompt, since it is produced without any vertex offsets from the initial collision. All charged lepton flavours are possible for the prompt lepton, Nevertheless, we will only consider the muonic channel, i.e. the HNL is produced alongside a $\mu^\pm$ -lepton in the following, since the qualitative behaviour is analogous for the other flavours. The following kinematic features of HNLs and prompt lepton are depicted in Figure 4.1:

The transverse momentum p_T is the momentum component perpendicular to the beam axis

$$p_T = \sqrt{p_x^2 + p_y^2} \quad \text{with } \hat{z} \text{ as the beam axis,}$$

and the pseudorapidity η is an observable parametrising the angle of a particle relative to the beam axis

$$\eta = -\ln \left[\tan \left(\frac{\theta}{2} \right) \right] \quad \text{with} \quad \theta = \arccos \left(\frac{p_z}{|\vec{p}|} \right).$$

The η distributions of both particles are symmetric around $\eta = 0$, meaning there is no preference for forward or backward production. The η distributions look the same for all particles involved in the HNL production and decay processes and thus, will not be shown again in the following sections.

The p_T distributions of the HNL and the prompt $\mu^\pm$ -lepton are shown in Figures 4.1d and 4.1c, respectively. Comparing the two simulations, we see that multijet merging (see Section 3.1.2) in SHERPA leads to better agreement with MG+P, especially in the high p_T tails of the HNL distributions. Nevertheless, we do still see deviations between MG+P and SHERPA with multijet merging at high p_T . This is expected, as higher order QCD effects play a bigger role at higher p_T values. In the MG+P samples, high p_T values are modelled using PYTHIA's parton shower which needs to be fine-tuned to the specific decay process. Thus, SHERPA enables us to more reliably simulate these effects using multijet merging (see Section 3.1.2). From now on, all plots showing SHERPA results will include multijet merging.

Overall, we see that the kinematics of the HNL production process align well between MG+P and SHERPA.

4.2 Validation of general decay parameters

In Table 4.1 all decay channels and their respective widths calculated using SHERPA are summarised.

In the following we will look at decay distributions which all HNLs have in common. Thus, we will only show them once, exemplary for the semileptonic decay $HNL \rightarrow \pi^\pm \mu^\mp$.

Decay channel	Γ in GeV	Charge-conjugated channel	Γ in GeV
$HNL \rightarrow e^- \nu_\mu \mu^+$	$7.1610166 \times 10^{-13}$	$HNL \rightarrow e^+ \bar{\nu}_\mu \mu^-$	$7.1540839 \times 10^{-13}$
$HNL \rightarrow \mu^- \nu_\mu \mu^+$	$6.9944034 \times 10^{-13}$	$HNL \rightarrow \mu^+ \bar{\nu}_\mu \mu^-$	$6.9961257 \times 10^{-13}$
$HNL \rightarrow \tau^- \nu_\mu \mu^+$	4.55022×10^{-17}	$HNL \rightarrow \tau^+ \bar{\nu}_\mu \mu^-$	$4.5475265 \times 10^{-17}$
$HNL \rightarrow \pi^+ \mu^-$	$3.4439291 \times 10^{-13}$	$HNL \rightarrow \pi^- \mu^+$	$3.4439291 \times 10^{-13}$
$HNL \rightarrow \rho^+ \mu^-$	$7.5518666 \times 10^{-13}$	$HNL \rightarrow \rho^- \mu^+$	$7.5501872 \times 10^{-13}$

Table 4.1: Decay widths Γ of the simulated HNL decay channels, calculated by SHERPA.

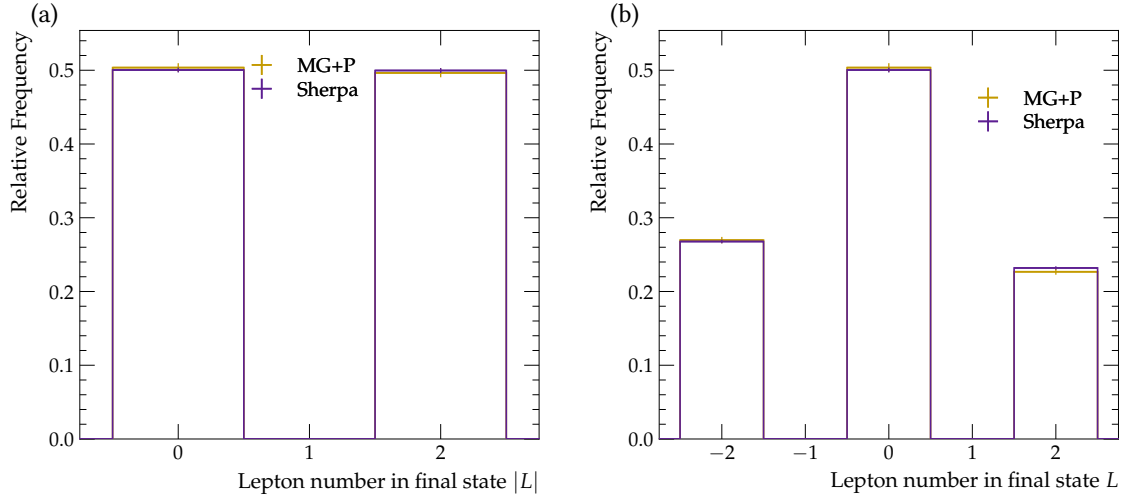


Figure 4.2: Lepton number L of the final state in HNL events: LNC events have $L = 0$, while LNV events have $|L| = 2$. L is shown in (a) absolute values and (b) signed values.

4.2.1 Lepton number conservation vs. violation

In Figure 4.2, the lepton numbers (L) in the final state are depicted, in absolute and signed values. The initial state is composed of two protons, i.e. $L_{in} = 0$. Hence, LNC is characterised by $L = 0$ in the final state, while LNV events have $|L| = 2$.

The branching ratios of HNL decays into positively and negatively charged leptons were set to be equal in the decay of the HNL. This results in the same number of $|L| = 0$ and $|L| = 2$ events, as can be seen in Figure 4.2a. In the production matrix element calculation, however, the cross-section was influenced by the probability of W^+ or W^- boson production in the initial proton-proton collision. As there are more W^+ bosons produced in proton-proton collisions at the LHC [85, 86], there is a slight preference for $L = -2$ over $L = +2$ events, as can be seen in Figure 4.2b. In the case of $L = 0$, this preference cancels out, as both W^+ and W^- bosons can produce LNC events with equal probability.

4.2.2 Vertex offsets

As the HNL is a long-lived particle it does not decay immediately after its production but travels a certain distance before decaying in a displaced vertex (DV). Thus, the vertex offset of the HNL, i.e. the travelled distance d in the lab frame before decay is depicted in Figure 4.3a. As expected for an exponential decay, d is a decreasing function. The typical linearity of an exponential decay depicted in a logarithmic plot is, however, not visible in d , since the HNL travels with different velocities near the speed of light in the lab frame. Thus, Lorentz time dilation has to be taken into account.

When considering the HNLs proper lifetime τ , i.e. the lifetime in the HNL's restframe (Figure

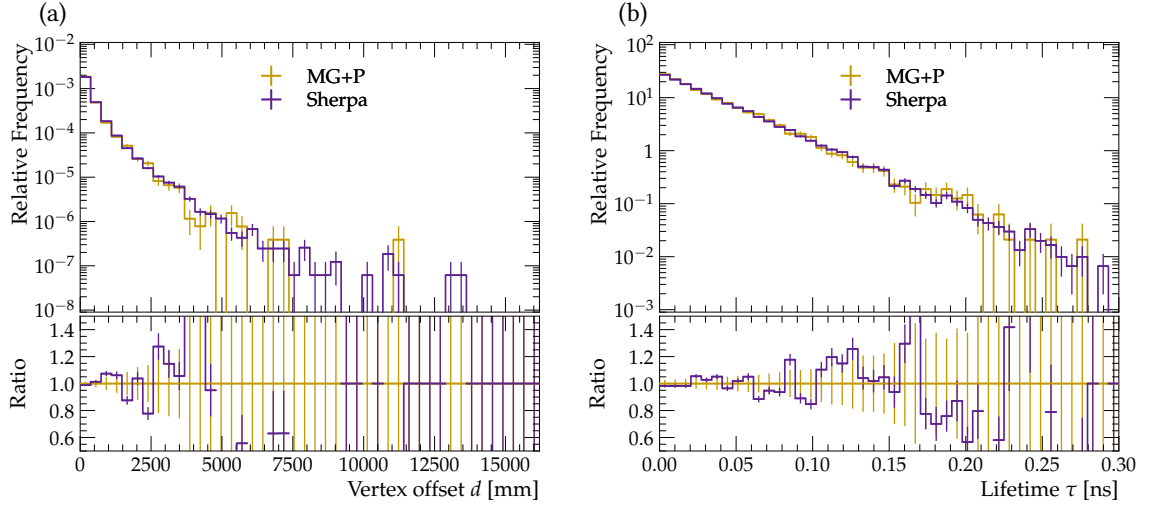


Figure 4.3: Vertex offset and lifetime of HNLs: (a) shows the vertex offset of the decaying HNL in the lab frame and (b) its lifetime in the HNL restframe.

4.3b), the linear behaviour of an exponential decay is restored using the Lorentz factor γ and the velocity v of the HNL in the lab frame:

$$d = \gamma v \tau \quad \text{with } \gamma = \frac{1}{\sqrt{1 - v^2}}.$$

The average proper lifetime, as set in the MG+P and SHERPA runcards, is $\tau = 10 \text{ mm} \approx 3.3 \times 10^{-11} \text{ s}$ in both simulation types.

4.3 Leptonic decay channel

In the following, we will consider the leptonic decay channels of the HNL in more detail. Besides validating the decays into different displaced lepton flavours, we will verify the equality of the two lepton number flow directions (see Section 3.4.2), exemplary for the $HNL \rightarrow e^\mp \nu_\mu \mu^\pm$ decay channel.

4.3.1 Leptonic decay channel $HNL \rightarrow e^\mp \nu_\mu \mu^\pm$

Validation using p_T

The decay channel $HNL \rightarrow e^\mp \nu_\mu \mu^\pm$ is facilitated purely through a $W^\pm$ boson, as the decay via a Z boson would require the charged leptons to be of the same flavour. Thus, we can validate the $W^\pm$ boson decays of SHERPA against MG+P using this channel.

In Figures 4.4, the p_T values of the three displaced leptons are depicted. From now on, the p_T values will be plotted using a linear y -axis, as the differences between LNC and LNV events can be better visualised this way.

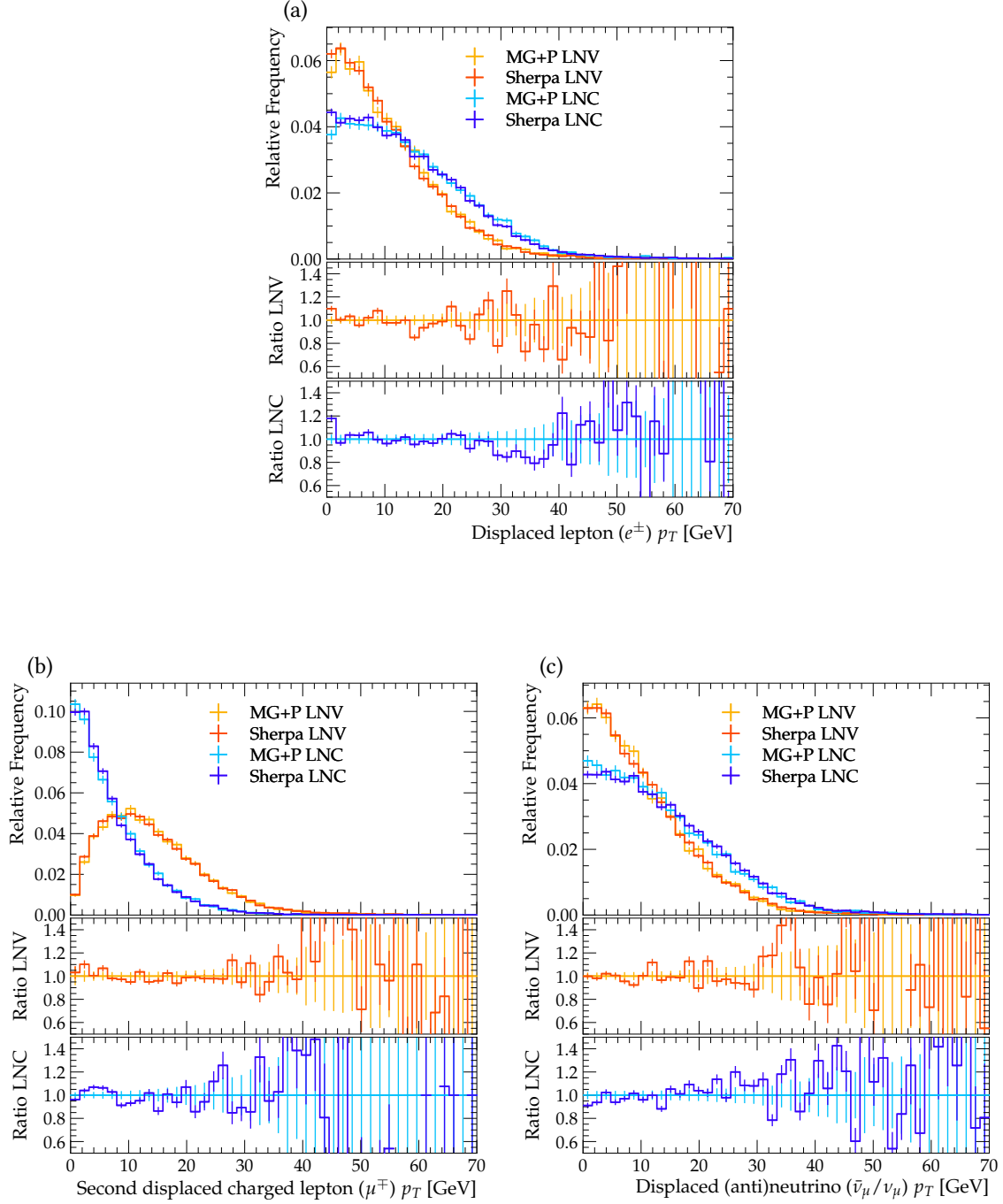


Figure 4.4: p_T distributions in the leptonic decay $HNL \rightarrow e^\pm \mu^\mp$: p_T distributions are shown for the (a) displaced lepton $e^\pm$, (b) second displaced lepton (charged, $\mu^\mp$) and (c) second displaced (anti)neutrino ($\nu/\bar{\nu}_\mu$)

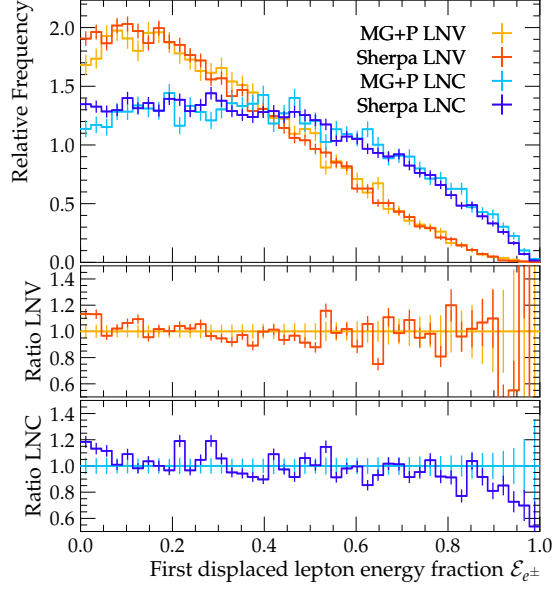


Figure 4.5: Angular distribution of $e^\pm$ in the leptonic decay $HNL \rightarrow e^\mp \nu_\mu \mu^\pm$: The figure plots the energy fraction $\mathcal{E}_e = \frac{E_e}{E_{HNL}}$, from which the angular distributions can be inferred.

Using Fermi's point like interaction (see Section 3.4) all displaced leptons are produced simultaneously in the decay of the HNL. However, we can still distinguish between the "first" displaced lepton produced in the $HNL \rightarrow e^\mp W^{\pm*}$ decay and the following displaced leptons, due to the $W^\pm$ boson decaying into a lepton and a neutrino of the same generation.

The differences in LNC and LNV events in the p_T can be explained by the effect of the different HNL helicities in the two decays and will be further analysed below. Overall, we can see that the SHERPA simulation reproduces the MG+P results well in the leptonic decay via a $W^\pm$ boson.

Energy fractions and spin correlations

As discussed in Section 2.7.3, spin correlation effects lead to different angular distributions of the decay products in LNC and LNV events. Their effects can be recognised in the distributions of the energy fractions $\mathcal{E}$ (see Section 2.7.3) depicted in Figure 4.5, where the energy fraction of the first displaced lepton $\mathcal{E}_{lep} = \frac{E_{lep}}{E_{HNL}}$ is plotted for LNC and LNV events separately.

Same sign prompt and displaced leptons (LNV) are preferentially emitted in the same direction, while opposite sign prompt and displaced leptons (LNC) are preferentially emitted in opposite directions. The distribution follows

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\mathcal{E}_{lep}} = \frac{1}{3} (1 - \mathcal{E}_{lep}) \left(5 + 5\mathcal{E}_{lep} - 4\mathcal{E}_{lep}^2 + P_{HNL} (1 + \mathcal{E}_{lep} - 8\mathcal{E}_{lep}^2) \right)$$

with the polarisation of the HNL P_{HNL} , with the HNL regarded as a particle [48].

Directions of lepton flow

Exemplary for this decay ($HNL \rightarrow e^\mp \nu_\mu \mu^\pm$), we also validate the implementation of Majorana particles in SHERPA by looking at the two possible lepton flow directions in the HNL decay. When having Majorana particles in an event, the Feynman rules are slightly changed in comparison to events where only Dirac particles are taking part. As was discussed in Section 3.4.2, there are two equivalent ways to calculate the Matrix element in these Majorana particle containing cases. In Figure 4.6, the p_T plots comparing the two directions in SHERPA are depicted, showing that, apart from statistical fluctuations, both ways of calculating the matrix element lead to the same results in the simulation. This is expected, since both directions of fermion flow, result in the same mathematical description of the matrix element (see Appendix A.3).

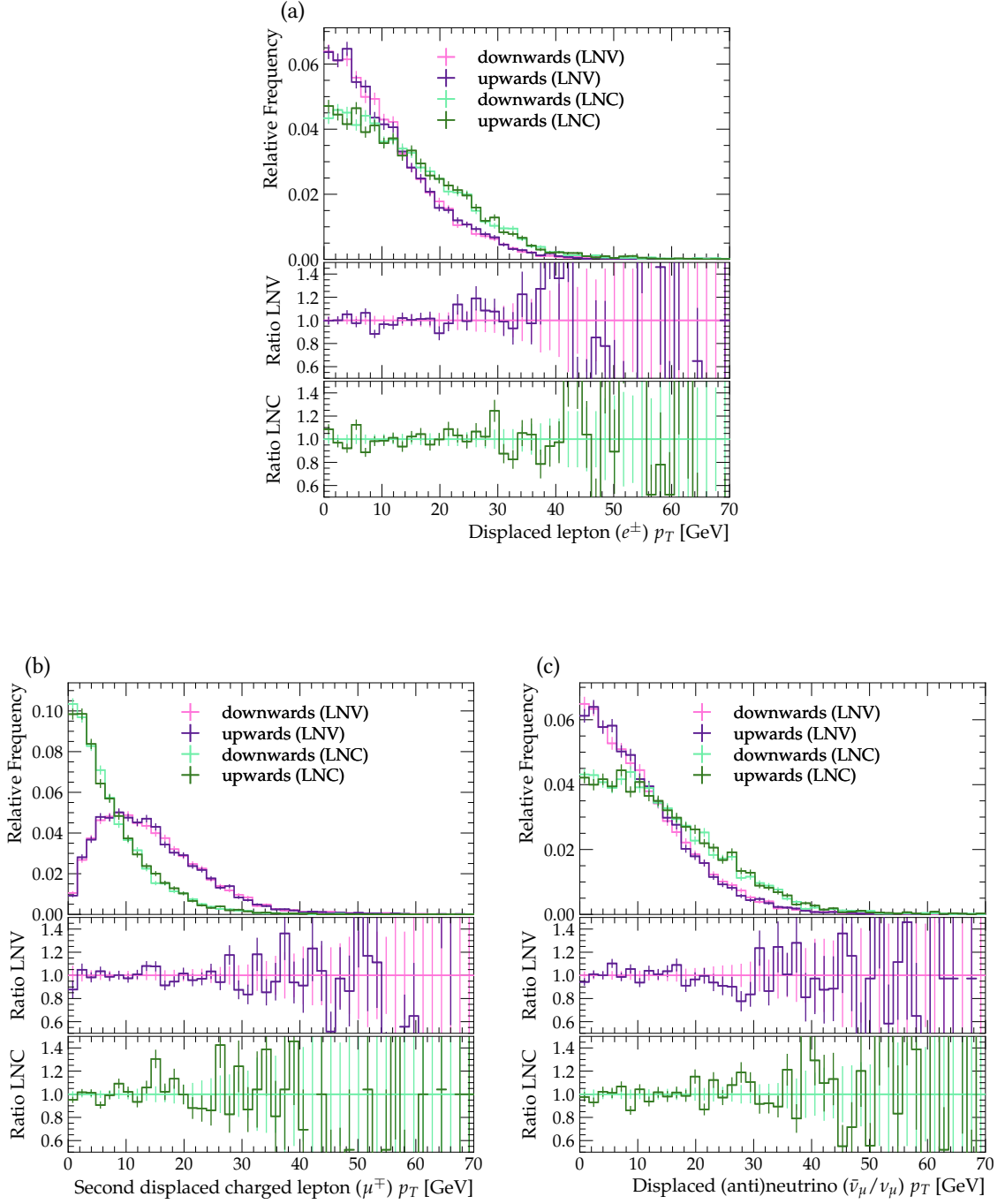


Figure 4.6: Comparing p_T distributions of different fermion flow directions in the leptonic decay $HNL \rightarrow e^\mp \nu_\mu \mu^\pm$: Choosing the direction of fermion flow to be "downwards" or "upwards" in the diagrams depicted in Tables 3.1 and 3.2, results in the same matrix elements. p_T distributions are shown for the (a) first displaced lepton ($e^\pm$), (b) second displaced charged lepton ($\mu^\mp$) and (c) second displaced (anti)neutrino ($\bar{\nu}_\mu/\nu_\mu$).

4.3.2 Leptonic decay channel $HNL \rightarrow \mu^\mp \nu_\mu \mu^\pm$

In order to validate SHERPA's performance regarding decays where $W^\pm$ and Z bosonic decays interfere, we consider the decay $HNL \rightarrow \mu^\mp \nu_\mu \mu^\pm$. In Figure 4.7, the p_T distributions of the three displaced leptons are depicted.

In this case, we cannot distinguish between the displaced leptons any more, i.e. whether it is a "first" or "secondary" displaced lepton. Thus, the allocation into the three plots in Figure 4.7 is done according to the charge of the displaced leptons, i.e. the p_T of the positively charged (μ^+), the negatively charged (μ^-) and neutral (ν or $\bar{\nu}$) displaced lepton. Again we see, that the SHERPA and MG+P simulations align. Albeit the Z boson having a much smaller coupling strength and thus only a small contribution to the total decay width of the HNL, we can still conclude that the $Z + W^\pm$ boson decays agree well between SHERPA and MG+P.

4.3.3 Leptonic decay channel $HNL \rightarrow \tau^\pm \nu_\mu \mu^\mp$

Lastly, we consider the decay channels $HNL \rightarrow \tau^\pm \nu_\mu \mu^\mp$. These decays are not possible to simulate in MG+P [3]. Thus, we can only show SHERPA results here.

When comparing to the decay channel including $e^\pm$ leptons instead of $\tau^\pm$ leptons, we see that the p_T of the displaced lepton is shifted to higher values in the case of the $\tau^\pm$ leptons, as expected due to the higher mass of the $\tau^\pm$ leptons. Furthermore, the LNC and LNV now look the same. This is due to the fact that the mass of the $\tau^\pm$ ($m_\tau \approx 1.777$ GeV) is much higher than the mass of the other charged leptons $m_\mu \approx 0.106$ GeV and $m_e \approx 0.5$ MeV [27]. Thus, the helicity and chirality states do not coincide any more for the $\tau^\pm$ and helicity suppression effects are much less pronounced in this decay channel.

Furthermore, the p_T values of the displaced $\mu^\pm$ and the neutrino/antineutrino, are much lower for the $\tau^\pm$ decay channel than for the $e^\pm$ decay channel. This is, again, due to the high mass of the $\tau^\pm$ leptons, leaving less energy to the other decay products of the HNL.

Overall, this shows that the decays of the HNL into $\tau^\pm$ leptons are now accessible to simulation using SHERPA, enabling us to study the decay of the HNL into all possible lepton flavours.

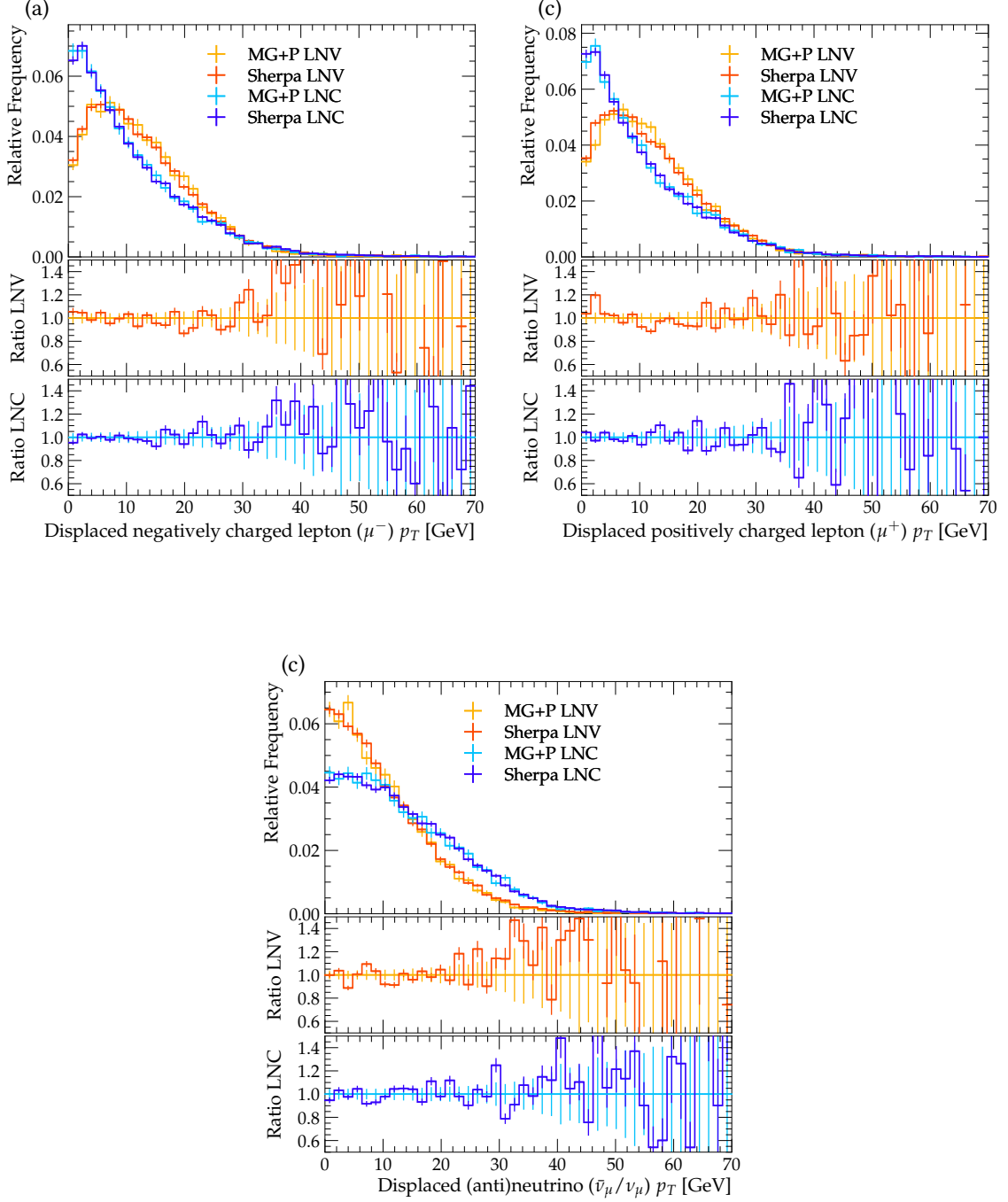


Figure 4.7: p_T distributions in the leptonic decay $HNL \rightarrow \mu^\pm \mu^\mp \nu_\mu$: p_T distributions are shown for the (a) displaced negatively charged particle (μ^-) (b) displaced positively charged particle (μ^+) and (c) displaced neutral particle ($\nu_\mu/\bar{\nu}_\mu$)

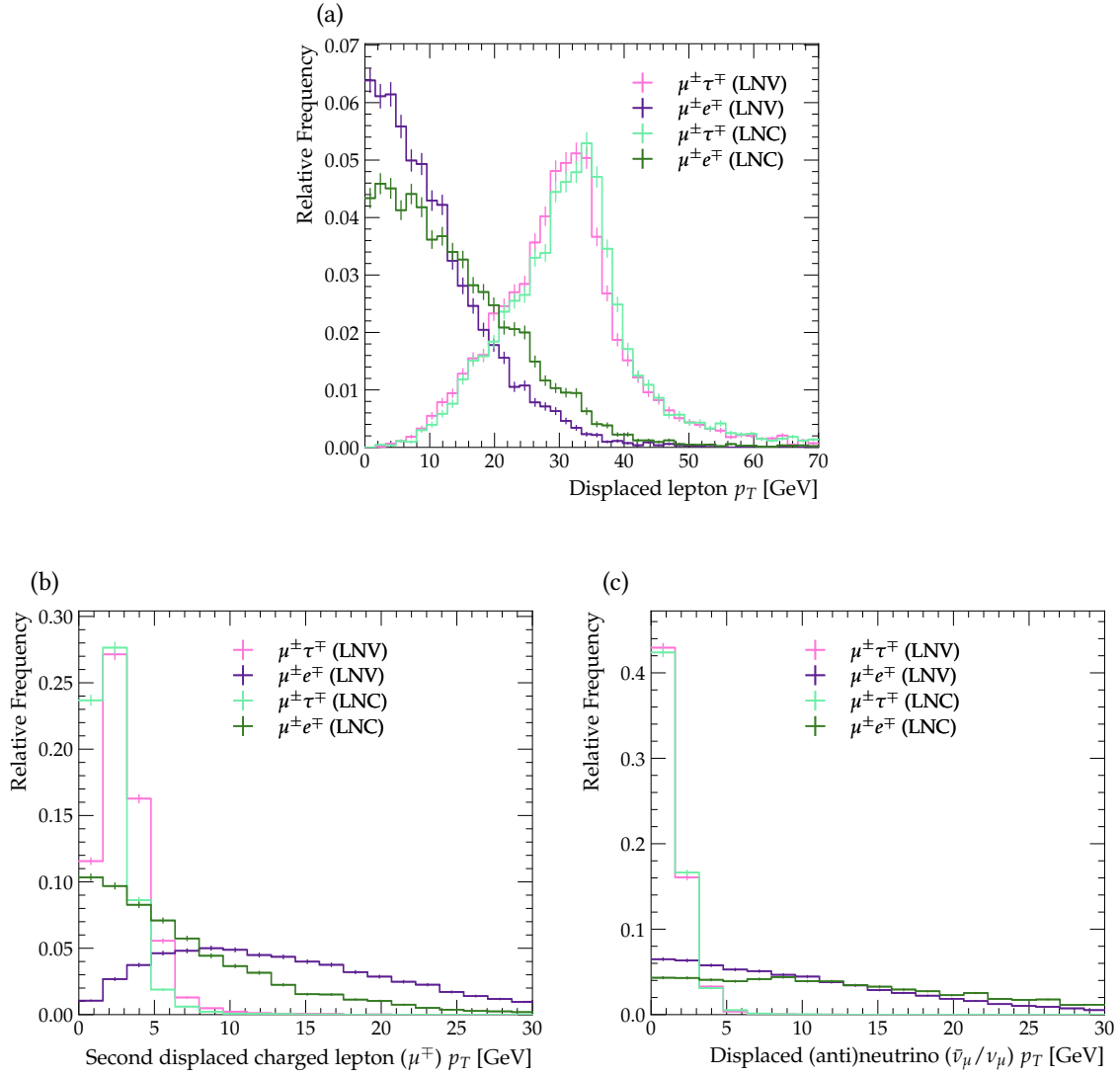


Figure 4.8: p_T distributions in the leptonic decay $HNL \rightarrow \tau^\pm \mu^\mp$: p_T distributions are shown for the (a) first displaced lepton ($\tau^\pm$ or $e^\pm$) (b) second, charged displaced lepton ($\mu^\pm$) and (c) second, displaced neutral lepton ($\nu_\mu/\bar{\nu}_\mu$)

4.4 Semileptonic decay channel

As discussed in Section 2.7.2, there are two possible semileptonic decay channels of the HNL via a $W^\pm$ boson. Their implementation into SHERPA will be validated in the following by, again, comparing kinematic distributions to MG+P simulations.

4.4.1 Semileptonic decay channel $HNL \rightarrow \pi^\pm \mu^\mp$

Validation using p_T

In Figure 4.9, the transverse momenta of the displaced $\mu^\pm$ -lepton and π -meson in the semileptonic decay are shown. Differences in p_T distributions are again due to the different helicity state of the decaying HNL. Both the SHERPA and MG+P simulations align well, showcasing that the new SHERPA simulation is able to produce events containing HNLs decaying semileptonically into $\pi^\pm$ correctly.

Energy fractions and spin correlations

Figure 4.10 shows the energy fractions $\mathcal{E}_\mu$ and the corresponding $\cos(\theta)$ of the displaced $\mu^\pm$ -leptons. Since there are only two decay products, the displaced leptons are emitted back-to-back in the HNLs restframe. Thus the distribution of $\mathcal{E}_{\pi^+}$ and θ_{π^+} are mirrored compared to $\mathcal{E}_\mu$

$$\mathcal{E}_\pi + \mathcal{E}_\mu = 1, \quad \theta_\mu = -\theta_\pi.$$

We can identify the expected angular distributions connected to L in the final state:

In LNV events, the displaced lepton is preferably emitted in the opposite direction of the HNL's direction of momentum in the HNL's restframe (peak at $\cos(\theta) = -1$ and $\mathcal{E}_\mu = 0$). LNC events, on the other hand, favour the emission in the same direction as the HNL's momentum in the HNL's restframe (peak at $\mathcal{E}_\mu = 1$). Thus, in the HNL's restframe, same sign leptons are emitted in the same direction, while opposite sign leptons are emitted in opposite directions.

As discussed in 2.7.3, the helicity of the decaying HNL is set by the LNC or LNV nature of the process. The LNC case corresponds to a negative helicity HNL decaying and the LNV case to a positive helicity HNL decaying [50]. The angular distribution of the displaced lepton in the decay of a polarised heavy lepton is thus given as stated in [48] for the τ^- and in [47] for the HNL:

$$\frac{1}{N} \frac{dN}{d\cos(\theta_\mu)} \propto 1 - P_{HNL} \cdot \cos(\theta_\mu) = 1 - P_{HNL} \cdot (2\mathcal{E}_\mu - 1) \quad (4.1)$$

where P_{HNL} is the polarisation of the decaying HNL.

In summary, we can firstly conclude that SHERPA and MG+P simulate these angular distributions caused by spin correlation effects in good agreement. Secondly, the differences in the energy

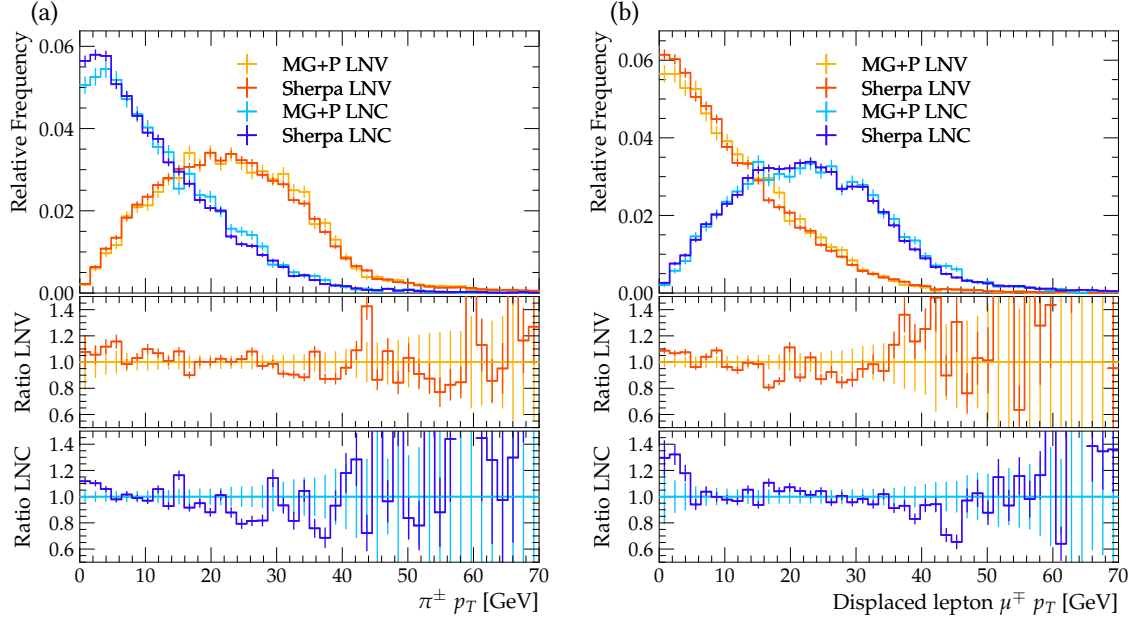


Figure 4.9: p_T distributions in the semileptonic decay $HNL \rightarrow \pi^\pm \mu^\mp$: p_T distributions are shown for the (a) $\pi^\pm$ meson and (b) displaced lepton ($\mu^\mp$)

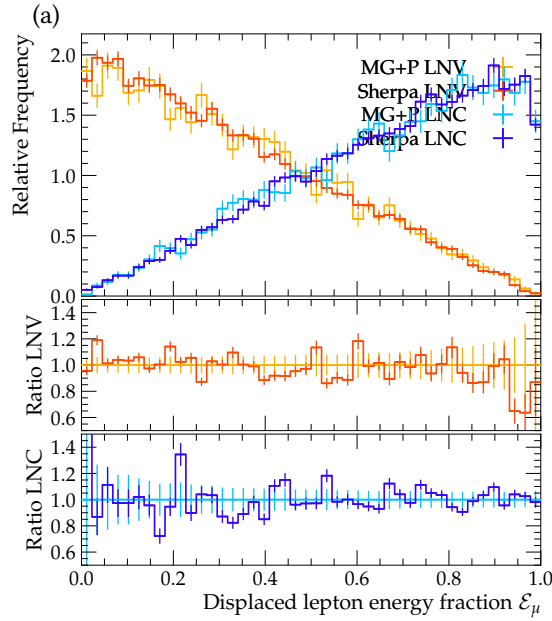


Figure 4.10: Angular distribution of $\mu^\mp$ in semileptonic decay $HNL \rightarrow \pi^\pm \mu^\mp$: Depicted is the energy fraction of the displaced lepton ($\mu^\mp$) $\mathcal{E}_\mu = \frac{E_\mu}{E_{HNL}}$.

fraction of the HNLs decay products are sensitive in distinguishing between LNC and LNV processes. This would prove especially helpful in experimental searches for HNLs, where the LN cannot be ascertained directly, due to high background levels or missing particles in the final state [47].

4.4.2 Semileptonic decay channel $HNL \rightarrow \mu^\mp \rho^\pm$

Validation using p_T

In Figure 4.11, the p_T distributions of the displaced $\mu^\mp$ and ρ -meson in the semileptonic decay are shown. As discussed in Section 3.4, the ρ -meson is not produced in SHERPA. Instead, its decay products, a $\pi^\pm$ and a π^0 mesons are produced directly, while including resonance effects via form factor models. Thus, for consistency, the p_T of the ρ -meson is reconstructed from its decay products $\pi^\pm$ and π^0 in both MG+P and SHERPA.

Both SHERPA and MG+P show the same behaviour for the LNC and LNV cases respectively, confirming that the new SHERPA simulation is able to produce events containing HNLs decaying semileptonically into $\rho^\pm$ correctly.

Comparing different SHERPA form factor models

In order to compare the two possible form factor models implemented in SHERPA, we compare the invariant mass of the ρ -meson reconstructed from its decay products

$$m_{\pi^+\pi^0} = \sqrt{(E_{\pi^+} + E_{\pi^0})^2 - (\vec{p}_{\pi^+} + \vec{p}_{\pi^0})^2}.$$

in both SHERPA and MG+P simulations in Figure 4.12.

Regarding the peak at $m_{\pi^+\pi^0} = 770$ MeV, for which the $\rho(770)$ resonance is responsible, both simulators align. Looking at the tails of the distribution, however, one can see a distinct difference: In MG+P the invariant mass of the ρ -meson cuts off abruptly at $m_{\pi^+\pi^0} = 1450$ MeV. This is due to the fact that the ρ -mesons were produced in MadGraph on-shell, before a Breit-Wigner width was applied manually (see Section 3.5). The two form factor models in SHERPA, i.e. the Resonance Chiral Theory (R χ T) and the Kühn-Santamaria model (KSM) both simulate higher invariant masses of the ρ -meson. Additionally, the KSM includes higher orders of ρ resonances. This is visible through the additional peaks at the masses of the ρ (1450) and ρ (1700) mesons, respectively.

Energy fractions and spin correlations

As before, we can analyse the energy fractions $\mathcal{E}$ to see, which angular distributions are favoured in the LNC and LNV decays respectively in order to verify SHERPA is able to reproduce these correctly.

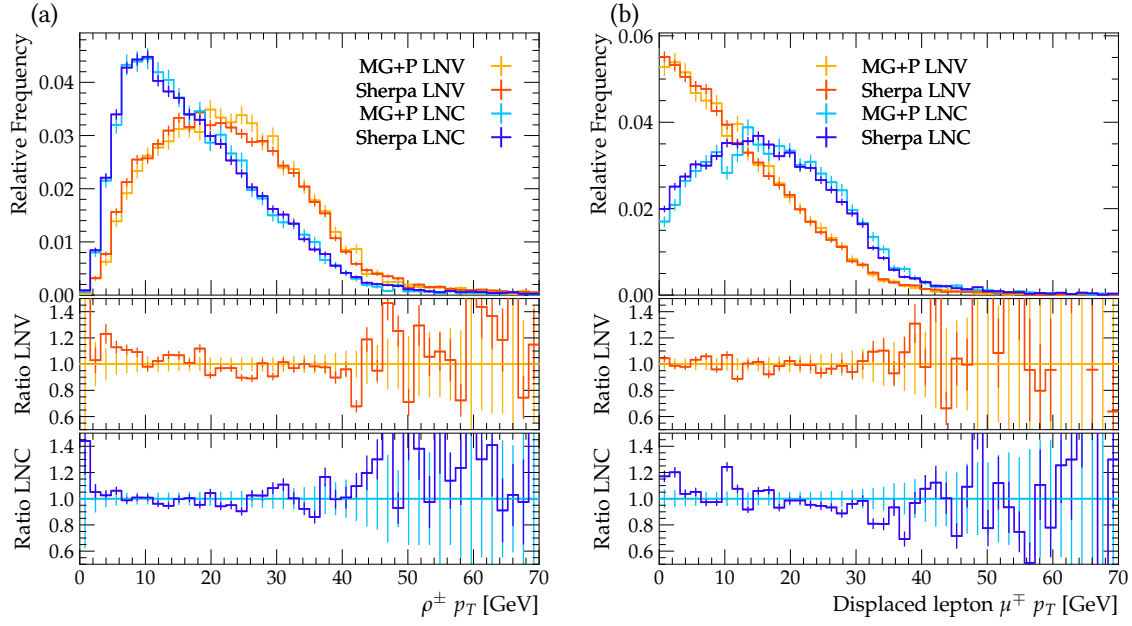


Figure 4.11: p_T distributions in the semileptonic decay $HNL \rightarrow \rho^\pm \mu^\mp$: Depicted are the p_T distributions of the (a) $\rho^\pm$ meson and (b) displaced lepton ($\mu^\mp$). The SHERPA samples were generated using the Kühn-Santamaria model.

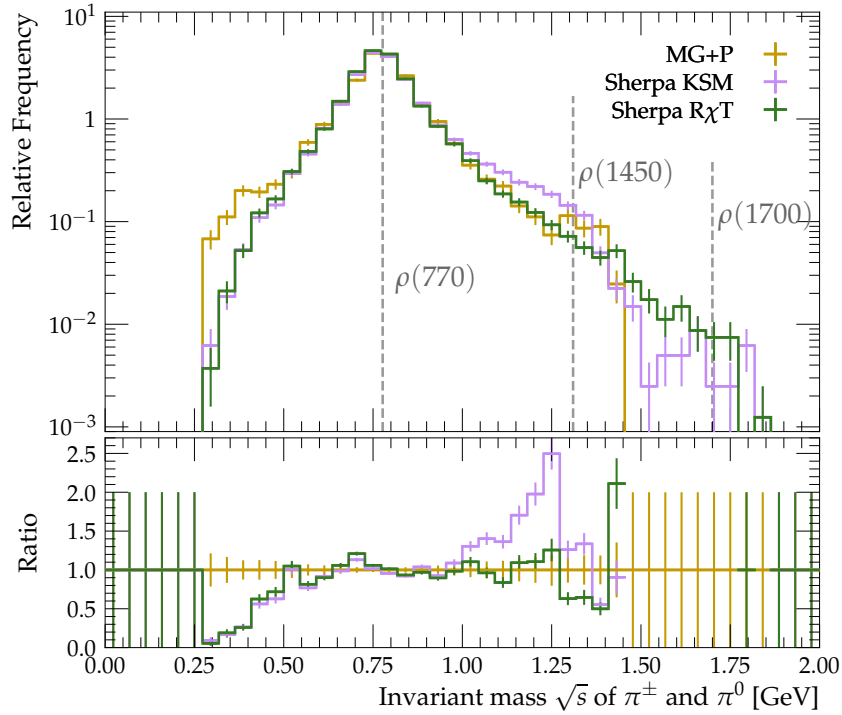


Figure 4.12: Invariant mass of the ρ -meson: SHERPA samples were generated using Resonant Chiral Theory (R χ T) and Kühn-Santamaria model (KSM). The grey dashed lines indicate the ρ resonances.

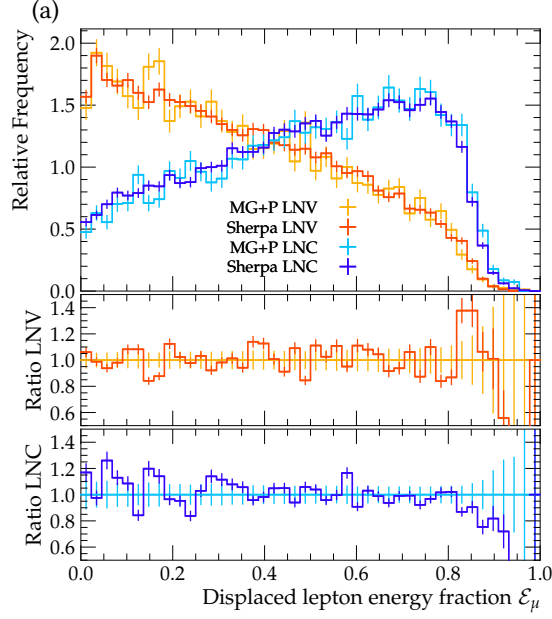


Figure 4.13: Angular distribution of $\mu^\mp$ in semileptonic decay $HNL \rightarrow \rho^\pm, \mu^\mp$: Depicted is the energy fraction of the displaced lepton ($\mu^\mp$) $\mathcal{E}_\mu = \frac{E_\mu}{E_{HNL}}$.

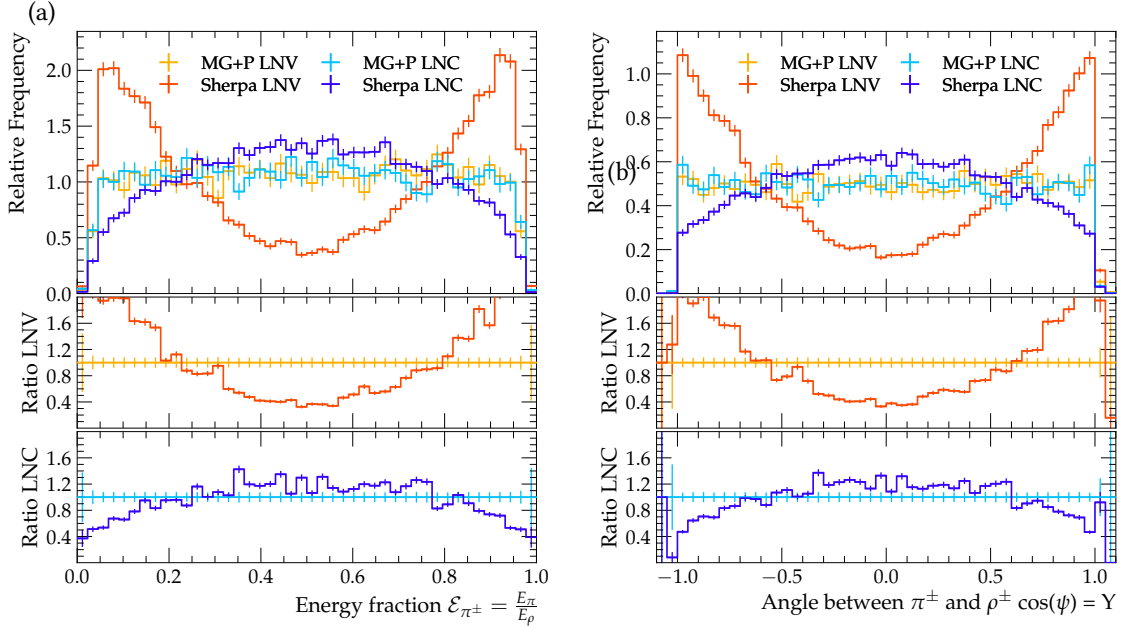


Figure 4.14: Angular distribution in the ρ -meson's decay $\rho^\pm \rightarrow \pi^\pm \pi^0$: In (a) the energy fraction of the $\pi^\pm$ is depicted $\mathcal{E}_{\pi^\pm} = \frac{E_{\pi^\pm}}{E_{\rho^\pm}}$. In (b) the observable $Y = \cos(\psi)$ is shown, with ψ being the angle between the $\rho^\pm$ and the $\pi^\pm$ in the ρ -meson's restframe.

In Figure 4.13 the $\mathcal{E}_\mu$ of the displaced $\mu^\pm$ -lepton with the HNL as the parent particle is depicted. From this distribution, we can see the same effects as above in the $HNL \rightarrow \pi^\pm \mu^\mp$ decay channel: In the LNC cases, the displaced lepton tends to be emitted in the same direction as the HNL momentum (peaks at $\mathcal{E}_\mu = 1$), while the ρ -meson is emitted in the opposite direction ($\mathcal{E}_\rho = \frac{m_\rho}{m_{HNL}}$). In the LNV decays the opposite angular distributions are observable. As before, we can relate this behaviour to the decay of a polarised HNL. When comparing to the decay of a polarised τ^- lepton which follows [87]

$$\begin{aligned} \frac{1}{N} \frac{dN}{d \cos(\theta_\rho)} &\propto \frac{1}{2} (1 - b P_{HNL} \cdot \cos(\theta_\mu)) = \frac{1}{2} (1 - b P_{HNL} \cdot (2 \mathcal{E}_\mu - 1)) \\ \text{with } b &= \frac{m_{HNL}^2 - 2m_\rho^2}{m_{HNL}^2 + 2m_\rho^2} \approx 0.54, \end{aligned} \quad (4.2)$$

we see good agreement with the simulation. Equation (4.2) of the angle θ between the prompt lepton and HNL is given analogously to the decay into a π -meson (see Equation (4.1)), with a correction term b accounting for the ρ -meson being a vector meson which can be longitudinally or transversely polarised.

These angular distributions can be seen both in the MG+P and SHERPA samples, further verifying that SHERPA is able to simulate these semileptonic HNL decays into ρ -mesons correctly.

The ρ -meson decays further into two π -mesons, for which we again plot the energy in comparison to their parent particle, the ρ -meson, in Figure 4.14. In this distribution, we see a clear difference between MG+P and SHERPA. This is due to the fact that the spin correlation information of the ρ -meson is lost in the MG+P simulation (see Section 3.5) and thus, the angular distributions of the π -mesons are isotropic.

In the ρ -meson's restframe both π -mesons are emitted in opposite directions. In SHERPA, we see that they tend to share the ρ -meson's energy equally (peaks at $\mathcal{E}_\pi = 0.5$) in the LNC process, meaning they are emitted orthogonally with respect to the ρ -meson momentum direction. In the LNV decays, one π meson carries most of the energy ($\mathcal{E}_\pi = \pm 1$), which corresponds to the π mesons being emitted (anti)parallel to the ρ -meson's momentum. These different angular distributions can be explained by a differently polarised ρ -meson in the two decay modes. We know that the angular distributions of the pions are related to the polarisation state of the ρ -meson through [48]

$$\frac{d\Gamma(\rho_T \rightarrow 2\pi)}{d \cos(\theta_\pi)} \propto \frac{1}{2} \sin^2(\theta) \quad (4.3)$$

$$\frac{d\Gamma(\rho_L \rightarrow 2\pi)}{d \cos(\theta_\pi)} \propto \cos^2(\theta) \quad (4.4)$$

with ρ_T indicating a transversely polarised ρ -meson, and ρ_L a longitudinally polarised ρ -meson.

Thus, in the LNC case, transversely polarised ρ -mesons were favoured, since the LNC-distribution in Figure 4.14a follows (4.3). For the LNV events, the ρ -meson was mostly longitudinally polarised, meaning it had a helicity of $h = 0$ (the LNV-distribution in Figure 4.14a follows (4.4)).

This behaviour matches, again, to the decay of a τ^- : A negative helicity lepton favours the decay into a transversely polarised ρ -meson, while a positive helicity lepton favours the decay into a longitudinally polarised ρ -meson [88]. The angle ψ between ρ -meson and π -meson can be ascertained by the following observable [87]

$$\Upsilon = \cos(\psi) = \frac{m_\rho}{\sqrt{m_\rho^2 - 4m_\pi^2}} \cdot \frac{E_{\pi^+} - E_{\pi^0}}{(P_{\pi^+} + P_{\pi^0})}$$

plotted in Figure 4.14b. This distribution follows, again, the case of a polarised τ^- decaying into a ρ -meson [49, 89].

Conclusion

In this work, we implemented the production and decay of heavy neutral leptons (HNLs) in the Monte Carlo event generator SHERPA. The HNLs were considered within the framework of the Type I Seesaw mechanism and were introduced as Majorana mass eigenstates that mix with the Standard Model (SM) neutrinos via small mixing angles. For the simulation of the HNLs, we used SHERPA's Universal FeynRules Output (UFO) interface was used to incorporate the interactions of the HNLs with SM particles.

During the simulation of HNLs in high-energy collisions, special attention has to be paid to account for the unique Majorana properties of the HNL, both in production and decay. This is handled in SHERPA by the matrix element generator COMIX and hadron decay module HADRONS++. Using HADRONS++ allowed us to simulate a displaced vertex for the HNL decay, which is a key feature in searches for long-lived particles like HNLs at collider experiments.

The angular distributions of the decay products were studied separately for two categories: lepton number conserving (LNC) and lepton number violating (LNV) processes. This distinction is essential for Majorana particles, since LNV events are forbidden in the SM and their observation would provide direct evidence for physics beyond the Standard Model. The simulated production channels were selected to match those proposed at the Large Hadron Collider (LHC) for HNL searches. The decay channels were chosen based on their branching ratios and the feasibility of detection in collider experiments. Therefore, this work studied leptonic decays where the HNL decays into charged leptons and neutrinos, and semileptonic decays into a π -meson or ρ -meson and a charged lepton.

We validated our samples by comparing kinematic distributions to the established simulations using MadGraph+PYTHIA (MG+P) [2]. For both production and decay, the kinematic distributions were found to be in good agreement between the two frameworks, supporting the validity of the SHERPA implementation for HNL studies. In particular, we verified that the angular distributions of the HNL decay products are consistent with theoretical expectations for the decay of polarised heavy leptons. In this context, LNC events correspond to the decay of negatively polarised HNLs, while LNV events involve positively polarised HNLs decaying. Moreover, we demonstrated additional features of the SHERPA simulation compared to MG+P: In SHERPA, spin correlations are consistently taken into account throughout the entire event generation. As a

result, the polarisation states of the ρ -meson in semileptonic decays are accessible. This is in contrast to MG+P, where spin information of the ρ -meson cannot be retained from production to decay. Furthermore, $\tau^\pm$ decays are implemented in SHERPA, allowing for a more comprehensive simulation of HNL decay chains, whereas in MG+P, the decay into $\tau^\pm$ leptons is not available.

In summary, SHERPA provides a reliable framework for the simulation of HNLs, accounting for their Majorana nature and the associated angular distributions in lepton number conserving and violating processes. Compared to the MG+P implementation, SHERPA additionally allows for an extended set of decay channels and the consistent treatment of spin correlations.

Future work could focus on validating the simulation over a broader range of HNL masses and mixing angles, as well as extending the available channels and making fixed target experiment setups accessible within SHERPA. This is particularly relevant in light of ongoing and planned experimental searches for HNLs at collider and fixed-target experiments, where accurate simulations are crucial for a reliable interpretation of experimental results [47, 90].

Appendices

APPENDIX A

Charge Conjugation and Fermion Flow for Majorana Neutrinos

A.1 Charge Conjugation Identities for Chiral Neutrino Fields

The following two relations must hold true:

$$(\bar{\nu}_L(x))^c (\nu_R(x))^c = \bar{\nu}_R(x) \nu_L(x) \quad (\text{A.1})$$

$$(\bar{\nu}_R(x))^c (\nu_L(x))^c = \bar{\nu}_L(x) \nu_R(x)$$

We will only show the first Equation, as the other line can be shown analogously.

For all fields $\psi(x)$, the following conditions can be applied:

$$\bar{\psi}(x) = \psi^\dagger(x) \gamma^0 \quad (\text{A.2})$$

$$\psi^c(x) = C \bar{\psi}^T(x) \quad (\text{A.3})$$

$$C \gamma^\mu C^{-1} = -(\gamma^\mu)^T \quad (\text{A.4})$$

$$C^\dagger = C^{-1} = -C \quad (\text{A.5})$$

$$(\gamma_0)^\dagger = \gamma_0 \quad (\text{A.6})$$

$$(\gamma^0)^2 = 1. \quad (\text{A.7})$$

First, we will reformulate the very left-hand side of Equation (A.1):

$$\begin{aligned}
 (\bar{\nu}_L(x))^c &= ((\nu_L(x))^c)^\dagger \gamma^0 \quad (\text{using (A.2)}) \\
 &= \left(C \bar{\nu}_L^T(x) \right)^\dagger \gamma^0 \quad (\text{using (A.3)}) \\
 &= \left(\bar{\nu}_L^T(x) \right)^\dagger C^\dagger \gamma^0 \\
 &= \left(\left(\nu_L^\dagger(x) \gamma^0 \right)^T \right)^\dagger C^\dagger \gamma^0 \\
 &= \nu_L^T(x) (\gamma^0)^T C^\dagger \gamma^0 \quad (\text{using (A.6)}) \\
 &= -\nu_L^T(x) (\gamma^0)^T C \gamma^0 \quad (\text{using (A.5)})
 \end{aligned}$$

The other part of the left-hand side, $(\nu_R(x))^c$, can be reexpressed as

$$\begin{aligned}
 (\nu_R(x))^c &= C \bar{\nu}_R^T(x) \quad (\text{using (A.3)}) \\
 &= C \left(\nu_R^\dagger(x) \gamma^0 \right)^T \quad (\text{using (A.2)}) \\
 &= C (\gamma^0)^T \left(\nu_R^\dagger(x) \right)^T \\
 &= -C^{-1} (\gamma^0)^T \left(\nu_R^\dagger(x) \right)^T \quad (\text{using (A.5)})
 \end{aligned}$$

Together, we have:

$$\begin{aligned}
 (\bar{\nu}_L(x))^c (\nu_R(x))^c &= +\nu_L^T(x) (\gamma^0)^T C \gamma^0 C^{-1} (\gamma^0)^T \left(\nu_R^\dagger(x) \right)^T \\
 &= \nu_L^T(x) (\gamma^0)^T (\gamma^0)^T (\gamma^0)^T \left(\nu_R^\dagger(x) \right)^T \quad (\text{using (A.4)}) \\
 &= \left(\nu_R^\dagger(x) \gamma^0 \gamma^0 \gamma^0 \nu_L(x) \right)^T \\
 &= (\bar{\nu}_R(x) \nu_L(x))^T \quad (\text{using (A.2) and (A.7)}) \\
 &= \bar{\nu}_R(x) \nu_L(x) \quad \text{Scalars are invariant under transposition}
 \end{aligned}$$

Thus, we have shown Equation (A.1).

A.2 Chirality of Charge-Conjugated Fermion Fields

First, we want to show that the antifermion excitations of a chiral fermion field have the opposite chirality. To achieve this, we will start of with a left-chiral field $\psi_L(x)$ containing exactly one particle

$$\psi_L(x) = \hat{\phi}_L(x) |\emptyset\rangle \quad \text{see Equation (2.3).}$$

Then, we apply charge conjugation to this field, to get a field that contains exactly one antiparticle excitation:

$$\begin{aligned}
 (\psi_L(x))^c &= C\bar{\psi}_L^T(x) \quad \text{using (A.3) and (A.2)} \\
 &= C\left((\psi_L(x))^\dagger \gamma^0\right)^T \\
 &= C\left((P_L\psi(x))^\dagger \gamma^0\right)^T \\
 &= C\left(\psi^\dagger(x)P_L^\dagger \gamma^0\right)^T \quad \text{since } (\gamma^5)^\dagger = \gamma^5 \\
 &= C\left(\psi^\dagger(x)\gamma^0 P_R\right)^T \quad \text{since } \gamma^5 \gamma^0 = -\gamma^0 \gamma^5 \\
 &= CP_R^T\left(\psi^\dagger(x)\gamma^0\right)^T \\
 &= CP_R\bar{\psi}^T(x) \quad \text{since } (\gamma^5)^T = (\gamma^5) \text{ and (A.2)} \\
 &= P_R C\bar{\psi}^T(x) \quad \text{since } [\gamma^5, C] = 0 \\
 &= P_R\psi^c(x) \\
 &= (\psi^c(x))_R
 \end{aligned}$$

A.3 Equivalence of Fermion Flow Directions in Majorana Matrix Elements

We will show that

$$\mathcal{M}_{\text{up}} = -\mathcal{M}_{\text{down}}$$

where $\mathcal{M}_{\text{down}}$ refers to the matrix elements as described in Table 3.1 and $\mathcal{M}_{\text{up}}$ to the ones described in 3.2 . Exemplary, we take a look at LNV part-part process.

To show this, we will utilise the following identities:

$$\mathcal{V}^T = C^{-1}\mathcal{V}'C$$

$$\mathcal{V}'^T = C^{-1}\mathcal{V}C$$

$$v^T(p) = \bar{u}(p)C^T = -\bar{u}(p)C$$

$$u^T(p) = \bar{v}(p)C^T = -\bar{v}(p)C$$

$$(\not{p} + m)^T = C^{-1}(-\not{p} + m)C$$

Additionally, we know, that the matrix element is a scalar and therefore invariant under transposition. We will use the matrix element $\mathcal{M}_{up}$ for the fermion flow direction shown in Table 3.2 as

a starting point.

$$\begin{aligned}
 \mathcal{M}_{\text{up}} &= \mathcal{M}_{\text{up}}^T = \left[\bar{u}(p_1) \mathcal{V} \frac{\not{p}_{12} + m}{p_{12}^2 + m^2} \mathcal{V}' v(p_2) \right]^T \\
 &= v^T(p_2) \mathcal{V}'^T \frac{(\not{p}_{12} + m)^T}{p_{12}^2 + m^2} \mathcal{V}^T \bar{u}^T(p_1) \\
 &= -\bar{u}(p_2) C C^{-1} \mathcal{V} C C^{-1} \frac{-\not{p}_{12} + m}{p_{12}^2 + m^2} C C^{-1} \mathcal{V}' C C^{-1} v(p_1) \\
 &= -\bar{u}(p_2) \mathcal{V} \frac{-\not{p}_{12} + m}{p_{12}^2 + m^2} \mathcal{V}' v(p_1) \\
 &= -\bar{u}(p_2) \mathcal{V} \frac{\not{p}_{21} + m}{p_{21}^2 + m^2} \mathcal{V}' v(p_1) \\
 &= -\mathcal{M}_{\text{down}}
 \end{aligned}$$

APPENDIX B

Example Runcards

The following shows exemplary run cards used for this thesis. Preamble and UFO parameters were the same for all decay channels and thus, will only be shown once for the leptonic decay.

B.1 Runcard for leptonic HNL decay

```
# --- Beam configuration

BEAMS: [2212, 2212]           # Proton--proton collisions
BEAM_ENERGIES: [6500.0, 6500.0] # Beam energies in GeV
                                # Corresponds to sqrt(s) = 13 TeV
EVENTS: 100000                # Total number of generated events
# --- Event output format
EVENT_OUTPUT:
  - HepMC3_GenEvent[output.hepmc] # Output in HepMC3 format
# --- Matrix-element generator
ME_GENERATORS: Comix           # Matrix-element generator
# --- Model definition
MODEL: SM_HeavyN_Meson_NLO     # UFO model including heavy neutral
                                leptons
UFO_HADRONS: [72]              # Additional particles treated as
                                hadrons in Sherpa
# --- Particle properties
PARTICLE_DATA:
  72:                          # Heavy Neutral Lepton (HNL),
                                # all parameters described here
                                # override the UFO parameters from
                                # below
                                # Mass in GeV
                                # Decay width in GeV
                                # Corresponds to a proper decay length
                                # of approximately 10 mm
                                Mass: 2.0
                                Width: 1.97e-14
# --- COMIX-specific configuration
COMIX:
```

```

WF_MODE: 0                                # Wave-function mode
                                           # WF_MODE: 0 (default) corresponds to
                                           # Table \ref{tab:ME_W_decay_down},
                                           # while WF_MODE: 1 corresponds to
                                           # Table \ref{tab:ME_W_decay_up}

# --- Hard-scattering processes
PROCESSES:
# Inclusive HNL production with up to one additional jet
- 93 93 -> 72 -13 93{1}:                  # pp -> N mu- + jet (Multijet-Merging)
                                           # 93 denotes any light quark (except
                                           # top) or gluon

    Order: {QCD: Any, QED: Any, NP: Any}
    CKKW: 20                              # CKKW merging scale in GeV
                                           # Separates ME jets from PS emissions

- 93 93 -> 72 13 93{1}:                  # pp -> N mu+ + jet
    Order: {QCD: Any, QED: Any, NP: Any}
    CKKW: 20

# --- Hadronic decay configuration -----
HADRON_DECAYS:
    Spin_Correlations: 1                  # If 1: spin correlations are taken
    into account
    Max_Proper_Lifetime: 101.0

    Channels:                            # Explicit decay-channel definition
    72:
        14,-13,11:                      # Decay into muon neutrino, antimuon
        and electron, order has to match Sherpas      internal ordering

        BR: [1.0, 0.0]                  # Branching ratio of this decay channel
        ME:
            - Current_ME:
                J1:
                    Type: VA_F_F_Majorana # Implements a V-A current
                    involving a Majorana fermion

                    Indices: [3,0]        # muon-, HNL (first entry is barred
                    spinor, while second entry is non barred spinor)
                    v: 1.0                 # Vector coupling (V)
                    a: -1.0                # Axial coupling (A)
                J2:
                    Type: VA_F_F
                                           # V-A current with Dirac leptons
                    Indices: [1,2]        # neutrino, antimuon

                    v: 1.0                 # Vector coupling (V)
                    a: -1.0                # Axial coupling (A)

```

```

-14, 13, -11:                # Charge-conjugate channel

BR: [1.0, 0.0]
ME:
  - Current_ME:
    J1:
      Type: VA_F_F_Majorana # Implements a V-A current
                           involving a Majorana fermion

      Indices: [3,0]      # muon-, HNL (kept as given in the
                           original snippet)
      v: 1.0              # Vector coupling (V)
      a: +1.0             # Axial coupling (A) sign switches due
                           to Majorana Feynman rules
    J2:
      Type: VA_F_F
      Indices: [2,1]      # muon, antineutrino
      v: 1.0
      a: -1.0

# --- UFO model parameters
UFO_PARAMS: |

Block LOOP
  1 9.118800e+01
Block NUMIXING                # Heavy-light neutrino mixing matrix
  1 1.0  # VeN1
  2 0.0  # VeN2
  3 0.0  # VeN3
  4 1.0  # VmuN1
  5 1.0  # VmuN2
  6 0.0  # VmuN3
  7 1.0  # VtauN1
  8 0.0  # VtauN2
  9 1.0  # VtauN3
Block SMINPUTS
  1 1.279400e+02
  2 1.174560e-05
  3 1.184000e-01
Block MASS                    # Particle masses (GeV)
  13 1.056584e-01
  23 9.118760e+01
  25 1.257000e+02
  72 3.000000e+02             # Heavy Neutral Lepton N1
  74 5.000000e+02             # Heavy Neutral Lepton N2
  76 1.000000e+03             # Heavy Neutral Lepton N3
DECAY 72 3.030000e-01         # Total decay width of N1

```

```
Block MESONBLOCK
  3 1.302000e-01
  7 1.700000e-01
 11 1.920000e-01
```

Listing B.1: Sherpa run card for the leptonic decay $HNL \rightarrow e^\pm \mu^\mp, \nu_\mu$

B.2 Runcard for semileptonic (π -meson) HNL decay

```
# --- Hadronic decay configuration
HADRON_DECAYS:
  Spin_Correlations: 1          # Include spin correlations in decays
  Max_Proper_Lifetime: 101.0
  Channels:                    # Explicit hadronic decay channels
    72:                         # Heavy Neutral Lepton (HNL)
      211,13:                  # pi^+, mu-
        BR: [1.0, 0.0]        # Branching ratio of this channel
        ME:
          - Current_ME:
            J1:
              Type: VA_F_F_Majorana
                                # Current involving a Majorana
                                # fermion
              Indices: [2,0]    # muon, HNL (ordering follows Sherpa
                                # convention)
              v: 1.0            # Vector coupling (V)
              a: -1.0           # Axial coupling (A)
            J2:
              Type: VA_0_P
                                # Pseudoscalar meson current
              Indices: [1]      # Pion
              fP: 0.1307        # Meson decay constant
              Vxx: 0.97377      # CKM matrix element
          PhaseSpace:
            - Dalitz_rho(770)+_12:
              Weight: 1.0
            -211,-13:          # pi^-, mu+
              BR: [1.0, 0.0]
              ME:
                - Current_ME:
                  J1:
                    Type: VA_F_F_Majorana # Current involving a Majorana
                    # fermion
                    Indices: [2,0]
                    v: 1.0
```

```

        a: +1.0          # Sign flip from Majorana Feynman
                        rules
J2:
  Type: VA_0_P
  Indices: [1]
  fP: 0.1307
  Vxx: 0.97377
PhaseSpace:
- Dalitz_rho(770)+_12:
  Weight: 1.0

```

Listing B.2: Hadronic HNL decay $HNL \rightarrow \pi^\pm \mu^\mp$

B.3 Runcard for semileptonic (ρ -meson) HNL decay

```

# --- Hadronic decay configuration
HADRON_DECAYS:          # Default decay model: HADRONS++
  Spin_Correlations: 1   # Include spin correlations in decays
  Max_Proper_Lifetime: 101.0 # Maximum proper lifetime in mm

Channels:               # Explicit hadronic decay channels
  72:                   # Heavy Neutral Lepton (HNL)

    211,111,13:         # pi^+, pi^0, mu-
      BR: [1.0, 0.0]    # Branching ratio of this channel
      ME:
        - Current_ME:
          J1:
            Type: VA_F_F_Majorana
                                # V-A current involving a Majorana
                                fermion
            Indices: [3,0]    # muon, HNL (ordering follows Sherpa
                                convention)
            v: 1.0            # Vector coupling (V)
            a: -1.0           # Axial coupling (A)
          J2:
            Type: VA_0_PP
                                # Hadronic two-pseudoscalar current
            Indices: [2,1]    # pi^0, pi^\pm
            RUNNING_WIDTH: 1
            FORM_FACTOR: 1    # 1: Kuehn-Santamaria model, 2:
                                Resonance Chiral Theory
            fpi: 0.1307       # Pion decay constant
            Vud: 0.97377      # CKM matrix element
            Mass_rho(770)+: 0.7769 # rho(770) resonance mass (GeV)
      PhaseSpace:
        - Dalitz_rho(770)+_12:

```

```

Weight: 1.0

-211,111,-13:                # pi^-, pi^0, mu+
BR: [1.0, 0.0]
ME:
  - Current_ME:
    J1:
      Type: VA_F_F_Majorana
                        # Charge-conjugate Majorana current
      Indices: [3,0]
      v: 1.0
      a: +1.0           # Sign flip from Majorana Feynman
                        rules
    J2:
      Type: VA_0_PP
      Indices: [2,1]
      RUNNING_WIDTH: 1
      FORM_FACTOR: 1
      fpi: 0.1307
      Vud: 0.97377
      Mass_rho(770)+: 0.7769
  PhaseSpace:
    - Dalitz_rho(770)+_12:
      Weight: 1.0

```

Listing B.3: Hadronic HNL decay $HNL \rightarrow \rho^\pm \mu^\mp$ and subsequent $\rho^\pm \rightarrow \pi^\pm \pi^0$

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DECLARATION ON AI ASSISTED LANGUAGE EDITING

I would like to acknowledge the use of AI-based large language models for assistance with grammar, wording, and stylistic refinement of written text. All scientific content, interpretations, and conclusions are entirely my own.

ERKLÄRUNG

Hiermit erkläre ich, dass ich die vorliegende Arbeit im Rahmen der Betreuung am Institut für Kern und Teilchenphysik ohne unzulässige Hilfe Dritter verfasst habe und alle verwendeten Quellen als solche gekennzeichnet habe.

Ort, Datum

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